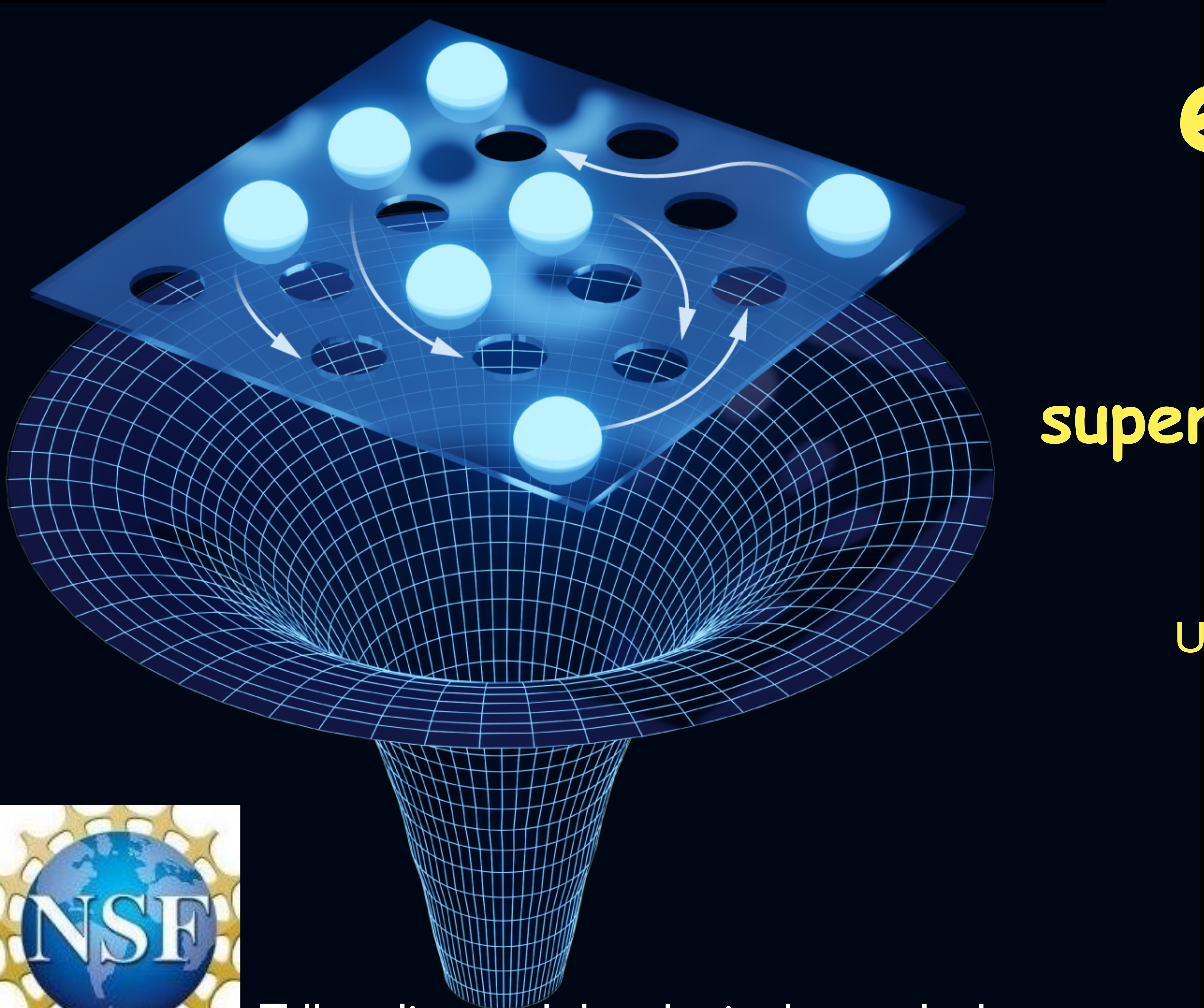
Quantum entanglement in nature:

superconductors and black holes



University of California, Santa Barbara
May 21, 2024

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS
HARVARD

Great discoveries in physics

Quantum entanglement (1865,1935)

Statistical mechanics (1870)

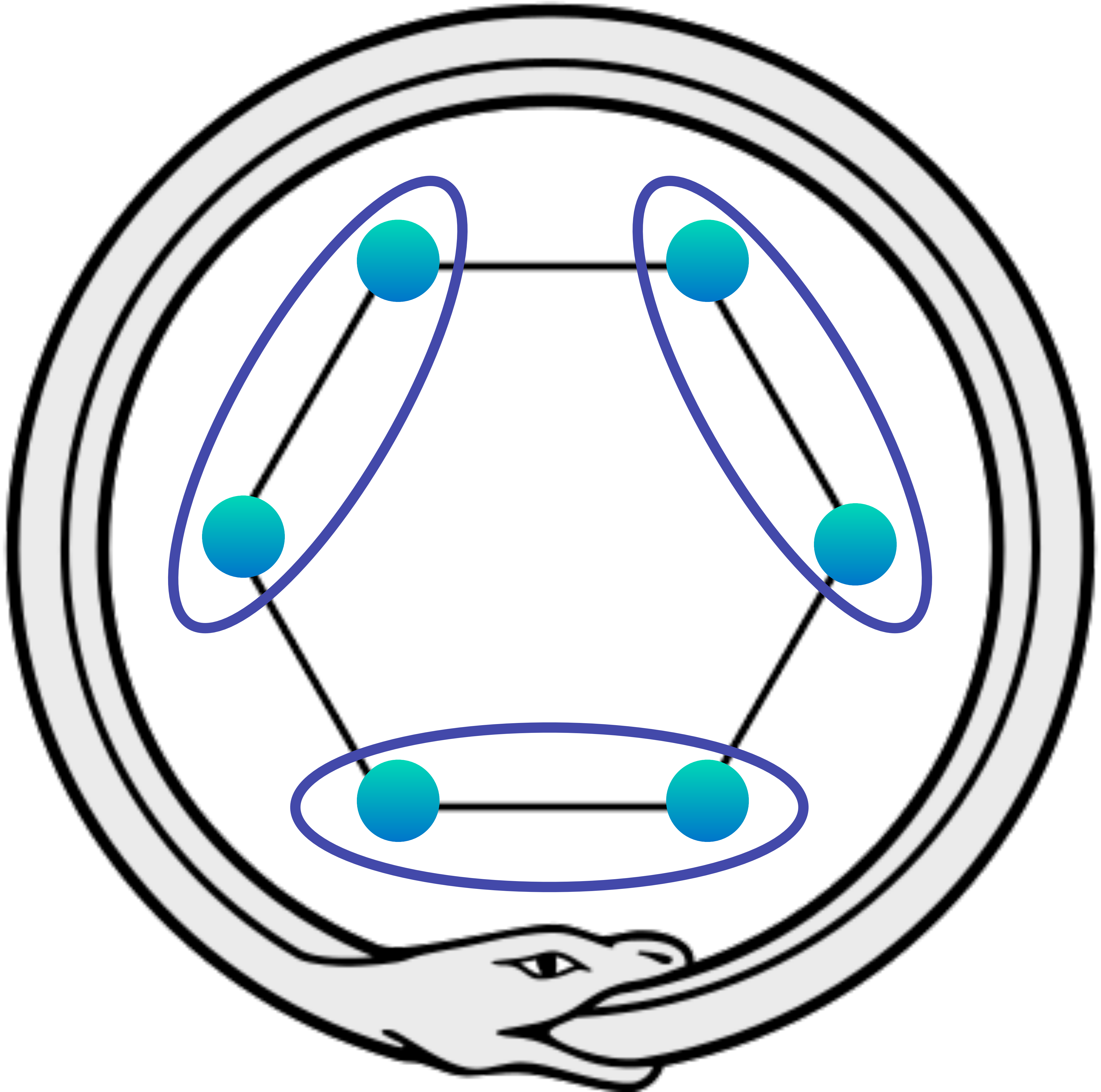
Superconductivity (1911)

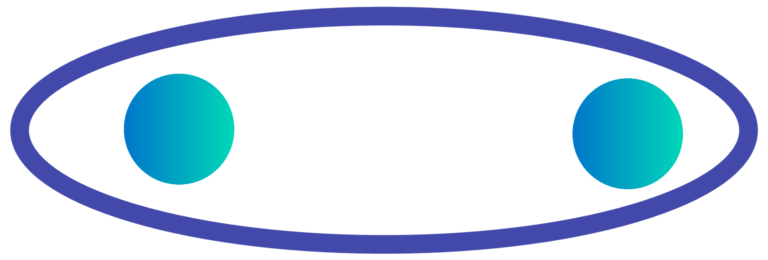
Black holes (1916)

Quantum entanglement (1865, 1935)

Kekule’s spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail *



 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Benzene

*Wikipedia

Kekule's spooky dream (1865)

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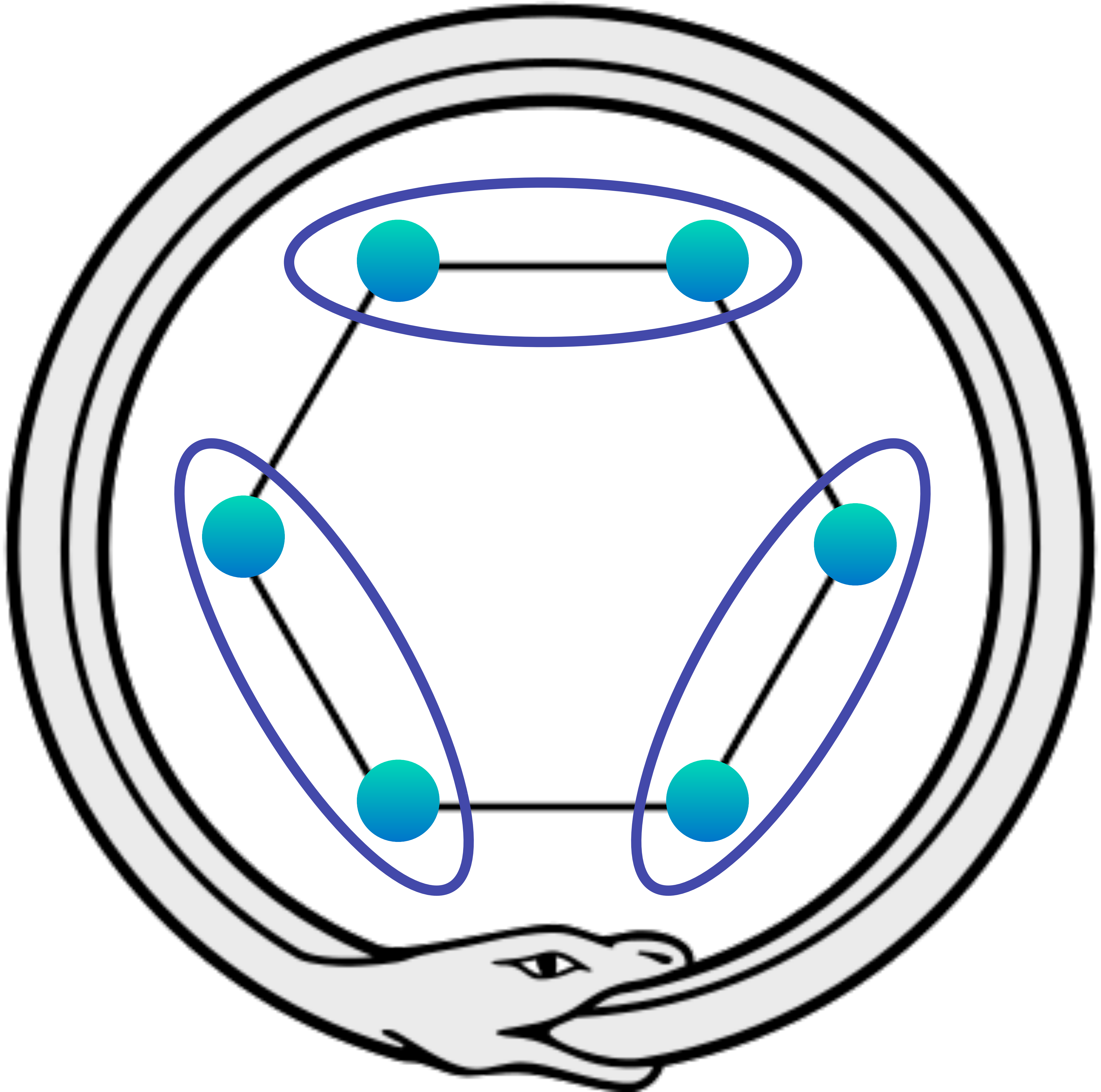
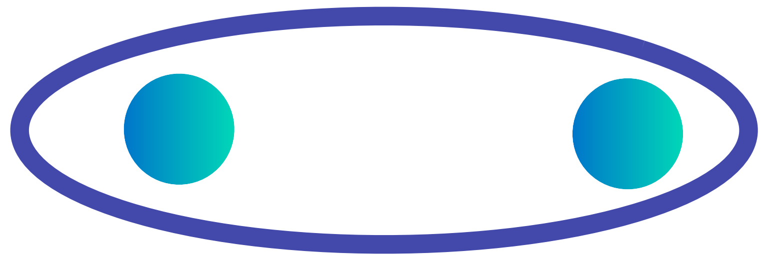


 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Benzene

*Wikipedia

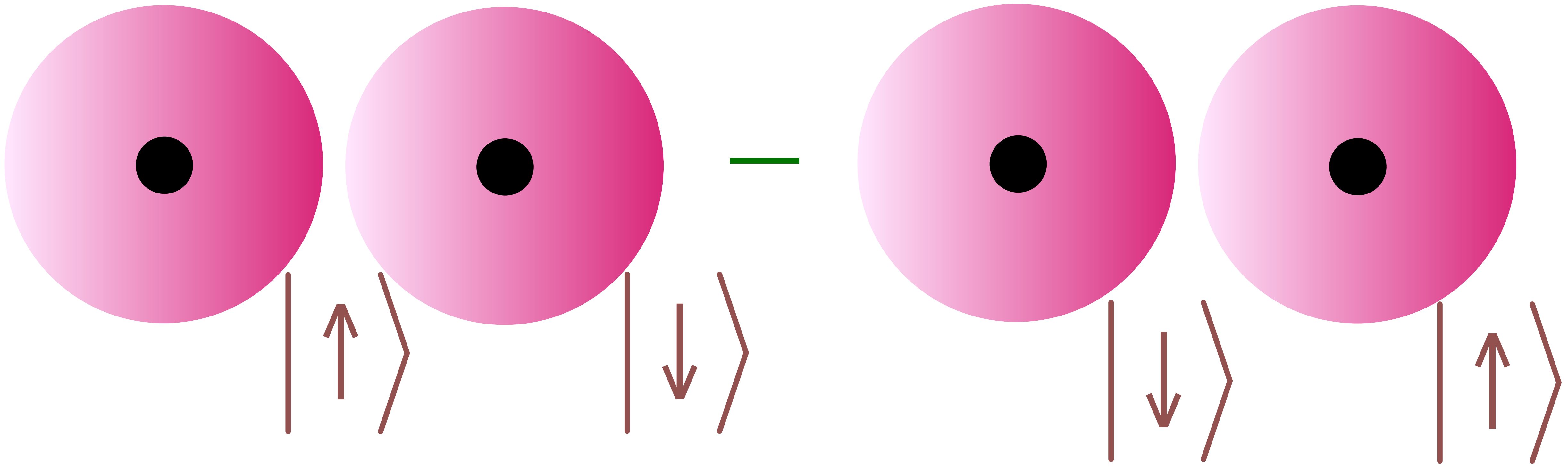
Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



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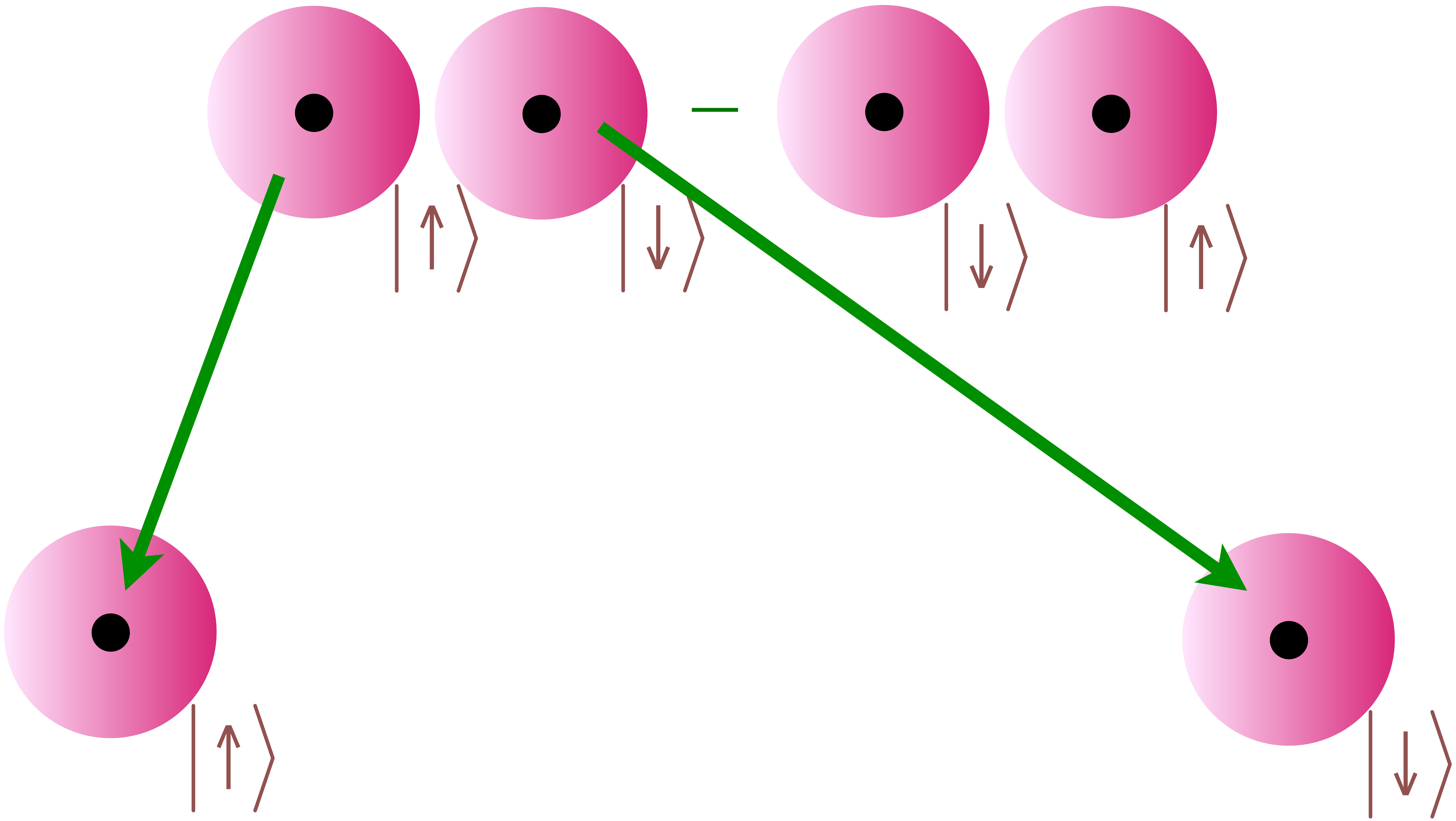

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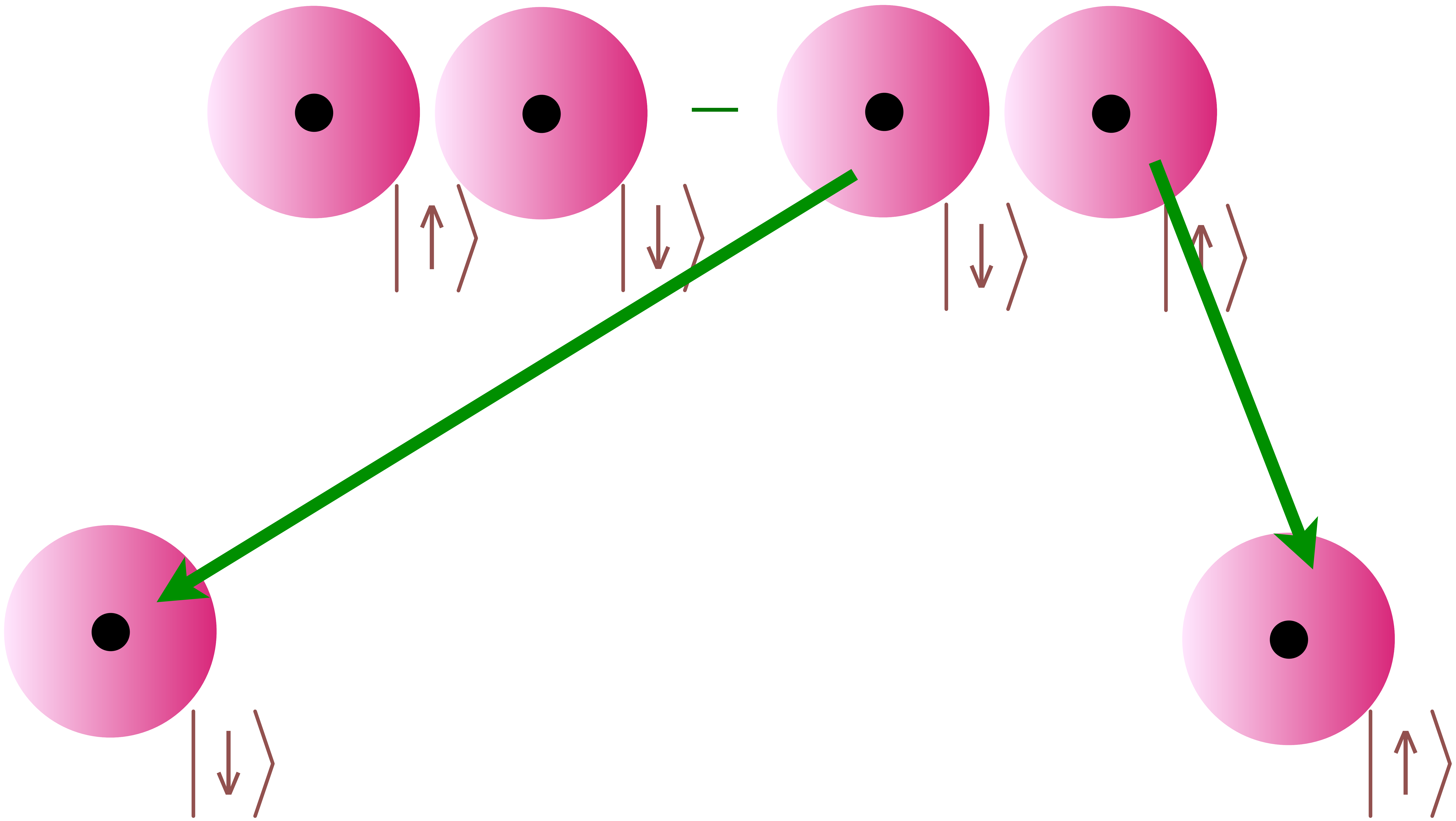

Quantum Entanglement

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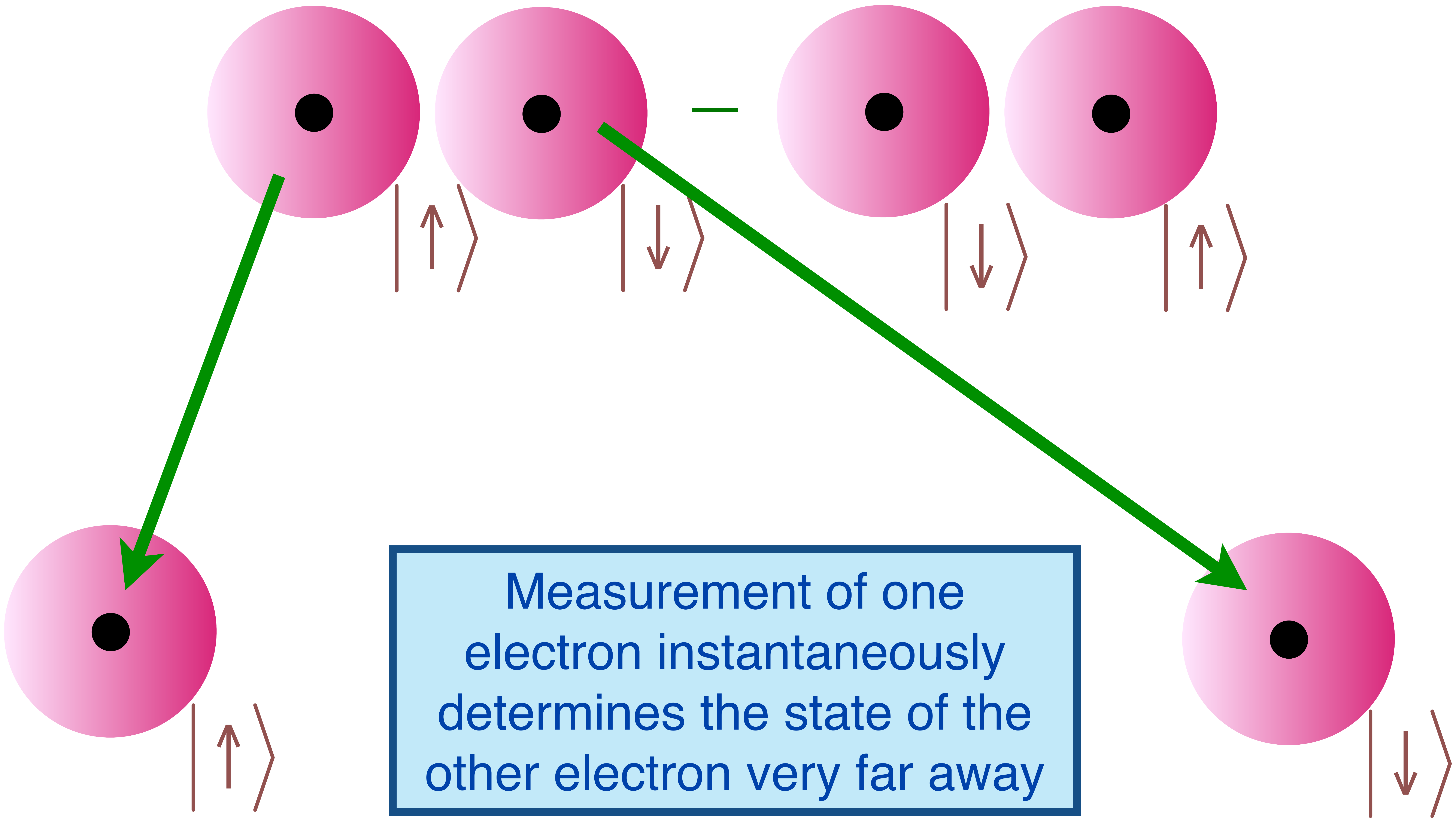
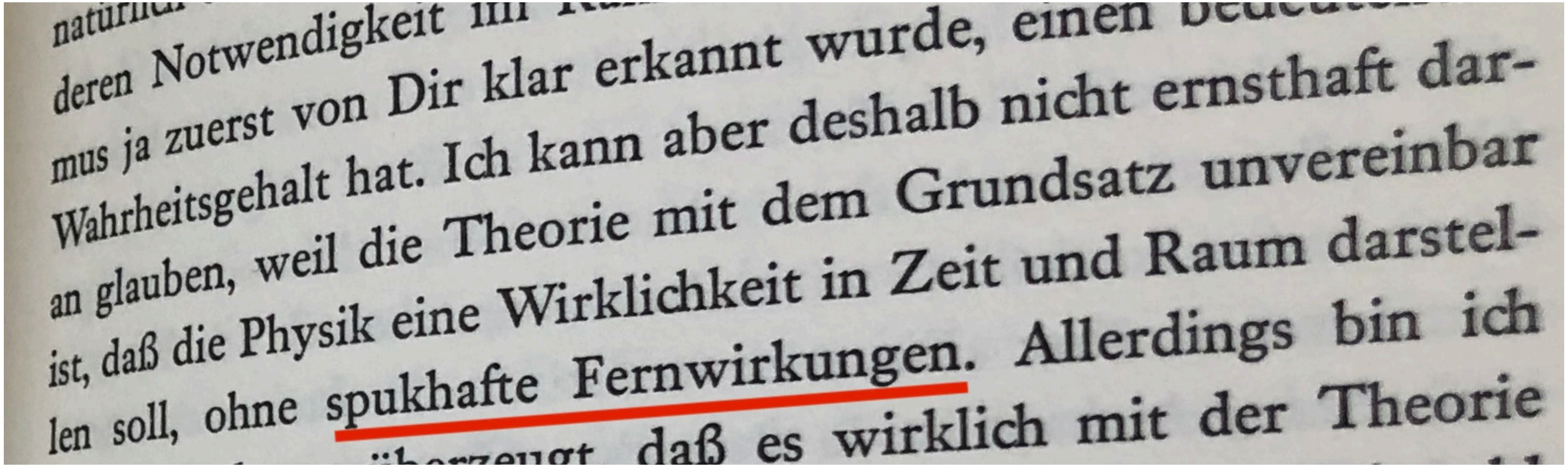


Spooky action at a distance !


deren Notwendigkeit im
mus ja zuerst von Dir klar erkannt wurde, einen
Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft dar-
an glauben, weil die Theorie mit dem Grundsatz unvereinbar
ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstel-
len soll, ohne spukhafte Fernwirkungen. Allerdings bin ich
überzeugt, daß es wirklich mit der Theorie

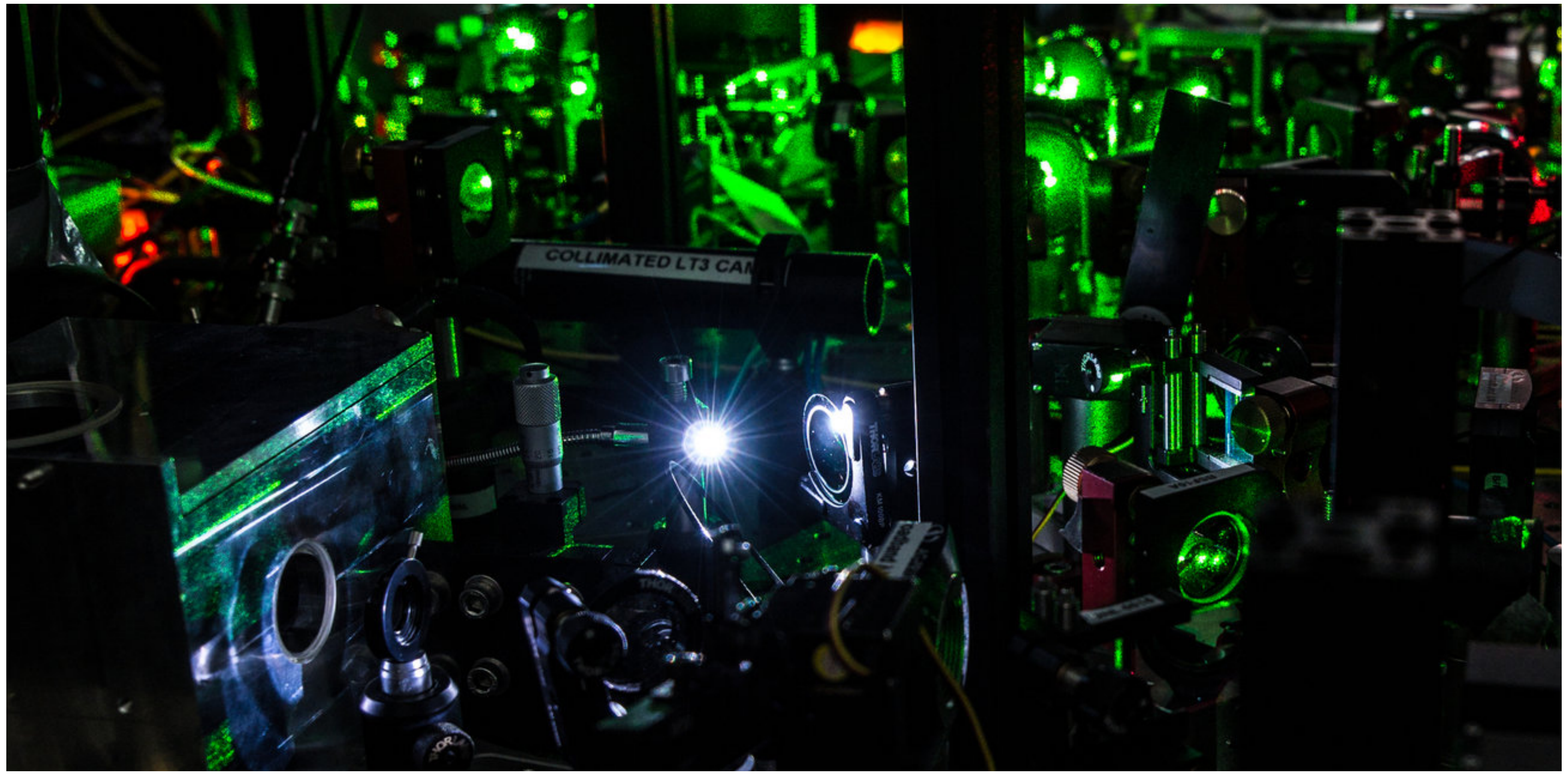
I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

Albert Einstein to Max Born, 3 March 1947

The New York Times

Sorry, Einstein. Quantum Study Suggests ‘Spooky Action’ Is Real.

By JOHN MARKOFF OCT. 21, 2015



Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

Quantum entanglement plays a central role in physics today:

1. Qubit devices (quantum computers)
2. Multi-particle, long-range, quantum entanglement is needed to understand modern quantum materials
3. Hawking's black hole quantum information problem

Great discoveries in physics

Quantum entanglement (1865,1935)

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Superconductivity (1911)

Black holes (1916)

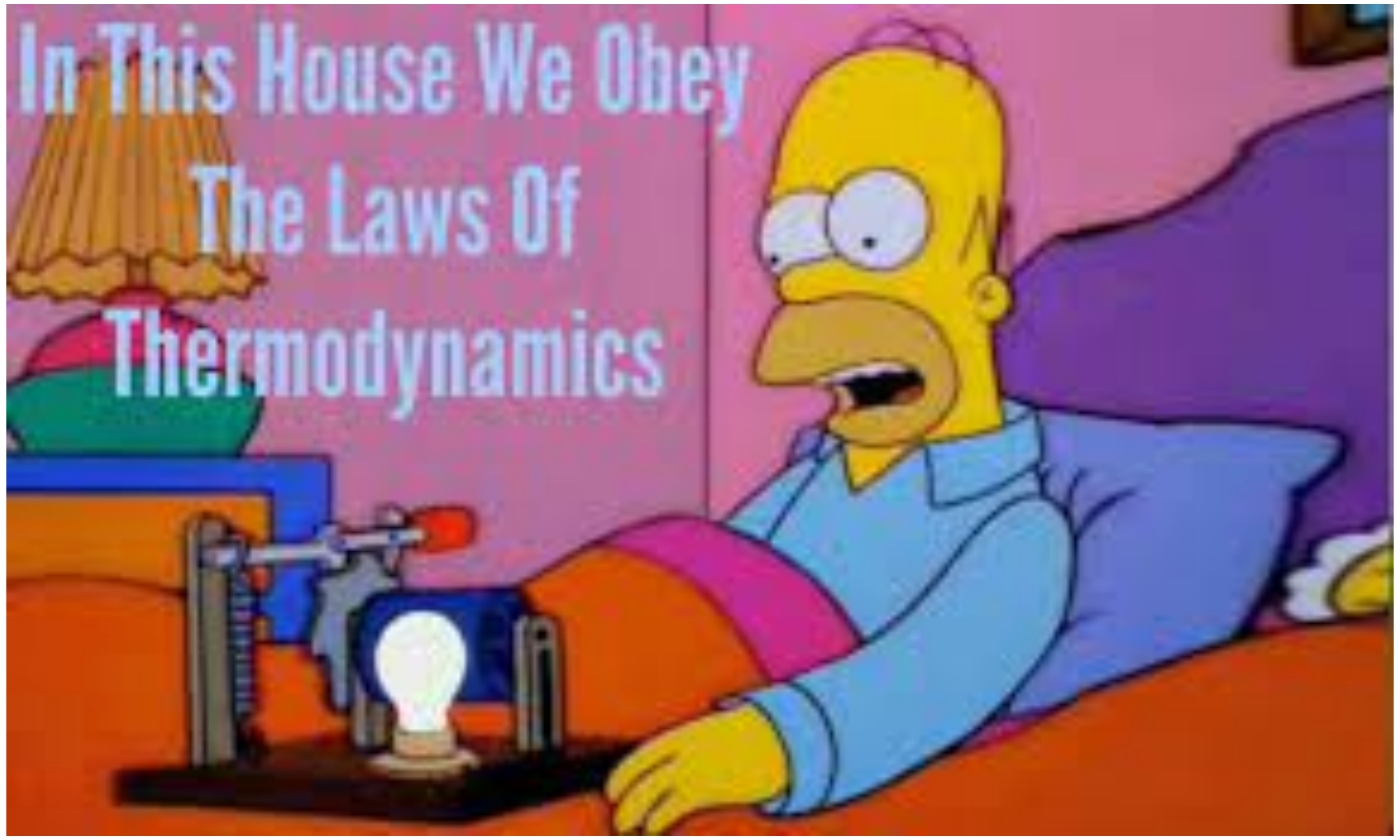
Statistical mechanics (1870)

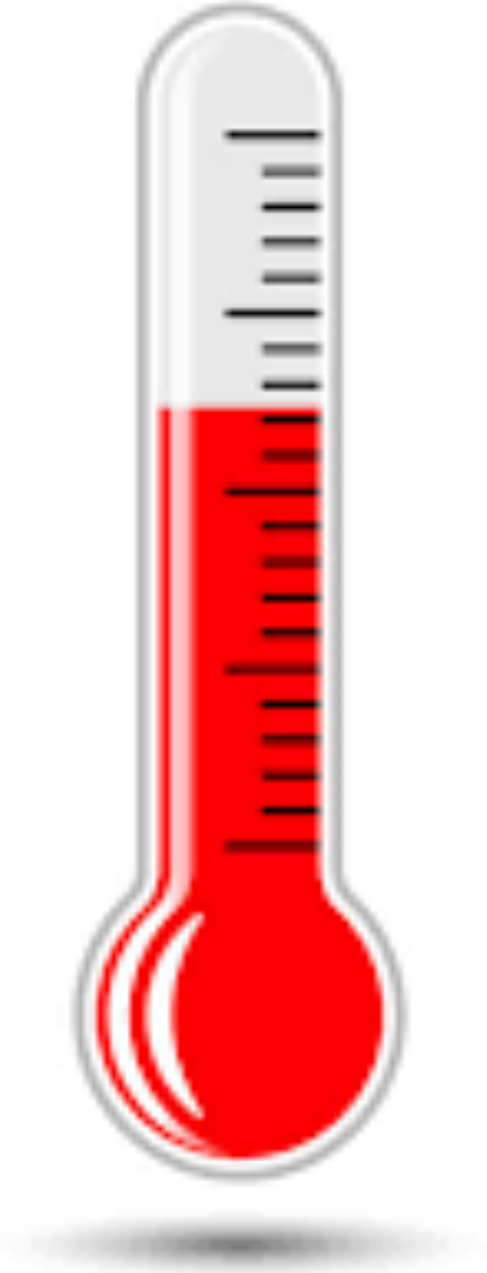
Clausius (1865):

Second Law of Thermodynamics:

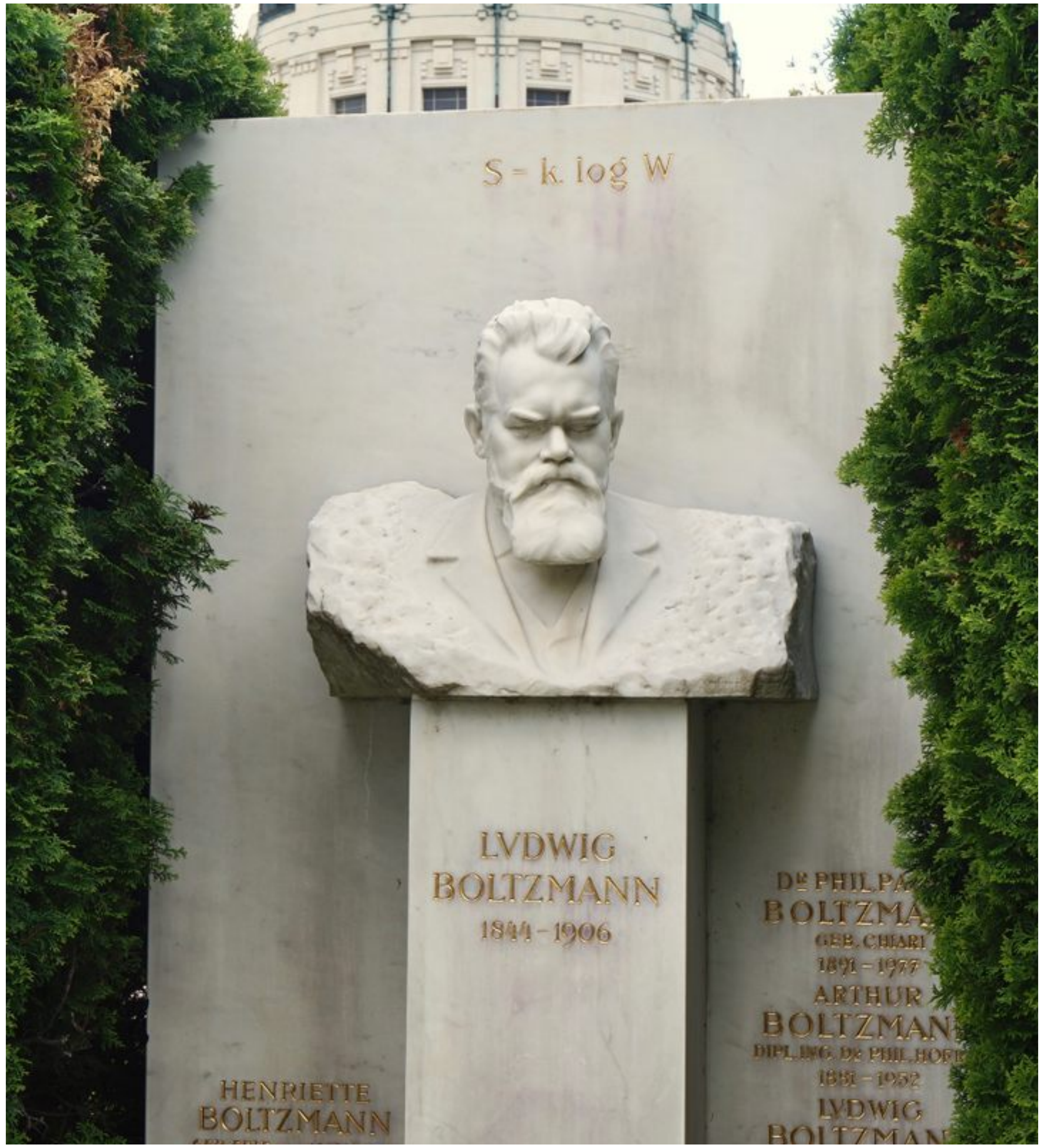
Every macroscopic system has an "entropy" which cannot decrease.

➜ No perpetual motion machines!





Temperature



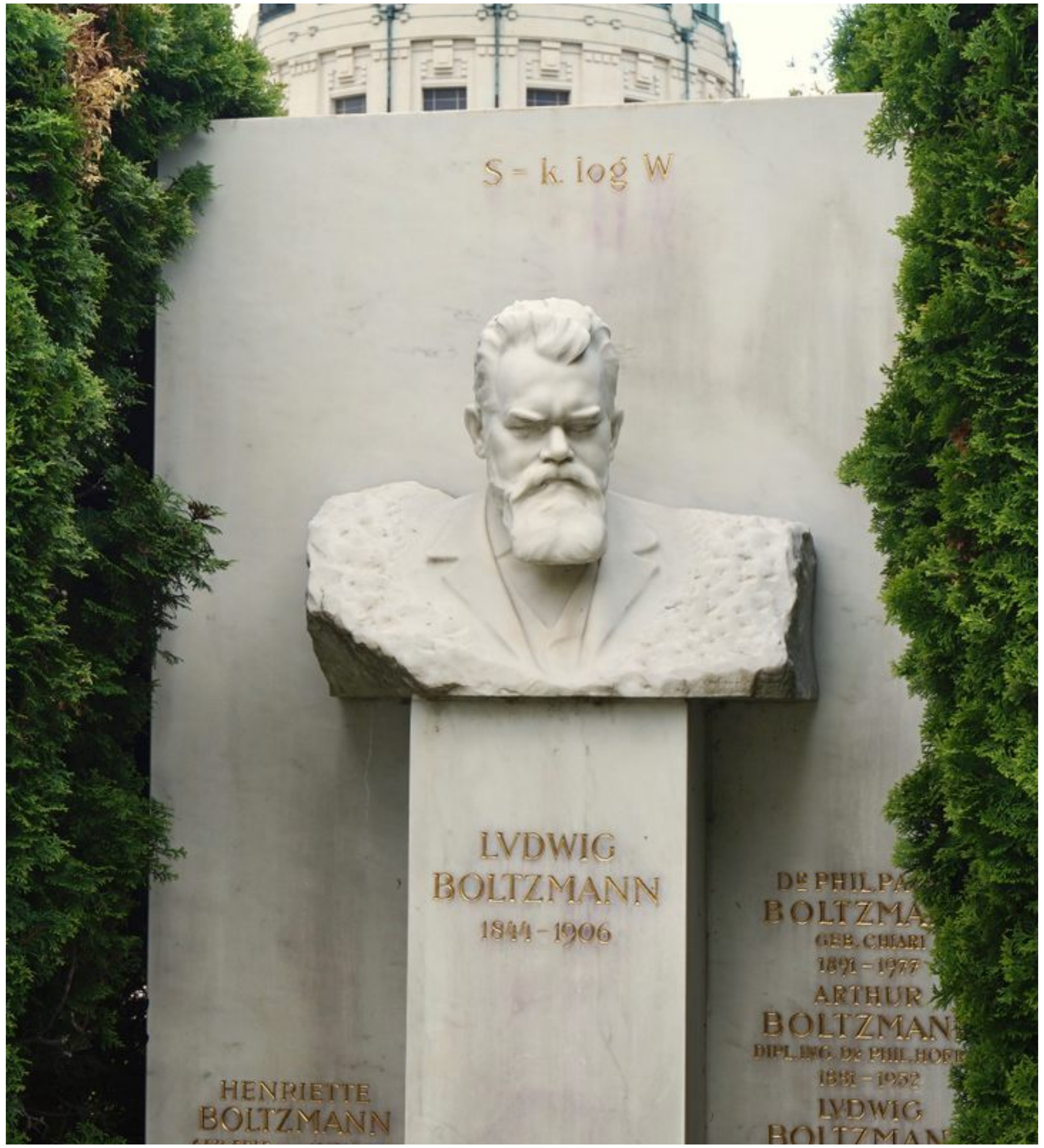
Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states $D(E) = \exp(S(E)/k_B)$

$$\frac{1}{T} = \frac{dS}{dE}$$



Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

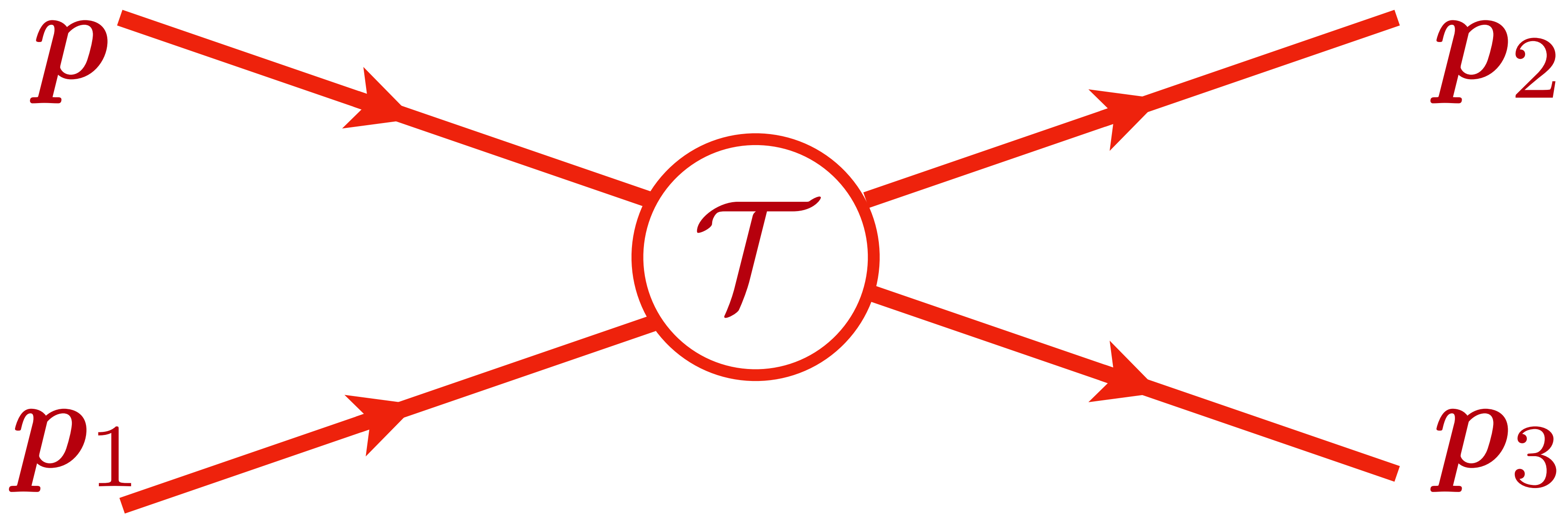
Boltzmann equation (1872)
Dilute classical gas

Molecular chaos: successive collisions are statistically independent

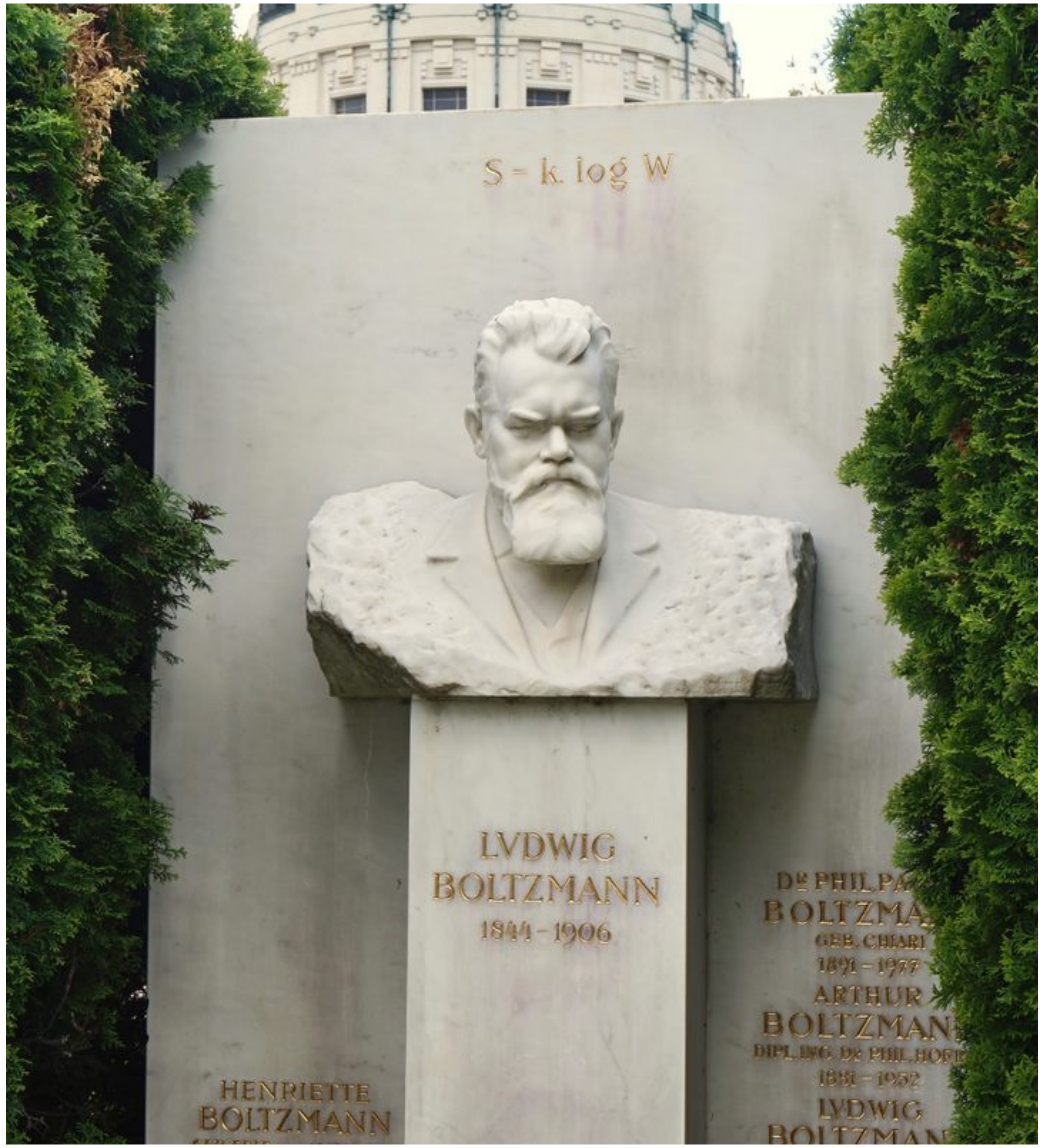
$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$

$$-2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$

$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} \right]$$



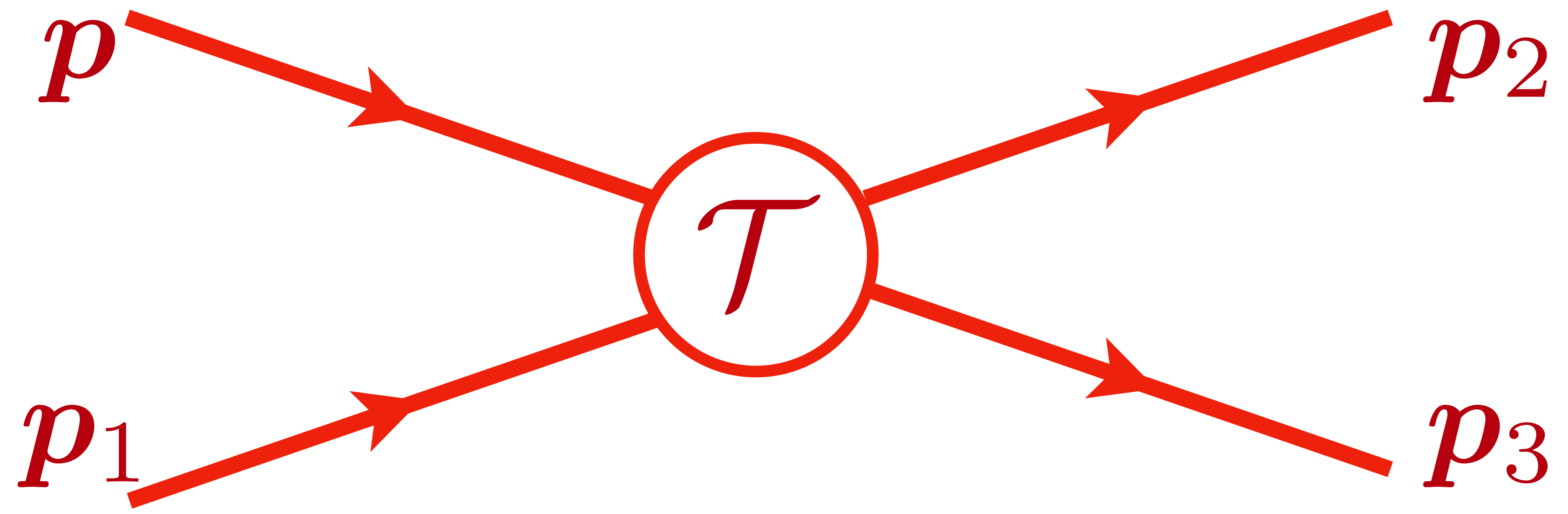
Quantum Boltzmann equation (Landau)
Dense gas of electrons



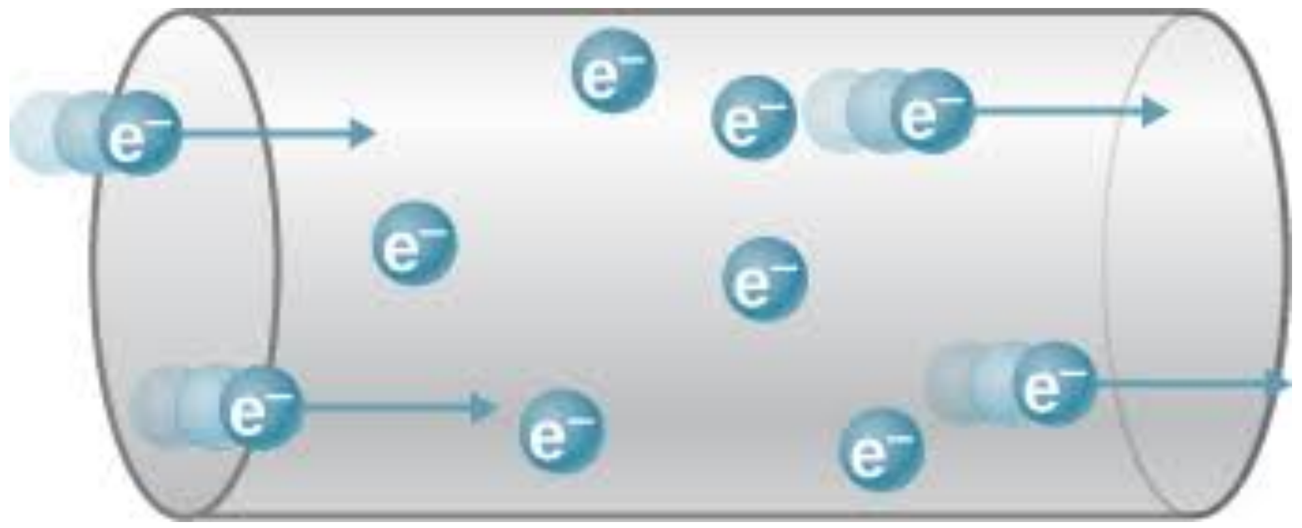
Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Neglects quantum interference (entanglement)
between successive collisions

$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$
$$- 2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$
$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} (1 - f_{\boldsymbol{p}_2})(1 - f_{\boldsymbol{p}_3}) - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} (1 - f_{\boldsymbol{p}})(1 - f_{\boldsymbol{p}_1}) \right]$$



Current flow with electrons in ordinary metals



Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

The long scattering time implies that individual electrons are well-defined.

