Long-range quantum entanglement in metals

Albanova and Nordita Colloquium
Stockholm
November 12, 2015

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Talk online: sachdev.physics.harvard.edu

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

PHYSICS HARVARD

1. Long-range entanglement in insulators

2. Theory of ordinary metals

   (a) Quasiparticles

   (b) Luttinger theorem for volume enclosed by Fermi surface

3. Fractionalized Fermi liquid

   Quasiparticles with a non-Luttinger volume in the pseudogap metal of the cuprate superconductors

4. Strange metals without quasiparticles

   Experiments in graphene, and charged black holes

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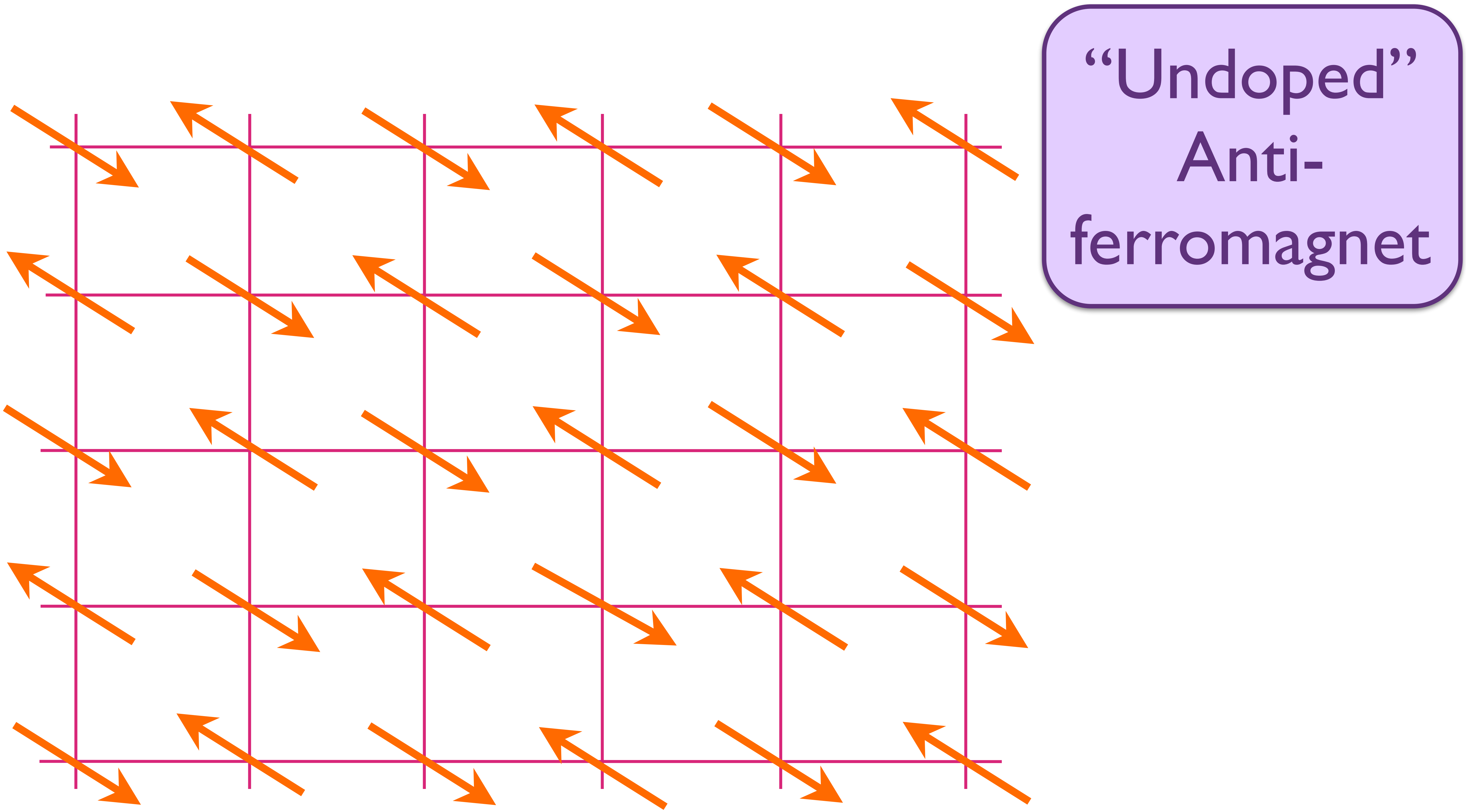
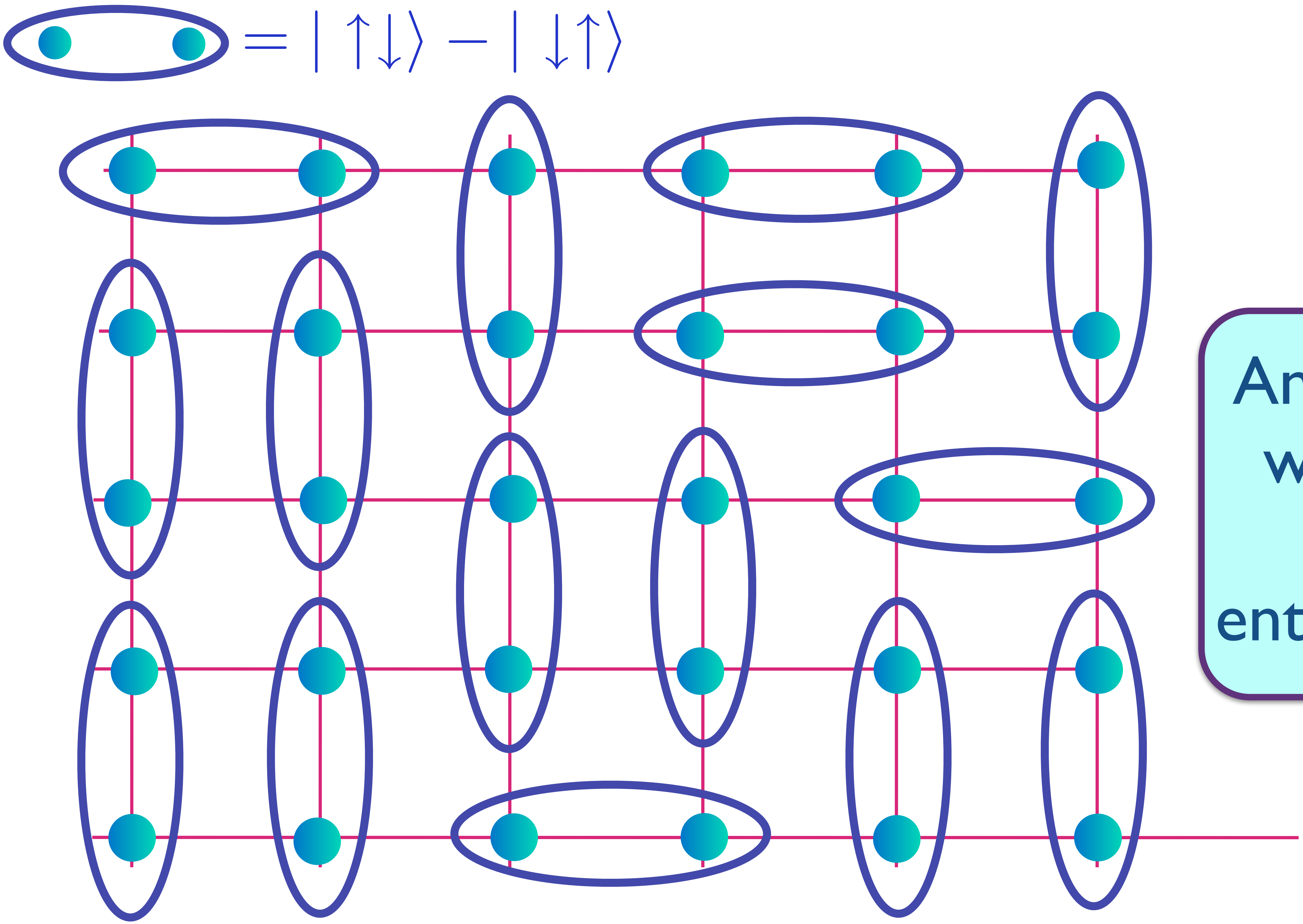

“Undoped”
Anti-
ferromagnet

Insulating spin liquid



An insulator with long-range entanglement

Baskaran, Zou, Anderson, Fradkin, Kivelson…

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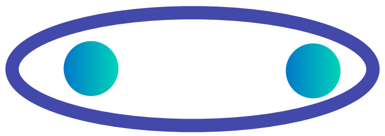
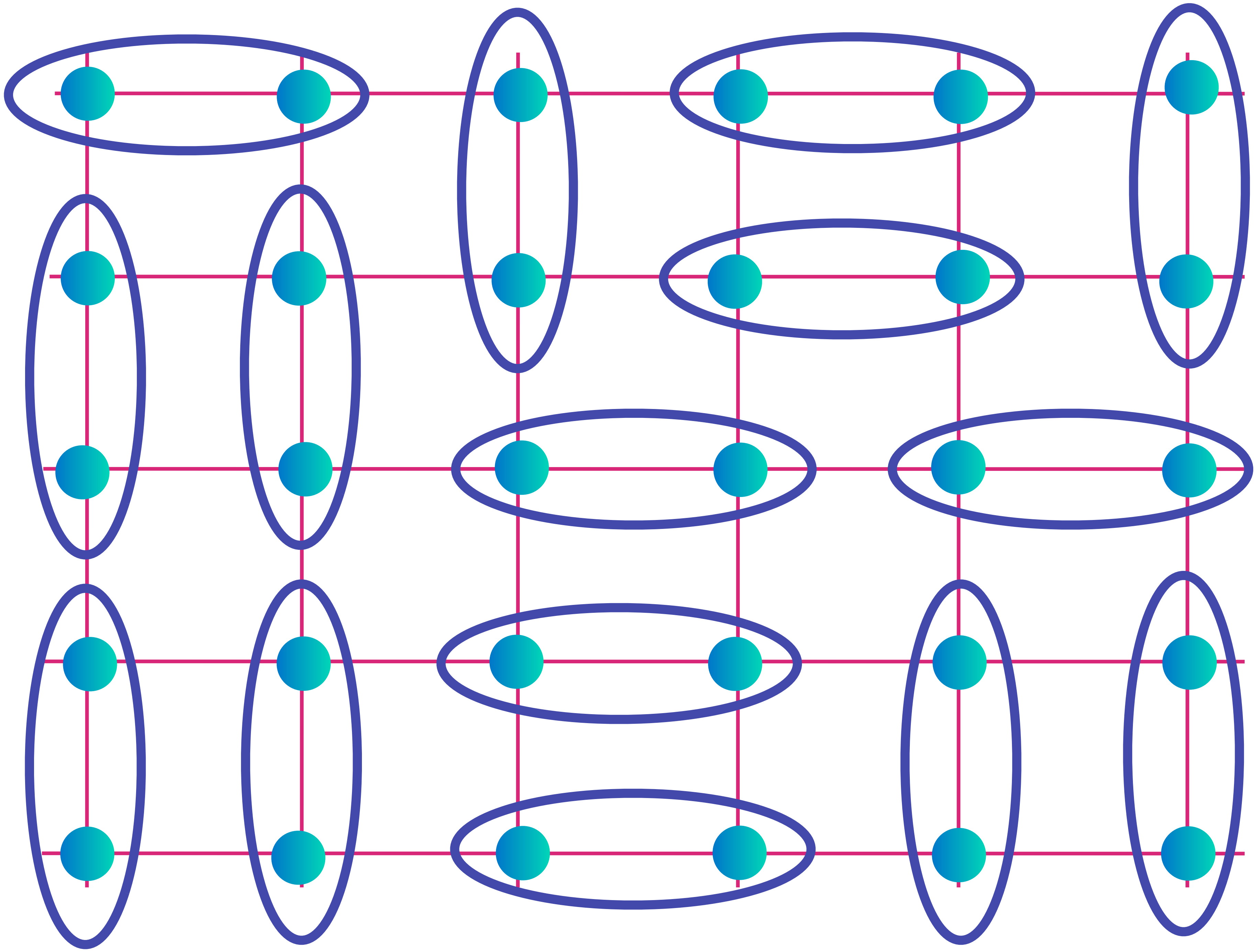


$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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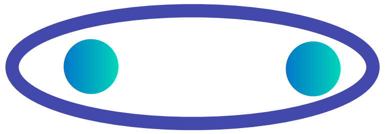

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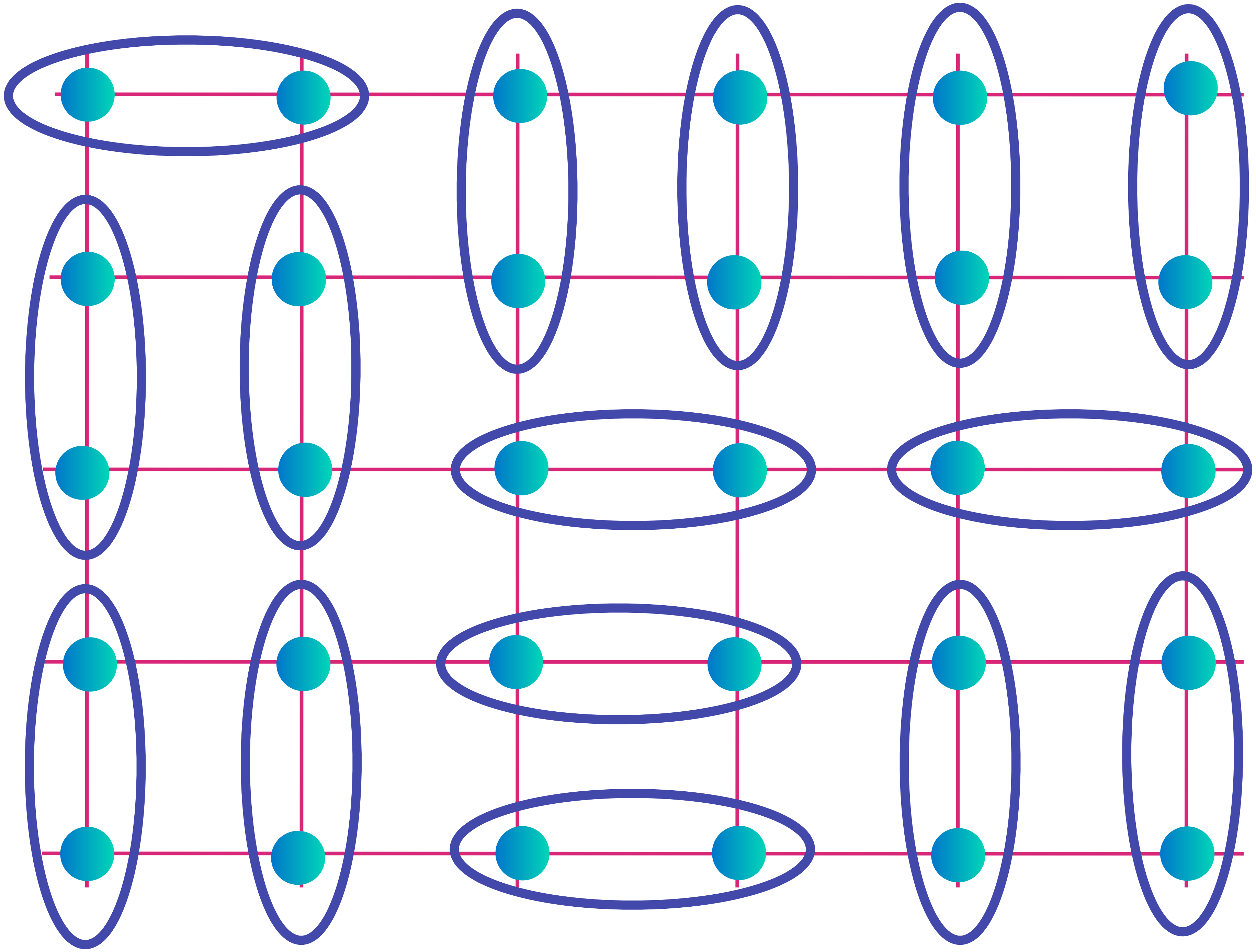

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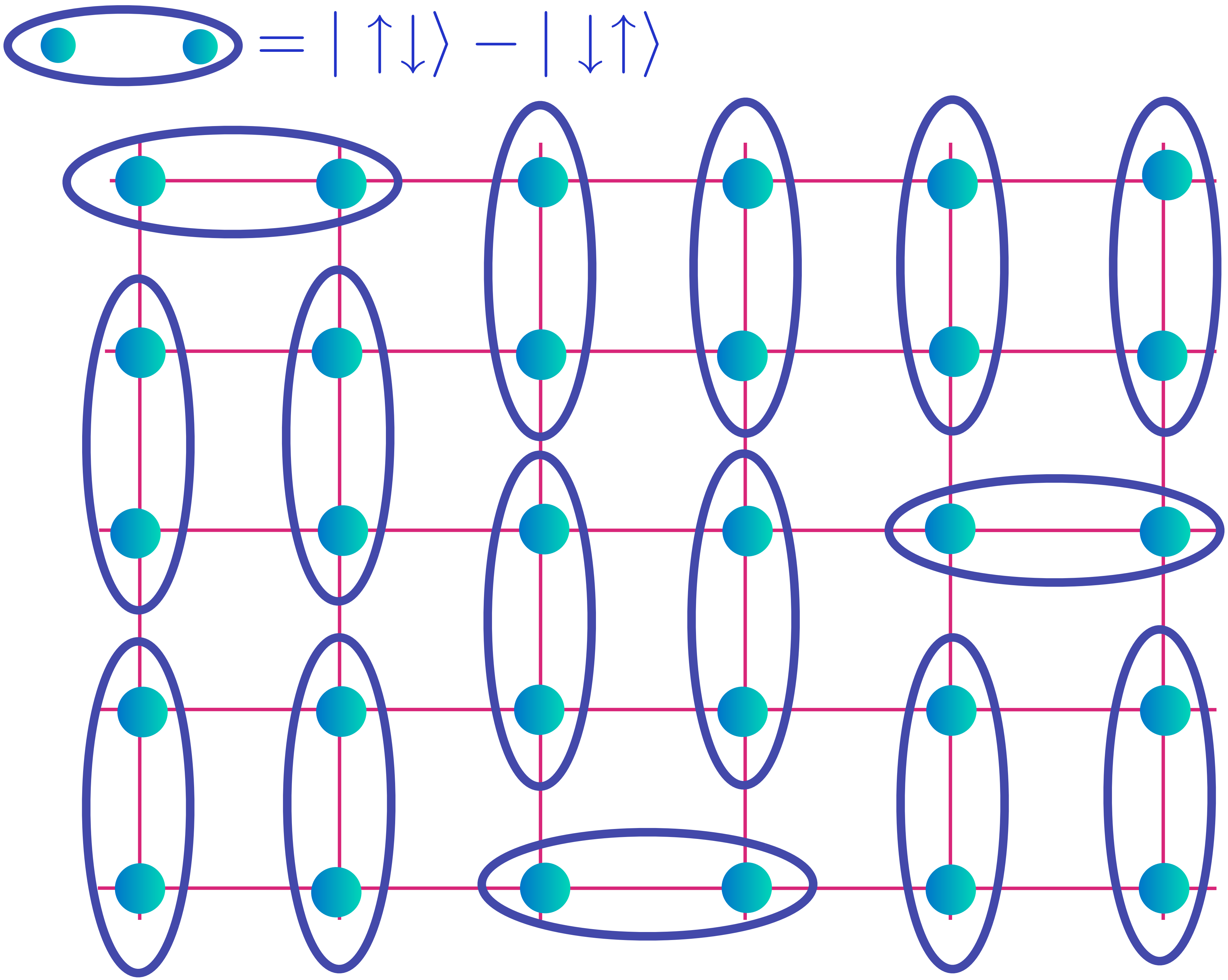

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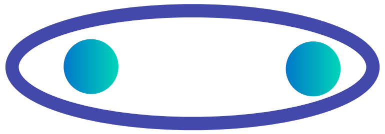

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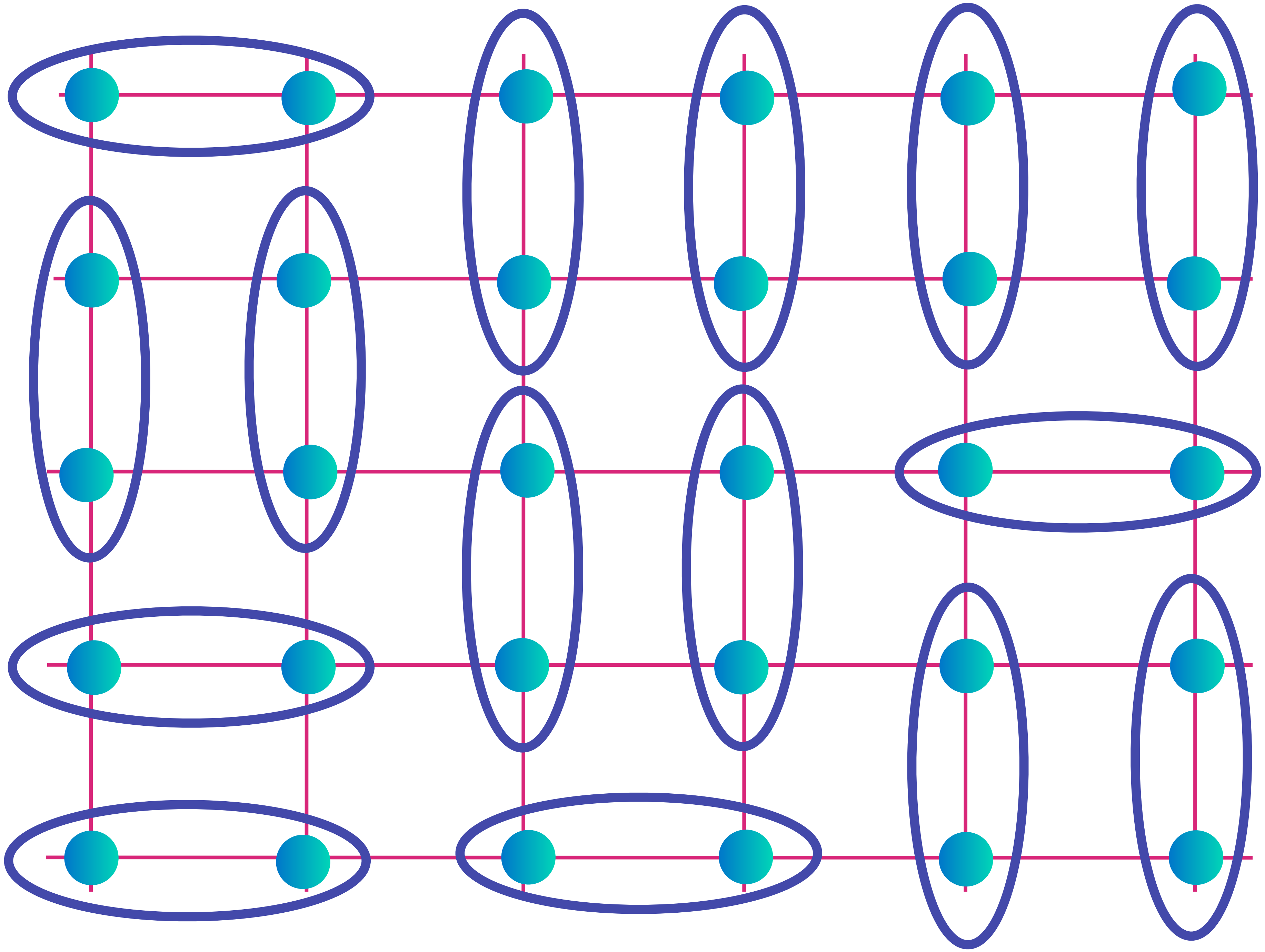

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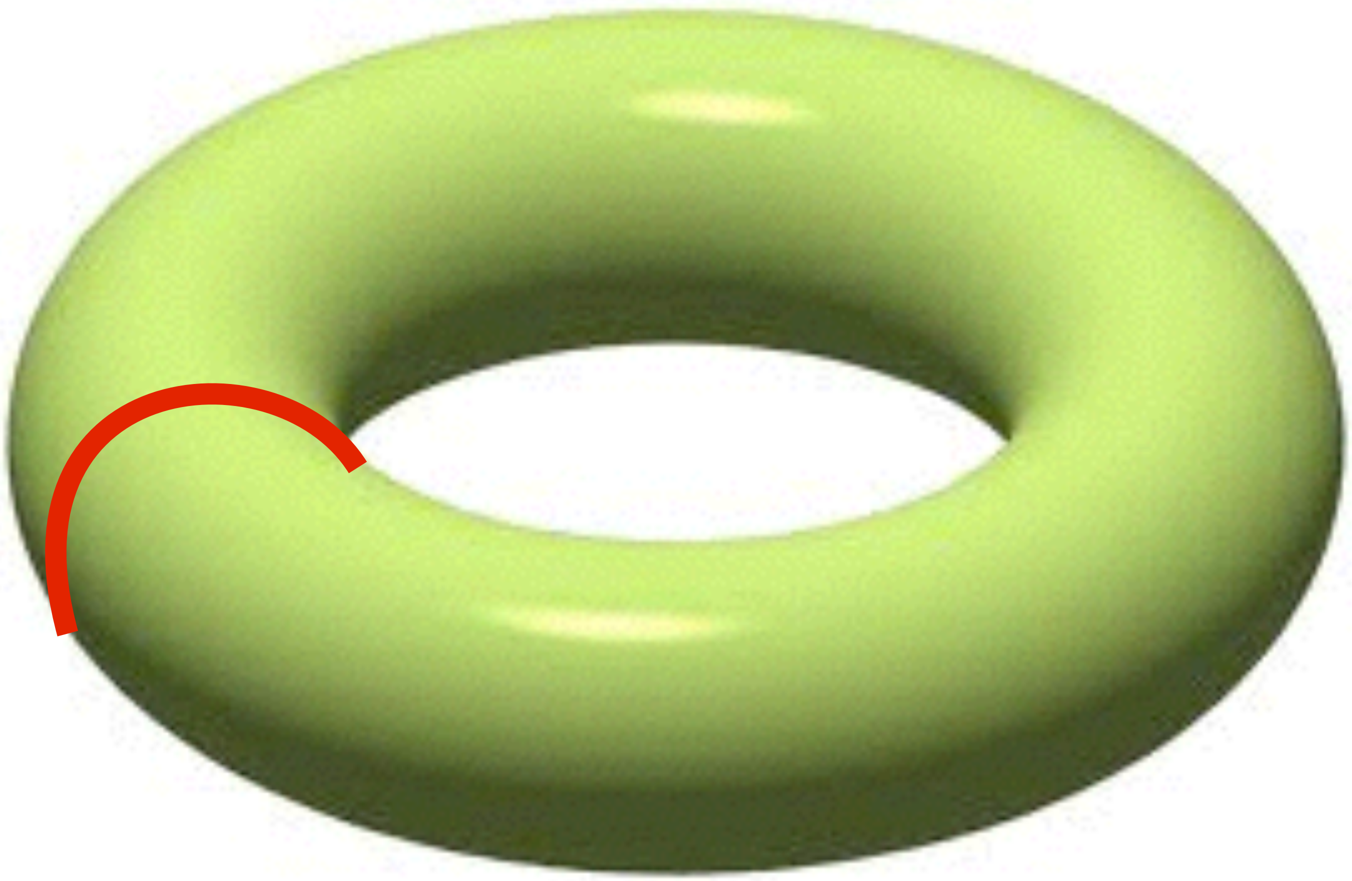
Ground state degeneracy

Place insulator on a torus;

Ground state degeneracy



Place insulator on a torus;

obtain "topological" states nearly degenerate with the ground state: number of dimers crossing red line is conserved modulo 2

N. Read and S. Sachdev, PRL **66**, 1773 (1991); X.-G. Wen PRB **44**, 2664 (1991)

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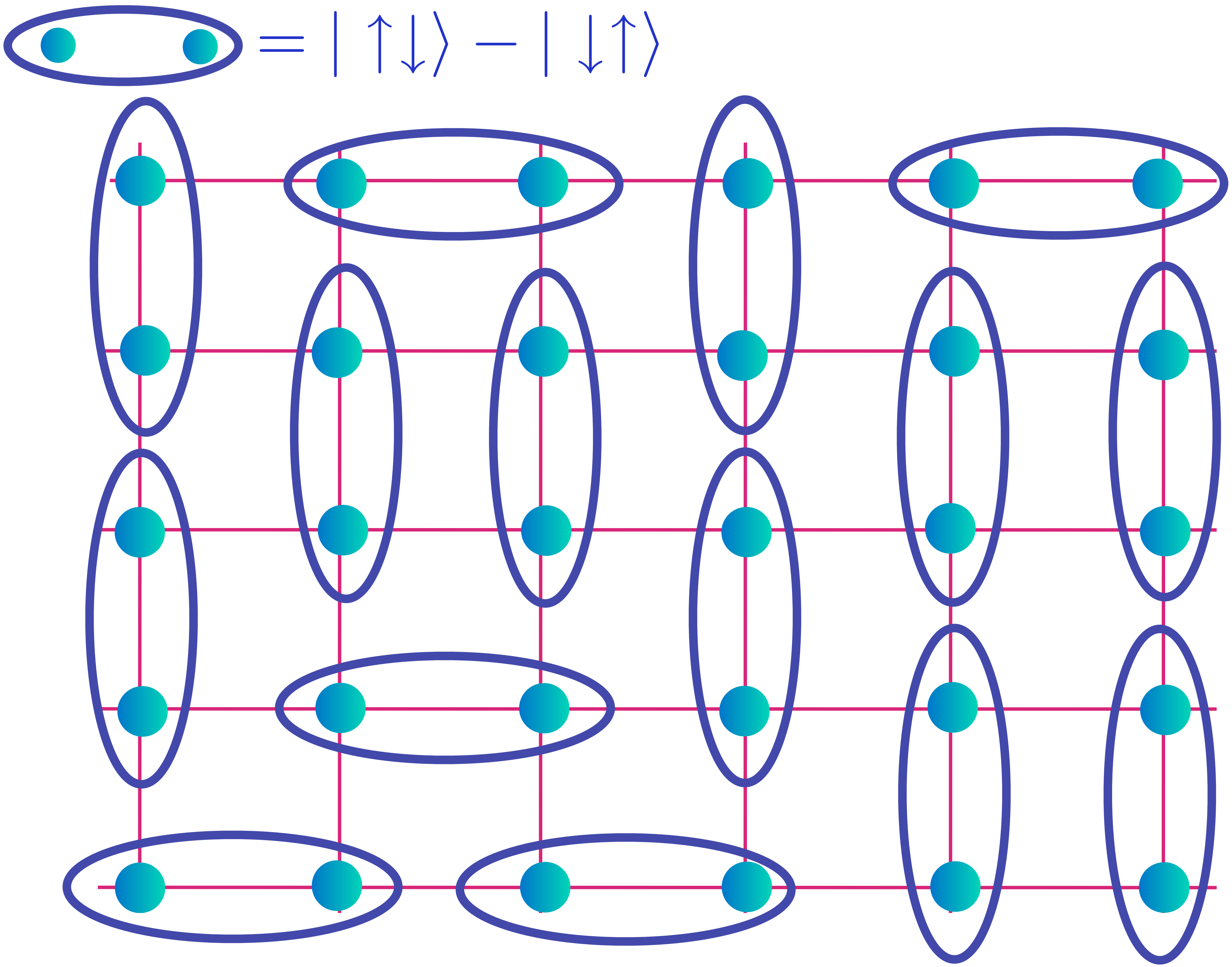

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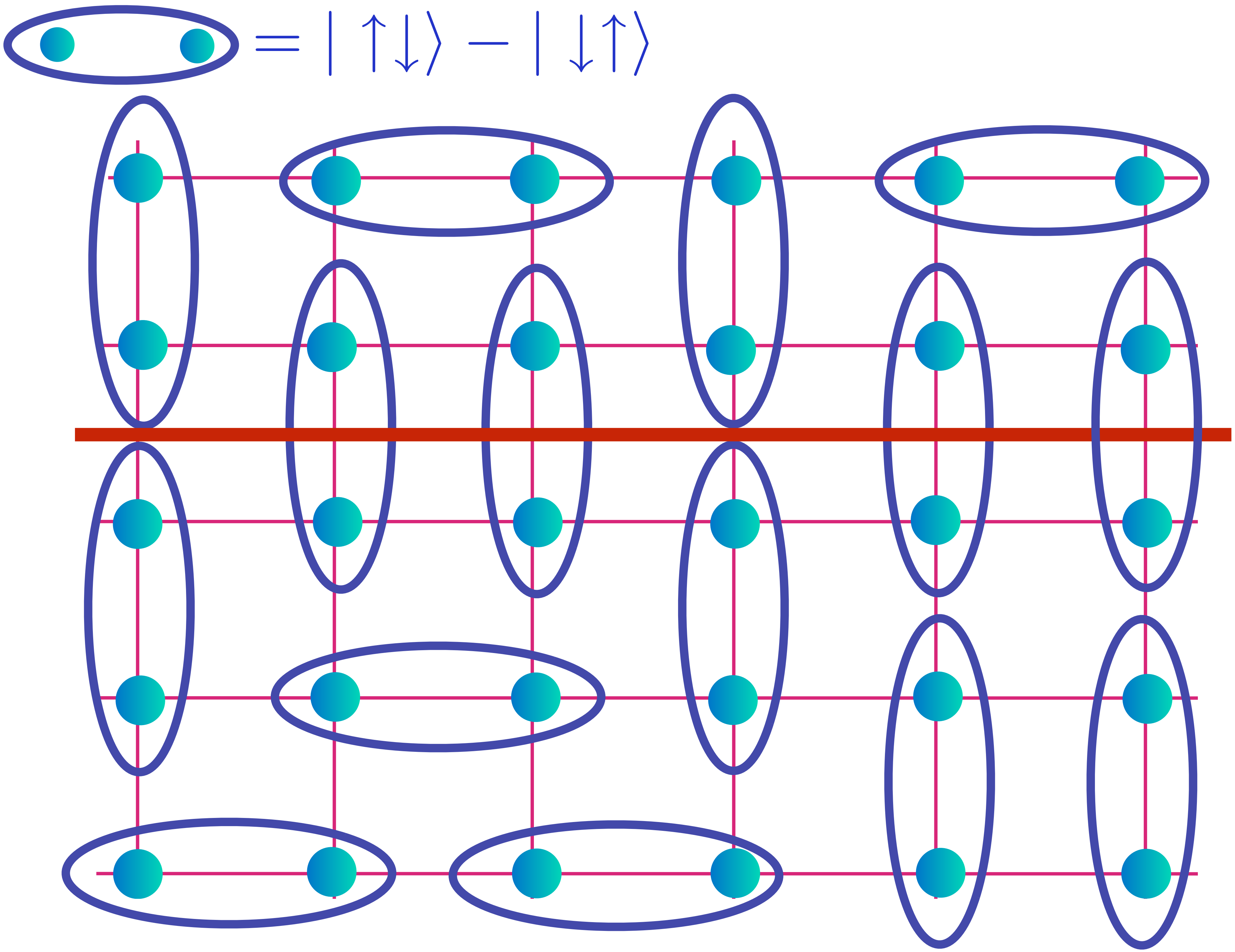

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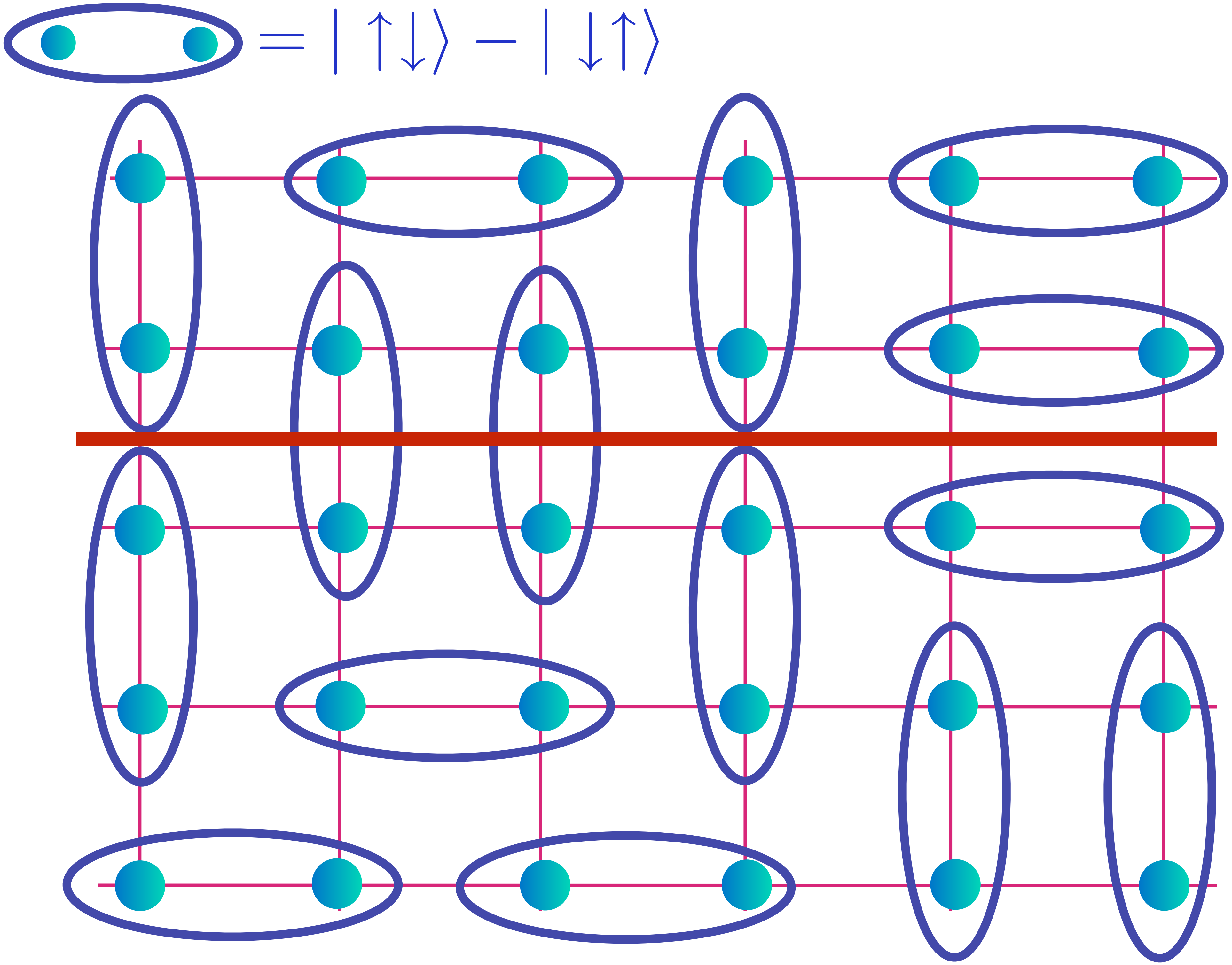

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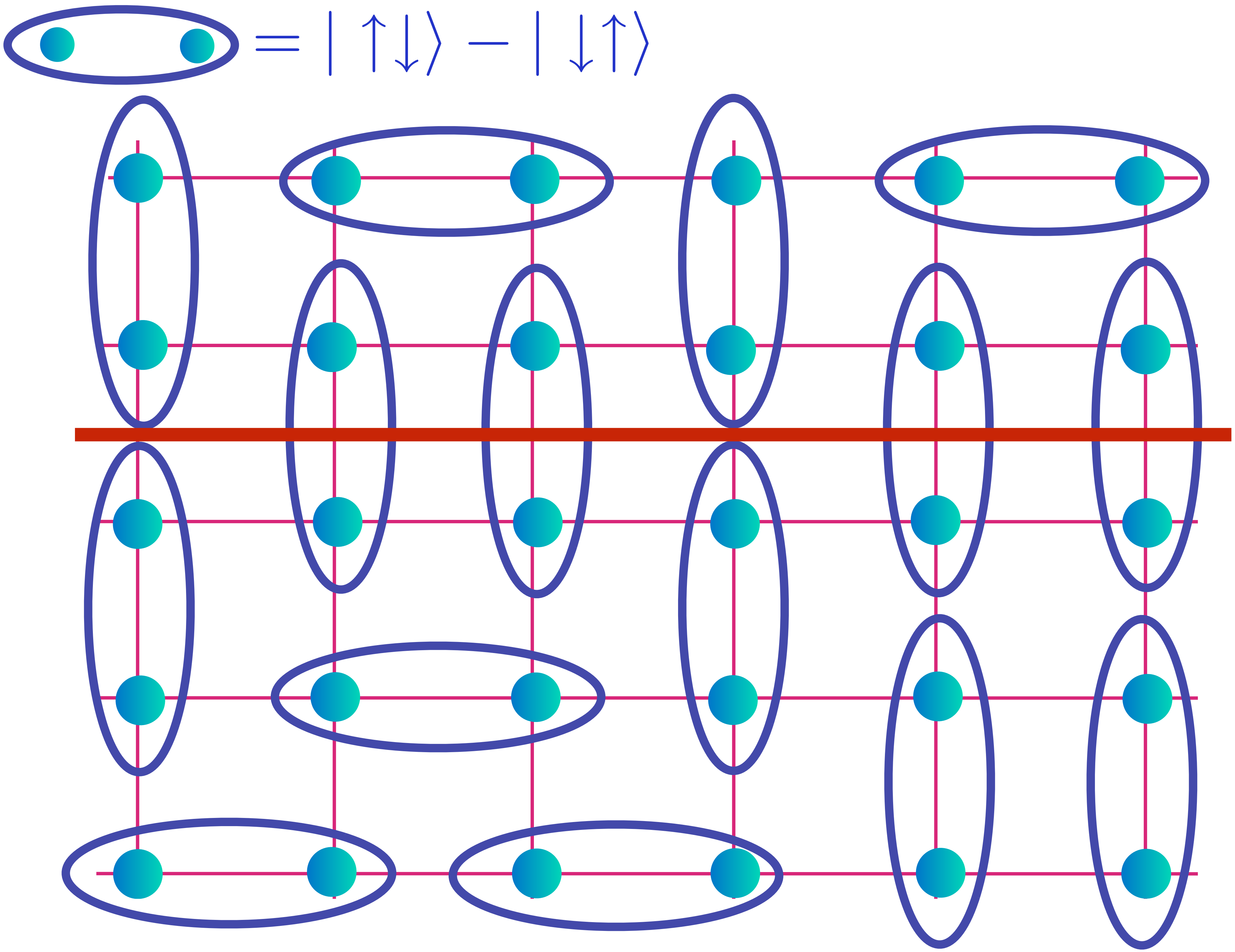

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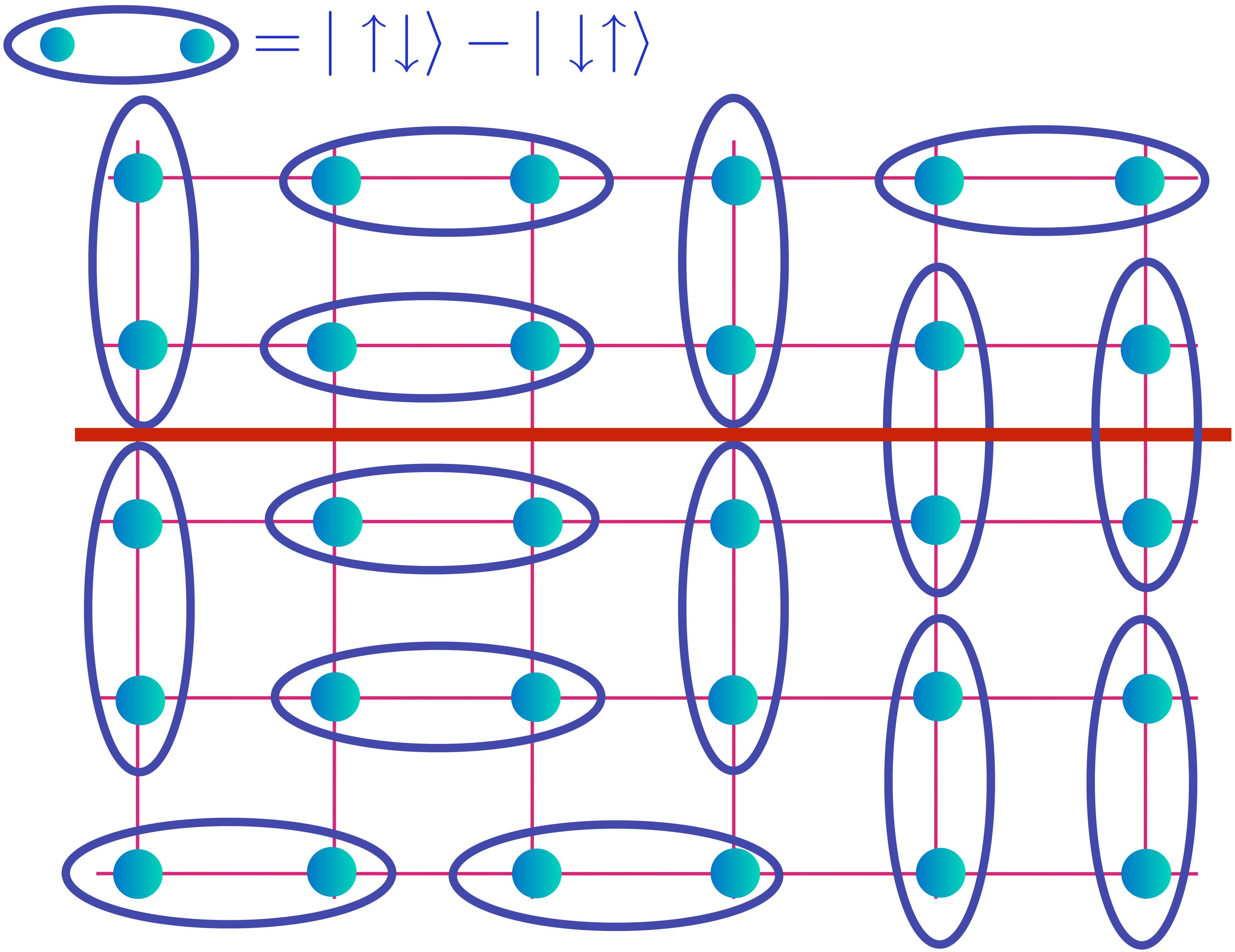

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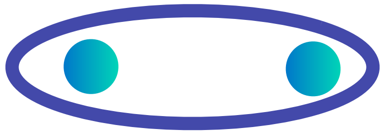

