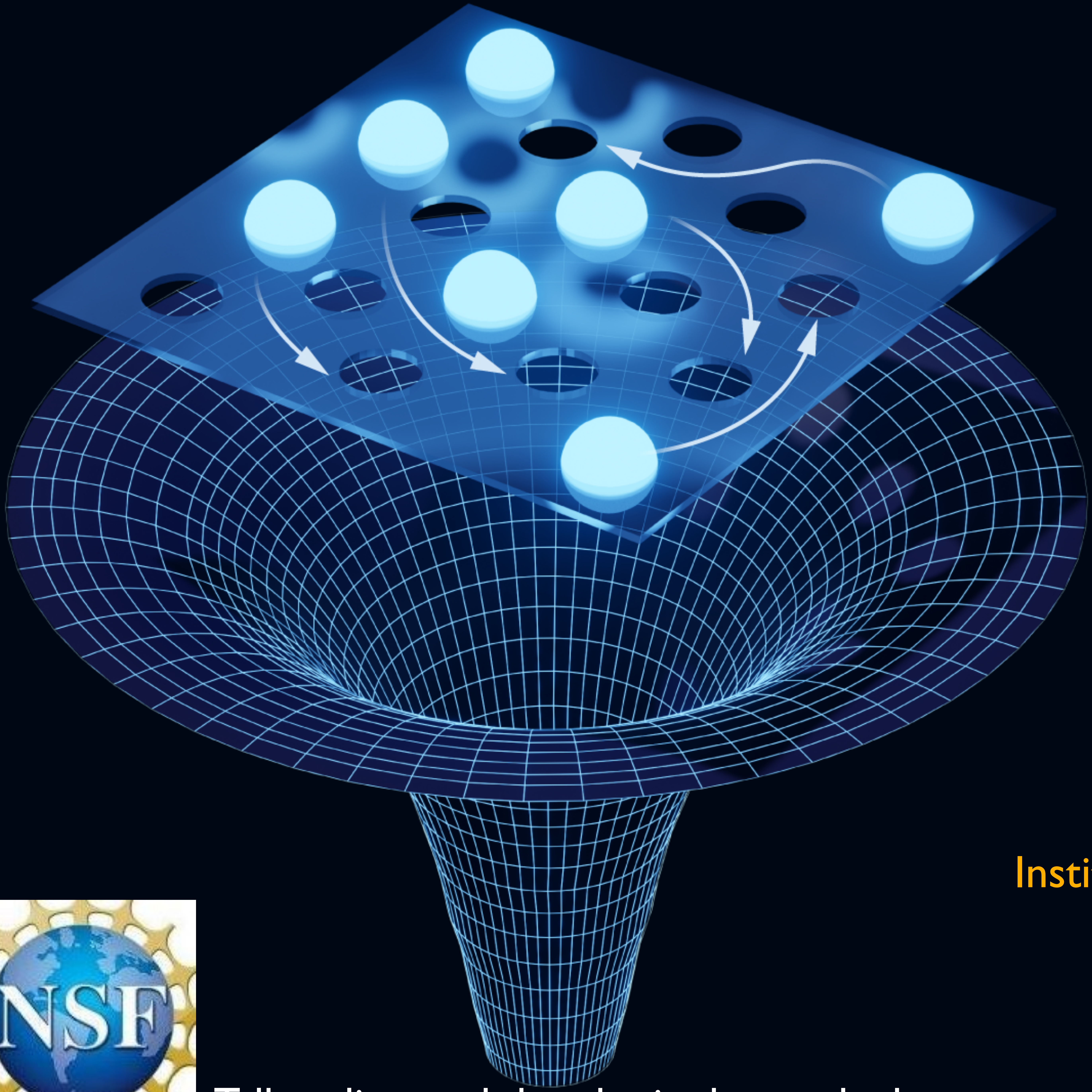


Quantum statistical mechanics of charged black holes and strange metals

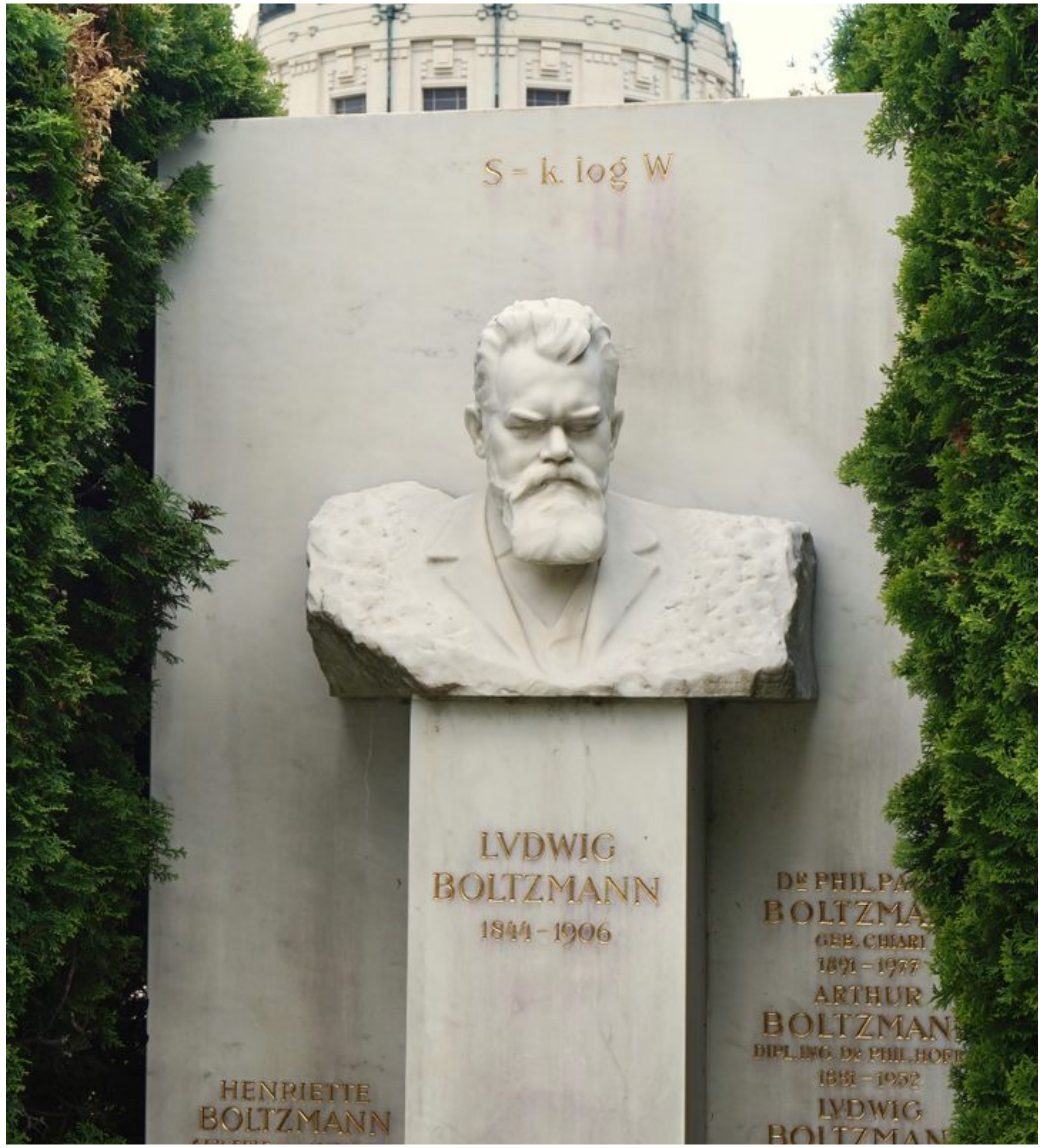
Review articles:
arXiv:2304.13744, 2305.01001

Institute for Theoretical Physics
KU Leuven
June 23, 2022

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

Foundations
by
Boltzmann

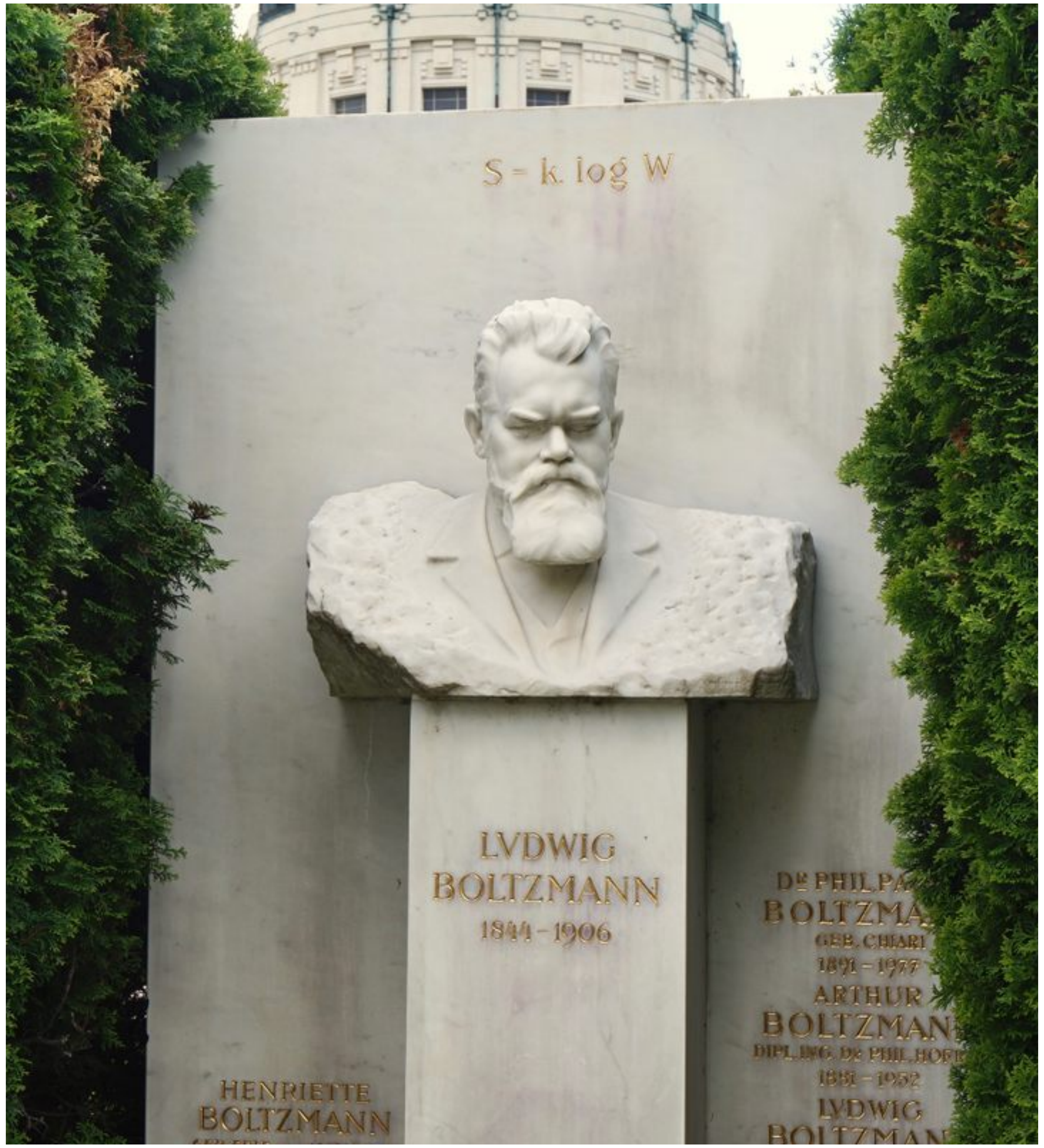


Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states $D(E) = \exp(S(E)/k_B)$



Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

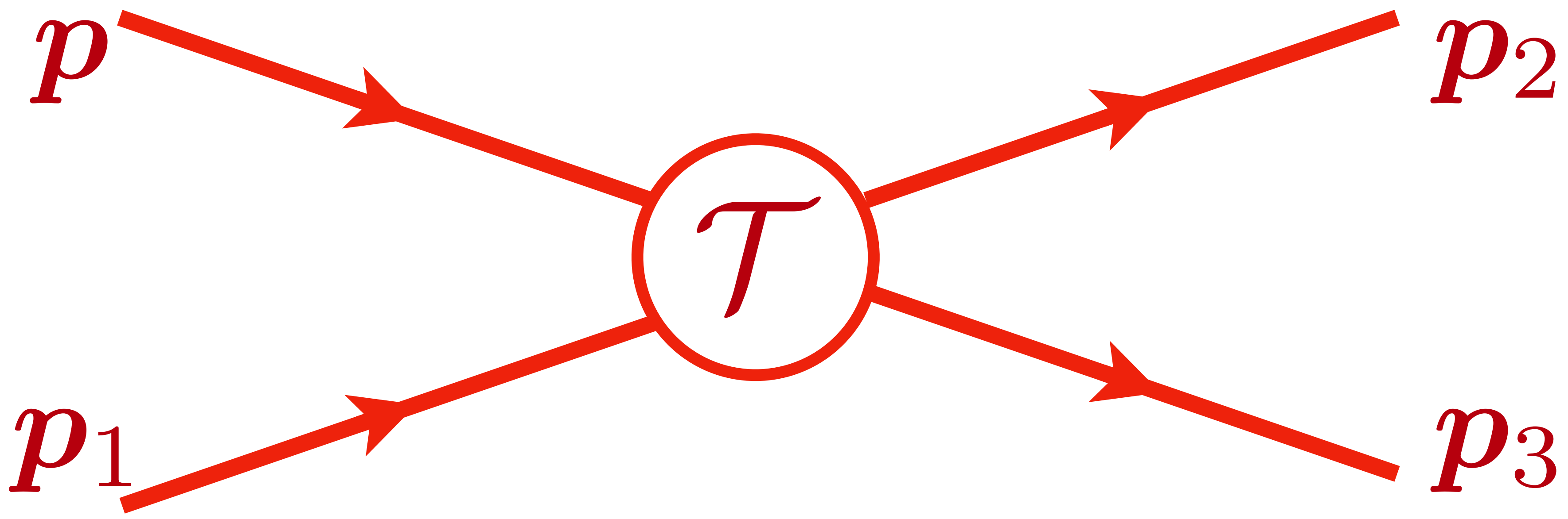
Boltzmann equation (1872)
Dilute classical gas

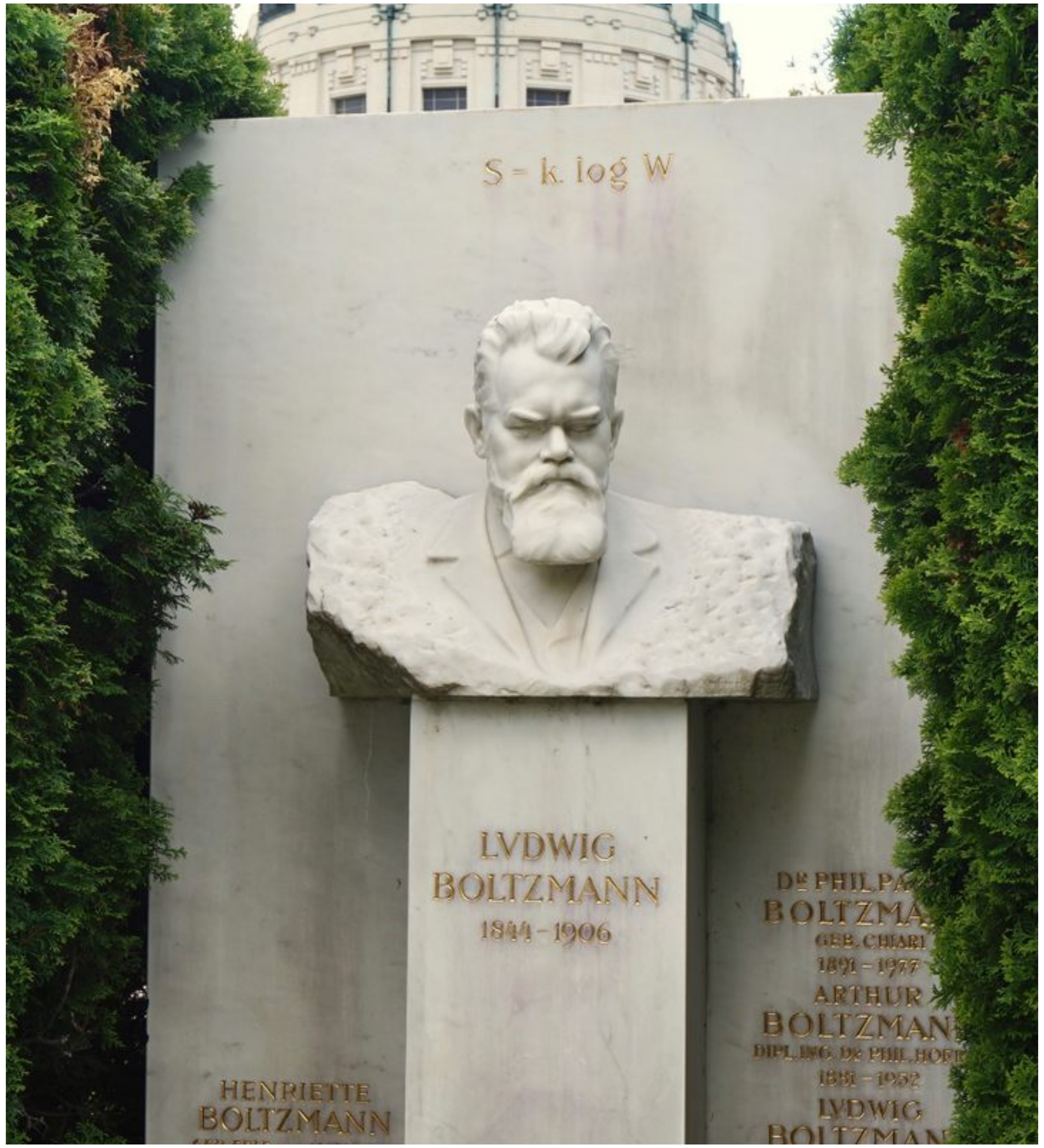
Molecular chaos: successive collisions are statistically independent

$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$

$$- 2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$

$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} \right]$$



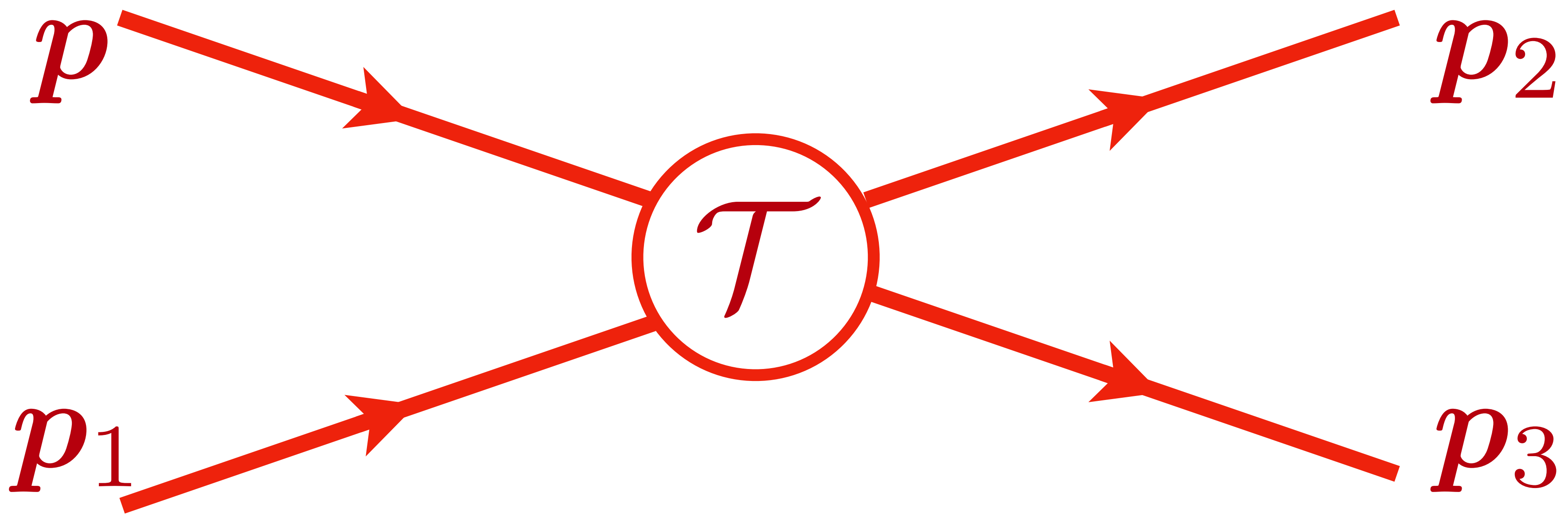


Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

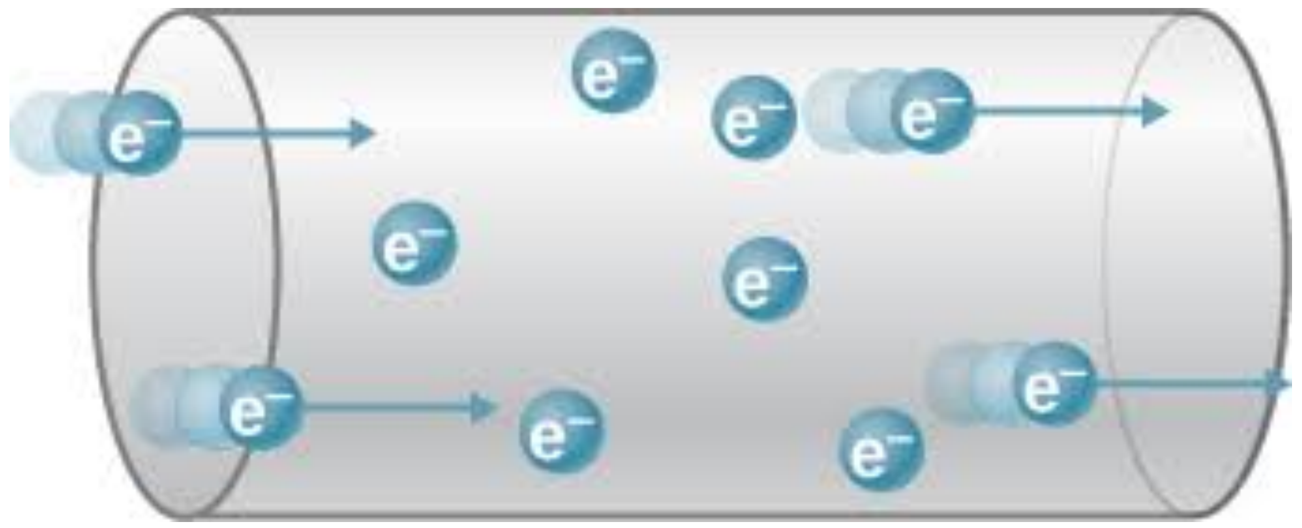
Quantum Boltzmann equation (Landau)
Dense gas of electrons

Neglects quantum interference (entanglement) between successive collisions

$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$
$$- 2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$
$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} (1 - f_{\boldsymbol{p}_2})(1 - f_{\boldsymbol{p}_3}) - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} (1 - f_{\boldsymbol{p}})(1 - f_{\boldsymbol{p}_1}) \right]$$



Current flow with electrons in Copper



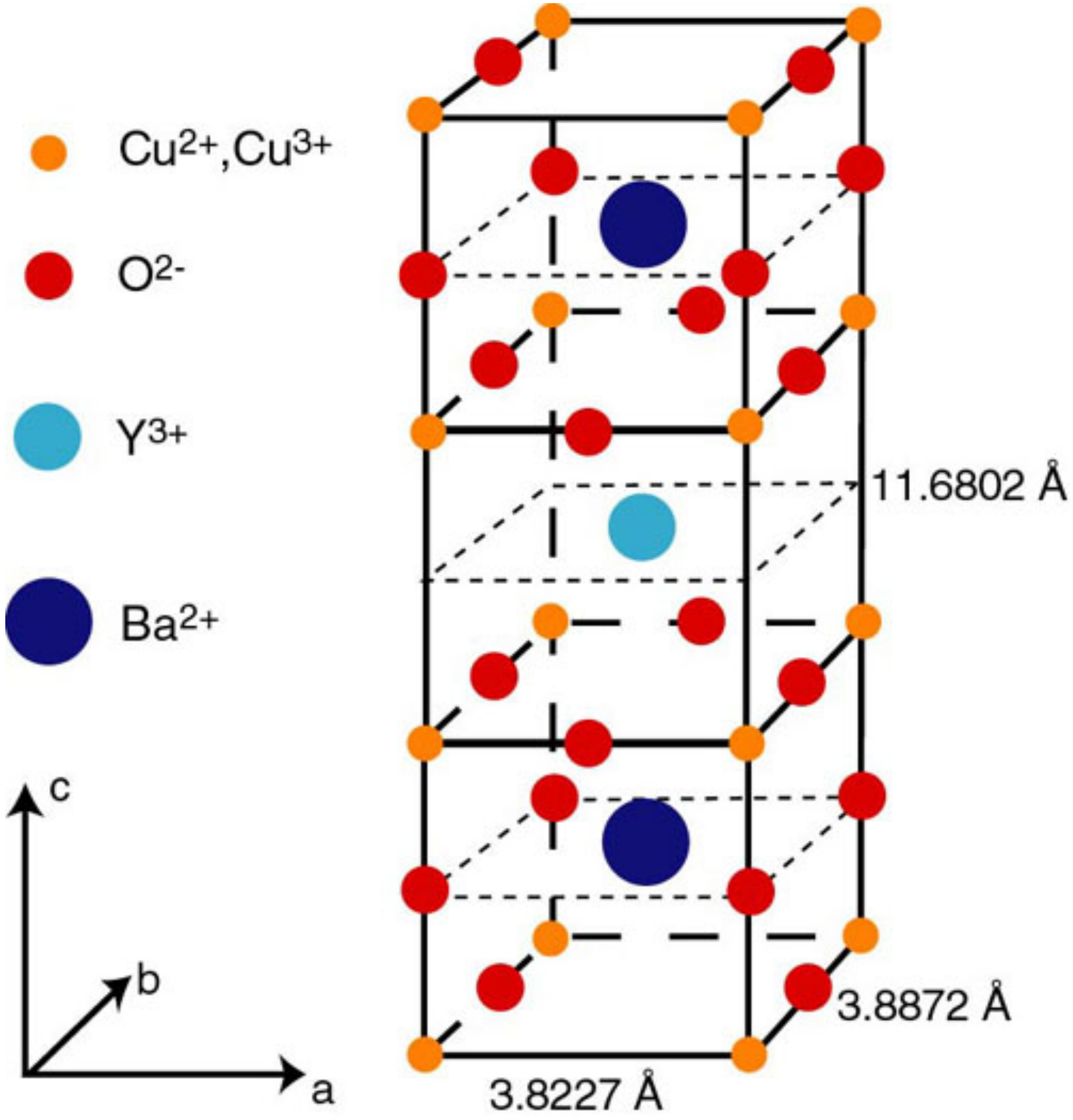
Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

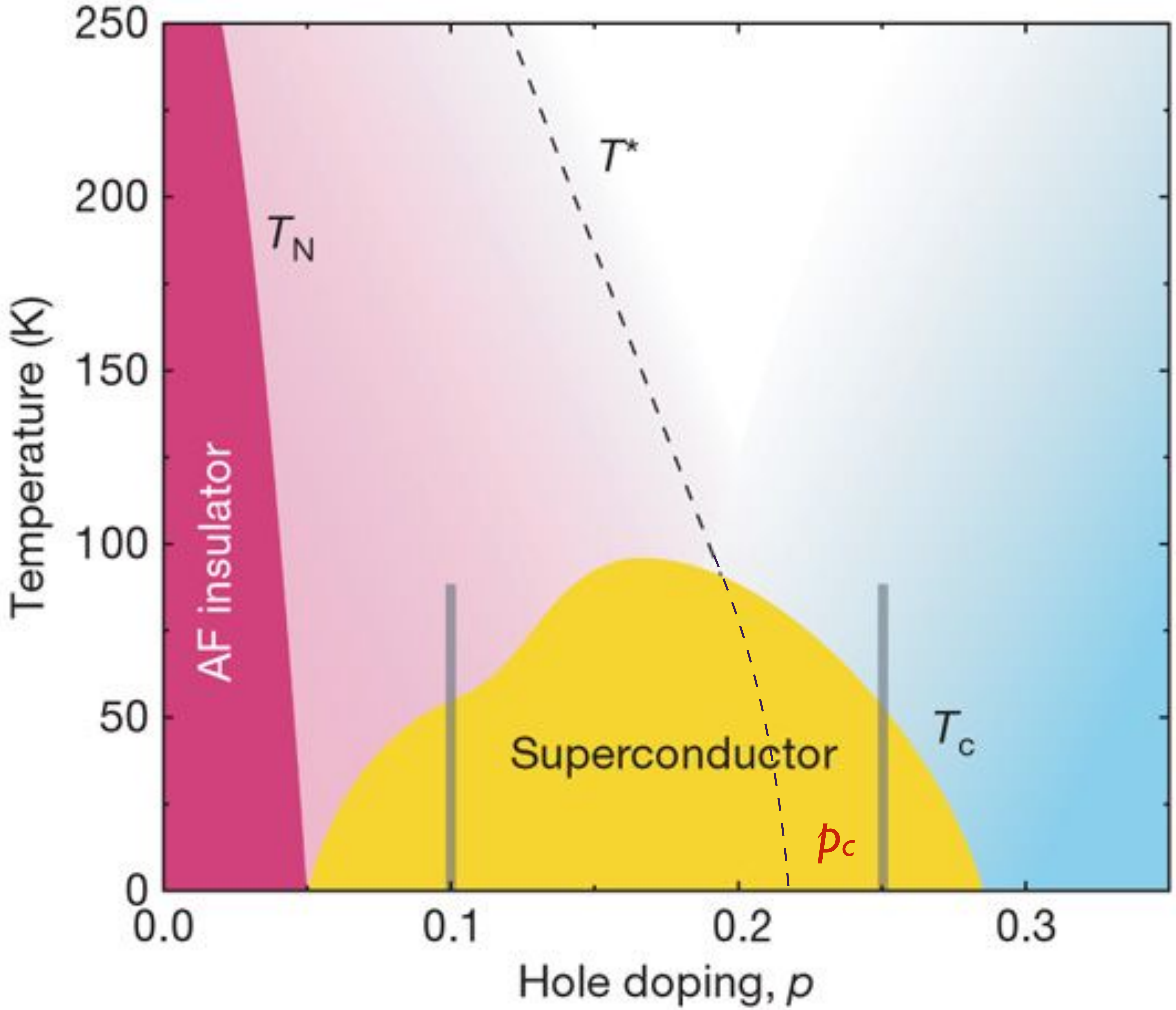
The long scattering time implies that individual electrons are well-defined.

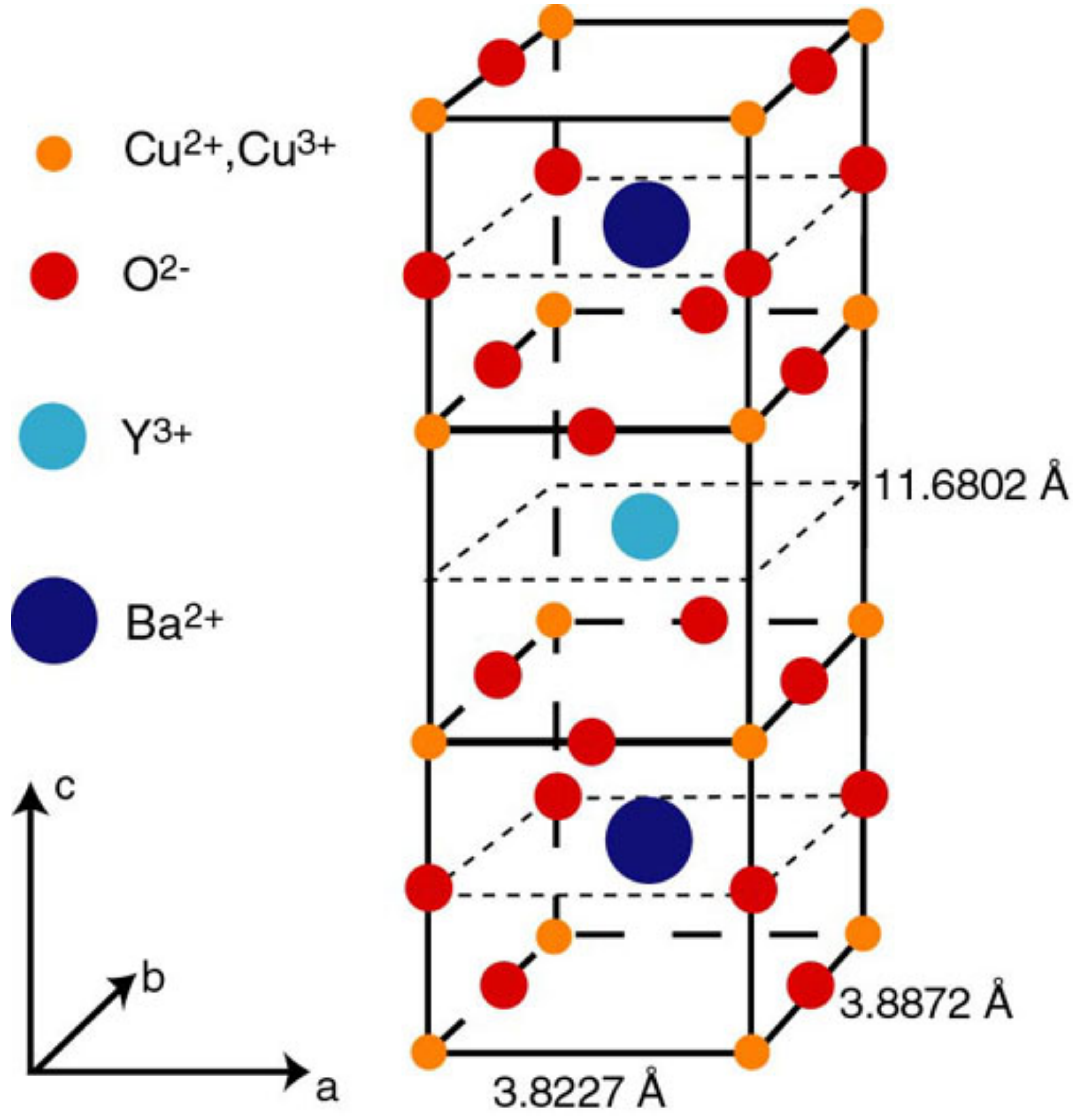
The motion of electrons is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.

Cuprate high temperature superconductors

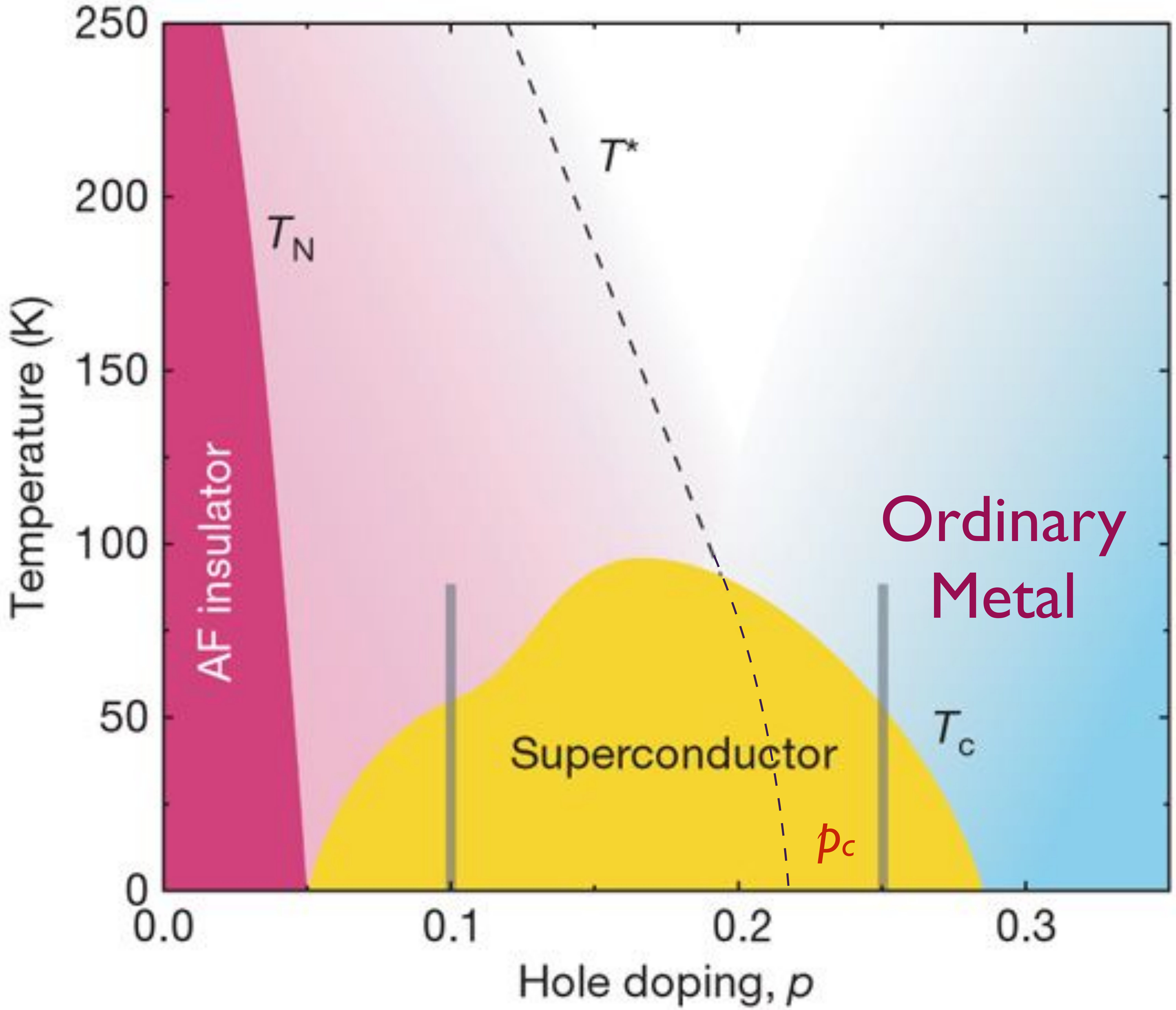


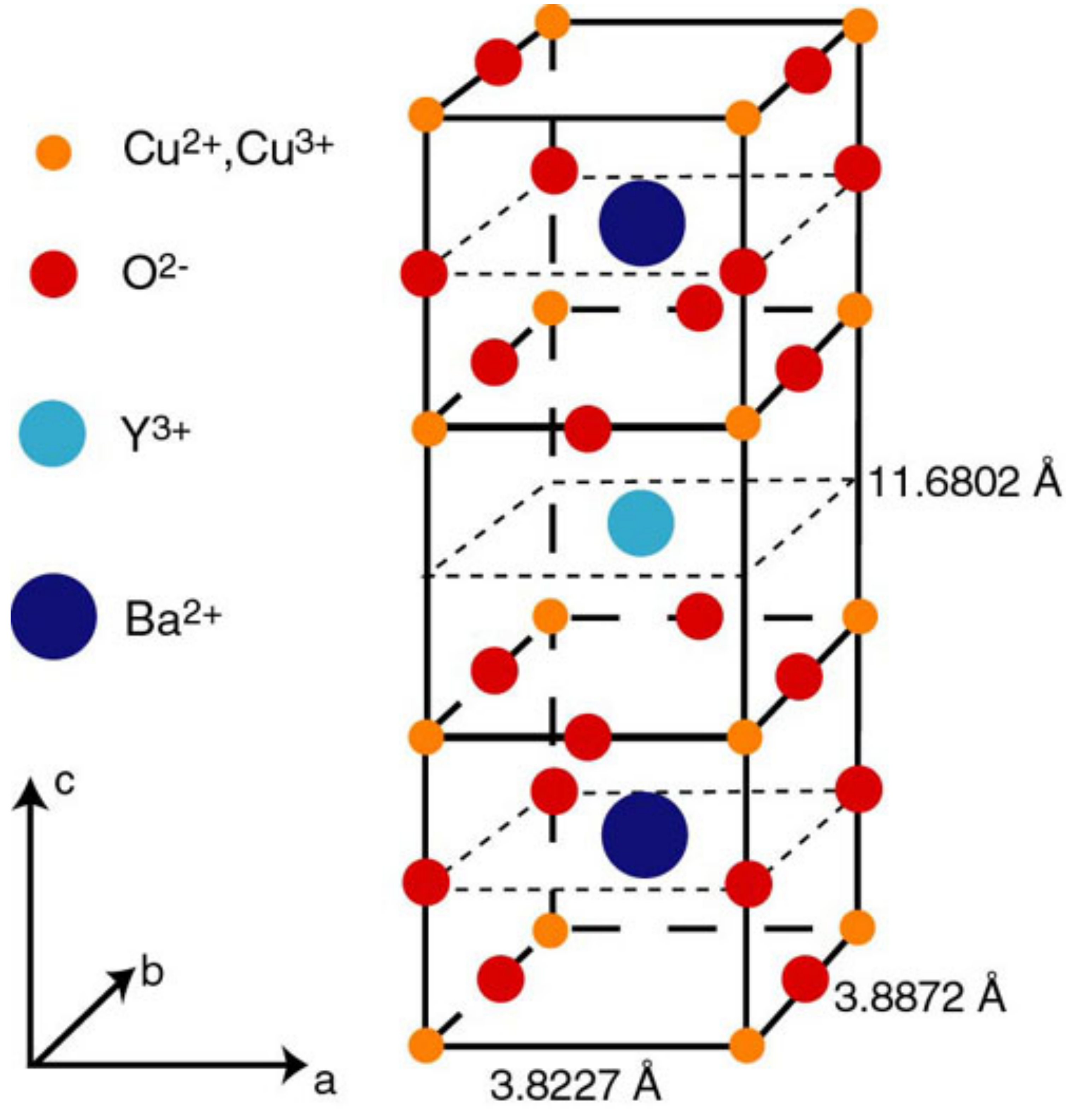
$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Superconductor
T_c
p_c

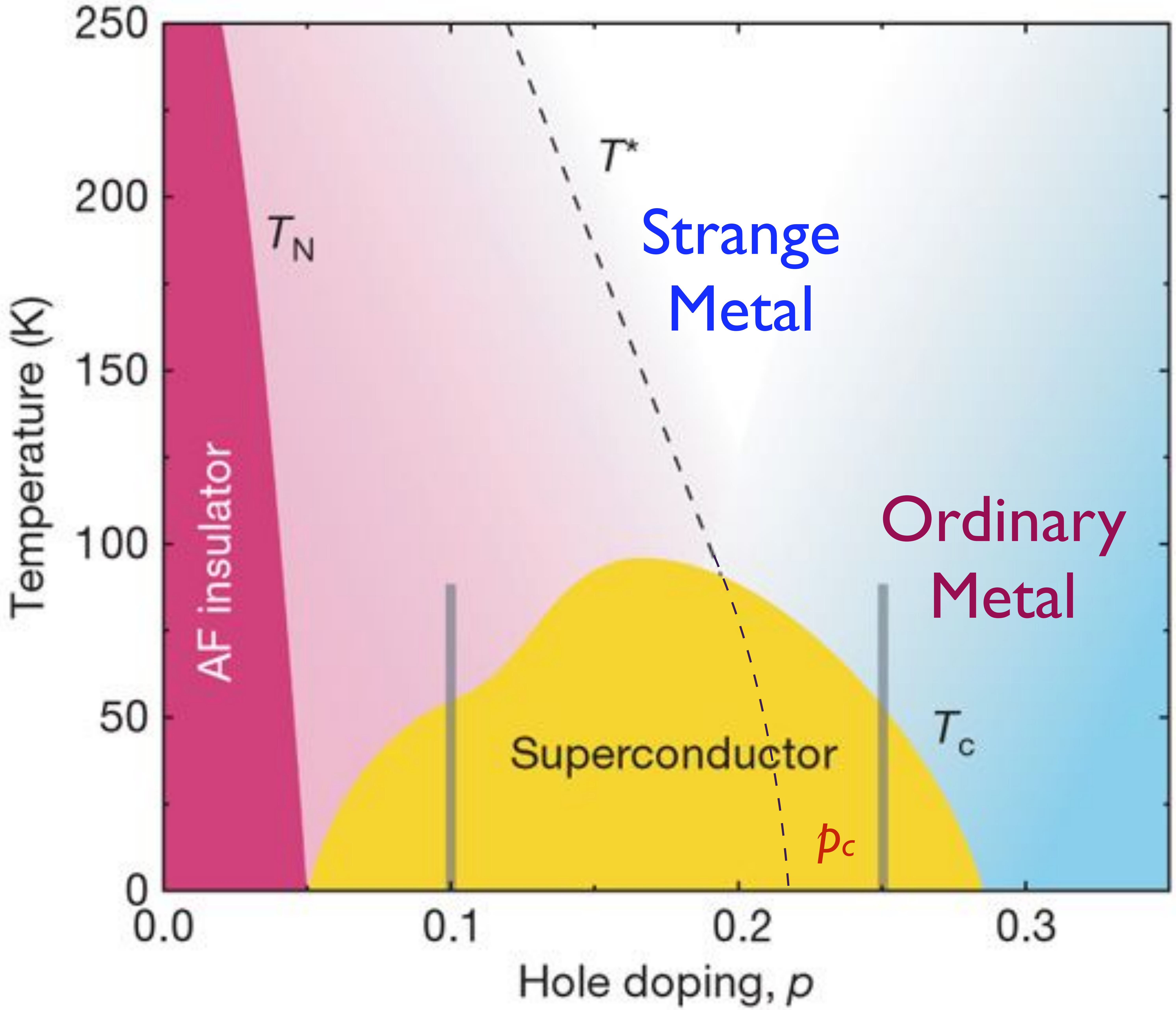

Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

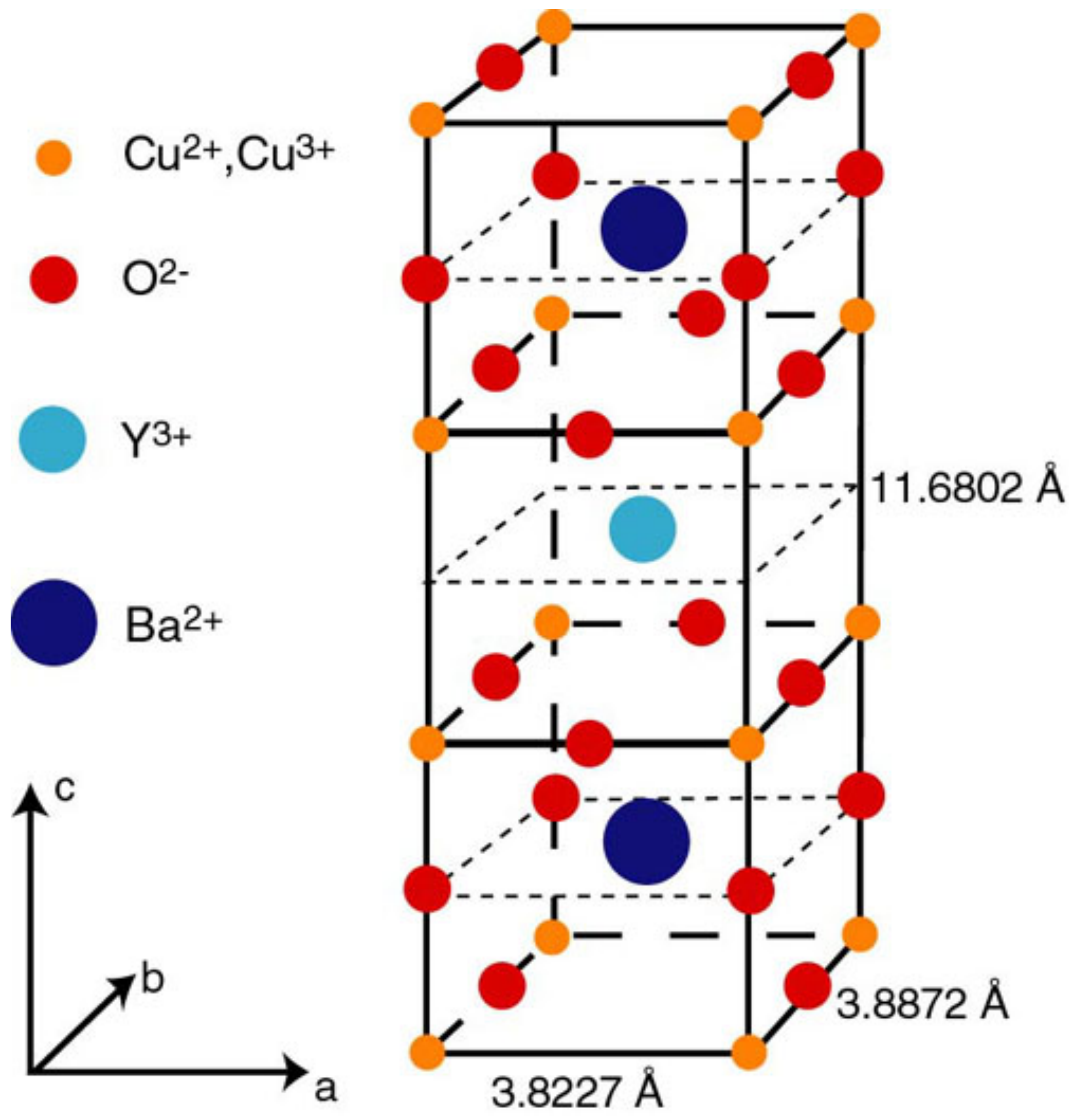
$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Ordinary Metal
Superconductor
T_c
p_c

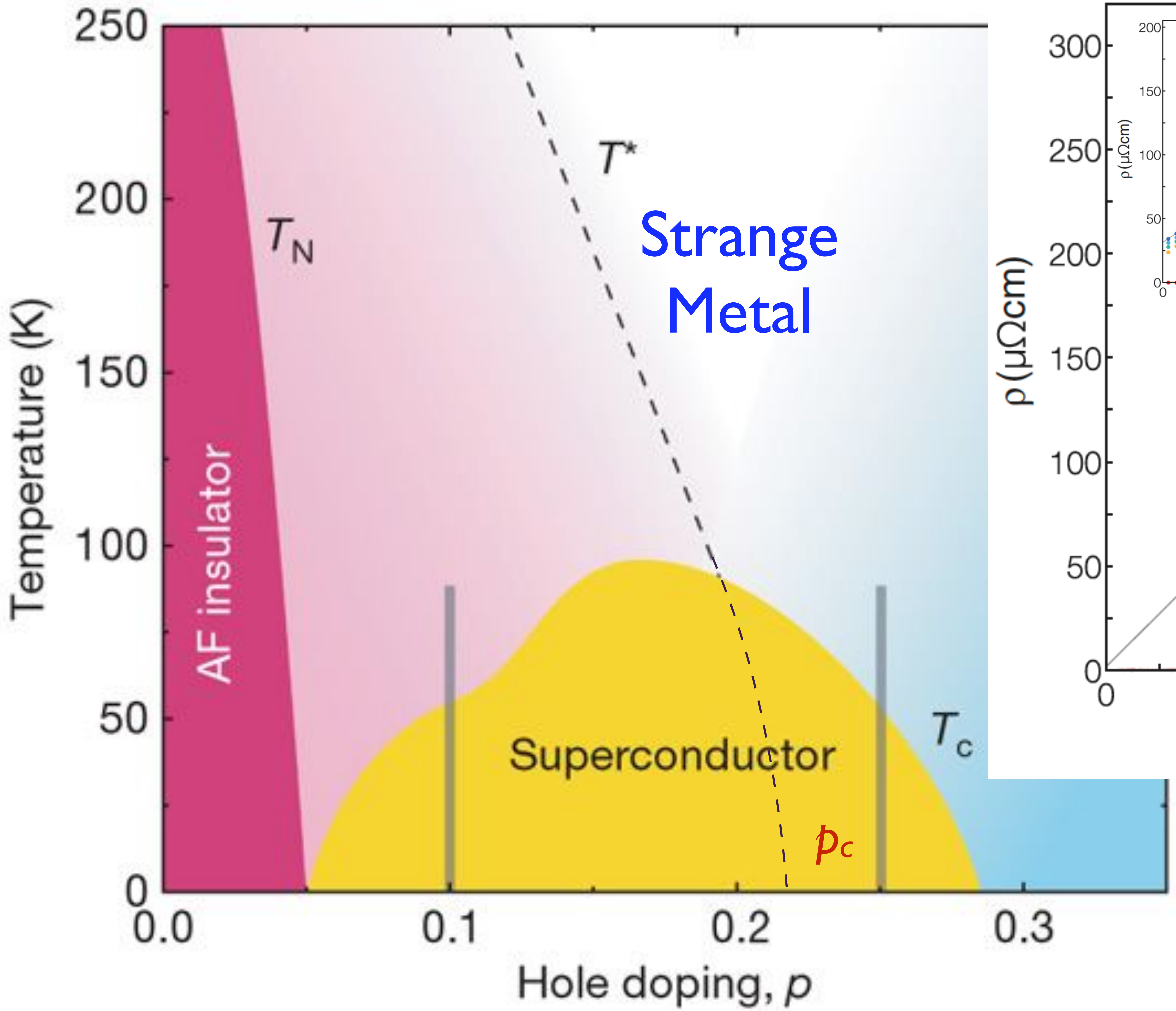

Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

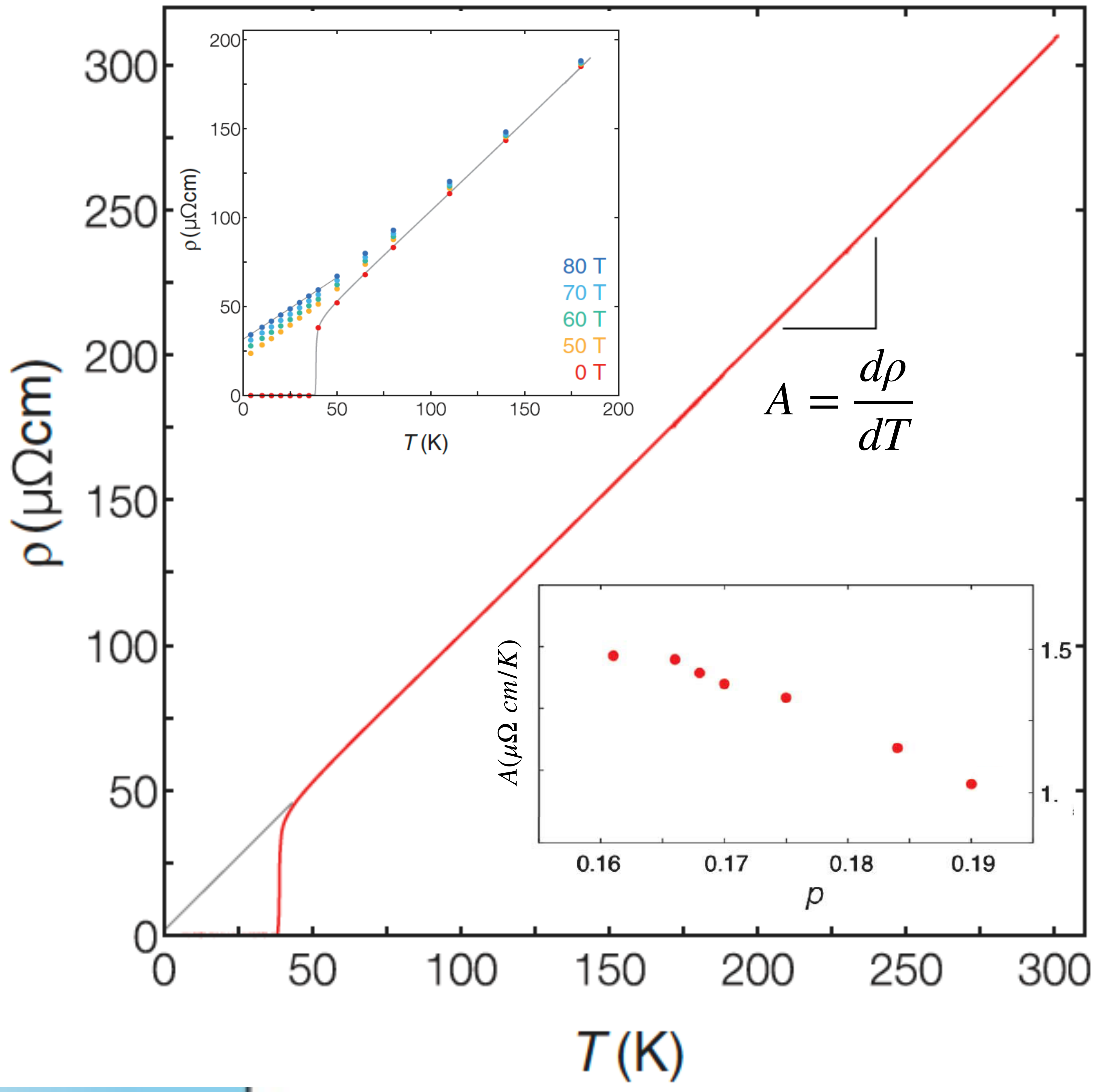
$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Ordinary Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$



