Deconfined criticality in the underdoped cuprates

Ribhu Kaul (Harvard), Yong-Baek Kim (Toronto)
Alexei Kolezhuk (Hannover), Michael Levin (Harvard)
Subir Sachdev (Harvard), T. Senthil (MIT)

Outline

1. Quantum transitions in dimerized antiferromagnets

   Landau-Ginzburg-Wilson (LGW) theory

2. Square lattice antiferromagnets

   VBS order and deconfined criticality

3. Doped square lattice antiferromagnets

   Fermi liquid states

4. Doped square lattice antiferromagnets

   Non-Fermi liquid holon metal

5. Instabilities of the holon metal

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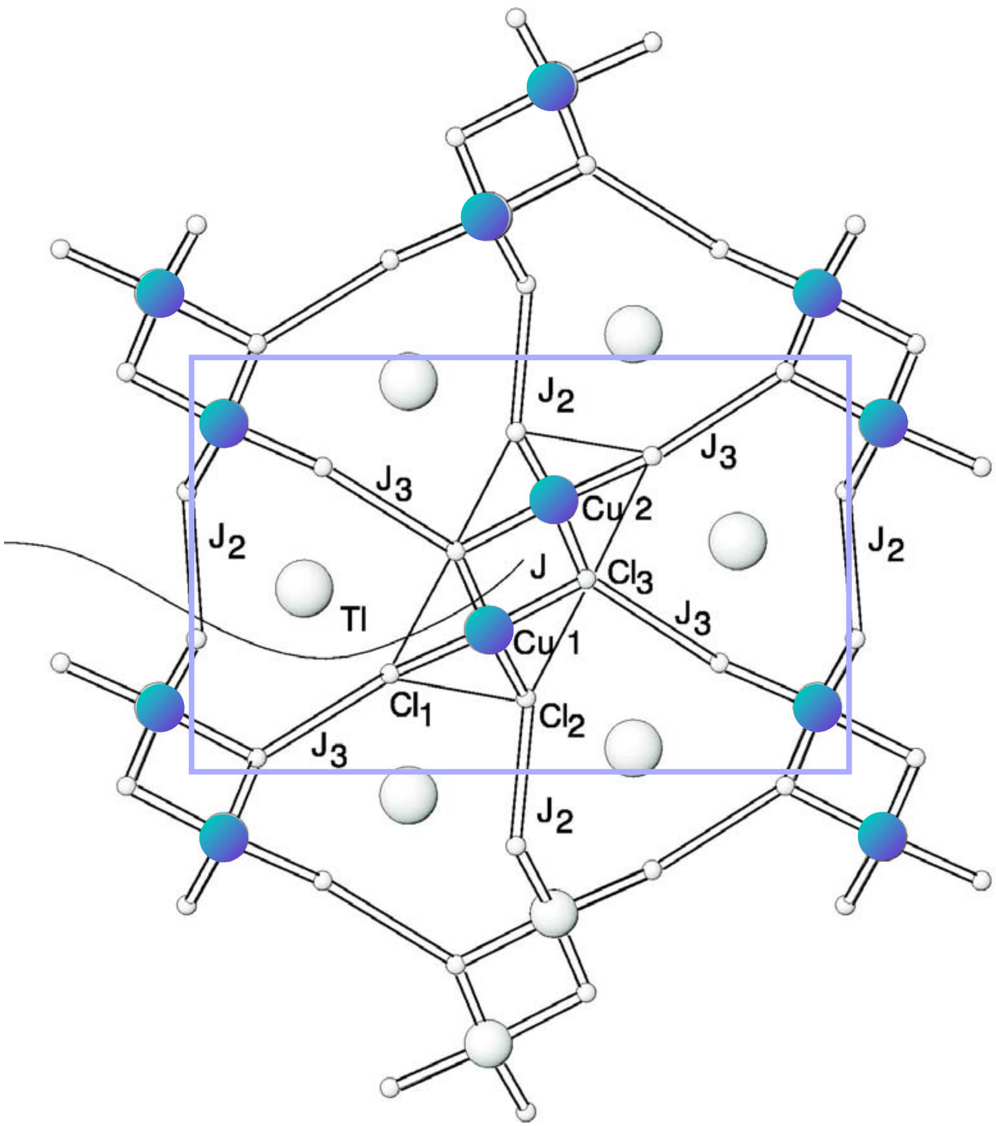
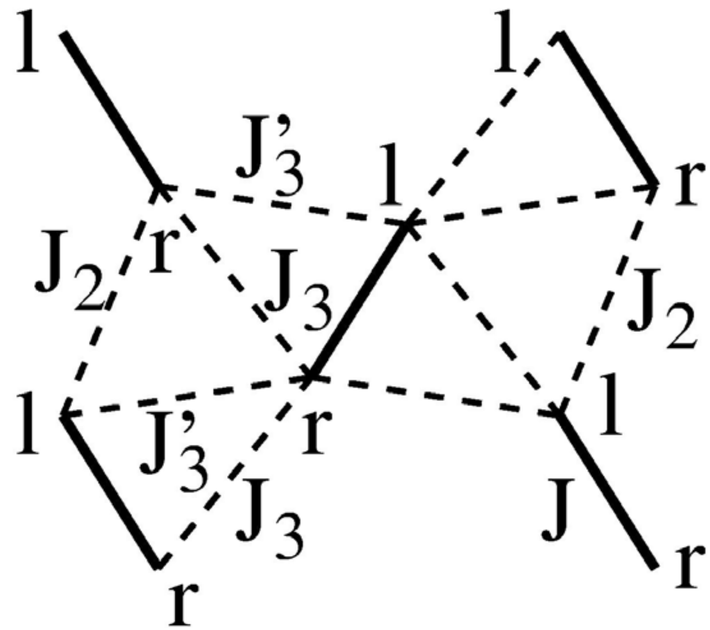
$TlCuCl_3$



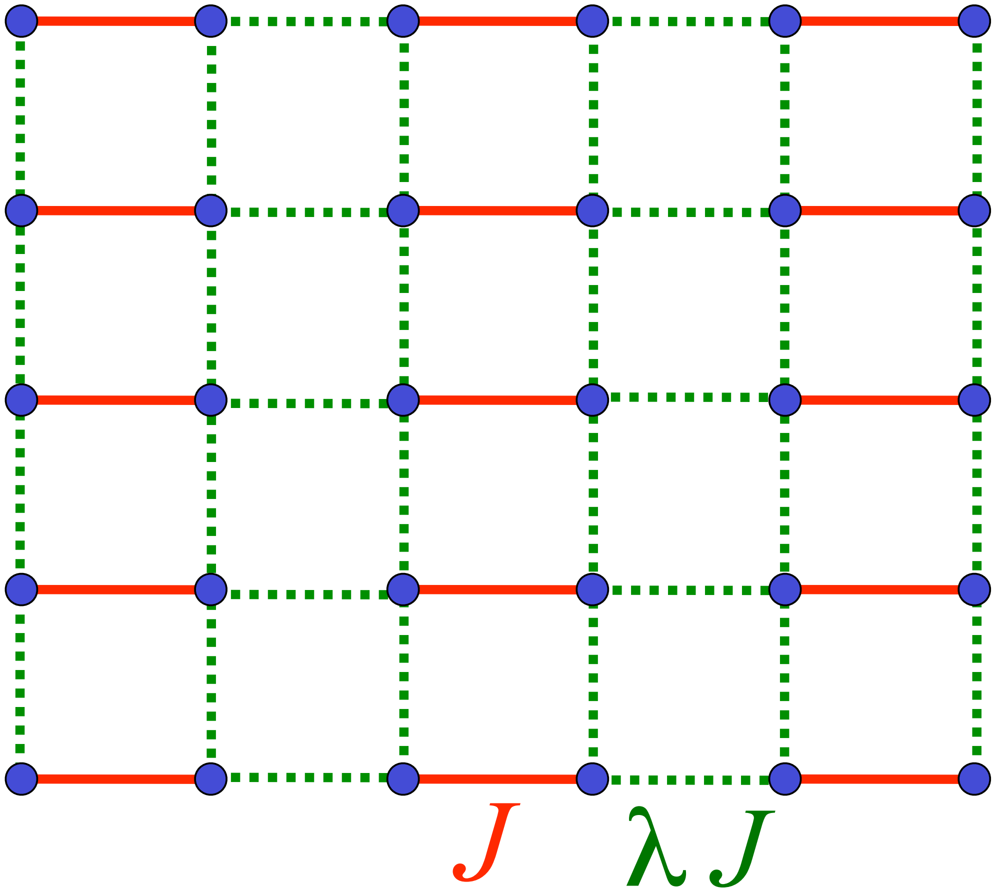


M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, cond-mat/0309440.

Coupled Dimer Antiferromagnet

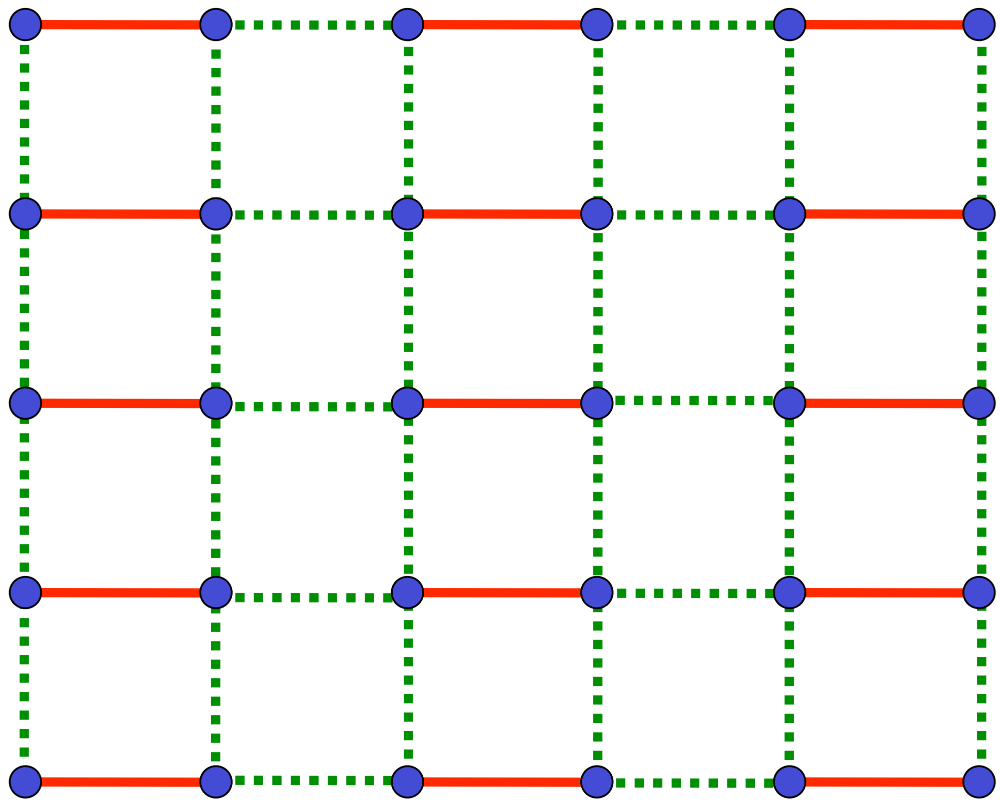
M. P. Gelfand, R. R. P. Singh, and D. A. Huse, *Phys. Rev. B* **40**, 10801-10809 (1989).
N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev*. B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).

S=1/2 spins on coupled dimers



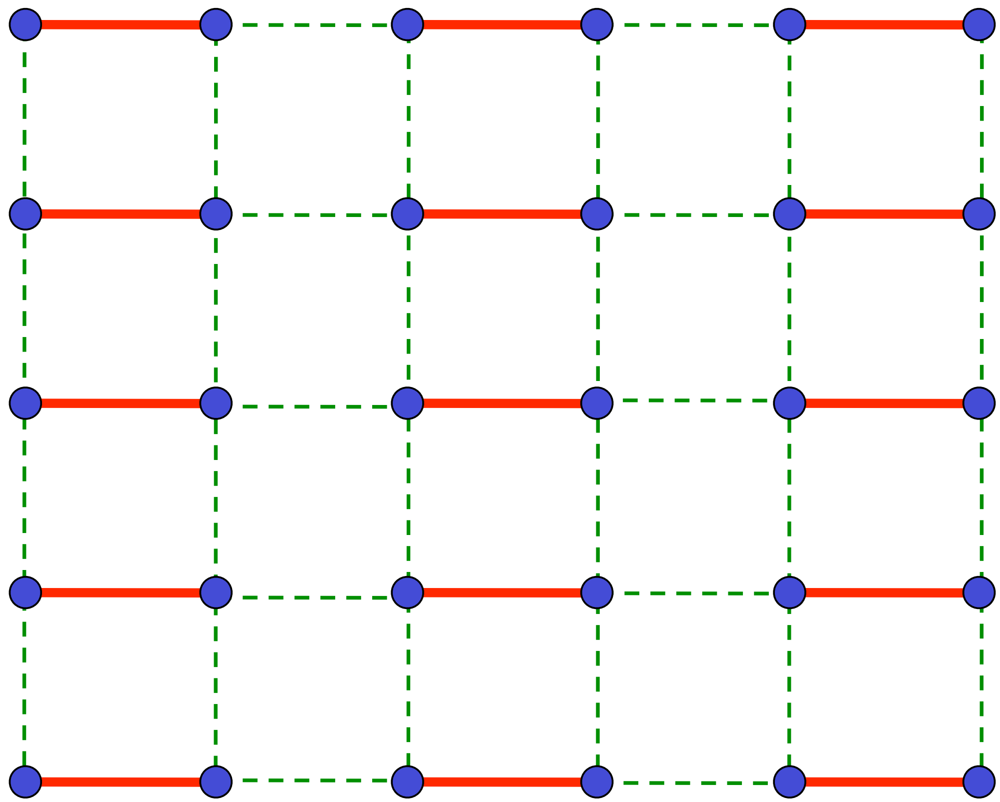
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \leq \lambda \leq 1$$



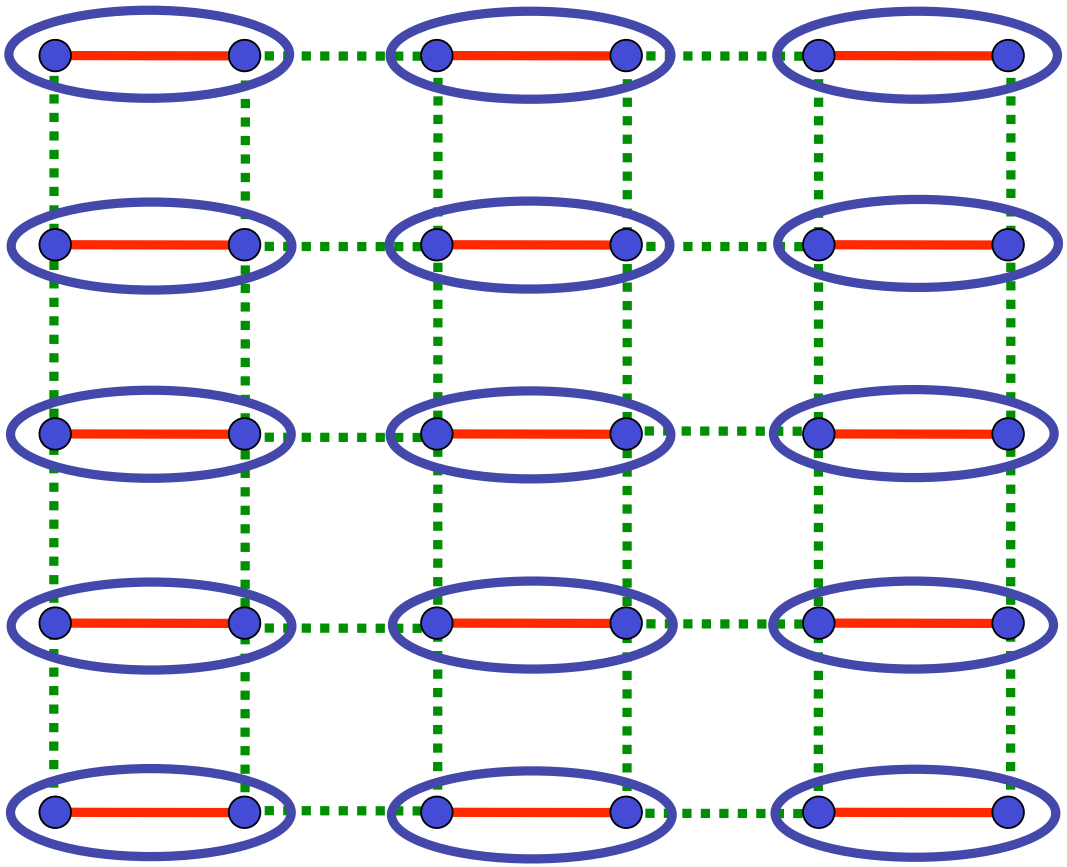
λ close to 0

Weakly coupled dimers

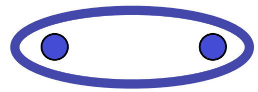


λ close to 0

Weakly coupled dimers



Paramagnetic ground state

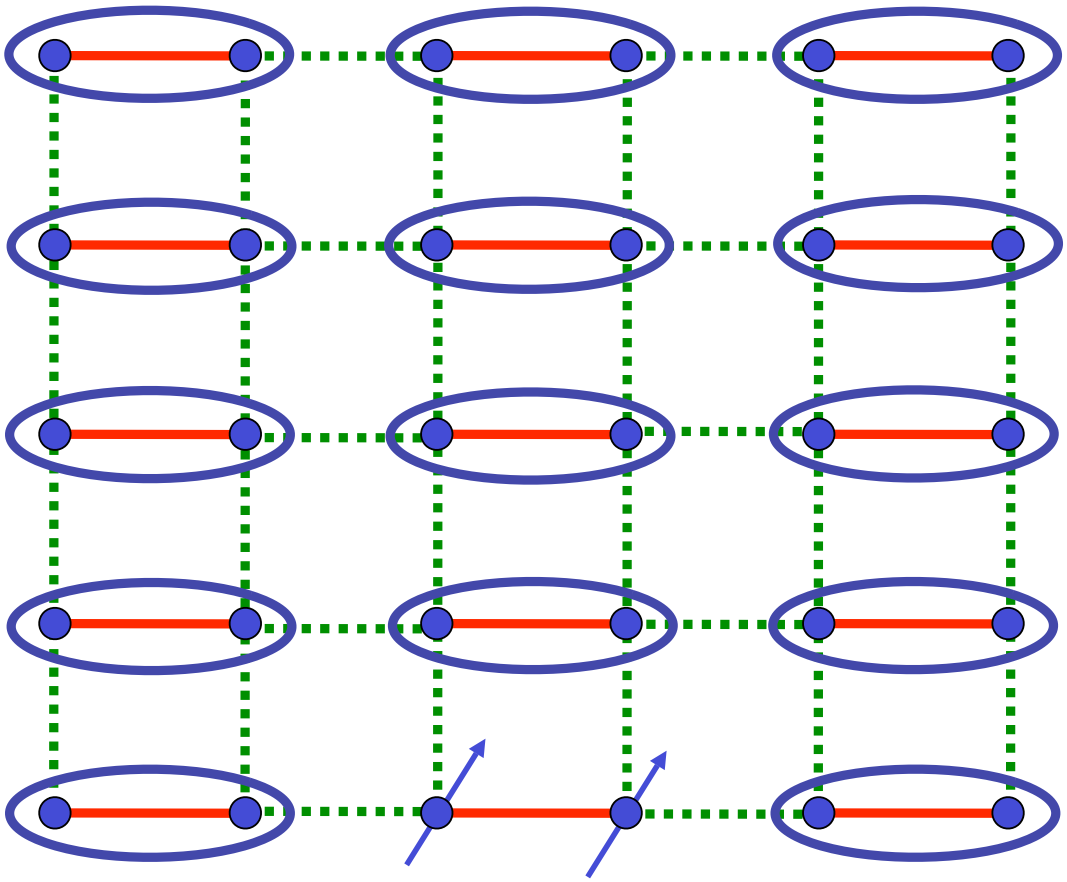


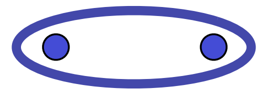
$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$$\left\langle \vec{S}_i \right\rangle = 0$$

λ close to 0

Weakly coupled dimers



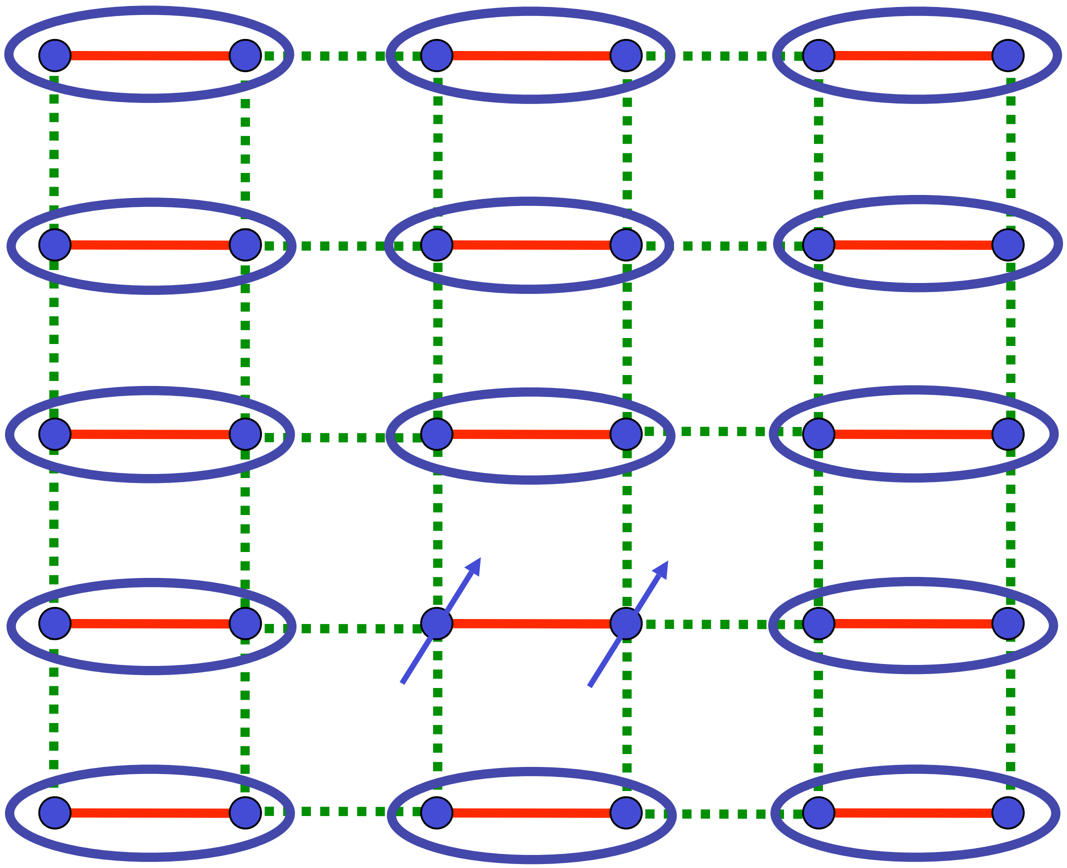


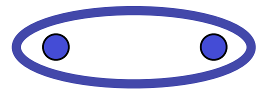
$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Excitation:
S=1 quasipartcle

