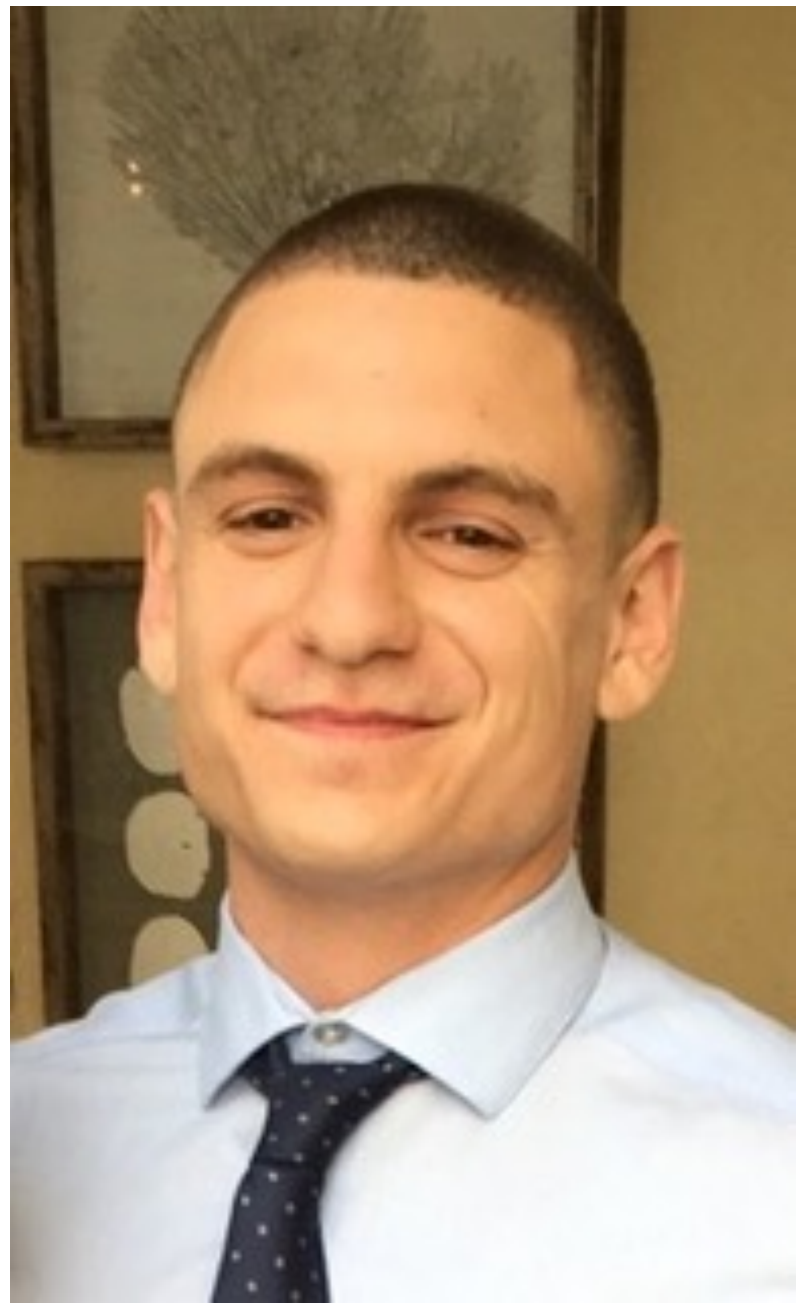
Universal theory of strange metals from spatially random interactions

Electron Correlations beyond the Quasiparticle Paradigm: Theory and Experiment
Conference at KITP, Santa Barbara
September 18, 2023

Subir Sachdev

PHYSICS
HARVARD

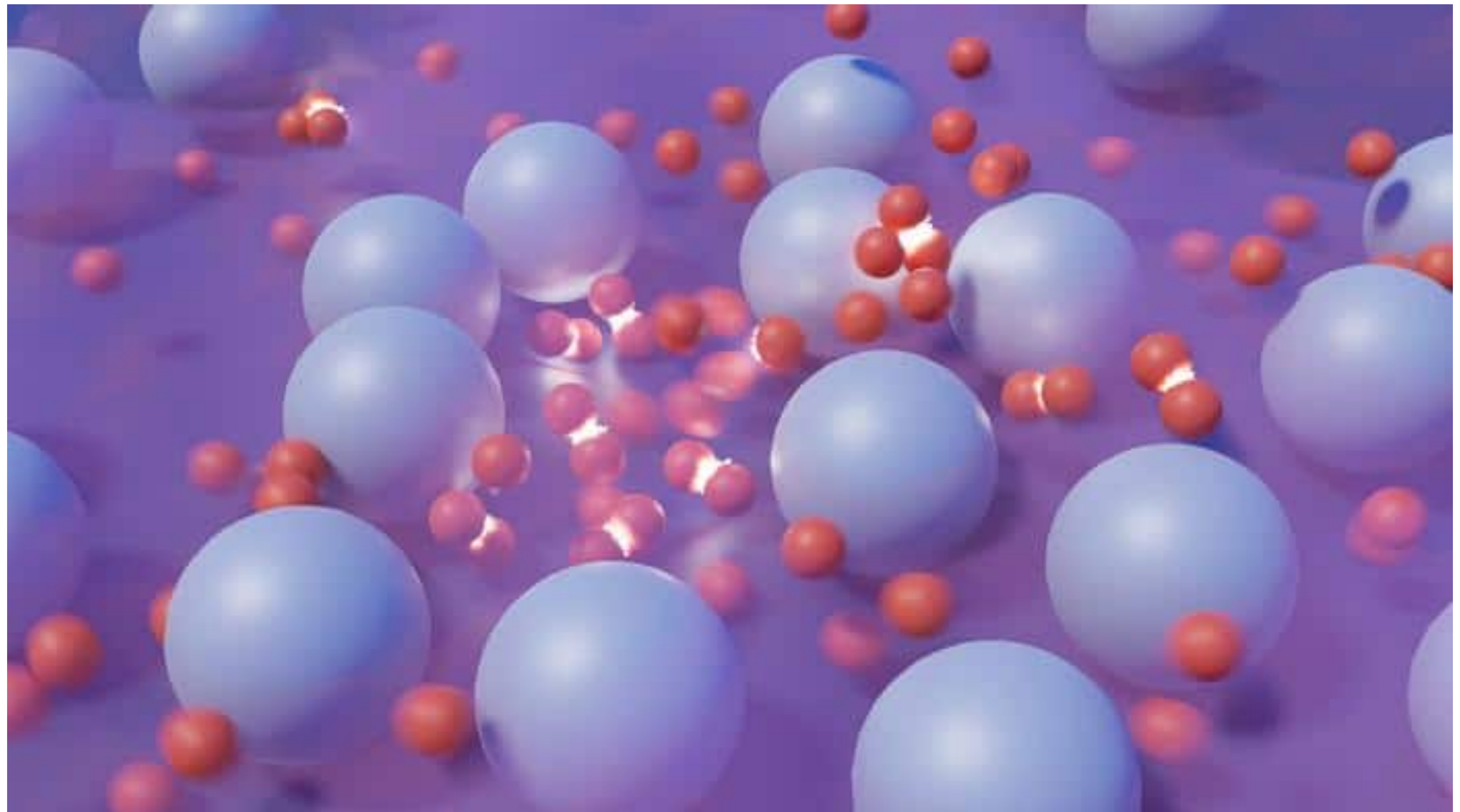
Talk online: sachdev.physics.harvard.edu



Aavishkar Patel
Flatiron Institute, NYC

Haoyu Guo
Cornell

Ilya Esterlis
Wisconsin



Universal theory of strange metals from spatially random interactions,
Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, and S. Sachdev,
Science **381**, 790 (2023)

Why is spatial disorder needed for a theory of strange metals at low-T?

Kohn's Theorem

PHYSICAL REVIEW VOLUME 123, NUMBER 4 AUGUST 15, 1961

Cyclotron Resonance and de Haas-van Alphen Oscillations of an Interacting Electron Gas*

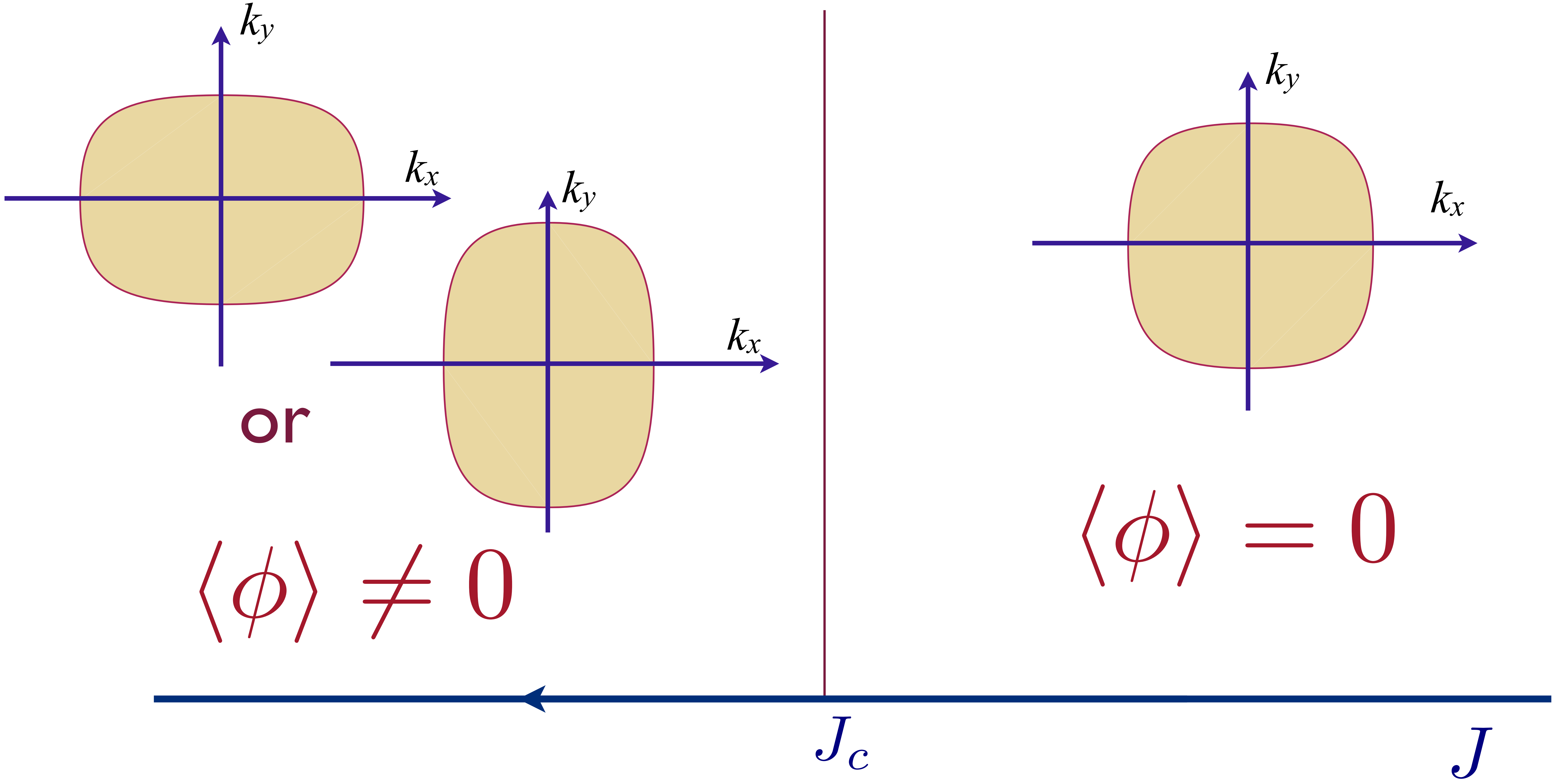
WALTER KOHN
University of California at San Diego, La Jolla, California
(Received April 5, 1961)

An electron gas with short-range interactions is considered in the presence of a uniform magnetic field. It is shown that (1) the cyclotron resonance frequency is independent of the interaction; (2) for a two-dimensional gas, the de Haas–van Alphen period is independent of the interaction. The low-lying excited states are briefly discussed.

In the absence of umklapp and impurities, the Fermi liquid is a *perfect metal*.

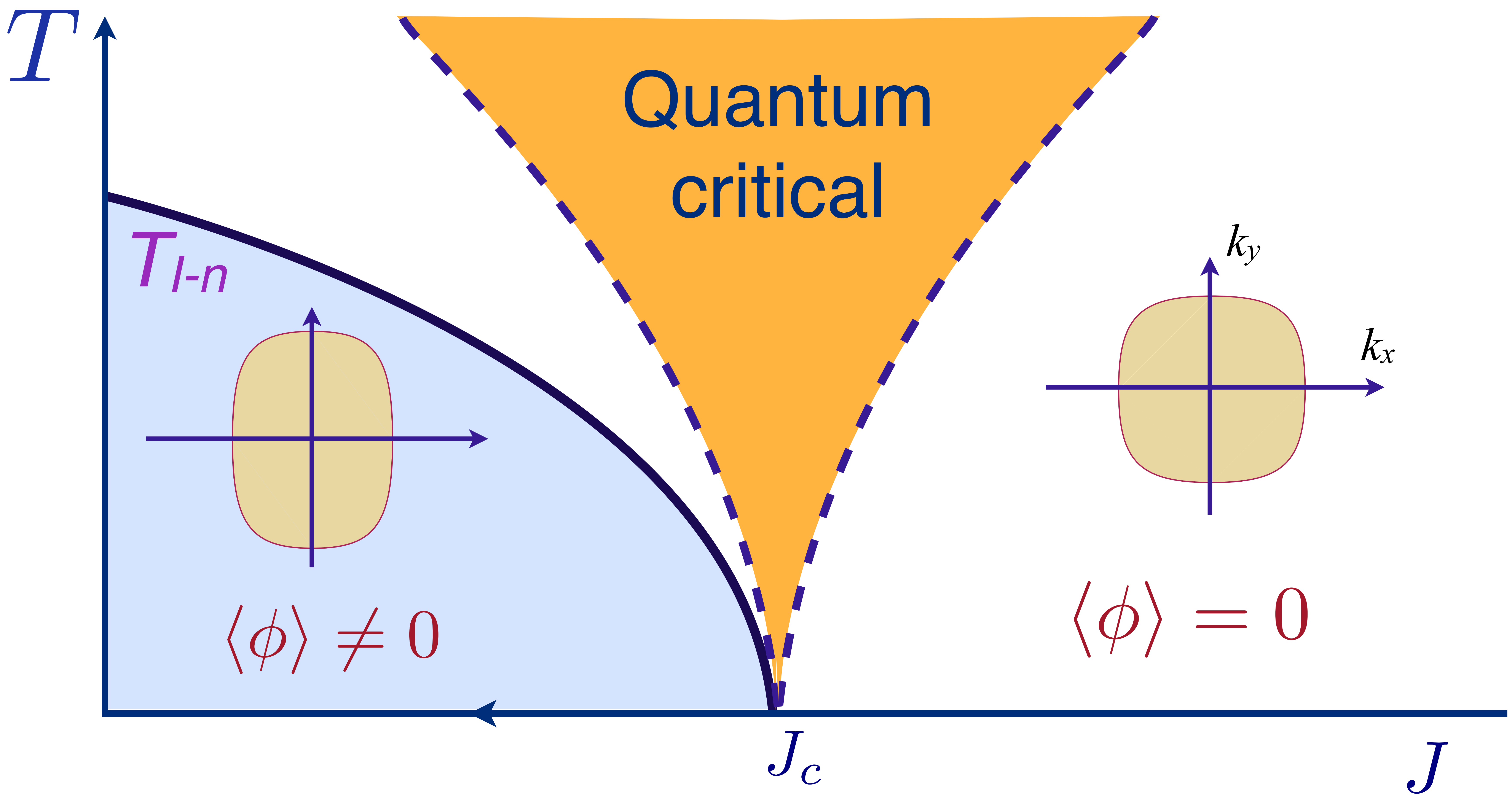
$$\sigma(\omega) = iD/(\omega - \omega_c)$$

Quantum criticality of Ising-nematic ordering in a metal



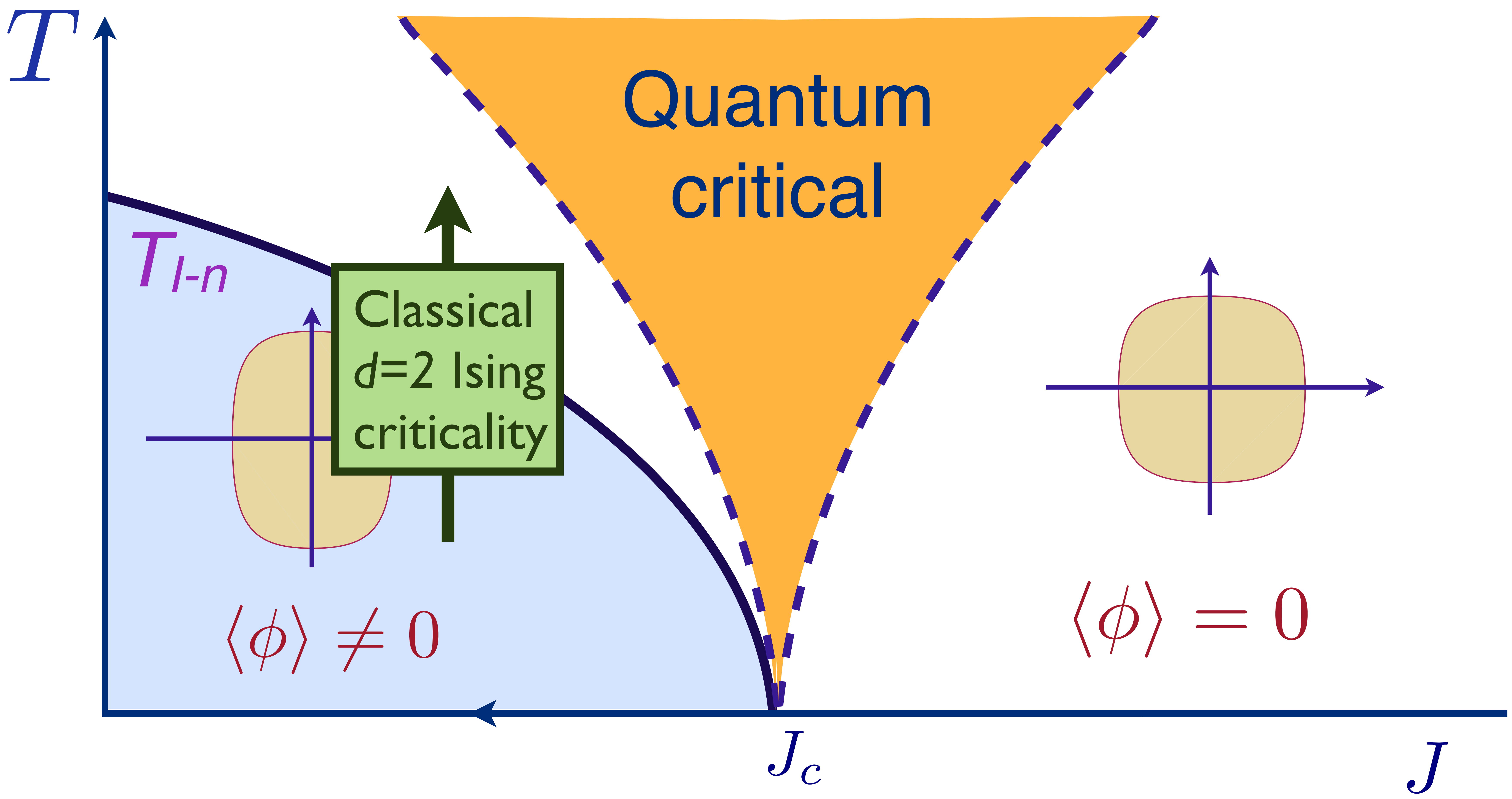
Pomeranchuk instability as a function of coupling J

