

Where is the quantum critical point in the cuprate superconductors ?

arXiv:0907.0008



Destruction of Neel order in the cuprates by electron doping,

R. K. Kaul, M. Metlitski, S. Sachdev, and C. Xu,

Physical Review B **78**, 045110 (2008).

Paired electron pockets in the underdoped cuprates,

V. Galitski and S. Sachdev,

Physical Review B **79**, 134512 (2009).

Competition between spin density wave order and
superconductivity in the underdoped cuprates,

Eun Gook Moon and S. Sachdev,

[arXiv:0905.2608](https://arxiv.org/abs/0905.2608)

Fractionalization of the spin density wave transition in metals

M. Metlitski, Y. Qi, S. Sachdev, and C. Xu

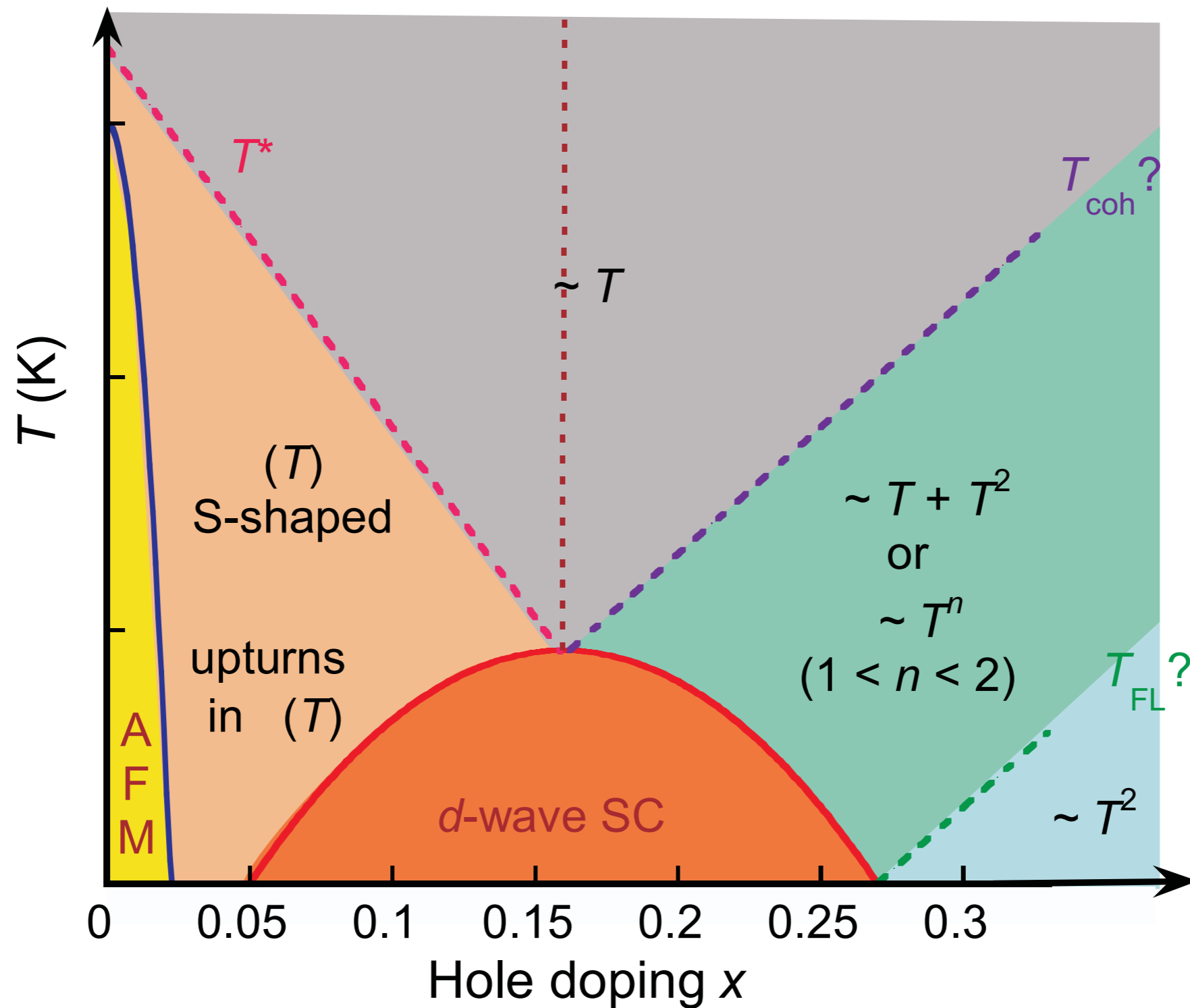
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PHYSICS



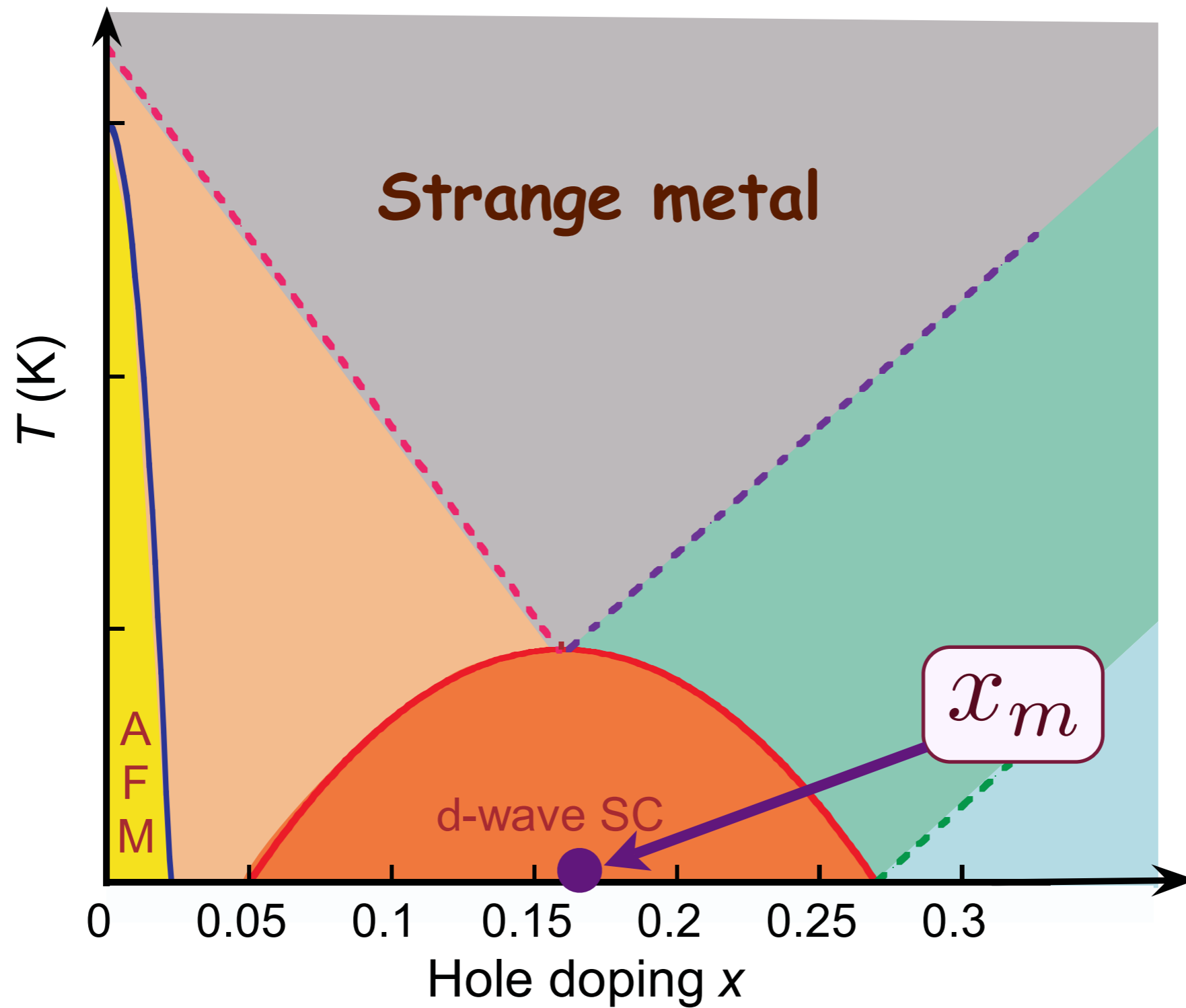
HARVARD

Crossovers in transport properties of hole-doped cuprates



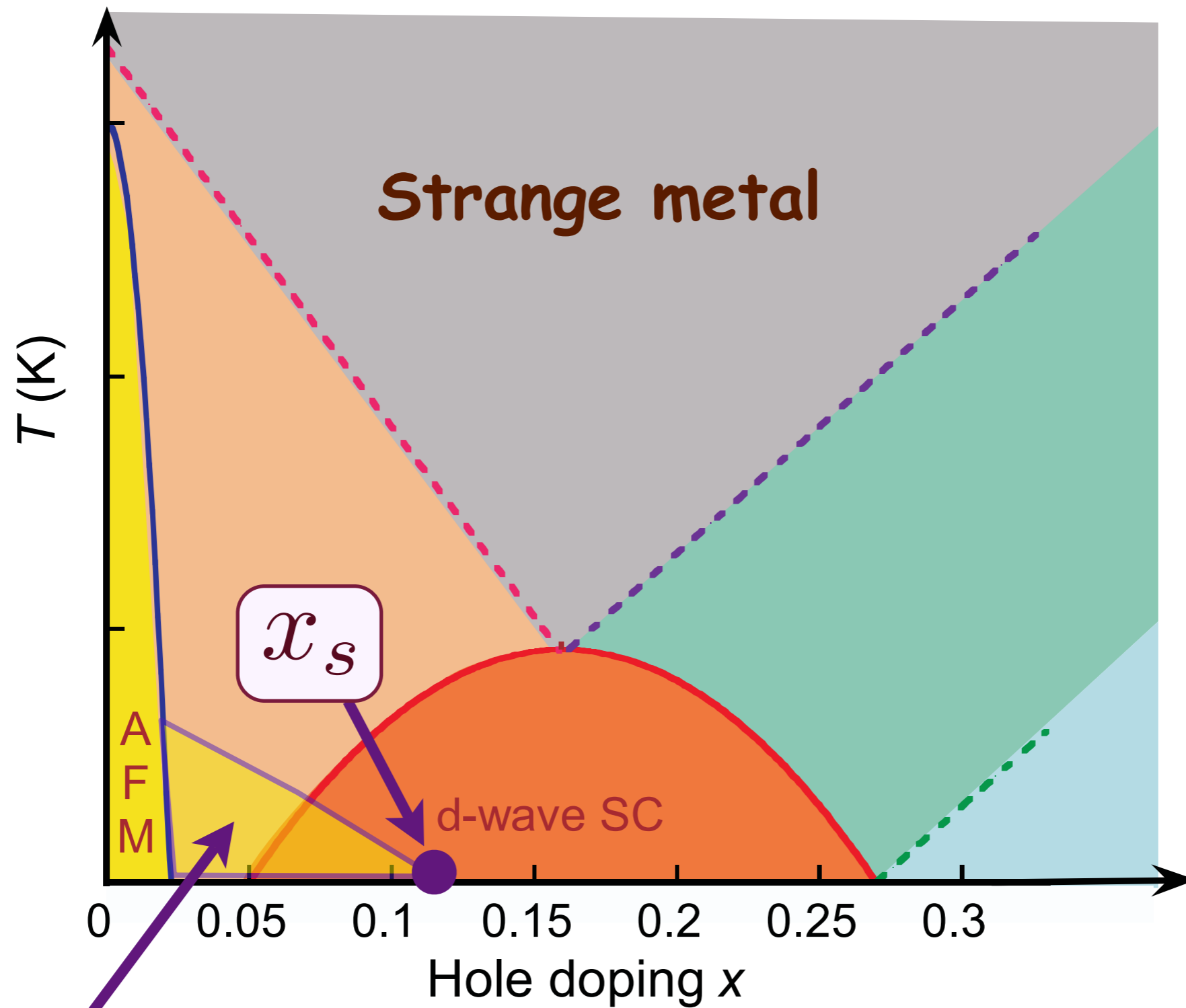
N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

Crossovers in transport properties of hole-doped cuprates



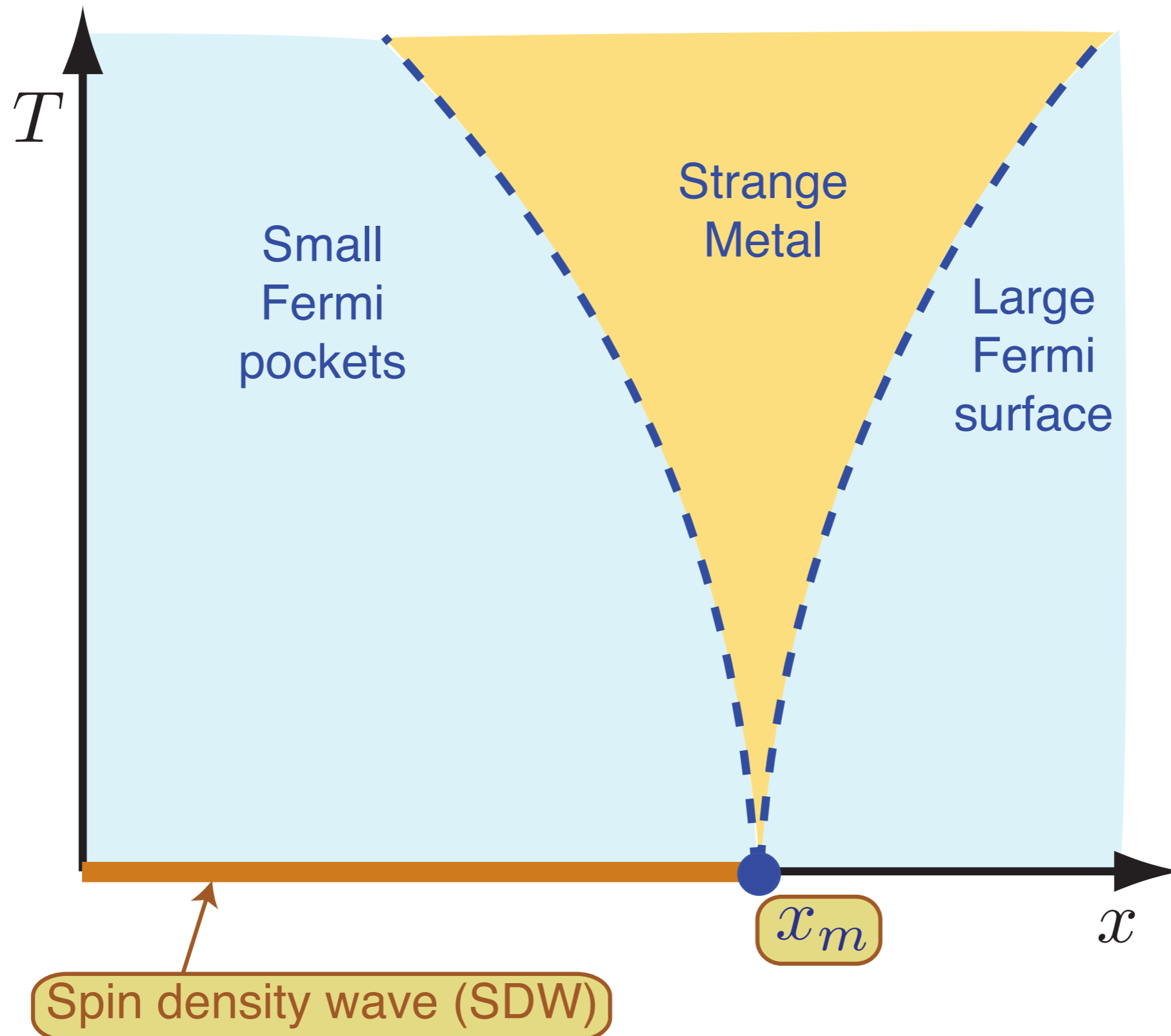
Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low T



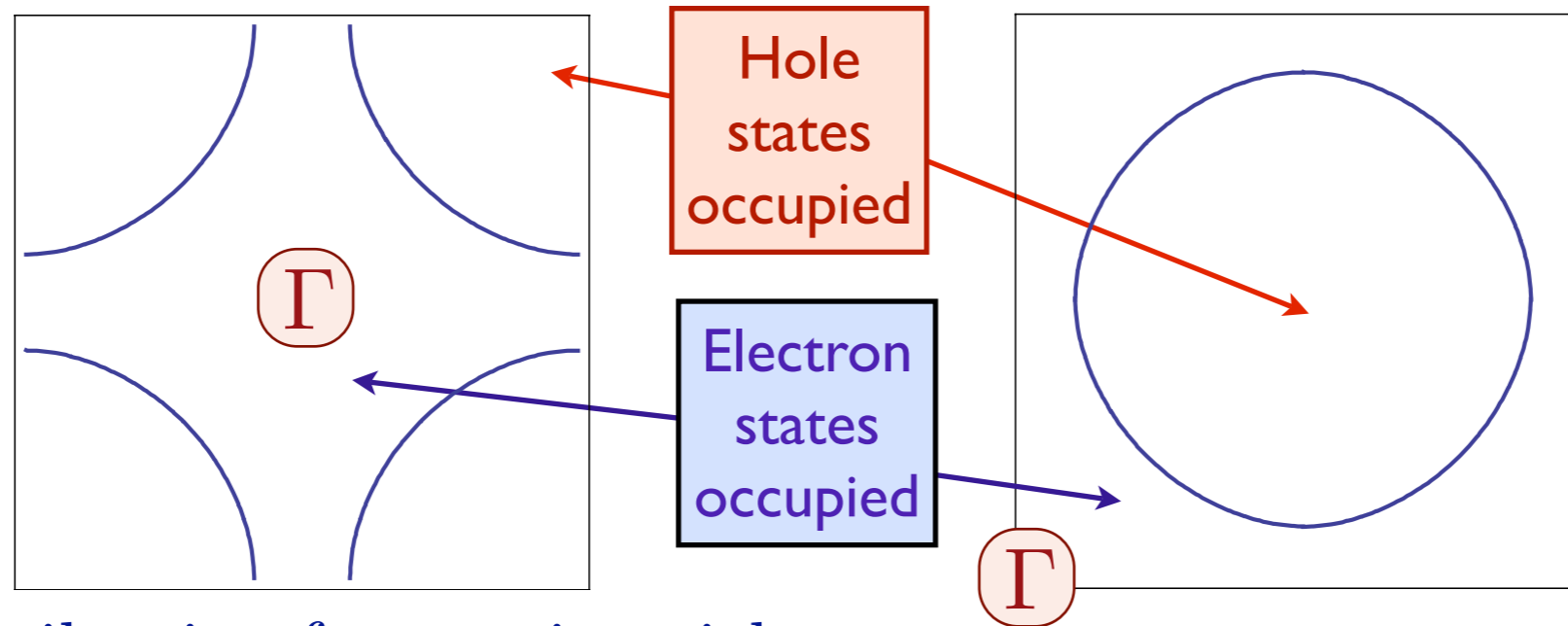
Spin and charge density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point
in metal at $x = x_m$

Fermi surfaces in electron- and hole-doped cuprates



Effective Hamiltonian for quasiparticles:

$$H_0 = - \sum_{i < j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \equiv \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

with t_{ij} non-zero for first, second and third neighbor, leads to satisfactory agreement with experiments. The area of the occupied electron states, \mathcal{A}_e , from Luttinger's theory is

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1 - x) & \text{for hole-doping } x \\ 2\pi^2(1 + p) & \text{for electron-doping } p \end{cases}$$

The occupied hole states form a closed Fermi surface; so it is their area, \mathcal{A}_h , which appears in quantum oscillation experiments: $\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$.

Spin density wave theory

In the presence of spin density wave order, $\vec{\varphi}$ at wavevector $\mathbf{K} = (\pi, \pi)$, we have an additional term which mixes electron states with momentum separated by \mathbf{K}

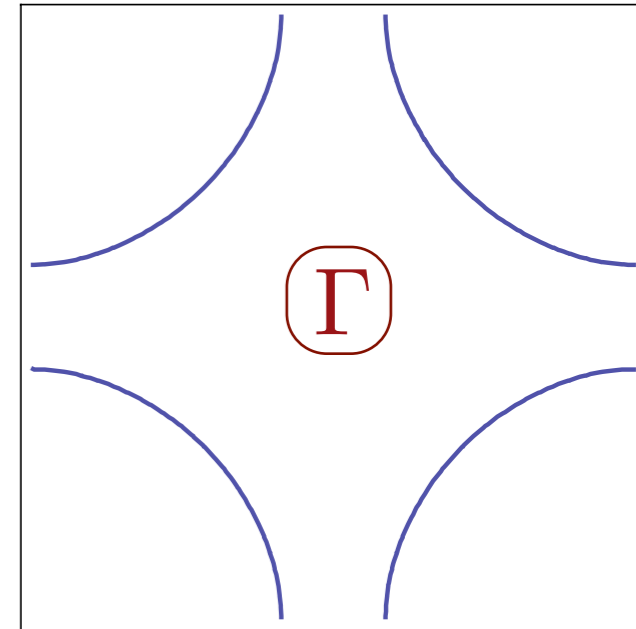
$$H_{\text{sdw}} = -\vec{\varphi} \cdot \sum_{\mathbf{k}, \alpha, \beta} c_{\mathbf{k}, \alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}, \beta}$$

where $\vec{\sigma}$ are the Pauli matrices. The electron dispersions obtained by diagonalizing $H_0 + H_{\text{sdw}}$ for $\vec{\varphi} = (0, 0, \varphi)$ are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right)^2 + \varphi^2}$$

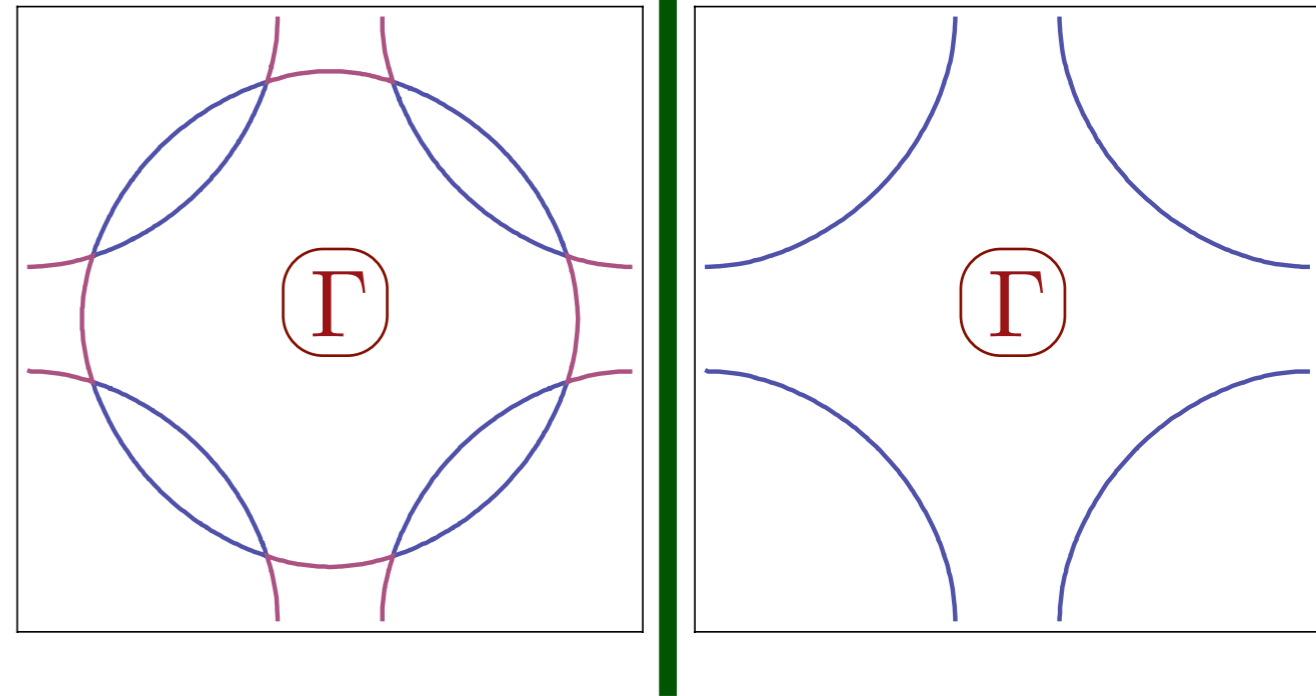
This leads to the Fermi surfaces shown in the following slides for hole doping.

Spin density wave theory in hole-doped cuprates



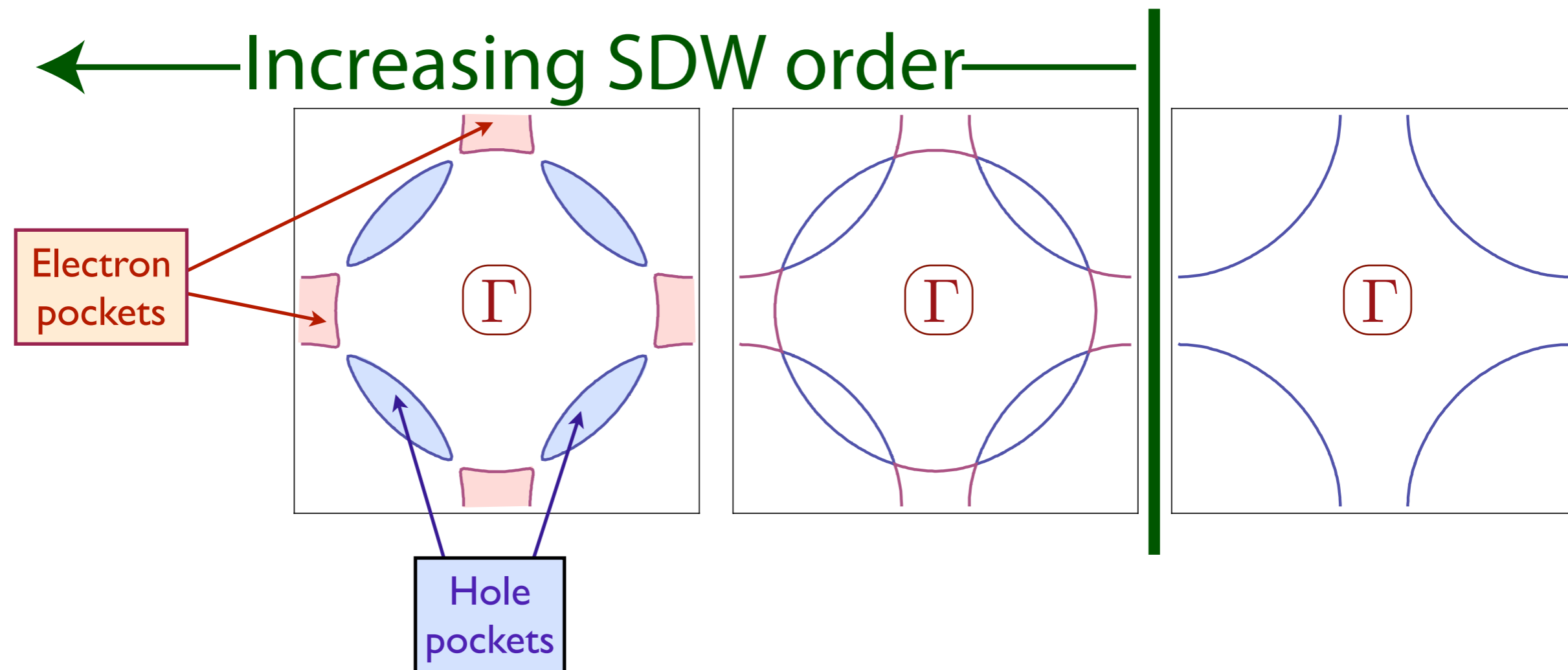
S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

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Spin density wave theory in hole-doped cuprates



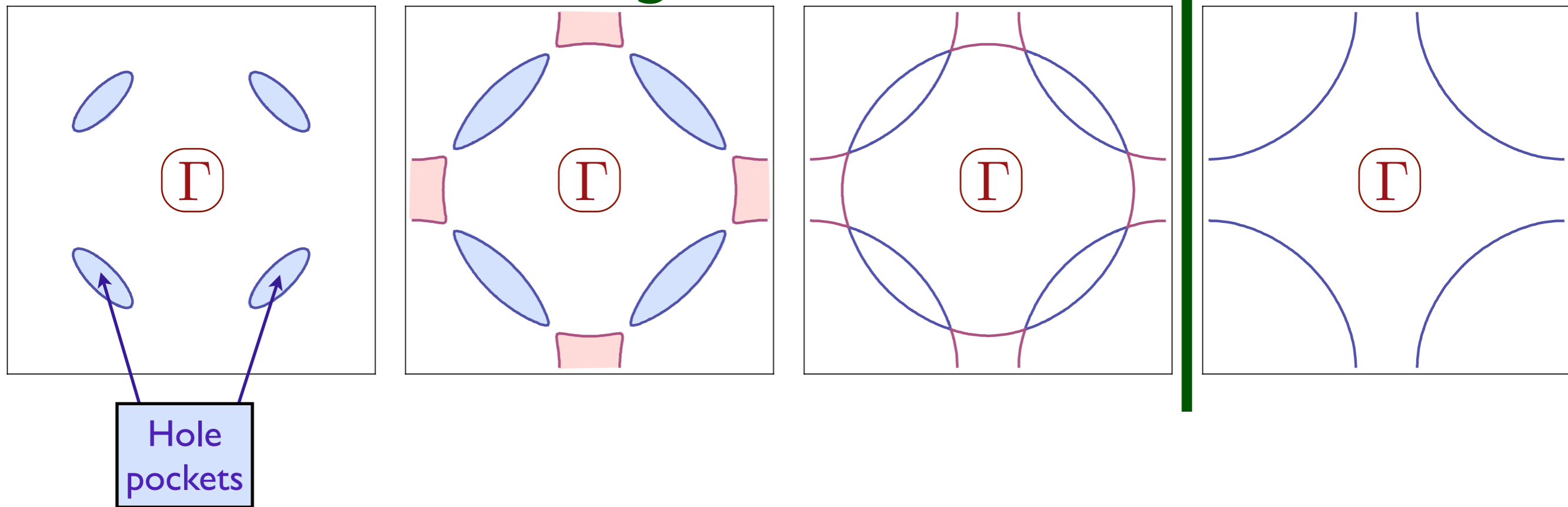
SDW order parameter is a vector, $\vec{\varphi}$, whose amplitude vanishes at the transition to the Fermi liquid.

