

# The pseudogap metal and its confinement transitions

Correlated Gapless Quantum Matter  
KITP, Santa Barbara  
May 21, 2024  
Subir Sachdev

Maine Christos, Zhu-Xi Luo, Henry Shackleton, Ya-Hui Zhang,  
Mathias Scheurer, and S. Sachdev, *Proc. Nat. Acad. Sci.* **120**, e2302701120 (2023)

Maine Christos and S. Sachdev, *npj Quantum Materials* **9**, 4 (2024)

M. Christos, H. Shackleton, S. Sachdev, and Zhu-Xi Luo, arXiv:2402.09502

Pietro M. Bonetti, Maine Christos, S. Sachdev (BCS), arXiv: 2405.08817



PHYSICS



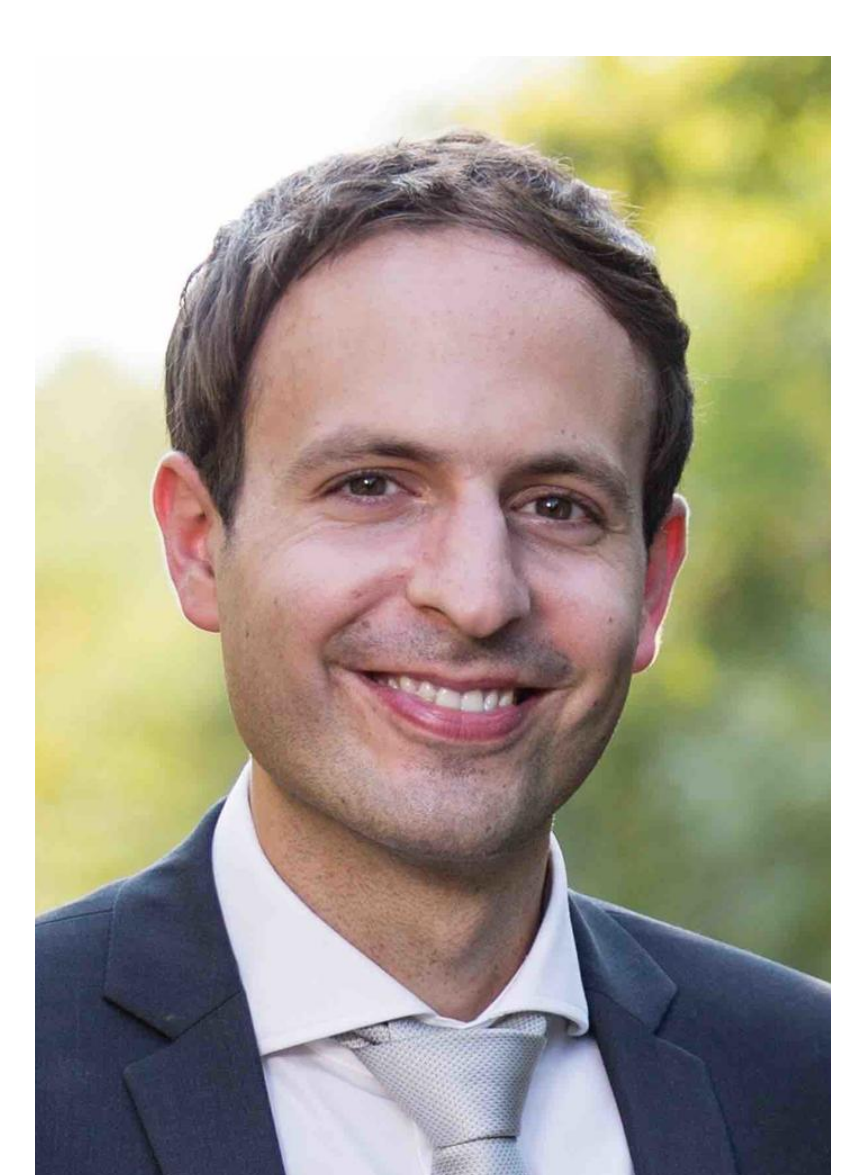
HARVARD



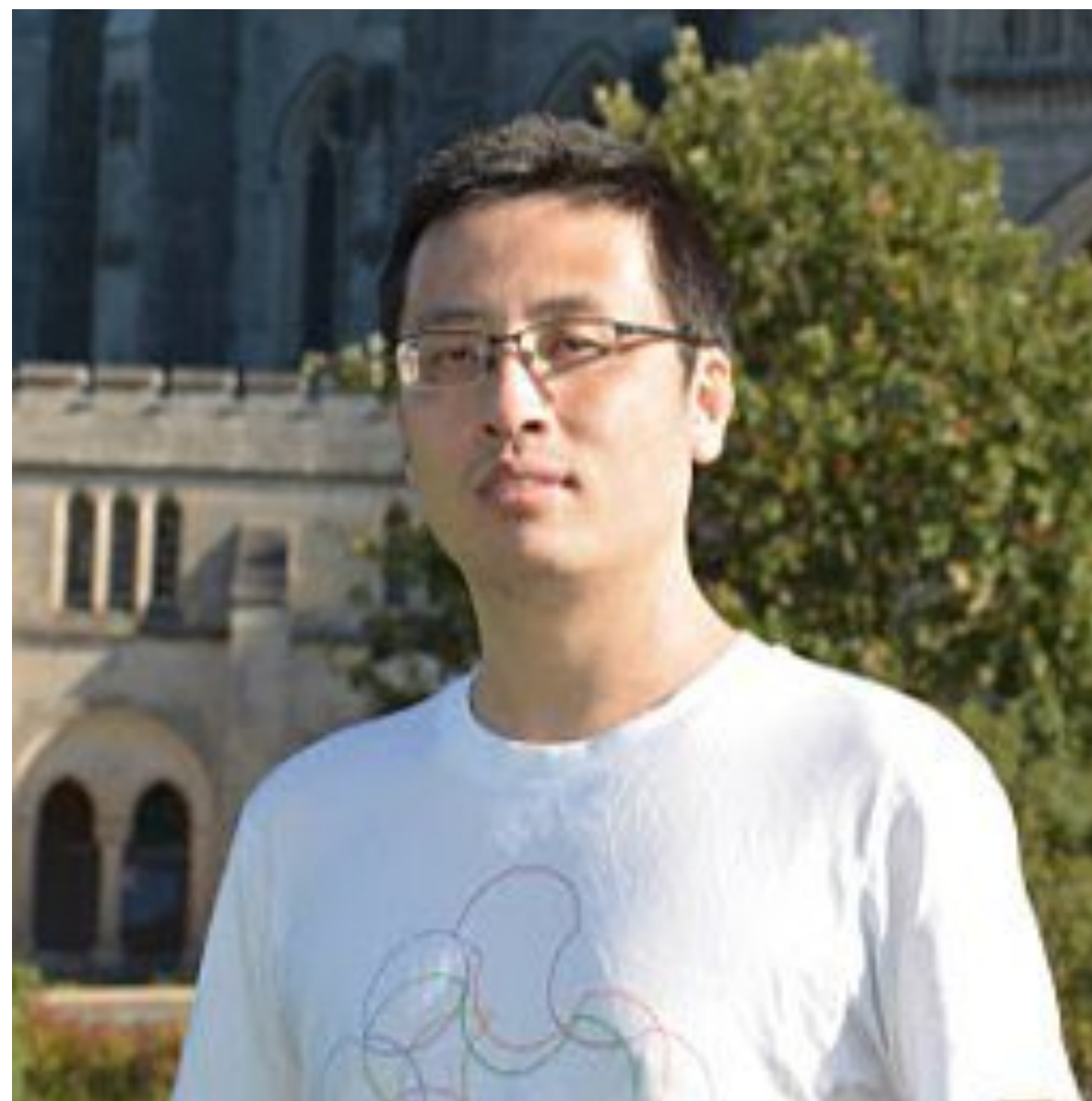
**Maine Christos**



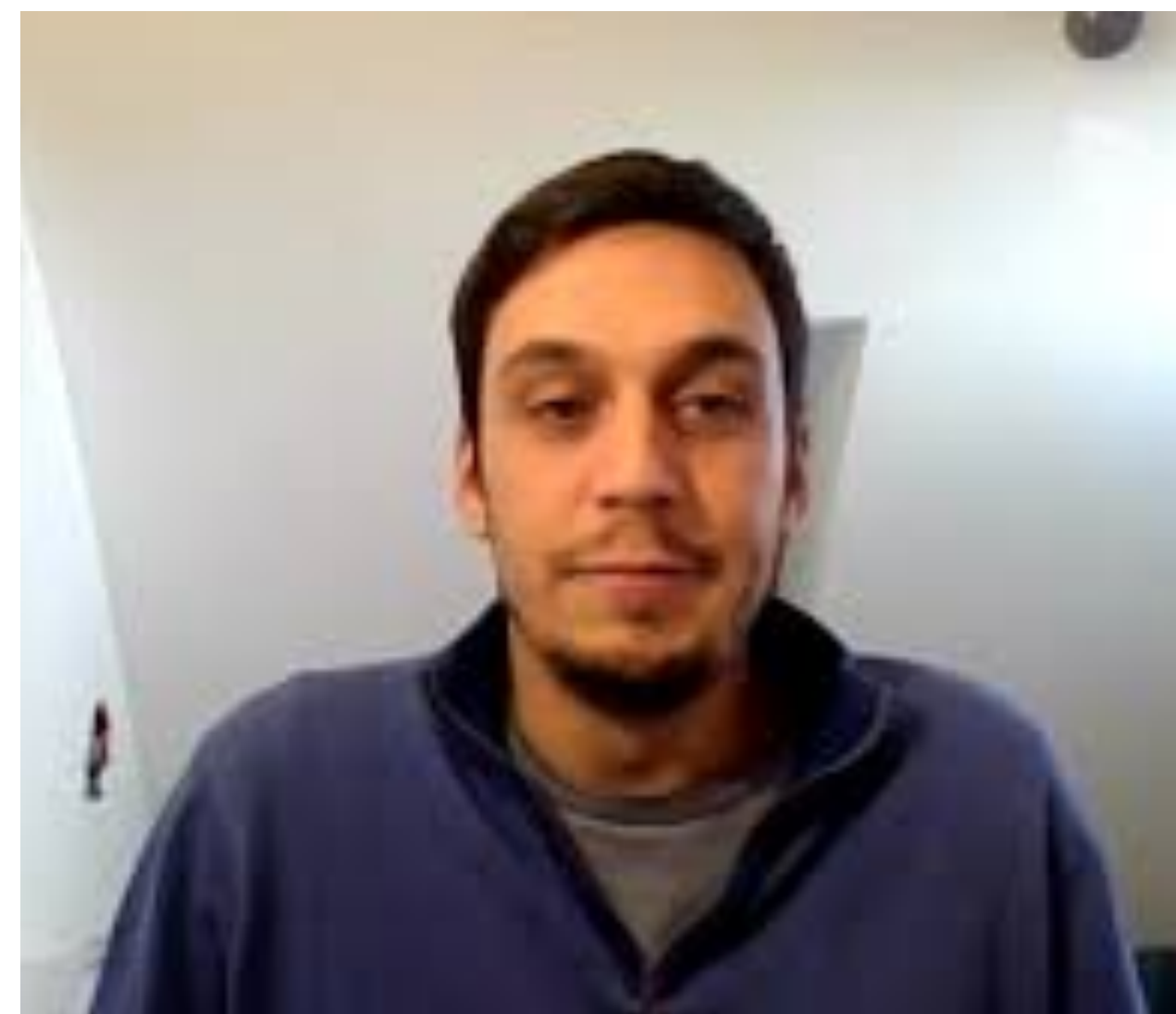
**Zhu-Xi Luo**



**Mathias Scheurer**



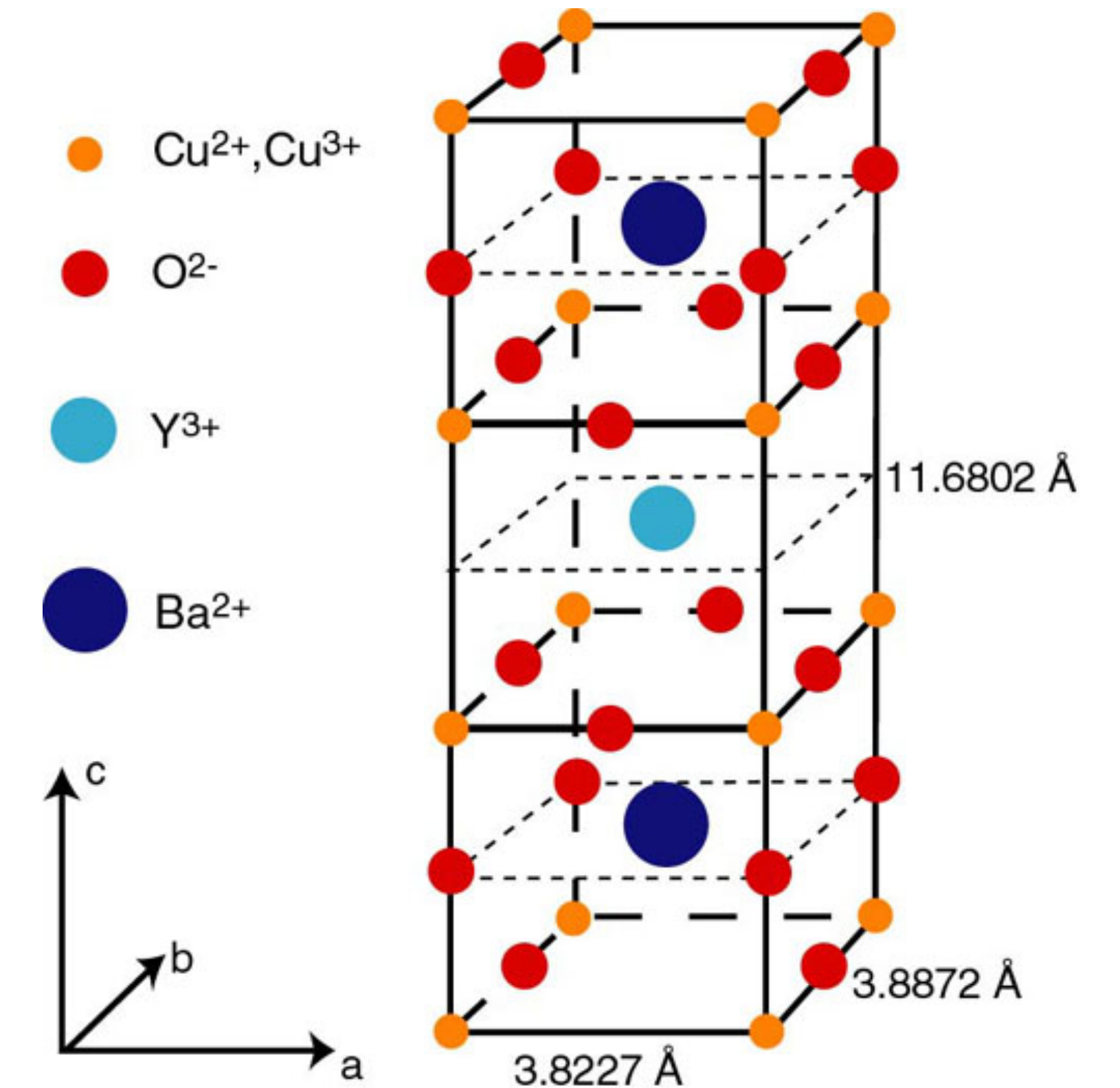
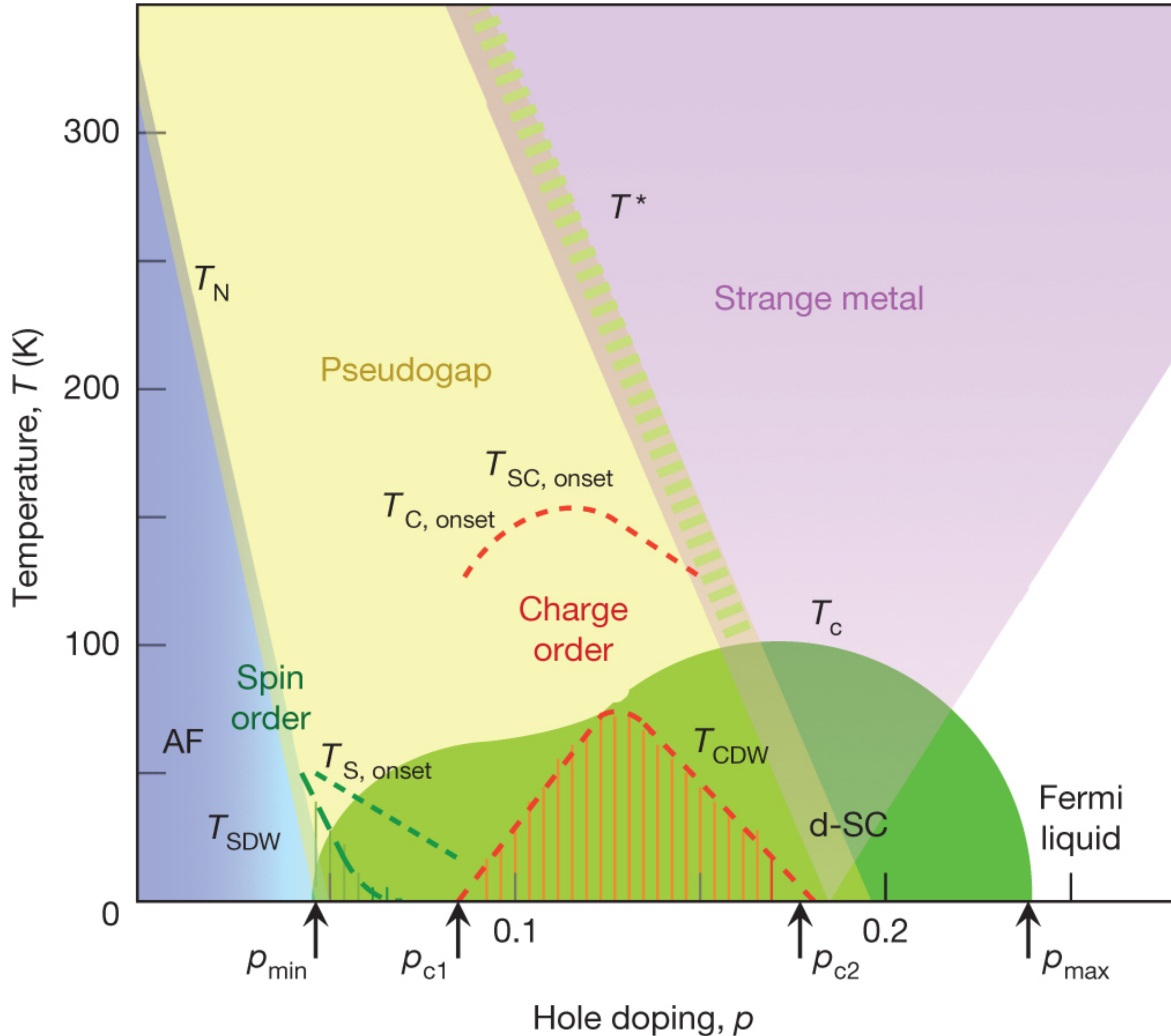
**Ya-Hui Zhang**

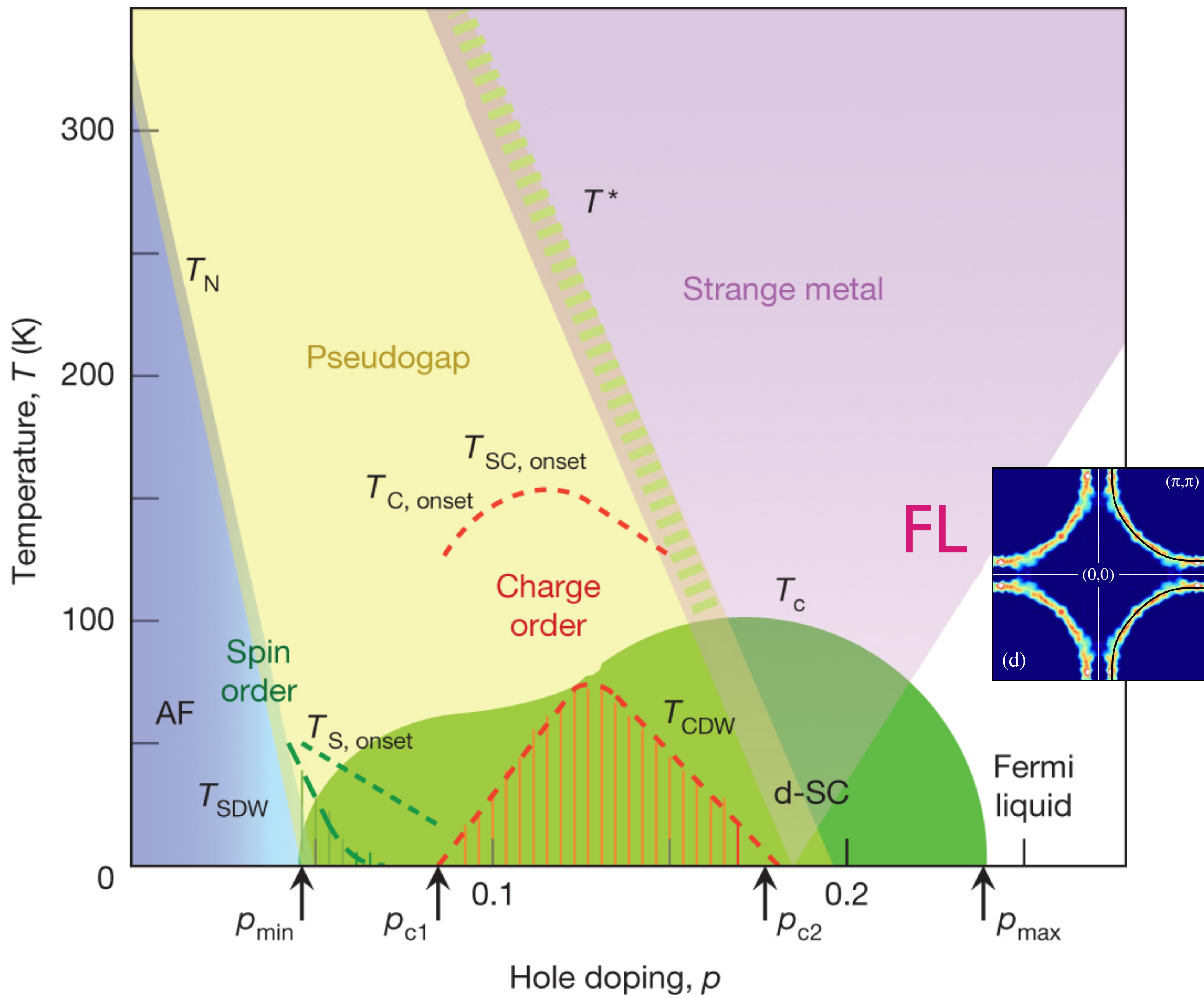


**Pietro Bonetti**



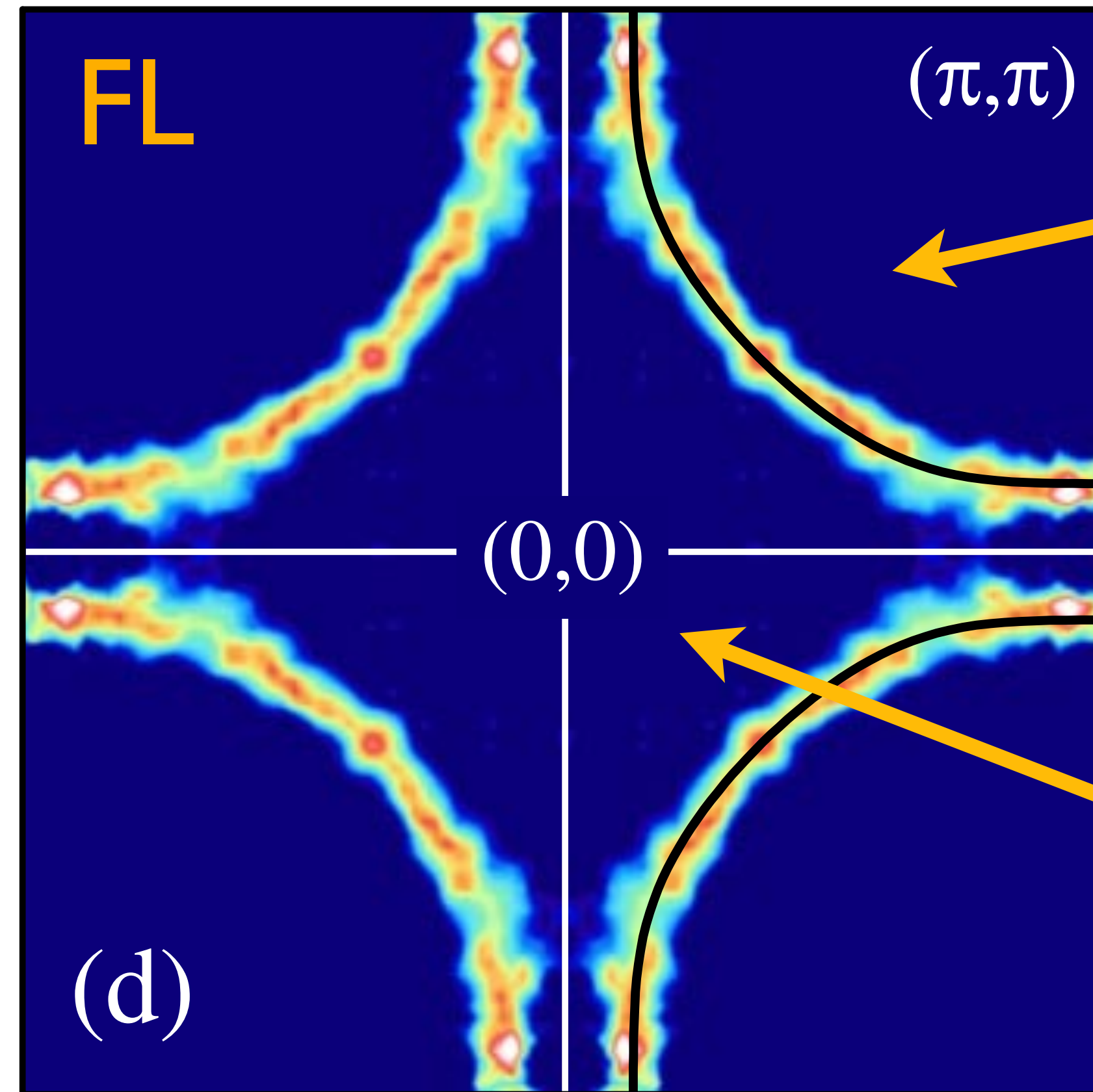
**Henry Shackleton**





Fermi liquid  
in the  
overdoped metal

# Photoemission at large $p$

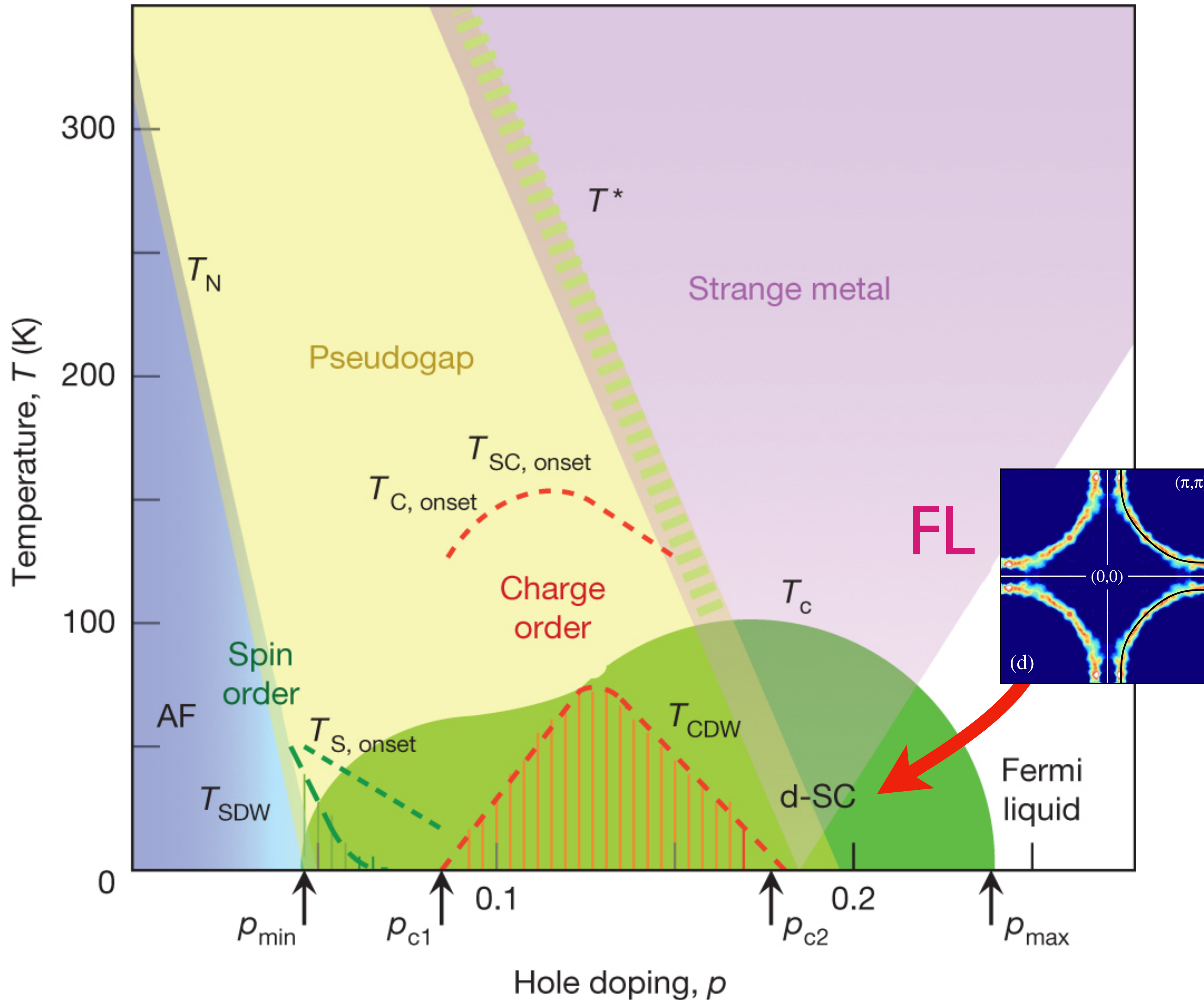


$1+p$  holes

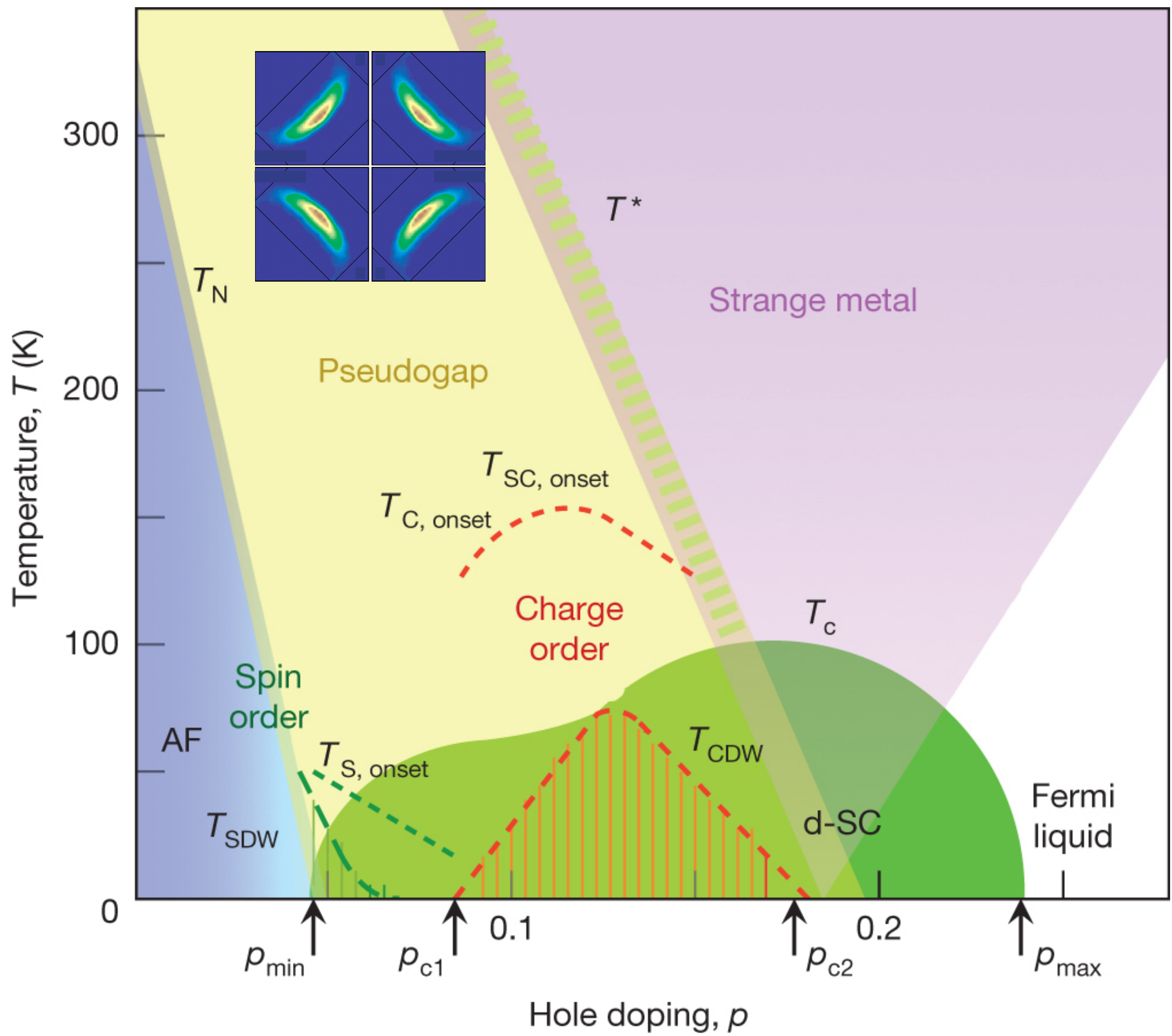
Overdoped  $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+\delta}$   
 $T_c = 30\text{K}$

$1-p$  electrons

$1+p$  mobile holes in a filled band of 2 electrons per site



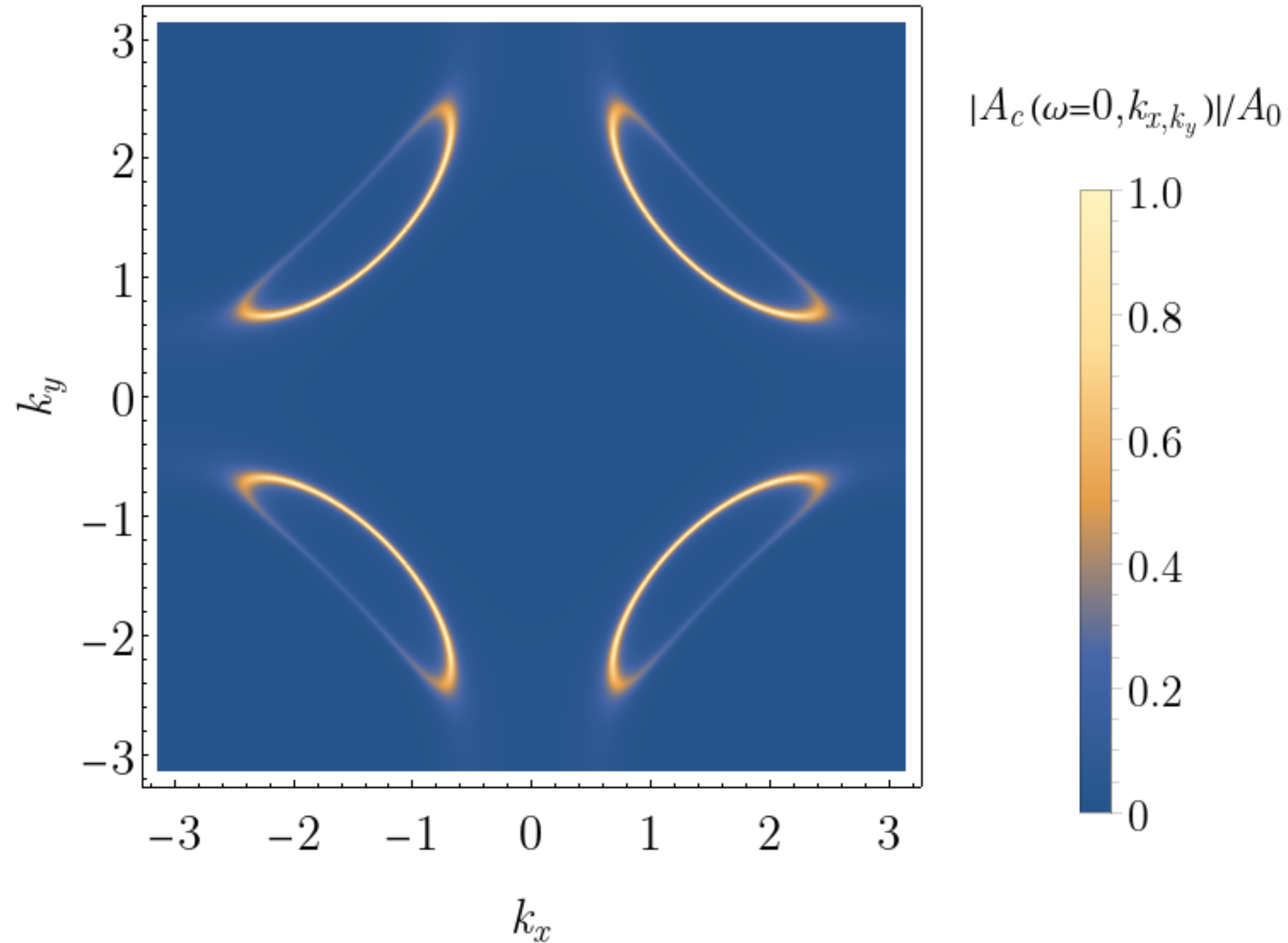
BCS-type theory of *d*-wave superconductivity (and charge order) induced by antiferromagnetic spin fluctuations.



Pseudogap metal with “Fermi arcs”

## “Backside problem I”

Hole pocket Fermi surfaces  
of size  $p$  with  
charge  $e$ , spin-1/2 quasiparticles



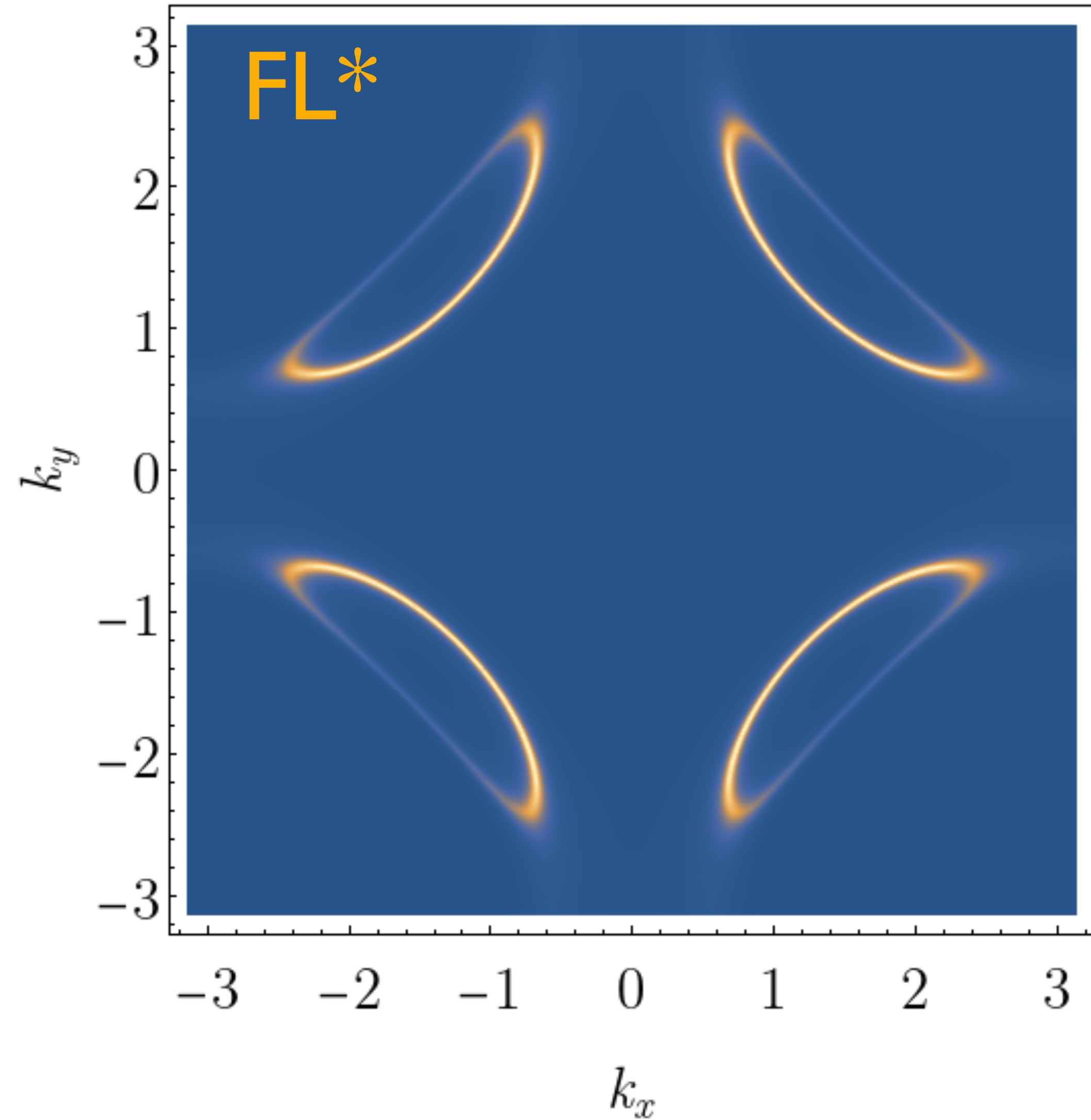
Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang,  
*Phys. Rev. B* **73**, 174501 (2006).  
T. D. Stanescu and G. Kotliar,  
*Phys. Rev. B* **74**, 125110 (2006).  
C. Berthod, T. Giamarchi, S. Biermann, and A. Georges,  
*Phys. Rev. Lett.* **97**, 136401 (2006).  
S. Sakai, Y. Motome, M. Imada,  
*Phys. Rev. Lett.* **102**, 056404 (2009).  
J. Skolimowski and M. Fabrizio,  
*Phys. Rev. B* **106**, 045109 (2022).