λ close to 0

Weakly coupled dimers





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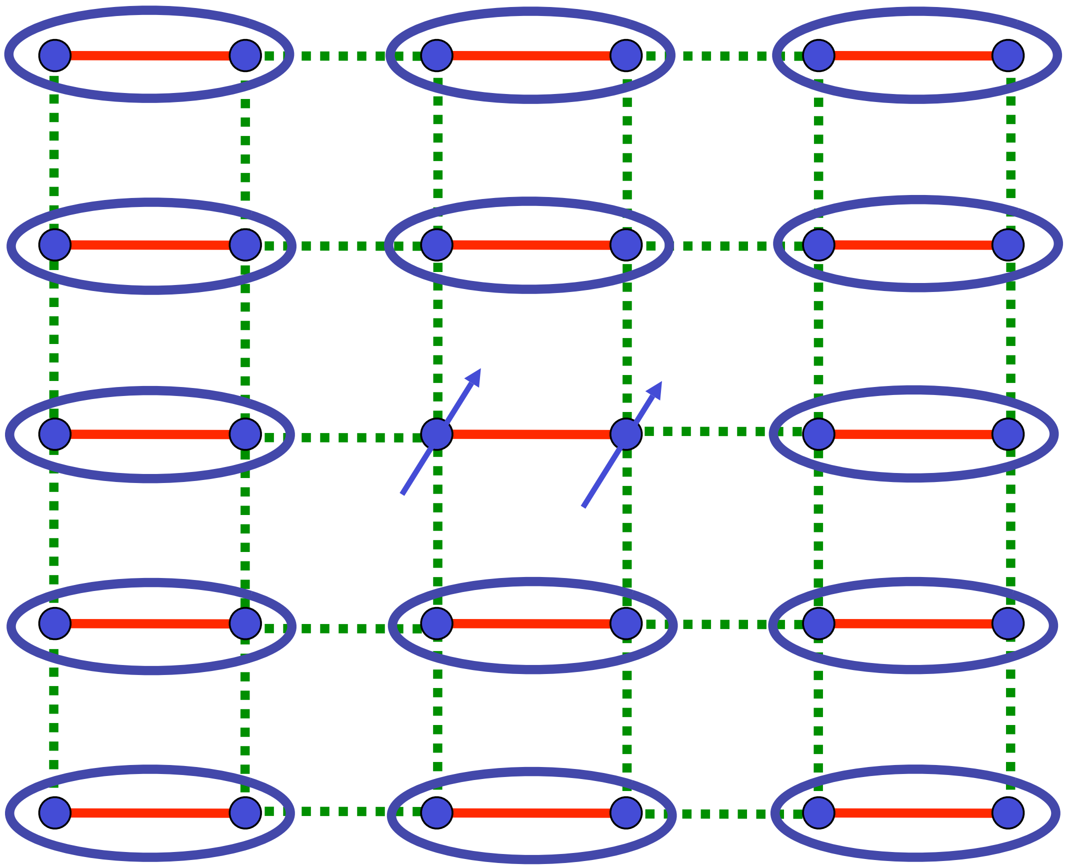

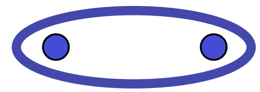

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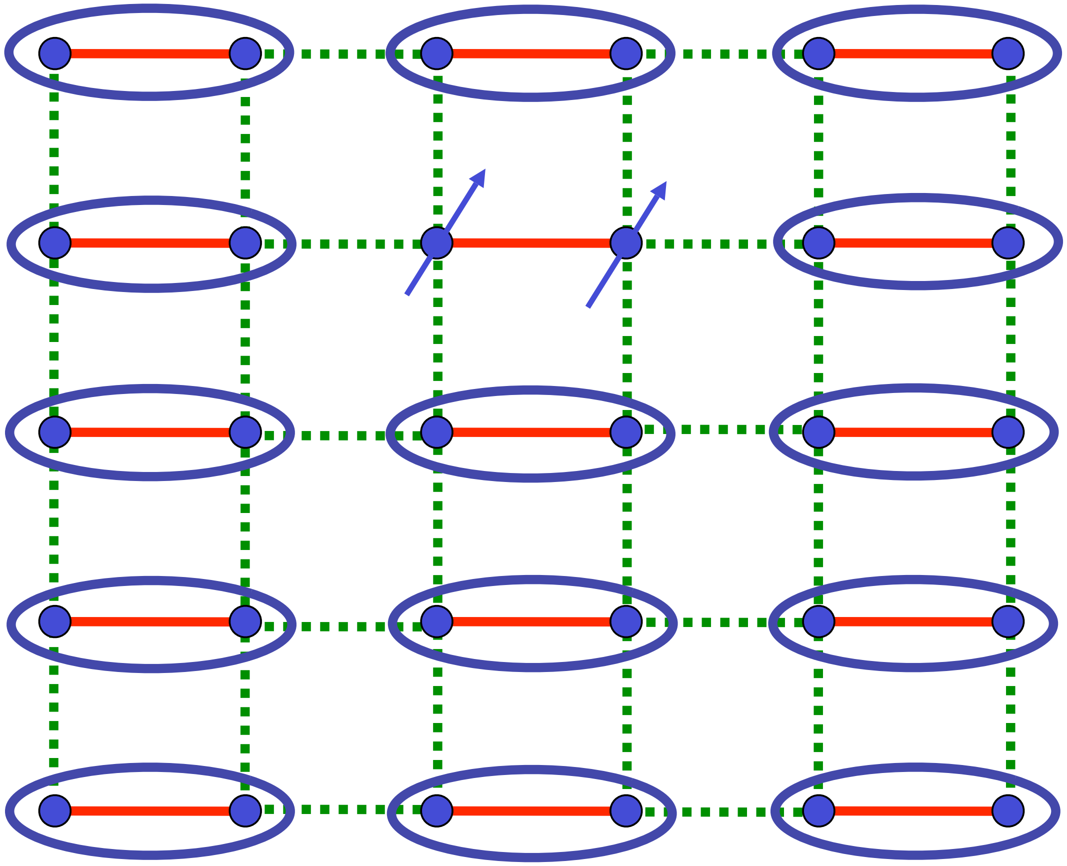

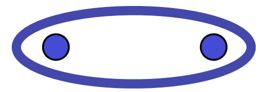

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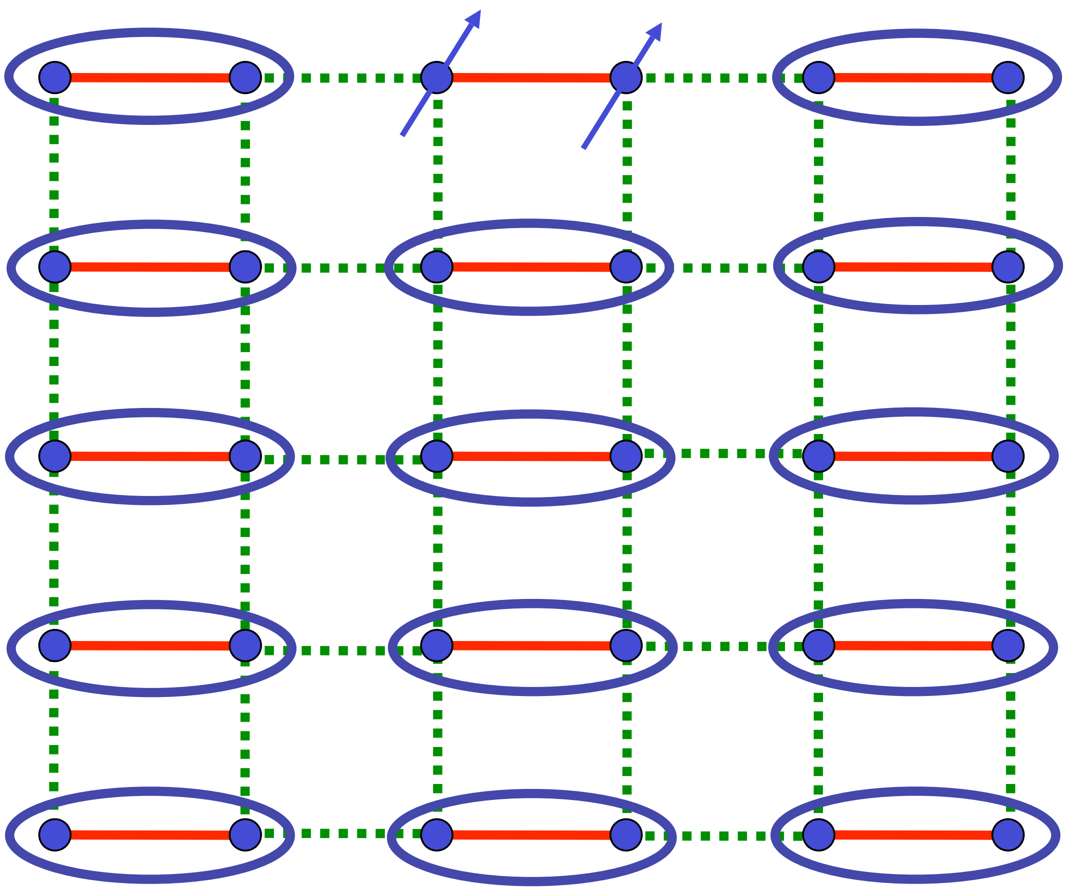

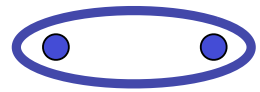

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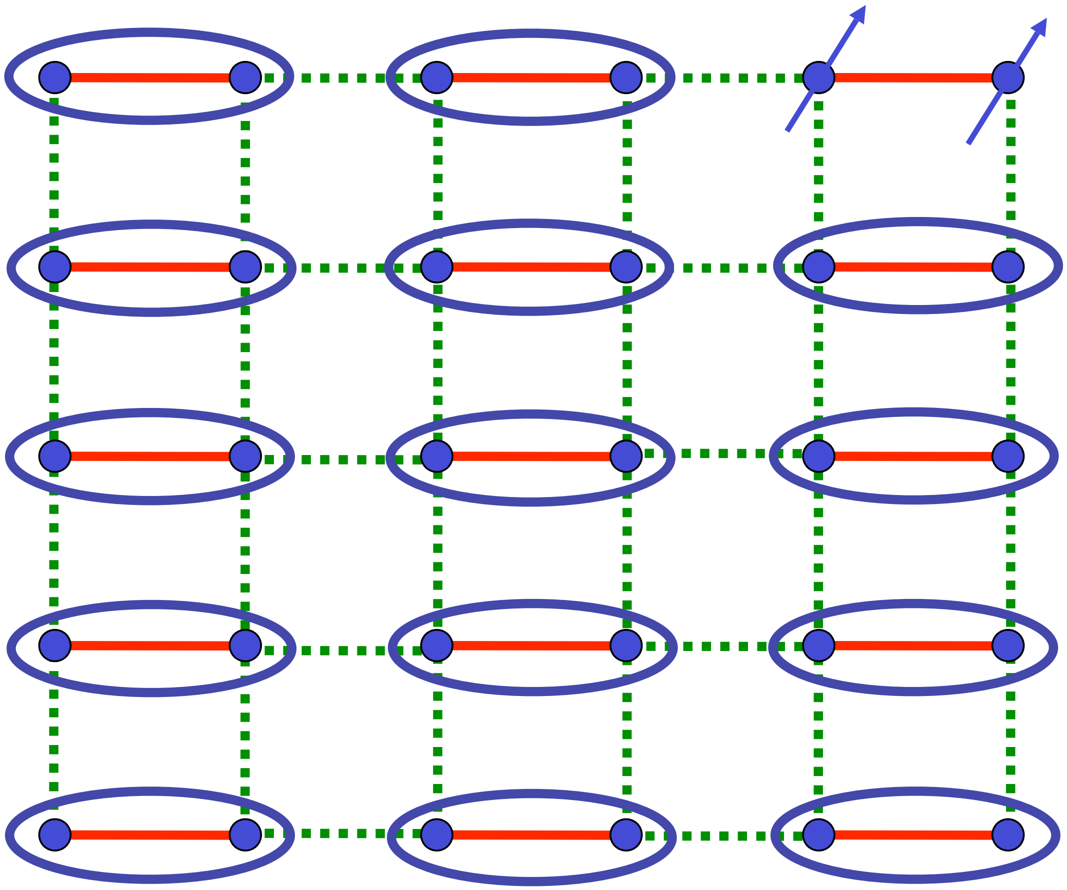
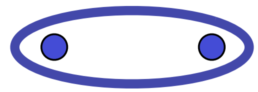




$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

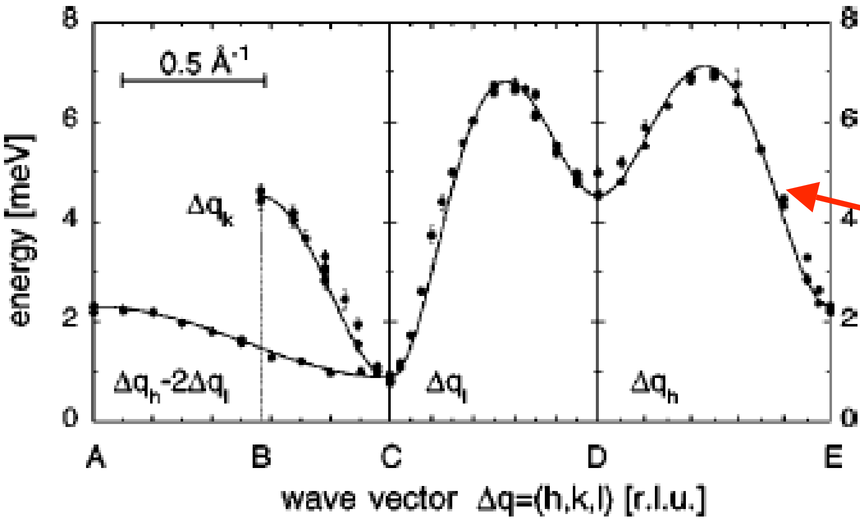
Excitation:
S=1 quasipartcle

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$$

$\Delta \rightarrow$ spin gap

$TlCuCl_3$



N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

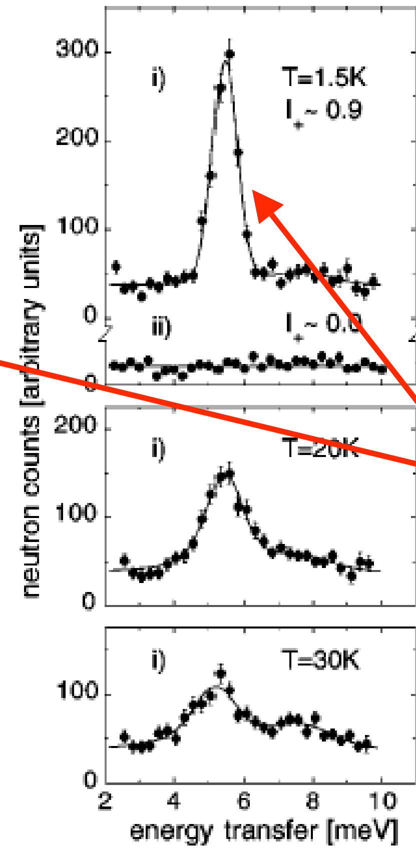
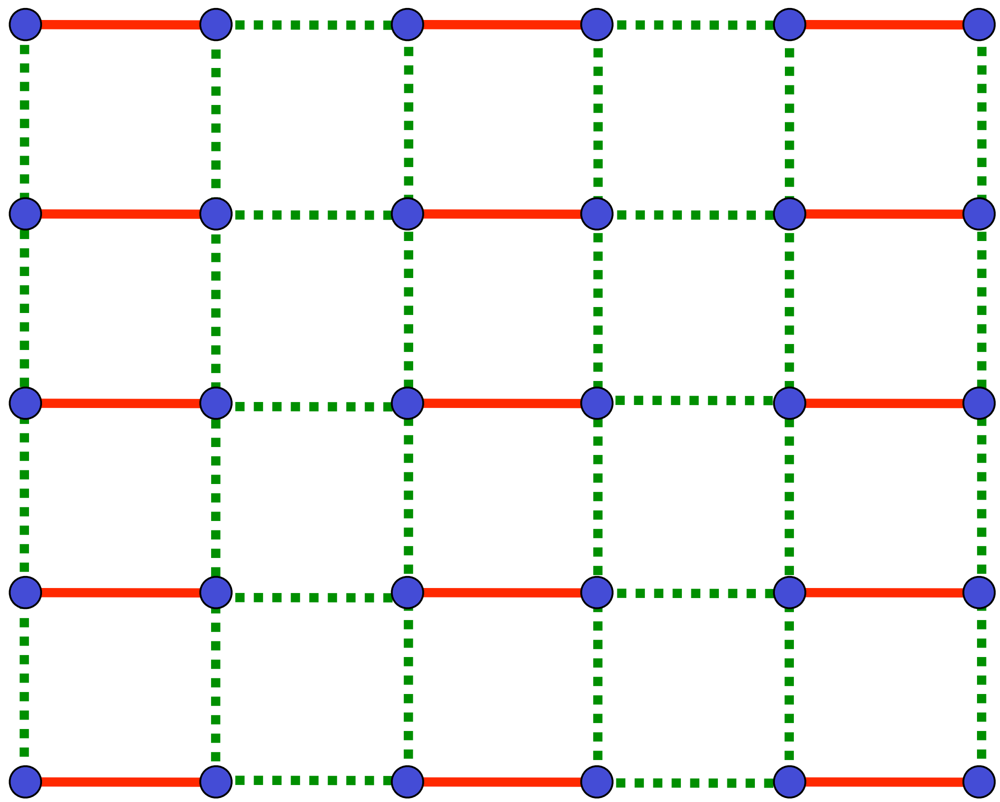


FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

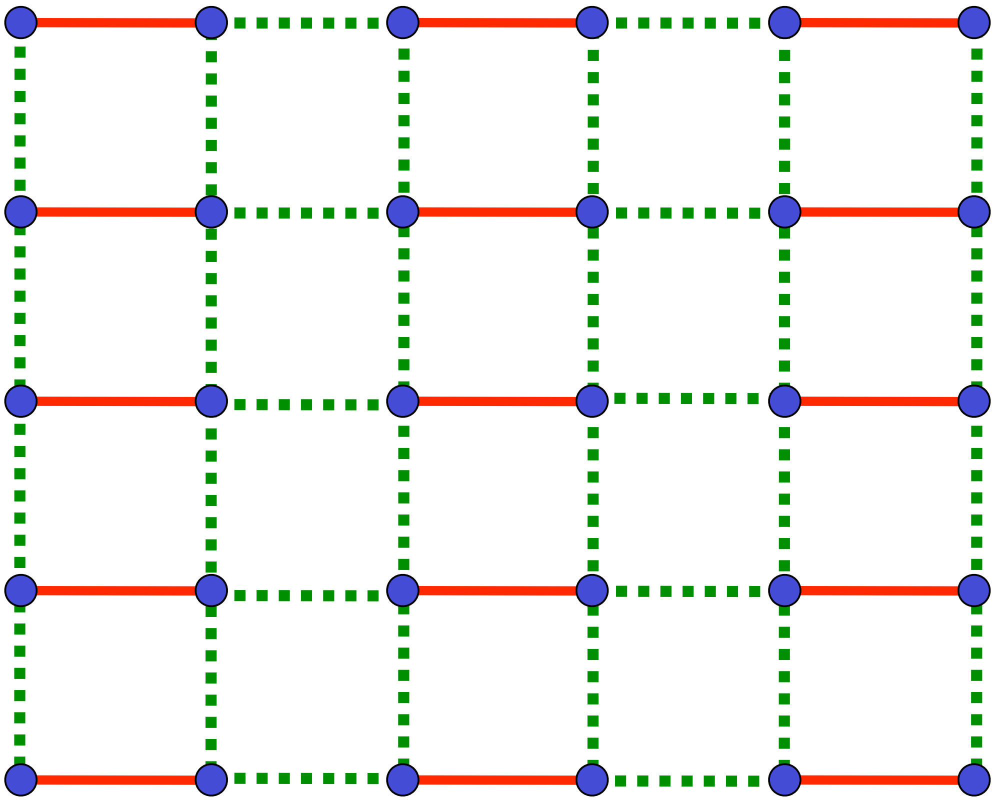
S=1 quasi-particle

Coupled Dimer Antiferromagnet



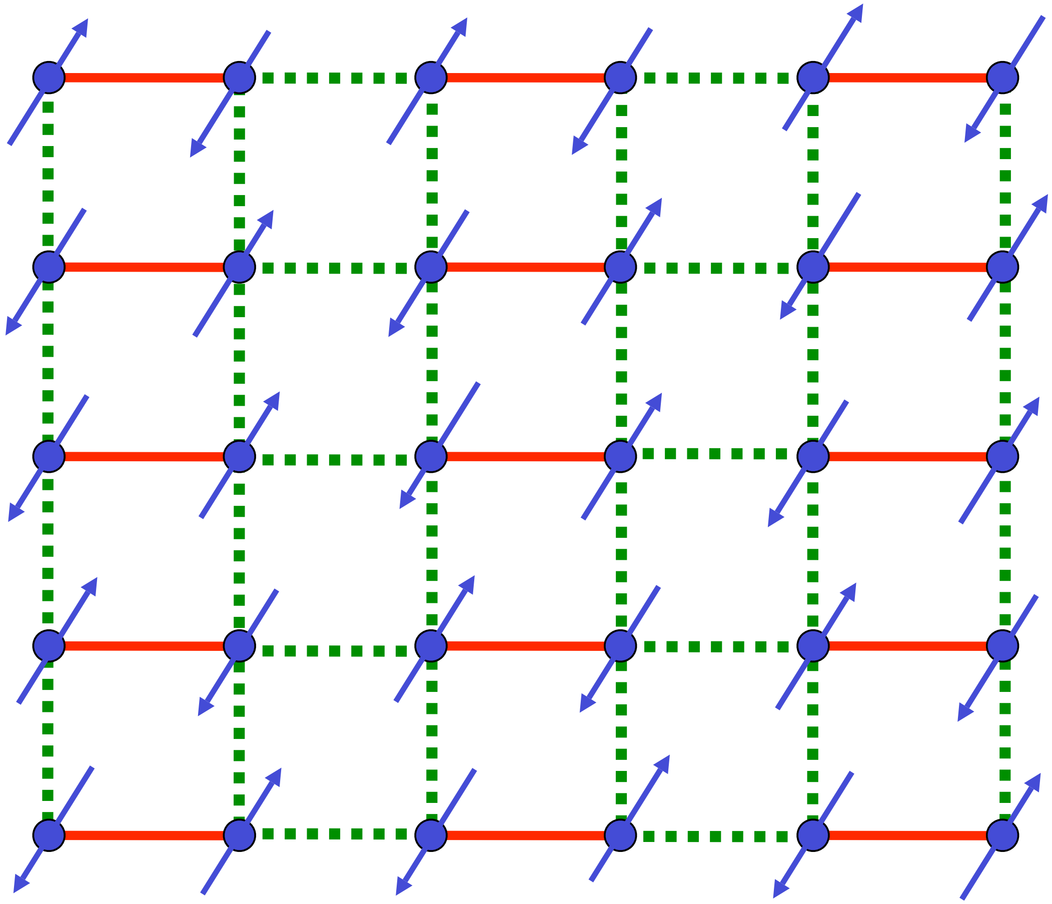
λ close to 1

Weakly dimerized square lattice



λ close to 1

Weakly dimerized square lattice



Excitations:
2 spin waves (***magnons***)

$$\varepsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$$

Ground state has long-range spin density wave (Néel) order at wavevector ***K***= (π,π)

$$\langle \vec{\varphi} \rangle \neq 0$$

spin density wave order parameter: $\vec{\varphi} = \eta_i \dfrac{\vec{S}_i}{S}$; $\eta_i = \pm 1$ on two sublattices