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Spin density wave theory in hole-doped cuprates

← Increasing SDW order →

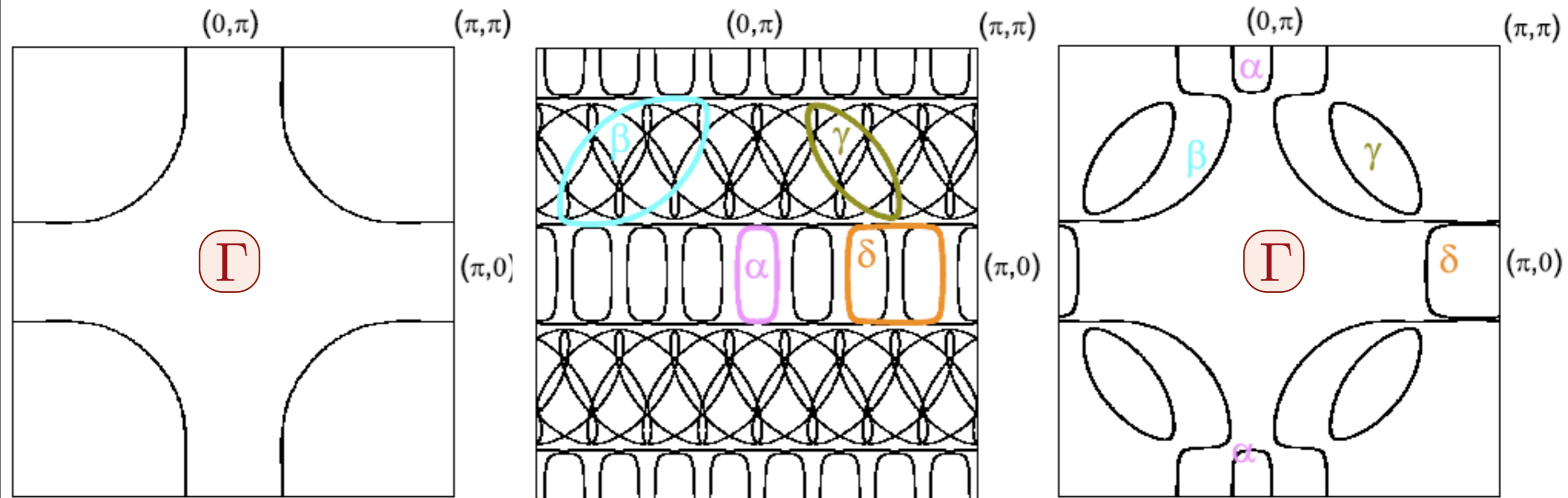


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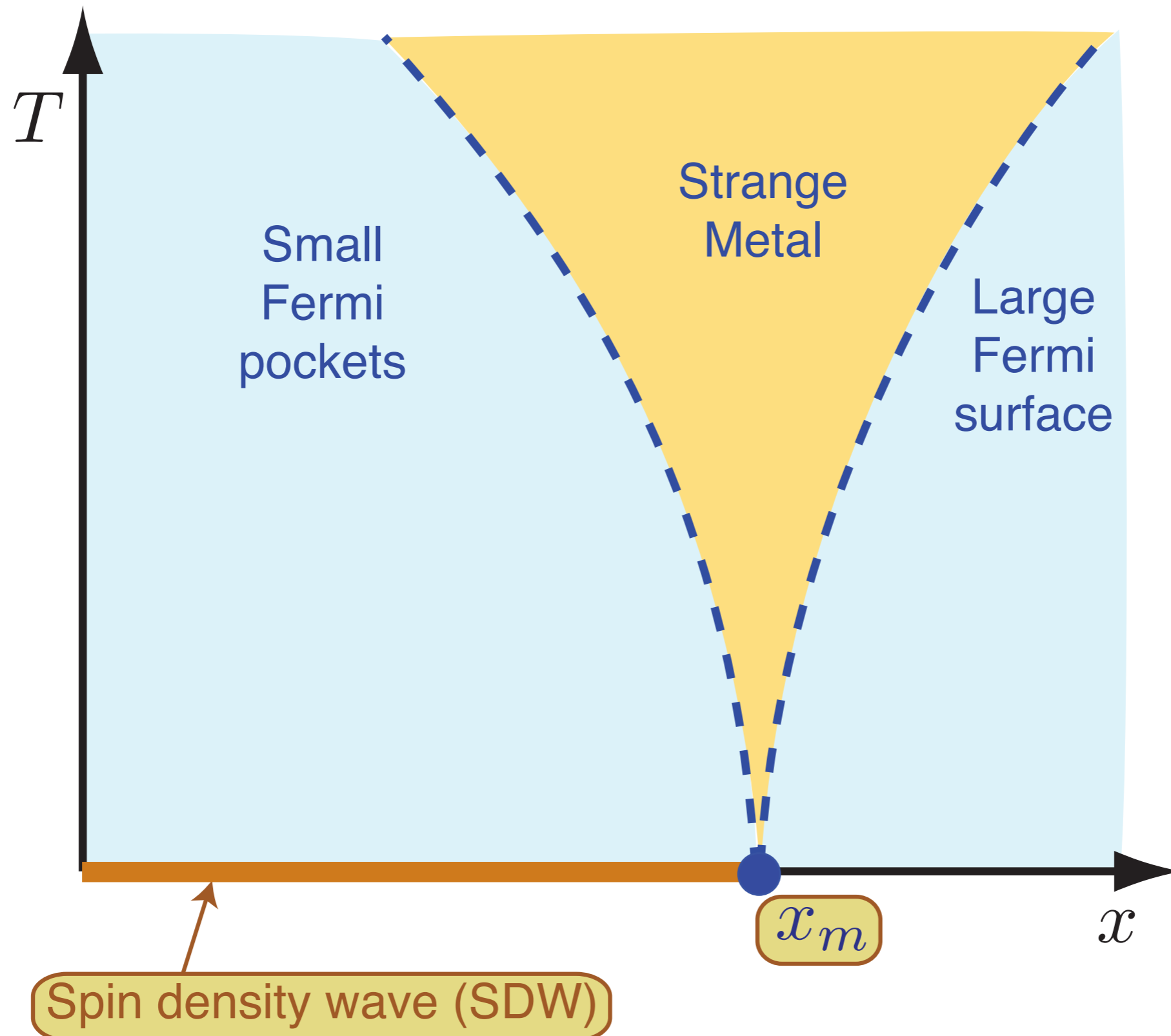


Incommensurate order in $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$

A. J. Millis and M. R. Norman, *Physical Review B* **76**, 220503 (2007).

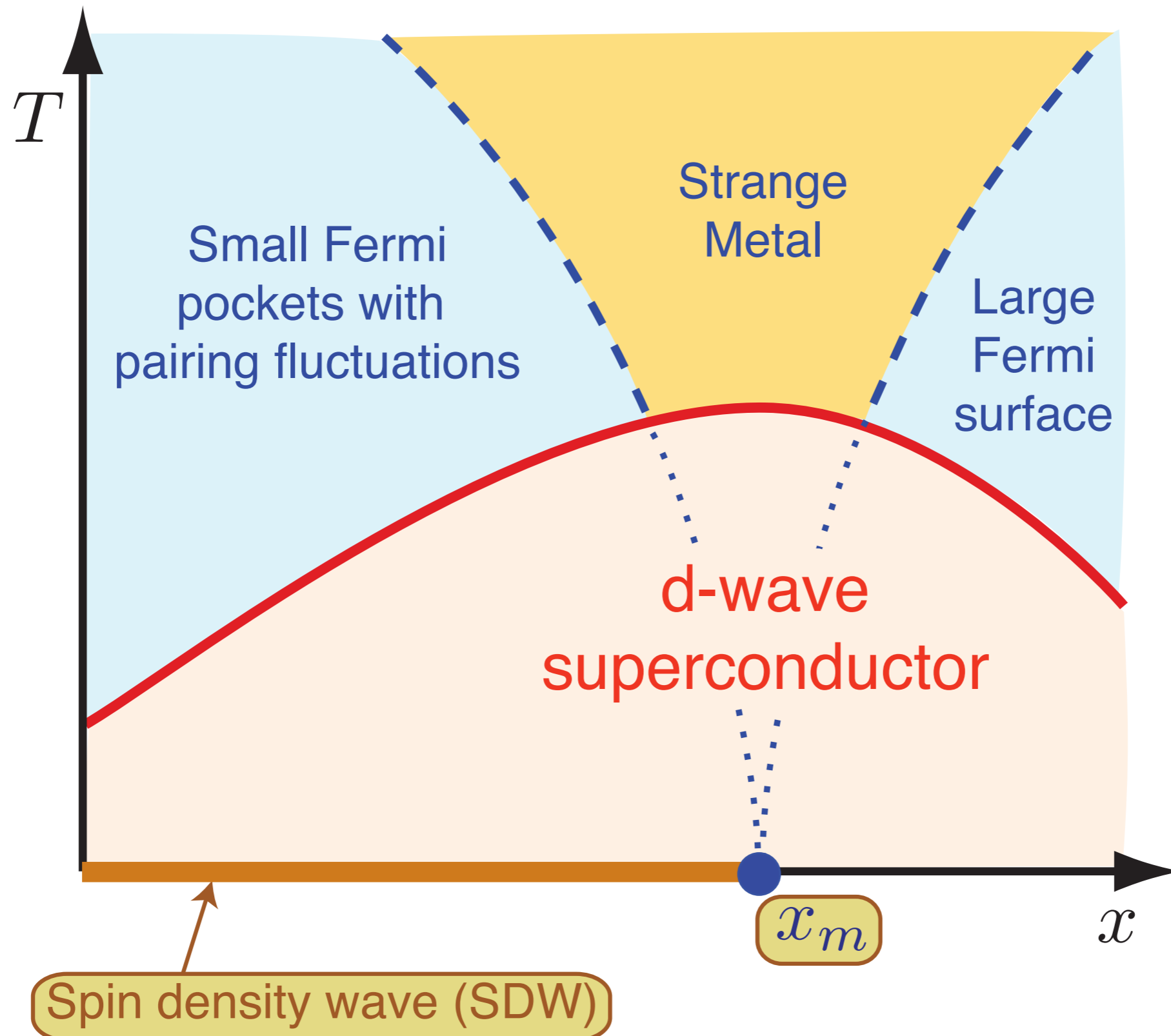
N. Harrison, *Physical Review Letters* **102**, 206405 (2009).

Theory of quantum criticality in the cuprates



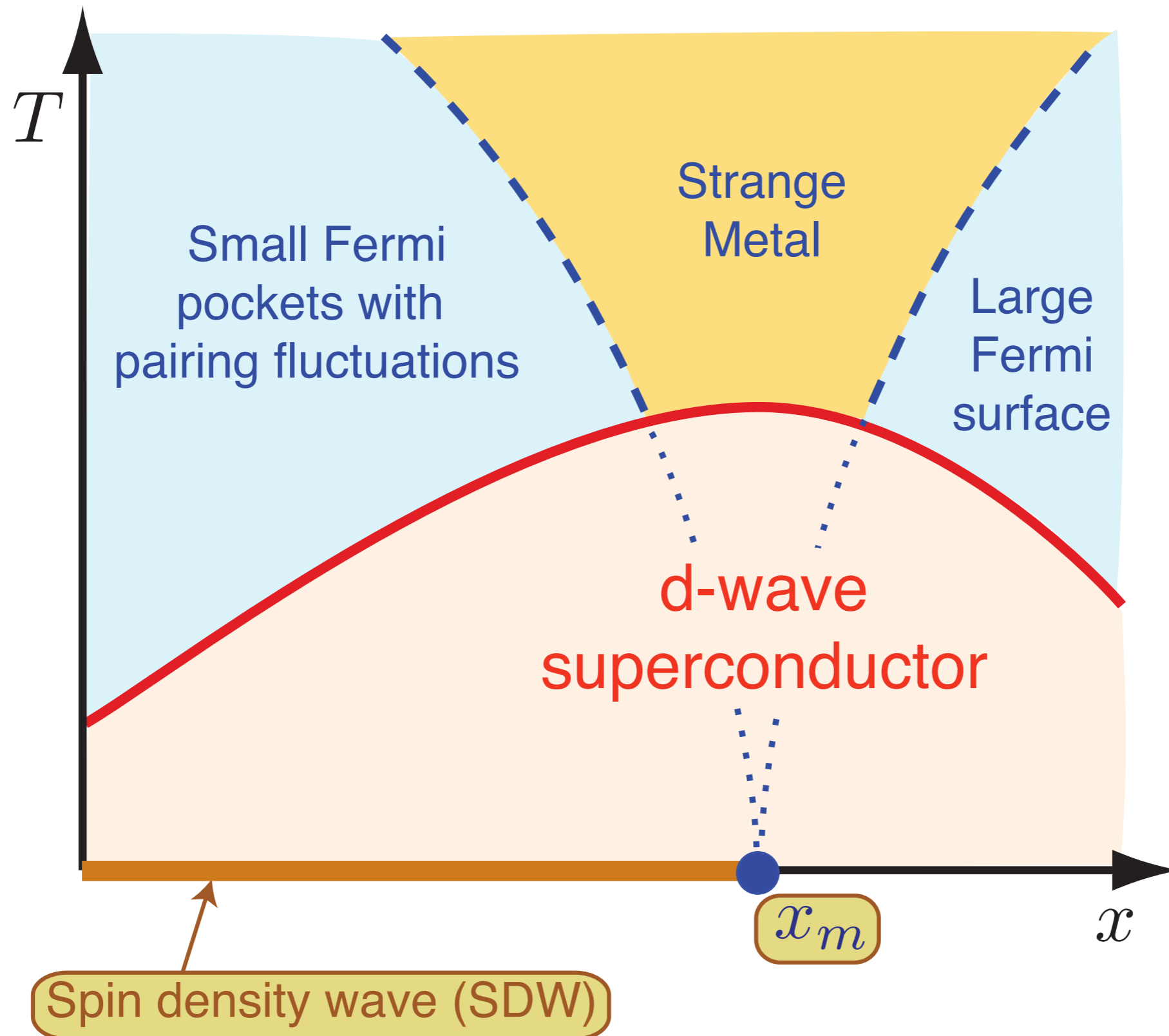
Underlying SDW ordering quantum critical point
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Theory of quantum criticality in the cuprates



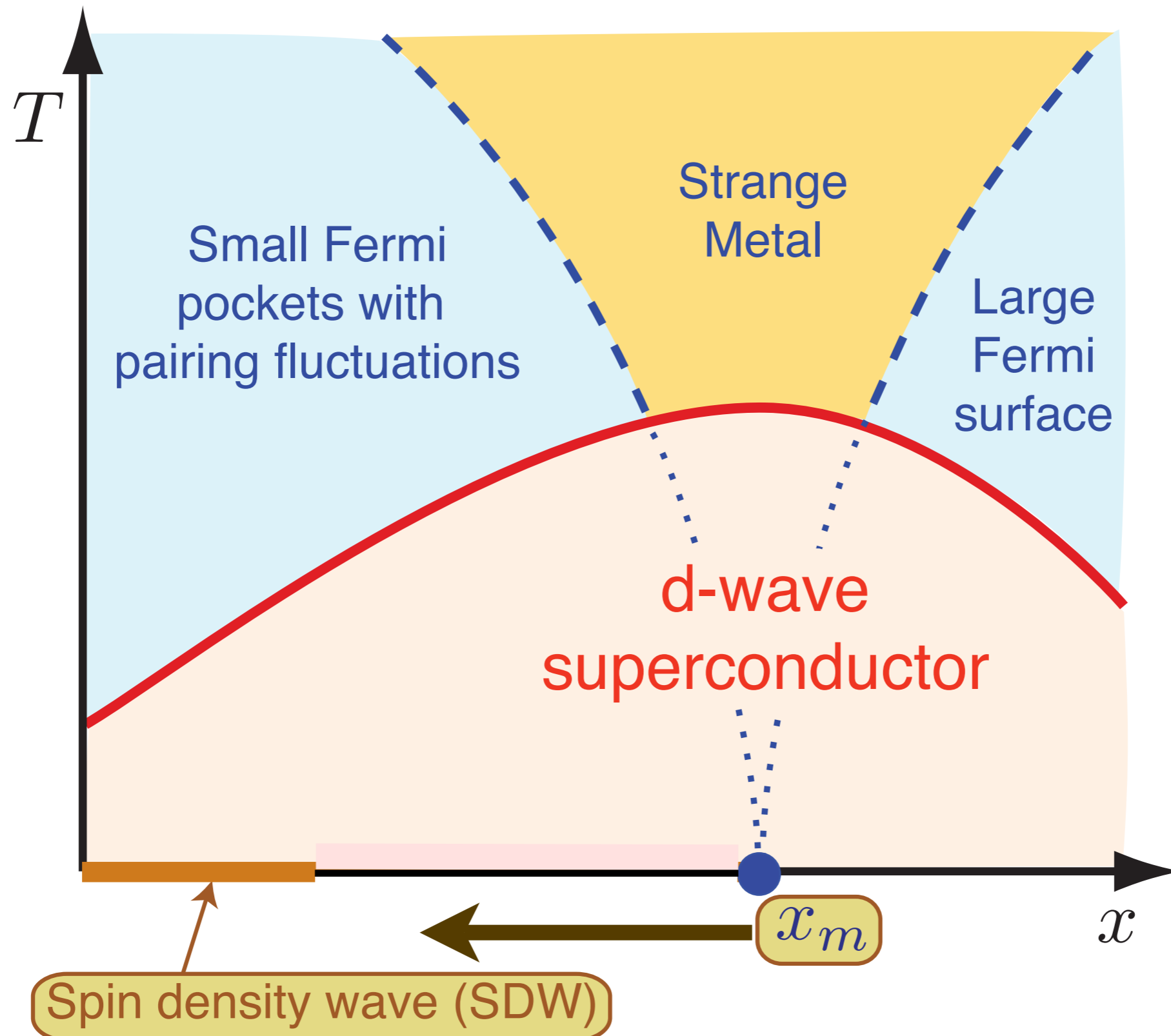
Onset of d -wave superconductivity
hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates



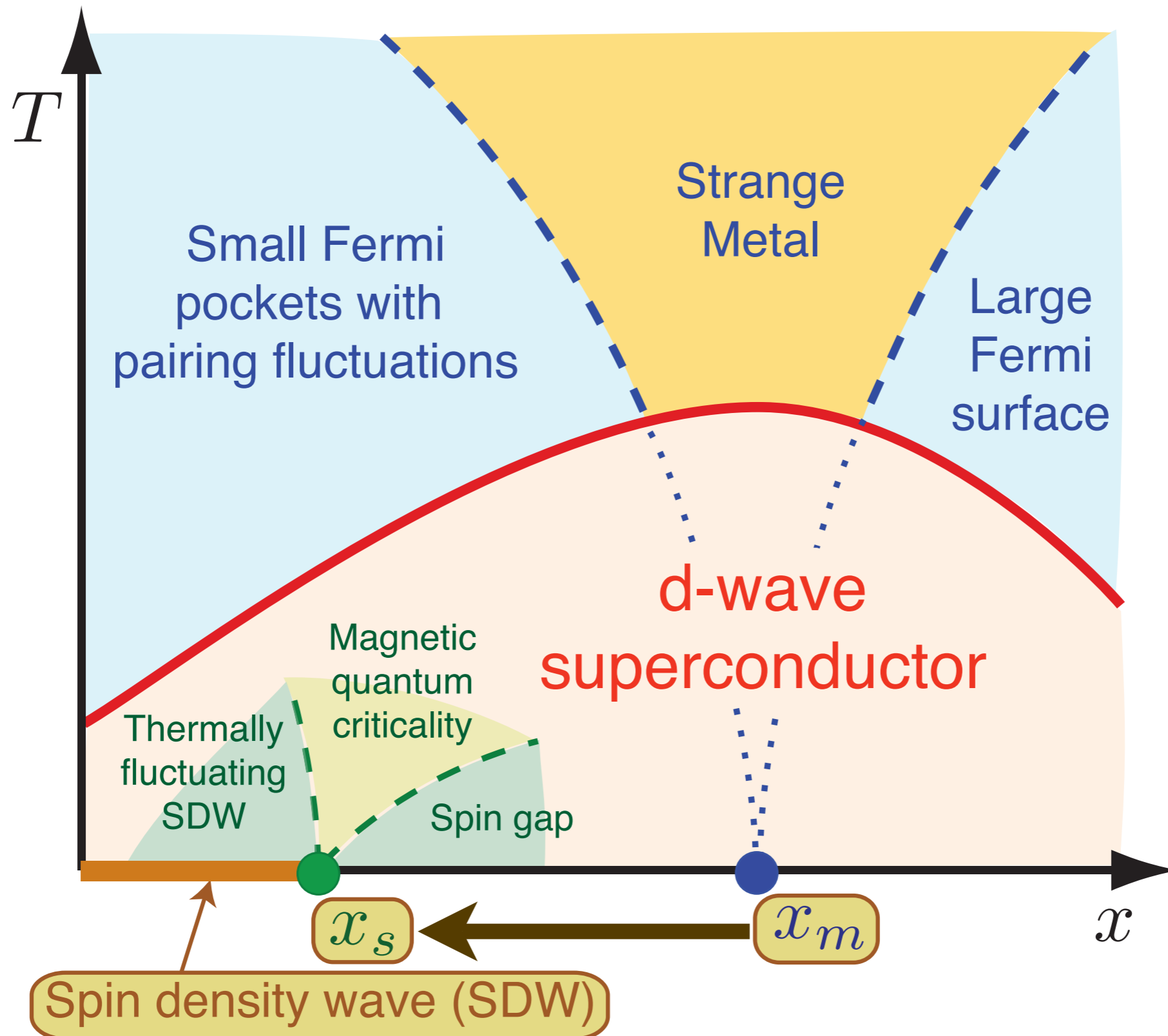
Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



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Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

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1. Phenomenological quantum theory of competition between superconductivity and SDW order
Survey of recent experiments
2. Superconductivity in the overdoped regime
BCS pairing by spin fluctuation exchange
3. Superconductivity in the underdoped regime
 $U(1)$ gauge theory of fluctuating SDW order
4. A unified theory
 $SU(2)$ gauge theory of transition from Fermi pockets to a large Fermi surface

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Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (ψ):

$$\mathcal{S} = \int d^2r d\tau \left[\frac{1}{2} (\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2} (\nabla_x \vec{\varphi})^2 + \frac{r}{2} \vec{\varphi}^2 + \frac{u}{4} (\vec{\varphi}^2)^2 + \kappa \vec{\varphi}^2 |\psi|^2 \right] + \int d^2r \left[|(\nabla_x - i(2e/\hbar c)\mathcal{A})\psi|^2 - |\psi|^2 + \frac{|\psi|^4}{2} \right]$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

See also E. Demler, W. Hanke, and S.-C. Zhang, *Rev. Mod. Phys.* **76**, 909 (2004);

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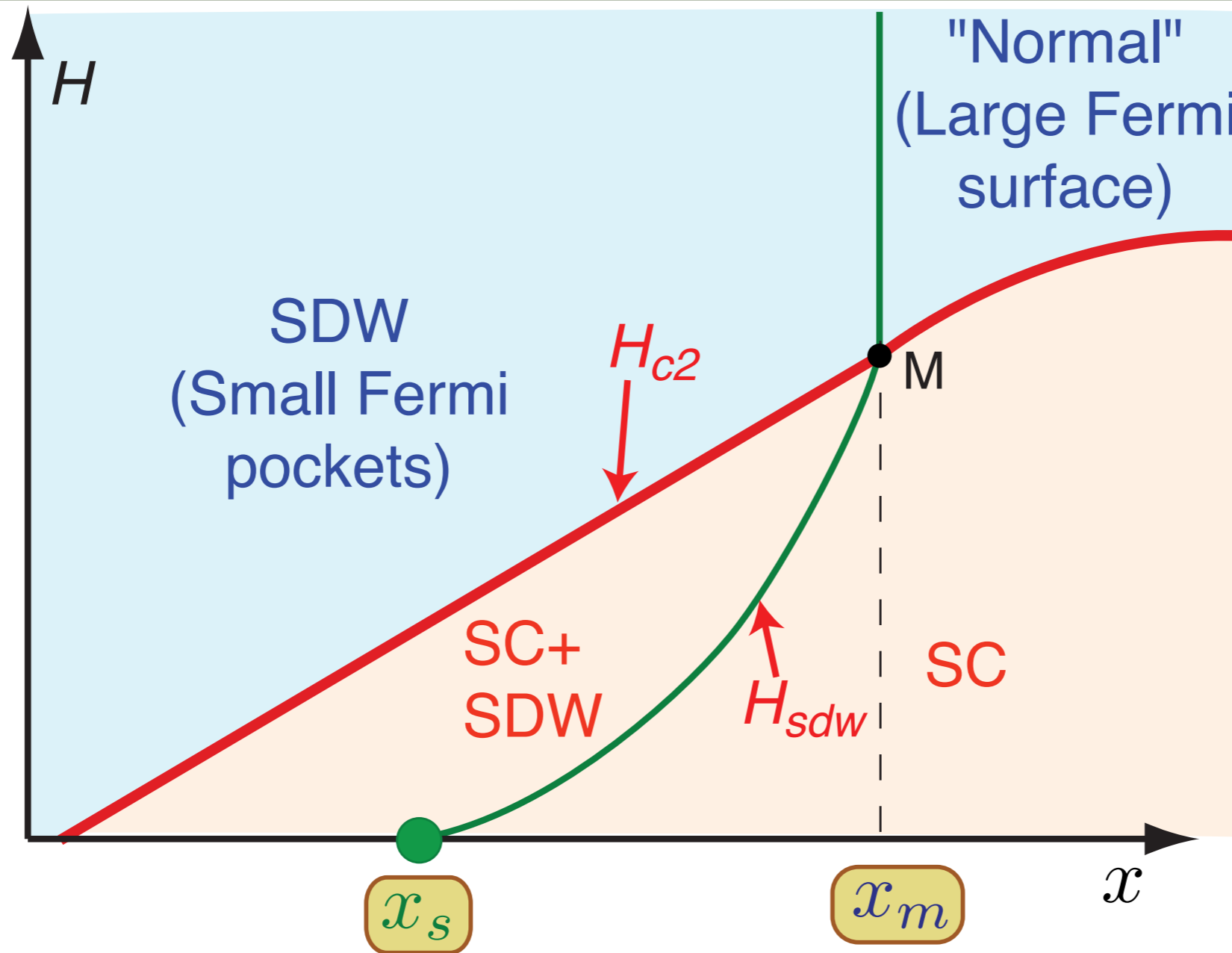
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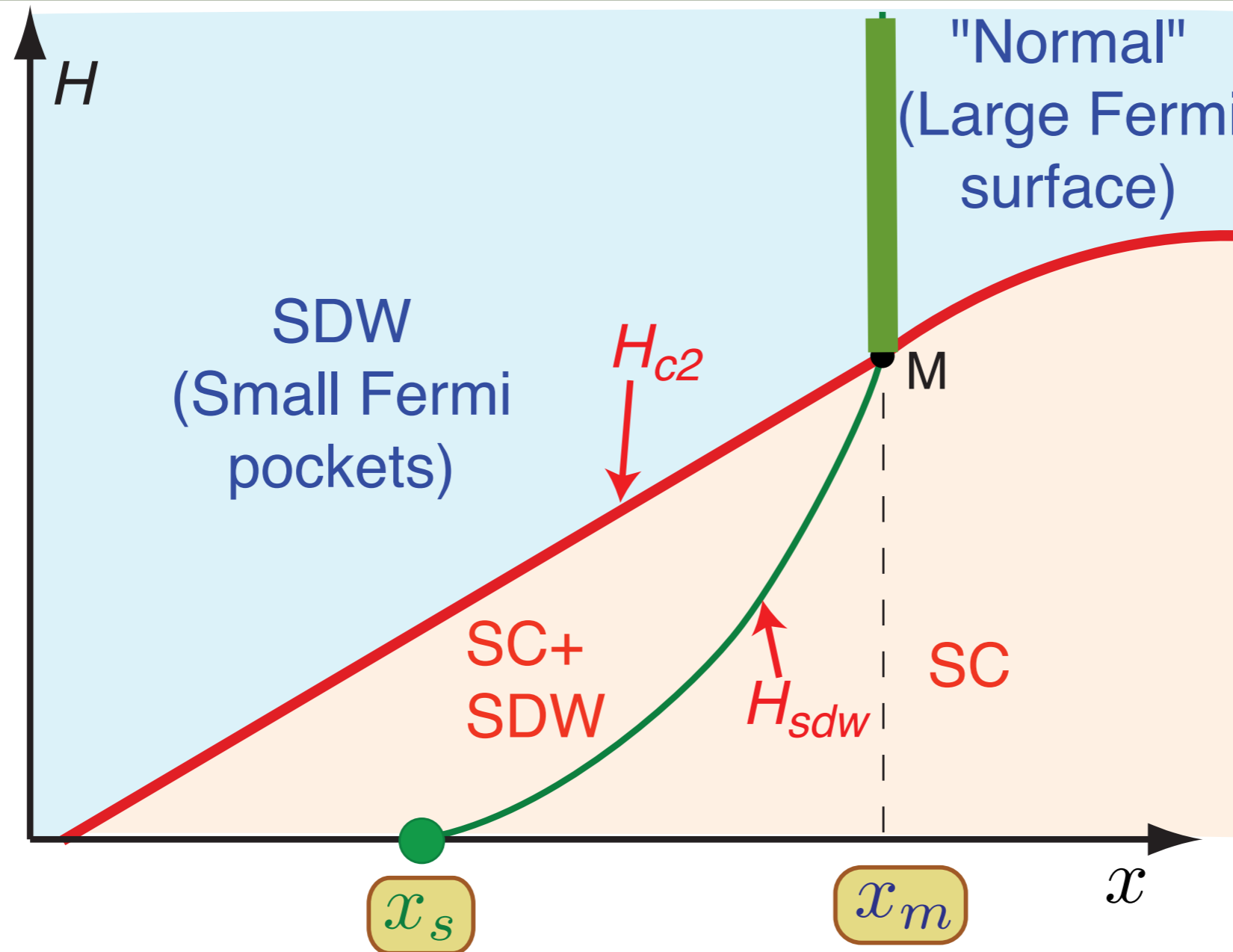
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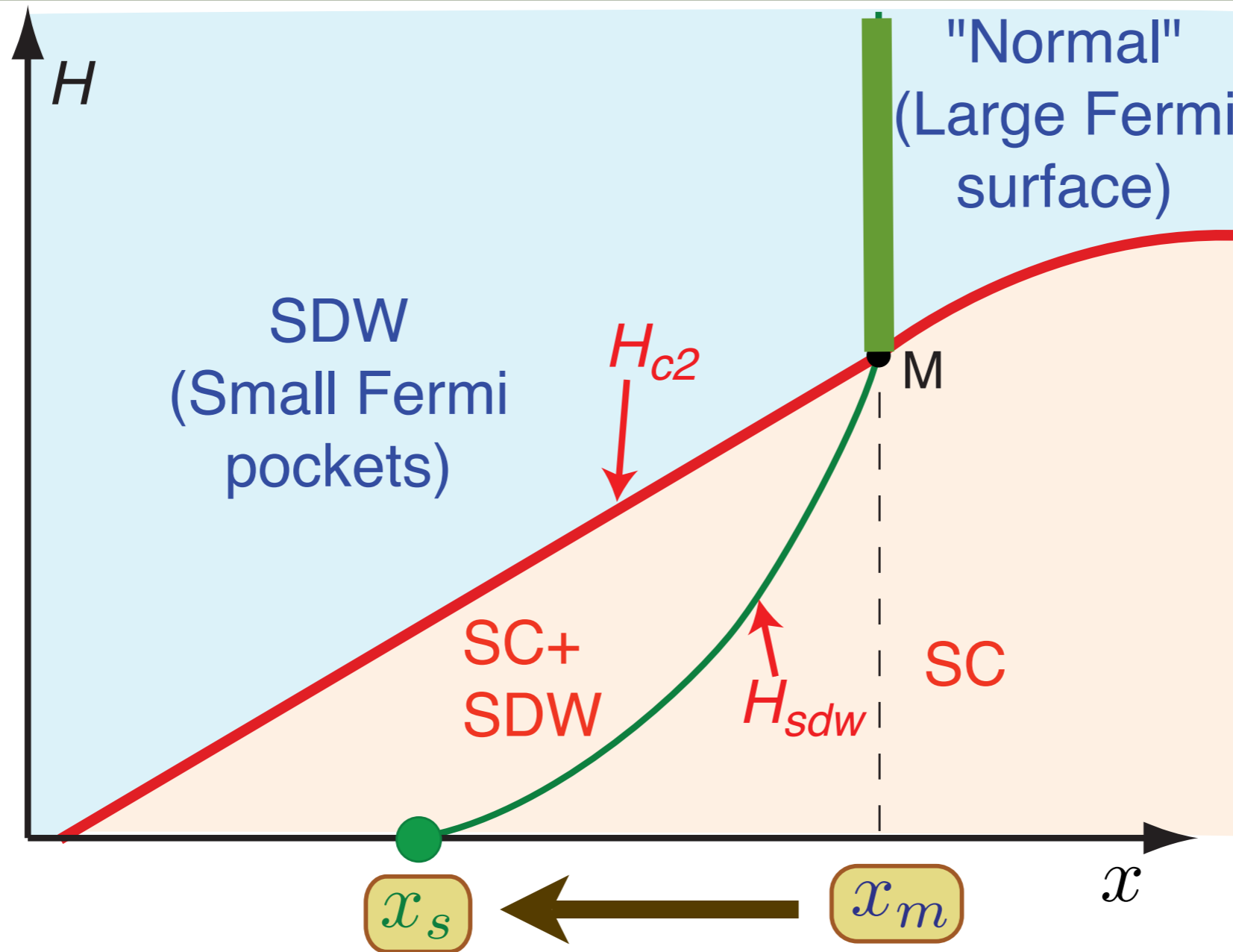