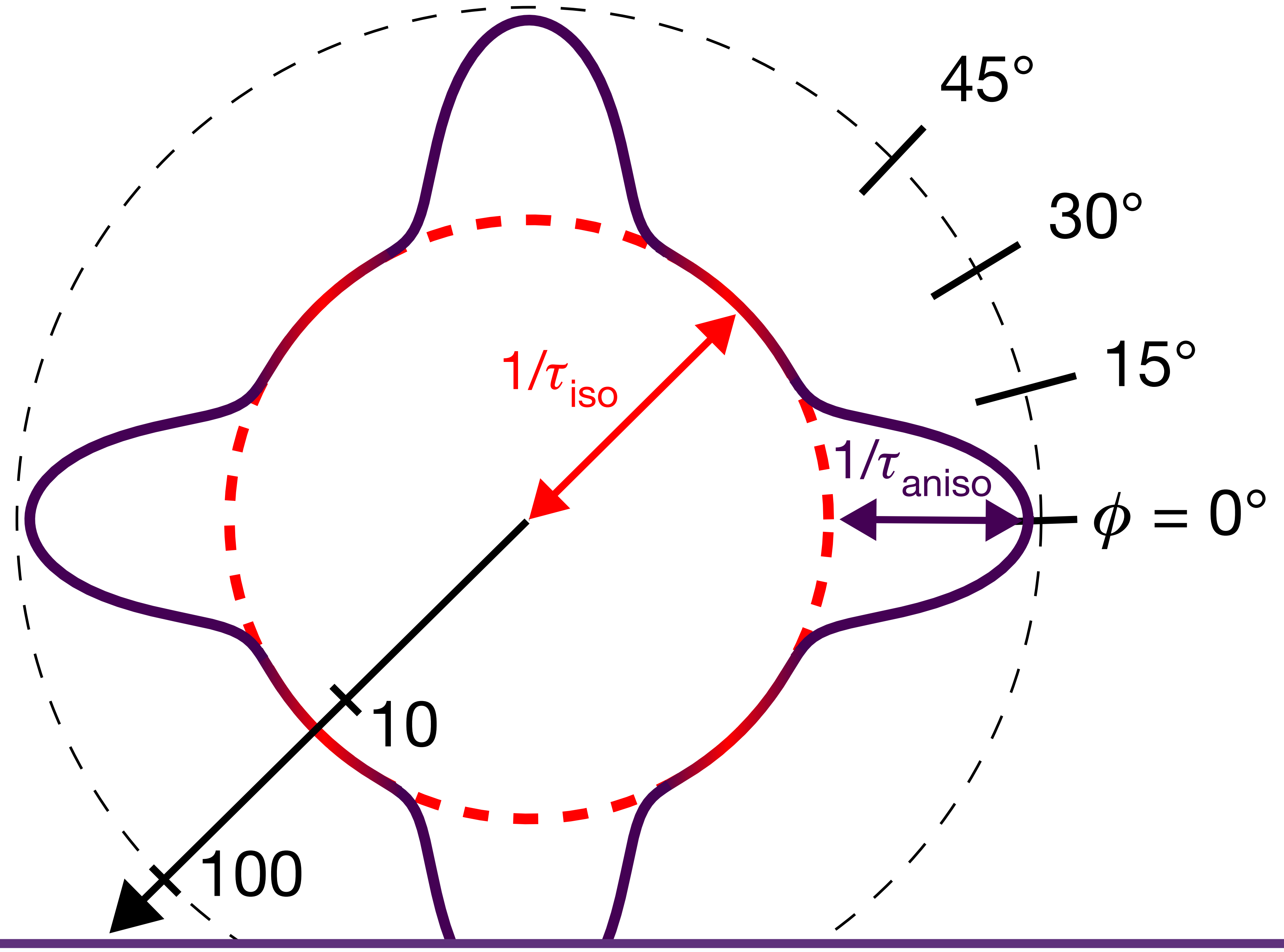


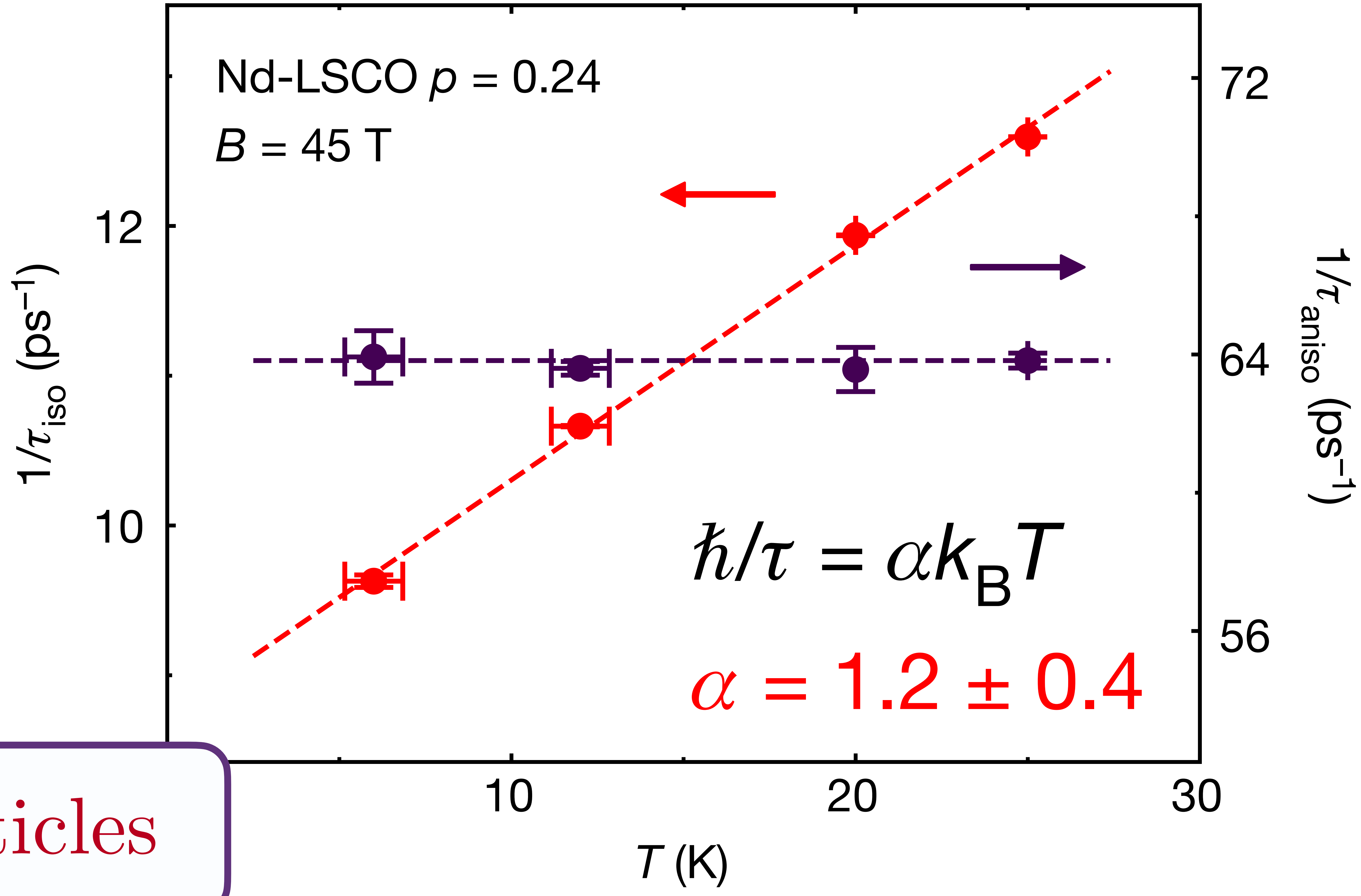
LSCO: Giraldo-Gallo et al. 2018

Linear-in temperature resistivity from an isotropic Planckian scattering rate

Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw





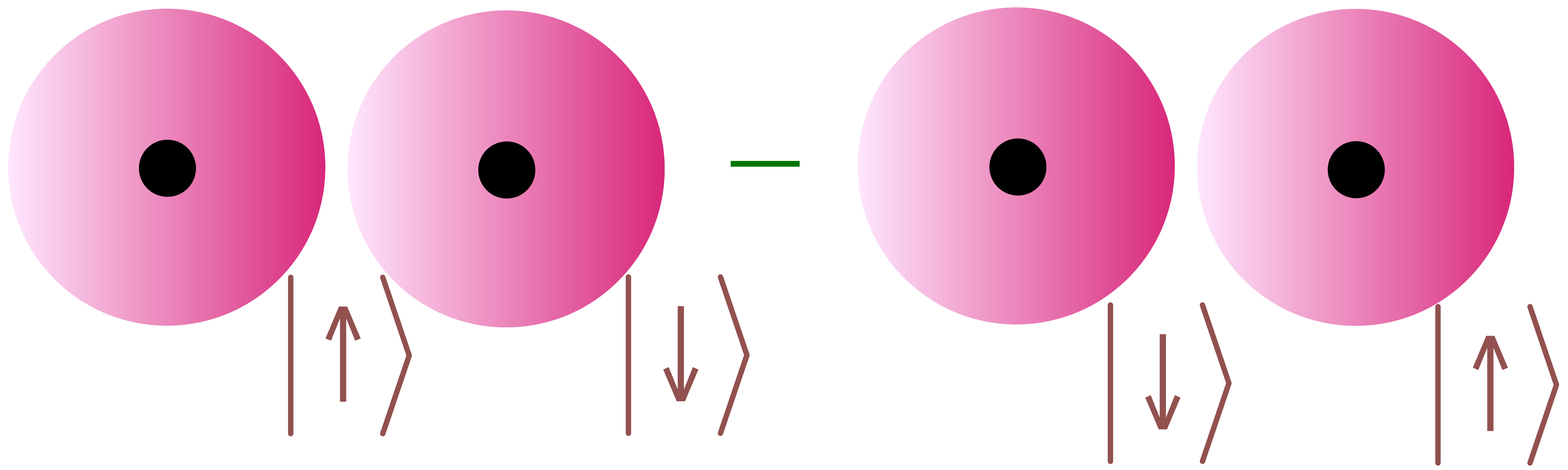
Current flow without quasiparticles

No Boltzmann-Landau quasiparticle description $\Rightarrow$
Many particle quantum entanglement
from quantum interference between “collisions”

Sachdev-Ye-Kitaev Model

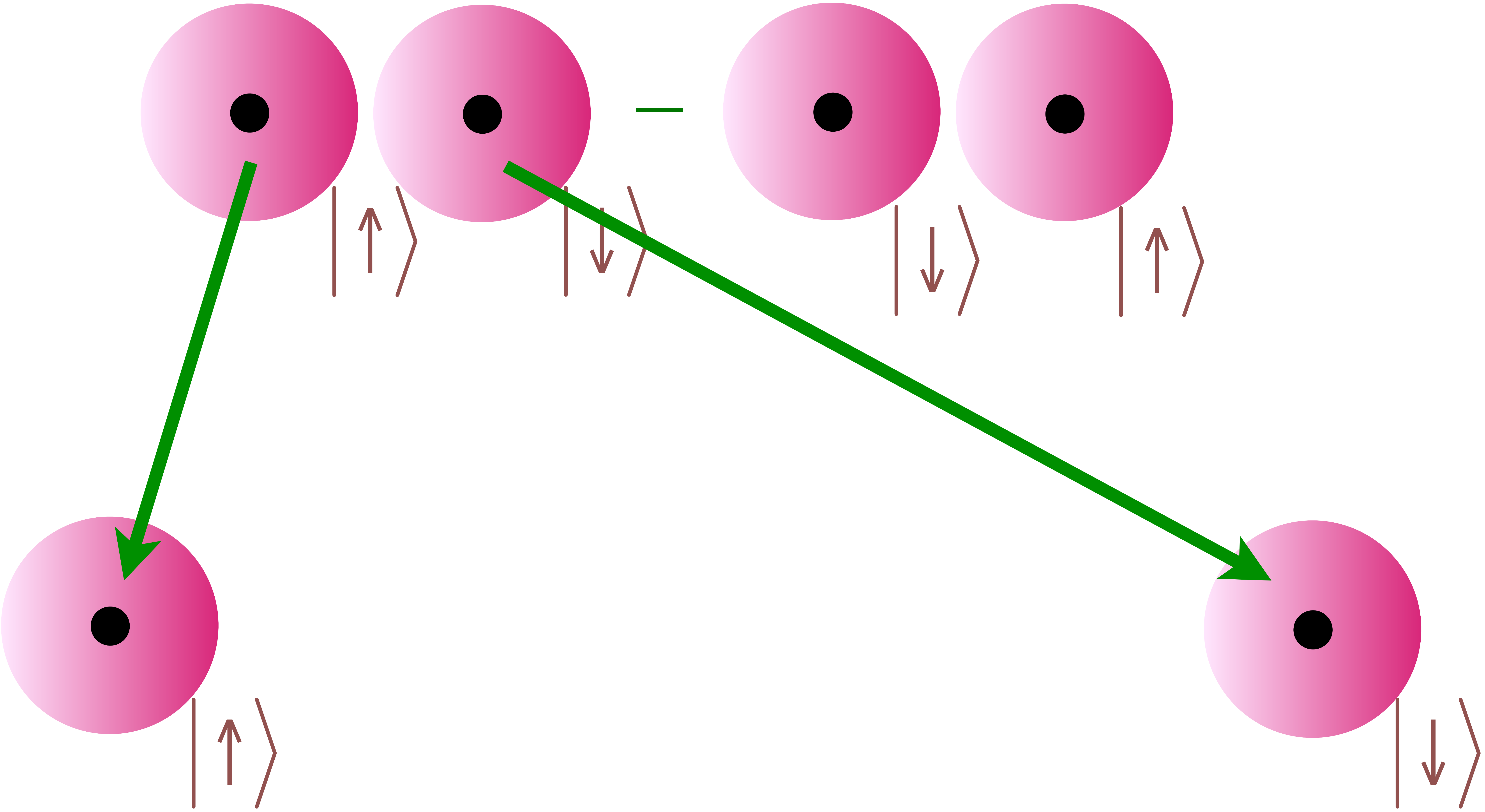
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



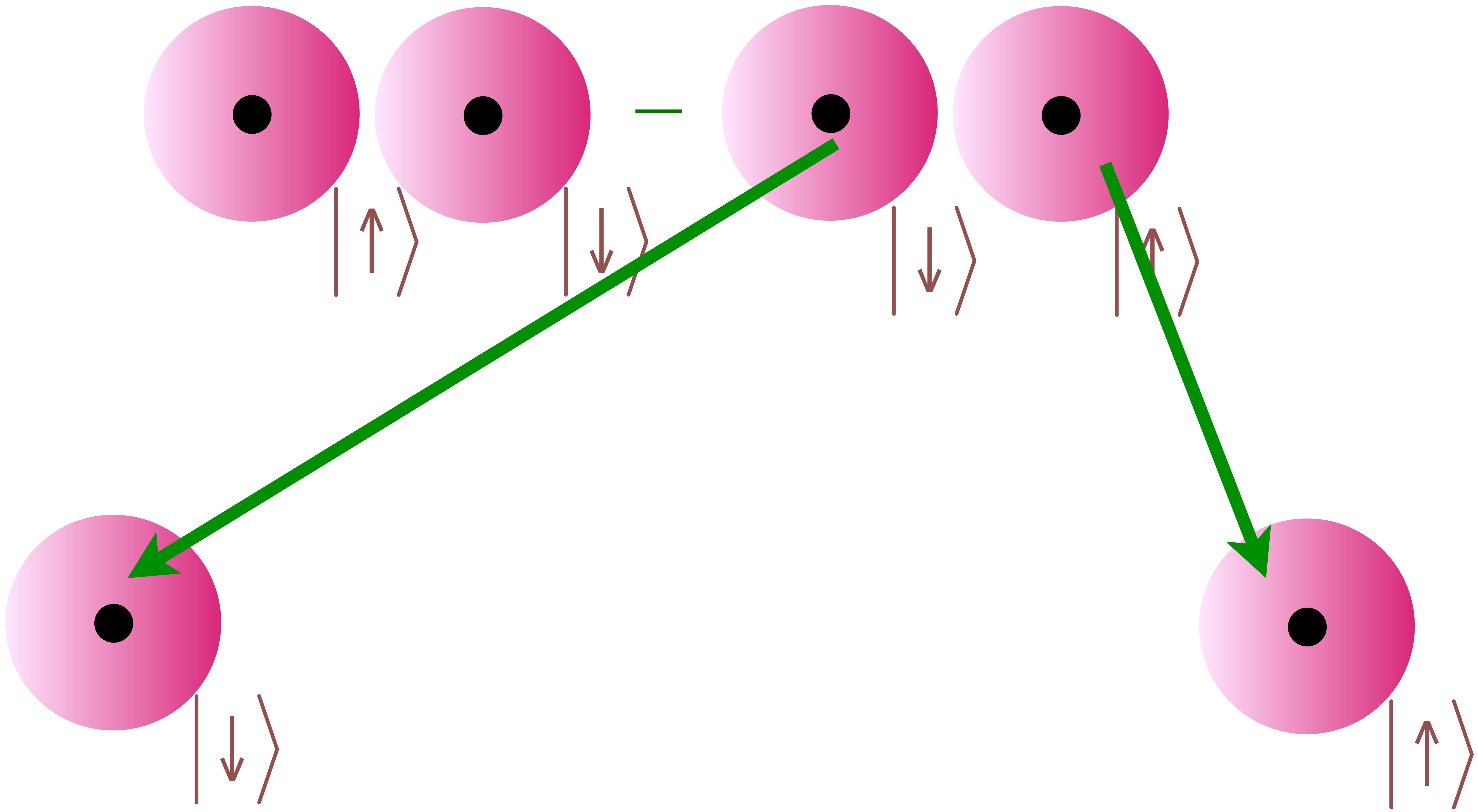
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



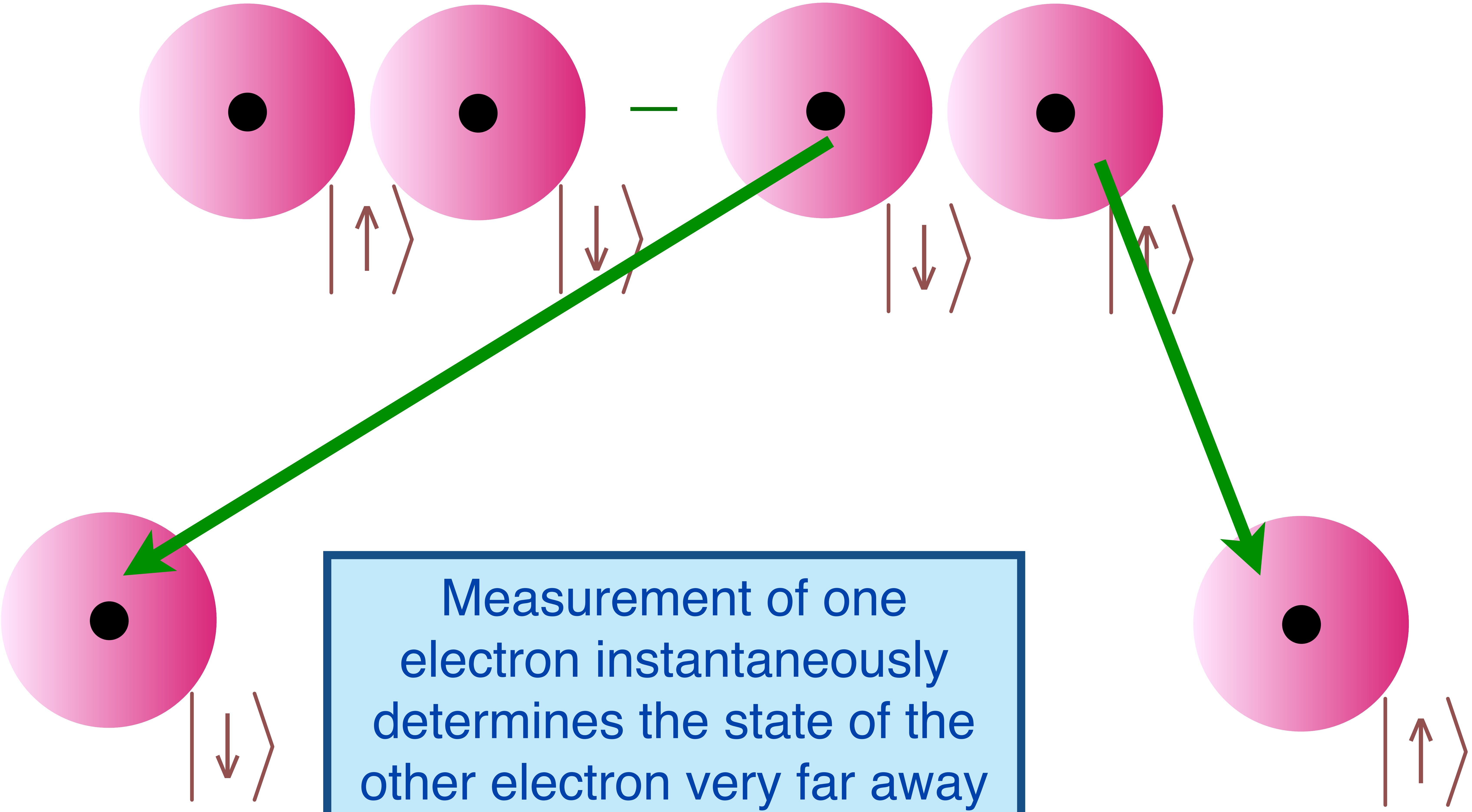
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



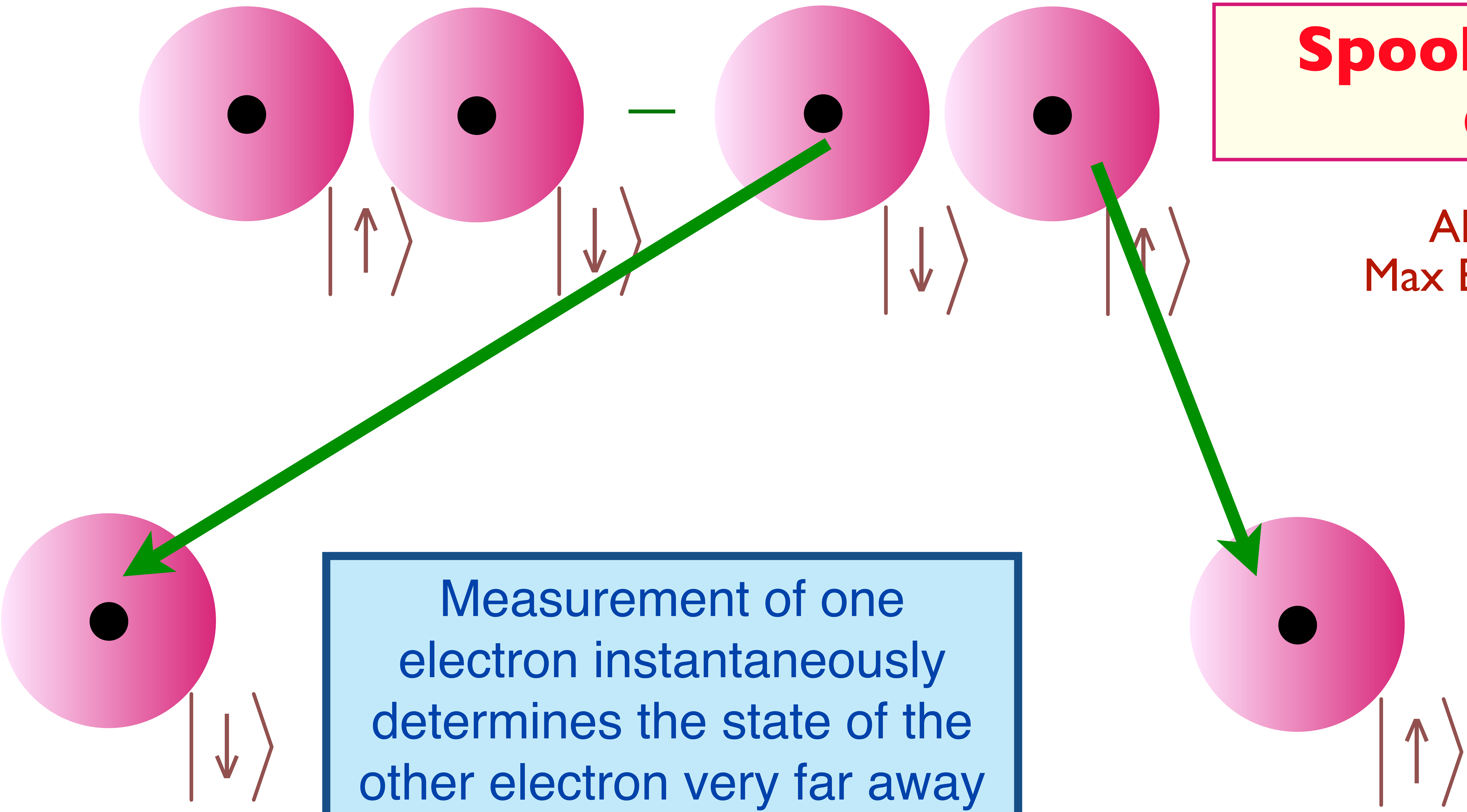
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



$|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle$

$|\downarrow\rangle$ $|\uparrow\rangle$

Spooky action at a distance

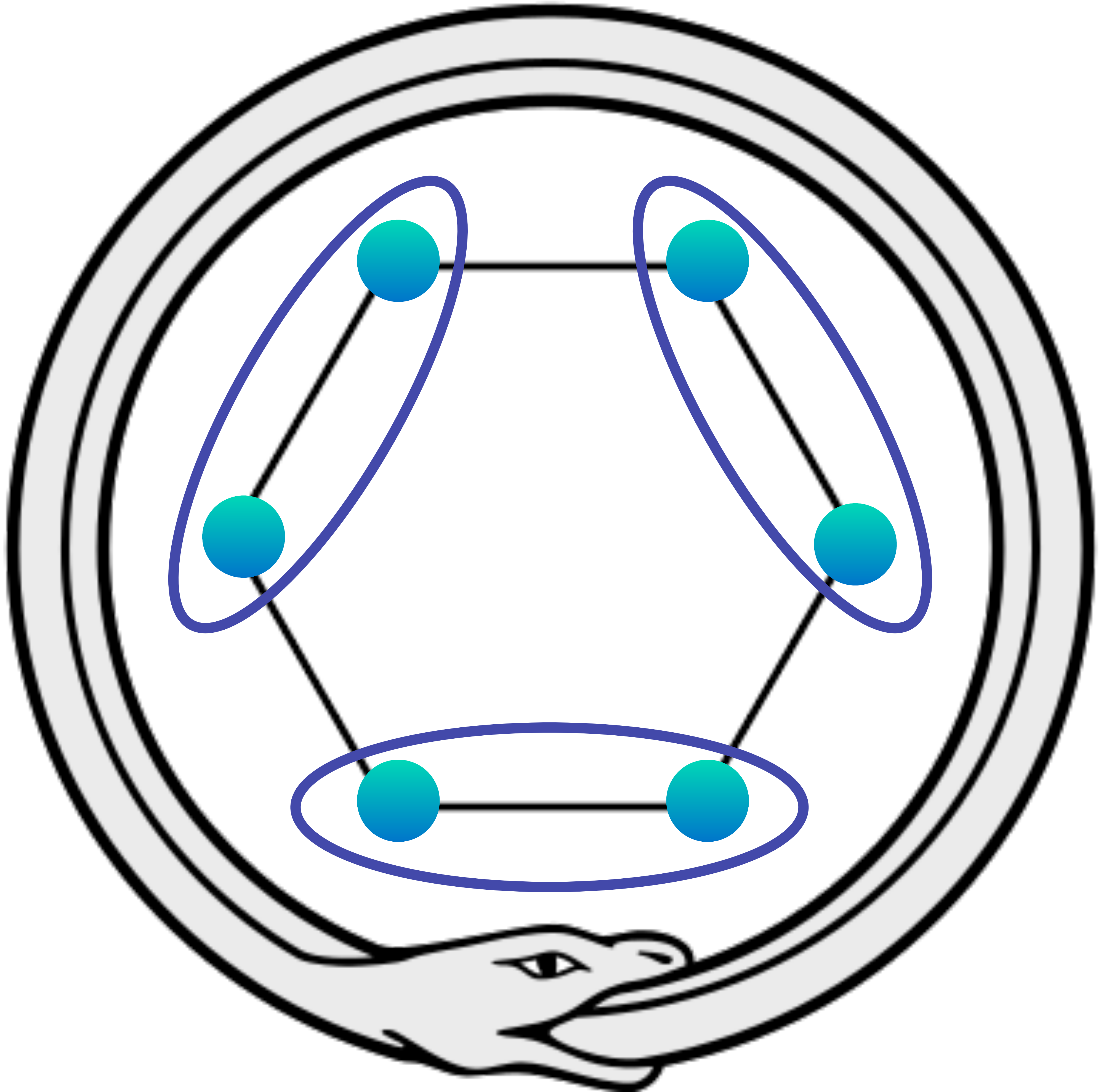
Albert Einstein to
Max Born, 3 March 1947

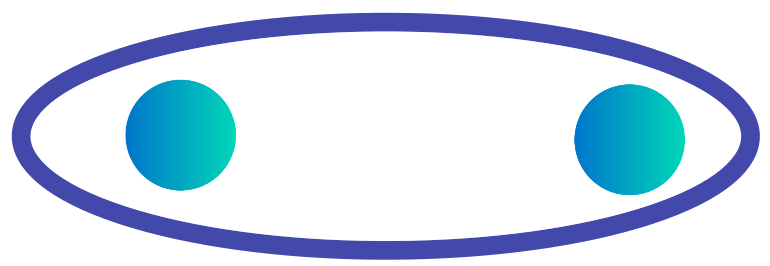
Measurement of one electron instantaneously determines the state of the other electron very far away

How about quantum entanglement of $3, 4, 5, \ldots \infty$ particles?

Kekule’s spooky dream (1865)

Here Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*



 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

*Wikipedia

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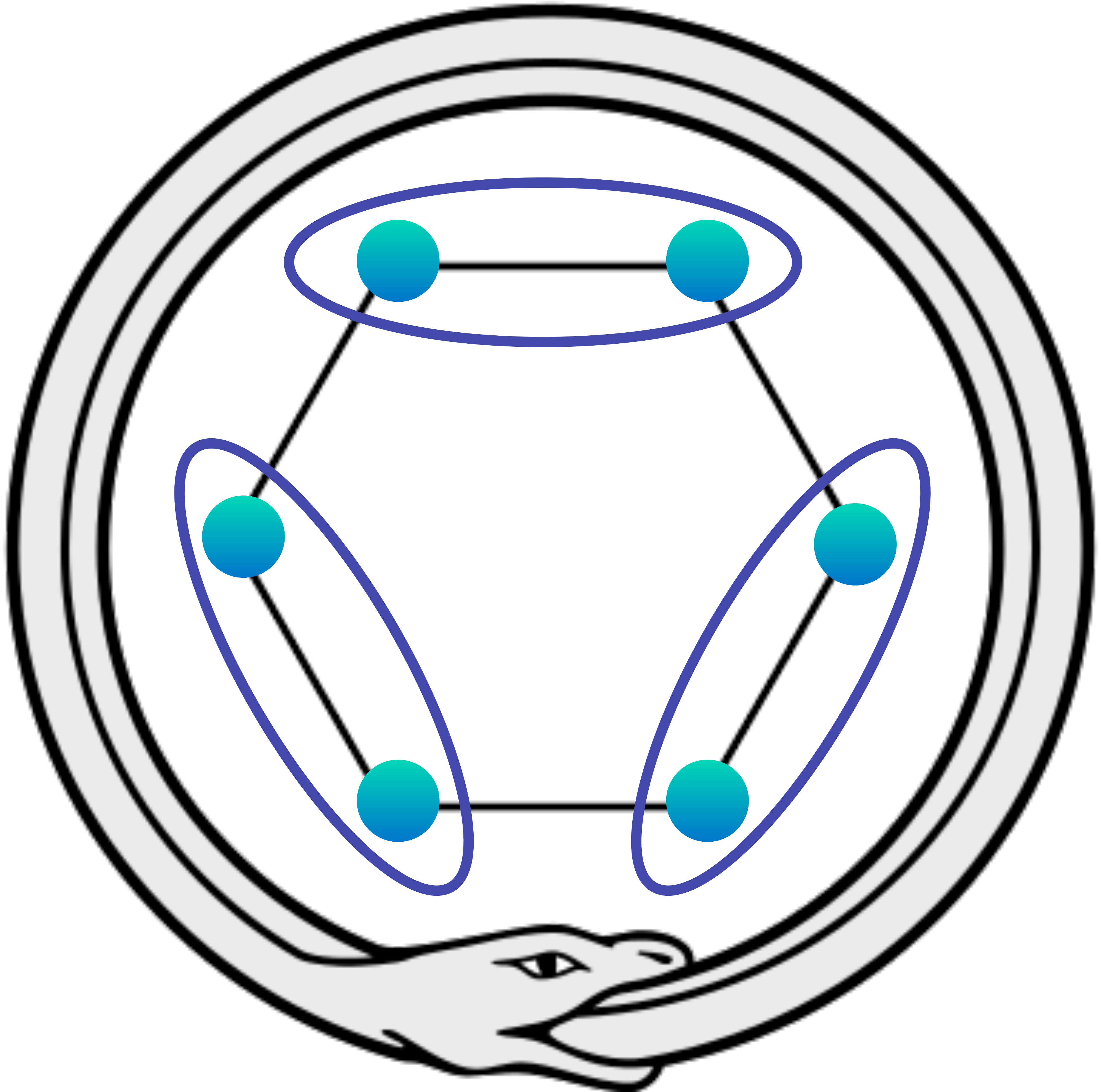

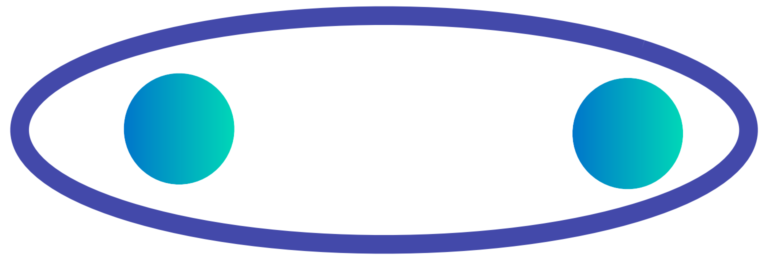

$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

*Wikipedia

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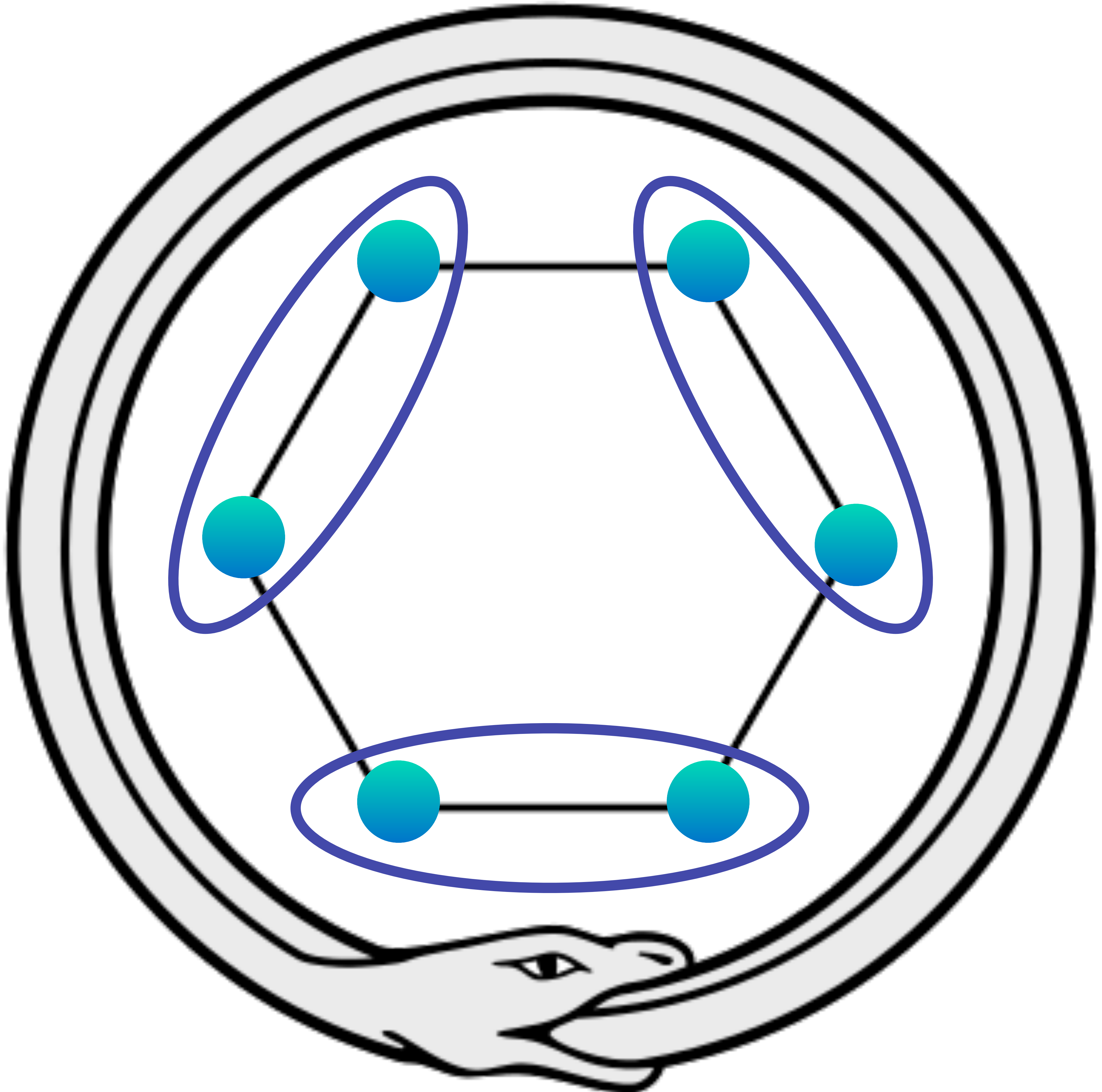
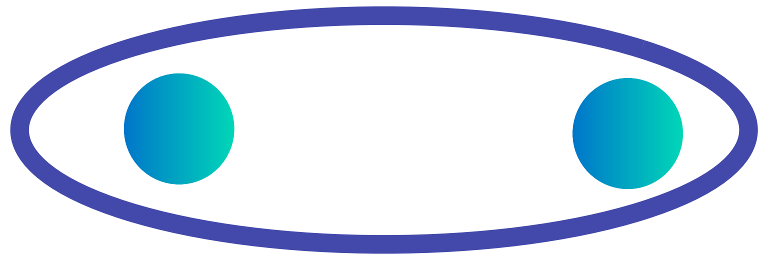




$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

*Wikipedia

My spooky dream: The Sachdev-Ye-Kitaev (SYK) model

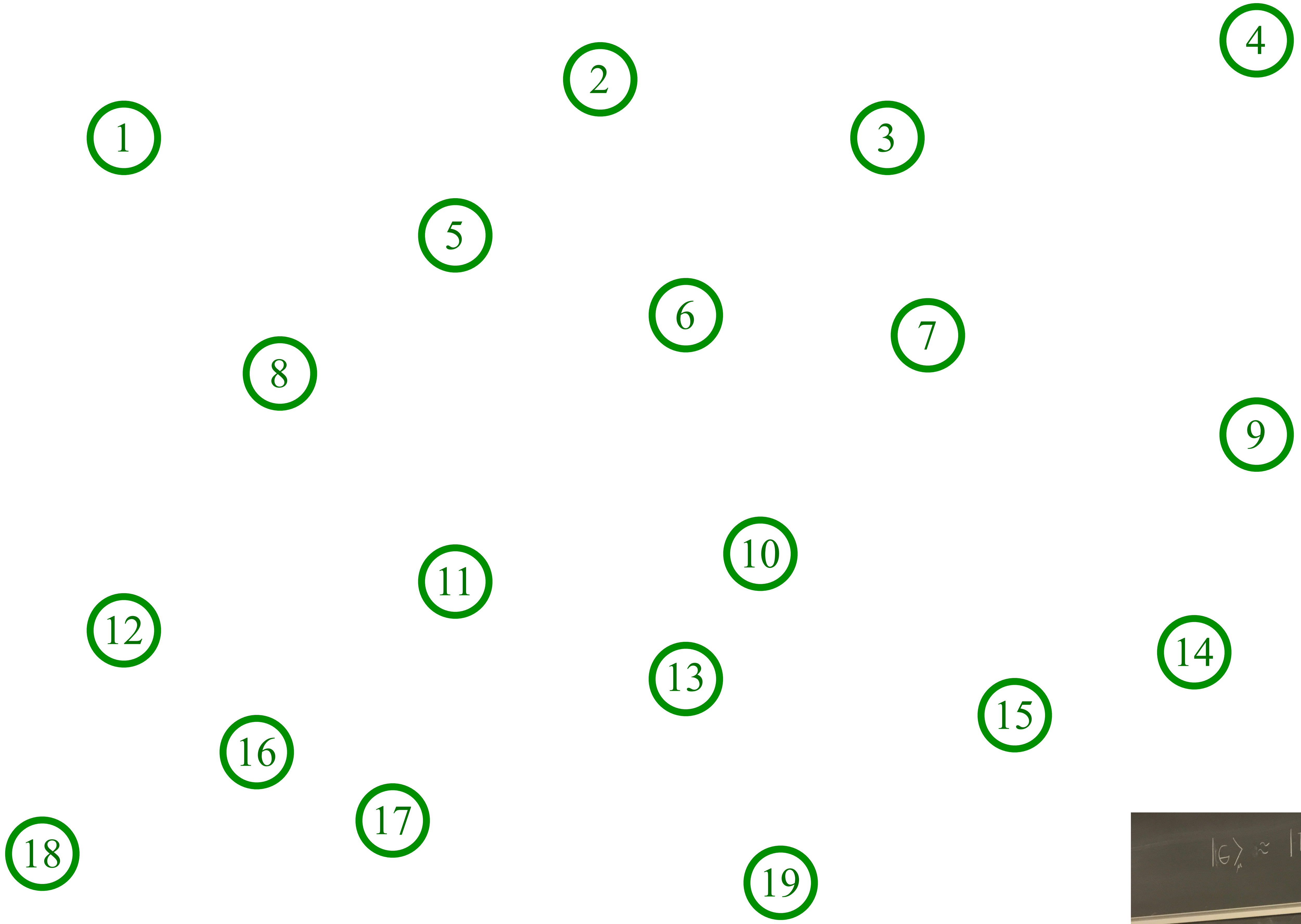
Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle quantum entanglement.

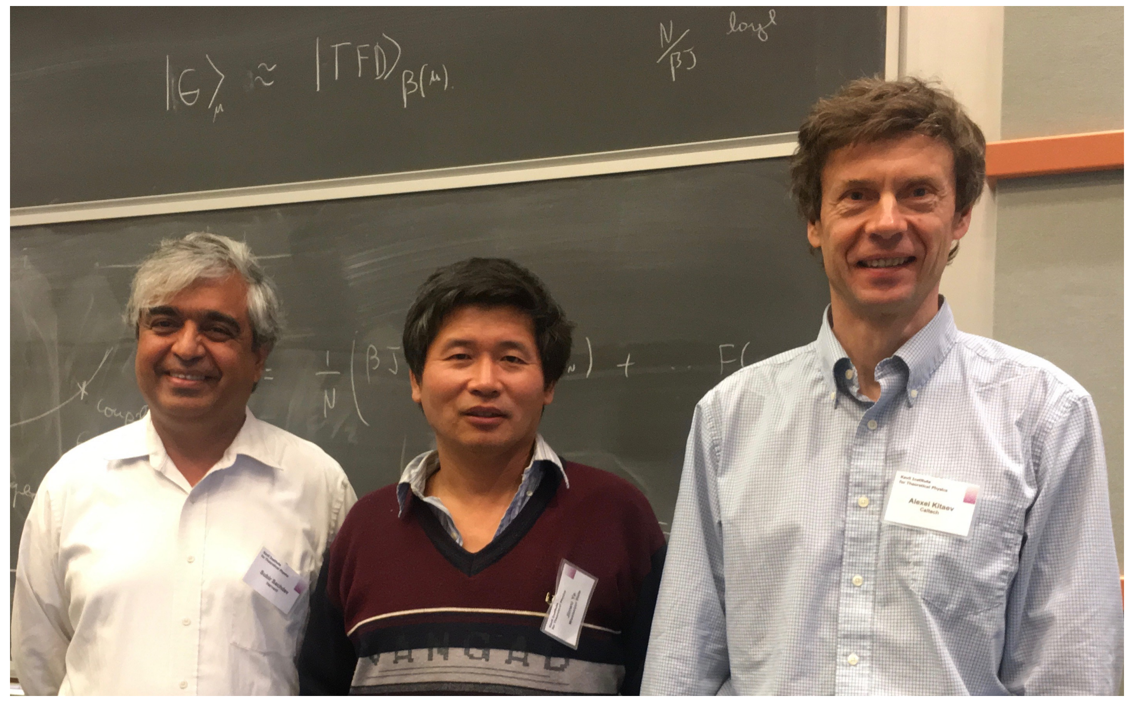
Yields a metal in which current is carried not by individual electrons, but by an entangled "quantum soup"

The SYK model

Sachdev, Ye (1993); Kitaev (2015)

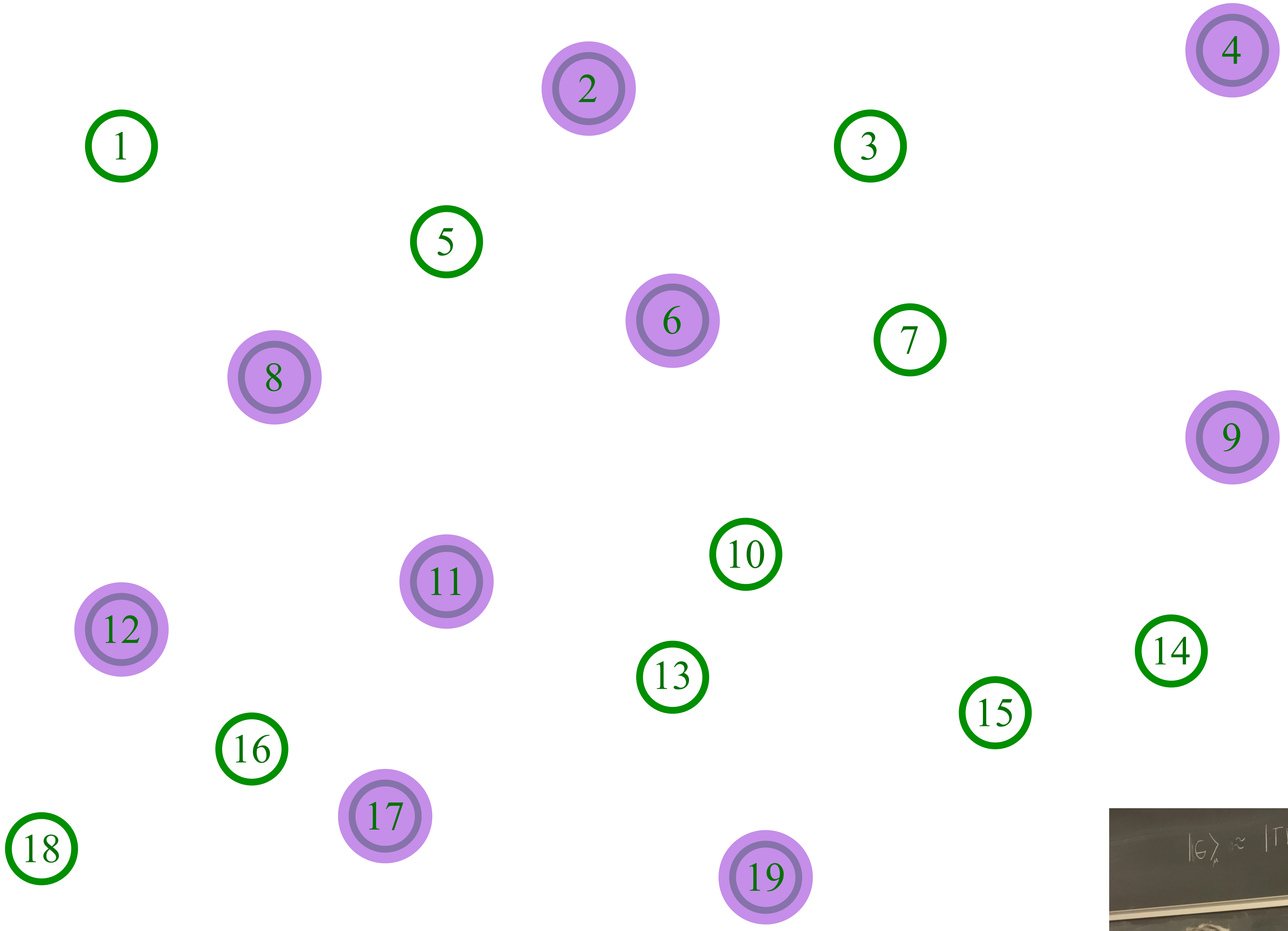


Pick a set of random positions

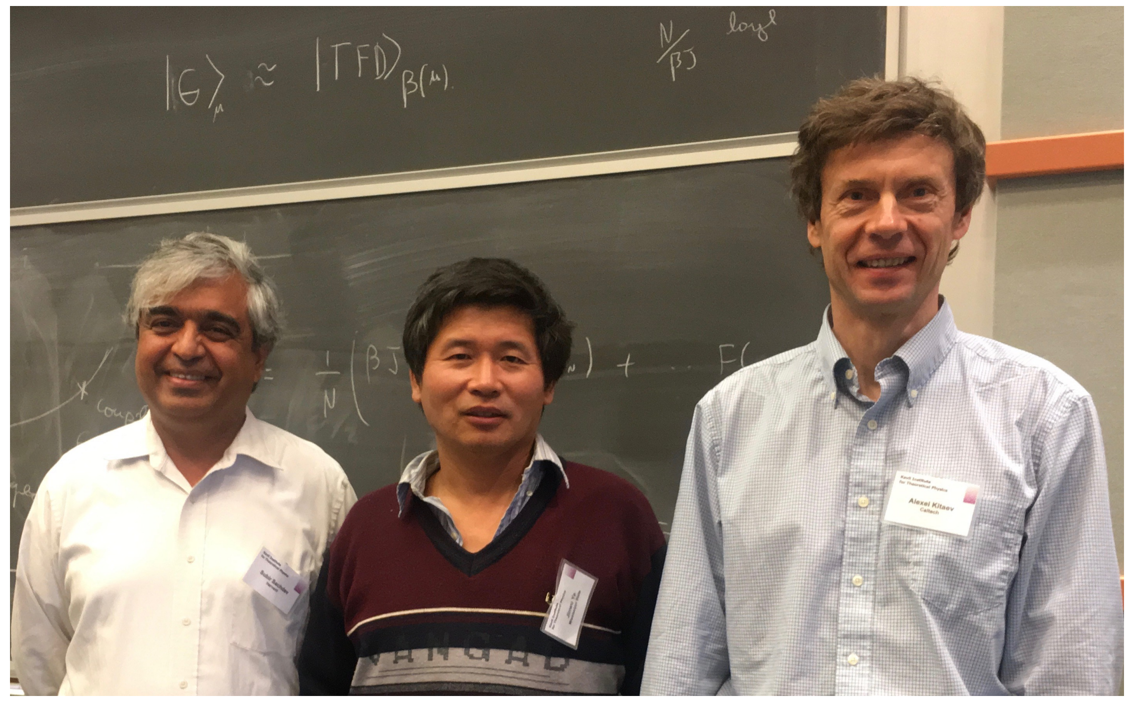


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



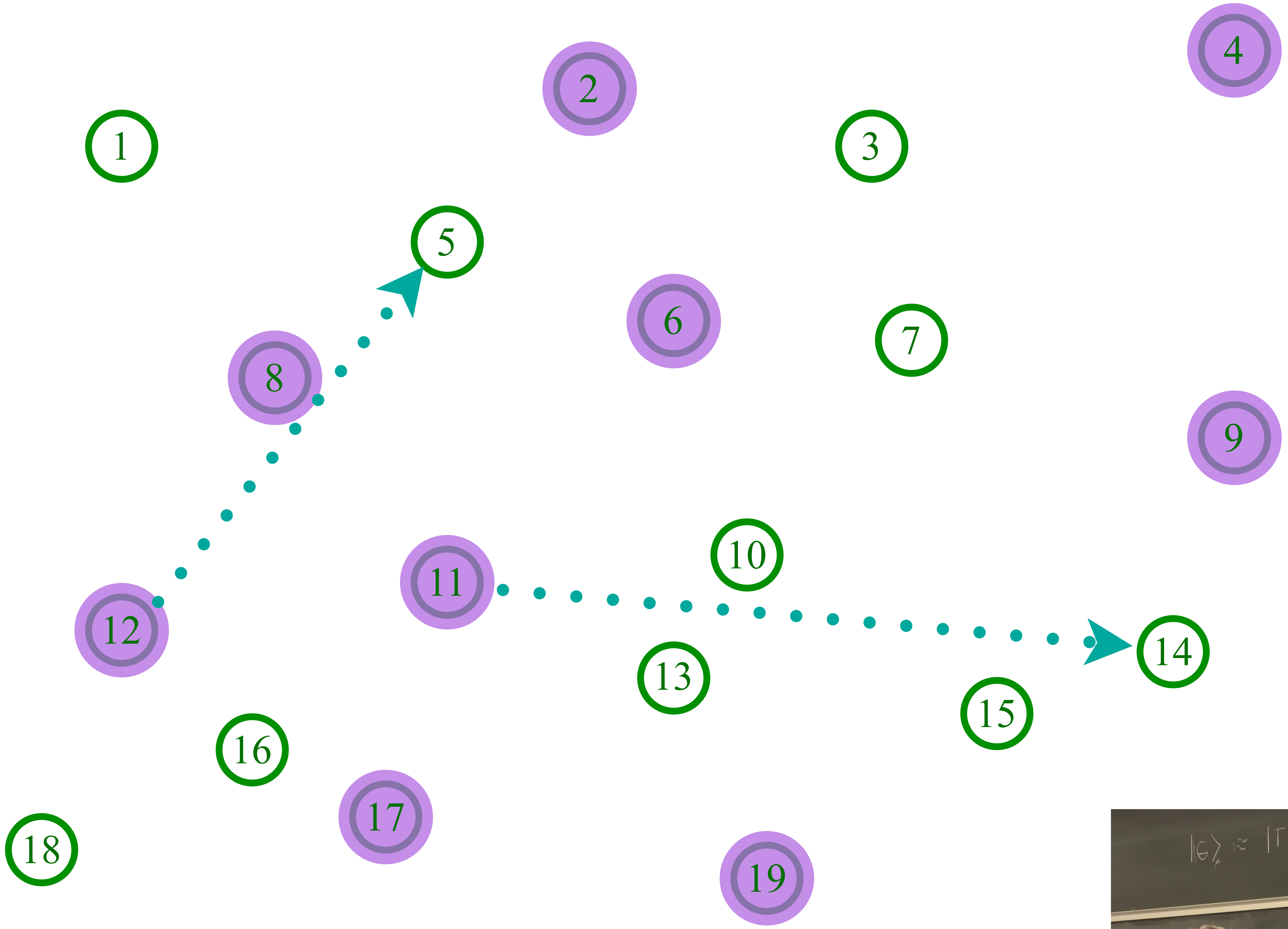
Place electrons randomly on some sites



The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



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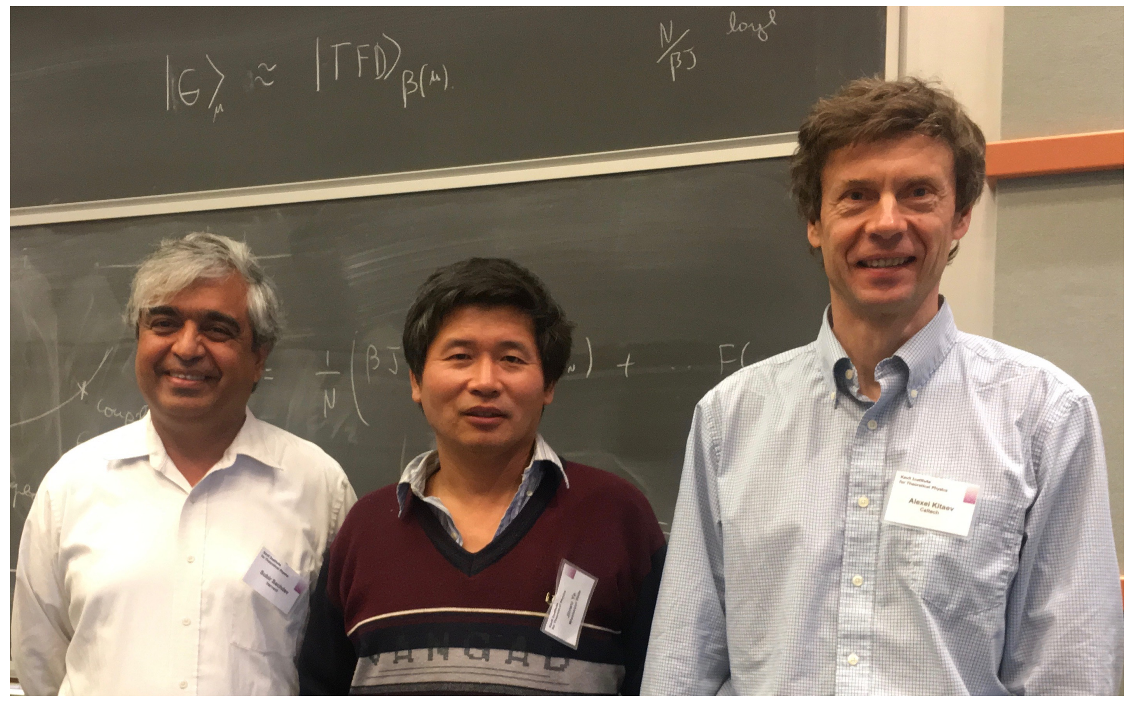
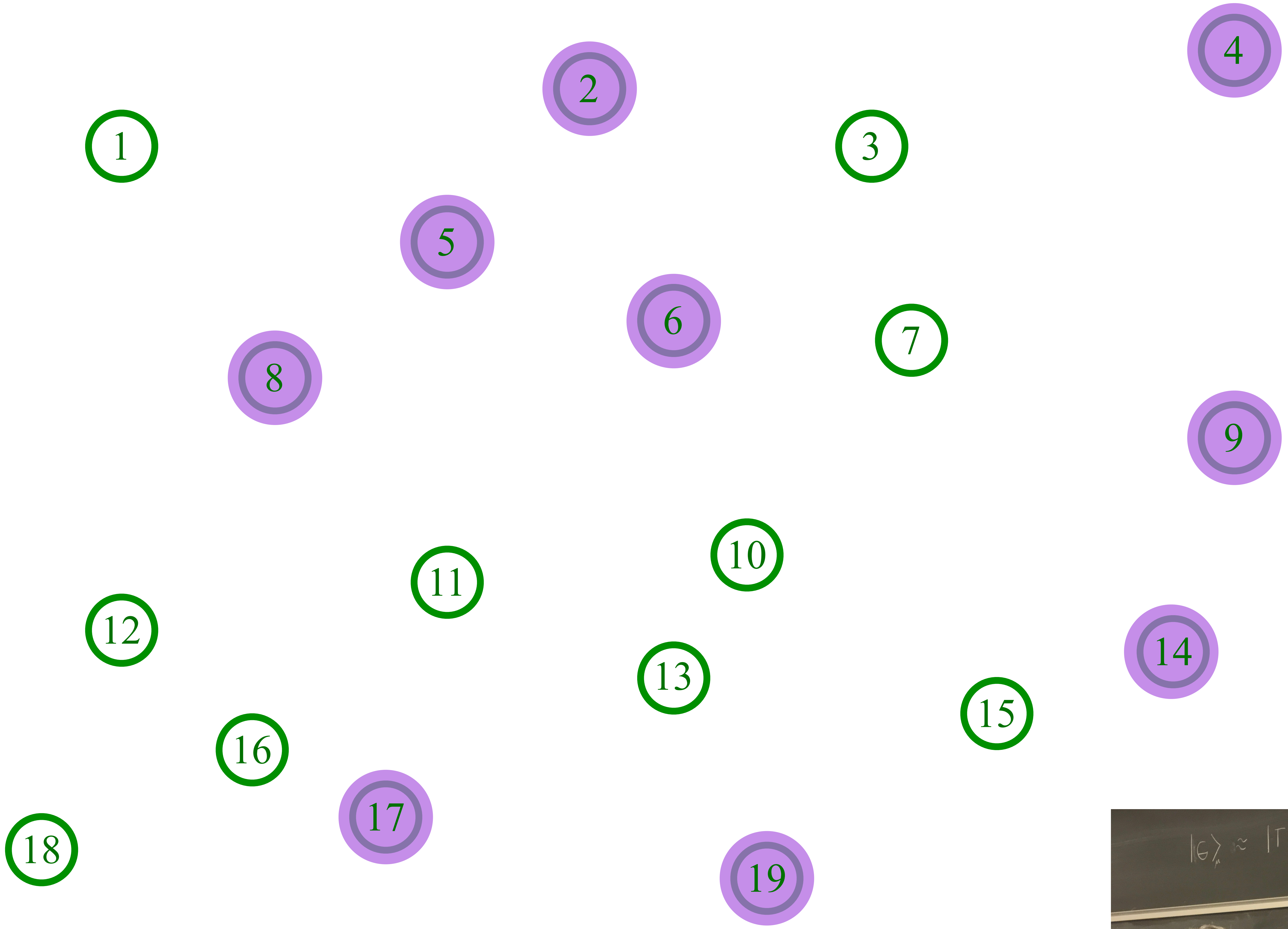