$TlCuCl_3$

Neutron Diffraction Study of the Pressure-Induced Magnetic Ordering in the Spin Gap System $TlCuCl_3$

Akira OOSAWA*, Masashi FUJISAWA[1], Toyotaka OSAKABE, Kazuhisa KAKURAI and Hidekazu TANAKA[2]

Advanced Science Research Center, Japan Atomic Energy Research Institute, Tokai, Ibaraki 319-1195
[1]*Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*
[2]*Research Center for Low Temperature Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*

(Received February 3, 2003)

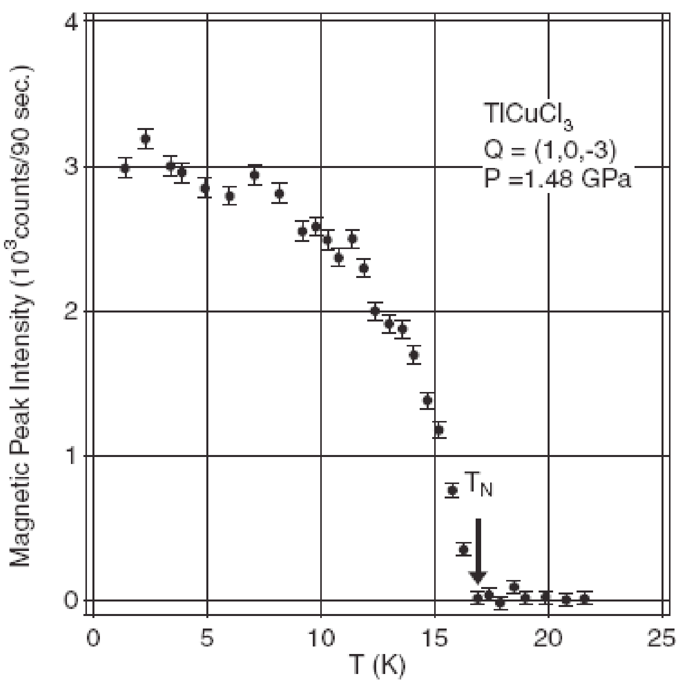


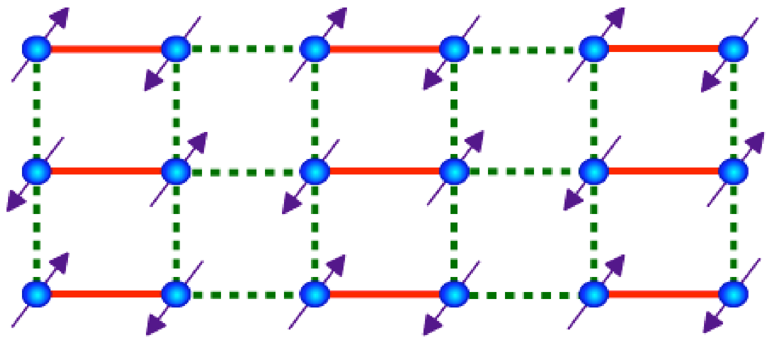
Fig. 3. Temperature dependence of the magnetic Bragg peak intensity for $\boldsymbol{Q} = (1, 0, -3)$ reflection measured at $P = 1.48$ GPa in $TlCuCl_3$.

J. Phys. Soc. Jpn **72**, 1026 (2003)

T=0

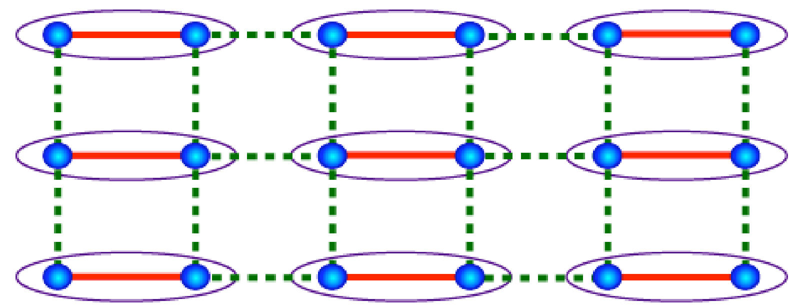
$\lambda_c = 0.52337(3)$

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002)



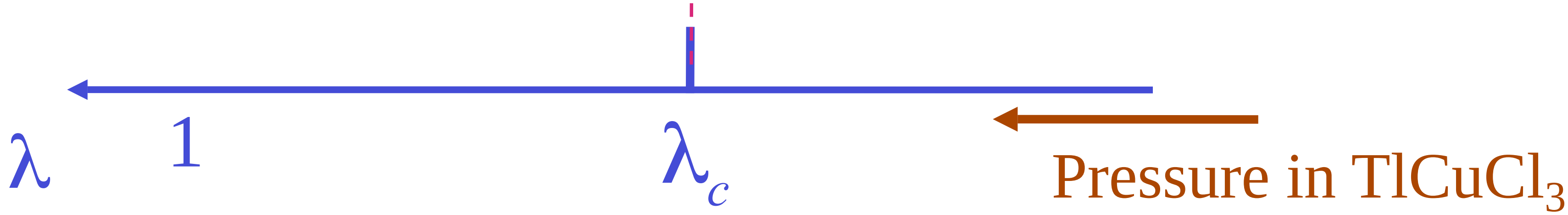
Néel state

$\langle \vec{\varphi} \rangle \neq 0$



Quantum paramagnet

$\langle \vec{\varphi} \rangle = 0$



LGW theory for quantum criticality

Landau-Ginzburg-Wilson theory: write down an effective action for the antiferromagnetic order parameter $\vec{\varphi}$ by expanding in powers of $\vec{\varphi}$ and its spatial and temporal derivatives, while preserving all symmetries of the microscopic Hamiltonian

$$\mathcal{S}_\varphi = \int d^2x d\tau \left[\frac{1}{2} \left(\left(\nabla_x \vec{\varphi} \right)^2 + c^2 \left(\partial_\tau \vec{\varphi} \right)^2 + \left(\lambda_c - \lambda \right) \vec{\varphi}^2 \right) + u \left(\vec{\varphi}^2 \right)^2 \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989)

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S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989)

For $\lambda < \lambda_c$, oscillations of $\vec{\varphi}$ about $\vec{\varphi} = 0$ constitute the *triplon* excitation

A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)

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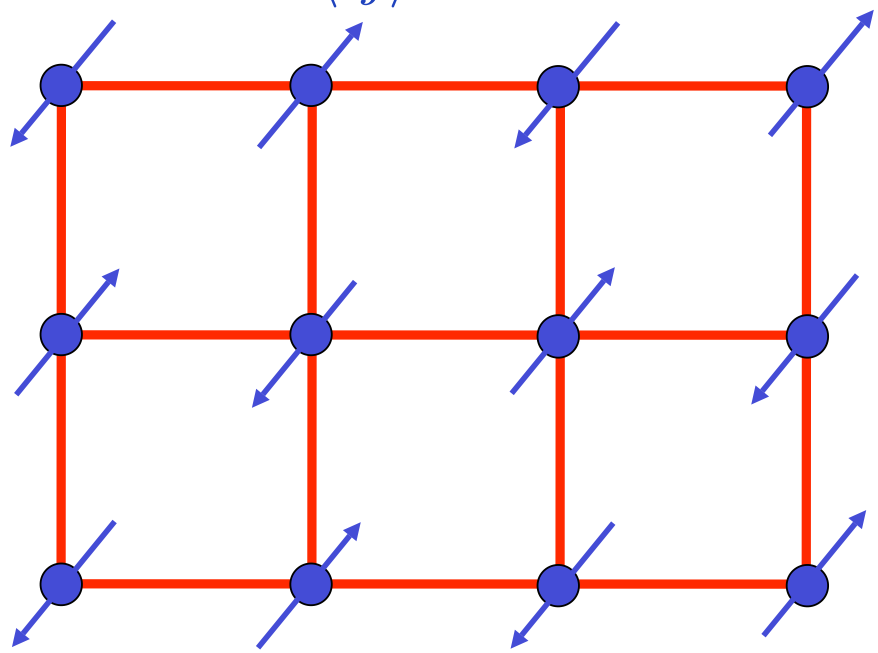
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

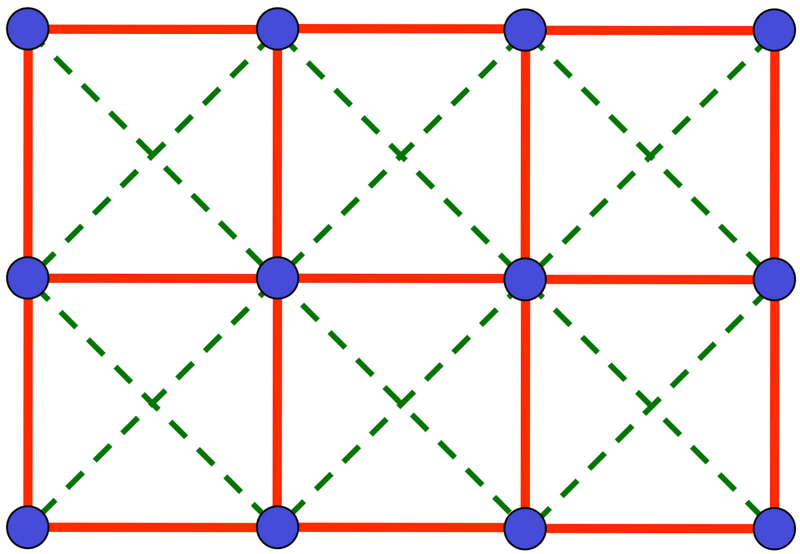
Order parameter $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$$\langle \vec{\varphi} \rangle \neq 0$$

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

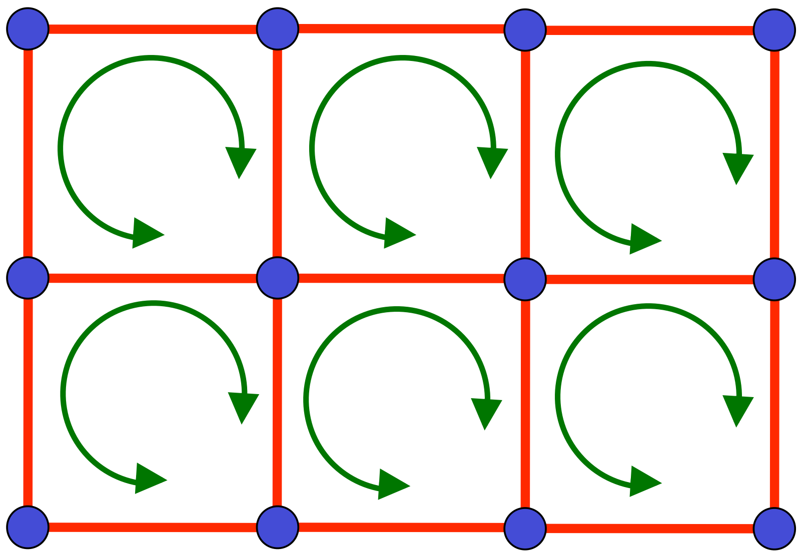


Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

What is the state with $\langle \vec{\varphi} \rangle = 0$?

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

What is the state with $\langle \vec{\varphi} \rangle = 0$?

Quantum theory for destruction of Neel order

Partition function on cubic lattice in spacetime

$$\mathcal{Z} = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g}\sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu}\right)$$

LGW theory: weights in partition function are those of a classical ferromagnet at a "temperature" g

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Quantum theory for destruction of Neel order

Coherent state path integral on cubic lattice in spacetime

$$\mathcal{Z} = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g}\sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i\mathcal{S}_{\text{Berry}}\right)$$

Modulus of weights in partition function: those of a classical ferromagnet at a "temperature" g

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Berry phases lead to large cancellations between different time histories

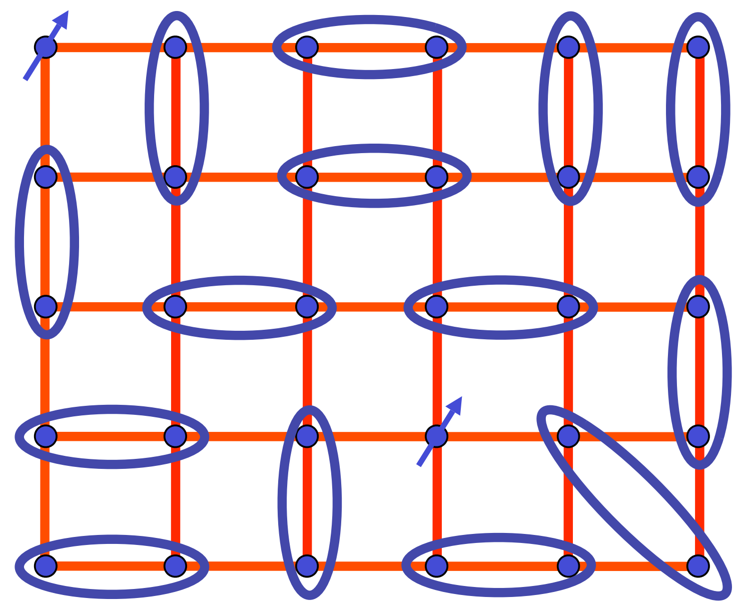
Quantum theory for destruction of Neel order

Partition function on cubic lattice

$$\mathcal{Z} = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g}\sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i\mathcal{S}_{\text{Berry}}\right)$$

Rewrite partition function in terms of spinors $z_{a\alpha}$, with $\alpha = \uparrow, \downarrow$ and

$$\vec{\varphi}_a = z_{a\alpha}^* \vec{\sigma}_{\alpha\beta} z_{a\beta}$$



S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Quantum theory for destruction of Neel order

Partition function on cubic lattice

$$\mathcal{Z} = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g}\sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i\mathcal{S}_{\text{Berry}}\right)$$

Partition function expressed as a gauge theory of spinor degrees of freedom

$$\mathcal{Z} = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(\sum_\alpha |z_{a\alpha}|^2 - 1\right)$$

$$\times \exp\left(\frac{1}{g}\sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + i\sum_a (2S)\eta_a A_{a\tau}\right)$$

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Large g effective action for the $A_{a\mu}$ after integrating $z_{\alpha\mu}$

$$\mathcal{Z} = \prod_{a,\mu} \int dA_{a\mu} \exp\left(\frac{1}{2e^2} \sum_{\square} \cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}\right) + i \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

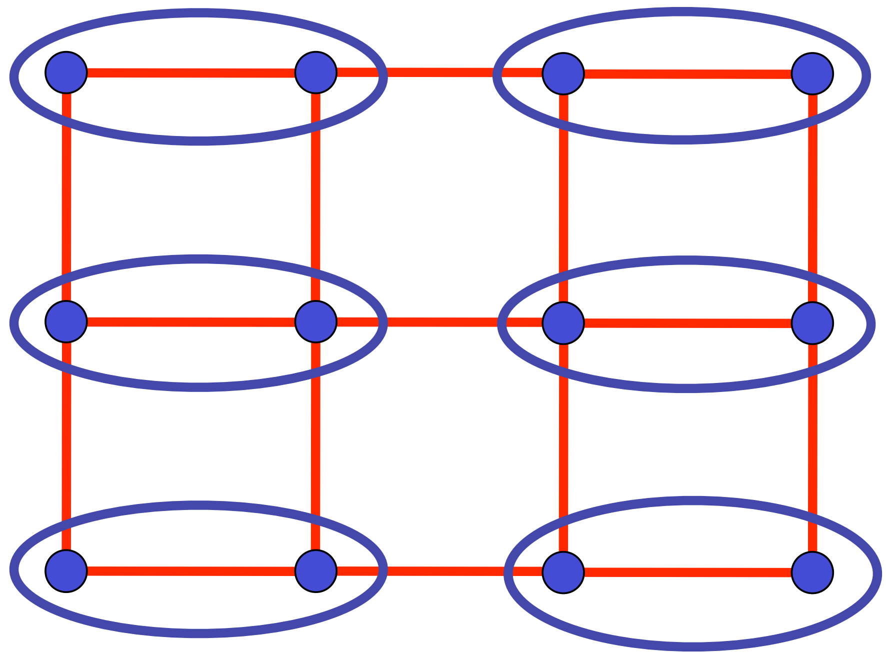
This is compact QED in 3 spacetime dimensions with static charges ± 1 on two sublattices.

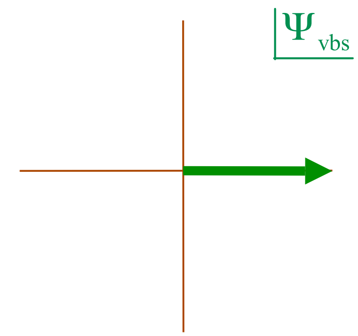
This theory can be reliably analyzed by a duality mapping.

The gauge theory is in a ***confining*** phase, and there is VBS order in the ground state. (Proliferation of monopoles in the presence of Berry phases).

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990).
K. Park and S. Sachdev, *Phys. Rev.* B **65**, 220405 (2002).

Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$

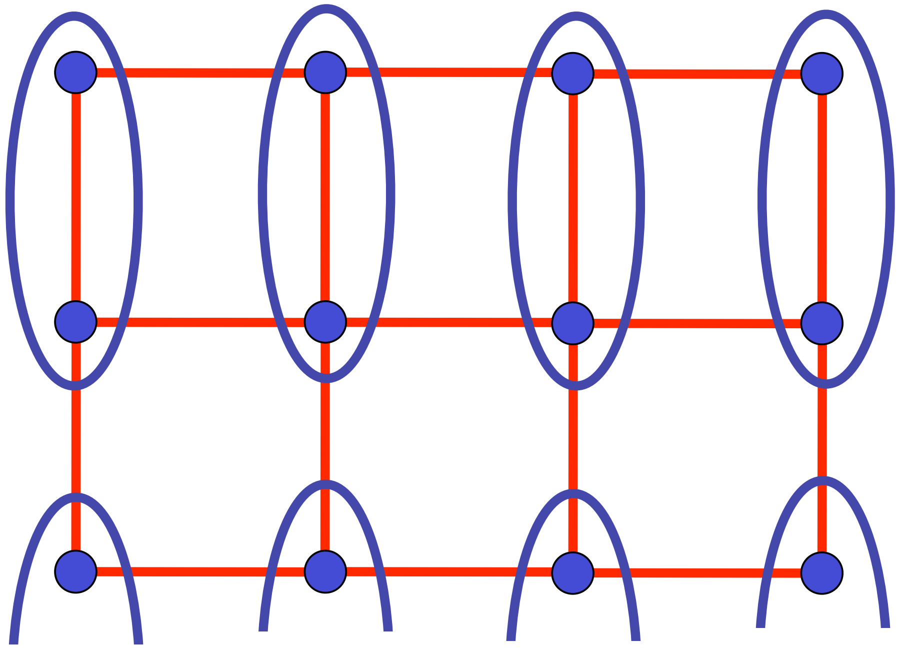


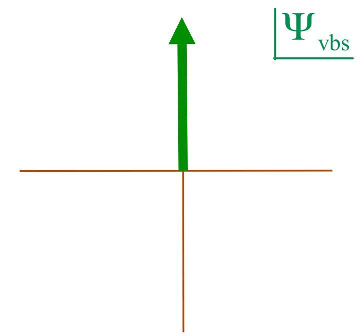


Such a state breaks the symmetry of rotations by $n\pi / 2$ about lattice sites, and has $\langle \Psi_{\text{vbs}} \rangle \neq 0$, where Ψ_{vbs} is the ***VBS order parameter***

$$\Psi_{\text{vbs}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i \arctan(\mathbf{r}_j - \mathbf{r}_i)}$$

Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$





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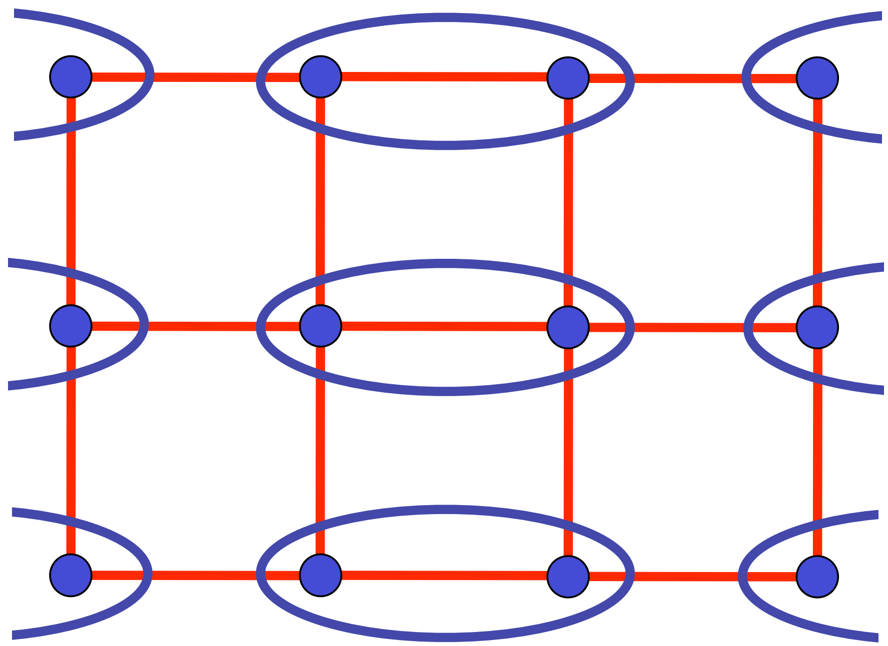
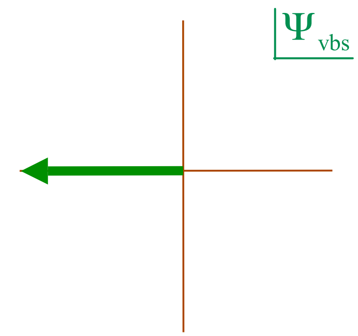
Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$





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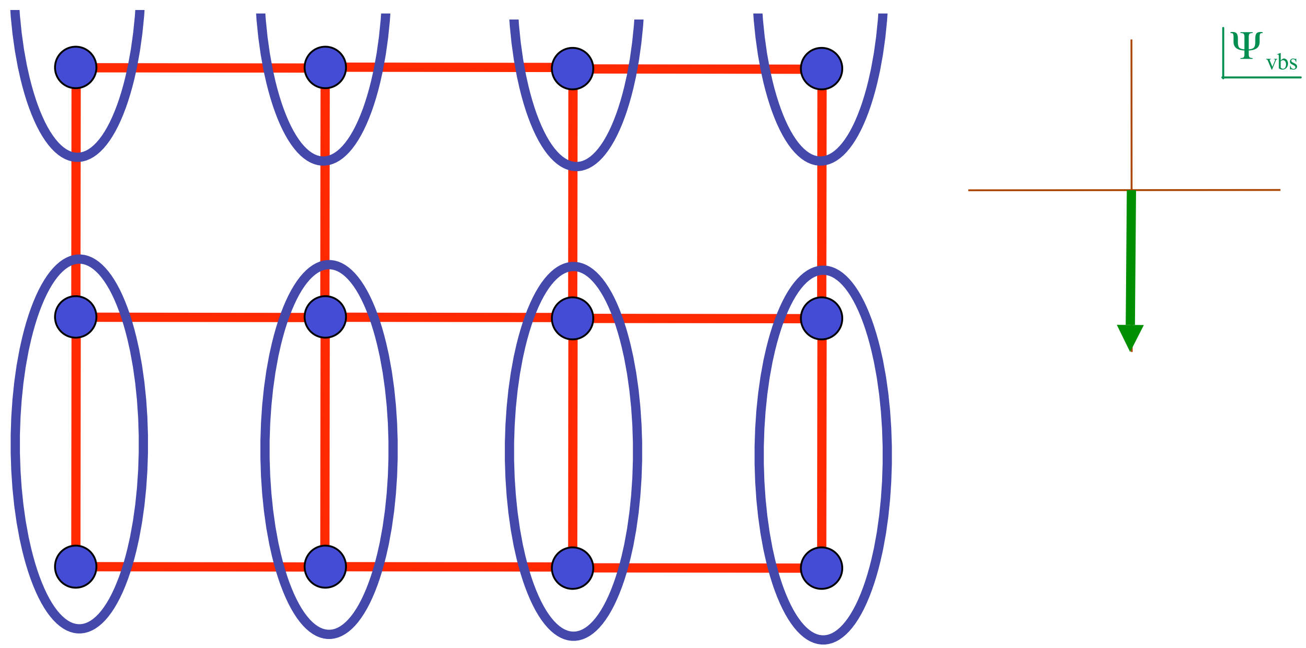
Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$



Such a state breaks the symmetry of rotations by $n\pi/2$ about lattice sites, and has $\langle \Psi_{vbs} \rangle \neq 0$, where Ψ_{vbs} is the ***VBS order parameter***

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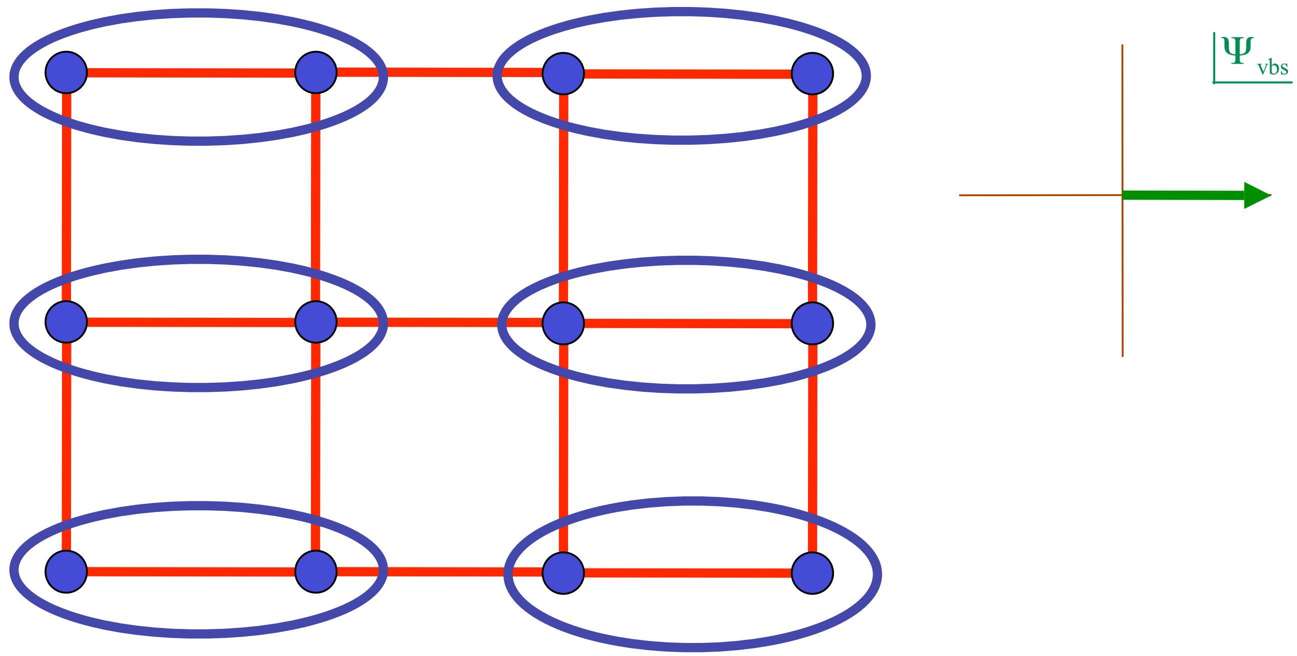

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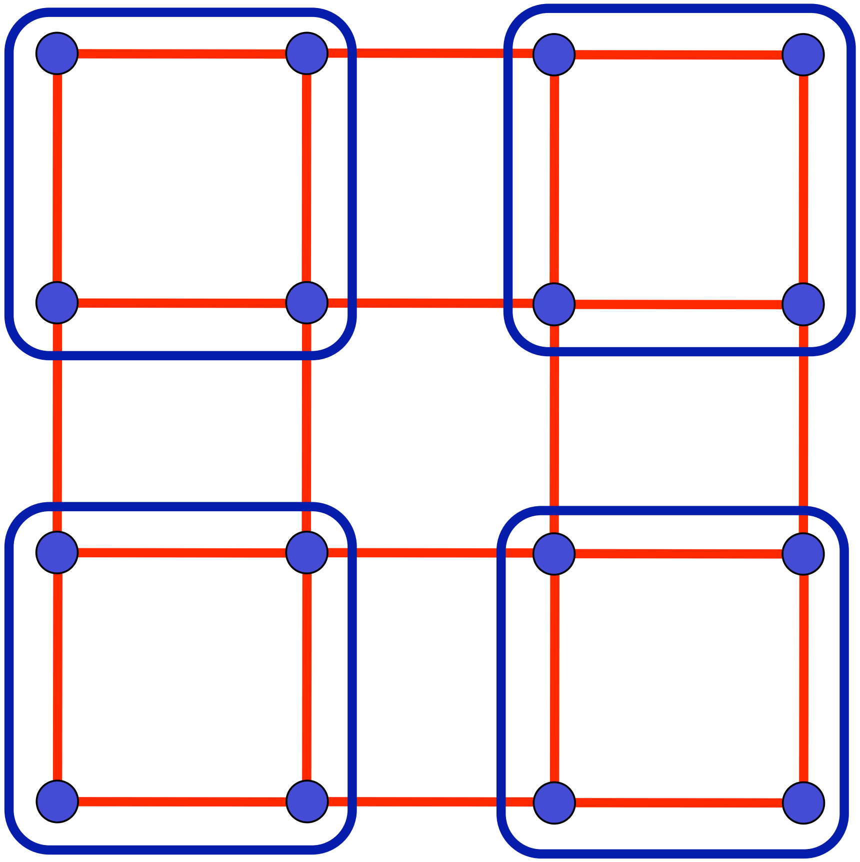
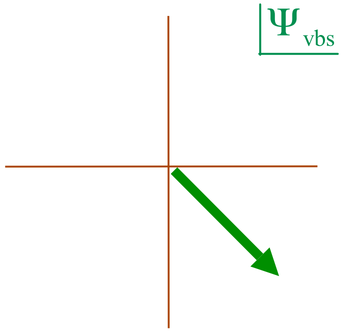
$$\Psi_{\mathrm{vbs}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i \arctan(\mathbf{r}_j - \mathbf{r}_i)}$$

Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$





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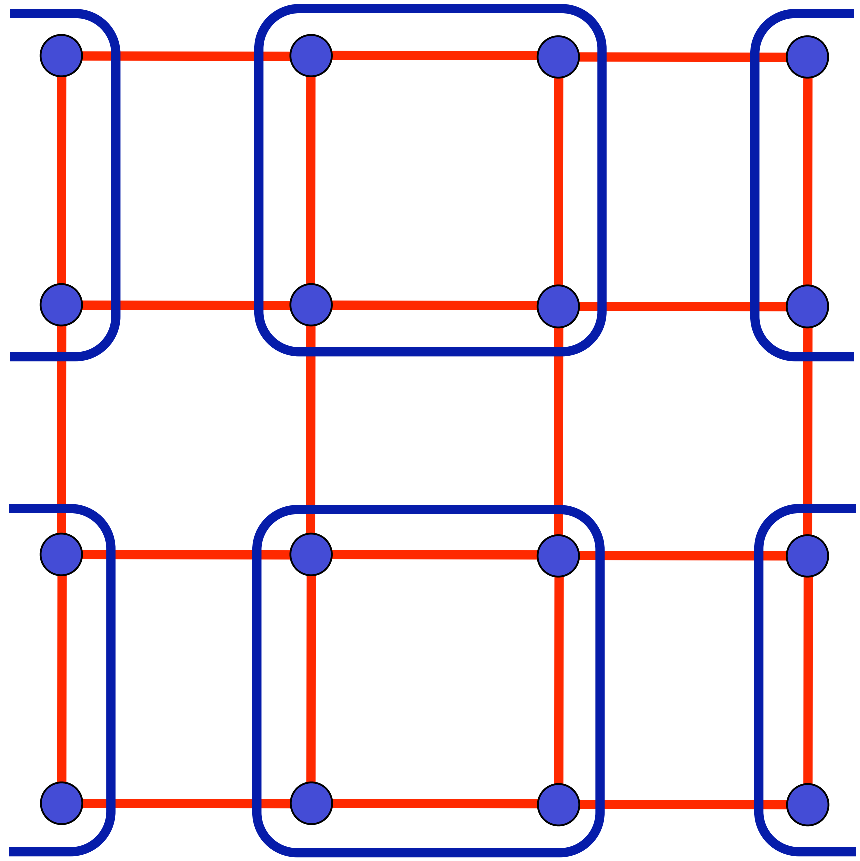

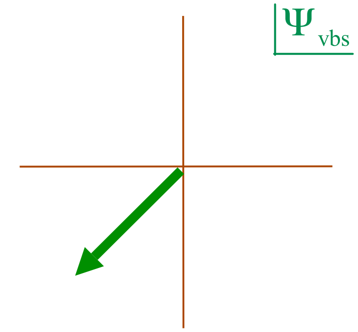

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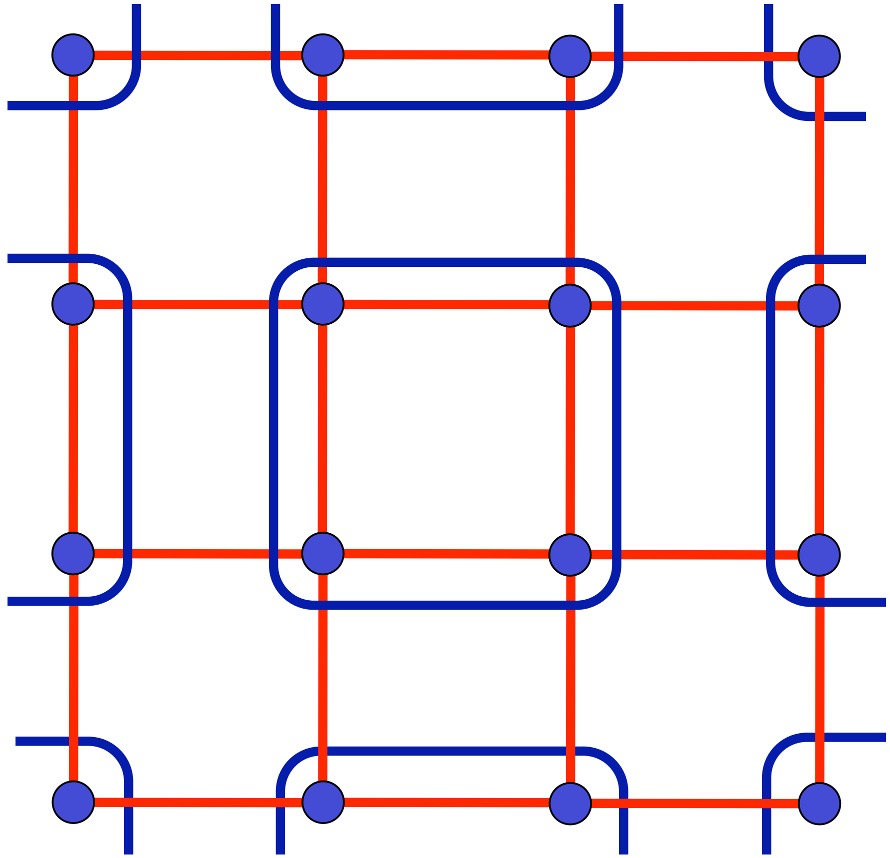

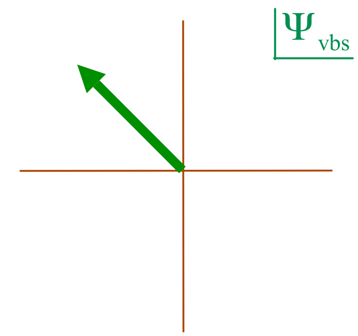

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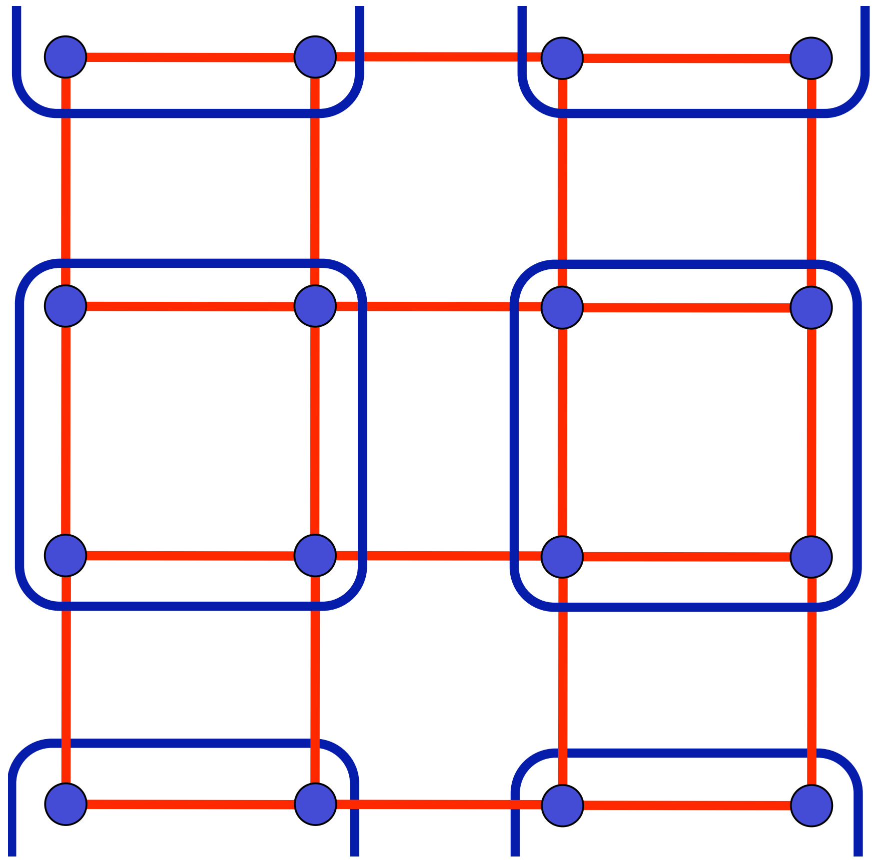

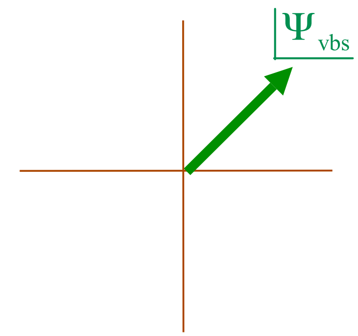

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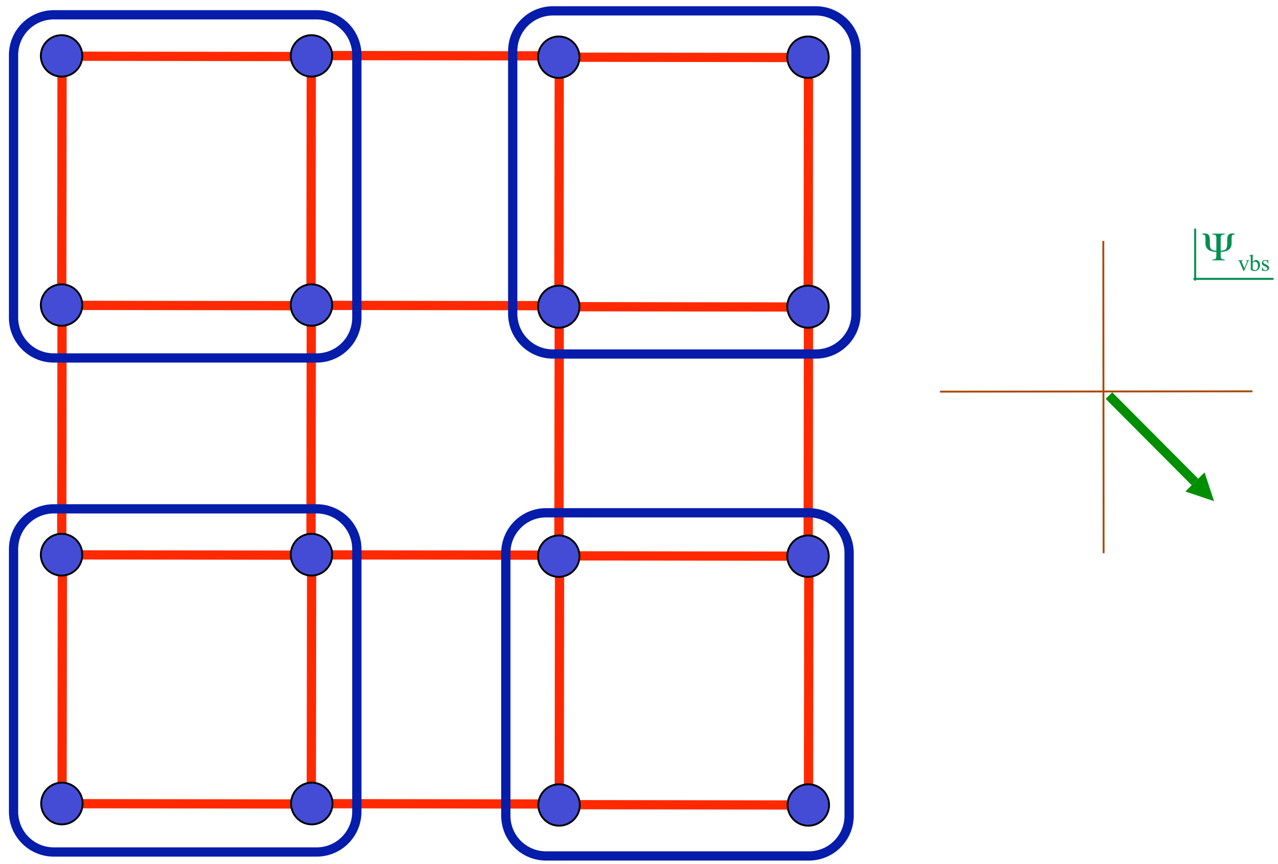