Great discoveries in physics

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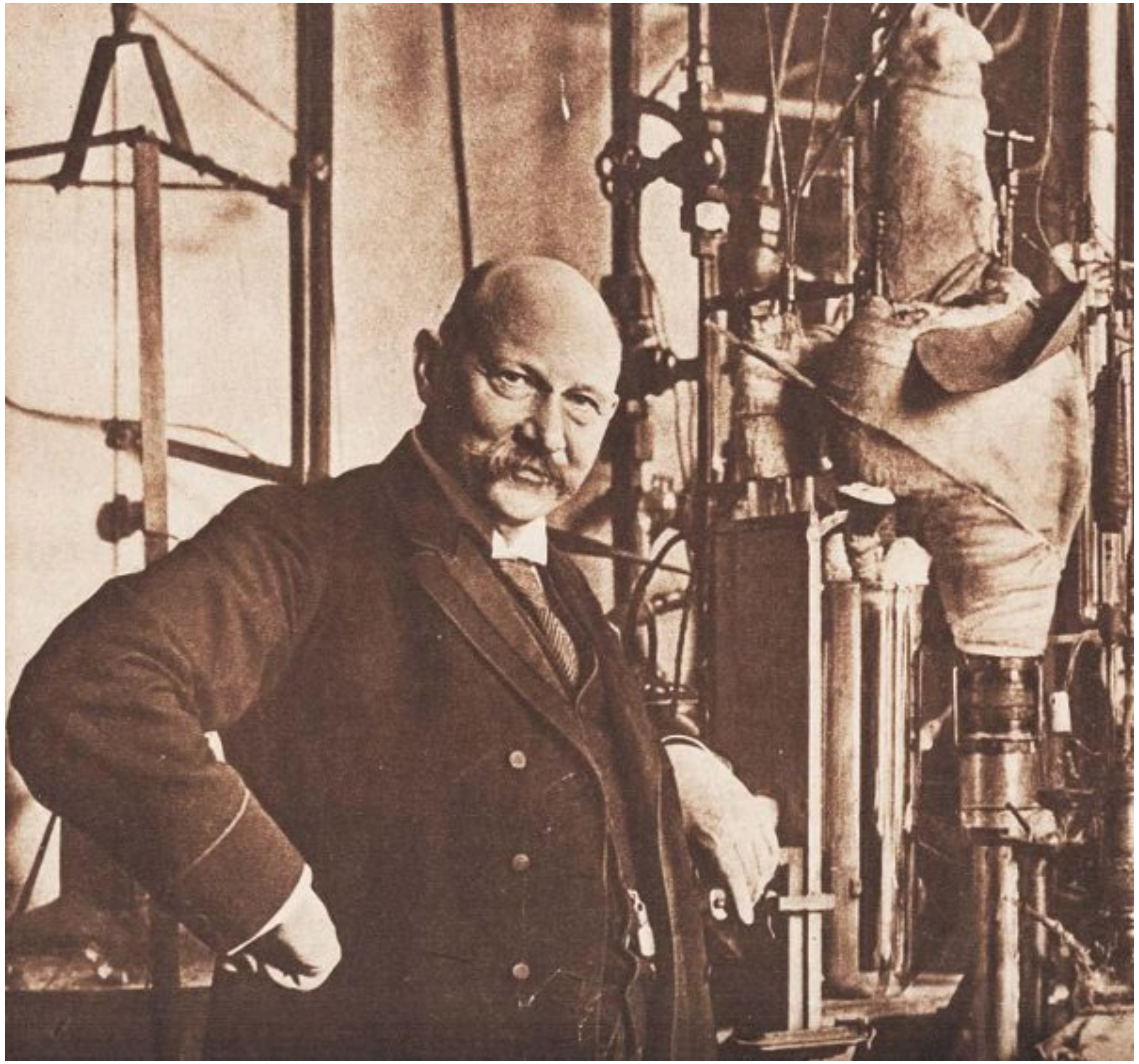
Superconductivity (1911)

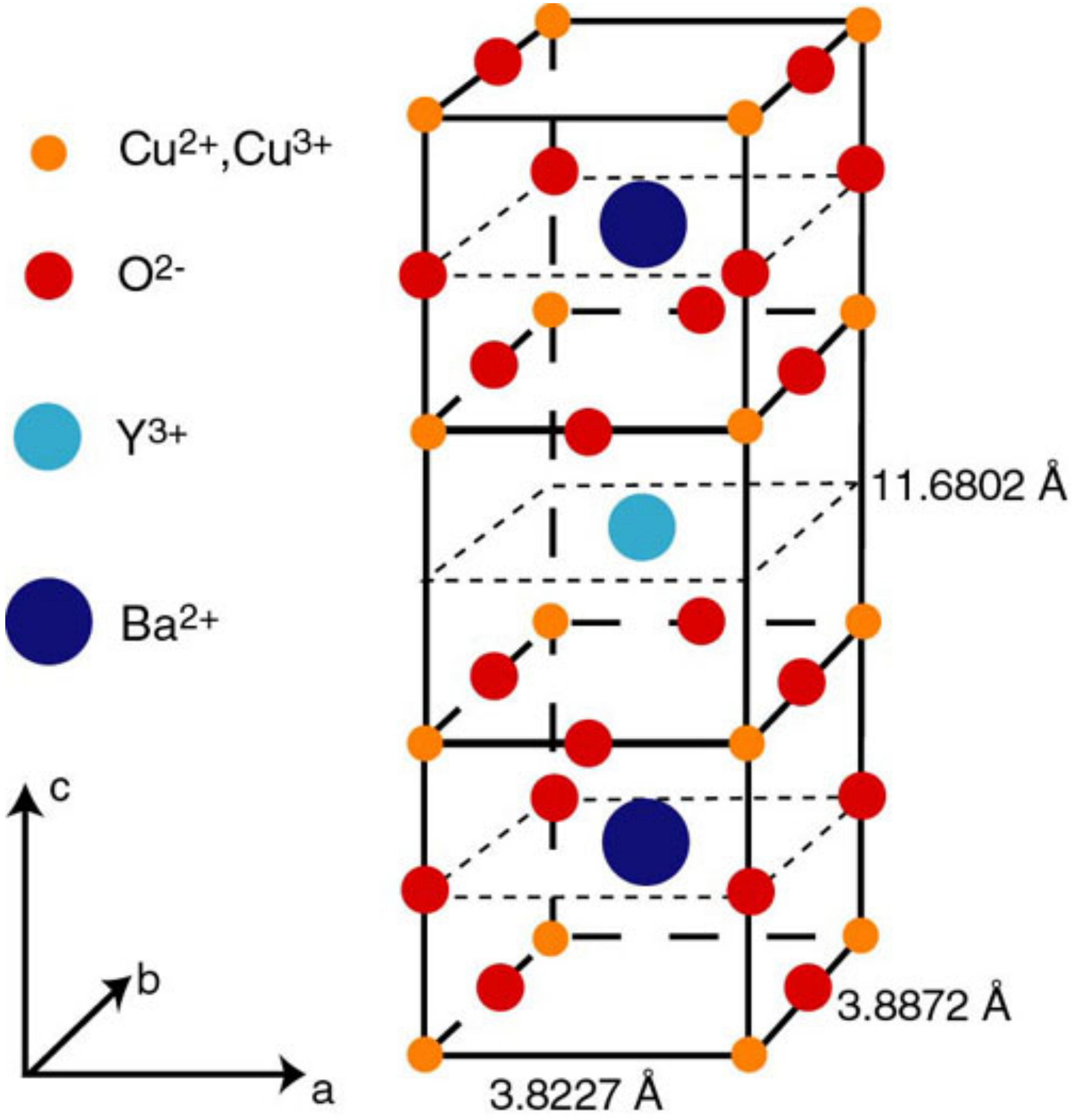
Black holes (1916)

Superconductors (1911- today)

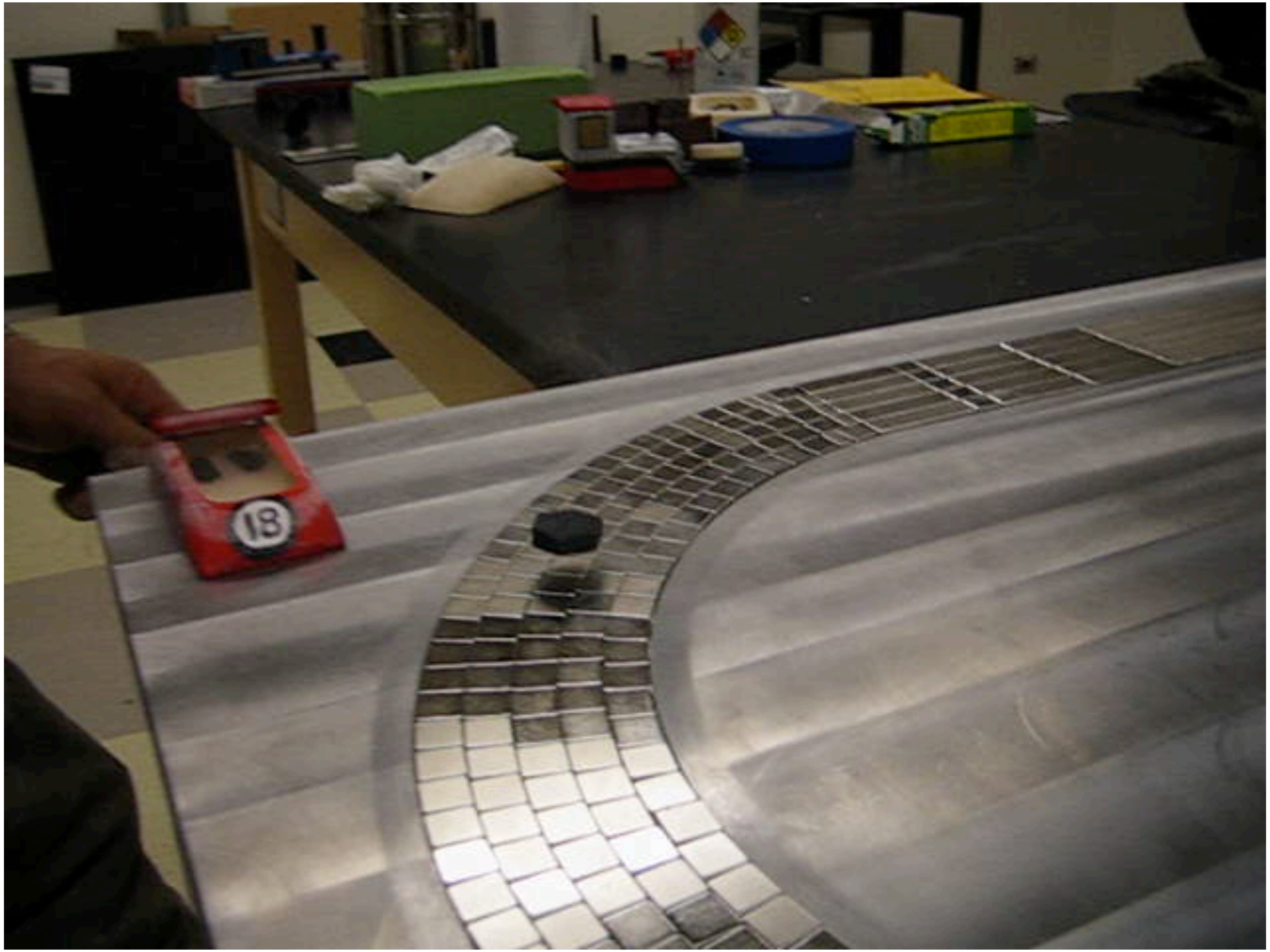


Kamerlingh Onnes 1911:
Mercury is a superconductor below -269 °C

Cuprate high temperature superconductors



$YBa_2Cu_3O_{6+x}$



Nd-Fe-B magnets, YBaCuO superconductor

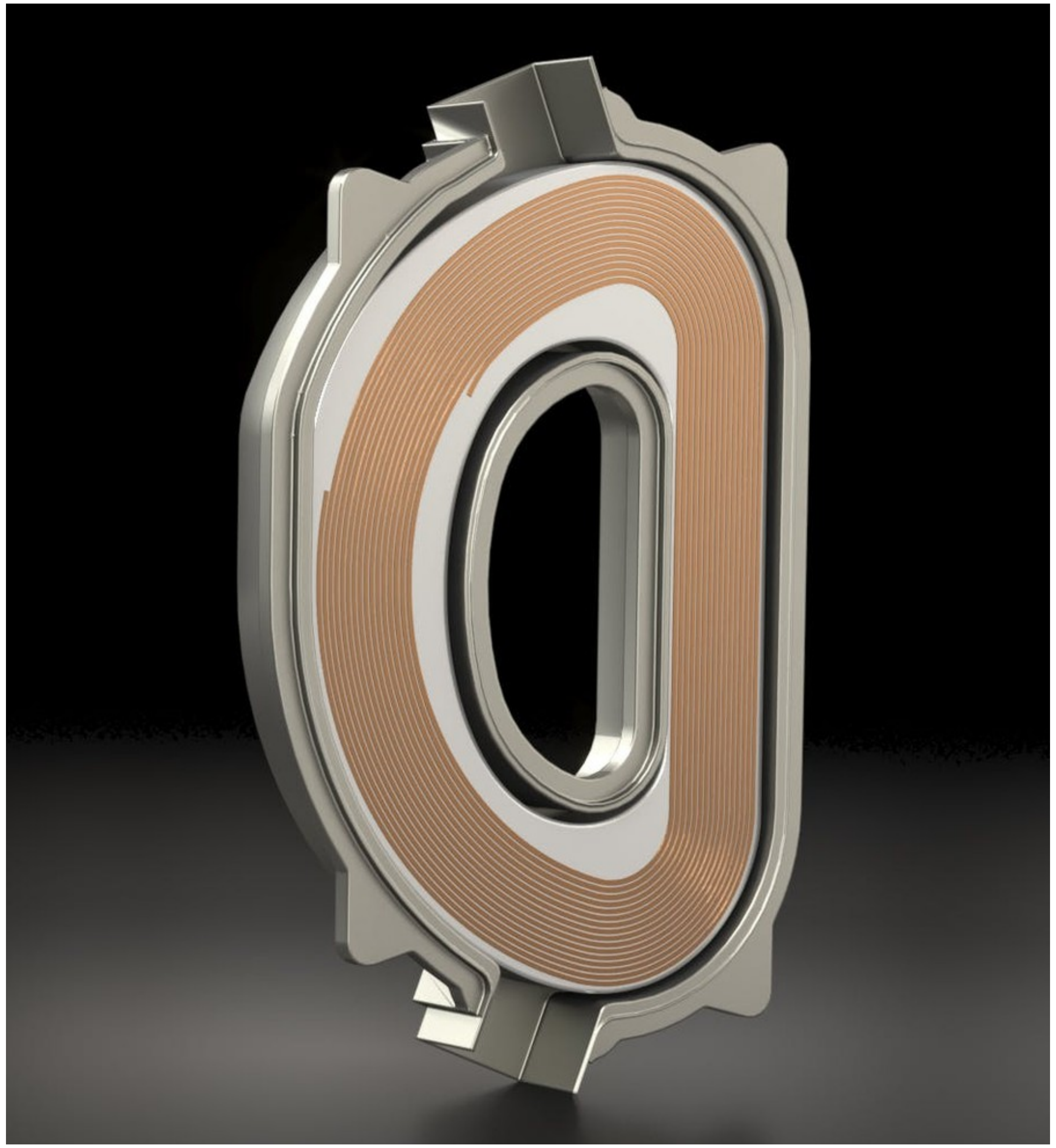
Julian Hetel and Nandini Trivedi, Ohio State University

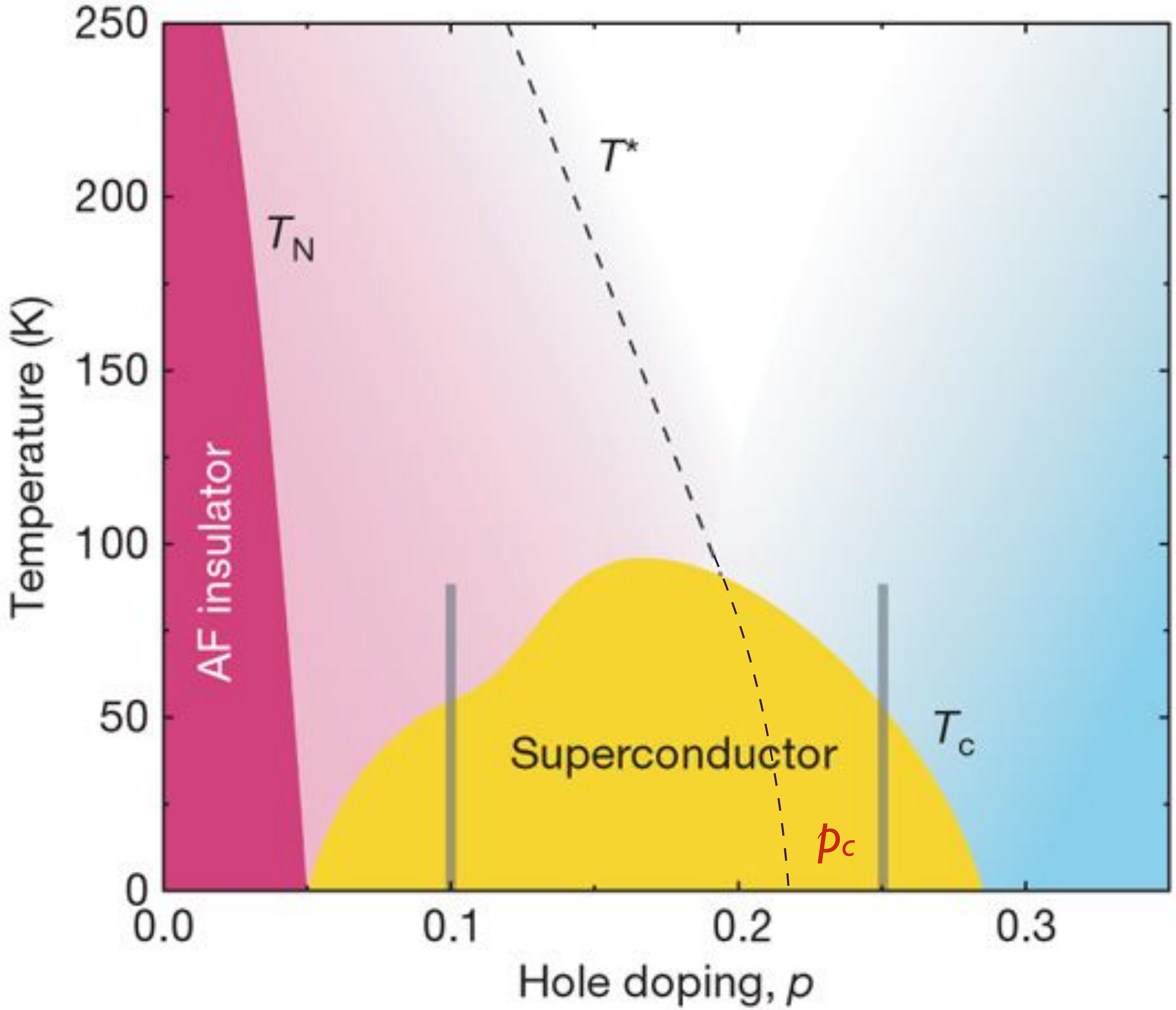
HTS Magnets: Enabling Technology

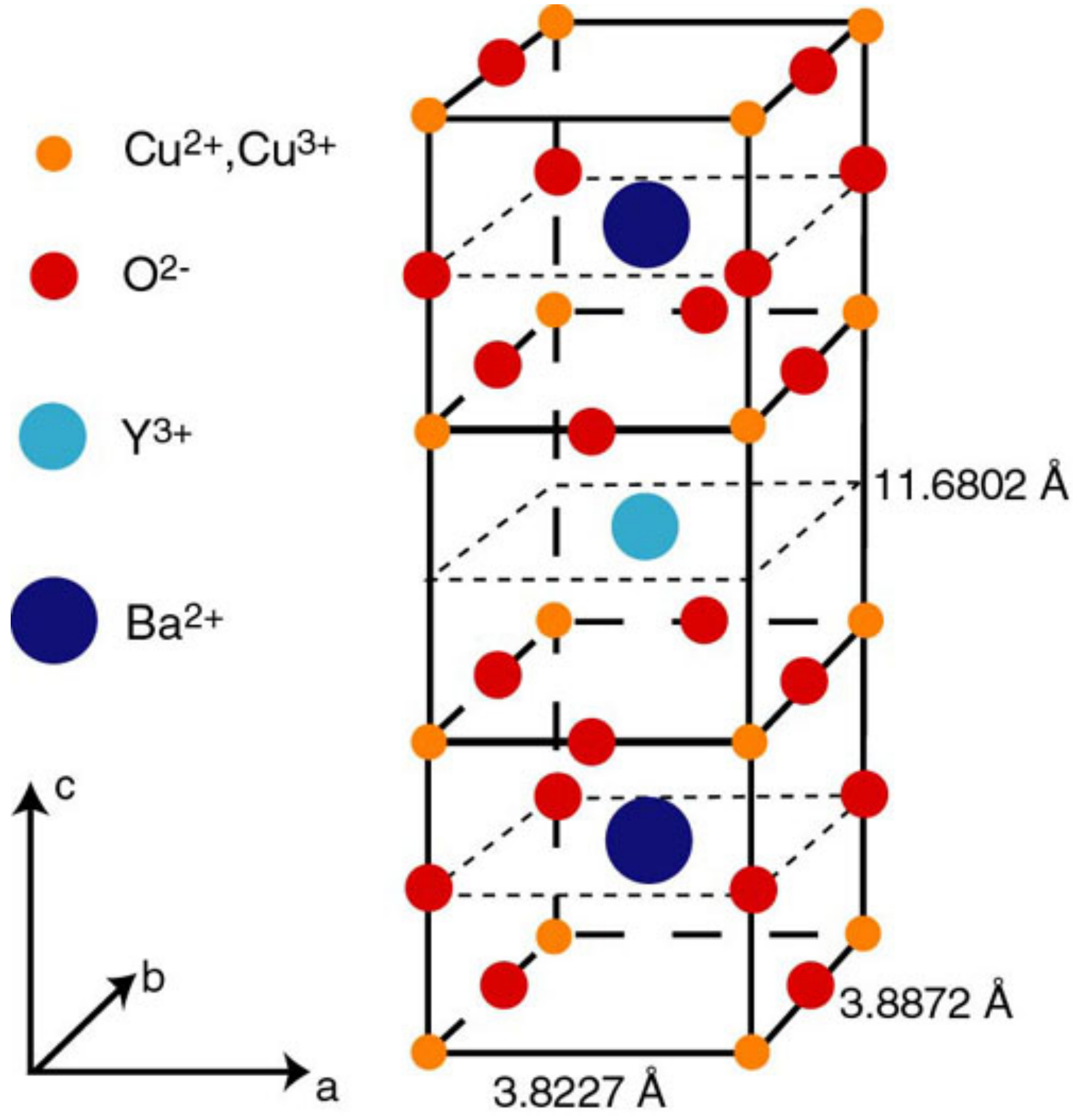
The surest path to limitless, clean, fusion energy

YBCO magnets allow for smaller, faster, and less expensive tokamaks for plasma fusion

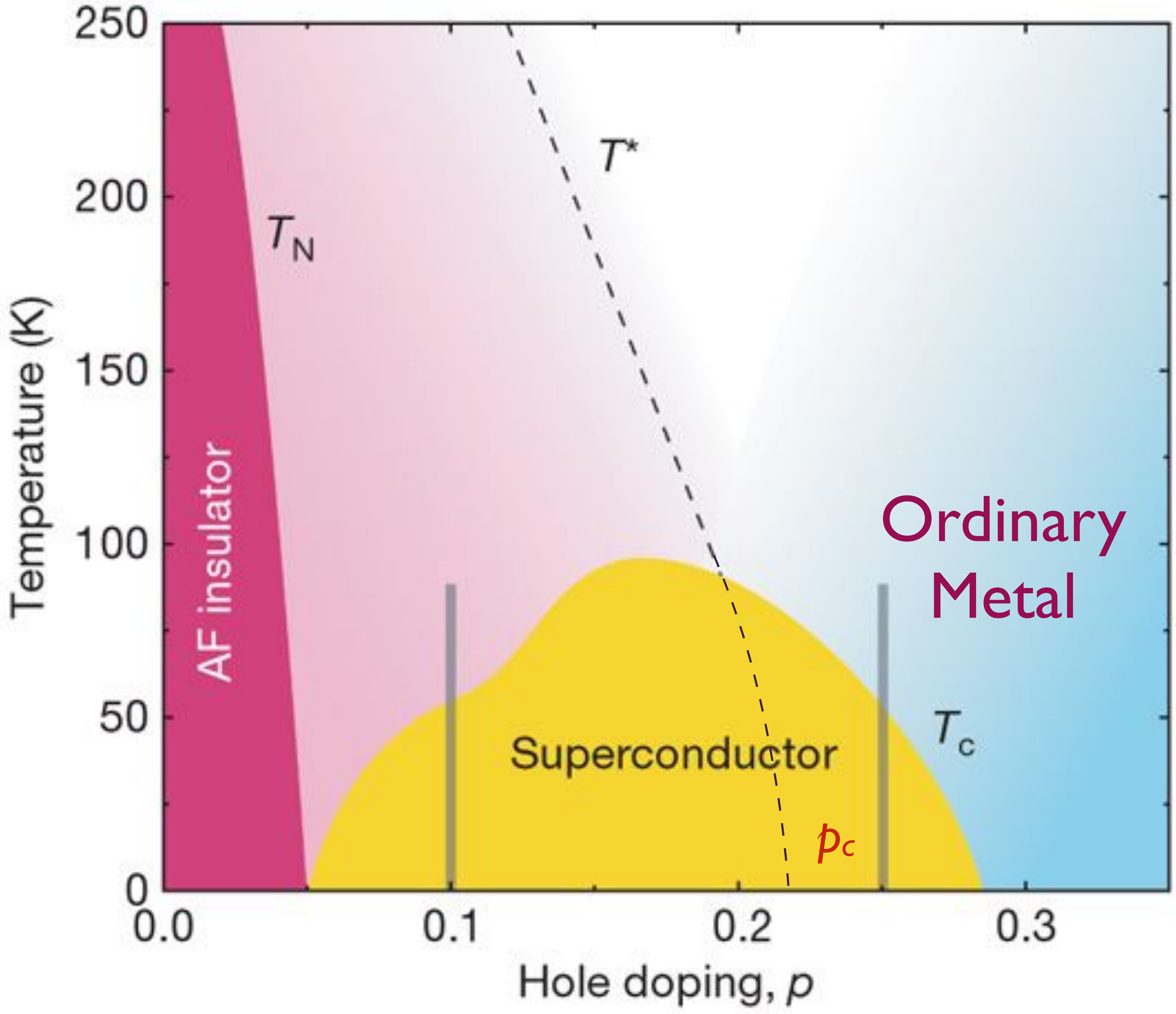
Commonwealth Fusion Systems

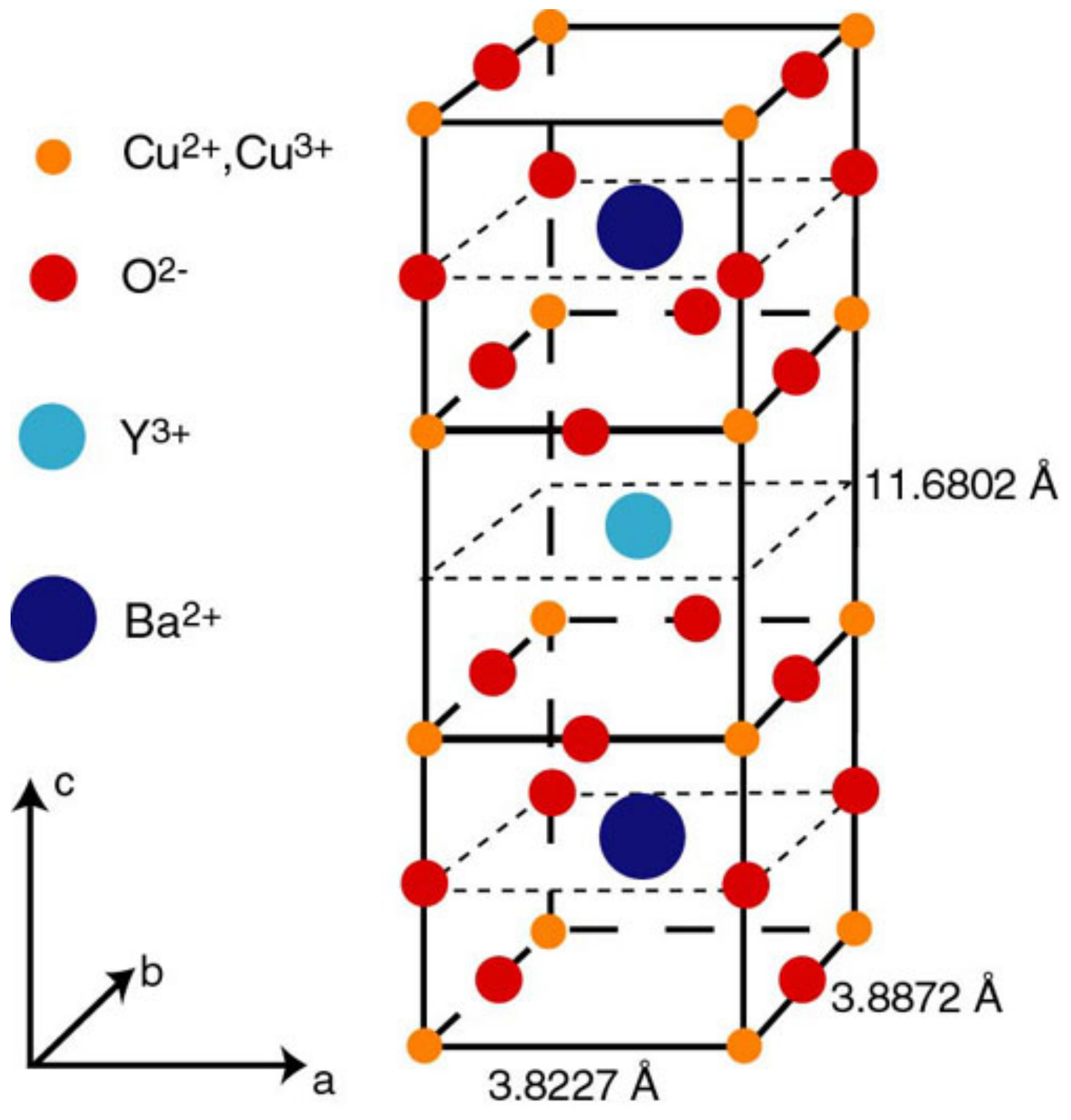



250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Superconductor
T_c
p_c

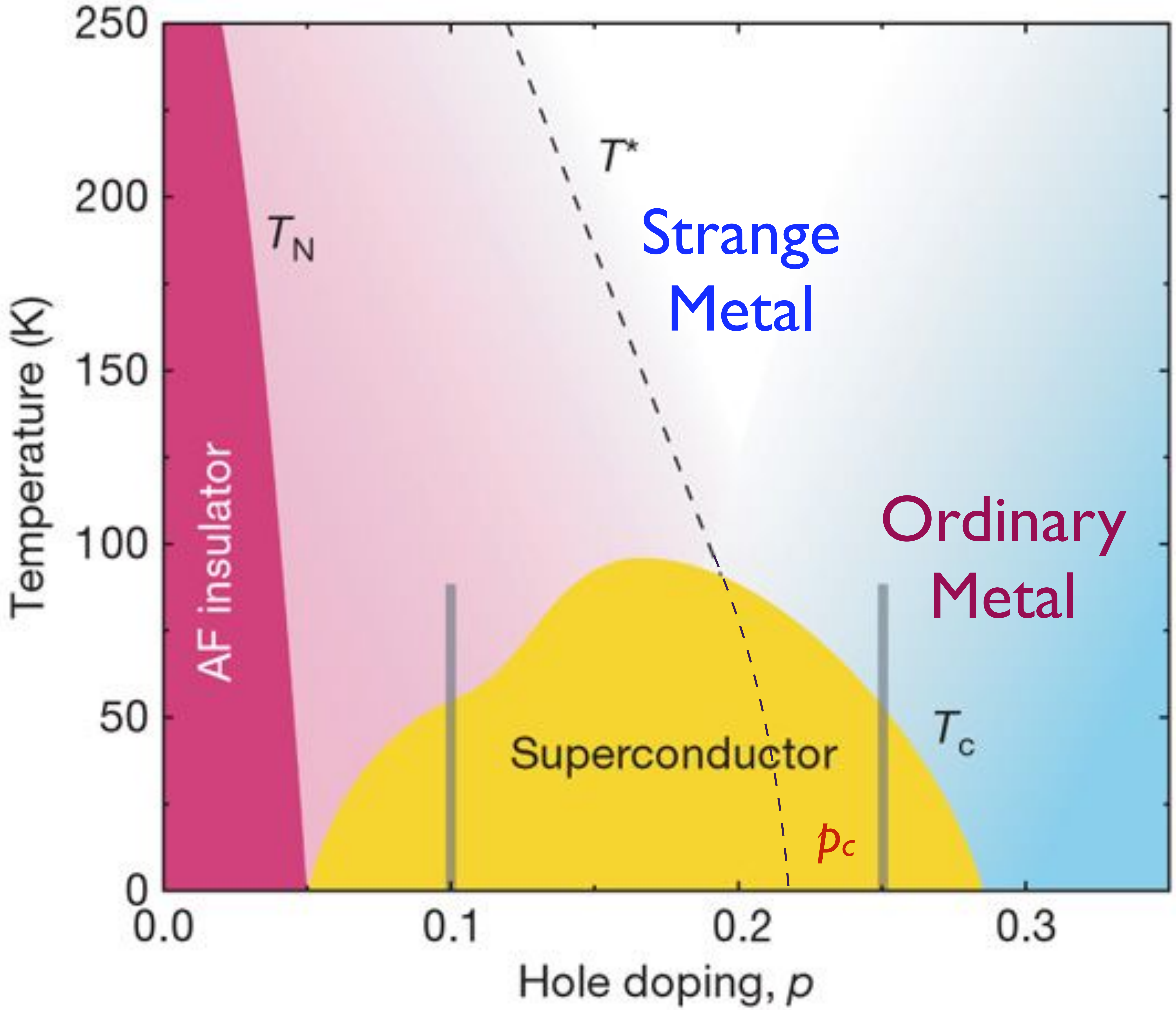
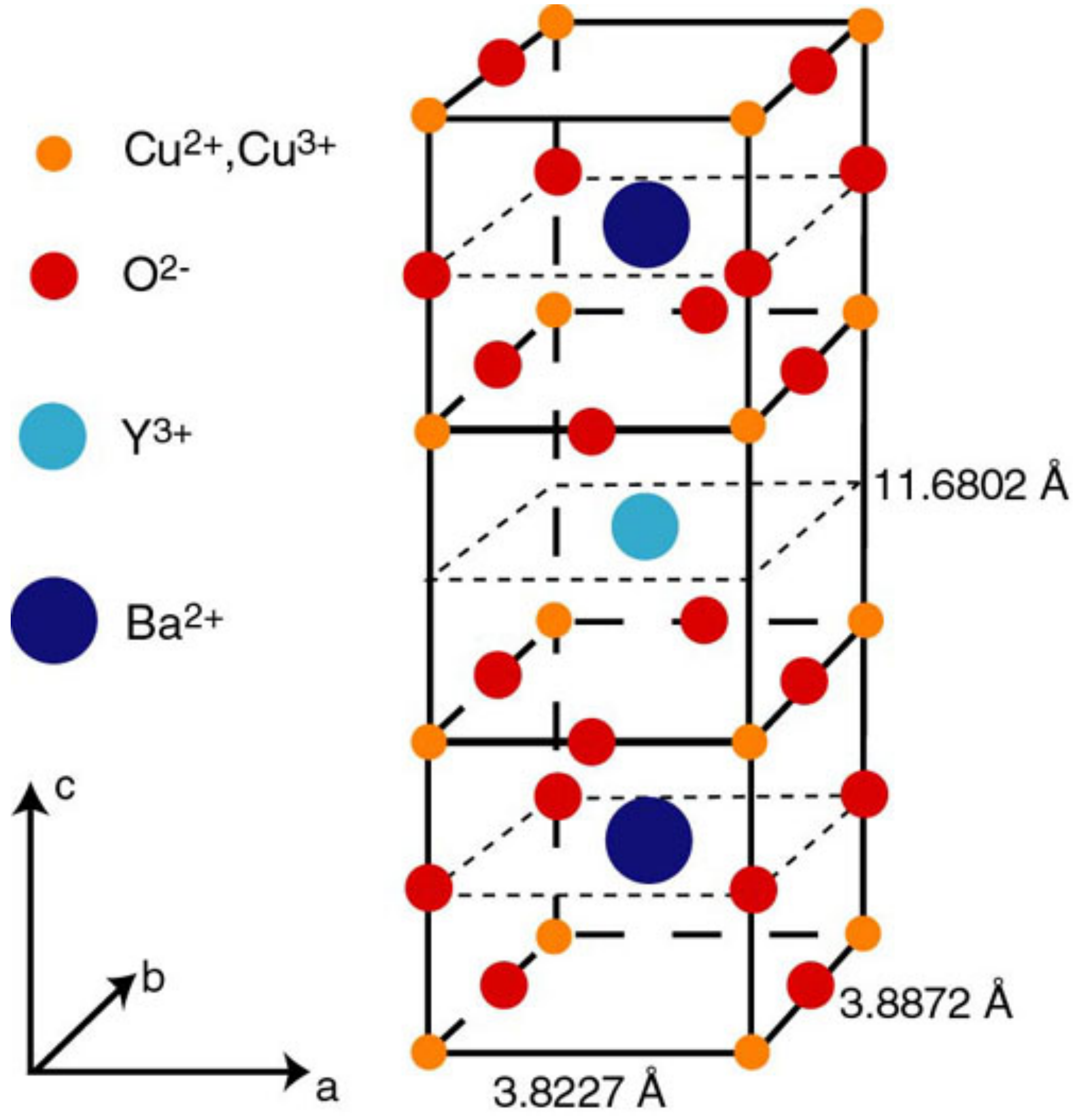

Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$





$$\mathrm{YBa_2Cu_3O_{6+\mathit{x}}}$$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Ordinary Metal
Superconductor
T_c
p_c

Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

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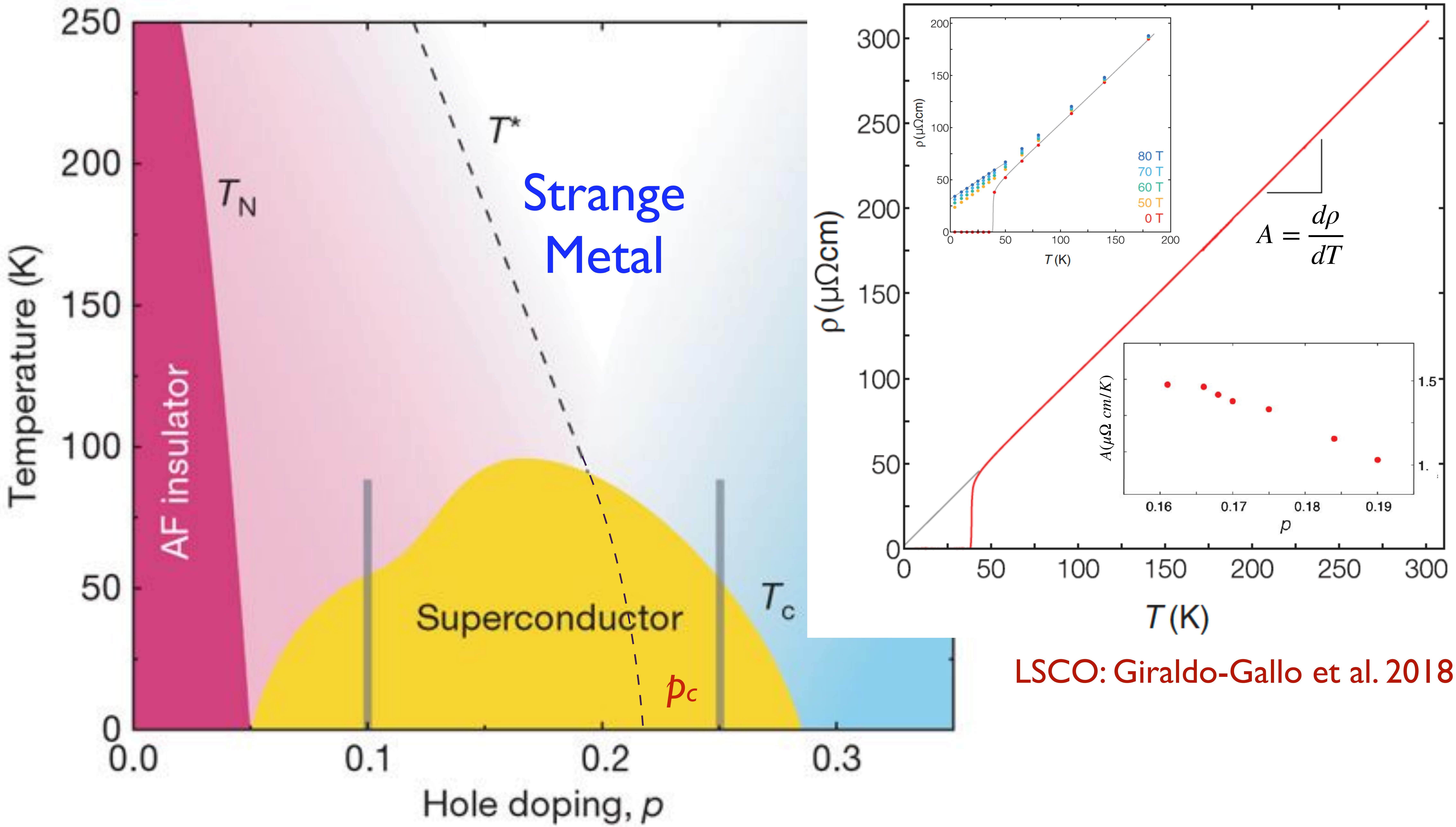


LSCO: Giraldo-Gallo et al. 2018

Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

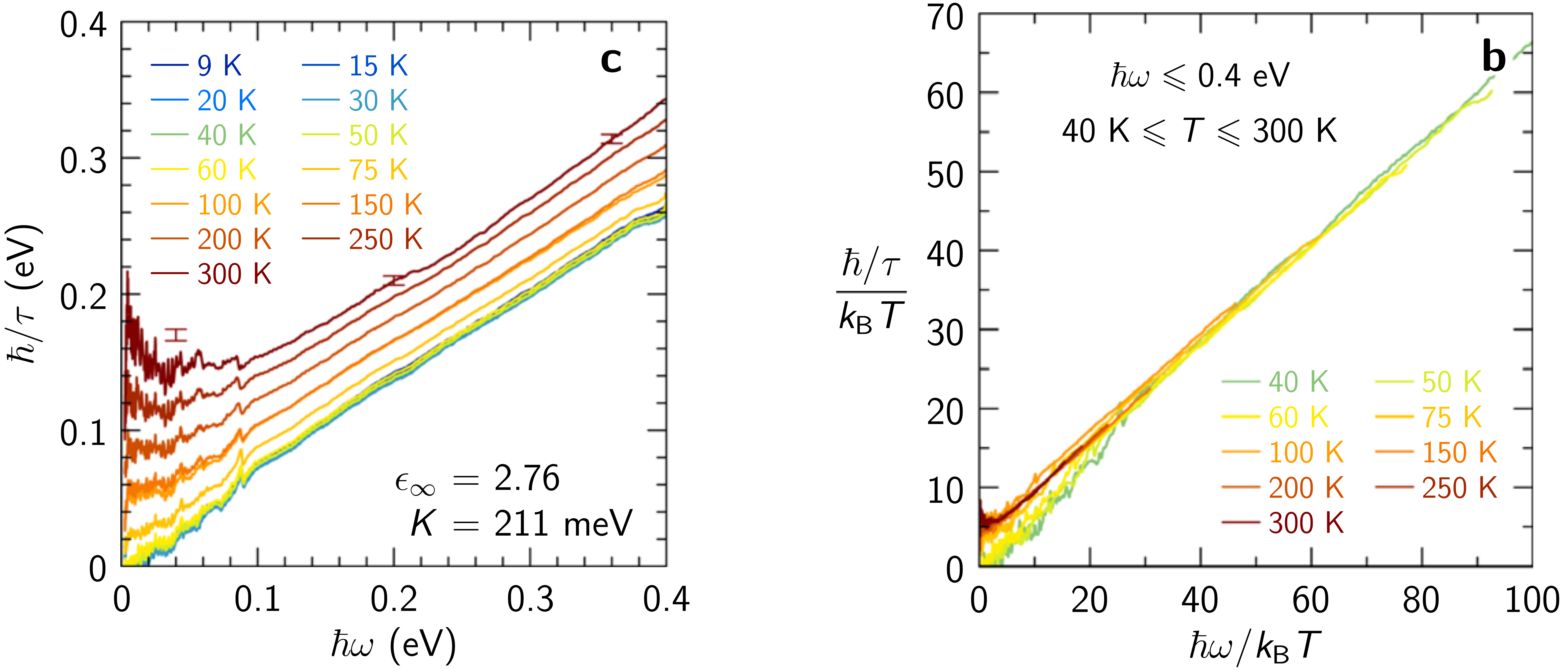
B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$

Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_BT}F\left(\frac{\hbar\omega}{k_BT}\right)$$



Great discoveries in physics

Quantum entanglement (1865,1935)

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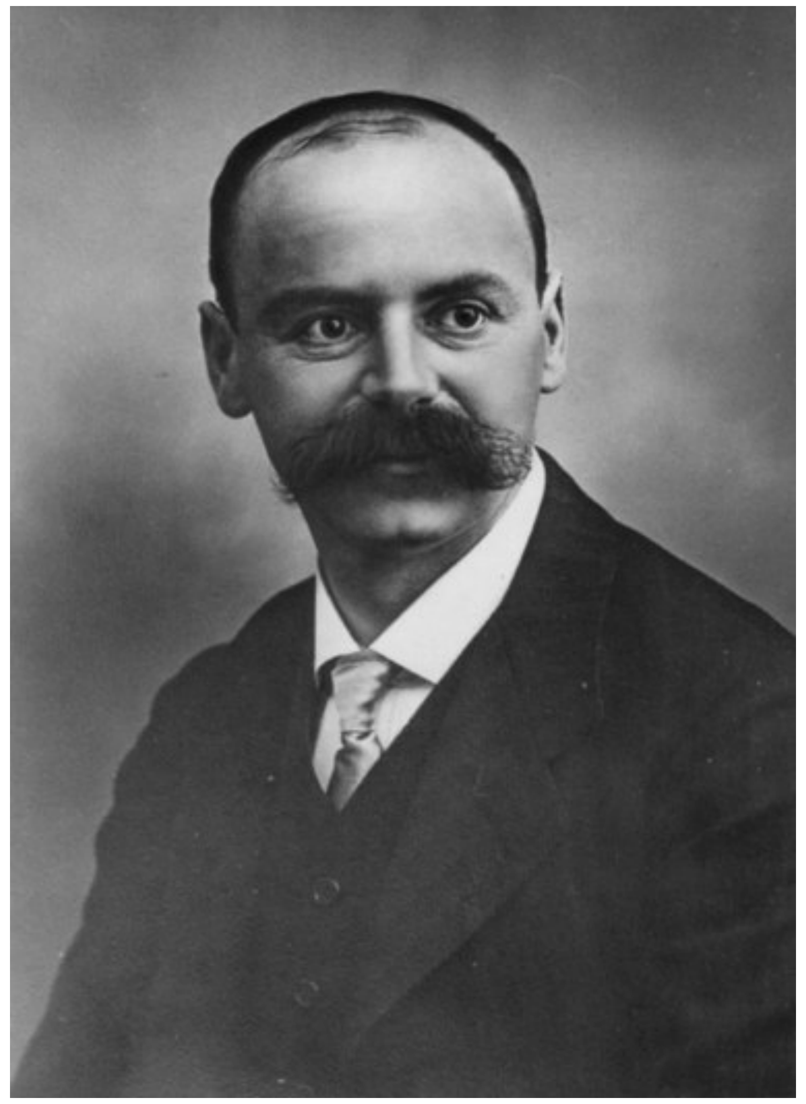
Black holes (1916)

Black holes (1916-today)

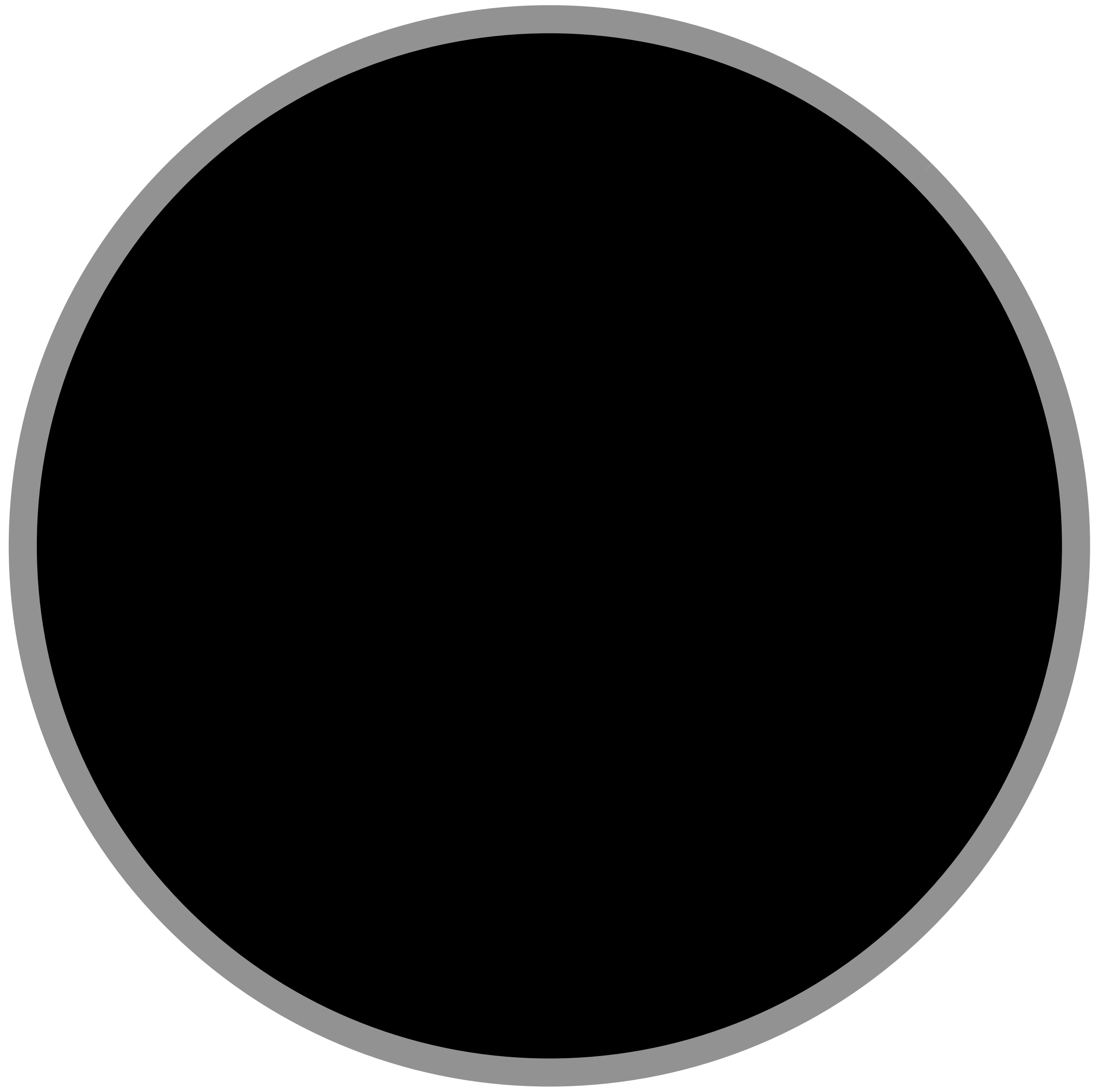
Black Holes

Objects so dense that light is gravitationally bound to them.



