Fermi surfaces and gauge-gravity duality

University of Kentucky, Feb 28, 2011

Lecture notes
arXiv:1010.0682
arXiv: 1012.0299

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PHYSICS
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Compressible quantum matter

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There are only a few established examples of such phases in condensed matter physics.
However, they appear naturally as duals of gravitational theories, and we want to interpret them in the gauge theory.

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All known examples of such phases have a *Fermi Surface*

(even in systems with only bosons in the Hamiltonian)

The Fermi surface

This is the locus of zero energy singularities in momentum space in the two-point correlator of fermions carrying charge $\mathcal{Q}$.

$$G^{-1}_{\text{fermion}}(k = k_F, \omega = 0) = 0.$$

Luttinger relation: The total "volume (area)" $\mathcal{A}$ enclosed by Fermi surfaces of fermions carrying charge $\mathcal{Q}$ is equal to $\langle \mathcal{Q} \rangle$. This is a *key* constraint which allows extrapolation from weak to strong coupling.

Examples of compressible phases and their Fermi surfaces

1. The Fermi liquid (FL)

2. Fermions coupled to gauge fields

3. Fermion-boson mixtures

4. The fractionalized Fermi liquid (FL*)

5. Theories similar to ABJM

6. Theories similar to $\mathcal{N} = 4$ SYM

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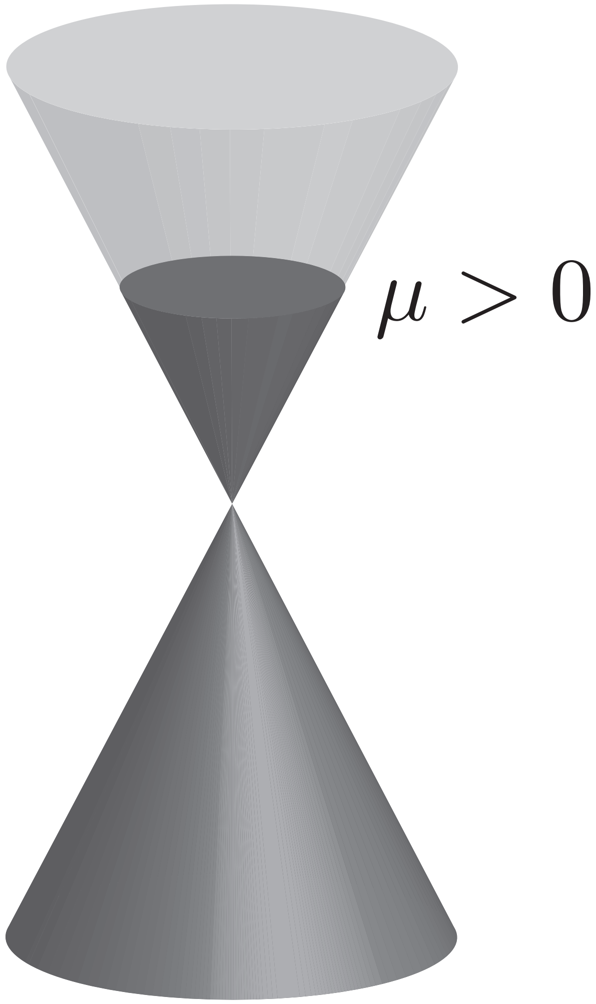
The Fermi Liquid (FL)

Most common example: electrons with short-range interactions (or screened long-range interactions), which are adiabatically connected to the non-interacting limit. The electron Green's function G_f has a pole which crosses zero energy at $k = k_F$, and the Fermi surface has the same area as the non-interacting case.

$$\mathcal{L} = \overline{f}\left(\partial_a - \mu\delta_{at}\right)\gamma^a f + 4 \text{ Fermi terms}$$



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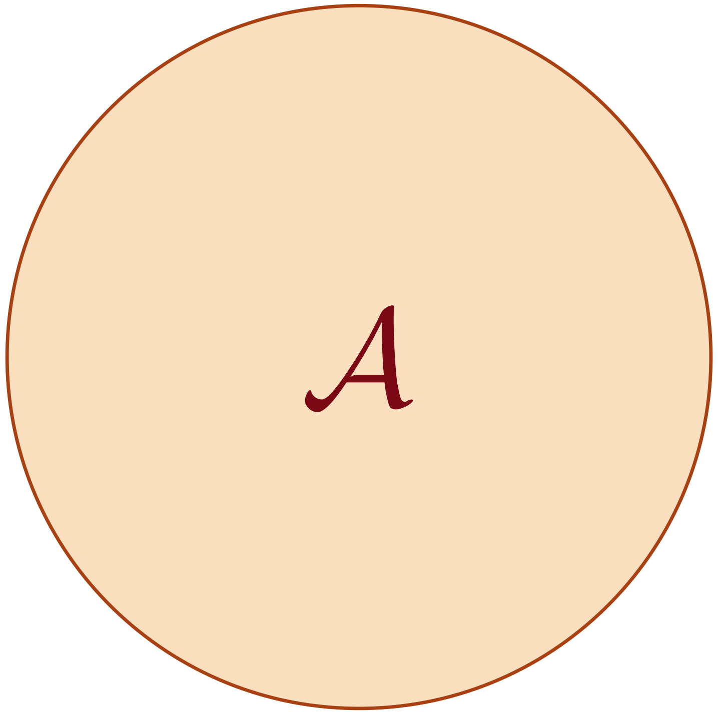
$$\mathcal{A} = \langle f^\dagger f \rangle = \langle \mathcal{Q} \rangle$$



$$G_f = \frac{1}{\omega - v_F(k - k_F) + i\omega^2}$$

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- Couple fermions to a dynamical gauge field A_a.
- Longitudinal gauge fluctuations are screened by the fermions. But transverse gauge fluctuations remain unscreened, and are Landau-damped by excitations near the Fermi surface. The theory of a Fermi surface coupled to transverse gauge fluctuations is *strongly coupled in two spatial dimensions*.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

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- The overdamped transverse gauge modes lead to "non-Fermi liquid" broadening of the fermion pole near the Fermi surface.

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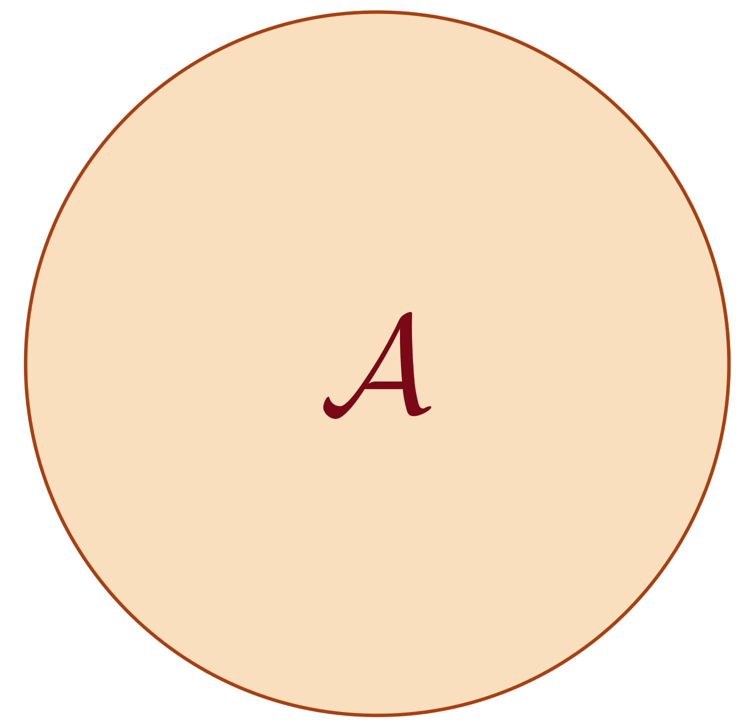
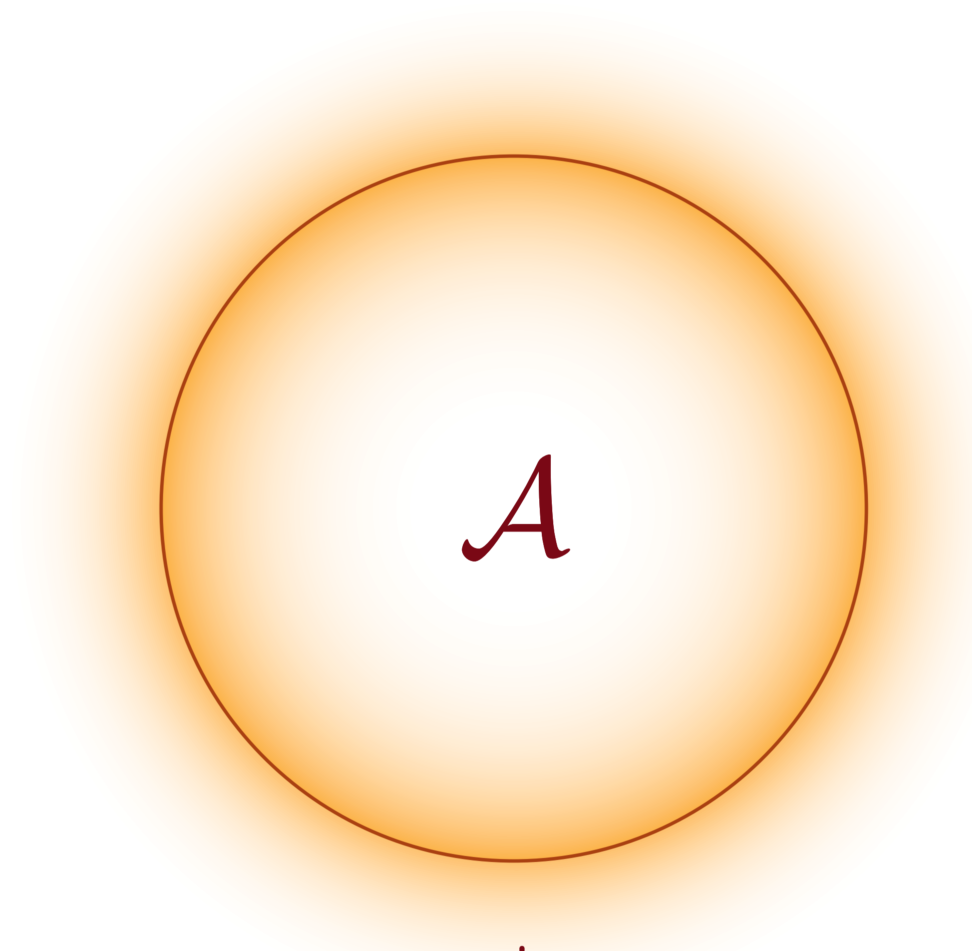


$$\mathcal{A} = \langle f^\dagger f \rangle = \langle \mathcal{Q} \rangle$$

$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f$$

- *The location of the Fermi surfaces is well defined, and the Luttinger relation applies as before.*



$$2\mathcal{A} = \left\langle f_\sigma^\dagger f_\sigma \right\rangle = \langle \mathcal{Q} \rangle$$