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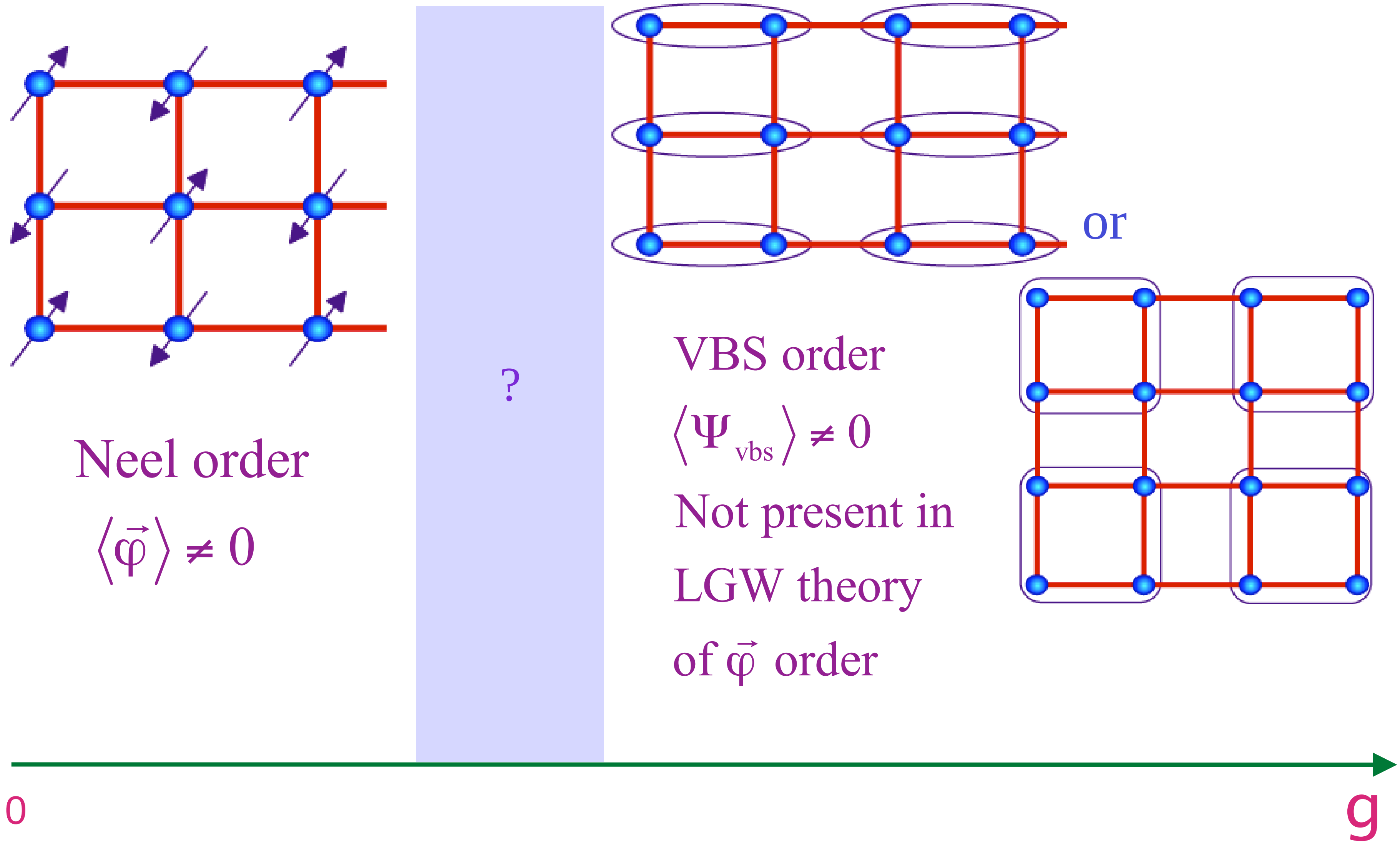
$$\mathcal{Z} = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(\sum_\alpha |z_{a\alpha}|^2 - 1\right) \exp\left(\frac{1}{g}\sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + i \sum_a \eta_a A_{a\tau}\right)$$



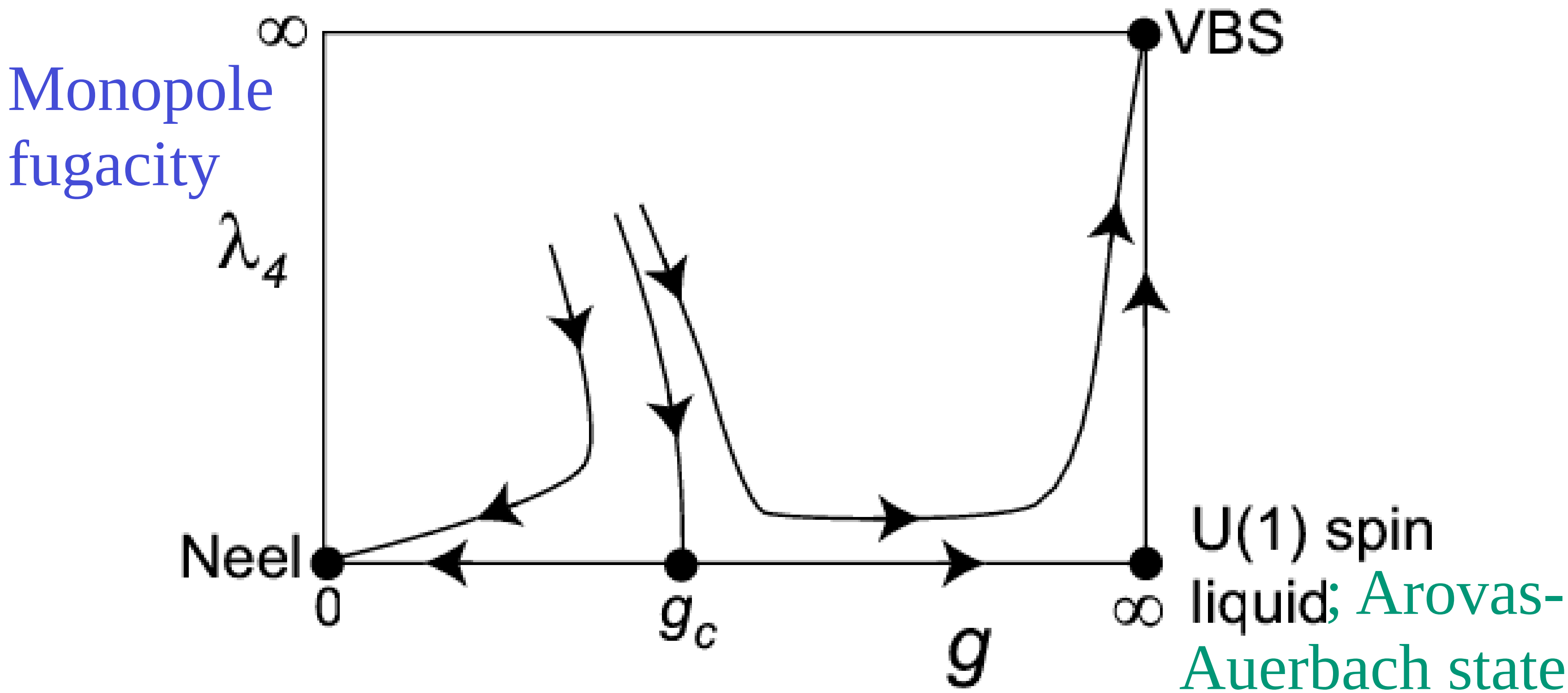
Theory of a second-order quantum phase transition between Neel and VBS phases

At the quantum critical point:

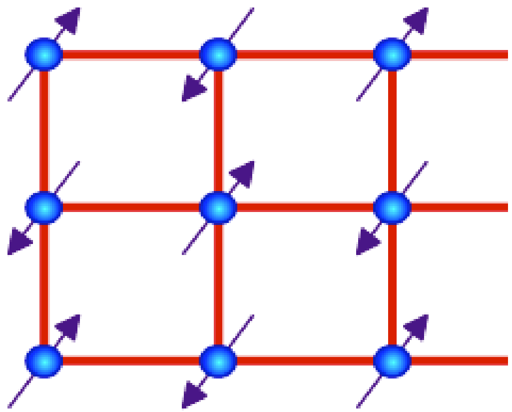
- $A_\mu \to A_\mu + 2\pi$ periodicity can be ignored

(Monopoles interfere destructively and are dangerously irrelevant).

- S=1/2 *spinons* z_α, with $\vec{\varphi} \sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$, are globally propagating degrees of freedom.

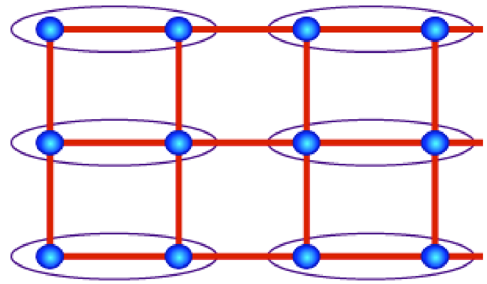

Monopole fugacity
λ_4
∞
VBS
Neel
0
g_c
g
∞
U(1) spin liquid; Arovas-Auerbach state

Phase diagram of S=1/2 square lattice antiferromagnet

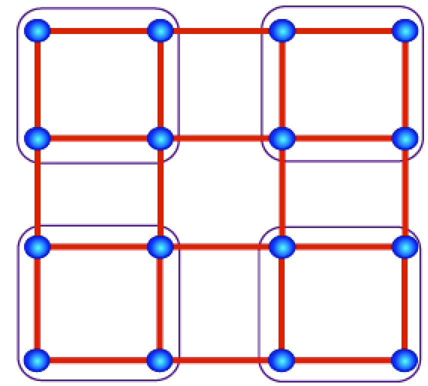


Neel order

$\langle \vec{\varphi} \rangle \sim \langle z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta \rangle \neq 0$



or



VBS order $\langle \Psi_{\text{vbs}} \rangle \neq 0$

(associated with condensation of monopoles in A_μ),

$S = 1/2$ spinons z_α confined,

$S = 1$ triplon excitations

Second-order critical point described by

$$\mathcal{S}_z = \int d^2 x d\tau \left[|(\partial_\mu - i A_\mu) z_\alpha|^2 + r |z_\alpha|^2 + \frac{u}{2} \left(|z_\alpha|^2 \right)^2 + \frac{1}{4e^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right]$$

at its critical point $r = r_c$ where A_μ is *non-compact*.

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Large scale Quantum Monte Carlo studies

Easy-plane model:

$$\mathcal{H}_{\mathrm{XY}} = 2J \sum_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y \right) - K \sum_{\langle ijkl \rangle} \left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+ \right)$$

A.W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett.* **89**, 247201 (2002); A.W. Sandvik and R.G. Melko, cond-mat/0604451.

SU(2)-invariant model:

$$\mathcal{H}_{\mathrm{SU(2)}} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4} \right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4} \right)$$

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).

Easy-plane model

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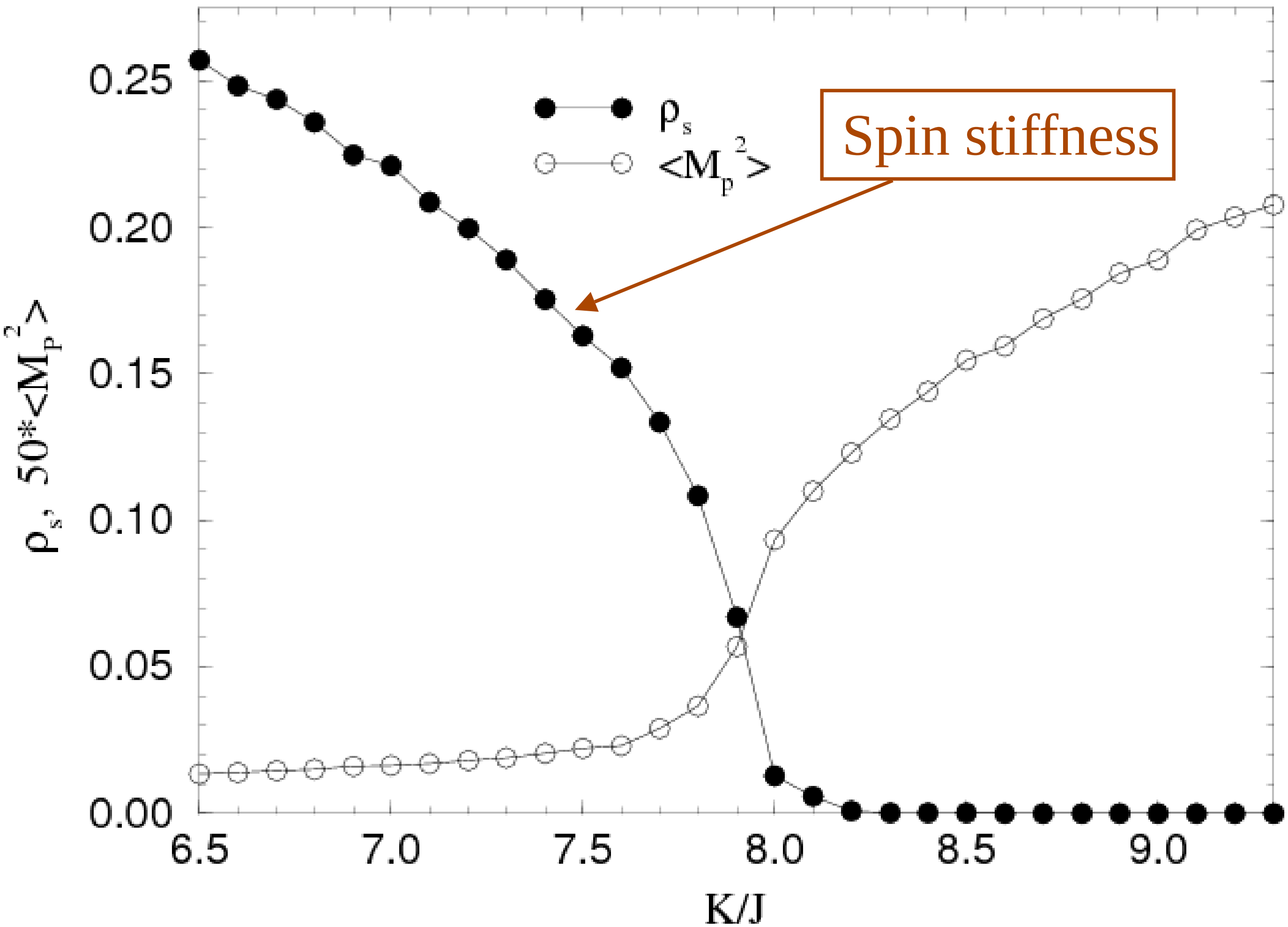

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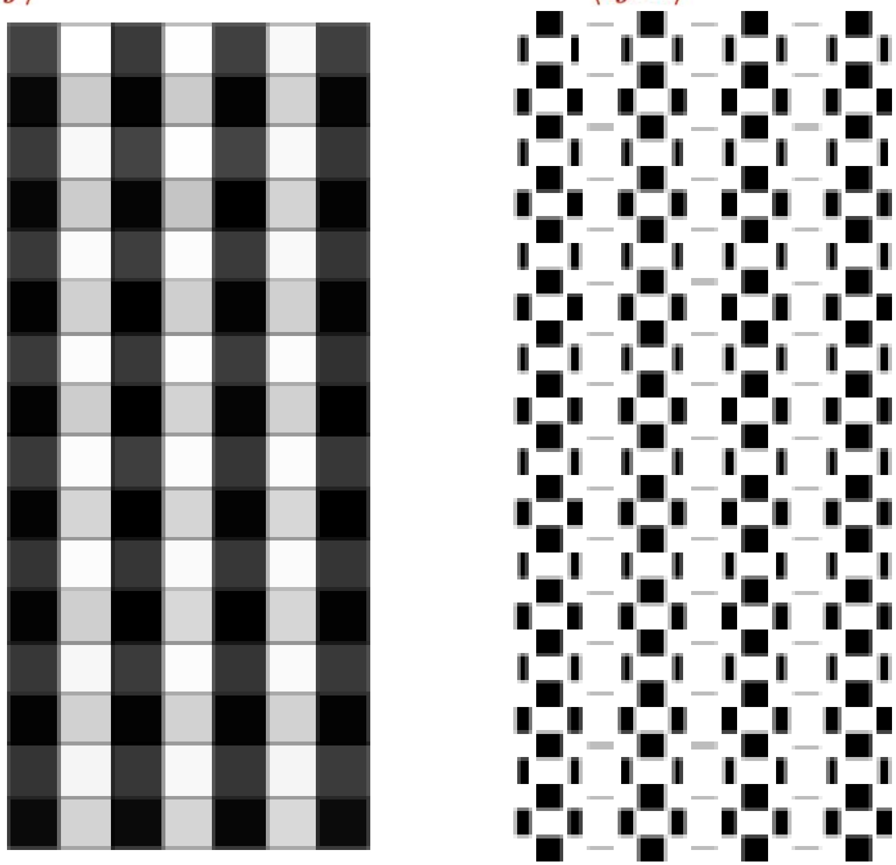
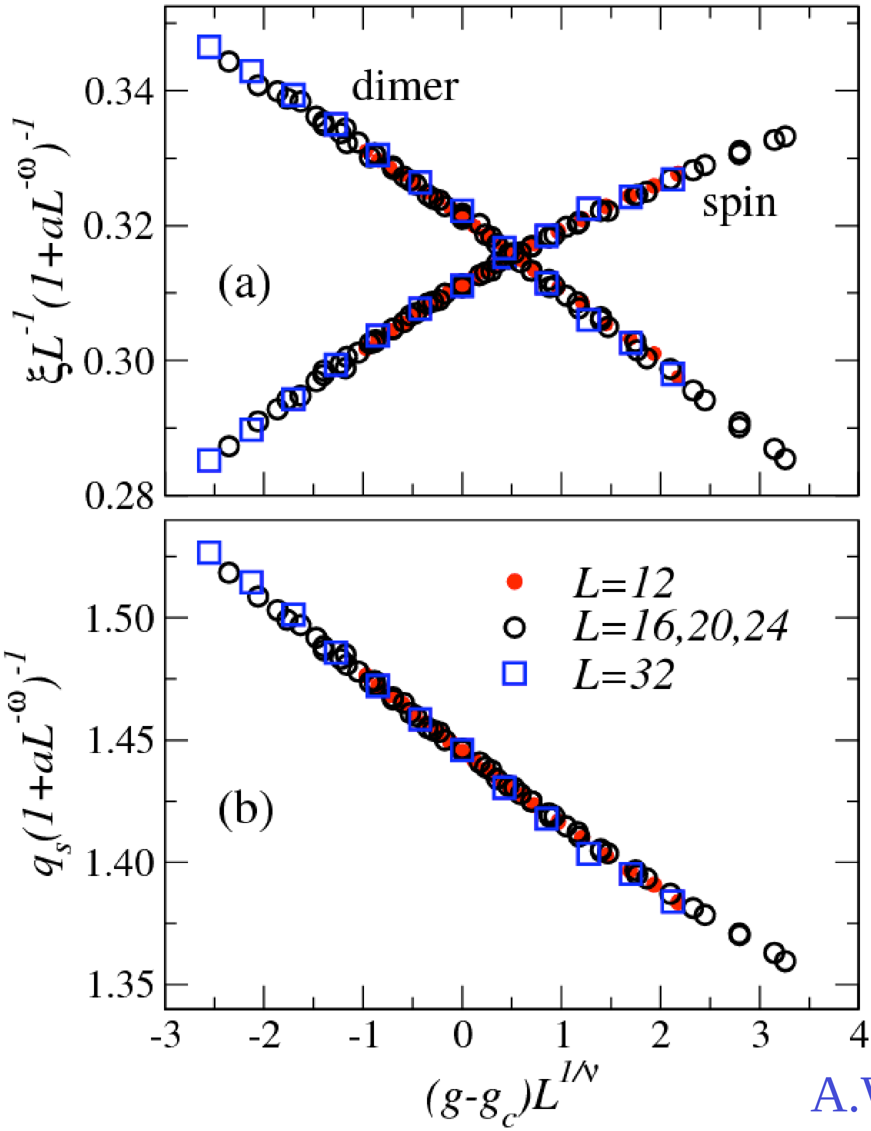


Valence bond solid (VBS) order in expectation values of plaquette and exchange terms

SU(2) invariant model

$$\mathcal{H}_{\mathrm{SU(2)}} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4}\right)\left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4}\right)$$



Strong evidence for a continuous "deconfined" quantum critical point

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).

