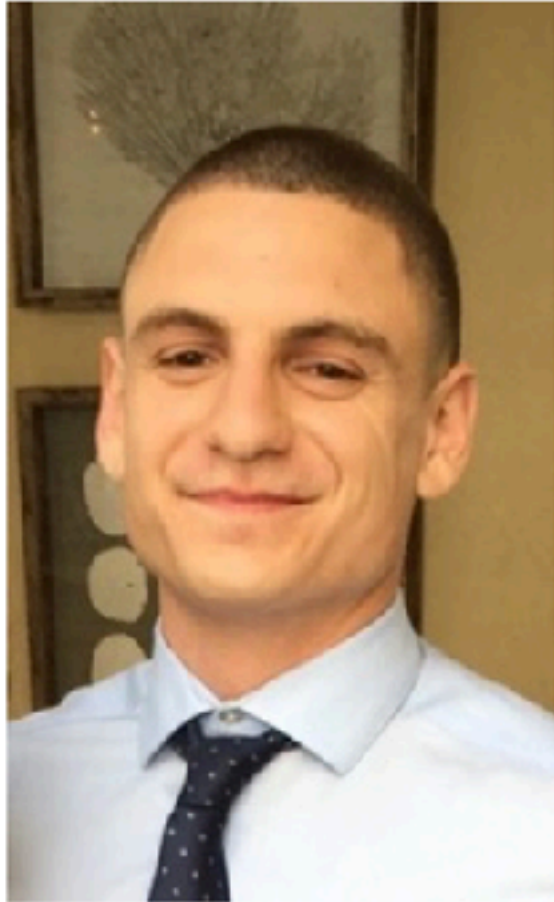


Universal theory for quantum phase transitions in two-dimensional metals with Harris disorder

Subir Sachdev

Theories, Experiments and Numerics on
Gapless Quantum Many-body Systems
KITP, Santa Barbara
May 14, 2024





Ilya Esterlis
Wisconsin



Haoyu Guo
Cornell



Aavishkar Patel
Flatiron



Chenyuan Li
Harvard → Rice



Davide Valentinis
KIT



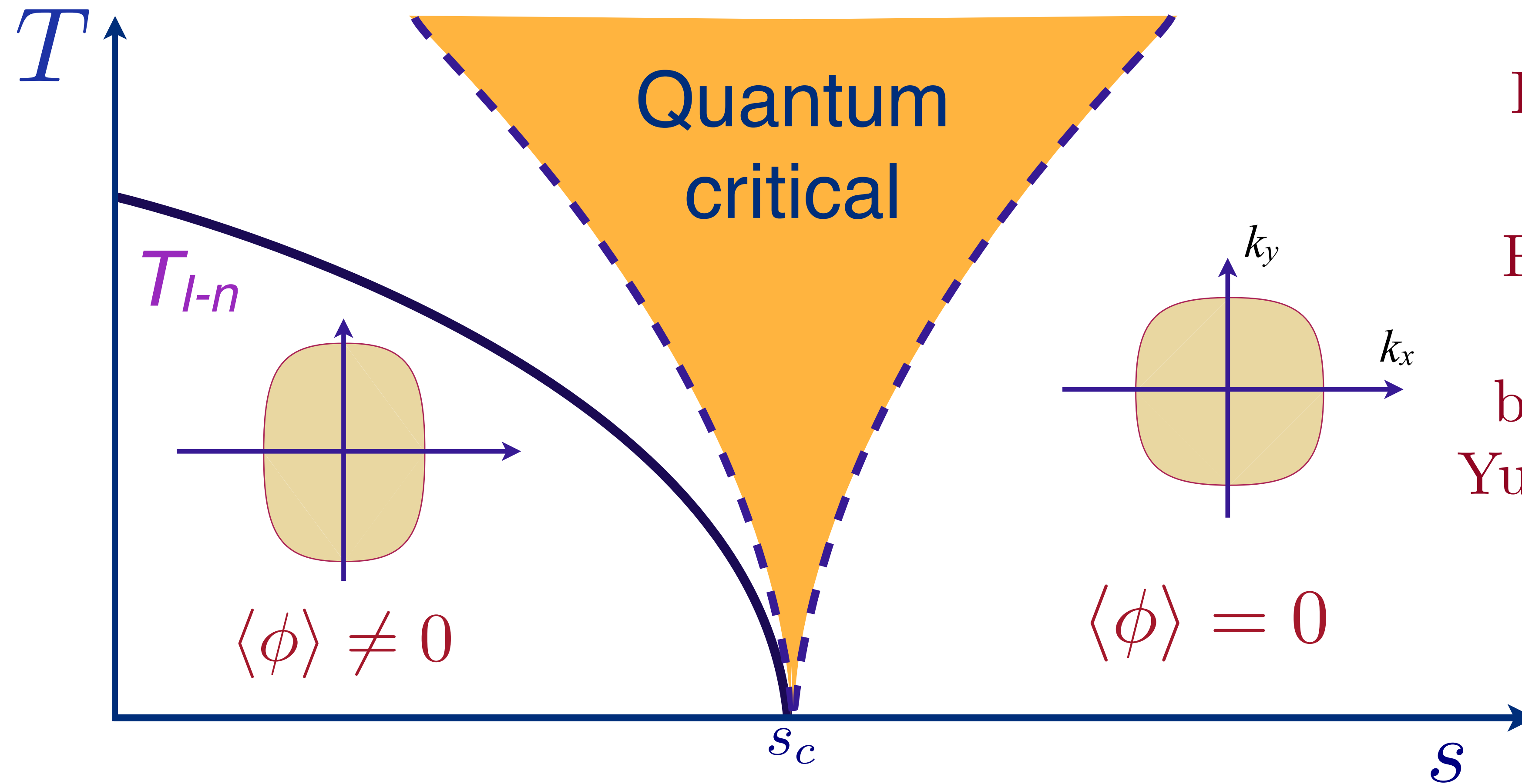
Joerg Schmalian
KIT



Peter Lunts
Harvard

Quantum phase transitions in two-dimensional metals

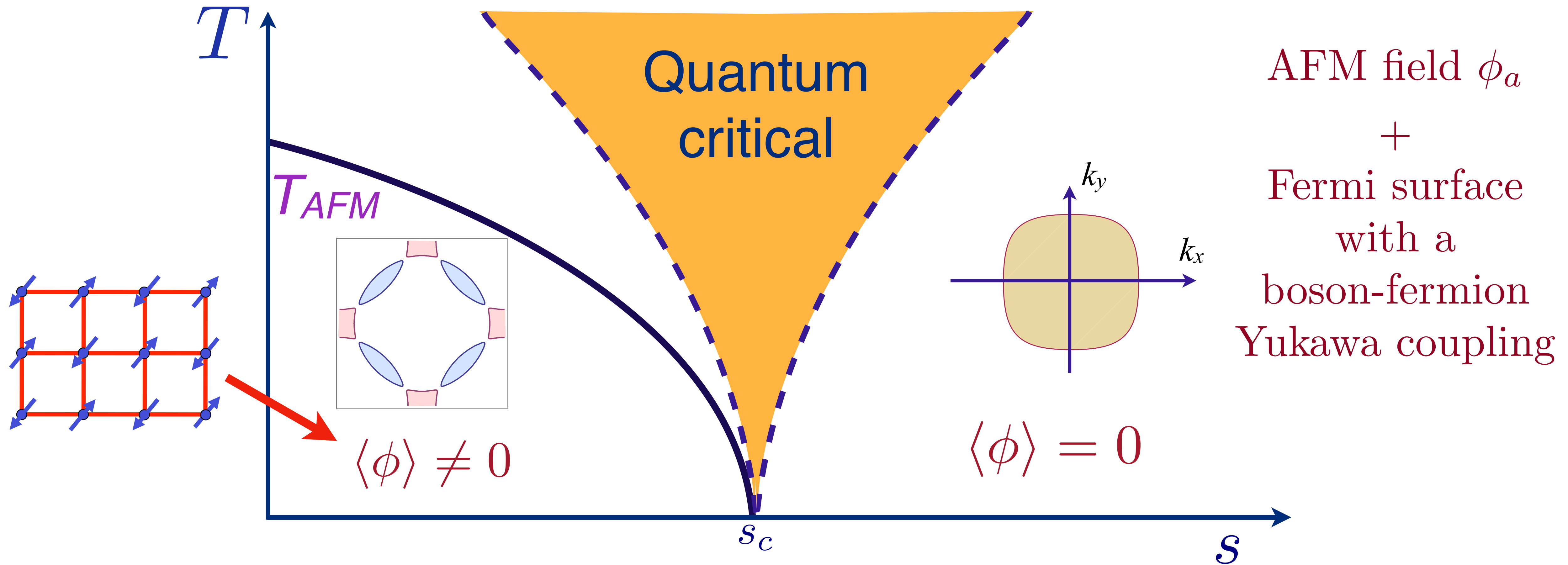
Type I: Fermi surface deformation



Ising field ϕ
+
Fermi surface
with a
boson-fermion
Yukawa coupling

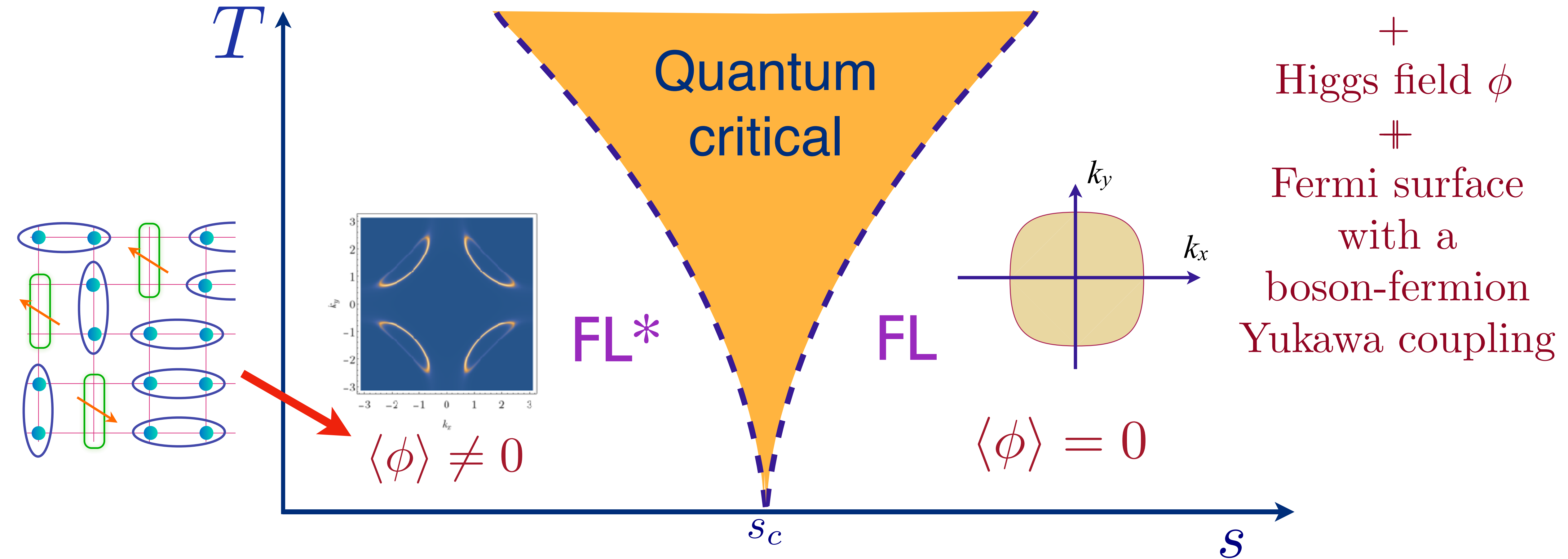
Quantum phase transitions in two-dimensional metals

Type II: Fermi surface reconstruction



Quantum phase transitions in two-dimensional metals

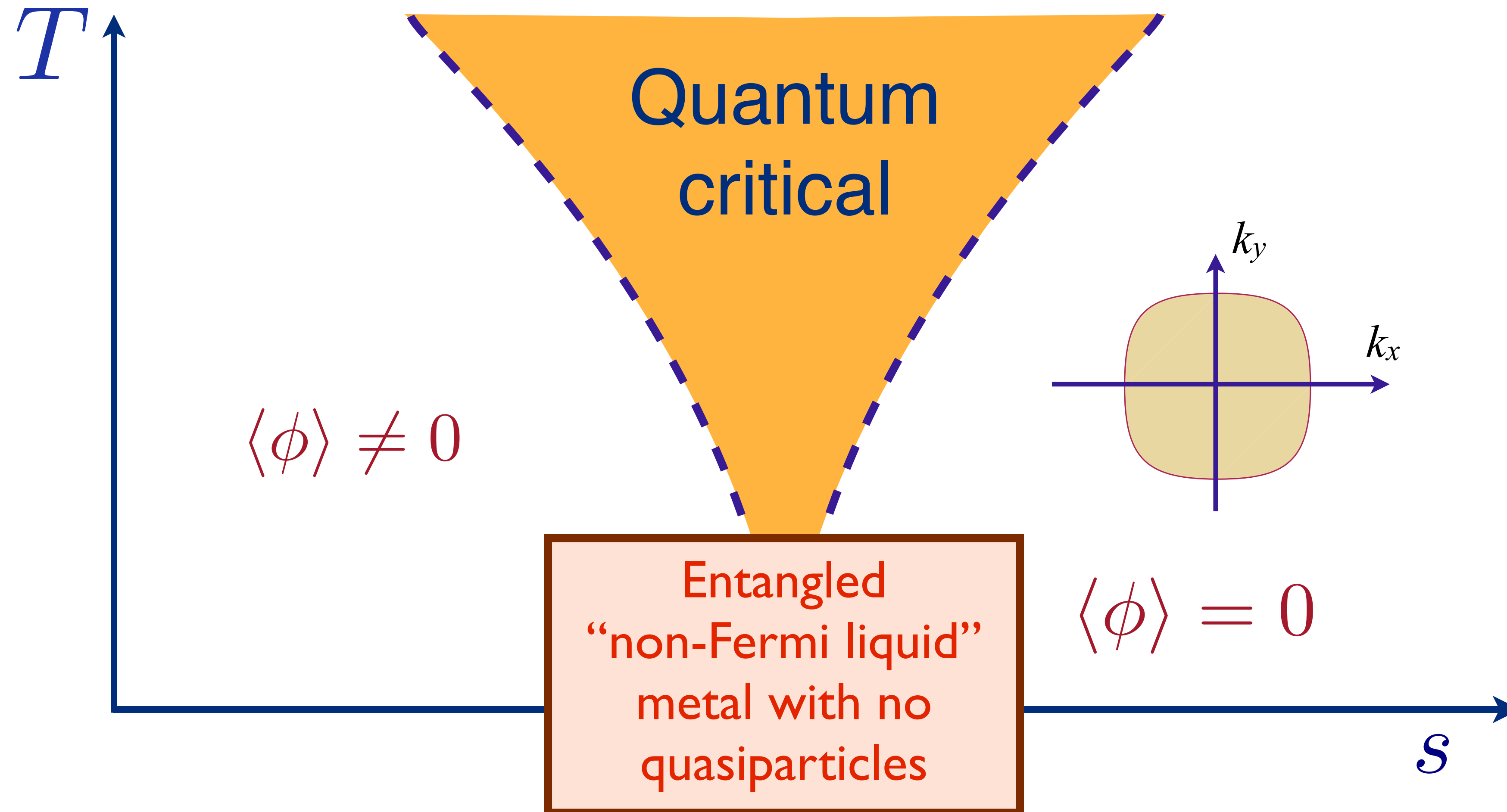
Type III: Fermi surface jump



Applies to hole-doped cuprates

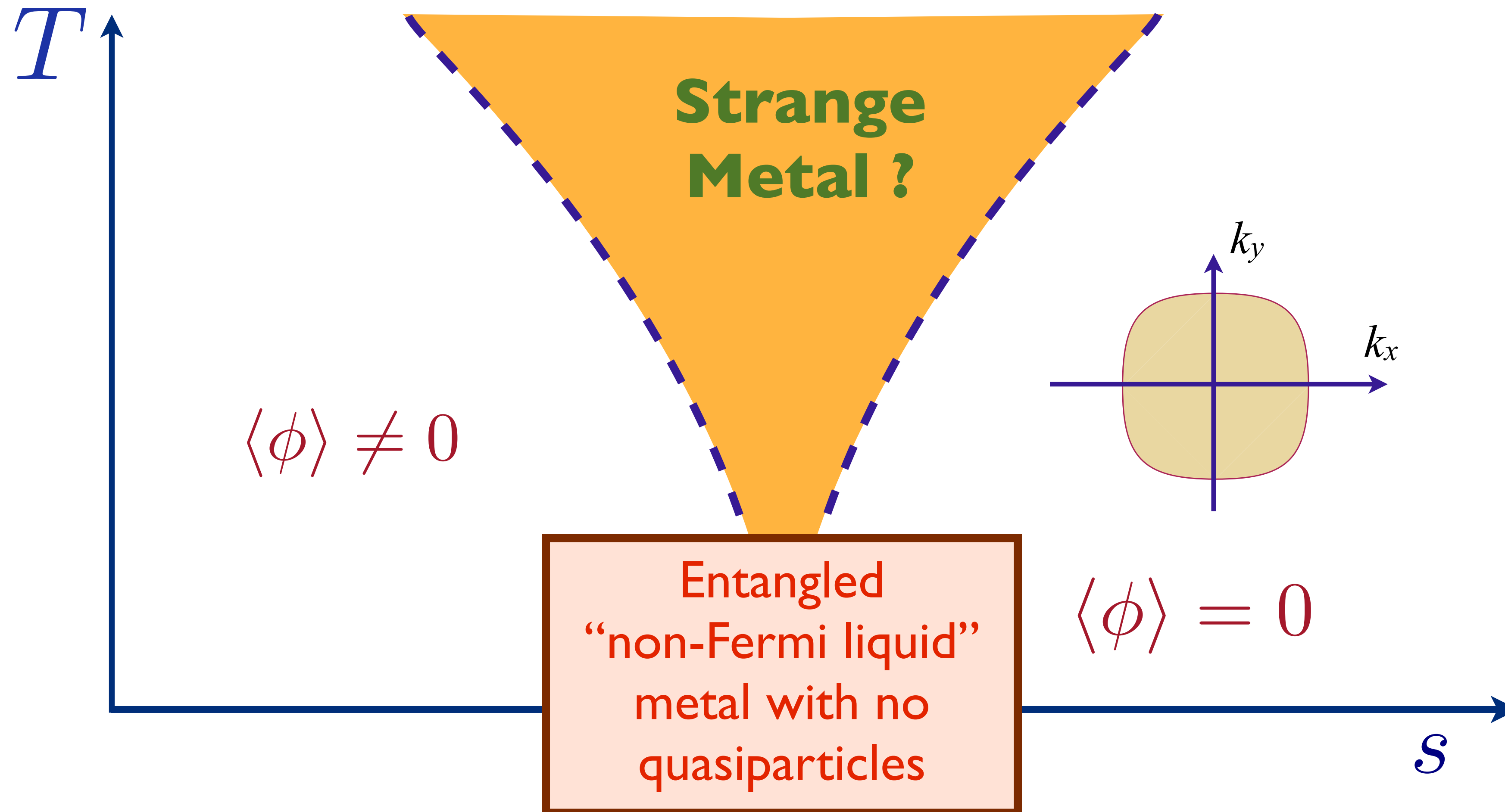
Quantum phase transitions in two-dimensional metals

Type I, II, or III



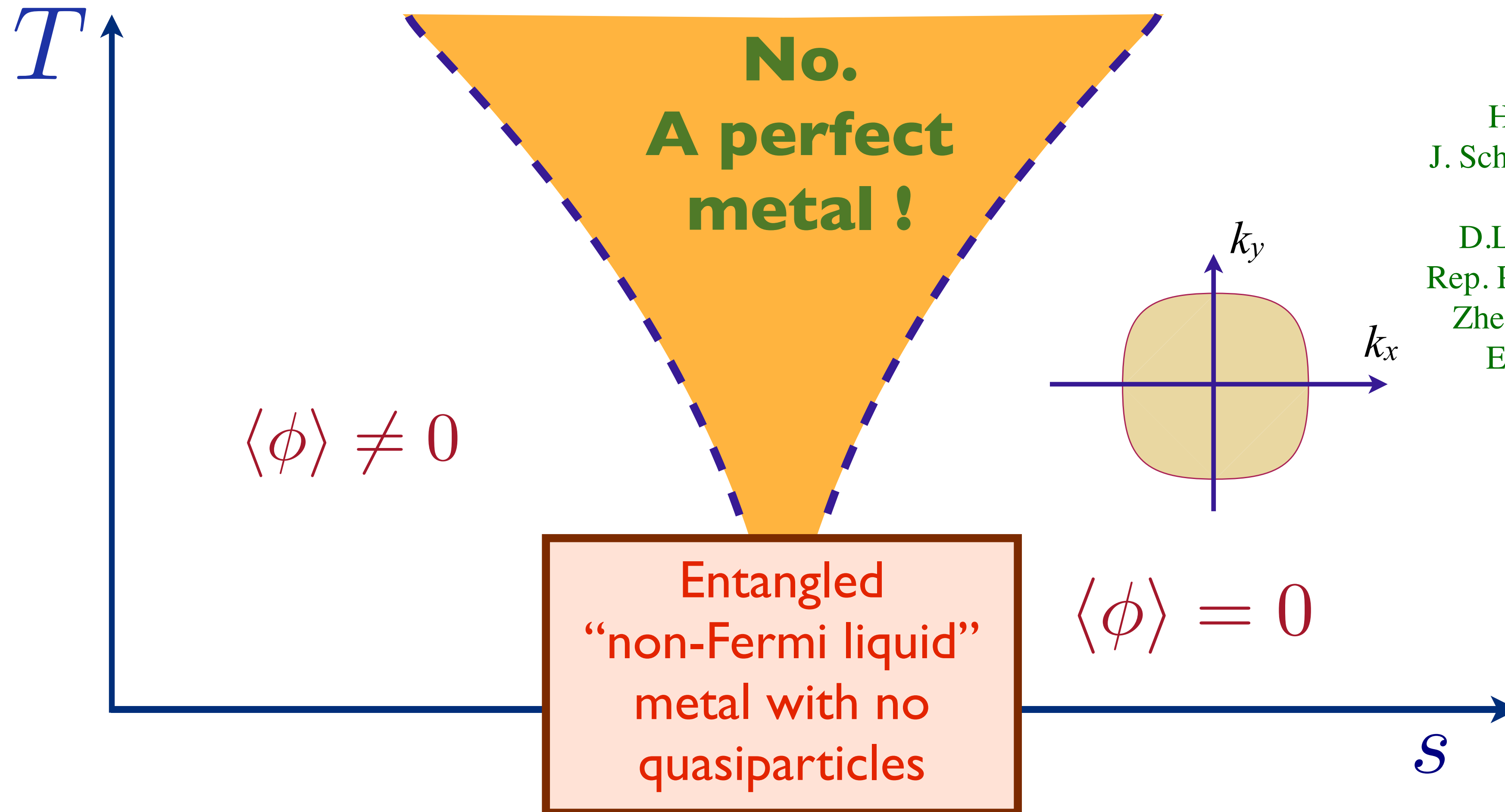
Quantum phase transitions in two-dimensional metals

Type I, II, or III



Quantum phase transitions in two-dimensional metals

Type I, II, or III



S. A. Hartnoll, P. K. Kovtun,
M. Muller, and S.S.
PRB **76**, 144502 (2007)

Haoyu Guo, Aavishkar Patel,
Ilya Esterlis, S.S.,
PRB **106**, 115151 (2022)

Haoyu Guo, Davide Valentinis,
J. Schmalian, S.S., Aavishkar Patel,
PRB **109**, 075162 (2024)

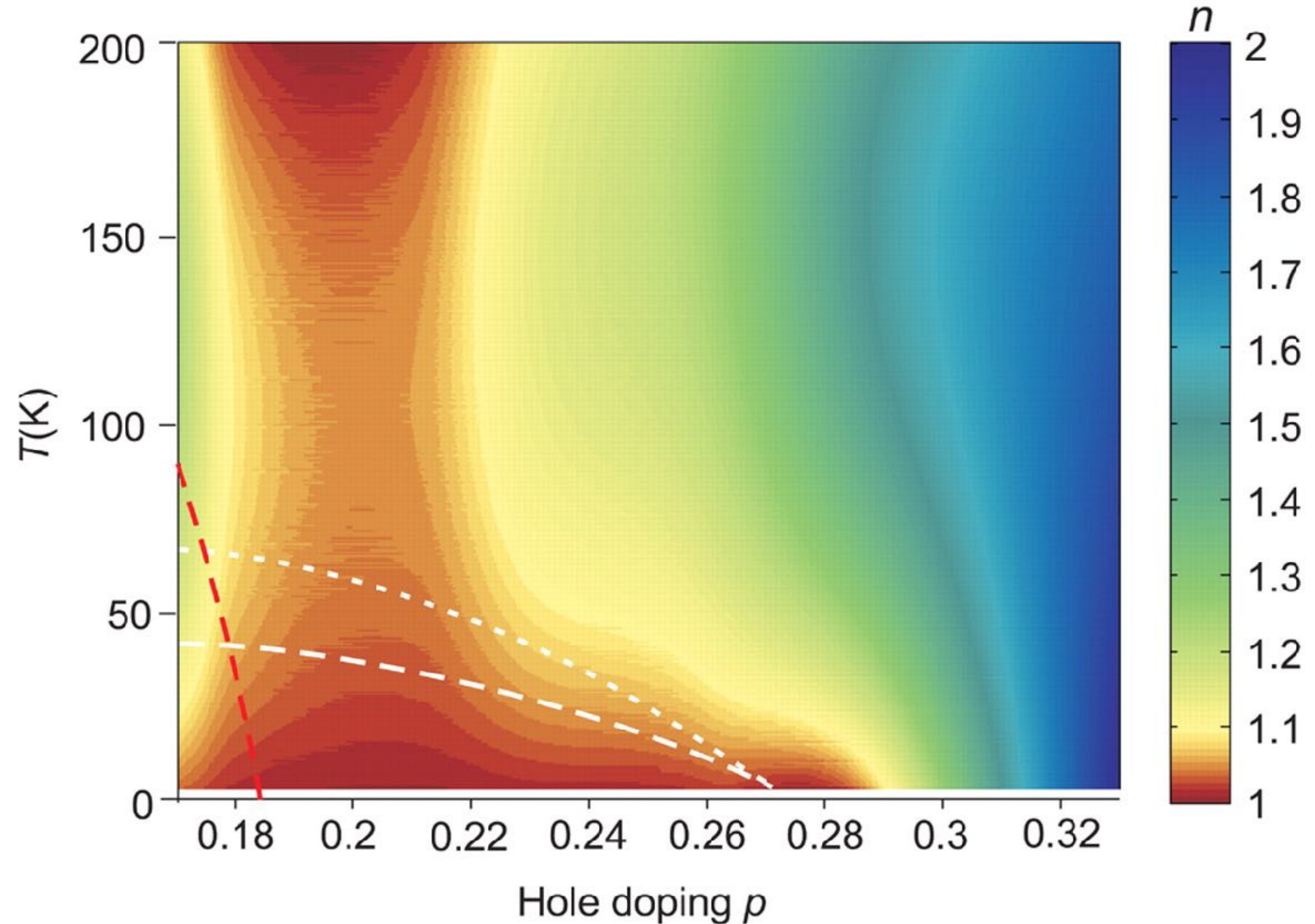
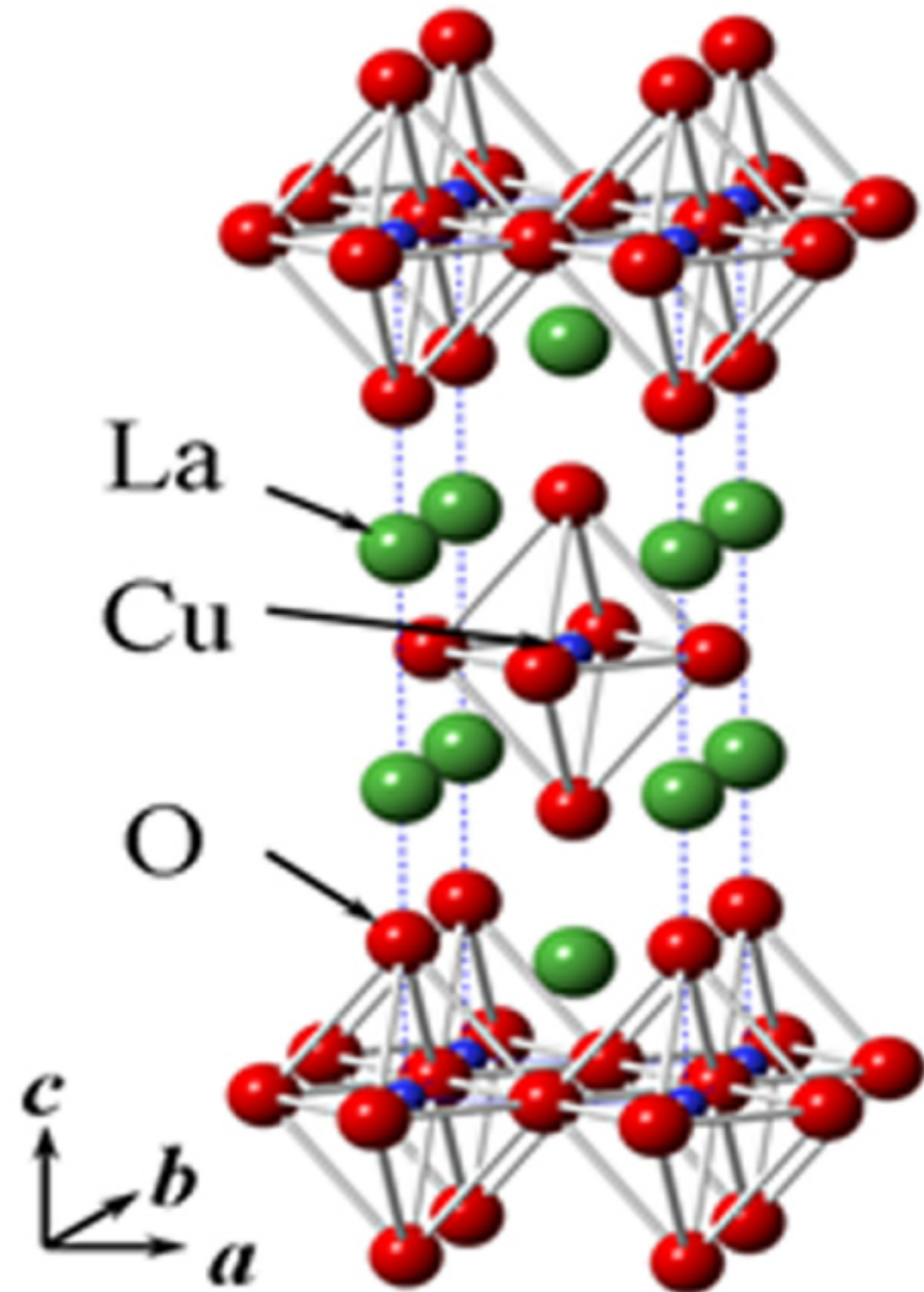
D.L. Maslov and A.V. Chubukov,
Rep. Prog. Phys. **80**, 026503 (2017)

Zhengyan Darius Shi, Dominic V.
Else, Hart Goldman, T. Senthil,
SciPost Phys. **14**, 113 (2023)

Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

R. A. Cooper,¹ Y. Wang,¹ B. Vignolle,² O. J. Lipscombe,¹ S. M. Hayden,¹ Y. Tanabe,³ T. Adachi,³ Y. Koike,³ M. Nohara,^{4*} H. Takagi,⁴ Cyril Proust,² N. E. Hussey^{1†}

SCIENCE VOL 323 603 2009

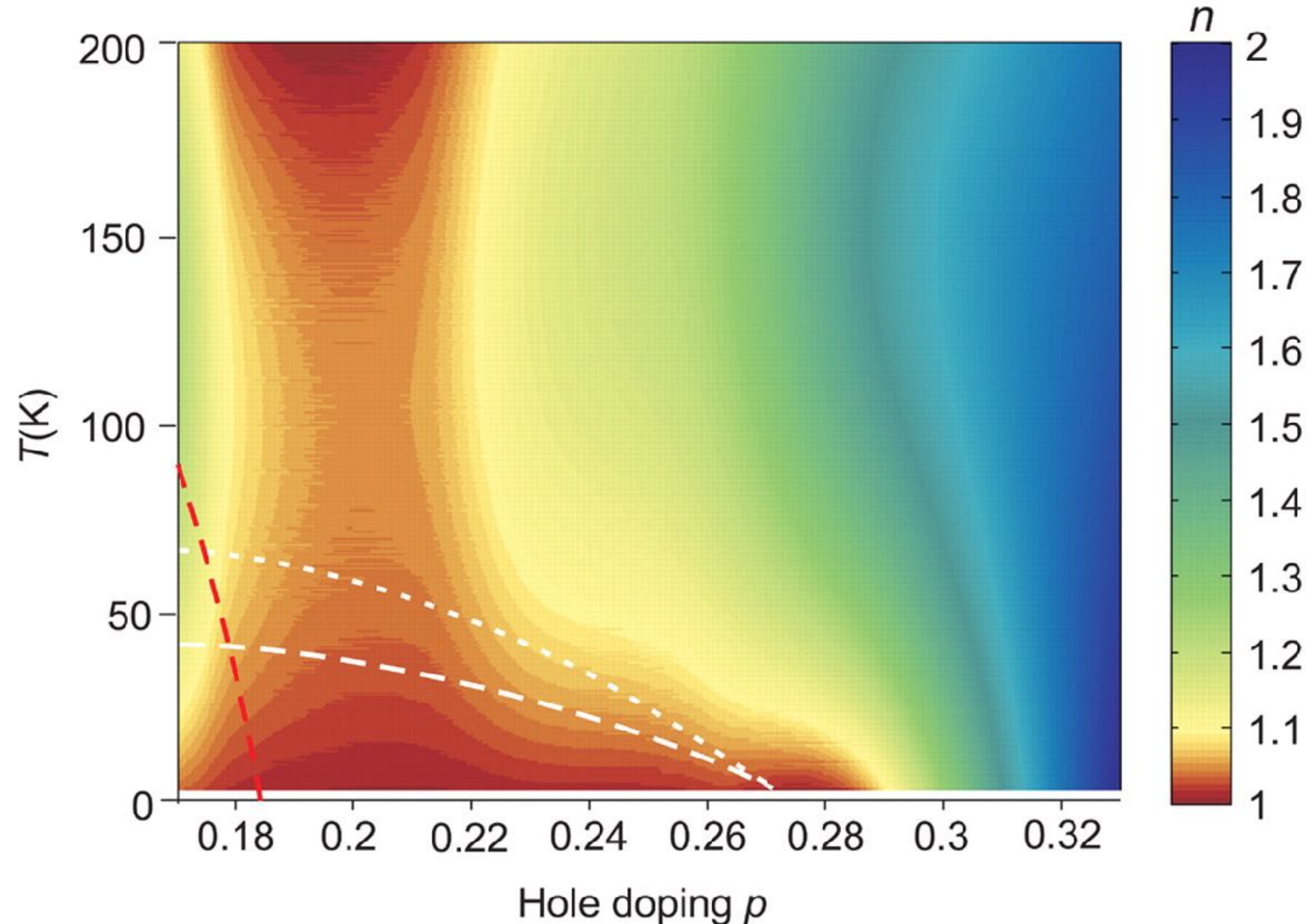
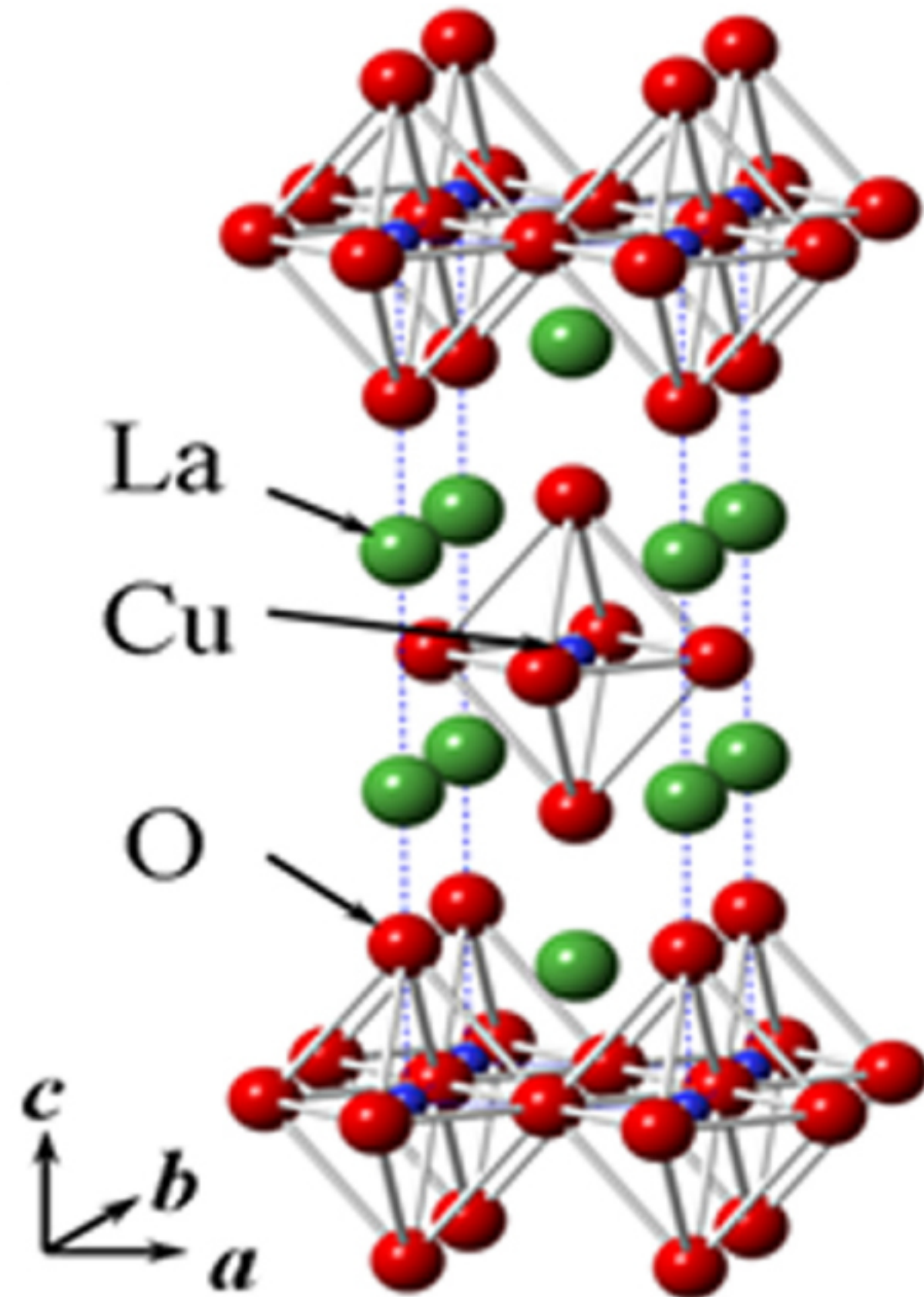