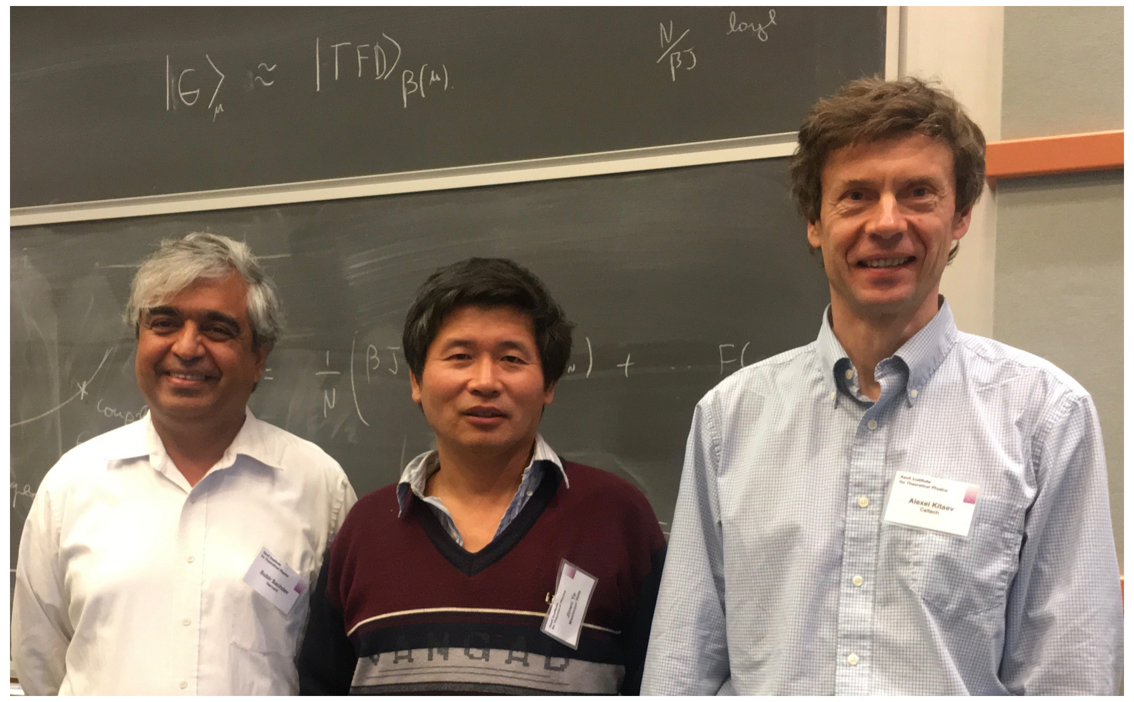
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



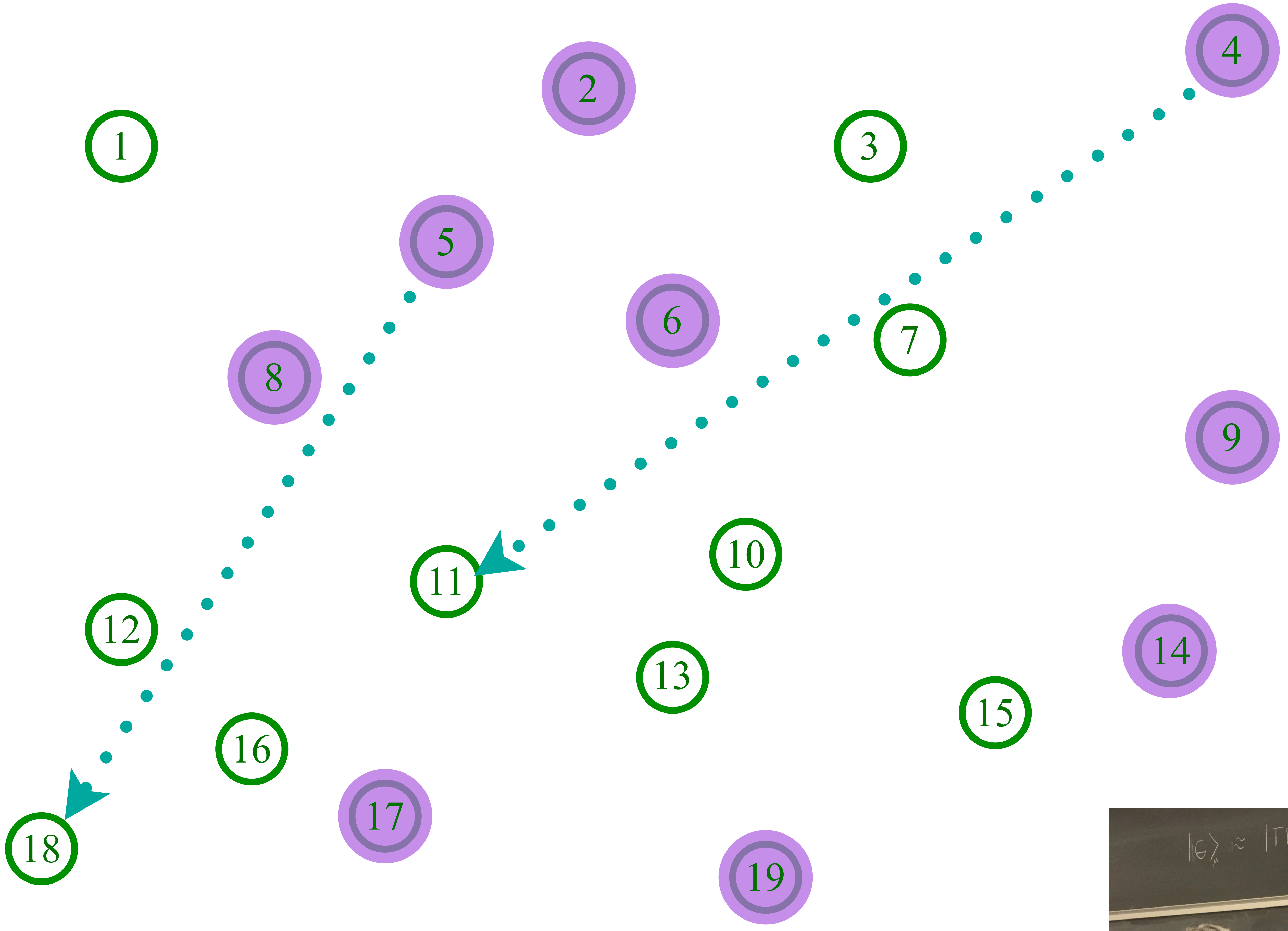
Entangle electrons pairwise randomly



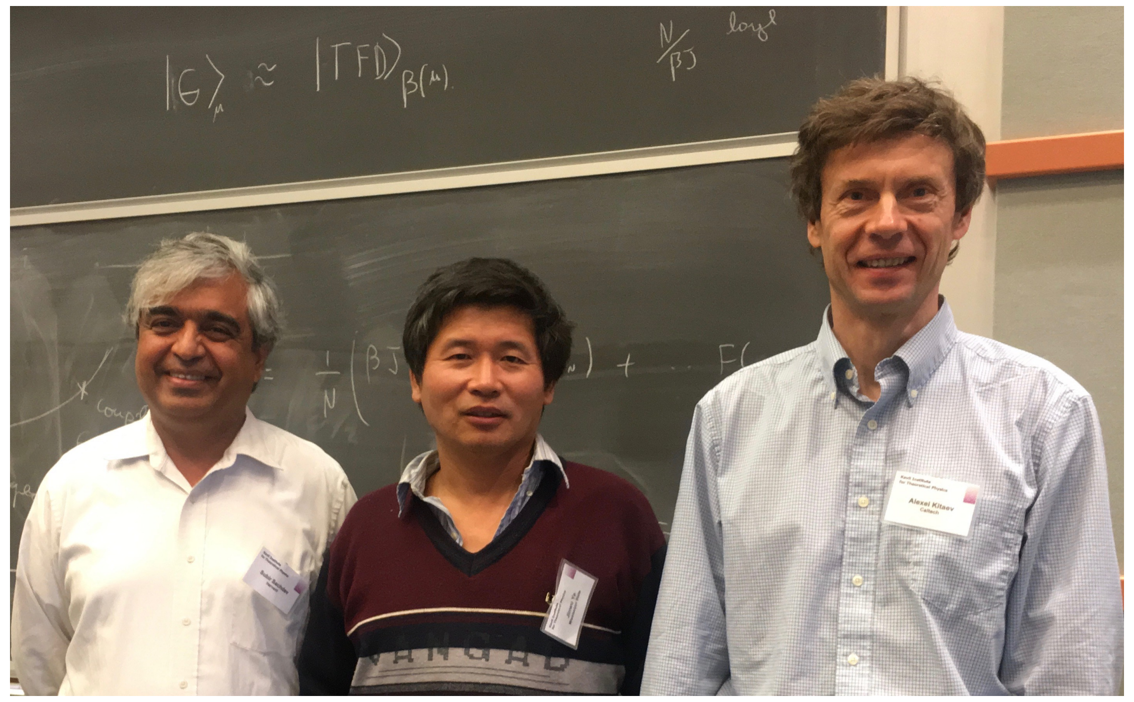
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



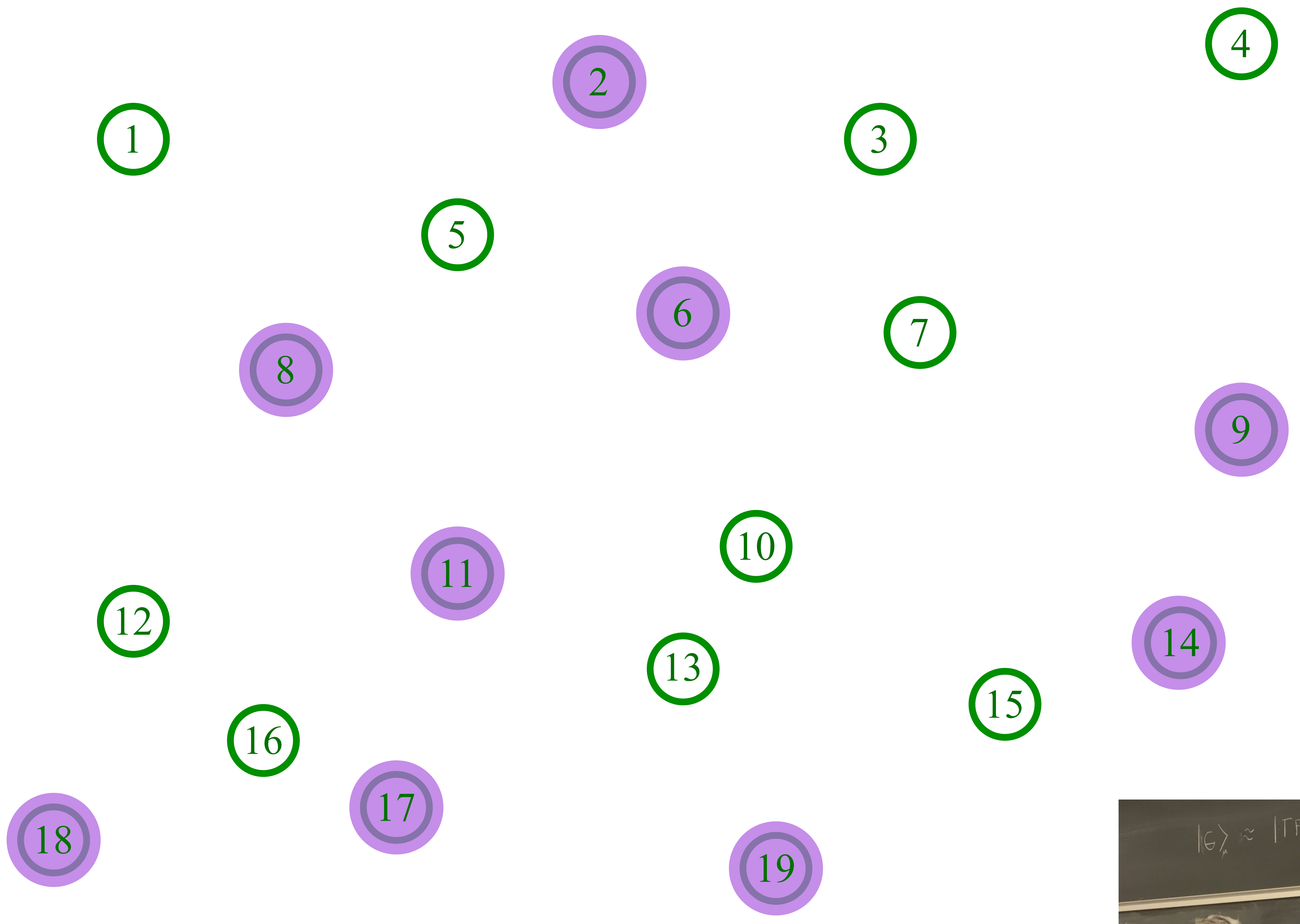
Entangle electrons pairwise randomly



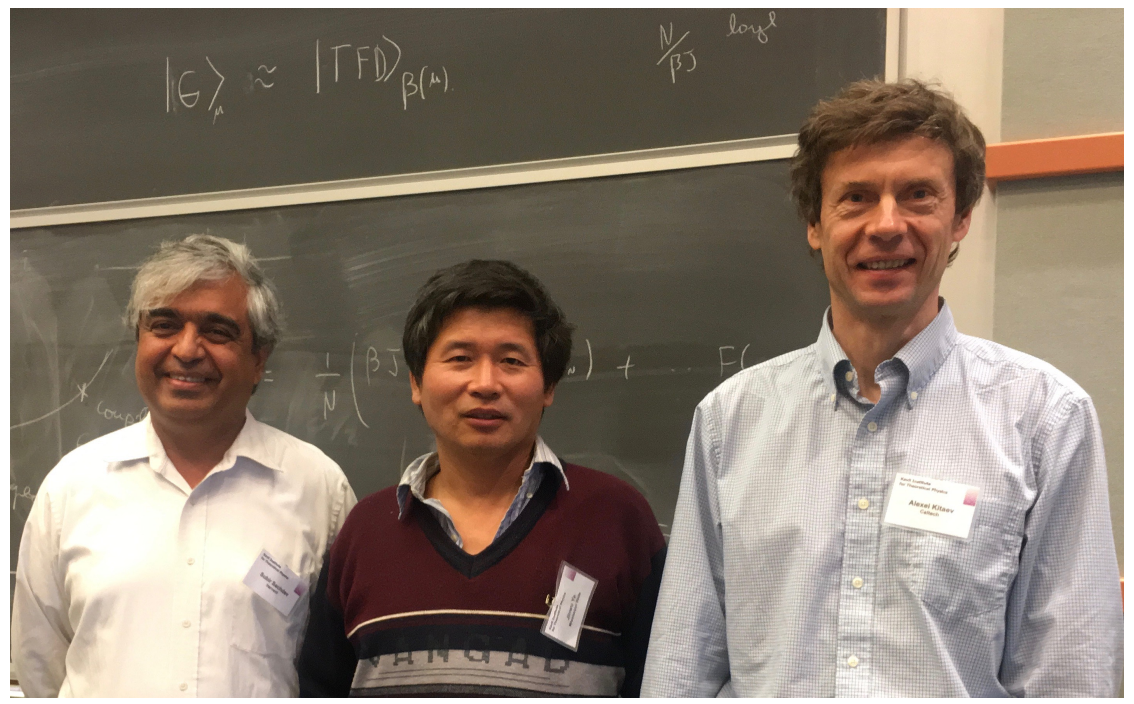
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



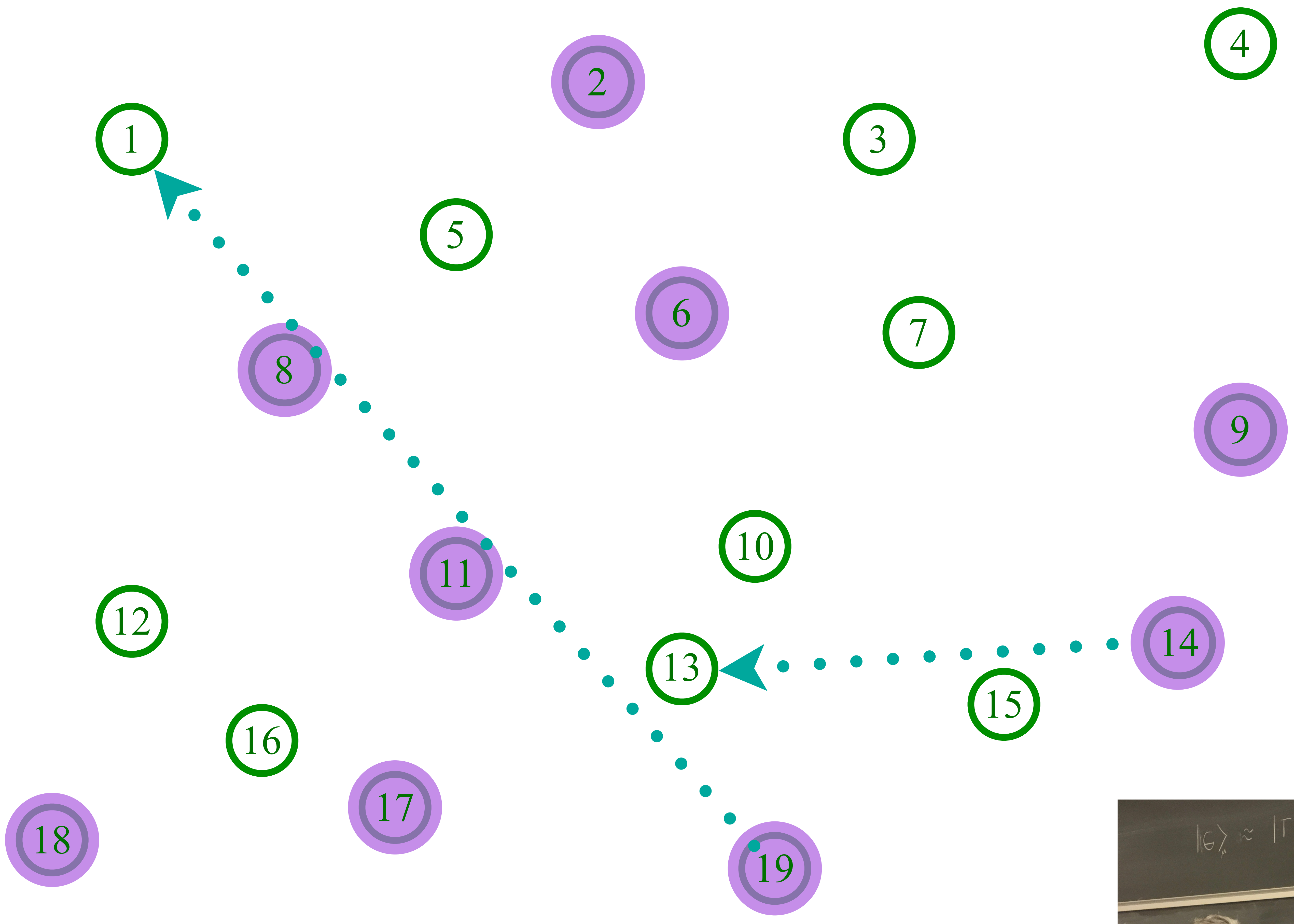
Entangle electrons pairwise randomly



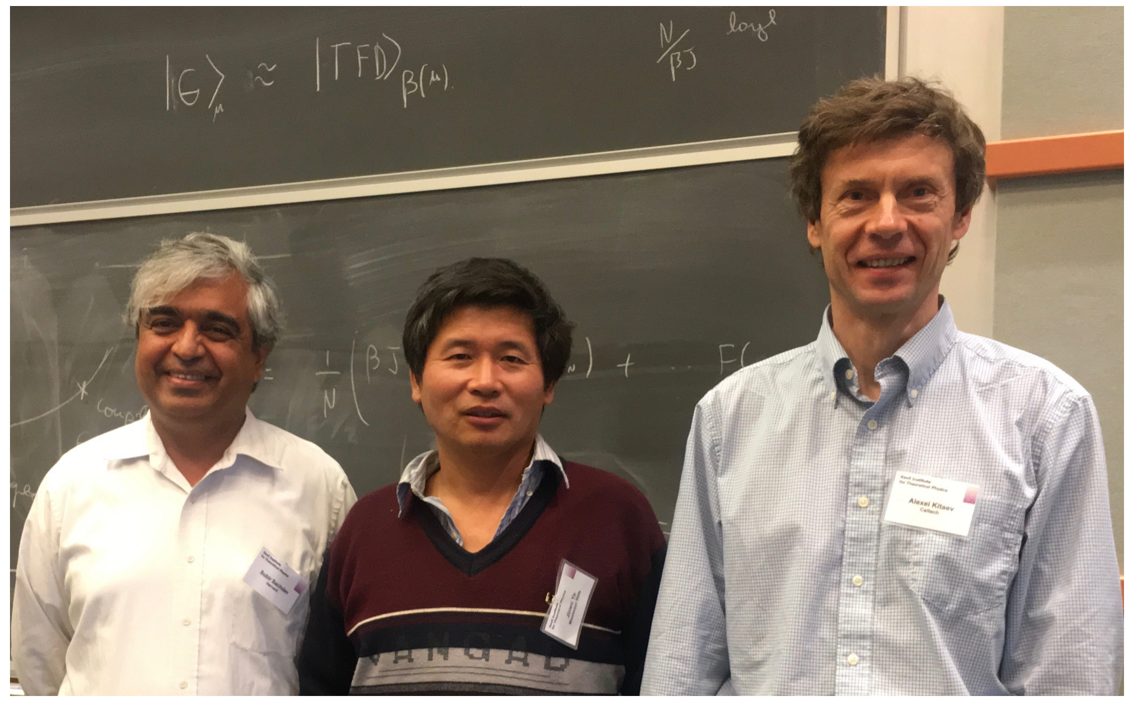
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



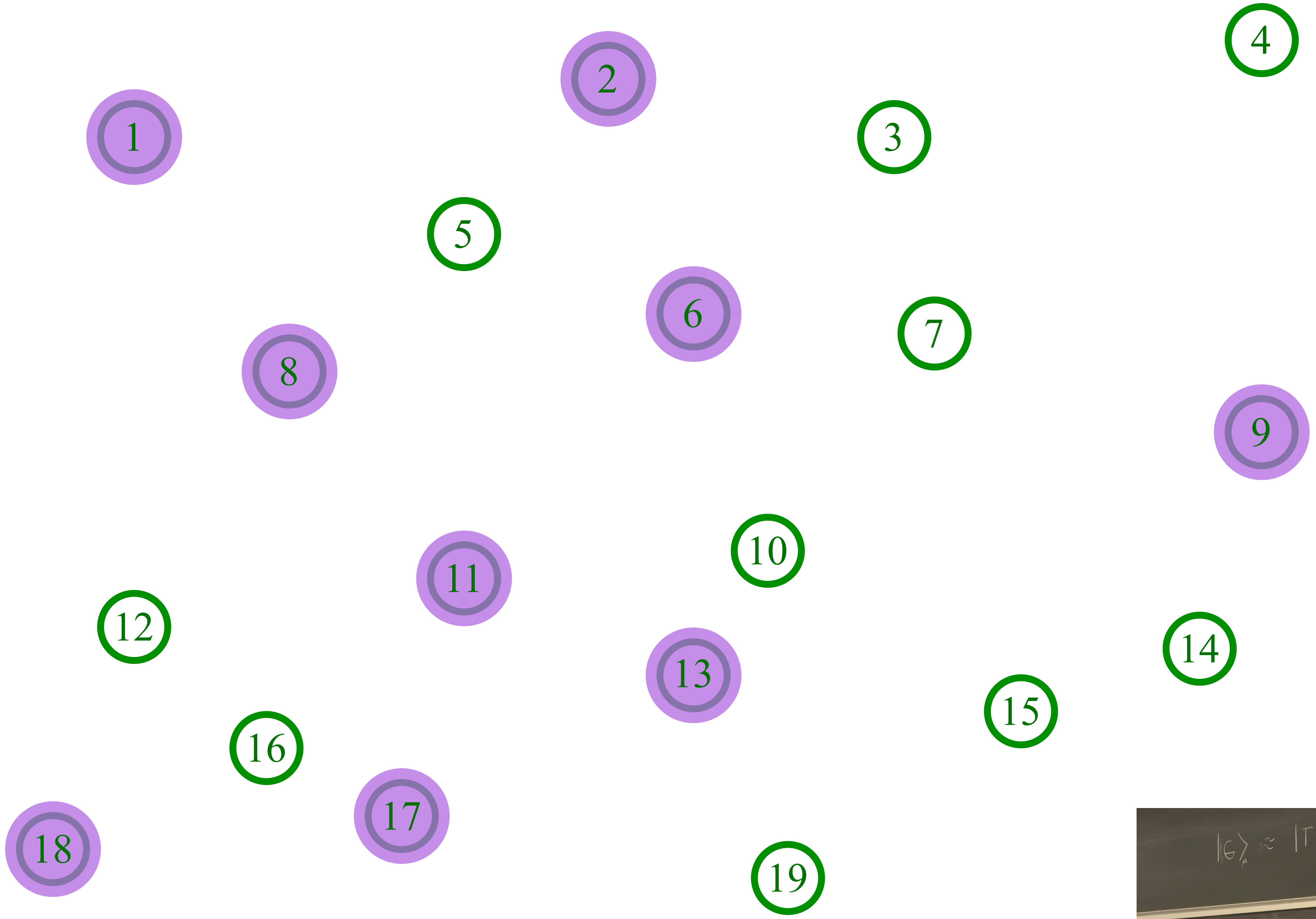
Entangle electrons pairwise randomly



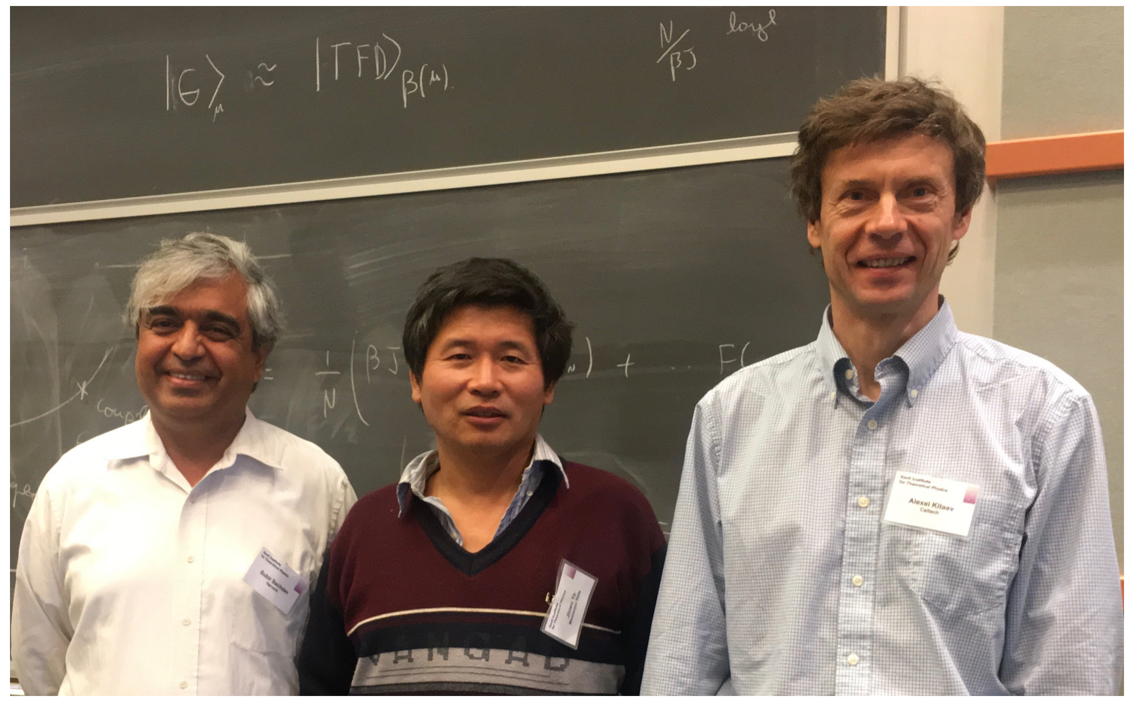
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



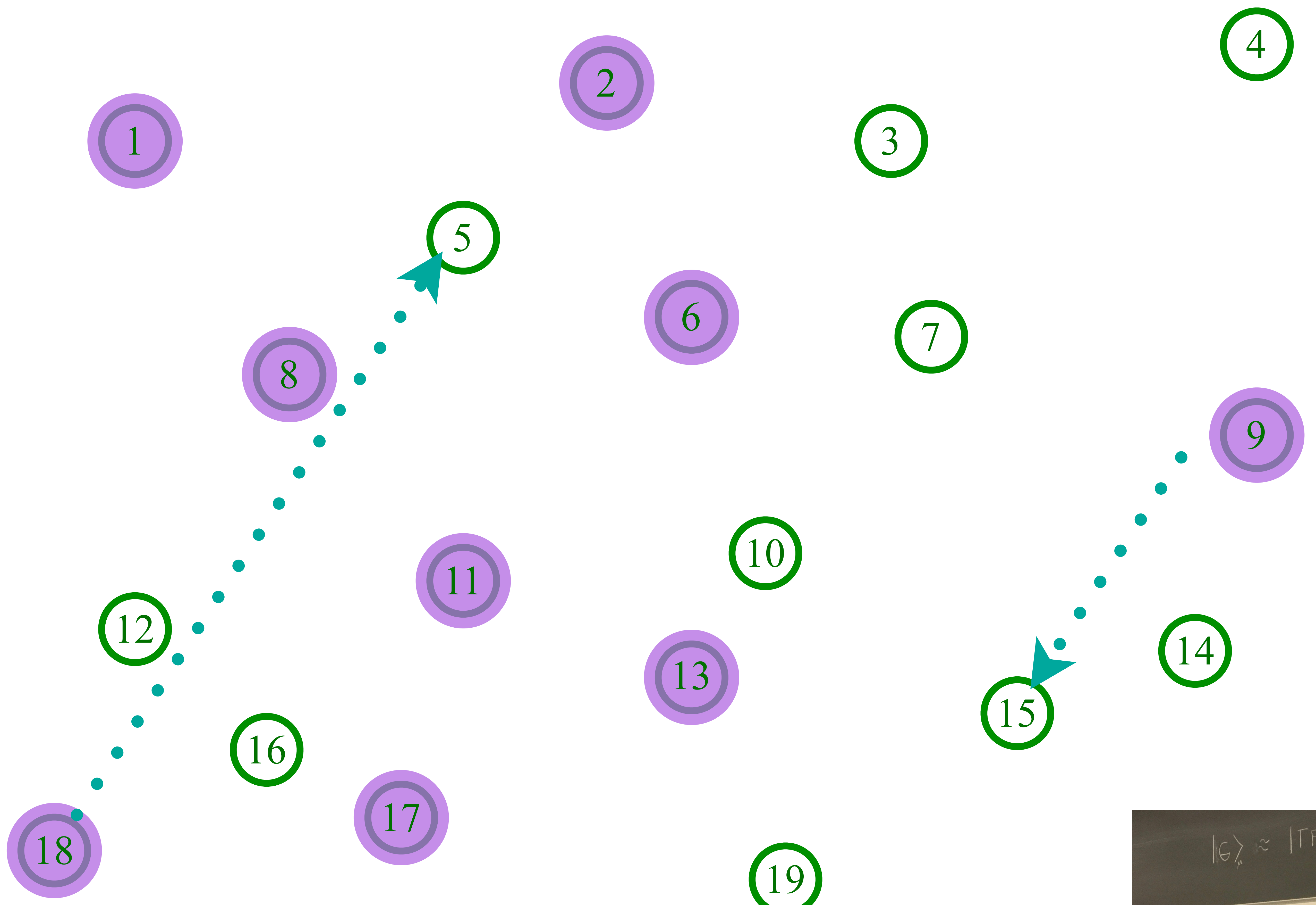
Entangle electrons pairwise randomly



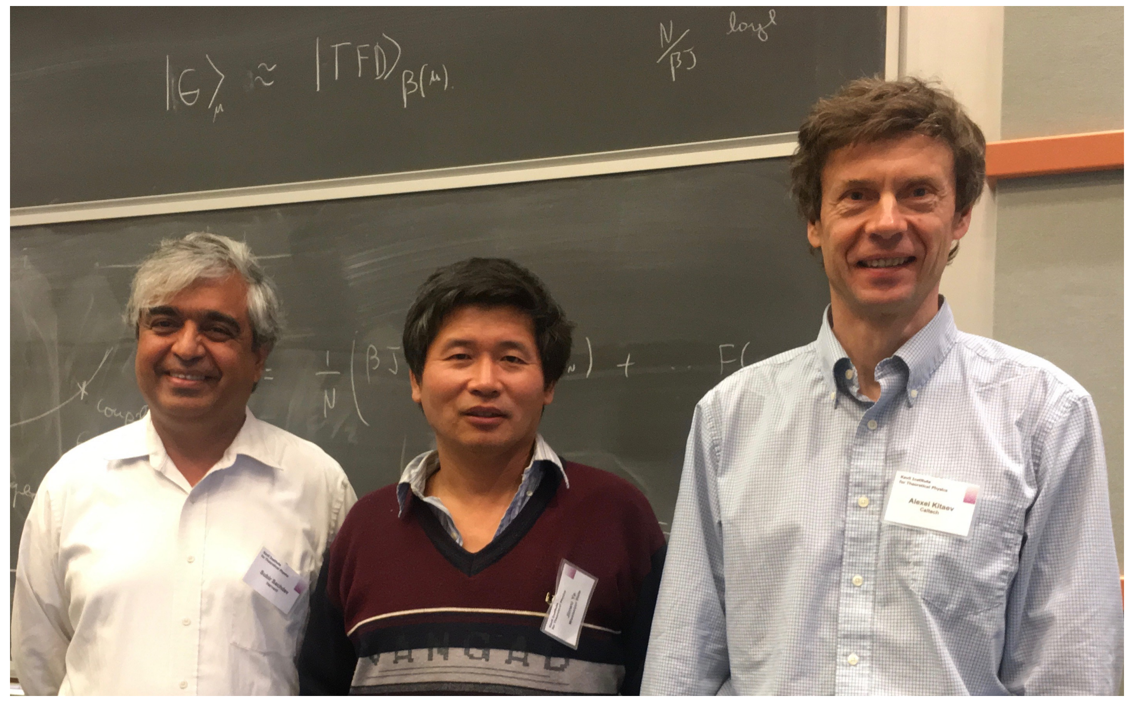
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



Entangle electrons pairwise randomly



The SYK model

Sachdev, Ye (1993); Kitaev (2015)

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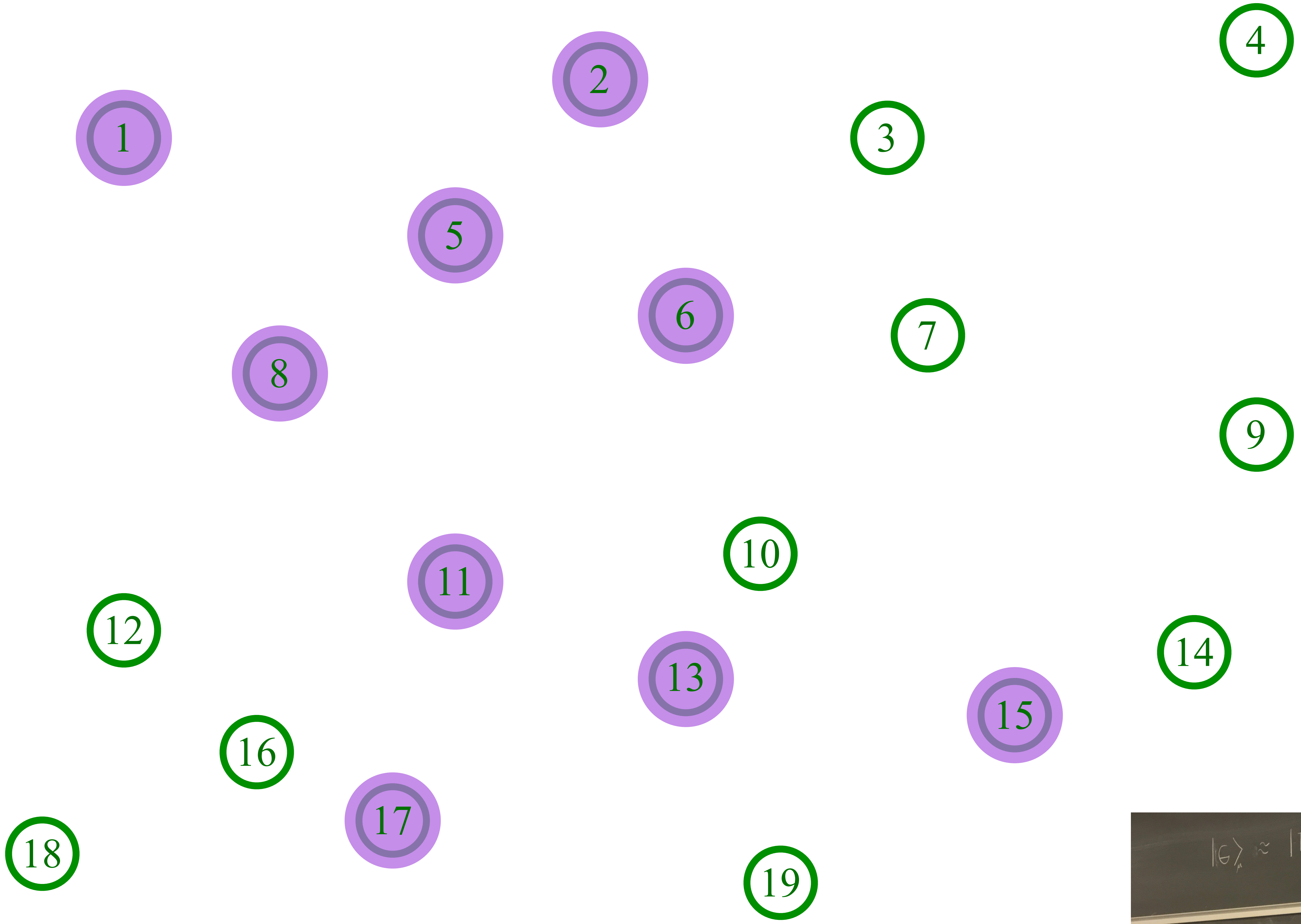
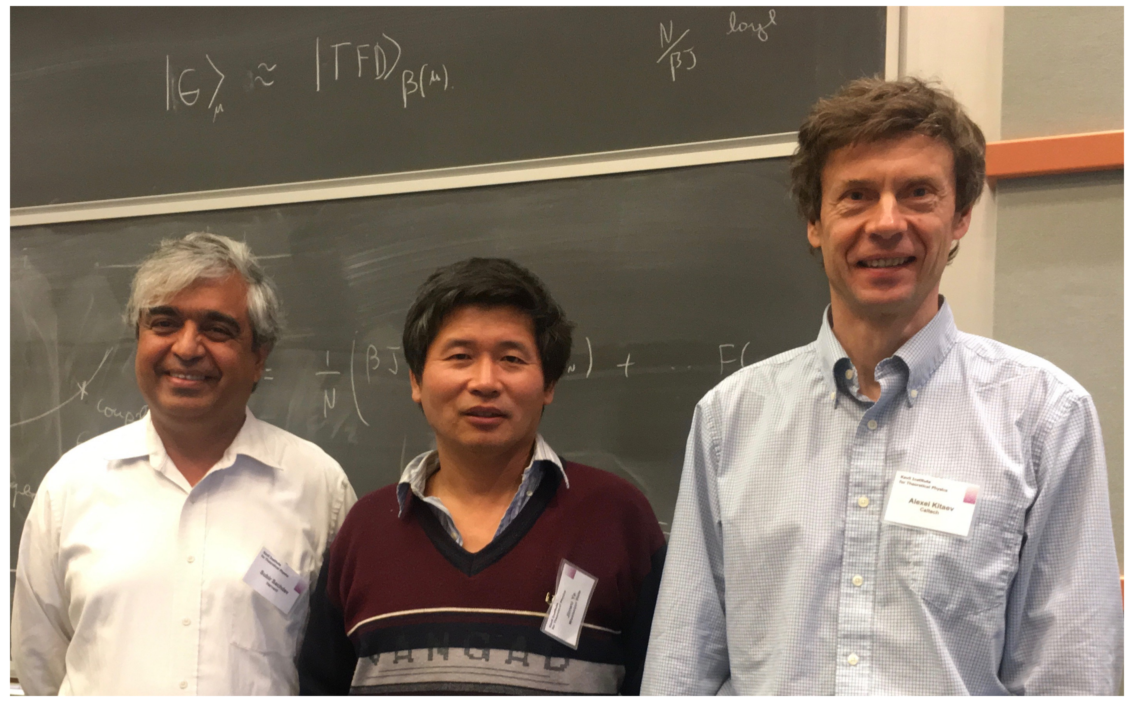


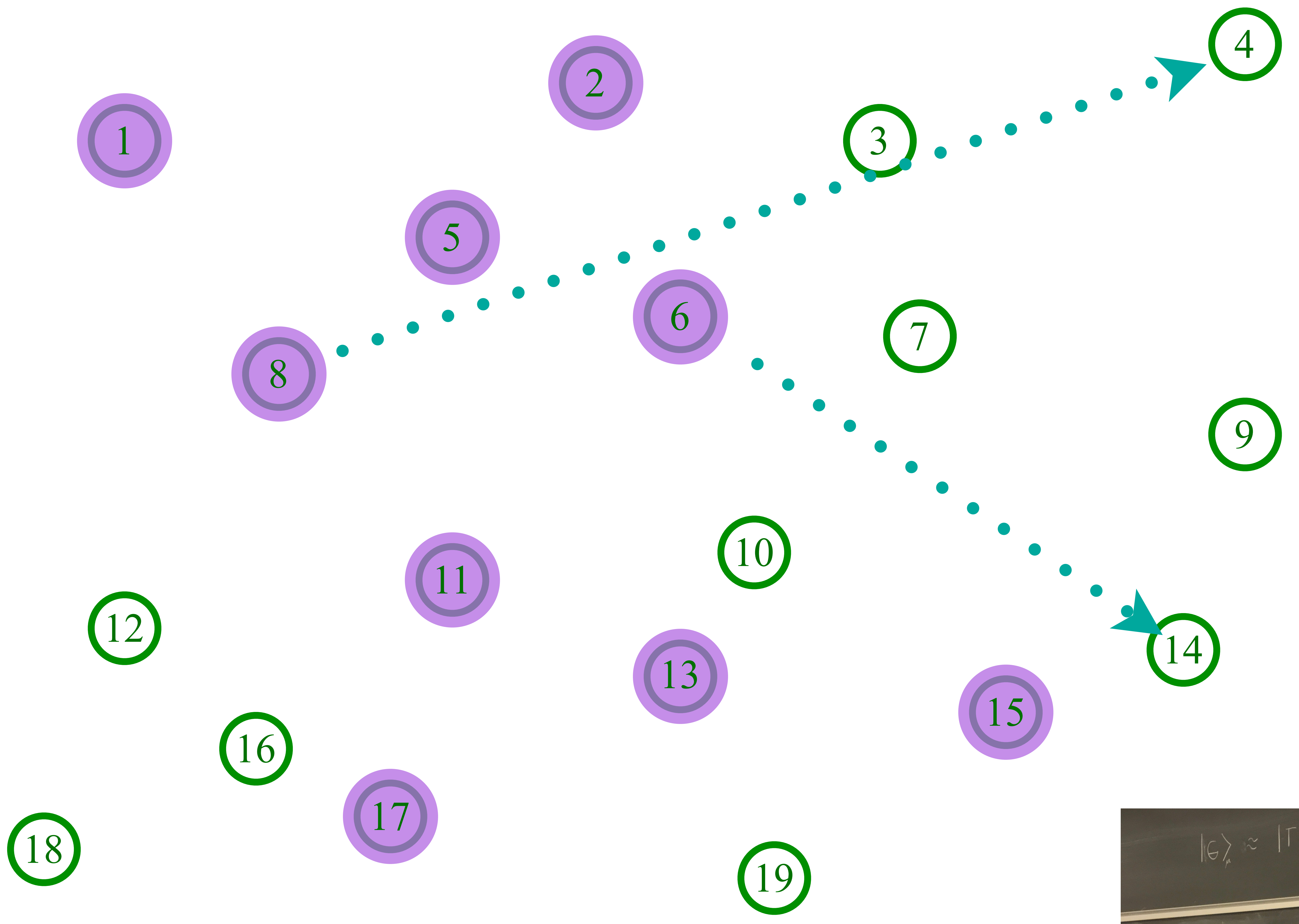
Entangle electrons pairwise randomly



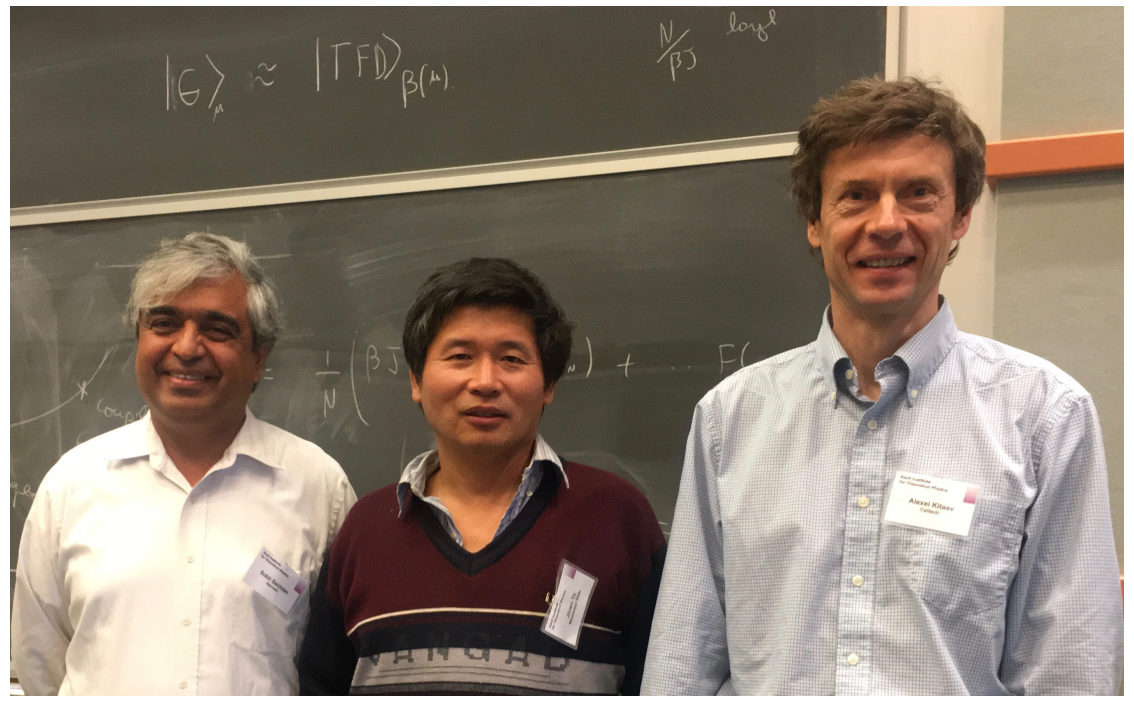
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



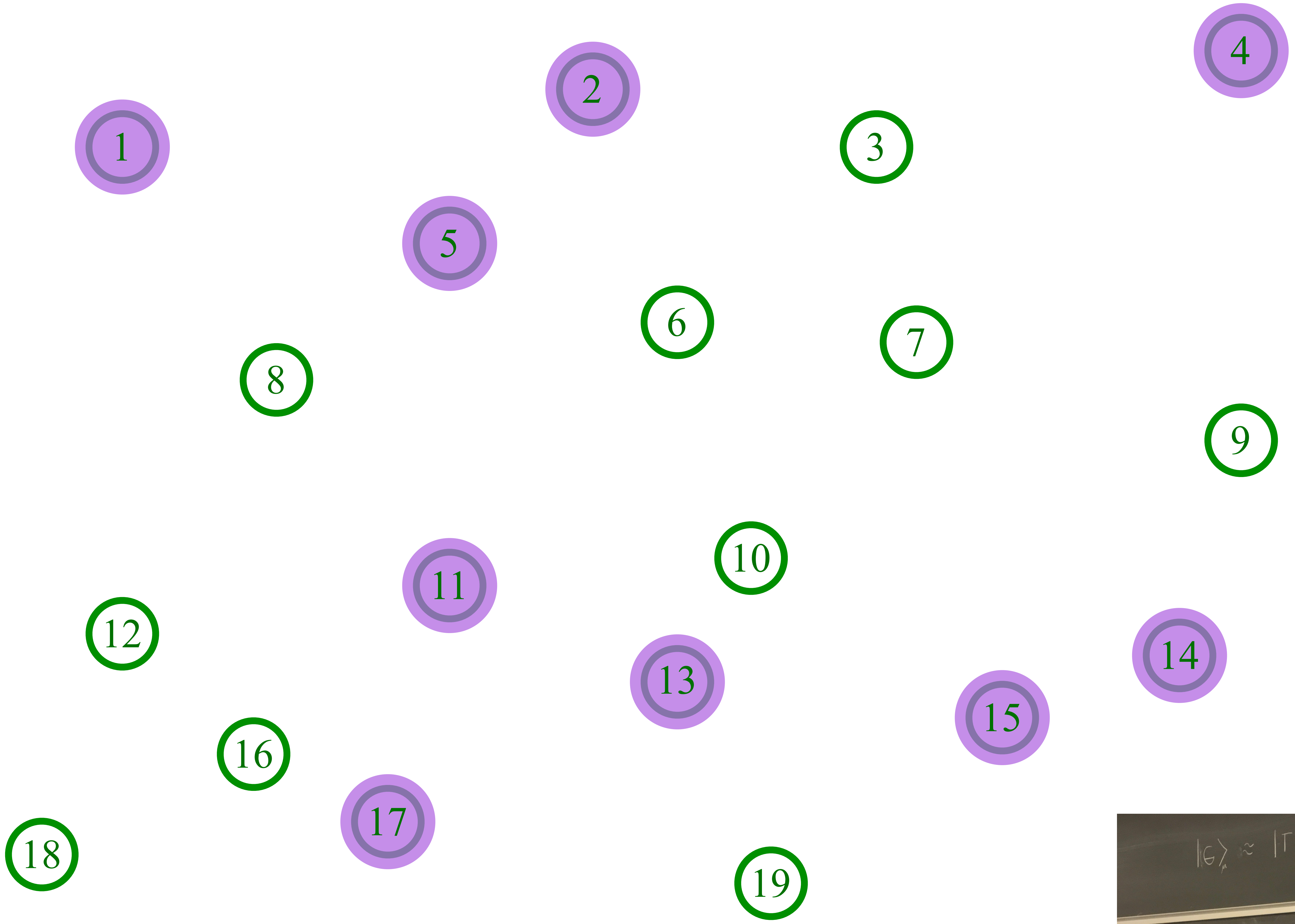
Entangle electrons pairwise randomly



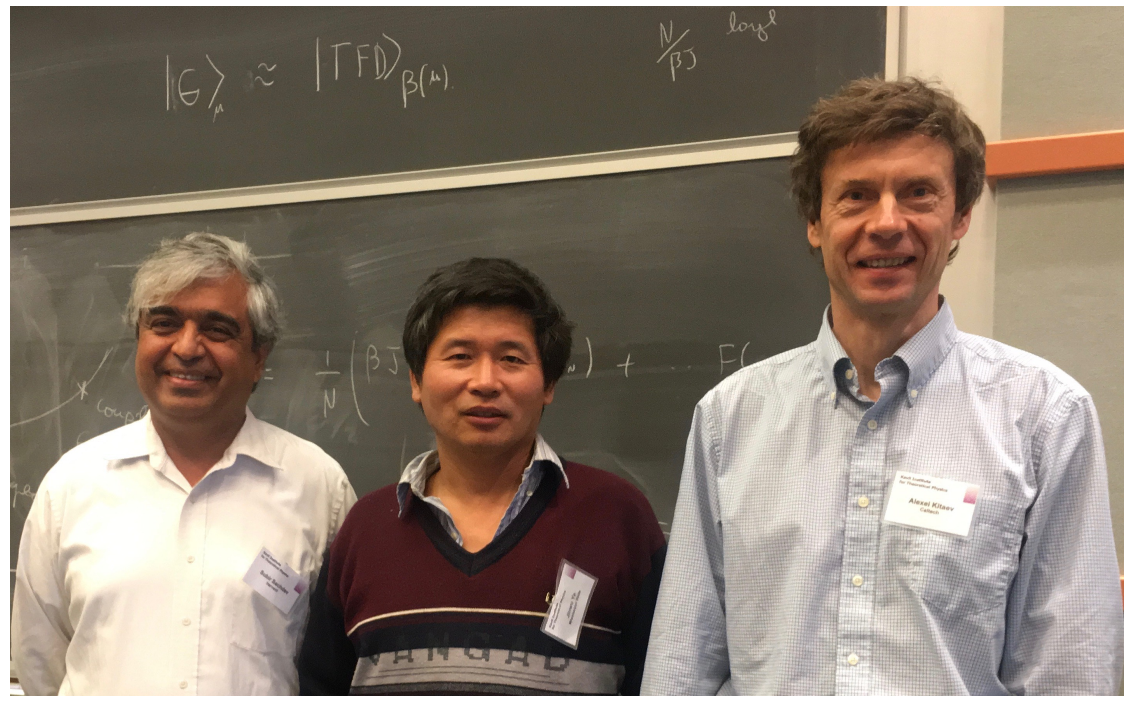
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{6,8;4,14}$



Entangle electrons pairwise randomly



The SYK model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta - \mu \sum_\alpha c^\dagger_\alpha c_\alpha$$

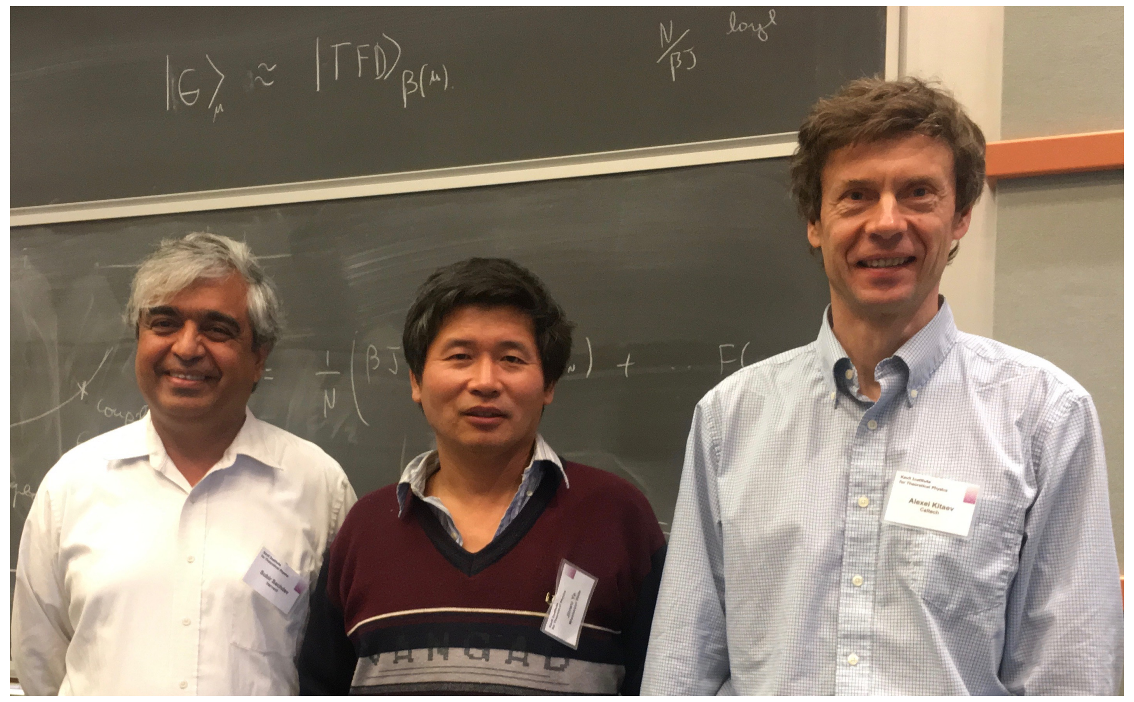
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c^\dagger_\beta + c^\dagger_\beta c_\alpha = \delta_{\alpha\beta}$$

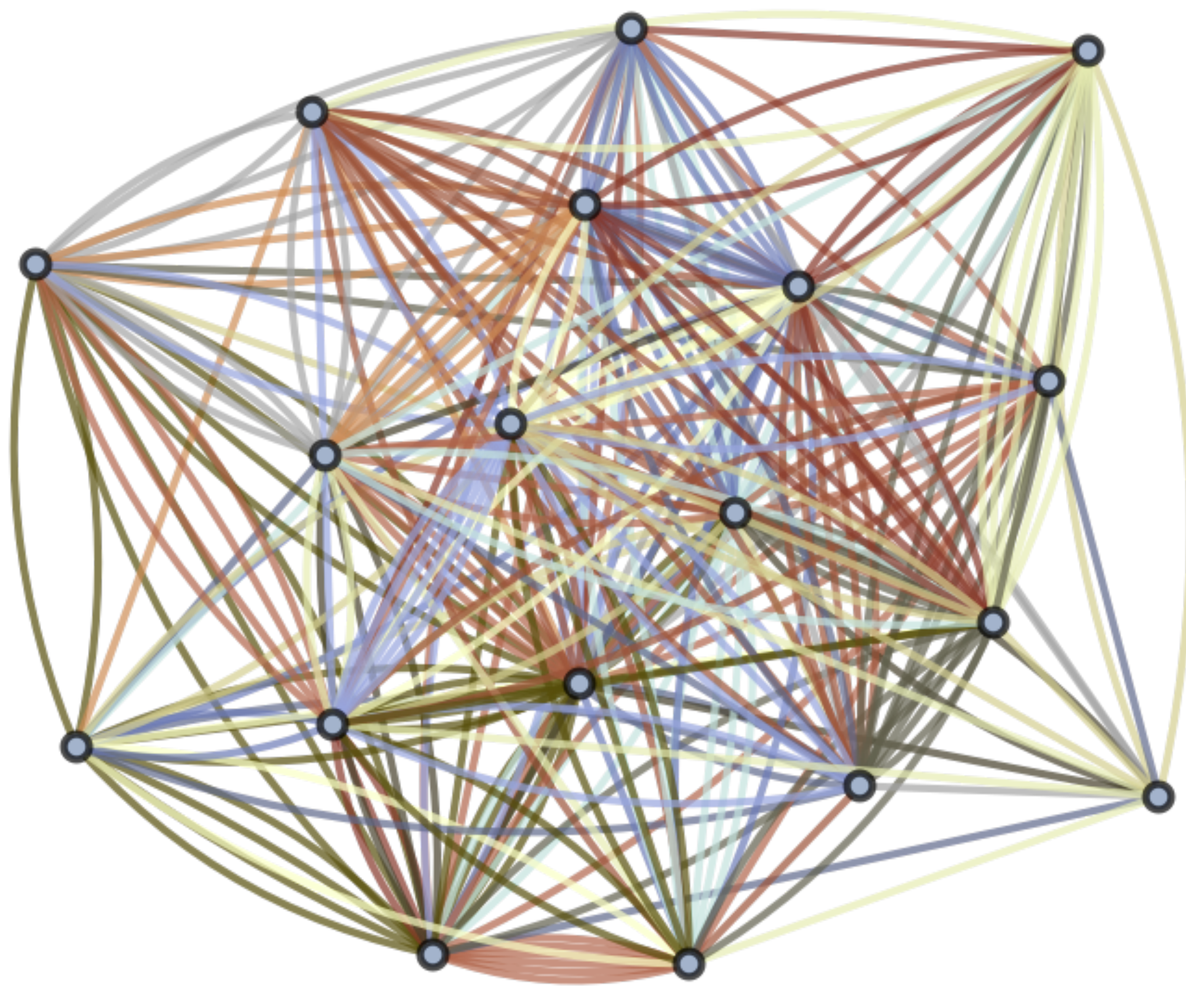
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c^\dagger_\alpha c_\alpha \, ; \quad [\mathcal{H}, \mathcal{Q}] = 0 \, ; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