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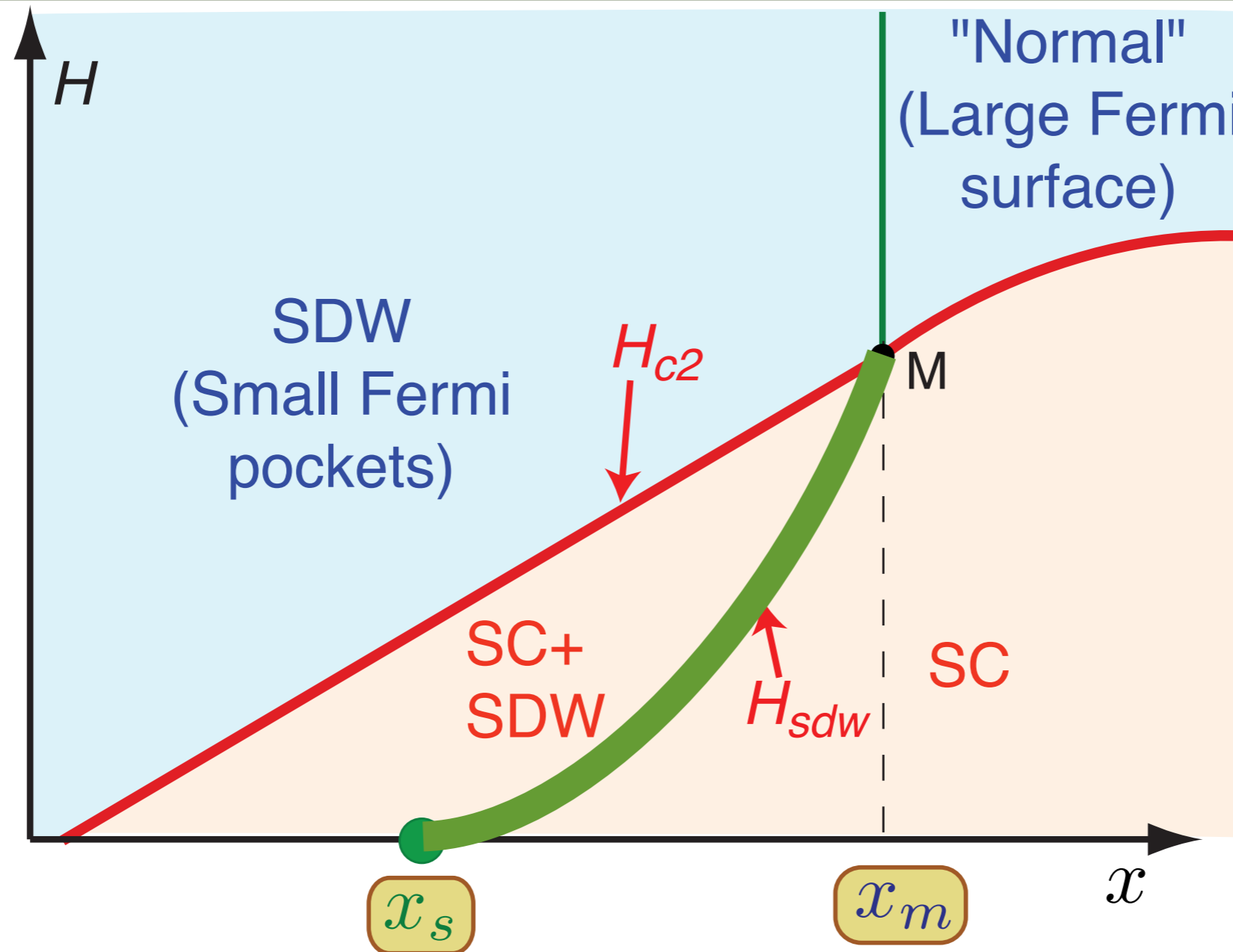
- SDW order is more stable in the metal than in the superconductor: $x_m > x_s$.

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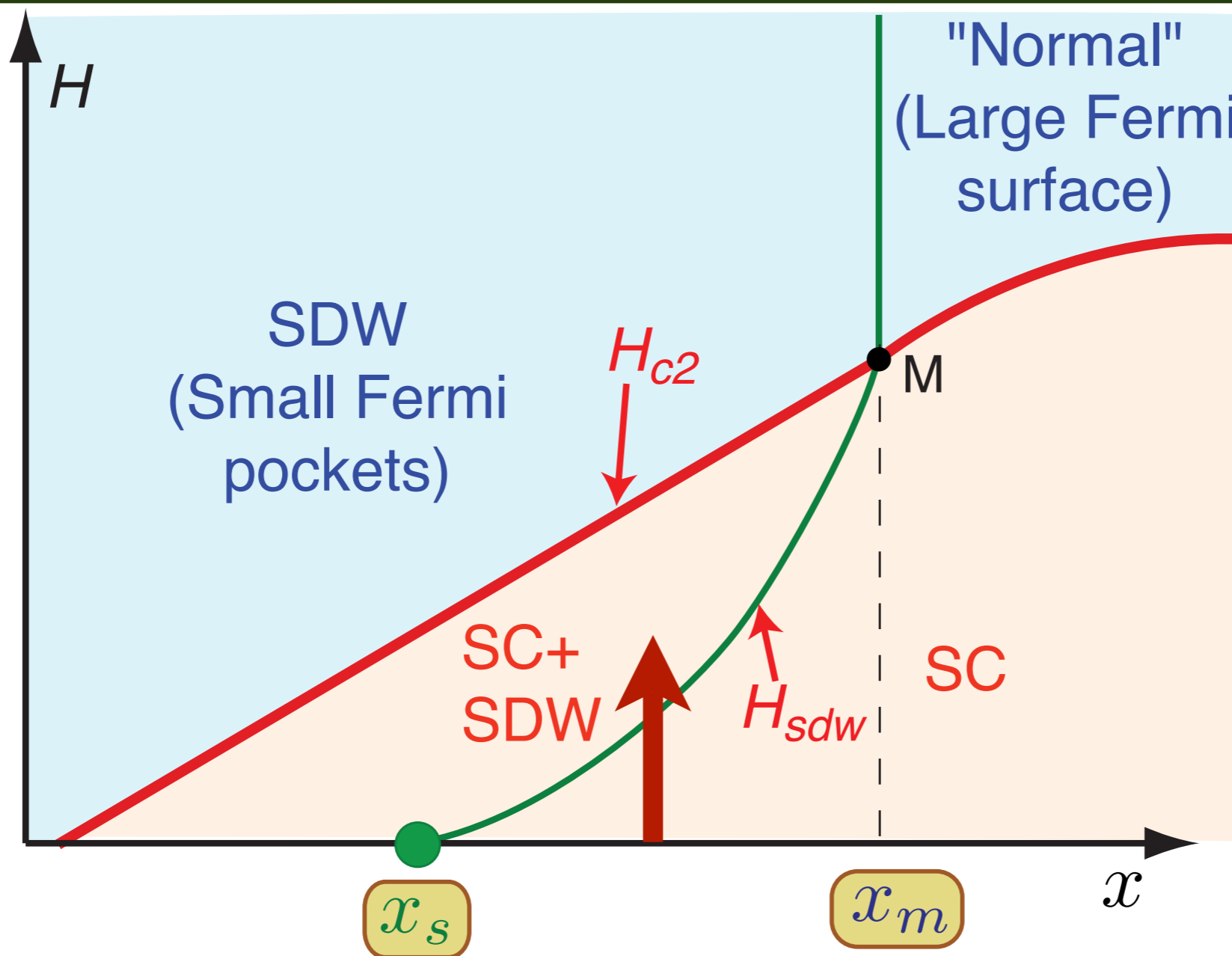
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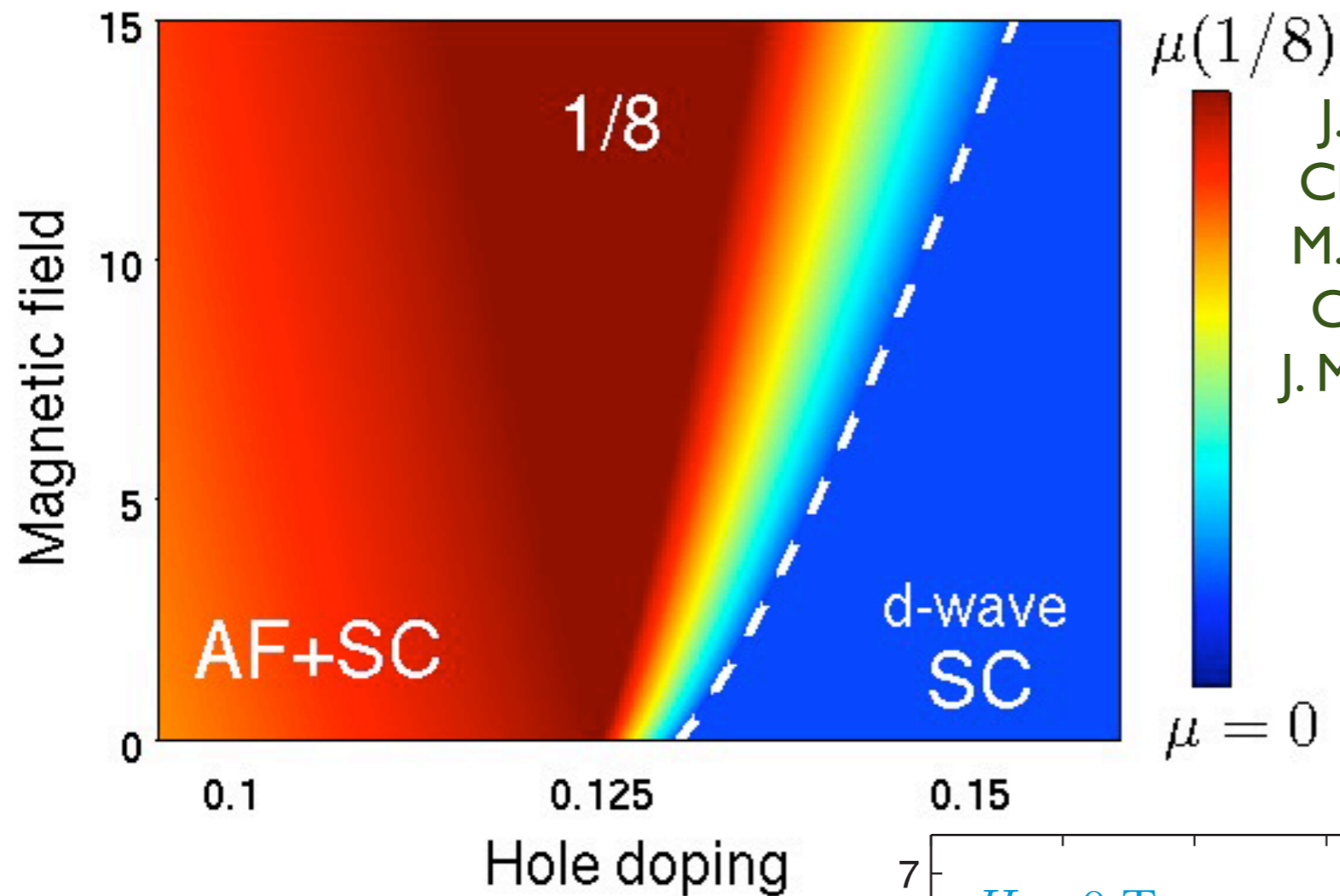


- For doping with $x_s < x < x_m$, SDW order appears at a quantum phase transition at $H = H_{sdw} > 0$.

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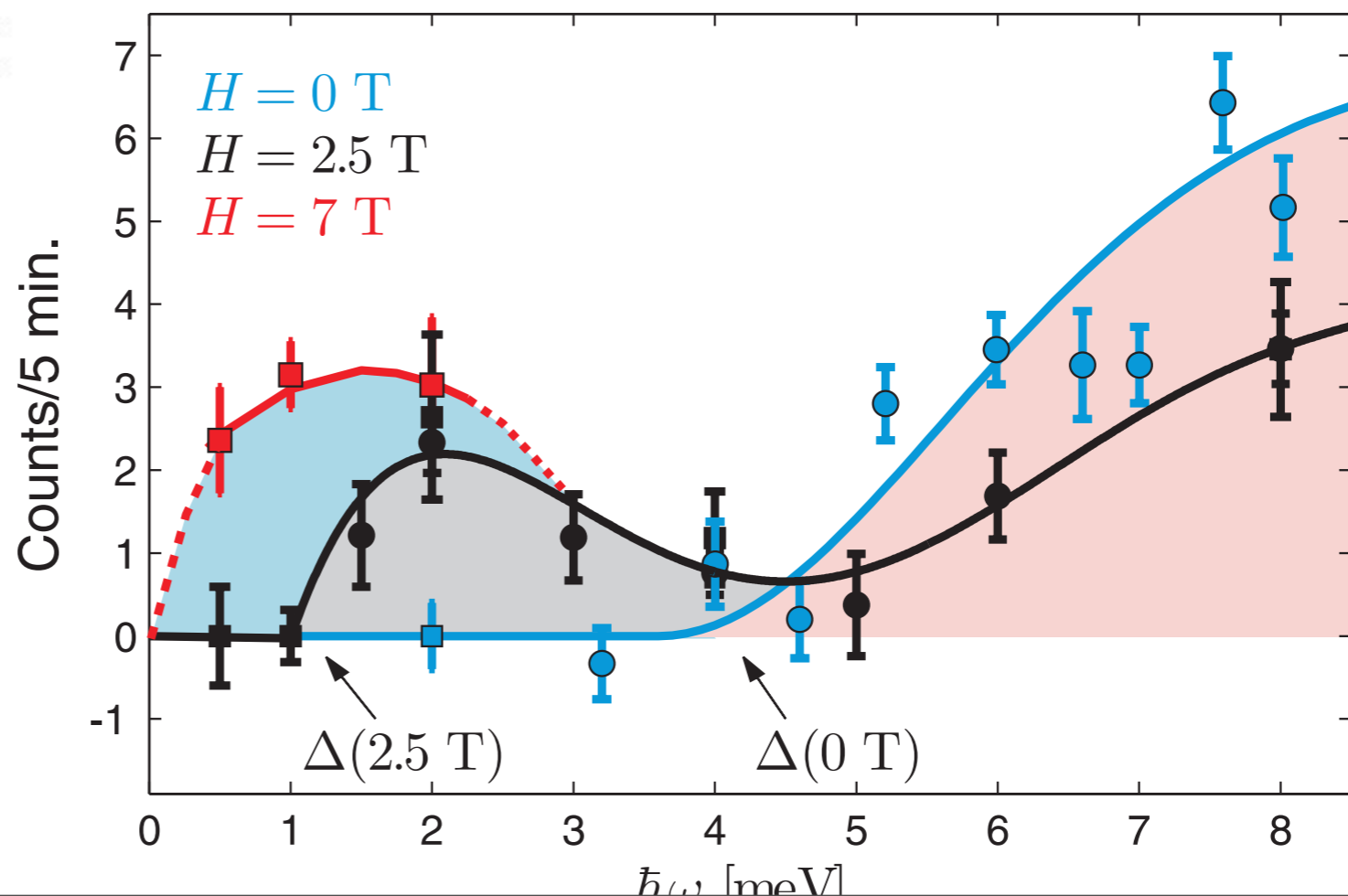


Neutron scattering on $\text{La}_{1.855}\text{Sr}_{0.145}\text{CuO}_4$
J. Chang et al., *Phys. Rev. Lett.* **102**, 177006 (2009).

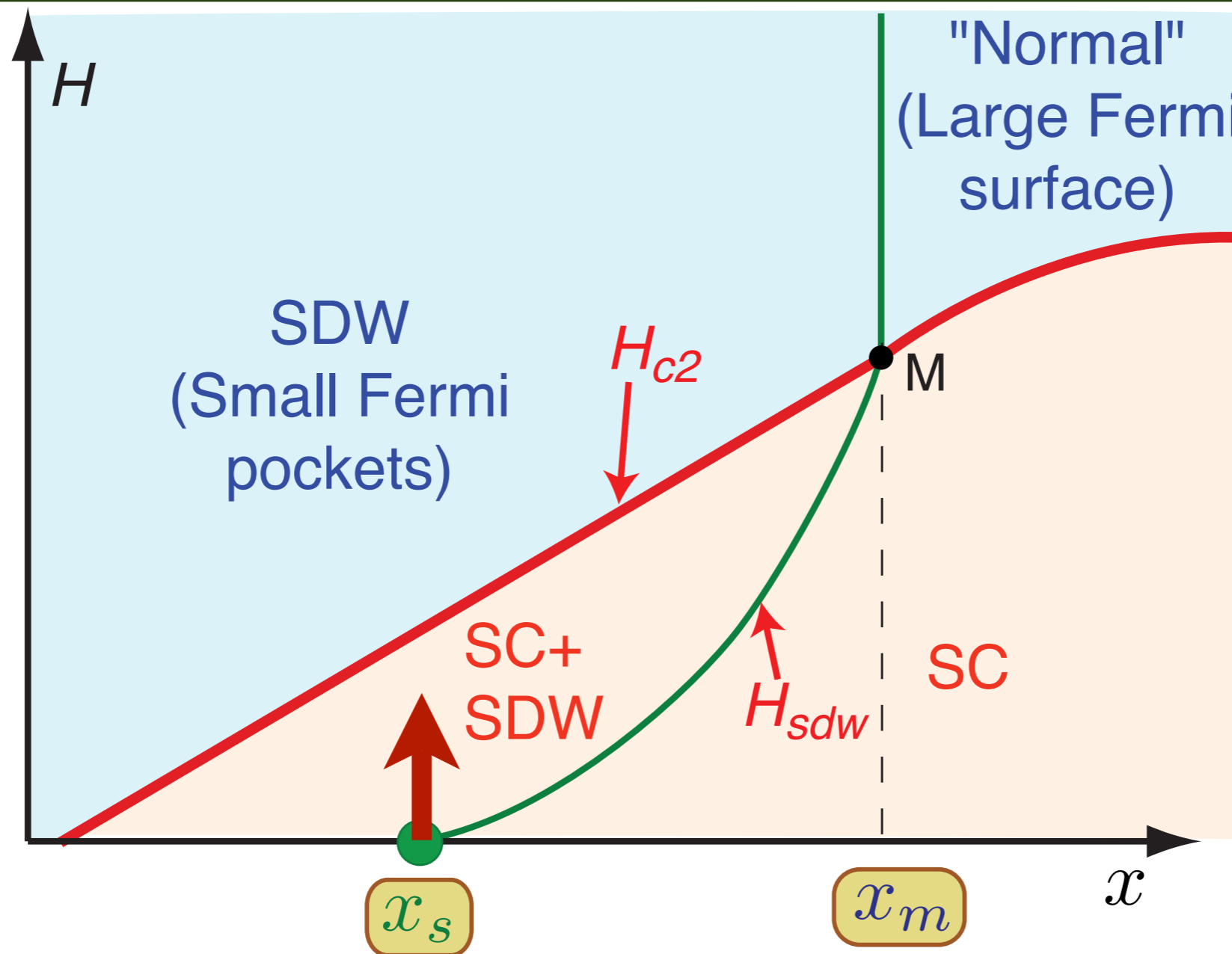


J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).

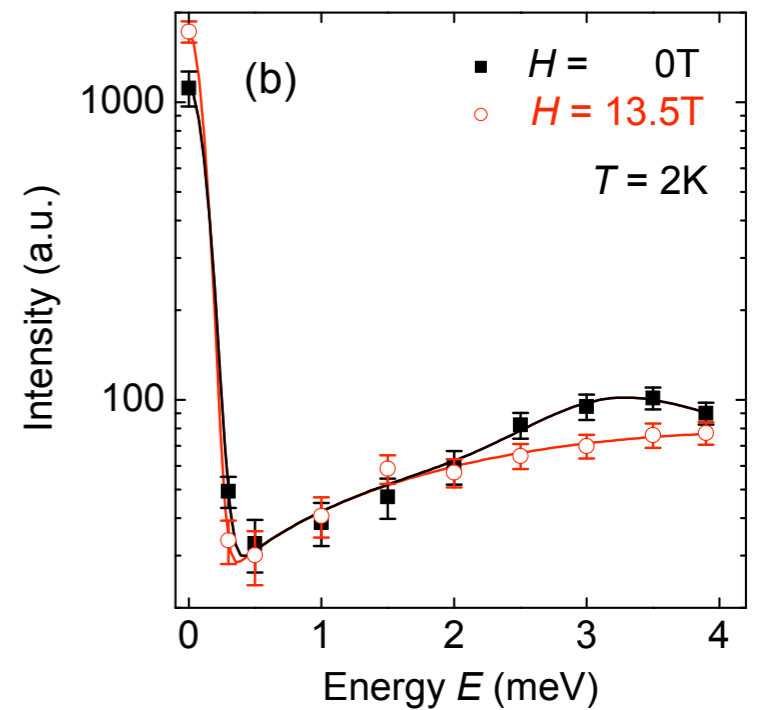
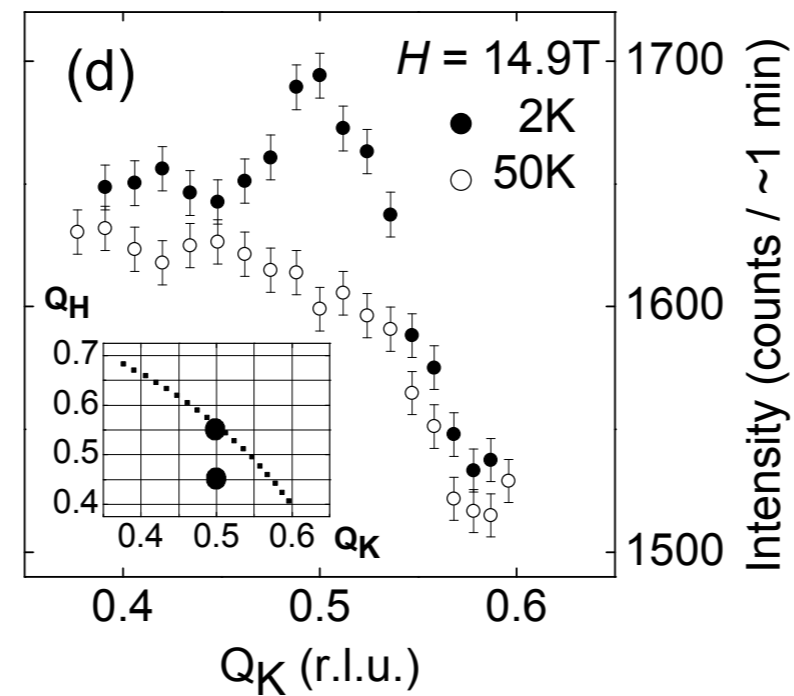
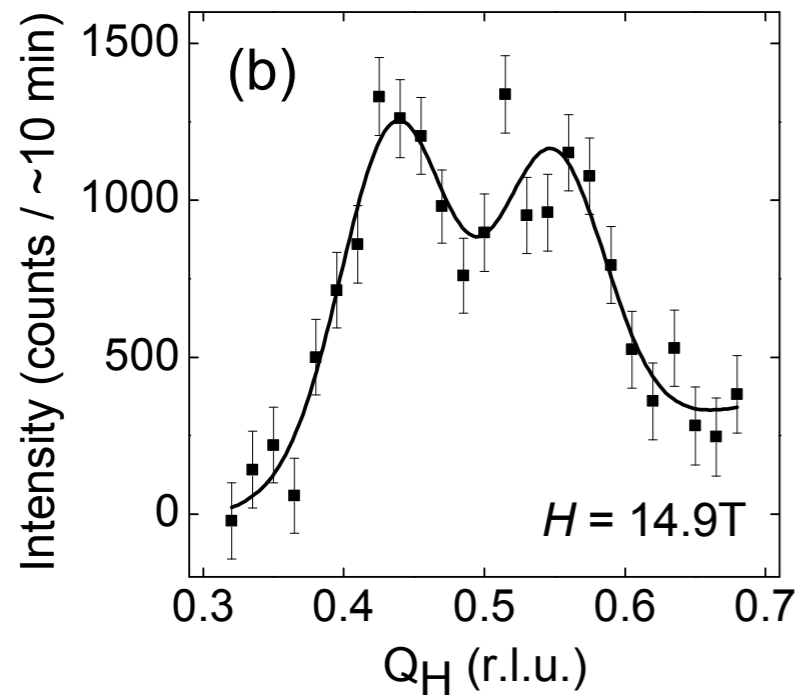
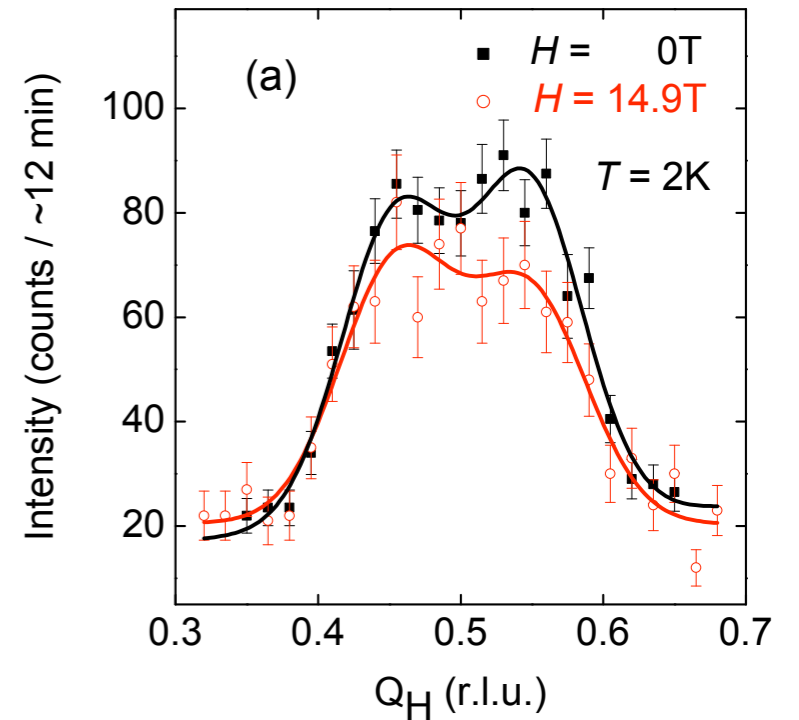
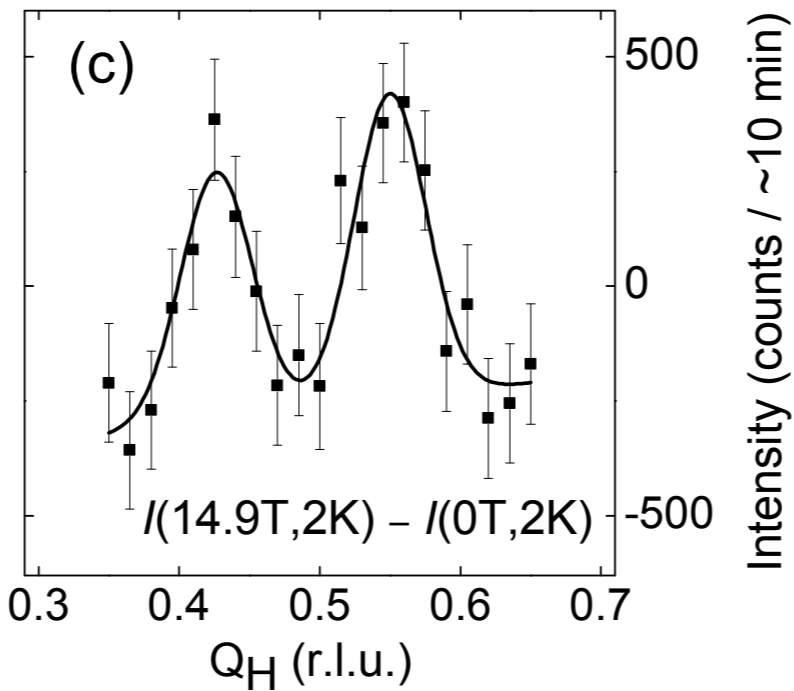
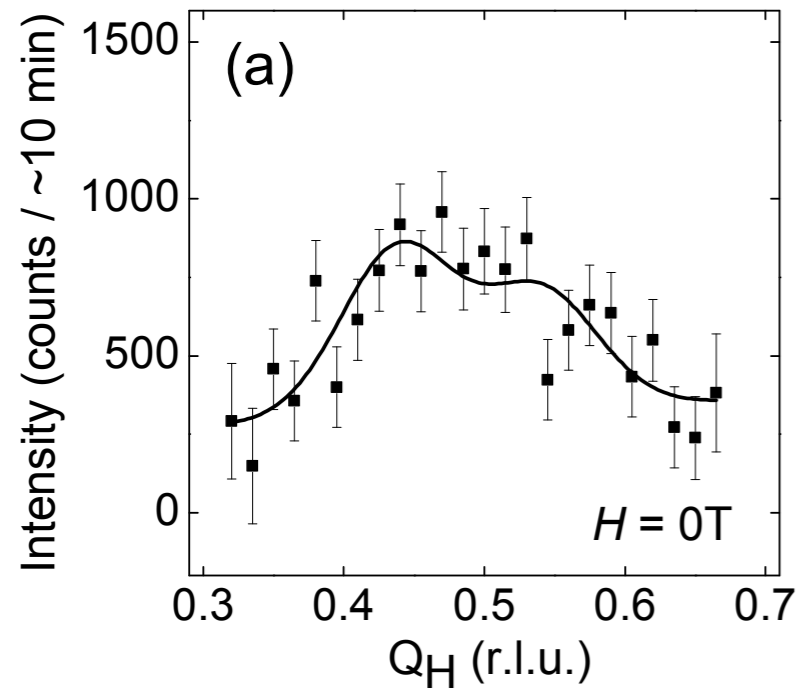
J. Chang, N. B. Christensen, Ch. Niedermayer, K. Lefmann, H. M. Roennow, D. F. McMorrow, A. Schneidewind, P. Link, A. Hiess, M. Boehm, R. Mottl, S. Pailhes, N. Momono, M. Oda, M. Ido, and J. Mesot, *Phys. Rev. Lett.* **102**, 177006 (2009).