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SCIENCE VOL 323 603 2009

Two-dimensional metals with Harris disorder



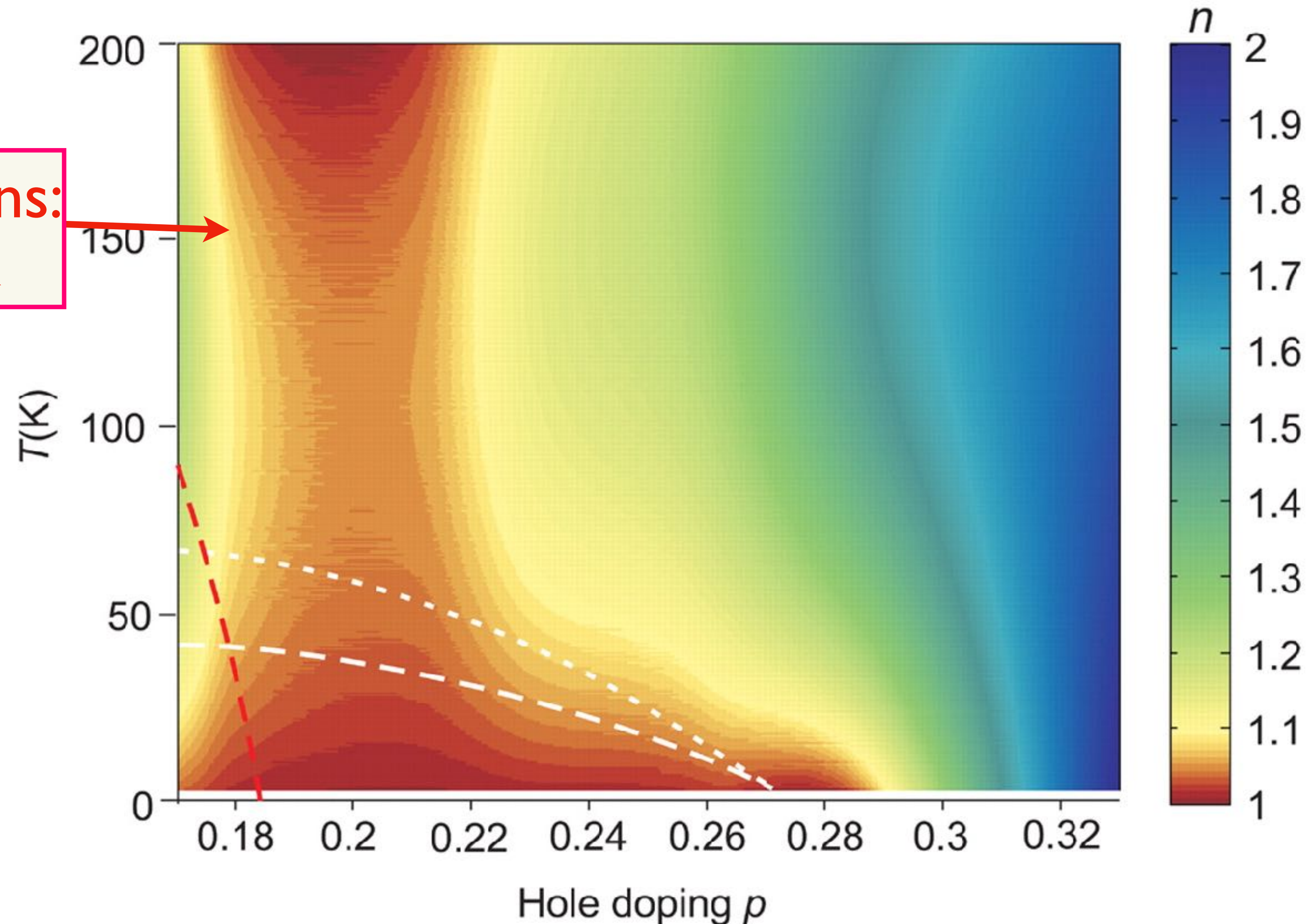
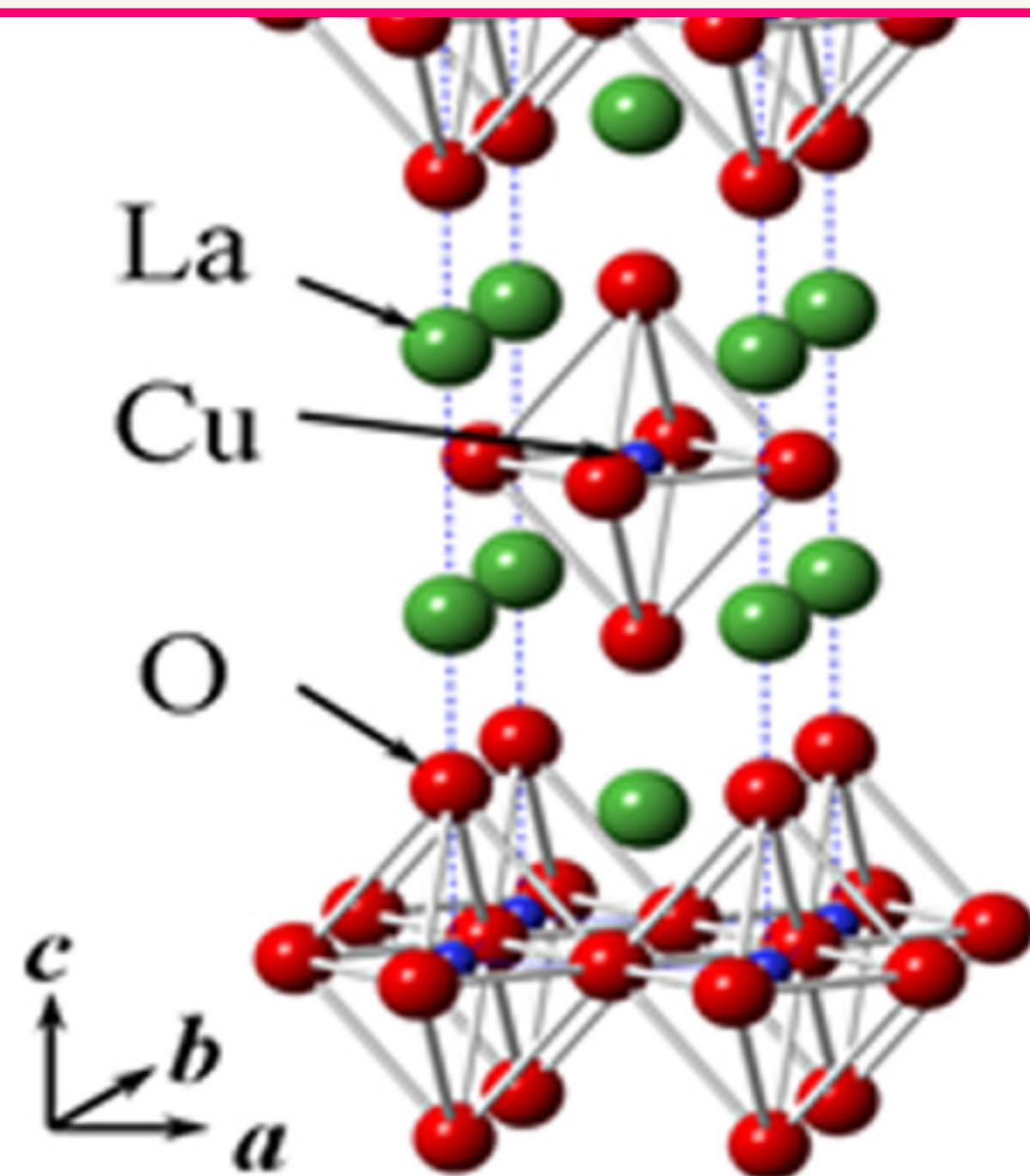
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SCIENCE VOL 323 603 2009

Two-dimensional metals with Harris disorder

Extended bosons and fermions:
physics of $d=2$ Yukawa-SYK



Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

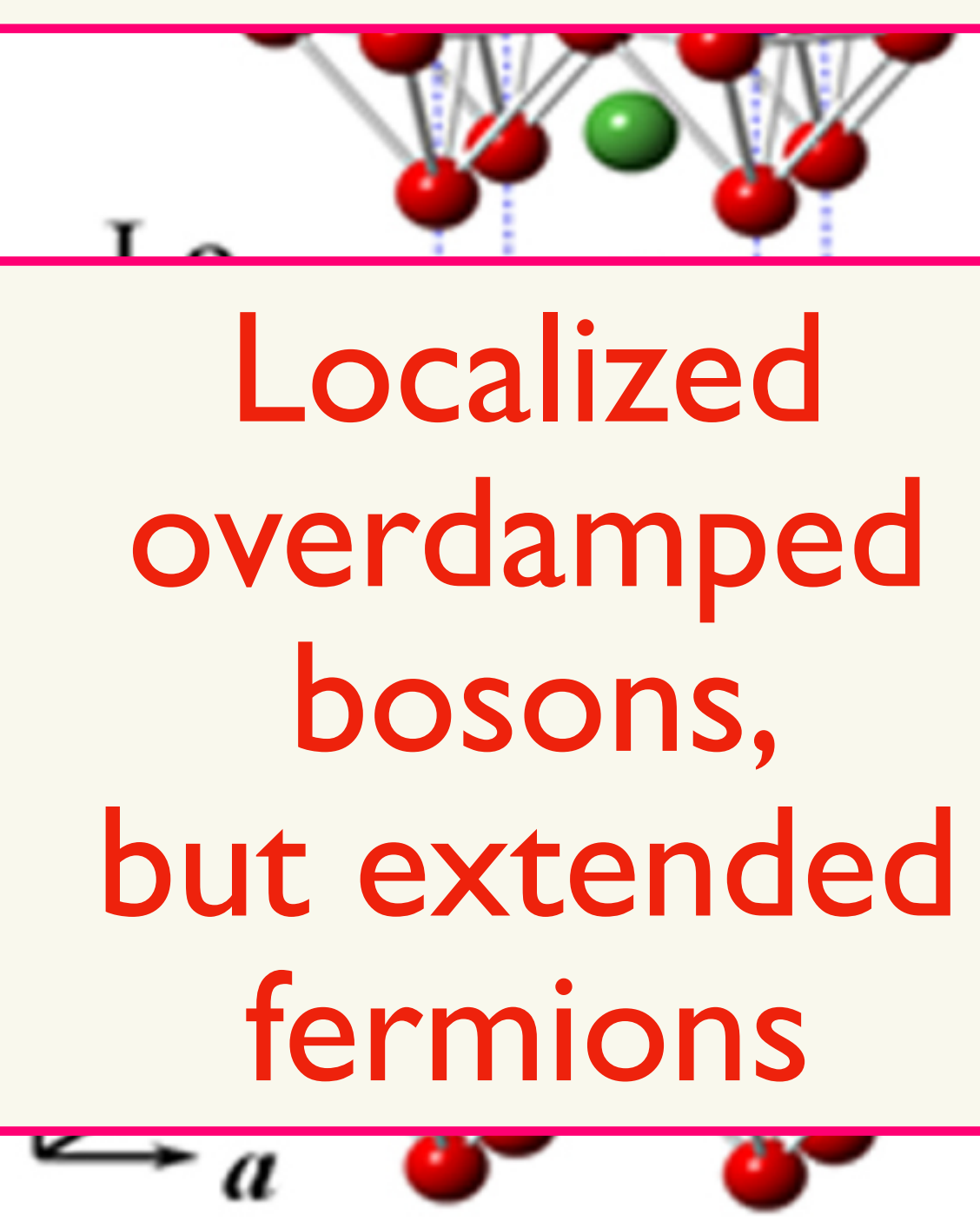
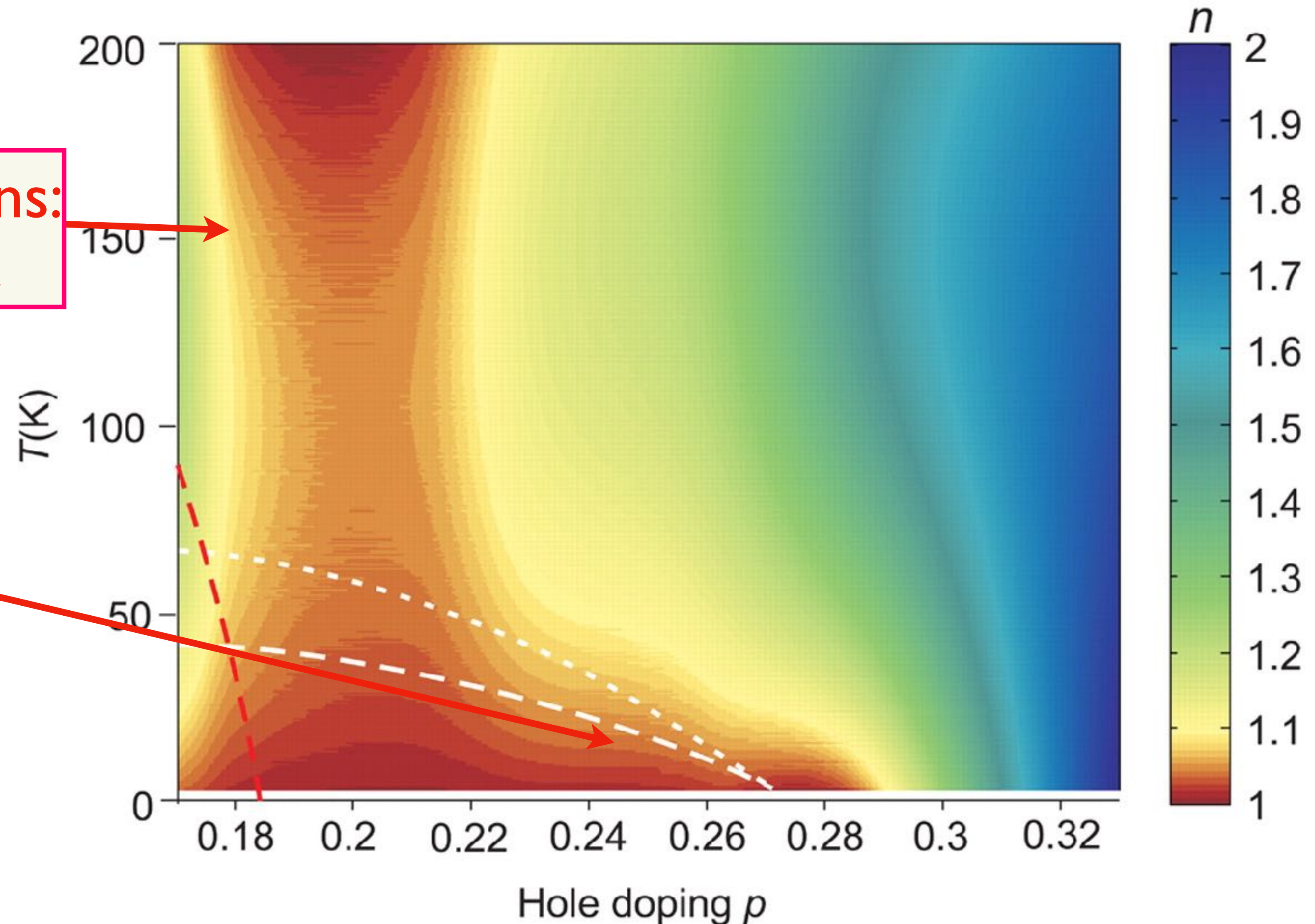
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SCIENCE VOL 323 603 2009

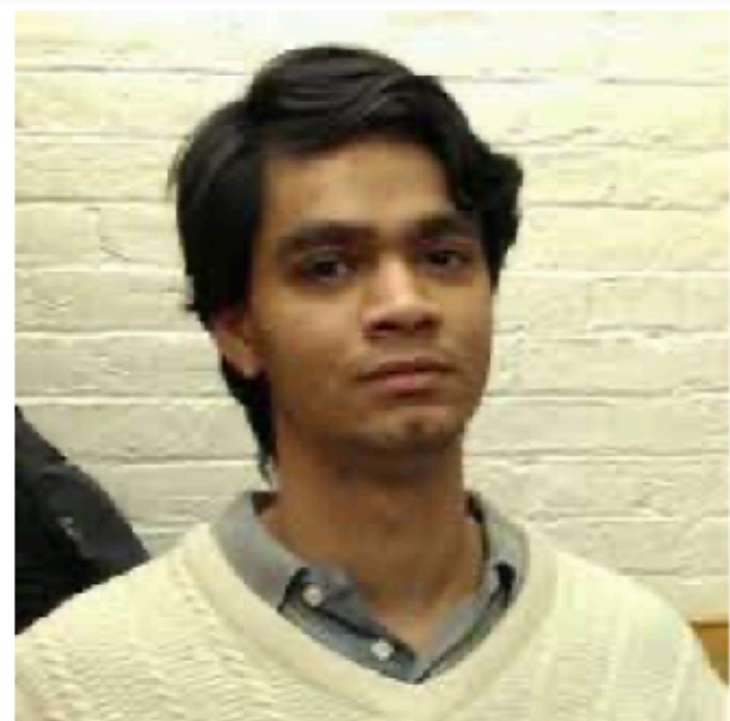
Two-dimensional metals with Harris disorder

Extended bosons and fermions:
physics of $d=2$ Yukawa-SYK

Localized overdamped bosons, but extended fermions



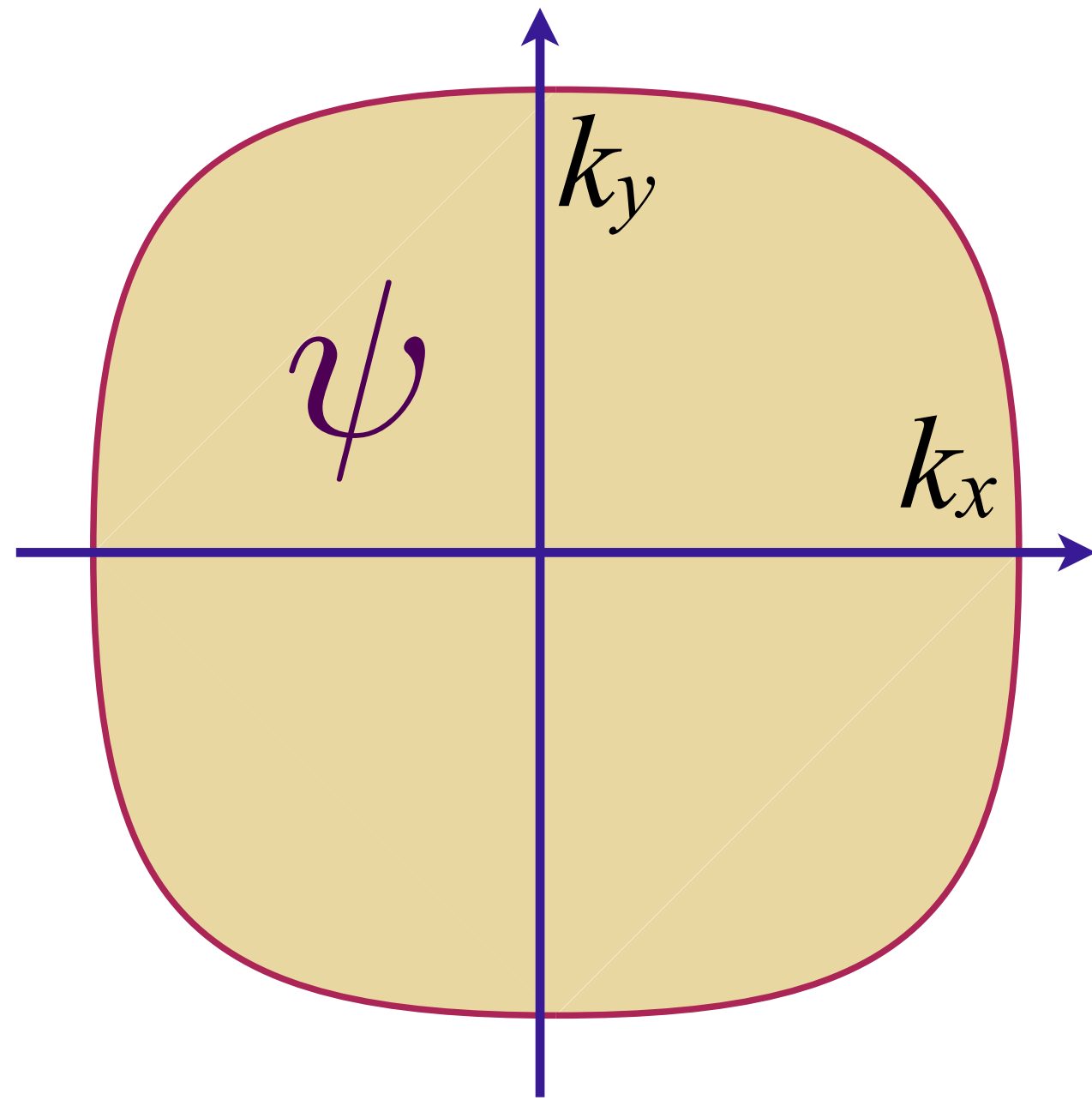
Overdamped boson localization from Harris disorder



Aavishkar A. Patel, Peter Lunts, S.S., PNAS **121**, e2402052121 (2024)

Fermi surface + critical boson with no spatial disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,

Higgs boson for Fermi-volume changing transition

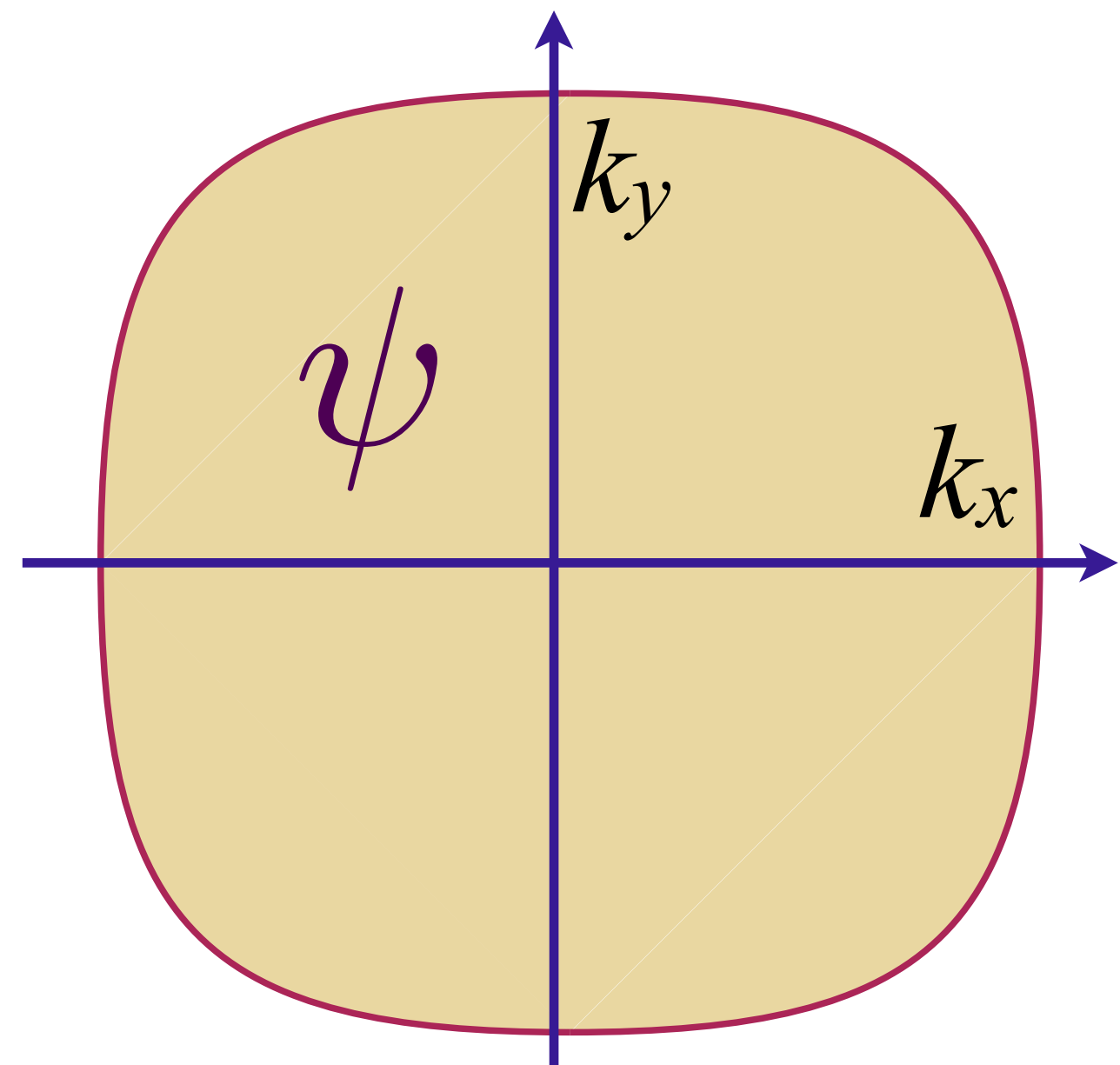
$$+s [\phi(\mathbf{r})]^2$$

$$+g \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

$$+K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4$$

Fermi surface + critical boson with potential disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
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Higgs boson for Fermi-volume changing transition

$$+s [\phi(\mathbf{r})]^2$$

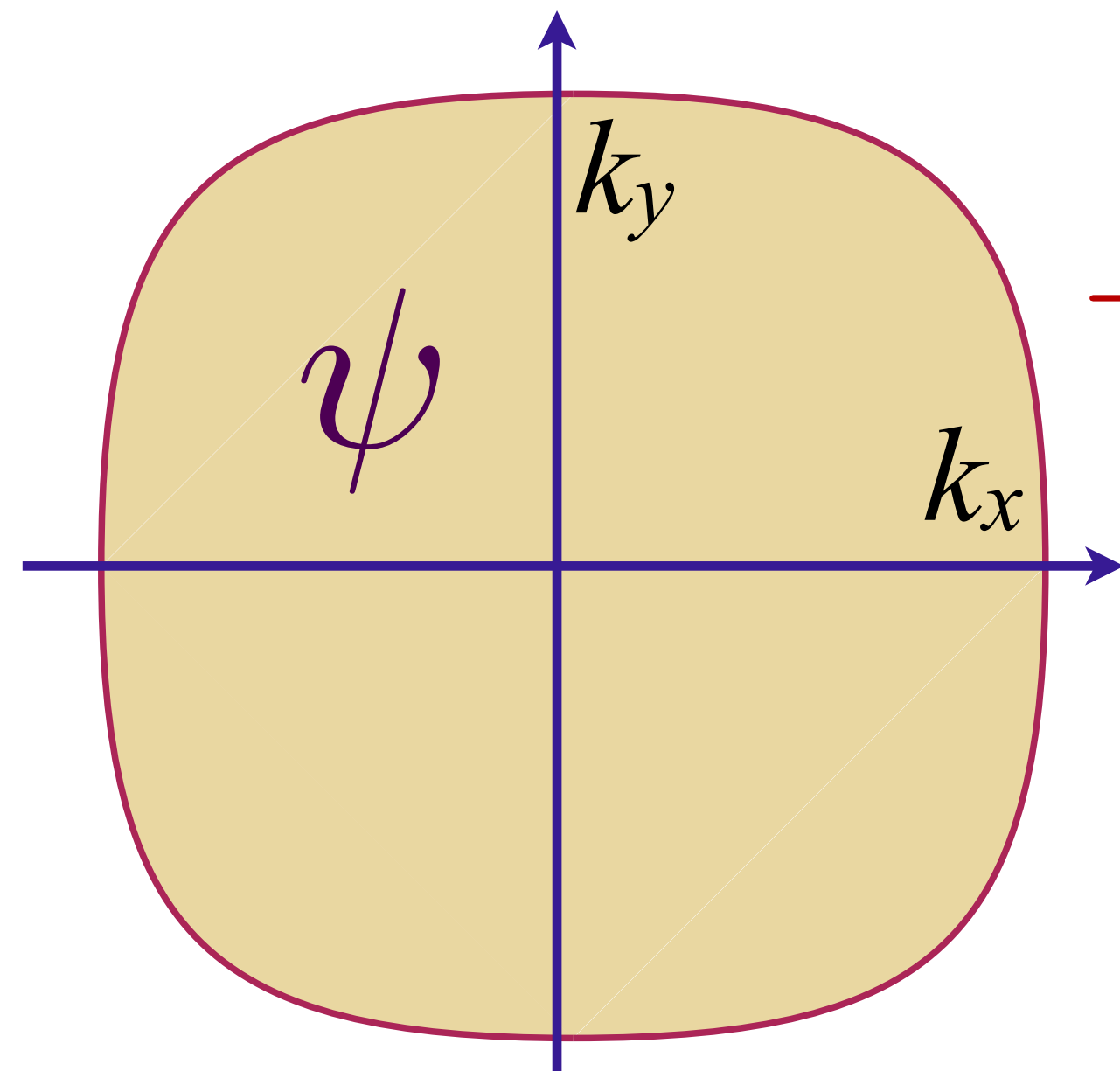
$$+g \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

$$+K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4 + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
 spin-density wave order,

Higgs boson for Fermi-volume changing transition

$$+ [s + \delta s(\mathbf{r})] [\phi(\mathbf{r})]^2 + +g \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

$$+ K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4 + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random mass $\delta s(\mathbf{r})$ with $\overline{\delta s(\mathbf{r})} = 0$, $\overline{\delta s(\mathbf{r})\delta s(\mathbf{r}')} = \delta s^2 \delta(\mathbf{r} - \mathbf{r}')$

RG analysis (Harris criterion) shows that $\delta s(\mathbf{r})$ is most relevant disorder.

Bosonic eigenmodes in random mass Hertz theory

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_\phi = \int d\tau \left[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \left(\frac{s + s'_j}{2} \phi_{ja}^2 + \frac{u}{4M} (\phi_{ja}^2)^2 \right) \right]$$
$$\mathcal{S}_{\phi d} = \frac{T}{2} \sum_{\Omega} \sum_j (\gamma |\Omega| + \Omega^2 / c^2) |\phi_{ja}(i\Omega)|^2,$$

where $a = 1 \dots M$ is a flavor index for an order parameter with $O(M)$ symmetry.