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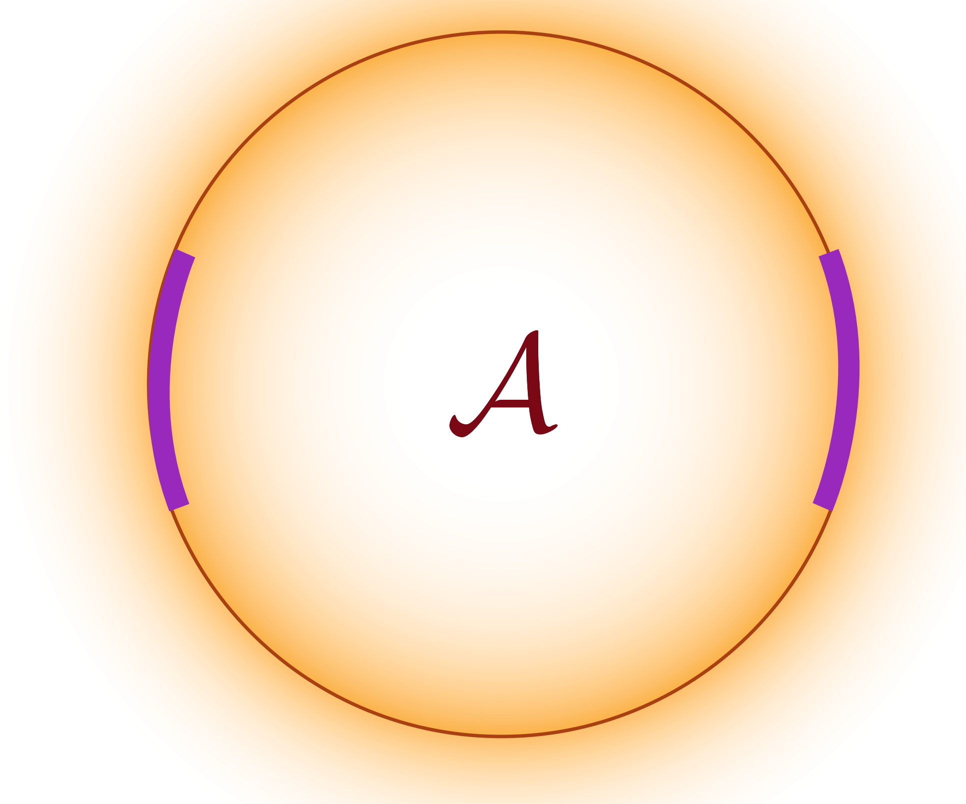
- *The location of the Fermi surfaces is well defined, and the Luttinger relation applies as before.*
- Fluctuations near the Fermi surface are described by a strongly-coupled two-patch theory. Ward identities allow consistent matching of the patches, and patches along different directions decouple in the low energy limit.



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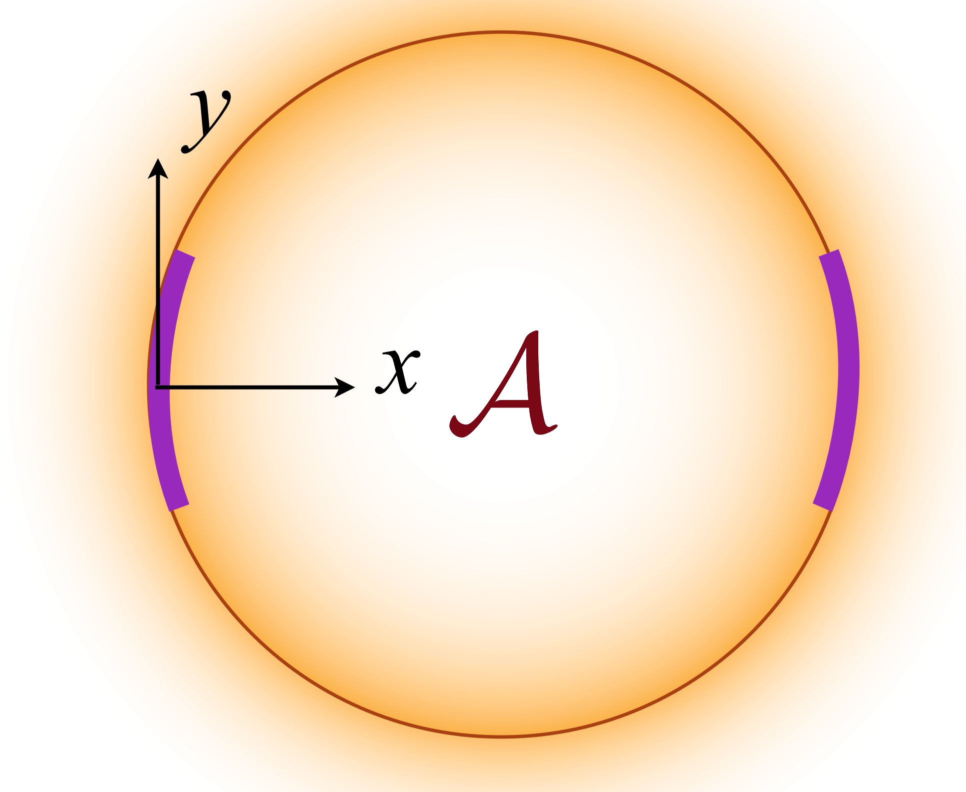
- Fluctuations near the Fermi surface are described by a strongly-coupled two-patch theory. Ward identities allow consistent matching of the patches, and patches along different directions decouple in the low energy limit.
- The singularity of the Green's function upon approaching the Fermi surface is described by the scaling form

$$G_f^{-1} = q^{1-\eta} F(\omega / q^{z/2})$$

where $q_x = k_x - k_F$, $q_y = k_y$, and $q = q_x + q_y^2/(2k_F)$, and η and z are anomalous exponents.



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Tuesday, March 1, 2011

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Consider mixture of fermions f and bosons b.

$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f + b^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_b} - \mu_b \right) b + s|b|^2 + -g\, b^\dagger f^\dagger f b + \ldots$$

Consider mixture of fermions f and bosons b.
There is a $\mathrm{U}(1) \times \mathrm{U}_b(1)$ symmetry
and 2 conserved charges:

$$\mathcal{Q} = f^\dagger f$$
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$$\begin{aligned} \mathcal{L} &= f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f \\ &+ b^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_b} - \mu_b \right) b + s|b|^2 + -g\, b^\dagger f^\dagger f b + \ldots \end{aligned}$$

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The 2 symmetries imply 2 Luttinger constraints. However, bosons at non-zero density invariably Bose condense at $T = 0$, and so $\mathrm{U}_b(1)$ is broken. So there is only the single constraint on the f Fermi surface. This describes mixtures of ^{3}He and ^{4}He.

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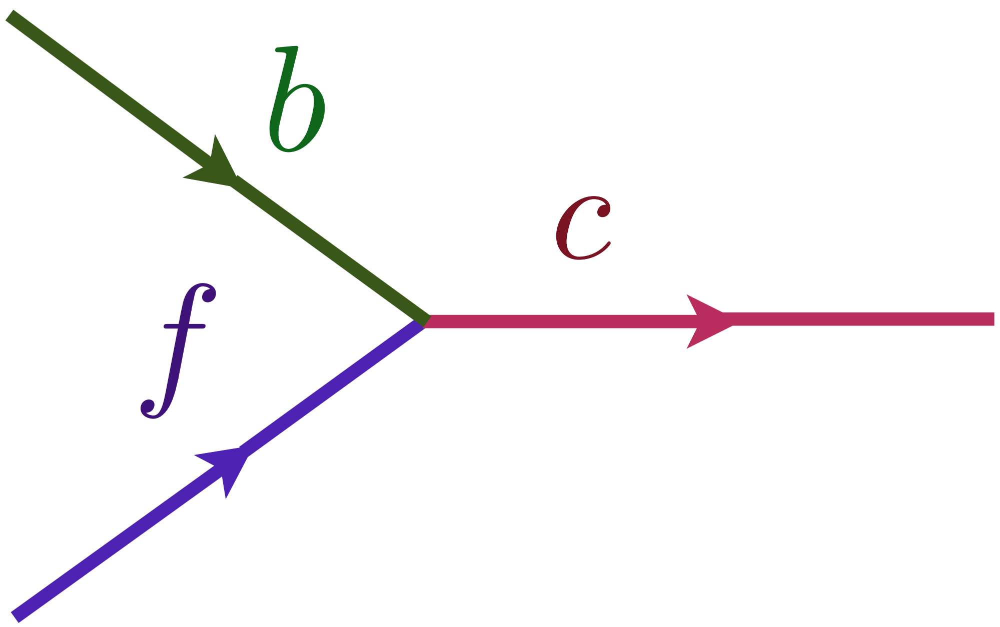
Superfluid: $\langle b \rangle \neq 0$
$\mathrm{U}_b(1)$ broken; $\mathrm{U}(1)$ unbroken

$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f + b^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_b} - \mu_b \right) b + s|b|^2 + -g\, b^\dagger f^\dagger f b + \ldots$$

Increase the coupling g until the boson, b, and fermion, f, can bind into a 'molecule', the fermion c.



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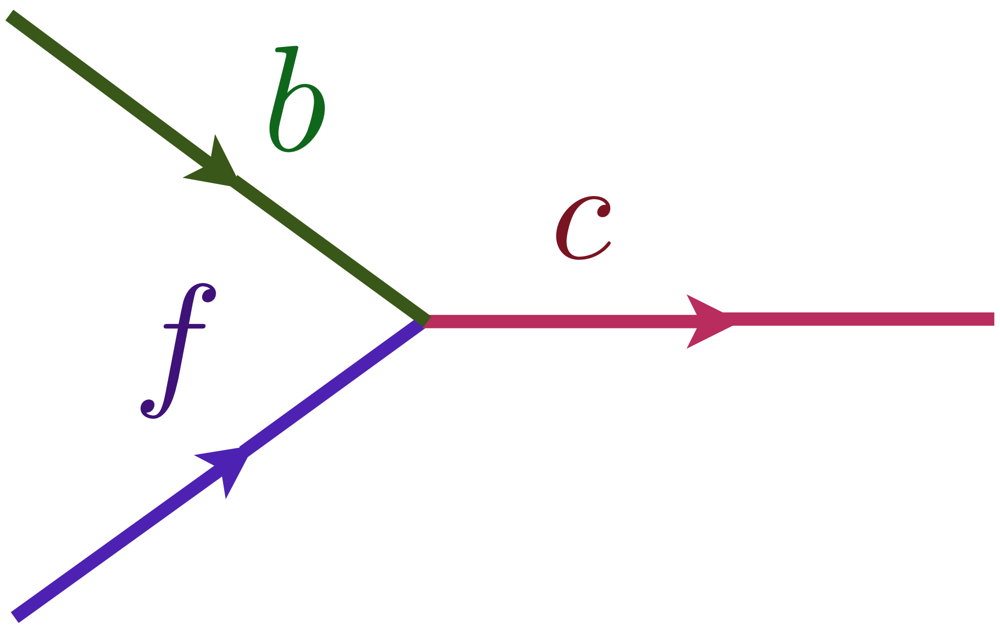
S. Powell, S. Sachdev, and H. P. Büchler, *Physical Review* B **72**, 024534 (2005)