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and J

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and J

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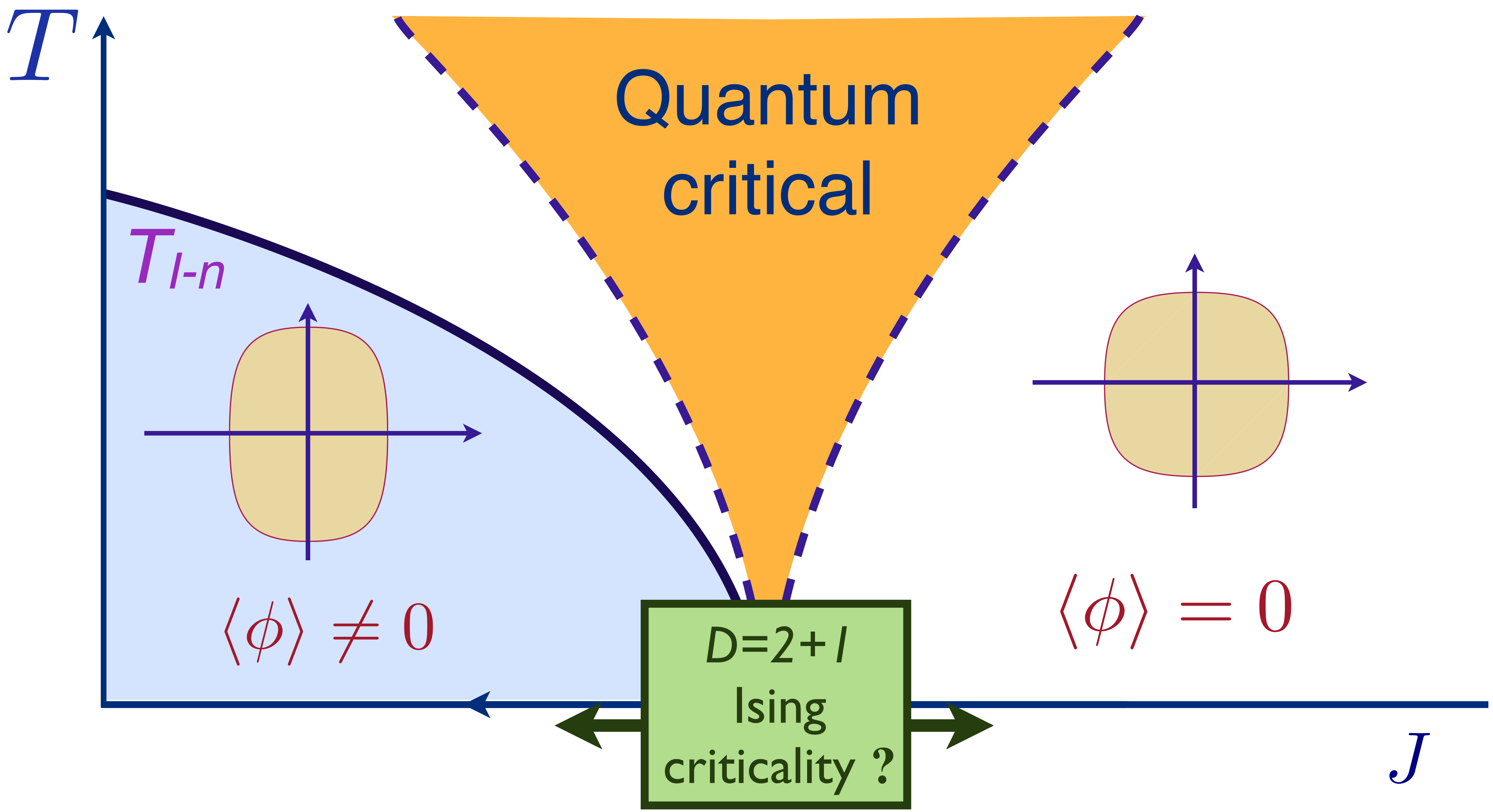

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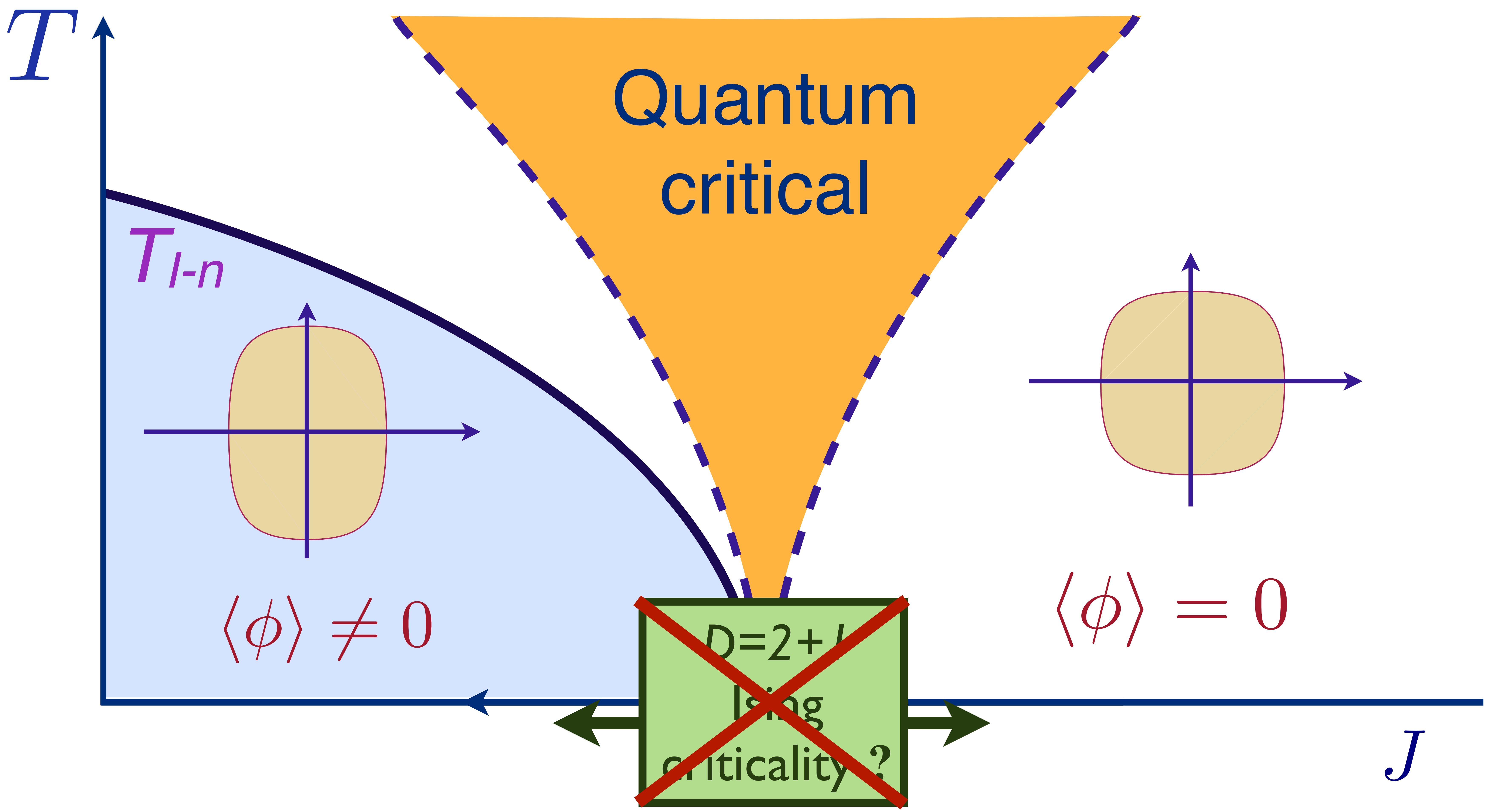

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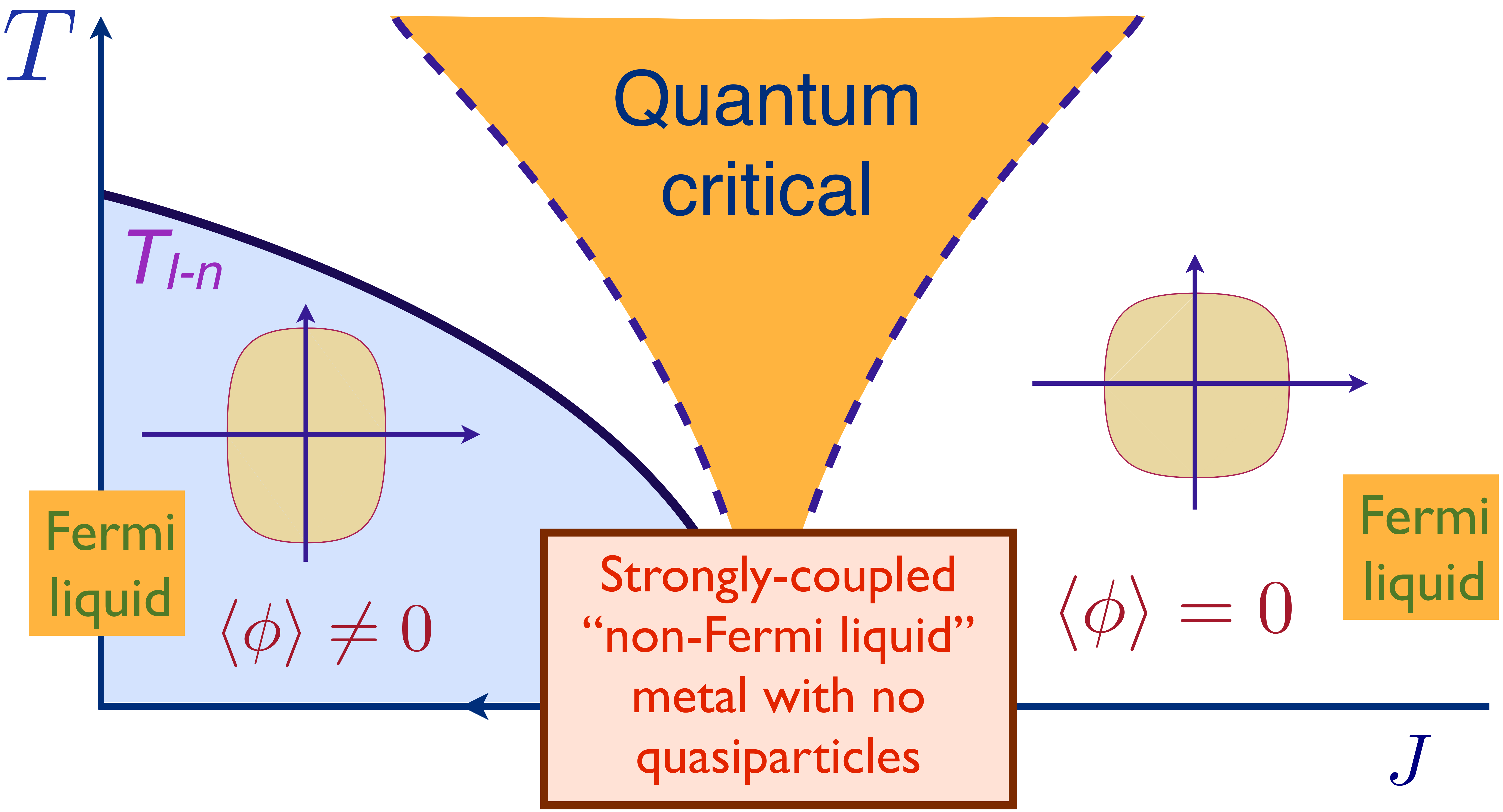

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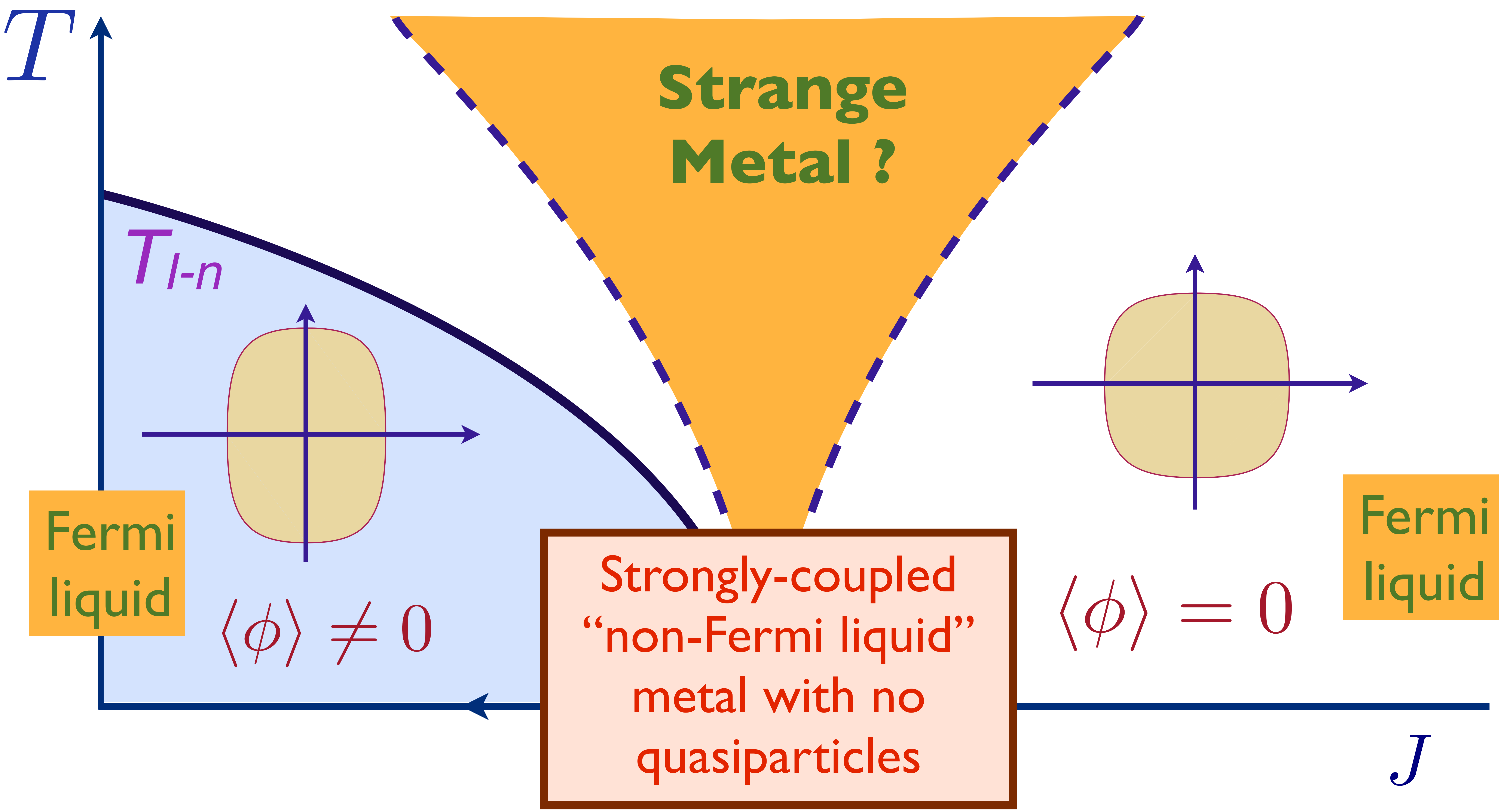

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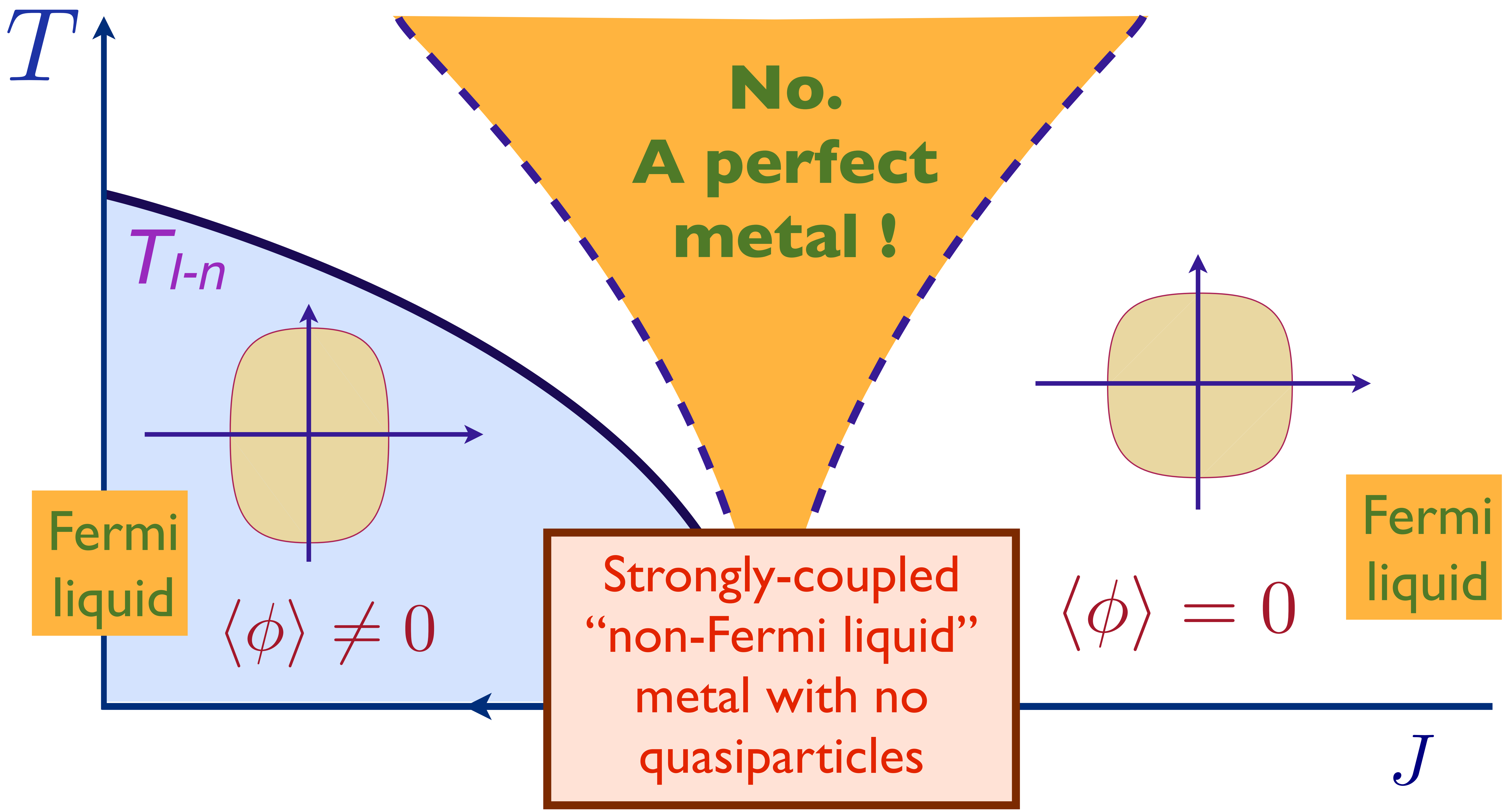
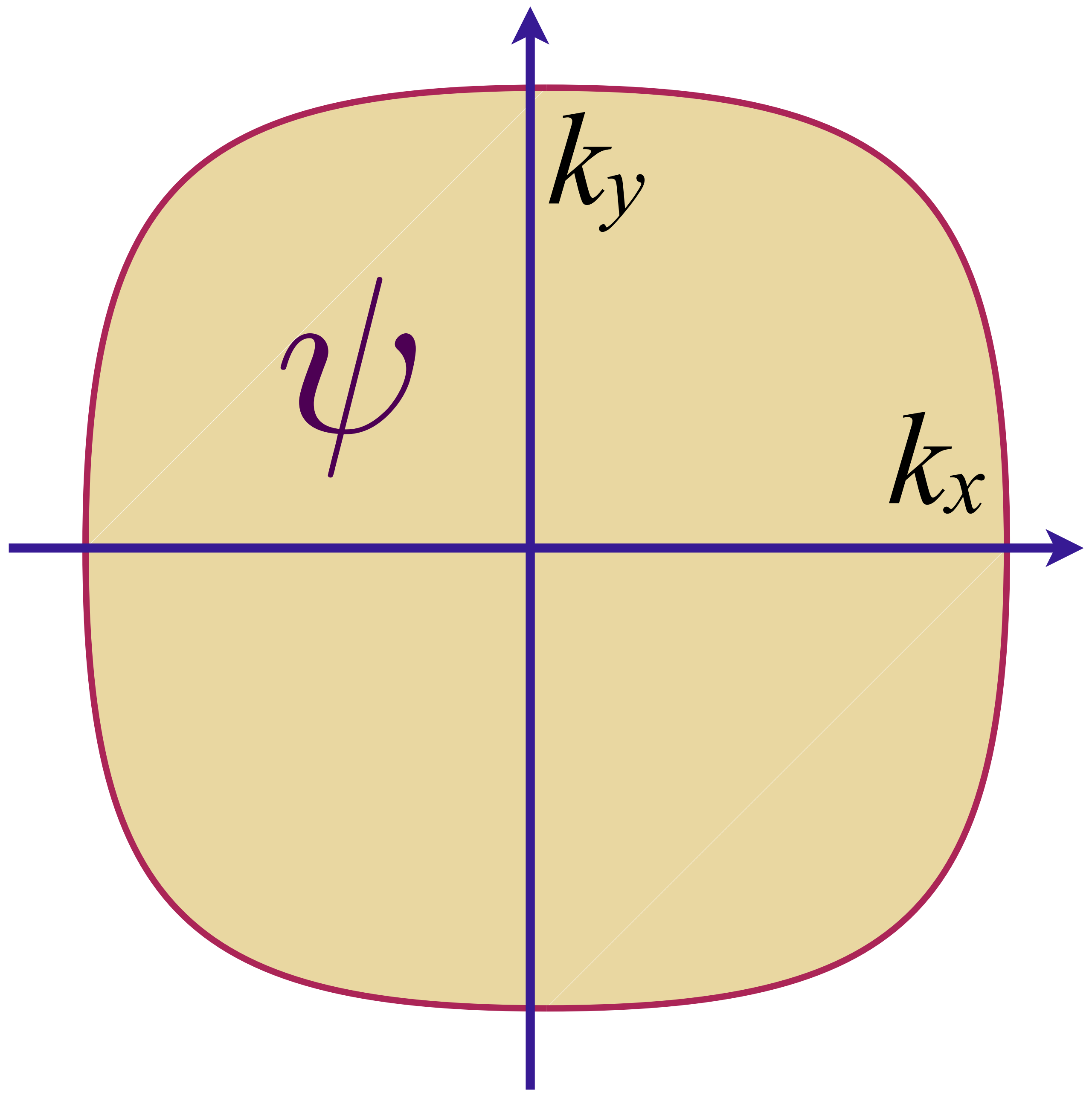