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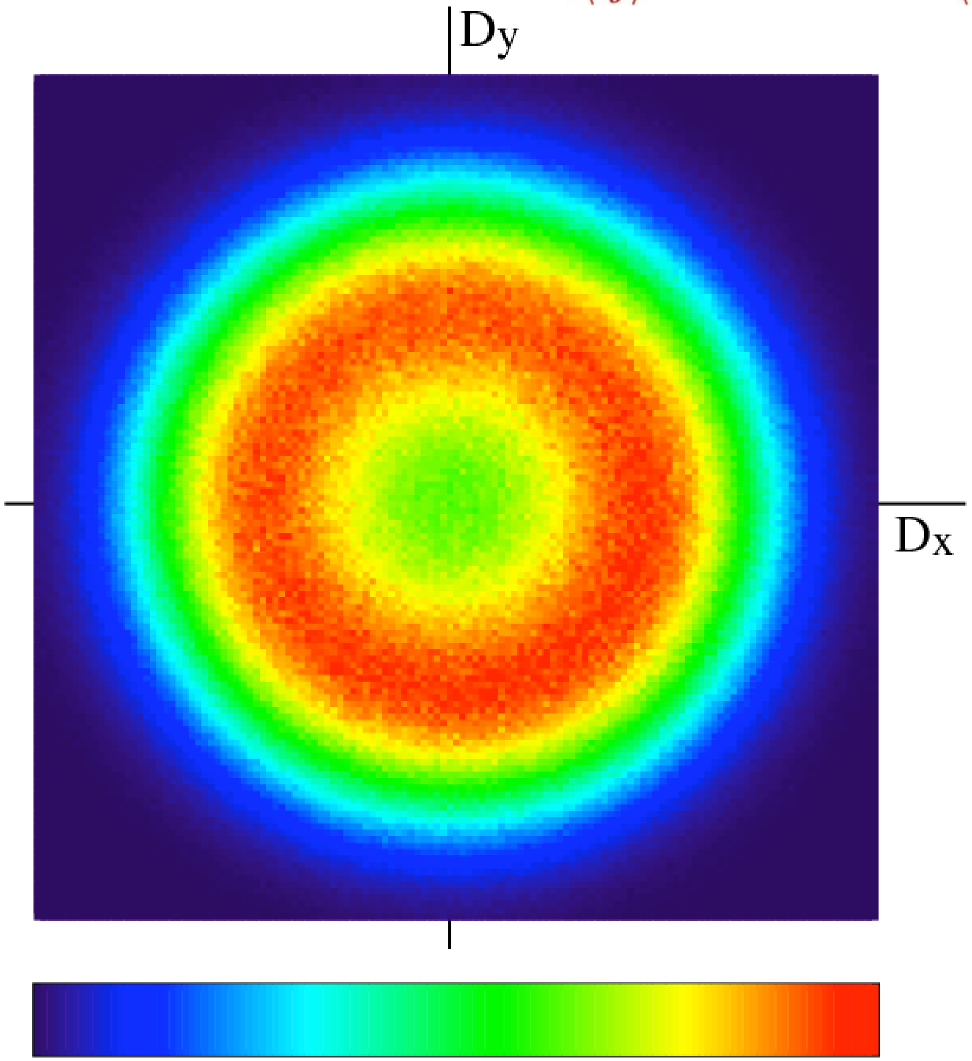

Probability distribution of VBS order Ψ at quantum critical point

Emergent circular symmetry is a consequence of a gapless photon excition

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).

Outline

1. Quantum transitions in dimerized antiferromagnets
   Landau-Ginzburg-Wilson (LGW) theory

2. Square lattice antiferromagnets
   VBS order and deconfined criticality

3. Doped square lattice antiferromagnets
   Fermi liquid states

4. Doped square lattice antiferromagnets
   Non-Fermi liquid holon metal

5. Instabilities of the holon metal

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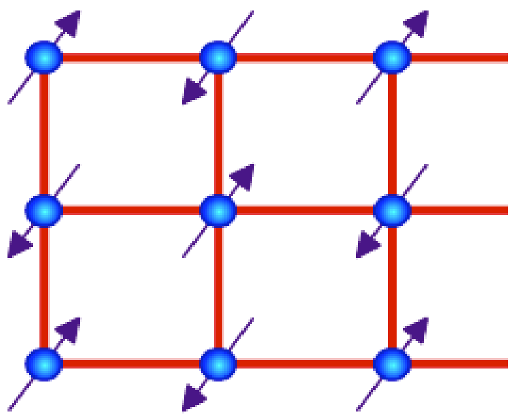
Hole dynamics in an antiferromagnet across a deconfined quantum critical point,
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil,
Physical Review B **75** , 235122 (2007)

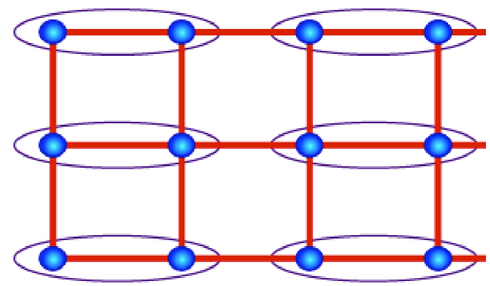
Algebraic charge liquids and the underdoped cuprates,
R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil,
arXiv:0706.2187

Phase diagram of S=1/2 square lattice antiferromagnet

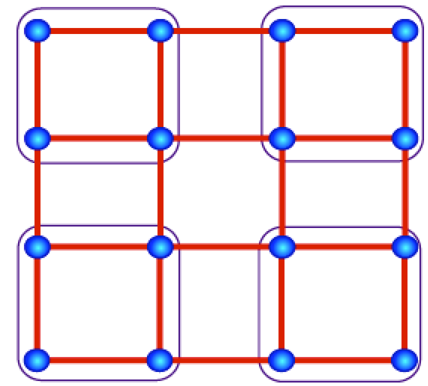


Neel order

$\langle \vec{\varphi} \rangle \sim \langle z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta \rangle \neq 0$



or



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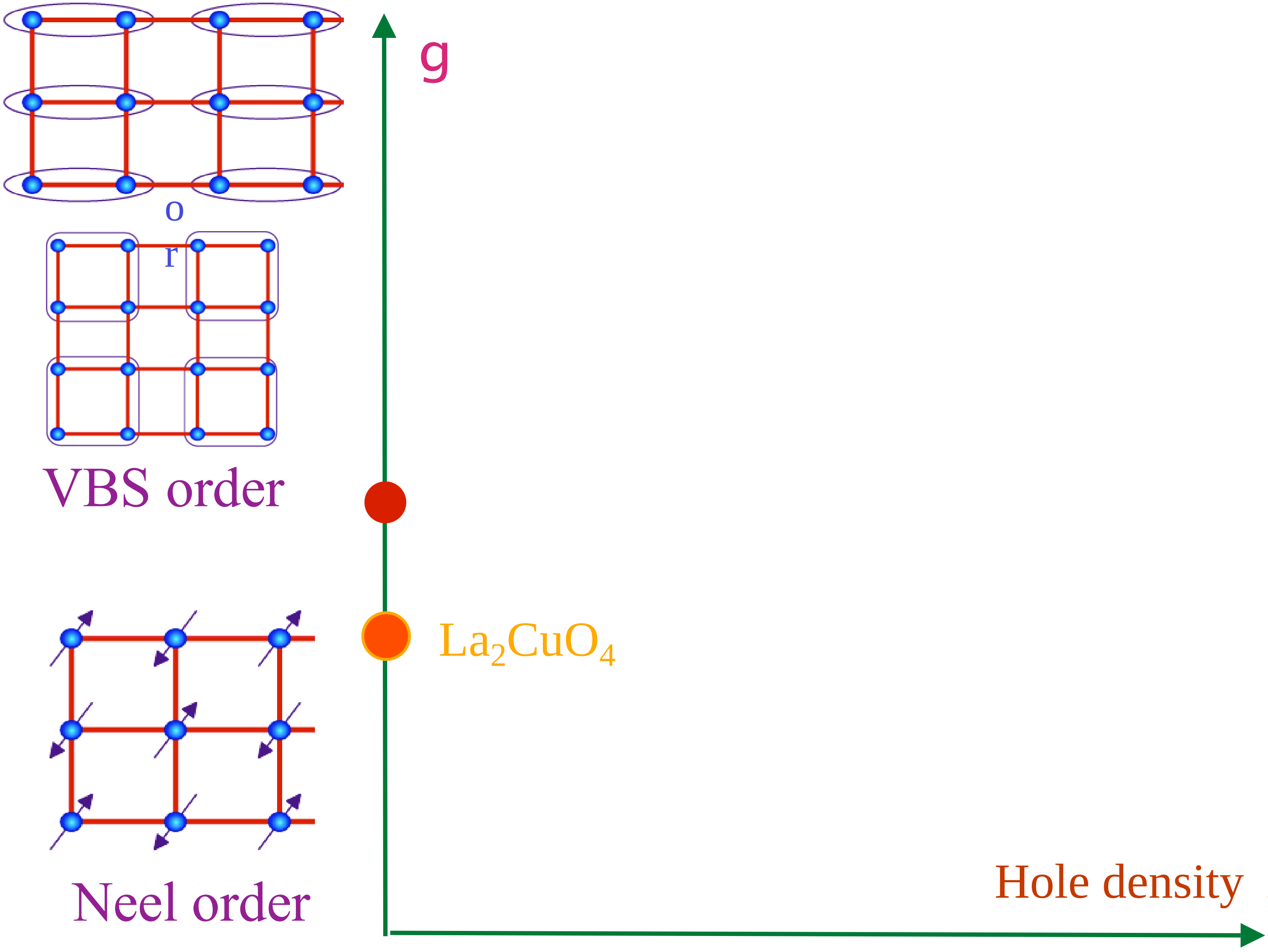
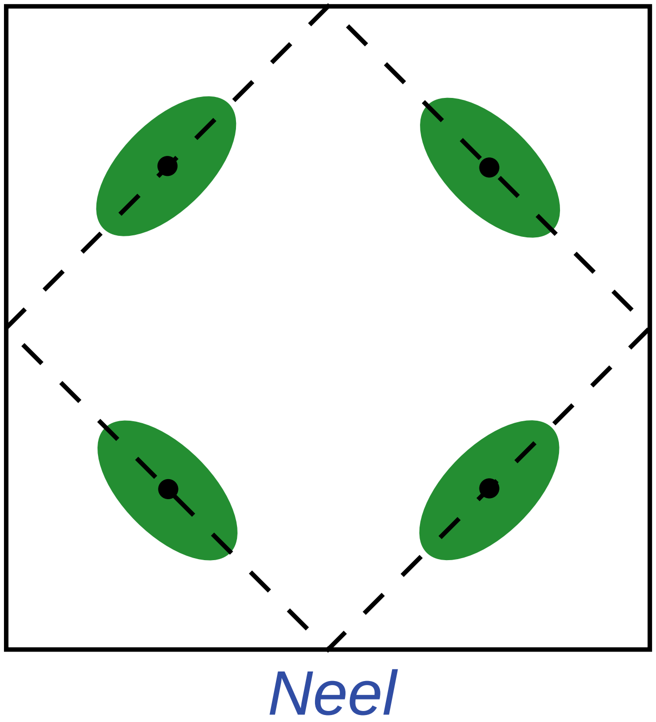
Phase diagram of doped antiferromagnets



Holes in the Neel state

Self-consistent spin-wave theory of a single hole dispersion in the Néel state shows that the energy minima are at $(\pm\pi/2, \pm\pi/2)$.



S. A. Trugman, Phys. Rev. B **37**, 1597 (1988).
C. L. Kane, P. A. Lee, and N. Read, Phys. Rev. B **39**, 6880 (1989).
S. Sachdev, Phys. Rev. B **39**, 12232 (1989).
V. Elser, D. A. Huse, B. I. Shraiman, and E. D. Siggia, Phys. Rev. B **41**, 6715 (1990).
Y. M. Li, D. N. Sheng, Z. B. Su, and L. Yu, Phys. Rev. B **45**, 5428 (1992).
Z. Y. Weng, V. N. Muthukumar, D. N. Sheng, and C. S. Ting, Phys. Rev. B **63**, 075102 (2001).

Holes in two valleys at $(\pi/2, \pm\pi/2)$. Each hole also has a sublattice index. So area of each pocket at hole density x is

$$\mathcal{A} = (2\pi)^2 x/4$$

In the Néel state, the sublattice index is the same as the spin index

• Begin with the representation of the quantum antiferromagnet as the lattice CP^1 model:

$$\mathcal{S}_z = -\frac{1}{g}\sum_{a,\mu} z_{a\alpha}^* e^{iA_{a\mu}} z_{a+\mu,\alpha} + i\sum_a \eta_a A_{a\tau}$$

• Write the electron operator at site r, $c_\alpha(r)$ in terms of **fermionic holon** operators $f_\pm$

$$c_\alpha(r) = \begin{cases} f_+^\dagger(r) z_{r\alpha} & \text{for } r \text{ on sublattice A} \\ \varepsilon_{\alpha\beta} f_-^\dagger(r) z_{r\beta}^* & \text{for } r \text{ on sublattice B} \end{cases}$$

Note that the holons f_s have charge s under the U(1) gauge field A_μ.

• Choose the dispersion, $\epsilon(\vec{k})$ of the $f_\pm$ in momentum space so that its minima are at $(\pm\pi/2, \pm\pi/2)$. *To avoid double-counting, these dispersions must be restricted to be within the diamond Brillouin zone.*

$$\mathcal{S}_f = \int d\tau \sum_{s=\pm} \int_\diamond \frac{d^2k}{4\pi^2} f_s^\dagger(\vec{k}) \left(\partial_\tau - isA_\tau + \epsilon(\vec{k} - s\vec{A}) \right) f_s(\vec{k})$$

• Include the hopping between opposite sublattices (Shraiman-Siggia term):

$$\begin{aligned} \mathcal{S}_t &= -t \sum_{\langle rr' \rangle} c_\alpha^\dagger(r) c_\alpha(r') + \text{h.c.} \\ &= -t \sum_{\langle rr' \rangle} (f_+^\dagger(r) z_{r\alpha})^\dagger \varepsilon_{\alpha\beta} f_-^\dagger(r') z_{r\beta}^* \end{aligned}$$

• Complete theory for doped antiferromagnet:

$$\mathcal{S} = \mathcal{S}_z + \mathcal{S}_f + \mathcal{S}_t$$

Holes in the Neel state

In the Néel state, the holon gauge charge $s = \pm$ is *identical* to the S_z quantum number.

This can be seen as an example of the "Meissner effect". Apply a magnetic field H along the direction of the Néel state, and choose the "Higgs" condensate $z_\alpha \propto (1, 0)$. Then this condensate leads to a term in the action

$$\propto \left[iA_\mu - (H/2)\delta_{\mu\tau}\right]^2$$

So the A_μ flux is "expelled", but about a non-zero minimum with $iA_\tau = H/2$. Now evaluating the magnetization, M, by taking the derivative of the free energy with respect to H, the coupling of A_τ to the fermions leads to a contribution

$$M = \frac{1}{2} \sum_s \int_\diamondsuit \frac{d^2k}{4\pi^2} f_s^\dagger(k) f_s(k)$$

So the f_s holons carry spin $s/2$ in the Néel state.

Holes in the VBS state

In the VBS state, the holon gauge charge $s = \pm$ is *decoupled* from the S_z quantum number.

- If we initially ignore the sublattice mixing, the holons and spinons bind to form $S = 1/2$ *holes*. The f_+ holon with bind with a spinon to give 2 hole species, and similarly for the f_- holon, for a total of *four* hole species.

- Sublattice mixing term will couple the holes with the same spin to yield new mixed eigenstates. This mixing has the effect of moving the dispersion minimum of the *hole* away from $(\pi/2, \pm\pi/2)$.

Holes in the VBS state

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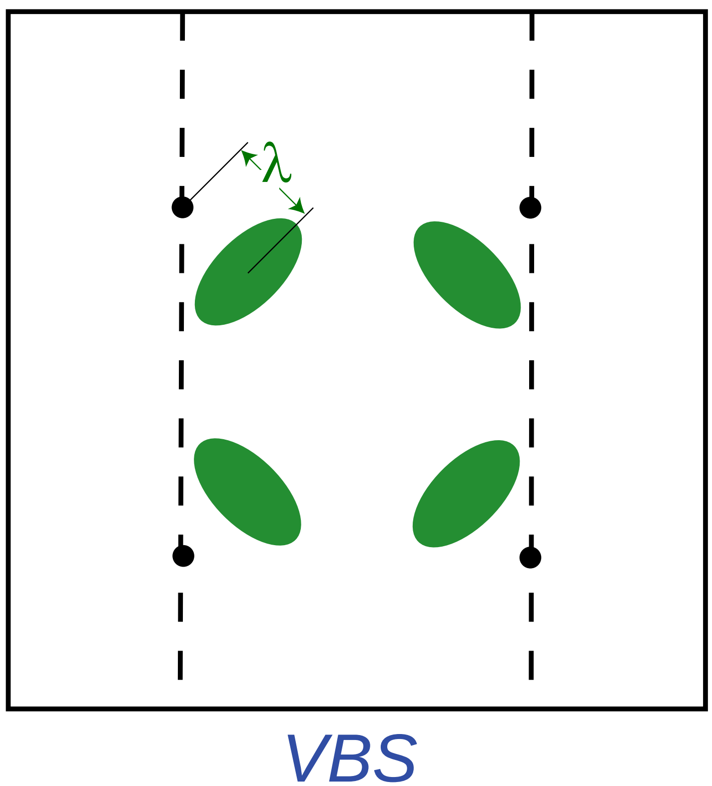
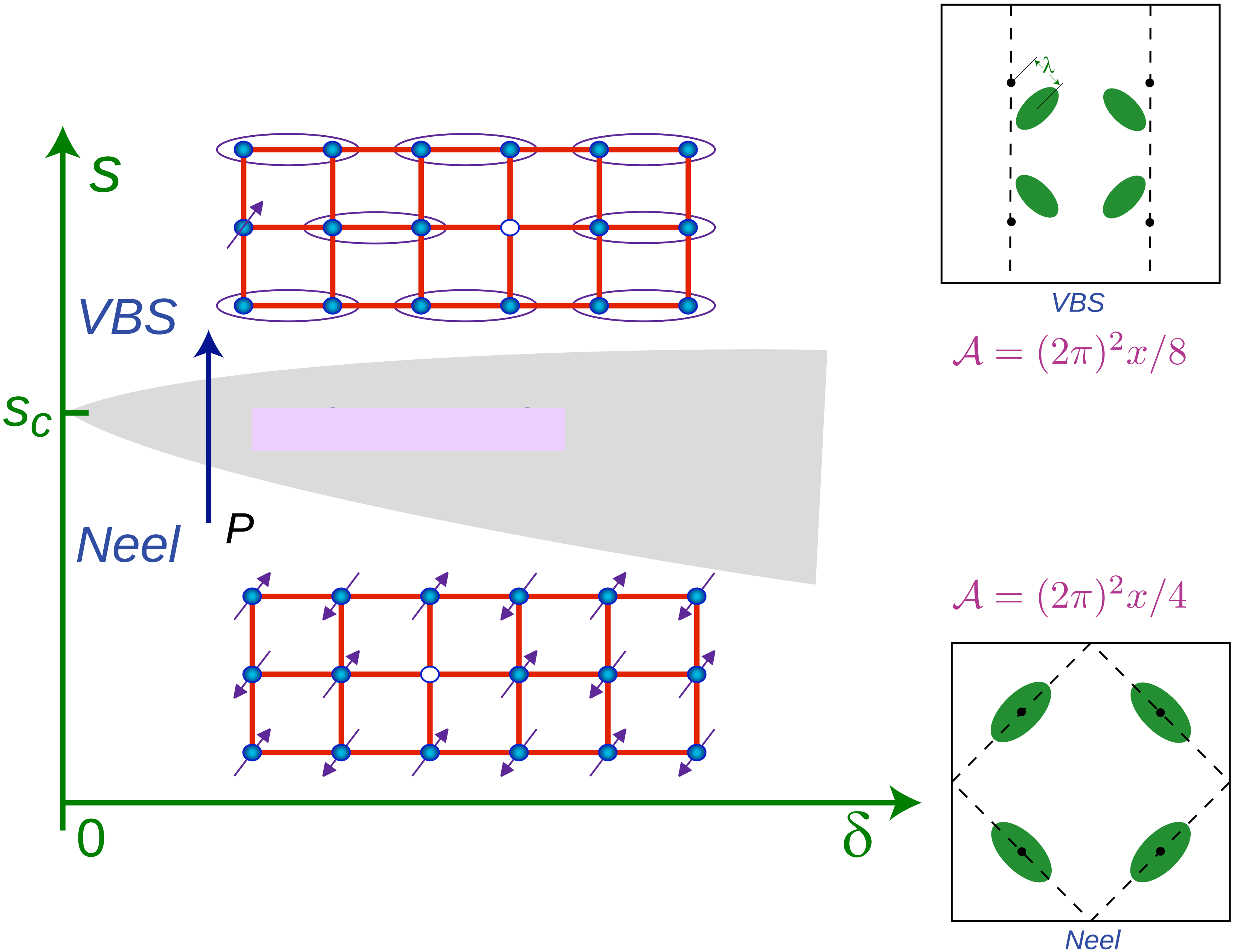


Area of each pocket, $\mathcal{A} = (2\pi)^2 x/8$

Phase diagram of lightly doped antiferromagnet



Pictorial explanation of factor of 2:

- In the Néel phase, sublattice index is identical to spin index. So for each valley and momentum, degeneracy of the hole state is 2.
- In the VBS state, the sublattice index and the spin index are distinct. So for each valley and momentum, degeneracy of the hole state is 4.