Increase the coupling g until the boson, b, and fermion, f, can bind into a 'molecule', the fermion c.
Decouple the interaction between b and f by a fermion c



$$Q = f^\dagger f$$
$$Q_b = b^\dagger b$$

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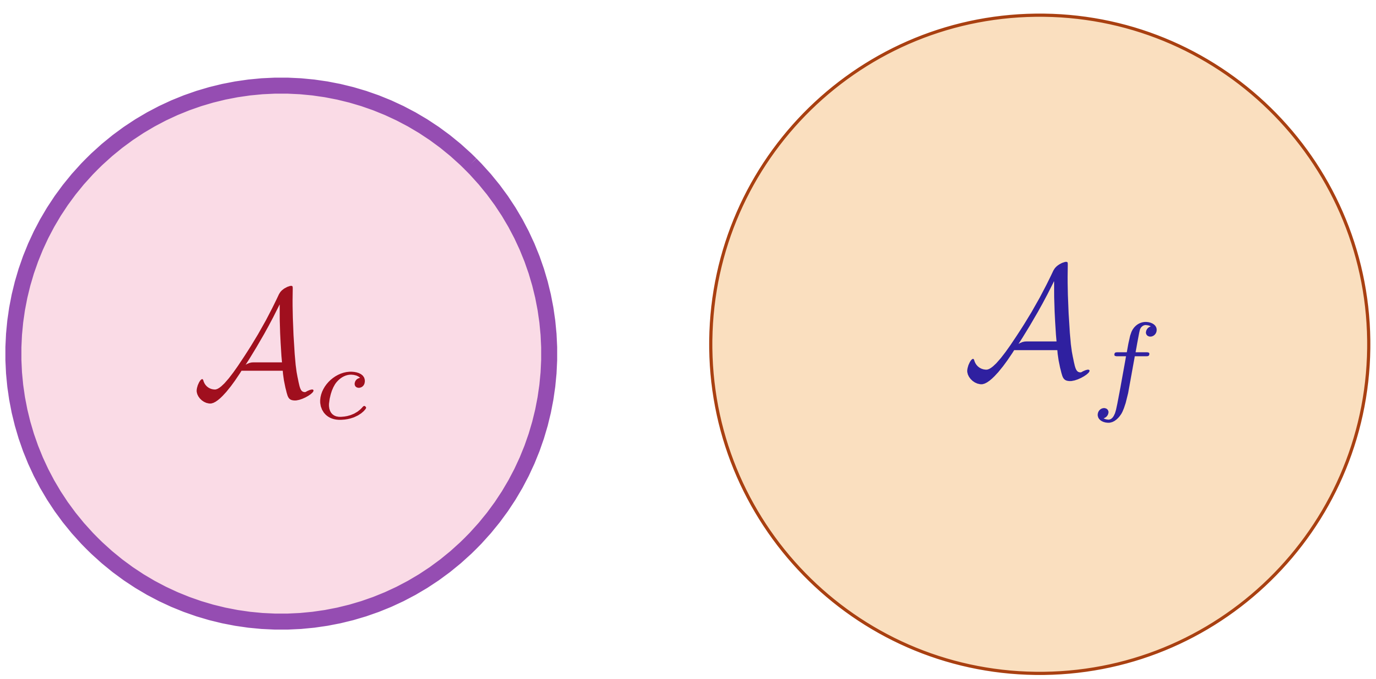
In a phase with $U_b(1)$ unbroken, there is a Luttinger relation for each conserved U(1) charge. However, the boson, b cannot have a Fermi surface in its Green's function, and so there is no area associated with it, although the boson density is *included* in the Luttinger relation

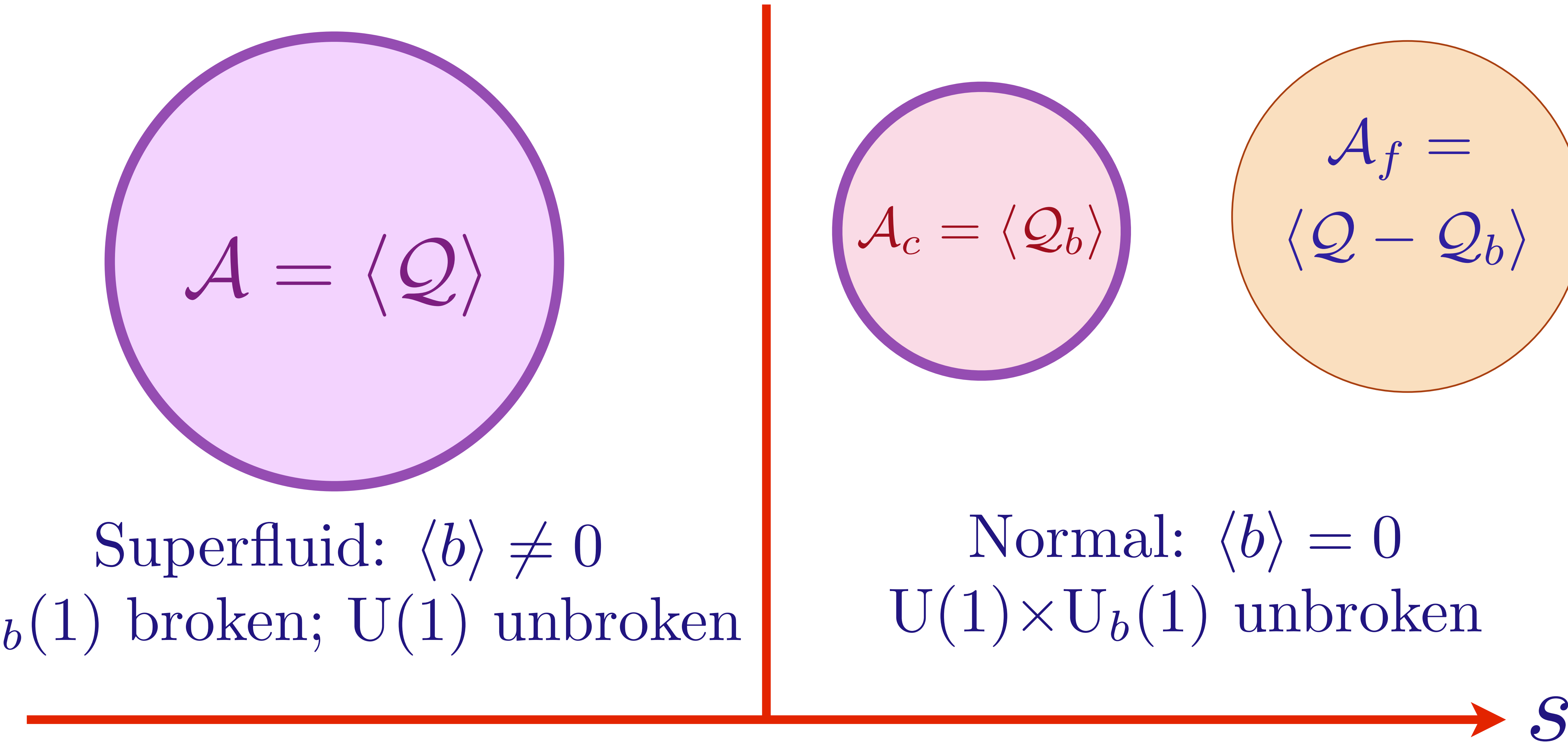
$$\begin{aligned} \mathcal{A}_c + \mathcal{A}_f &= \langle f^\dagger f \rangle = \langle \mathcal{Q} \rangle \\ \mathcal{A}_c &= \langle b^\dagger b \rangle = \langle \mathcal{Q}_b \rangle \end{aligned}$$



The b bosons have bound with f fermions to form c "molecules"

S. Powell, S. Sachdev, and H. P. Büchler, *Physical Review* B **72**, 024534 (2005)

Phase diagram of boson-fermion mixture



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(Need a background neutralizing charge)

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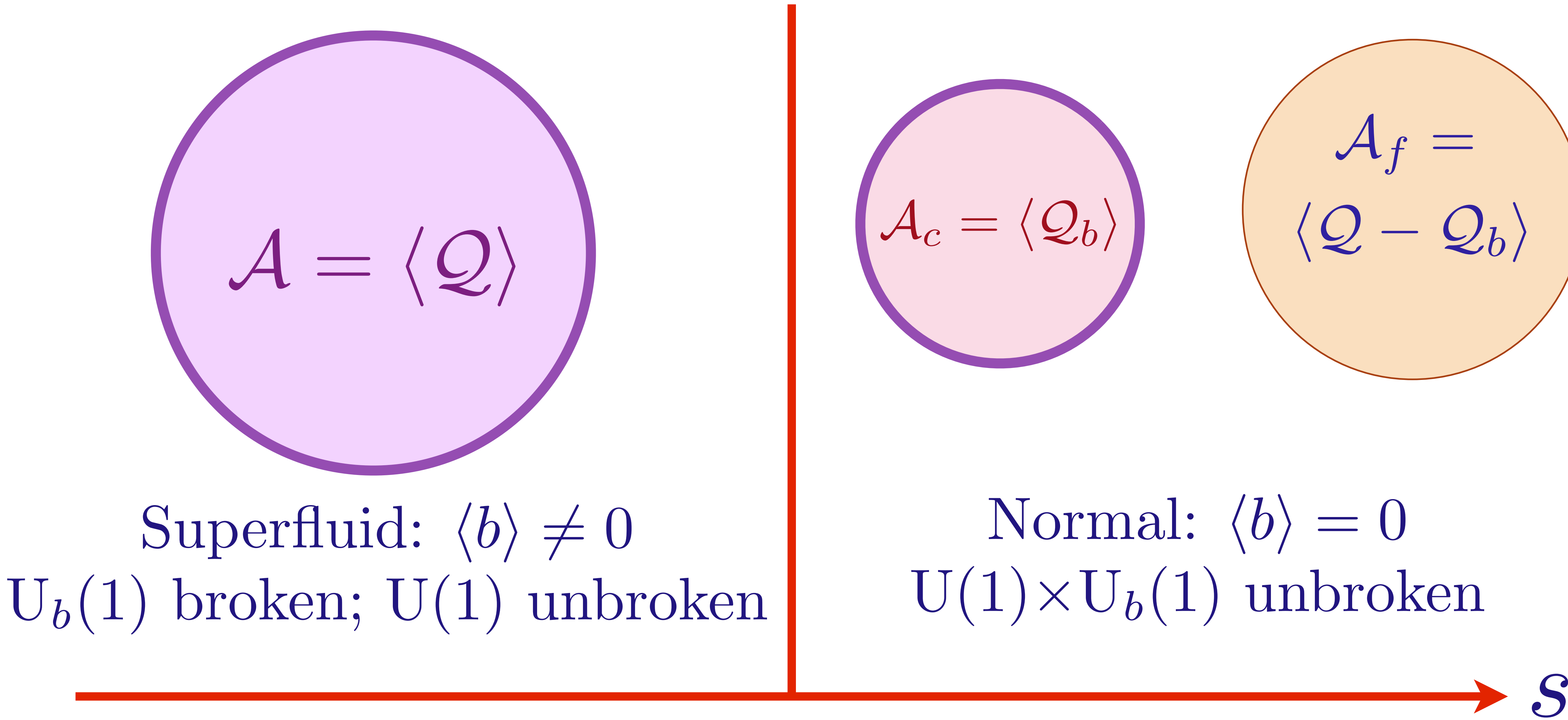