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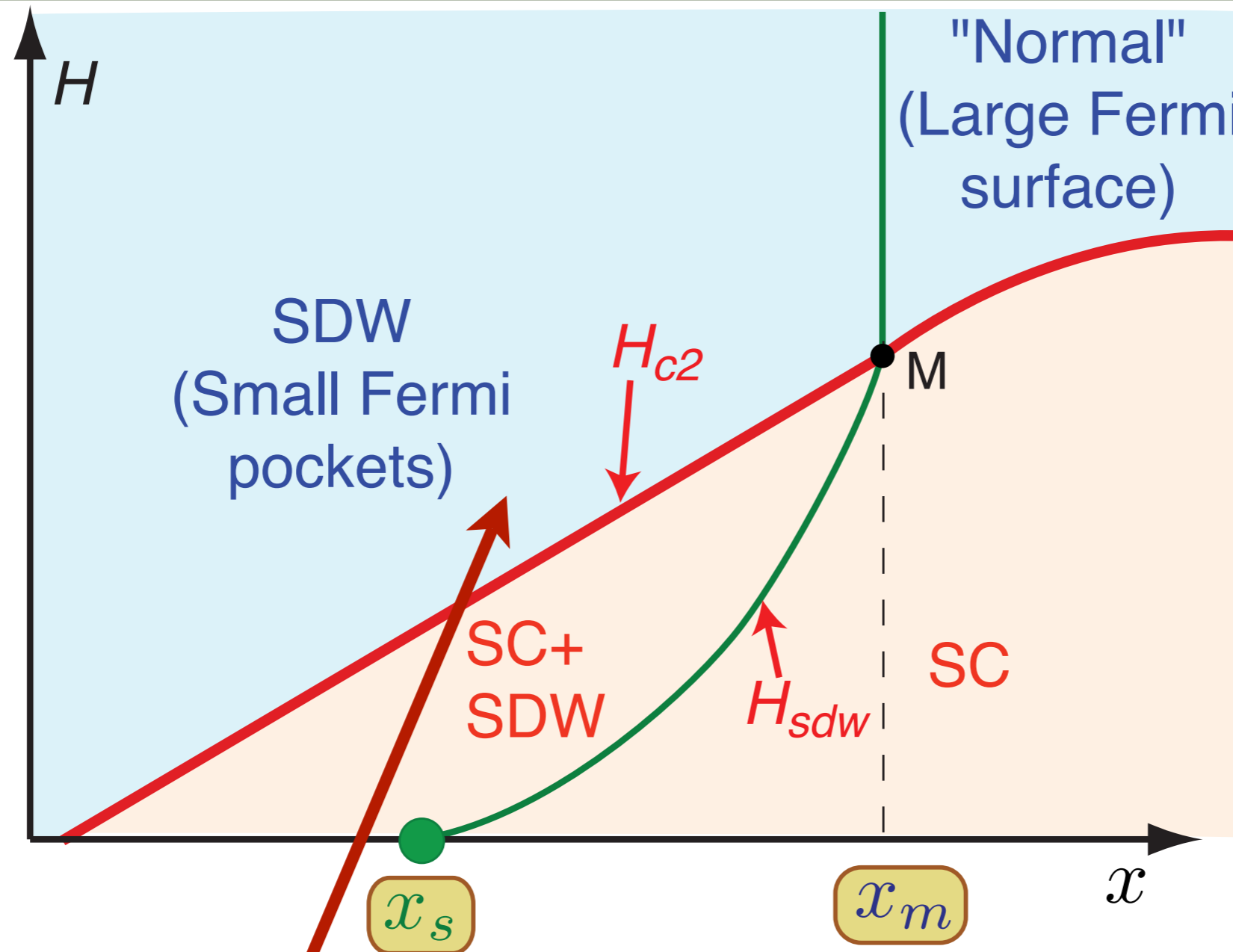


Neutron scattering on $\text{YBa}_2\text{Cu}_3\text{O}_{6.45}$
D. Haug *et al.*, *Phys. Rev. Lett.* **103**, 017001 (2009).



D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *Phys. Rev. Lett.* **103**, 017001 (2009)

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order



Quantum oscillations without Zeeman splitting

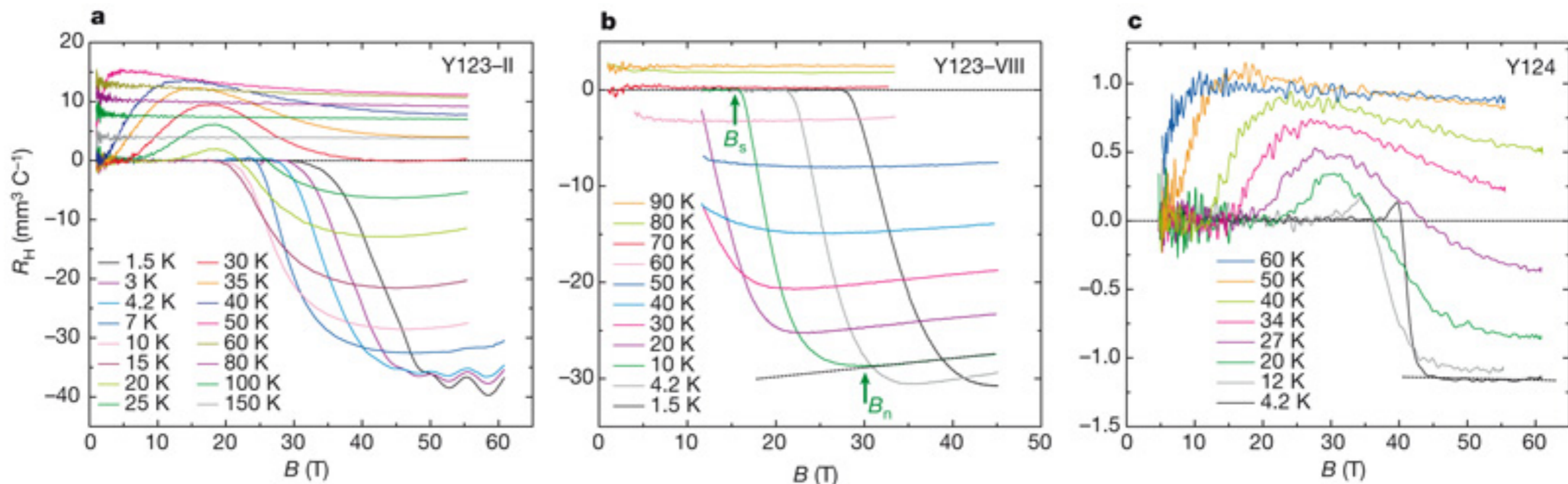
N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaïson, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007)

Quantum oscillations

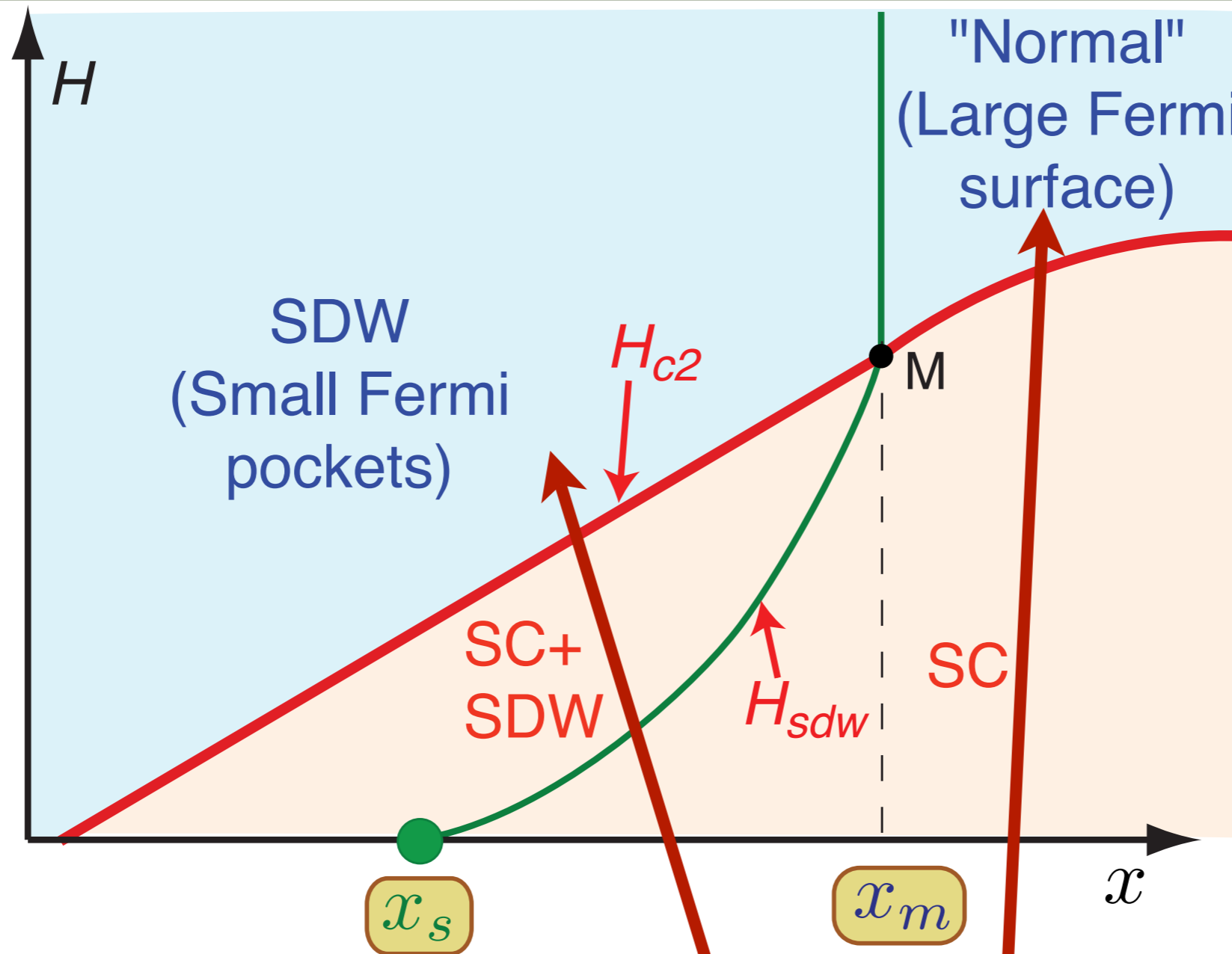
Electron pockets in the Fermi surface of hole-doped high- T_c superconductors

David LeBoeuf¹, Nicolas Doiron-Leyraud¹, Julien Levallois², R. Daou¹, J.-B. Bonnemaïson¹, N. E. Hussey³, L. Balicas⁴, B. J. Ramshaw⁵, Ruixing Liang^{5,6}, D. A. Bonn^{5,6}, W. N. Hardy^{5,6}, S. Adachi⁷, Cyril Proust² & Louis Taillefer^{1,6}

Nature **450**, 533 (2007)

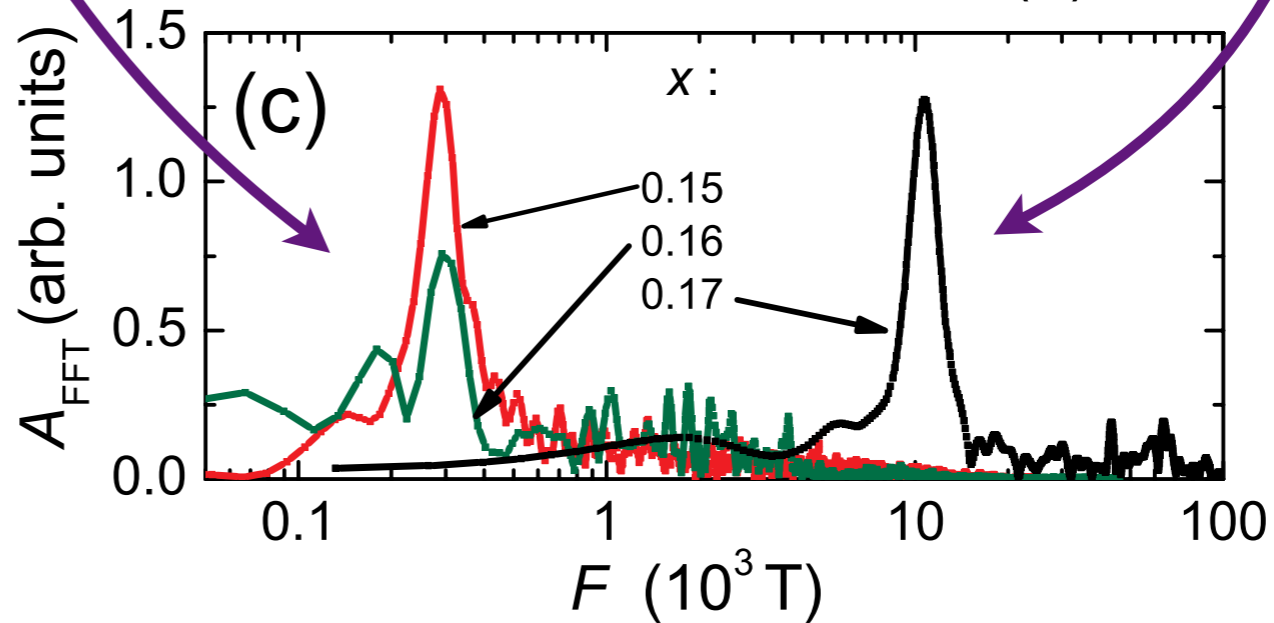
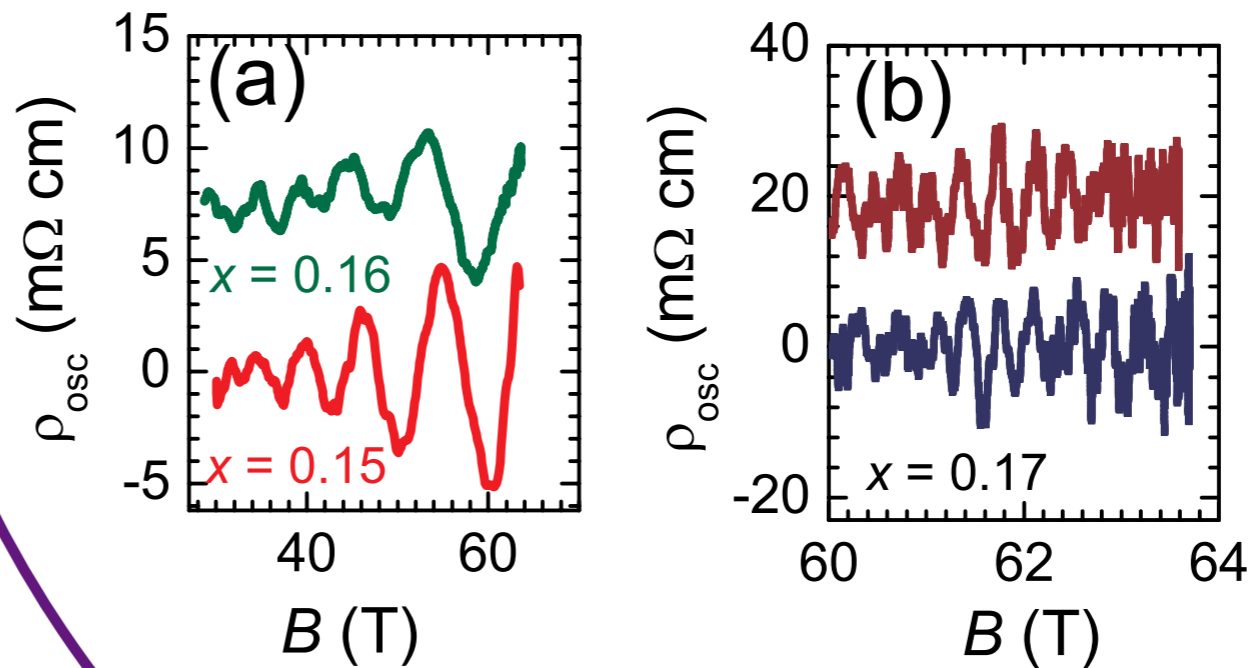
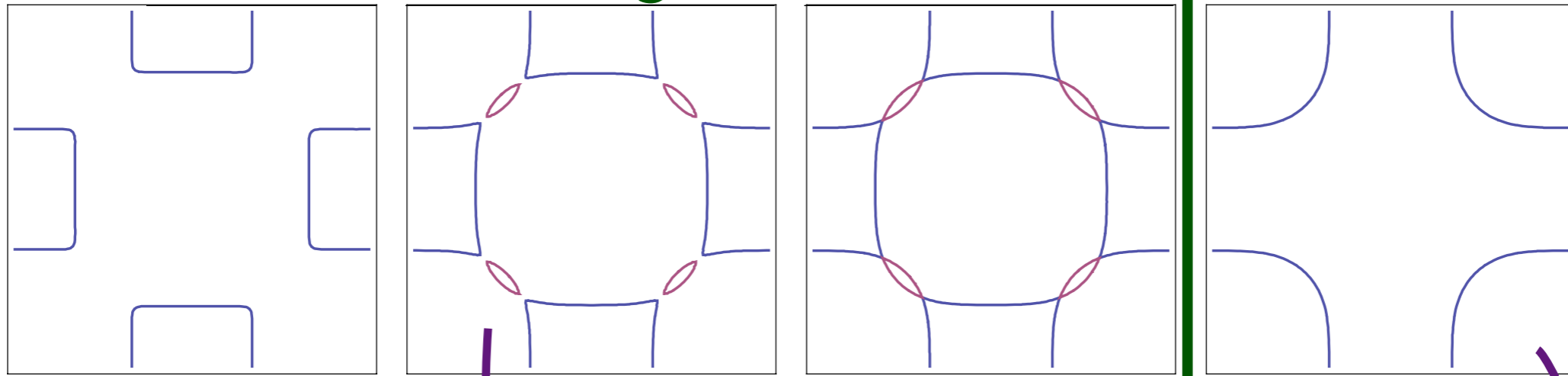


Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order



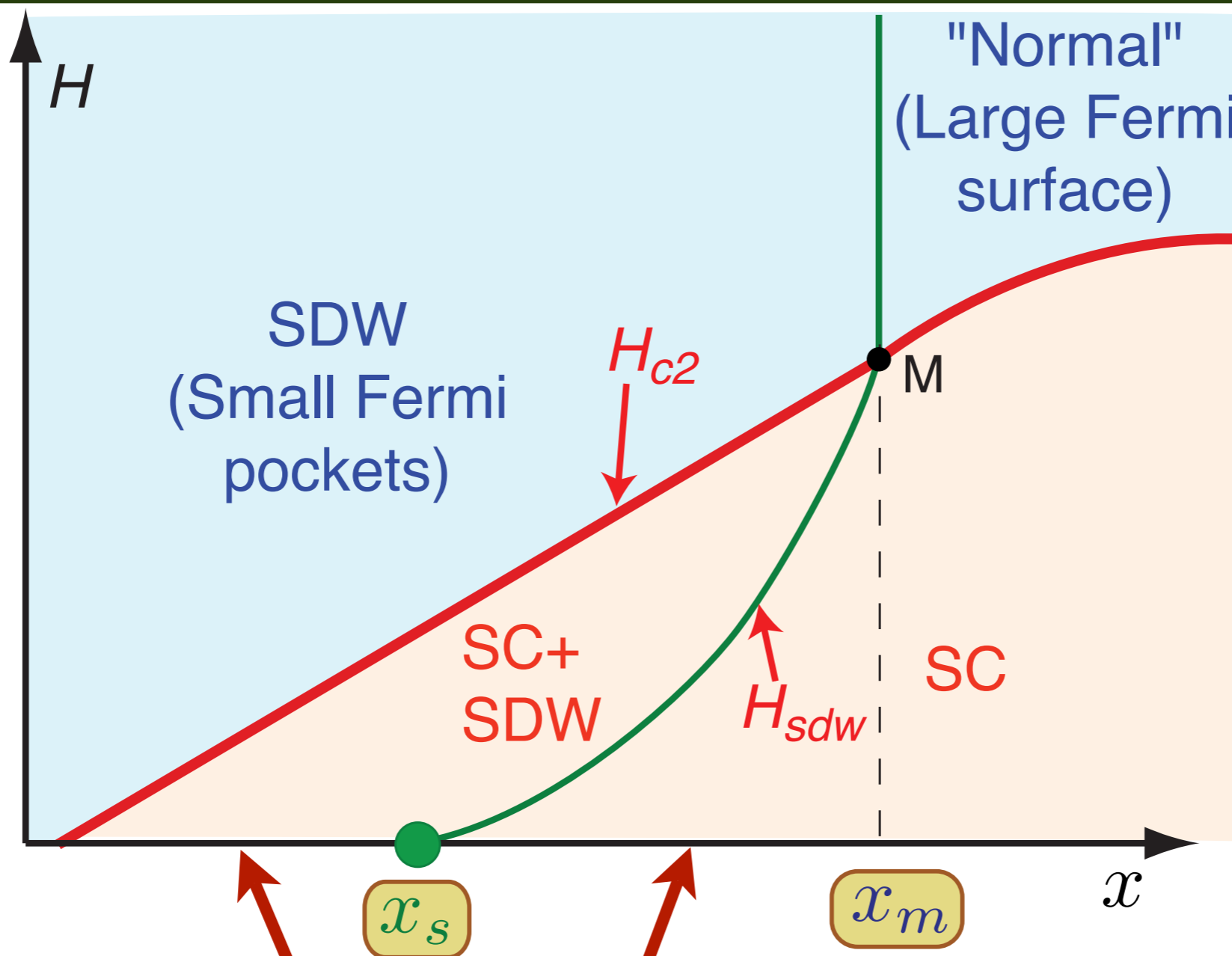
Change in frequency of quantum oscillations in electron-doped materials identifies $x_m = 0.165$

← Increasing SDW order →

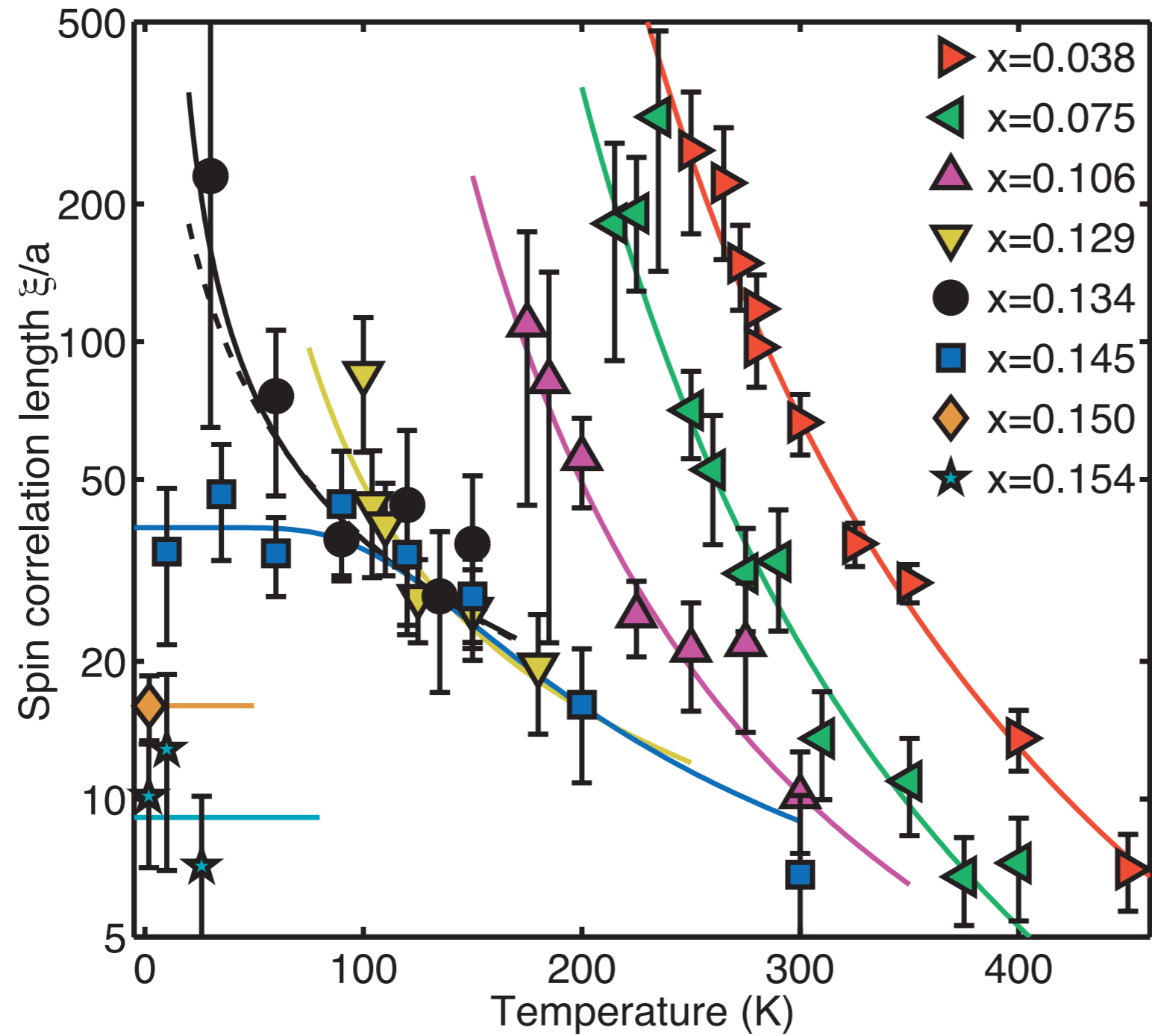


T. Helm, M.V. Kartsovni,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
R. Gross, arXiv:0906.1431

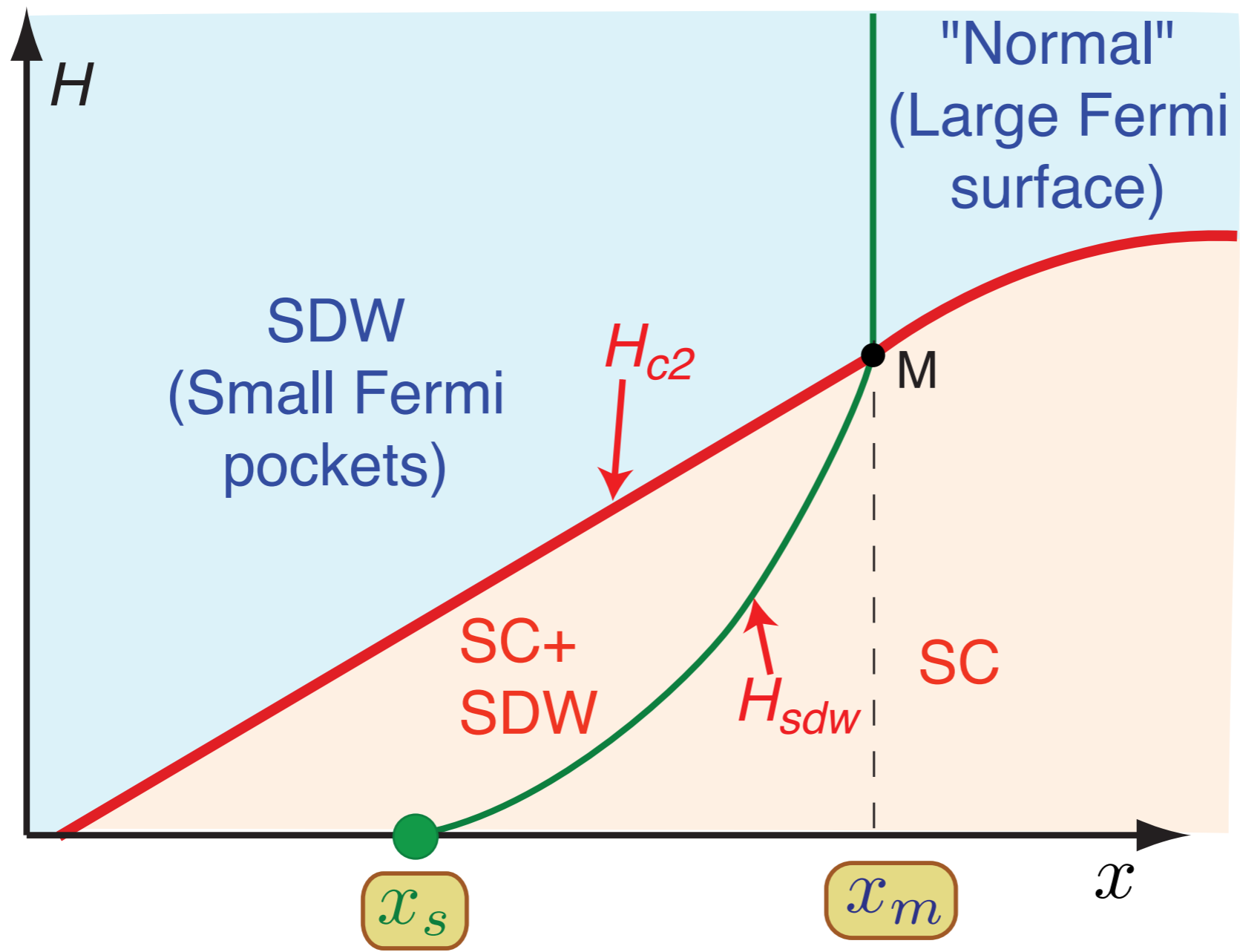
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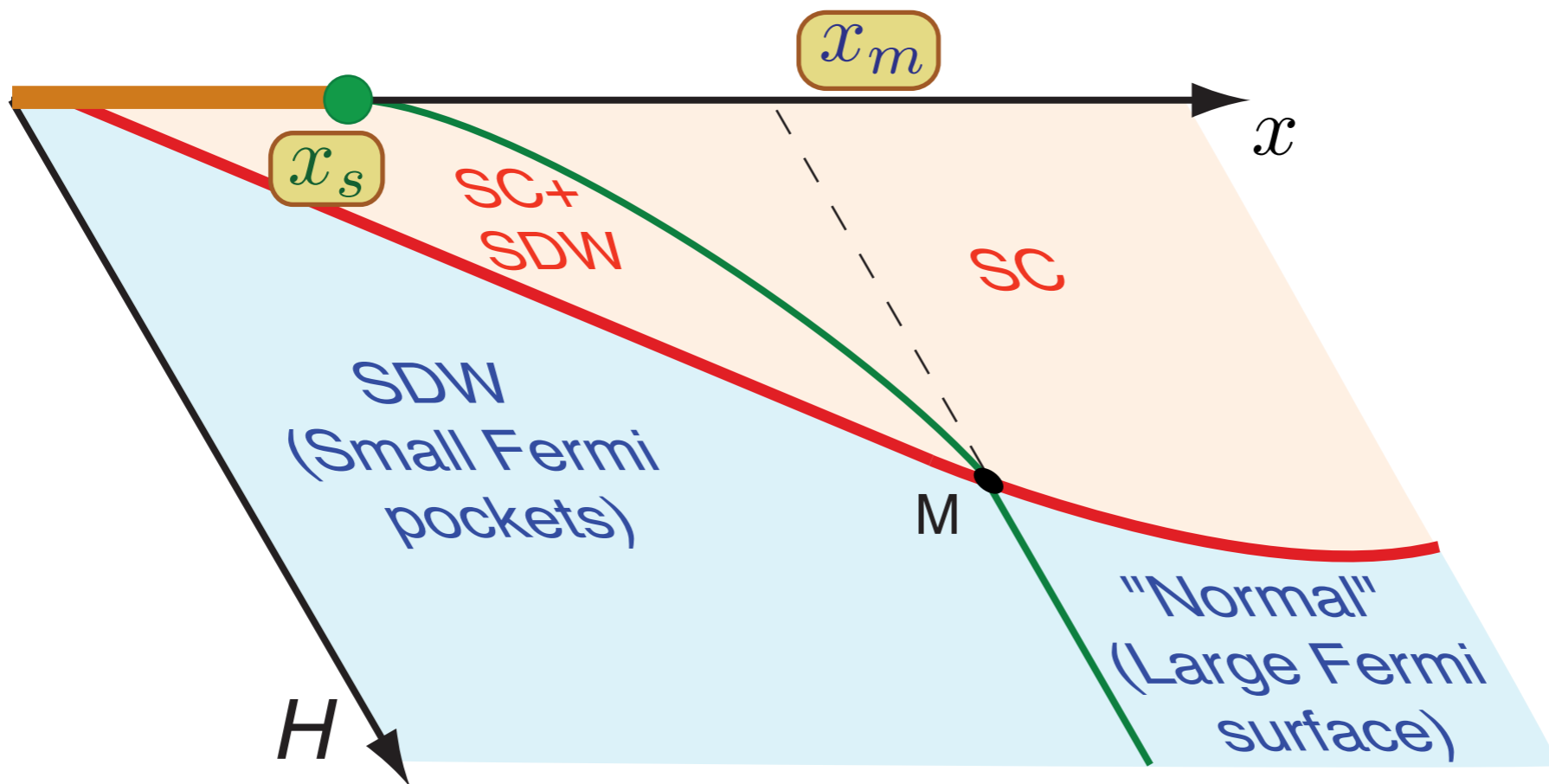