The Fermi surface does not enclose  
the Luttinger volume  $(1 + p)$ .

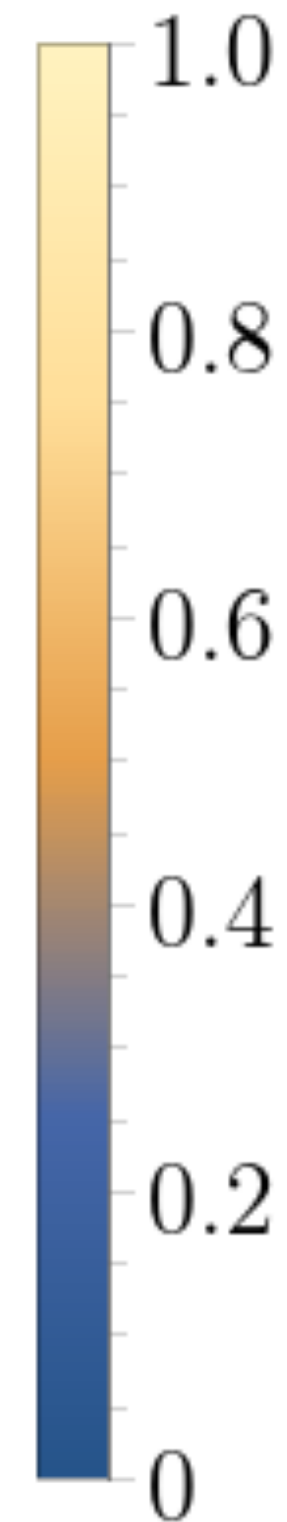
Y. Qi and S. Sachdev *Phys. Rev. B* **81**, 115129 (2010)

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and  
S. Sachdev, *Phys. Rev. B* **105**, 075146 (2022)

## “Backside problem I”



$$|A_c(\omega=0, k_x, k_y)|/A_0$$



Hole pocket Fermi surfaces  
of size  $p$  with  
charge  $e$ , spin-1/2 quasiparticles  
+  
'spectator'  
square lattice spin liquid  
*at half-filling.*

FL\*: Spin liquid is *required* because  
the Fermi surface does not enclose  
the Luttinger volume  $(1 + p)$ .

M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000)

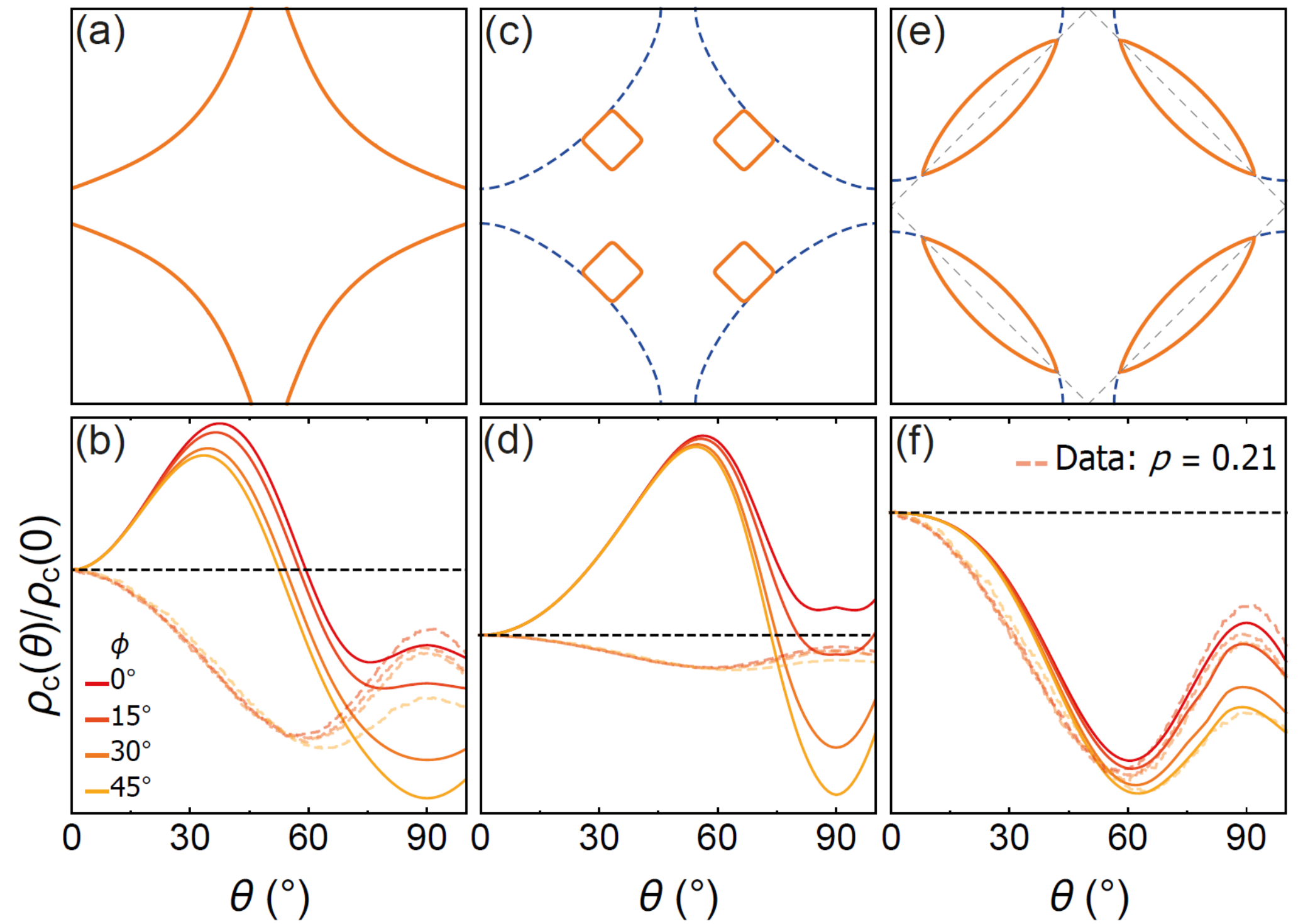
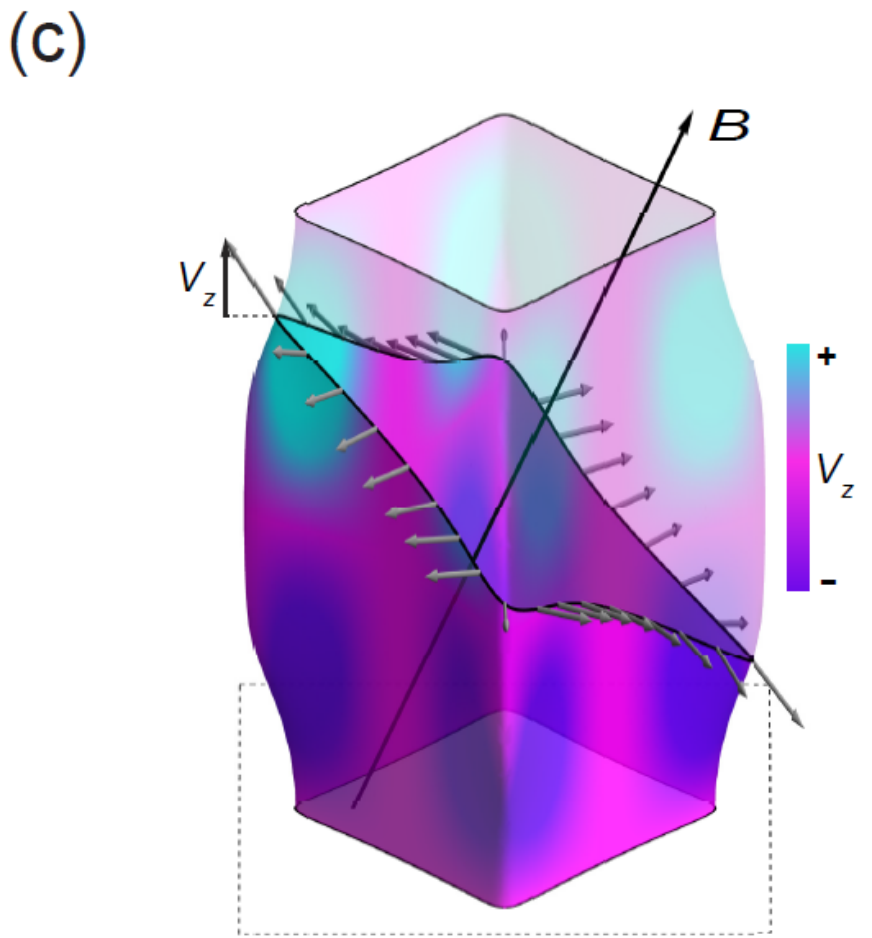
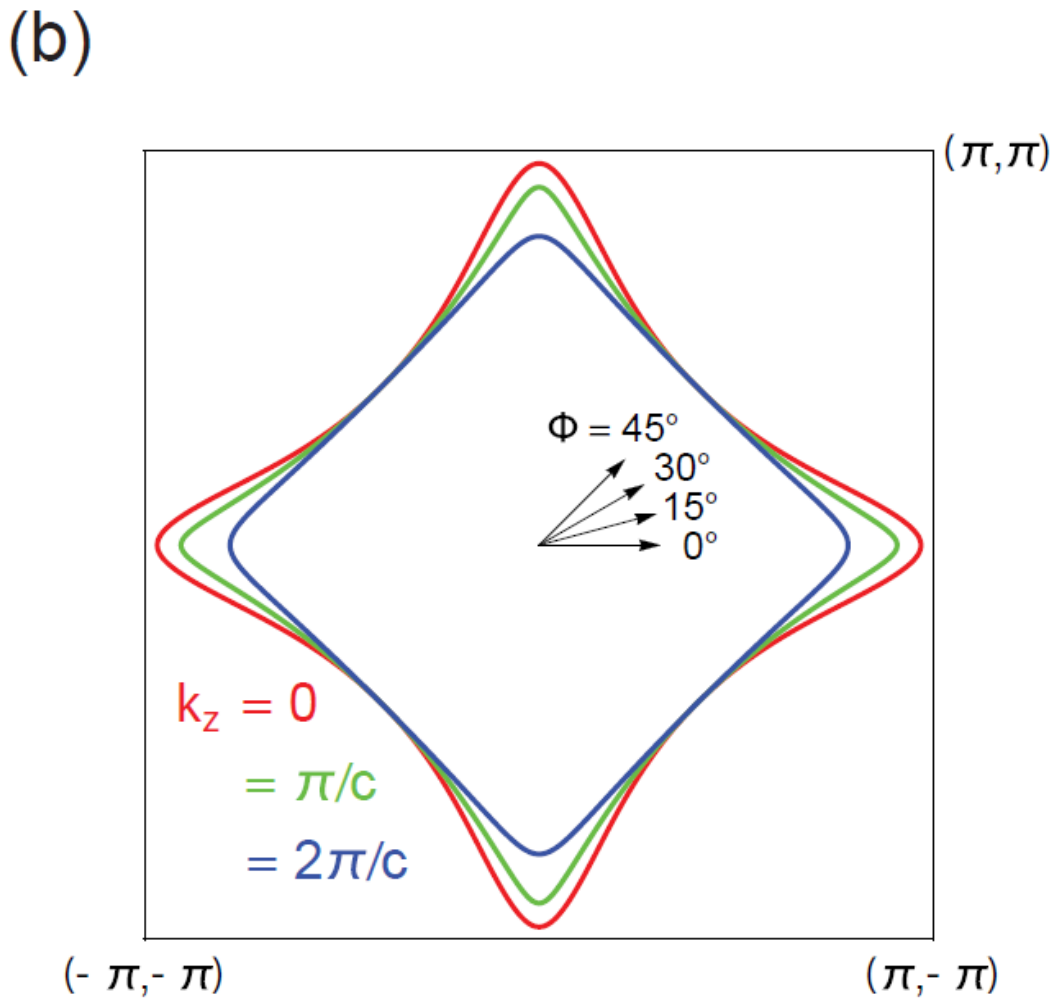
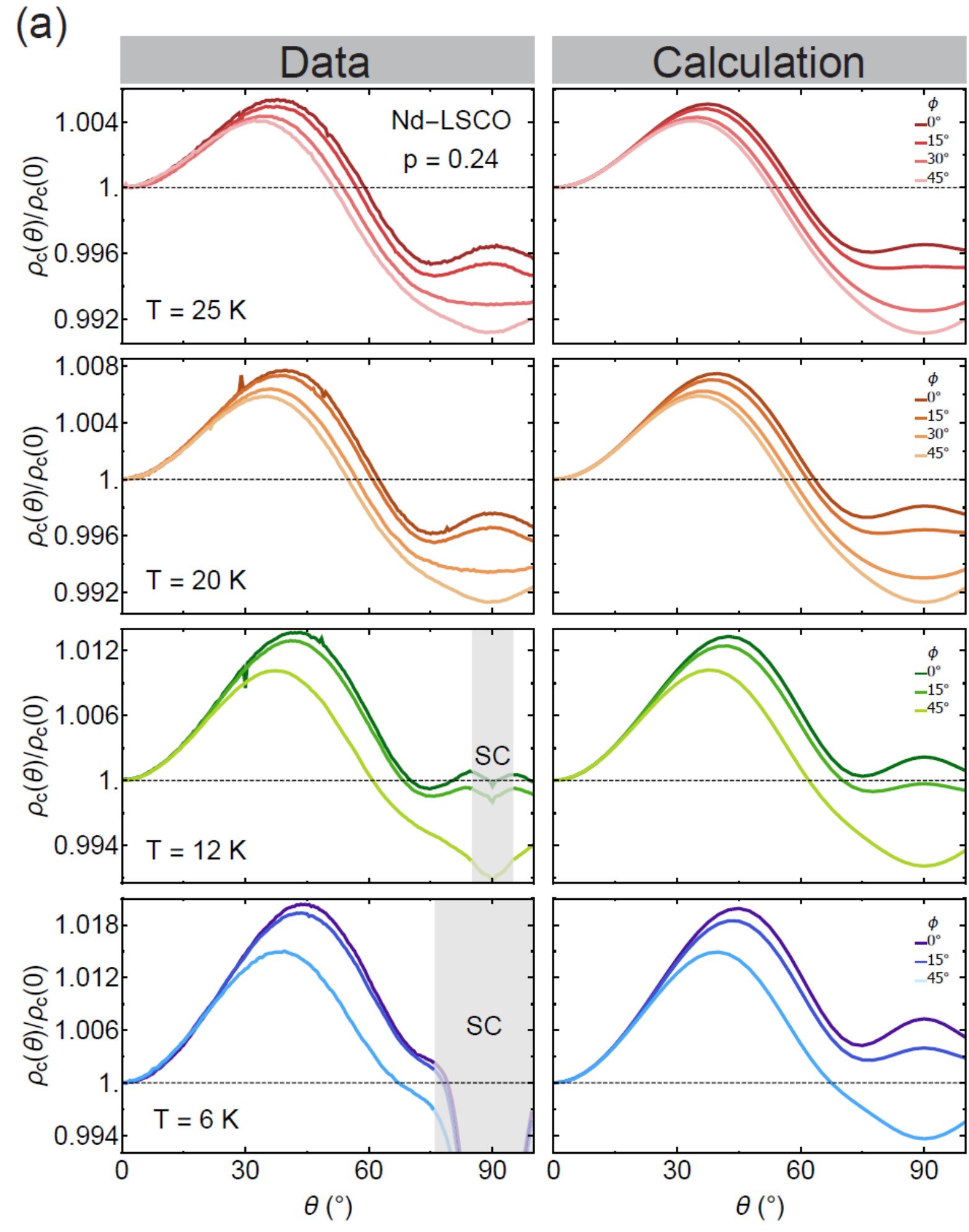
T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

A. Paramekanti and A. Vishwanath, *Phys. Rev. B* **70**, 245118 (2004)

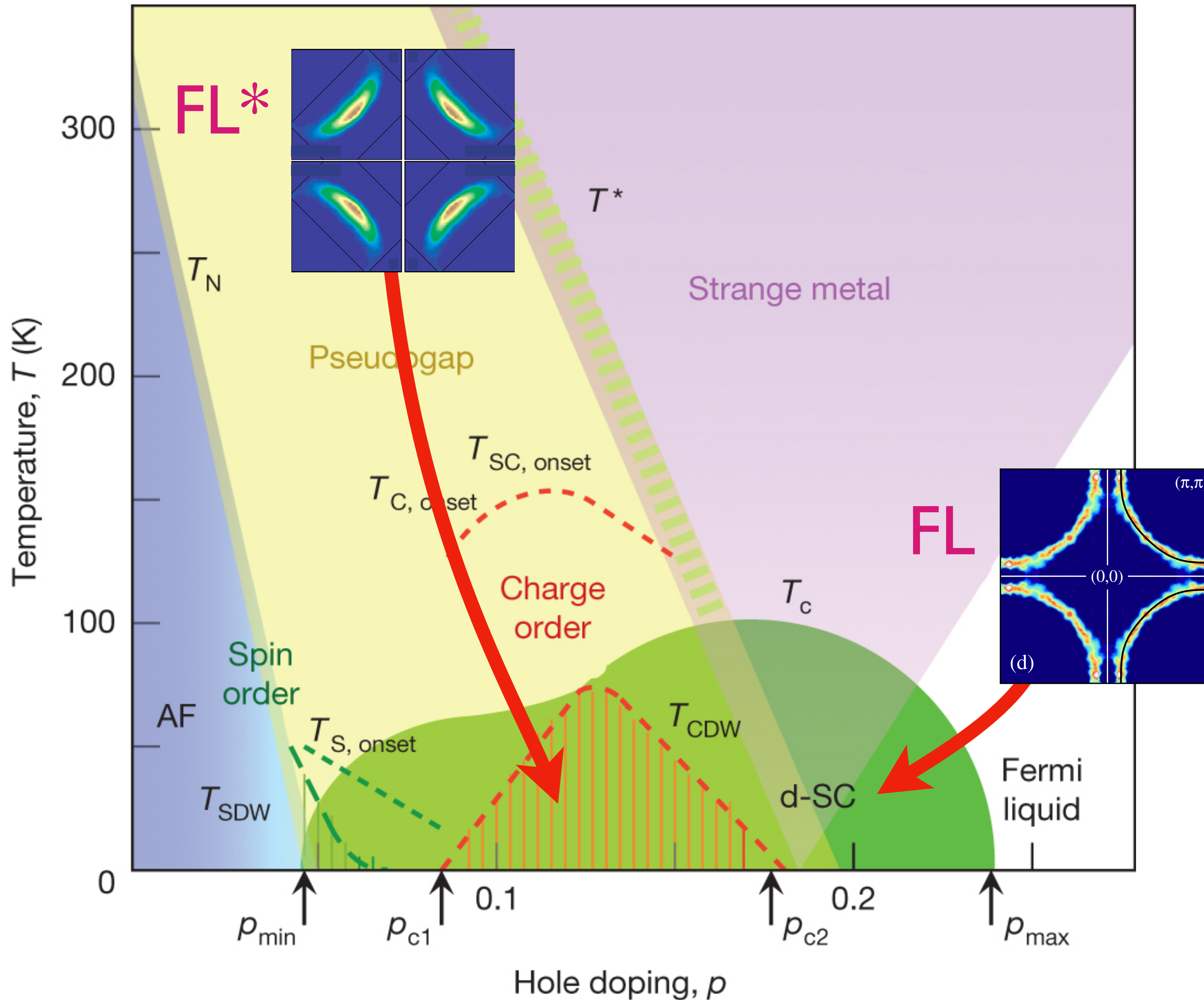
# Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, *Nature Physics* **18**, 558 (2022)



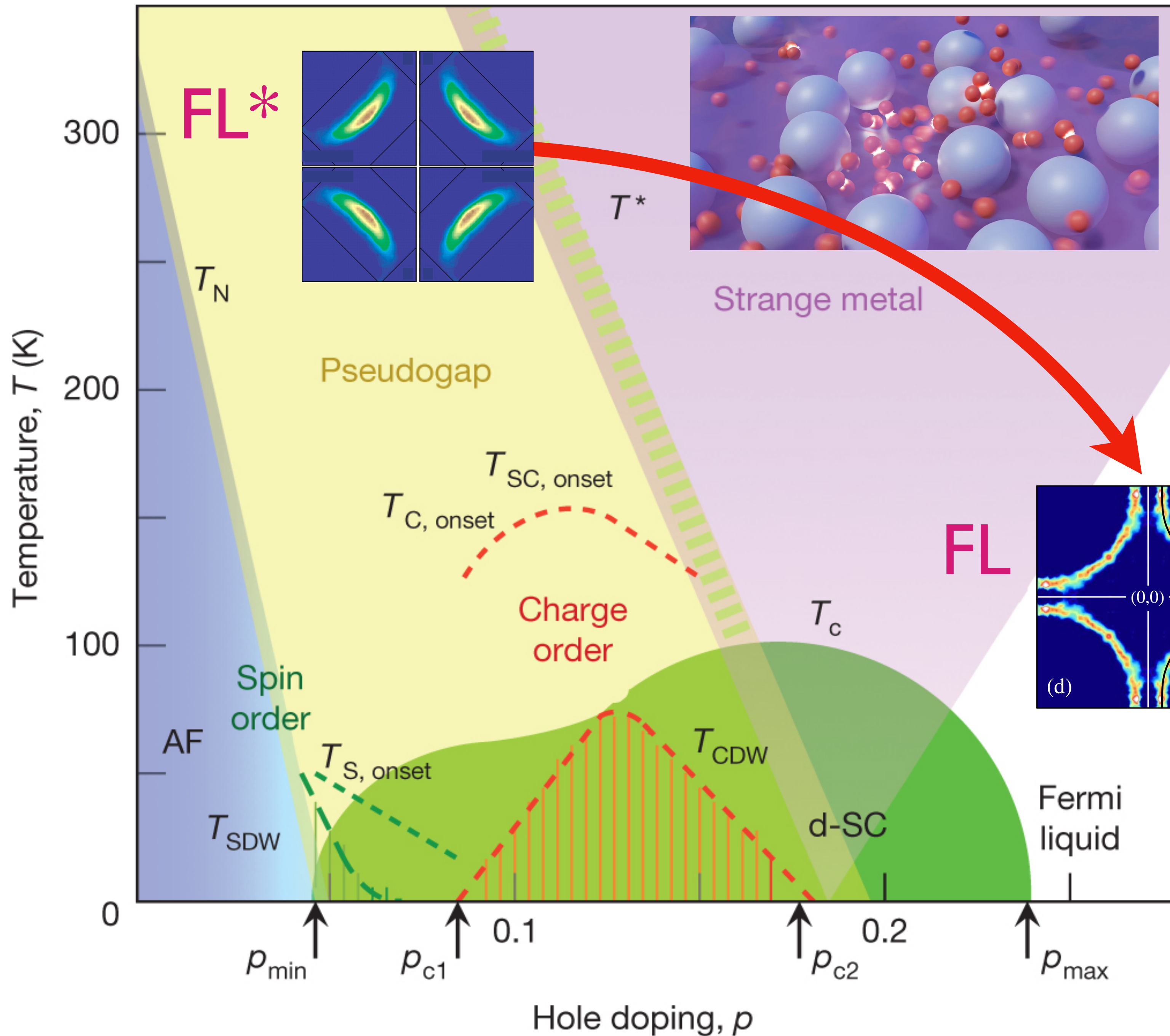
$p < p_c$  Reconstructed Fermi surface

$p > p_c$  Large Fermi surface



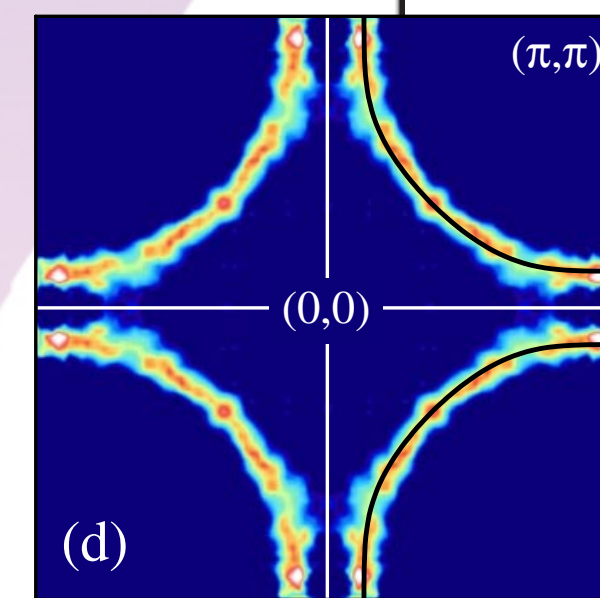
Obtain  $d$ -wave superconductor and charge order from a theory of *confinement* instabilities of FL\*.

The resulting low  $T$  ordered states should be adiabatically connected to the corresponding states obtained from instabilities of FL.



Universal  
Yukawa-SYK theory  
for quantum-criticality  
of confinement transition  
from FL\* to FL  
in the presence of disorder

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S.,  
*Science* **381**, 790 (2023)



1. Confinement transitions of the Kondo lattice
2. Ancilla theory of  $FL^*$  in a single band model
3. Confinement of  $FL^*$  with Neel-VBS DQCP
4. Backside problem 2:  
Nodal fermionic quasiparticles in d-wave SC
5. Backside problem 3:  
Quantum oscillations in CDW state

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Quantum oscillations in CDW state

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + \sum_i J_K c_{i\sigma}^\dagger \frac{\tau_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \mathbf{S}_i + \sum_{\langle ij \rangle} J_H \mathbf{S}_i \cdot \mathbf{S}_j$$

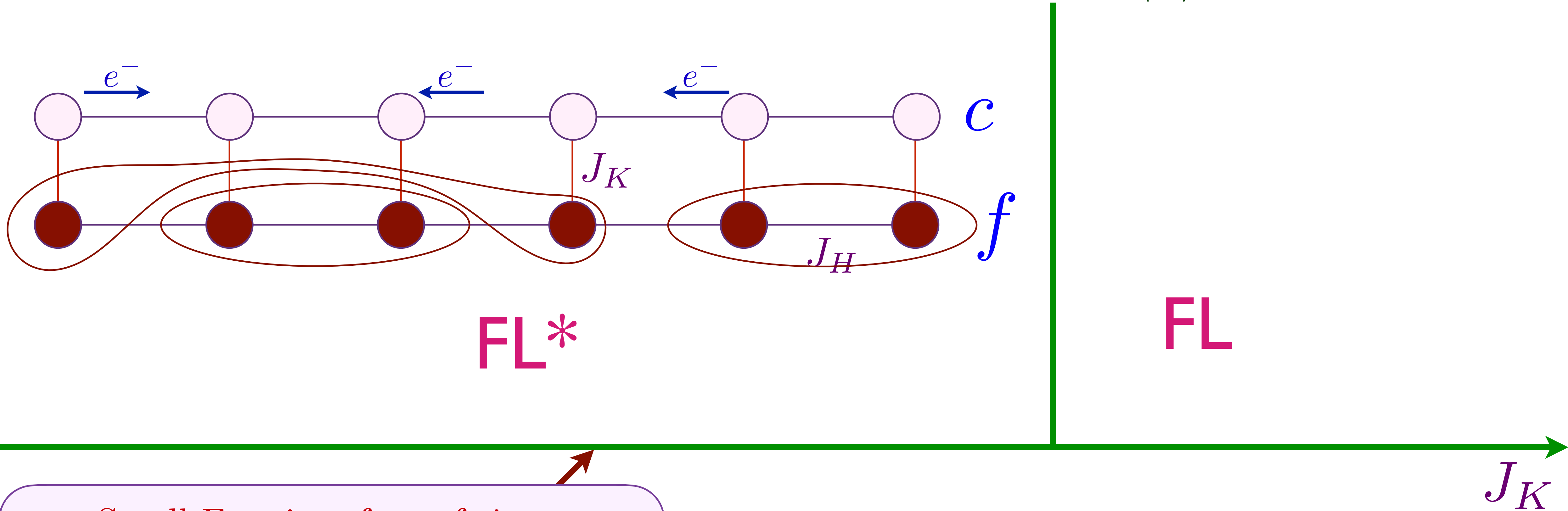
FL\*

FL

$J_K$



$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + \sum_i J_K c_{i\sigma}^\dagger \frac{\tau_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \mathbf{S}_i + \sum_{\langle ij \rangle} J_H \mathbf{S}_i \cdot \mathbf{S}_j$$



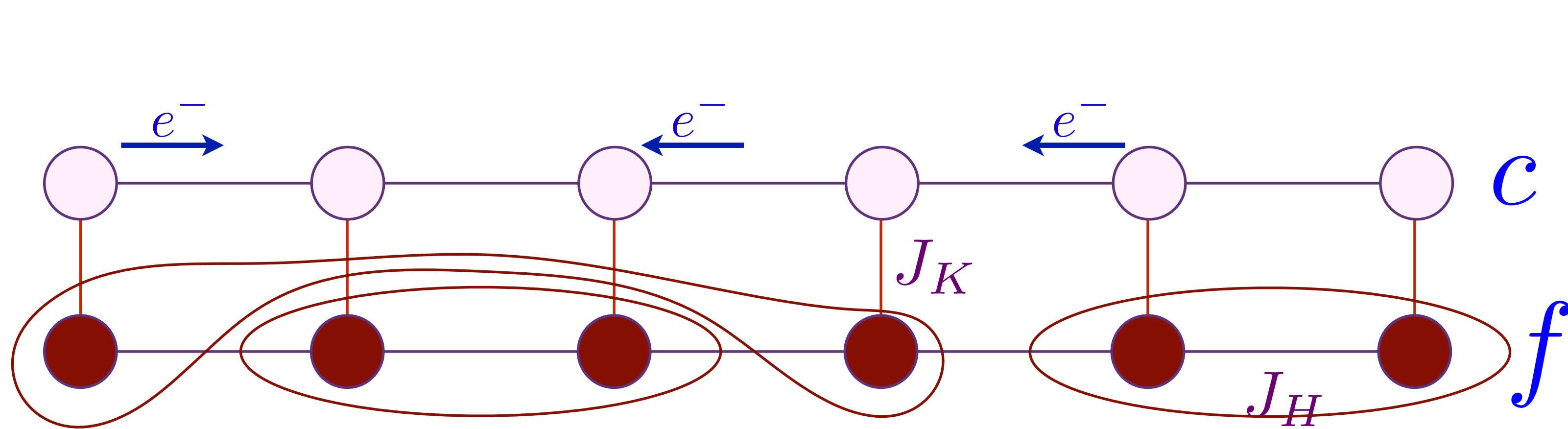
Small Fermi surface of size  $p$

$|\text{FL}^*\rangle = [\text{Projection onto one } f \text{ per site}]$

$\boxtimes$  |Slater determinant of  $f$

$\otimes$  |Slater determinant of  $c$

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + \sum_i J_K c_{i\sigma}^\dagger \frac{\tau_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \mathbf{S}_i + \sum_{\langle ij \rangle} J_H \mathbf{S}_i \cdot \mathbf{S}_j$$



**FL\***

}  $\langle B \rangle \neq 0$

**FL**

Fractionalized excitations of spin liquid are confined by the condensation of a Higgs boson  $B \sim f_\alpha^\dagger c_\alpha$ .

Small Fermi surface of size  $p$

$|\text{FL}^*\rangle = [\text{Projection onto one } f \text{ per site}]$   
 $\boxtimes |\text{Slater determinant of } f\rangle$   
 $\otimes |\text{Slater determinant of } c\rangle$

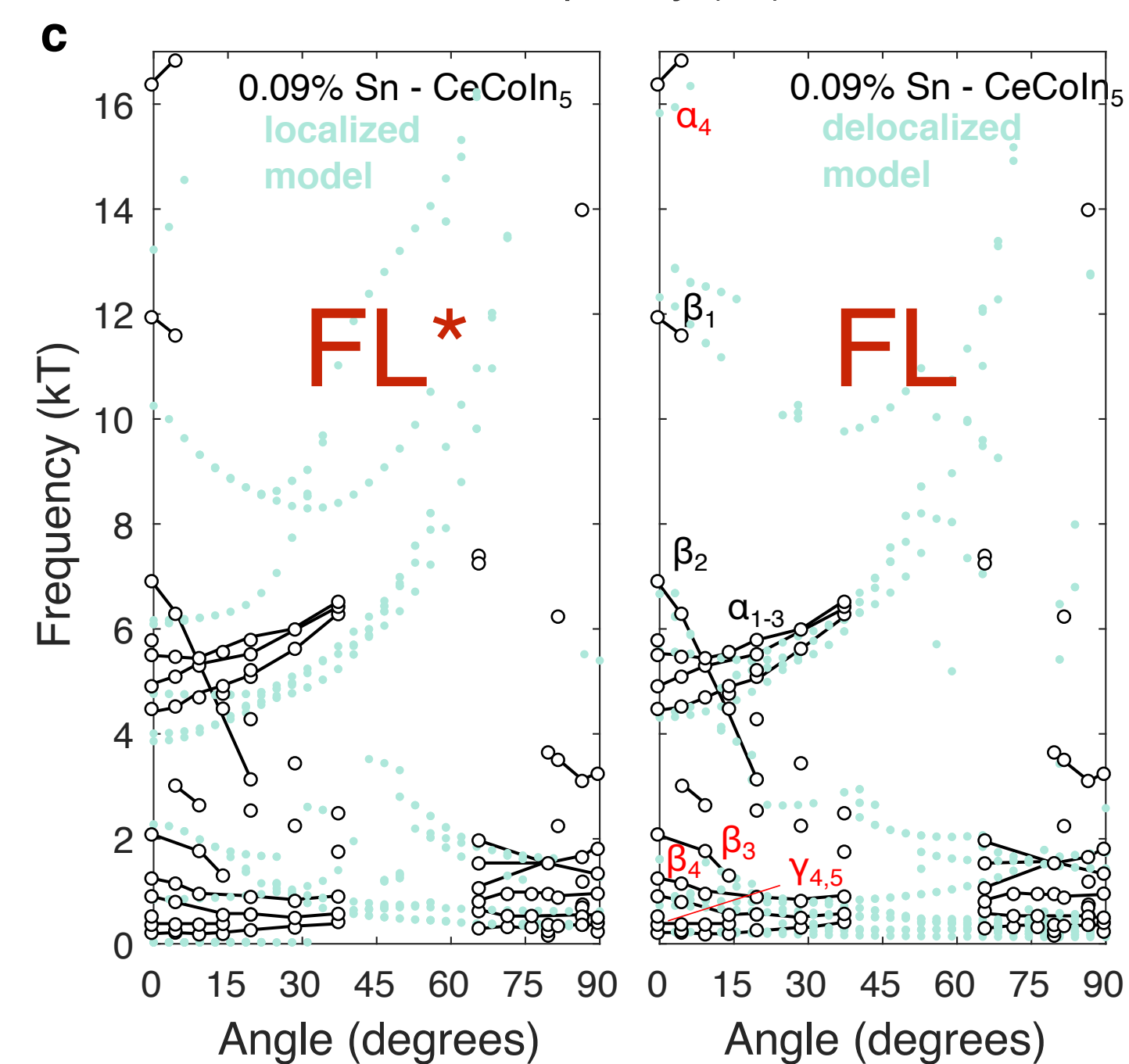
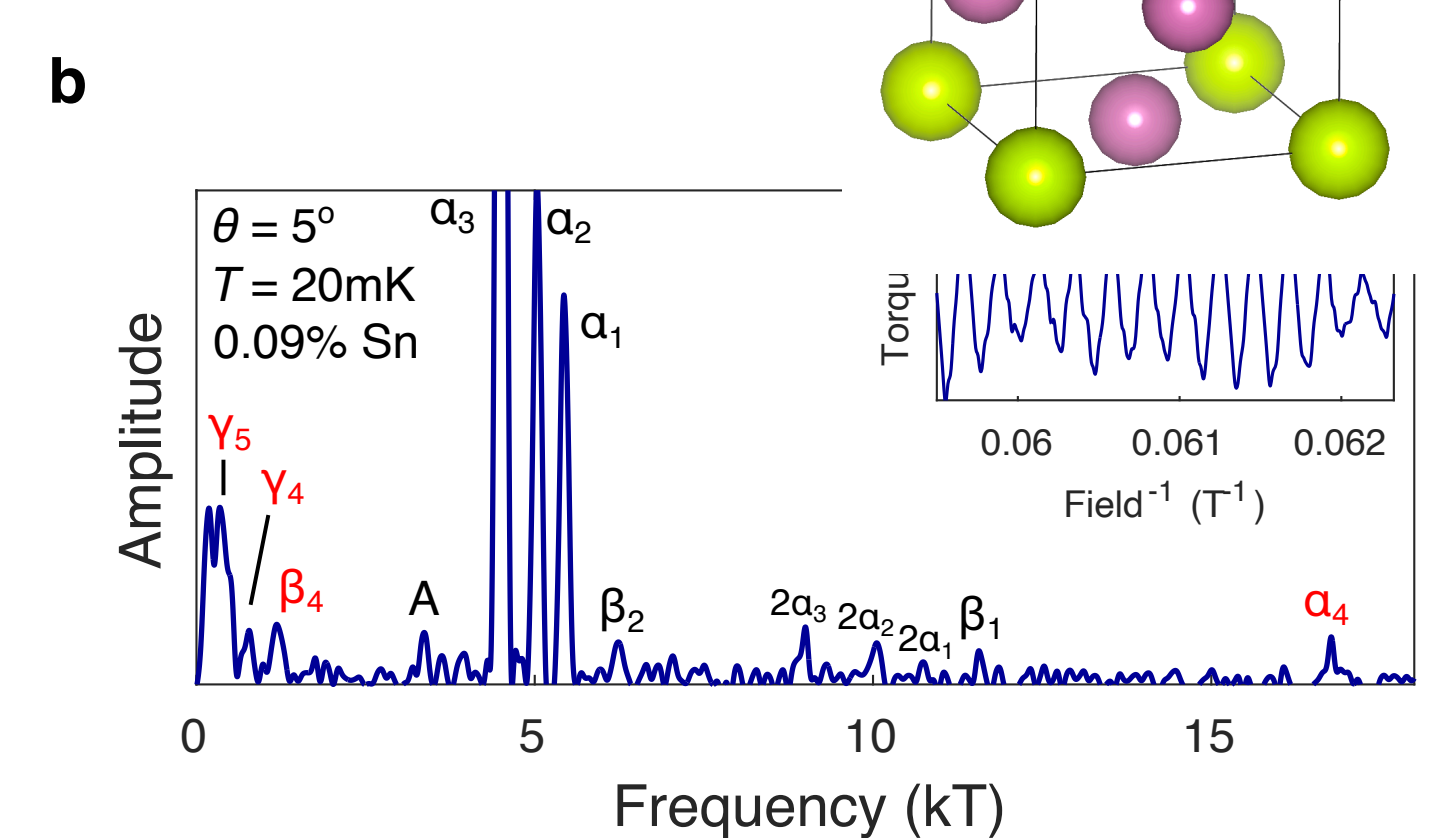
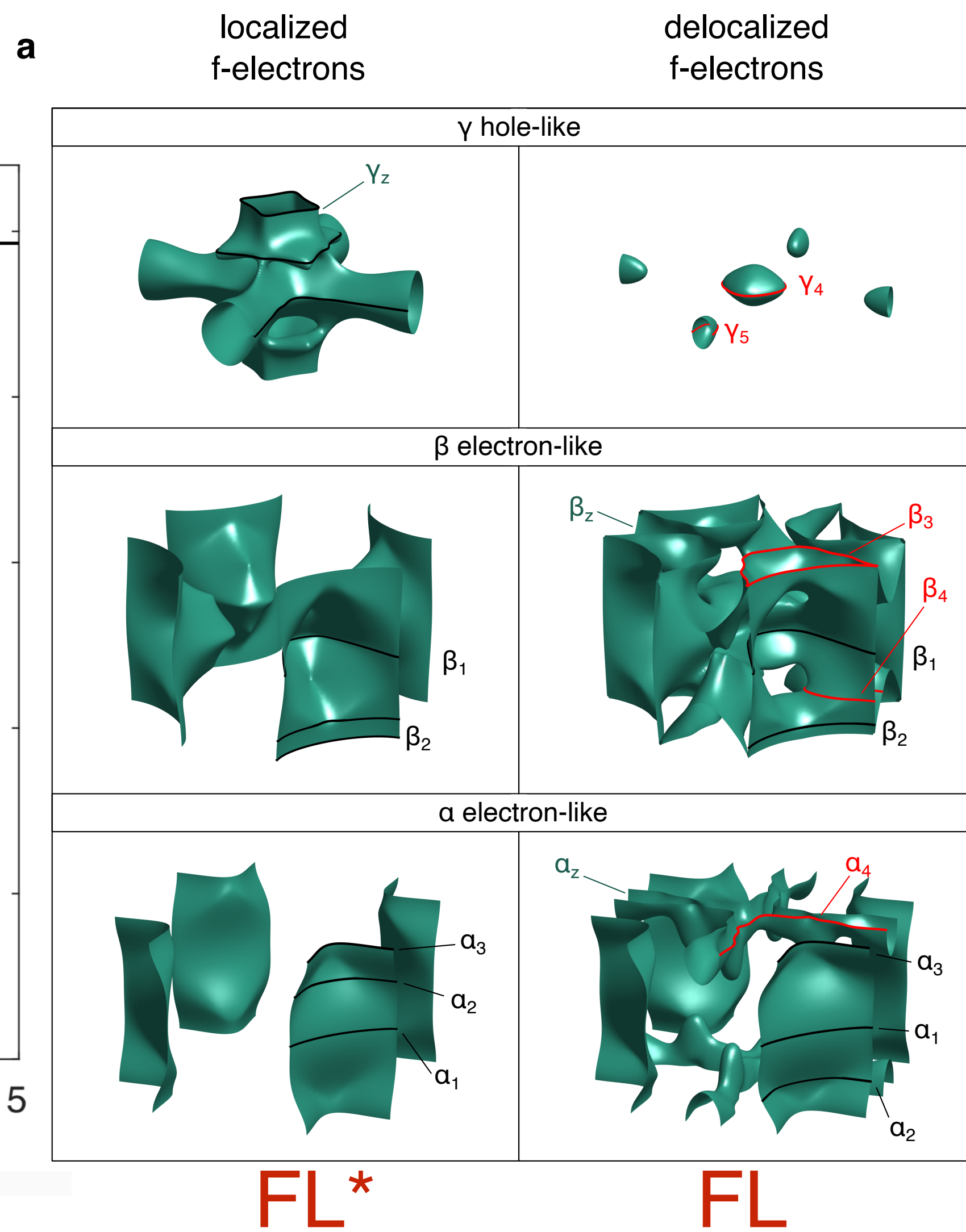
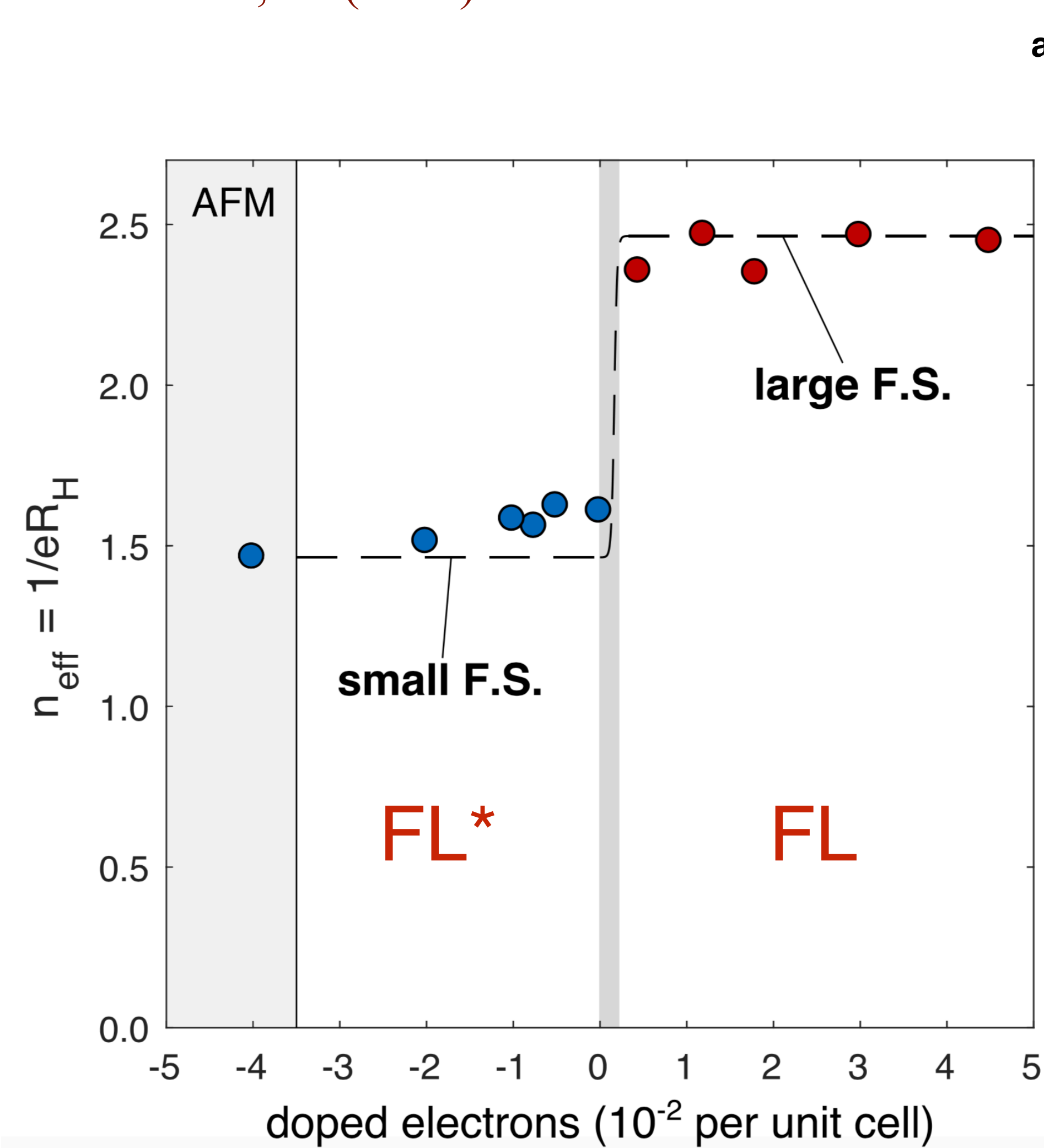
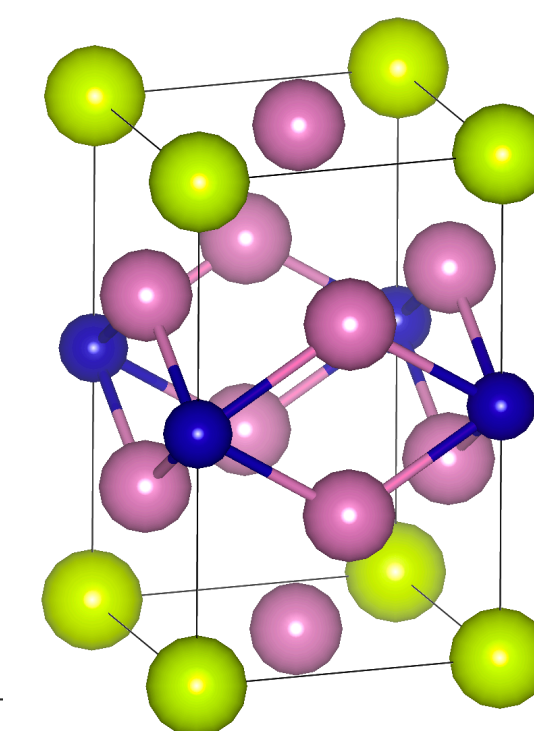
Large Fermi surface of size  $1 + p$