Karl Schwarzschild (1916)

Horizon radius $R = \dfrac{2GM}{c^2}$



G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

The supermassive black hole lurking at the heart of the Milky Way – Sagittarius A* contains about 4.3 million solar masses

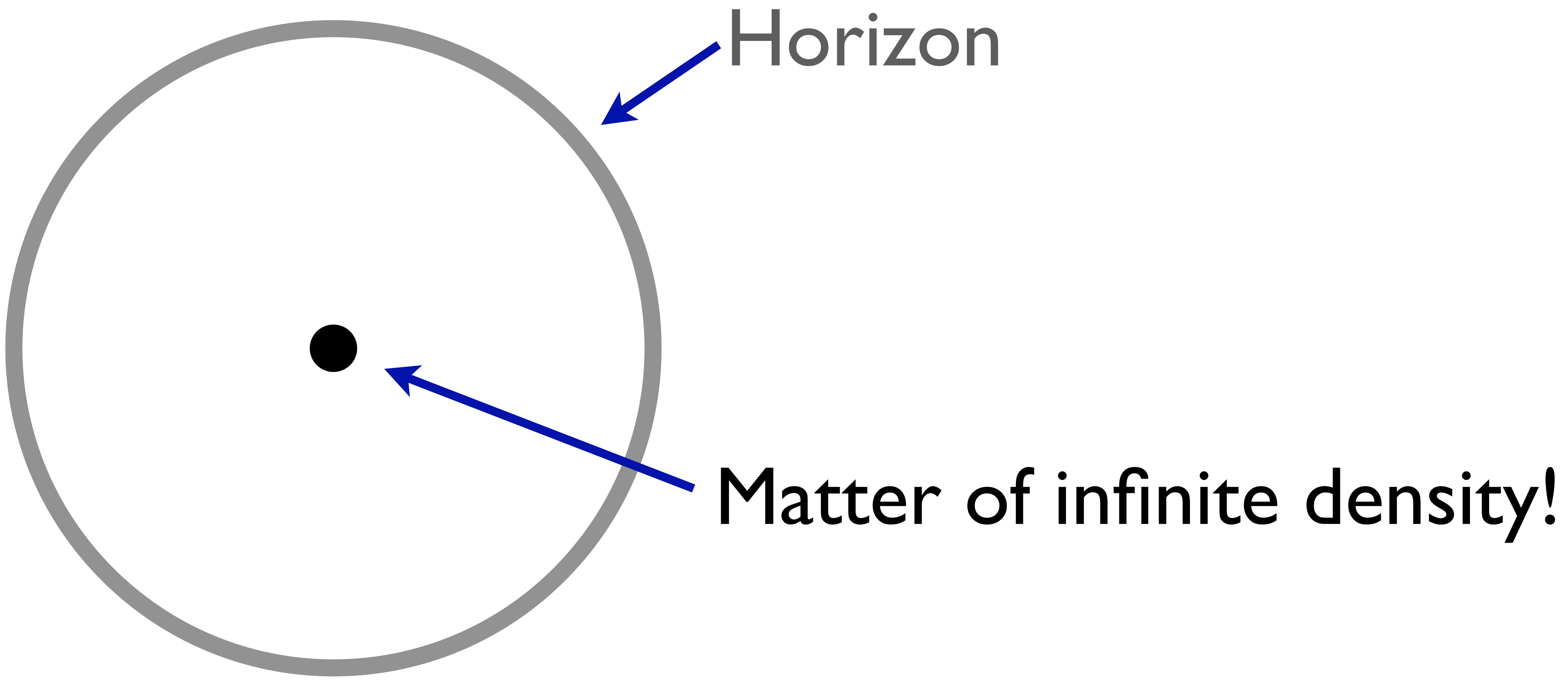
$$R = 1.3 \times 10^{11}\mathrm{m}$$

$$\approx \text{earth's orbit}$$

Event Horizon Telescope
May 12, 2022

What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



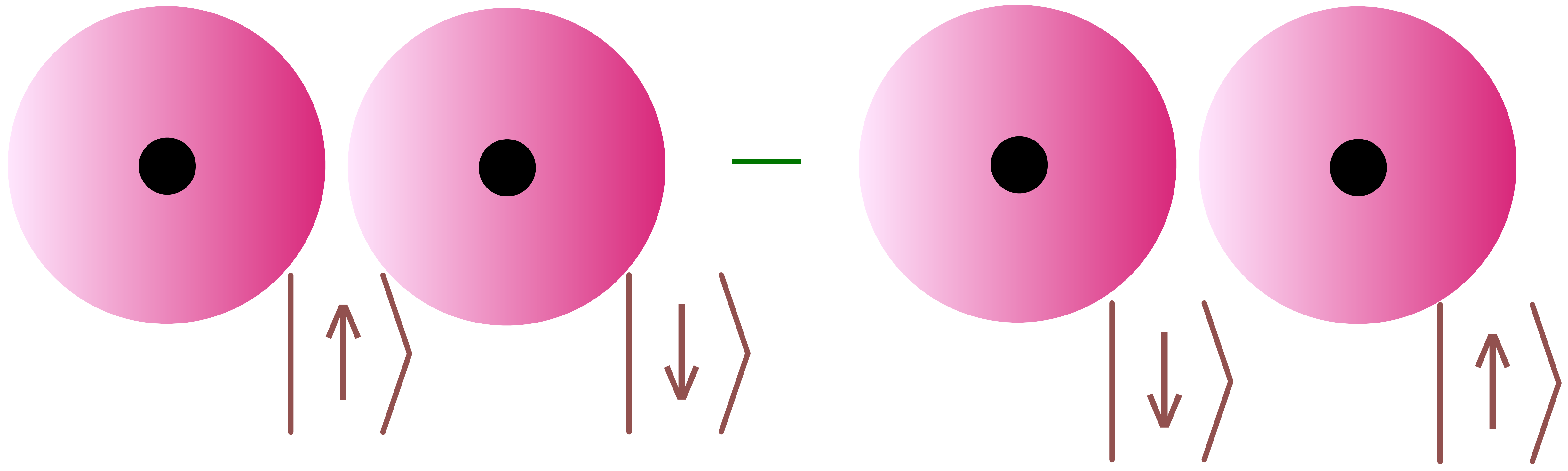
What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.

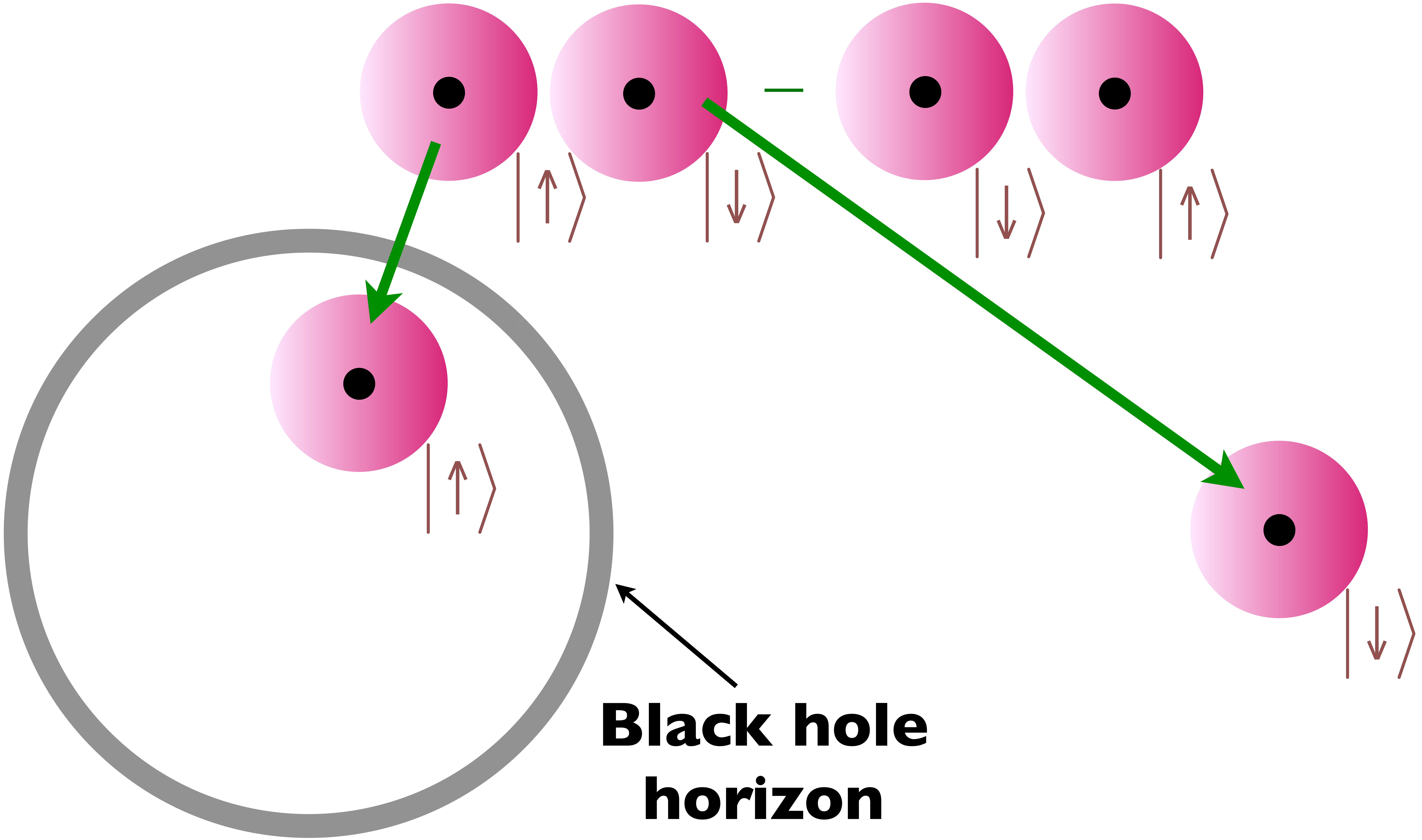
This singularity convinced many early on that black holes were unphysical solutions of Einstein's equations, and did not exist in our universe.

In any case, it was clear that quantum theory should be applied to the collapsed matter, but no one knew how to.

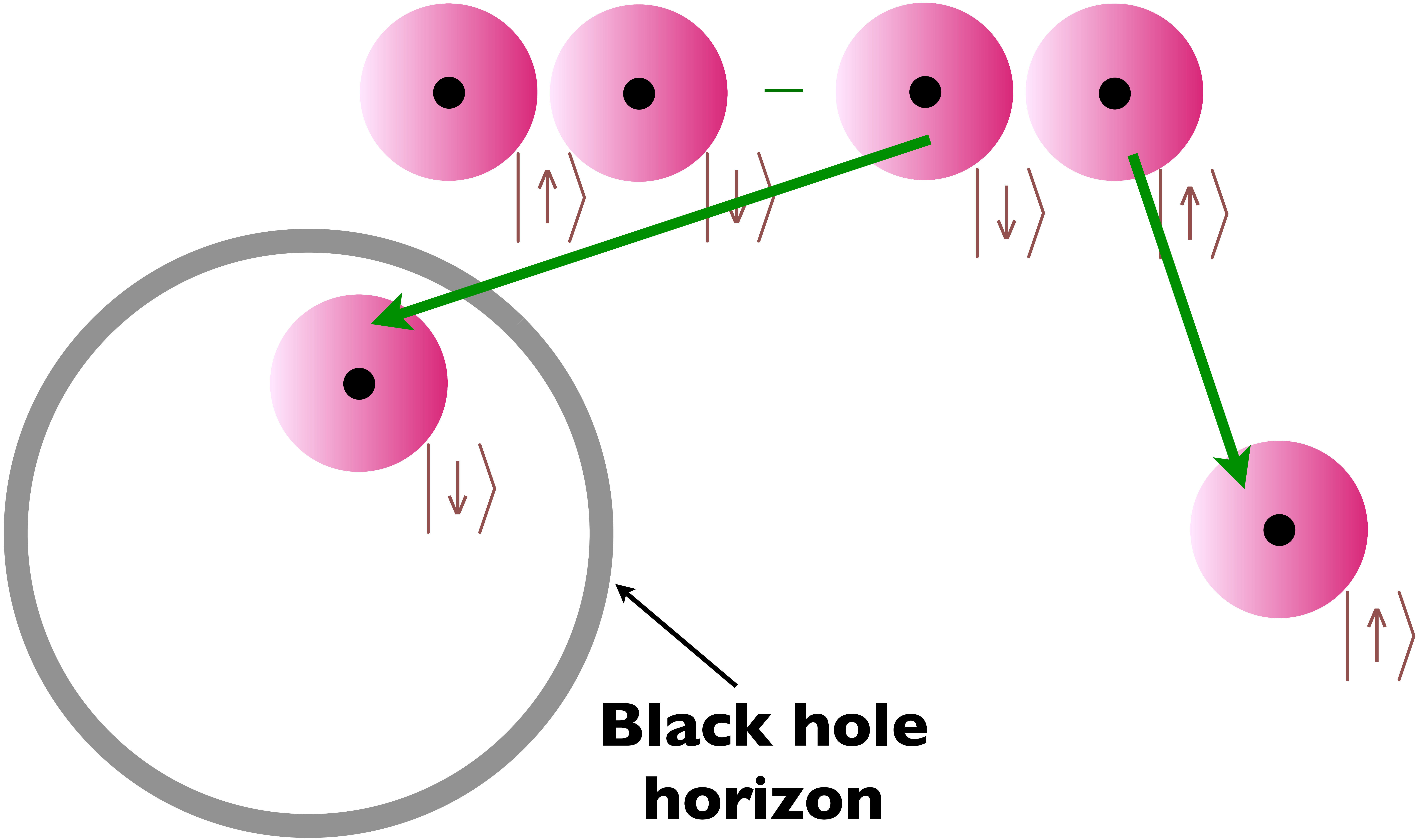
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon



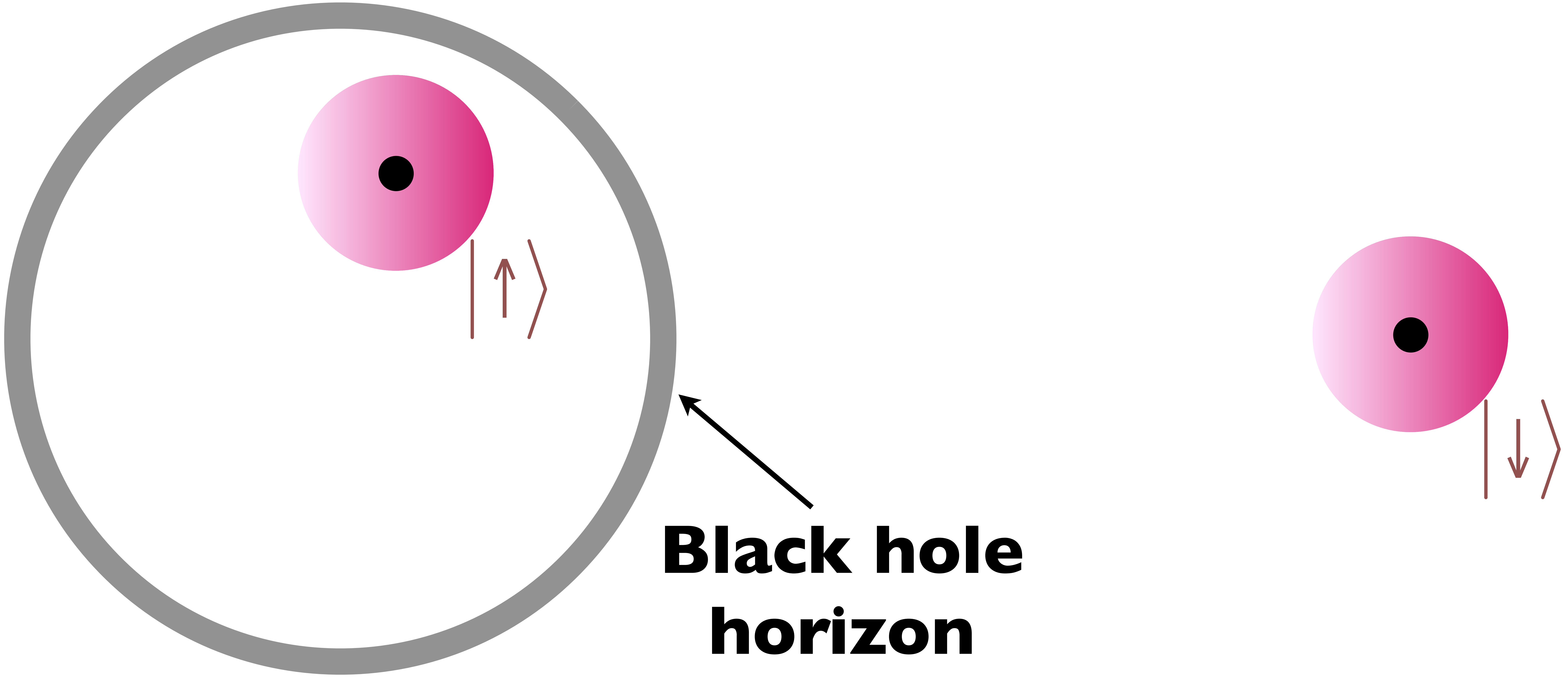
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

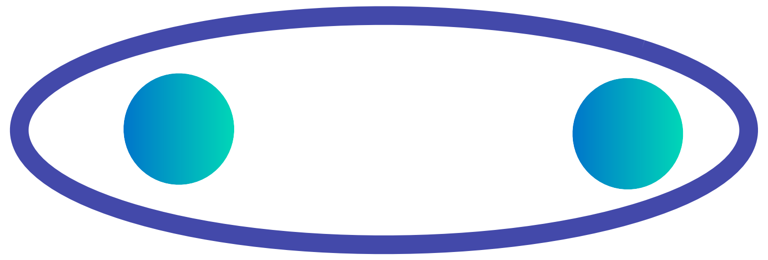
Hawking (1975): Black holes have a temperature and an entropy!

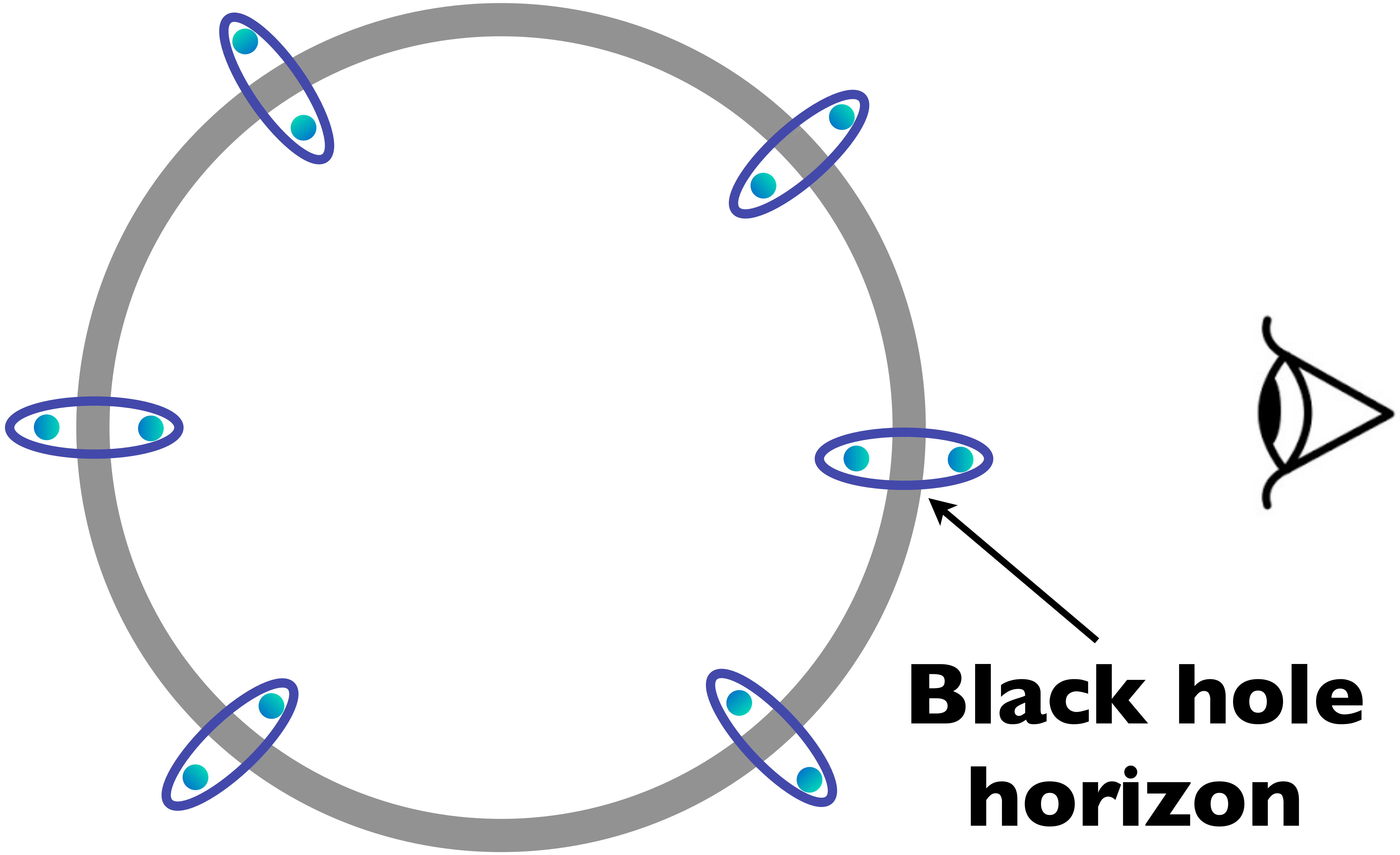
To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.



Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



By computations *outside* the black hole, Hawking obtained

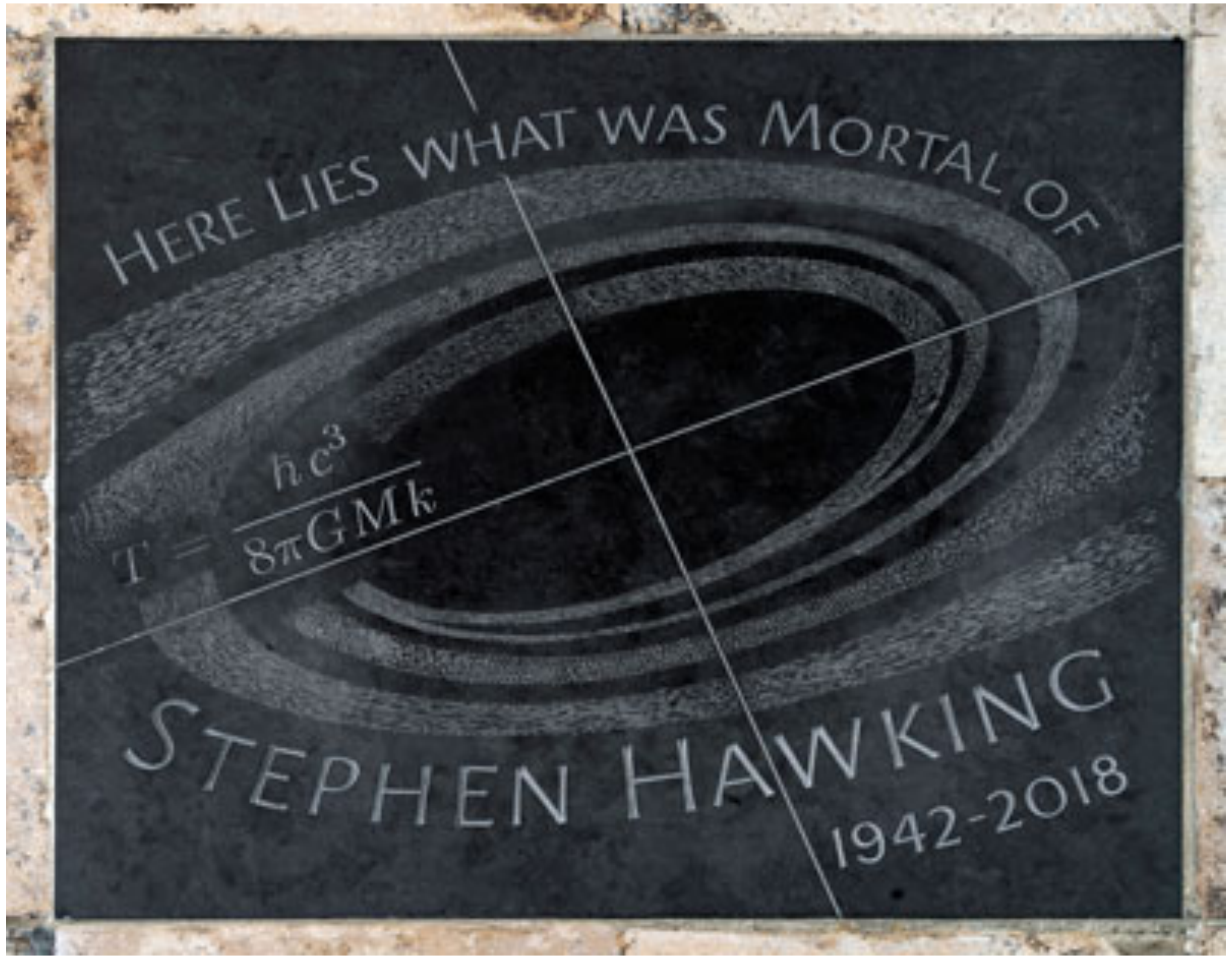
$$S = \frac{k_B A c^3}{4 G \hbar}$$

where A is area of the black hole horizon.

All other systems have entropy proportional to their volume.

What is inside a black hole ???

Hawking (1975): when viewed from the outside, black holes have an entropy and a temperature, and slowly evaporate like any thermal object



$$T = \frac{\hbar c^3}{8\pi G M k_B}$$

$$S = \frac{k_B A c^3}{4G\hbar}$$

J. D. Bekenstein, PRD **7**, 2333 (1973); S.W. Hawking, Nature **248**, 30 (1974)

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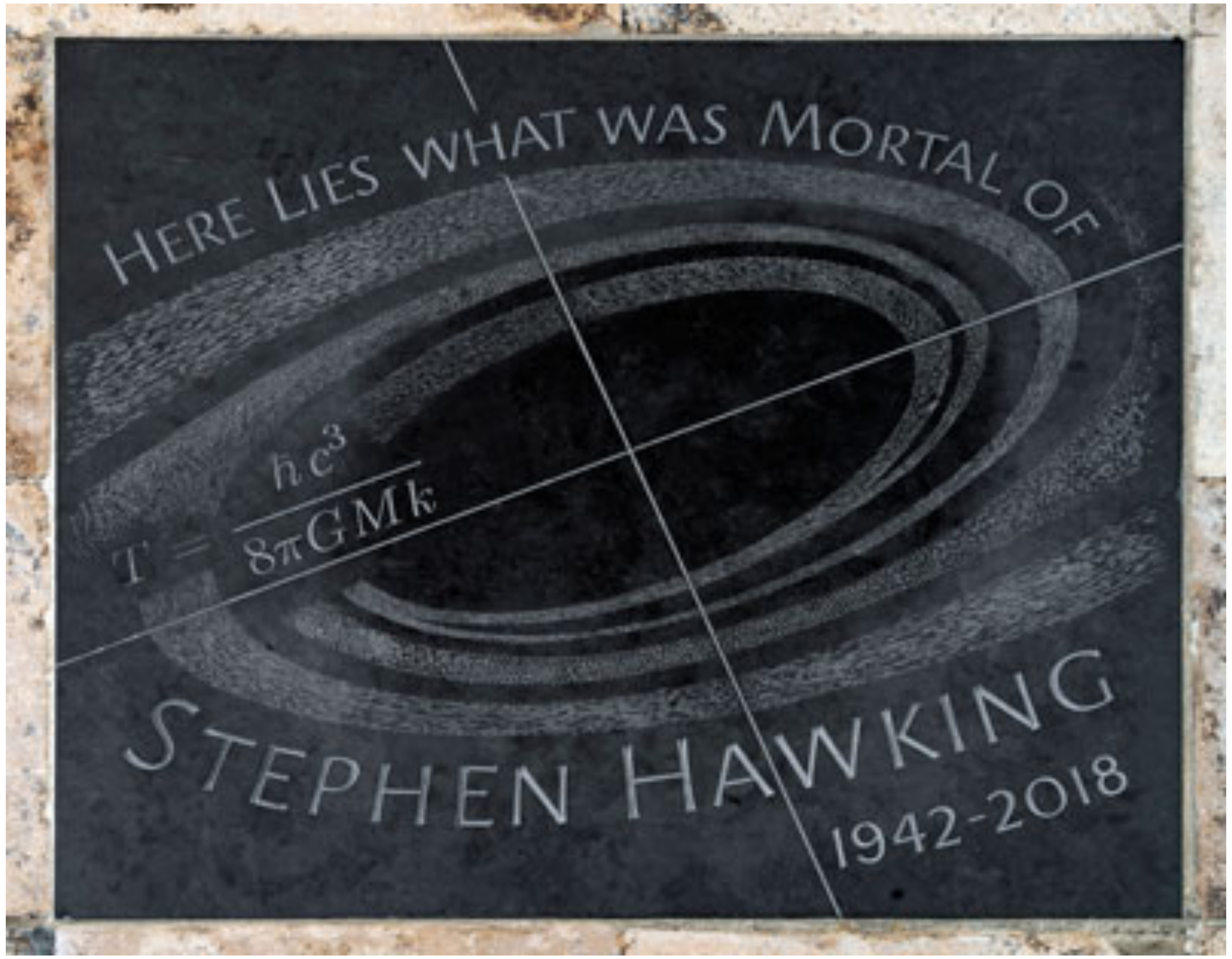

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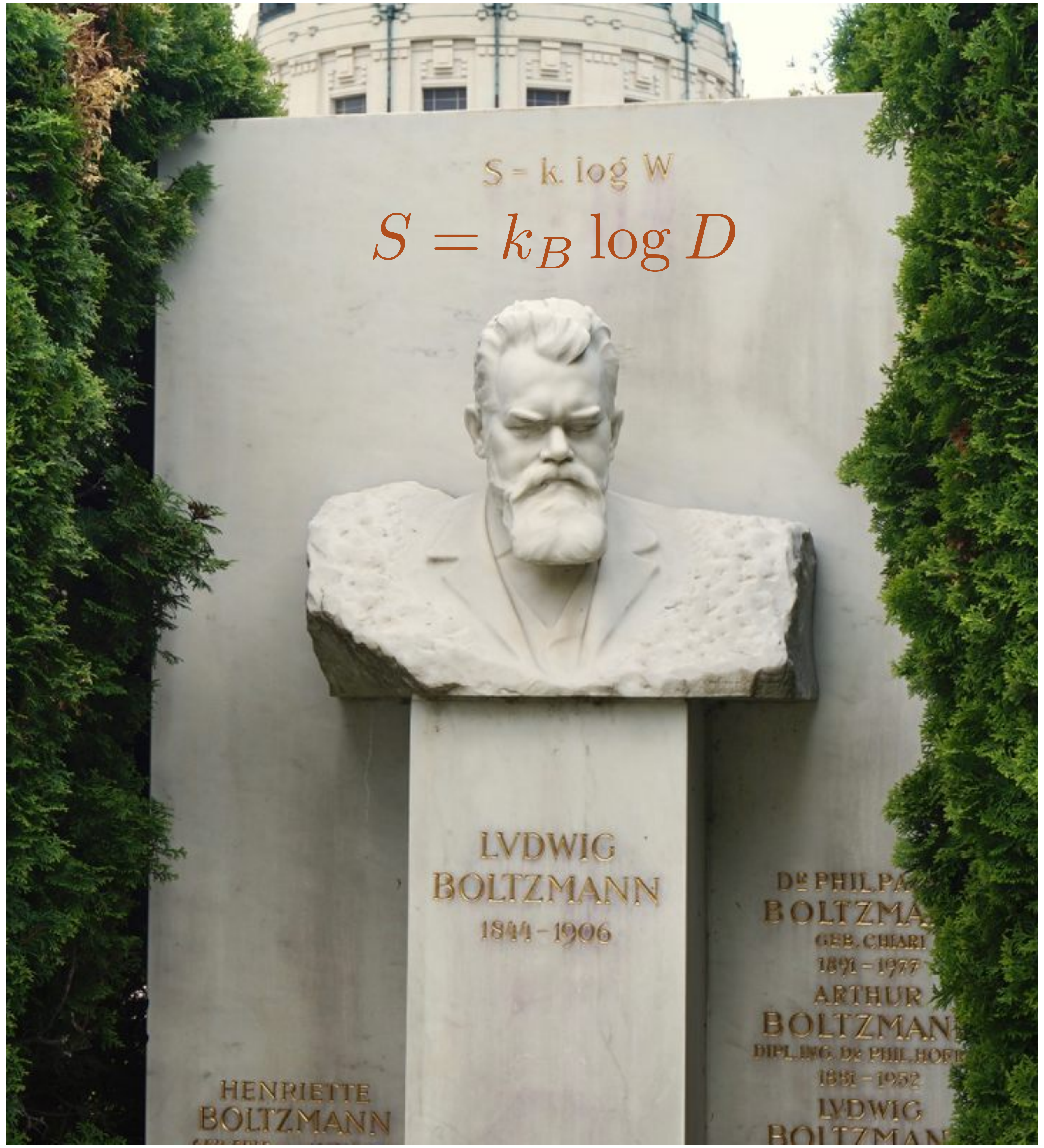
$$S = \frac{k_B A c^3}{4G\hbar}$$

$$\tau_{\mathrm{ring-down}} \sim \frac{\hbar}{k_B T}$$

Planckian dynamics!

J. D. Bekenstein, PRD **7**, 2333 (1973); S.W. Hawking, Nature **248**, 30 (1974); C.V. Vishveshwara, Nature **227**, 936 (1970)



Ludwig Boltzmann
20 February 1844 - September 5, 1906
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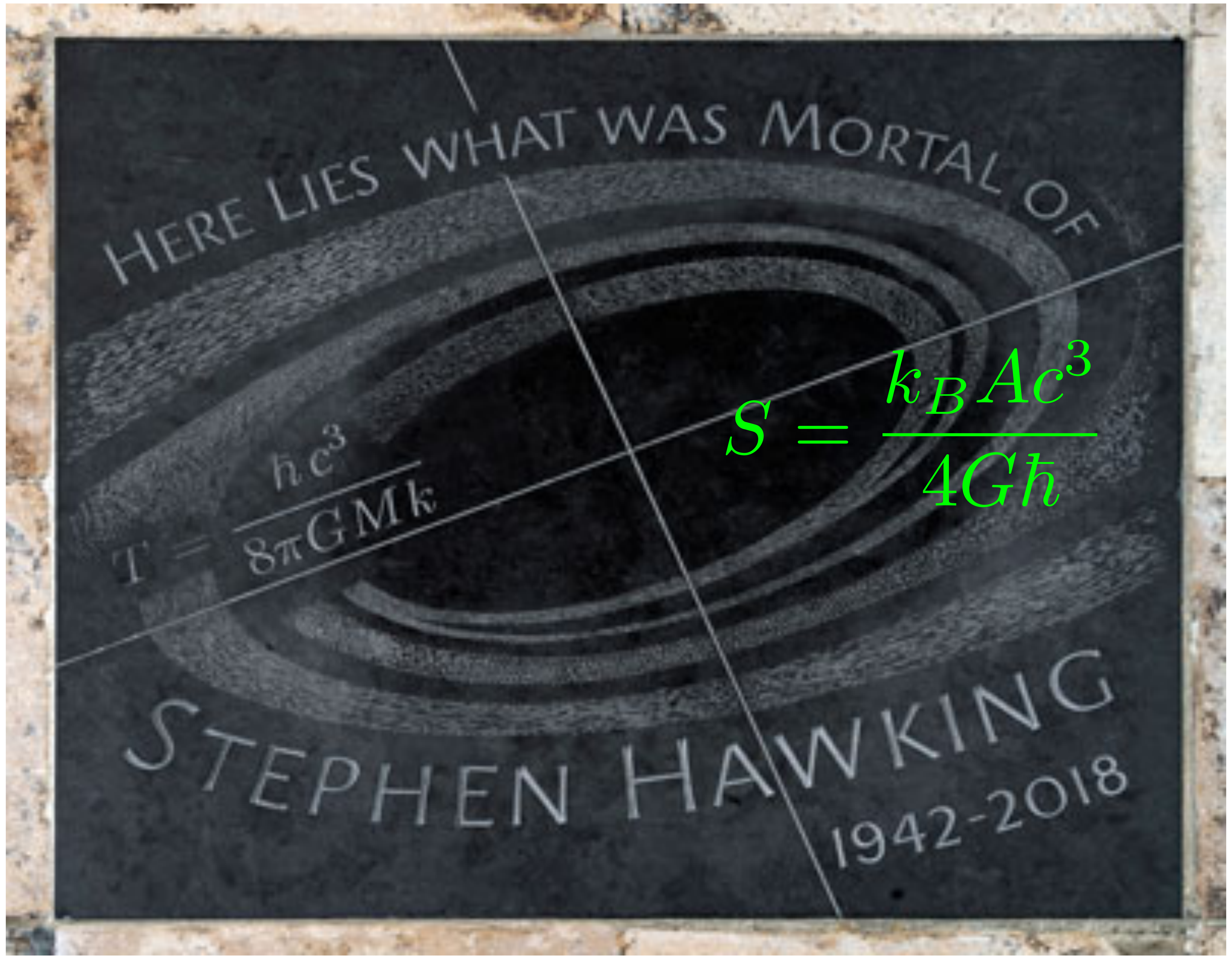

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J. D. Bekenstein, PRD **7**, 2333 (1973); S.W. Hawking, Nature **248**, 30 (1974); C.V. Vishveshwara, Nature **227**, 936 (1970)

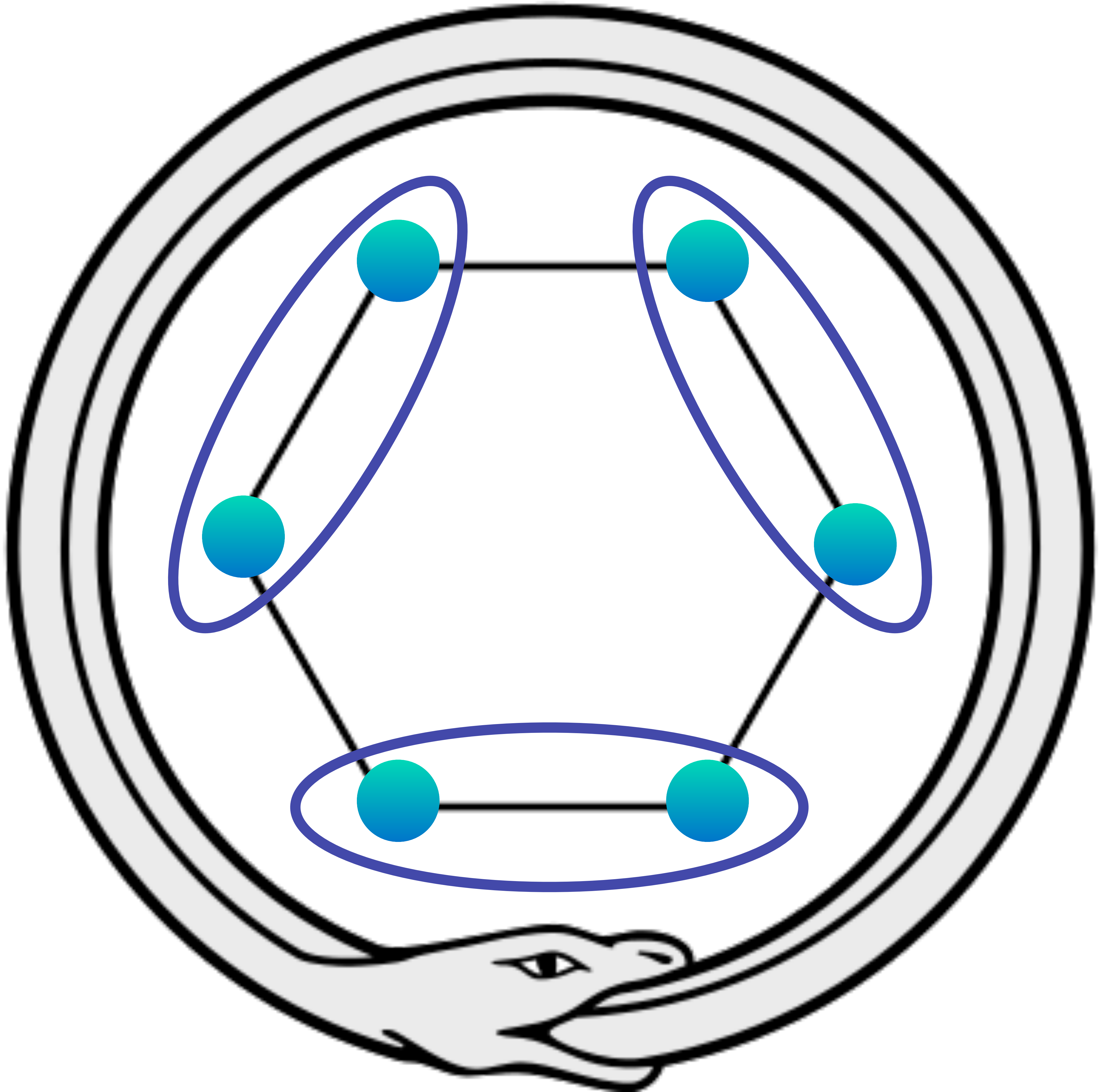
Needed,
to solve open problems in the theory of superconductivity and black holes:

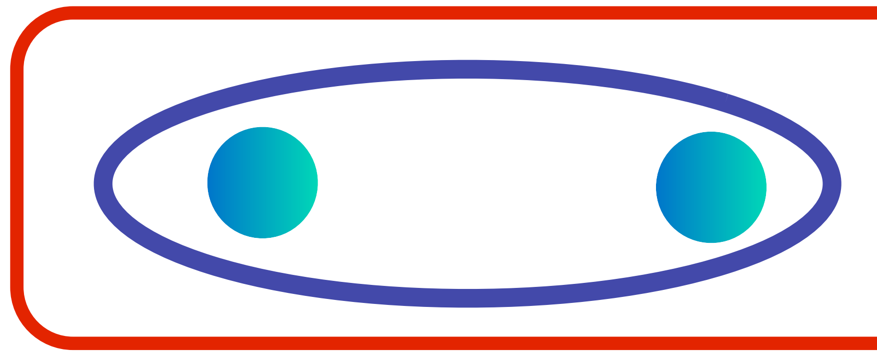
A solvable model of quantum entanglement of $3, 4, 5, \ldots \infty$ particles