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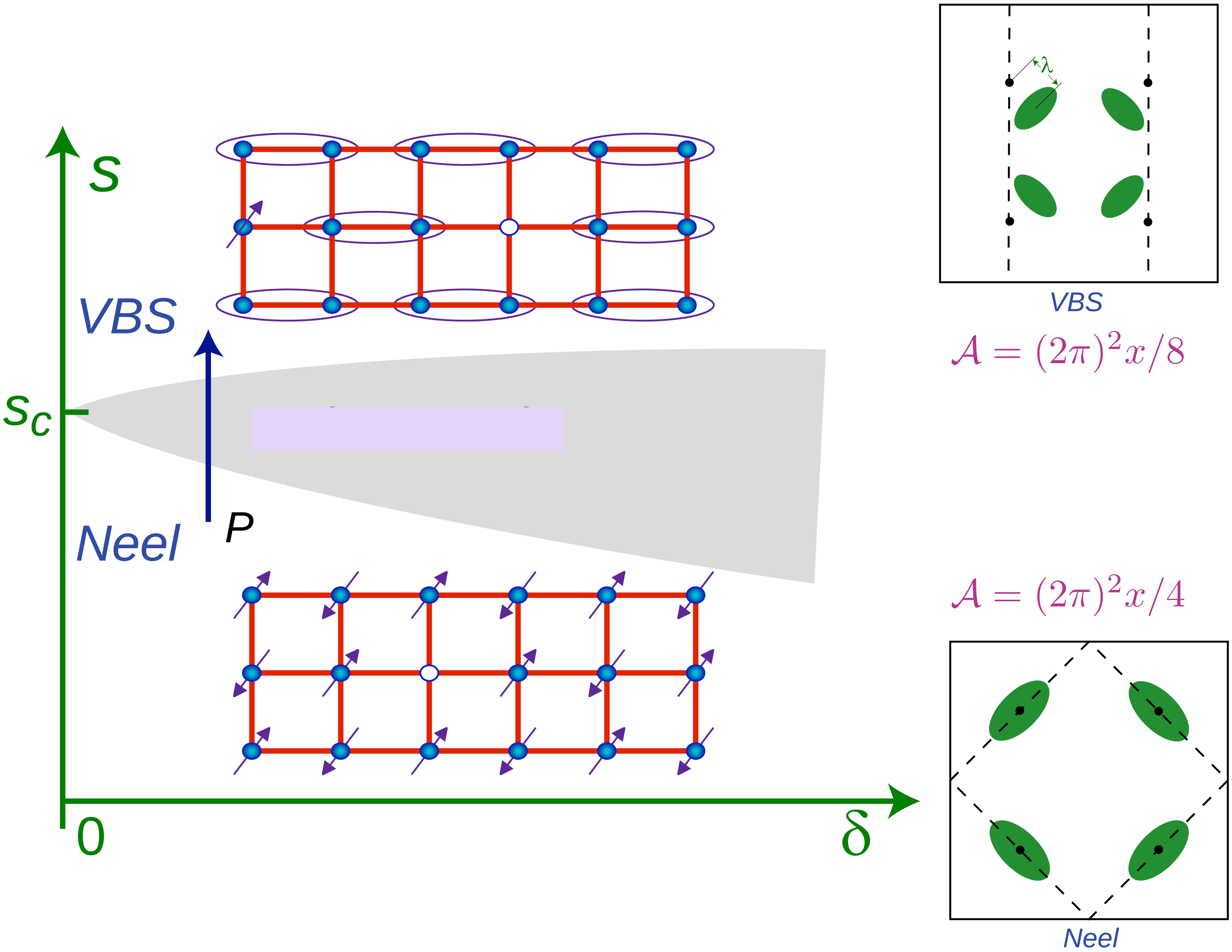
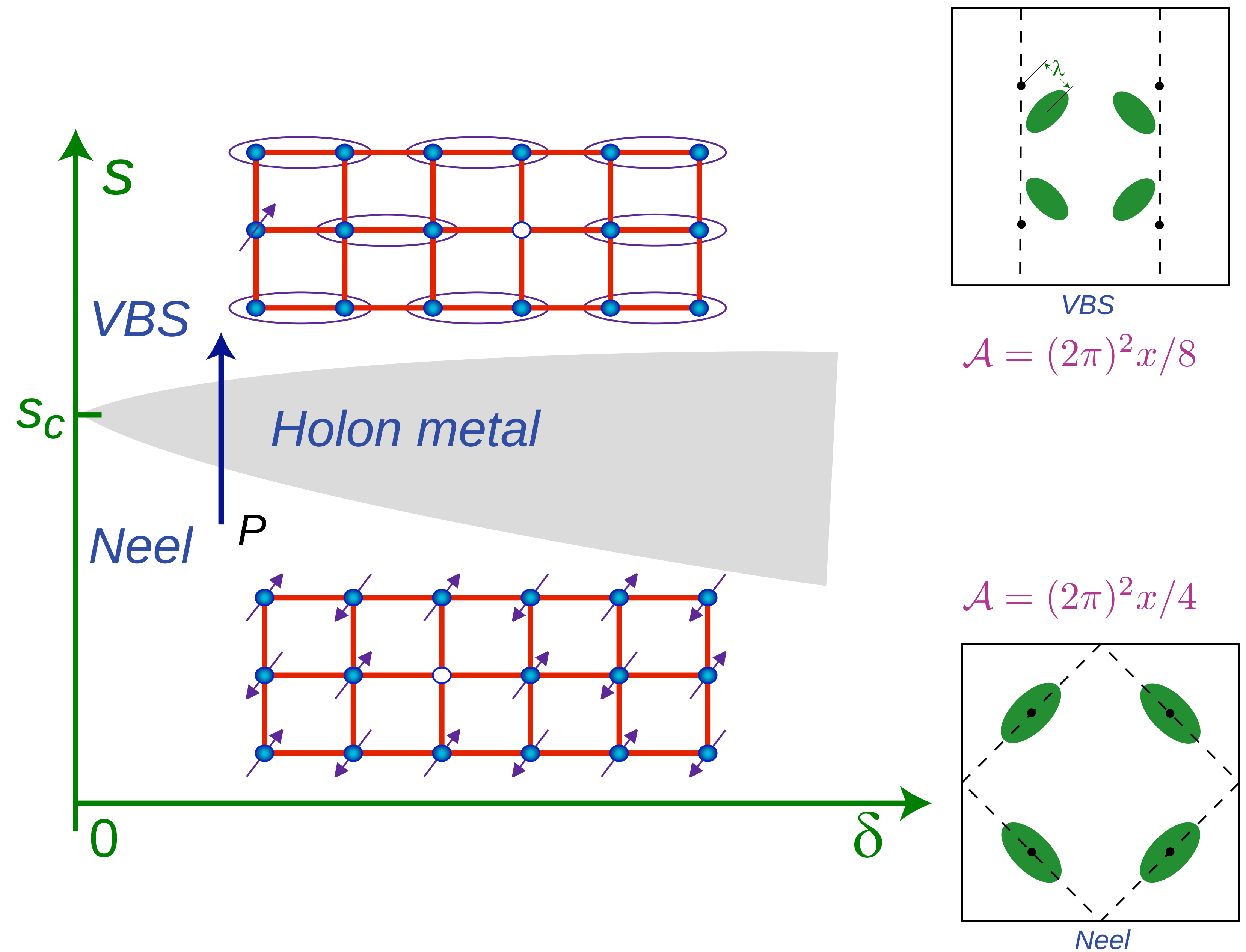
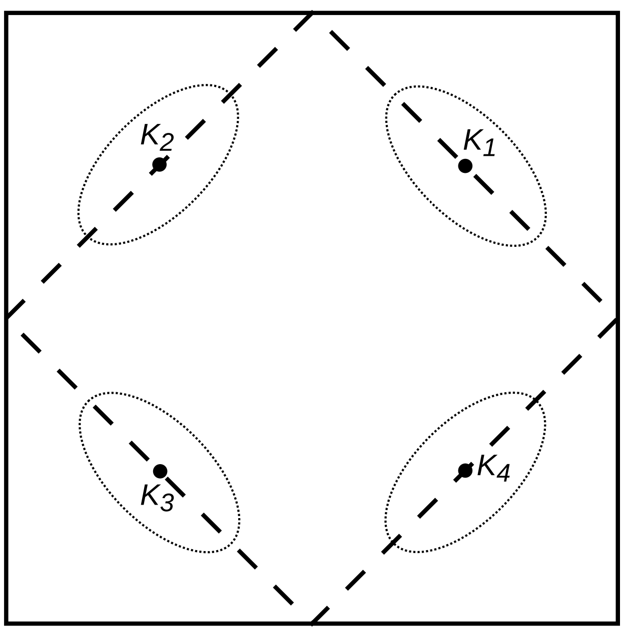


Phase diagram of lightly doped antiferromagnet



A new non-Fermi liquid phase:
The holon metal
An algebraic *charge* liquid.

- Ignore compactness in A_μ and Berry phase term.
- Neutral spinons z_α are gapped.
- Charge e fermions f_s form Fermi surfaces and carry charges $s = \pm 1$ under the U(1) gauge field A_μ.
- Quasi-long range order in a variety of VBS and pairing correlations.



Area of each Fermi pocket,

$$\mathcal{A} = (2\pi)^2 x/4.$$

The Fermi pocket will show sharp magnetoresistance oscillations, but it is invisible to photoemission.

Effective action for holons and spinons in holon metal

Spinon dynamics

$$\mathcal{S}_{\text{holon metal}}$$
$$= \int d\tau \sum_r \frac{1}{g} |(\partial_\tau - iA_\tau) z_{r\alpha}|^2 - \frac{1}{g'} \sum_{r,r'} \left(z^*_{r\alpha} e^{iA_{rr'}} z_{r'\alpha} + \text{c.c} \right)$$
$$\int d\tau \sum_{v,s} \int_\diamond \frac{d^2k}{4\pi^2} f_s^\dagger(\vec{k}) \left(\partial_\tau - isA_\tau + \epsilon(\vec{k} - s\vec{A}) \right) f_s(\vec{k})$$
$$- t \sum_{\langle rr' \rangle} (f_+^\dagger(r) z_{r\alpha})^\dagger \epsilon_{\alpha\beta} f_-^\dagger(r') z^*_{r\beta}$$

Holon dynamics

Sublattice mixing - attractive force between holons and spinons

Outline

1. Quantum transitions in dimerized antiferromagnets
   Landau-Ginzburg-Wilson (LGW) theory

2. Square lattice antiferromagnets
   VBS order and deconfined criticality

3. Doped square lattice antiferromagnets
   Fermi liquid states

4. Doped square lattice antiferromagnets
   Non-Fermi liquid holon metal

5. Instabilities of the holon metal

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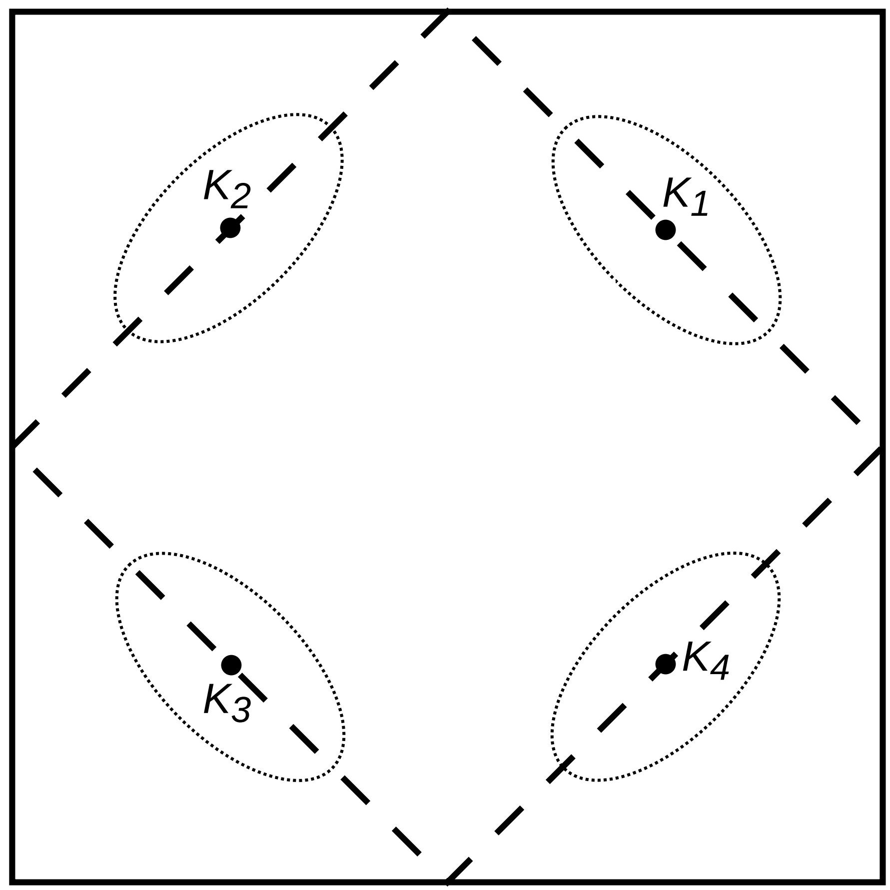
Instabilities of the holon metal

- Holon-spinon binding from sublattice mixing

- Holon-holon pairing leading to *d*-wave superconductivity

Resulting phases are also algebraic charge liquids

Holon-spinon binding from sublattice mixing



Effective action for holons and spinons in holon metal

Spinon dynamics

$$\mathcal{S}_{\text{holon metal}}$$

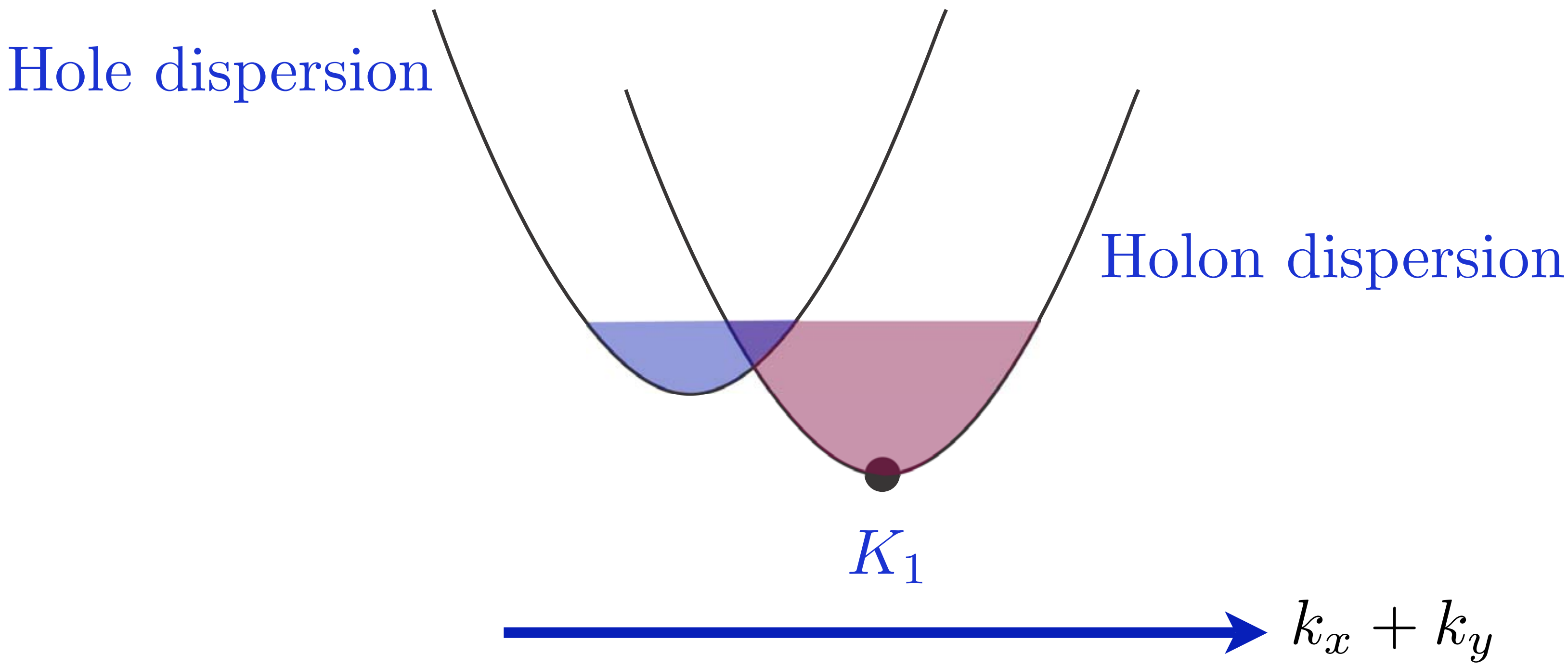
$$= \int d\tau \sum_{r} \frac{1}{g} |(\partial_\tau - iA_\tau) z_{r\alpha}|^2 - \frac{1}{g'} \sum_{r,r'} \left(z^*_{r\alpha} e^{iA_{rr'}} z_{r'\alpha} + \text{c.c} \right)$$

$$\int d\tau \sum_{v,s} \int_{\Diamond} \frac{d^2k}{4\pi^2} f^\dagger_s(\vec{k}) \left(\partial_\tau - isA_\tau + \epsilon(\vec{k} - s\vec{A}) \right) f_s(\vec{k})$$

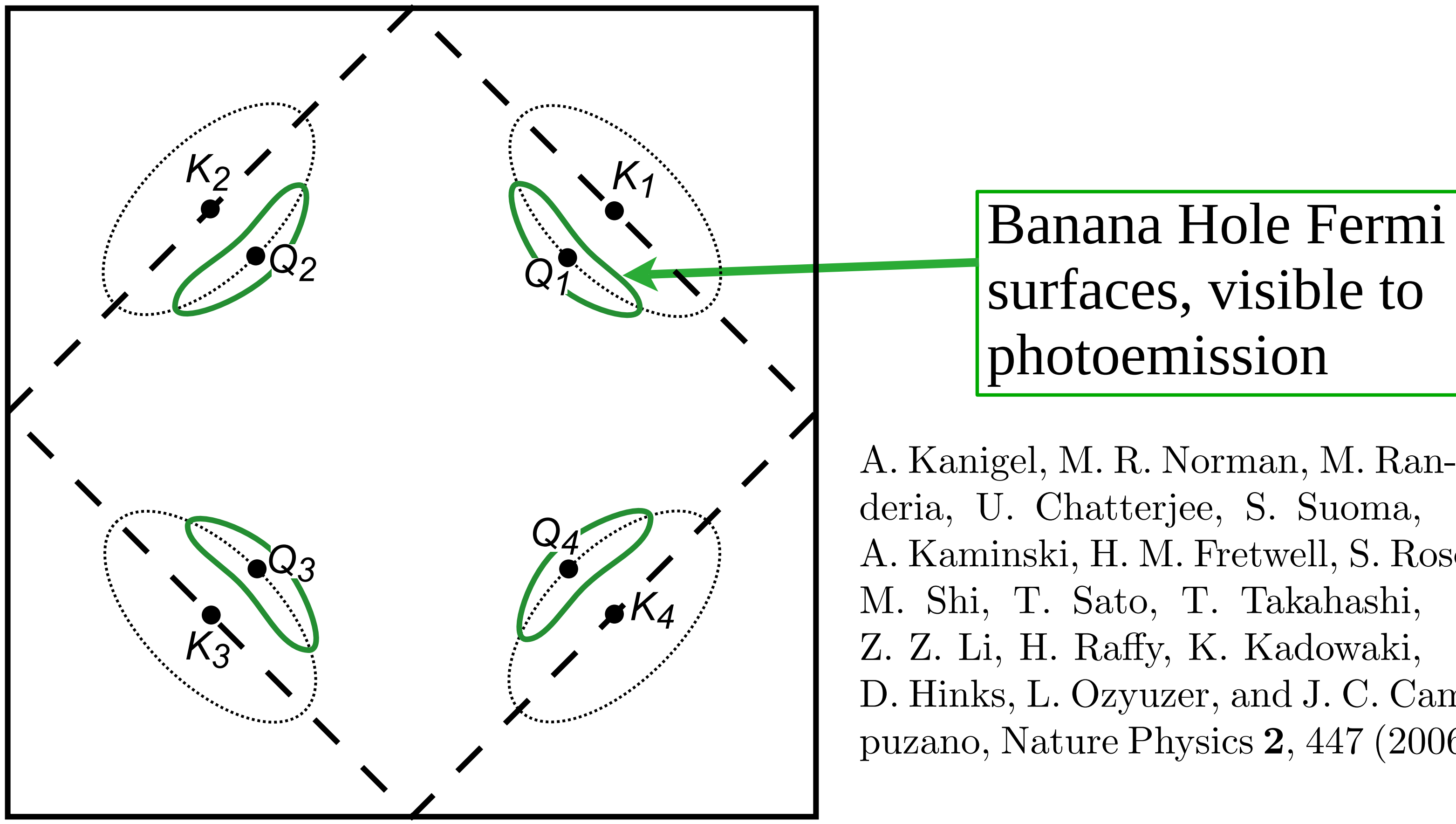
$$- t \sum_{\langle rr' \rangle} (f^\dagger_+(r) z_{r\alpha})^\dagger \epsilon_{\alpha\beta} f^\dagger_-(r') z^*_{r\beta}$$

Holon dynamics

Sublattice mixing - attractive force between holons and spinons


Hole dispersion
Holon dispersion
K_1
$k_x + k_y$

Holon-spinon binding from sublattice mixing

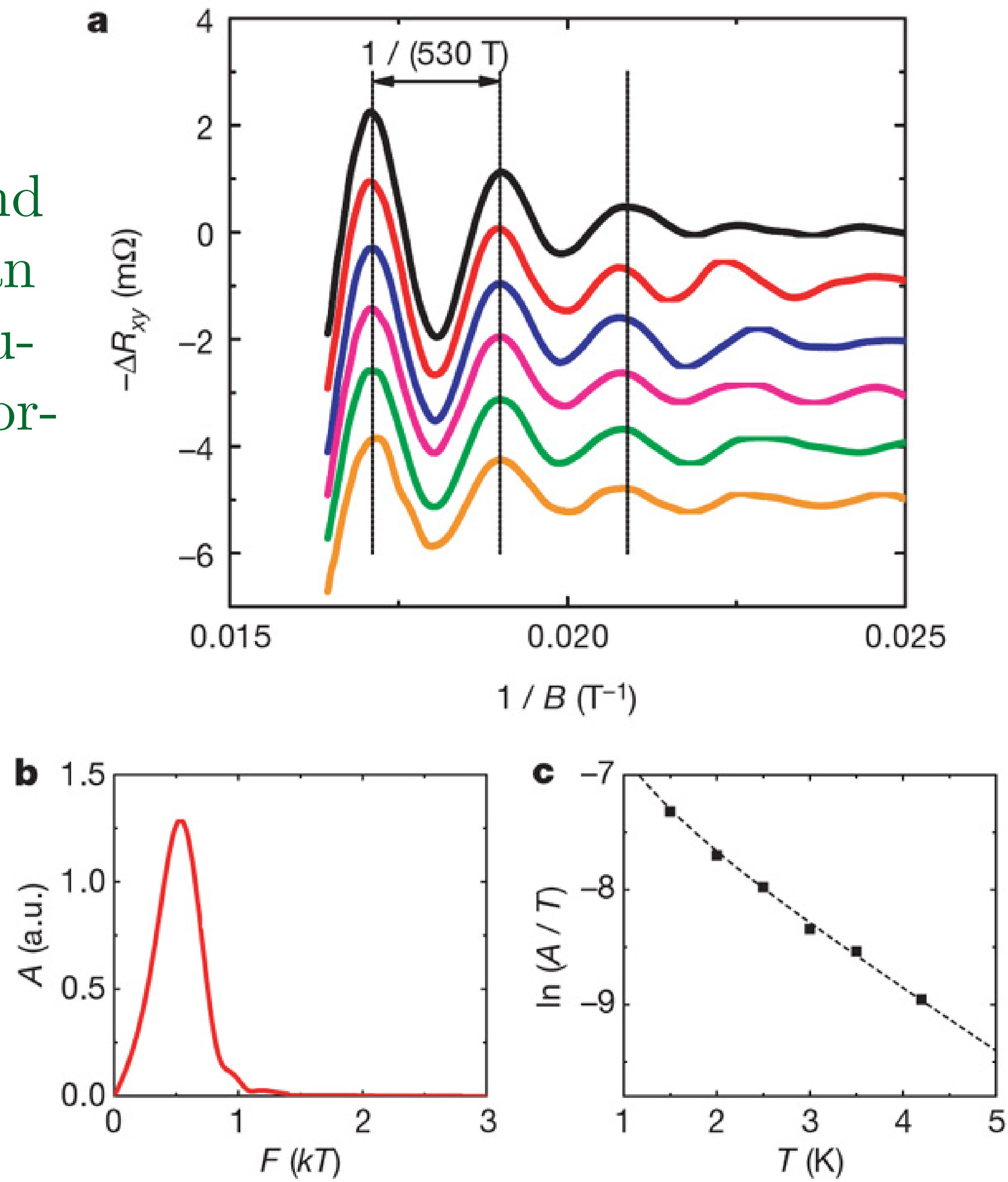


A. Kanigel, M. R. Norman, M. Randeria, U. Chatterjee, S. Suoma, A. Kaminski, H. M. Fretwell, S. Rosenkranz, M. Shi, T. Sato, T. Takahashi, Z. Z. Li, H. Raffy, K. Kadowaki, D. Hinks, L. Ozyuzer, and J. C. Campuzano, Nature Physics **2**, 447 (2006).