$|\text{HFL}\rangle = [\text{Projection onto one } f \text{ per site}]$   
 $\boxtimes |\text{Slater determinant of } (c, f)\rangle$

$J_K$

# Evidence for a delocalization quantum phase transition without symmetry breaking in $\text{CeCoIn}_5$

Nikola Maksimovic, Daniel H. Eilbott, Tessa Cookmeyer.....Ehud Altman, Alessandra Lanzara, James G. Analytis, *Science* **375**, 76 (2021)



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3. Confinement of  $FL^*$  with Neel-VBS DQCP

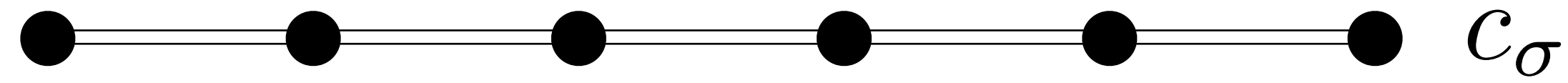
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Nodal fermionic quasiparticles in d-wave SC

5. Backside problem 3:

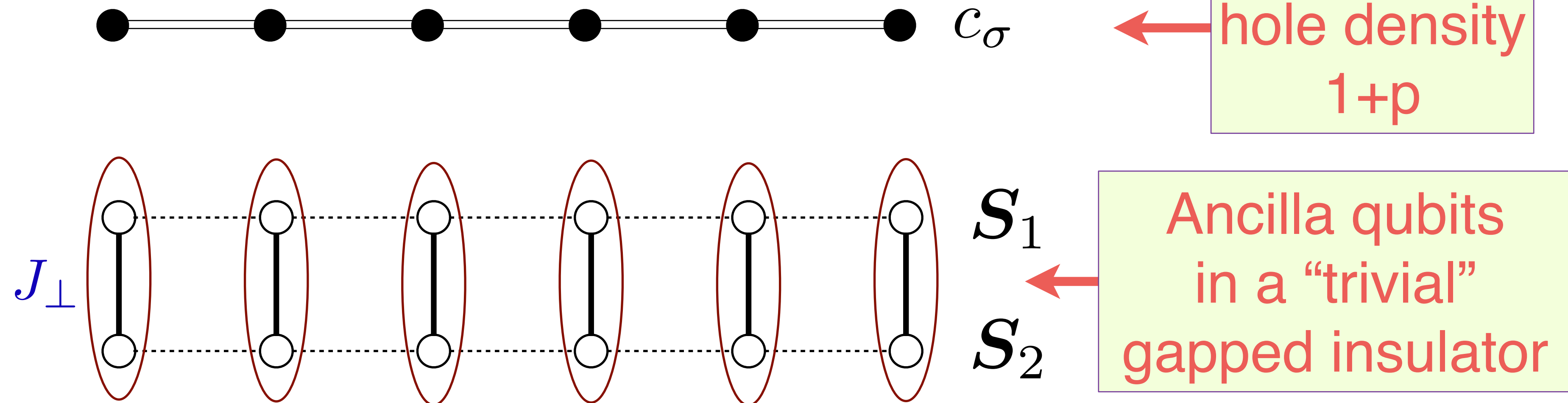
Quantum oscillations in CDW state

# Ancilla theory of the Hubbard model



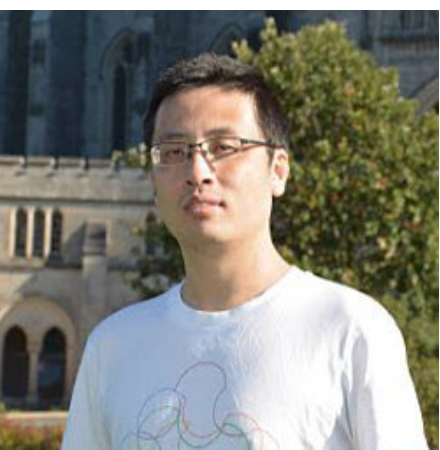
Hubbard  
model of  
hole density  
 $1+p$

# Ancilla theory of the Hubbard model

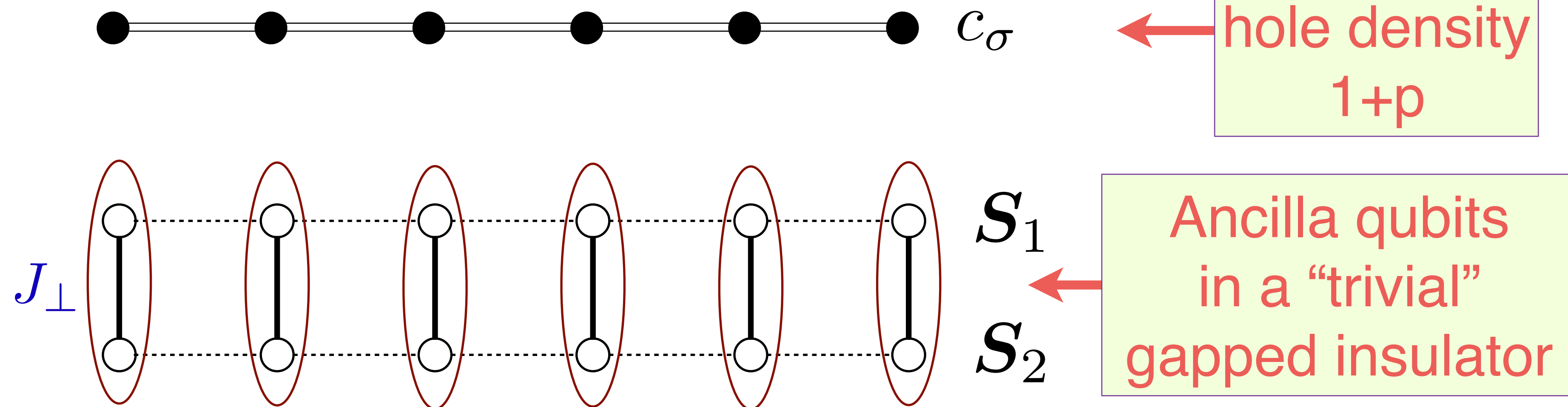


$$\mathcal{H}_{\text{Hubbard}} + \mathcal{H}_{\text{trivial insulator}}$$

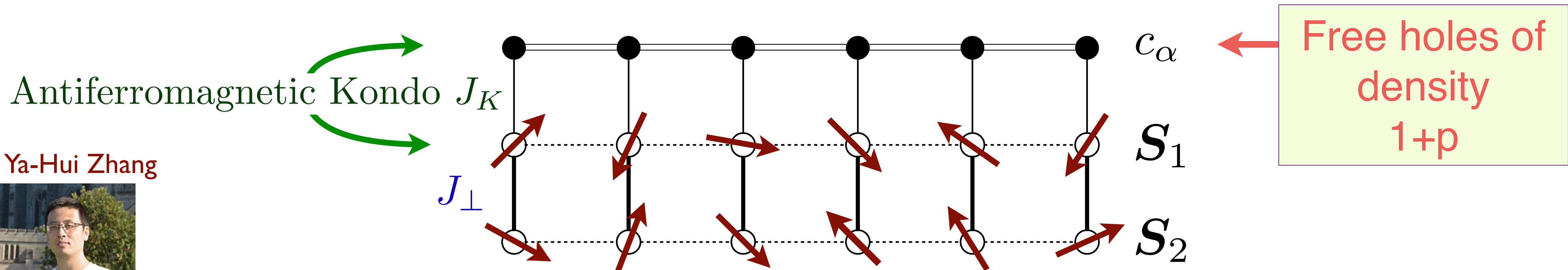
Ya-Hui  
Zhang



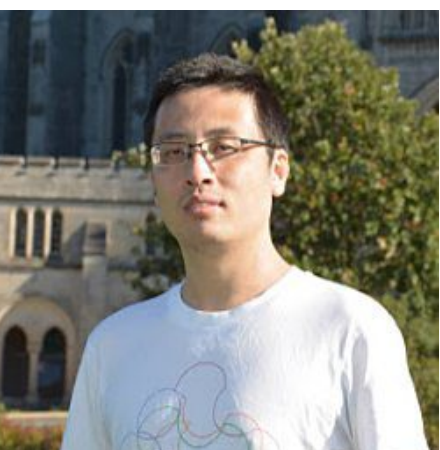
# Ancilla theory of the Hubbard model



$$U (\mathcal{H}_{\text{Hubbard}} + \mathcal{H}_{\text{trivial insulator}}) U^{-1} = \mathcal{H}_{\text{ancilla}}$$

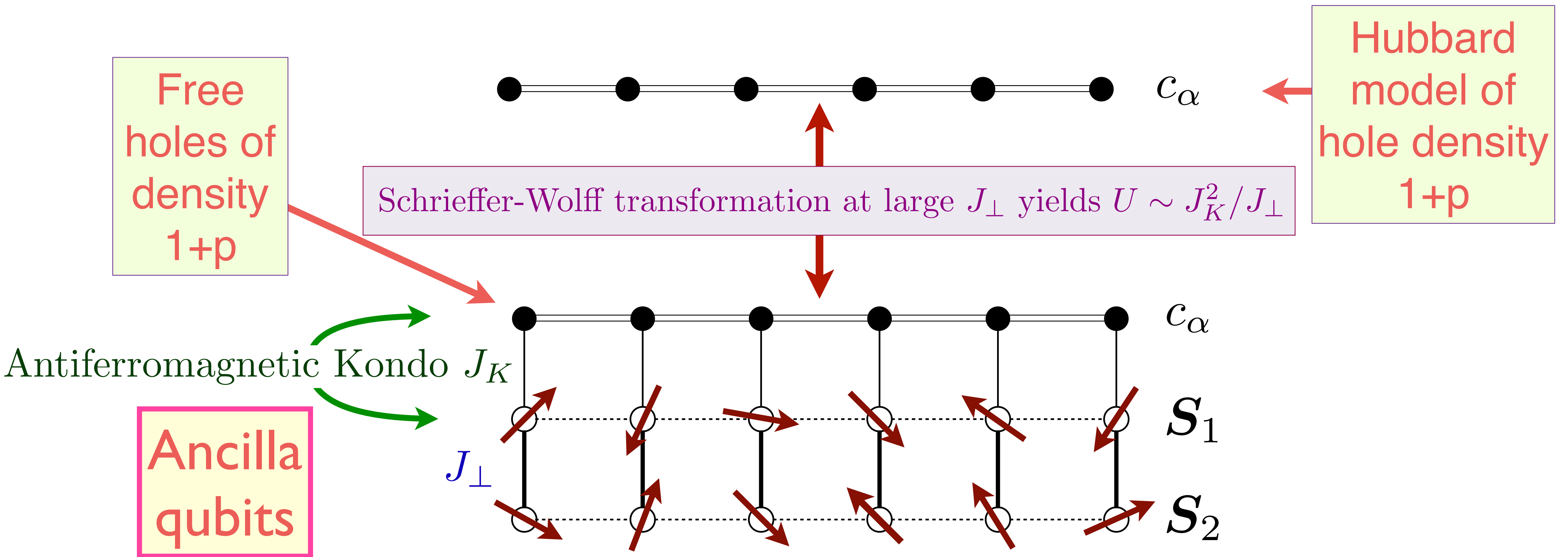


Ya-Hui Zhang

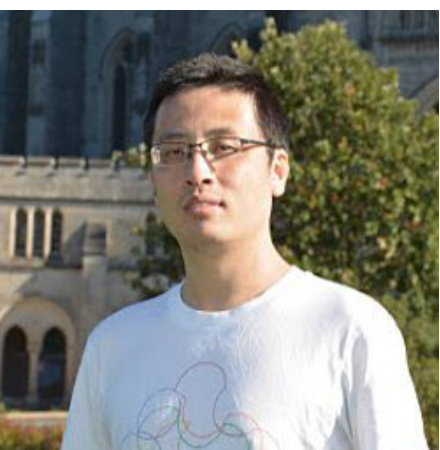


# Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev,  
*Phys. Rev. Res.* **2**, 023172 (2020)



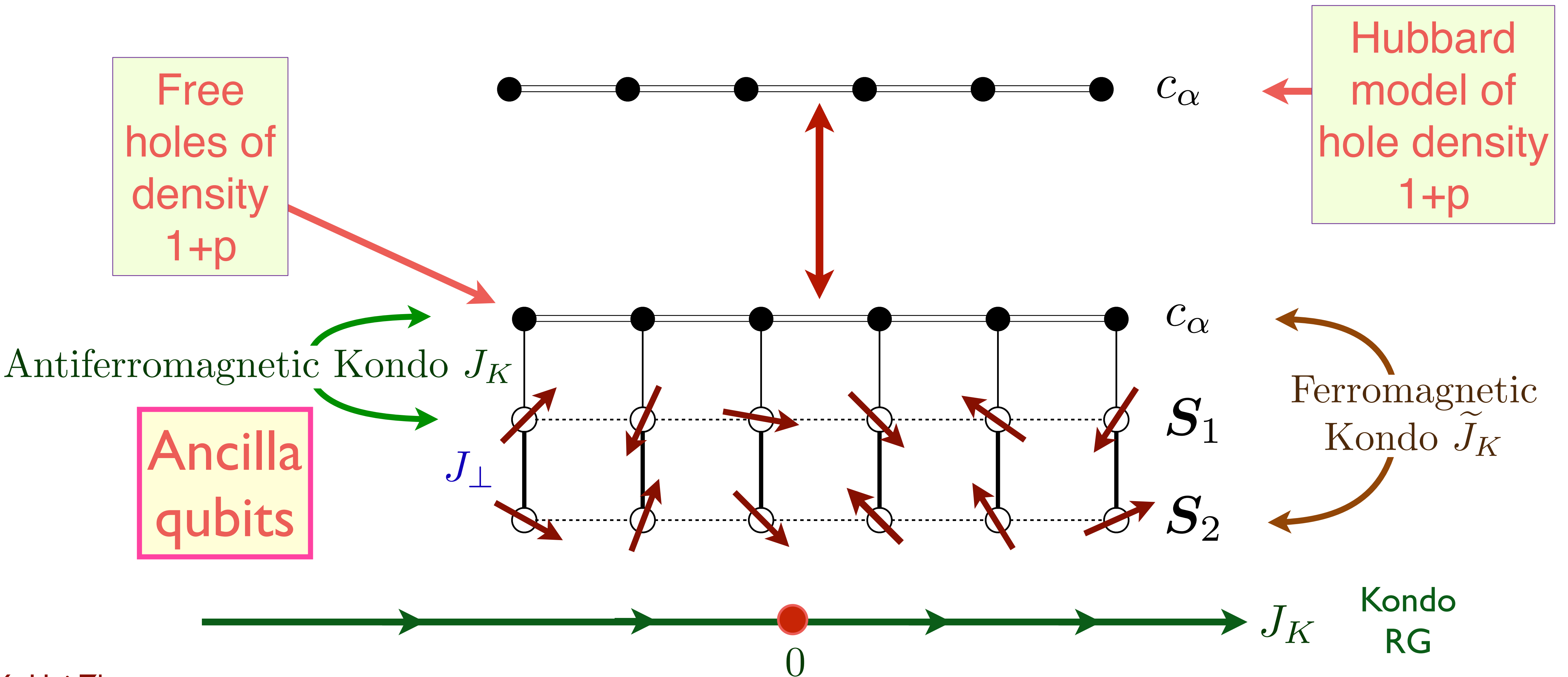
Ya-Hui Zhang



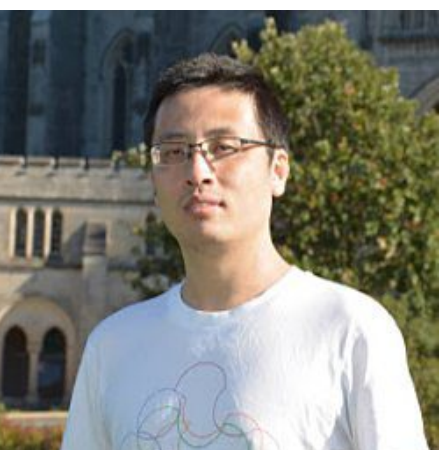
$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^{\dagger} c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^{\dagger} \frac{\sigma_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \mathbf{S}_{1i} + J_{\perp} \sum_i \mathbf{S}_{1i} \cdot \mathbf{S}_{2i}$$

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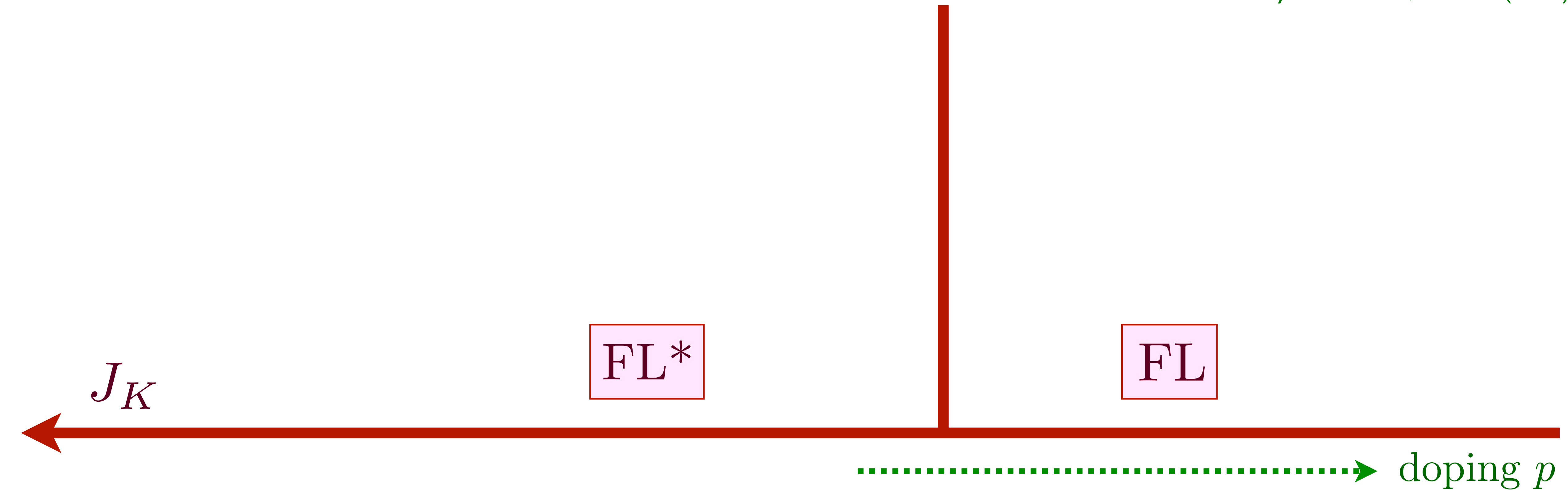
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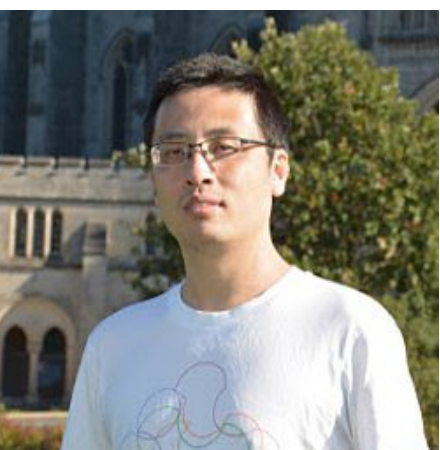
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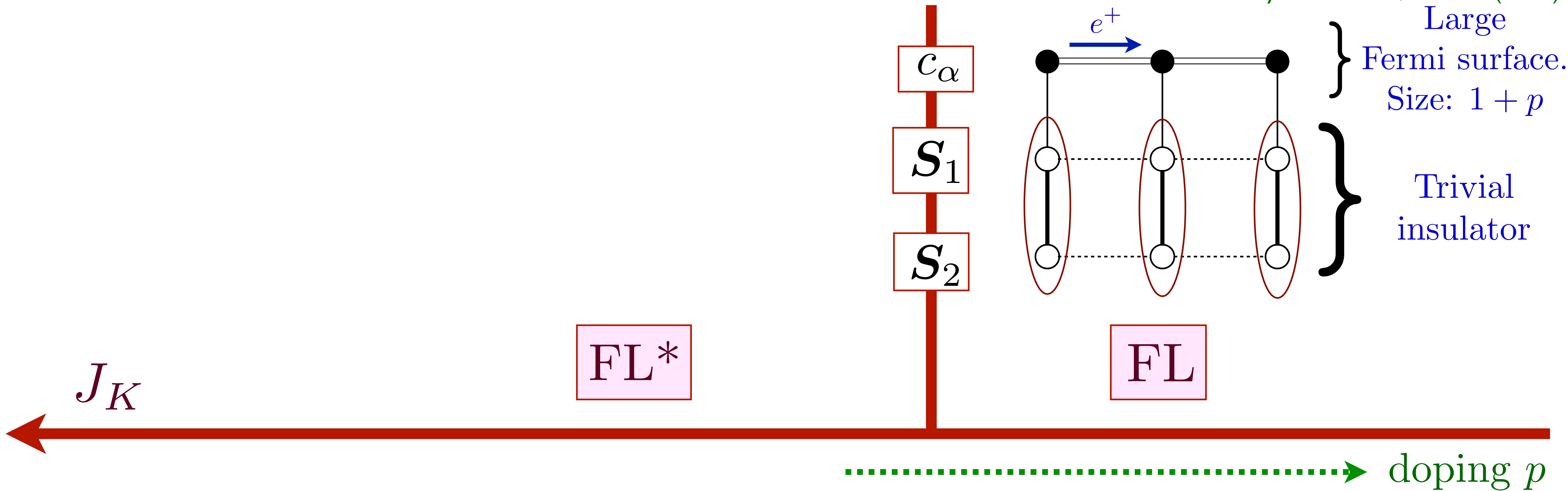


Ya-Hui Zhang

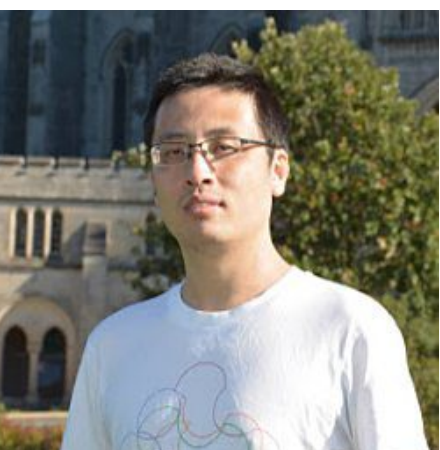


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Ya-Hui Zhang



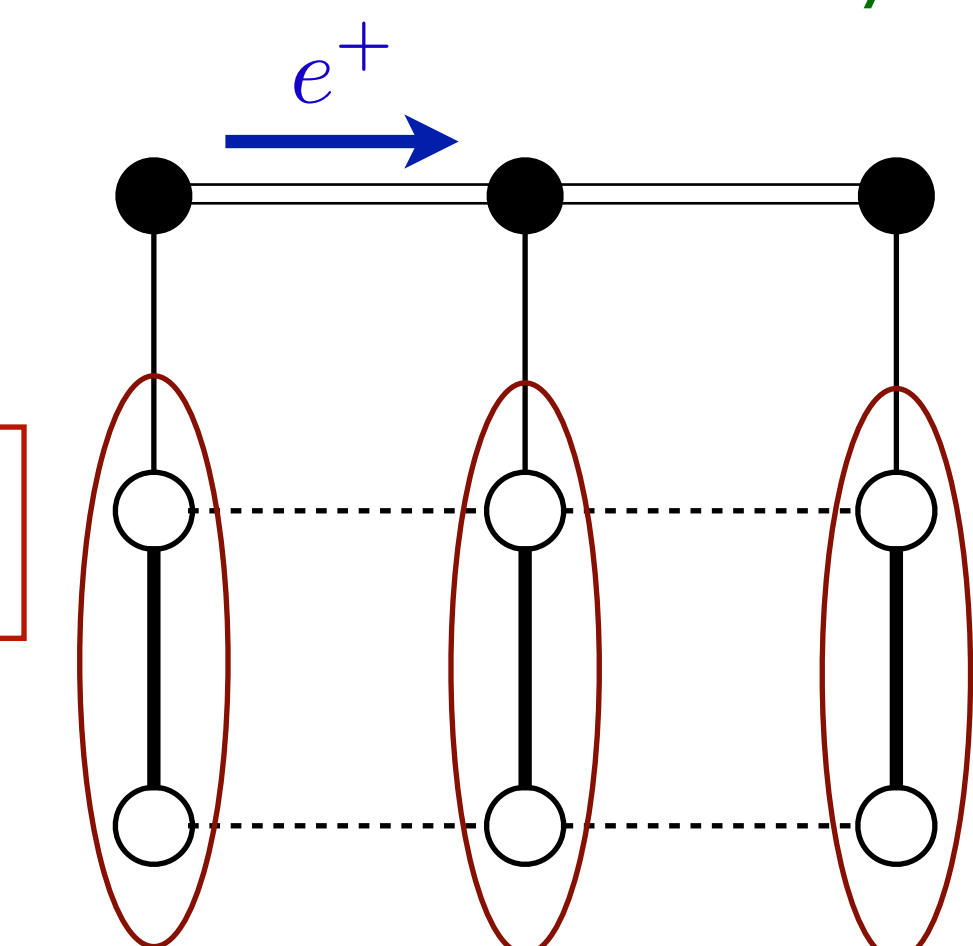
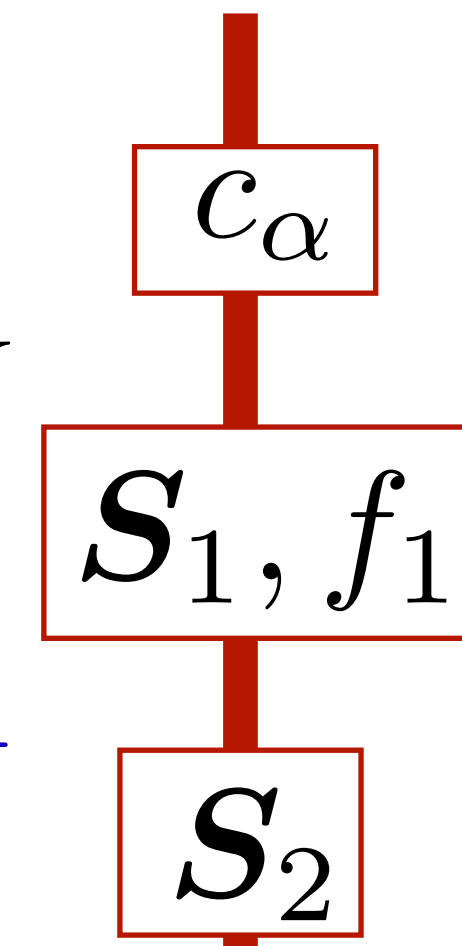
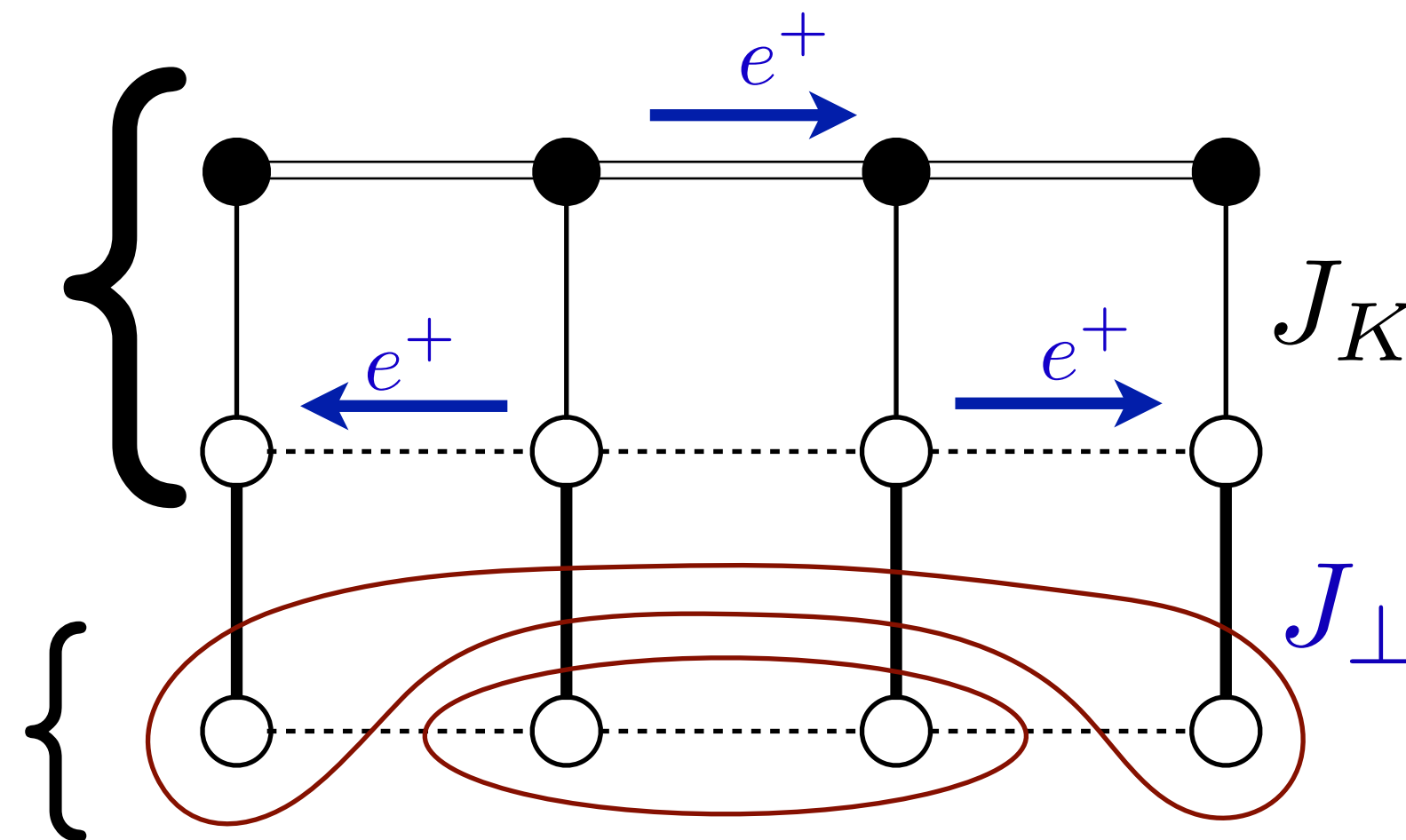
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Ya-Hui Zhang and S. Sachdev,  
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Kondo lattice heavy  
 Fermi liquid.  
 Size  $1 + p + 1$   
 $= p \pmod{2}$ .  
*Small Fermi surface!*

$$\langle \Phi \rangle \neq 0$$

Spin liquid



Large  
 Fermi surface.  
 Size:  $1 + p$

Trivial  
 insulator

FL\*

FL

$J_K$

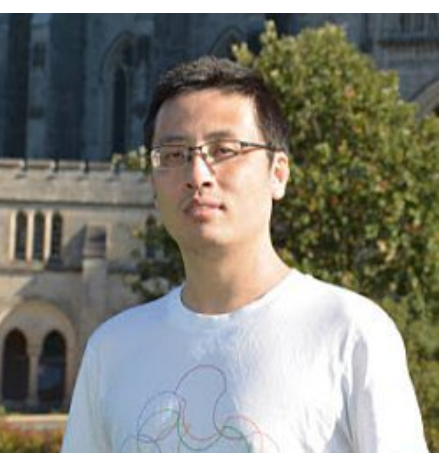
doping  $p$

Pseudogap metal =  
 Kondo Lattice Heavy  
 Fermi Liquid  
 $\oplus$   
 Spin Liquid

Fractionalized excitations of layer  $S_1$  confined  
 by condensation of Higgs boson  $\Phi \sim f_{1\alpha}^\dagger c_\alpha$ .