The Sachdev-Ye-Kitaev model of many-particle entanglement

Kekule's spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*

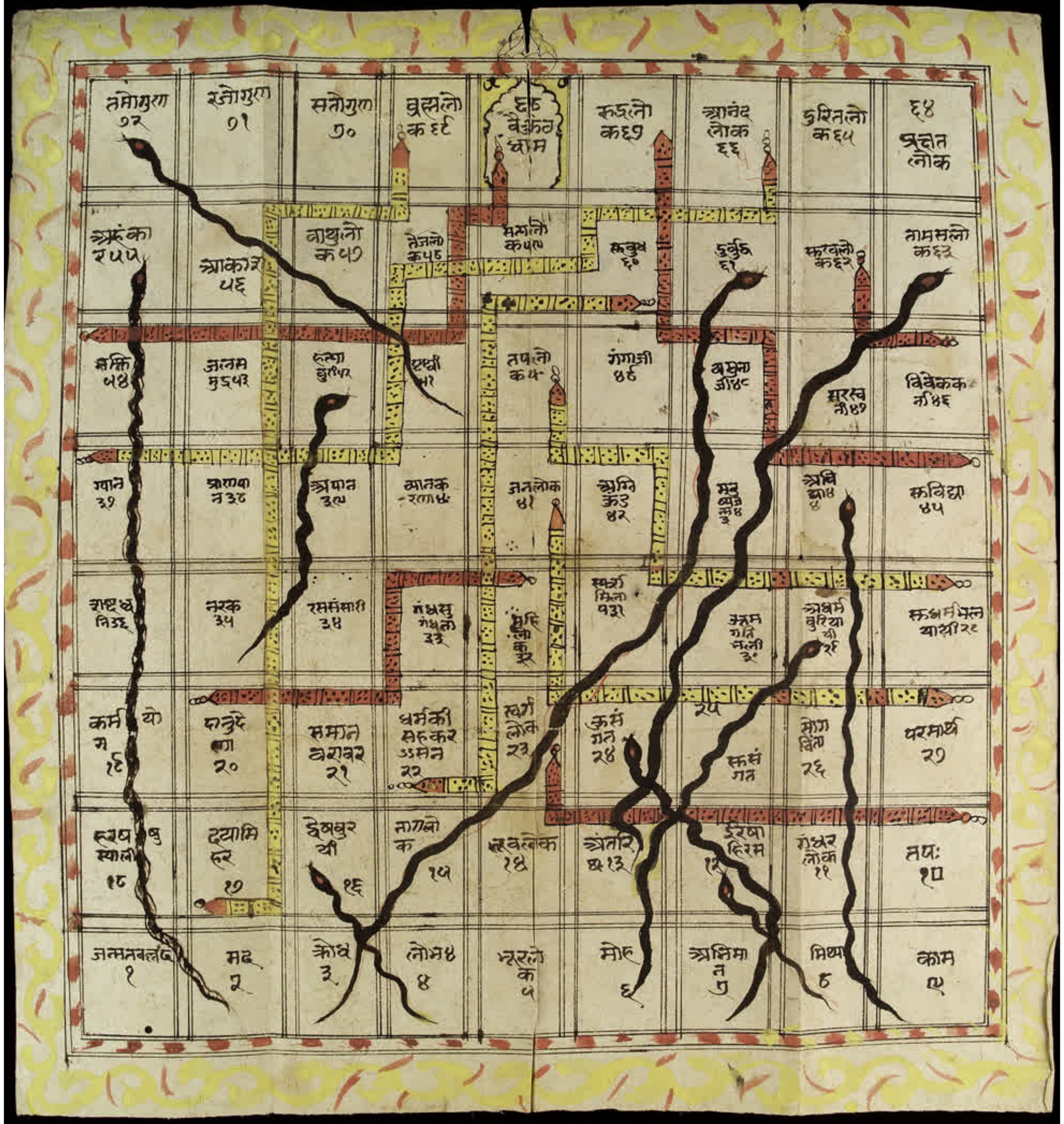




$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Benzene

*Wikipedia



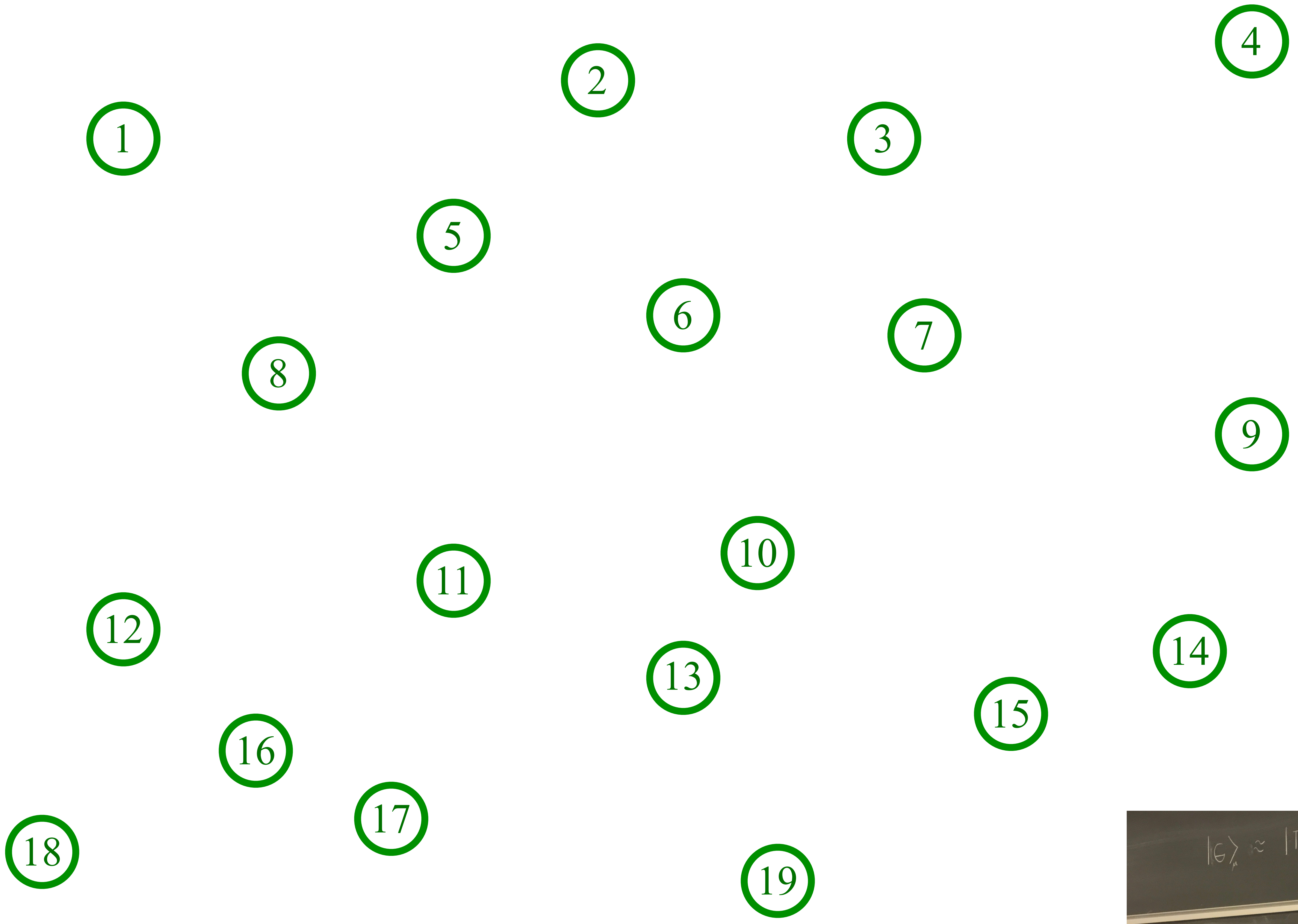
My spooky dream*

Ancient Indian game of Snakes and Ladders

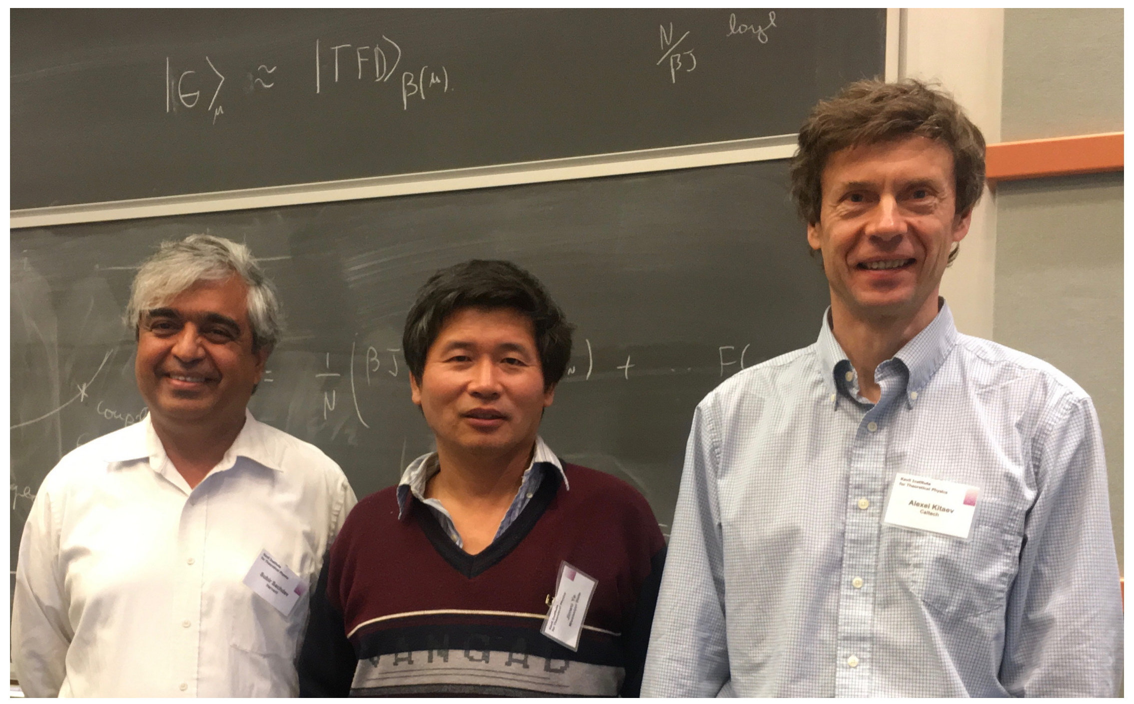
*Not true

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

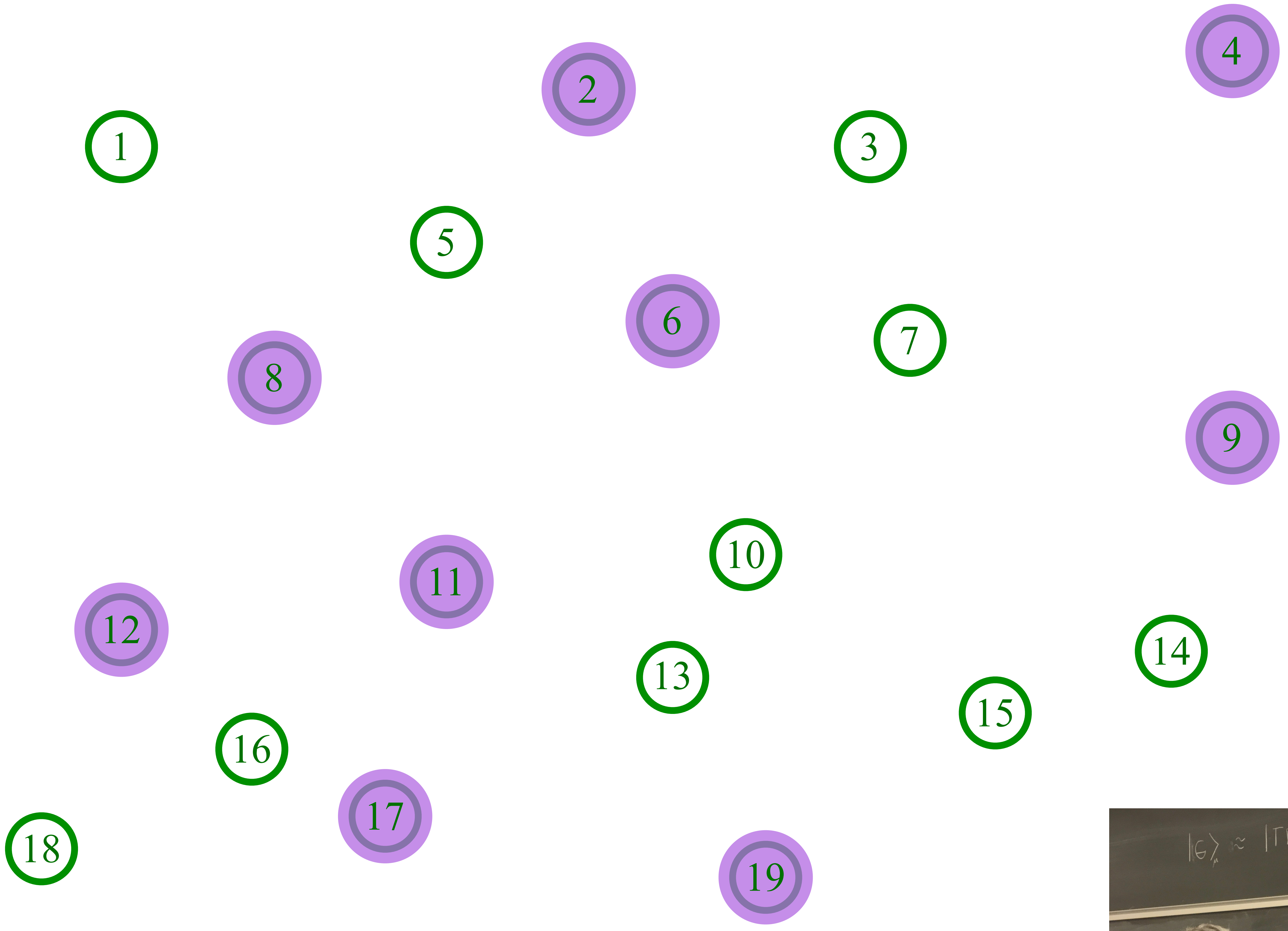


Pick a set of random positions

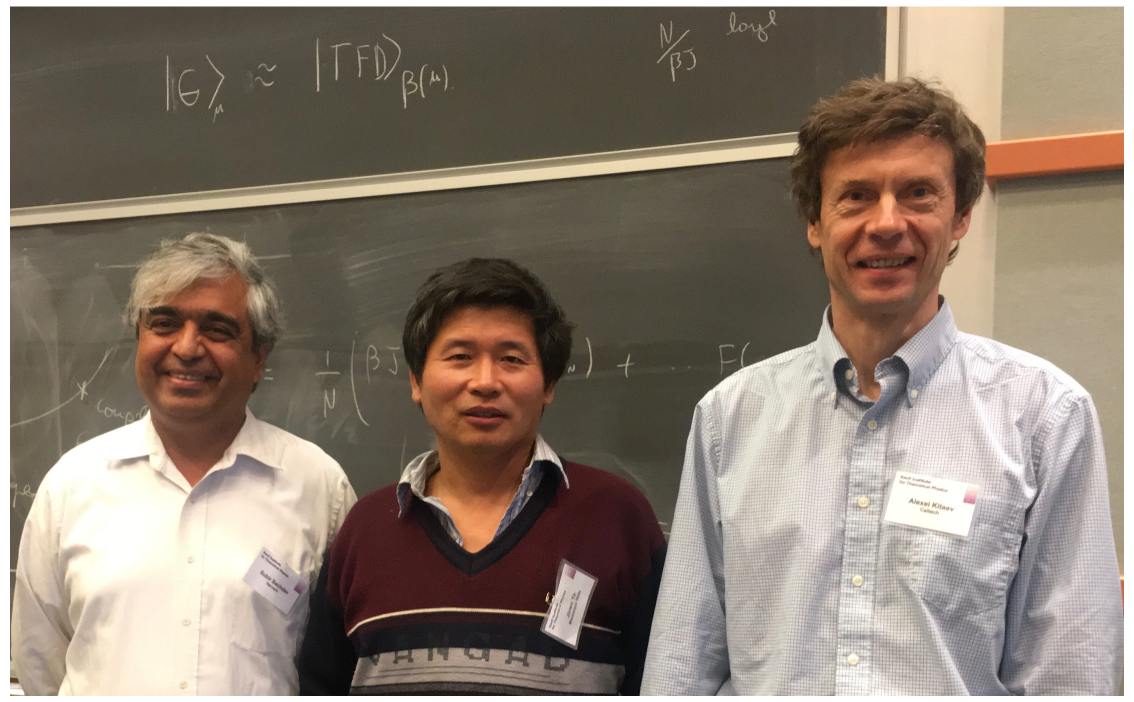


The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)



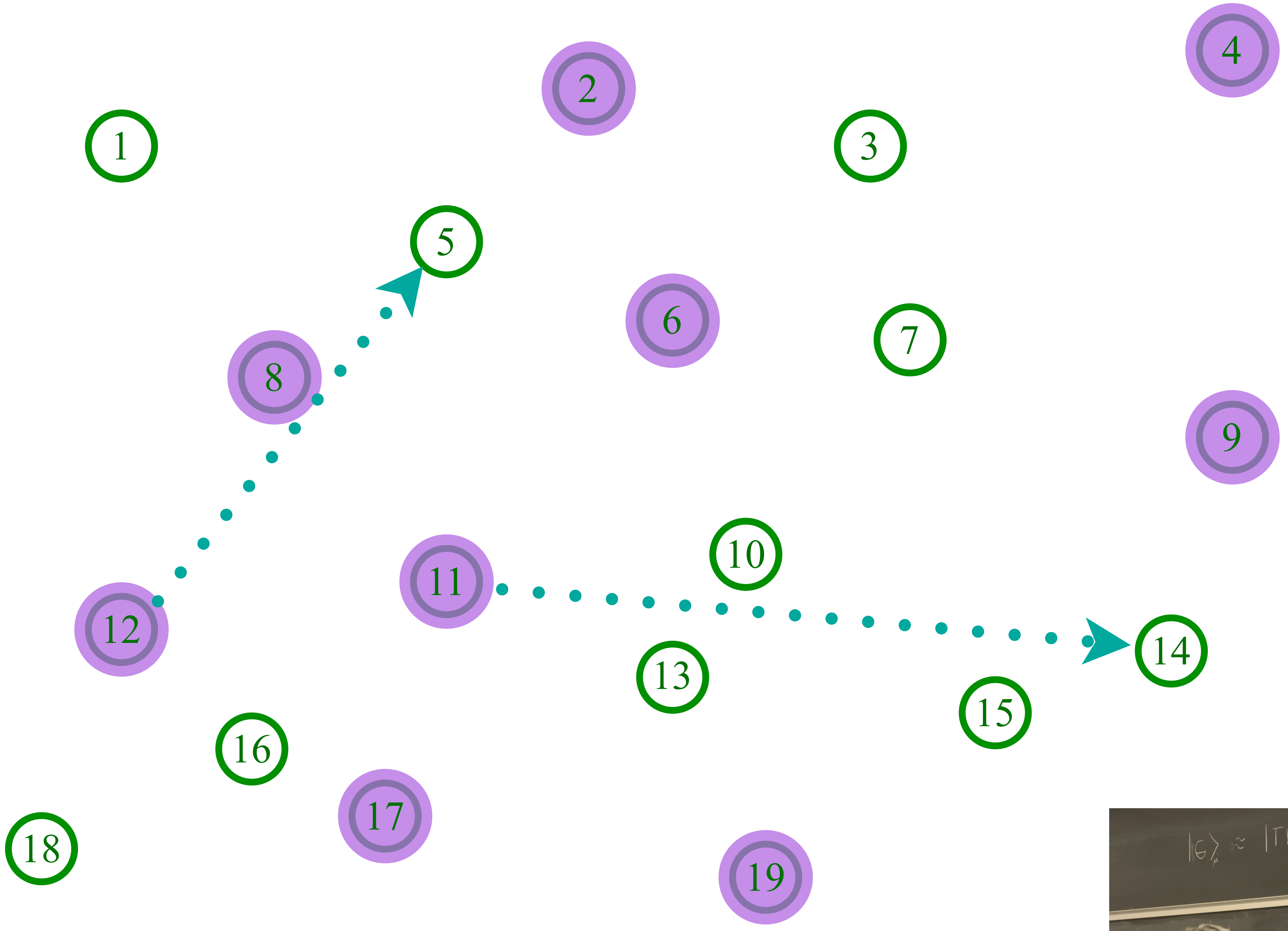
Place electrons randomly on some sites



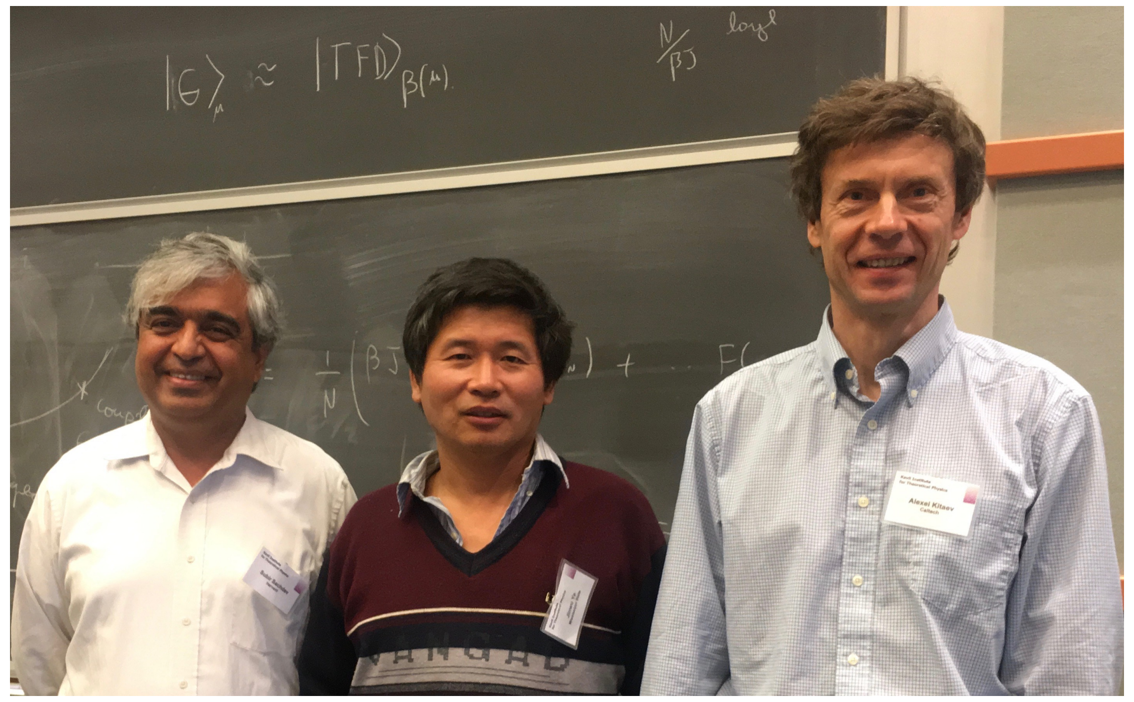
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



Place electrons randomly on some sites



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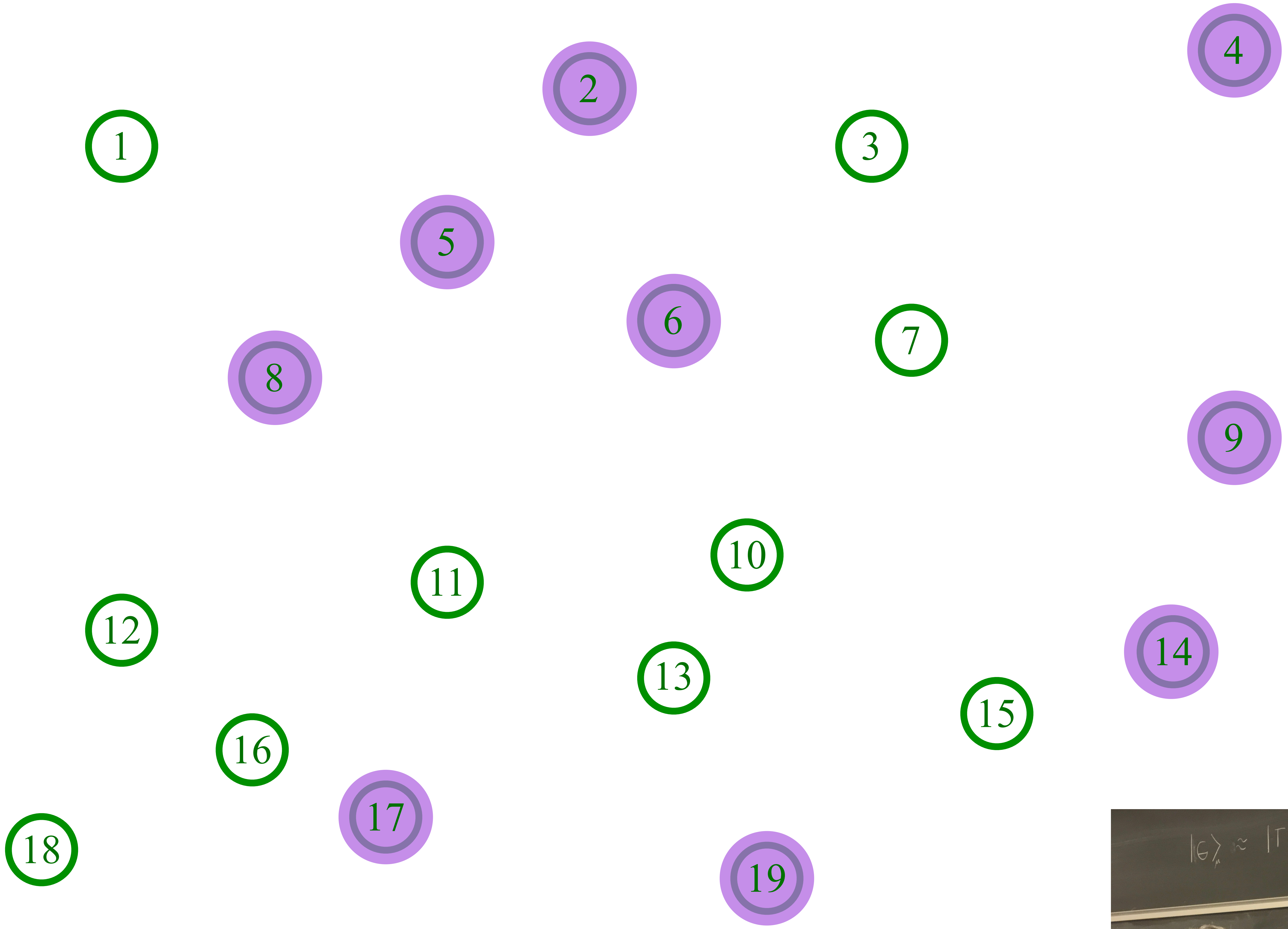
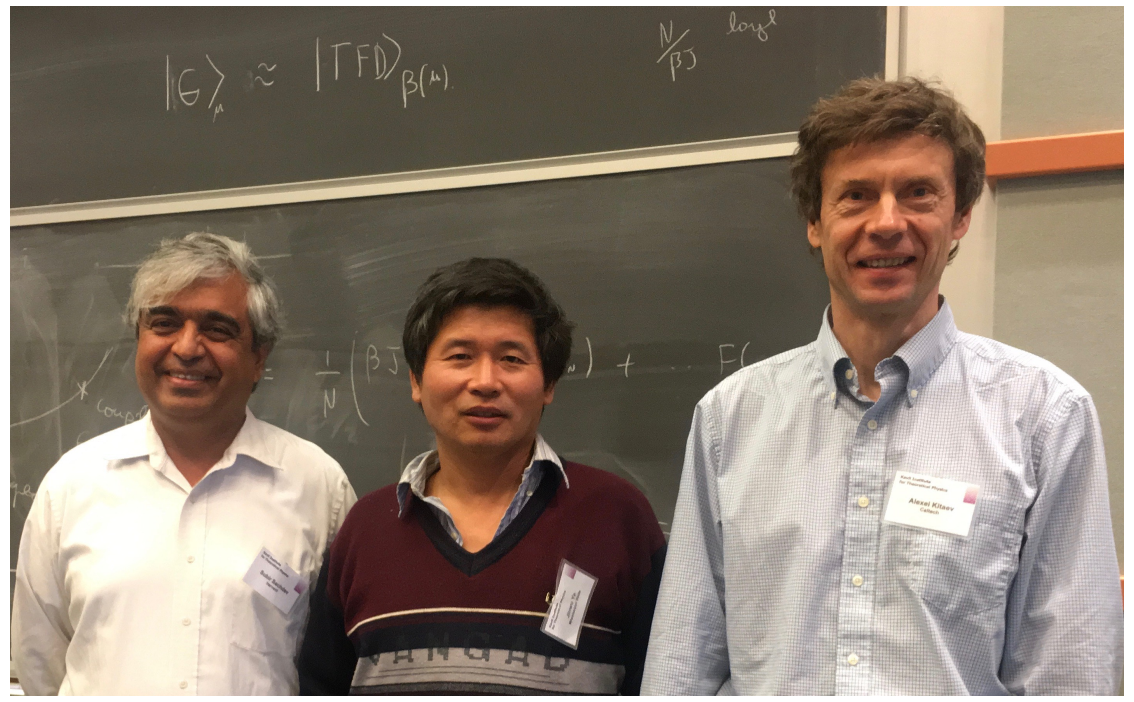


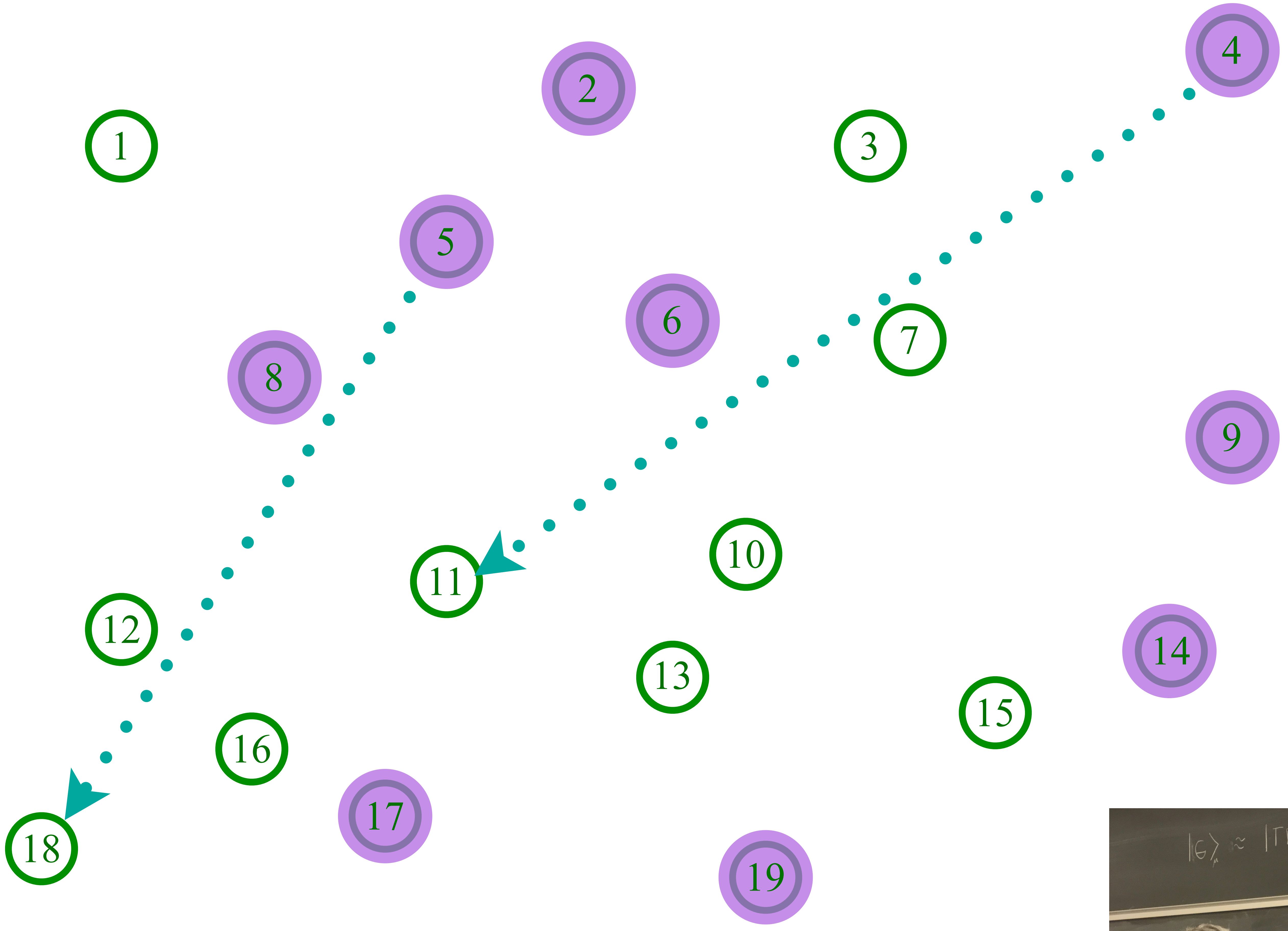
Entangle electrons pairwise randomly



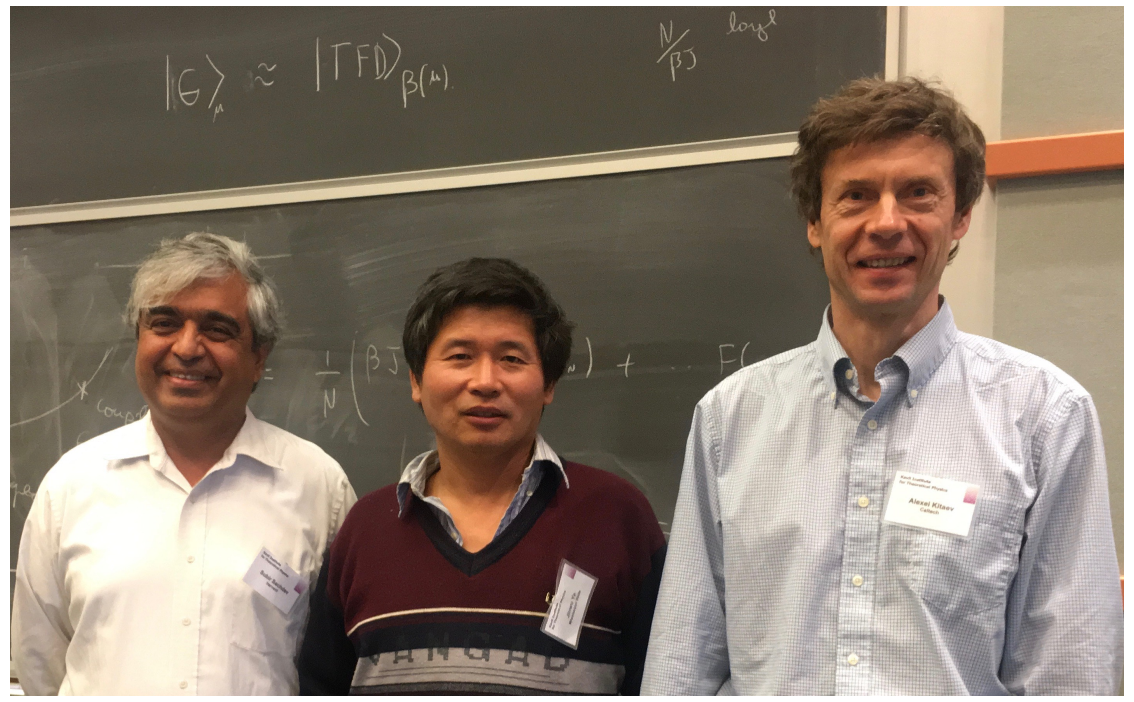
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$U_{4,5;11,18}$



Entangle electrons pairwise randomly



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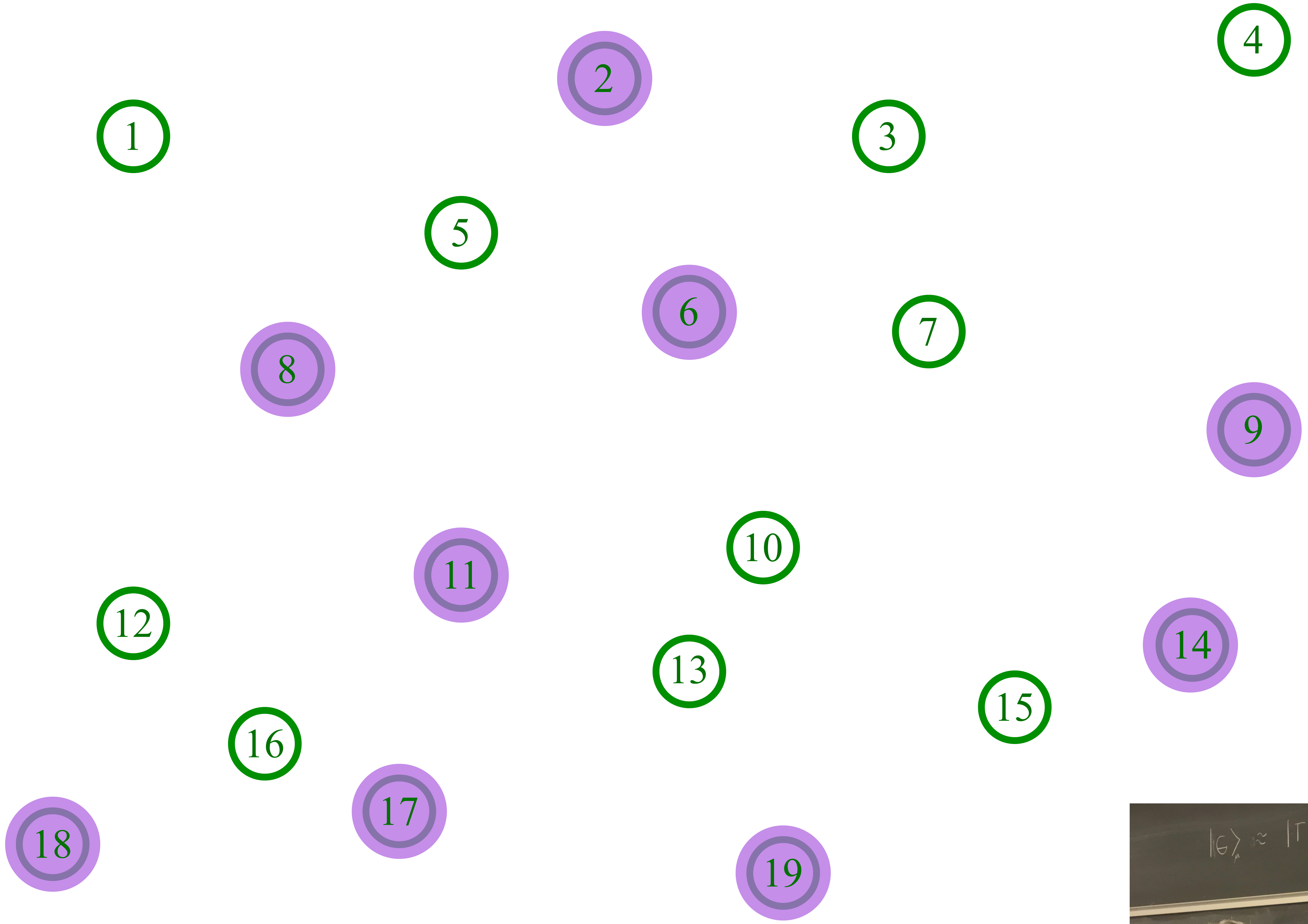
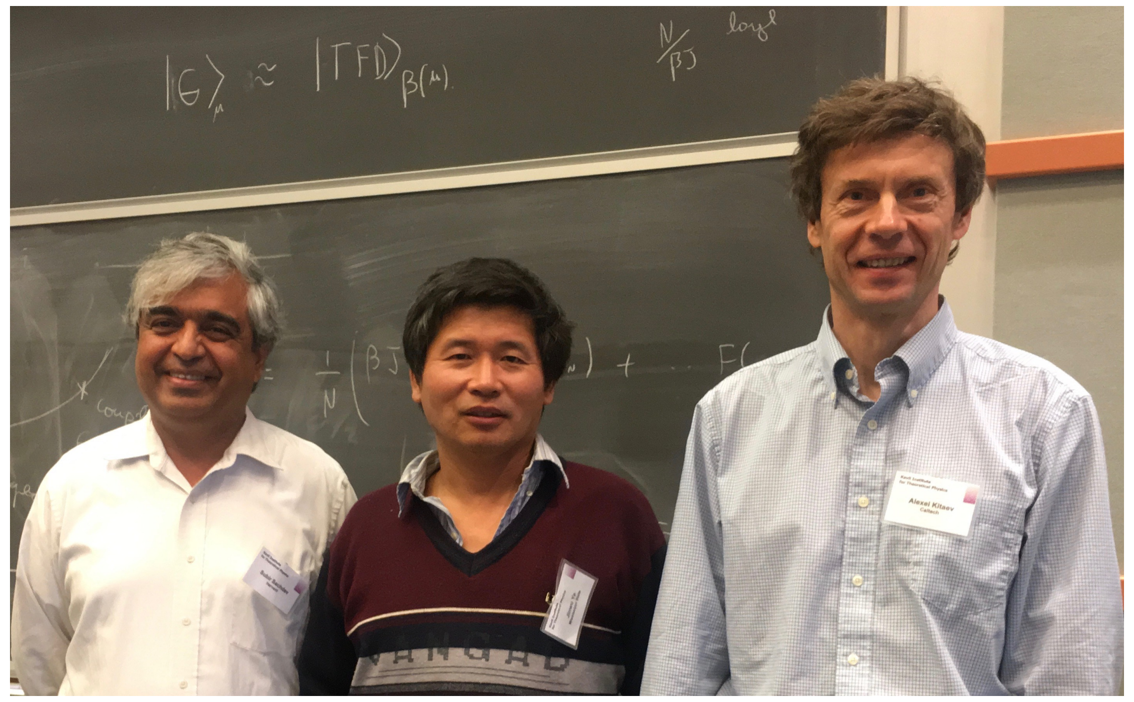
Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



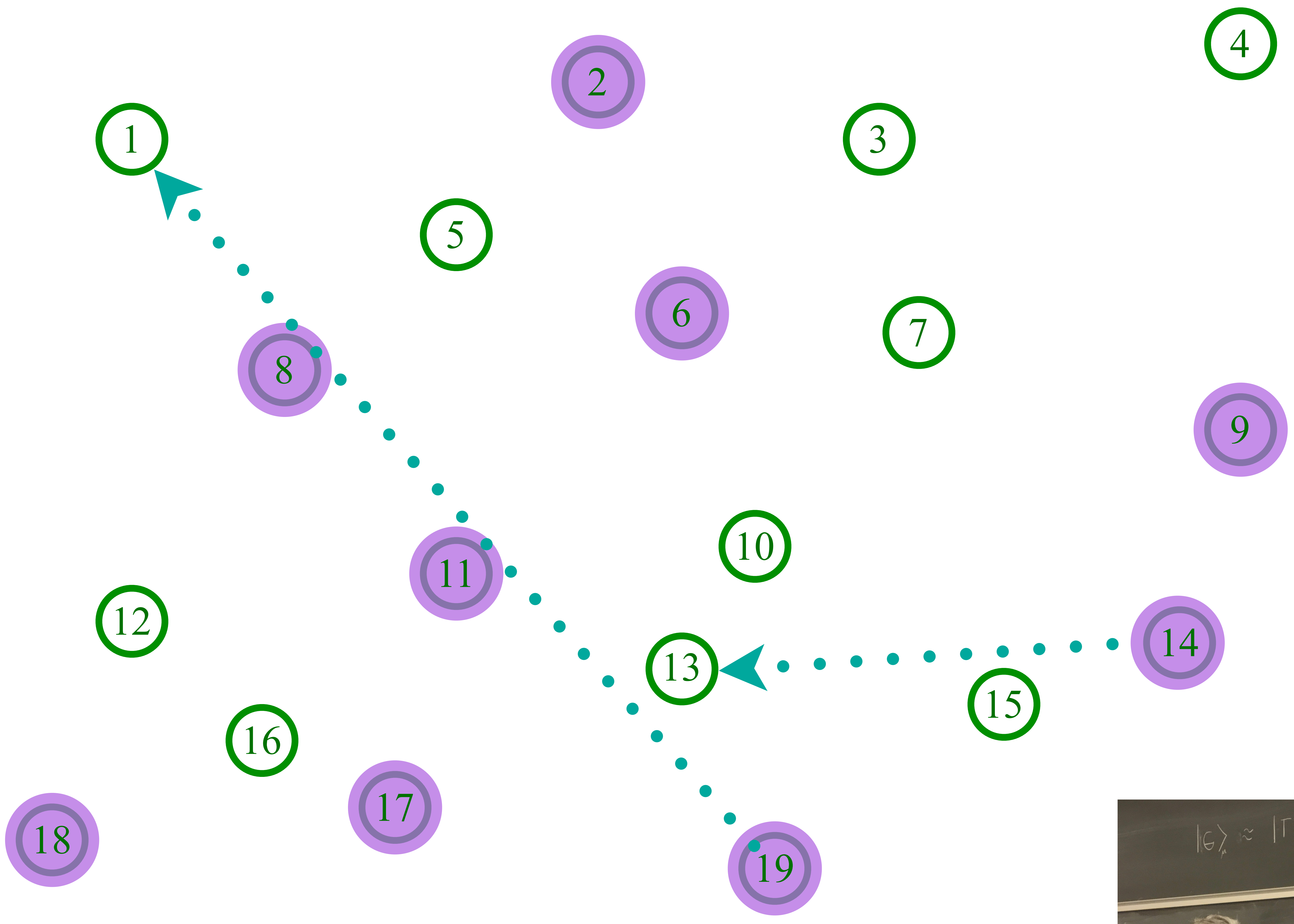
Entangle electrons pairwise randomly



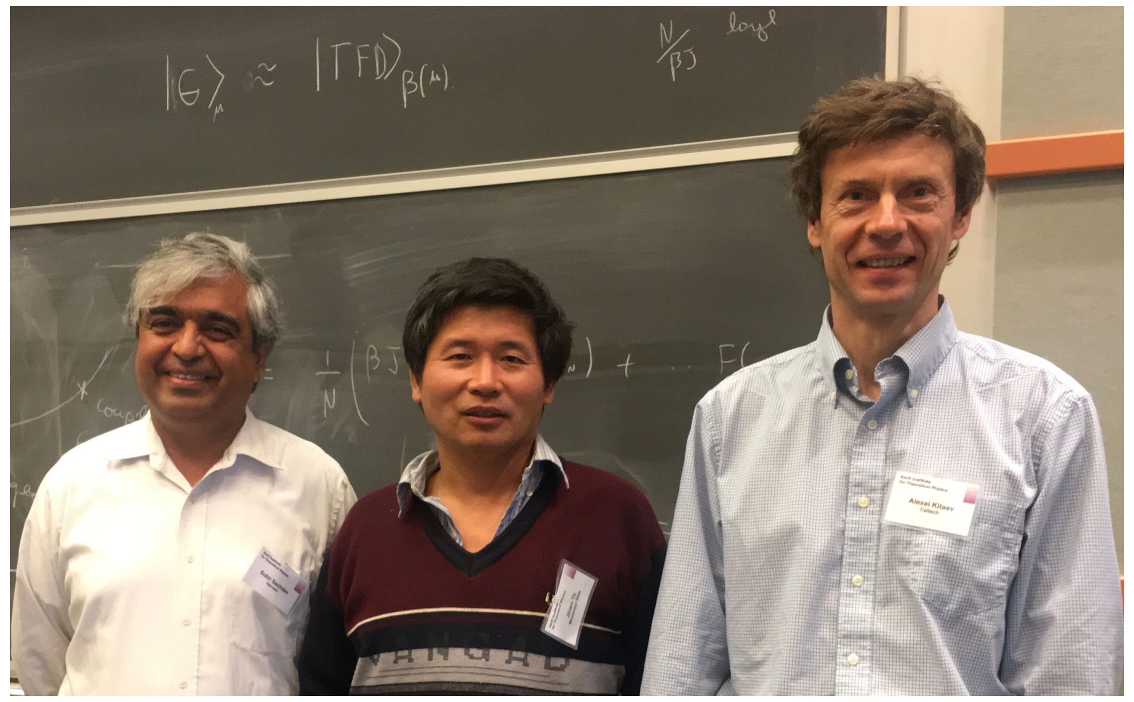
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



Entangle electrons pairwise randomly



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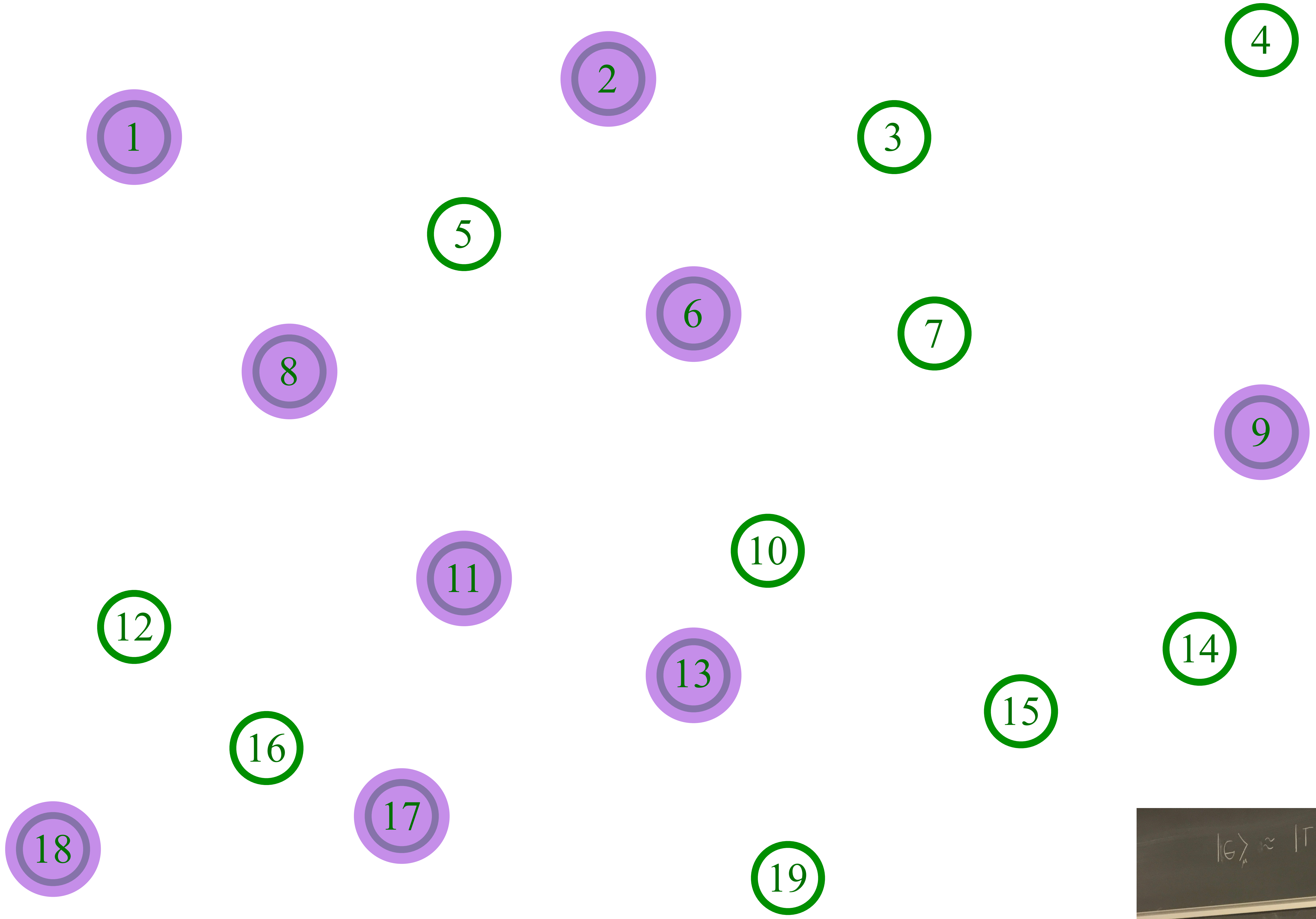
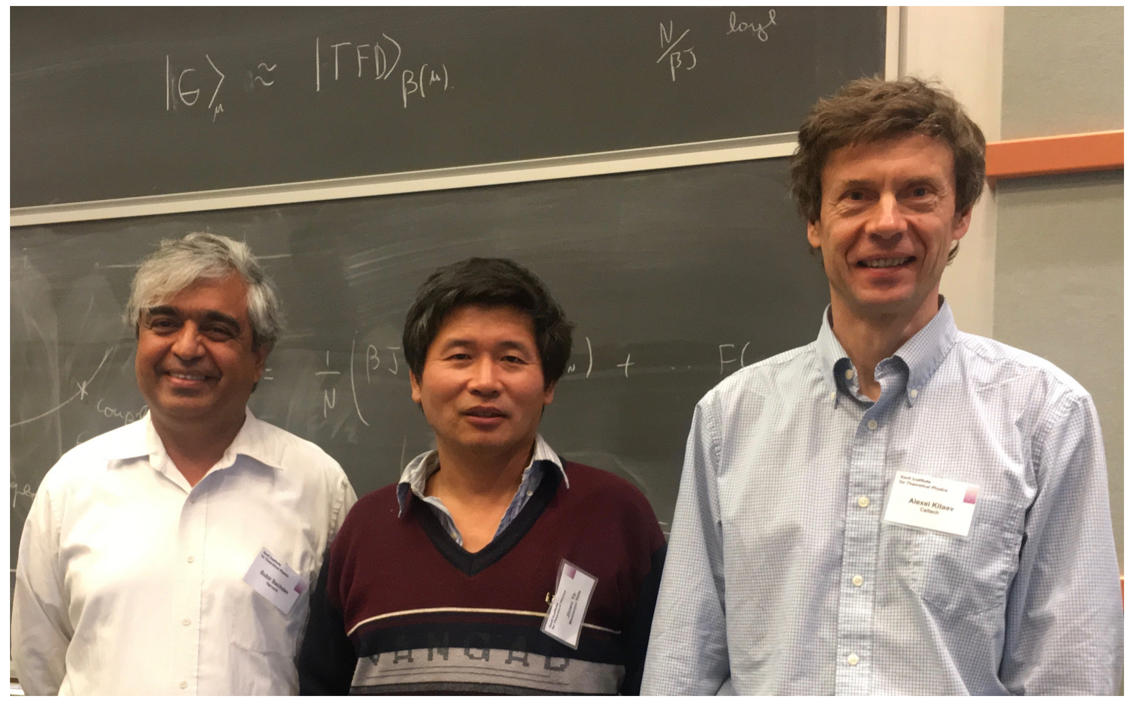


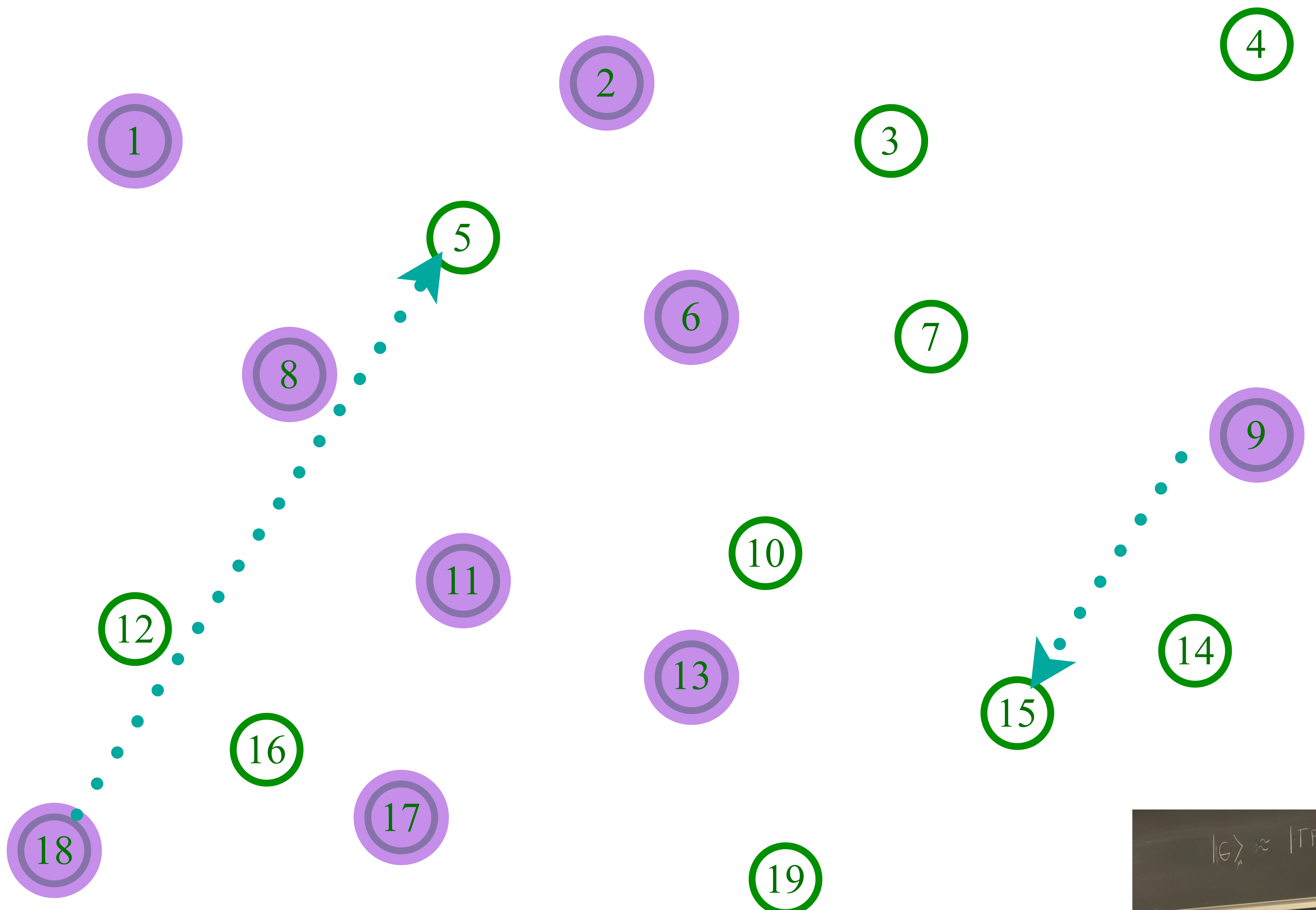
Entangle electrons pairwise randomly



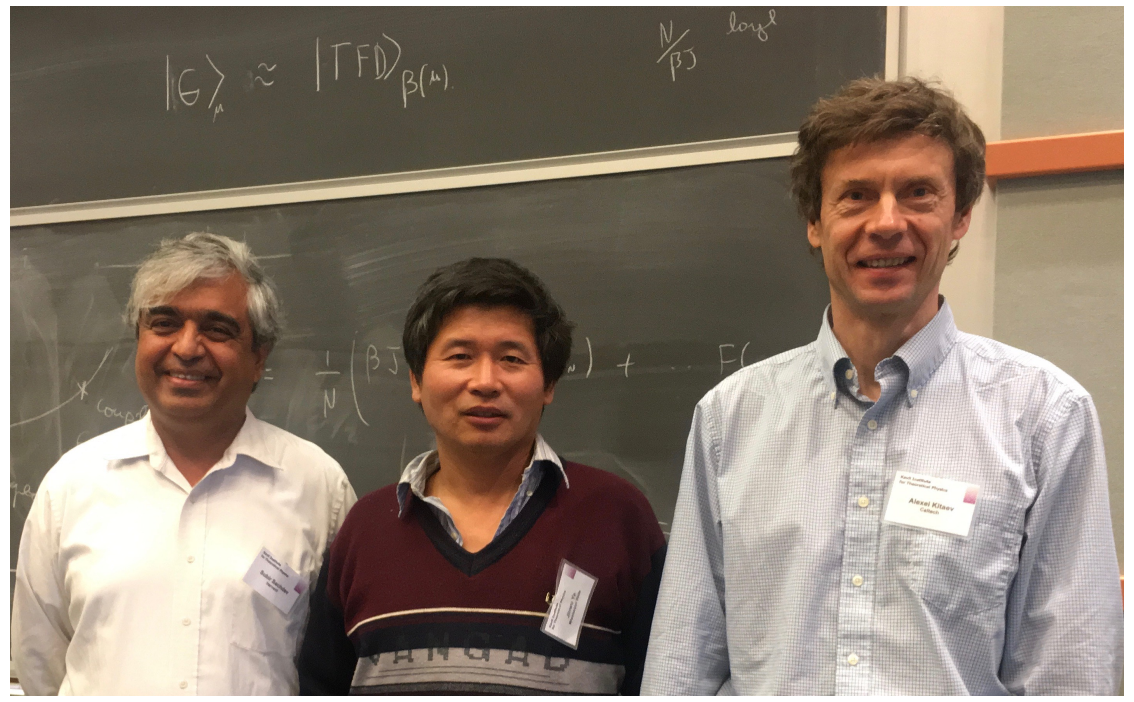
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