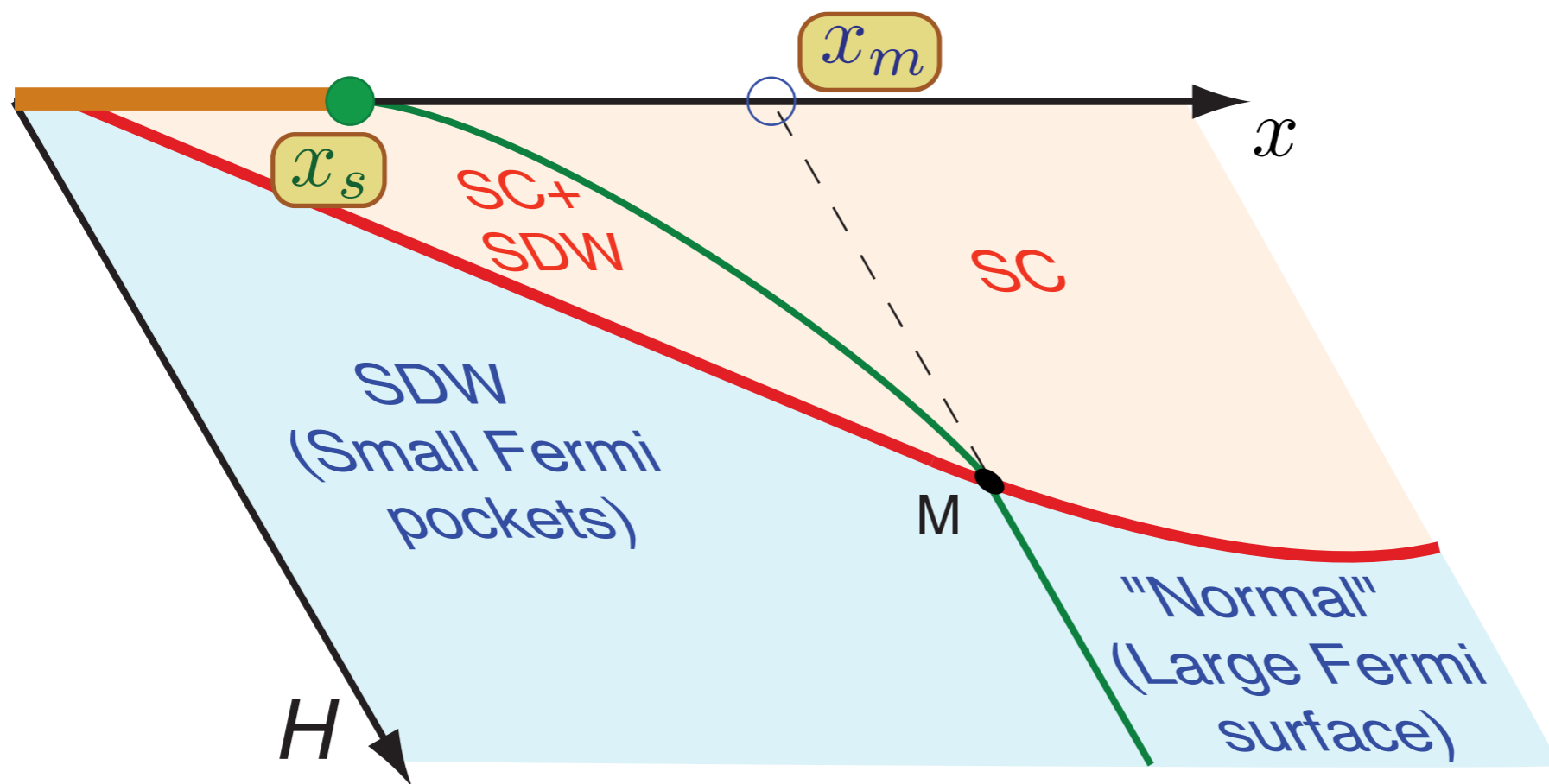
Neutron scattering at $H=0$ in **same** material identifies $x_s = 0.14 < x_m$

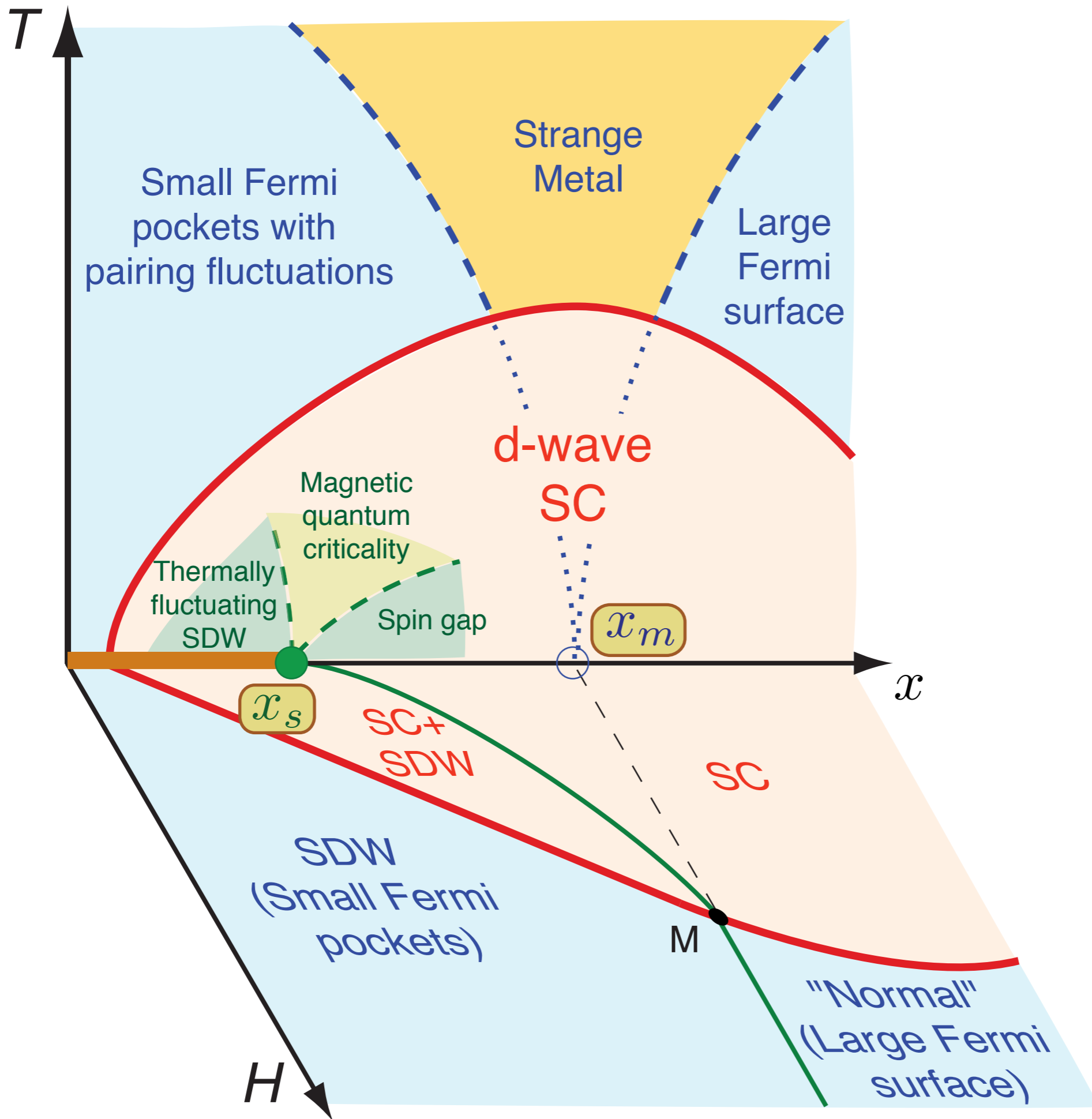


E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven,
Nature **445**, 186 (2007).









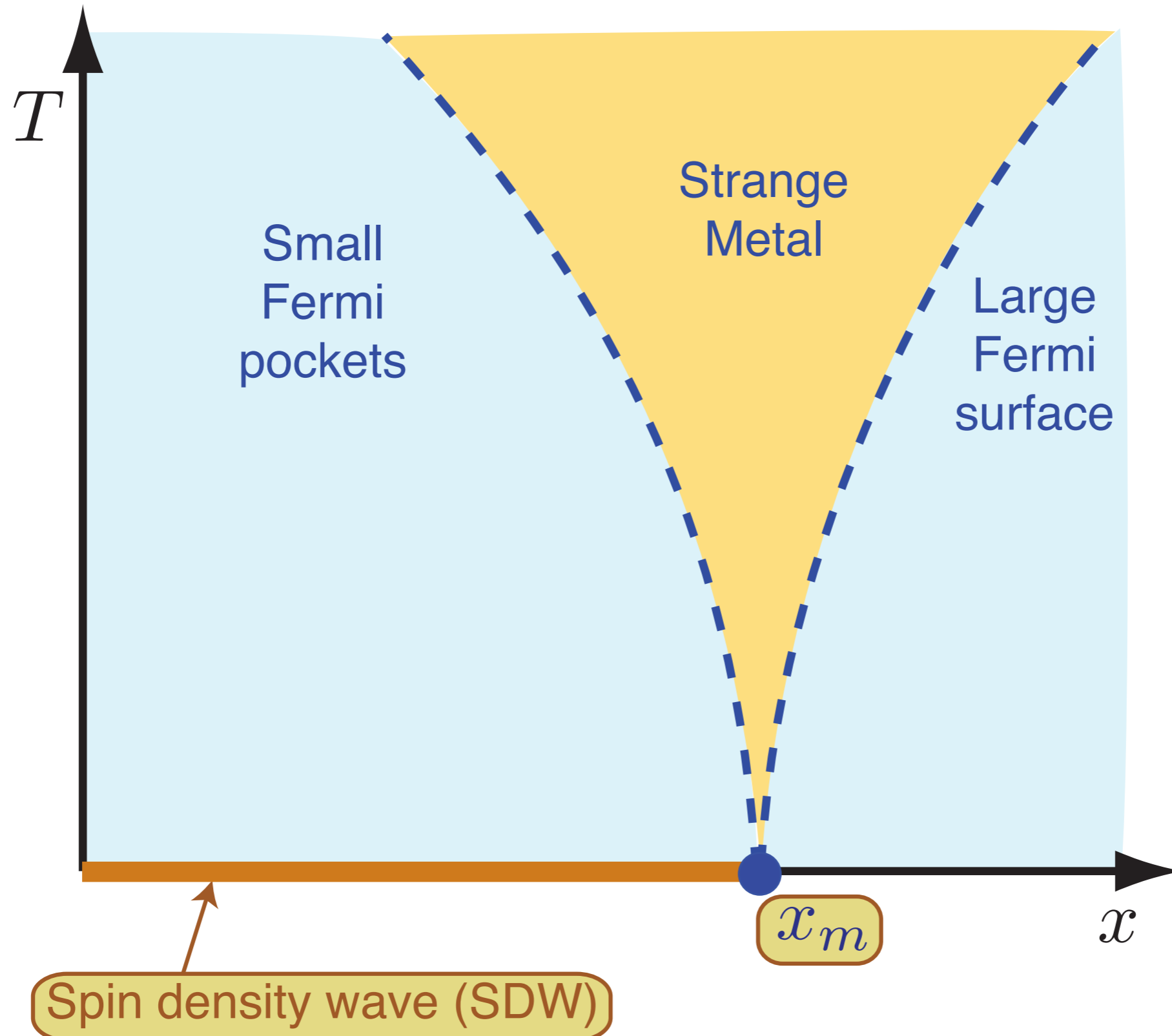
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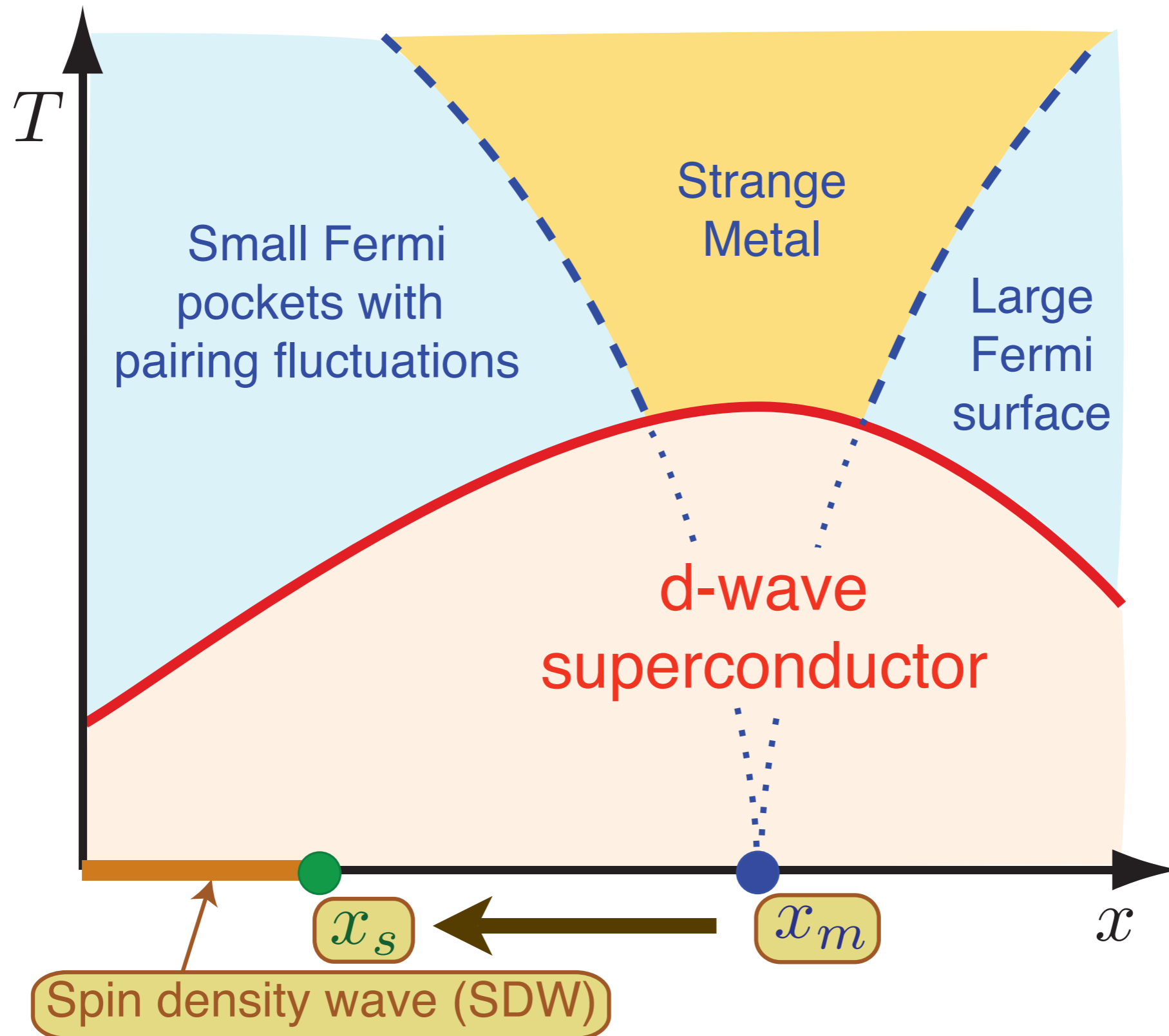
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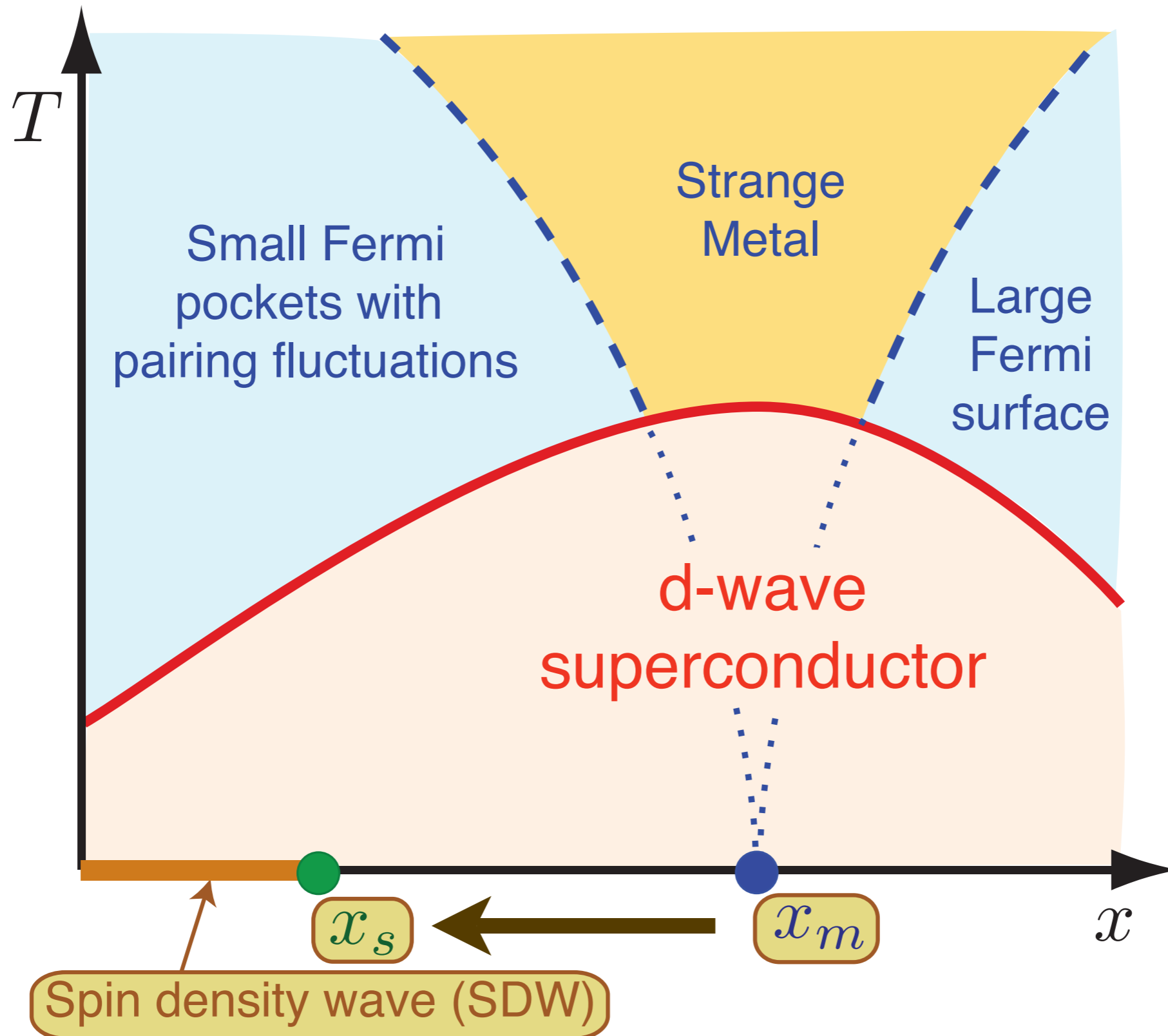
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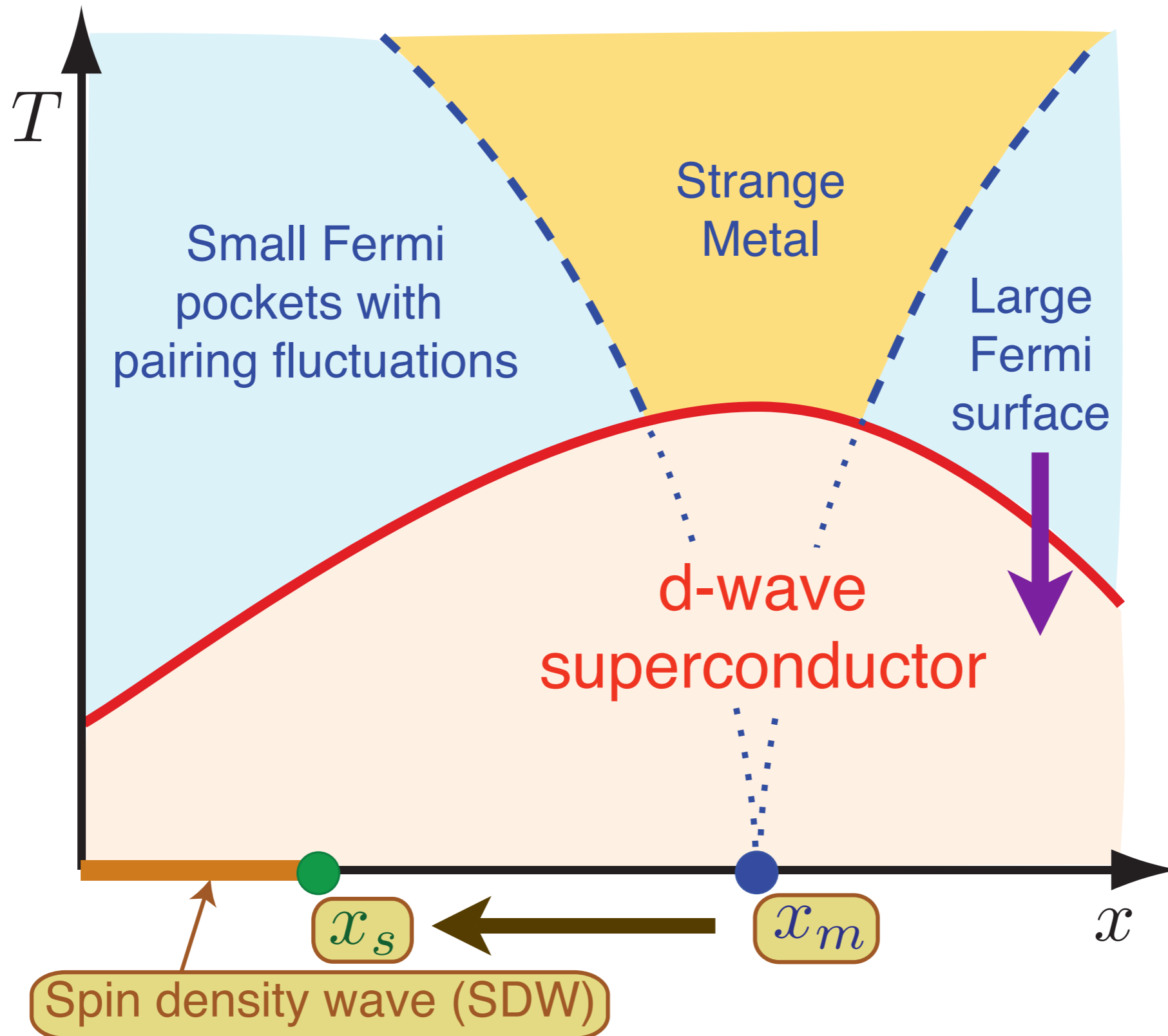
Onset of d -wave superconductivity hides the critical point at $x = x_m$ and moves it to $x = x_s < x_m$

Theory of quantum criticality in the cuprates



Theory of the onset of d -wave superconductivity from a large Fermi surface

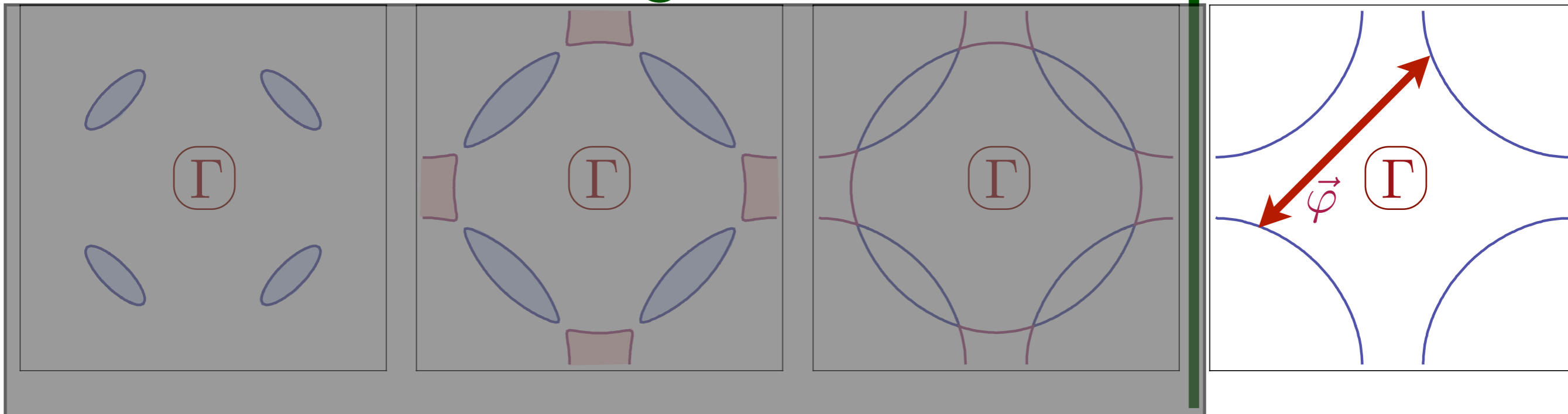
Theory of quantum criticality in the cuprates



Theory of the onset of *d*-wave superconductivity from a large Fermi surface

Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates

← Increasing SDW order →



Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.