The SYK model

The (averaged) partition function can be written as path integral over the bilocal fermion Green's function $G(\tau_1, \tau_2) \sim \frac{1}{N}\sum_\alpha c_\alpha(\tau_1) c_\alpha^\dagger(\tau_2)$

$$\overline{\mathcal{Z}} = \int \mathcal{D}G(\tau_1, \tau_2) \exp\left(-N S_{\text{eff}}[G]\right)$$

The large N saddle point equation $\delta S_{\text{eff}}/\delta G = 0$ for $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ is

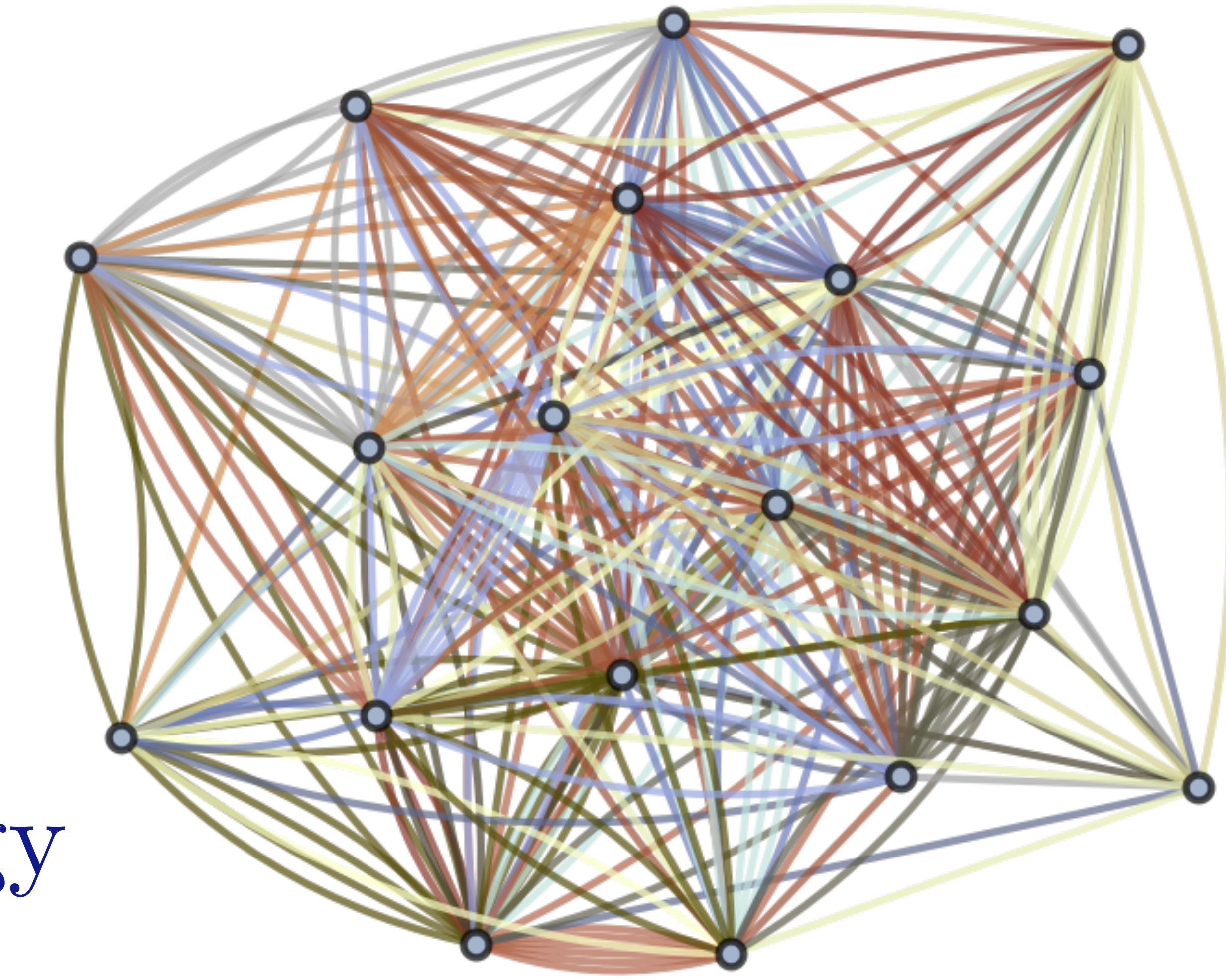
$$G_s(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma_s(\tau) = -U^2 G_s^2(\tau) G_s(-\tau)$$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

Time reparameterization symmetry:

At frequencies $\ll U$, the path integral for is invariant under time reparametrization $f(\sigma)$

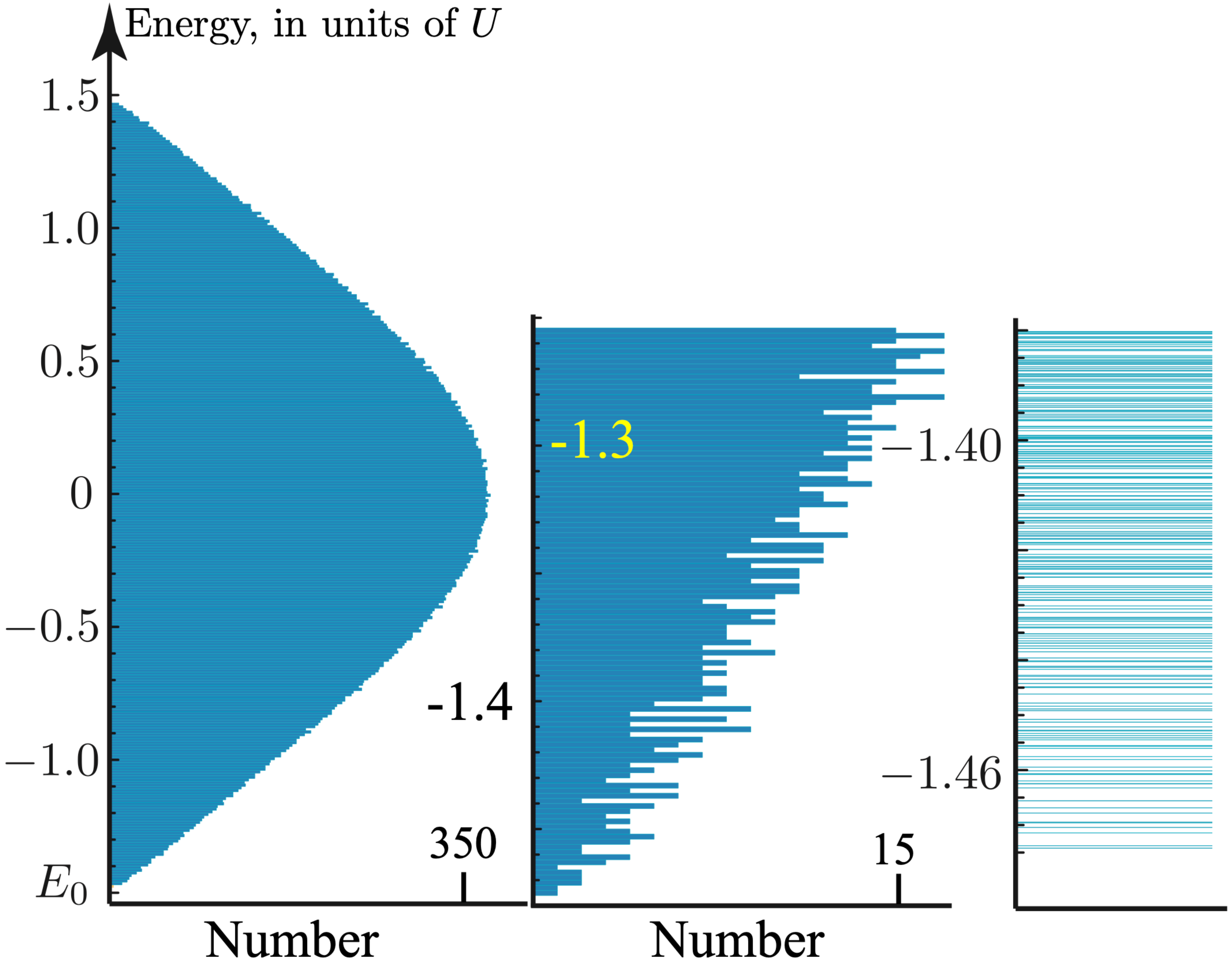
$$\tau = f(\sigma)$$
$$G(\tau_1, \tau_2) = \left[f'(\sigma_1) f'(\sigma_2)\right]^{-1/4} \widetilde{G}(\sigma_1, \sigma_2)$$



There is also an emergent U(1) gauge symmetry. Hints that the low energy theory is quantum gravity+electromagnetism!

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



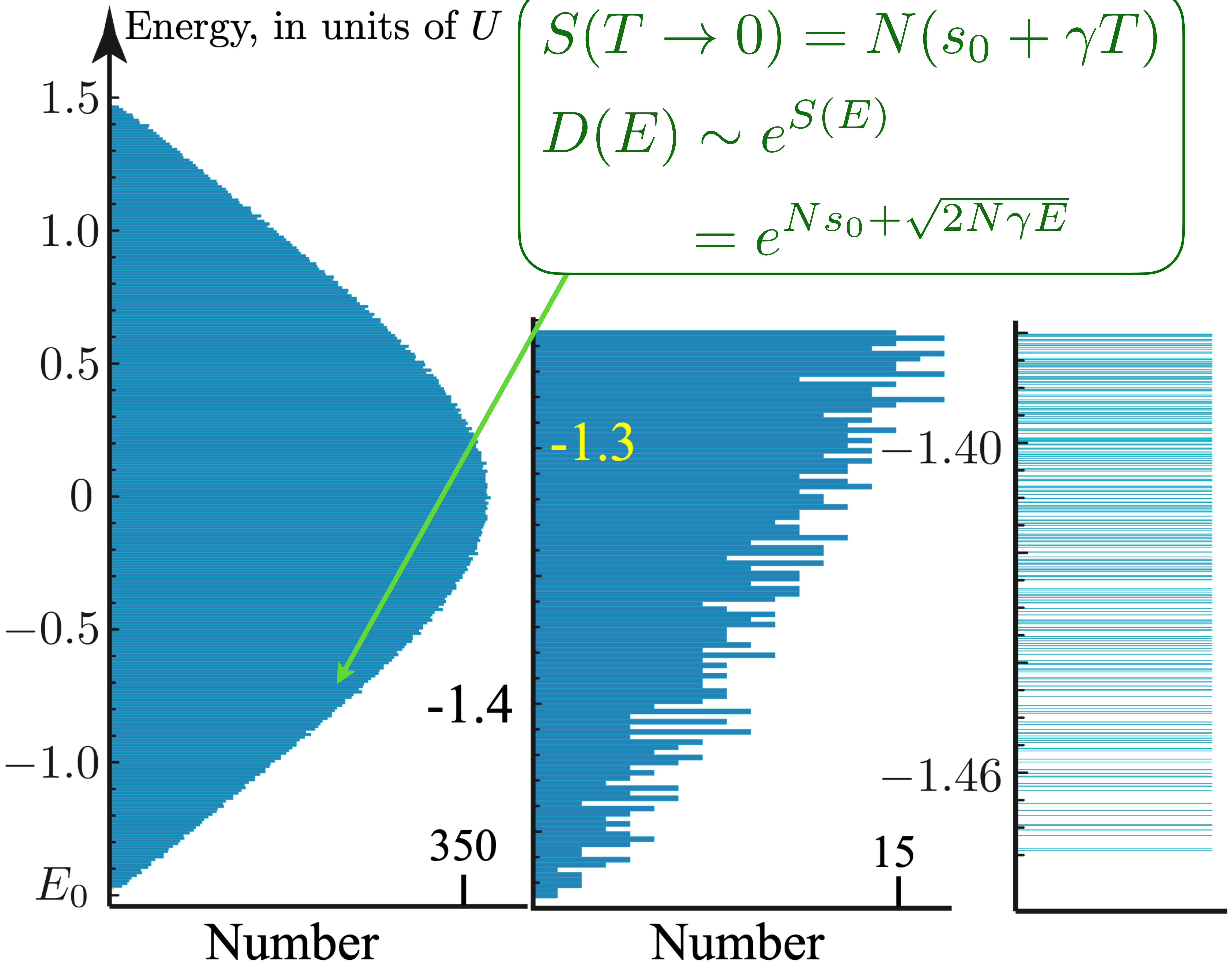
Complex SYK model

(Numerics: G. Tarnopolsky)

Many-body density of states

Boltzmann

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



$$S(T \to 0) = N(s_0 + \gamma T)$$
$$D(E) \sim e^{S(E)}$$
$$= e^{N s_0 + \sqrt{2N\gamma E}}$$

At $\mathcal{Q} = 1/2$

$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4}$$
$$= 0.46484769917\ldots.$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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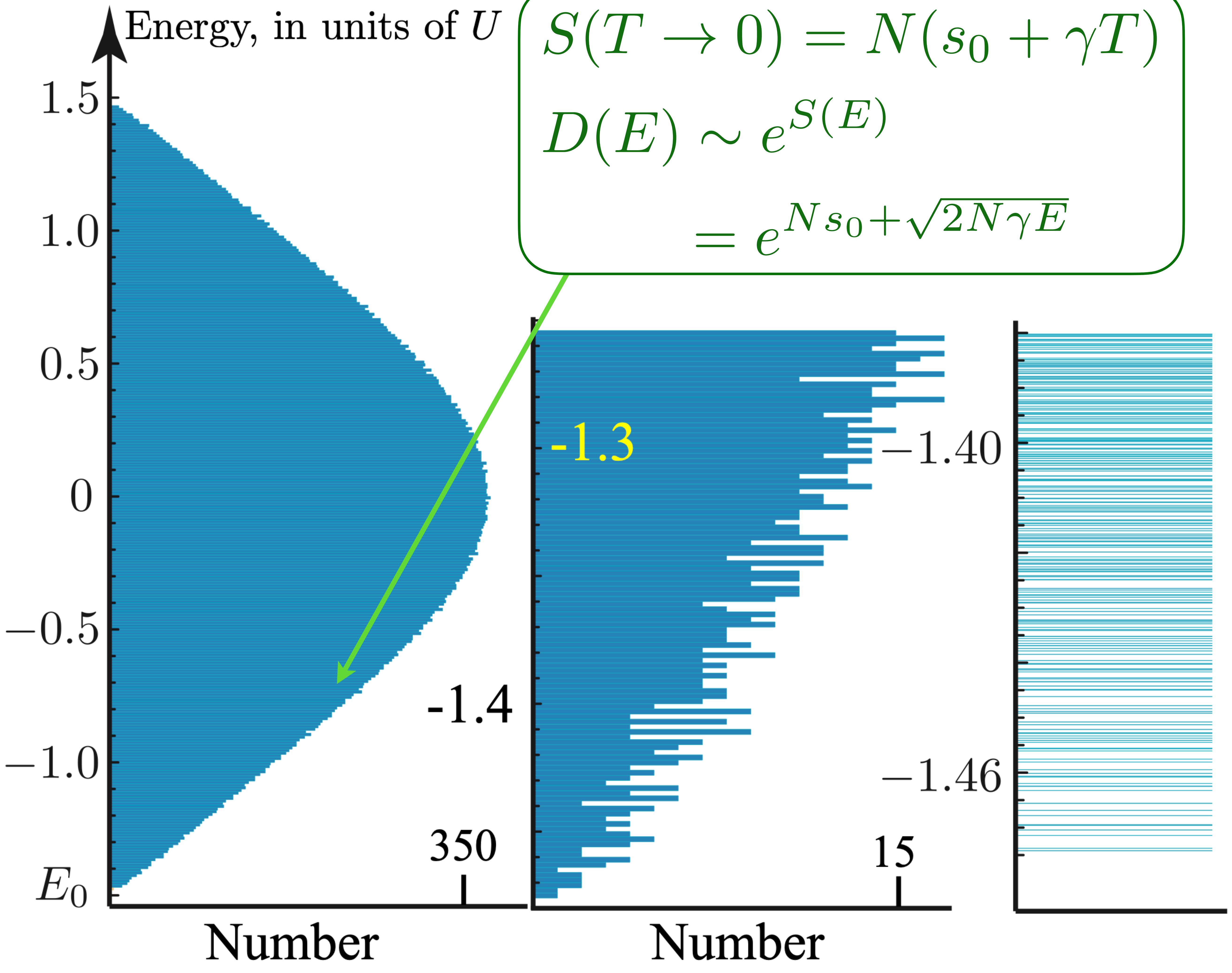

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A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Energy level spacing $\sim e^{-N s_0}$!

No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

Complex SYK model

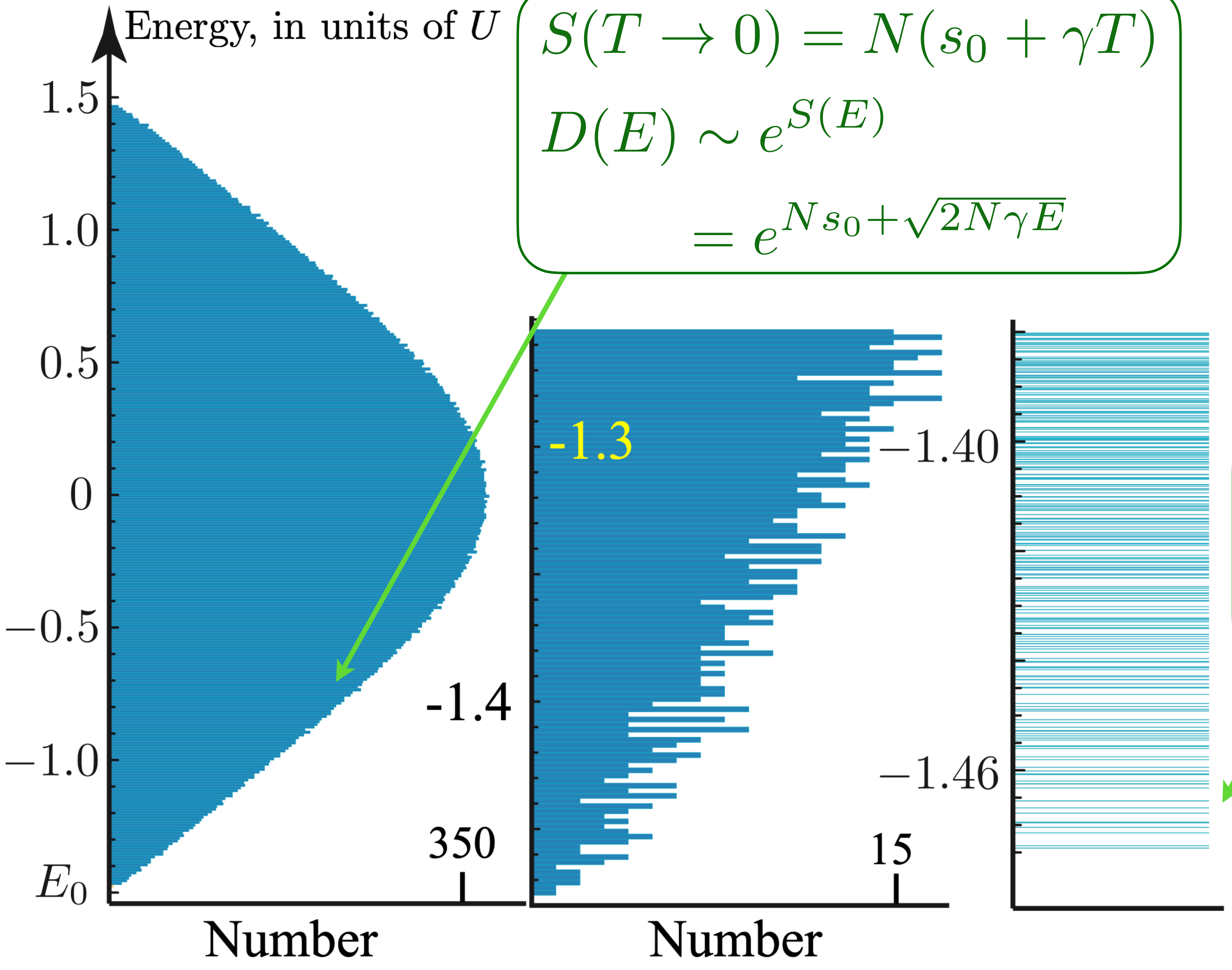
(Numerics: G. Tarnopolsky)

Many-body density of states

Beyond Boltzmann

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

At $\mathcal{Q} = 1/2$



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A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

$$D(E) \sim$$
$$N^{-1} \exp\left(N s_0\right) \sinh(\sqrt{2N\gamma E})$$

J. S. Cotler et al., JHEP 05 (2017) 118

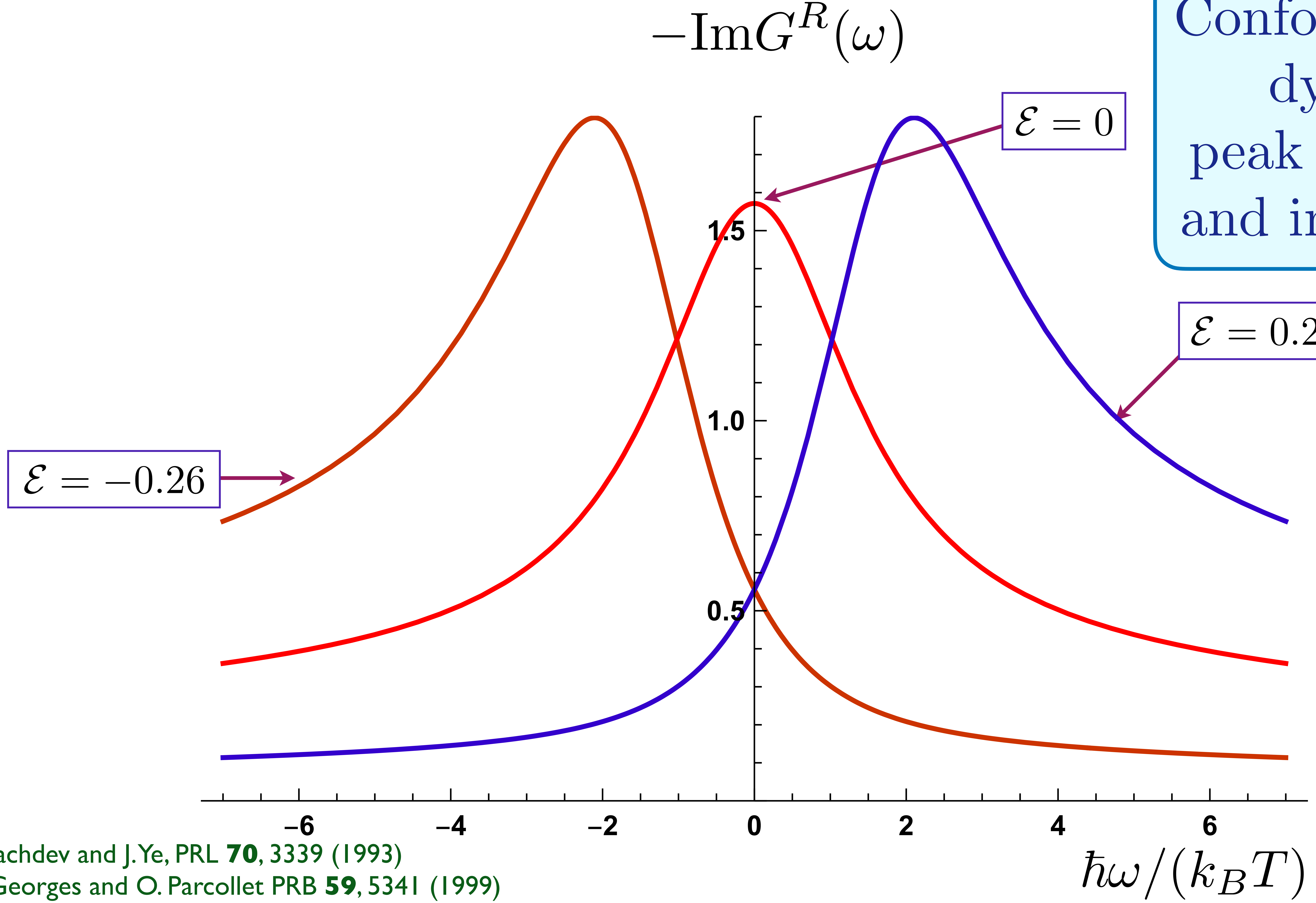
Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, JHEP 02 (2020) 157

Complex SYK model

(Numerics: G. Tarnopolsky)

The SYK model

Conformal 'Planckian' dynamics with peak width $\sim k_B T/\hbar$ and independent of U



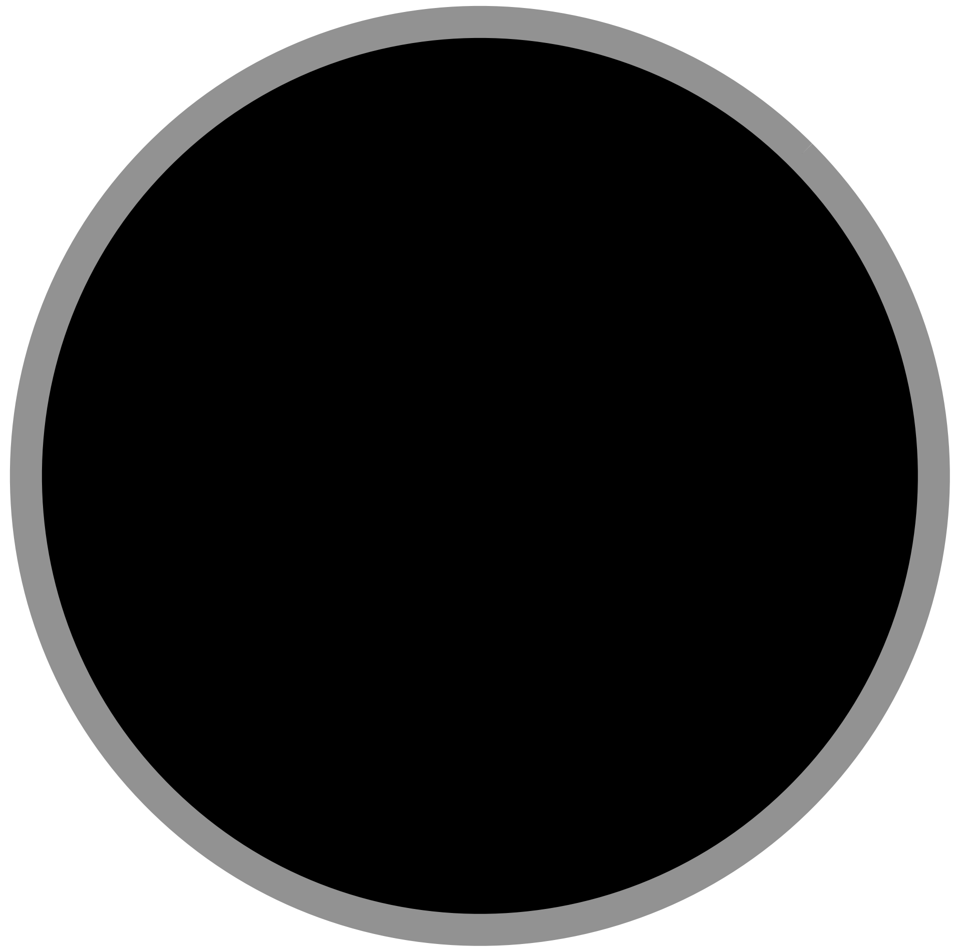
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

Quantum black holes

Black Holes

Objects so dense that light is gravitationally bound to them.

Horizon radius $R = \frac{2GM}{c^2}$

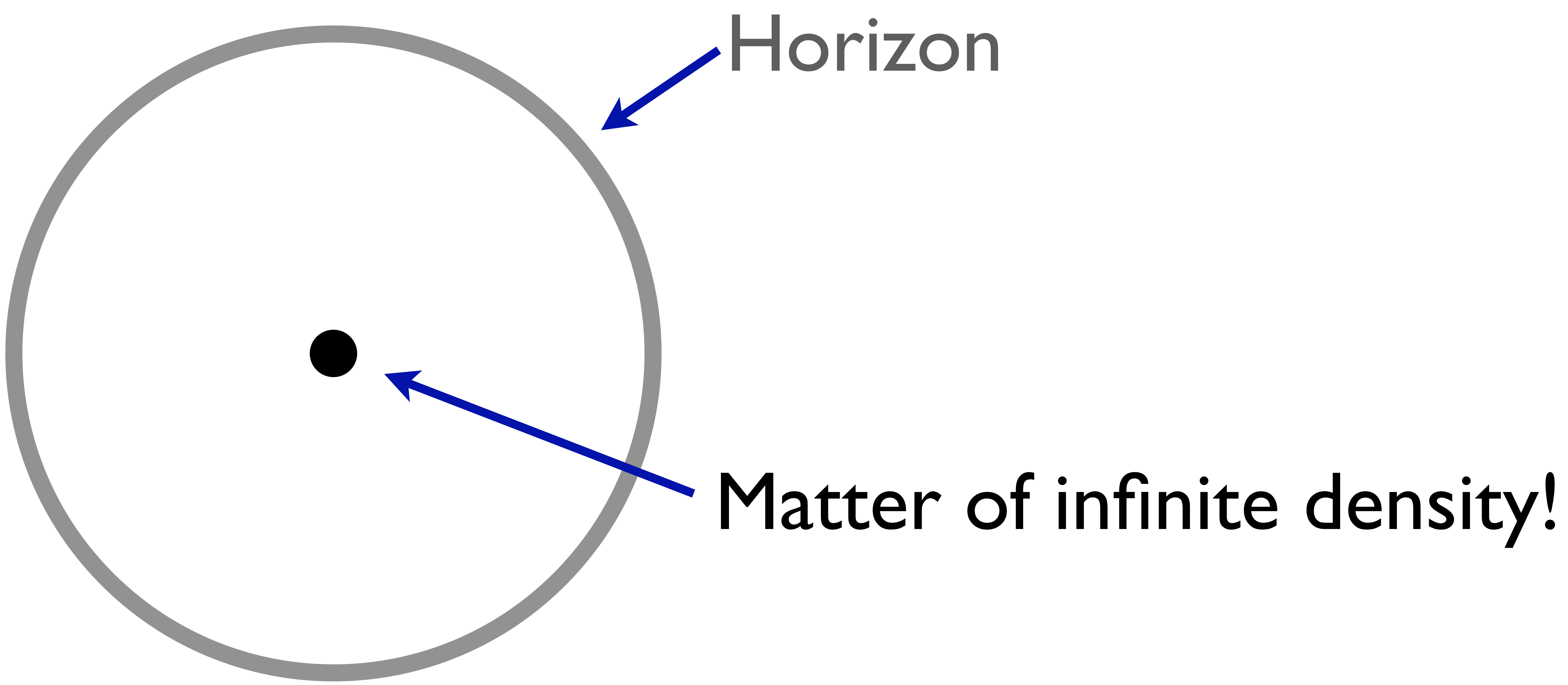


G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



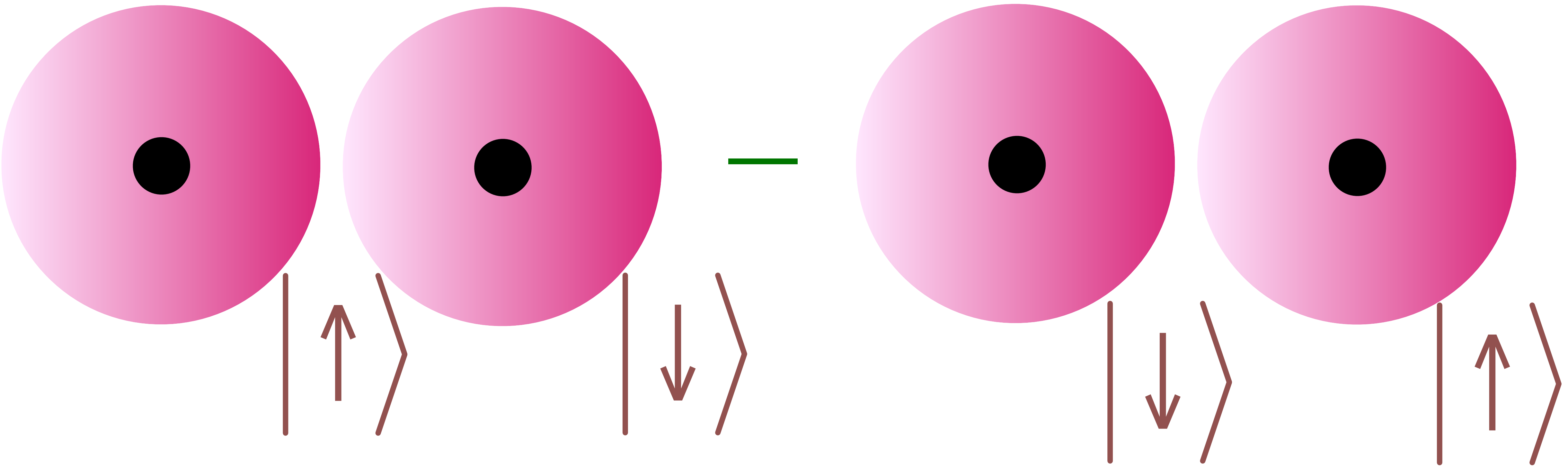
What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.

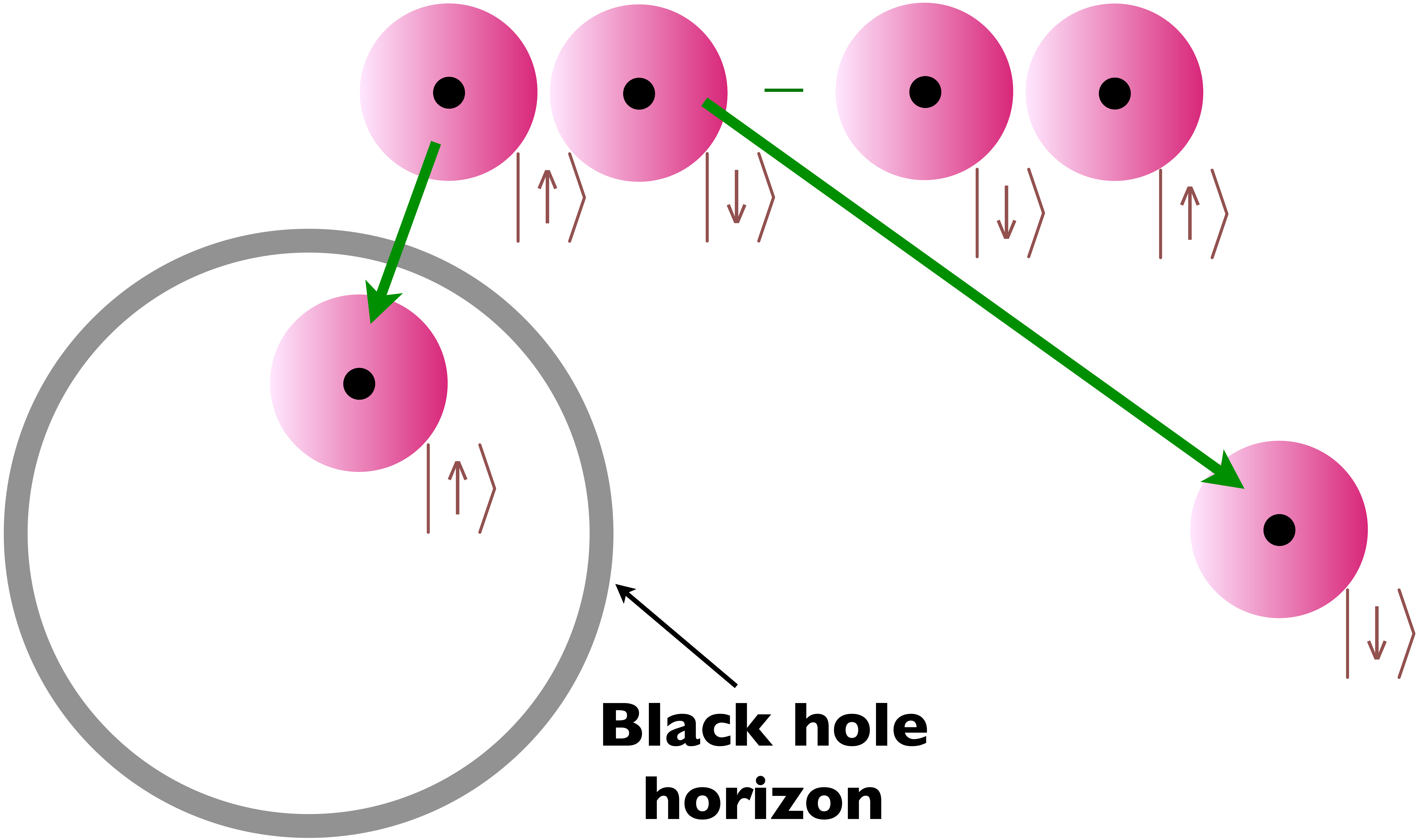
This singularity convinced many early on that black holes were unphysical solutions of Einstein's equations, and did not exist in our universe.

In any case, it was clear that quantum theory should be applied to the collapsed matter, but no one knew how to.

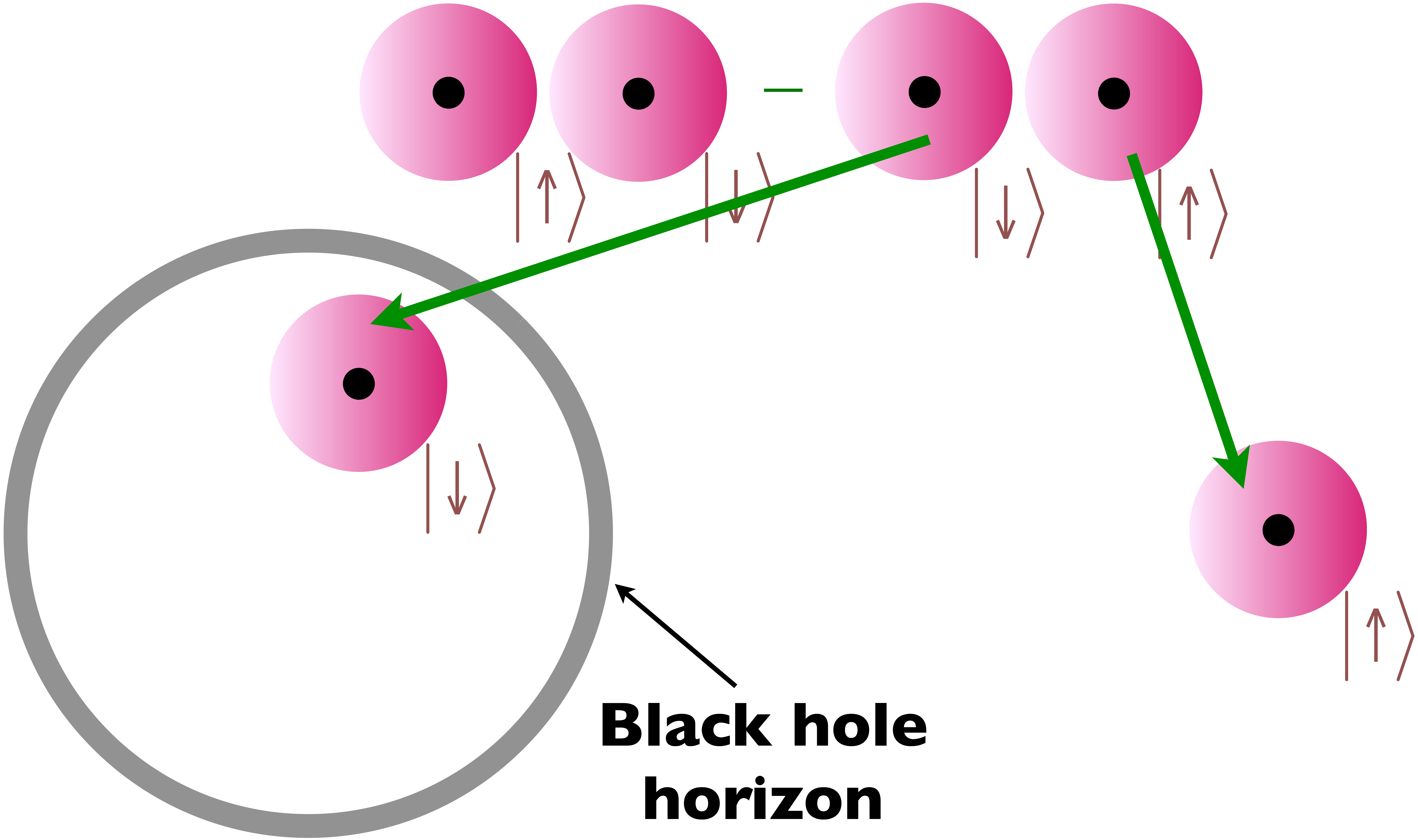
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

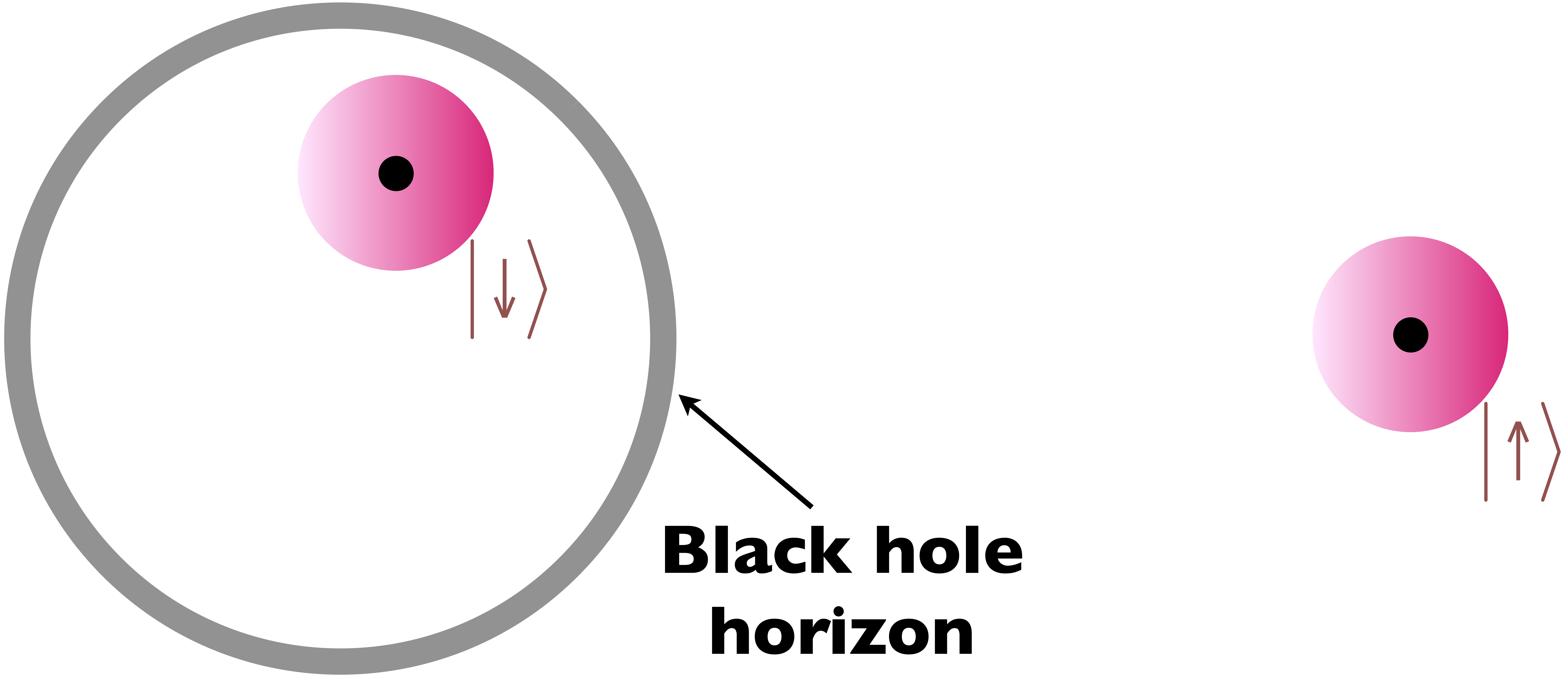


Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole



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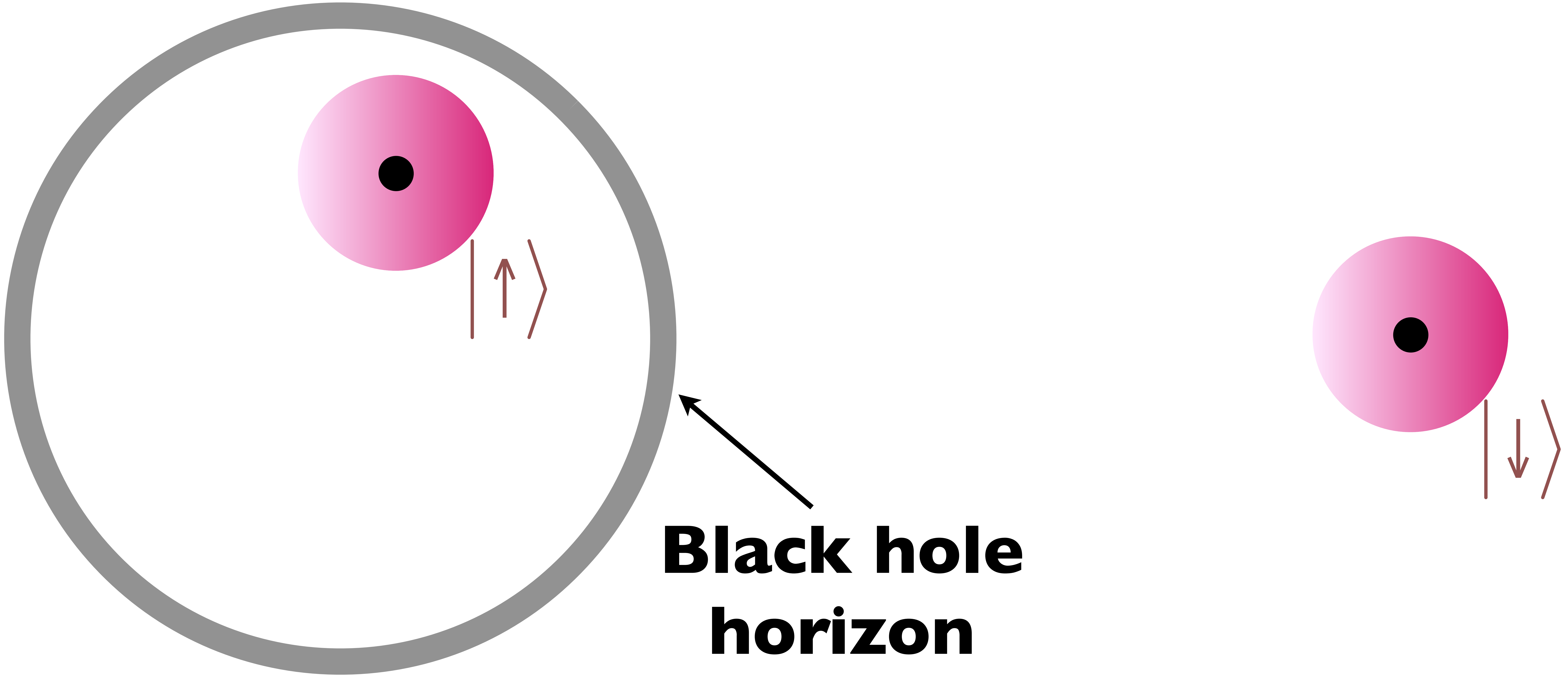
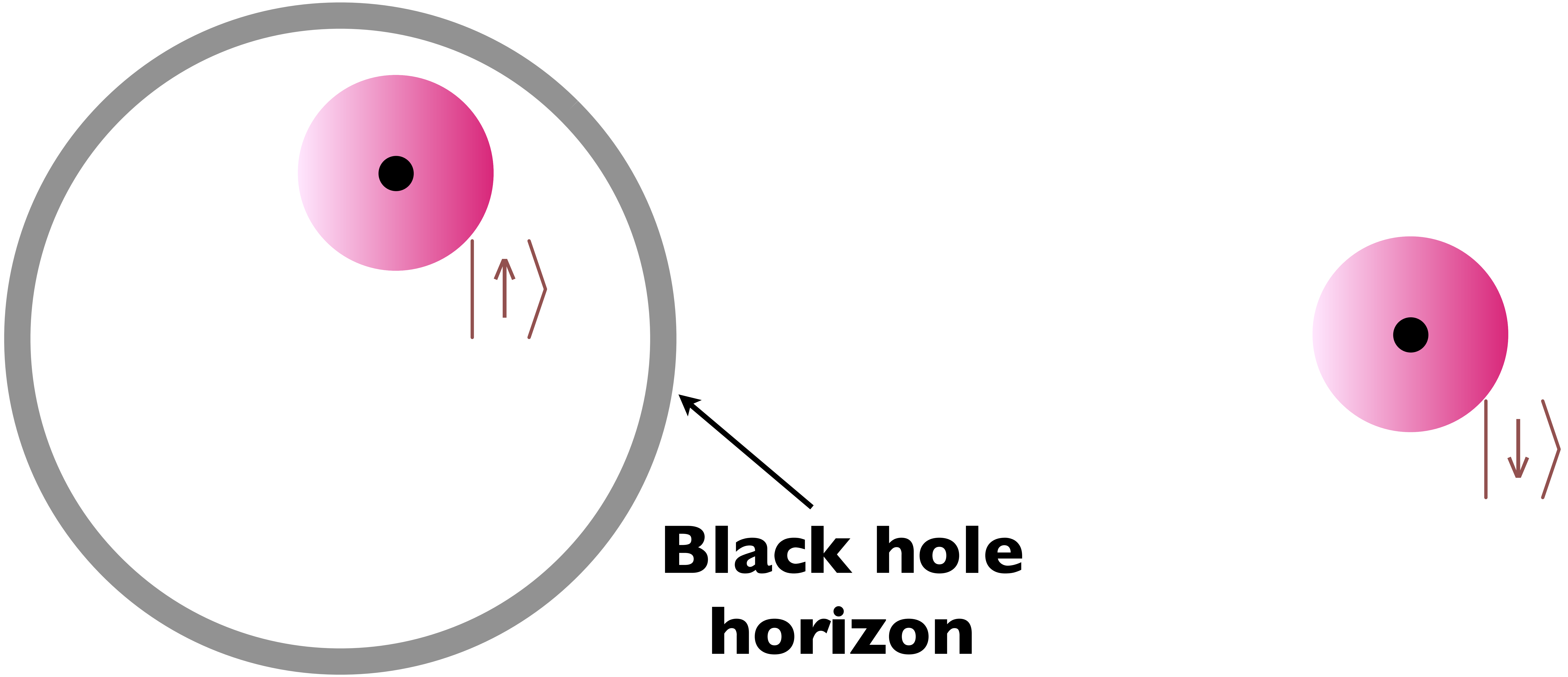


Quantum Entanglement across a black hole horizon

Hawking (1975): Black holes have a temperature and an entropy!

To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.



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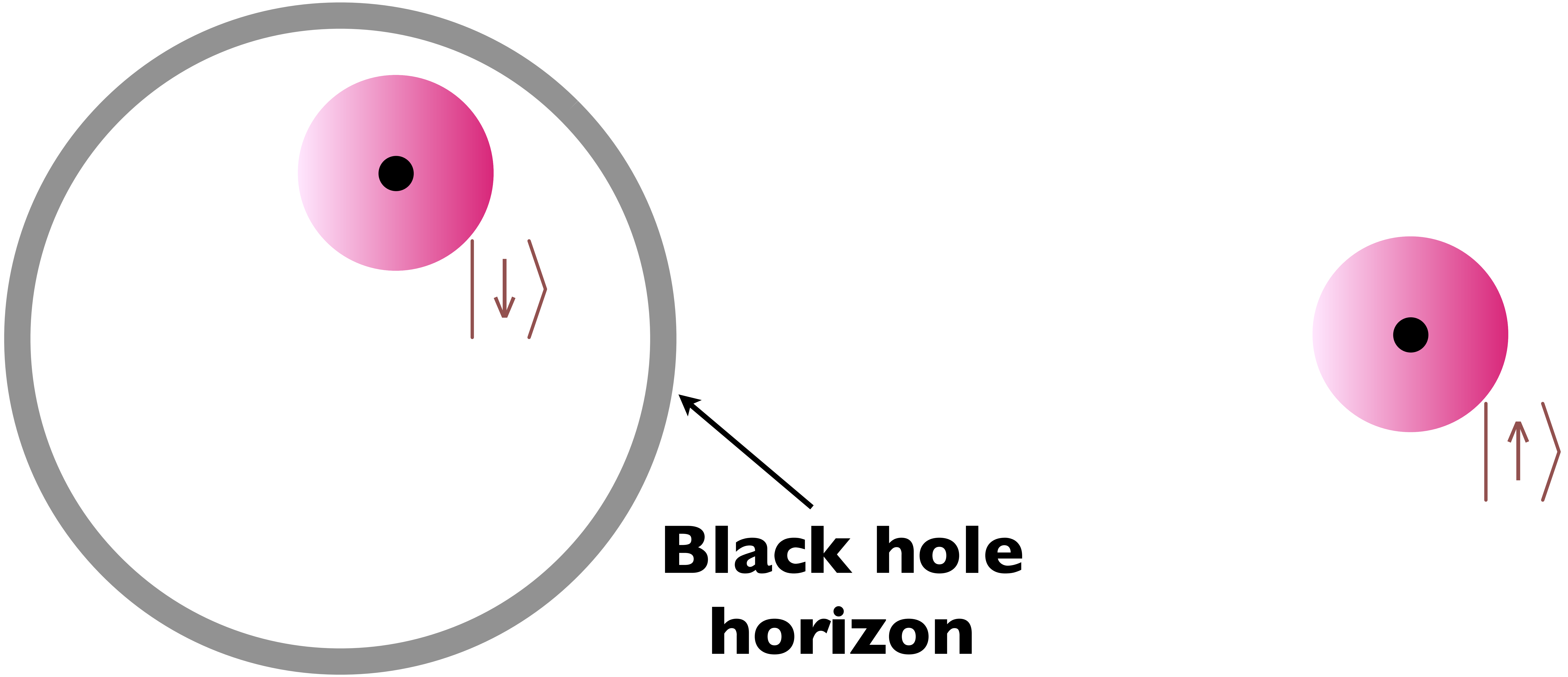
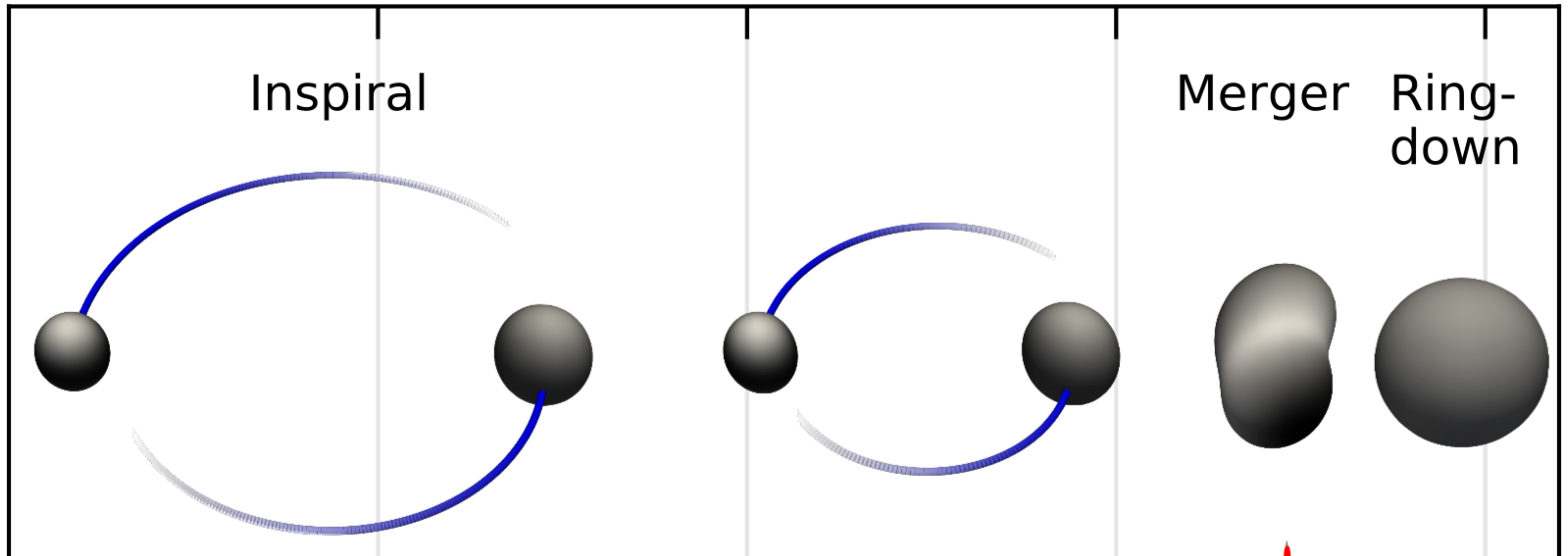


Quantum black holes

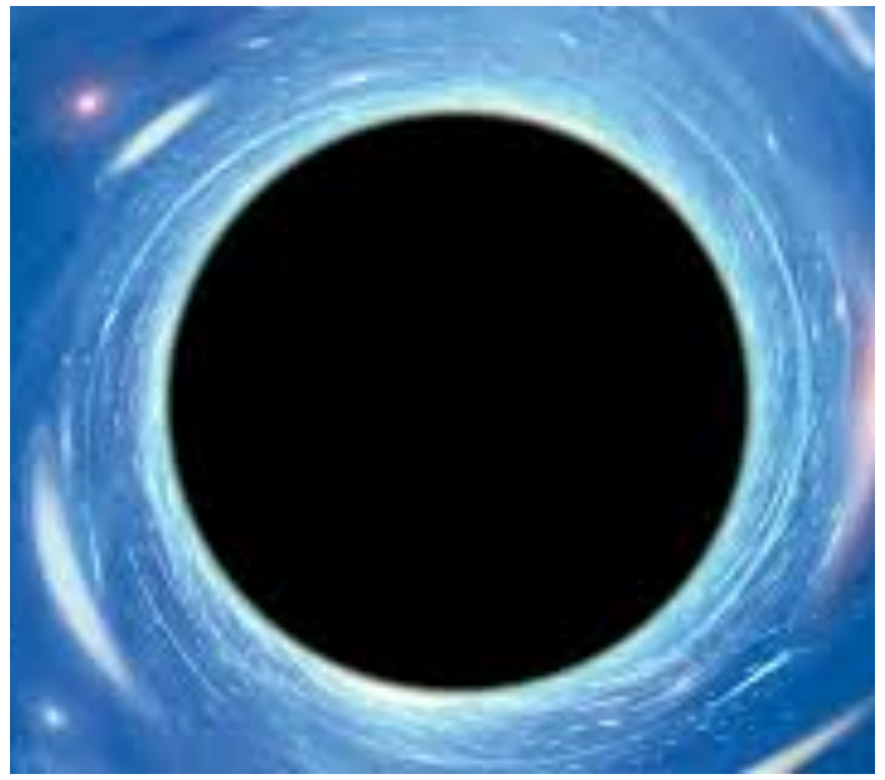
- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- The entropy is proportional to their surface area. $S = A k_B c^3/(4G\hbar)$.
- They relax to thermal equilibrium in a time $\sim 8\pi G M/c^3 = \hbar/(k_B T_H)$ which is Planckian!

