

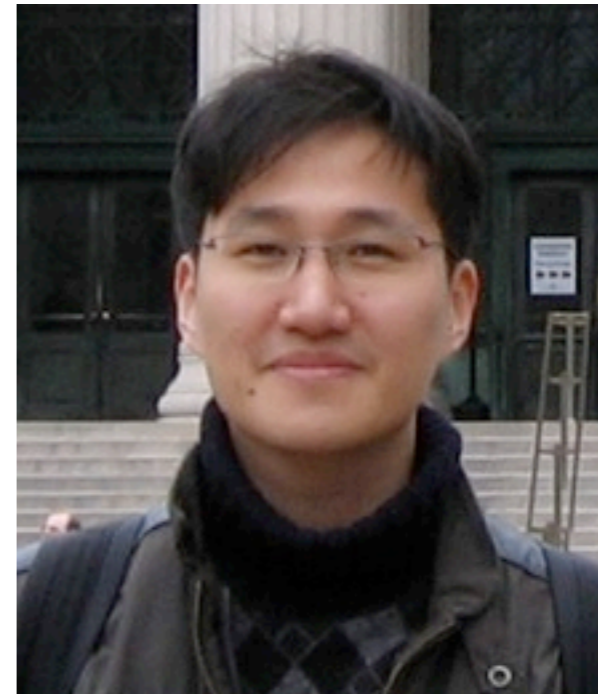
The phase diagrams of the high temperature superconductors

Talk online: sachdev.physics.harvard.edu





Max Metlitski, Harvard



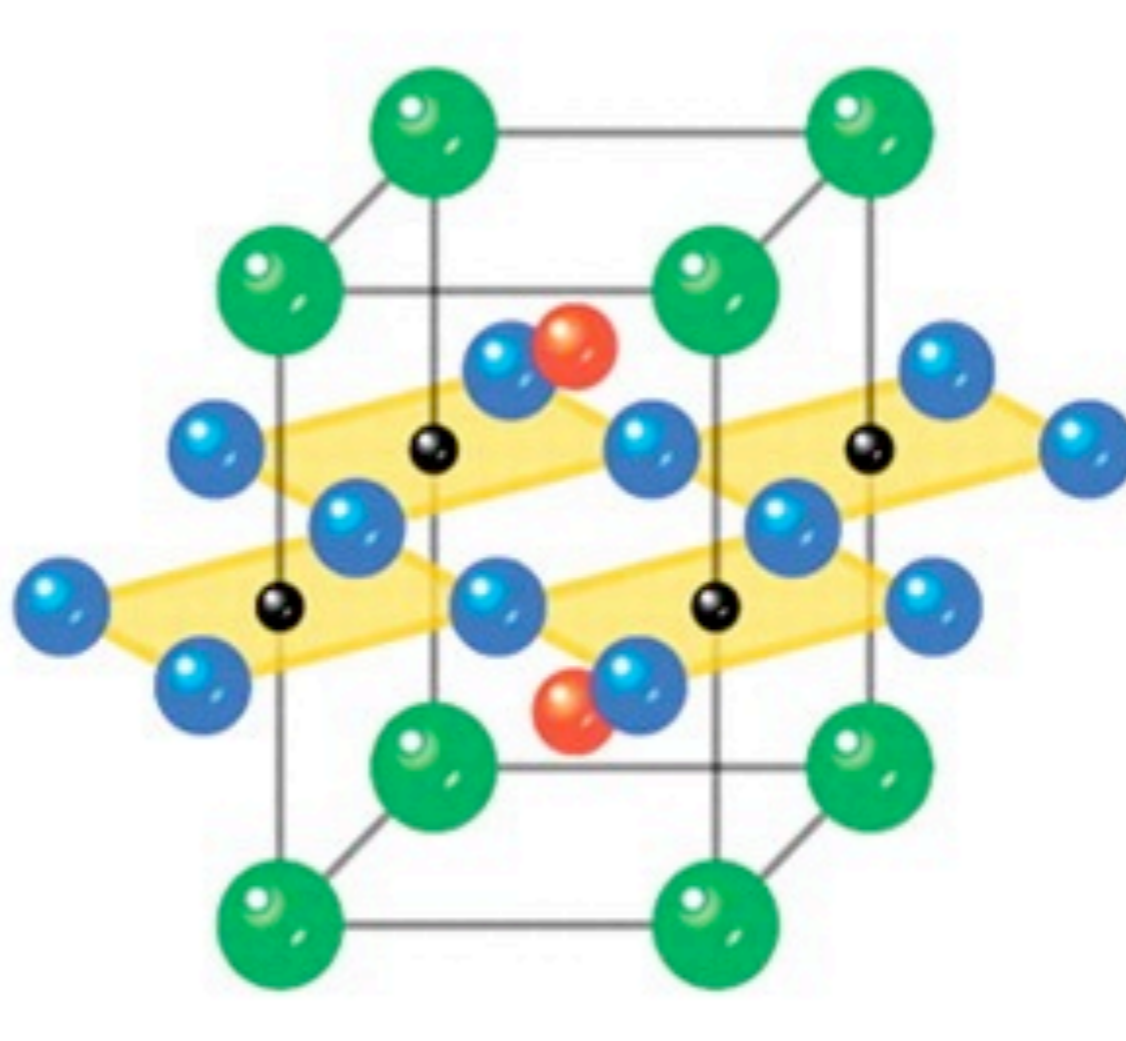
Eun Gook Moon, Harvard



The cuprate superconductors

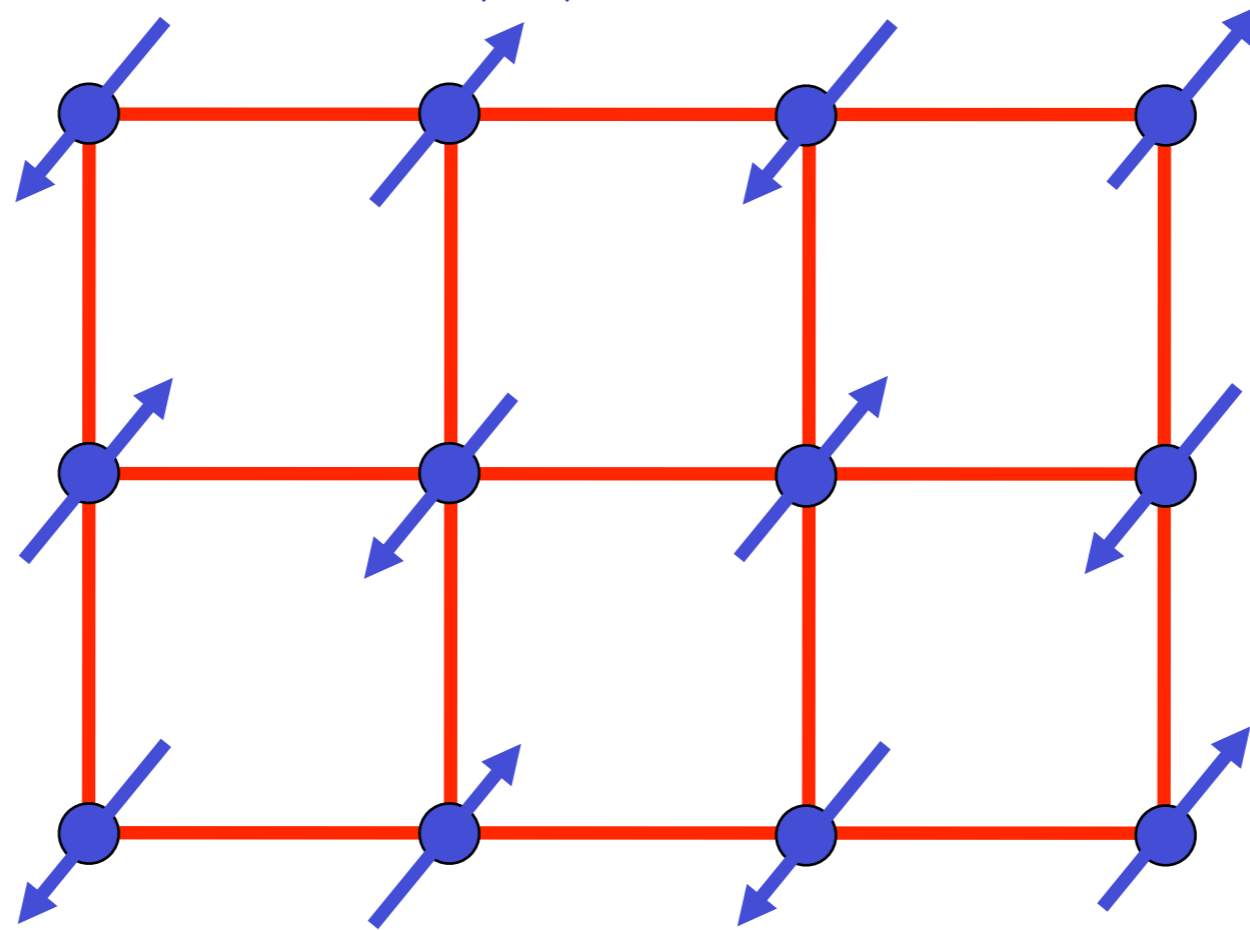
Na-CCOC

- Cu
- Ca/Na
- O
- Cl



Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

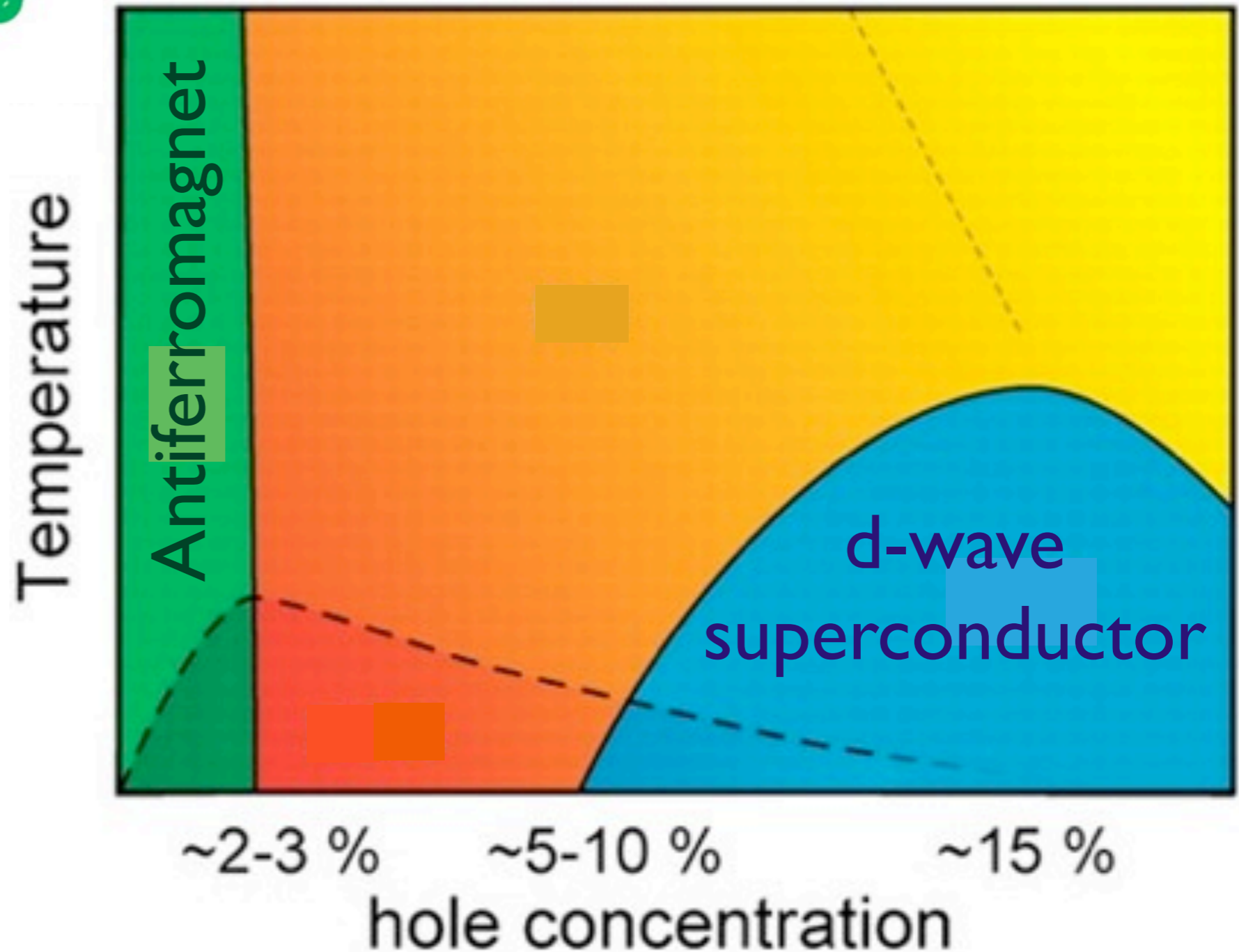
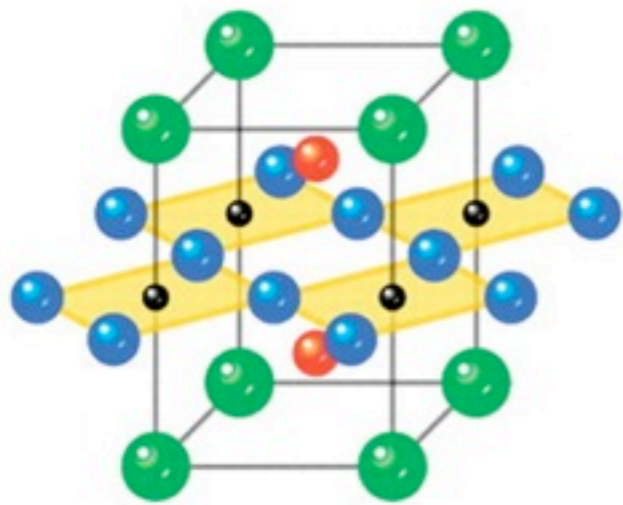
$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

The cuprate superconductors

Na-CCOC

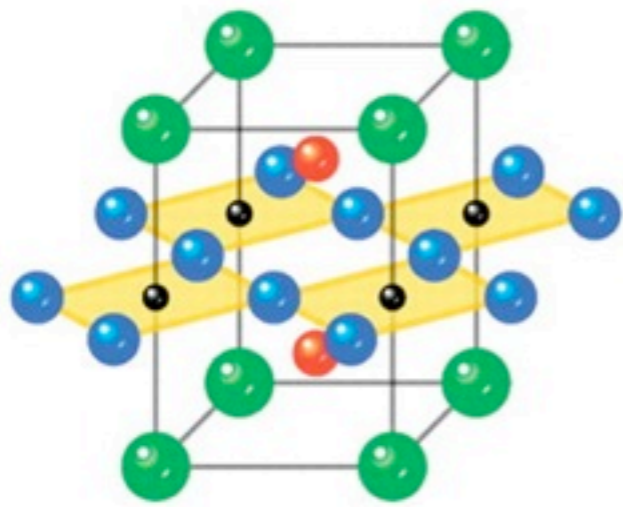
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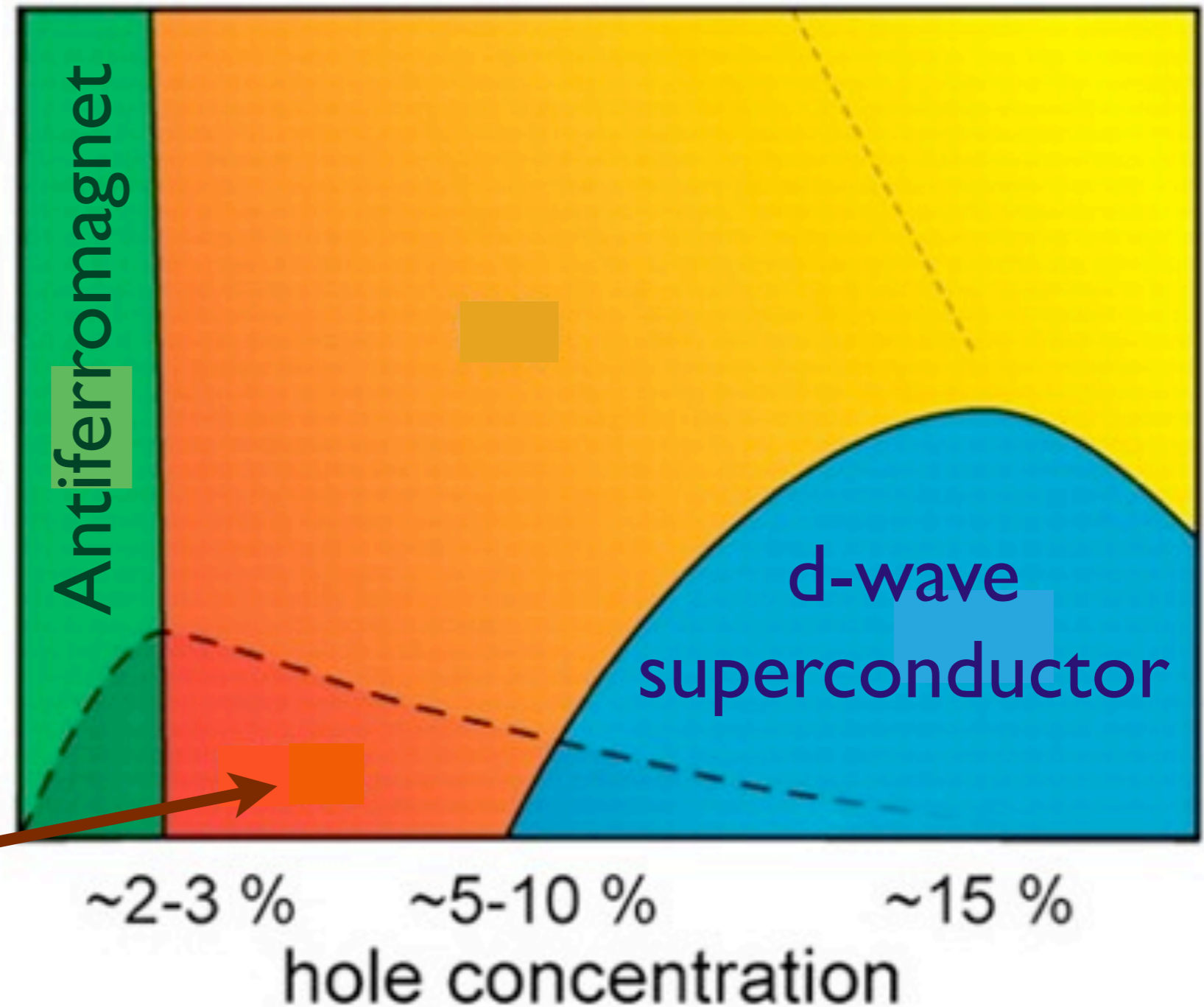
The cuprate superconductors

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Temperature

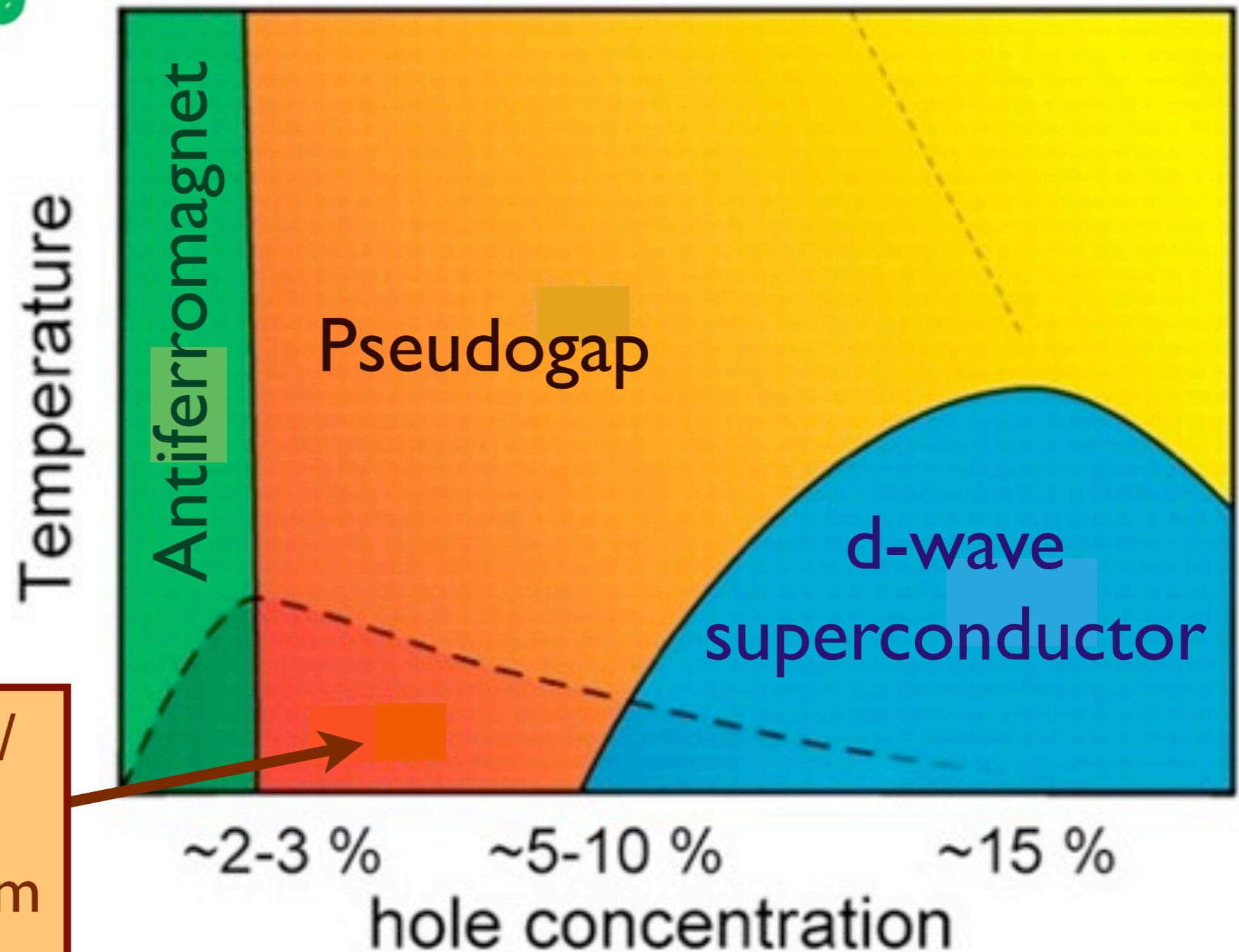
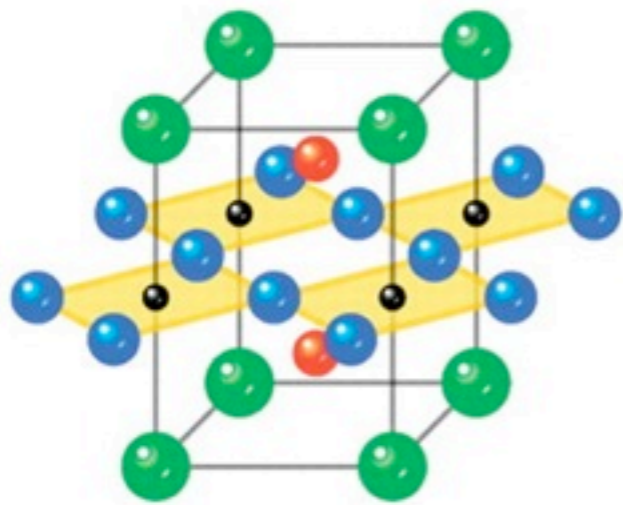


Incommensurate/
disordered
antiferromagnetism
and charge order

The cuprate superconductors

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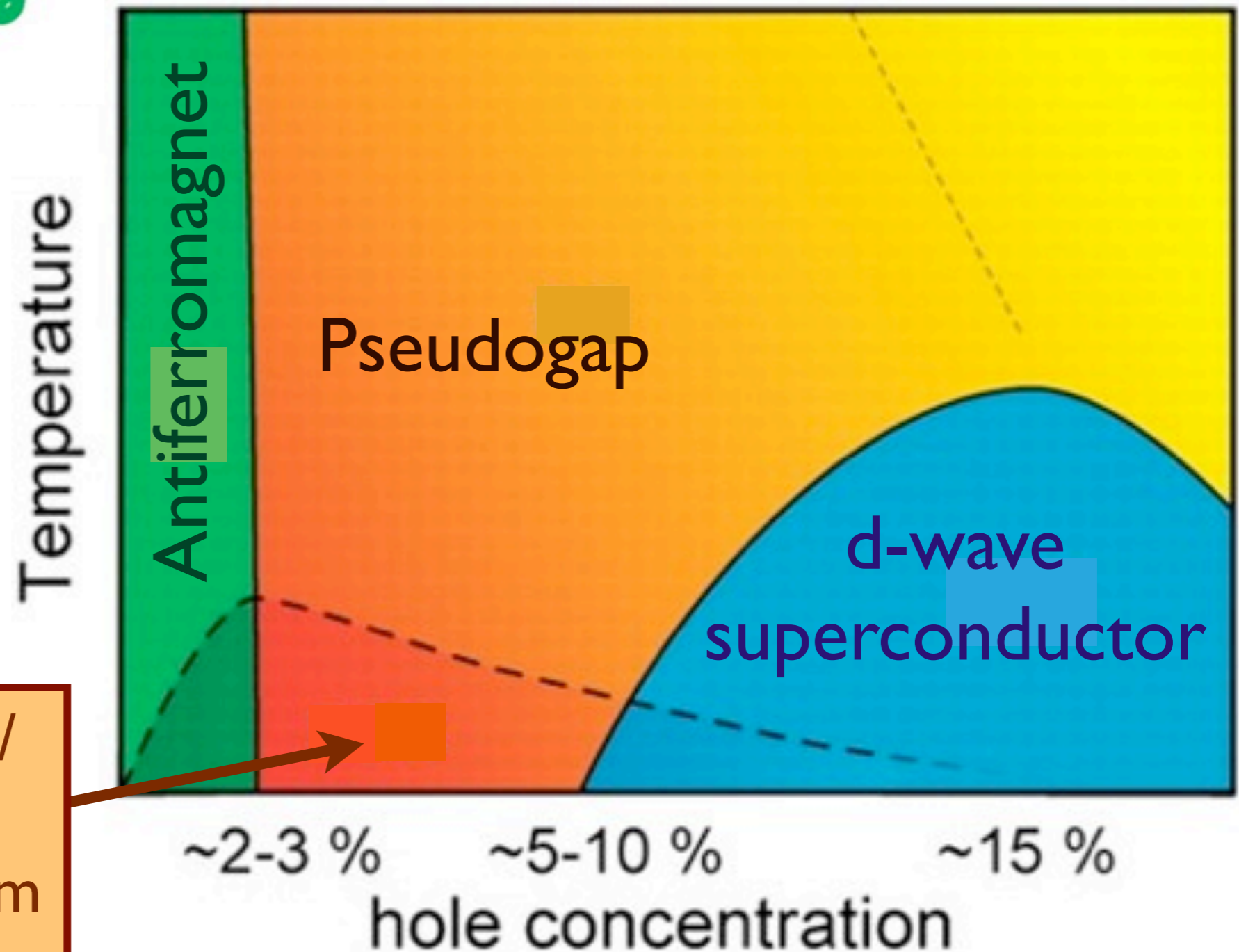
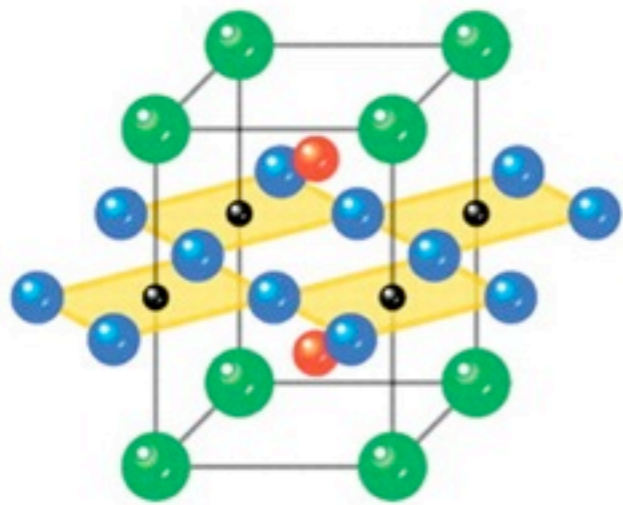


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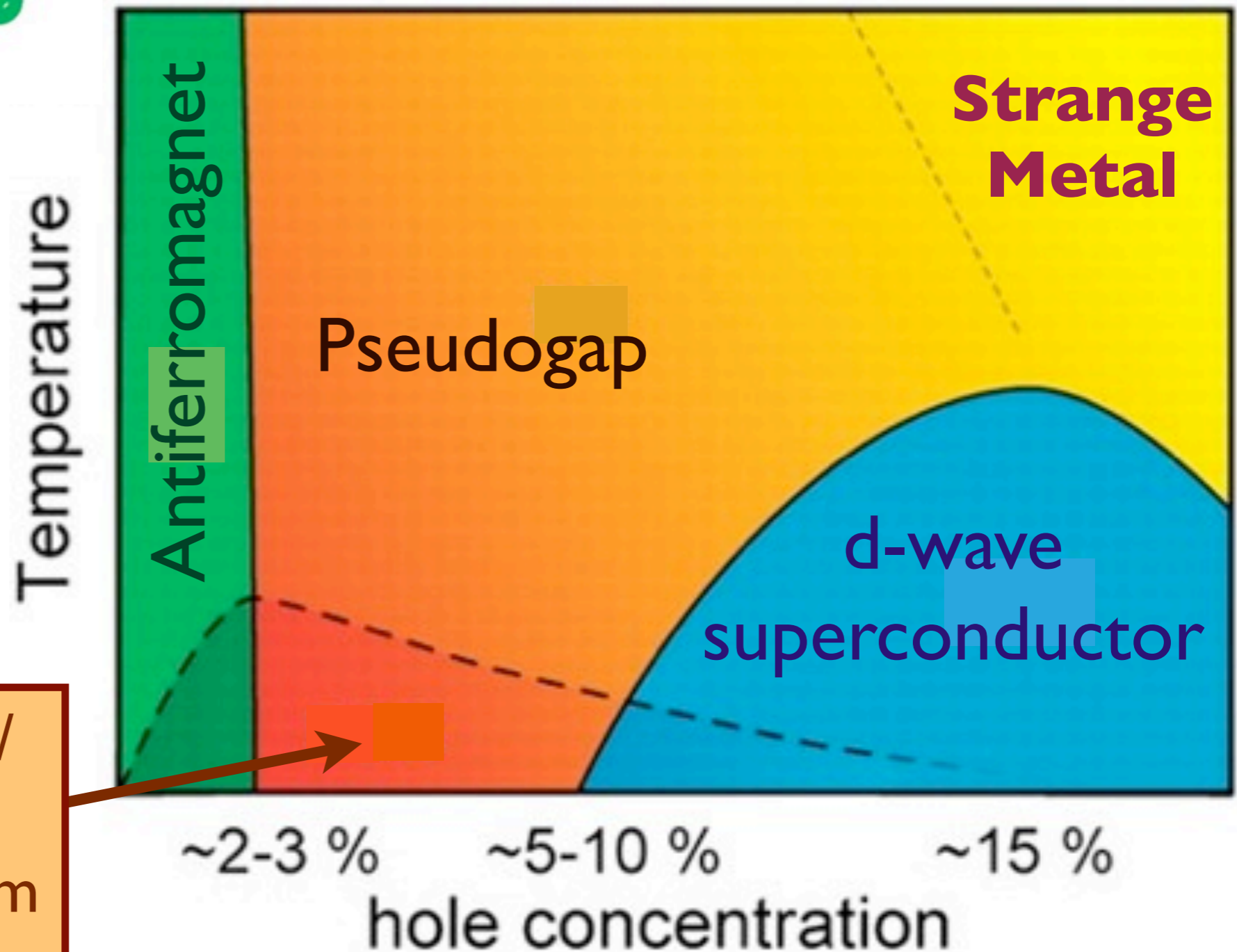
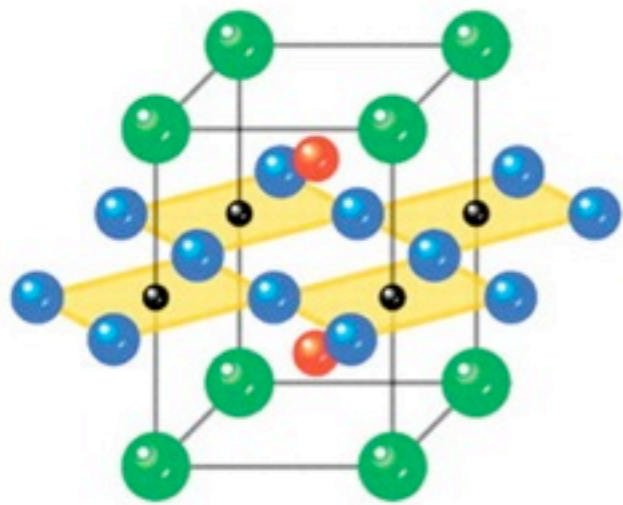


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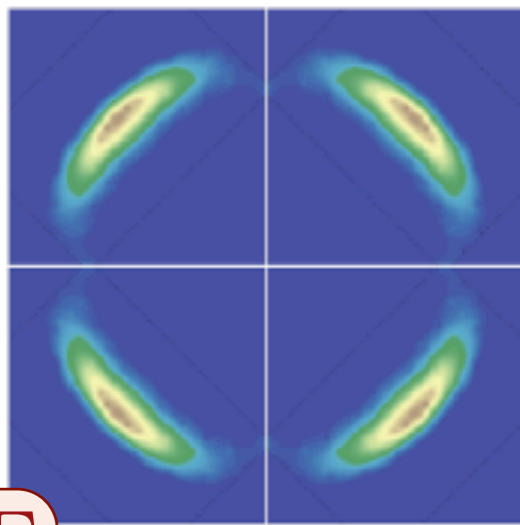
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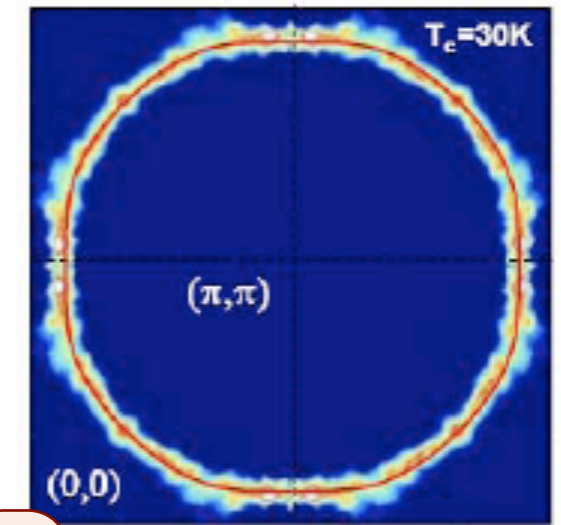
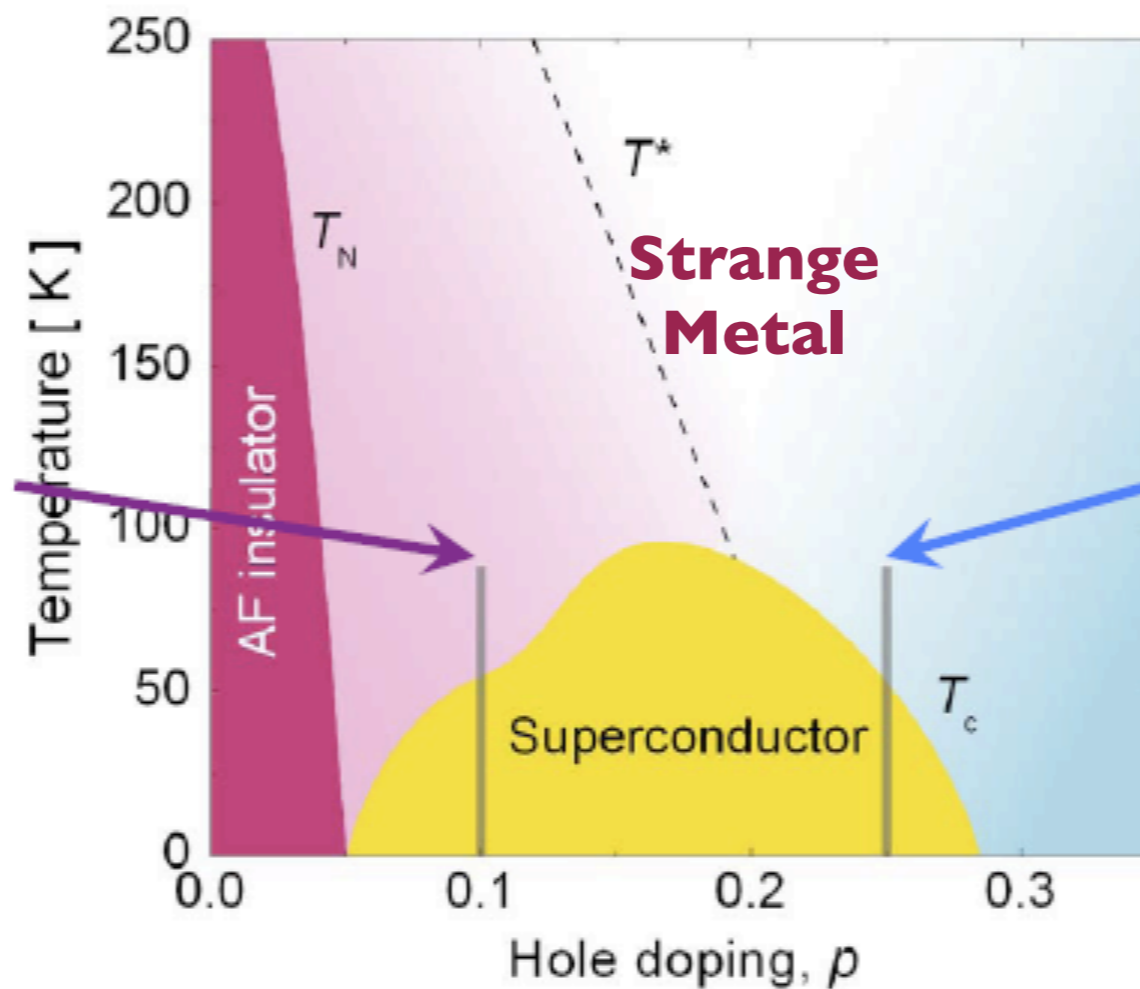
Incommensurate/
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Γ

K.M. Shen et al., Science 2005

Smaller hole
Fermi-pockets

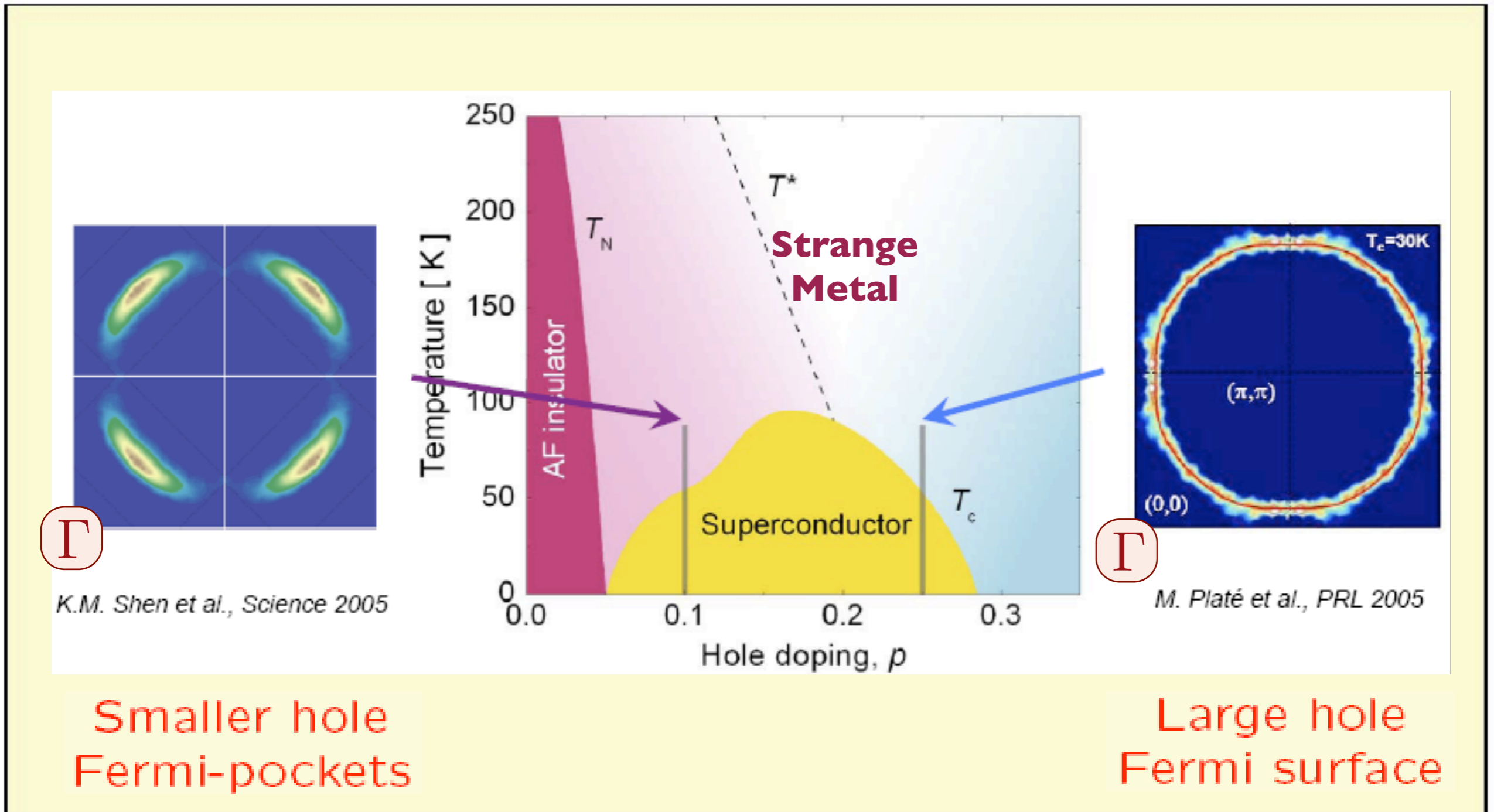


Γ

M. Platé et al., PRL 2005

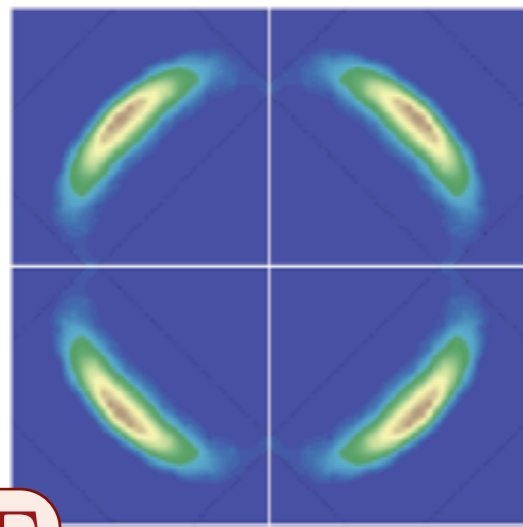
Large hole
Fermi surface

Key Ingredients:



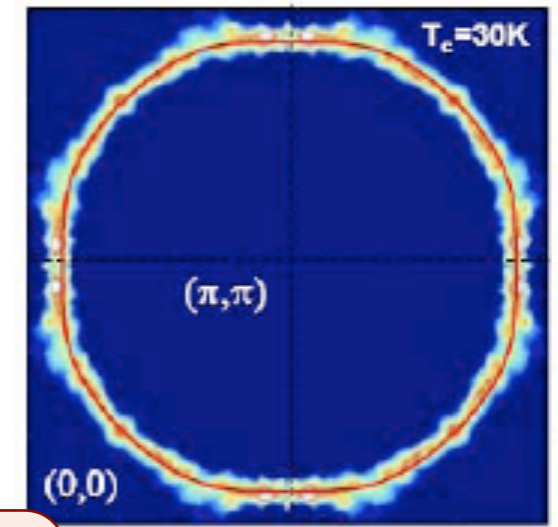
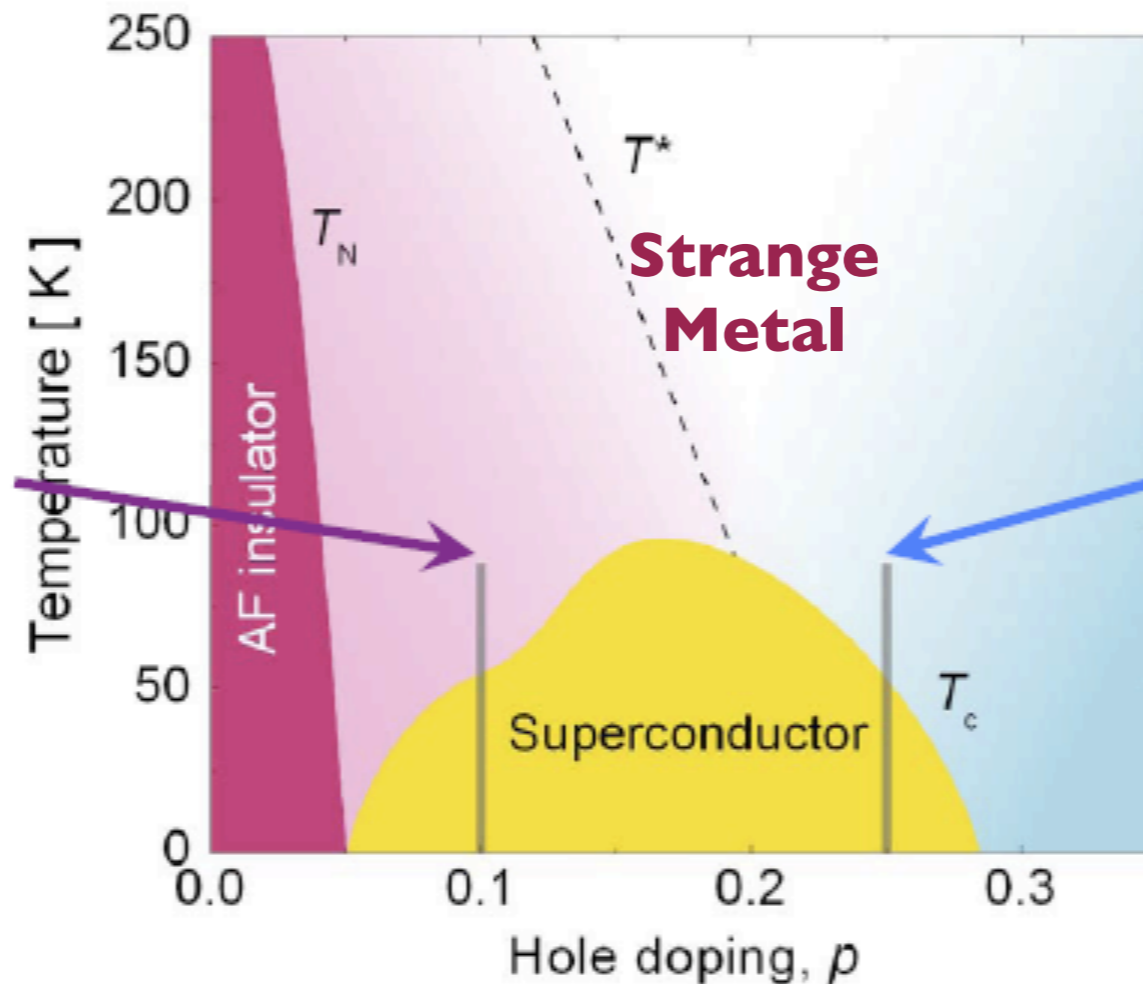
Key Ingredients:

Antiferromagnetism (AF)
Spin density wave (SDW)



K.M. Shen et al., Science 2005

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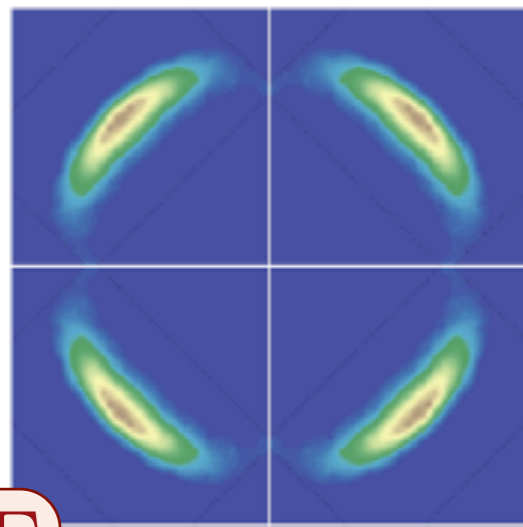
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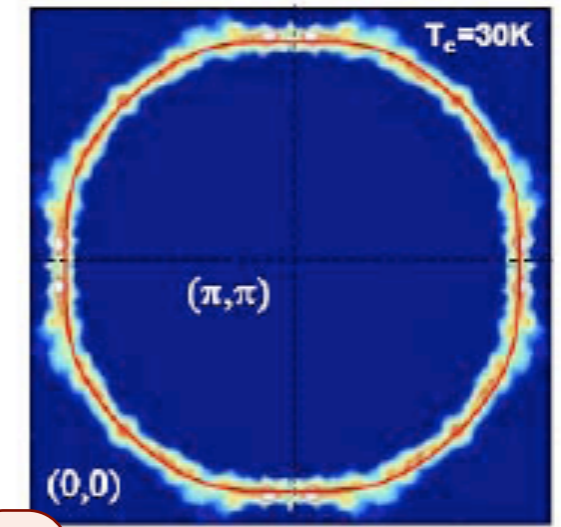
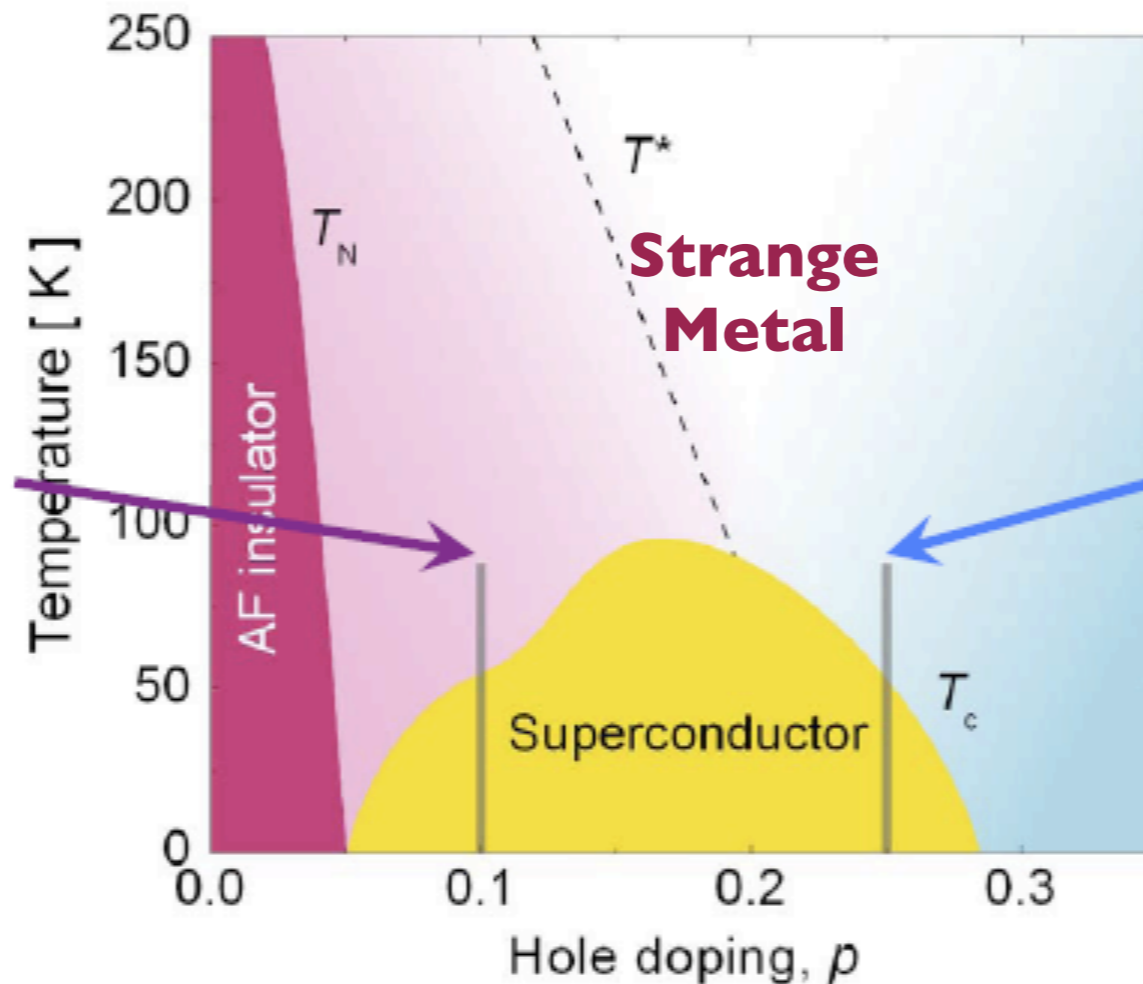
d-wave superconductivity

Antiferromagnetism (AF)
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K.M. Shen et al., Science 2005

Smaller hole
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M. Platé et al., PRL 2005

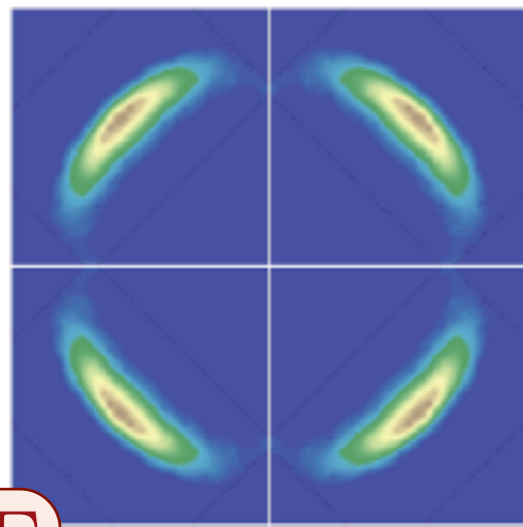
Large hole
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Key Ingredients:

d-wave superconductivity

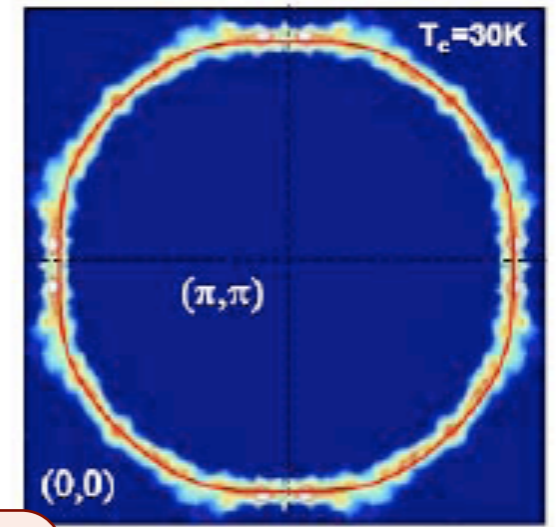
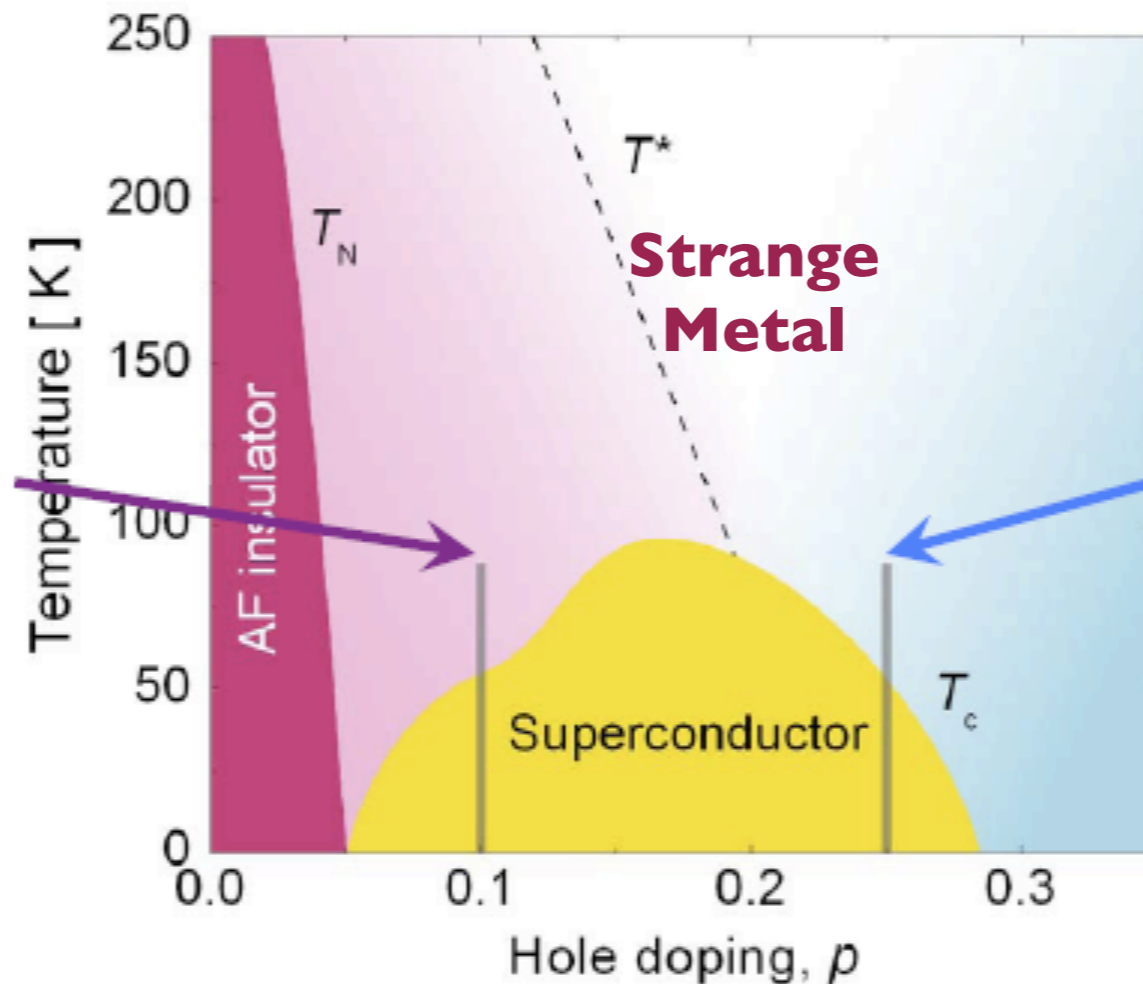
Antiferromagnetism (AF)
Spin density wave (SDW)

Fermi surface change



Γ

K.M. Shen et al., Science 2005

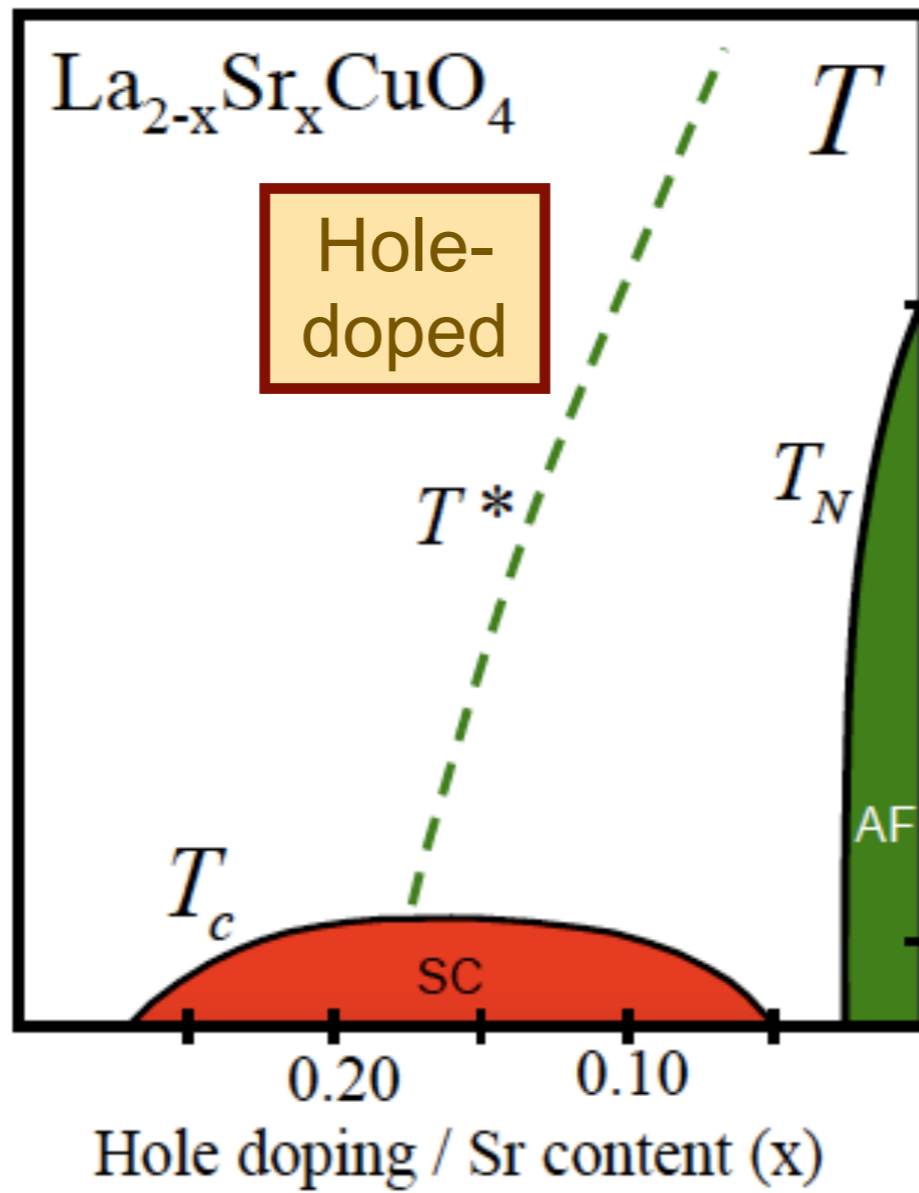


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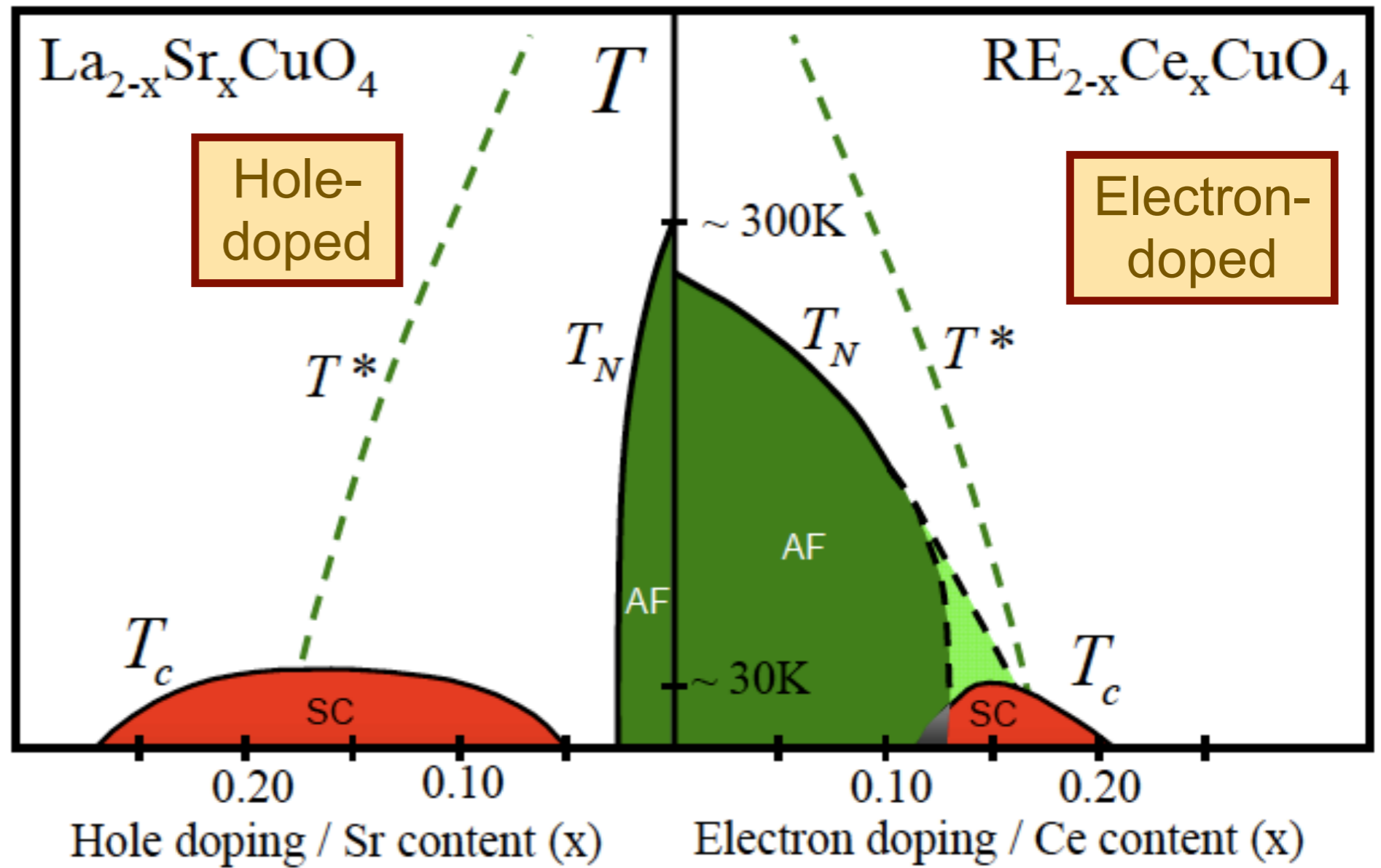
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Electron-doped cuprate superconductors



Electron-doped cuprate superconductors

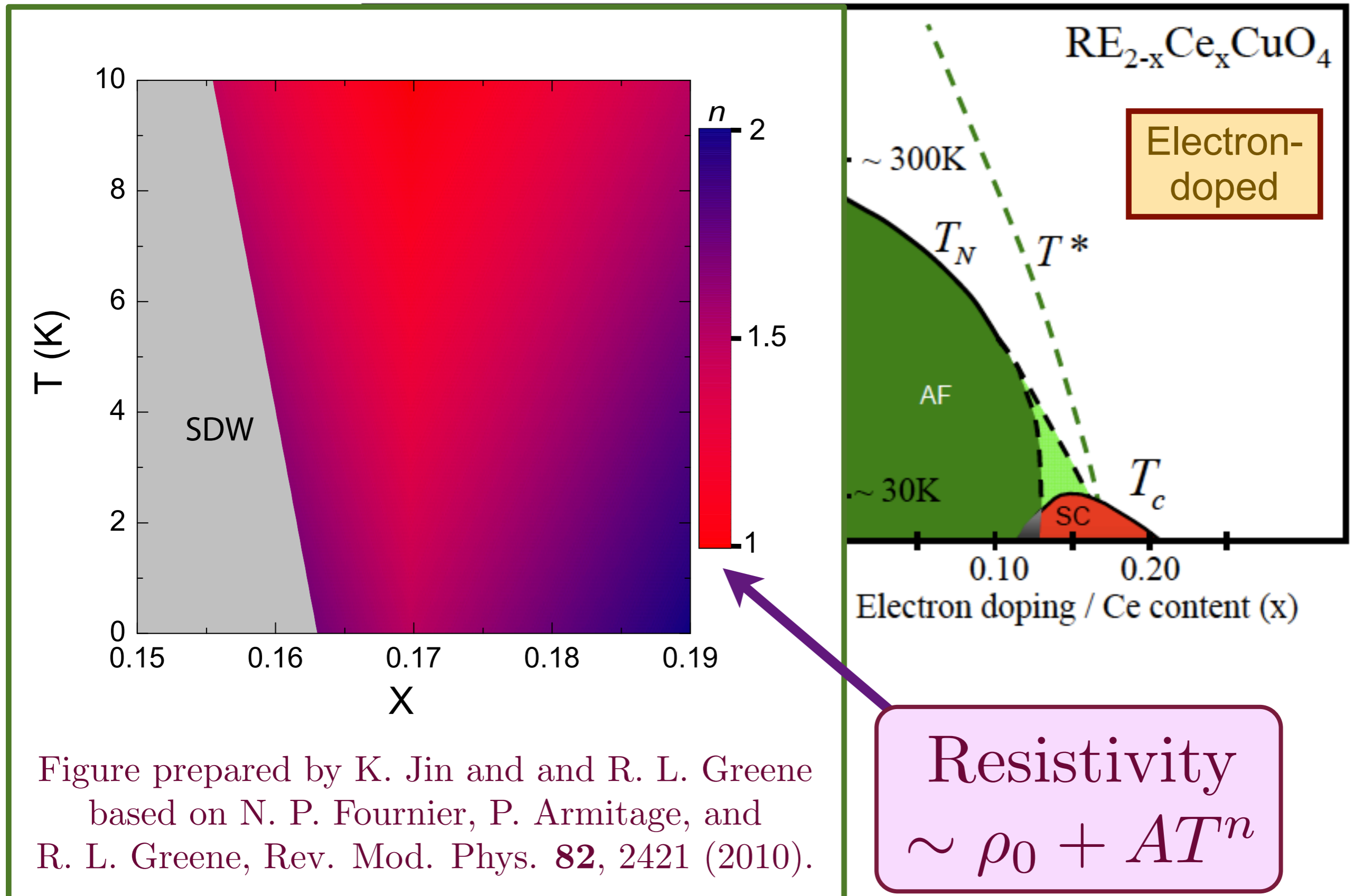


Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).

Resistivity
 $\sim \rho_0 + AT^n$

Electron-doped cuprate superconductors

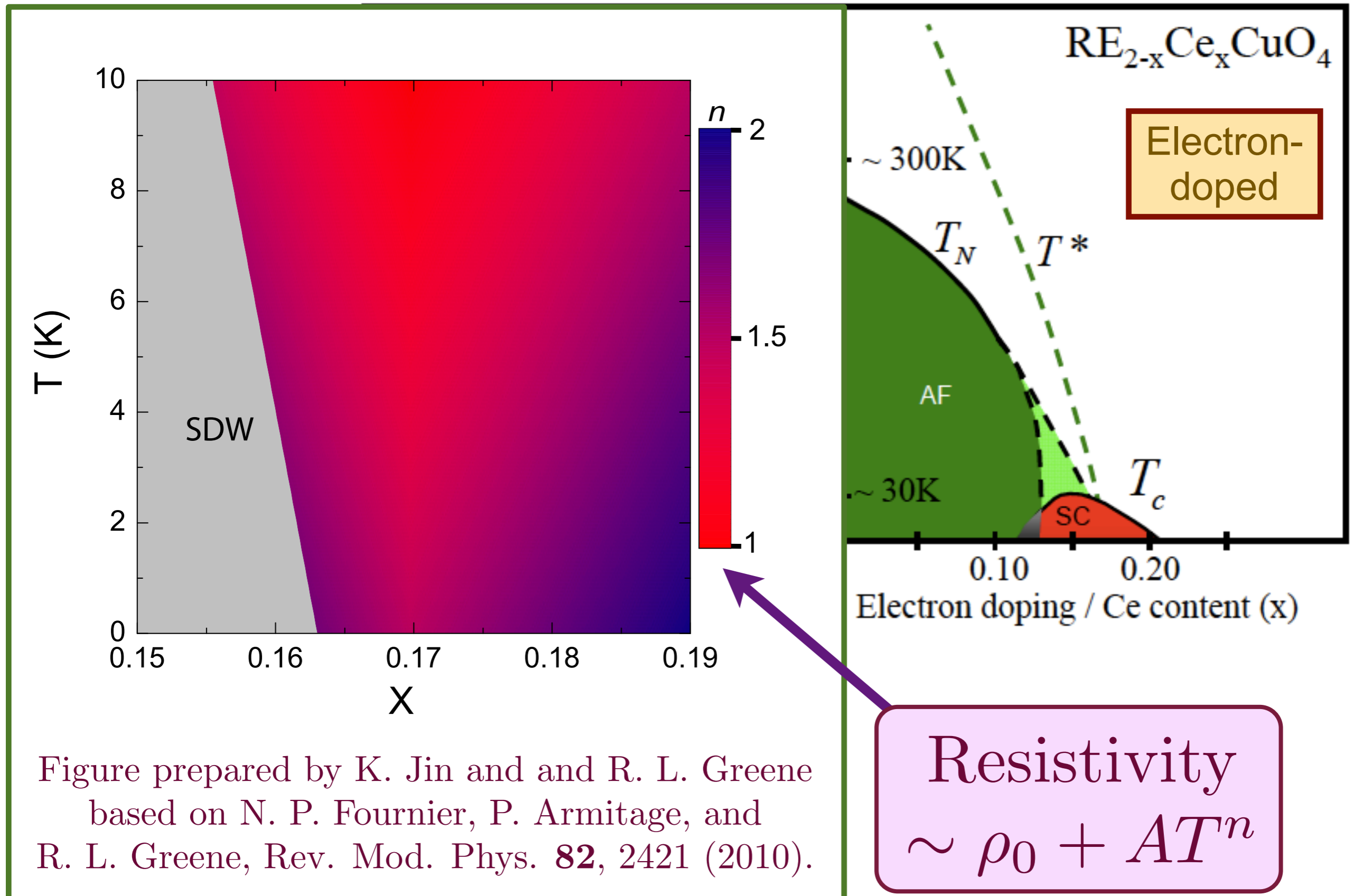


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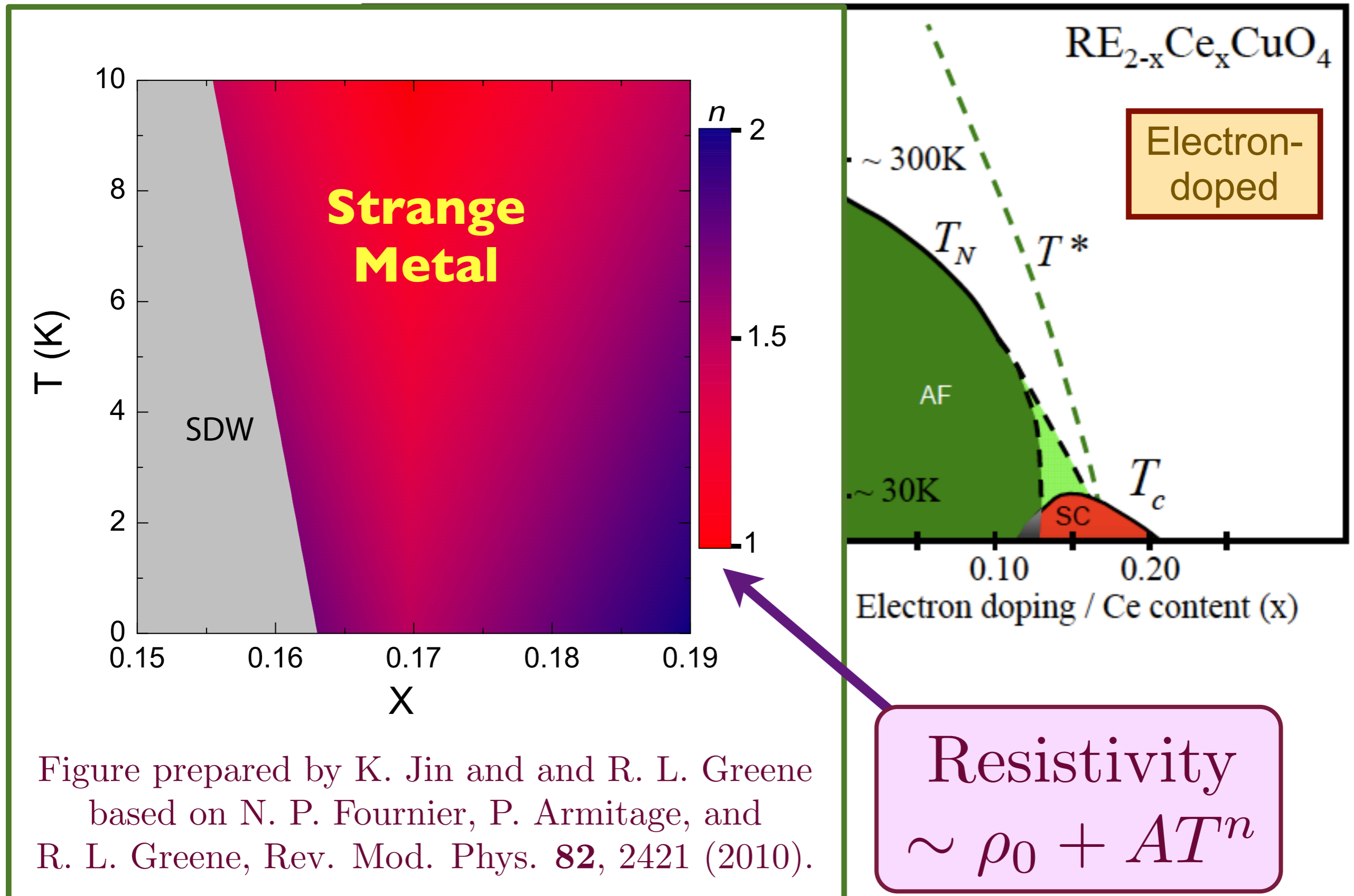
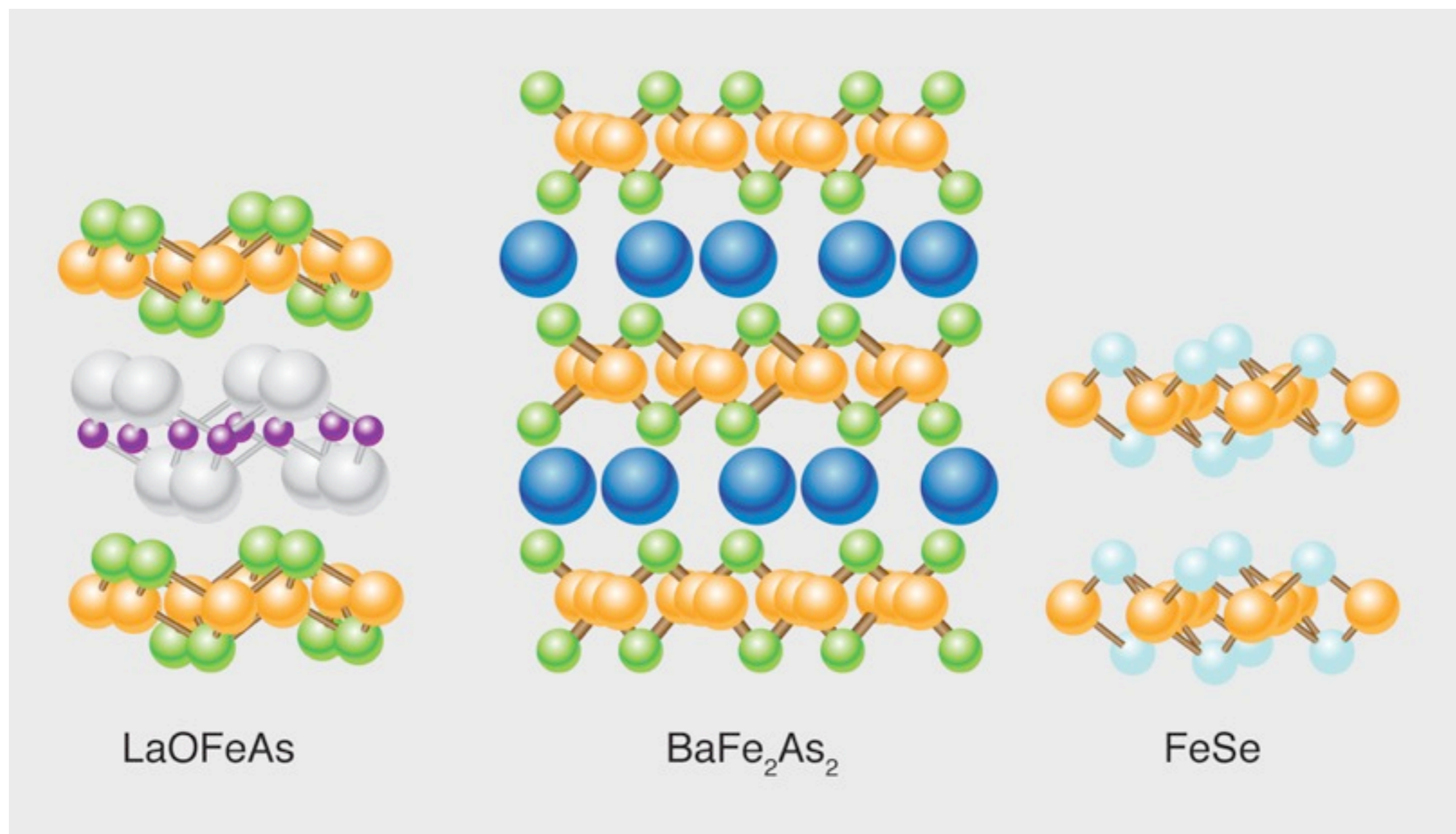