The lowest energy holons will bind the spinons to form a Fermi surface of "molecules" – a "banana" Fermi surface of $S = 1/2$ holes. Now we have

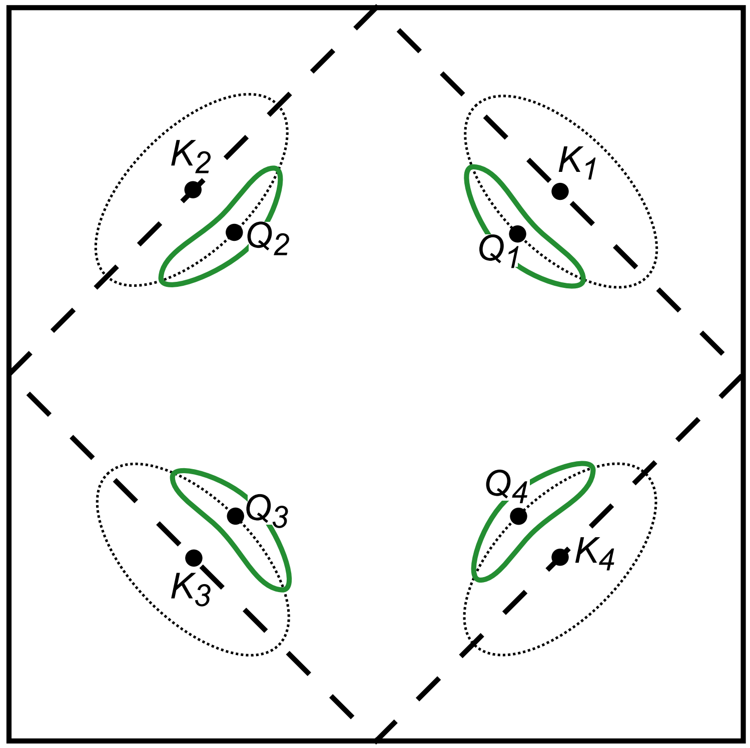
$$2\mathcal{A}_{\text{banana}} + \mathcal{A}_{\text{holon}} = (2\pi)^2 x/4$$

Quantum oscillations and the Fermi surface in an underdoped high T_c superconductor (ortho-II ordered $YBa_2Cu_3O_{6.5}$).



N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007)

Holon-spinon binding from sublattice mixing



Recent experiments have observed magnetoresistance oscillations at an area $\mathcal{A} = (2\pi)^2(0.076)/4$ for $x = 0.1$. We propose these are due to holon Fermi surfaces. The deficit is taken up by the hole "banana" Fermi surfaces, which we predict will lead to magnetoresistance oscillations at a frequency associated with the area $\mathcal{A}_{\text{banana}} = (2\pi)^2(0.012)/4$.

arXiv:0707.0057

Title: Quantum Oscillations in the Underdoped Cuprate YBa2Cu4O8

Authors: E. A. Yelland, J. Singleton, C. H. Mielke, N. Harrison, F. F. Balakirev, B. Dabrowski, J. R. Cooper

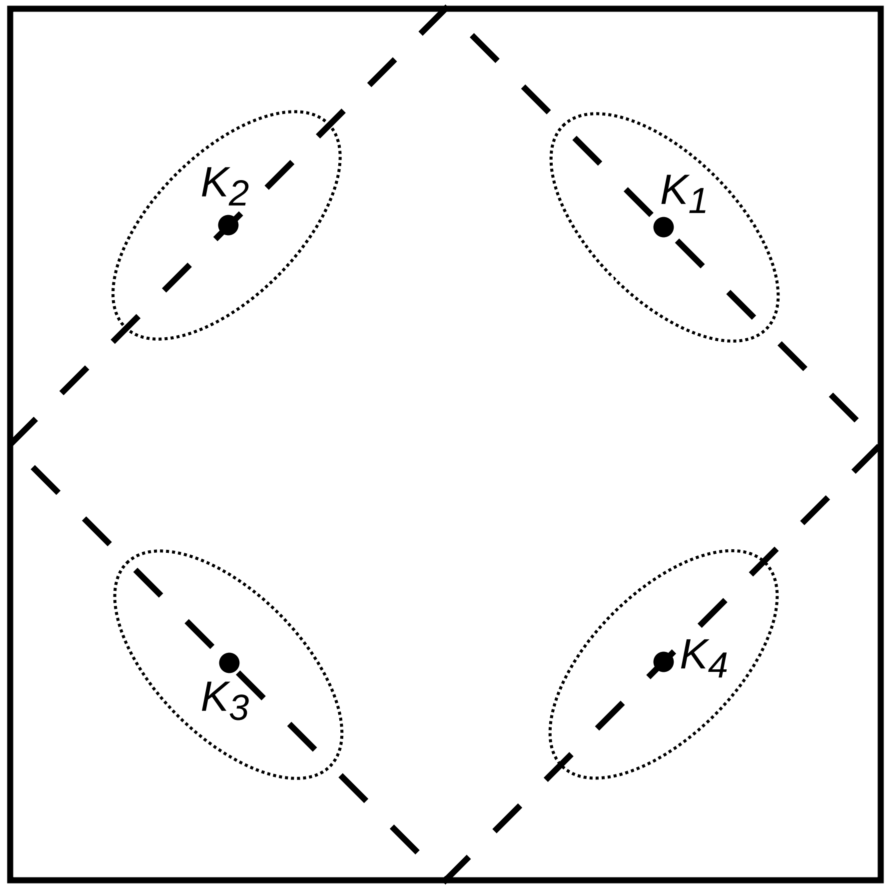
We report the observation of quantum oscillations in the underdoped cuprate superconductor YBa2Cu4O8 using a tunnel-diode oscillator technique in pulsed magnetic fields up to 85T. There is a clear signal, periodic in inverse field, with frequency 660+/-15T and some evidence for the presence of two components of slightly different frequency. The quasiparticle mass is m*=3.0 +/-0.3m_e. In conjunction with the results of Doiron-Leyraud et al. for YBa2Cu3O6.5, the present measurements suggest that Fermi surface pockets are a general feature of underdoped copper oxide planes and provide information about the doping dependence of the Fermi surface.

Holon pairing leading to *d*-wave superconductivity

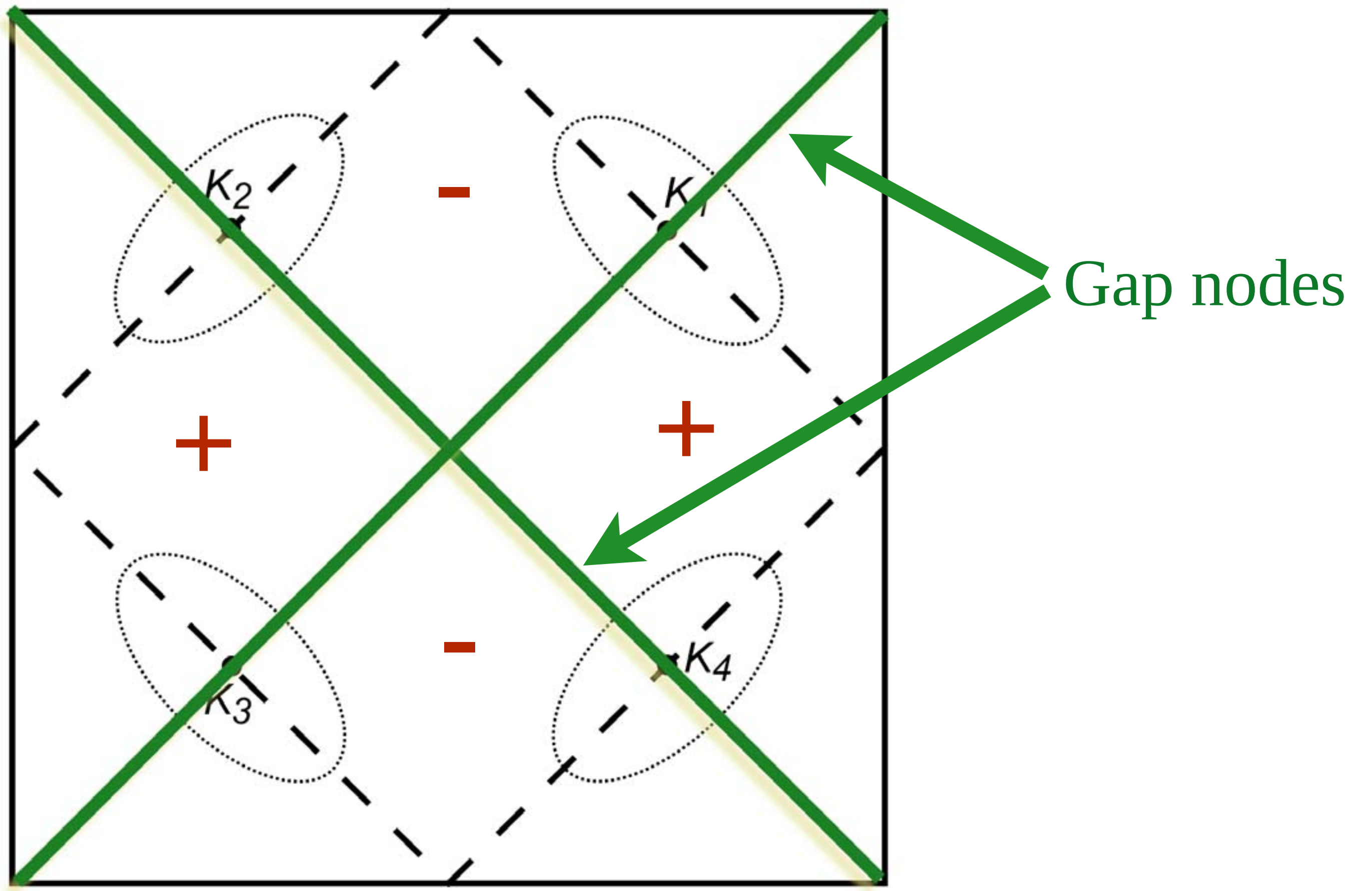
- First consider holon pairing in the Neel state, where holon=hole.

- This was studied in V. V. Flambaum, M. Yu. Kuchiev, and O. P. Sushkov, *Physica* C **227**, 267 (1994); V. I. Belincher *et al.*, *Phys. Rev.* B **51**, 6076 (1995). They found *p*-wave pairing of holons, induced by spin-wave exchange from the sublattice mixing term $\mathcal{S}_t$. This corresponds to *d*-wave pairing of physical electrons

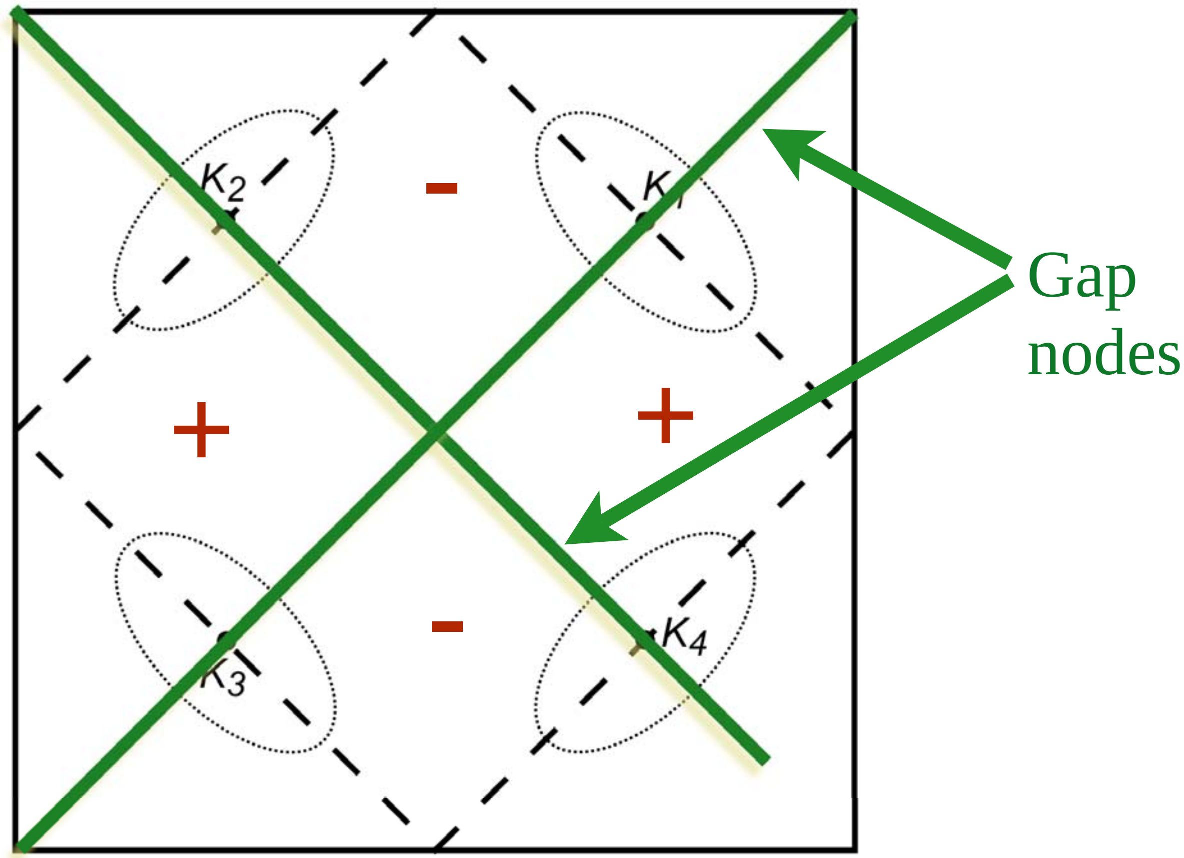
Holon pairing leading to *d*-wave superconductivity



Holon pairing leading to *d*-wave superconductivity



Holon pairing leading to *d*-wave superconductivity



We assume the same pairing holds across a transition involving loss of long-range Néel order. The resulting phase is another algebraic charge liquid - the *holon superconductor*. This superconductor has gapped spinons with no electrical charge, and spinless, nodal Bogoliubov-Dirac quasiparticles. Both the spinons and nodal fermions are coupled to the U(1) gauge field A_μ.

Low energy theory of holon superconductor

4 two-component Dirac quasiparticles coupled to a U(1) gauge field

$$\mathcal{S}_{\text{holon superconductor}} = \int d\tau d^2 r \left[\frac{1}{2e_0^2} \left(\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda \right)^2 + \sum_{i=1}^{4} \psi_i^\dagger \left(D_\tau - i v_F (\partial_x - i A_x) \tau^x - i v_F (\partial_y - i A_y) \tau^y \right) \psi_i \right]$$

Low energy theory of holon superconductor

External vector potential $\vec{A}$ couples as

$$\mathcal{H}_A = \vec{j} \cdot \vec{A}$$

where

$$j_x = v_F \left(\psi_3^\dagger \psi_3 - \psi_1^\dagger \psi_1 \right) \quad , \quad j_x = v_F \left(\psi_4^\dagger \psi_4 - \psi_2^\dagger \psi_2 \right)$$

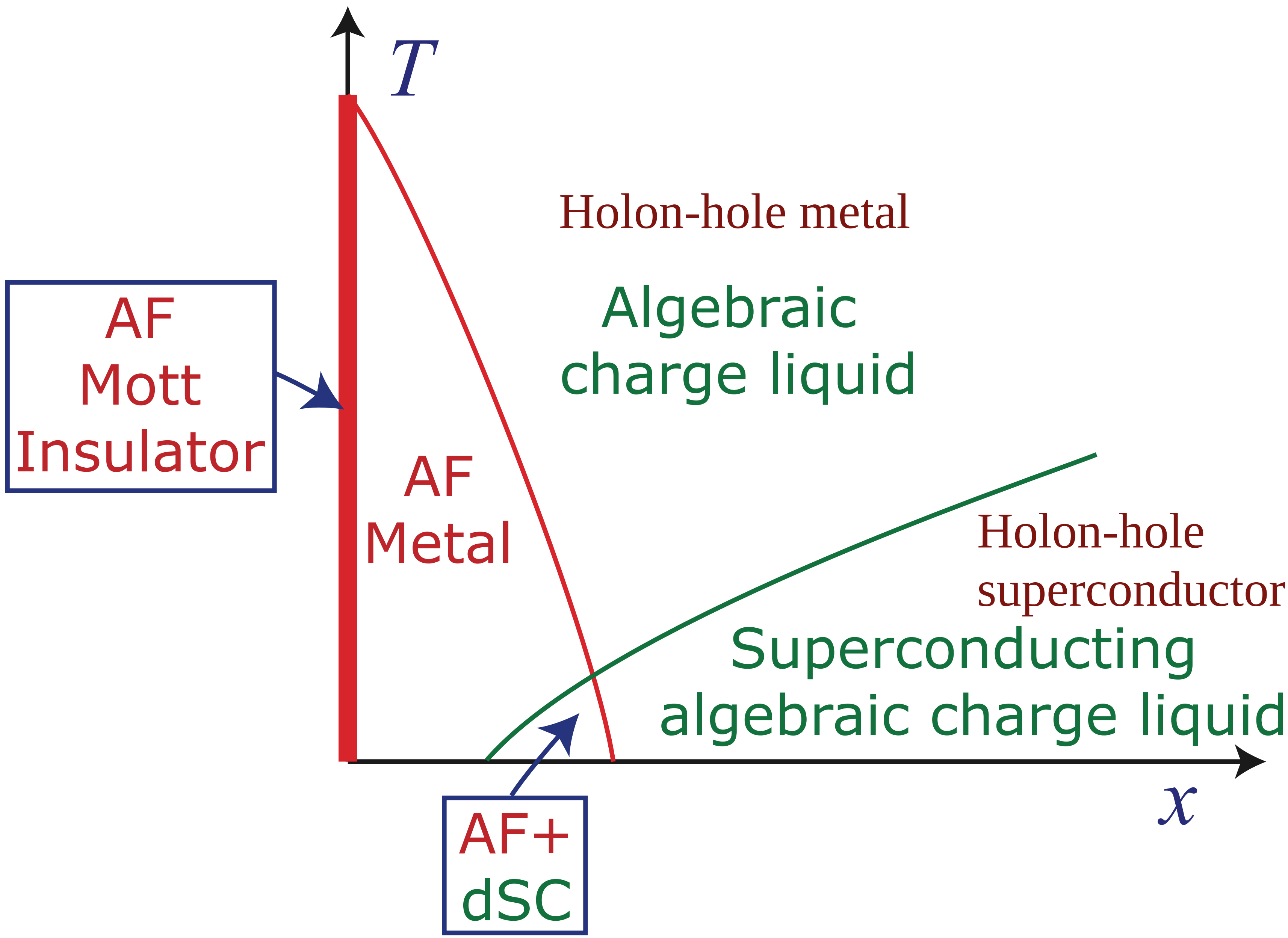
are conserve charges of $\mathcal{S}_{\text{holon superconductor}}$.

Fundamental property: The superfluid density, ρ_s, has the following x and T dependence:

$$\rho_s(x, T) = cx - \mathcal{R} k_B T$$

where c is a non-universal constant and $\mathcal{R}$ is a universal constant obtained in a $1/N$ expansion ($N = 4$ is the number of Dirac fermions):

$$\mathcal{R} = 0.4412 + \frac{0.074}{N} + \ldots$$


T
Holon-hole metal
Algebraic
charge liquid
AF
Mott
Insulator
AF
Metal
Holon-hole
superconductor
Superconducting
algebraic charge liquid
x
AF+
dSC

Conclusions

1. Good evidence for deconfined criticality in insulating square lattice antiferromagnets

2. *Algebraic charge liquids* appear naturally upon doping antiferromagnets in the vicinity of such a transition.

3. The holon-hole metal is proposed as an explanation of magnetoresistance oscillations in underdoped cuprates

4. The holon superconductor has a superfluid density whose doping and temperature dependence matches observations.