Quantum criticality of metals and high temperature superconductivity

50th Karpacz Winter School of Theoretical Physics
Quantum Criticality in Condensed Matter:
Phenomena, Materials and Ideas in Theory and Experiment
March 3, 2014

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

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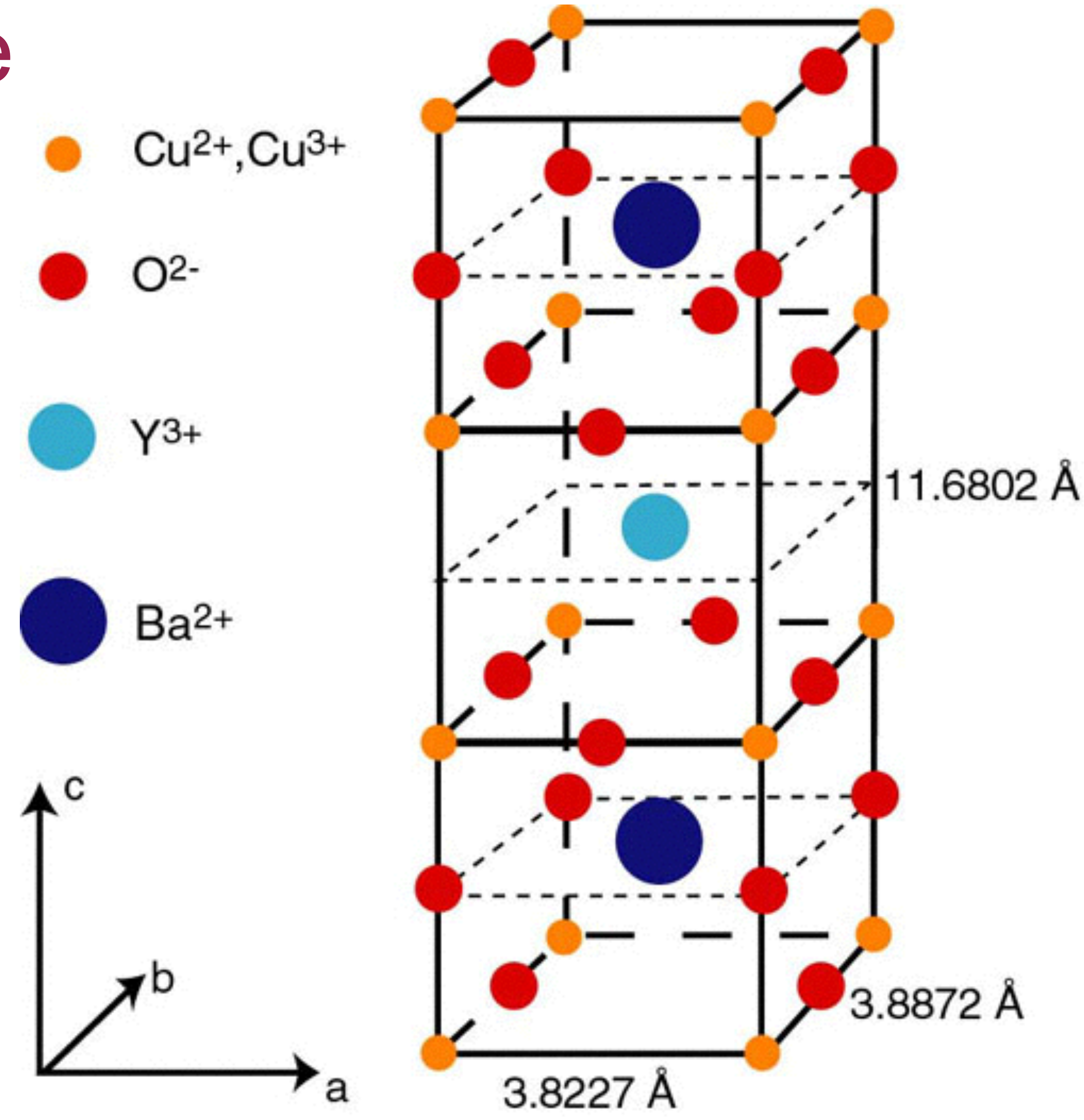
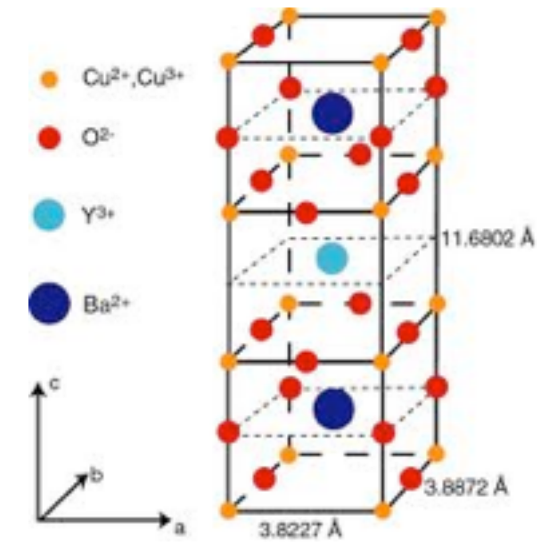
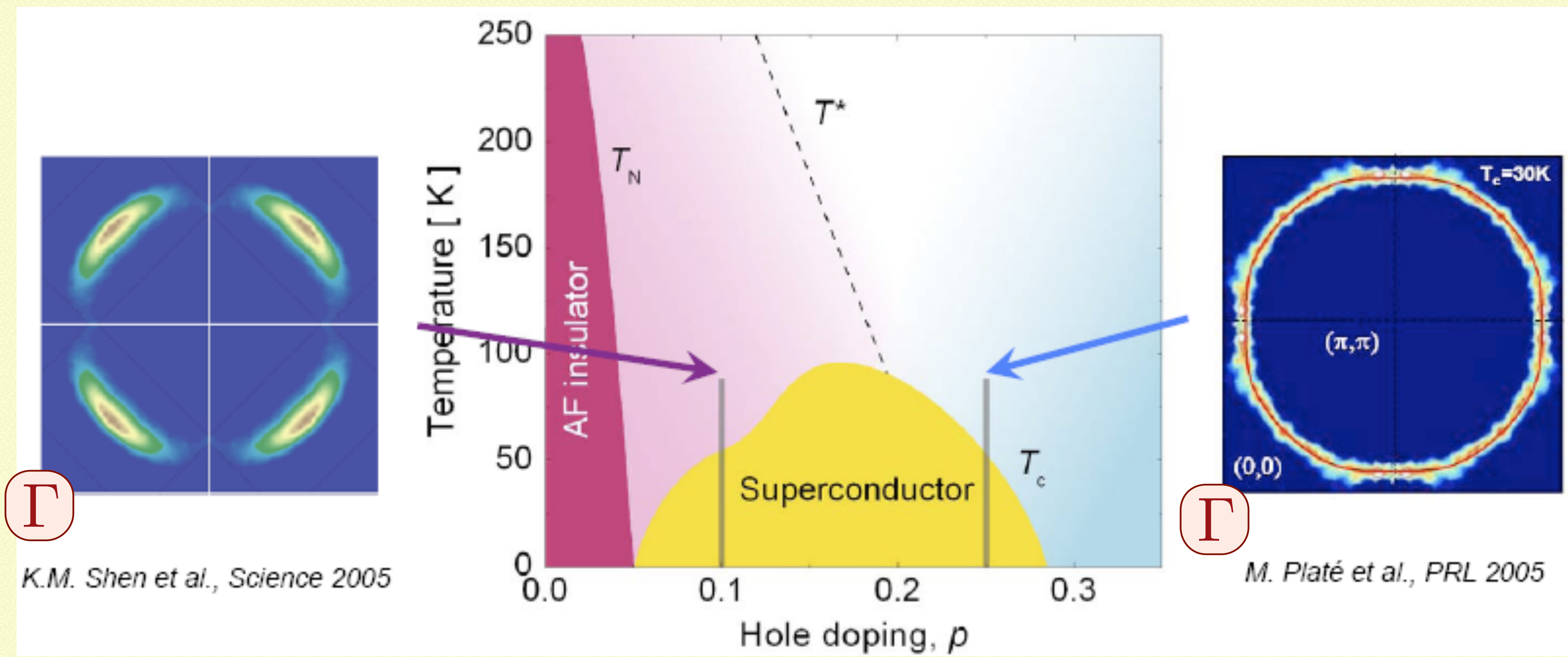
High temperature superconductors



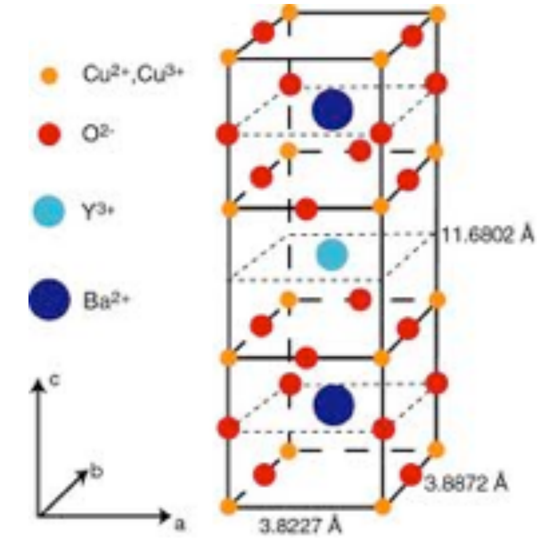
$$YBa_2Cu_3O_{6+x}$$

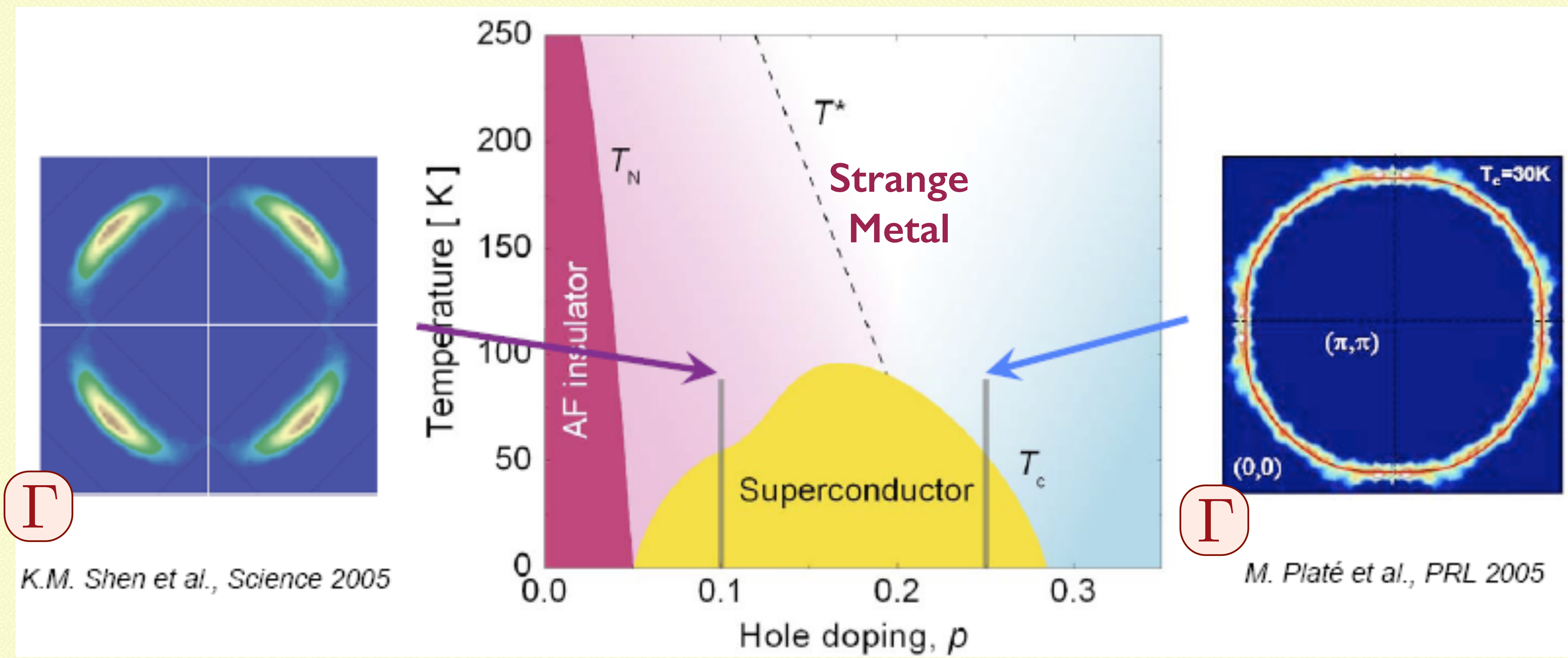




Smaller hole
Fermi-pockets

Large hole
Fermi surface





Smaller hole Fermi-pockets

Large hole Fermi surface

Direct observation of competition between superconductivity and charge density wave order in $YBa_2Cu_3O_{6.67}$

NATURE PHYSICS | VOL 8 | DECEMBER 2012 |

J. Chang[1,2]★, E. Blackburn[3], A. T. Holmes[3], N. B. Christensen[4], J. Larsen[4,5], J. Mesot[1,2], Ruixing Liang[6,7], D. A. Bonn[6,7], W. N. Hardy[6,7], A. Watenphul[8], M. v. Zimmermann[8], E. M. Forgan[3] and S. M. Hayden[9]

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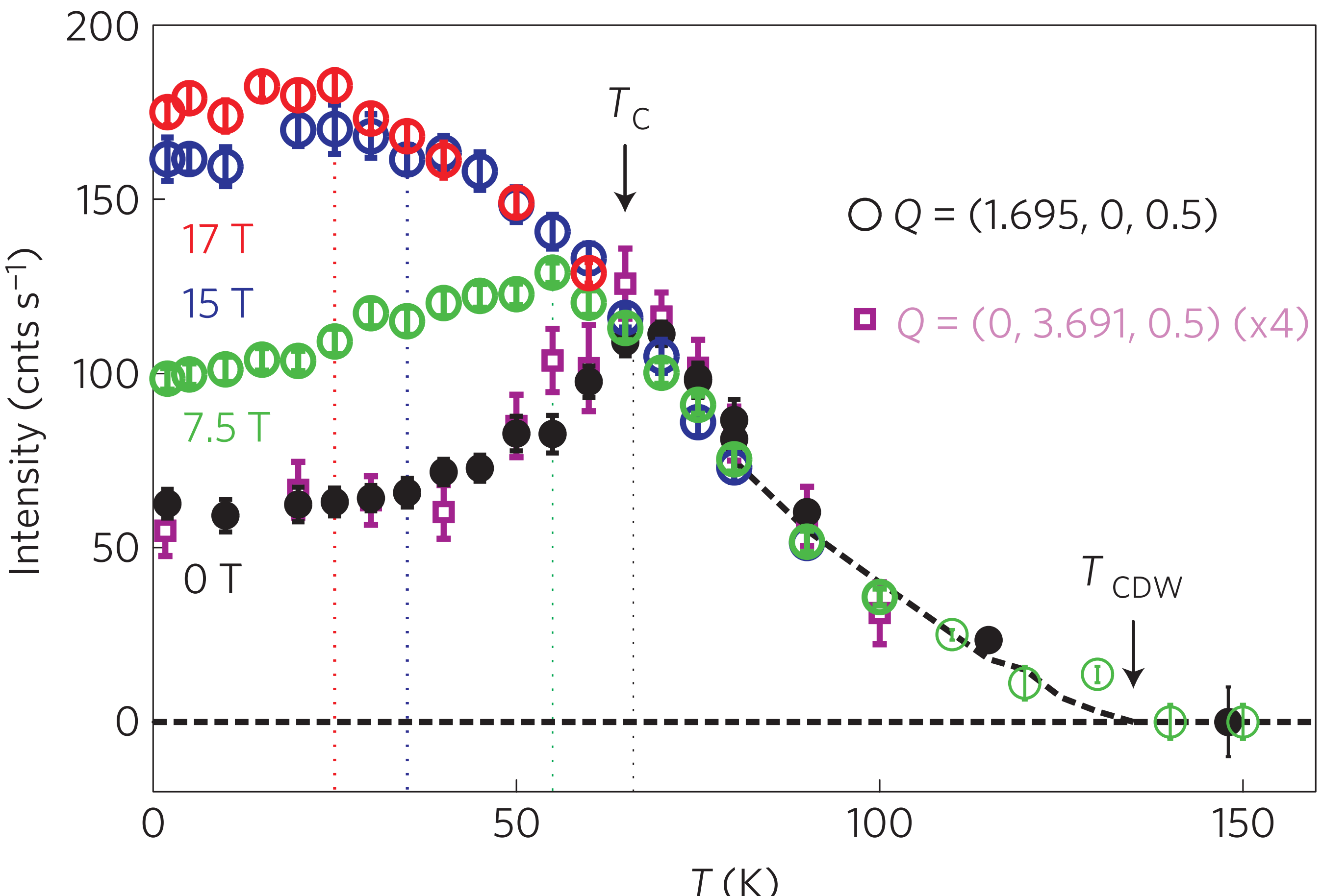

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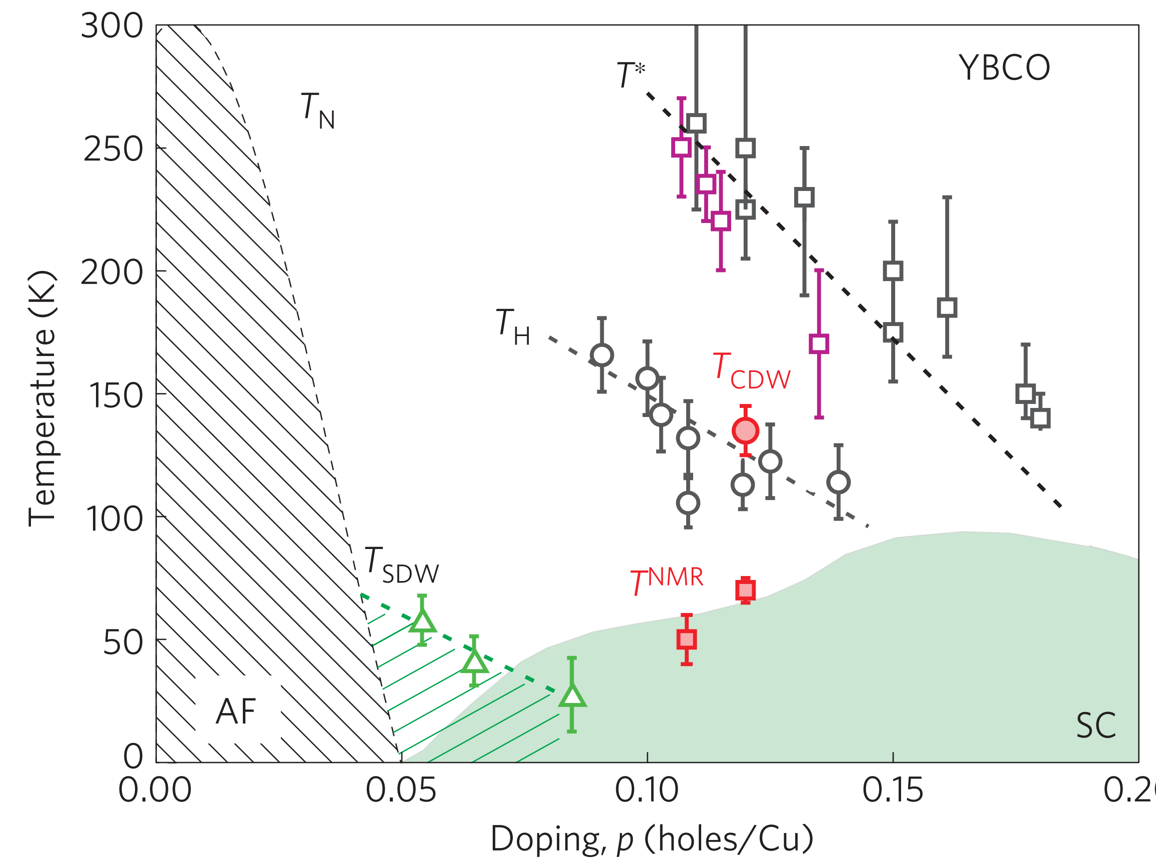