Fractionalized excitations of layer  $S_2$  remain deconfined

Ya-Hui  
 Zhang



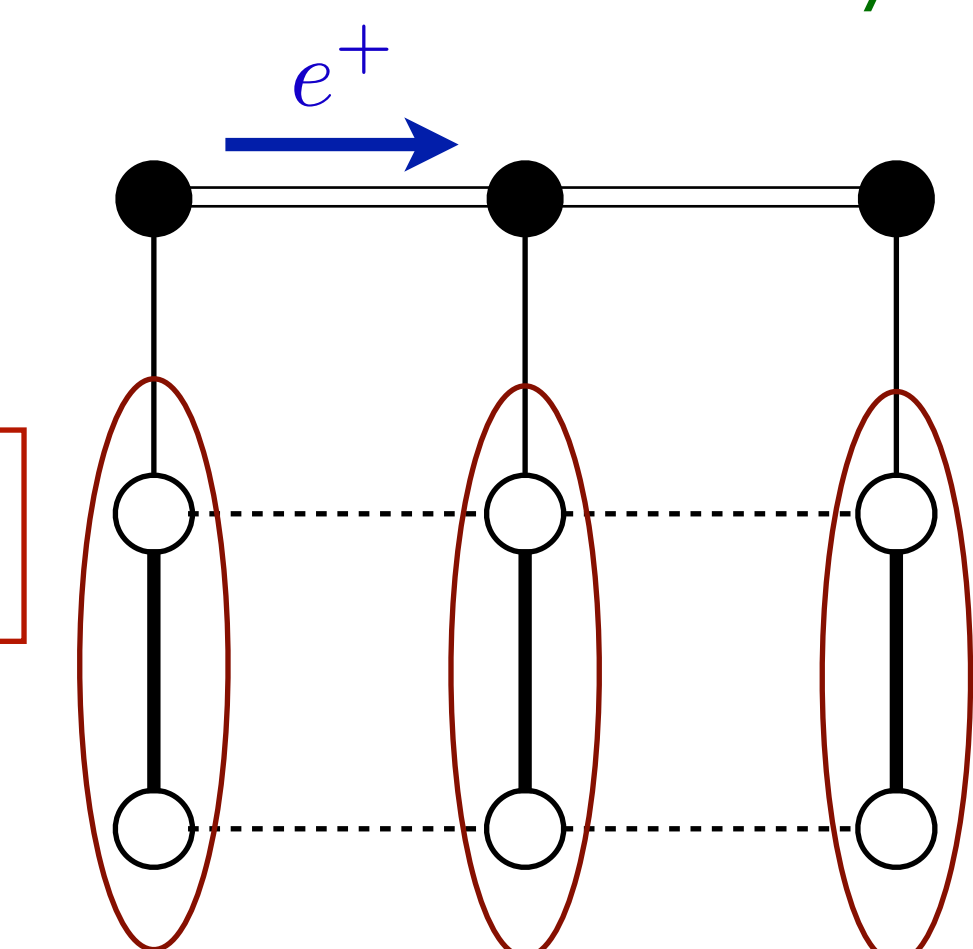
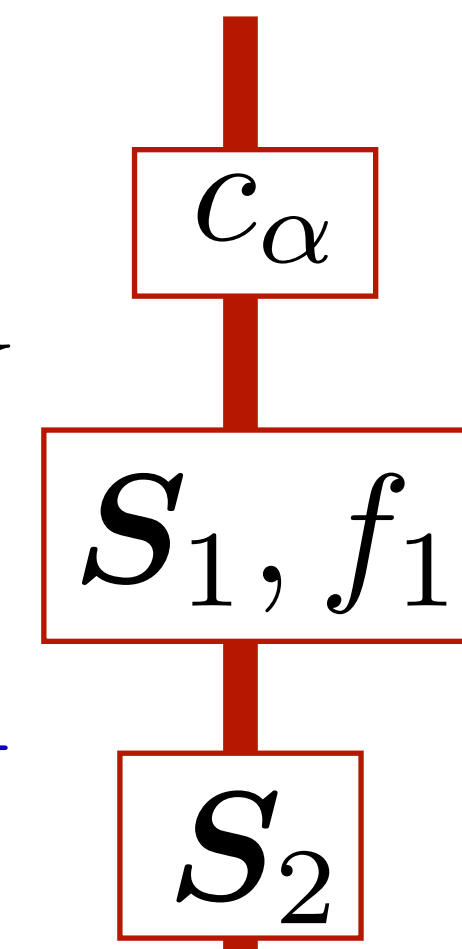
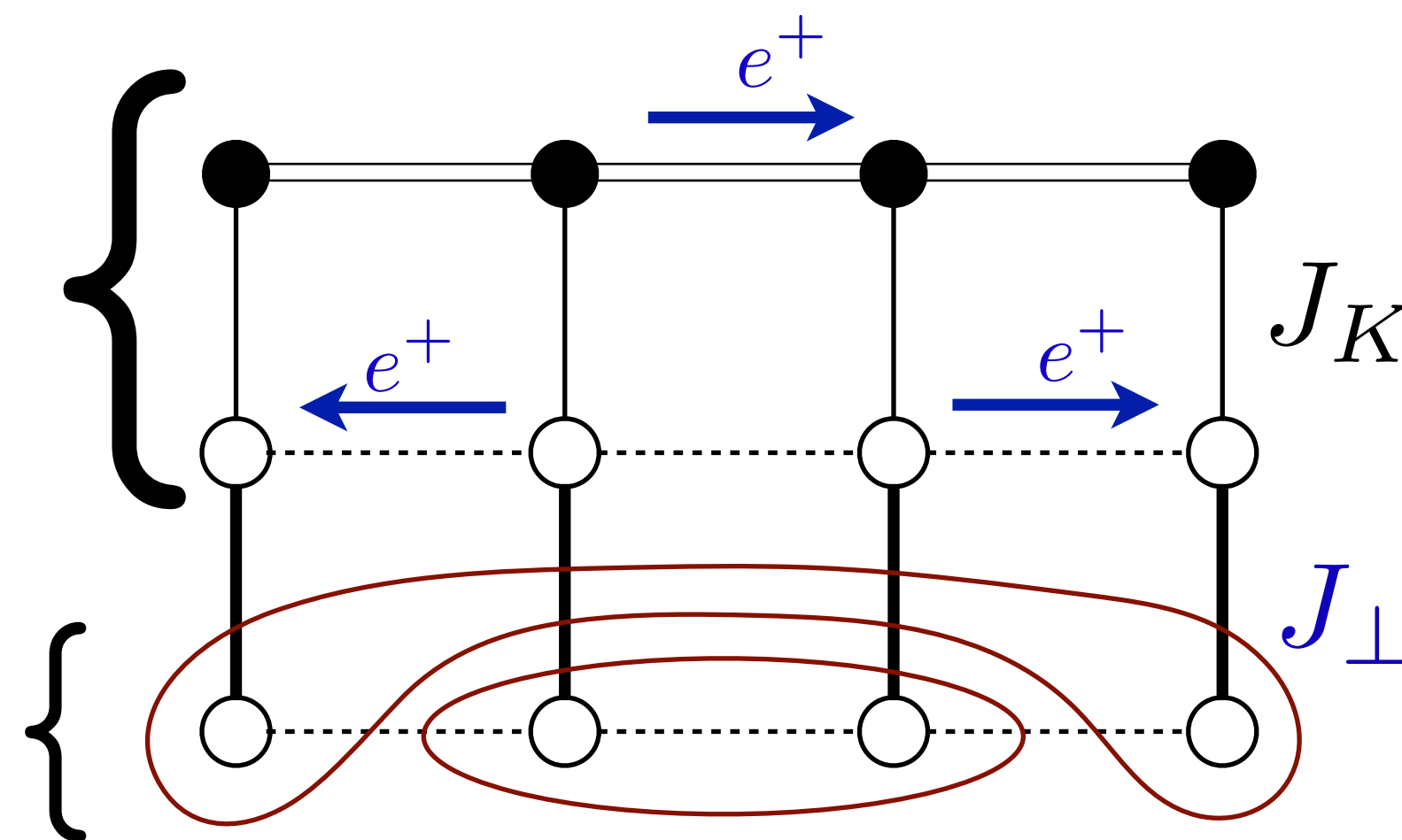
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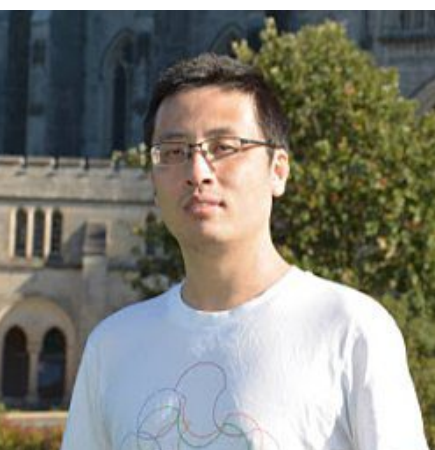
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Pseudogap metal =  
 Kondo Lattice Heavy  
 Fermi Liquid  
 $\oplus$   
 Spin Liquid

$$|FL^*\rangle = [\text{Projection onto rung singlets of } S_1, S_2] \\
\otimes |\text{Slater determinant of } (c, f_1)\rangle \\
\otimes |\text{Spin liquid of } S_2\rangle$$

Replacement for “vanilla” Gutzwiller-projected Fermi liquid in the underdoped regime

Ya-Hui  
 Zhang

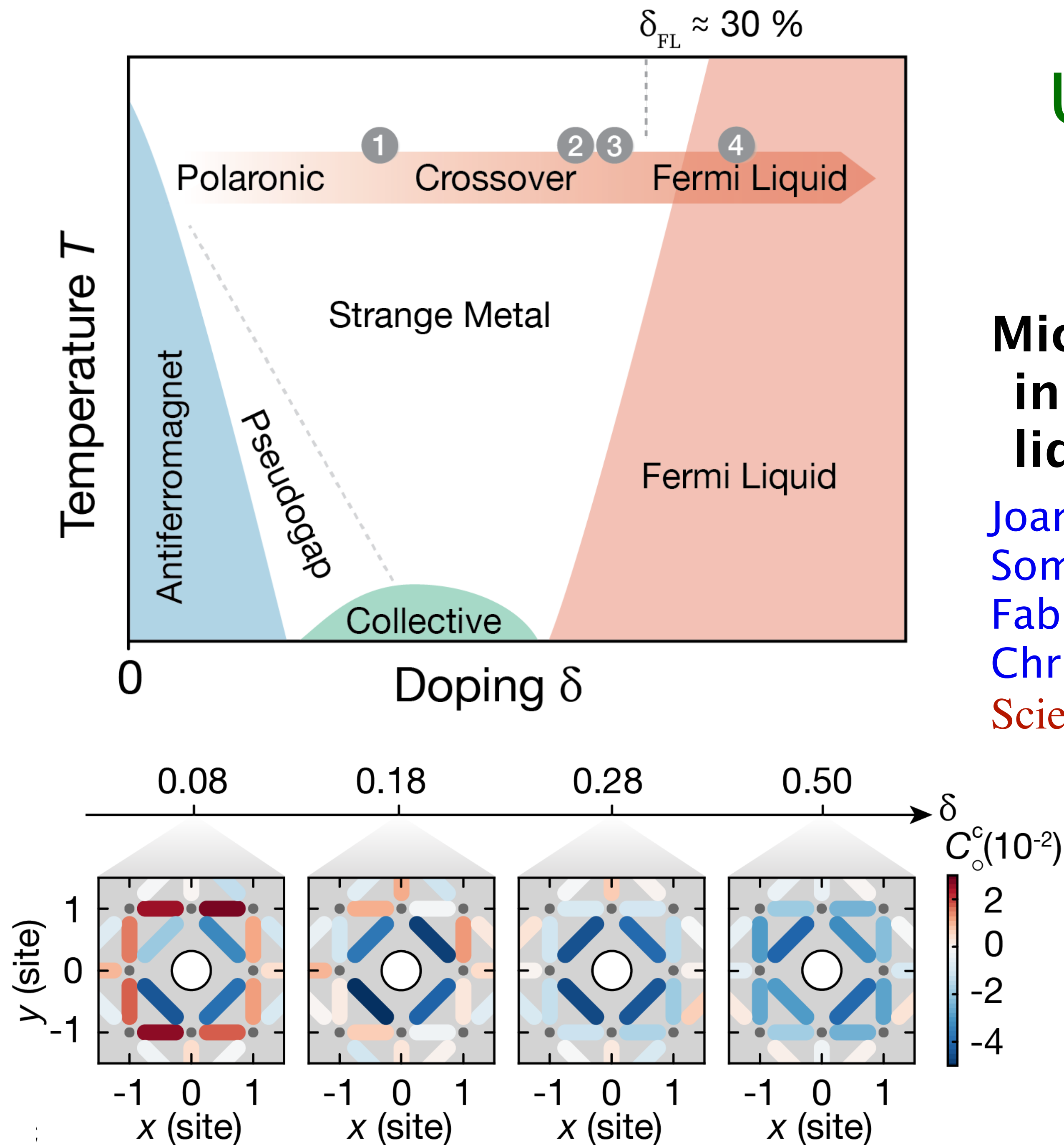


# Ultracold fermionic atoms in optical lattices

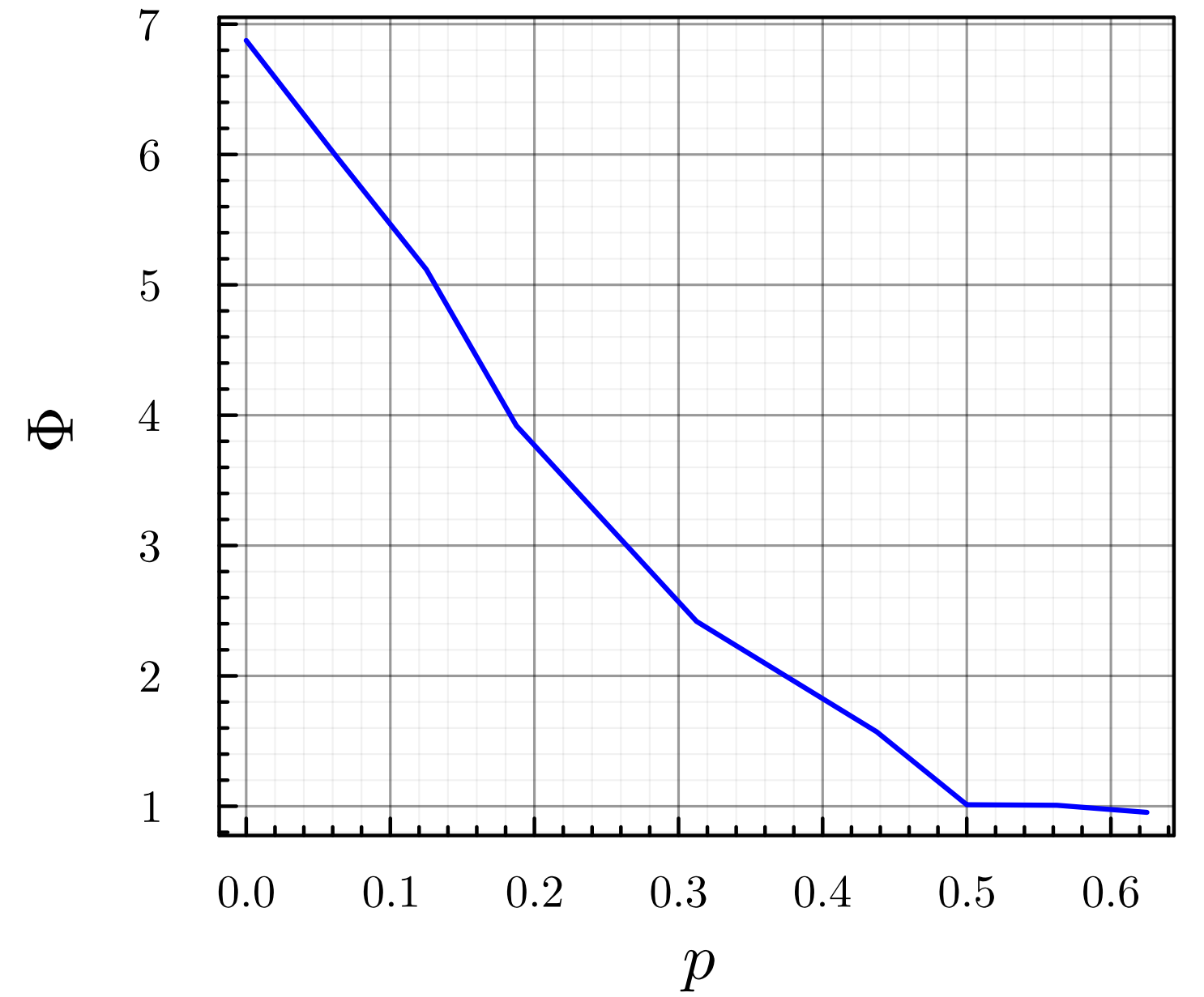
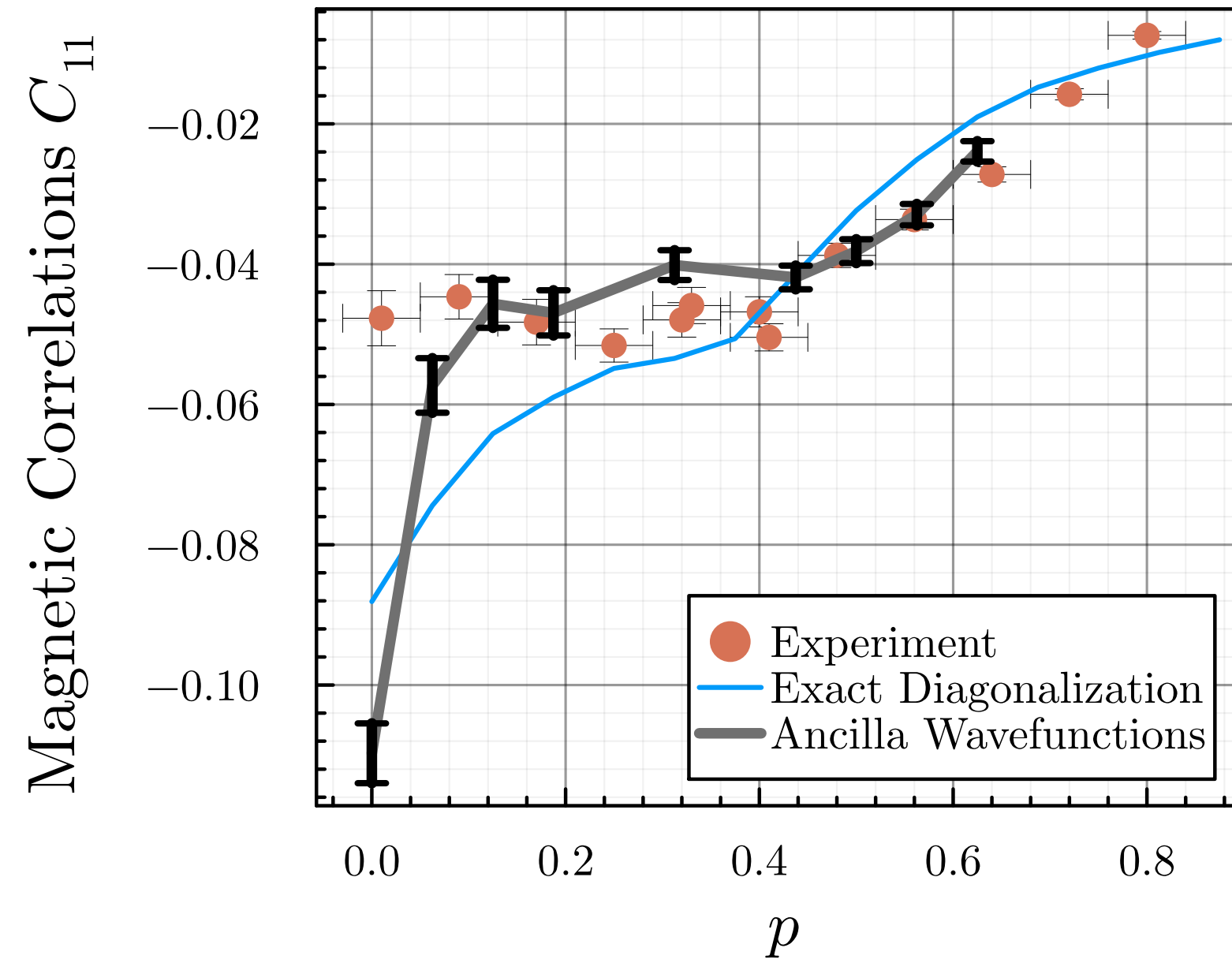
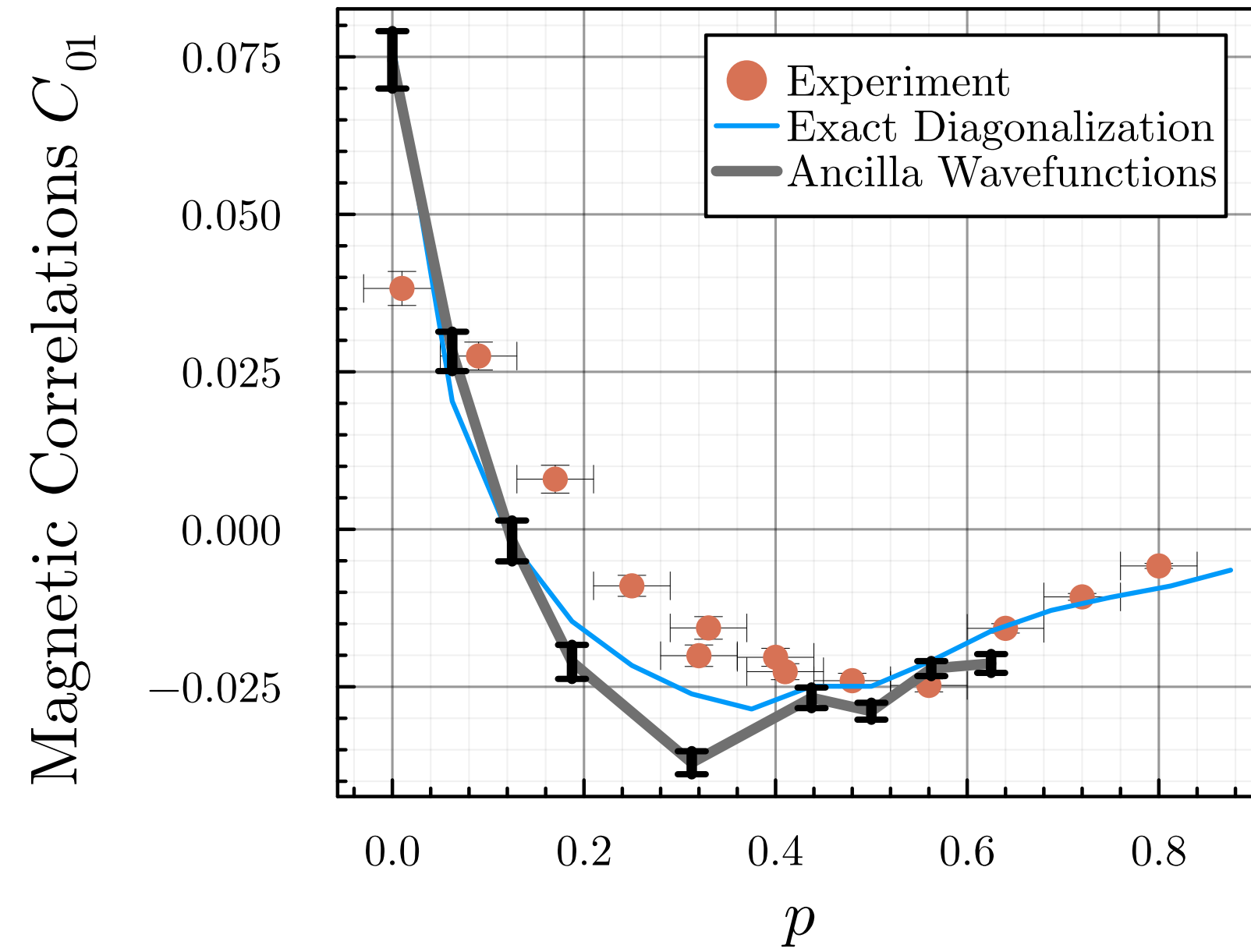
## Microscopic evolution of doped Mott insulators from polaronic metal to Fermi liquid

Joannis Koepsell, Dominik Bourgund, Pimonpan Sompet, Sarah Hirthe, Annabelle Bohrdt, Yao Wang, Fabian Grusdt, Eugene Demler, Guillaume Salomon, Christian Gross, Immanuel Bloch

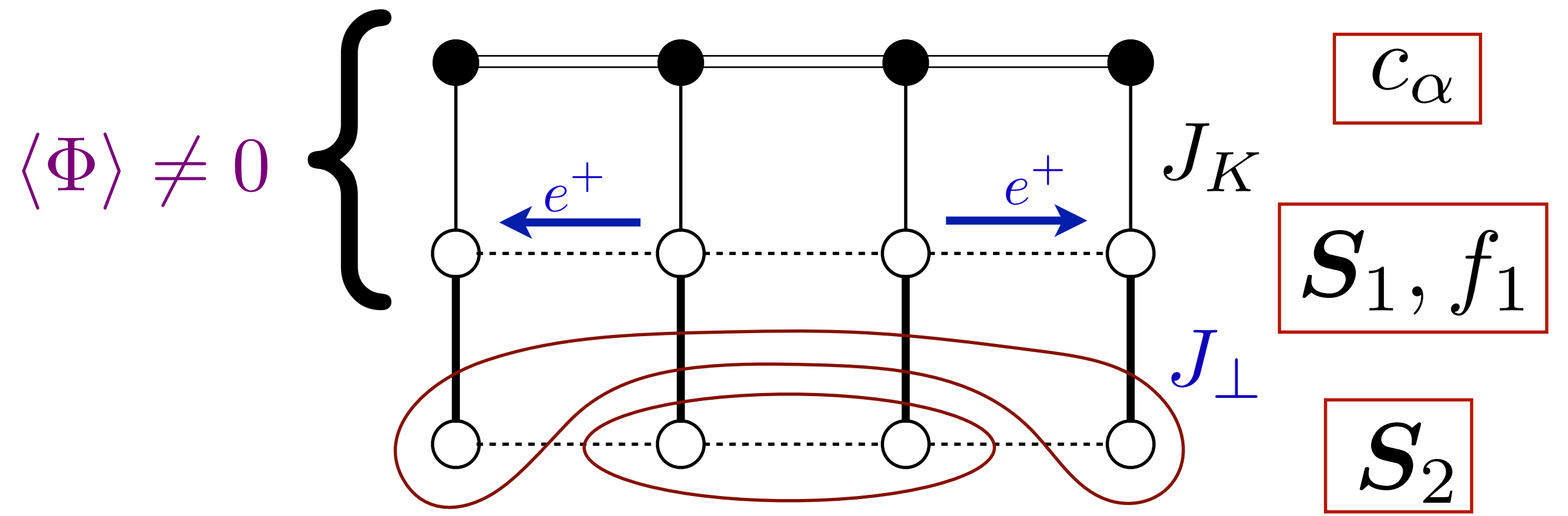
*Science* **374** (2021) 82



Max Planck Institute of  
Quantum Optics,  
Garching



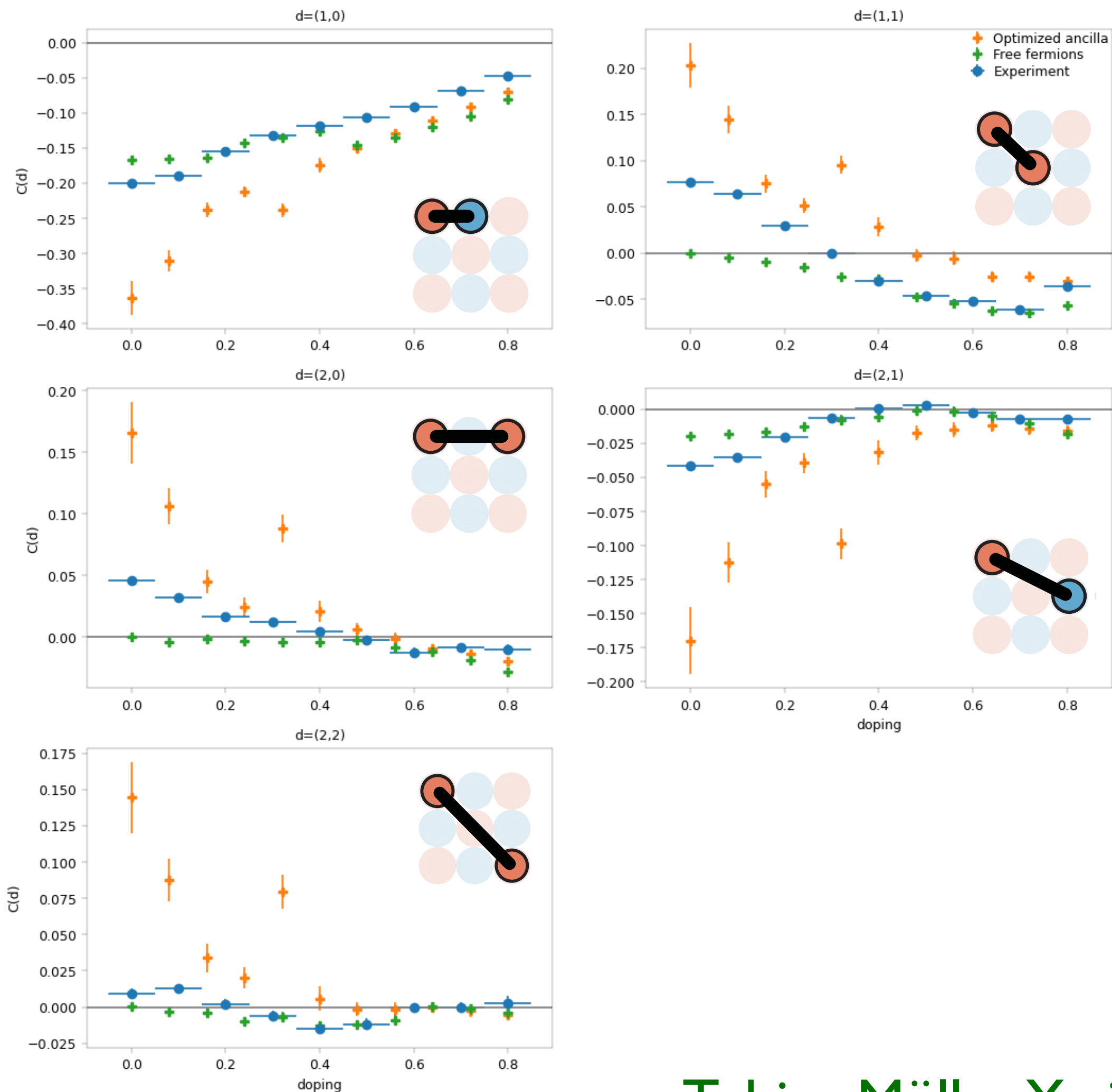
Higgs boson  $\Phi \sim f_{1\alpha}^\dagger c_\alpha$ .



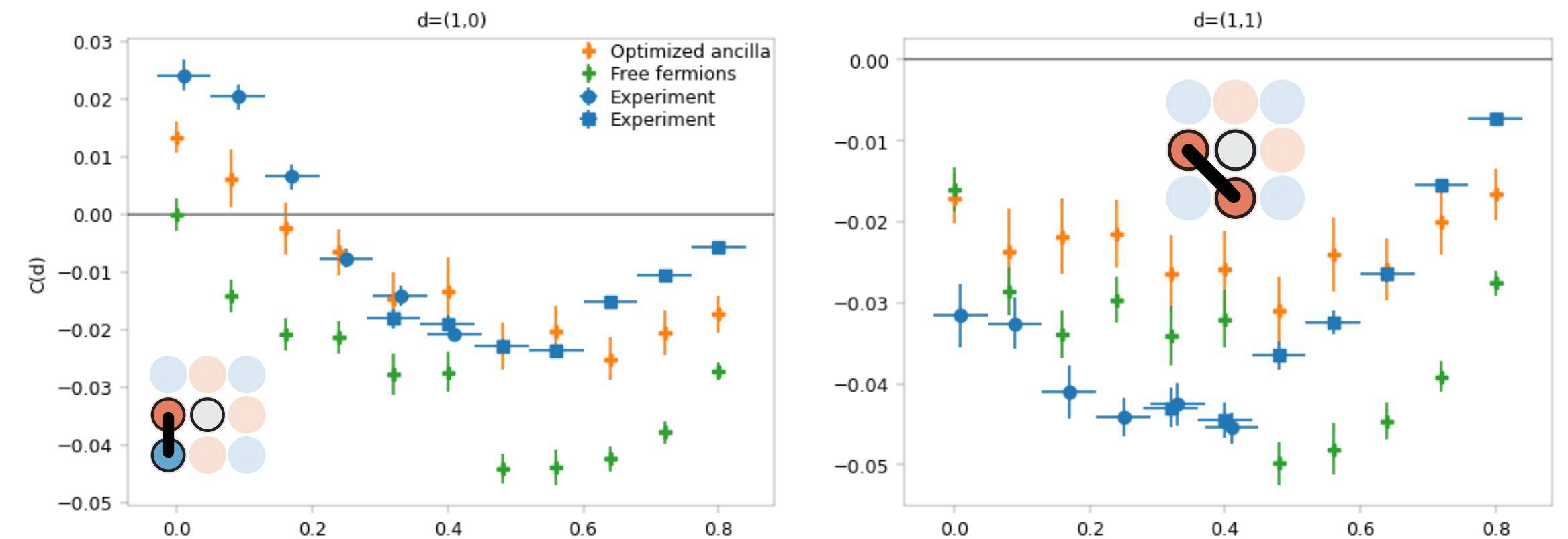
H. Shackleton and Shiwei Zhang, to appear

# Results for Connected Correlators: Comparison with experiment

## Bare Spin-Spin Correlators



## Hole-induced Spin-Spin Correlators



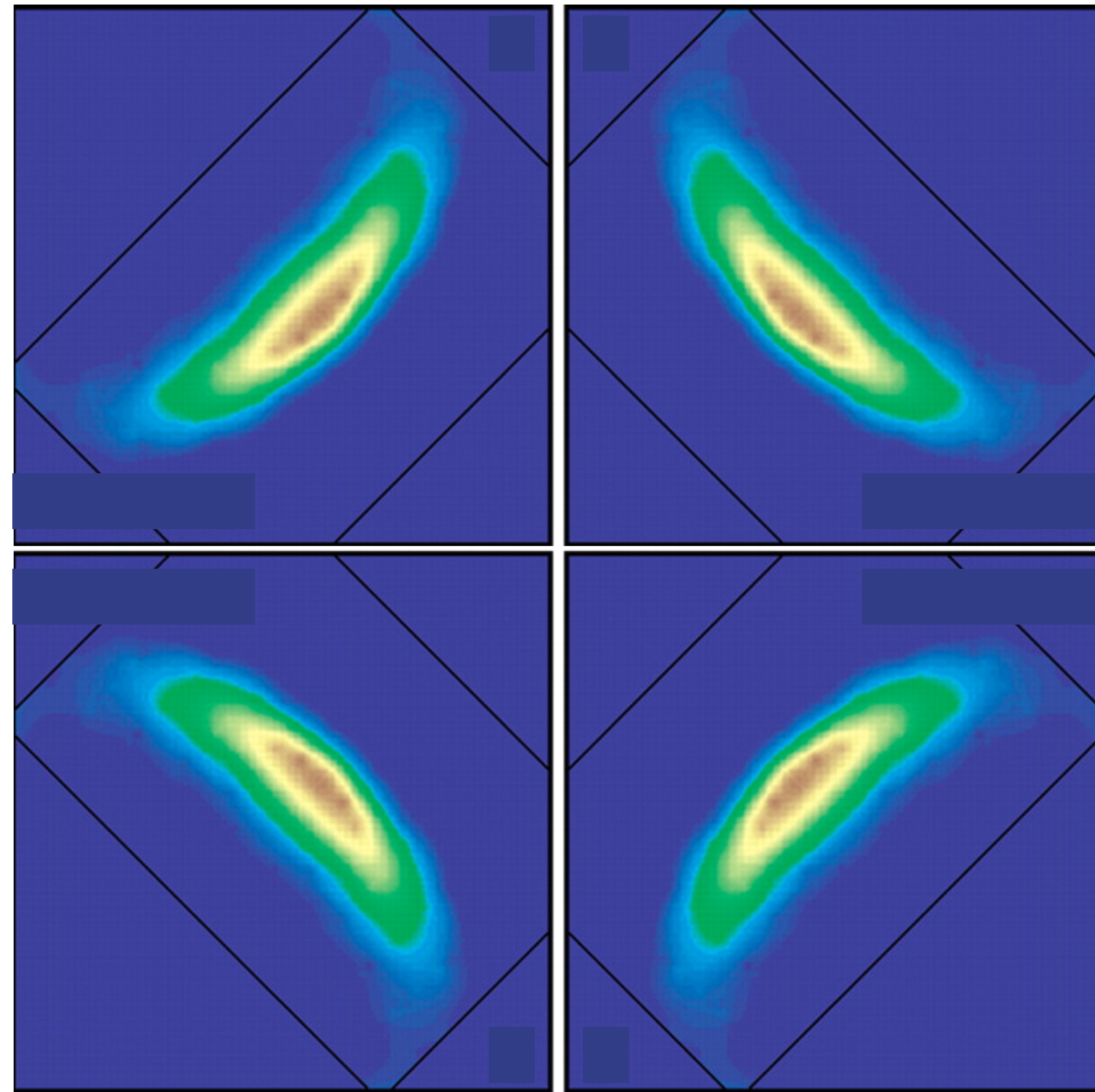
Preliminary data

- \*Test different spin-liquid ansätze for second ancilla layer
- \*Improve sampling of wave-functions
- \*Better statistics to obtain more reliable error-bars

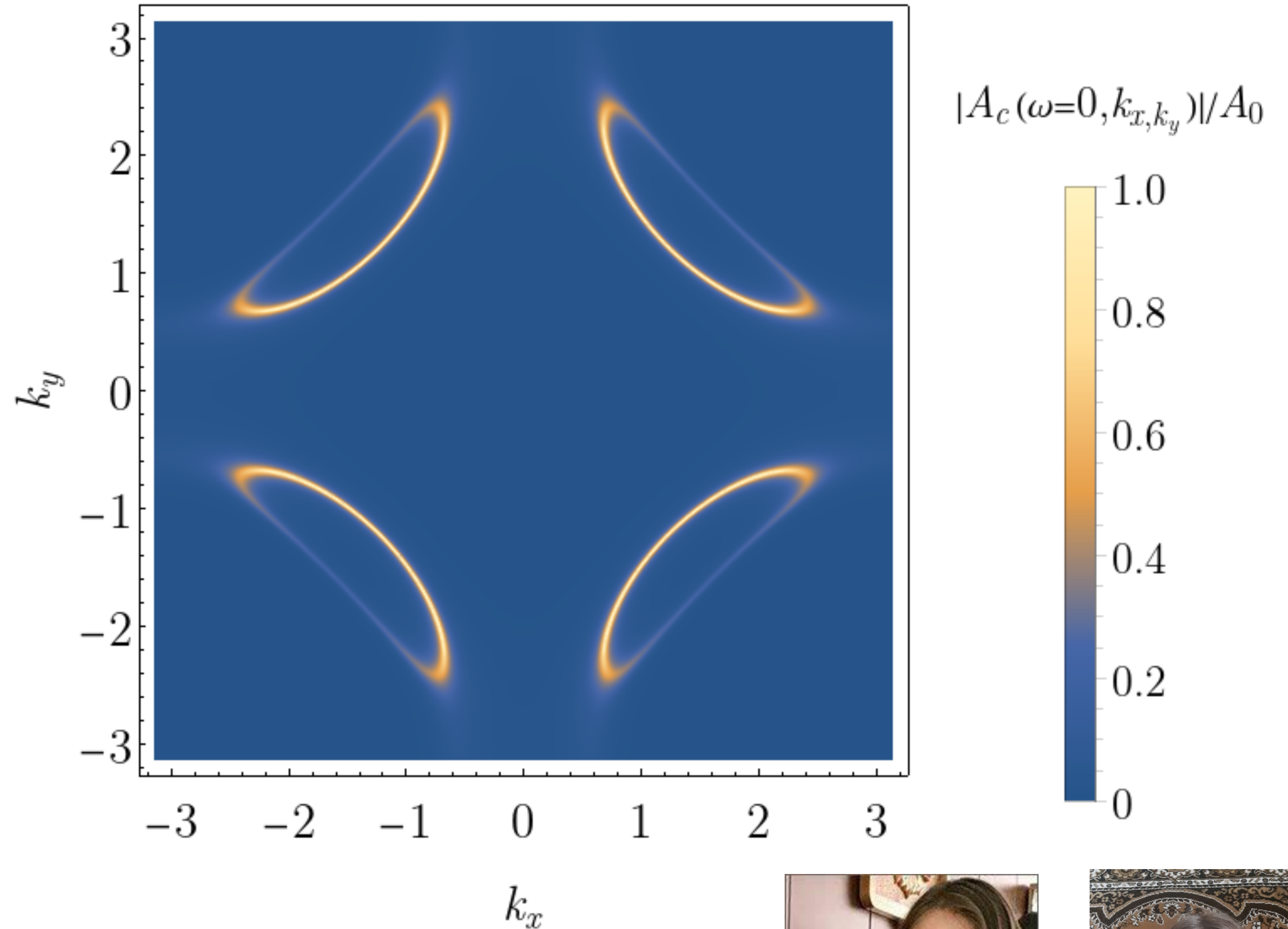
Tobias Müller, Yasir Iqbal, S.S., Ronny Thomale to appear



# Photoemission at small $p$



$\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$   
at  $x = 0.10$

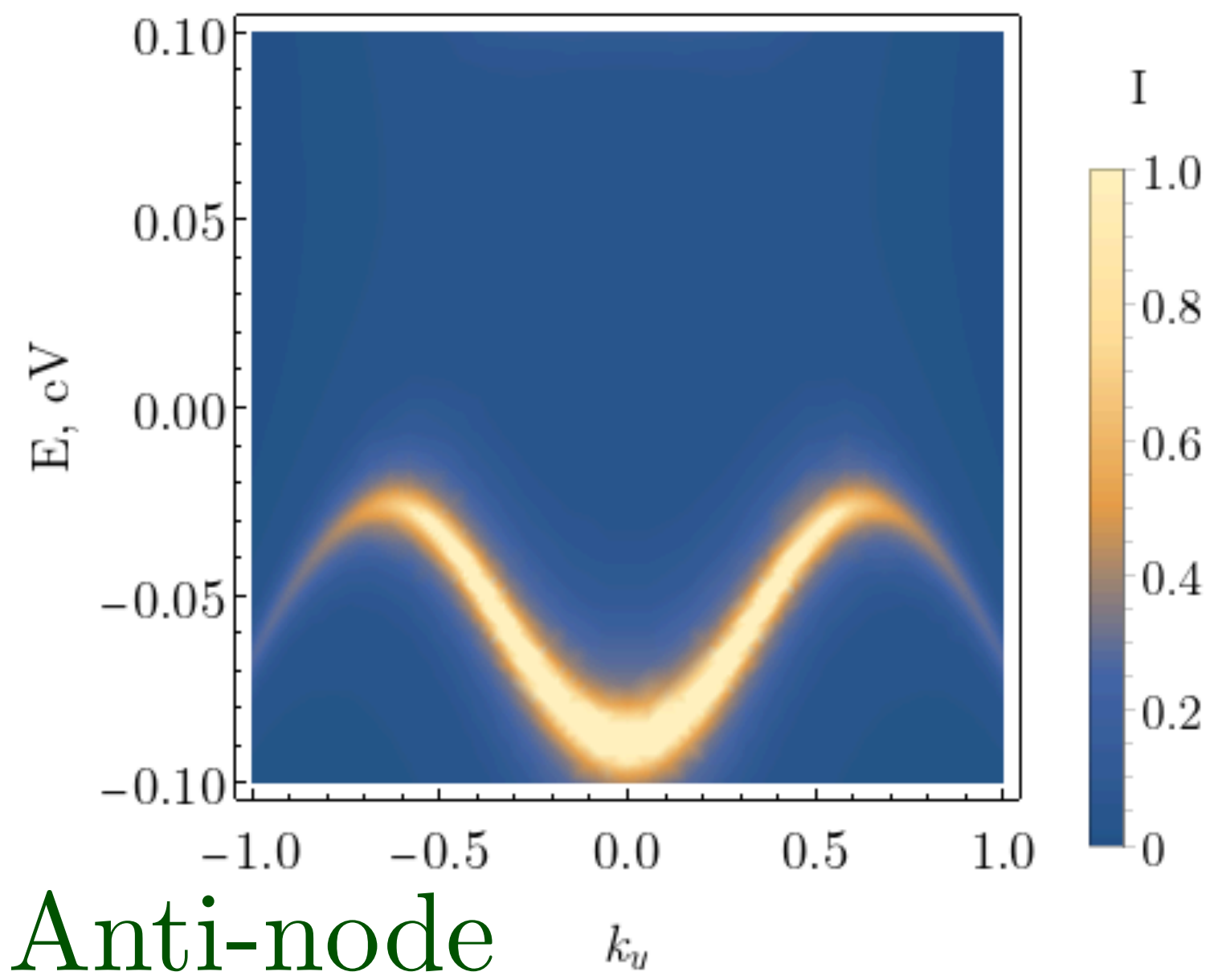


“*Fermi arcs*”