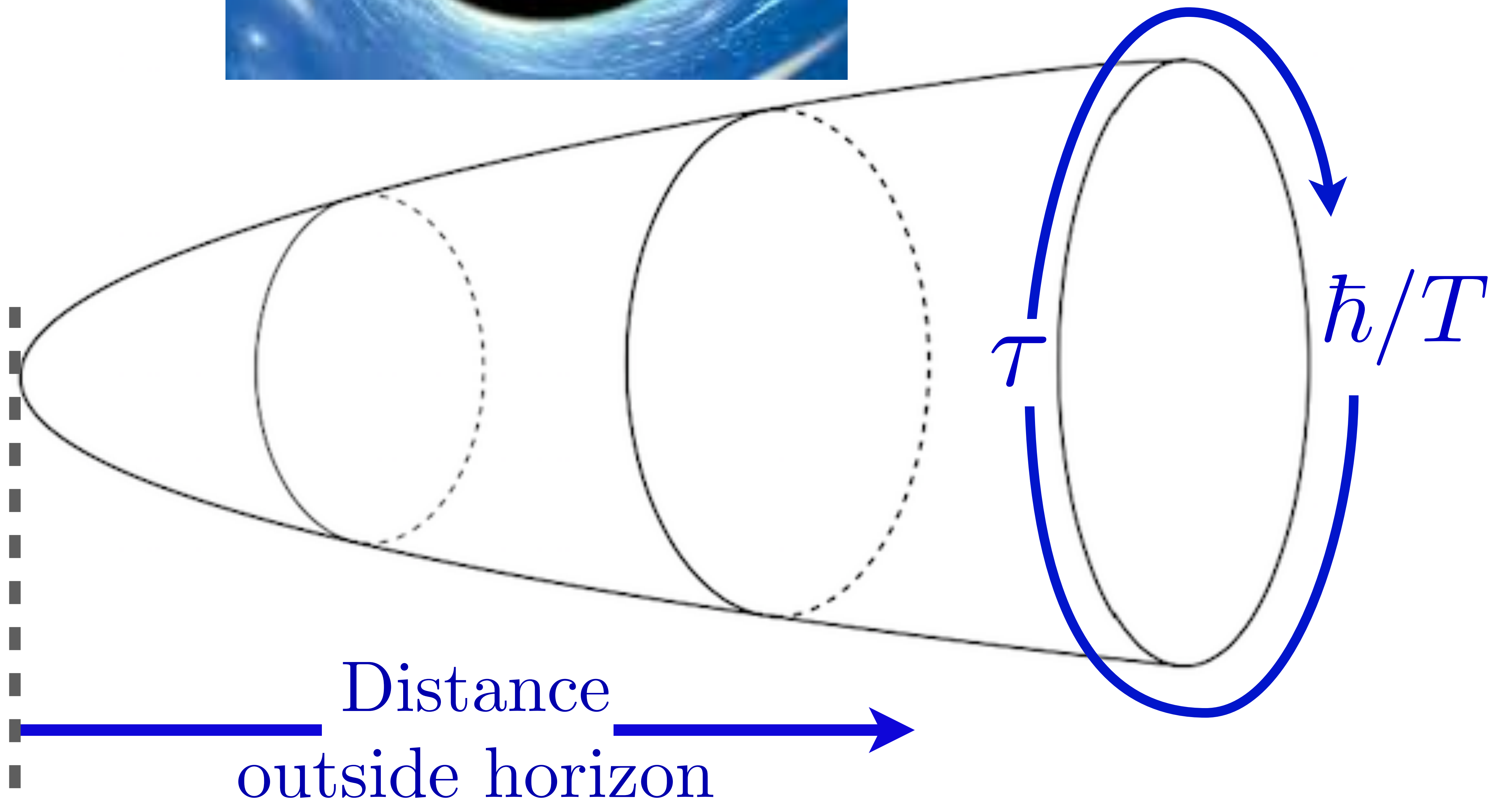


J. D. Bekenstein, PRD **7**, 2333 (1973)
S. W. Hawking, Nature **248**, 30 (1974)
C.V. Vishveshwara, Nature **227**, 936 (1970)

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\mathcal{Z}(\mathcal{Q},T) = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu},A_\mu]\right)$$





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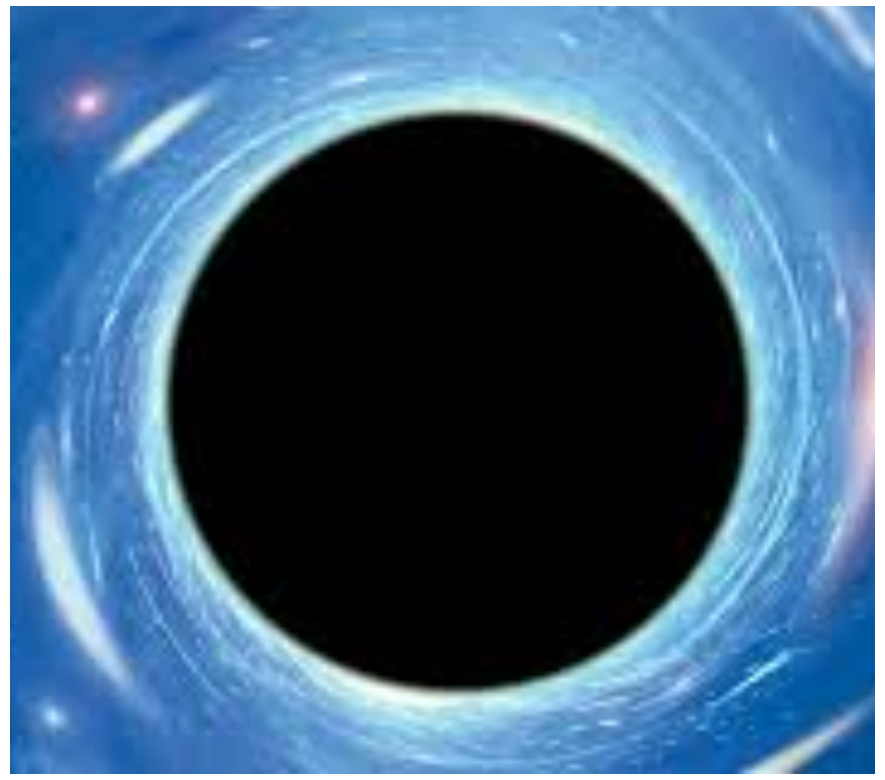
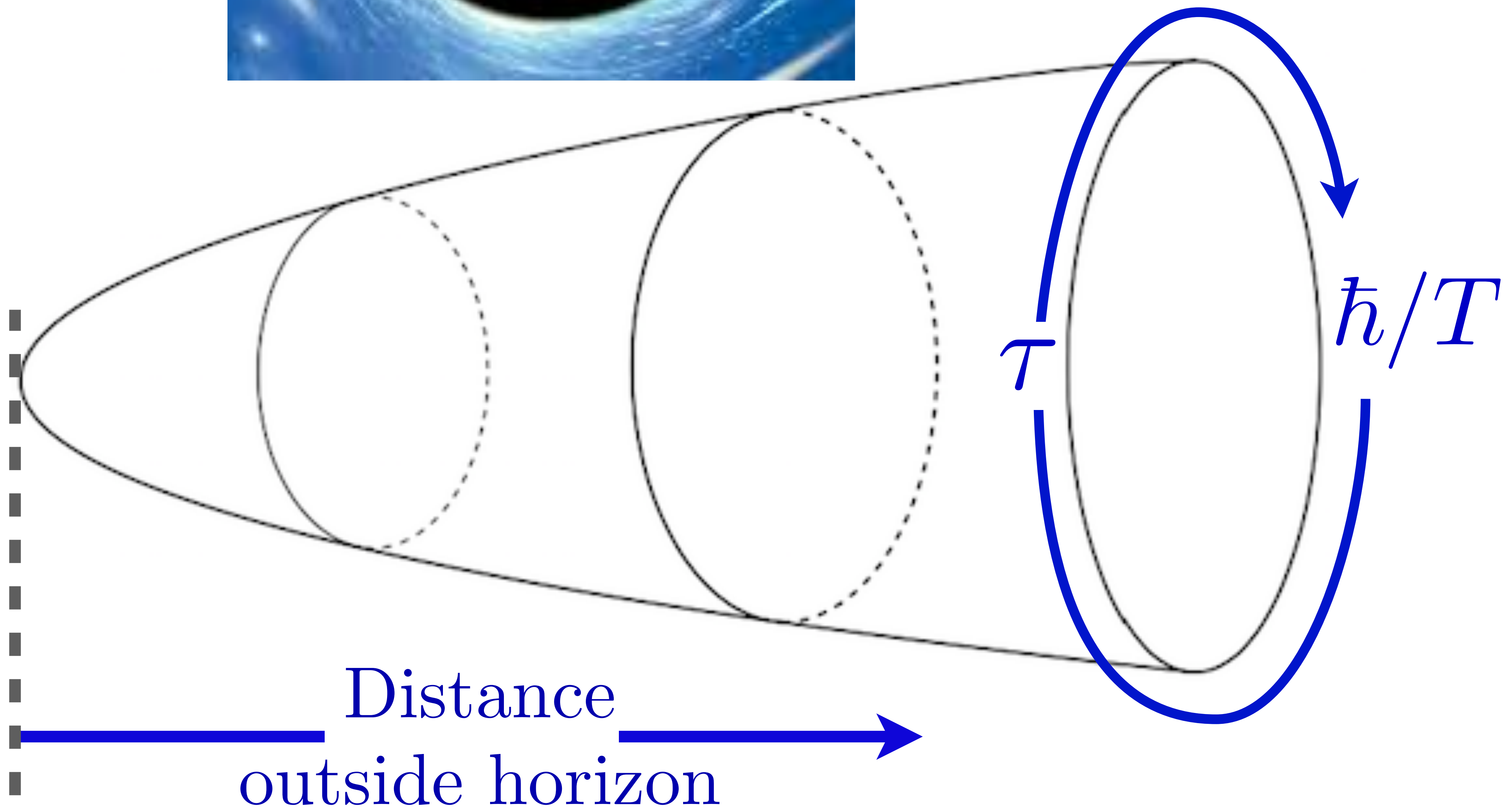
$$= \exp\Big(S_{BH}\Big) \times \left(\quad ....????... \quad \right)$$

Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)

$$S_{BH}(T \to 0, \mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2} T}{\hbar c}\right)$$



$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T = 0$.



Obtained from the saddle-point of the gravity path integral in the imaginary time spacetime outside the black hole.

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

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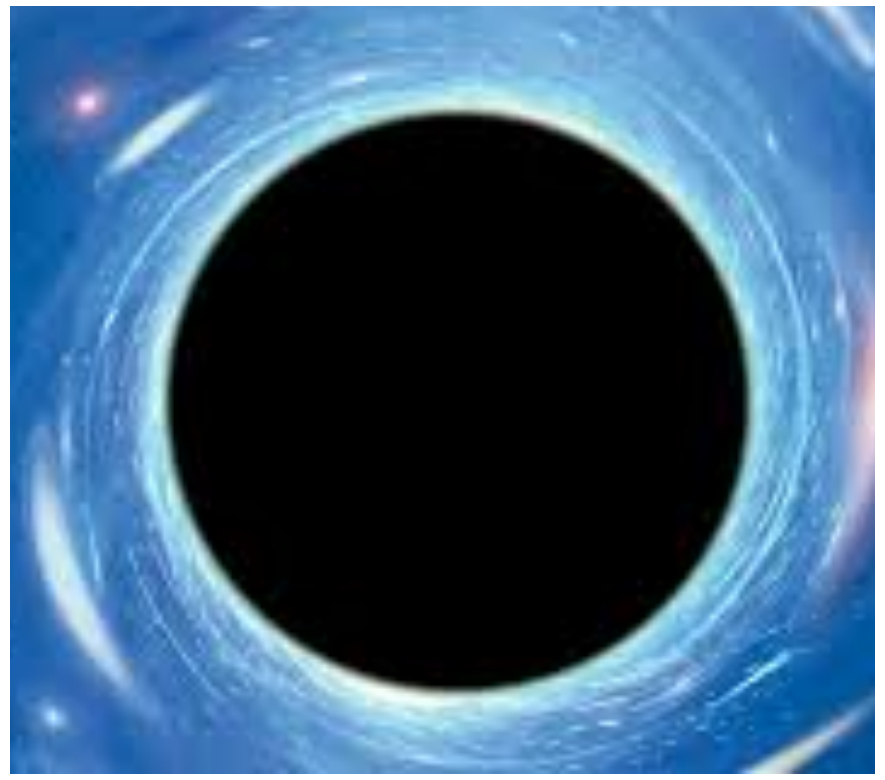

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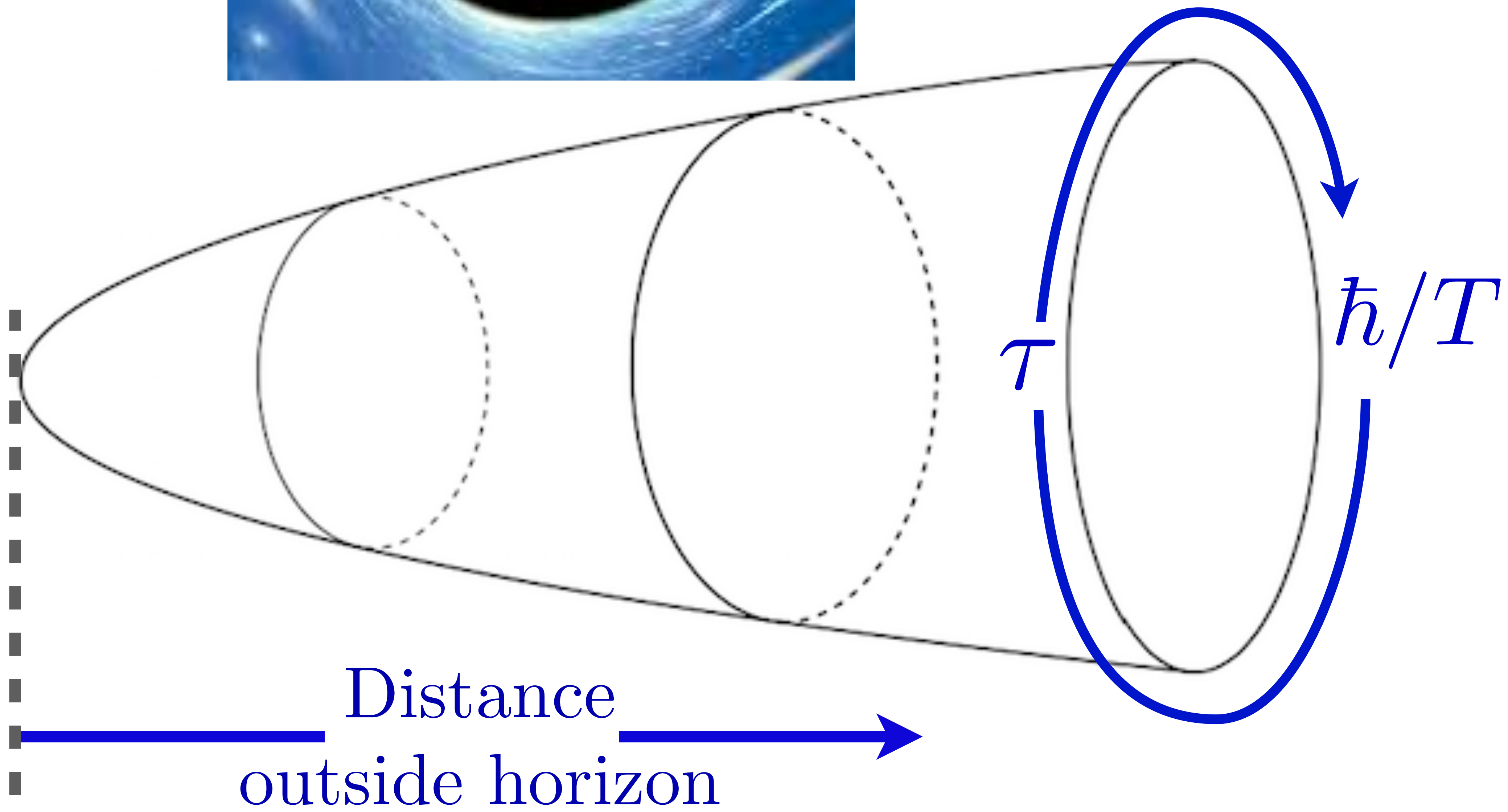


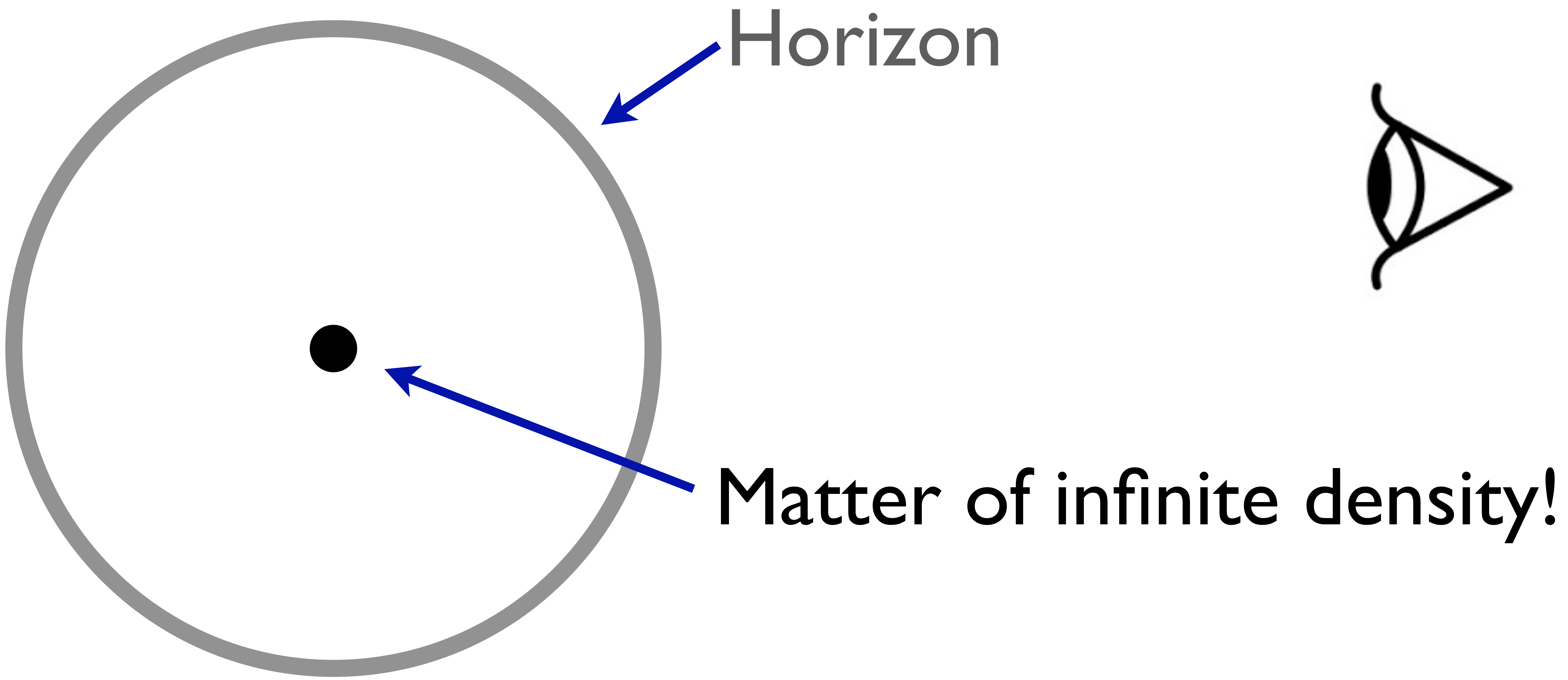
Note the similarity to the large *N* entropy of the SYK model !
(along with other similarities)

Sachdev PRL 2010

Quantum Black Holes

Hawking obtained the black hole entropy by semiclassical computations for an observer *outside* the black hole horizon.

This allowed Hawking to avoid the contradictions associated with the singularity at the center of the black hole.



Quantum Black Holes

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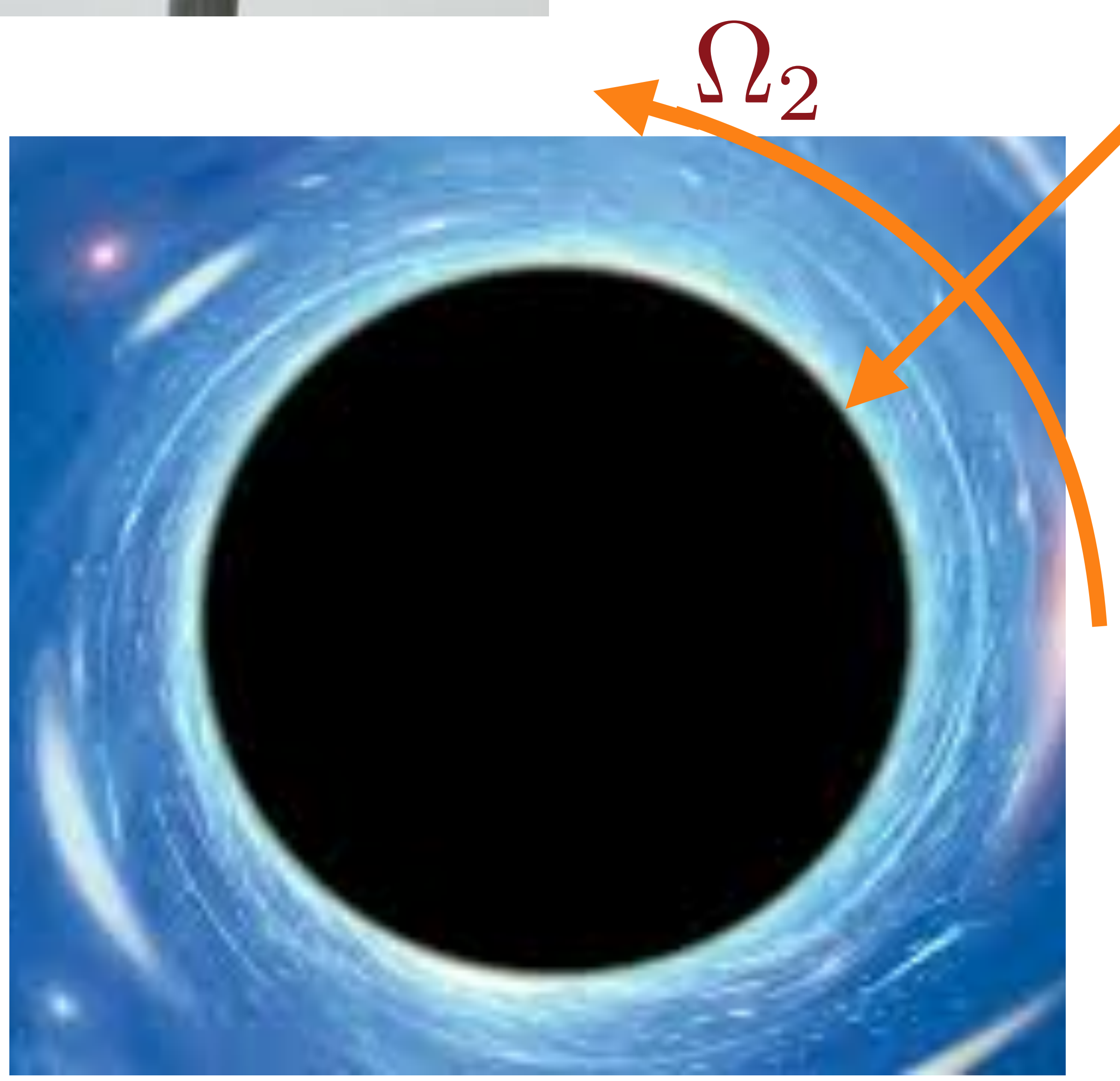
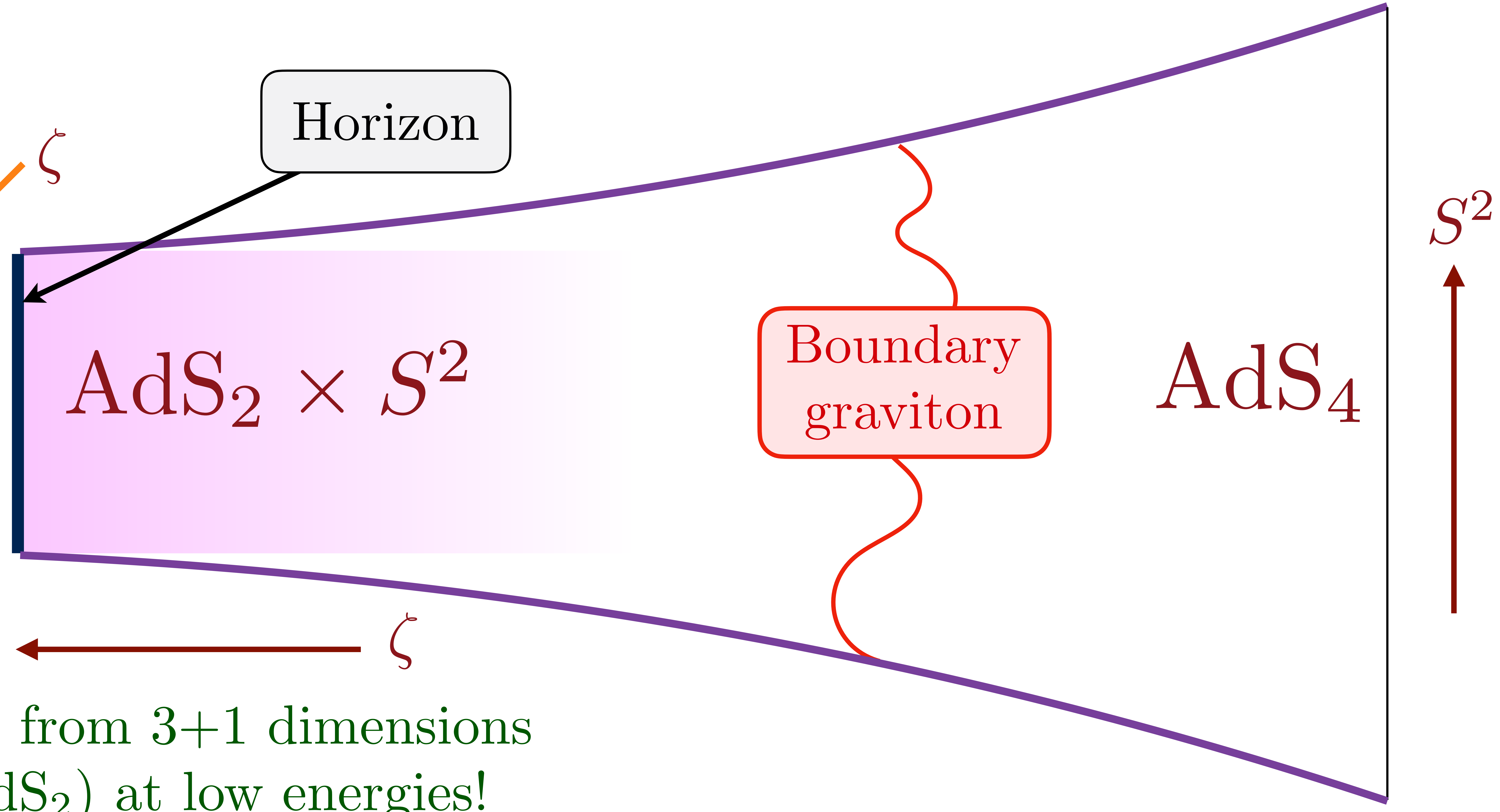
Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* matches the Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics ?

From the SYK model to a quantum theory of charged black holes

Reissner-Nordstrom black hole of Einstein-Maxwell theory





Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!
The isometry group of AdS_2 is the 0+1 dimensional conformal group SL(2,R).

Faulkner, Liu, McGreevy, Vegh 2009; Sachdev 2010

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Saddle-point:

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$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

Maldacena, Stanford, Yang (2016)

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$$\approx \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(1+1)}_{\text{JT gravity of AdS}_2\text{+boundary graviton}}[g_{\mu\nu},A_\mu]\right)$$

Saddle-point:

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$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)\exp\left(-\frac{1}{\hbar} I_{\text{SYK}}[\text{time reparameterizations } f(\tau),\text{ phase rotations } \phi(\tau)]\right)$$

Saddle-point:

$$S_{BH}(T\to 0,\mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1+\frac{2(\pi A_0)^{1/2}T}{\hbar c}+\dots\right)$$

$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T=0$.

Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019); Iliesiu, Turaci (2020)

Quantum simulation of charged black holes by the SYK model

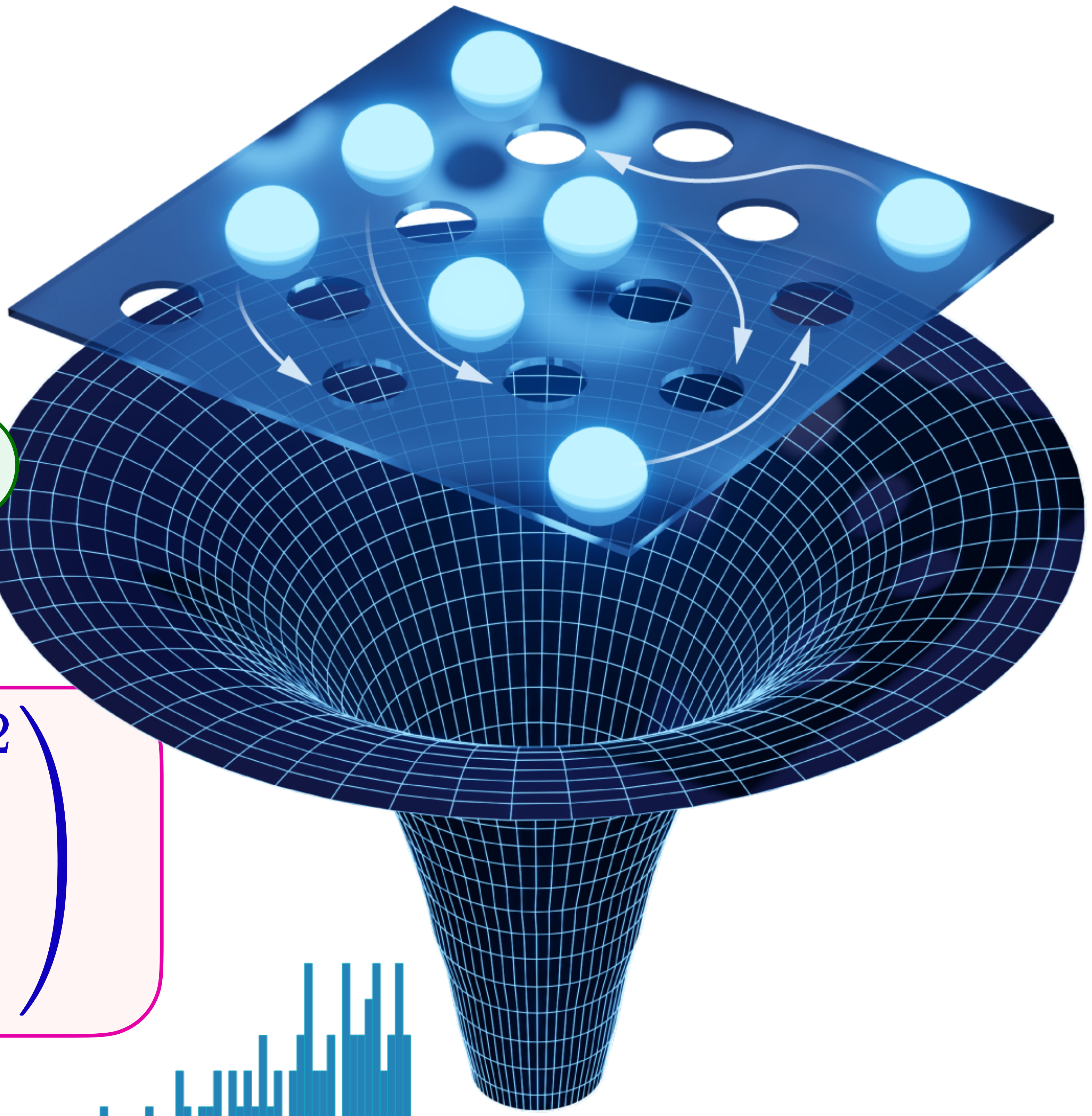
- For generic charged black holes in 3+1 dimensions, the SYK model yields, in terms of $A_0 = 2GQ^2/c^4$ the horizon area at $T = 0$:

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

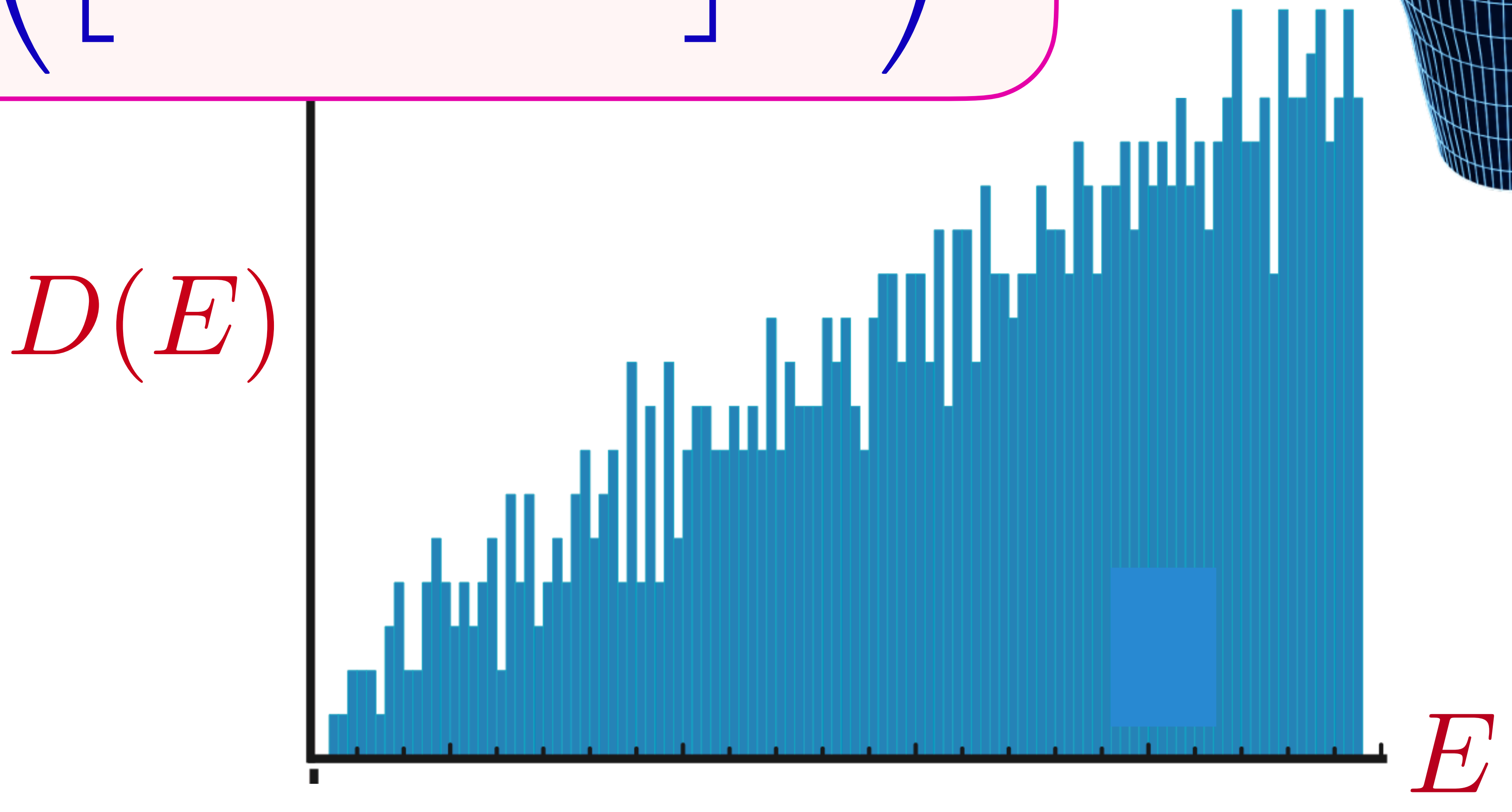
Iliesiu, Murthy, Turiaci (2022)

Bekenstein-Hawking

Developments from the SYK model



There is no degeneracy, but an exponentially small level spacing down to the ground state.



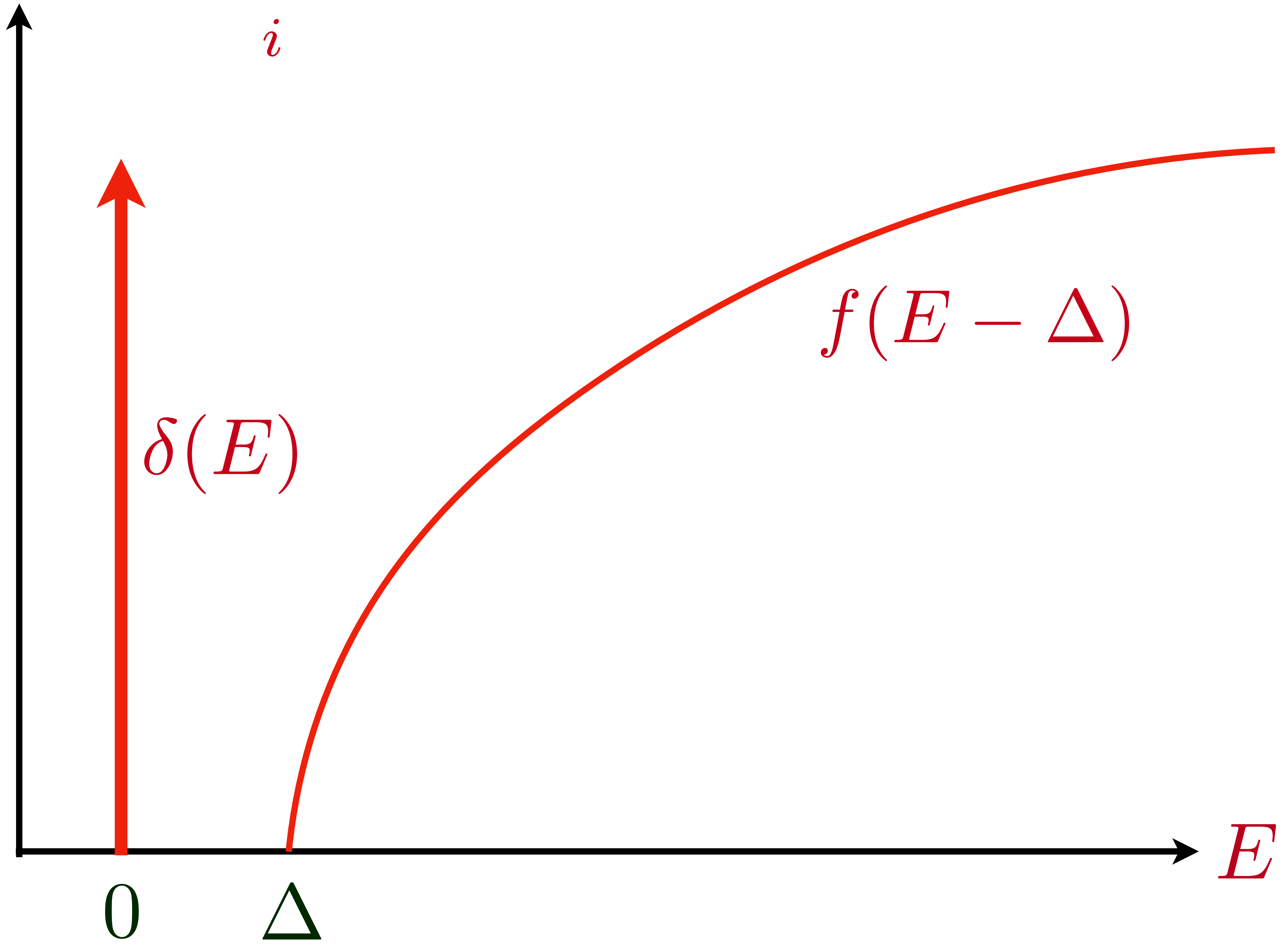
String theory of charged black holes

- With sufficient low energy supersymmetry, string theory yields:

$$D(E) = \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\delta(E) + \theta(E-\Delta) f(E-\Delta) + \dots$$

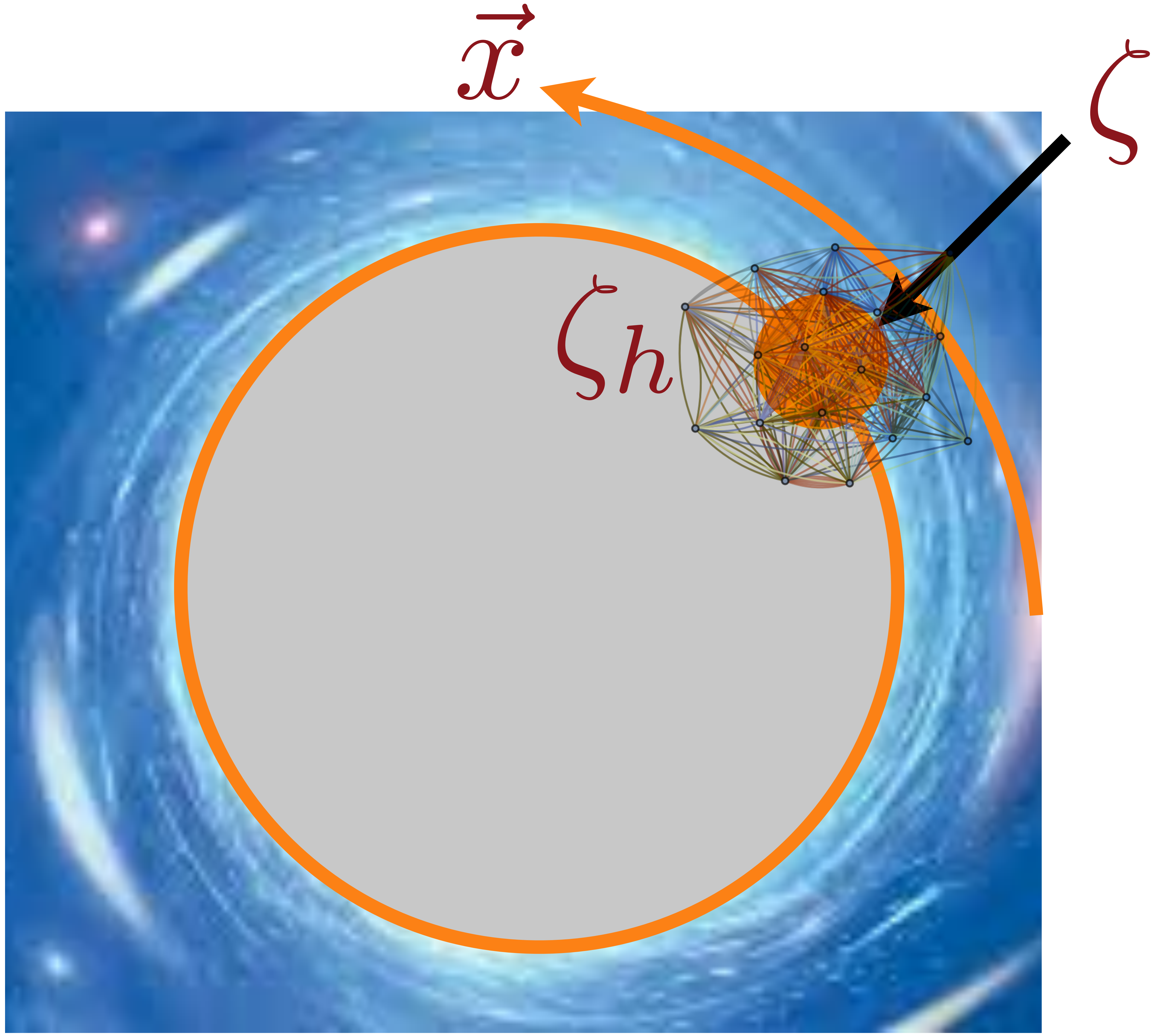
There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state.

$$D(E) = \sum_i \delta(E - E_i)$$



M. Heydeman, L.V. Iliesiu, G. J. Turiaci, and W. Zhao, 2020
L.V. Iliesiu, S. Murthy, G. J. Turiaci, 2022

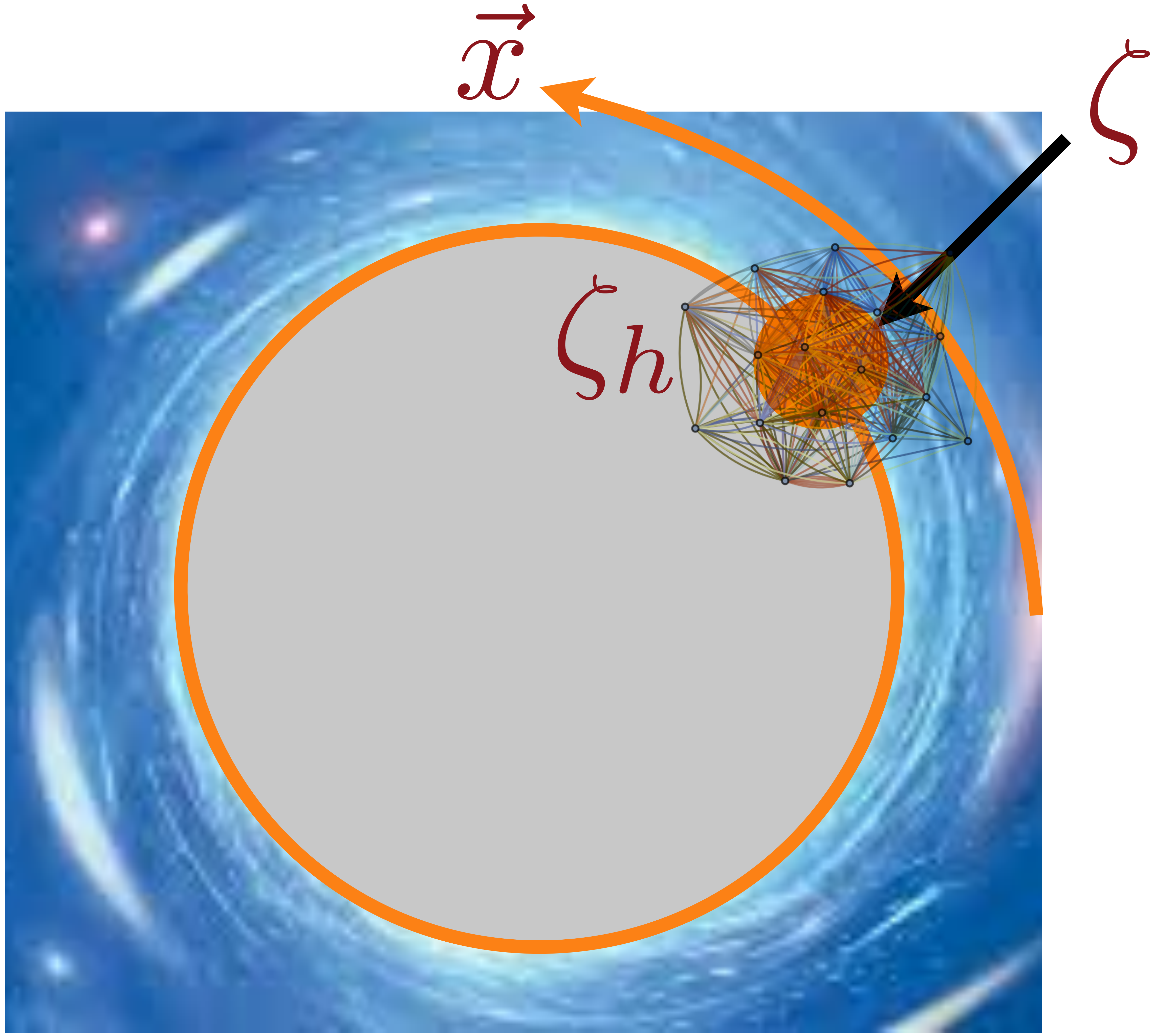
Quantum simulation of charged black holes by the SYK model



The SYK provides the needed realization of the black hole interior, and its density of quantum states matches gravitational entropy computations for charged black holes !

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

Quantum simulation of charged black holes by the SYK model



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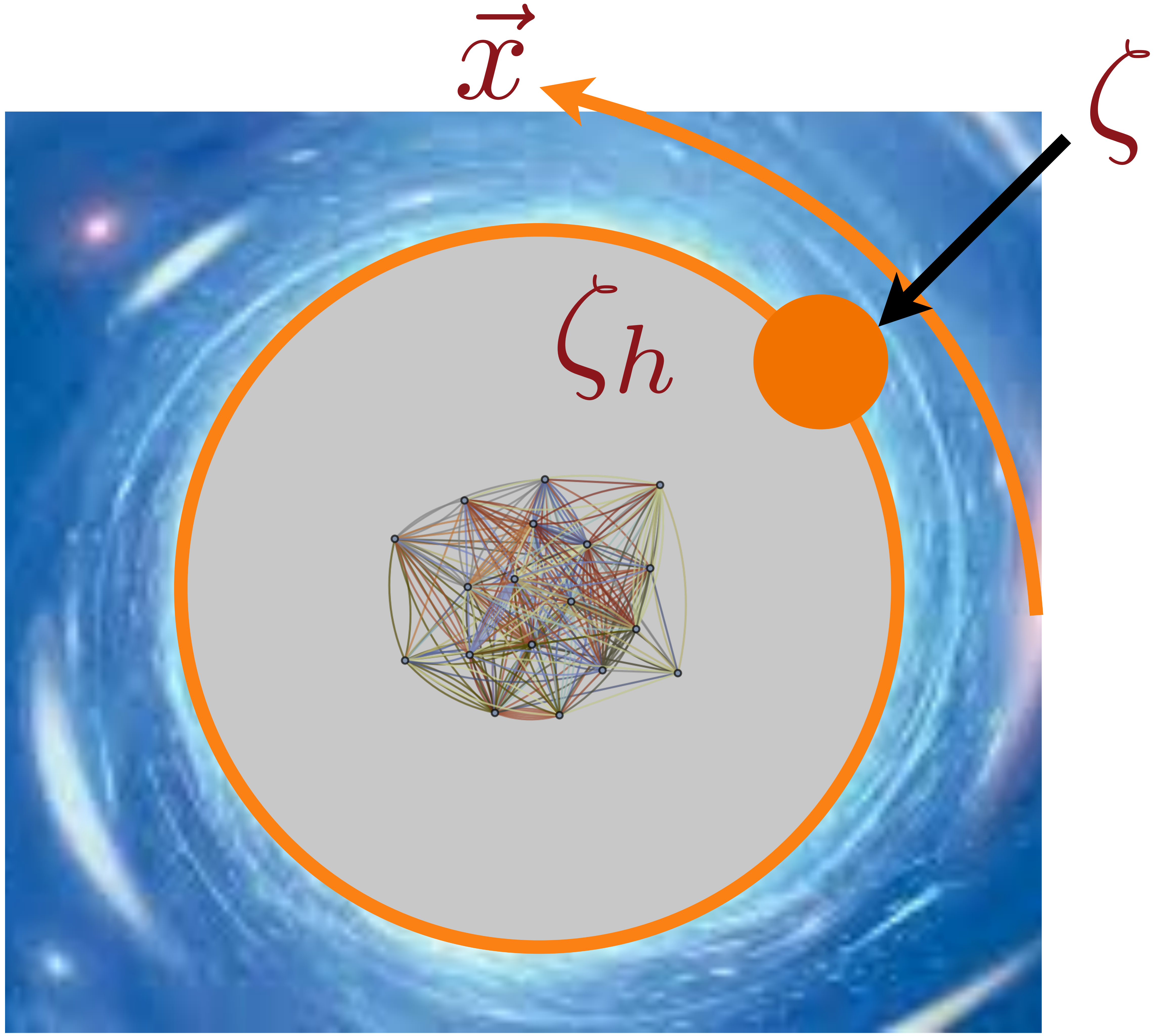

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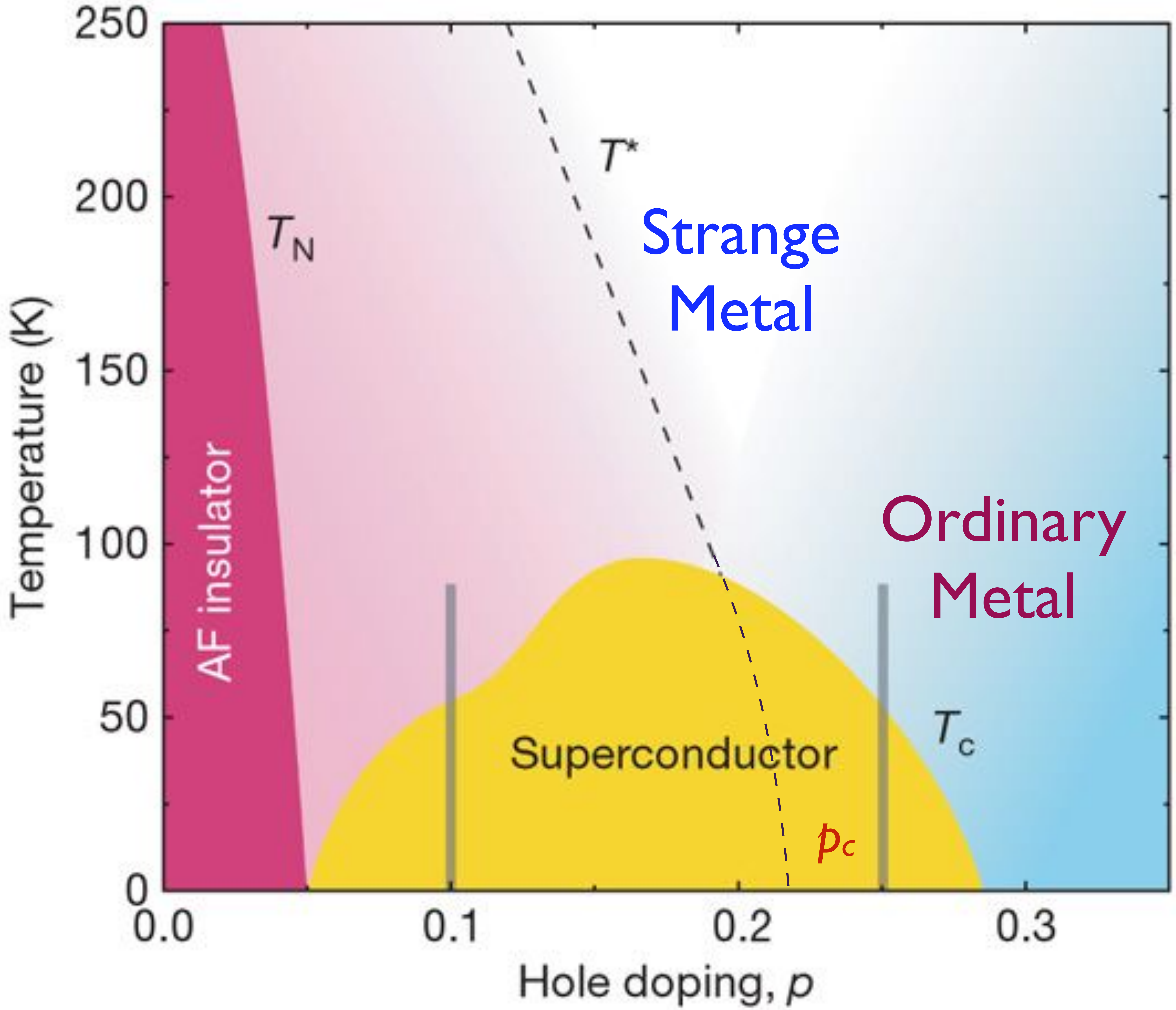
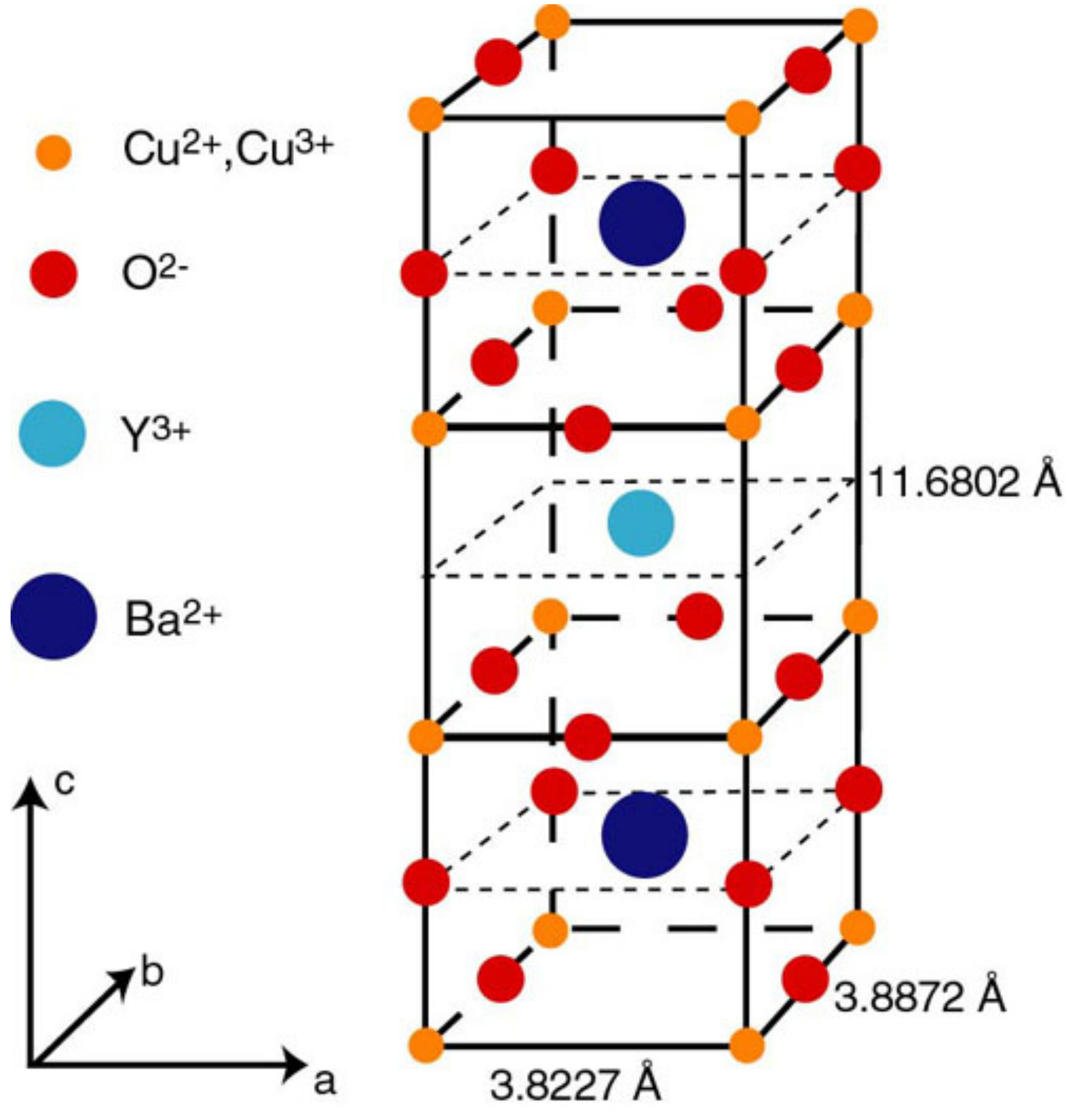
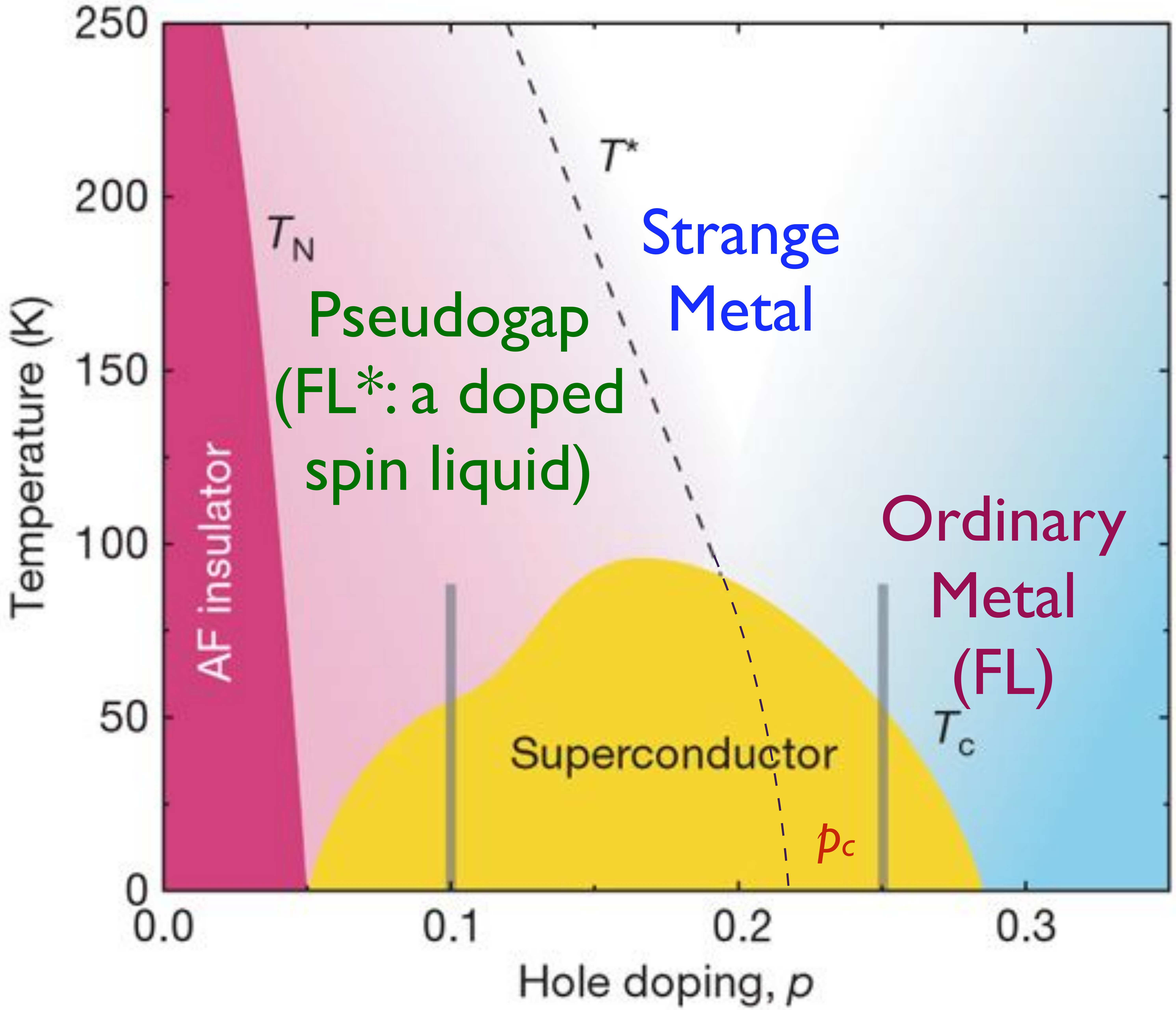
D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