Entangle electrons pairwise randomly



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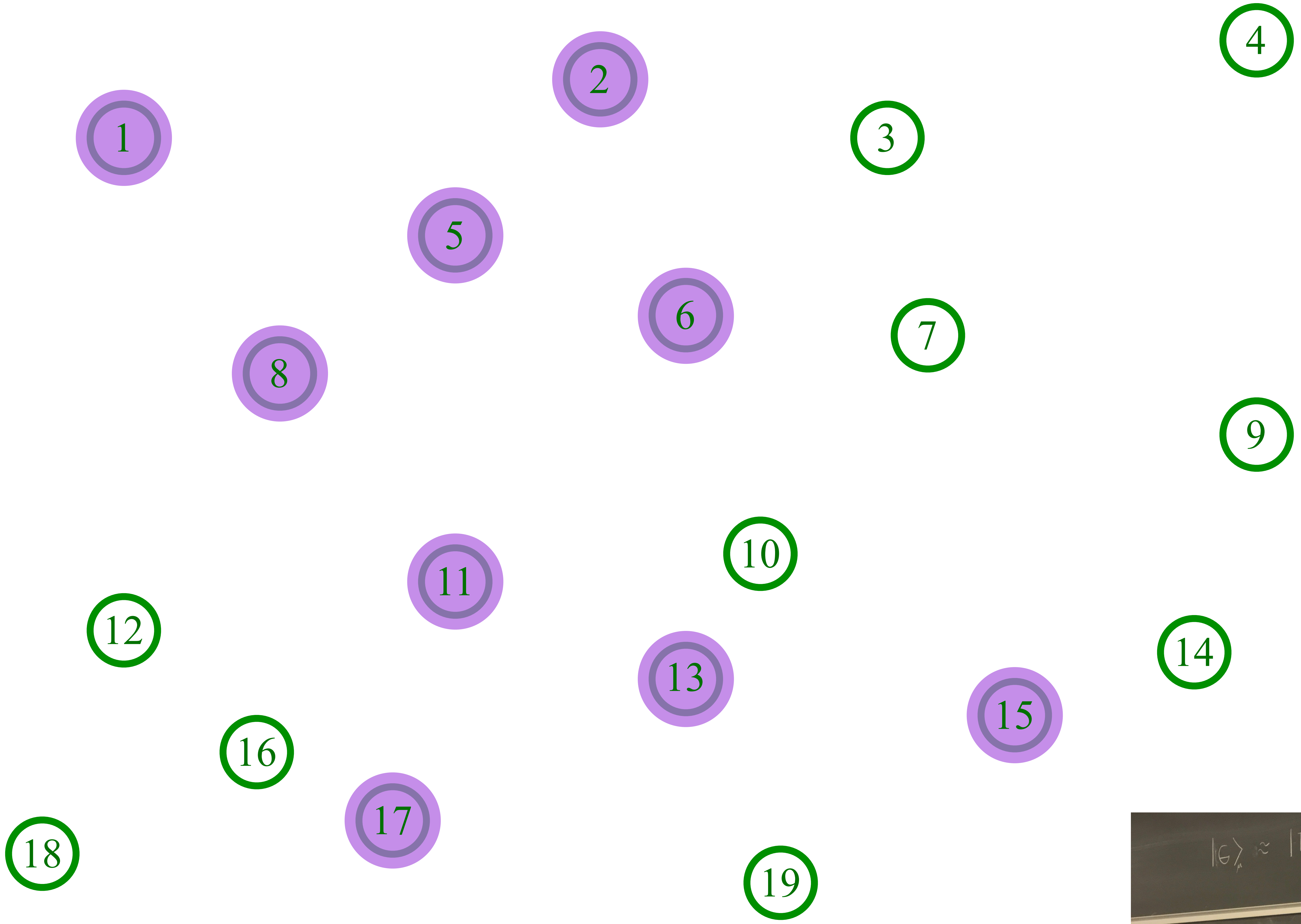

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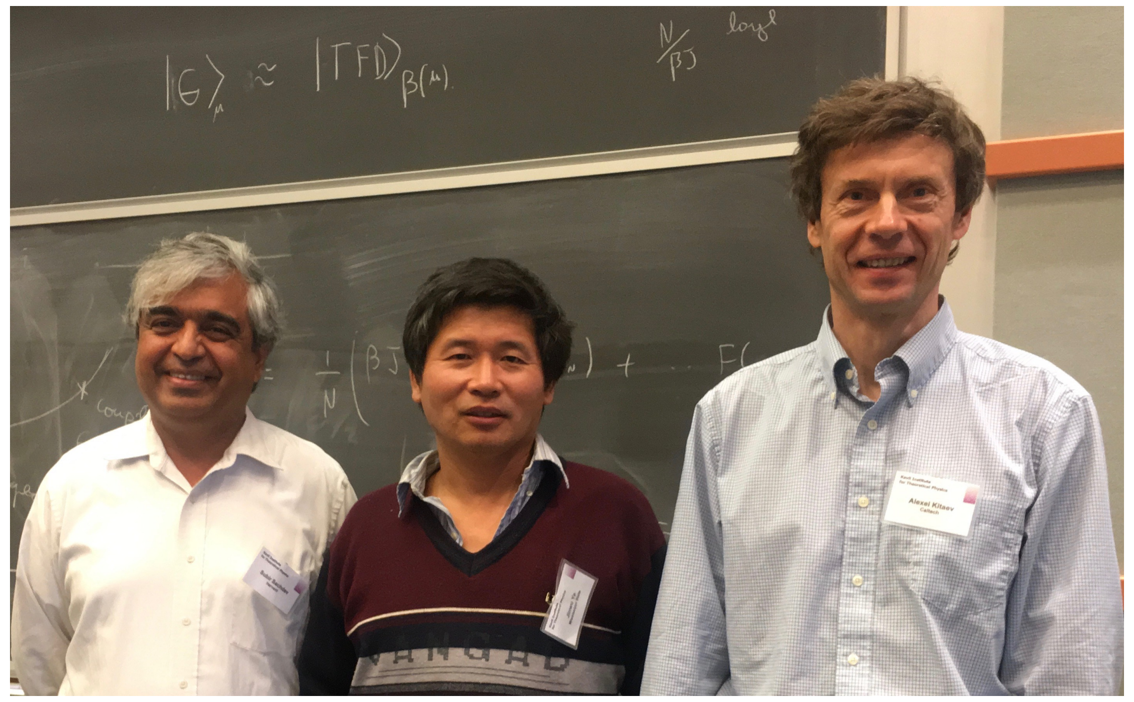
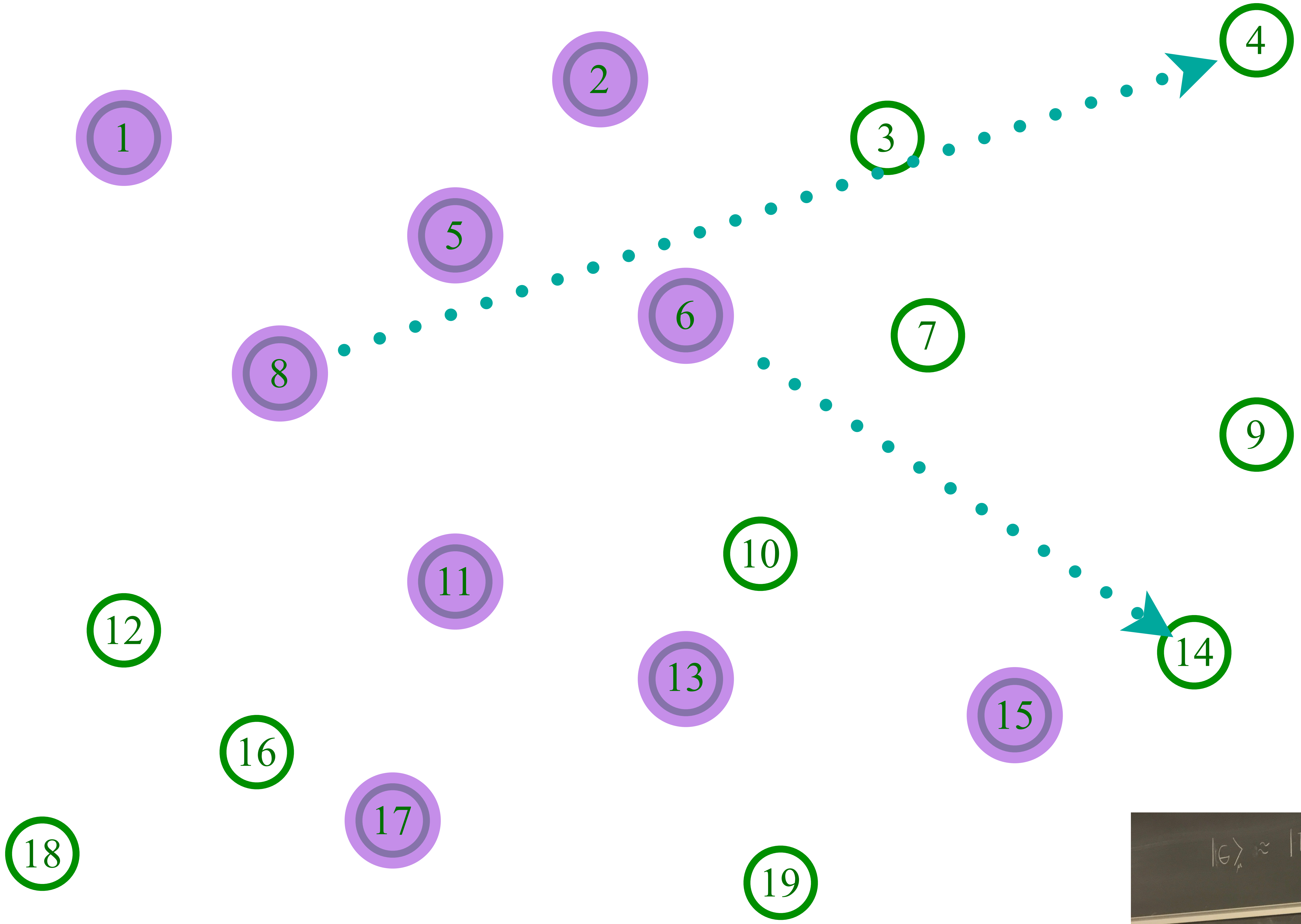


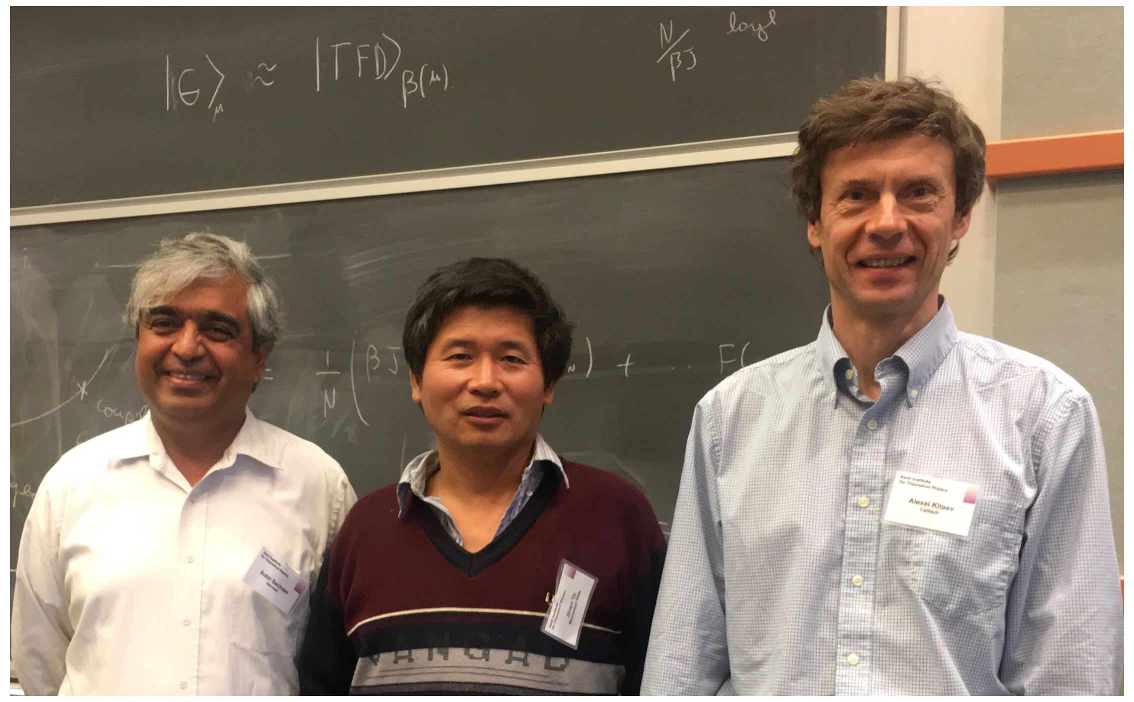
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



Entangle electrons pairwise randomly



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Sachdev, Ye (1993); Kitaev (2015)

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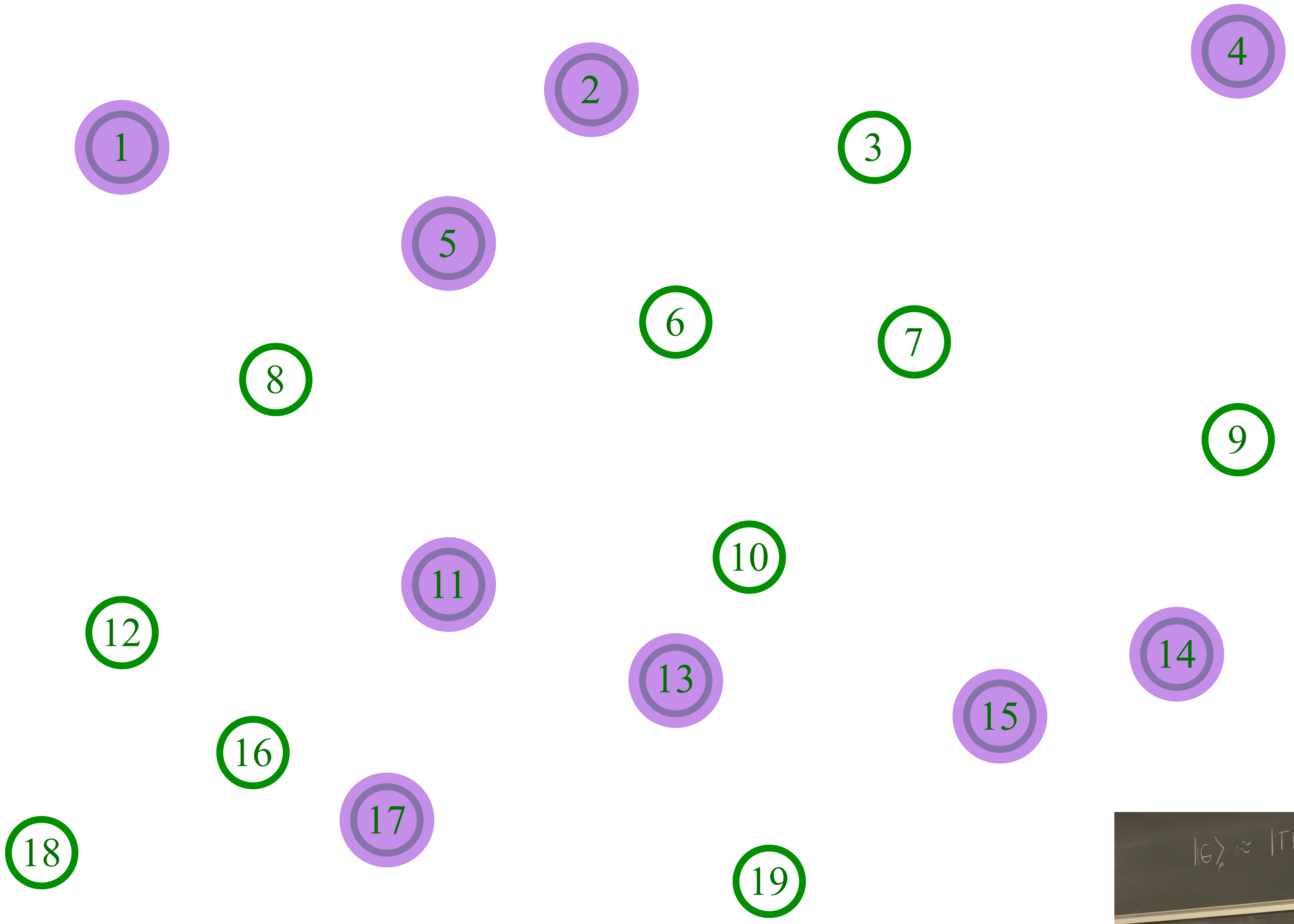
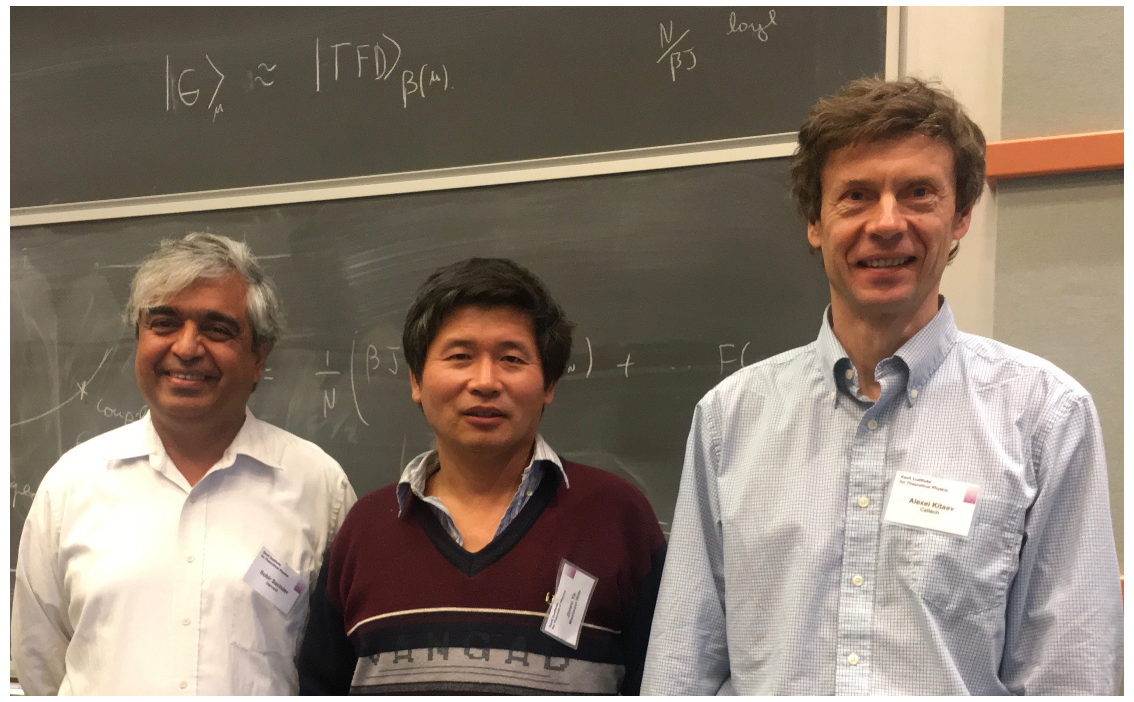


Entangle electrons pairwise randomly



The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle quantum entanglement.

No quasiparticles: yields a metal in which current is carried not by individual electrons, but by an entangled "quantum soup"

The SYK model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

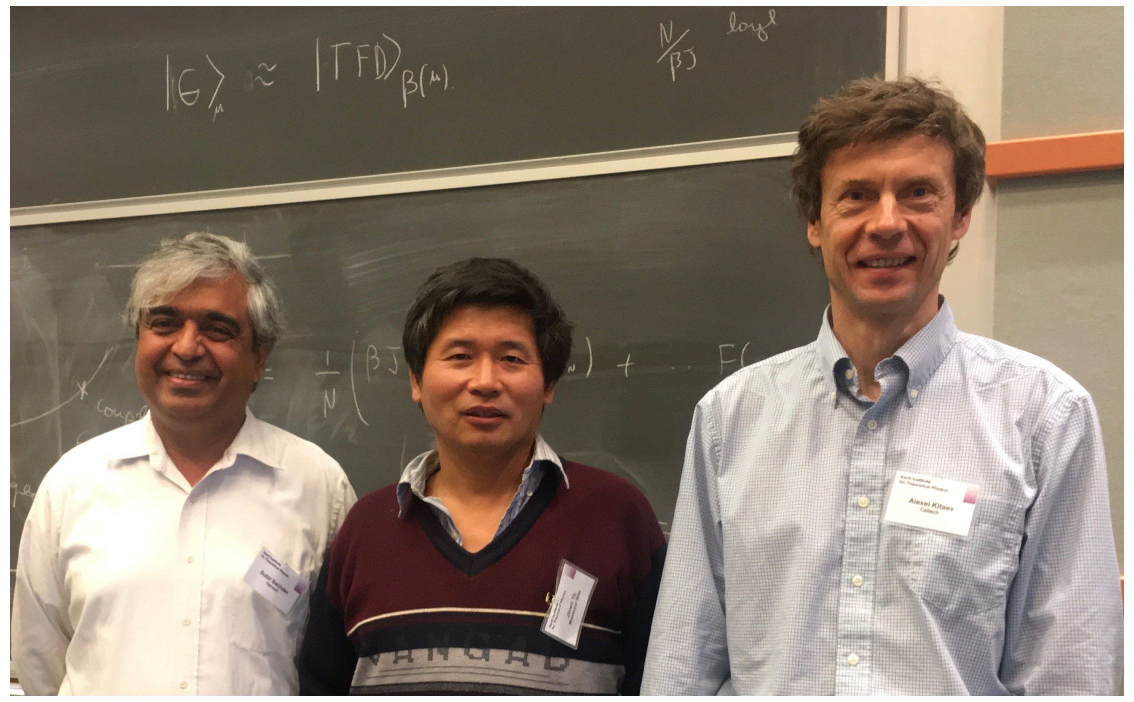
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

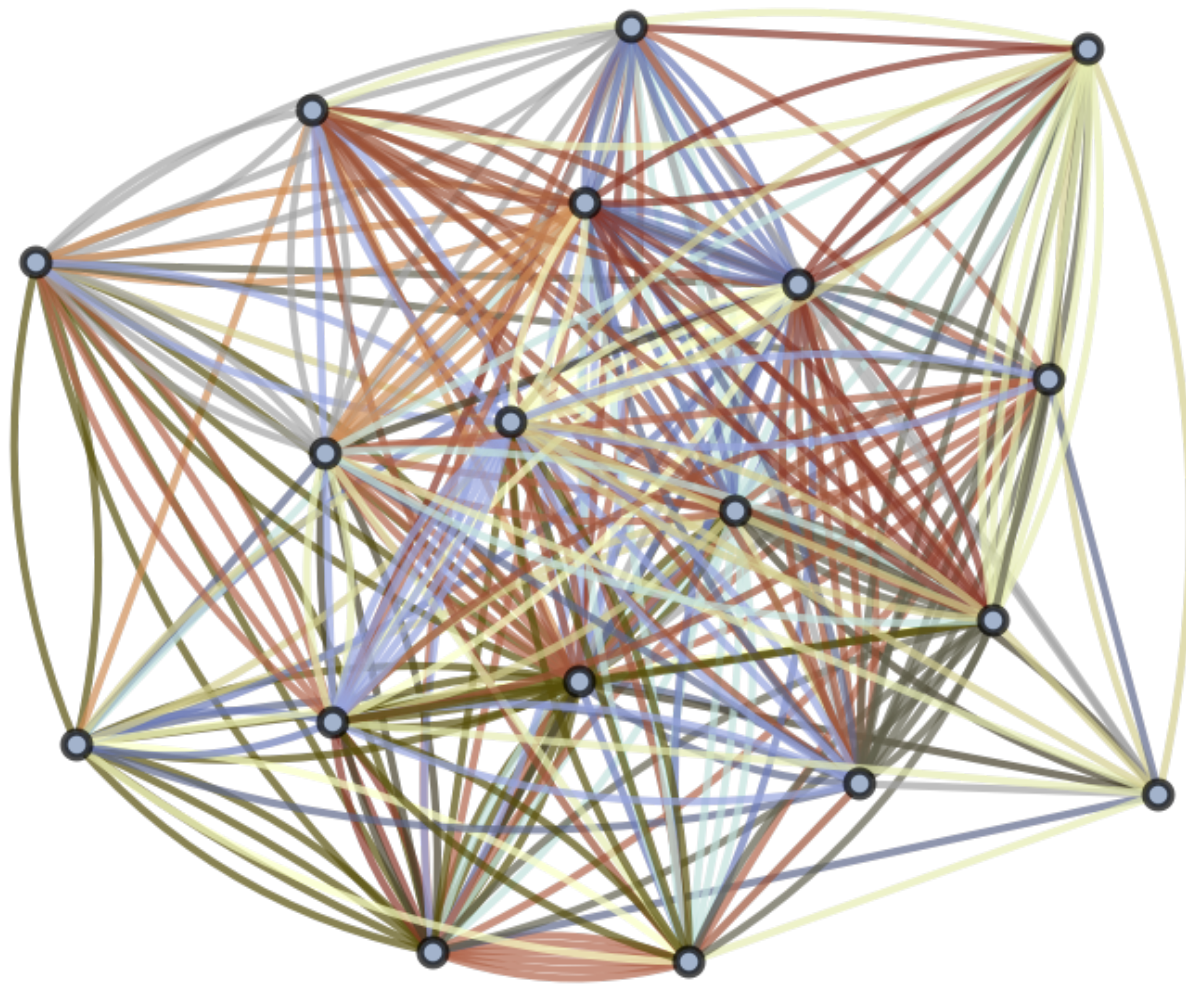
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha \, ; \quad [\mathcal{H}, \mathcal{Q}] = 0 \, ; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



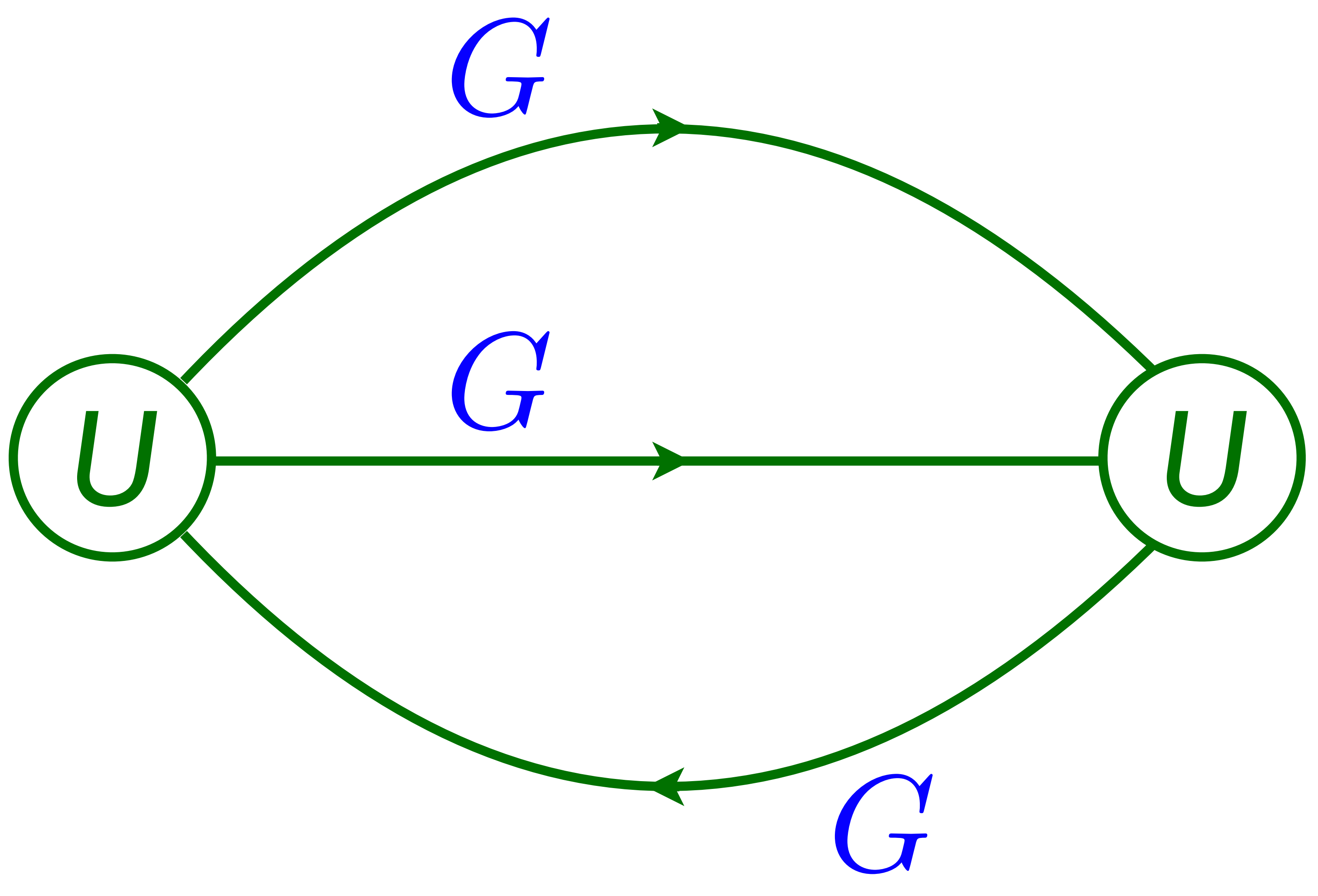


The SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations for the fermion Green's function $G_s(\tau) = -\sum_\alpha \left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle / N$ in the large N limit:

$$G_s(i\omega) = \frac{1}{i\omega + \mu - \Sigma_s(i\omega)} \quad , \quad \Sigma_s(\tau) = -U^2 G_s^2(\tau) G_s(-\tau)$$

$$G_s(\tau = 0^-) = \mathcal{Q}.$$

$\Sigma =$ 

S. Sachdev and J. Ye,
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$$G_s(\tau = 0^-) = \mathcal{Q}.$$

1. Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)

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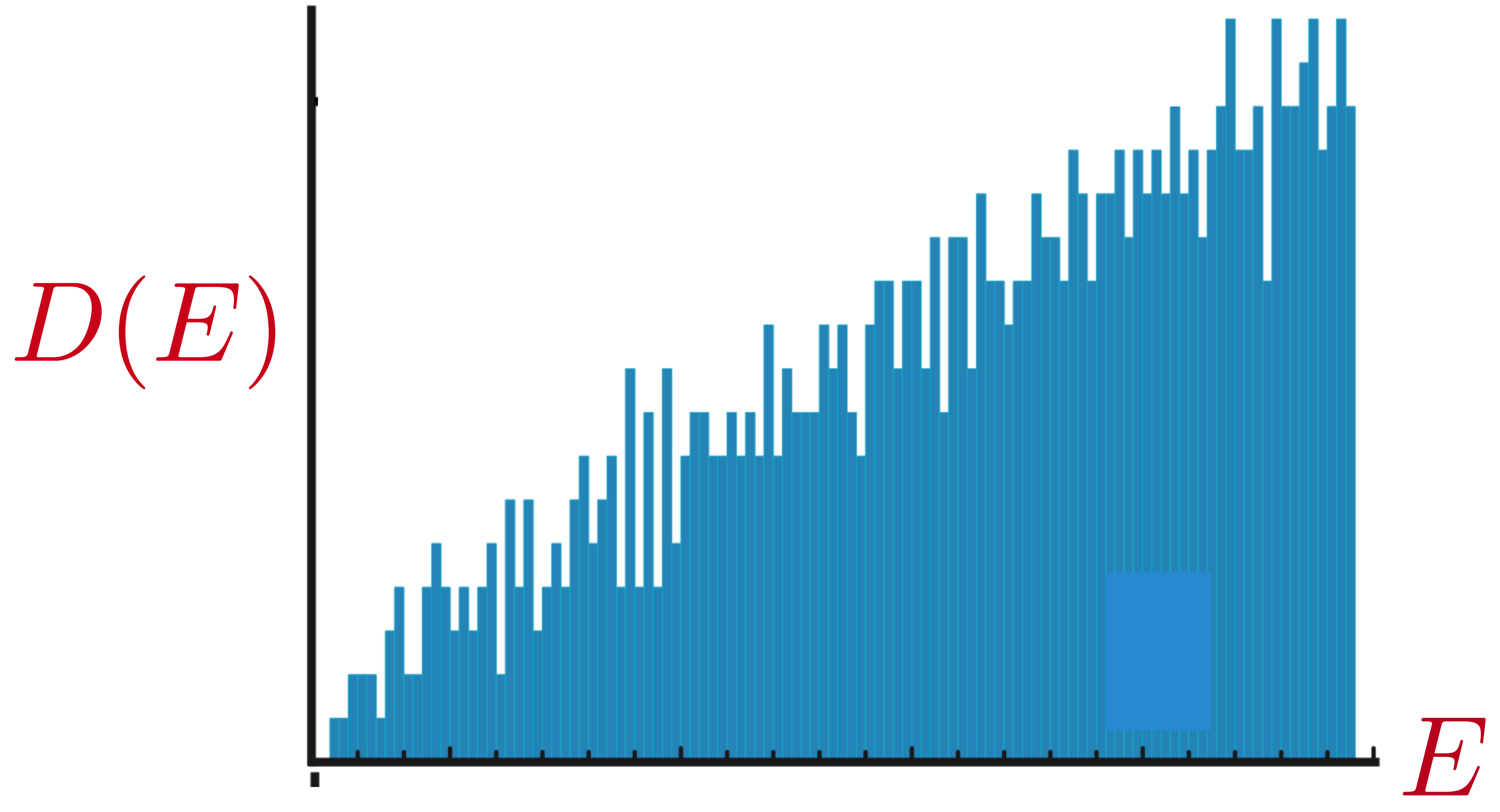
$$G_s(\tau = 0^-) = \mathcal{Q}.$$

2. Zero temperature entropy

$$\lim_{T \to 0} \lim_{N \to \infty} \frac{1}{N} S(T) = s_0 \quad , \quad D(E \to 0) \sim e^{N s_0}$$

$s_0 = 0.4648476991708051074 9\ldots$ for $\mathcal{Q} = 1/2$.



A. Georges, O. Parcollet, and S. Sachdev, Physical Review B **63**, 134406 (2001)

The SYK model

The (averaged) partition function can be written as path integral over the bilocal fermion Green's function $G(\tau_1, \tau_2) \sim \frac{1}{N} \sum_\alpha c_\alpha(\tau_1) c_\alpha^\dagger(\tau_2)$

A. Georges, O. Parcollet, S. Sachdev 2001
A. Kitaev, 2015
J. Maldacena, D. Stanford 2016

$$\overline{\mathcal{Z}} = \int \mathcal{D}G(\tau_1, \tau_2) \exp\left(-N S_{\text{eff}}[G]\right)$$

The large N saddle point equation $\delta S_{\text{eff}} / \delta G = 0$ yields $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$.

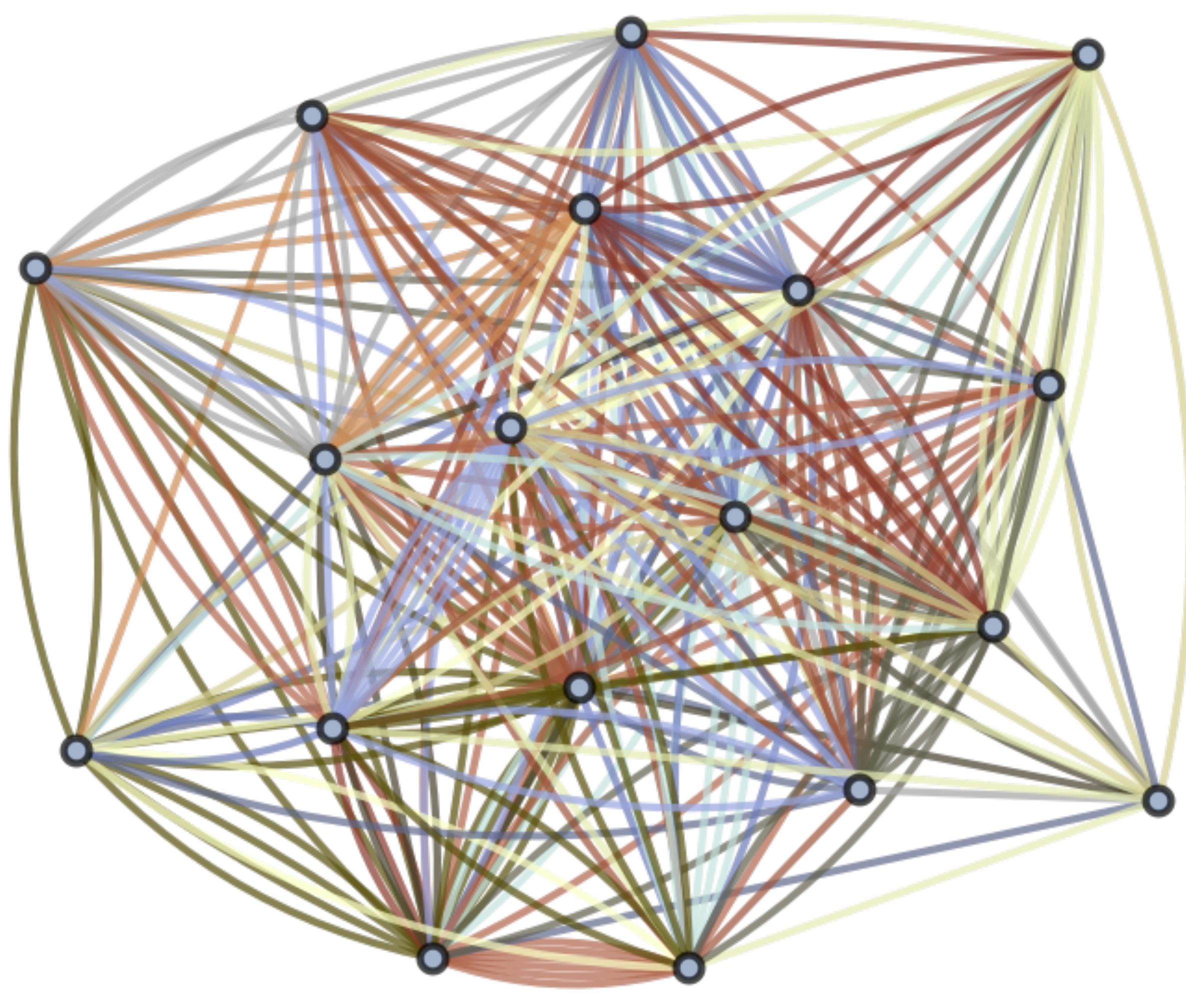
3. Time reparameterization symmetry:

At frequencies $\ll U$, the path integral for is invariant under time reparametrization $f(\sigma)$

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \, \widetilde{G}(\sigma_1, \sigma_2)$$

There is also an emergent U(1) gauge symmetry. Hints that the low energy theory is quantum gravity+electromagnetism!



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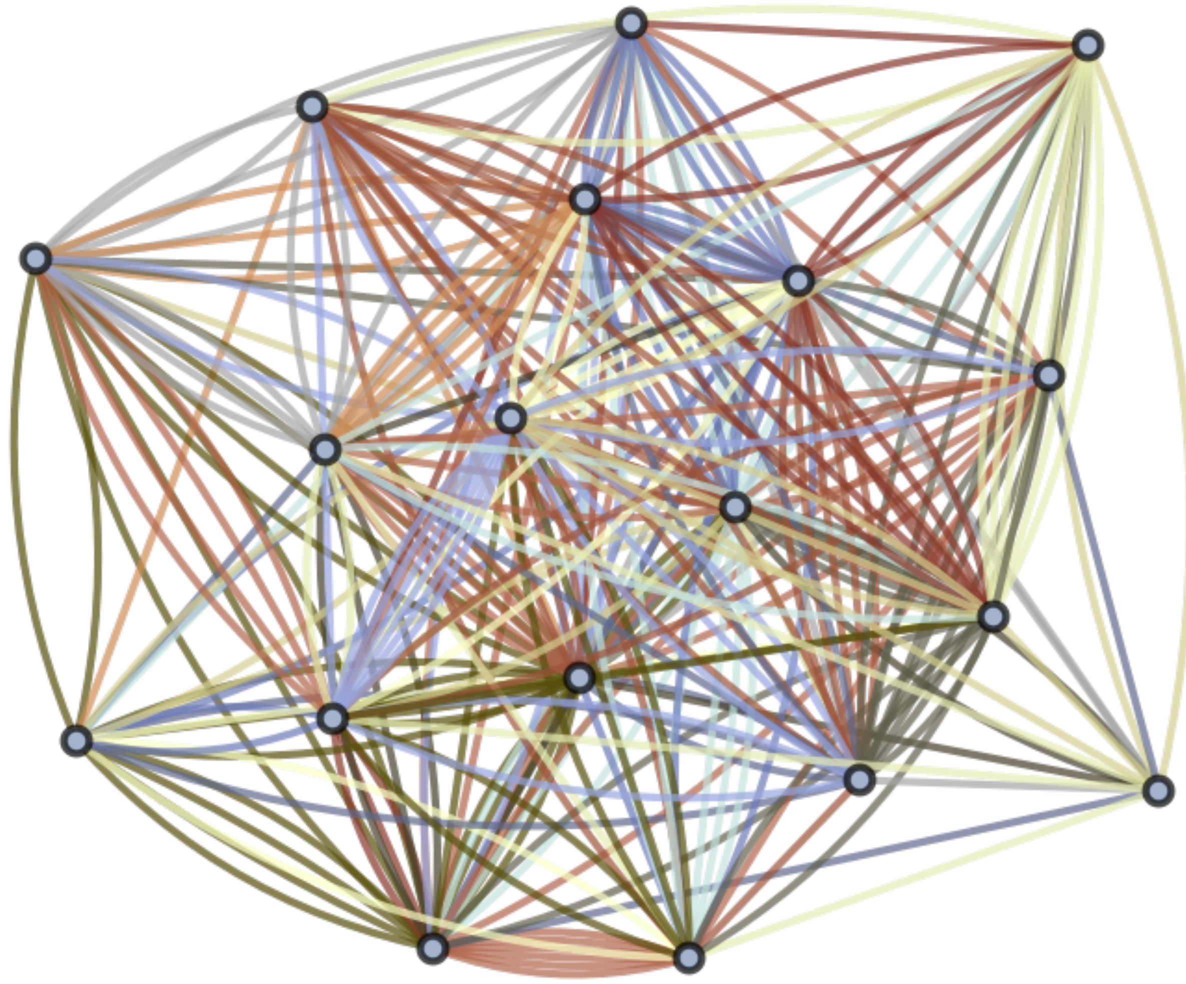
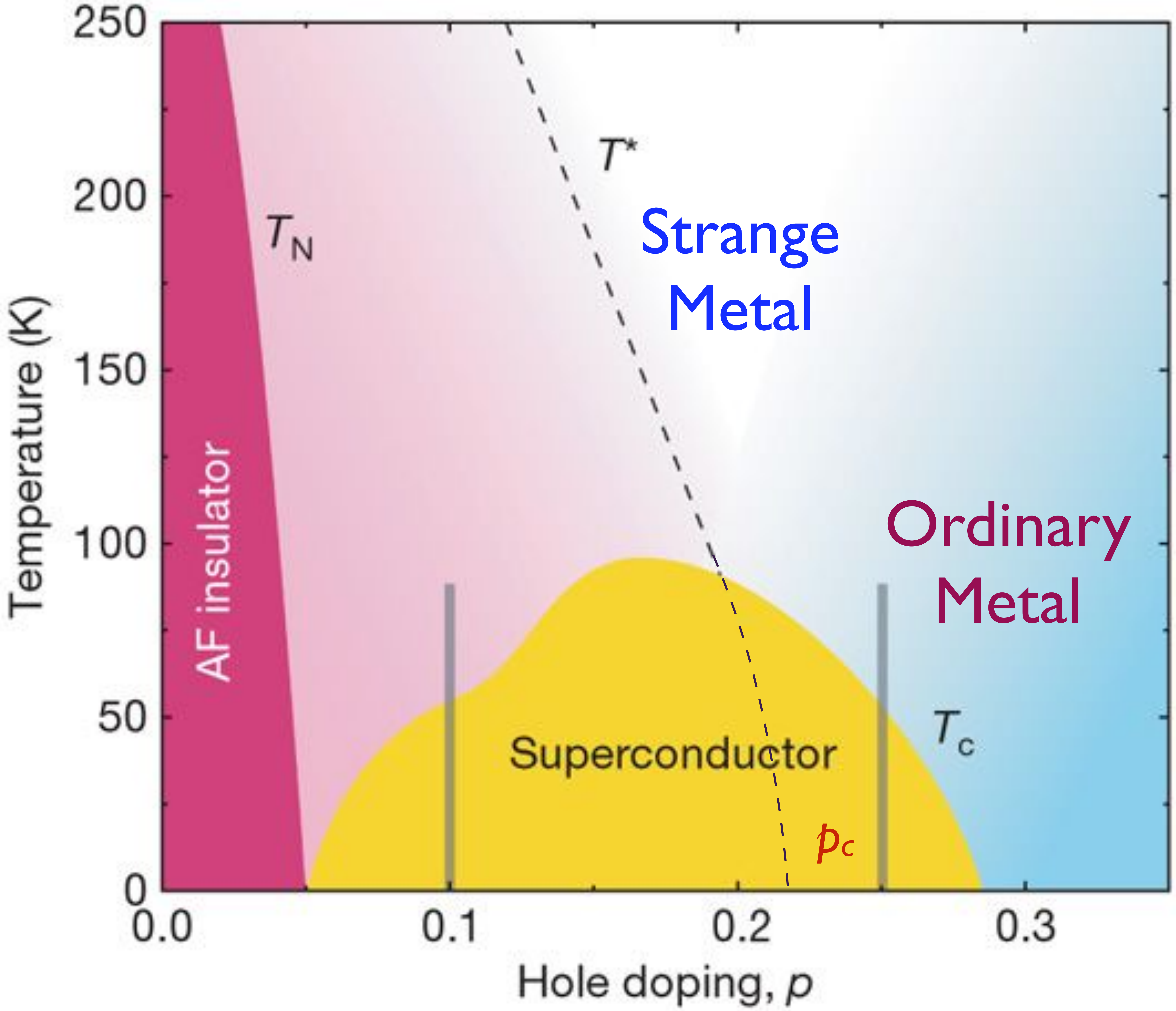
There is also an emergent U(1) gauge symmetry. Hints that the low energy theory is quantum gravity+electromagnetism!