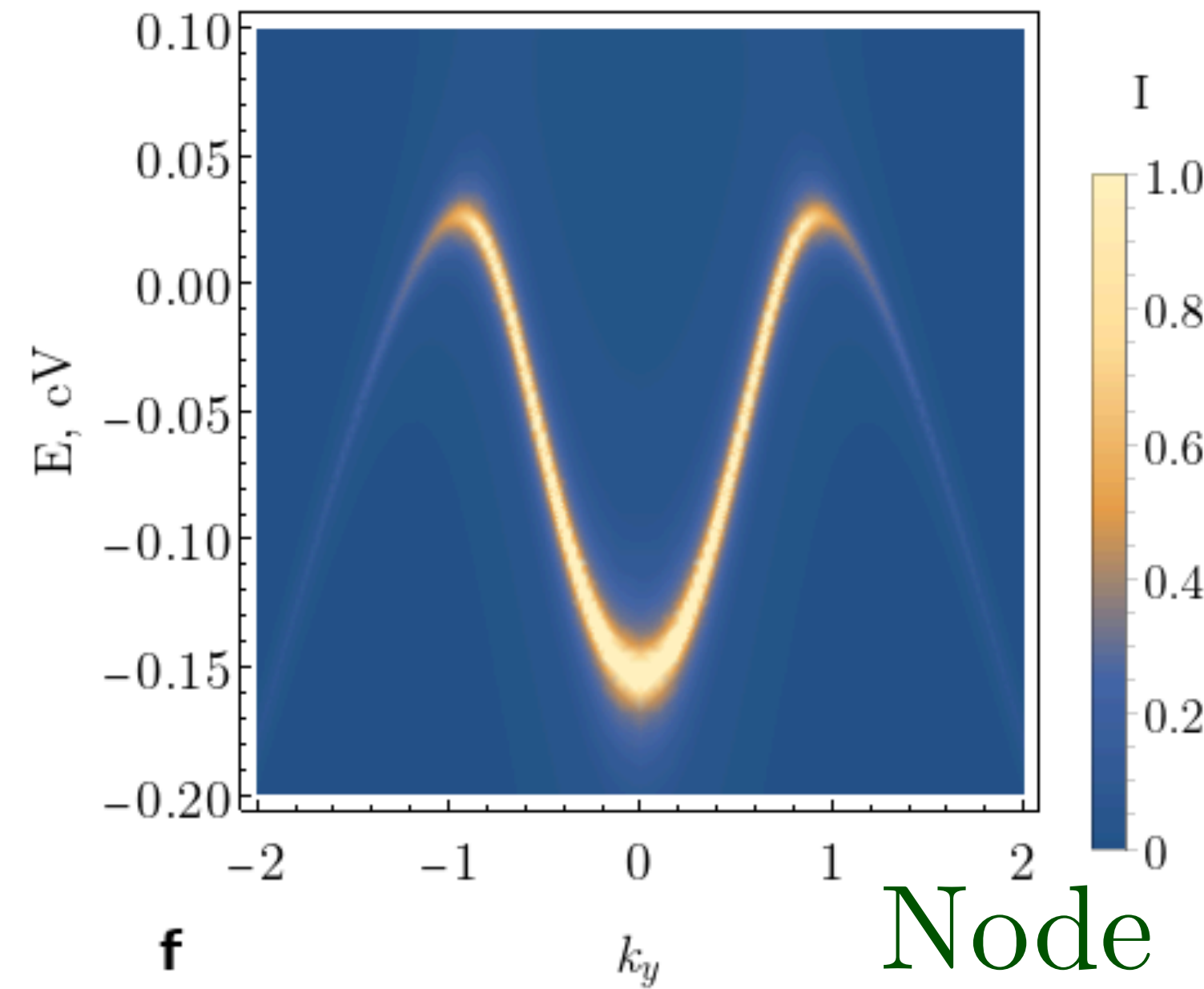


FL\* in a  
**one-band** model

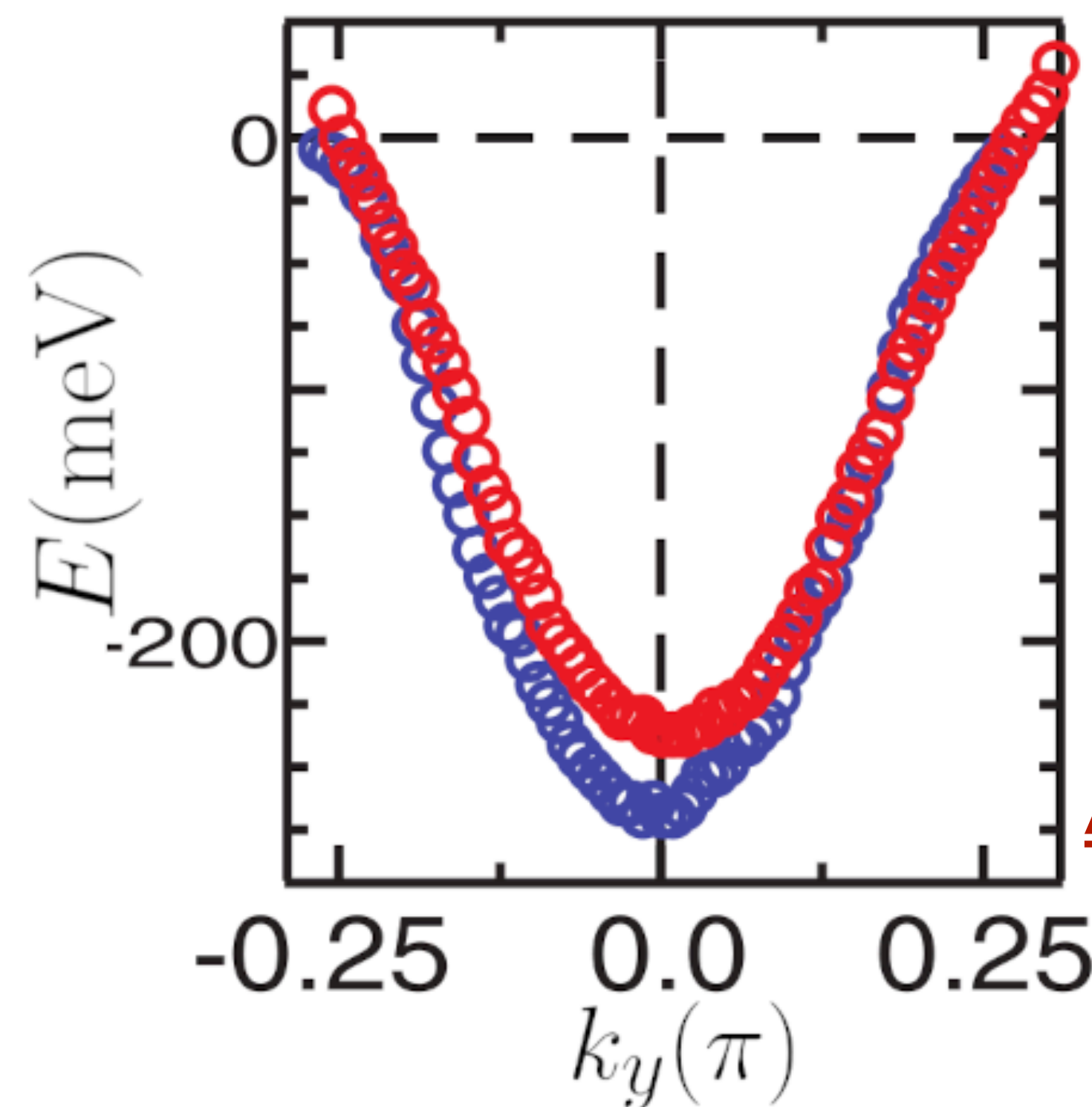
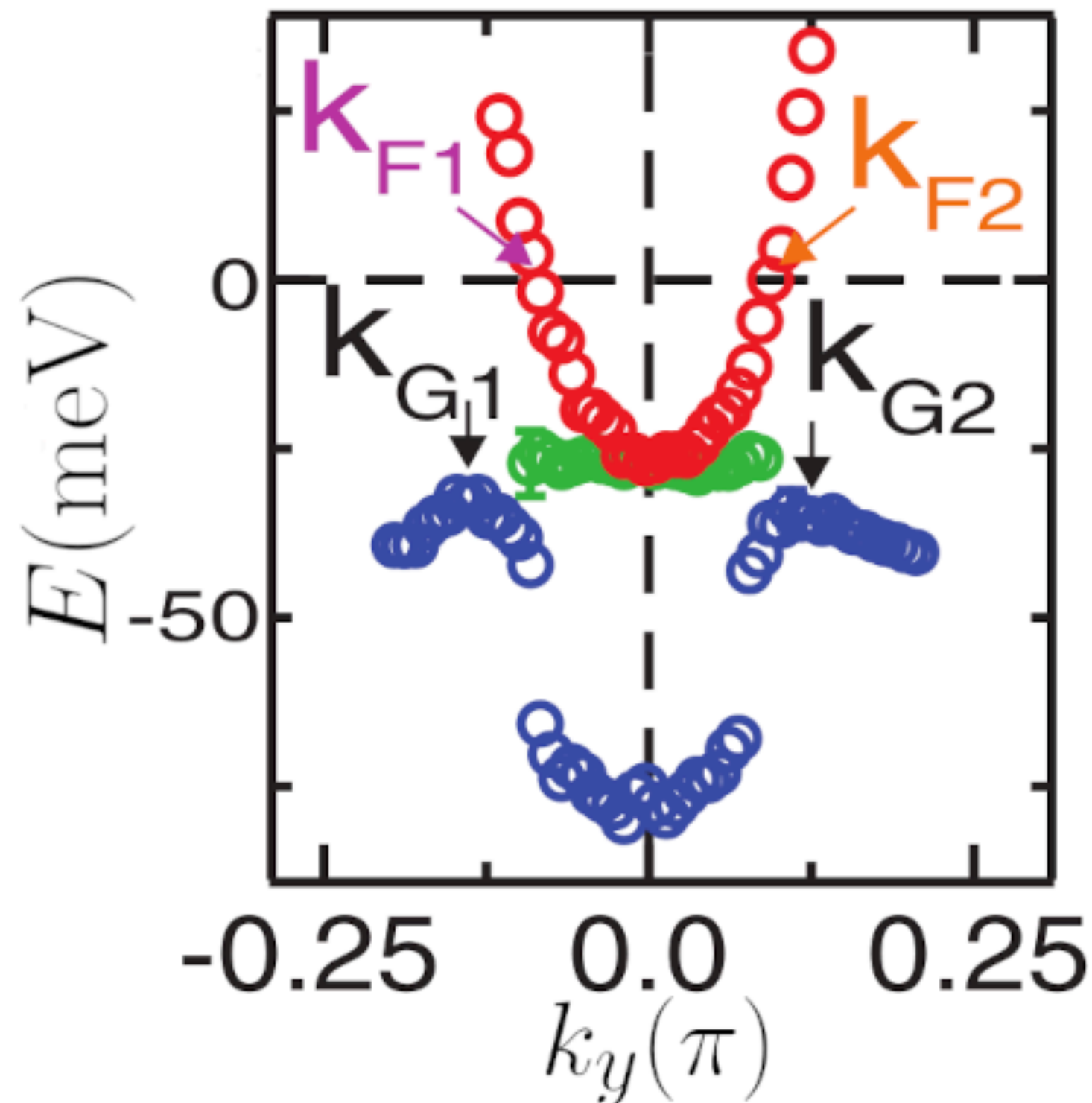
Second ancilla layer is needed  
to describe MDC and EDC



Anti-node



Node



R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

ARPES on  
Bi2201



1. Confinement transitions of the Kondo lattice

2. Ancilla theory of  $FL^*$  in a single band model

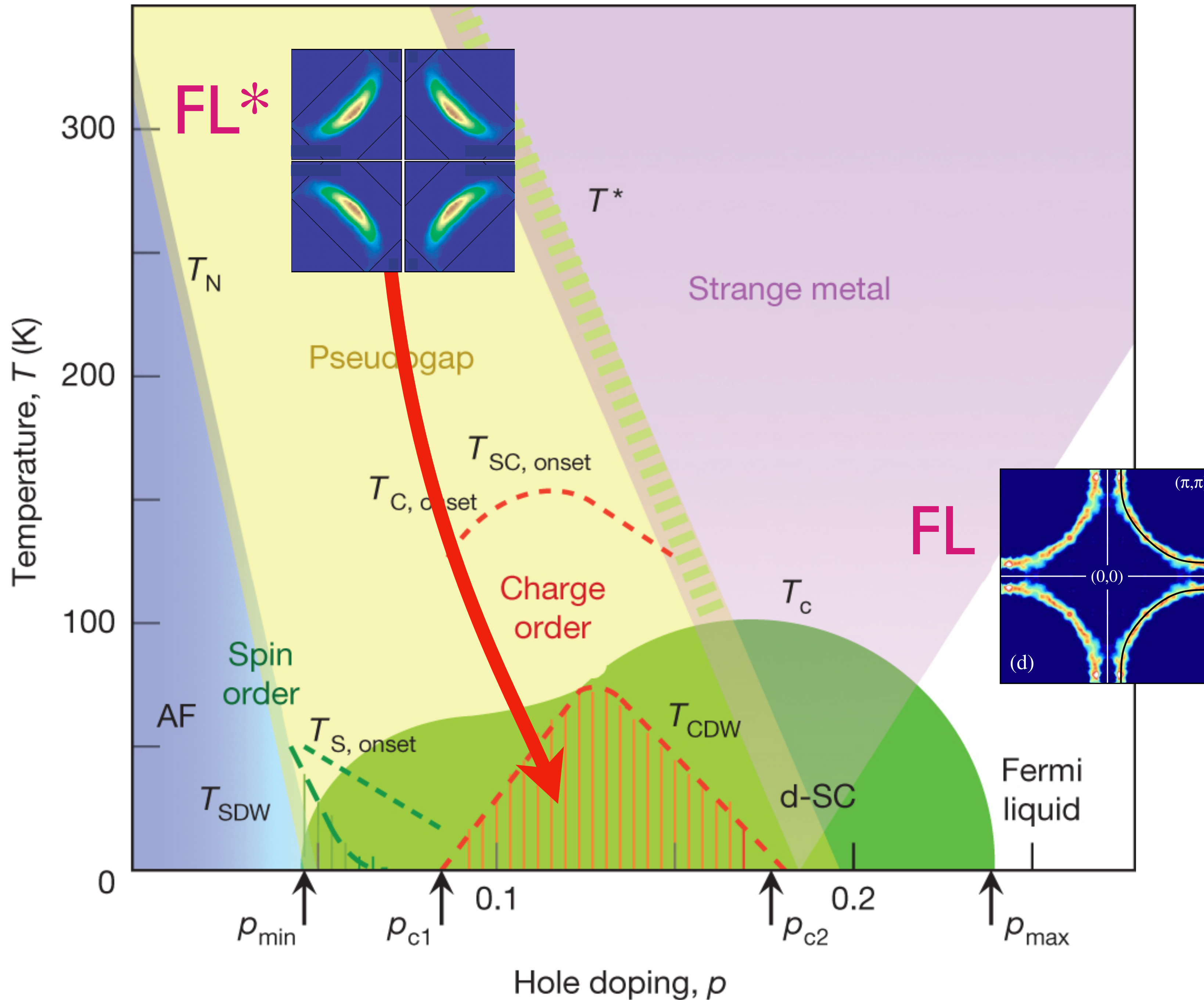
3. Confinement of  $FL^*$  with Neel-VBS DQCP

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5. Backside problem 3:

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Obtain *d*-wave superconductor and charge order from a theory of *confinement* instabilities of FL\*.

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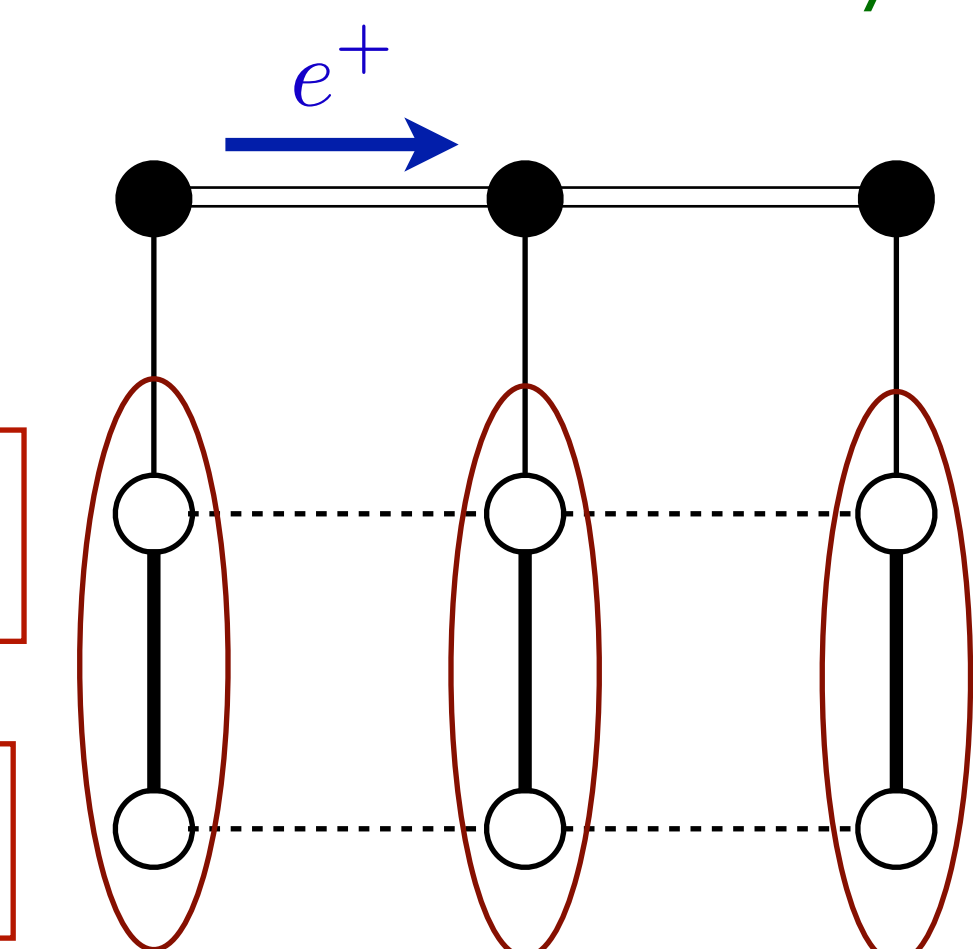
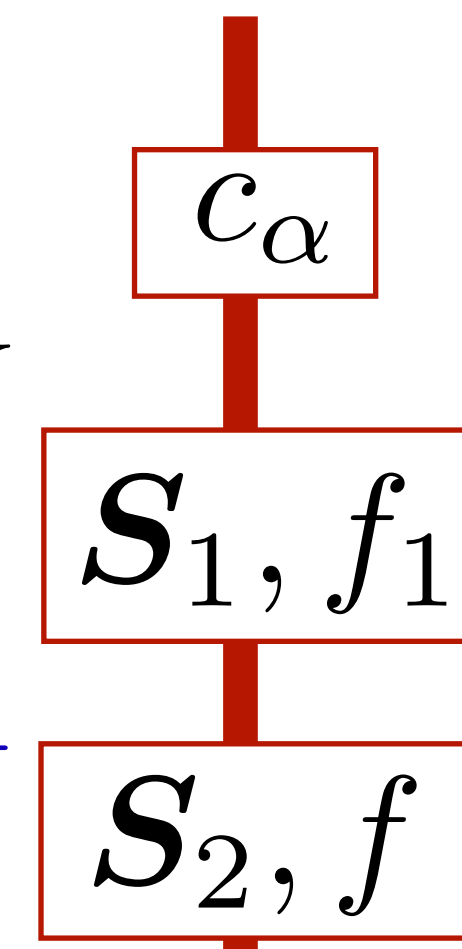
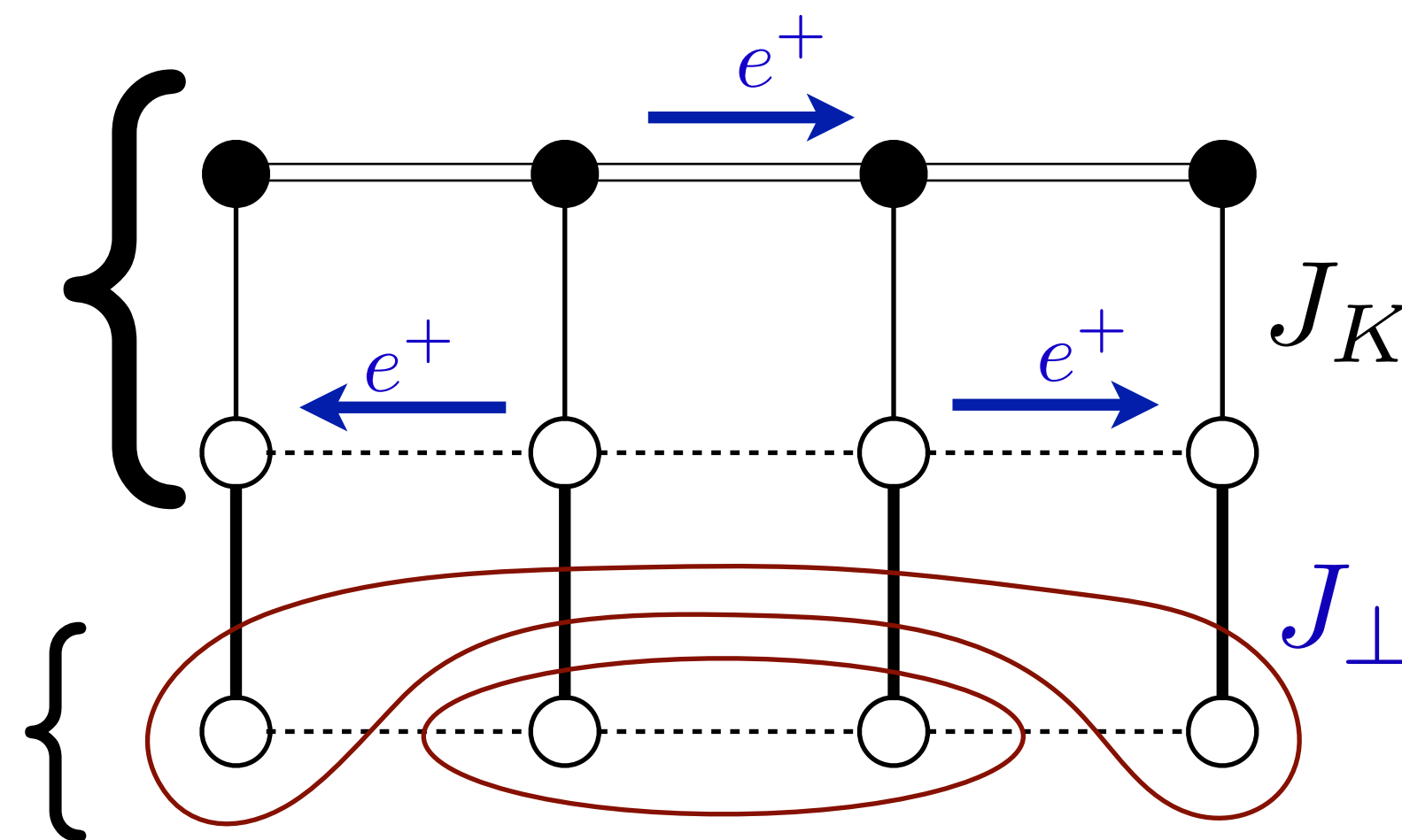
# Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev,  
*Phys. Rev. Res.* **2**, 023172 (2020)

Kondo lattice heavy  
 Fermi liquid.  
 Size  $1 + p + 1$   
 $= p \pmod{2}$ .  
 Small Fermi surface!

$$\langle \Phi \rangle \neq 0$$

Spin liquid



Large  
 Fermi surface.  
 Size:  $1 + p$

Trivial  
 insulator

FL\*

FL

$J_K$

doping  $p$

1. Choose Néel-VBS DQCP spin liquid:  $f_\alpha$  spinons moving in  $\pi$ -flux coupled to  $SU(2)_N$  gauge field. Note  $\Phi \sim f_{1\alpha}^\dagger c_\alpha$  is condensed.

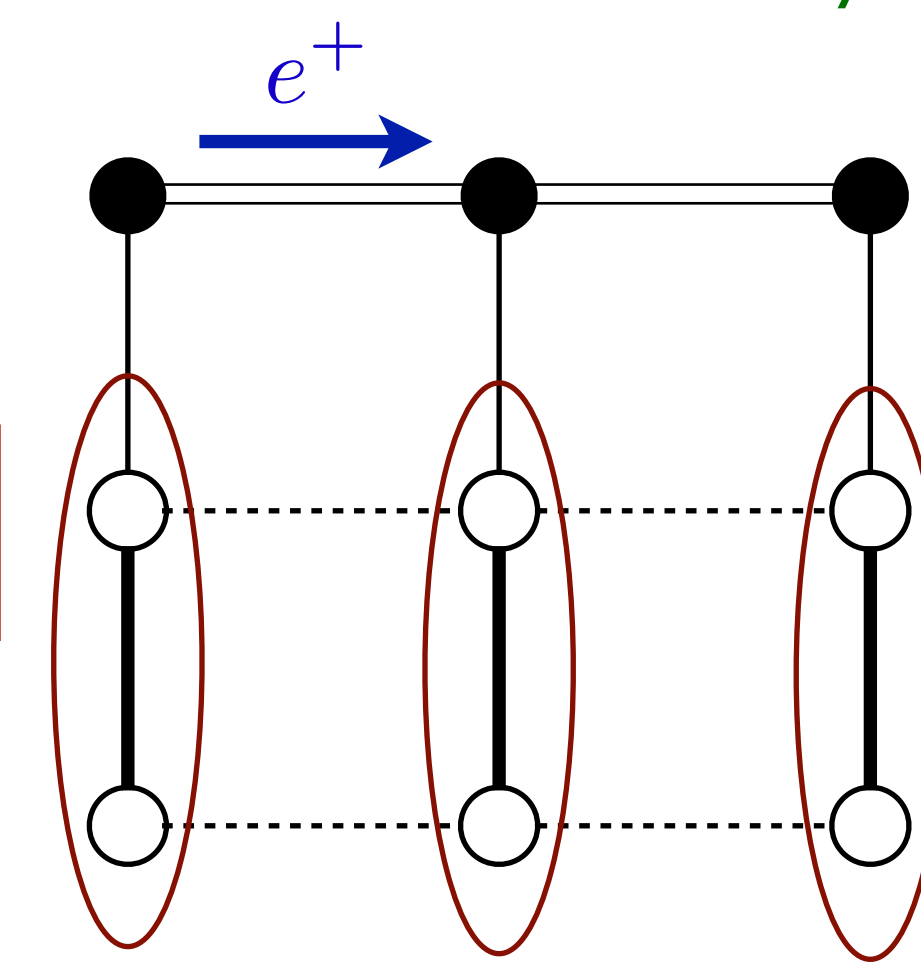
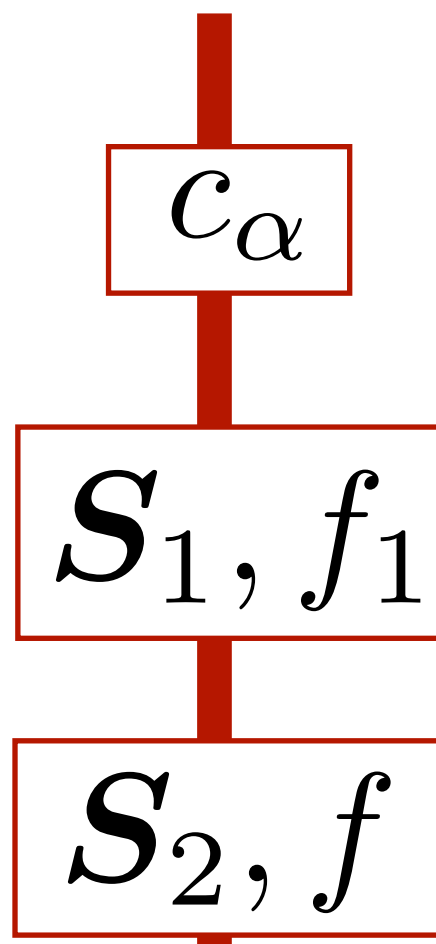
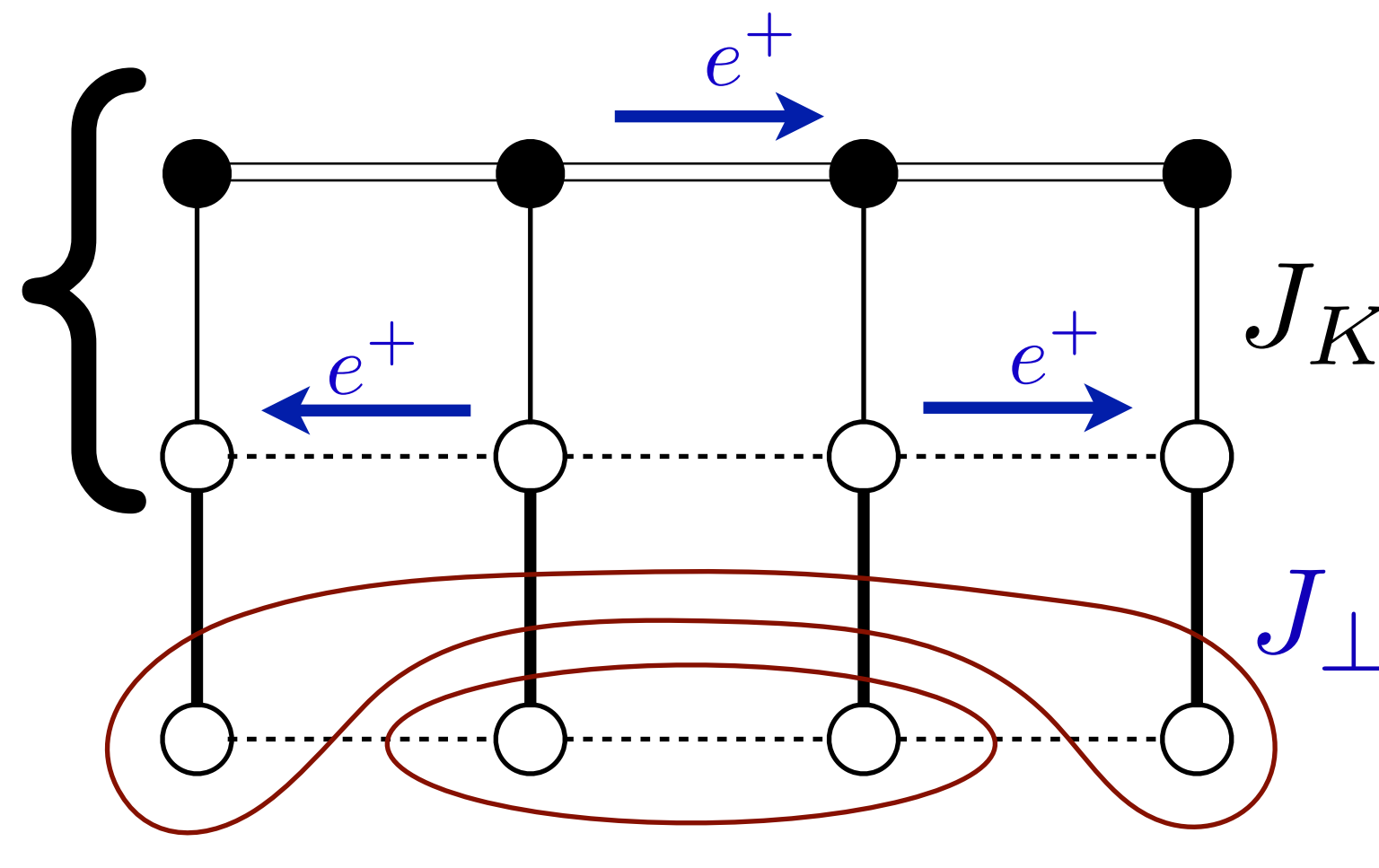
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$B$



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2. Confine spin liquid by condensing charge  $e$ ,  $SU(2)_N$  fundamental, Higgs boson

$$B \sim \begin{pmatrix} f_{1\alpha}^\dagger f_\alpha \\ \varepsilon_{\alpha\beta} f_{1\alpha}^\dagger f_\beta^\dagger \end{pmatrix}$$

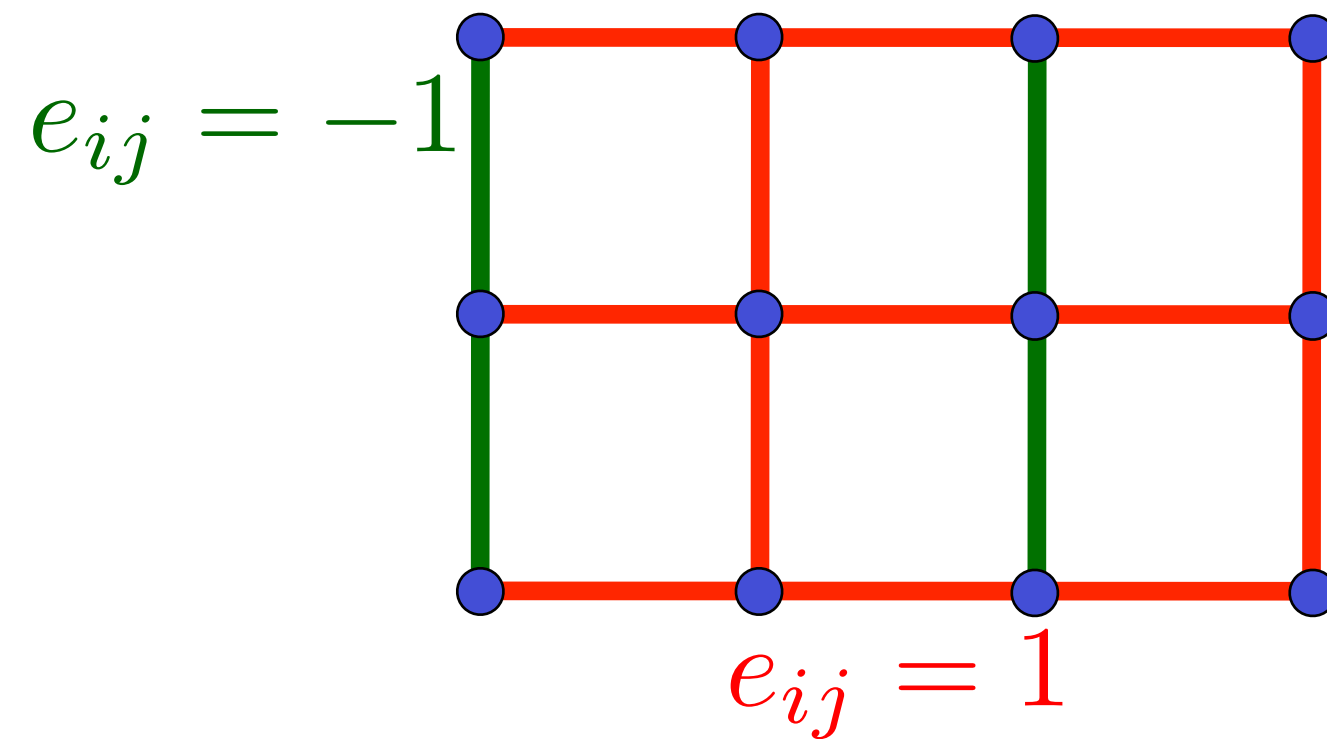
Boson with same quantum numbers in X.-G. Wen and P.A. Lee, PRL **76**, 503 (1996)

# Confinement of $SU(2)_N$ gauge theory by charge fluctuations

- Begin with the  $\pi$ -flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left( f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right) = iJ \sum_{\langle ij \rangle} e_{ij} \left( \Psi_i^\dagger U_{ij} \Psi_j - \Psi_j^\dagger U_{ji} \Psi_i \right); \quad \Psi_i = \begin{pmatrix} f_{i\uparrow}^\dagger \\ f_{i\downarrow}^\dagger \end{pmatrix}$$

$H_f$  is invariant under  $SU(2)$  rotations in spin and  $SU(2)_N$  rotations in Nambu space;  $U_{ij}$  is the  $SU(2)_N$  gauge field.

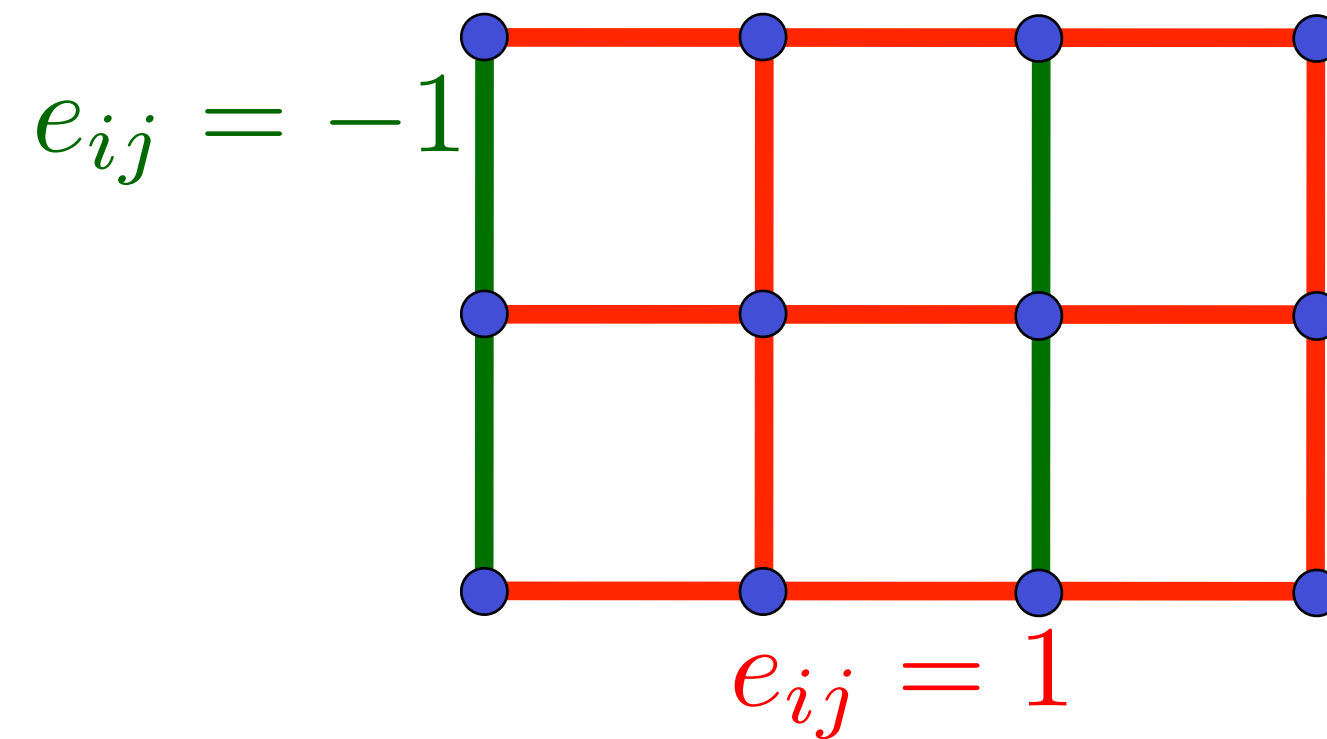


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- Introduce a charge  $e$ ,  $SU(2)_N$  fundamental boson  $B_i$  such that the composite of  $B_i$  and  $\Psi_i$  is an electron. The projective symmetries require

$$H_B = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left( B_i^\dagger U_{ij} B_j - B_j^\dagger U_{ji} B_i \right) + \dots$$

# Confinement of $SU(2)_N$ gauge theory by charge fluctuations

$$\mathcal{L}(B) = H_B + \frac{u}{2} \sum_i \rho_i^2 + V_1 \sum_i \rho_i (\rho_{i+\hat{x}} + \rho_{i+\hat{y}}) + g \sum_{\langle ij \rangle} |\Delta_{ij}|^2$$

$$+ J_1 \sum_{\langle ij \rangle} Q_{ij}^2 + K_1 \sum_{\langle ij \rangle} J_{ij}^2.$$

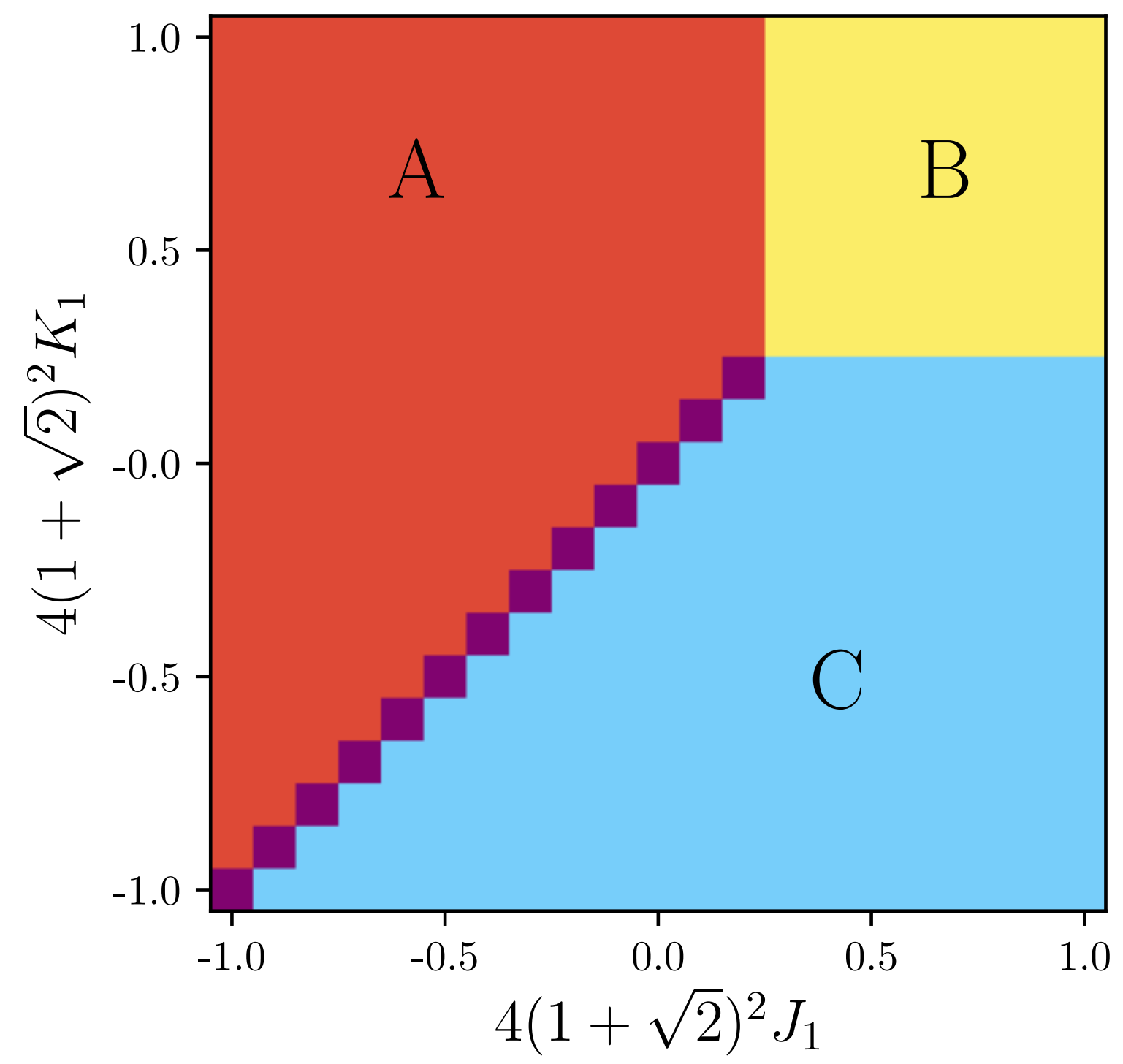
site charge density:  $\langle c_{i\alpha}^\dagger c_{i\alpha} \rangle \sim \rho_i = B_i^\dagger B_i$

bond density:  $\langle c_{i\alpha}^\dagger c_{j\alpha} + c_{j\alpha}^\dagger c_{i\alpha} \rangle \sim Q_{ij} = Q_{ji} = \text{Im} \left( B_i^\dagger e_{ij} U_{ij} B_j \right)$

bond current:  $i \langle c_{i\alpha}^\dagger c_{j\alpha} - c_{j\alpha}^\dagger c_{i\alpha} \rangle \sim J_{ij} = -J_{ji} = \text{Re} \left( B_i^\dagger e_{ij} U_{ij} B_j \right)$

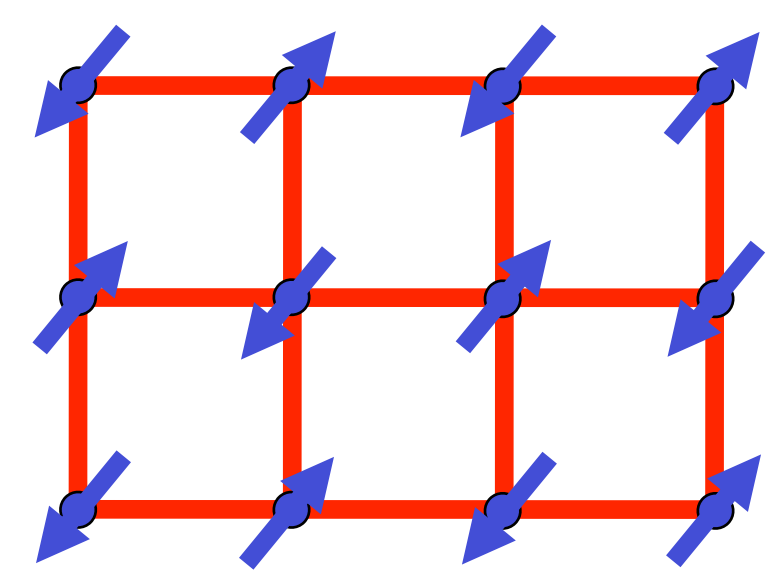
Pairing:  $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim \Delta_{ij} = \Delta_{ji} = \varepsilon_{ab} B_{ai} e_{ij} U_{ij} B_{bj}.$

# Global phase diagram of $SU(2)_N$ gauge theory

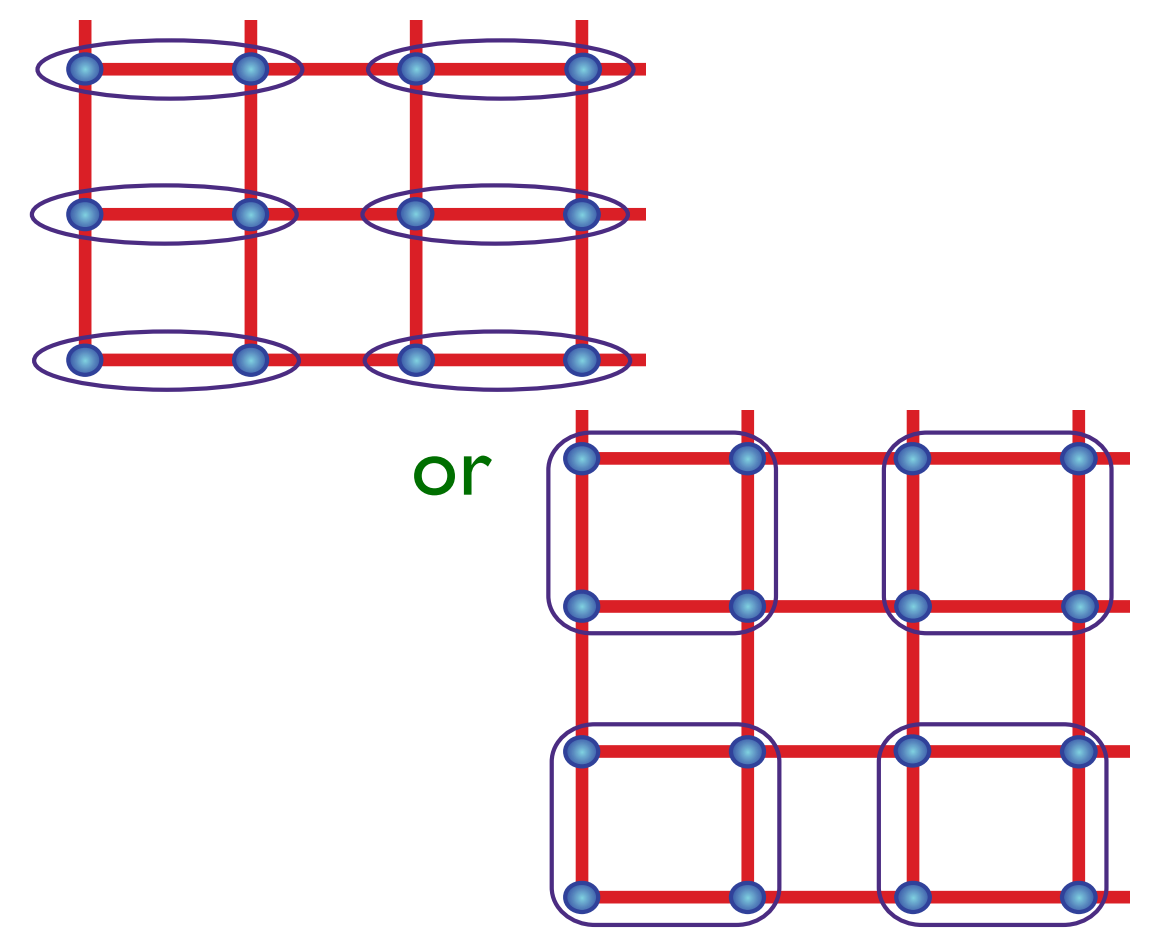


$\langle B \rangle \neq 0$

$\langle B \rangle = 0$



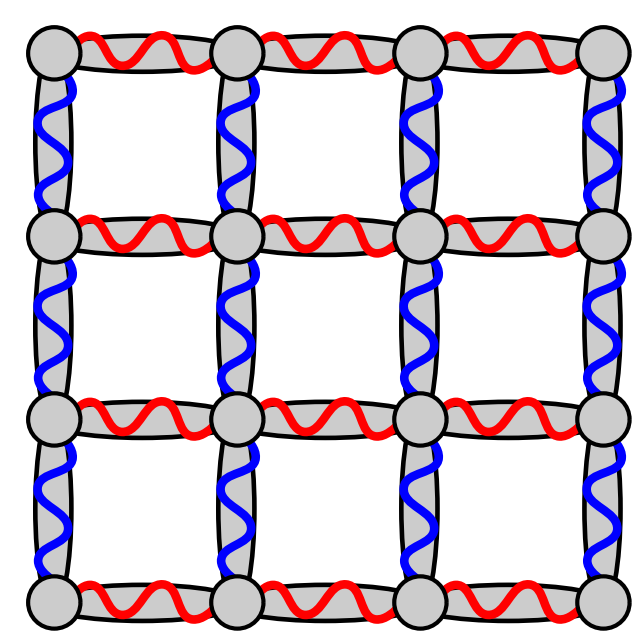
Confining phase:  
Néel order



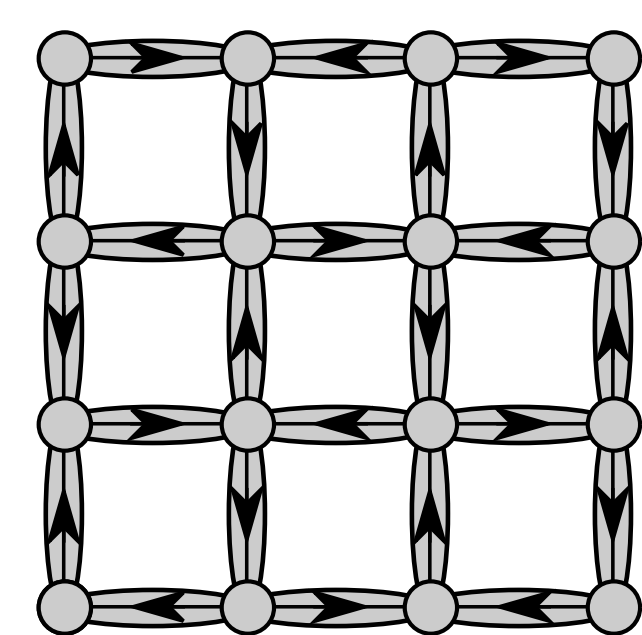
Confining phase:  
VBS order



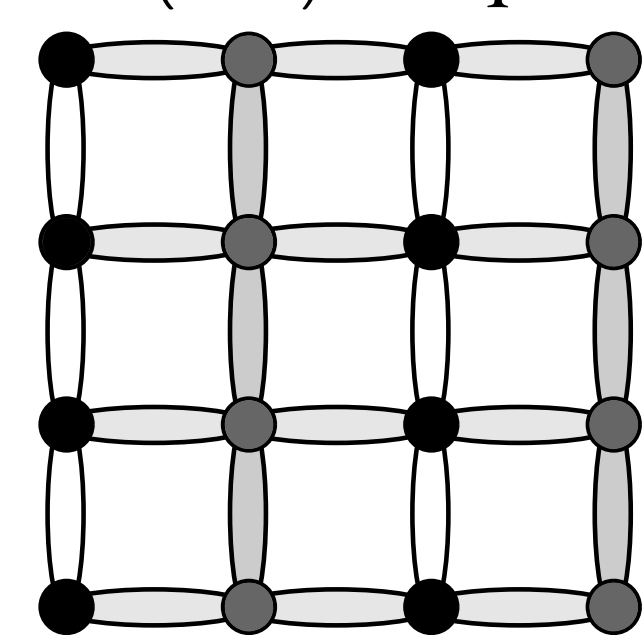
**Phase B**  
*d*-wave SC



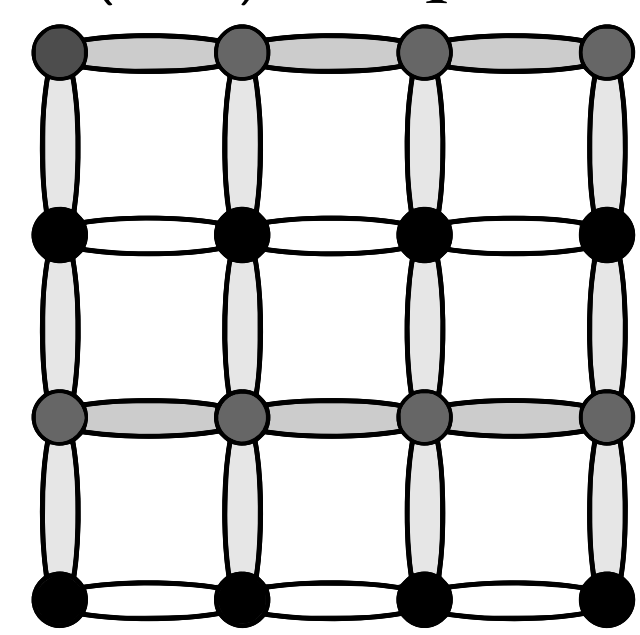
**Phase C**  
*d*-density



**Phase A**  
 $(\pi, 0)$  stripe

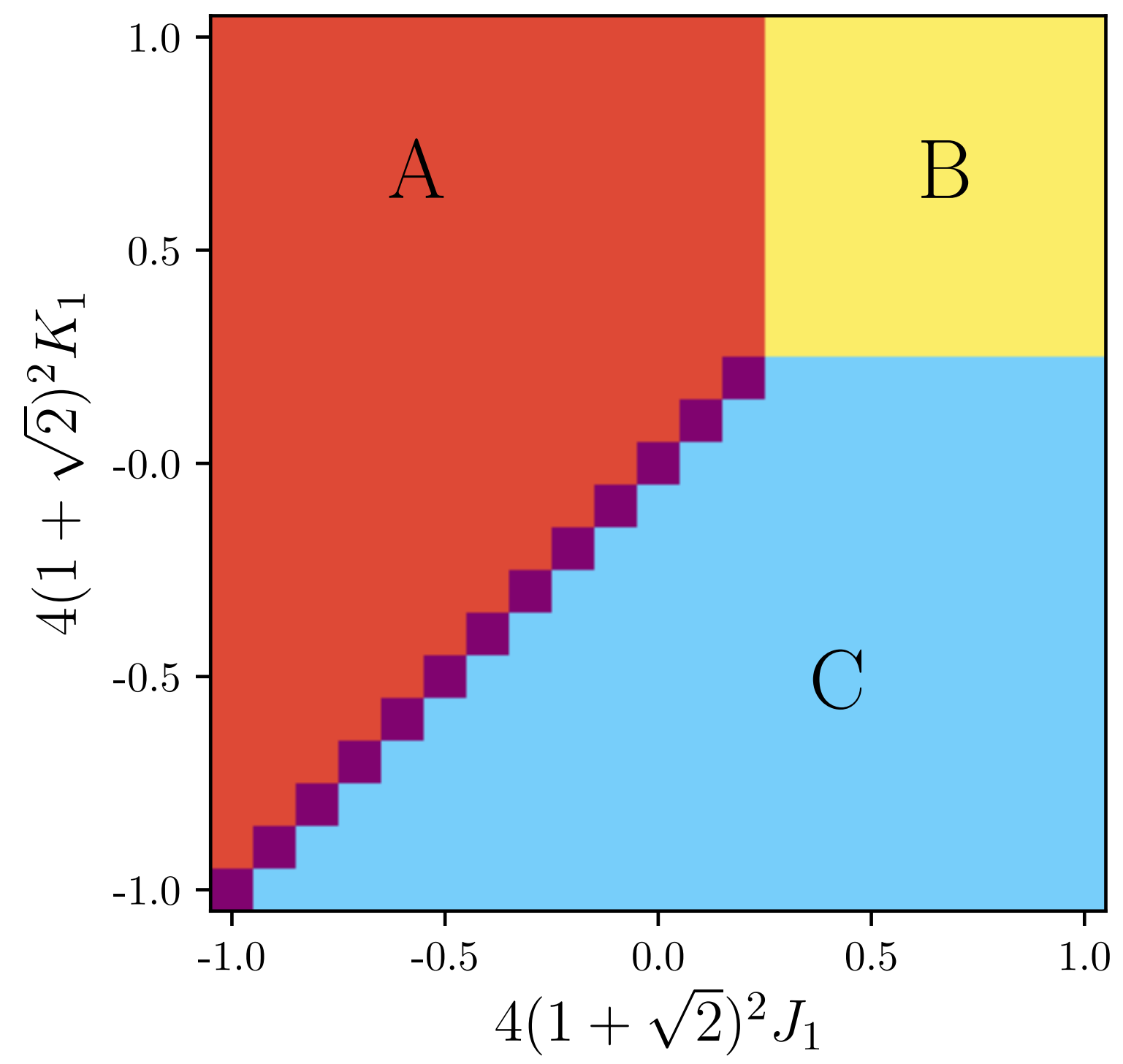


**Phase A**  
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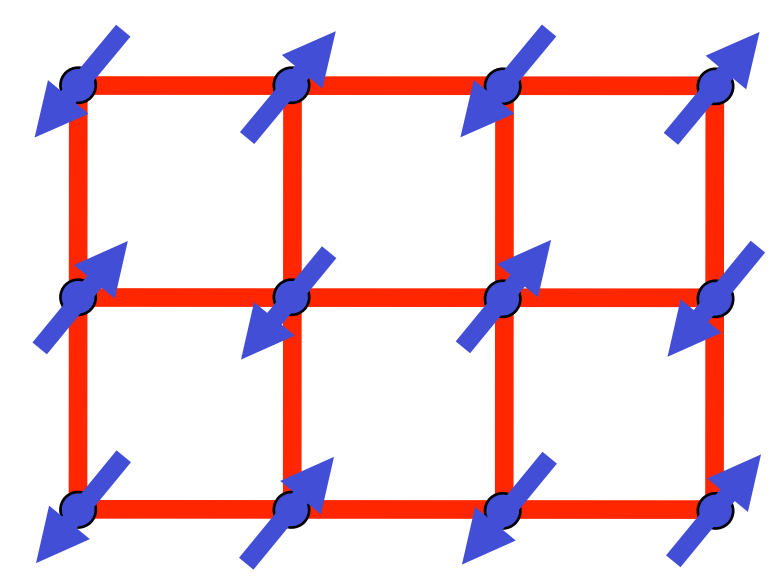
M. Christos, Zhu-Xi Luo,  
H. Shackleton, Ya-Hui Zhang,  
M. Scheurer, and S. S., *PNAS*  
**120**, e2302701120 (2023)

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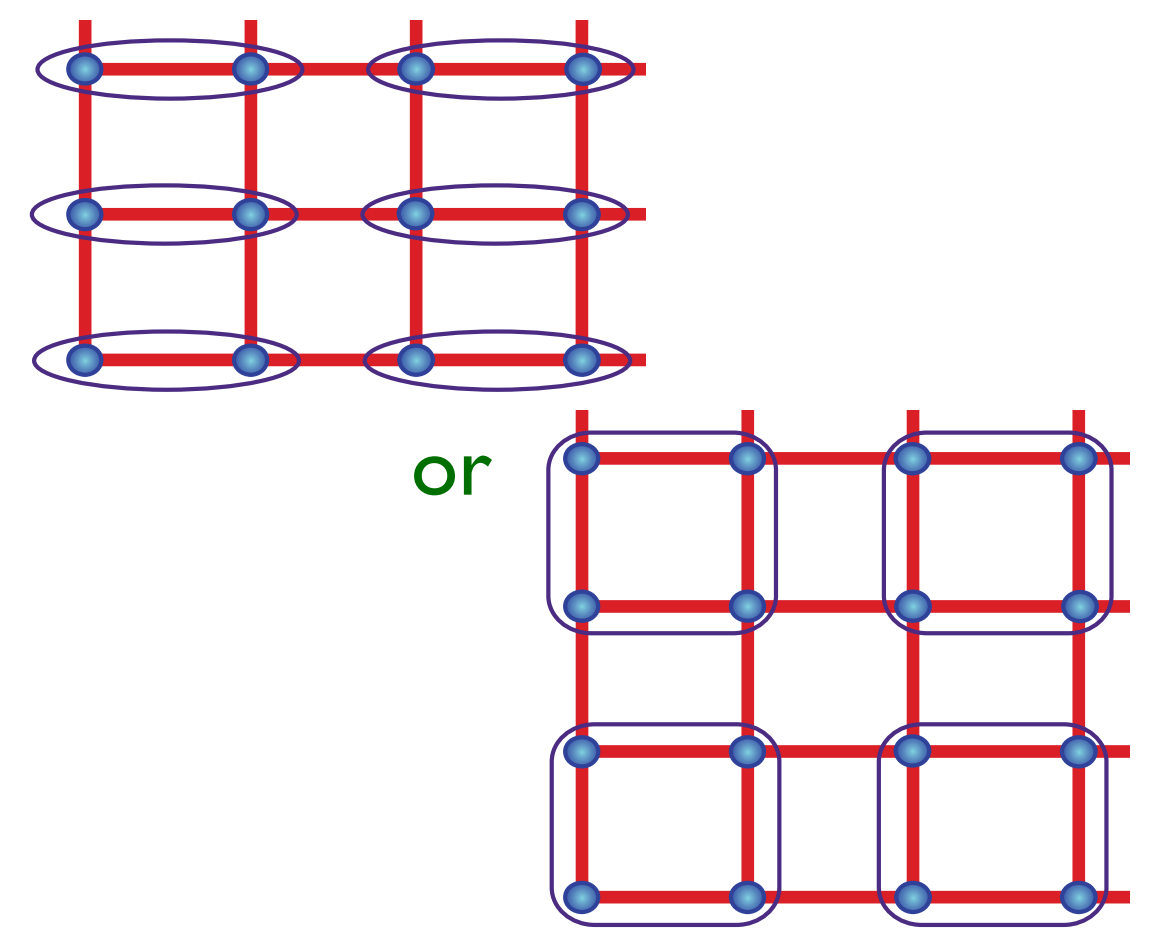


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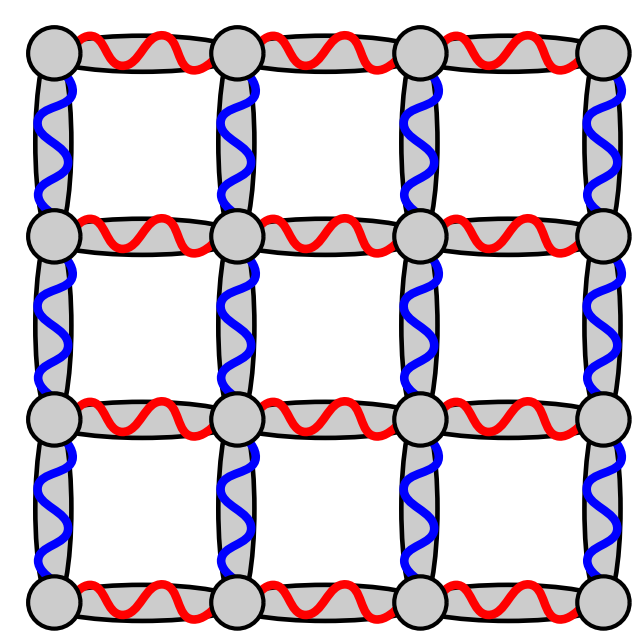
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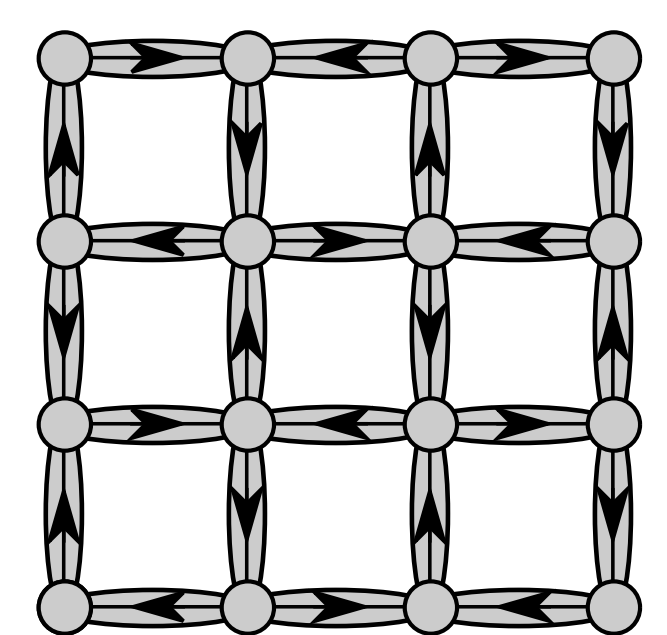
Confining phase:  
VBS order



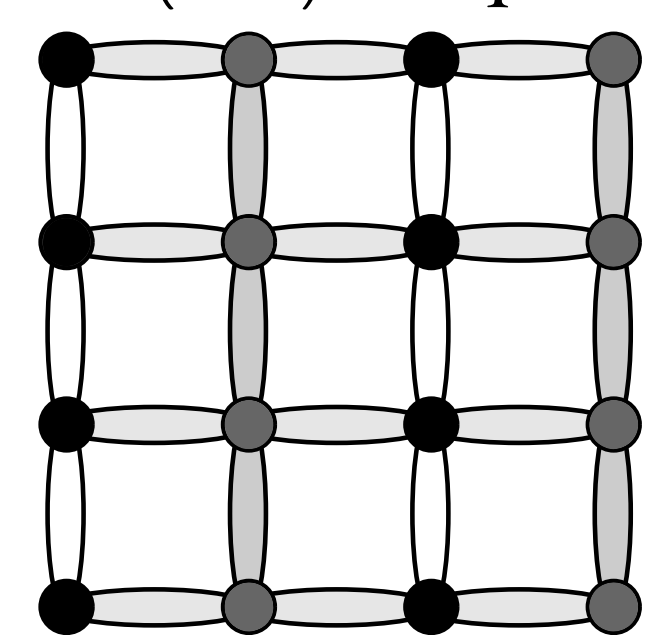
**Phase B**  
*d*-wave SC



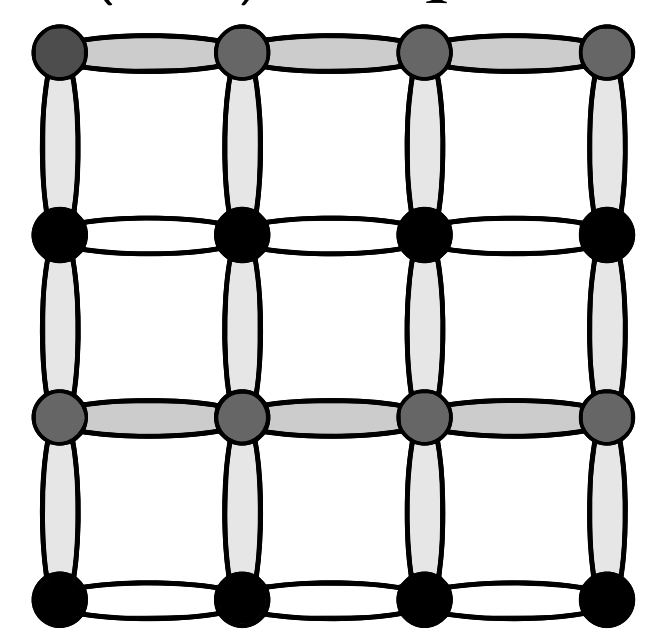
**Phase C**  
*d*-density



**Phase A**  
 $(\pi, 0)$  stripe

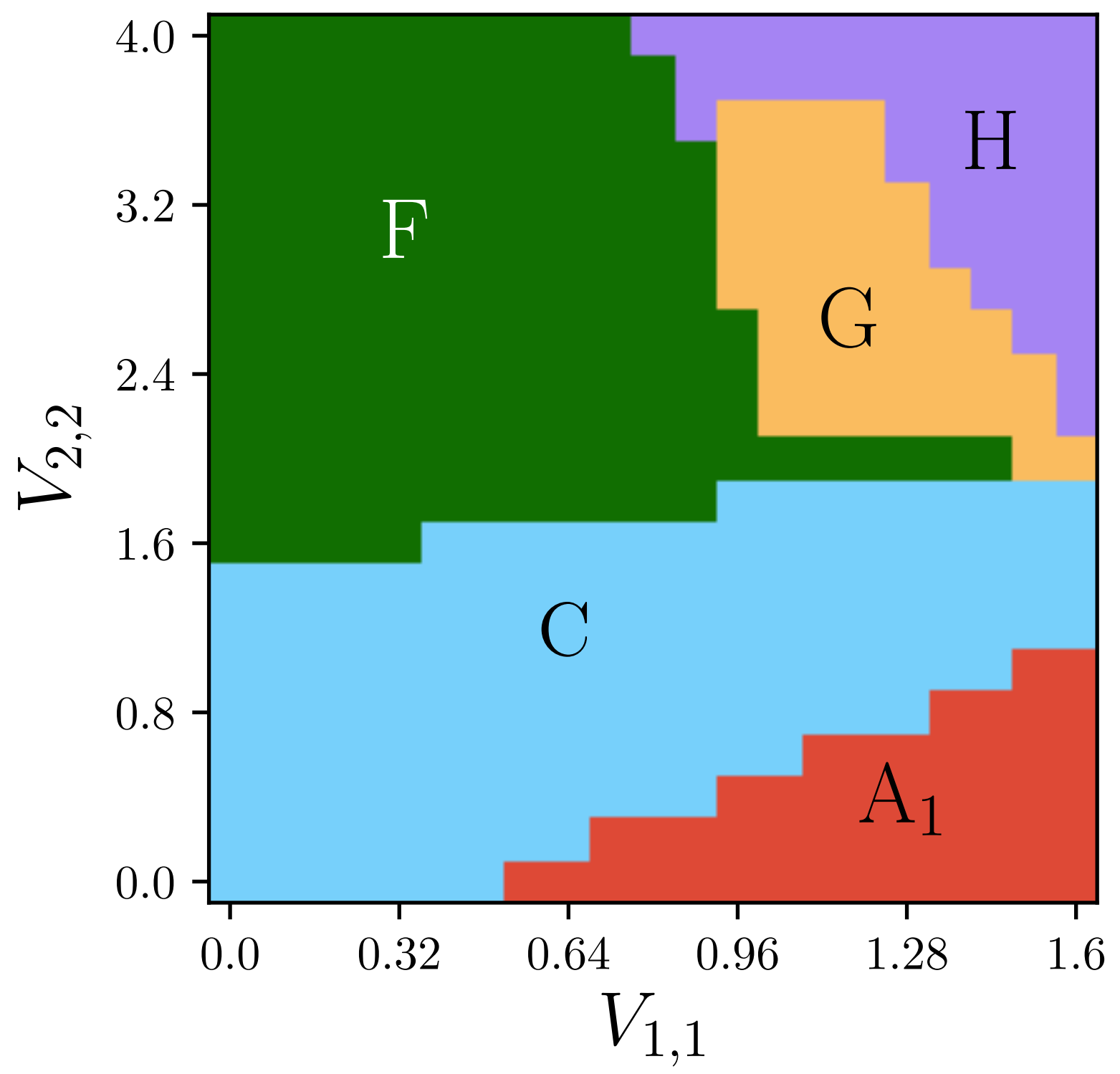


**Phase A**  
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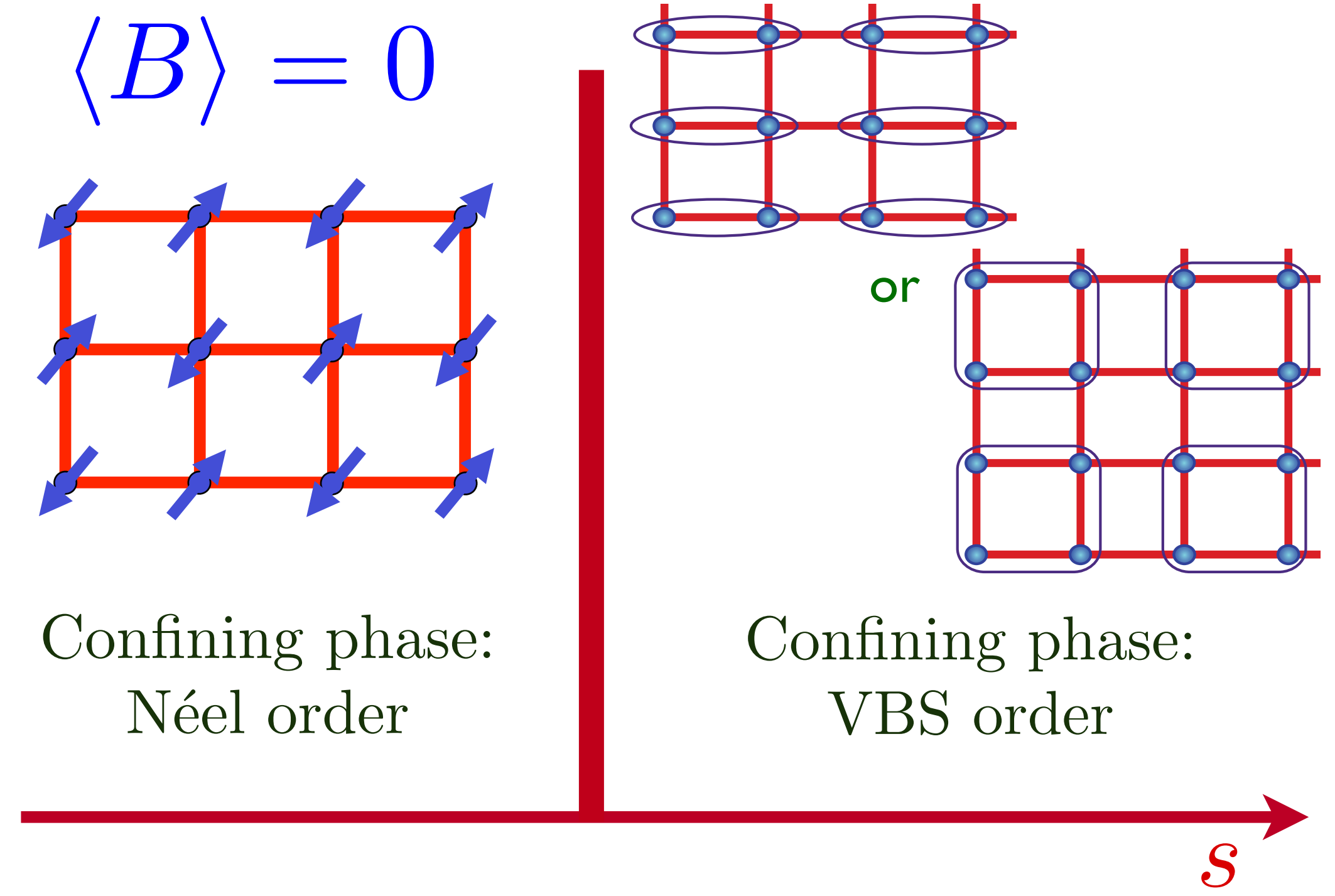
At half filling:  
possible CFT/DQCP  
with  $N_f = 2$  Dirac fermions  
and  $N_b = 2$  complex scalars  
coupled to  $SU(2)_N$  gauge field.  
M. Christos, H. Shackleton, S. S.,  
and Zhu-Xi Luo, arXiv:2402.09502

# Global phase diagram of $SU(2)_N$ gauge theory

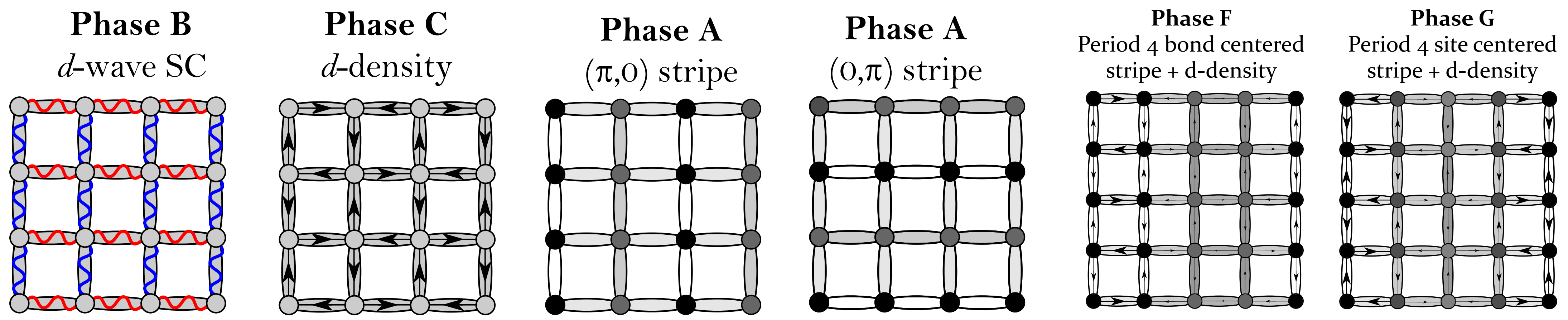


$\langle B \rangle \neq 0$

Including further-neighbor couplings in  $B$



$r$



M. Christos,  
Zhu-Xi Luo,  
H. Shackleton,  
Ya-Hui Zhang,  
M. Scheurer, and  
S. S., *PNAS* **120**,  
e2302701120  
(2023)

1. Confinement transitions of the Kondo lattice

2. Ancilla theory of  $FL^*$  in a single band model

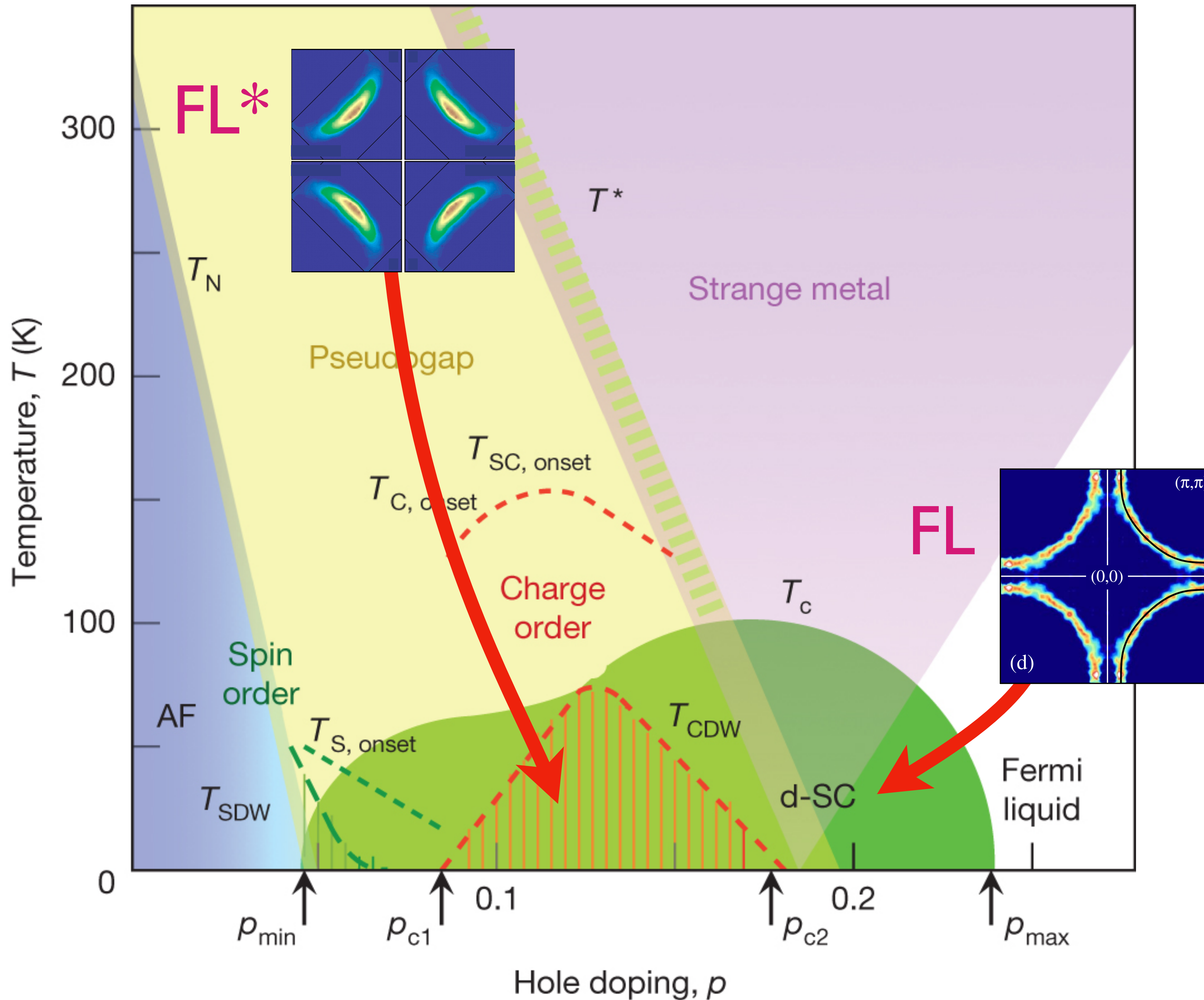
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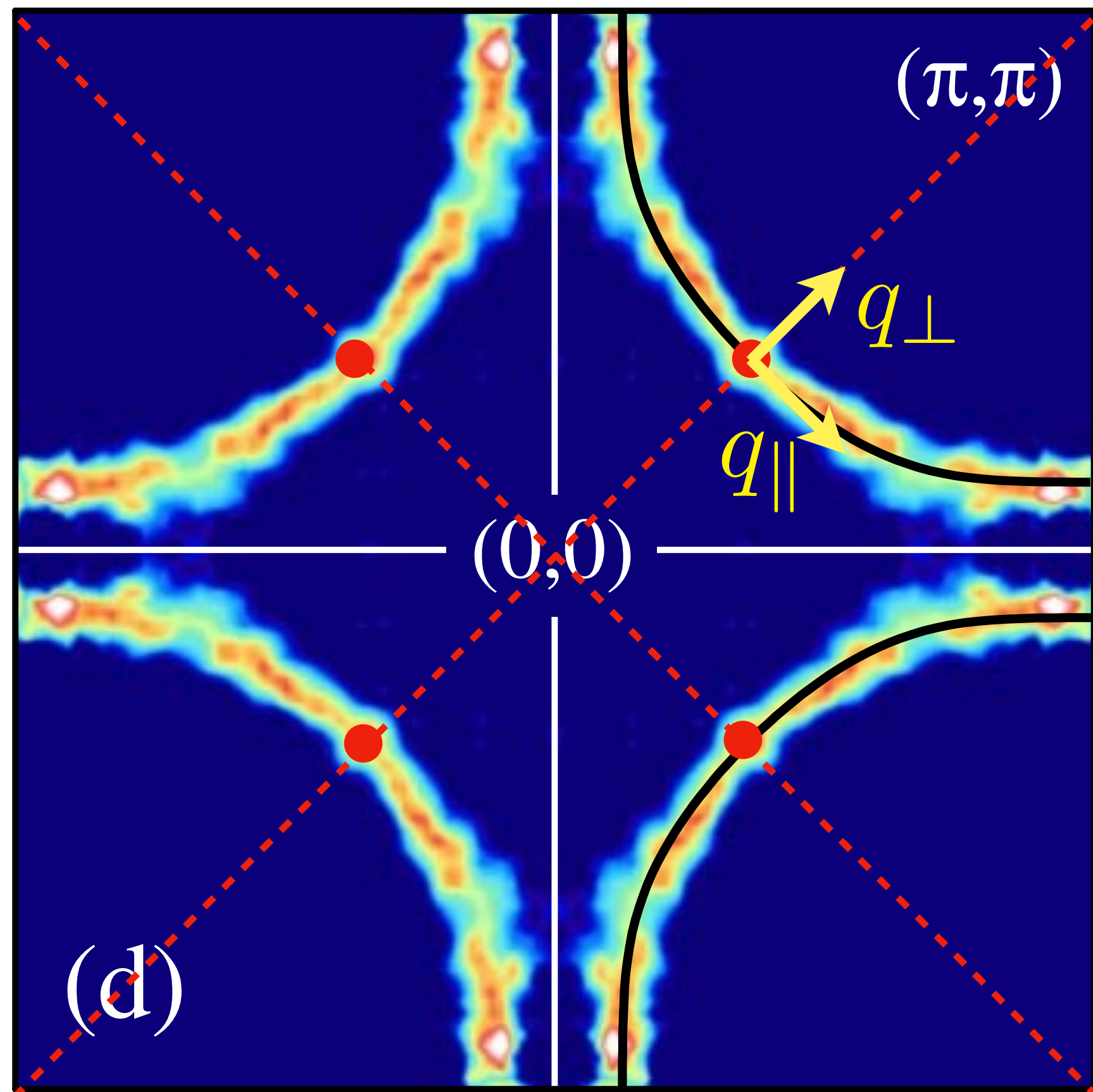
Quantum oscillations in CDW state



Obtain *d*-wave superconductor and charge order from a theory of *confinement* instabilities of FL\*.