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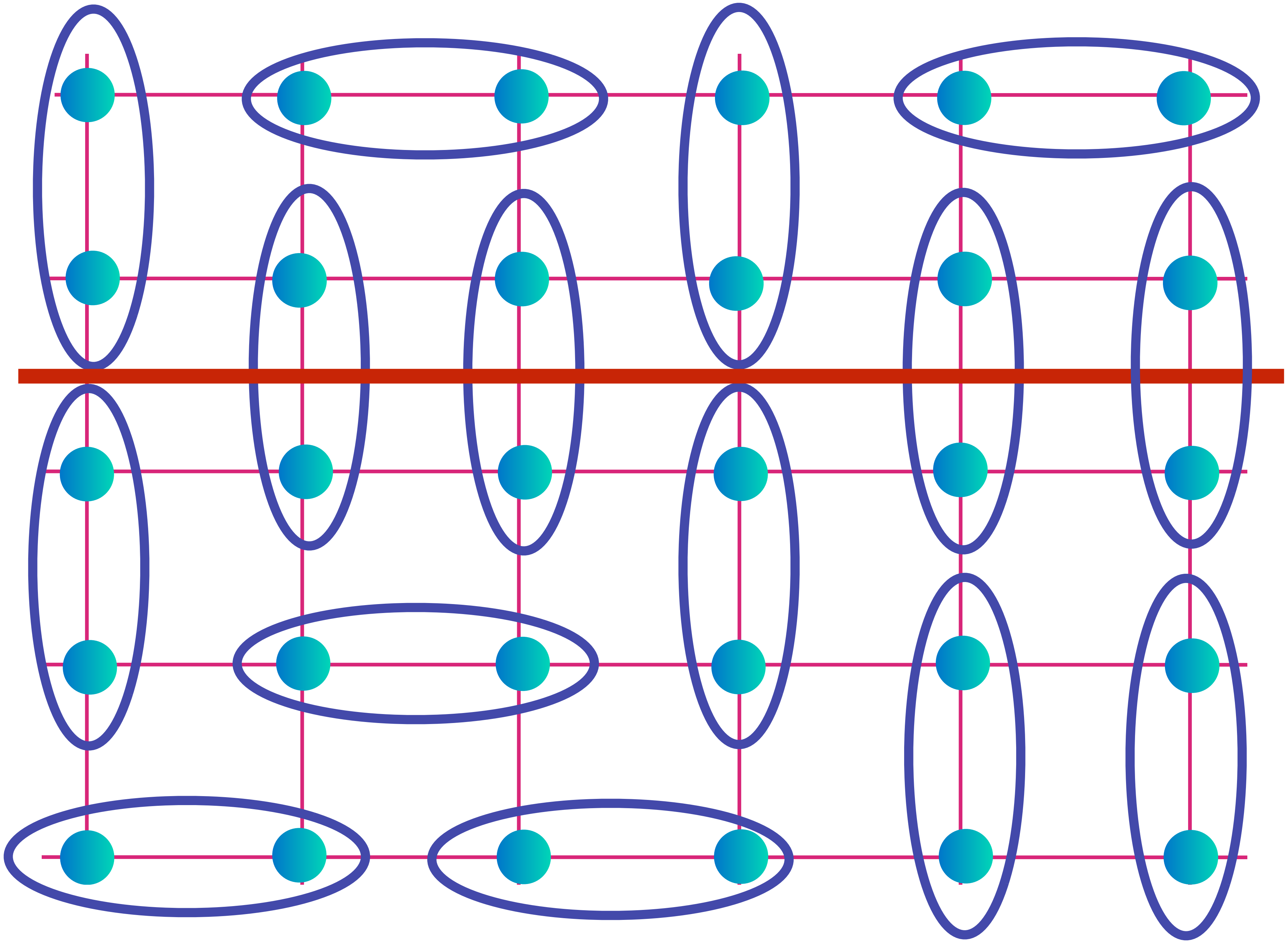

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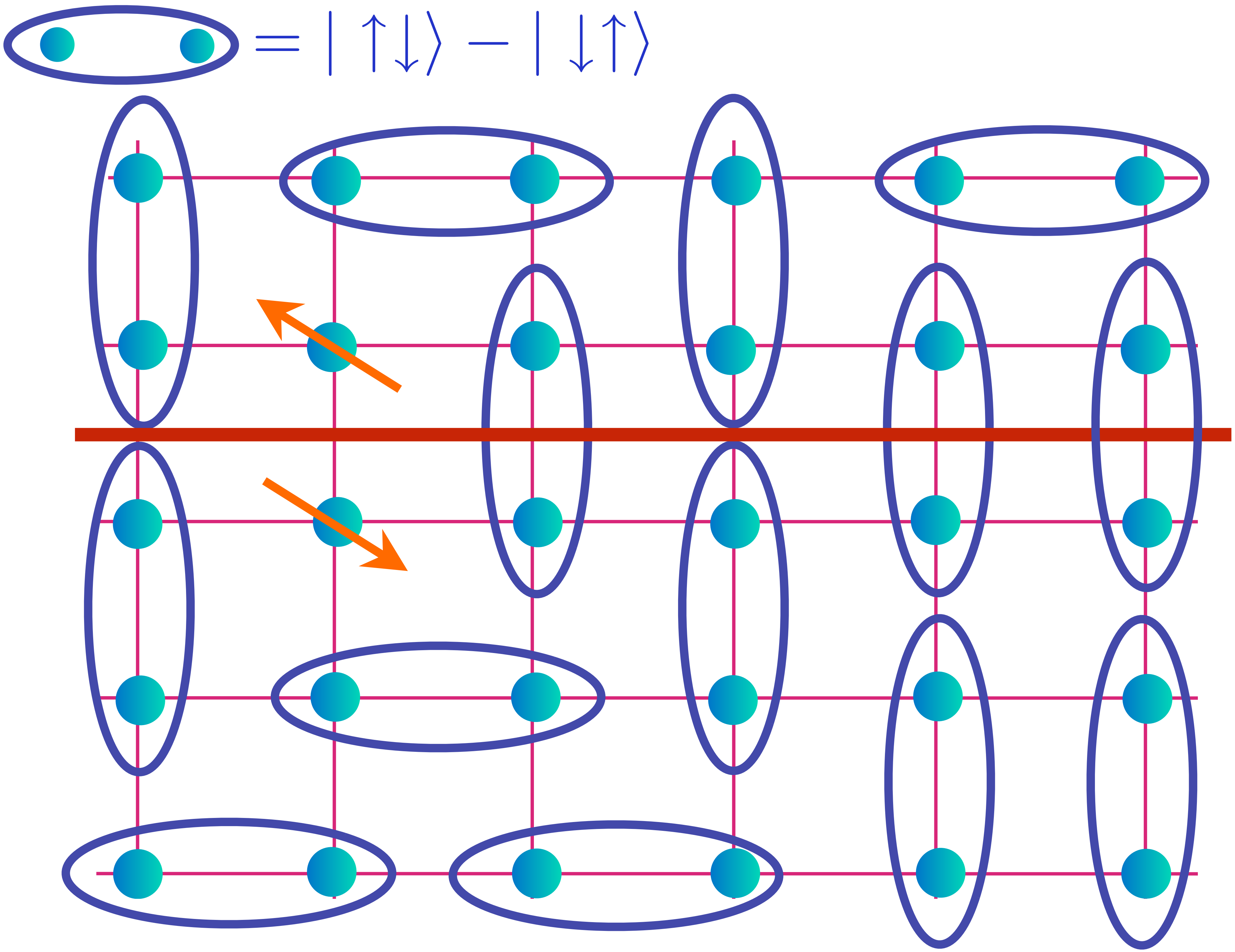

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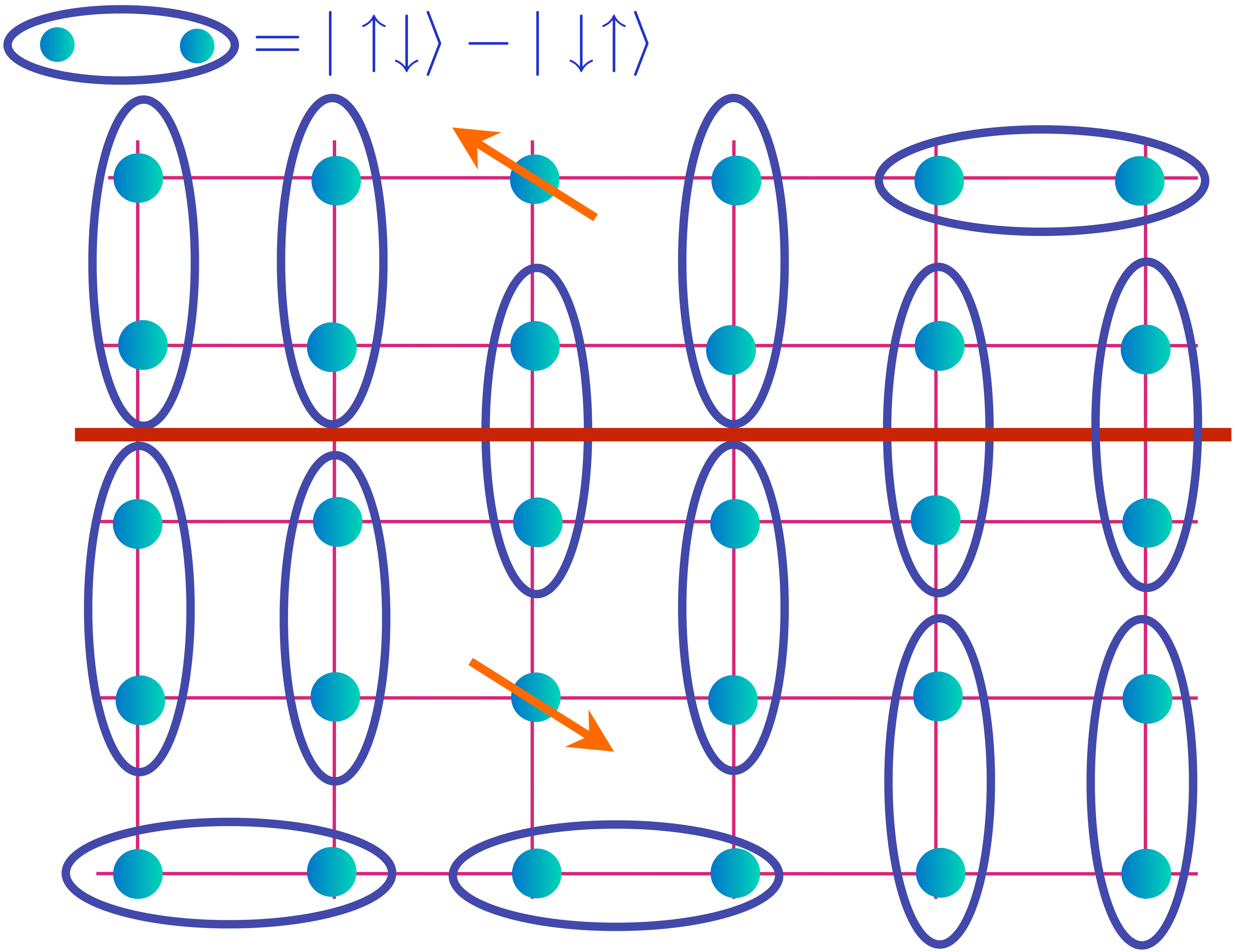

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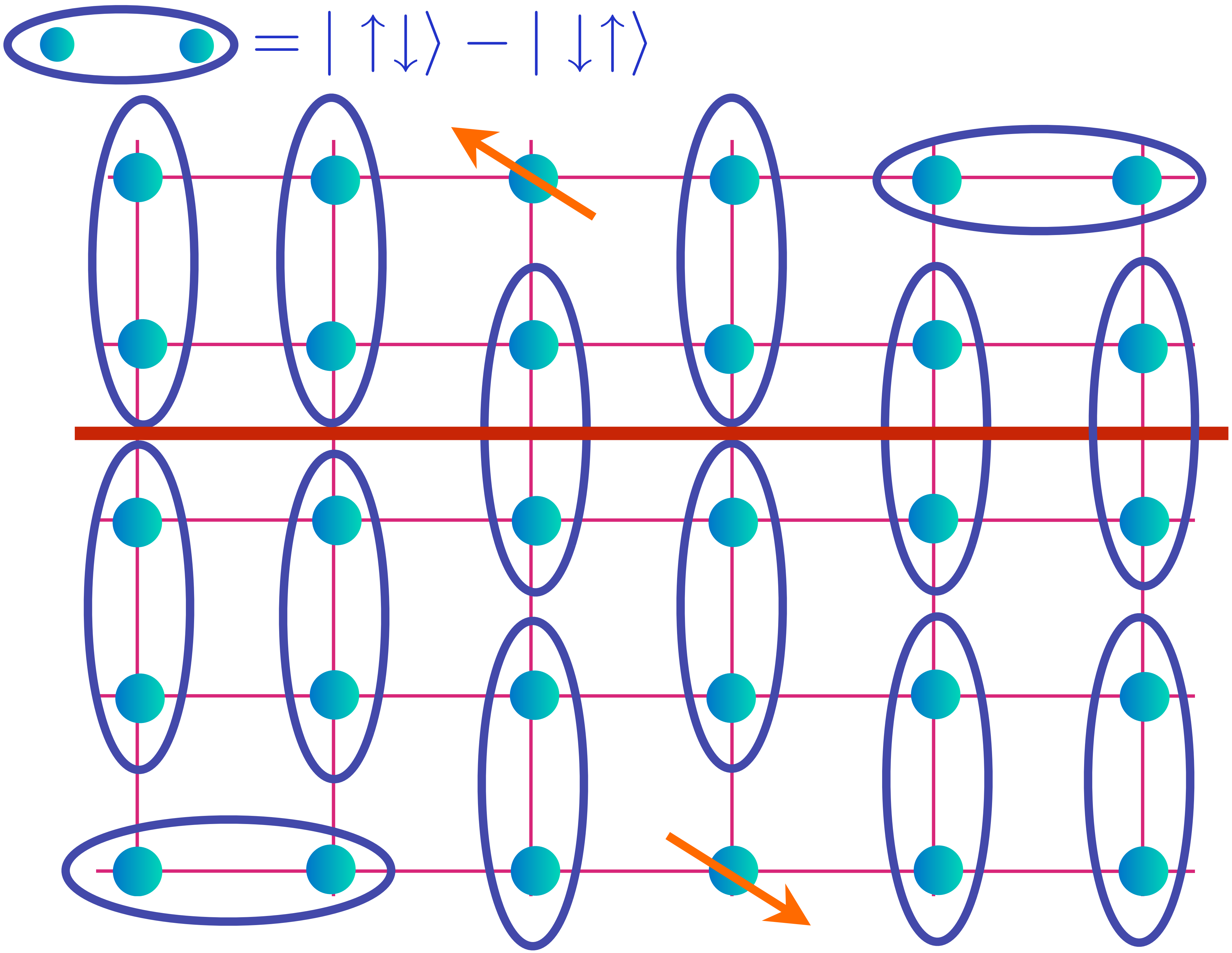

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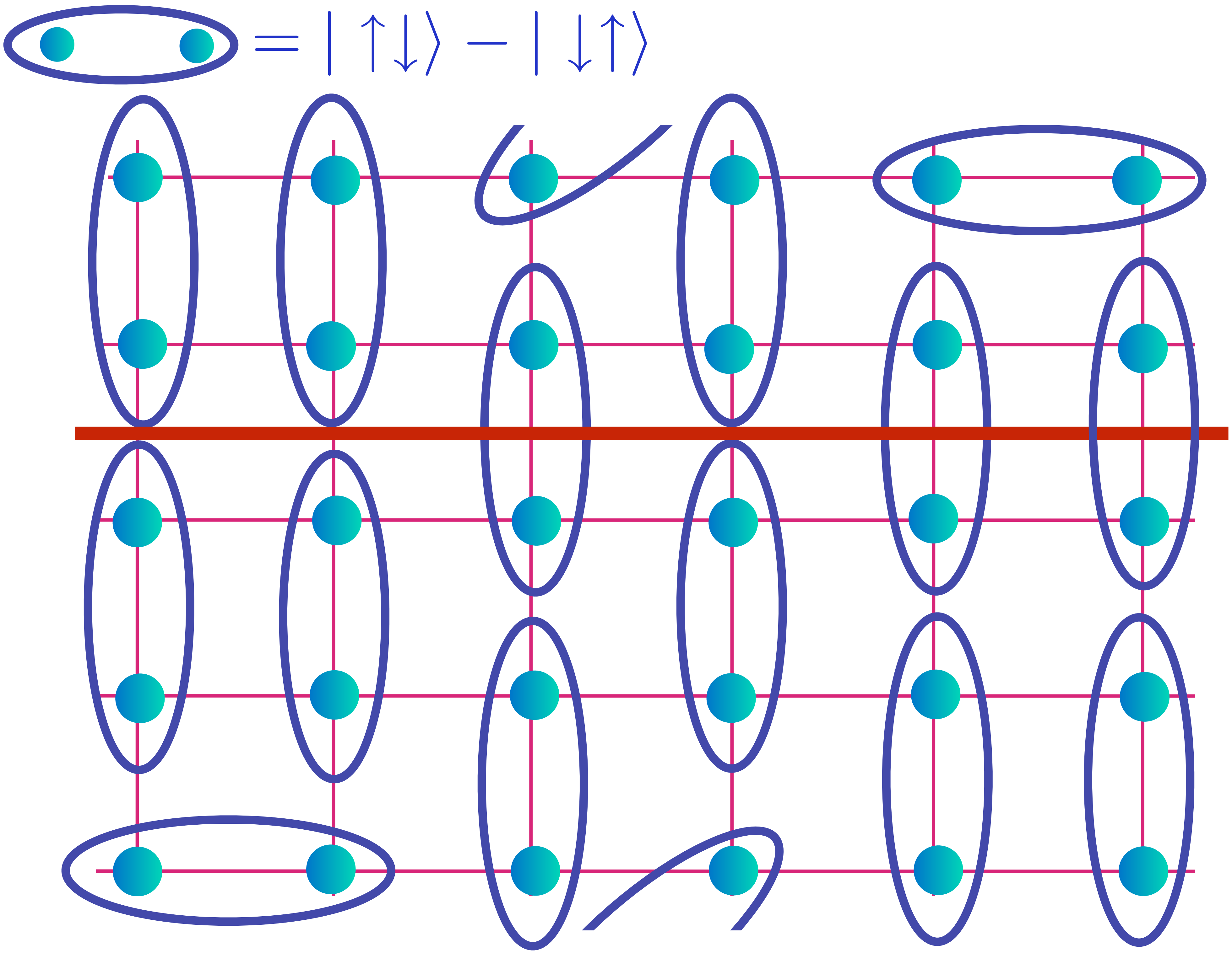

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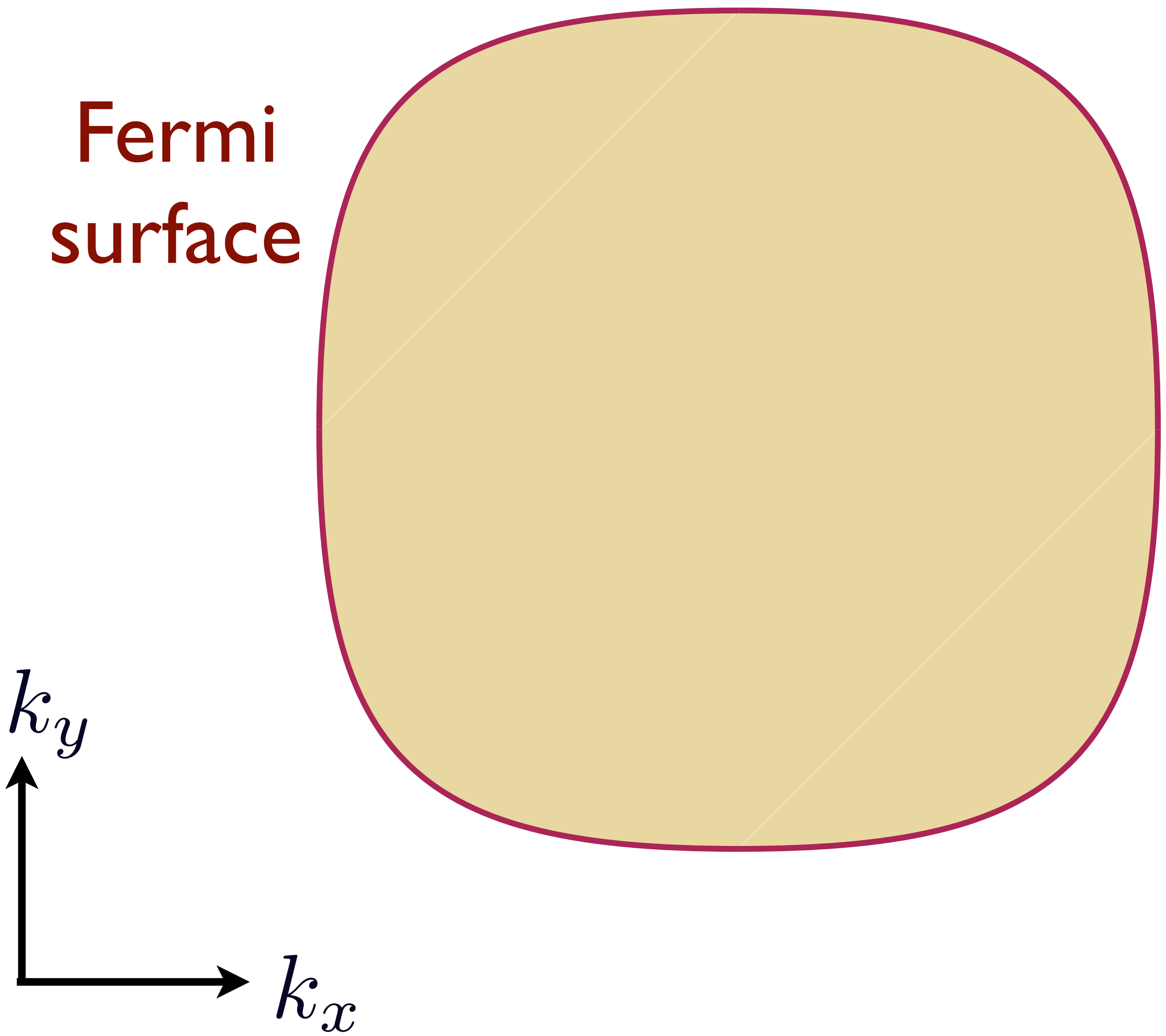
Ordinary metals: the Fermi liquid



- Fermi surface separates empty and occupied states in momentum space.

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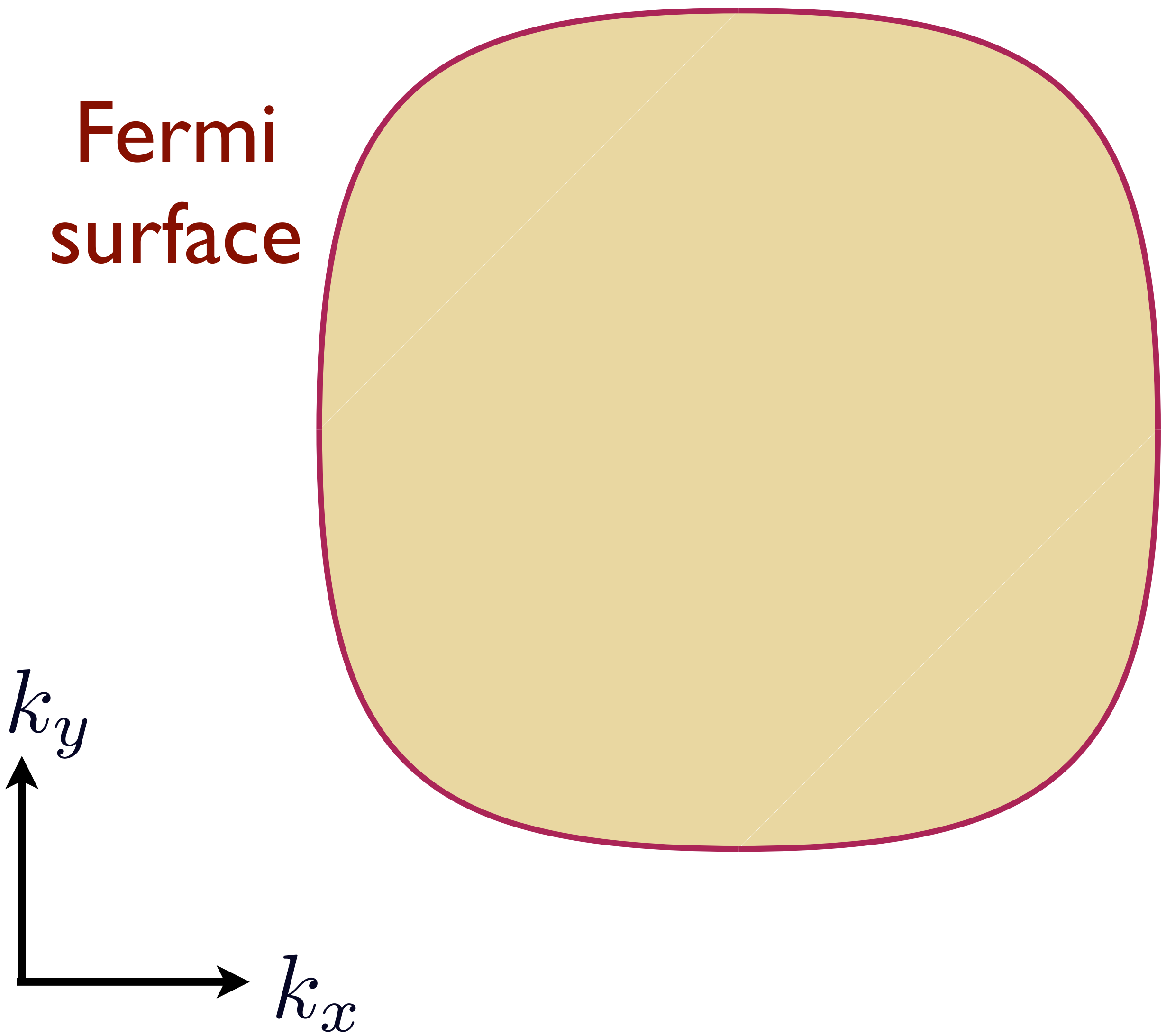

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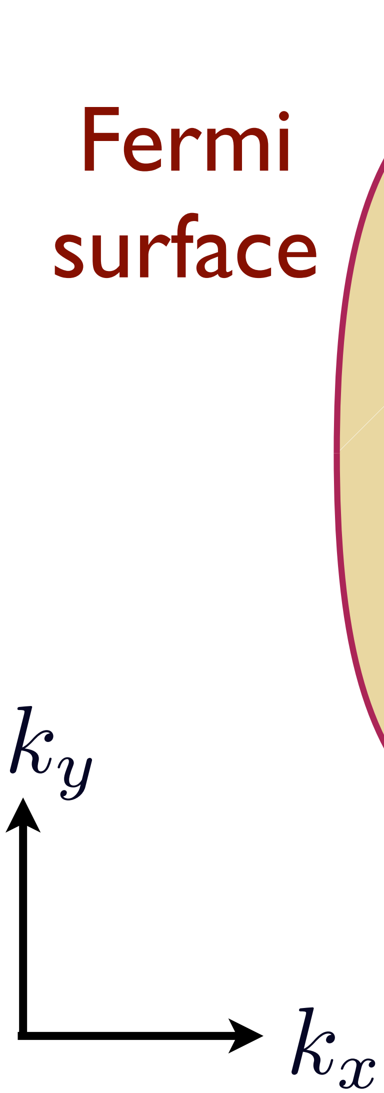

- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface $= \mathcal{Q}$, the electron density. Momenta of low energy excitations fixed by density of *all* electrons.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.

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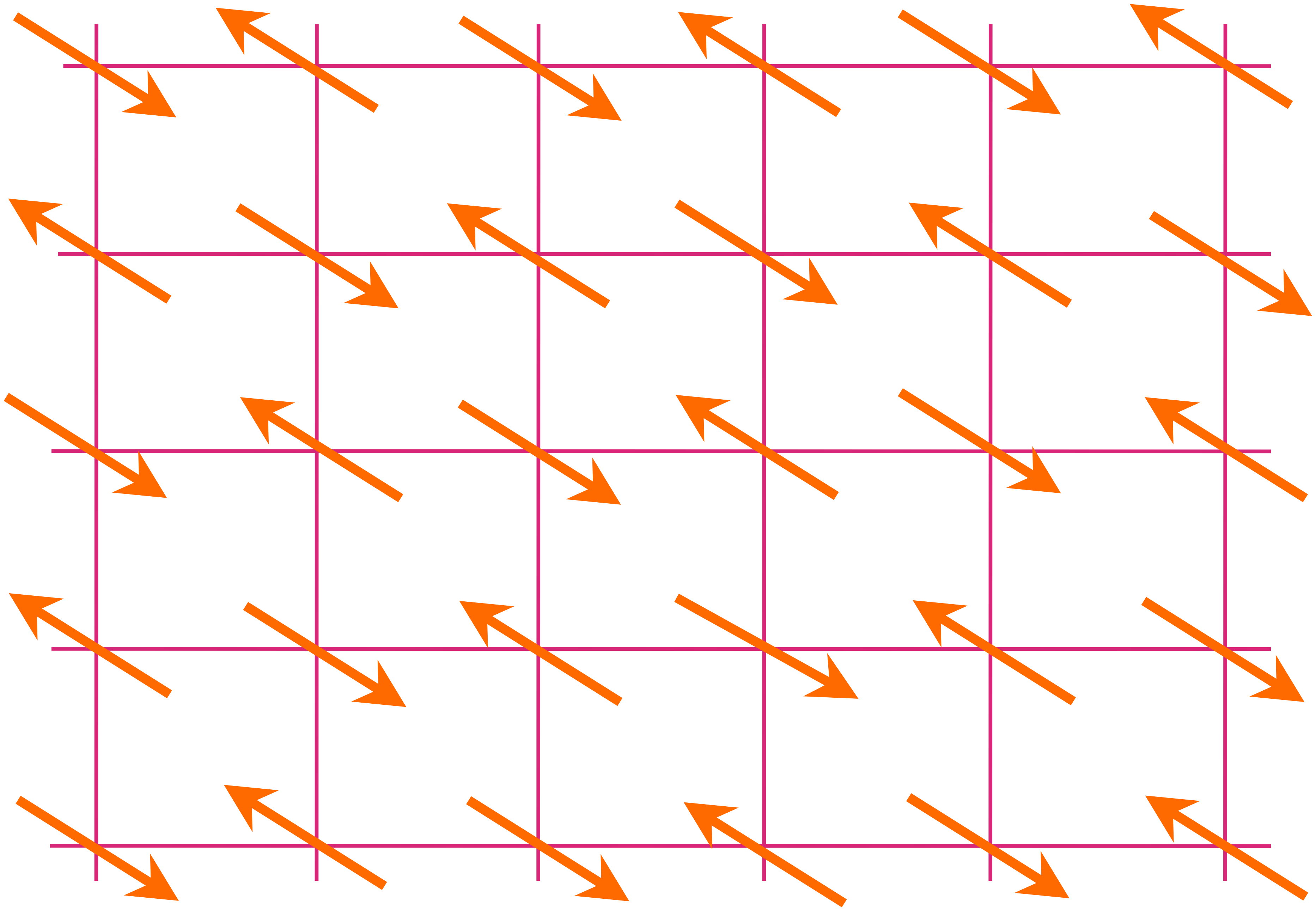
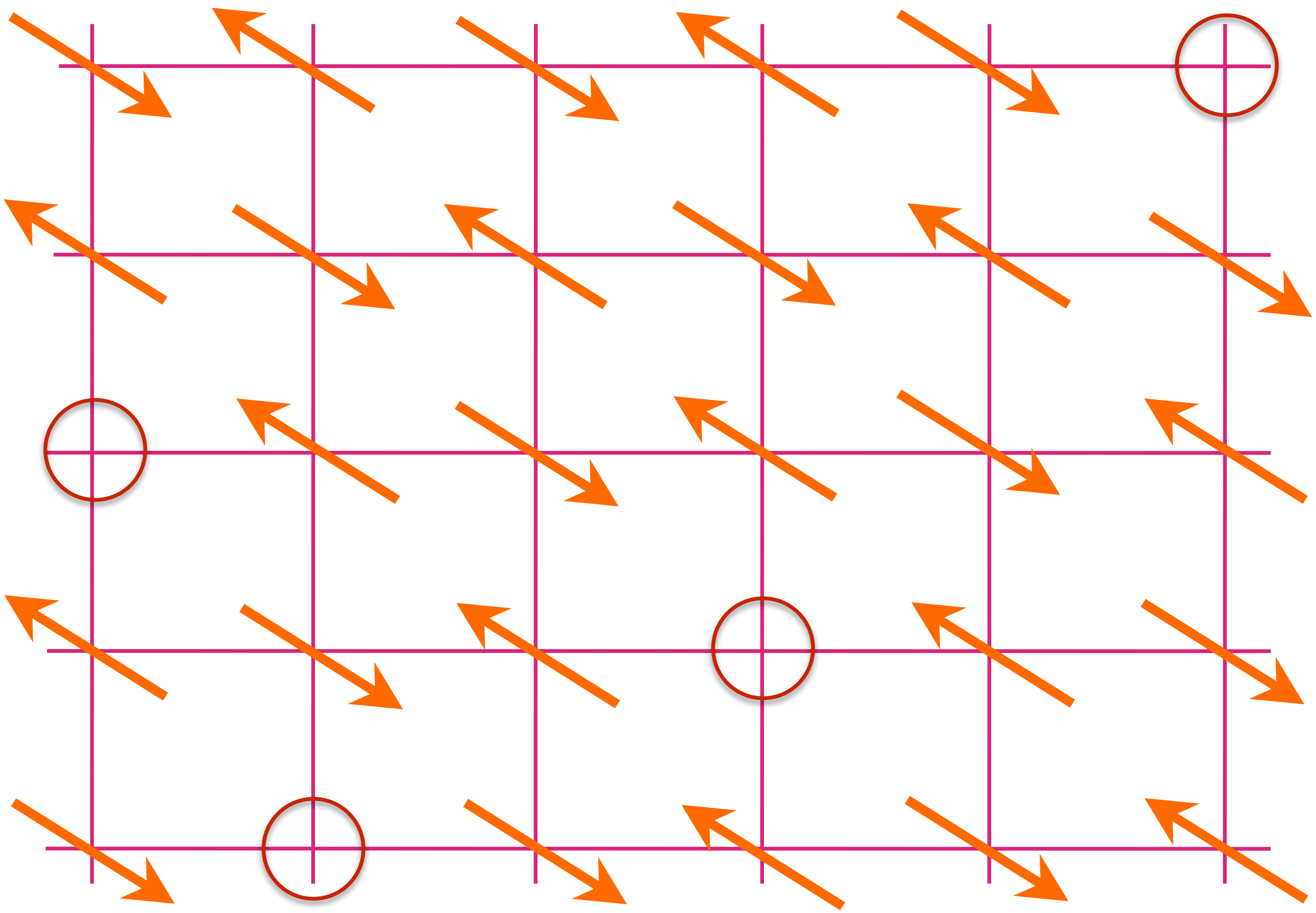
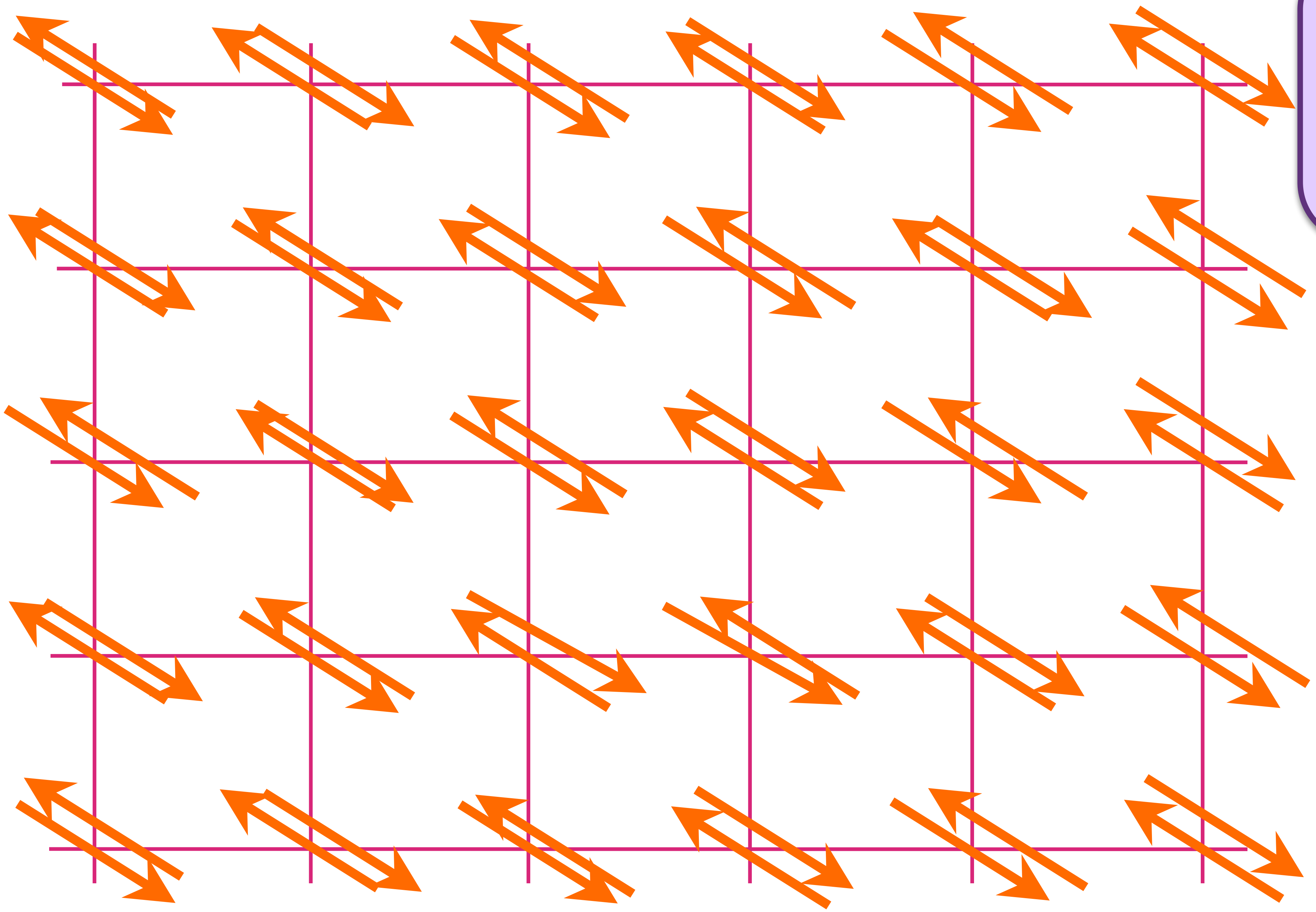


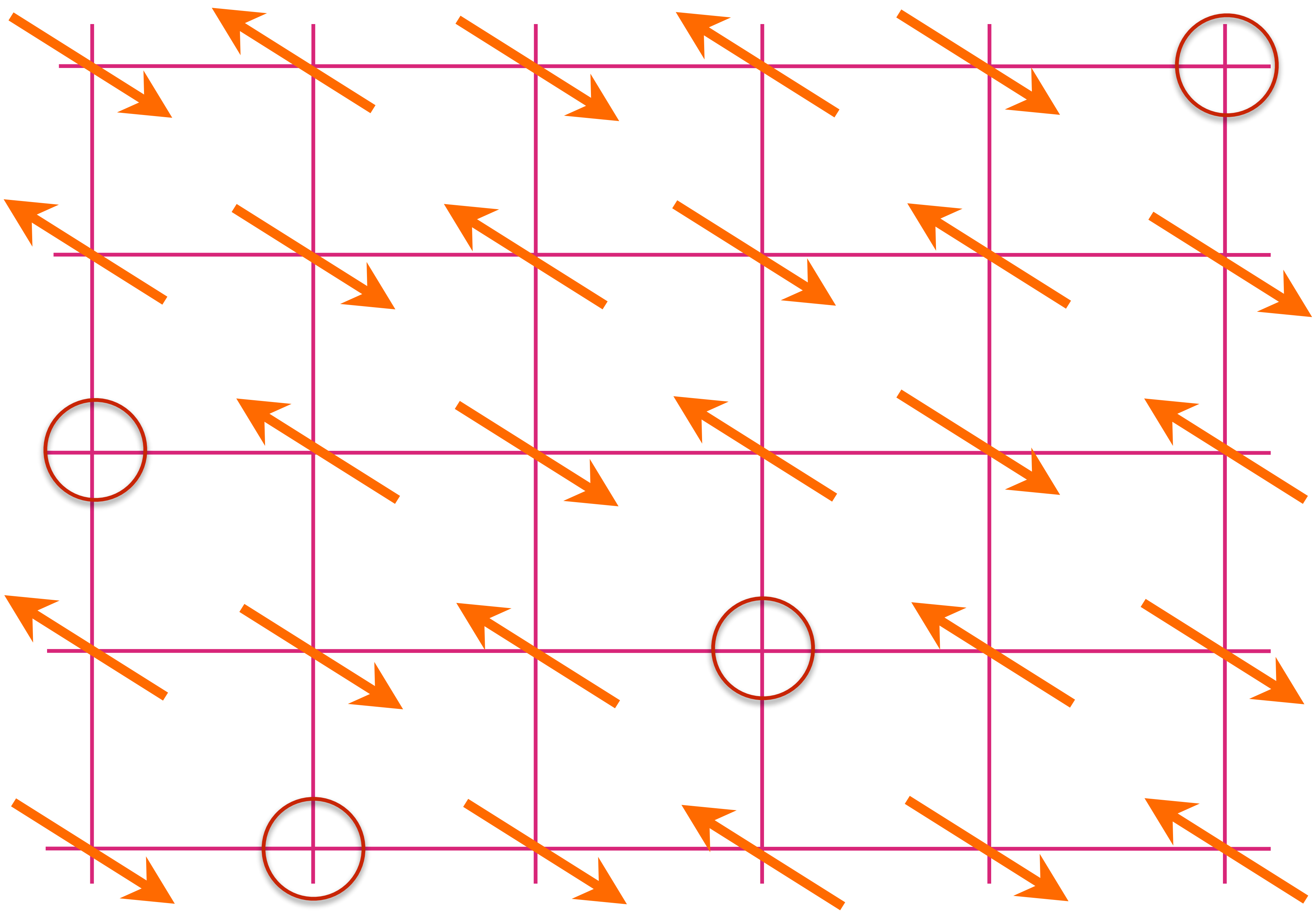
“Undoped” Anti-ferromagnet



Anti-ferromagnet with p holes per square

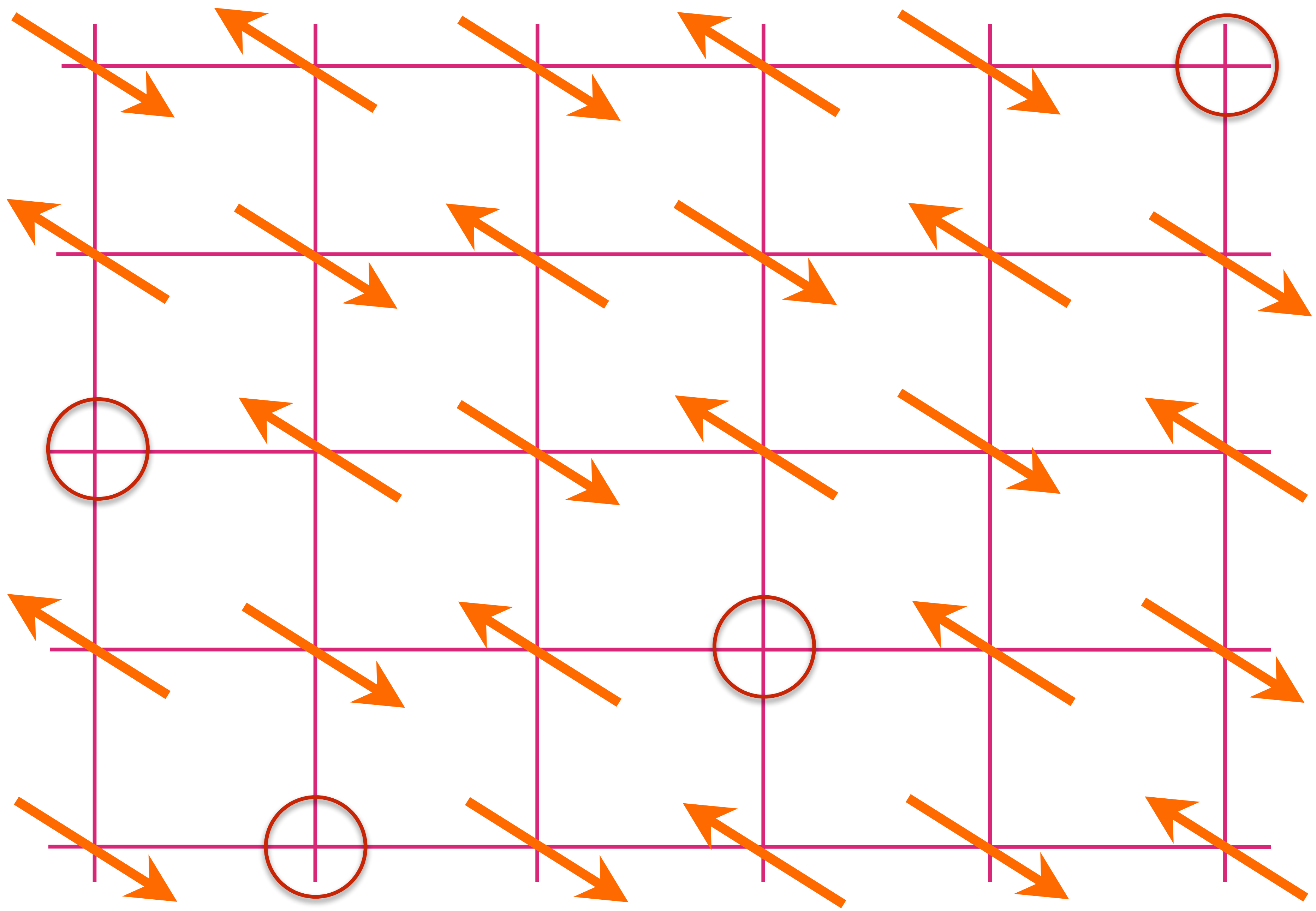


Filled Band



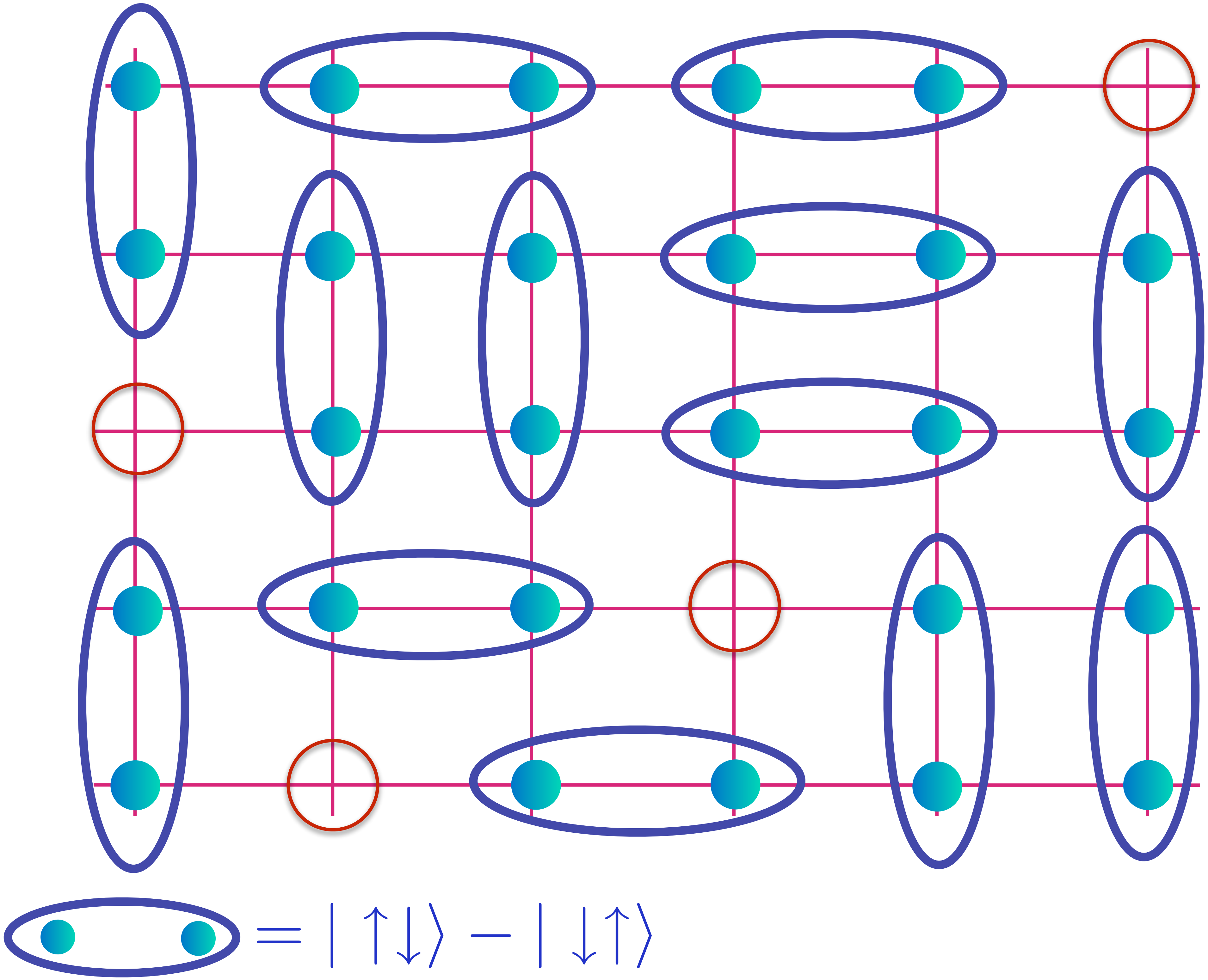
Anti-ferromagnet with p holes per square