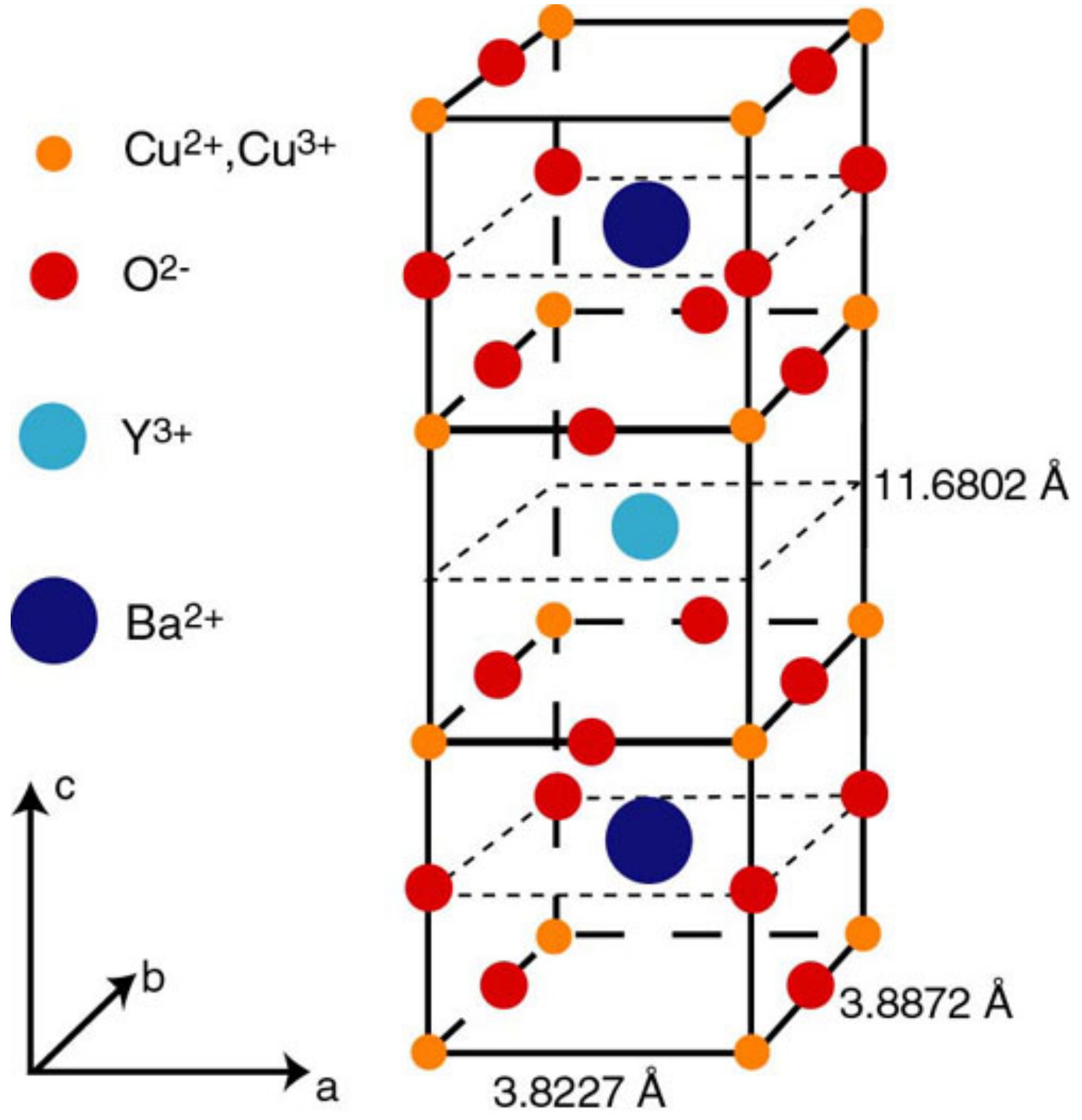
4. Conformal symmetry of saddle point:



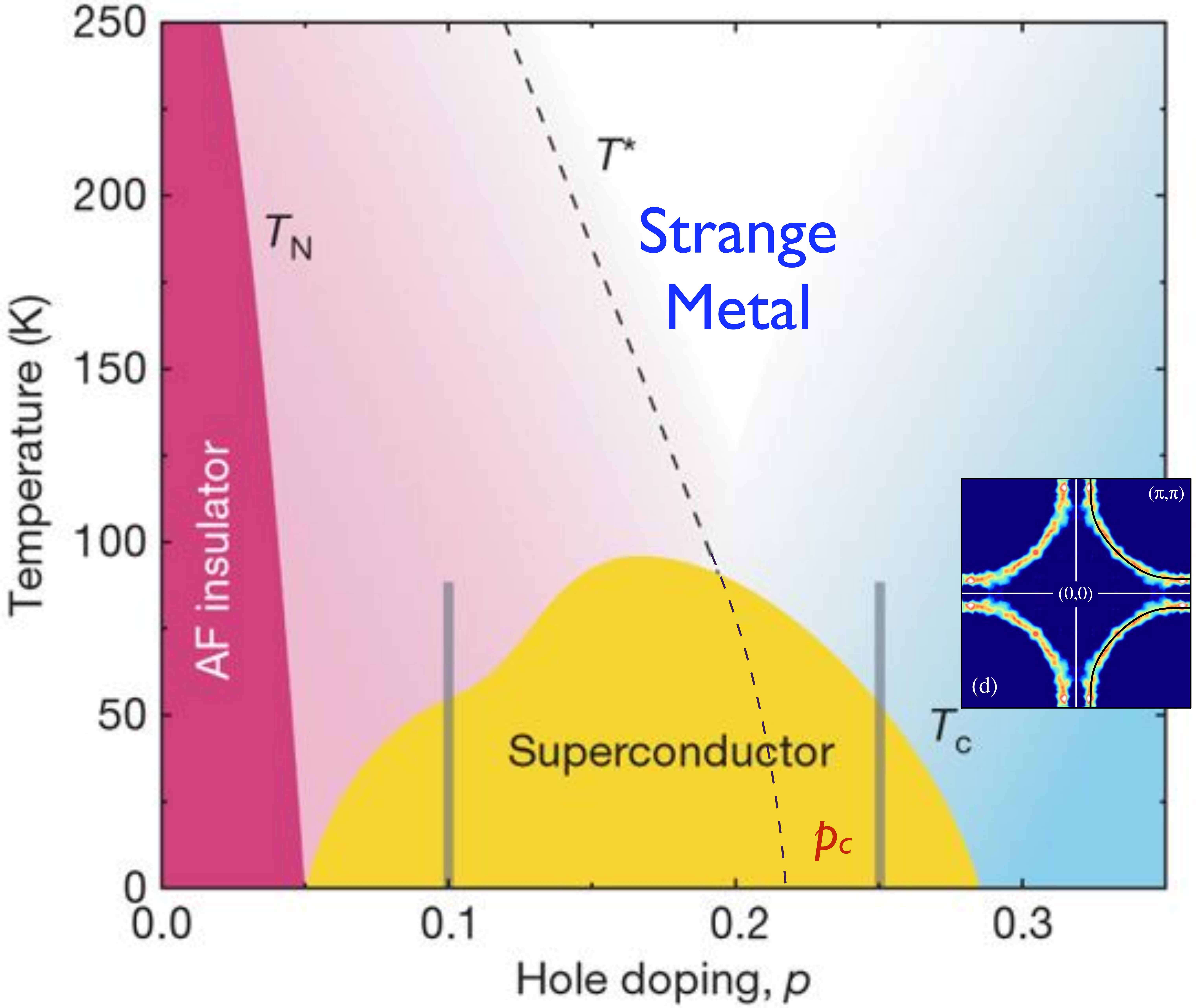
$$G_s(\tau_1 - \tau_2) = \widetilde{G}_s(\sigma_1 - \sigma_2) \text{ for } \tau = \frac{a\sigma + b}{c\sigma + d}, \; ad - bc = 1\,.$$

The SYK model
and
superconductivity


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Strange
Metal
Ordinary
Metal
Superconductor
T_c
p_c

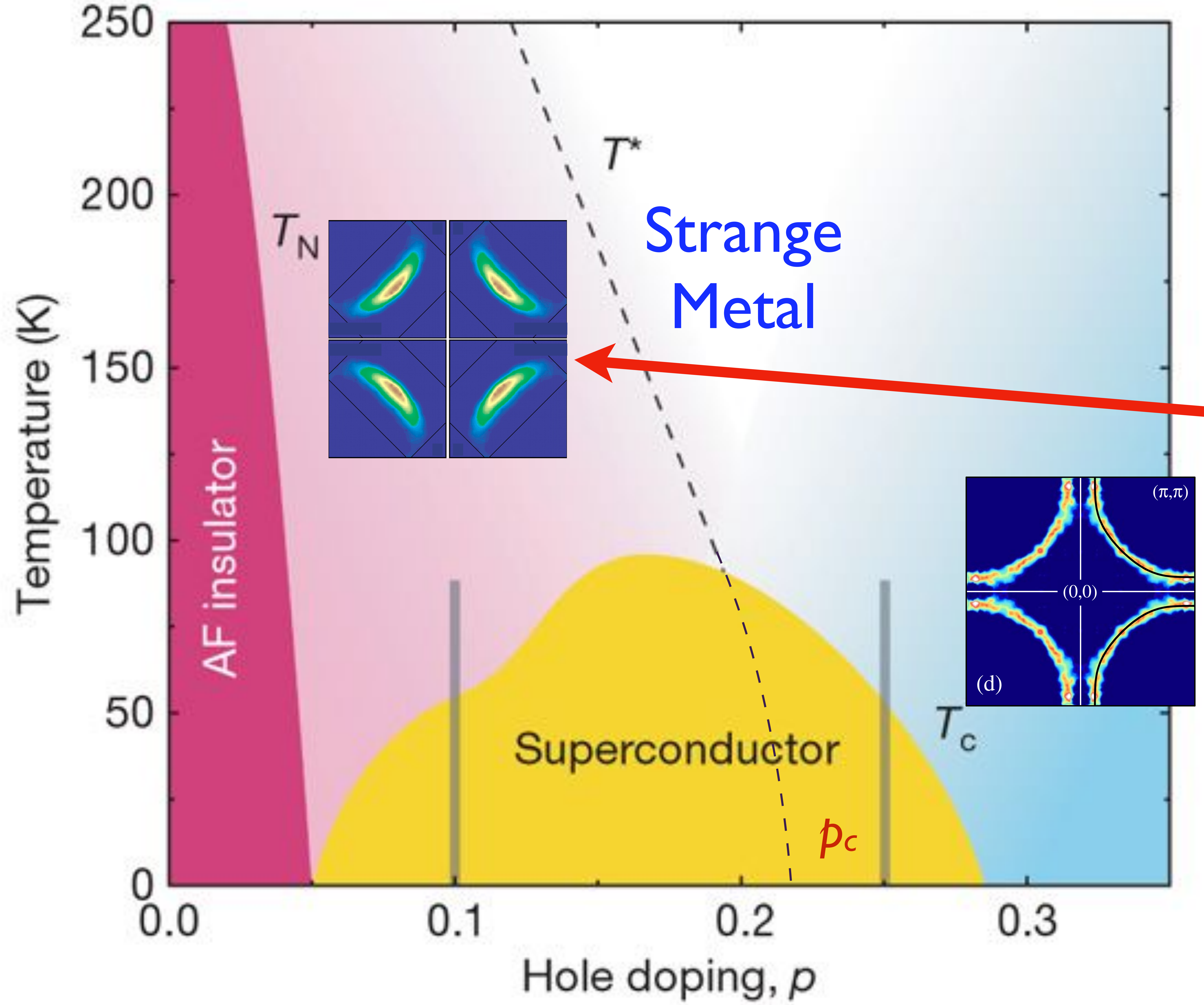

Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$

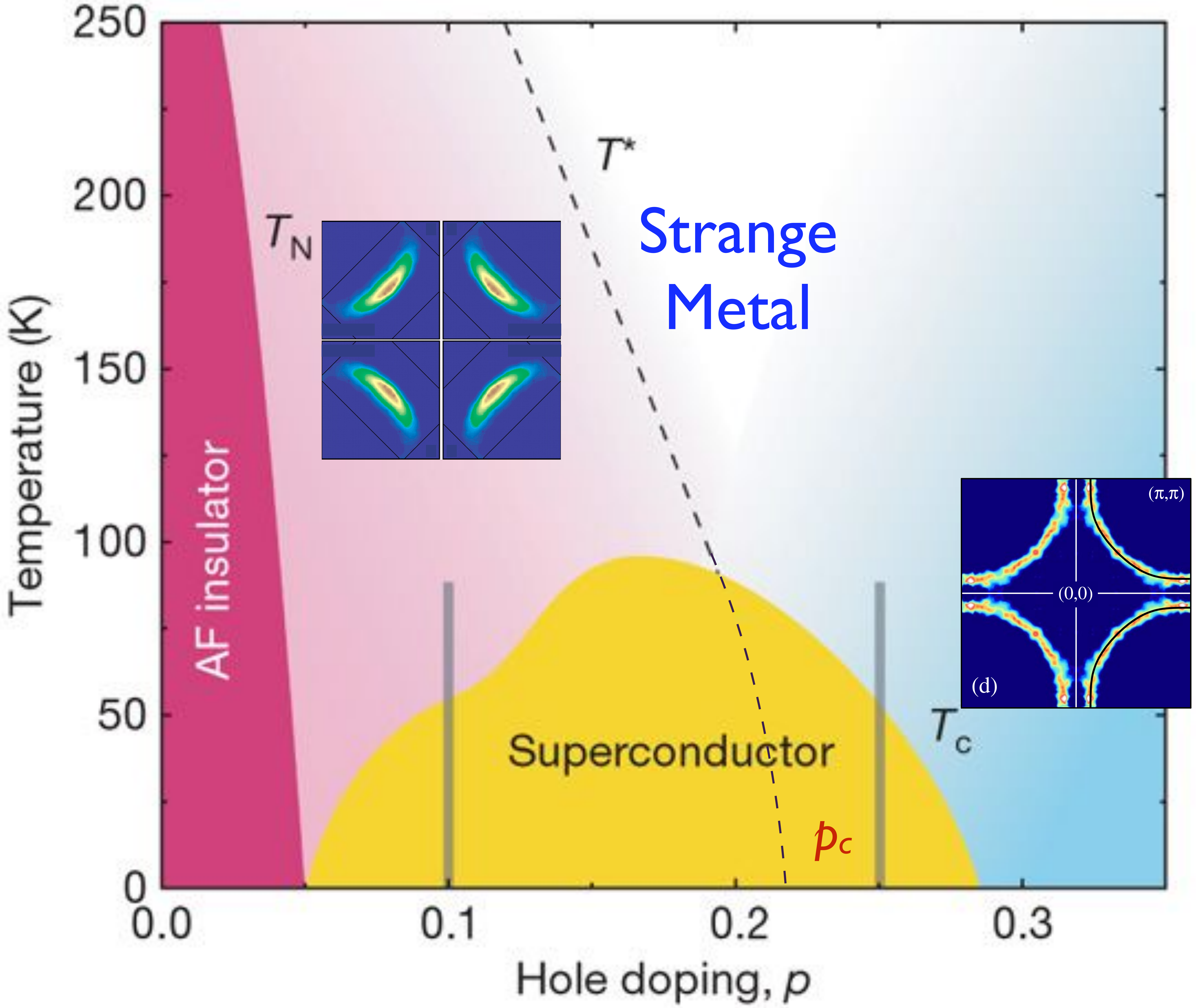


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Superconductor
T_c
p_c
(π,π)
(0,0)
(d)

Fermi surface
as expected
in a model
of free electrons



"Pseudogap metal" Fermi surface modified by electron-electron interactions

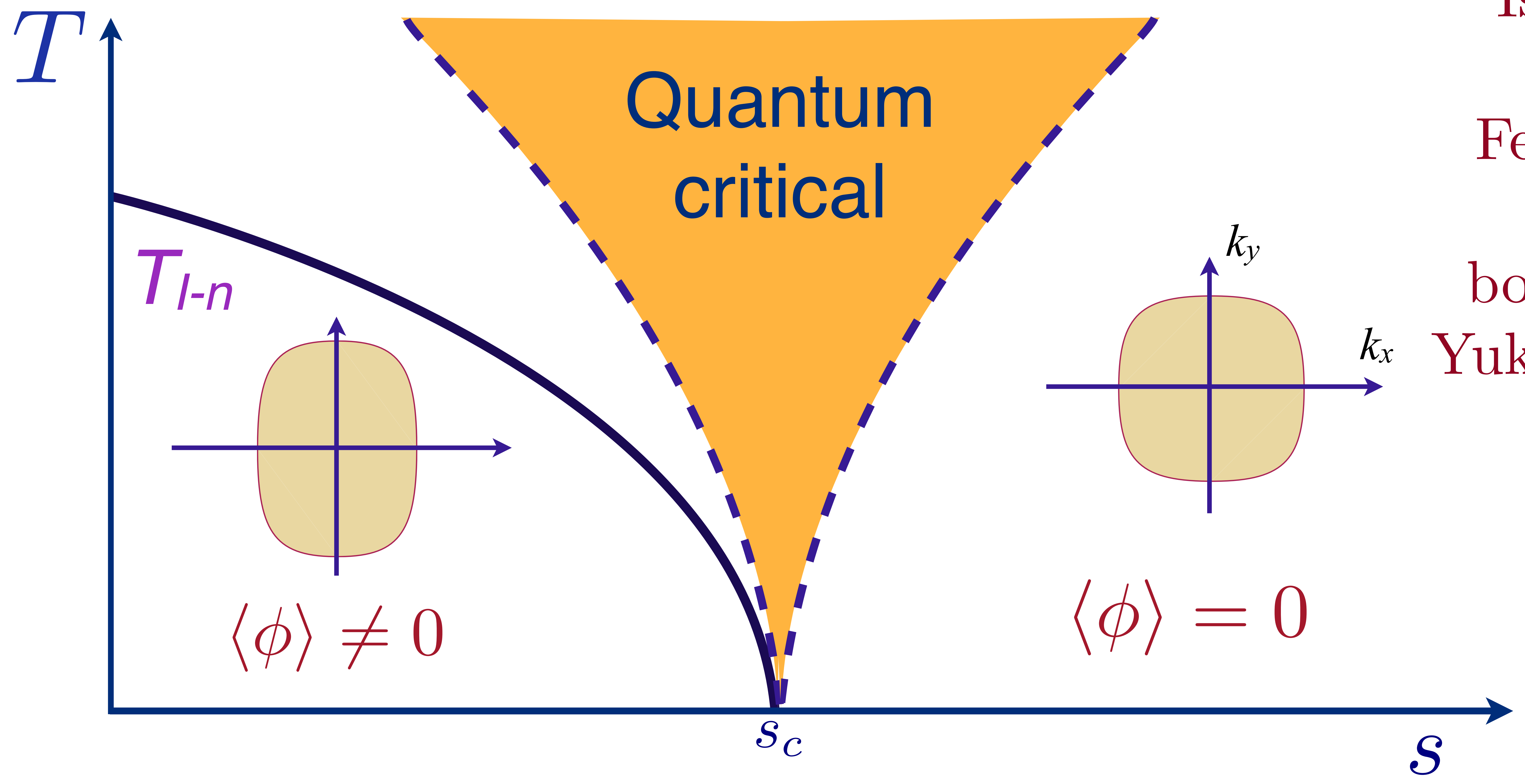


View the strange metal as a property of a $T = 0$ quantum phase transition involving change in the Fermi surface.

The onset of superconductivity may "hide" this quantum transition.

Hertz 1976, Millis 1993

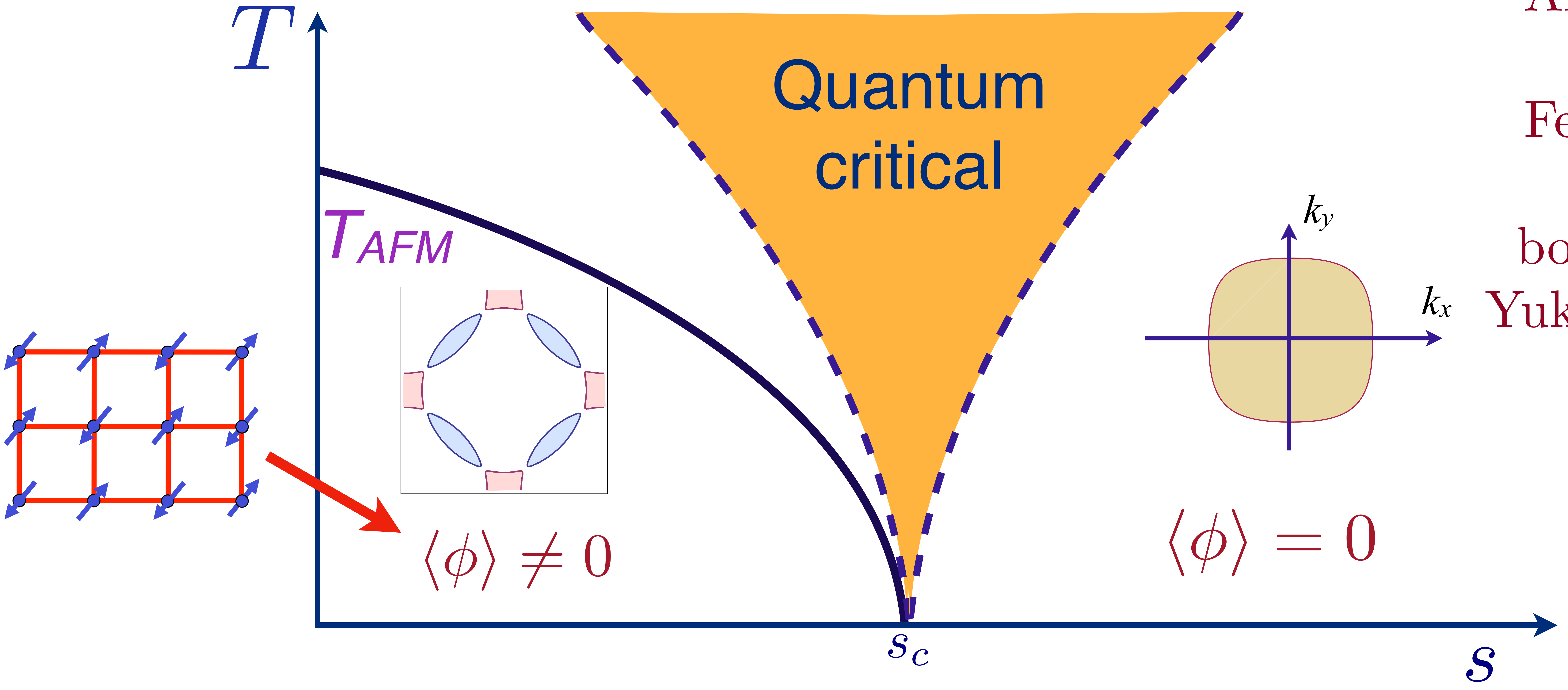
Type 1: Fermi surface distortion



Ising field ϕ
$+$
Fermi surface
with a
boson-fermion
Yukawa coupling

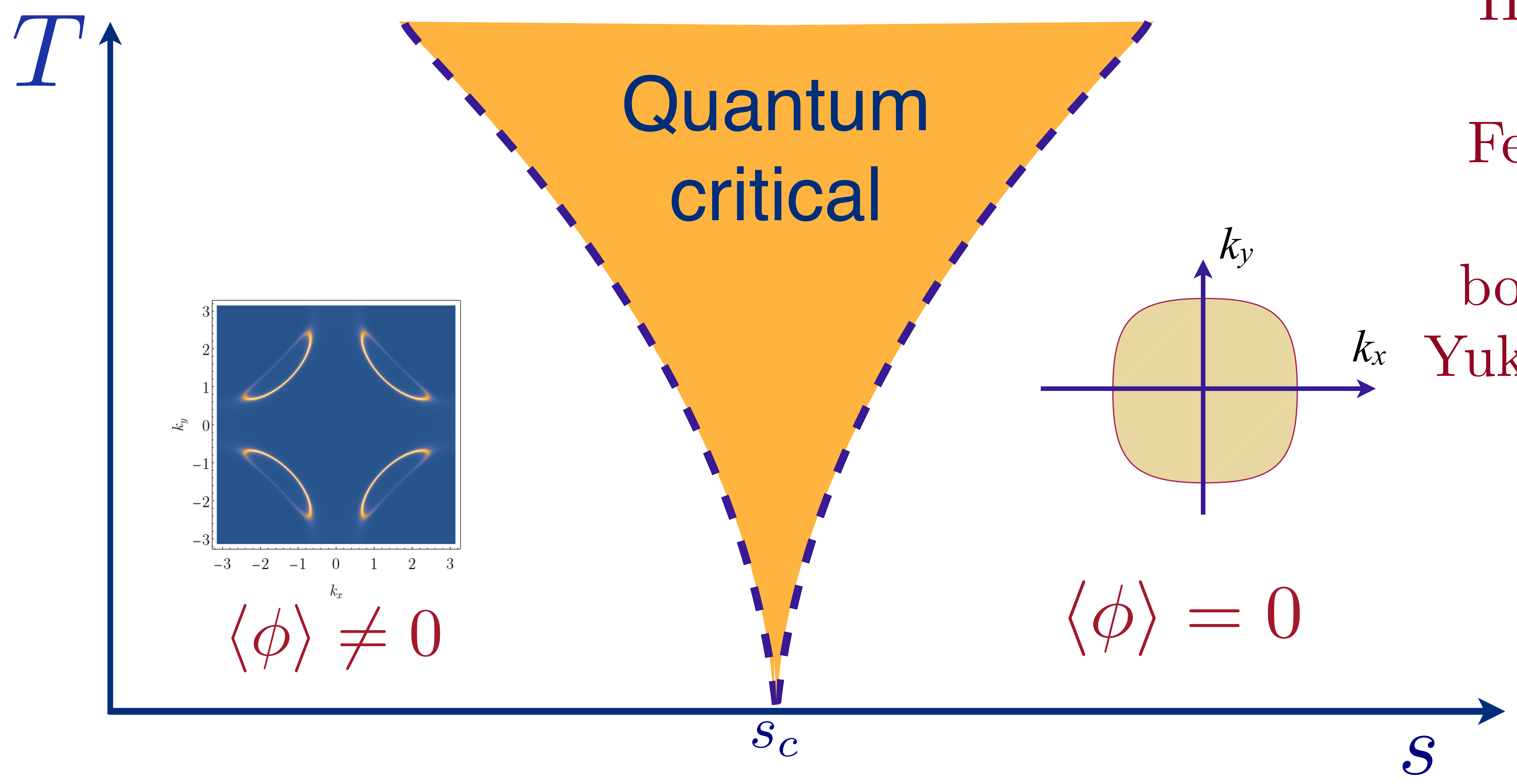
Hertz 1976, Millis 1993

Type II: Fermi surface reconstruction



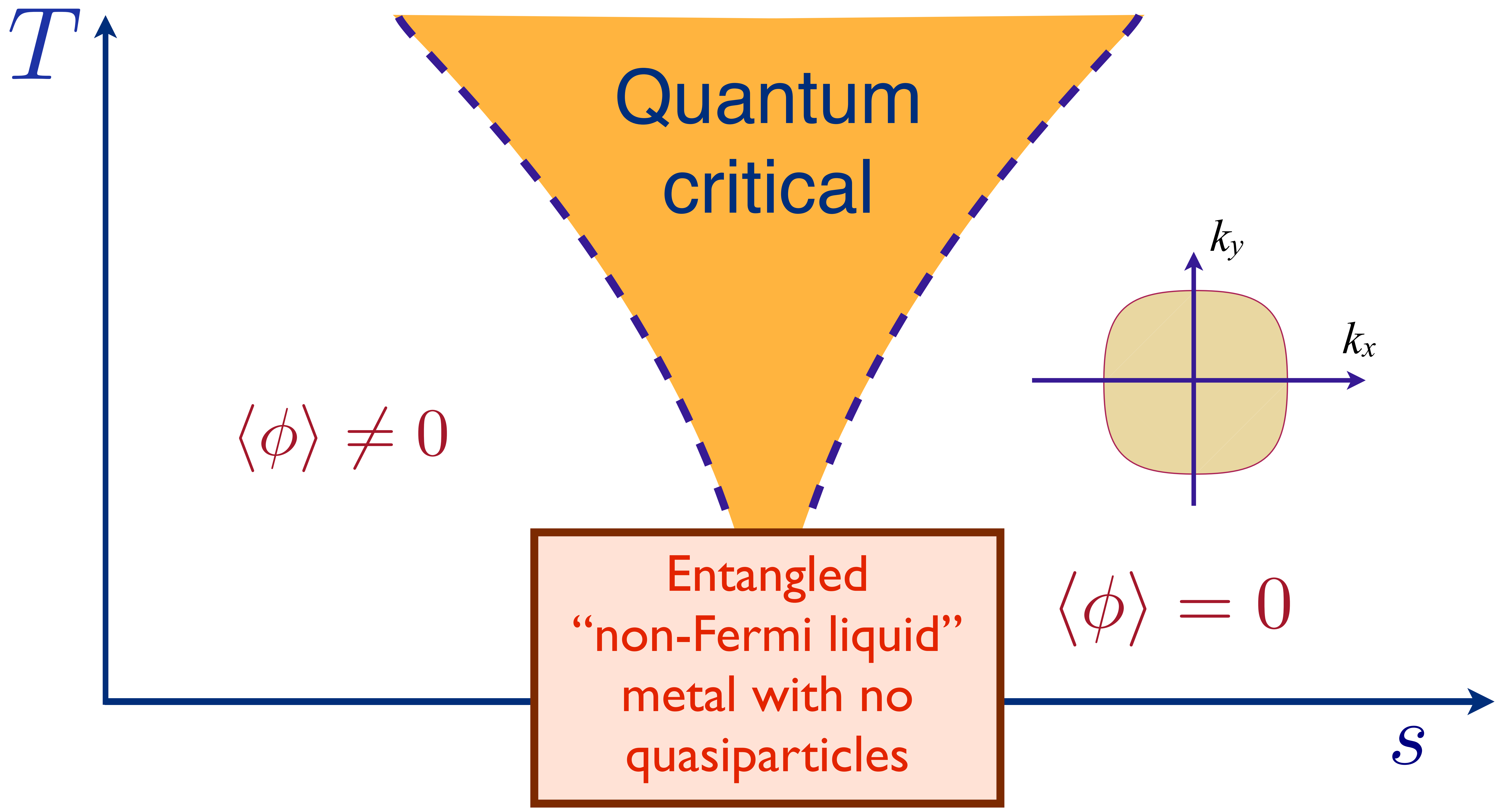
AFM field ϕ_a
$+$
Fermi surface
with a
boson-fermion
Yukawa coupling

Type III: Fermi surface jump

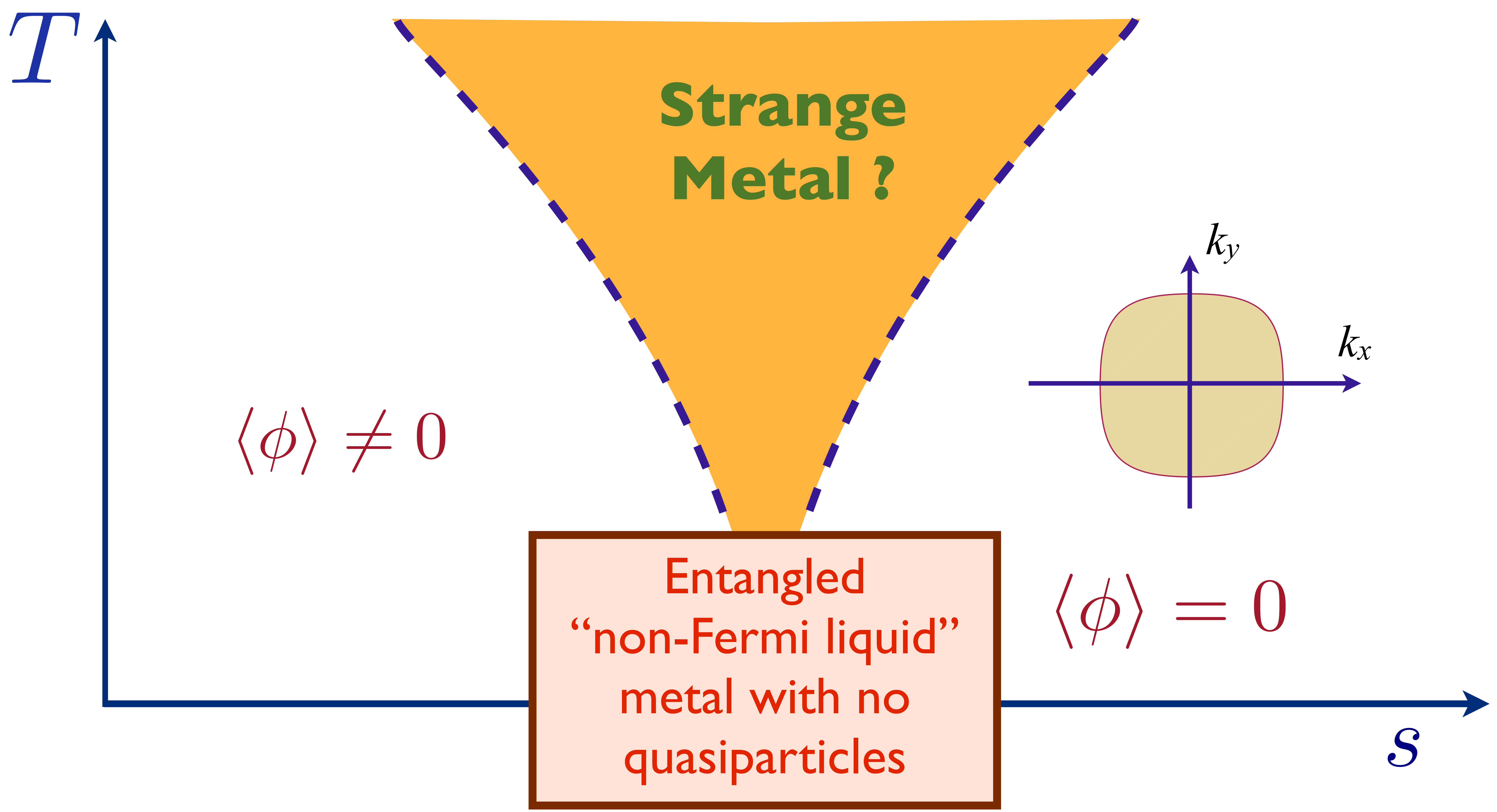


Spin liquid
+
Higgs field ϕ
+
Fermi surface with a boson-fermion Yukawa coupling

Type I, II, or III

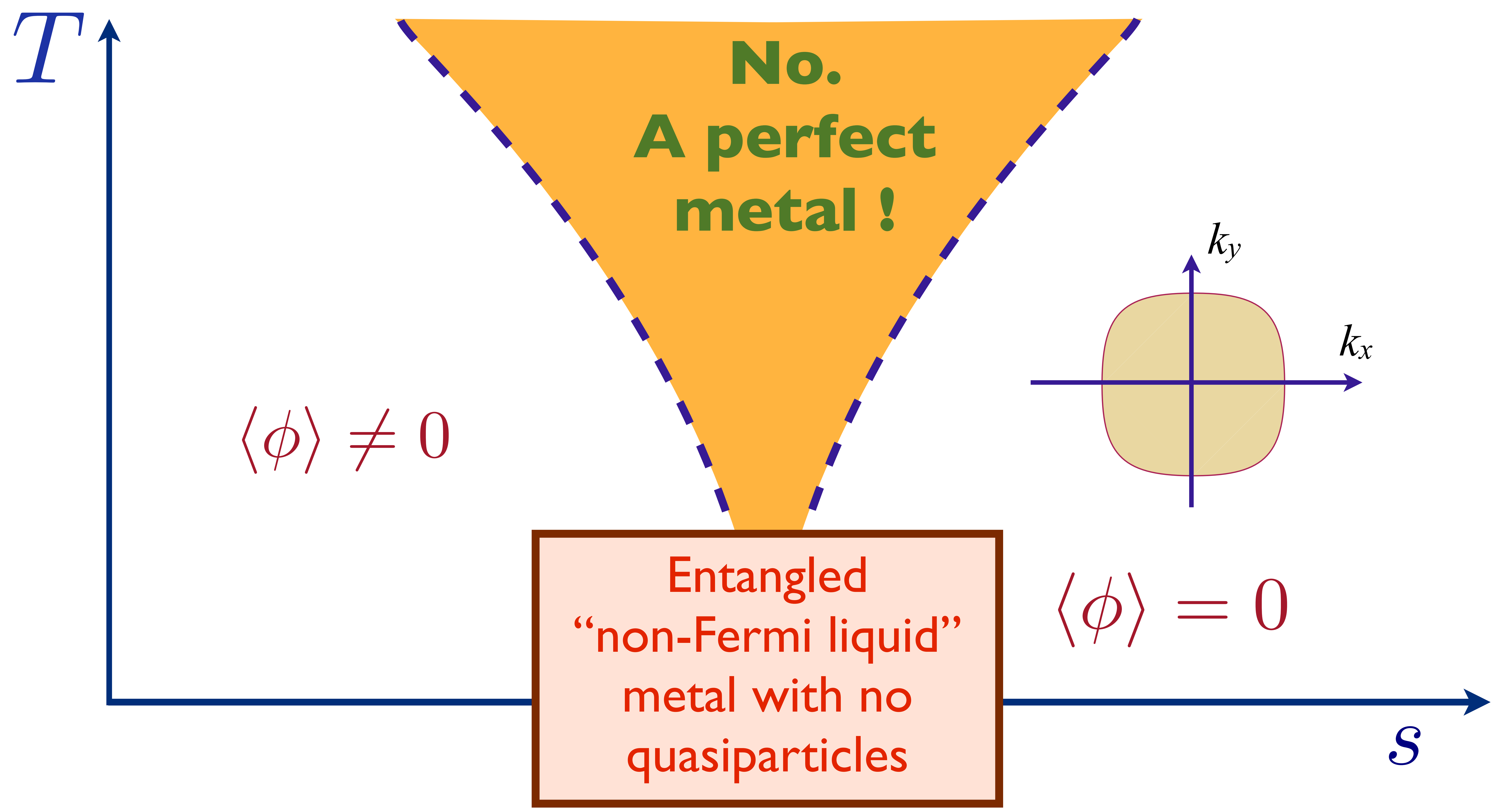


Type I, II, or III



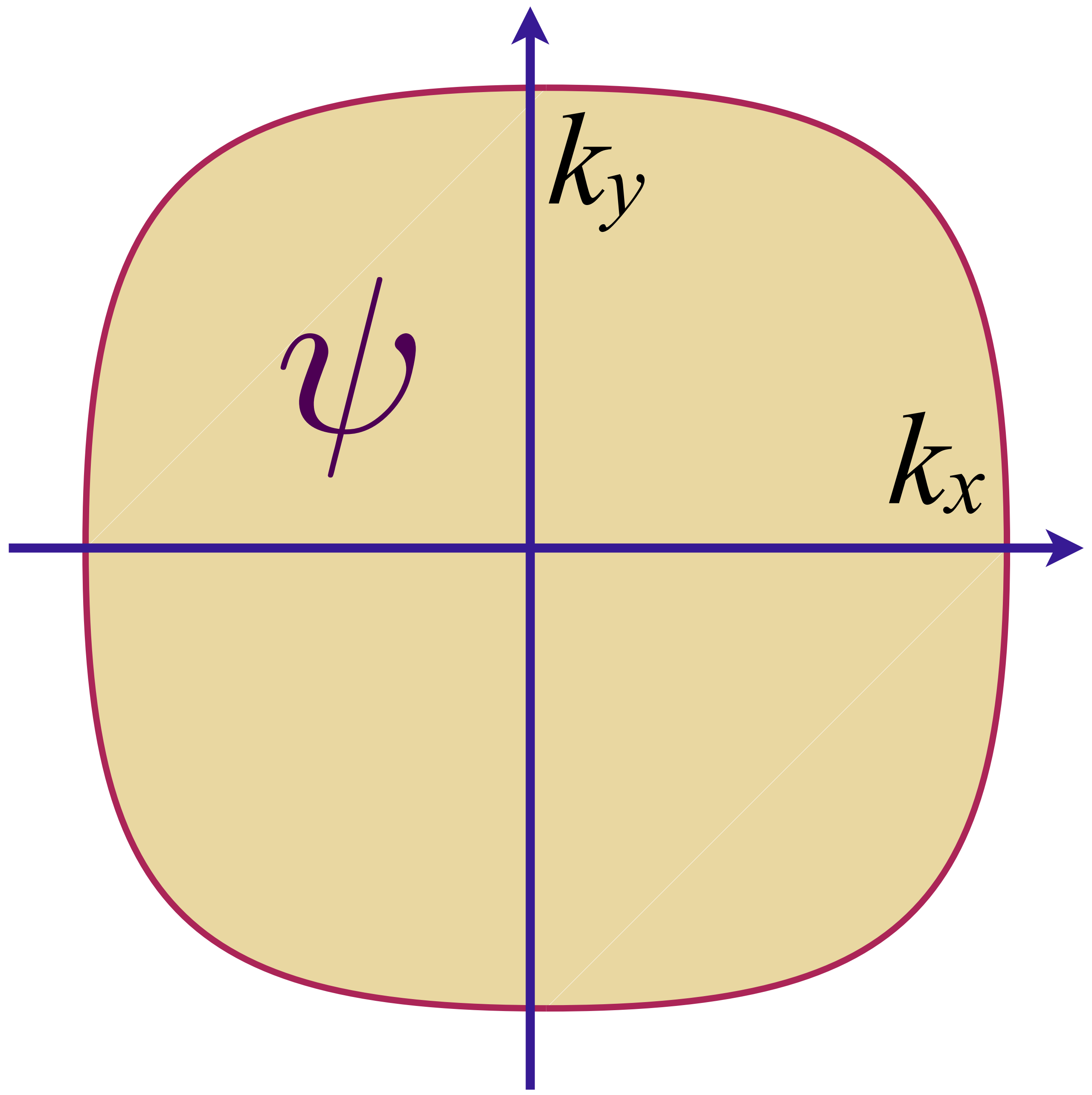
Type I, II, or III

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S., PRB **106**, 115151 (2022)
Haoyu Guo, Davide Valentinis, J. Schmalian, S.S., Aavishkar Patel, PRB **109**, 075162 (2024)
D.L. Maslov and A.V. Chubukov, Rep. Prog. Phys. **80**, 026503 (2017)
Zhengyan Darius Shi, Dominic V. Else, Hart Goldman, T. Senthil, SciPost Phys. **14**, 113 (2023)



Fermi surface + critical boson with no spatial disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



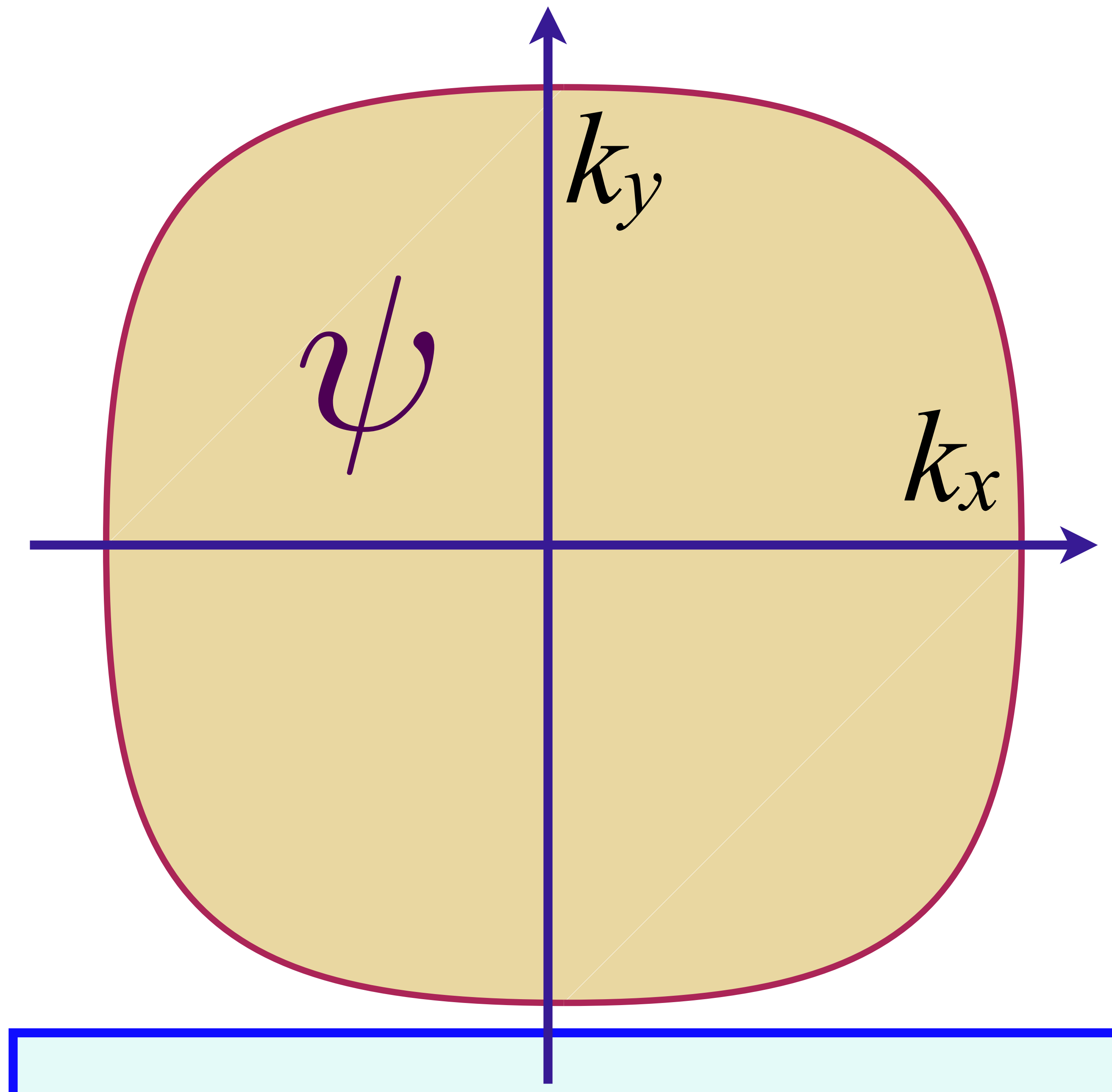
A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume changing transition

$$+s\,[\phi(\boldsymbol{r})]^2 \qquad + g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4$$

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume changing transition

$$+s\,[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 + v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatial randomness in position of quantum critical point,
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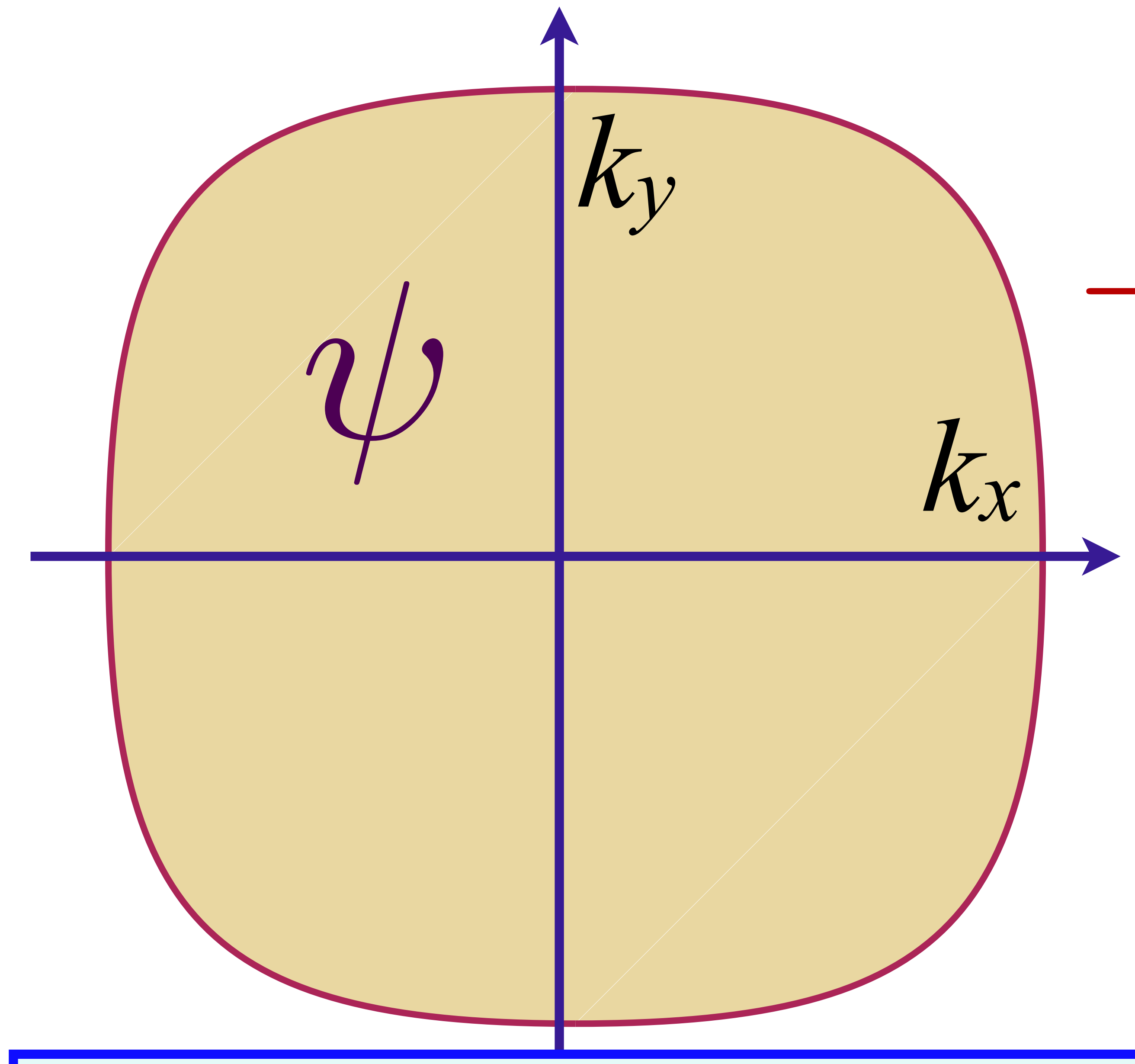

$$+[s + \delta s(\boldsymbol{r})]\,[\phi(\boldsymbol{r})]^2 + \qquad + g\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 \; + v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

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Spatial randomness in position of quantum critical point,
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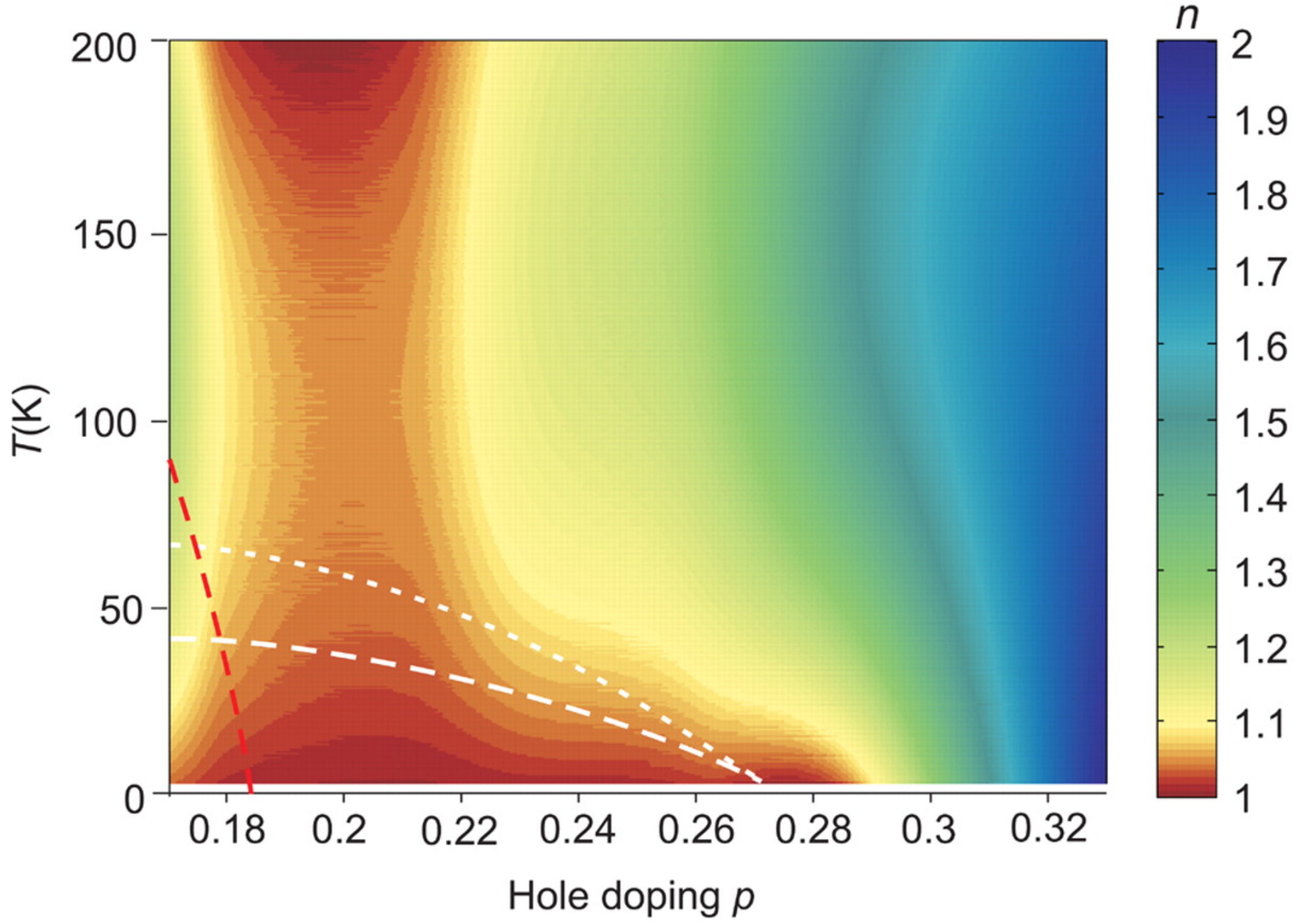
Universal theory of quantum phase transitions in disordered metals

Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4*] H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

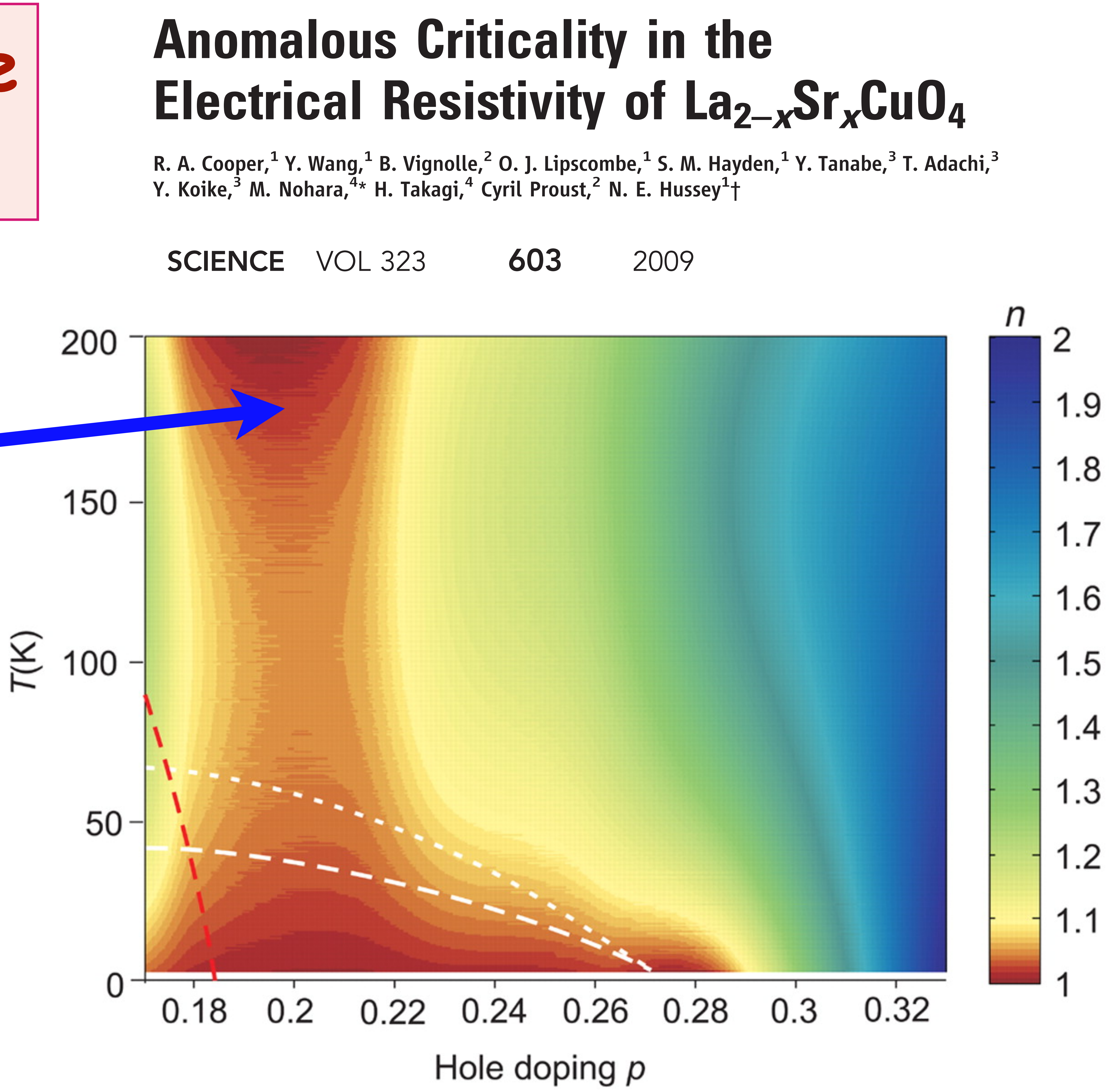
SCIENCE VOL 323 **603** 2009



Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023);
Aavishkar A. Patel, Peter Lunts, S.S., PNAS **121**, e2402052121 (2024);
Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, to appear

Universal theory of quantum phase transitions in disordered metals

Fermions and bosons are extended,
and we can apply
SYK-self-averaging theory to
a two-dimensional 'Yukawa-SYK' model
with a Fermi surface and
random interactions between
the bosons and fermions.