Pairing by SDW fluctuation exchange

We now allow the SDW field $\vec{\varphi}$ to be dynamical, coupling to electrons as

$$H_{\text{sdw}} = - \sum_{\mathbf{k}, \mathbf{q}, \alpha, \beta} \vec{\varphi}_{\mathbf{q}} \cdot c_{\mathbf{k}, \alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}+\mathbf{q}, \beta}.$$

Exchange of a $\vec{\varphi}$ quantum leads to the effective interaction

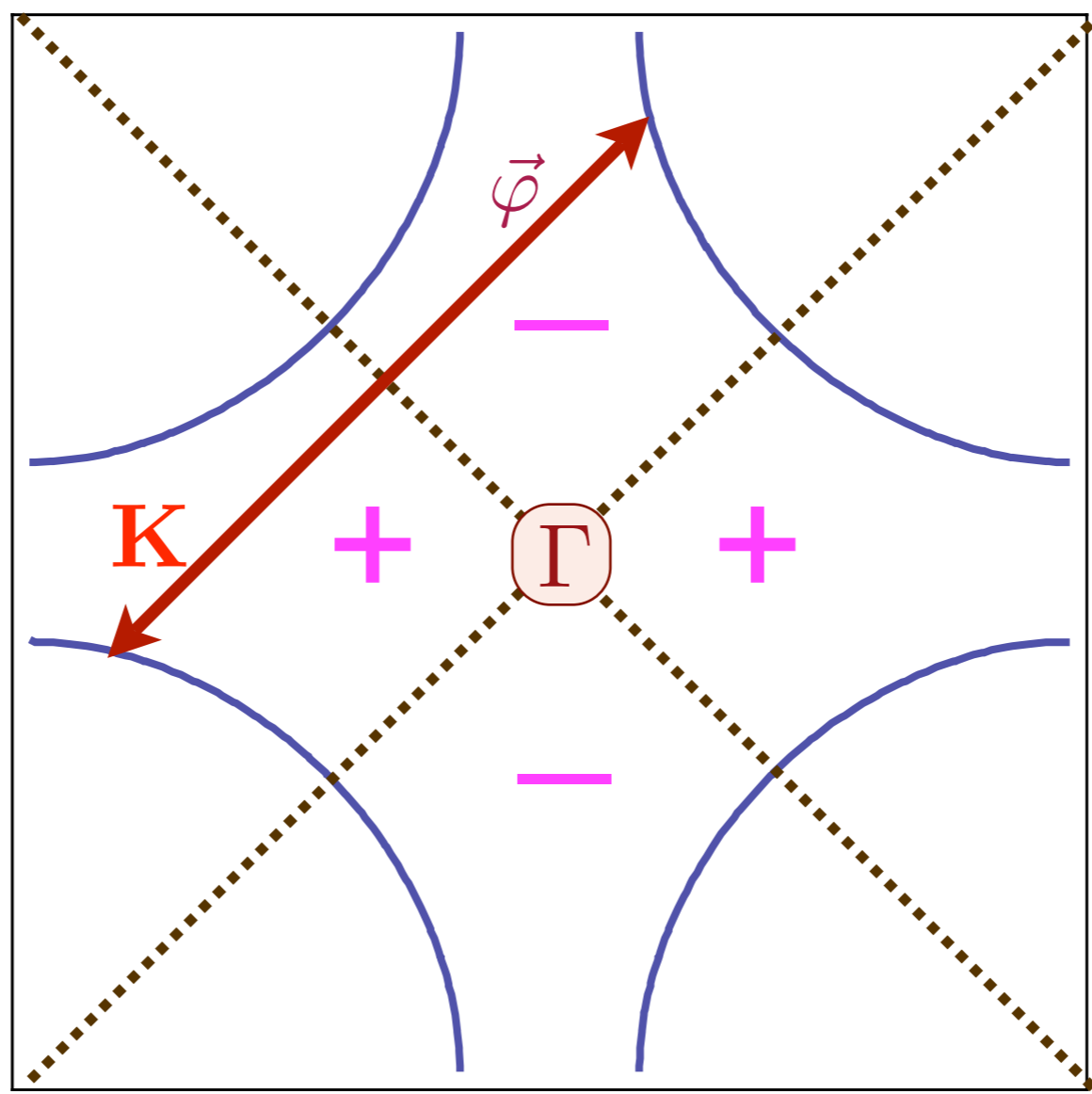
$$H_{ee} = -\frac{1}{2} \sum_{\mathbf{q}} \sum_{\mathbf{p}, \gamma, \delta} \sum_{\mathbf{k}, \alpha, \beta} V_{\alpha\beta, \gamma\delta}(\mathbf{q}) c_{\mathbf{k}, \alpha}^{\dagger} c_{\mathbf{k}+\mathbf{q}, \beta} c_{\mathbf{p}, \gamma}^{\dagger} c_{\mathbf{p}-\mathbf{q}, \delta},$$

where the pairing interaction is

$$V_{\alpha\beta, \gamma\delta}(\mathbf{q}) = \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} \frac{\chi_0}{\xi^{-2} + (\mathbf{q} - \mathbf{K})^2},$$

with $\chi_0 \xi^2$ the SDW susceptibility and ξ the SDW correlation length.

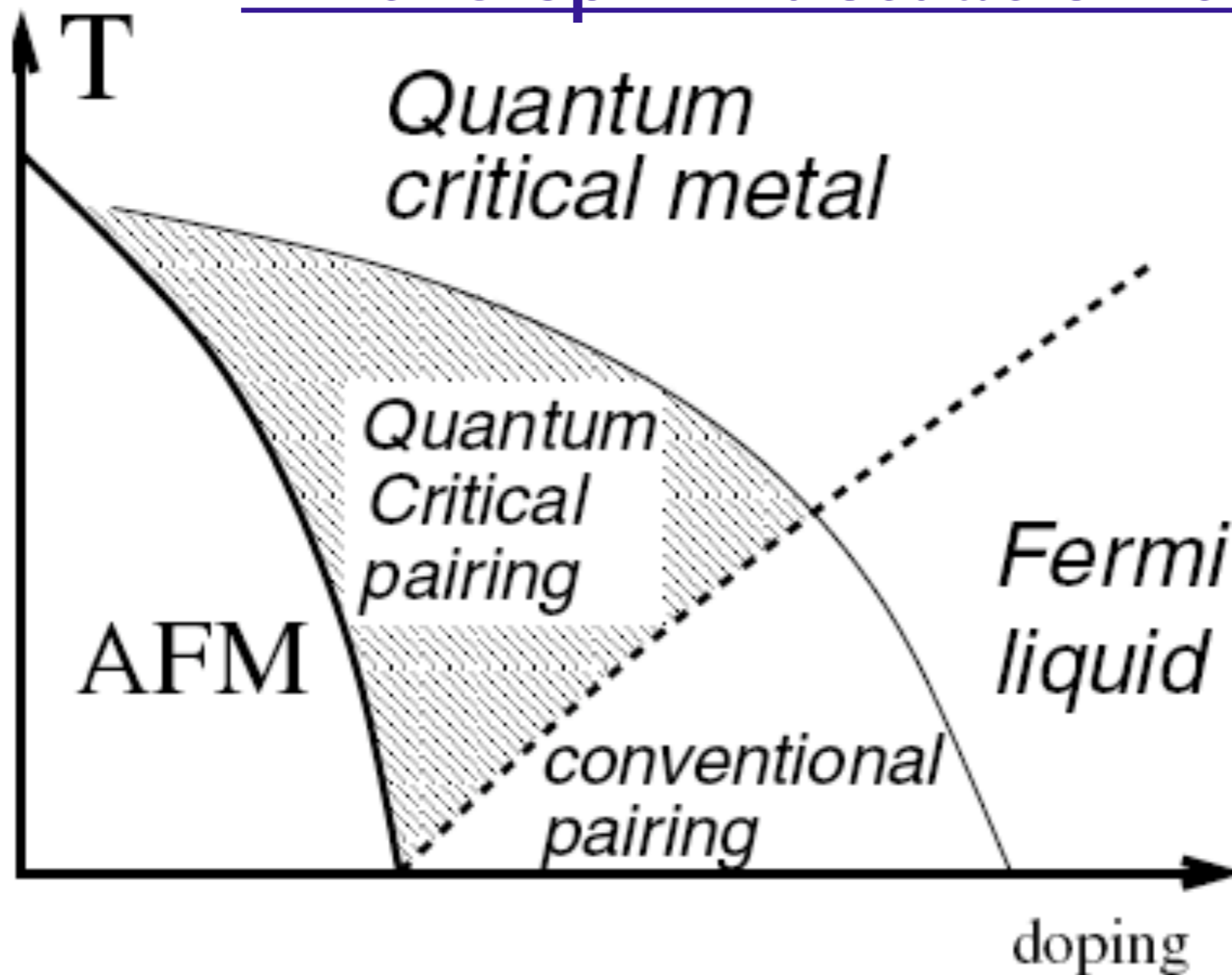
d -wave pairing of the large Fermi surface



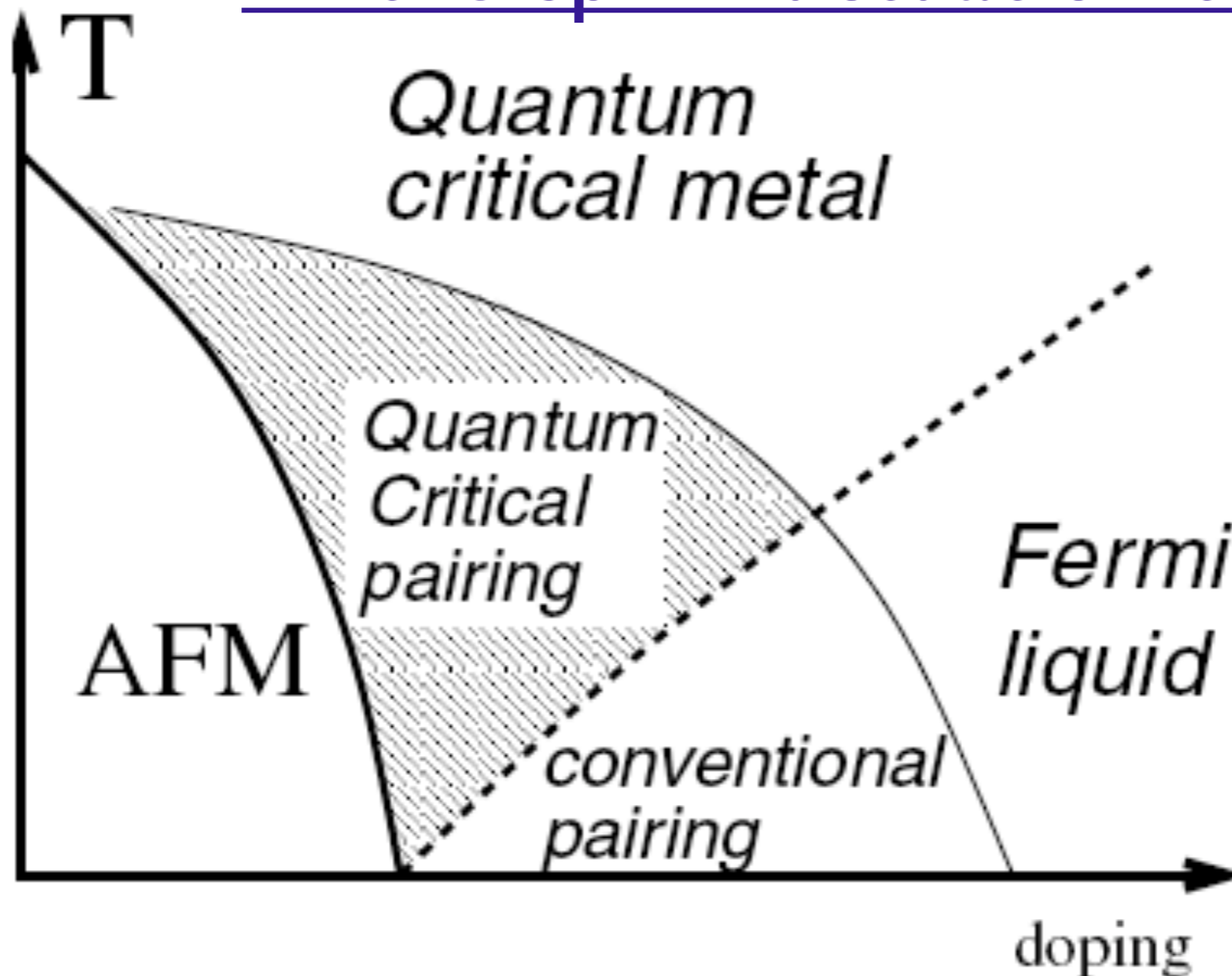
$$\langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \propto \Delta_{\mathbf{k}} = \Delta_0 (\cos(k_x) - \cos(k_y))$$

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

Approaching the onset of antiferromagnetism in the spin-fluctuation theory



Approaching the onset of antiferromagnetism in the spin-fluctuation theory



- T_c increases upon approaching the SDW transition. SDW and SC orders do not compete, but attract each other.
- No simple mechanism for nodal-anti-nodal dichotomy.

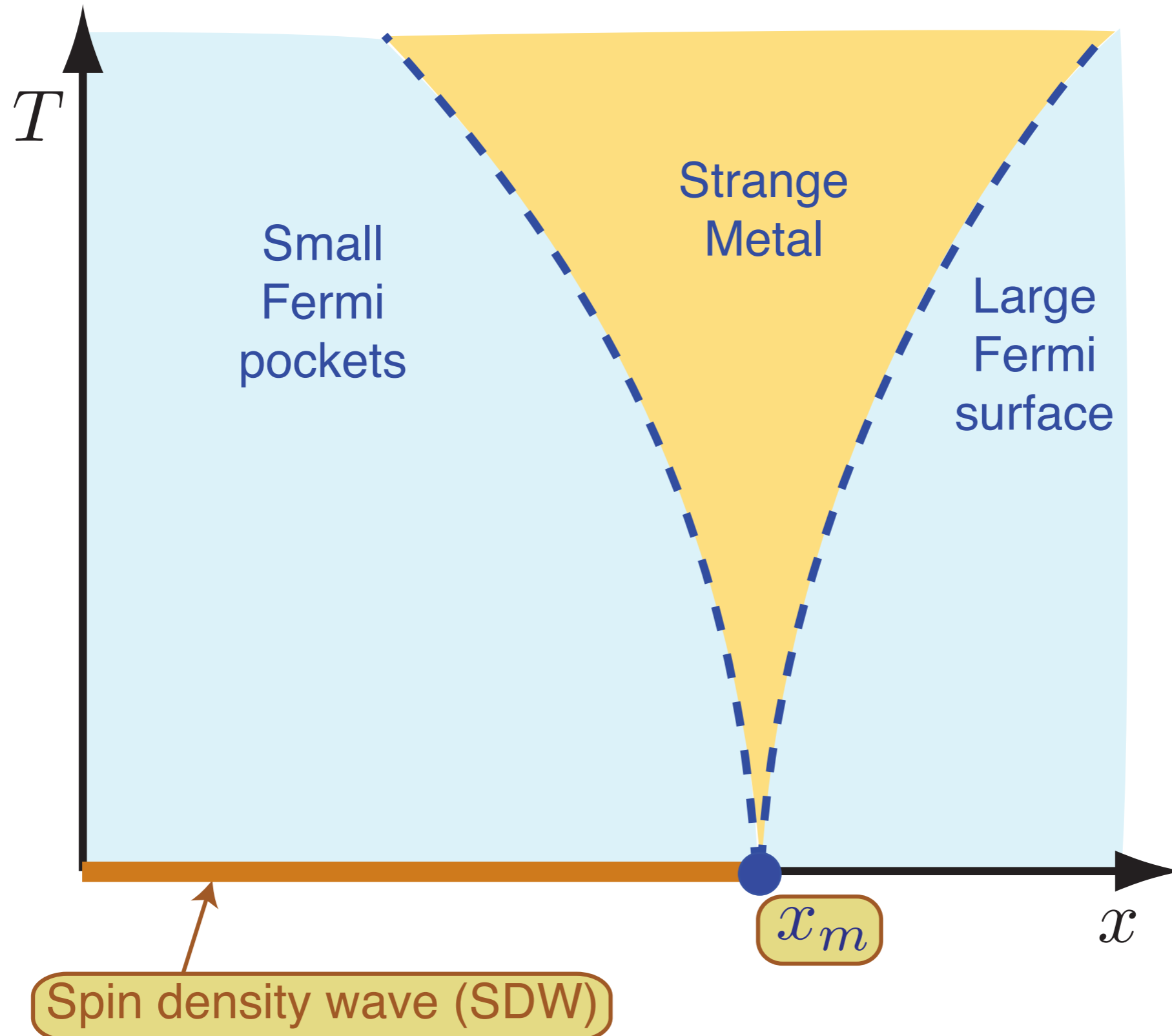
Outline

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2. Superconductivity in the overdoped regime
BCS pairing by spin fluctuation exchange
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 $SU(2)$ gauge theory of transition from Fermi pockets to a large Fermi surface

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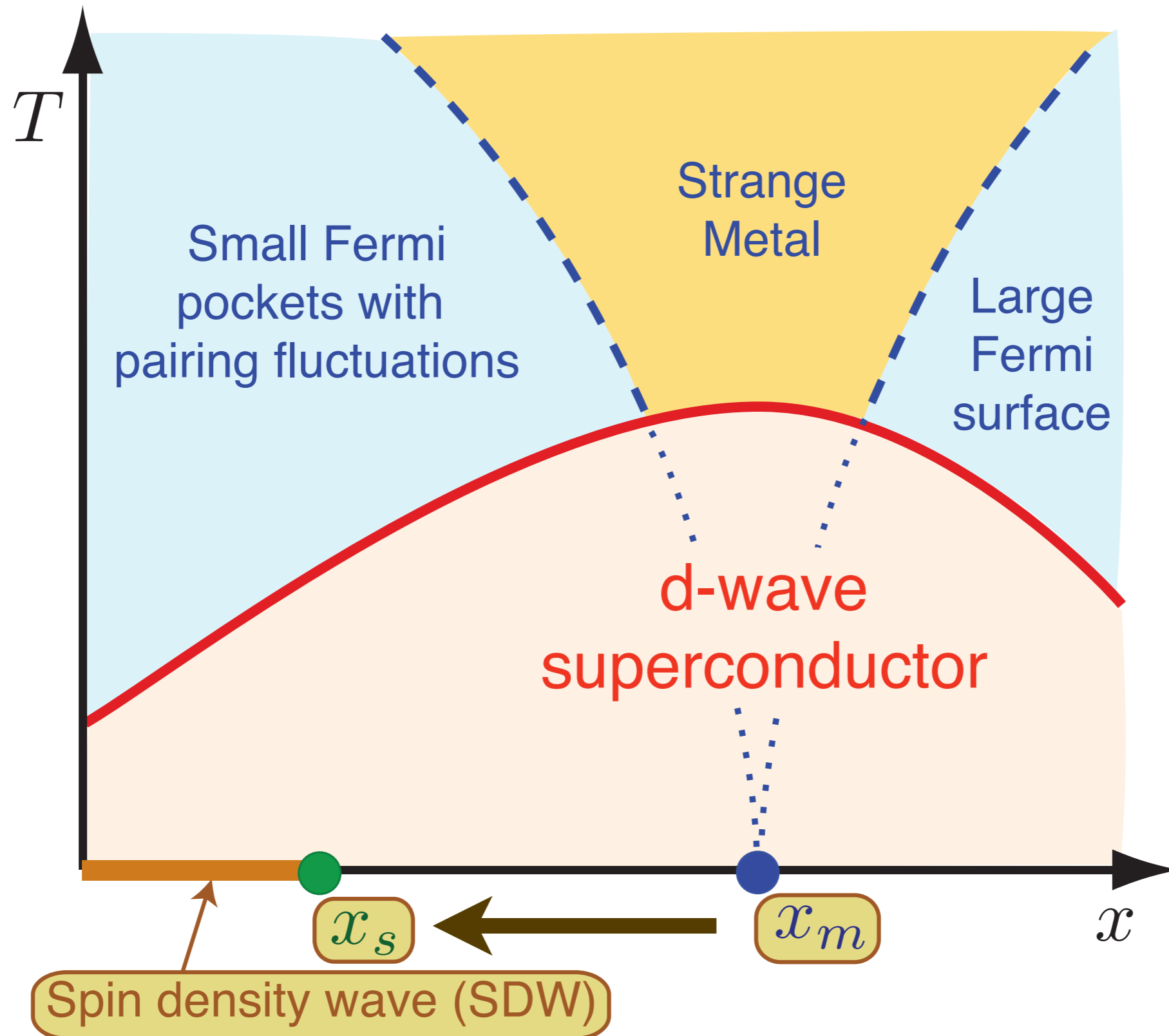
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Theory of quantum criticality in the cuprates



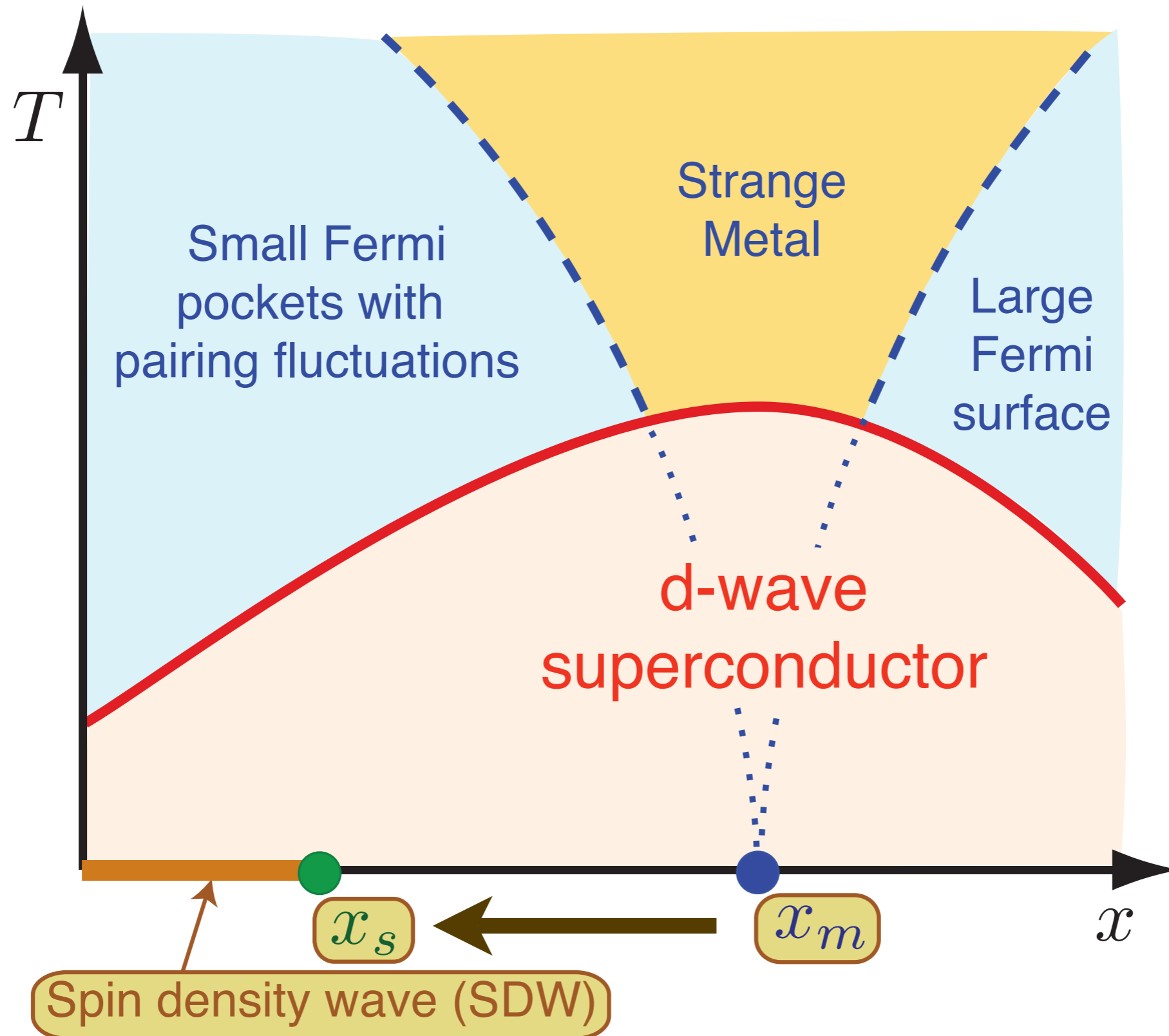
Underlying SDW ordering quantum critical point
in metal at $x = x_m$

Theory of quantum criticality in the cuprates



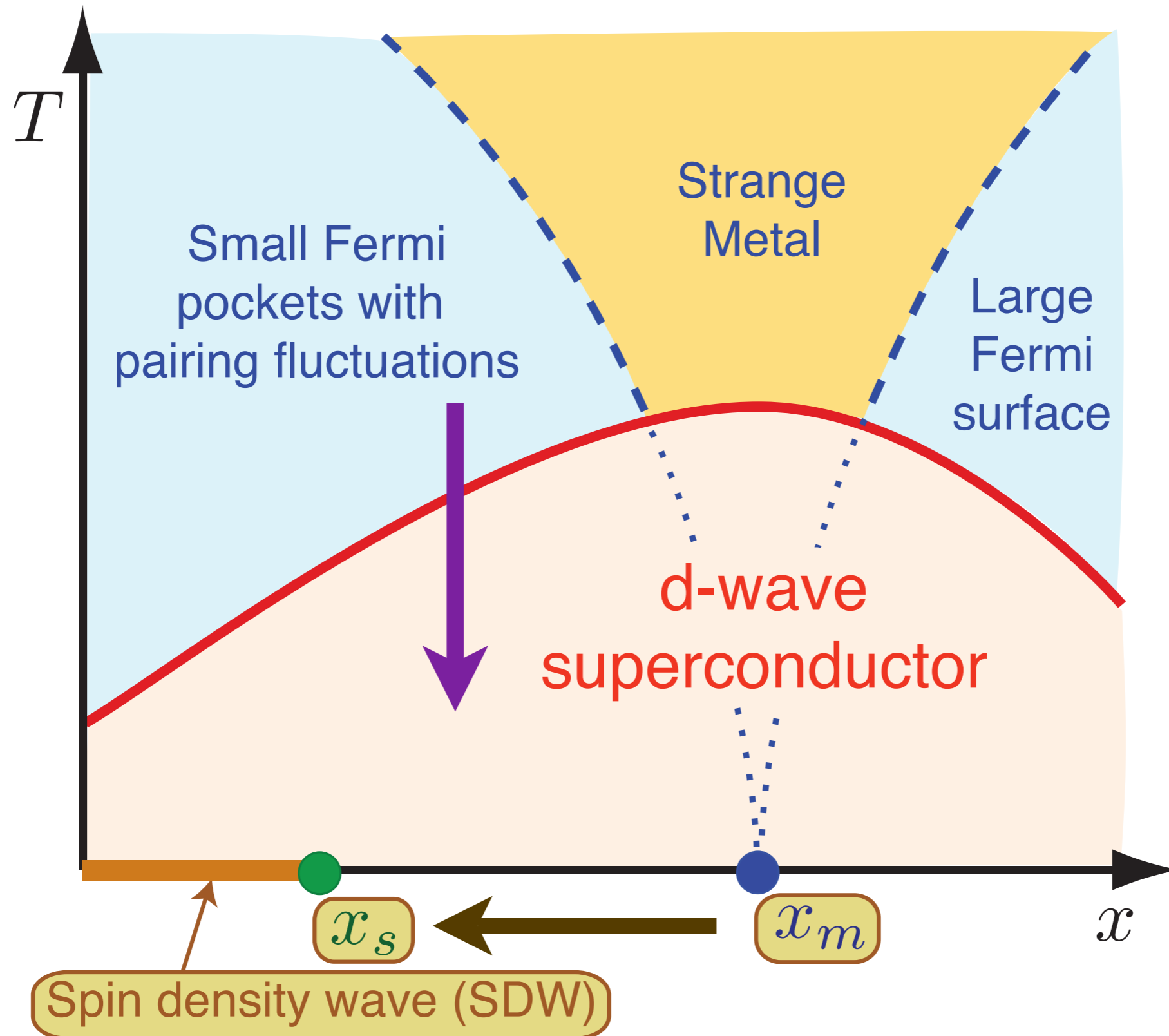
Onset of d -wave superconductivity hides the critical point at $x = x_m$ and moves it to $x = x_s < x_m$

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Theory of the onset of d -wave superconductivity from small Fermi pockets

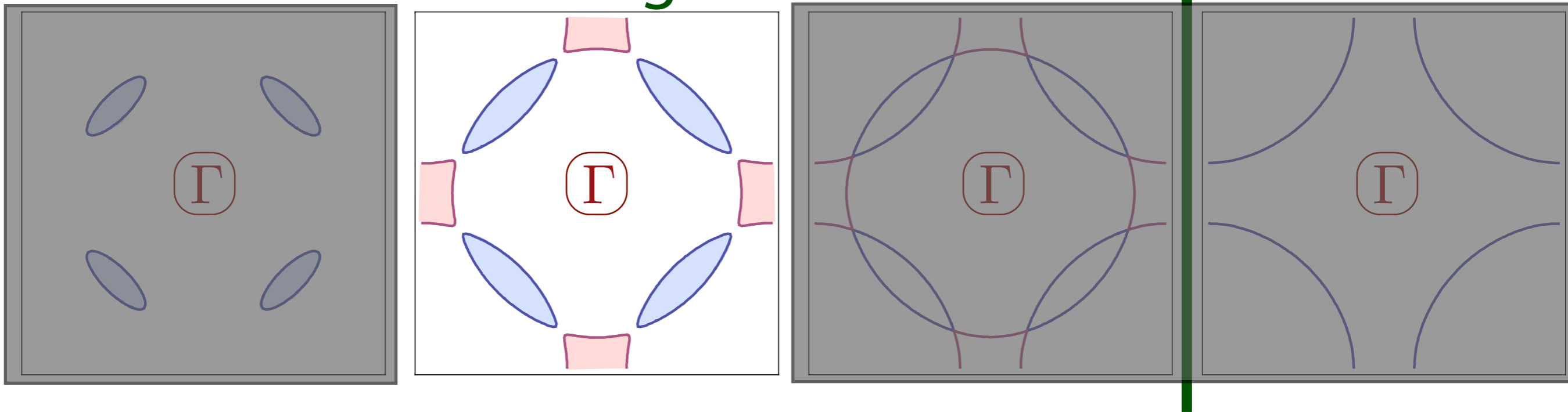
Theory of quantum criticality in the cuprates



Theory of the onset of d -wave superconductivity from small Fermi pockets

Theory of underdoped cuprates

← Increasing SDW order →



Begin with SDW ordered state, and rotate to a frame polarized along the local orientation of the SDW order $\hat{\varphi}$

$$\begin{pmatrix} c_{\uparrow} \\ c_{\downarrow} \end{pmatrix} = R \begin{pmatrix} \psi_{+} \\ \psi_{-} \end{pmatrix} ; \quad R^{\dagger} \hat{\varphi} \cdot \vec{\sigma} R = \sigma^z ; \quad R^{\dagger} R = 1$$

Theory of underdoped cuprates

$$\text{With } R = \begin{pmatrix} z_{\uparrow} & -z_{\downarrow}^* \\ z_{\downarrow} & z_{\uparrow}^* \end{pmatrix}$$

the theory is invariant under the U(1) gauge transformation

$$z_{\alpha} \rightarrow e^{i\theta} z_{\alpha} \quad ; \quad \psi_{+} \rightarrow e^{-i\theta} \psi_{+} \quad ; \quad \psi_{-} \rightarrow e^{i\theta} \psi_{-}$$

and the SDW order is given by

$$\hat{\vec{\varphi}} = z_{\alpha}^* \vec{\sigma}_{\alpha\beta} z_{\beta}$$

Theory of underdoped cuprates

Starting from the “SDW-fermion” model
with Lagrangian

$$\begin{aligned}\mathcal{L} = & \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha} \\ & - E_{sdw} \sum_i c_{i\alpha}^\dagger \hat{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i} \\ & + \frac{1}{2t} \left(\partial_\mu \hat{\varphi} \right)^2\end{aligned}$$