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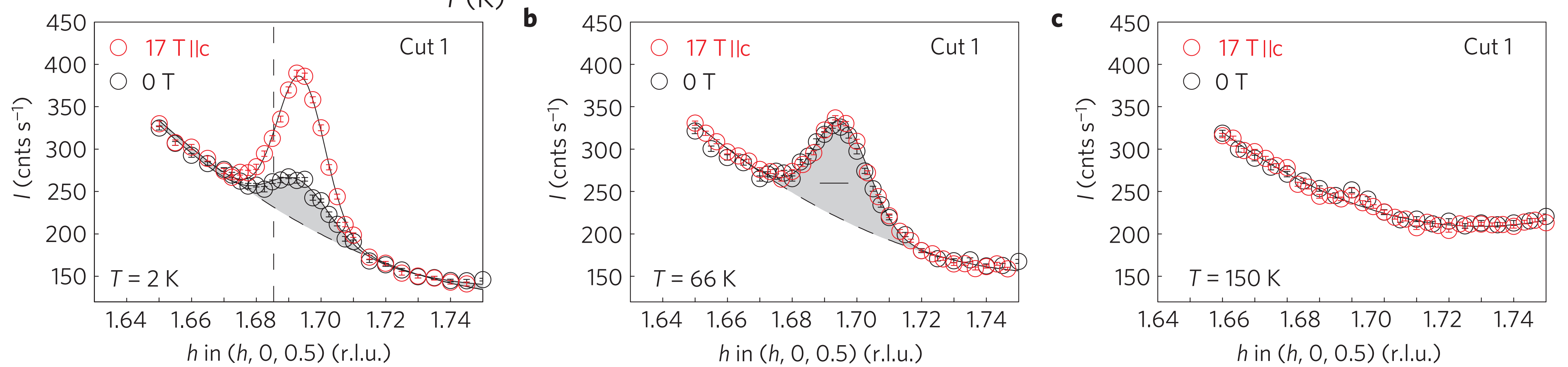
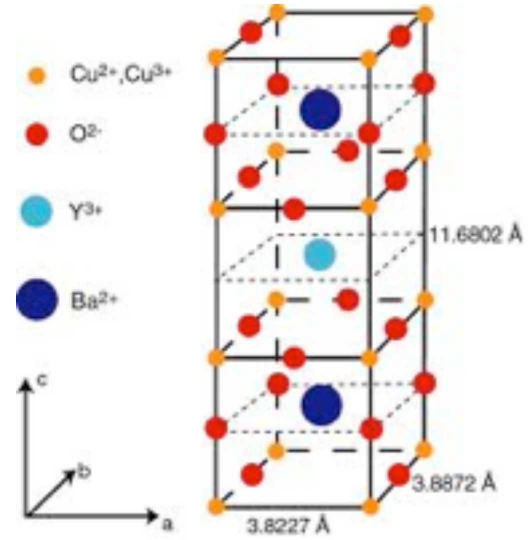
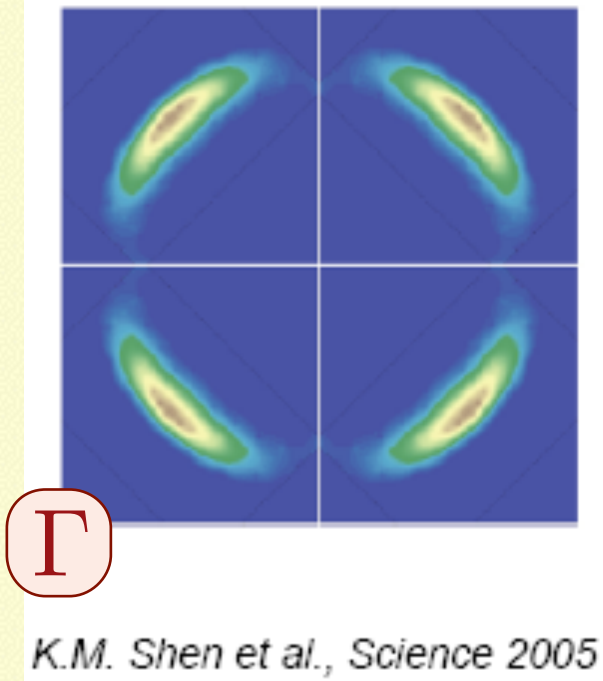
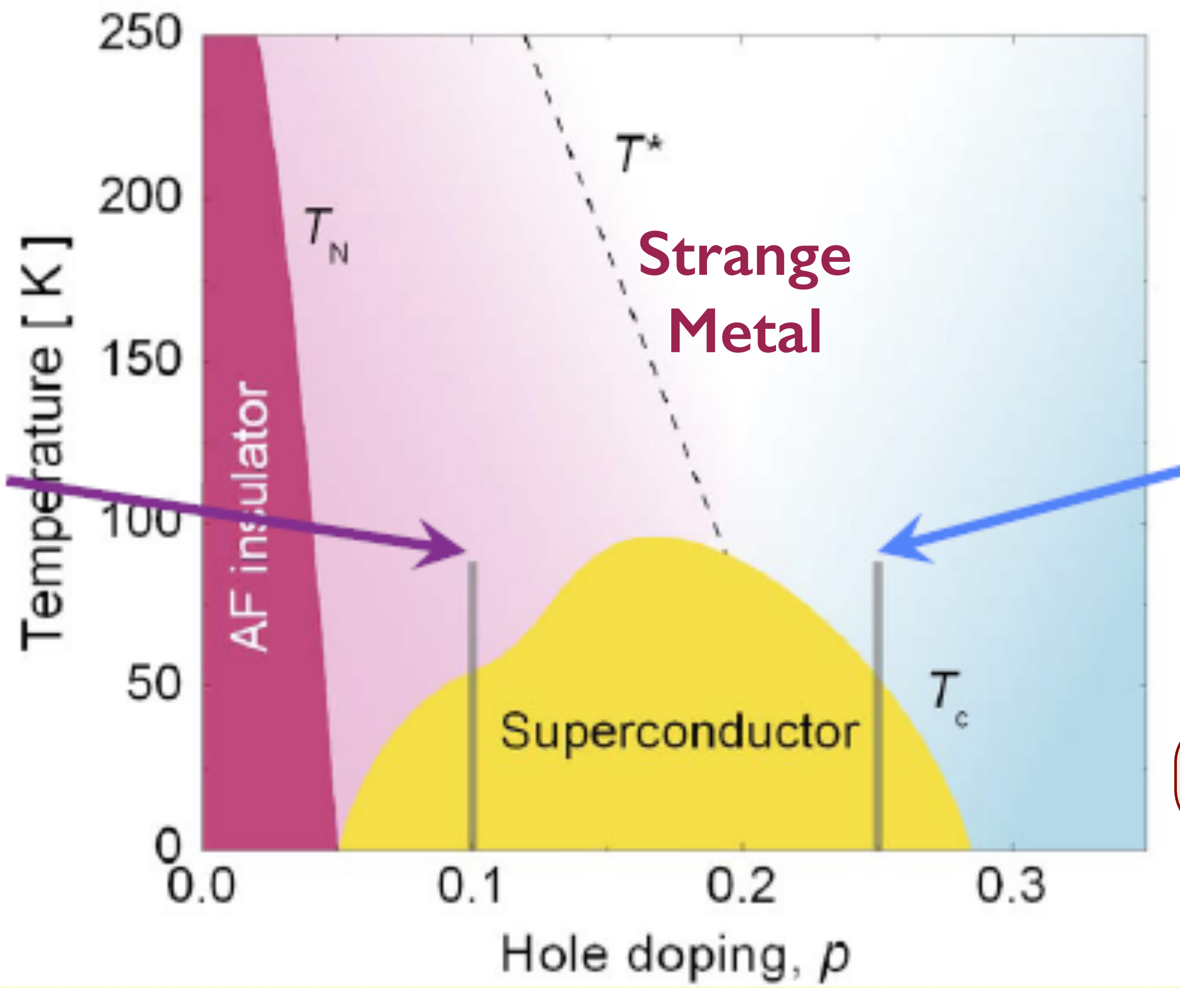
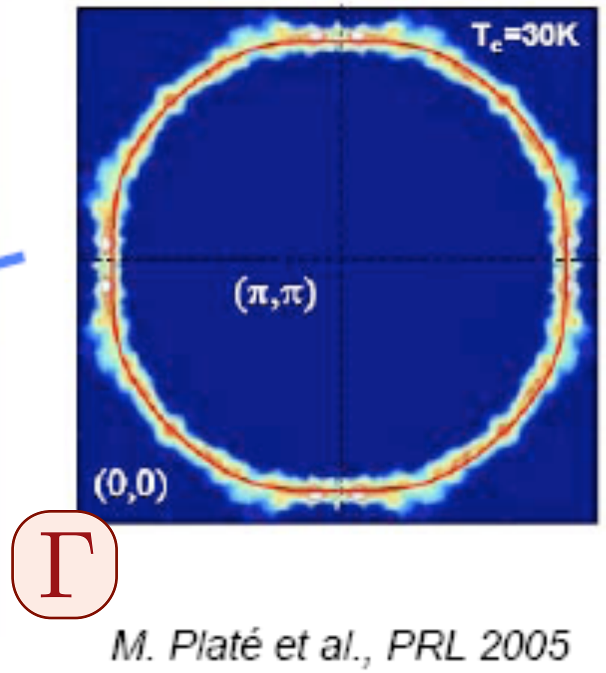










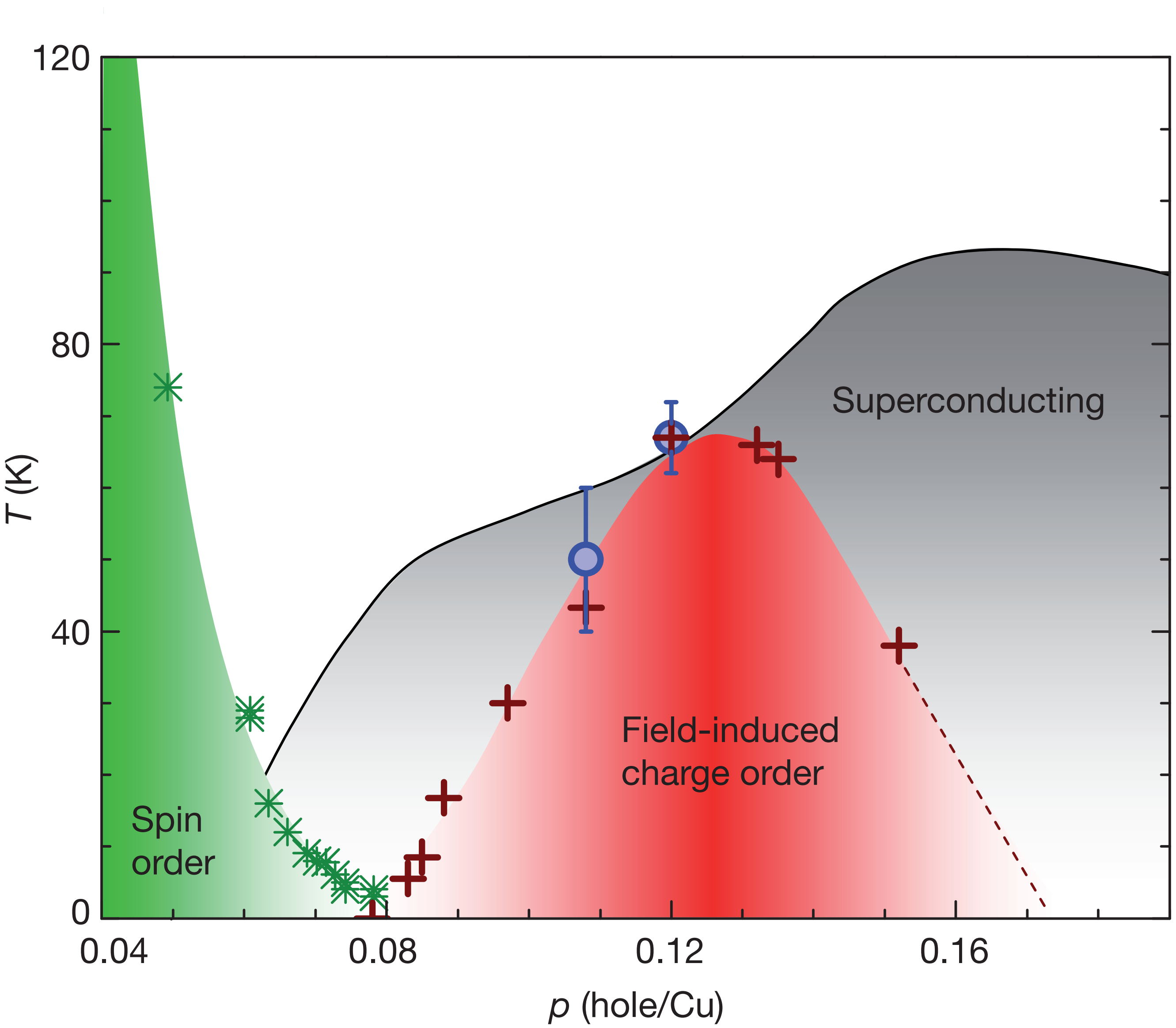
Smaller hole Fermi-pockets

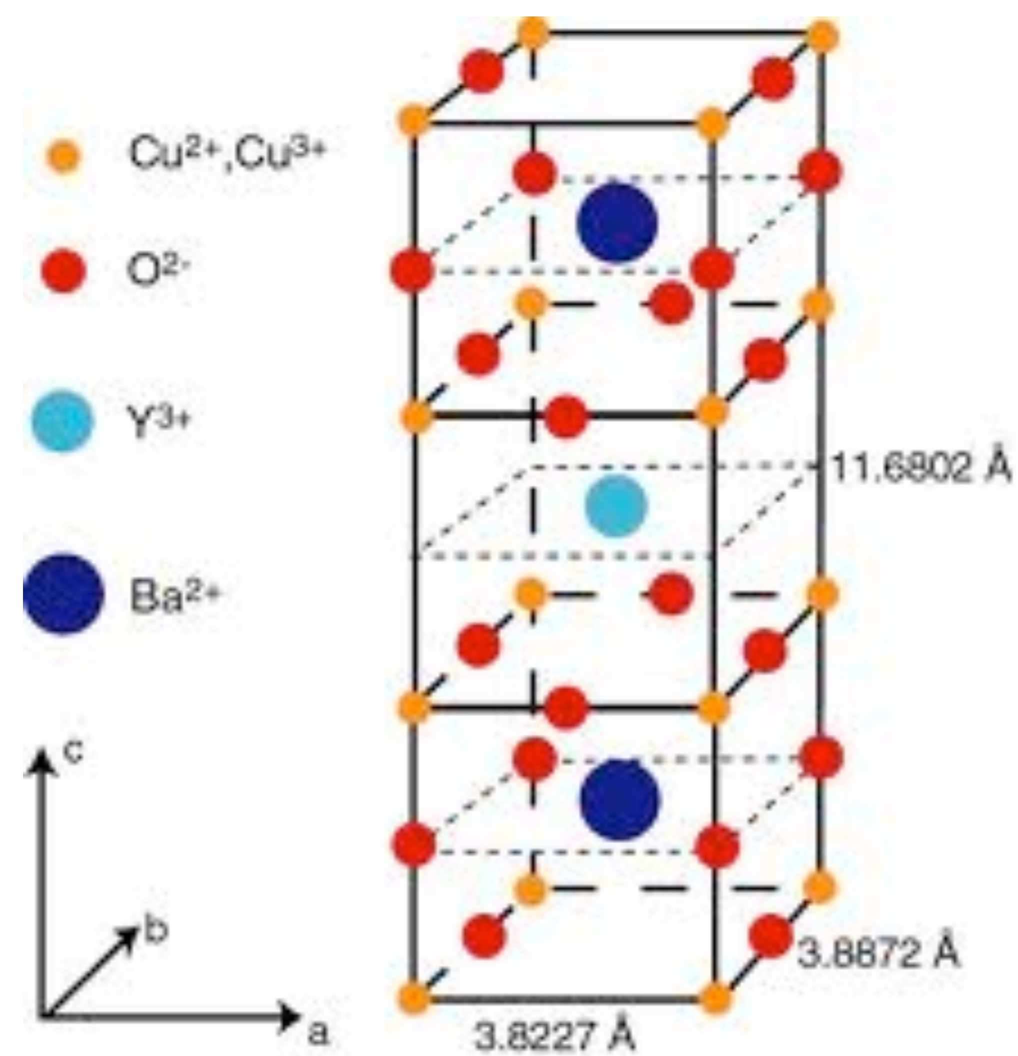
Large hole Fermi surface

Magnetic-field-induced charge-stripe order in the high-temperature superconductor $YBa_2Cu_3O_y$

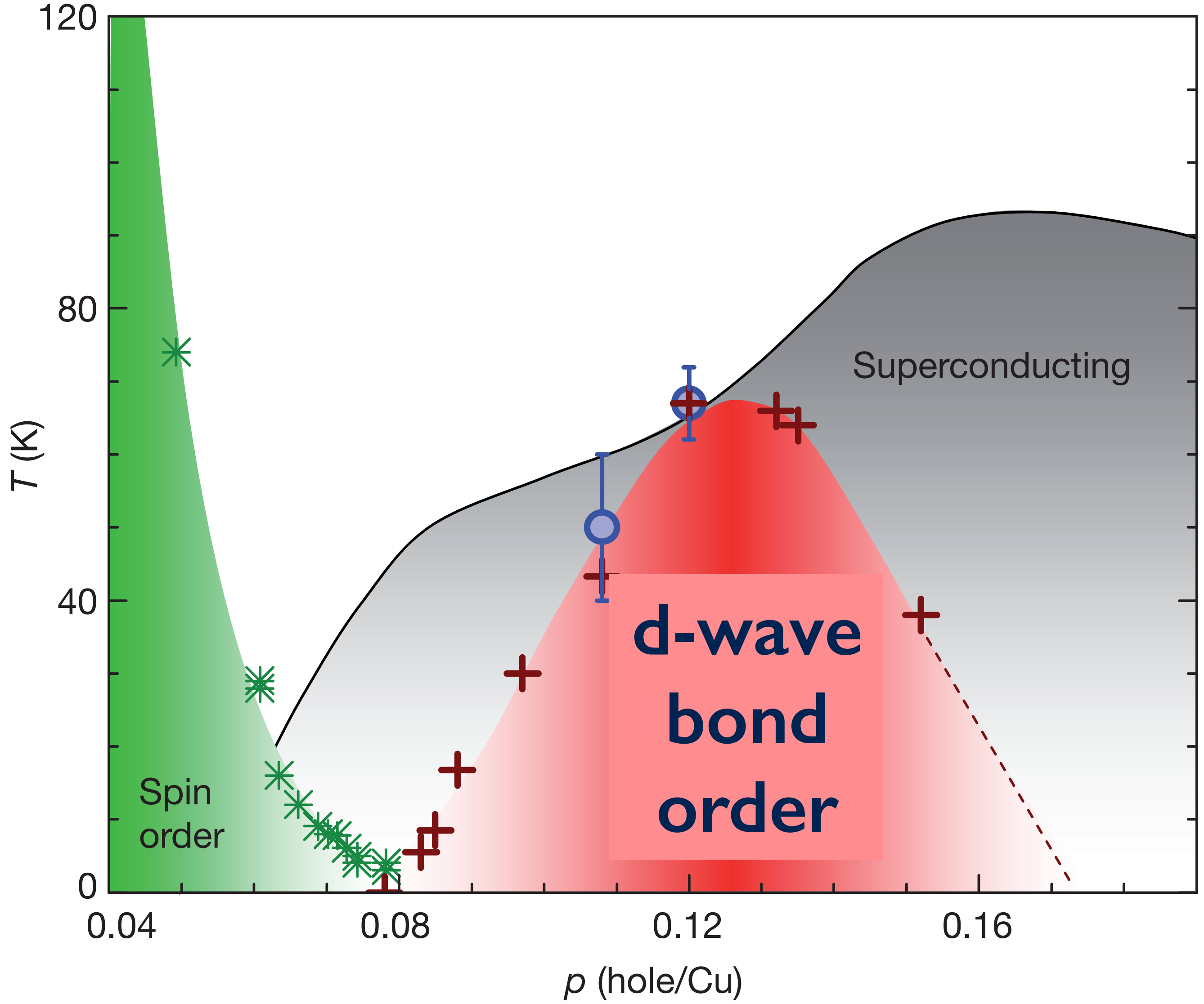
Tao Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Mladen Horvatić[1], Claude Berthier[1], W. N. Hardy[2,3], Ruixing Liang[2,3], D. A. Bonn[2,3] & Marc-Henri Julien[1]

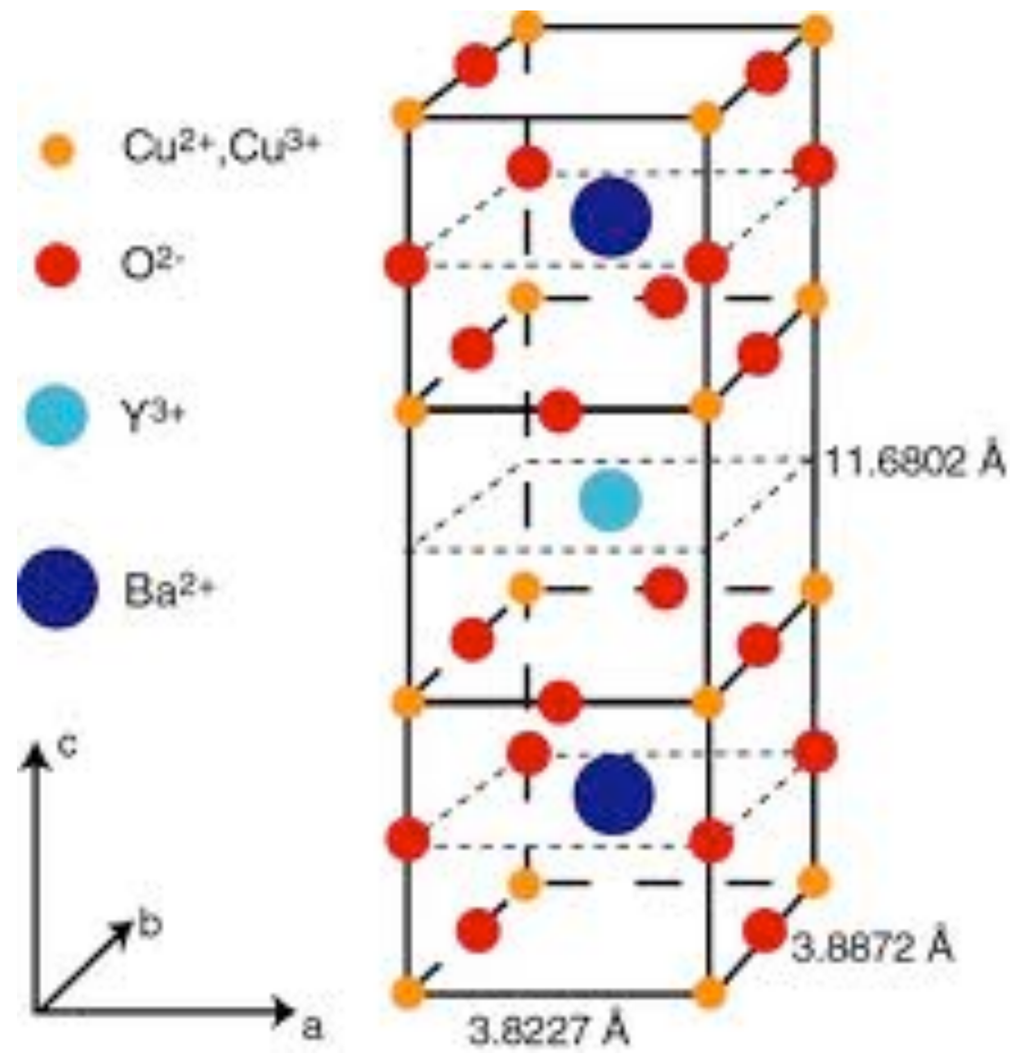
8 SEPTEMBER 2011 | VOL 477 | NATURE | 191