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Fermions remain extended,
but bosons localize, and
SYK-self-averaging theory does *not* apply.
'Griffiths phase' with overdamped localized bosons
coupling to extended fermions

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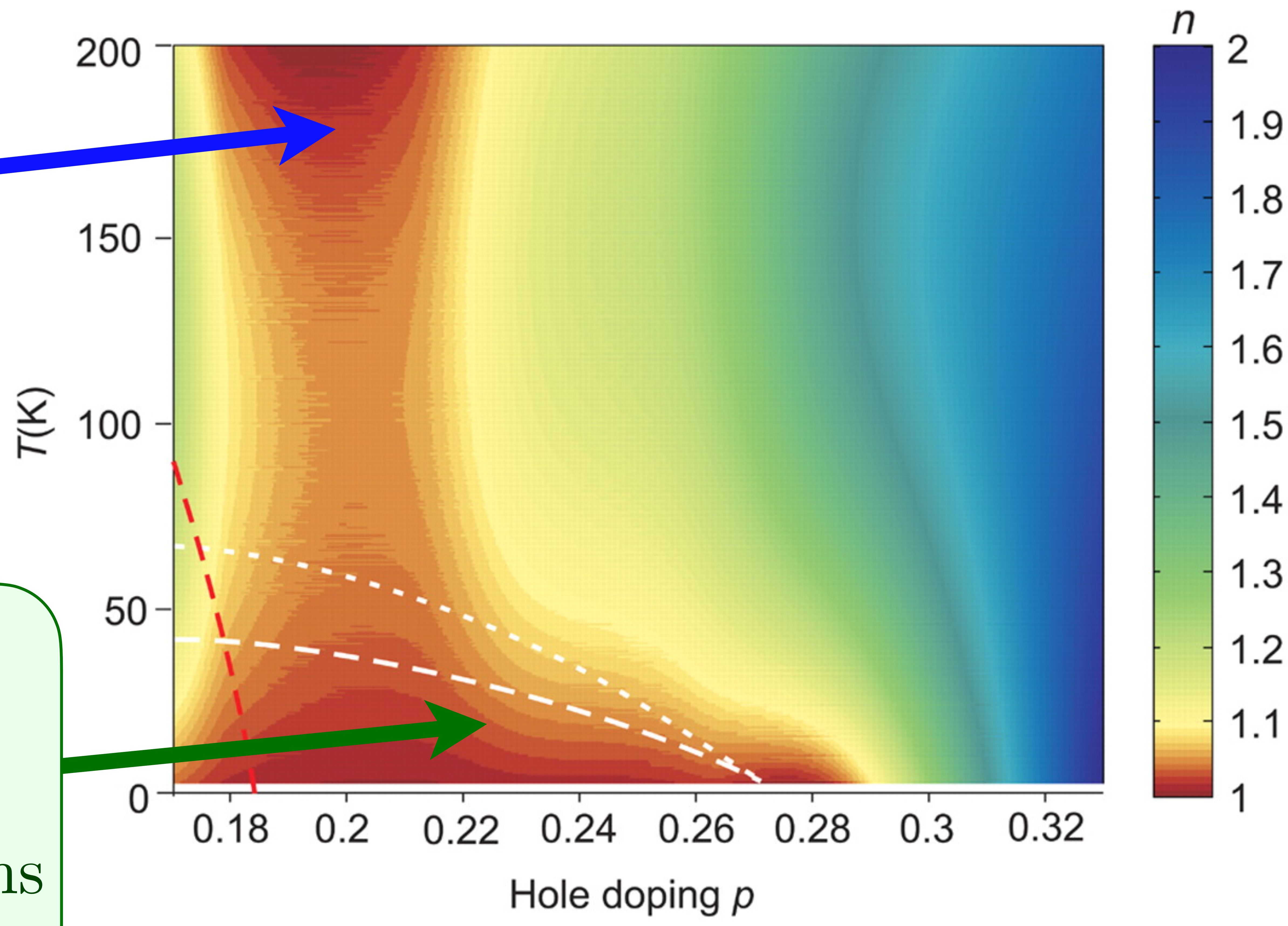

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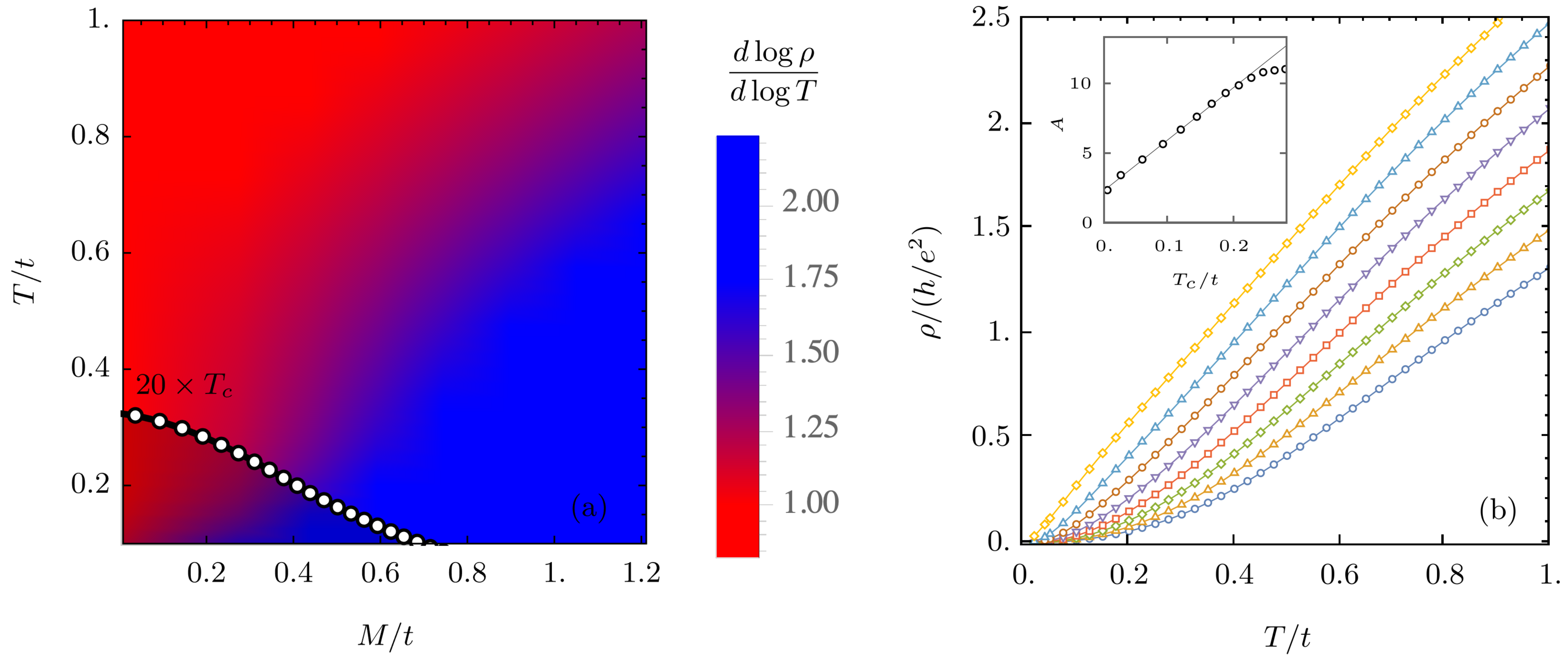
Strange metal and superconductor in the two-dimensional Yukawa-SYK model

(Self-averaging theory)

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, to appear

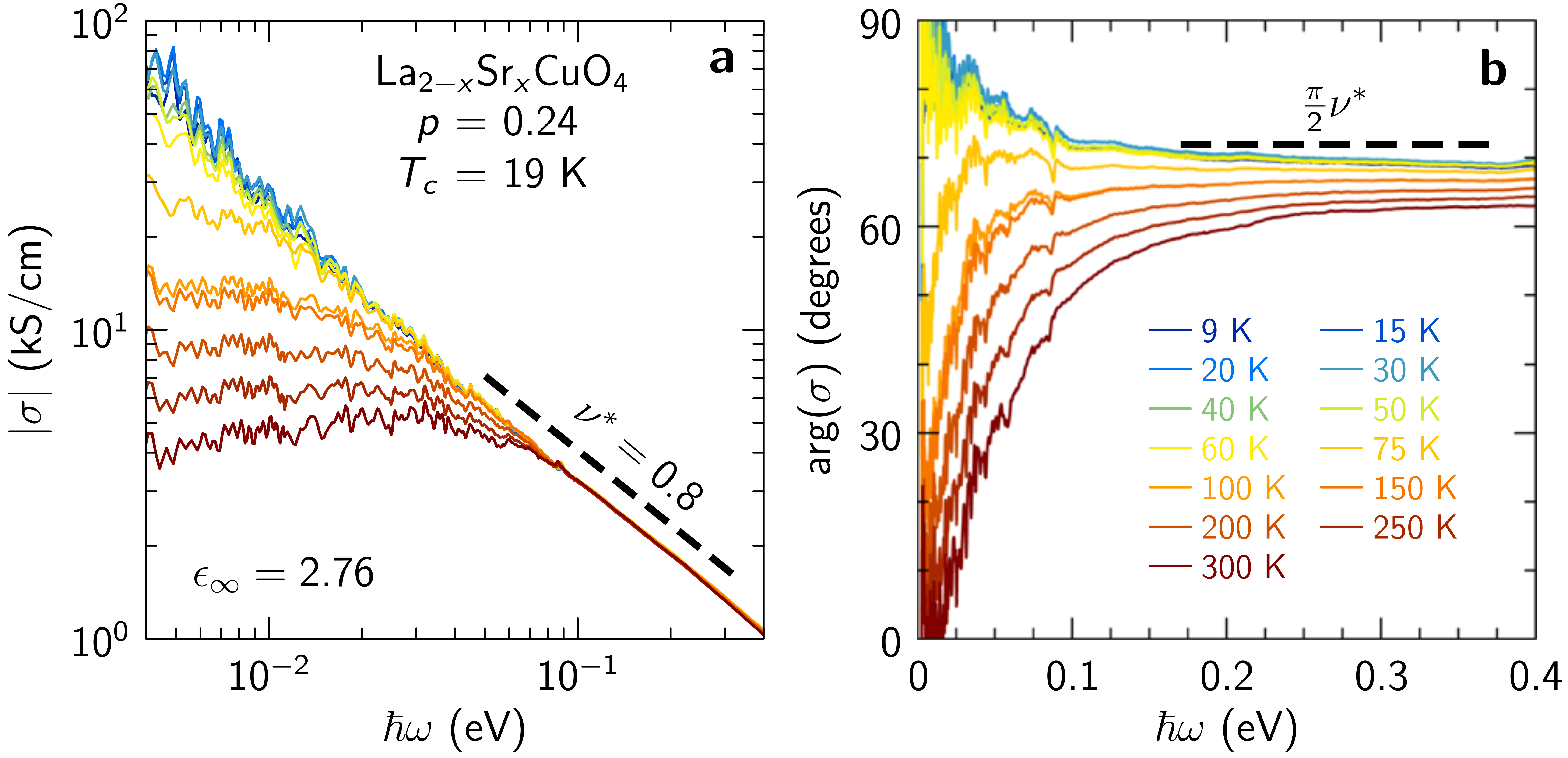


Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$



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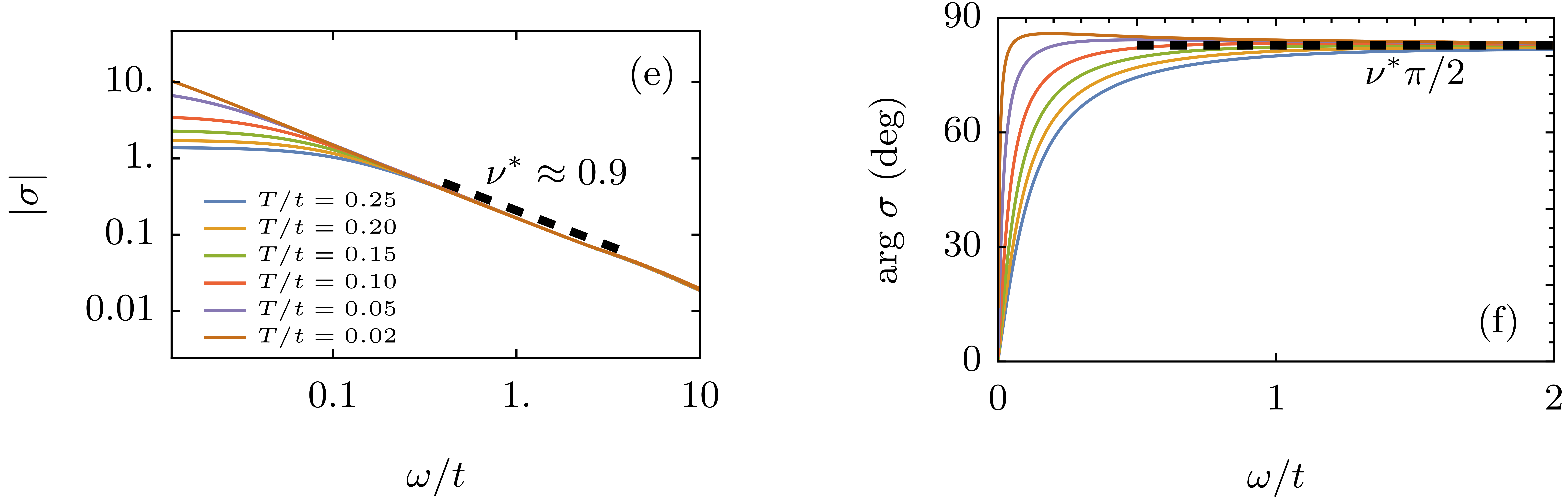

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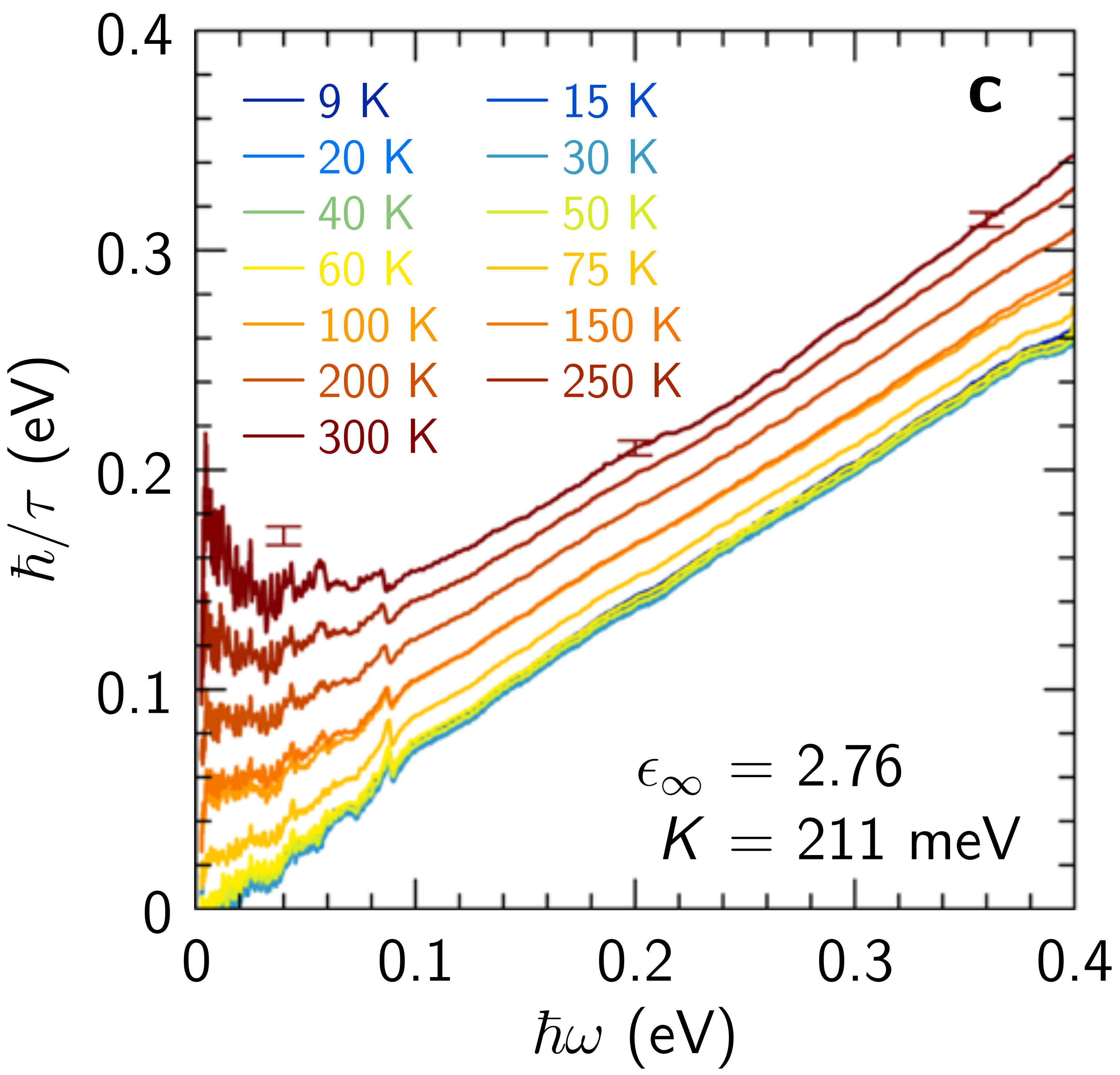

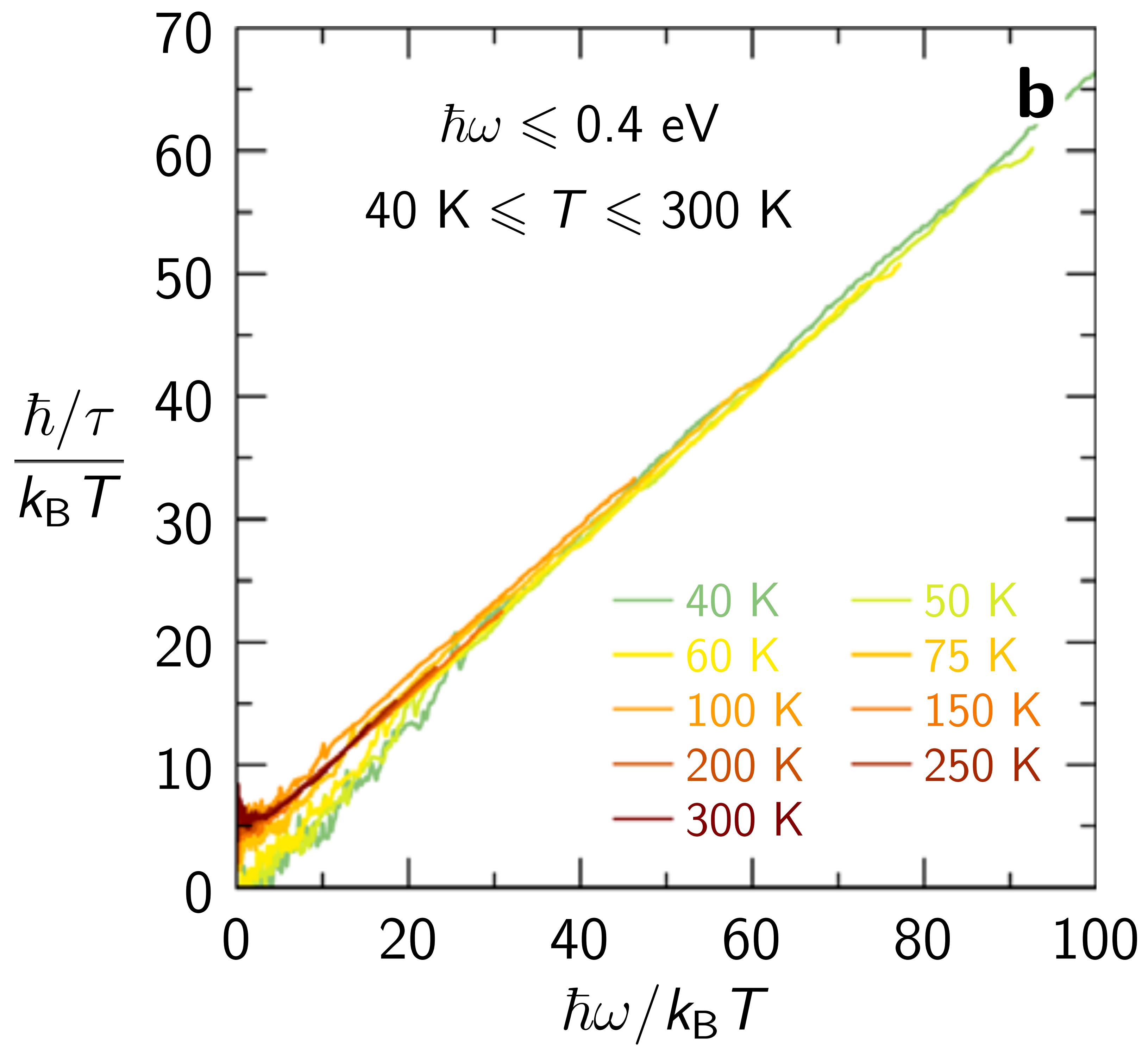

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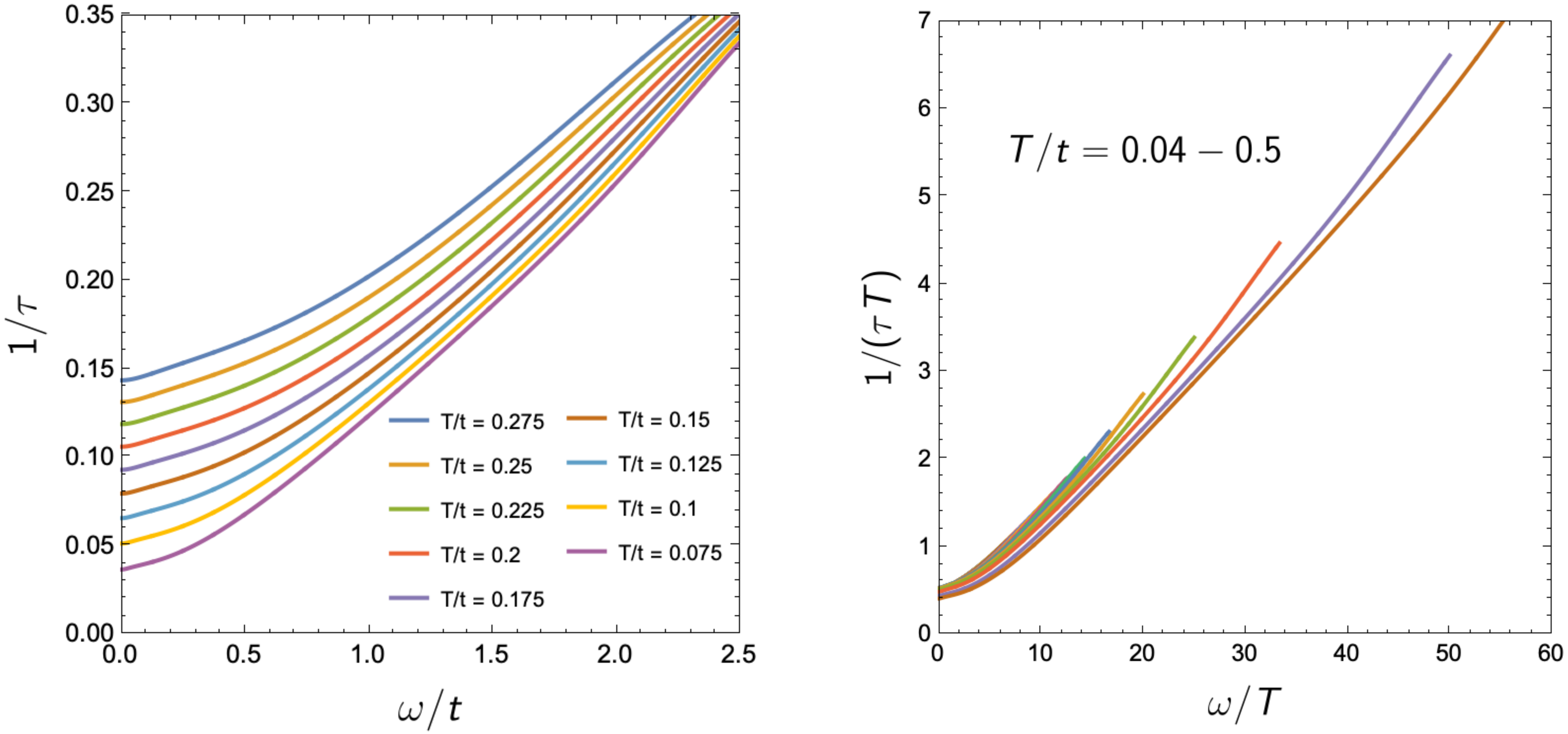

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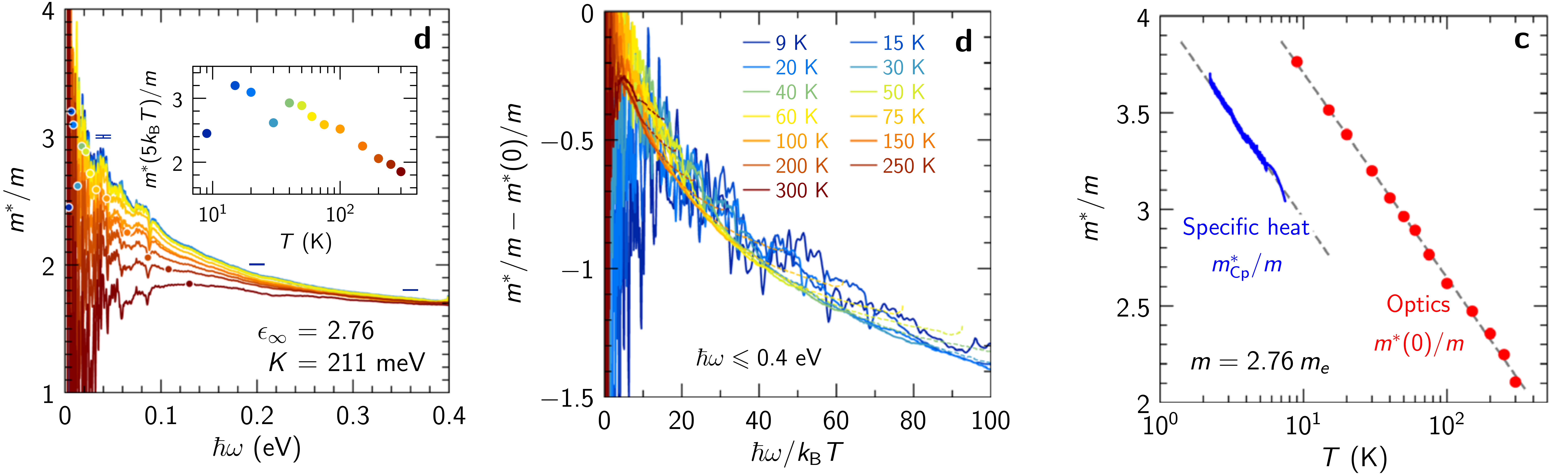

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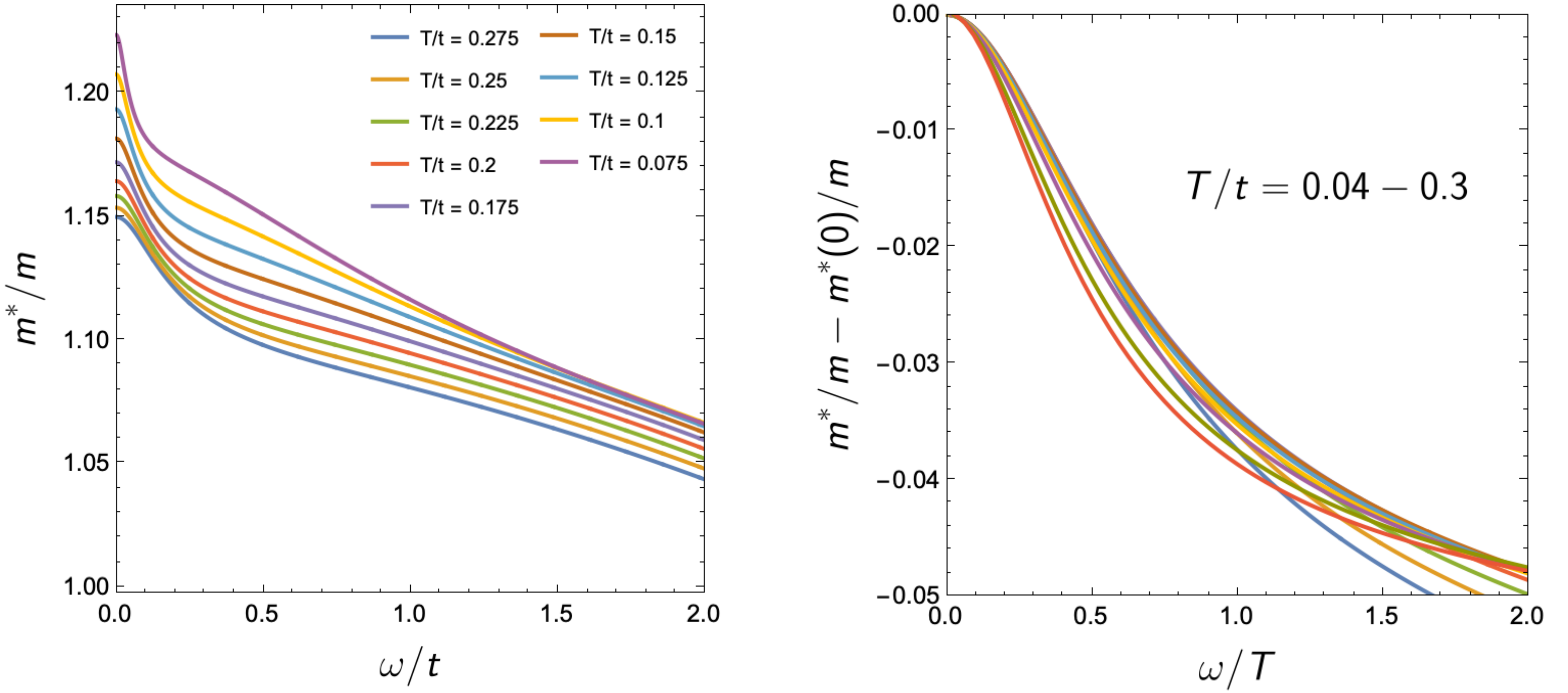
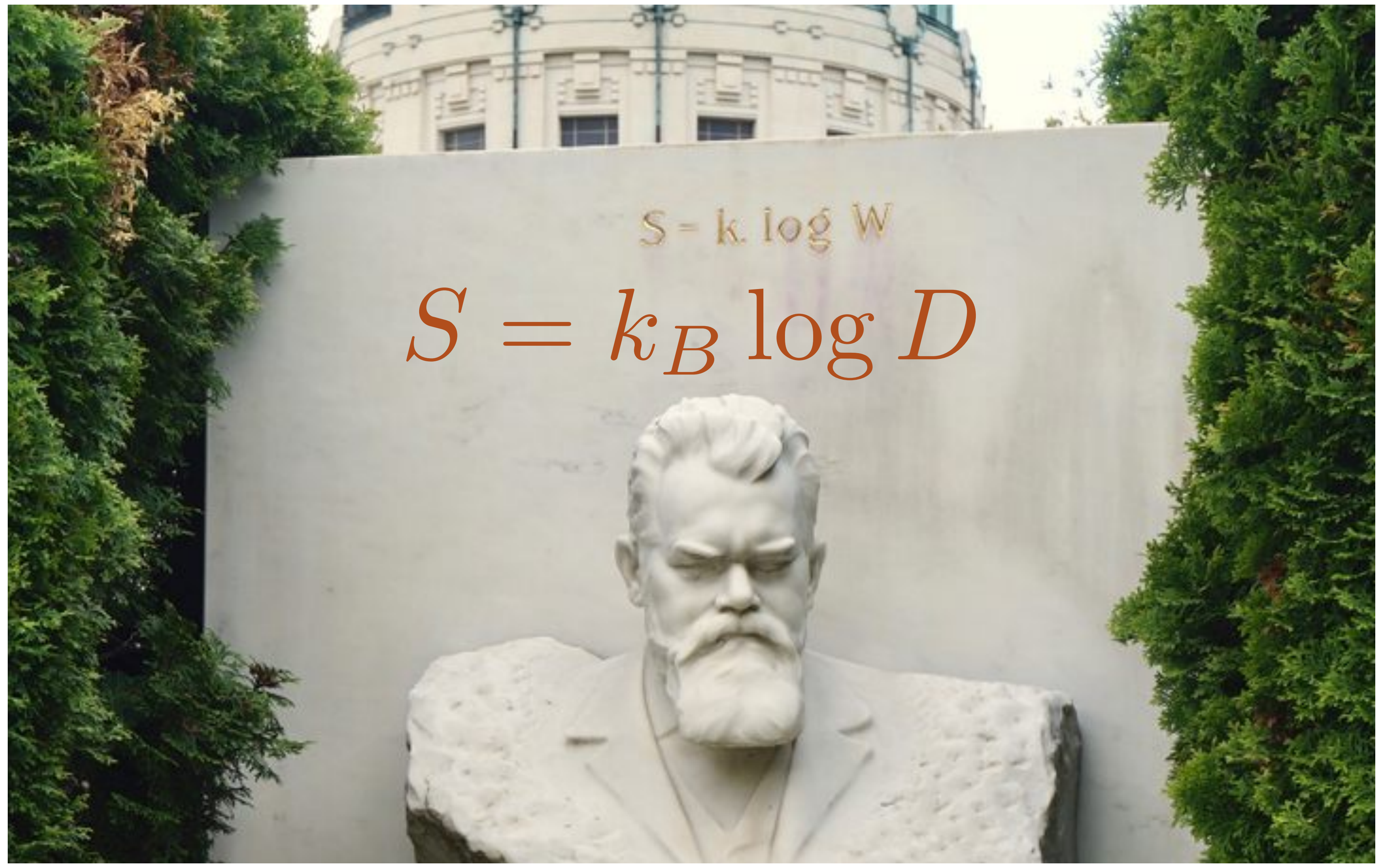
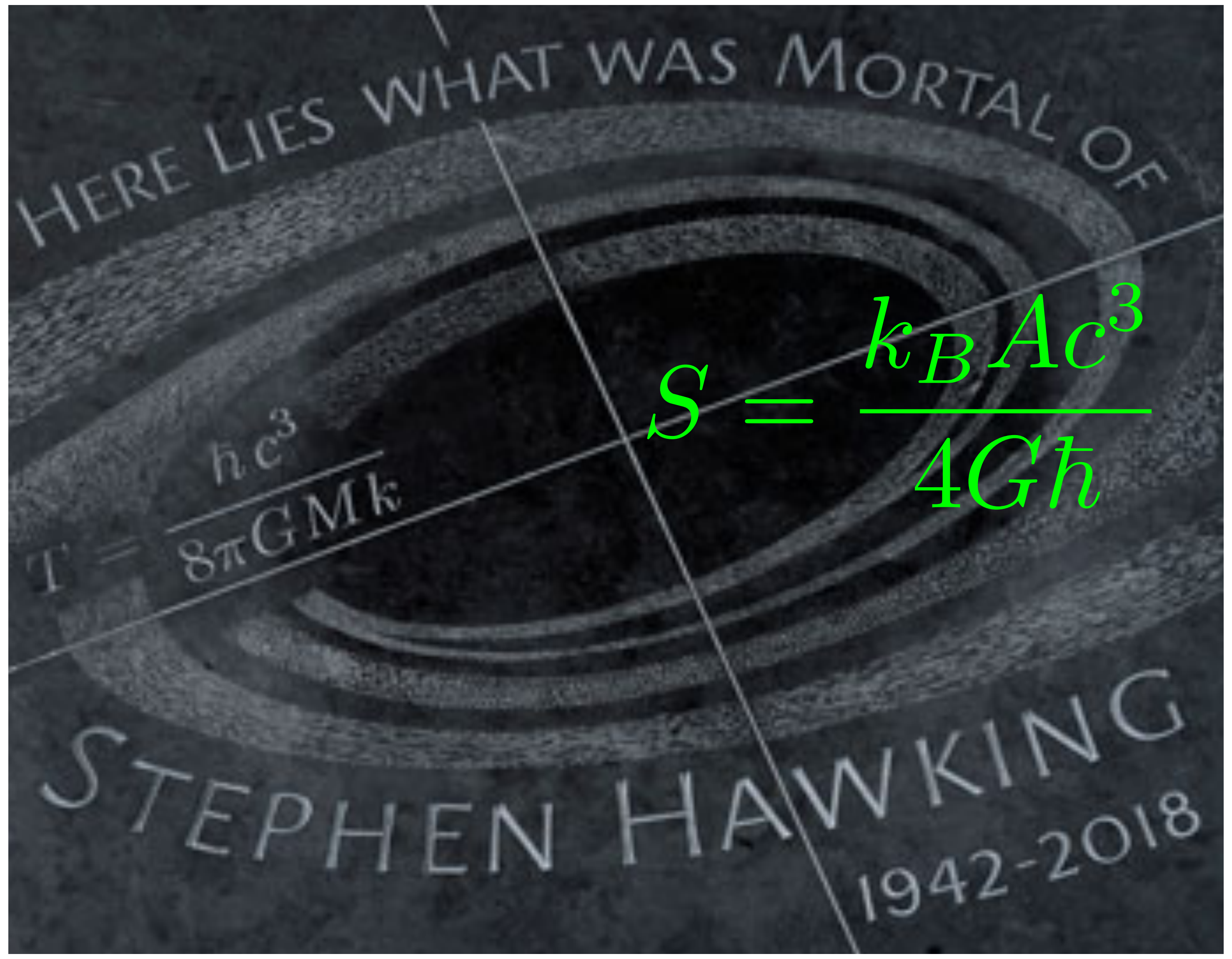


The SYK model, and black holes

Quantum Black Holes

- Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* $D(E)$ [the quantum analog of Boltzmann's W] matches Hawking's entropy, in accordance with Boltzmann's principles of statistical mechanics, $S(E) = k_B \log D(E)$?





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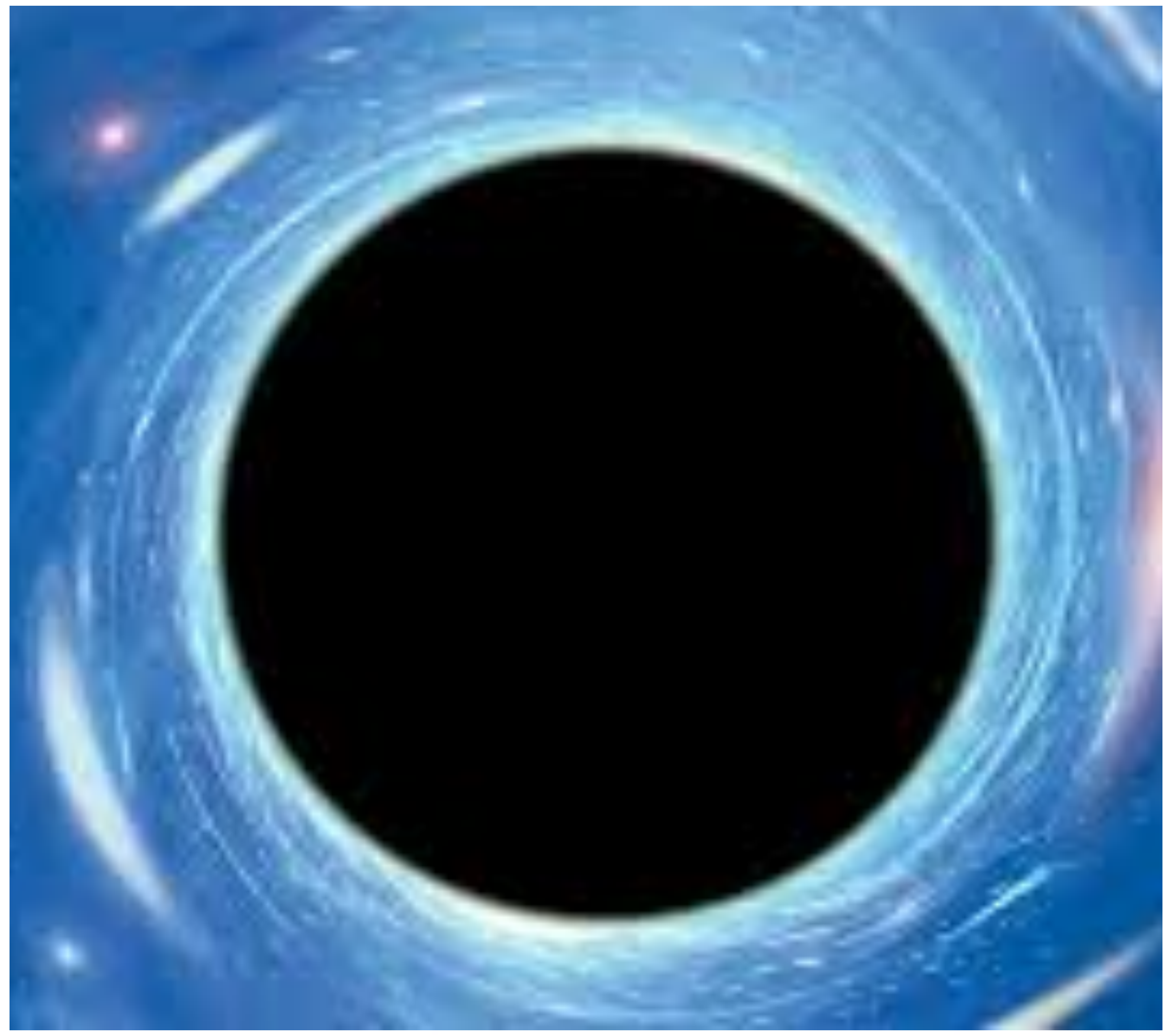
- Answer from string theory for 'supersymmetric' charged black holes: $D(E) = e^S \delta(E)$ *i.e.* all the states required by Hawking's entropy have exactly the same energy.

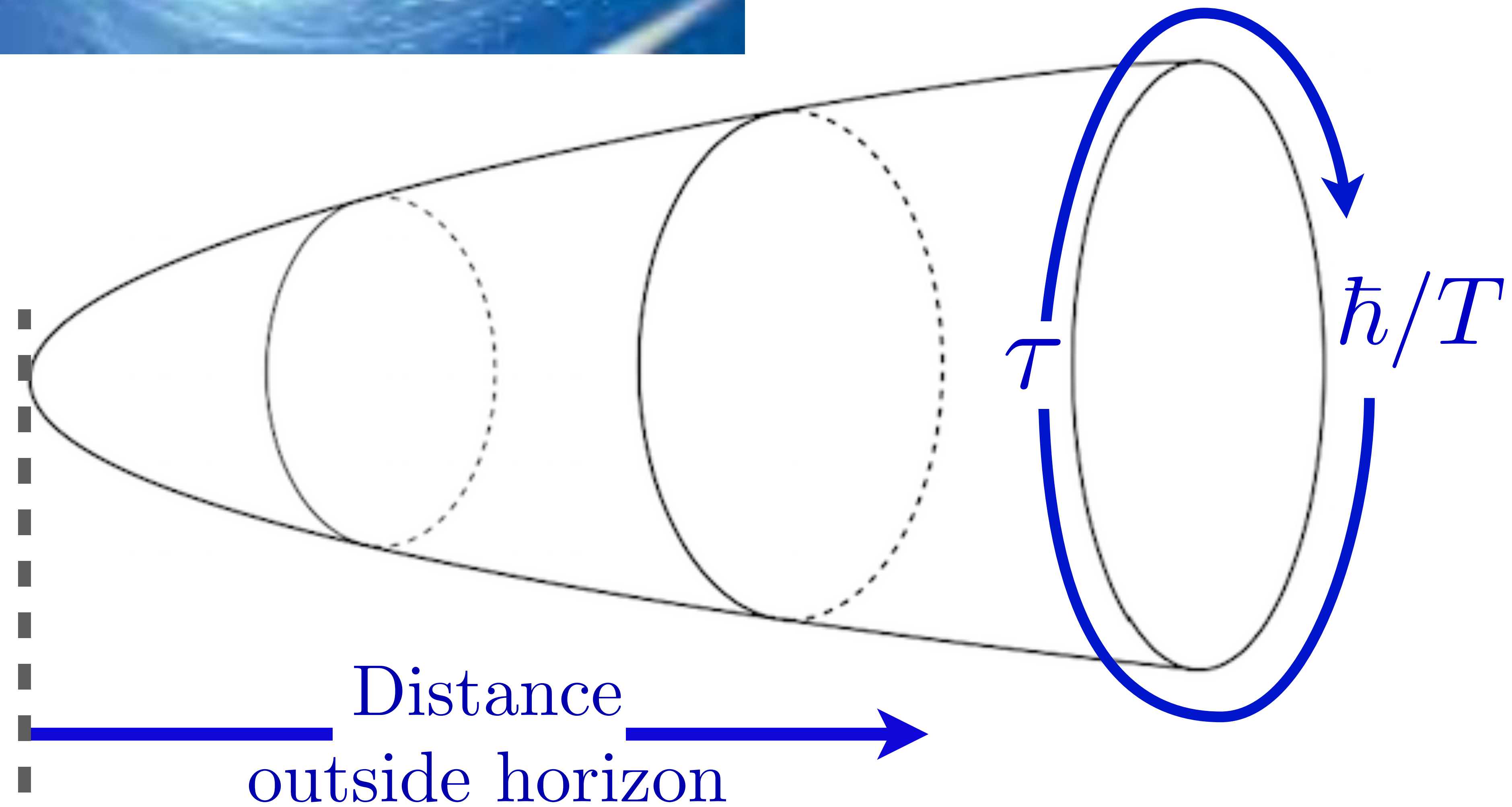
Strominger, Vafa (1996)

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\mathcal{Z}(\mathcal{Q},T) = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu}, A_\mu]\right)$$



A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD **60**, 064018 (1999)



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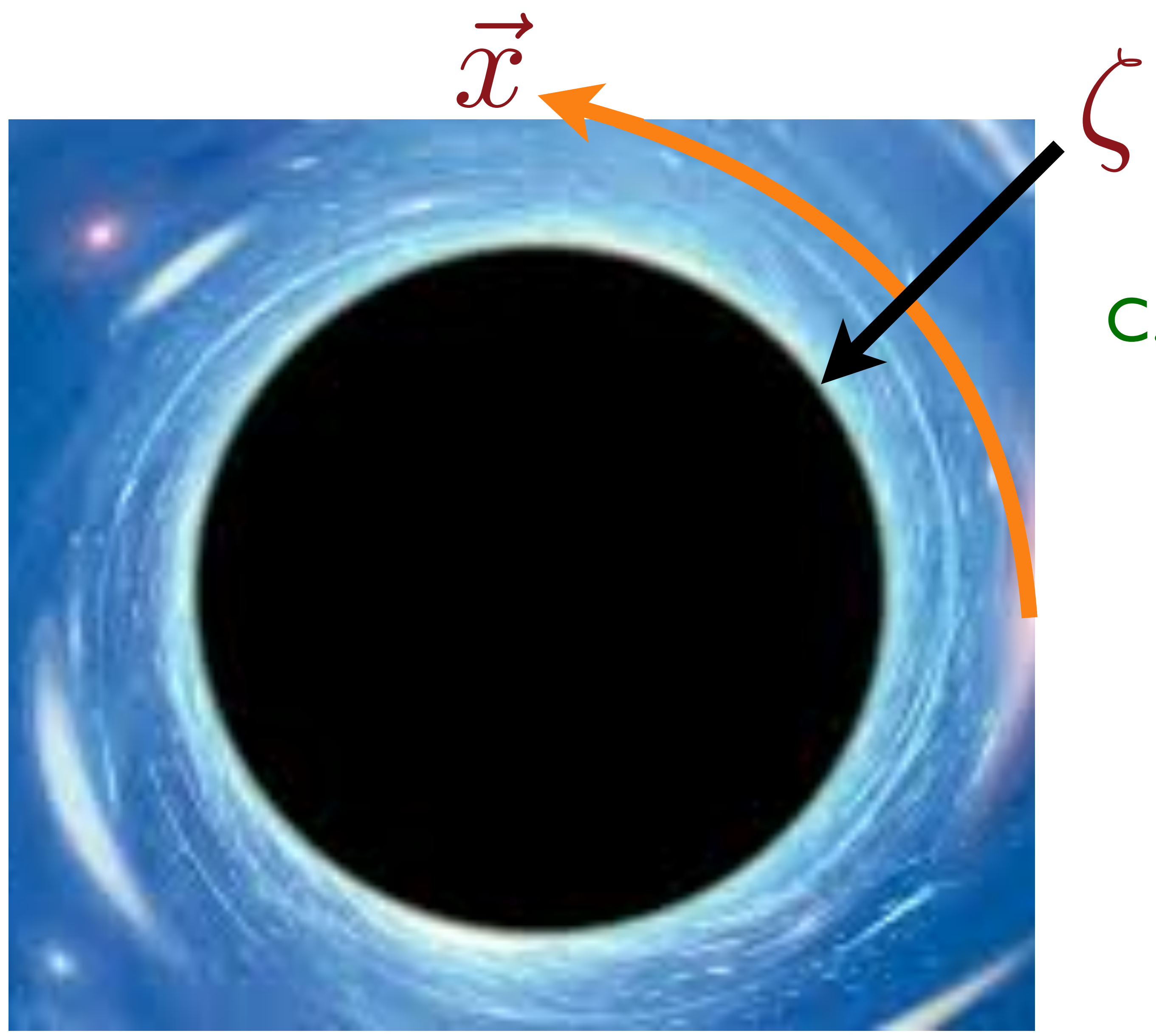


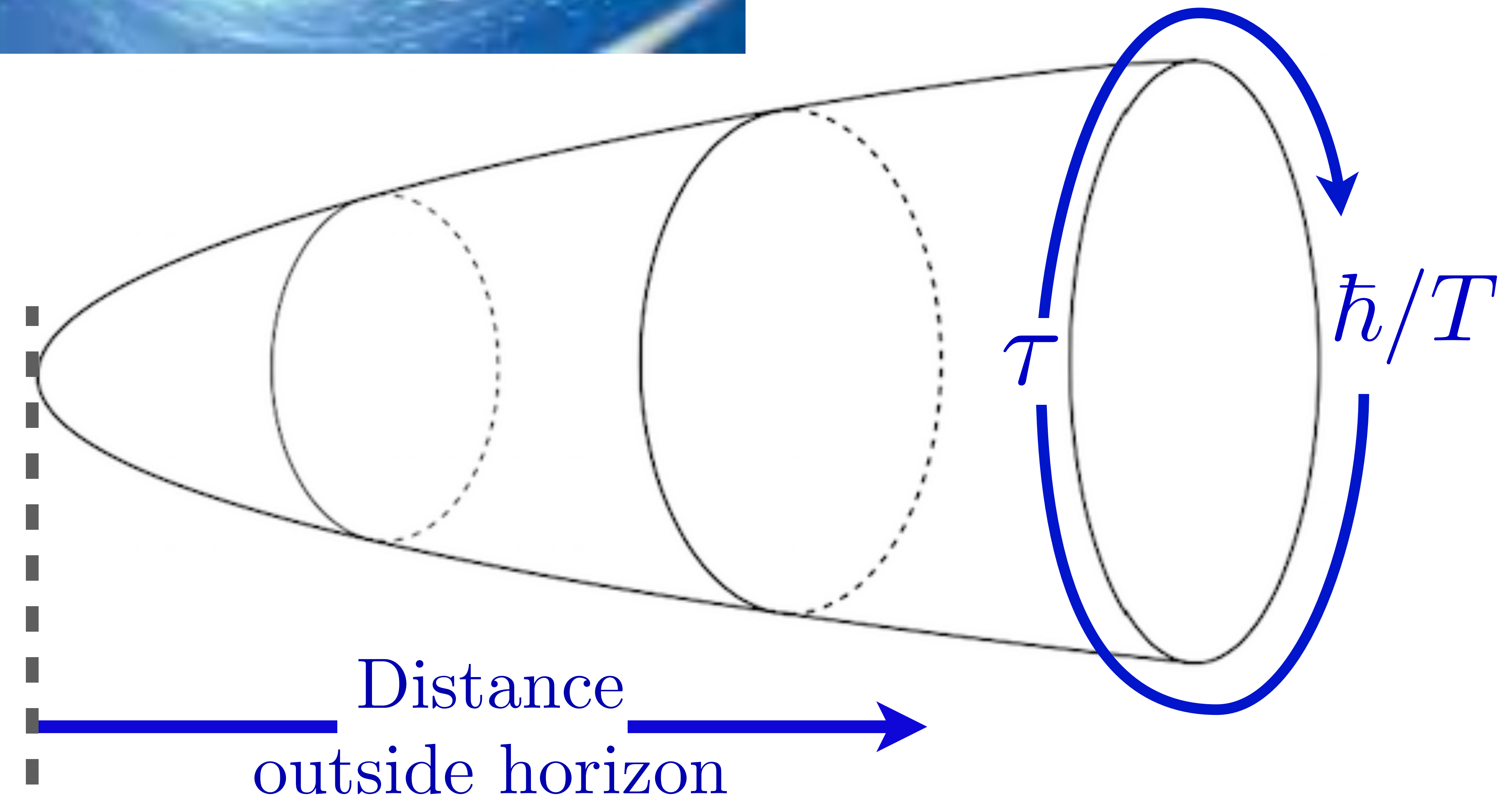
A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD **60**, 064018 (1999)

The near-horizon metric is $\text{AdS}_2 \times S^2$, and the AdS_2 quantum fluctuations dominate at low energies.
The AdS_2 metric in Euclidean time is

$$ds^2 = \frac{d\tau^2 + d\zeta^2}{\zeta^2}\,,$$

which has the same conformal symmetry as the SYK saddle point!