From the SYK model to a universal theory of strange metals

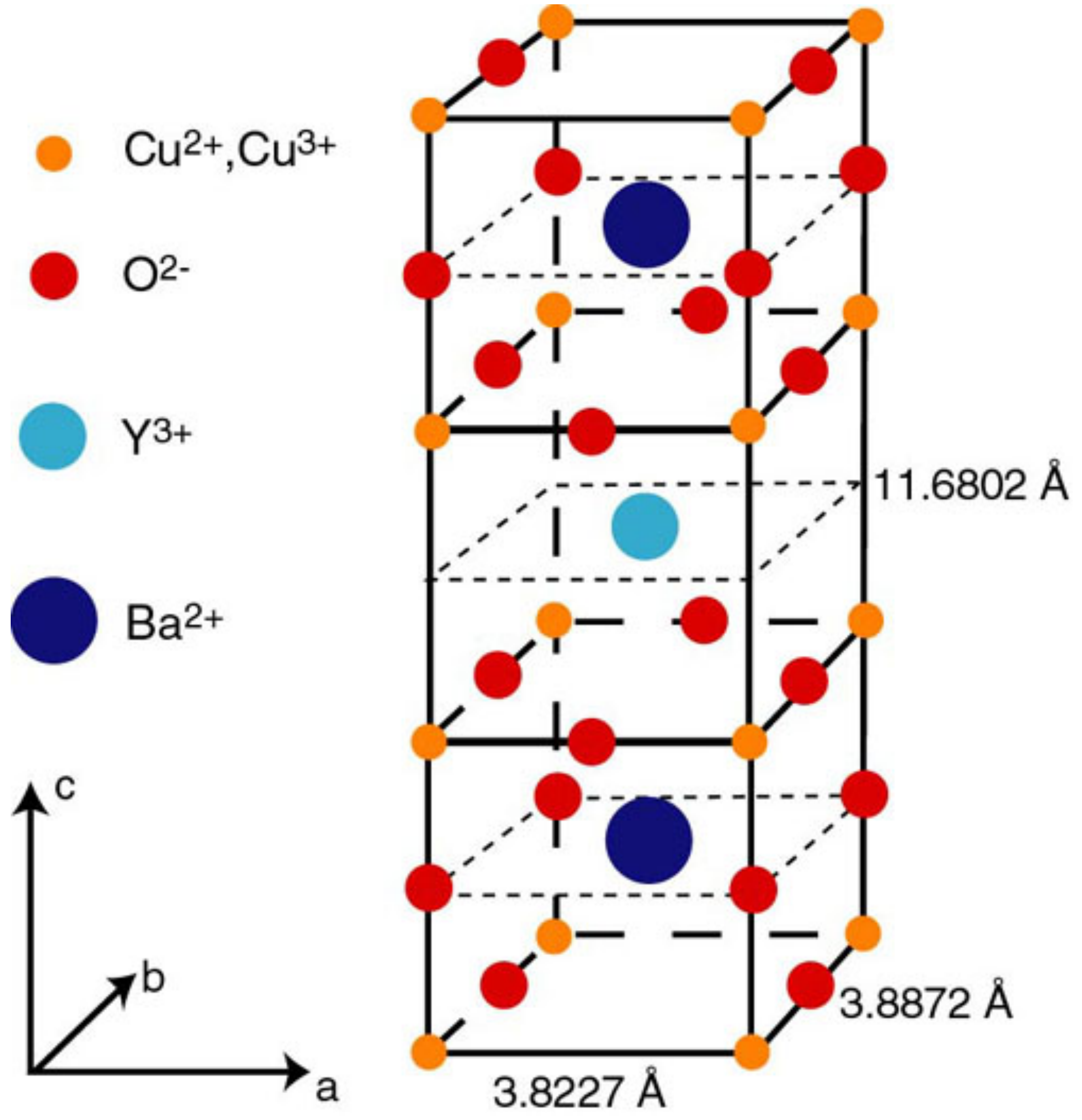

250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Strange Metal
Ordinary Metal
Superconductor
T_c
p_c


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$



250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Pseudogap
(FL*: a doped
spin liquid)
Strange
Metal
Ordinary
Metal
(FL)
Superconductor
T_c
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
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11.6802 Å
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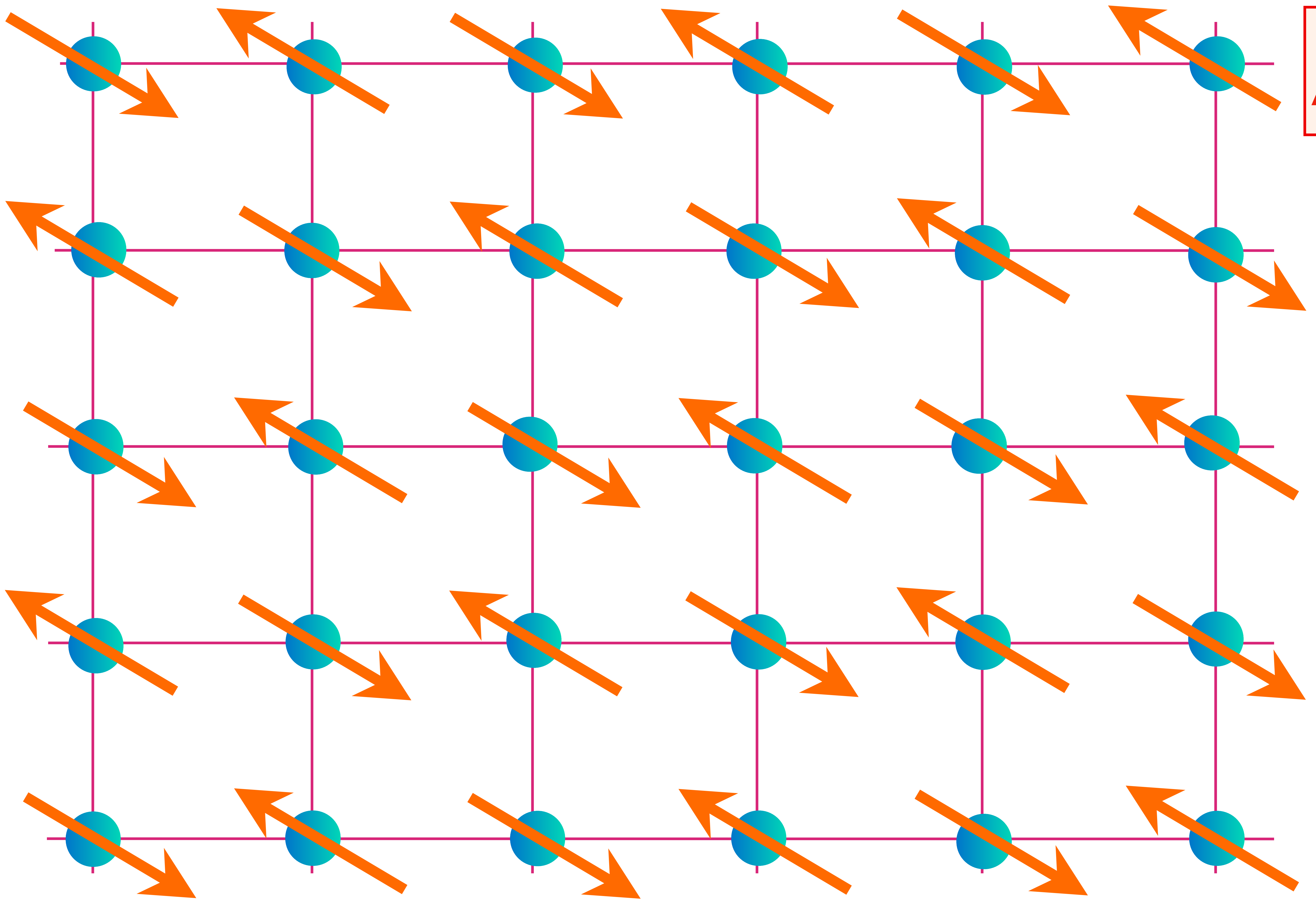
$YBa_2Cu_3O_{6+x}$

The dance of electrons on Cu atoms in YBCO

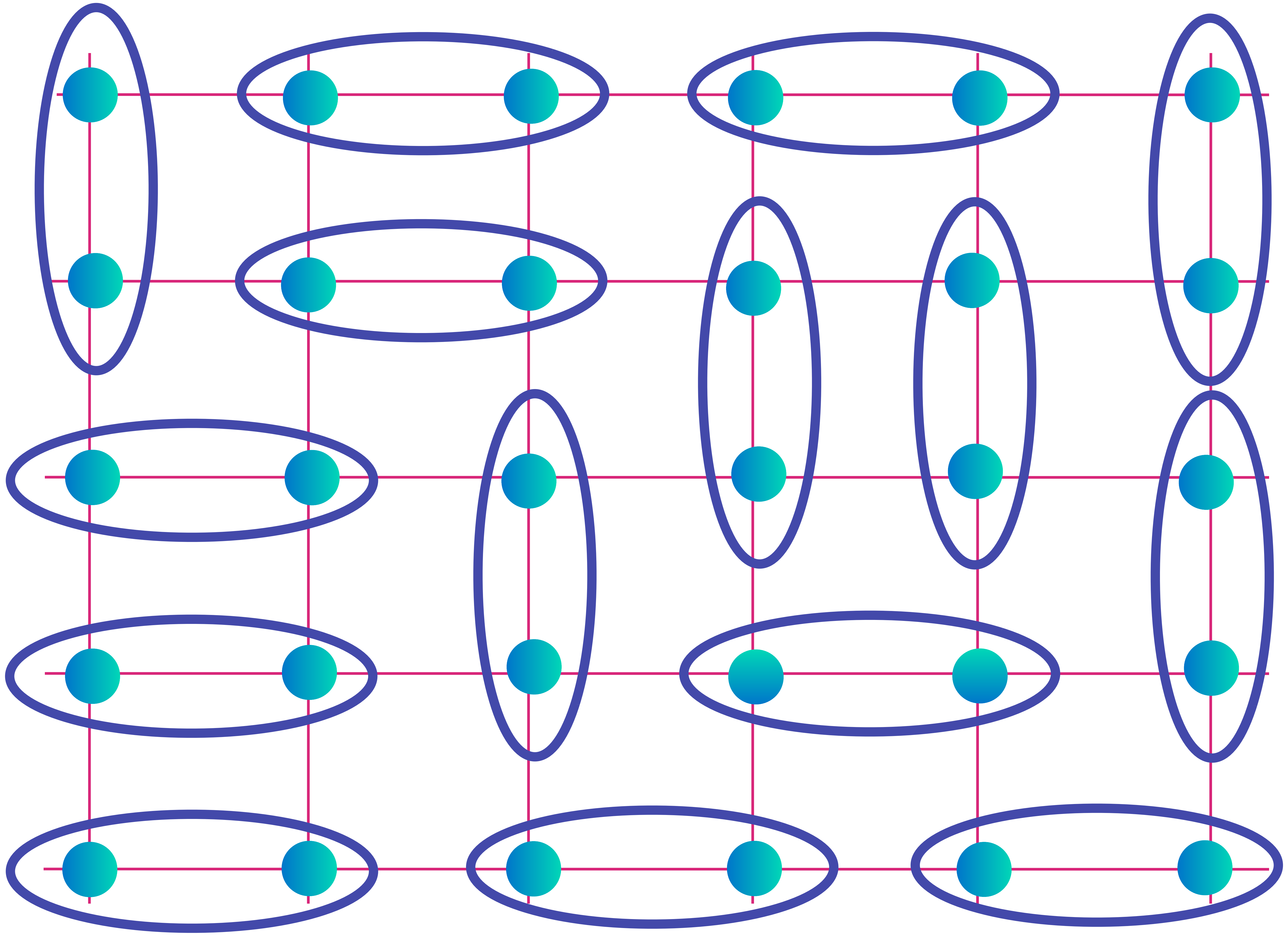


Antiferromagnetism

All nearest-neighbor pairs of electrons have opposite spins

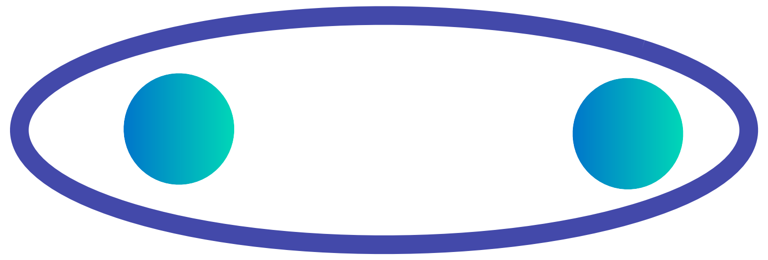
The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)



Spin liquid

Electrons form entangled pairs, and the pairs entangle across the entire sample


$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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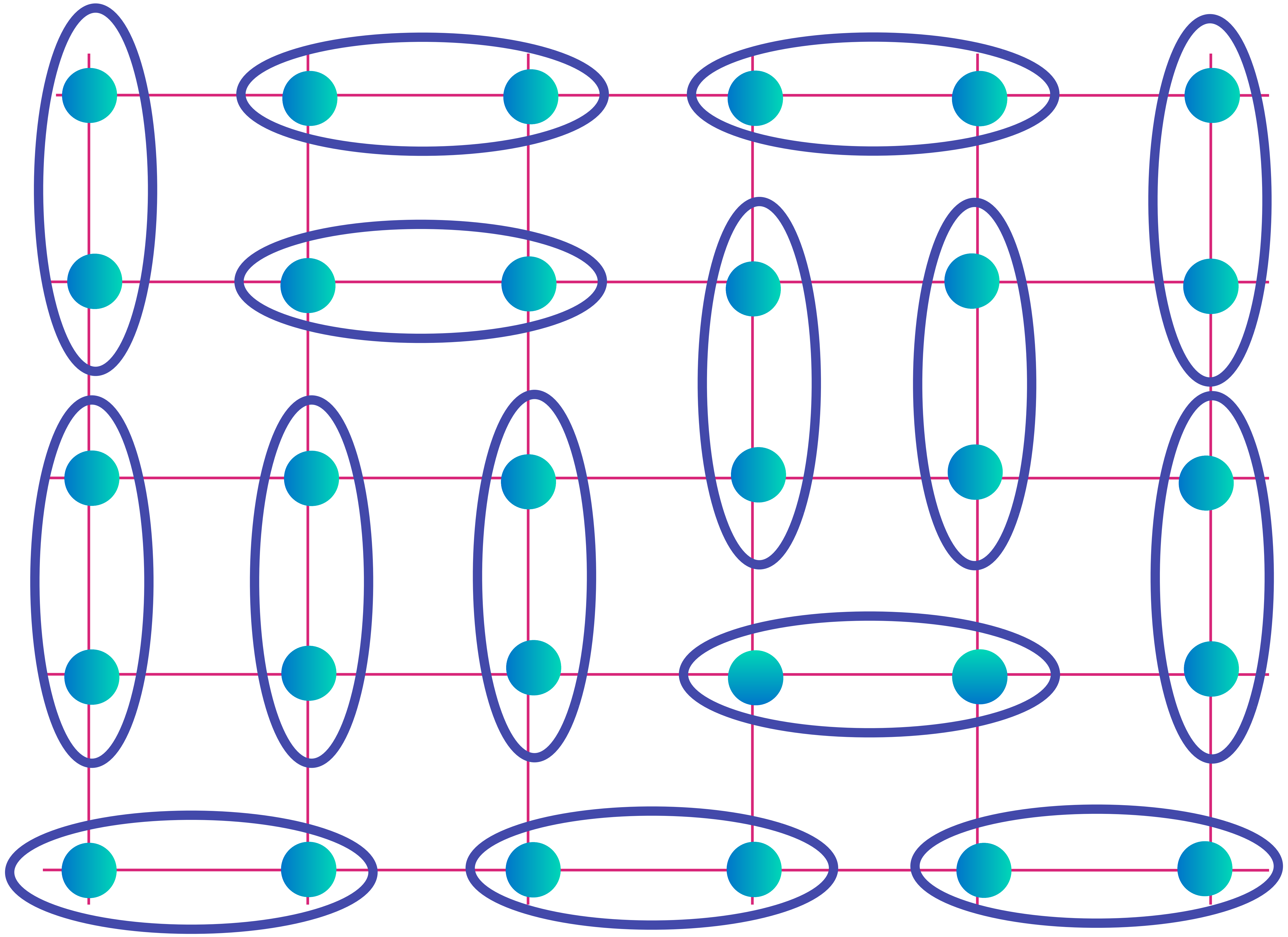
P.W. Anderson (1973)

Spin liquid



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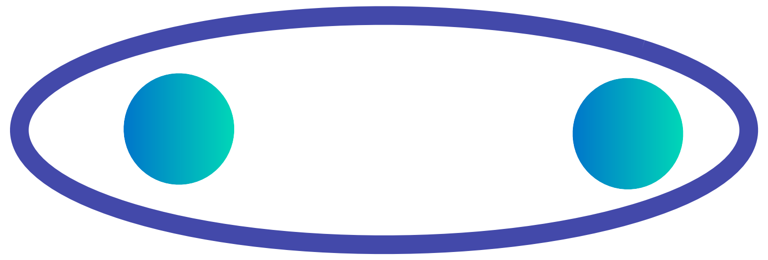

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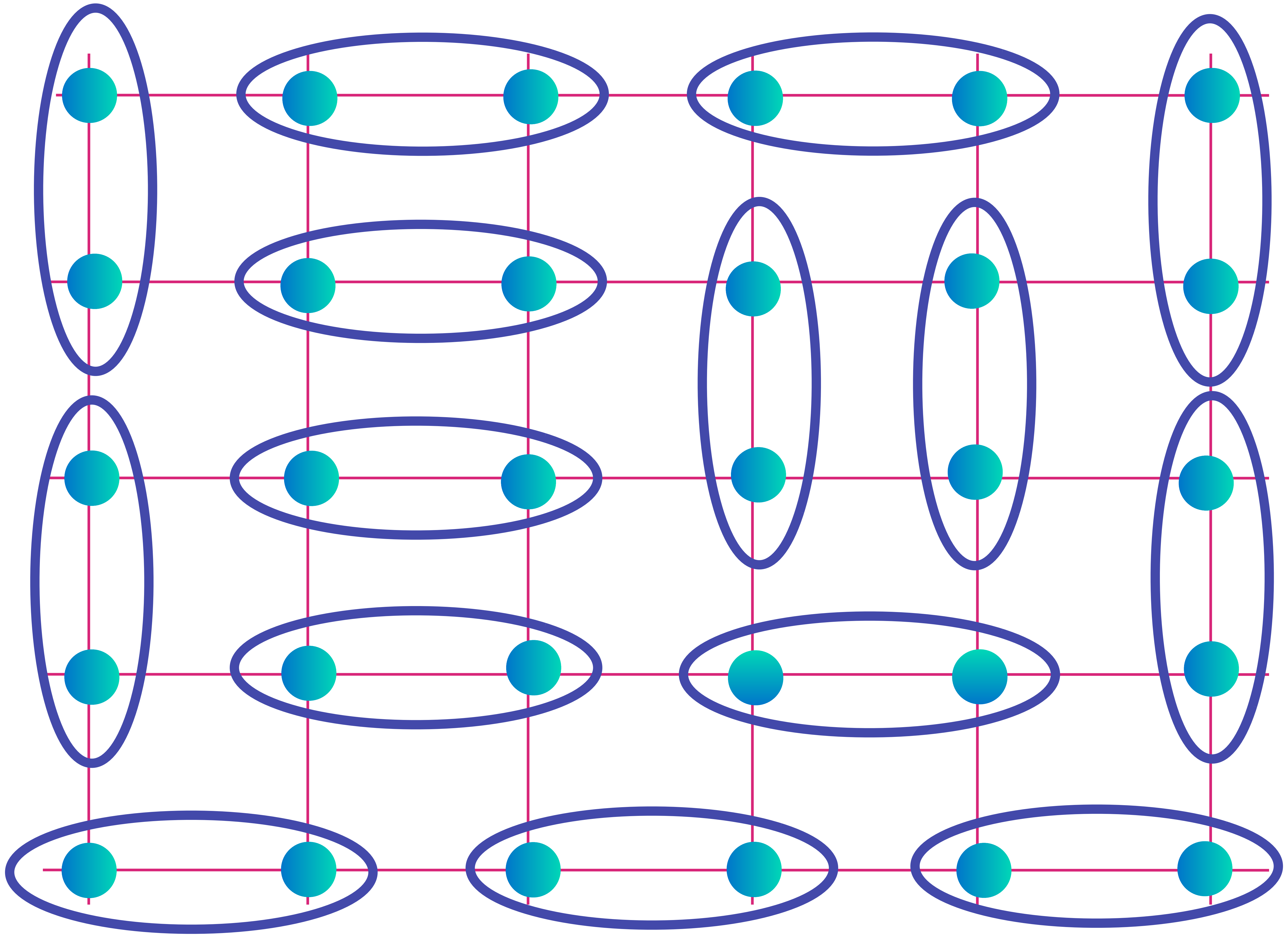

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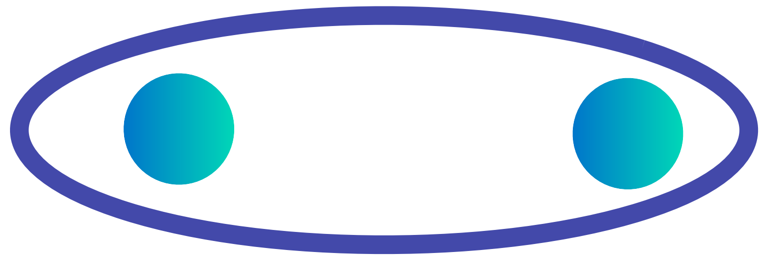 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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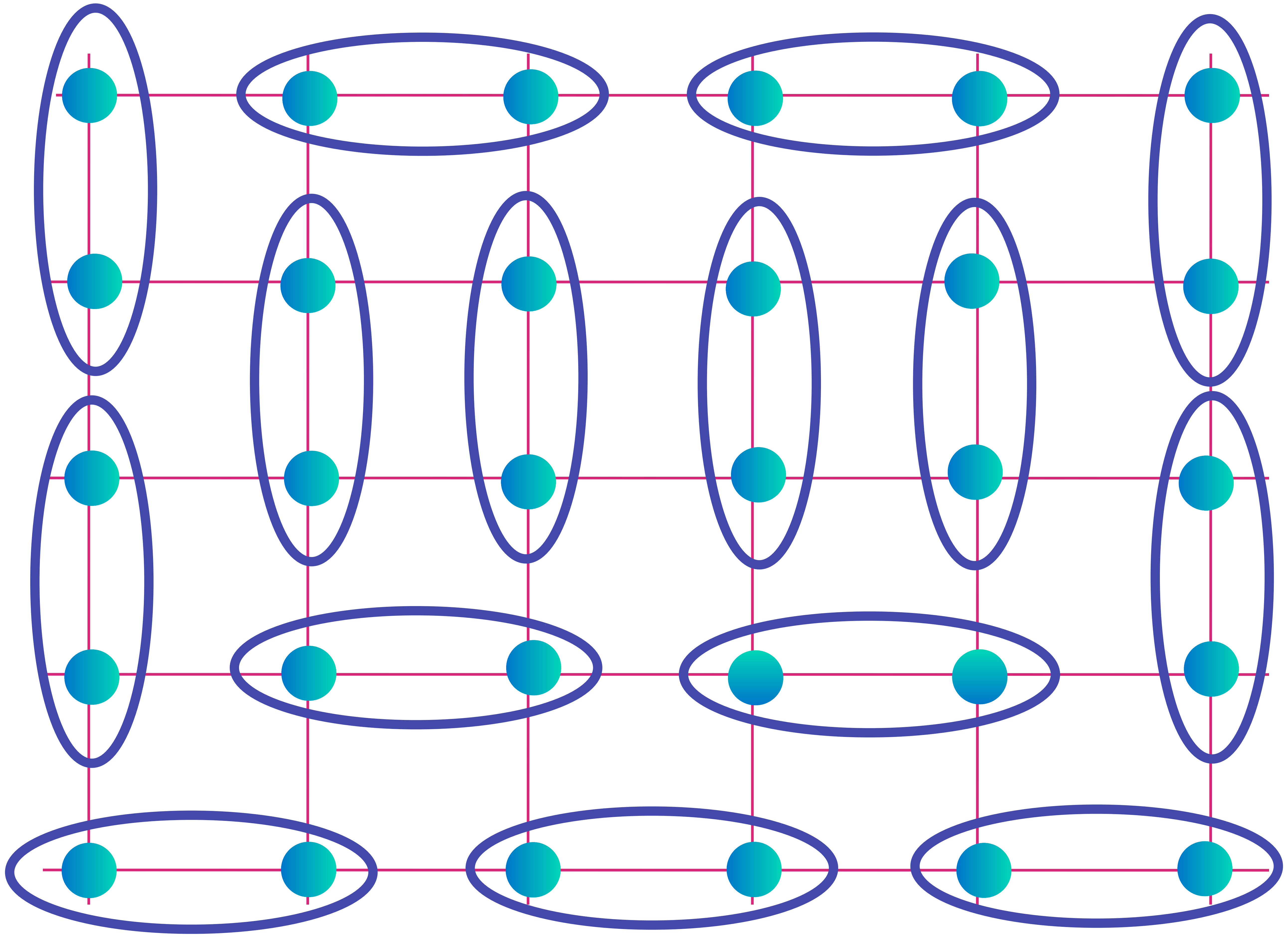

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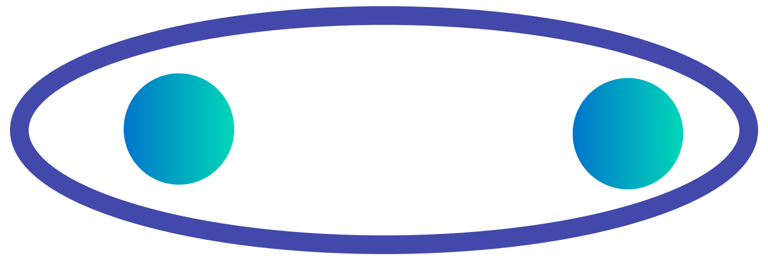

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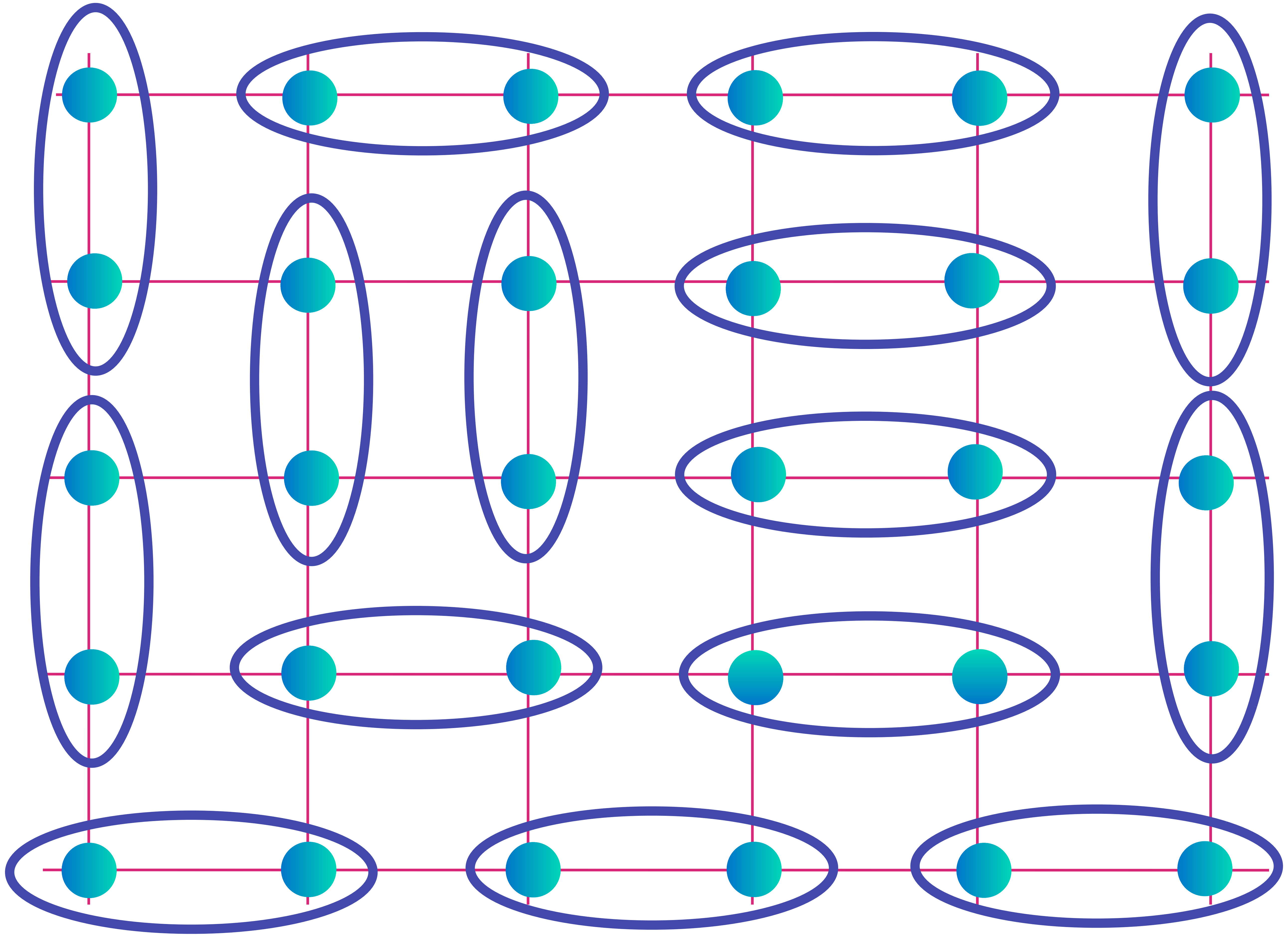

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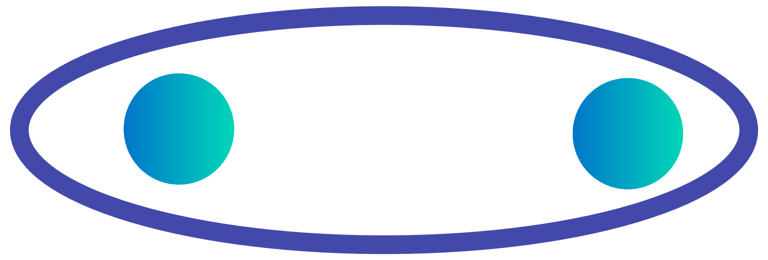

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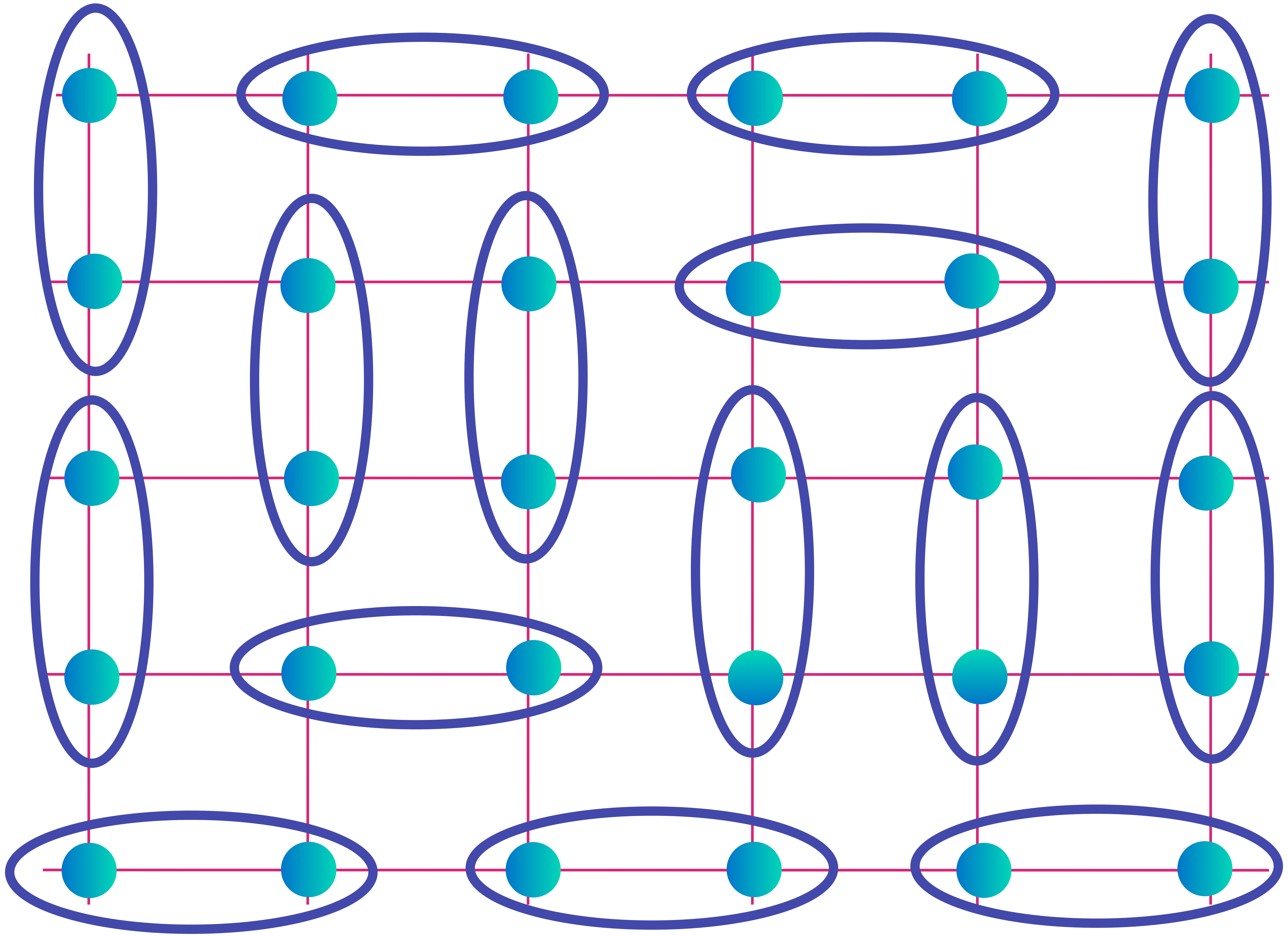

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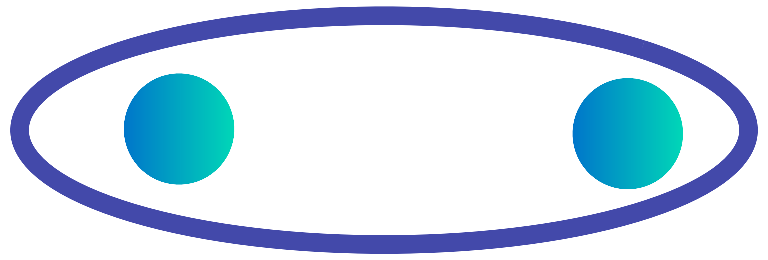

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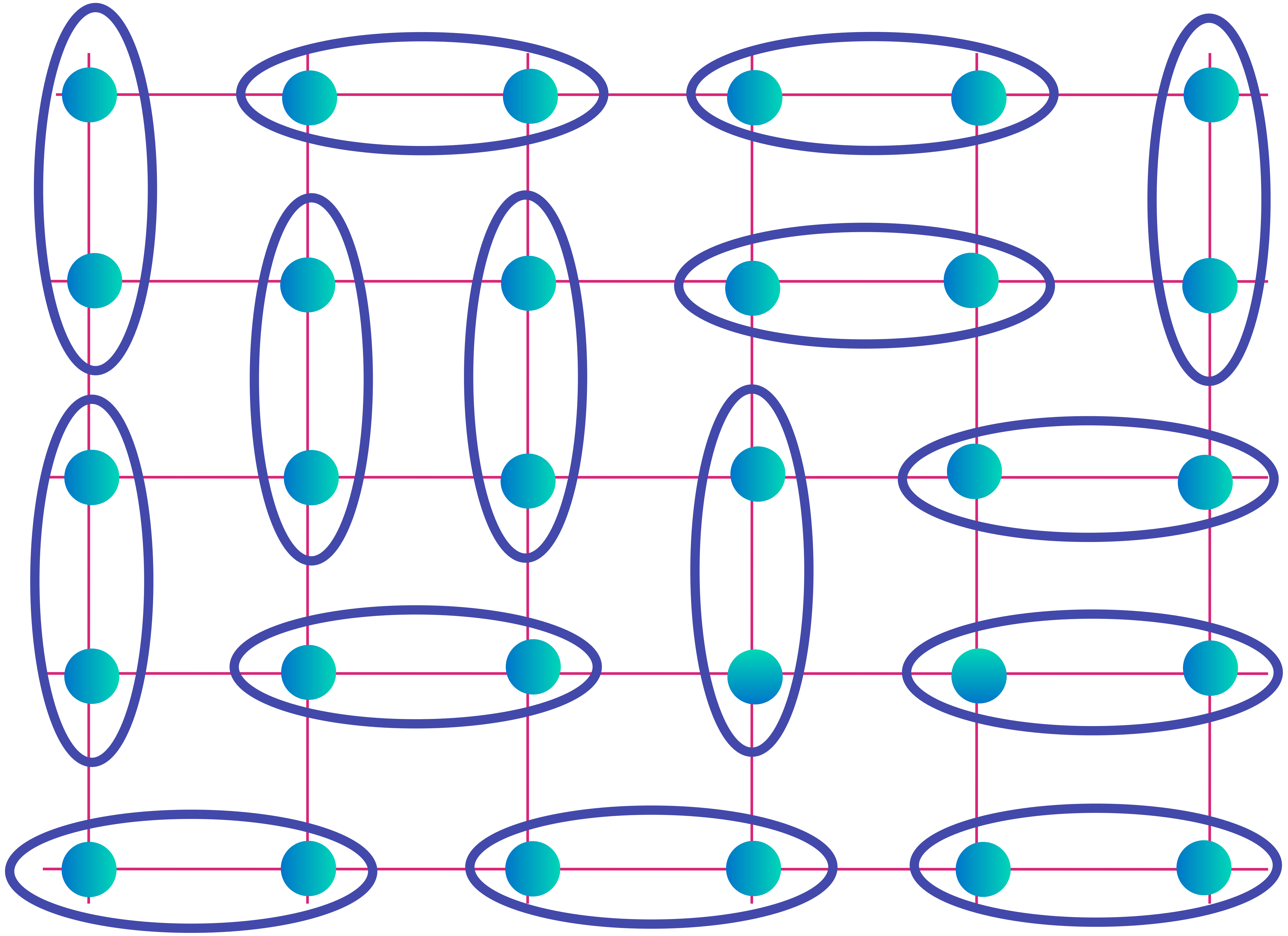

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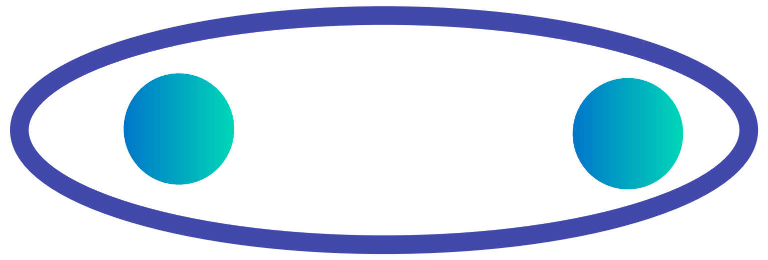

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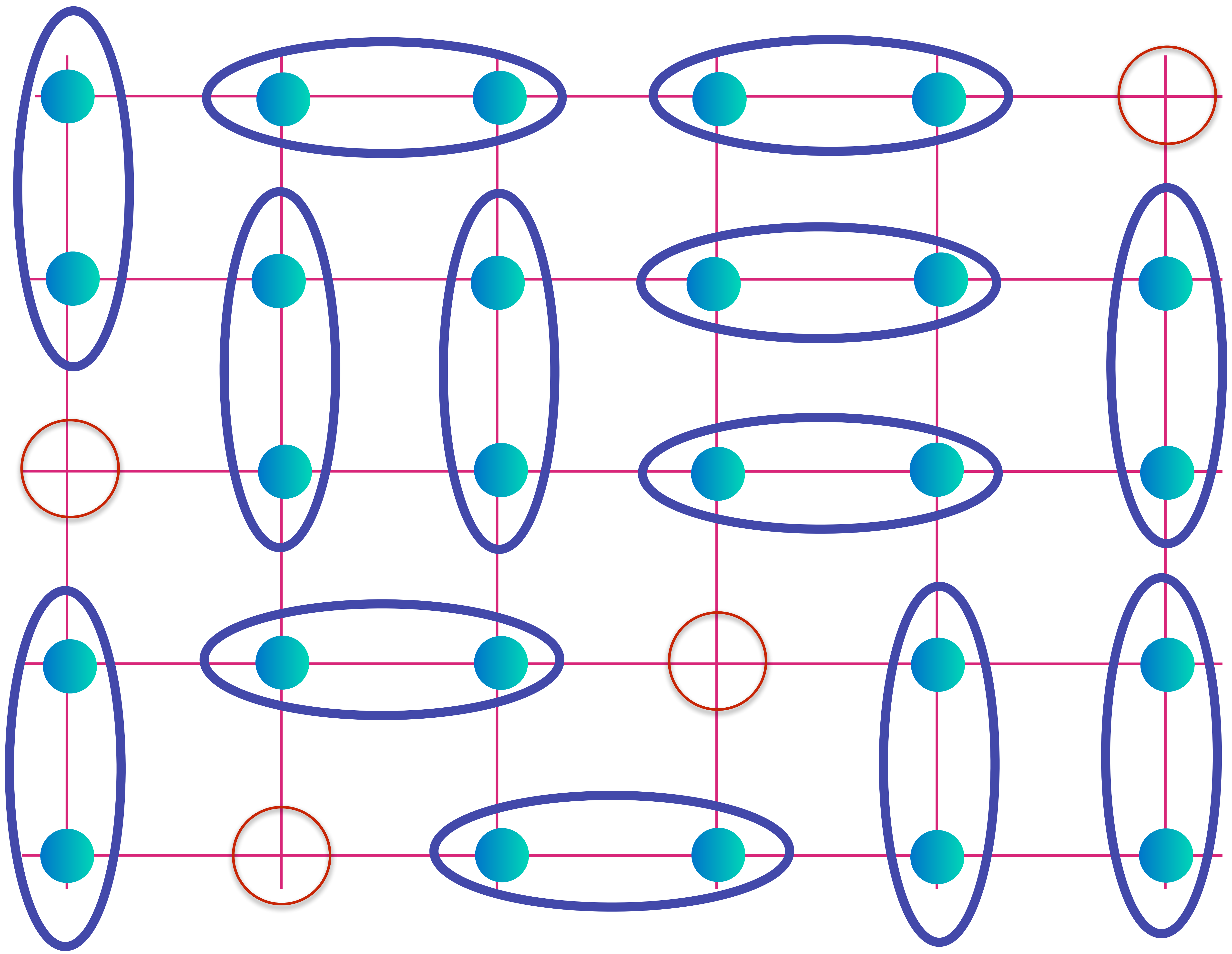
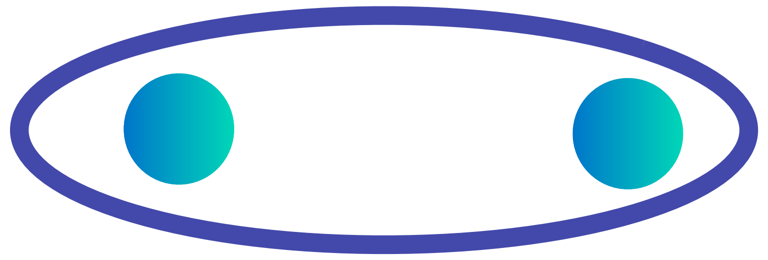


Holon metal

Spin liquid with density *p* of spinless, charge +e "holons"



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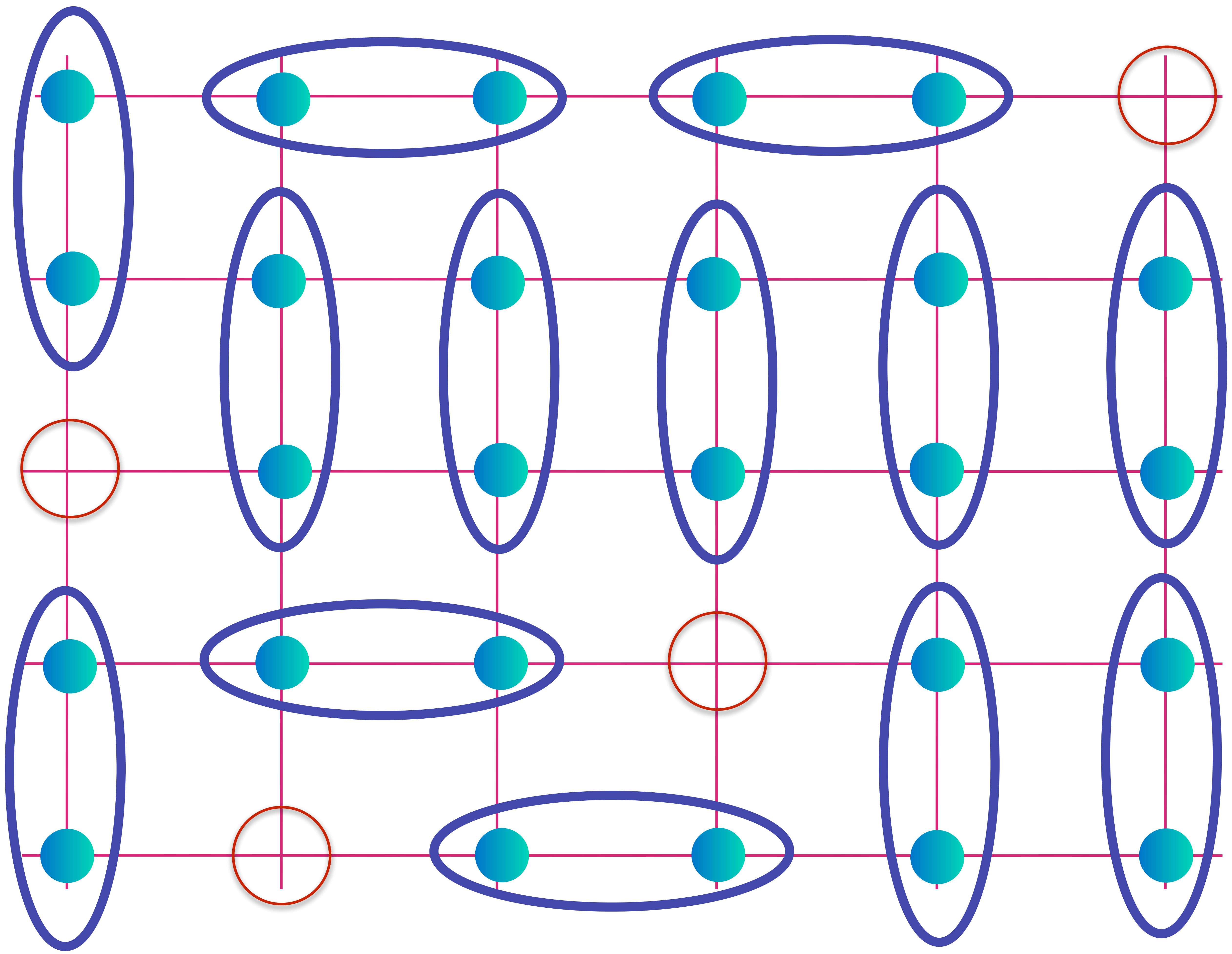

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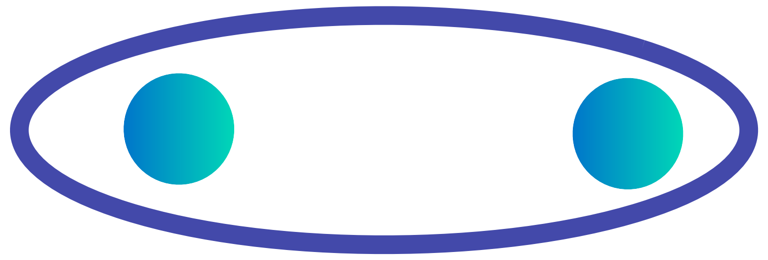

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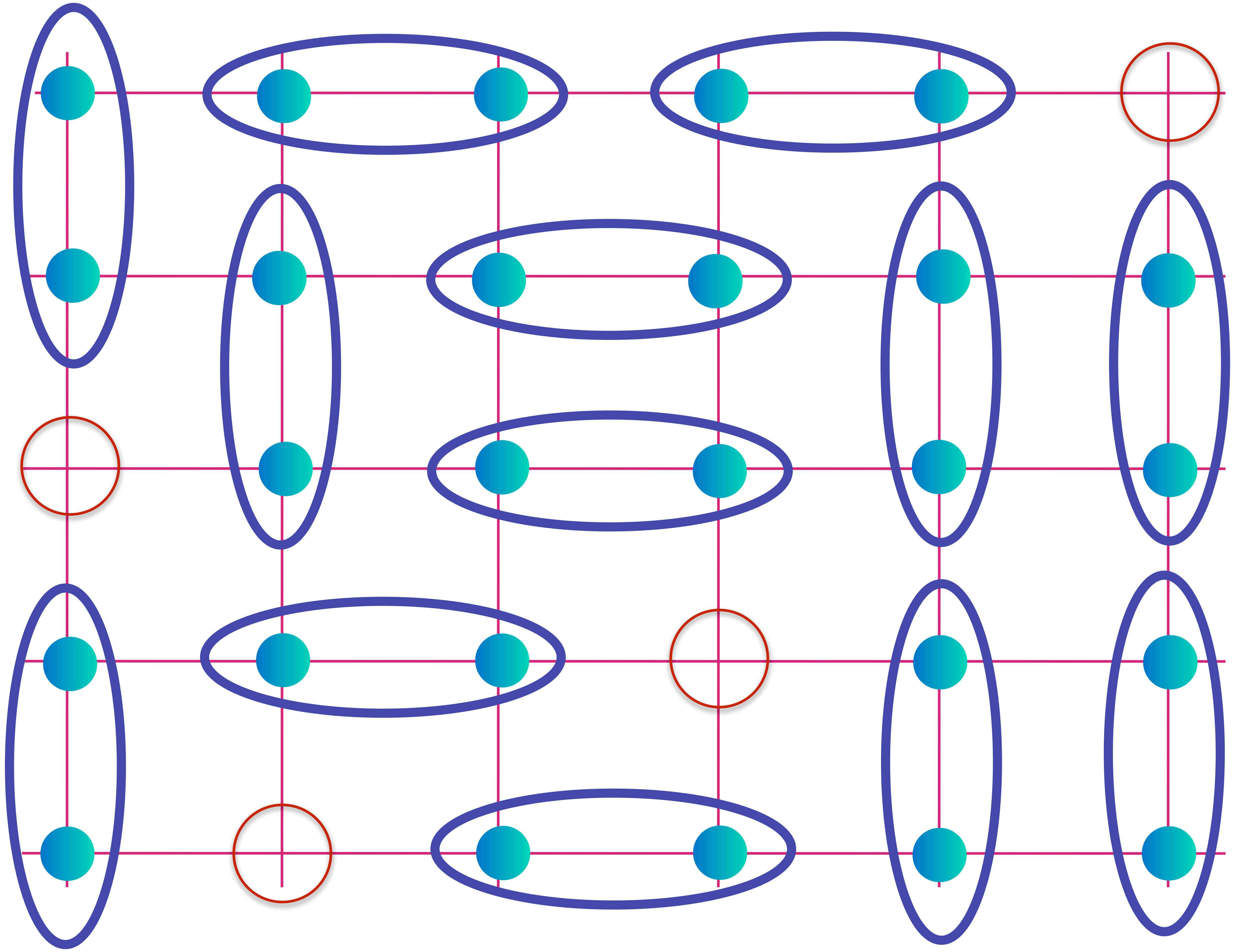

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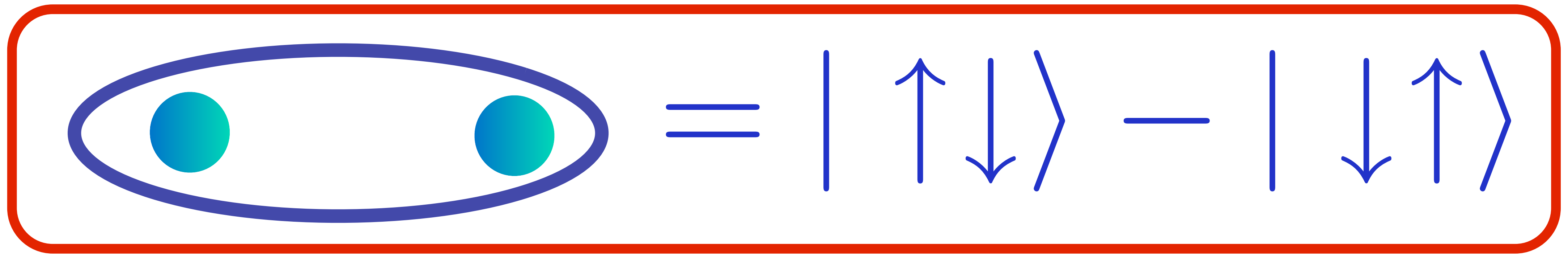

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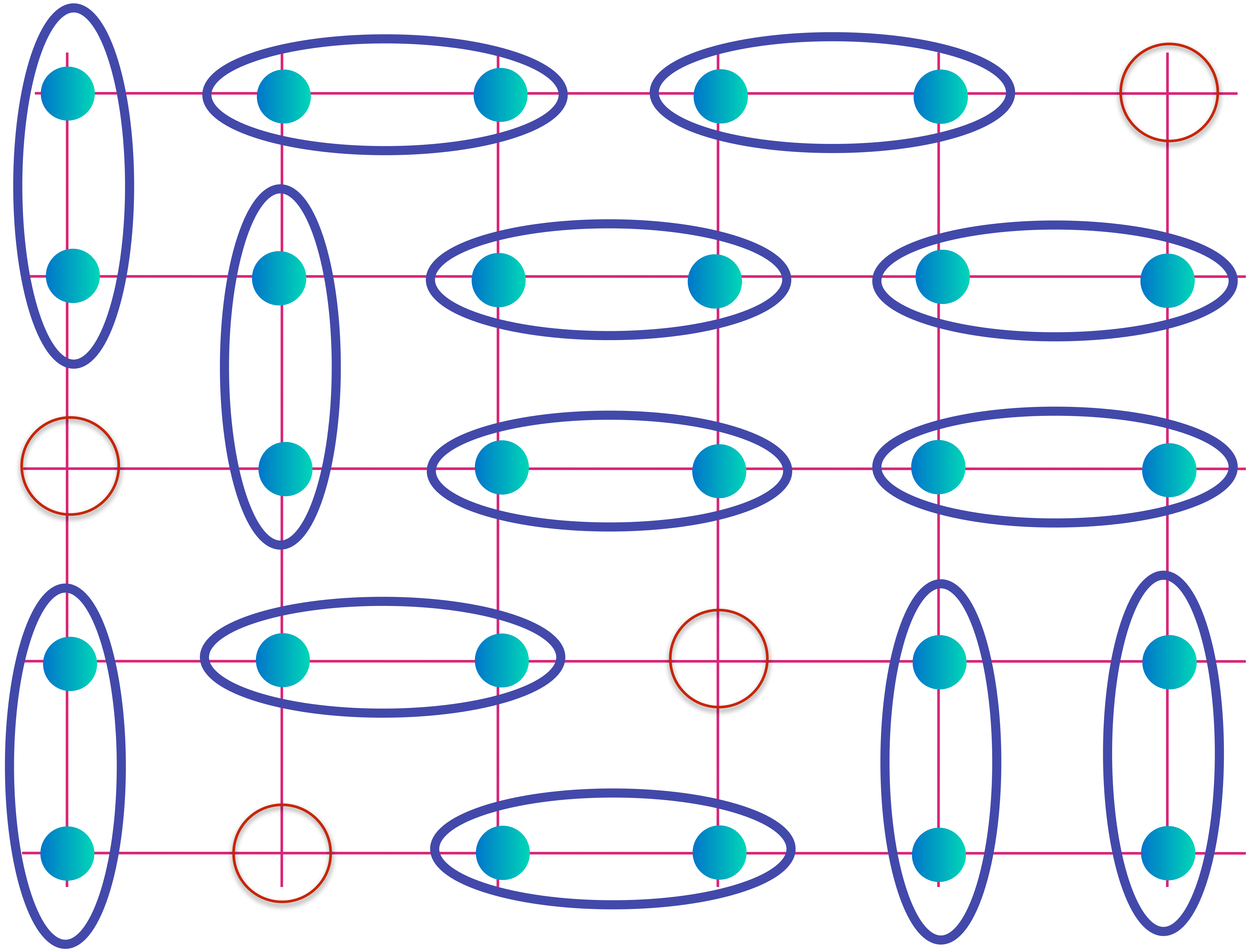

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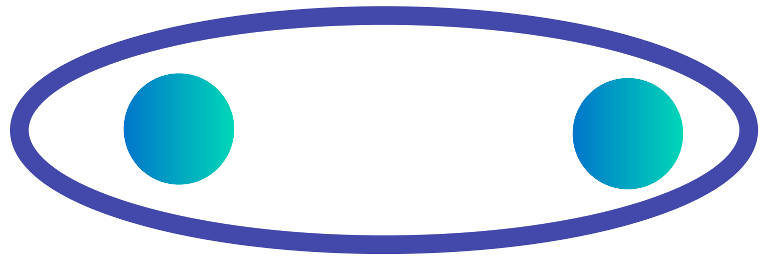 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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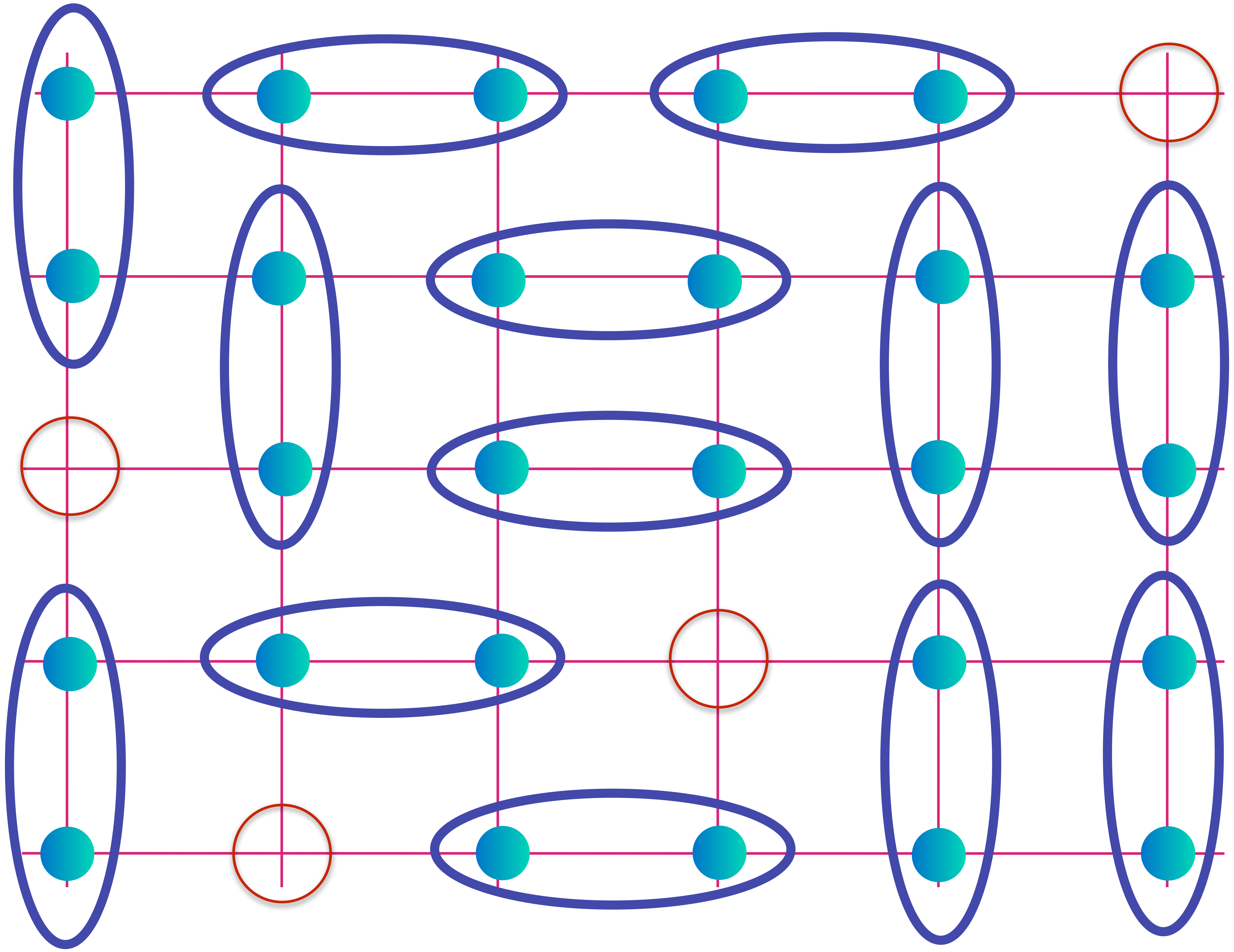

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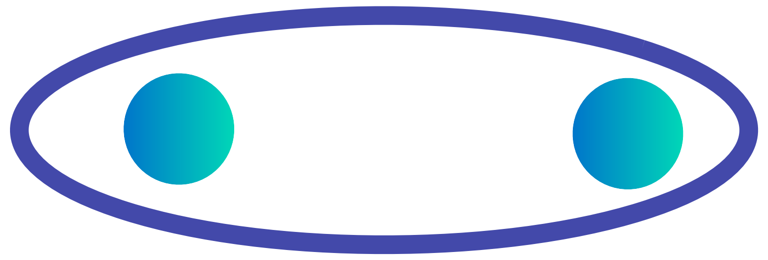

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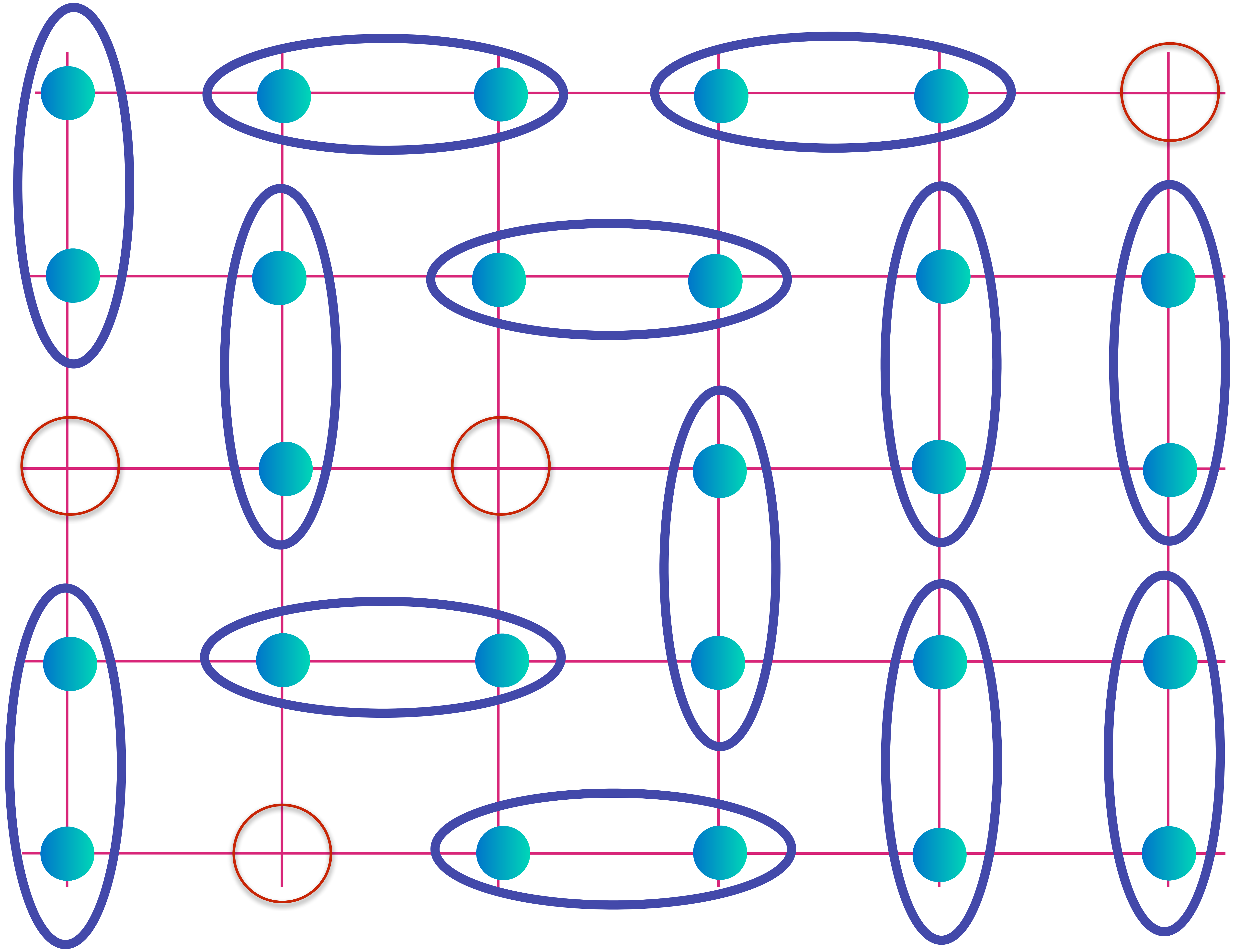

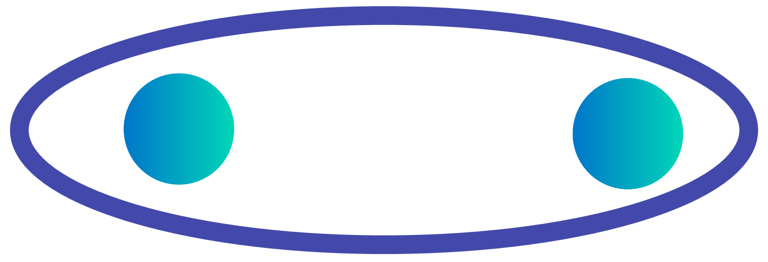 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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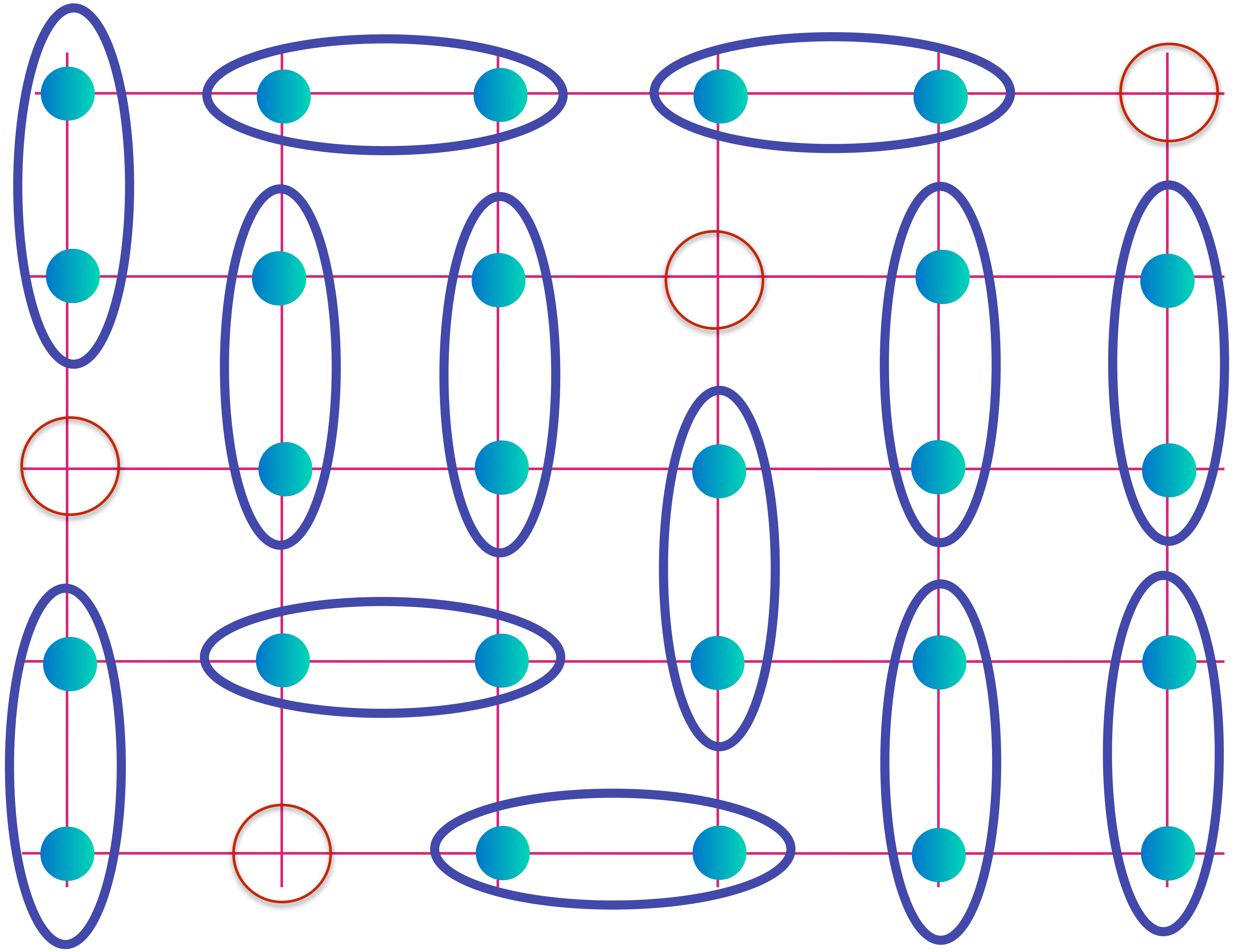

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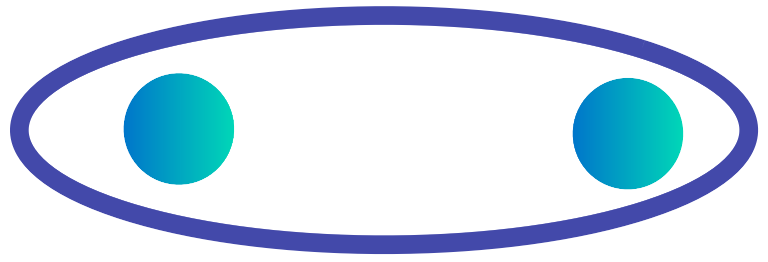 $= | \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle$

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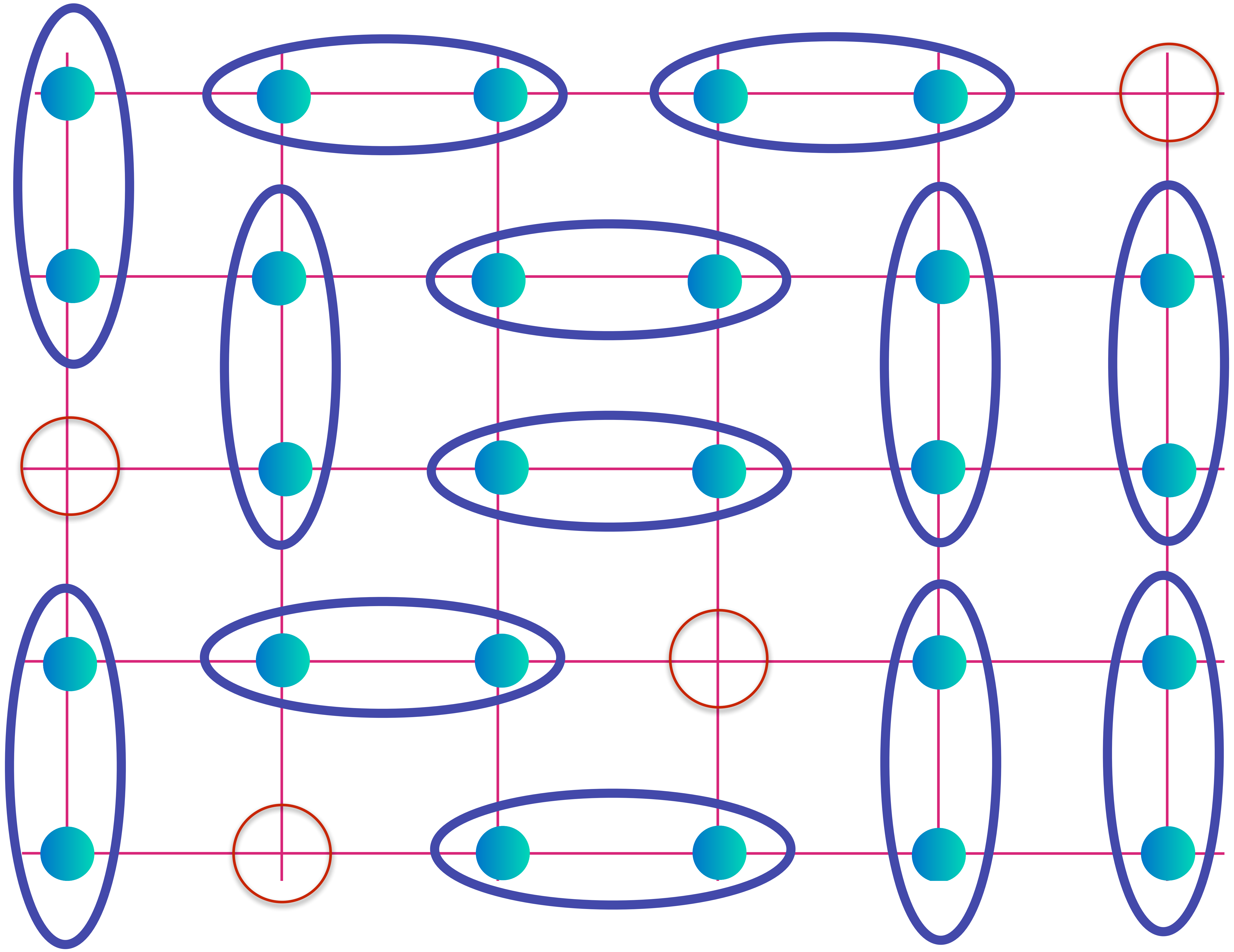

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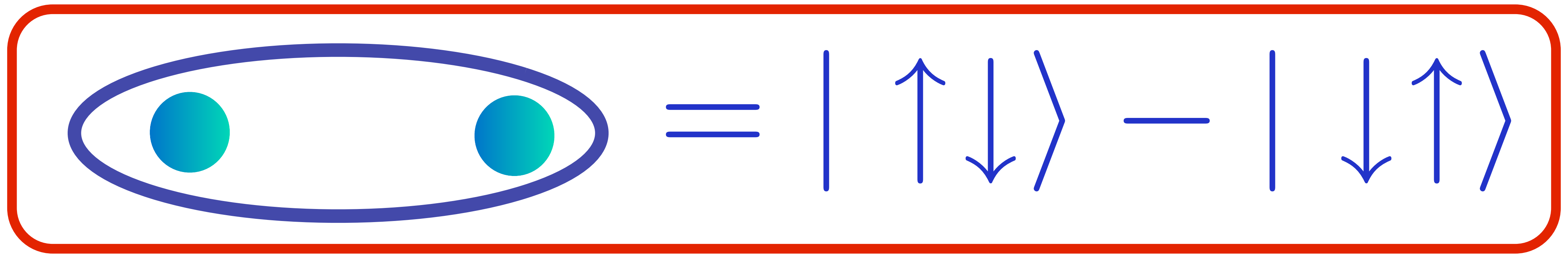

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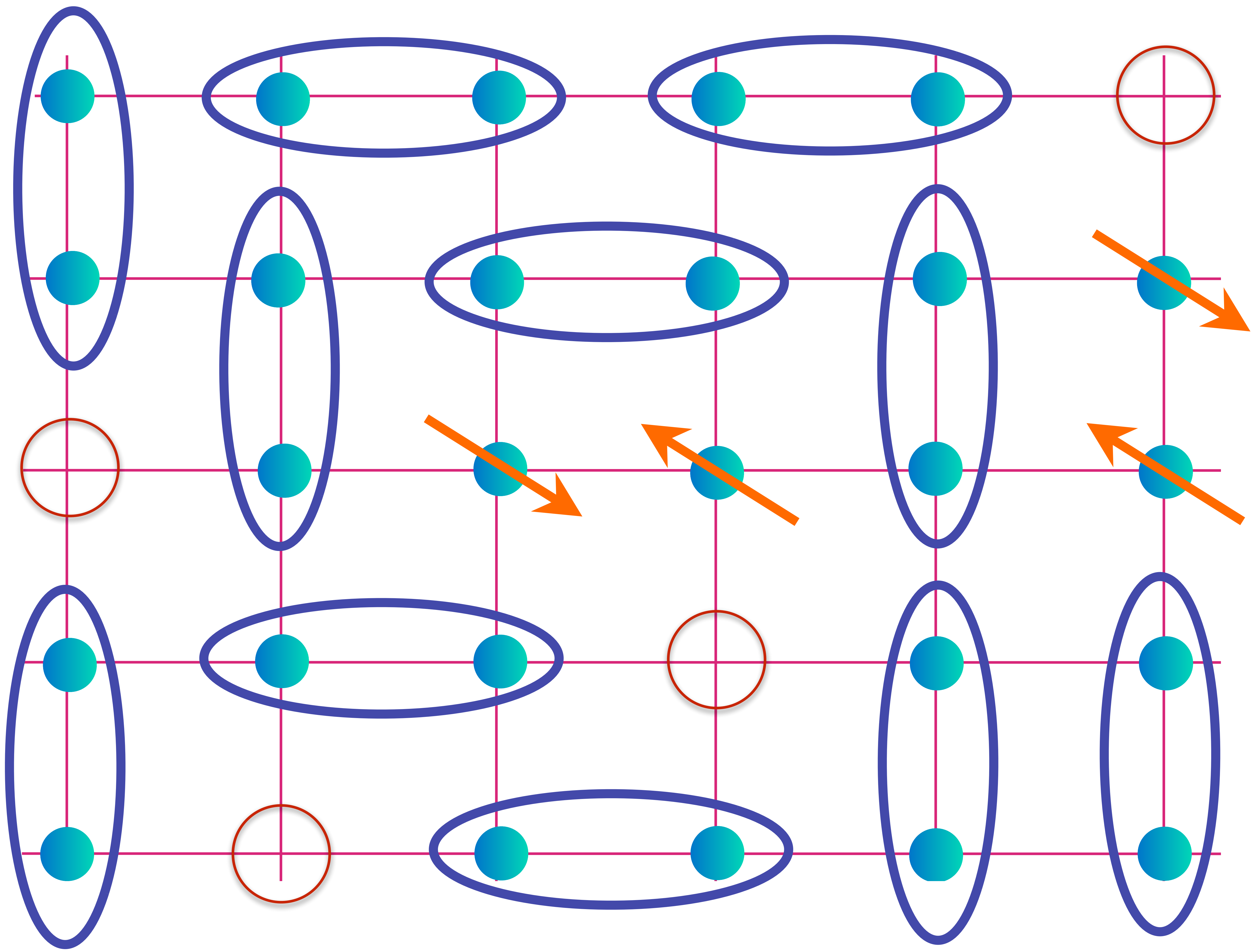
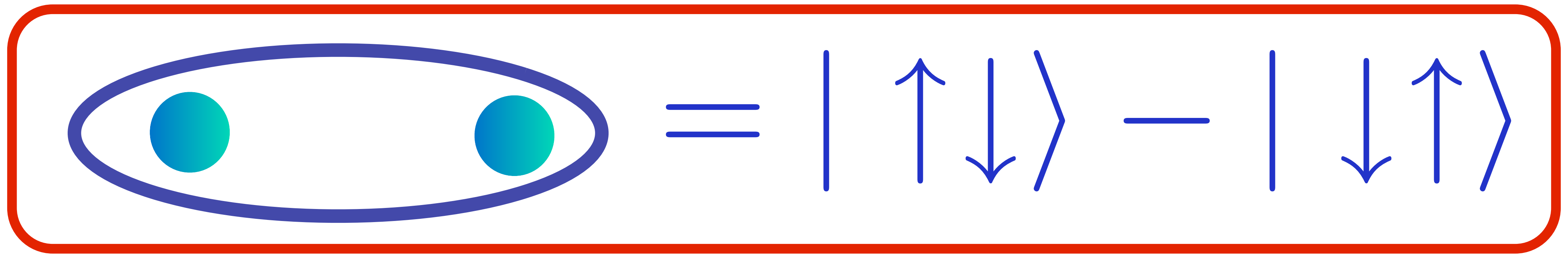


Holon metal

Spin liquid with density p of spinless, charge +e “holons” and charge 0, spin-1/2 “spinons”



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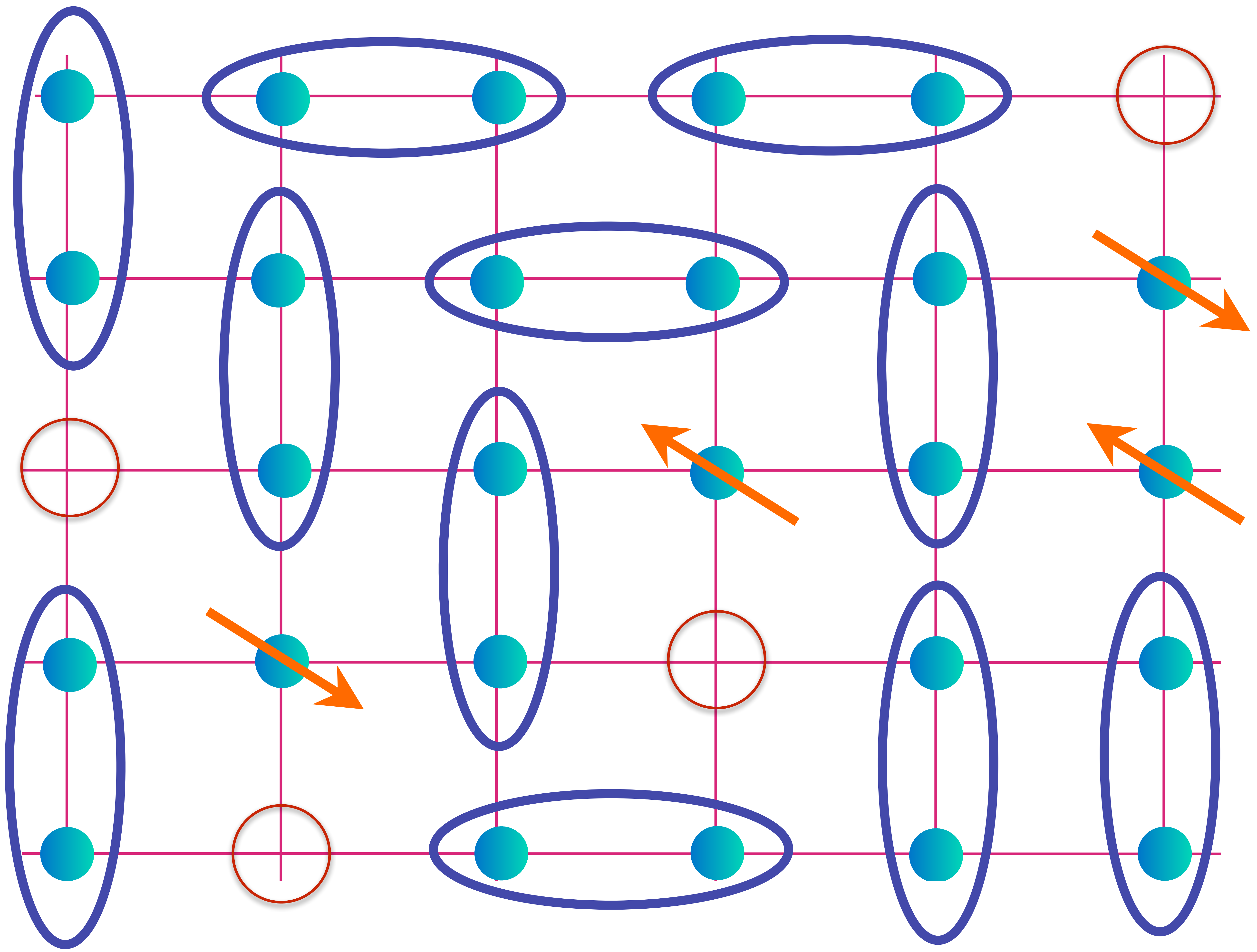

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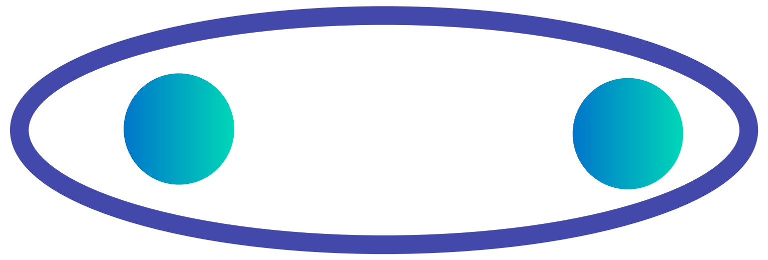

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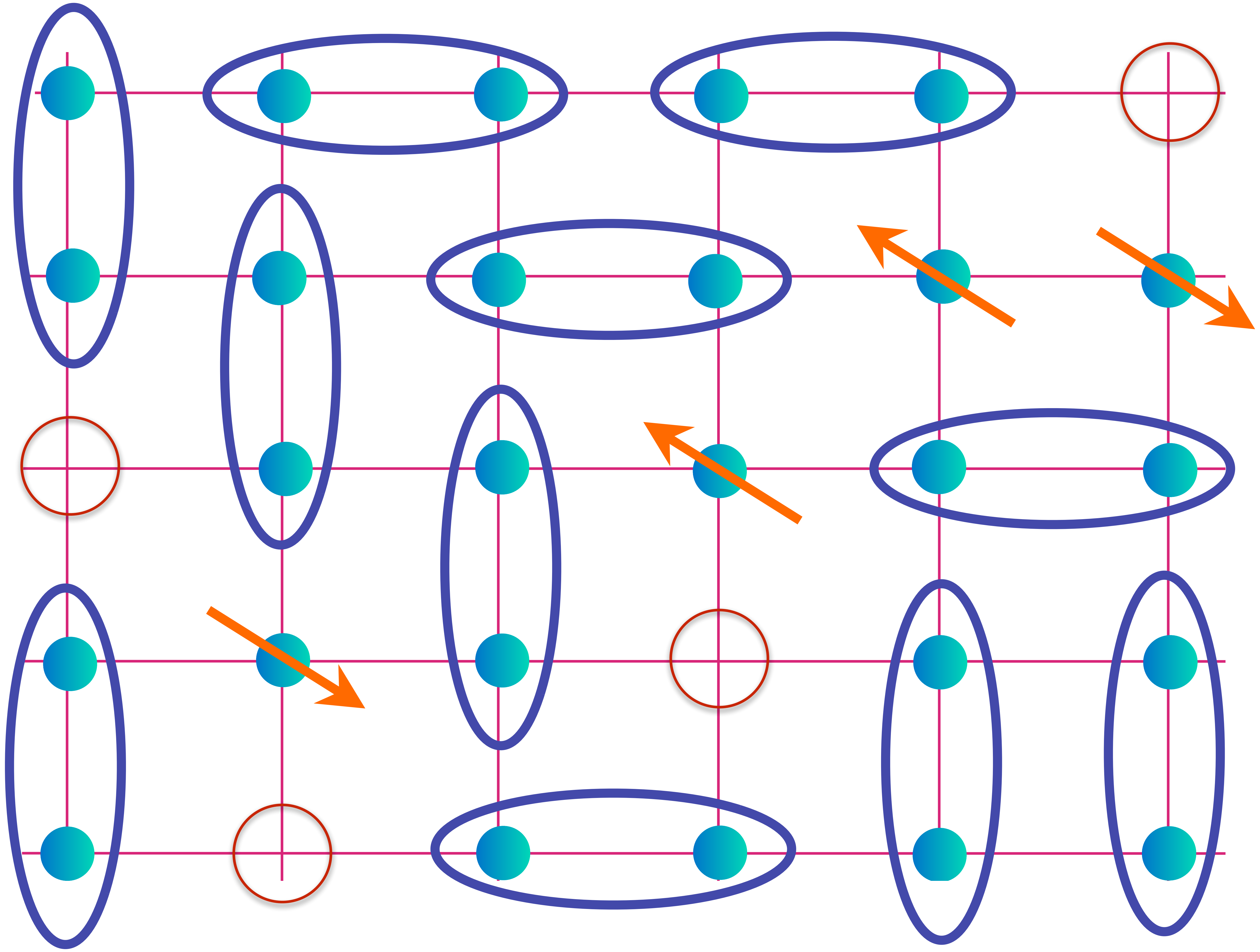

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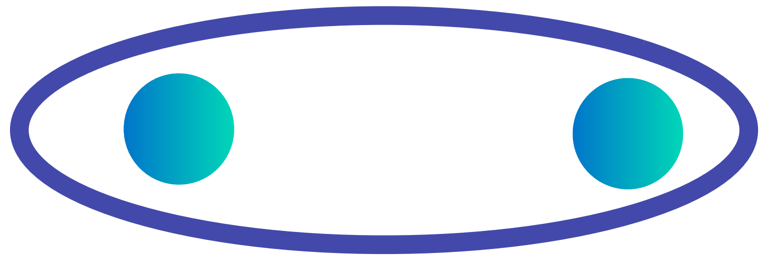

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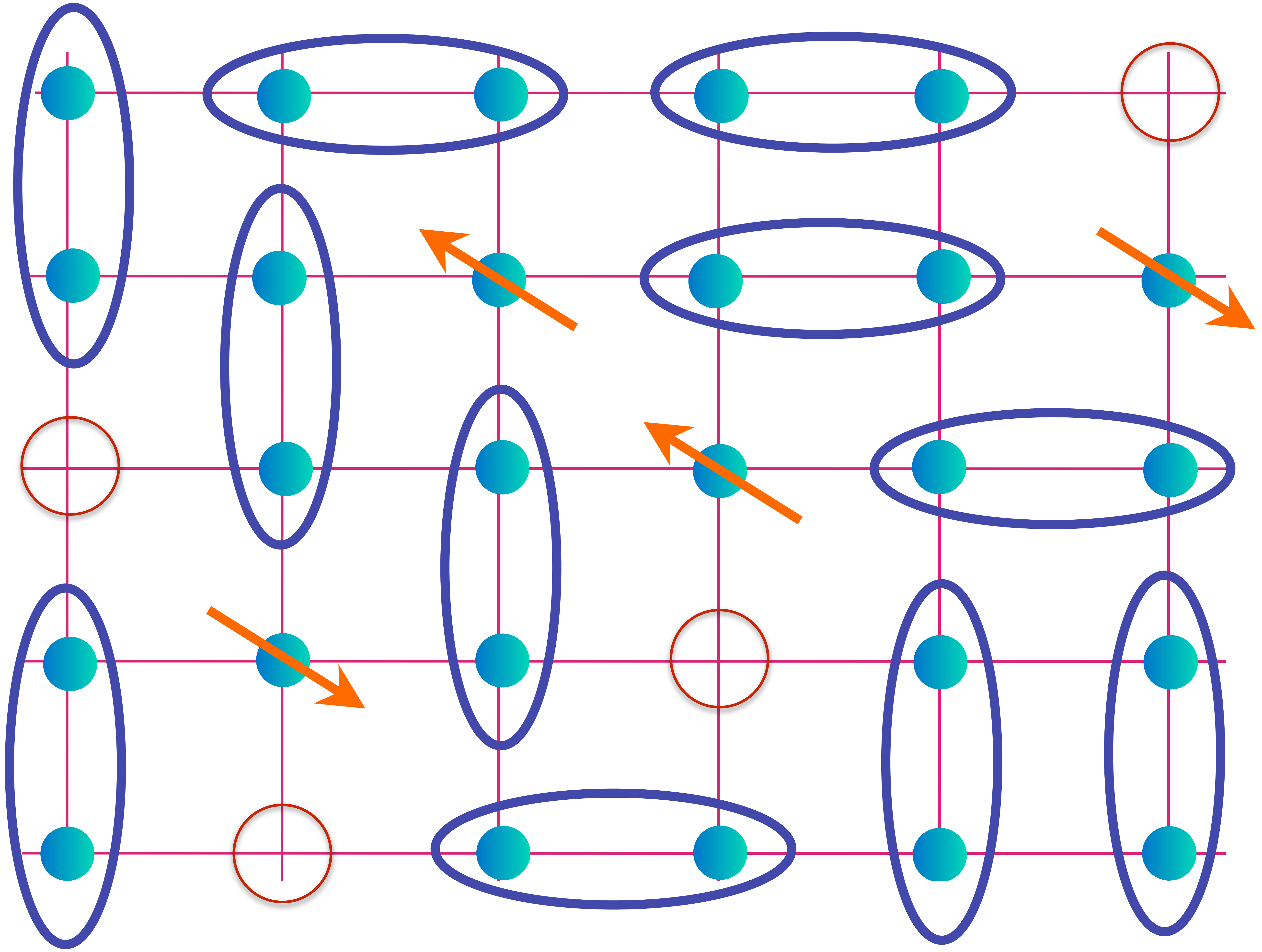

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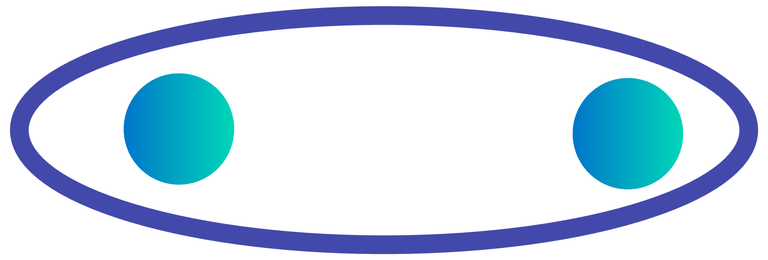 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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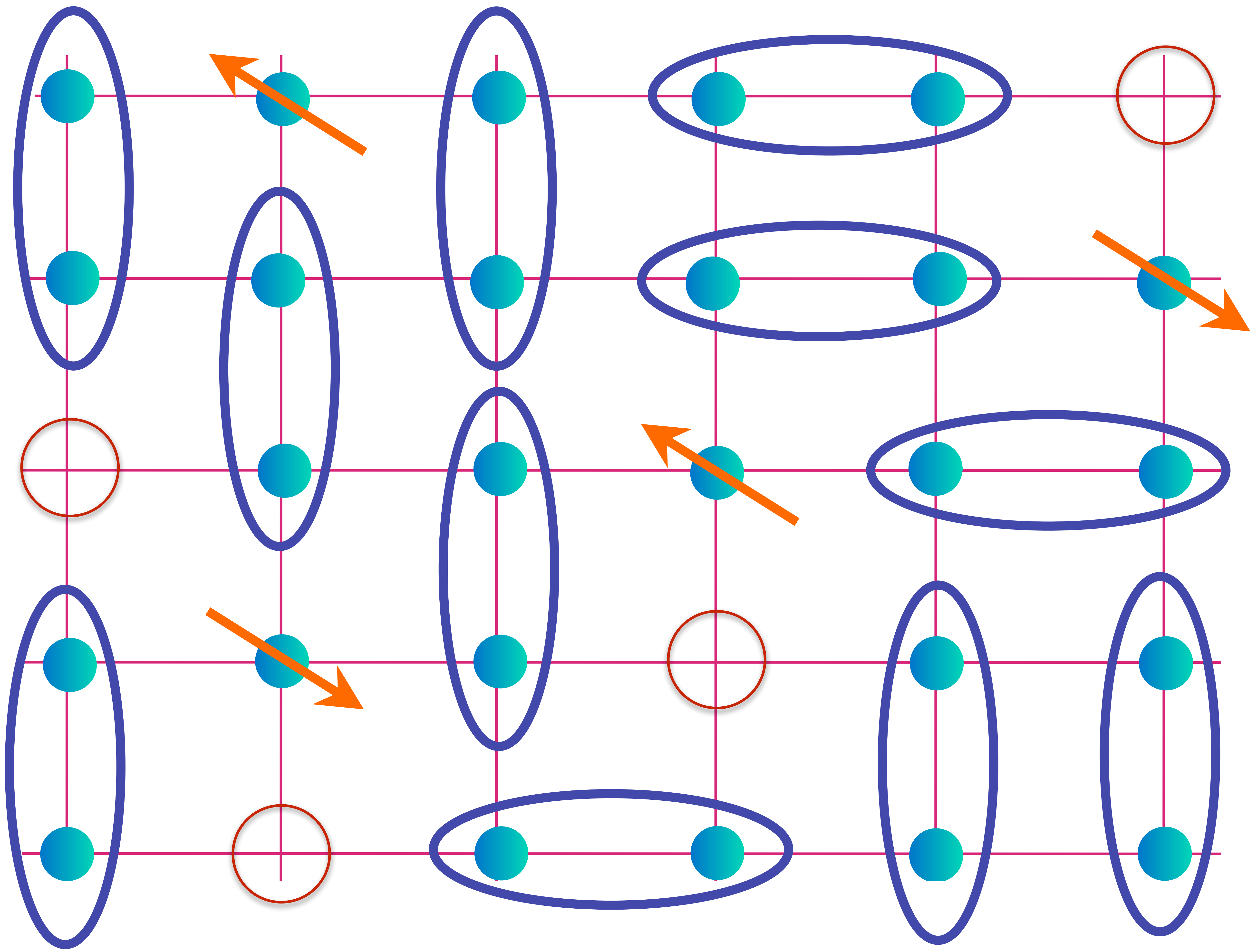

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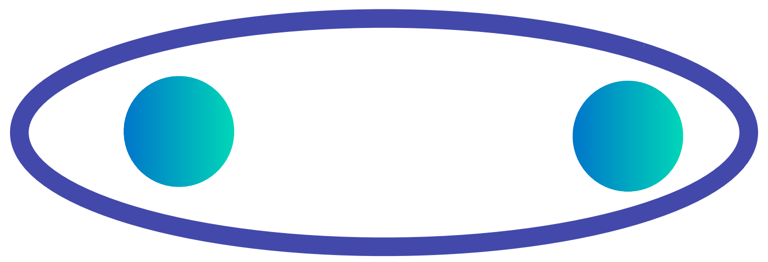

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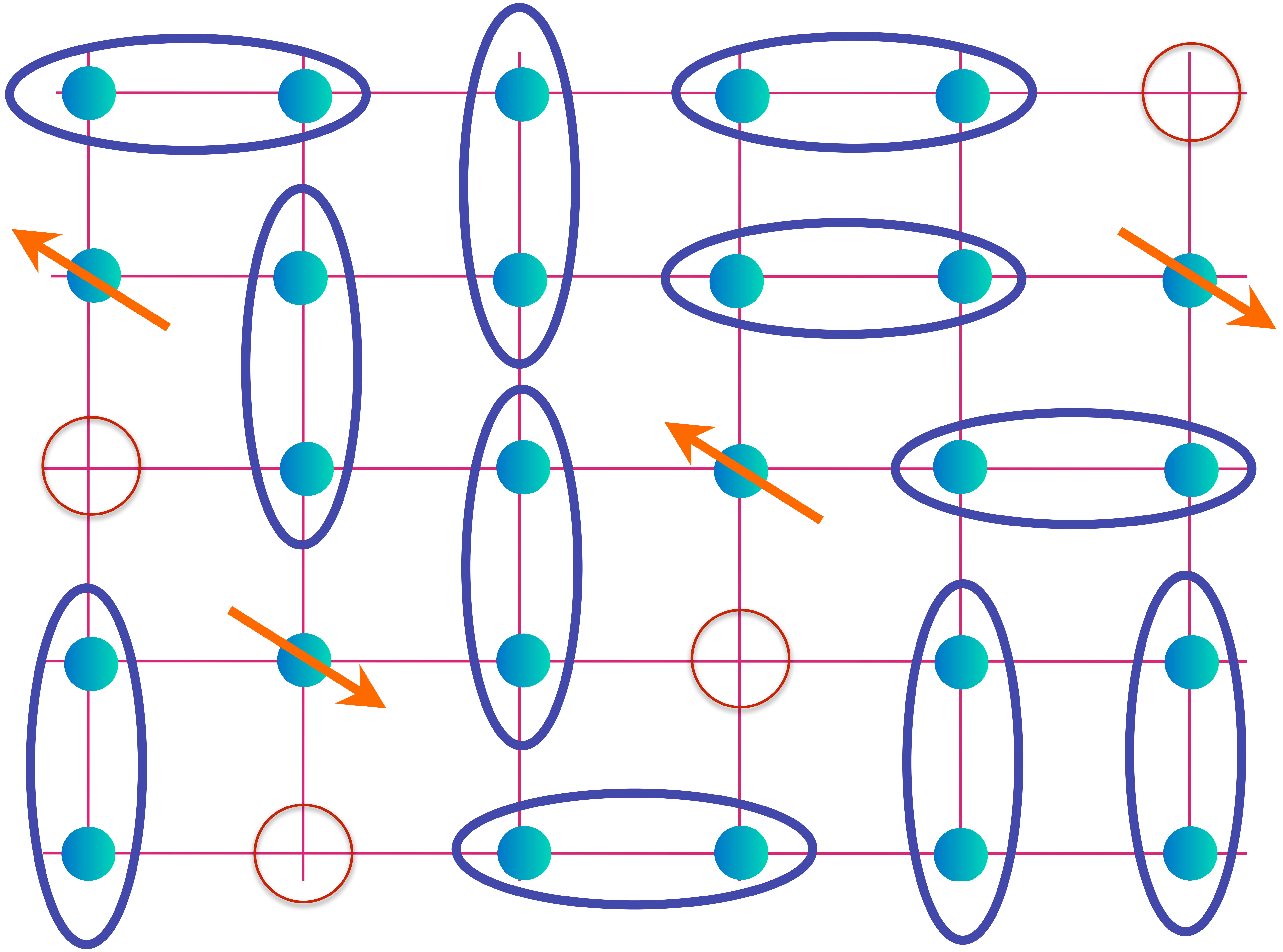

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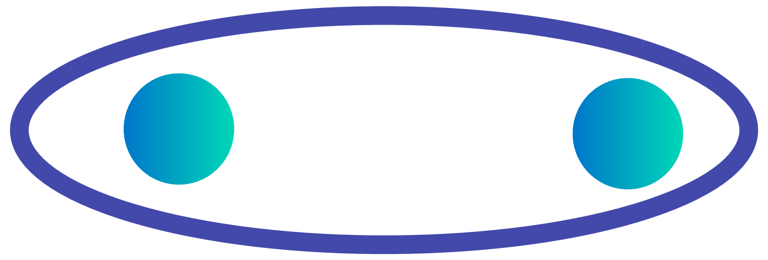

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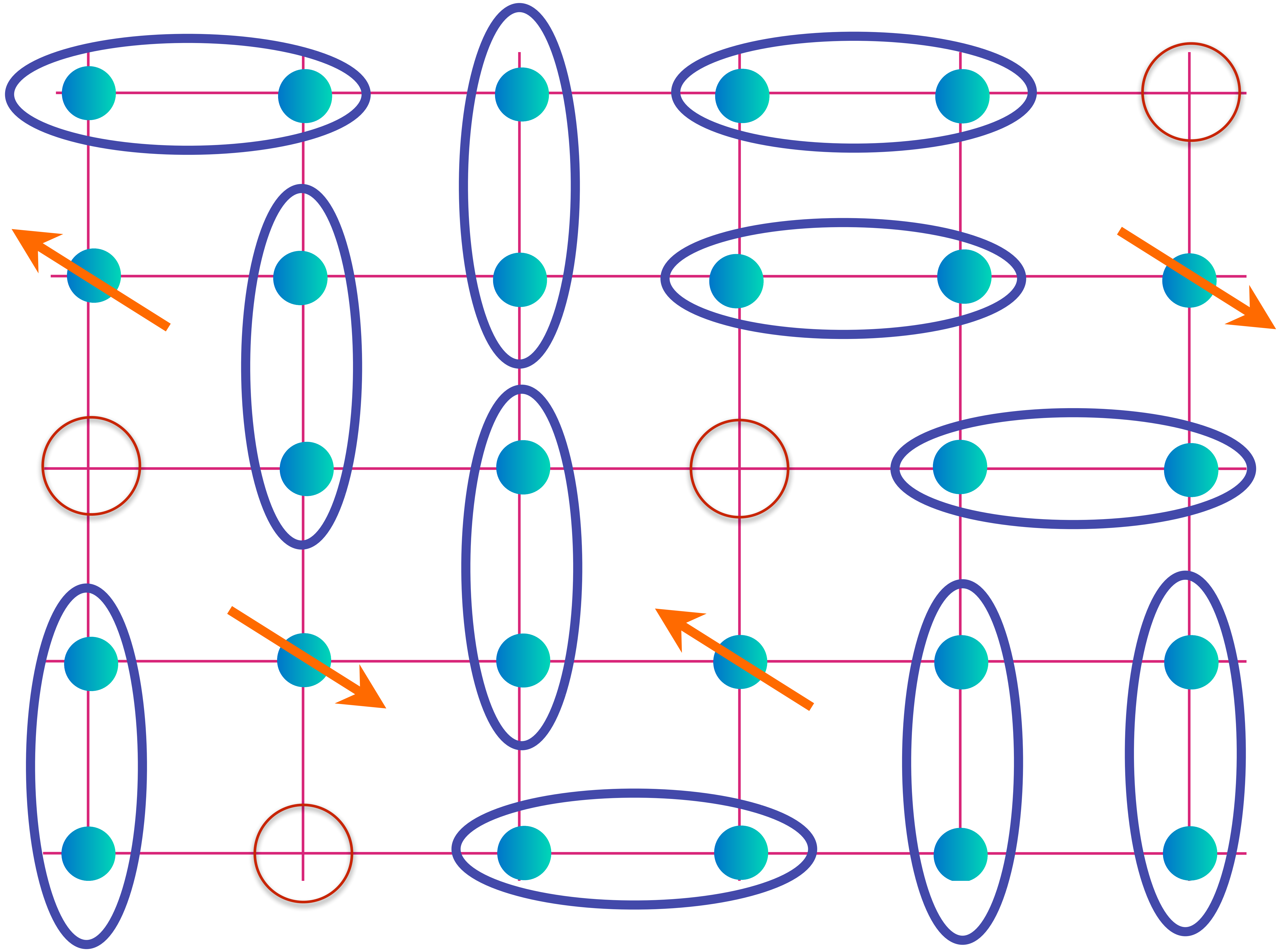

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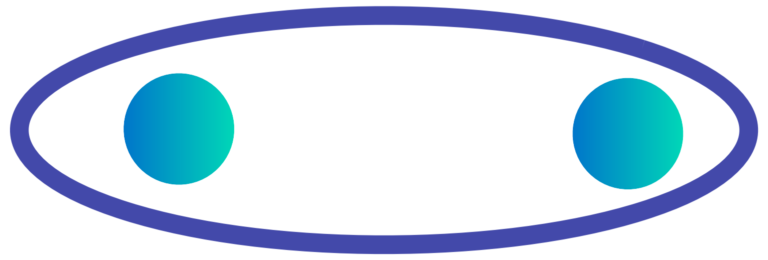

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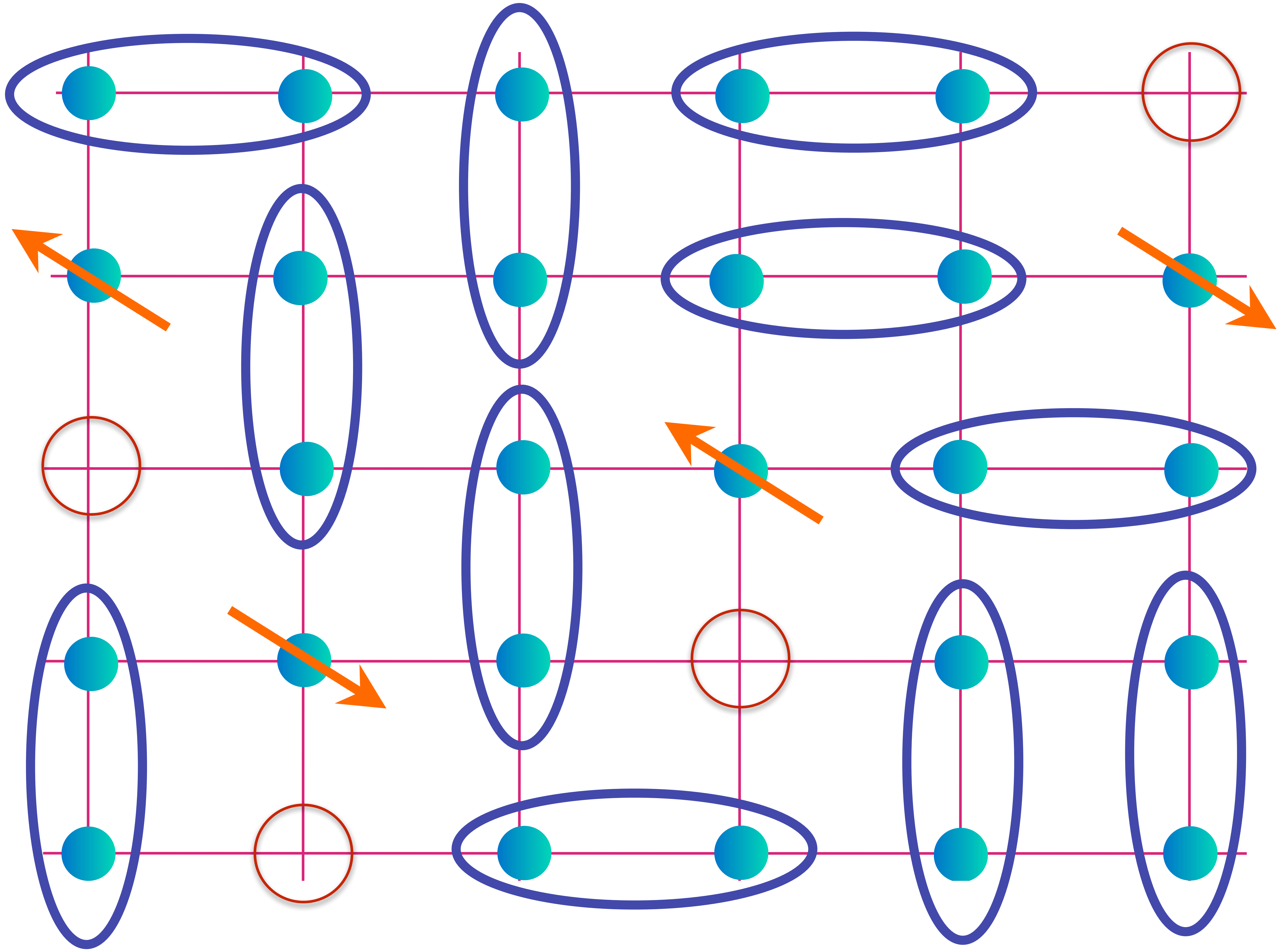
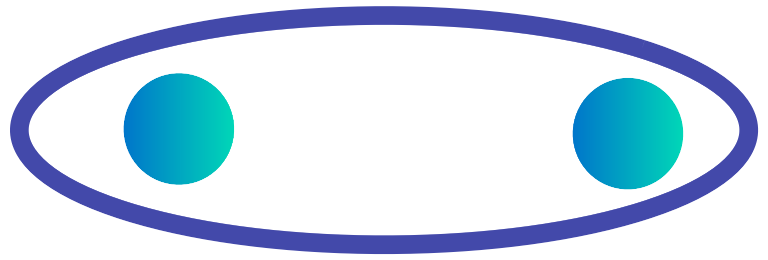
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

The dance of electrons on Cu atoms in YBCO



Holon metal

Spin liquid with density p of spinless, charge +e “holons” and charge 0, spin-1/2 “spinons”

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
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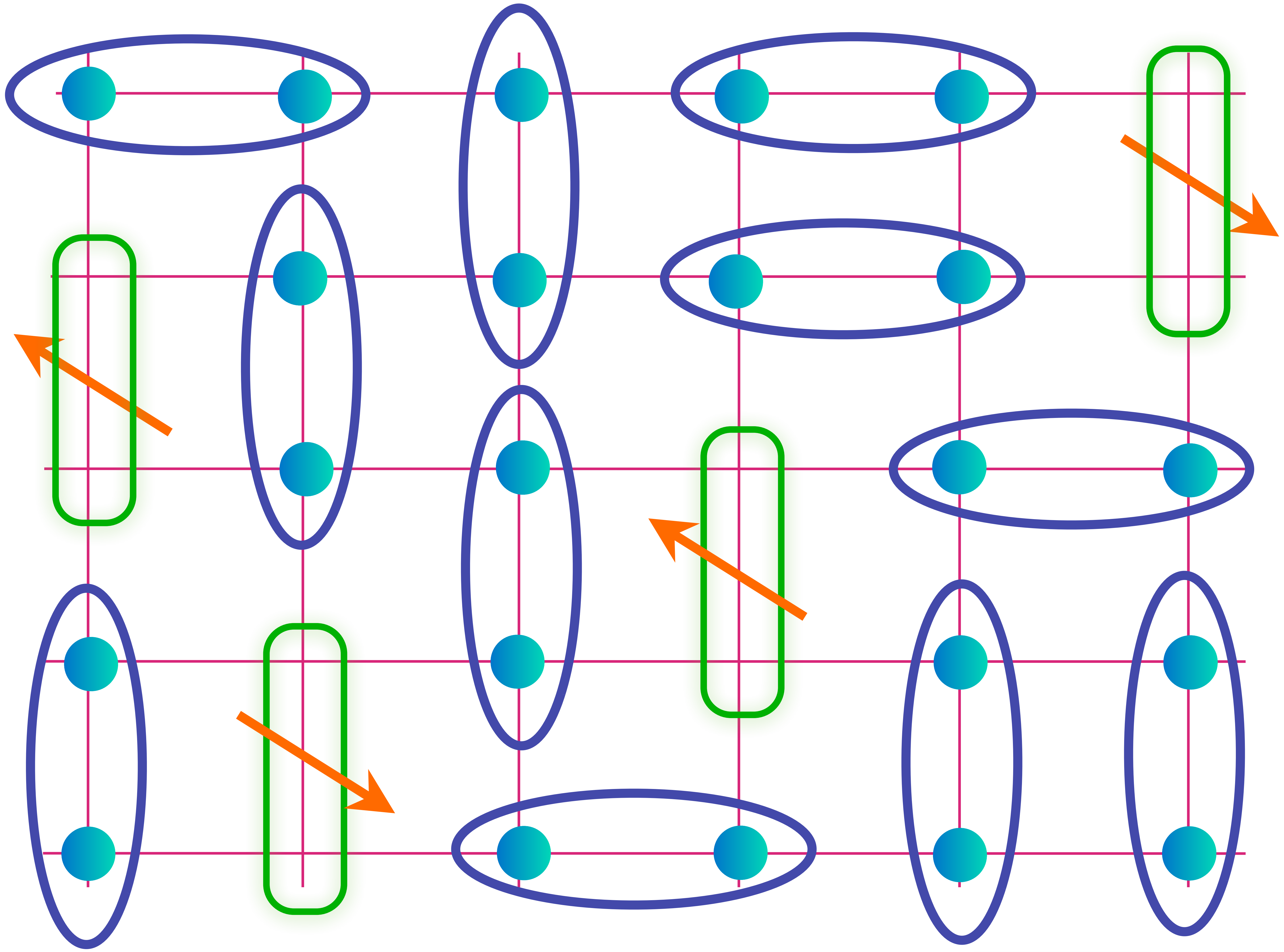
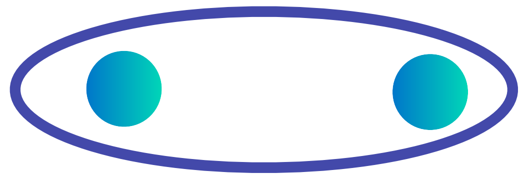
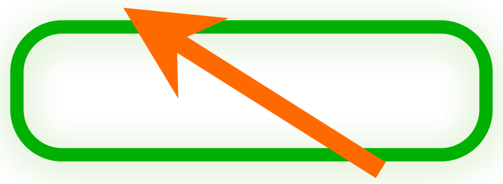


FL*

Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996); T. Senthil, S. Sachdev, and M. Vojta, PRL **90**, 216403 (2003)

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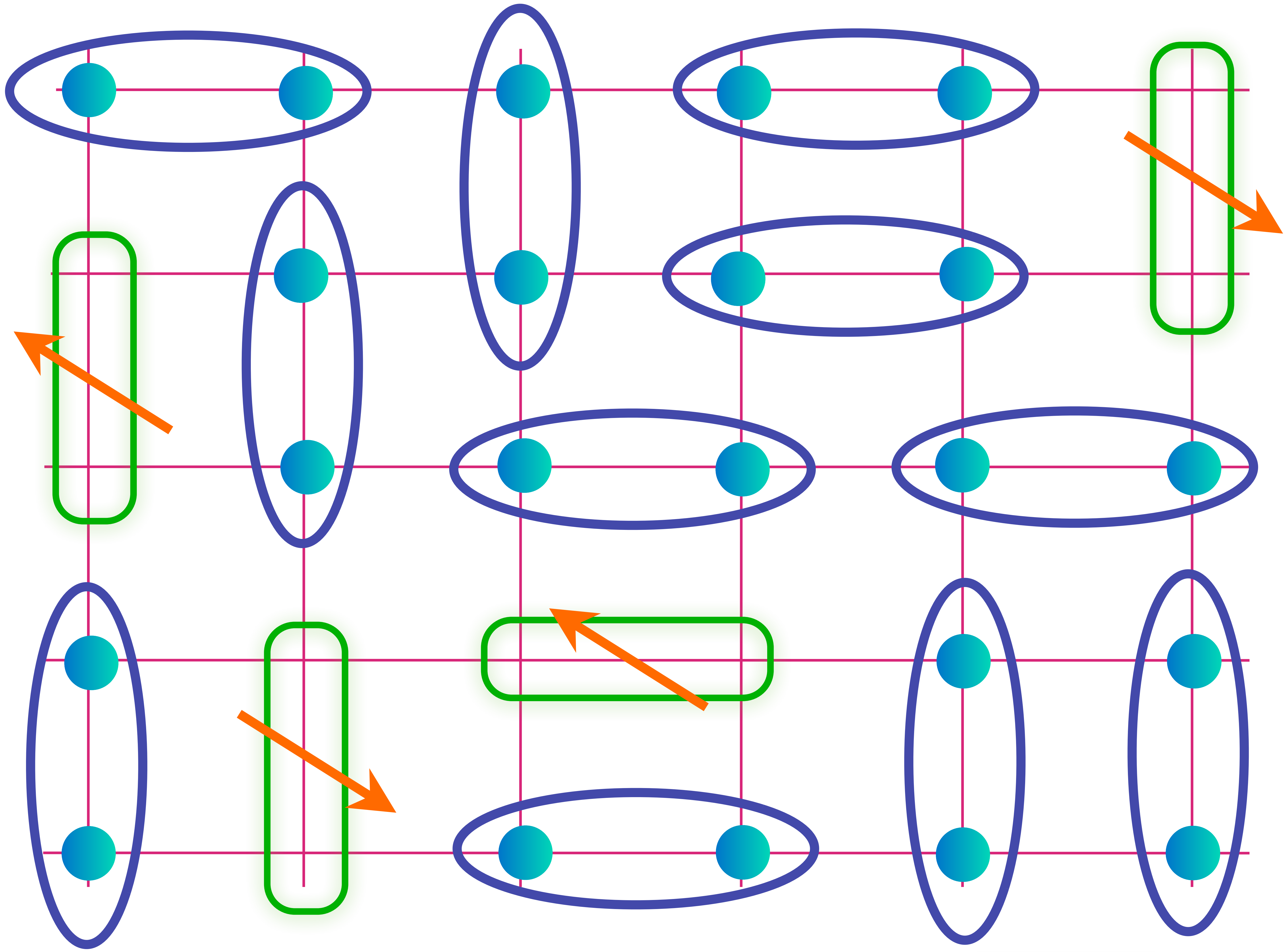

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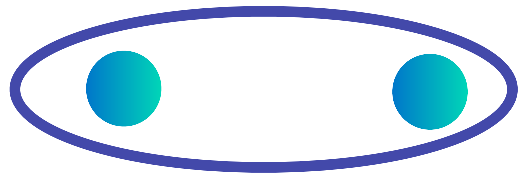
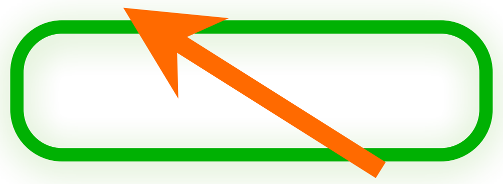
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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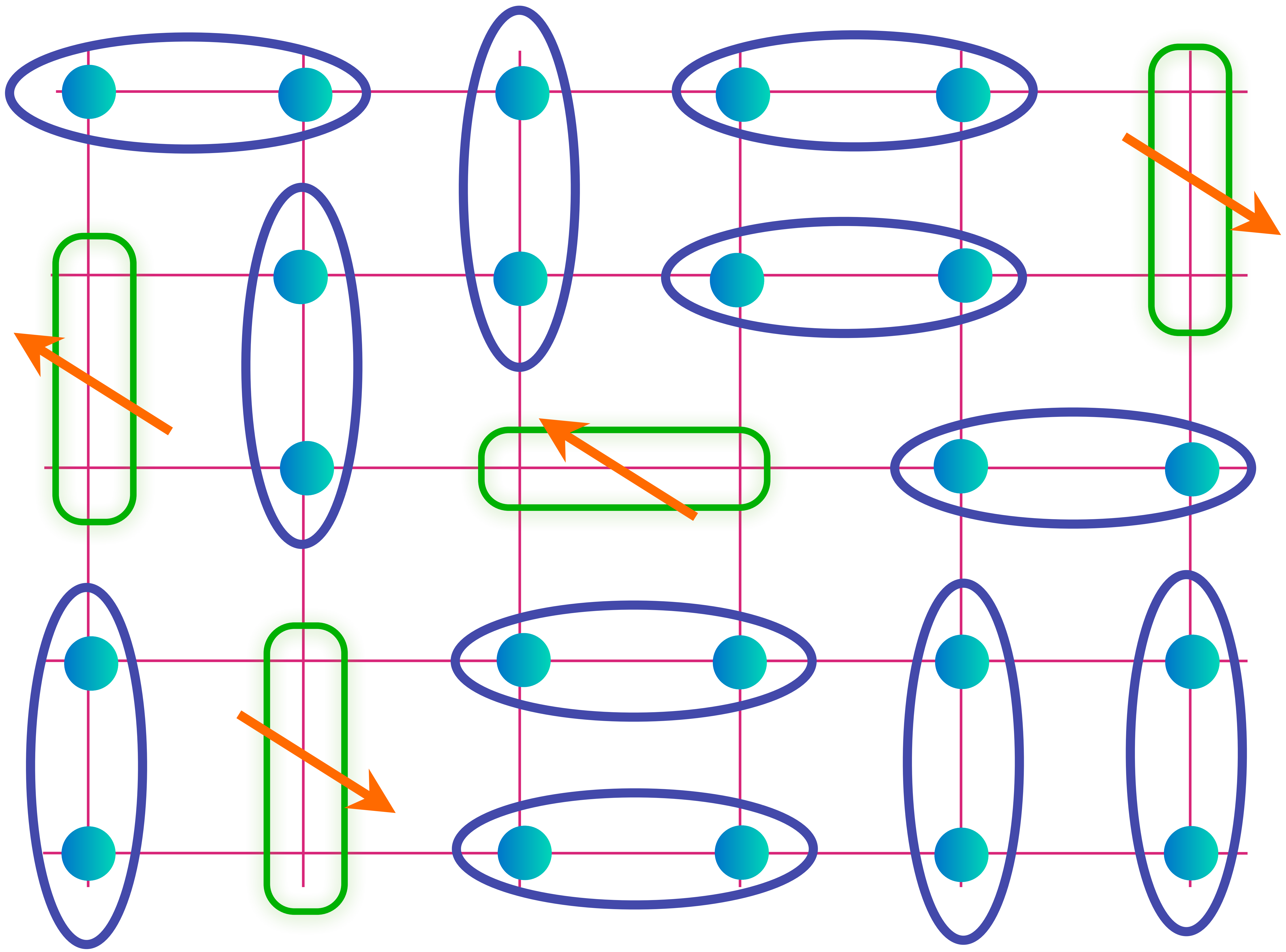

FL*

Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

$= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996); T. Senthil, S. Sachdev, and M. Vojta, PRL **90**, 216403 (2003)

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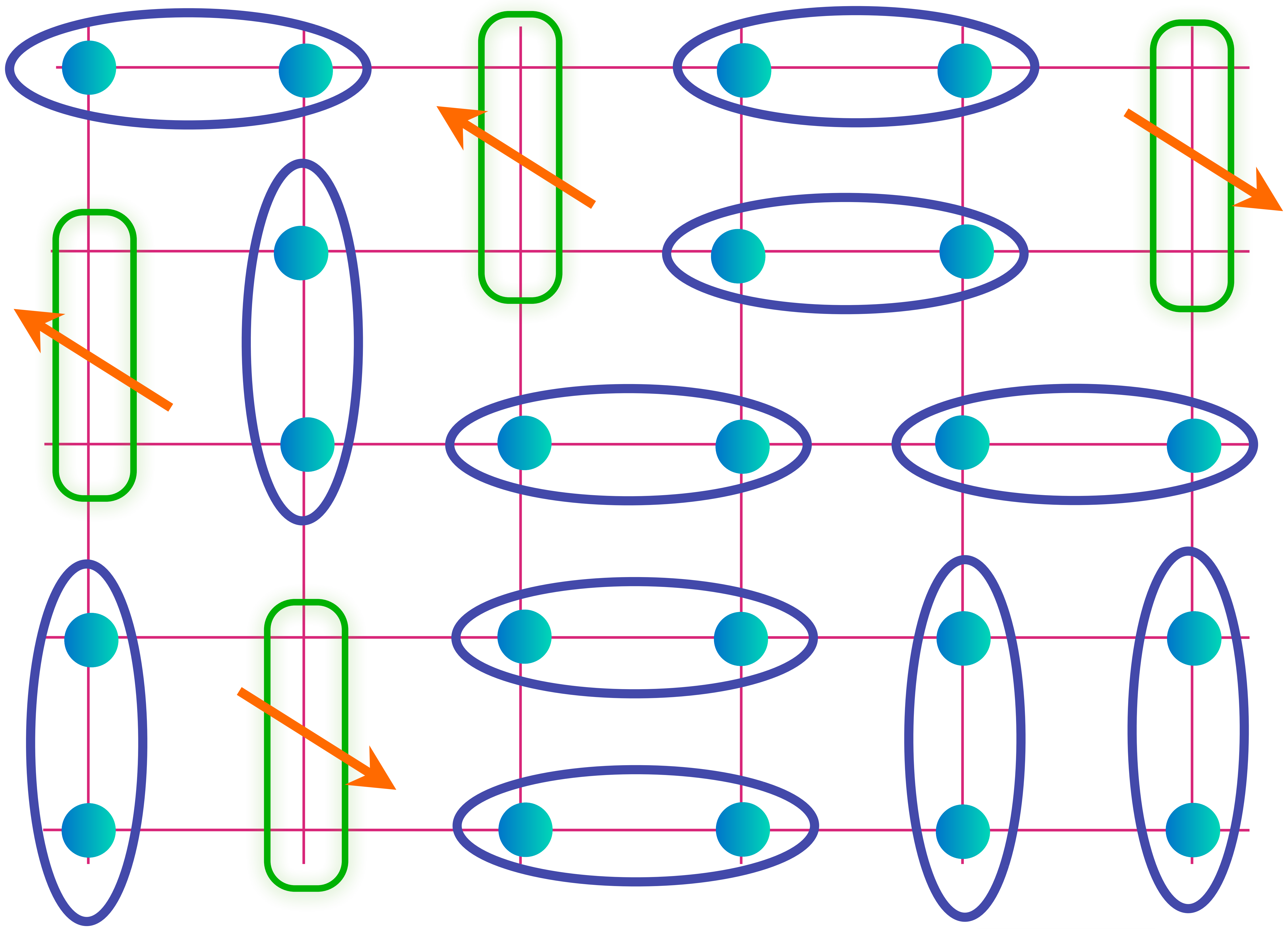

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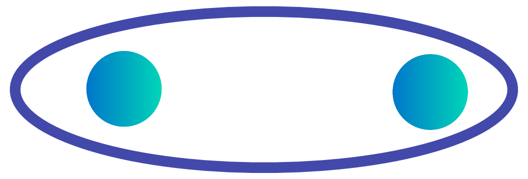
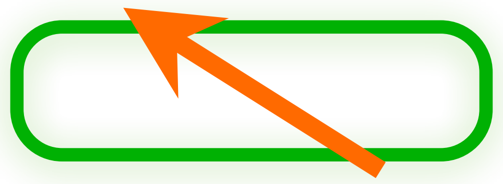
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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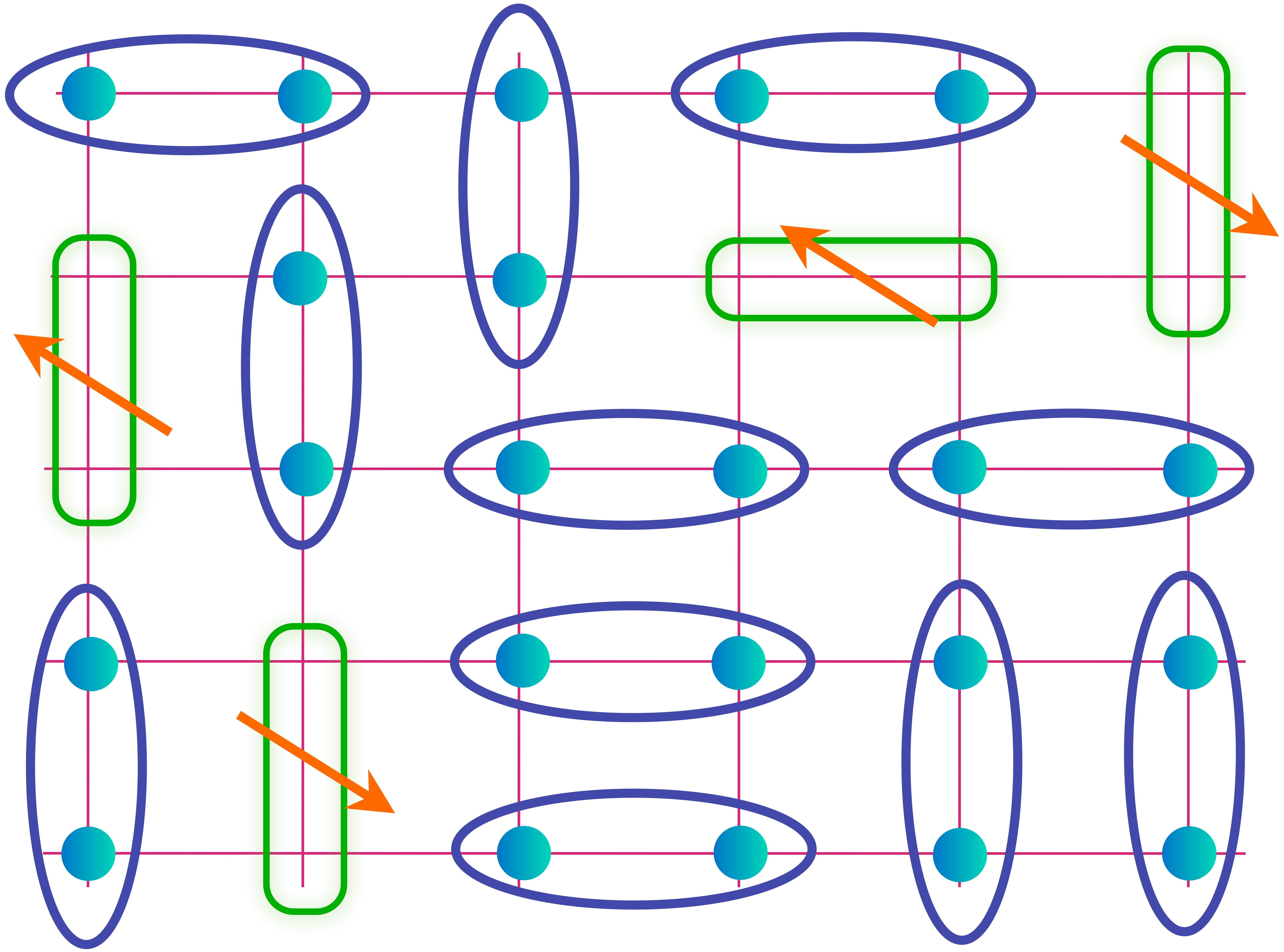

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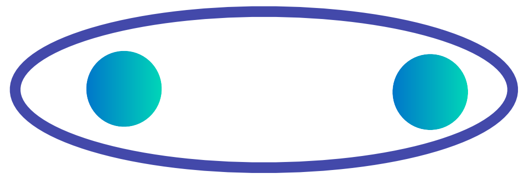
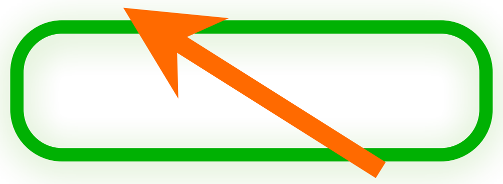
Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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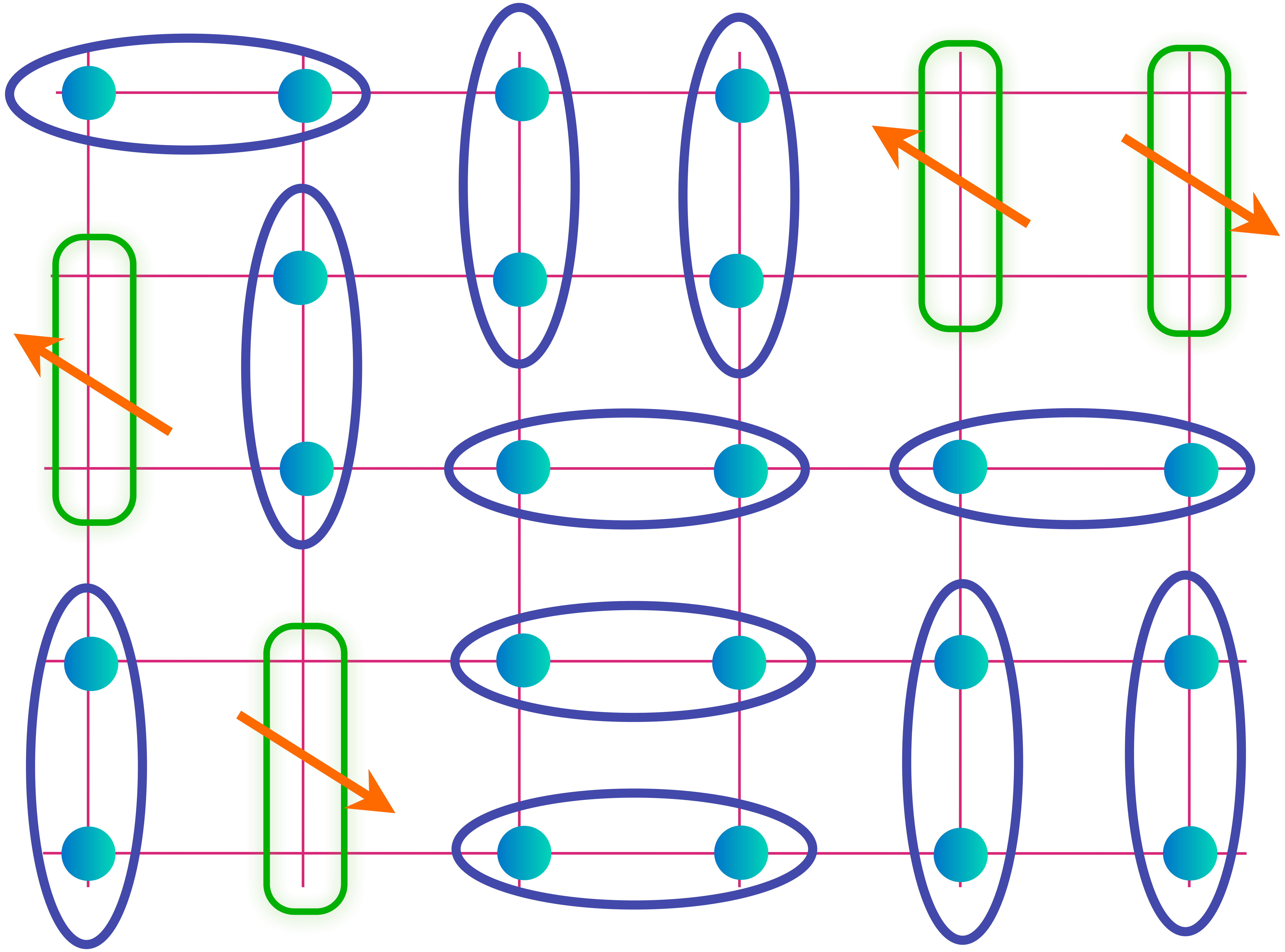

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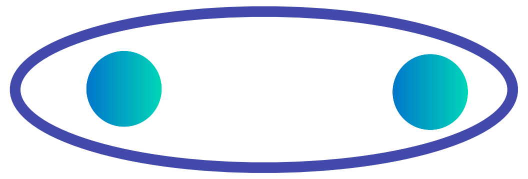
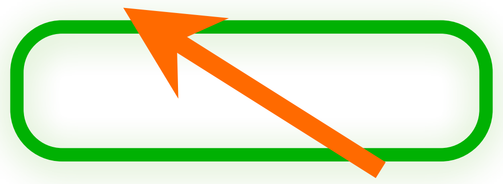
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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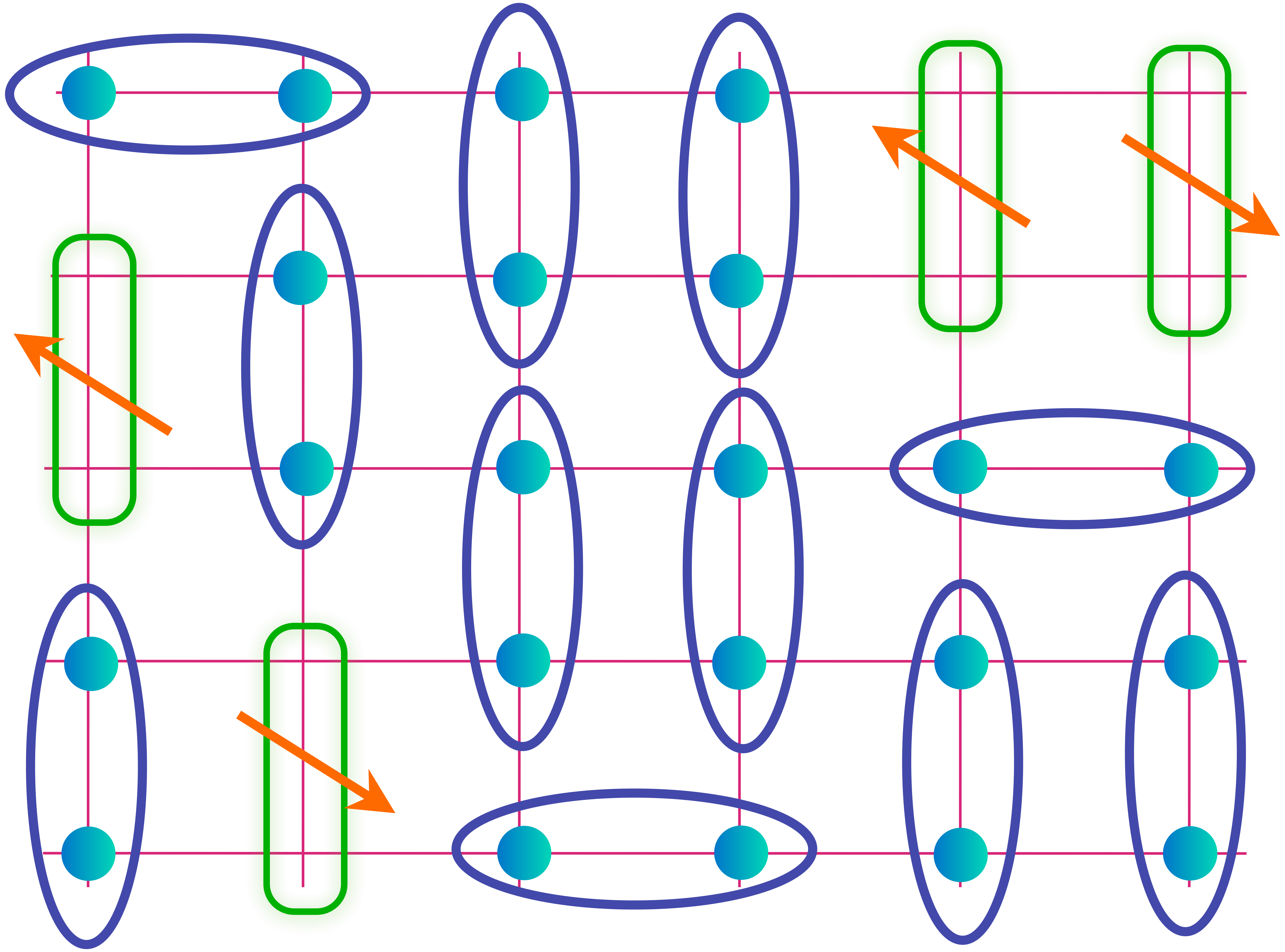

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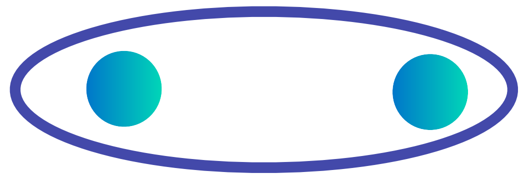
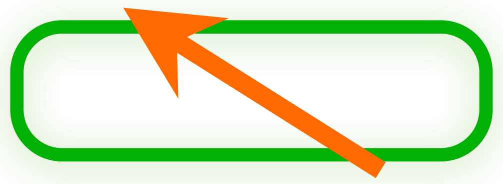
Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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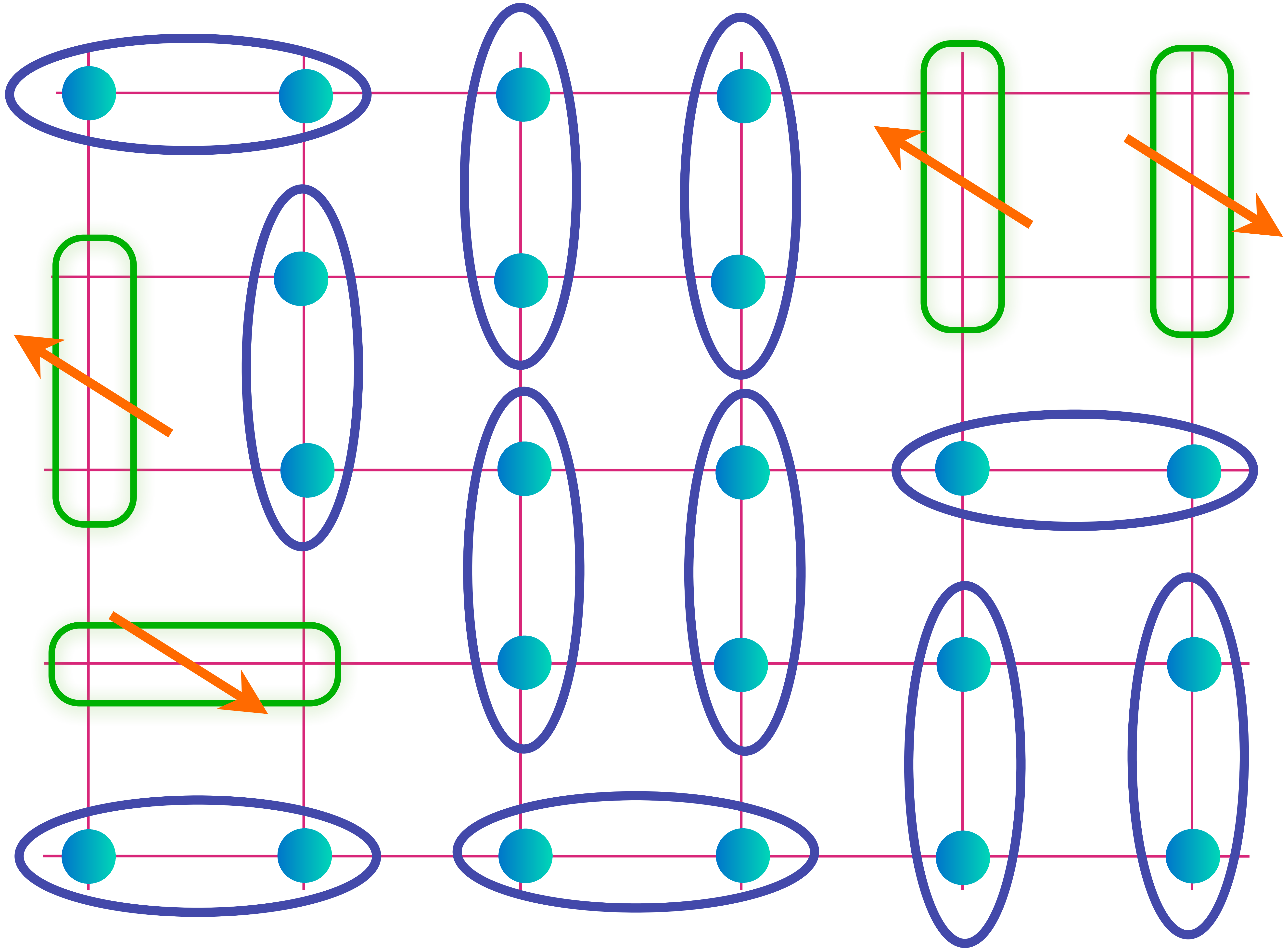

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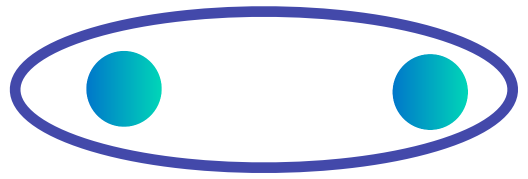
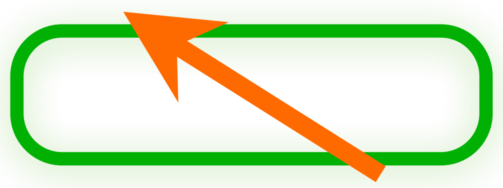


$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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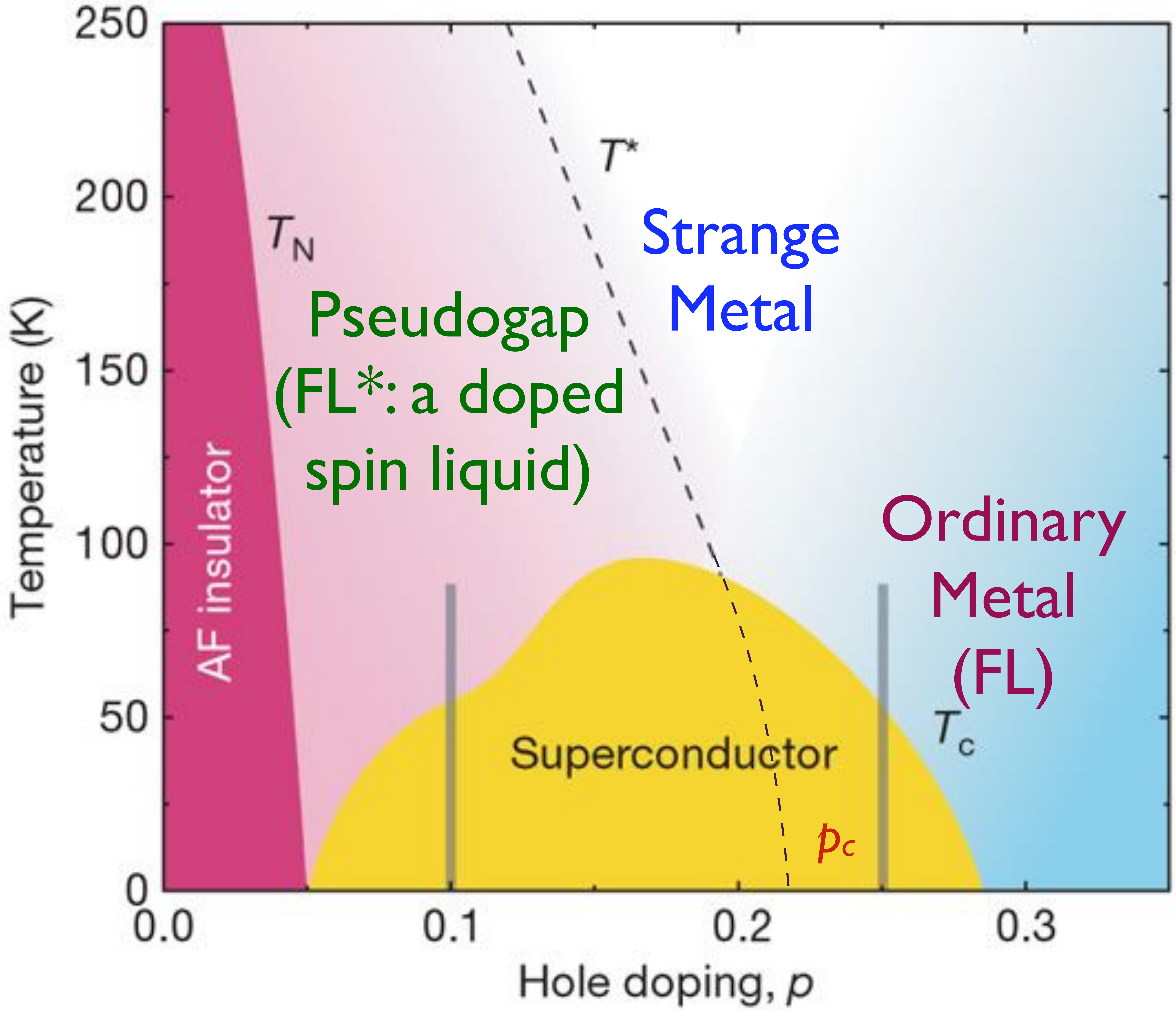
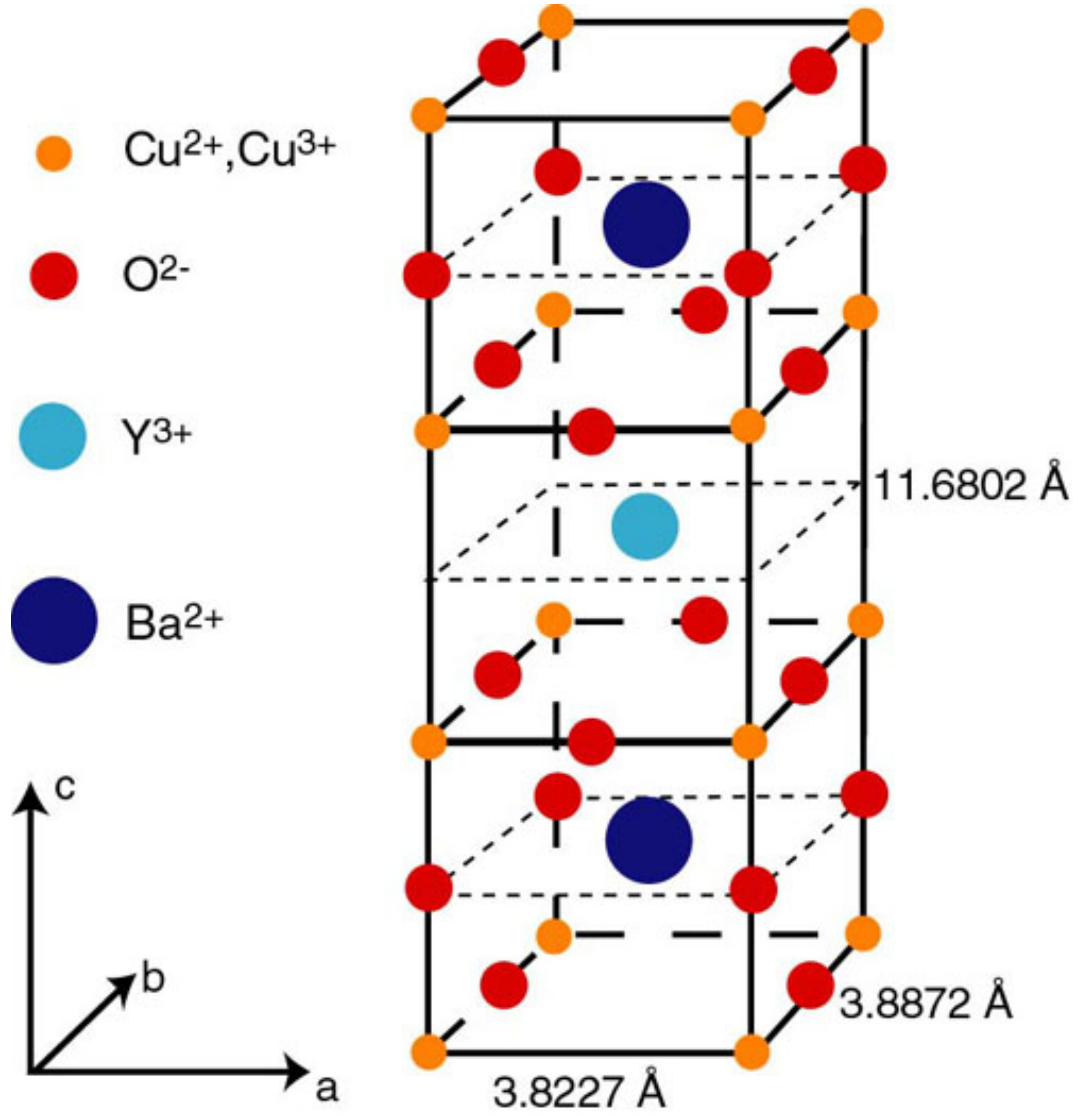
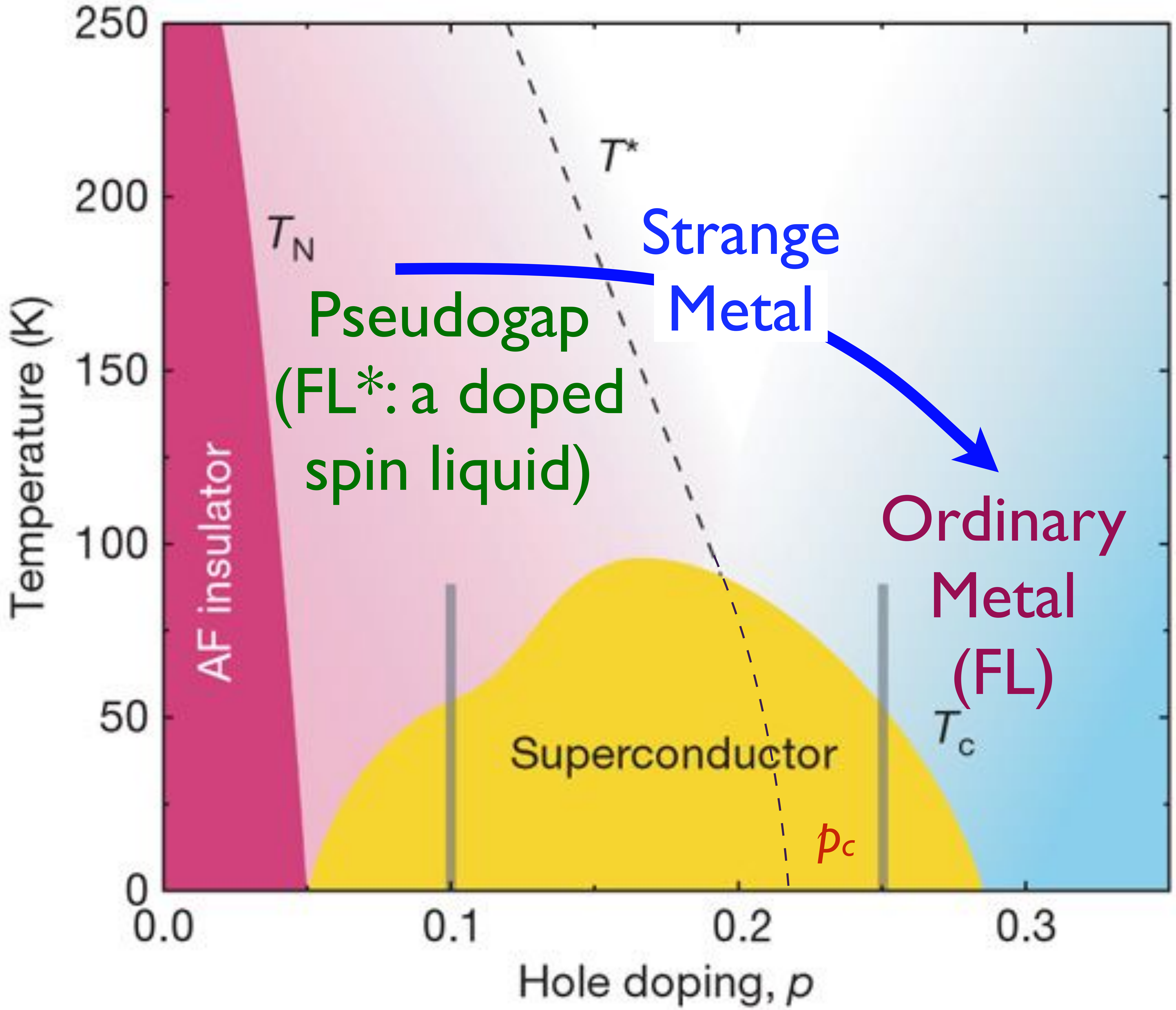


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Pseudogap
(FL*: a doped
spin liquid)
Strange
Metal
Ordinary
Metal
(FL)
T_c
Superconductor
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$



250
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T_c
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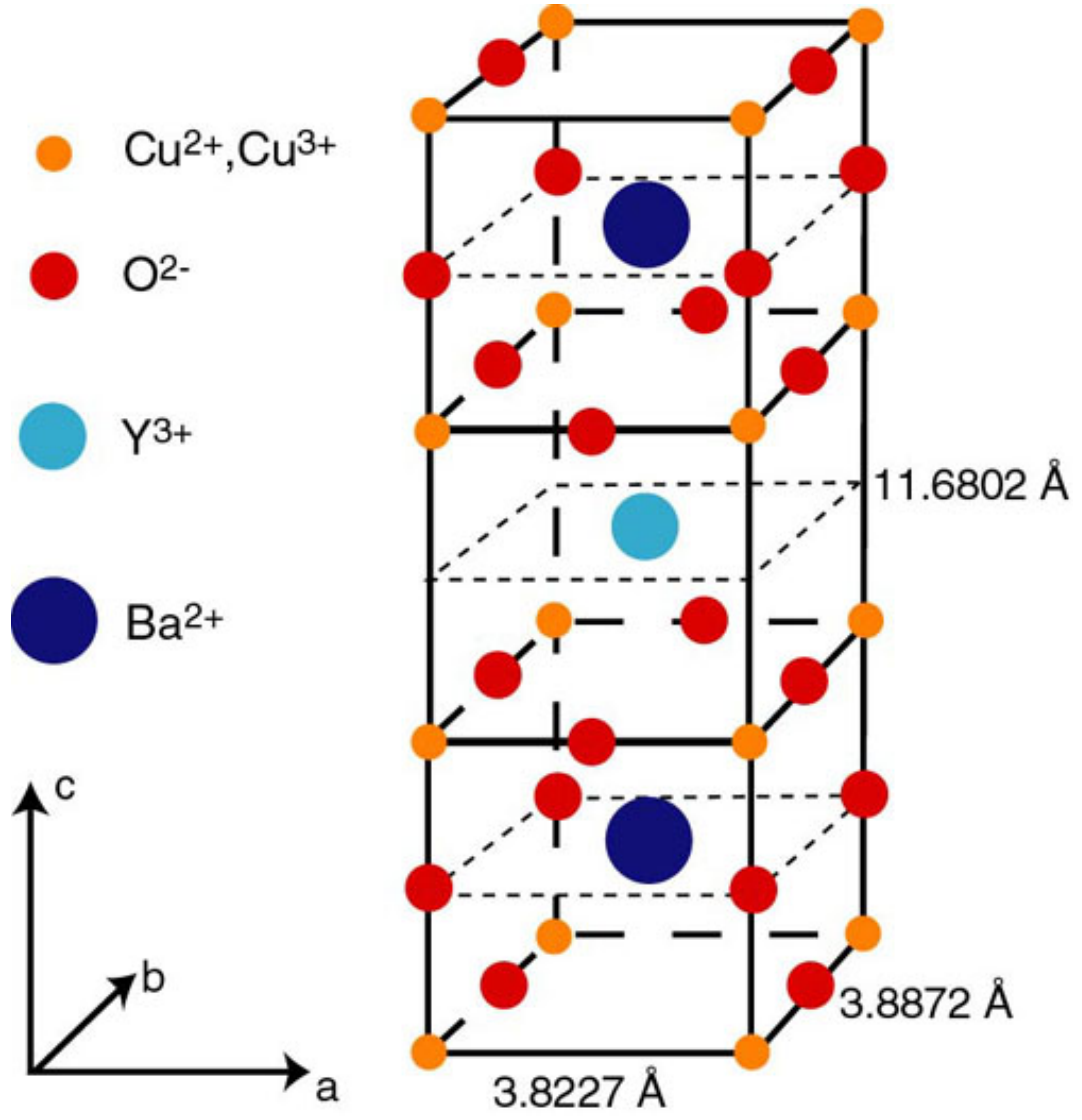

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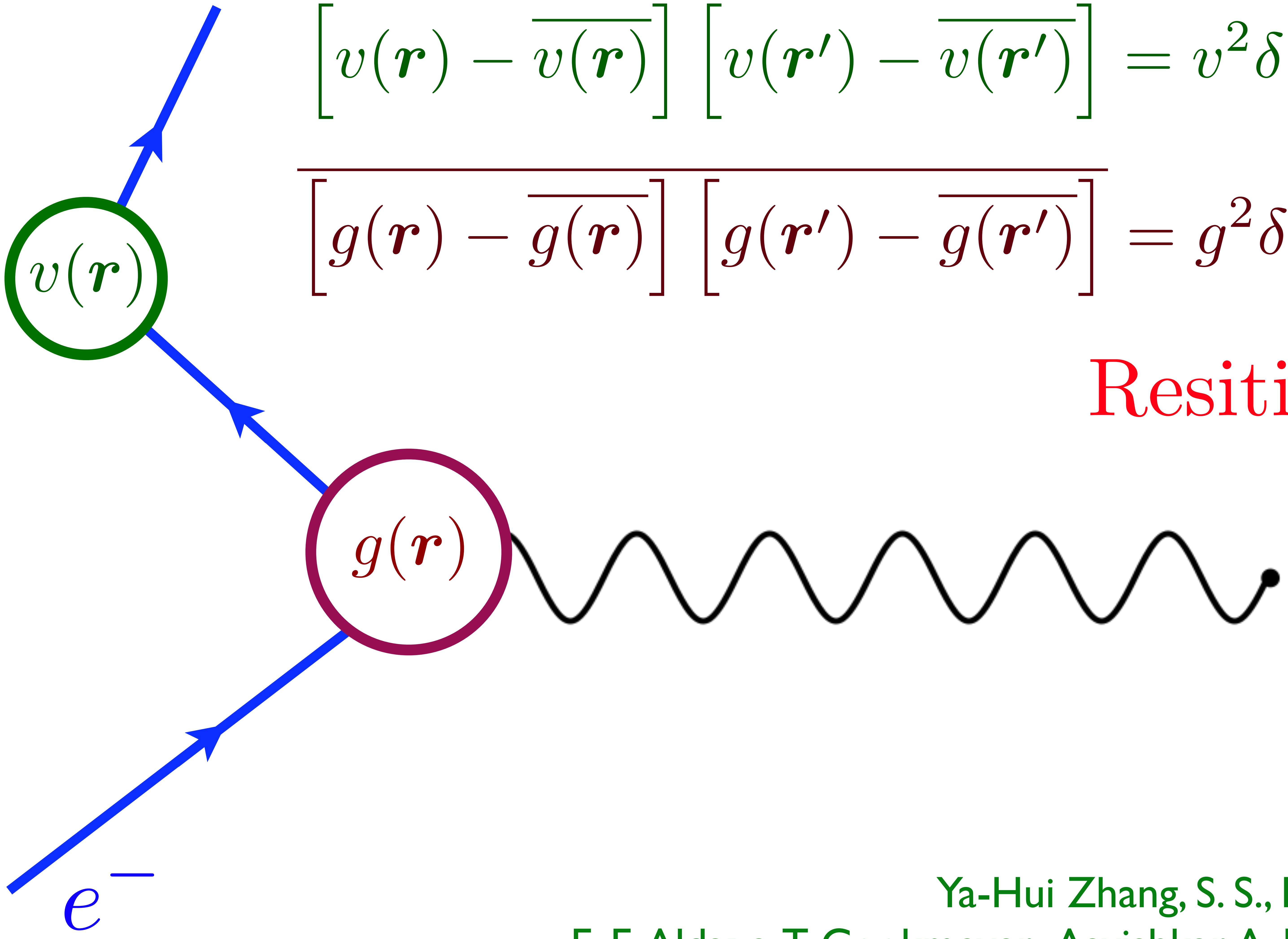
The dance of electrons on Cu atoms in YBCO

$$\overline{\left[v(\boldsymbol{r}) - \overline{v(\boldsymbol{r})}\right]\left[v(\boldsymbol{r}') - \overline{v(\boldsymbol{r}')}\right]} = v^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$$

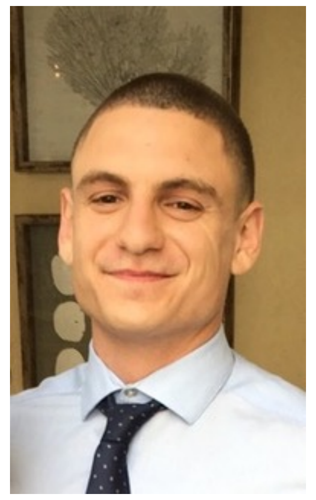
$$\overline{\left[g(\boldsymbol{r}) - \overline{g(\boldsymbol{r})}\right]\left[g(\boldsymbol{r}') - \overline{g(\boldsymbol{r}')}\right]} = g^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$$

Random interactions as in SYK model



Resitivity $\rho(T) \sim v^2 + g^2 T$





Ya-Hui Zhang, S. S., PRR **2**, 023172 (2020); PRB **102** , 155124 (2020)
E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, PRB **105**, 235111 (2022)
Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990, *Science*, to appear

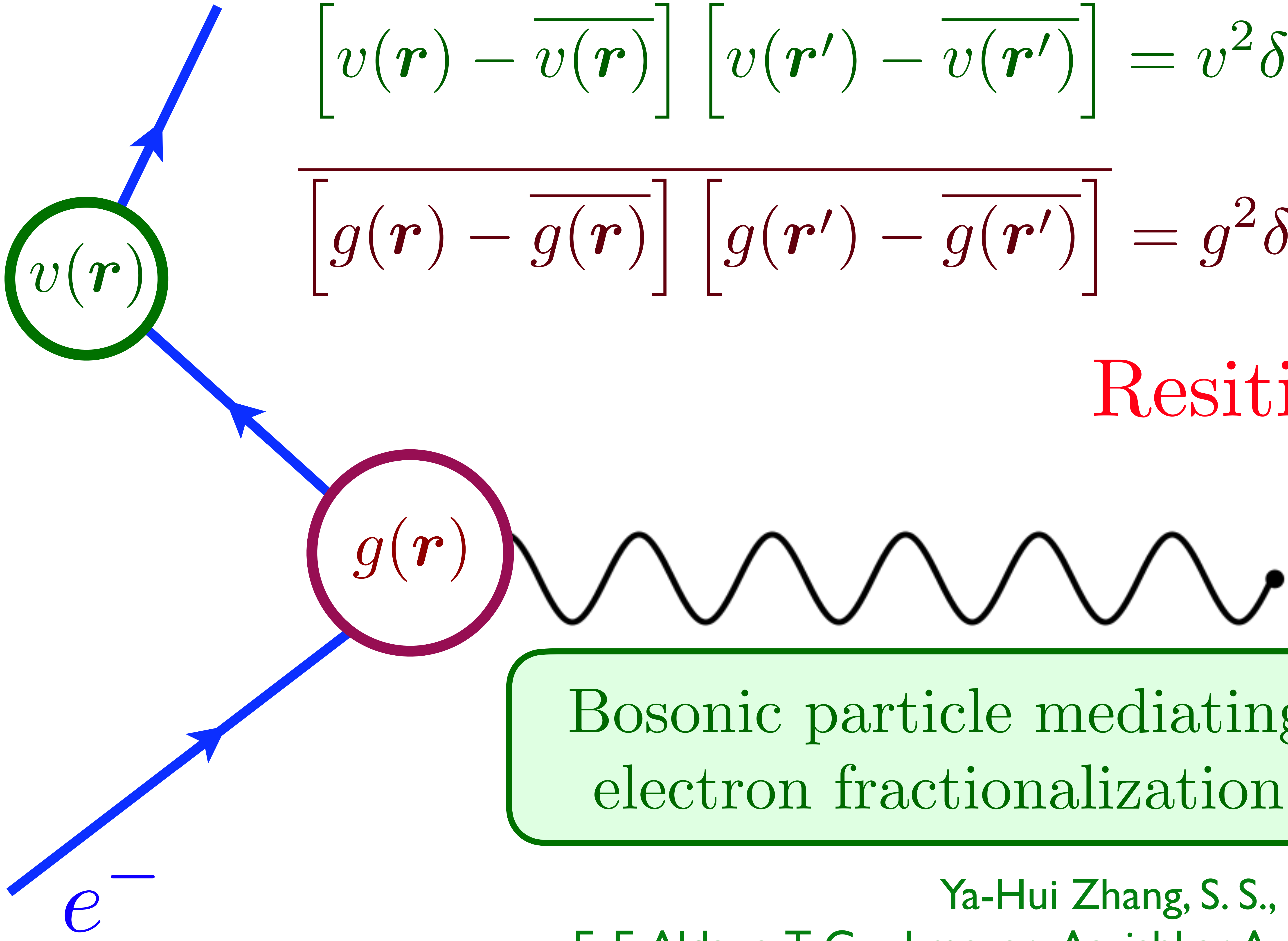
The dance of electrons on Cu atoms in YBCO

$$\overline{\left[v(\boldsymbol{r}) - \overline{v(\boldsymbol{r})}\right]\left[v(\boldsymbol{r}') - \overline{v(\boldsymbol{r}')}\right]} = v^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$$

$$\overline{\left[g(\boldsymbol{r}) - \overline{g(\boldsymbol{r})}\right]\left[g(\boldsymbol{r}') - \overline{g(\boldsymbol{r}')}\right]} = g^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$$

Random interactions as in SYK model

Resitivity $\rho(T) \sim v^2 + g^2 T$



Bosonic particle mediating electron fractionalization

Ya-Hui Zhang, S. S., PRR **2**, 023172 (2020); PRB **102** , 155124 (2020)
E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, PRB **105**, 235111 (2022)
Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990, *Science*, to appear

Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

3. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon......A. Georges, arXiv:2205.04030

4. Photoemission: nearly “marginal Fermi liquid” electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau(\omega)} \sim |\omega| \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

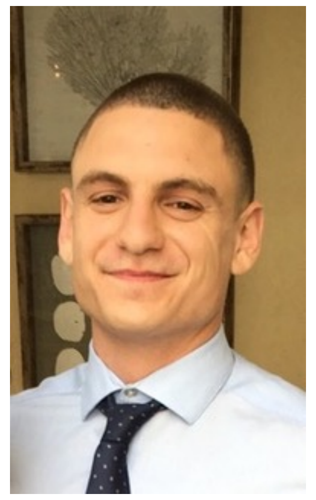
T.J. Reber....D. Dessau, Nature Communications **10**, 5737 (2019)

Summary

- SYK: a solvable toy model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

- Toy SYK model captures the correct universal low energy quantum theory of charged black holes, and provides a Hamiltonian realization of black hole microstates.





- Linear-T resistivity, $T \ln(1/T)$ specific heat, $\sim 1/\omega$ optical conductivity, and marginal Fermi liquid electron spectrum *all* arise from a SYK-like model with spatially random interactions in a two-dimensional quantum-critical metal.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S., Science to appear, arXiv: 2203.

The many faces of multi-particle entanglement

- Absence of quasiparticles, as in the SYK model and the strange metal

- Fractionalization and new emergent particles, as in spin liquids.

- Higher temperature superconductivity (?)

- A quantum theory of the interior of a black hole.