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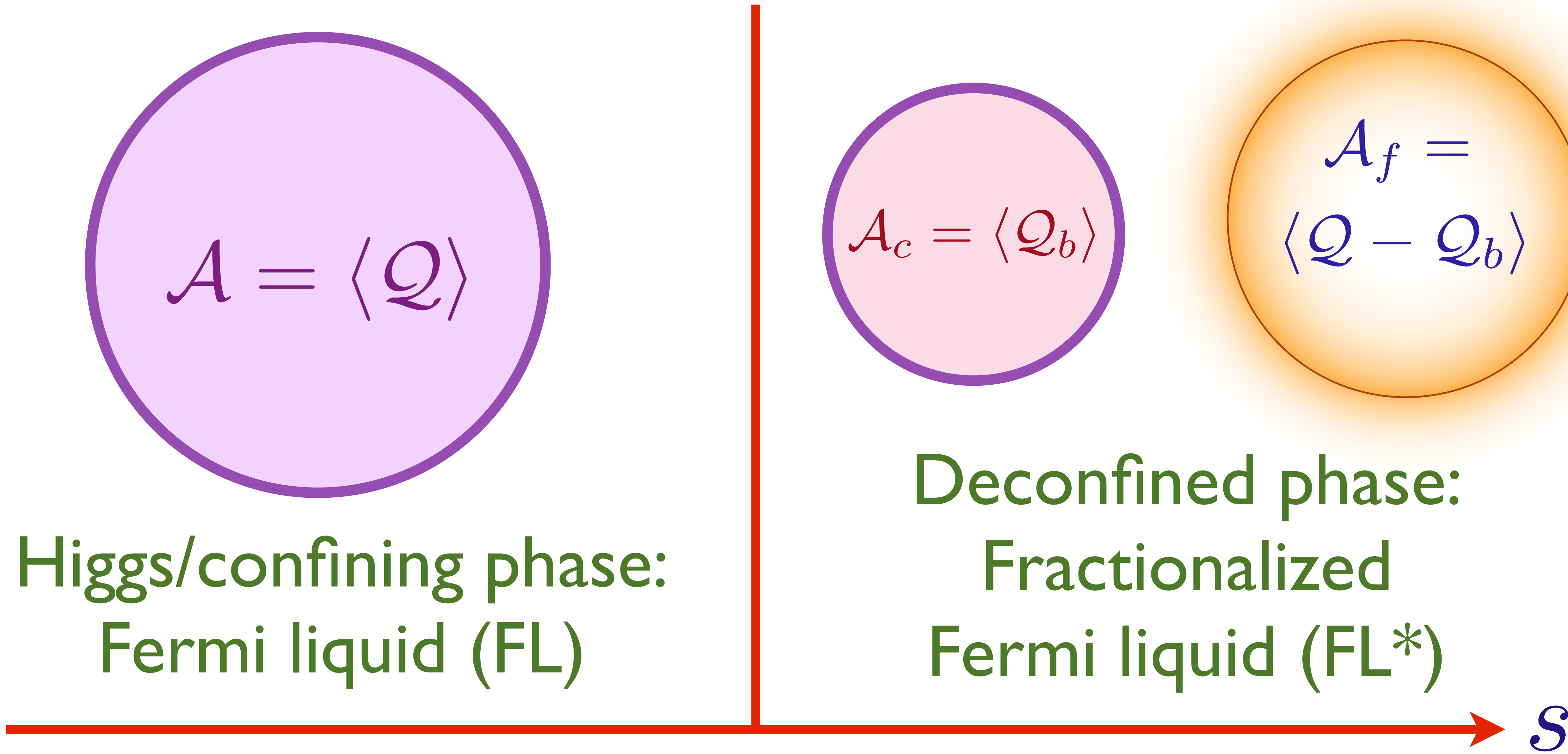
T. Senthil, M. Vojta, and S. Sachdev, *Physical Review* B **69**, 035111 (2004)

Phase diagram of U(1) gauge theory



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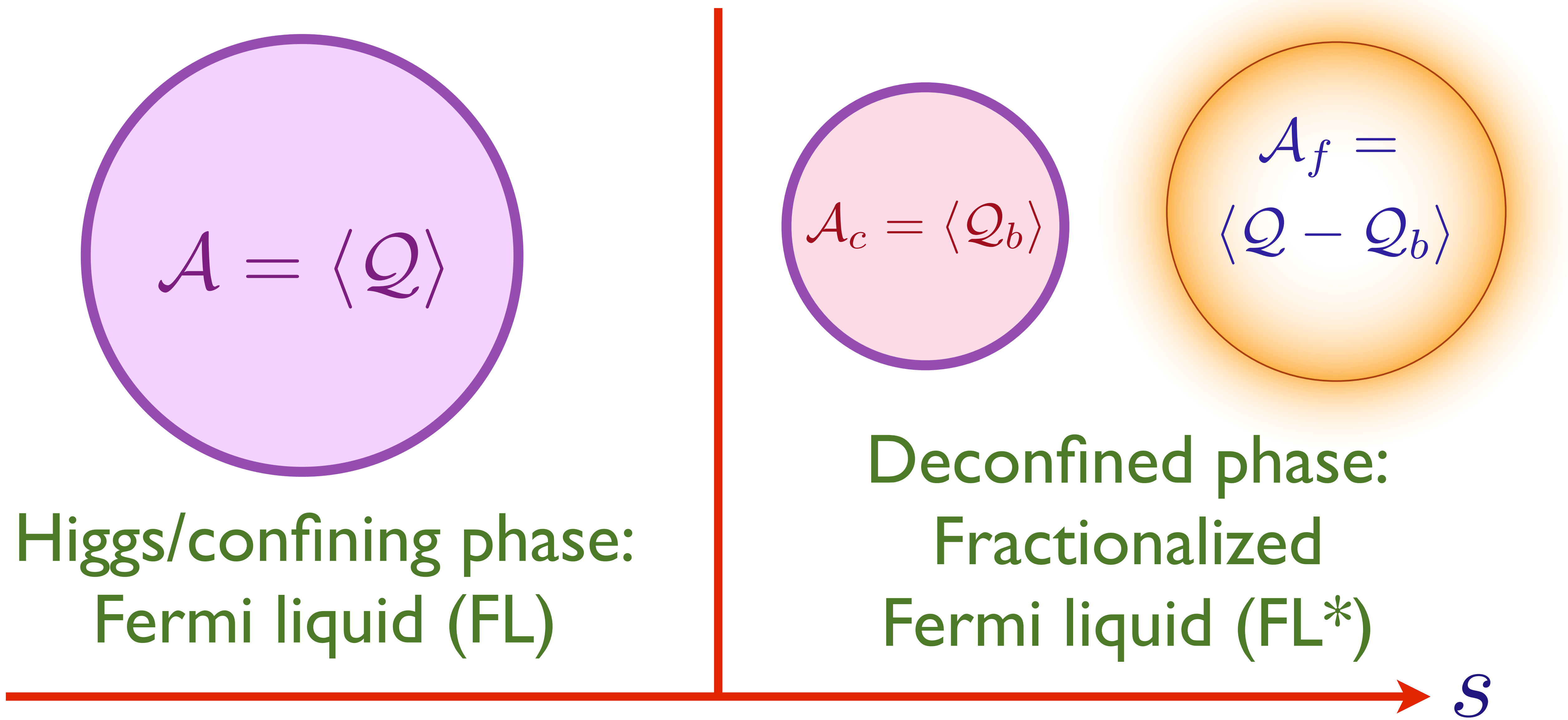

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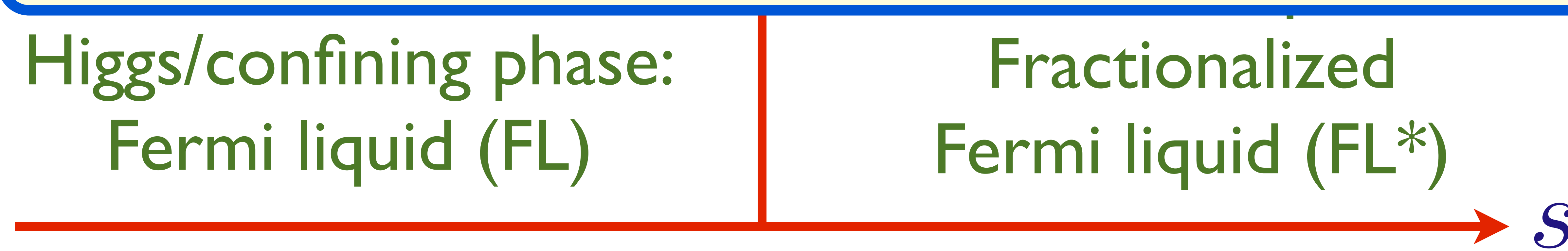
Phase diagram of U(1) gauge theory

- FL phase: Fermi surface of gauge-neutral fermions encloses total global charge $\mathcal{Q}$
- FL* phase: Fermi surface of gauge neutral fermions encloses only part of the global charge $\mathcal{Q}$



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ABJM theory in D=2+1 dimensions

- $4N^2$ Weyl fermions carrying fundamental charges of $\mathrm{U}(N)\times\mathrm{U}(N)\times\mathrm{SU}(4)_R$.
- $4N^2$ complex bosons carrying fundamental charges of $\mathrm{U}(N)\times\mathrm{U}(N)\times\mathrm{SU}(4)_R$.
- $\mathcal{N} = 6$ supersymmetry

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Adding a chemical potential coupling to a SU(4) charge breaks supersymmetry and SU(4) invariance

Theory similar to ABJM

- U(1) gauge invariance and U(1) global symmetry
- Fermions, f_+ and f_-, carry U(1) gauge charges ± 1, and global U(1) charge 1.
- Bosons, b_+ and b_-, carry U(1) gauge charges ± 1, and global U(1) charge 1.
- No supersymmetry

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- Bosons, b_+ and b_-, carry U(1) gauge charges ± 1, and global U(1) charge 1.
- No supersymmetry
- Fermions, c, gauge-invariant bound states of fermions and bosons carrying global U(1) charge 2.

Theory similar to ABJM

$$
\begin{aligned}
\mathcal{L} &= f_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m} - \mu \right] f_\sigma \\
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&+ \frac{u}{2} \left(b_\sigma^\dagger b_\sigma \right)^2 - g_1 \left(b_+^\dagger b_-^\dagger f_- f_+ + \mathrm{H.c.} \right)
\end{aligned}
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The index $\sigma = \pm 1$

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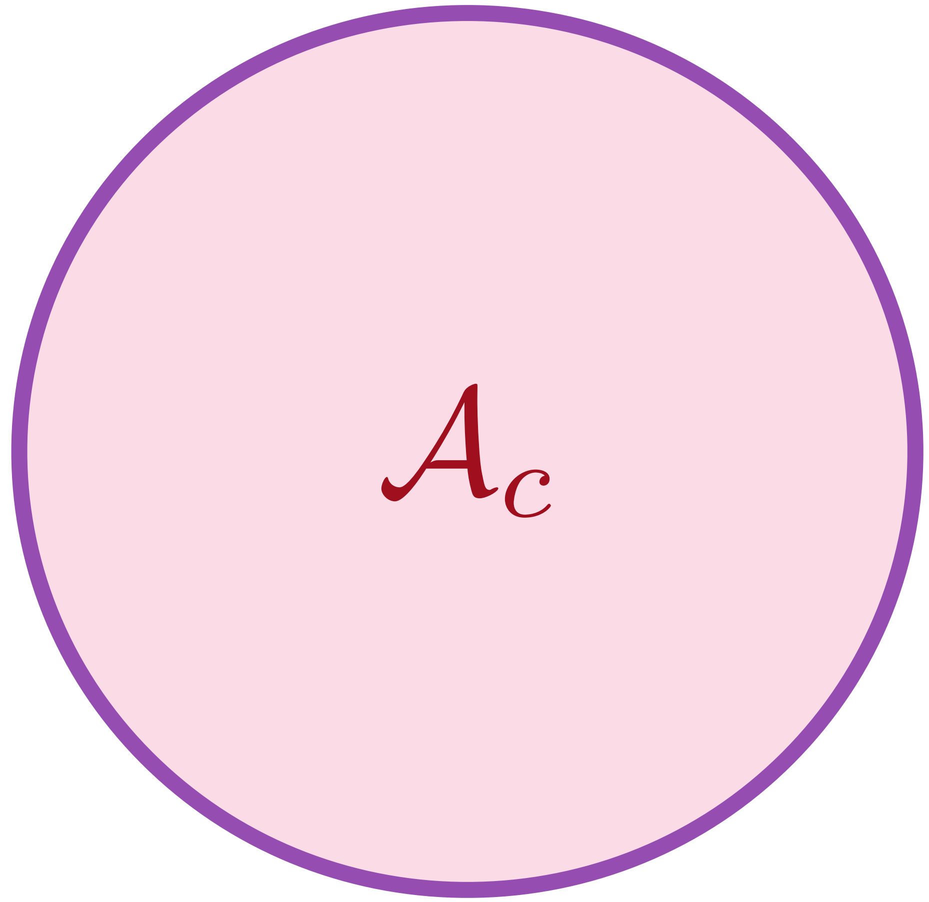
&\quad + \frac{u}{2} \left(b_\sigma^\dagger b_\sigma \right)^2 - g_1 \left(b_+^\dagger b_-^\dagger f_- f_+ + \text{H.c.} \right) \\
&+ c^\dagger \left[\partial_\tau - \frac{\nabla^2}{2m_c} + \epsilon_2 - 2\mu \right] c \\
&\quad - g_2 \left[c^\dagger \left(f_+ b_- + f_- b_+ \right) + \text{H.c.} \right]
\end{aligned}
$$

The index $\sigma = \pm 1$, and $\epsilon_{1,2}$ are tuning parameters of phase diagram

$$
\mathcal{Q} = f_\sigma^\dagger f_\sigma + b_\sigma^\dagger b_\sigma + 2c^\dagger c
$$

Phases of ABJM-like theories

$$\langle b_\pm \rangle = 0$$



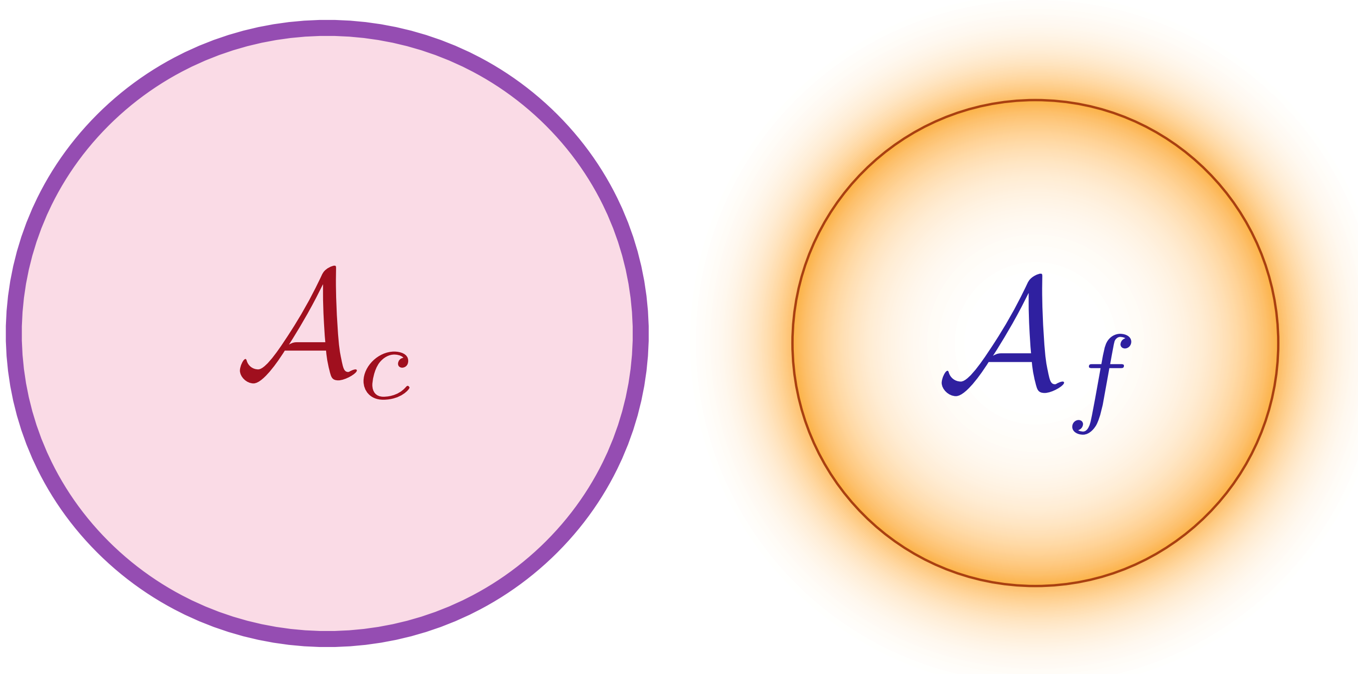
$$2\mathcal{A}_c = \langle \mathcal{Q} \rangle$$

Fermi liquid (FL) of gauge-neutral particles

U(1) gauge theory is in confining phase

Phases of ABJM-like theories

$$\langle b_\pm \rangle = 0$$



$$2\mathcal{A}_c + 2\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

Fractionalized Fermi liquid (FL*)

U(*1*) gauge theory is in deconfined phase

Phases of ABJM-like theories

$$\langle b_\pm \rangle \neq 0$$

$$\langle b_+ b_- \rangle \neq 0$$

No constraint on Fermi surface area, which can be zero

Superconductor

U(*1*) gauge theory is in Higgs phase and global U(1) is broken

Examples of compressible phases and their Fermi surfaces

1. The Fermi liquid (FL)

2. Fermions coupled to gauge fields

3. Fermion-boson mixtures

4. The fractionalized Fermi liquid (FL*)

5. Theories similar to ABJM

6. Theories similar to $\mathcal{N} = 4$ SYM

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$\mathcal{N} = 4$ SYM in D=3+1 dimensions

- SU(N) gauge invariance and SO(6) global symmetry
- Fermions carry adjoint gauge charges and are SO(6) spinors
- Bosons carry adjoint gauge charges and are SO(6) fundamentals. Bosons are paired fermions.
- $\mathcal{N} = 4$ supersymmetry

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Adding a chemical potential coupling to a SO(6) charge breaks supersymmetry and SO(6) invariance

Theory similar to $\mathcal{N} = 4$ SYM in a chemical potential

- SU(N) gauge invariance and U(1) global symmetry

- Fermions, f_α, ($\alpha = 1 \ldots N^2 - 1$) carry adjoint gauge charges and U(1) charge 1.

- Bosons, b_α, carry adjoint gauge charges and U(1) charge 2. Bosons are paired f_α fermions.

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- No supersymmetry
- Fermions, c, (analog of baryons), gauge-invariant bound states of b and f, carry U(1) charge 3.

$$\mathcal{Q} = f_\alpha^\dagger f_\alpha + 2 b_\alpha^\dagger b_\alpha + 3 c^\dagger c$$

Theory similar to $\mathcal{N} = 4$ SYM in a chemical potential

$$H_f = \sum_{k,a} \frac{k^2}{2m_3} f_\alpha^\dagger f_\alpha - \mu \sum_k \left(\sum_\alpha f_\alpha^\dagger f_\alpha + 2 \sum_\alpha b_\alpha^\dagger b_\alpha + 3 c^\dagger c \right)$$

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Theory similar to $\mathcal{N} = 4$ SYM in a chemical potential

$$
\begin{aligned}
H_f &= \sum_{k,a} \frac{k^2}{2m_3} f^\dagger_\alpha f_\alpha - \mu \sum_k \left(\sum_\alpha f^\dagger_\alpha f_\alpha + 2 \sum_\alpha b^\dagger_\alpha b_\alpha + 3 c^\dagger c \right) \\
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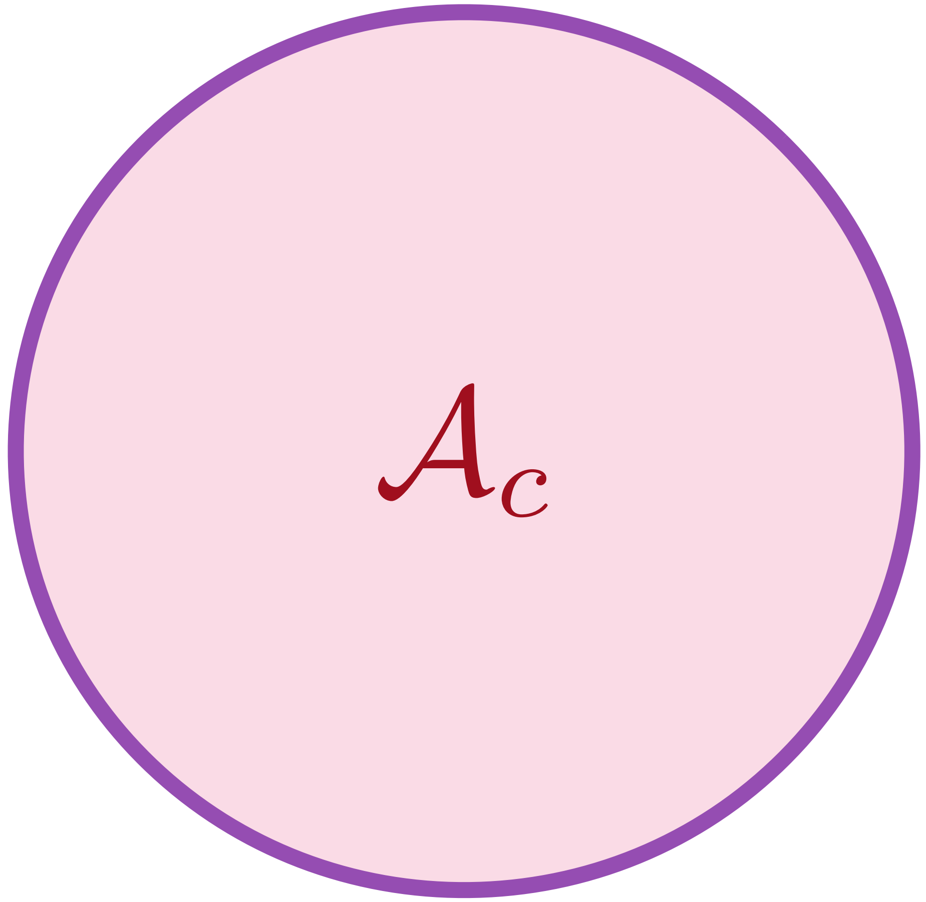
$$H_{\rm int} = g \int d^d x \left(\epsilon_{\alpha\beta\gamma} b^\dagger_\alpha f_\beta f_\gamma + \text{c.c.} \right) + \lambda \int d^d x \left(c^\dagger b_\alpha f_\alpha + \text{c.c.} \right),$$

The indices, $\alpha, \beta, \gamma = 1 \ldots N^2 - 1$, the structure constants of SU((N)) are $\epsilon_{\alpha\beta\gamma}$, and $\varepsilon_{1,2}$ are parameters tuning between possible phases. The SU(N) gauge fields are not shown, and are included as usual by covariantizing derivatives.

$$\mathcal{Q} = f^\dagger_\alpha f_\alpha + 2 b^\dagger_\alpha b_\alpha + 3 c^\dagger c$$

Phases of SYM-like theories

$$\langle b_\alpha \rangle = 0$$



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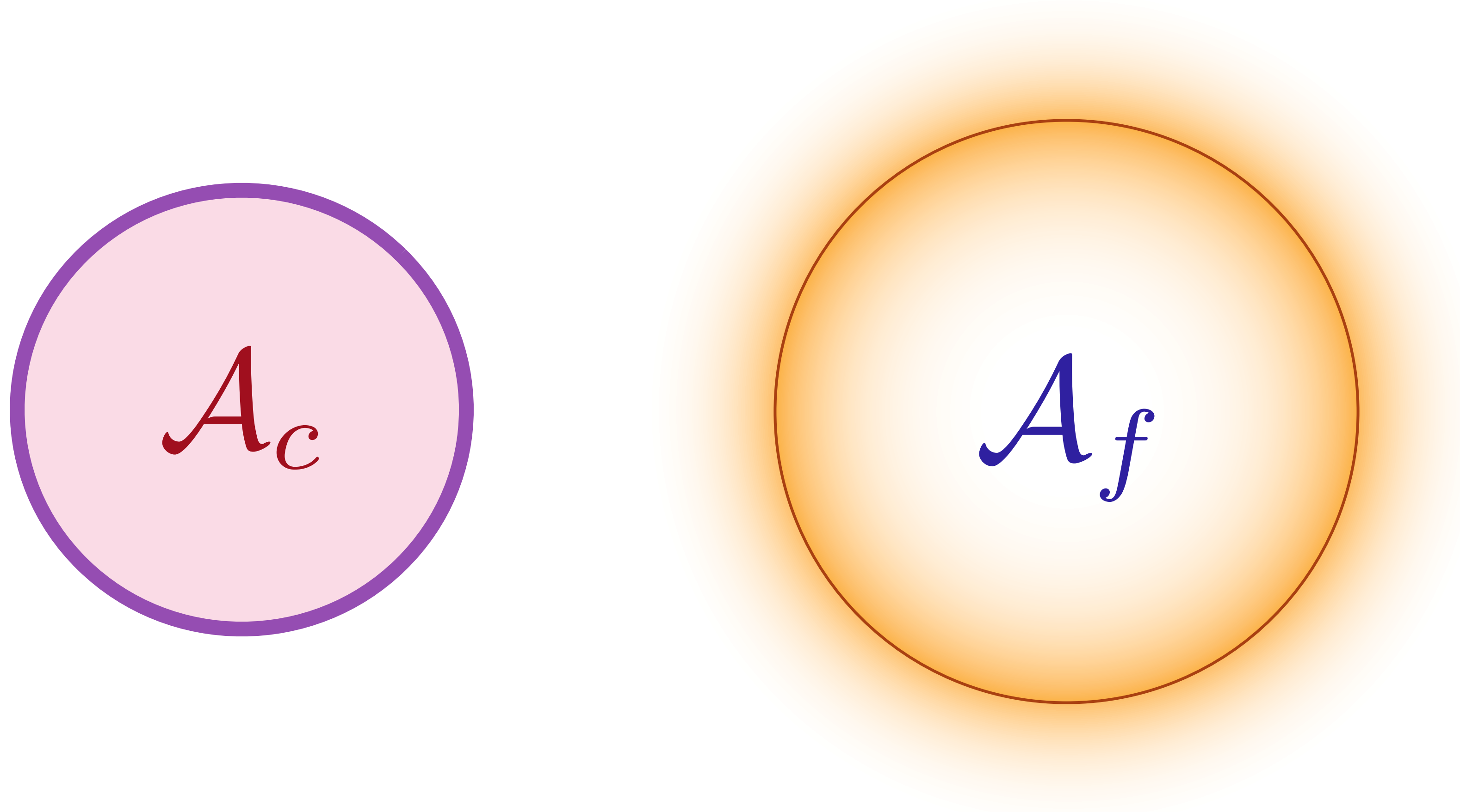

$$3\mathcal{A}_c + (N^2 - 1)\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

Fractionalized Fermi liquid (FL*)

SU(*N*) gauge theory is in deconfined phase

Phases of SYM-like theories

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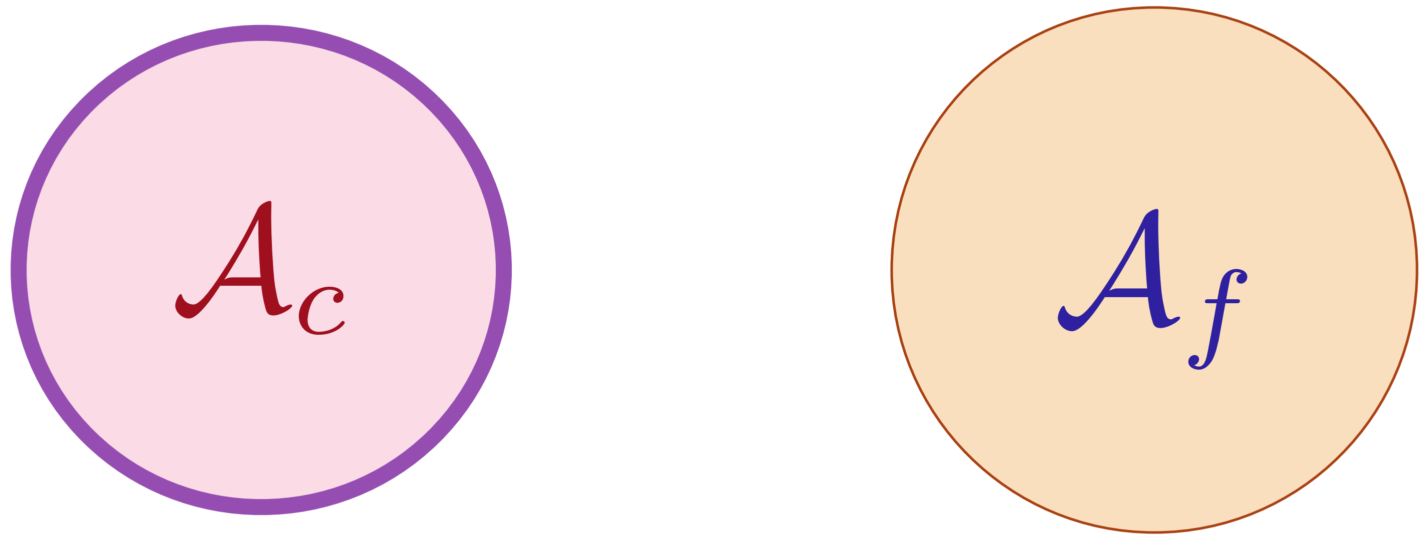

No constraint on Fermi surface areas

Color Superconductor

SU(*N*) gauge theory is in Higgs phase

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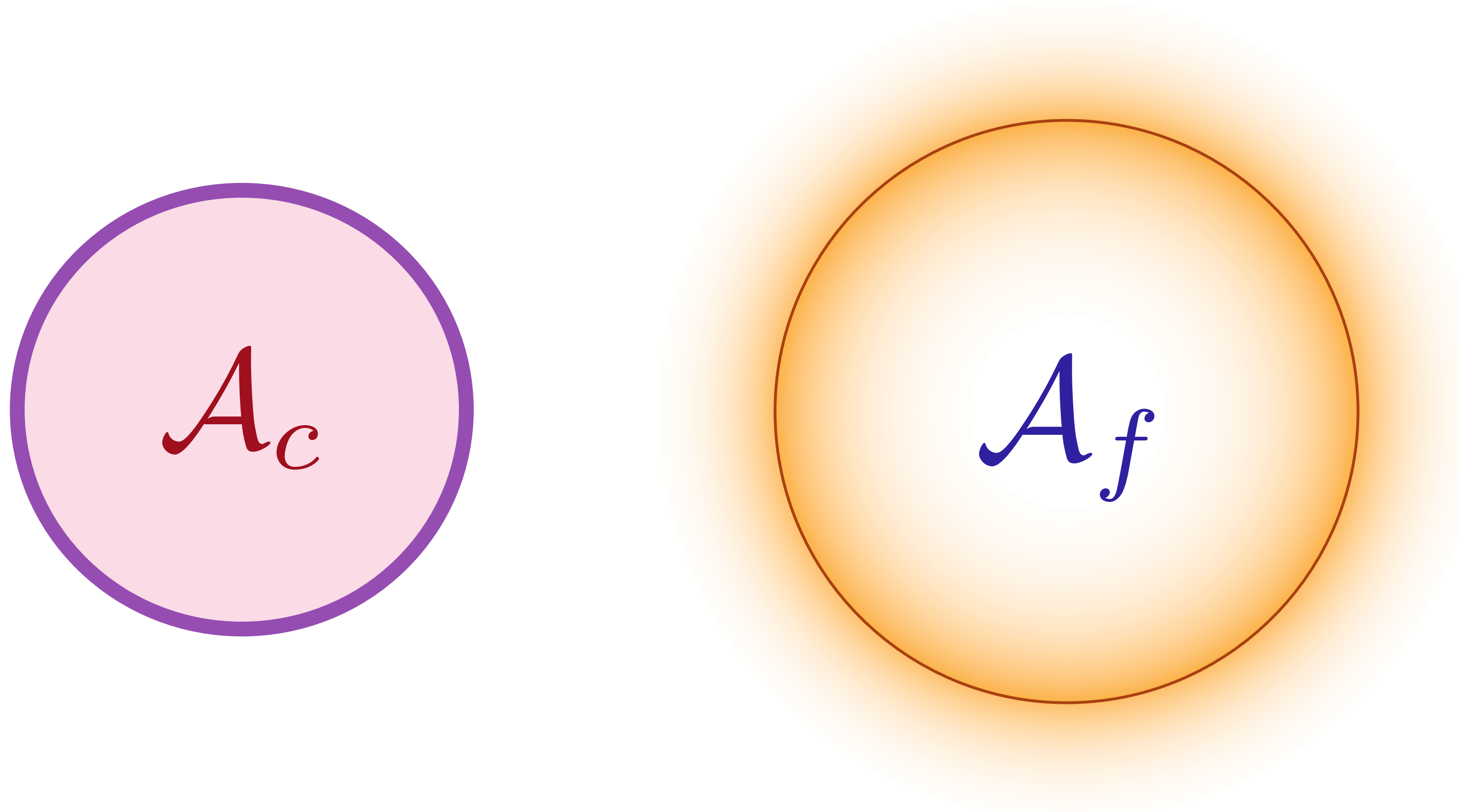

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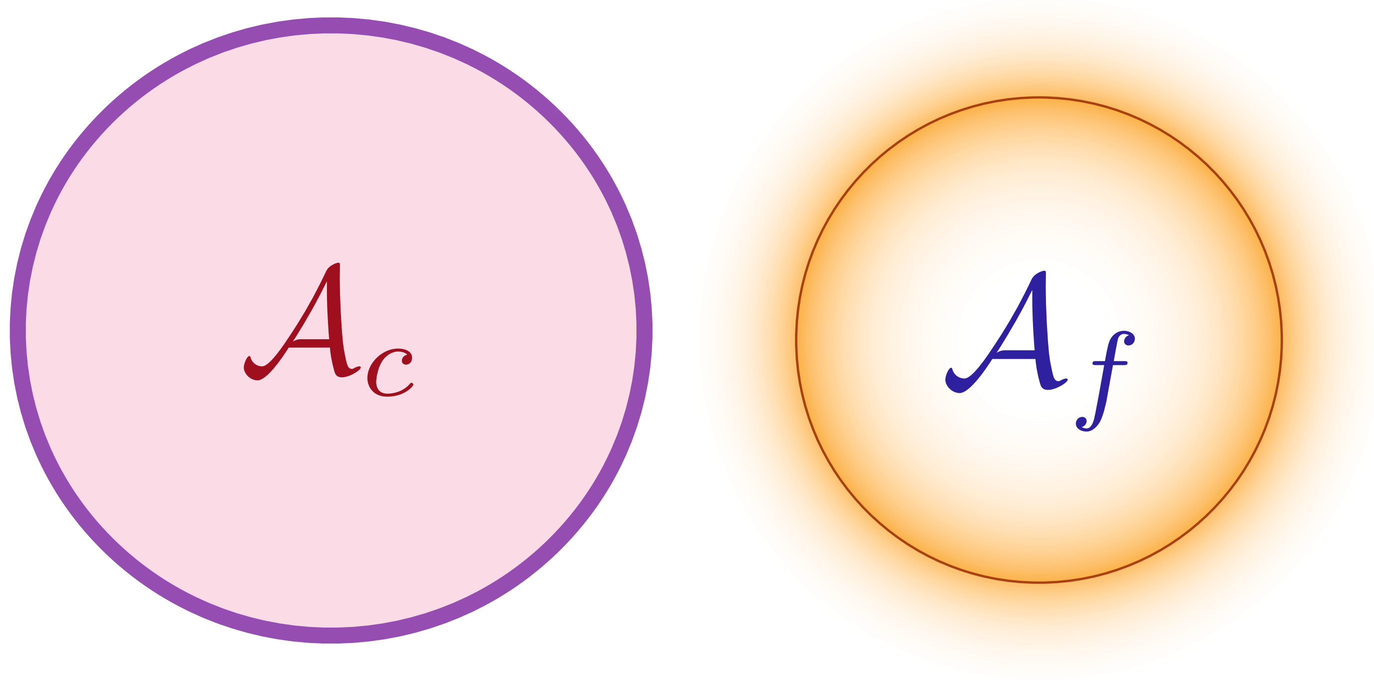

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Phases of ABJM-like theories

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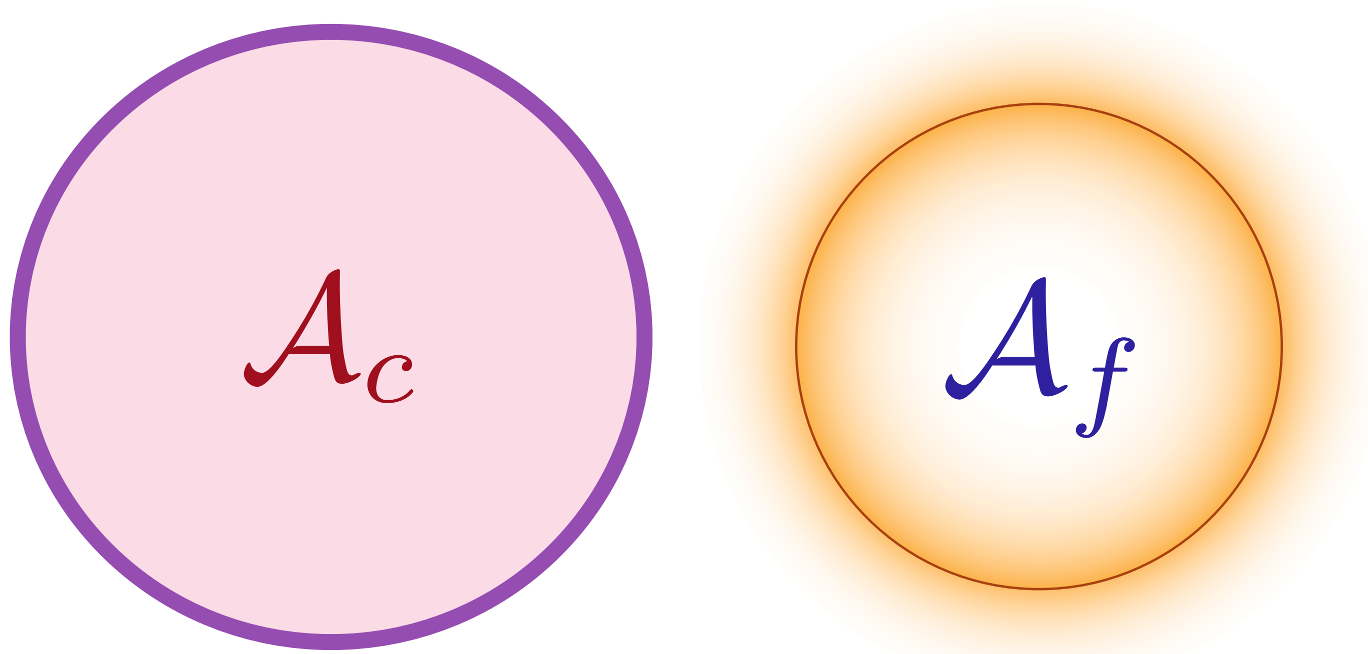

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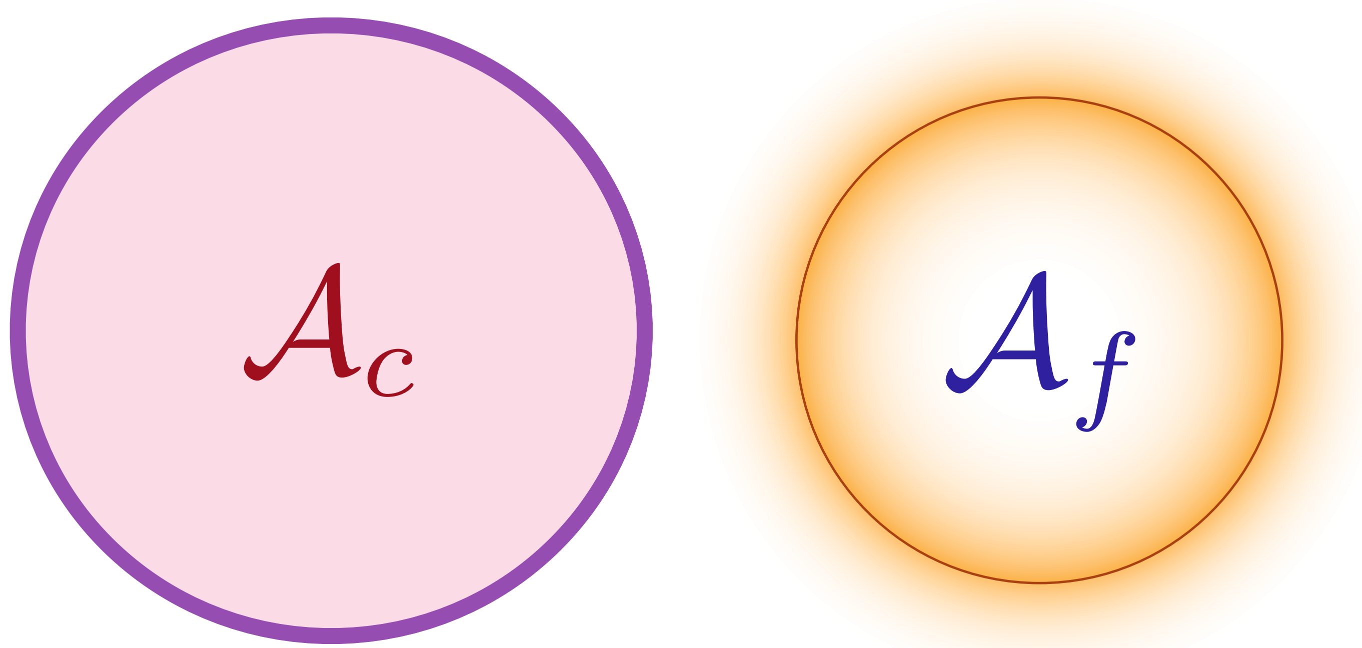

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Fractionalized Fermi liquid (FL*)

U(*1*) gauge theory is in deconfined phase

Claim: this is the phase underlying recent holographic theories of compressible metallic states.
However, a number of artifacts appear in the classical gravity approximation.

Gauge-gravity duality
and
impurity mean-field theories

Gauge-gravity duality

- Begin with a CFT e.g. the SYM theory with a SO(6) global symmetry

- The CFT is dual to a gravity theory on $AdS_5 \times S^5$

Gauge-gravity duality

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- In the Einstein-Maxwell theory, the chemical potential leads at T=0 to an extremal Reissner-Nordtrom black hole in the AdS_5 spacetime.
- The near-horizon geometry of the RN black hole is $AdS_2 \times R^3$. This factorization leads to finite ground state entropy density

AdS theory of finite density quantum matter

Features of AdS Einstein-Maxwell theory of non-zero density quantum matter, not expected in the final theory:

- Non-zero ground state entropy density.

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Features of AdS Einstein-Maxwell theory of non-zero density quantum matter, not expected in the final theory:

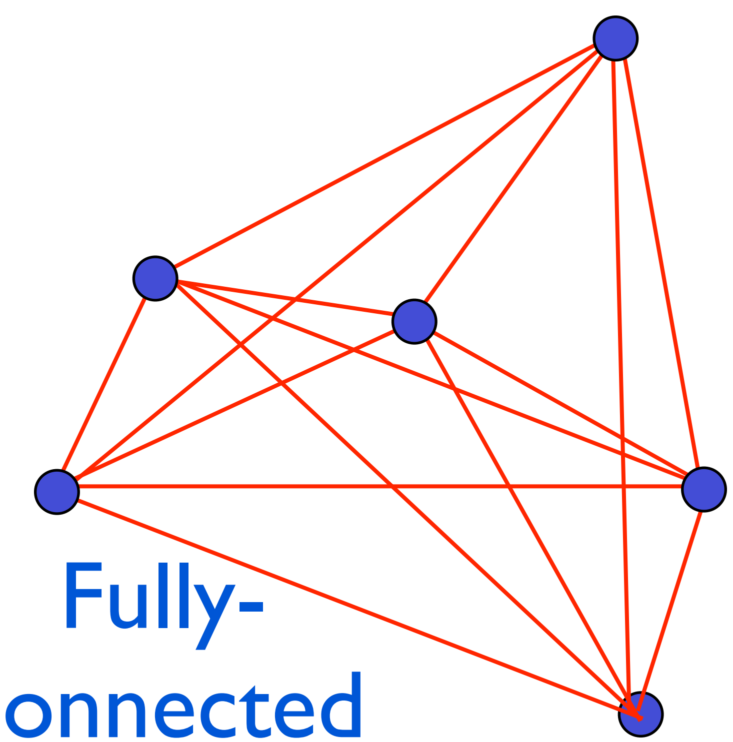
- Non-zero ground state entropy density.
- Single fermion self energies are momentum independent, and their singular behavior is the same on and off the Fermi surface.
- Low energy singularities are described by "conformal quantum mechanics": a 0+1 dimensional defect in a d+1 dimensional CFT. This is linked to the factorization of the near-horizon metric to $\text{AdS}_2 \times R^d$,

Solution of lattice models

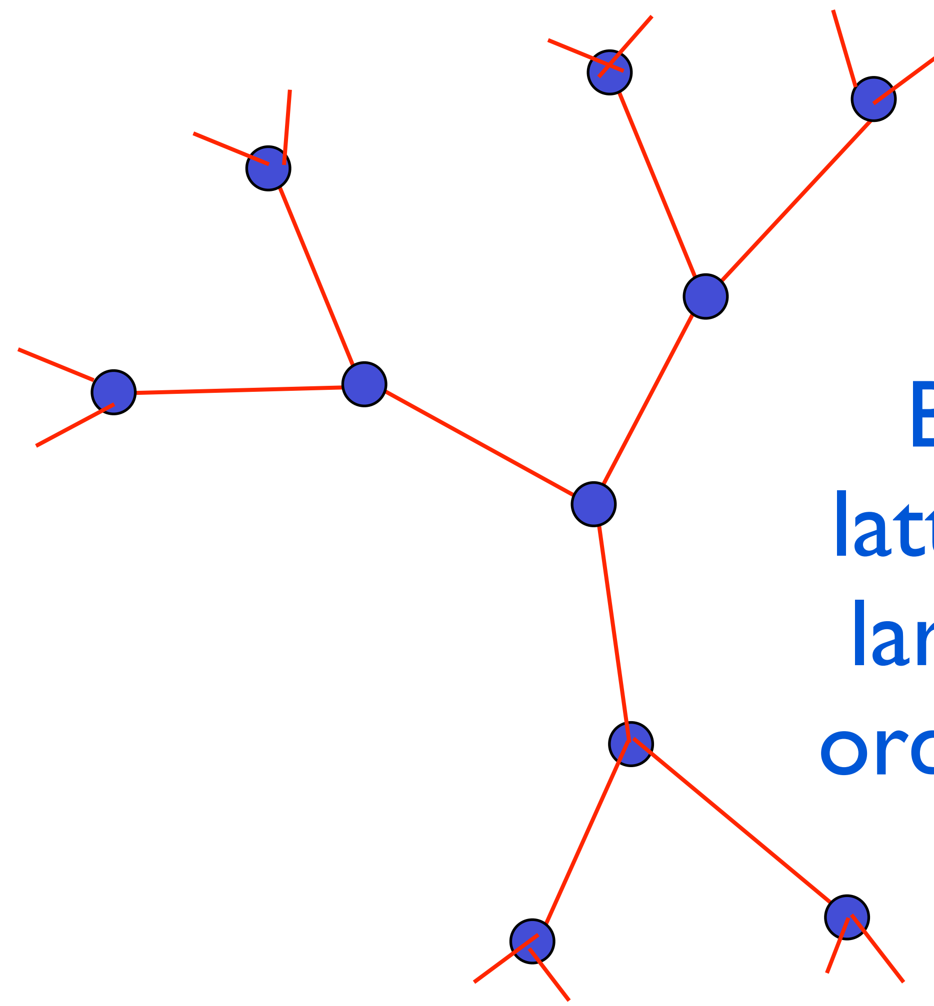
Place U(1) gauge theory theory on a lattice, integrate out b and A_a, to obtain Kondo lattice Hamiltonian

$$H = -\sum_{i<j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + \sum_{i<j} J_H(i,j) \vec{S}_i \cdot \vec{S}_j + J_K \sum_i \vec{S}_i \cdot c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

where $\vec{S}_i = f^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} f_\beta$



Fully-connected cluster



Bethe lattice for large co-ordination

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993).
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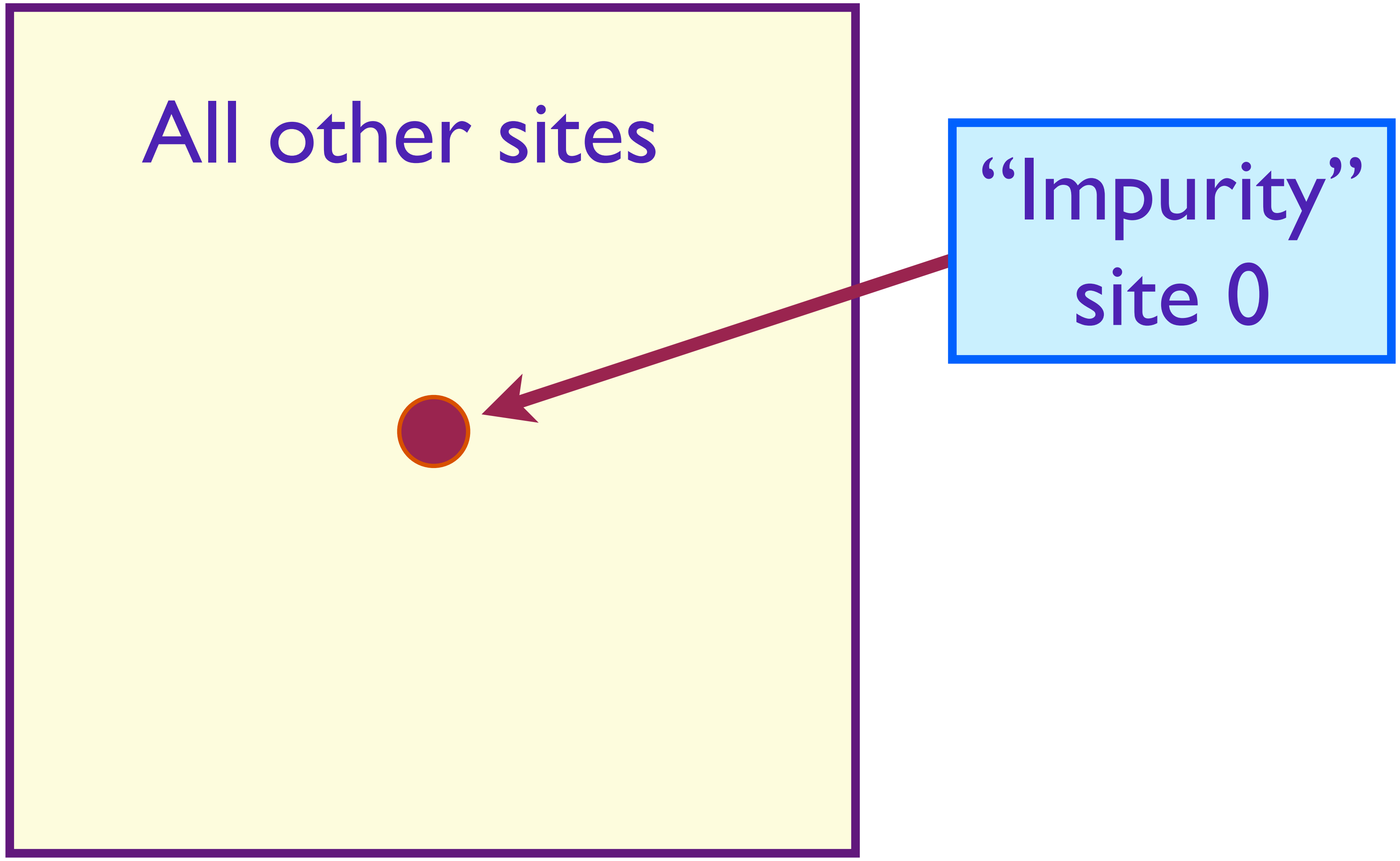
Solution of lattice models



$$\mathcal{L} = \mathcal{L}_{\rm imp}[c_0, f_0] + c_0^\dagger F_{\rm bulk} + F_{\rm bulk}^\dagger c_0 + \mathcal{L}_{\rm bulk}$$

Has to be combined with a *self-consistency condition* between correlators on the impurity and the bulk.

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Obtain both FL and FL* phases;

properties of the FL* phase:

- The ground state has a non-zero entropy density

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> These features, and the resulting fermion correlator and transport properties, co-incide with those obtained (for general Δ) using the holographic $AdS_2 \times R^d$ theory defined on the extremal horizon of the Reissner-Nordstrom black hole (T. Faulkner, H. Liu, J. McGreevy and D. Vegh, arXiv:0907.2694)
>
> S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010).

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Conclusions

- Compressible quantum matter is characterized by Fermi surfaces.

- Fermi surfaces can be removed from the Luttinger count if the fermions acquire gauge charges

- Phases of a strongly-coupled gauge theory: Fermi liquids (FL) and fractionalized Fermi liquids (FL*)

Conclusions

- Mean field Kondo lattice models capture the physics of holographic metals with a $AdS_2 \times R^d$ geometry

- Needed: Holographic theory for FL* or related compressible phases, without a factorized geometry. Challenge: detect Fermi surfaces of fermions with gauge charges