Theory of underdoped cuprates

we obtain a U(1) gauge theory of

- fermions ψ_p with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,

$$\mathcal{L} = \sum_{\mathbf{k}, p=\pm} \left[\psi_{\mathbf{k}p}^\dagger \left(\frac{\partial}{\partial \tau} - ipA_\tau + \varepsilon_{\mathbf{k}-p\mathbf{A}} \right) \psi_{\mathbf{k}p} - E_{sdw} \psi_{\mathbf{k}p}^\dagger p \psi_{\mathbf{k}+\mathbf{K},p} \right]$$

Theory of underdoped cuprates

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- relativistic complex scalars z_α with charge 1, representing the orientational fluctuations of the SDW order

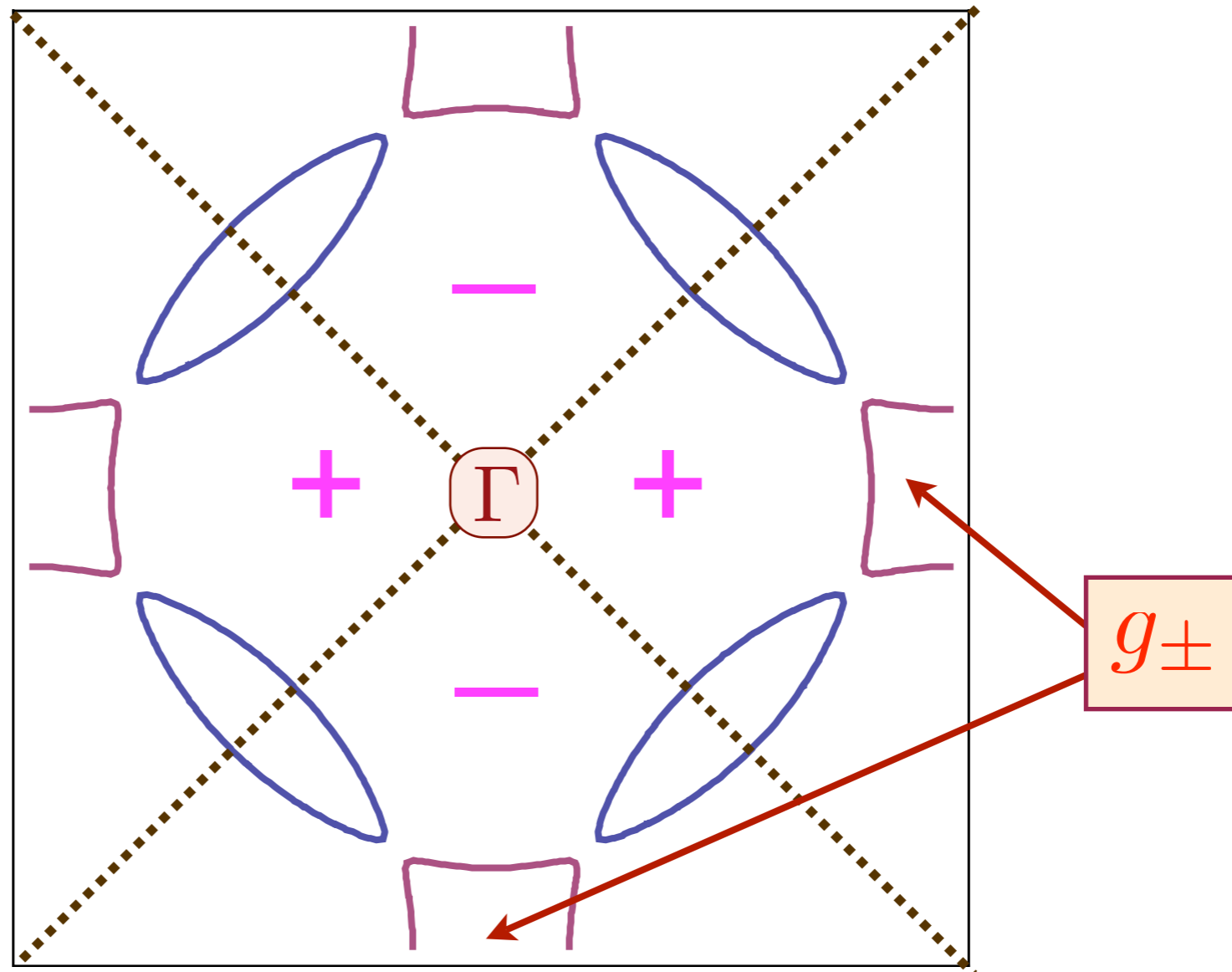
$$\mathcal{L} = \sum_{\mathbf{k}, p=\pm} \left[\psi_{\mathbf{k}p}^\dagger \left(\frac{\partial}{\partial \tau} - ipA_\tau + \varepsilon_{\mathbf{k}-p\mathbf{A}} \right) \psi_{\mathbf{k}p} - E_{sdw} \psi_{\mathbf{k}p}^\dagger p \psi_{\mathbf{k}+\mathbf{K}, p} \right]$$
$$+ \frac{1}{t} \left[|(\partial_\tau - iA_\tau)z_\alpha|^2 + v^2 |\nabla - i\mathbf{A})z_\alpha|^2 + i\lambda(|z_\alpha|^2 - 1) \right]$$

Theory of underdoped cuprates

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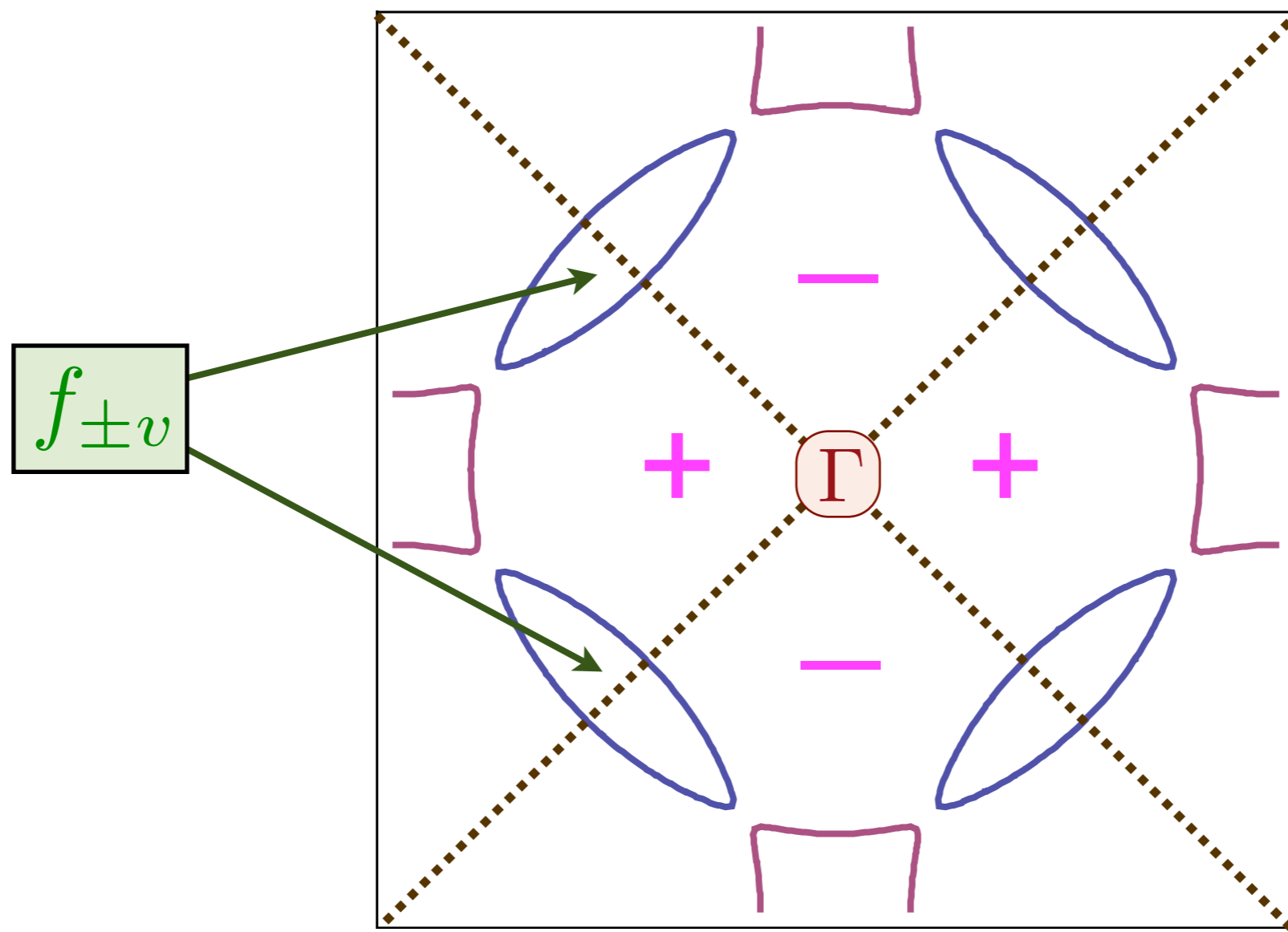
- fermions ψ_p with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,
- relativistic complex scalars z_α with charge 1, representing the orientational fluctuations of the SDW order
- Monopoles carrying Berry phases; onset of superconductivity leads to confinement via condensation of monopoles, which induces charge order.

Strong pairing of the g_{\pm} electron pockets



$$\langle g_+ g_- \rangle = \Delta$$

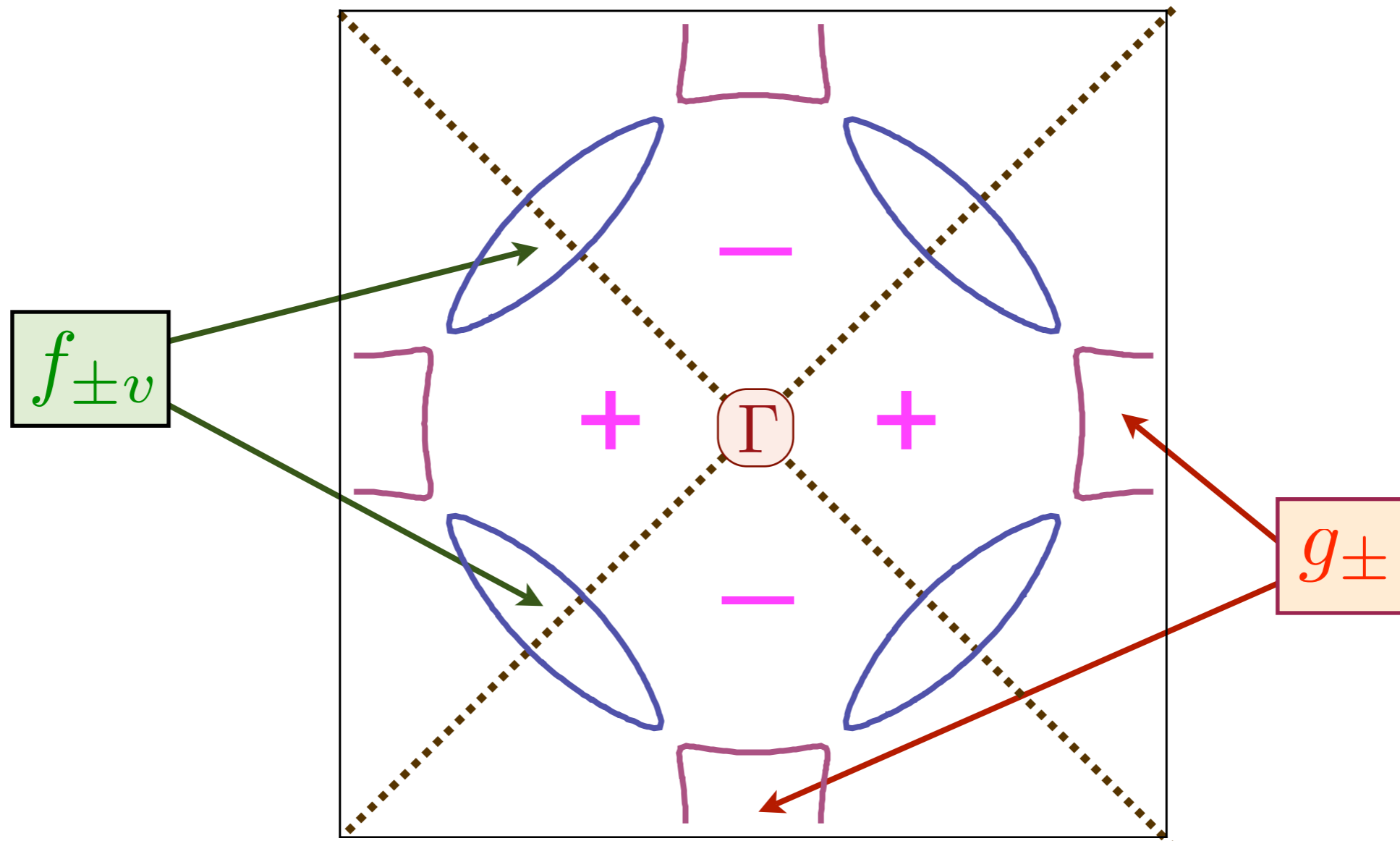
Weak pairing of the f_{\pm} hole pockets



$$\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k}) \rangle \sim (k_x - k_y) J \langle g_+ g_- \rangle;$$

$$\langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k}) \rangle \sim (k_x + k_y) J \langle g_+ g_- \rangle;$$

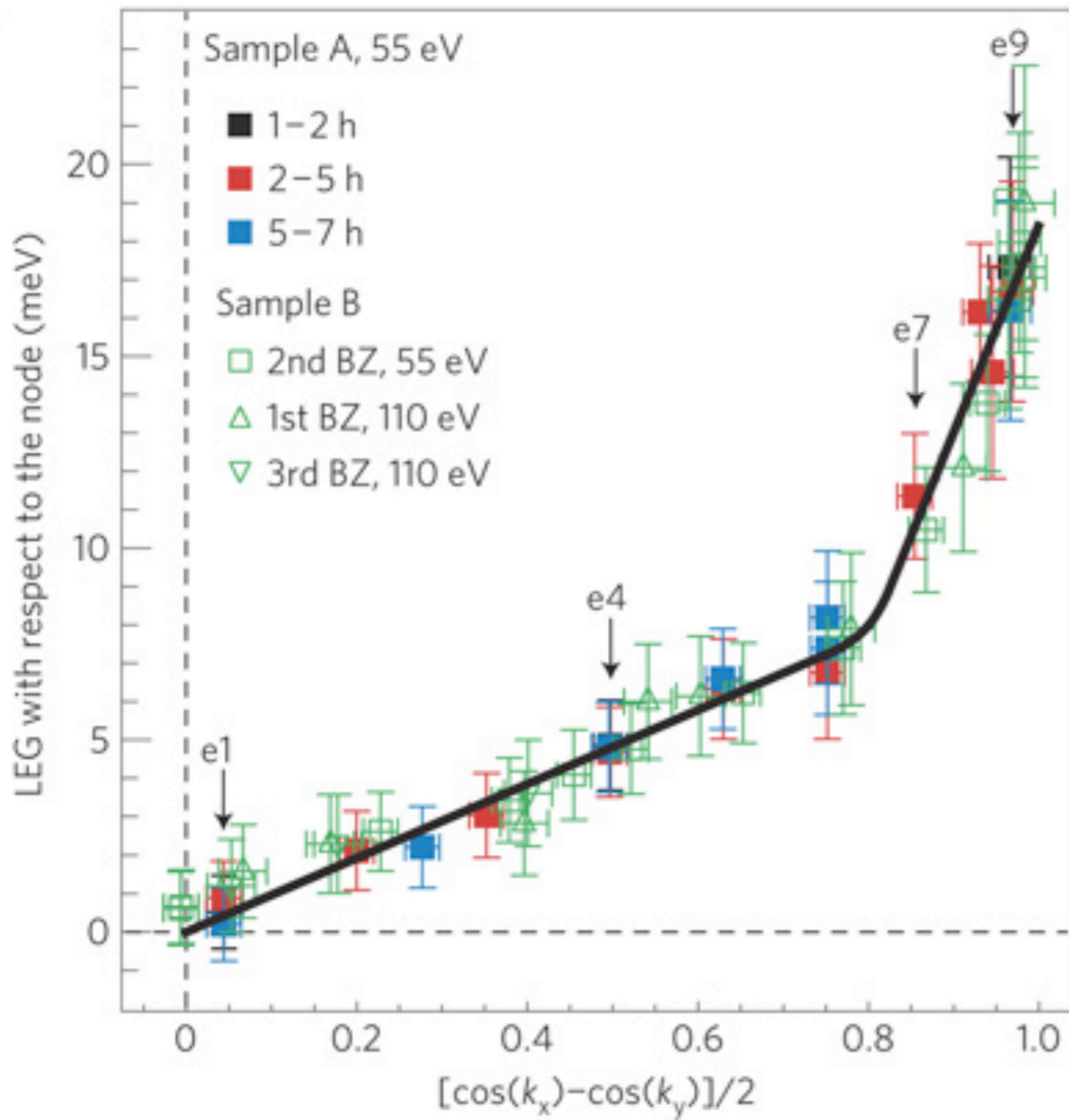
$$\langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k}) \rangle = 0,$$



d -wave pairing of the electrons is associated with

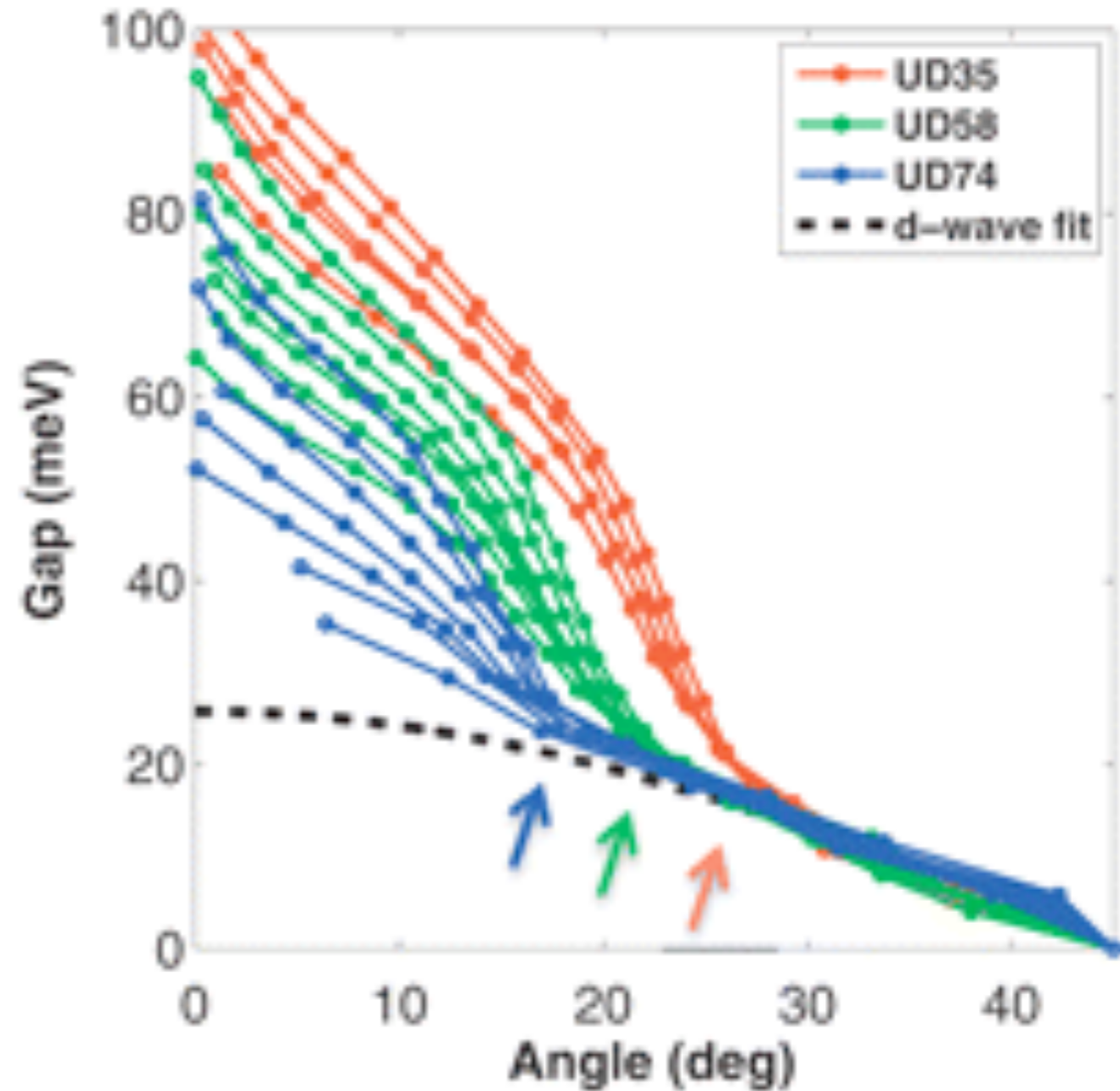
- **Strong s -wave** pairing of g_{\pm}
- **Weak p -wave** pairing of $f_{\pm v}$.

Photoemission in LBCO

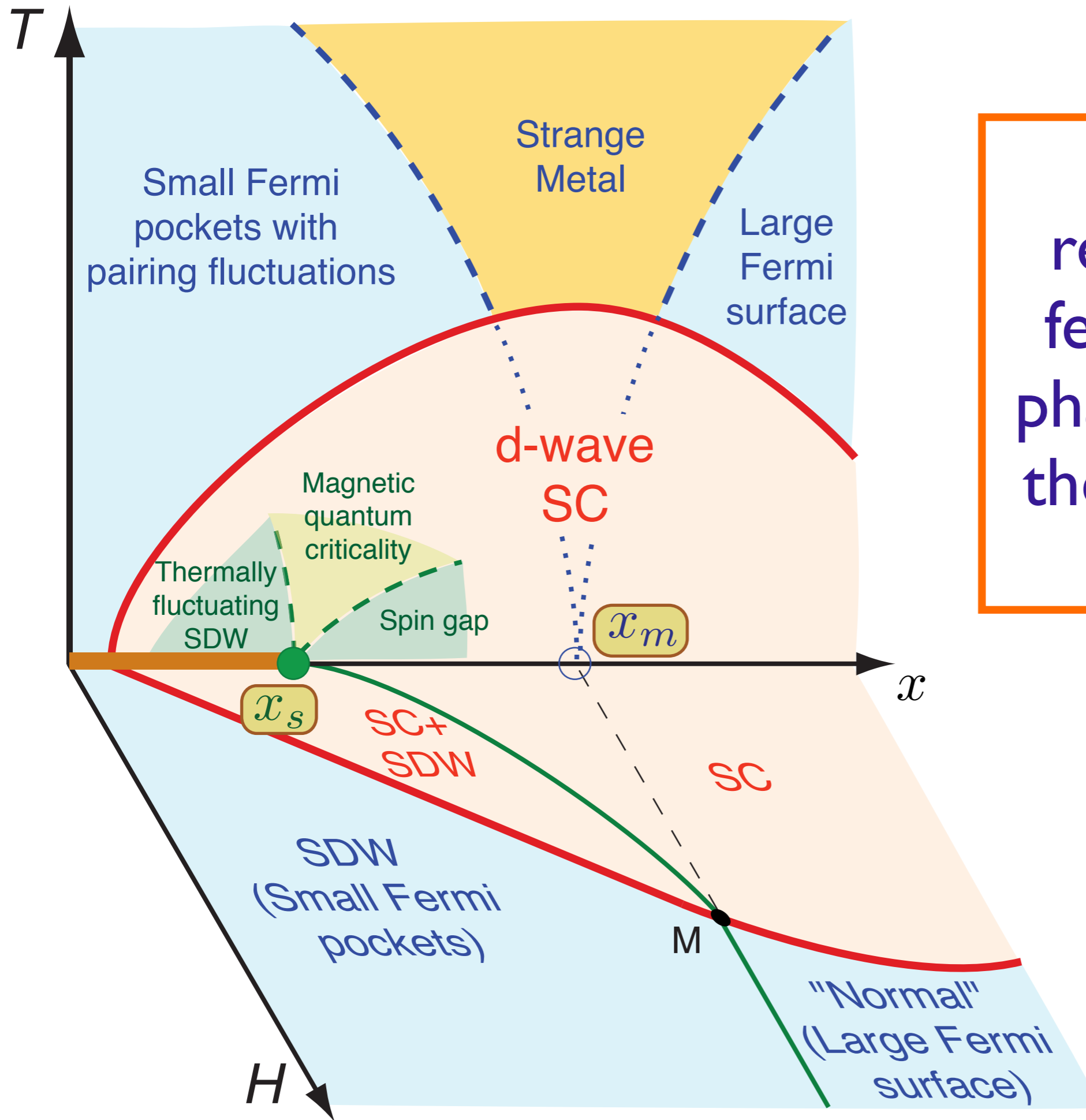


R.-H. He, K. Tanaka, S.-K. Mo, T. Sasagawa, M. Fujita, T. Adachi, N. Mannella, K. Yamada, Y. Koike, Z. Hussain and Z.-X. Shen, *Nature Physics* **5**, 119 (2008)

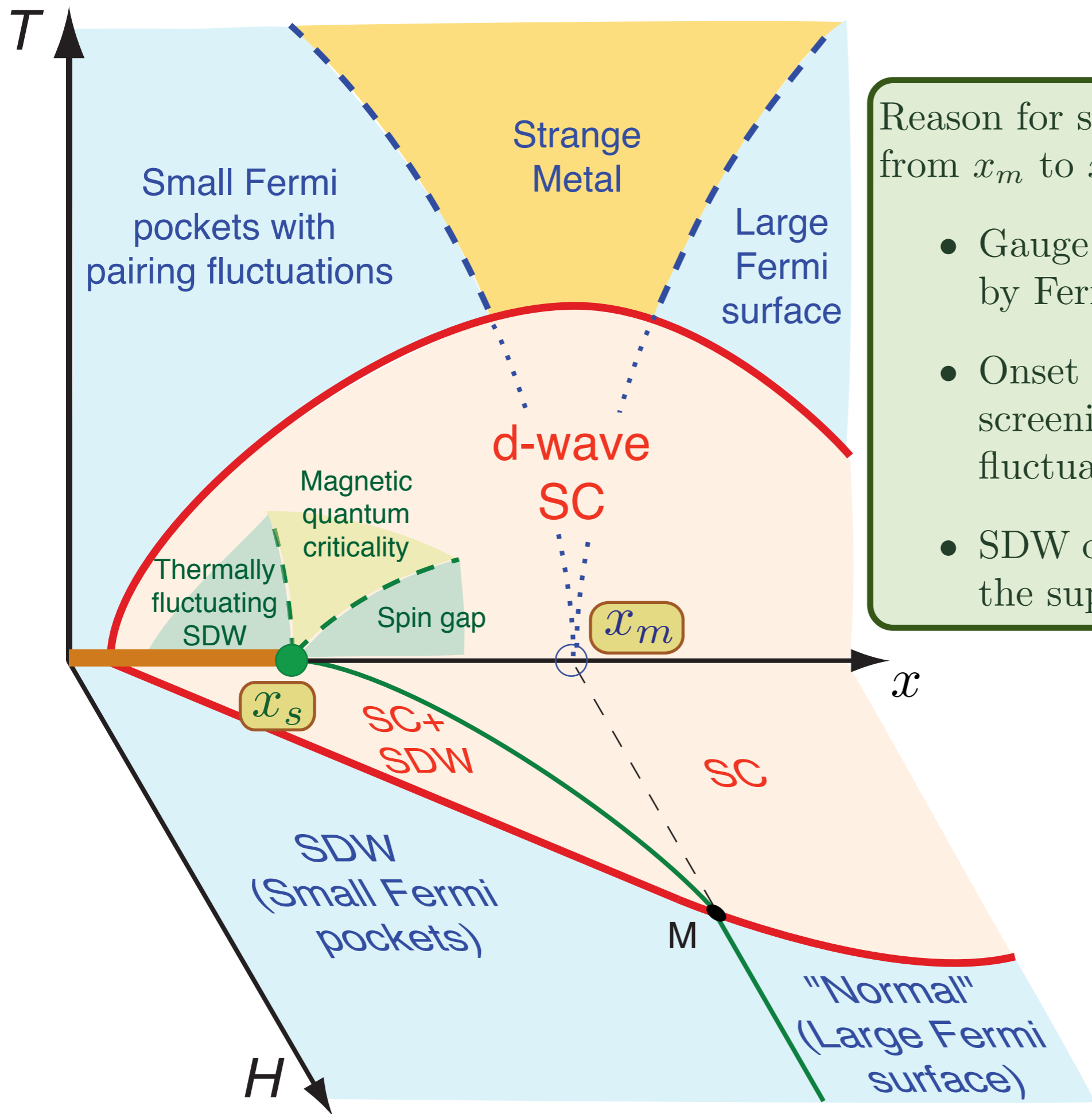
STM in BSCCO



A. Pushp, C.V. Parker, A. N. Pasupathy, K. K. Gomes, S. Ono, J. Wen, Z. Xu, G. Gu, and A. Yazdani, *Science* **324**, 1689 (2009)



Theory
 reproduces all
 features of the
 phase diagram in
 the underdoped
 regime



Reason for shift in onset of SDW from x_m to x_s :

- Gauge fluctuations are screened by Fermi surface in metal
- Onset of pairing suppresses screening, and enhances gauge fluctuations
- SDW order is suppressed in the superconductor

Outline

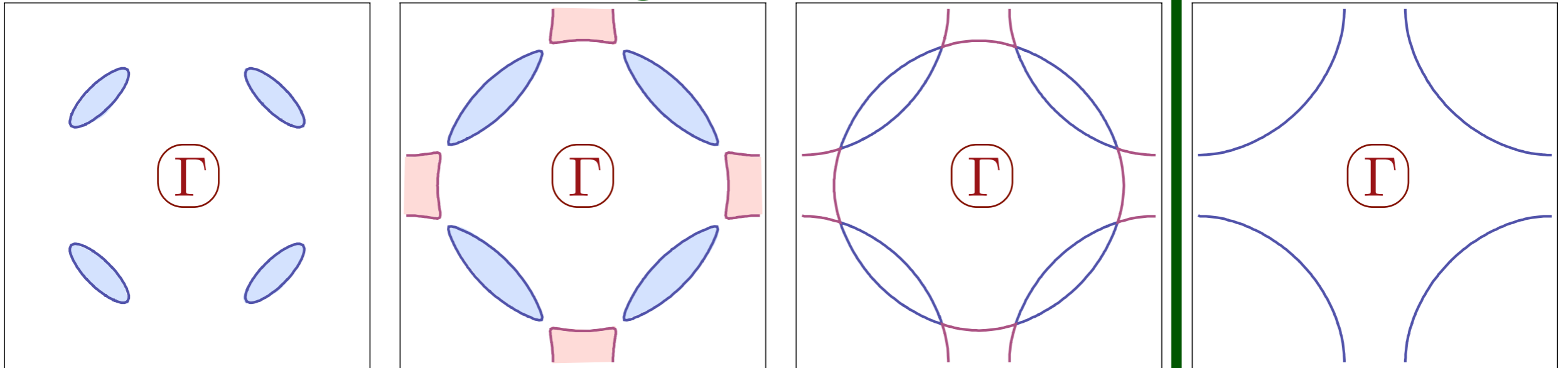
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Unified theory