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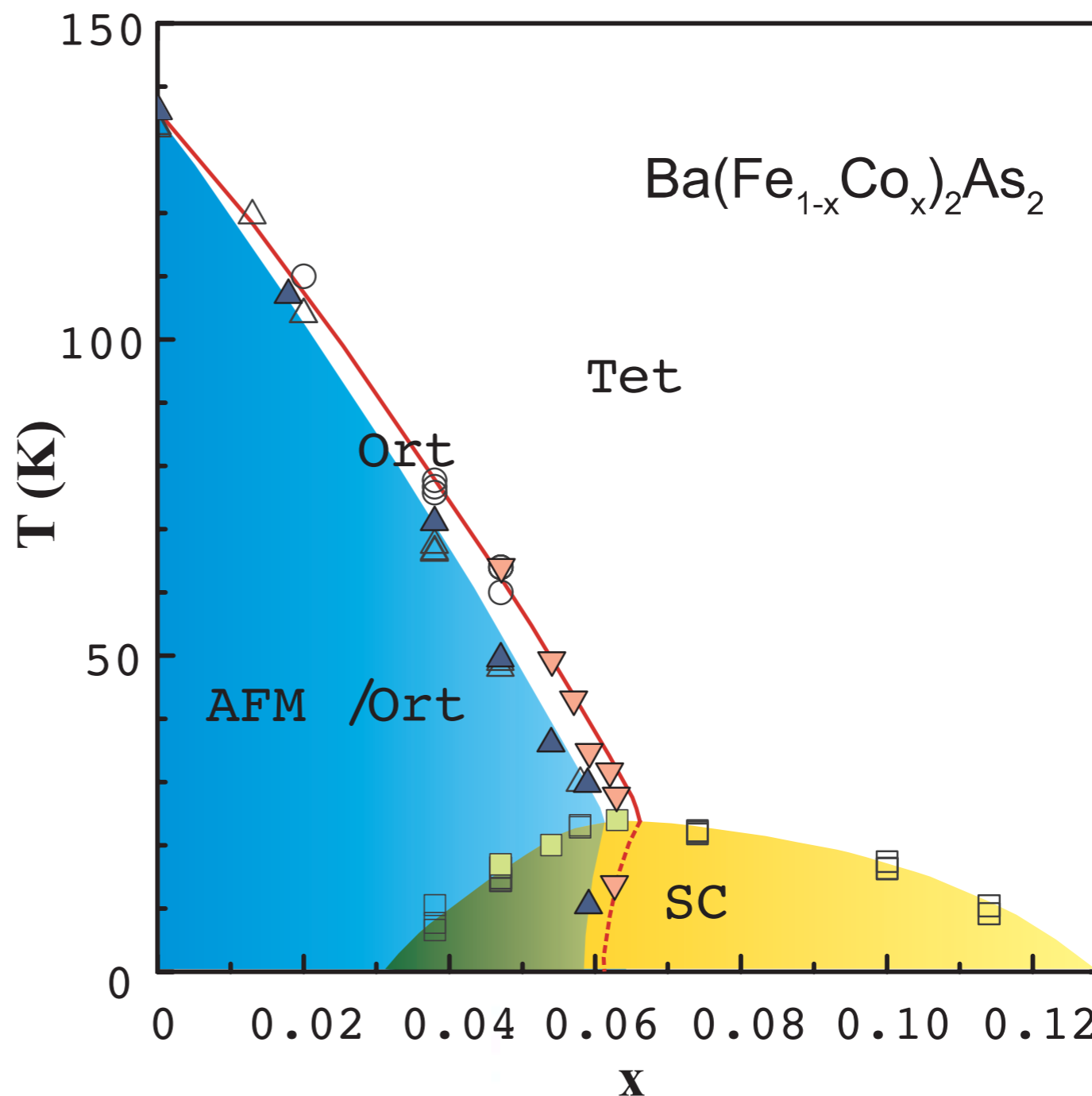
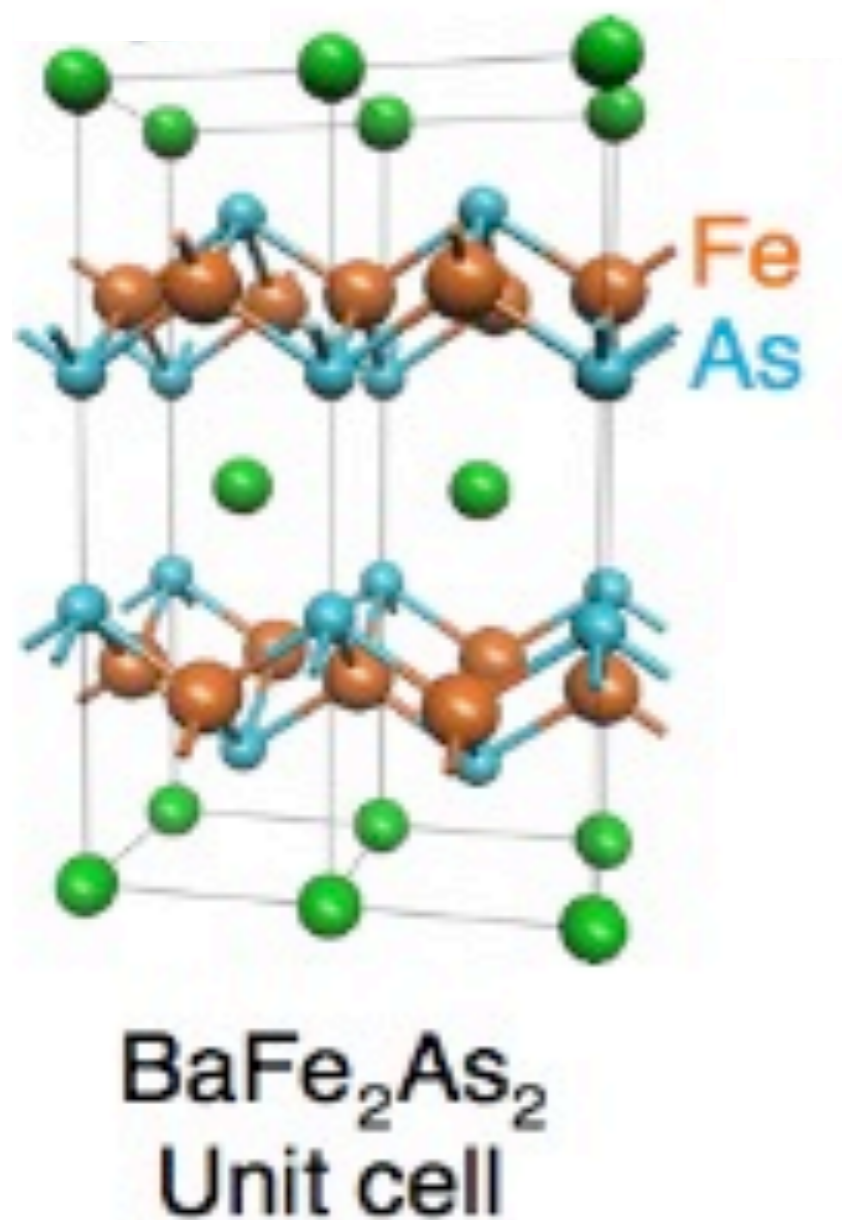
Iron pnictides:

a new class of high temperature superconductors



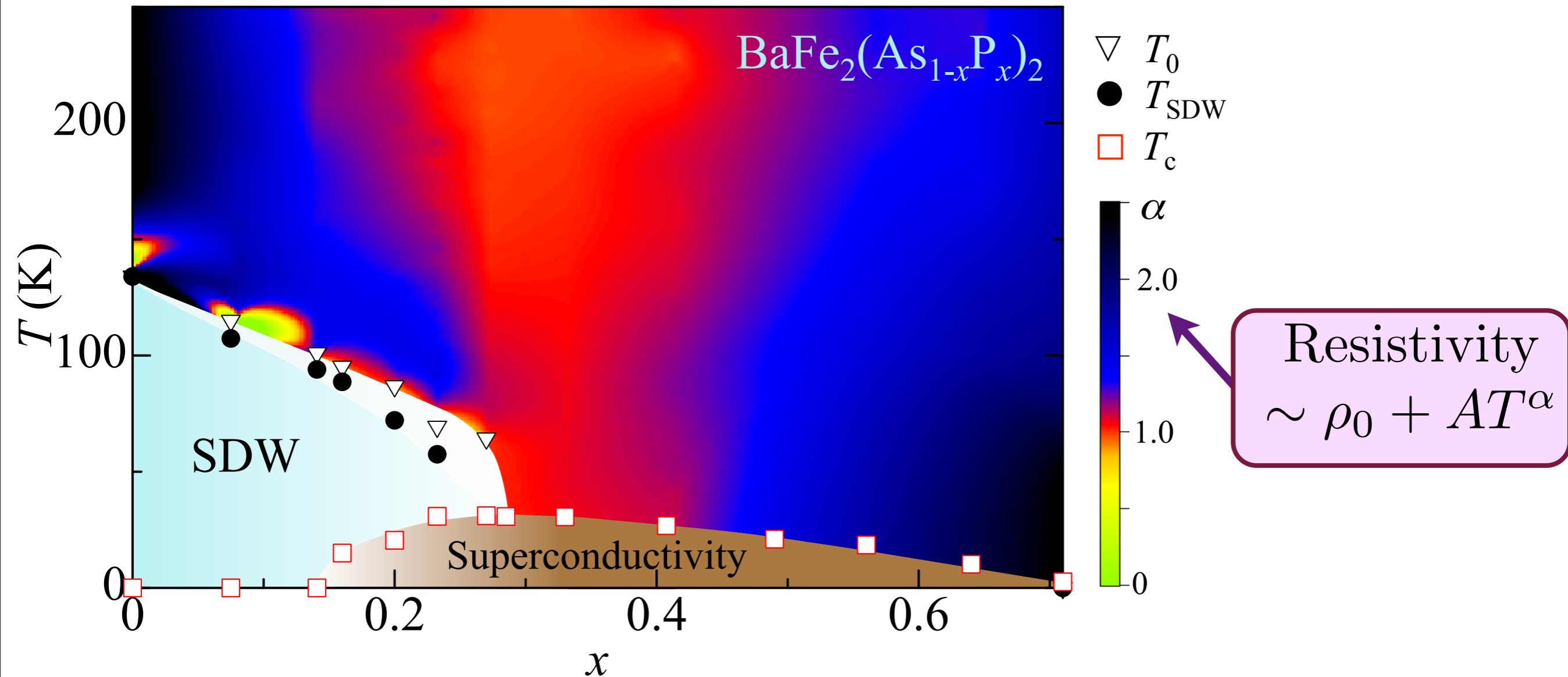
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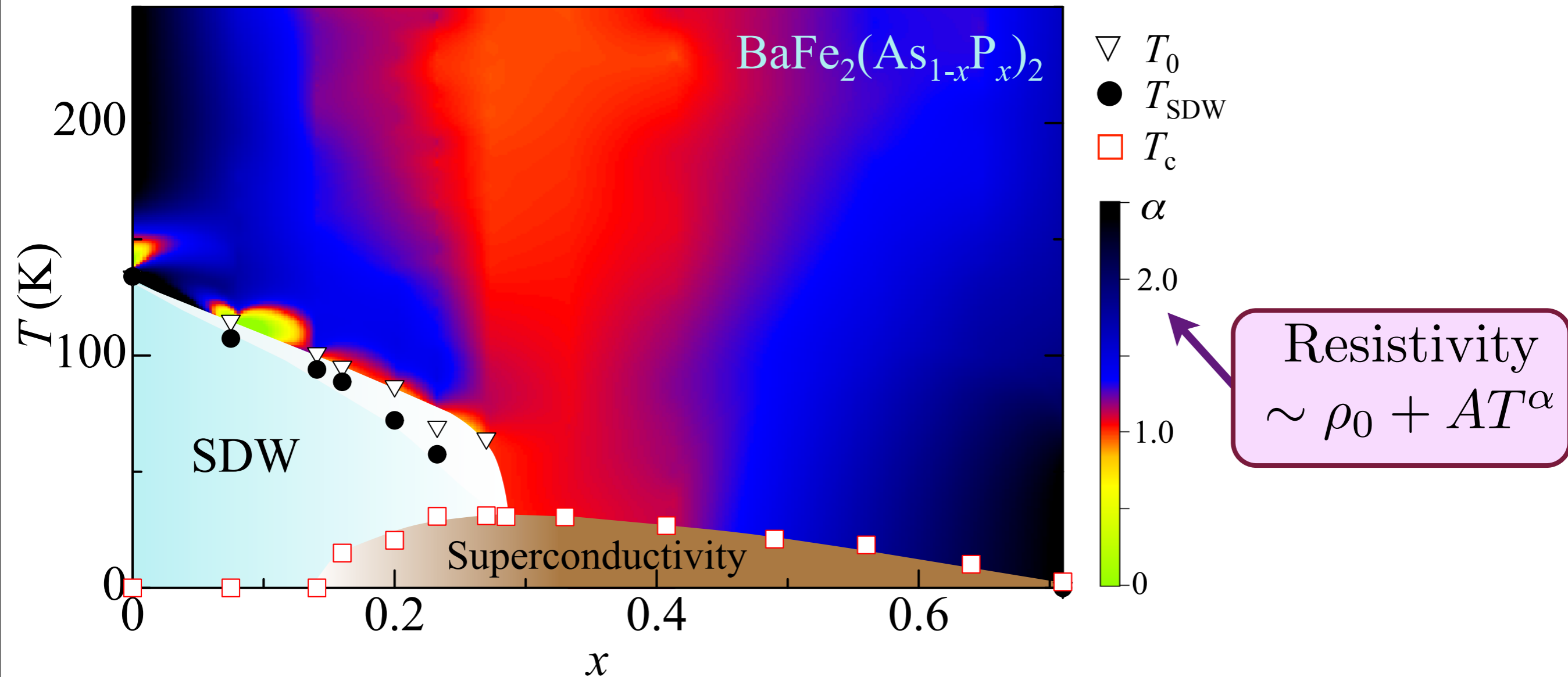
S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni,
S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman,
Physical Review Letters **104**, 057006 (2010).

Temperature-doping phase diagram of the iron pnictides:



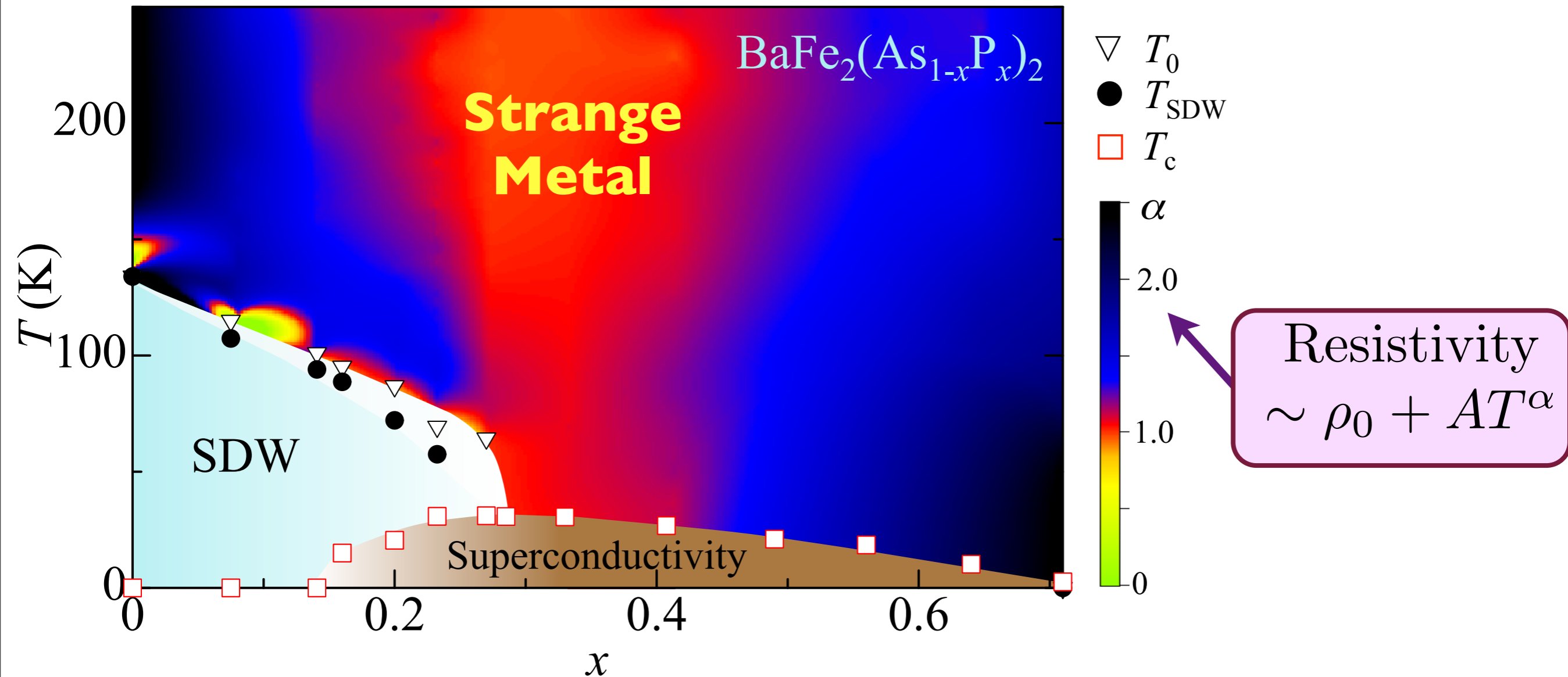
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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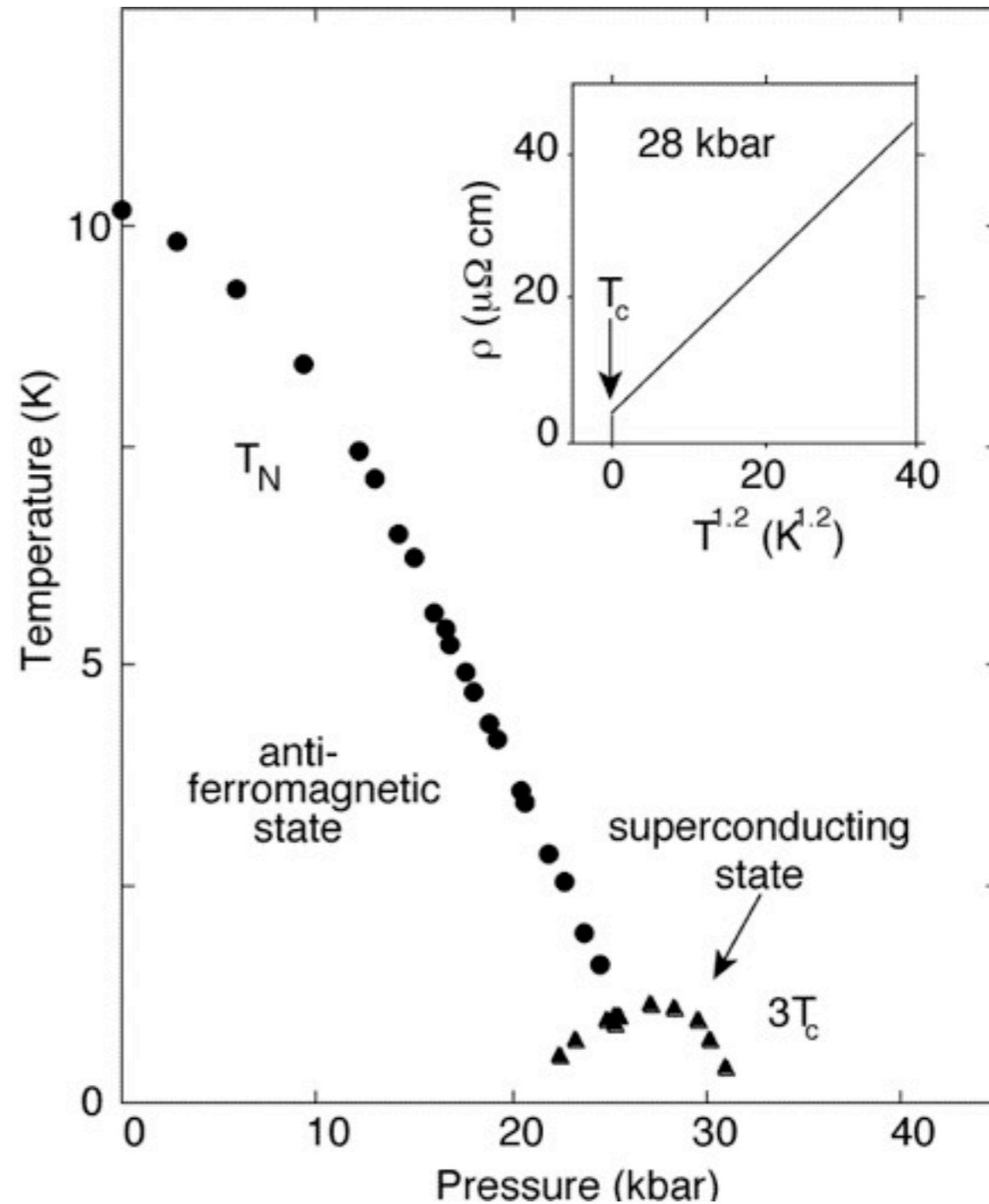
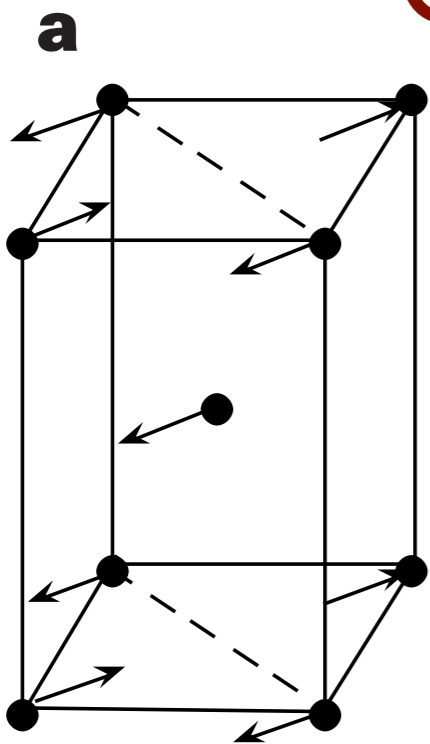
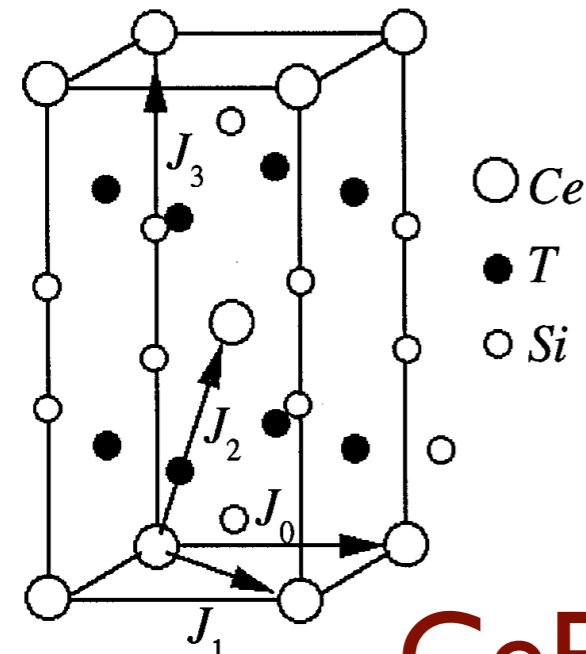
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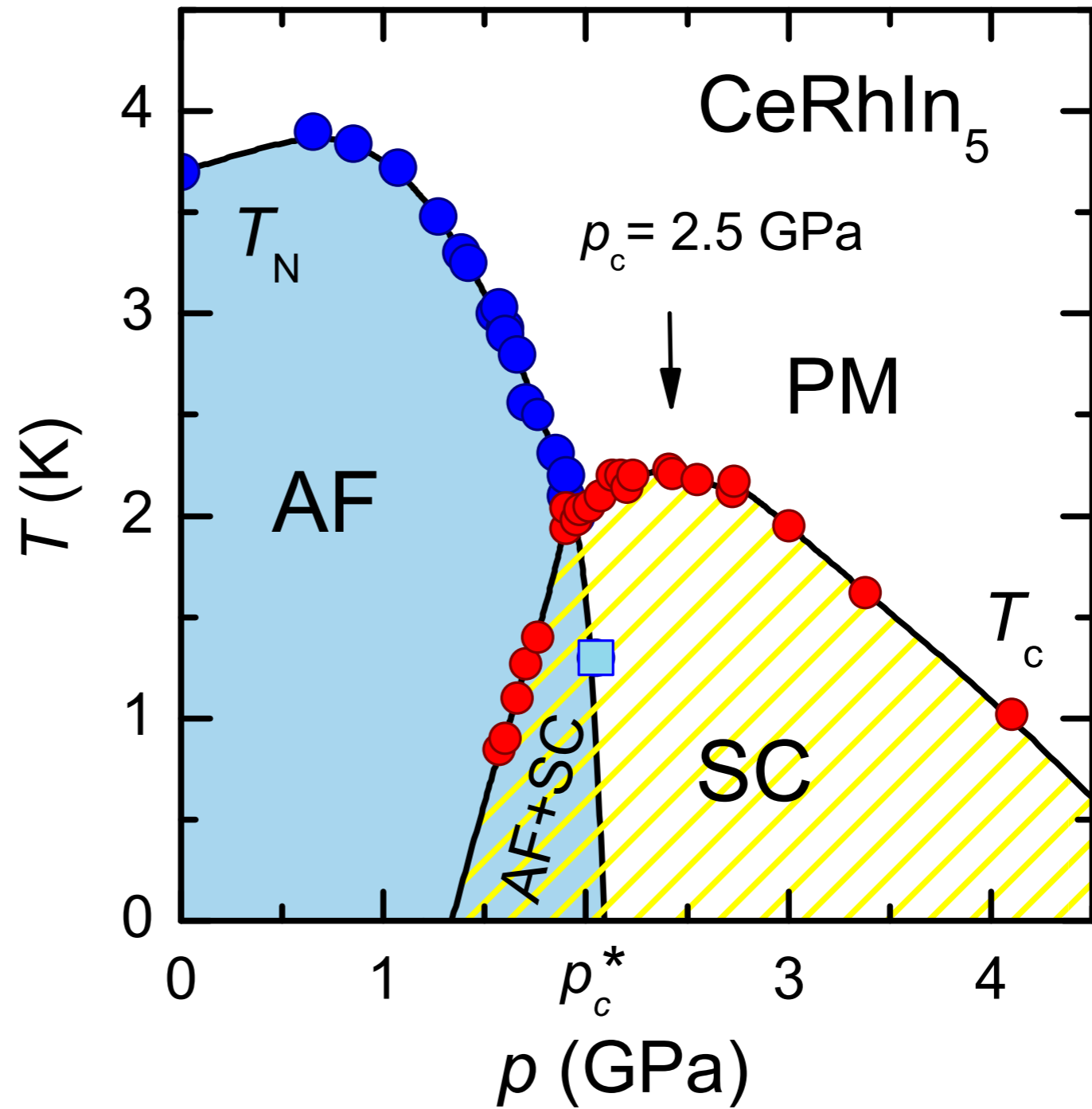
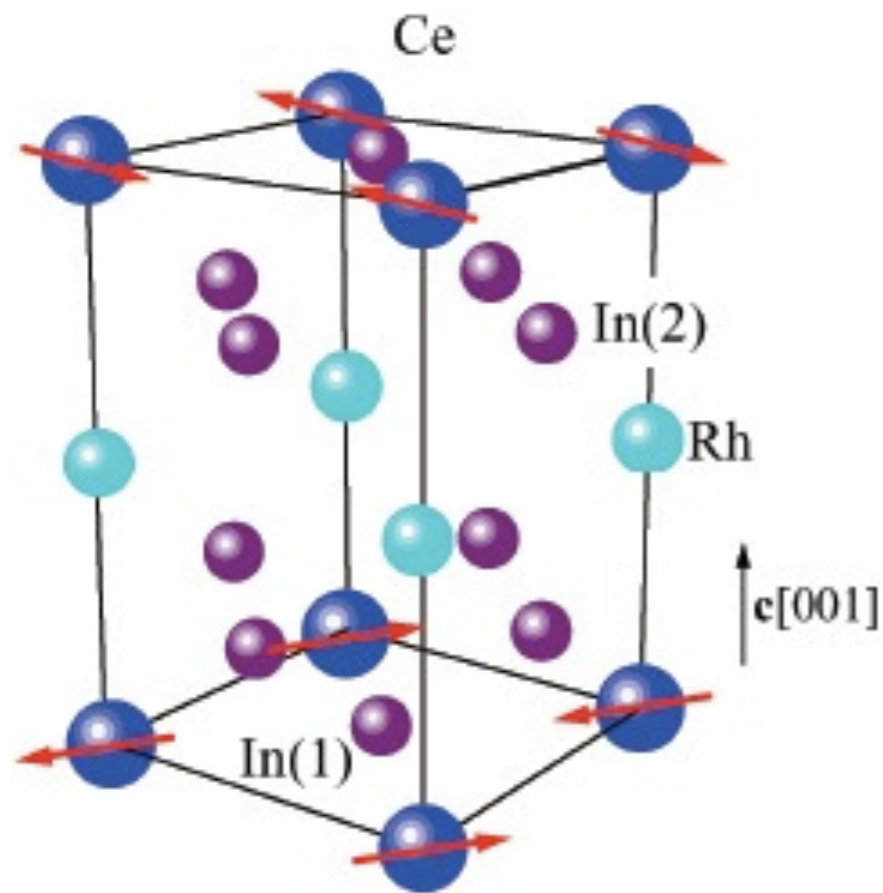
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Lower T_c superconductivity in the heavy fermion compounds



N. D. Mathur, F. M. Grosche, S. R. Julian, I. R. Walker, D. M. Freye, R. K. W. Haselwimmer, and G. G. Lonzarich, *Nature* **394**, 39 (1998)

Lower T_c superconductivity in the heavy fermion compounds



G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

Questions

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

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- *If so, why is there no antiferromagnetism in the hole-doped cuprates near the point where the superconductivity is strongest ?*
- *What is the physics of the strange metal ?*

Outline

1. Loss of antiferromagnetism in an insulator

Coupled-dimer antiferromagnets and quantum criticality

2. Onset of antiferromagnetism in a metal

From large Fermi surfaces to Fermi pockets, d-wave superconductivity, and competing orders

3. Strongly-coupled quantum criticality in metals

Fermi surfaces and gapless bosons

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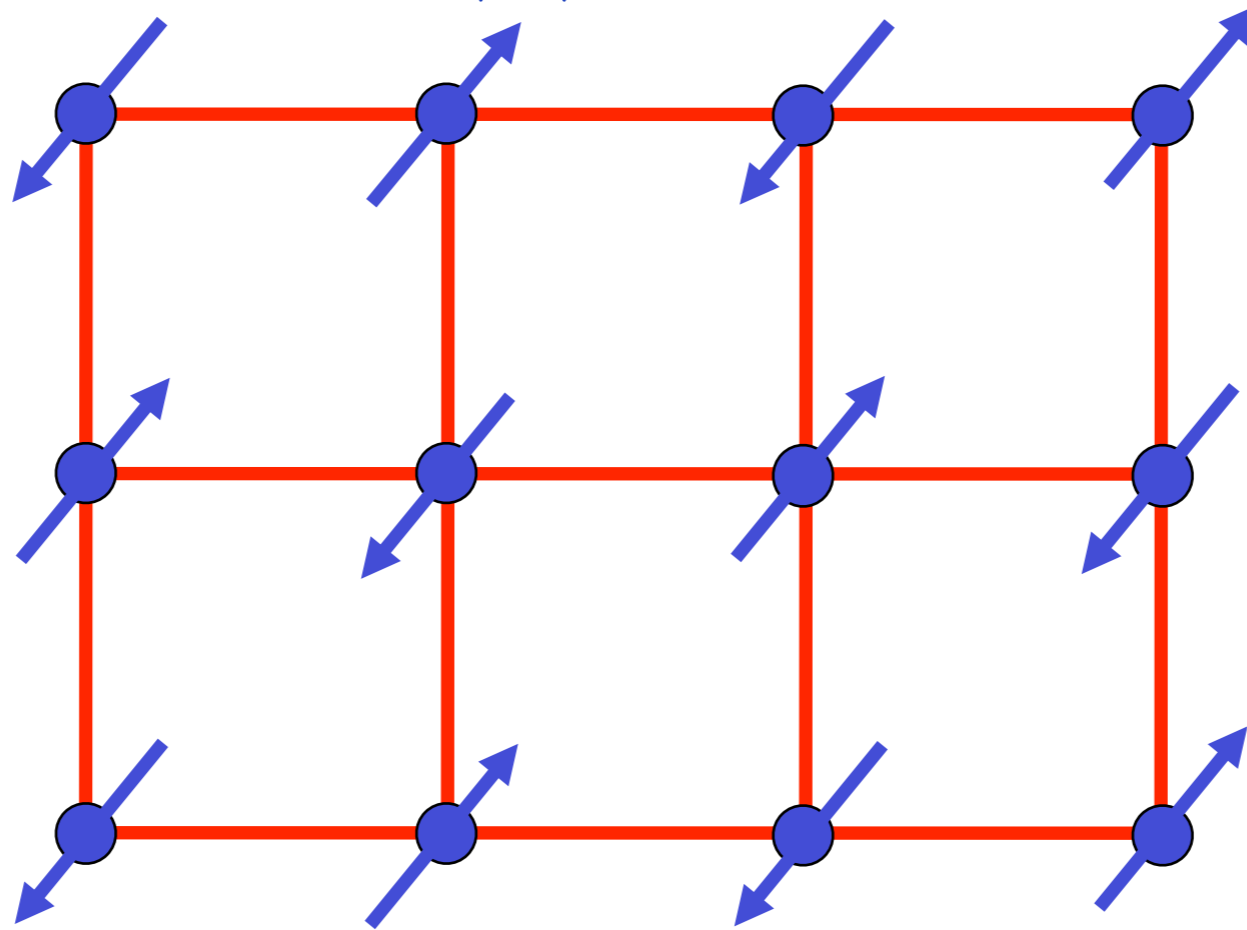
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Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

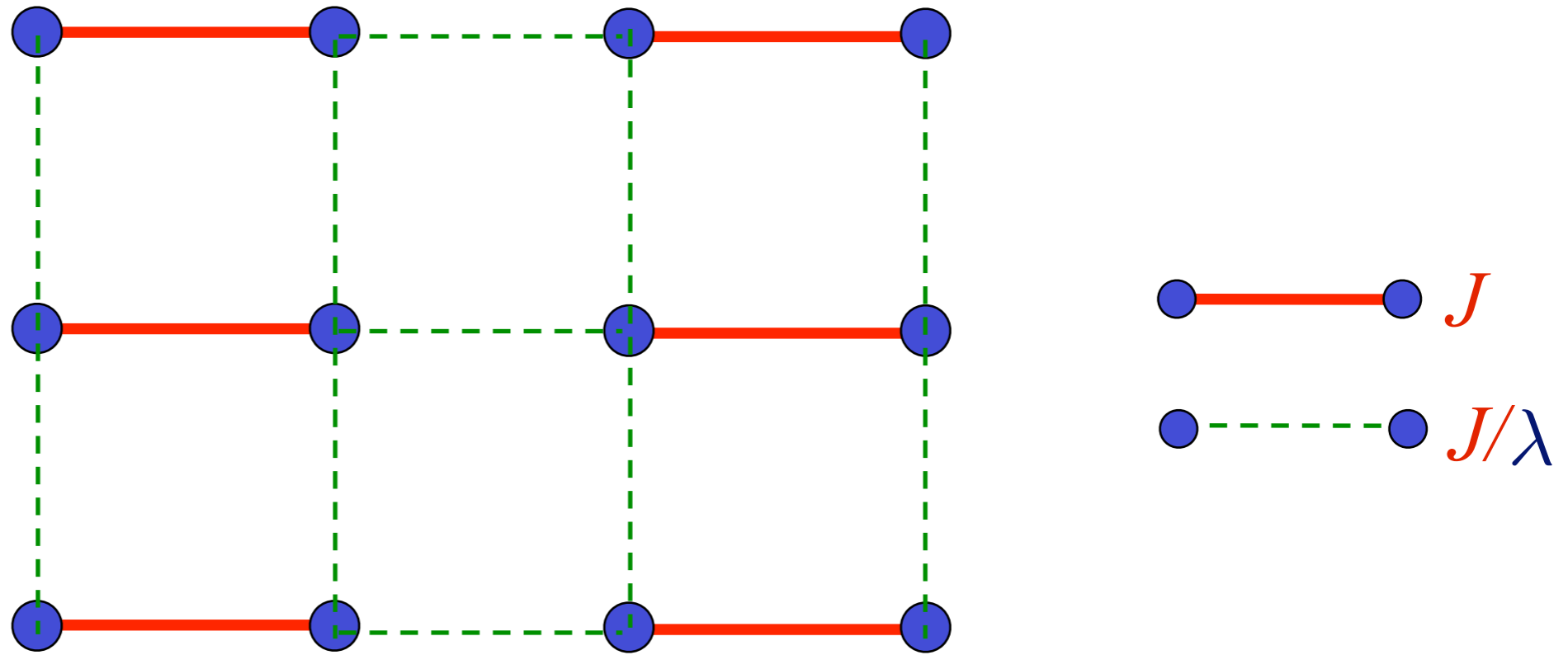
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

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Square lattice antiferromagnet

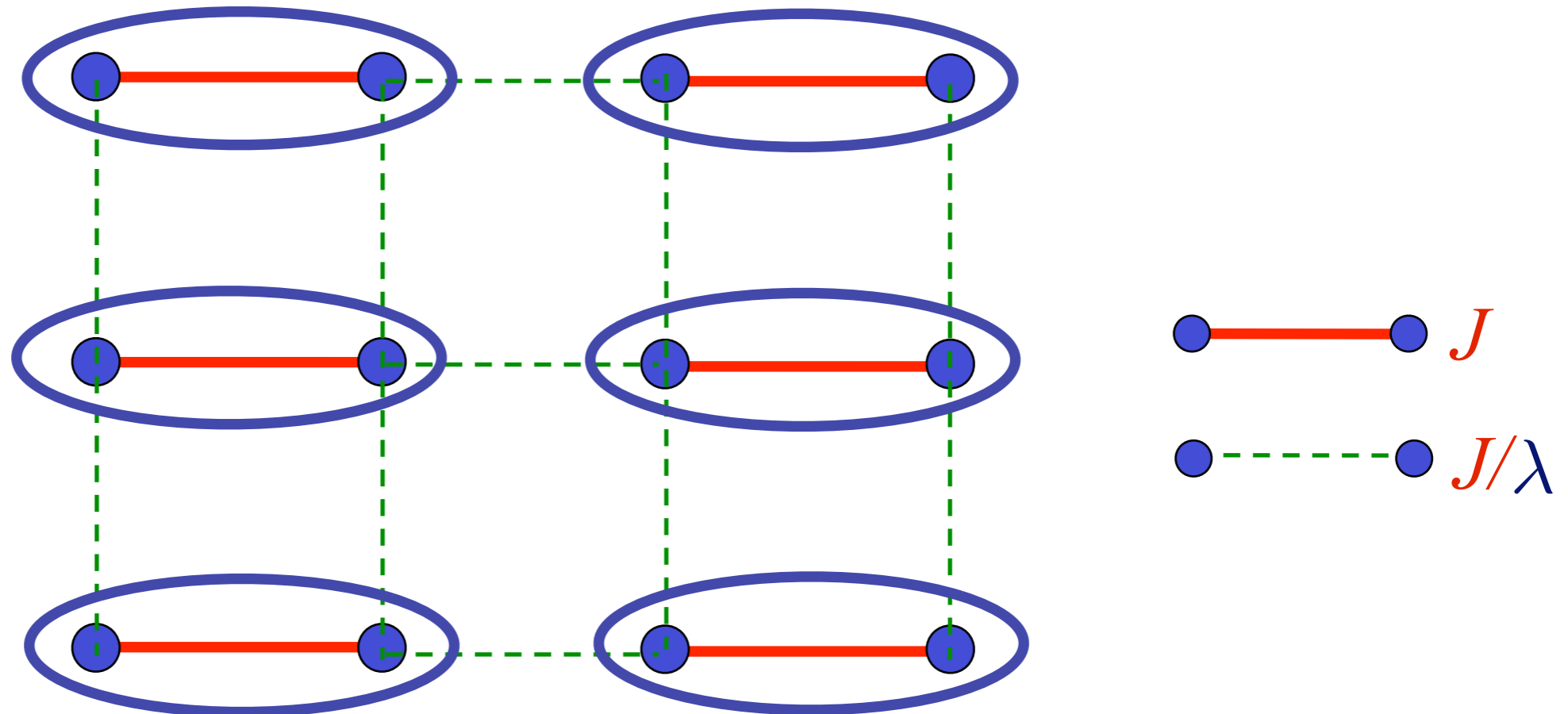
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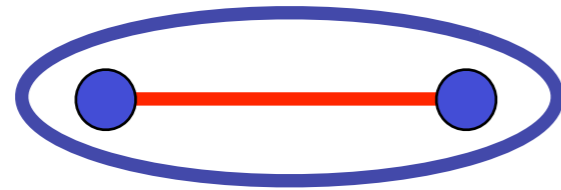
Weaken some bonds to induce spin entanglement in a new quantum phase

Square lattice antiferromagnet

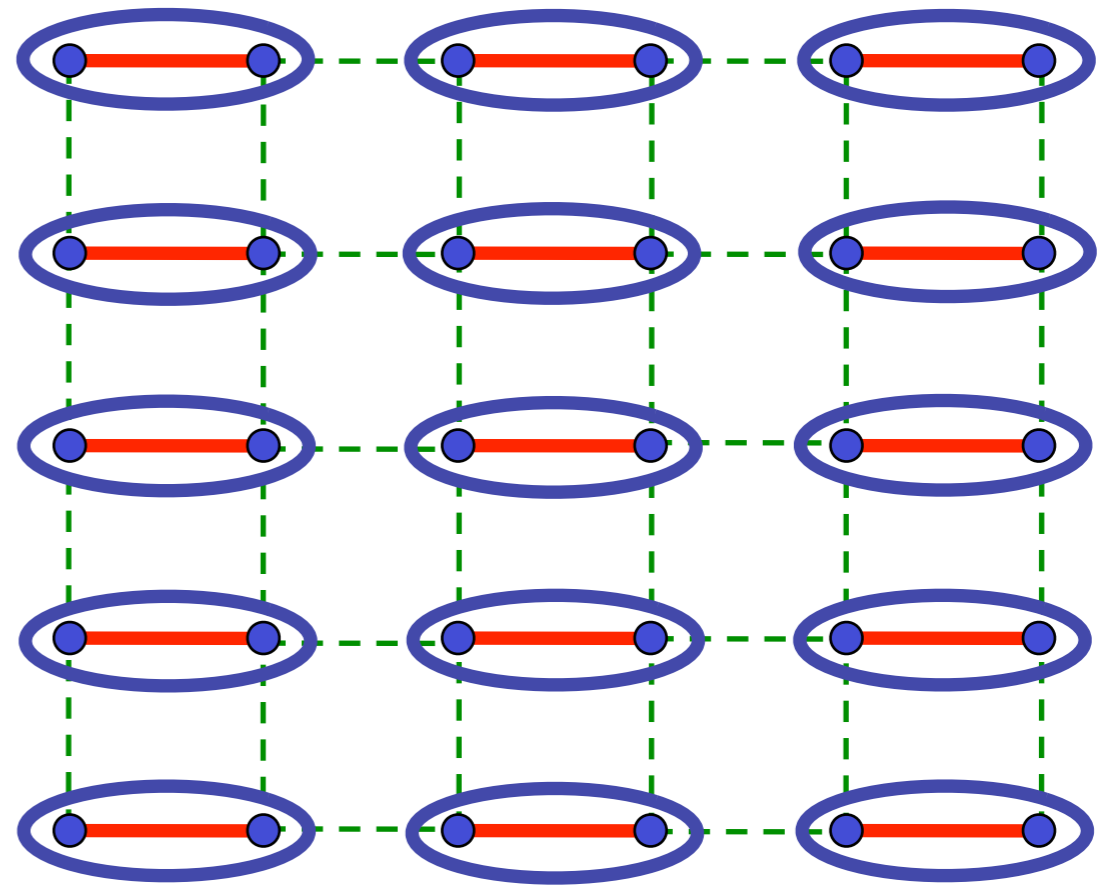
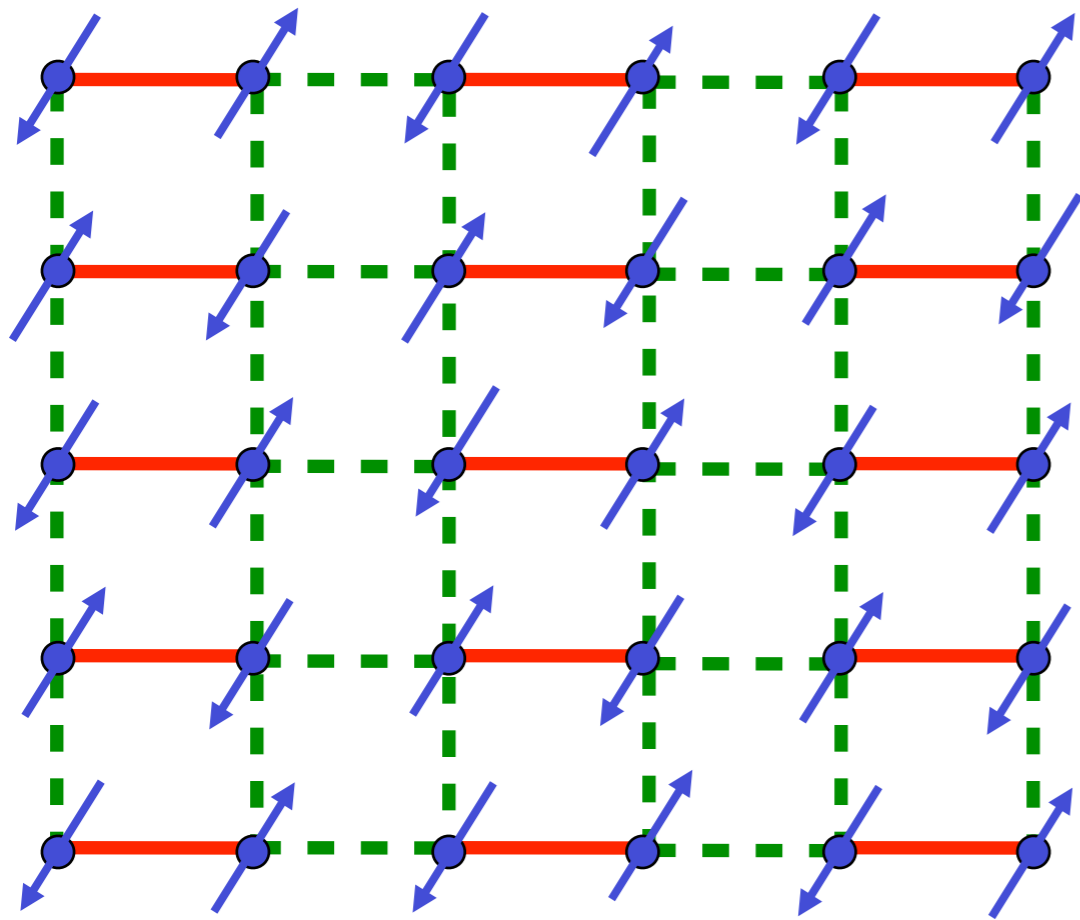
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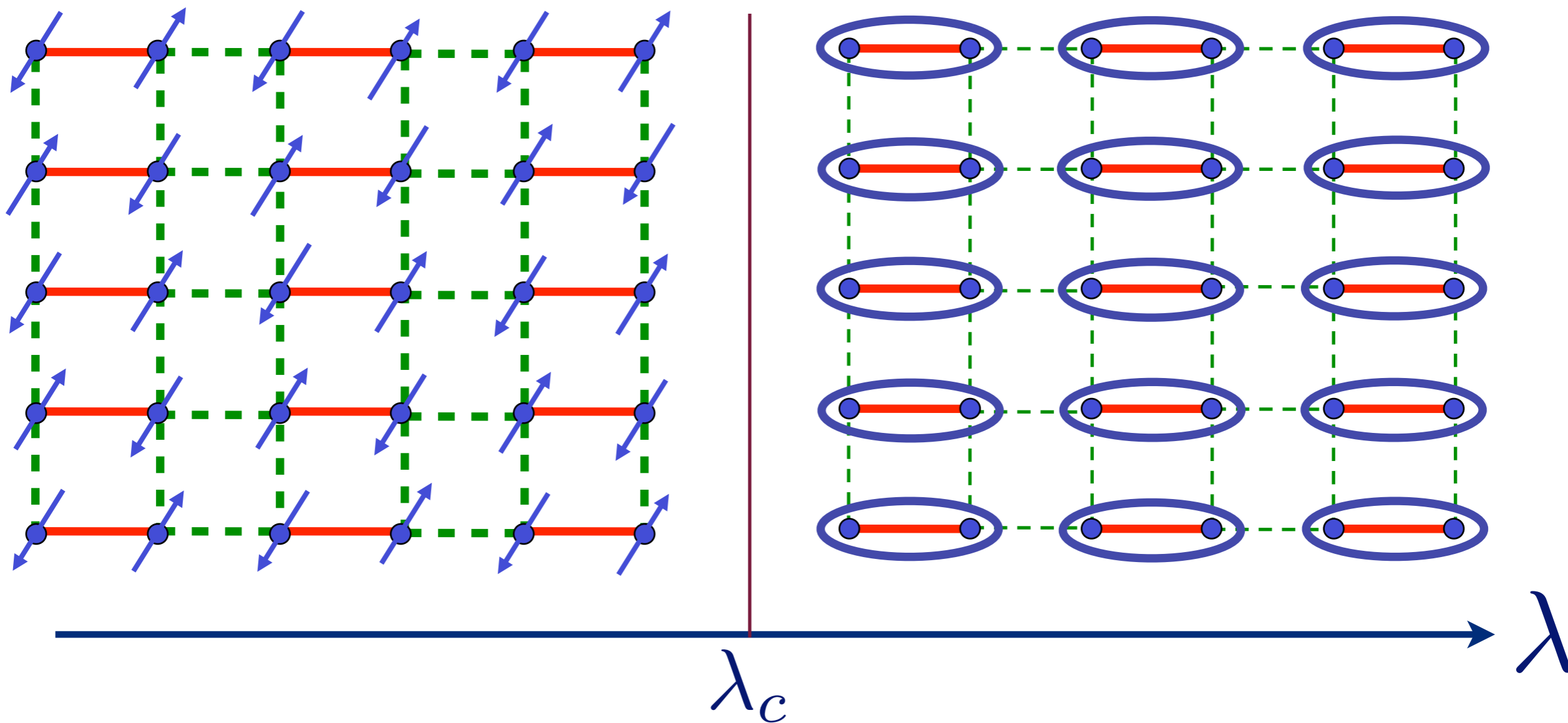
Ground state is a “quantum paramagnet”
with spins locked in valence bond singlets


$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

$$\text{Diagram of two blue dots connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



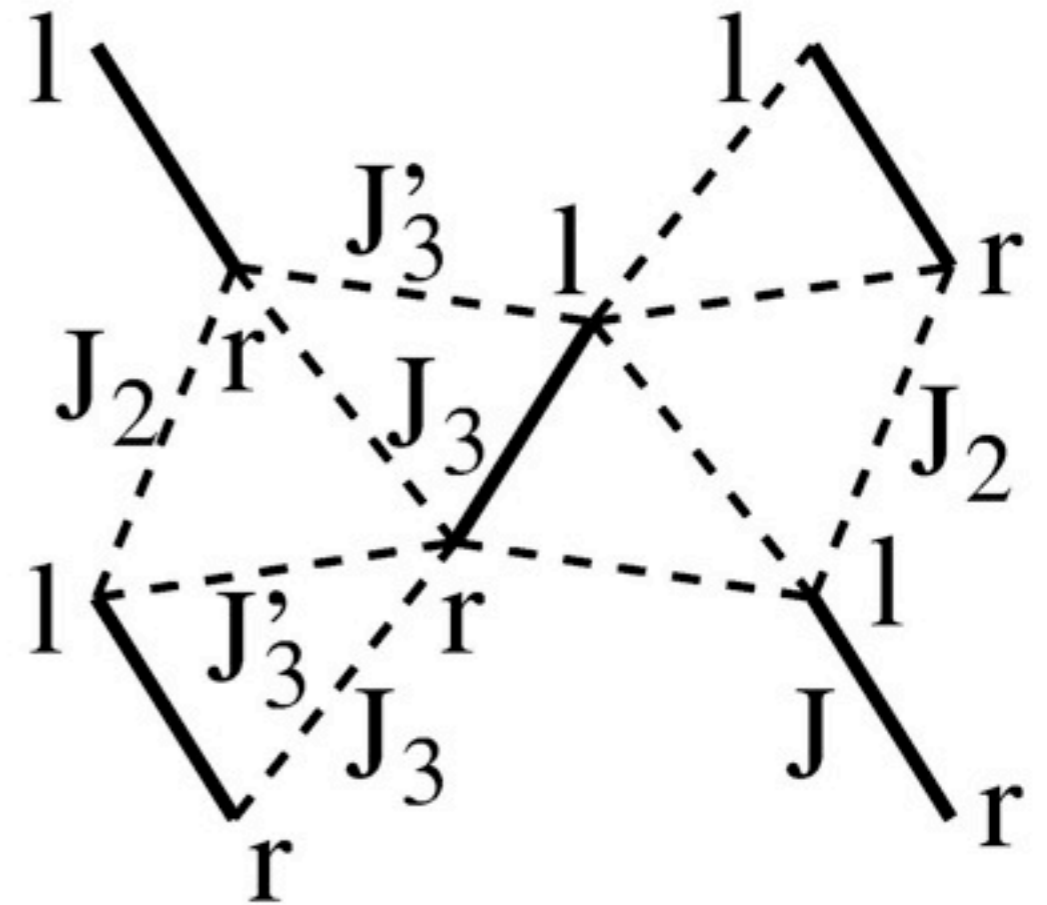
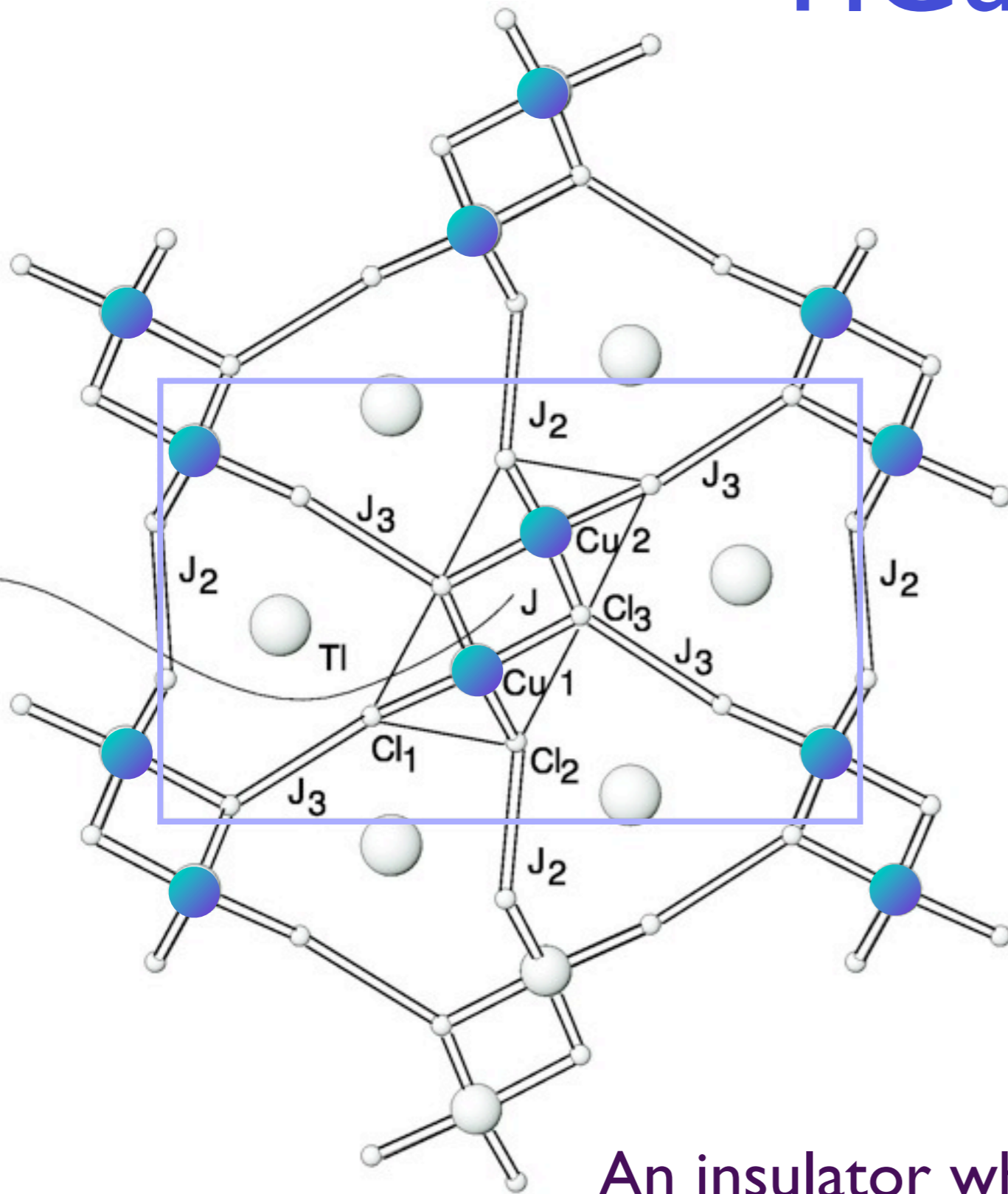
$$\text{Diagram of two blue spheres connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



Pressure in TlCuCl_3

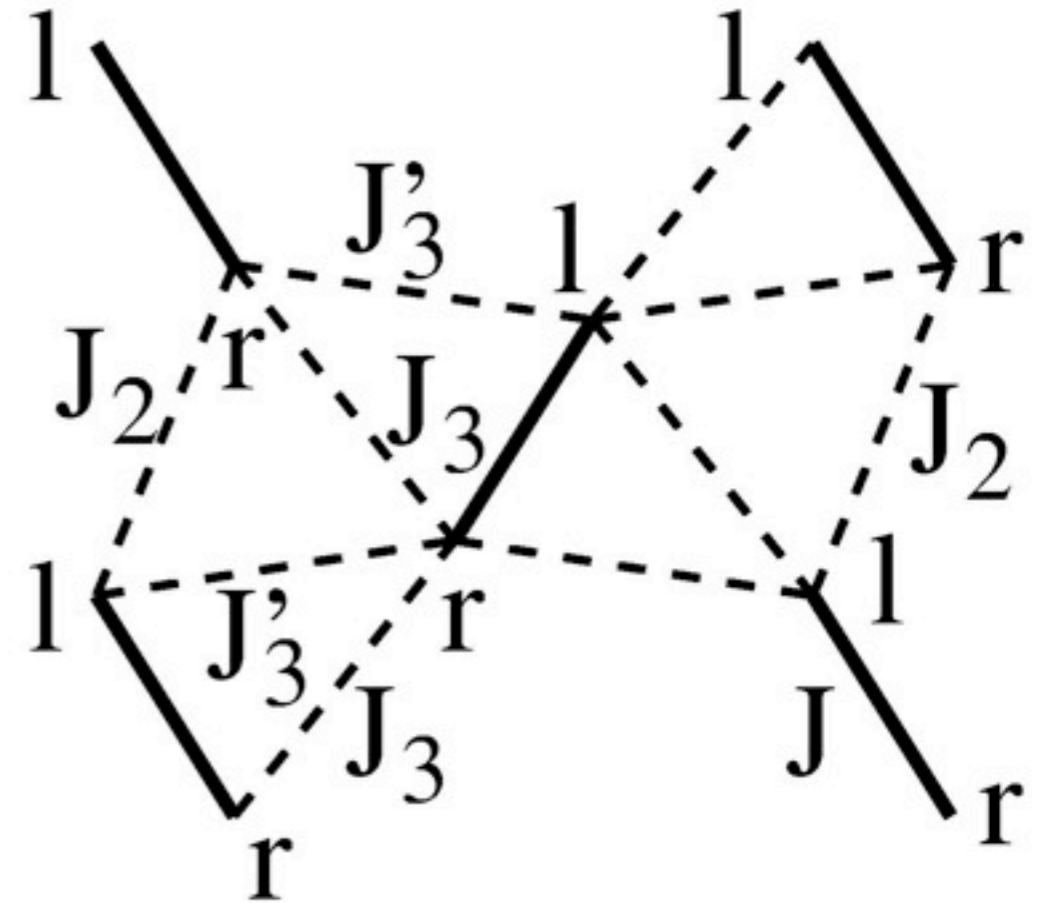
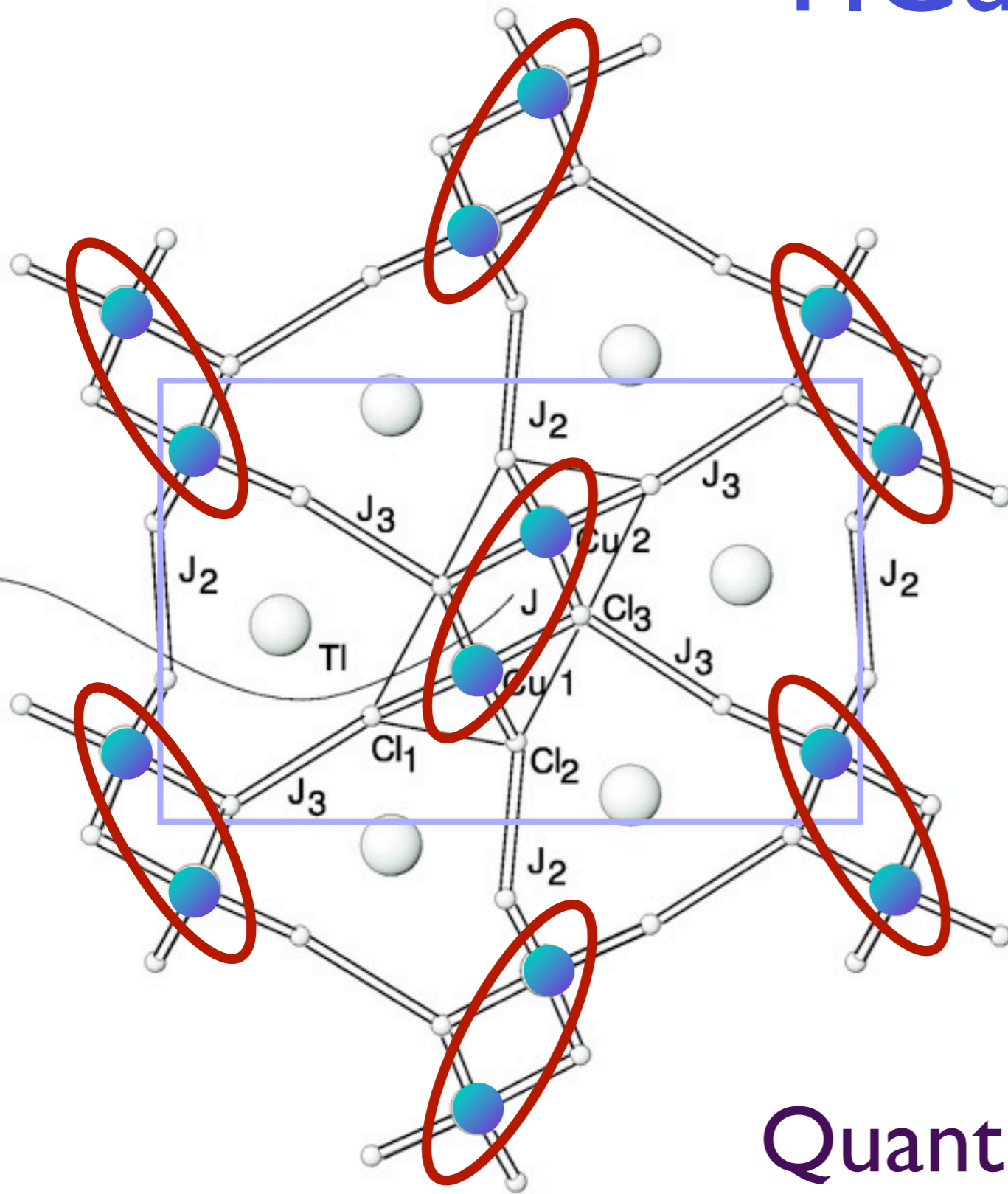
A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka,
Journal of the Physical Society of Japan, **73**, 1446 (2004).

TlCuCl₃



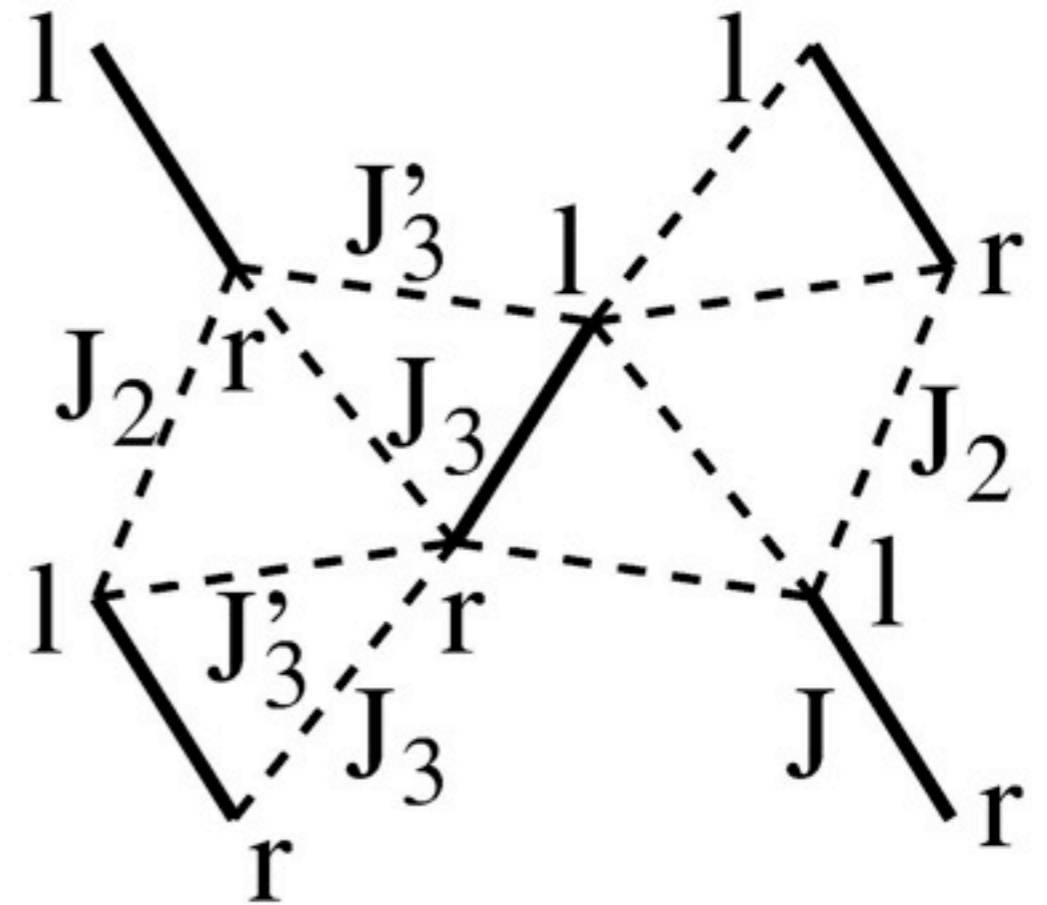
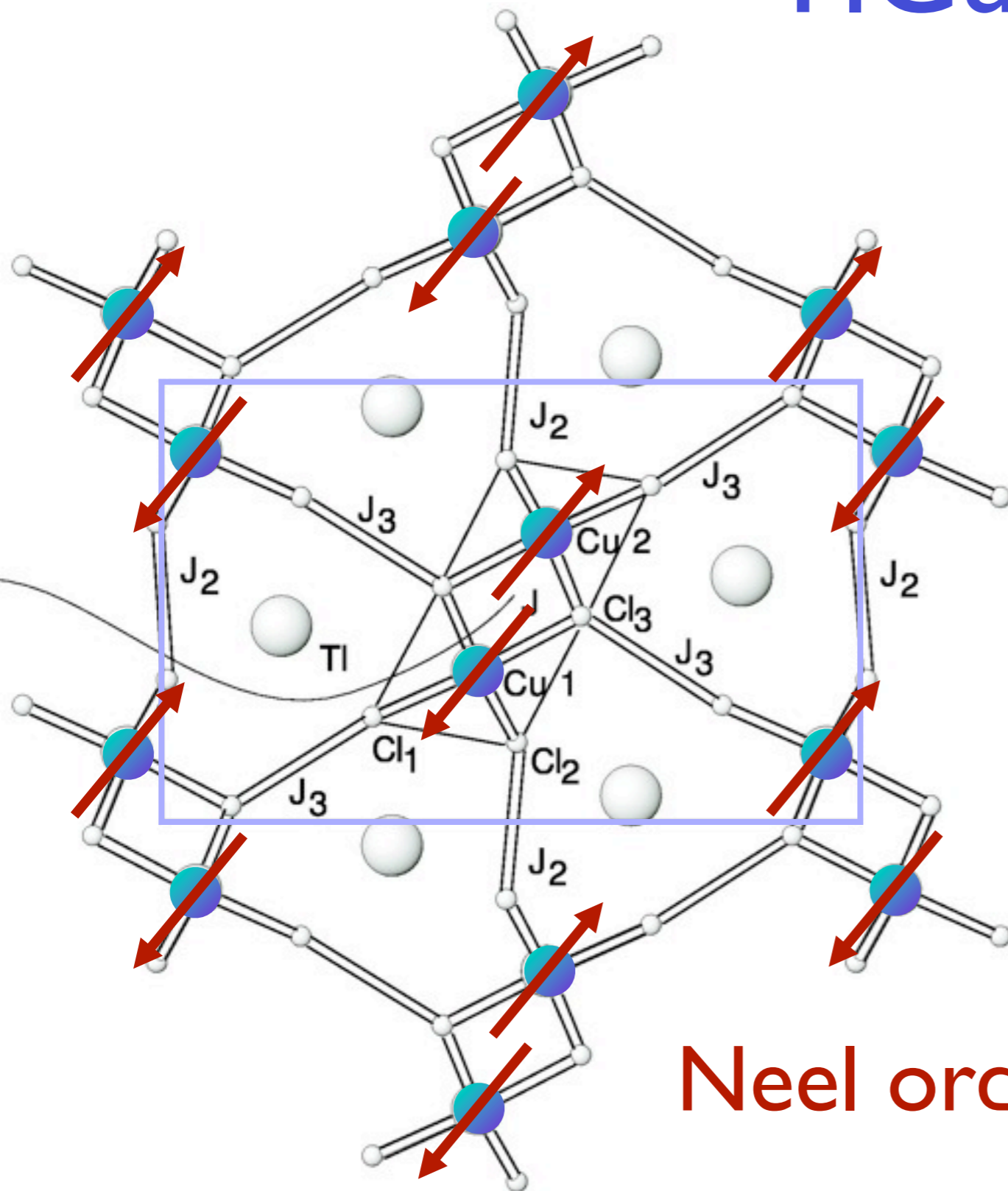
An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

TlCuCl₃



Quantum paramagnet at
ambient pressure

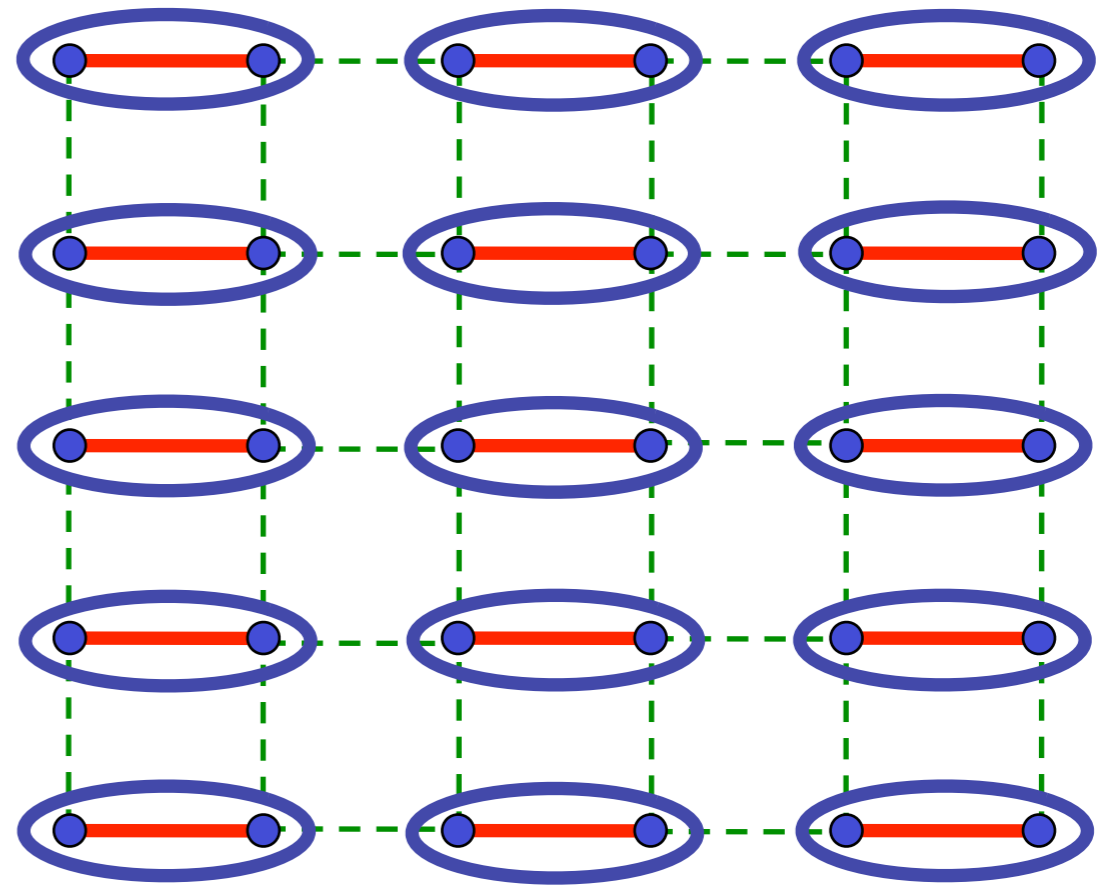
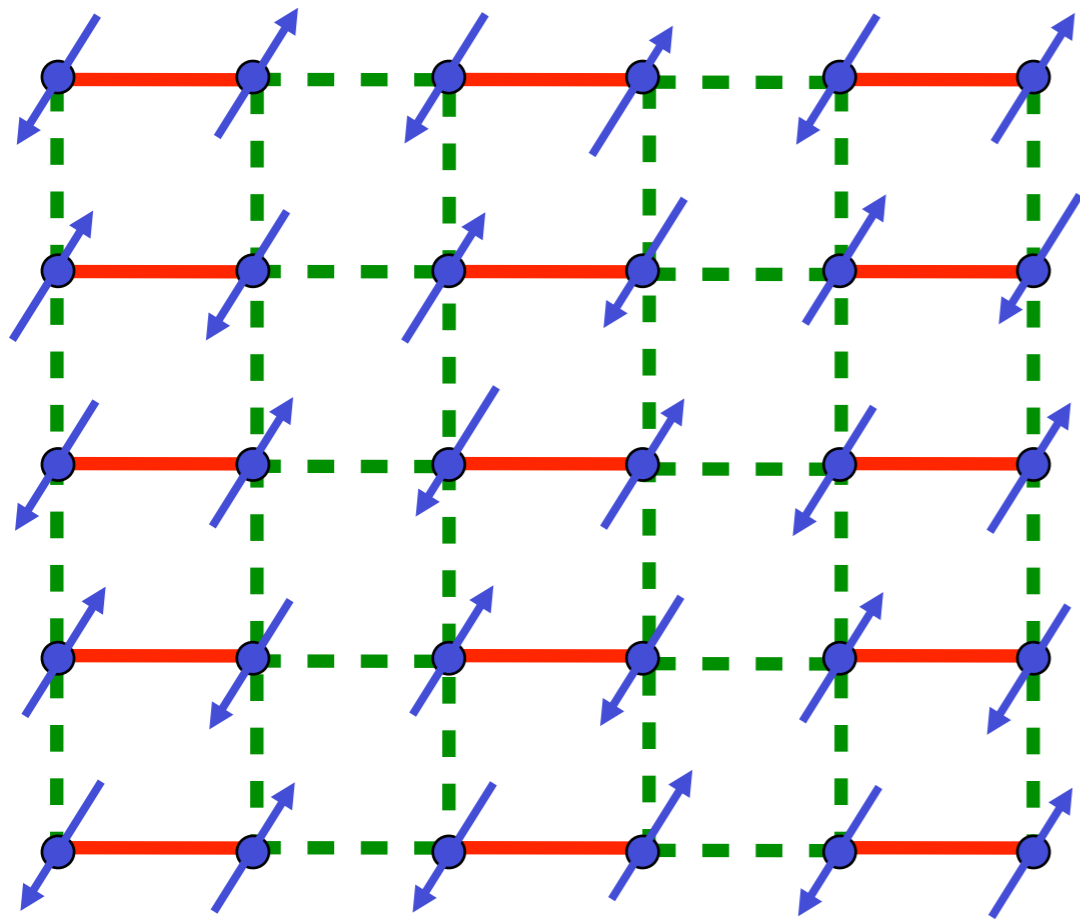
TlCuCl₃



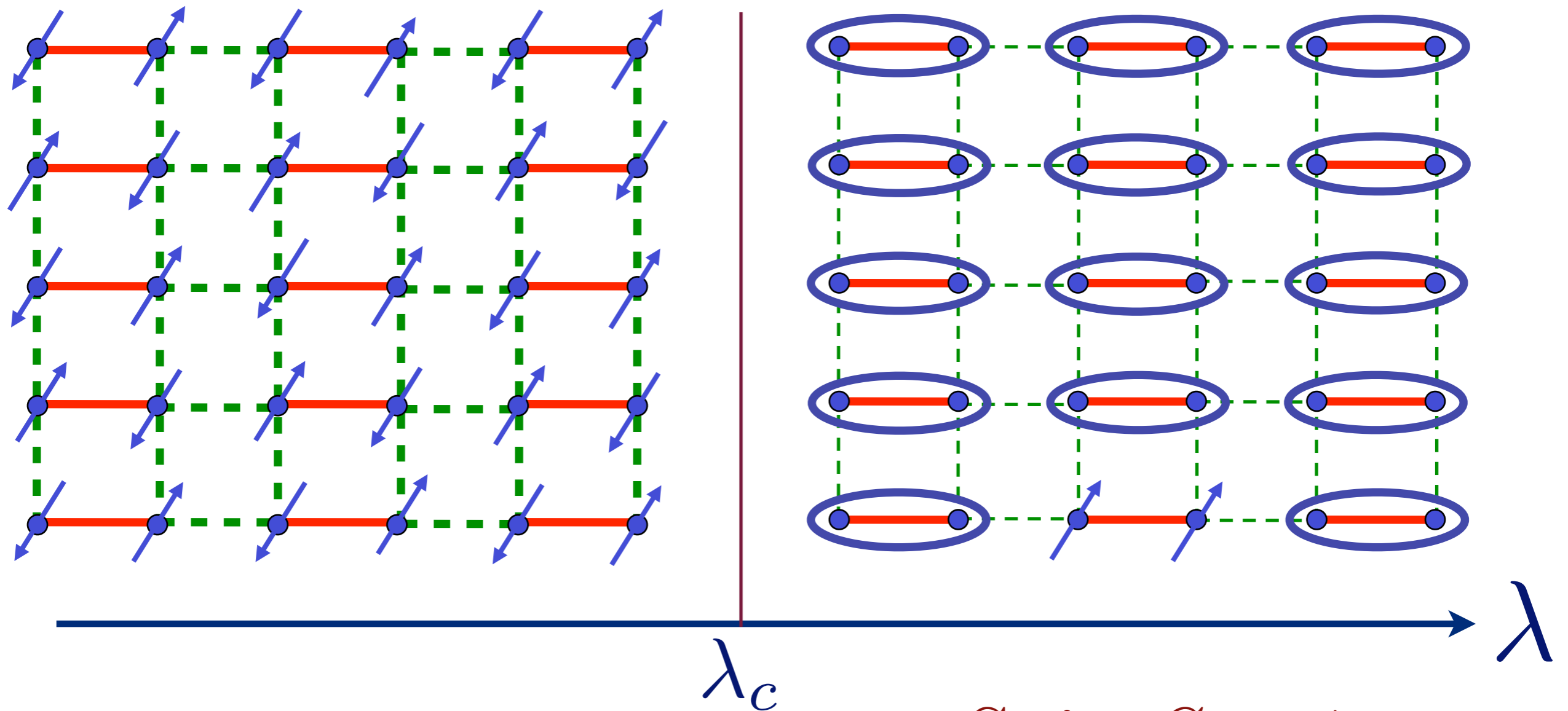
Neel order under pressure

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka,
Journal of the Physical Society of Japan, **73**, 1446 (2004).

$$\text{Diagram of two blue dots connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

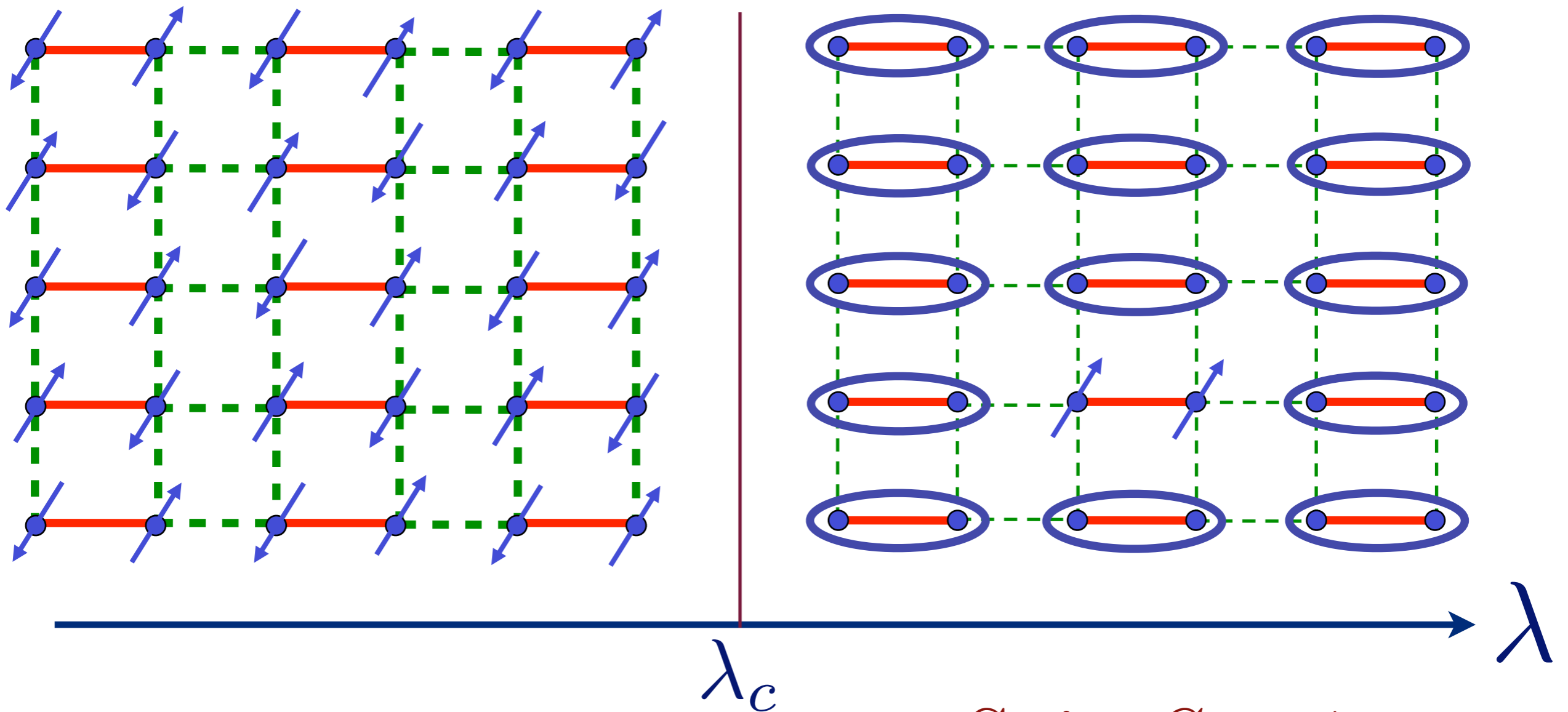


Excitation spectrum in the paramagnetic phase



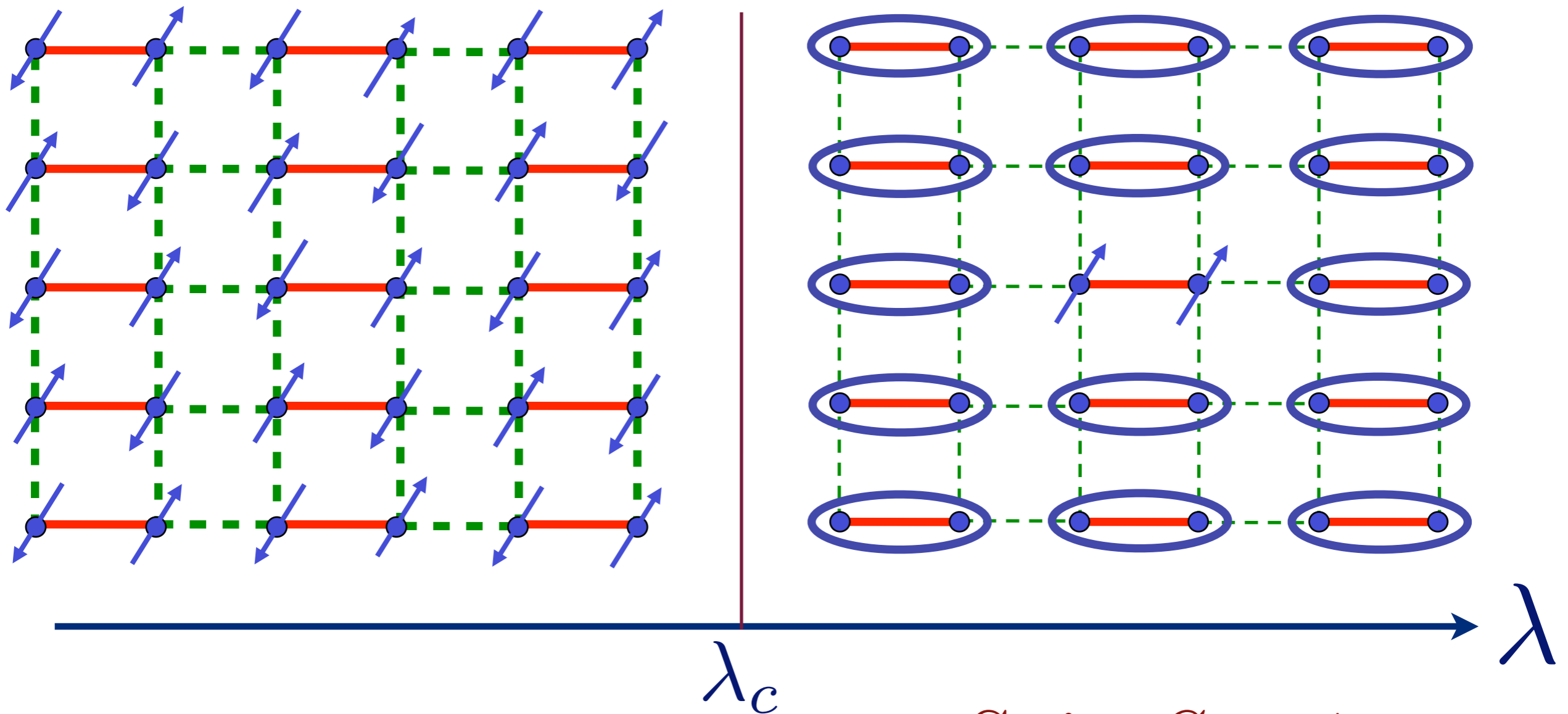
Spin $S = 1$
“triplon”

Excitation spectrum in the paramagnetic phase



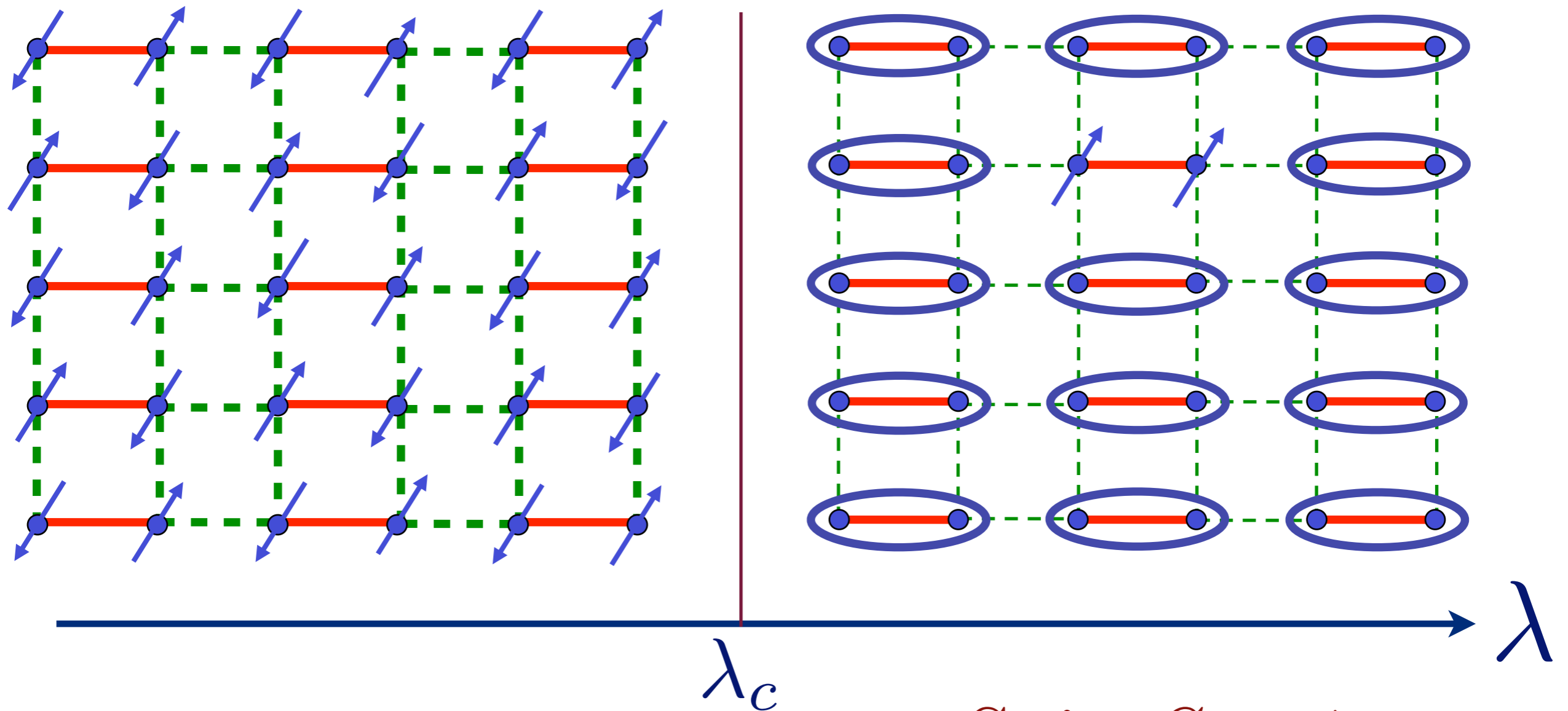
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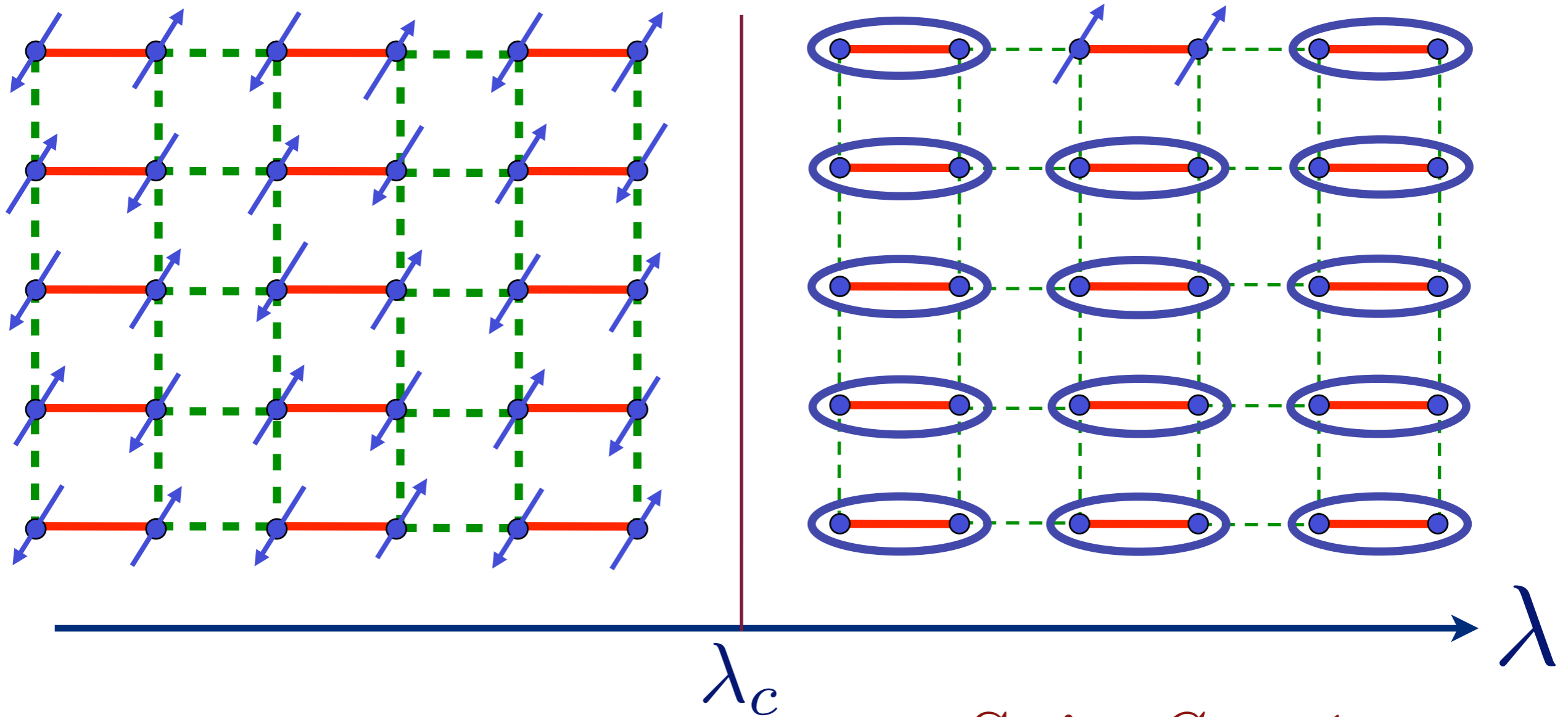
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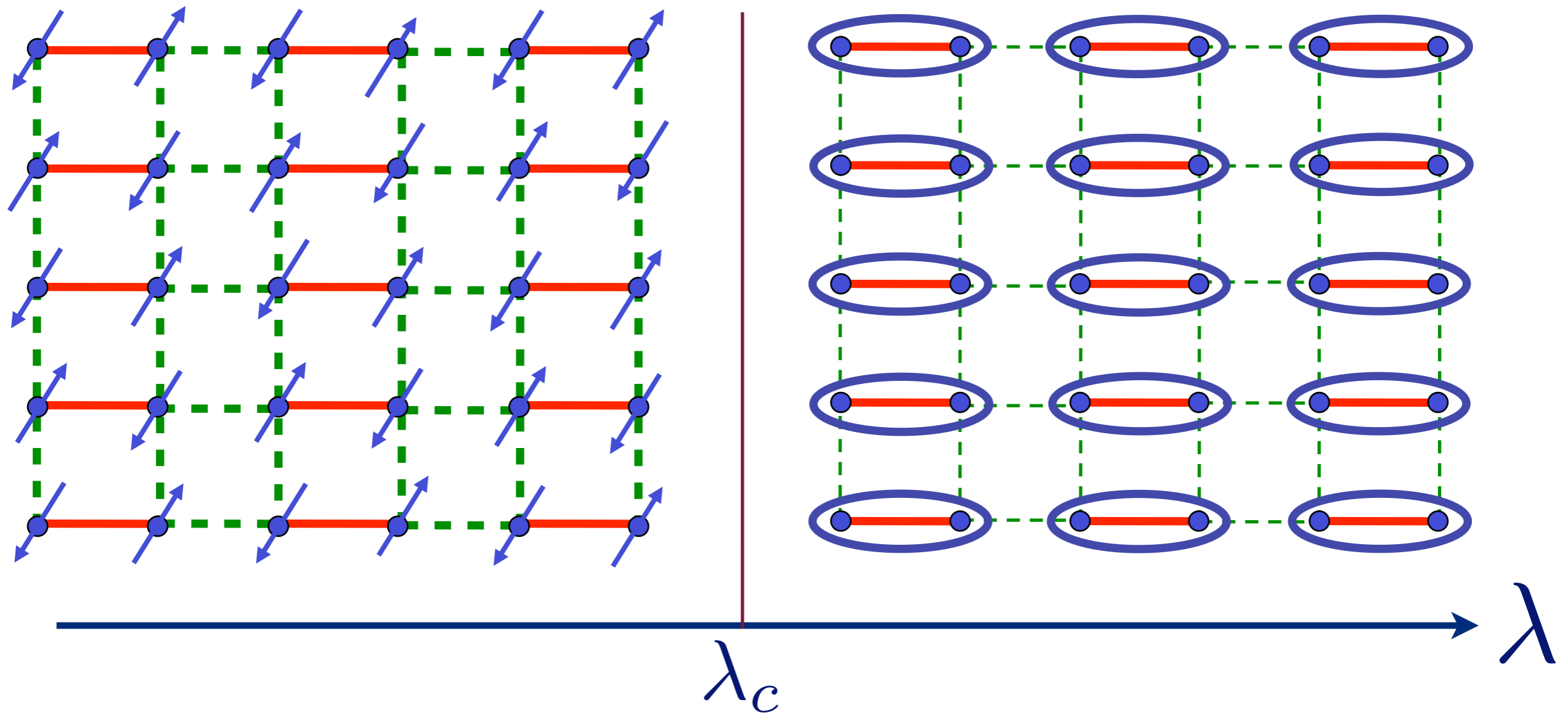
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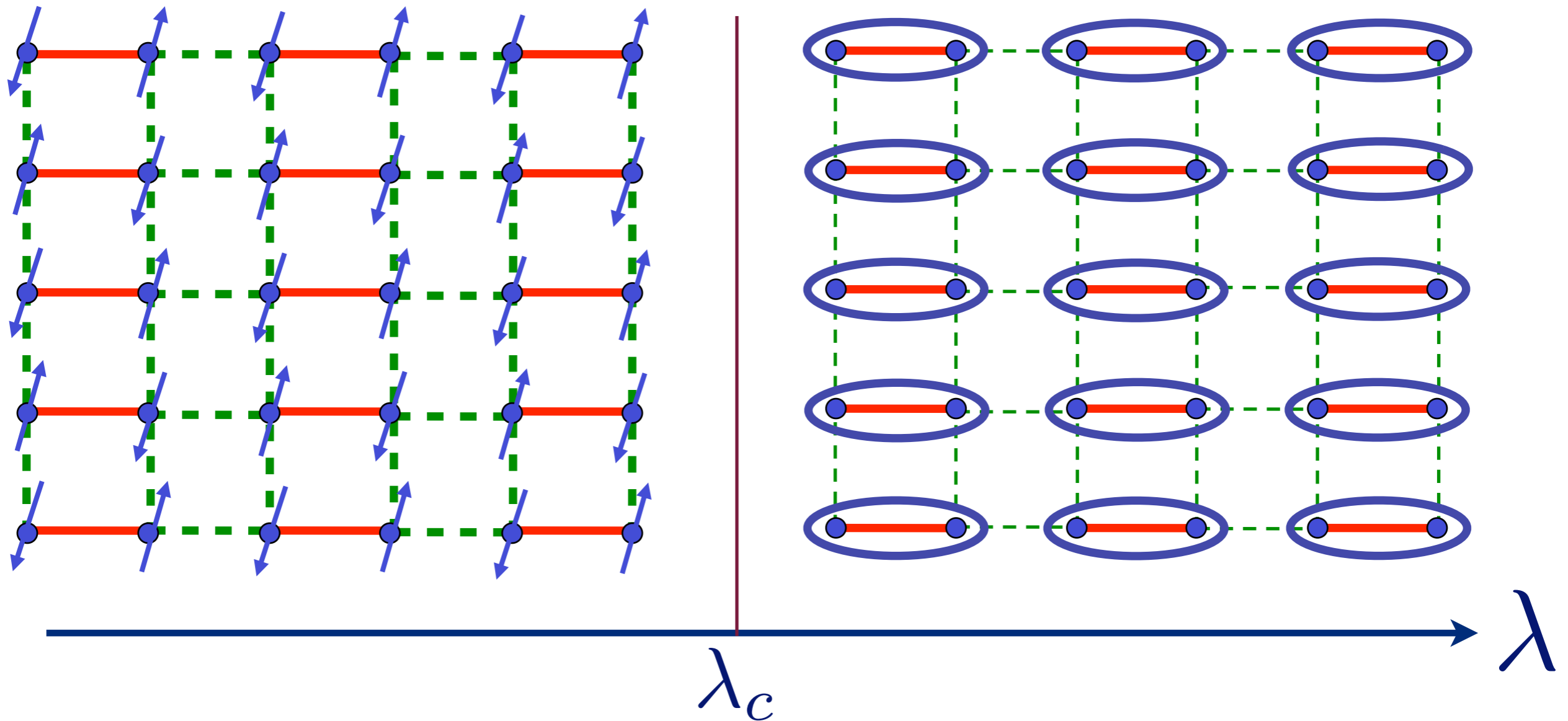
Spin $S = 1$
“triplon”

Excitation spectrum in the Néel phase



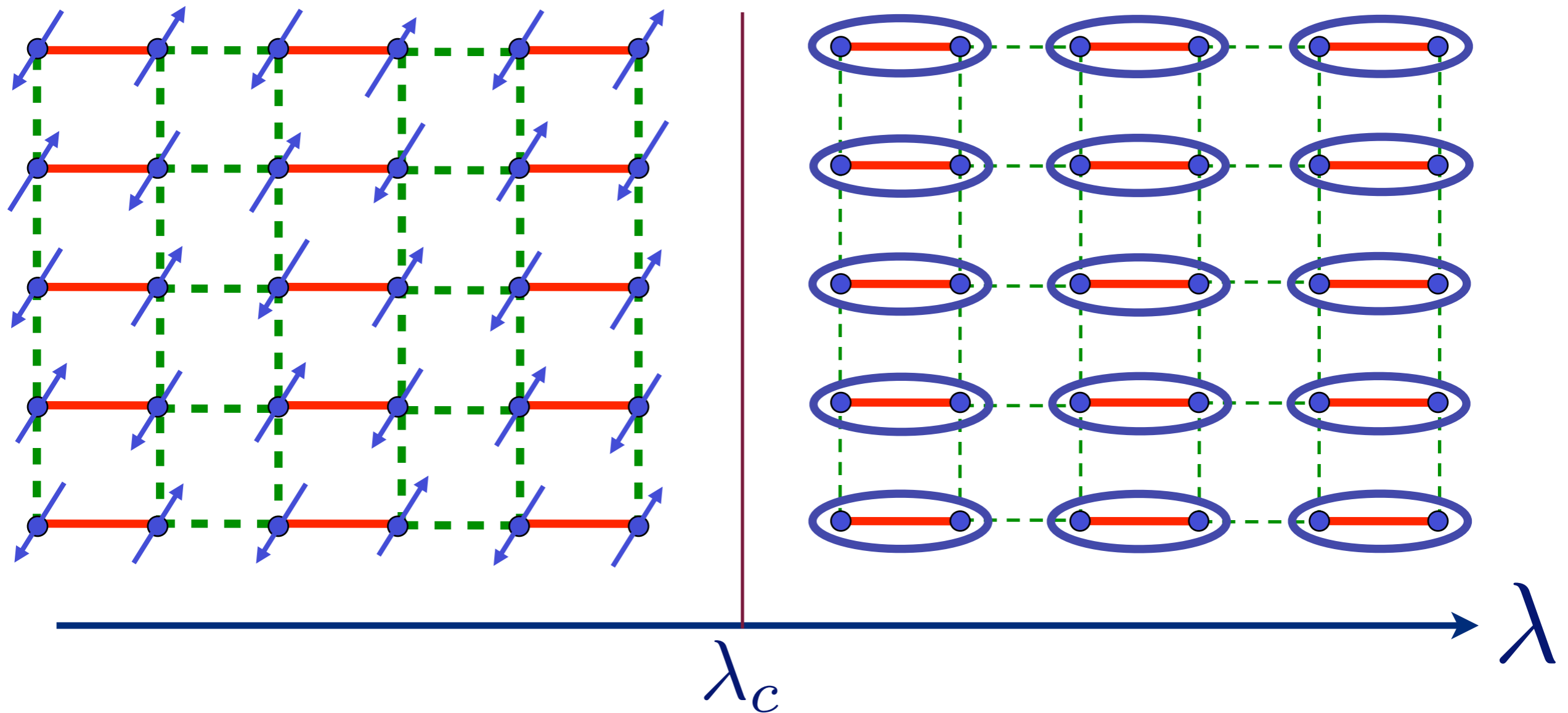
Spin waves

Excitation spectrum in the Néel phase



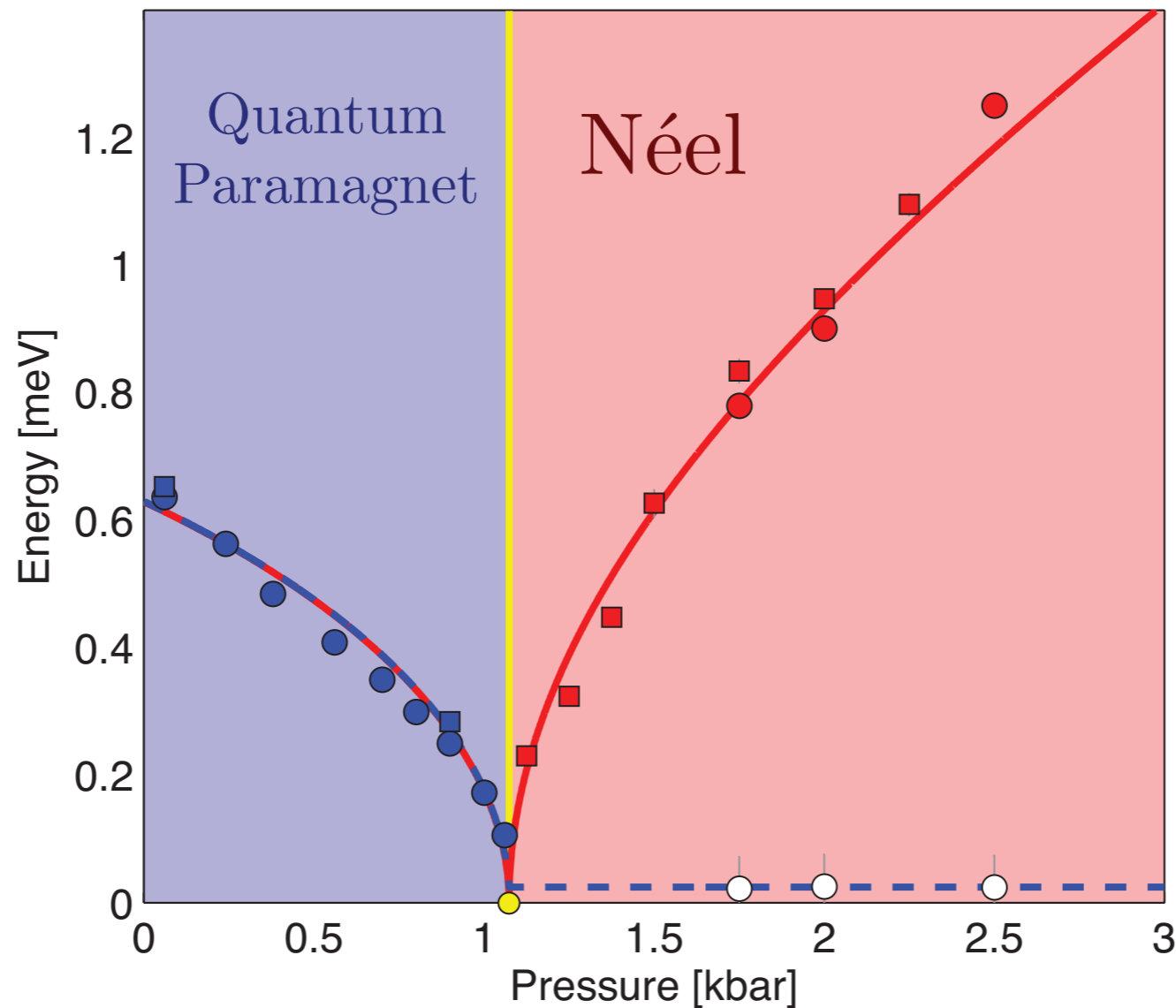
Spin waves

Excitation spectrum in the Néel phase



Spin waves

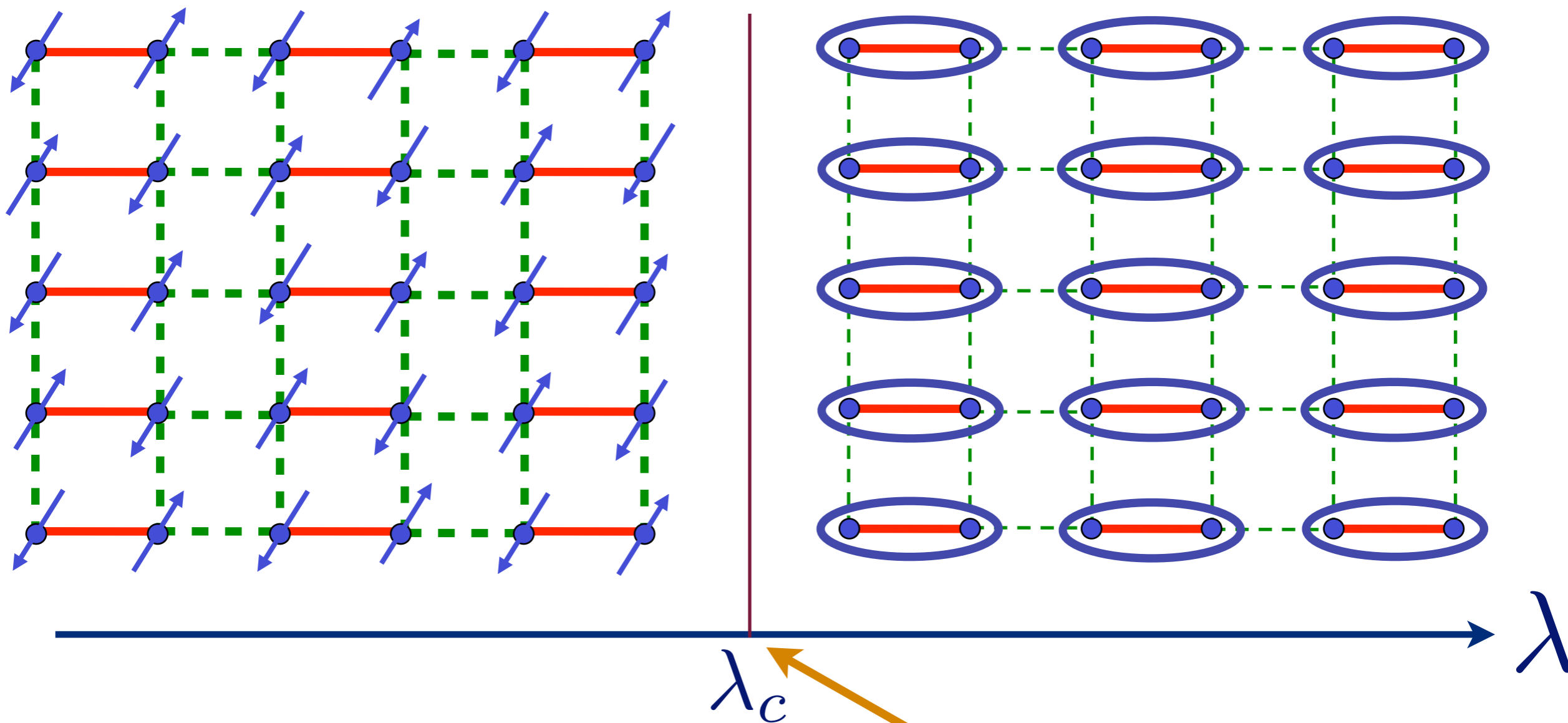
TlCuCl₃ with varying pressure



Observation of $3 \rightarrow 2$ low energy modes,
emergence of new Higgs particle in the Néel phase.

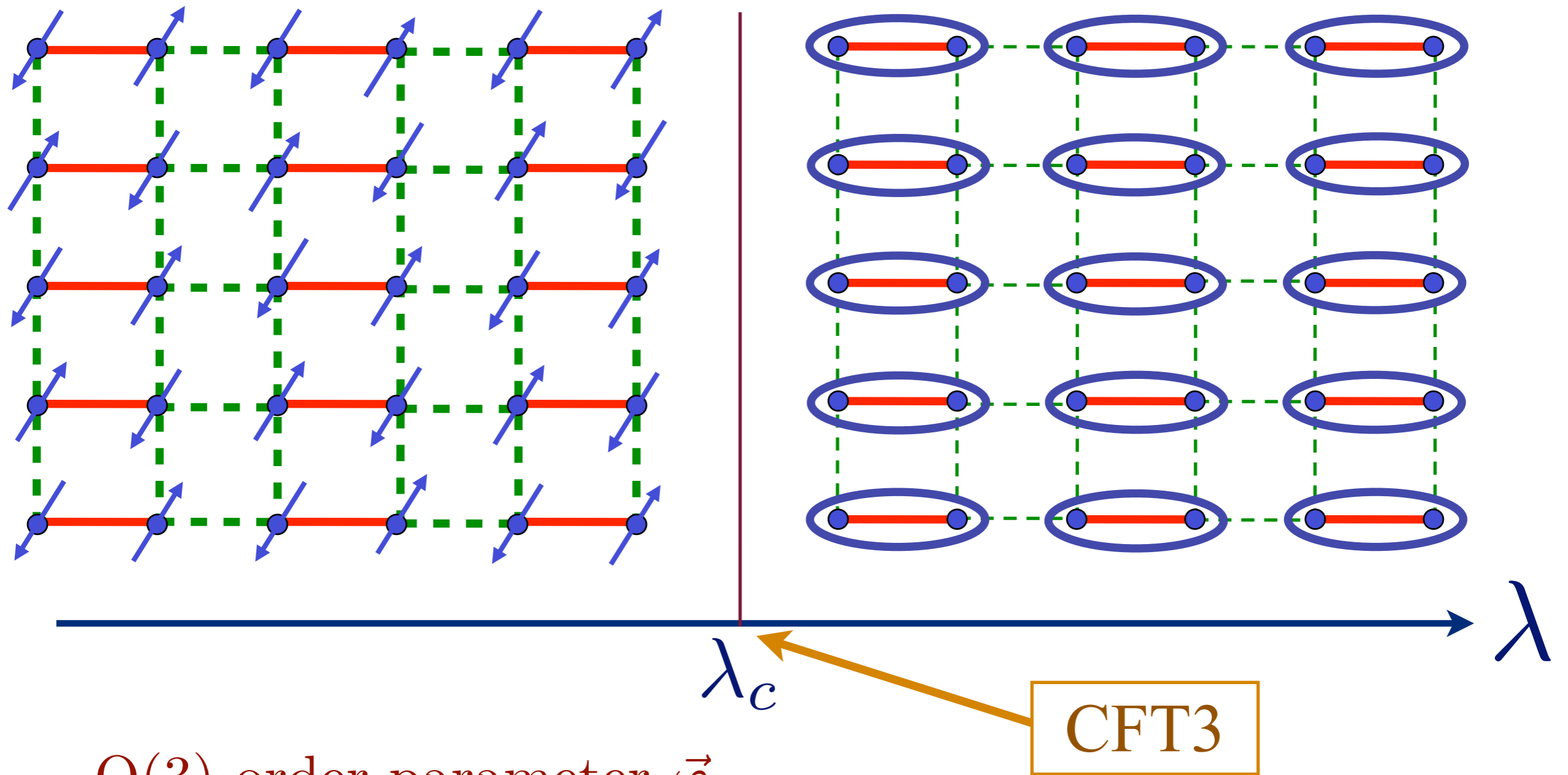
Christian Rüegg, Bruce Normand, Masahige Matsumoto, Albert Furrer,
Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya,
Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

$$\text{[Diagram of two blue dots connected by a red line, enclosed in a blue oval]} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



Quantum critical point with non-local entanglement in spin wavefunction

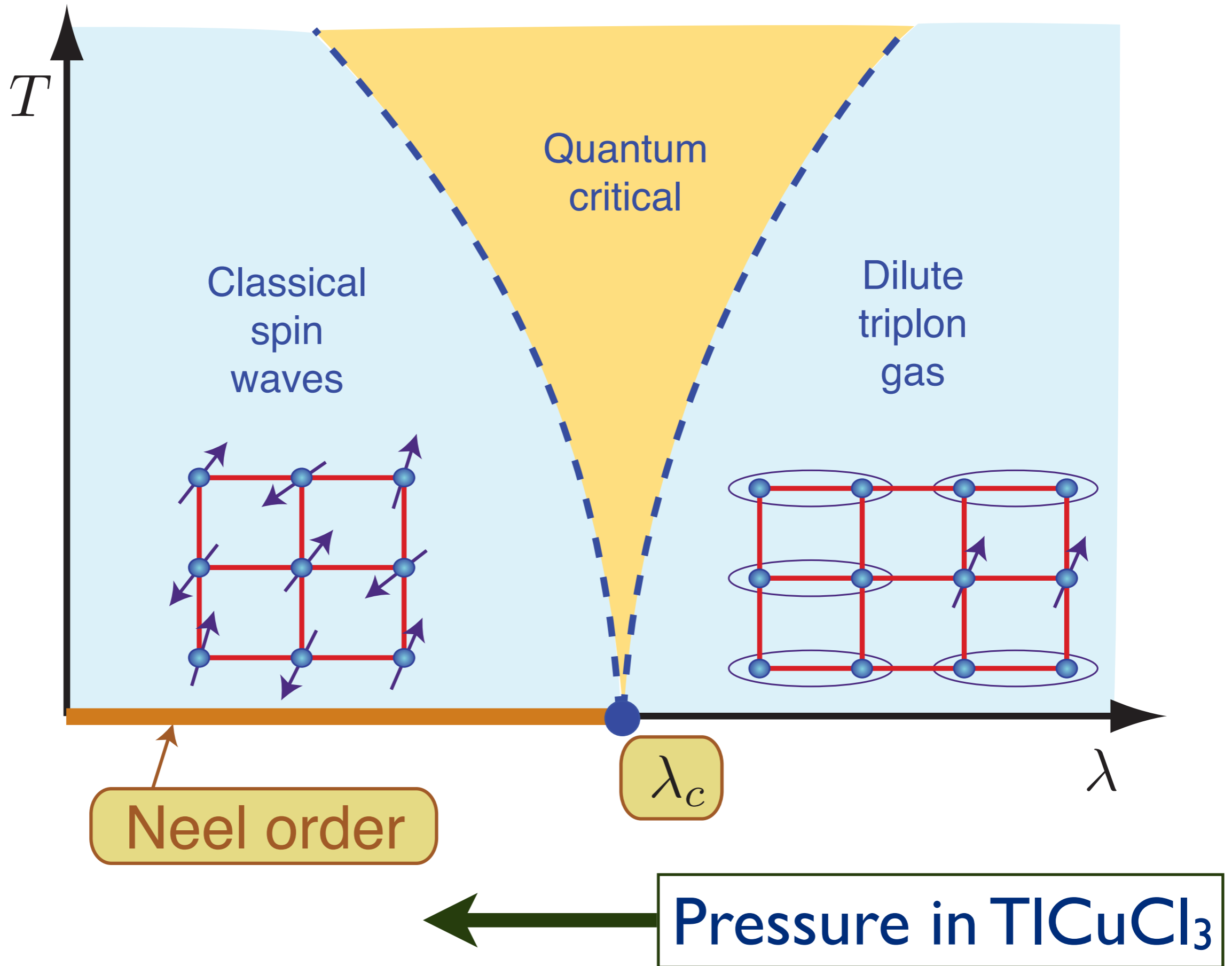
Description using Landau-Ginzburg field theory



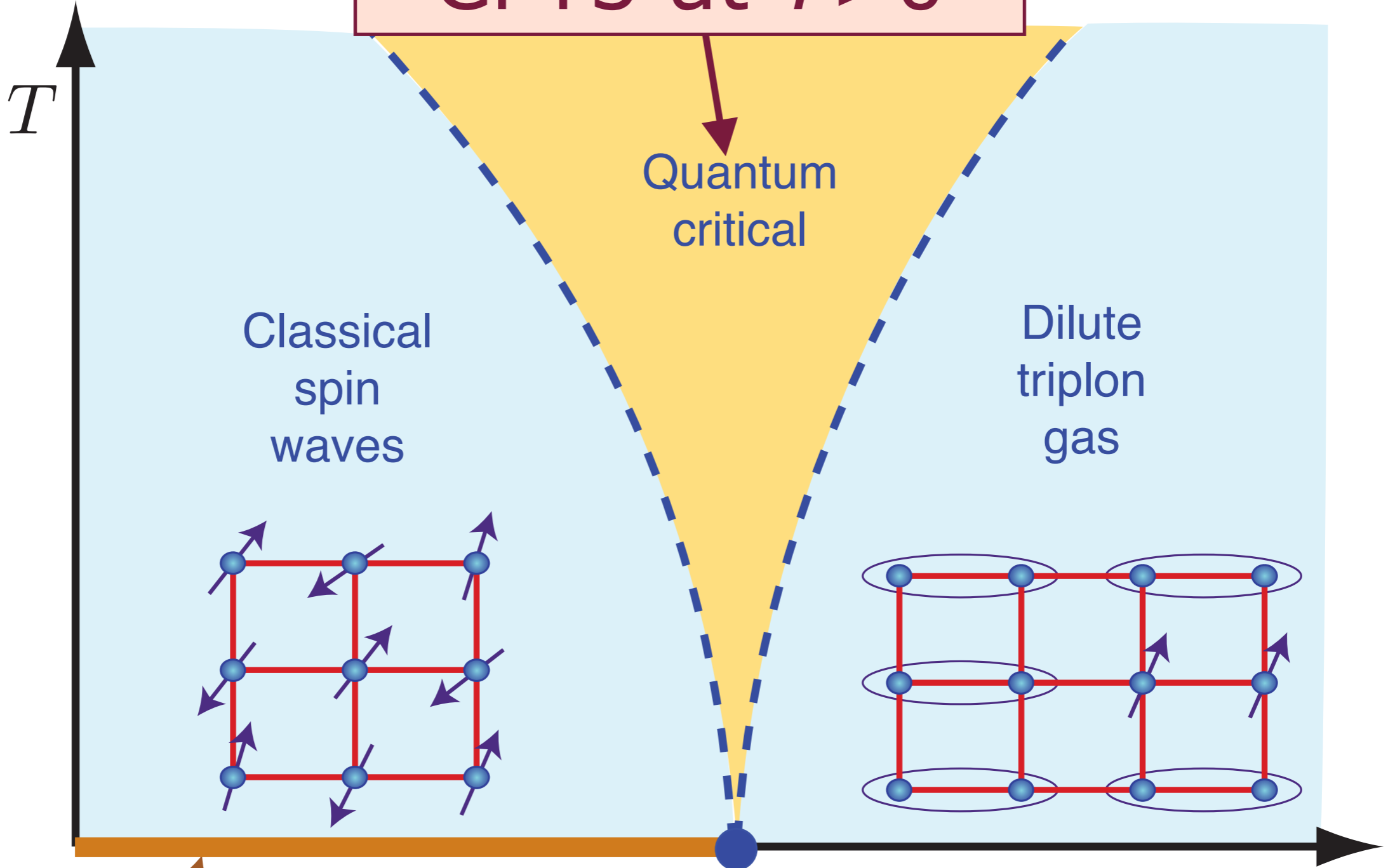
$O(3)$ order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \right]$$

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).



CFT3 at $T > 0$



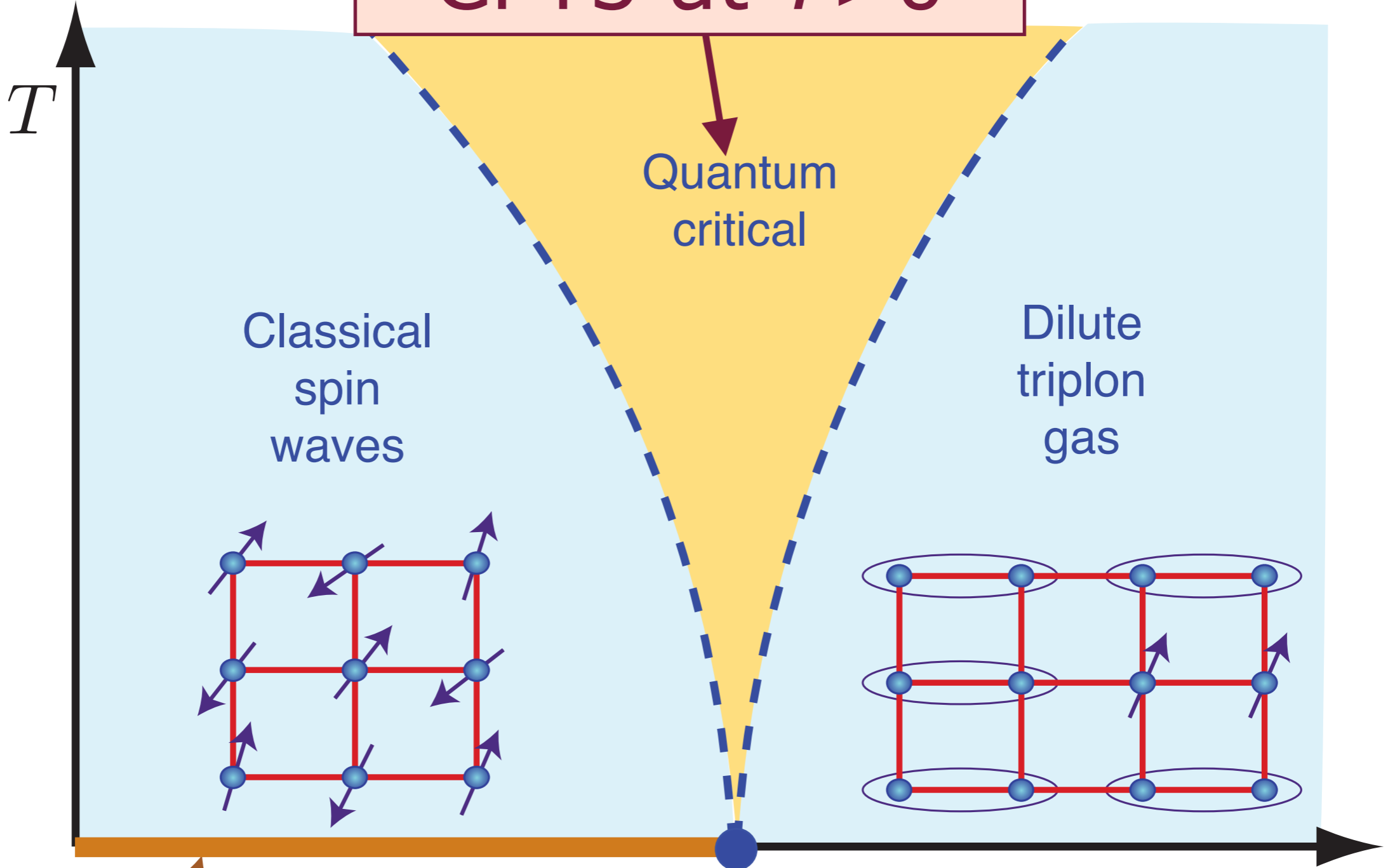
Neel order

λ_c

Pressure in $TlCuCl_3$



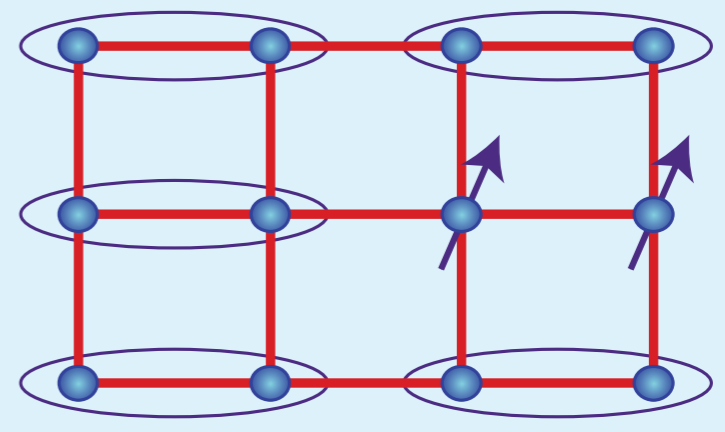
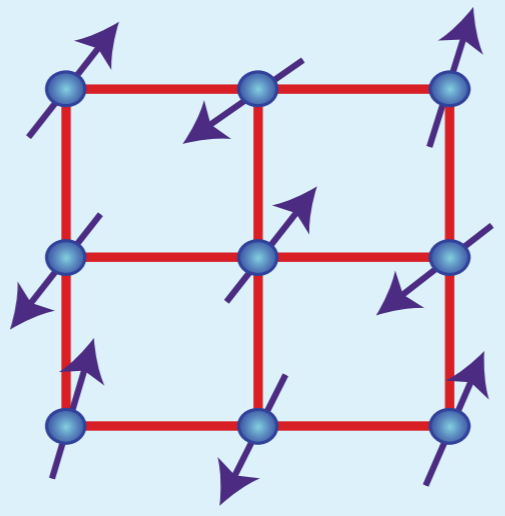
CFT3 at $T > 0$



Classical spin waves

Dilute triplon gas

Quantum critical



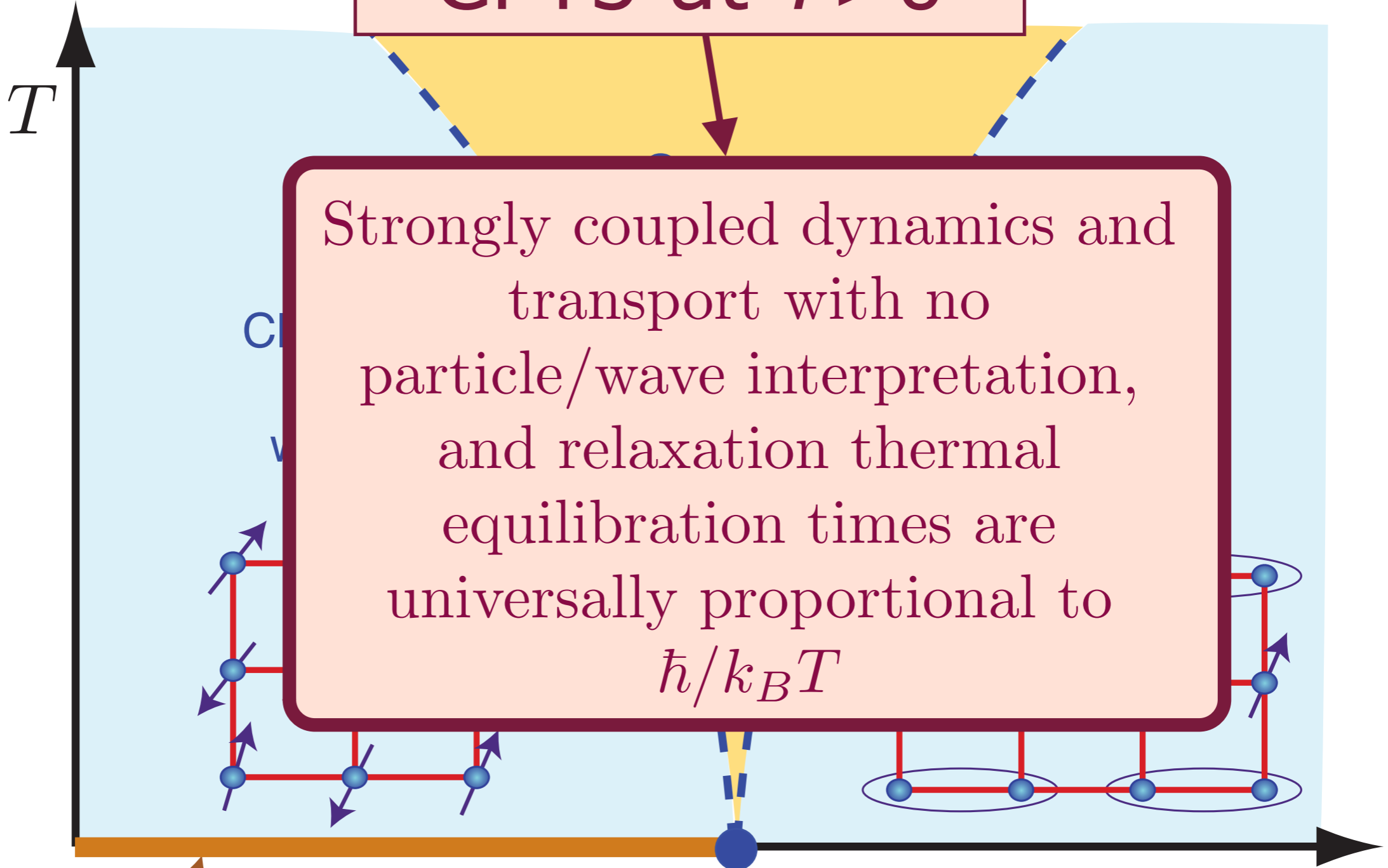
λ_c

Neel order

Pressure in $TlCuCl_3$



CFT3 at $T > 0$



Strongly coupled dynamics and transport with no particle/wave interpretation, and relaxation thermal equilibration times are universally proportional to $\hbar/k_B T$

Neel order

λ_c

Pressure in $TlCuCl_3$

Outline

1. Loss of antiferromagnetism in an insulator

Coupled-dimer antiferromagnets and quantum criticality

2. Onset of antiferromagnetism in a metal

From large Fermi surfaces to Fermi pockets, d-wave superconductivity, and competing orders

3. Strongly-coupled quantum criticality in metals

Fermi surfaces and gapless bosons

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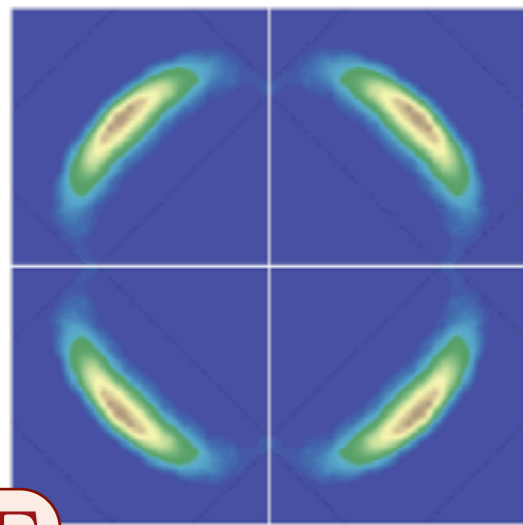
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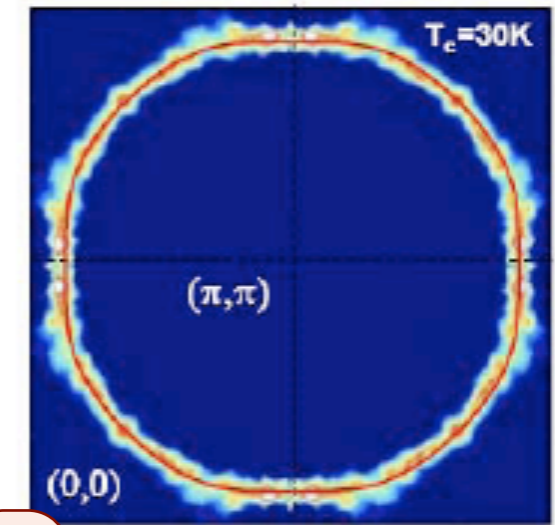
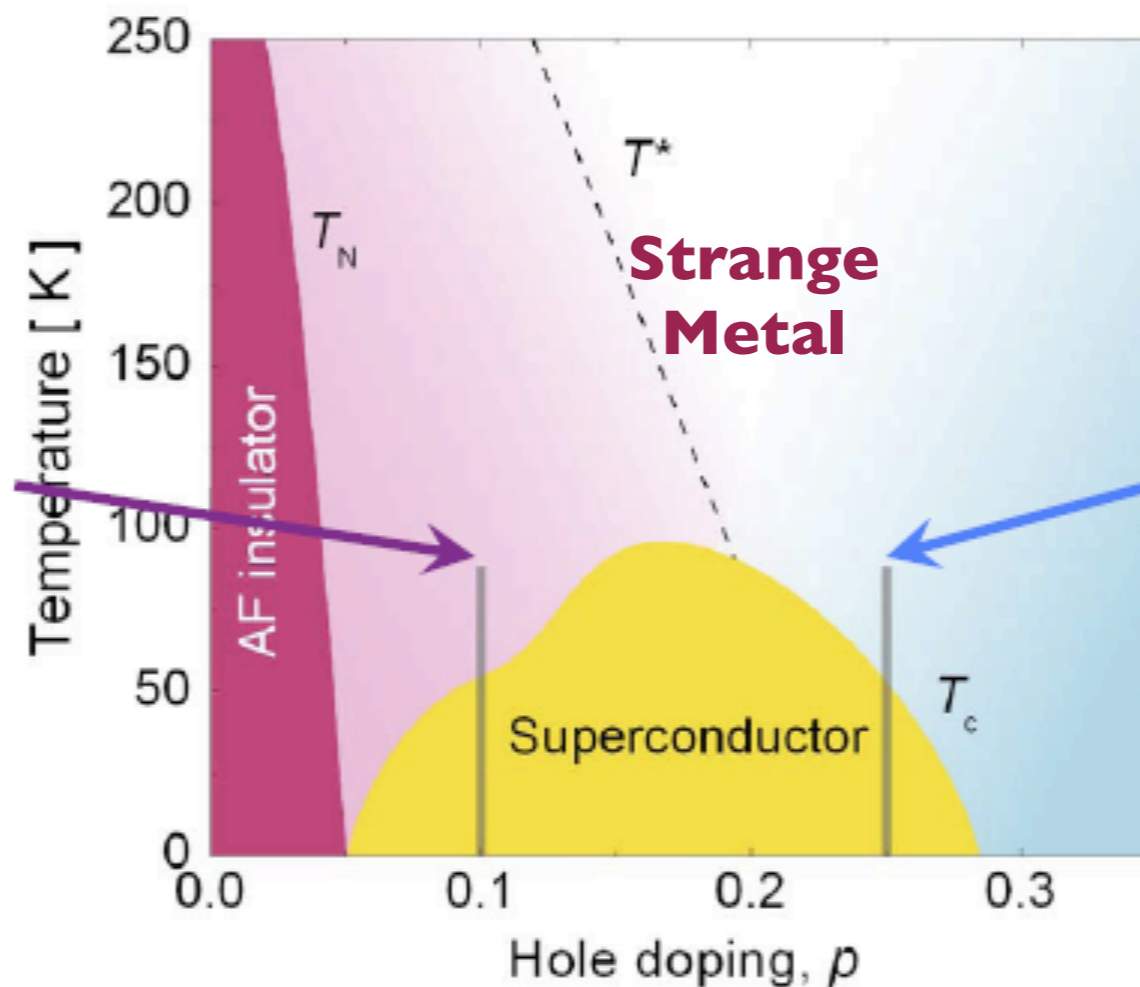
Fermi surfaces and gapless bosons

Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



Γ

K.M. Shen et al., Science 2005



Γ

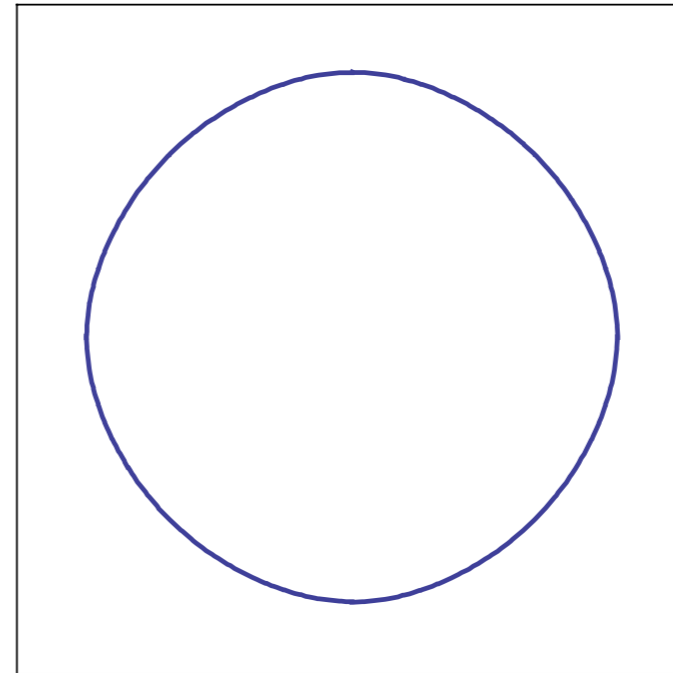
M. Platé et al., PRL 2005

Smaller hole
Fermi-pockets

Large hole
Fermi surface

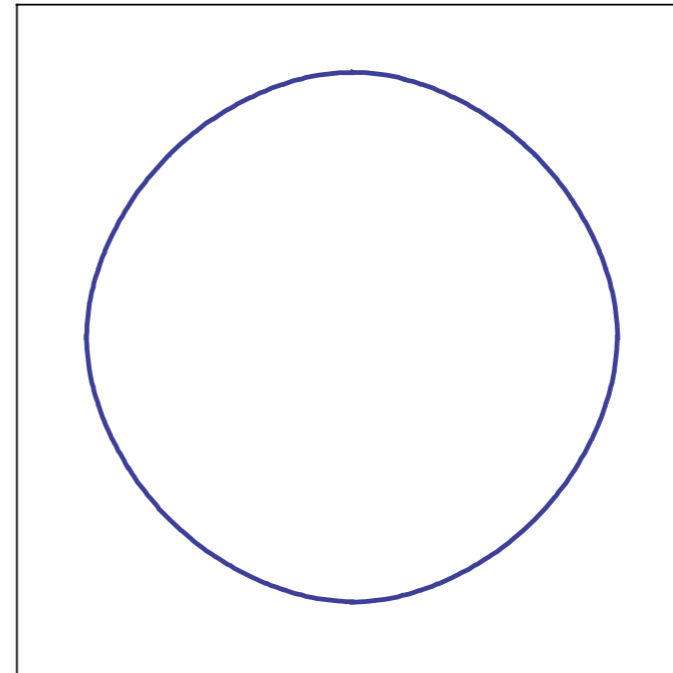
Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface

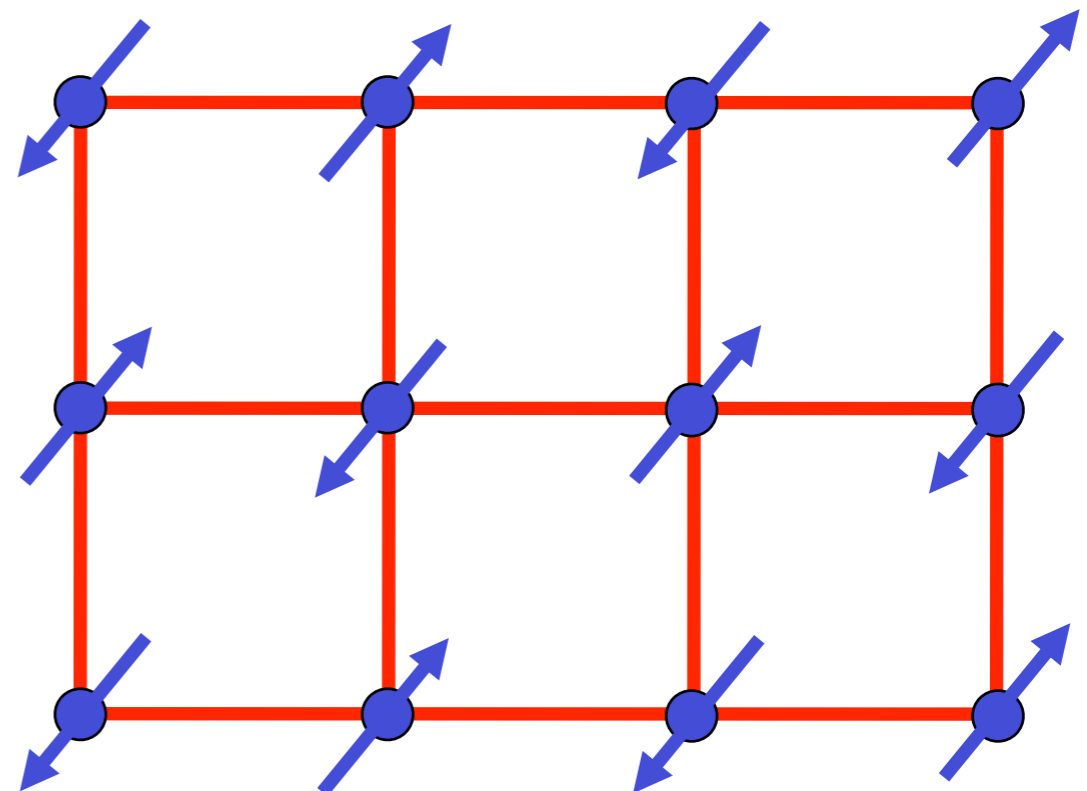


Fermi surface+antiferromagnetism

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+

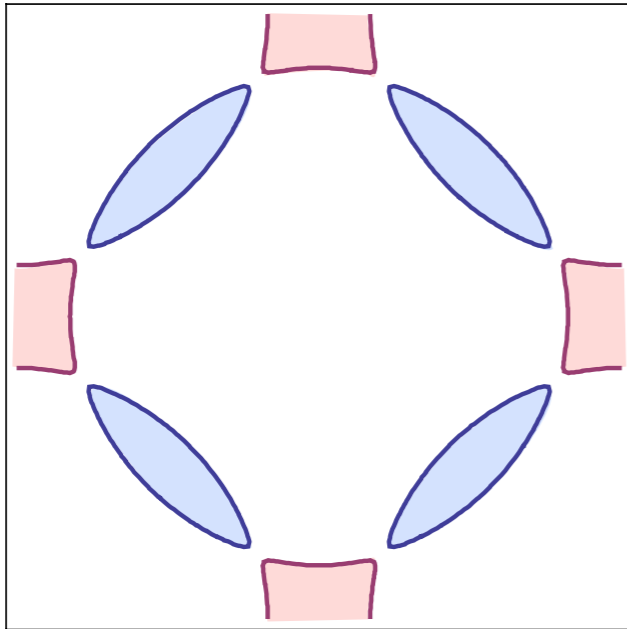


The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

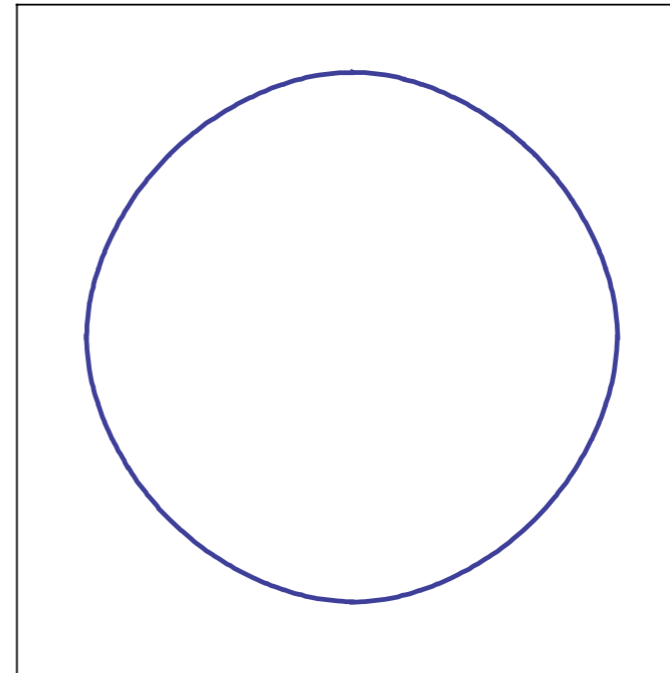
where \mathbf{K} is the ordering wavevector.

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets



$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

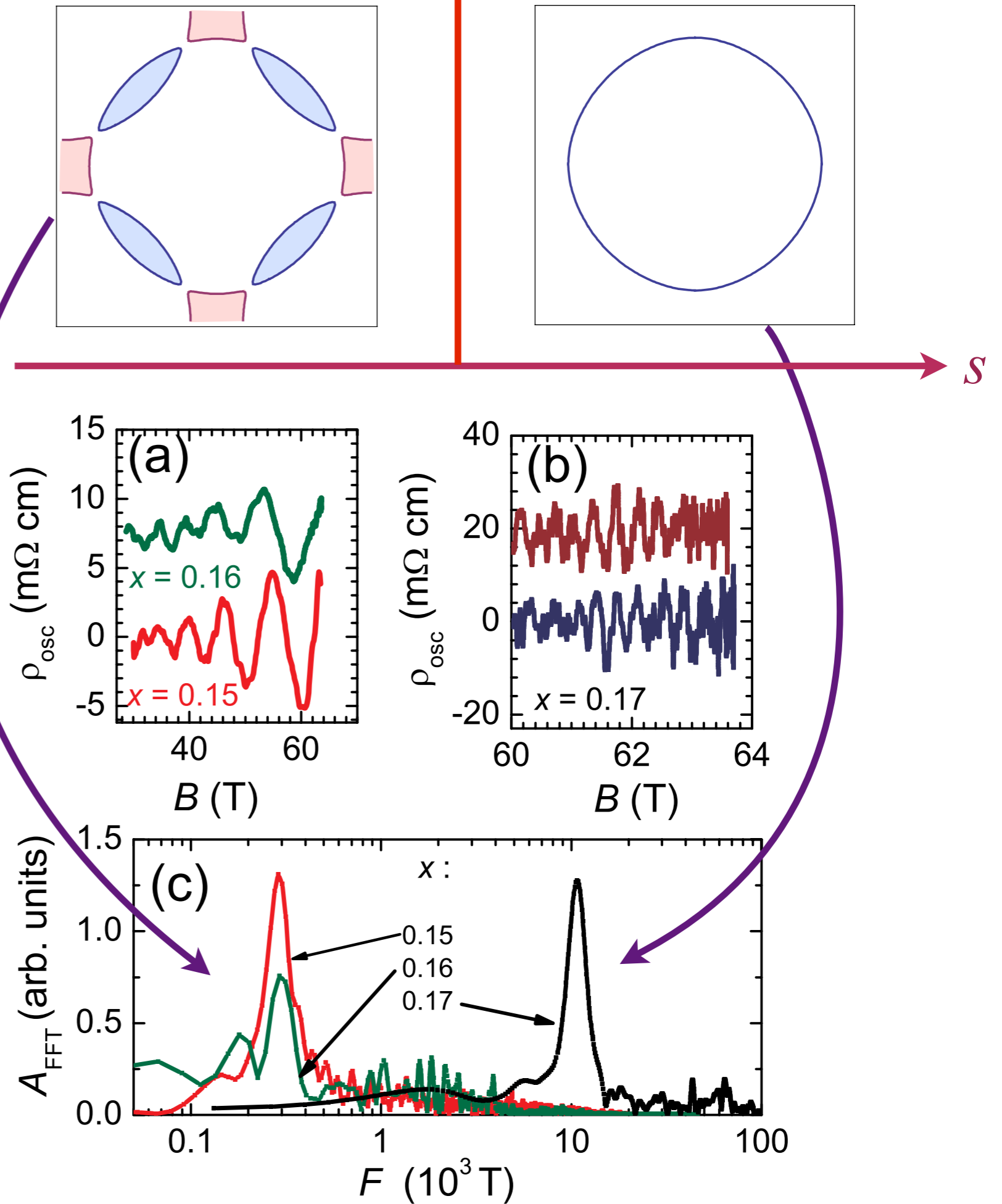
S

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

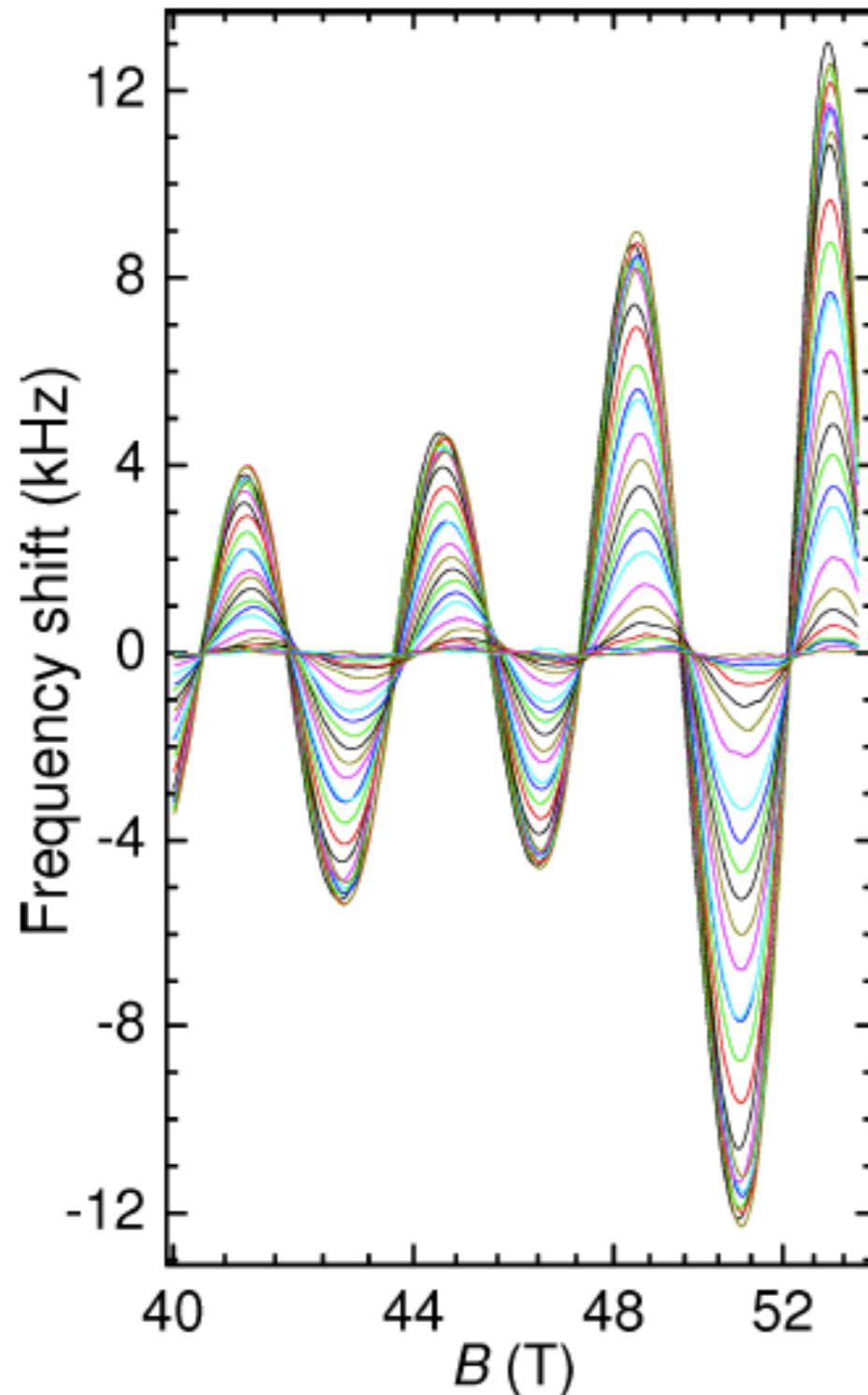
Quantum oscillations



T. Helm, M.V. Kartsovnik,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
and R. Gross,
Phys. Rev. Lett. **103**, 157002 (2009).



Evidence for small Fermi pockets

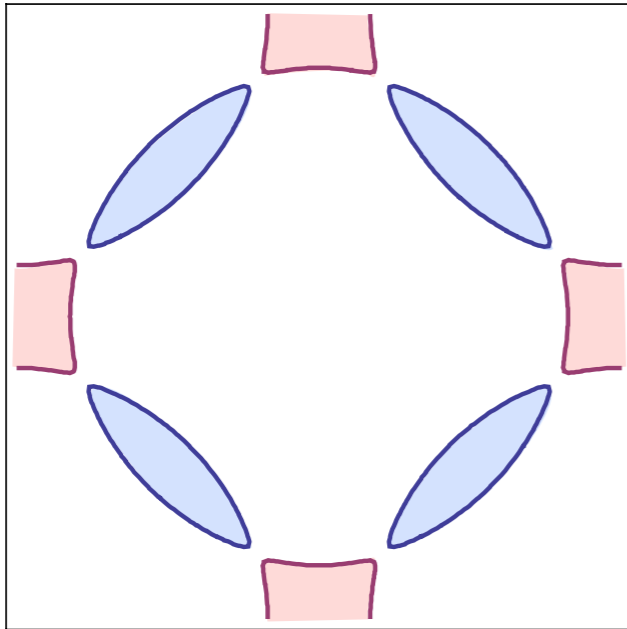


Suchitra E. Sebastian, N. Harrison,
M. M. Altarawneh, Ruixing Liang, D.A. Bonn,
W. N. Hardy, and G. G. Lonzarich
Physical Review B **81**, 140505(R) (2010)

Original observation:
N. Doiron-Leyraud, C. Proust,
D. LeBoeuf, J. Levallois,
J.-B. Bonnemaïson, R. Liang,
D.A. Bonn, W. N. Hardy,
and L. Taillefer,
Nature **447**, 565 (2007)

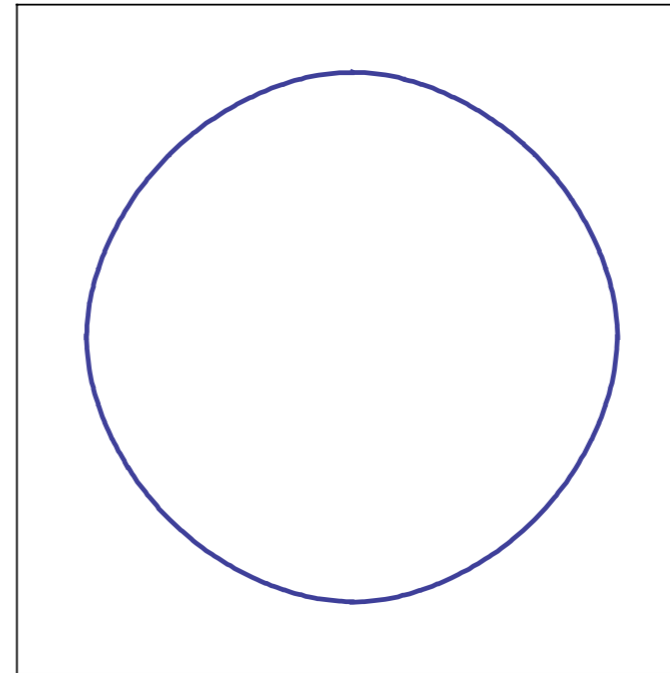
FIG. 2: Magnetic quantum oscillations measured in $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ with $x \approx 0.56$ (after background polynomial subtraction). This restricted interval in $B = |\mathbf{B}|$ furnishes a dynamic range of ~ 50 dB between $T = 1$ and 18 K. The actual T values are provided in Fig. 3.

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets



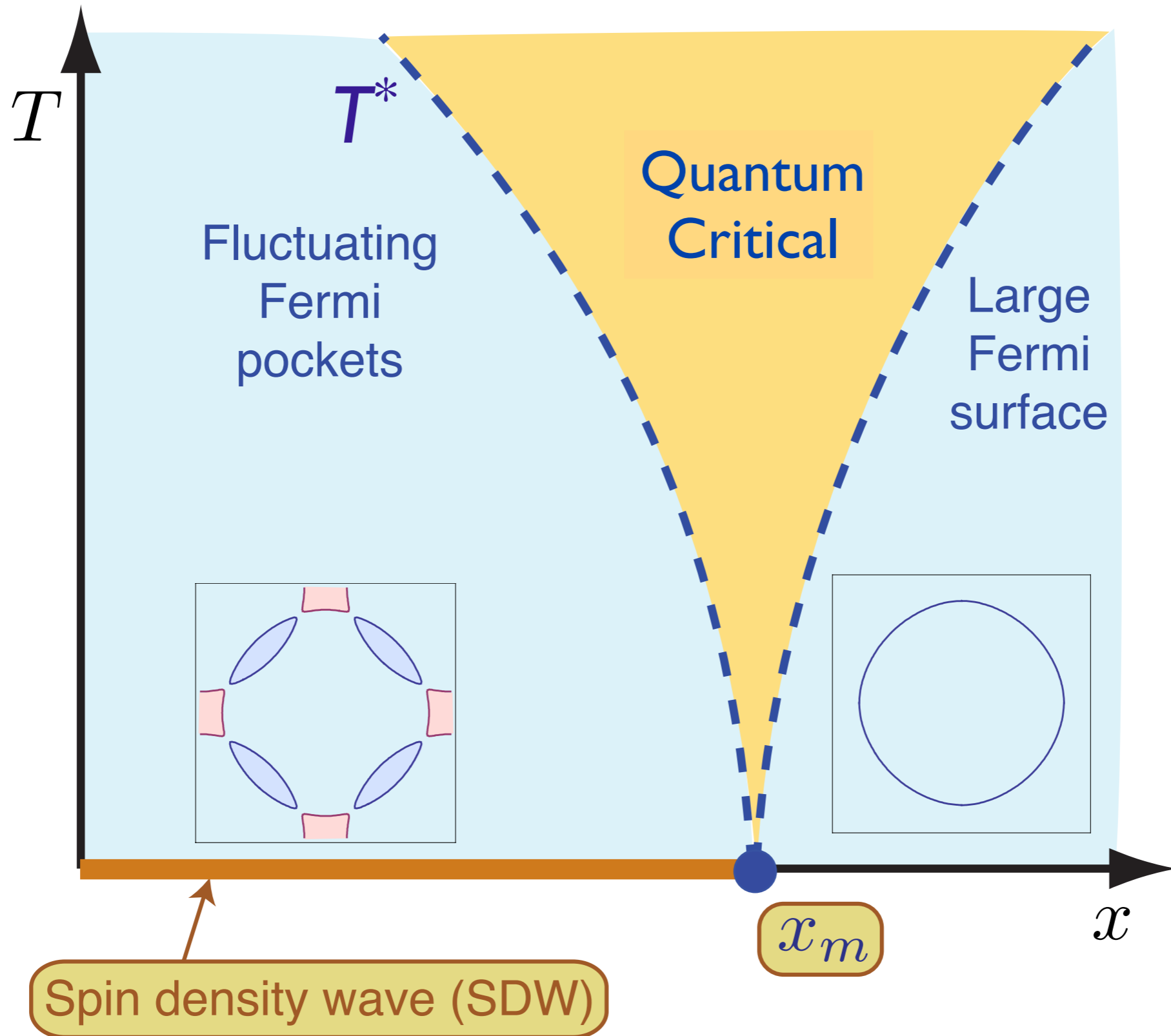
$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

S

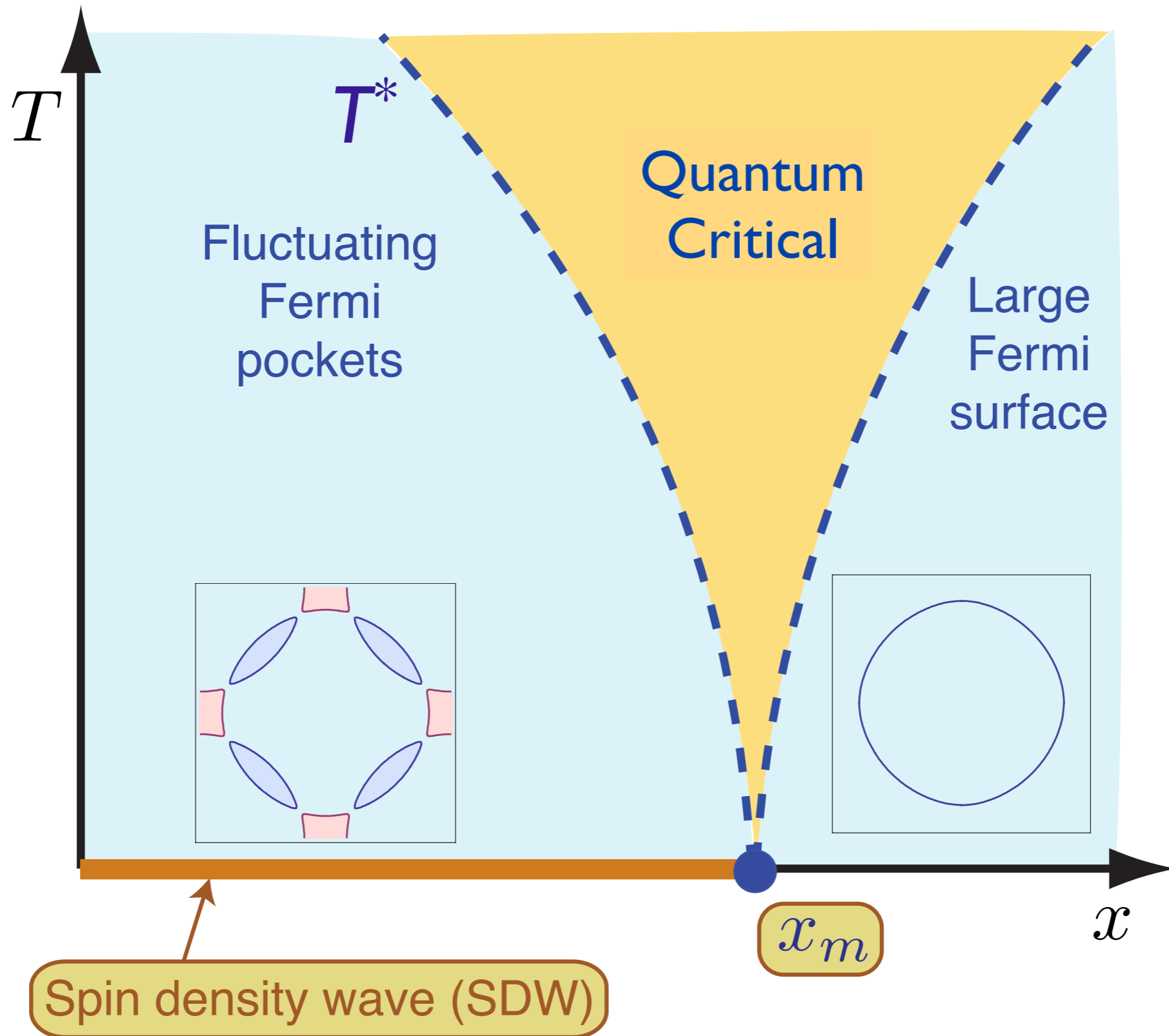
S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Theory of quantum criticality in the cuprates



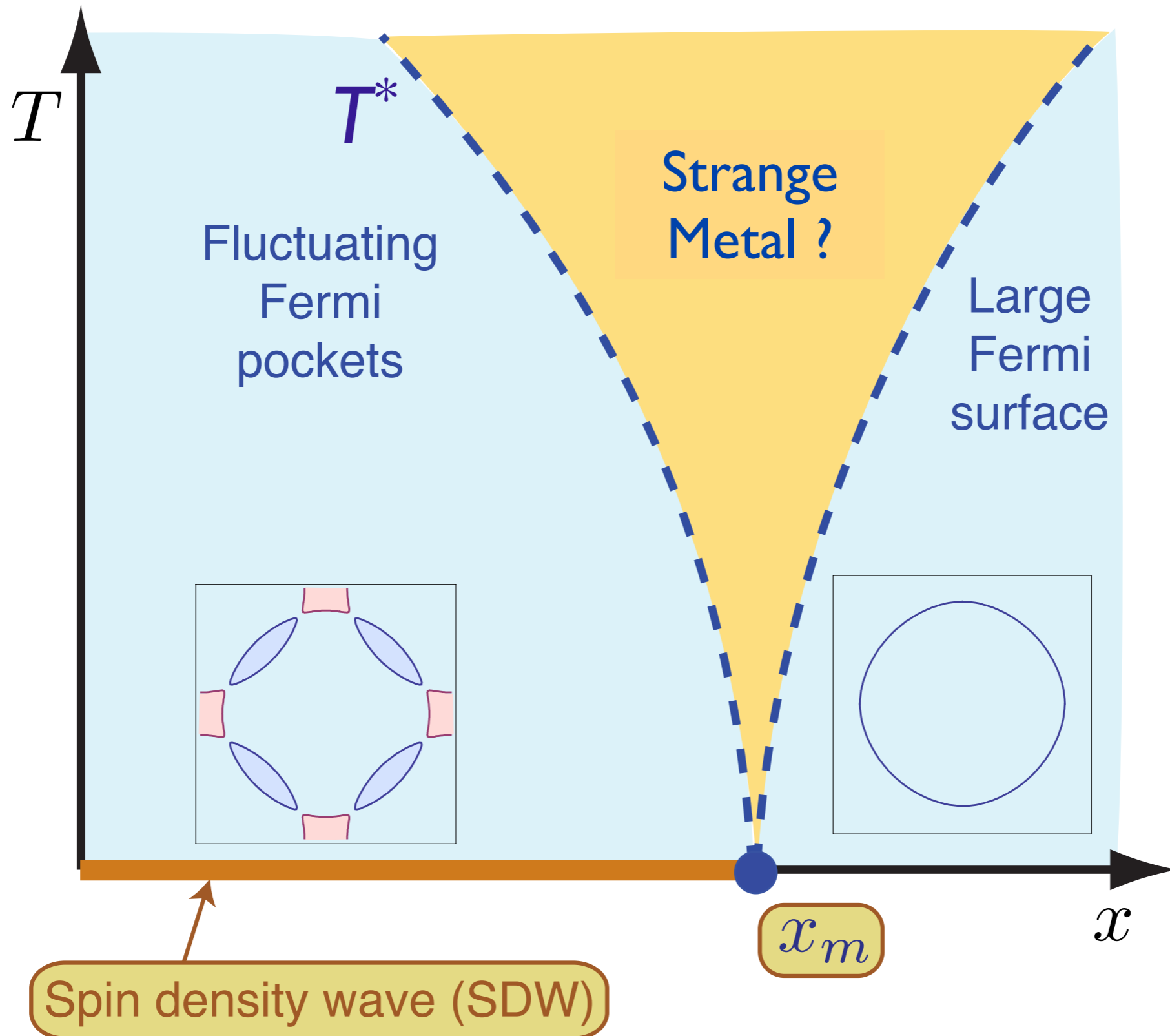
Underlying SDW ordering quantum critical point
in metal at $x = x_m$

Theory of quantum criticality in the cuprates



Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

Theory of quantum criticality in the cuprates



Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

***d*-wave pairing near a spin-density-wave instability**

D. J. Scalapino, E. Loh, Jr.,* and J. E. Hirsch†

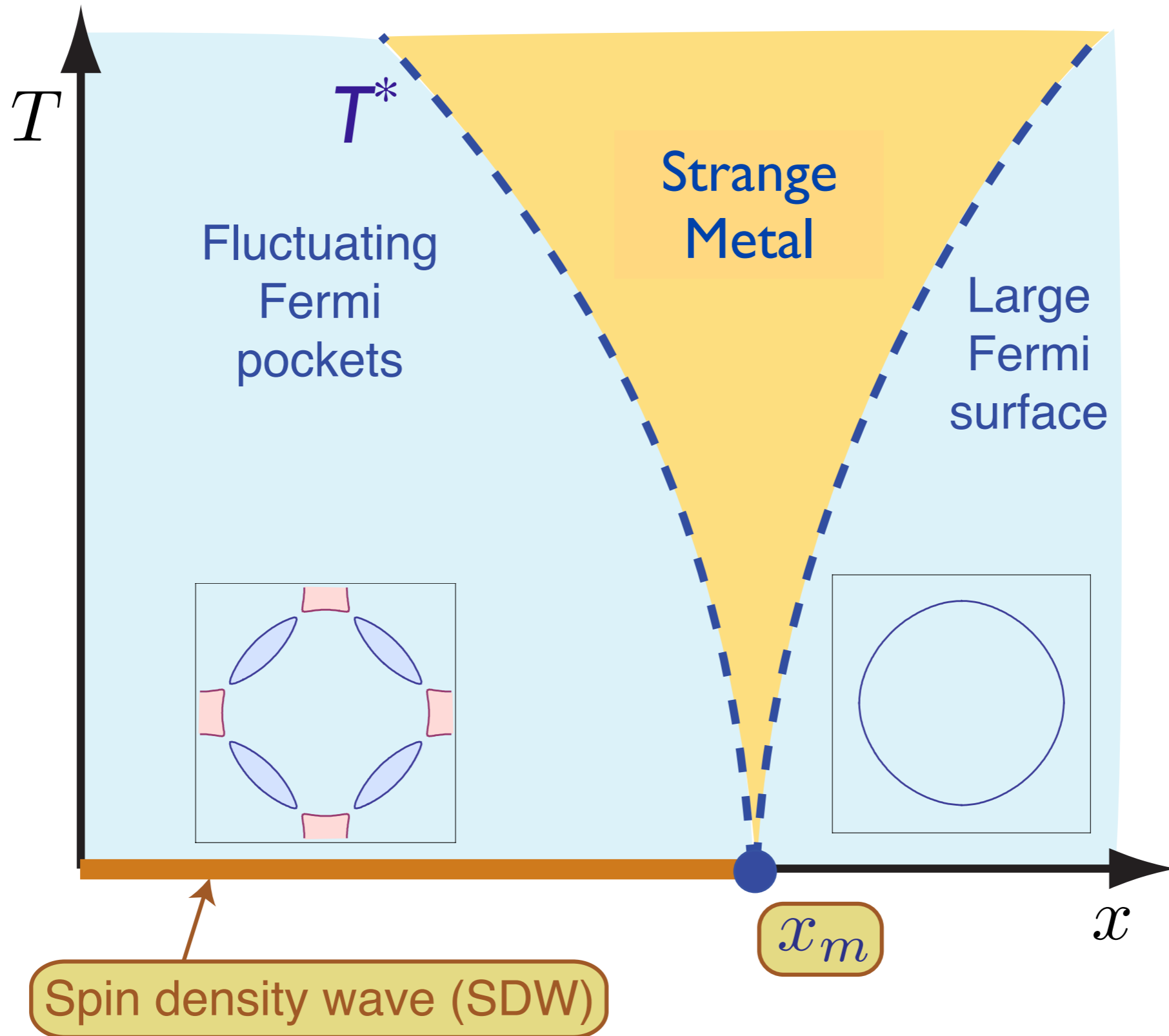
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

(Received 23 June 1986)

We investigate the three-dimensional Hubbard model and show that paramagnon exchange near a spin-density-wave instability gives rise to a strong singlet *d*-wave pairing interaction. For a cubic band the singlet ($d_{x^2-y^2}$ and $d_{3z^2-r^2}$) channels are enhanced while the singlet (d_{xy}, d_{xz}, d_{yz}) and triplet *p*-wave channels are suppressed. A unique feature of this pairing mechanism is its sensitivity to band structure and band filling.

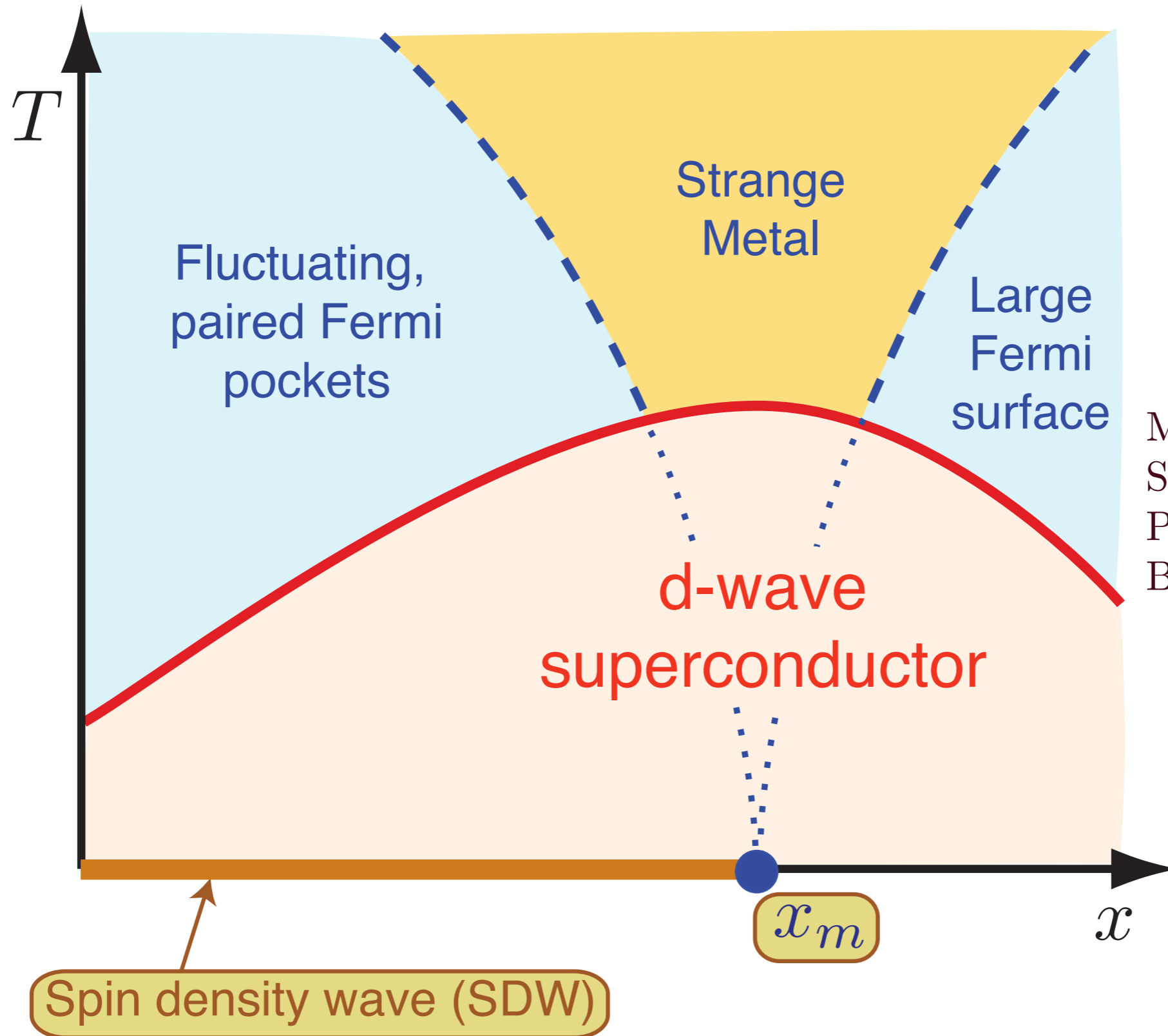
Physical Review B **34**, 8190 (1986)

Theory of quantum criticality in the cuprates



Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

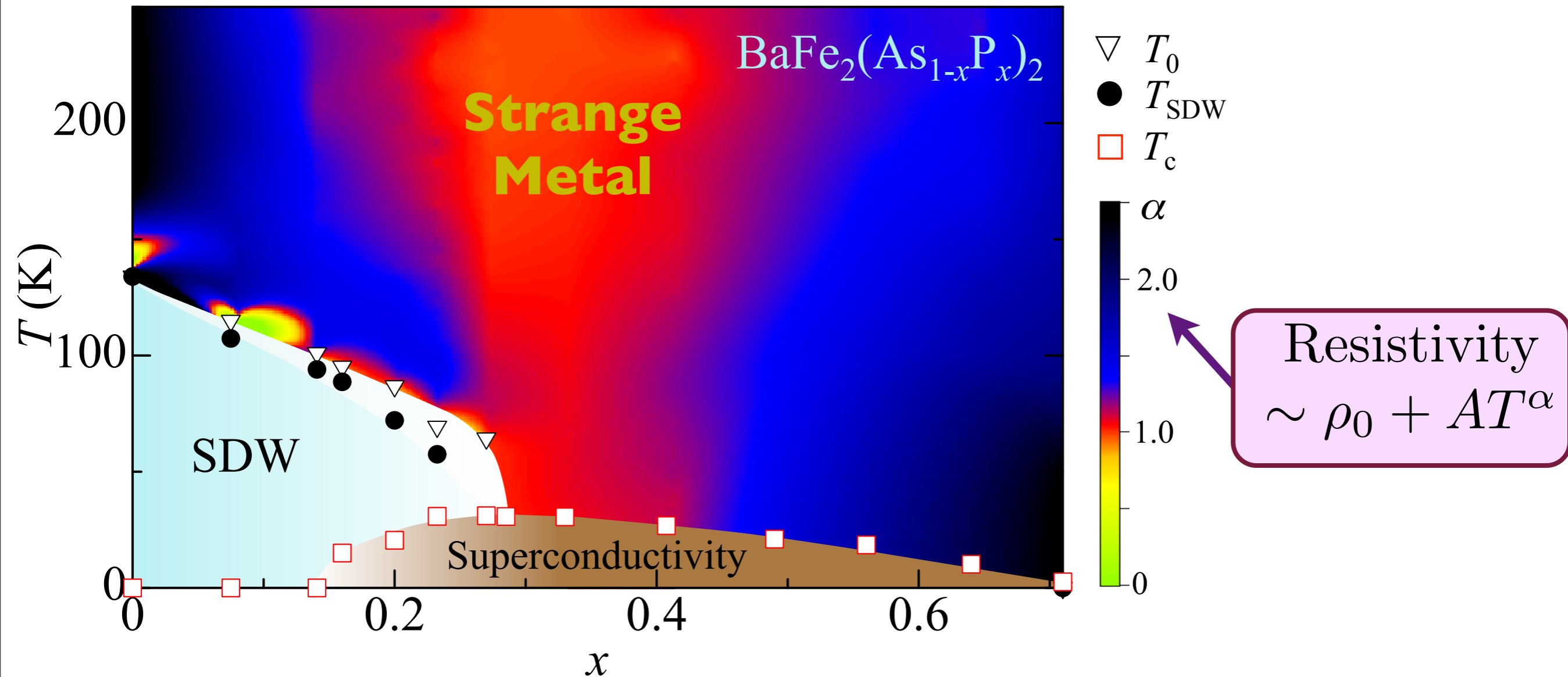
Theory of quantum criticality in the cuprates



M. A. Metlitski and
S. Sachdev,
Physical Review
B **82**, 075128 (2010)

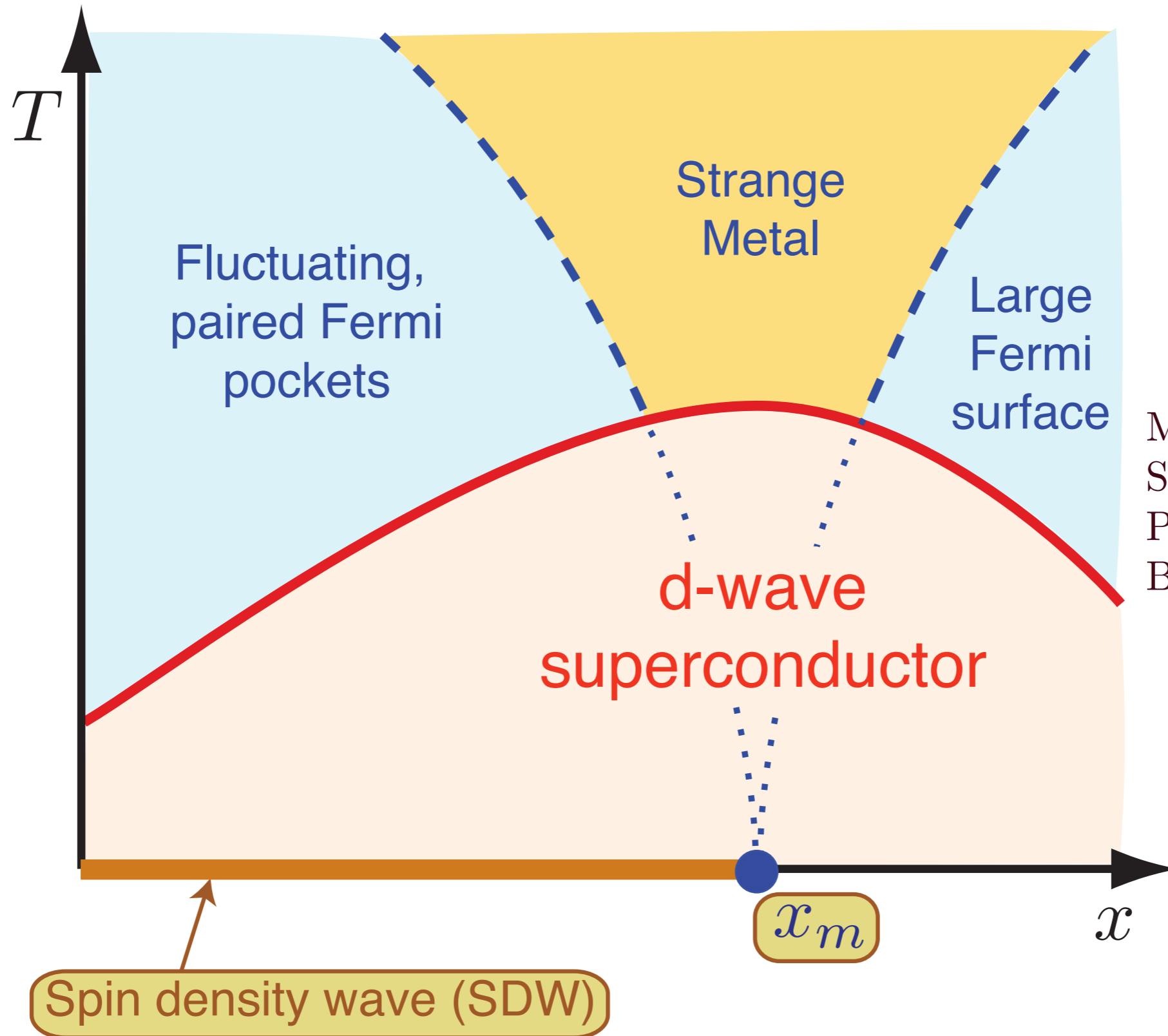
SDW quantum critical point is unstable to *d*-wave superconductivity
This instability is stronger than that in the BCS theory

Temperature-doping phase diagram of the iron pnictides:



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

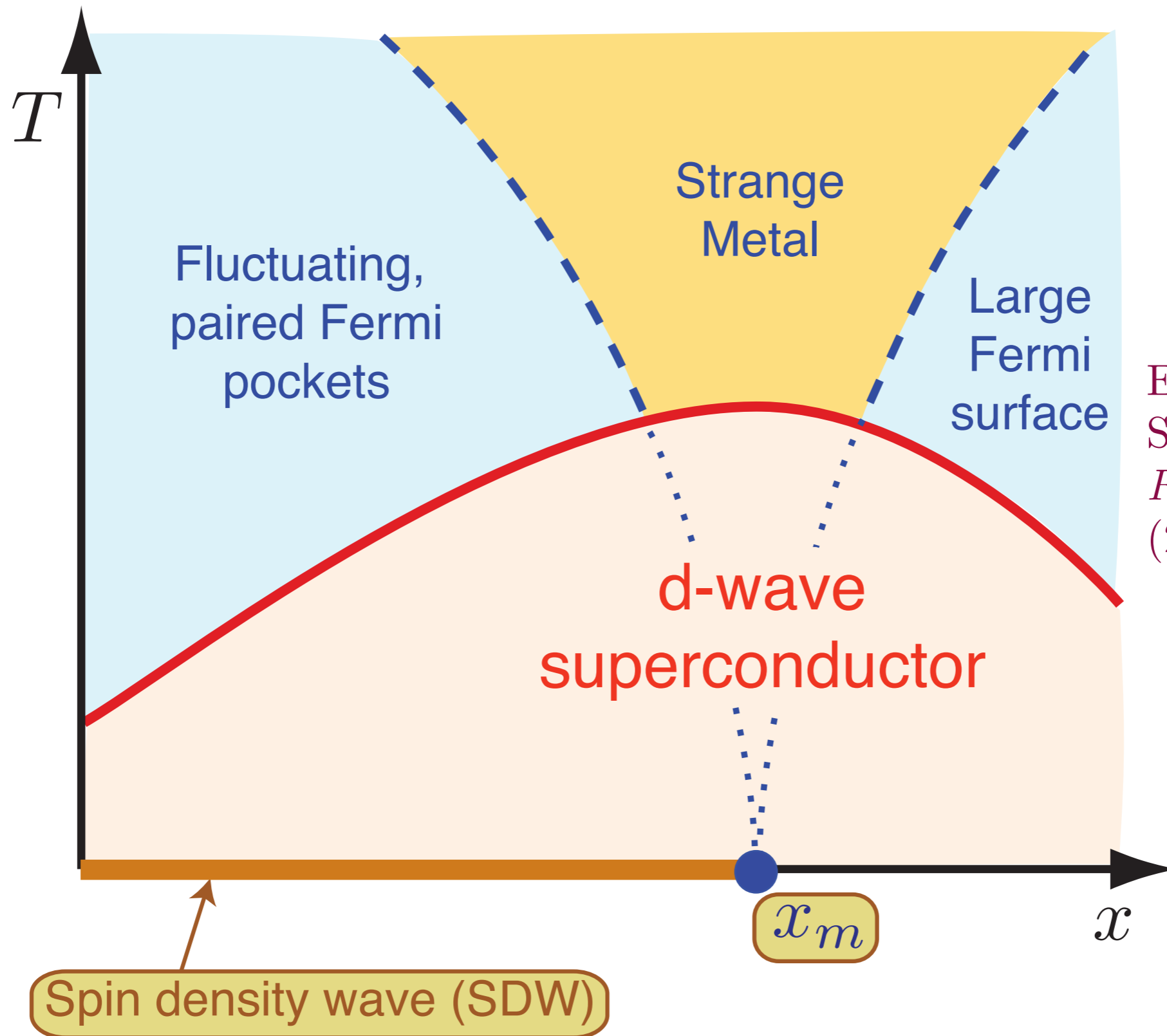
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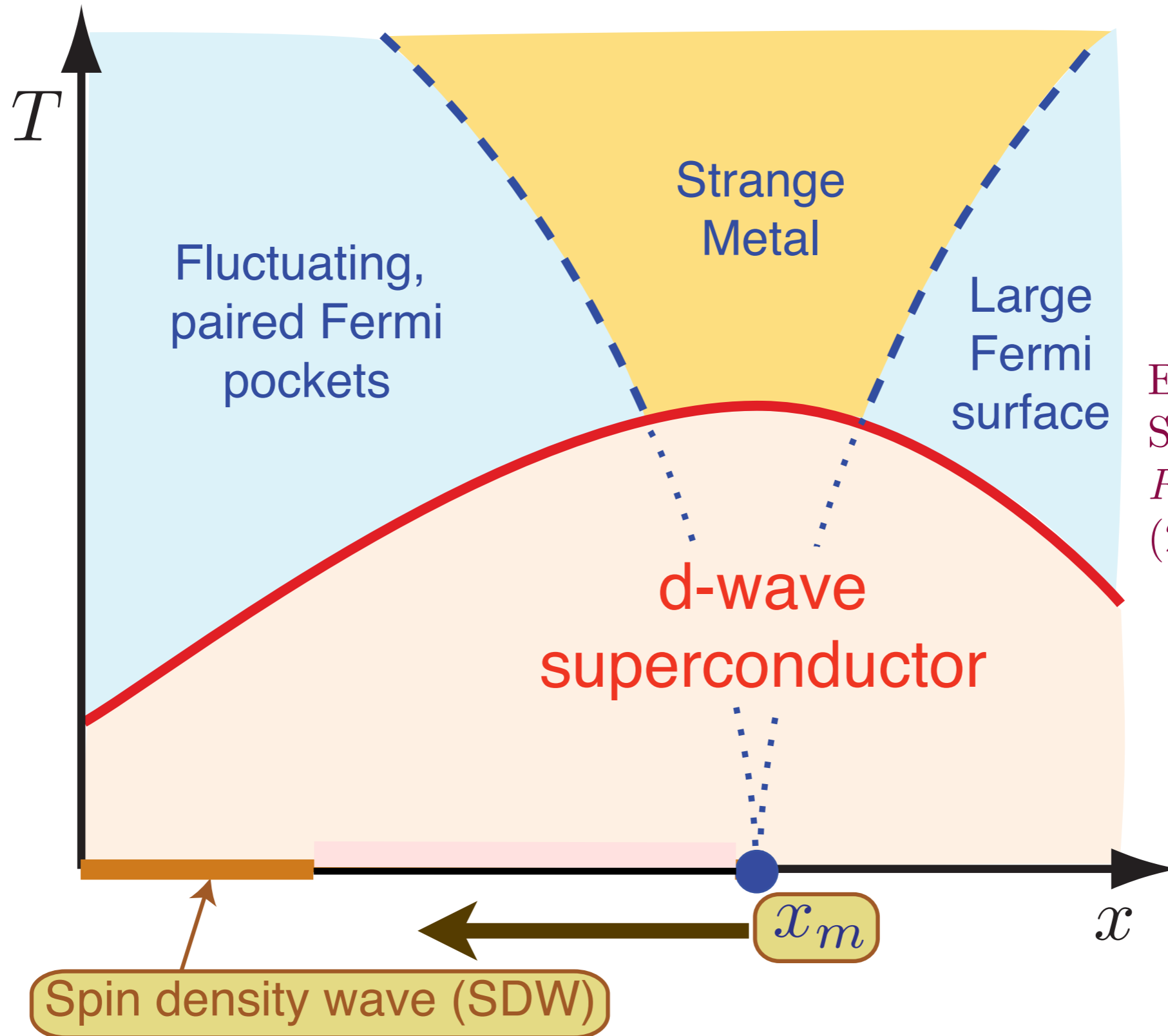
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

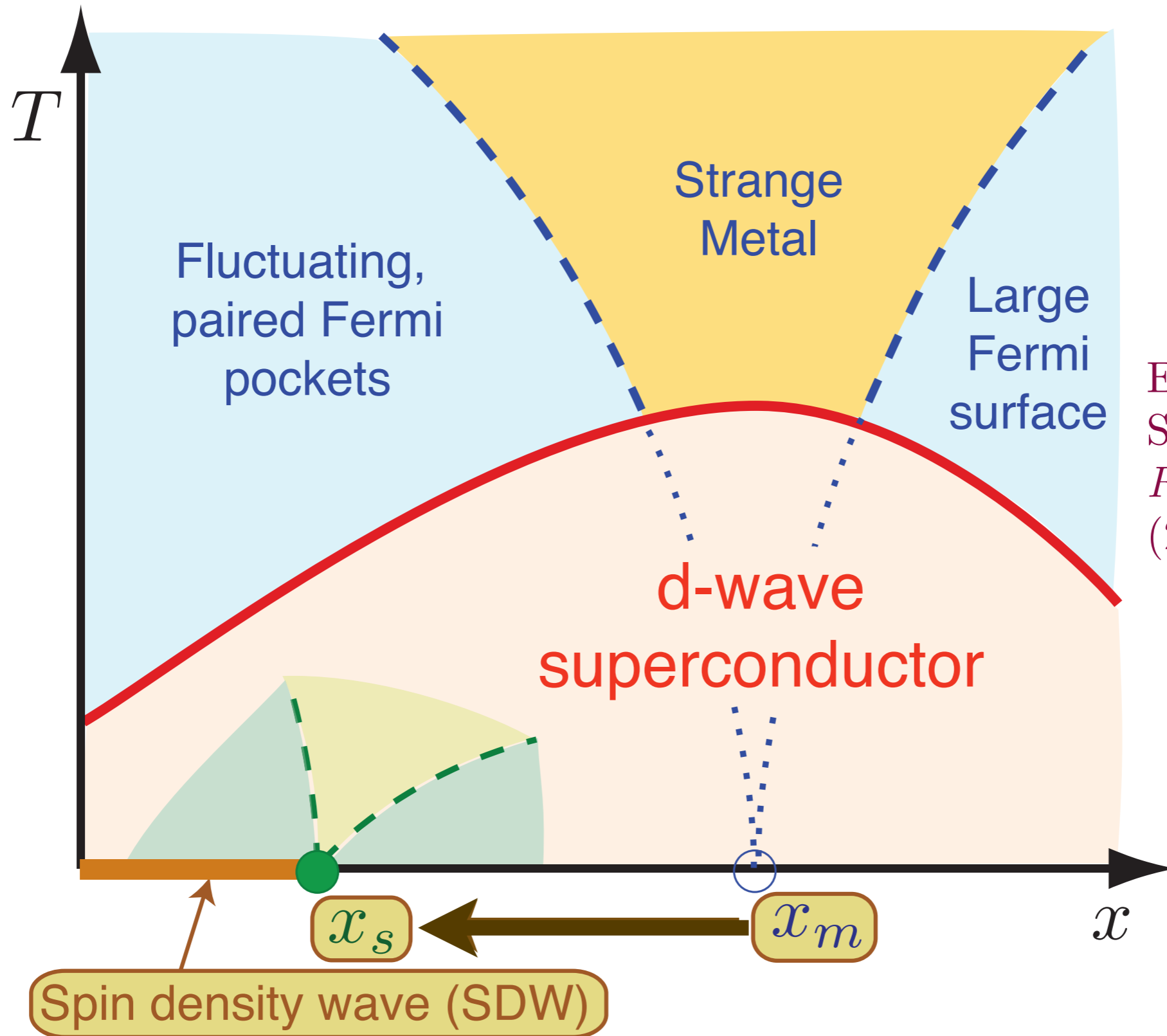
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E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

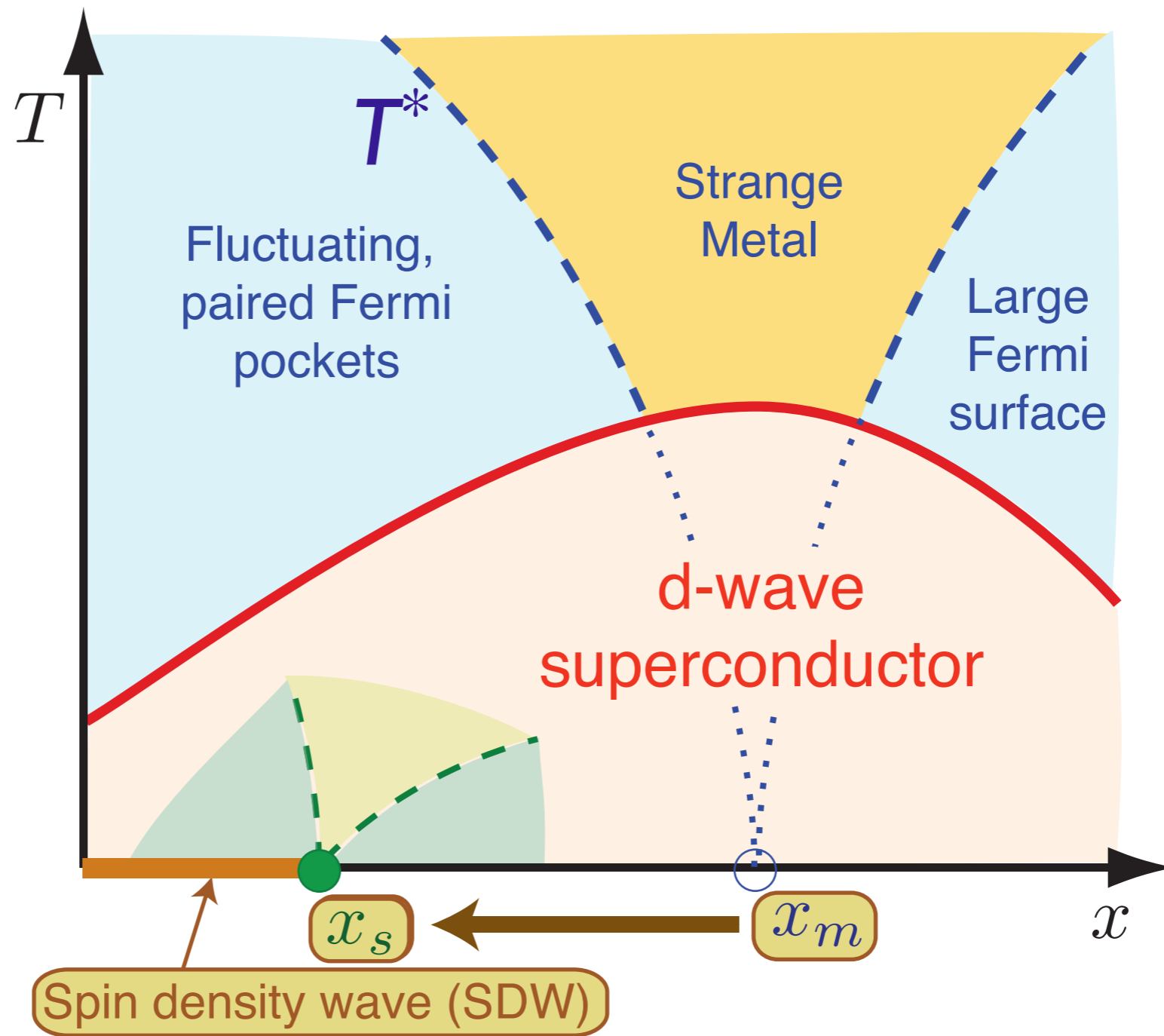
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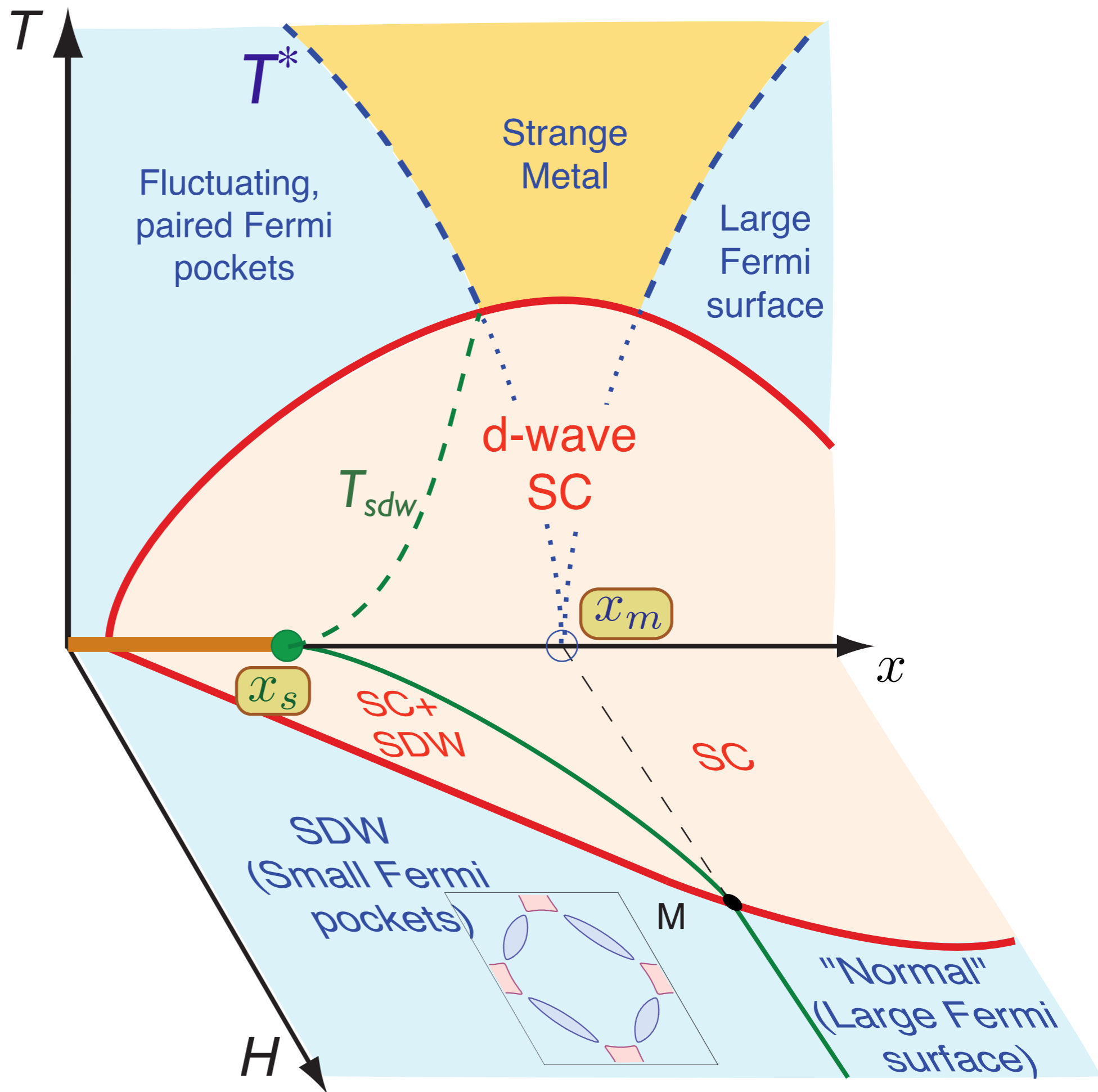


E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

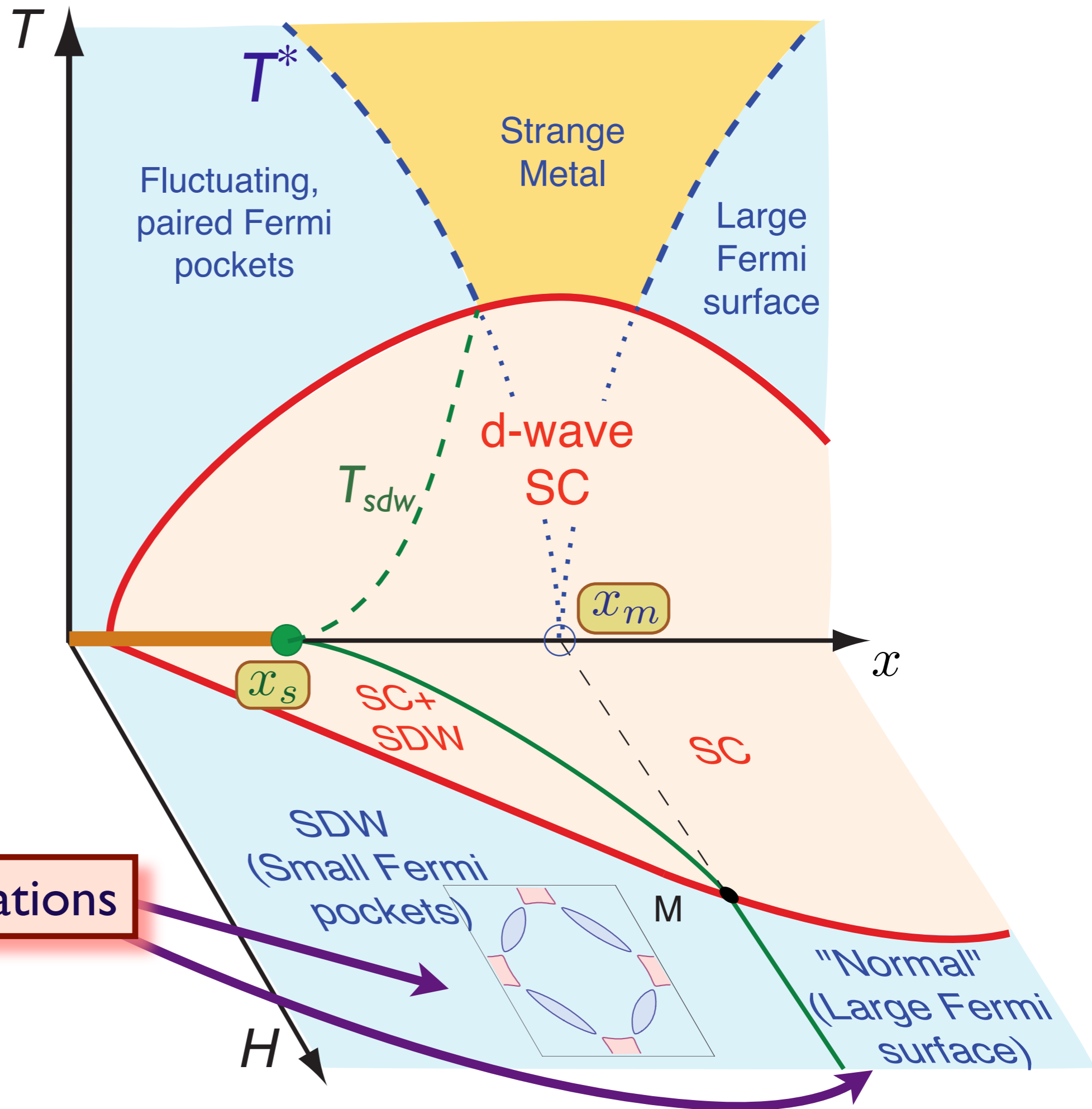
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E. Demler, S. Sachdev
and Y. Zhang, *Phys.
Rev. Lett.* **87**,
067202 (2001).



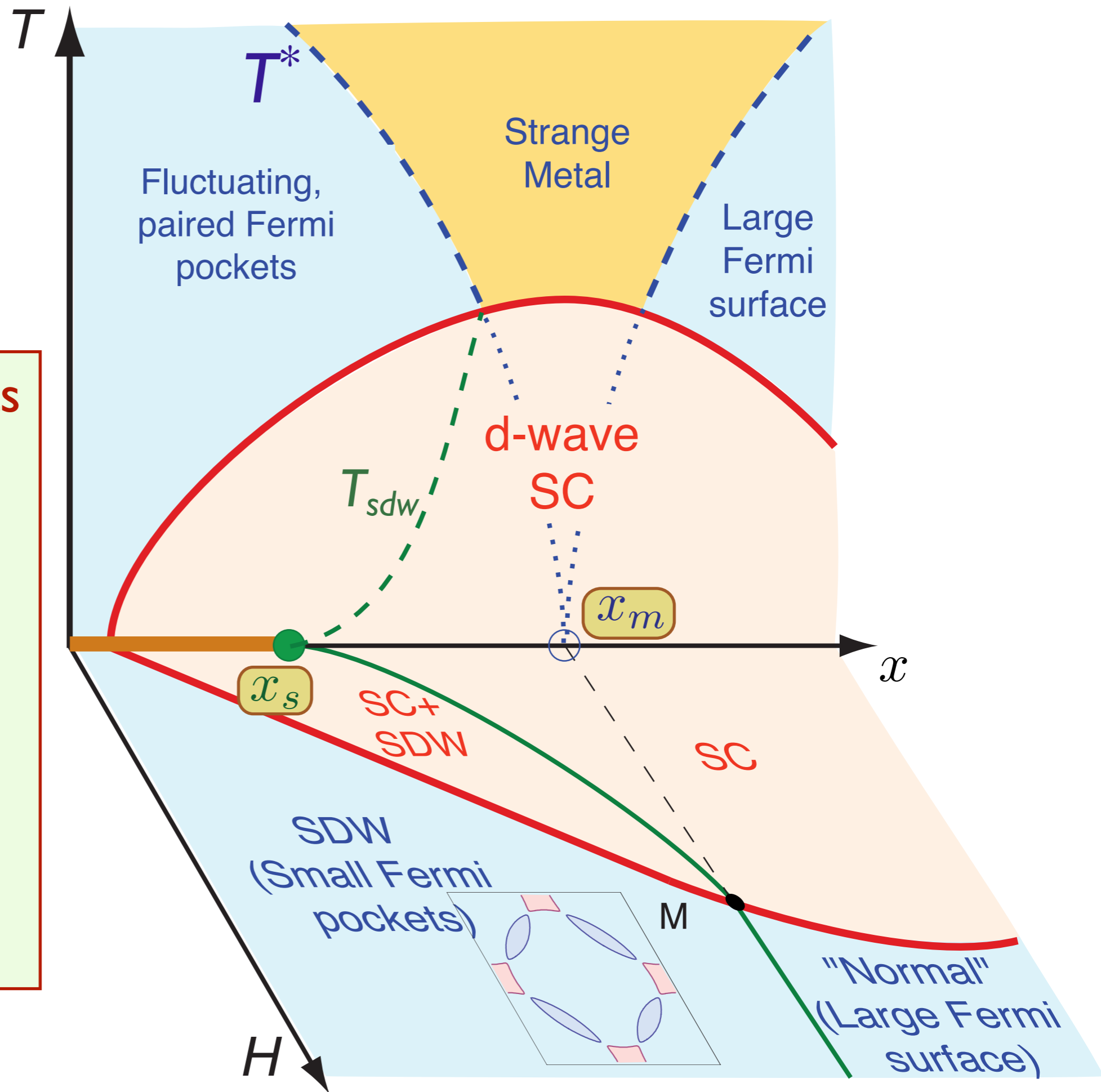
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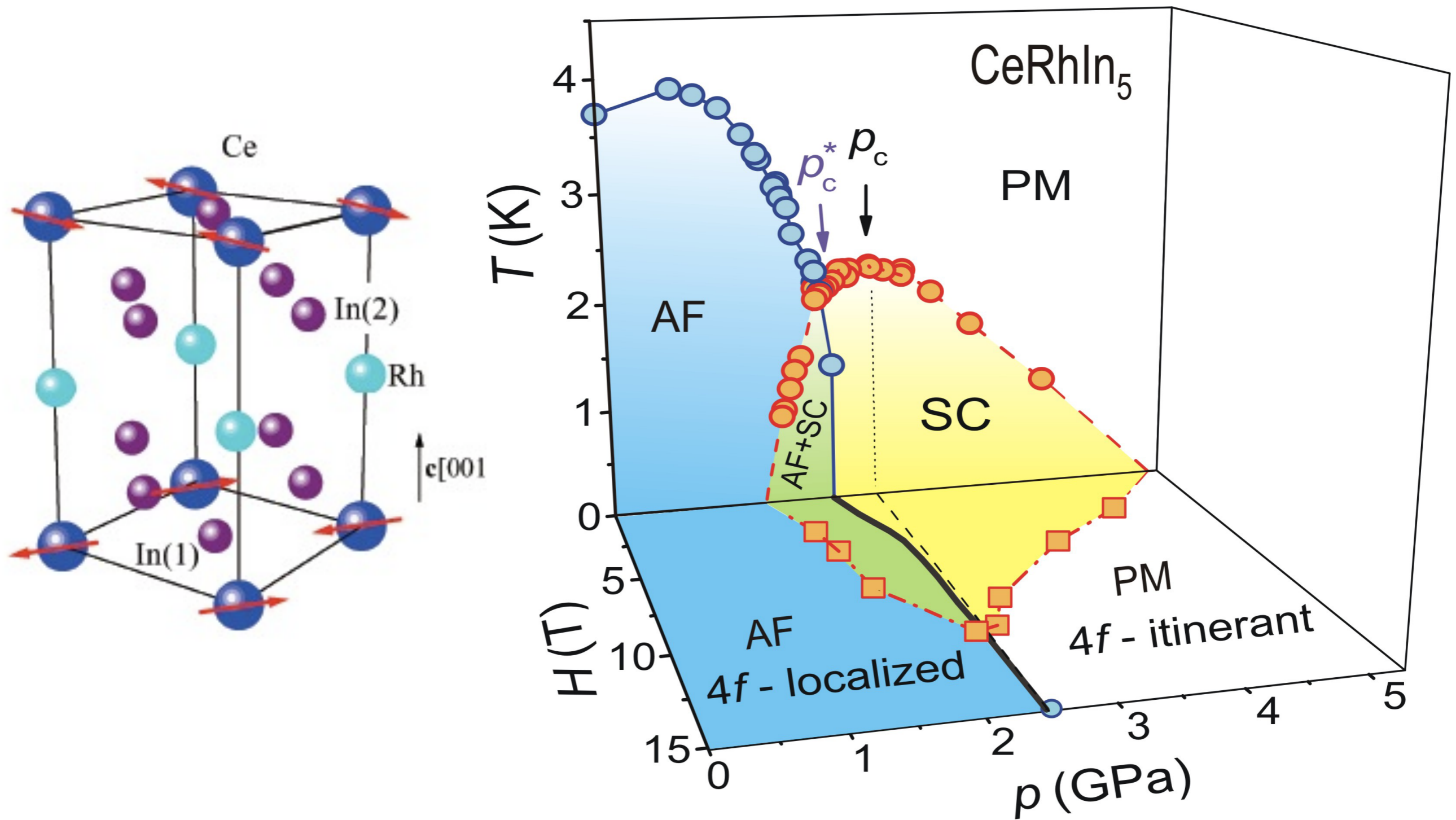
Quantum oscillations

E. Demler, S. Sachdev
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Rev. Lett.* **87**,
067202 (2001).

Many experiments
have presented
evidence for the
predicted
green quantum
phase transition
line
from SC to SC
+SDW in a
magnetic field



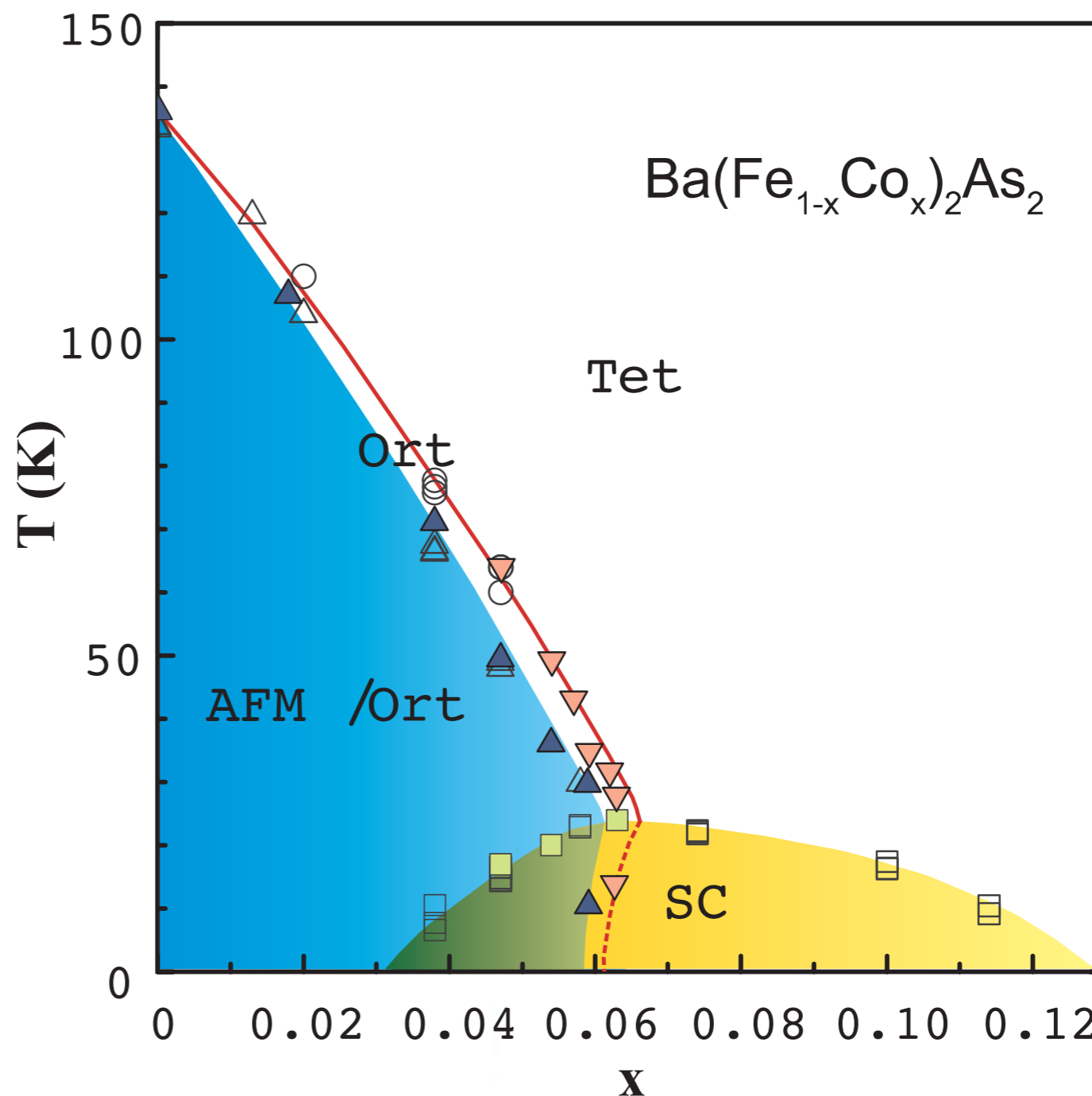
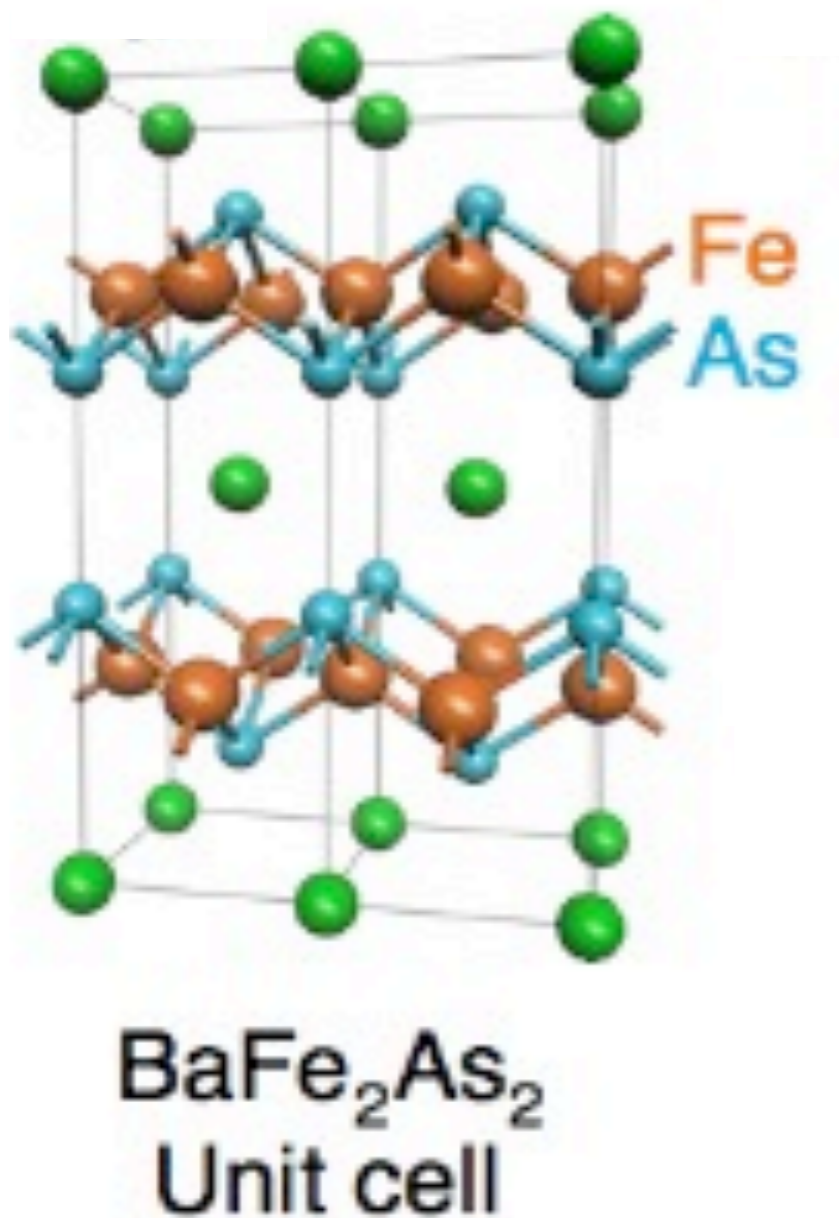
Similar phase diagram for CeRhIn₅



G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

Iron pnictides:

a new class of high temperature superconductors



S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni,
S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman,
Physical Review Letters **104**, 057006 (2010).

Outline

1. Loss of antiferromagnetism in an insulator

Coupled-dimer antiferromagnets and quantum criticality

2. Onset of antiferromagnetism in a metal

From large Fermi surfaces to Fermi pockets, d-wave superconductivity, and competing orders

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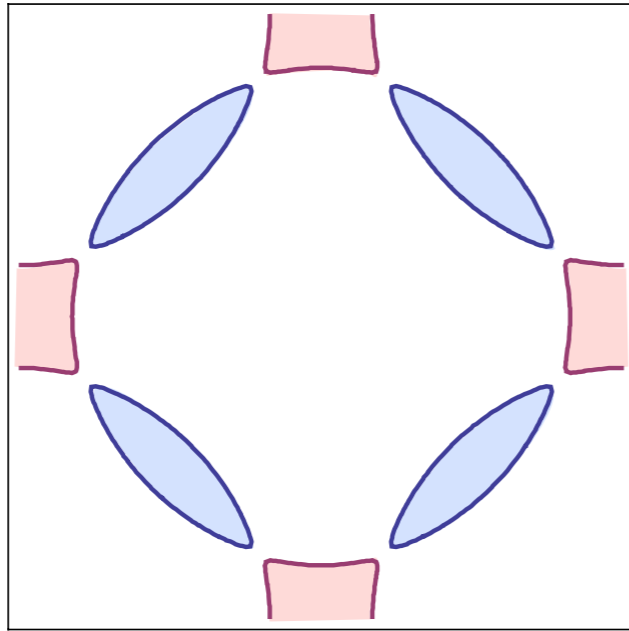
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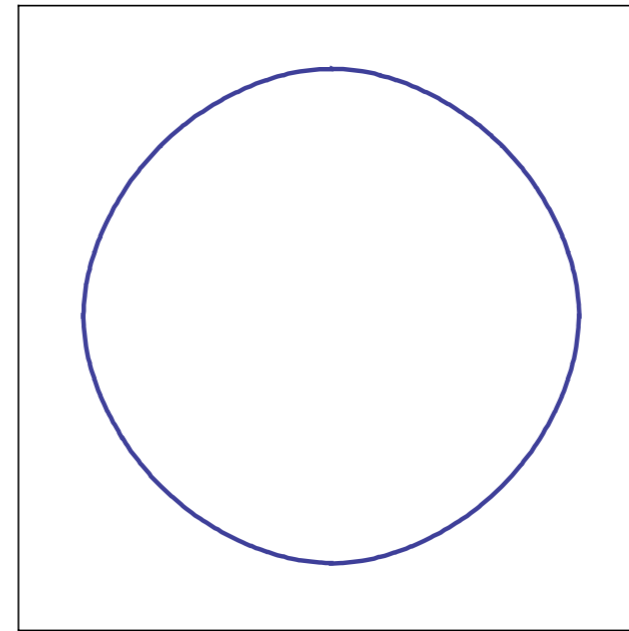
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$$\langle \vec{\varphi} \rangle \neq 0$$



Metal with electron
and hole pockets

$$\langle \vec{\varphi} \rangle = 0$$

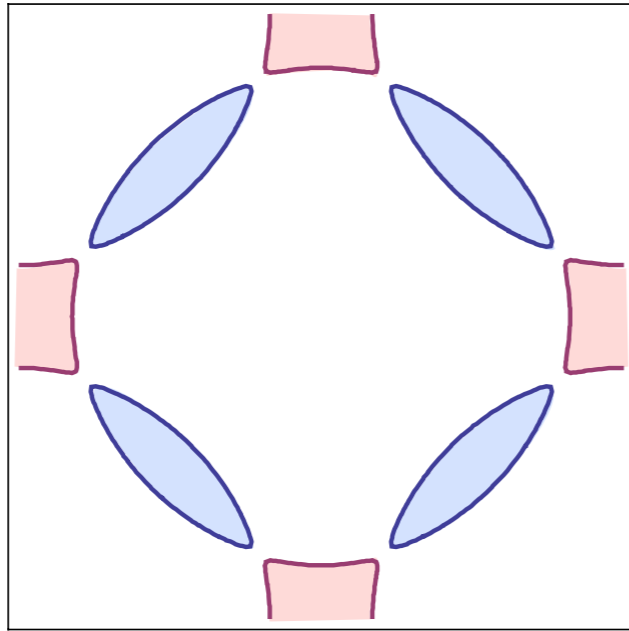


Metal with "large"
Fermi surface

S

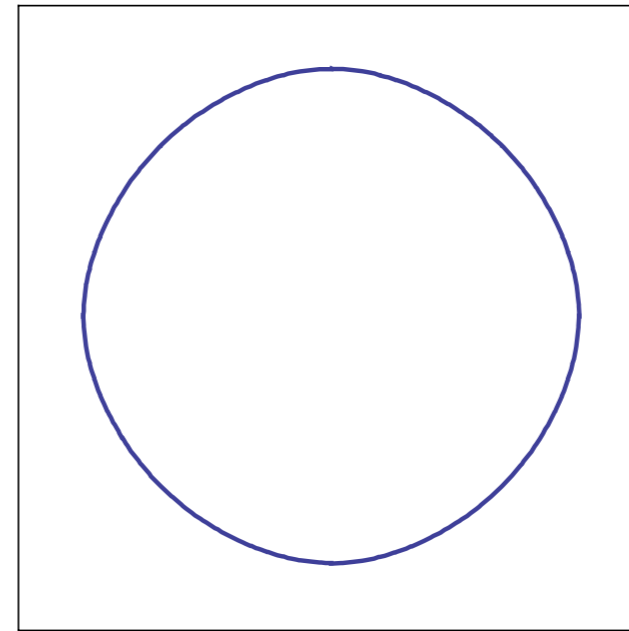
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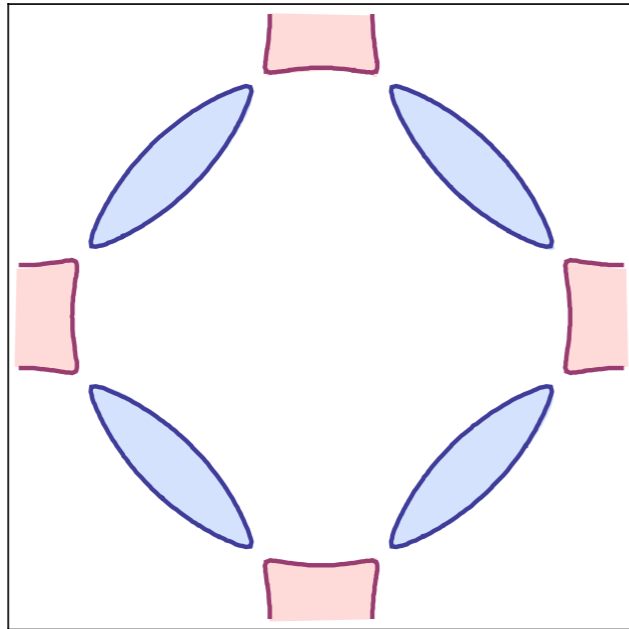


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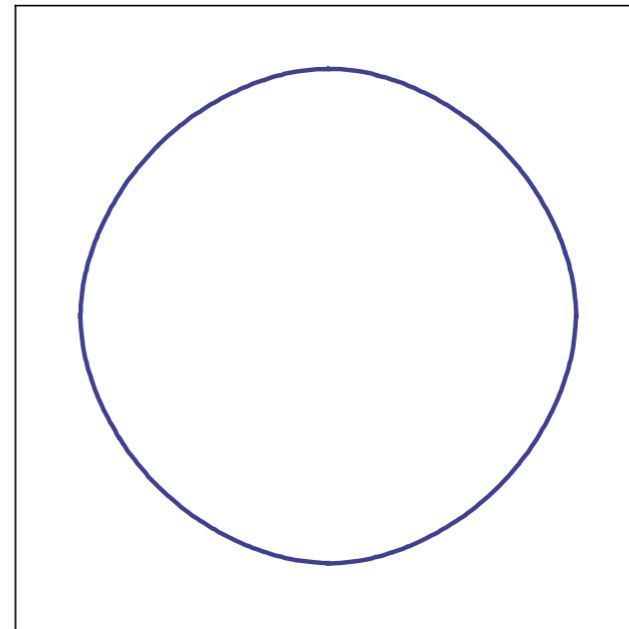
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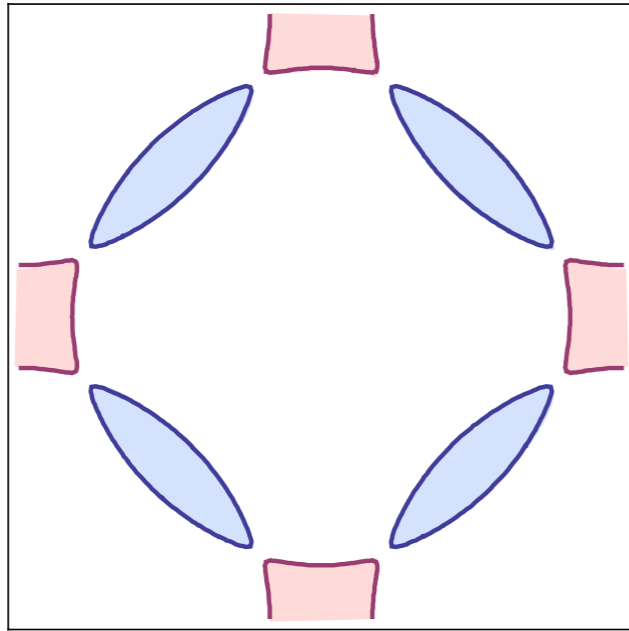


Metal with "large"
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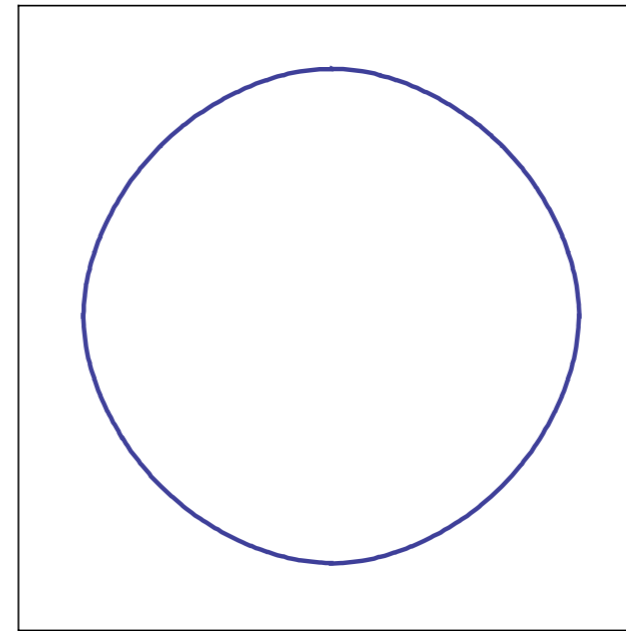
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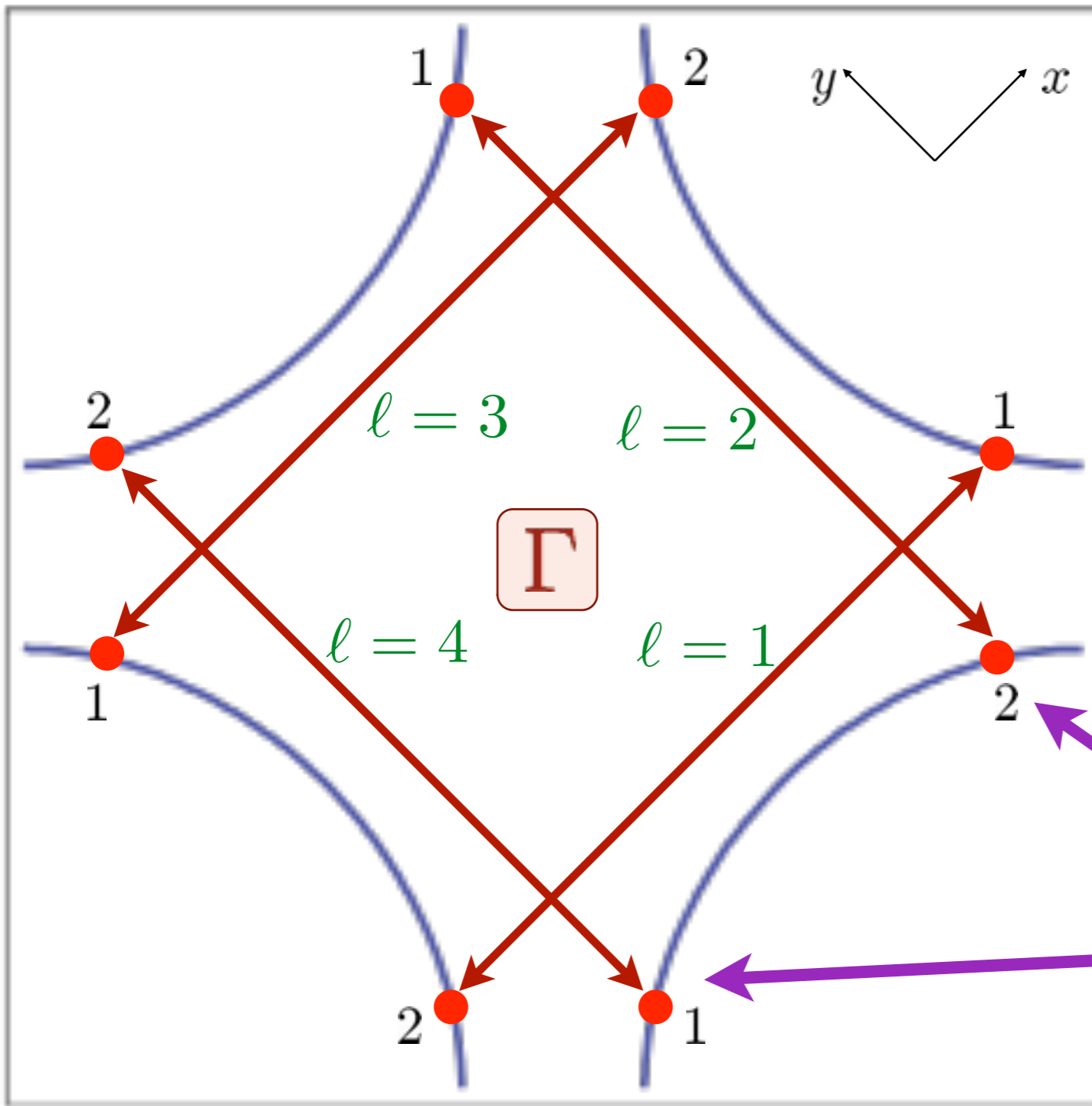


Metal with “large” Fermi surface

$\rightarrow S$

- Quantum critical theory is strongly-coupled in two (but not higher) spatial dimensions (but not a CFT).

M.A. Metlitski and S. Sachdev, *Physical Review B* **82**, 075128 (2010)



Low energy fermions
at hot spots $\mathbf{k} = \mathbf{k}_l$:
 $\psi_{1\alpha}^l, \psi_{2\alpha}^l$
 $l = 1, \dots, 4.$
 with $c_{\mathbf{k}+\mathbf{k}_l} = \psi^l(\mathbf{k})$

$$\mathcal{L}_f = \psi_{1\alpha}^{l\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^l \cdot \nabla_r) \psi_{1\alpha}^l + \psi_{2\alpha}^{l\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^l \cdot \nabla_r) \psi_{2\alpha}^l$$

$$\mathbf{v}_1^{l=1} = (v_x, v_y), \quad \mathbf{v}_2^{l=1} = (-v_x, v_y)$$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter:
$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$$

“Yukawa” coupling:
$$\mathcal{L}_c = -\lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1\beta}^\ell \right)$$

Results of RG analysis at 2+ loops

- The Hertz-Millis-Moriya procedure is valid in $d = 3$, but breaks down strongly in $d = 2$. (*cf.* Abanov-Chubukov)

M. A. Metlitski and S. Sachdev,
Physical Review B **82**, 075127 (2010)

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M. A. Metlitski and S. Sachdev,
Physical Review B **82**, 075127 (2010)

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- The $1/N$ expansion (N is the number of hot-spots) initially appears to be a genus expansion (*cf.* Sung-Sik Lee), but even this breaks down at 5 loops.

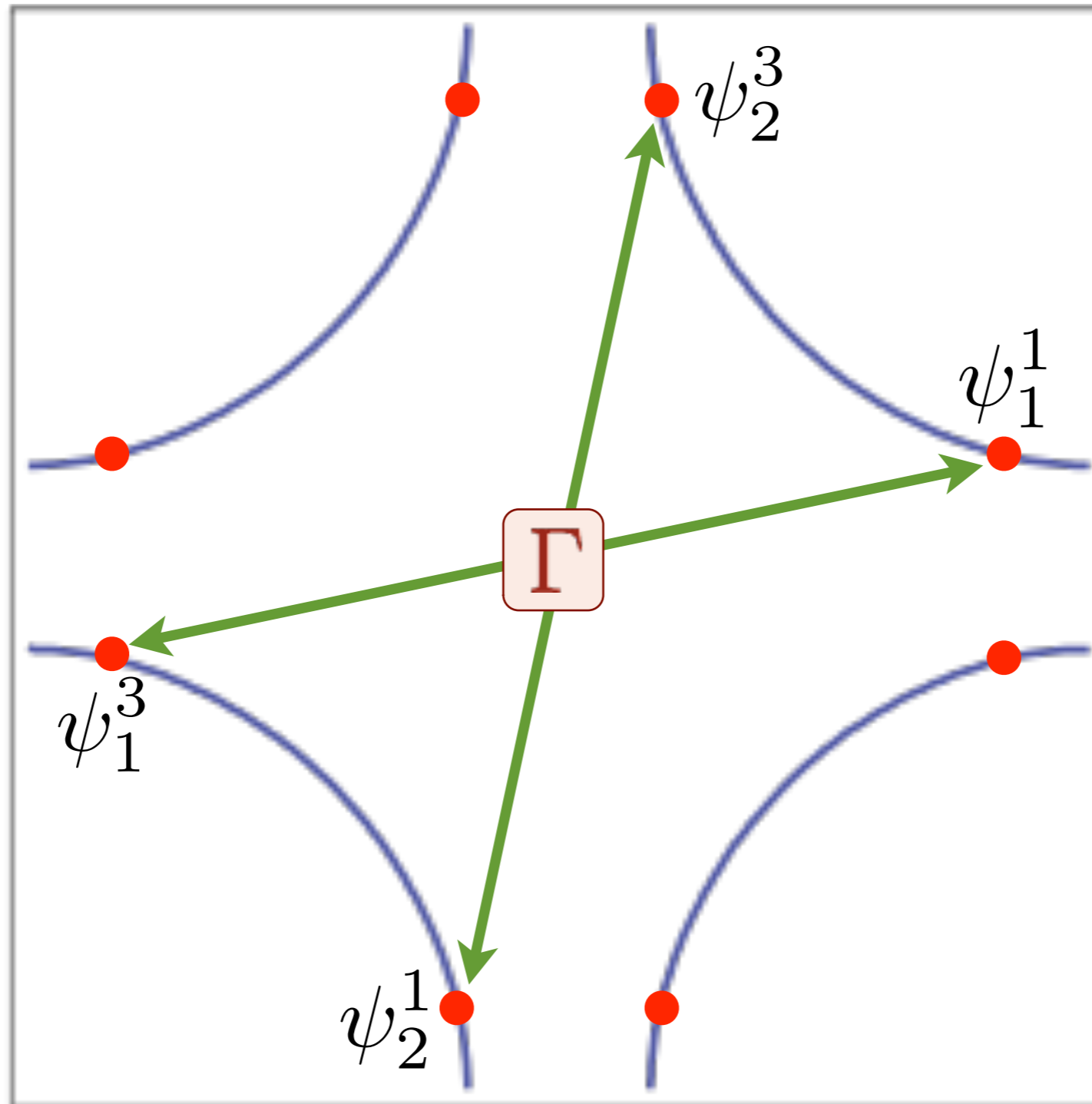
M. A. Metlitski and S. Sachdev,
Physical Review B **82**, 075127 (2010)

Results of RG analysis at 2+ loops

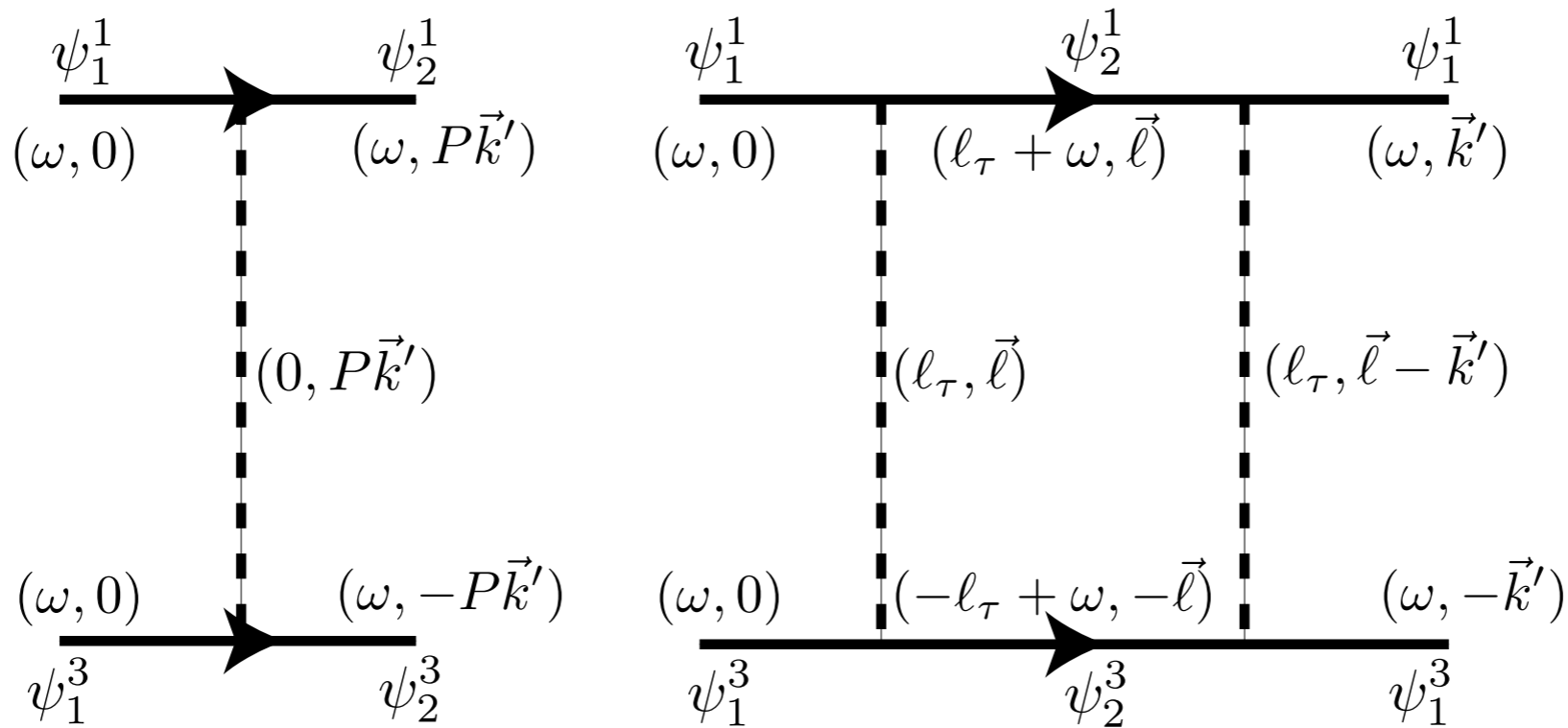
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M. A. Metlitski and S. Sachdev,
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d-wave pairing



Pairing order parameter: $\varepsilon^{\alpha\beta} \left(\psi_{1\alpha}^3 \psi_{1\beta}^1 - \psi_{2\alpha}^3 \psi_{2\beta}^1 \right)$



Need fermion Green's functions on Fermi surface near hot spots:

$$G(\omega, \vec{p}) \sim \frac{\mathcal{Z}(p_{\parallel})}{i\omega - v_F(p_{\parallel})p_{\perp}}.$$

Near the hot spot we have $v_F \sim \mathcal{Z} \sim p_{\parallel}$. The pairing interaction is enhanced at one loop by the factor

$$\left[1 + \frac{\alpha}{\pi(\alpha^2 + 1)} \log^2 \frac{\vec{k}'^2}{\gamma|\omega|} \right]$$

where $\alpha = v_y/v_x$ is of order unity.

M. A. Metlitski and S. Sachdev,
Physical Review B **82**, 075127 (2010)

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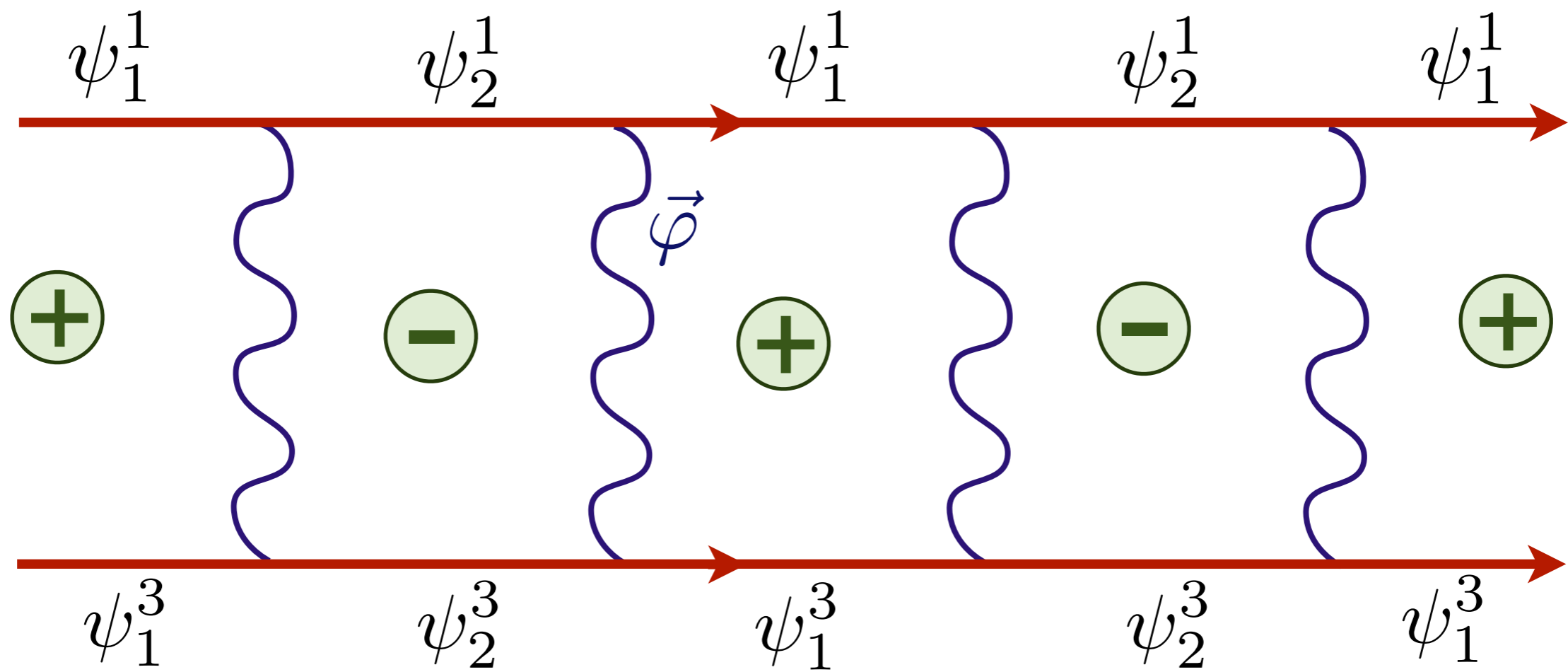
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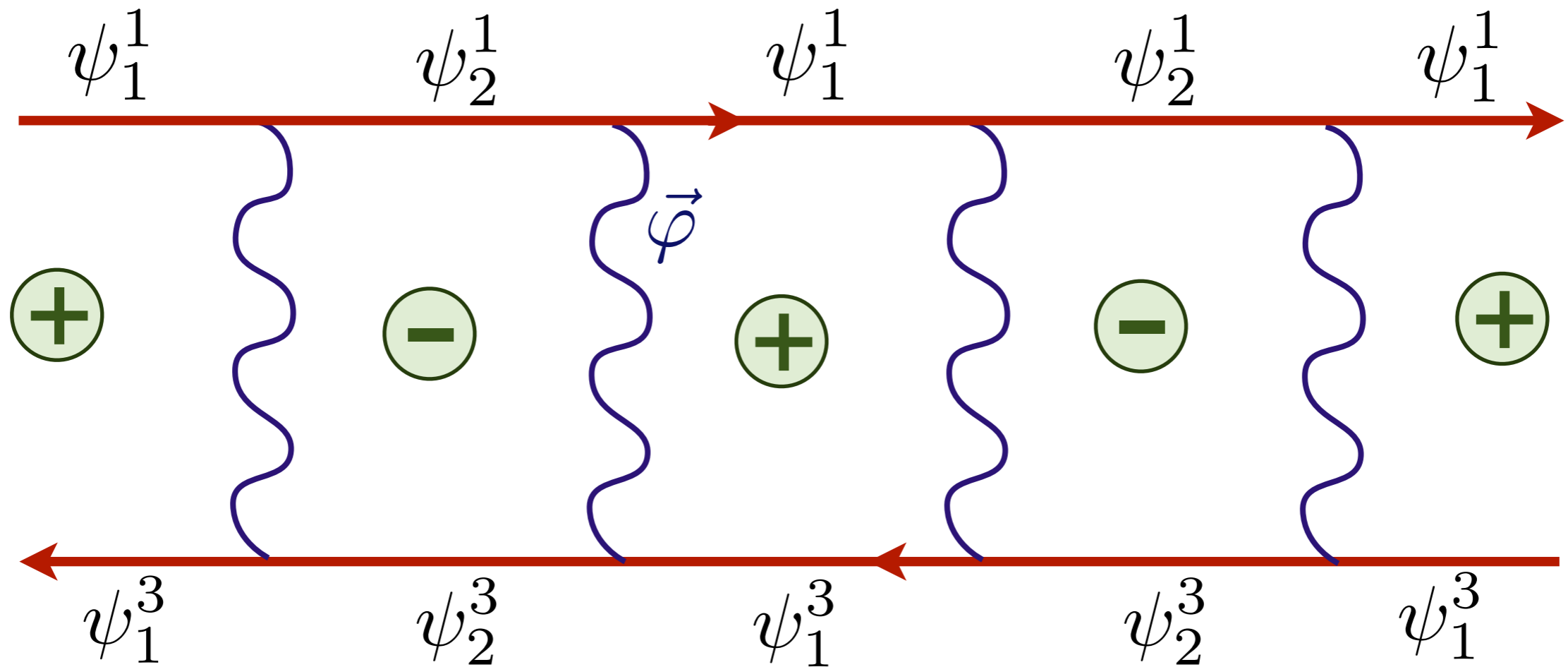
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M. A. Metlitski and S. Sachdev,
Physical Review B **82**, 075127 (2010)

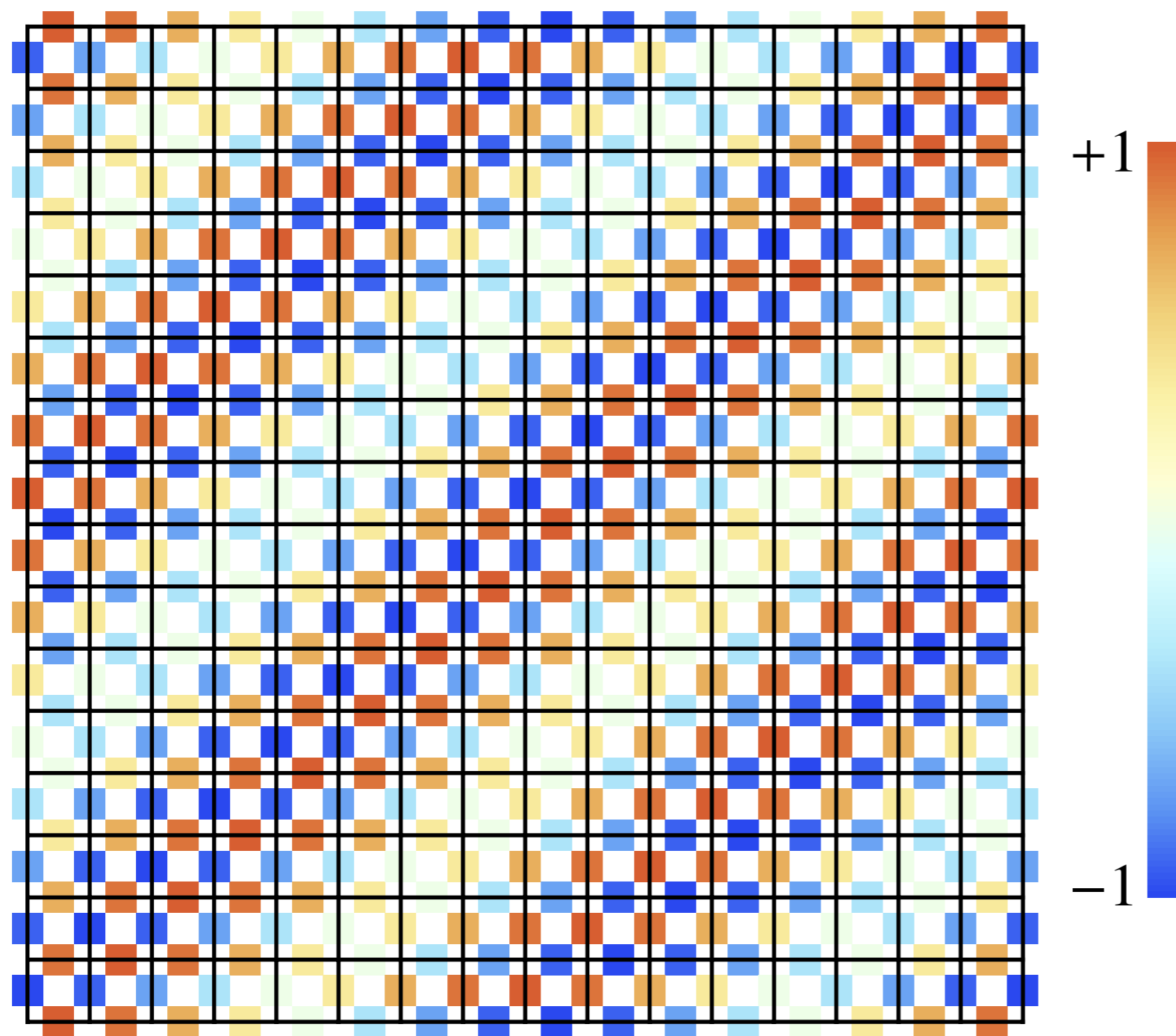


d-wave pairing instability in particle-particle channel



Bond density wave (with local Ising-nematic order) instability in particle-hole channel.

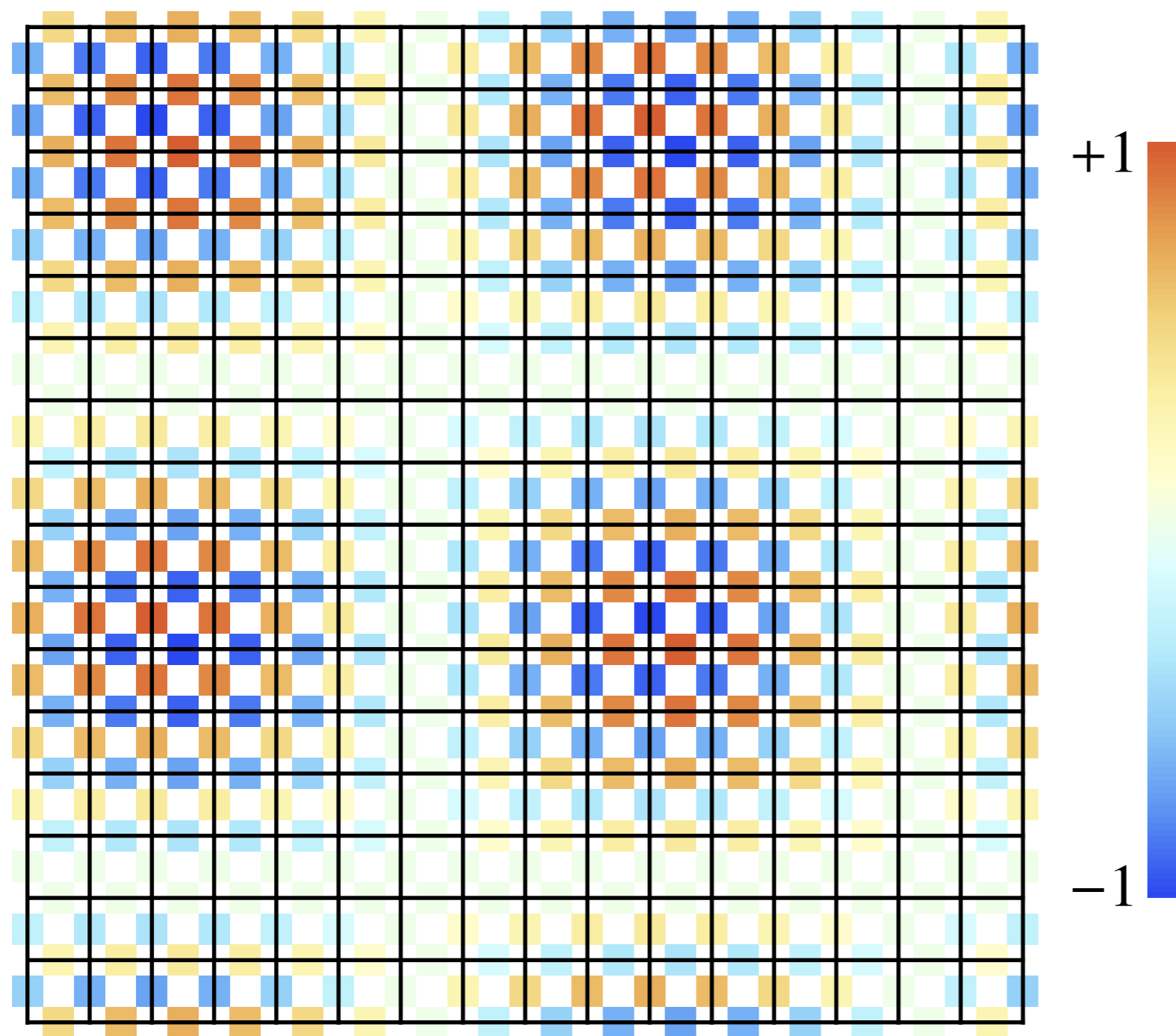
Nearly as strong as pairing instability because of a pseudospin symmetry of low energy theory



“Bond density”
measures amplitude
for electrons to be
in spin-singlet
valence bond:
VBS order

No modulations on sites: $\langle c_{\mathbf{r}\alpha}^\dagger c_{\mathbf{s}\alpha} \rangle$ is non-zero only for $\mathbf{r} \neq \mathbf{s}$. Modulated bond-density wave with local Ising-nematic ordering:

$$\left\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$



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Questions

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*
- *If so, why is there no antiferromagnetism in the hole-doped cuprates near the point where the superconductivity is strongest ?*
- *What is the physics of the strange metal ?*

Questions and answers

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Proposal: strongly-coupled quantum criticality of Fermi surface change in a metal