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where $a = 1 \dots M$ is a flavor index for an order parameter with $O(M)$ symmetry. Analyze in a self-consistent quadratic theory, treating disorder numerically exactly

$$\bar{\mathcal{S}}_\phi = \int d\tau \left[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \frac{\bar{s}'_j}{2} \phi_{ja}^2 \right]$$

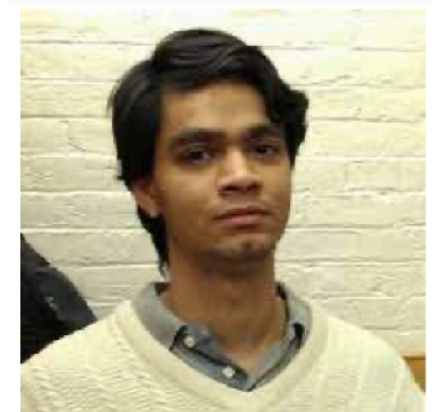
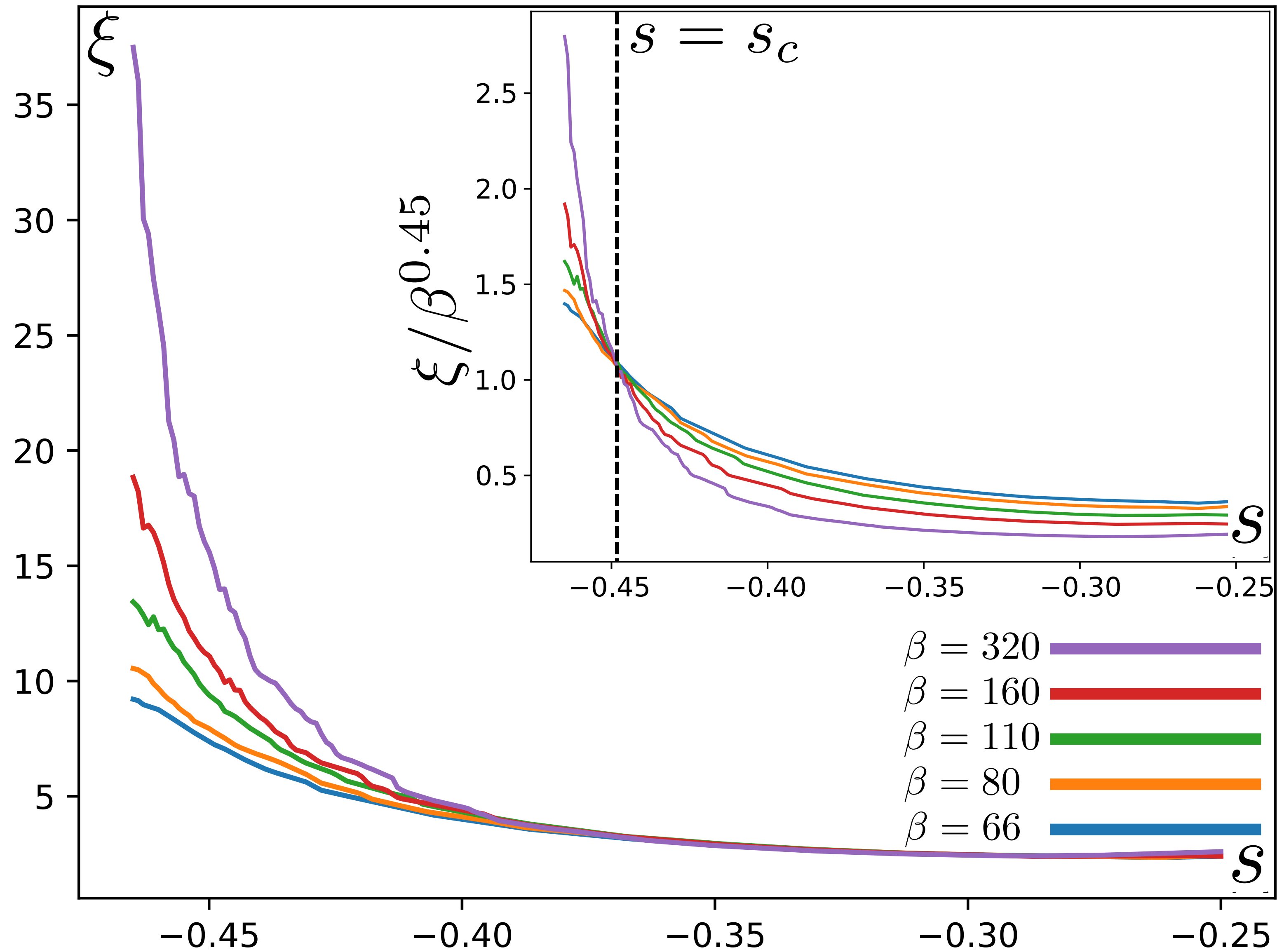
Similar analysis in $d = 1$ works very well
A. Del Maestro, B. Rosenow, M. Müller and S. Sachdev,
Phys. Rev. Lett. **101**, 035701 (2008).

$$\bar{s}'_j = s + s'_j + \frac{u}{M} \sum_a \langle \phi_{ja}^2 \rangle_{\bar{\mathcal{S}}_\phi + \mathcal{S}_{\phi d}} = s + s'_j + uT \sum_{\Omega} \sum_{\alpha} \frac{\psi_{\alpha i} \psi_{\alpha j}}{\gamma|\Omega| + \Omega^2/c^2 + e_{\alpha}}$$

where e_{α} and $\psi_{\alpha j}$ are eigenvalues and eigenfunctions of the ϕ quadratic form in $\bar{\mathcal{S}}_\phi$, labeled by the index $\alpha = 1 \dots L^2$ for a $L \times L$ sample.

Bosonic eigenmodes in random mass Hertz theory

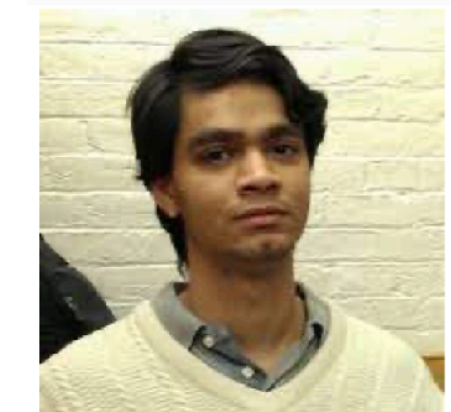
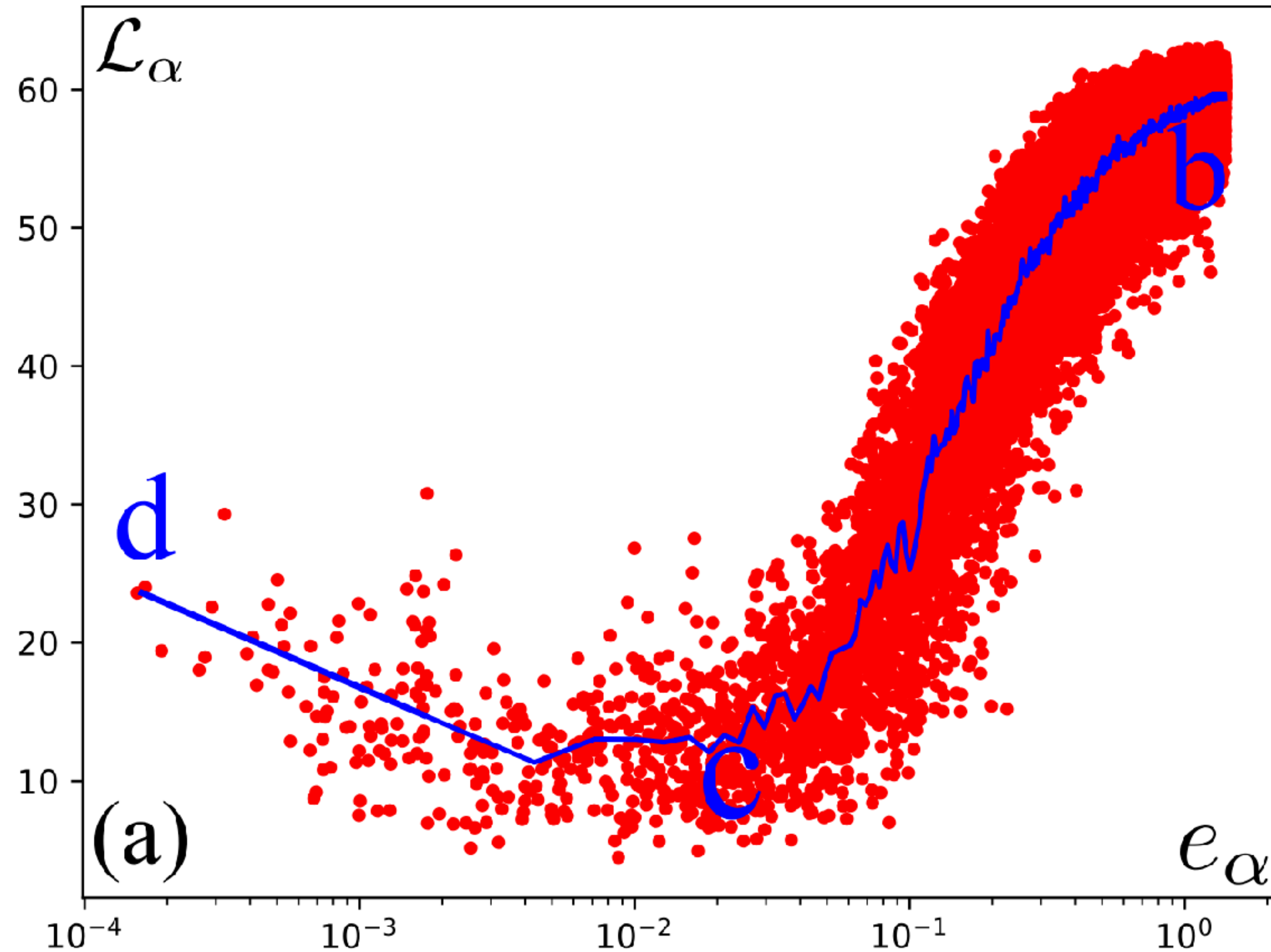
ϕ correlation length ξ



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length \mathcal{L}_α

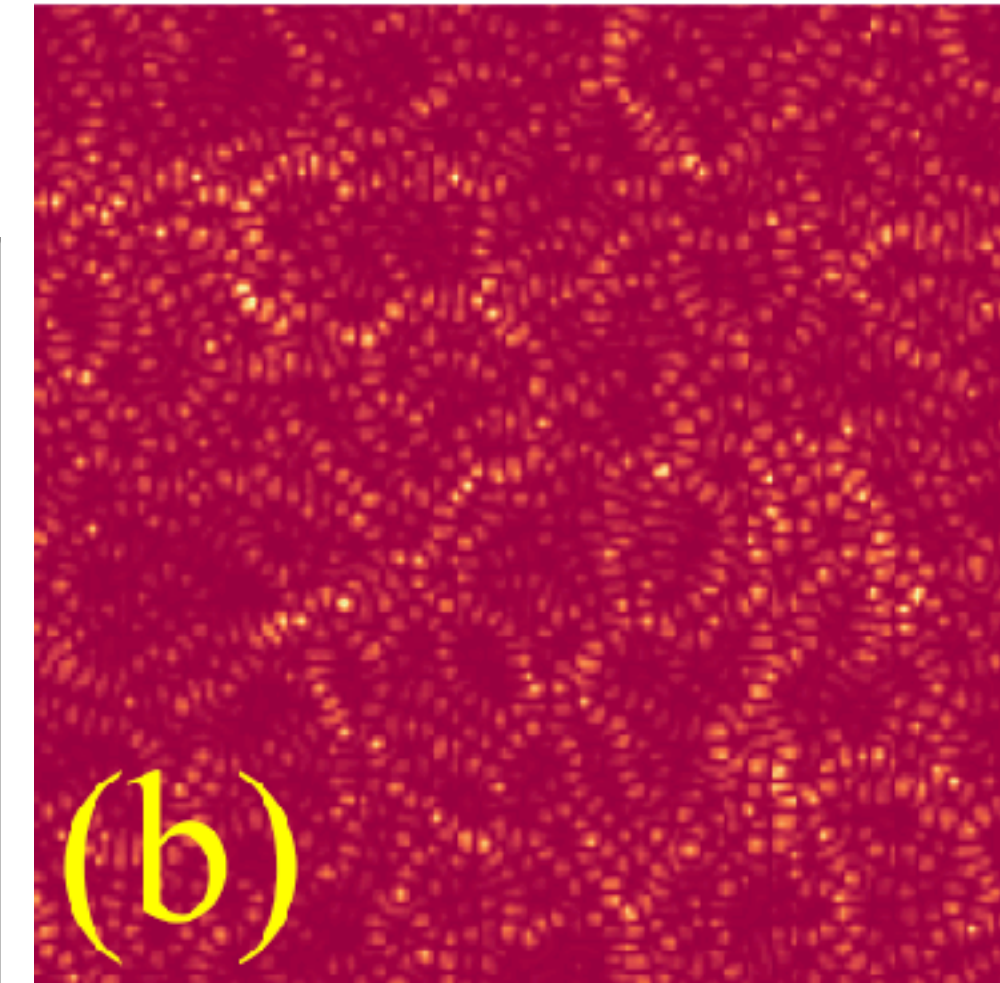
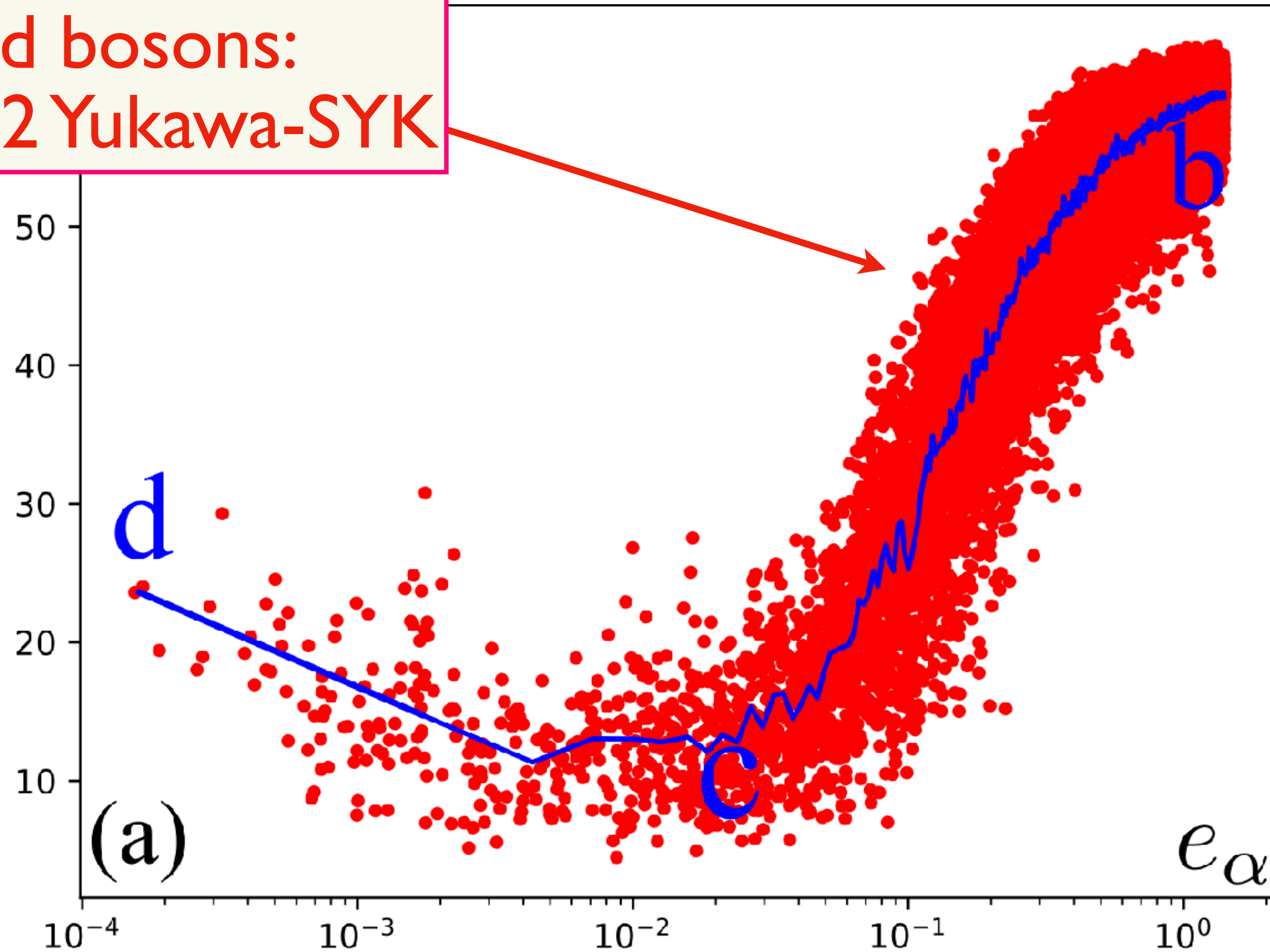


Aavishkar A. Patel,
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PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length \mathcal{L}_α

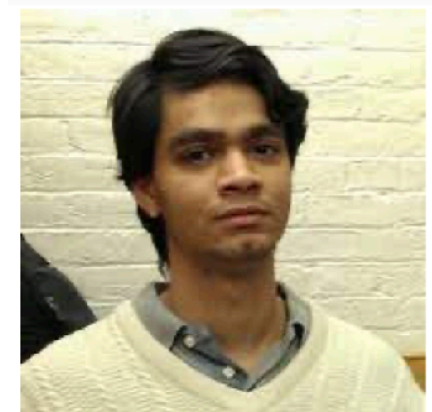
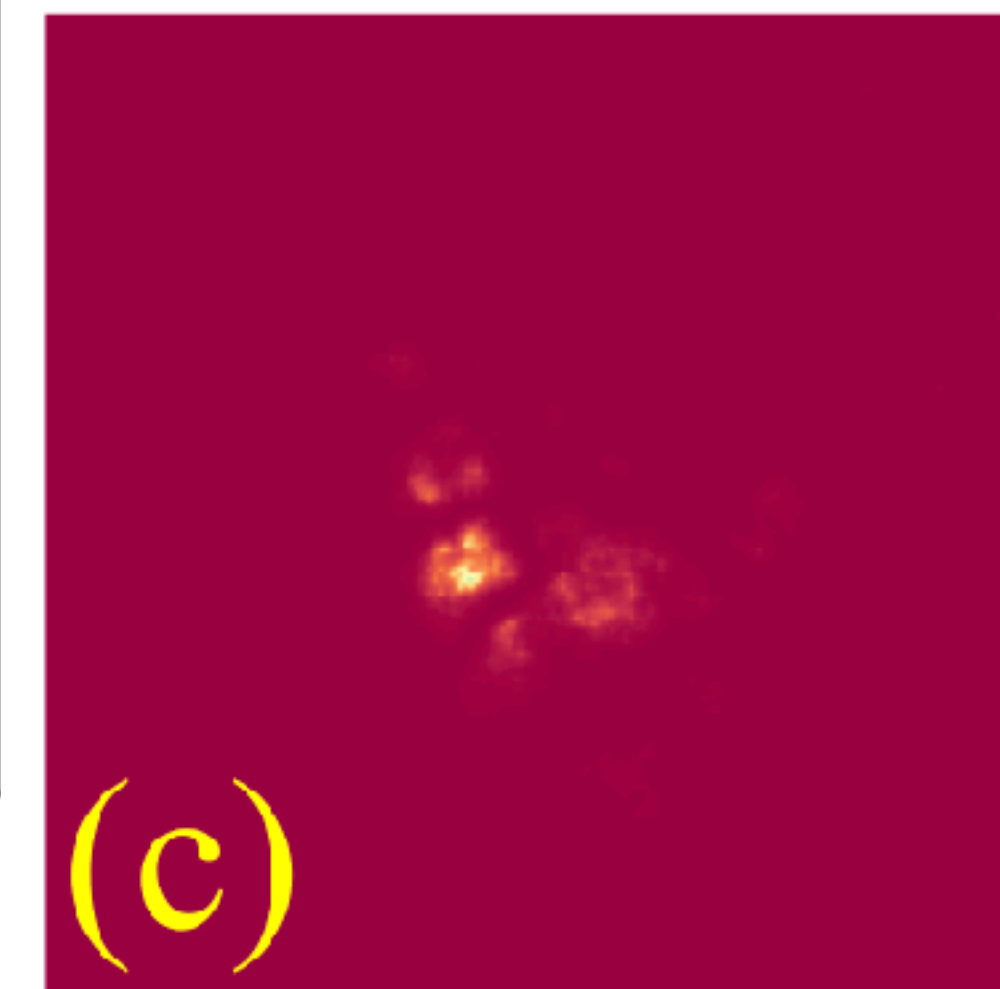
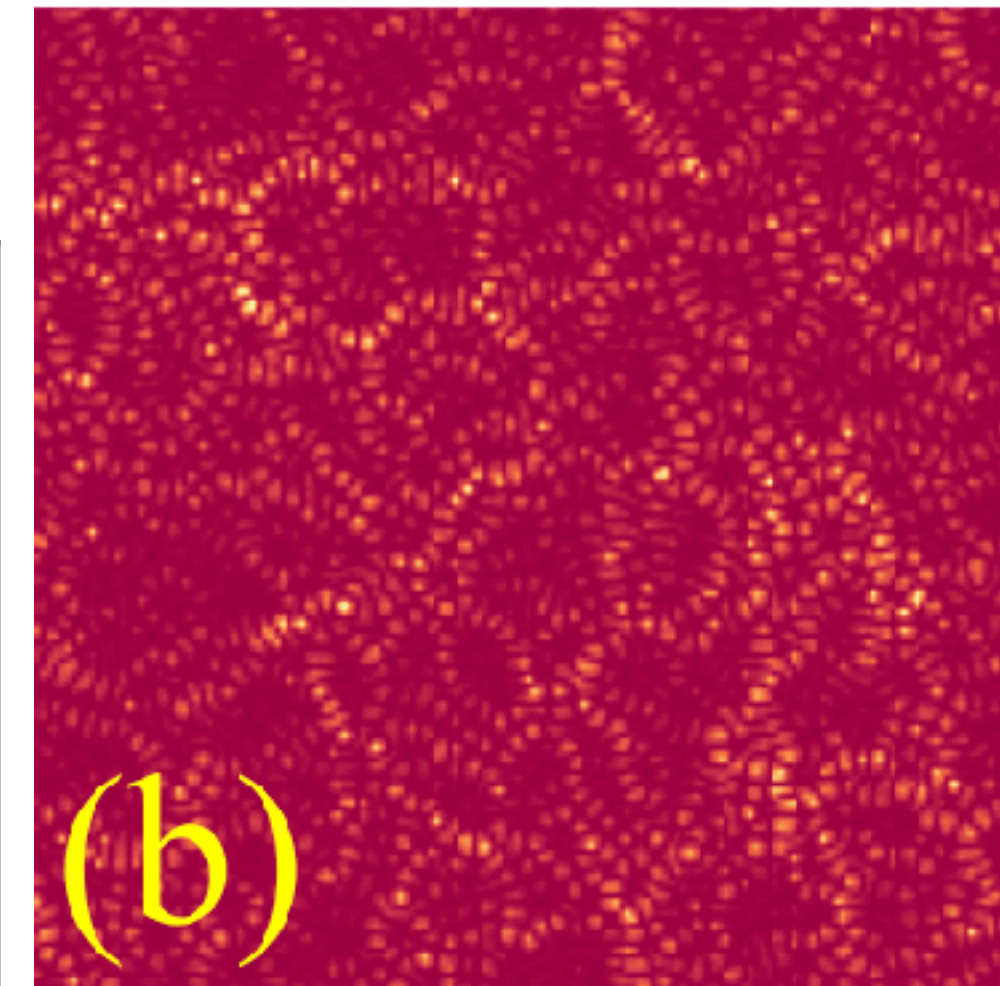
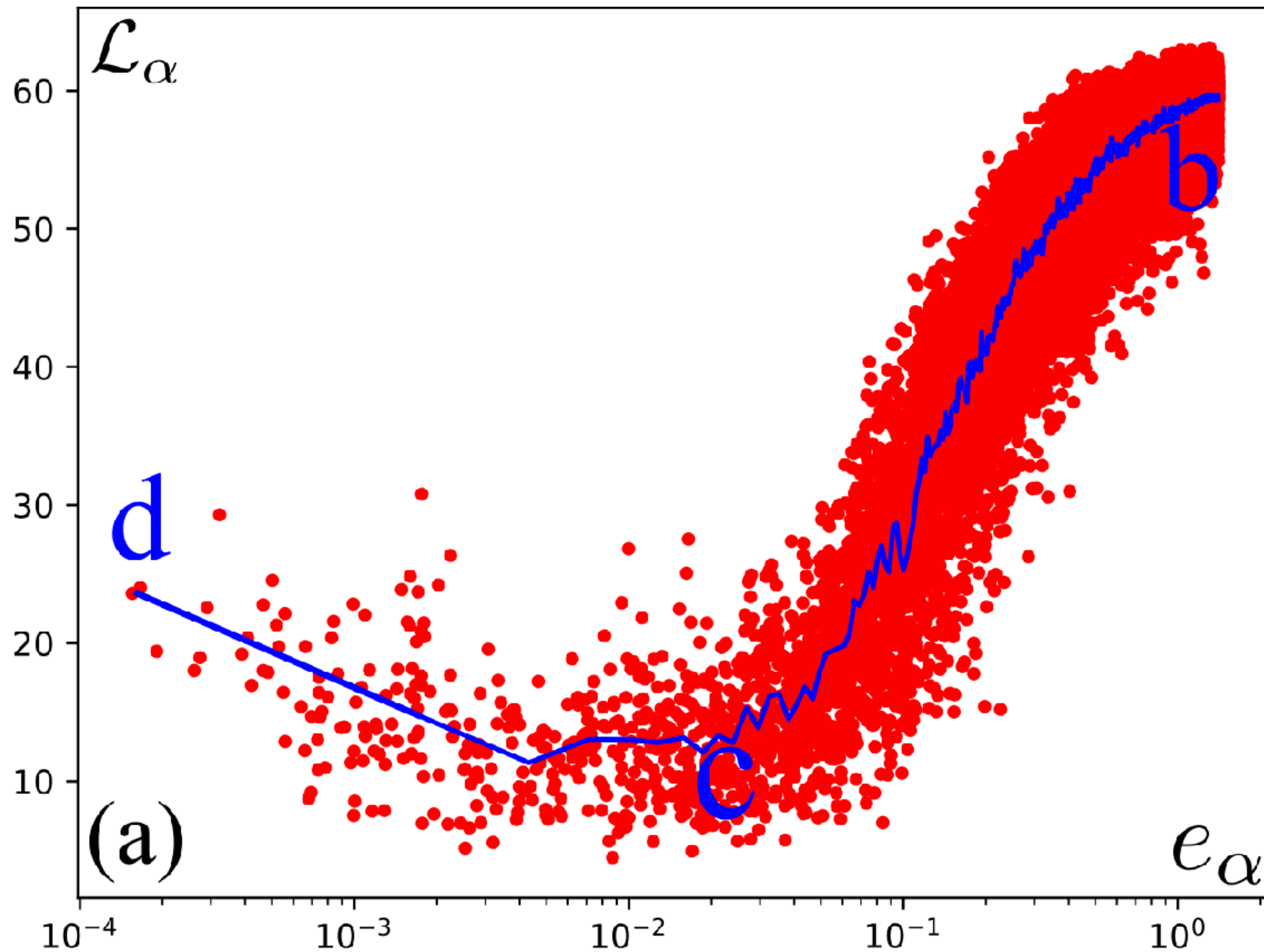
Extended bosons:
physics of $d=2$ Yukawa-SYK



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS to appear,
arXiv:2312.06751

Bosonic eigenmodes in random mass Hertz theory

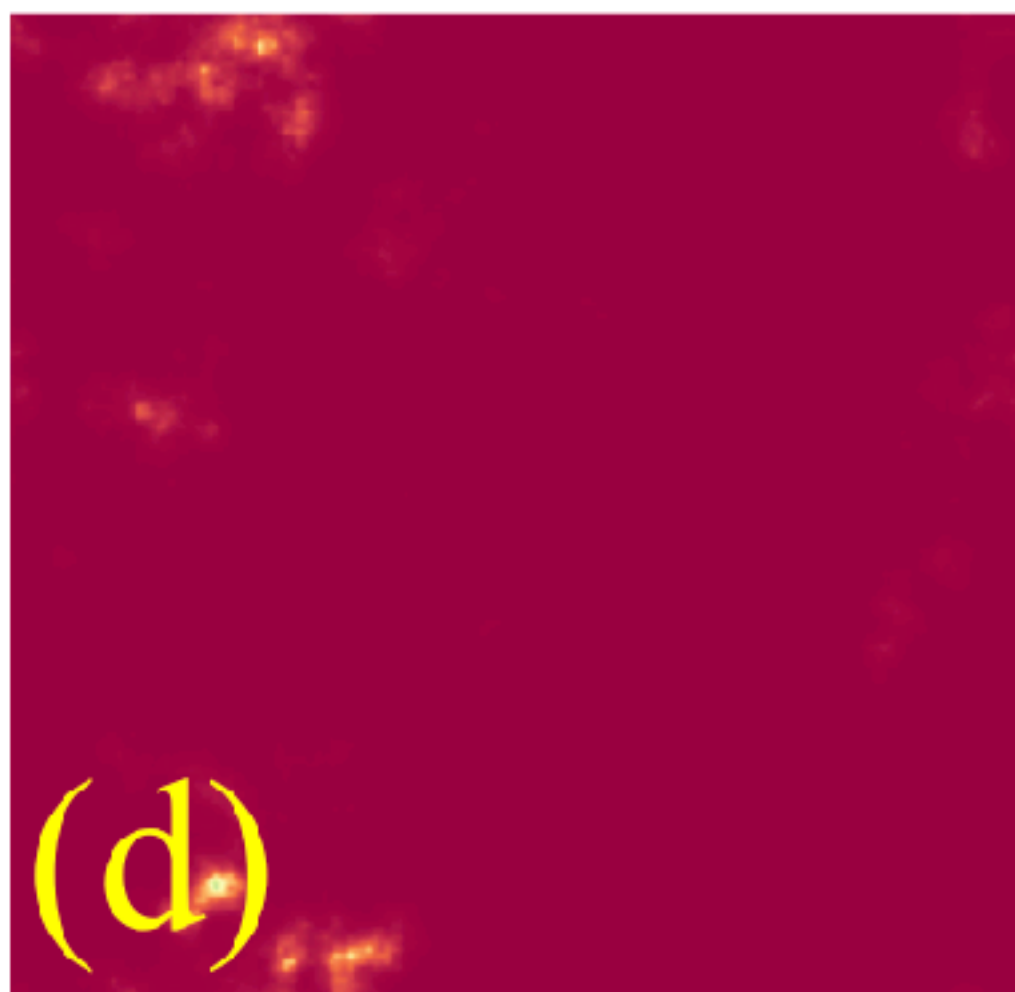
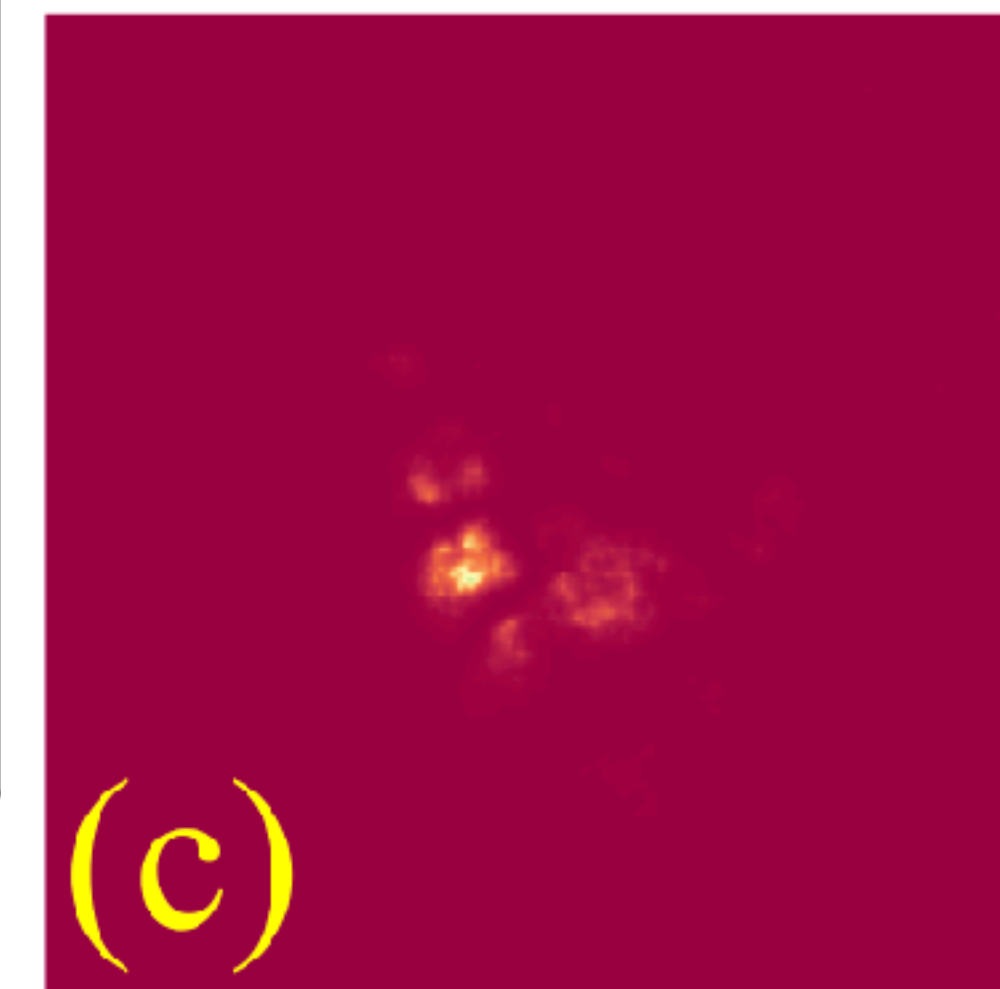
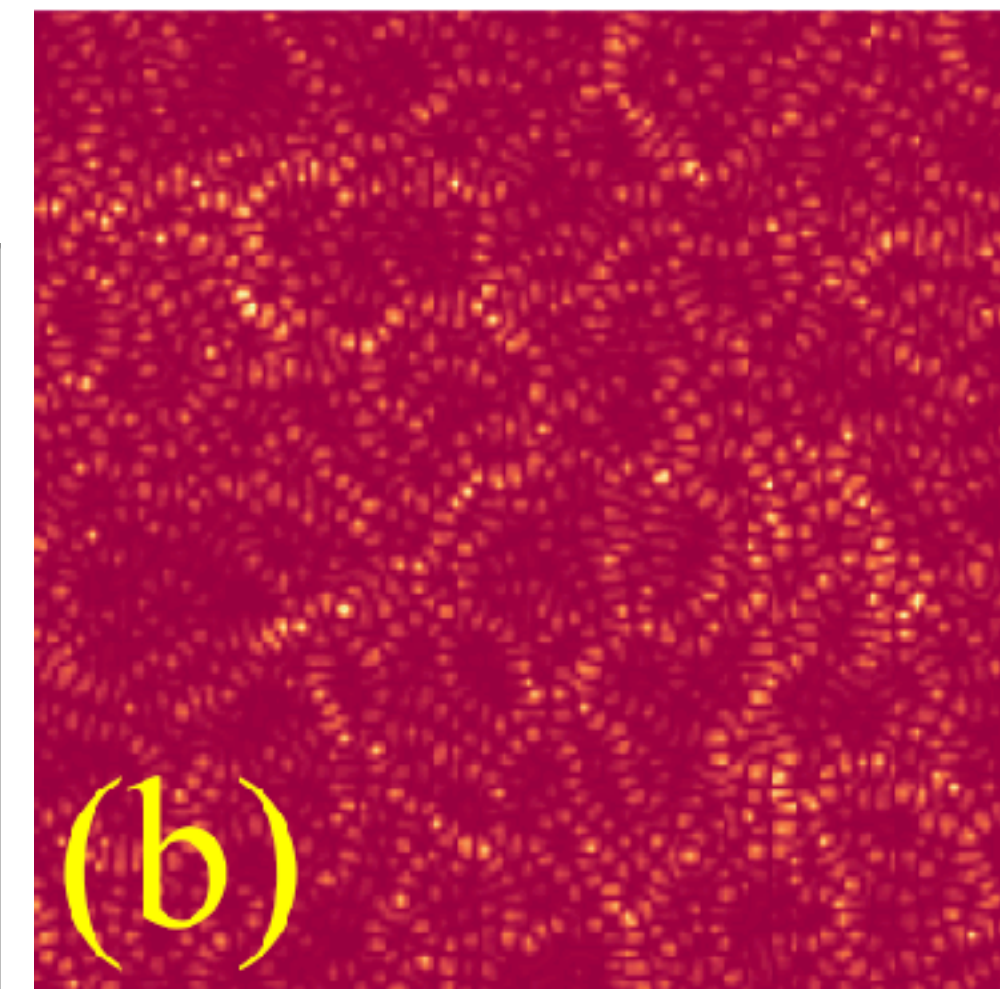
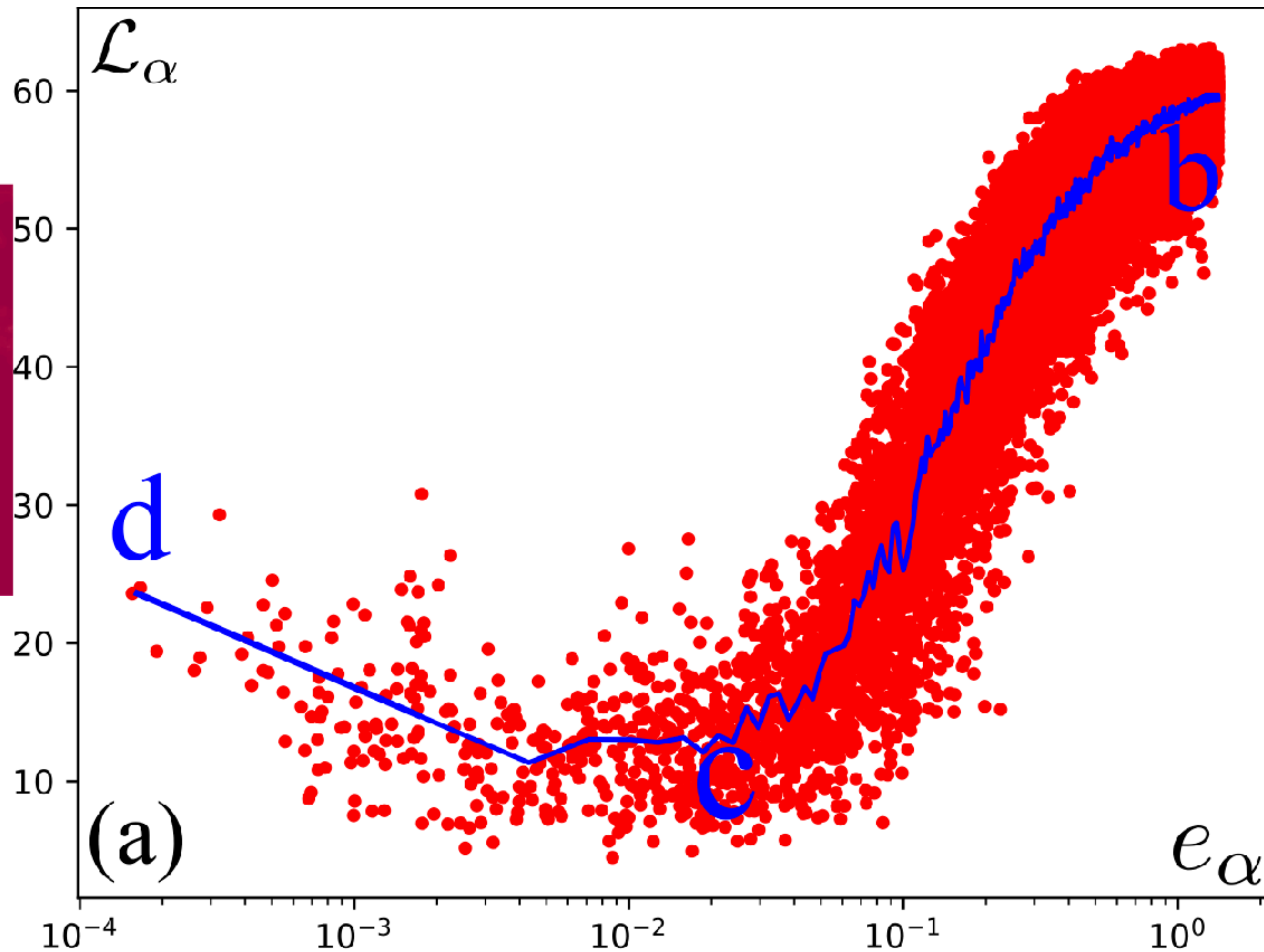
ϕ eigenmodes localization length \mathcal{L}_α



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length \mathcal{L}_α

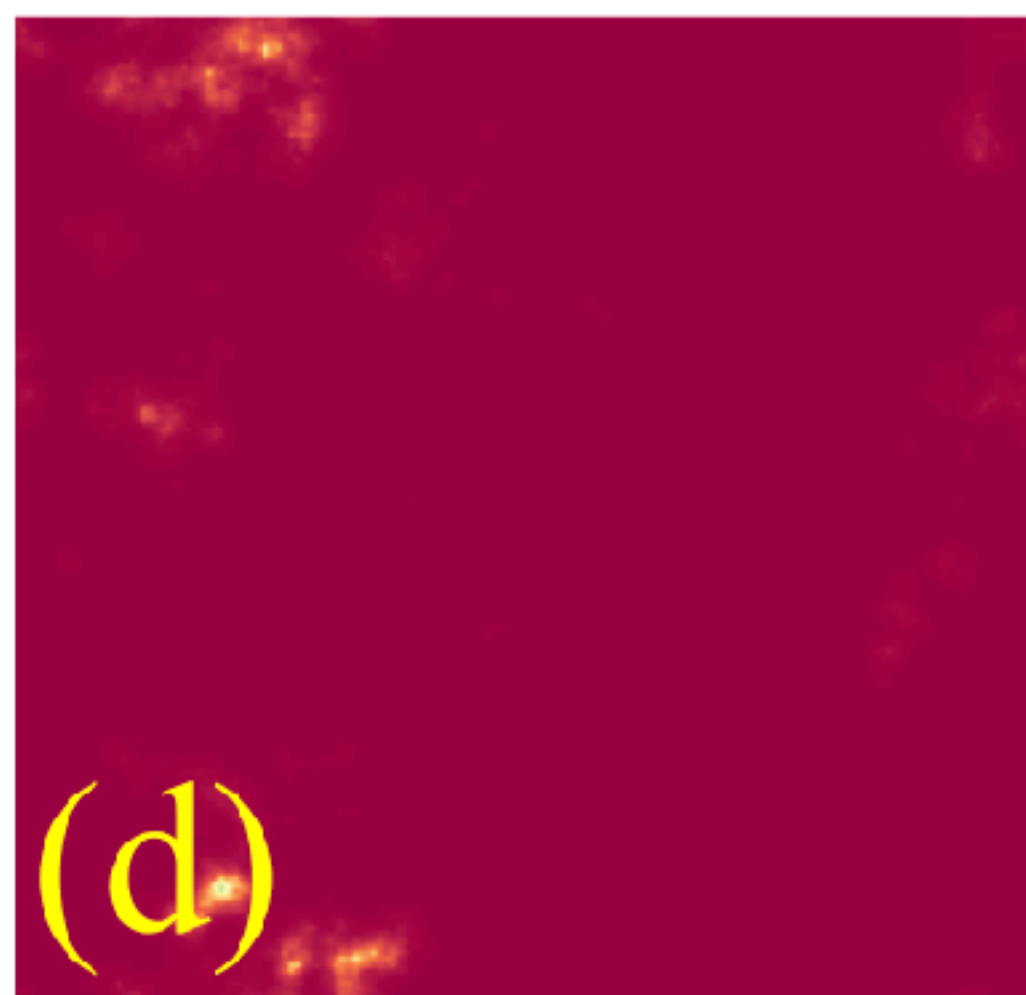
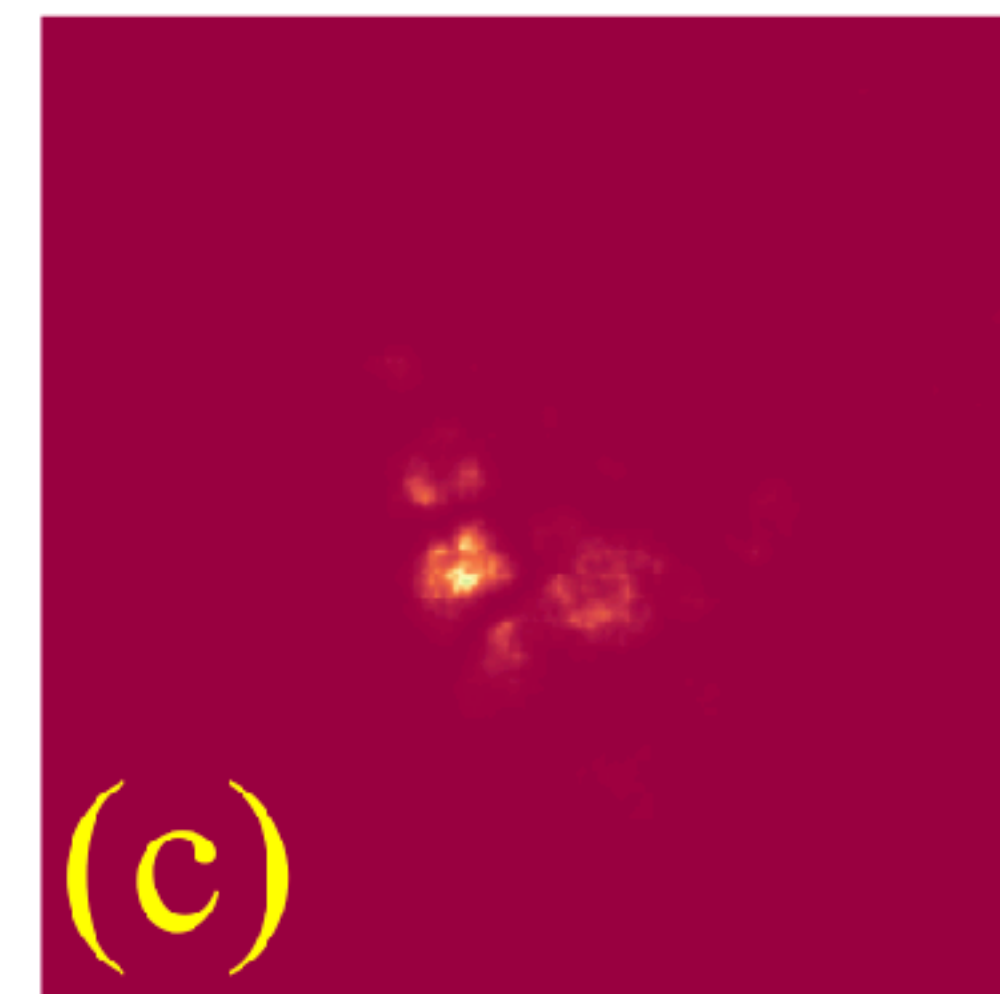
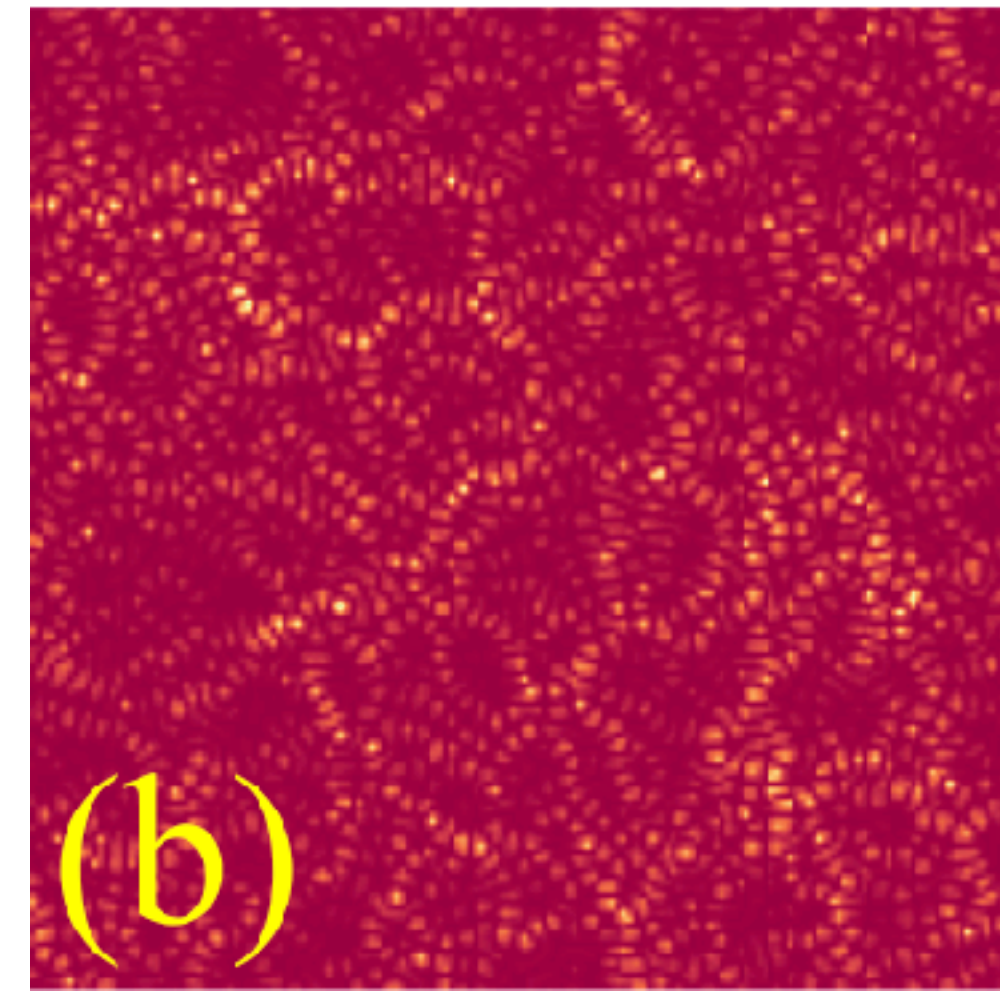
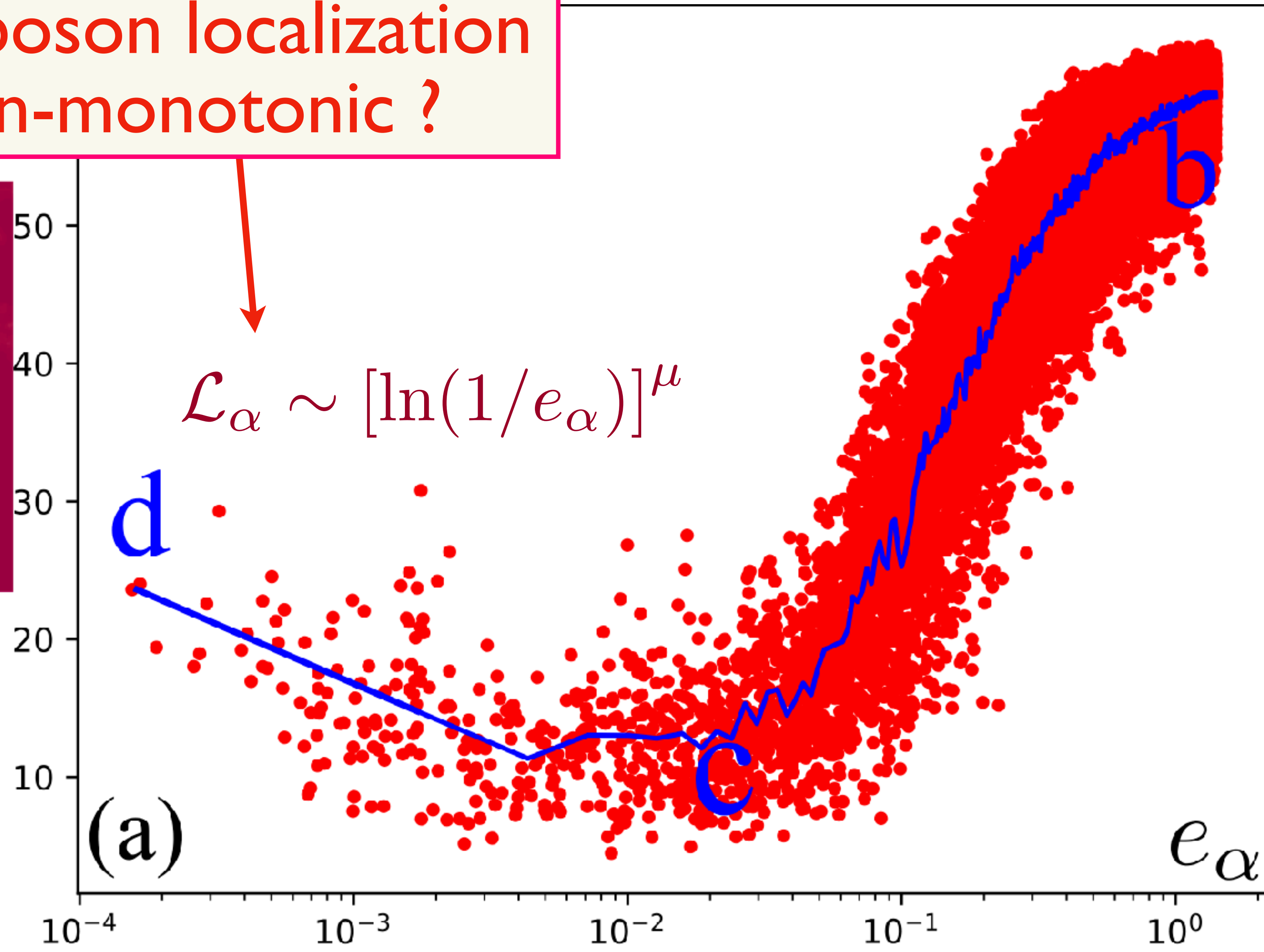


Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Landau-damped bosonic eigenmodes with random mass

ϕ eigenmodes localization length \mathcal{L}_α

Why is the boson localization length non-monotonic?



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta

Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

$$\mathcal{S}_b = \int d\tau \left(- \sum_{\langle ij \rangle} J_{ij} \phi_{ia} \phi_{ja} + \sum_j \left[\frac{s_j}{2} \phi_{ja}^2 + \frac{u}{4} (\phi_{ja}^2)^2 \right] \right) + \frac{T\gamma}{2} \sum_{\omega_n} \sum_j |\omega_n| |\phi_{ja}(\omega_n)|^2$$

Effects of Dissipation on a Quantum Critical Point with Disorder

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Strong disorder RG identical to that for the RTFIM

$$\tilde{J}_{ij} = J_{ij} + \frac{J_{i2}J_{2j}}{s_2}$$

$$\tilde{s}_2 = 2 \frac{s_2 s_3}{J_{23}}$$

$$H_{\text{RTFIM}} = - \sum_{\langle ij \rangle} J_{ij} Z_i Z_j - \sum_j s_j X_j$$

Numerically studied in $d=2$ by
O. Motrunich, S.-C. Mau, D.A. Huse and D.S. Fisher,
PRB **61** (2000) 1160

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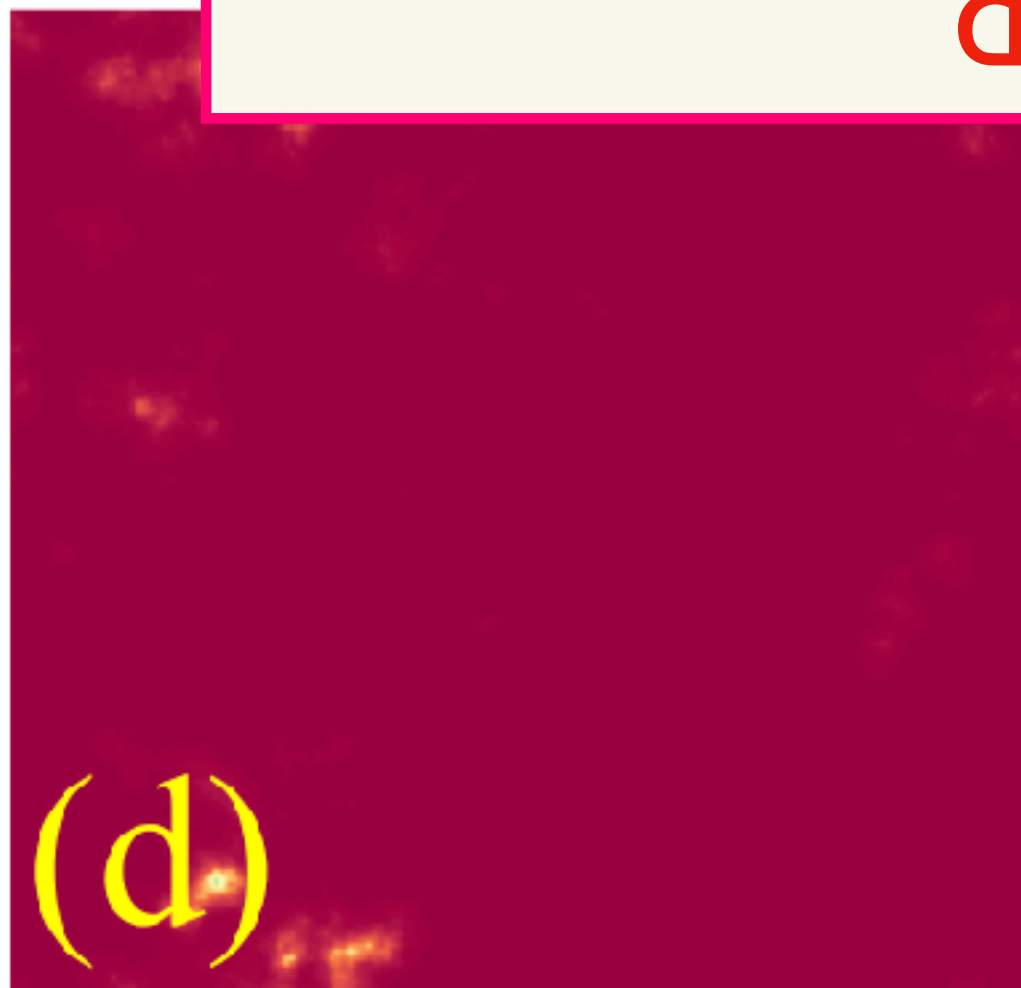
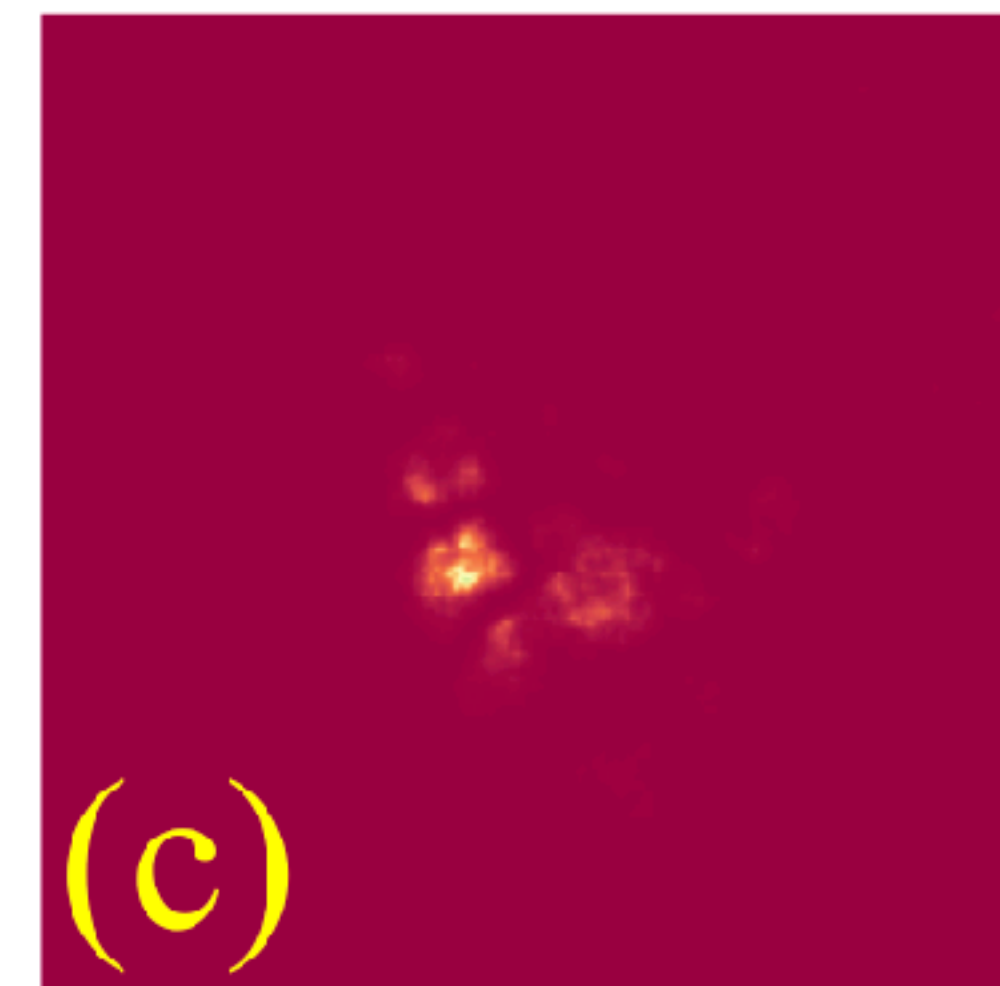
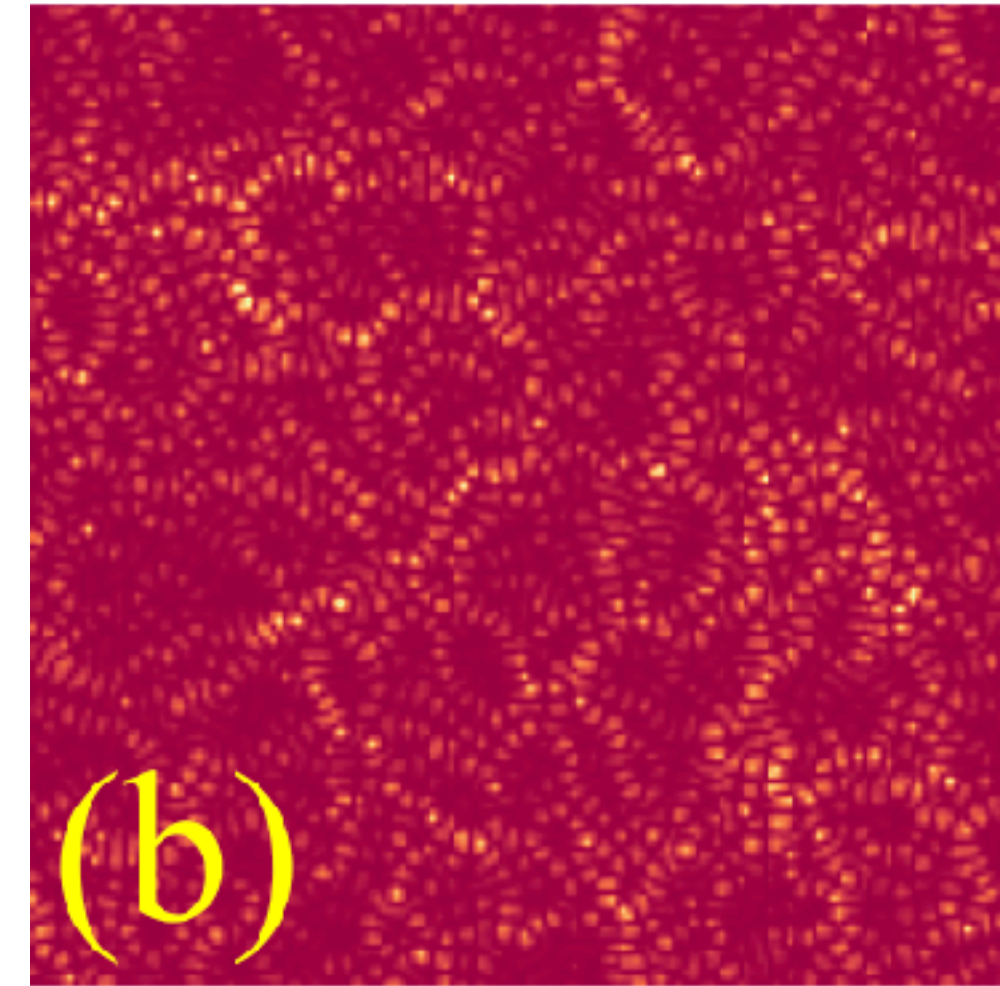
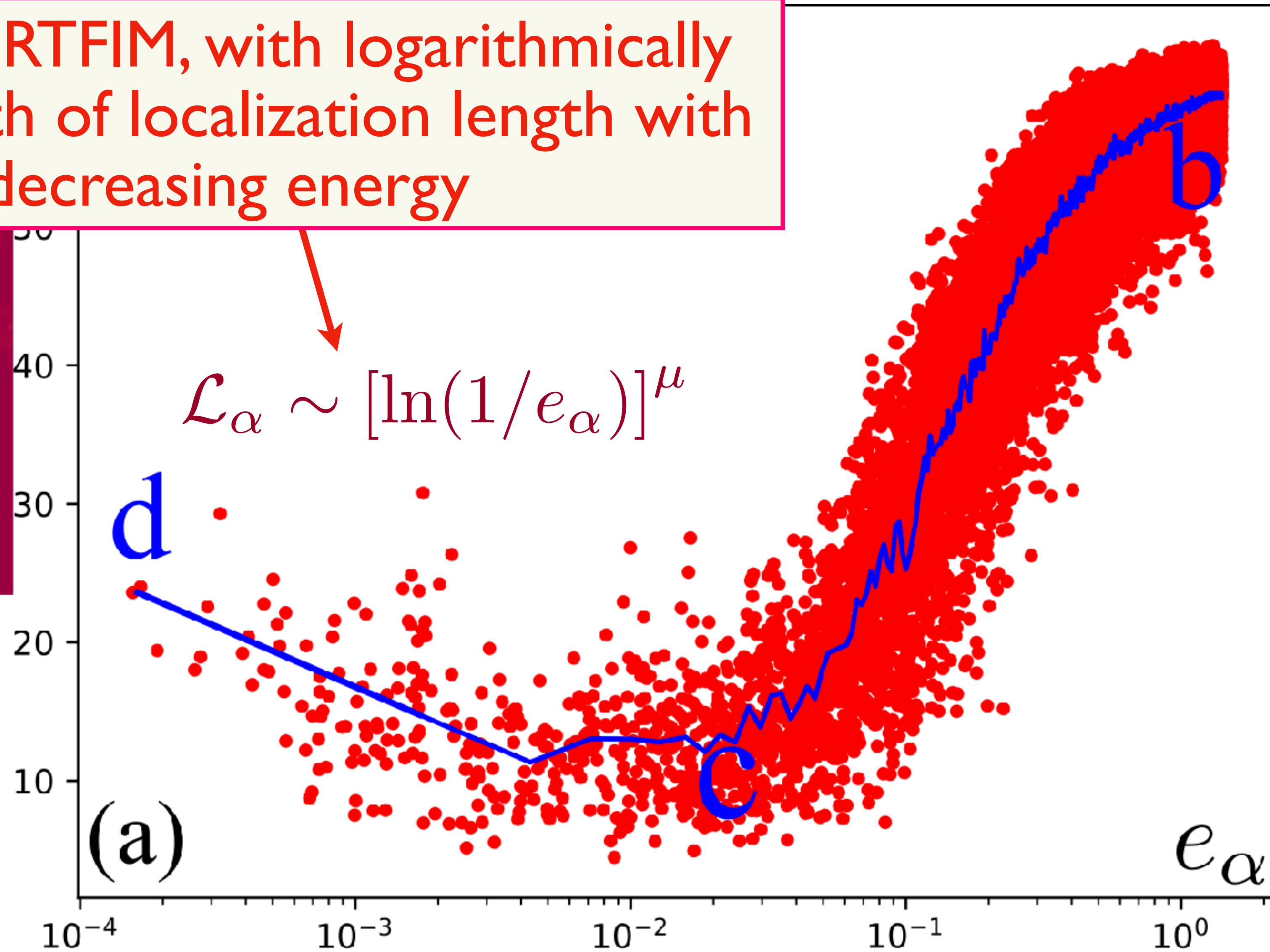
PRB **61** (2000) 1160

Key fact: Similarity of classical $O(N \geq 2)$ chain with $1/r^2$ interactions, and classical Ising chain with short-range interactions.

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length \mathcal{L}_α

Physics of RTFIM, with logarithmically slow growth of localization length with decreasing energy

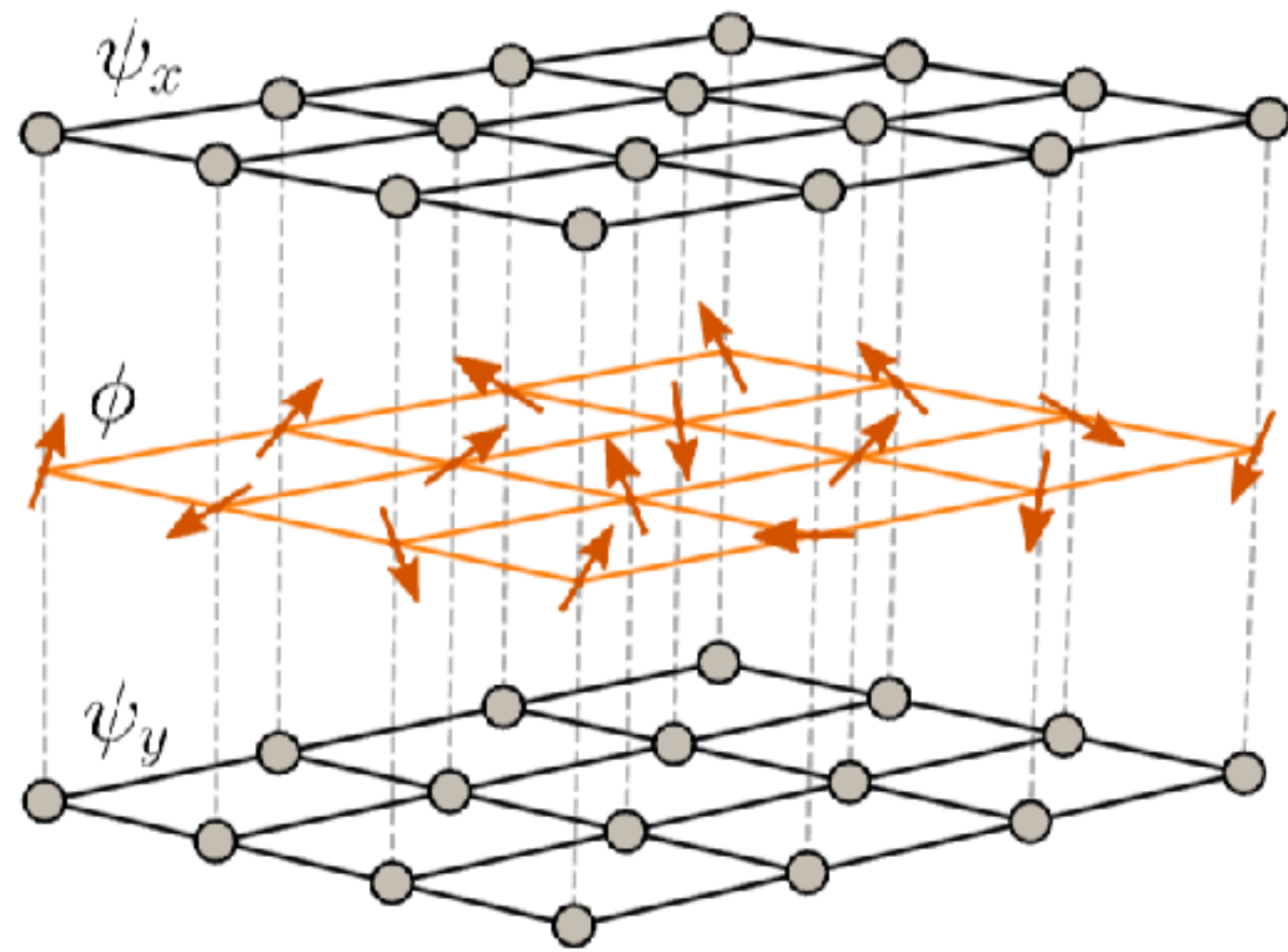


Model for sign-free QMC

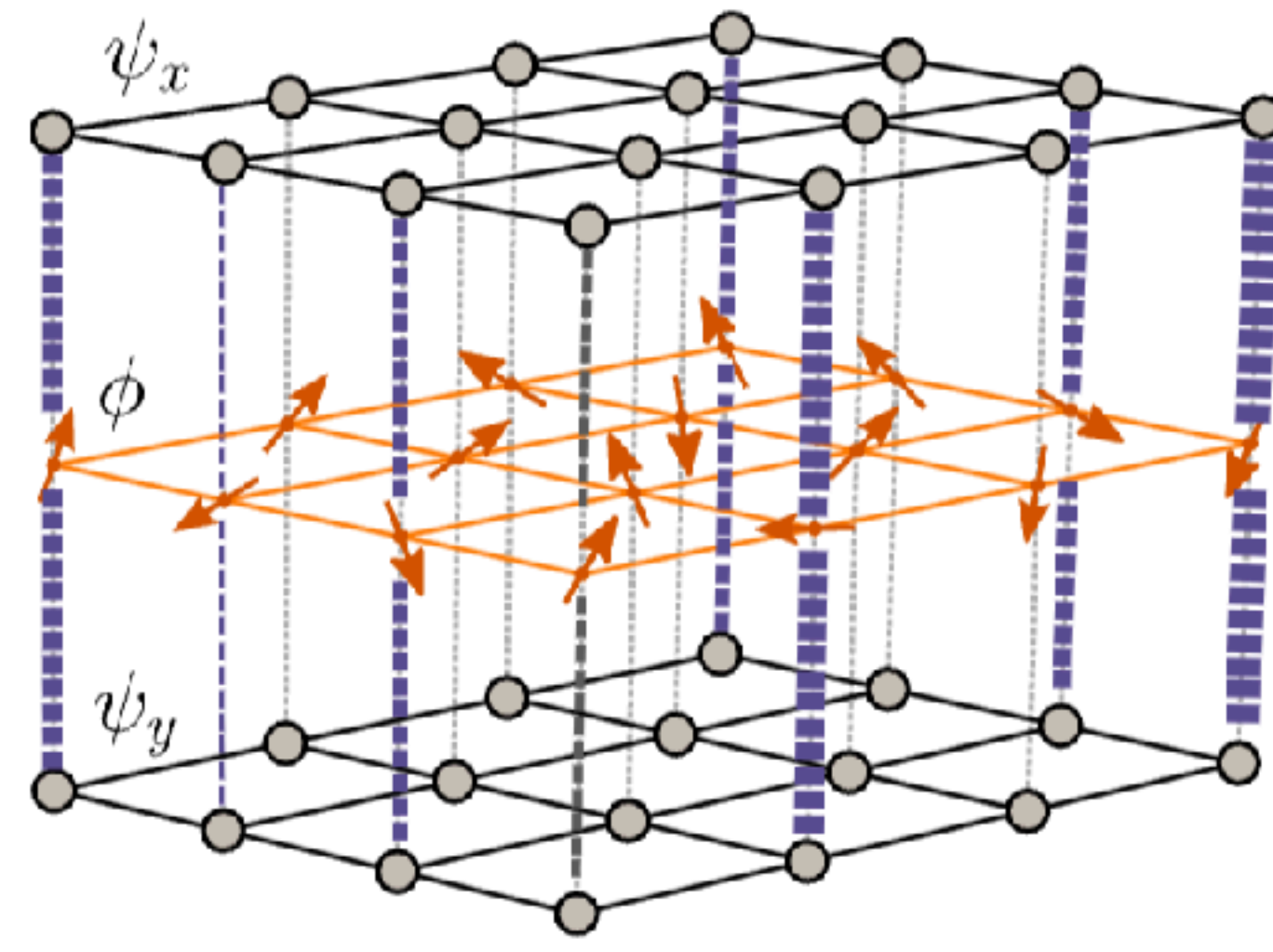
$$\mathcal{S} = \int d\tau \sum_{\sigma=\uparrow,\downarrow} \sum_{\alpha=0,1} \sum_{j=1}^2 \sum_{\vec{r},\vec{r}'} \psi_{\alpha,\sigma,j,\vec{r}}^\dagger [\partial_\tau - (-1)^\alpha \mu - \delta_{\vec{r},\vec{r}'} - t_{\alpha,\vec{r},\vec{r}'}] \psi_{\alpha,\sigma,j,\vec{r}'}$$

$$+ \int d\tau \sum_{\vec{r}} \left[\frac{1}{c^2} (\partial_\tau \vec{\phi}_{\vec{r}})^2 + \frac{1}{2} (\nabla \vec{\phi}_{\vec{r}})^2 + \frac{r}{2} (\vec{\phi}_{\vec{r}})^2 + \frac{u}{4} (\vec{\phi}_{\vec{r}})^4 \right] + \sum_{\sigma,\sigma'=\uparrow,\downarrow} \sum_{j=1}^2 \int d\tau \sum_{\vec{r}} \boxed{g'(\vec{r}) e^{i\vec{Q}_{AF}\cdot\vec{r}}} \vec{\phi}_{\vec{r}} \cdot \left[\psi_{0,\sigma,\vec{r}}^\dagger \vec{\tau}_{\sigma,\sigma'} \psi_{1,\sigma',\vec{r}} + \text{H.c.} \right].$$

$$g = 0$$

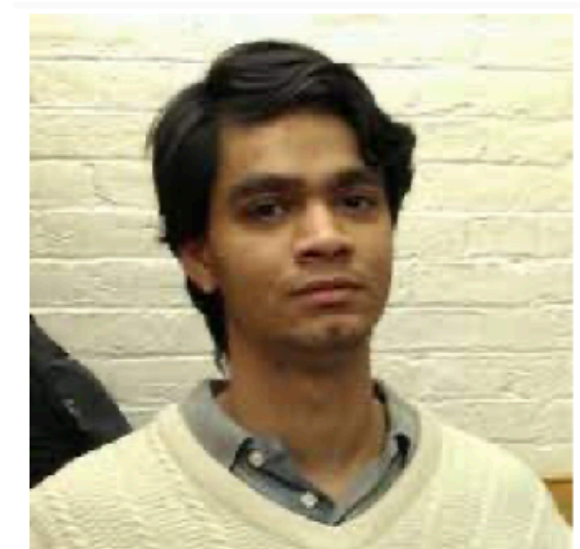


coupling $g = \text{const}$



coupling $g = \text{spatially random}$

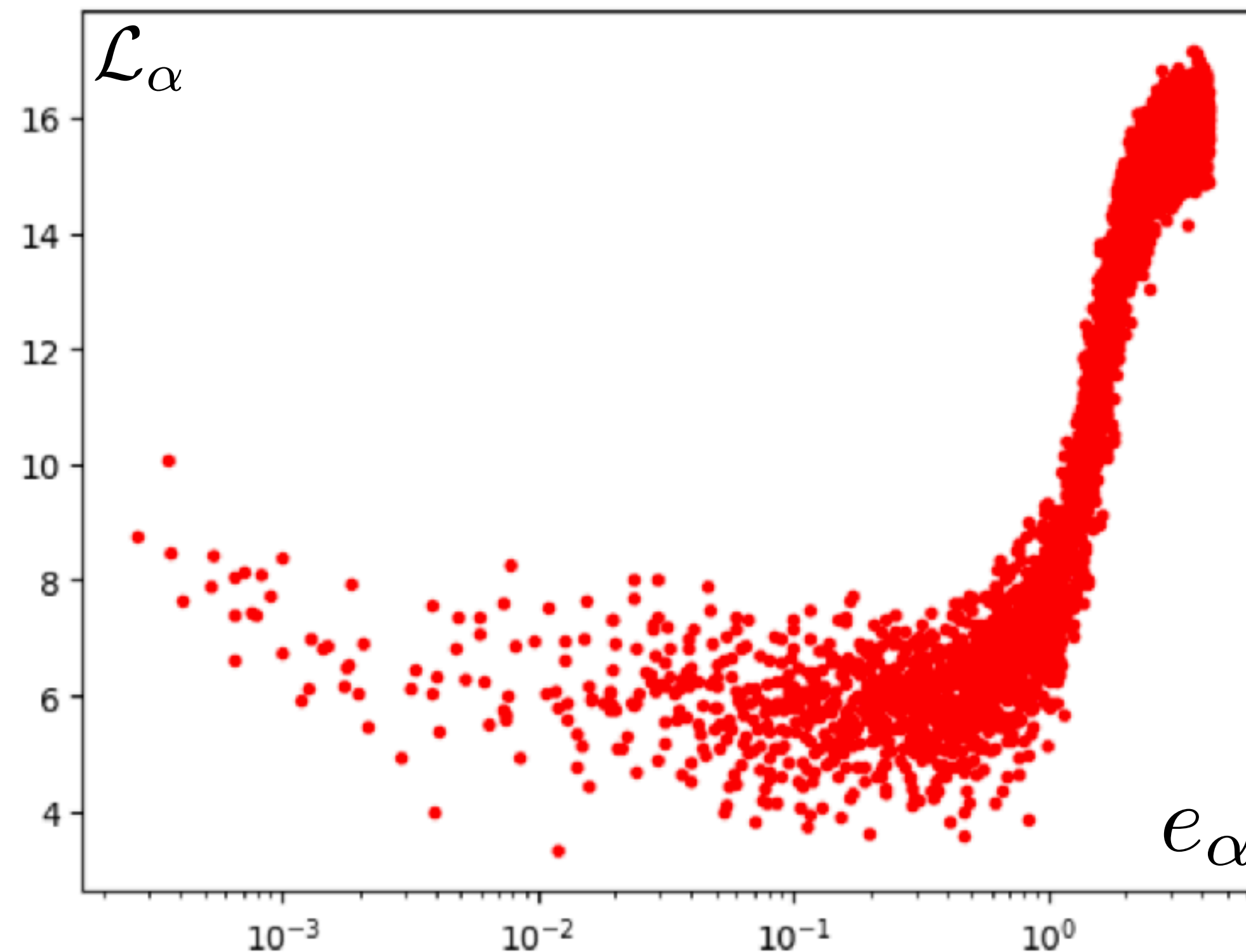
Two-band structure: Berg, Metlits Sachdev, Science 338 1606-1609 (2012).



Aavishkar A. Patel, Peter Lunts, M. Albergo **(to appear)**

Boson eigenmodes at $\Omega = 0$ from QMC

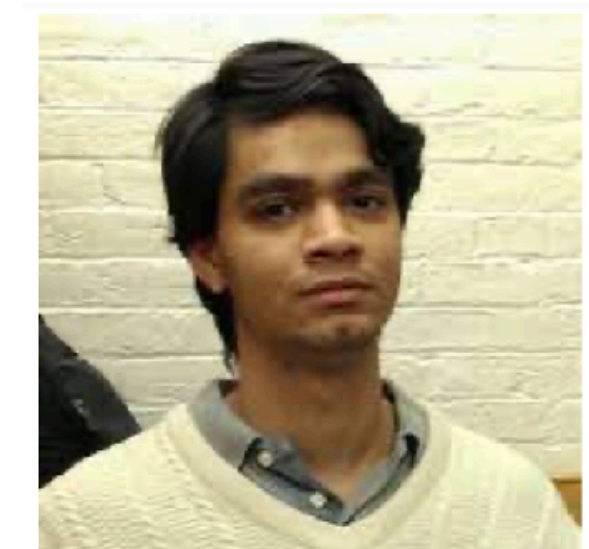
- Localization length vs eigenvalue



10 disorder samples, $L = 40$, $\beta t = 66$, $g'^2 = t/2$, $\lambda \approx \lambda_c$

- Same qualitative features as large- M : localization followed by slow delocalization as energy is reduced.

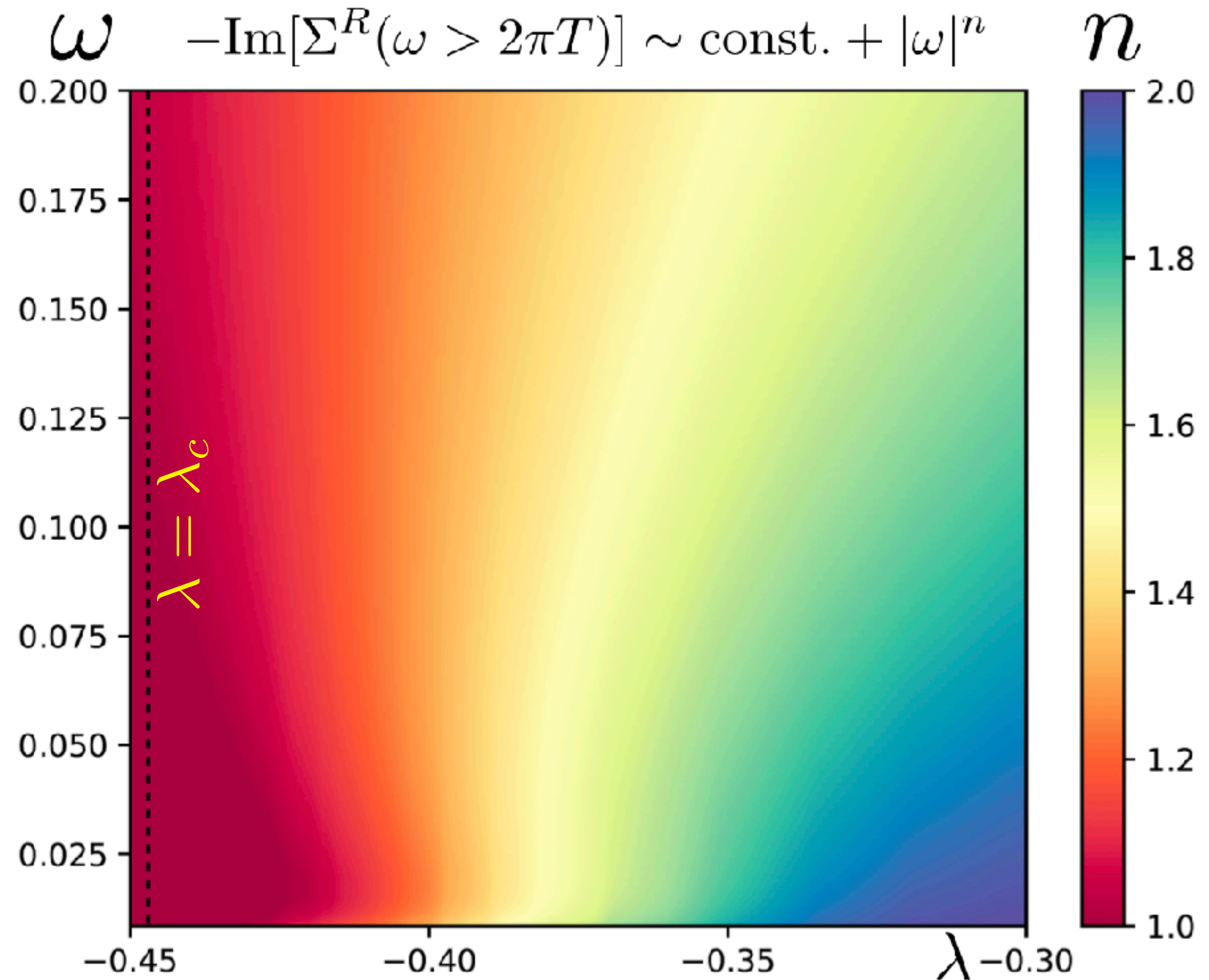
Aavishkar A. Patel, Peter Lunts, M. Alberg (to appear)



Bosonic eigenmodes in random mass Hertz theory

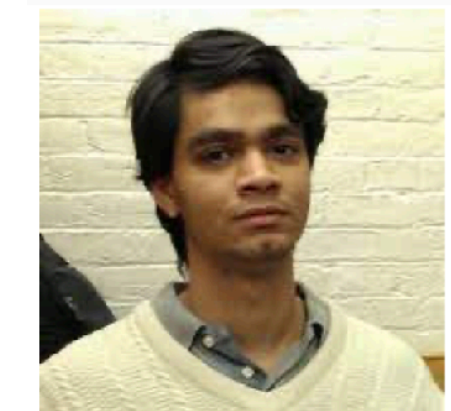
Transport scattering rate

$$\Sigma(i\omega) = -i\pi g'^2 \mathcal{N}_0 \frac{T}{L^2} \sum_{\alpha, \Omega} \frac{\text{sgn}(\omega + \Omega)}{\gamma|\Omega| + \Omega^2/c^2 + e_\alpha}.$$



$L = 160, \beta = 800, 10$ disorder samples

Extended region in λ with $n \approx 1$ - a strange metal *phase*

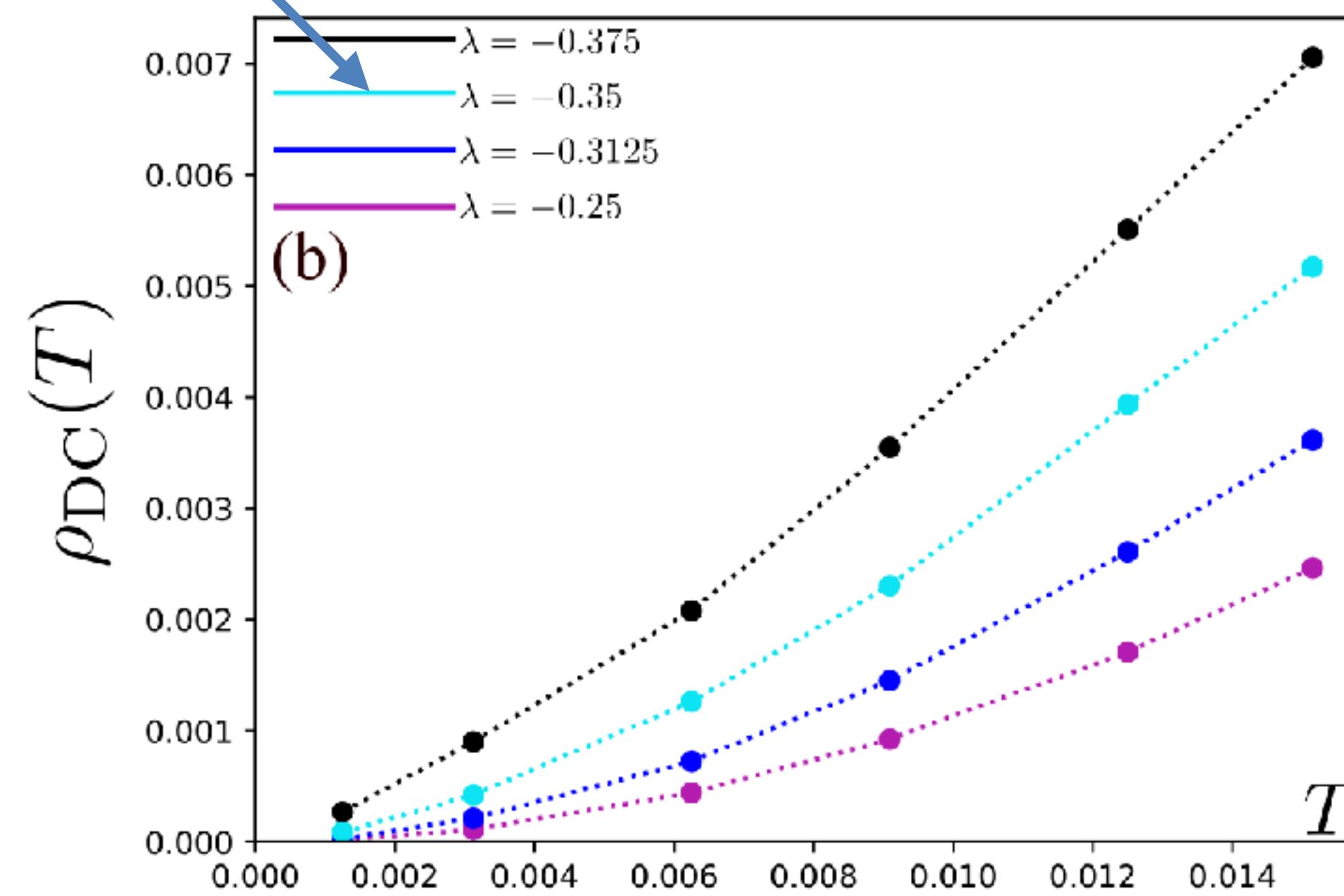
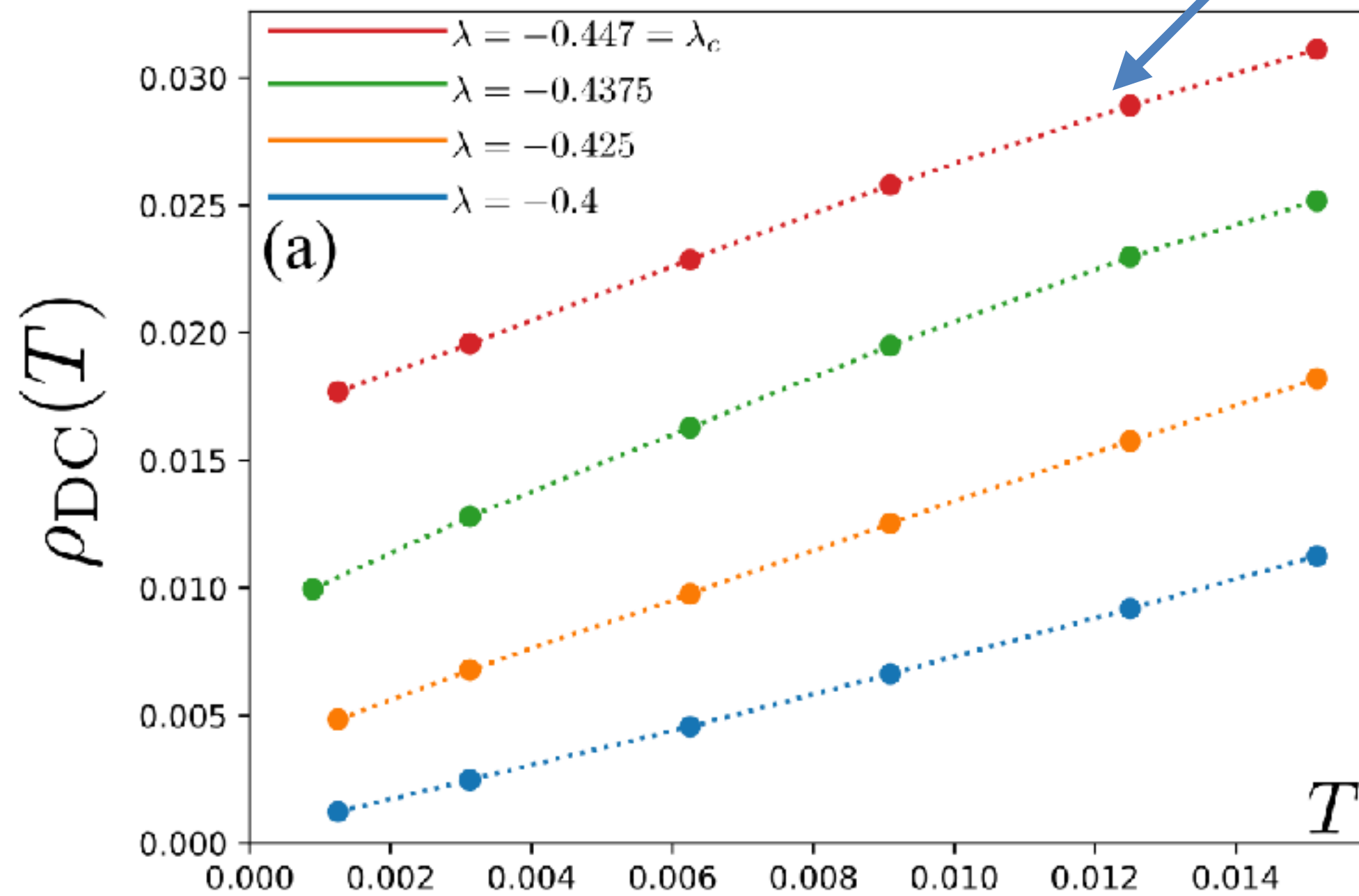
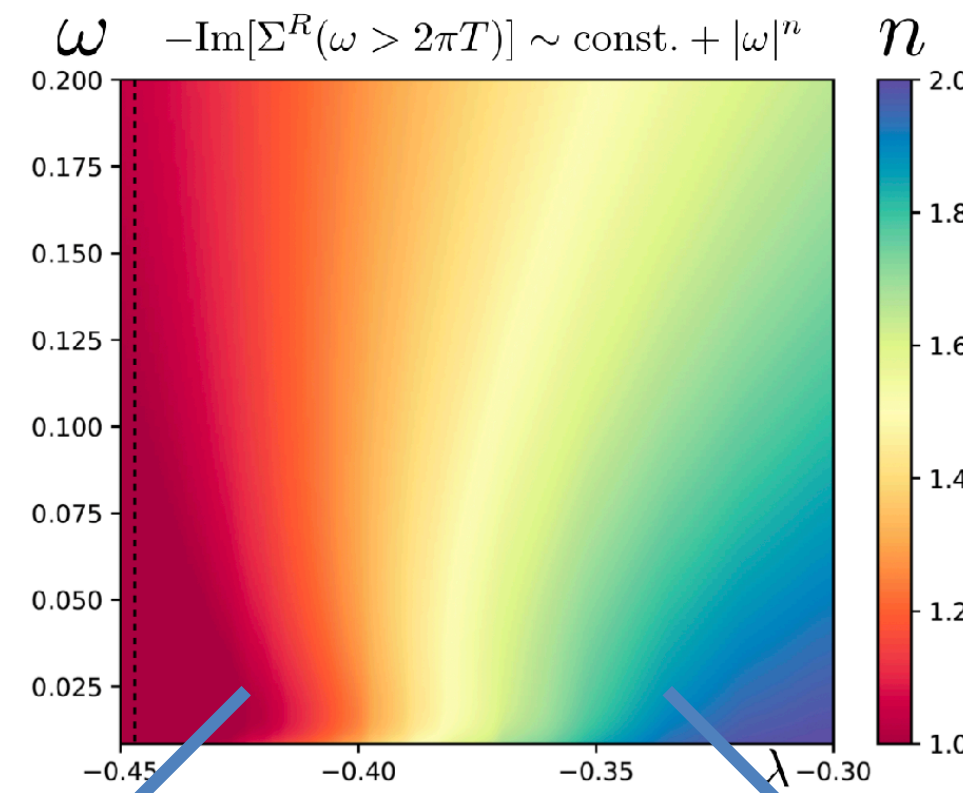


Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

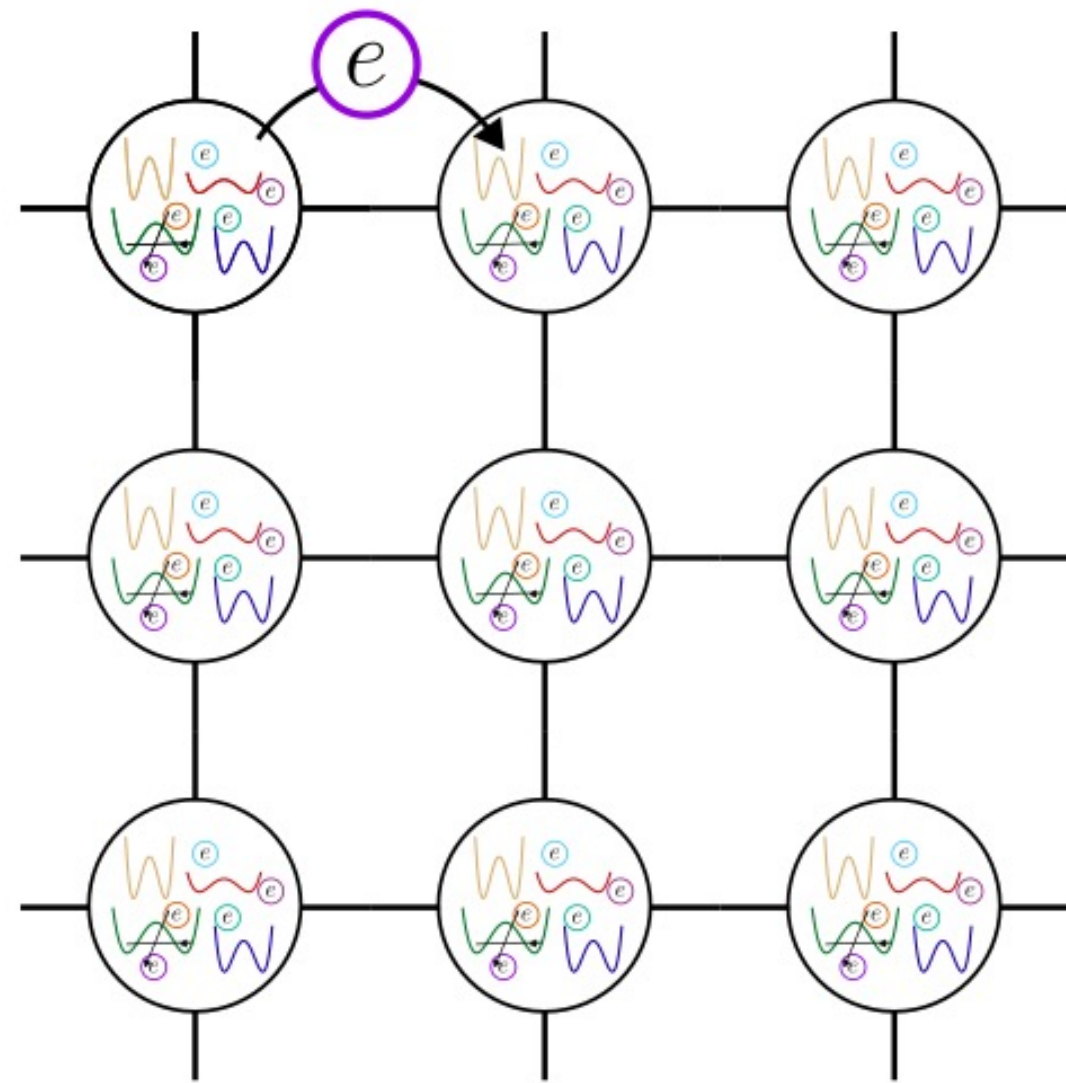
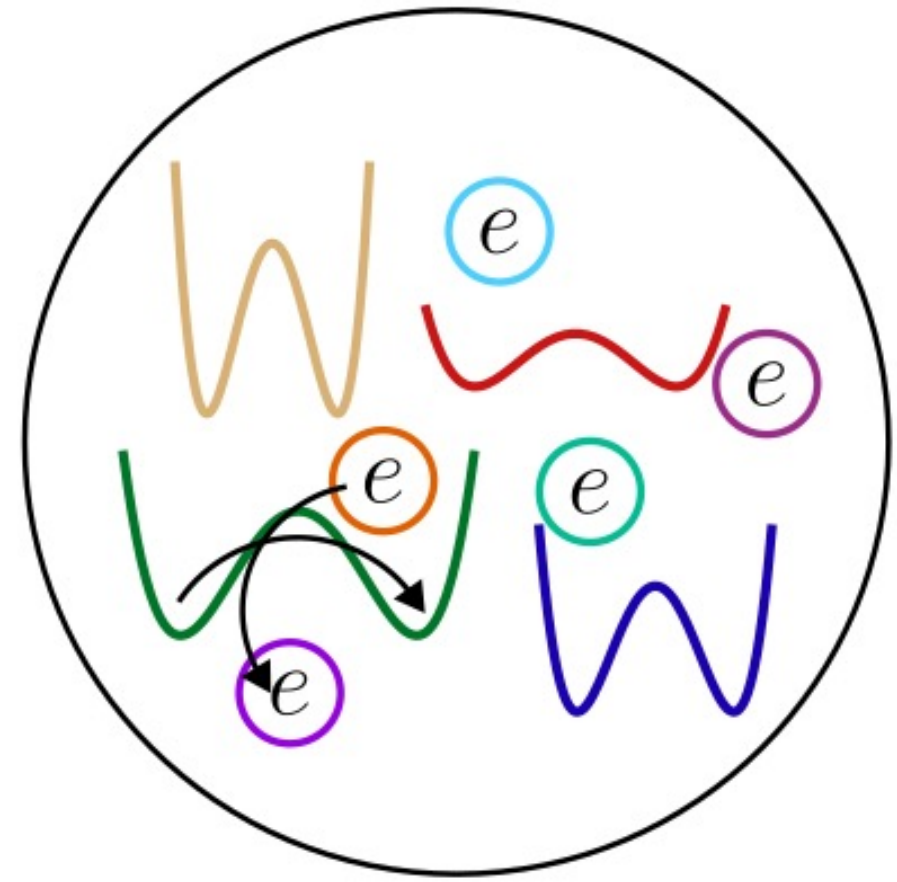


DC resistivity



Extended region in λ with T -linear resistivity - a strange metal *phase*

Tunable non-Fermi liquid phase in electronic glasses

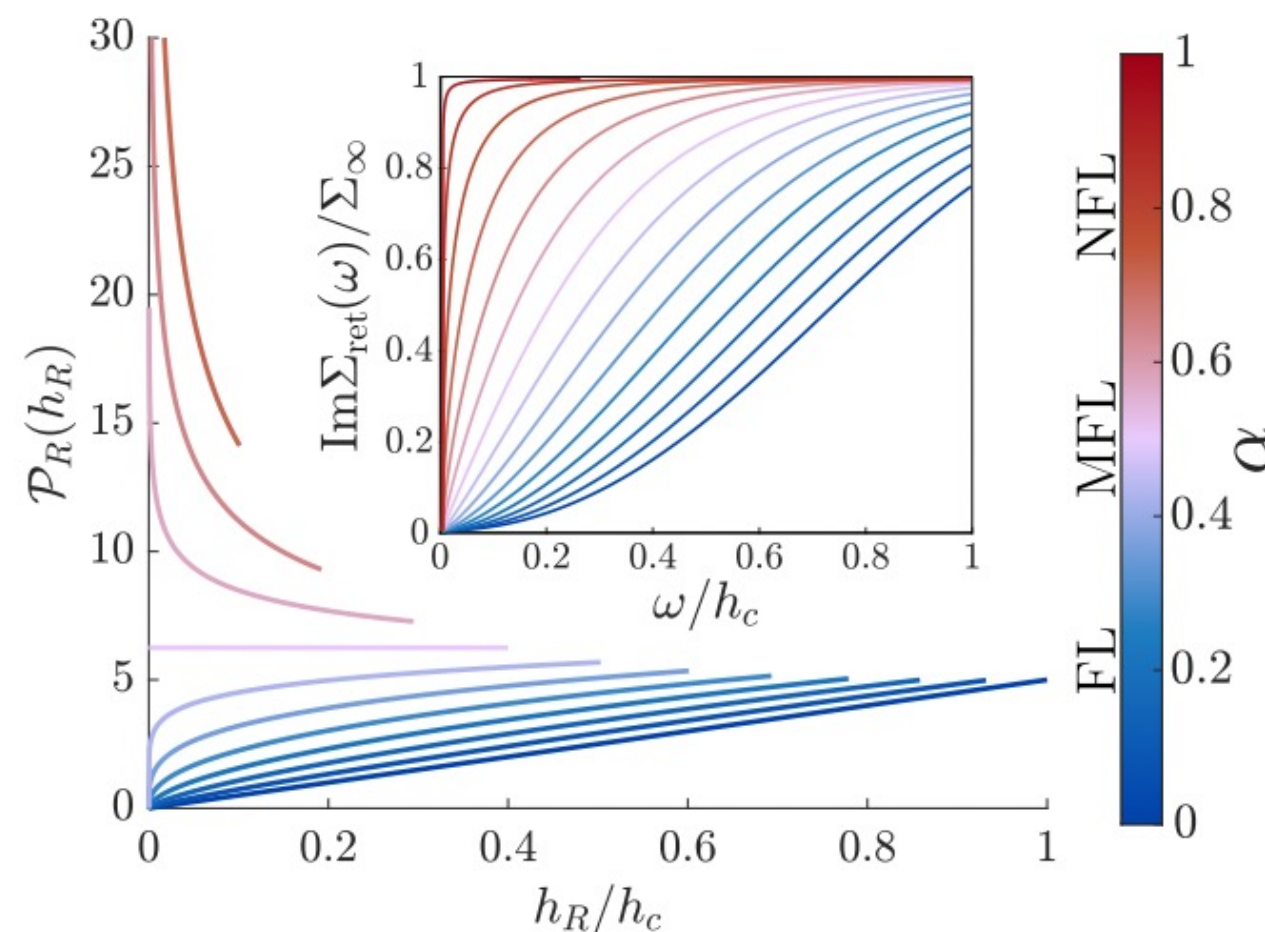


generalization of the Sachdev-Ye-Kitaev approach to two-level systems in glasses



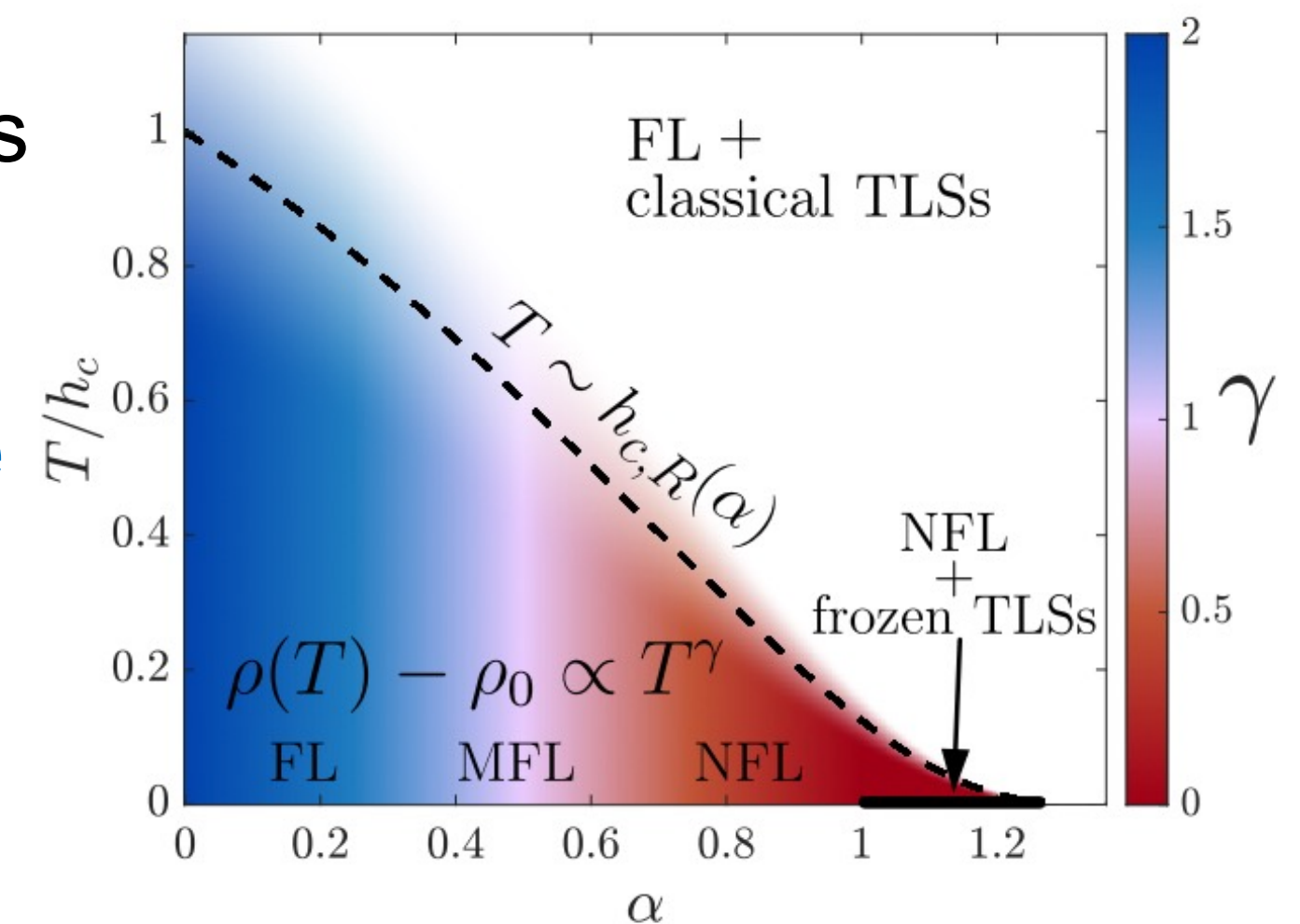
controlled approach to obtain a tunable Non-Fermi liquid state

strong renormalizations of the TLS distribution function



non-Fermi liquid physics in electronic glasses

Is this relevant to the physics of correlated quantum materials?

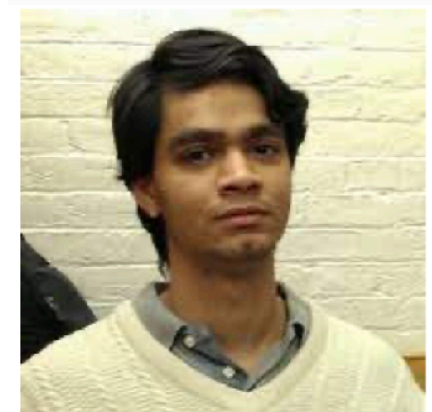
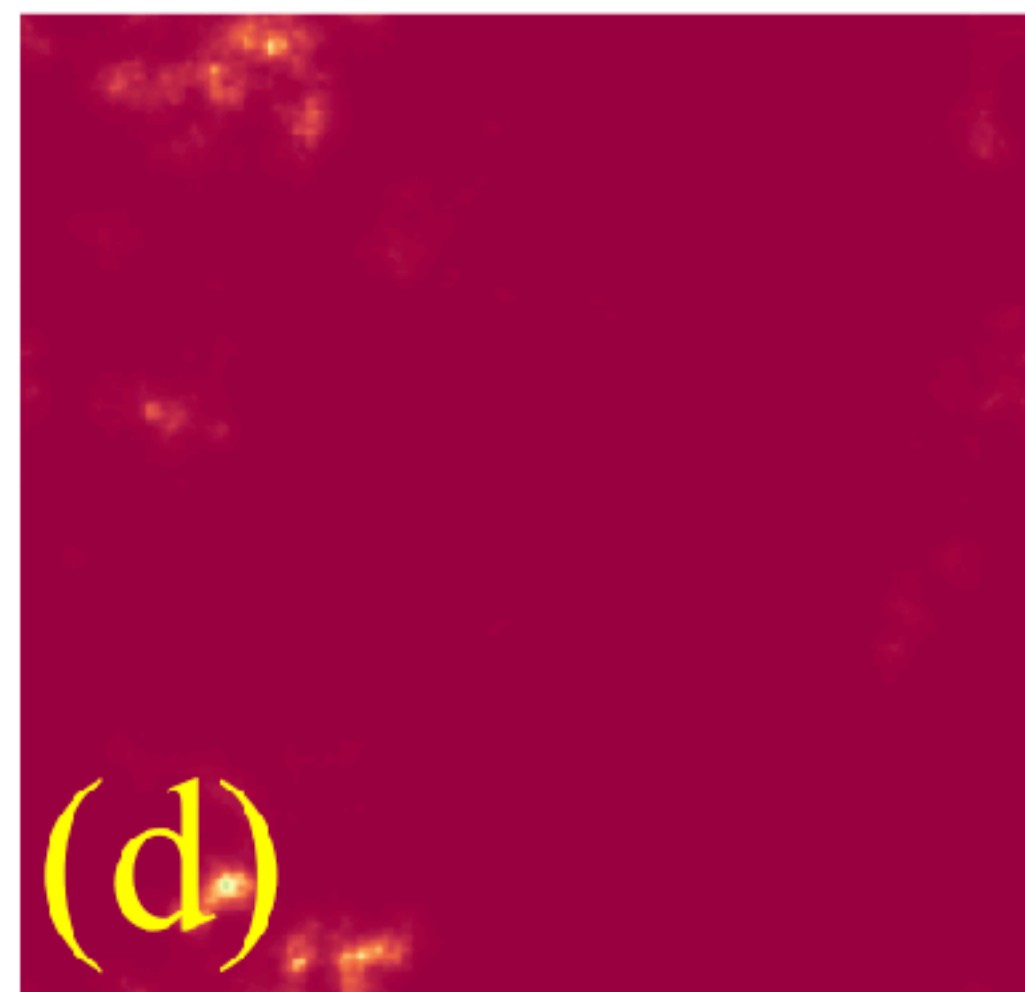
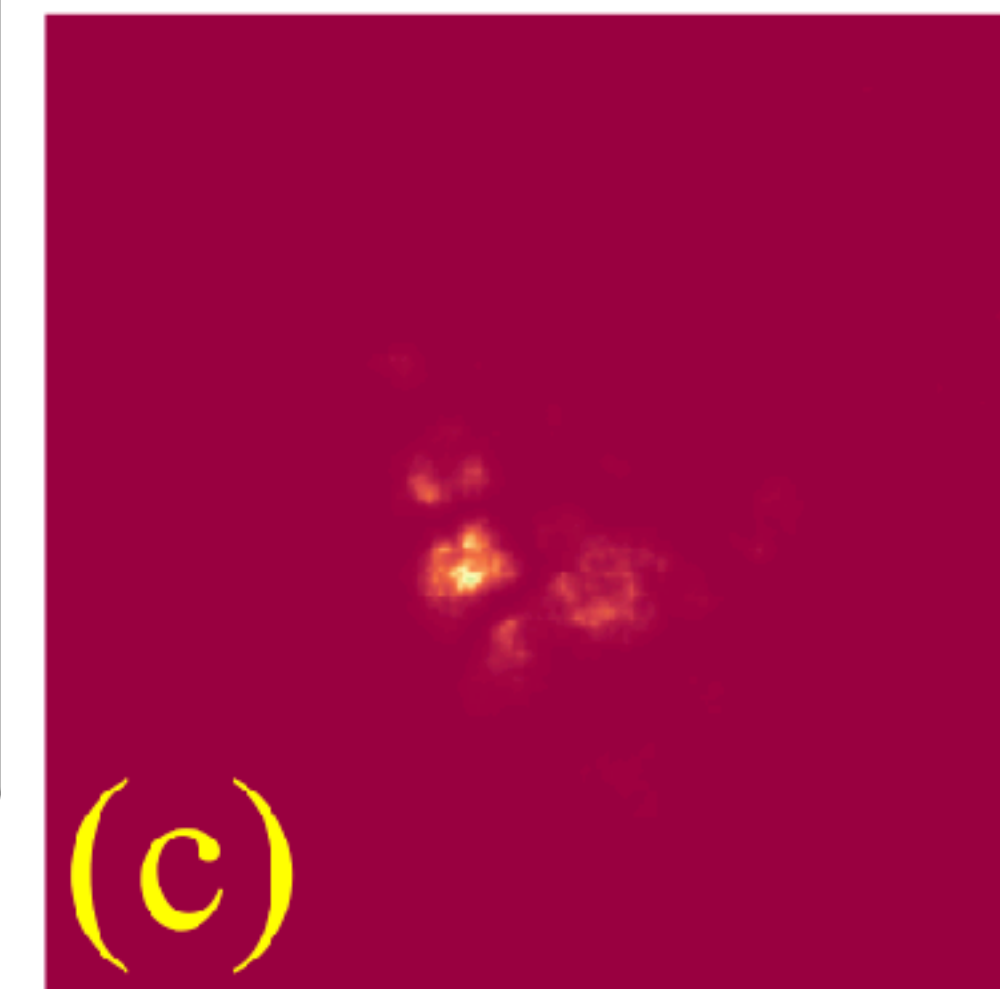
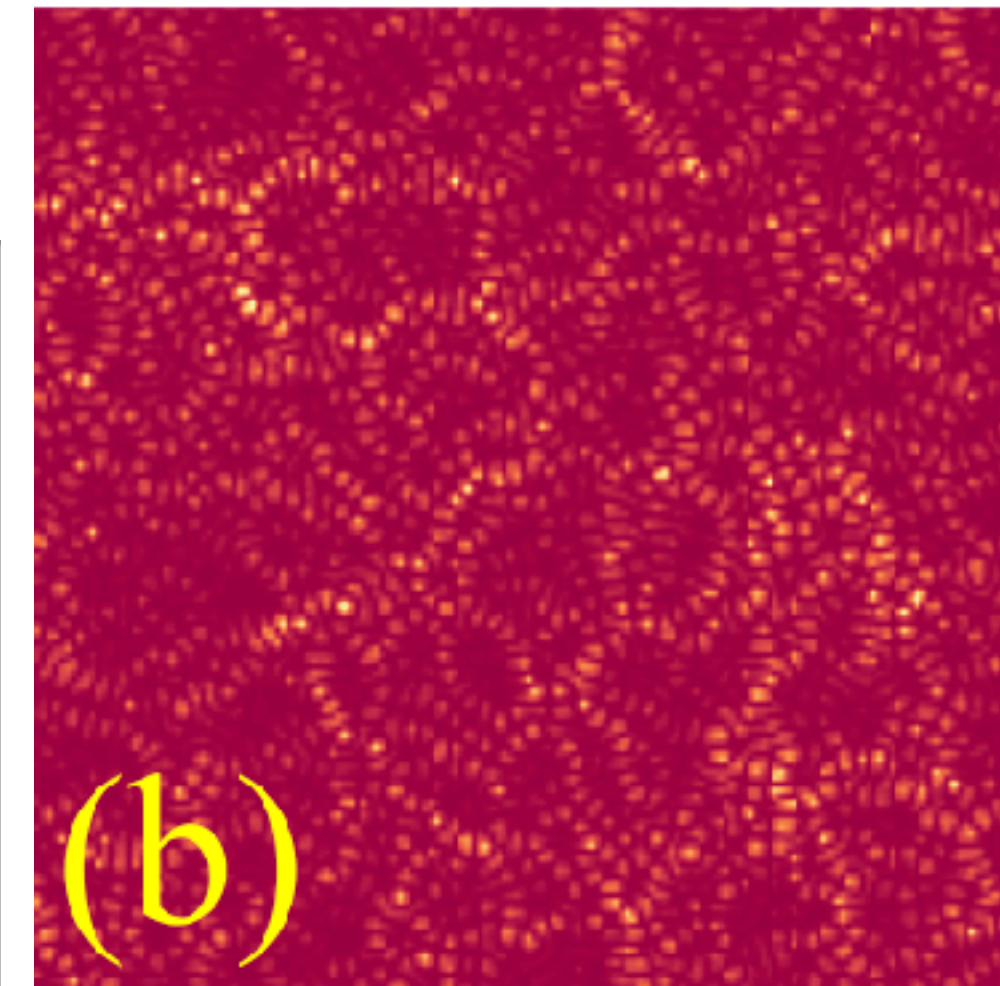
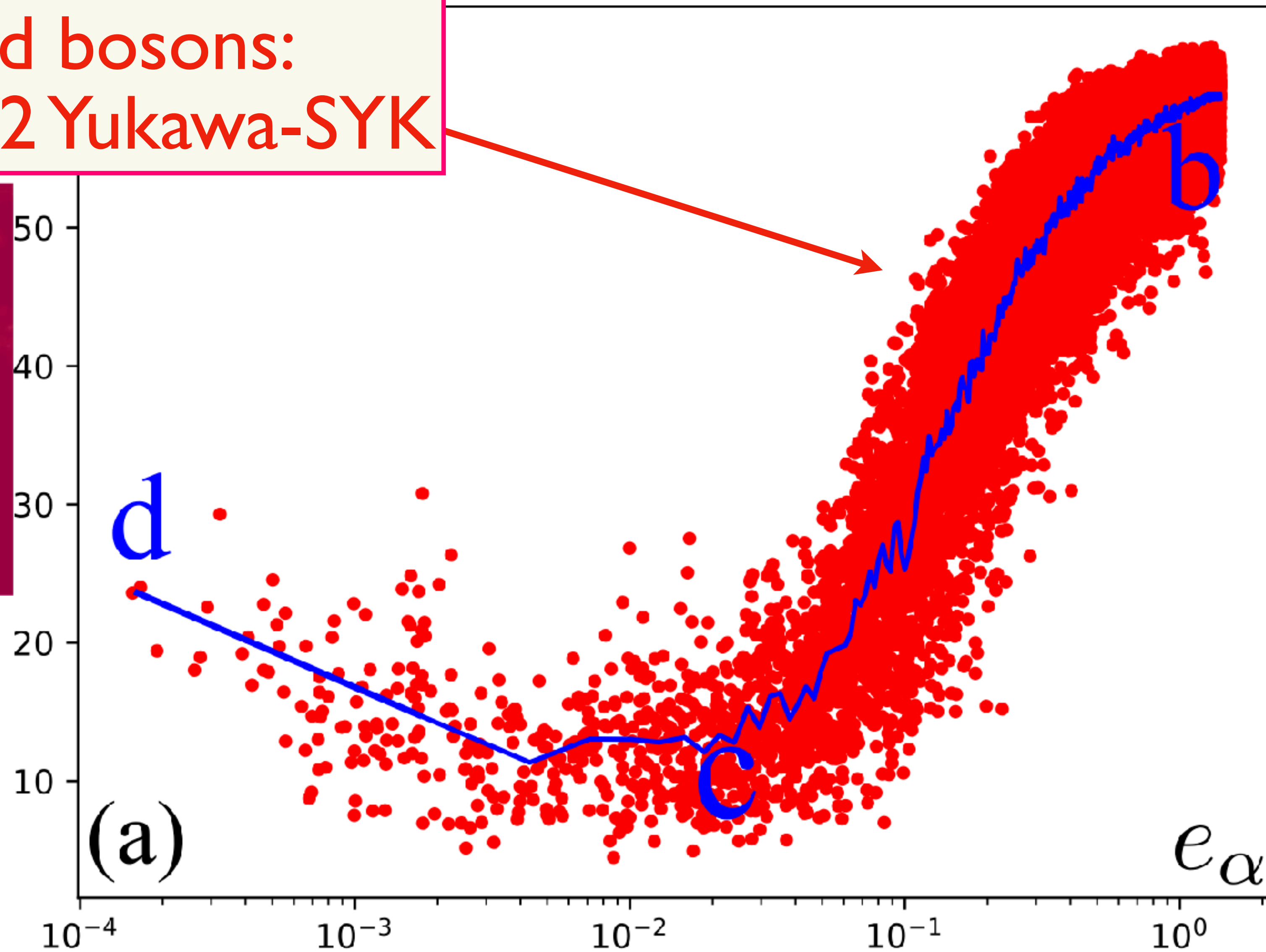


Mapping of extended
fermions+bosons regime to
 $d=2$ Yukawa-SYK model

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length \mathcal{L}_α

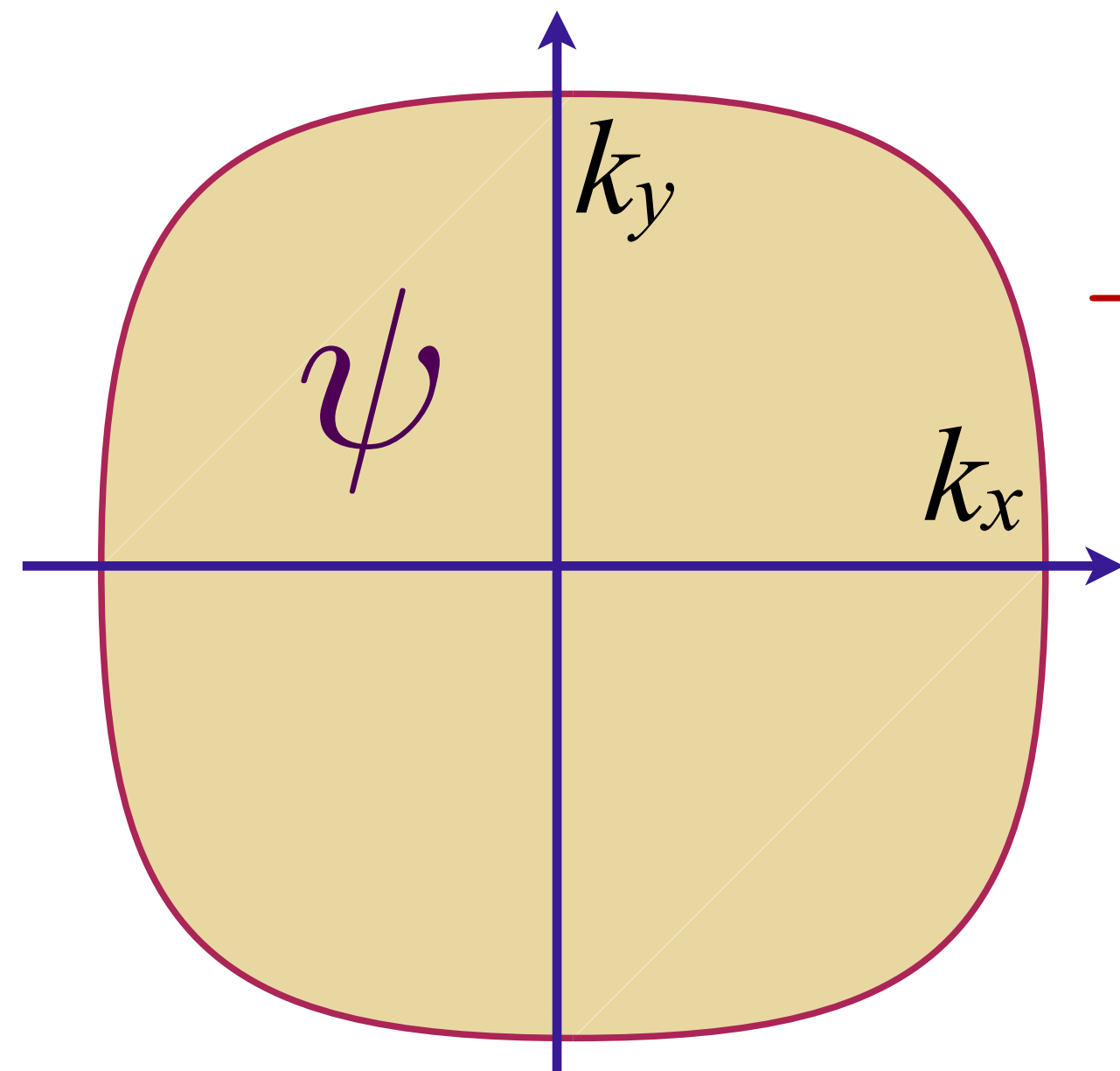
Extended bosons:
physics of $d=2$ Yukawa-SYK



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
 spin-density wave order,

Higgs boson for Fermi-volume changing transition

$$+ [s + \delta s(\mathbf{r})] [\phi(\mathbf{r})]^2 + +g \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

$$+ K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4 + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

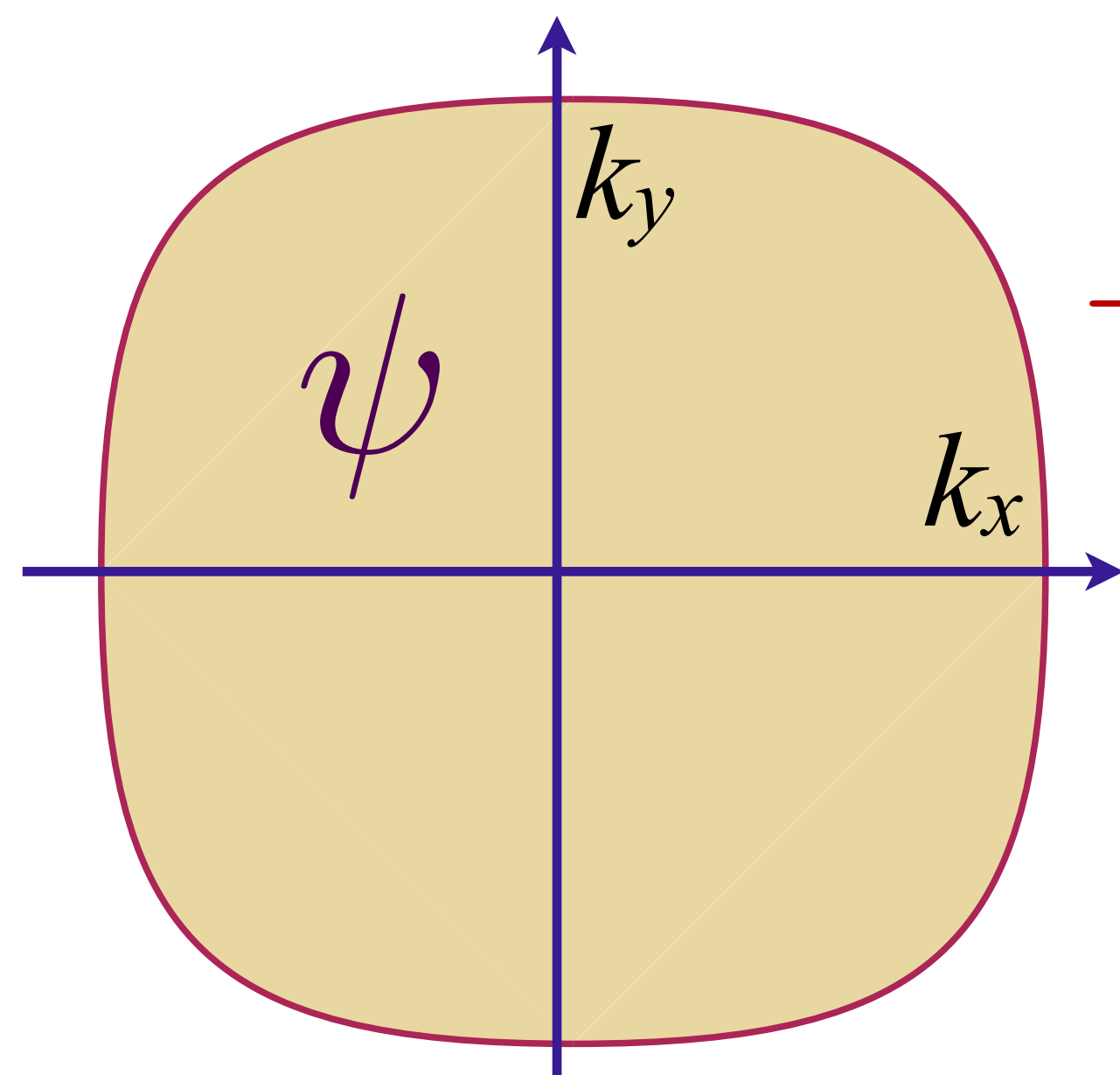
Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random mass $\delta s(\mathbf{r})$ with $\overline{\delta s(\mathbf{r})} = 0$, $\overline{\delta s(\mathbf{r})\delta s(\mathbf{r}')} = \delta s^2 \delta(\mathbf{r} - \mathbf{r}')$

RG analysis (Harris criterion) shows that $\delta s(\mathbf{r})$ is most relevant disorder.

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Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

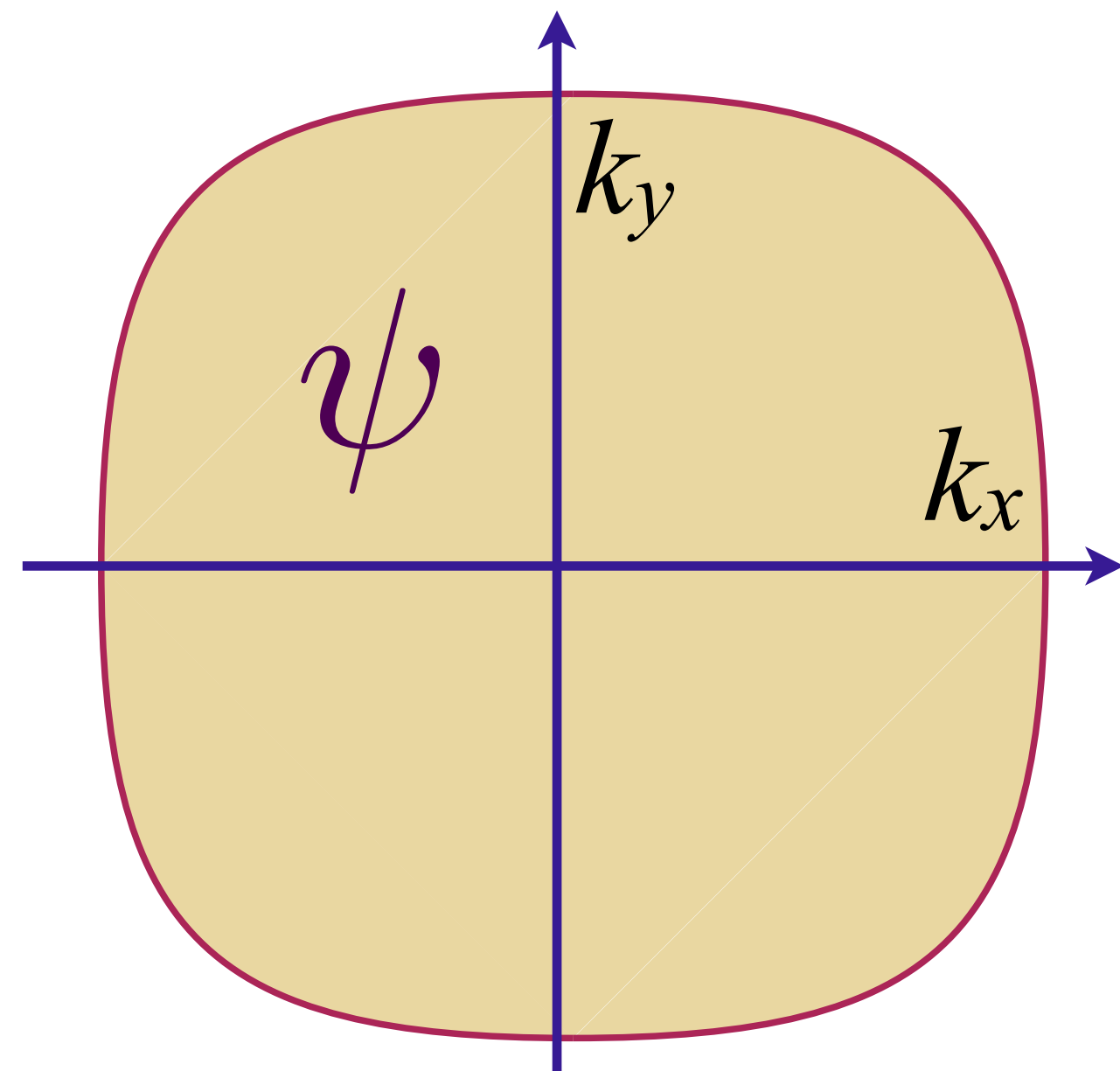
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 Rescale $\phi(\mathbf{r})$ to obtain a theory with $\delta s(\mathbf{r}) = 0$.

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Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

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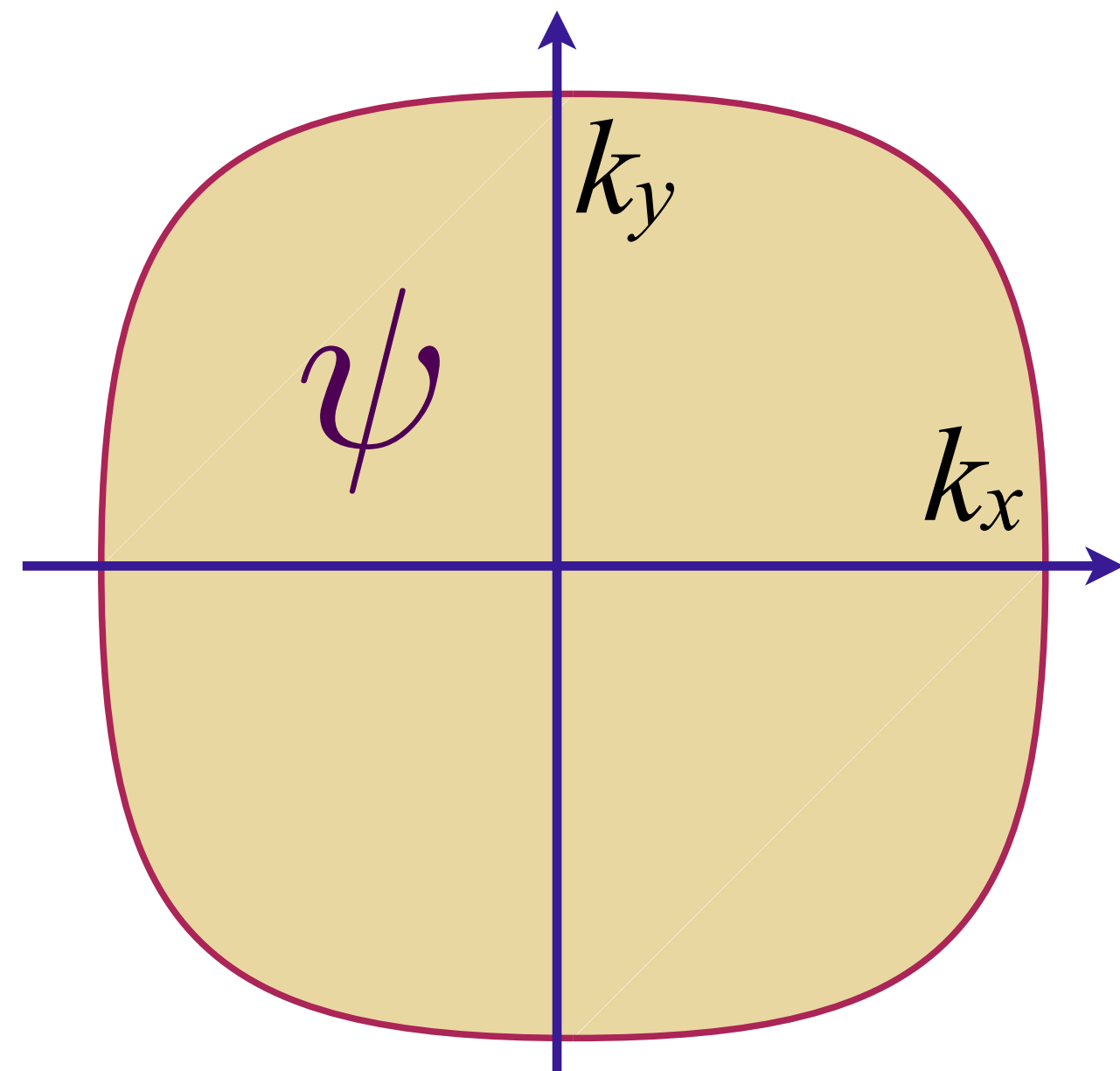
Spatially random Yukawa coupling $g'(\mathbf{r})$ with $\overline{g'(\mathbf{r})} = 0$, $\overline{g'(\mathbf{r})g'(\mathbf{r}')} = g'^2 \delta(\mathbf{r} - \mathbf{r}')$

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Analyze such a theory in a self-averaging manner as in the Yukawa-SYK model.
 Should be applicable as long as eigenmodes of $\phi(\mathbf{r})$ are extended.

Fermi surface + critical boson with potential and interaction disorder

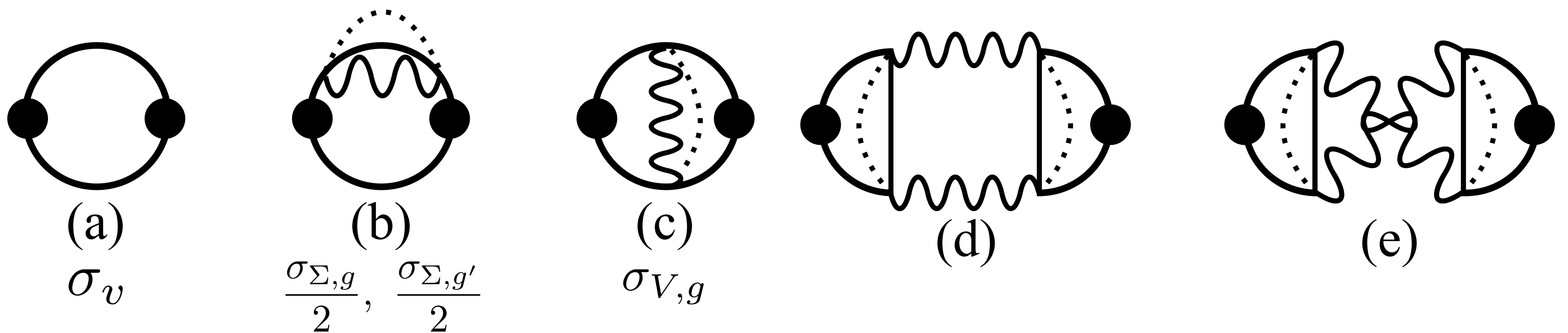
SYK-type self-consistent equations

$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}) + v^2 G(\tau, \mathbf{r}) \delta^2(\mathbf{r}) + g'^2 G(\tau, \mathbf{r}) D(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}) - g'^2 G(-\tau, \mathbf{r}) G(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$



+ all ladders and bubbles.....

Conductivity:

Fermi surface + critical boson with potential and interaction disorder

Electron Green's function:
$$G(\omega) \sim \frac{1}{\omega \frac{m^*(\omega)}{m} - \varepsilon(\mathbf{k}) + i \left(\frac{1}{\tau_e} + \frac{1}{\tau_{\text{in}}(\omega)} \right) \text{sgn}(\omega)}$$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi} \left(\frac{g^2}{v^2} + g'^2 \right) \ln(\Lambda/\omega)$$

Conductivity:
$$\sigma(\omega) \sim \frac{1}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m_{\text{trans}}^*(\omega)}{m}}$$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2 |\omega| \quad ; \quad \frac{m_{\text{trans}}^*(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

B. Michon.....A. Georges, Nat. Commun. **14**, 3033 (2023)

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.

Residual resistivity is determined by v^2 ; Linear-in- T resistivity determined by g'^2 ;

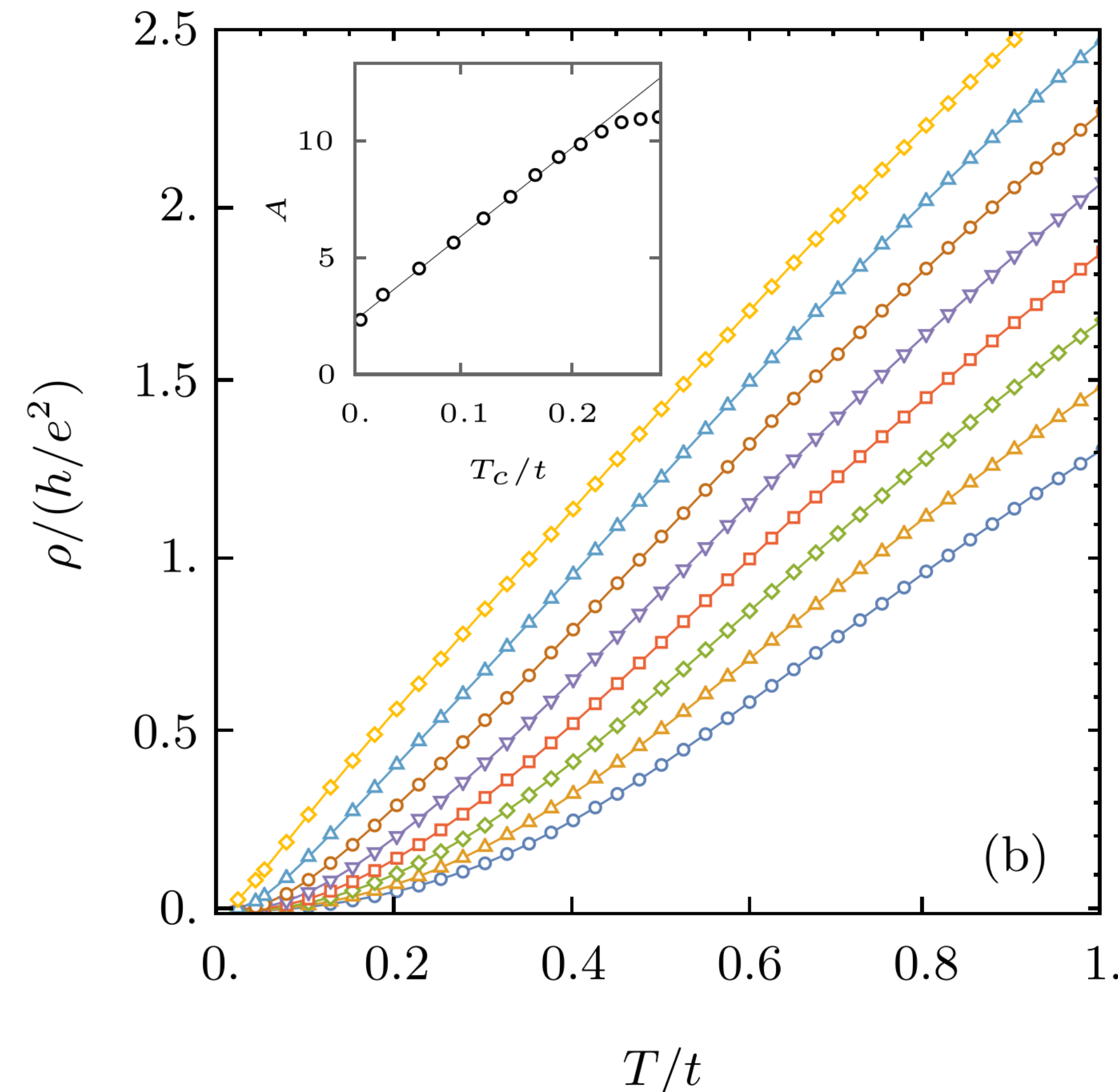
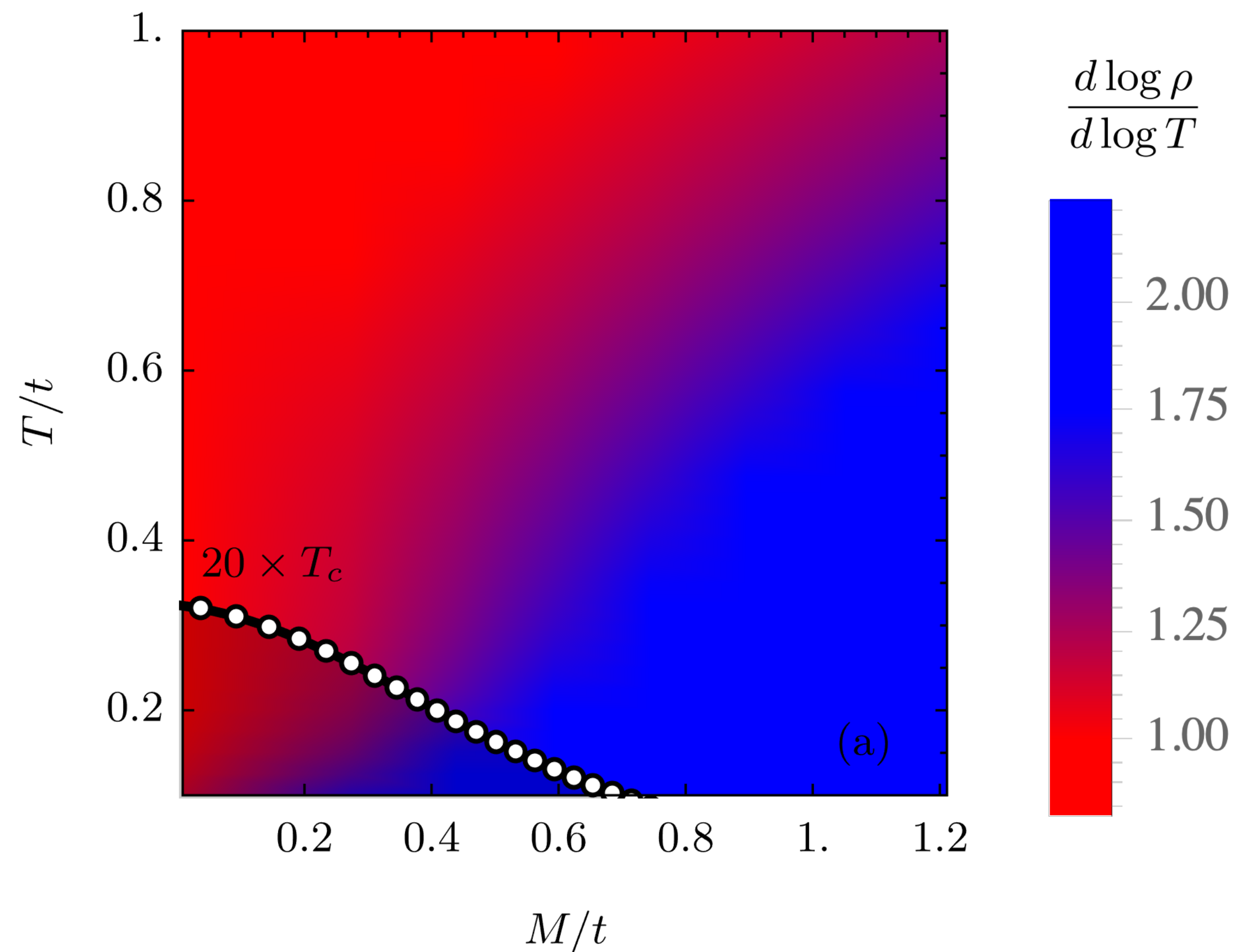
Transport insensitive to g ;

Strange metal and superconductor

$g = 0$

in the two-dimensional Yukawa-SYK model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, to appear

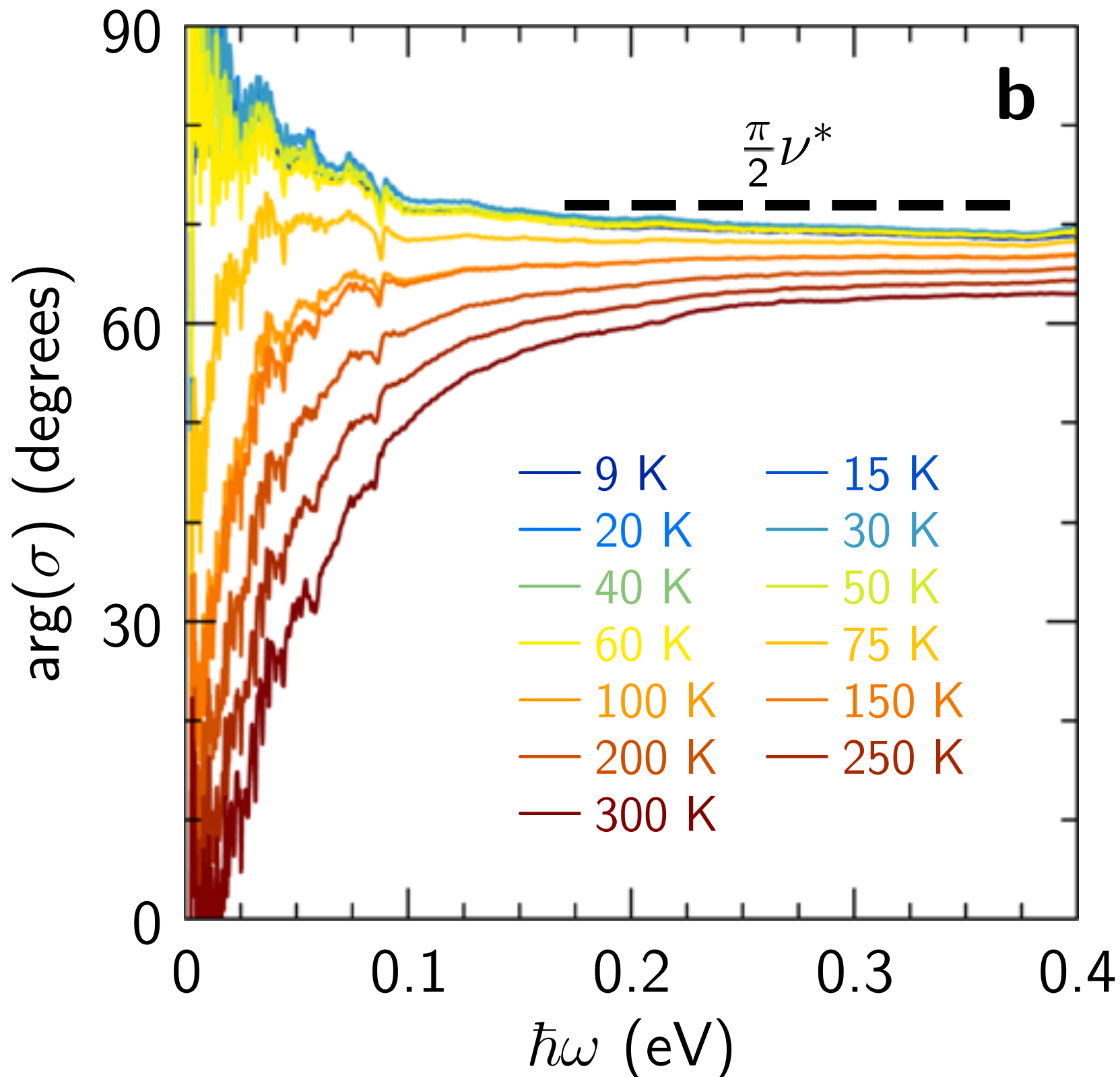
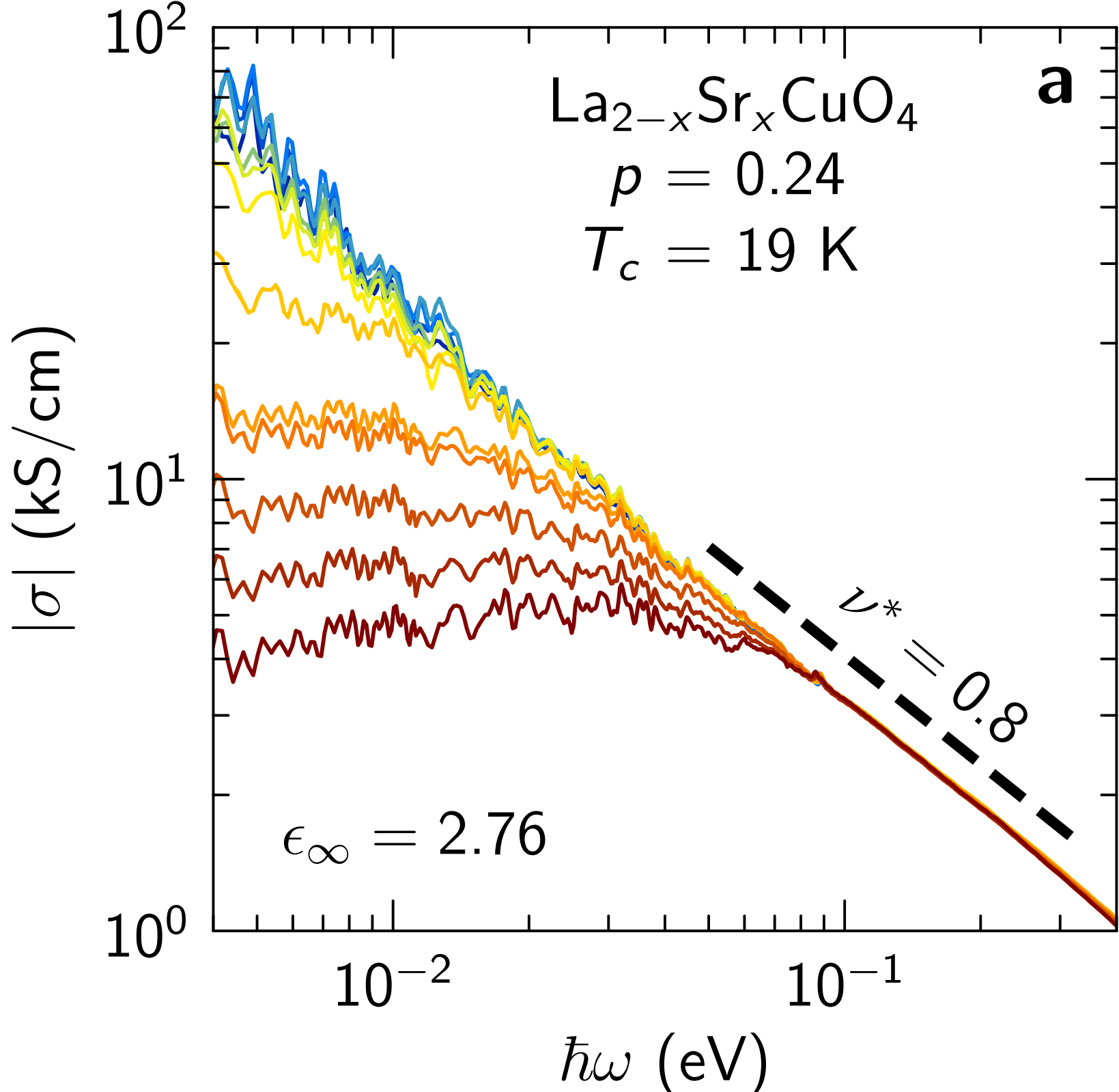


Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i \frac{e^2 K / (\hbar d_c)}{\hbar\omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$



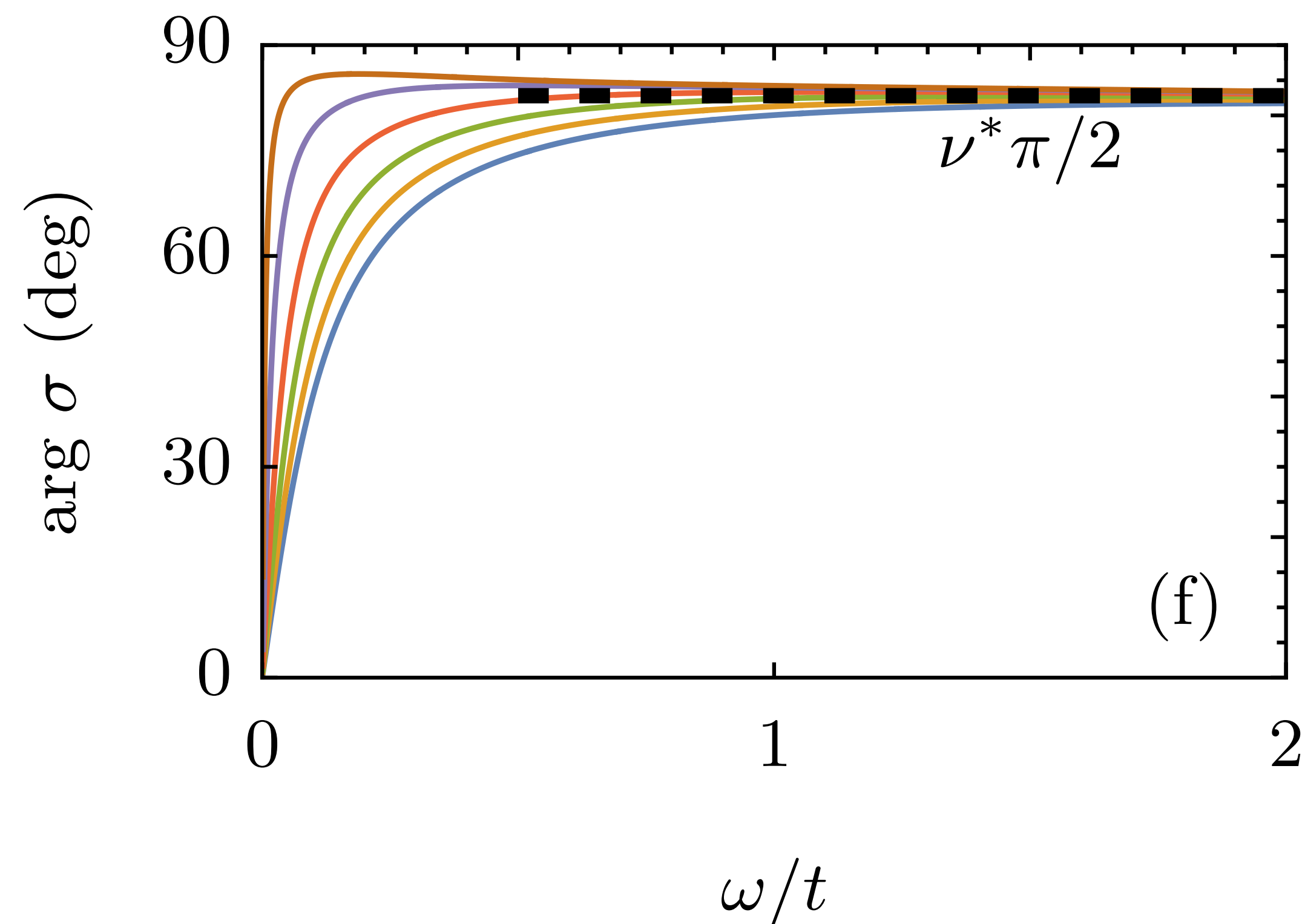
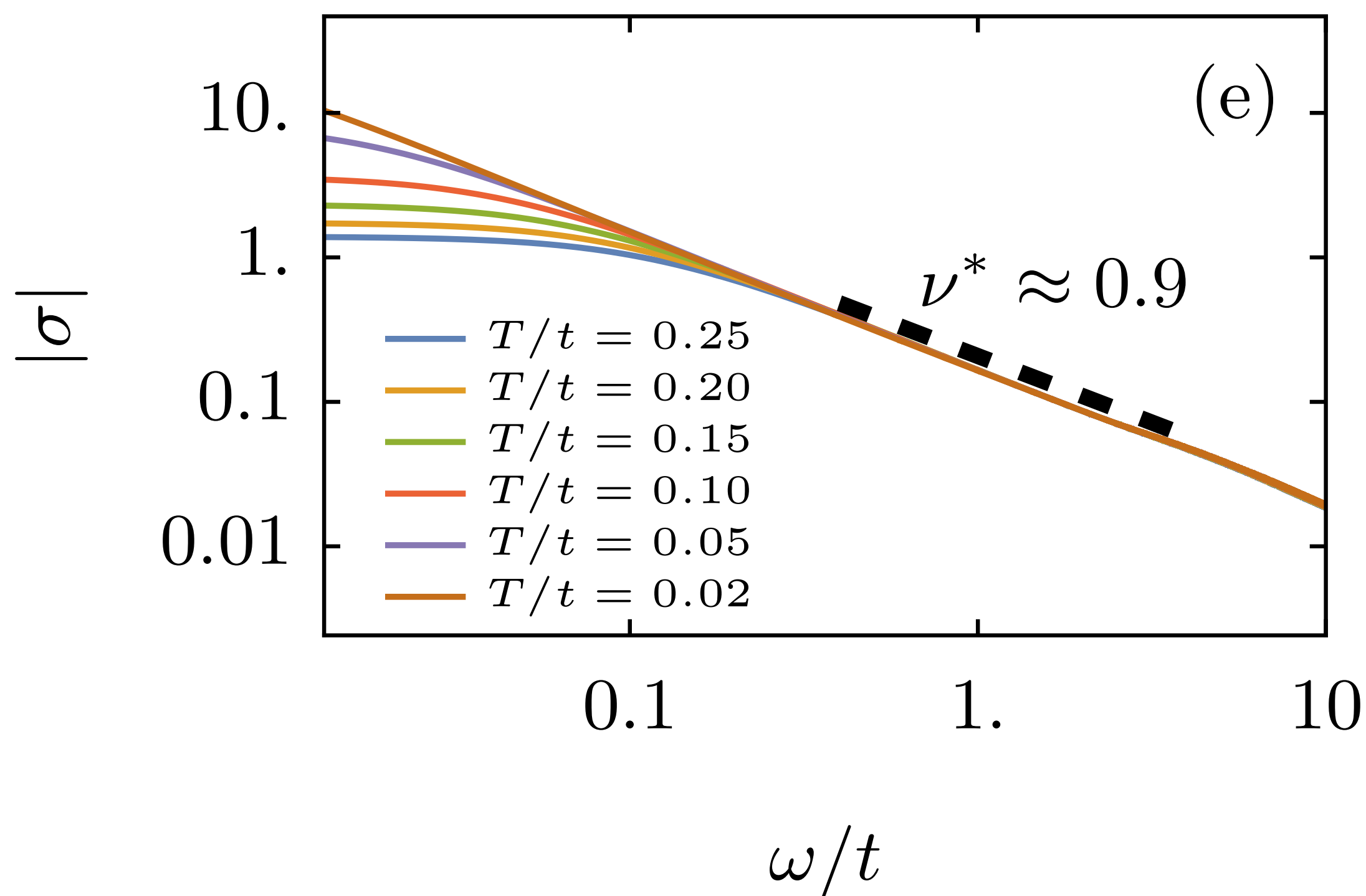
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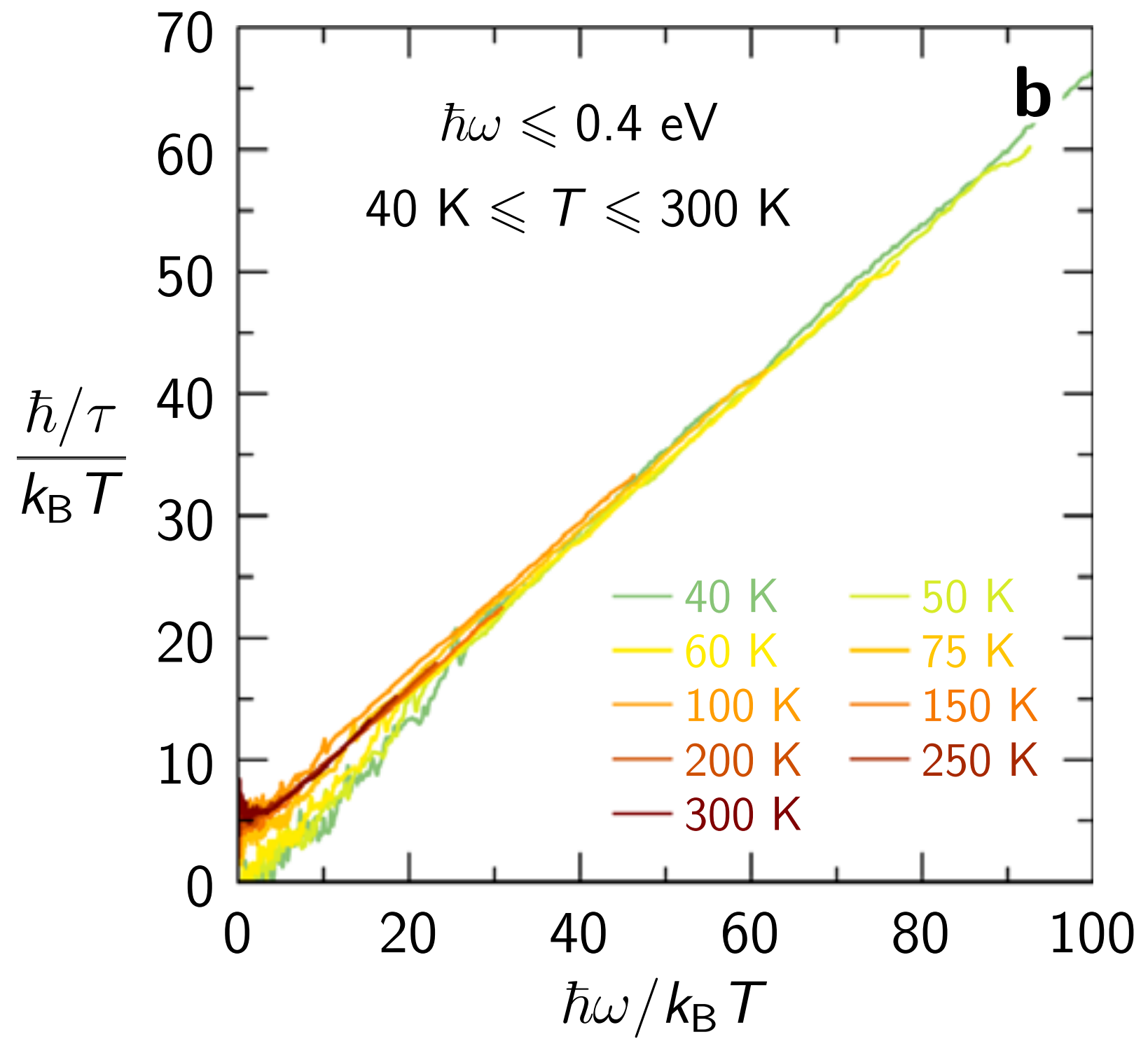
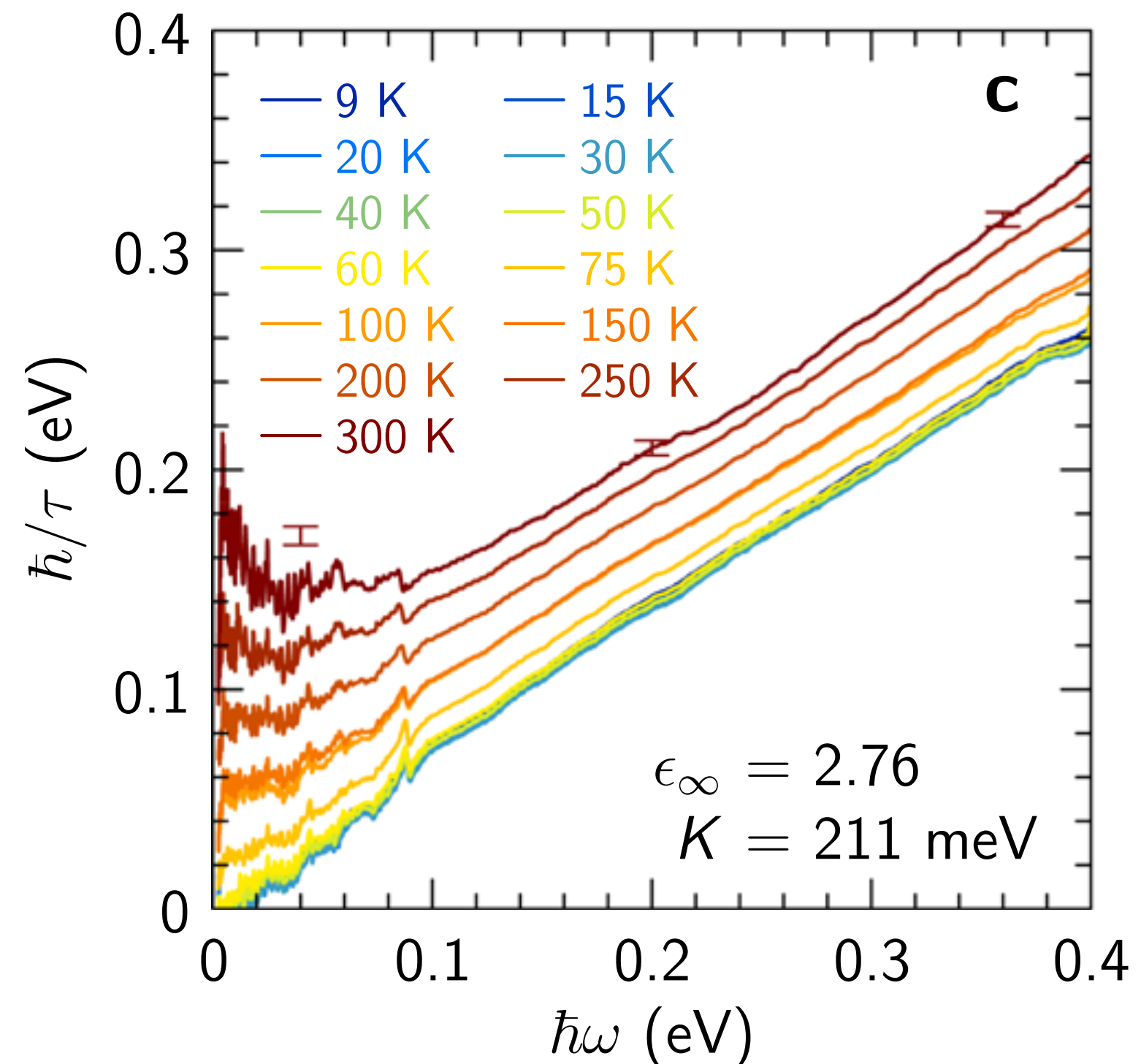


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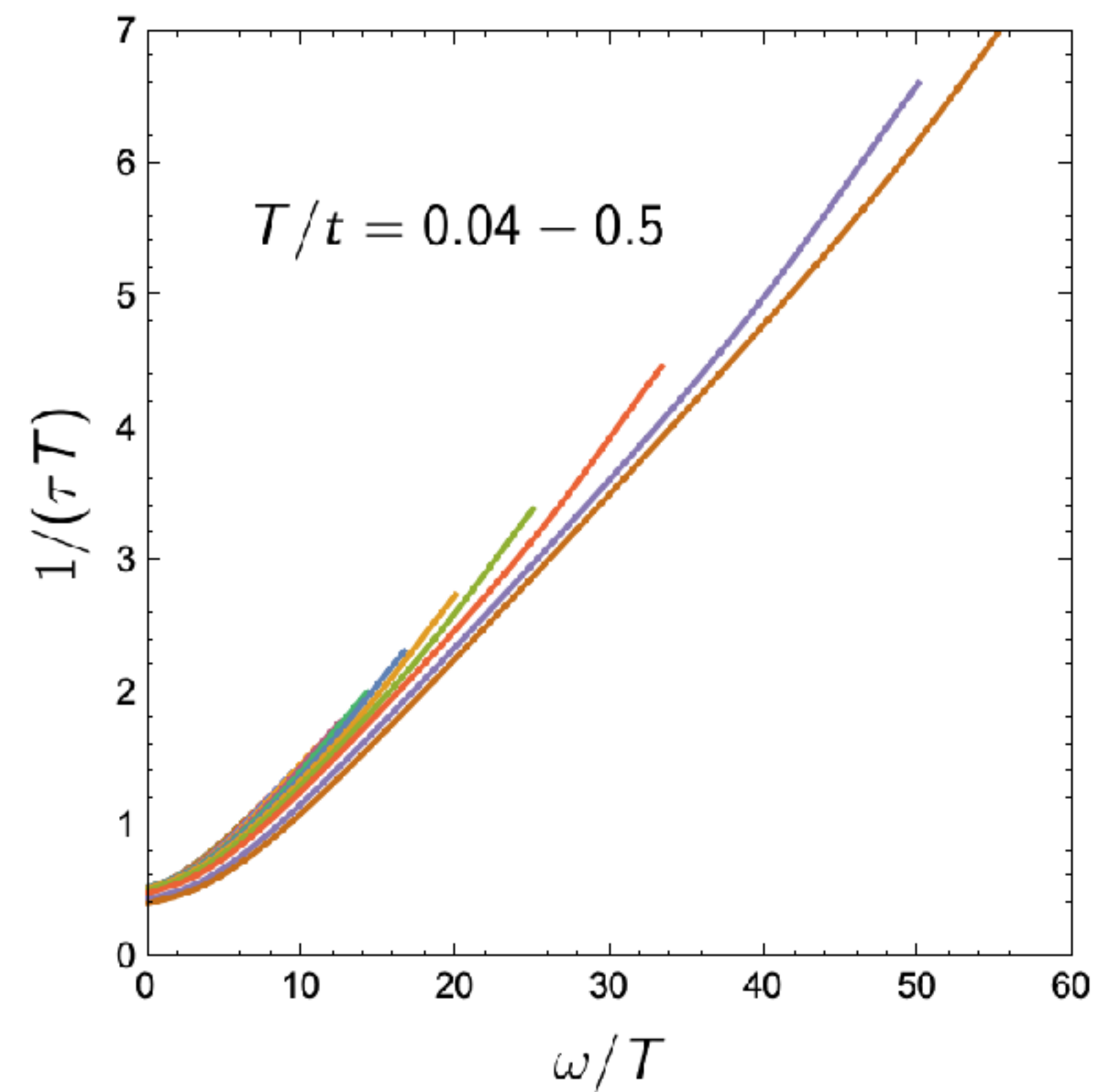
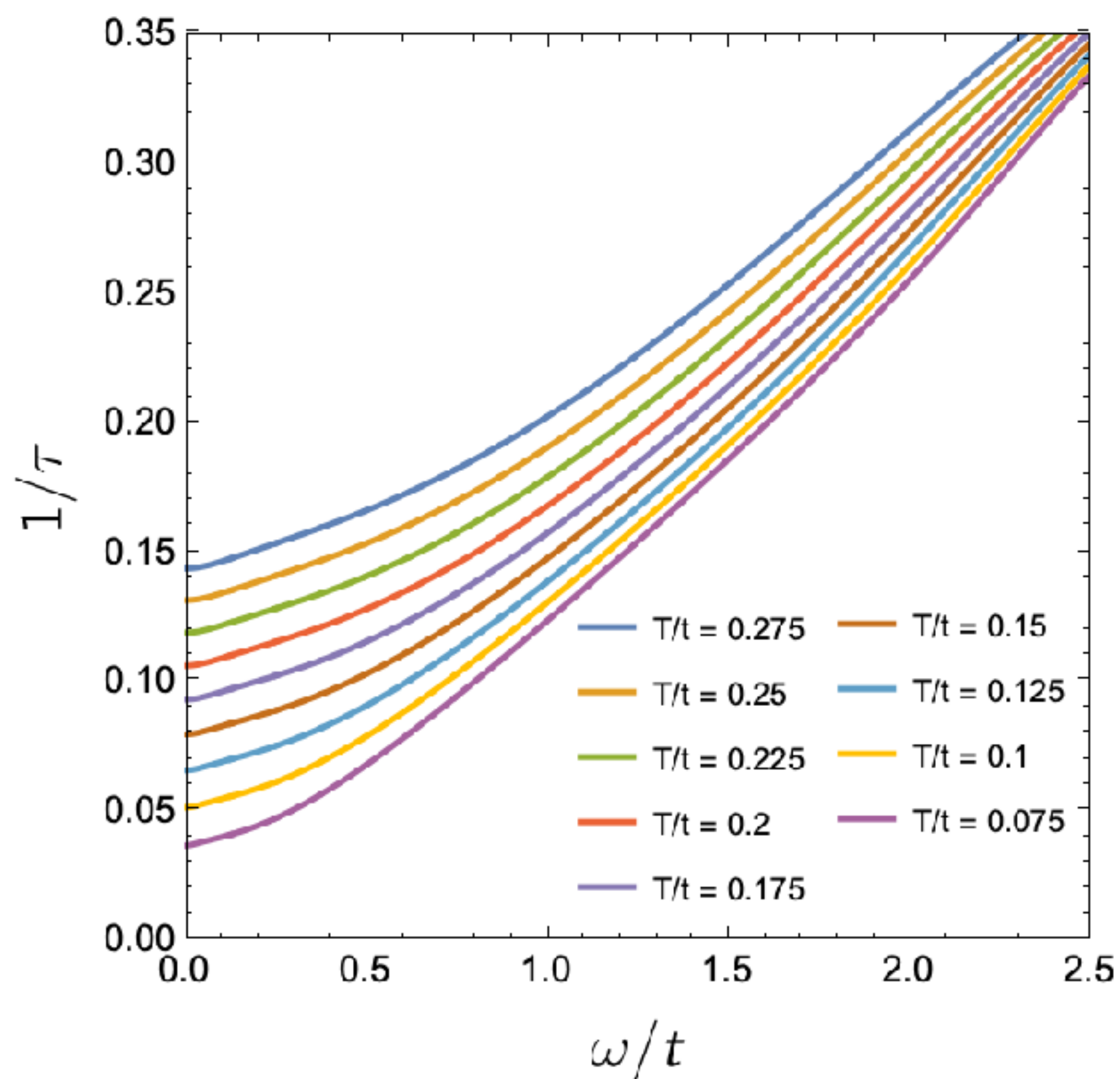
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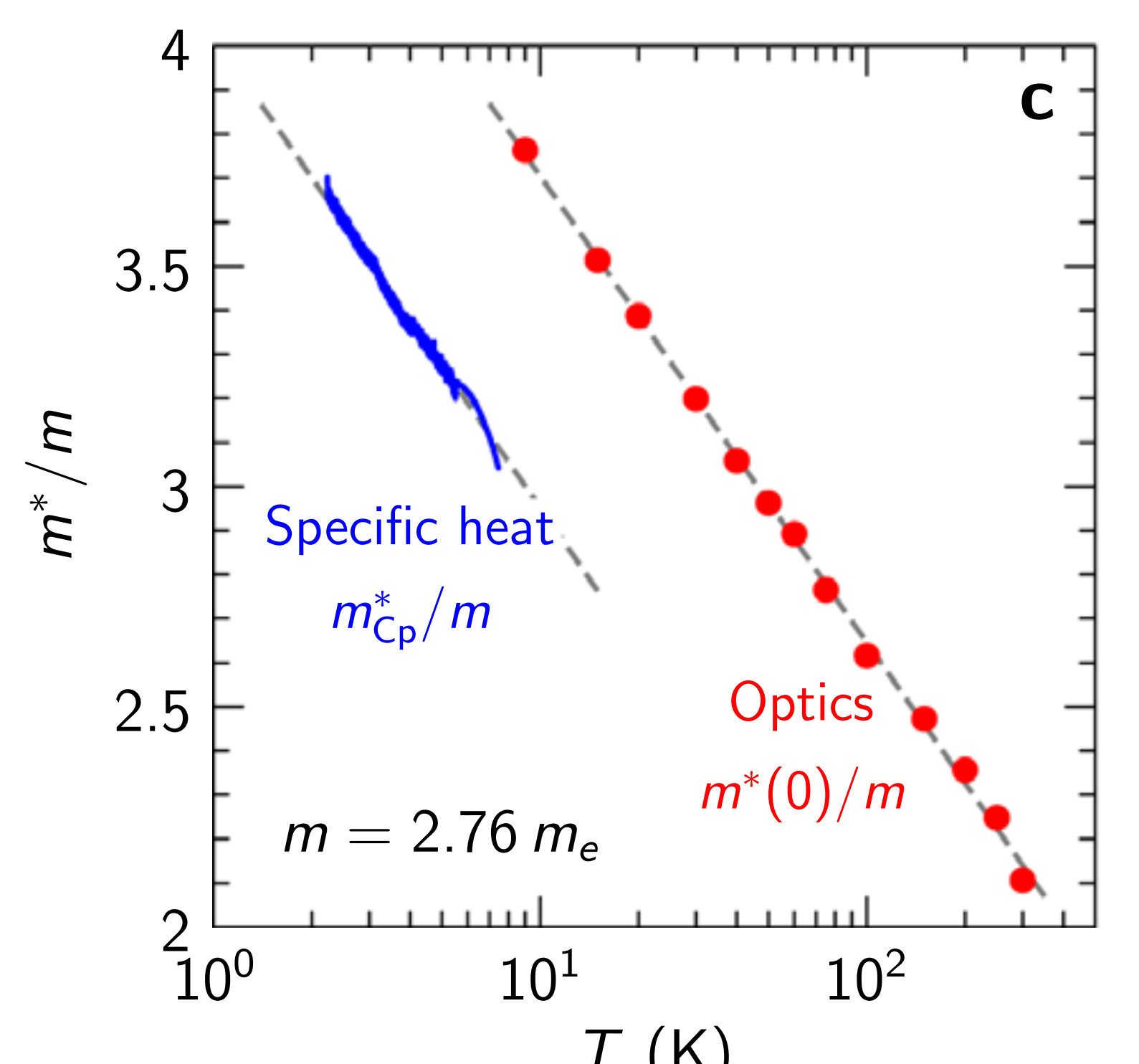
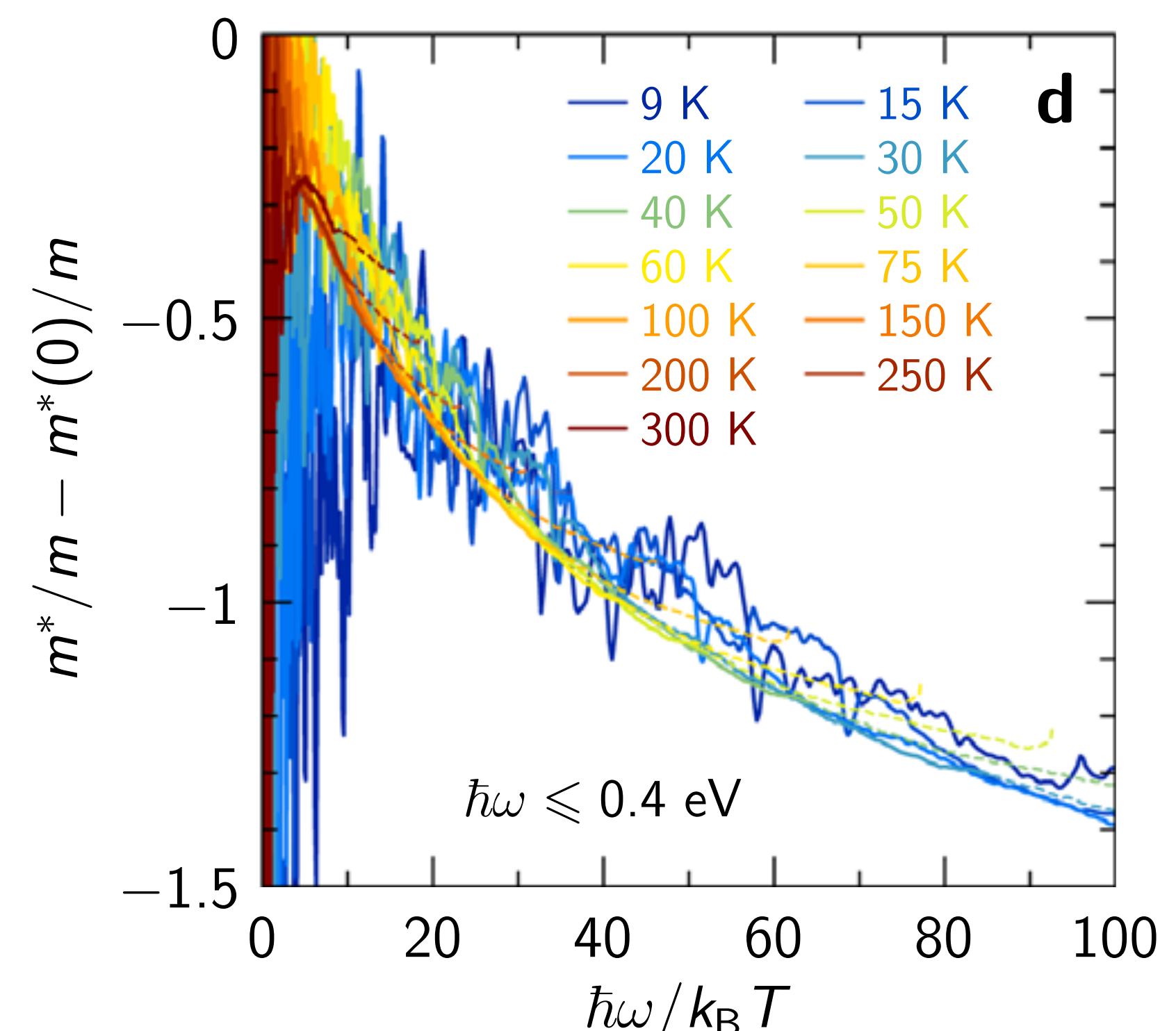
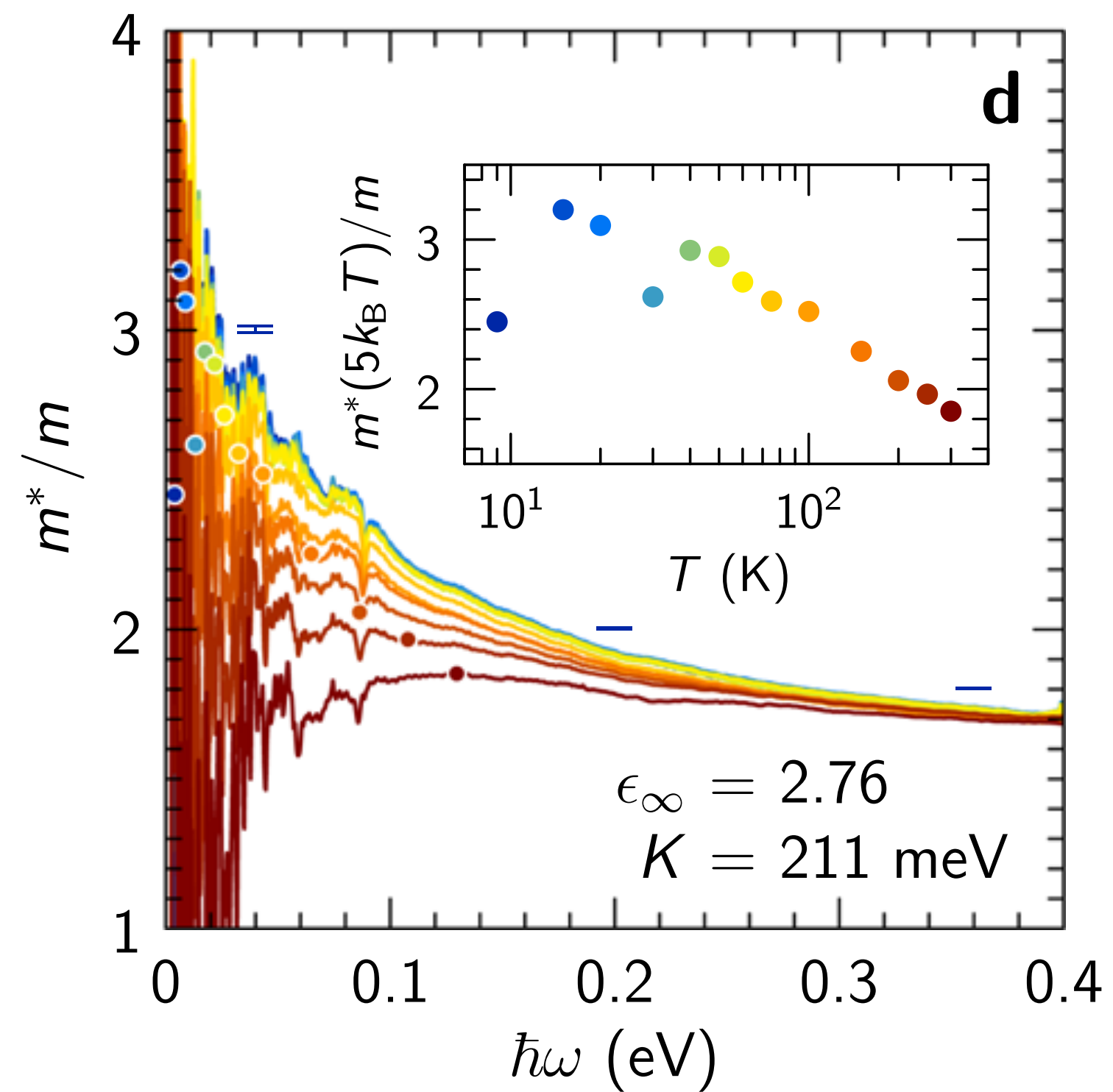


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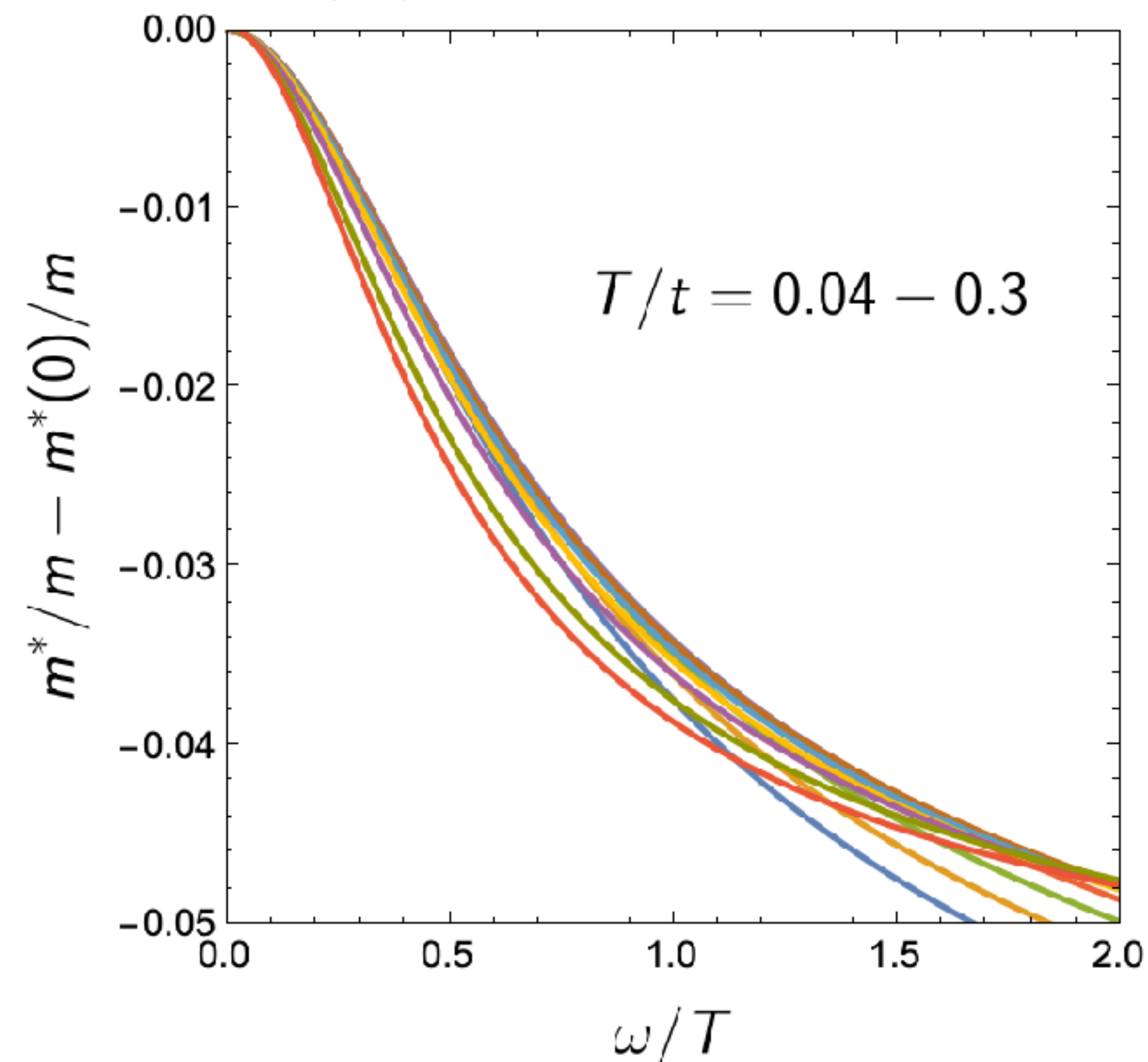
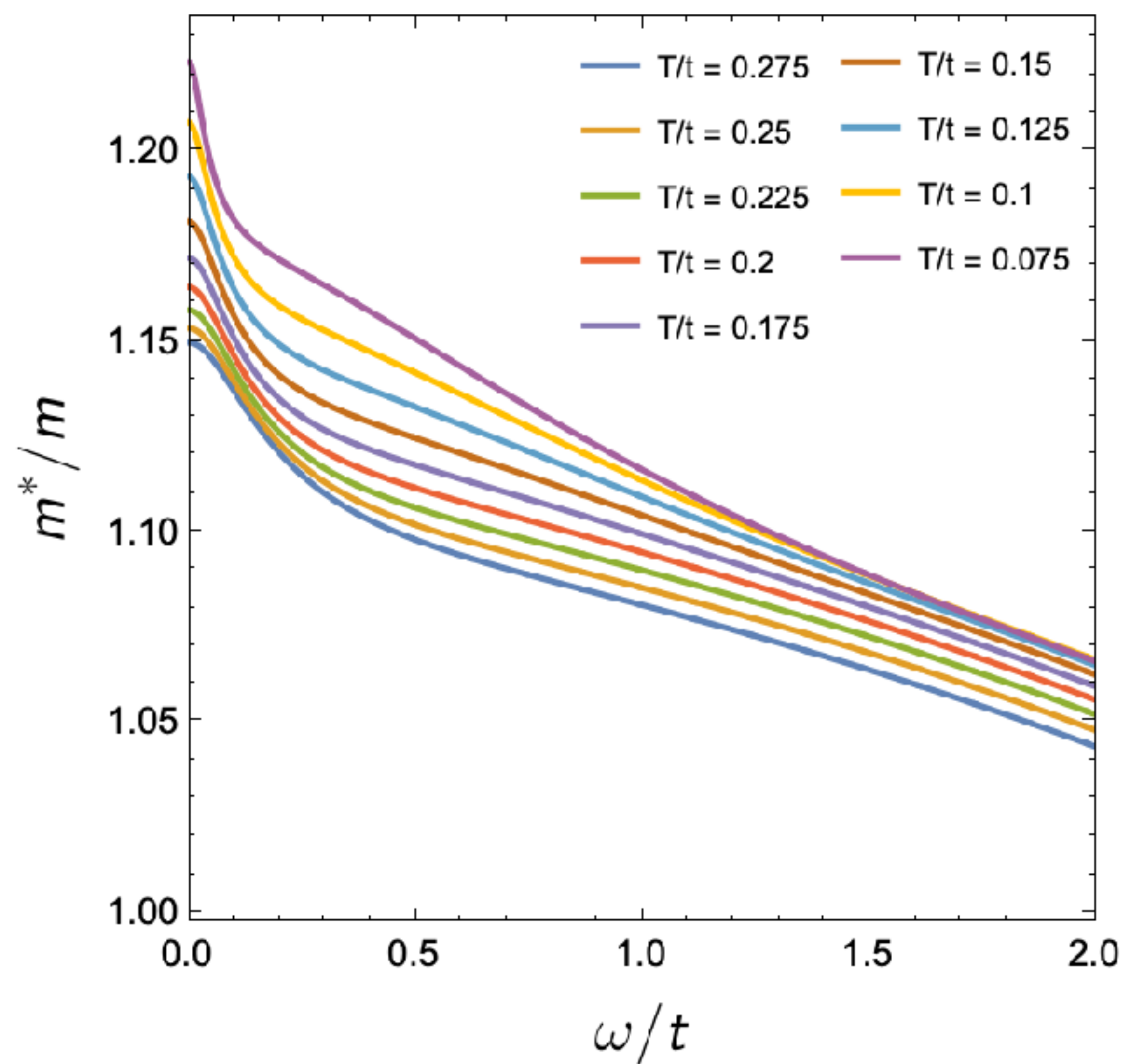
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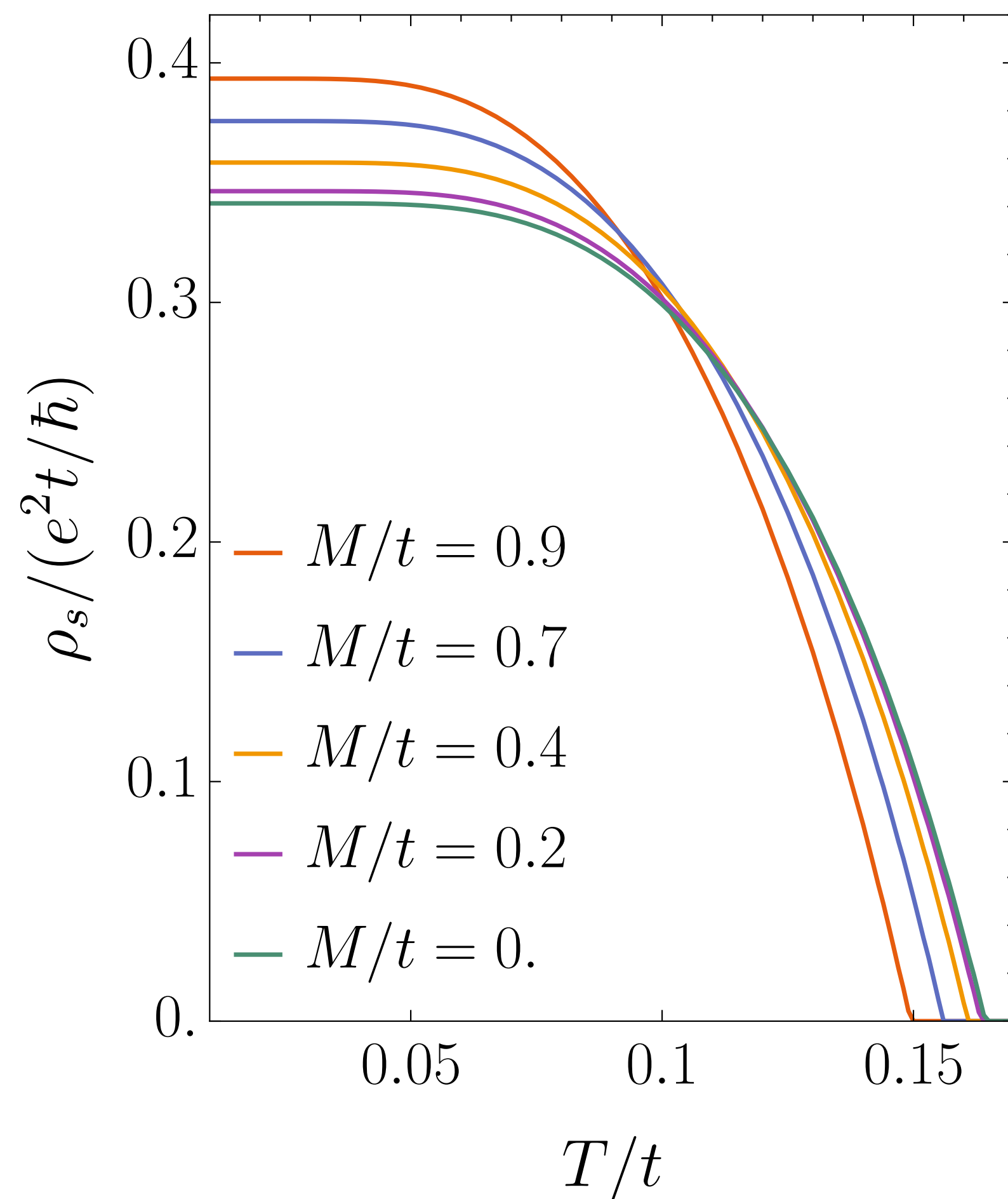


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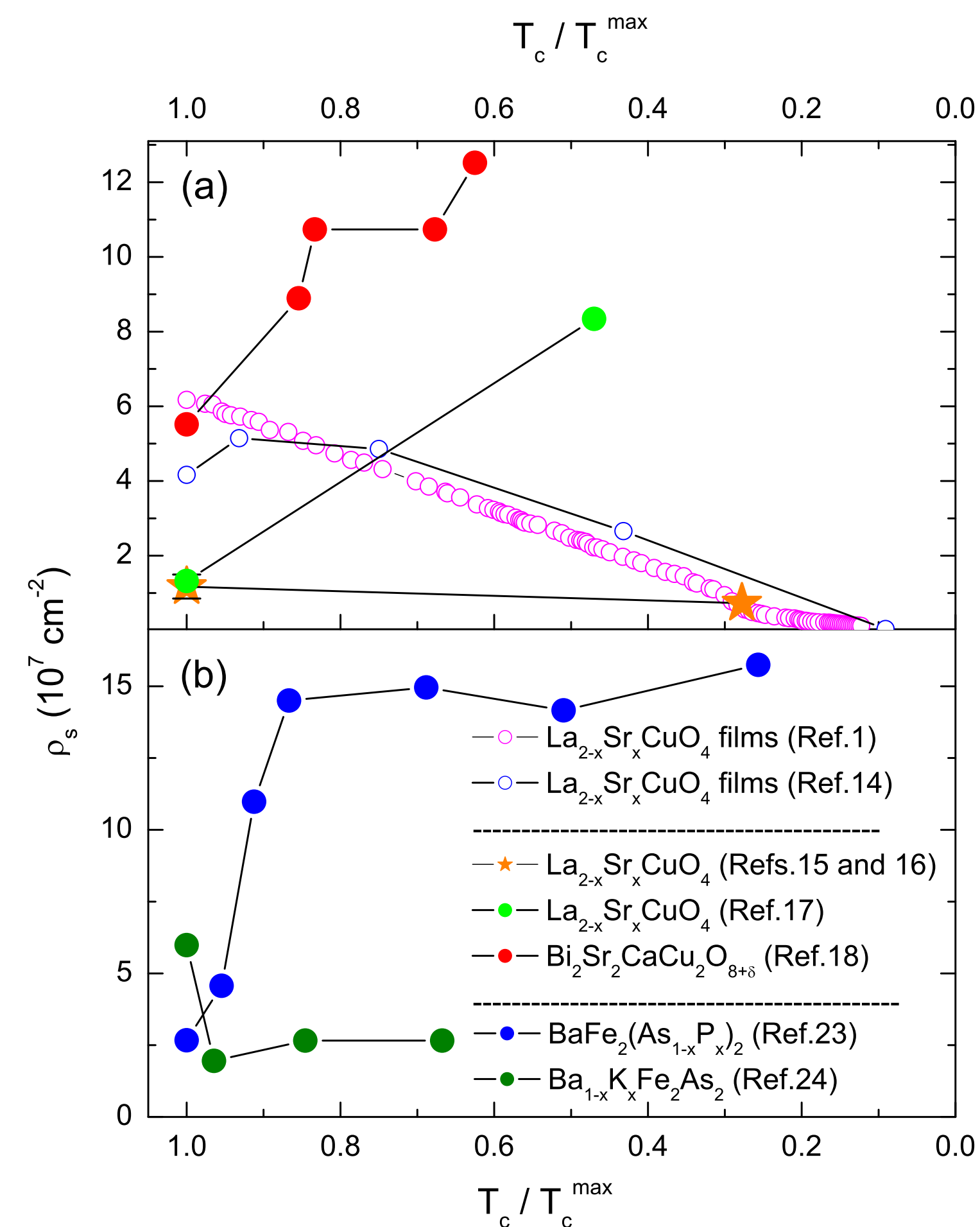
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Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentini, Jorg Schmalian, S.S., Ilya Esterlis, to appear



Superfluid stiffness ρ_s at $T = 0$
increases as T_c decreases



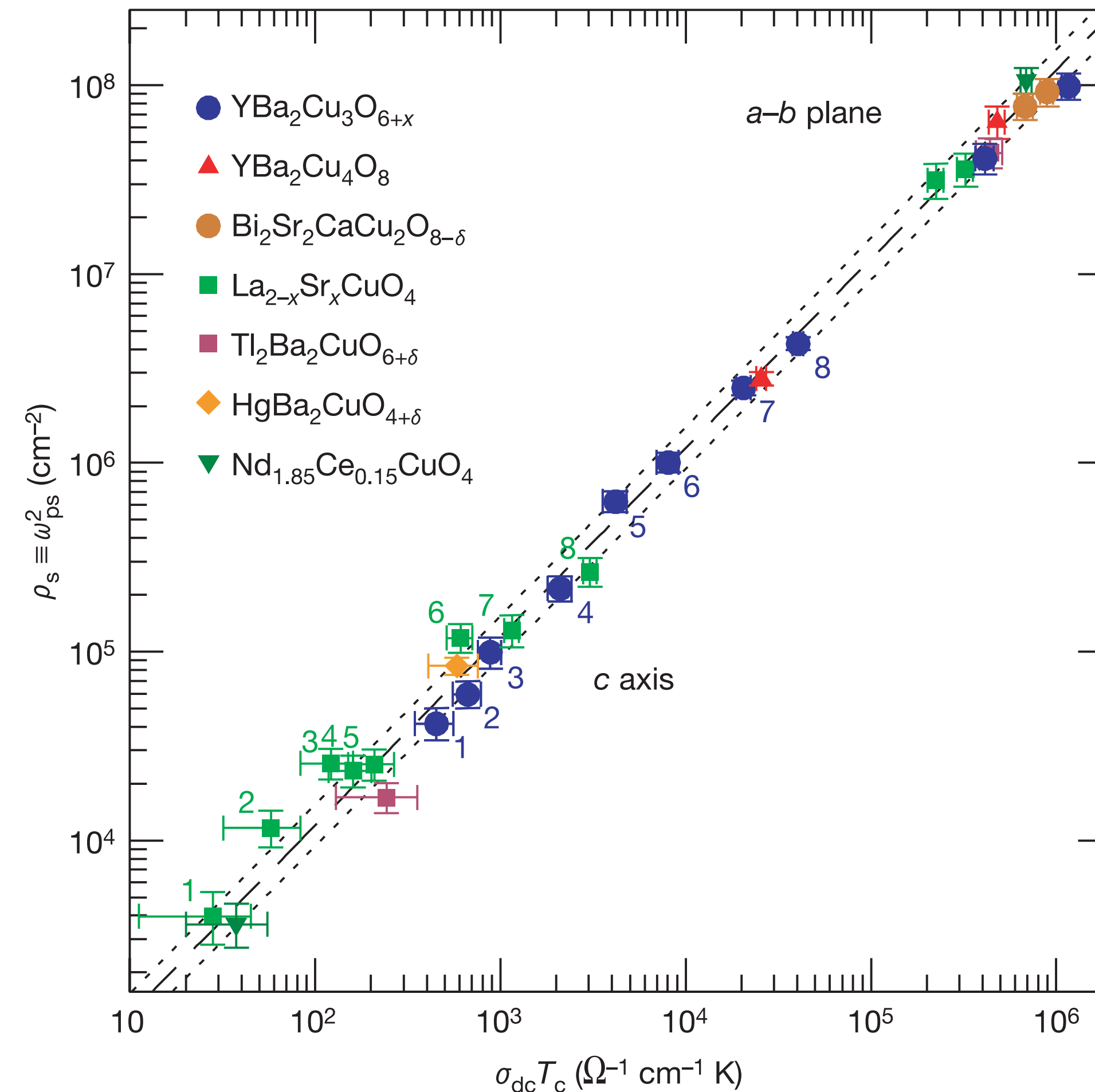
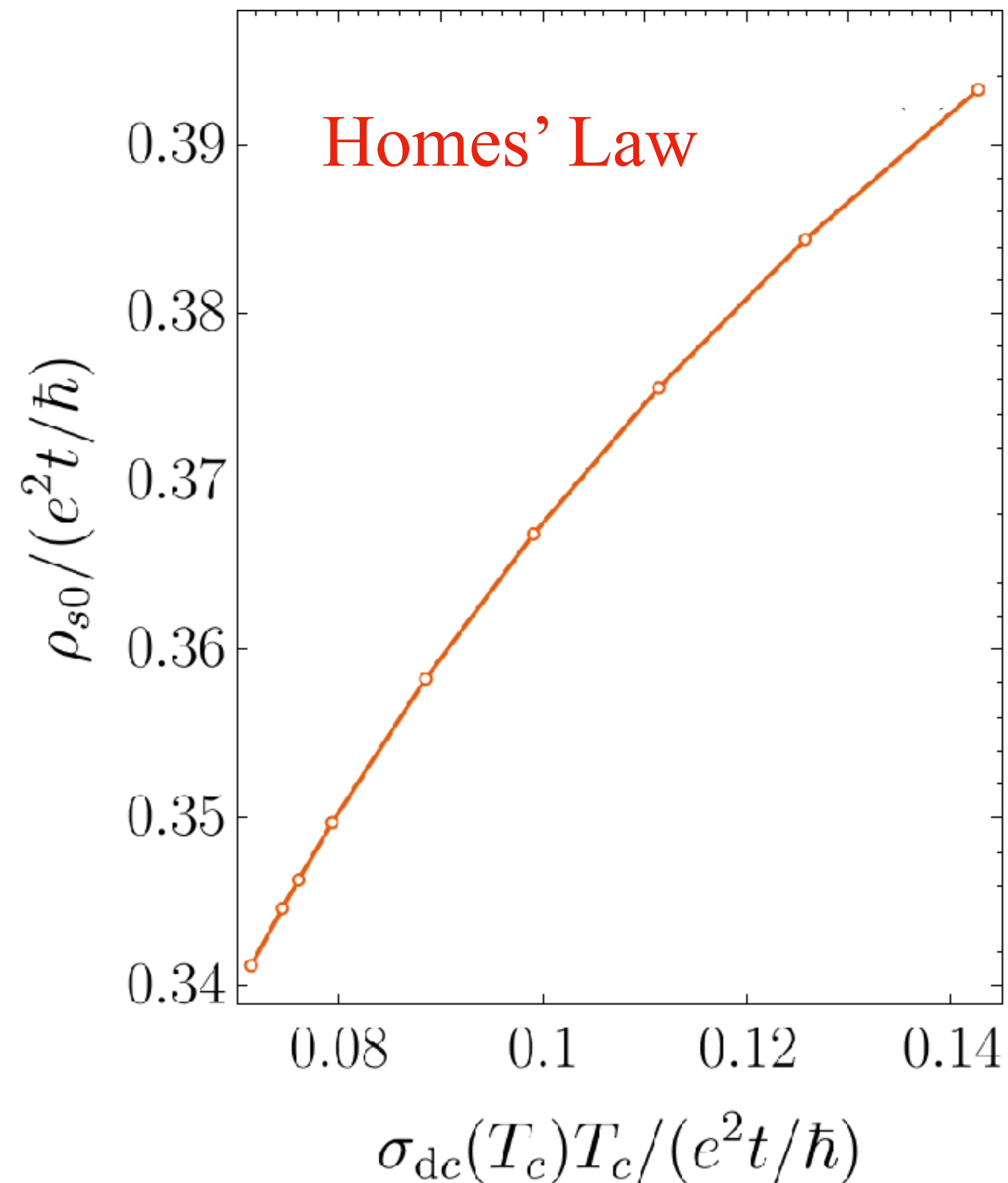
S.V. Dordevic and C.C. Homes,
PRB **105**, 214514 (2022)

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C.C. Homes et al. Nature 430, 539 (2004)

Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

R. A. Cooper,¹ Y. Wang,¹ B. Vignolle,² O. J. Lipscombe,¹ S. M. Hayden,¹ Y. Tanabe,³ T. Adachi,³ Y. Koike,³ M. Nohara,^{4*} H. Takagi,⁴ Cyril Proust,² N. E. Hussey^{1†}

SCIENCE VOL 323 603 2009

Two-dimensional metals with Harris disorder

Extended bosons and fermions:
physics of $d=2$ Yukawa-SYK

Localized overdamped bosons, but extended fermions