← Increasing SDW order →



The parameterization $\begin{pmatrix} c_{\uparrow} \\ c_{\downarrow} \end{pmatrix} = R \begin{pmatrix} \psi_{+} \\ \psi_{-} \end{pmatrix}$

is actually invariant under a $SU(2)$ gauge transformation

$$\begin{pmatrix} \psi_{+} \\ \psi_{-} \end{pmatrix} \rightarrow U \begin{pmatrix} \psi_{+} \\ \psi_{-} \end{pmatrix} ; \quad R \rightarrow RU^{\dagger}$$

Unified theory

The theory has $SU(2)_{\text{gauge}} \otimes SU(2)_{\text{spin}} \otimes U(1)_{\text{em charge}} \otimes (\text{lattice space group})$ invariance,

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- fermion ψ transforming as $(\mathbf{2}, \mathbf{1}, 1)$, and with dispersion $\varepsilon_{\mathbf{k}}$ from the band structure,

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- relativistic real scalar N transforming as $(\mathbf{3}, \mathbf{1}, 0)$, measuring the local SDW amplitude,

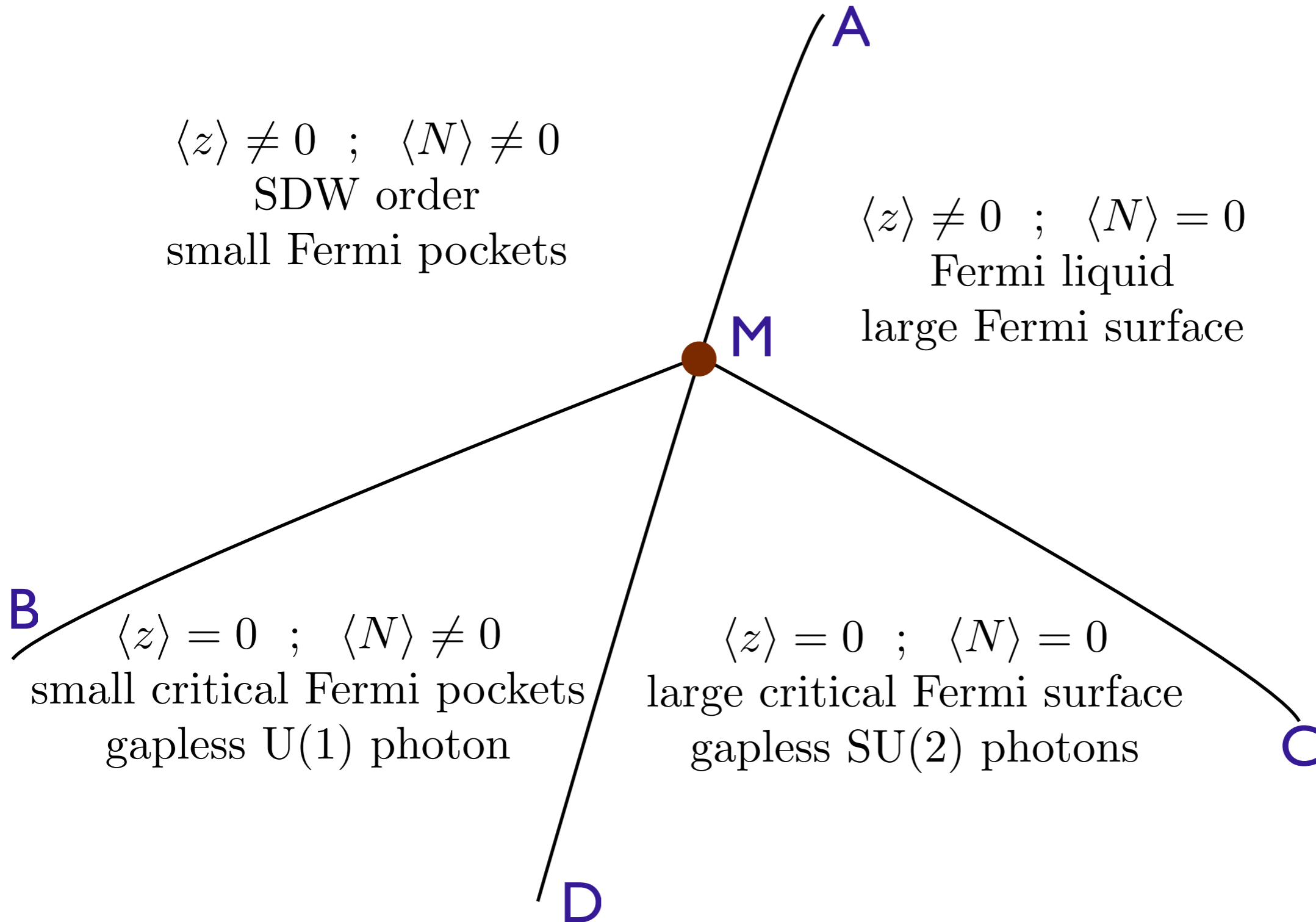
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- relativistic real scalar N transforming as $(\mathbf{3}, \mathbf{1}, 0)$, measuring the local SDW amplitude,
- a Yukawa coupling between N and ψ , which $\sim e^{i\mathbf{K}\cdot\mathbf{r}}$ because of space group transformations.

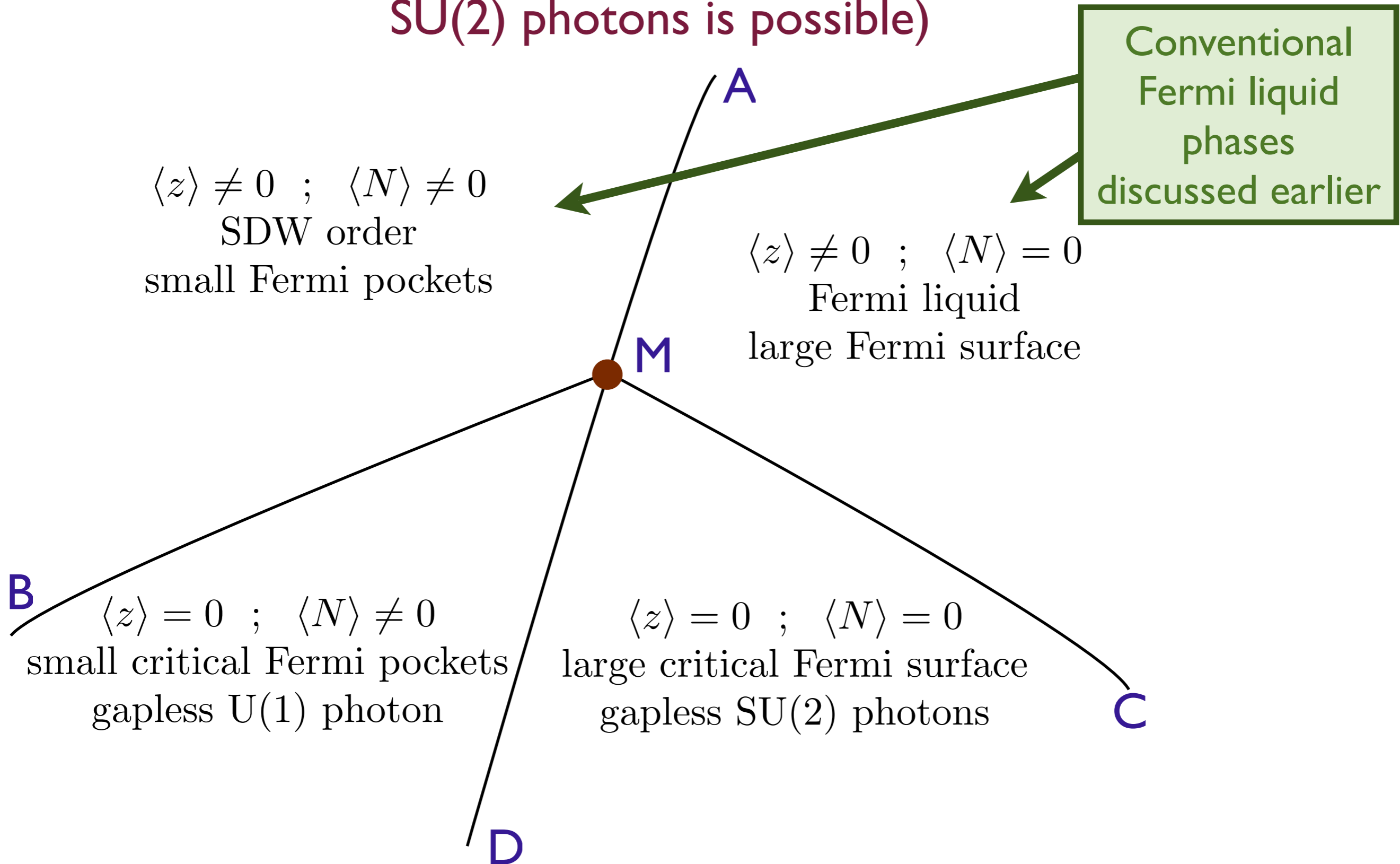
Unified theory

Conjectured phase diagram (assuming a phase with gapless SU(2) photons is possible)



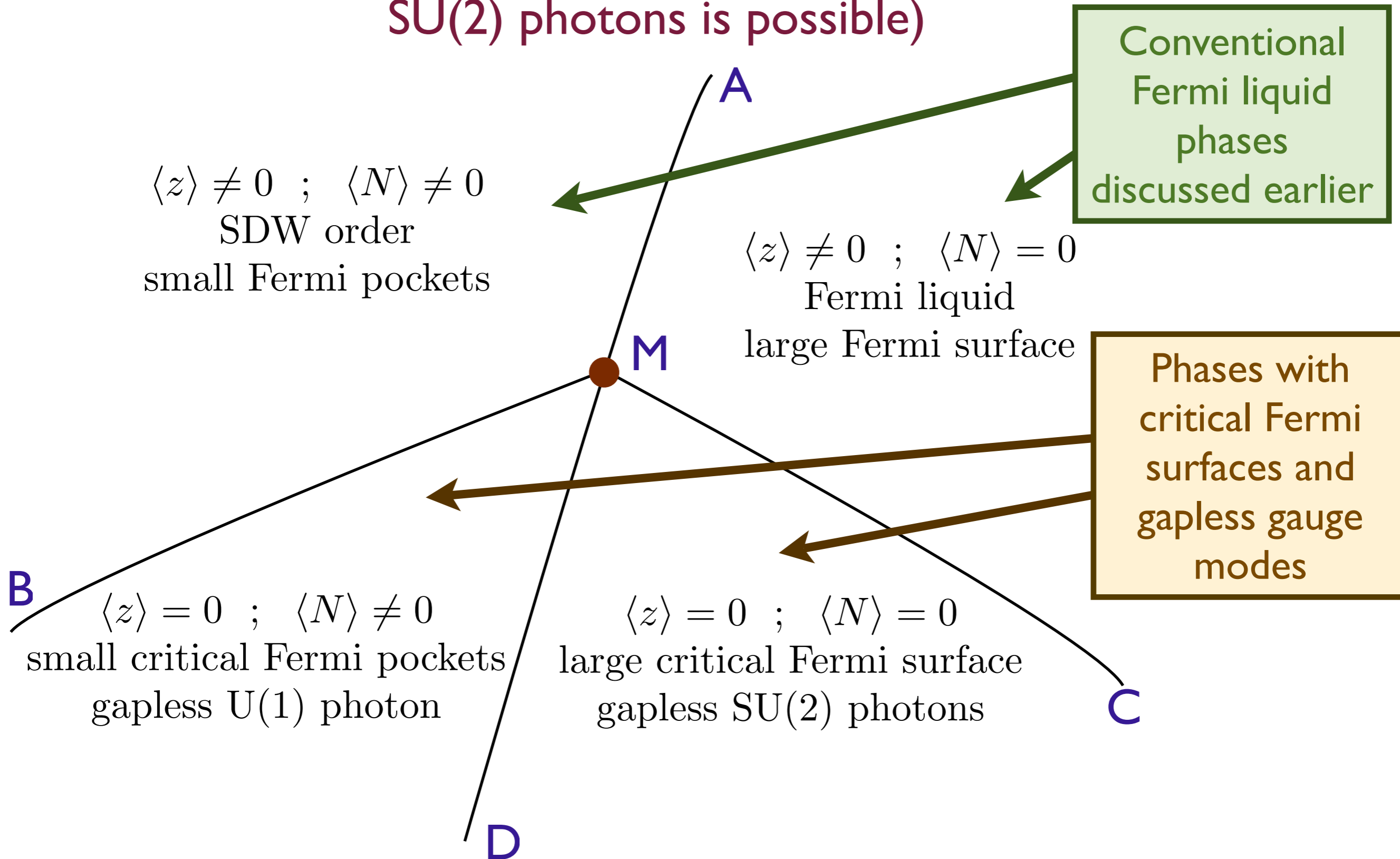
Unified theory

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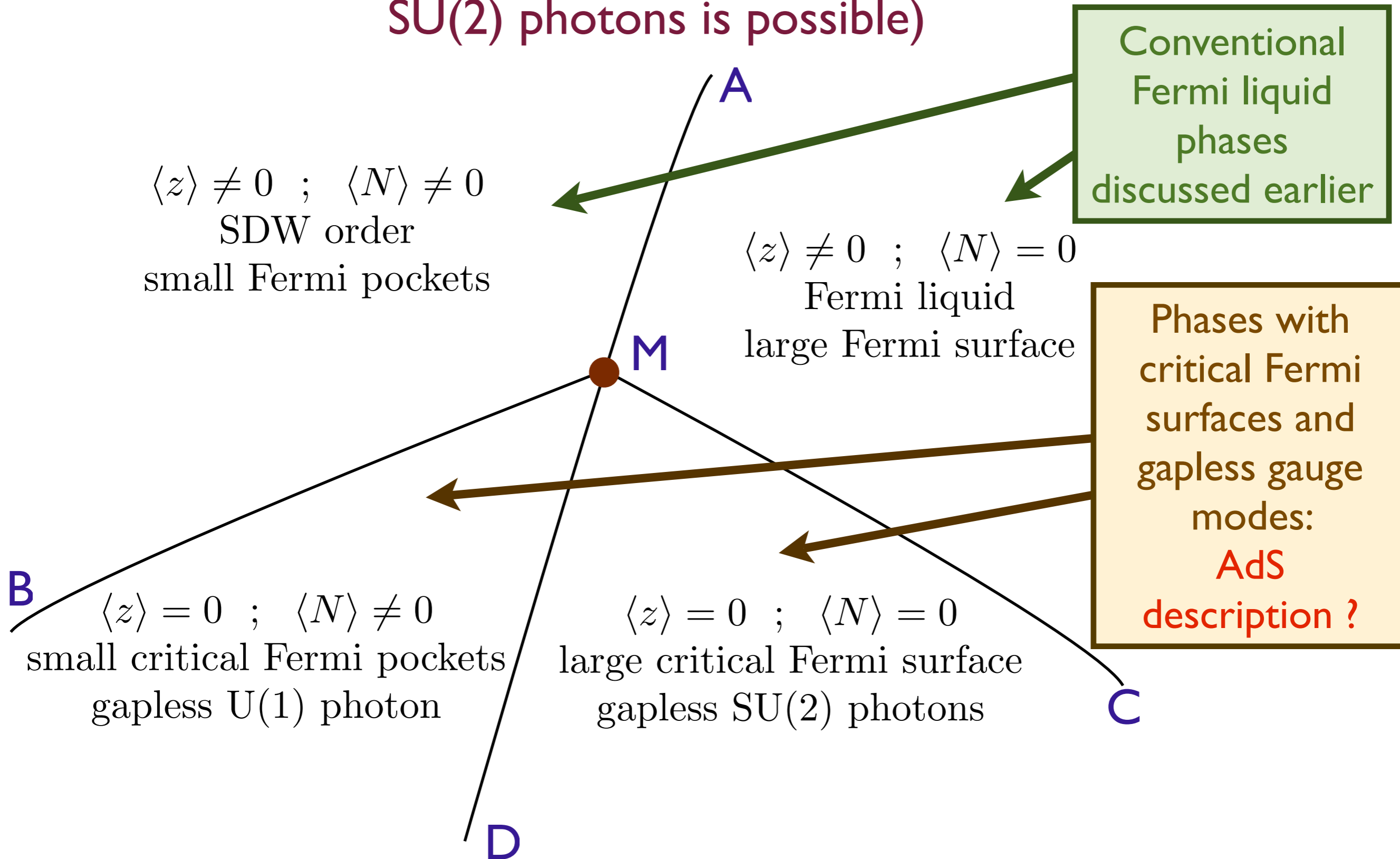
Unified theory

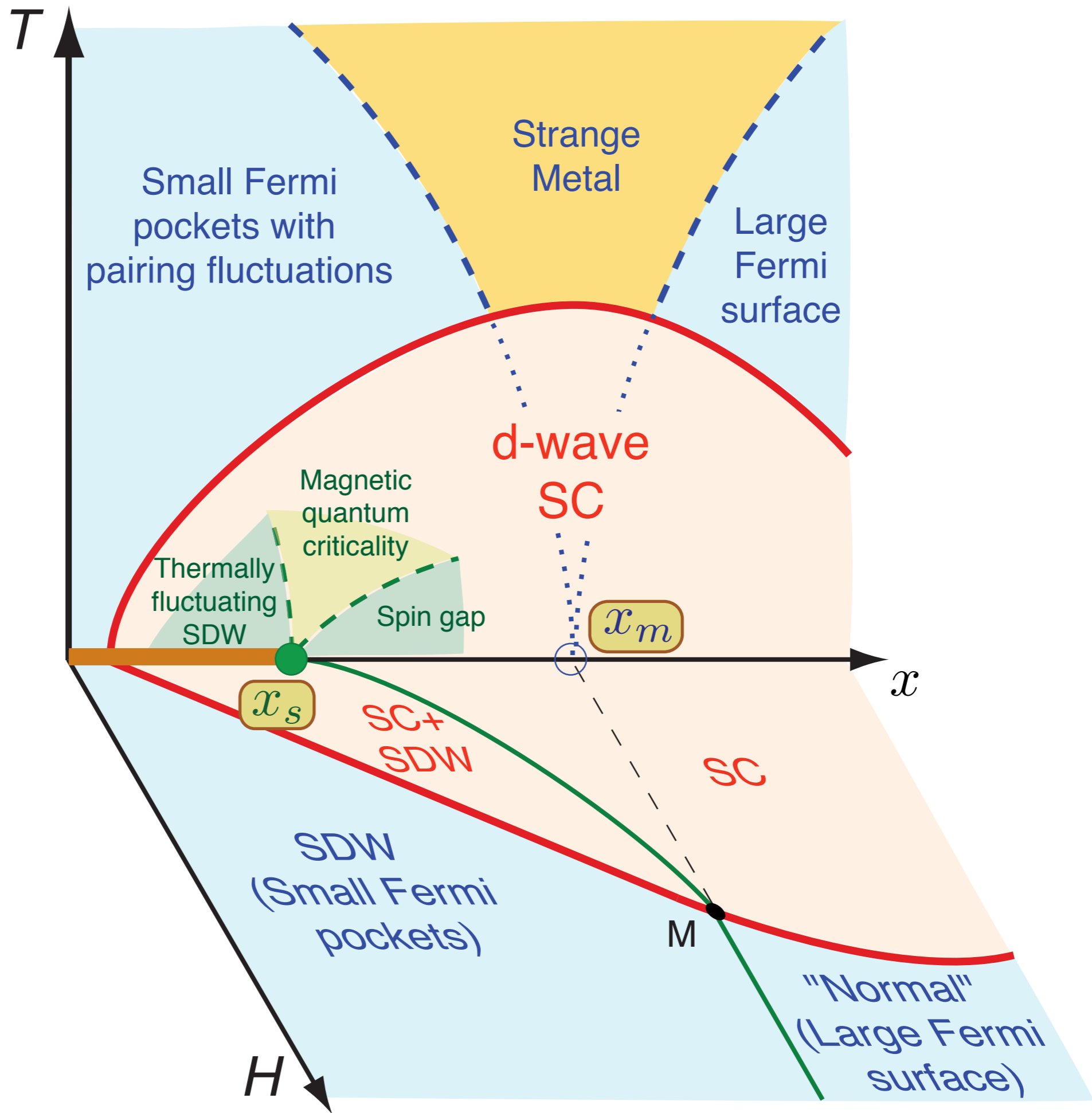
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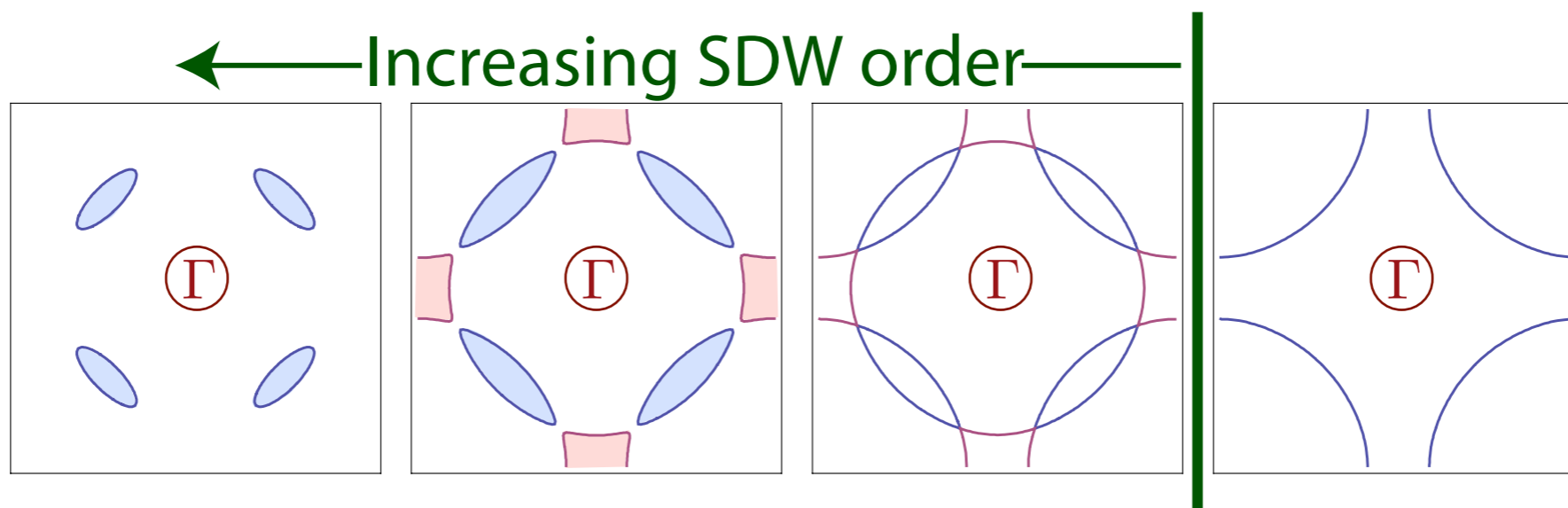
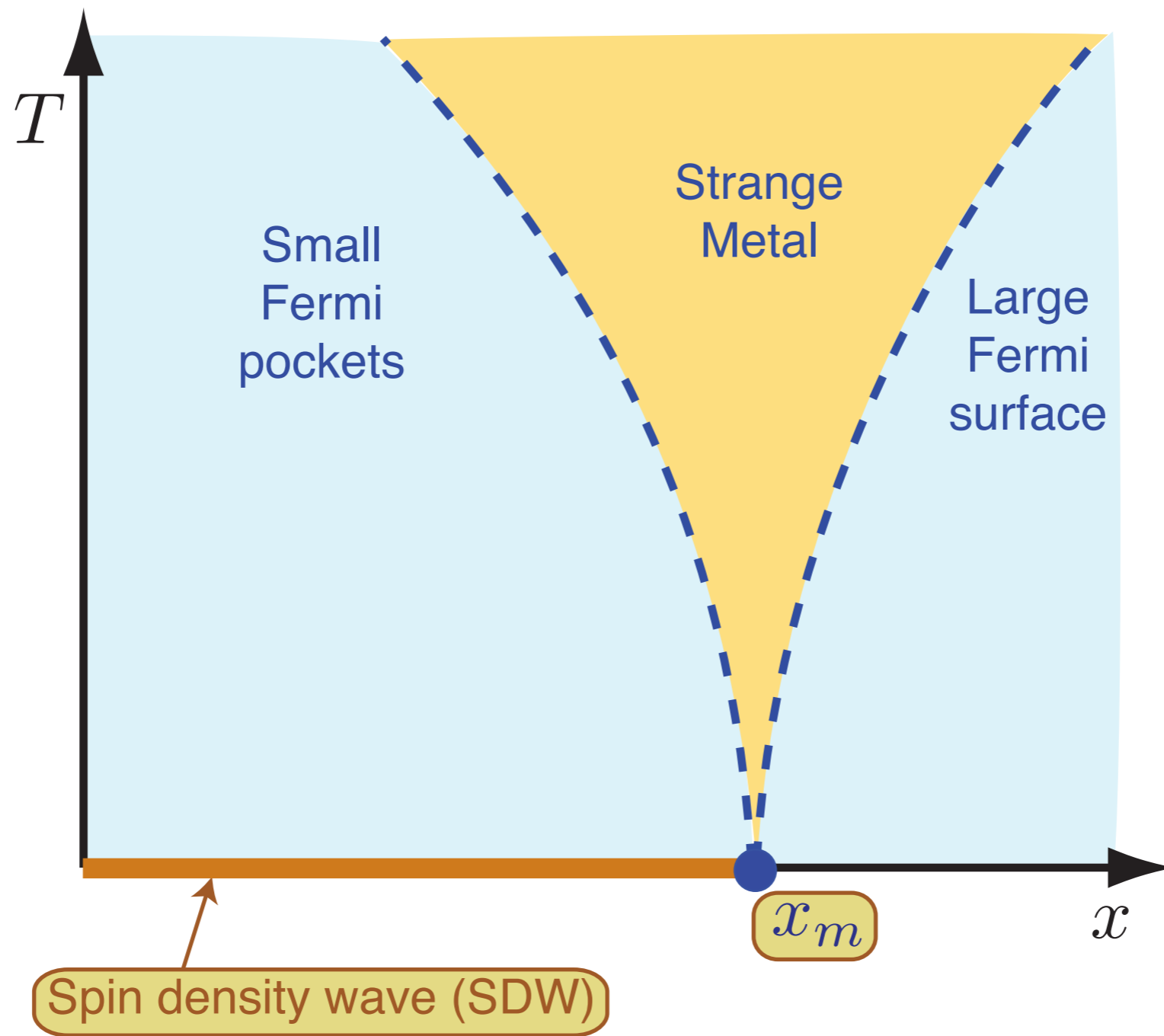


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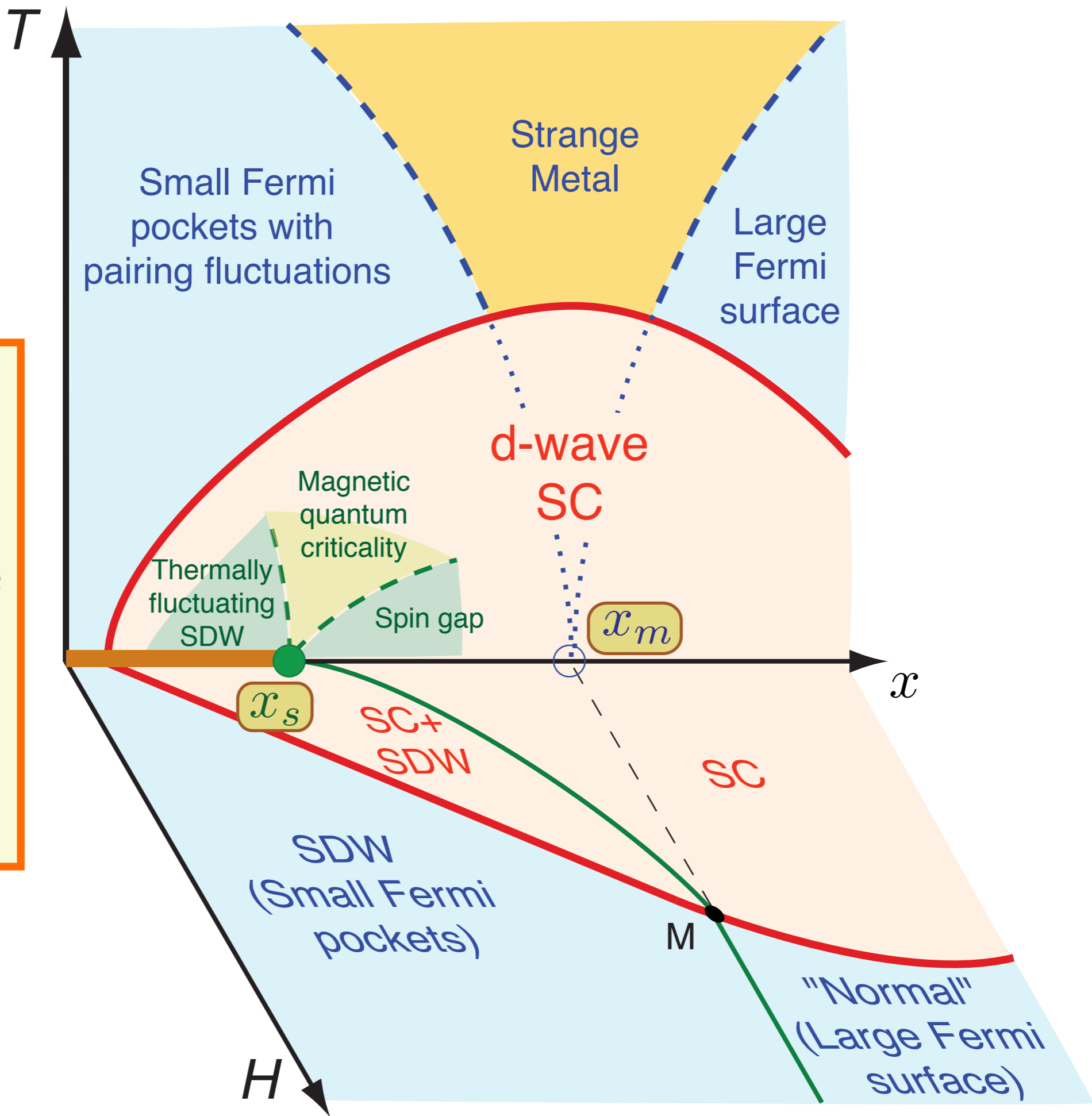
Conjectured phase diagram (assuming a phase with gapless SU(2) photons is possible)







U(I) theory reproduces all features of the phase diagram in the underdoped regime



Elusive optimal doping
quantum critical point has
been “hiding in plain sight”.

It is shifted to lower doping
by the onset of
superconductivity