Phase diagram as a function of T and J

Fermi surface

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



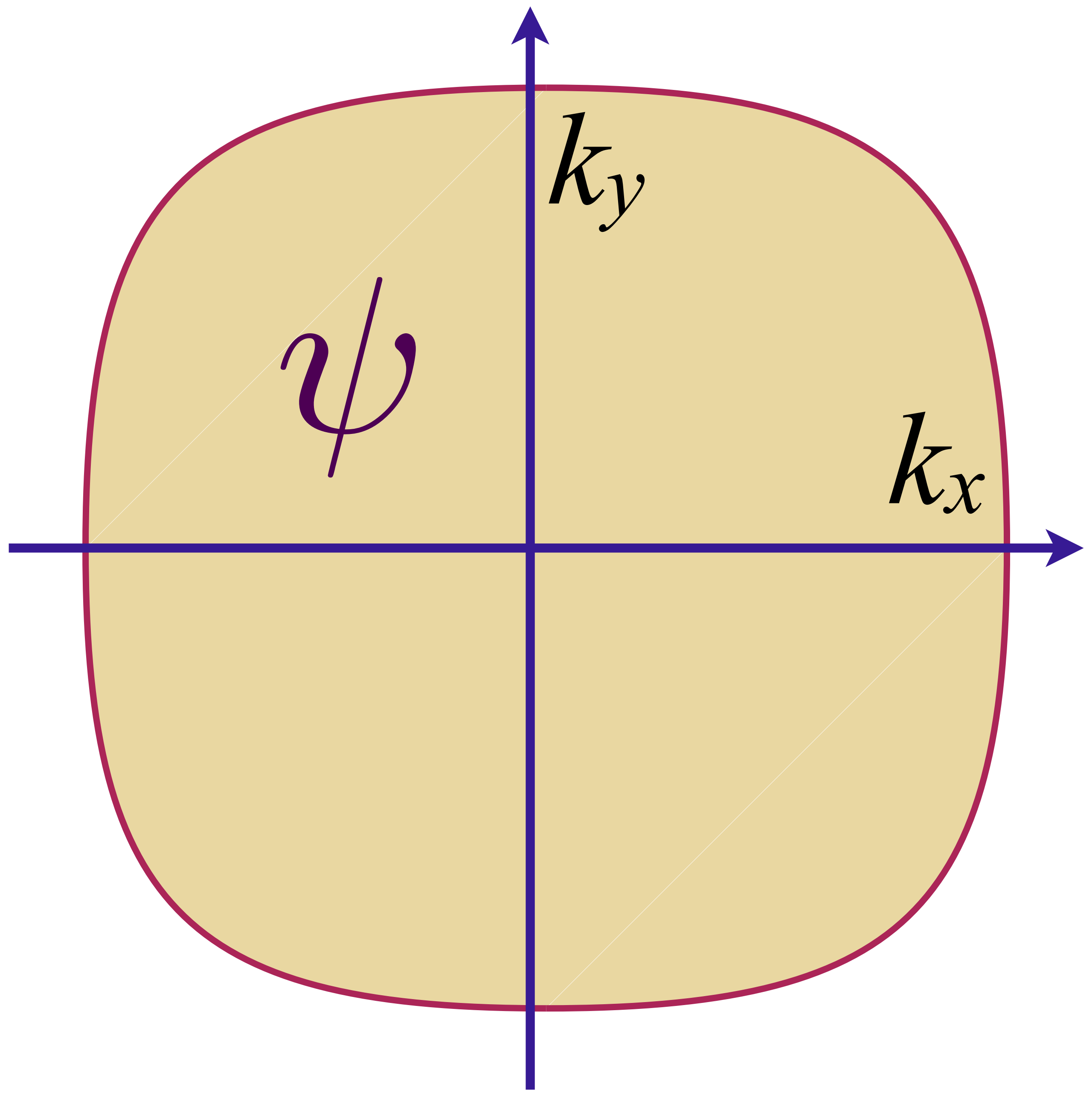
$$- J \, \psi^\dagger(\boldsymbol{r}) \psi^\dagger(\boldsymbol{r}) \psi(\boldsymbol{r}) \psi(\boldsymbol{r})$$

Fermi surface + critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ

e.g. Ising-nematic order



$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

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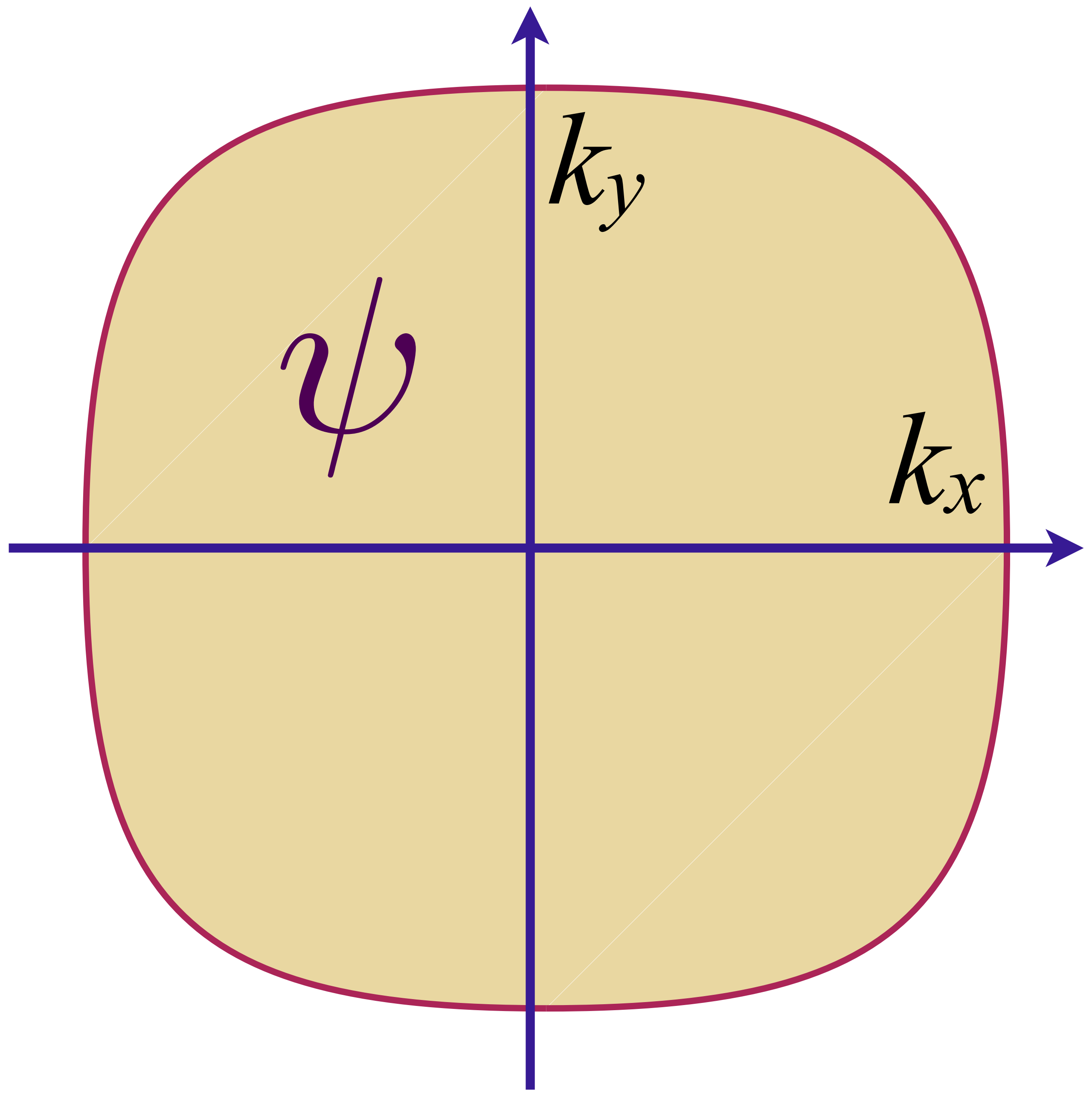
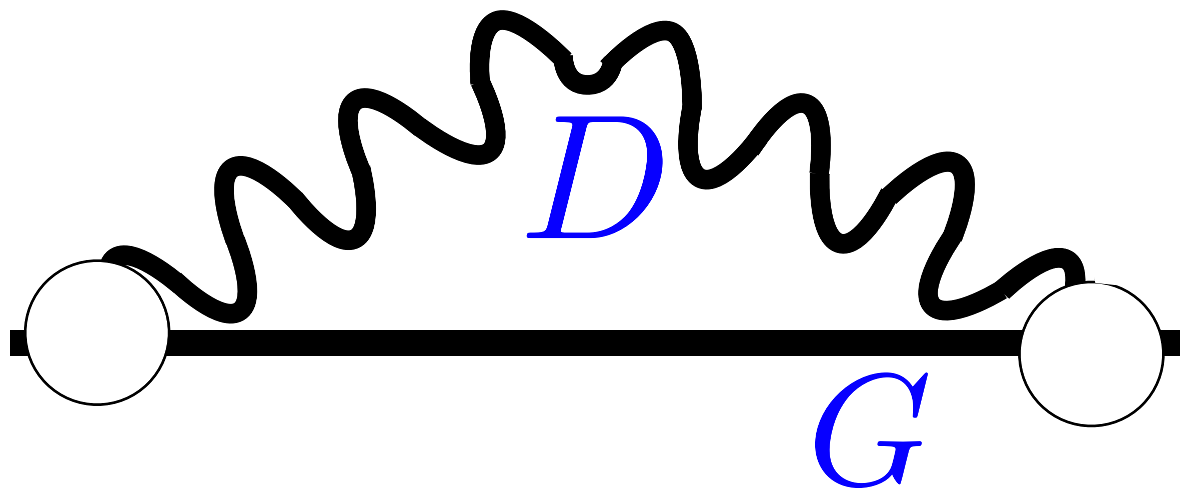
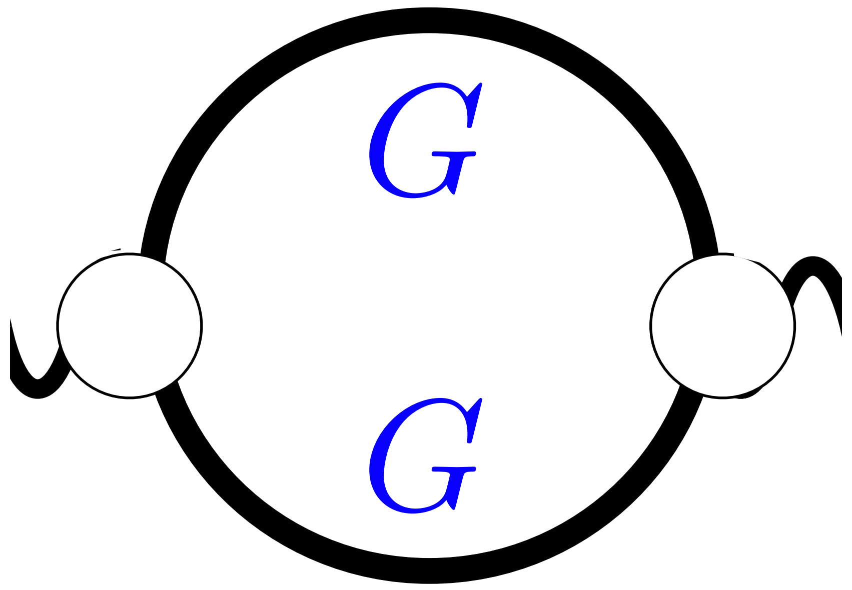


$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$\Sigma =$ 

$\Pi =$ 

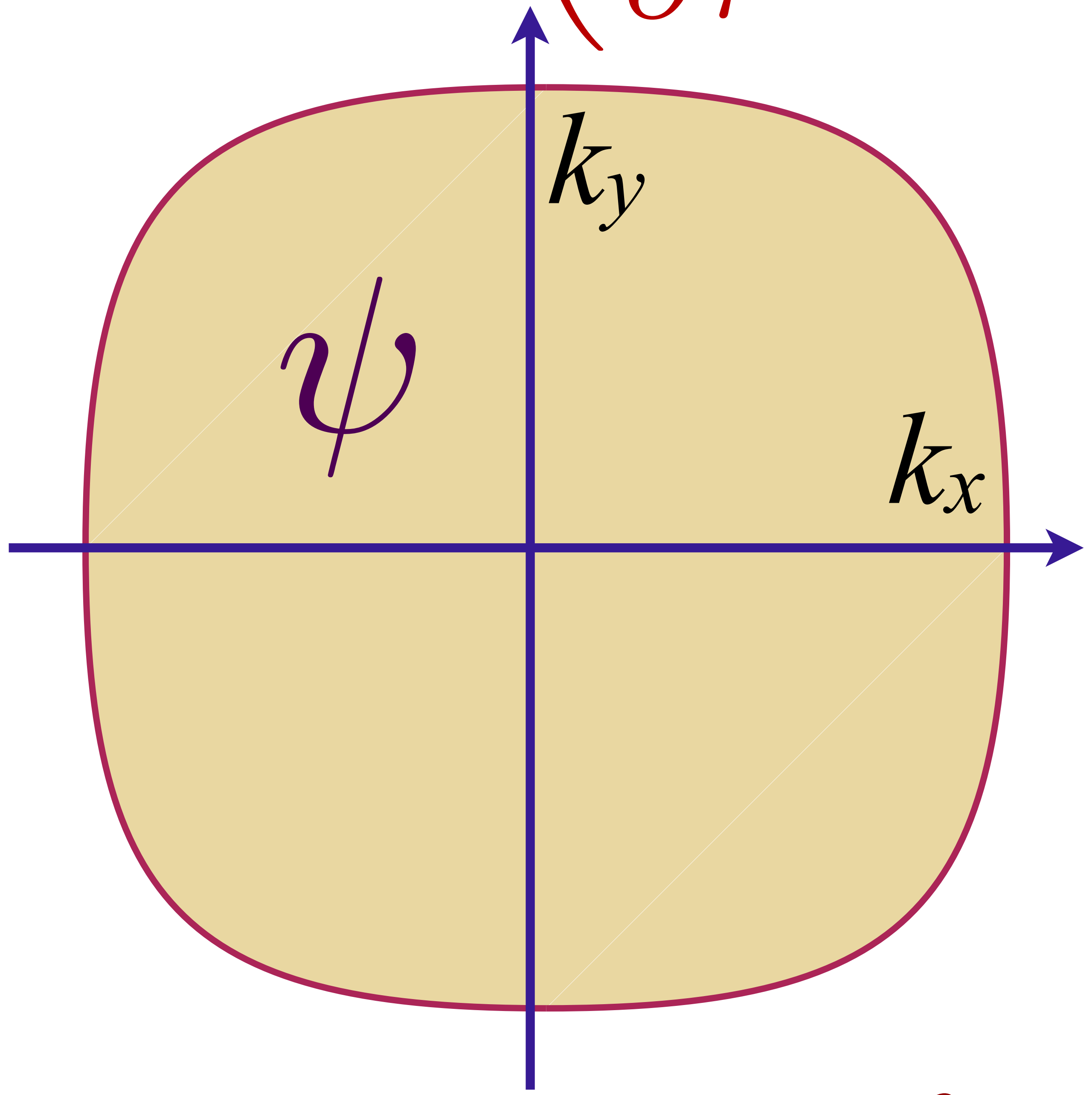
Solution of Migdal-Eliashberg equations for electron (G) and boson (D) Green's functions at small ω:

P.A. Lee (1989)

$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\text{sgn}(\omega)|\omega|^{2/3}, \quad G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)}, \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma|\Omega|/q}$$

Fermi surface + critical boson

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Transport—a perfect metal!

Conservation of momentum and fermion-boson drag imply:

$$\mathrm{Re}\,[\sigma(\omega)] = D\delta(\omega) + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
D. L. Maslov, V. I. Yudson, and A. V. Chubukov PRL **106**, 106403 (2011)
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S.S. PRB **94**, 045133 (2016)

Fermi surface + critical boson

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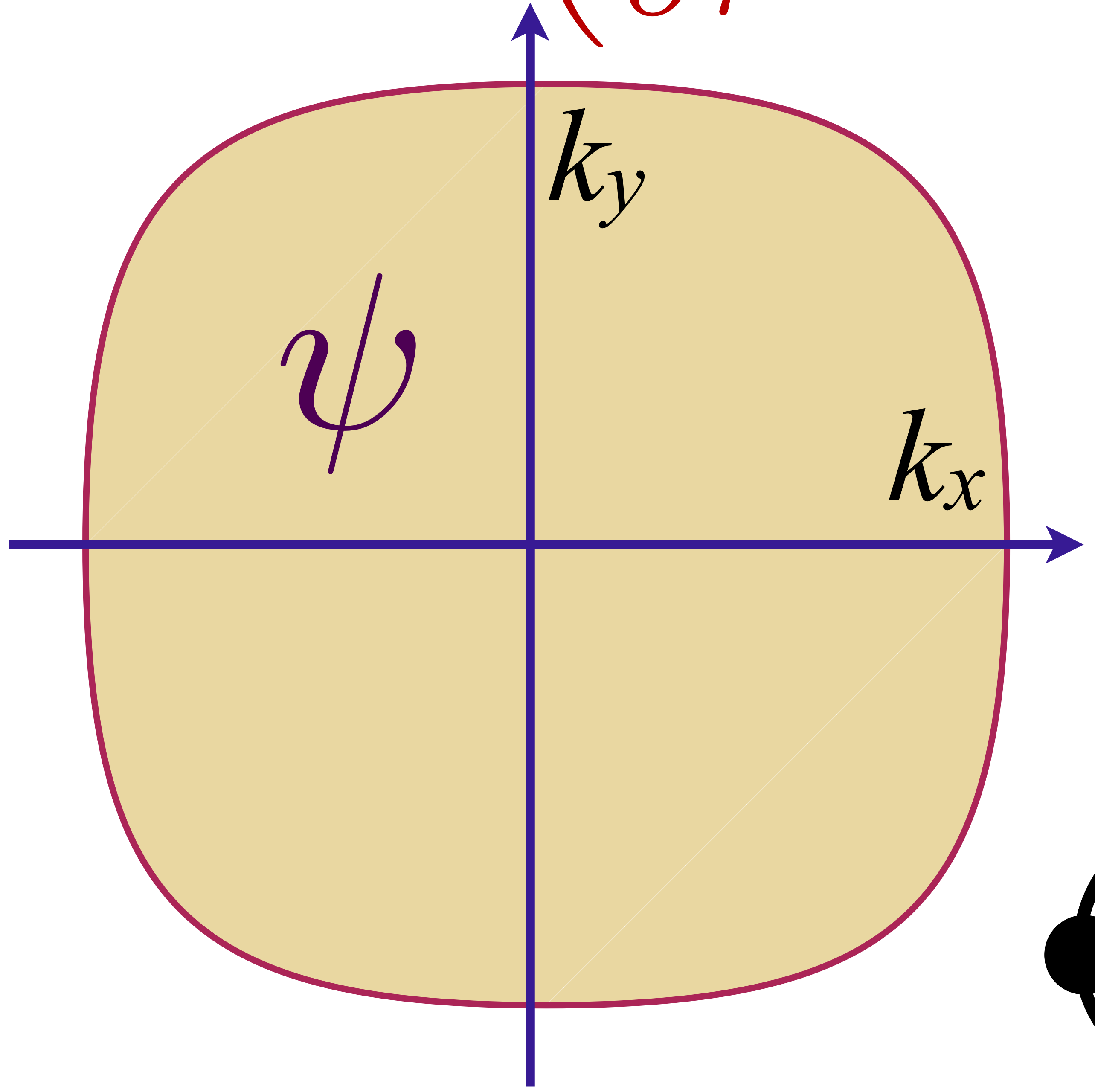


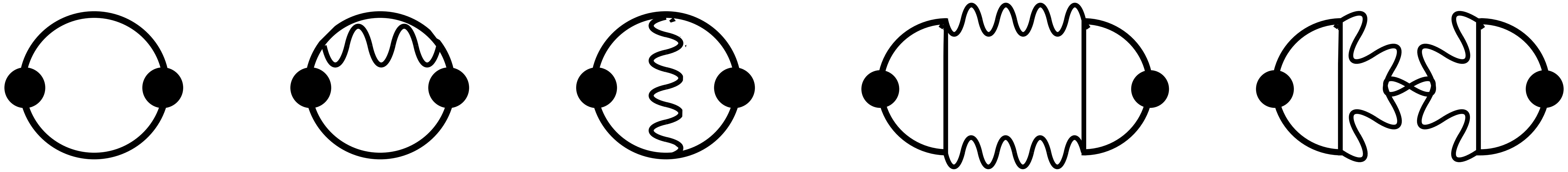
$$\mathrm{Re}\left[\sigma(\omega)\right] = C\,|\omega|^{-2/3}$$

a critical boson ϕ

e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Optical conductivity—Diagrams



Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, P. A. Lee, PRB **50**, 17917 (1994).

Fermi surface + critical boson

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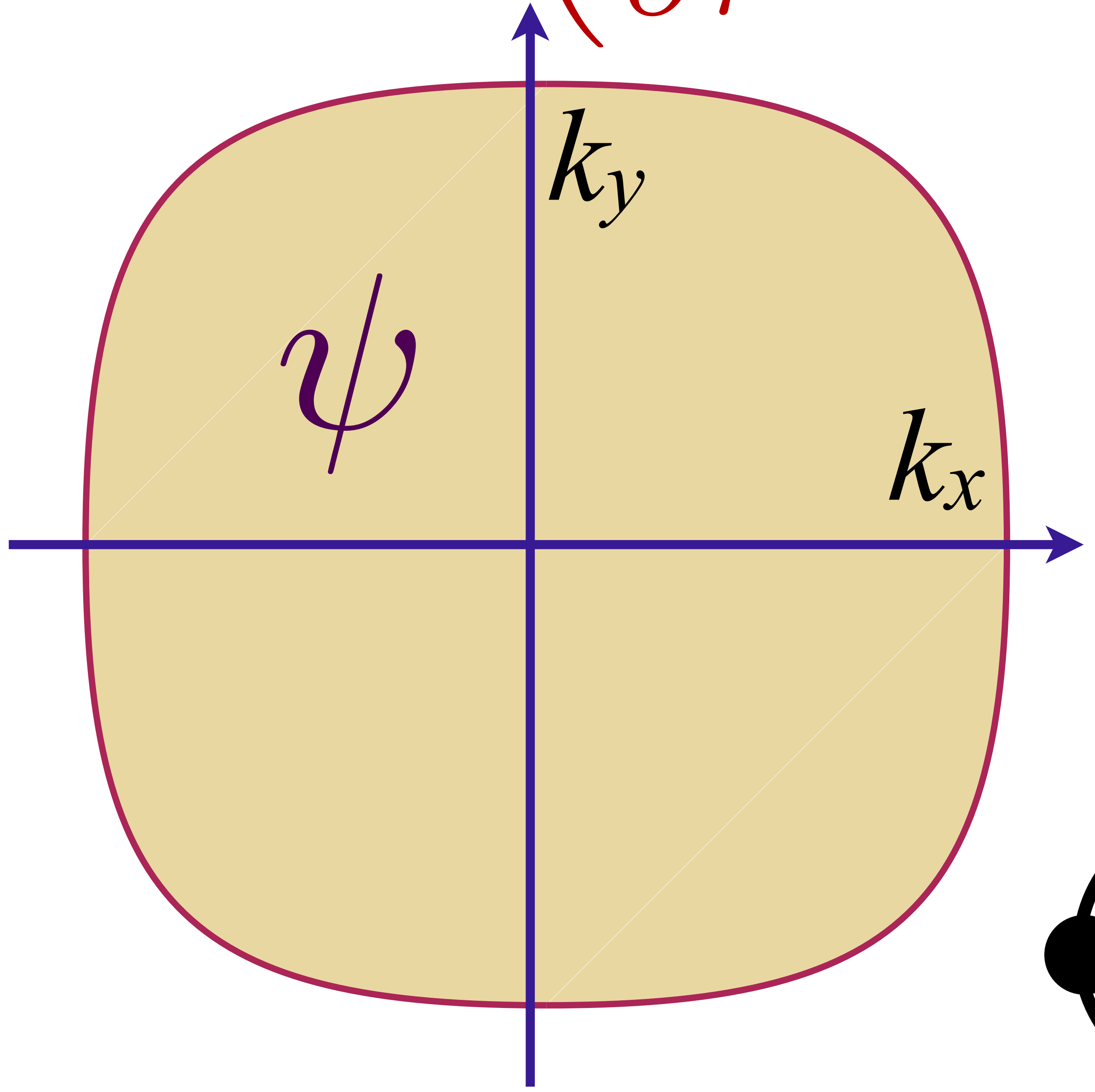

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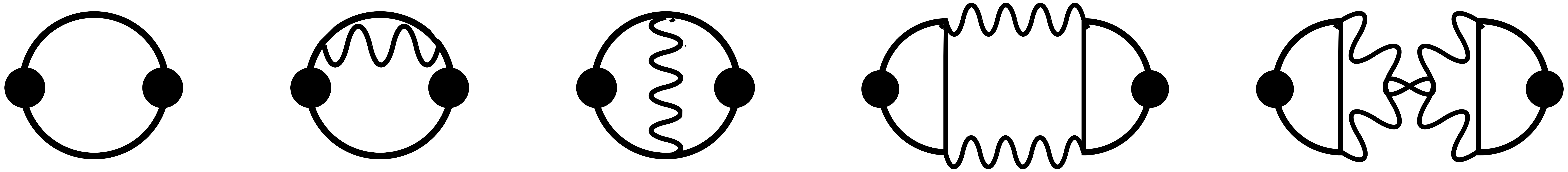


$$\mathrm{Re}\,[\sigma(\omega)] = C\,|\omega|^{-2/3}$$

Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, P. A. Lee, PRB **50**, 17917 (1994).

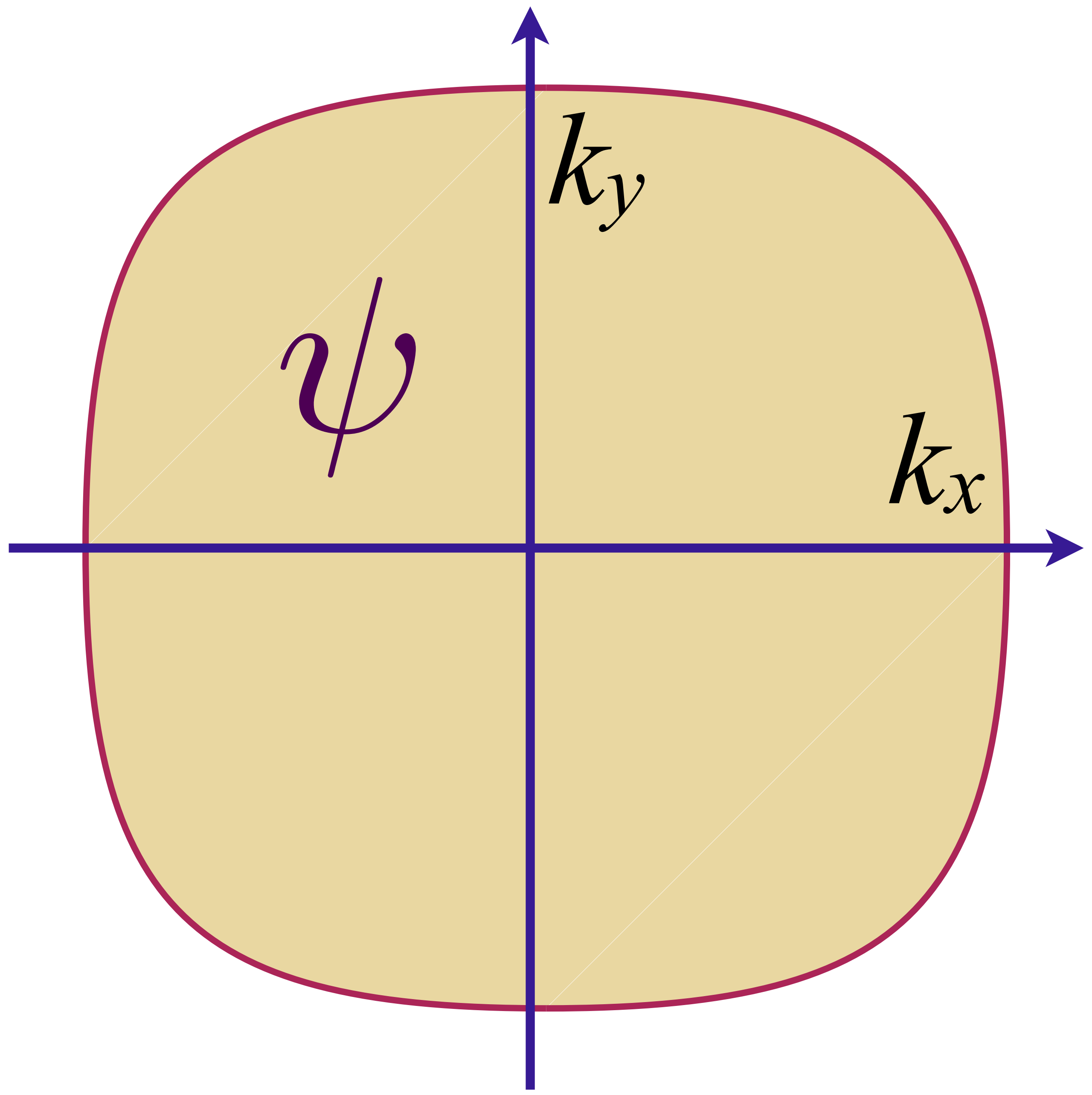
$$C = 0\,; \quad \sigma(\omega) = iD/(\omega - \omega_c) + \omega^0 + \ldots$$

Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S., PRB **106**, 115151 (2022)
Haoyu Guo, Davide Valentinis, J. Schmalian, S.S., Aavishkar Patel, arXiv:2308.01956
D.L. Maslov and A.V. Chubukov, Rep. Prog. Phys. **80**, 026503 (2017)
Zhengyan Darius Shi, Dominic V. Else, Hart Goldman, T. Senthil, SciPost Phys. **14**, 113 (2023)

Universal theory of strange metals

Fermi surface + critical boson with potential disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ

e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Fermi surface + critical boson with potential disorder

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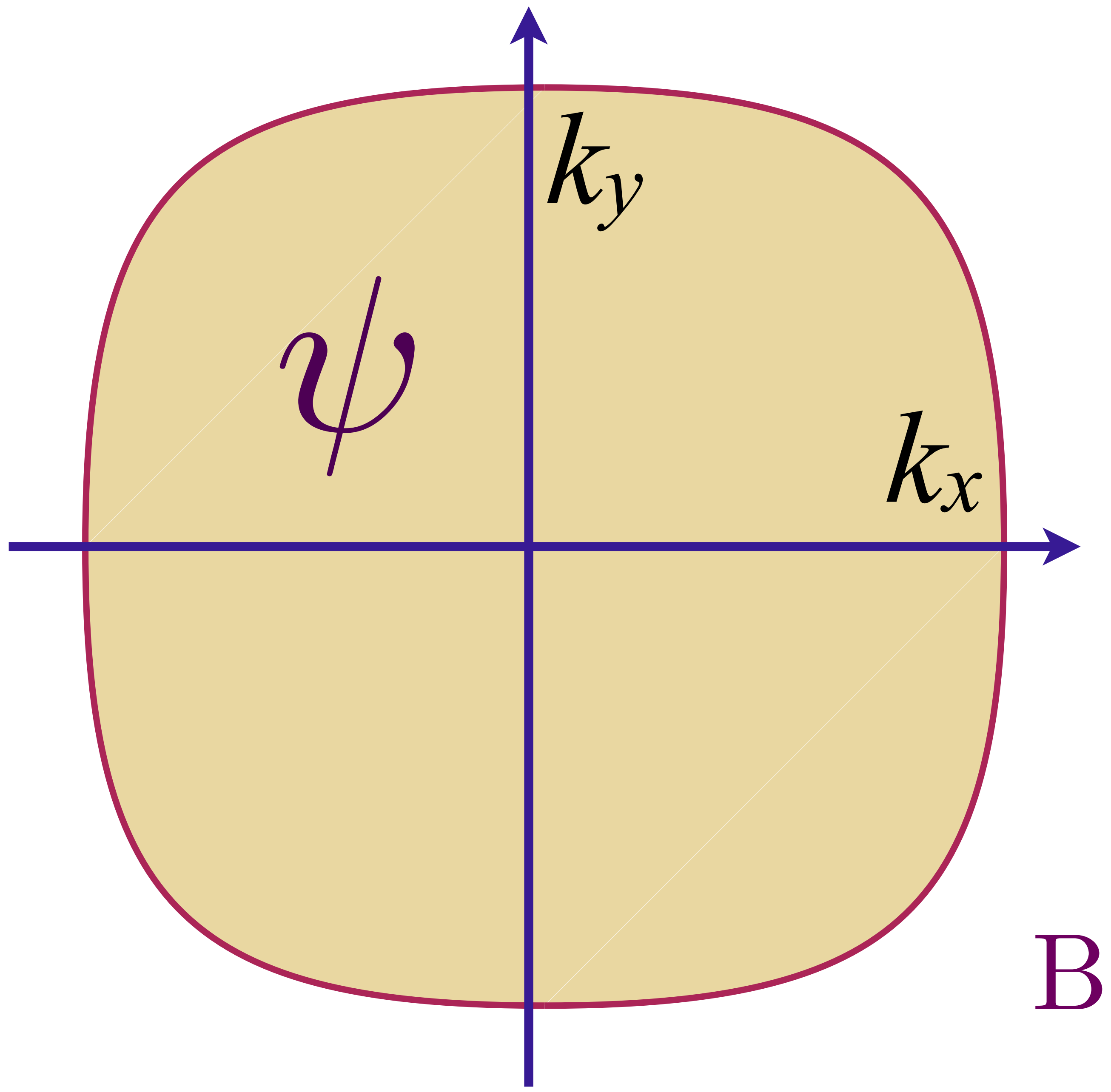

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Boson self energy: $\Pi \sim -\dfrac{g^2}{v^2}|\Omega|, \qquad D(q, i\Omega) = \dfrac{1}{q^2 + \gamma|\Omega|}$

Fermion self energy: $\Sigma(i\omega) \sim -iv^2 \mathrm{sgn}(\omega) - i\dfrac{g^2}{v^2}\omega \ln(1/|\omega|); \quad \dfrac{1}{\tau_{\rm in}(\varepsilon)} \sim |\varepsilon|$

Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat

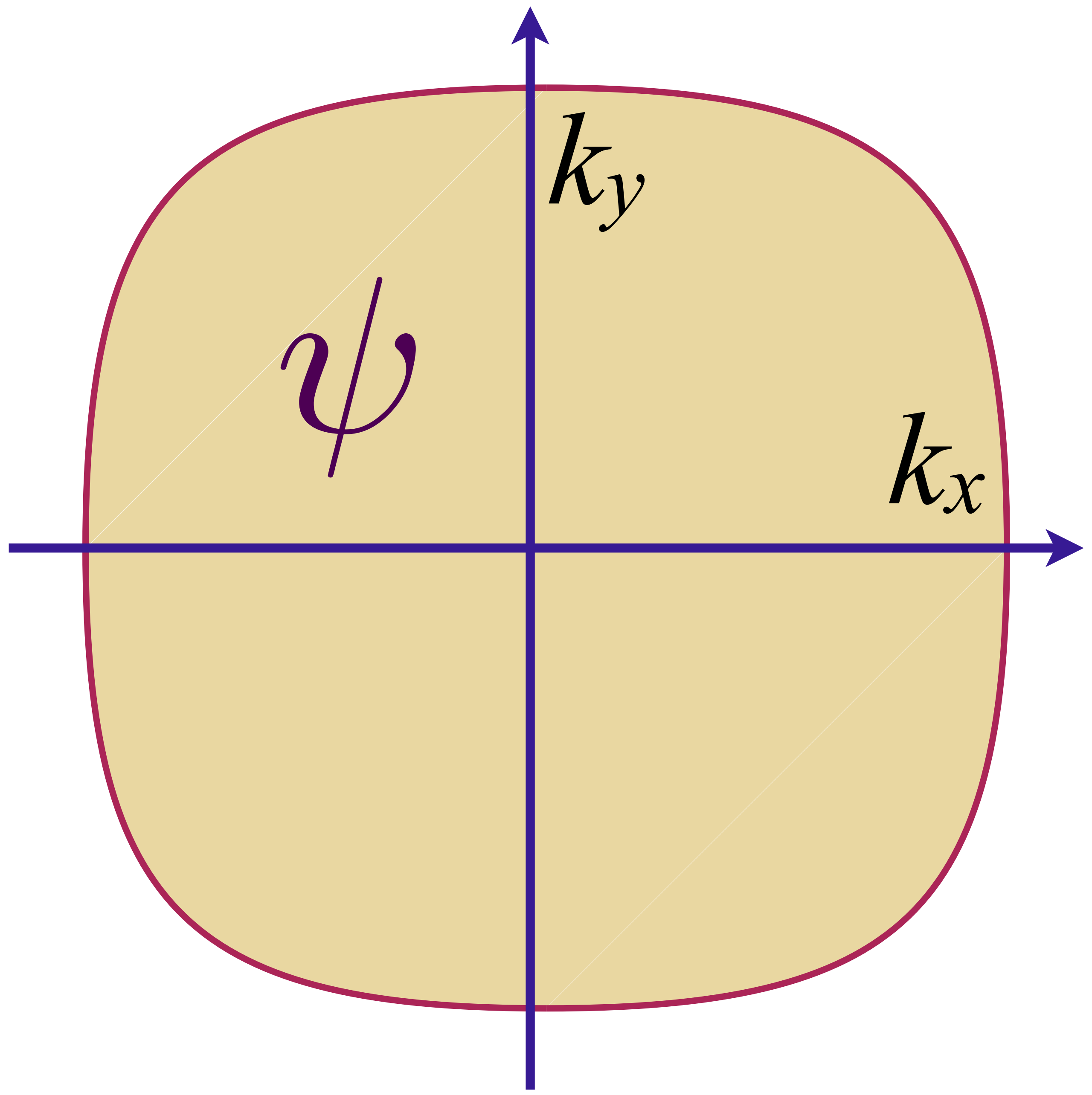
Halperin Lee Read (1993)