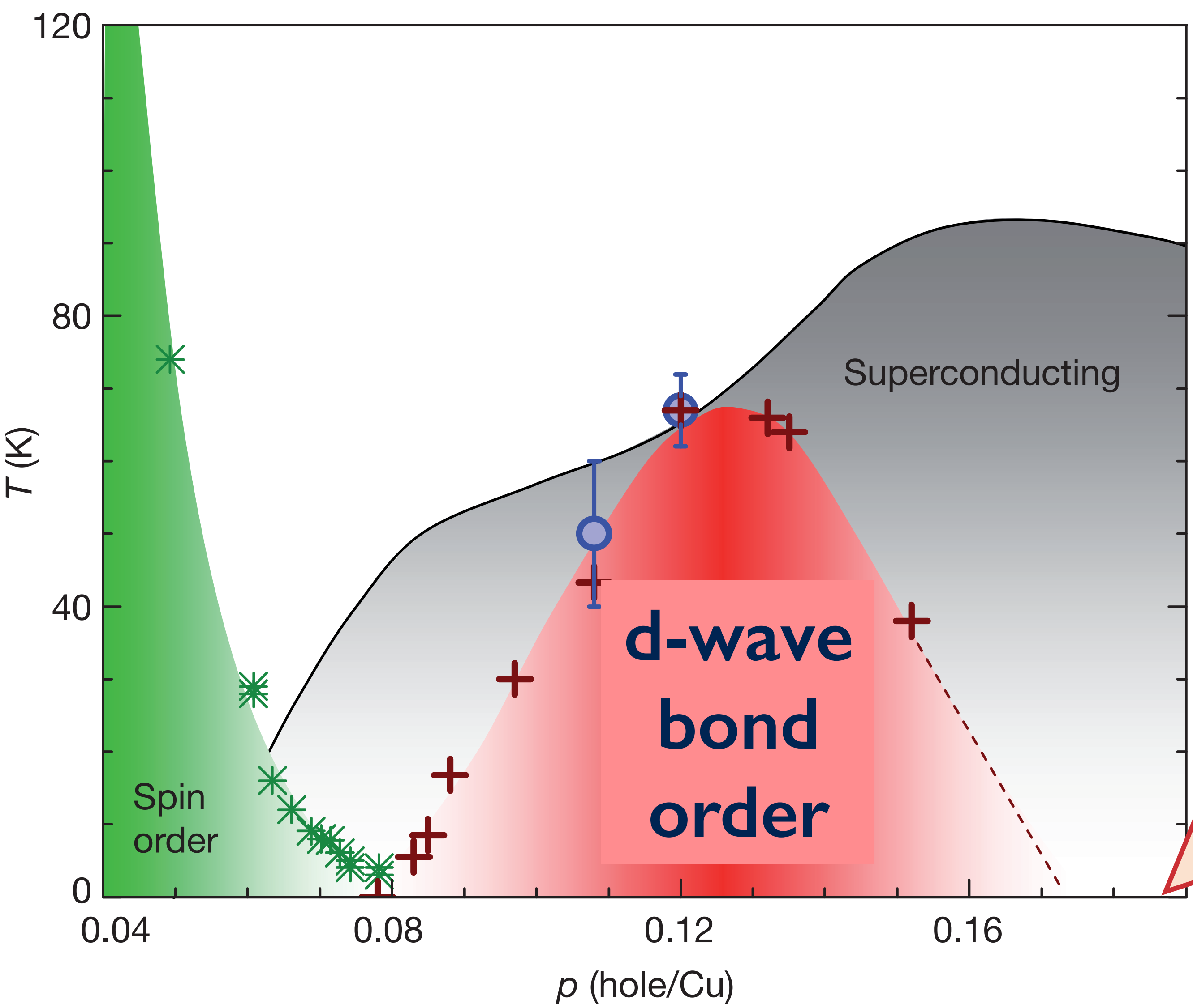


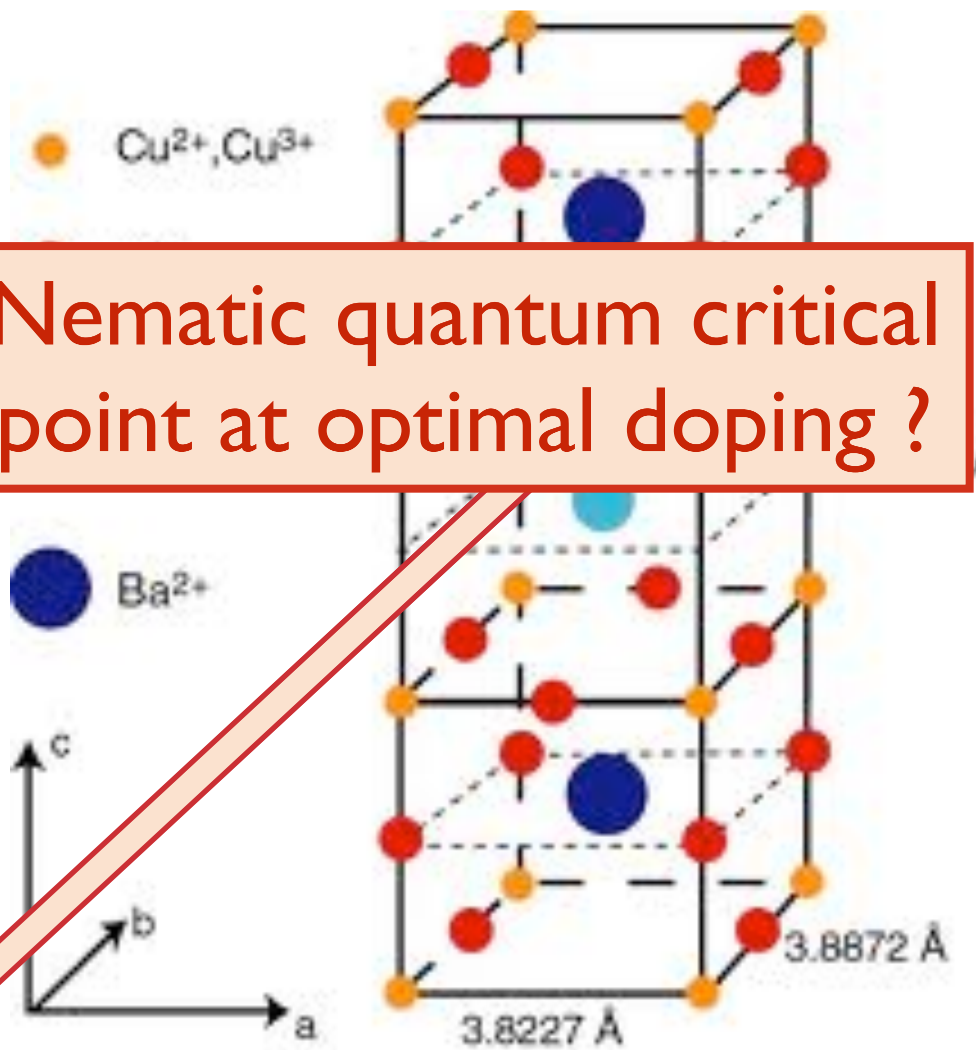


M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)
M. Vojta and O. Rosch, Physical Review B **77**, 094504 (2008)
M. Metlitski and S. Sachdev, Physical Review B **82**, 075128 (2010)
S. Sachdev and R. La Placa, Physical Review Letters **111**, 027202 (2013)
J. D. Sau and S. Sachdev, Physical Review B **89**, 075129 (2014)
A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.4807
A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.6311









Nematic quantum critical point at optimal doping ?

Outline

1. Antiferromagnetism in metals and *d*-wave superconductivity

2. Competing order: *d*-wave bond order

3. Nematic quantum criticality and the strange metal

4. The pseudogap regime of the hole-doped cuprate superconductors
   Angular fluctuations of a multicomponent order

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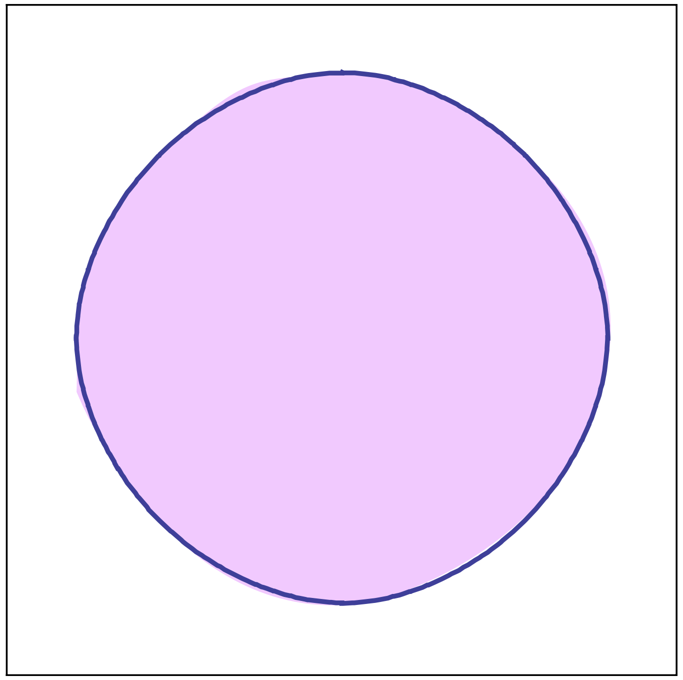
Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface



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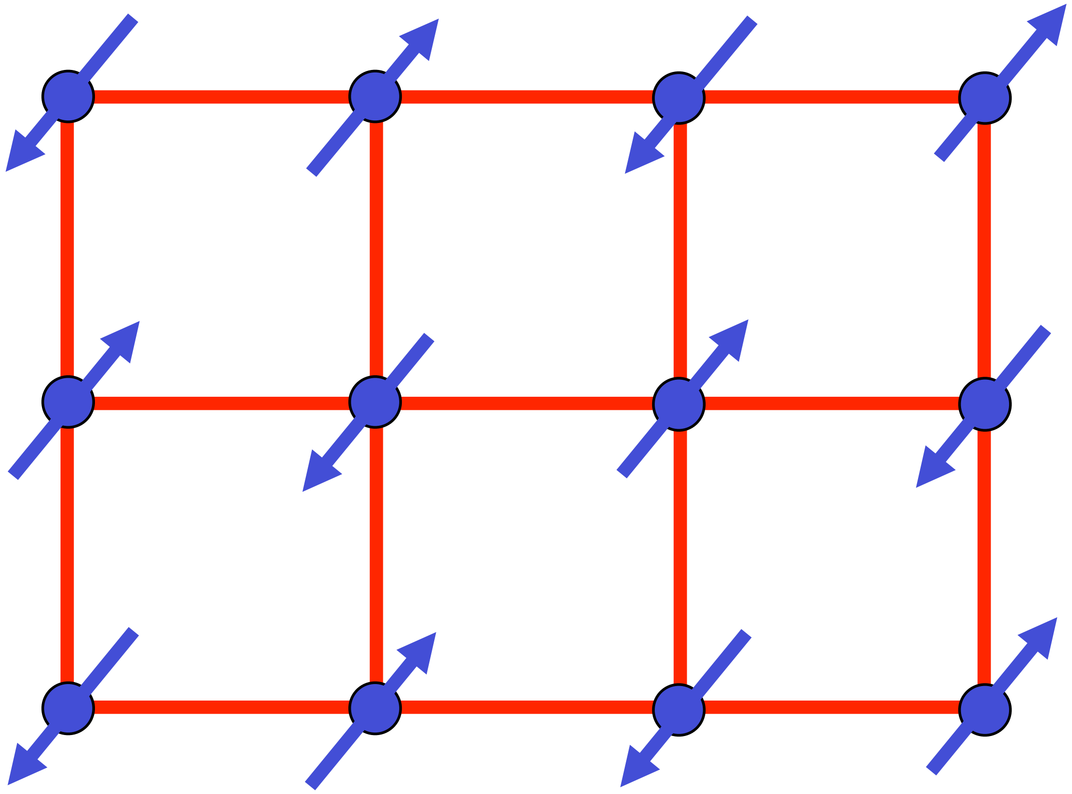


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

The Hubbard Model

$$H = -\sum_{i<j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c^\dagger_{i\alpha} c_{i\alpha}$$

$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

Spin index $\alpha = \uparrow, \downarrow$

$$n_{i\alpha} = c^\dagger_{i\alpha} c_{i\alpha}$$

$$c^\dagger_{i\alpha} c_{j\beta} + c_{j\beta} c^\dagger_{i\alpha} = \delta_{ij} \delta_{\alpha\beta}$$
$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0$$

The Hubbard Model

Decouple *U* term by a Hubbard-Stratanovich transformation

$$\mathcal{S} = \int d^2 r d\tau \left[\mathcal{L}_c + \mathcal{L}_\varphi + \mathcal{L}_{c\varphi}\right]$$

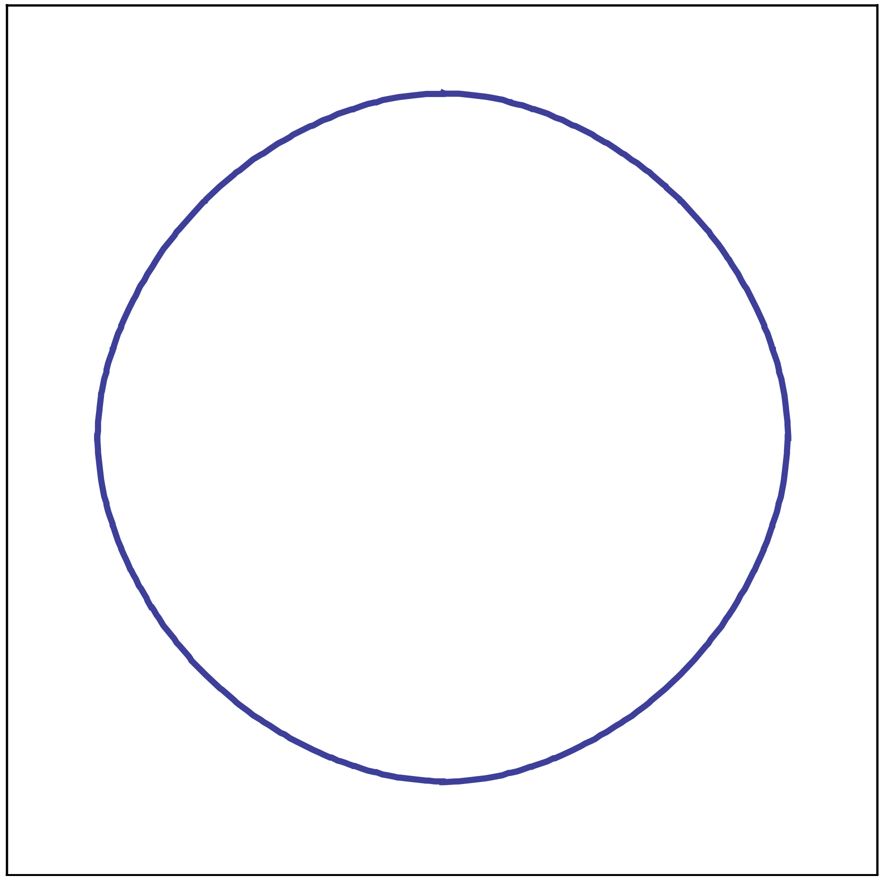
$$\mathcal{L}_c = c_a^\dagger \varepsilon(-i\boldsymbol{\nabla}) c_a$$

$$\mathcal{L}_\varphi = \frac{1}{2}(\boldsymbol{\nabla}\varphi_\alpha)^2 + \frac{r}{2}\varphi_\alpha^2 + \frac{u}{4}\left(\varphi_\alpha^2\right)^2$$

$$\mathcal{L}_{c\varphi} = \lambda\,\varphi_\alpha\, e^{i\mathbf{K}\cdot\mathbf{r}}\, c_a^\dagger\, \sigma_{ab}^\alpha\, c_b.$$

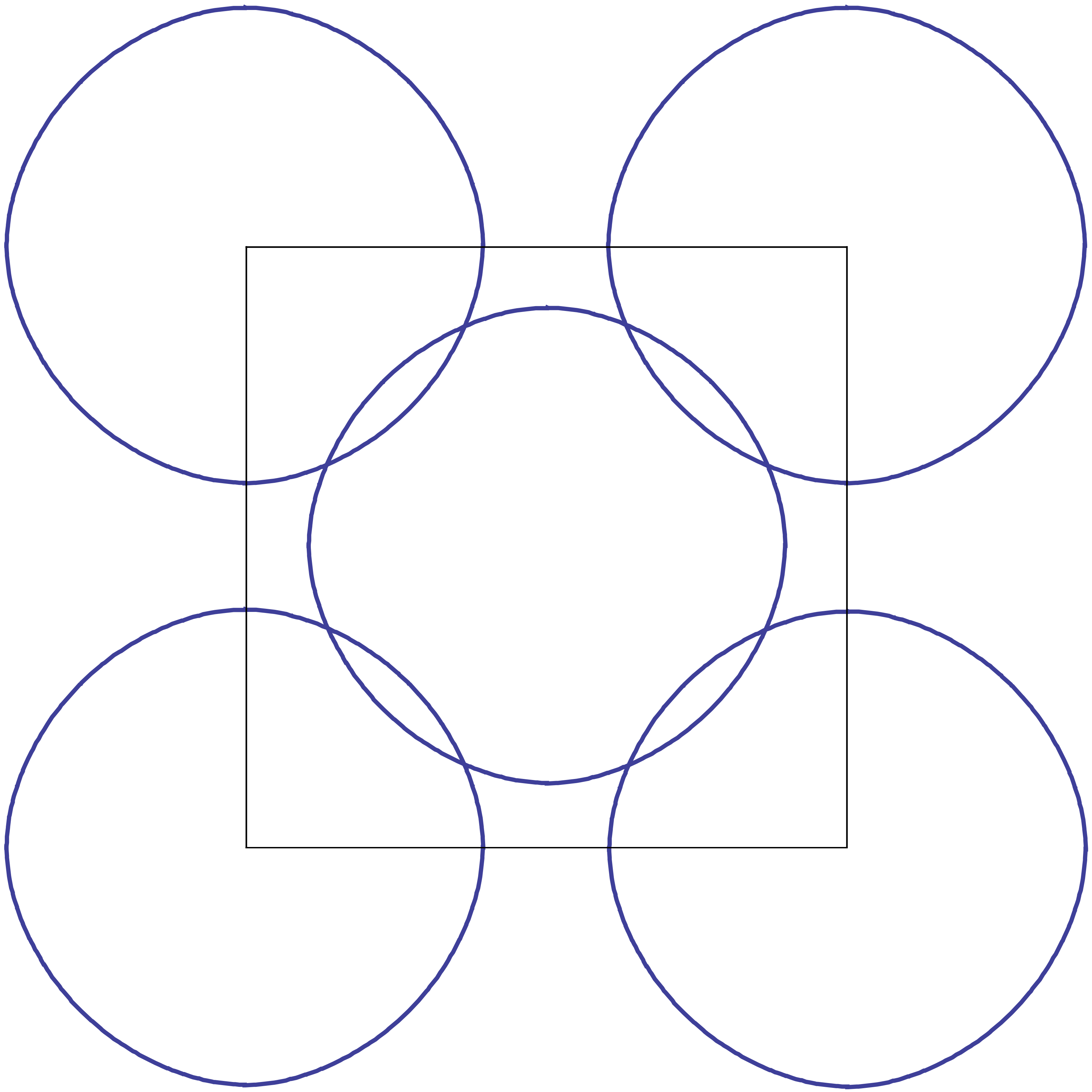
"Yukawa" coupling between fermions and antiferromagnetic order: $\lambda^2 \sim U$, the Hubbard repulsion

Fermi surface+antiferromagnetism



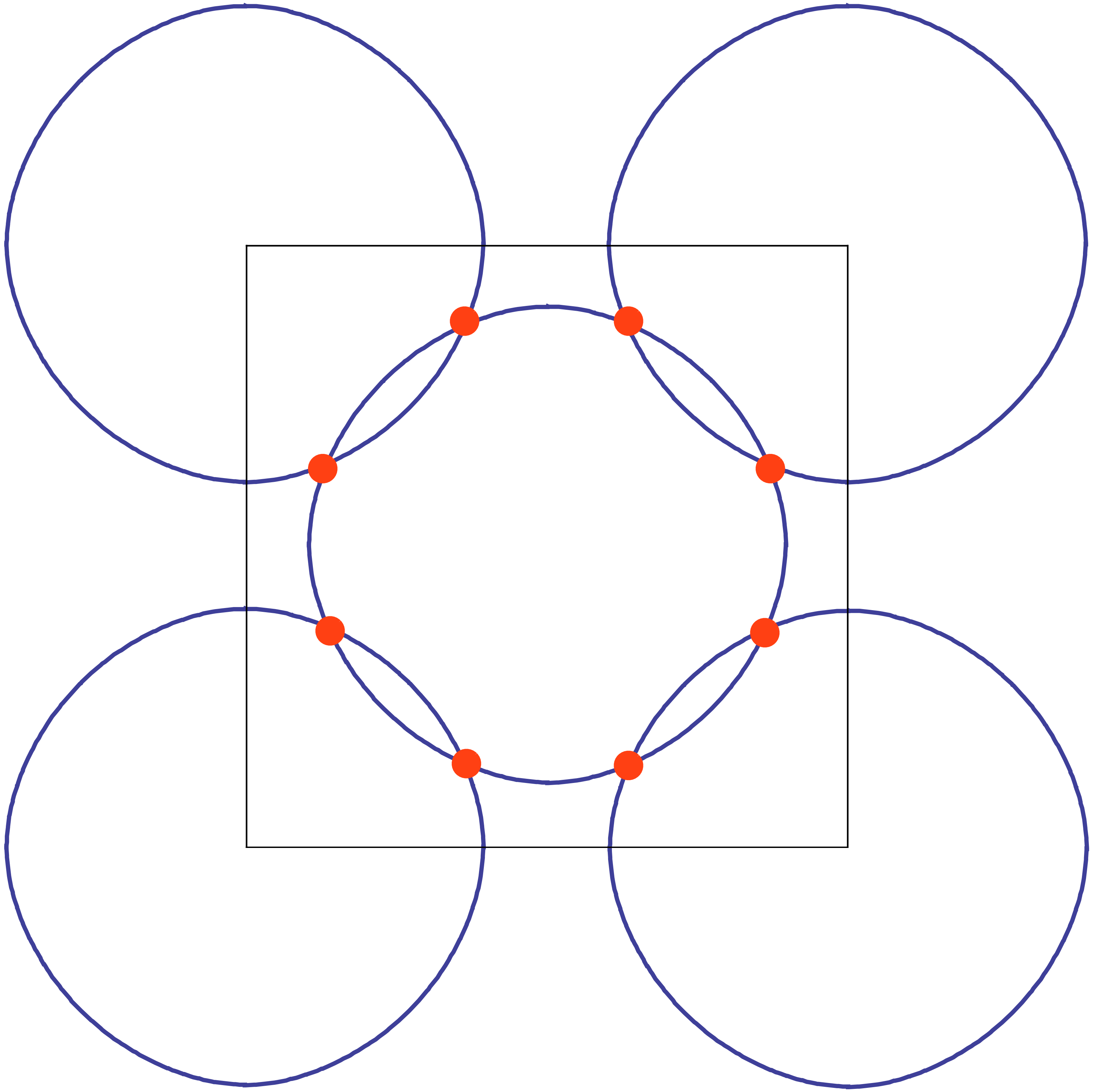
Metal with “large” Fermi surface

Fermi surface+antiferromagnetism



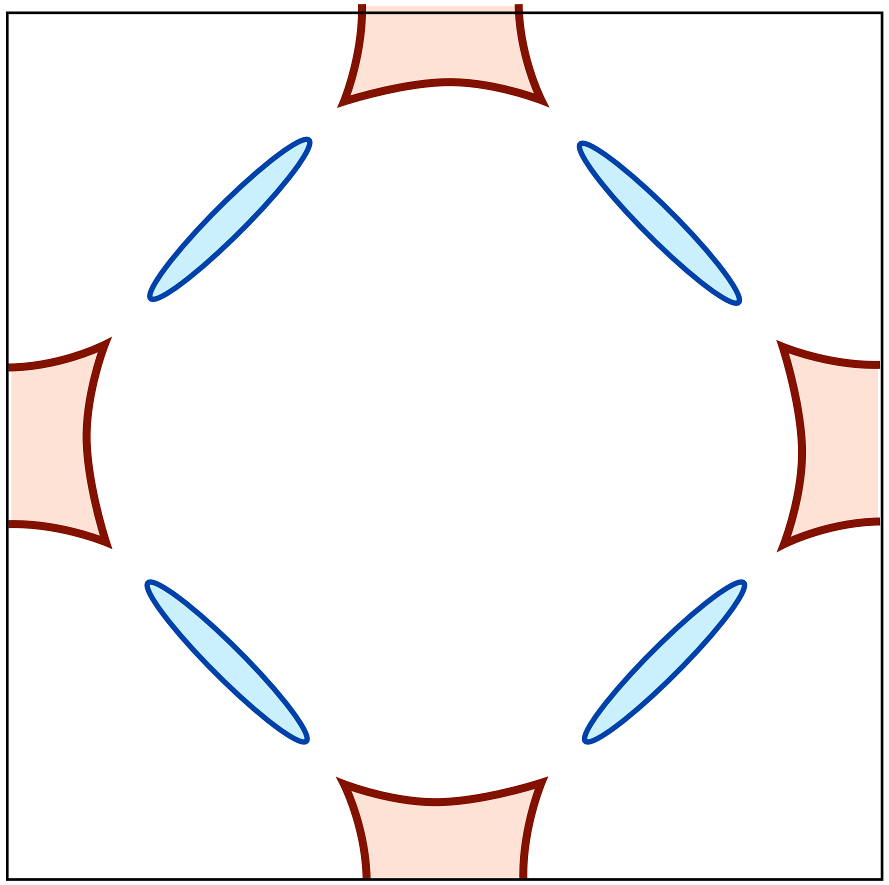
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



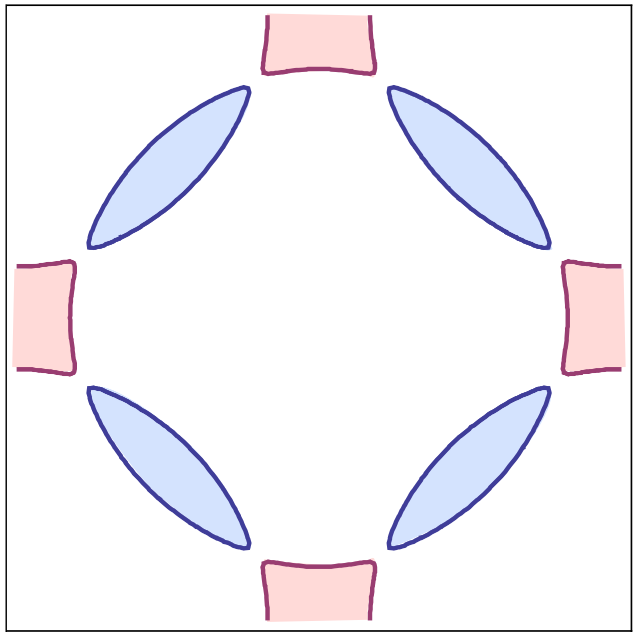
"Hot" spots

Fermi surface+antiferromagnetism



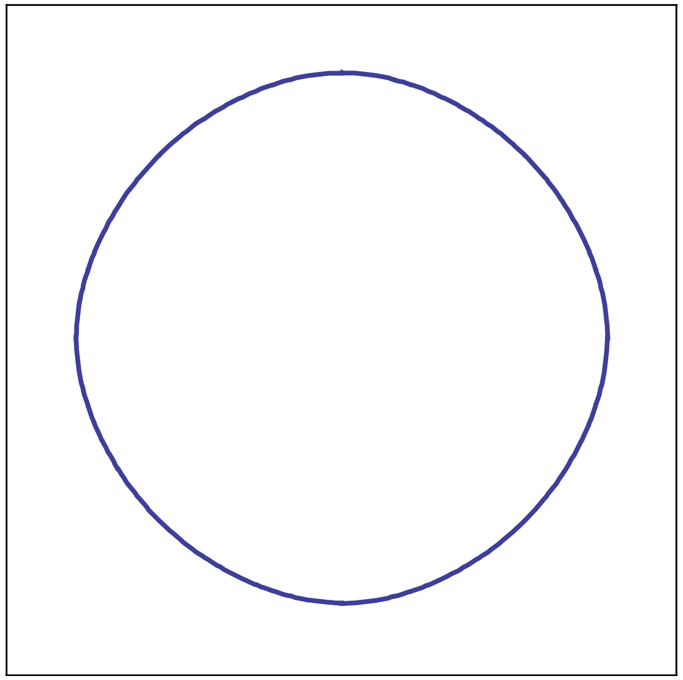
Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$

Quantum phase transition with onset of antiferromagnetism in a metal



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron and hole pockets



$$\langle \vec{\varphi} \rangle = 0$$

Metal with "large" Fermi surface

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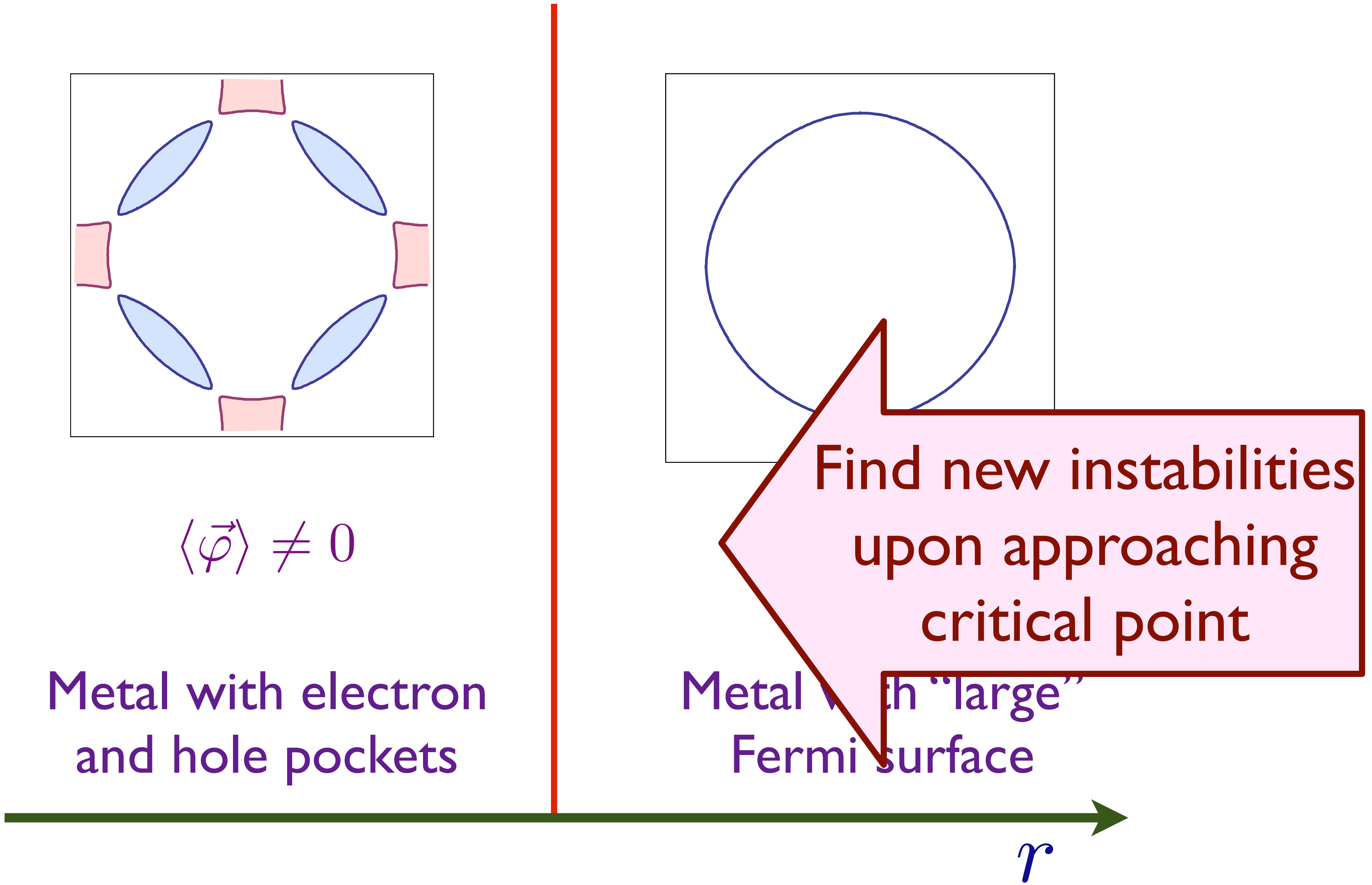
Quantum phase transition with onset of antiferromagnetism in a metal



Pairing by SDW fluctuation exchange

We now allow the SDW field $\vec{\varphi}$ to be dynamical, coupling to electrons as

$$H_{\rm sdw} = - \sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\varphi}_{\mathbf{q}} \cdot c^\dagger_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}+\mathbf{q},\beta}.$$

Exchange of a $\vec{\varphi}$ quantum leads to the effective interaction

$$H_{ee} = -\frac{1}{2} \sum_{\mathbf{q}} \sum_{\mathbf{p},\gamma,\delta} \sum_{\mathbf{k},\alpha,\beta} V_{\alpha\beta,\gamma\delta}(\mathbf{q}) c^\dagger_{\mathbf{k},\alpha} c_{\mathbf{k}+\mathbf{q},\beta} c^\dagger_{\mathbf{p},\gamma} c_{\mathbf{p}-\mathbf{q},\delta},$$

where the pairing interaction is

$$V_{\alpha\beta,\gamma\delta}(\mathbf{q}) = \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} \frac{\chi_0}{\xi^{-2} + (\mathbf{q} - \mathbf{K})^2},$$

with $\chi_0 \xi^2$ the SDW susceptibility and ξ the SDW correlation length.

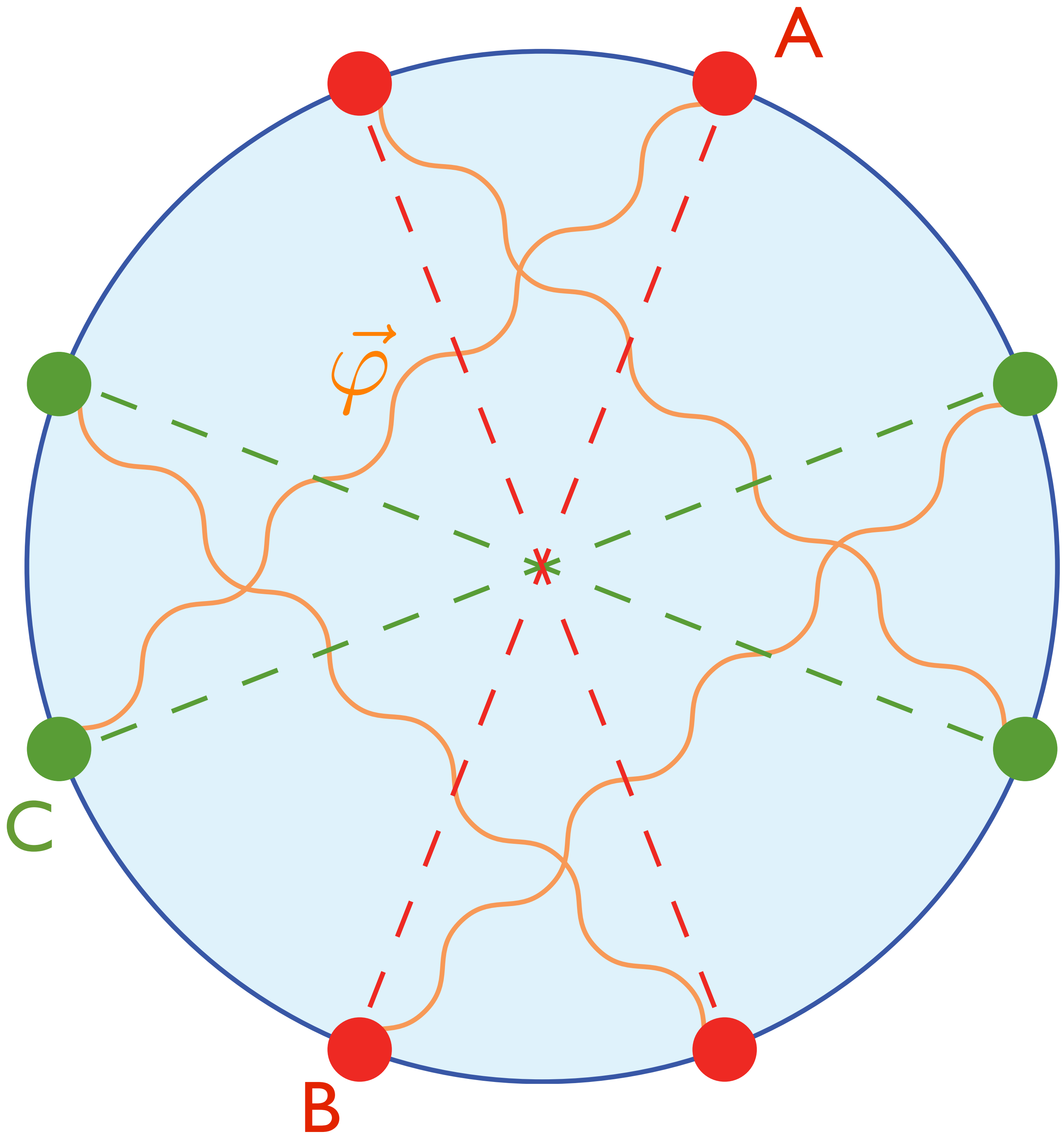
BCS Gap equation

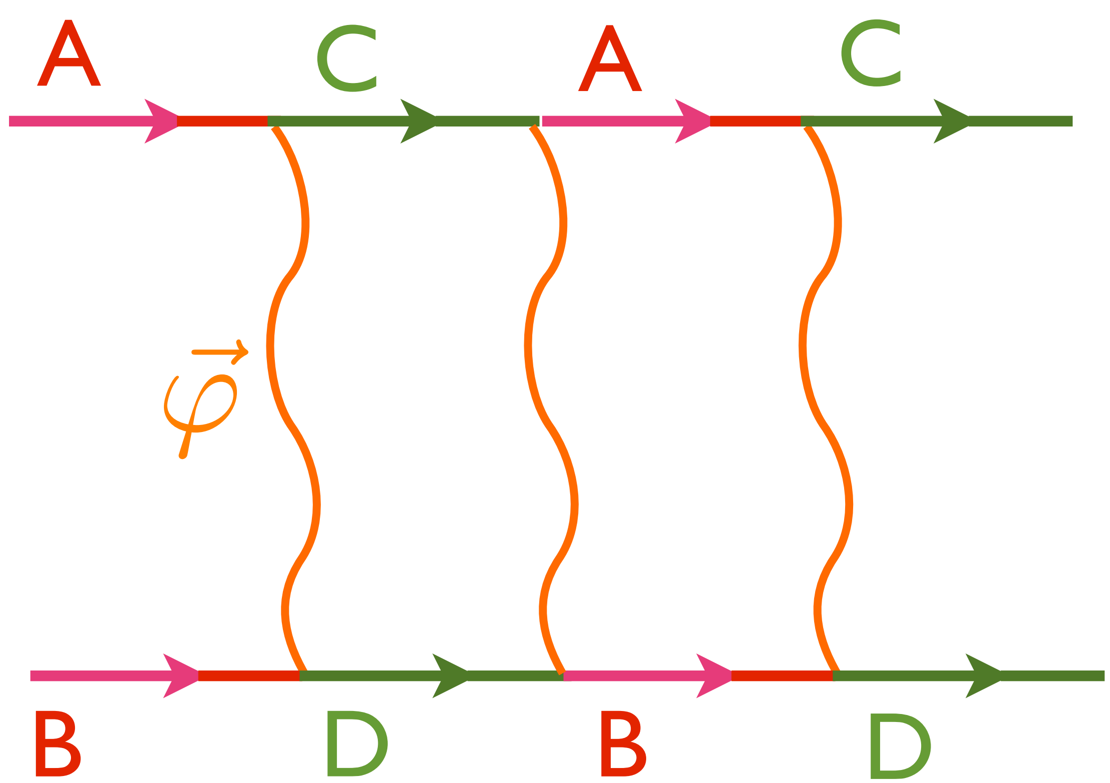
In BCS theory, this interaction leads to the 'gap equation' for the pairing gap $\Delta_{\mathbf{k}} \propto \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle$.

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{p}} \left(\frac{3\chi_0}{\xi^{-2} + (\mathbf{p} - \mathbf{k} - \mathbf{K})^2} \right) \frac{\Delta_{\mathbf{p}}}{2\sqrt{\varepsilon_{\mathbf{p}}^2 + \Delta_{\mathbf{p}}^2}}$$

Non-zero solutions of this equation require that $\Delta_{\mathbf{k}}$ and $\Delta_{\mathbf{p}}$ have opposite signs when $\mathbf{p} - \mathbf{k} \approx \mathbf{K}$.

Pairing "glue" from antiferromagnetic fluctuations





V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
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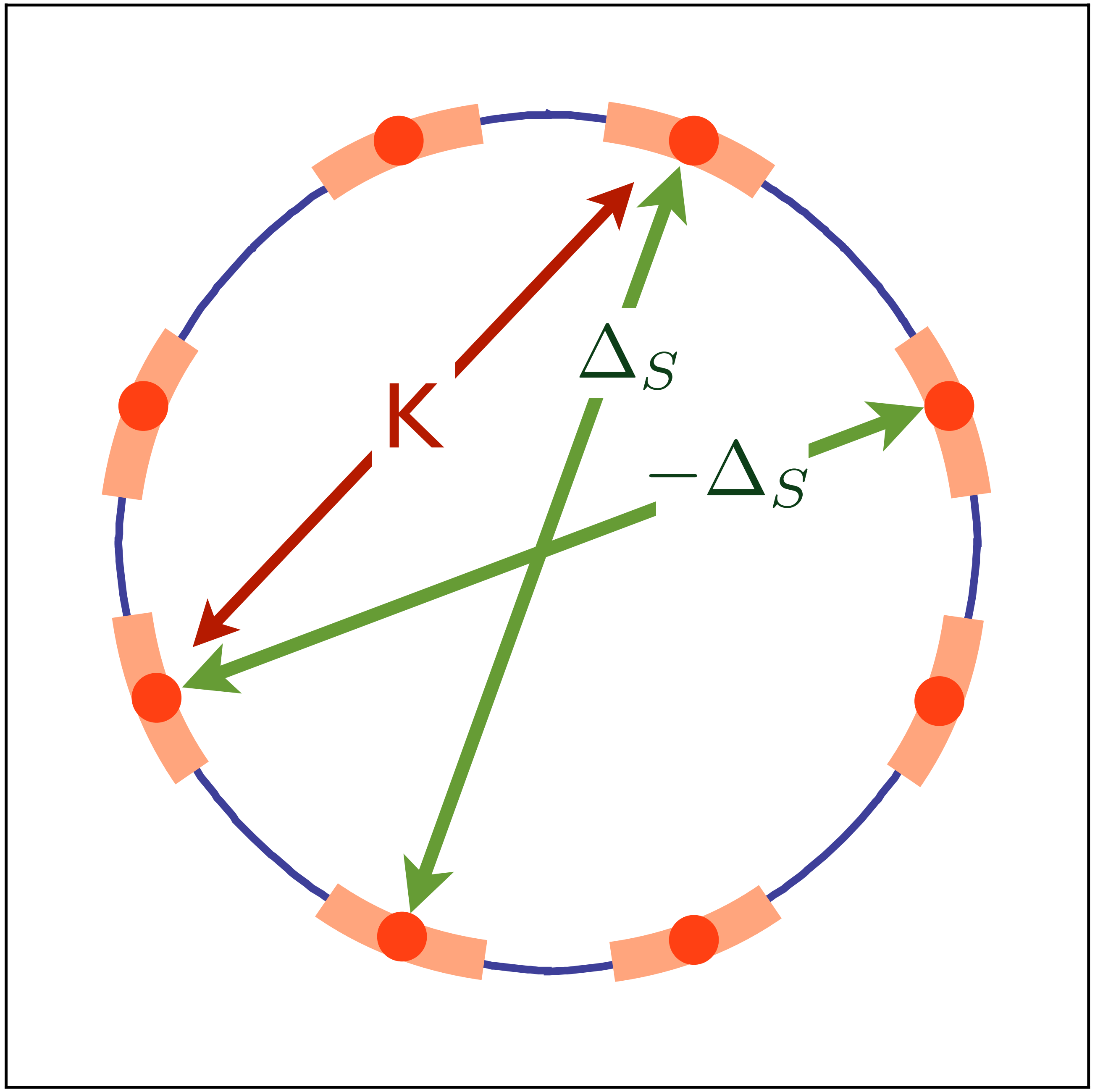

d-wave superconductor: particle-particle pairing at *and near* hot spots, with sign-changing pairing amplitude

Near the antiferromagnetic critical point, the coupling becomes infinitely strong:

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- Pairing glue becomes stronger.

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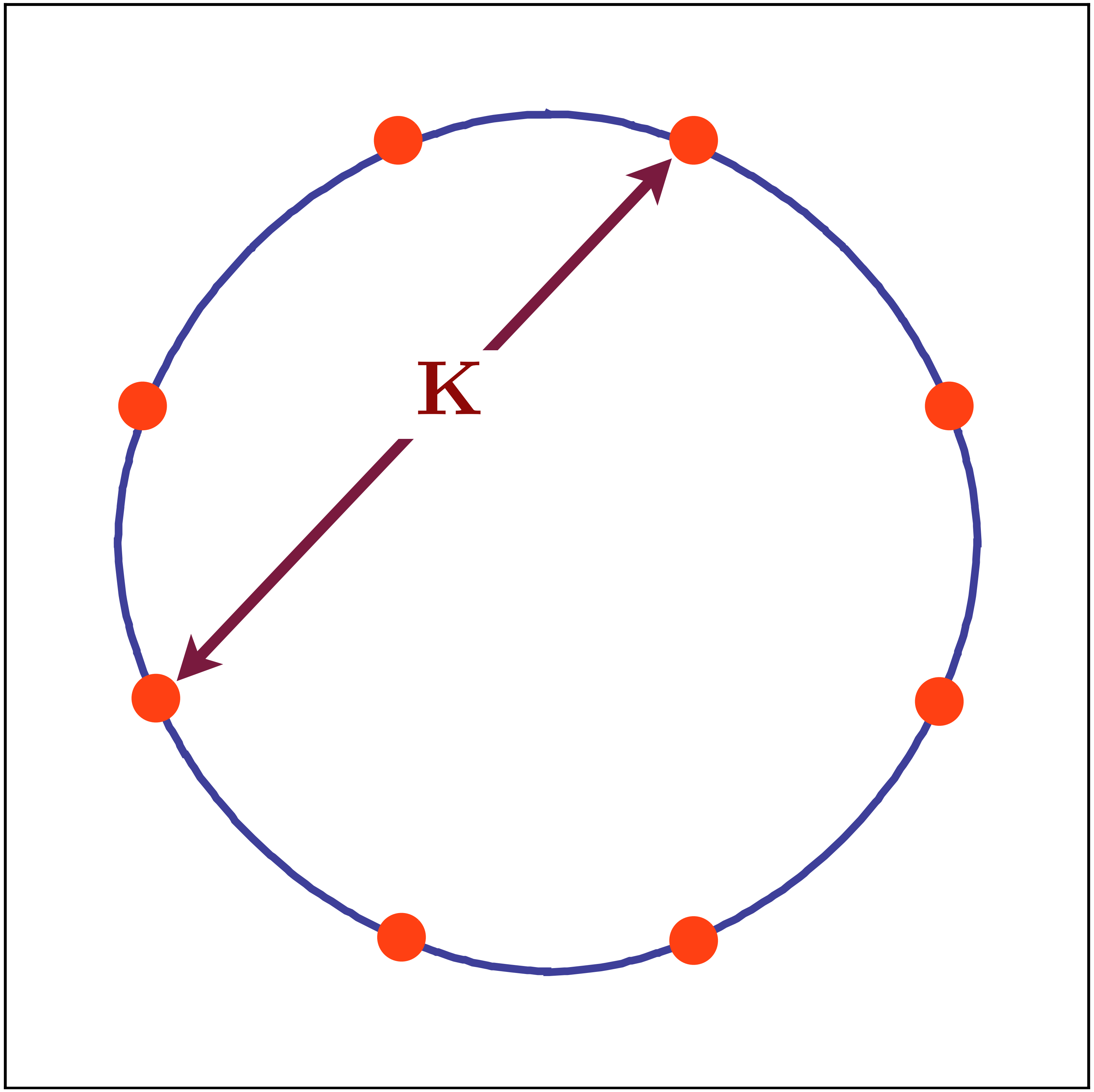
- Pairing glue becomes stronger.

- There is stronger fermion-boson scattering, and fermionic quasi-particles lose their integrity.

- An instability to charge-density-wave/bond order can become nearly degenerate with super-conductivity if the Fermi-surface is not too curved.

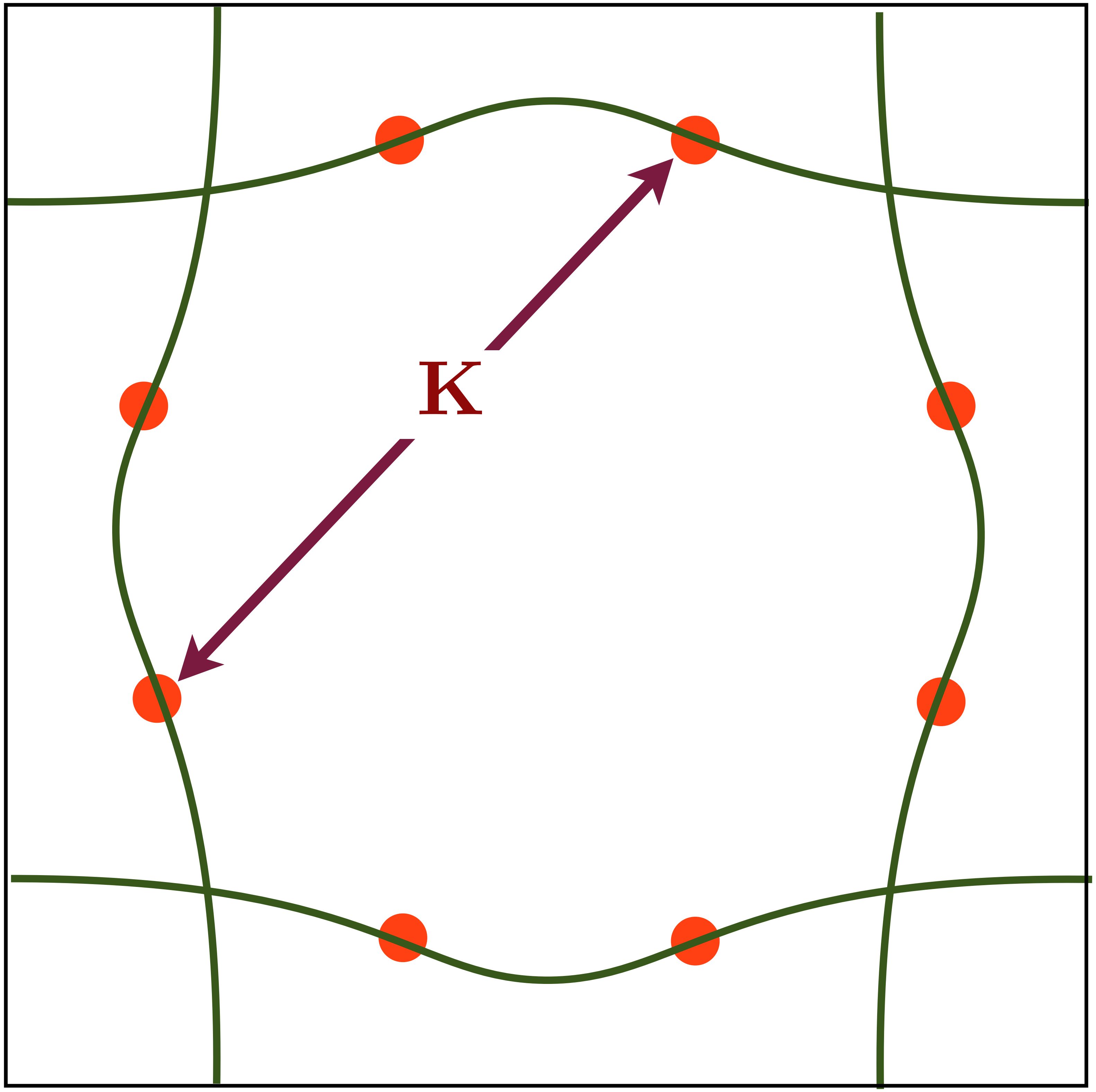
M.A. Metlitski and S. Sachdev,
Phys. Rev. B **85**, 075127 (2010)

QMC for the onset of antiferromagnetism



Hot spots in a single band model

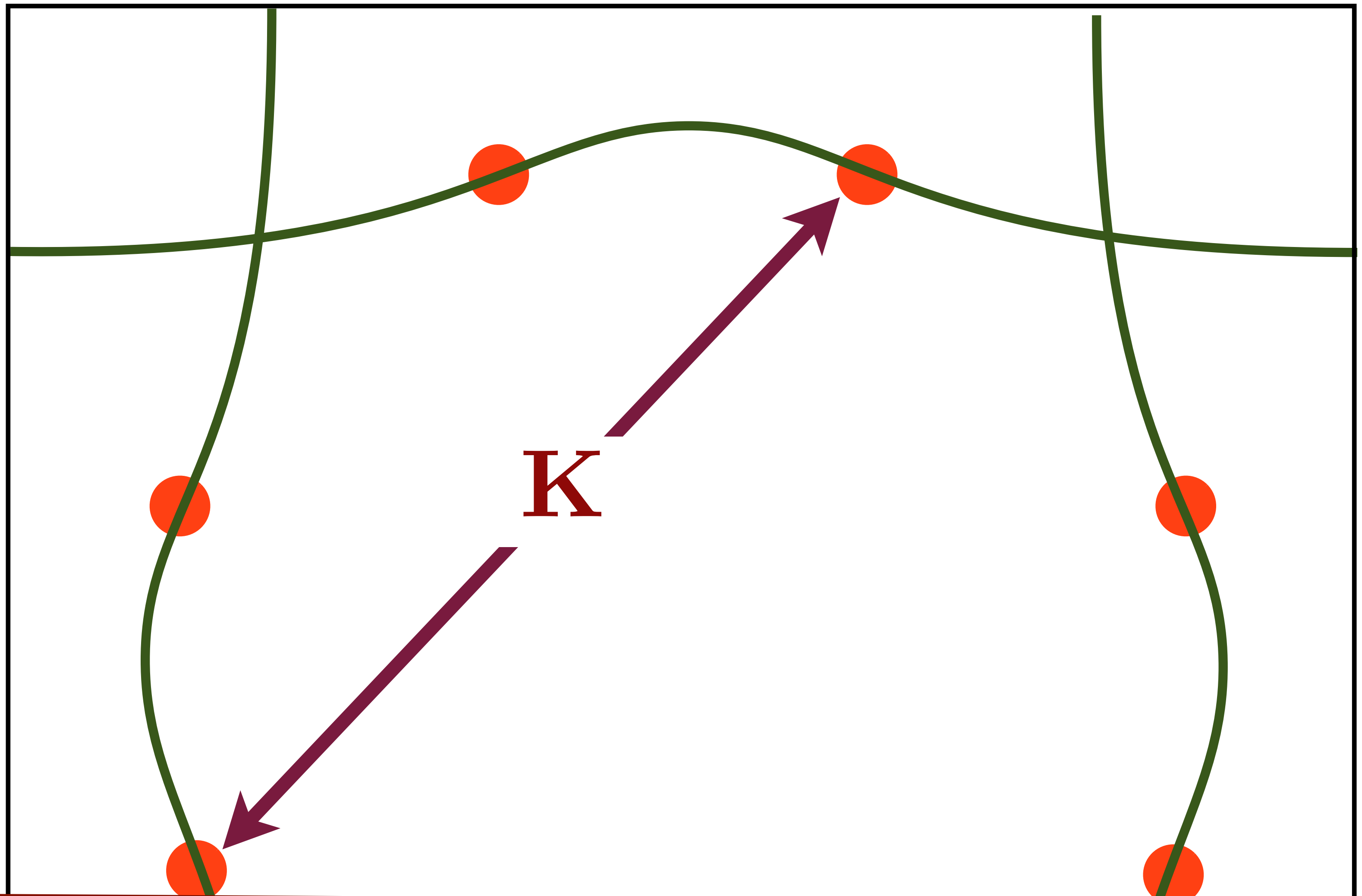
QMC for the onset of antiferromagnetism



E. Berg,
M. Metlitski, and
S. Sachdev,
Science **338**, 1606
(2012).

Hot spots in a two band model

QMC for the onset of antiferromagnetism

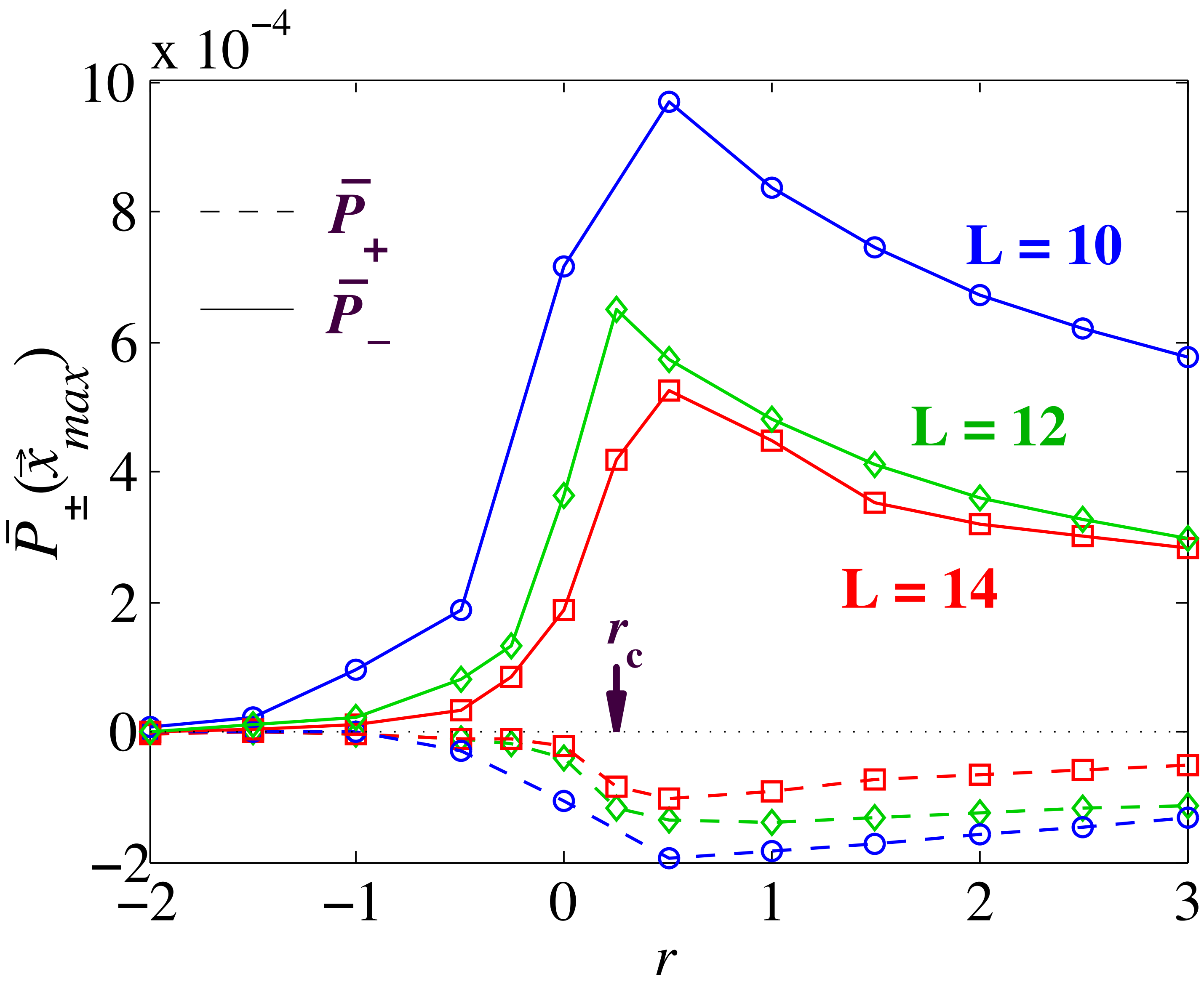


E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

No sign problem in fermion determinant Monte Carlo !

Determinant is positive because of Kramer's degeneracy, and no additional symmetries are needed; holds for arbitrary band structure and band filling, provided *K* only connects hot spots in distinct bands

Sign-problem-free Quantum Monte Carlo for antiferromagnetism in metals



s/d pairing amplitudes P_+/P_-
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

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Pseudospin symmetry of the exchange interaction

$$H_J = \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

with $\vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c^\dagger_{i\downarrow} \end{pmatrix} \quad , \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c^\dagger_{i\uparrow} \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} \sum_{i<j} J_{ij} \left(\Psi^\dagger_{i\alpha a} \vec{\sigma}_{\alpha\beta} \Psi_{i\beta a} \right) \cdot \left(\Psi^\dagger_{j\gamma b} \vec{\sigma}_{\gamma\delta} \Psi_{j\delta b} \right)$$

where a, b are the Nambu indices. This form makes explicit the symmetry under *independent* SU(2) pseudospin transformations on each site

$$\Psi_{i\alpha a} \to U_{i,ab} \Psi_{i\alpha b}$$

This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of H_J.

I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)
P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)

Pseudospin symmetry of the exchange interaction

$$H_{tJ} = -\sum_{i,j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + V \sum_{\langle ij \rangle} n_i n_j$$

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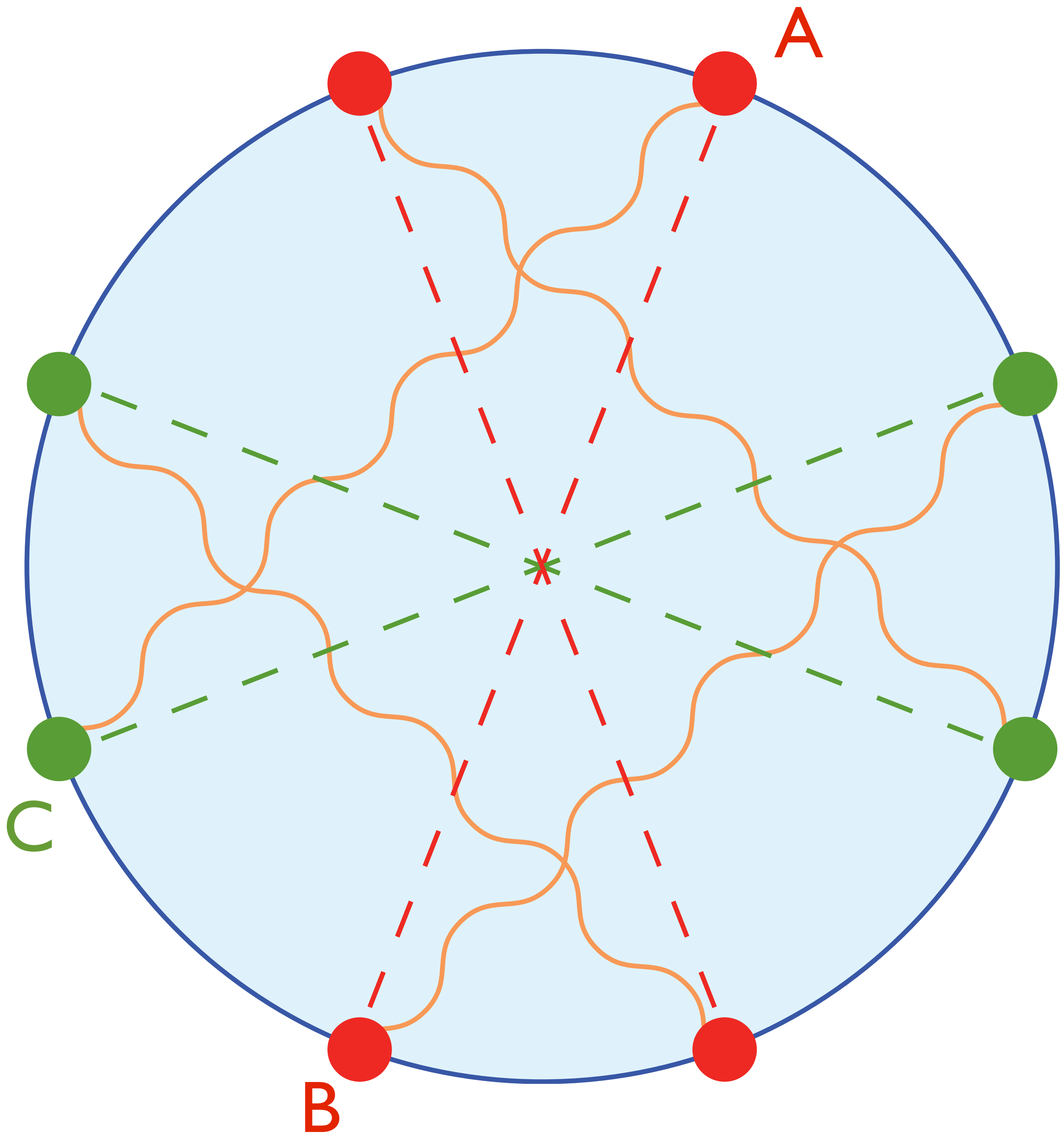
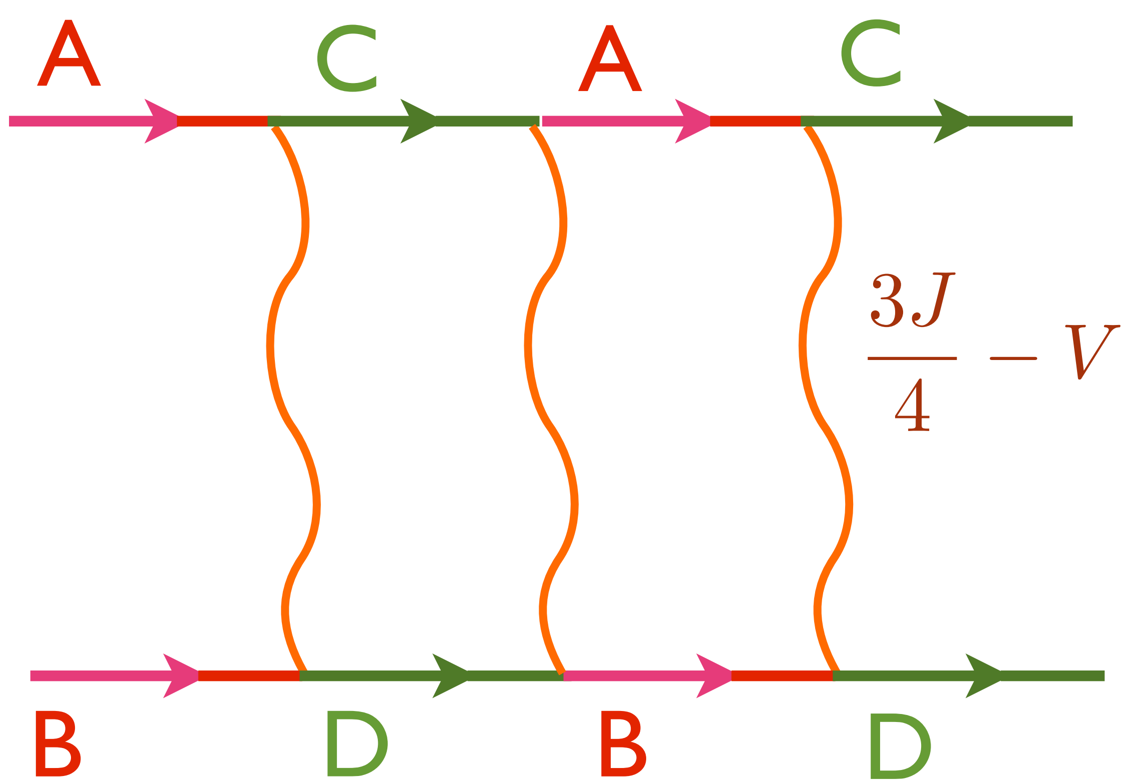
We will find important consequences of the pseudospin symmetry in ordinary metals with antiferromagnetic correlations.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Pairing "glue" from antiferromagnetic fluctuations





V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
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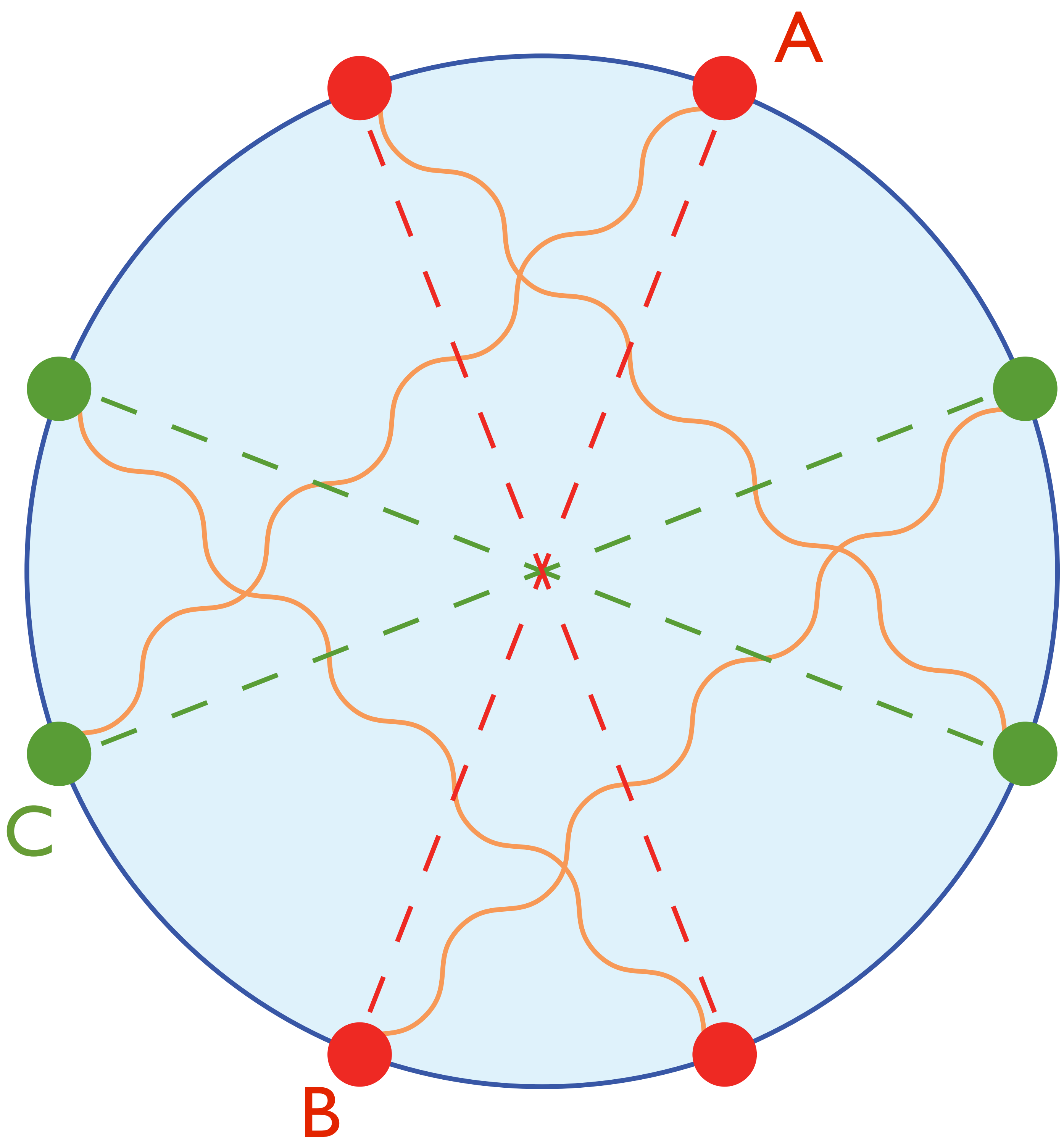
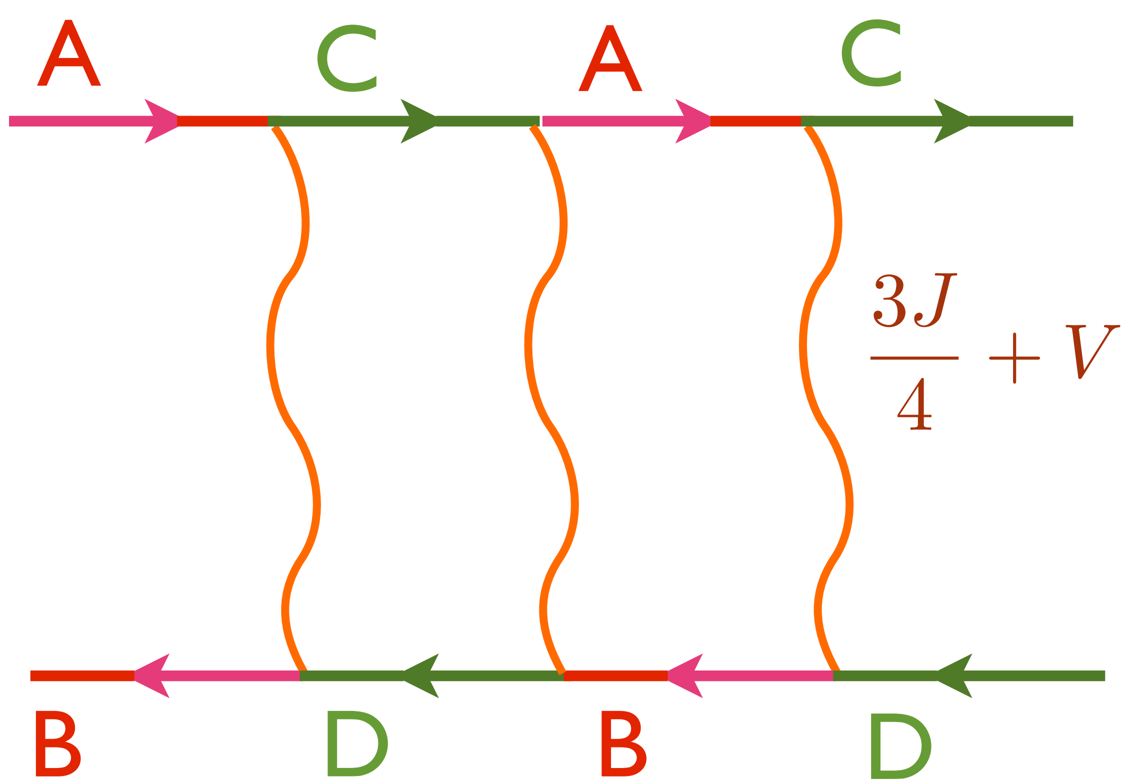
Same "glue" leads to particle-hole pairing





M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$

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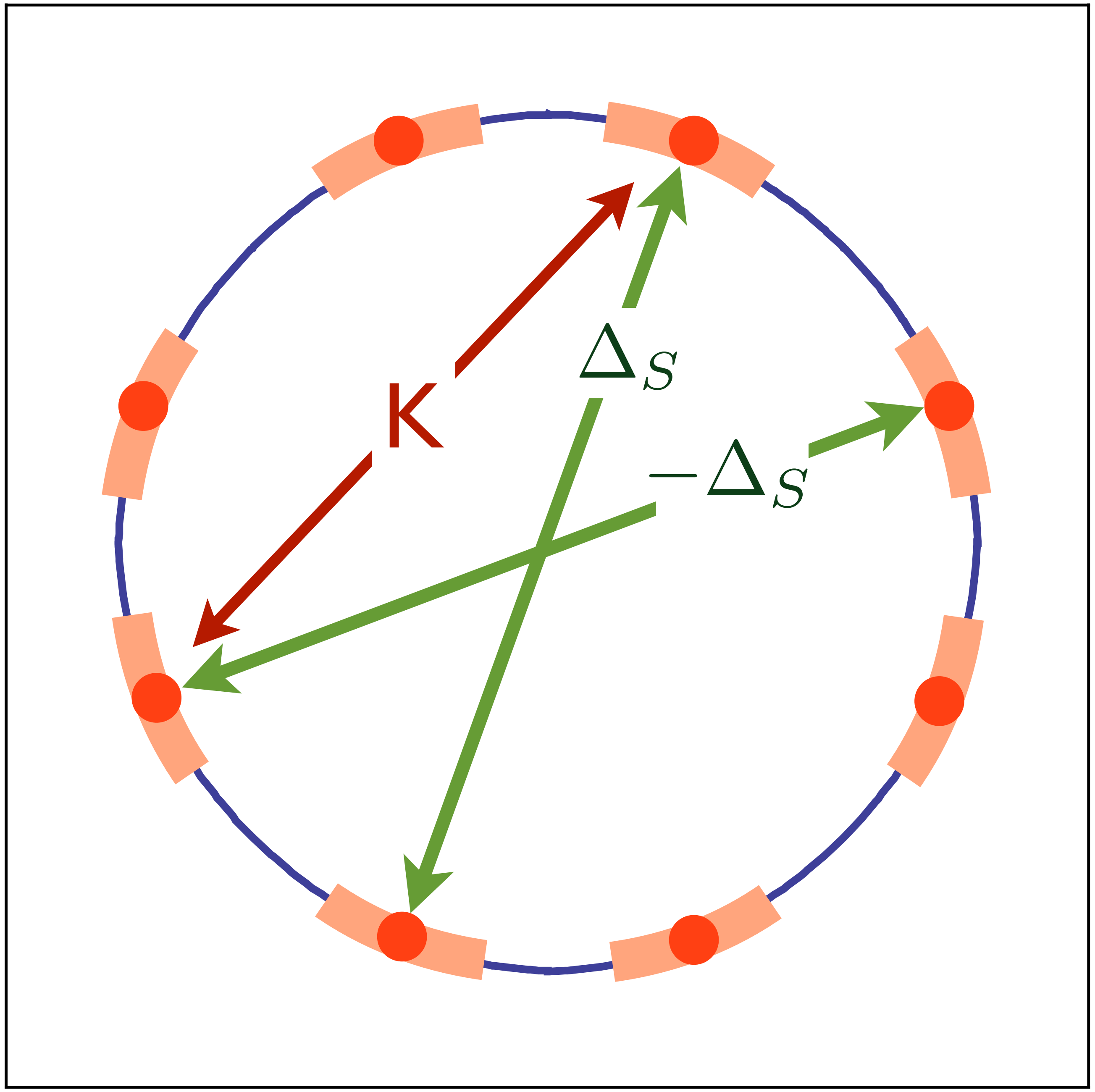
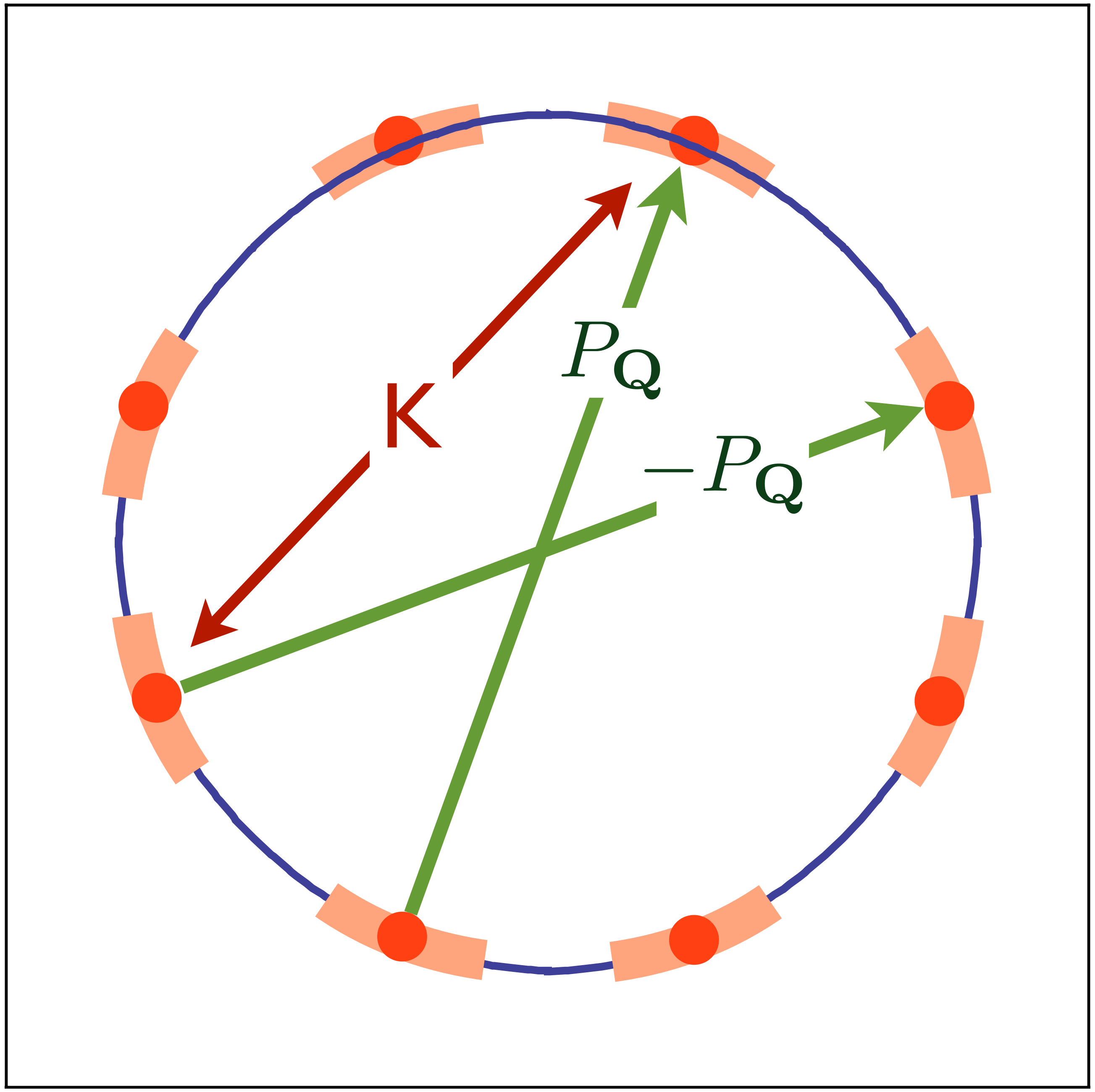


d-wave superconductor: particle-particle pairing at *and near* hot spots, with sign-changing pairing amplitude

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = P_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

After pseudospin rotation on *half* the hot-spots

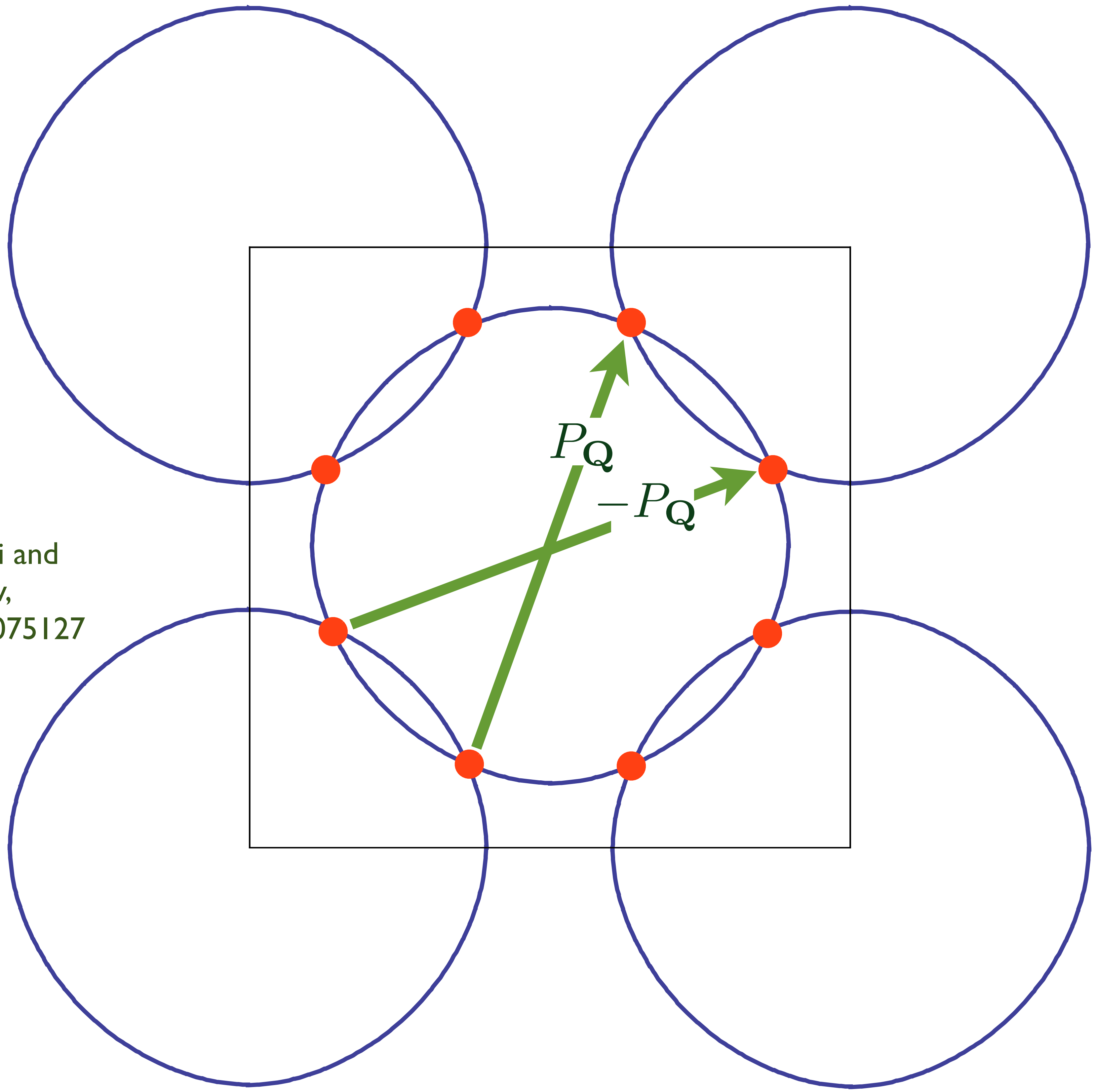


$\mathbf{Q}$ is '$2k_F$' wavevector

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Incommensurate d-wave bond order: particle-hole pairing at *and near* hot spots, with sign-changing pairing amplitude

Incommensurate *d*-wave bond order



M. A. Metlitski and
S. Sachdev,
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(2010)

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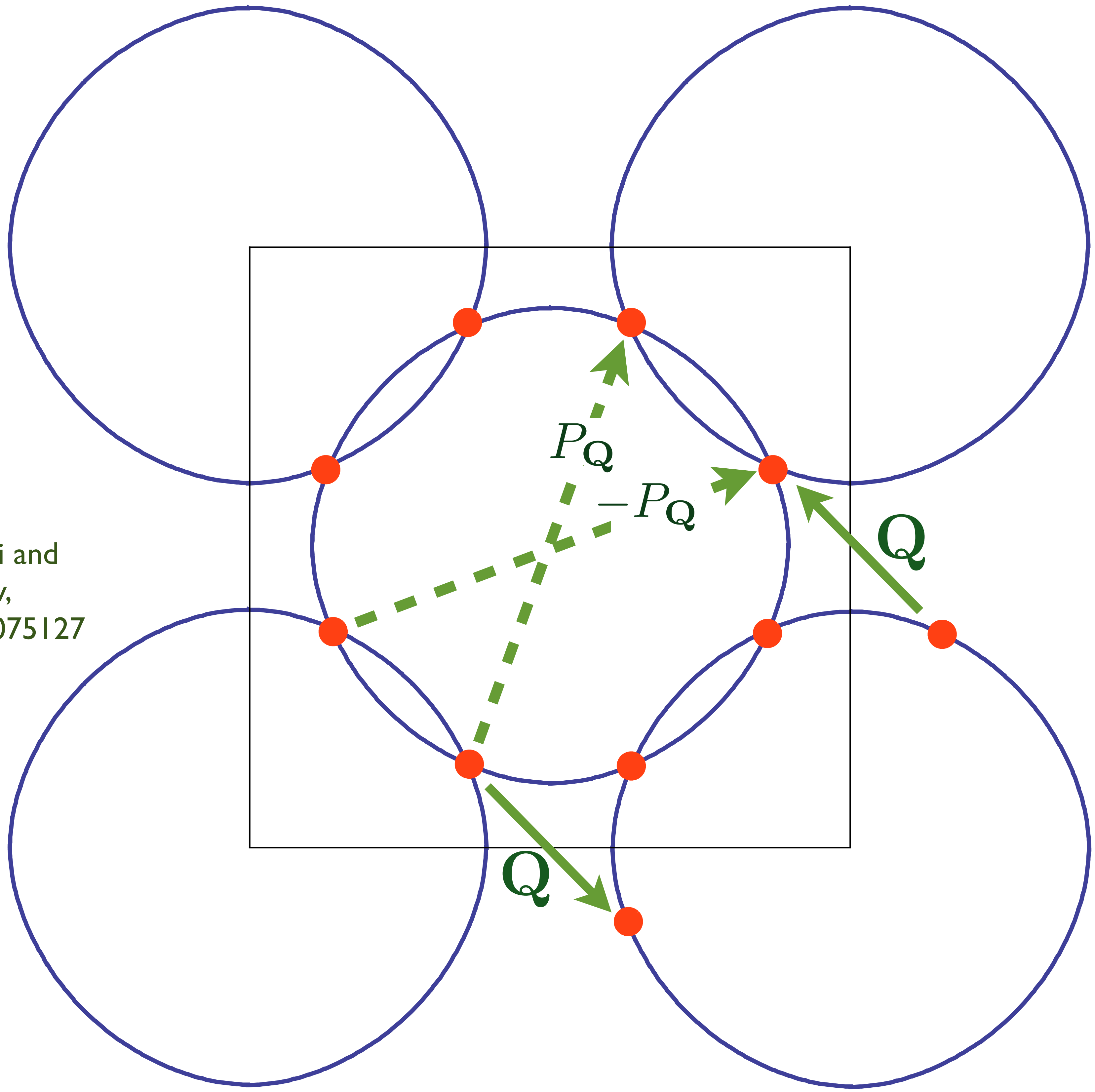

M. A. Metlitski and
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Bond, charge, and current order

Consider modulation in a bilocal variable at the Cu sites $\mathbf{r}_i$ and $\mathbf{r}_j$

$$\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle \sim \left[\sum_{\mathbf{k}} P_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

relative co-ord.

average co-ord.

The wavevector $\mathbf{Q}$ is associated with a modulation in the *average* co-ordinate $(\mathbf{r}_i + \mathbf{r}_j)/2$: this determines the wavevector of the X-ray scattering peak.

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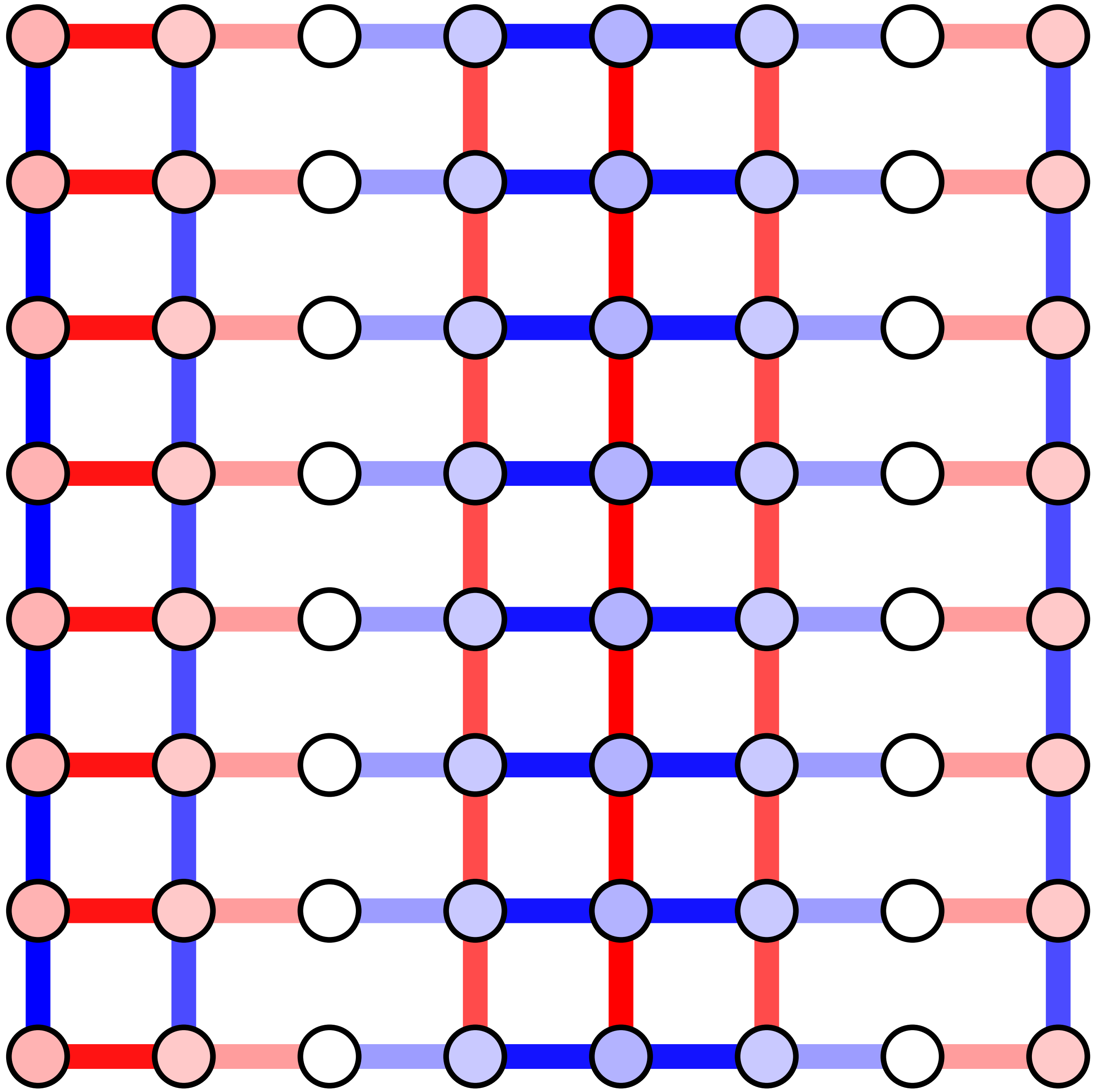
States with spontaneous currents have $\mathcal{P}_p$ non-zero: they break time-reversal

Bond, charge, and current order

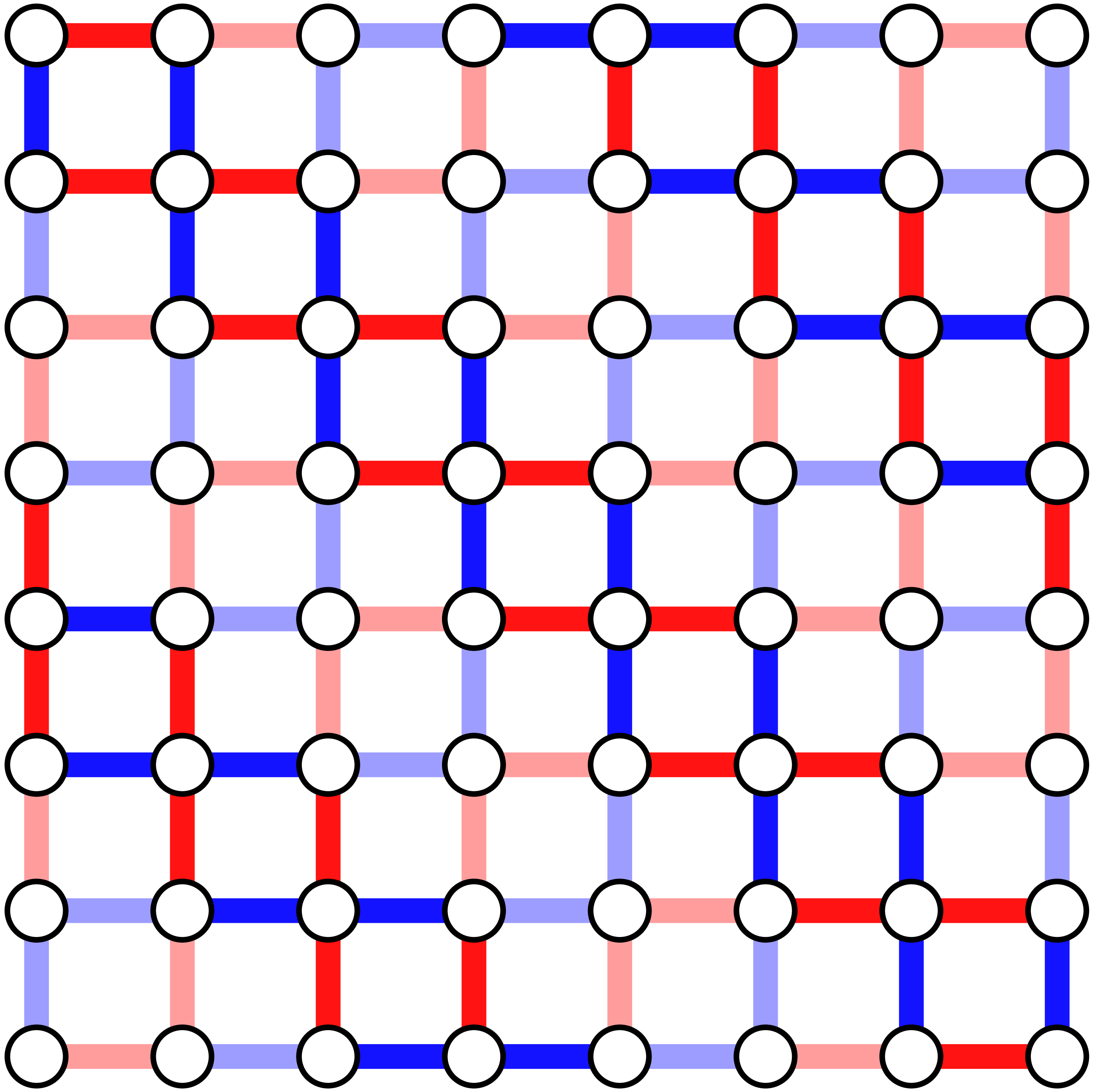
Plot of $P_{ij} = \left\langle c_{i\alpha}^\dagger c_{j\alpha} \right\rangle$ for $i = j$, and i, j nearest neighbors.



Predominantly d-wave bond order at $\mathbf{Q} = (\pi/4, 0)$

Bond, charge, and current order

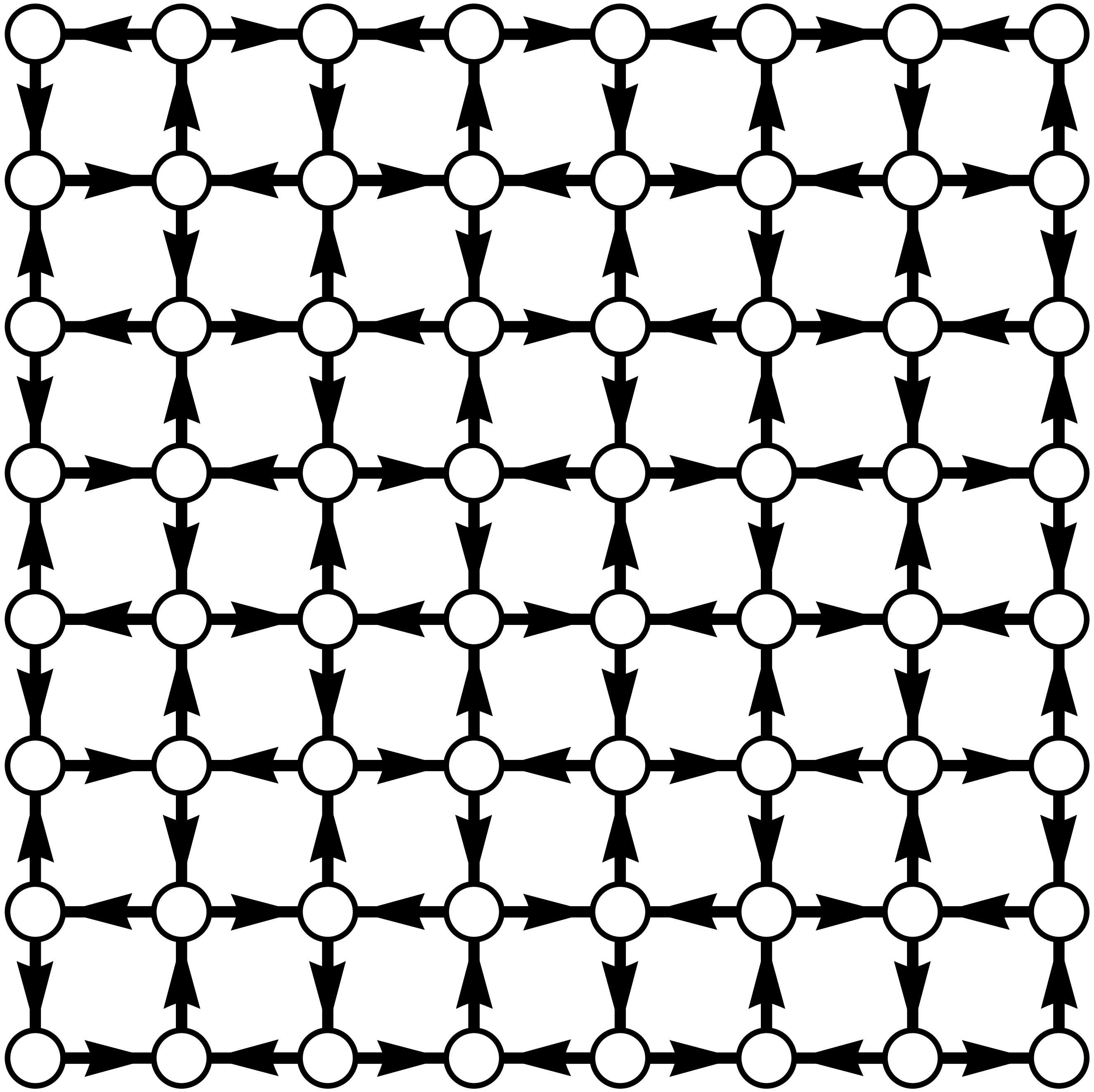
Plot of $P_{ij} = \left\langle c_{i\alpha}^\dagger c_{j\alpha} \right\rangle$ for $i = j$, and i, j nearest neighbors.



d-wave bond order at $\mathbf{Q} = (\pi/4, \pi/4)$

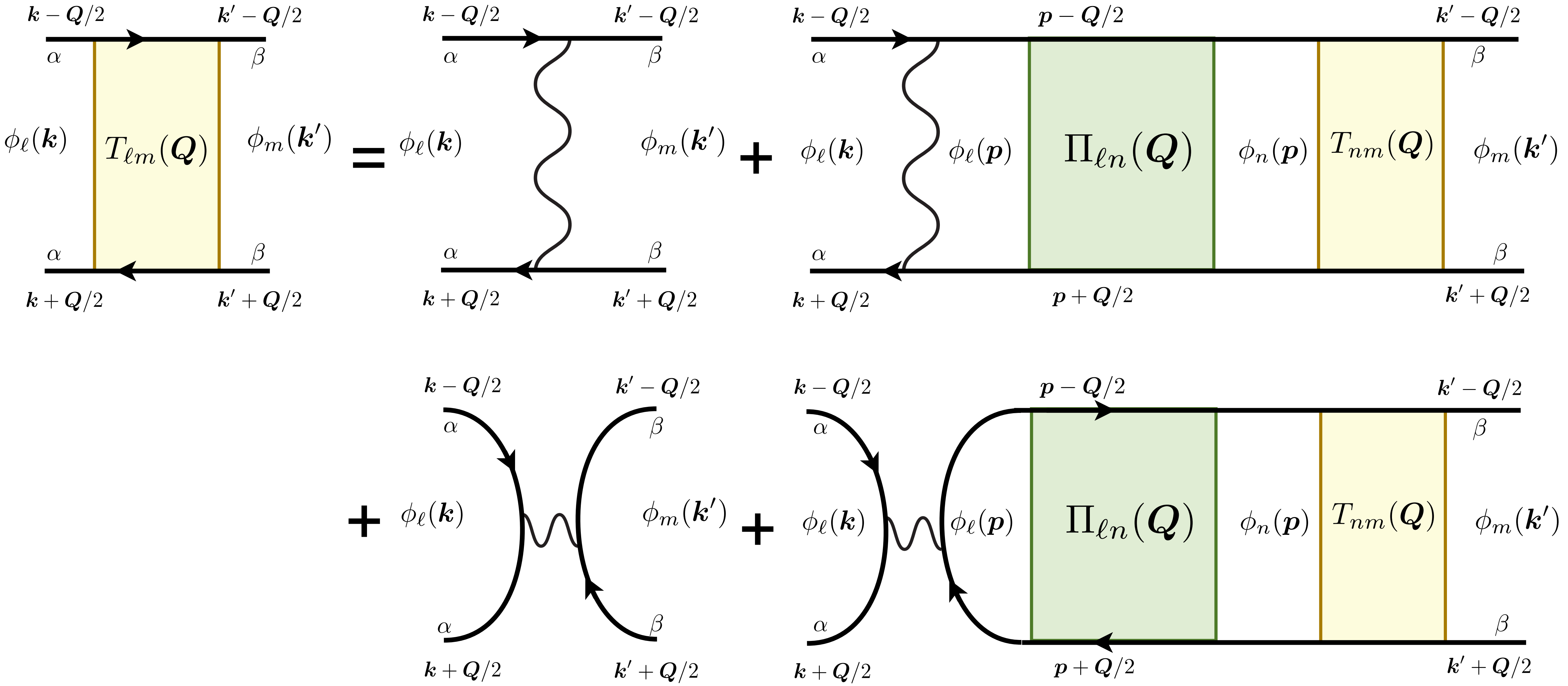
Bond, charge, and current order

Plot of $P_{ij} = \left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle$ for $i = j$, and i, j nearest neighbors.



p-wave current order at $\mathbf{Q} = (\pi, \pi)$;
This state is also known as "d-density wave" (*unfortunately!*),
and "staggered-flux (SF)".

Computation of $\mathcal{P}_\ell$



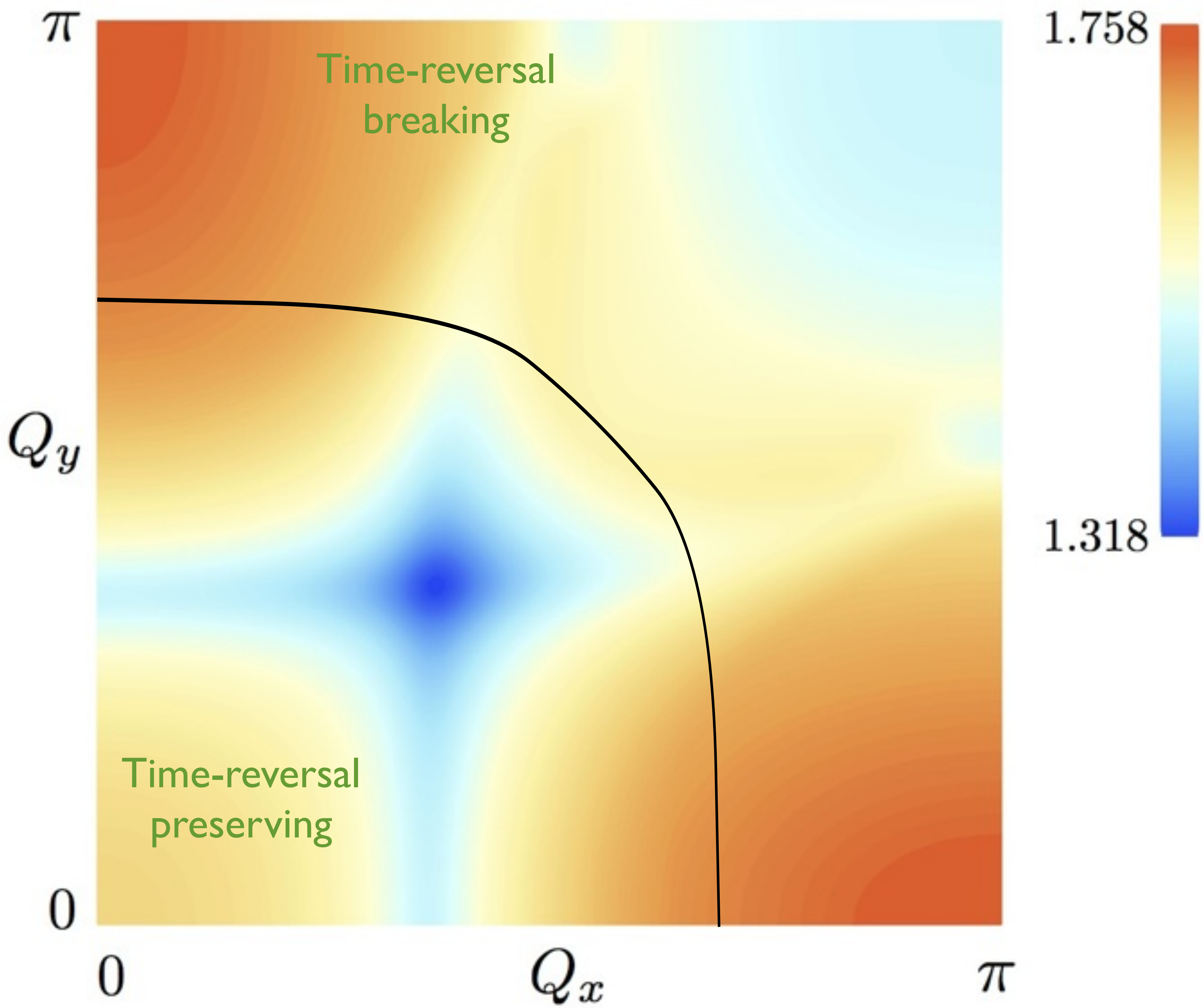
Find the lowest eigenvalues $\lambda(\mathbf{Q})$, and corresponding eigenvectors, of the matrix

$$\delta_{\ell m} - \frac{1}{2}\left(\frac{3}{4}\mathcal{J}_\ell + \mathcal{V}_\ell\right)\Pi_{\ell m}(\mathbf{Q}) + \delta_{\ell,0} W(\mathbf{Q})\Pi_{0m}(\mathbf{Q}), \text{ where}$$

$$\Pi_{\ell m}(\mathbf{Q}) = 2\sum_{\mathbf{k}} \phi_\ell(\mathbf{k})\phi_m(\mathbf{k}) \frac{f(\varepsilon(\mathbf{k}-\mathbf{Q}/2)) - f(\varepsilon(\mathbf{k}+\mathbf{Q}/2))}{\varepsilon(\mathbf{k}+\mathbf{Q}/2) - \varepsilon(\mathbf{k}-\mathbf{Q}/2)} \text{ and}$$

$$W(\mathbf{Q}) \equiv \sum_{\ell} \mathcal{V}_\ell \phi_\ell(0)\phi_\ell(\mathbf{Q})$$

S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013); A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.6311



Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $P_{\mathbf{Q}}(\mathbf{k})$ and this leads to the order

$$\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle = \left[\sum_{\mathbf{k}} P_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

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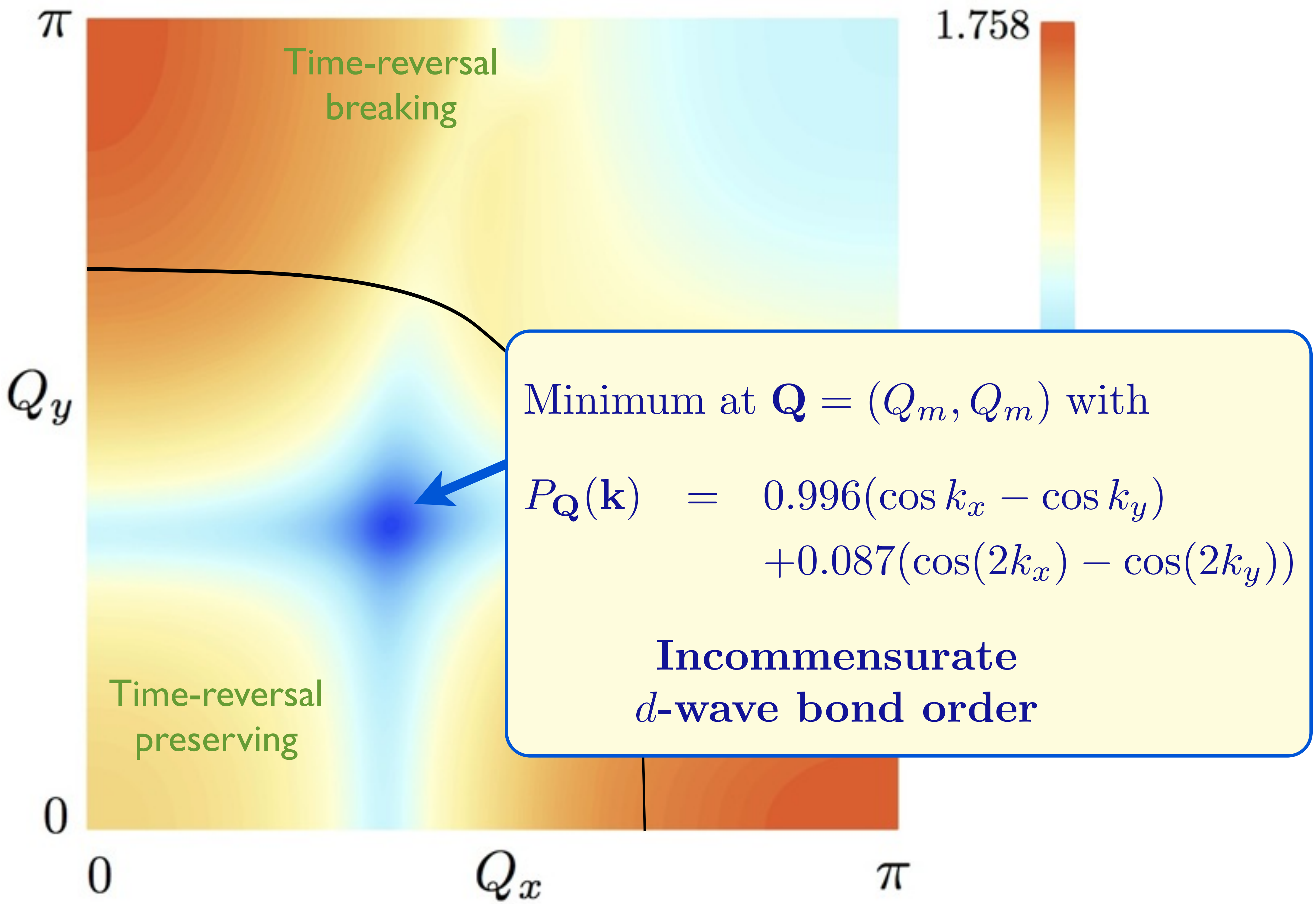

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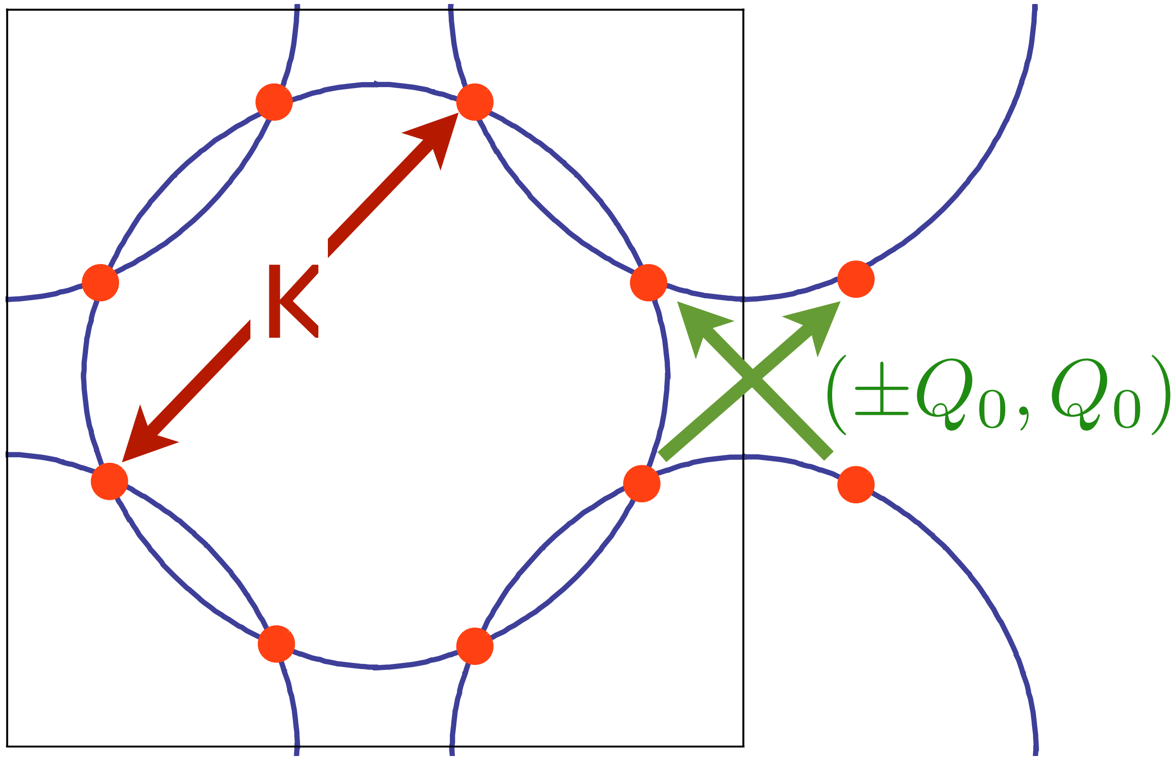
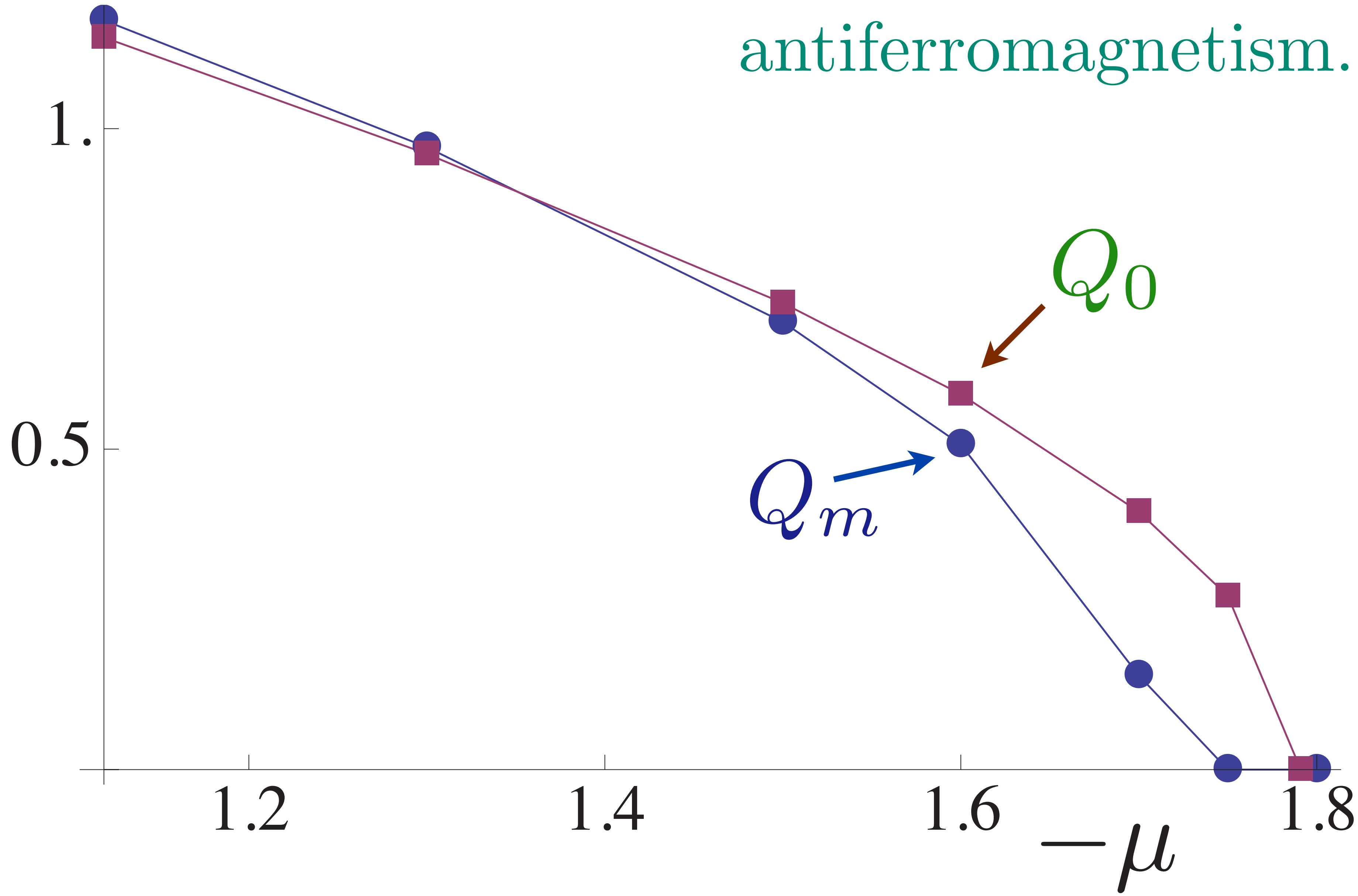
Remarkable agreement between the value of Q_m from Hartree-Fock in a metal with short-range *incommensurate* spin correlations, and the value of Q_0 from hot spots of *commensurate* antiferromagnetism.





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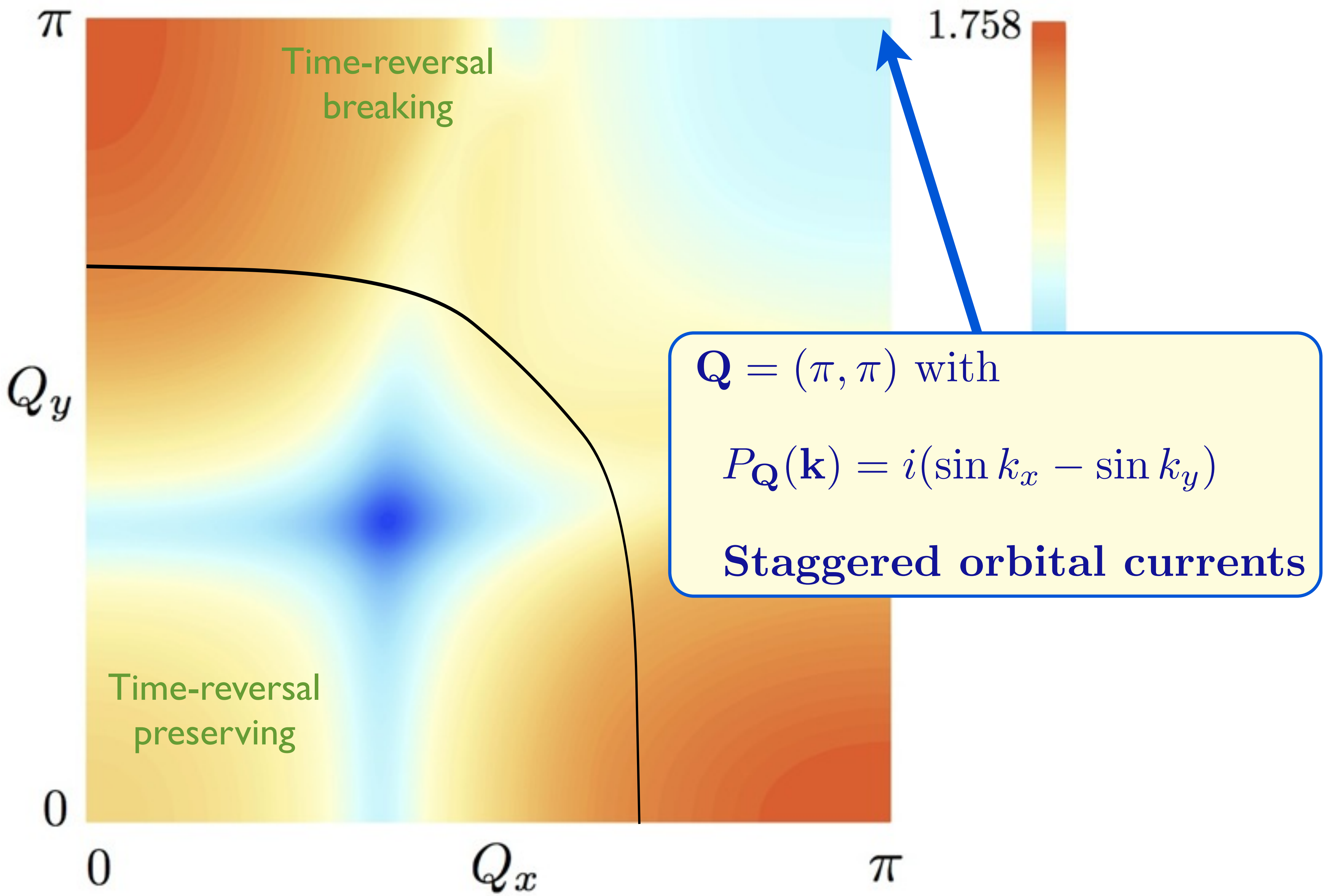

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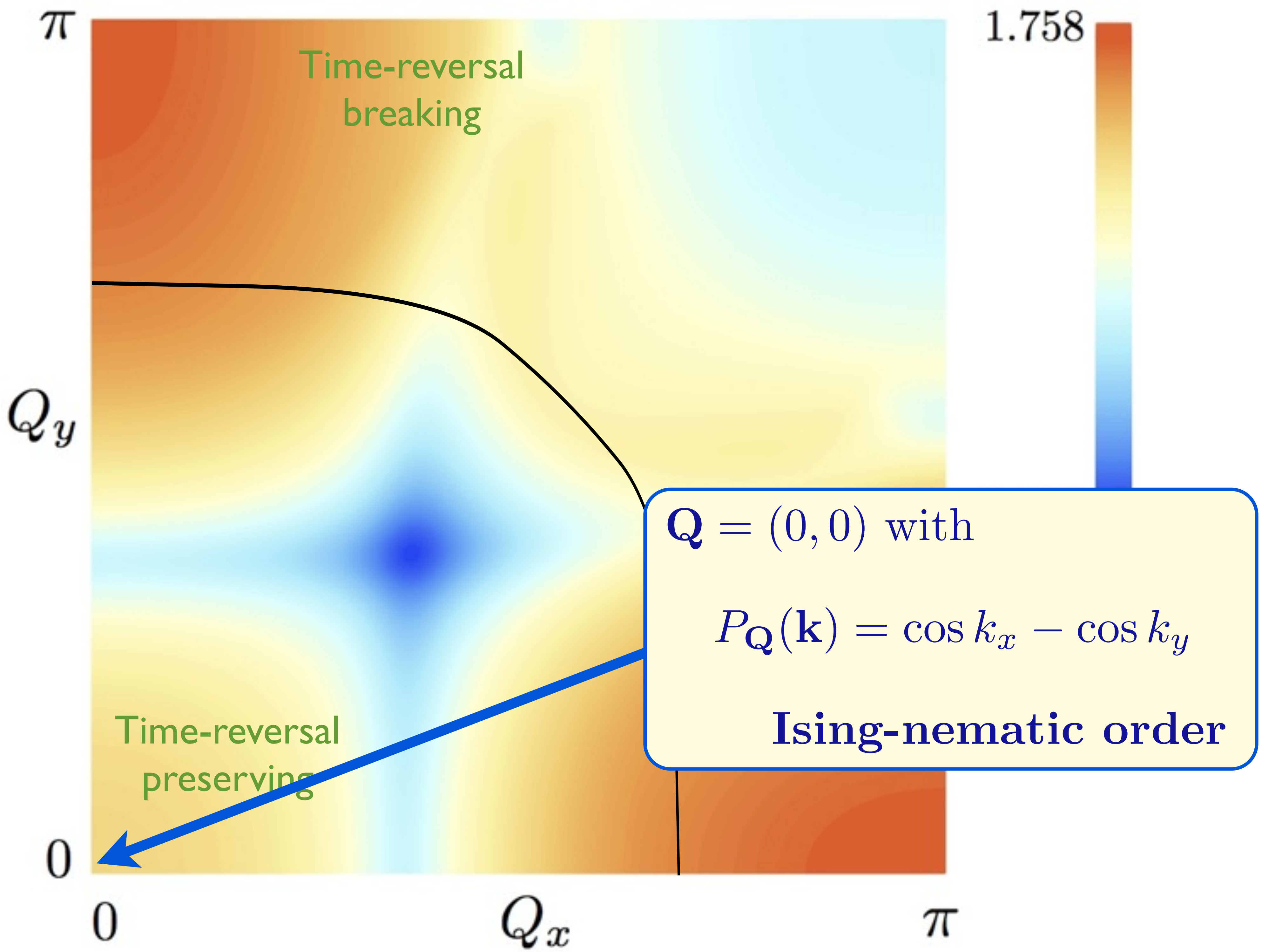

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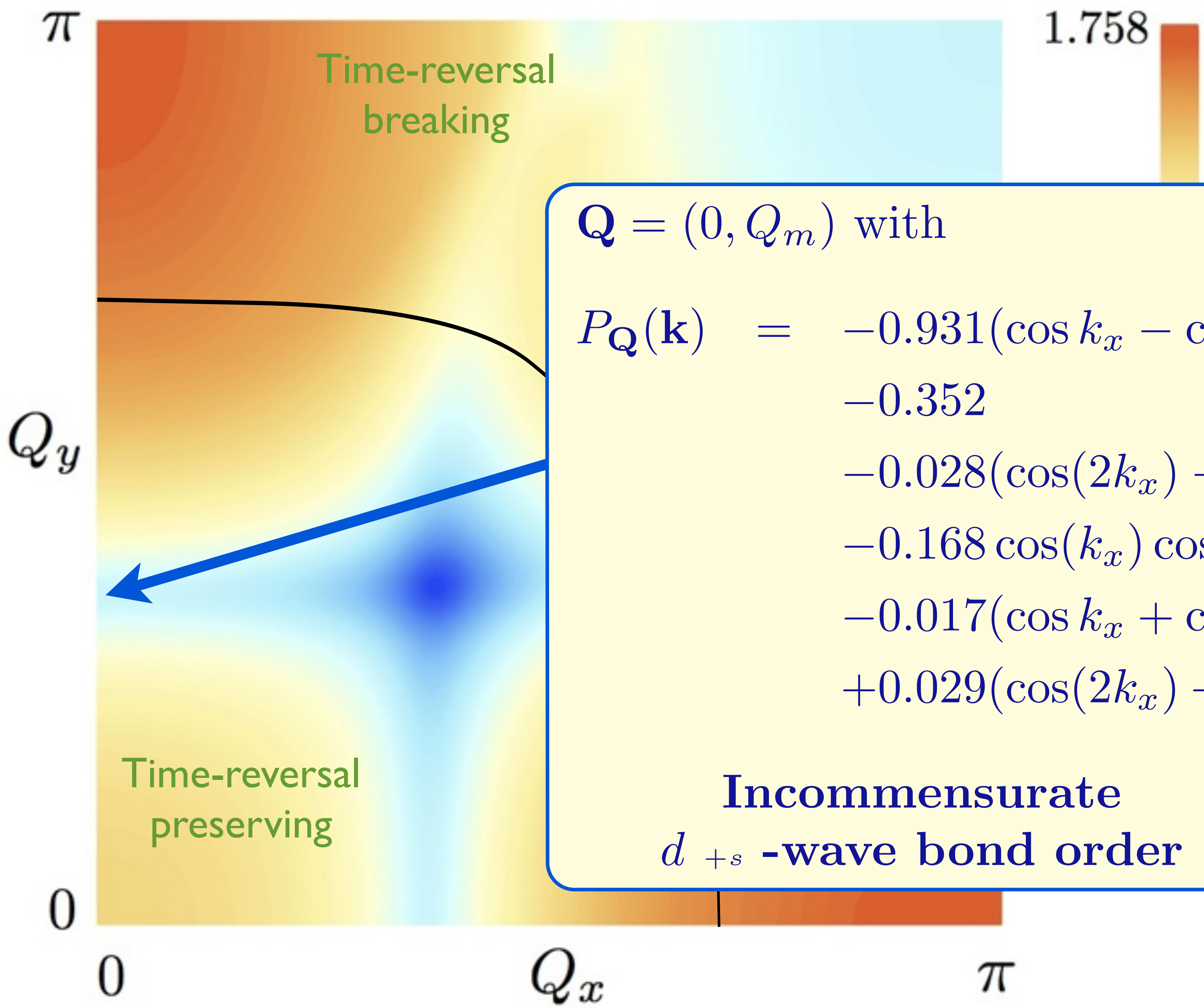


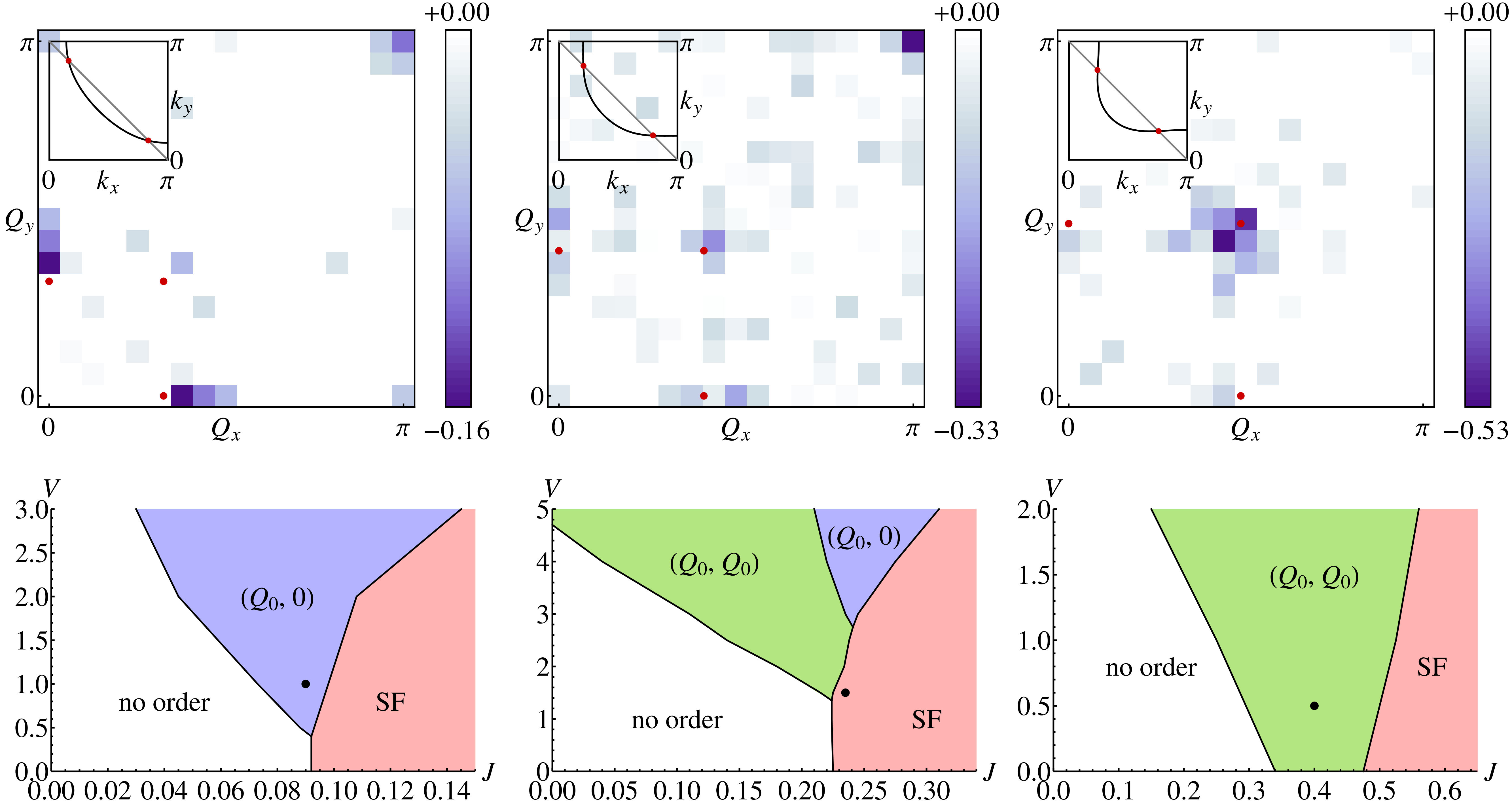
$\mathbf{Q} = (0, Q_m)$ with

$$
\begin{aligned}
P_{\mathbf{Q}}(\mathbf{k}) \; = \; & -0.931(\cos k_x - \cos k_y) \\
& -0.352 \\
& -0.028(\cos(2k_x) - \cos(2k_y)) \\
& -0.168 \cos(k_x) \cos(k_y) \\
& -0.017(\cos k_x + \cos k_y) \\
& +0.029(\cos(2k_x) + \cos(2k_y))
\end{aligned}
$$

Incommensurate
d_{+s} **-wave bond order**

Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $P_{\mathbf{Q}}(\mathbf{k})$ and this leads to the order $\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle = \left[\sum_{\mathbf{k}} P_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$

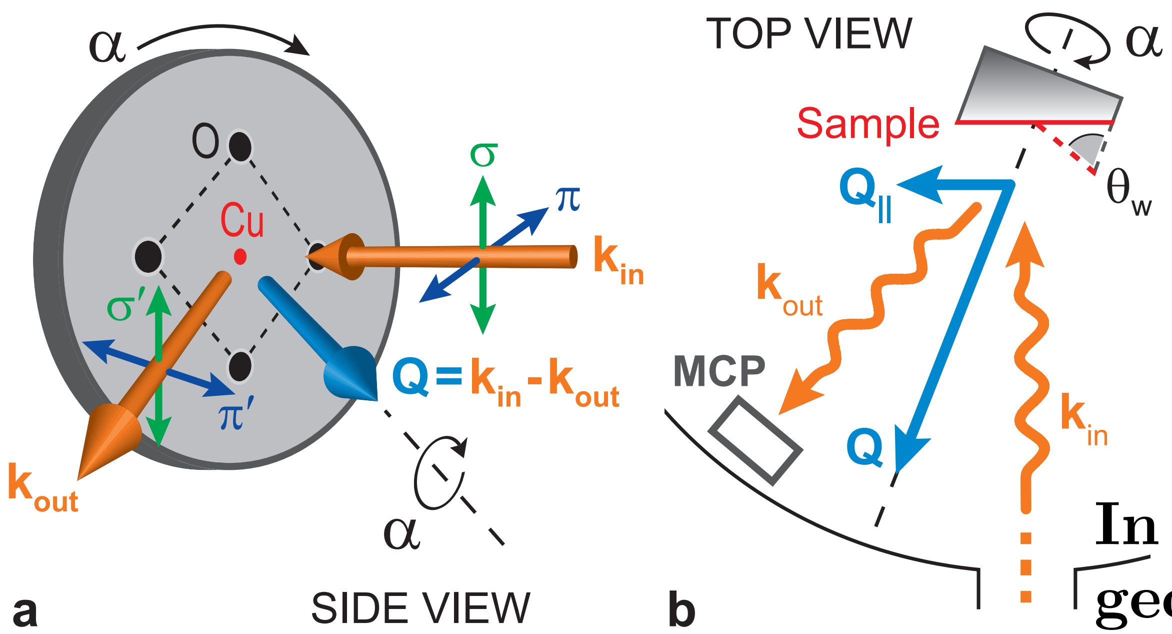
Results of a variational Monte Carlo computation on a wavefunction with double-occupancy projected out.
A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.4807



The ordering at $(Q_0, 0)$ is predominantly d-wave, while that at (Q_0, Q_0) is purely d-wave.

The symmetry of charge order in the cuprates

R. Comin, R. Sutarto, F. He, E. da Silva Neto, L. Chauviere, A. Frano, R. Liang, W.N. