

# Understanding the cuprate phase diagram from a theory of the pseudogap metal

Quantum Materials With and Without Quasiparticles  
KITP, Santa Barbara  
October 5, 2023  
Subir Sachdev

Maine Christos, Zhu-Xi Luo, Henry Shackleton, Ya-Hui Zhang,  
Mathias Scheurer, and S. Sachdev, *Proc. Nat. Acad. Sci.* **120**, e2302701120 (2023)

Alexander Nikolaenko, Jonas v. Milczewski, Darshan G. Joshi,  
and S. Sachdev, *Phys. Rev. B* **108**, 045123 (2023)

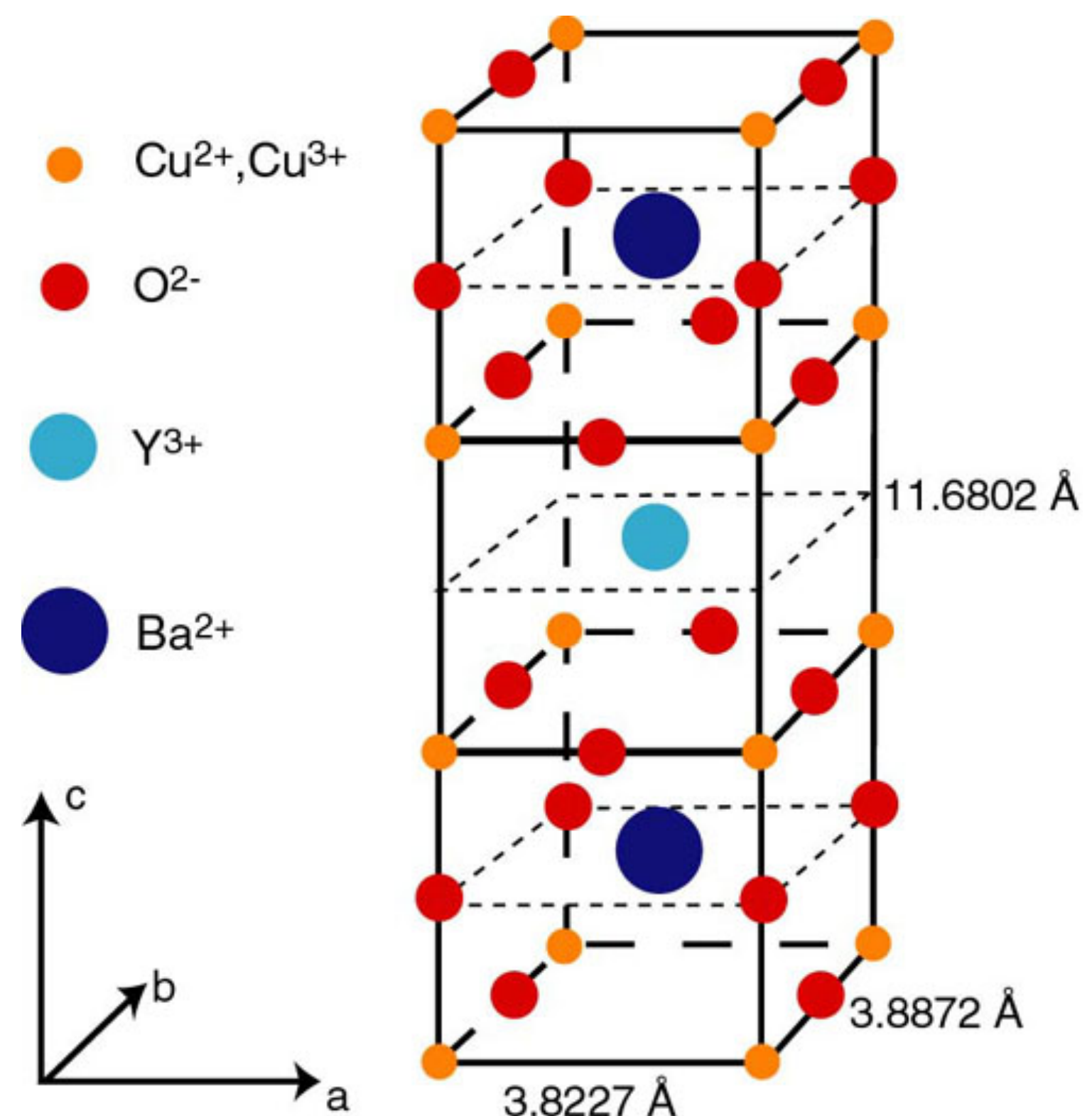
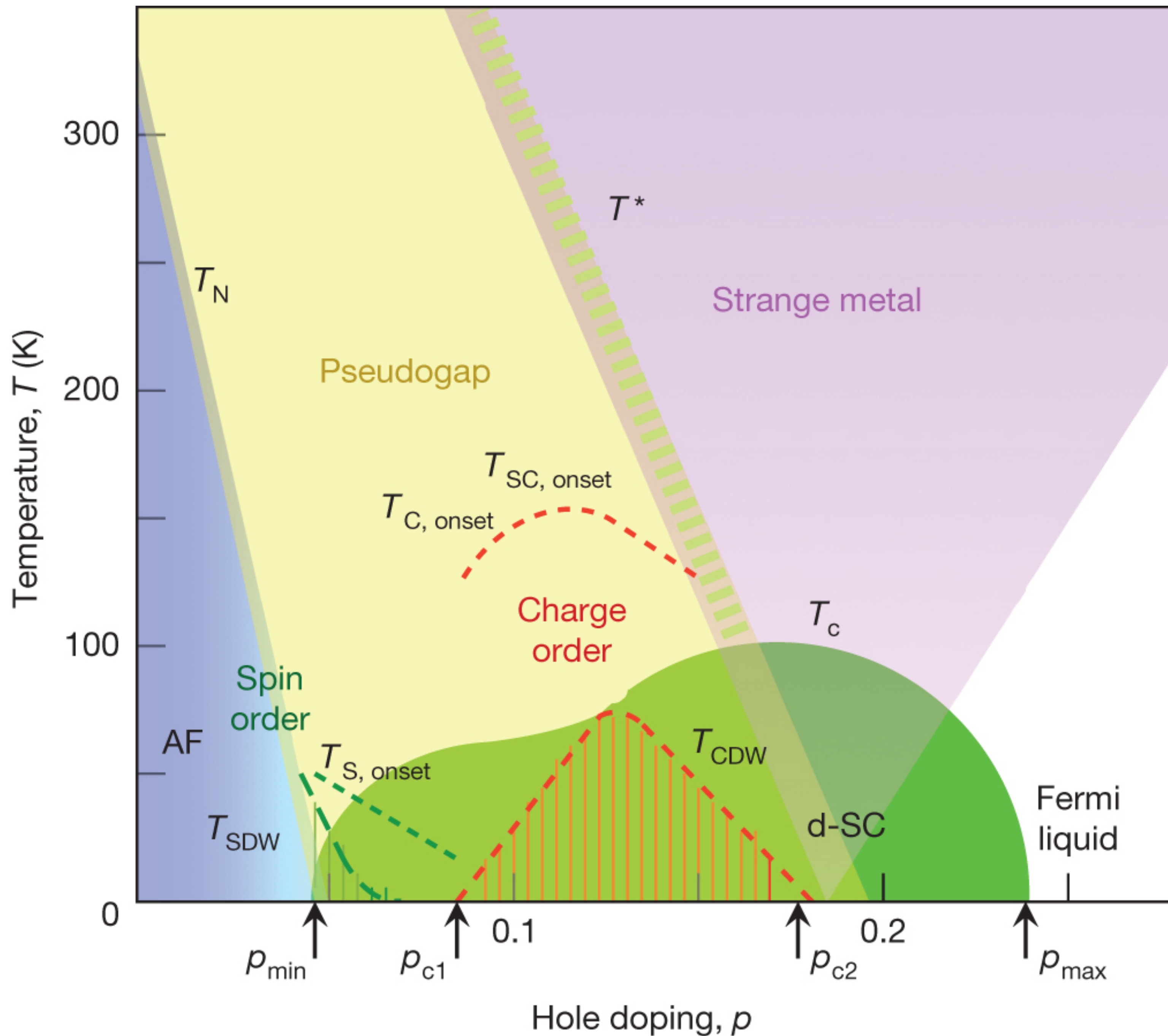
Maine Christos and S. Sachdev, [arXiv:2308.03835](https://arxiv.org/abs/2308.03835)

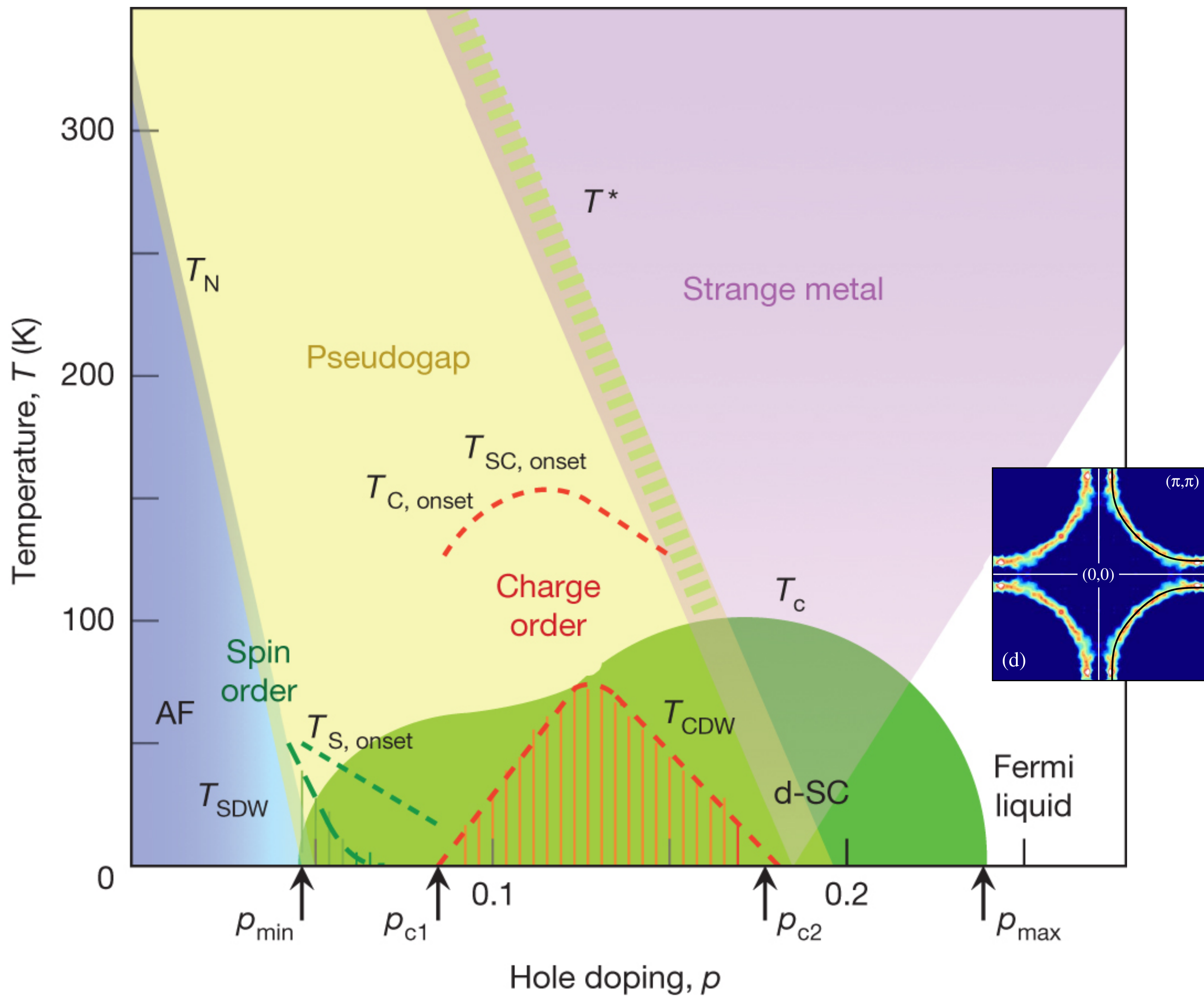


PHYSICS

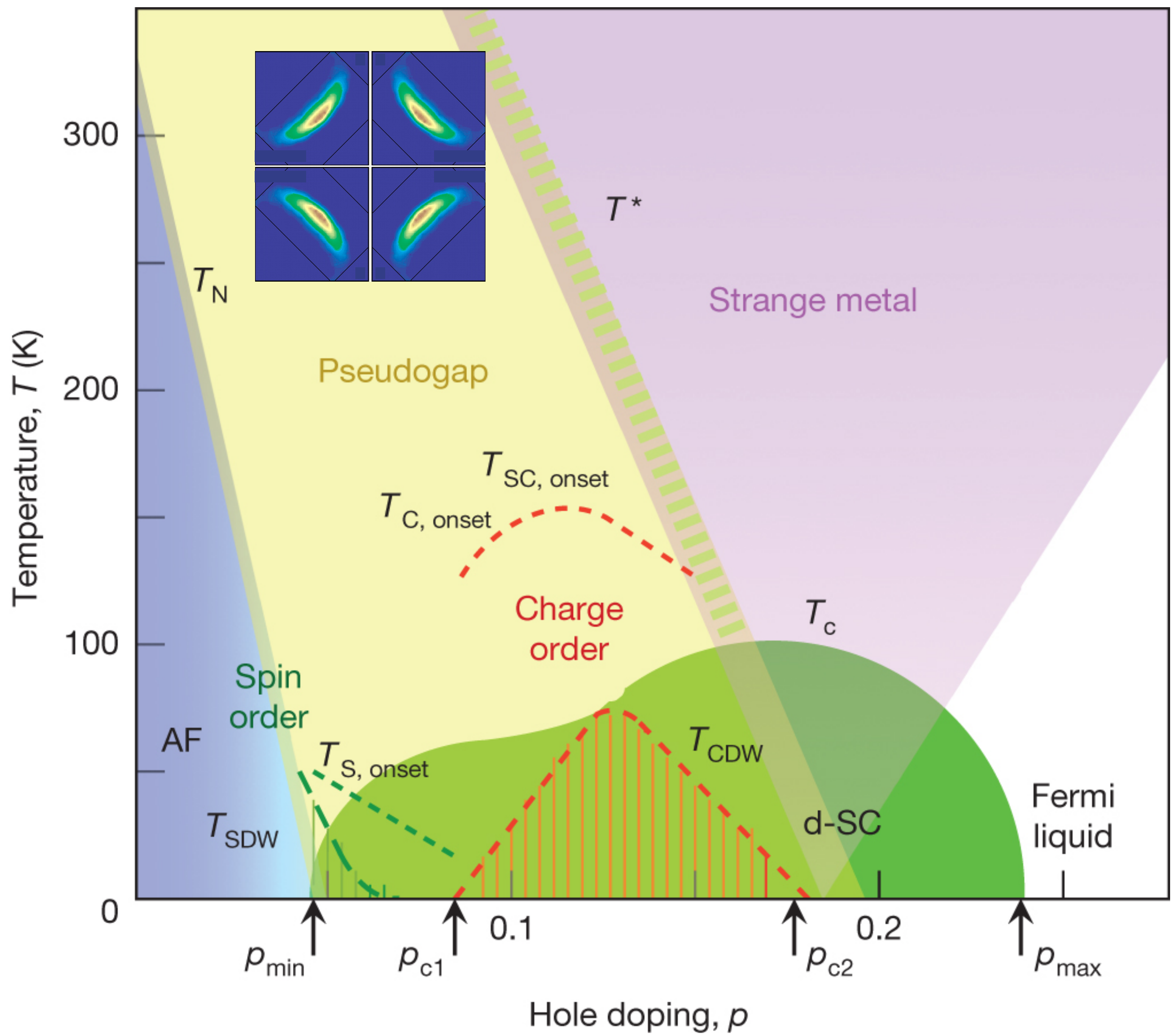


HARVARD

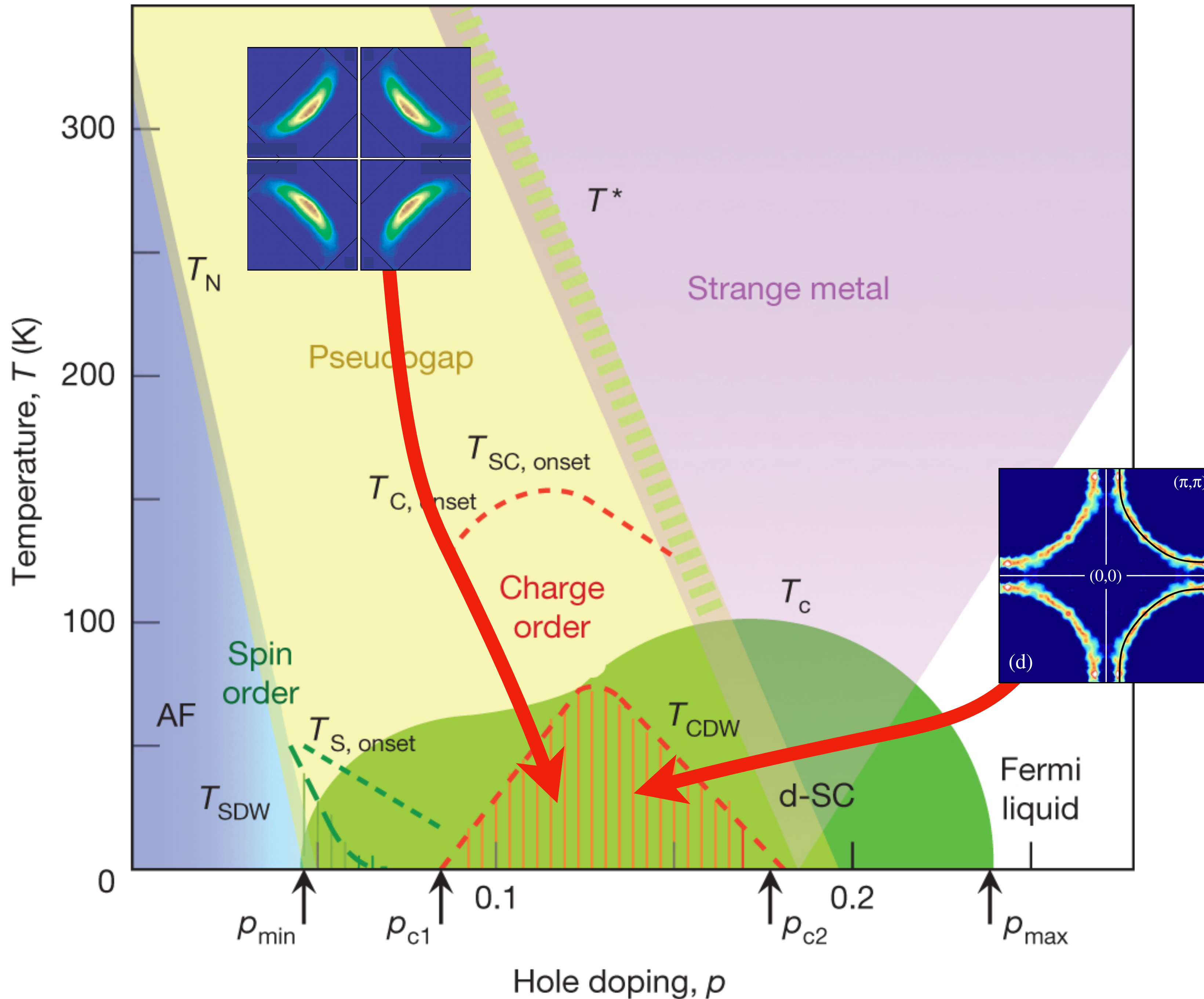




Fermi liquid  
in the  
overdoped metal

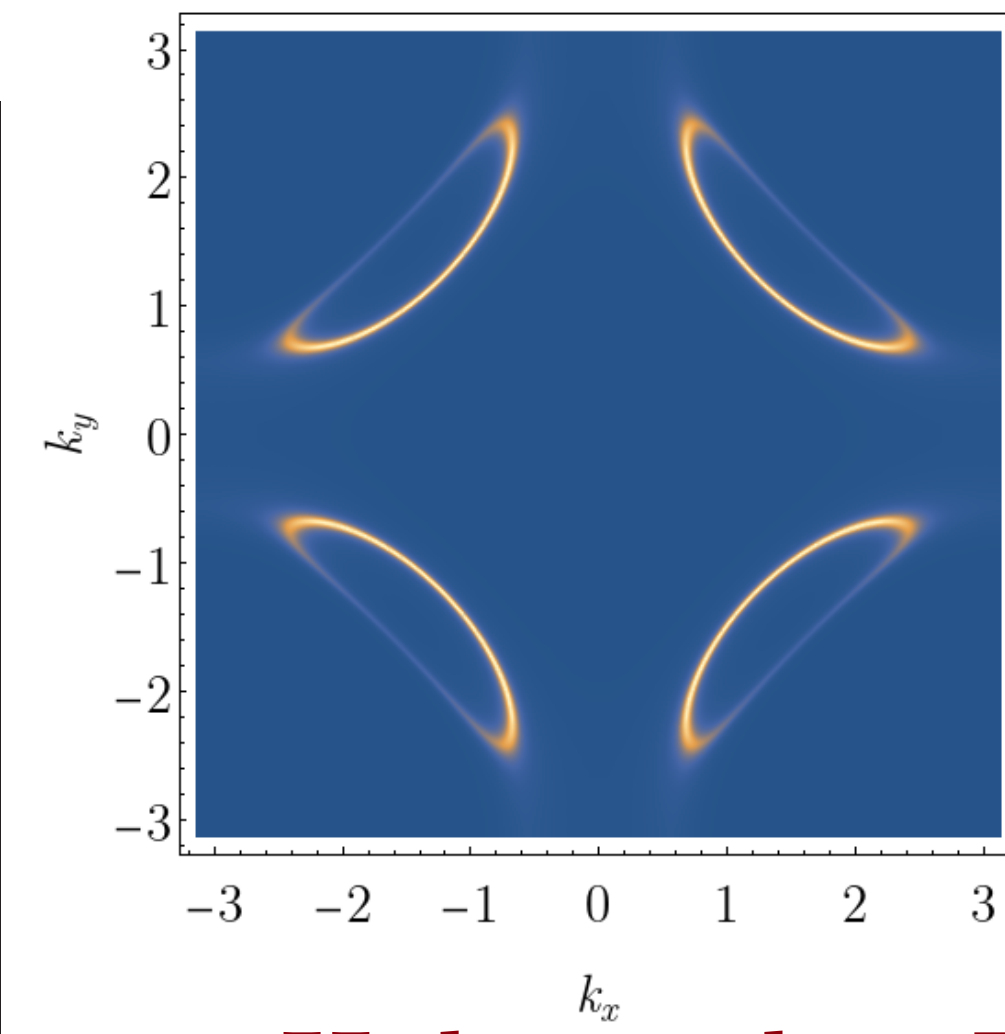
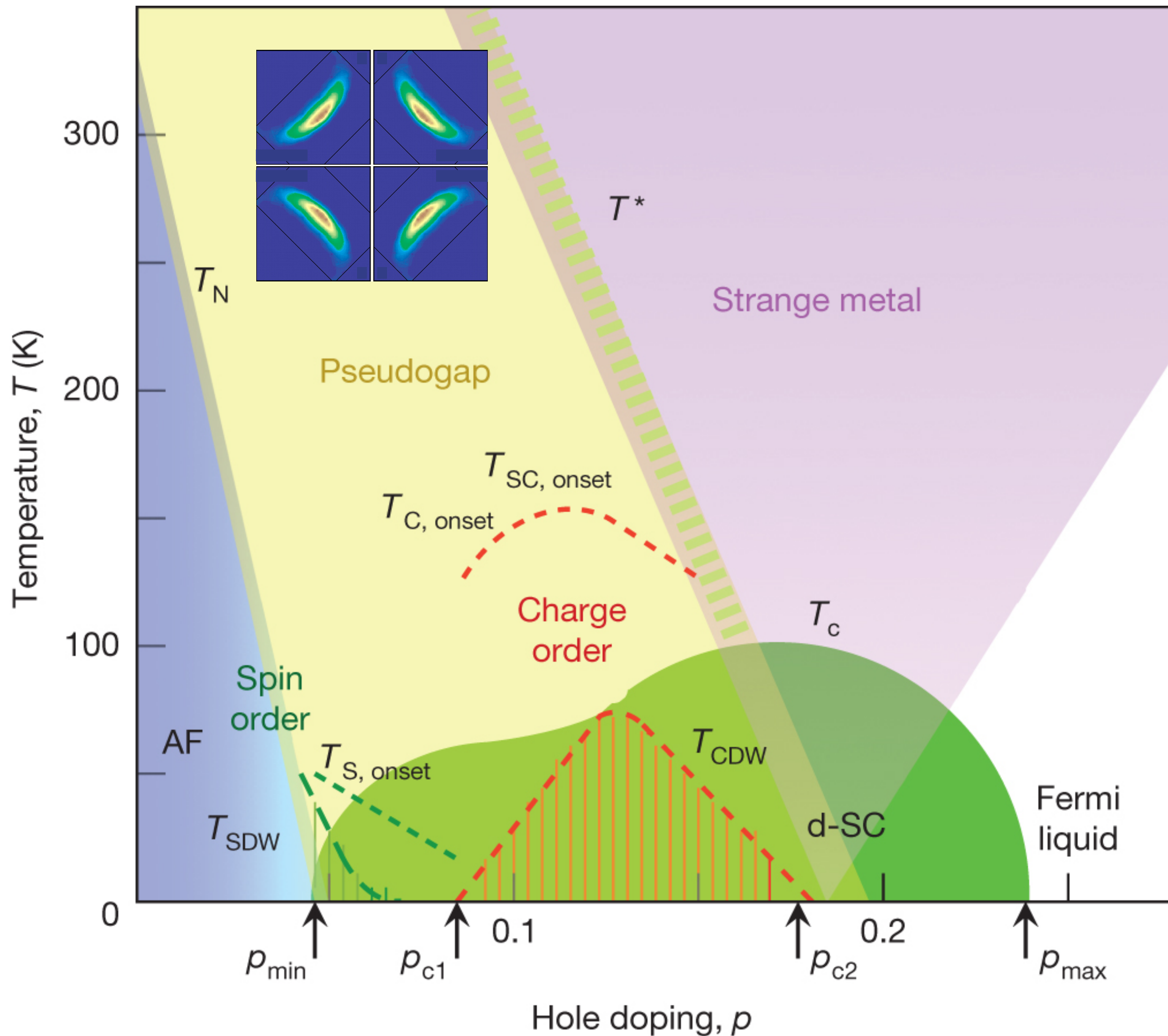


Theory for  
“pseudogap metal”  
with “Fermi arcs”?



Build a theory for the phase diagram from a theory of the pseudogap metal as a ‘metastable’  $T = 0$  quantum phase.

Lowest  $T$  phases obtained from pseudogap metal should connect smoothly to conventionally order phases obtained from the Fermi liquid.



Y. Qi and S. Sachdev  
*Phys. Rev. B* **81**,  
115129 (2010)

E. Mascot,  
A. Nikolaenko,  
M. Tikhanovskaya, Ya-  
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S. Sachdev, *Phys. Rev. B*  
**105**, 075146 (2022)

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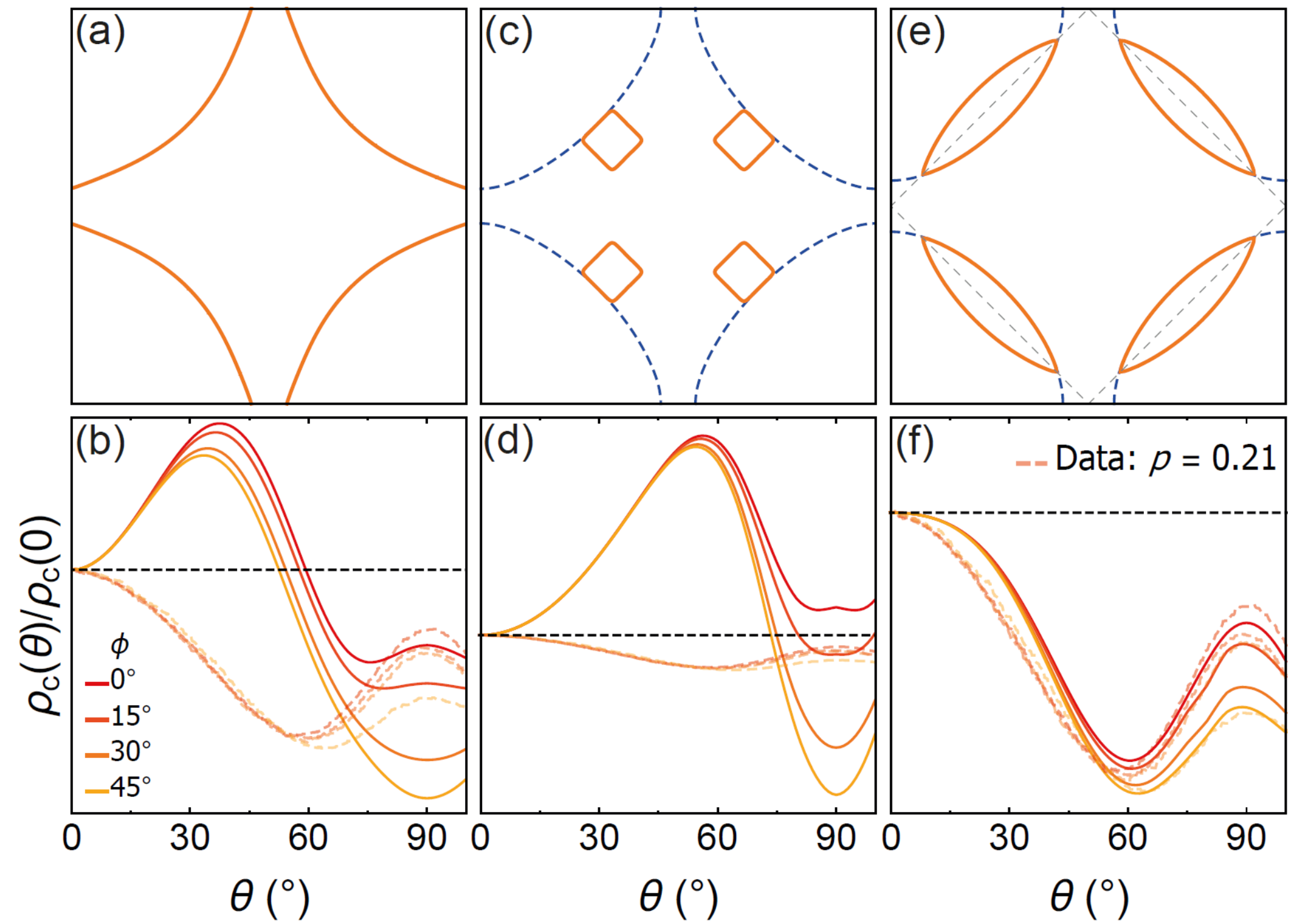
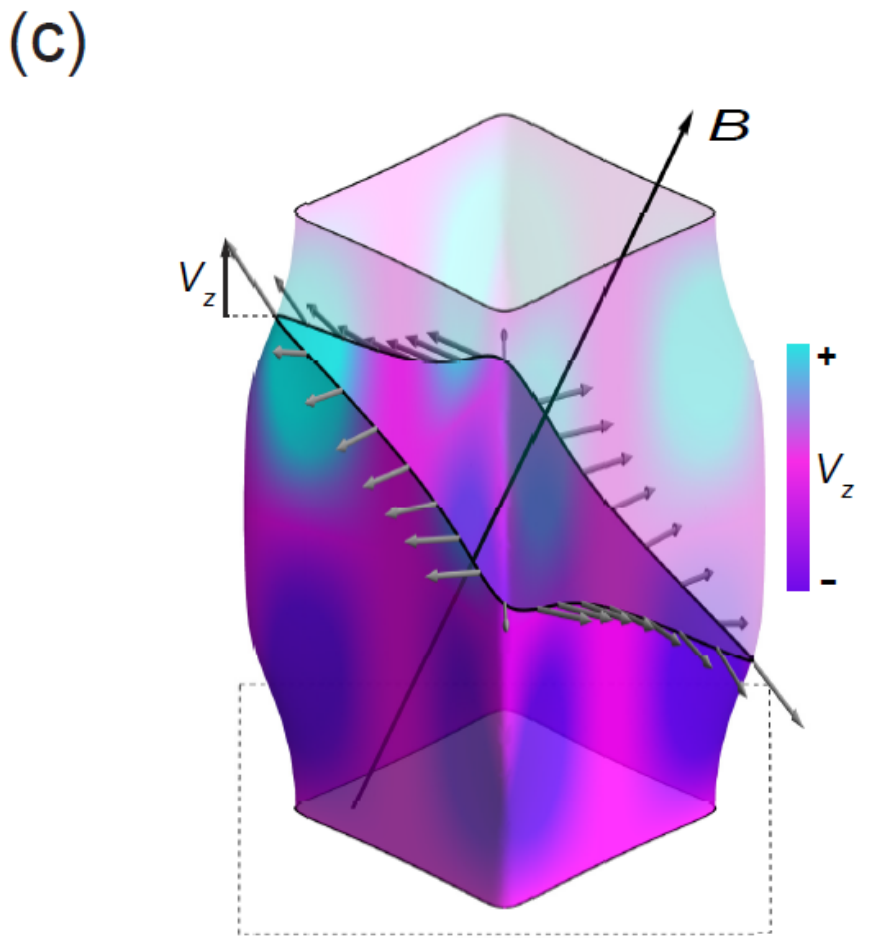
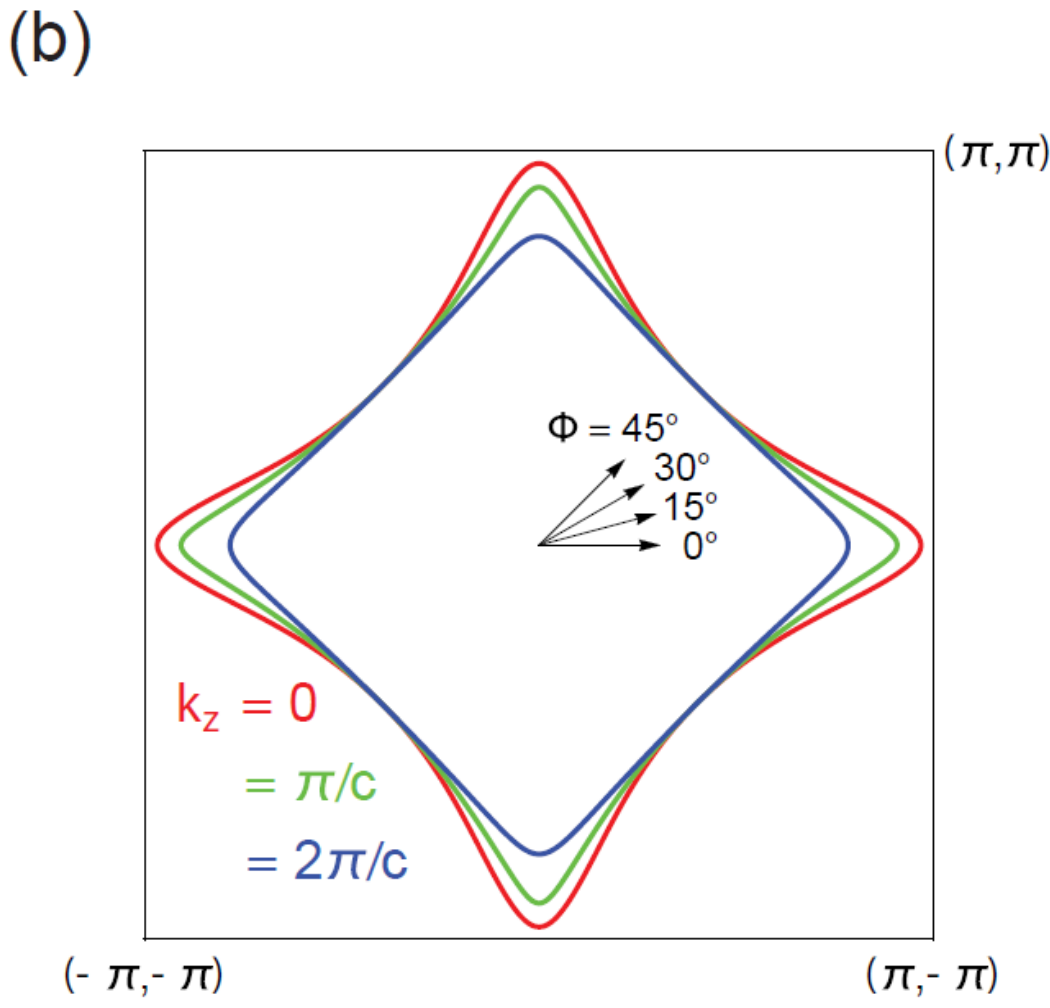
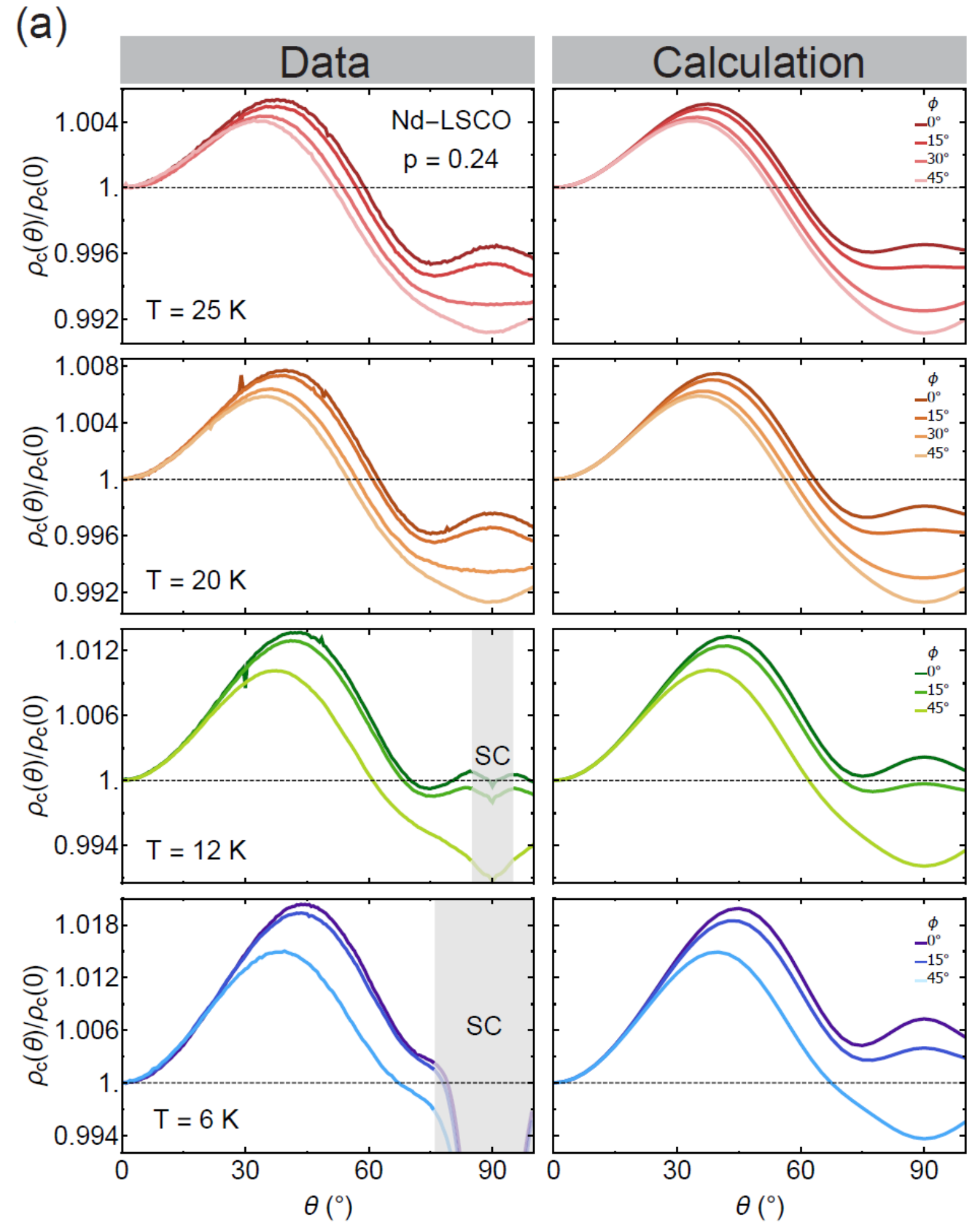
S. Sakai, Y. Motome, M. Imada,  
*Phys. Rev. Lett.* **102**, 056404 (2009).

J. Skolimowski and M. Fabrizio,  
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Jinchao Zhao, Gabriele La Nave, Philip Phillips,  
arXiv:2304.04787.

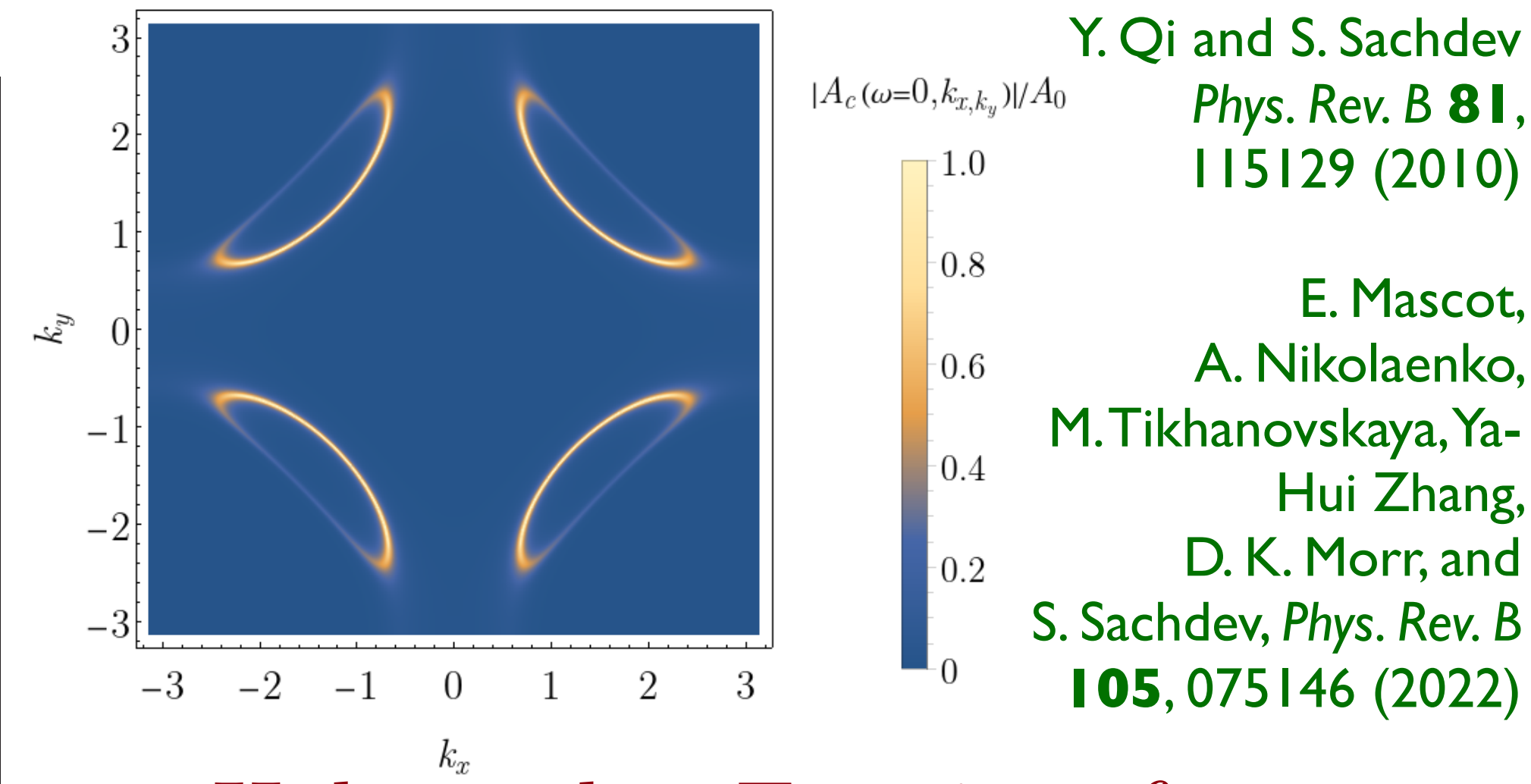
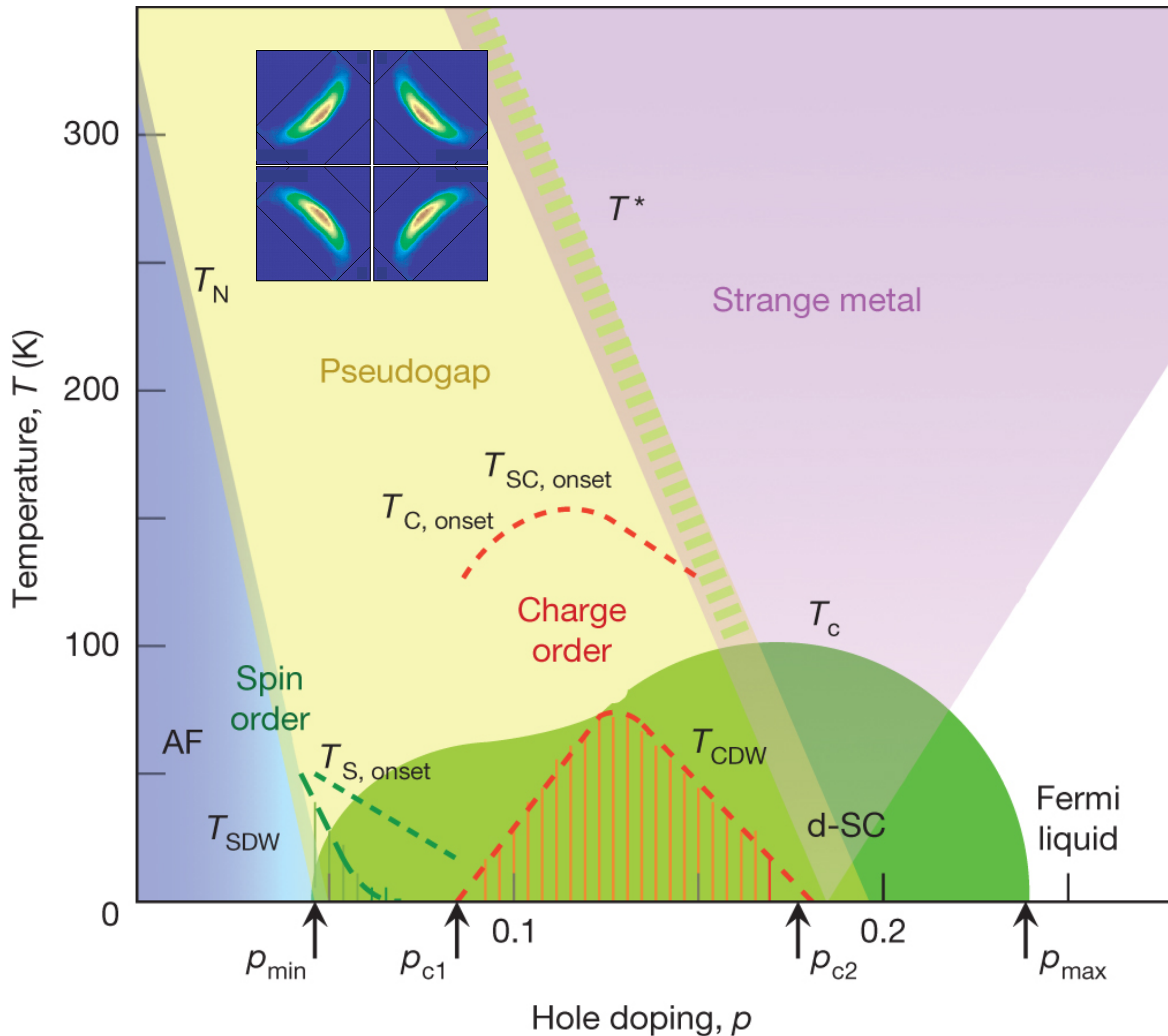
# Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, *Nature Physics* **18**, 558 (2022)



$p > p_c$  Large Fermi surface

$p < p_c$  Reconstructed Fermi surface



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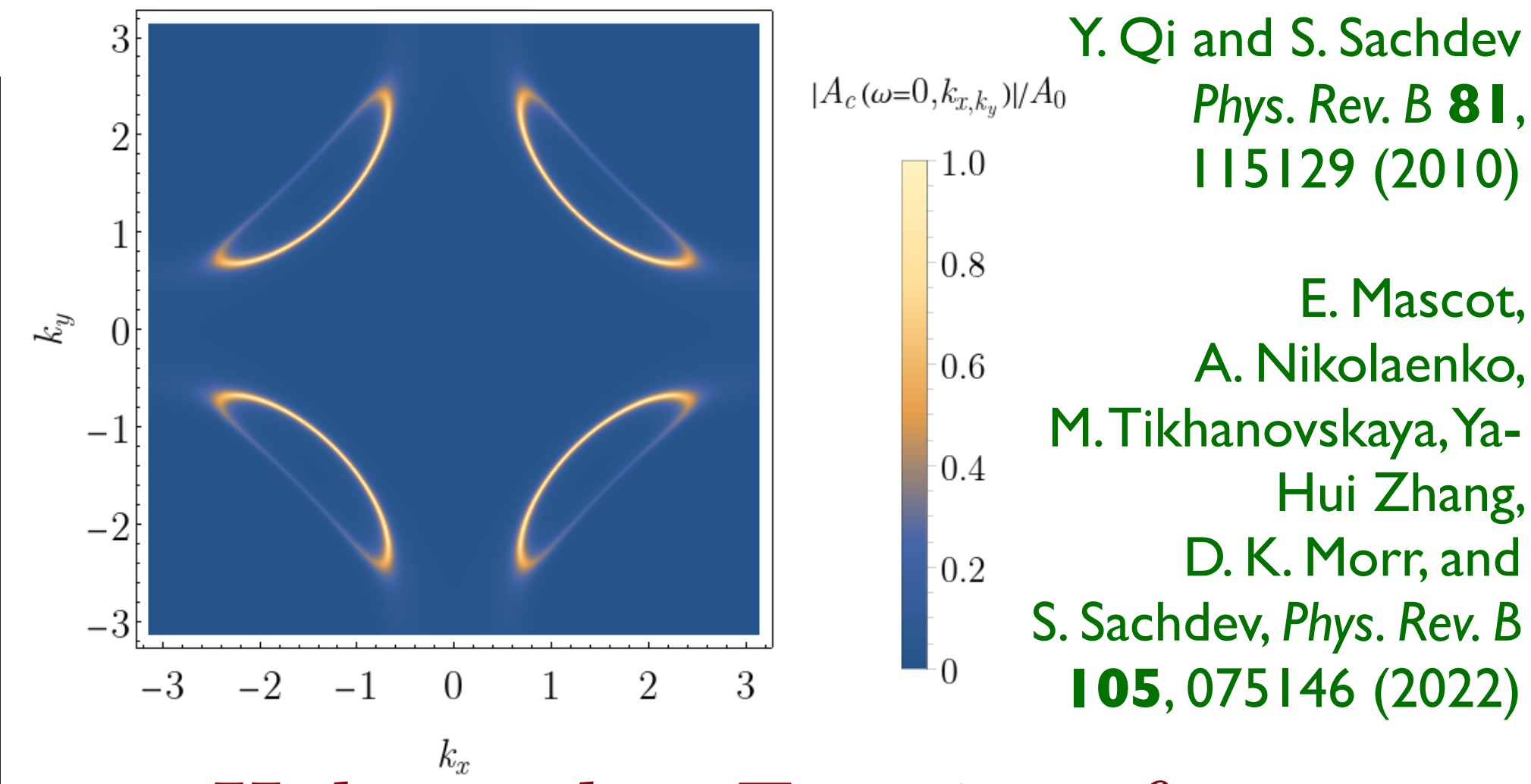
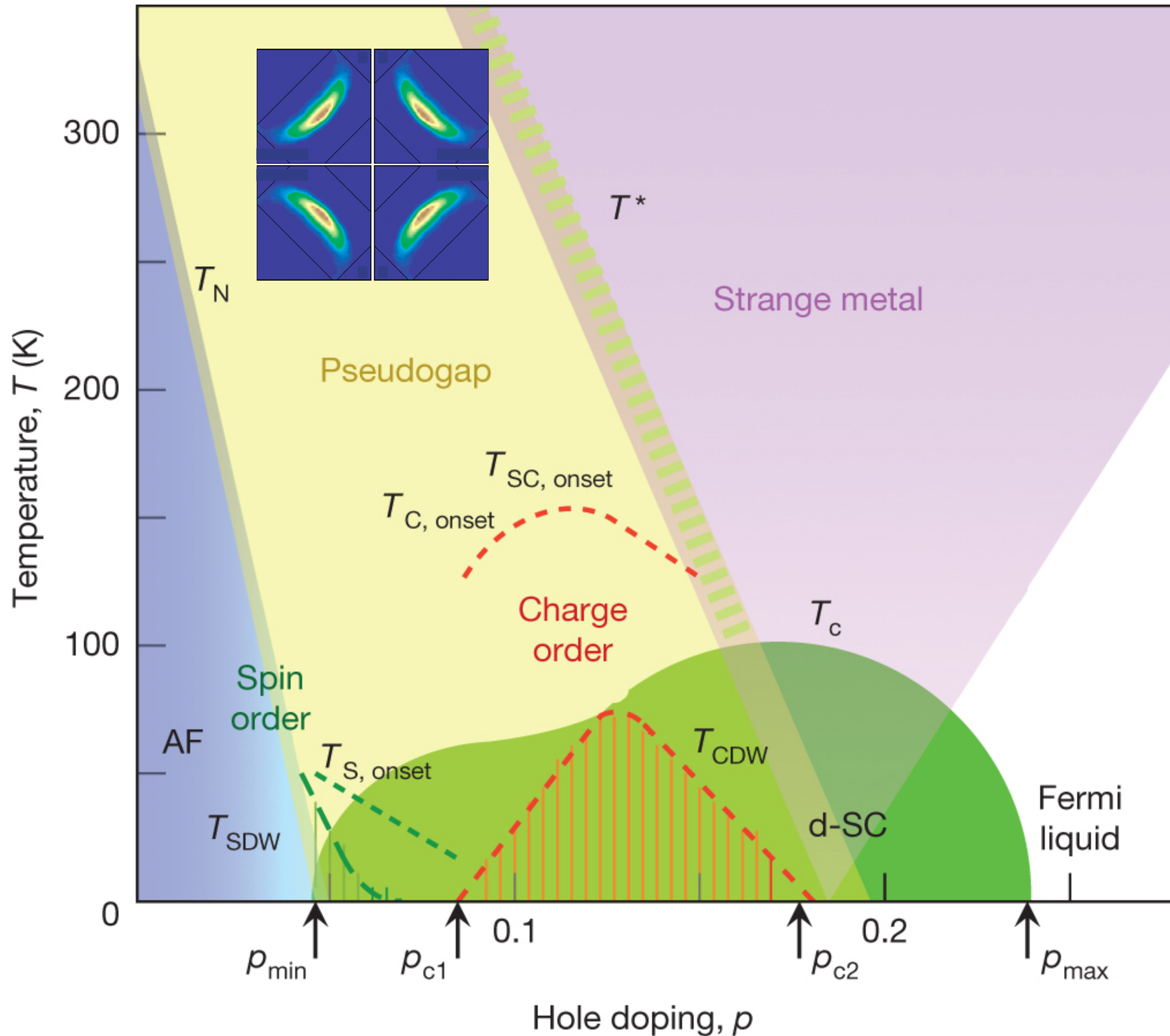
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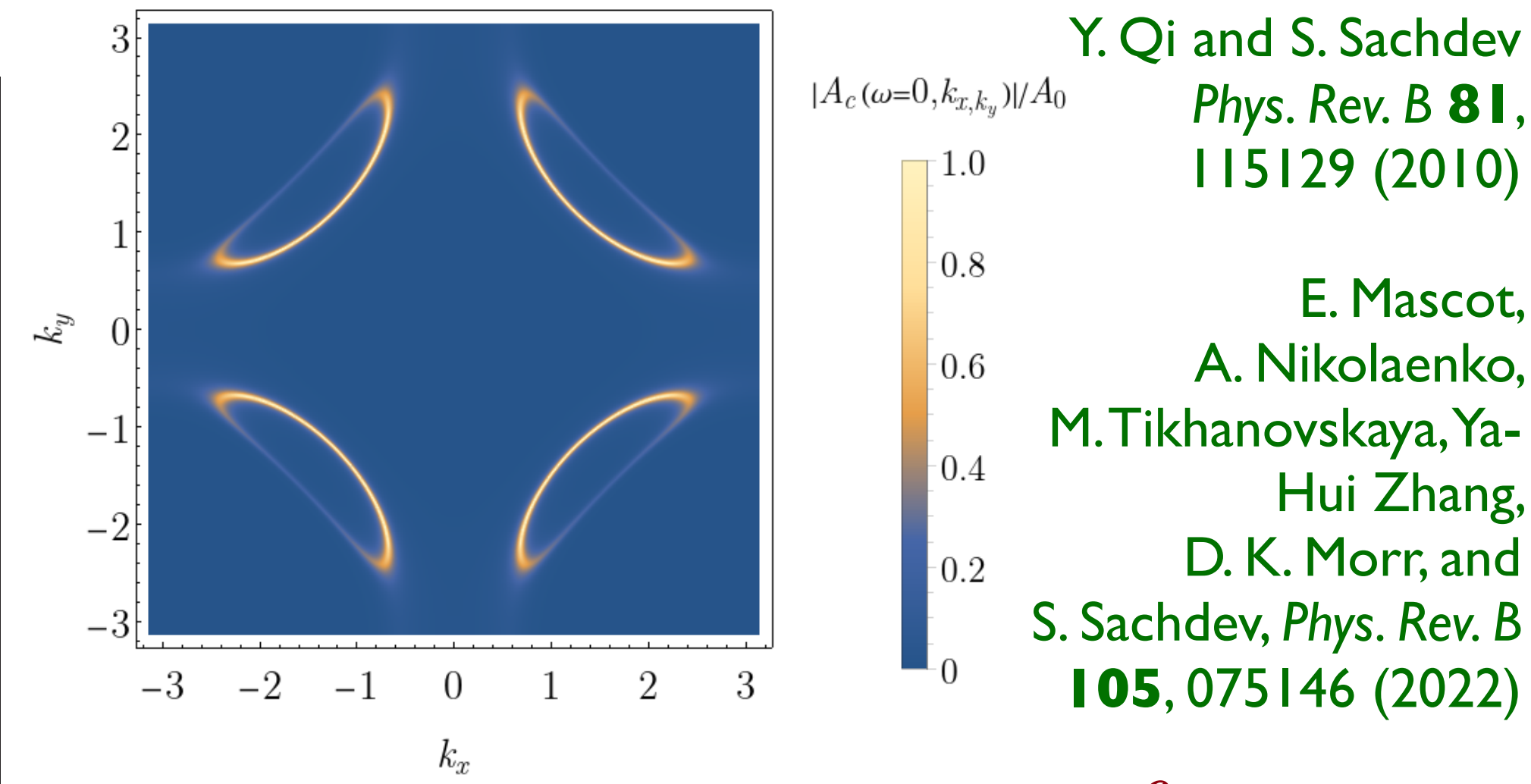
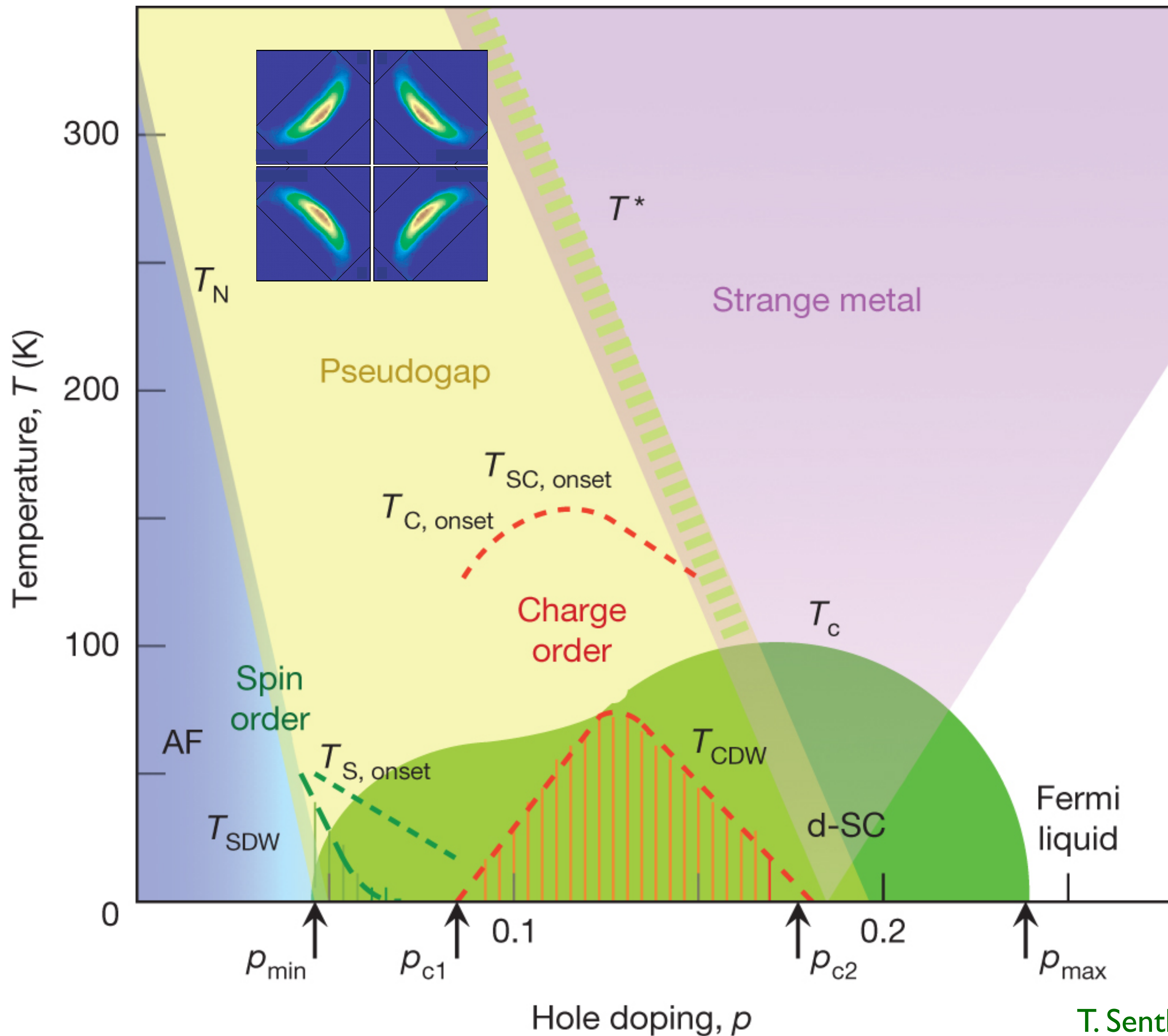
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Green function zeros....



Hole pocket Fermi surfaces  
of size  $p$  with  
charge  $e$ , spin-1/2 quasiparticles  
+  
'spectator'  
square lattice spin liquid  
at half-filling.

FL\*: Spin liquid is *required* because  
the Fermi surface does not enclose  
the Luttinger volume  $(1 + p)$ .

From FL\*

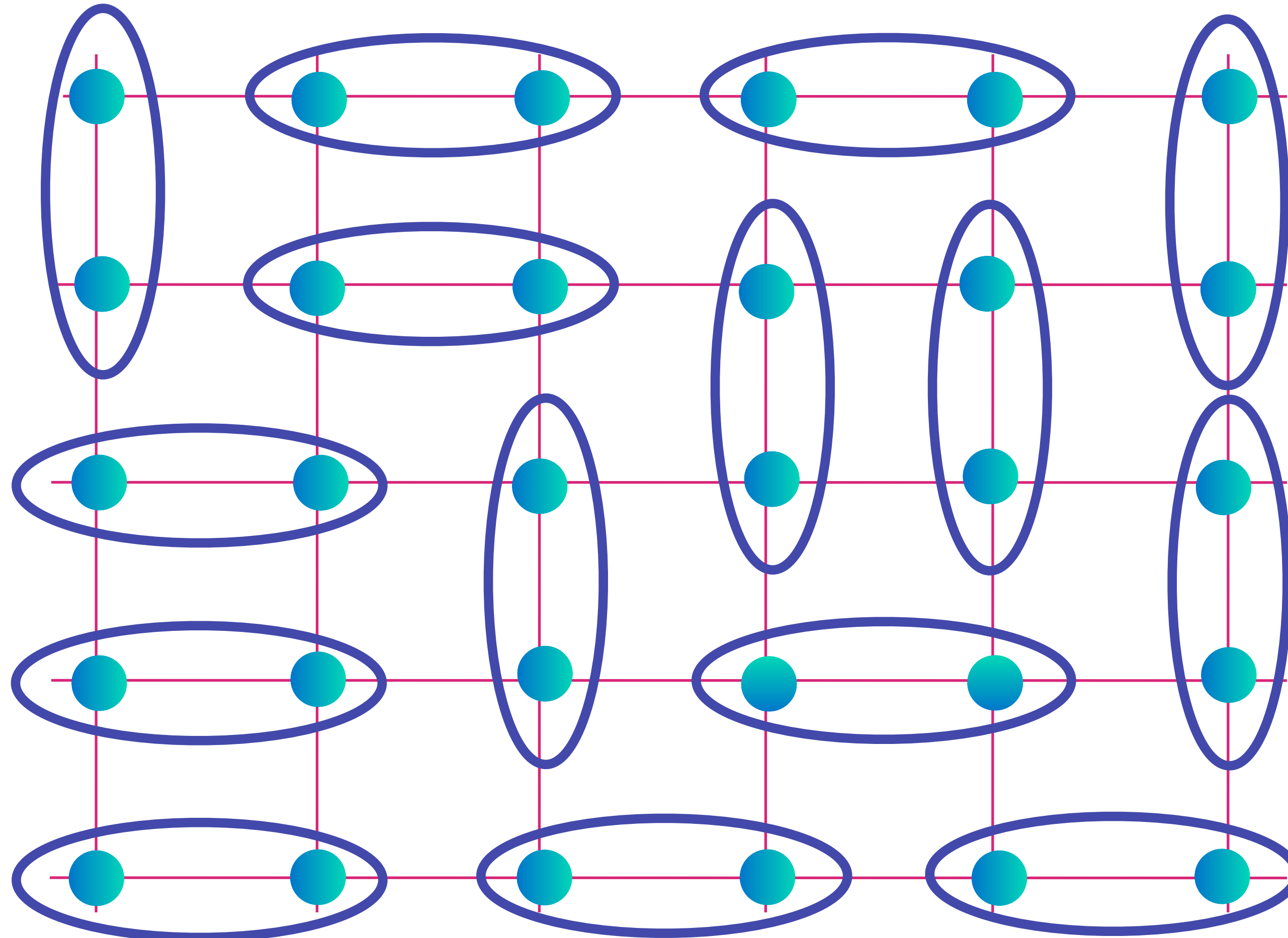
to

a cuprate phase diagram

# The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid



Electrons form entangled pairs, and the pairs entangle across the entire sample

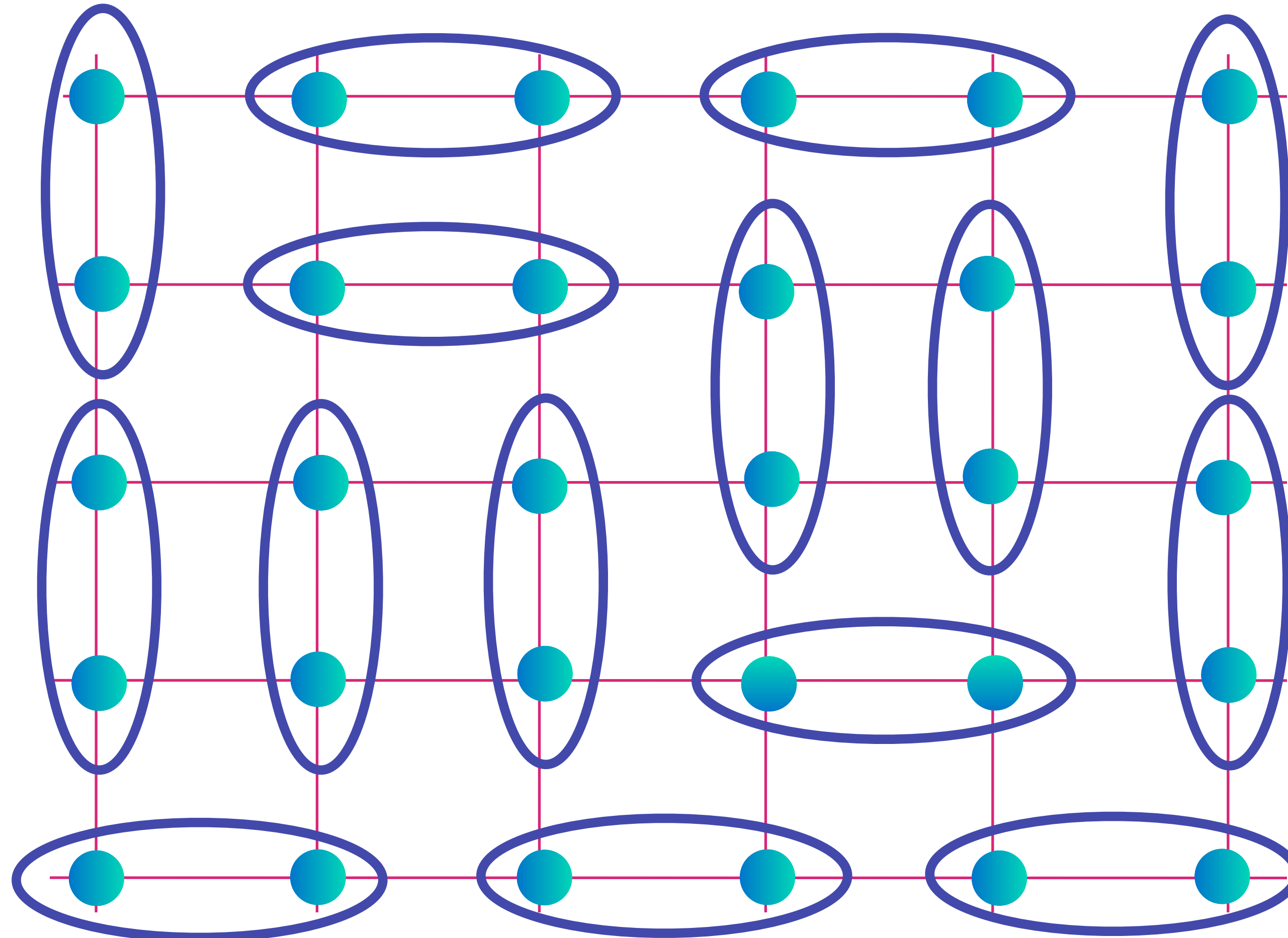
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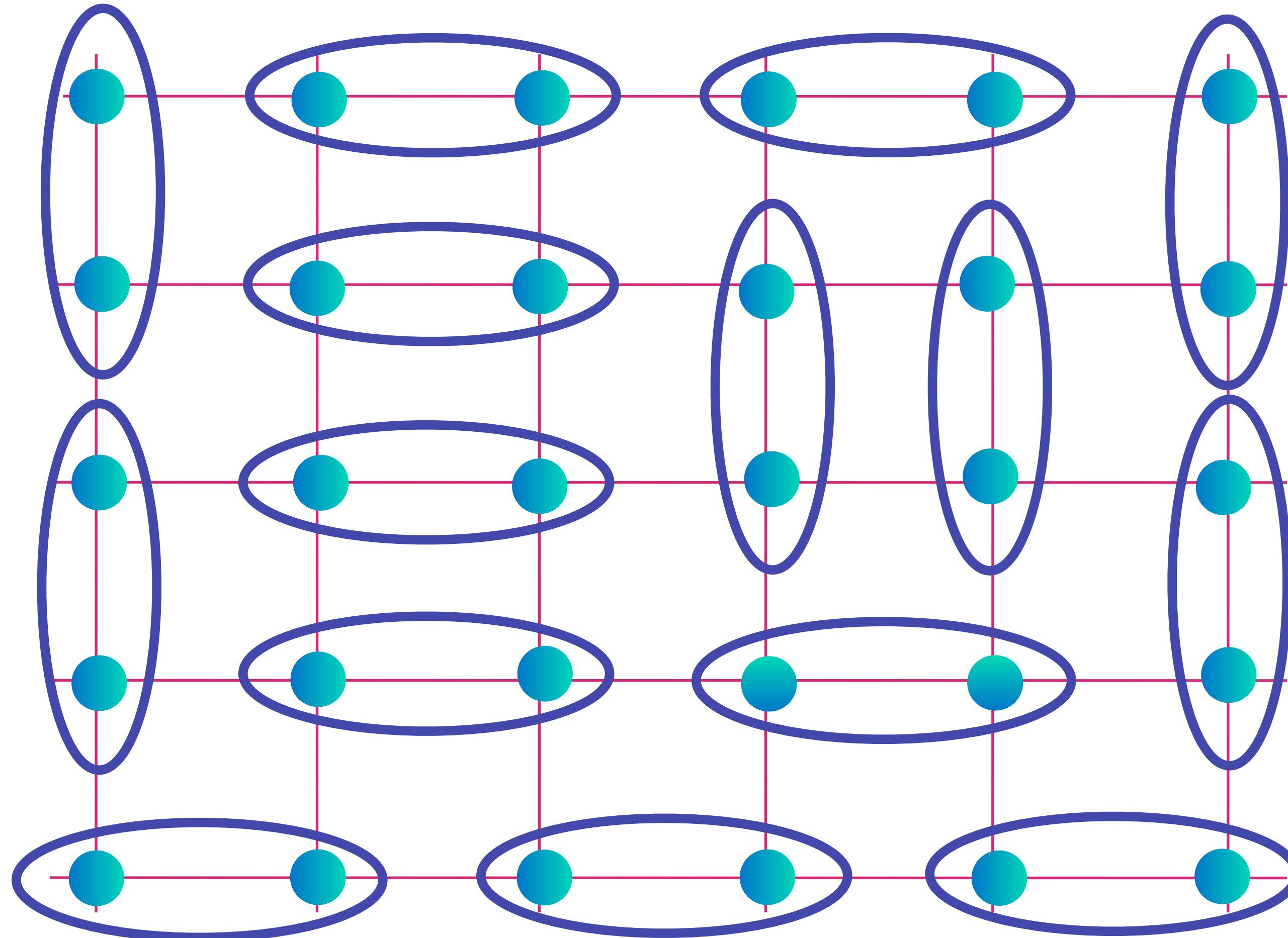
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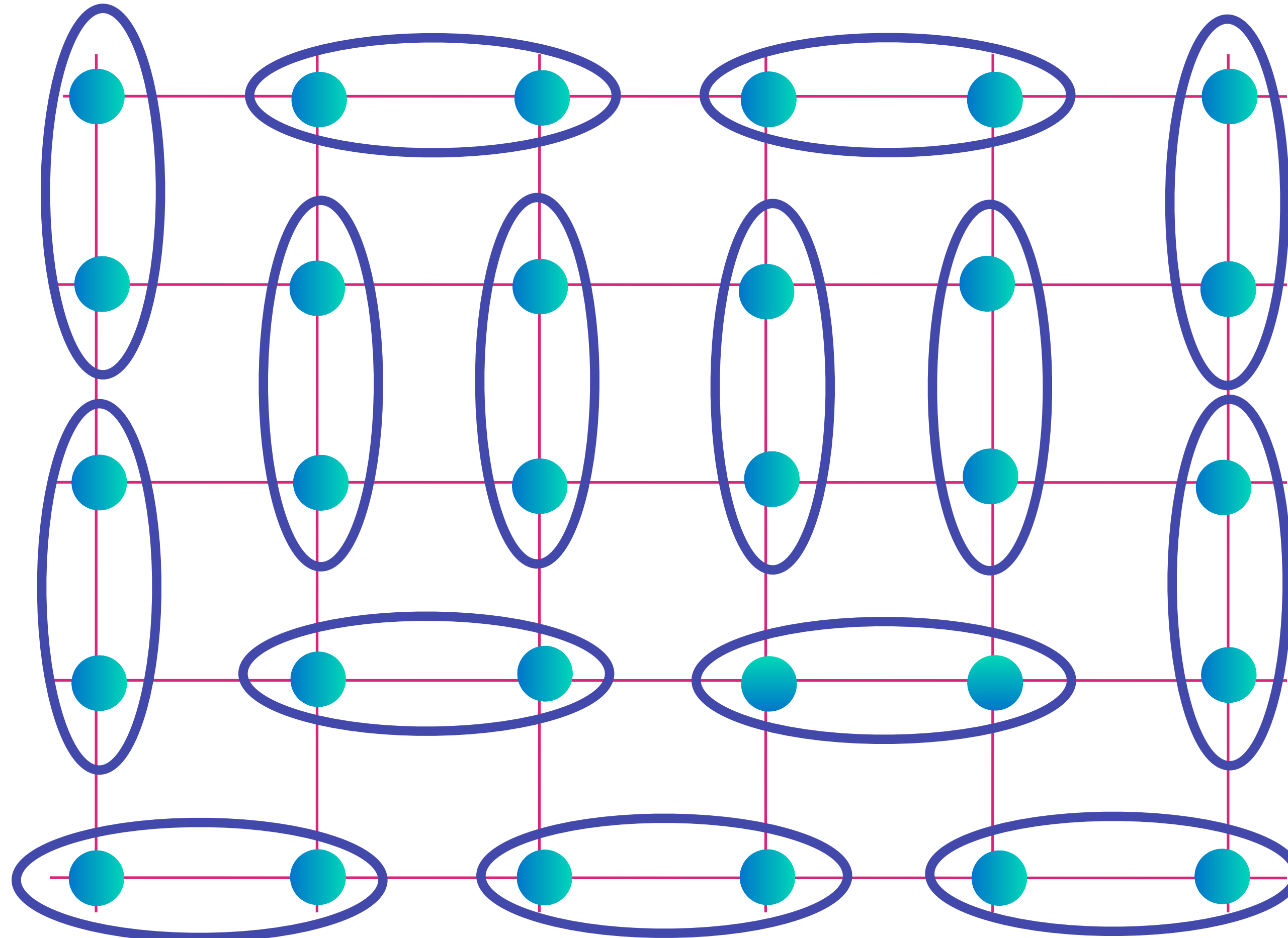
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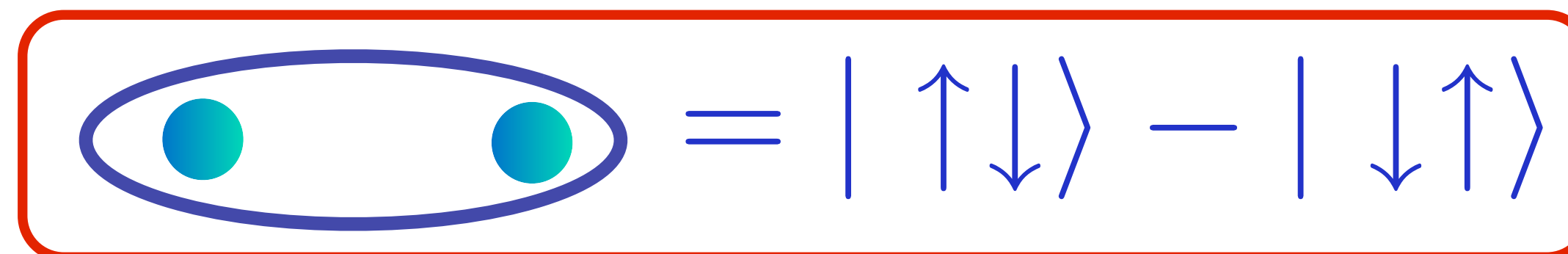
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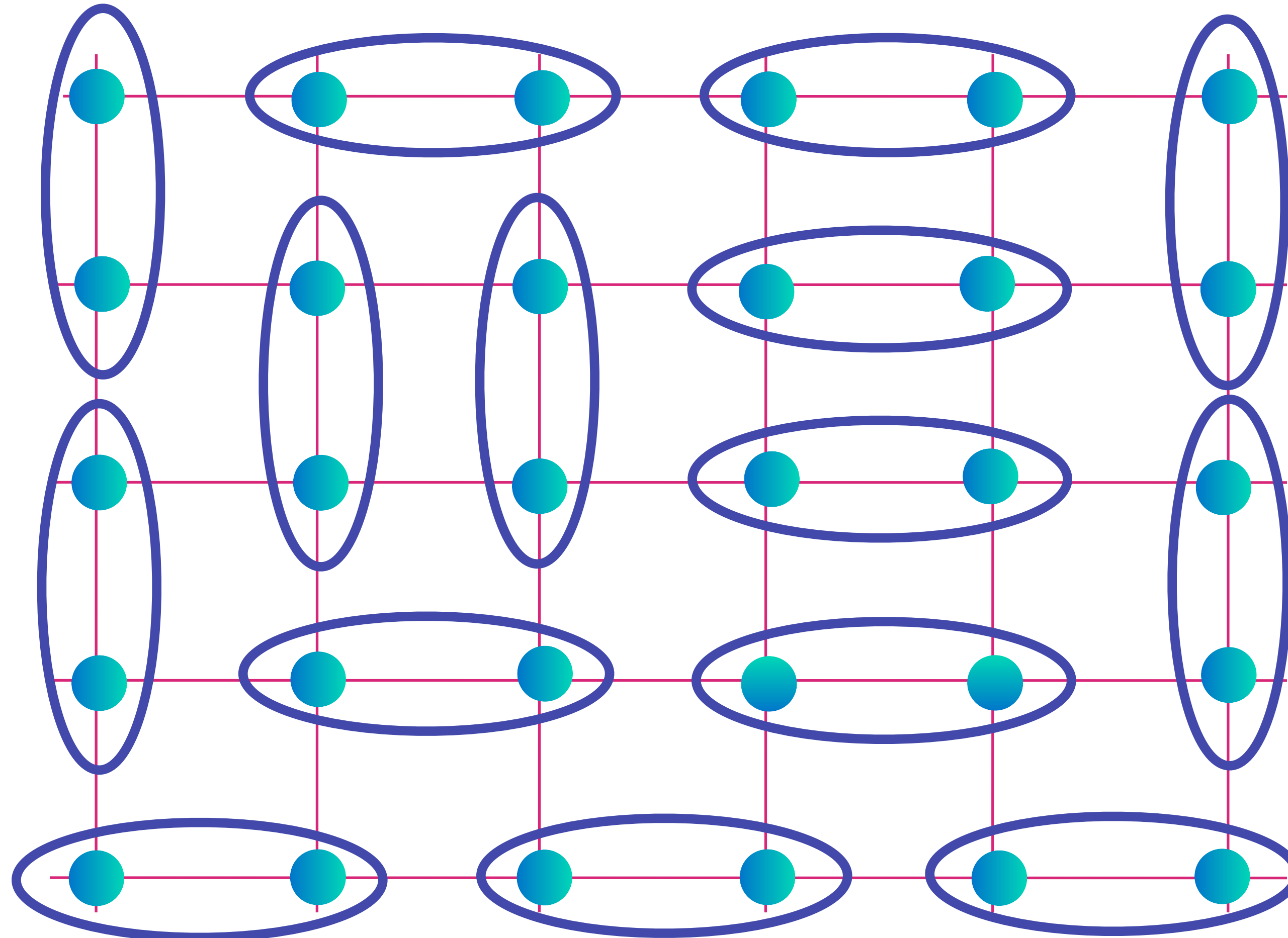


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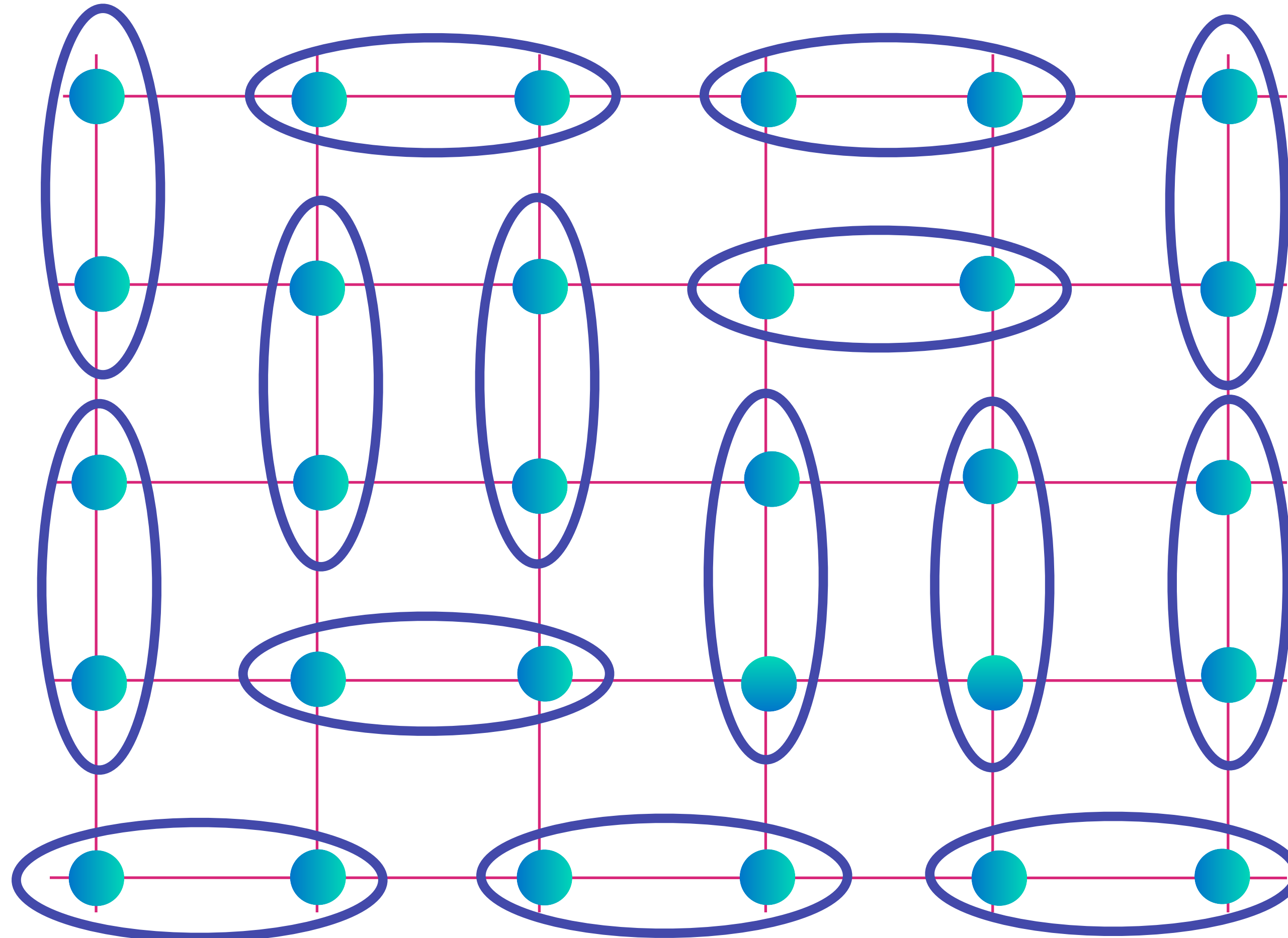
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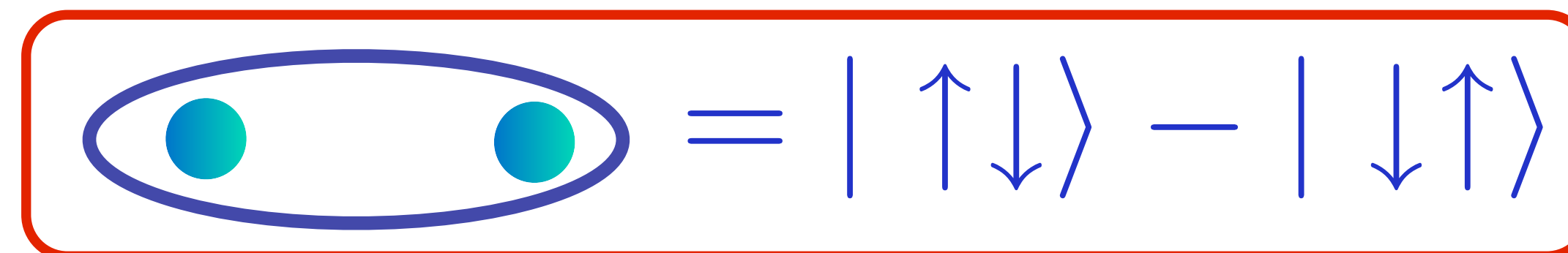
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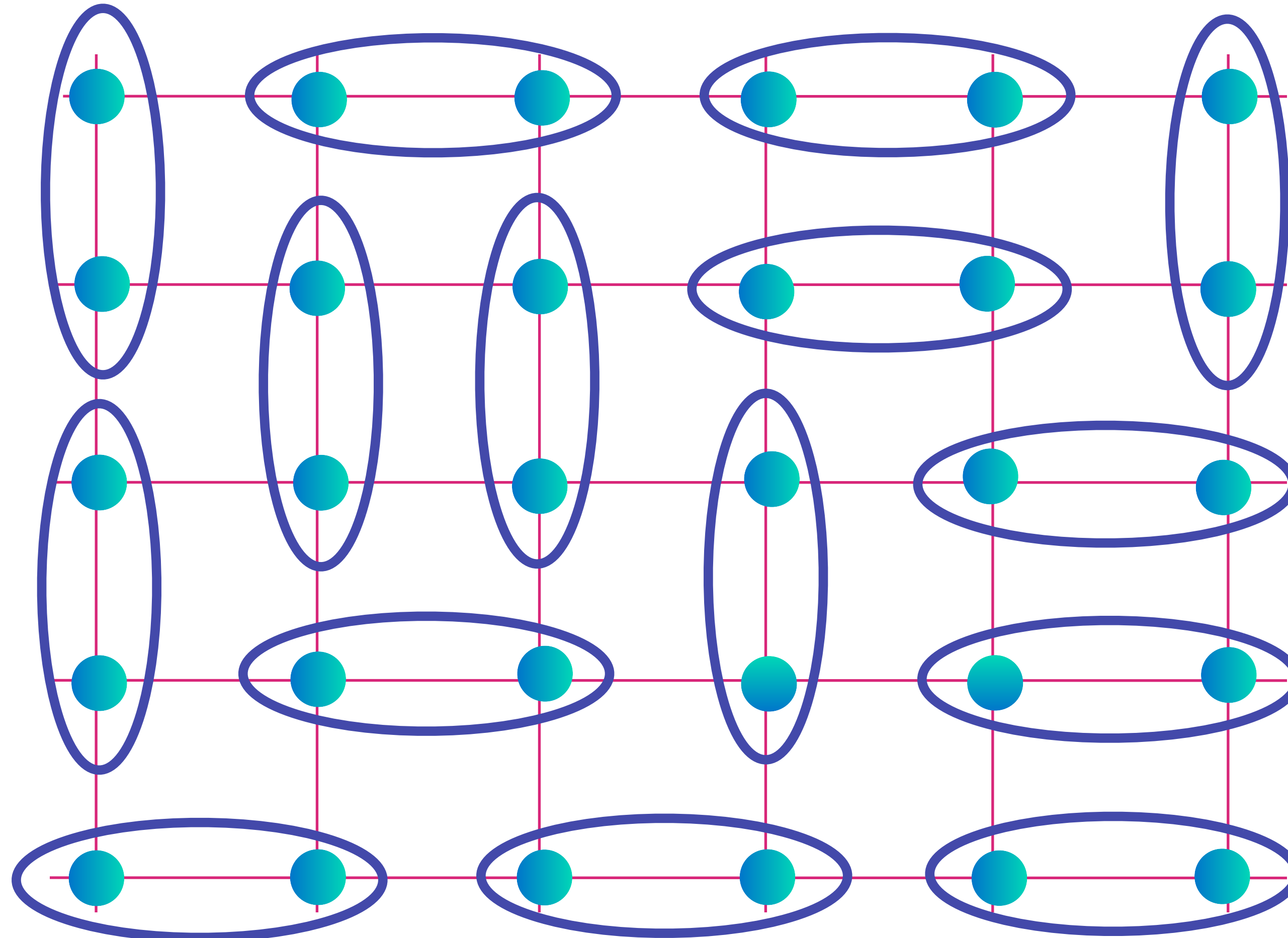


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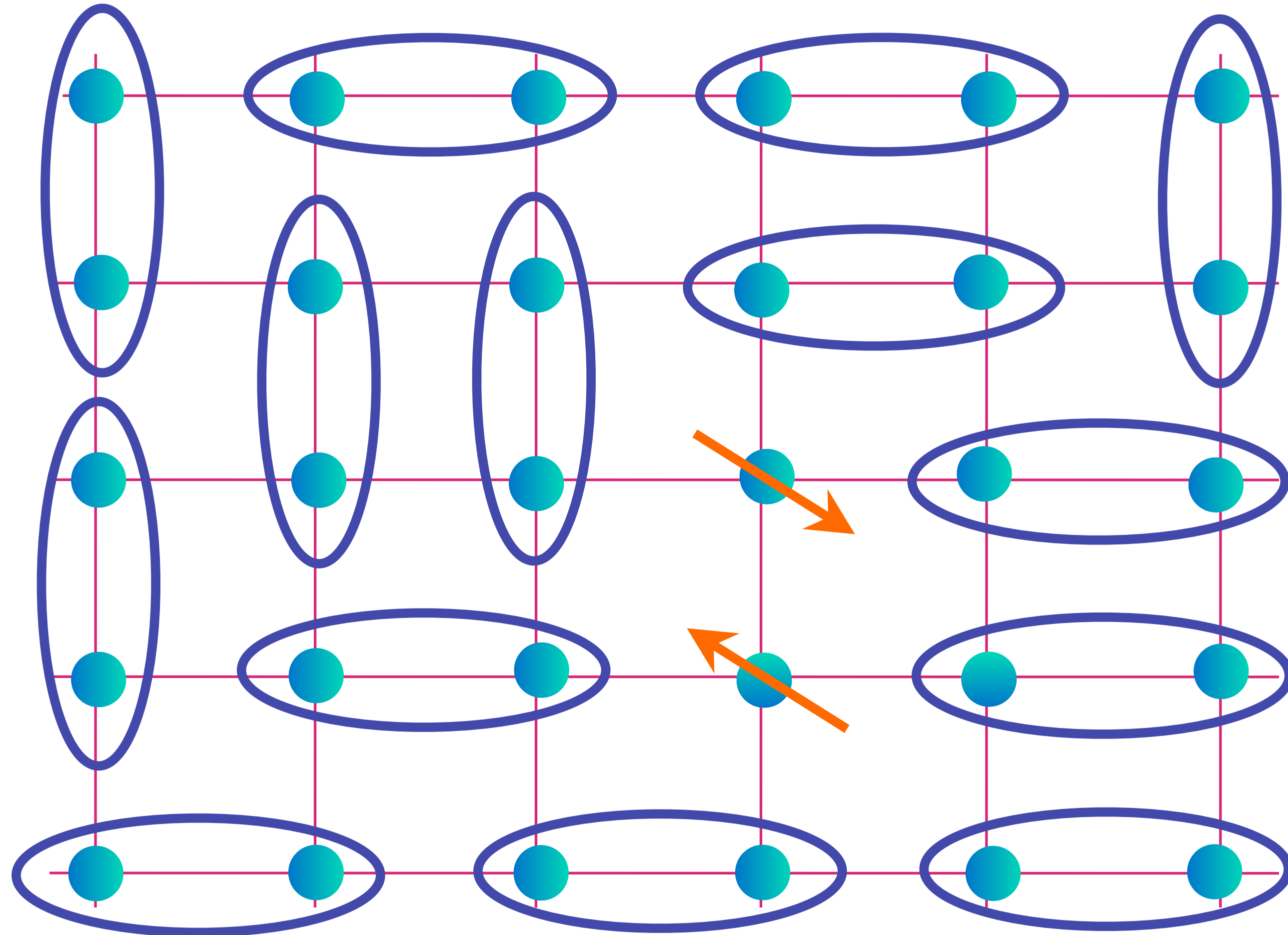


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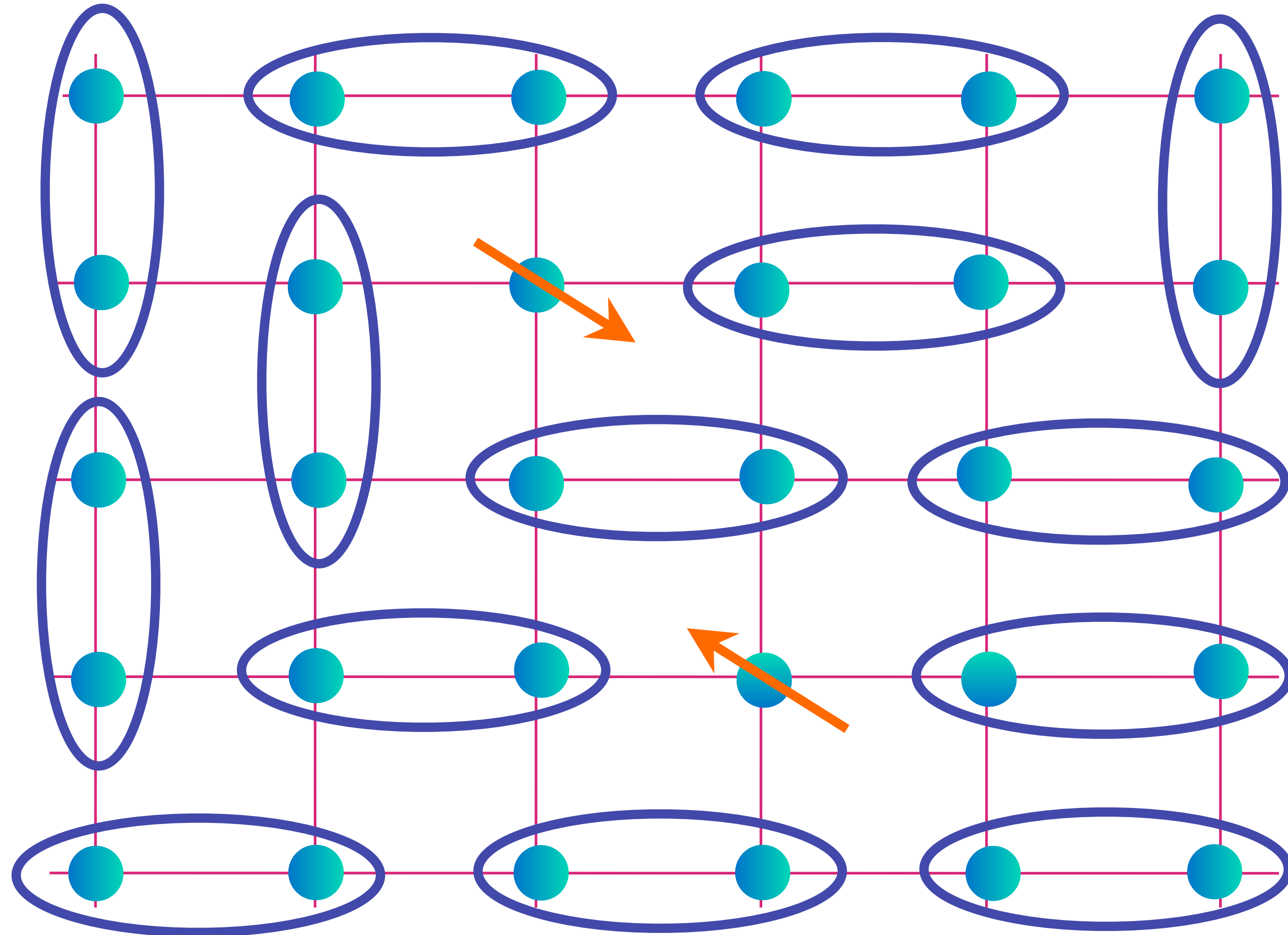
Spin liquid

Fractionalized  
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excitations  
with spin  $S=1/2$   
and charge 0.

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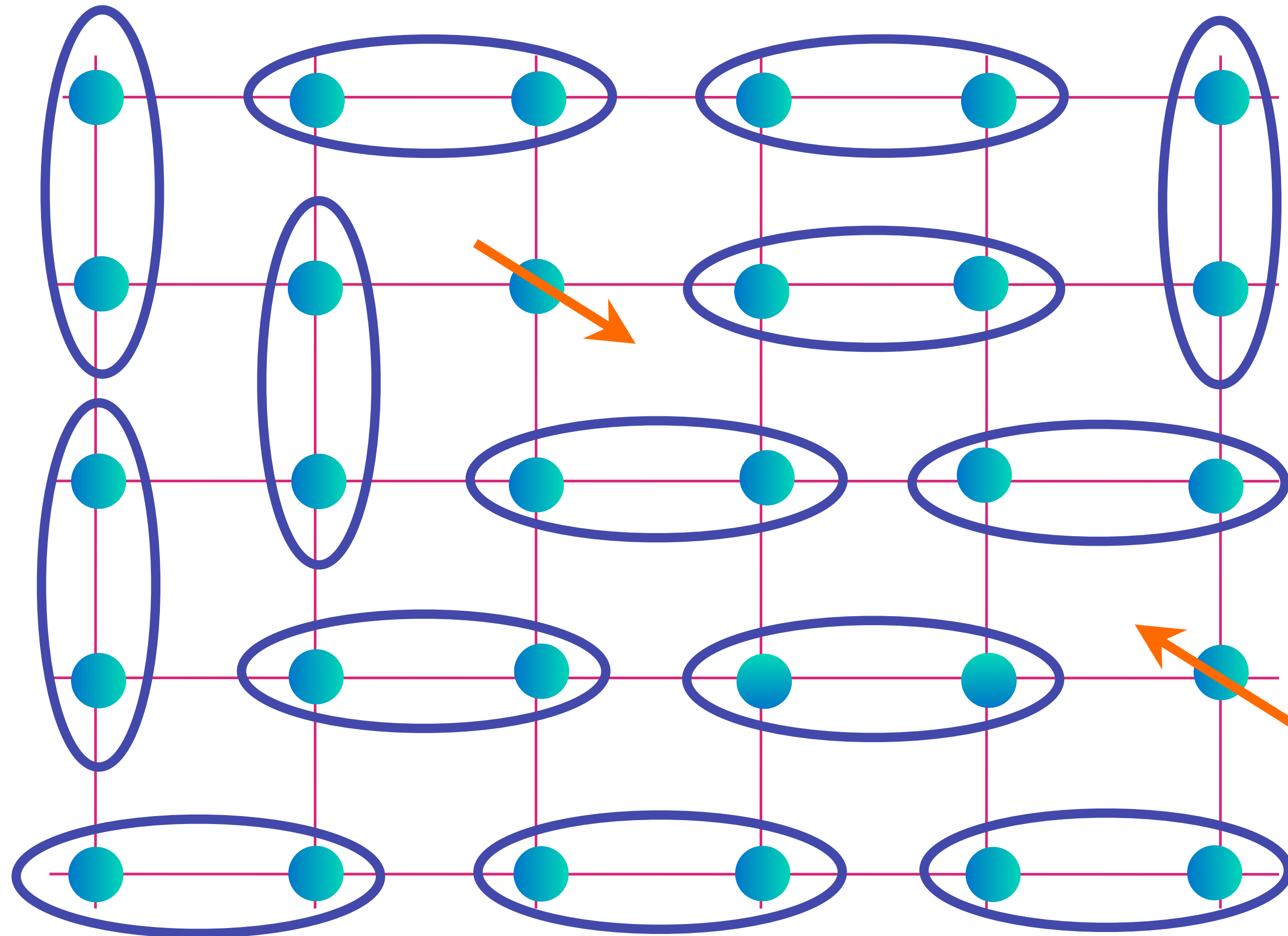
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Fractionalized spinon excitations with spin  $S=1/2$  and charge 0.

$$\text{Oval with two electrons} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

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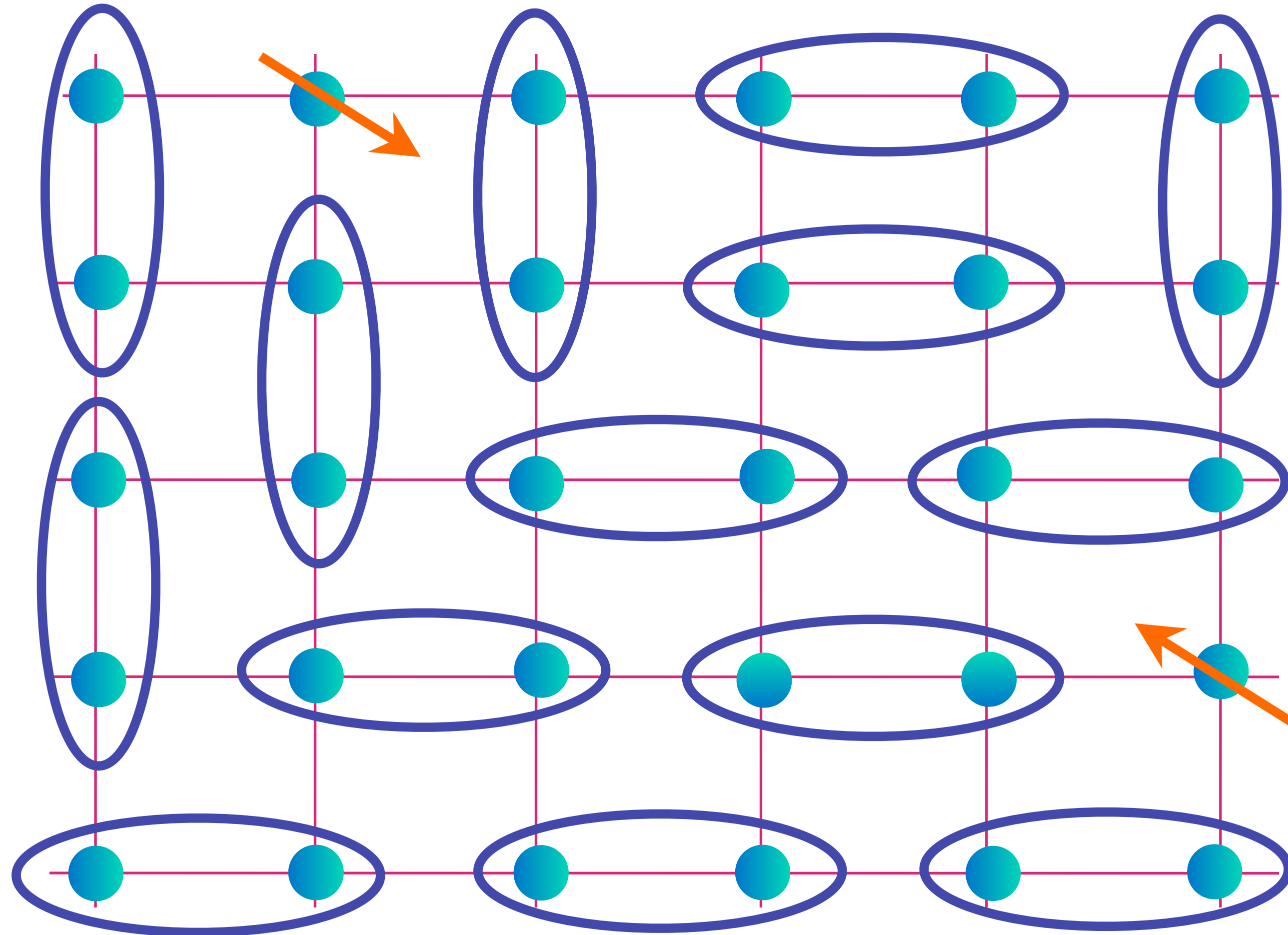
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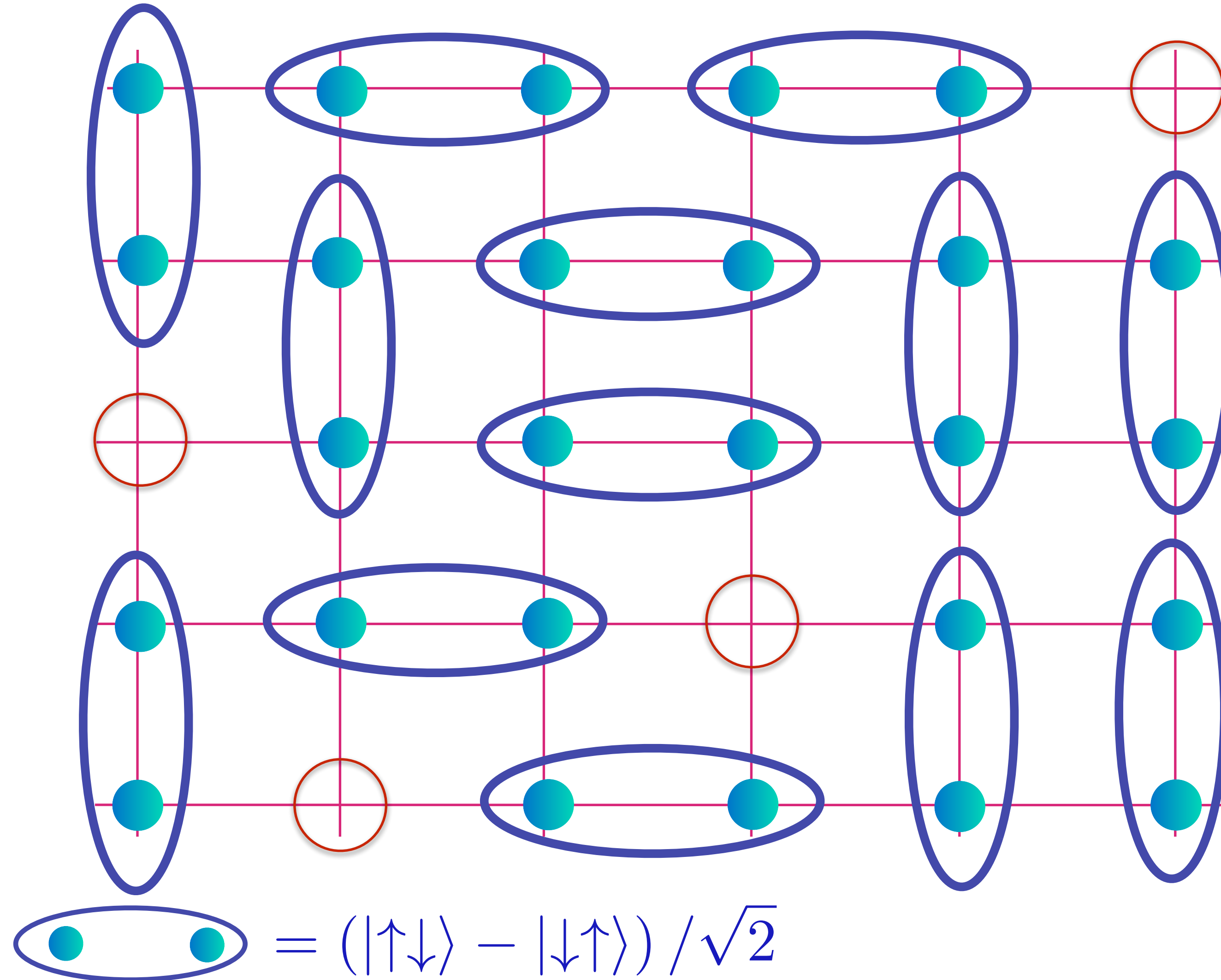
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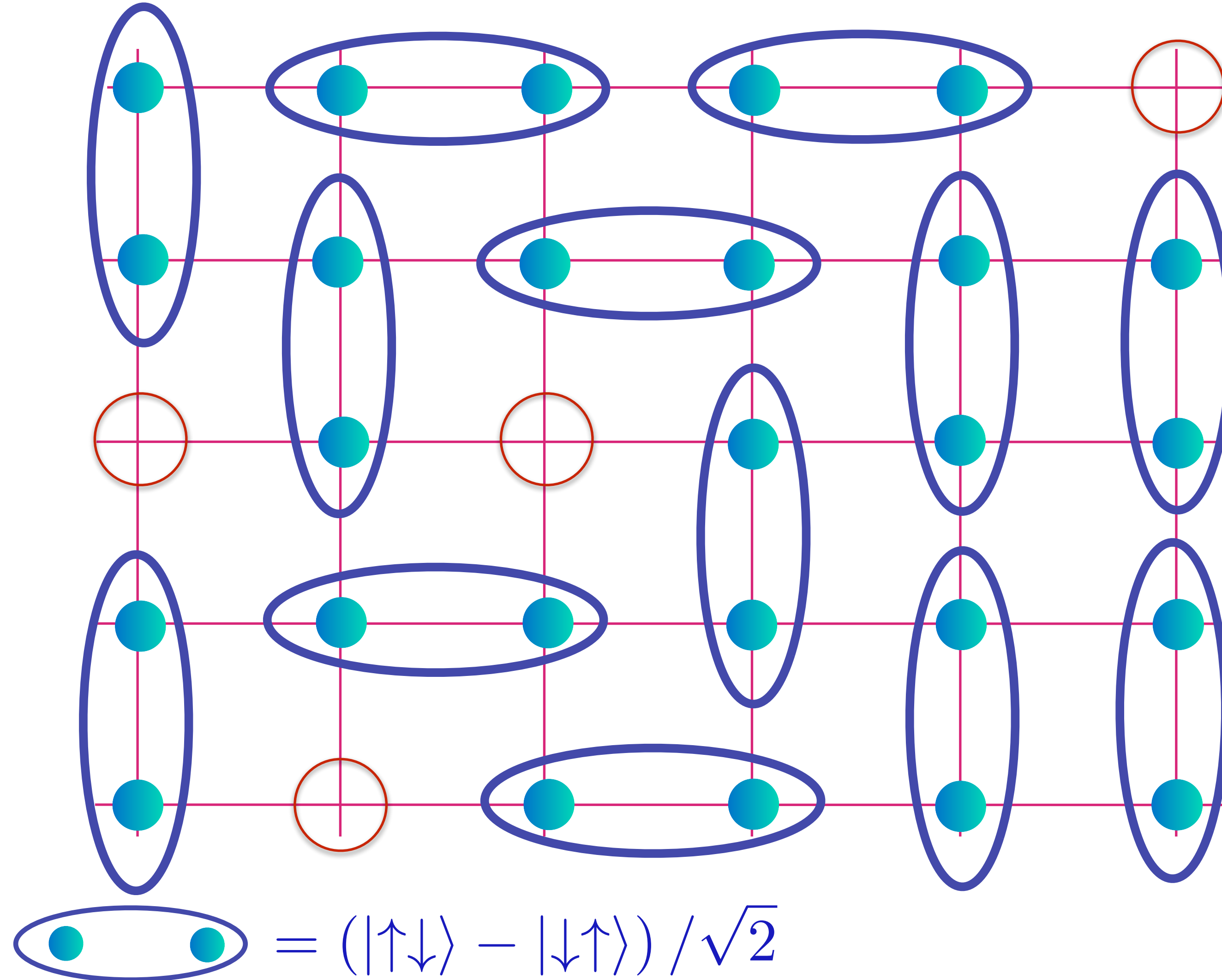
Fractionalized  
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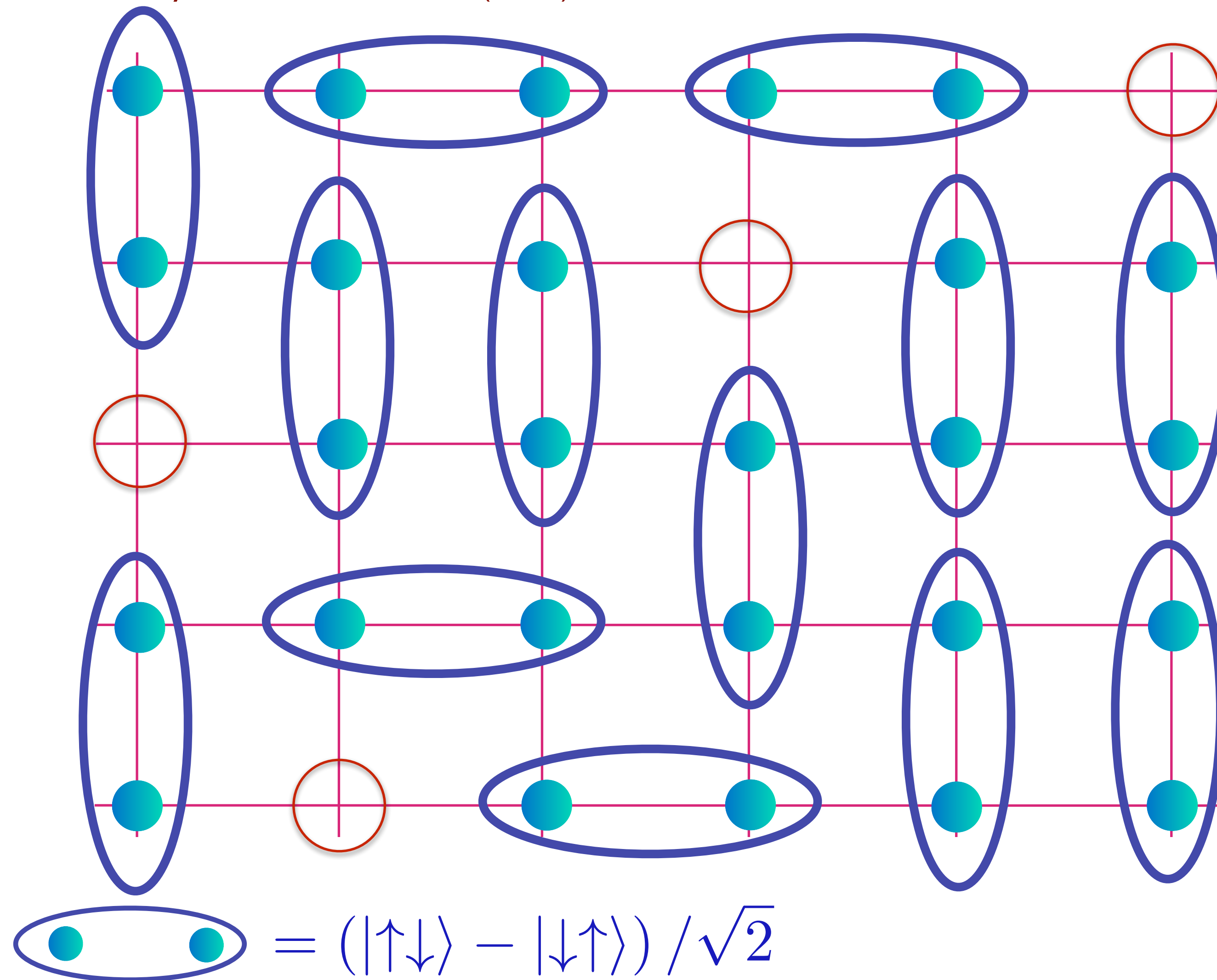
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# ~~Holon metal~~

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If each holon is a fermion, we obtain a Fermi surface of holons of size  $p$

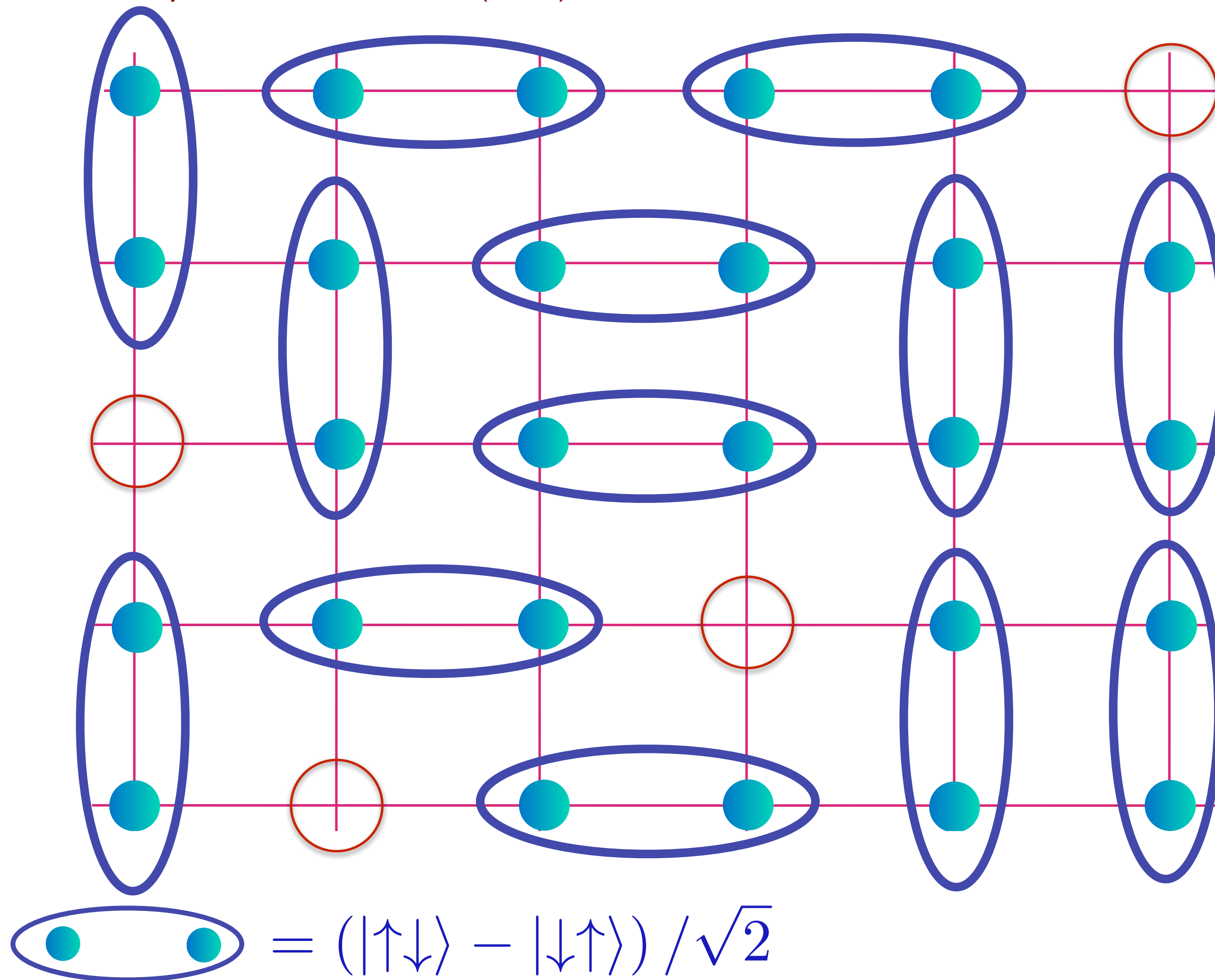
Cannot explain pseudogap metal

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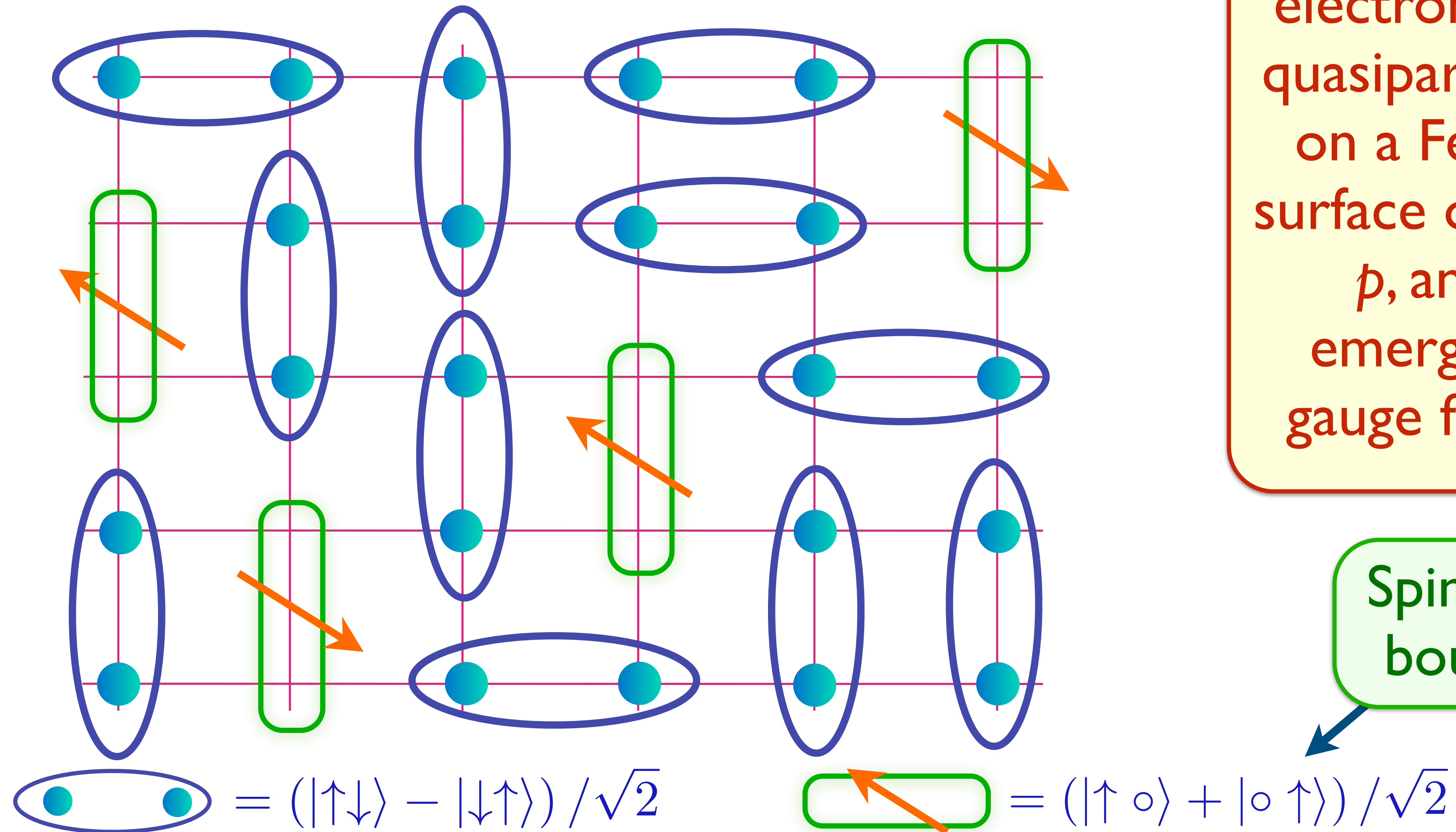
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# FL\* in a **one-band** model

S. Sachdev *Phys. Rev. B* **49**, 6770 (1994); X.-G. Wen and P.A. Lee *Phys. Rev. Lett.* **76**, 503 (1996)

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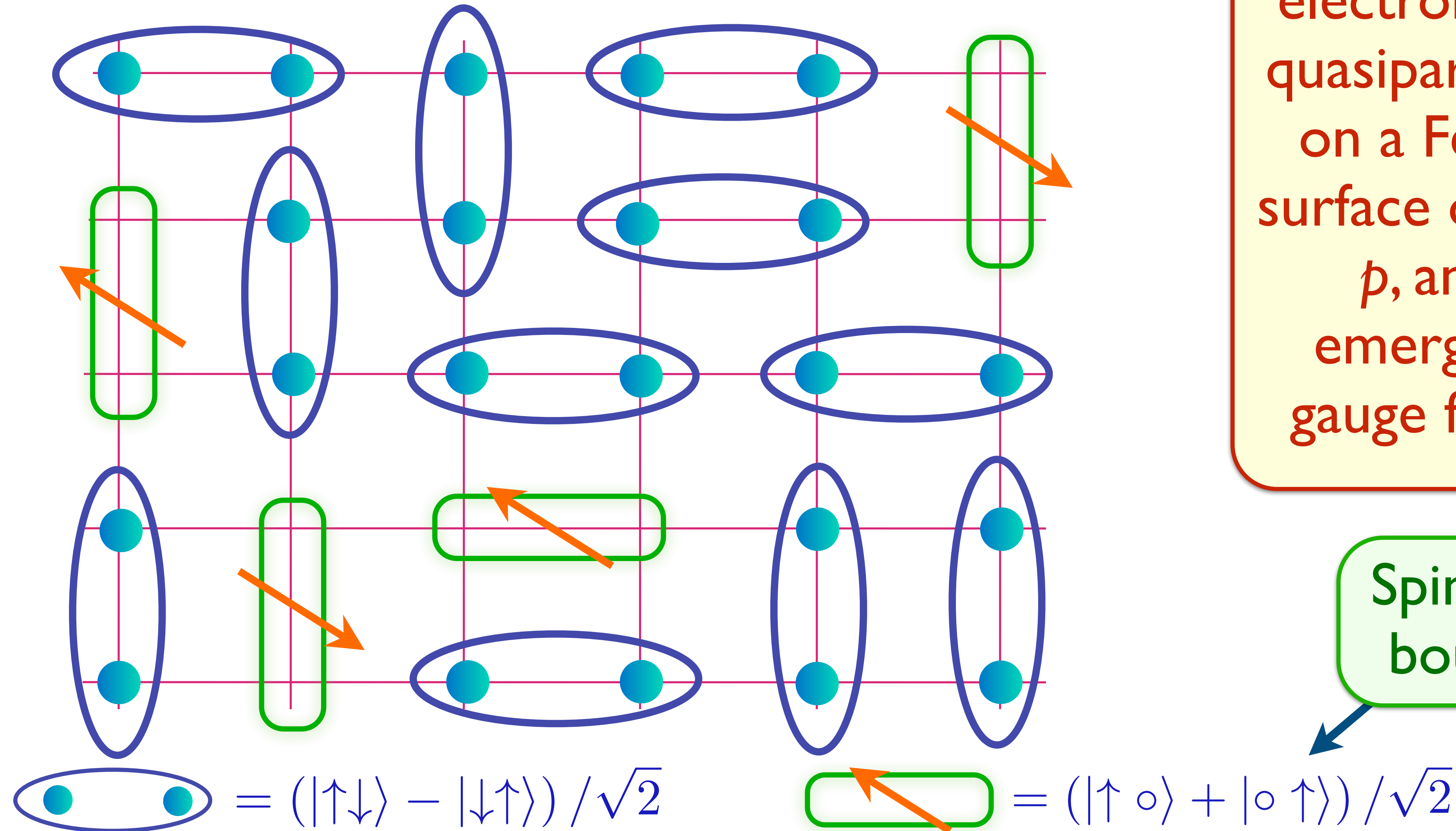
Metal with electron-like quasiparticles on a Fermi surface of size  $p$ , and emergent gauge fields

Spinon-holon bound state

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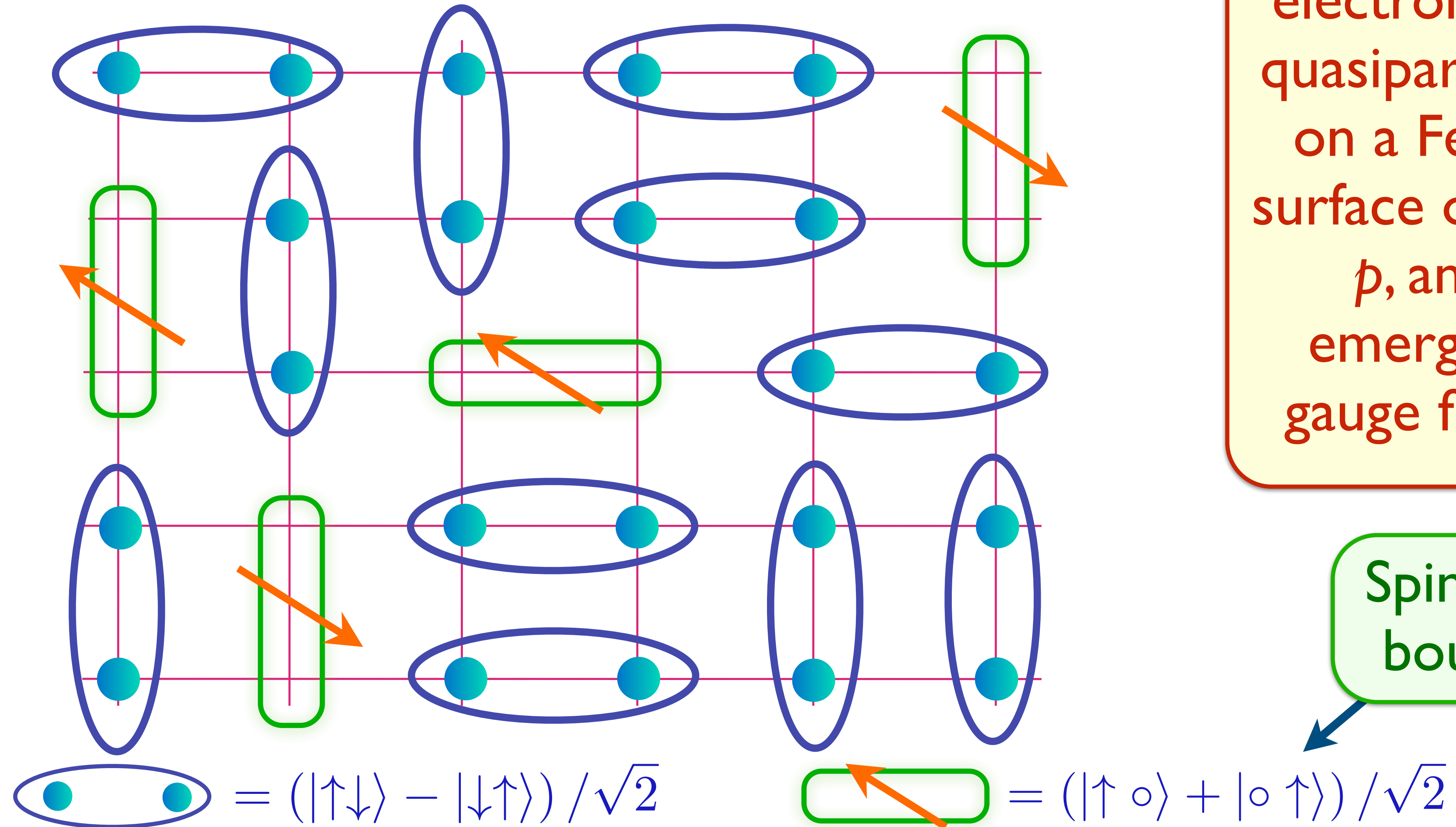
Metal with electron-like quasiparticles on a Fermi surface of size  $p$ , and emergent gauge fields

Spinon-holon bound state

# FL\* in a **one-band** model

S. Sachdev *Phys. Rev. B* **49**, 6770 (1994); X.-G. Wen and P.A. Lee *Phys. Rev. Lett.* **76**, 503 (1996)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, *Phys. Rev. B* **75**, 235122 (2007)



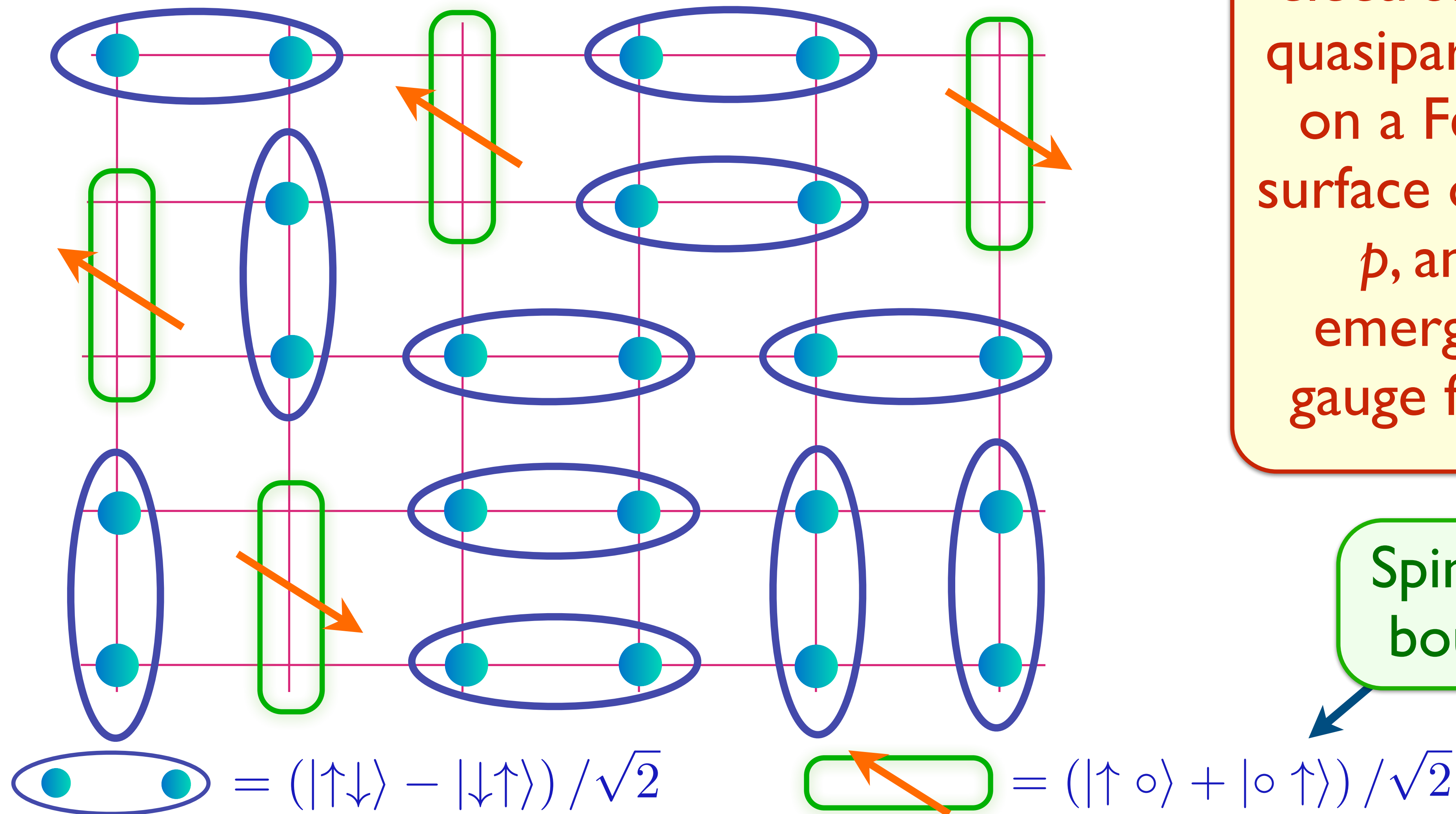
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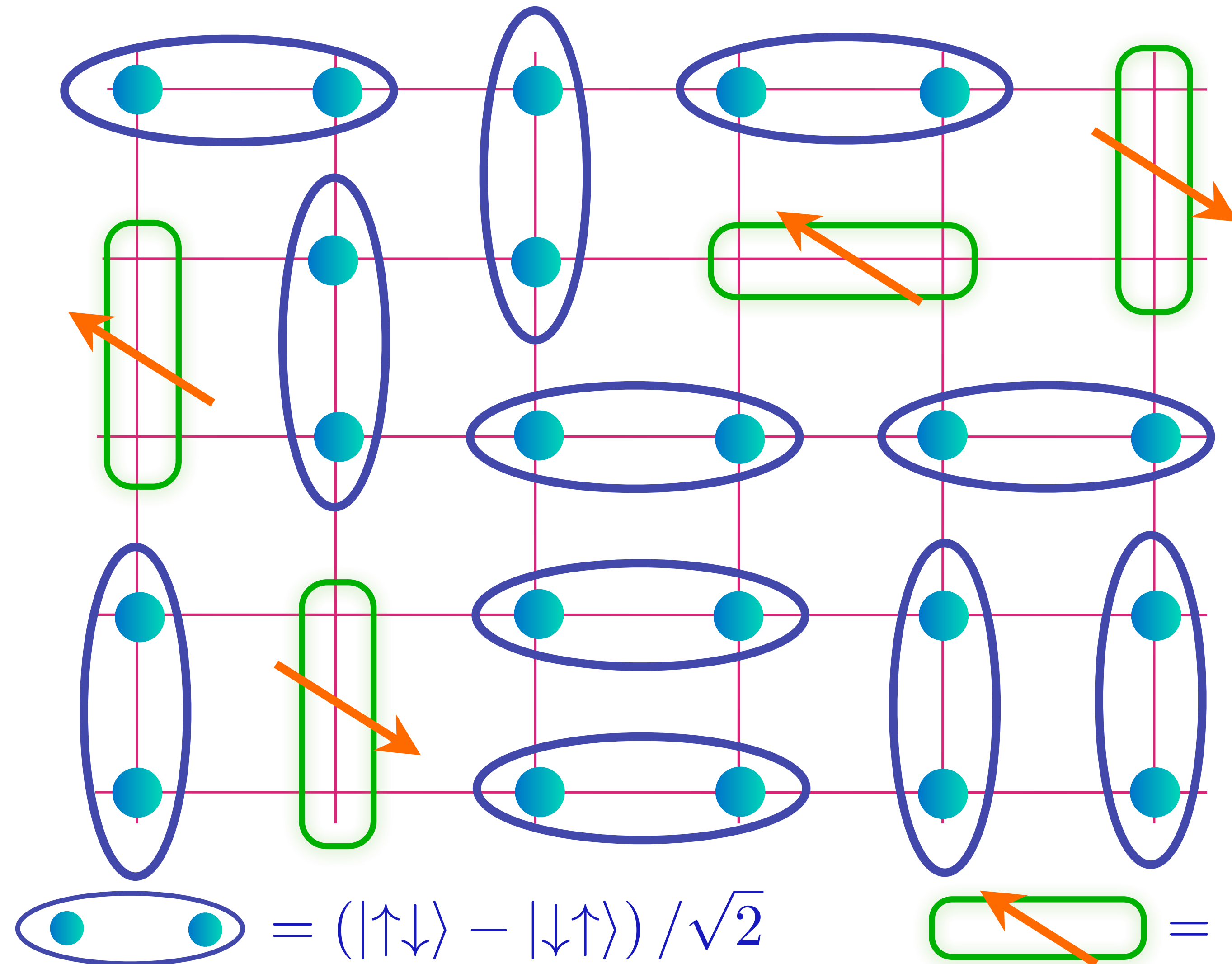
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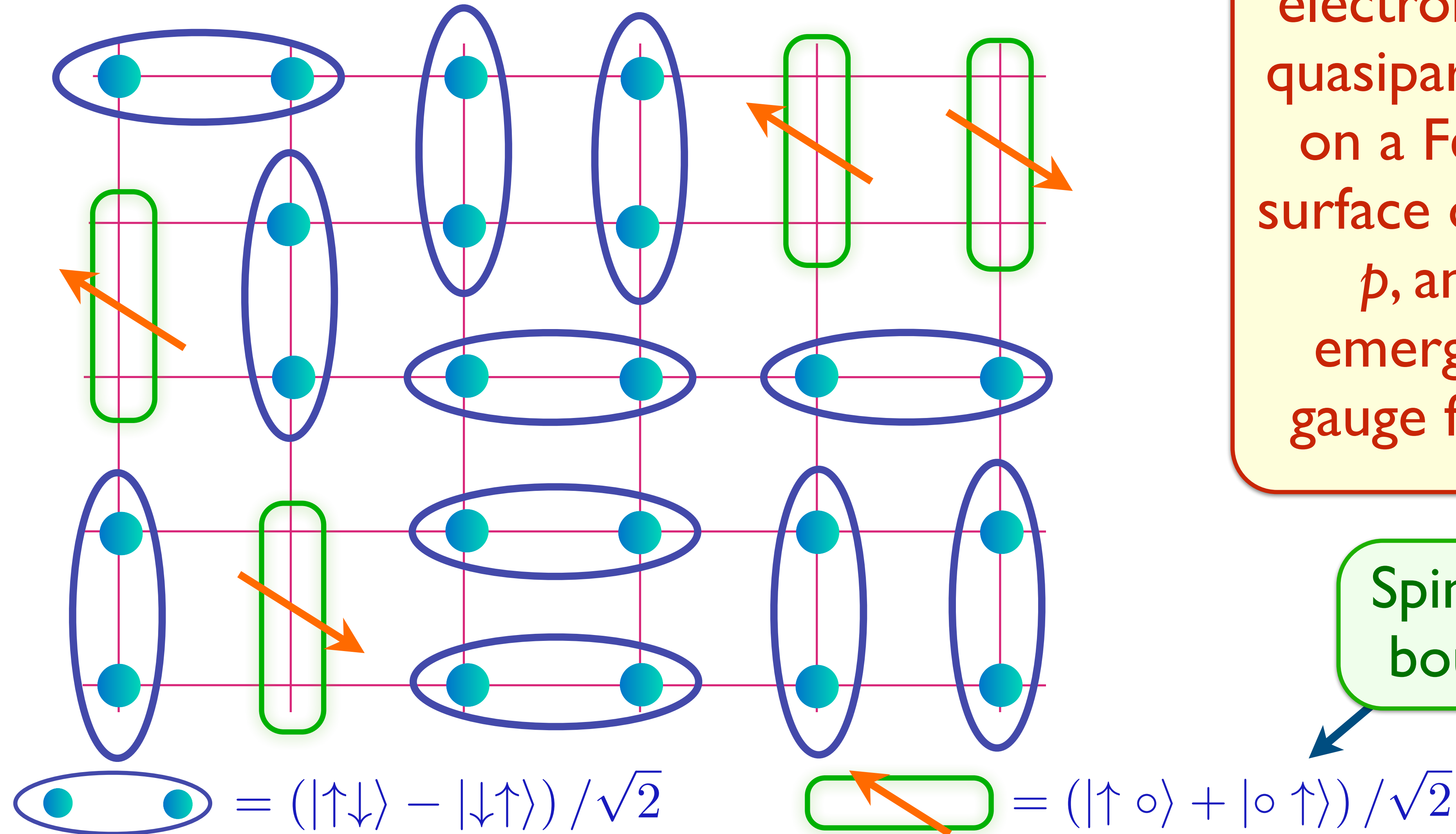
Metal with electron-like quasiparticles on a Fermi surface of size  $p$ , and emergent gauge fields

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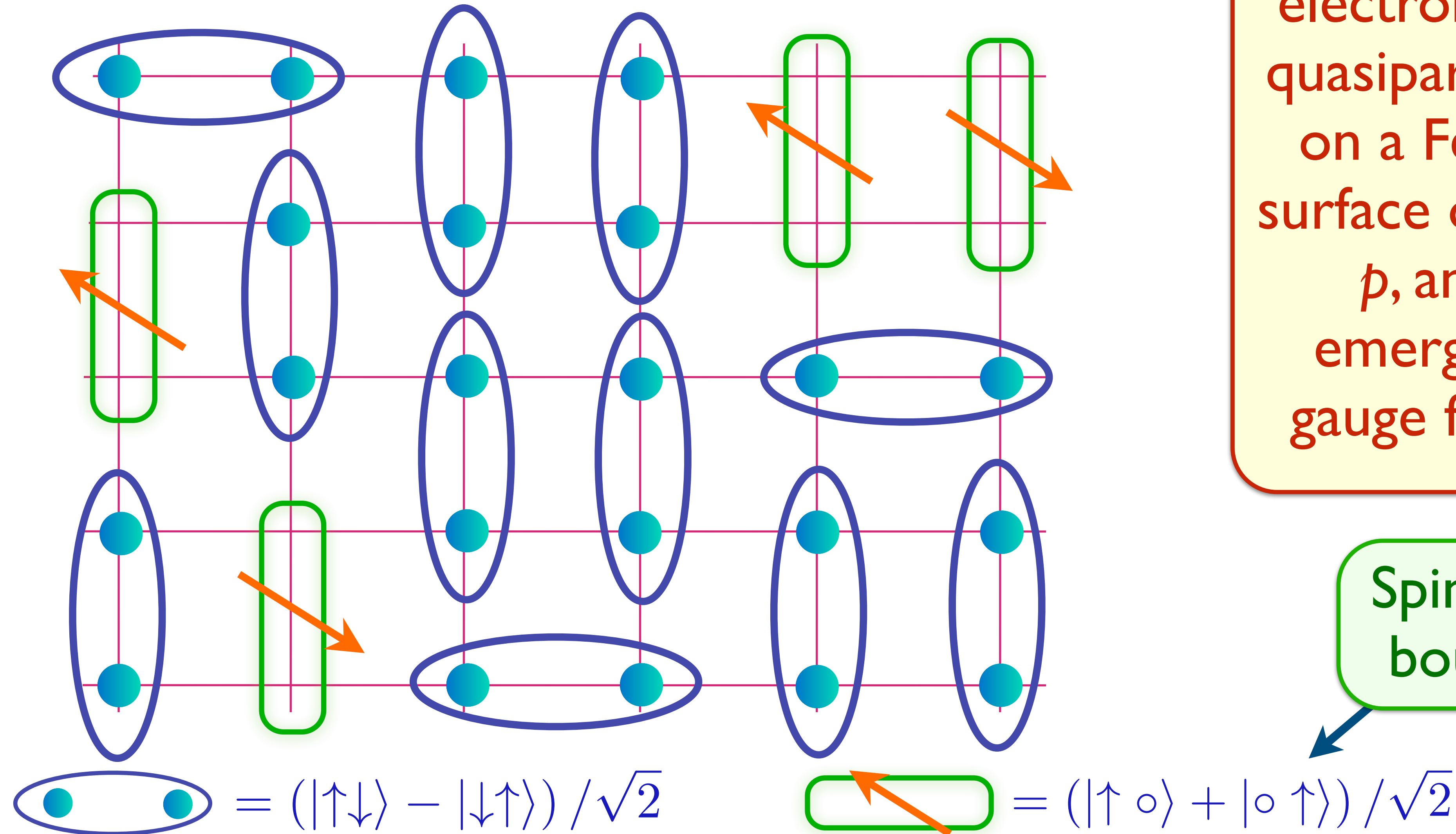


Metal with electron-like quasiparticles on a Fermi surface of size  $p$ , and emergent gauge fields

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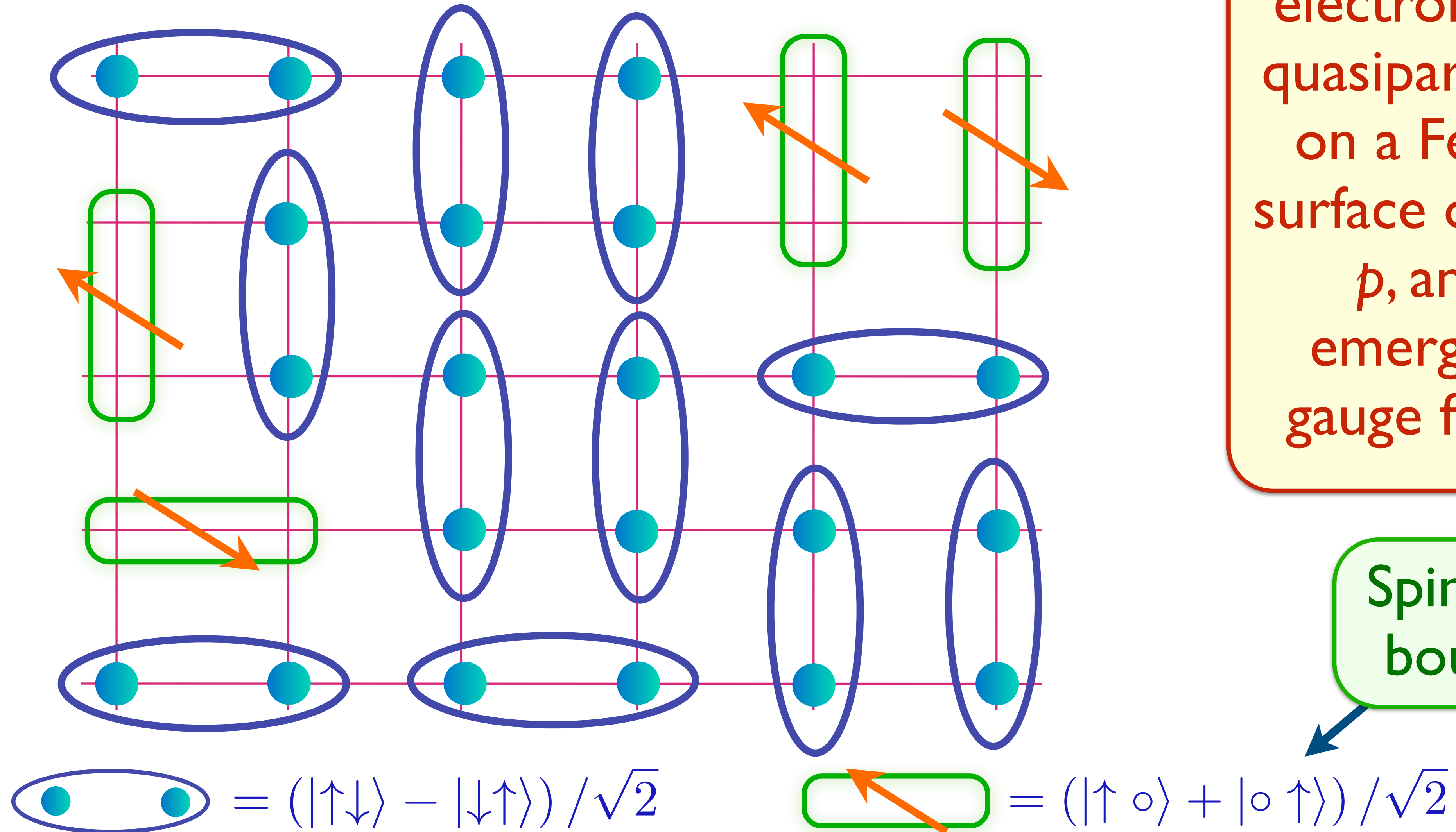
Metal with electron-like quasiparticles on a Fermi surface of size  $p$ , and emergent gauge fields

Spinon-holon bound state

# FL\* in a **one-band** model

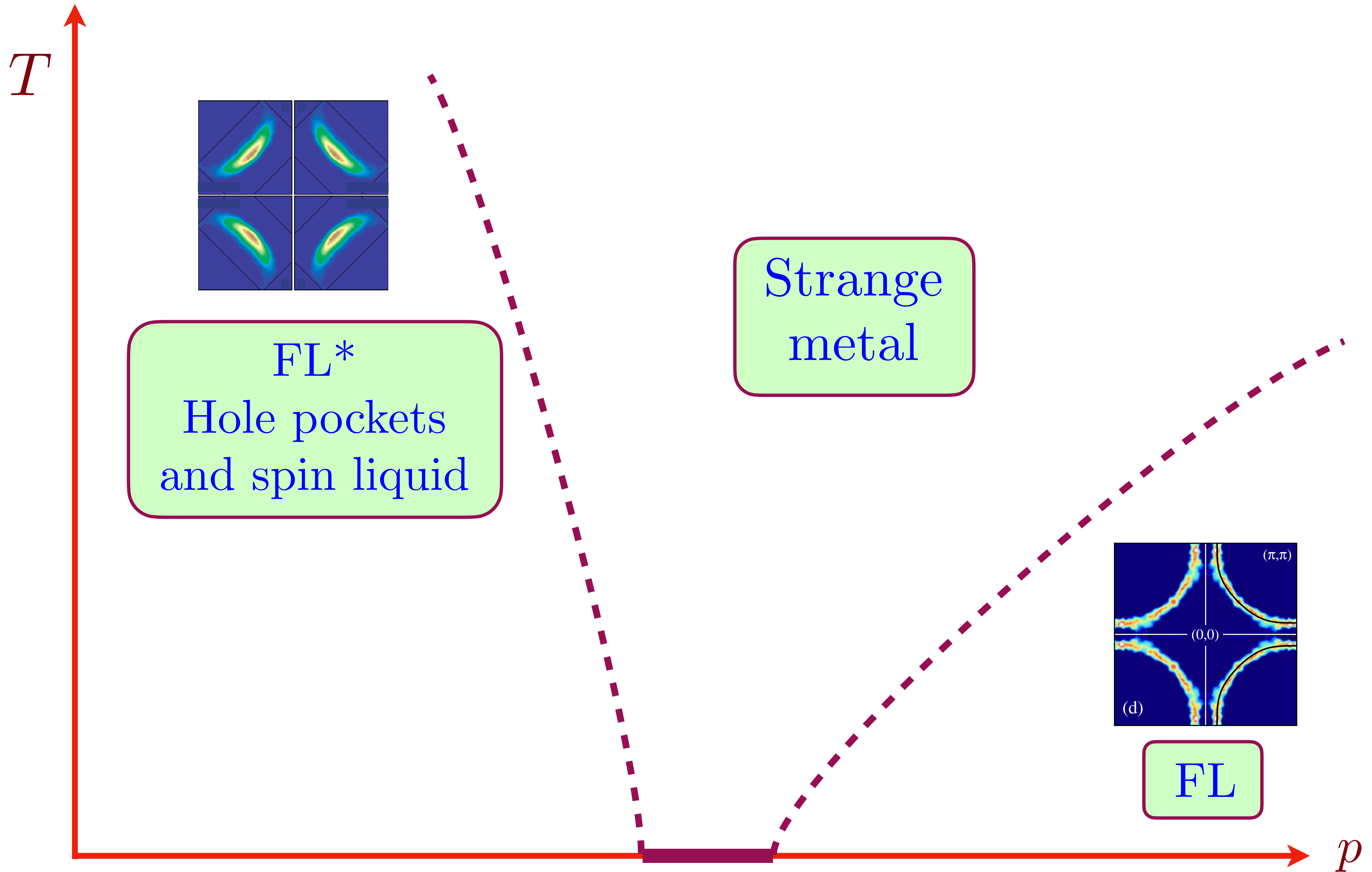
S. Sachdev *Phys. Rev. B* **49**, 6770 (1994); X.-G. Wen and P.A. Lee *Phys. Rev. Lett.* **76**, 503 (1996)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, *Phys. Rev. B* **75**, 235122 (2007)

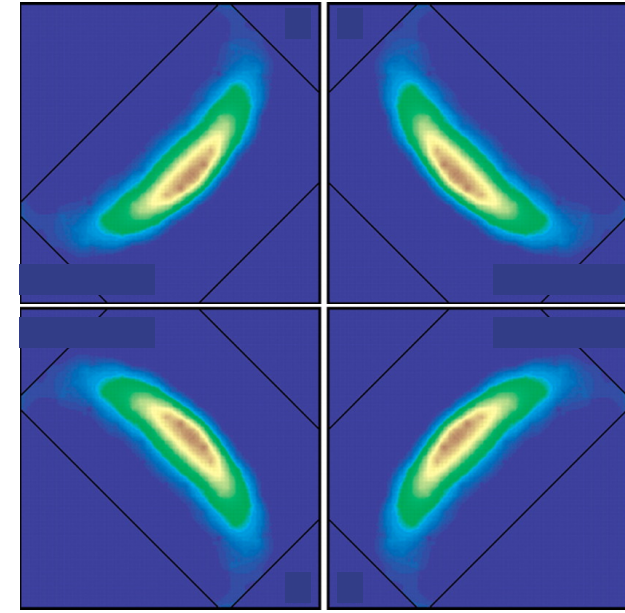


Metal with electron-like quasiparticles on a Fermi surface of size  $p$ , and emergent gauge fields

Spinon-holon bound state

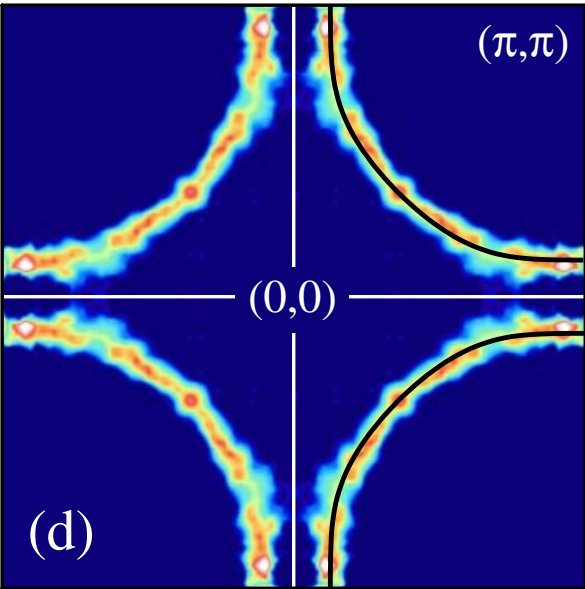


$T$



$FL^*$   
Hole pockets  
and spin liquid

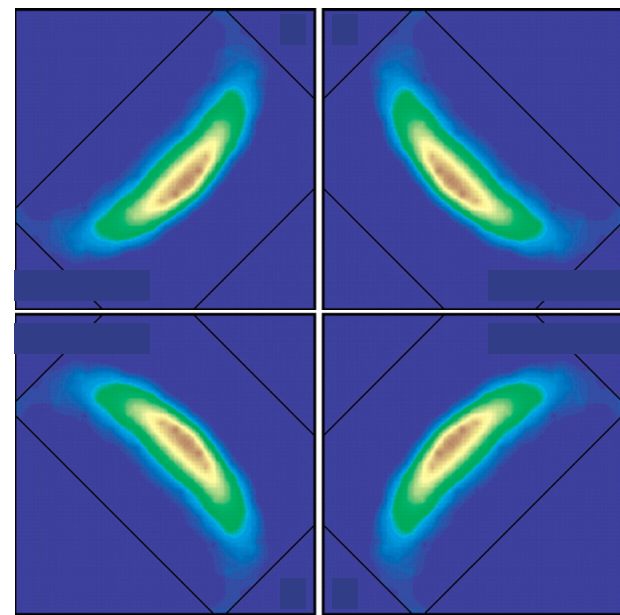
Strange  
metal



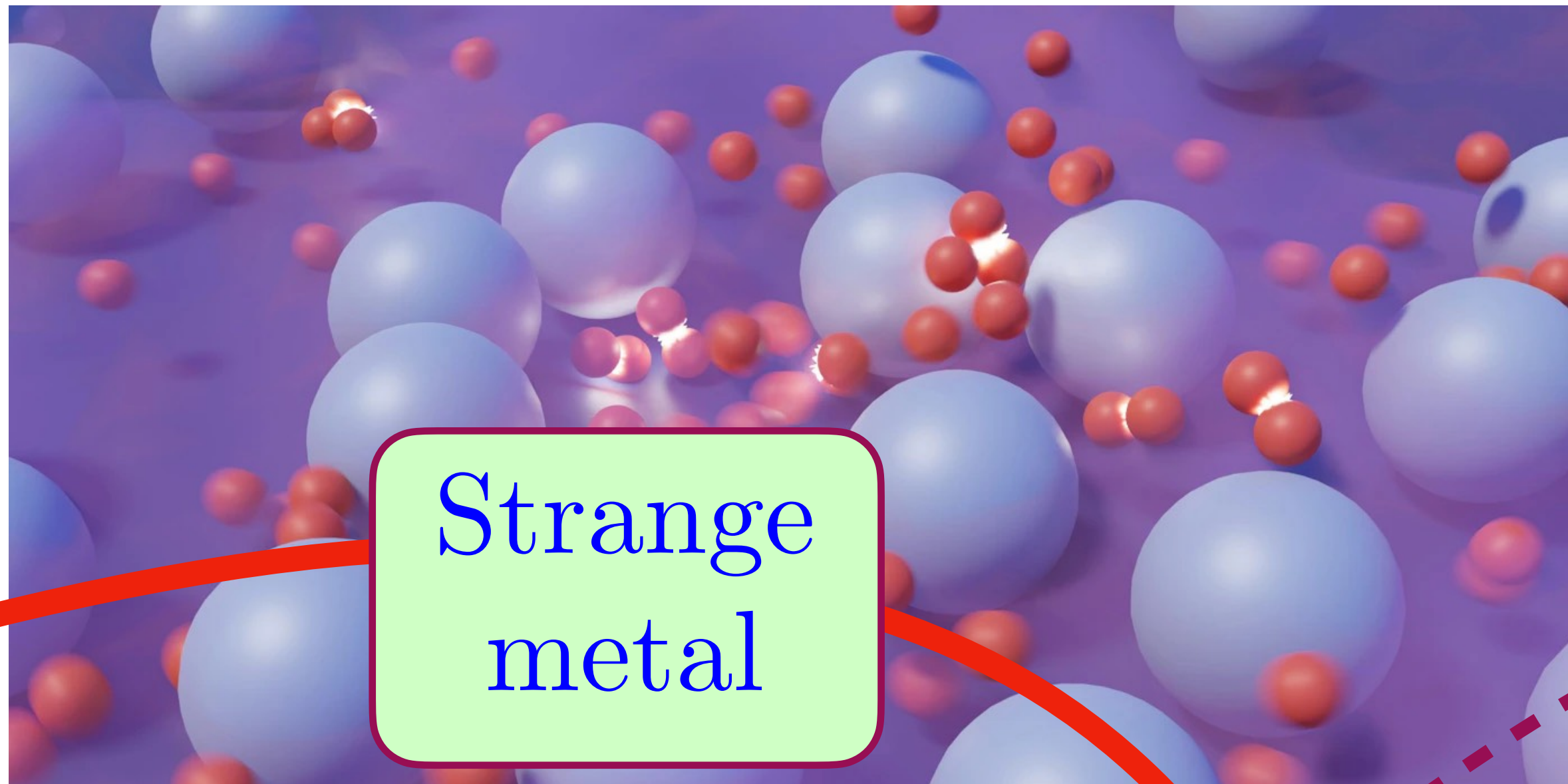
$FL$

$p$

$T$

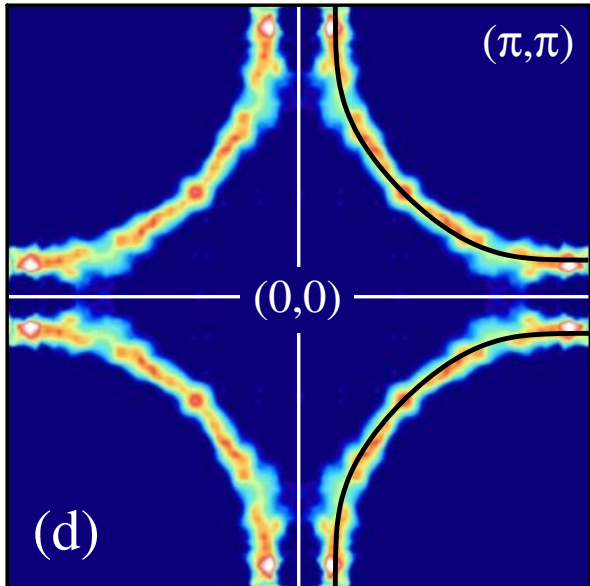


FL\*  
Hole pockets  
and spin liquid



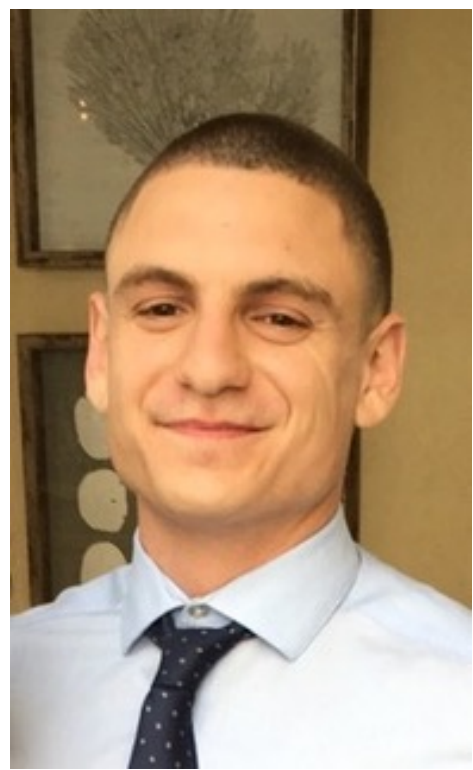
Strange  
metal

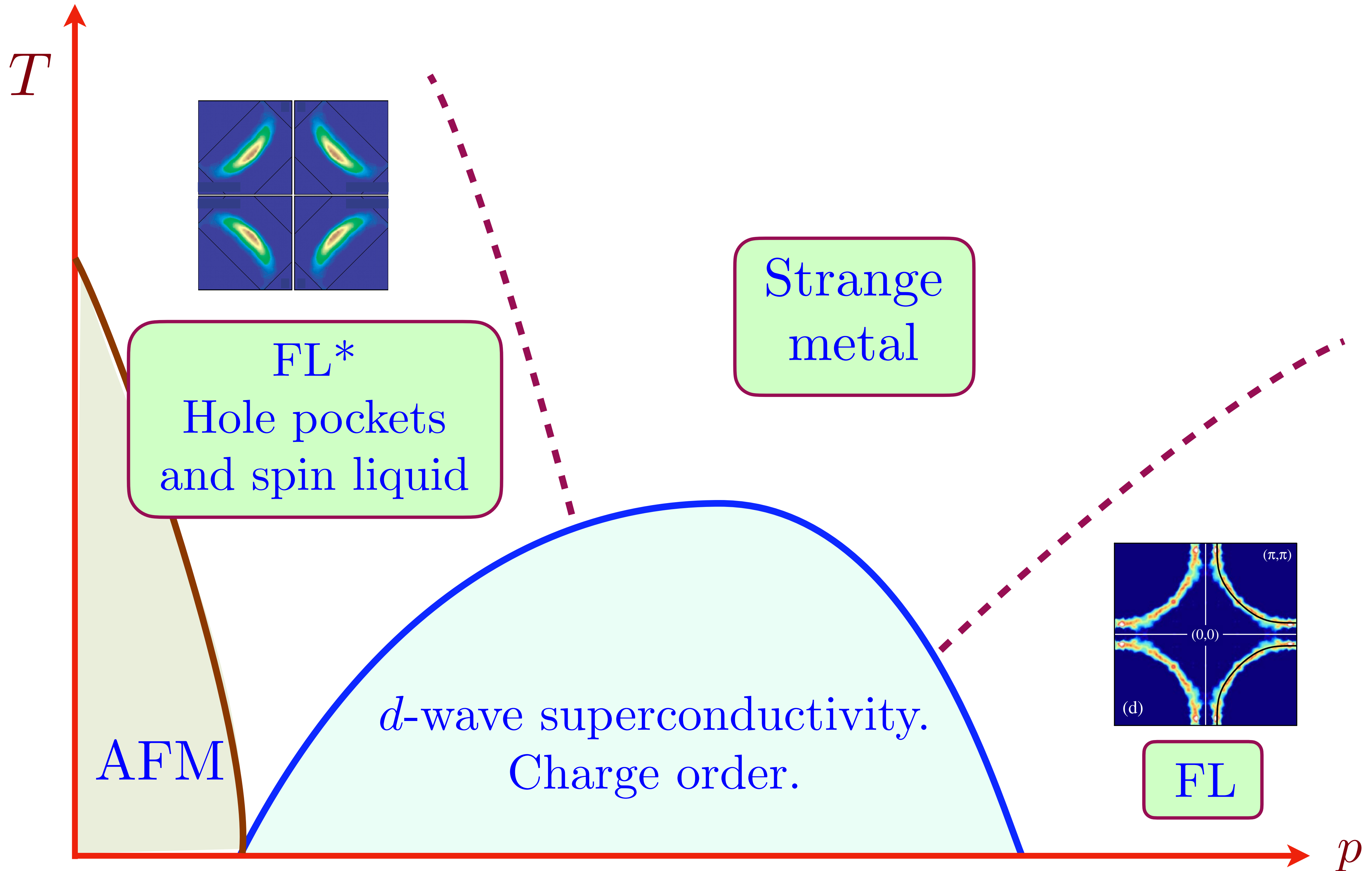
Patel,  
Haoyu  
Guo,  
Esterlis,  
Sachdev,  
Science  
**381**, 790  
(2023)

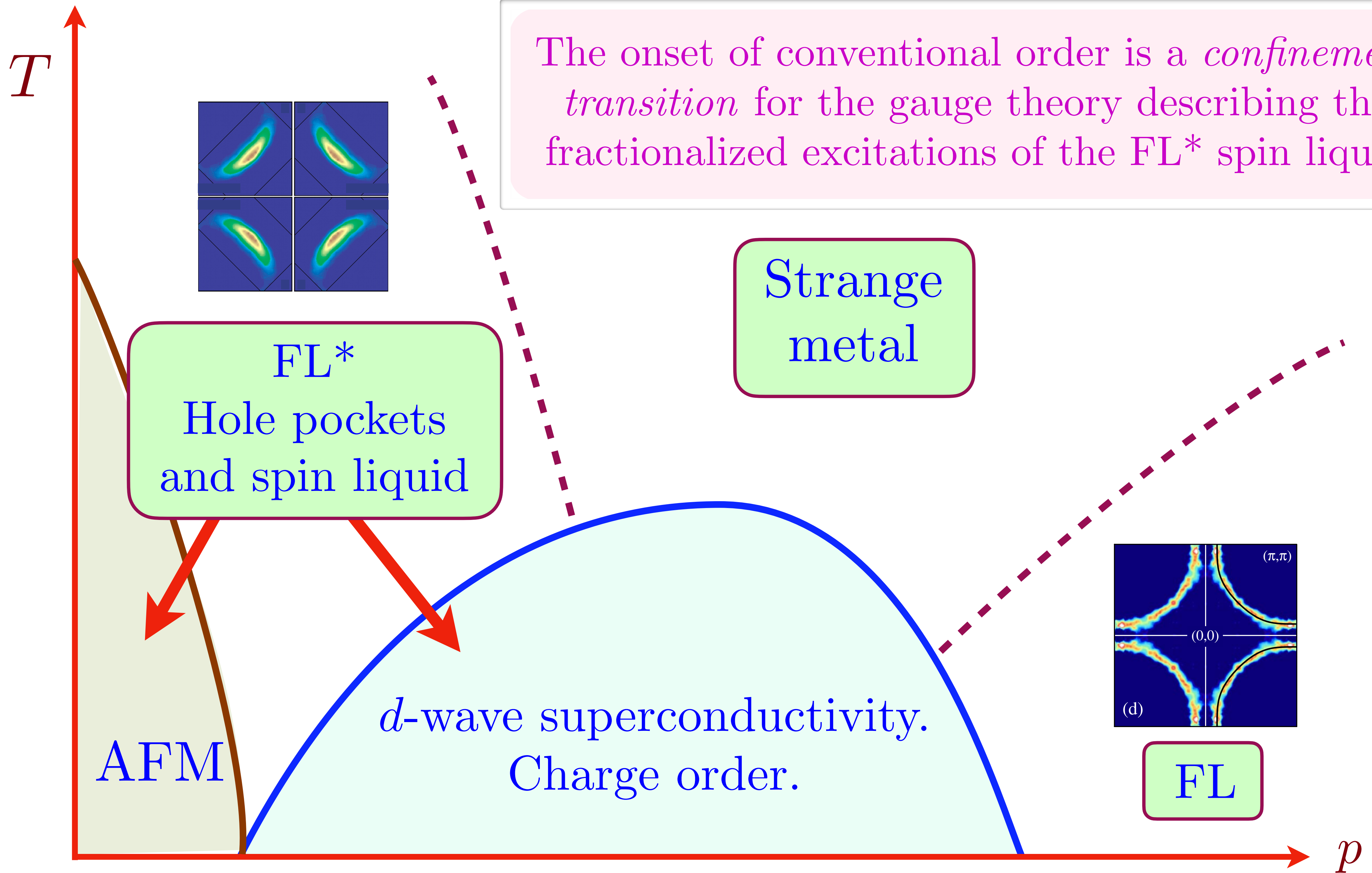


FL

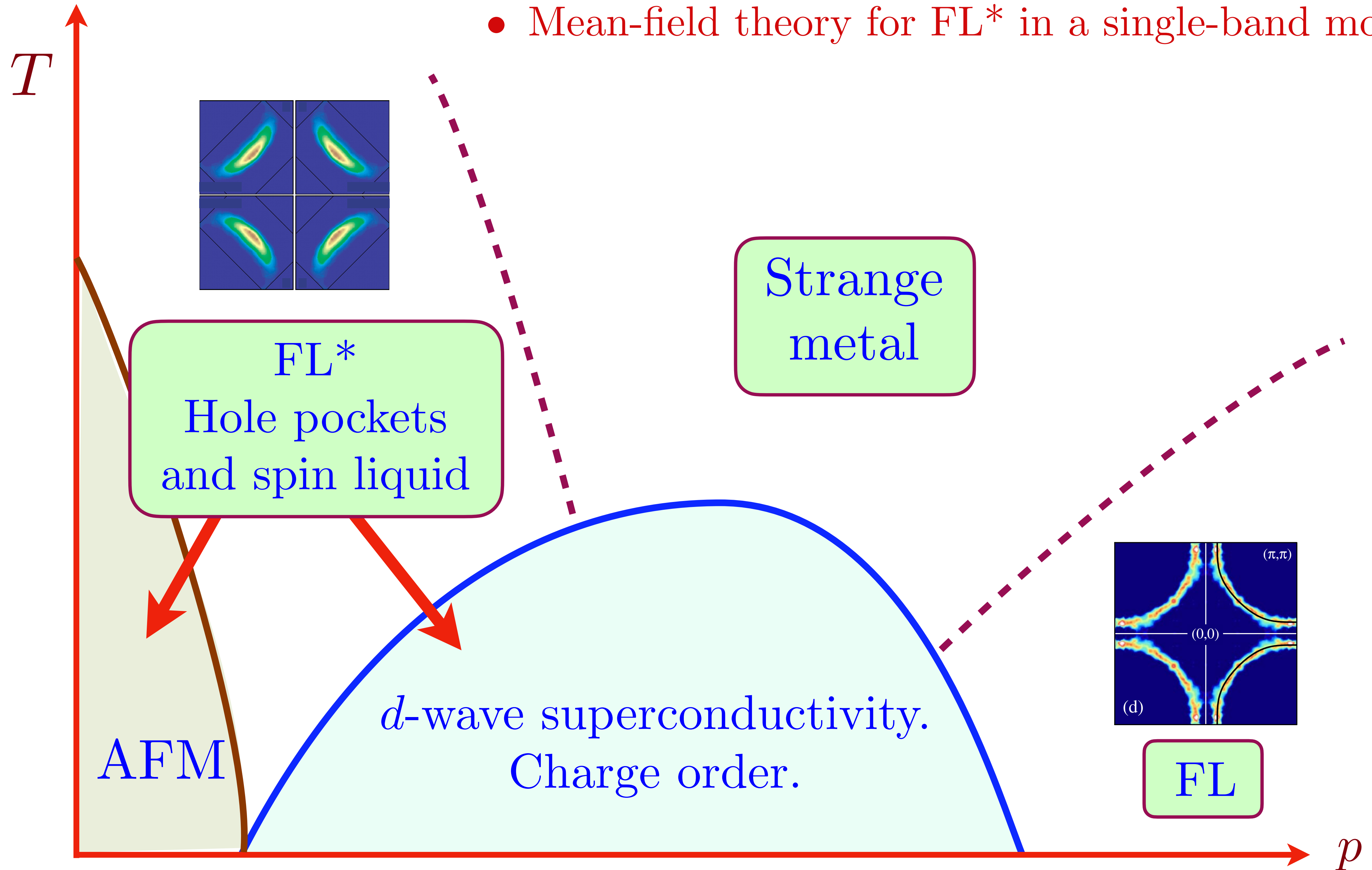
$p$

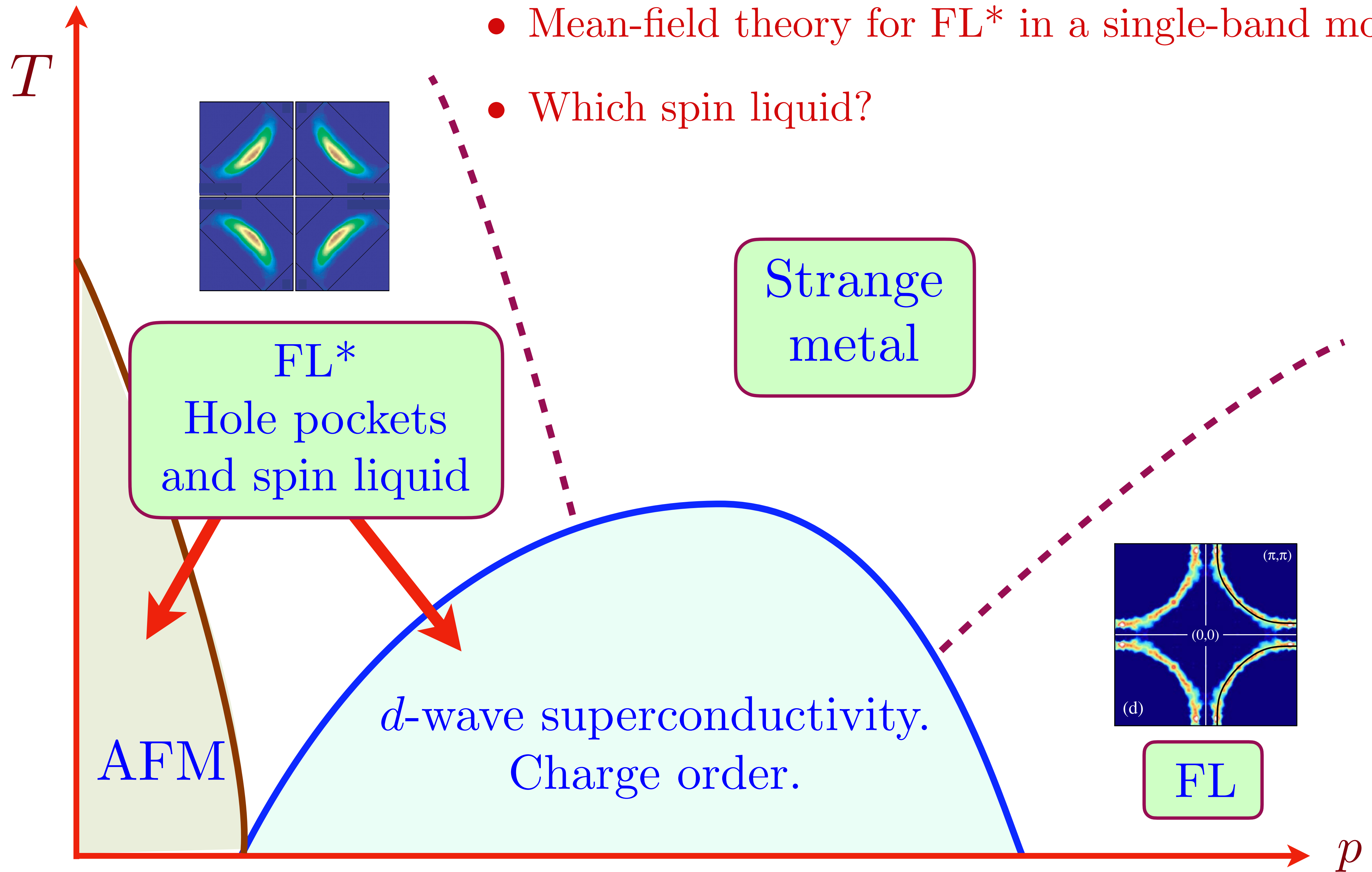






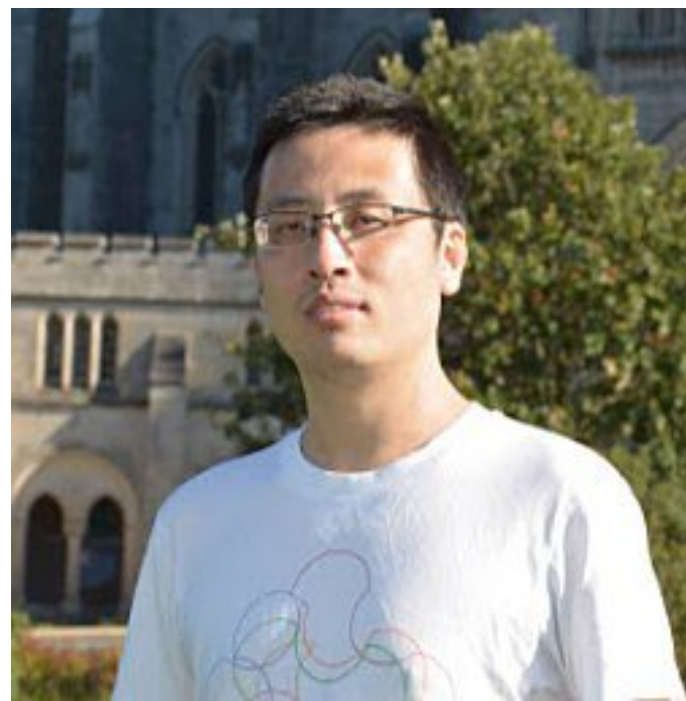
- Mean-field theory for FL\* in a single-band model?





- Mean-field theory for FL\* in a single-band model?
- Which spin liquid?

# Ancilla theory of $FL^*$ in a single-band model



Ya-Hui Zhang

# Paramagnon theory of the Hubbard model

$$H = - \sum_{i < j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i \left( n_{i\uparrow} - \frac{1}{2} \right) \left( n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c_{i\sigma}^\dagger c_{i\sigma}$$

We use the operator equation (valid on each site  $i$ ):

$$U \left( n_\uparrow - \frac{1}{2} \right) \left( n_\downarrow - \frac{1}{2} \right) = -\frac{2U}{3} \mathbf{S}^2 + \frac{U}{4}$$

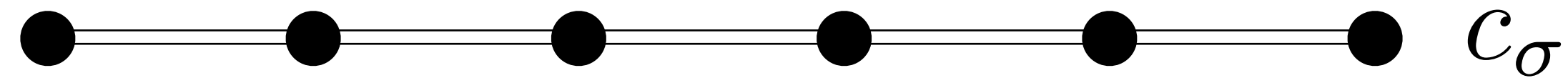
Then we decouple the interaction via

$$\exp \left( \frac{2U}{3} \sum_i \int d\tau \mathbf{S}_i^2 \right) = \int \mathcal{D}\Phi_i(\tau) \exp \left( - \sum_i \int d\tau \left[ \frac{3}{8U} \Phi_i^2 - \Phi_i \cdot c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \right] \right)$$

This yields the ‘Scalapino-Pines-Chubukov-Schmalian...’ theory for a ‘paramagnon quantum rotor’  $\Phi_i$  coupled to otherwise free fermions  $c_{i\sigma}$ .

$\Phi_i$  is the creation/annihilation operator for charge 0, spin  $S = 1$  particle.

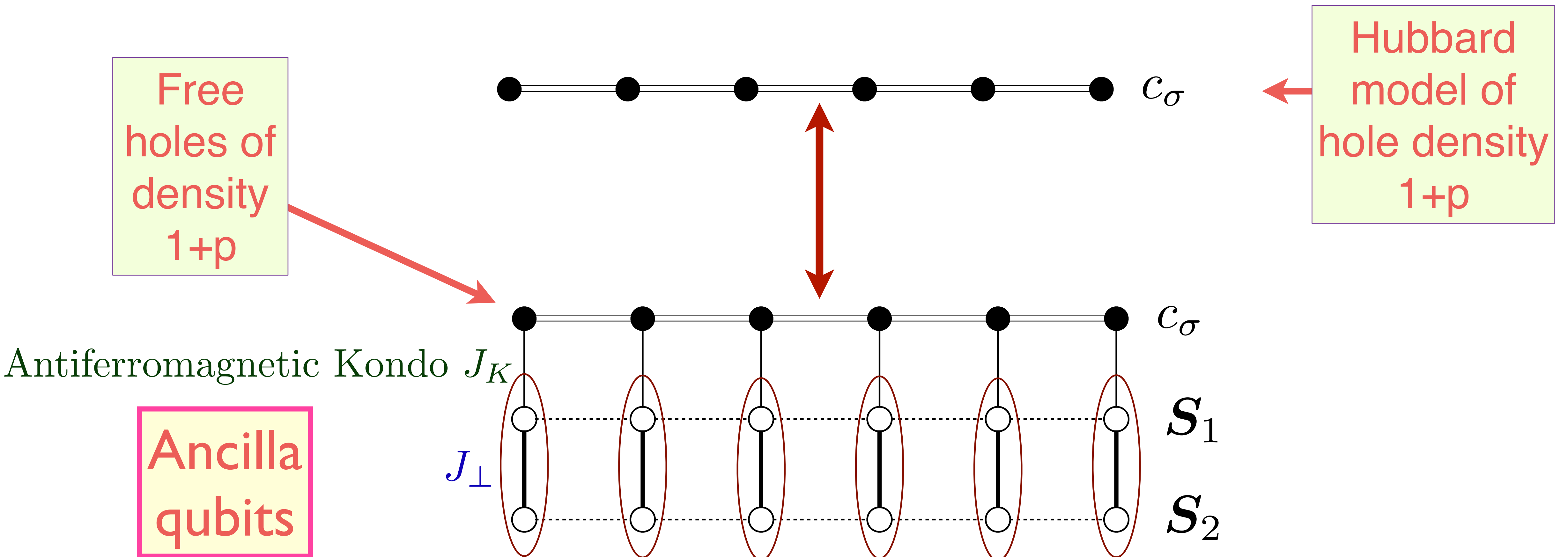
# Ancilla theory of the Hubbard model



Hubbard  
model of  
hole density  
 $1+p$

# Ancilla theory of the Hubbard model

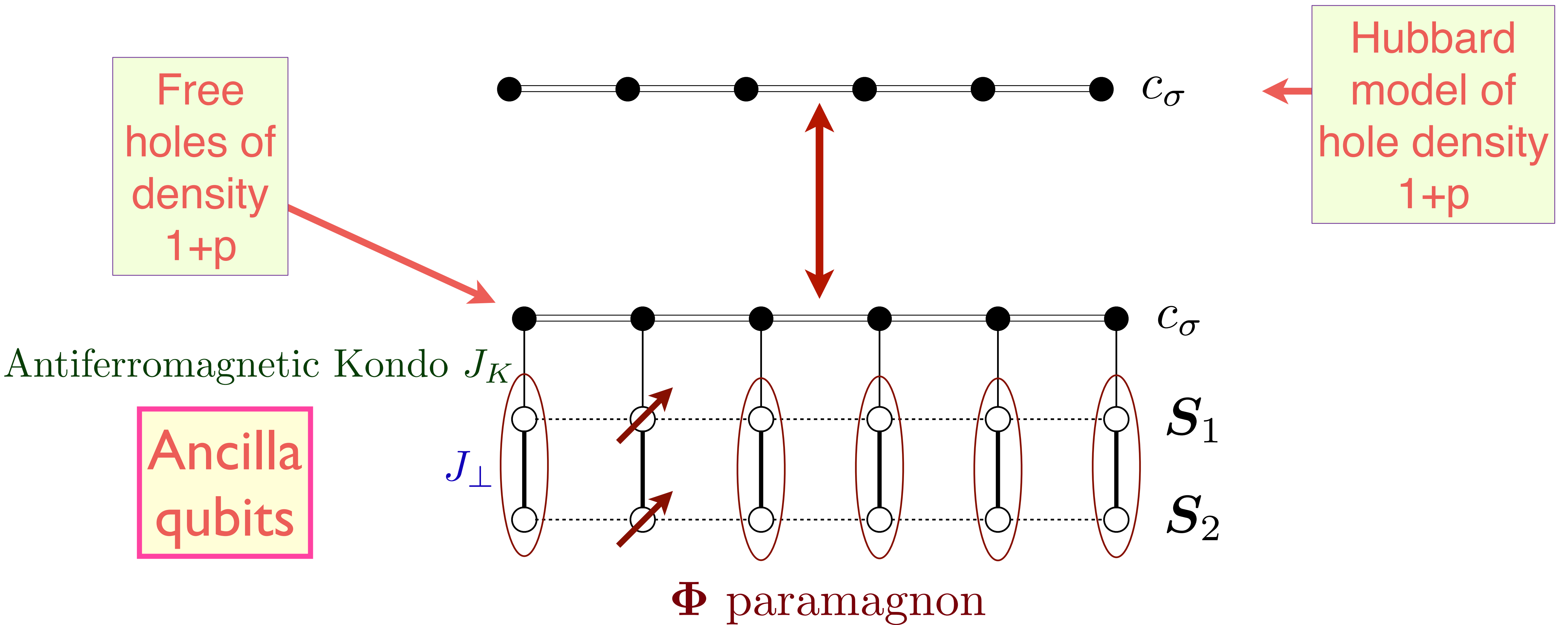
Ya-Hui Zhang and S. Sachdev,  
*Phys. Rev. Res.* **2**, 023172 (2020)



$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^{\dagger} c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^{\dagger} \frac{\sigma_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \mathbf{S}_{1i} + J_{\perp} \sum_i \mathbf{S}_{1i} \cdot \mathbf{S}_{2i}$$

# Ancilla theory of the Hubbard model

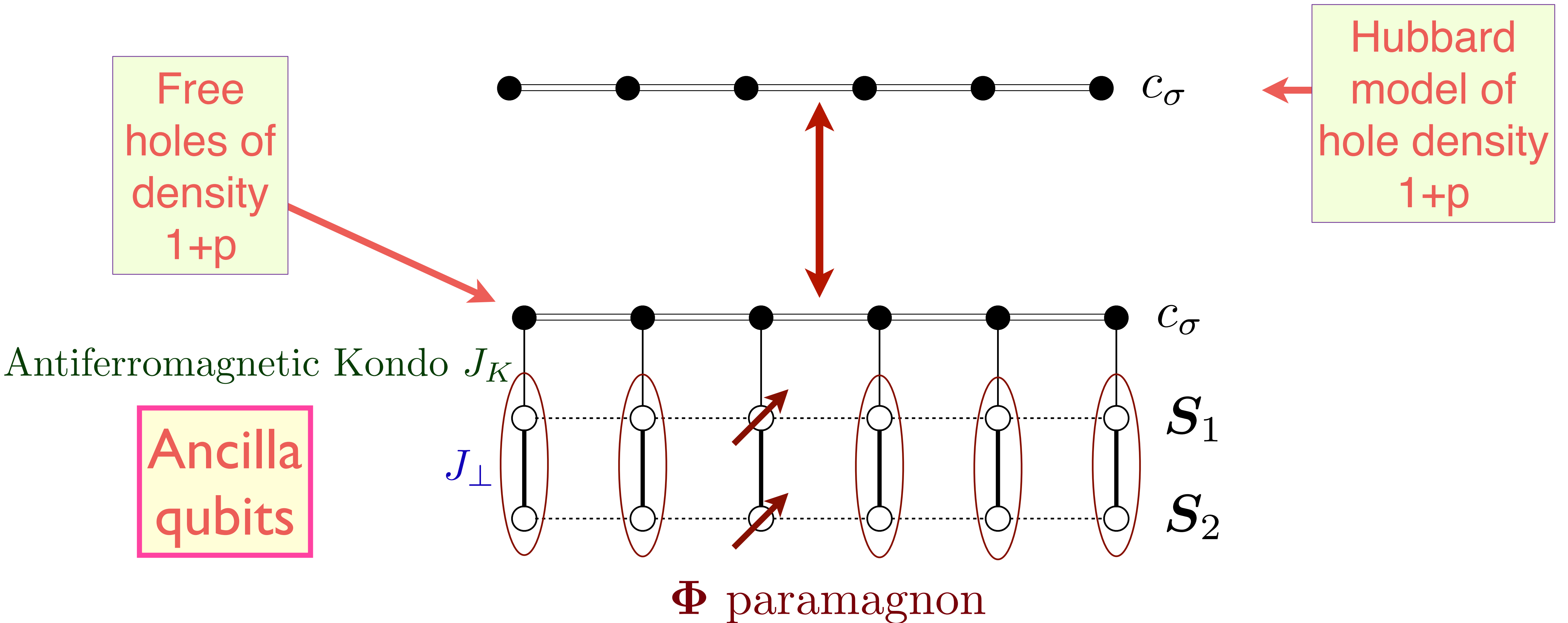
Ya-Hui Zhang and S. Sachdev,  
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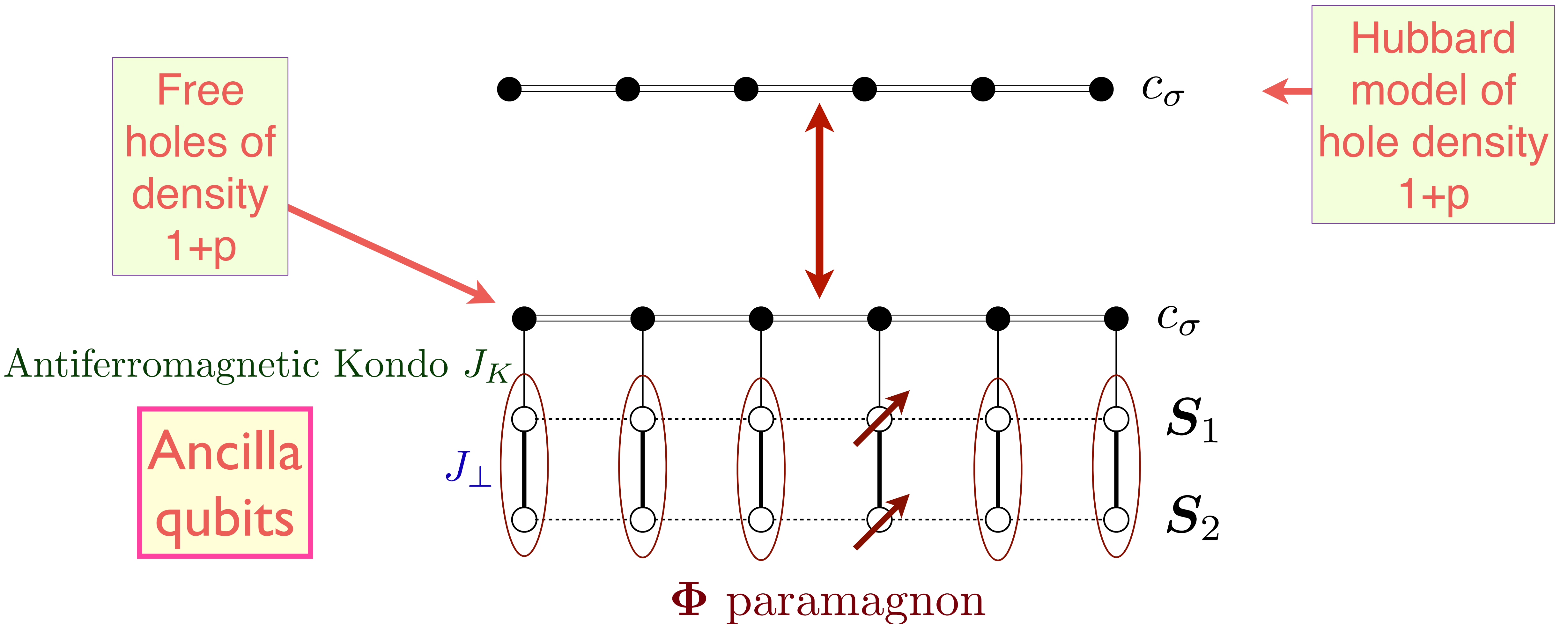
Ya-Hui Zhang and S. Sachdev,  
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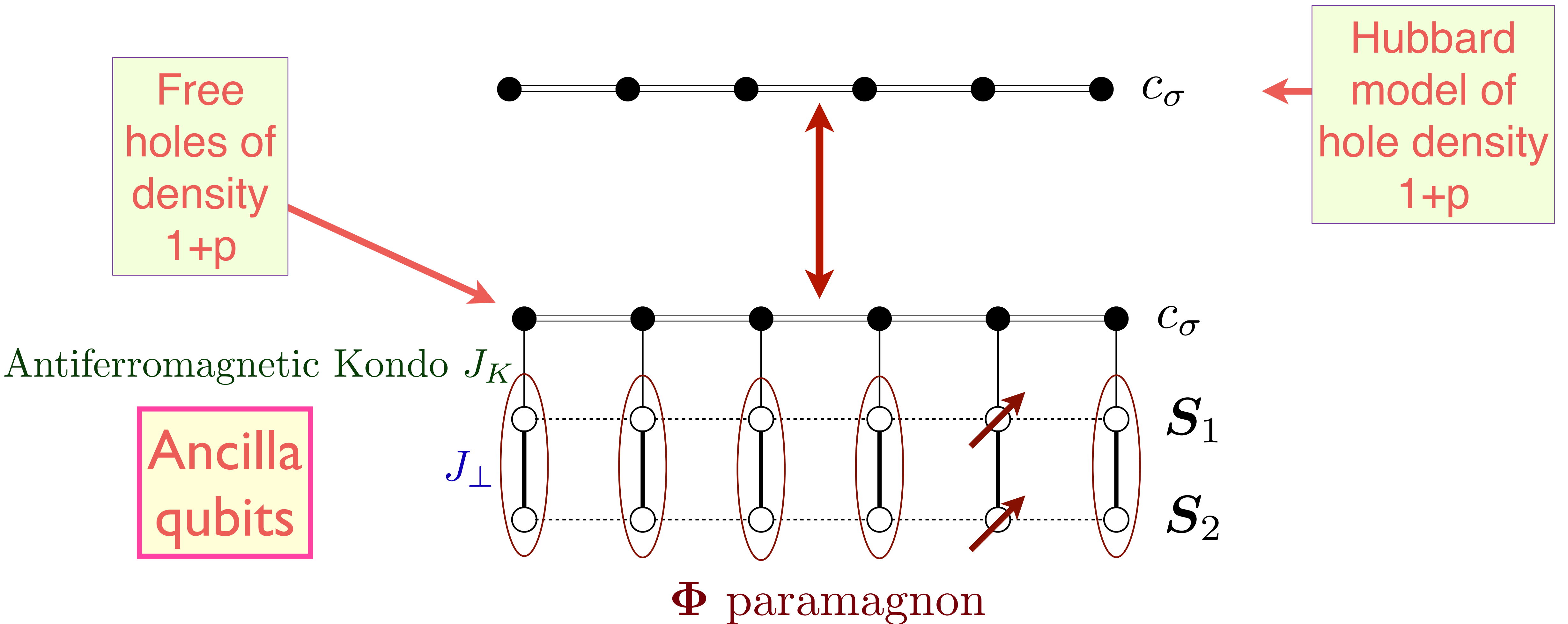
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# Ancilla theory of the Hubbard model

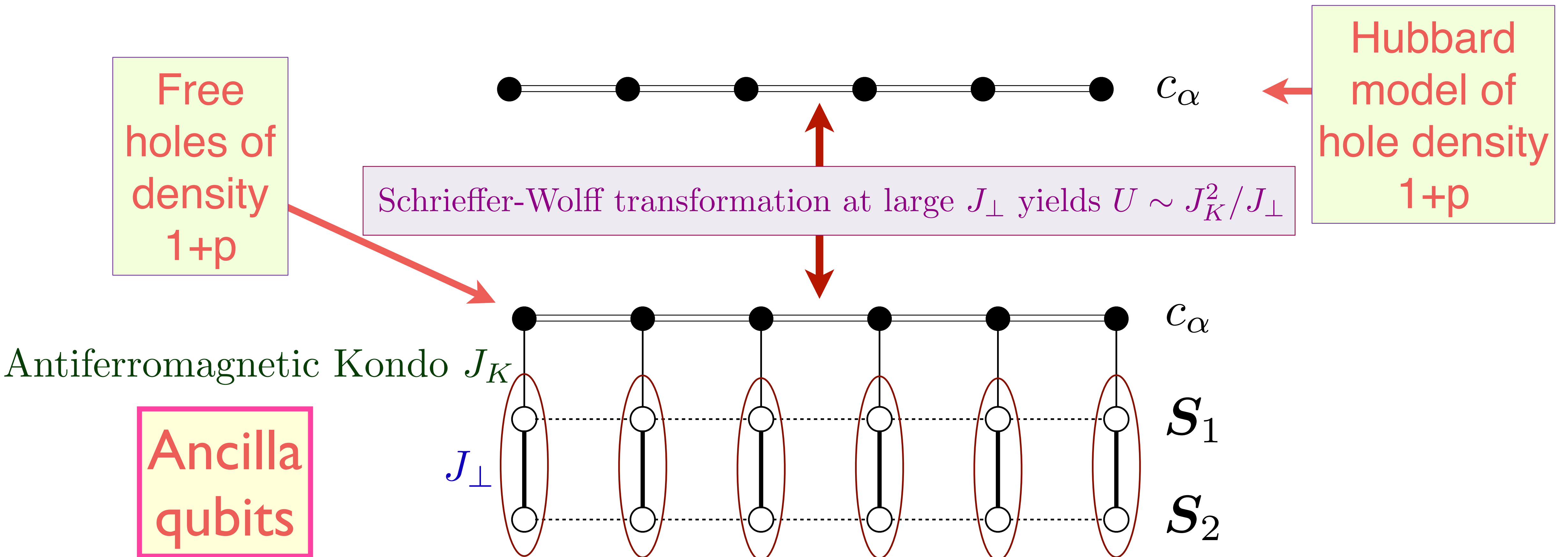
Ya-Hui Zhang and S. Sachdev,  
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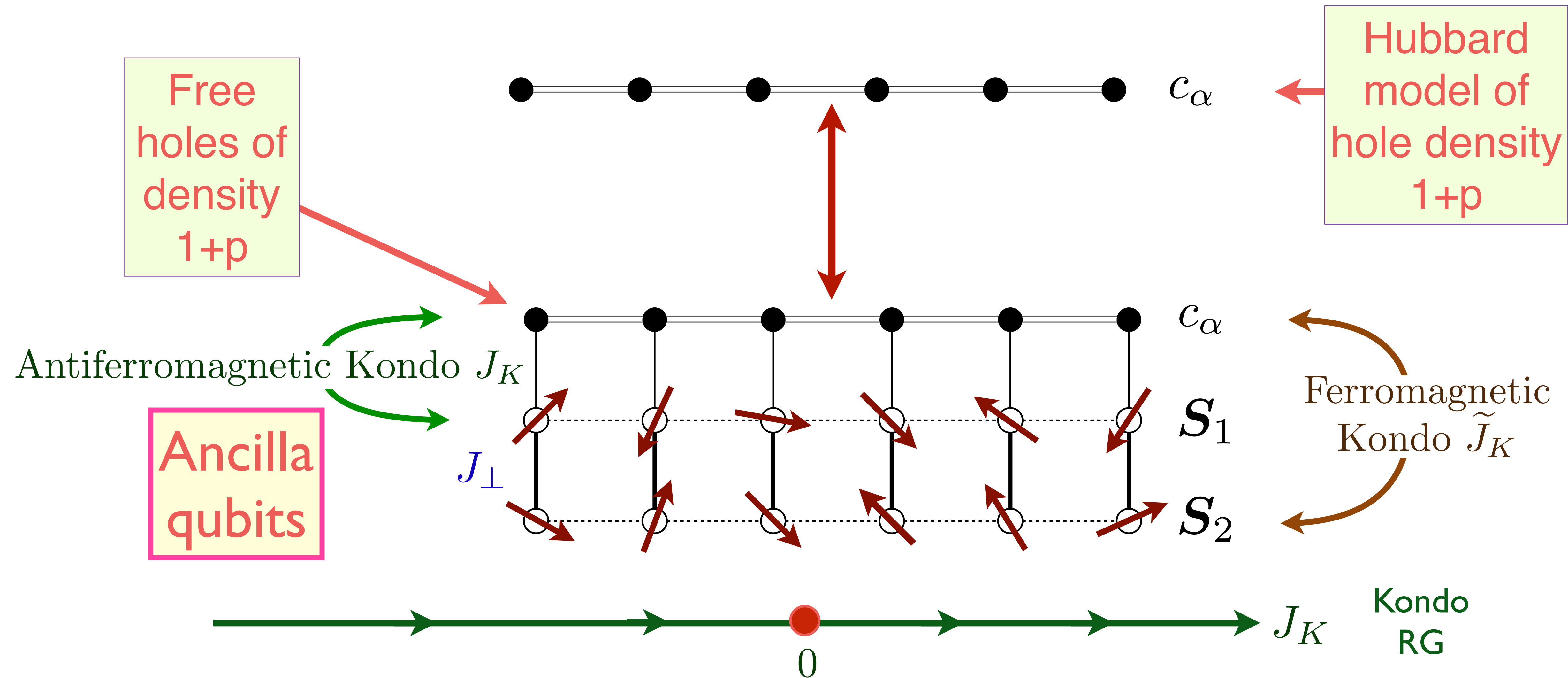
Ya-Hui Zhang and S. Sachdev,  
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# Ancilla theory of the Hubbard model

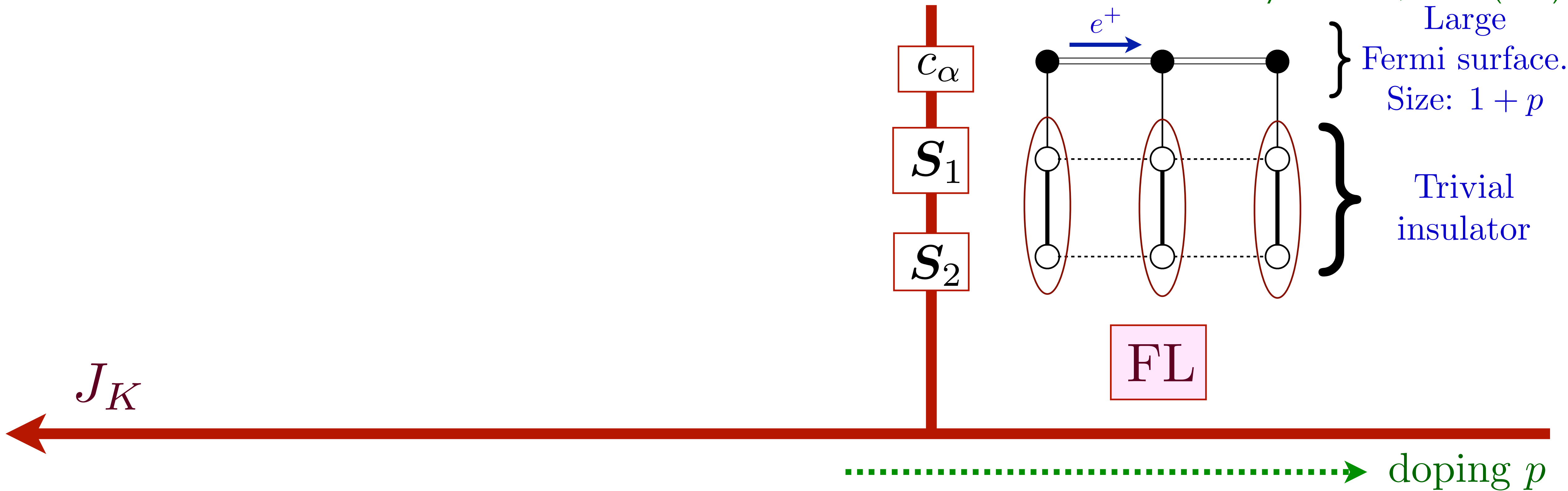
Ya-Hui Zhang and S. Sachdev,  
*Phys. Rev. Res.* **2**, 023172 (2020)



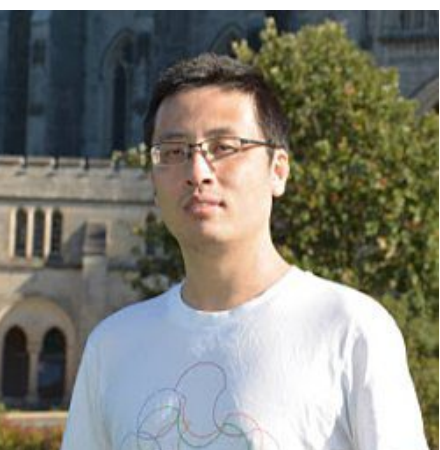
$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^{\dagger} c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^{\dagger} \frac{\sigma_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \mathbf{S}_{1i} + J_{\perp} \sum_i \mathbf{S}_{1i} \cdot \mathbf{S}_{2i}$$

# Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev,  
*Phys. Rev. Res.* **2**, 023172 (2020)



Ya-Hui  
Zhang

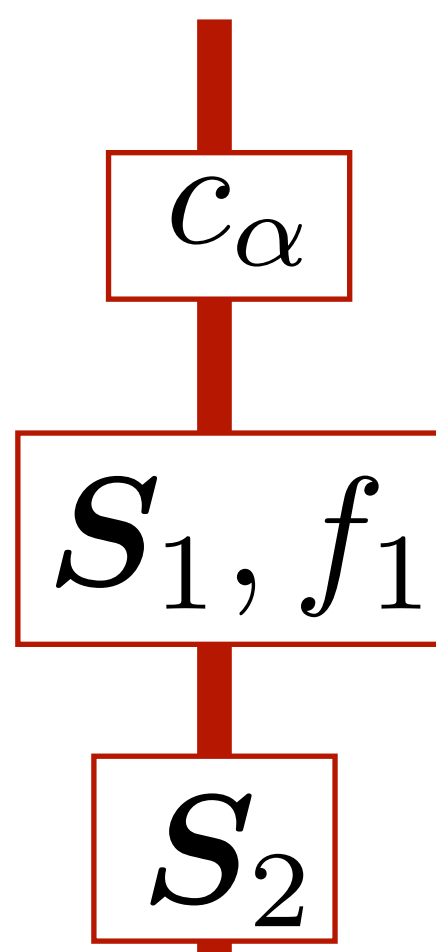
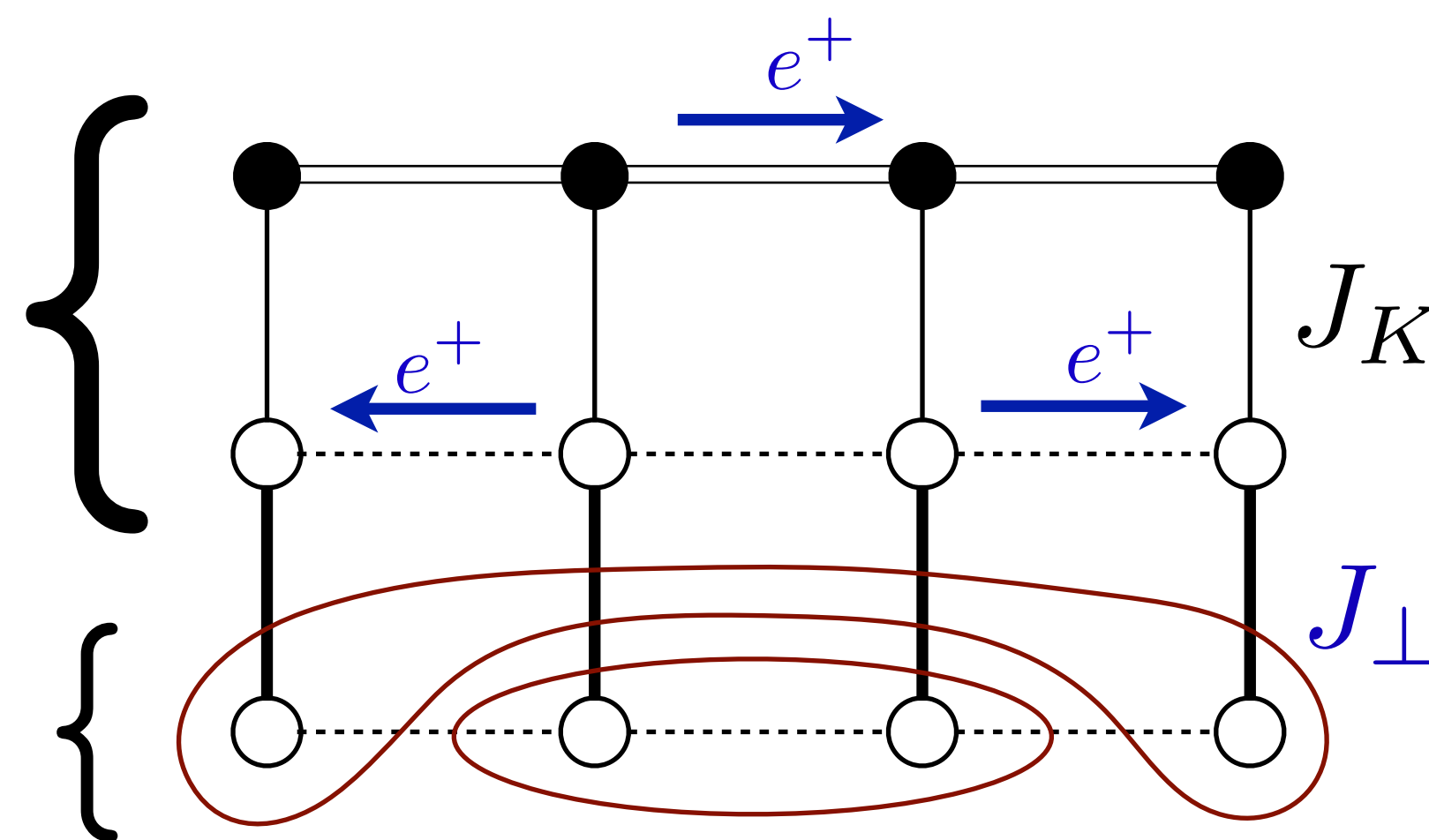


# Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev,  
*Phys. Rev. Res.* **2**, 023172 (2020)

Kondo lattice  
 heavy Fermi liquid.  
 Size  $1 + p + 1$   
 $= p \pmod{2}$ .  
*Small* Fermi surface!

Spin liquid



Large  
 Fermi surface.  
 Size:  $1 + p$

Trivial  
 insulator

$J_K$

FL\*

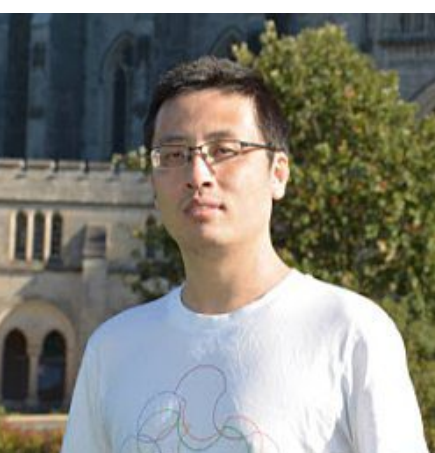
FL

doping  $p$

Pseudogap metal =  
 Kondo Lattice Heavy  
 Fermi Liquid  
 $\oplus$   
 Spin Liquid

$$|FL^*\rangle = [\text{Projection onto rung singlets of } \mathcal{S}_1, \mathcal{S}_2] \\
\otimes |\text{Slater determinant of } (c, f_1)\rangle \\
\otimes |\text{Spin liquid of } \mathcal{S}_2\rangle$$

Ya-Hui  
 Zhang



# Ancilla theory of the Hubbard model



Henry Shackleton

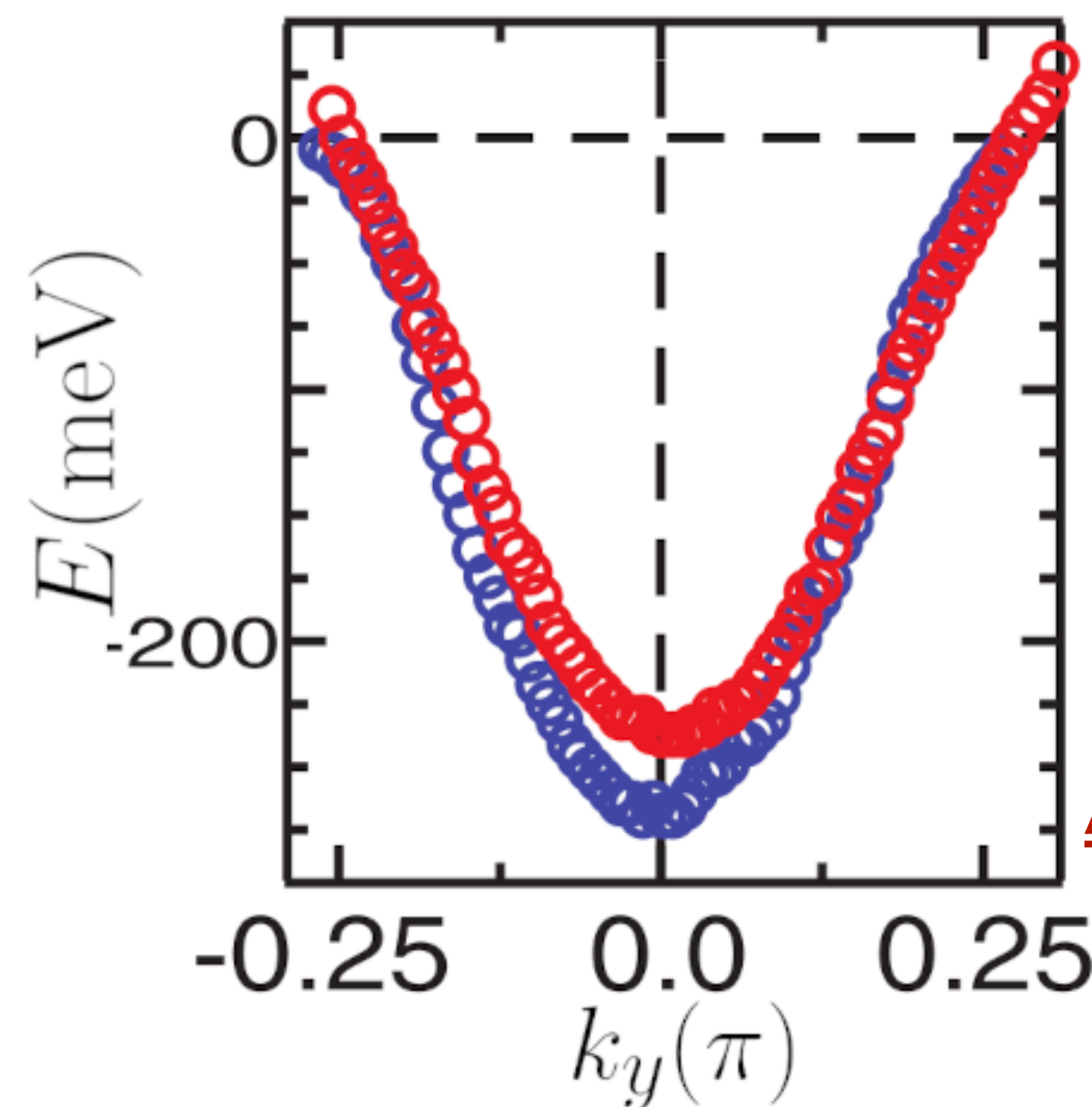
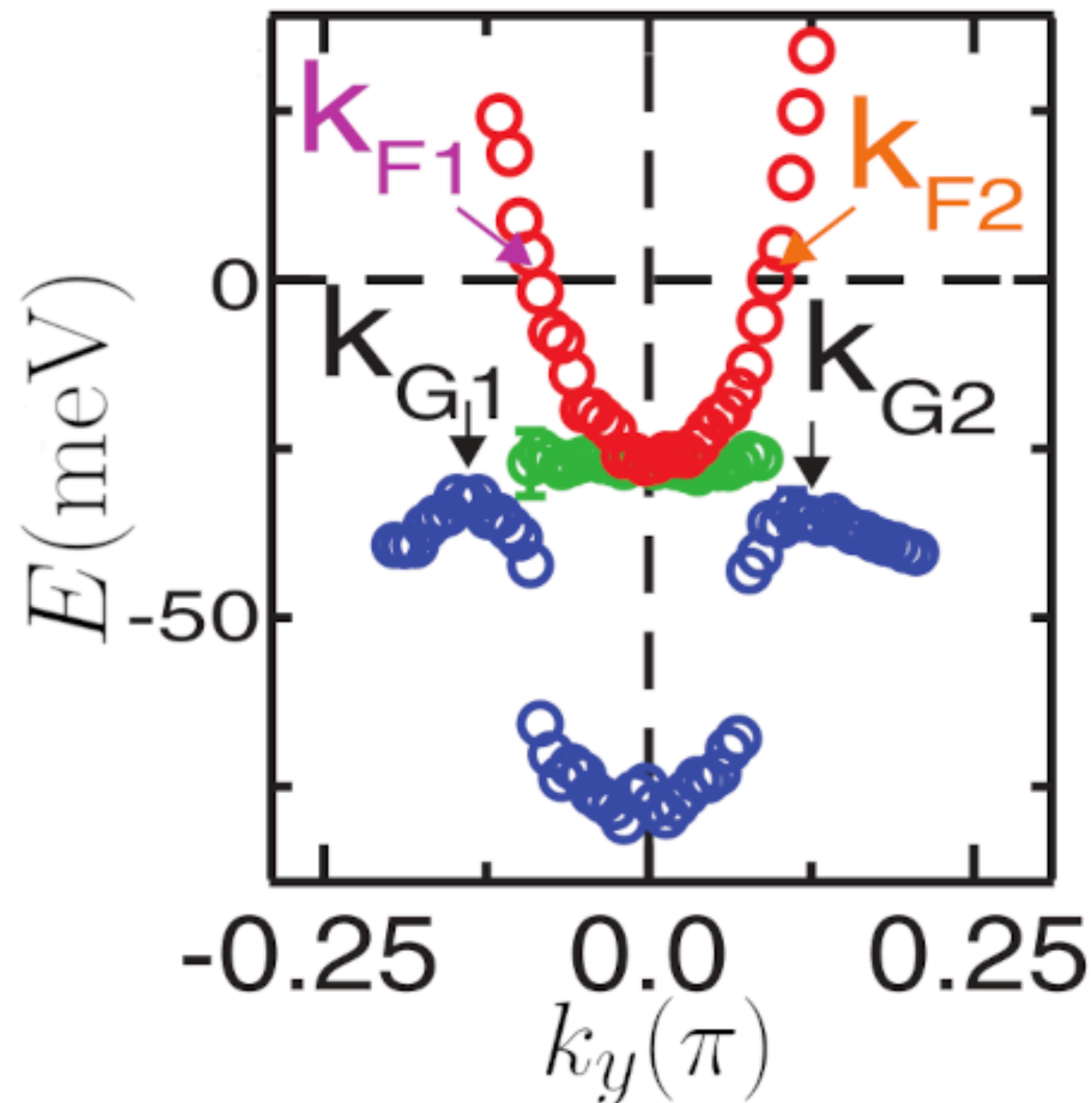
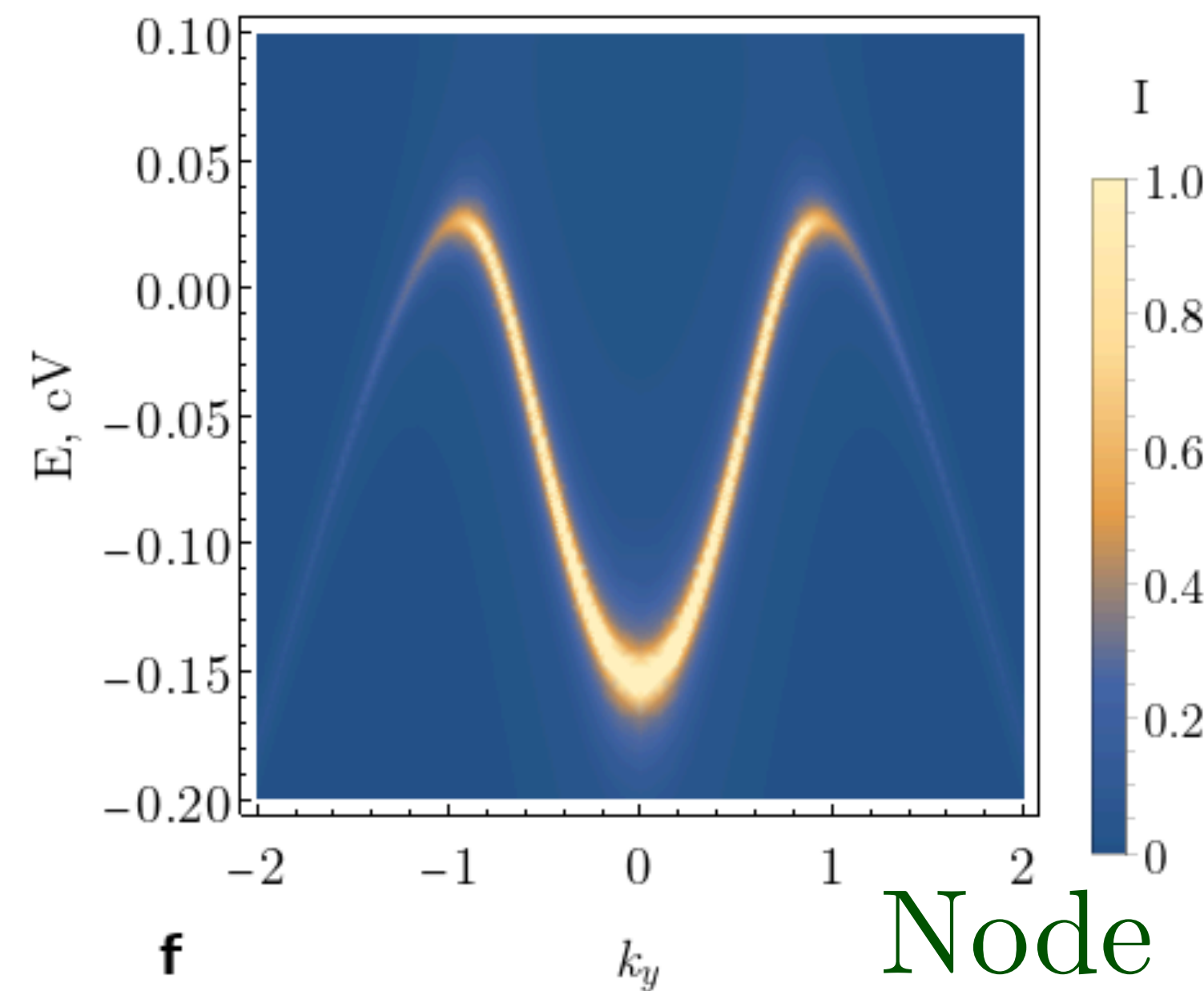
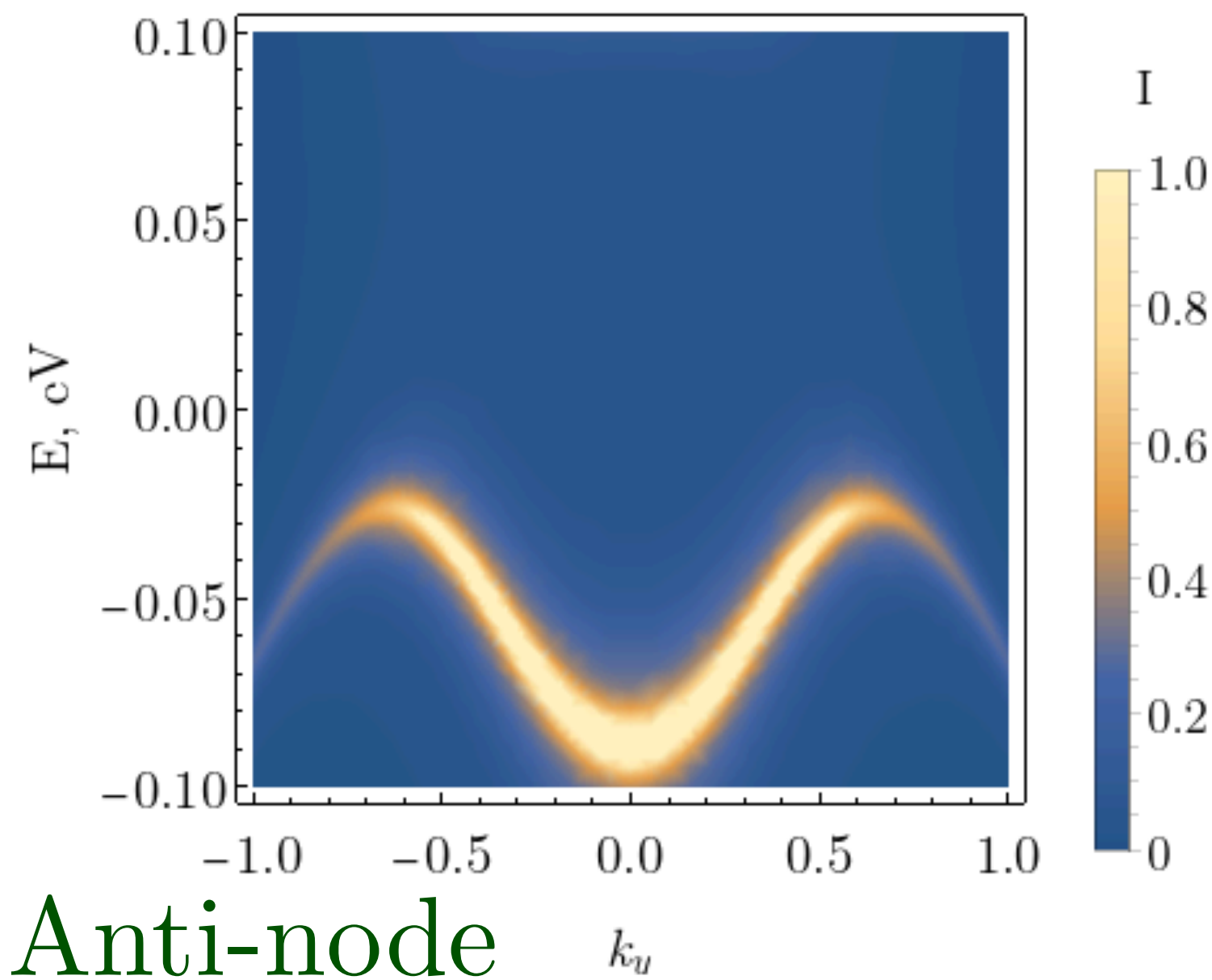
To appear: numerical results from this variational wavefunction

Pseudogap metal =  
Kondo Lattice Heavy  
Fermi Liquid  
 $\oplus$   
Spin Liquid

$|\text{FL}^*\rangle = [\text{Projection onto rung singlets of } \mathbf{S}_1, \mathbf{S}_2]$   
 $\bowtie |\text{Slater determinant of } (c, f_1)\rangle$   
 $\otimes |\text{Spin liquid of } \mathbf{S}_2\rangle$

# FL\* in a one-band model

Broadening by second ancilla layer is needed to describe MDC and EDC

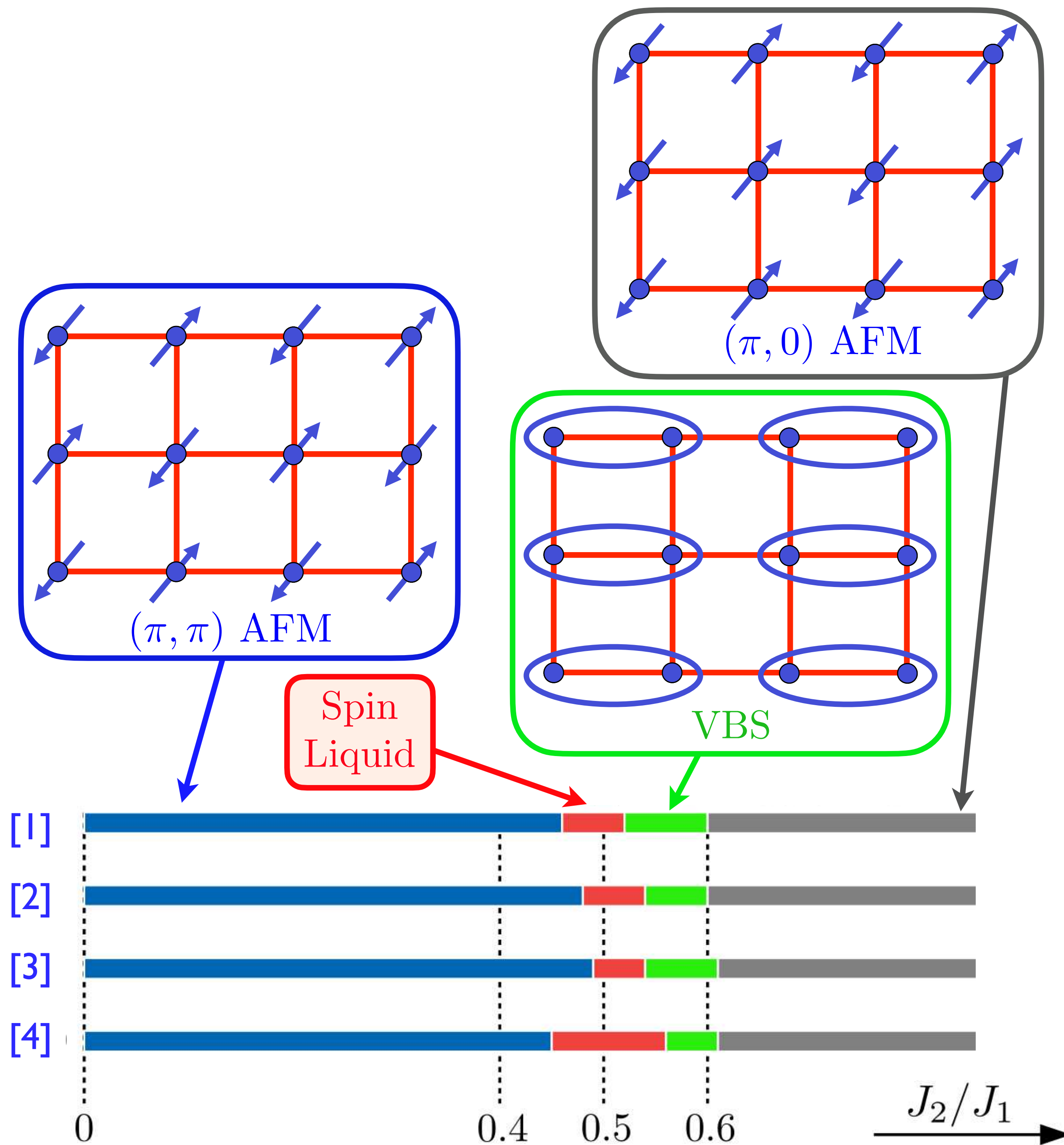


He, Hashimoto, Karapetyan, Koralek, Hinton, Testaud, Nathan, Yoshida, Yao, Tanaka, Meevasana, Moore, Lu, Mo, Ishikado, Eisaki, Hussain, Devereaux, Kivelson, Orenstein, Kapitulnik, and Shen, *Science* **331**, 1579 (2011)

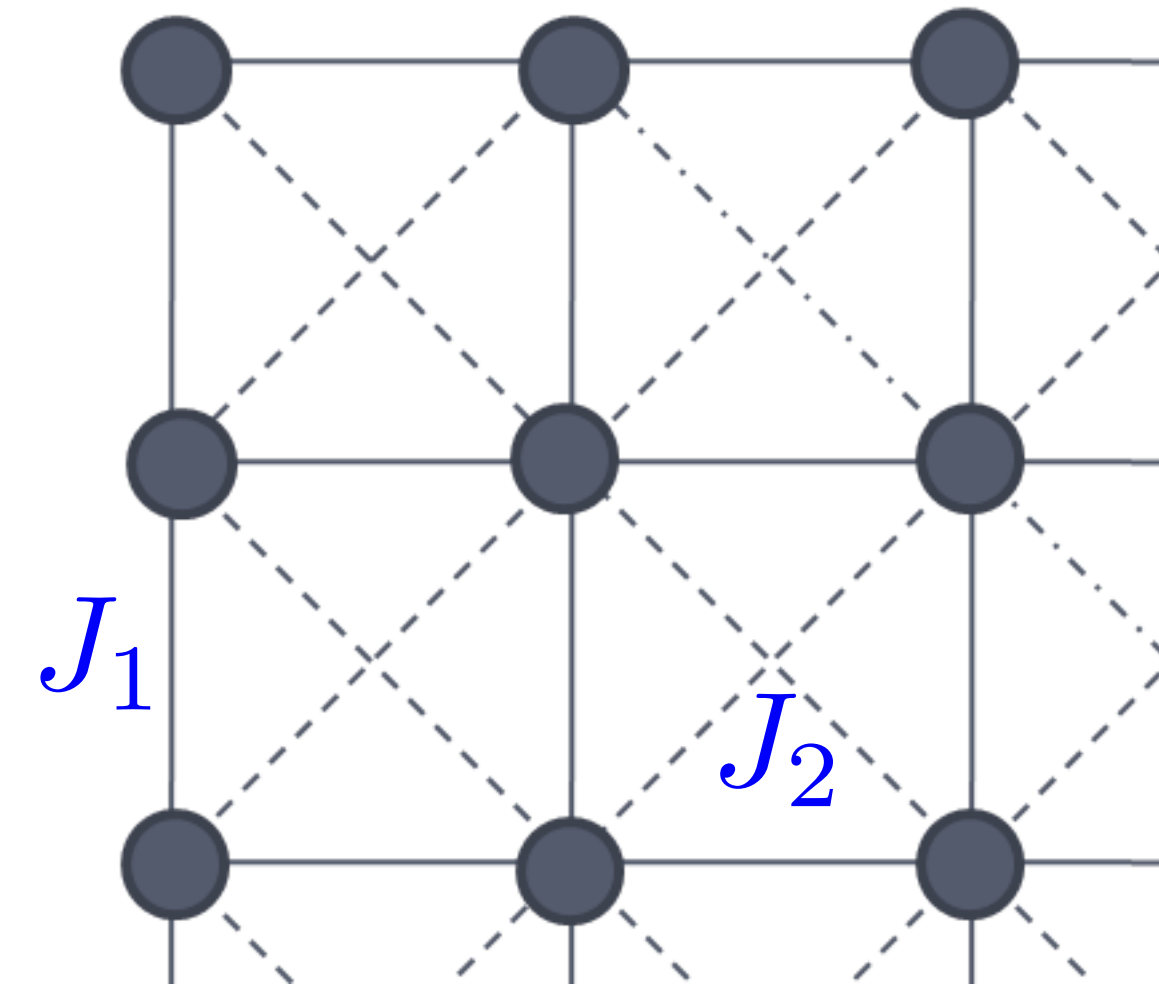
ARPES on Bi2201



The  $CP^1/\pi$ -flux spin liquid

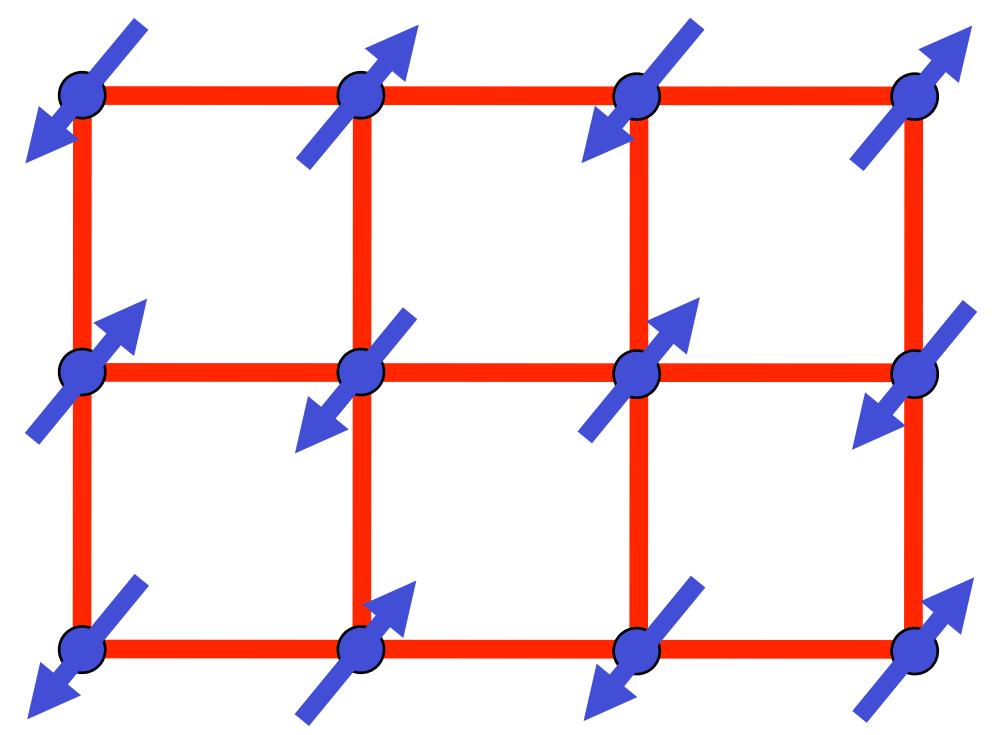


$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



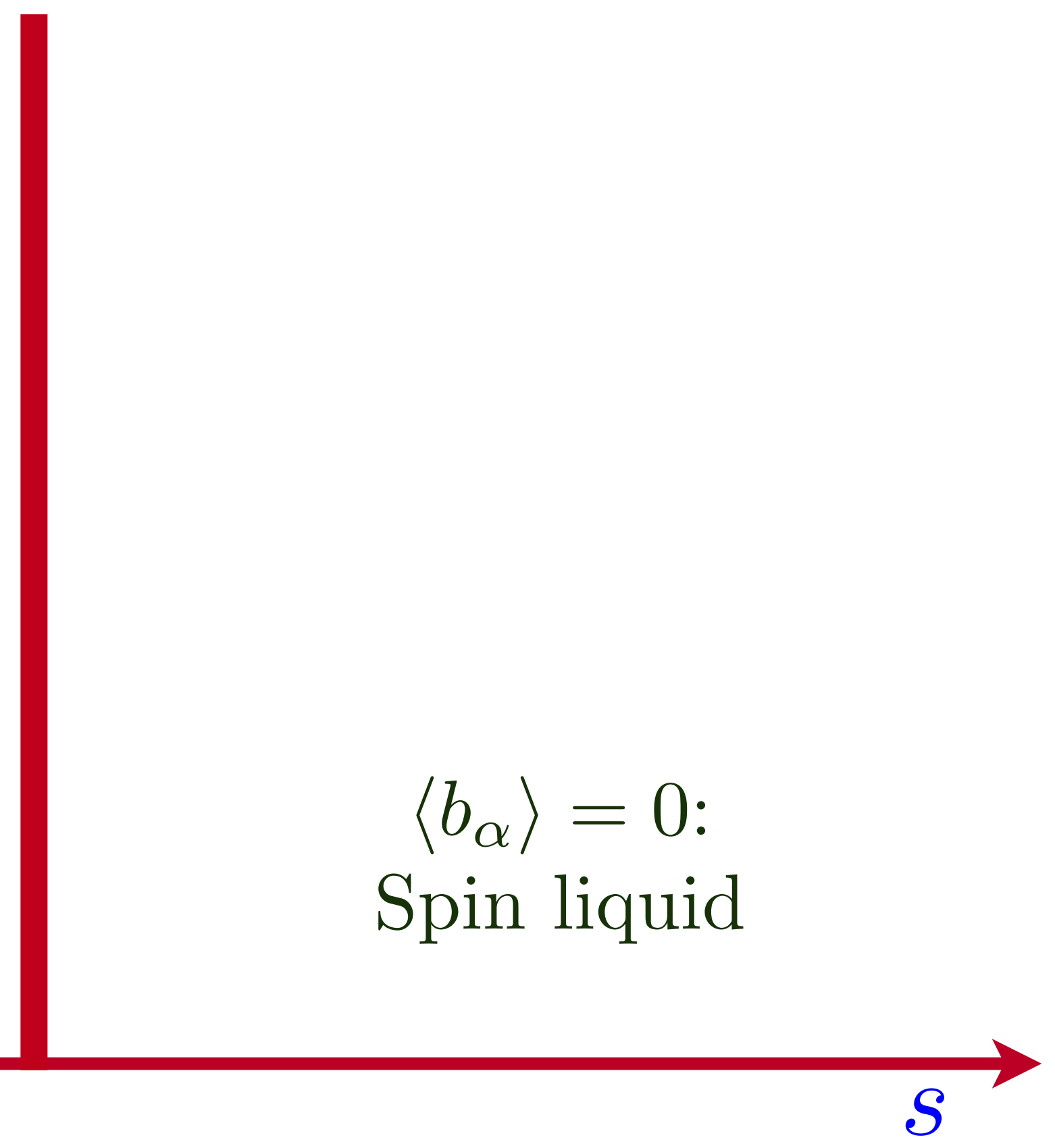
1. L. Wang and A. W. Sandvik, *Phys. Rev. Lett.* **121**, 107202 (2018)
2. F. Ferrari and F. Becca, *Phys. Rev. B* **102**, 014417 (2020)
3. Y. Nomura and M. Imada, *Phys. Rev. X* **11**, 031034 (2021)
4. W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, *Science Bulletin* **67**, 1034 (2022)

Insulating  $S=1/2$  antiferromagnet



$\langle b_\alpha \rangle \neq 0$ :  
Néel order

$\langle b_\alpha \rangle = 0$ :  
Spin liquid



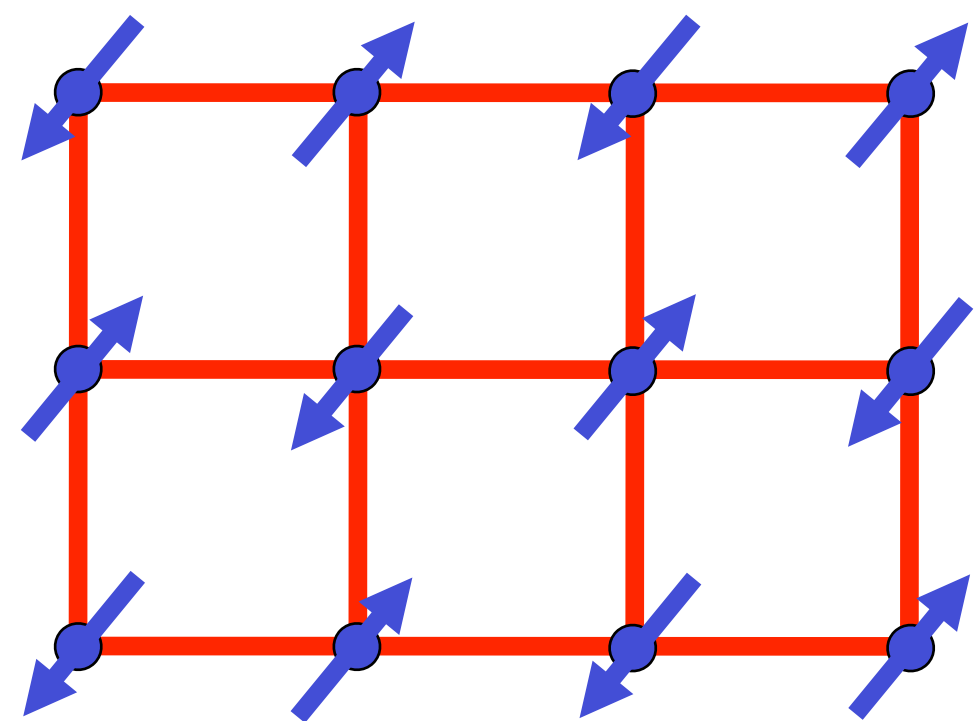
$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Schwinger bosons

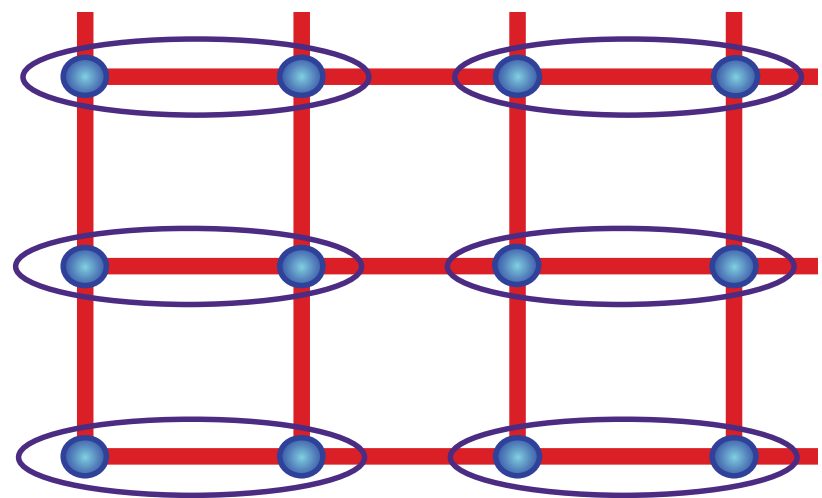
$$\mathbf{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}, \quad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^\dagger b_{i\alpha} = 1$$

Mean-field spin liquid  
with gapped bosonic spinons.

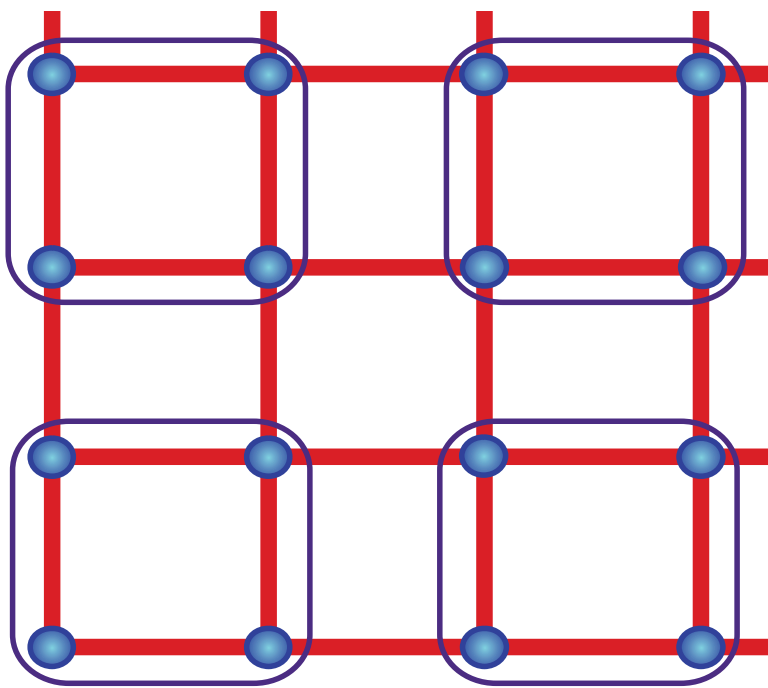
# Insulating $S=1/2$ antiferromagnet



Spin liquid



or



Higgs phase,  $\langle z_\alpha \rangle \neq 0$ :  
Néel order

Confining phase,  $\langle z_\alpha \rangle = 0$ :  
VBS order



$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Schwinger bosons

$$\mathbf{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}, \quad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^\dagger b_{i\alpha} = 1$$

Mean-field spin liquid  
with gapped bosonic spinons.

Low energy  $\mathbb{C}P^1$  U(1) gauge theory

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}$$

$$\mathcal{L} = |(\partial_\mu - ia_\mu)z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

# $\mathbb{C}P^1$ U(1) gauge theory

$S=1/2$   
square  
lattice anti-  
ferromagnet

$\mathbb{C}P^1$  U(1) gauge theory

$S=1/2$   
square  
lattice anti-  
ferromagnet

SU(2) gauge theory of  $N_f = 2$   
fundamental, massless, Dirac fermions.

Obtained from a saddle-point of  
fermionic spinons moving in  $\pi$ -flux.

I. Affleck and J.B. Marston, *Phys. Rev. B* **37**, 3774 (1988)

SO(5) non-linear  $\sigma$ -model  
of Néel/VBS orders  
with  $k = 1$  WZW term

$\mathbb{CP}^1$  U(1) gauge theory

$S=1/2$   
square  
lattice anti-  
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SU(2) gauge theory of  $N_f = 2$   
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SO(5) non-linear  $\sigma$ -model  
of Néel/VBS orders  
with  $k = 1$  WZW term

Many numerical works show that deconfined critical theory applies over a substantial length scale, but ultimately confines at the longest distances.

Anders W. Sandvik *Phys. Rev. Lett.* **98**, 227202 (2007)

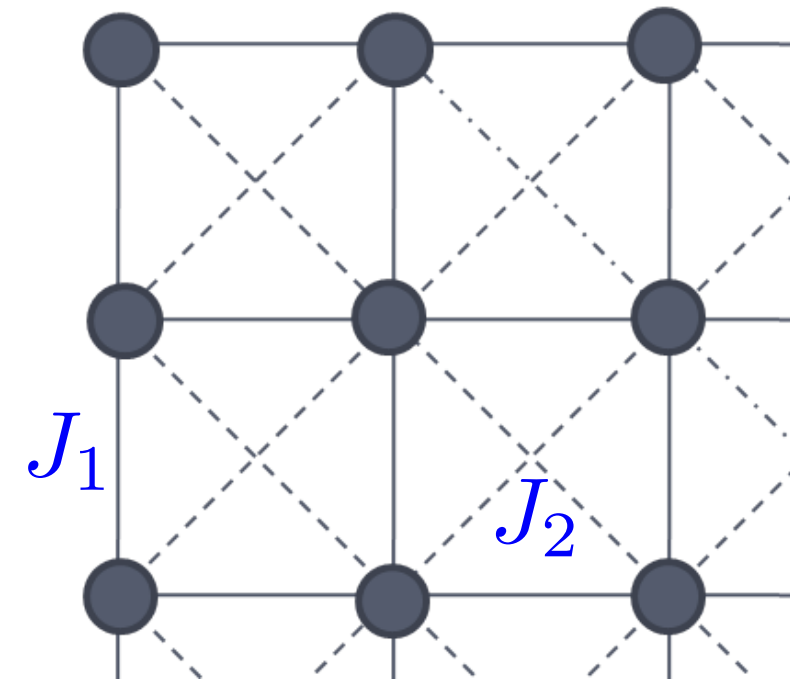
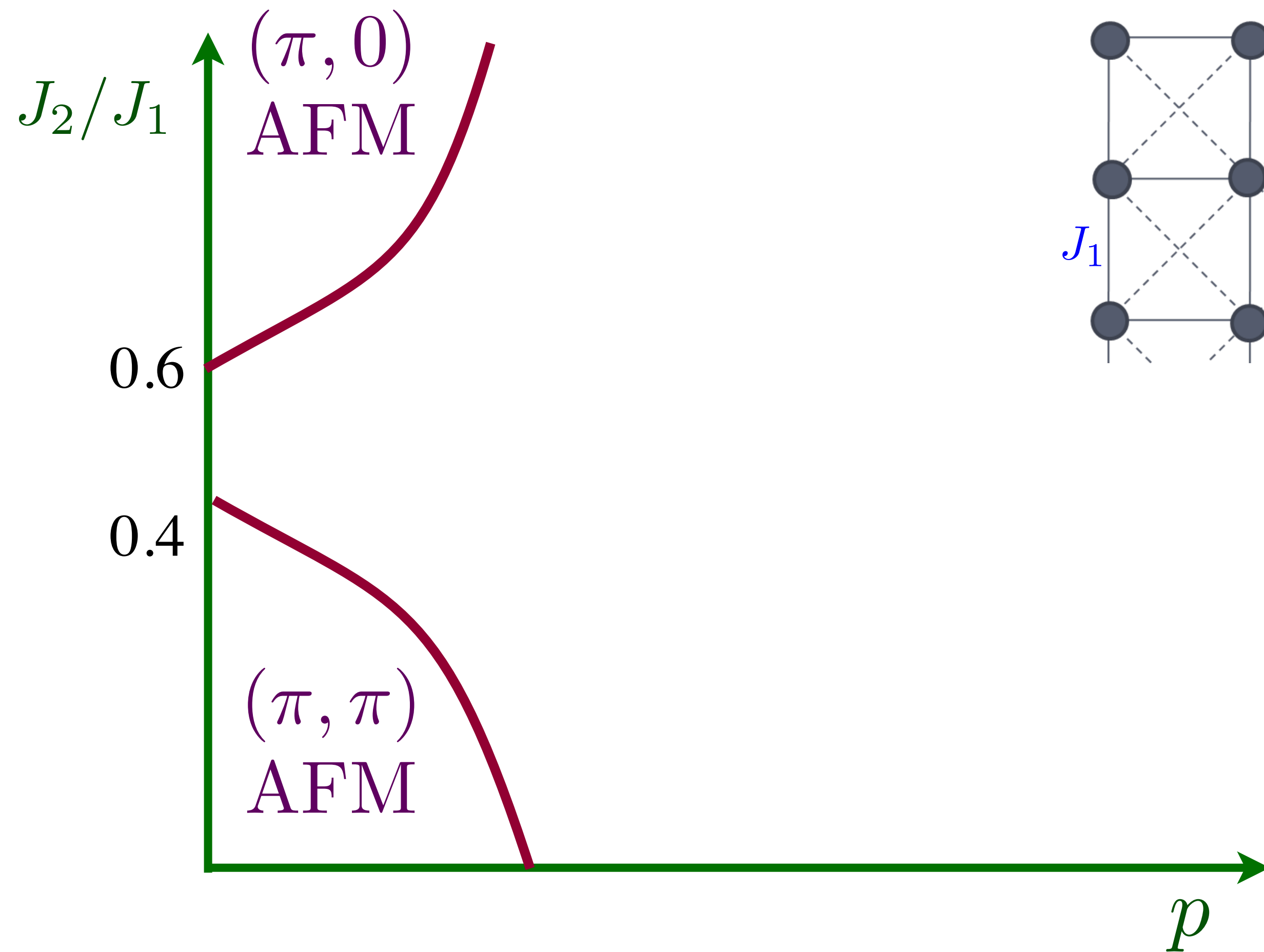
Adam Nahum, P. Serna, J. T. Chalker, M. Ortuño, and A. M. Somoza, *Phys. Rev. Lett.* **115**, 267203 (2015)

Z. Zhou, L. Hu, W. Zhu, and Yin-Chen He, arXiv:2306.16435

# High Temperature Superconductivity in a Lightly Doped Quantum Spin Liquid

Hong-Chen Jiang <sup>1,\*</sup> and Steven A. Kivelson <sup>2</sup>

*Phys. Rev. Lett.* **127**, 097002 (2021)



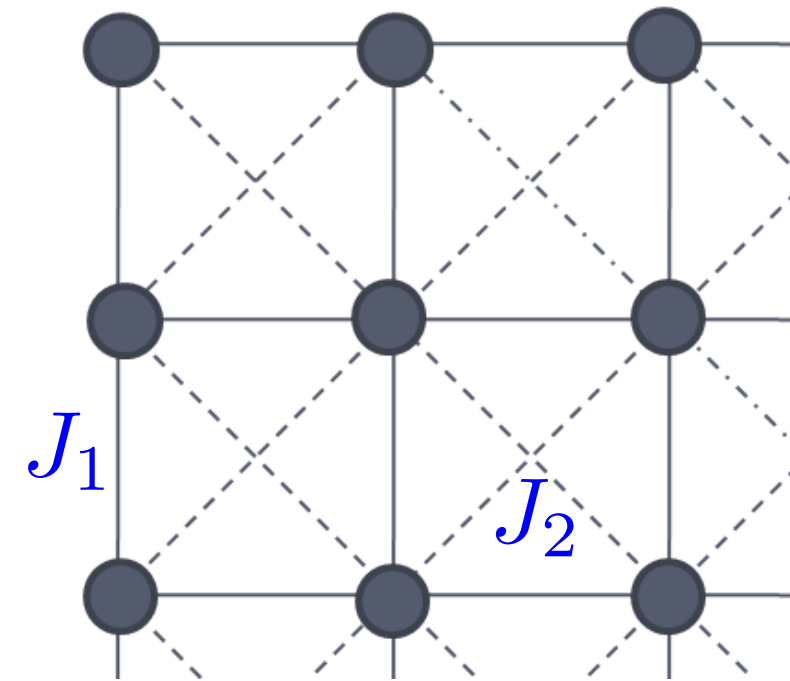
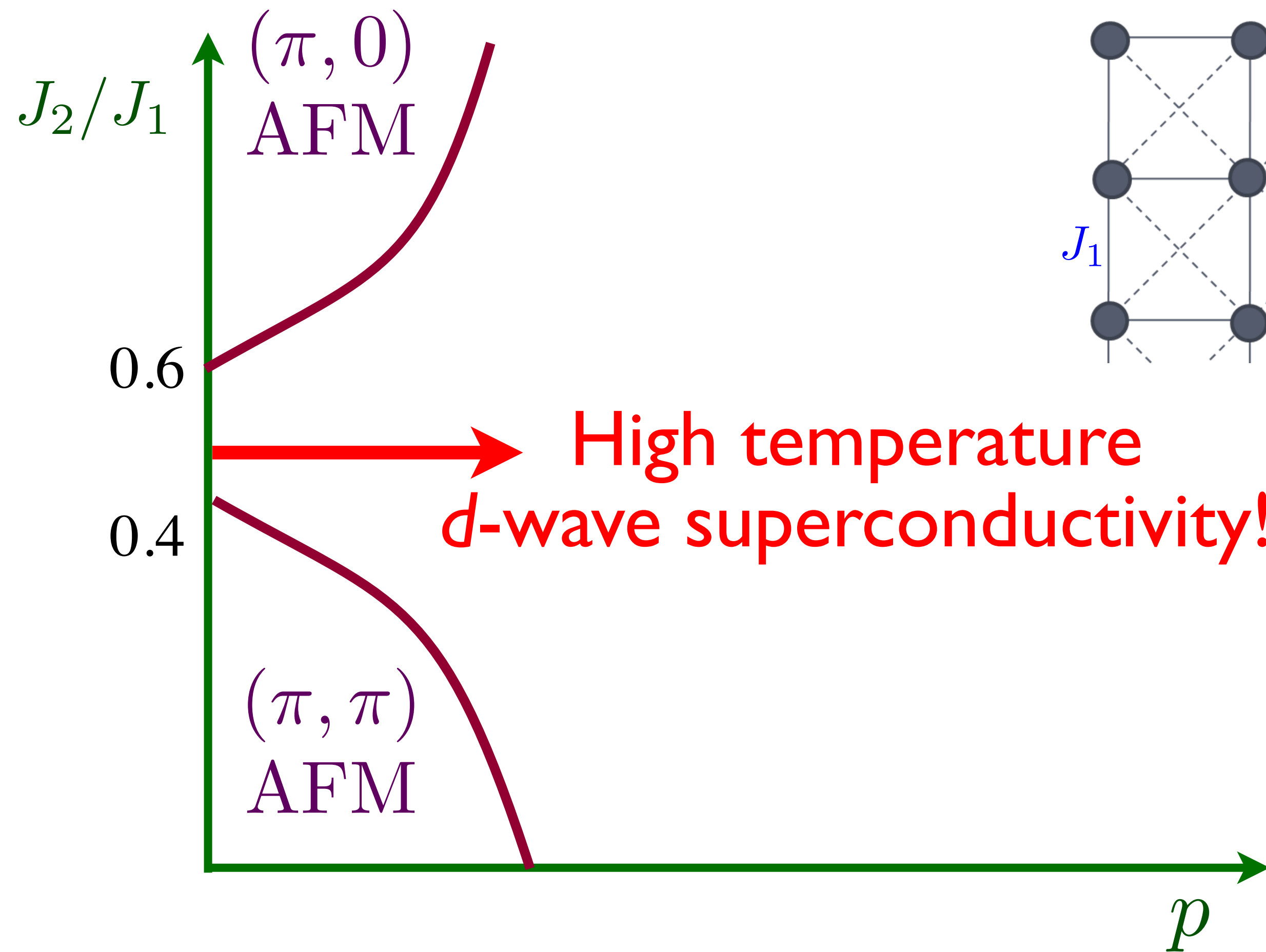
Superconducting valence bond fluid in  
lightly doped 8-leg  $t$ - $J$  cylinders  
Hong-Chen Jiang, Steven A. Kivelson, and  
Dung-Hai Lee, arXiv:2302.11633

Upon increasing the cylinder width from 4 to 8, we observed a significant strengthening of the quasi-long-range superconducting correlations, and a dramatic suppression of any “competing” charge-density-wave order. Extrapolating from the observed behavior of the width 8 cylinders, we speculate that the system has a nodeless d-wave superconducting ground-state in the 2D limit.

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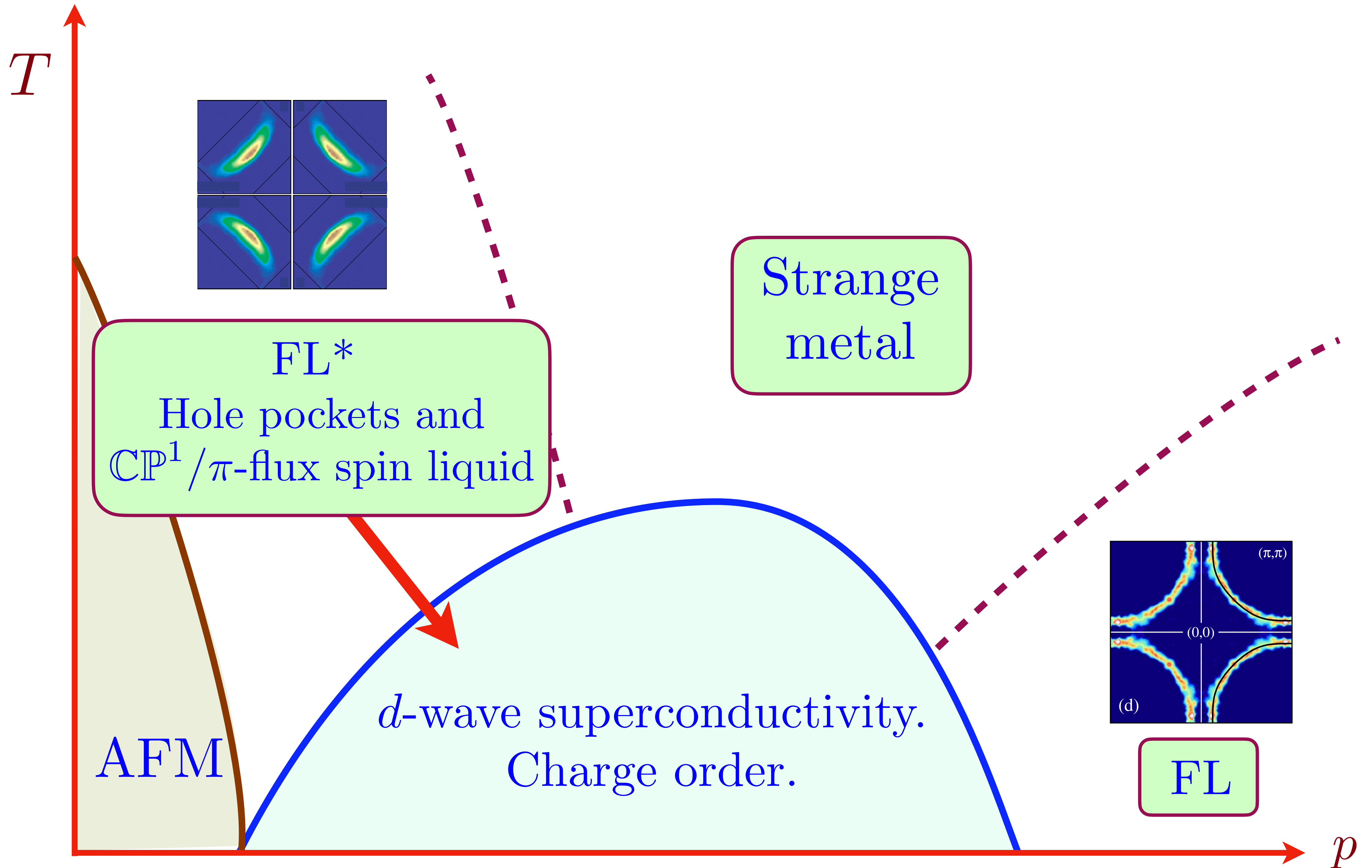
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From  $CP^1/\pi$ -flux  $FL^*$

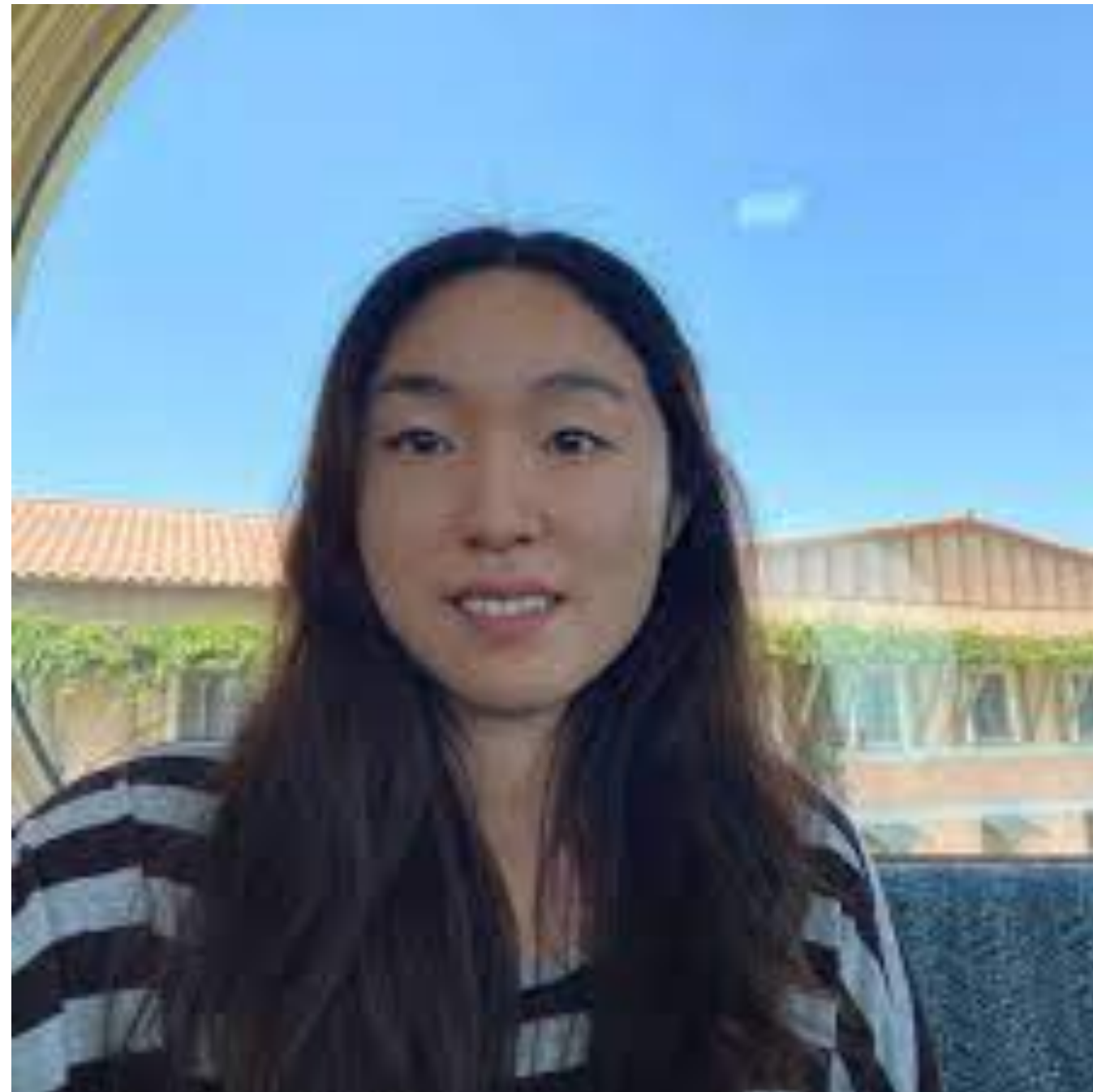
to

d-wave superconductivity

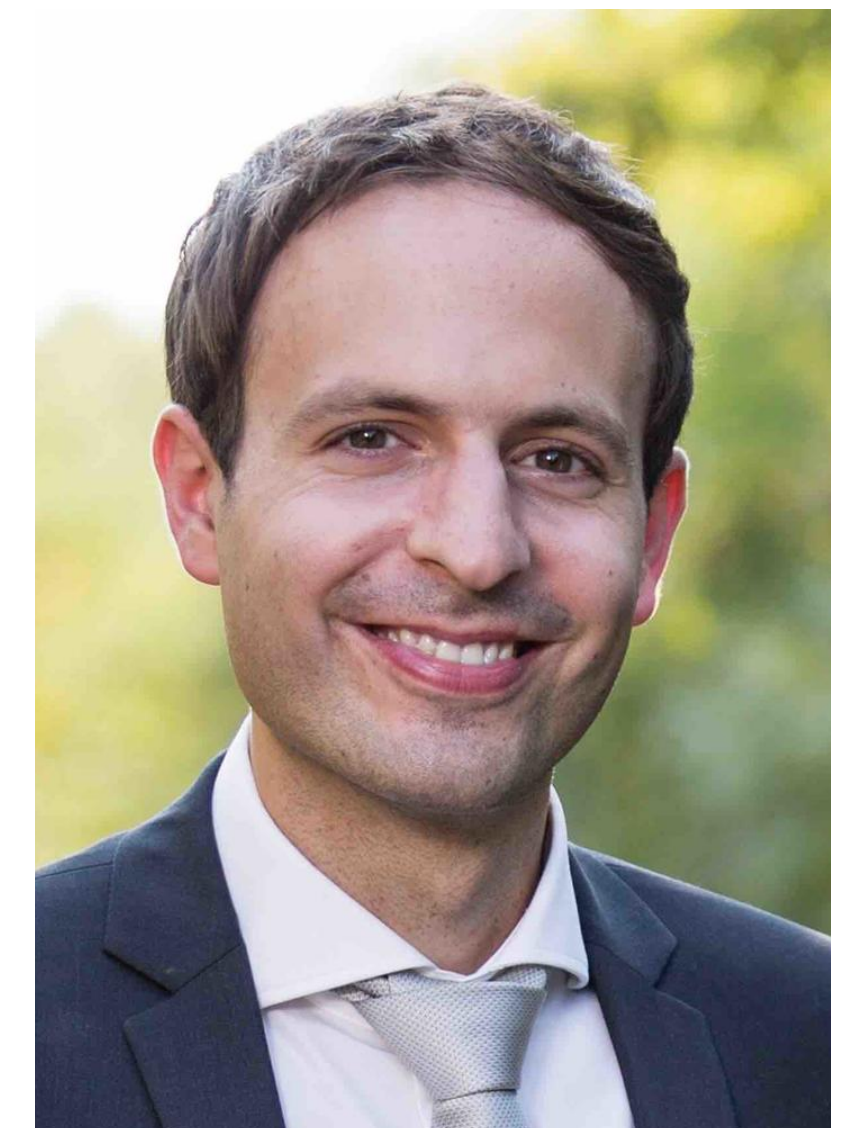




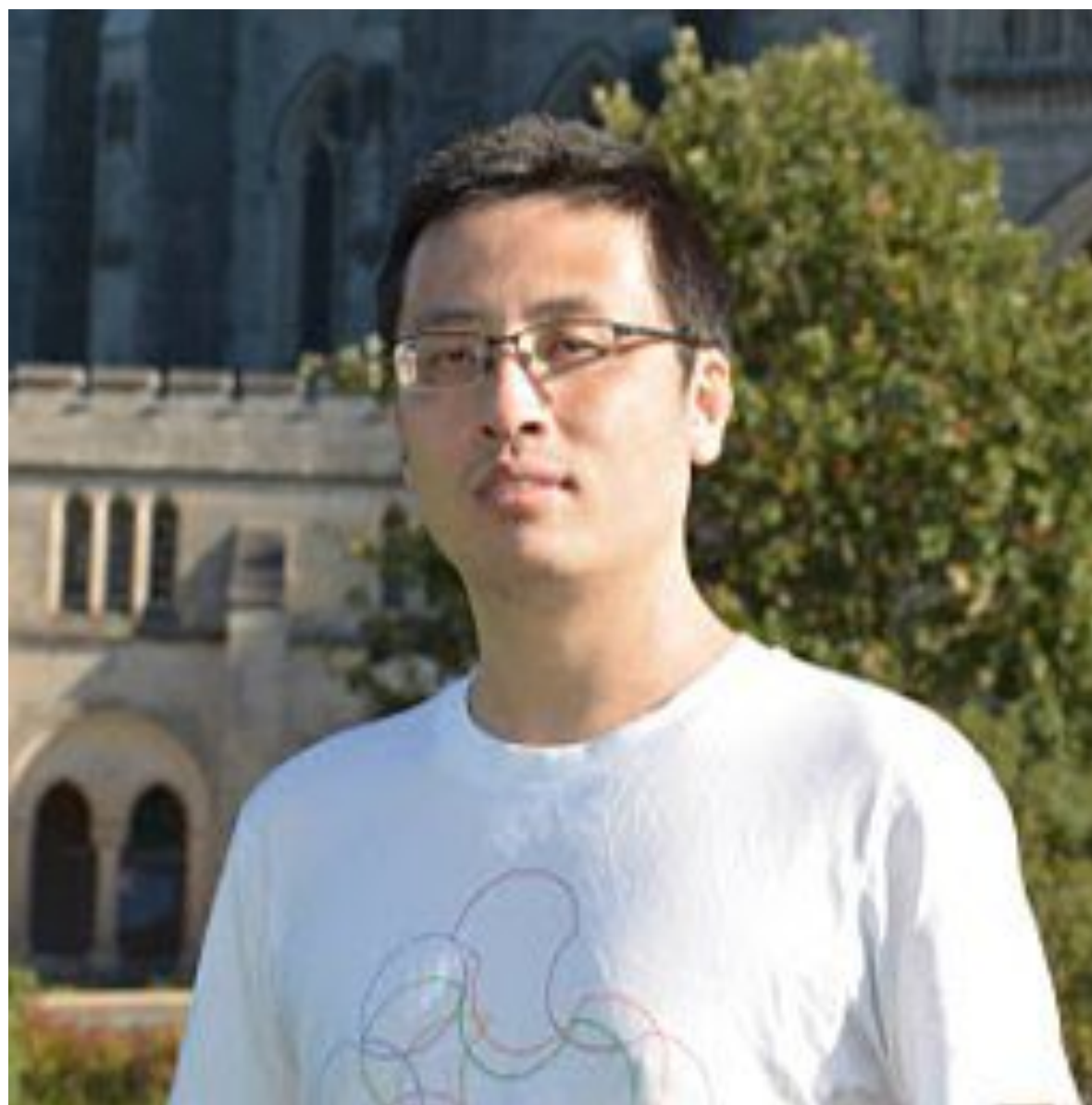
**Maine Christos**



**Zhu-Xi Luo**



**Mathias Scheurer**



**Ya-Hui Zhang**

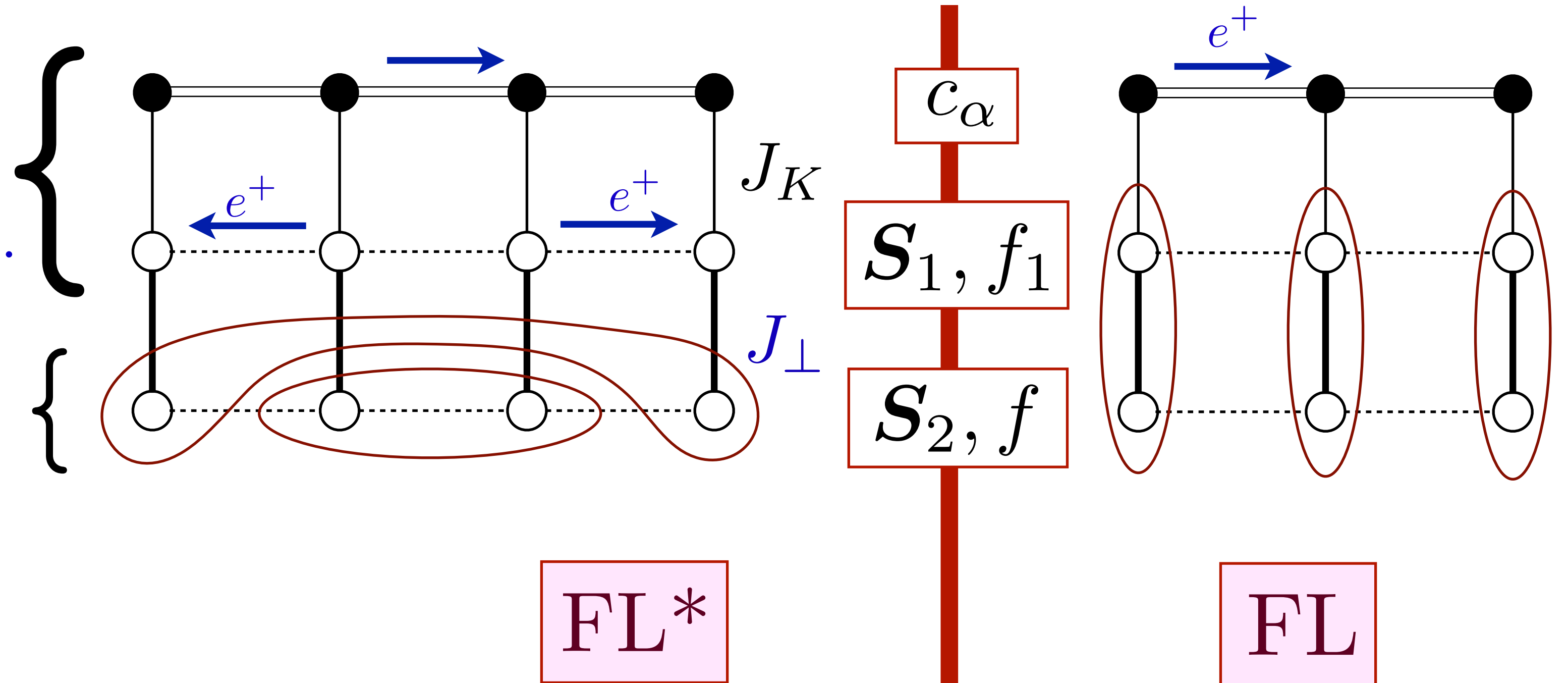


**Henry Shackleton**

# Ancilla theory of the Hubbard model

Kondo lattice  
heavy Fermi liquid.  
Size  $1 + p + 1 = p \pmod{2}$ .

$\pi$ -flux spin liquid  
of  $f_\alpha$  with  $SU(2)_N$   
gauge field



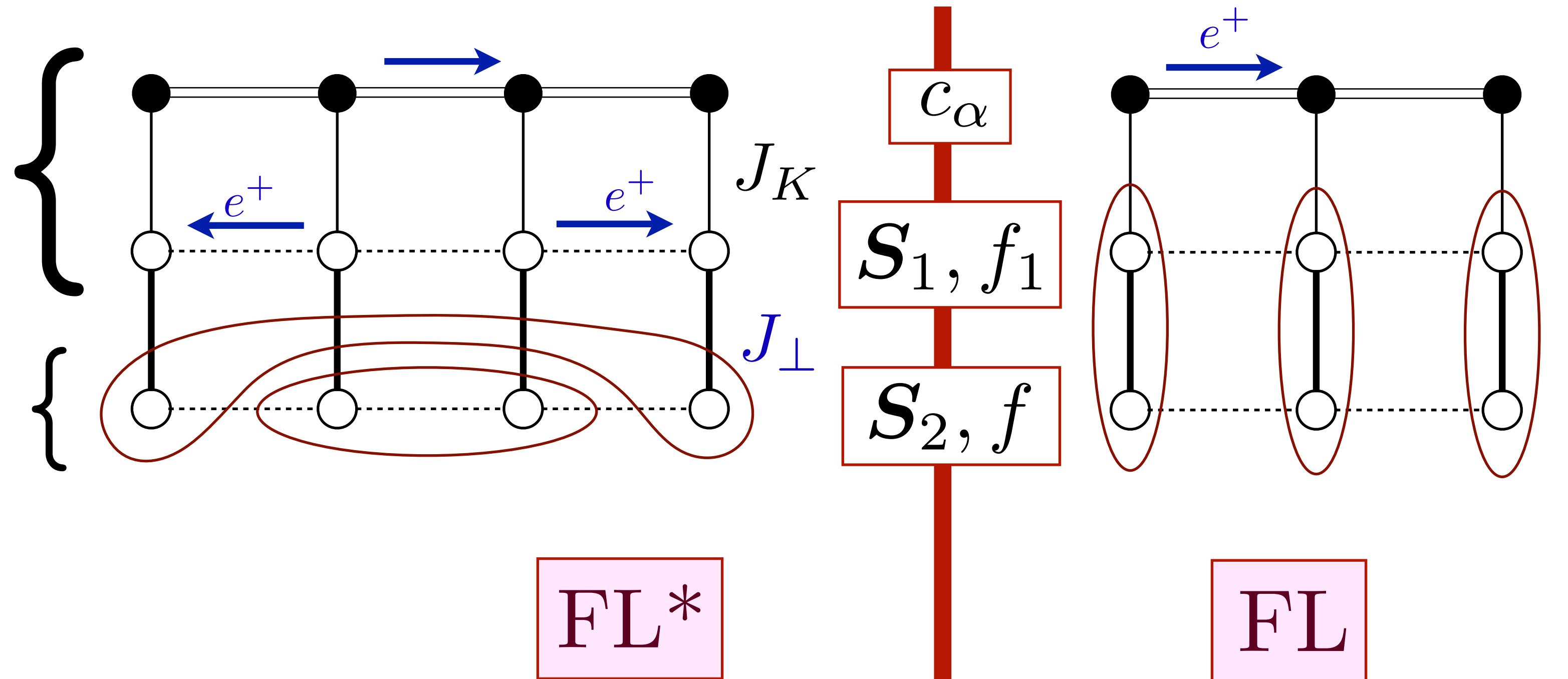
$J_K$

doping  $p$

# Ancilla theory of the Hubbard model

Higgs field 1  
 $\Phi \sim c_\alpha^\dagger f_{1\alpha}$   
 $\langle \Phi \rangle \neq 0$

$\pi$ -flux spin liquid  
of  $f_\alpha$  with  $SU(2)_N$   
gauge field



$J_K$

doping  $p$

Higgs field  $\Phi$  drives  $FL^*$ -strange metal- $FL$

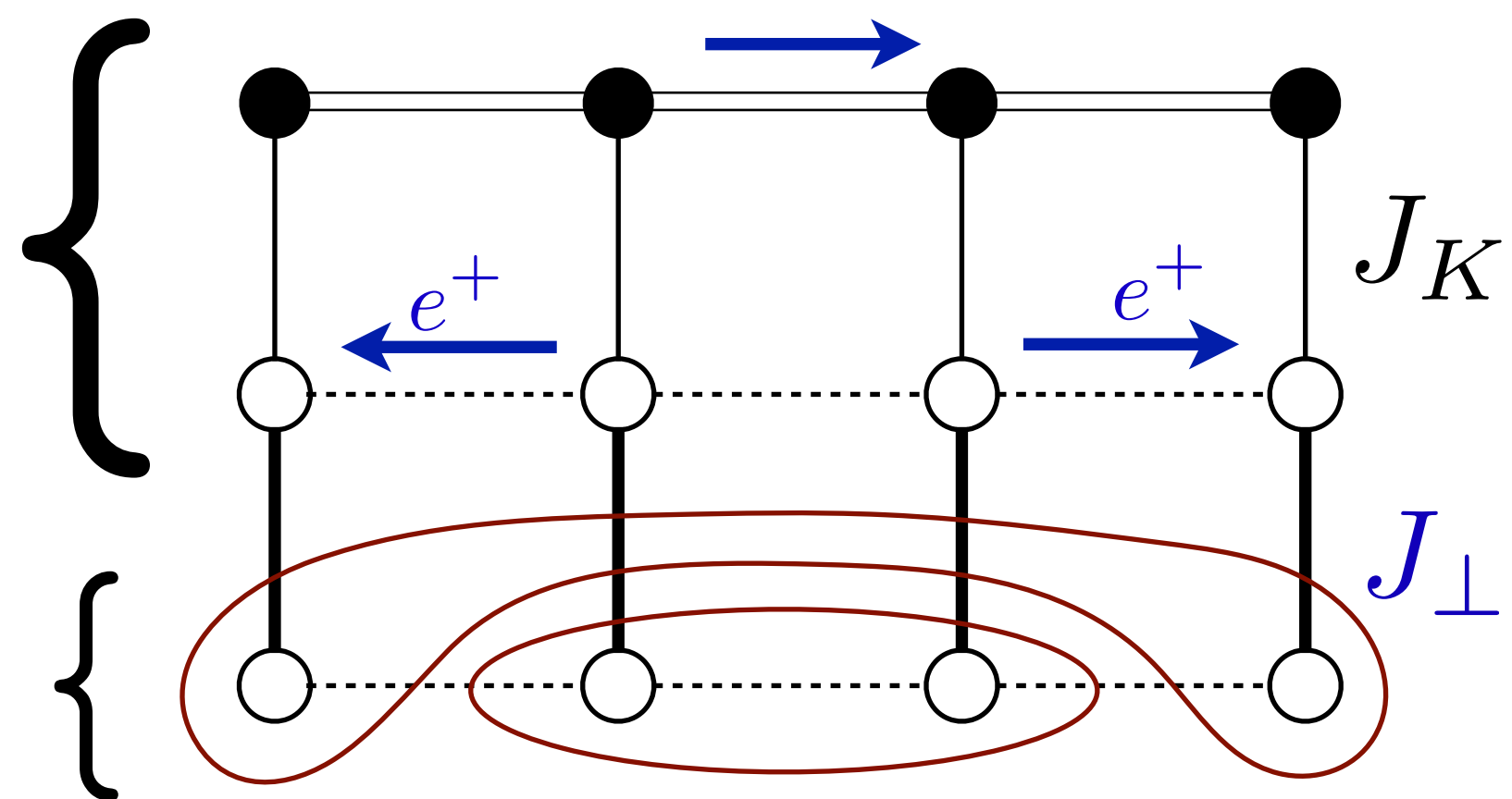
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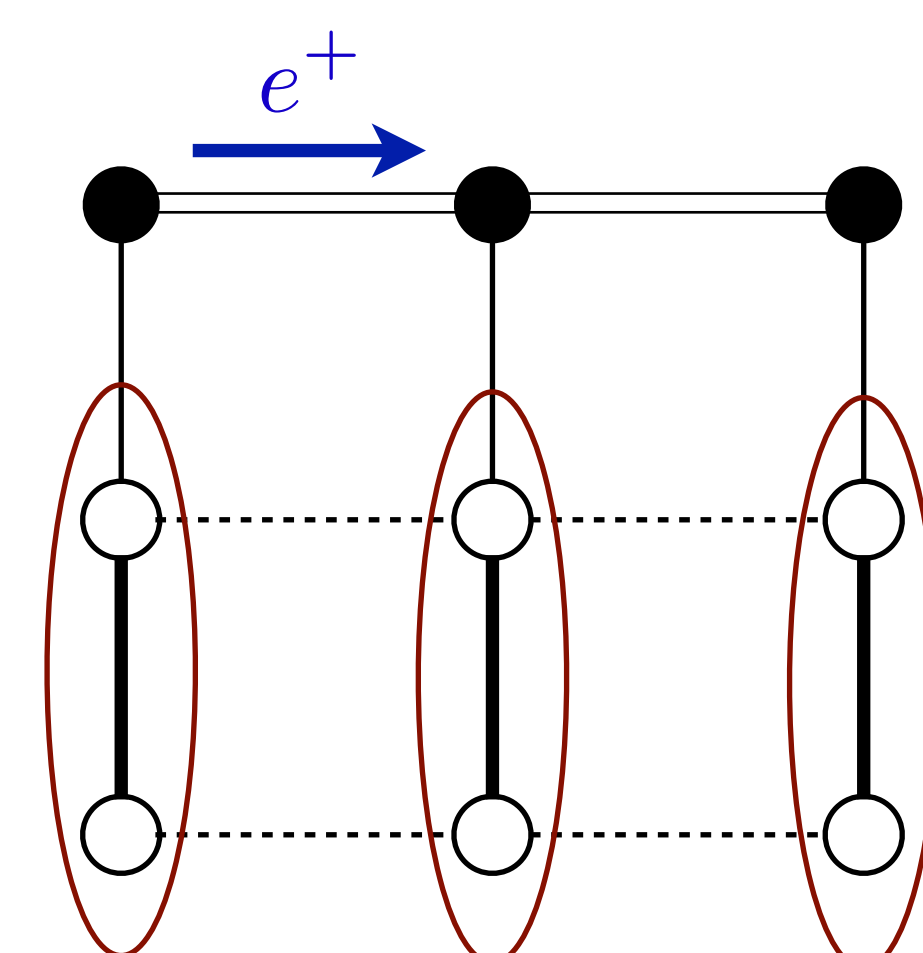
$\pi$ -flux spin liquid  
of  $f_\alpha$  with  $SU(2)_N$   
gauge field



$c_\alpha$

$S_1, f_1$

$S_2, f$



FL\*

FL

Higgs field 2  
Charge  $e$ ,  $SU(2)_N$  fundamental

$$B \sim \begin{pmatrix} f_{1\alpha}^\dagger f_\alpha \\ \varepsilon_{\alpha\beta} f_{1\alpha}^\dagger f_\beta^\dagger \end{pmatrix}$$

$J_K$

doping  $p$

$B$  has same quantum numbers as boson in X.-G. Wen and P.A. Lee, *Phys. Rev. Lett.* **76**, 503 (1996)

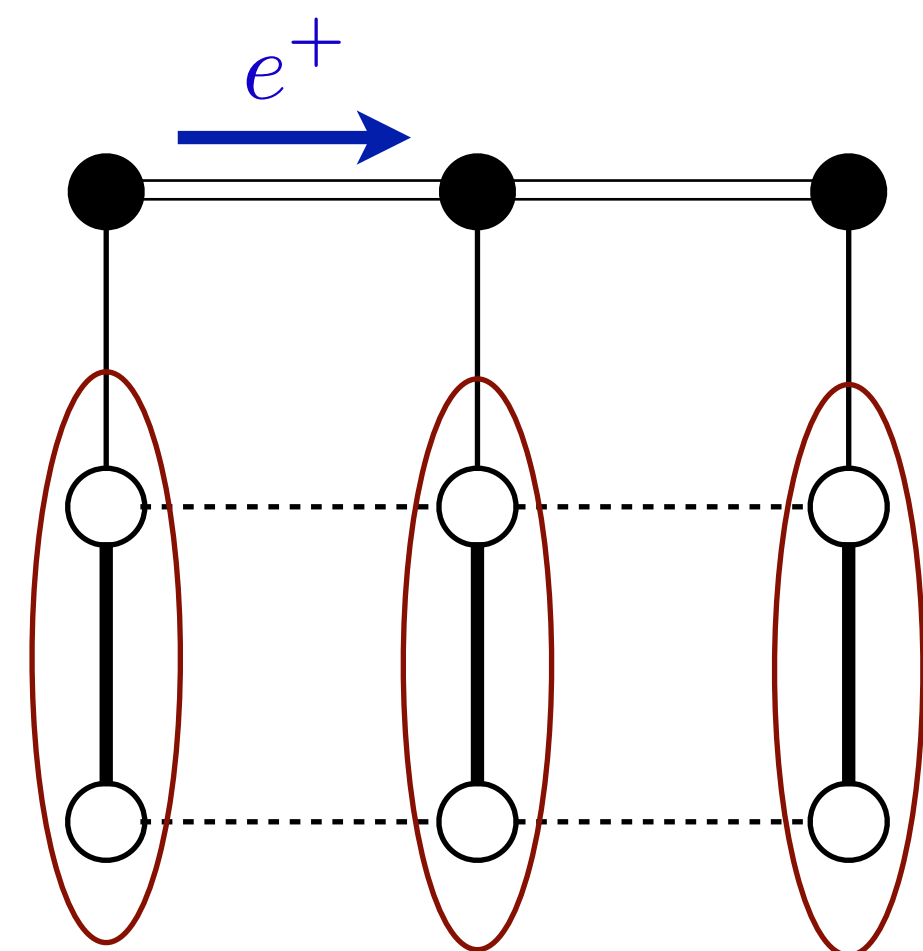
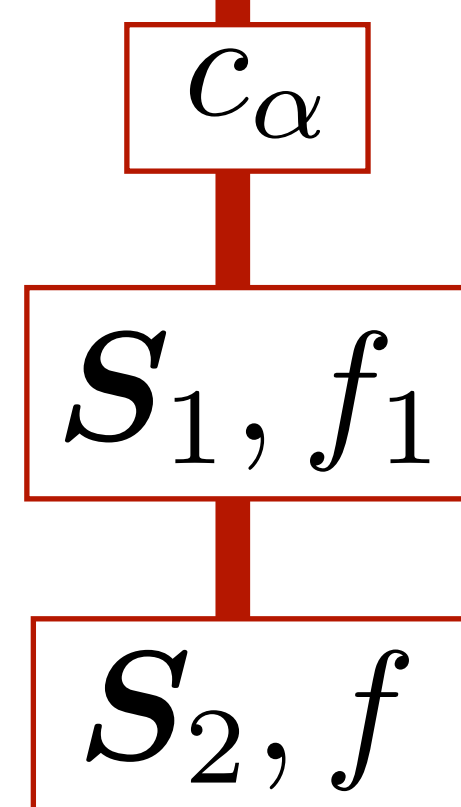
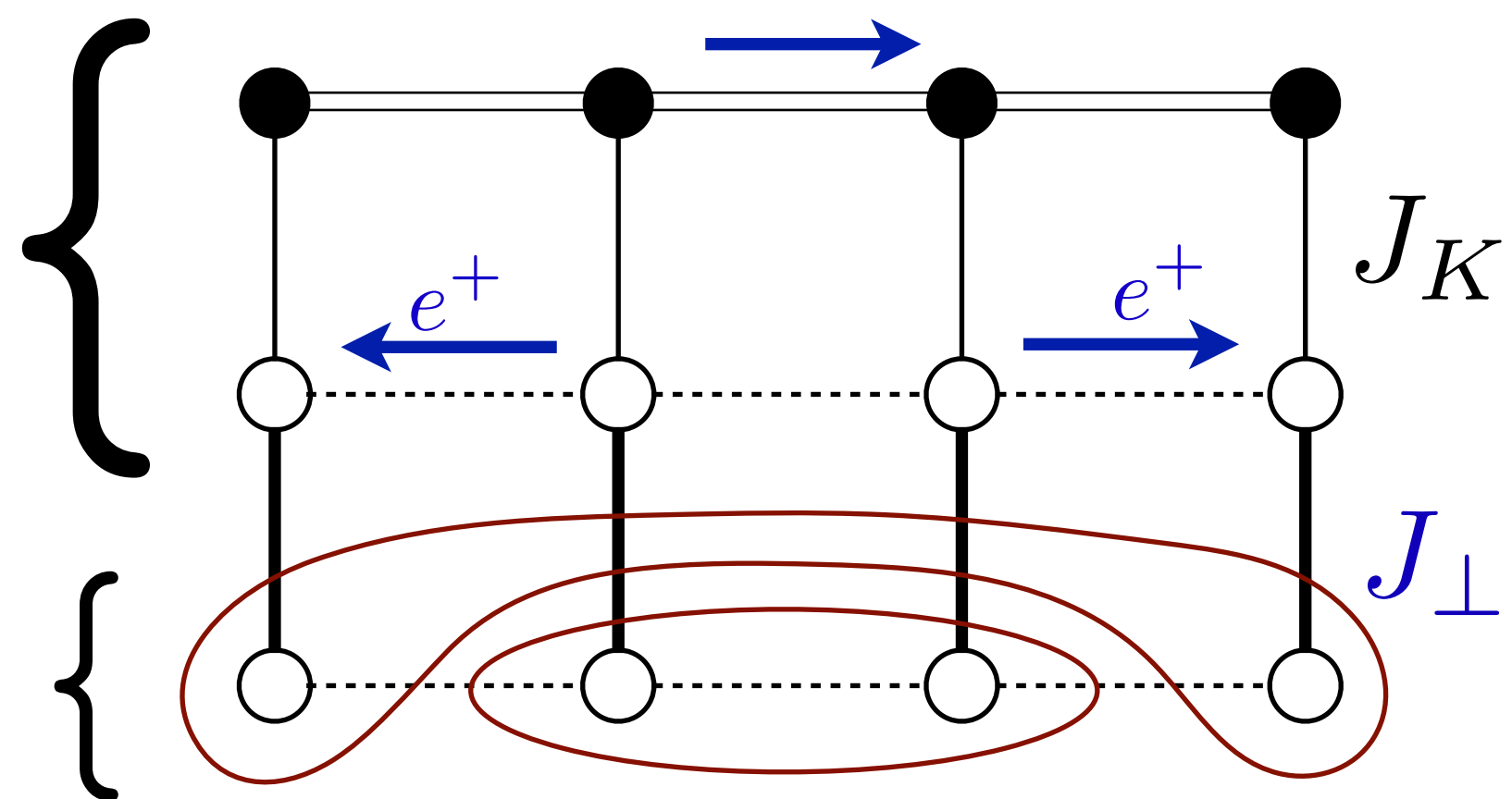
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gauge field



FL\*

FL

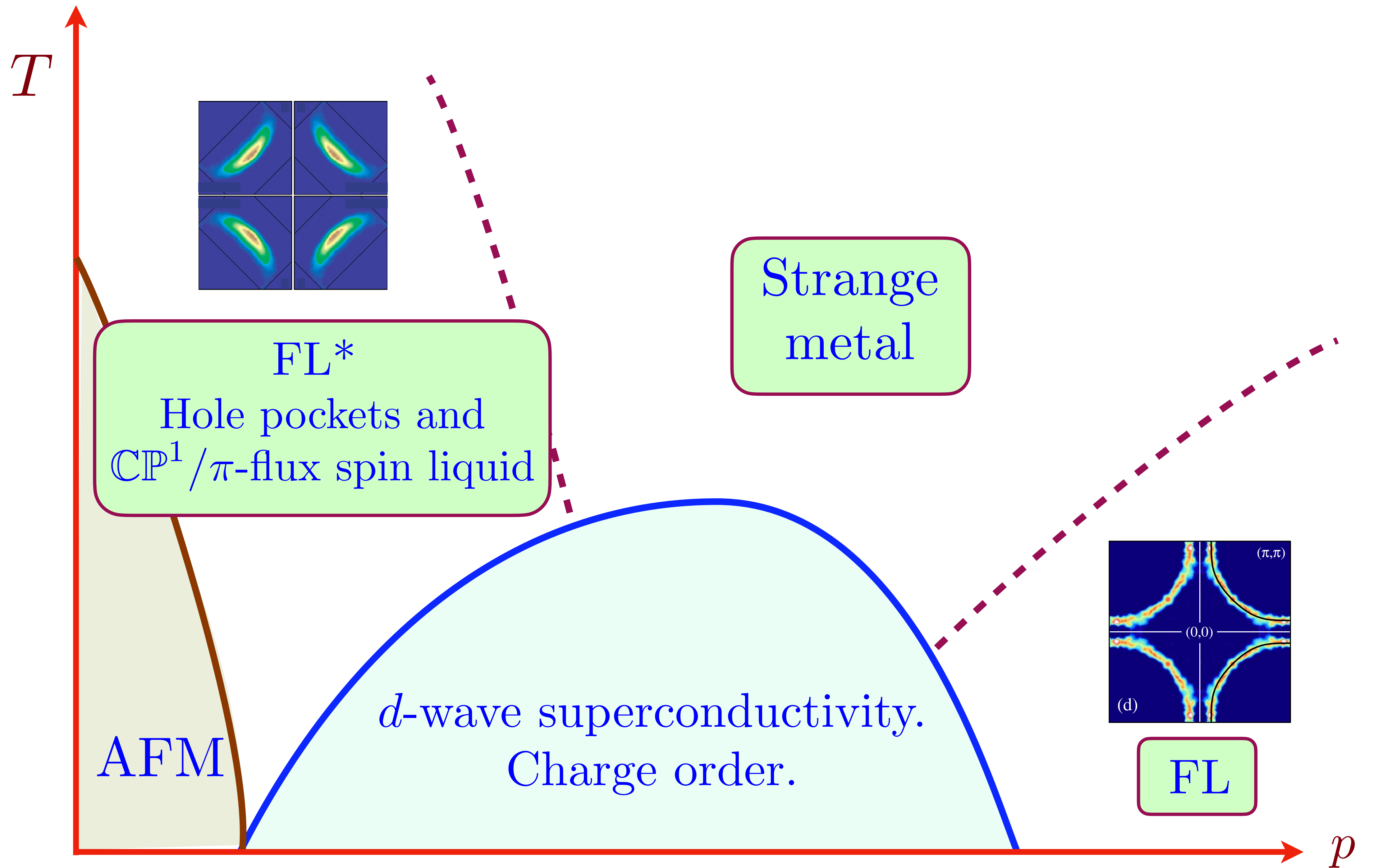
Higgs field 2  
Charge  $e$ ,  $SU(2)_N$  fundamental

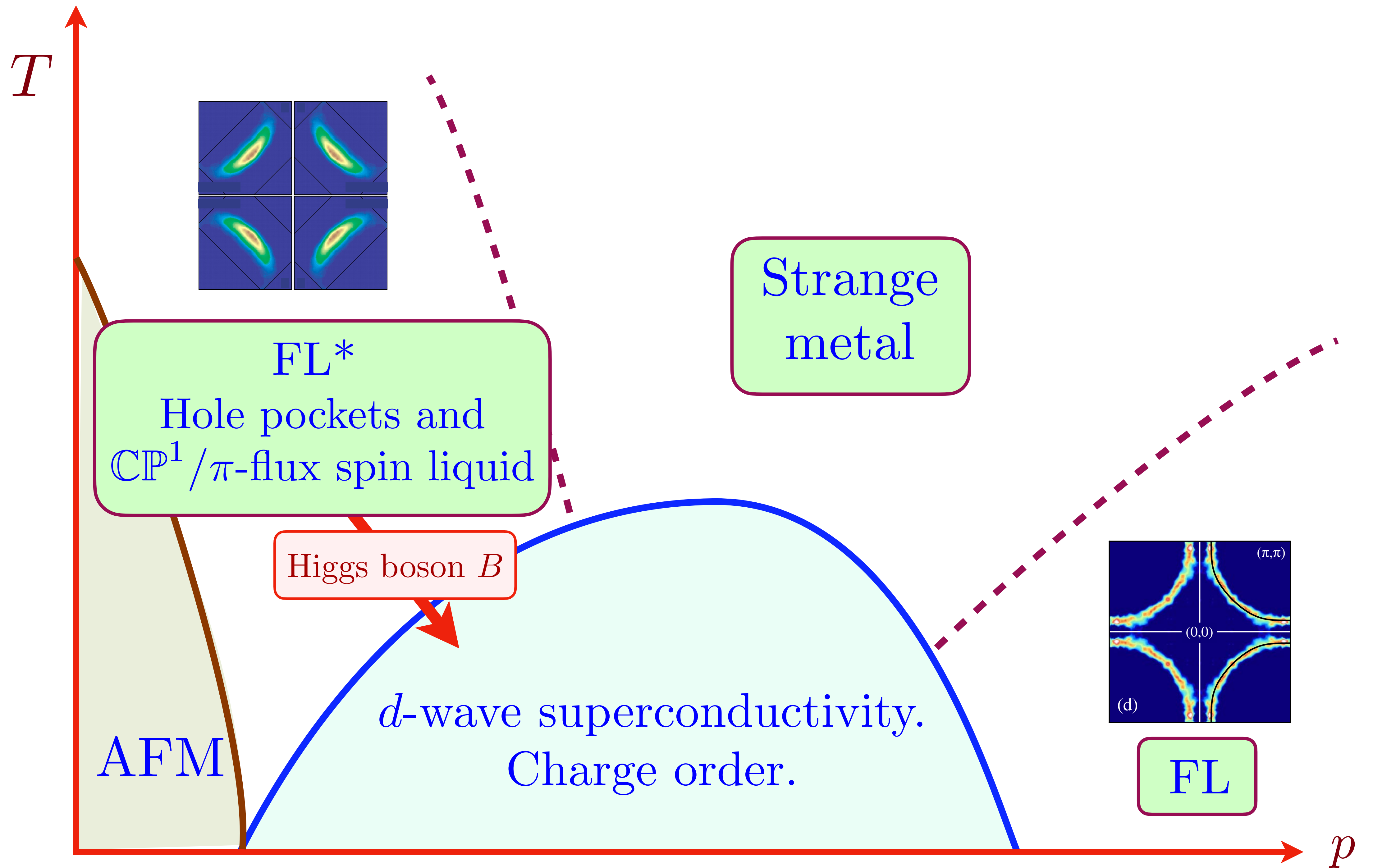
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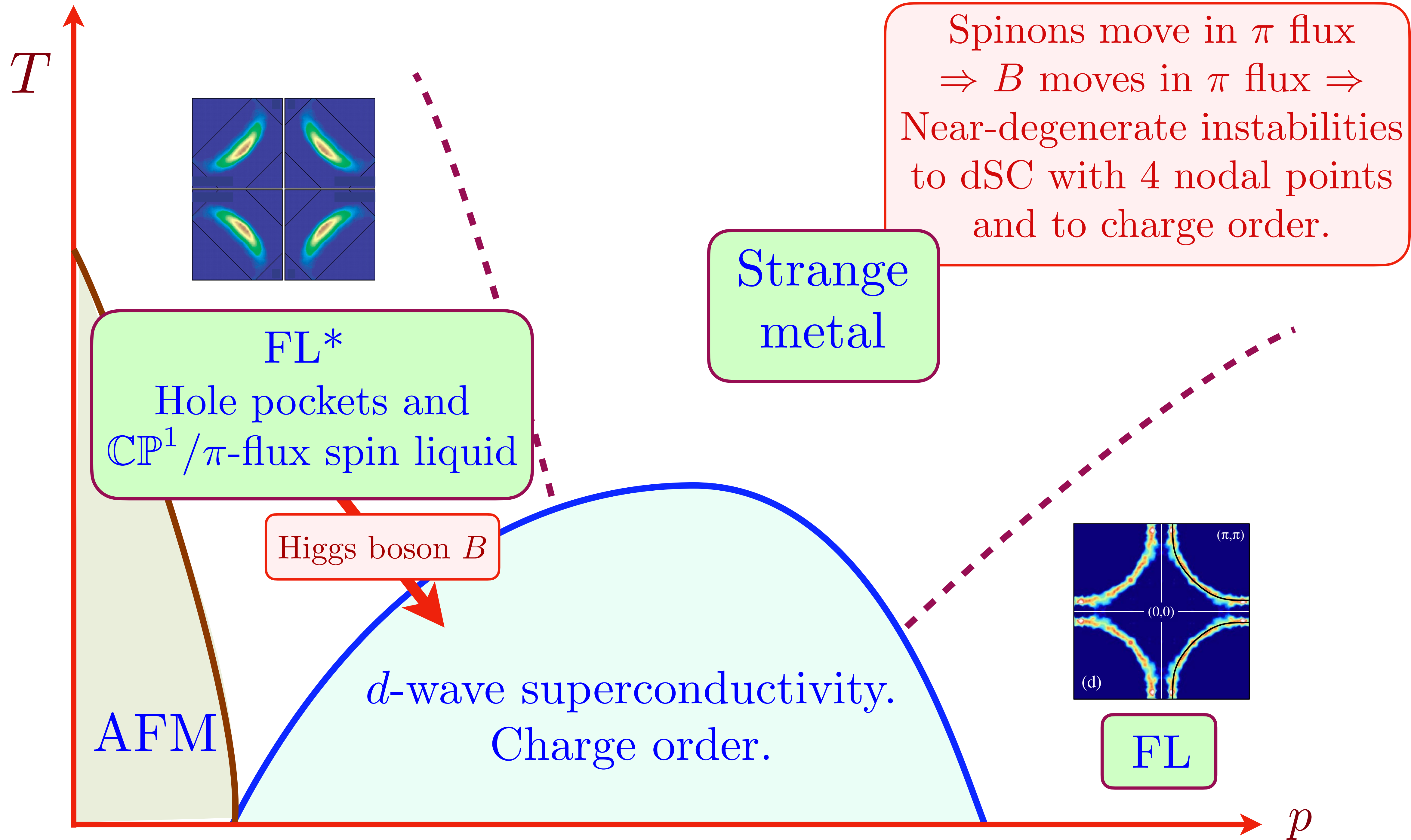
$J_K$

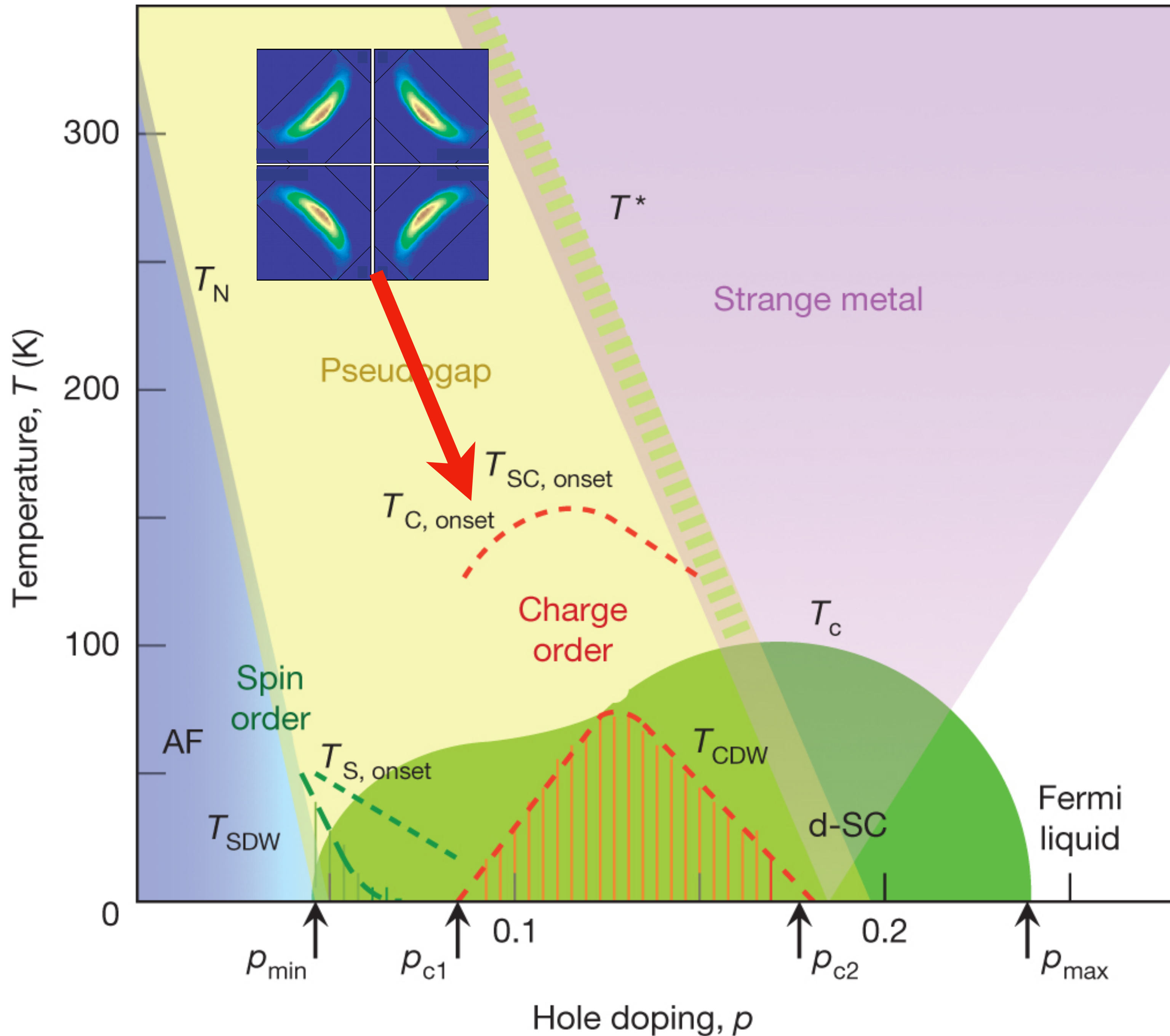
doping  $p$

Higgs field  $B$  drives FL\*-dSC/CDW





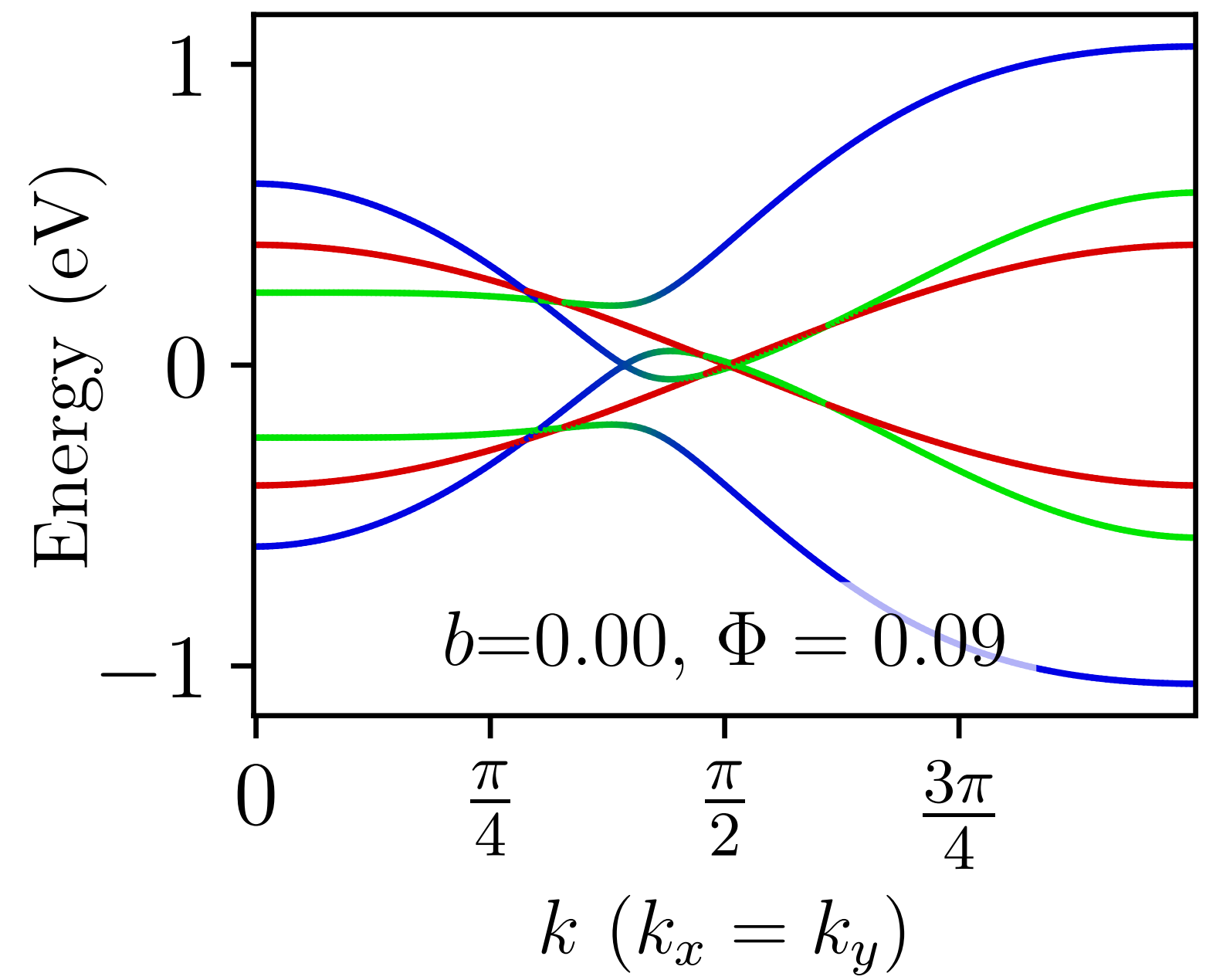
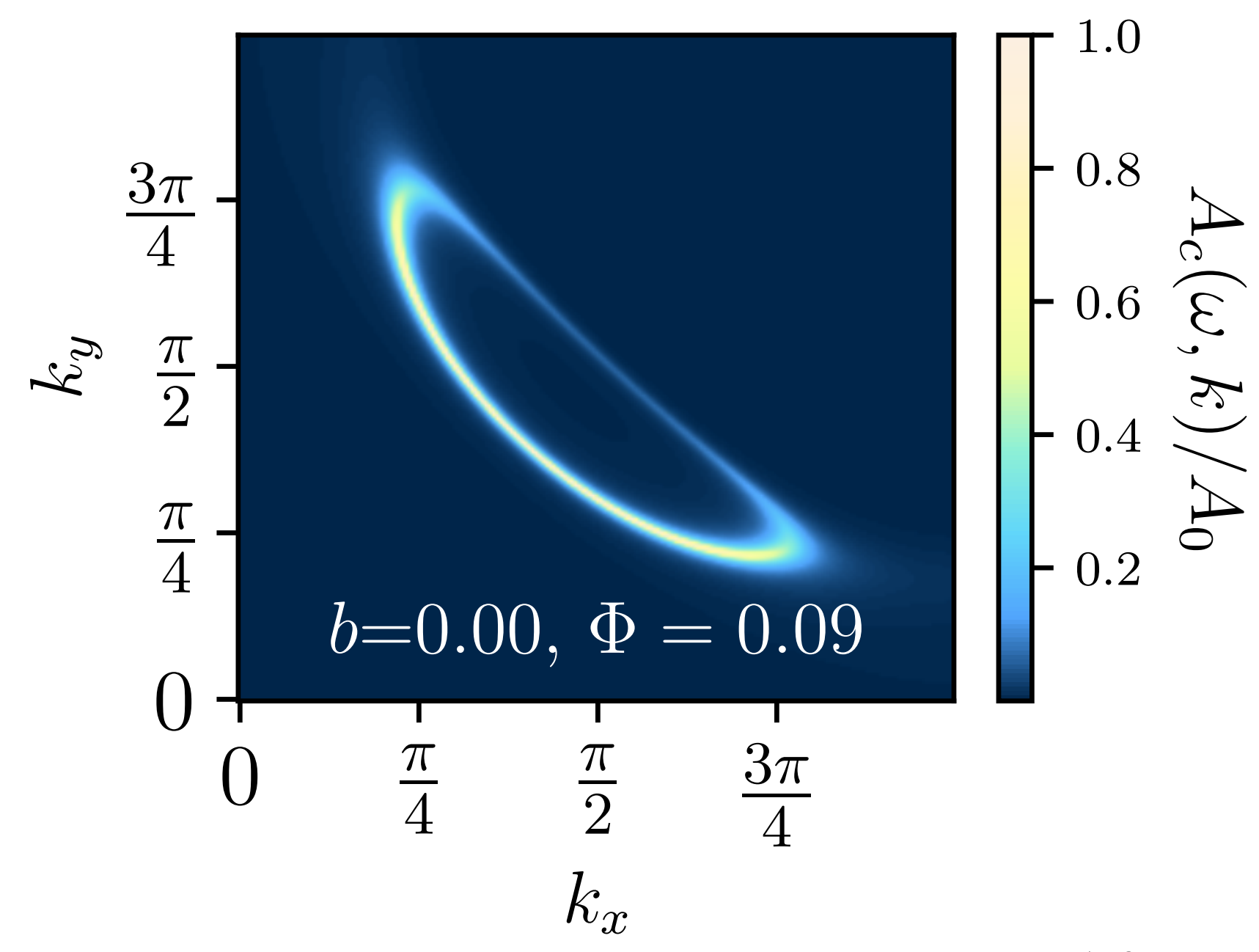




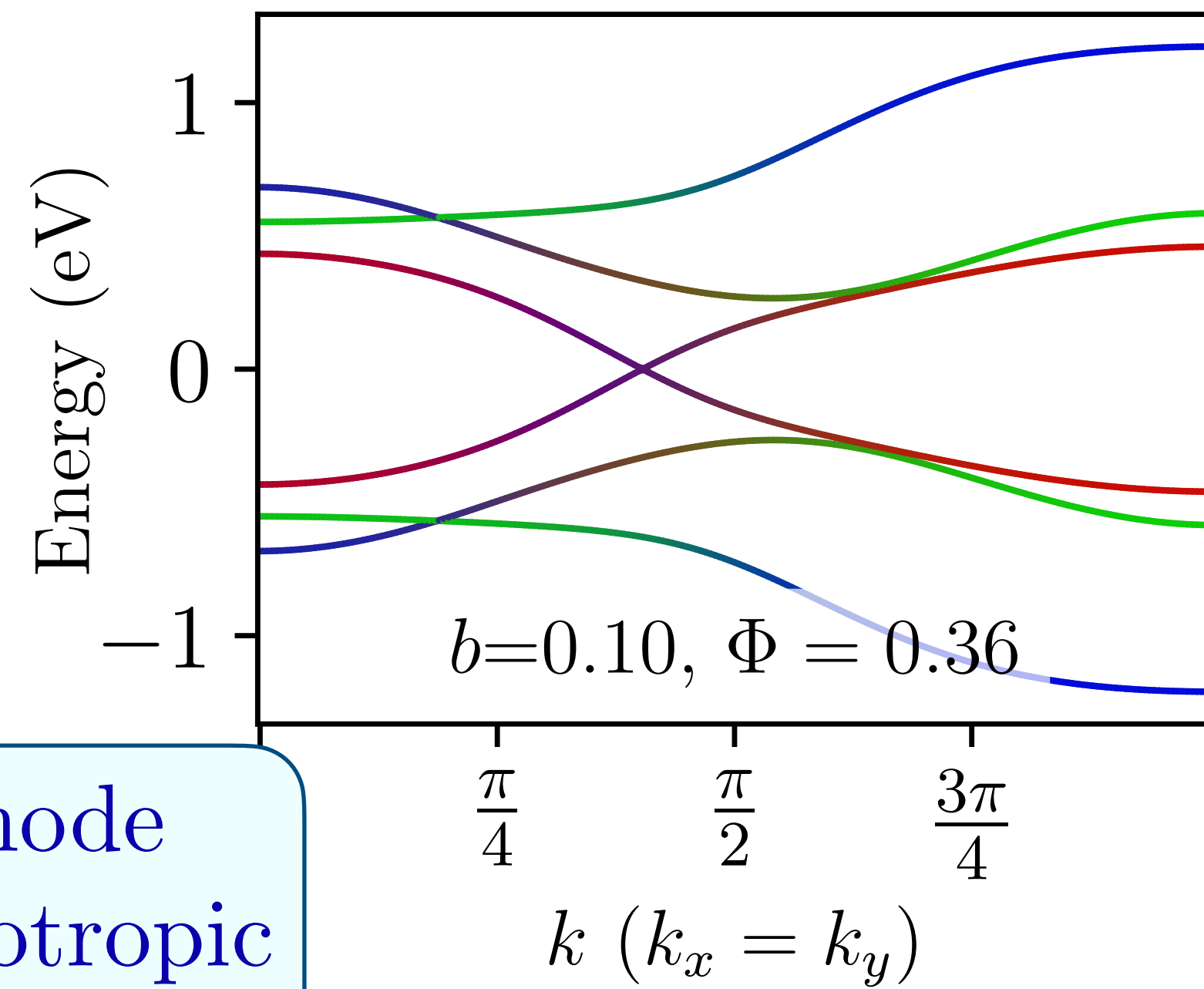
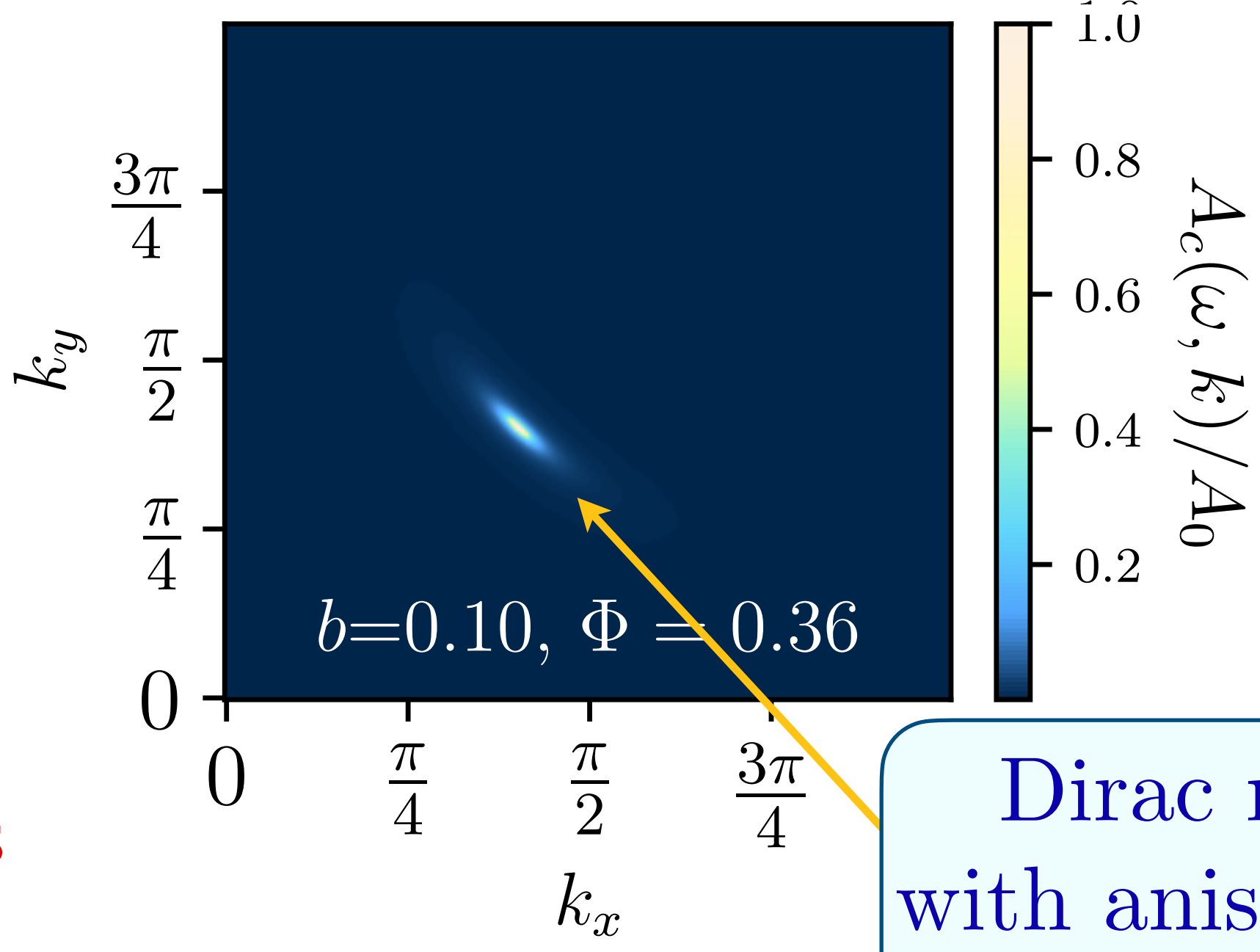
Spinons move in  $\pi$  flux  
 $\Rightarrow B$  moves in  $\pi$  flux  $\Rightarrow$   
Near-degenerate instabilities  
to dSC with 4 nodal points  
and to charge order.

**Predictions**

# Electron spectral density in hole-doped cuprates



FL\*

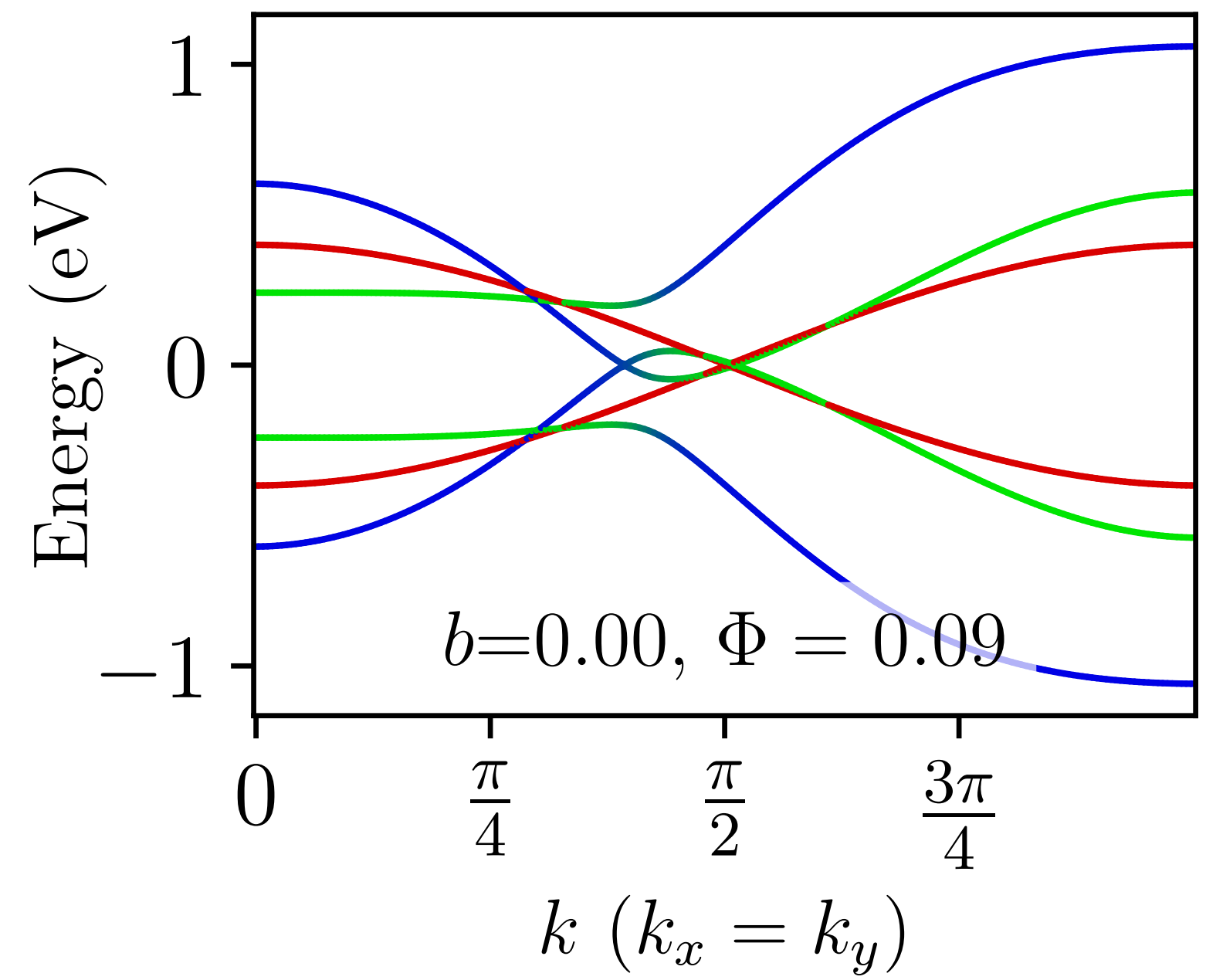
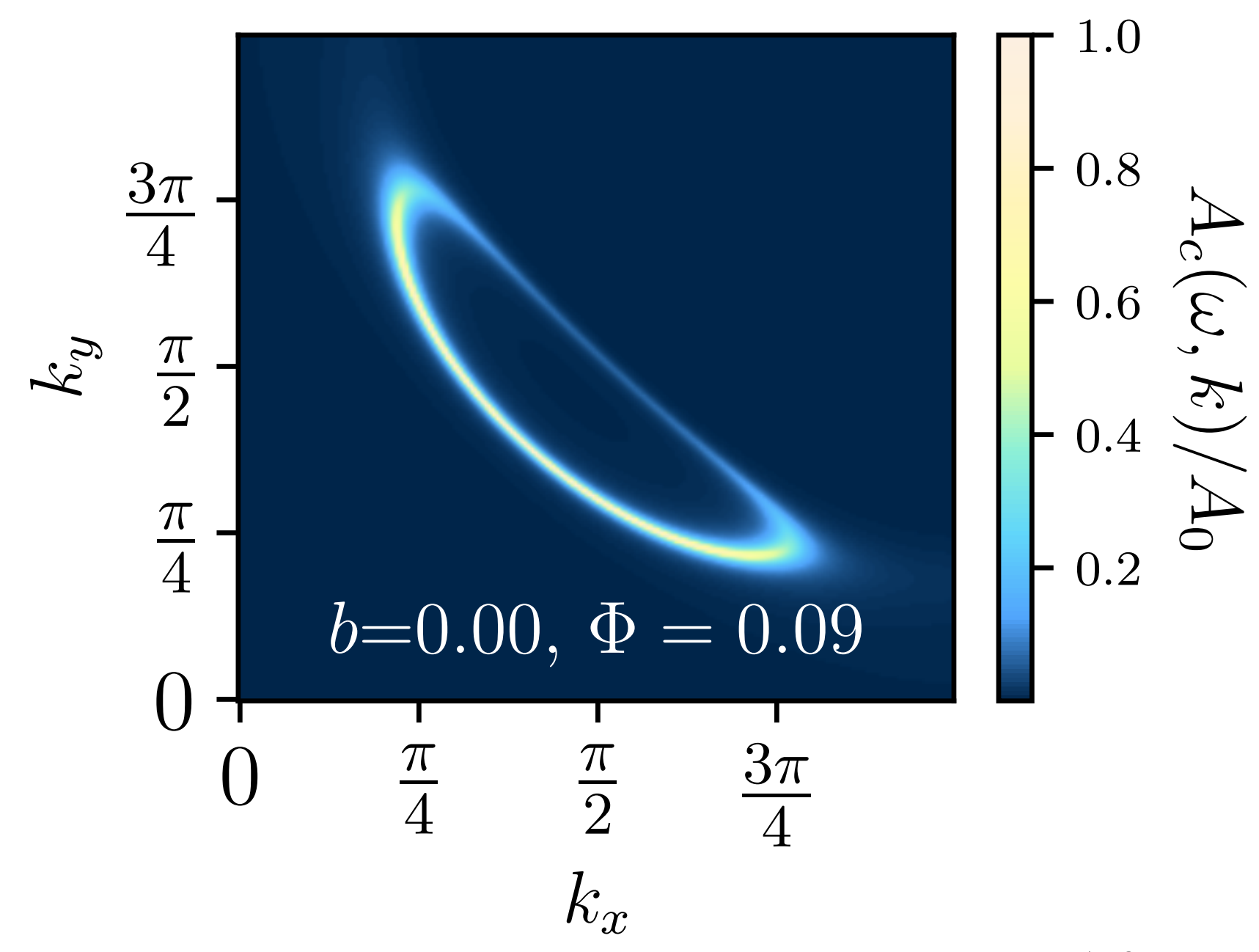


dSC

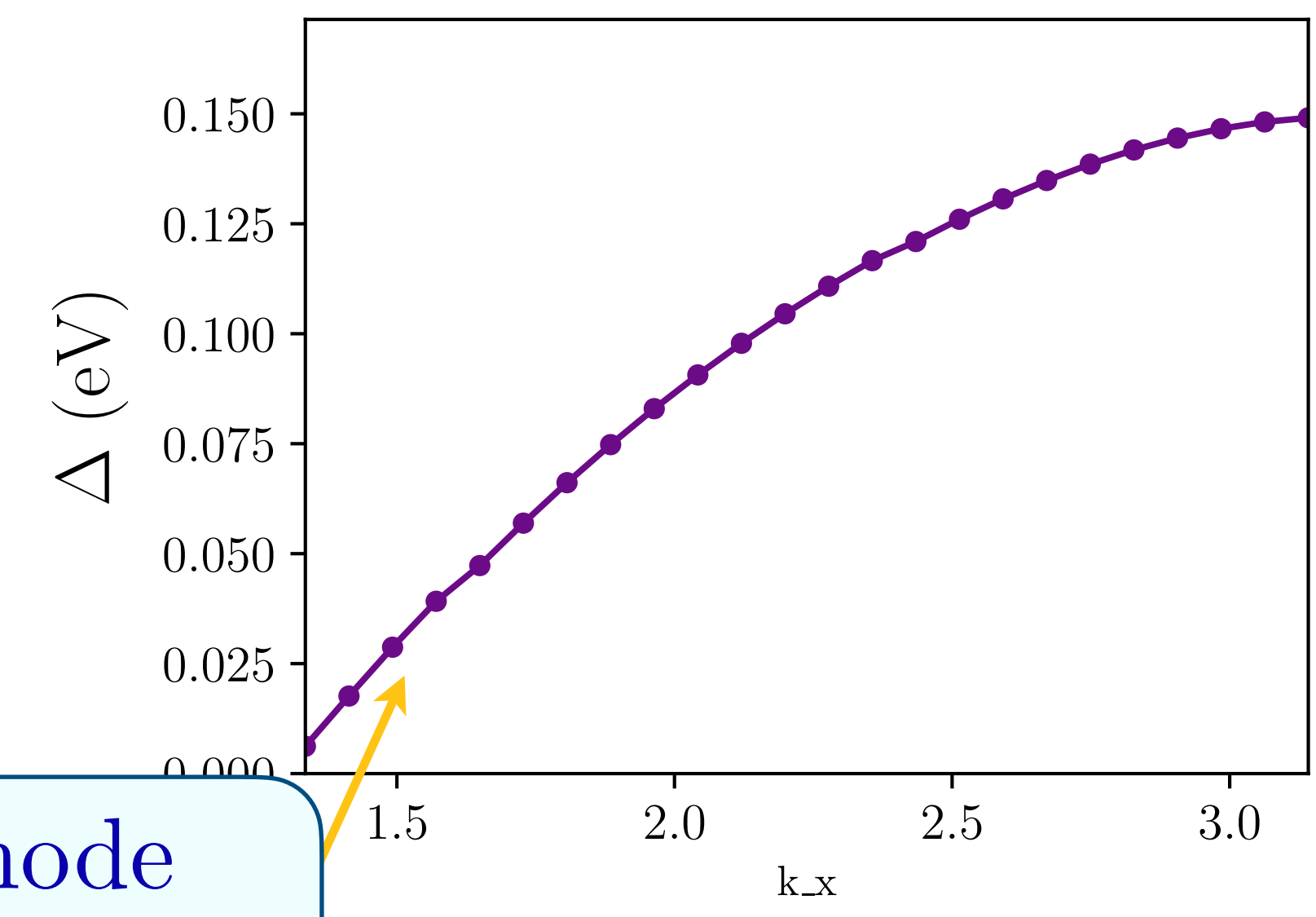
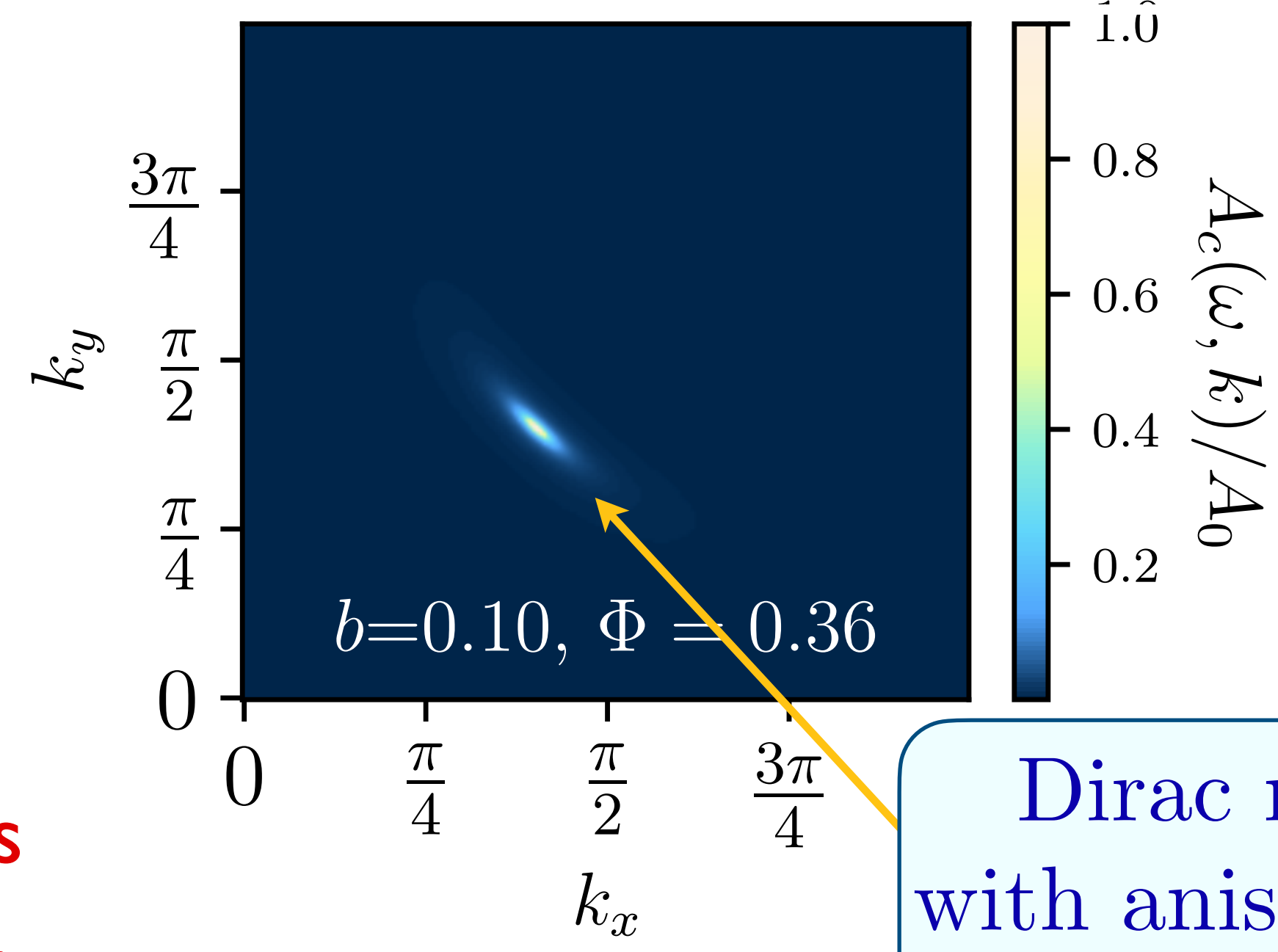
Dirac node with anisotropic velocities

Maine Christos and S.Sachdev, arXiv:2308.03835

# Electron spectral density in hole-doped cuprates



FL\*

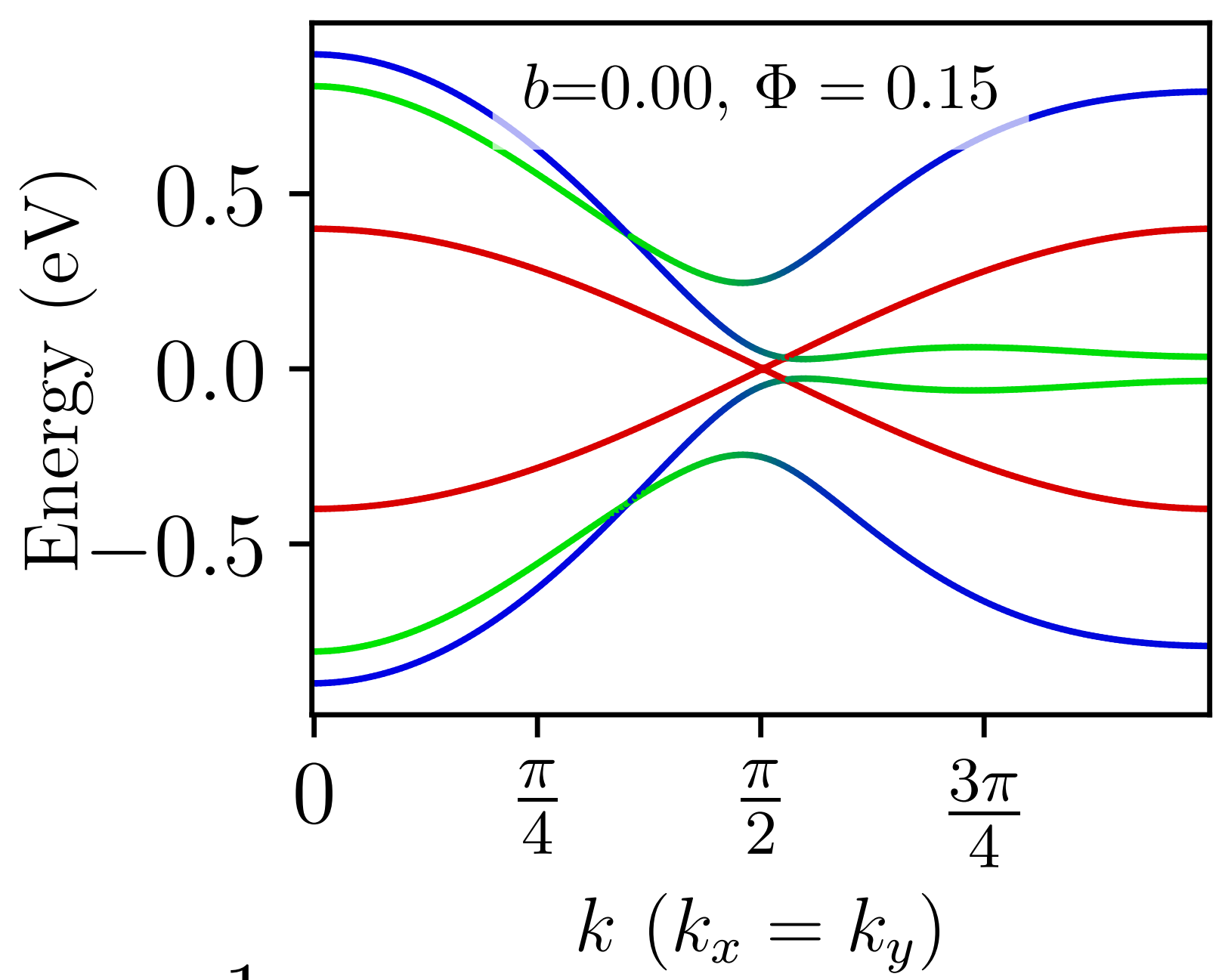
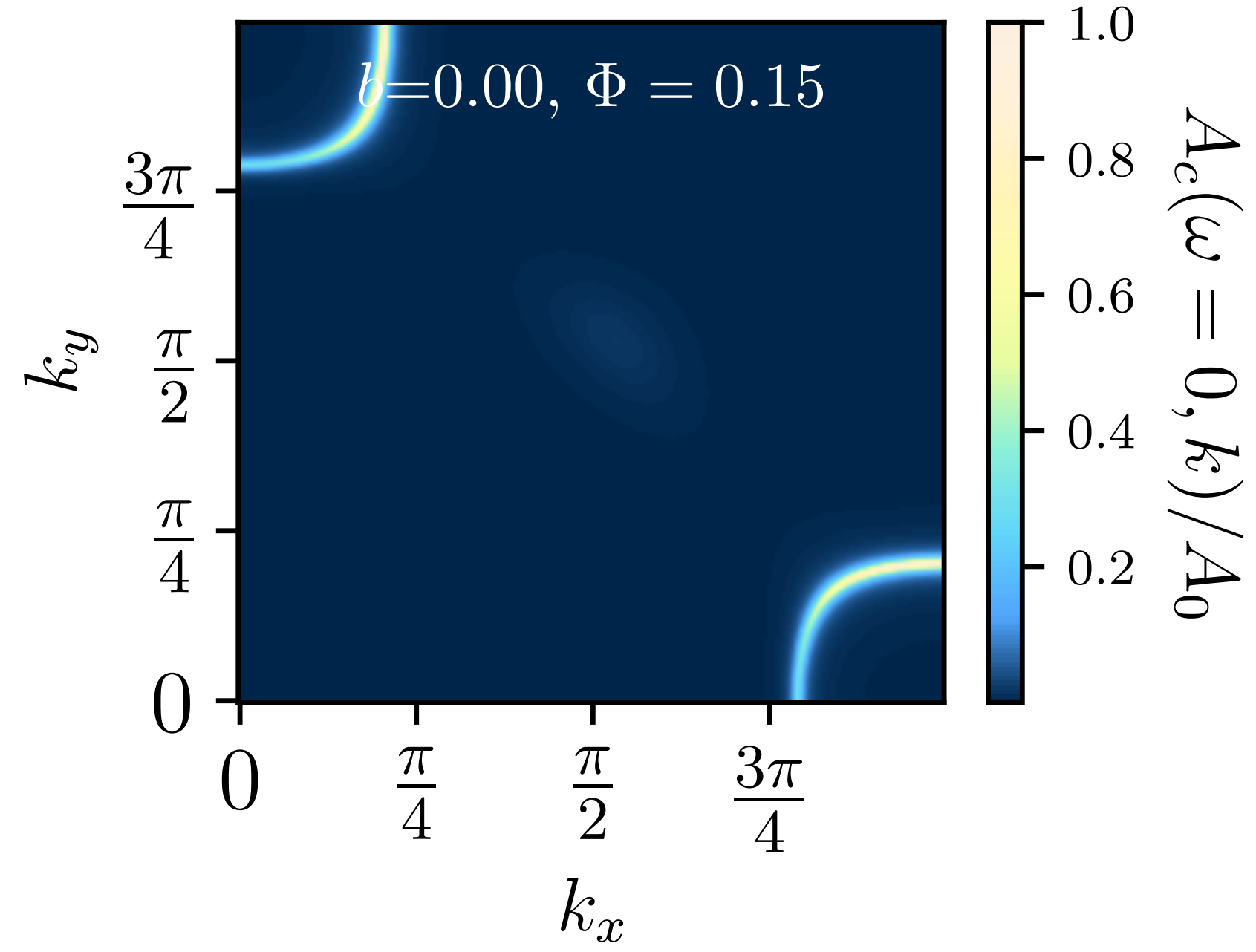


Dirac node with anisotropic velocities

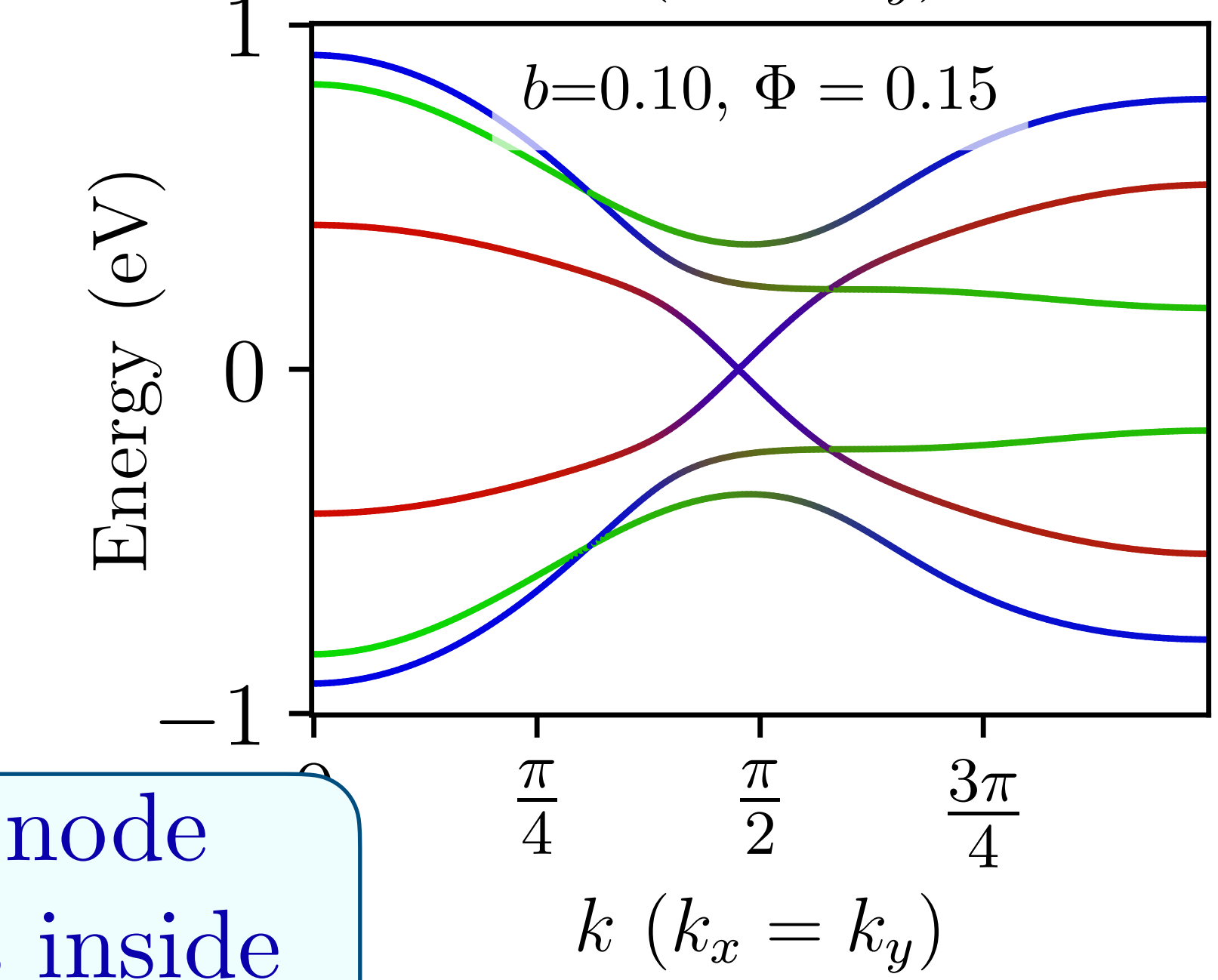
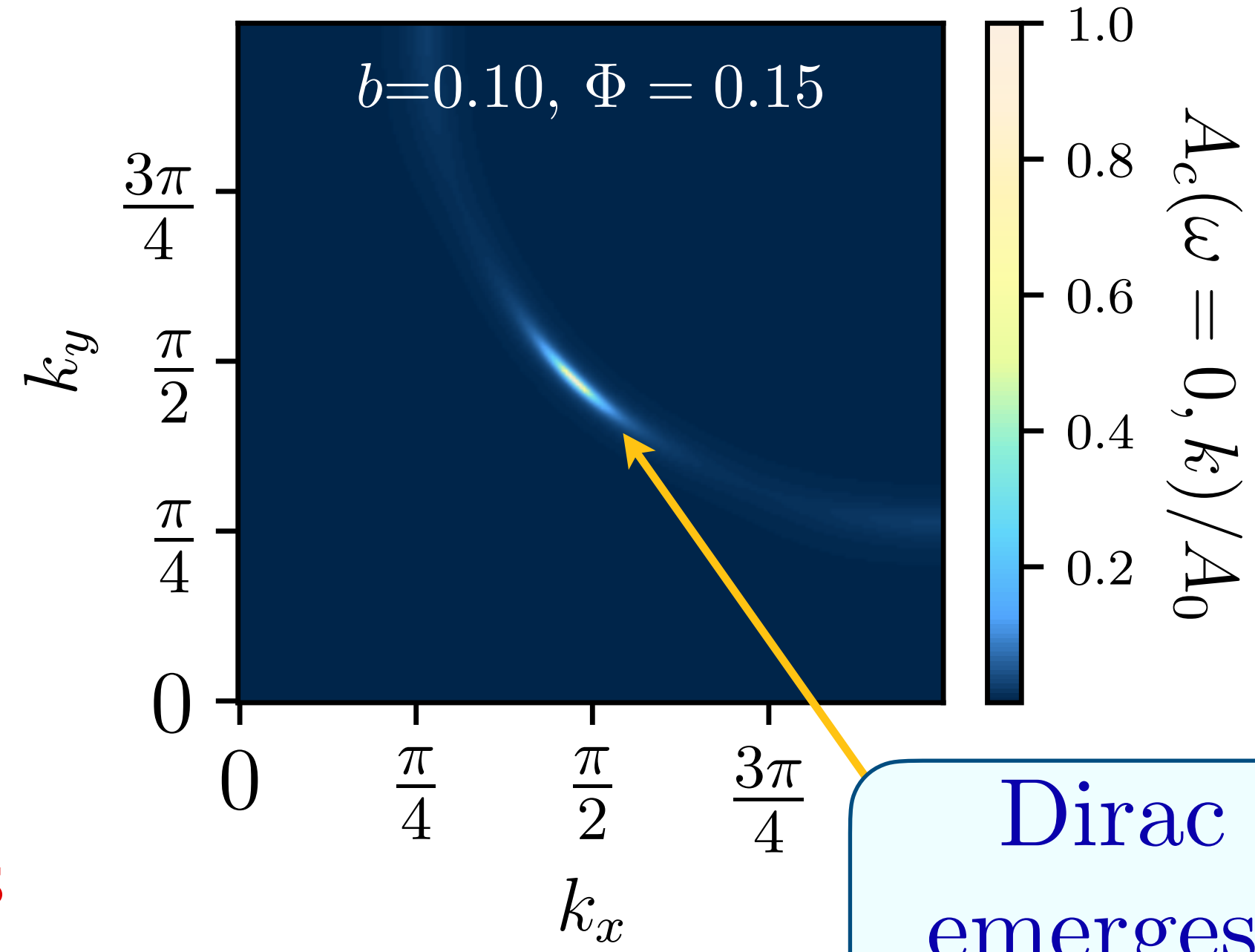
dSC

Maine Christos and S.Sachdev, arXiv:2308.03835

# Electron spectral density in electron-doped cuprates



FL\*



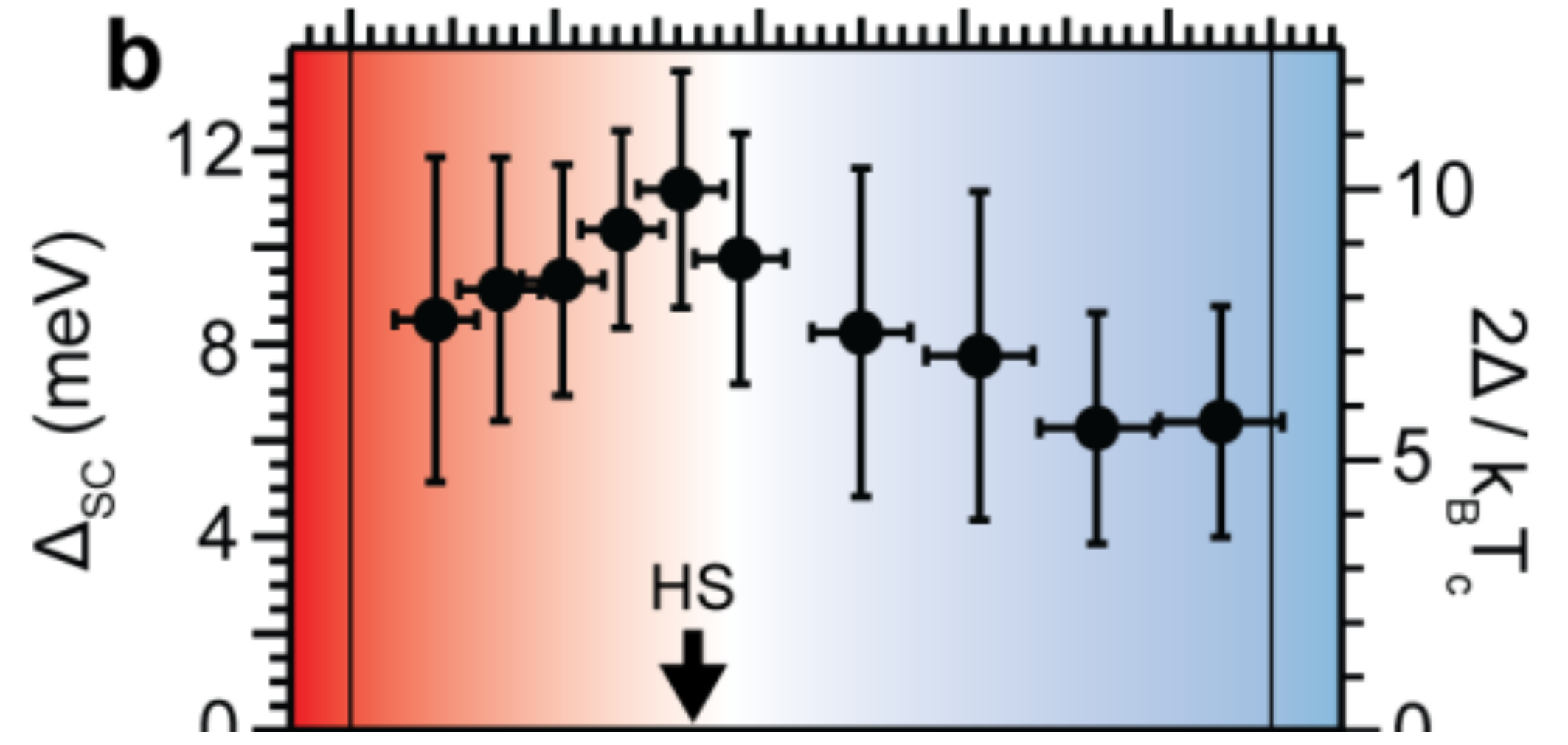
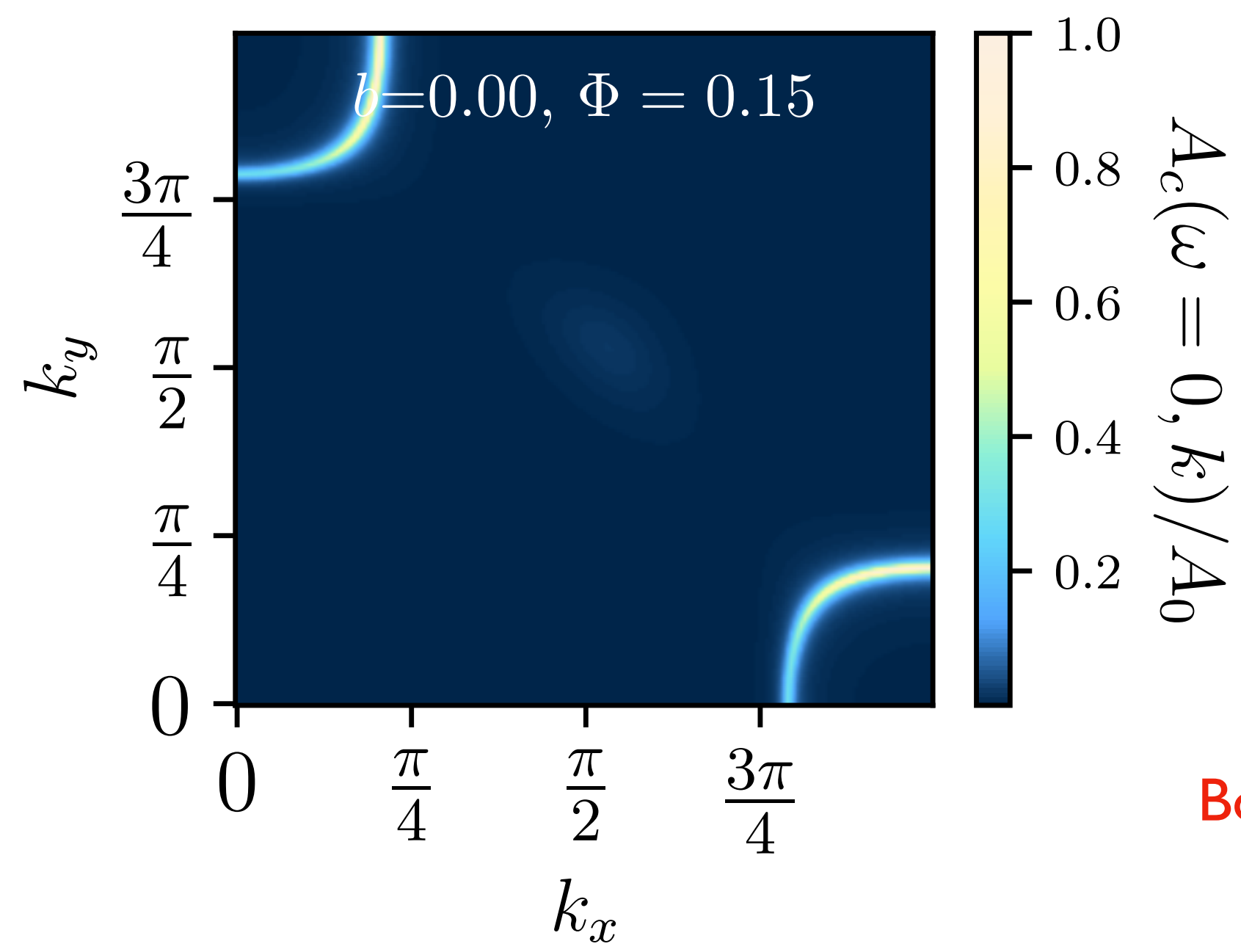
dSC

Dirac node emerges inside normal state gap

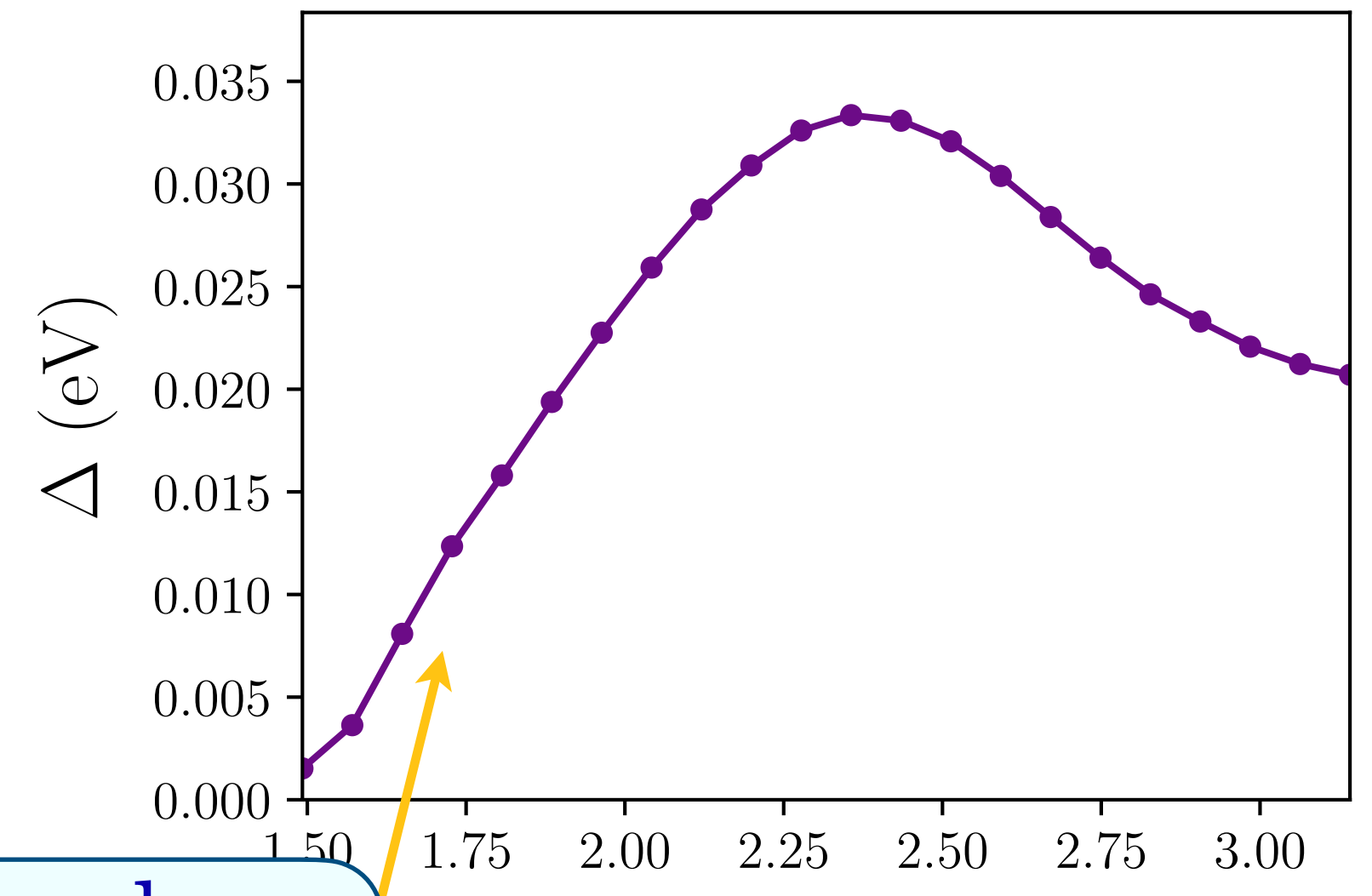
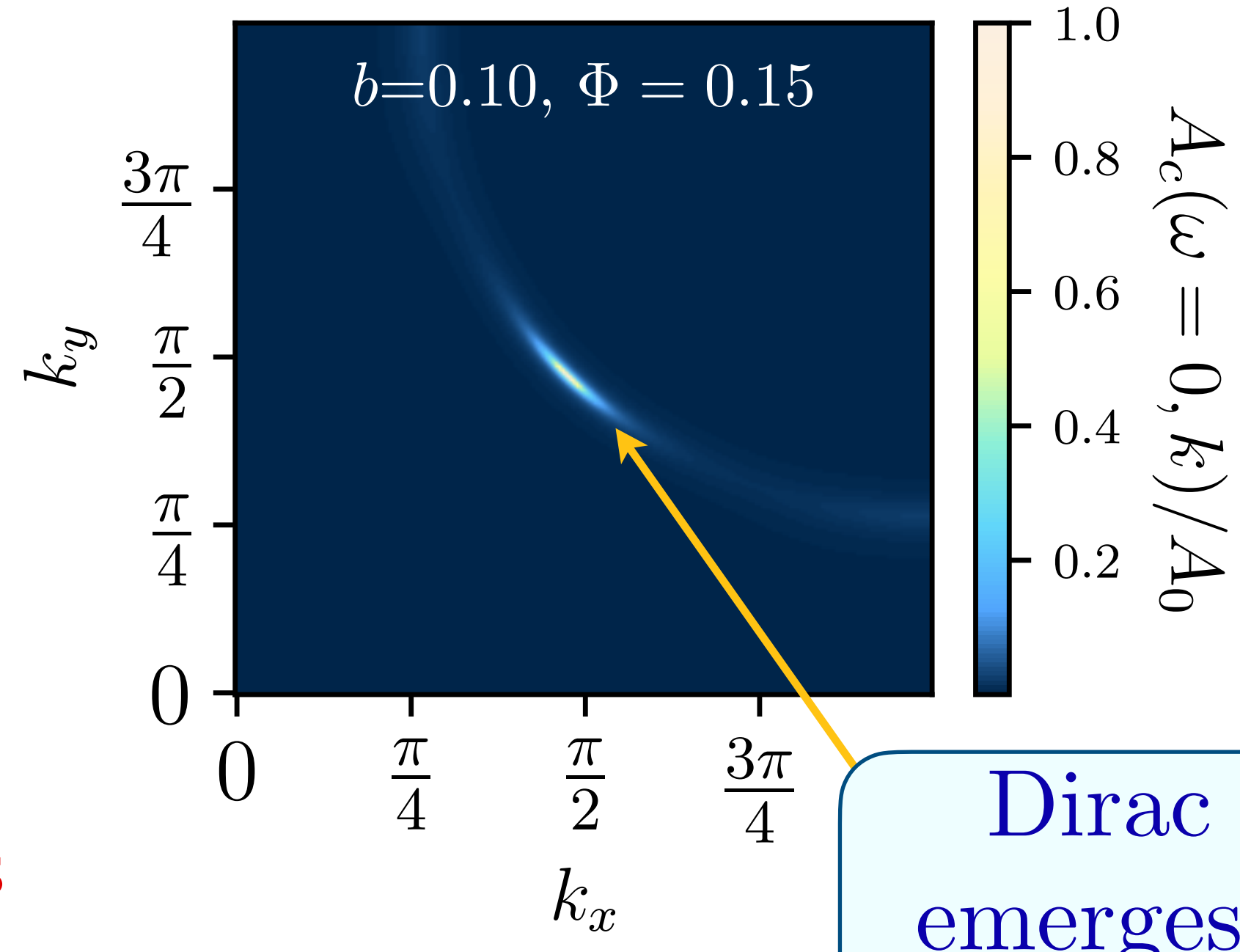


Maine Christos and S.Sachdev, arXiv:2308.03835

# Electron spectral density in electron-doped cuprates



Bogoliubov Quasiparticle on the Gossamer Fermi Surface in Electron-Doped Cuprates, Ke-Jun Xu.....Z.-X. Shen, Nature Physics (2023)



Dirac node emerges inside normal state gap

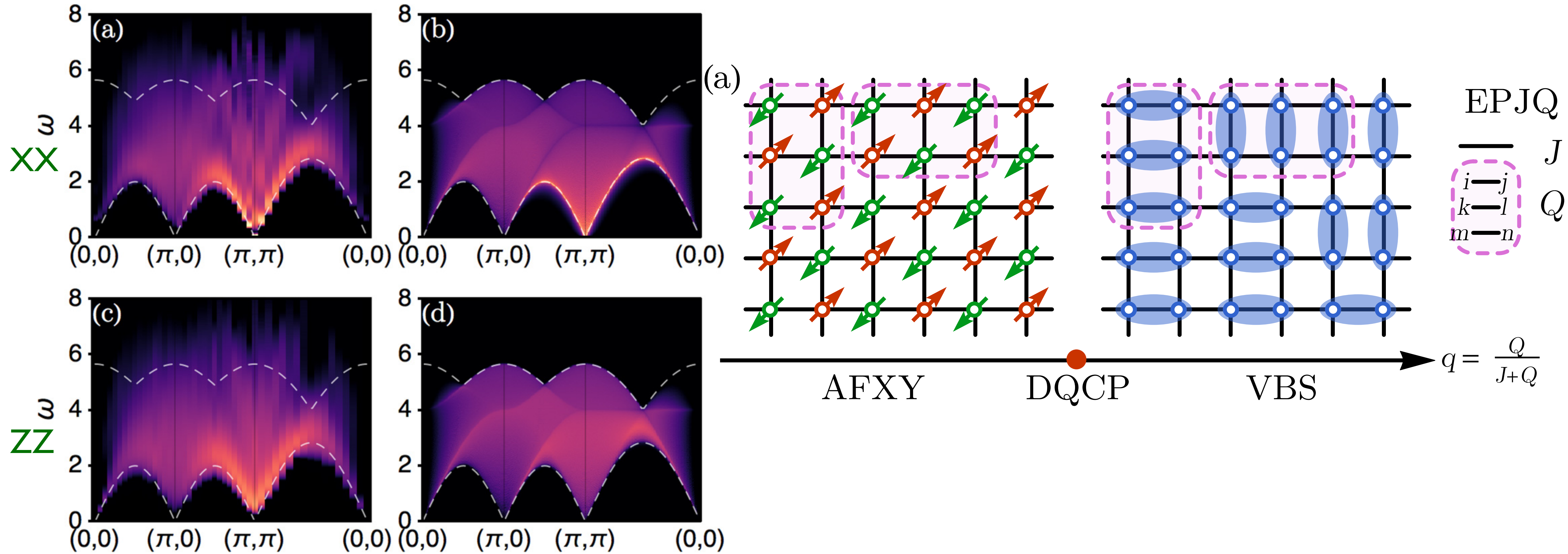
dSC

Maine Christos and S.Sachdev, arXiv:2308.03835

# Observable by neutron scattering in pseudogap ?

QMC

Free fermion  
spinons in  $\pi$ -flux



Summary

$\mathbb{CP}^1$  U(1) gauge theory

Insulating  
 $S=1/2$  anti-  
ferromagnet

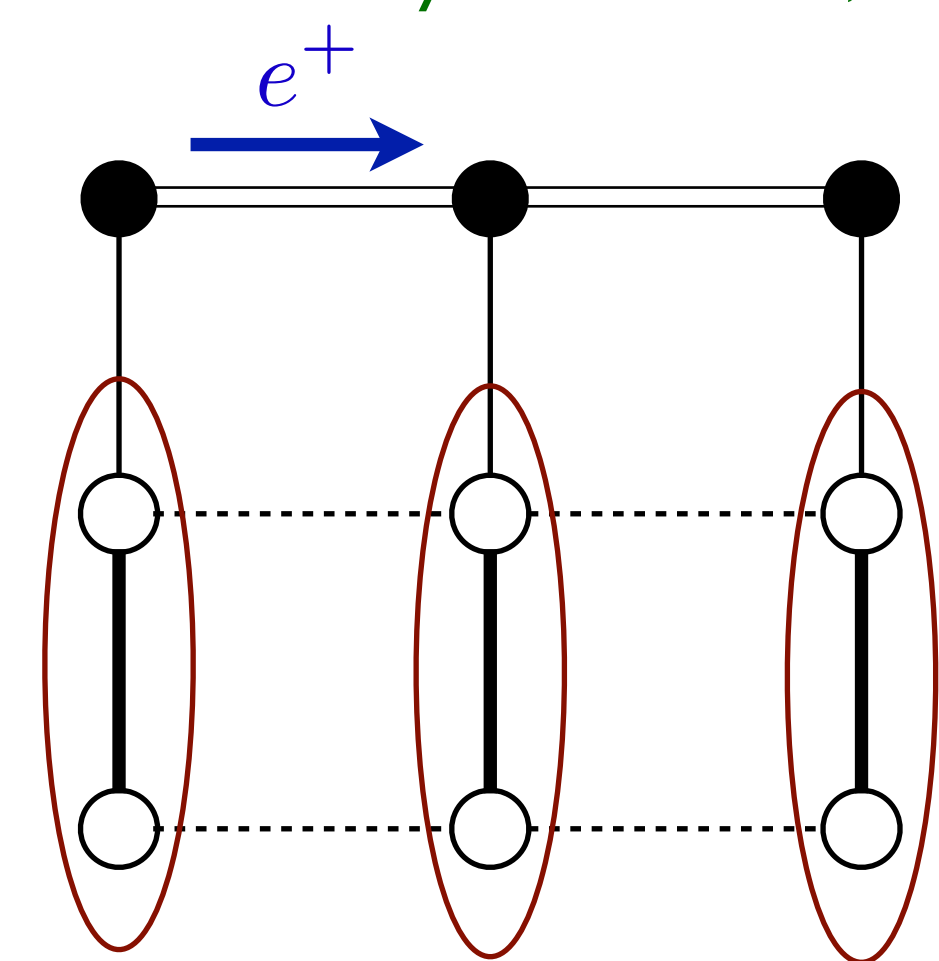
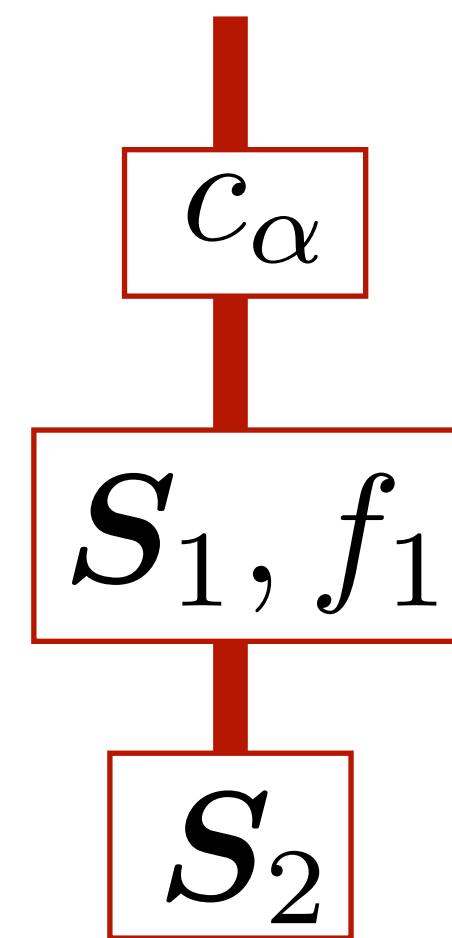
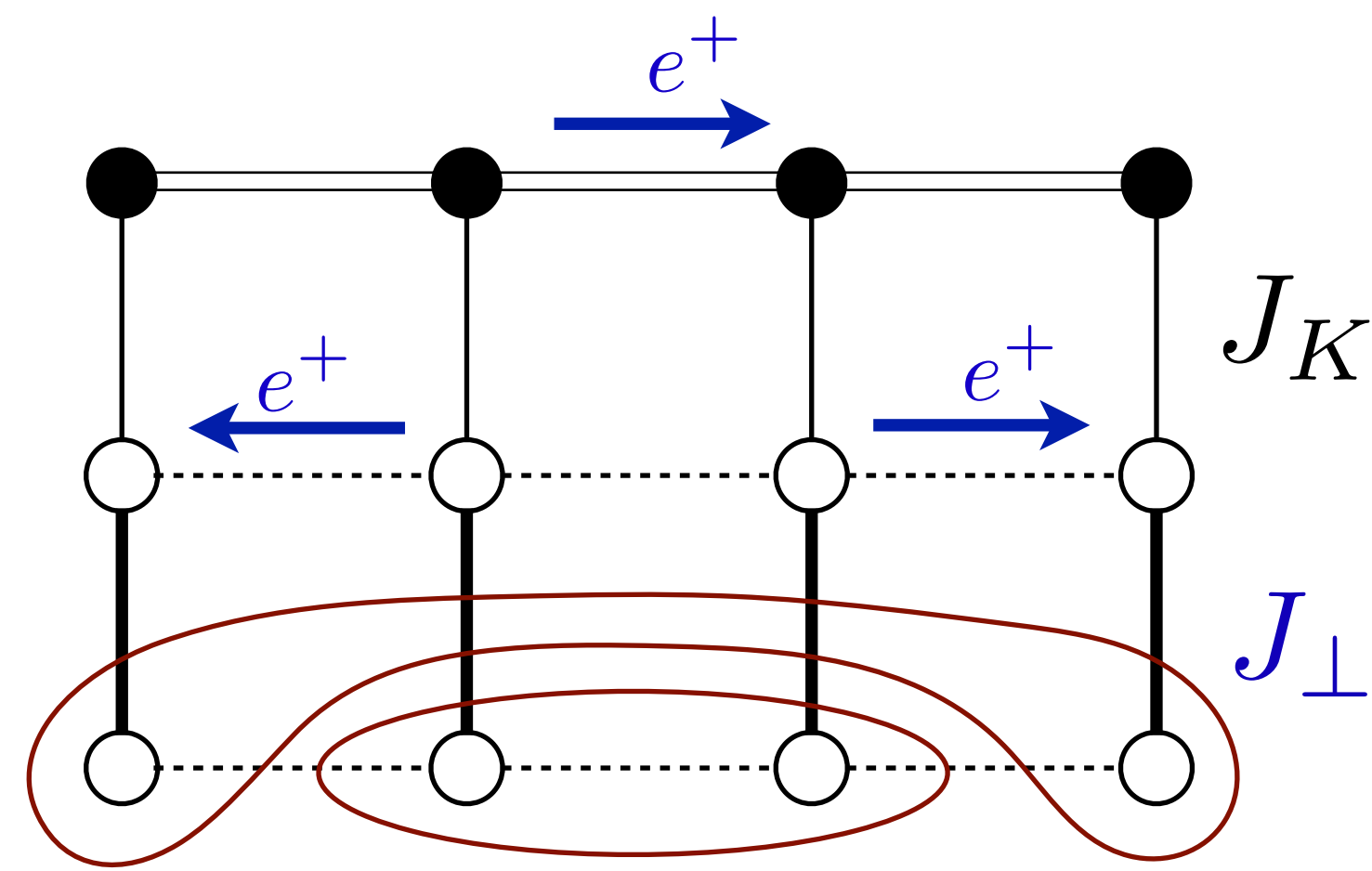
SU(2) gauge theory of  $N_f = 2$   
fundamental, massless, Dirac fermions.

Obtained from a saddle-point of  
fermionic spinons moving in  $\pi$ -flux.

SO(5) non-linear  $\sigma$ -model  
of Néel/VBS orders  
with  $k = 1$  WZW term

# Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev,  
*Phys. Rev. Res.* **2**, 023172 (2020)



FL\*

FL

$J_K$

doping  $p$

Pseudogap metal =  
Kondo Lattice Heavy Fermi Liquid  
 $\oplus$   
Spin Liquid