Fermi surface + critical boson with potential disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ

e.g. Ising-nematic order



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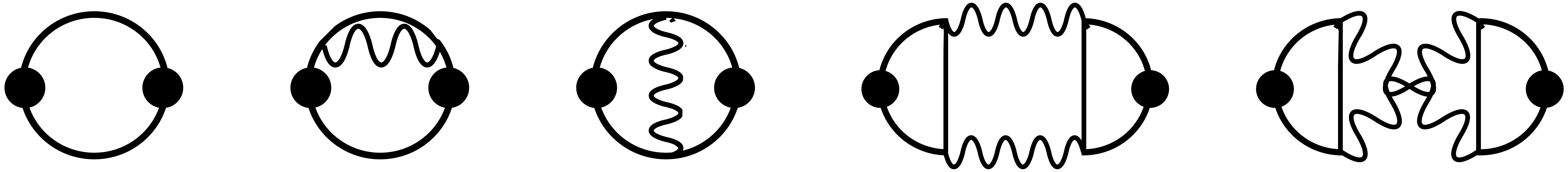


Conductivity: $\sigma(\omega) \sim \dfrac{1}{\dfrac{1}{\tau_{\text{trans}}} - i\omega}$; $\dfrac{1}{\tau_{\text{trans}}} \sim v^2$

MFL self-energy cancels in transport.

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:

Potential disorder v

A marginal Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:

Potential disorder v

A marginal Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:

Interaction disorder g'

A marginal Fermi liquid AND strange metal transport

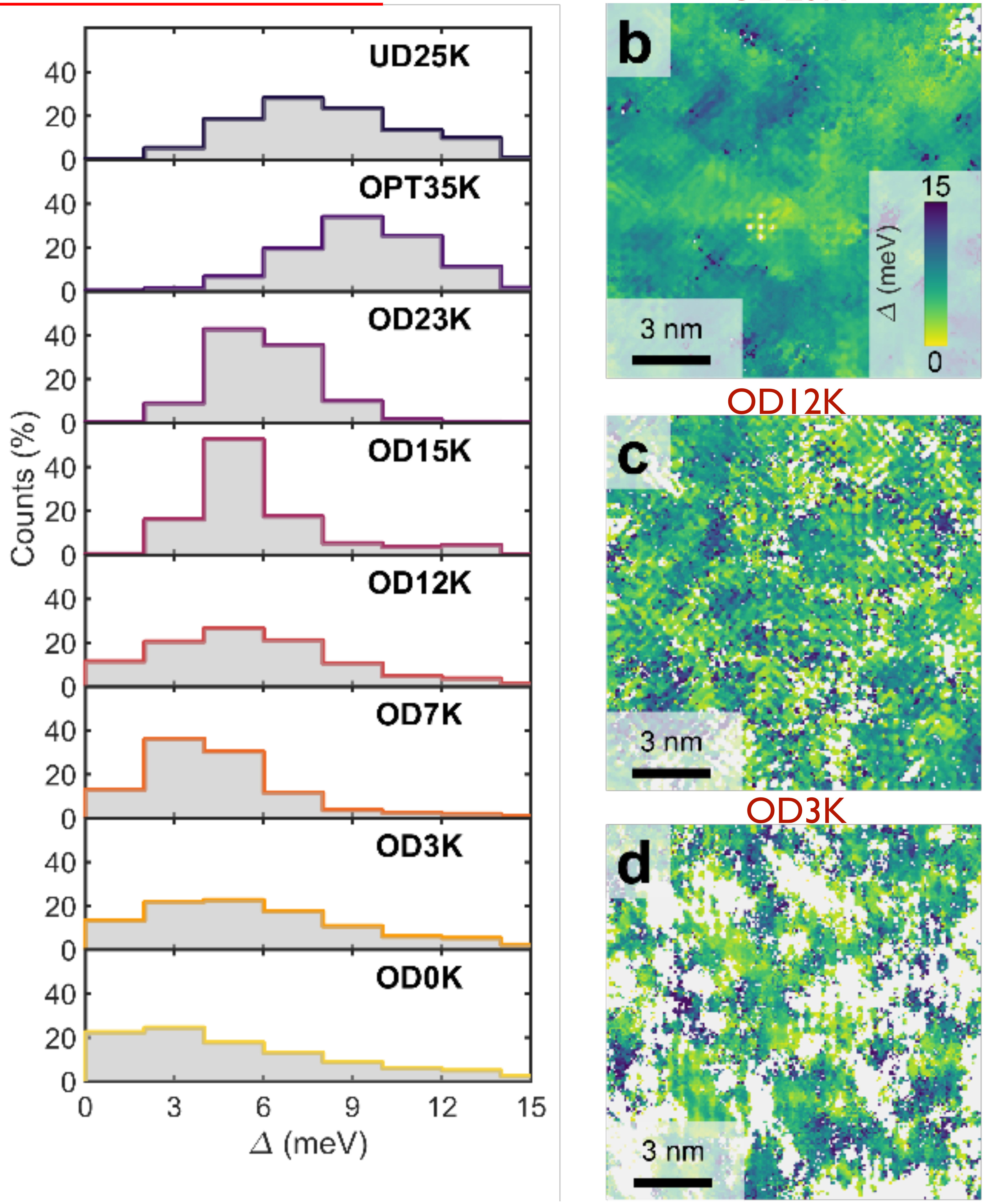
Spatially random interactions!

Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(Pb,Bi)_2Sr_2CuO_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge, Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos, Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan

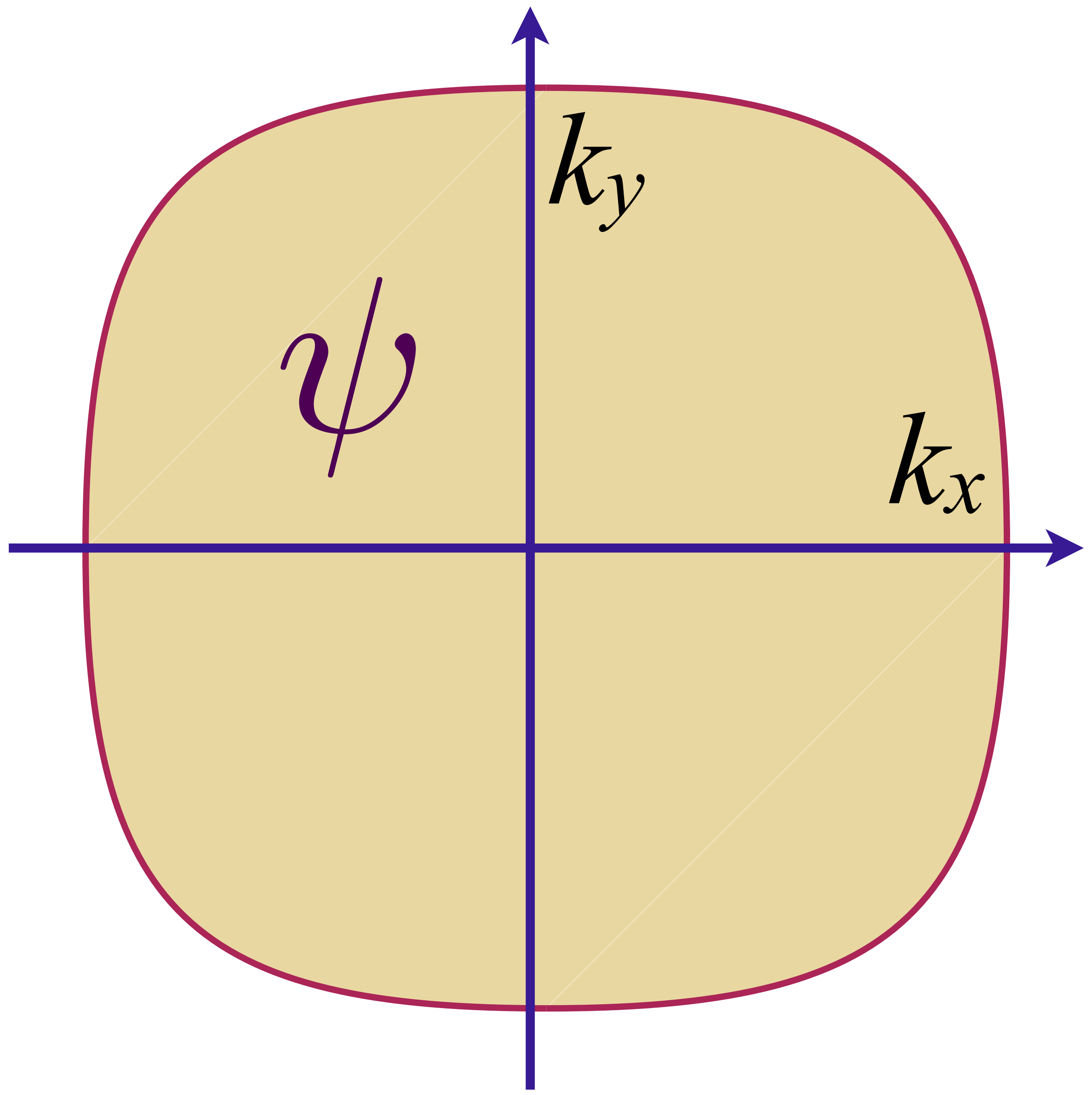
Nature Materials ***22***, *703 (2023)*

In the Hubbard model, disorder in the hopping t_{ij} generates disorder in the exchange interaction $J_{ij} = \dfrac{t_{ij}^2}{U}$.



Fermi surface + critical boson with potential disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ

e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Fermi surface + critical boson with potential and interaction disorder

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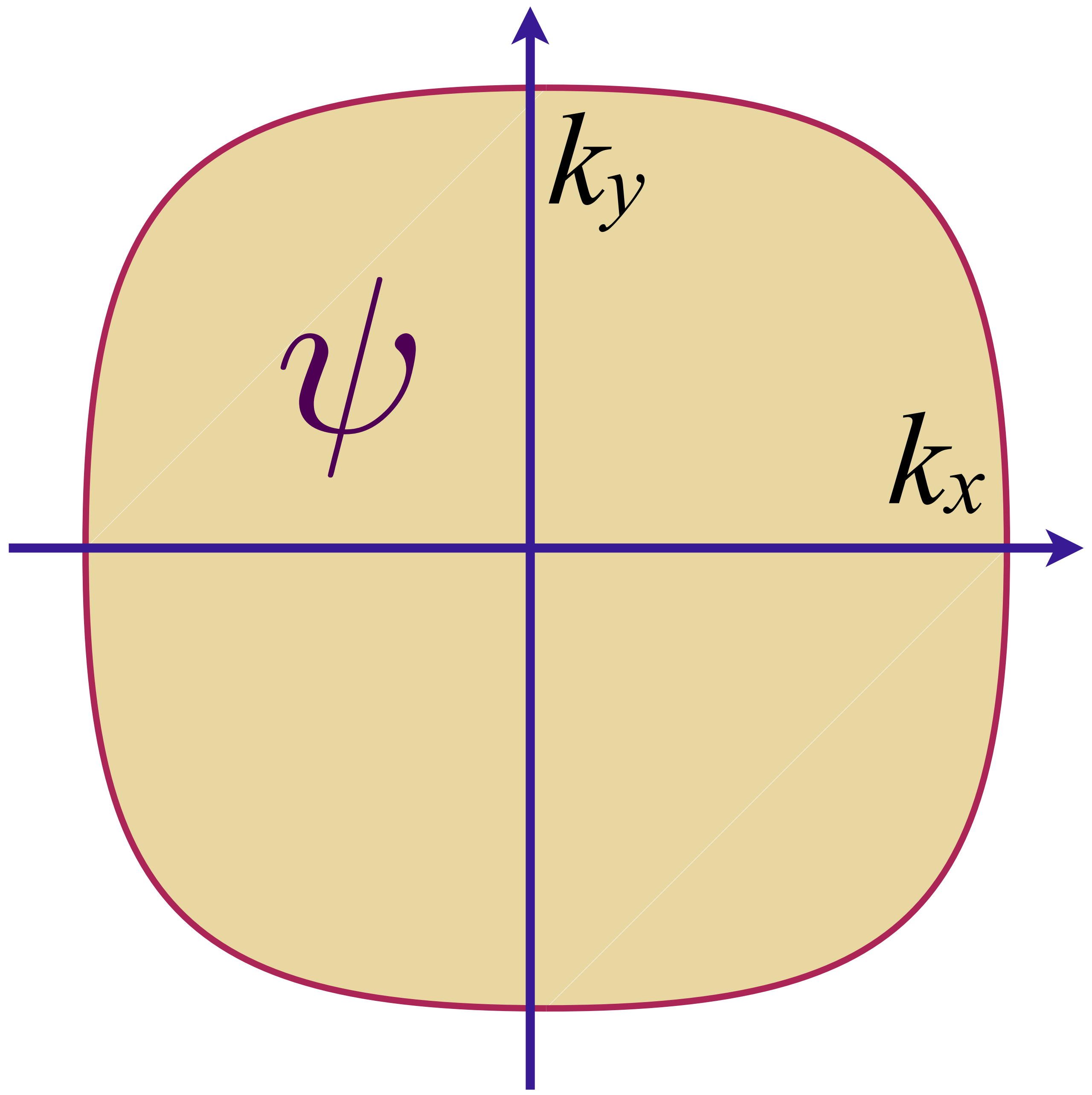


a critical boson ϕ

e.g. Ising-nematic order

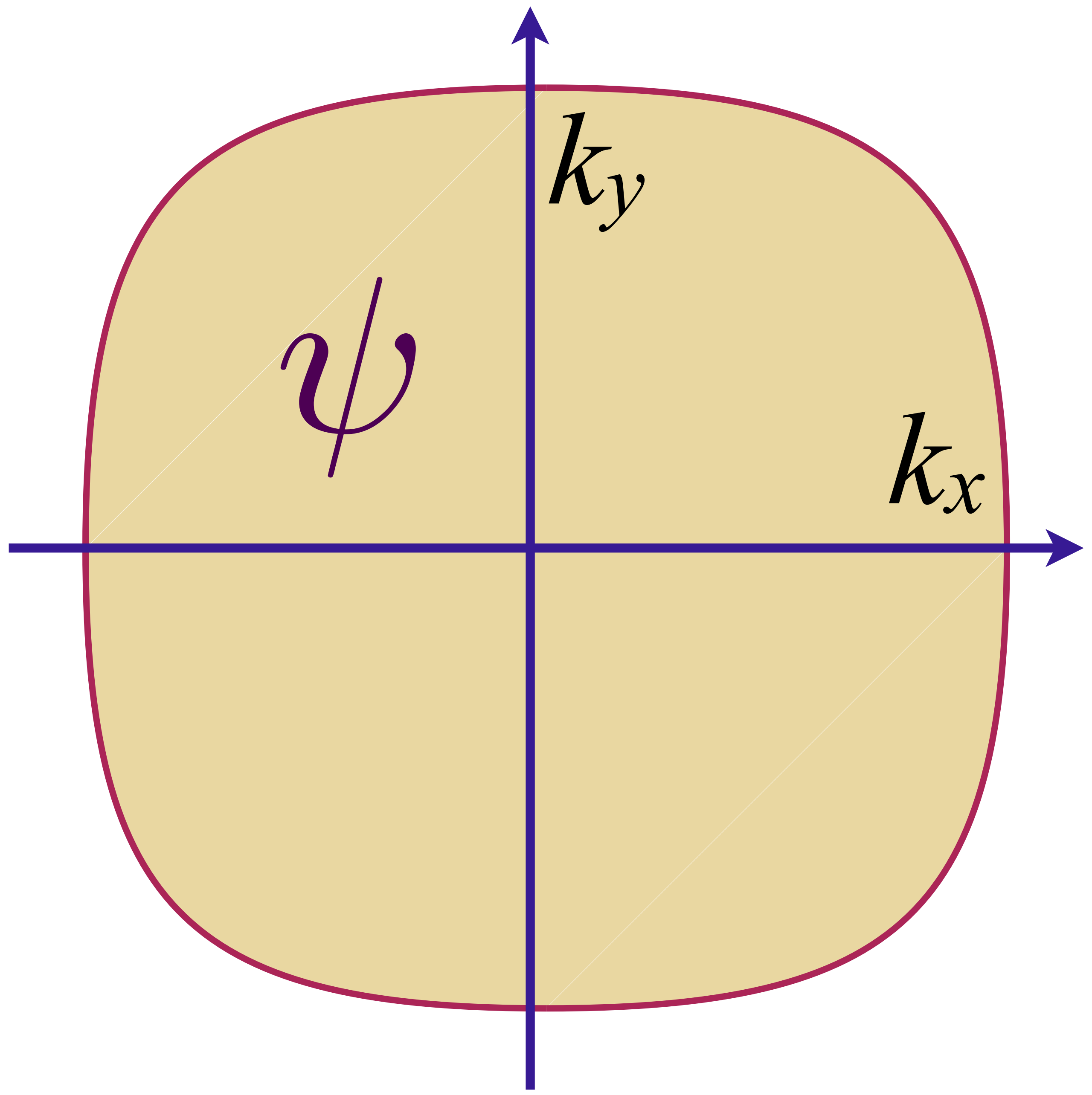
$$\frac{[\phi(\boldsymbol{r})]^2}{J + J'(\boldsymbol{r})} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

ϕ^2 "mass" disorder $J'(\boldsymbol{r})$ is strongly relevant;
rescale ϕ to move disorder to the Yukawa coupling;

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$

E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, PRB **105**, 235111 (2022)

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

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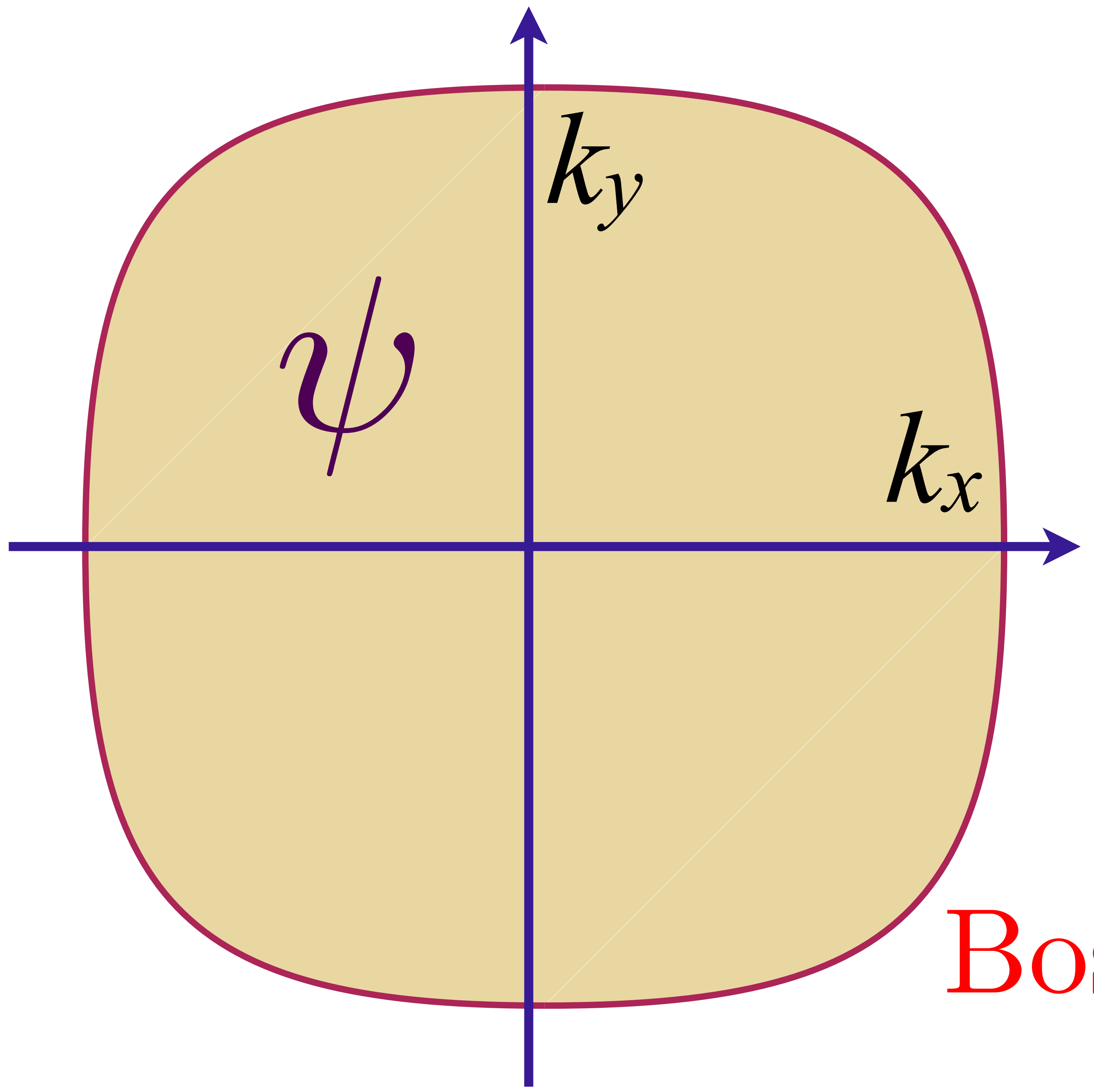

$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

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Boson Green's function: $D(q, i\Omega) \sim 1/(q^2 + \gamma|\Omega|)$

Fermion self energy:

$$\Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i \left(\frac{g^2}{v^2} + g'^2 \right) \omega \ln(1/|\omega|); \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega|$$

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat

Fermi surface + critical boson with potential and interaction disorder

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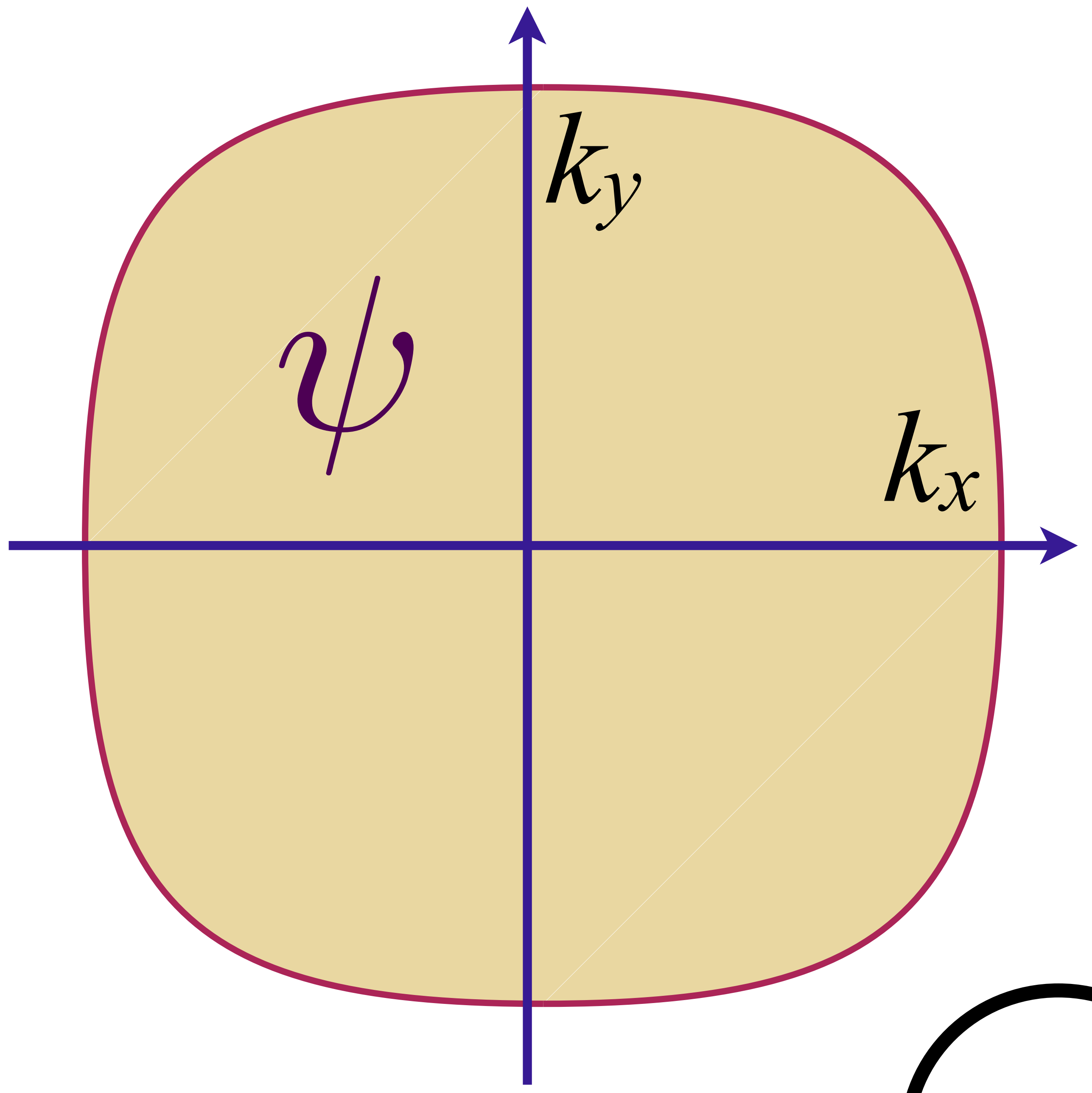
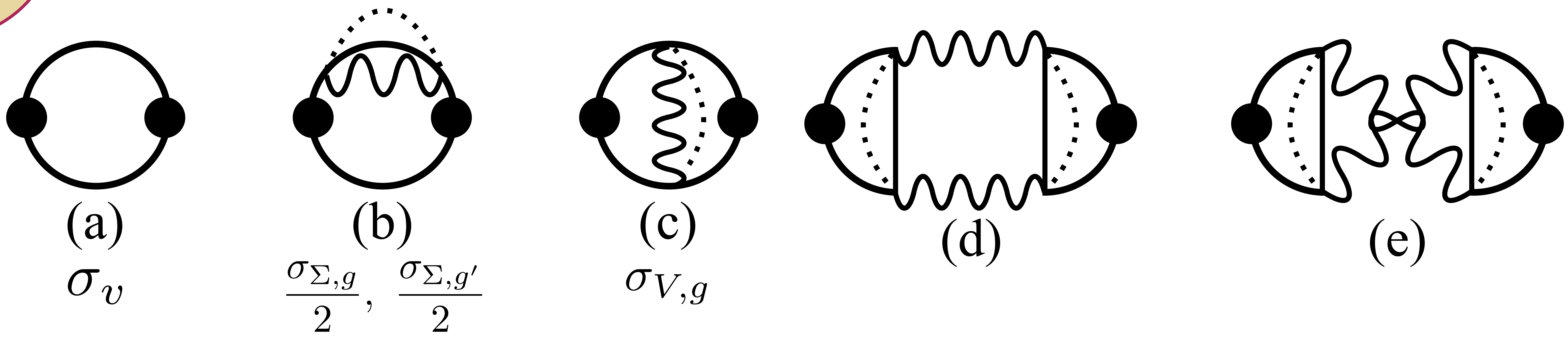


$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Conductivity:



+ all ladders and bubbles…..

Fermi surface + critical boson with potential and interaction disorder

$$\text{Conductivity: } \sigma(\omega) \sim \frac{1}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \, \dfrac{m^*_{\rm trans}(\omega)}{m}}$$

$$\frac{1}{\tau_{\rm trans}(\omega)} \sim v^2 + g'^2 |\omega| \quad ; \quad \frac{m^*_{\rm trans}(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

$$\text{Electron Green's function: } G(\omega) \sim \frac{1}{\omega \, \dfrac{m^*(\omega)}{m} - \varepsilon(\boldsymbol{k}) + i \left(\dfrac{1}{\tau_e} + \dfrac{1}{\tau_{\rm in}(\omega)} \right) \mathrm{sgn}(\omega)}$$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi} \left(\frac{g^2}{v^2} + g'^2 \right) \ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2; Transport insensitive to g; Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.

Observable properties:

1. Resistivity $\rho(T) \sim v^2 + g'^2 T$.

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2. Optical conductivity

$$\sigma(\omega) = \frac{K}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m^*_{\text{trans}}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon……A. Georges, Nat. Commun. **14**, 3033 (2023)

Observable properties:

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B. Michon……A. Georges, Nat. Commun. **14**, 3033 (2023)

3. Photoemission: nearly marginal Fermi liquid electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim |\omega| \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

T.J. Reber….D. Dessau, Nature Communications **10**, 5737 (2019)

Observable properties:

1. Resistivity $\rho(T) \sim v^2 + g'^2 T$.

2. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon……A. Georges, Nat. Commun. **14**, 3033 (2023)

3. Photoemission: nearly marginal Fermi liquid electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim |\omega| \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

T.J. Reber….D. Dessau, Nature Communications **10**, 5737 (2019)

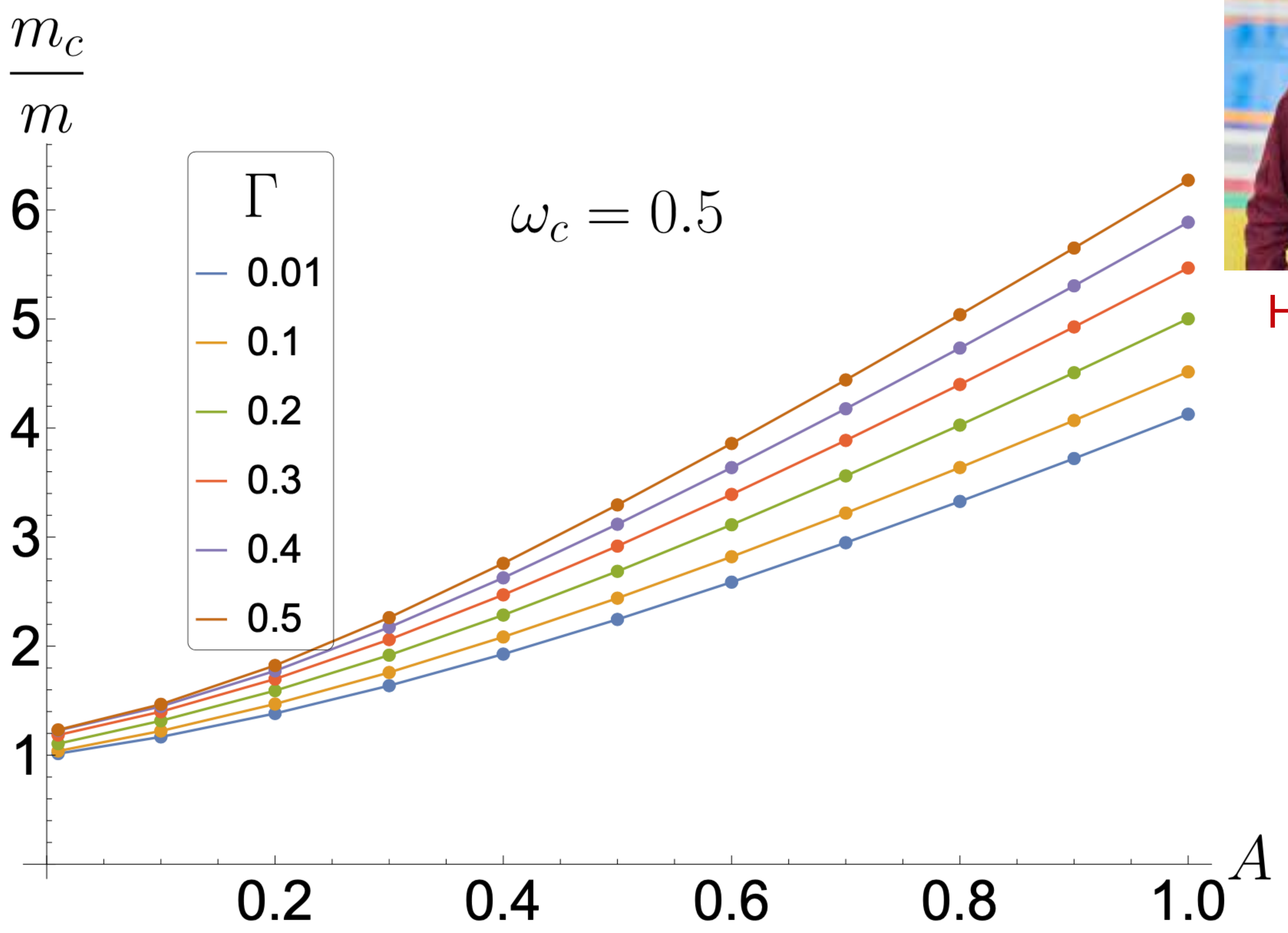
4. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, RMP (2022)

Observable properties:

5. Cyclotron resonance.

Haoyu Guo,
Davide Valentinis,
Jorg Schmalian,
S.S.,
Aavishkar Patel,
arXiv:2308.01956



Haoyu Guo

FIG. 8: Cyclotron mass renormalization $\frac{m_c}{m} \equiv \frac{\omega_c}{\omega_c^*}$ at different values of potential disorder (represented by scattering rate Γ) and interaction disorder (represented by A). The bare cyclotron frequency is $\omega_c = 0.5$. When $\omega_c > \Gamma$, the cyclotron mass is only weakly renormalized by Γ, but is more sensitive to A.

Observable properties:

5. Cyclotron resonance.

Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Aavishkar Patel, arXiv:2308.01956

6. Shot noise. Fano factor:

$$F = \frac{S}{2\sigma_r WV/L} = \frac{1}{3}\left(\frac{1+\pi g'^2 V/(4(2\pi)^2 v^2)}{1+\pi g'^2 V/(2(2\pi)^2 v^2)}\right) .$$

Alex Nikolaenko, S.S., Aavishkar Patel, arXiv:2305.02336

Alexander Nikolaenko

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Alex Nikolaenko, S.S., Aavishkar Patel, arXiv:2305.02336

7. Magnetotransport: Hall effect and magnetoresistance

Davide Valentinis, Haoyu Guo, Chenyuan Li,Ilya Esterlis, S.S., Jorg Schmalian, Aavishkar Patel, to appear...

Davide Valentinis

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Alex Nikolaenko, S.S., Aavishkar Patel, arXiv:2305.02336

7. Magnetotransport: Hall effect and magnetoresistance

Davide Valentinis, Haoyu Guo, Chenyuan Li,Ilya Esterlis, S.S., Jorg Schmalian, Aavishkar Patel, to appear…

8. Non-linear optics.

Serhii Kryhin, S.S., Pavel Volkov, to appear…

Observable properties:

9. Spectral functions and superfluid density upon onset of superconductivity.

Chenyuan Li, Davide Valentinis, Haoyu Guo, S.S., Jorg Schmalian, Aavishkar Patel, Ilya Esterlis to appear…

Chenyuan Li

Observable properties:

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Chenyuan Li, Davide Valentinis, Haoyu Guo, S.S., Jorg Schmalian, Aavishkar Patel, Ilya Esterlis to appear…

10. Thermoelectric response.

Peter Lunts, S.S., Aavishkar Patel…………..

Peter Lunts

Observable properties:

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Chenyuan Li, Davide Valentinis, Haoyu Guo, S.S., Jorg Schmalian, Aavishkar Patel, Ilya Esterlis to appear…

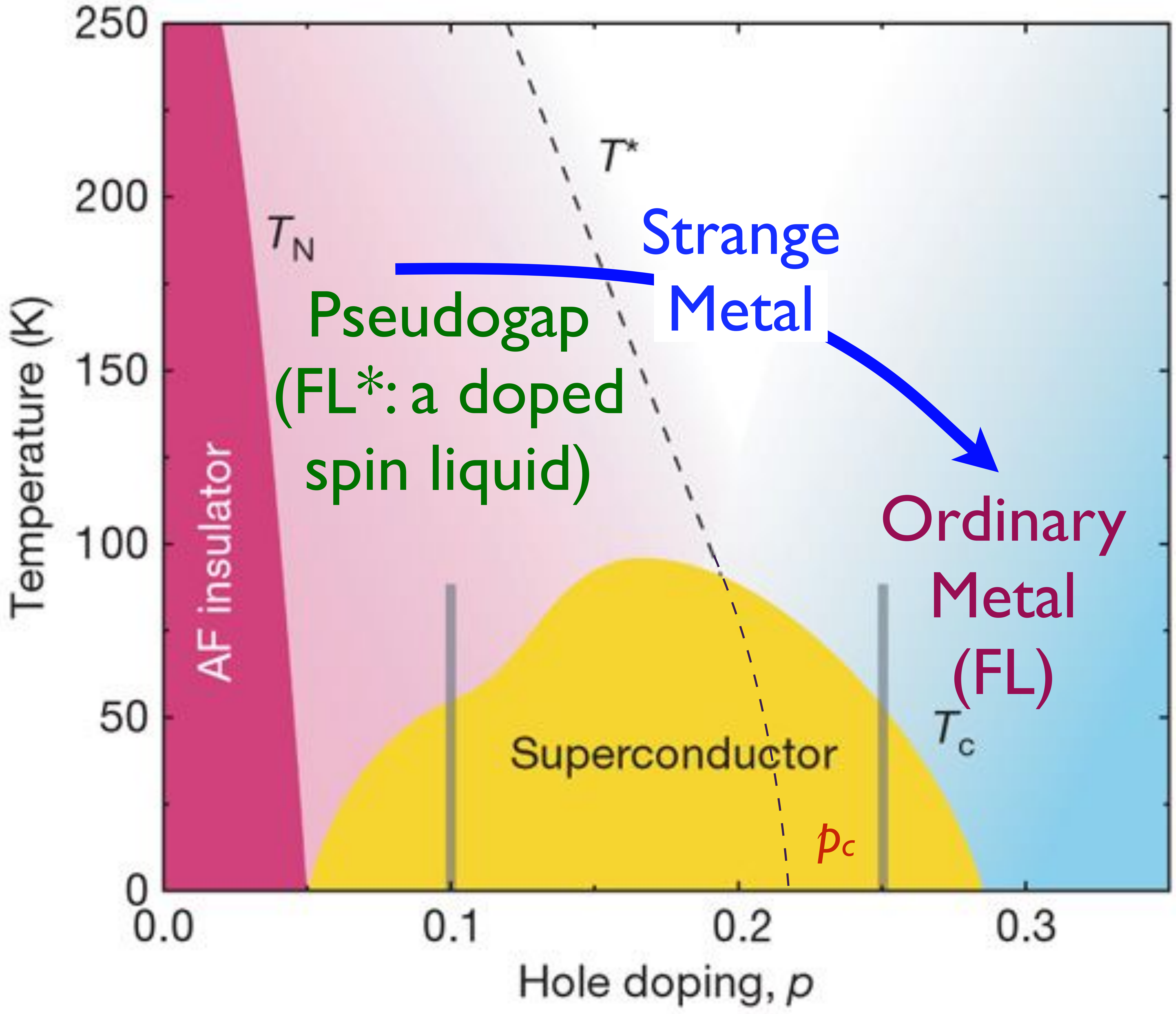
10. Thermoelectric response.

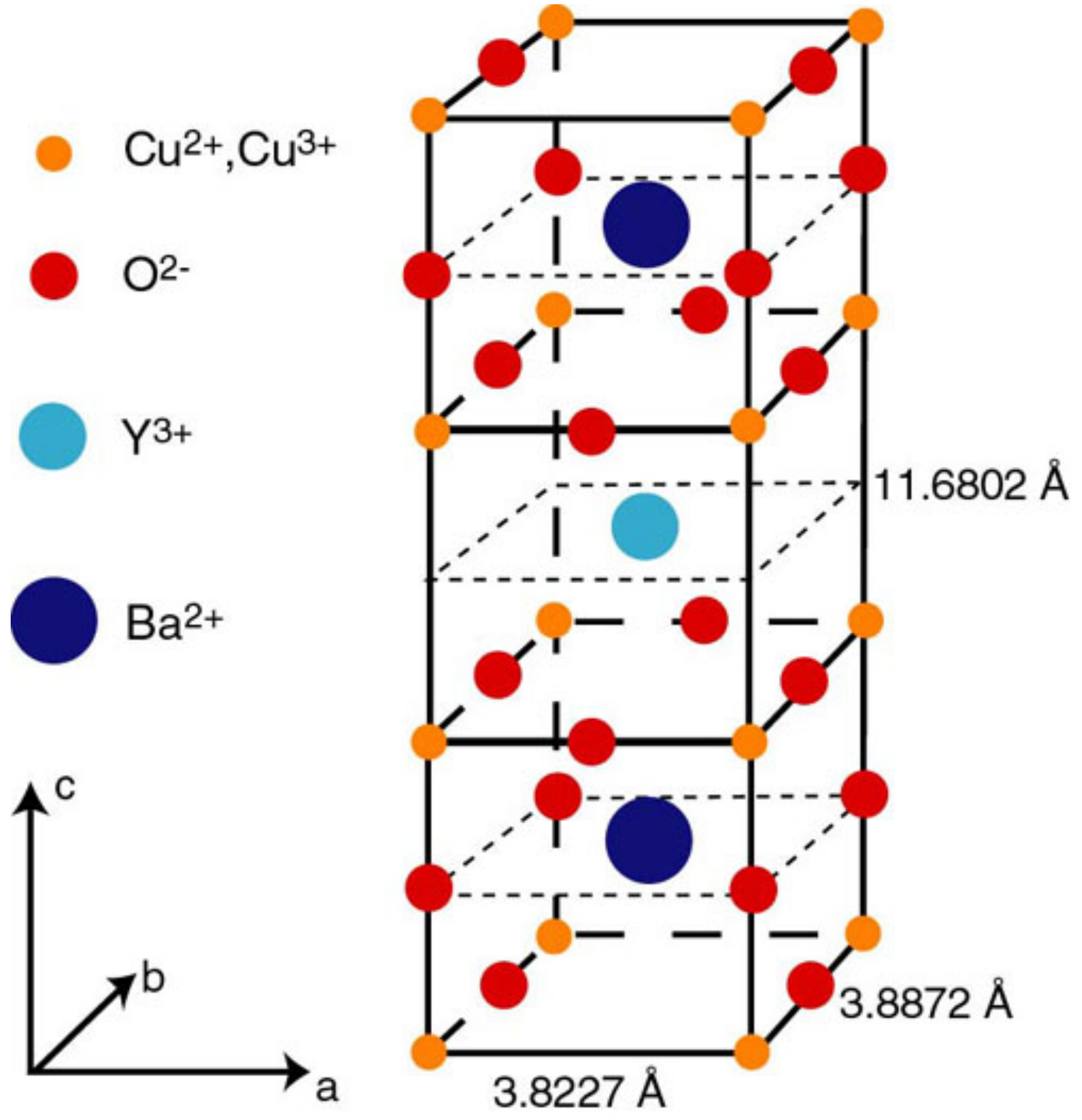
Peter Lunts, S.S., Aavishkar Patel…………..

11. Dynamic density fluctuations (for EELS).

Xuepeng Wang, Debanjan Chowdhury, Phys. Rev. B **107**, 125157 (2023)

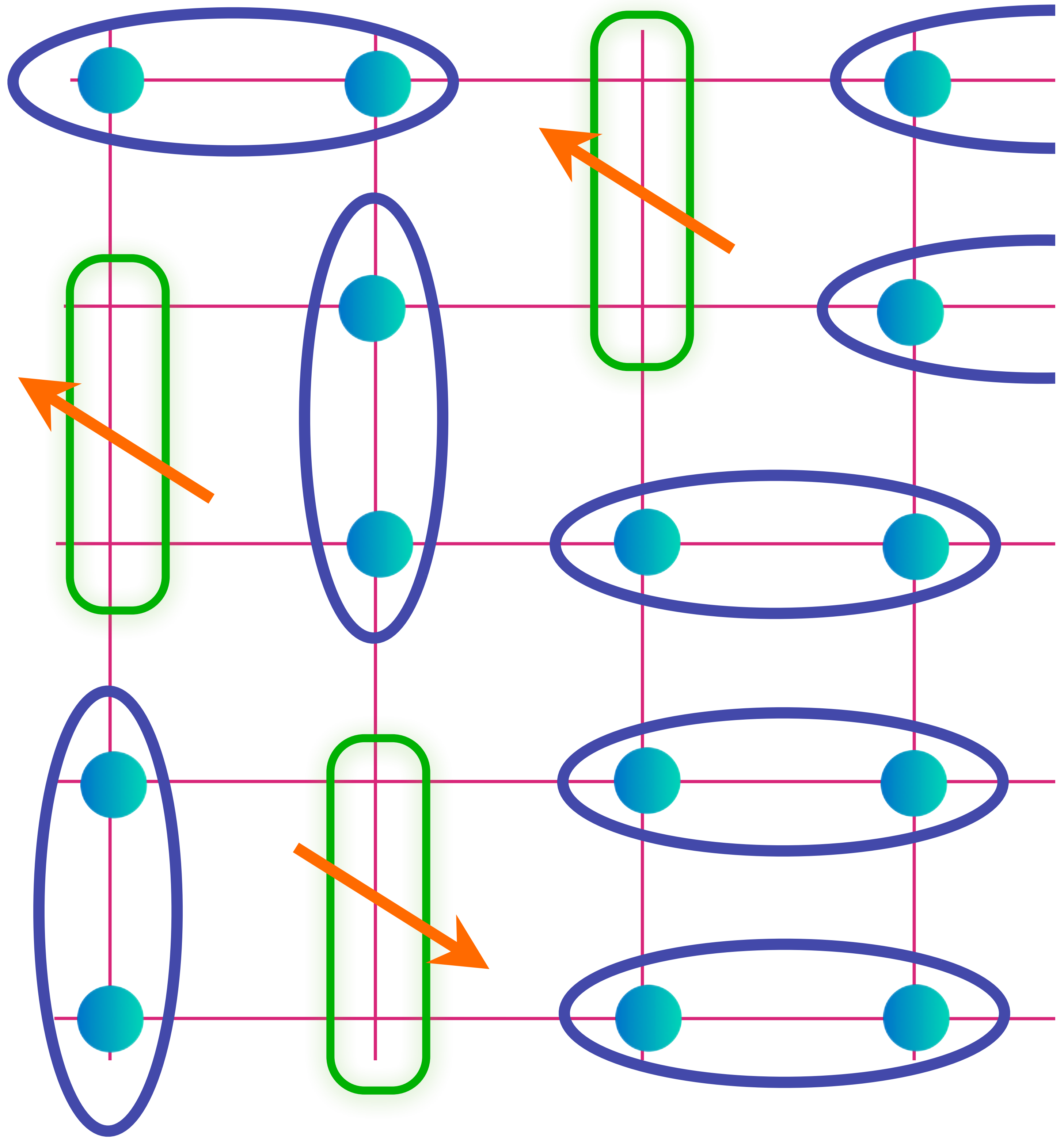
Application to the cuprates


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Strange
Metal
Pseudogap
(FL*: a doped
spin liquid)
Ordinary
Metal
(FL)
Superconductor
T_c
p_c

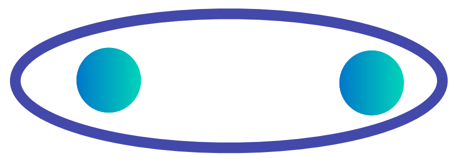

Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

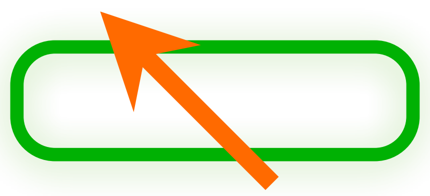
$YBa_2Cu_3O_{6+x}$

Zhang and Sachdev *Phys. Rev. Research* **2**, 023172 (2020); *Phys. Rev. B* **102**, 155124 (2020)



Φ Higgs boson with the fundamental gauge charge of an emergent SU(2) gauge field.

 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

Large Fermi surface of size $1 + p$

Small Fermi surface of size p $+$ spin liquid.

FL*

$\langle \Phi \rangle \neq 0$

FL

$\langle \Phi \rangle = 0$

doping p

QMC results: beyond SYK-large N

Aavishkar Patel

Michael Albergo

Peter Lunts

Model for QMC

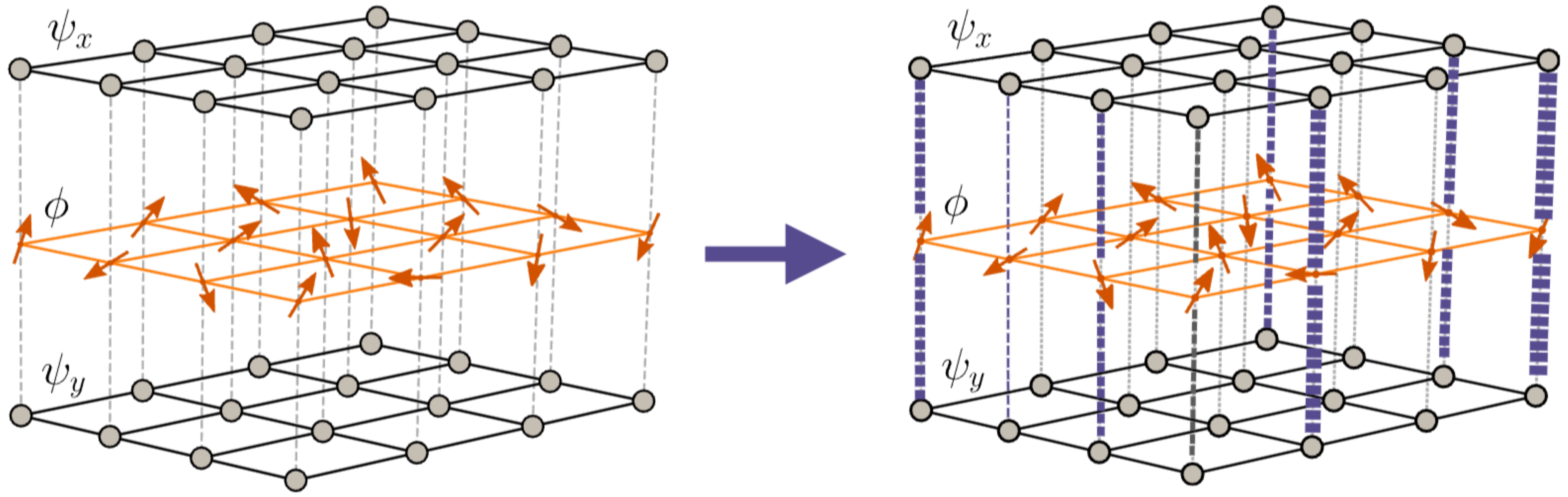
- Sign-problem free hybrid quantum Monte Carlo with g' interactions.
- Set $v, g = 0$ in order to isolate effects of g' (next phase of project will consider $g+g'$)

$$\mathcal{S} = \int d\tau \sum_{\sigma=\uparrow,\downarrow} \sum_{\alpha=x,y} \sum_{j=1}^{2} \sum_{\vec{r},\vec{r}'} \psi^{\dagger}_{\alpha,\sigma,j,\vec{r}} \left[\partial_\tau - (-1)^{\alpha}\mu - \delta_{\vec{r},\vec{r}'} - t_{\alpha,\vec{r},\vec{r}'}\right] \psi_{\alpha,\sigma,j,\vec{r}'}$$

$$+ \int d\tau \sum_{\vec{r}} \left[\frac{1}{c^2}(\partial_\tau \vec{\phi}_{\vec{r}})^2 + \frac{1}{2}(\nabla \vec{\phi}_{\vec{r}})^2 + \frac{r}{2}(\vec{\phi}_{\vec{r}})^2 + \frac{u}{4}(\vec{\phi}_{\vec{r}})^4\right] + \sum_{\sigma,\sigma'=\uparrow,\downarrow} \sum_{j=1}^{2} \int d\tau \sum_{\vec{r}} \boxed{g'(\vec{r}) e^{i\vec{Q}_{\mathrm{AF}}\cdot\vec{r}}} \vec{\phi}_{\vec{r}} \cdot \left[\psi^{\dagger}_{x,\sigma,j,\vec{r}} \vec{\tau}_{\sigma,\sigma'} \psi_{y,\sigma',j,\vec{r}} + \mathrm{H.c.}\right].$$

Two-band structure: Berg, Metlitski, Sachdev, Science **338** 1606-1609 (2012).

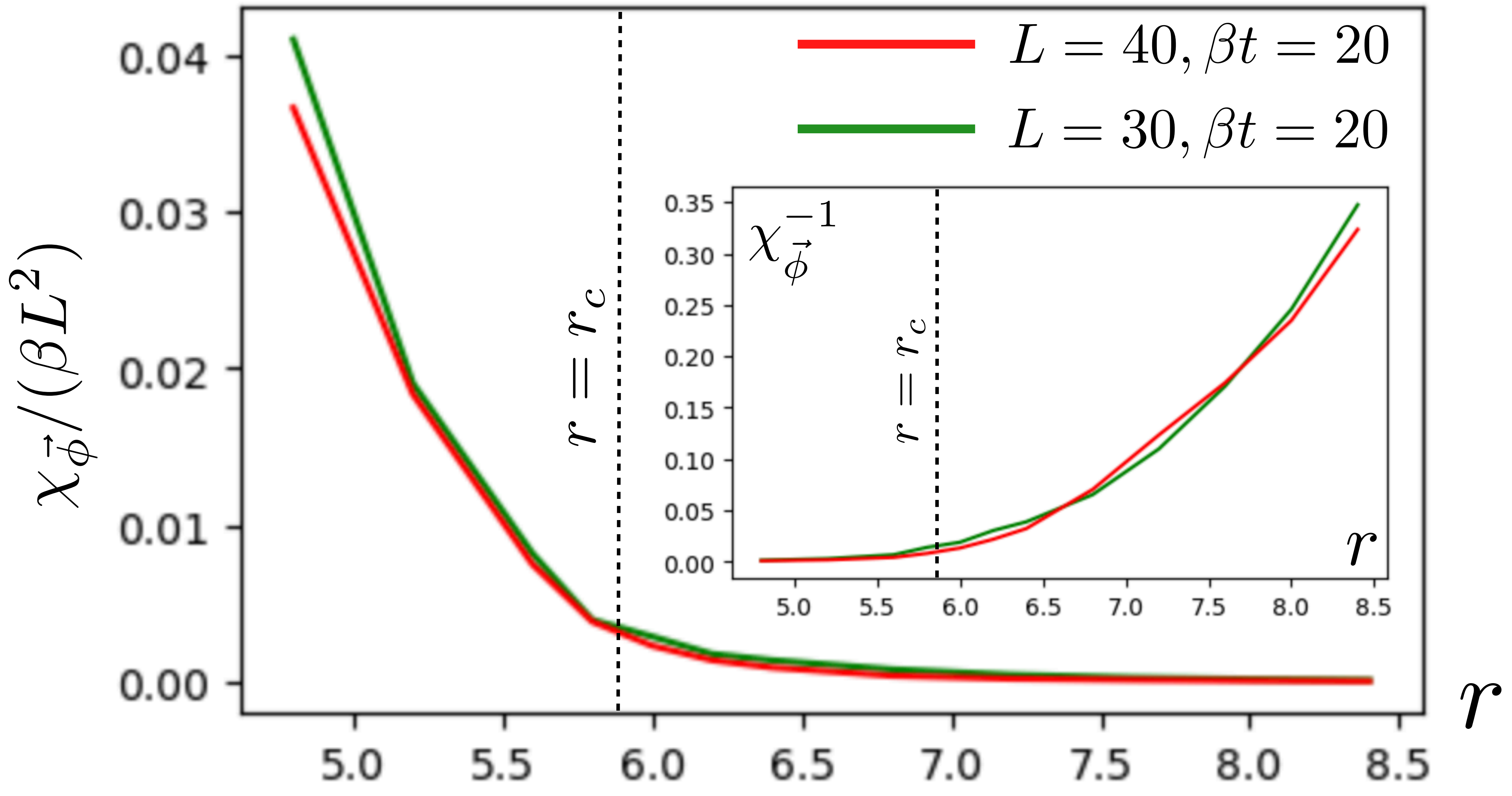
(Corresponds to random *anti*-ferromagnetic interactions)

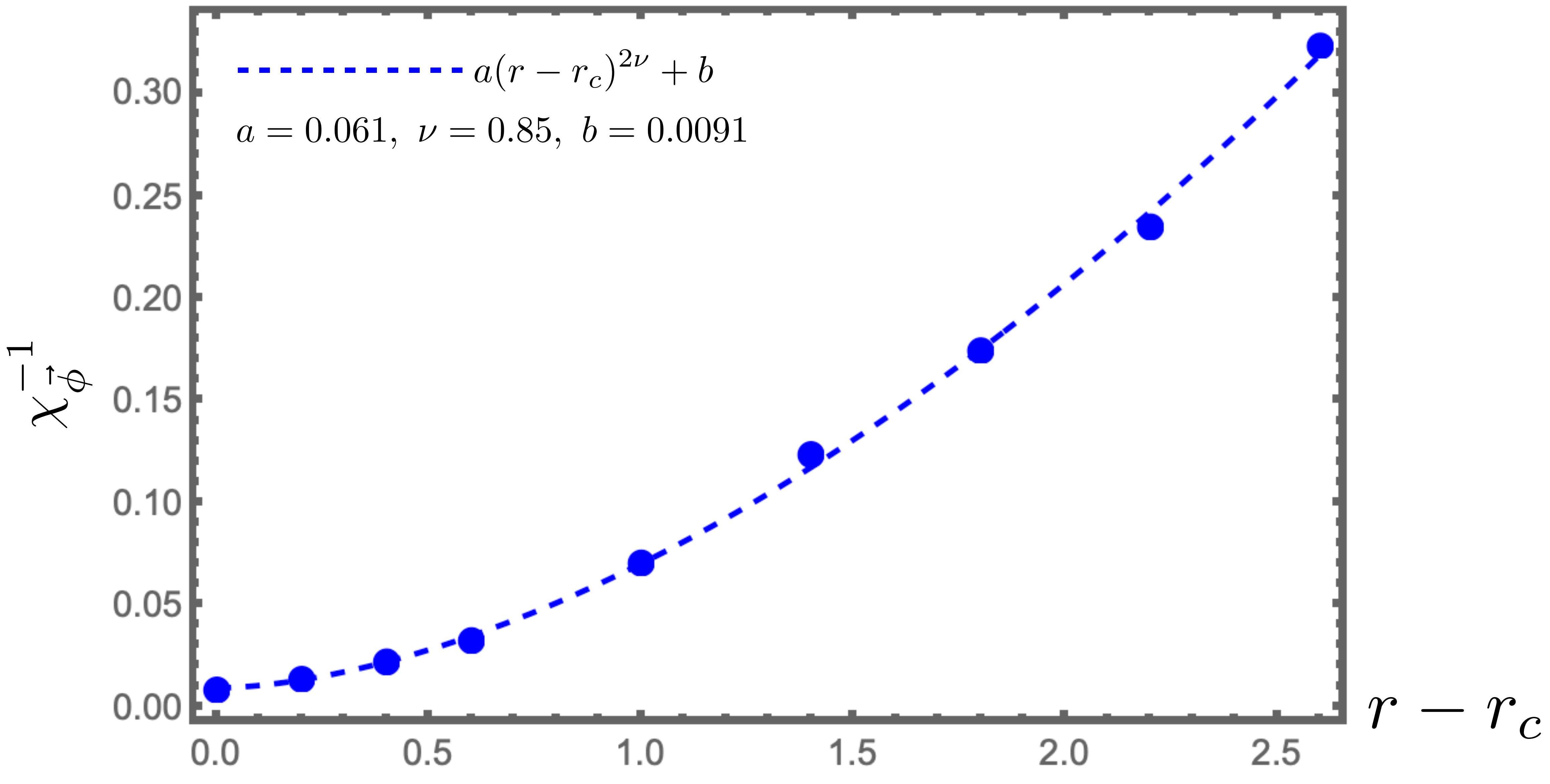


coupling g = const   coupling g = spatially random

Aavishkar Patel, Michael Albergo, Peter Lunts (to appear)

Some results: phase transition





- $\vec{\phi}$ develops quasi-long range order ($d=2$), for $r<r_c$.

- Boson mass dependence on r gives an estimate of correlation length exponent $\nu \sim 0.85$ at $\beta t = 20$ (for a boson dispersion that is quadratic at small q).

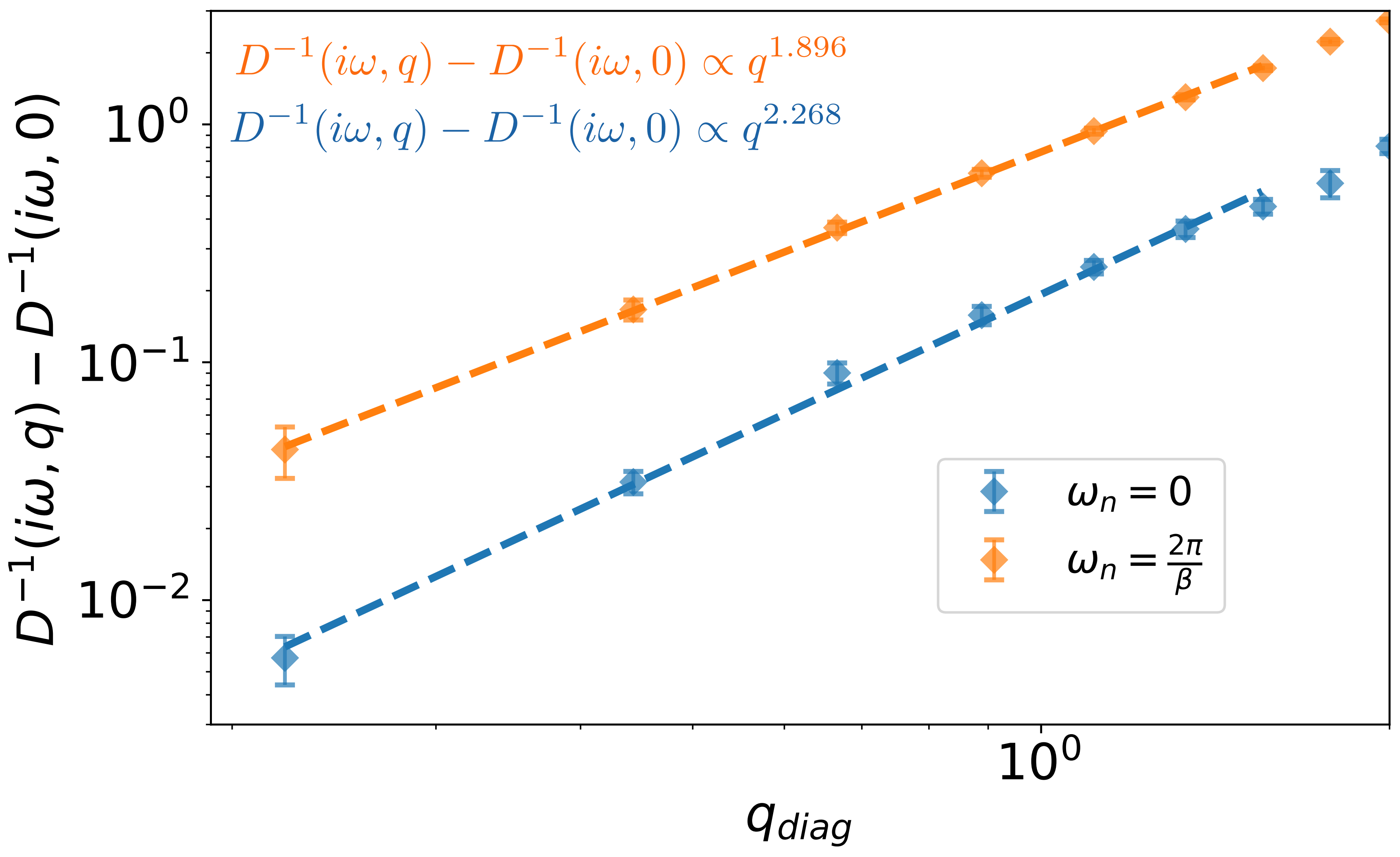
Coupling $g'^2 = t = 2E_F/5 = W/6.$

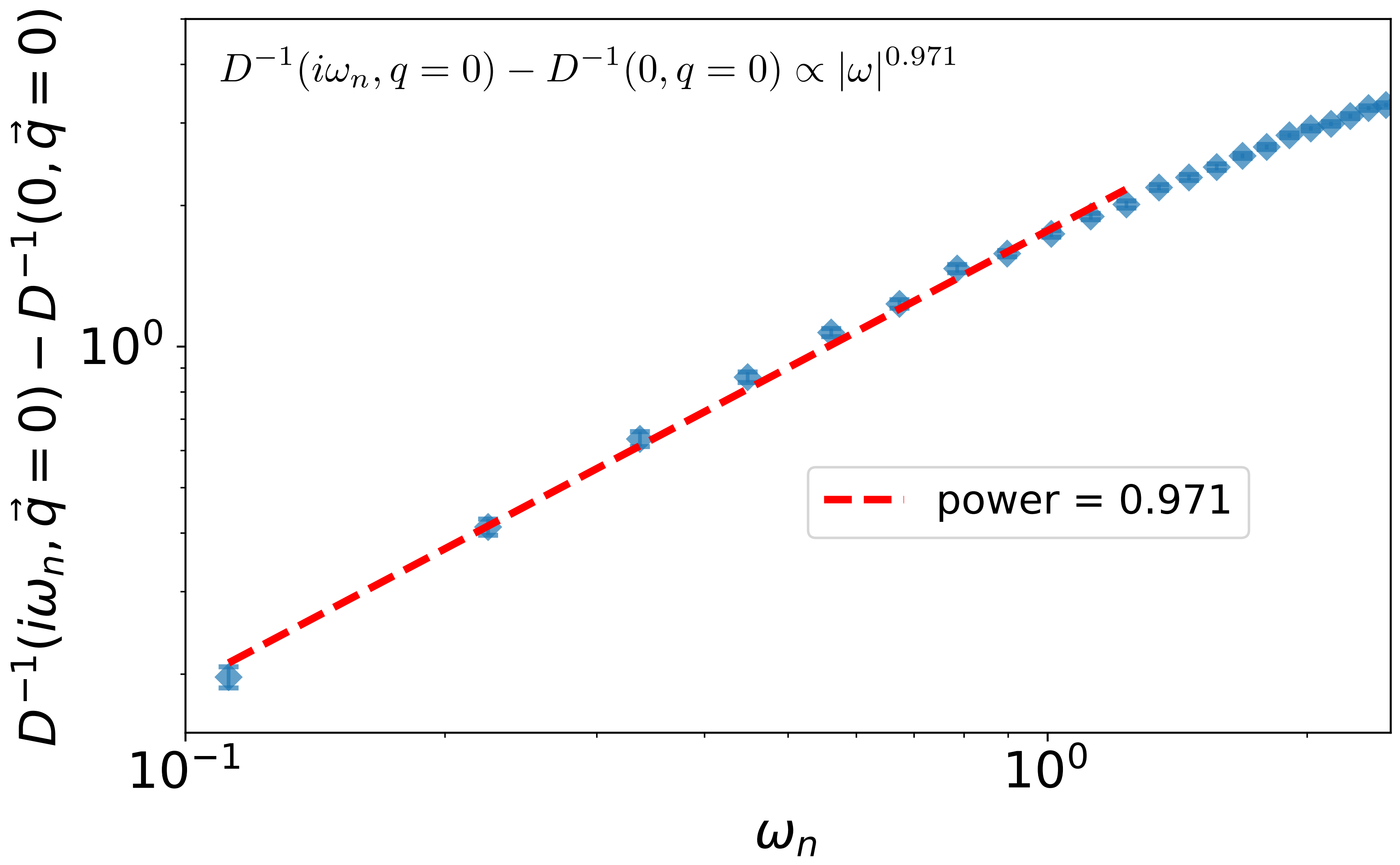
Linear dimension L

Quenched disorder average over 10 samples.

Aavishkar Patel, Michael Albergo, Peter Lunts (to appear)

Some results: boson propagator





- Momentum dependence of inverse $\vec{\phi}$ propagator is close to q^2 at small q (small deviations in exponent are due to lattice effects).
- Frequency dependence of inverse $\vec{\phi}$ propagator is consistent with $|\omega|$ Landau damping at low frequencies.

$$D(i\omega > 0, q) \sim \frac{1}{q^2 + \gamma|\omega|}$$

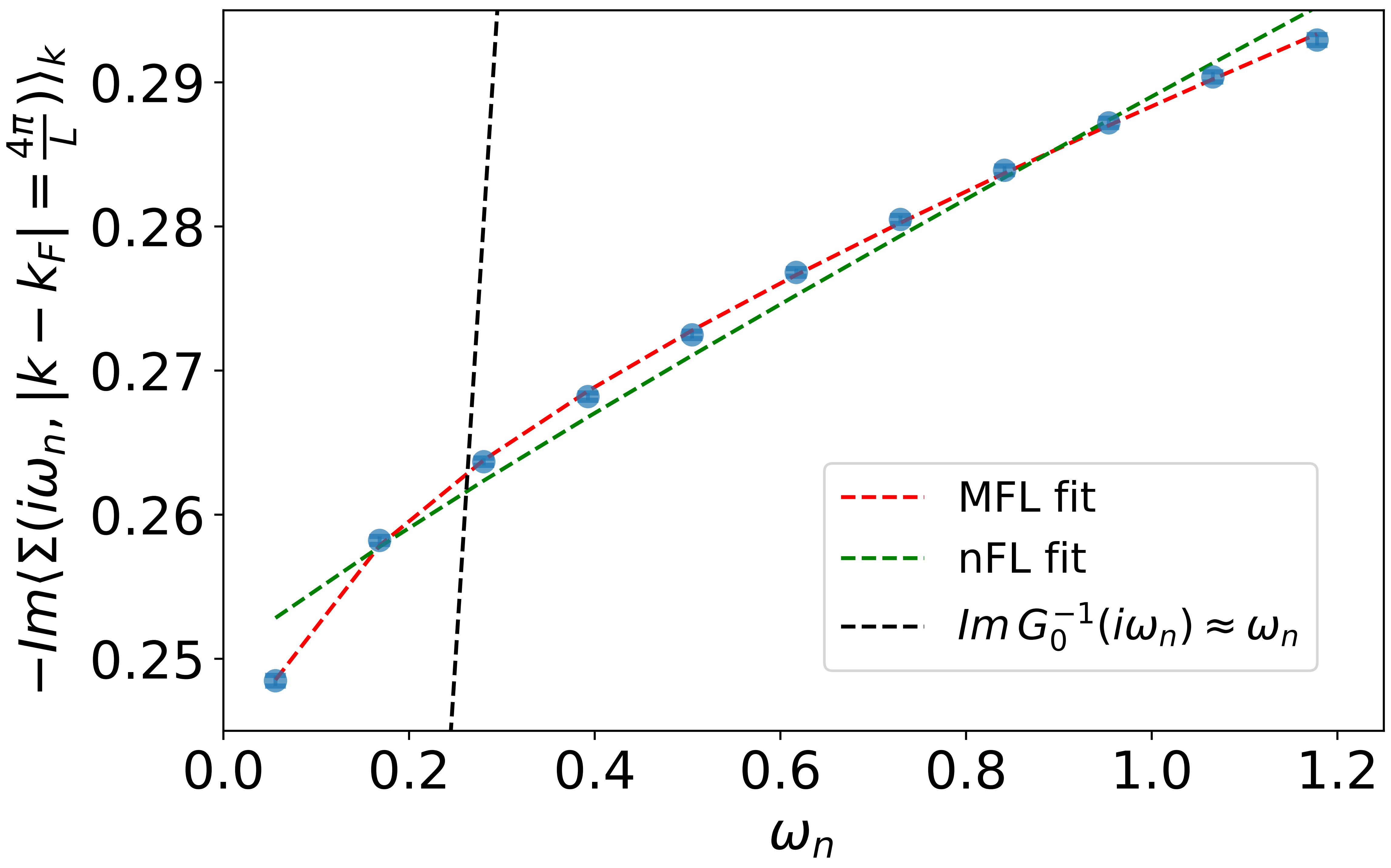
$$D(i\omega = 0, q) \sim \frac{1}{q^2}$$

$$\beta t = 56 \;\; L = 40 \;\; g'^2 = t = 2E_F/5 = W/6 \quad r \approx r_c$$

Quenched disorder average over 22 samples

Aavishkar Patel, Michael Albergo, Peter Lunts (to appear)

Some results: fermion self energy



$\beta t = 56 \;\; L = 40 \;\; g'^2 = t = 2E_F/5 = W/6 \;\; r \approx r_c$

Quenched disorder average over 22 samples

MFL fit: $a(\omega \ln(1 + b/\omega) + b \ln(1 + \omega/b)) + c + d\omega$

$a = 0.195, b = 0.039, c = 0.234, d = 0.021$

Marginal FL with finite UV cutoff b, and a constant residual c that is T-independent

nFL fit: $a\omega^{1-\epsilon} + c + d\omega$

$a = 0.093, c = 0.25, \epsilon = 0.1, d = 0.0$

Fit to nFL with exponent $1-\epsilon$ is bad for small ϵ

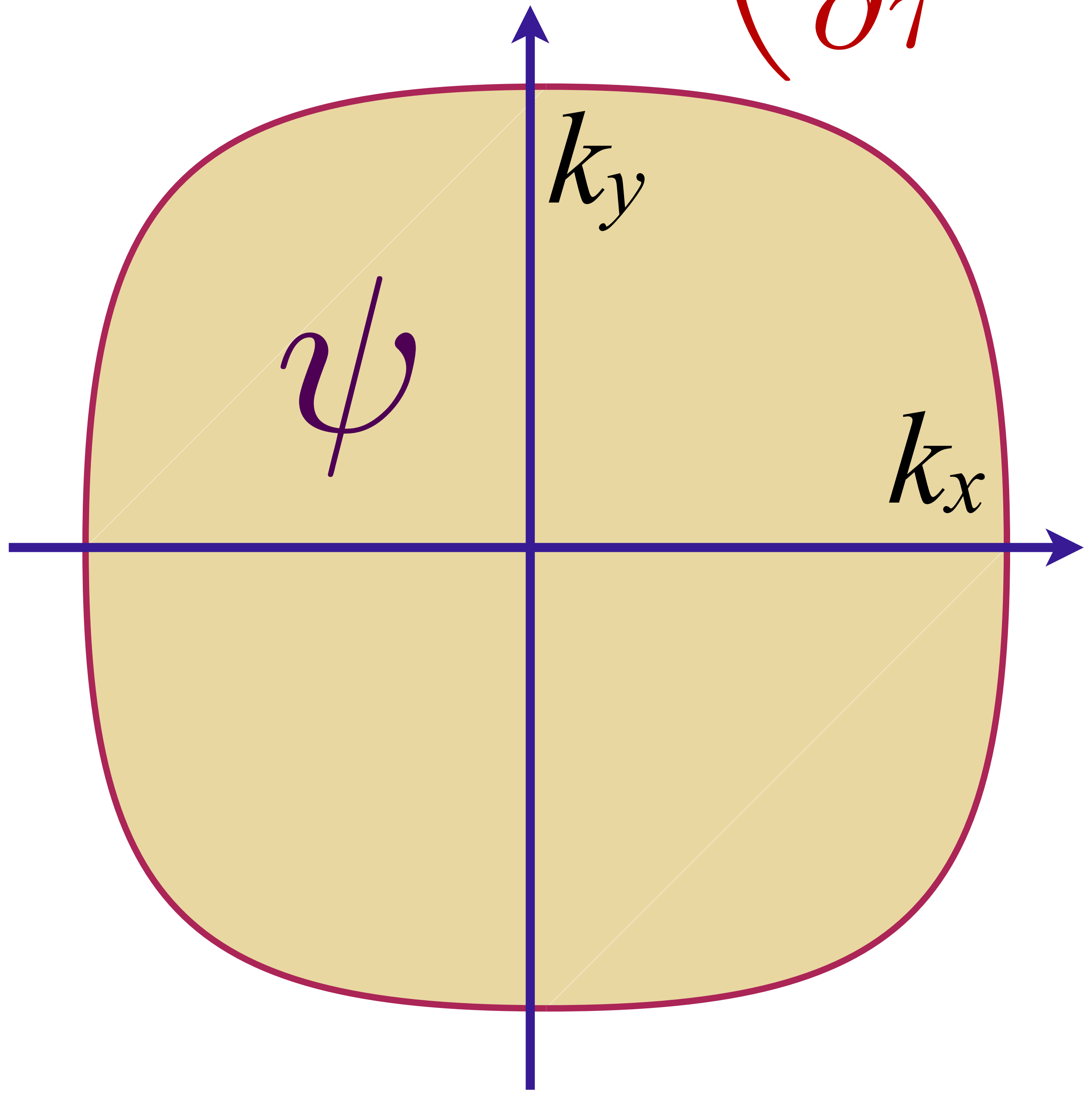
- Frequency scaling of Σ averaged over contour close to FS is compatible with marginal FL form at low frequencies.

Aavishkar Patel, Michael Albergo, Peter Lunts (to appear)

Universal theory of strange metals

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume changing transition

$$\mathcal{L}_\phi = s[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r}) + v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$