But relative to the band insulator, there are $1+p$ holes per square, and so a Fermi liquid has a Fermi surface of size $1+p$



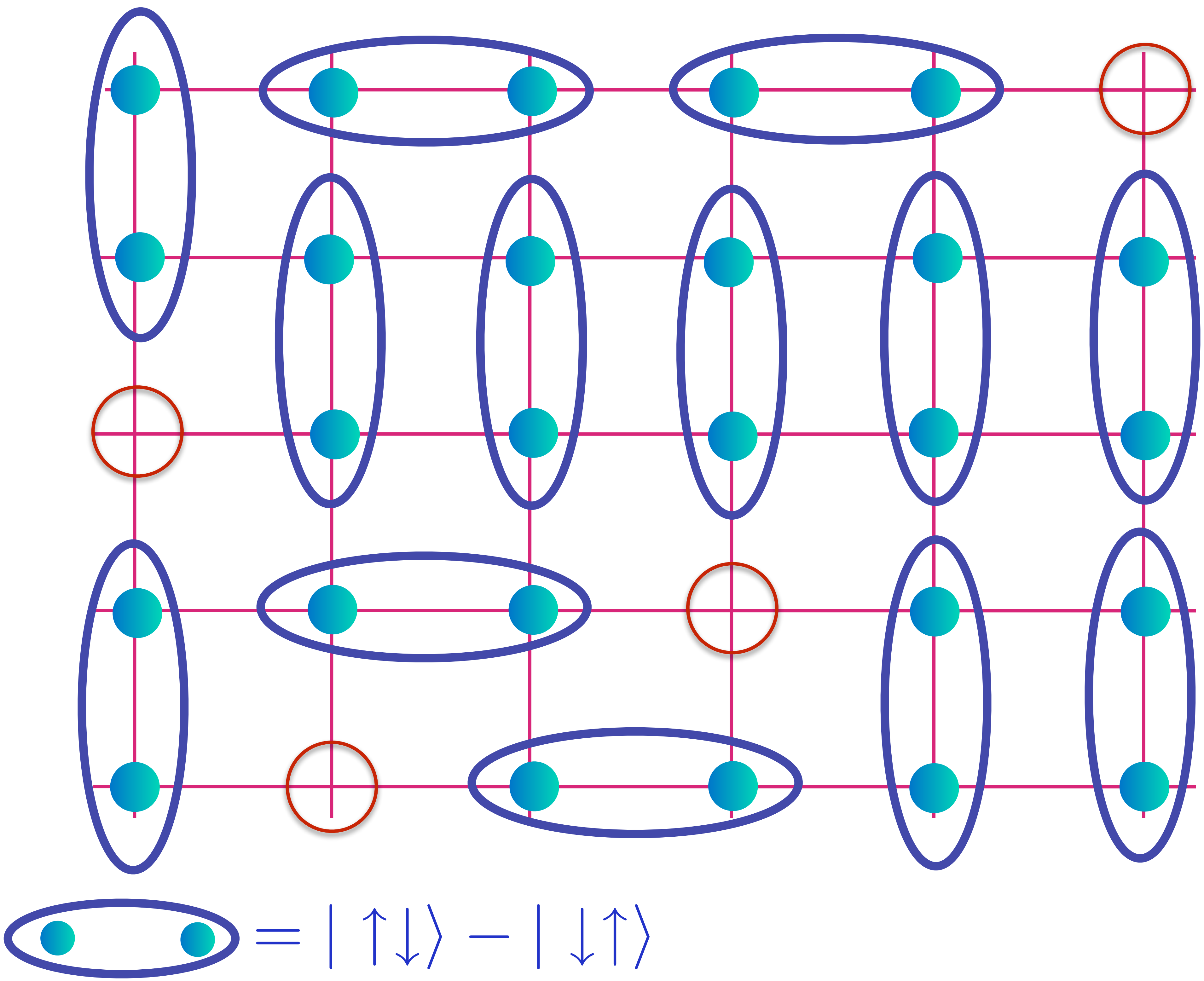
Anti-ferromagnet with p holes per square

Can we get a Fermi surface of size p?
(and full square lattice symmetry)



Spin liquid with density p of spinless, charge +e "holons". These can form a Fermi surface of size p, but this is not visible in electron photo-emission

Baskaran, Zou, Anderson, Fradkin, Kivelson…



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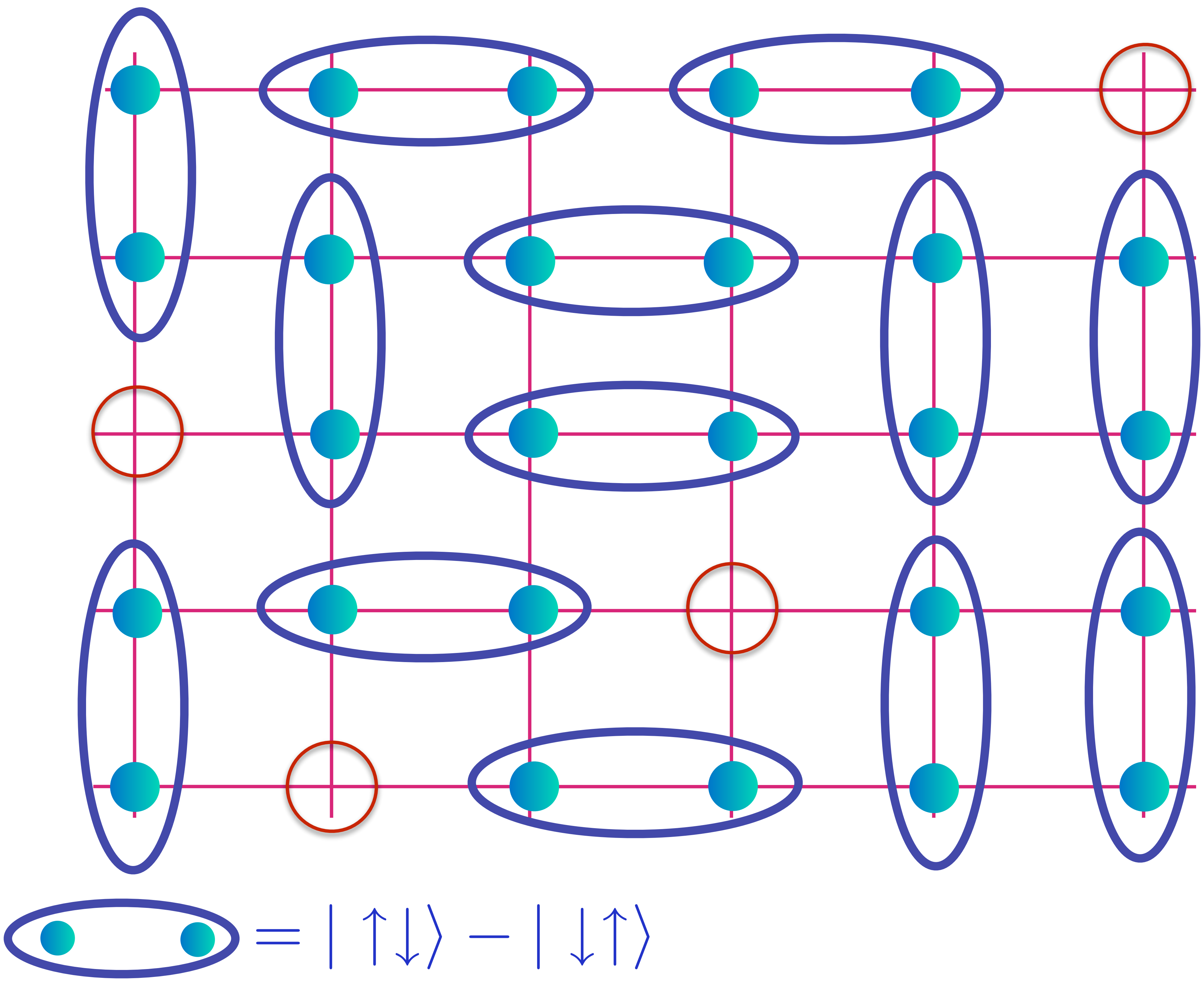

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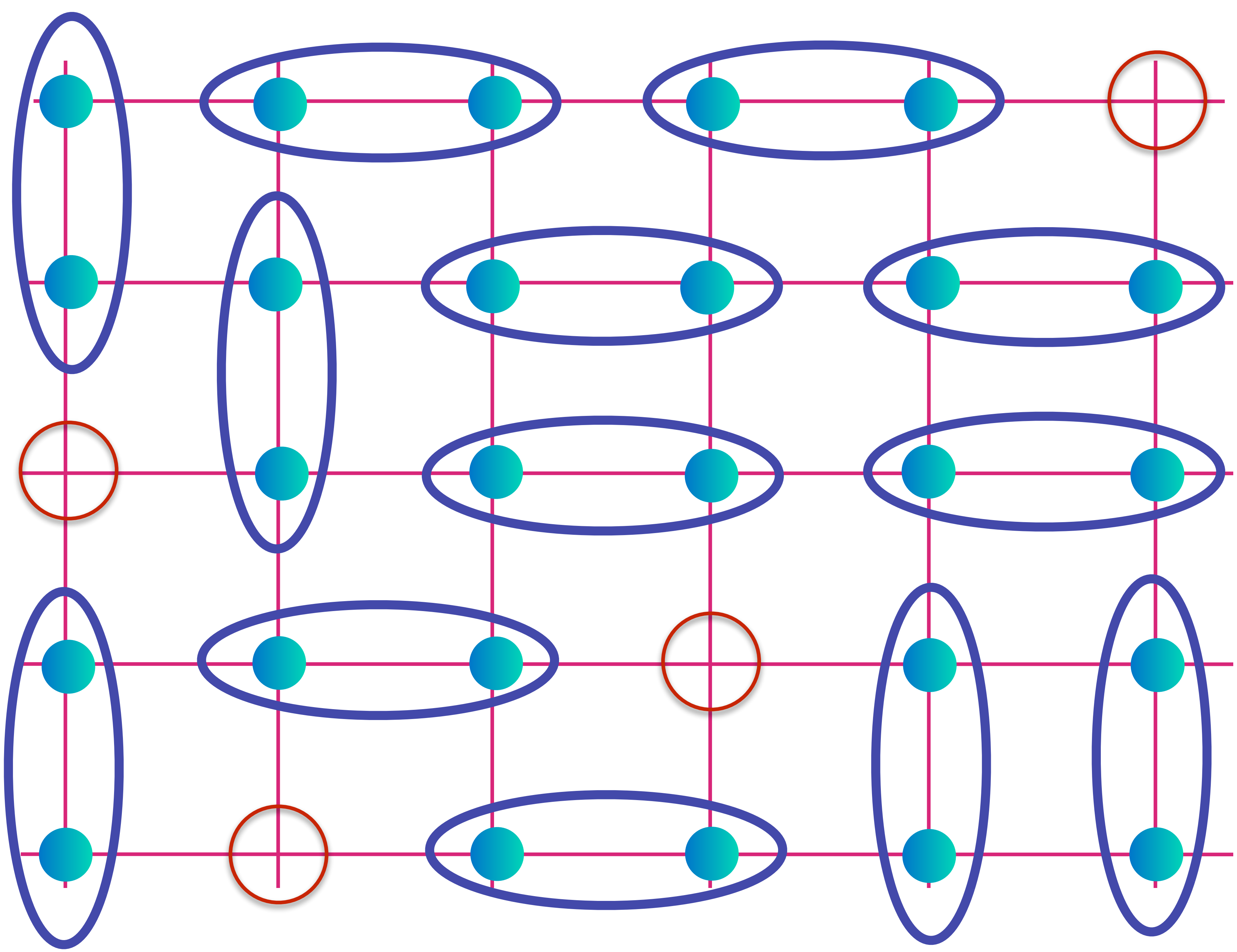

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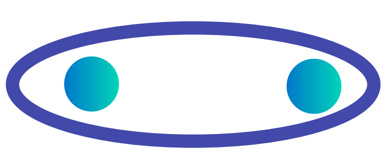

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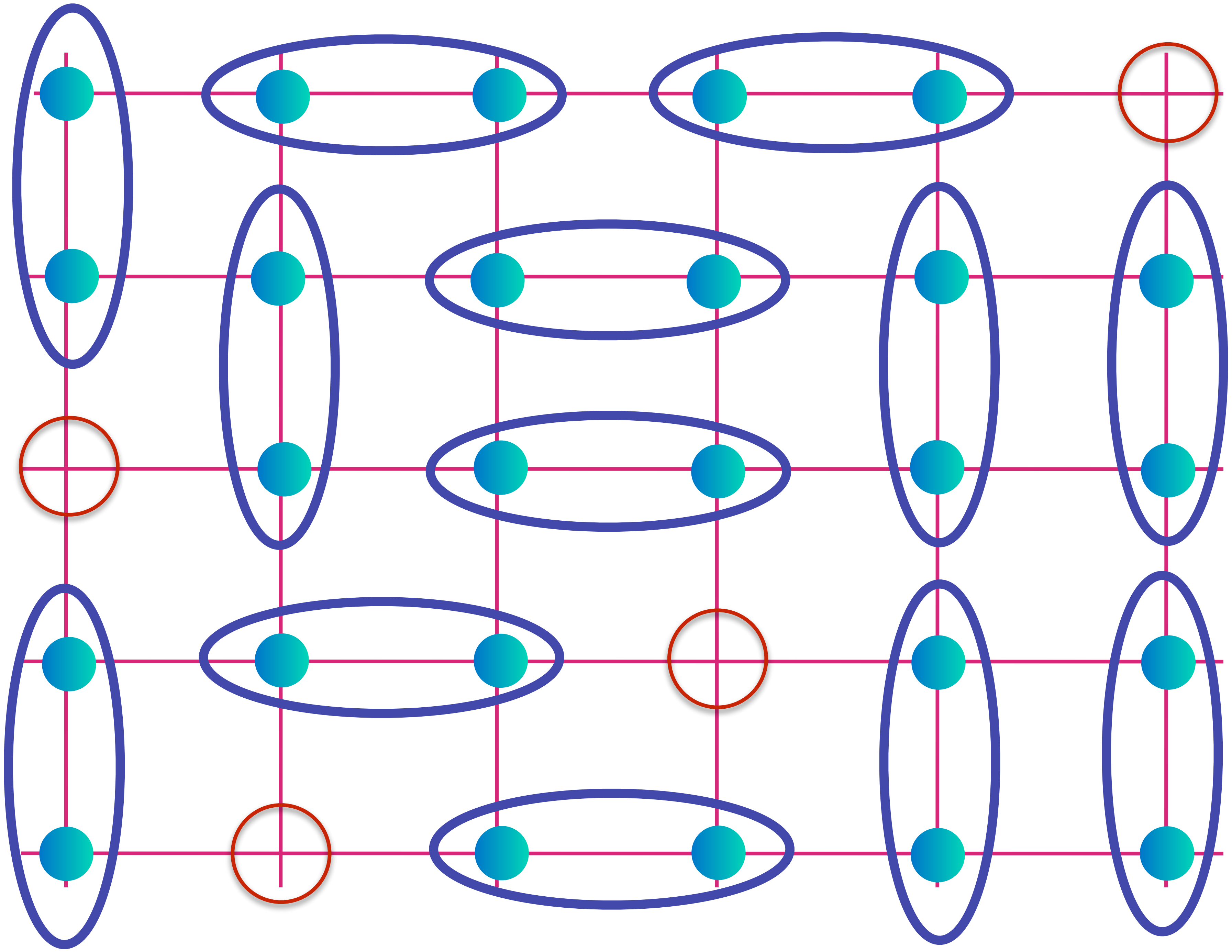

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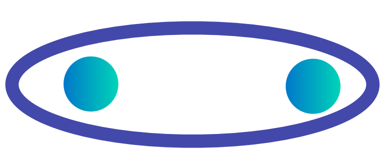 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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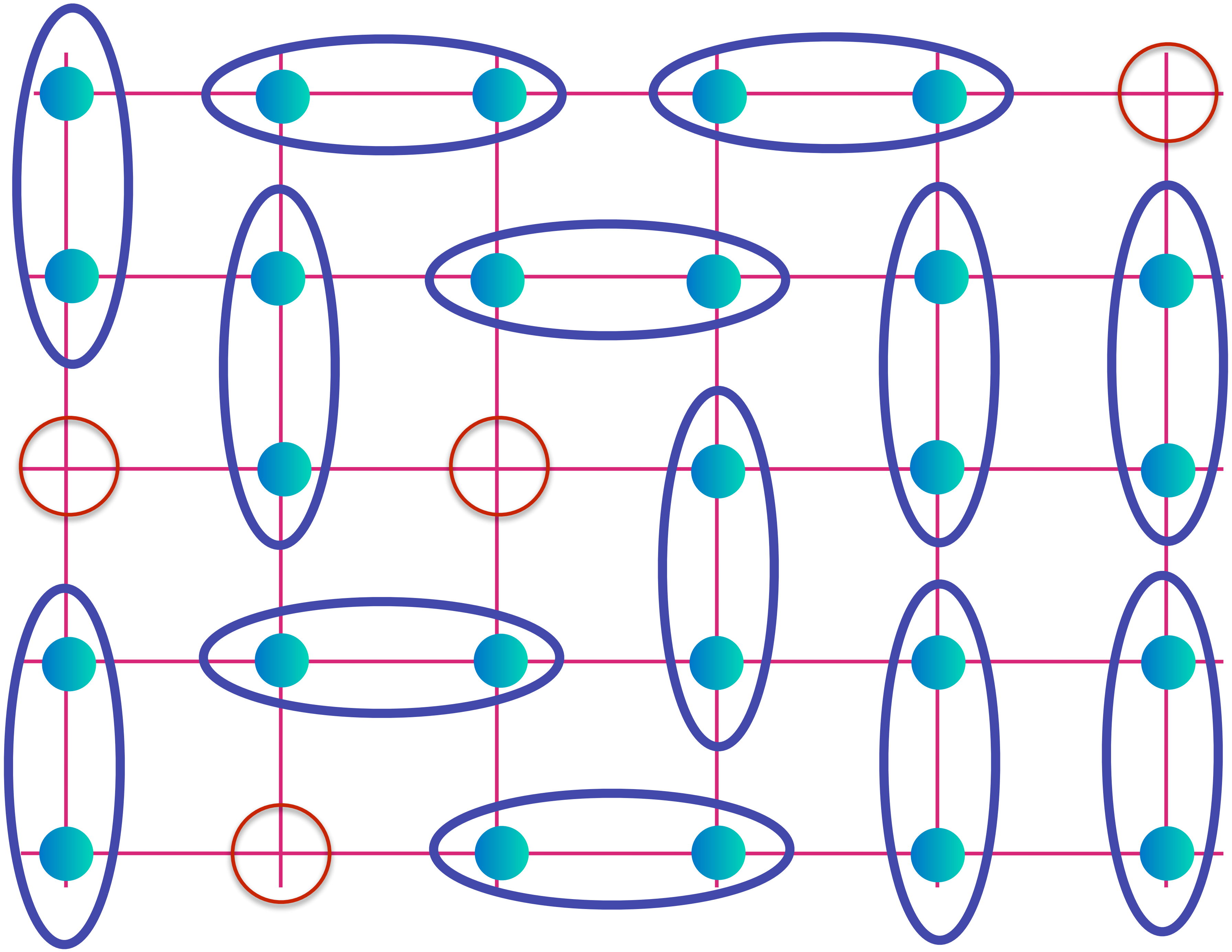

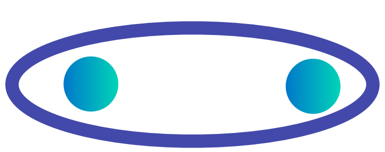 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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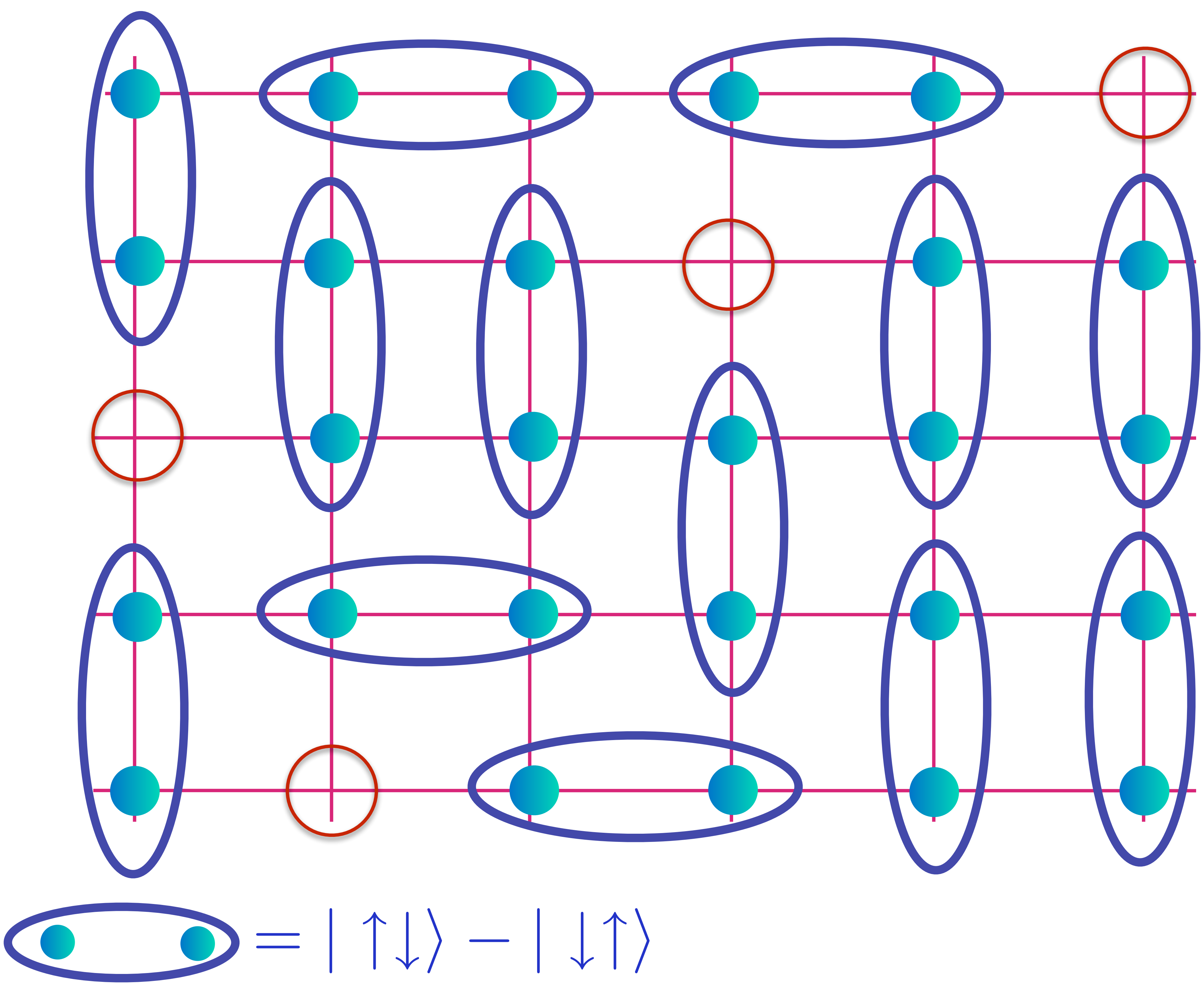

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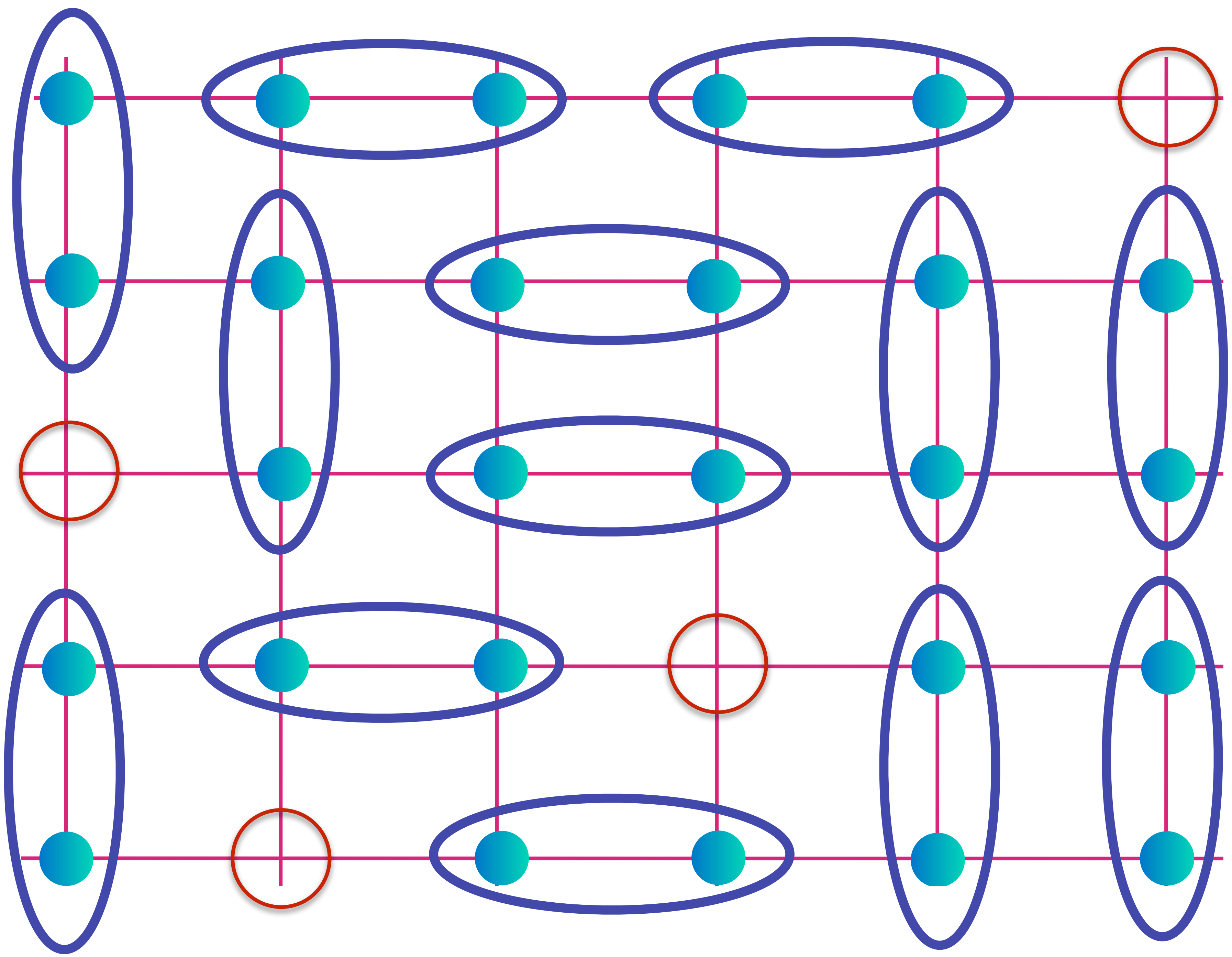

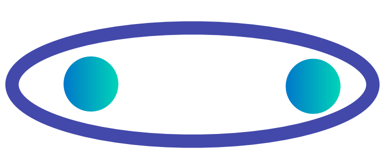 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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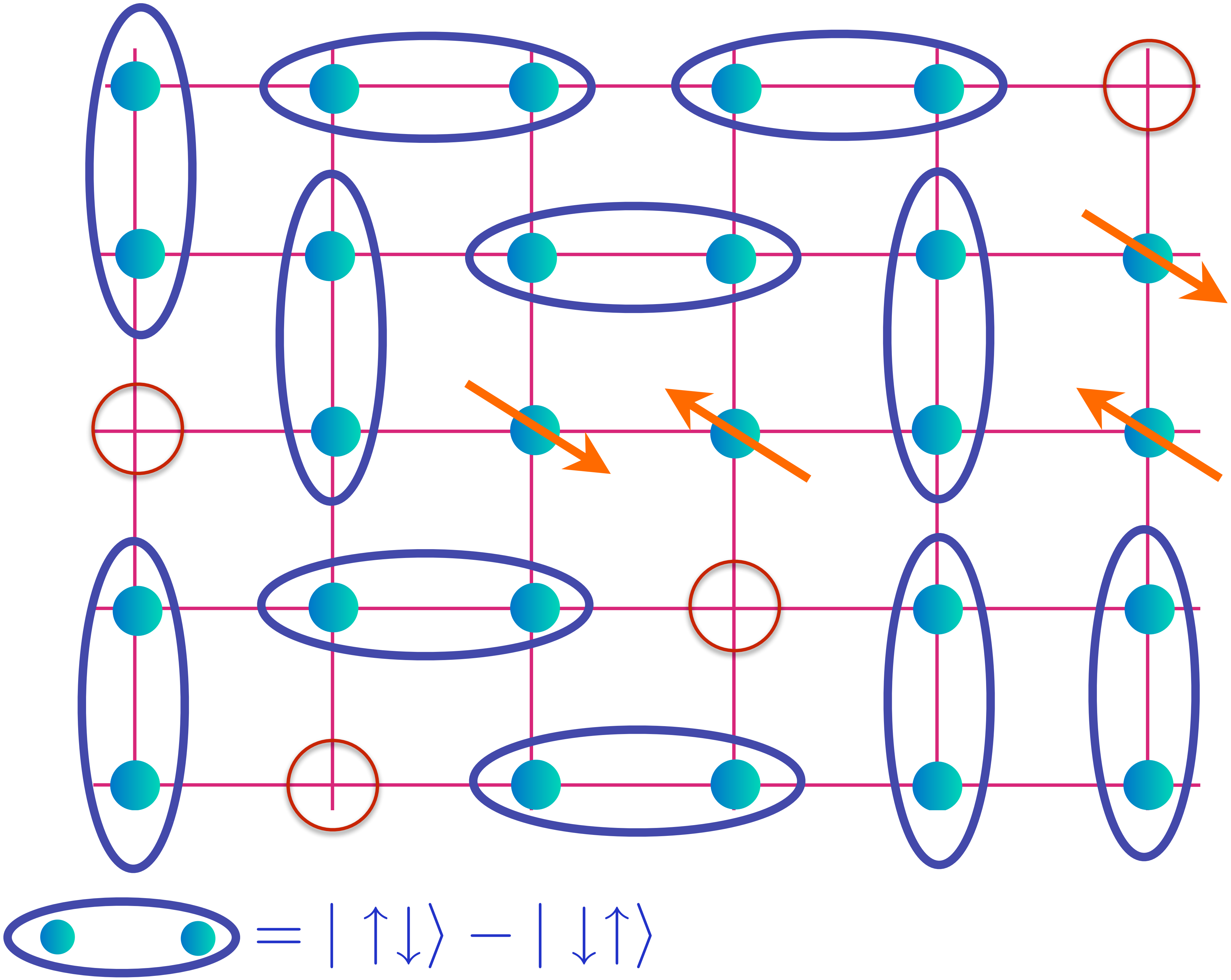
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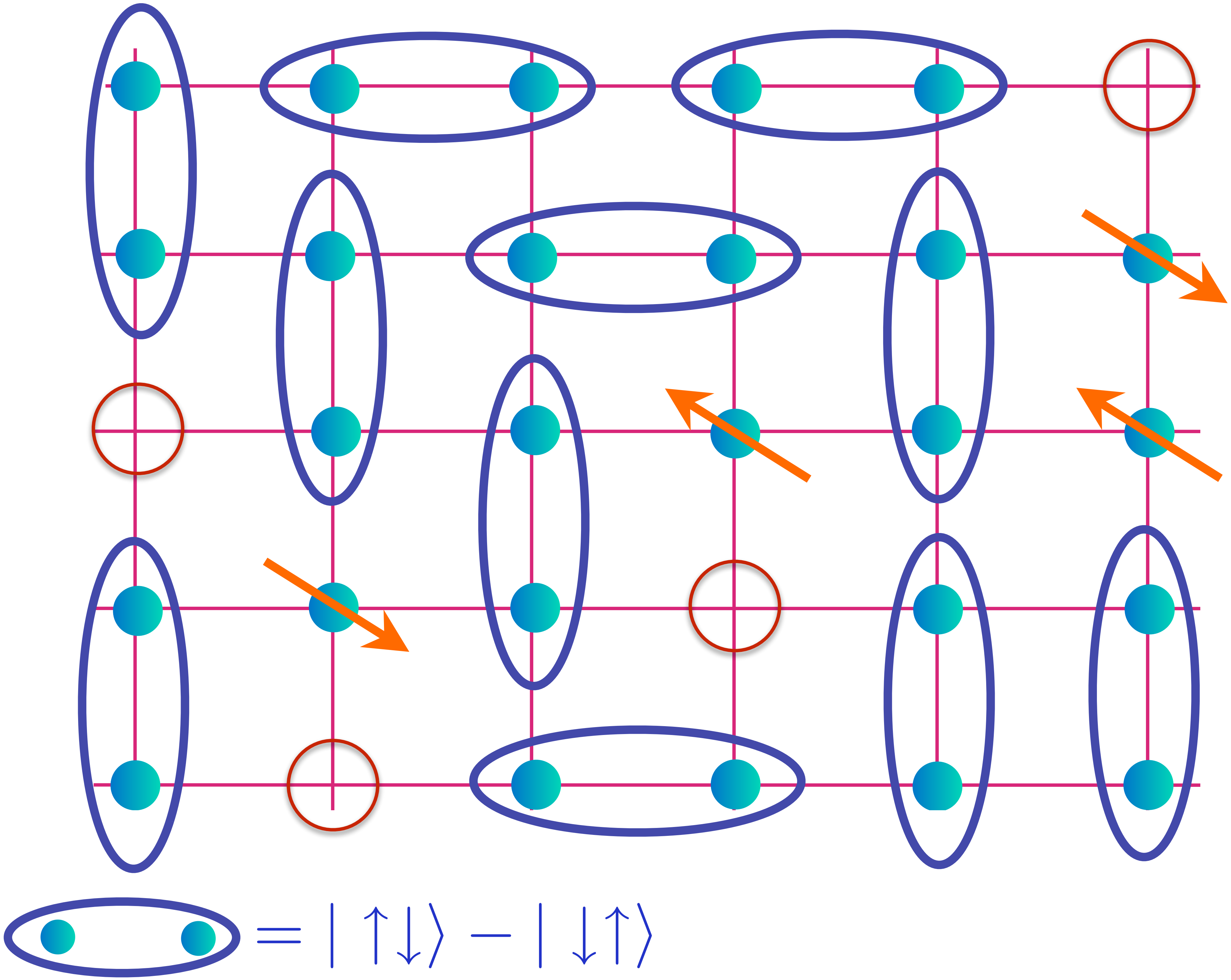

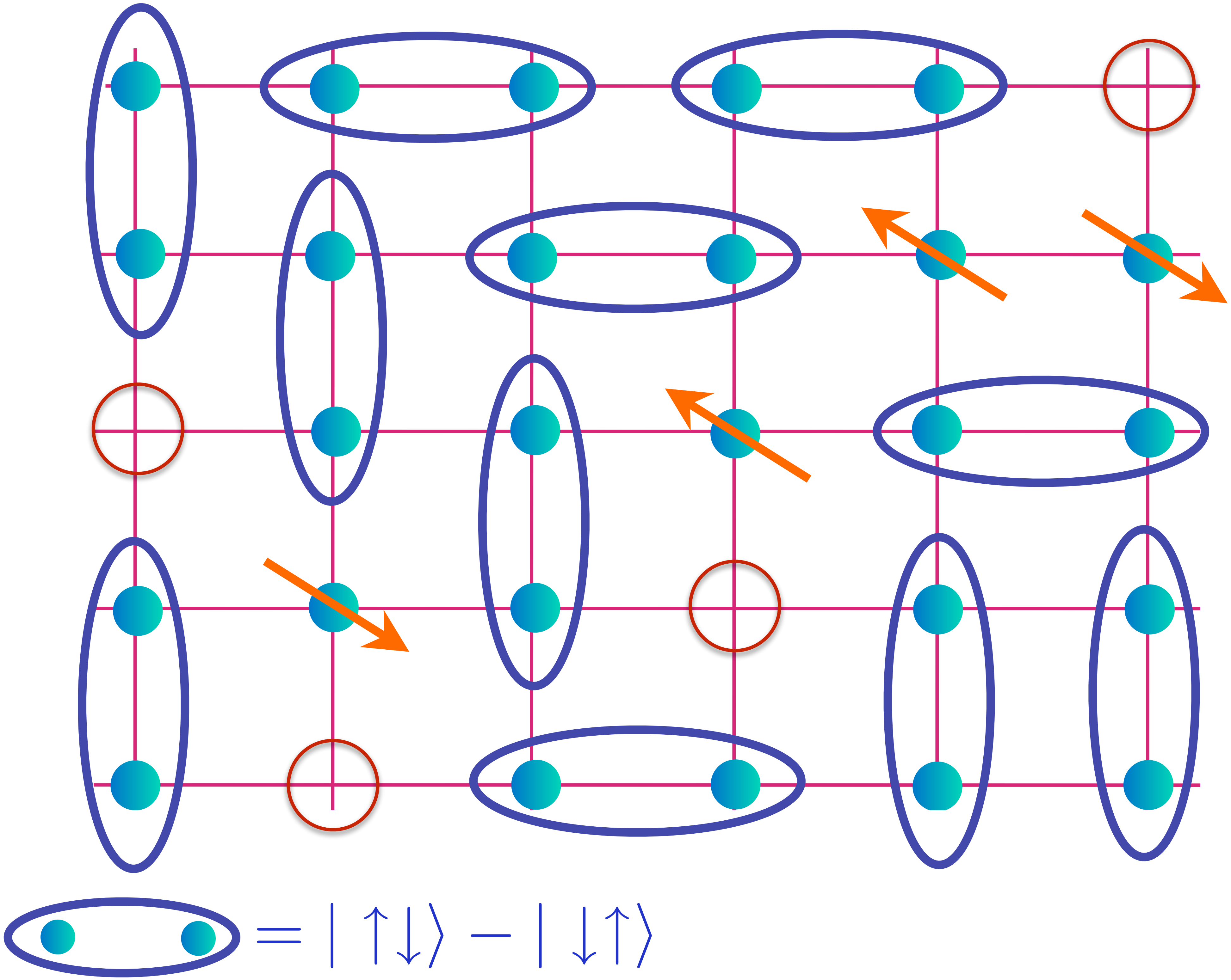
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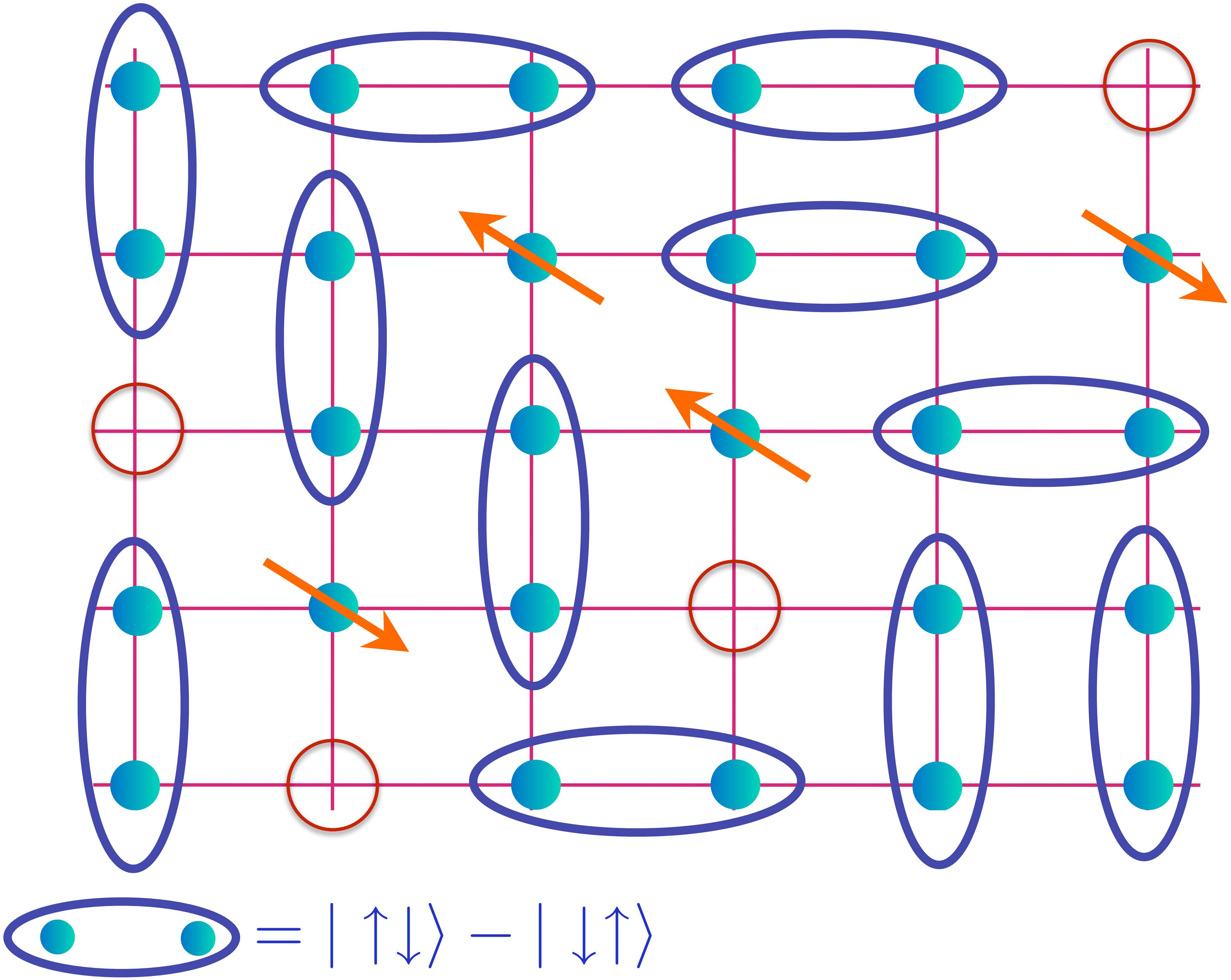
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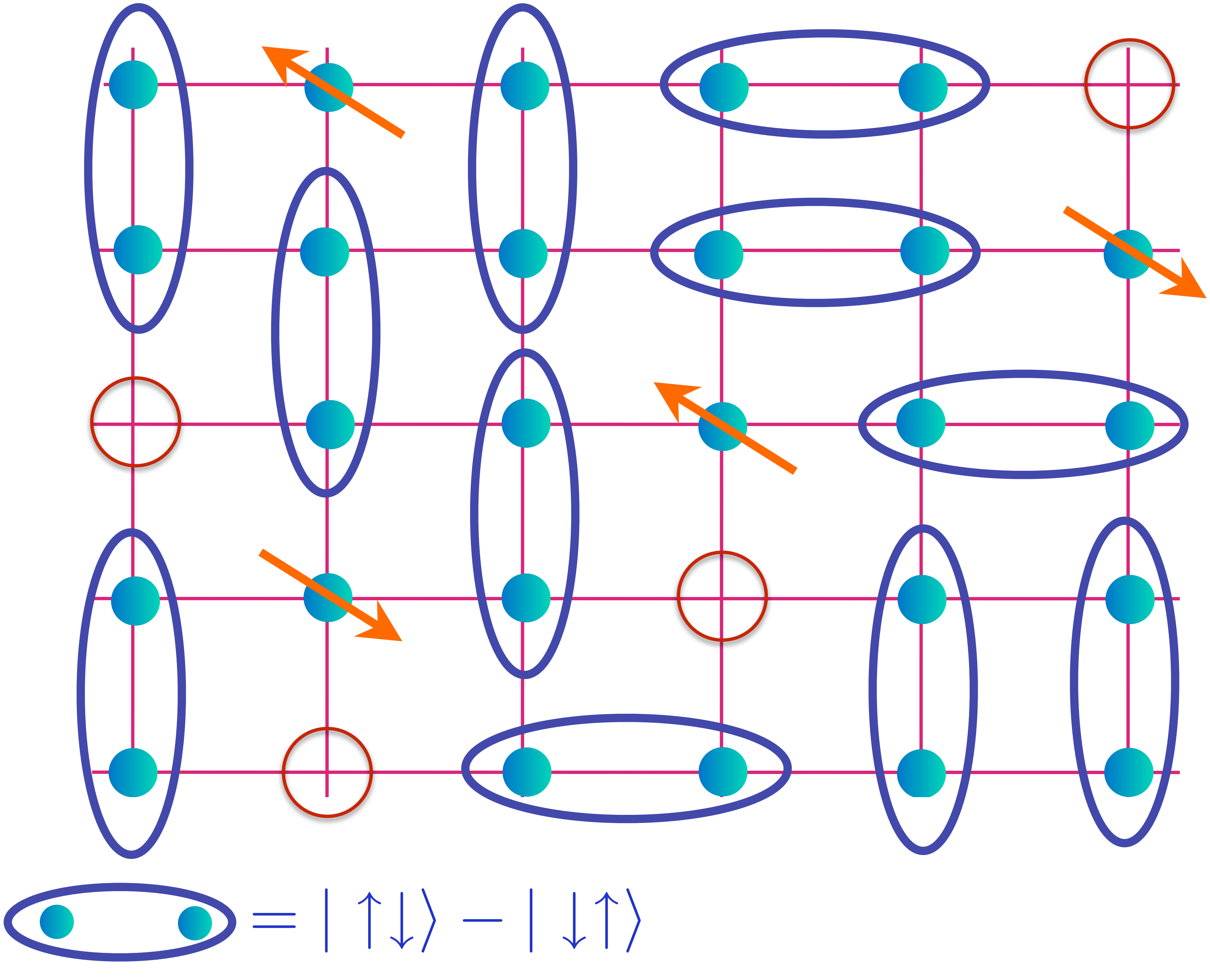
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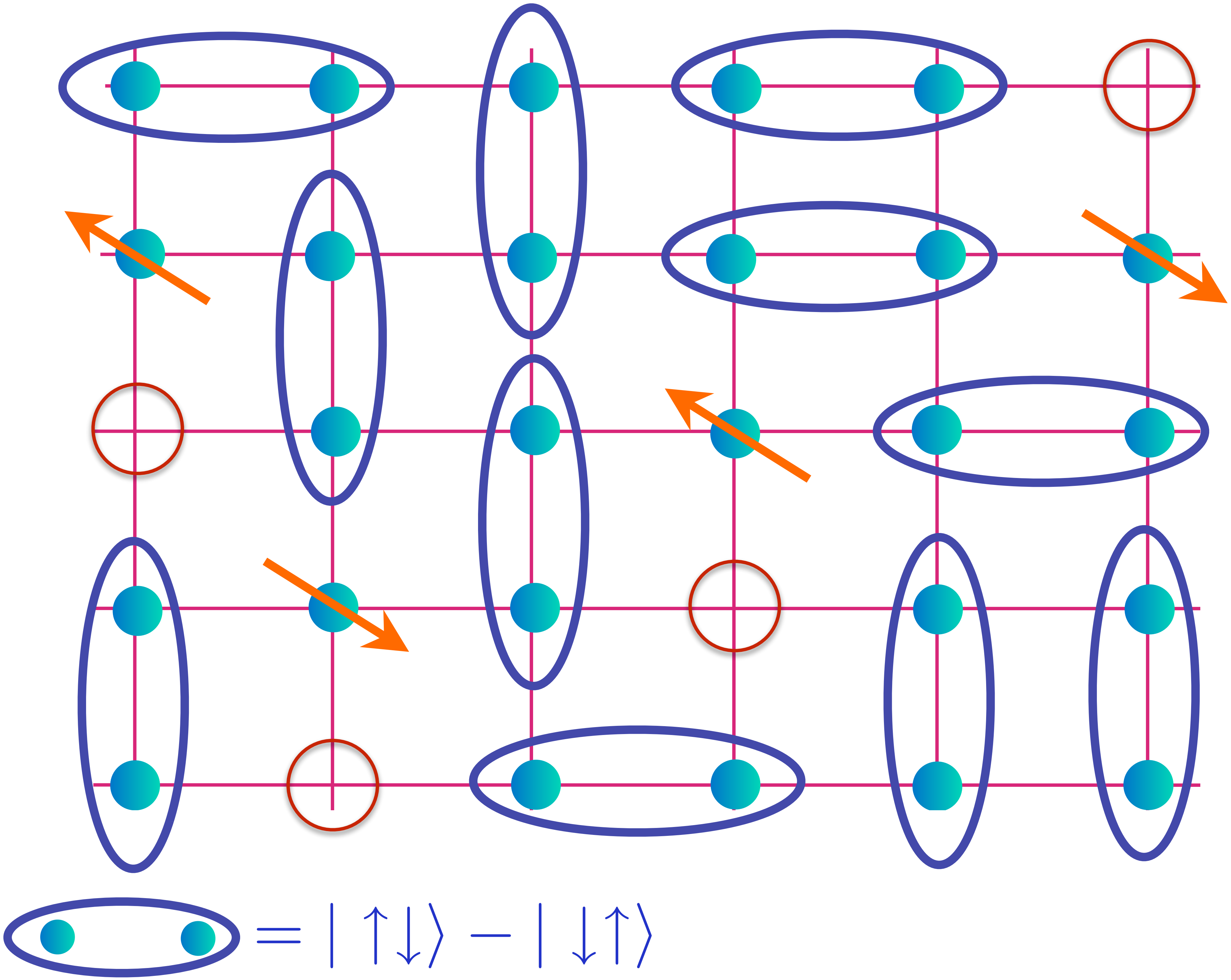
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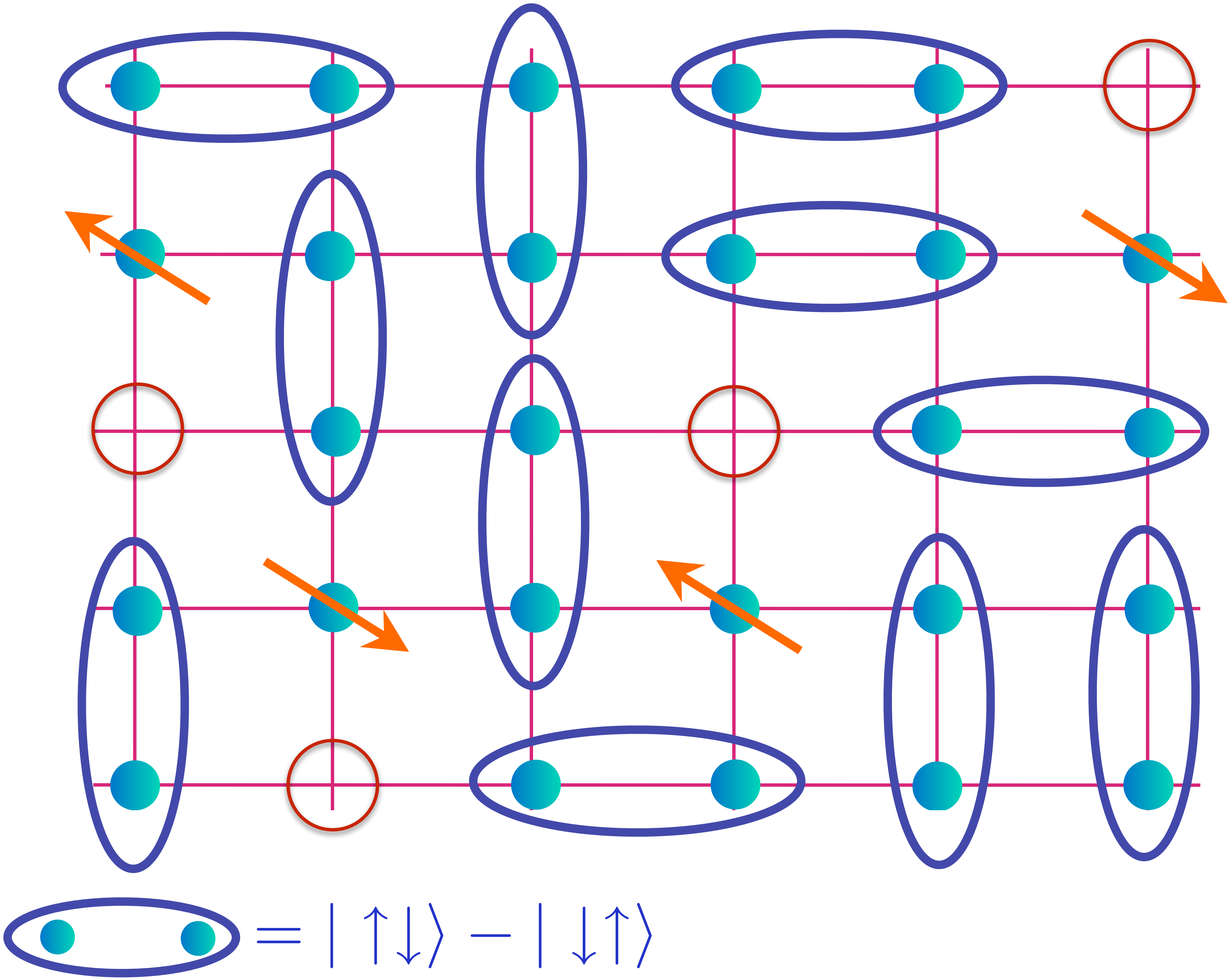
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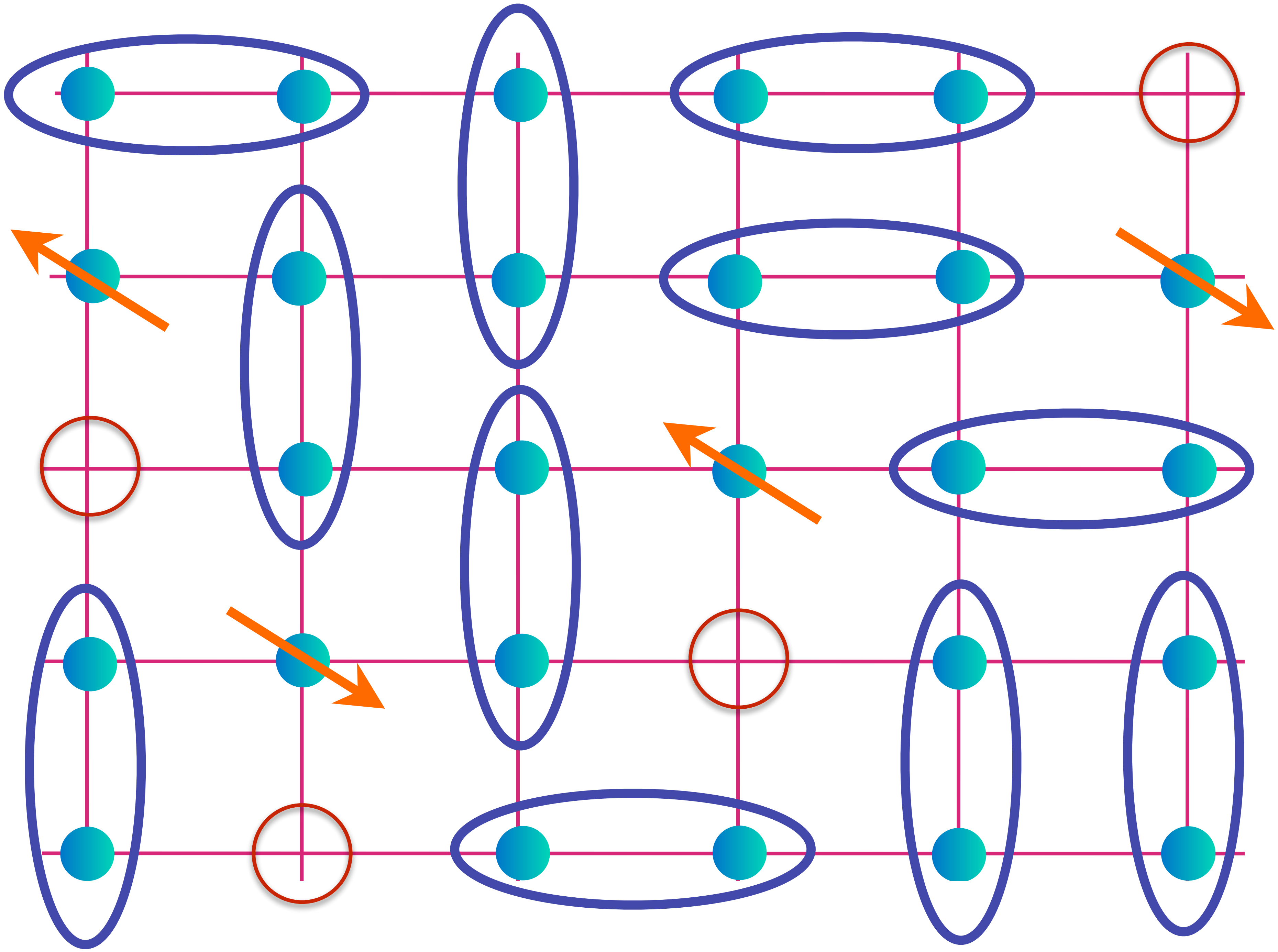
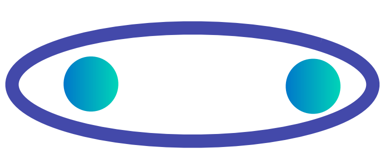
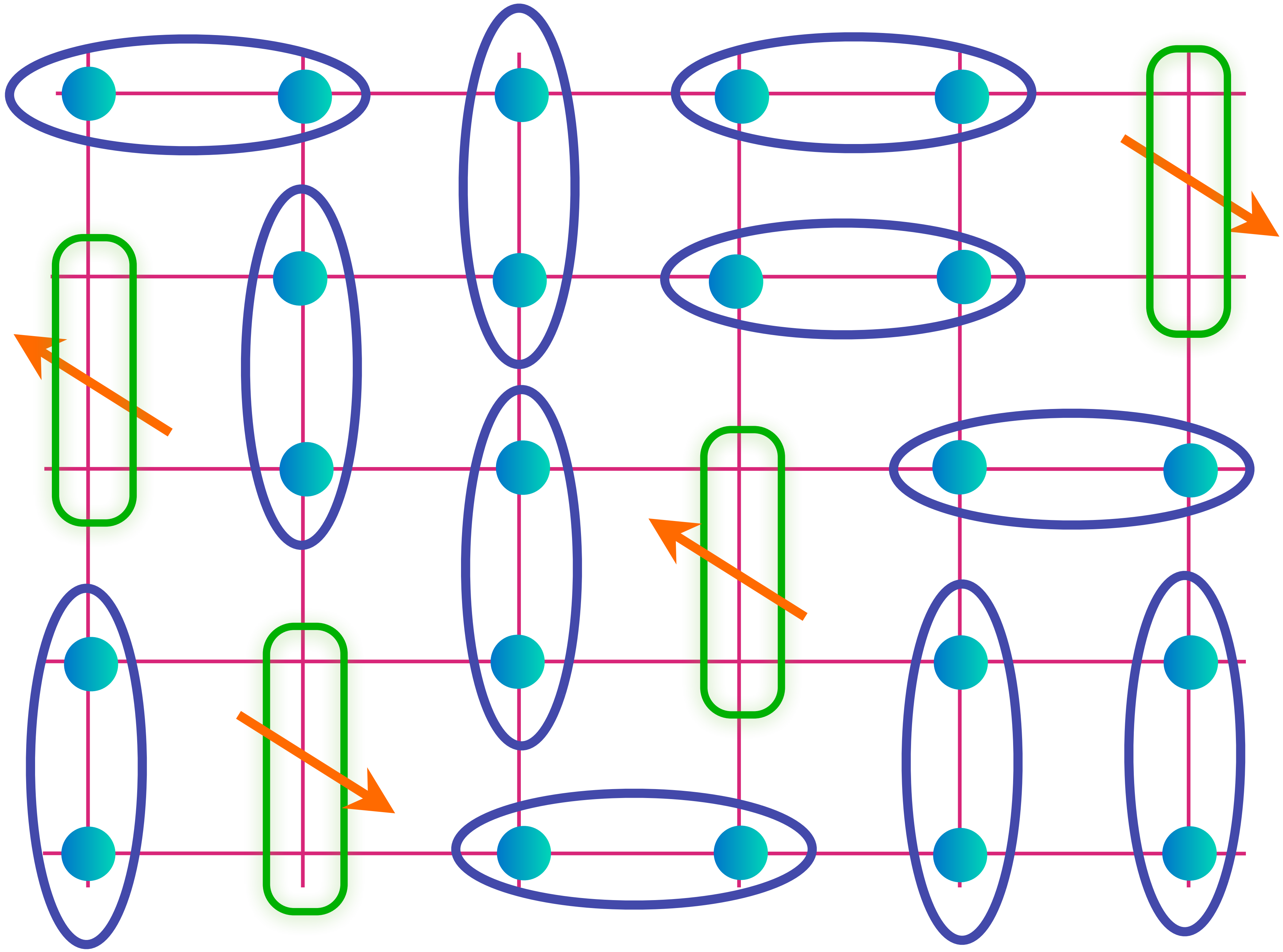


 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Fractionalized Fermi liquid (FL*)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p visible in electron photo-emission

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M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)

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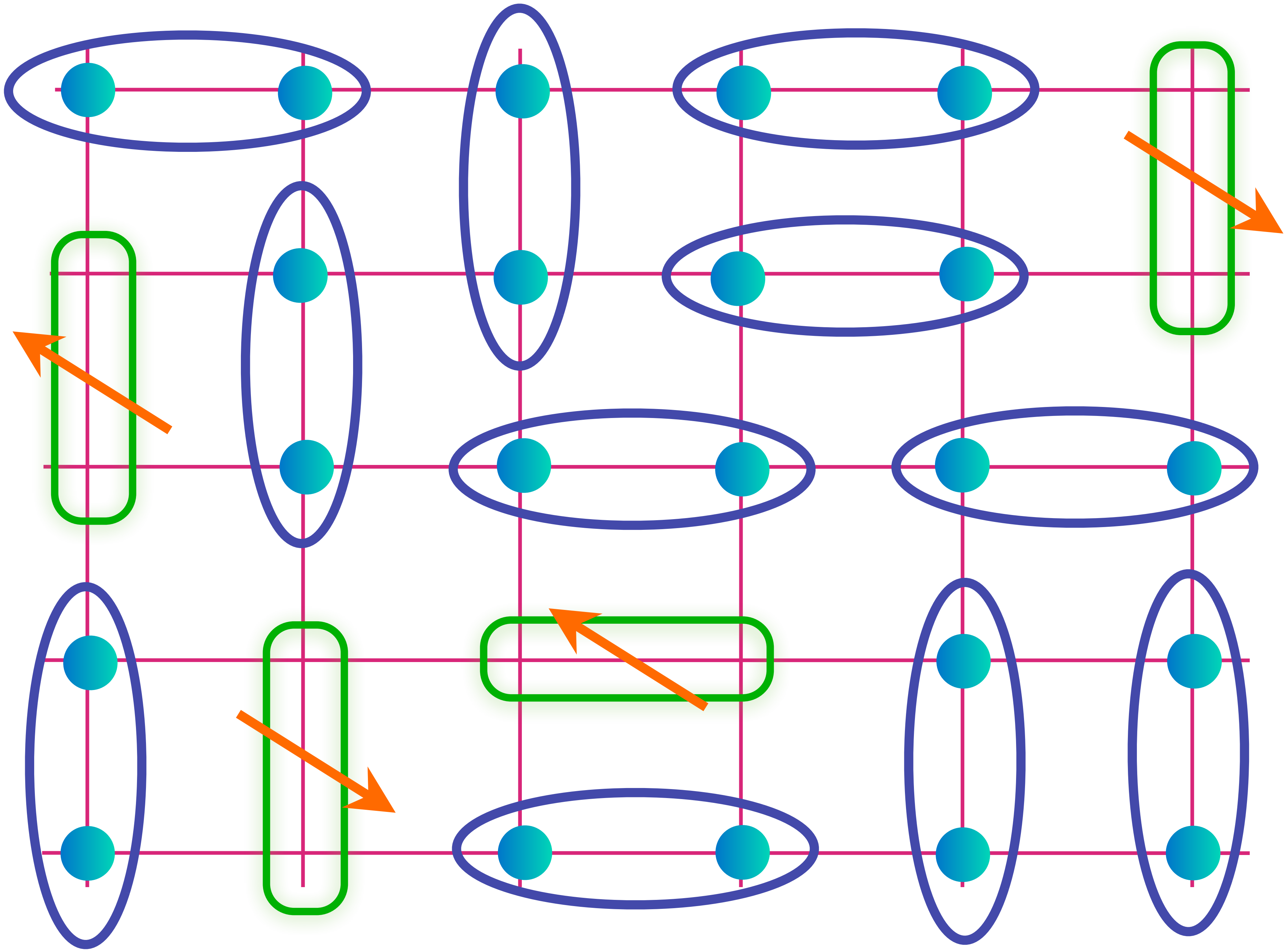

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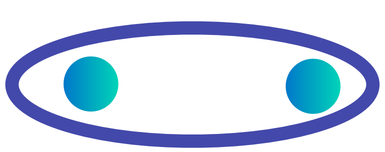

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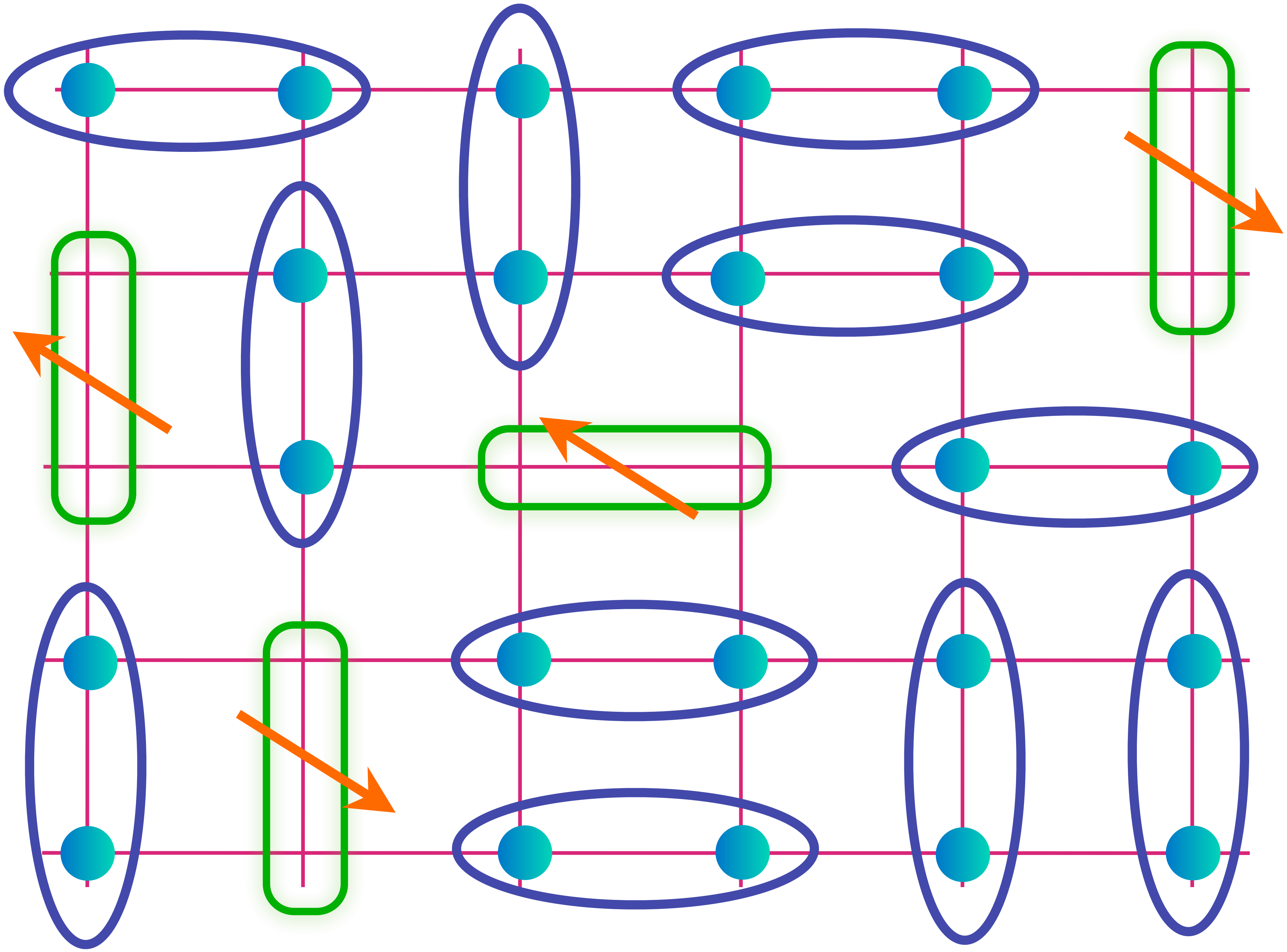
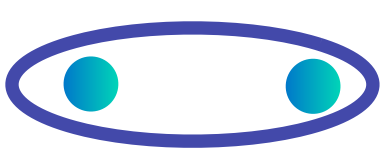


Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p visible in electron photo-emission



$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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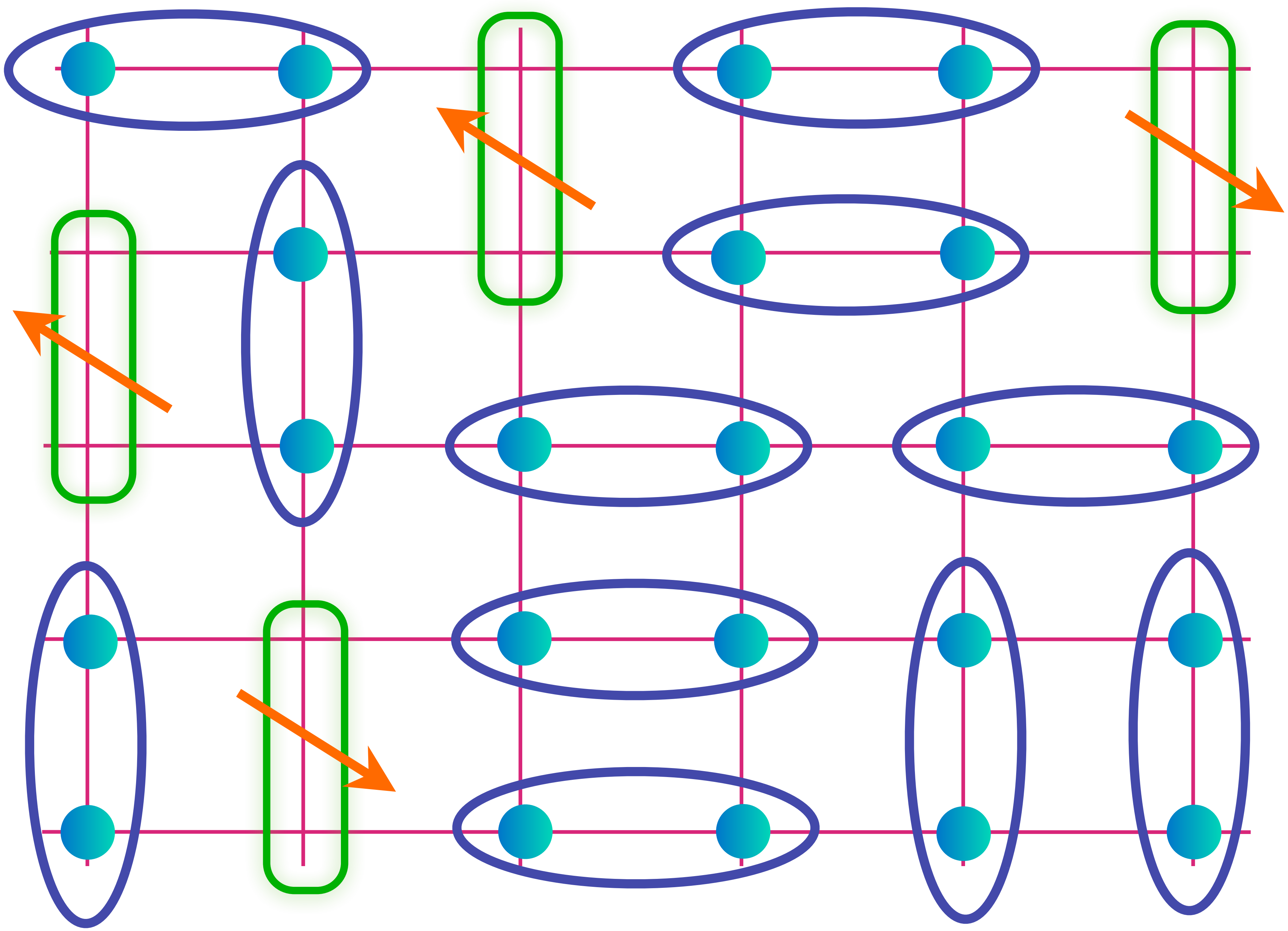

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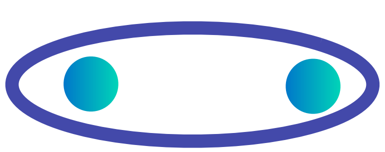 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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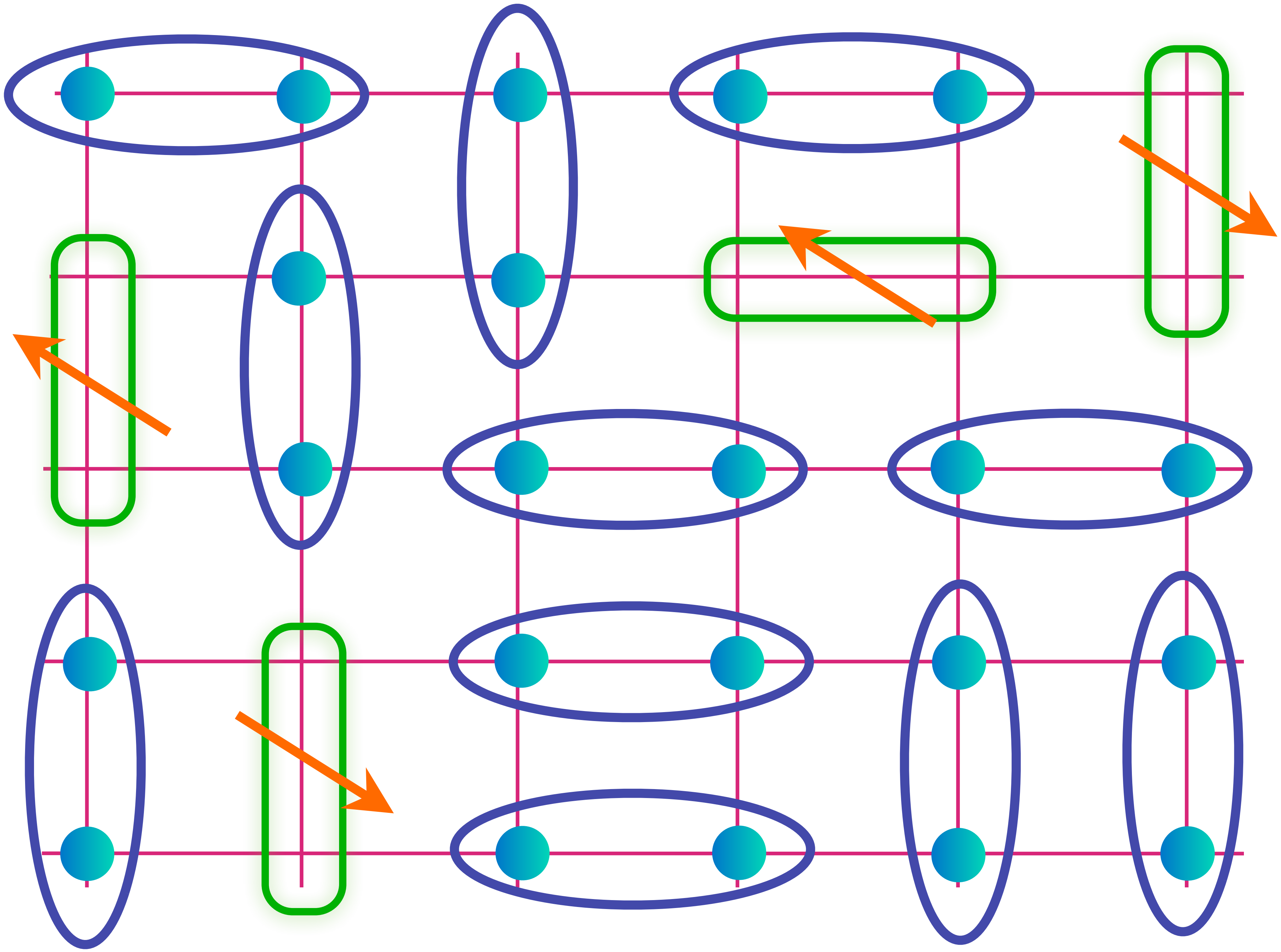

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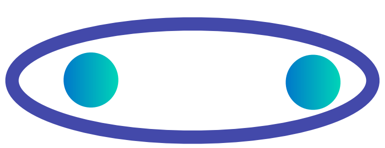 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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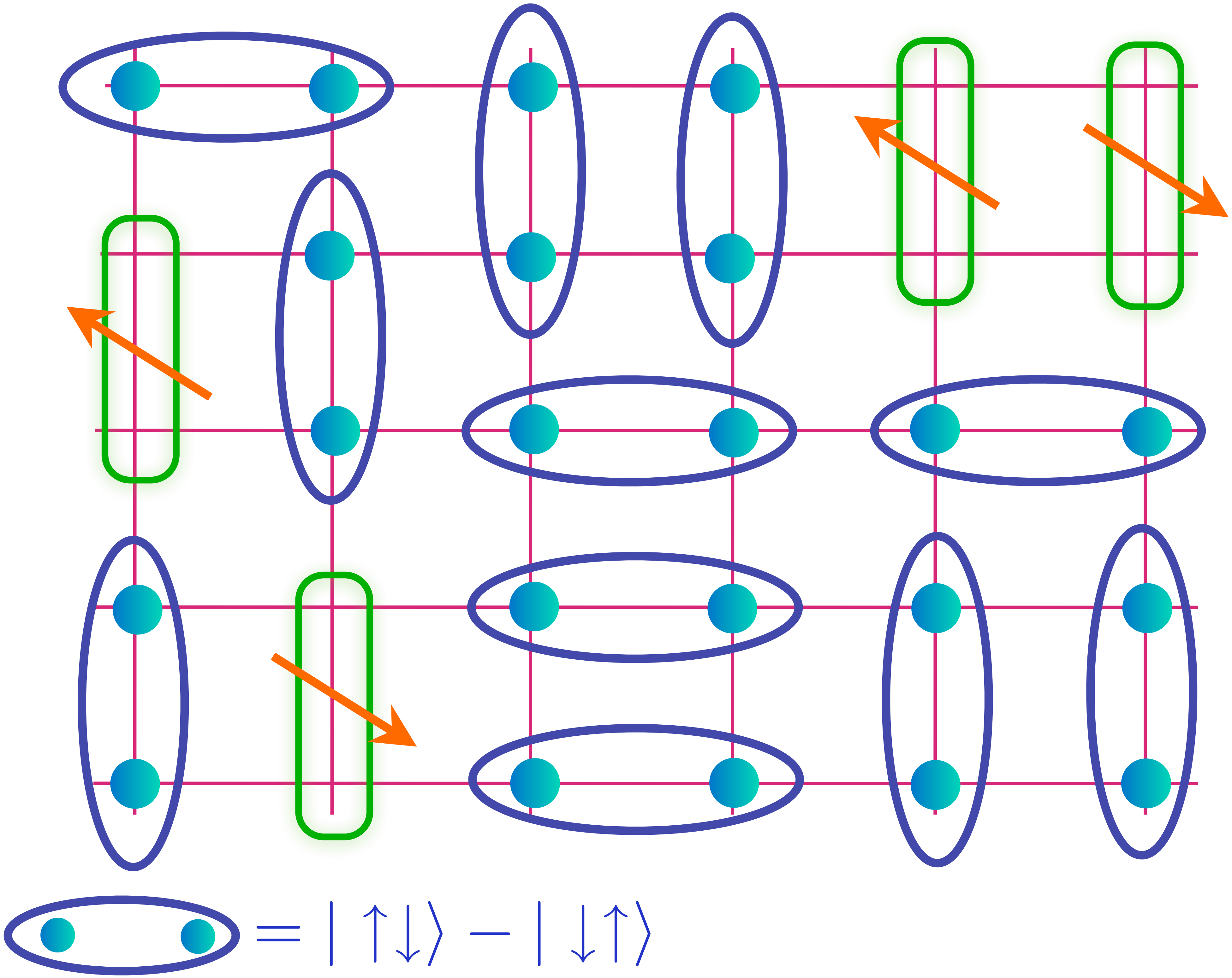

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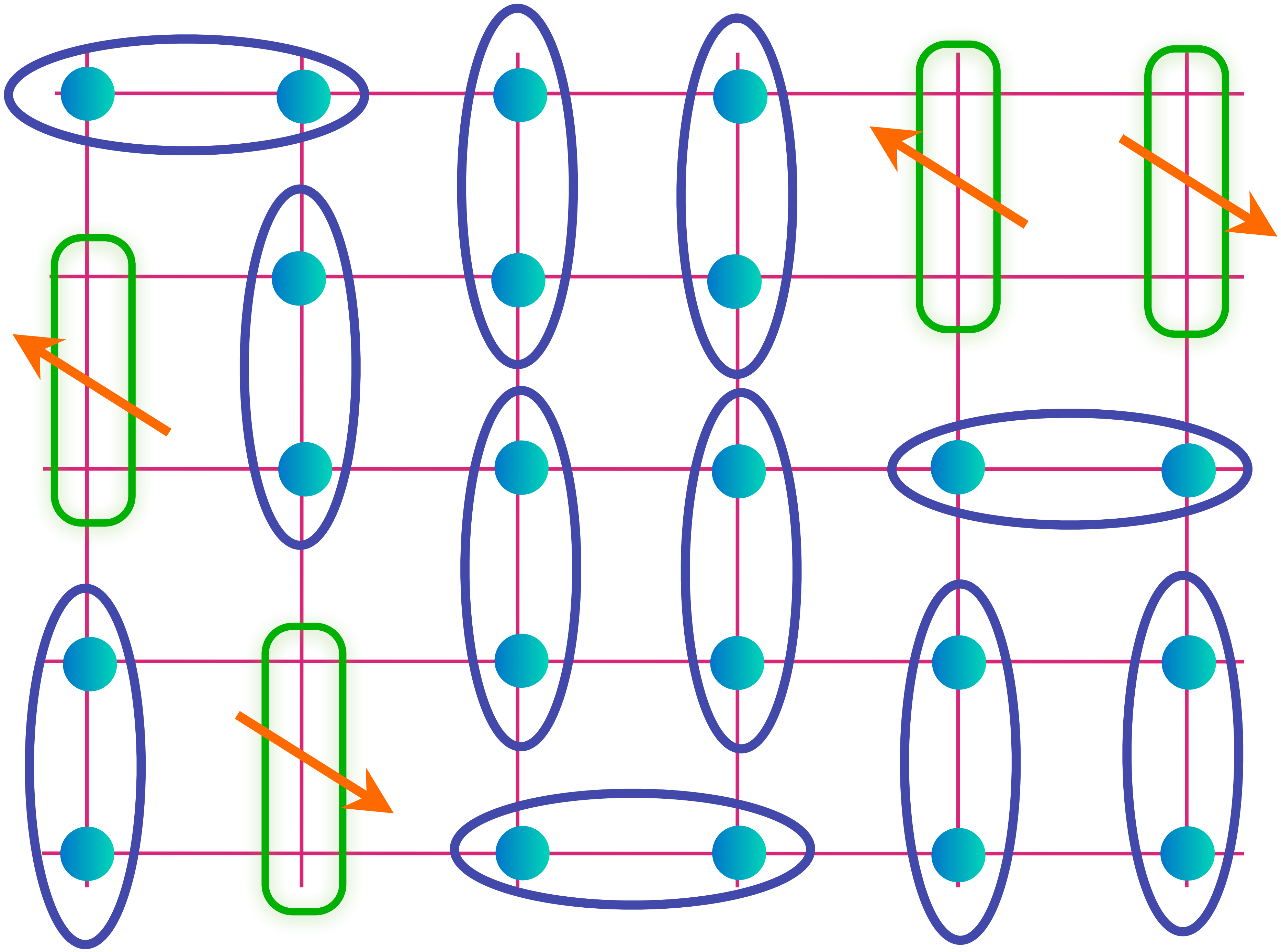

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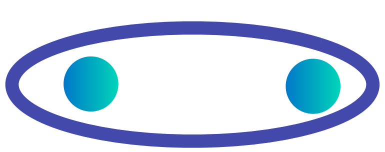 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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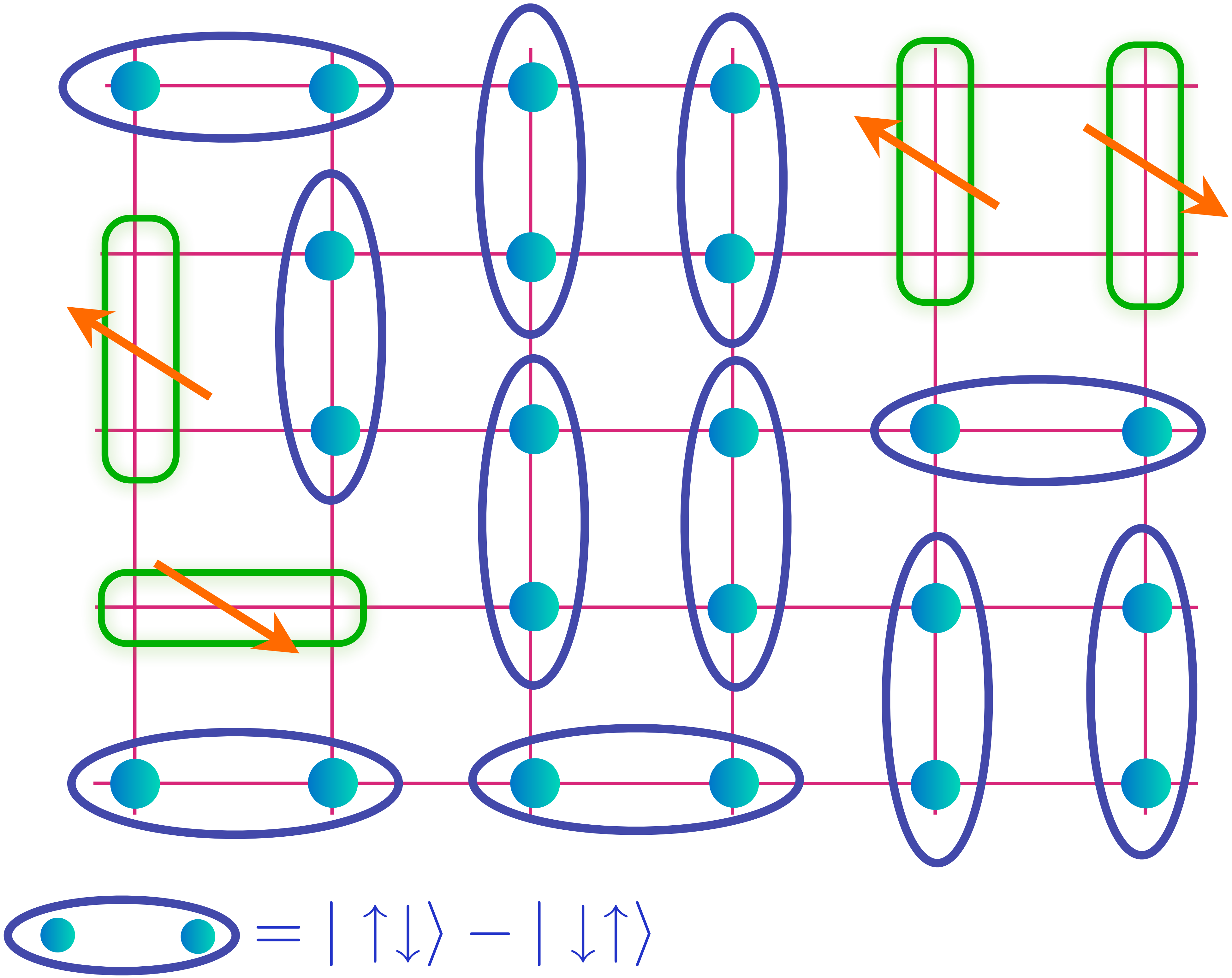

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M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)

Ground state degeneracy

Place FL*
on a torus:

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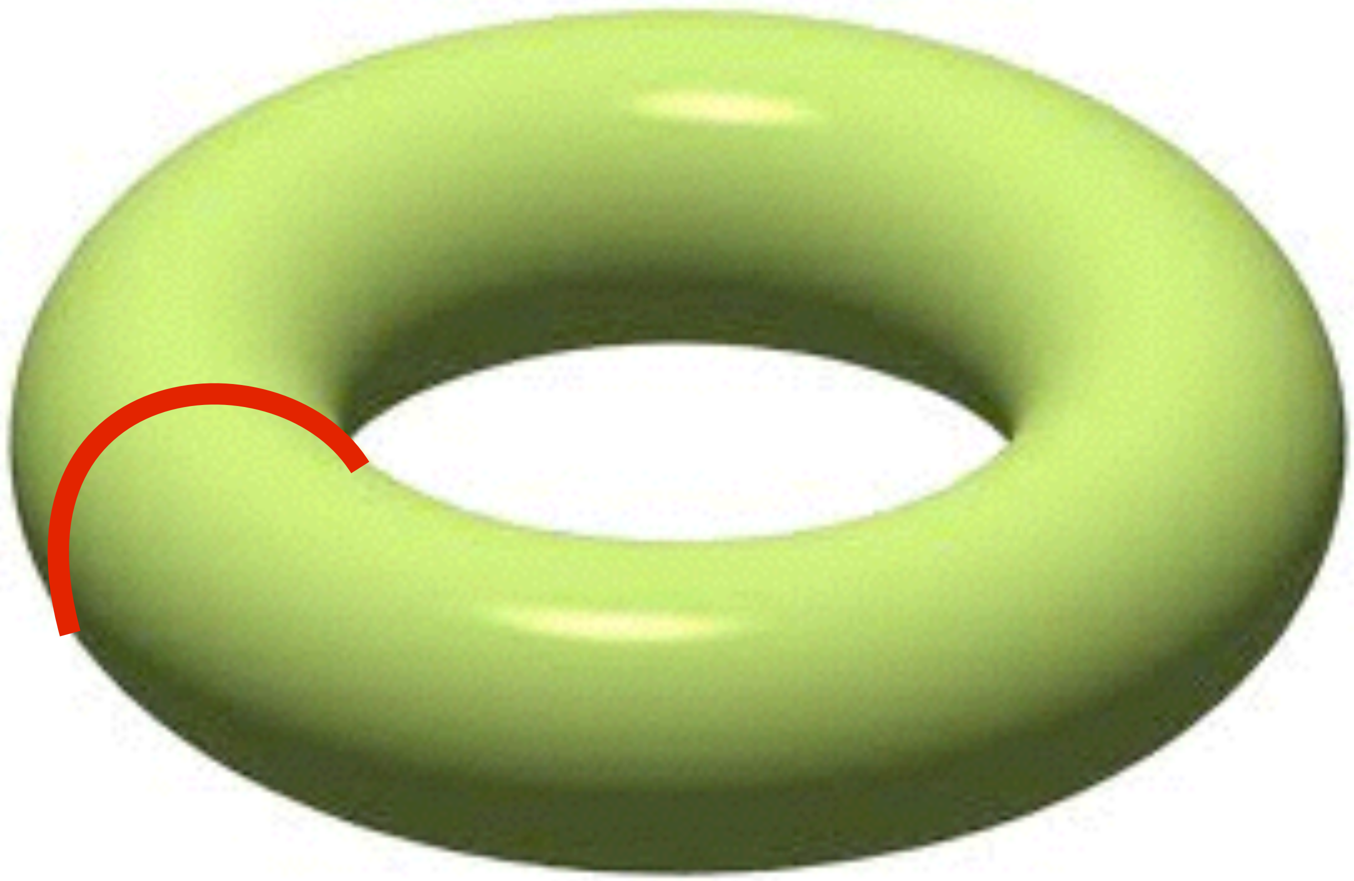

Place FL* on a torus:

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T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

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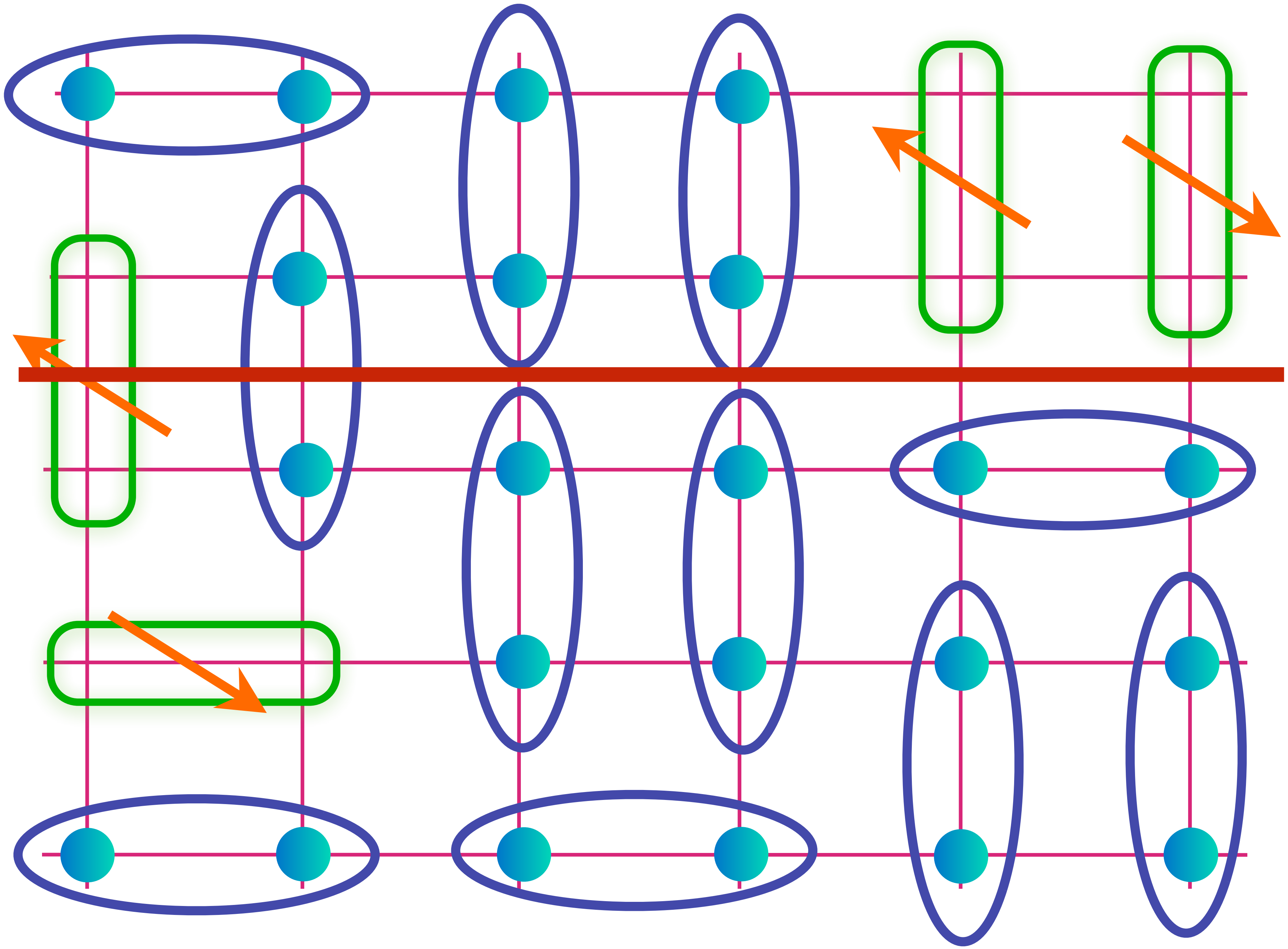

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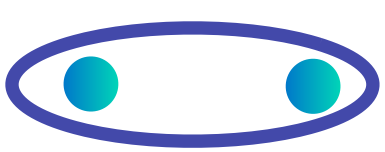 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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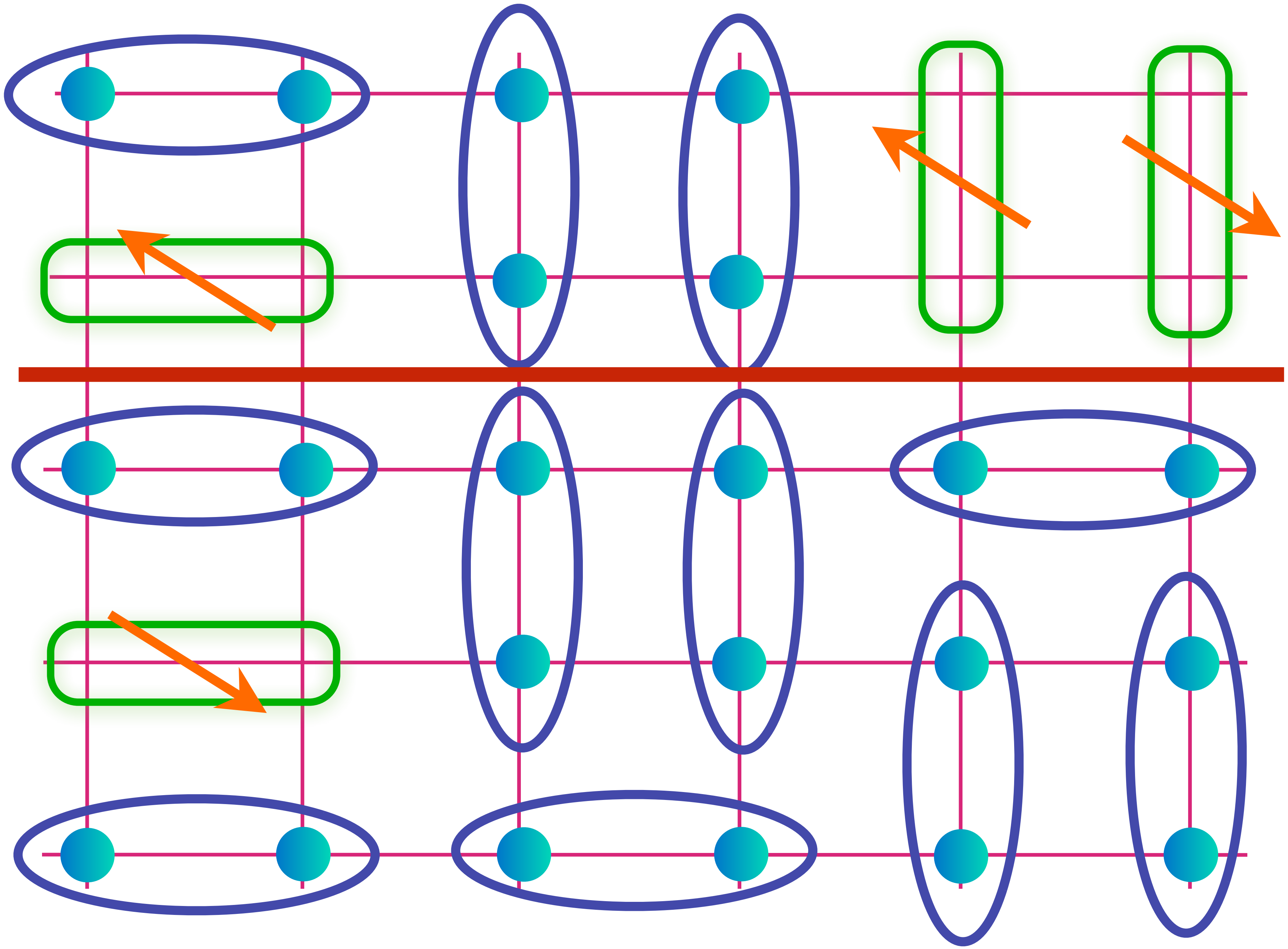

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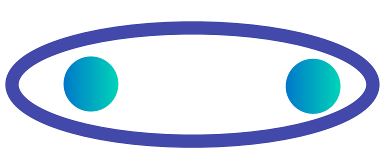

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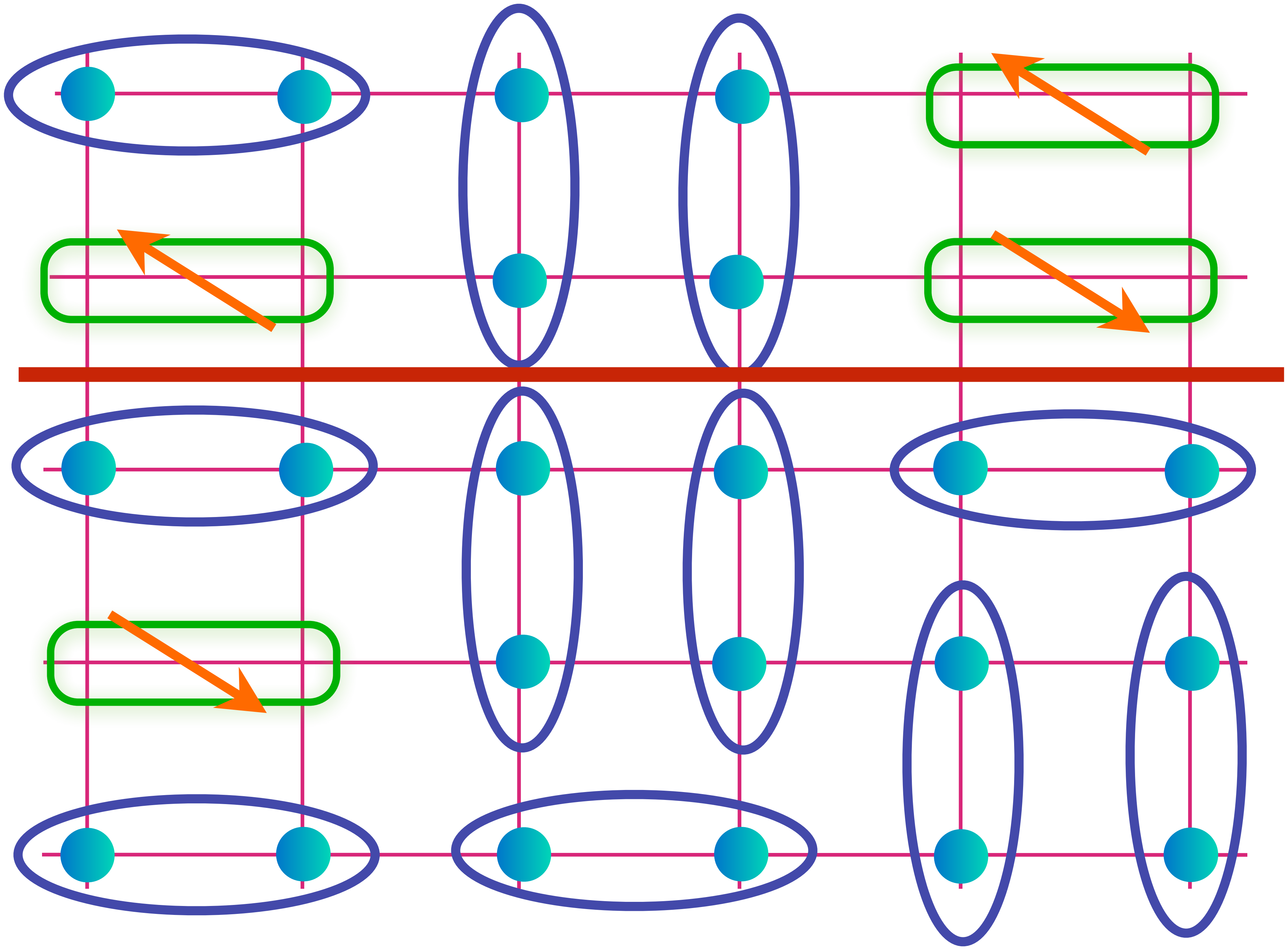

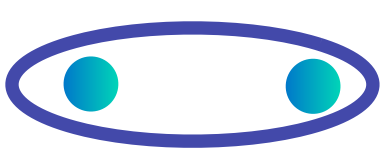 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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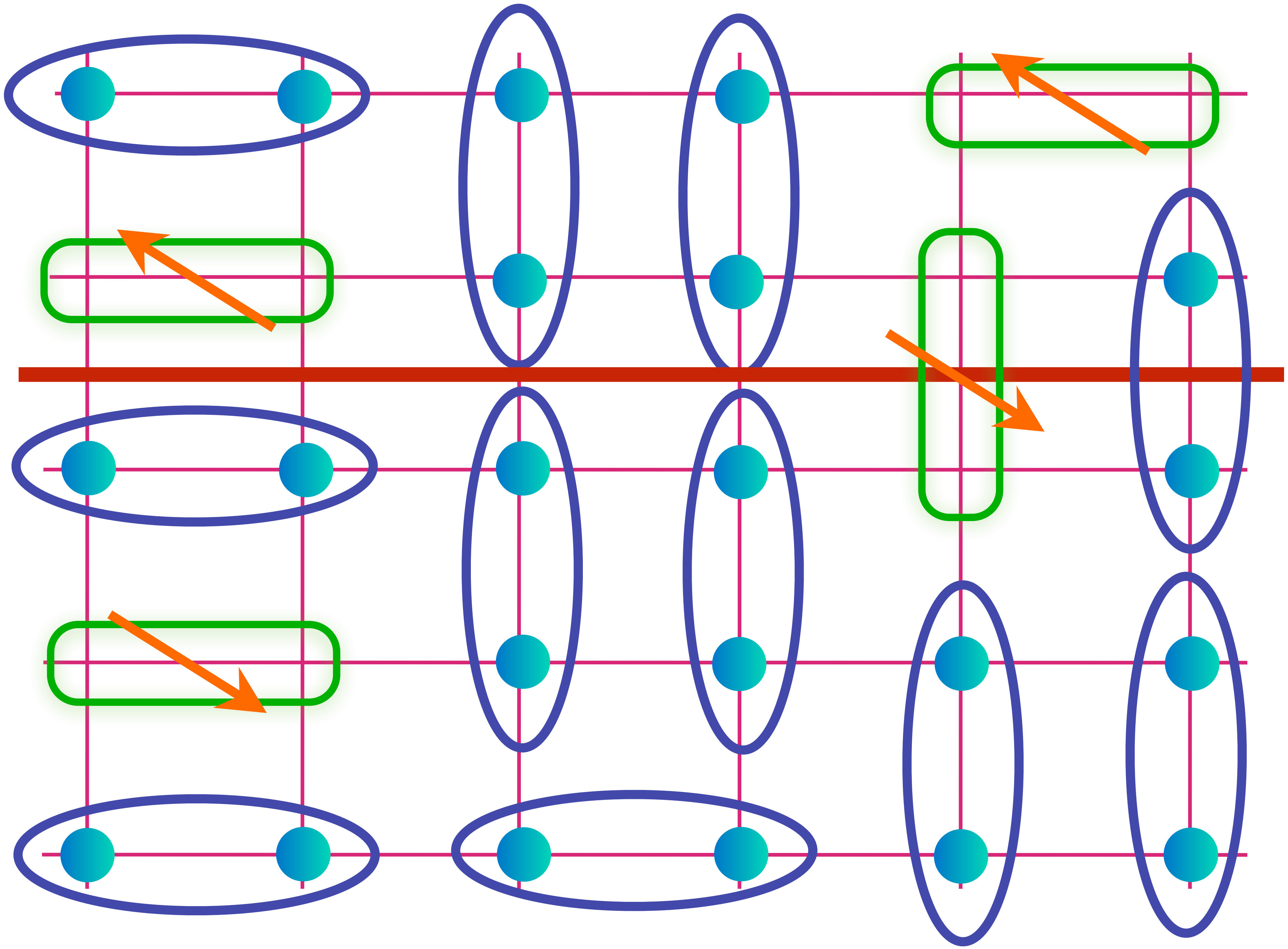

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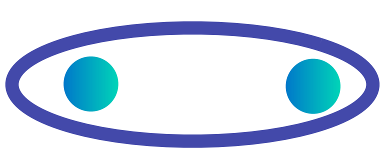

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Fractionalized Fermi liquid (FL*)

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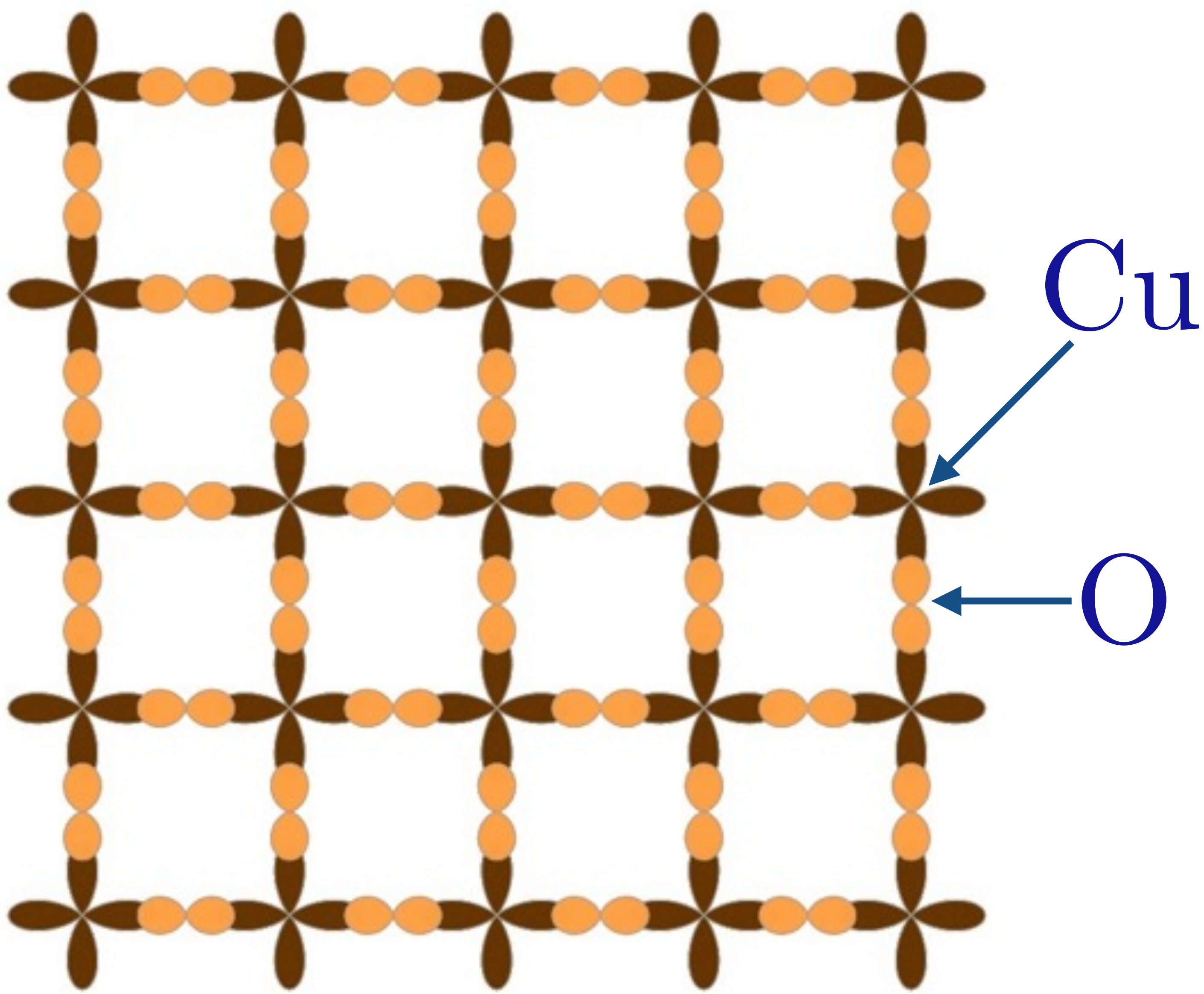
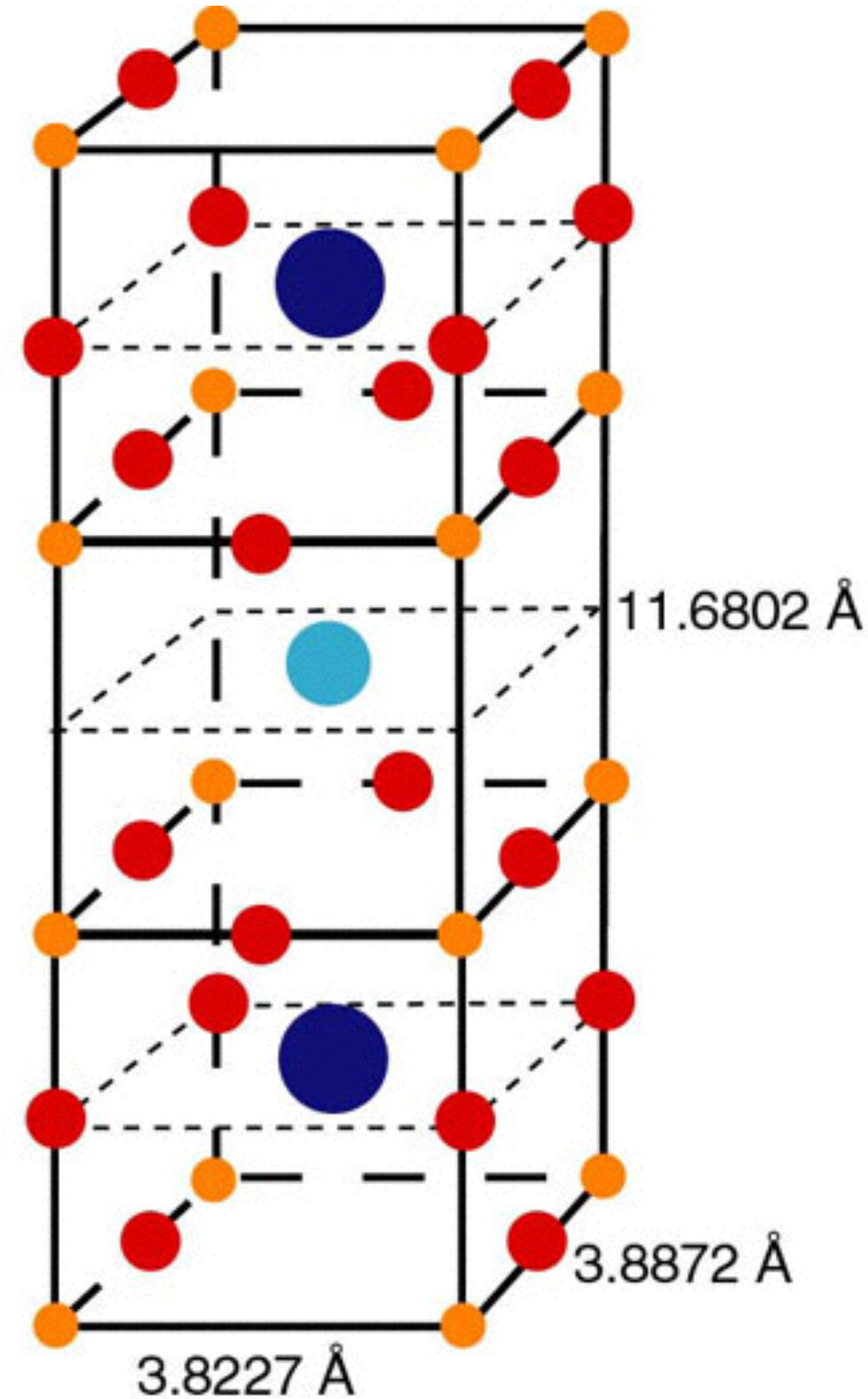
There is a general and fundamental relationship between these two characteristics.

M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000)
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

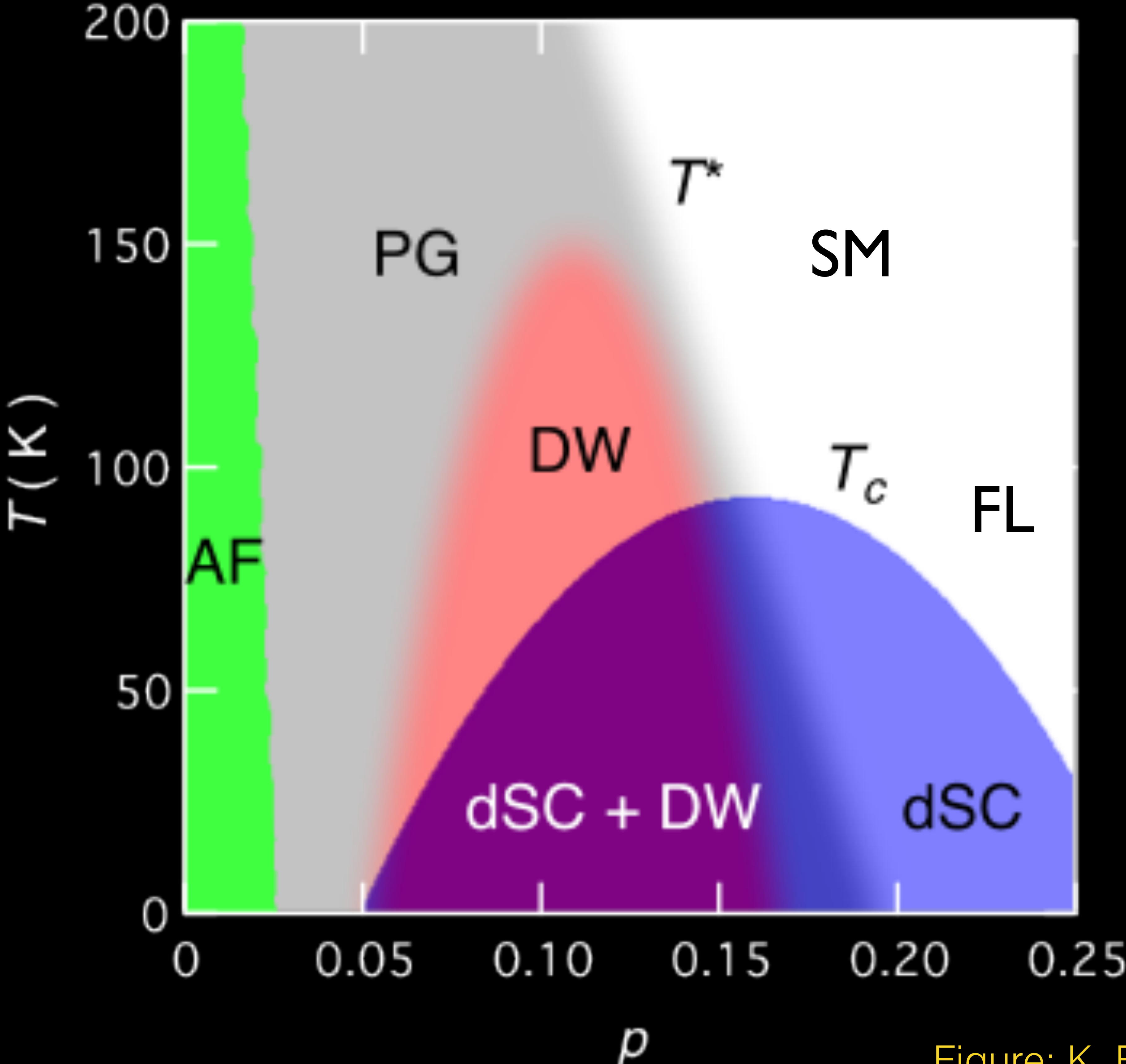
High temperature superconductors



CuO_2 plane



$YBa_2Cu_3O_{6+x}$



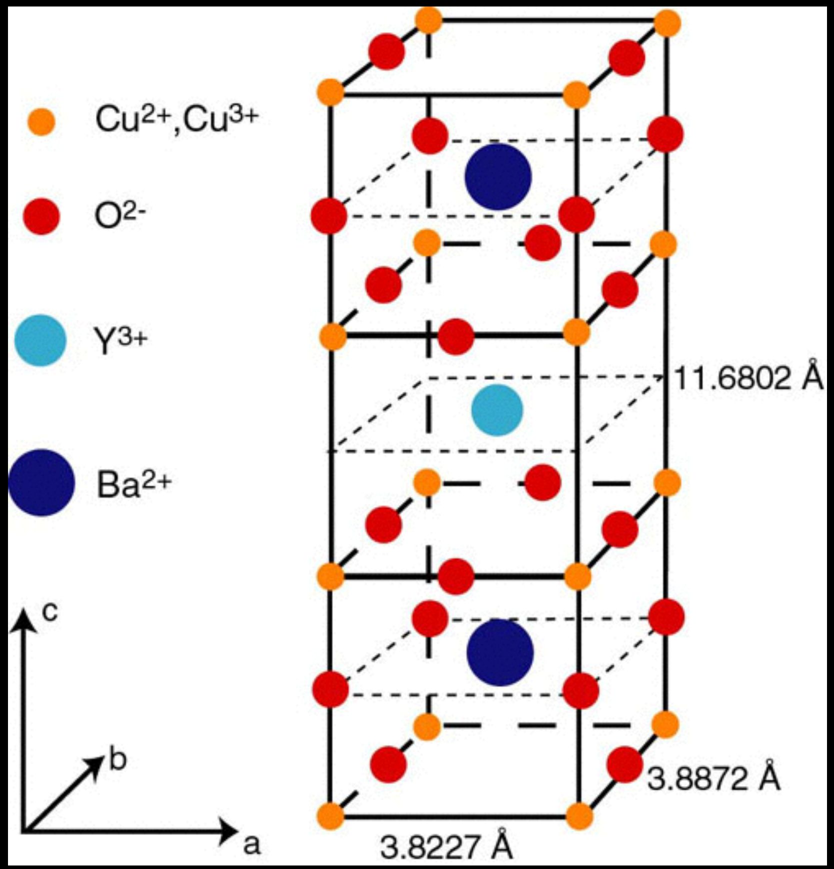


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Figure: K. Fujita and J. C. Seamus Davis

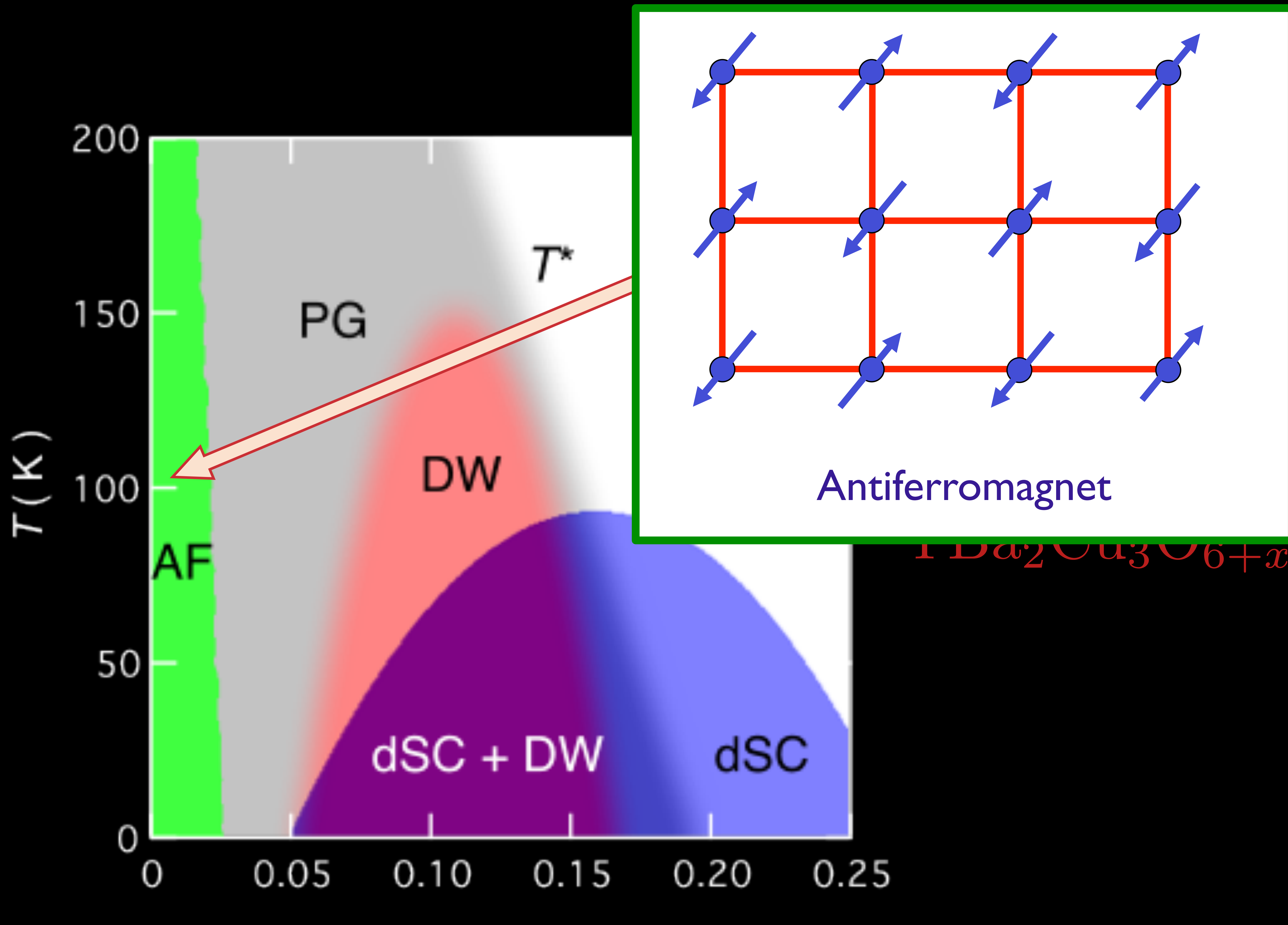
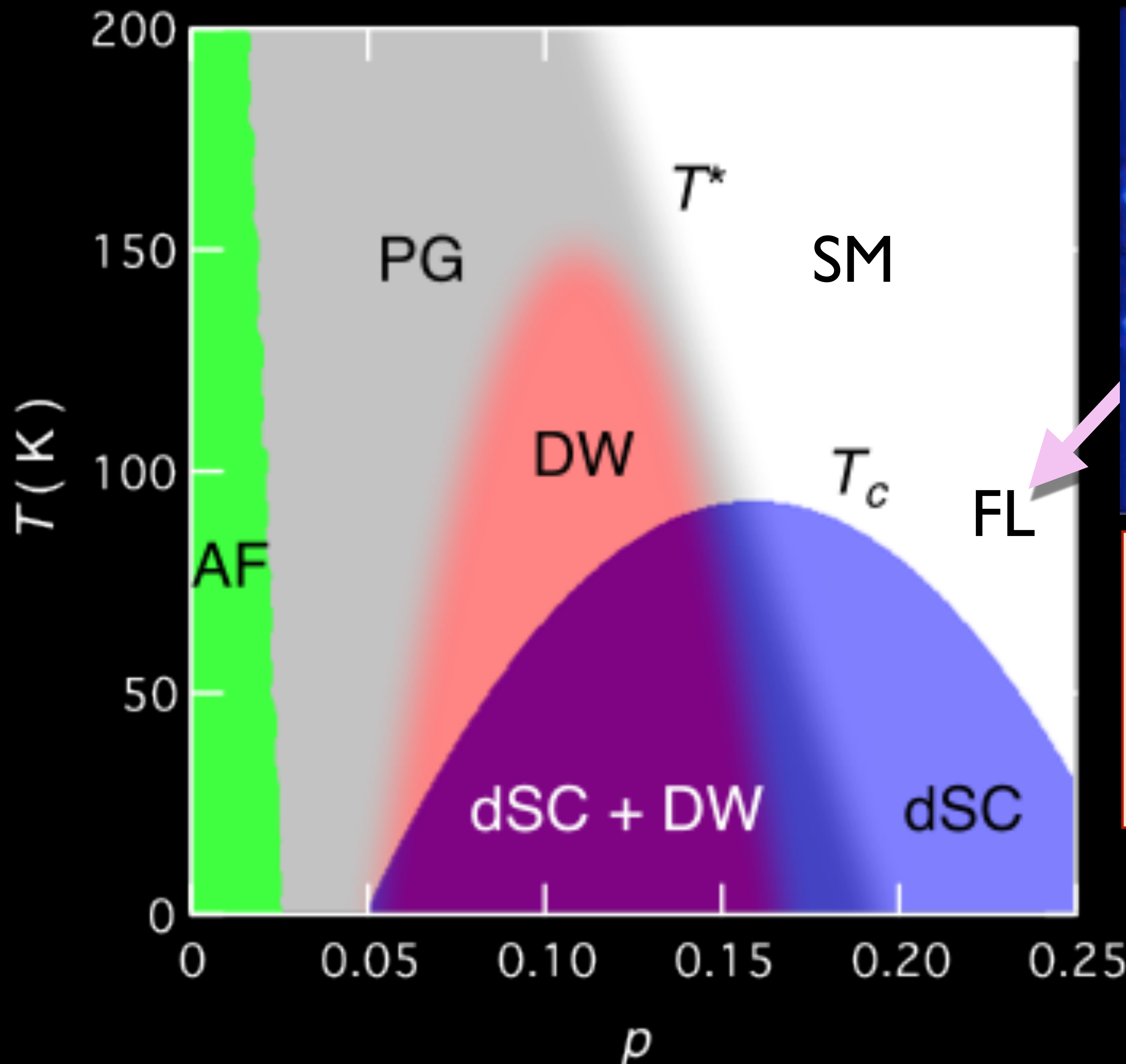
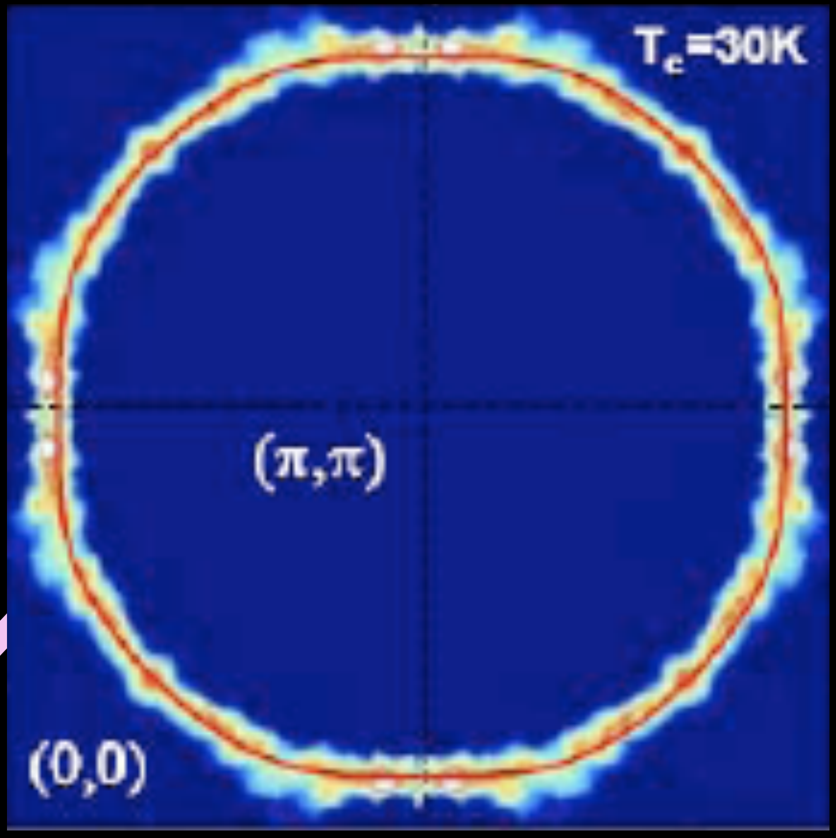


Figure: K. Fujita and J. C. Seamus Davis

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

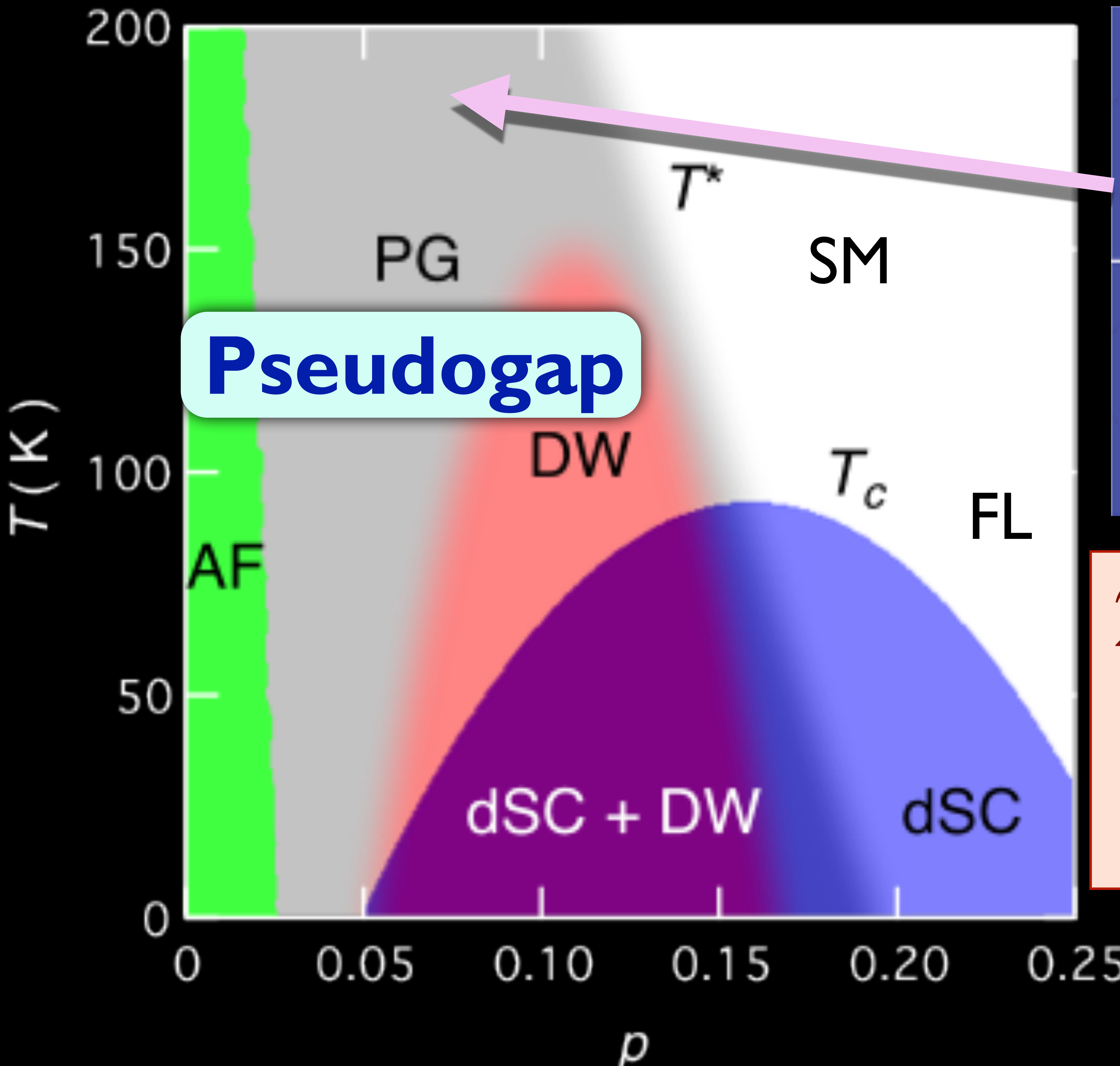


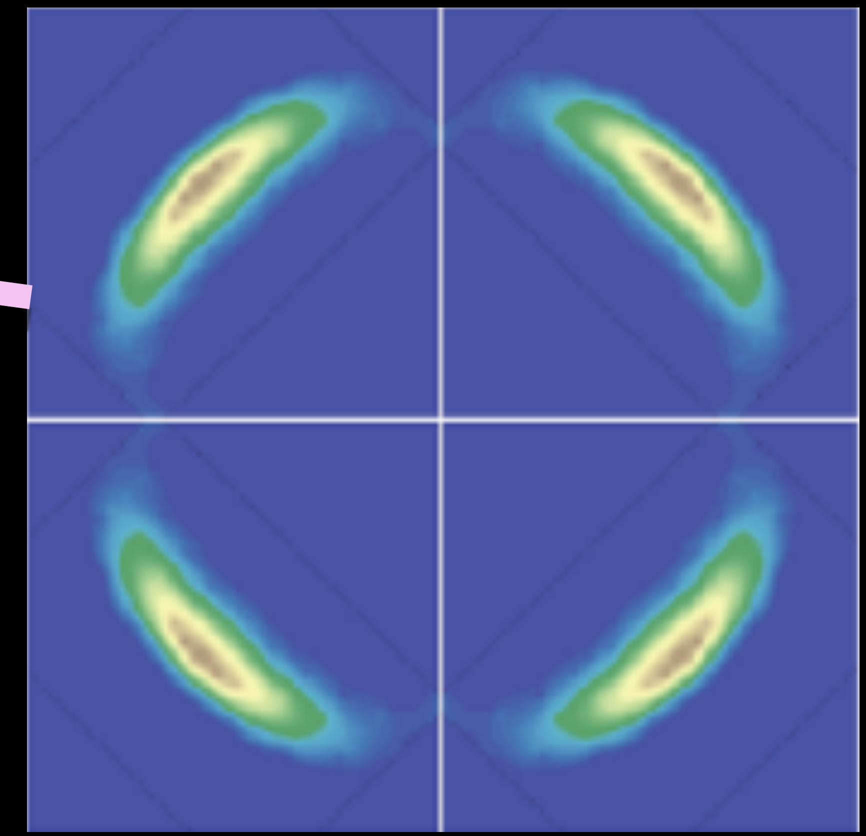


1. Conventional metal

Area enclosed by Fermi surface =1+p

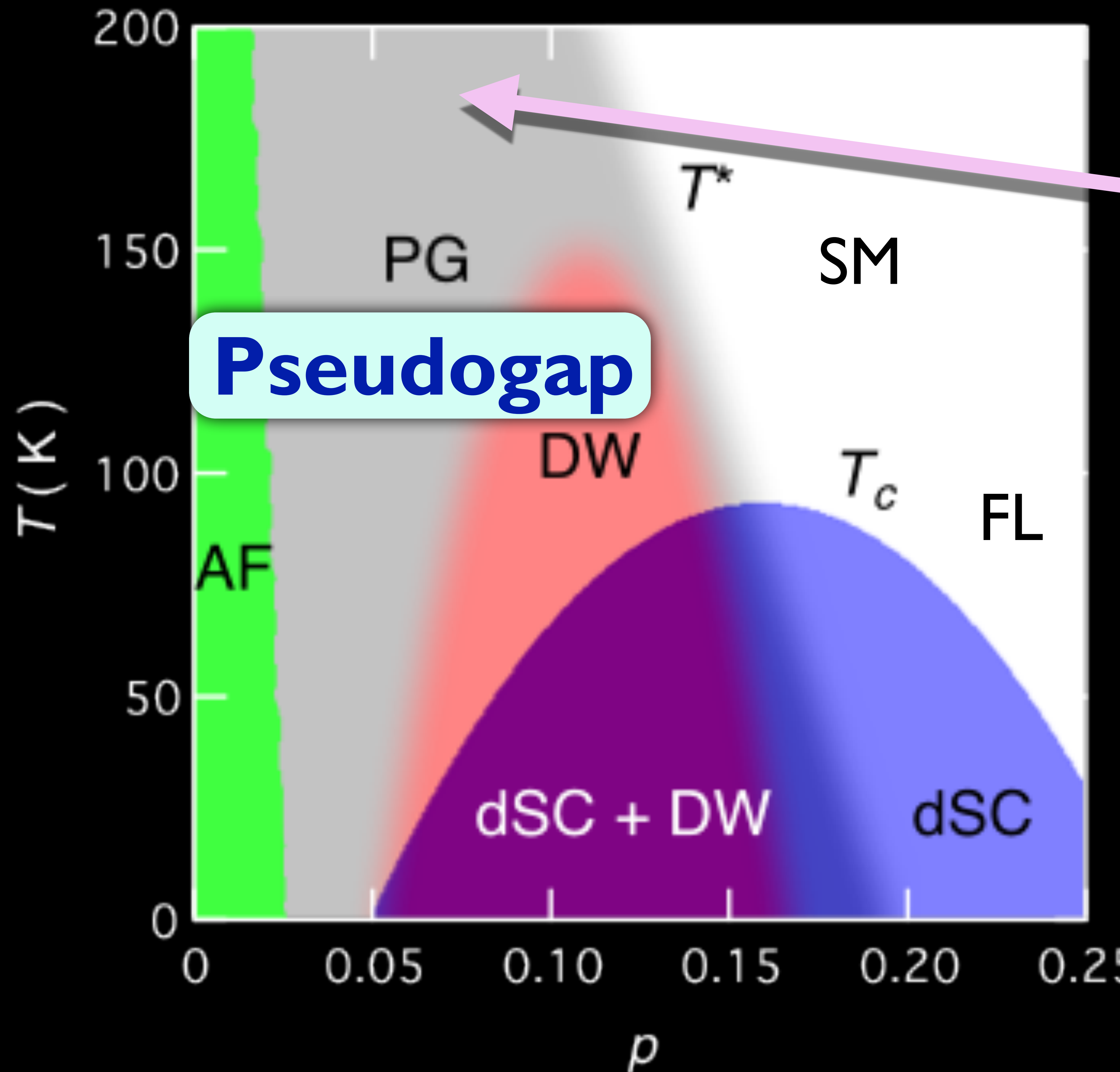
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

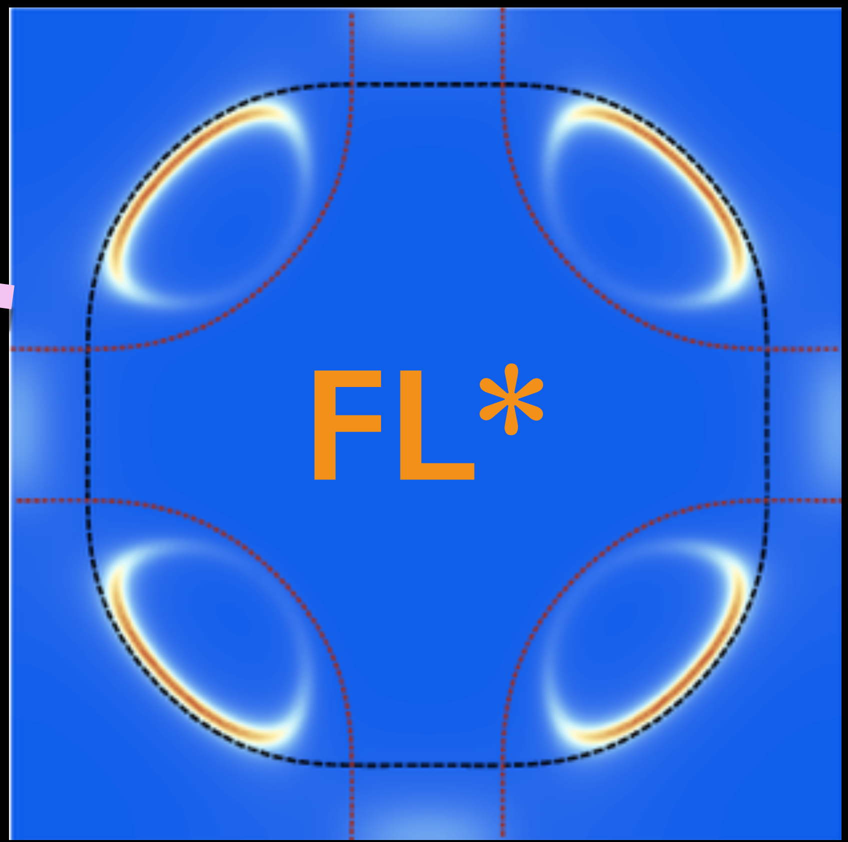




2. Pseudogap metal at low p

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)





A new metal — a fractionalized Fermi liquid (FL*) — with electron-like quasiparticles on a Fermi surface of size p

Recent evidence for pseudogap metal as FL*

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- Optical conductivity $\sim 1/(-i\omega + 1/\tau)$ with $1/\tau \sim \omega^2 + T^2$, with carrier density p (Mirzaei *et al.*, PNAS **110**, 5774 (2013)).

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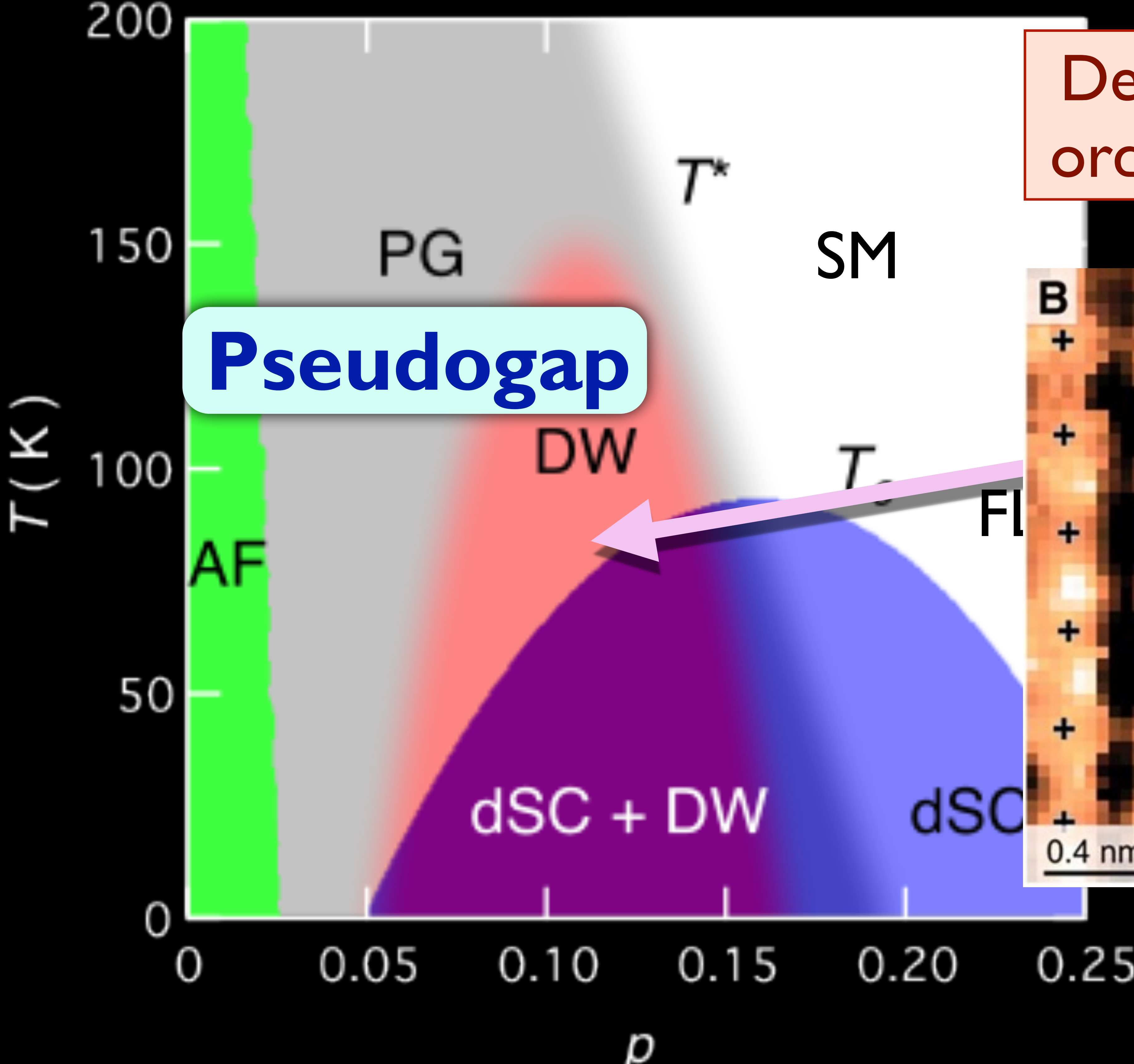
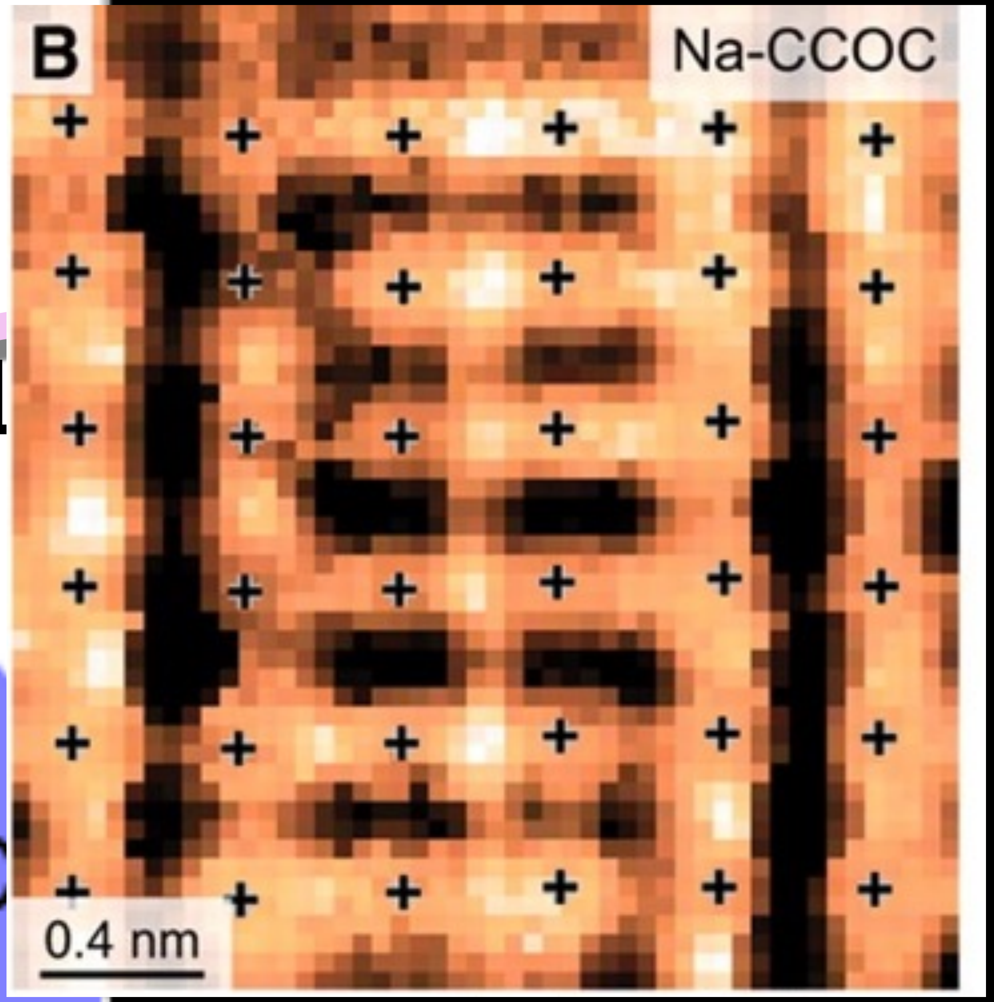
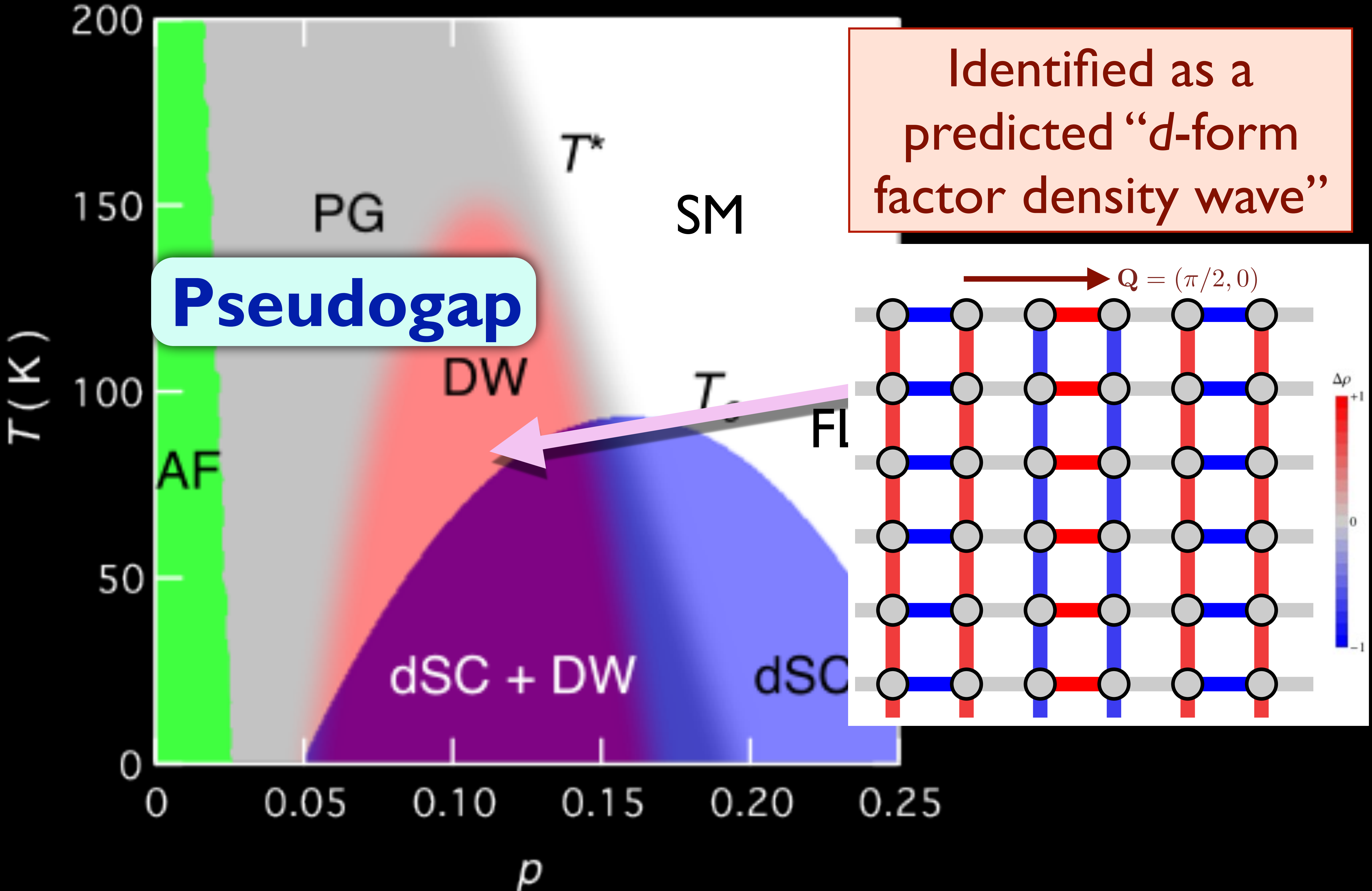
Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



Density wave (DW) order at low T and p



M. A. Metlitski and S. Sachdev, PRB **82**, 075128 (2010). S. Sachdev R. La Placa, PRL **111**, 027202 (2013).
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)



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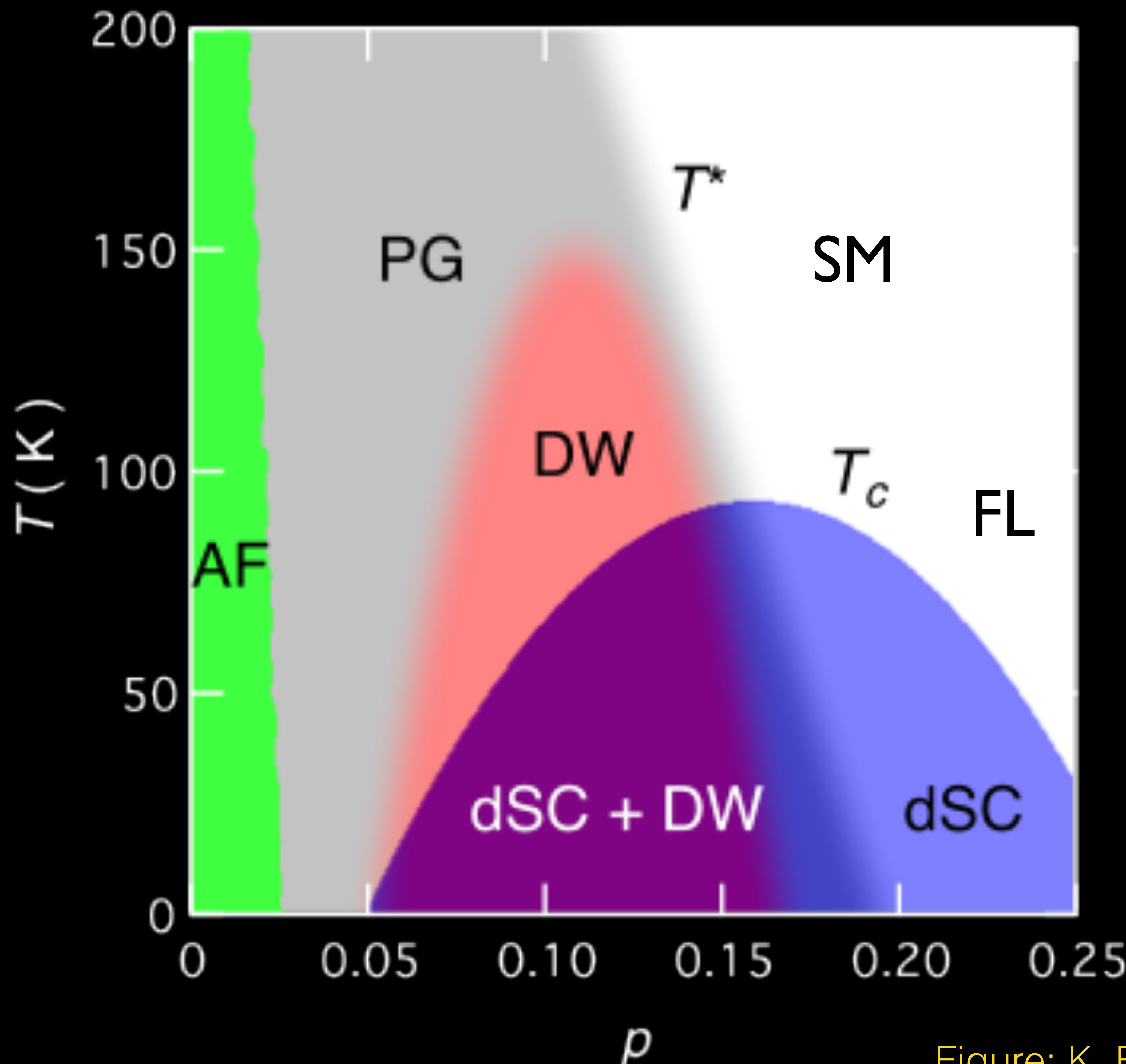
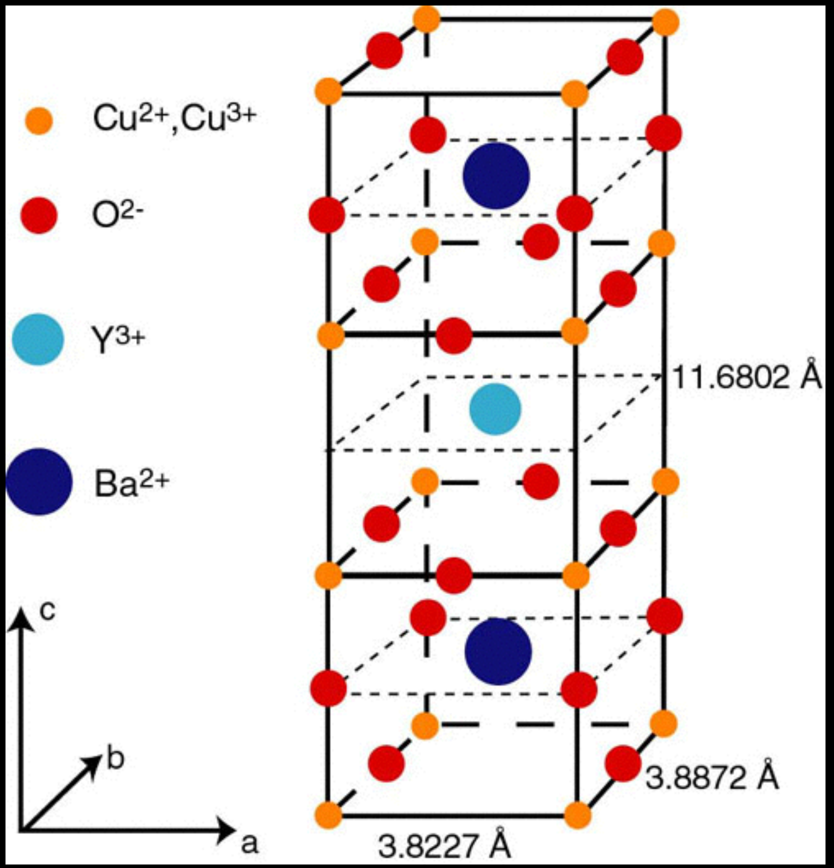
- T-independent Hall positive resistance R_H corresponding to carrier density p in the higher temperature pseudogap (Ando *et al.*, PRL **92**, 197001 (2004)) *and* in recent measurements at high fields, low T, and around $p \approx 0.16$ in YBCO (Proust-Taillefer-UBC collaboration, Badoux *et al.*, submitted).



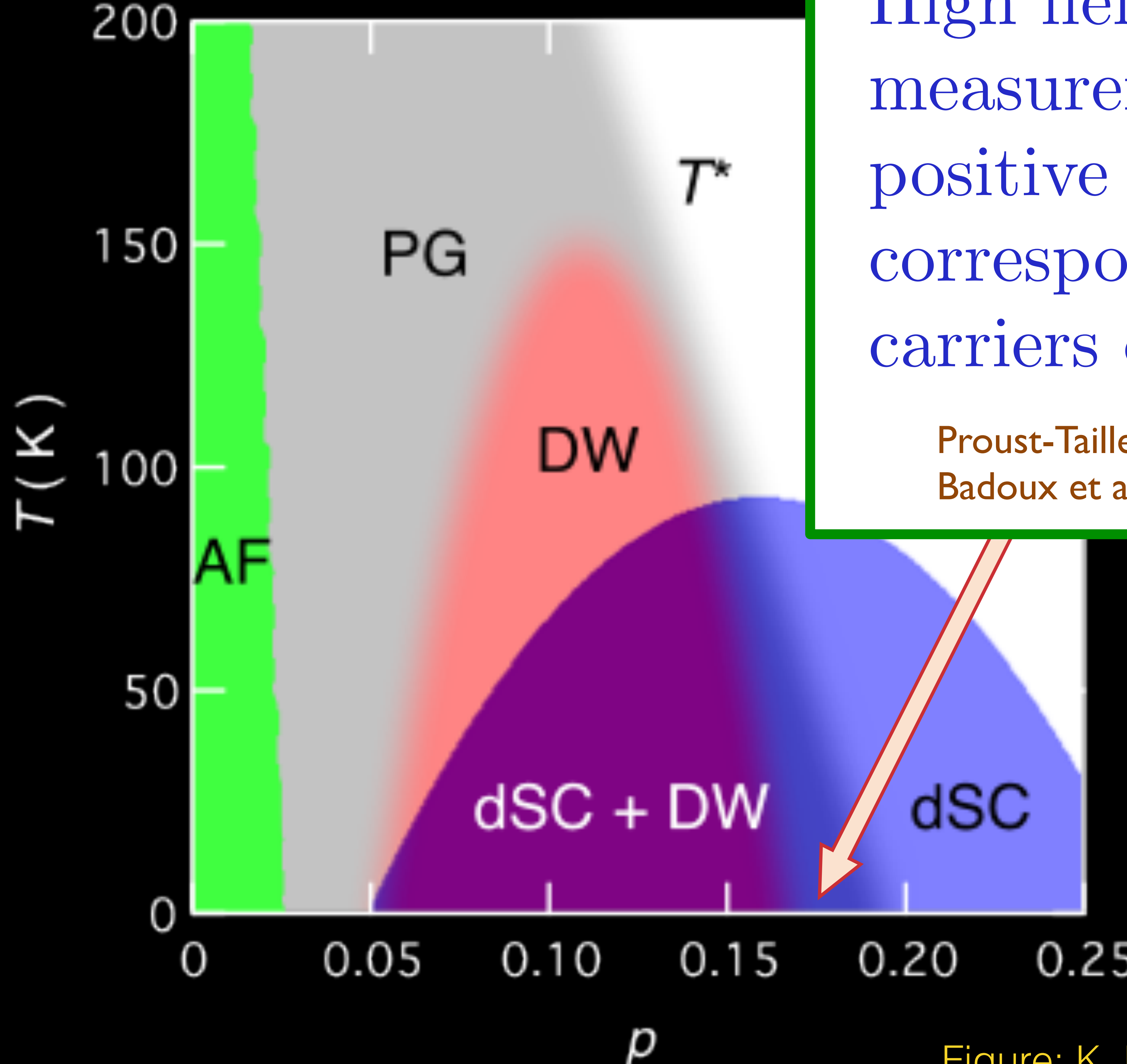


$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

High field, low T measurements show a positive Hall resistance corresponding to carriers of density p

Proust-Taillefer-UBC collaboration, Badoux et al., submitted



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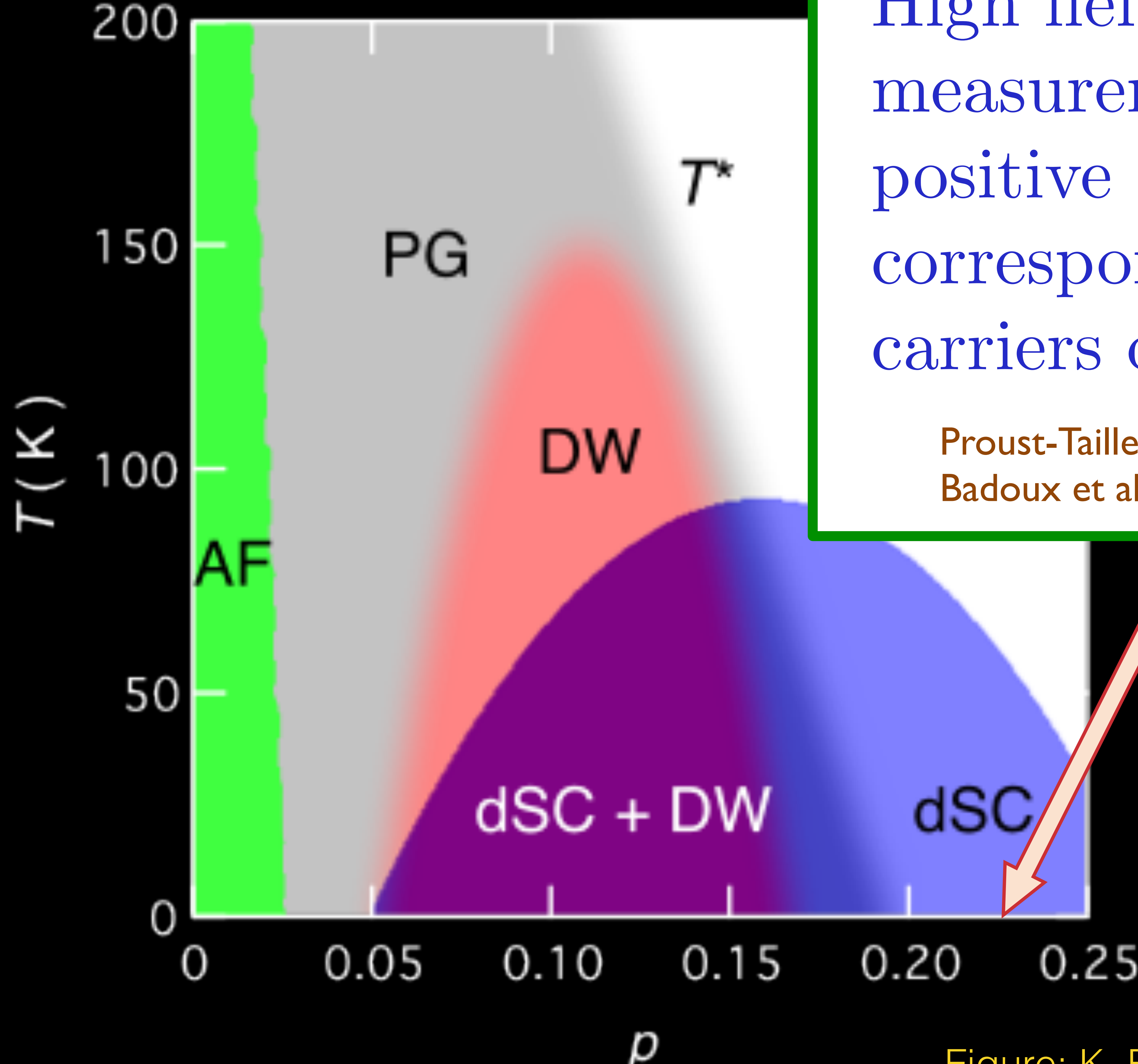

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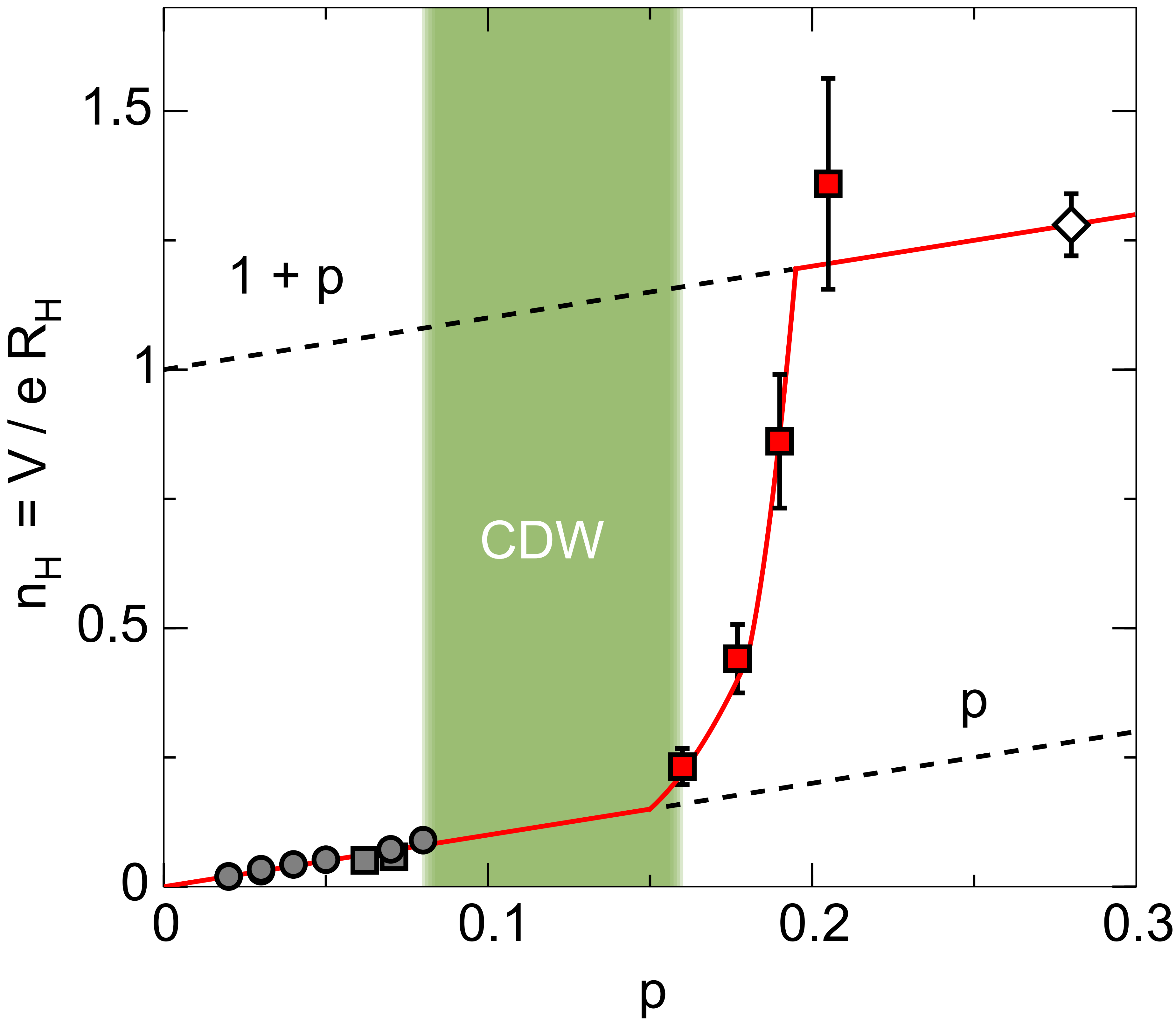

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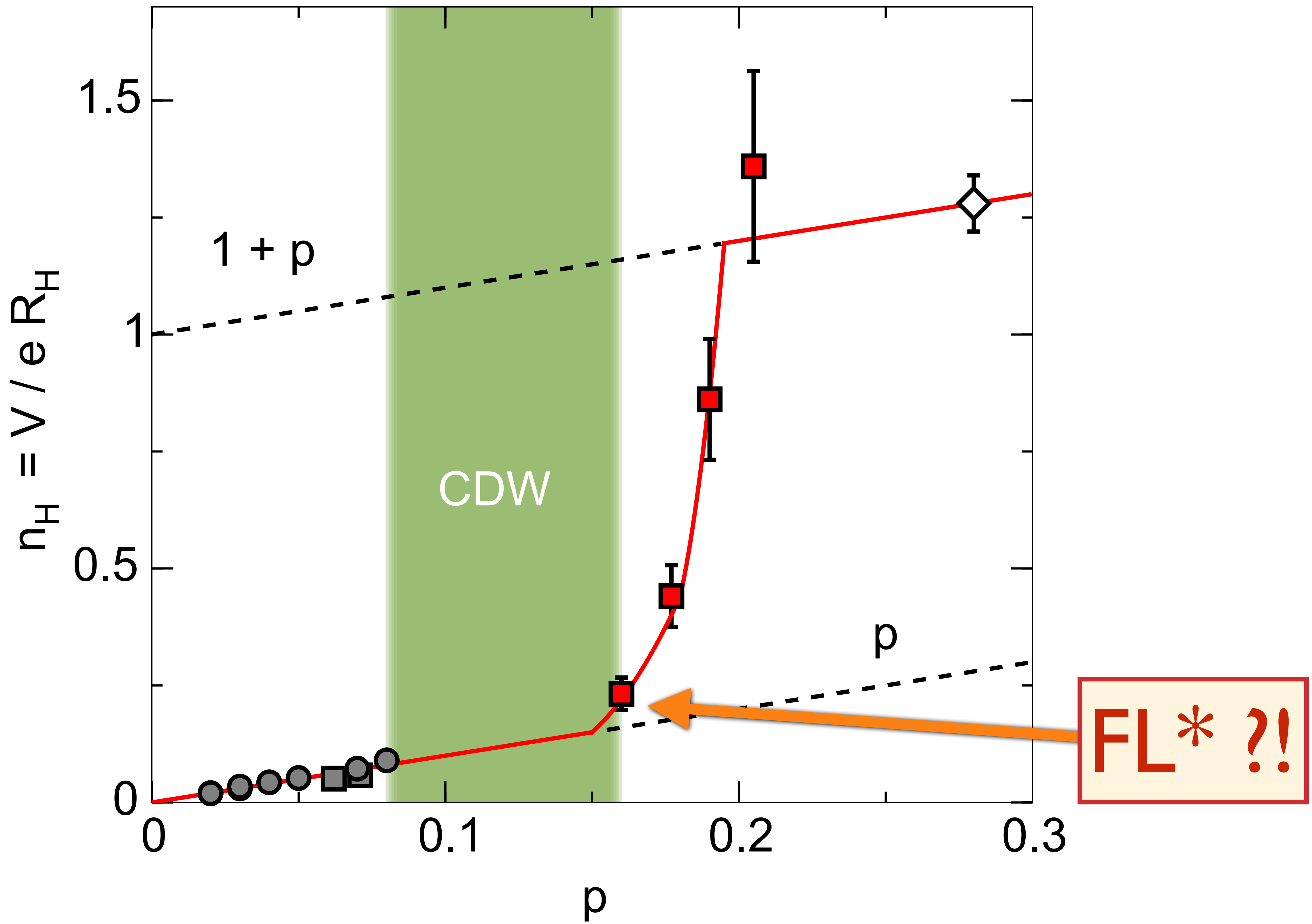

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1. Long-range entanglement in insulators

2. Theory of ordinary metals

   (a) Quasiparticles

   (b) Luttinger theorem for volume enclosed by Fermi surface

3. Fractionalized Fermi liquid

   Quasiparticles with a non-Luttinger volume in the pseudogap metal of the cuprate superconductors

4. Strange metals without quasiparticles

   Experiments in graphene, and charged black holes

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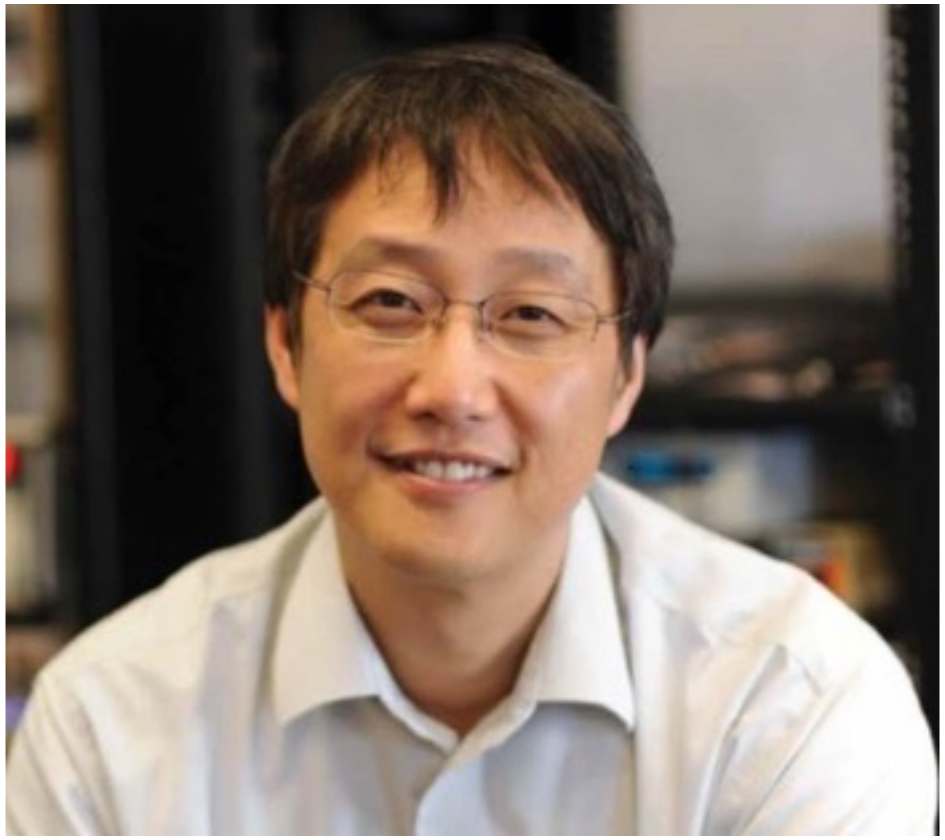
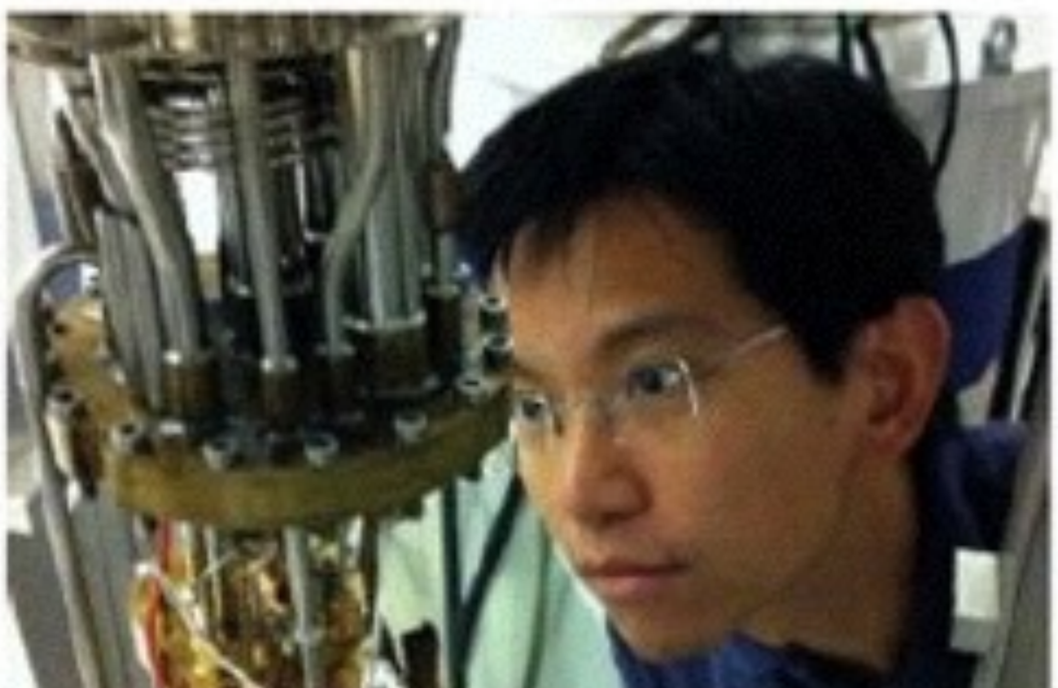
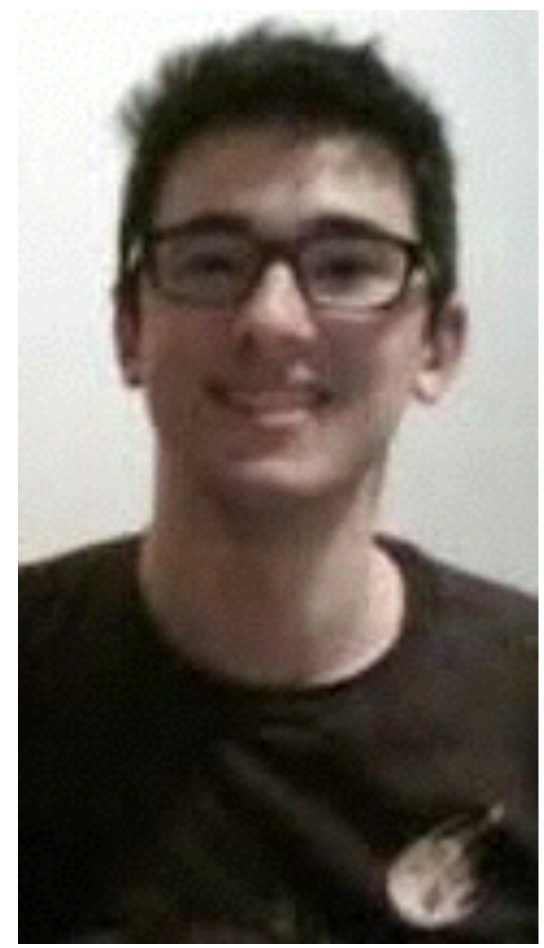


Philip Kim

Jesse Crossno

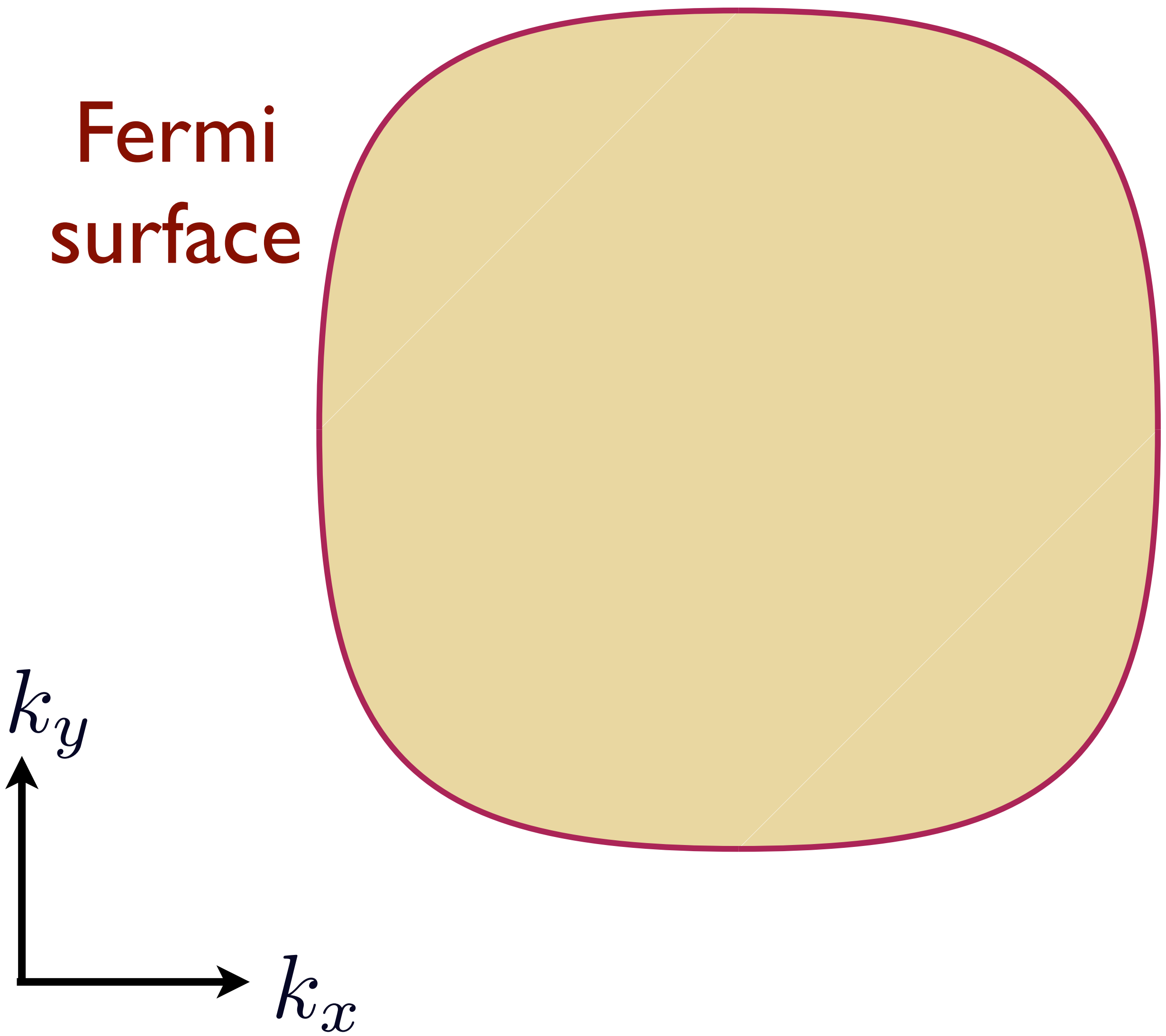


Kin Chung Fong



Andrew Lucas

Ordinary metals: the Fermi liquid



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- Area enclosed by Fermi surface $= \mathcal{Q}$, the electron density. Momenta of low energy excitations fixed by density of *all* electrons.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.

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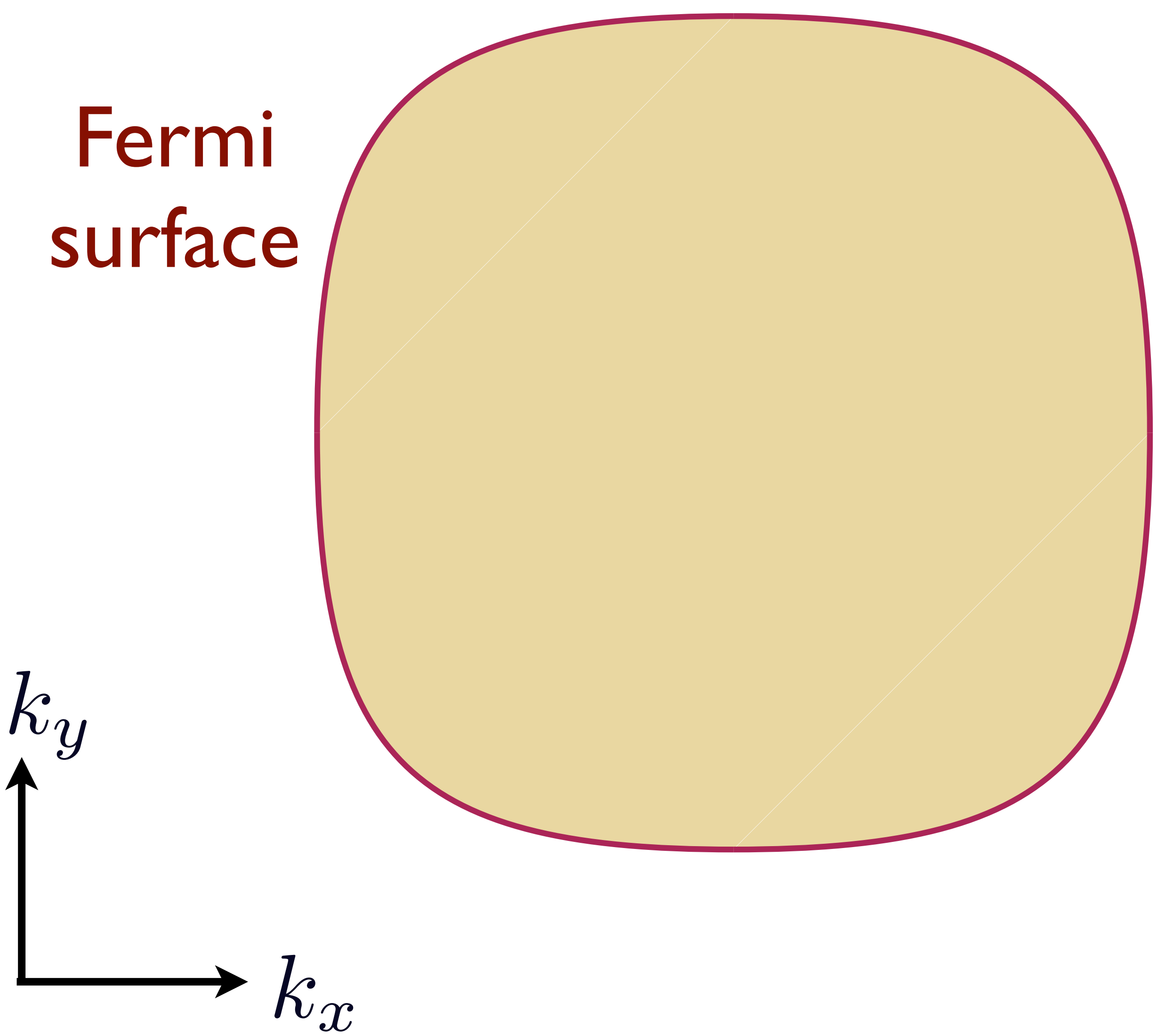

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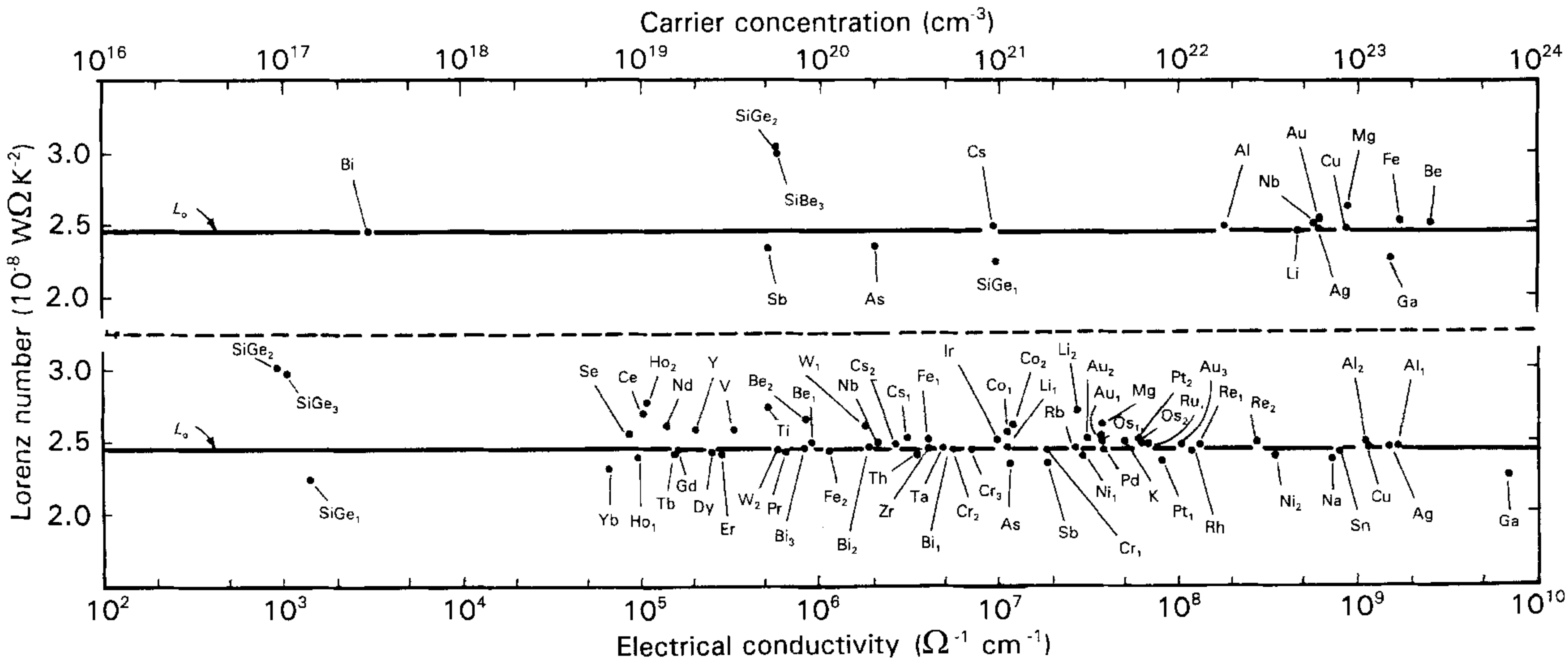
- $$\frac{\text{(Thermal conductivity)}}{T\,\text{(Electrical conductivity)}} = \frac{\pi^2 k_B^2}{3e^2}$$

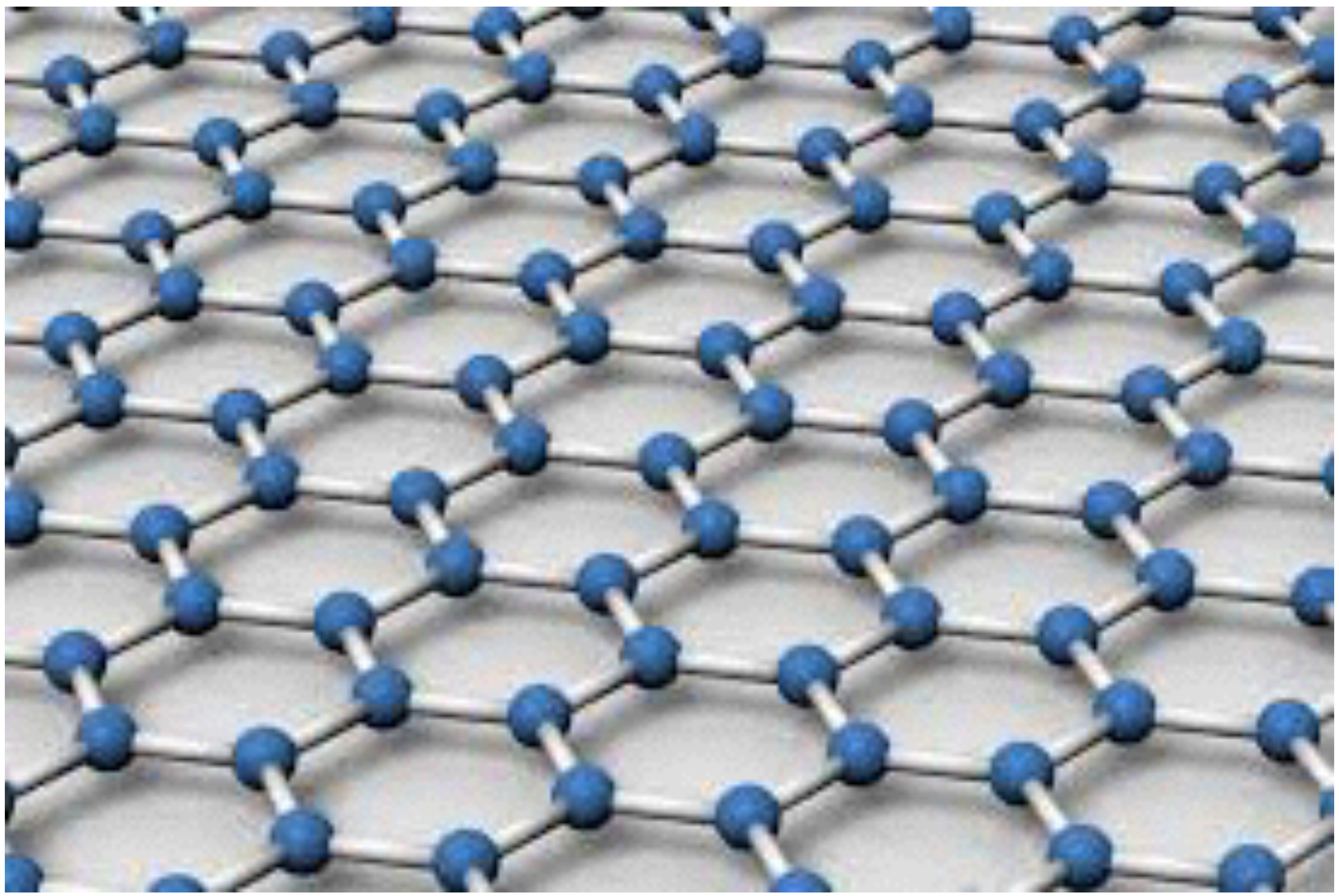
- Wiedemann-Franz law in a Fermi liquid:

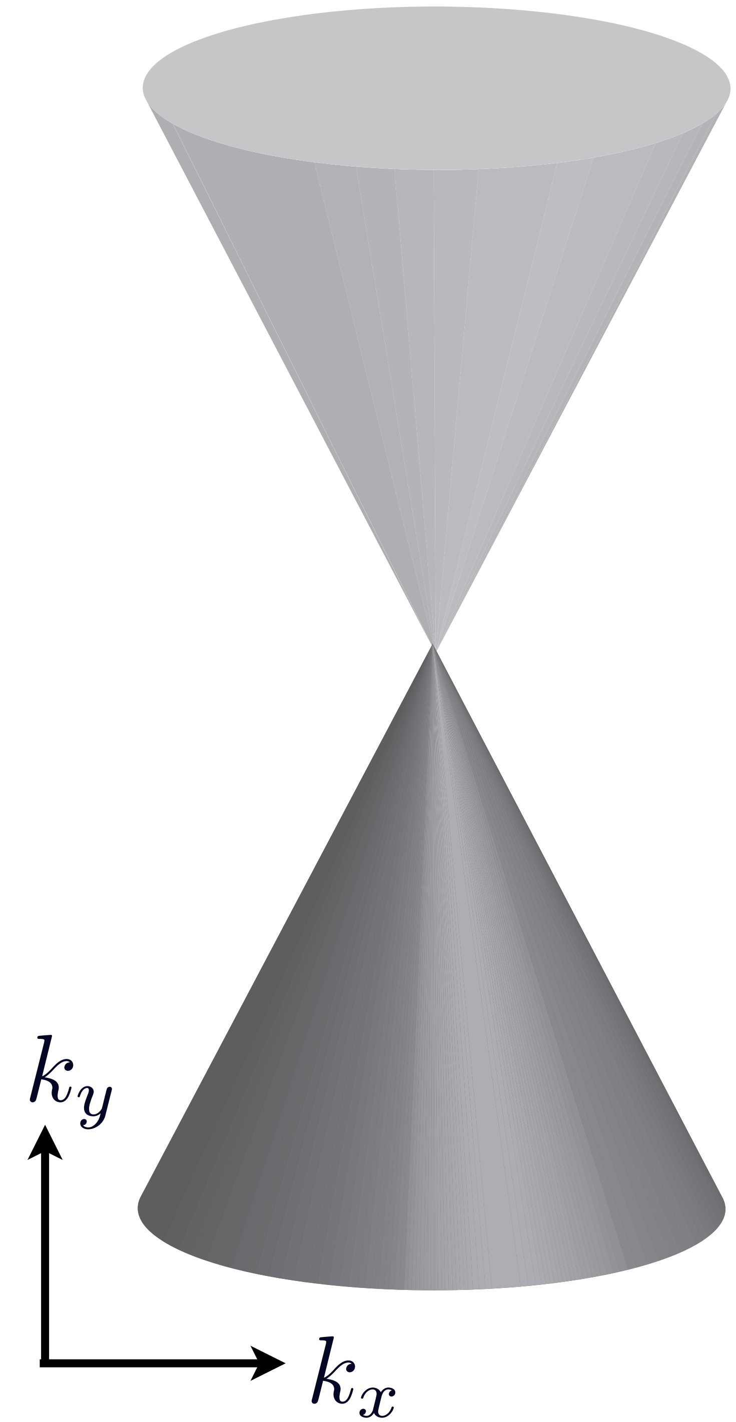
$$\frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8}\,\frac{\mathrm{W}\cdot\Omega}{\mathrm{K}^2}.$$



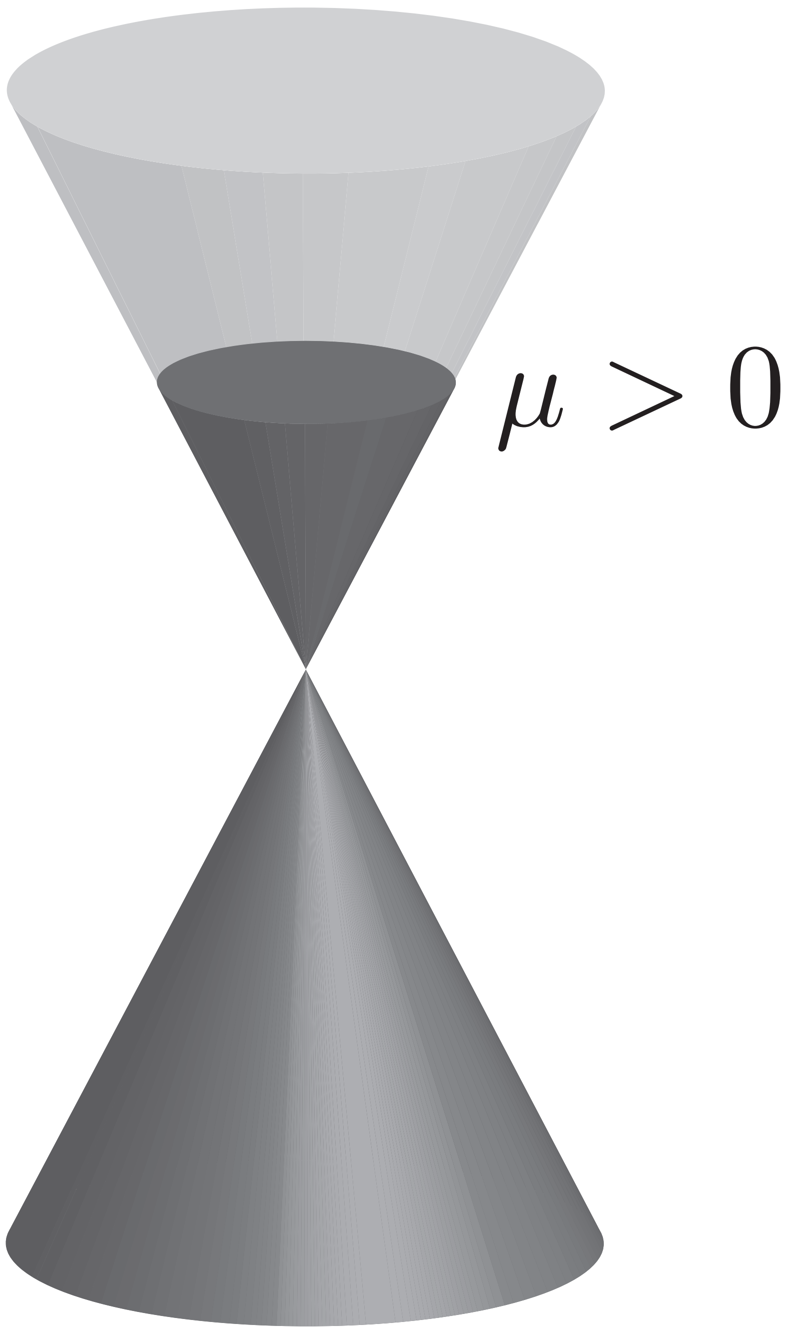
G. S. Kumar, G. Prasad, and R.O. Pohl, J. Mat. Sci. **28**, 4261 (1993)

Graphene





Graphene



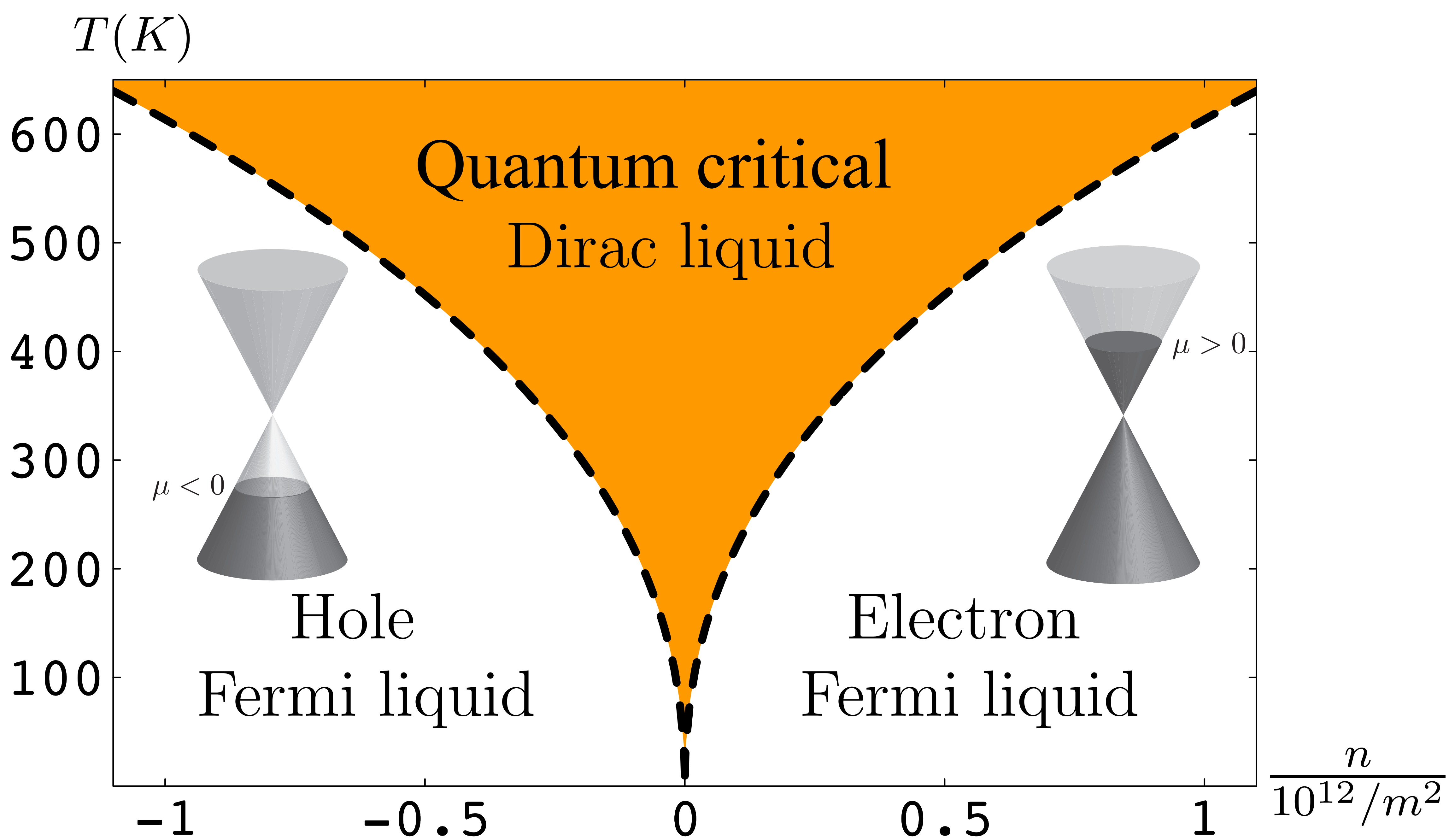
Electron
Fermi surface

Graphene

Hole
Fermi surface

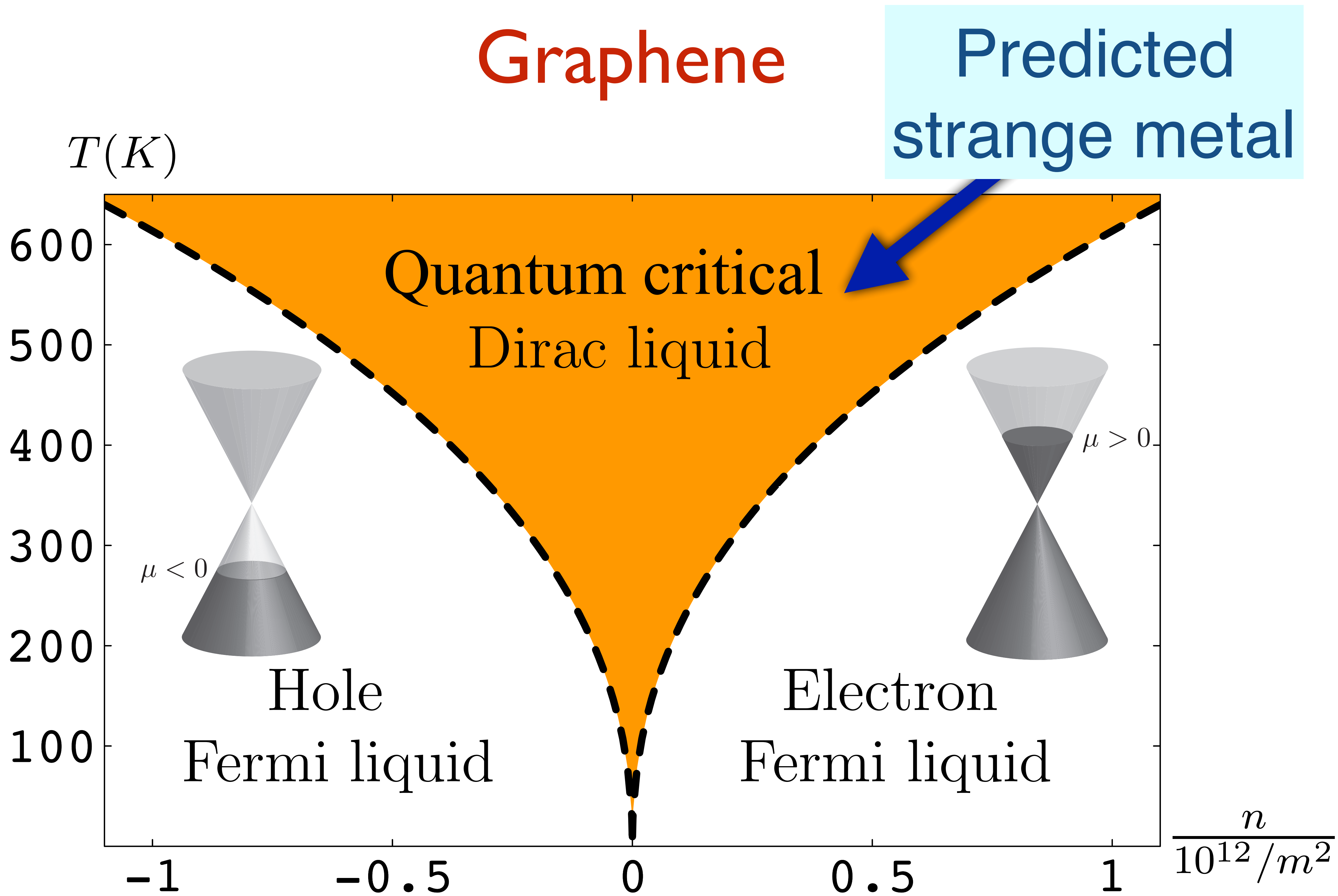
Electron
Fermi surface

Graphene



D. E. Sheehy and J. Schmalian, PRL **99**, 226803 (2007)
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Graphene



M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Key properties of a strange metal

- No quasiparticle excitations

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- Shortest possible "collision time", or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$

S. Sachdev, Quantum Phase Transitions (1999)

Key properties of a strange metal

- No quasiparticle excitations
- Shortest possible "collision time", or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$
- Continuously variable density, $\mathcal{Q}$ (conformal field theories are usually at fixed density, $\mathcal{Q} = 0$)

Transport in Strange Metals

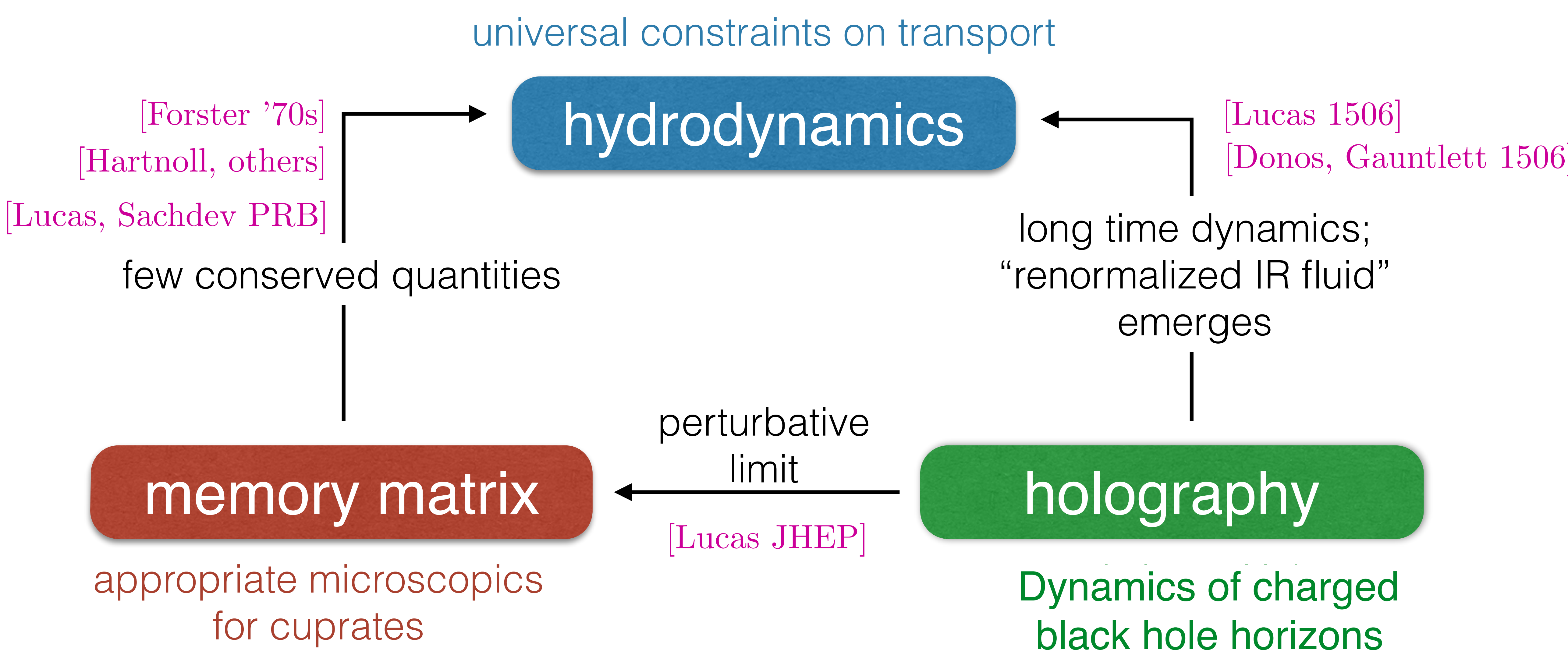
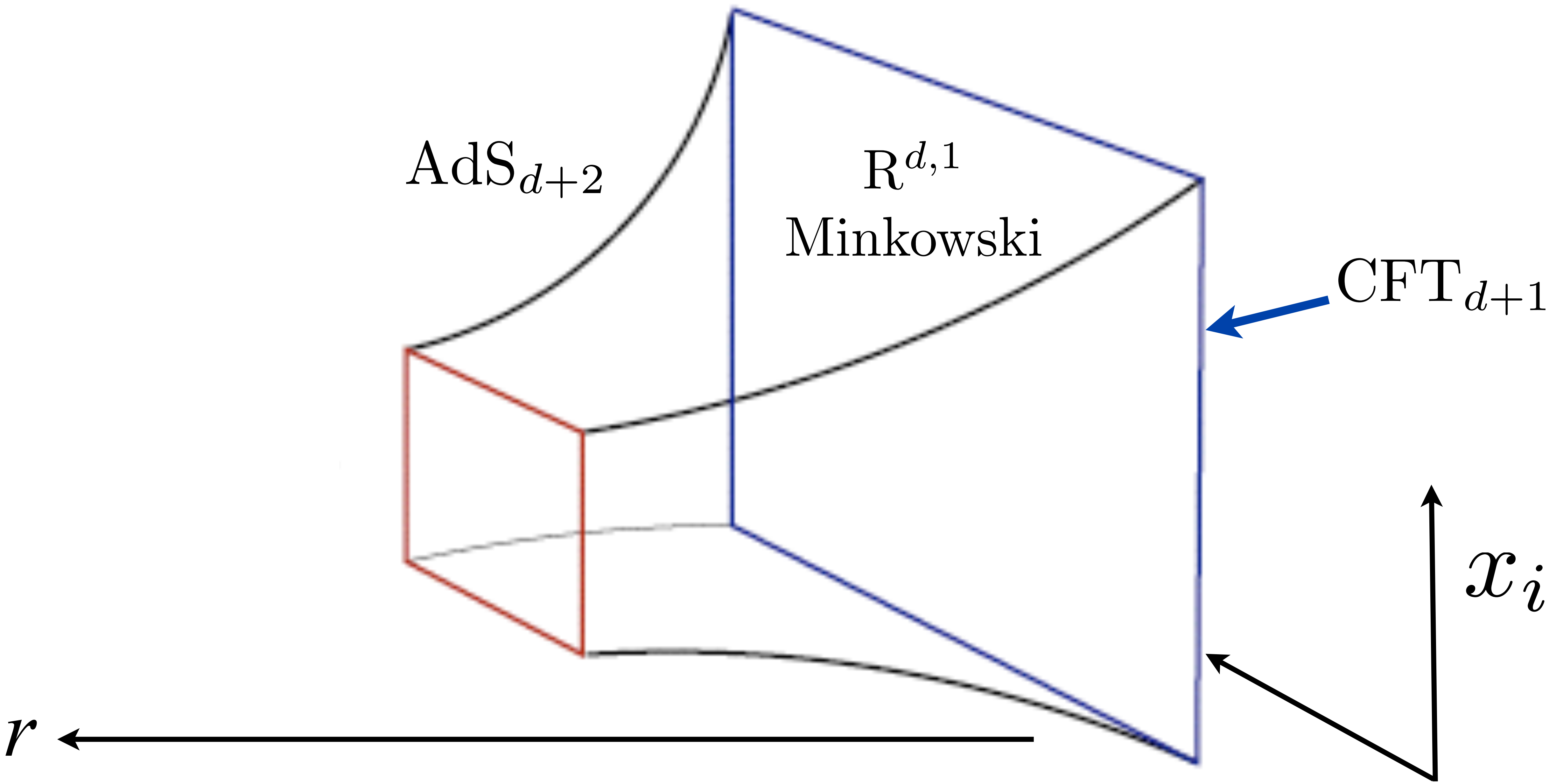


figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

AdS/CFT correspondence at zero temperature

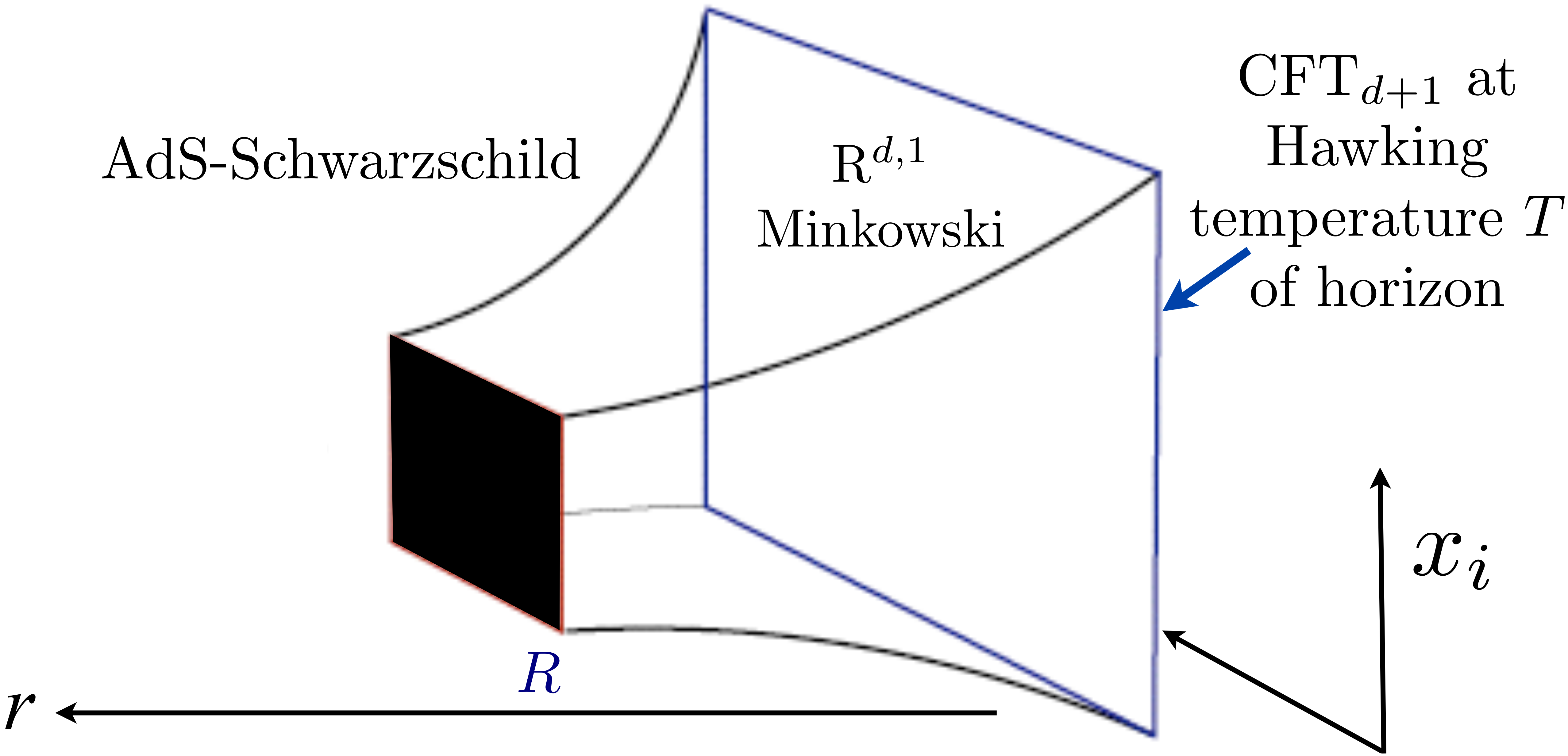
Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2}\right)\right]$



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[dr^2 - dt^2 + d\vec{x}^2\right]$$

AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(\mathcal{R} + \frac{d(d+1)}{L^2} \right) \right]$

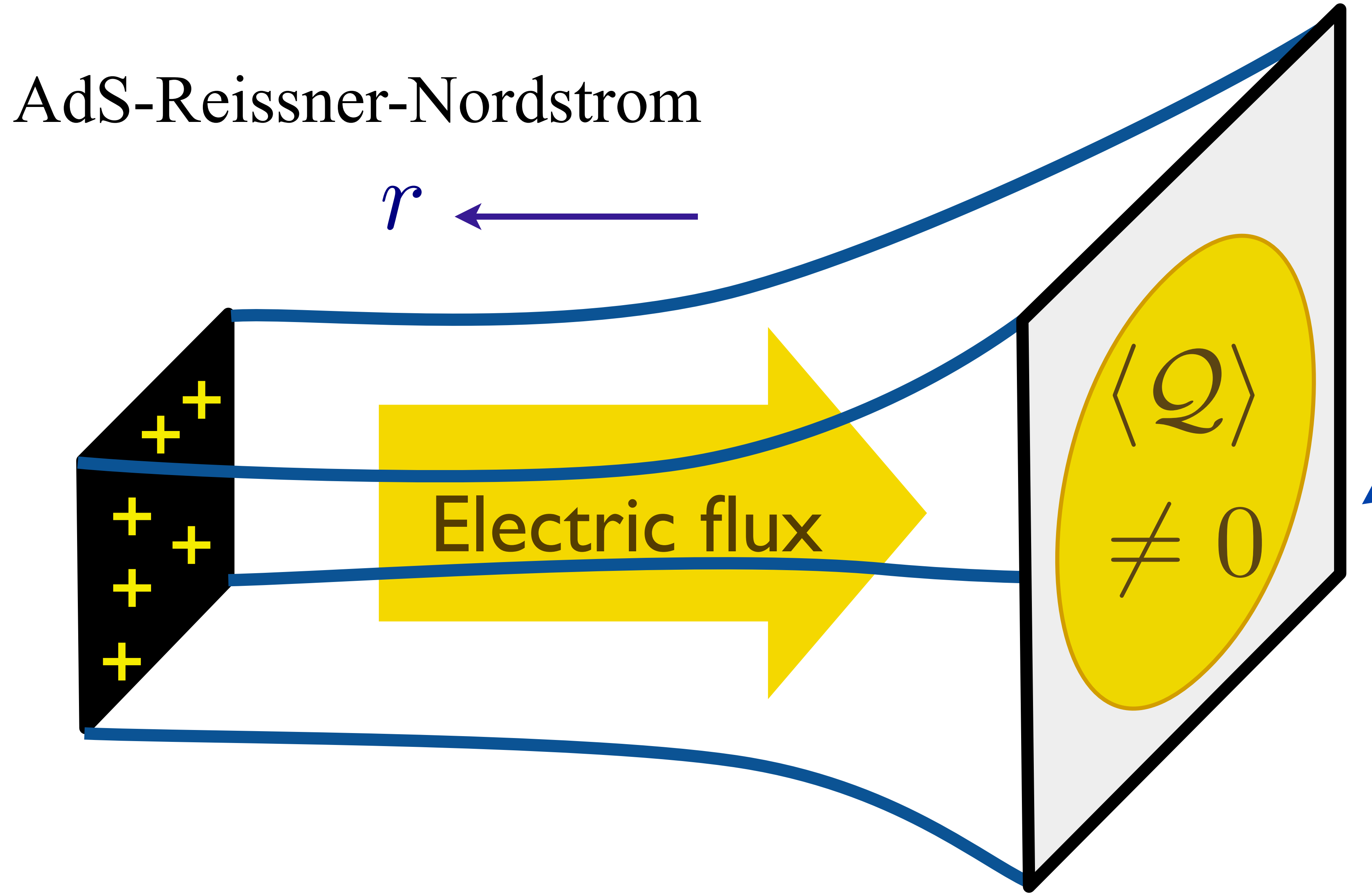


Entropy density of CFT$_{d+1}$, $\mathcal{S} \sim T^d$

Bekenstein-Hawking entropy density, $\mathcal{S}_{\rm BH} \sim T^d$

Charged black branes

Einstein-Maxwell theory $$\mathcal{S}_{EM} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2}F^2\right)\right]$$



Quantum matter on the boundary with a variable charge density $\mathcal{Q}$ of a global U(1) symmetry.

Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, $\mathcal{Q}$, at $T = 0$ which does not have any quasiparticle excitations.

A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, 99

Transport in Strange Metals

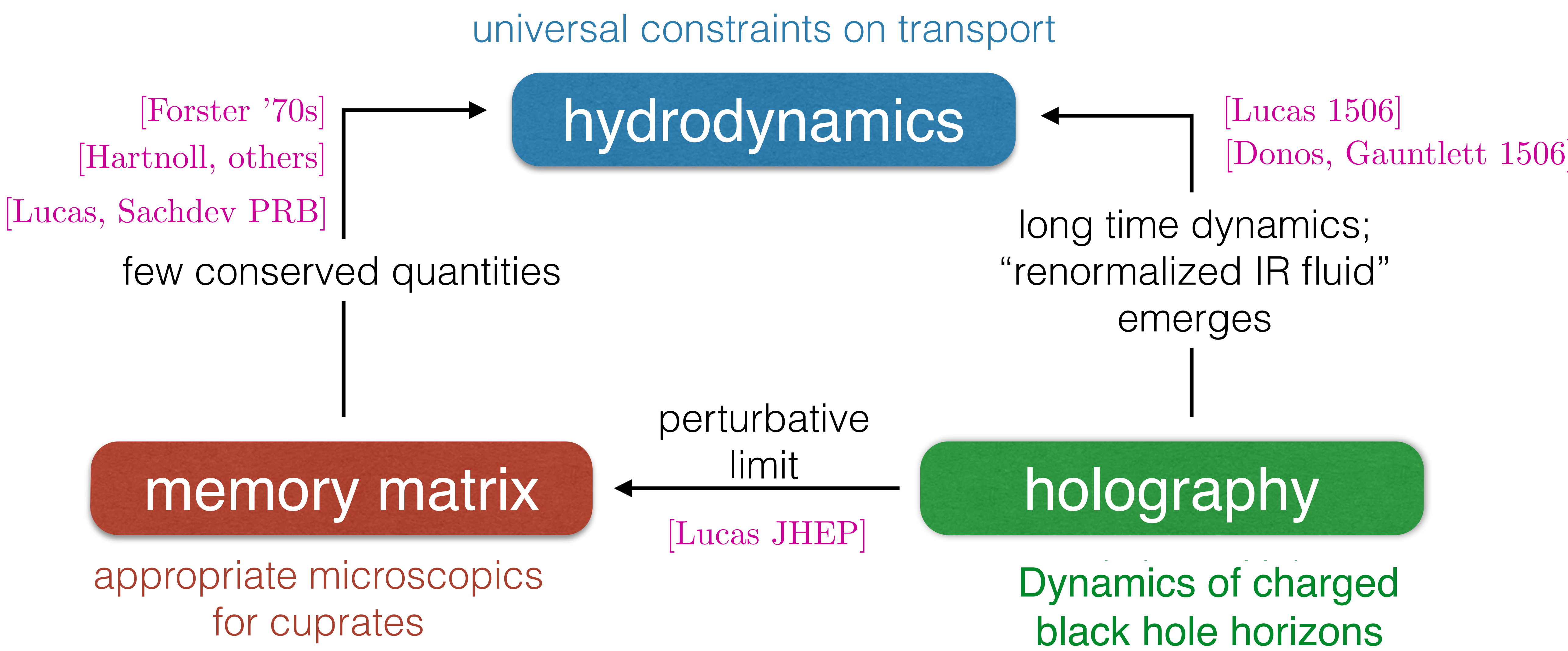


figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

Prediction for transport in the graphene strange metal

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa/(T\sigma)$, where κ is the thermal conductivity, and σ is the conductivity, is given by $L = \pi^2 k_B^2/(3e^2)$.

Prediction for transport in the graphene strange metal

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa/(T\sigma)$, where κ is the thermal conductivity, and σ is the conductivity, is given by $L = \pi^2 k_B^2/(3e^2)$.

For a strange metal with a "relativistic" Hamiltonian, hydrodynamic, holographic, and memory function methods yield

$$\sigma = \sigma_Q \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H} \sigma_Q}\right) \quad , \quad \kappa = \frac{v_F^2 \mathcal{H} \tau}{T} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H} \sigma_Q}\right)^{-1}$$

$$L = \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H} \sigma_Q}\right)^{-2},$$

where $\mathcal{H}$ is the enthalpy density, $\tau_{\rm imp}$ is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, "quantum critical" conductivity. Note that the limits $\mathcal{Q} \to 0$ and $\tau_{\rm imp} \to \infty$ do not commute.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2,*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] and Kin Chung Fong[5,†]

[1] *Department of Physics, Harvard University, Cambridge, MA 02138, USA*
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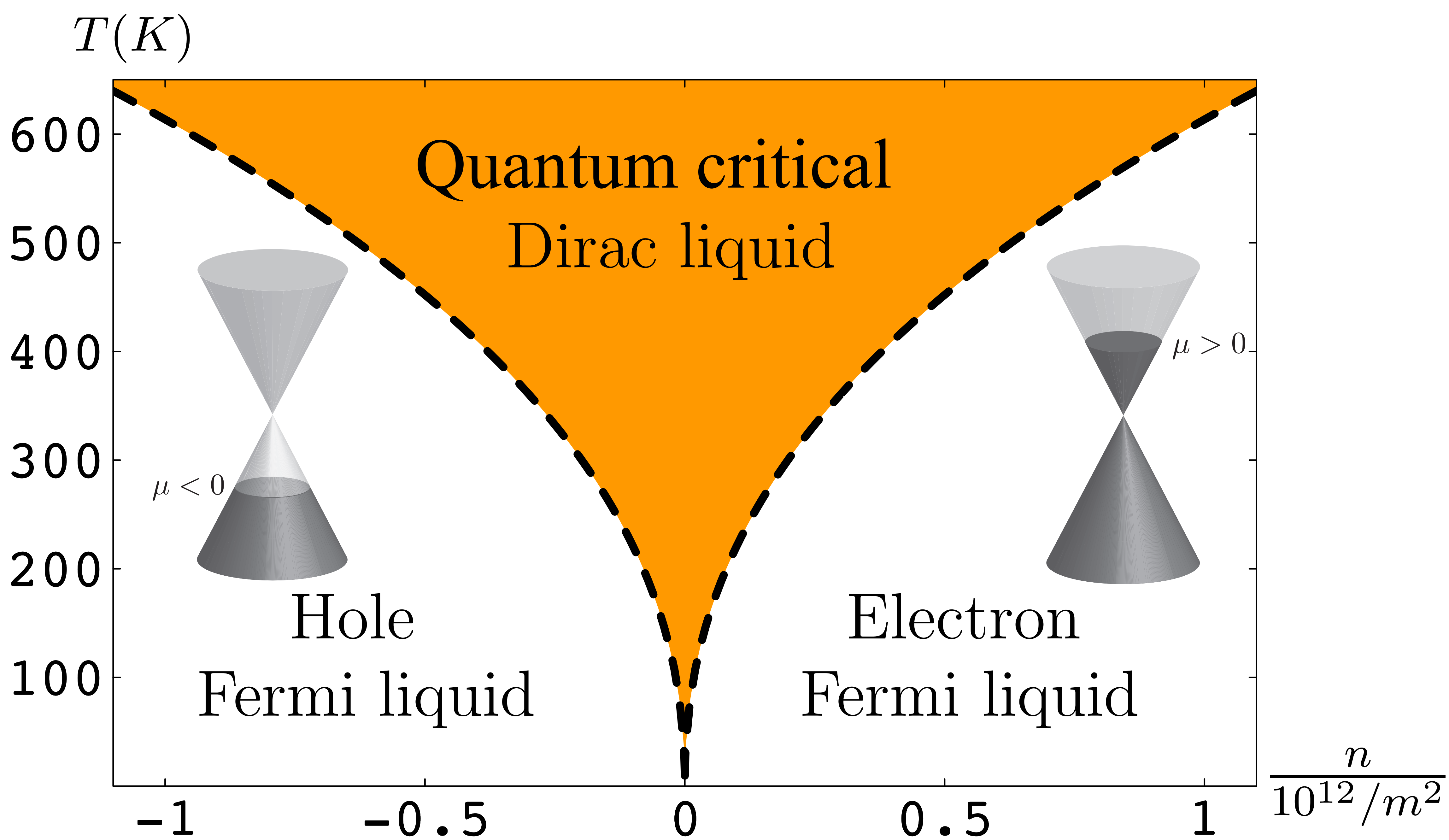
[4] *National Institute for Materials Science, Namiki 1-1, Tsukuba, Ibaraki 305-0044, Japan*
[5] *Raytheon BBN Technologies, Quantum Information Processing Group, Cambridge, Massachusetts 02138, USA*

(Dated: September 28, 2015)

Interactions between particles in quantum many-body systems can lead to collective behavior described by hydrodynamics. One such system is the electron-hole plasma in graphene near the charge neutrality point which can form a strongly coupled Dirac fluid. This charge neutral plasma of quasi-relativistic fermions is expected to exhibit a substantial enhancement of the thermal conductivity, due to decoupling of charge and heat currents within hydrodynamics. Employing high sensitivity Johnson noise thermometry, we report the breakdown of the Wiedemann-Franz law in graphene, with a thermal conductivity an order of magnitude larger than the value predicted by Fermi liquid theory. This result is a signature of the Dirac fluid, and constitutes direct evidence of collective motion in a quantum electronic fluid.

arXiv:1509.04713

Graphene



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M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
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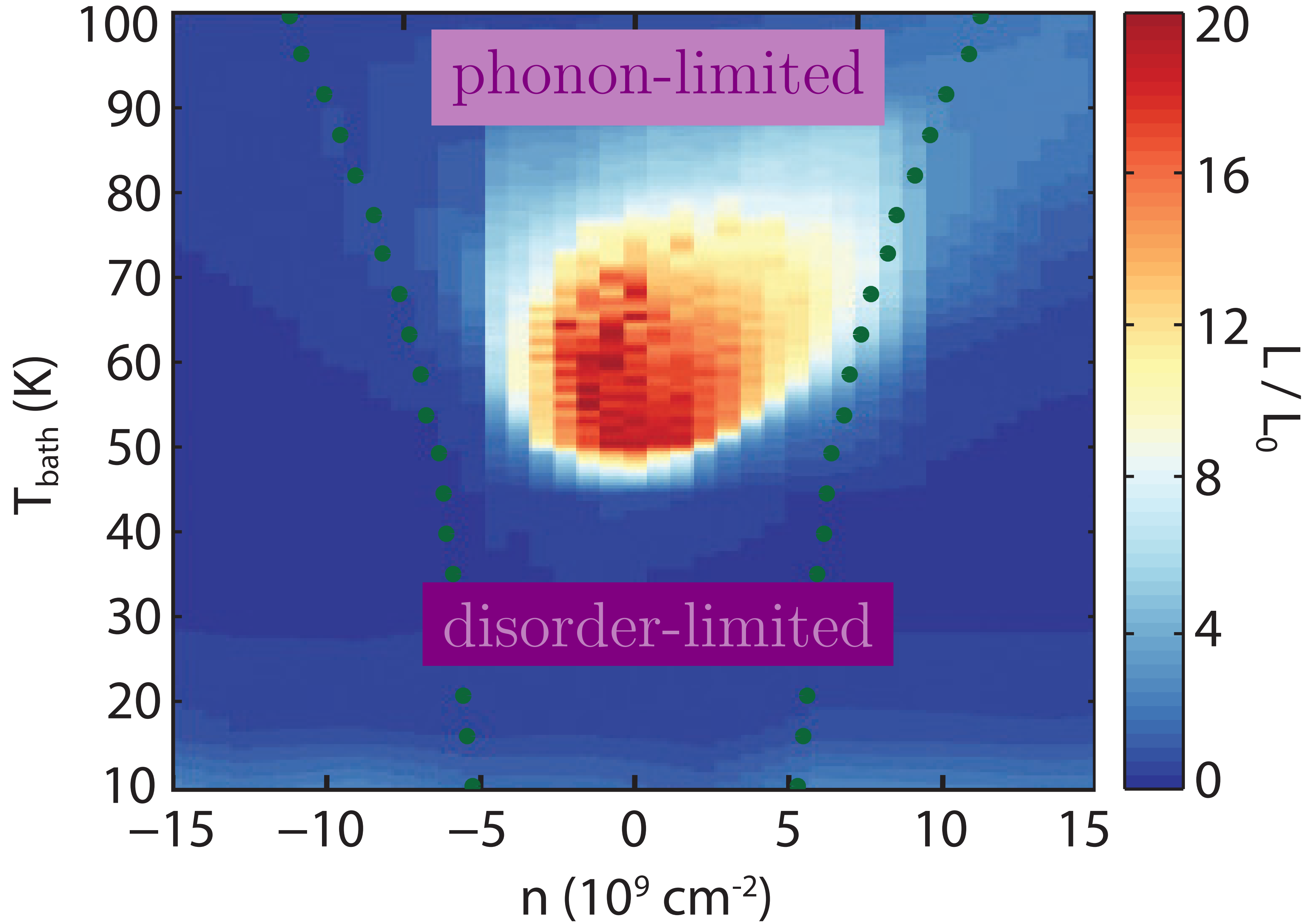
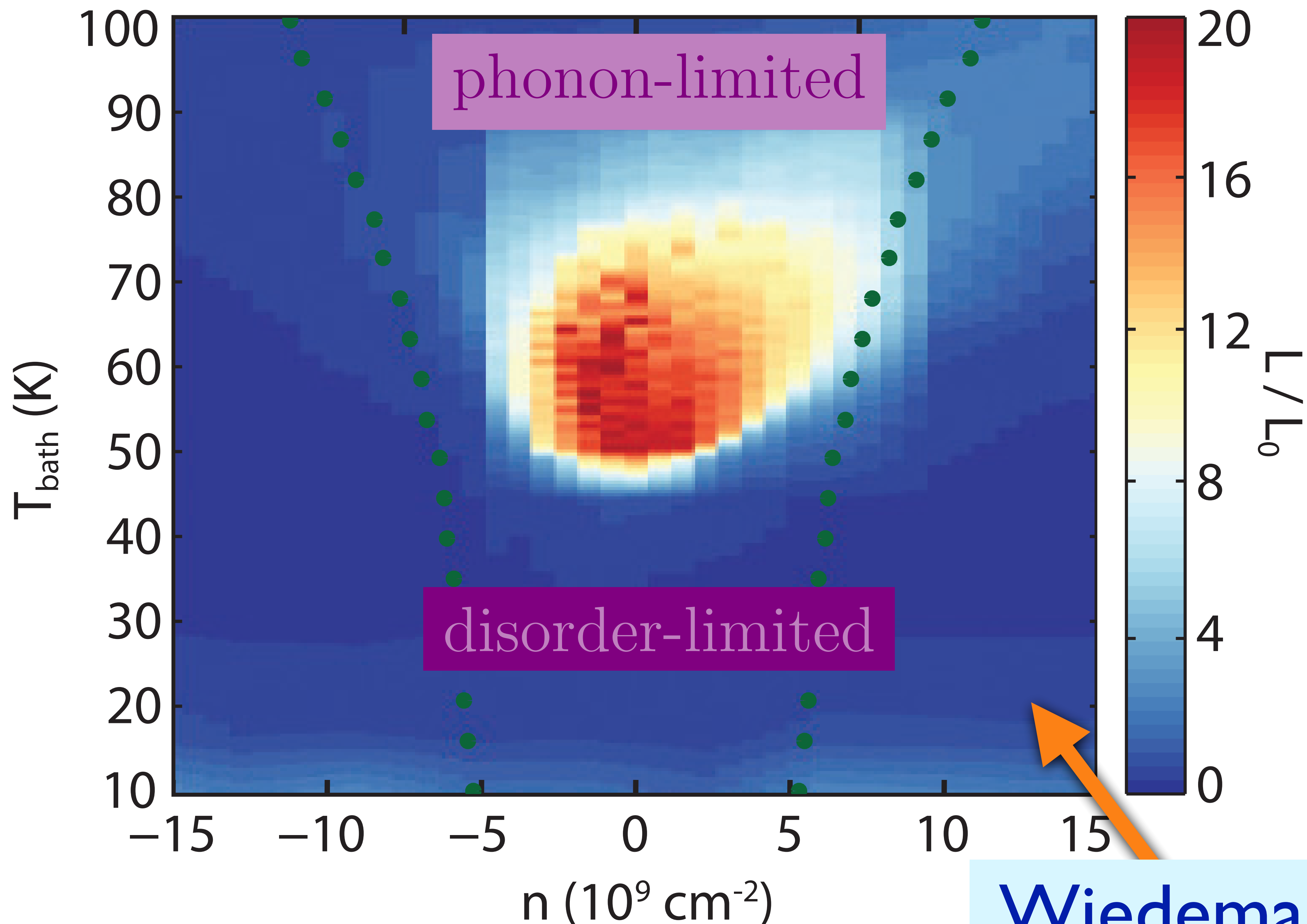
Strange metal in graphene



J. Crossno *et al.* arXiv:1509.04713

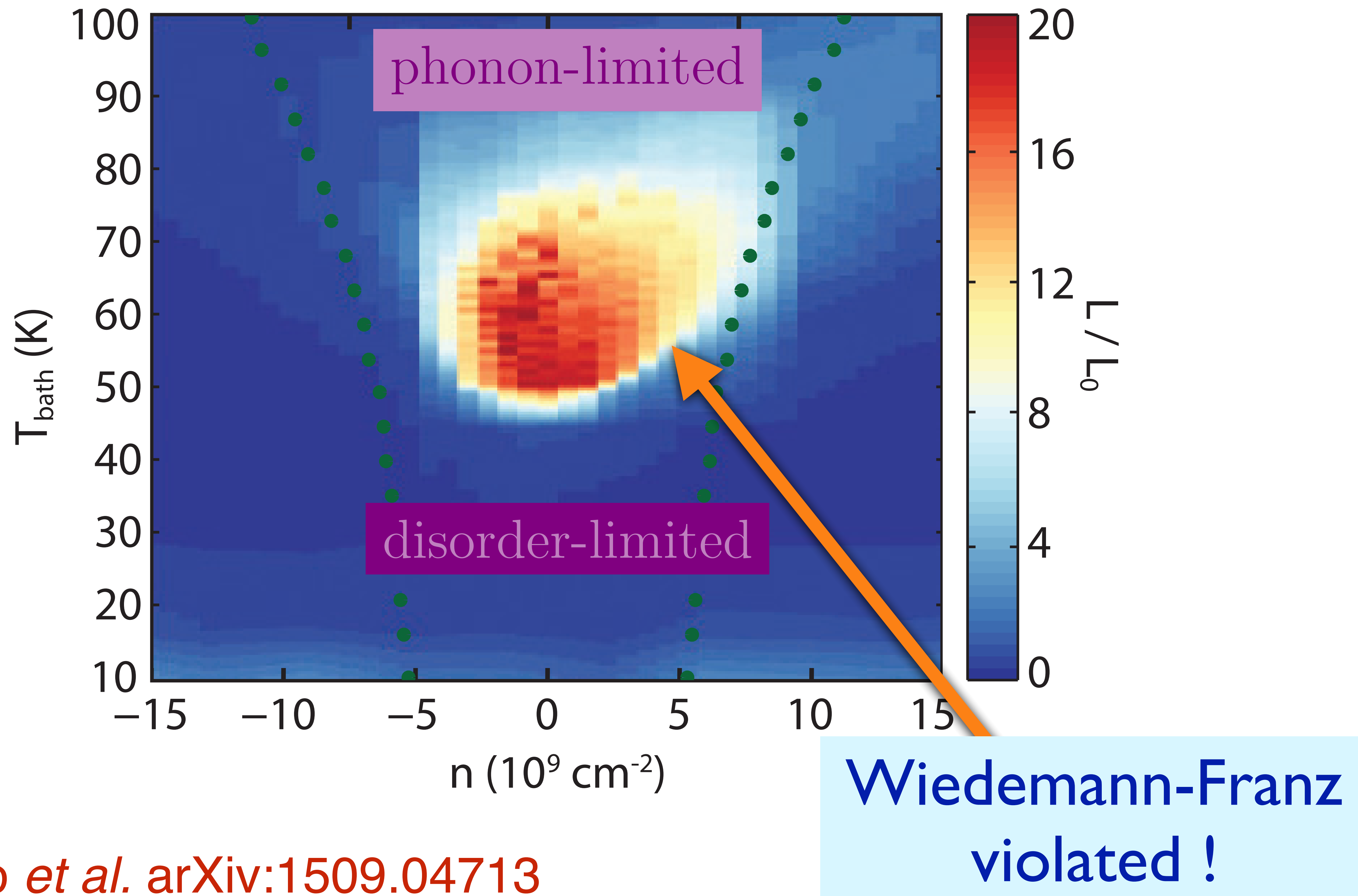
Strange metal in graphene



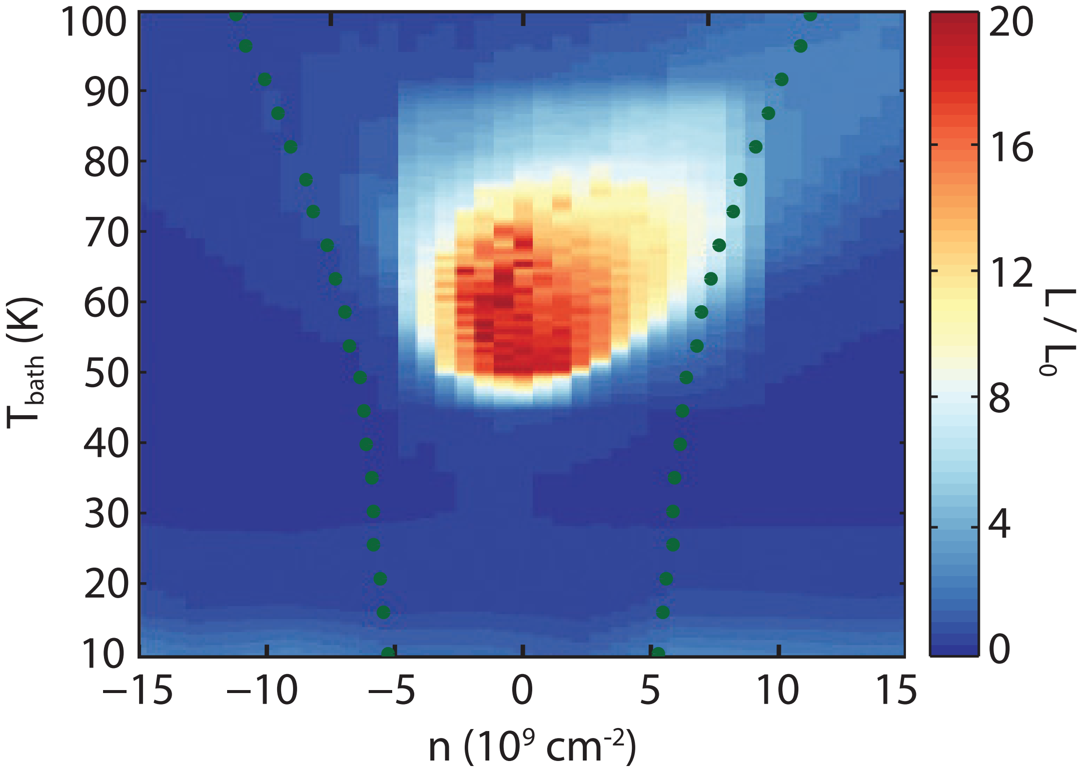
Wiedemann-Franz obeyed

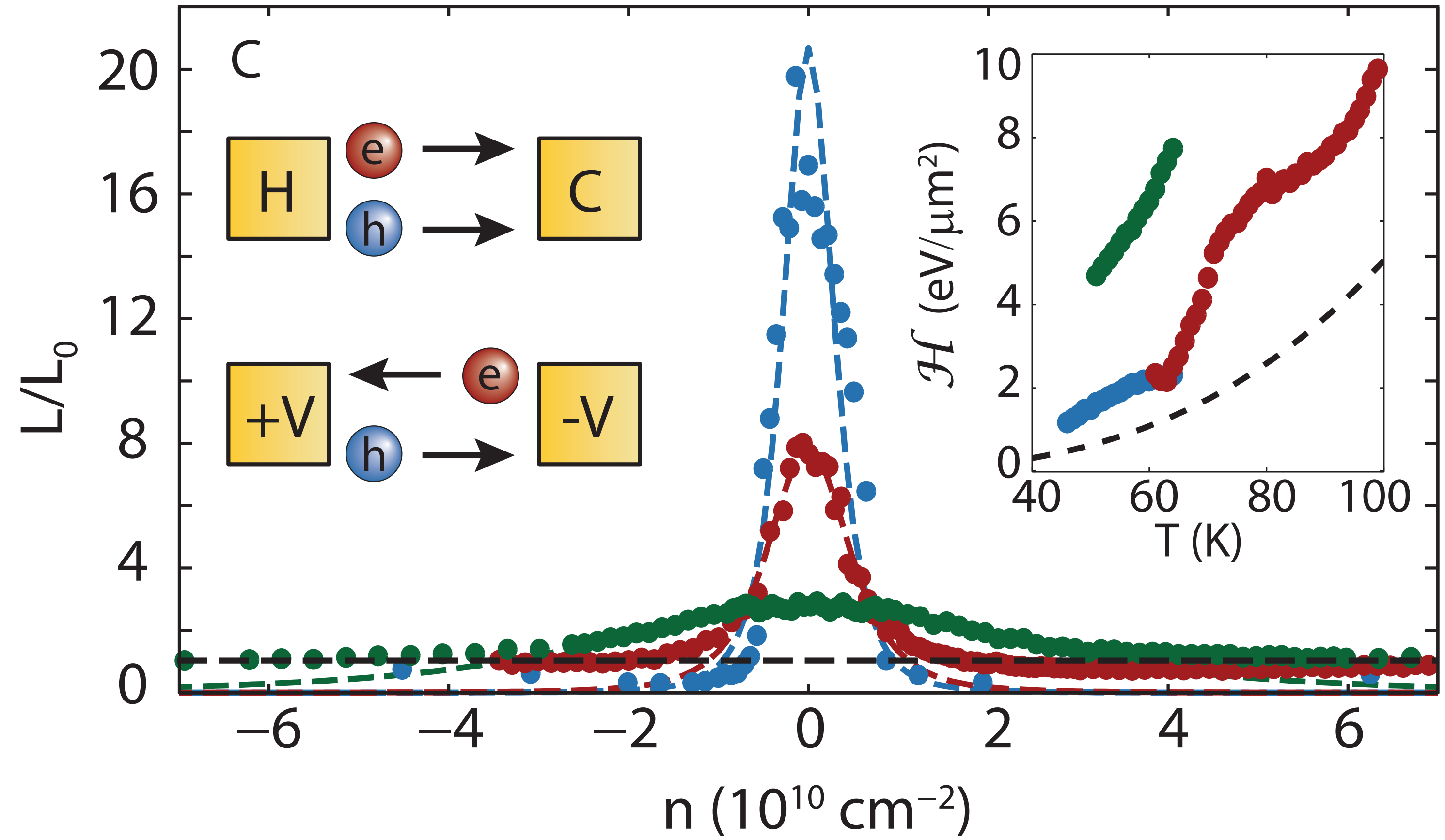
J. Crossno *et al.* arXiv:1509.04713

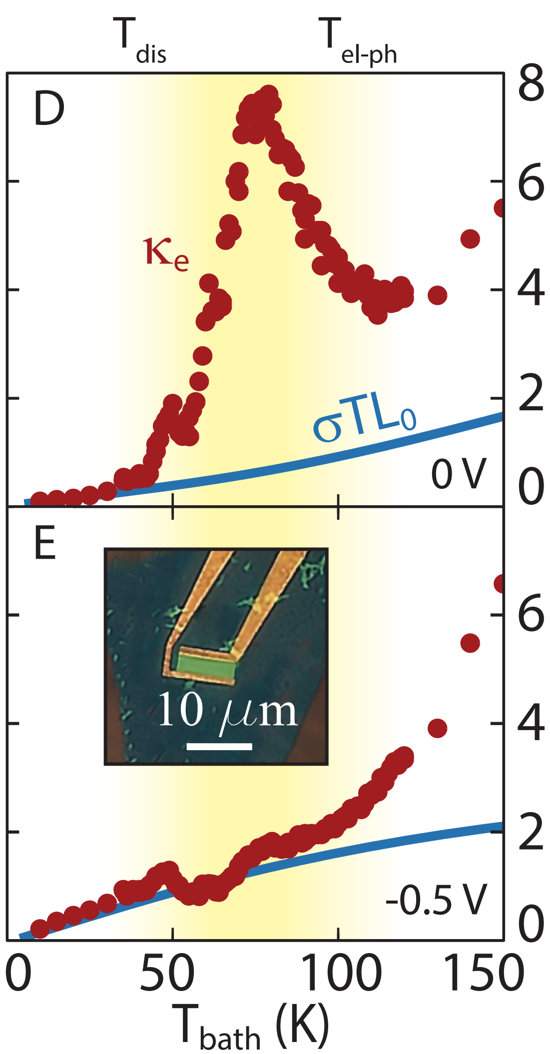
Strange metal in graphene



J. Crossno *et al.* arXiv:1509.04713



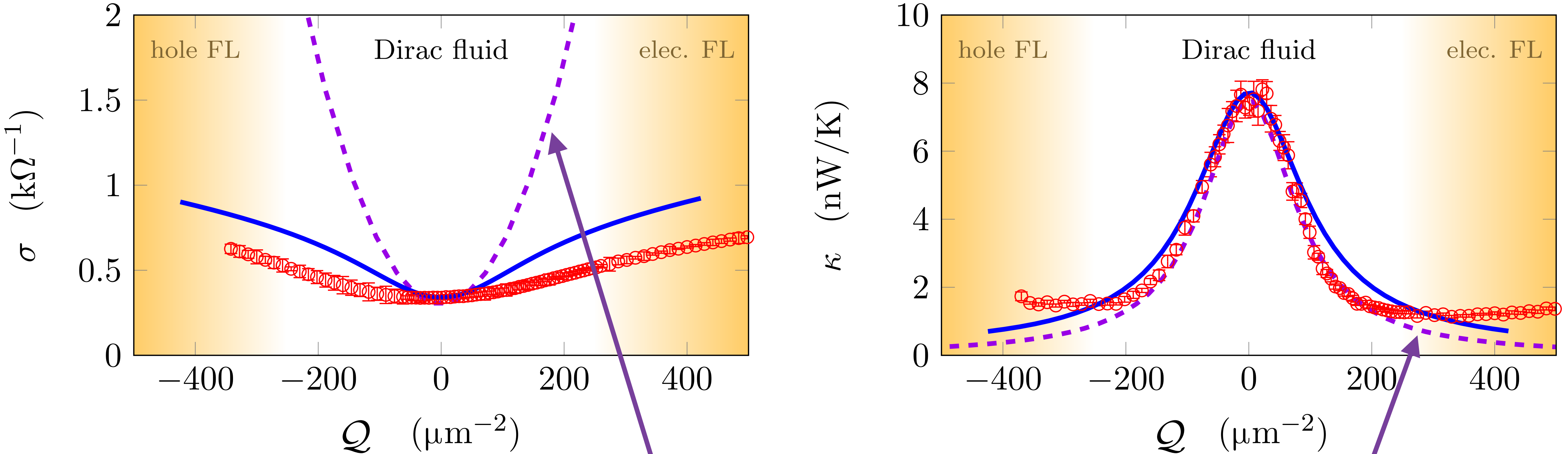




$$\text{Lorentz ratio } L = \kappa/(T\sigma)$$

$$= \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \frac{1}{\left(1 + e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}/(\mathcal{H}\sigma_Q)\right)^2}$$

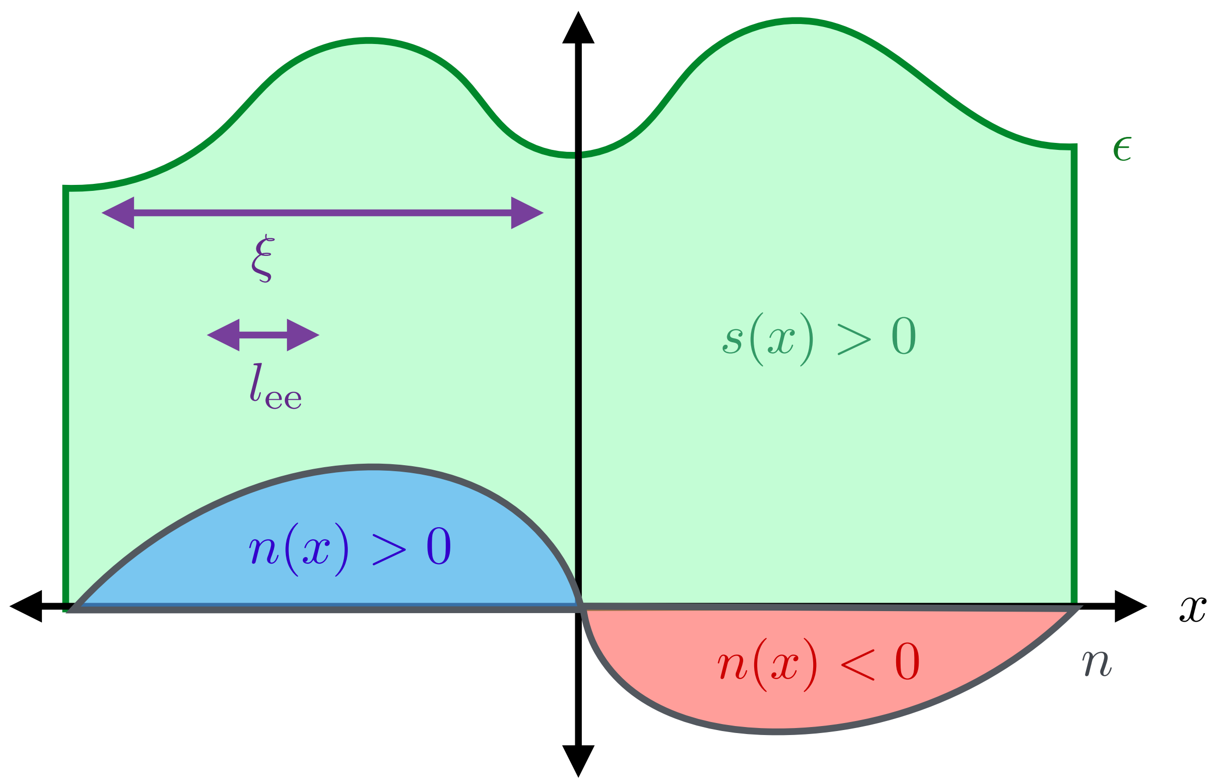
J. Crossno *et al.* arXiv:1509.04713



Comparison to theory with a single momentum relaxation time τ_{imp}. Best fit of density dependence to thermal conductivity does not capture the density dependence of electrical conductivity

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738

Non-perturbative treatment of disorder

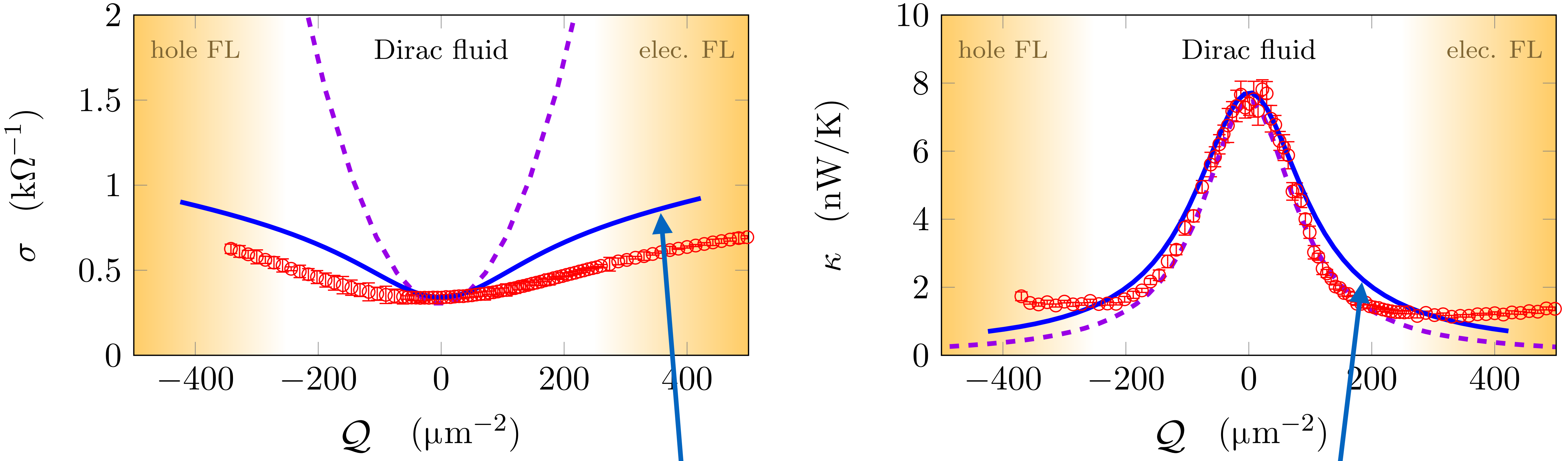


Note
$n \equiv \mathcal{Q}$

Figure 3: A cartoon of a nearly quantum critical fluid where our hydrodynamic description of transport is sensible. The local chemical potential $\mu(\mathbf{x})$ always obeys $|\mu| \ll k_B T$, and so the entropy density s/k_B is much larger than the charge density $|n|$; both electrons and holes are everywhere excited, and the energy density ϵ does not fluctuate as much relative to the mean. Near charge neutrality the local charge density flips sign repeatedly. The correlation length of disorder ξ is much larger than l_{ee}, the electron-electron interaction length.

Numerically solve the hydrodynamic equations of Hartnoll, Kovtun, Müller, Sachdev (PRB 76, 144502 (2007)) but in the presence of a x-dependent chemical potential. The thermoelectric transport properties will then depend upon the value of the shear viscosity, η.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738



Solution of the hydrodynamic equations in the presence of a space-dependent chemical potential.
Best fit of density dependence to thermal conductivity now gives a better fit to the density dependence of the electrical conductivity (for $\eta/s \approx 10$). The T dependencies of other parameters also agree well with expectation.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738

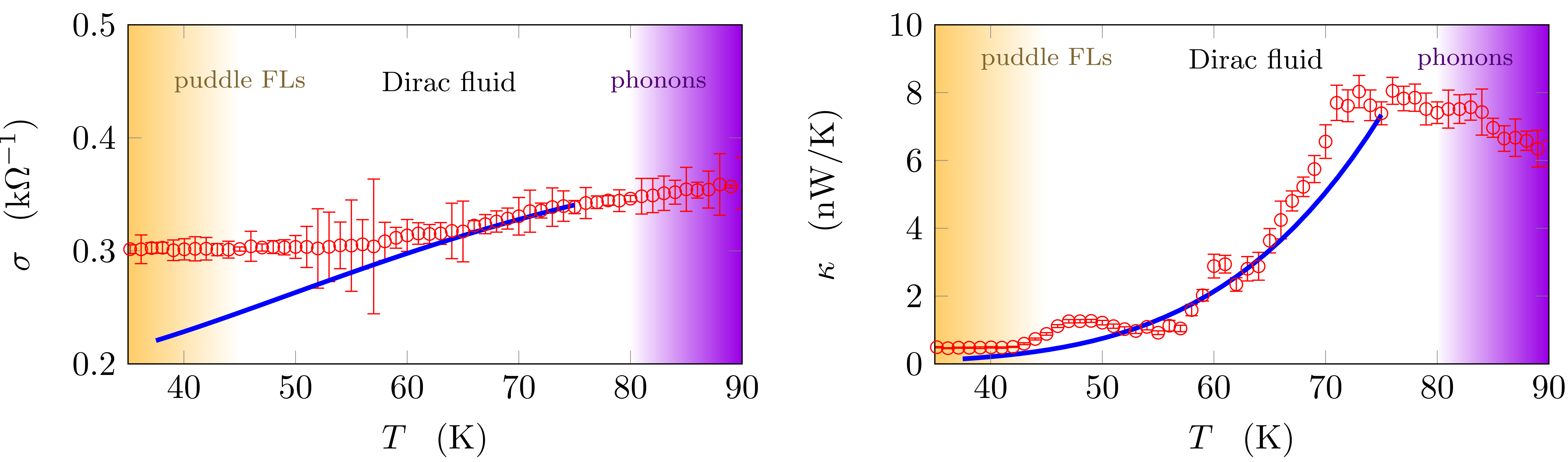


Figure 2: A comparison of our hydrodynamic theory of transport with the experimental results of [33] in clean samples of graphene at the charge neutrality point ($n = 0$). We use no new fit parameters compared to Figure 1. The yellow shaded region denotes where Fermi liquid behavior is observed; the purple shaded region denotes the likely onset of electron-phonon coupling.

Solution of the hydrodynamic equations in the presence of a space-dependent chemical potential.
Best fit of density dependence to thermal conductivity now gives a better fit to the density dependence of the electrical conductivity (for $\eta/s \approx 10$). The T dependencies of other parameters also agree well with expectation.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738

Strange metal in graphene

arXiv:1509.04165

Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin[1], I. Torre[2,3], R. Krishna Kumar[1,4], M. Ben Shalom[1,5], A. Tomadin[6], A. Principi[7], G. H. Auton[5], E. Khestanova[1,5], K. S. Novoselov[5], I. V. Grigorieva[1], L. A. Ponomarenko[1,4], A. K. Geim[1], M. Polini[3,6]

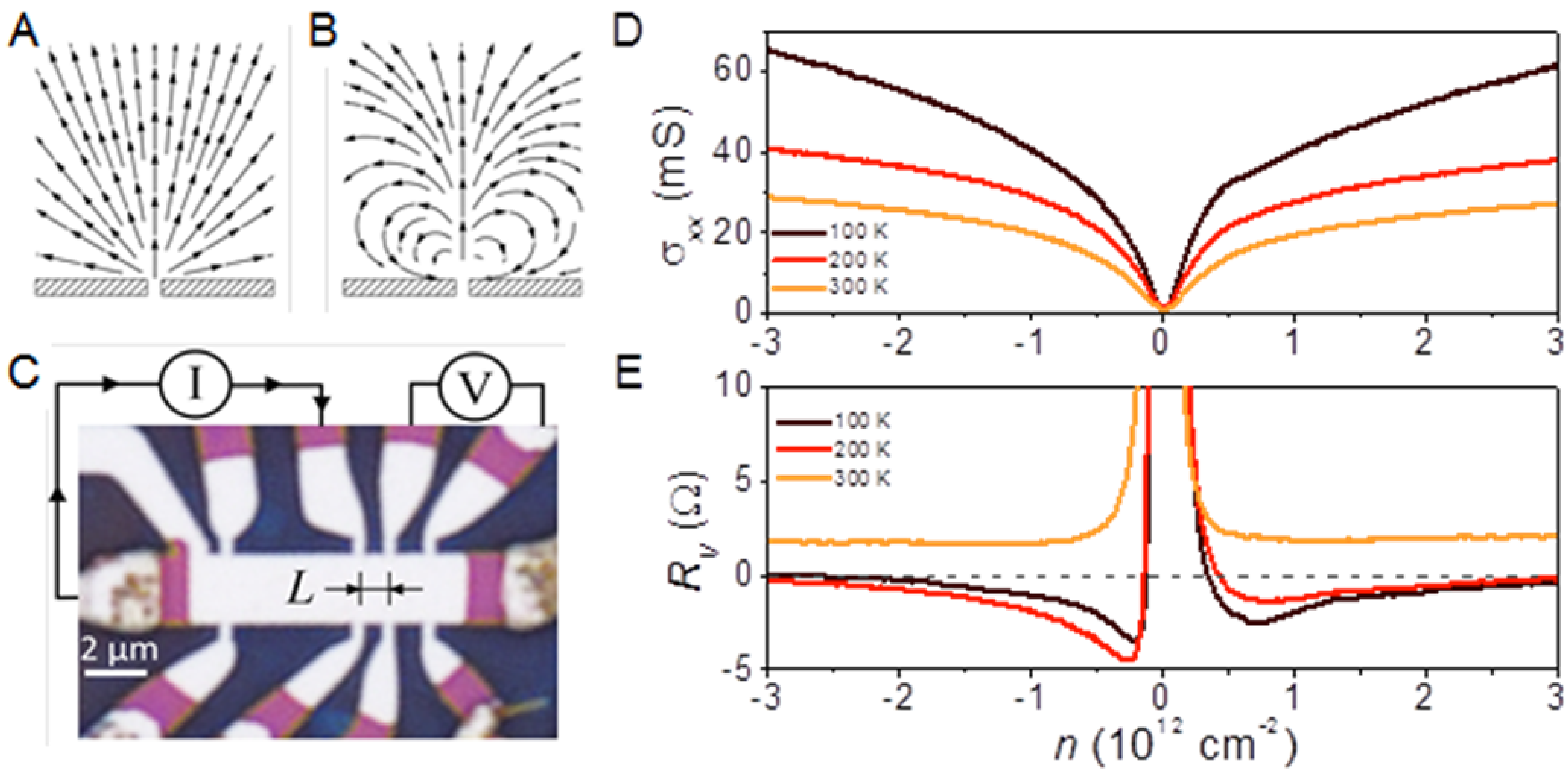


Figure 1. Viscous backflow in doped graphene. (**a,b**) Steady-state distribution of current injected through a narrow slit for a classical conducting medium with zero ν (a) and a viscous Fermi liquid (b). (**c**) Optical micrograph of one of our SLG devices. The schematic explains the measurement geometry for vicinity resistance. (**d,e**) Longitudinal conductivity σ_{xx} and R_{V} for this device as a function of n induced by applying gate voltage. $I = 0.3$ μA; $L = 1$ μm. For more detail, see Supplementary Information.

1. Long-range entanglement in insulators

2. Theory of ordinary metals

   (a) Quasiparticles

   (b) Luttinger theorem for volume enclosed by Fermi surface

3. Fractionalized Fermi liquid

   Quasiparticles with a non-Luttinger volume in the pseudogap metal of the cuprate superconductors

4. Strange metals without quasiparticles

   Experiments in graphene, and charged black holes