The resulting low *T* ordered states should be adiabatically connected to the corresponding states obtained from instabilities of FL.

FL → dSC



BCS/Bogoliubov quasiparticles  
in a *d*-wave superconductor

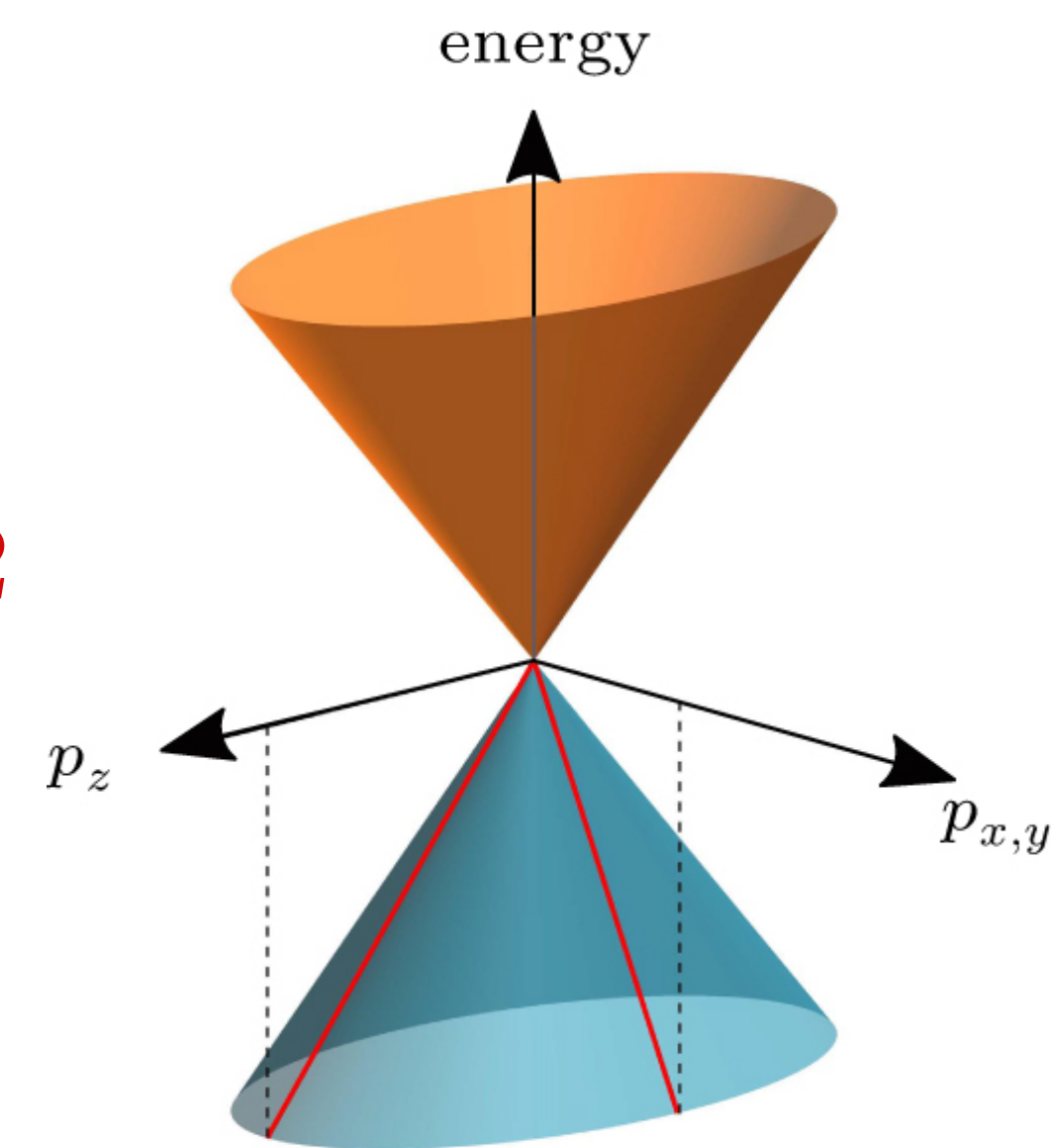
$$E_{\mathbf{k}} = \left( \varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2 \right)^{1/2}$$

$$\Delta_{\mathbf{k}} = \Delta_0 (\cos k_x - \cos k_y)$$

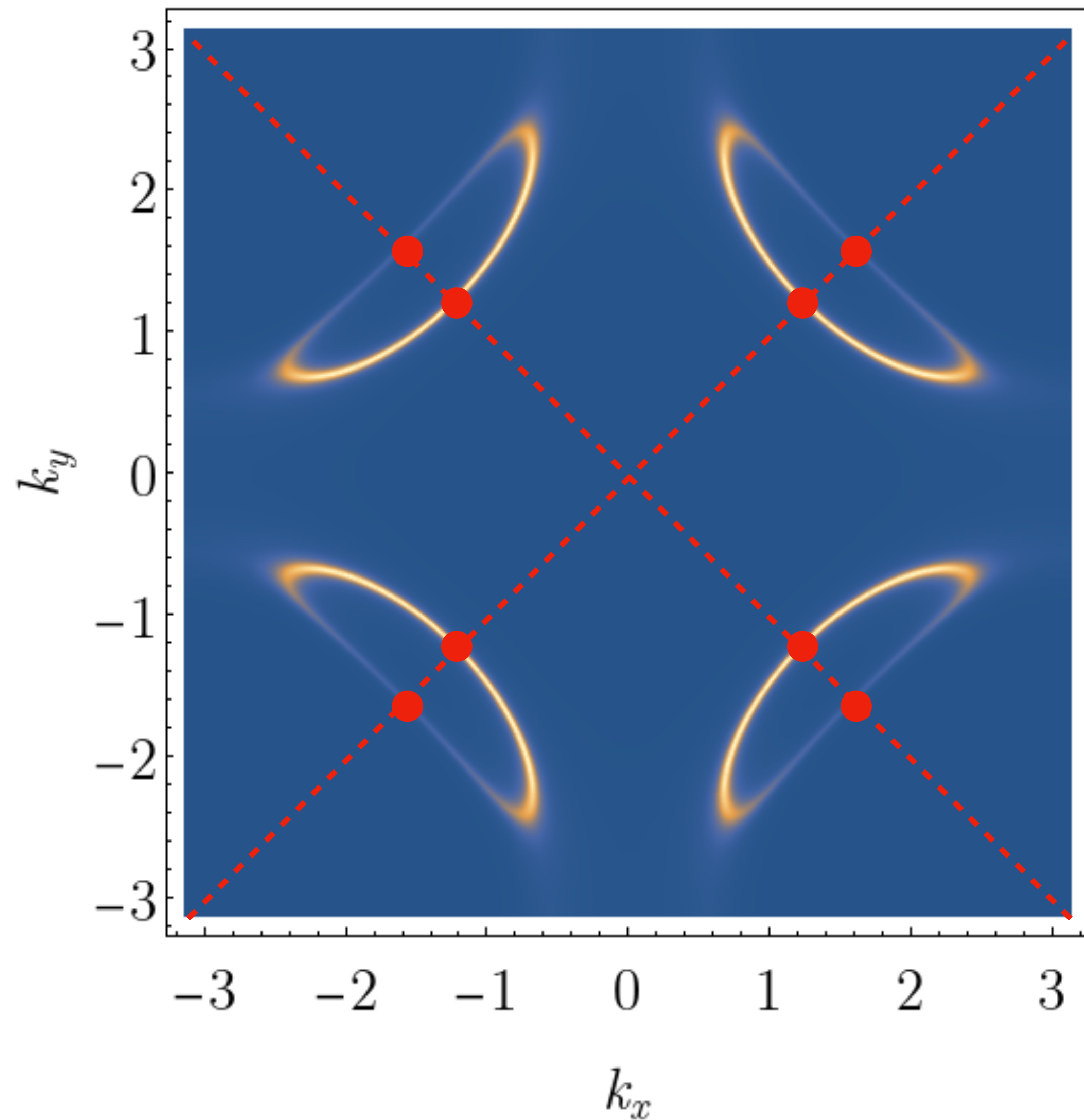
4 nodal points where

$$E_{\mathbf{k}_0 + \mathbf{q}} = \left( v_F^2 q_{\perp}^2 + v_{\Delta}^2 q_{\parallel}^2 \right)^{1/2}$$

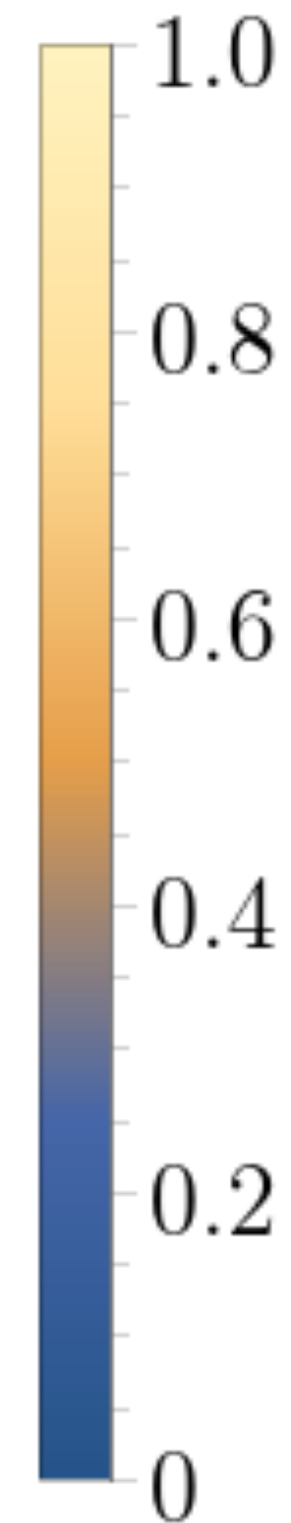
with  $v_F \gg v_{\Delta}$ .



FL\* → dSC\*



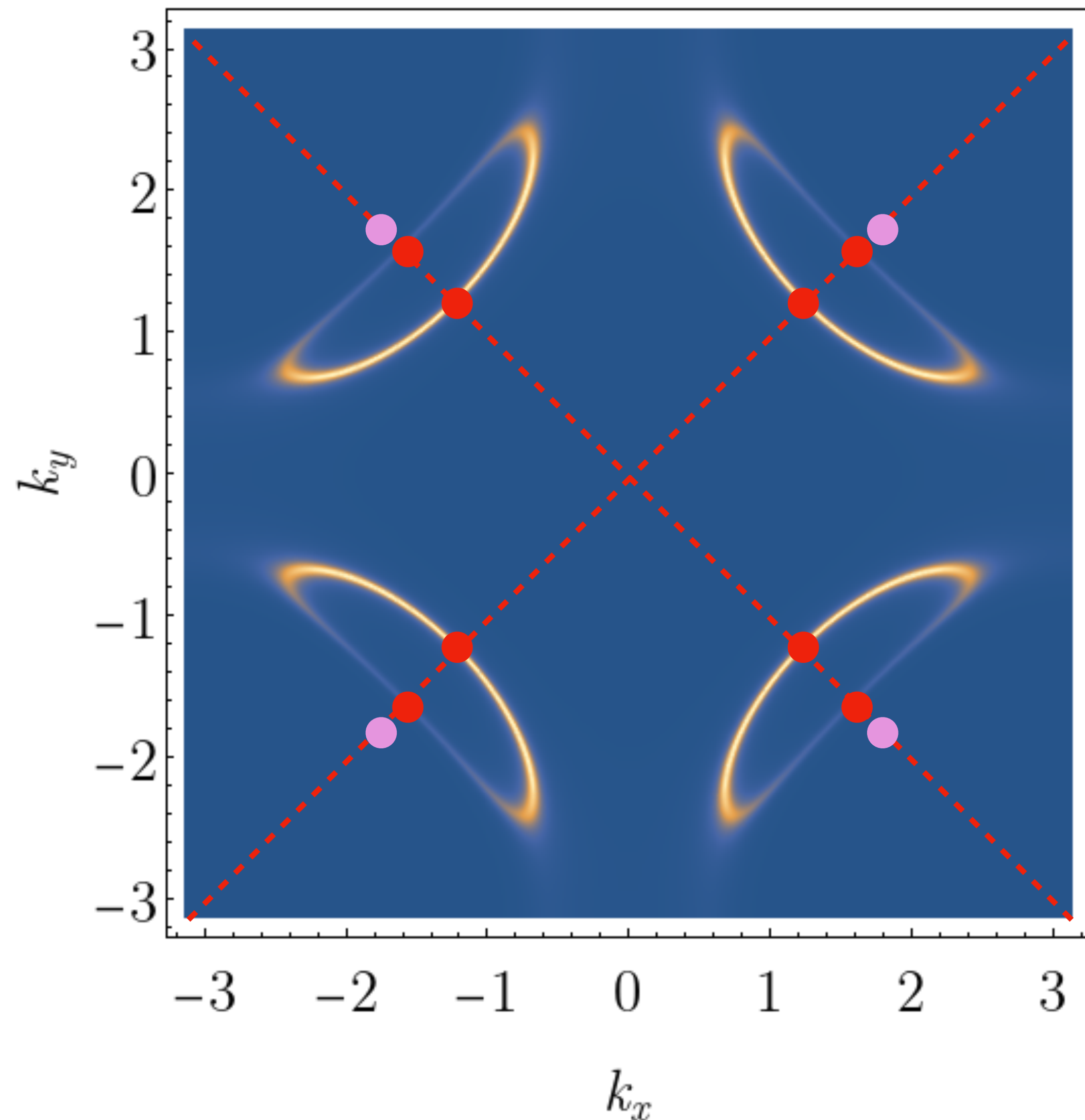
$|A_c(\omega=0, k_x, k_y)|/A_0$



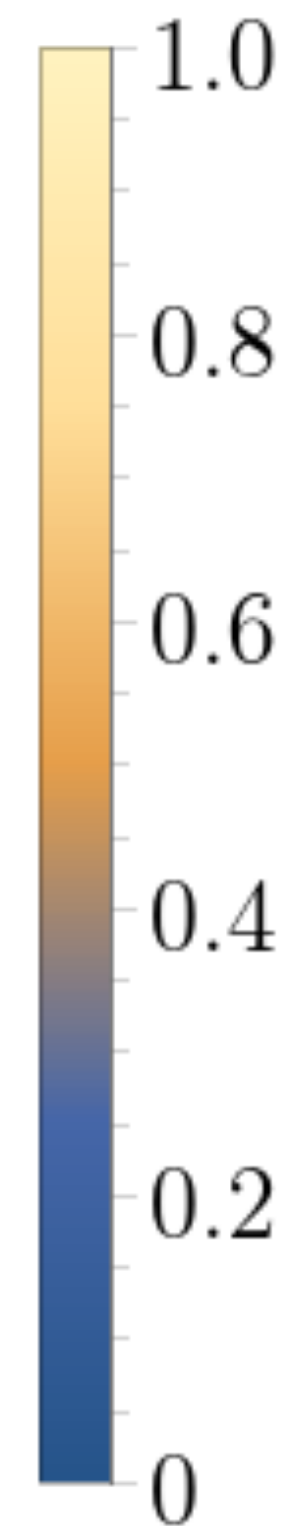
$$E_{\mathbf{k}} = (\varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2)^{1/2}$$
$$\Delta_{\mathbf{k}} = \Delta_0 (\cos k_x - \cos k_y)$$

Adding *d*-wave pairing  
to the hole pockets  
leads to 8 nodal points???

$FL^* \rightarrow dSC^*$

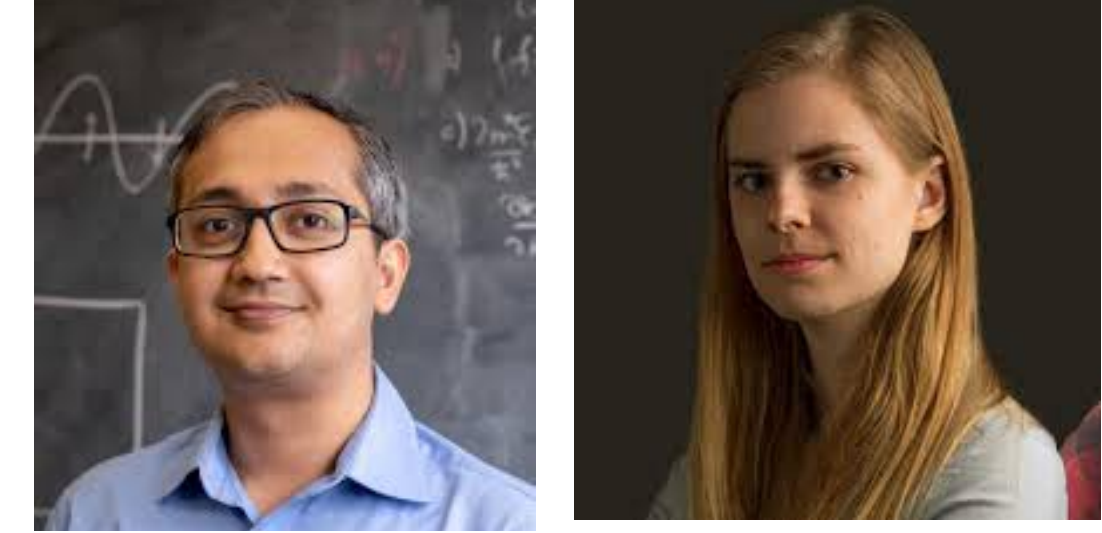


$|A_c(\omega=0, k_x, k_y)|/A_0$

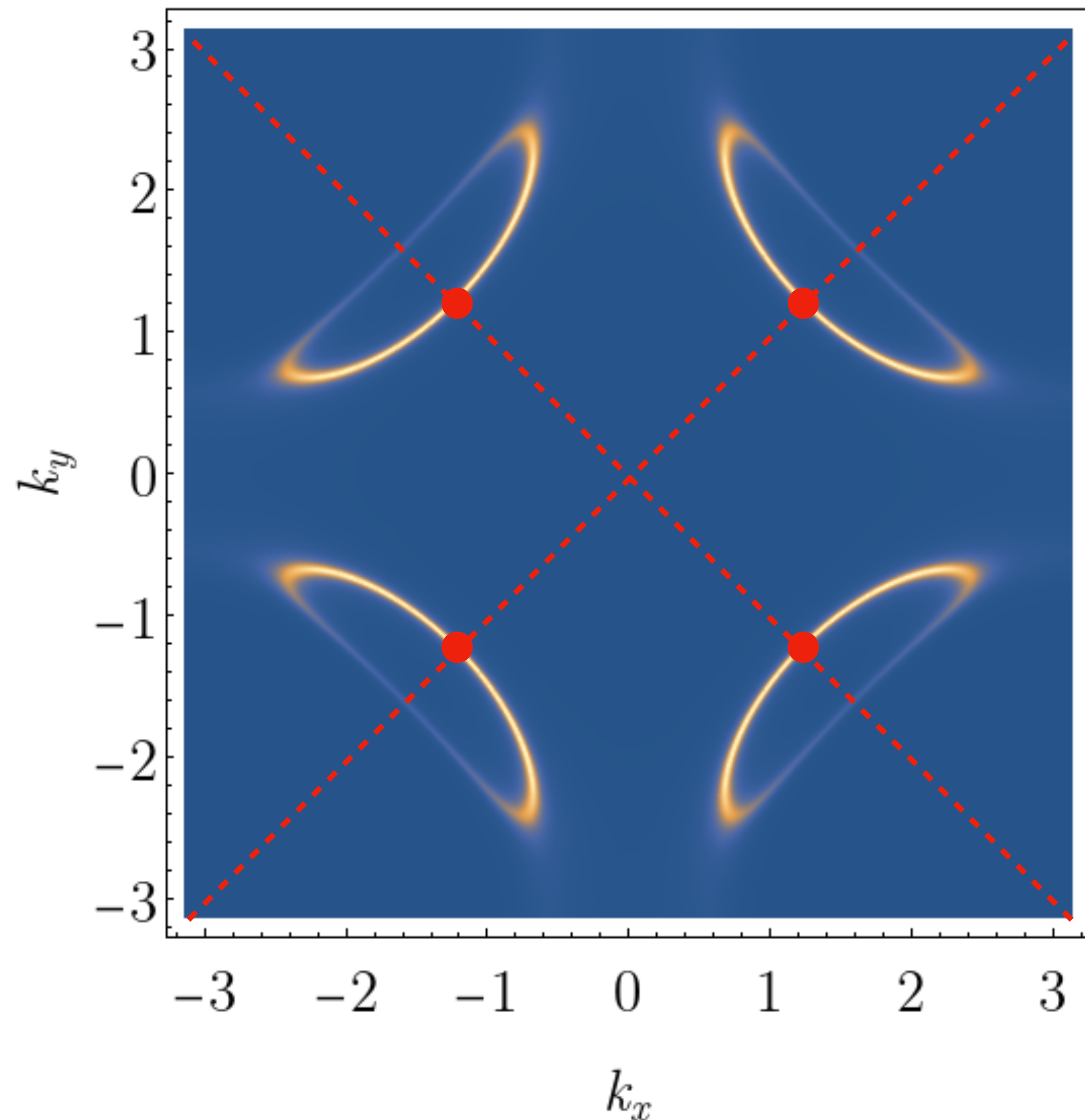


8 nodal points from  
the Fermi pockets  
and  
4 nodal points from  
the  $\pi$ -flux spin liquid

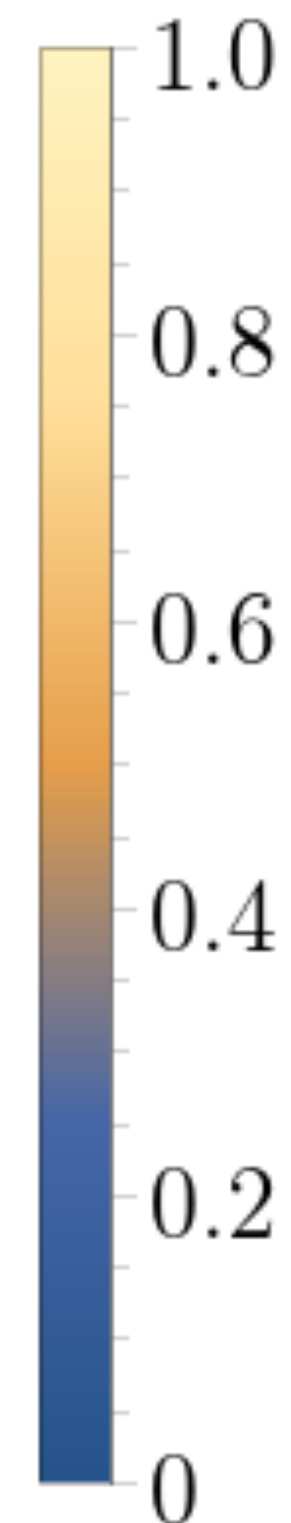
Shubhayu Chatterjee and S. Sachdev,  
PRB **94**, 205117 (2016)  
Maine Christos and S.Sachdev,  
npj Quantum Materials **9**, 4 (2024)



FL\* → dSC



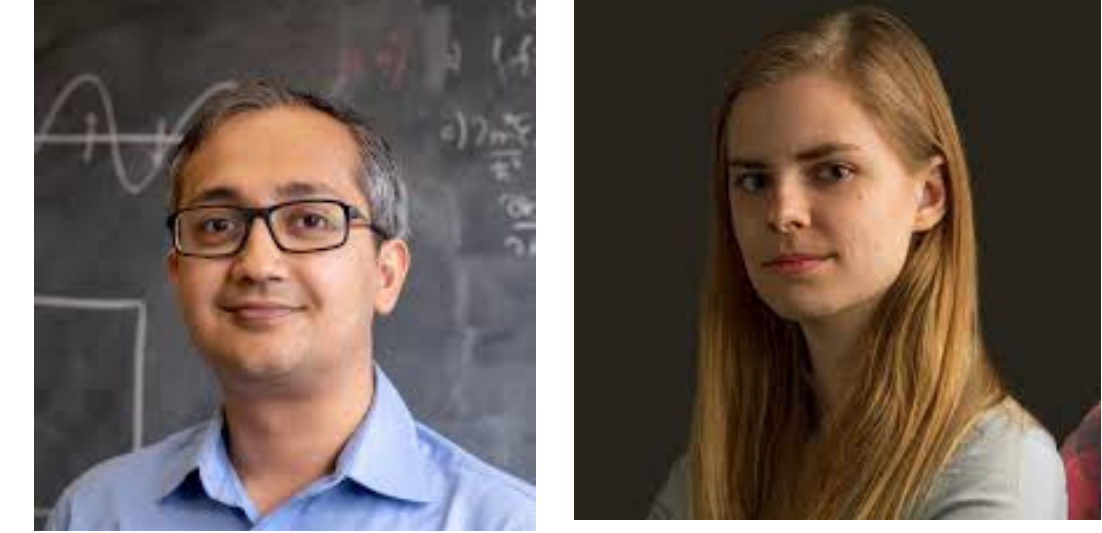
$$|A_c(\omega=0, k_x, k_y)|/A_0$$



The  $B$  Higgs condensate allows spinons and Bogoliubov quasiparticles to hybridize:

8 nodal points annihilate each other, leaving 4 nodal points with anisotropic velocities, just as in a BCS  $d$ -wave state.

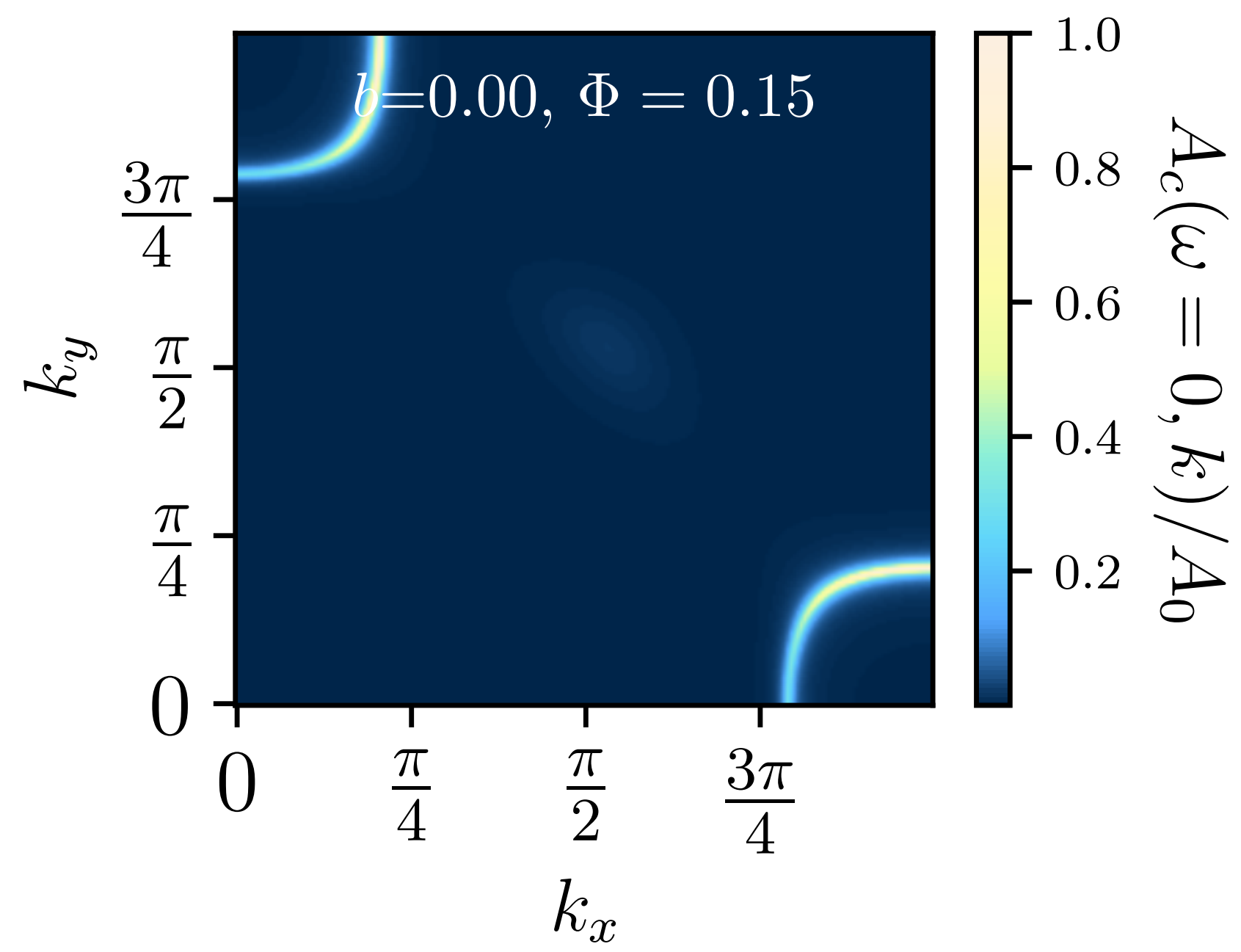
Shubhayu Chatterjee and S. Sachdev,  
PRB **94**, 205117 (2016)  
Maine Christos and S. Sachdev,  
npj Quantum Materials **9**, 4 (2024)



# Electron spectral density in electron-doped cuprates



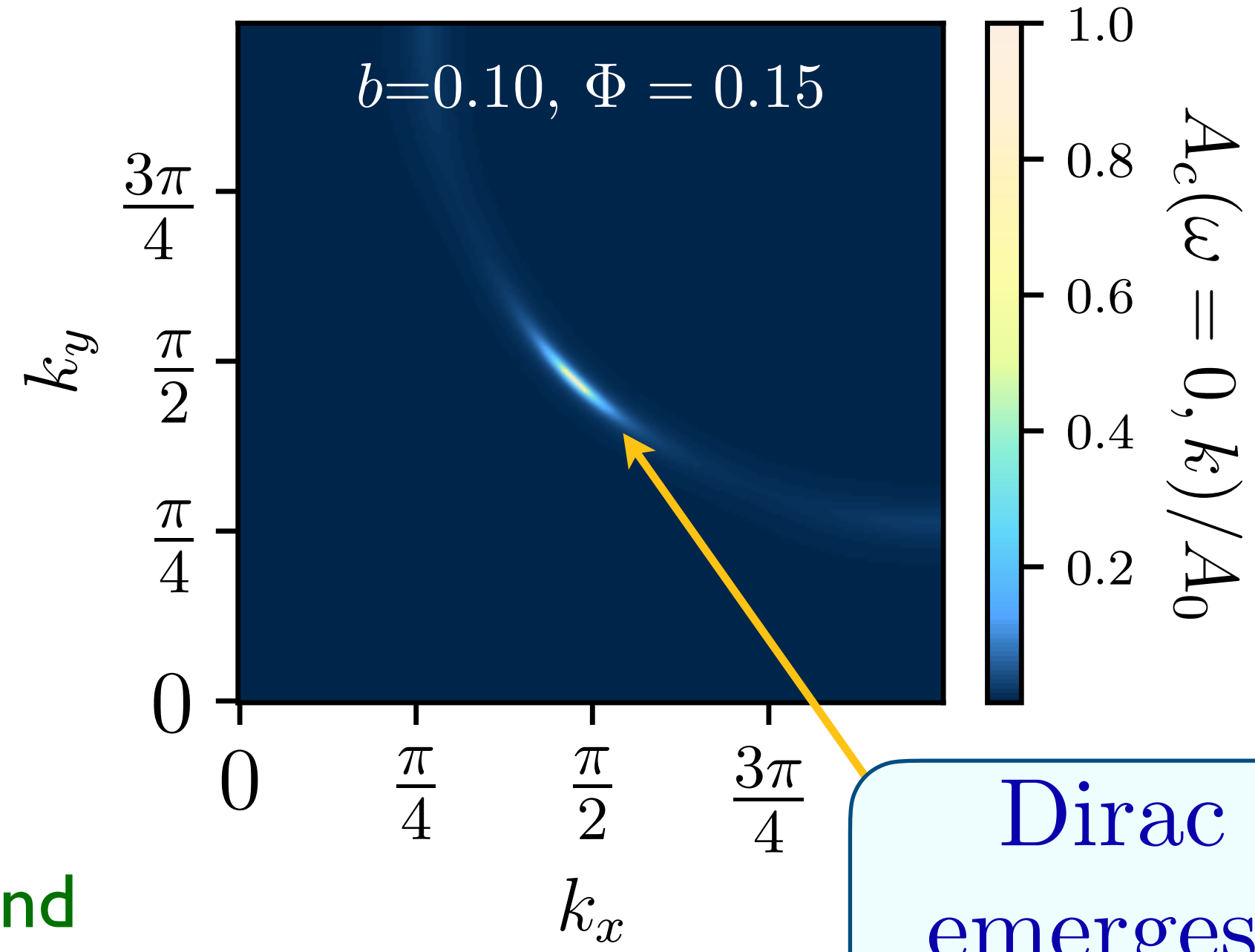
Maine Christos and S.Sachdev, npj Quantum Materials **9**, 4 (2024)



FL\*

*d*-wave superconductor obtained by condensing charge-*e*, SU(2) fundamental boson.

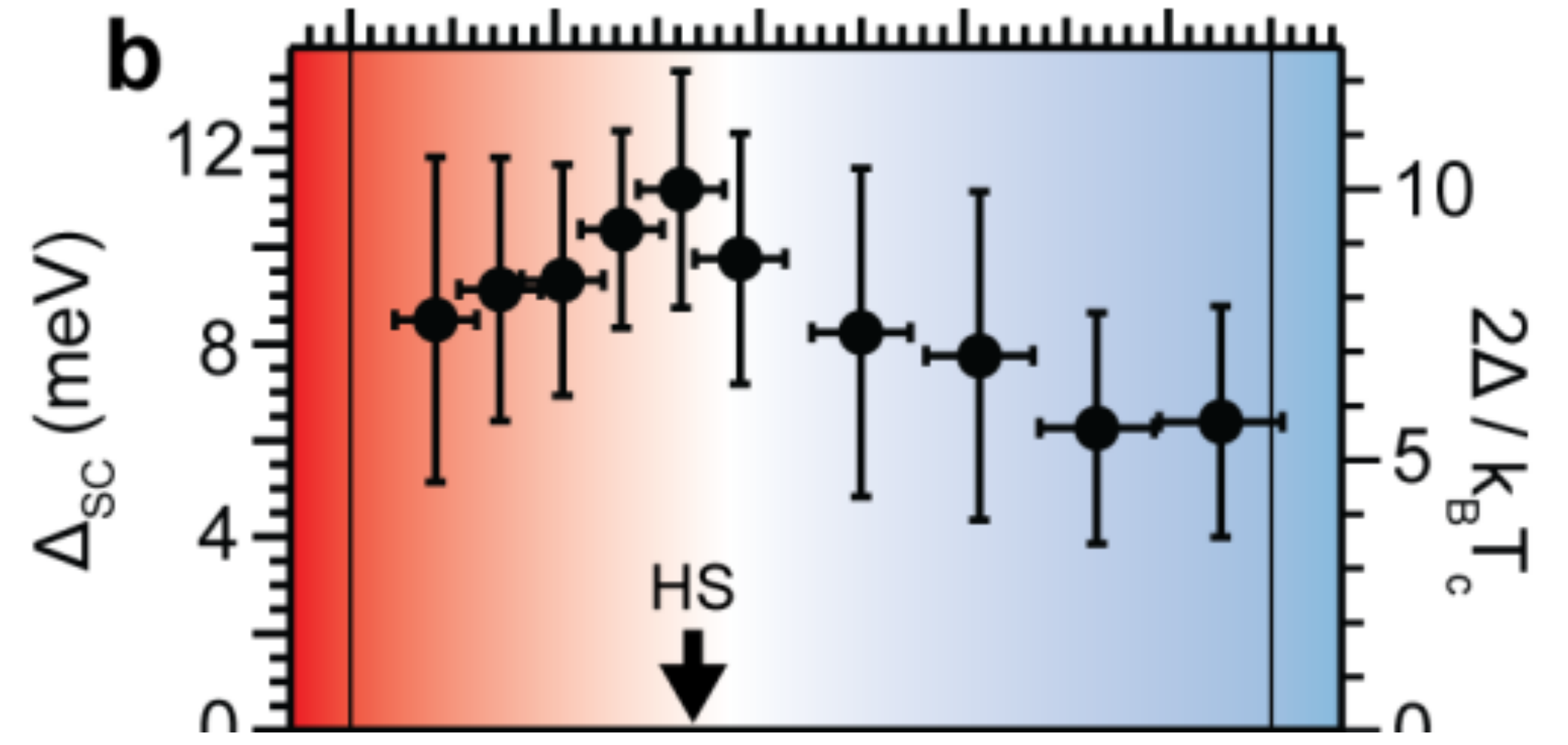
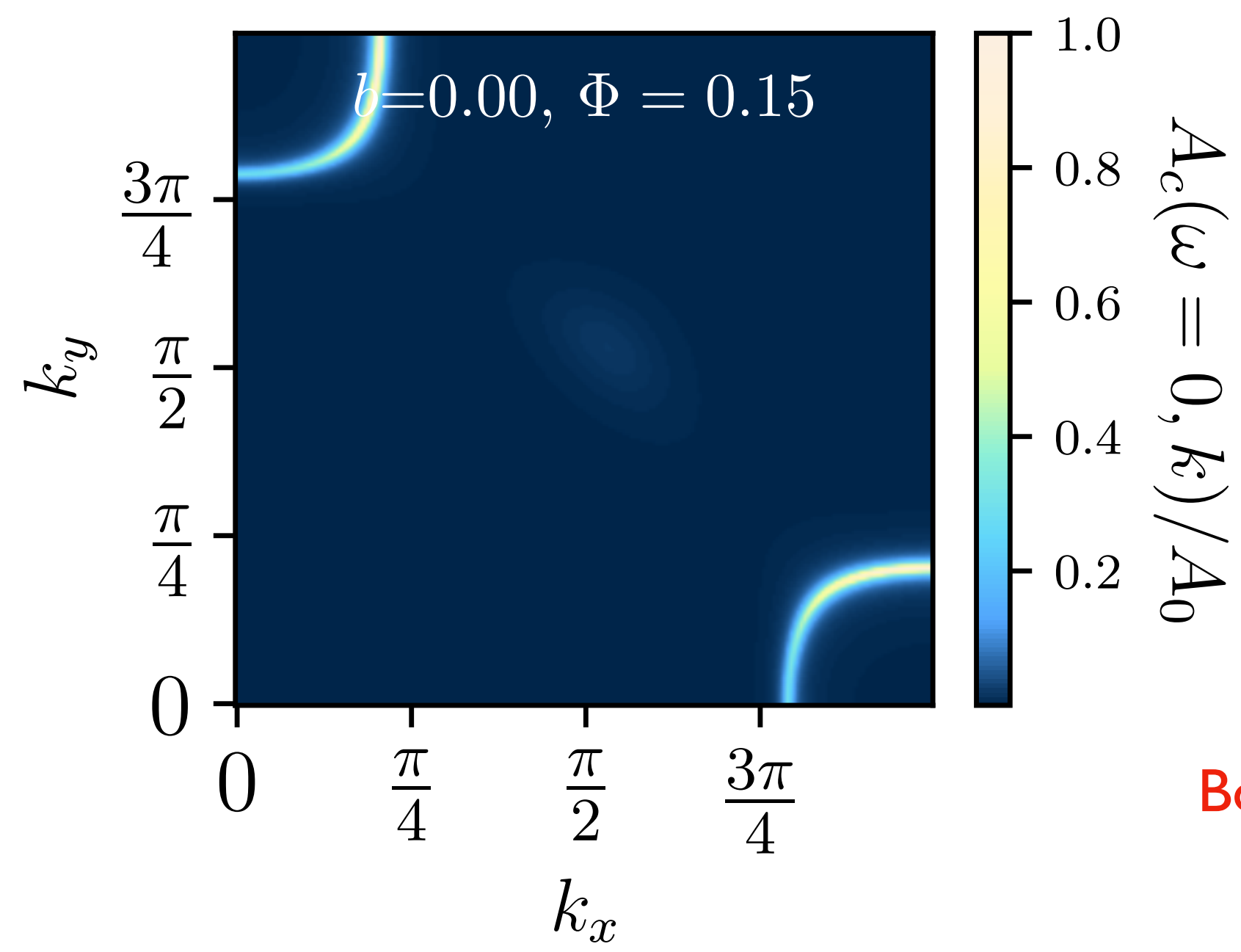
Nodes of the *d*-wave superconductor are remnants of the spinons of the  $\pi$ -flux state.



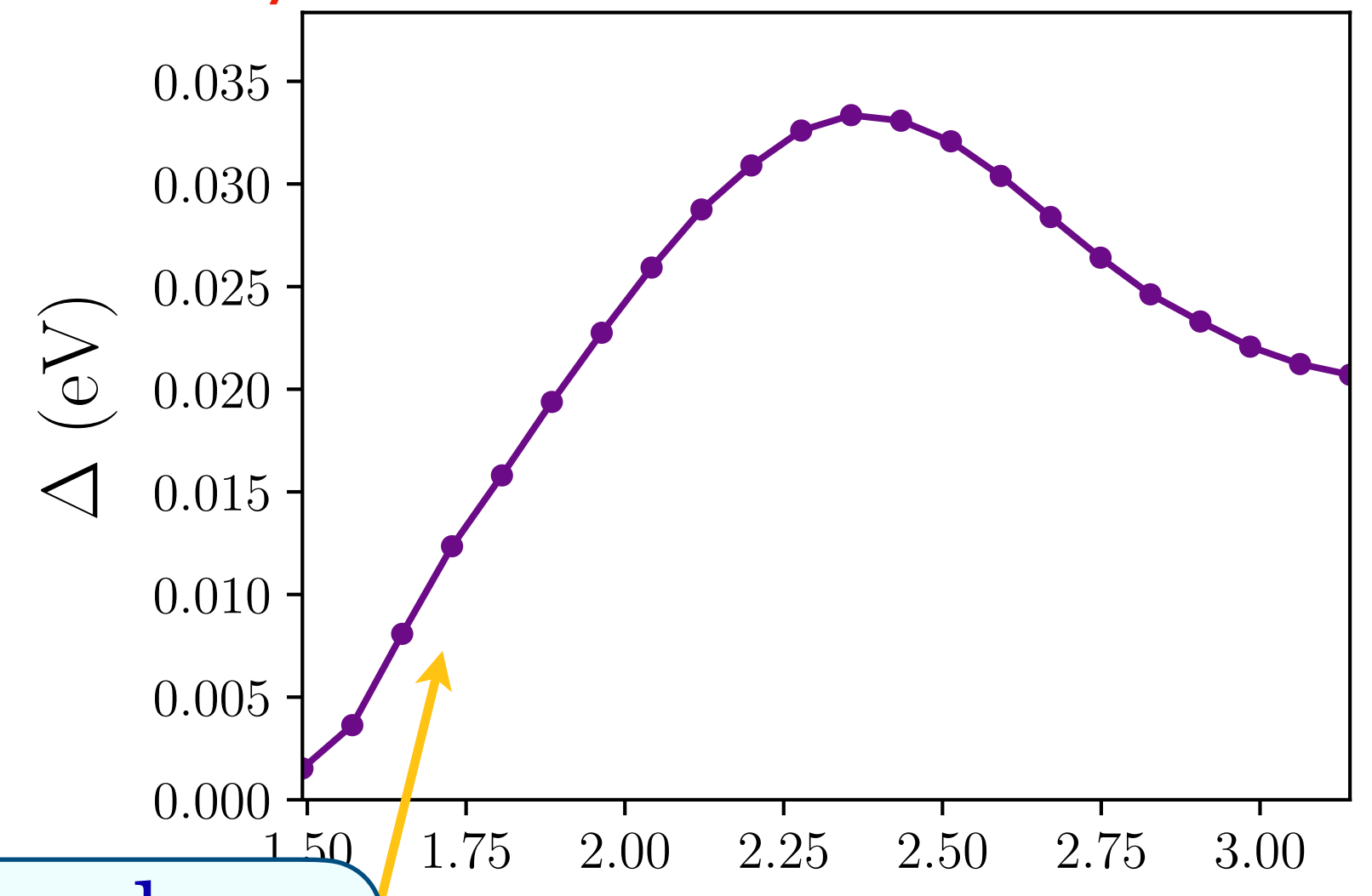
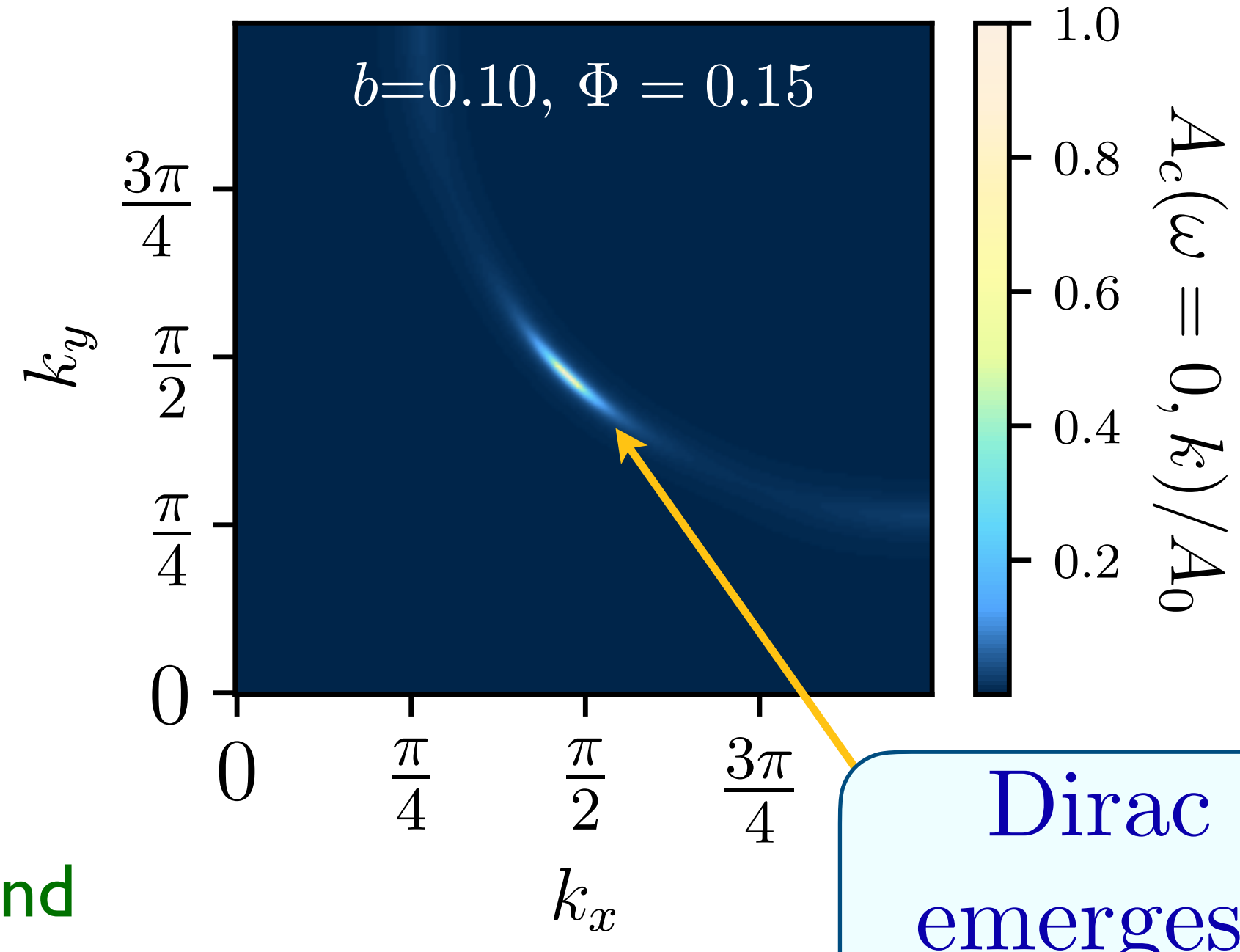
dSC

Dirac node emerges inside normal state gap

# Electron spectral density in electron-doped cuprates



Bogoliubov Quasiparticle on the Gossamer Fermi Surface in Electron-Doped Cuprates, Ke-Jun Xu.....Z.-X. Shen, arXiv:2308.05313; Nature Physics



Dirac node emerges inside normal state gap

Maine Christos and S.Sachdev, npj Quantum Materials **9**, 4 (2024)

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2. Ancilla theory of  $FL^*$  in a single band model

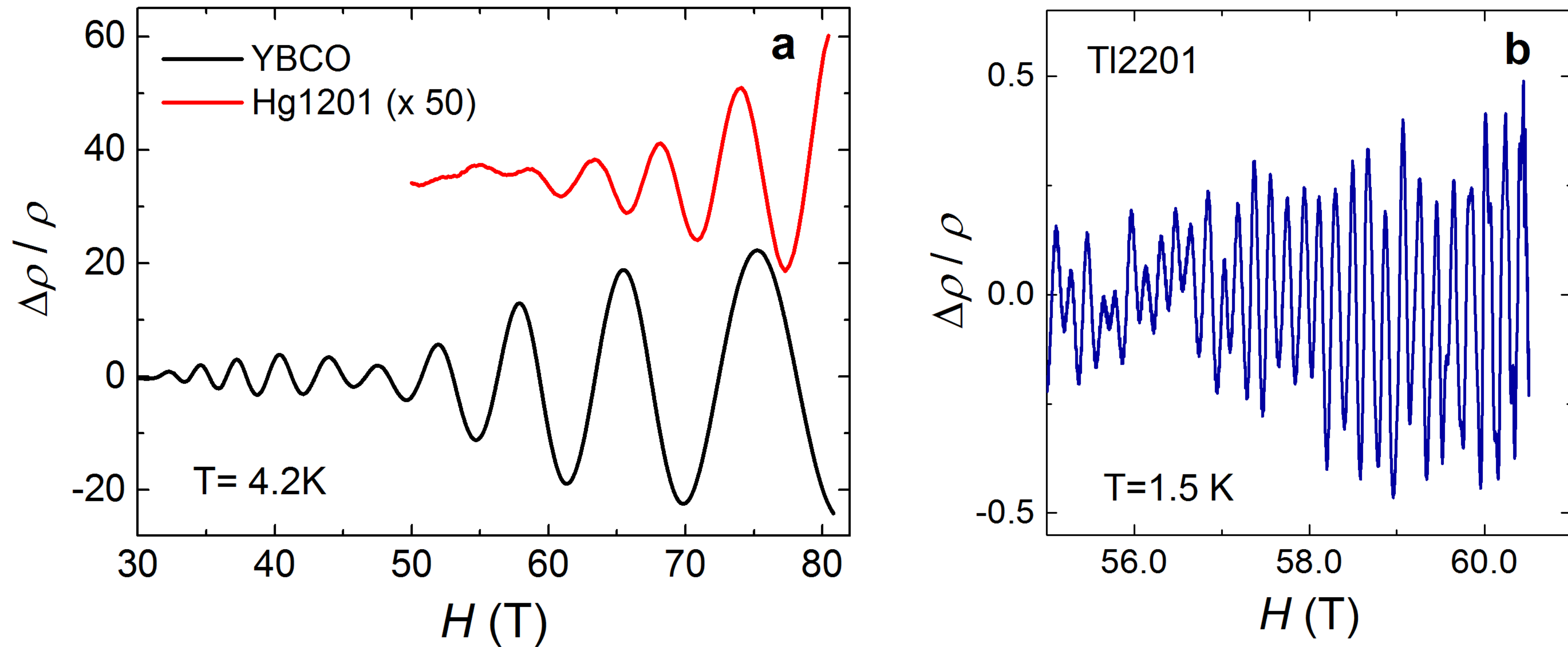
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Quantum oscillations in CDW state



**Figure 4**

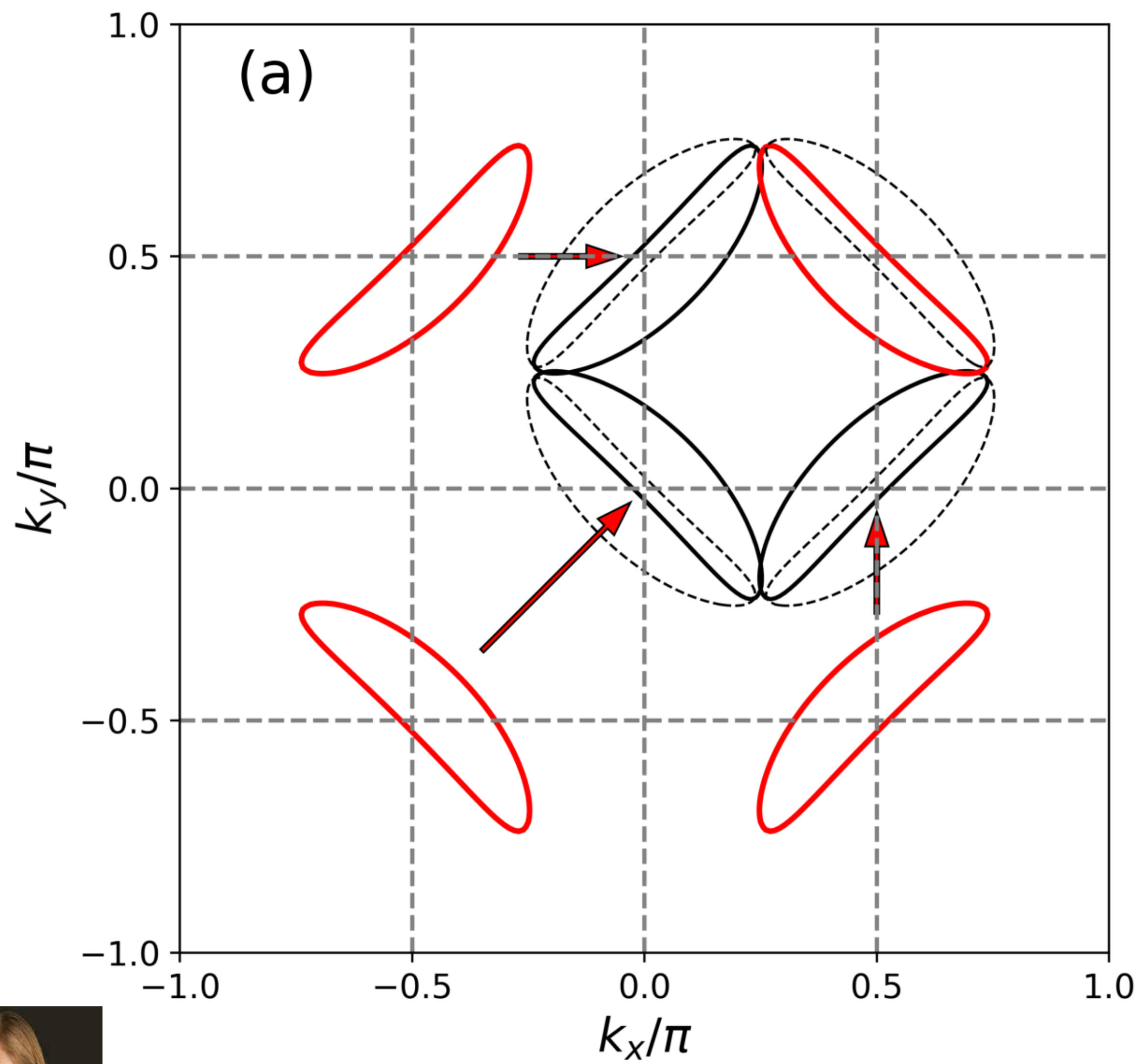
**Quantum oscillations in cuprates.** **a)** Underdoped YBCO ( $p = 0.11$ ,  $T_c = 62$  K) (black (22)) and Hg1201 ( $p \simeq 0.1$ ,  $T_c = 72$  K) (red (33),  $\times 50$ ). **b)** Overdoped Tl2201 ( $p \simeq 0.3$ ,  $T_c \approx 10$  K) (28).

22. Vignolle B, et al. 2013. *C. R. Physique* 14:39–52

28. Vignolle B, et al. 2008. *Nature* 455:952–955

33. Barisic N, et al. 2013. *Nat. Phys.* 9:761–764

FL\*



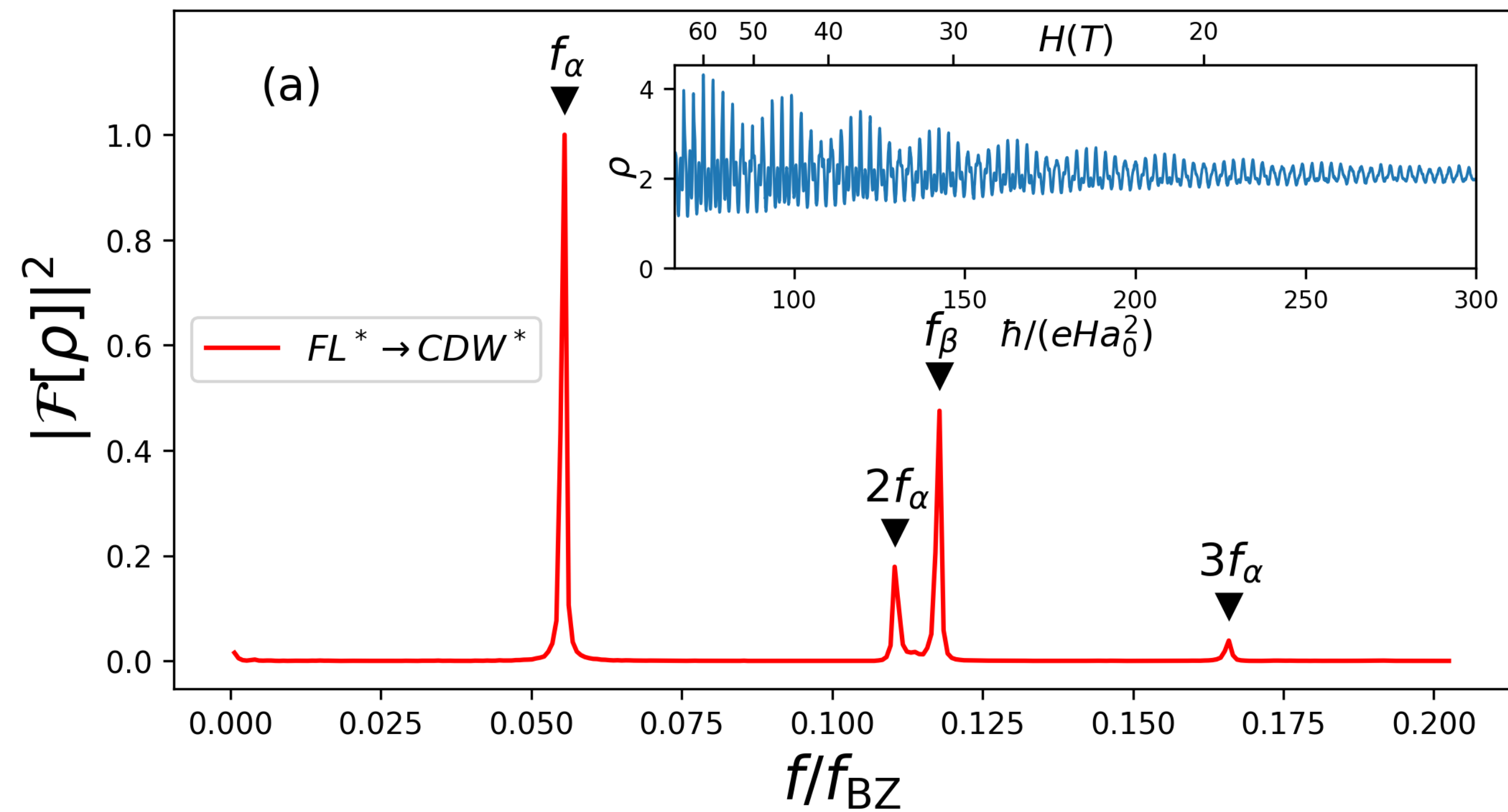
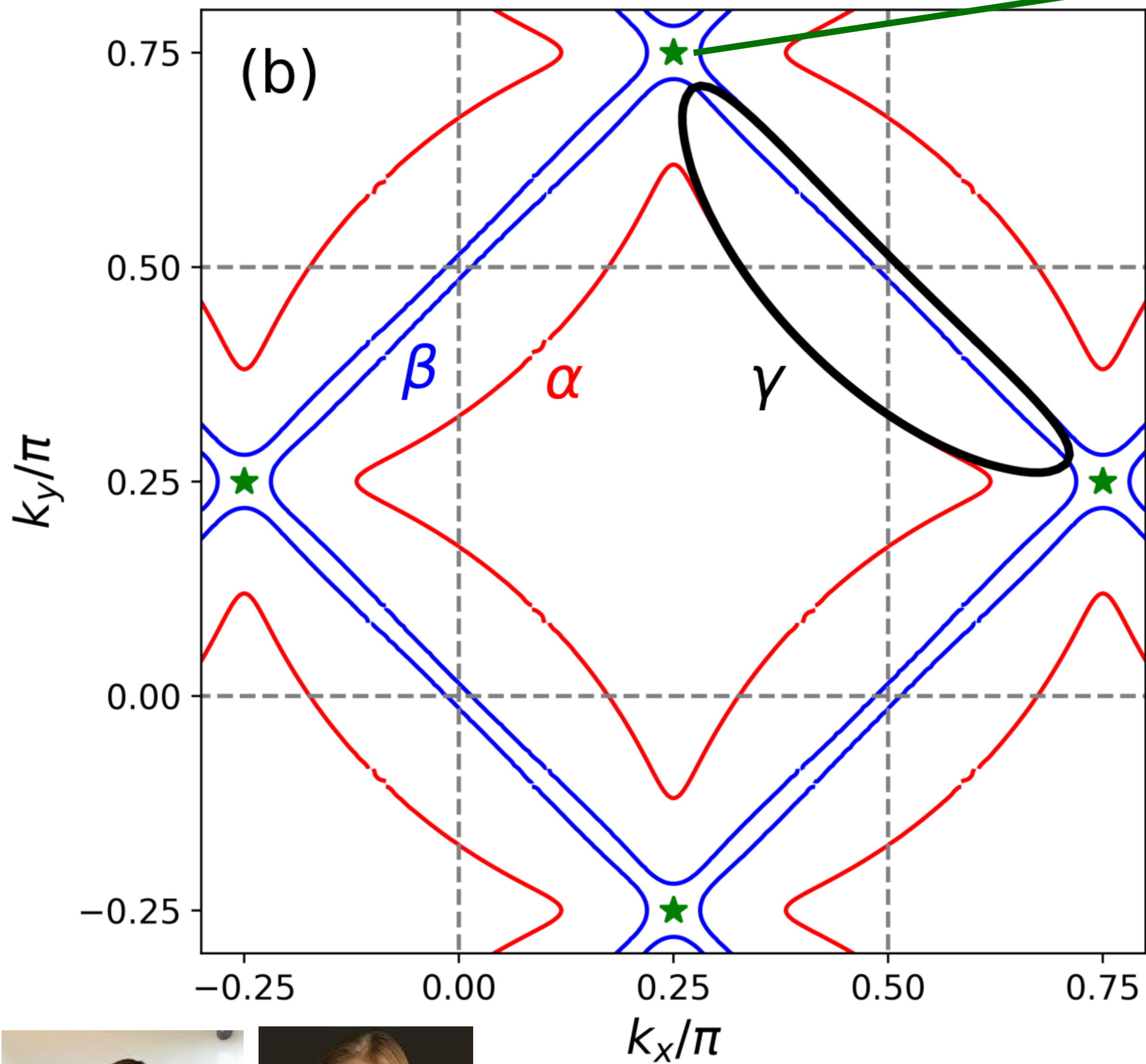
N. Harrison and S. Sebastian  
electron pocket  
(PRL **106**, 226402 (2011))



Pietro Bonetti, Maine Christos and S.S. (**BCS**), arXiv:2405.08817

$FL^* \rightarrow CDW^*$

Spinon



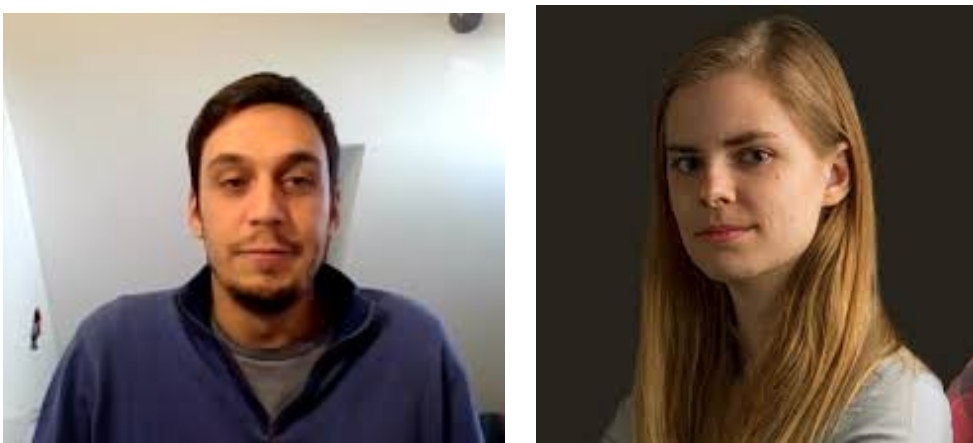
Computation does not account for spinons.

$\alpha$  and  $\beta$  pockets

show clear quantum oscillations.

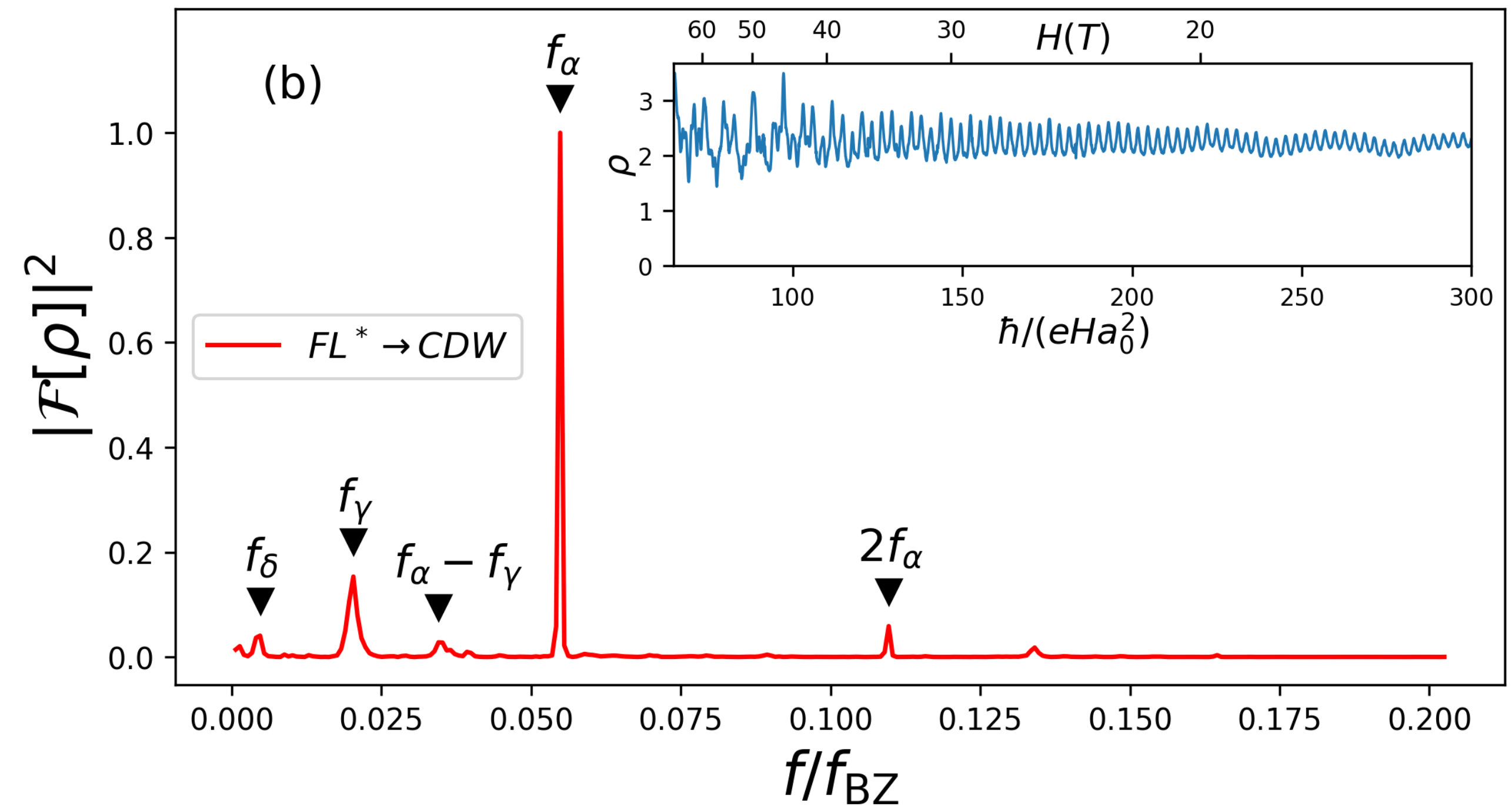
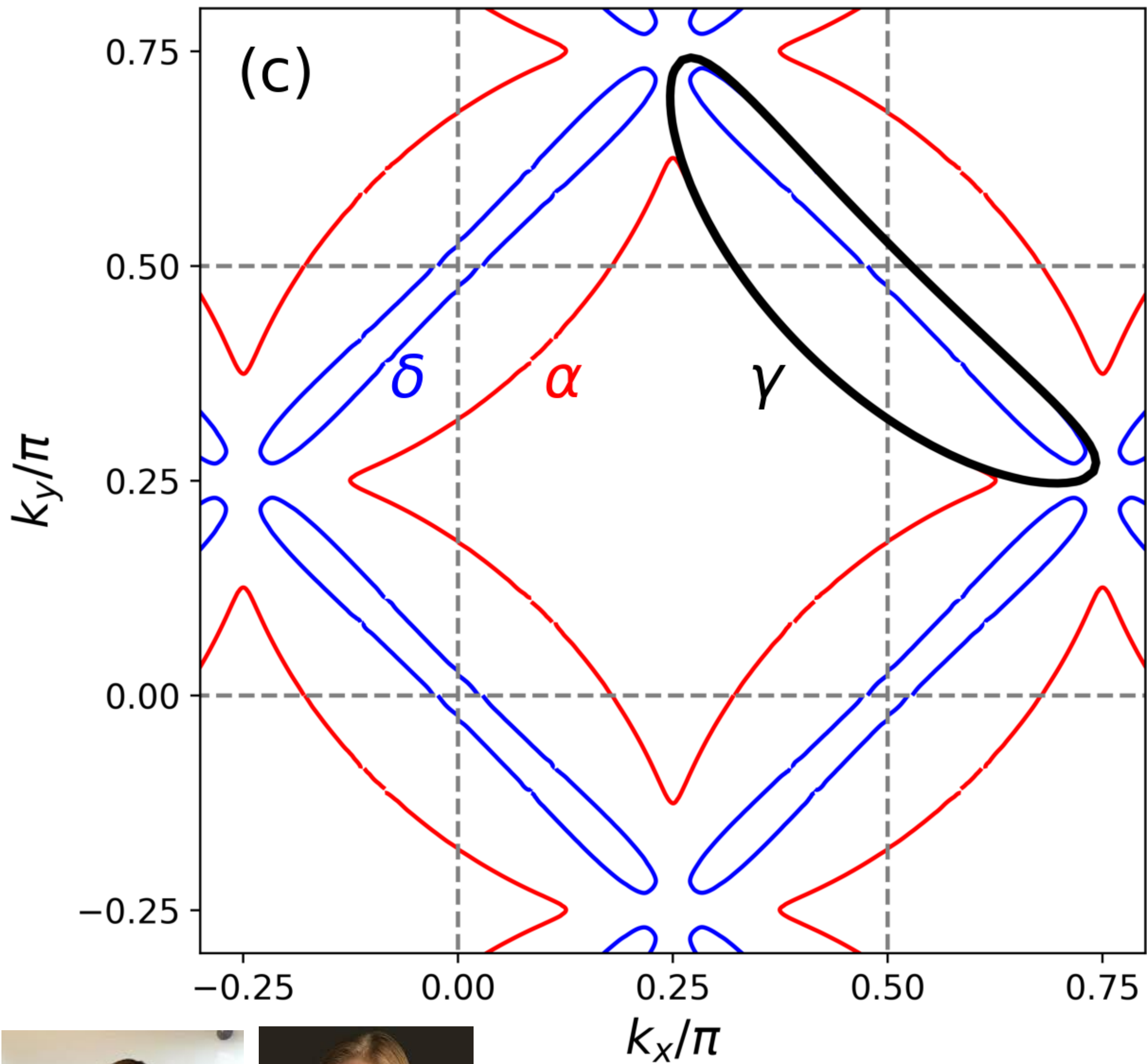
Long Zhang and Jia-Wei Mei,

EPL **114**, 47008 (2016)

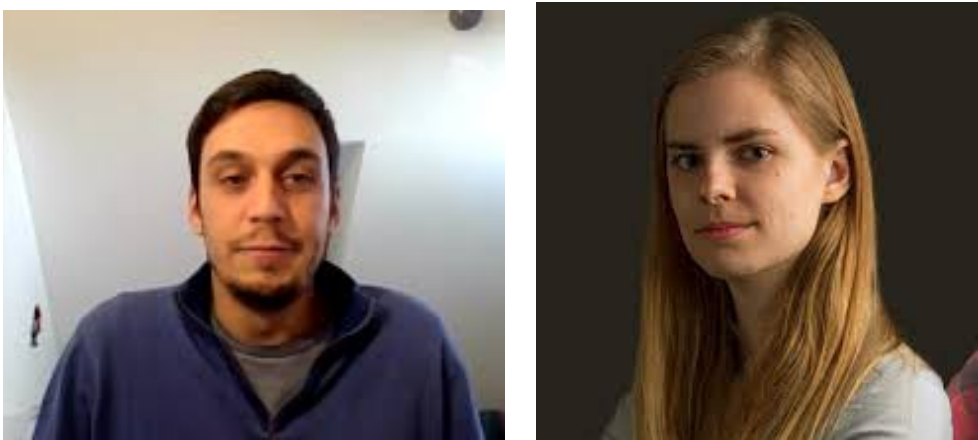


Pietro Bonetti, Maine Christos and S.S. (**BCS**), arXiv:2405.08817

# FL\* → CDW

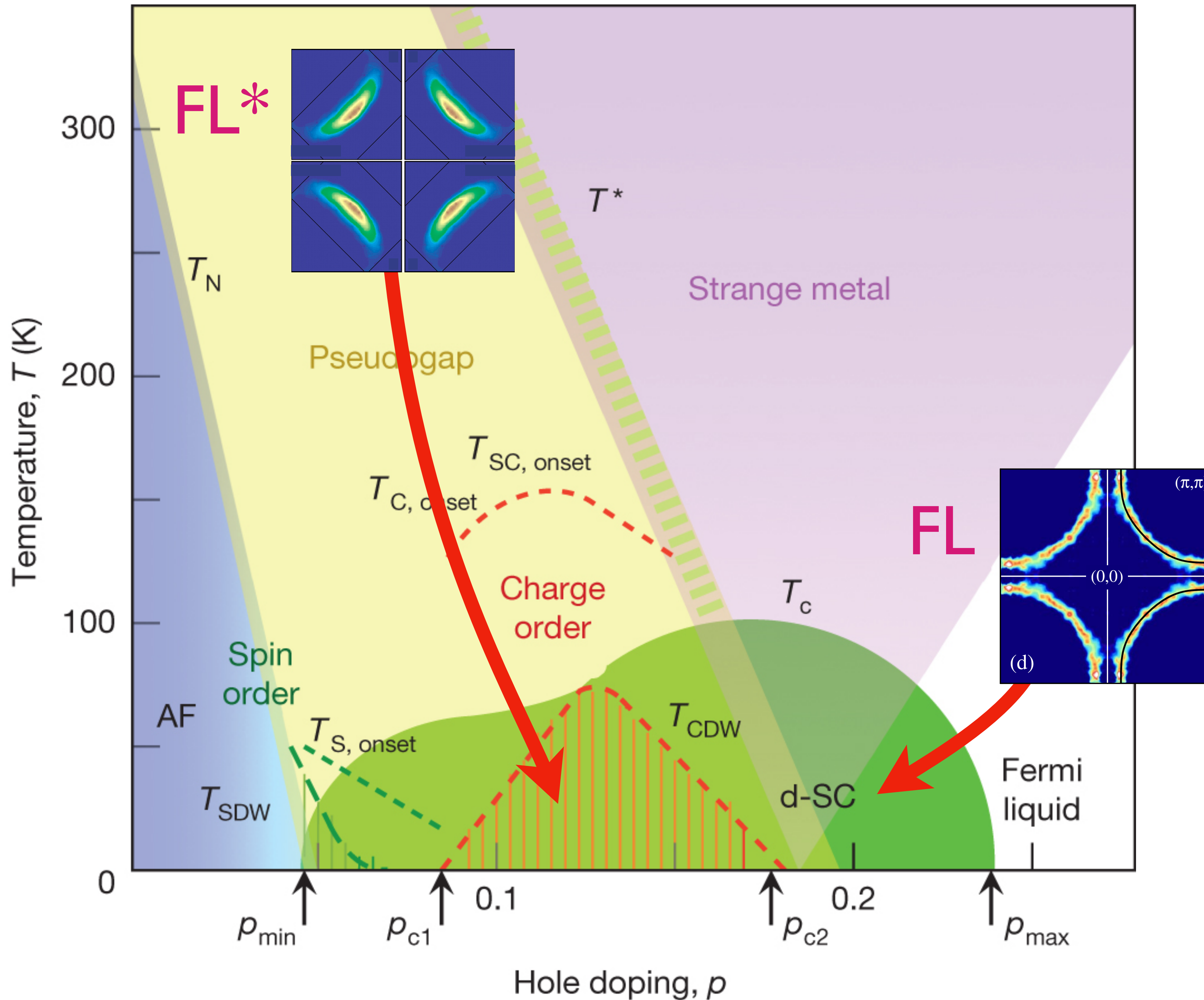


Period 4 CDW obtained by condensing  $B$ .  
 SU(2) gauge theory allows CDW with  
 vanishing  $\Delta_{ij}$  and  $J_{ij}$ .  
 Mixing between electrons and  
 spinons removes  $\beta$  pocket.



Pietro Bonetti, Maine Christos and S.S. (**BCS**), arXiv:2405.08817

Summary



Obtain *d*-wave superconductor and charge order from a theory of *confinement* instabilities of FL\*.

The resulting low *T* ordered states should be adiabatically connected to the corresponding states obtained from instabilities of FL.

Ancilla theory of pseudogap metal with hole pockets and underlying SU(2) gauge theory of  $\pi$ -flux spin liquid yields:

- Theory for Fermi arcs in hole-doped pseudogap metal.
- **ADMR** in pseudogap.
- Anti-nodal and nodal electronic dispersion.
- ***d*-wave superconductor with 4 nodal points in both electron- and hole-doped cuprates.**
- Near-equality of dSC and charge order onset temperatures
- **Multipoint correlators measured by cold atom experiments**
- Theory for strange metal in the crossover from FL\* to FL
- **Theory for quantum oscillations in high magnetic field**