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d}\,, \quad ad - bc = 1\,.$$



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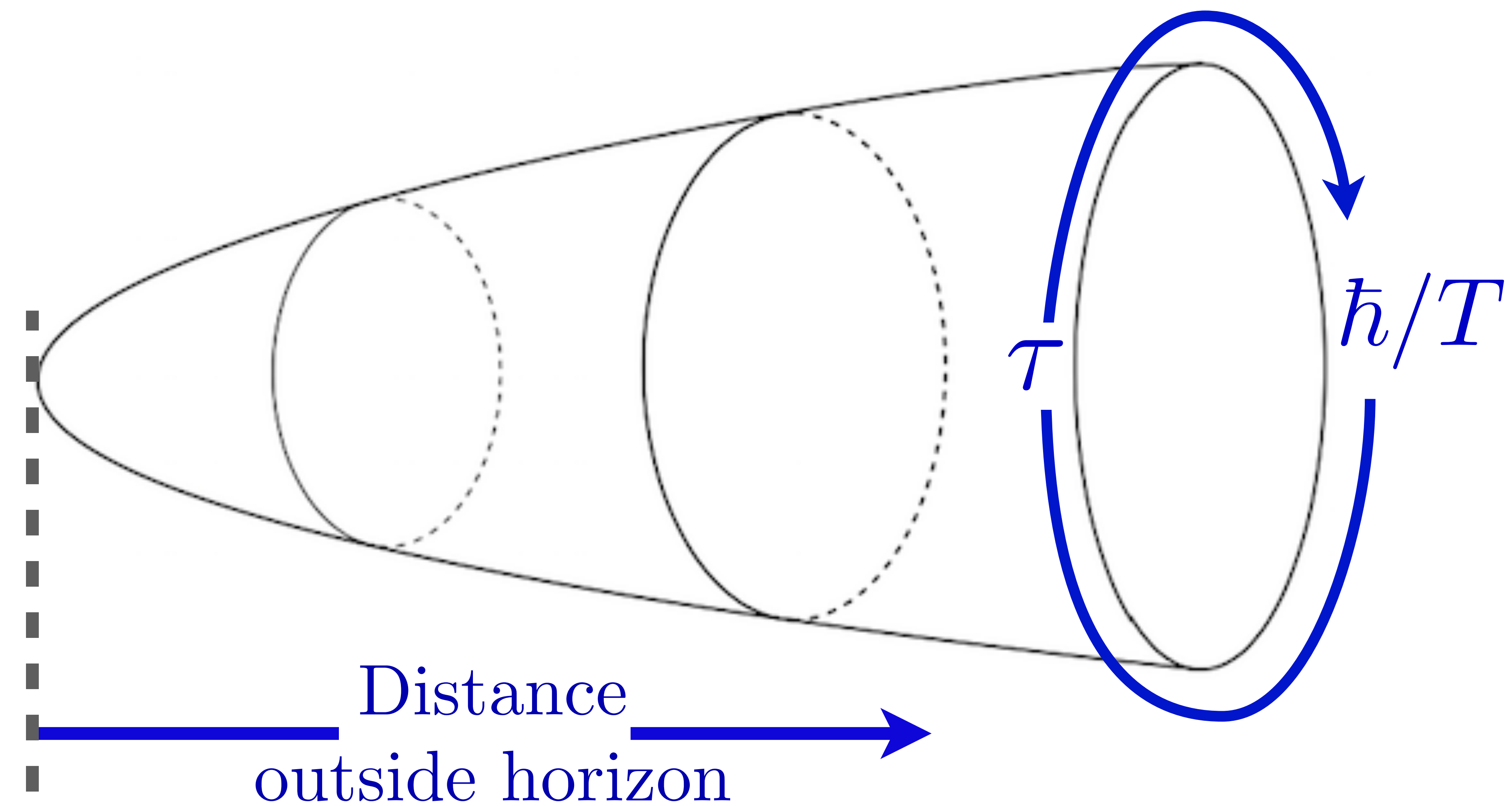
$$\approx \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(1+1)}_{\text{JT gravity of AdS}_2\text{+boundary graviton}}[g_{\mu\nu}, A_\mu]\right)$$

$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T = 0$.



Maldacena, Stanford, Yang (2016)

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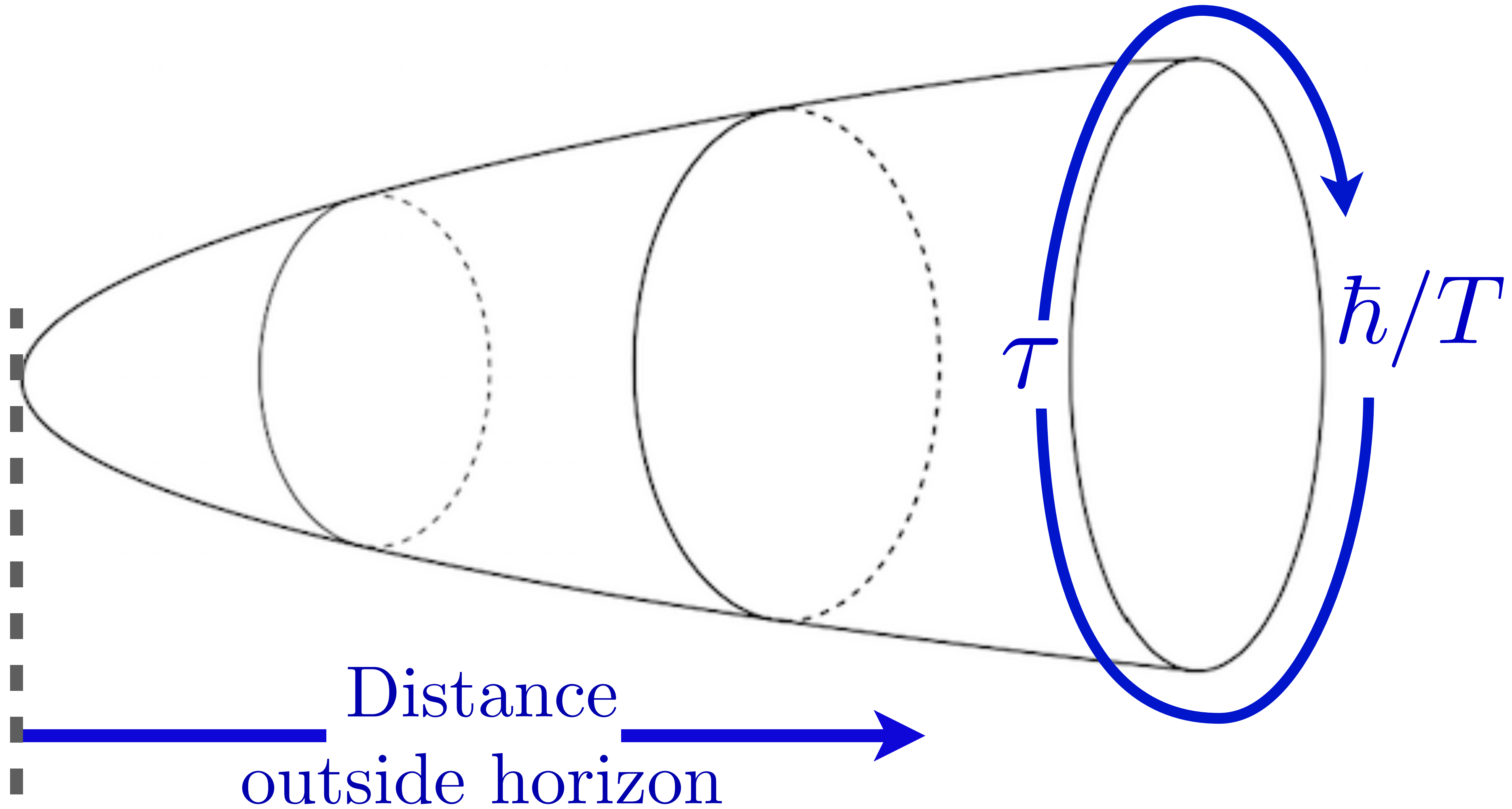
$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)\exp\left(-\frac{1}{\hbar}I_{\text{SYK}}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)]\right)$$

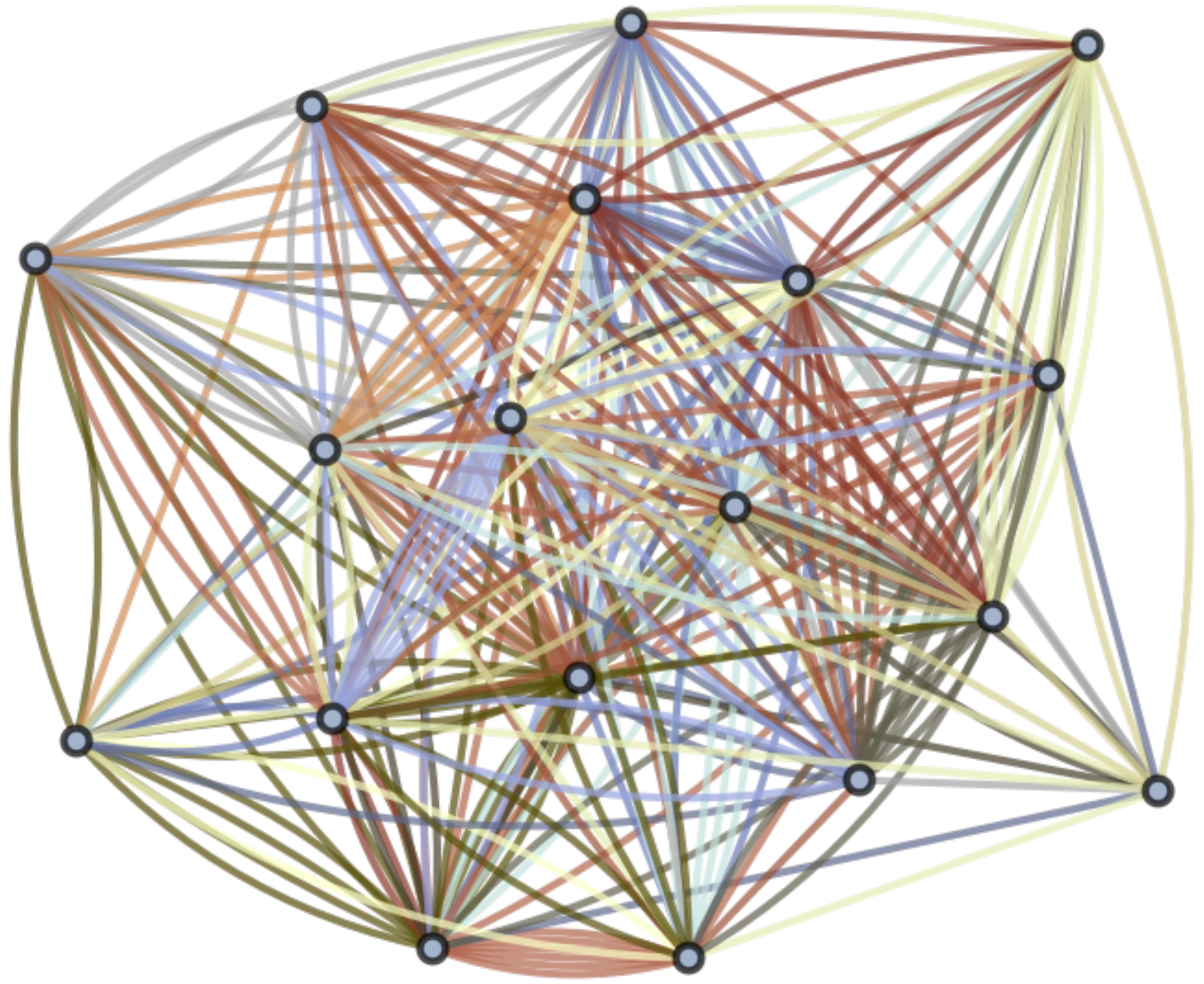
The path integral over the action I_{SYK} can be evaluated exactly,
and leads to a computation of $D(E)$

$$\mathcal{Z}(\mathcal{Q},T) = \int dE D(E)\exp\left(-\frac{E}{k_B T}\right)$$



Maldacena, Stanford, Yang (2016); Cotler *et al.* (2017)

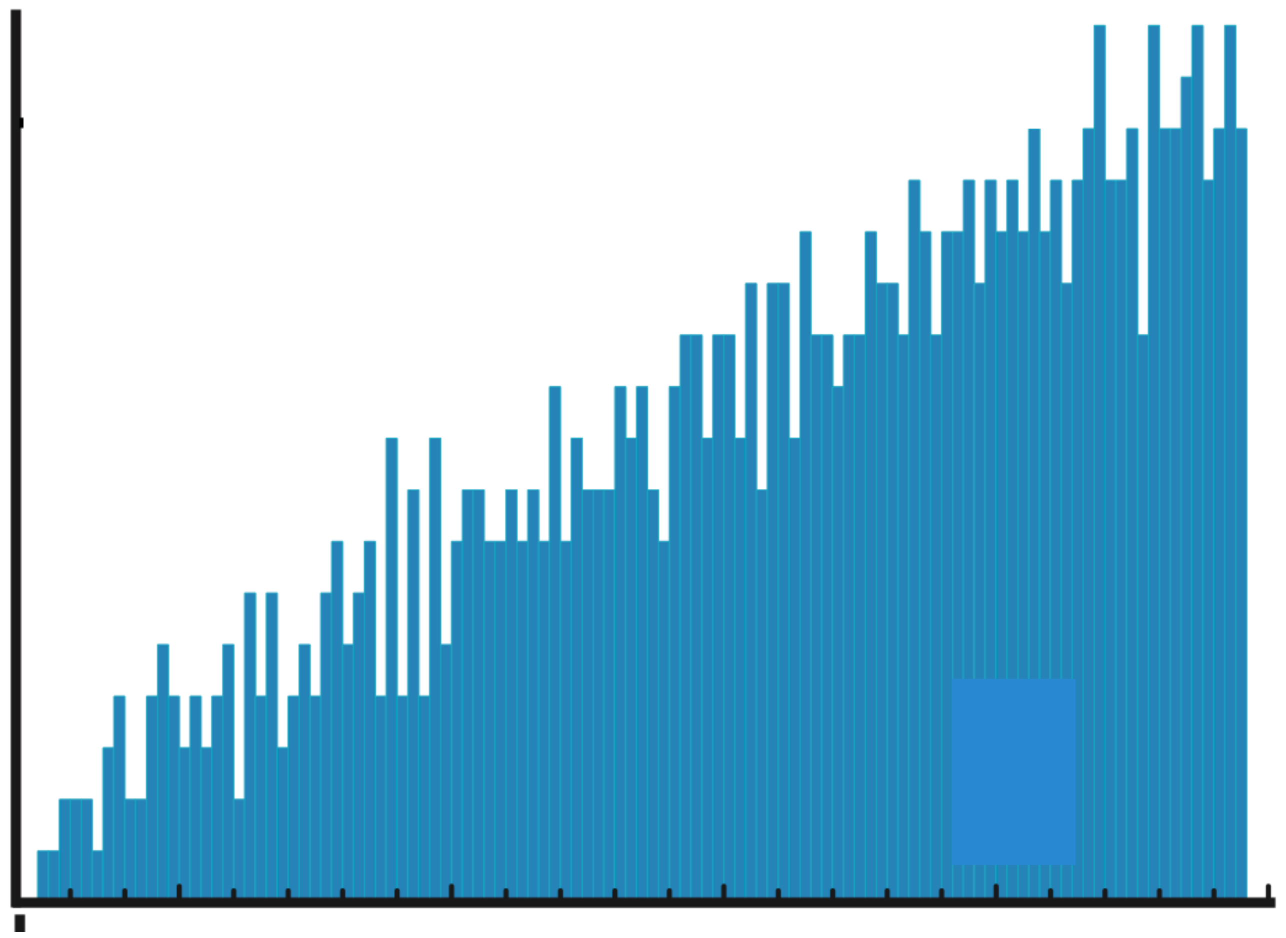
The Sachdev-Ye-Kitaev (SYK) model



- Density of quantum states of the SYK model with N sites

$$D(E) \sim \frac{1}{N} \exp\left(N s_0\right) \sinh\left(\left[2N\gamma E\right]^{1/2}\right)$$

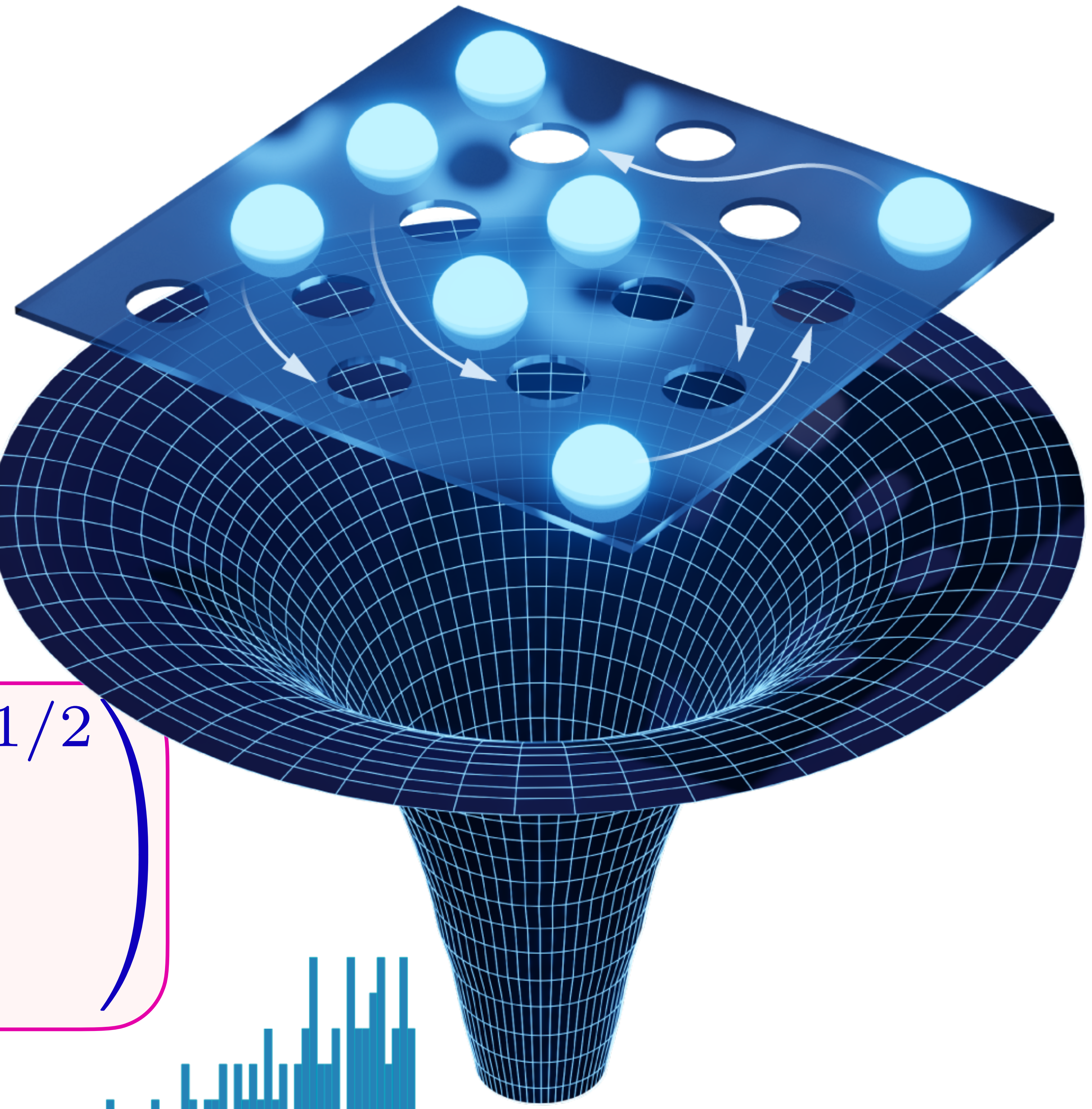
where $s_0 = 0.4648476991708051 0749\ldots$
and $\lim_{N\to\infty} \frac{1}{N} S(T) = s_0 + \gamma T$



D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

D(E) of charged black holes from the SYK model

- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T=0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy E is

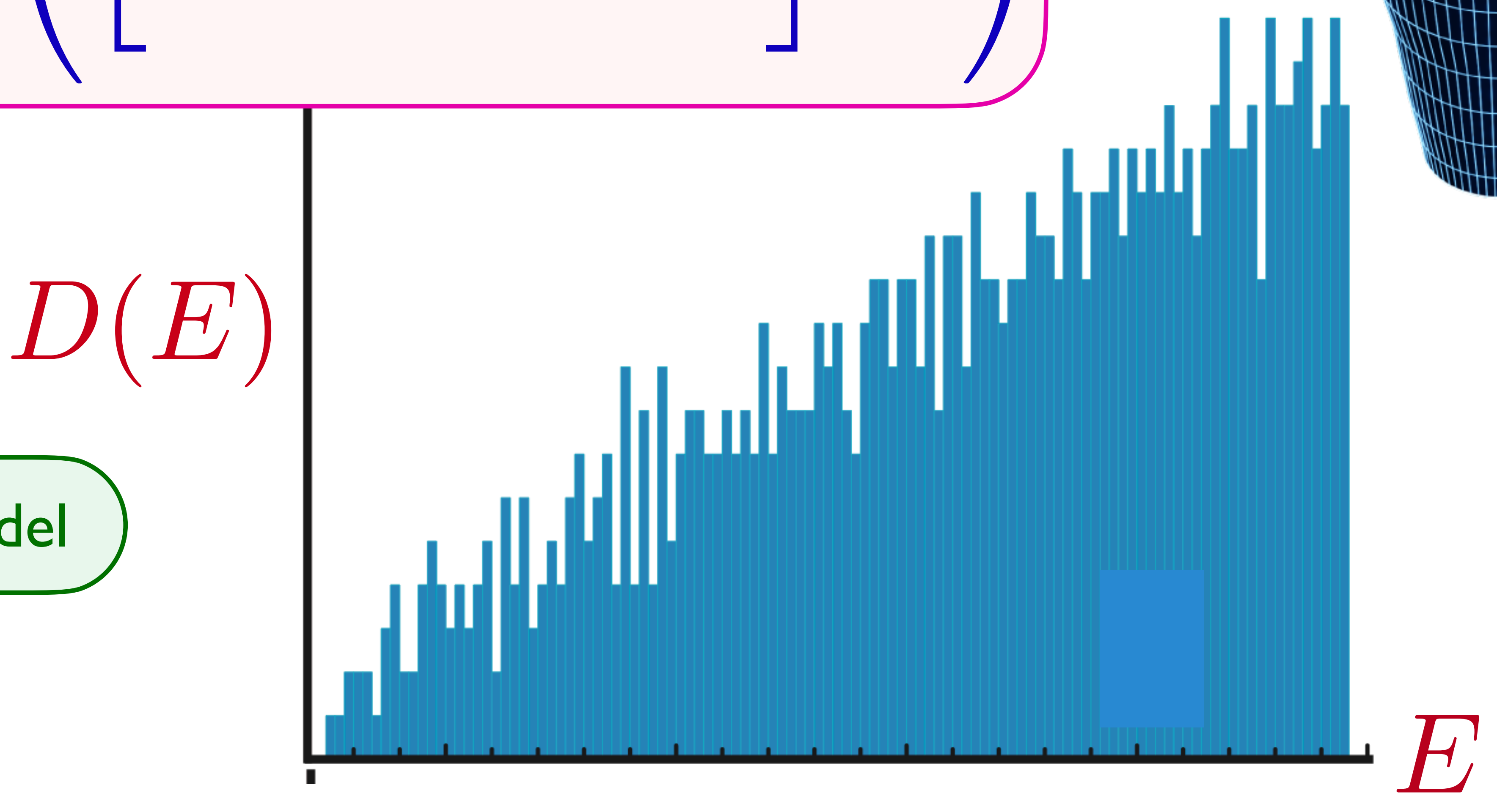


Bekenstein-Hawking

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi}{A_0}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

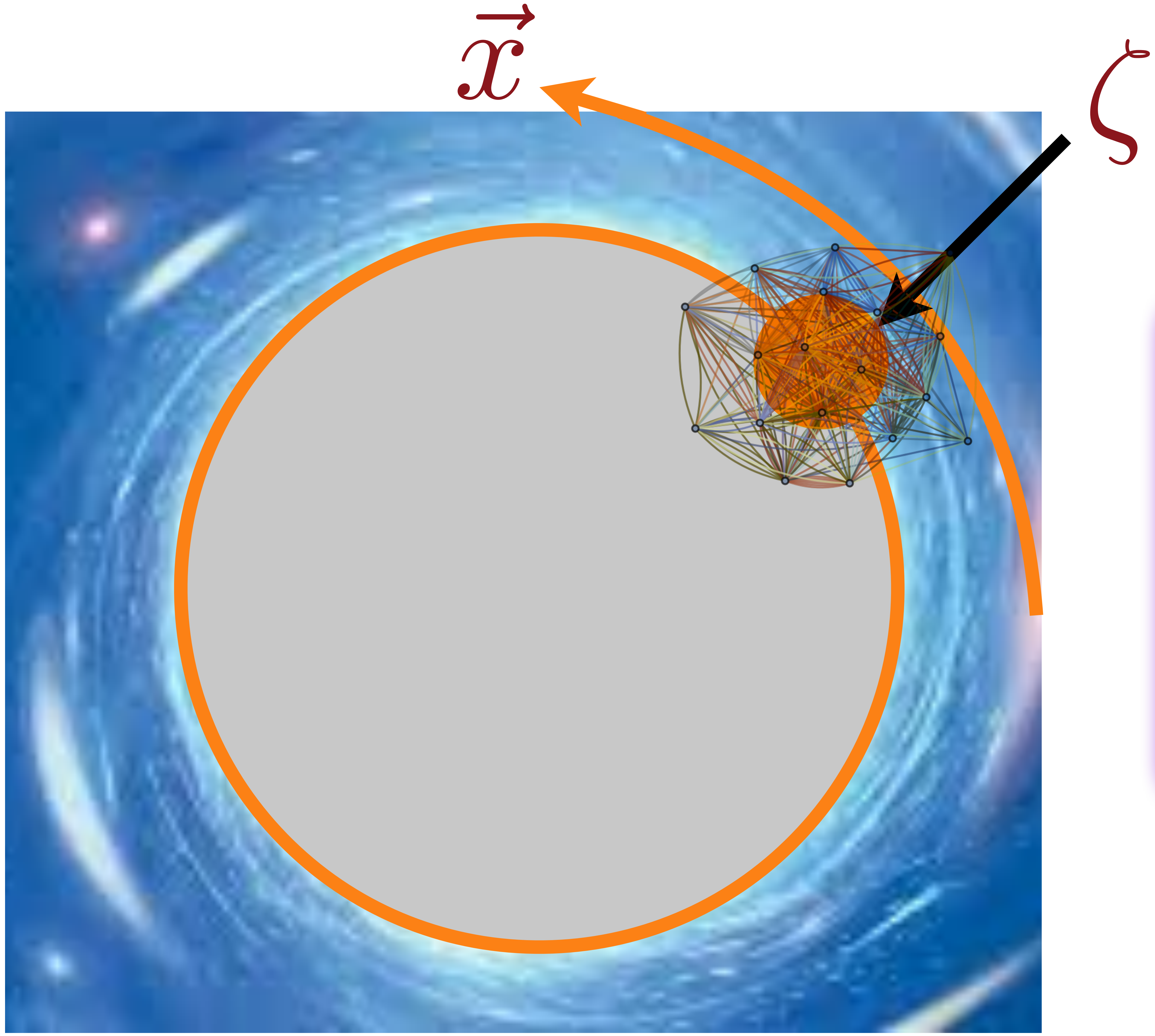
Iliesiu, Murthy, Turiaci (2022)

Developments from the SYK model



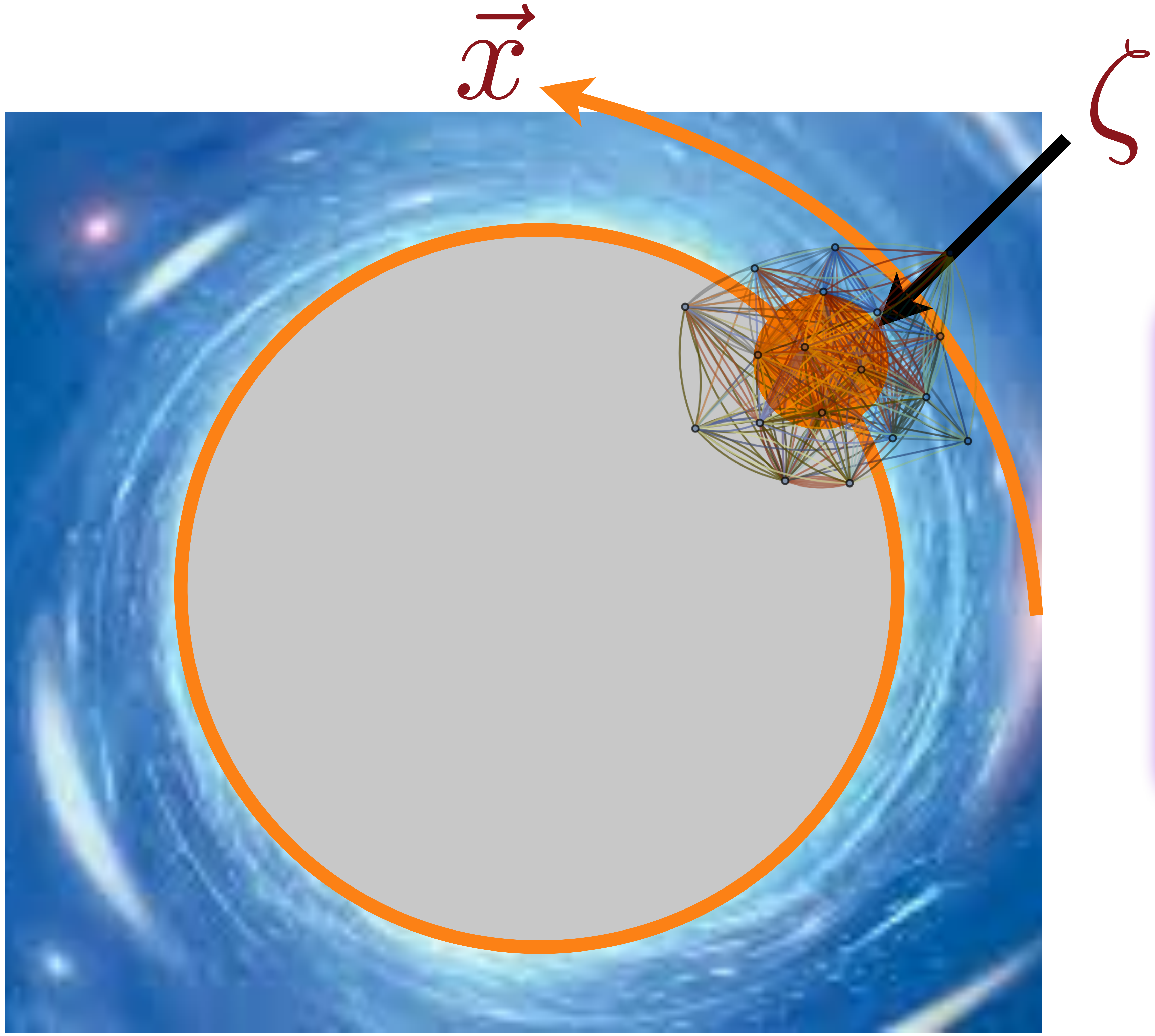
D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

Quantum simulation of charged black holes by the SYK model



The SYK model simulates the interior of the black hole to an outside observer

Quantum simulation of charged black holes by the SYK model



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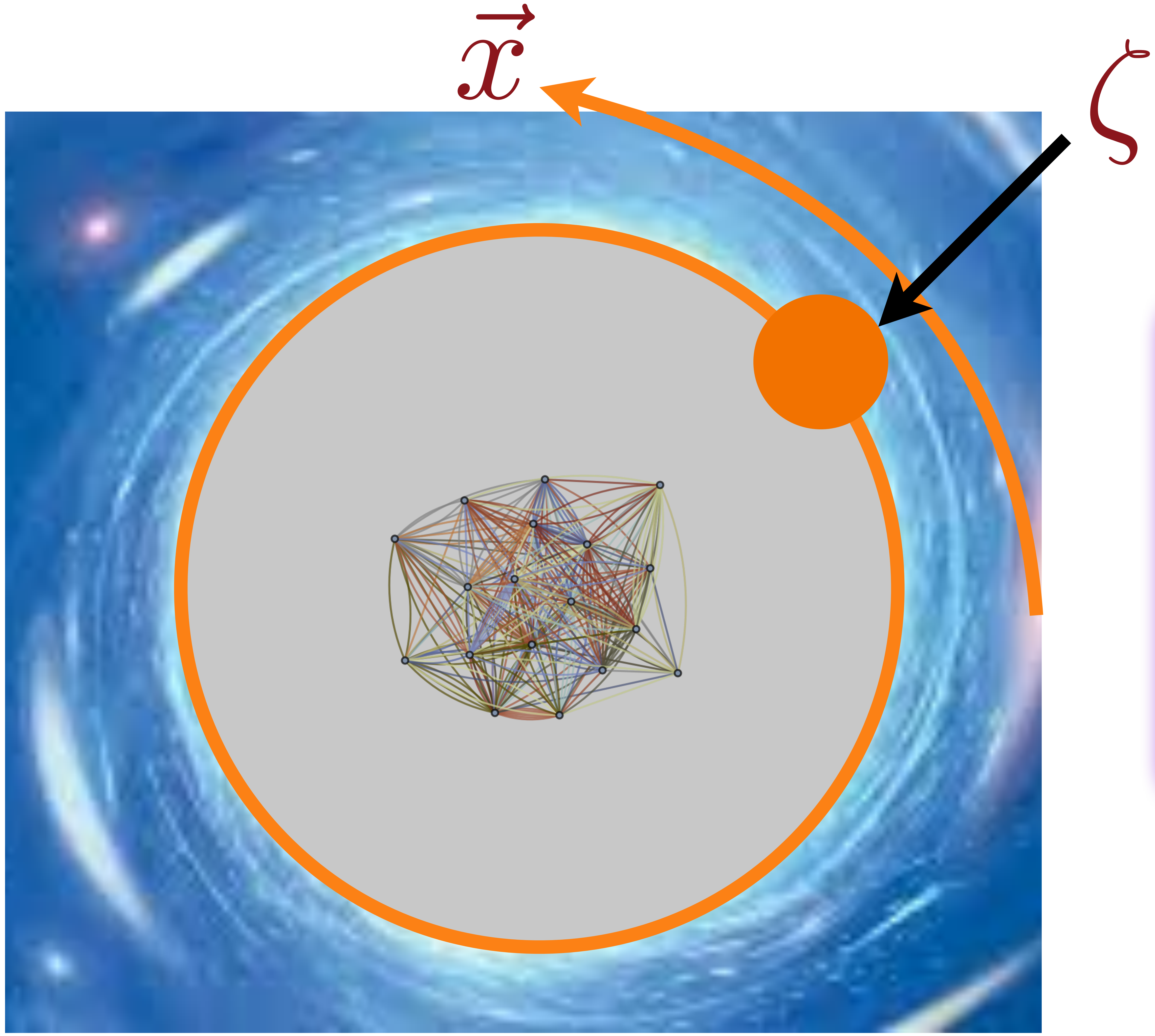

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Recap

Great discoveries in physics

Quantum entanglement (1865,1935)

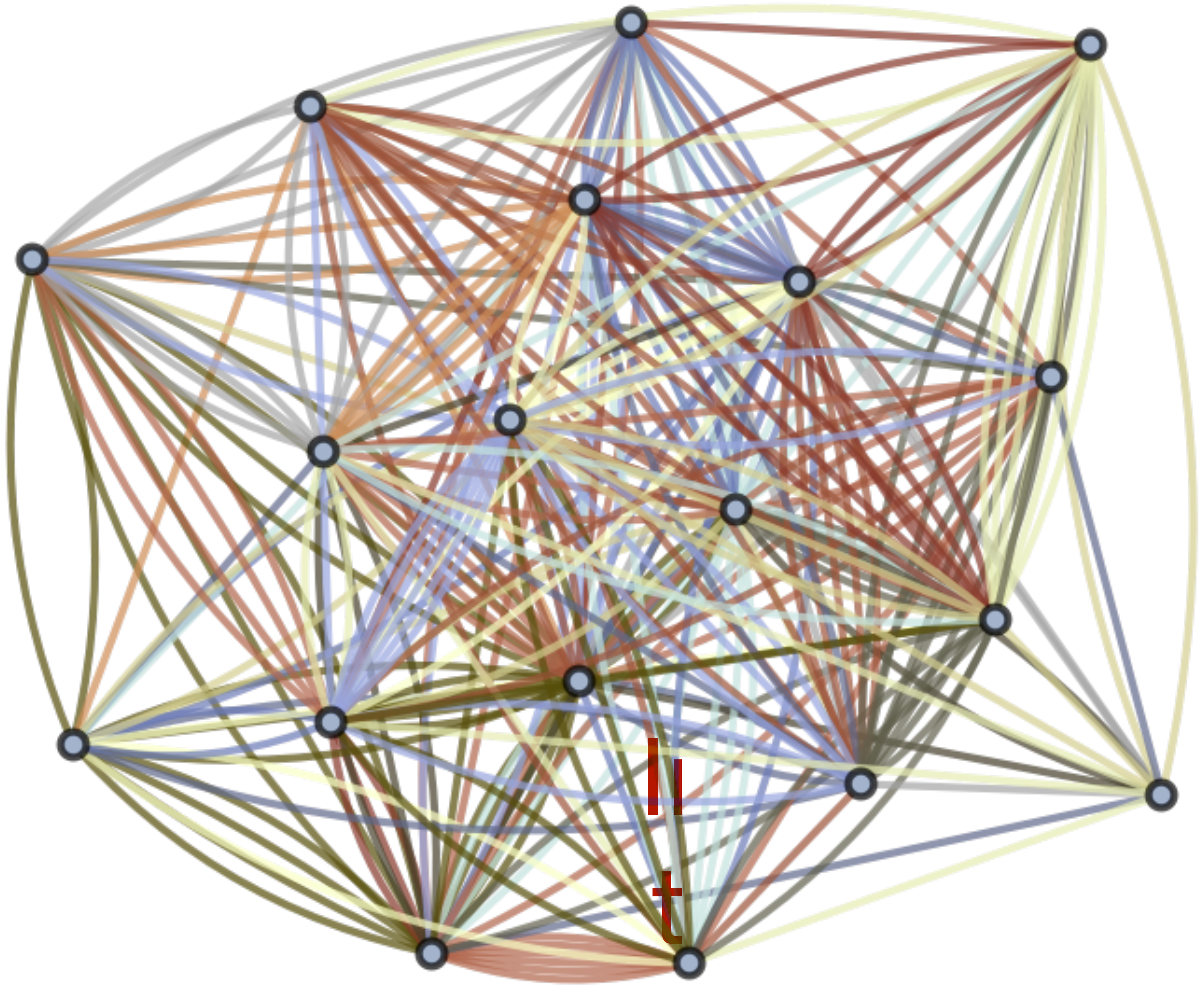
Statistical mechanics (1870)

Superconductivity (1911)

Black holes (1916)

The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



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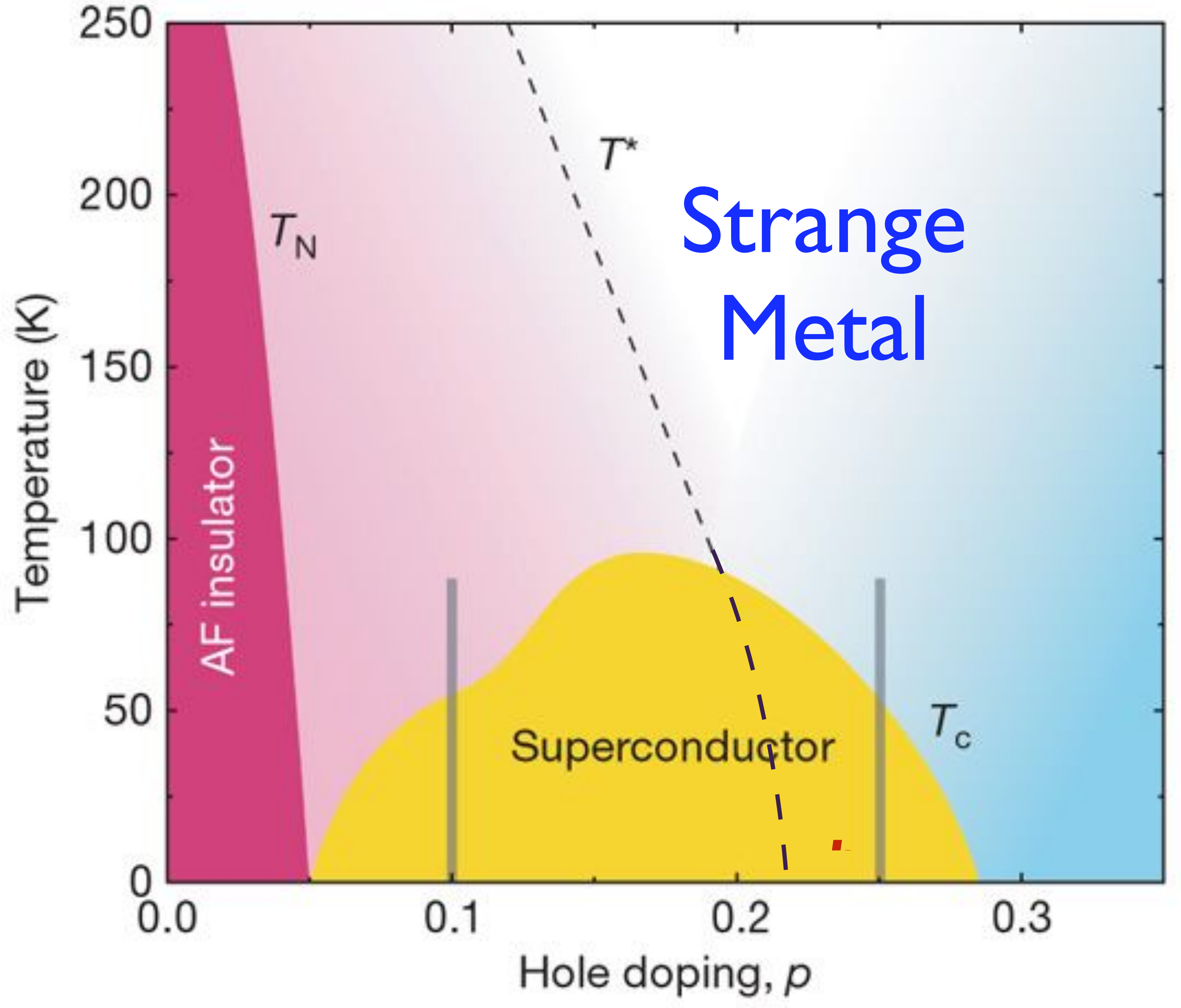
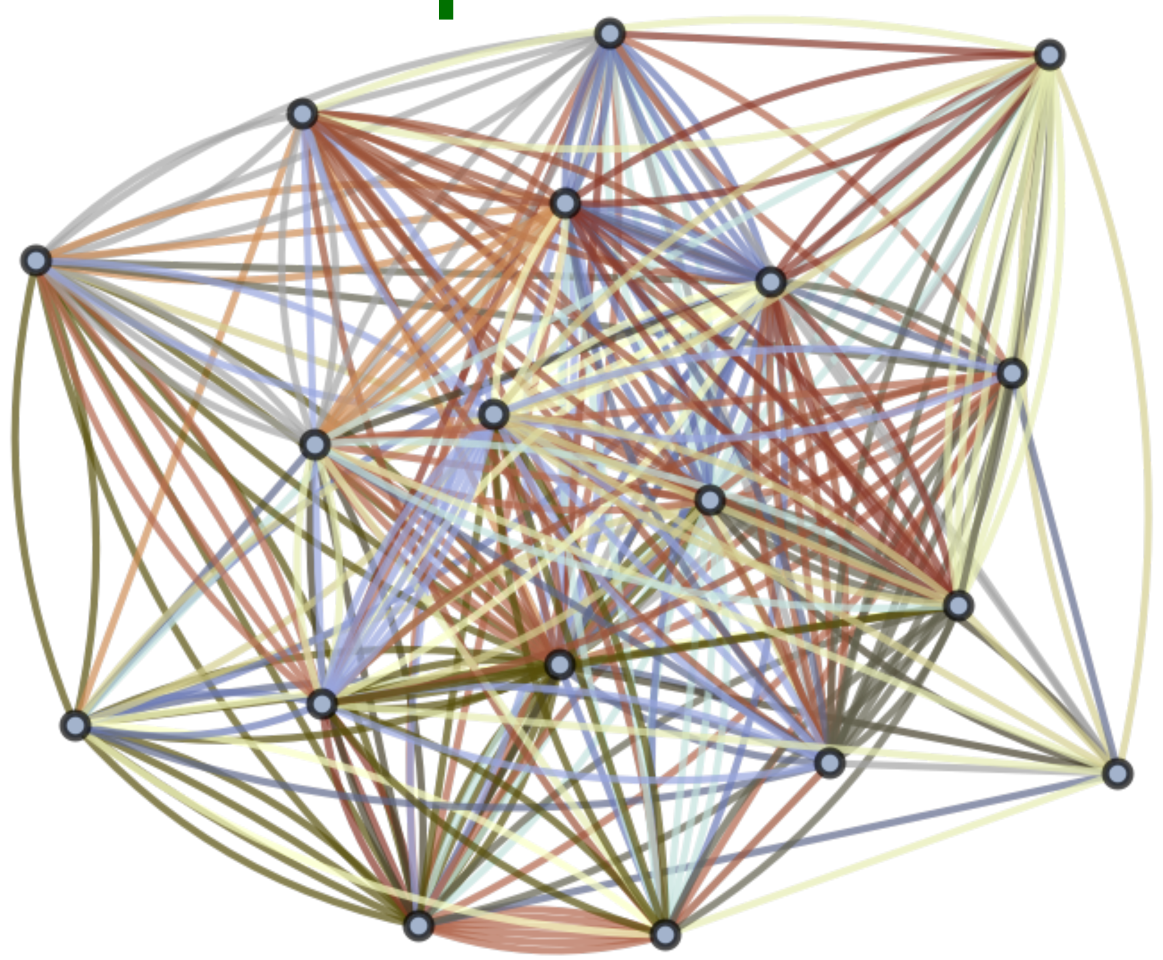


A self-averaging two-dimensional, Yukawa-SYK theory describes the **strange metal** behavior of numerous quantum materials



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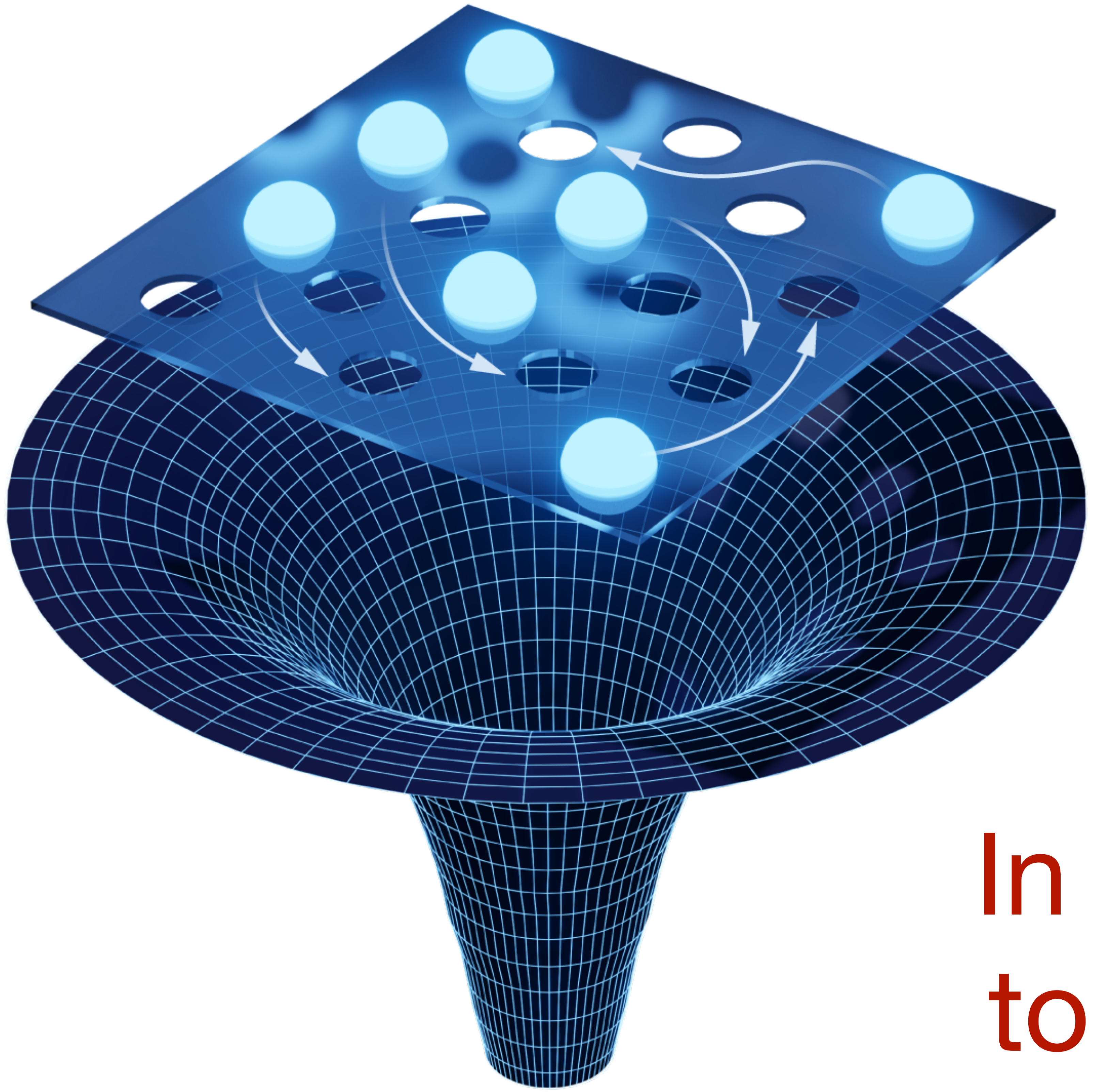

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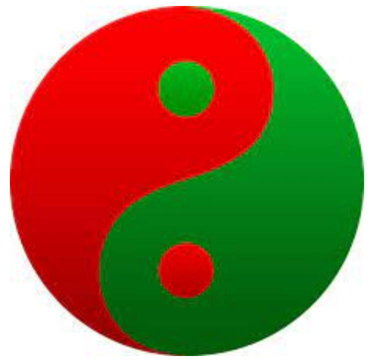

In a *dual* set of variables the SYK model has led to the computation of the low energy density of states of ***charged black holes***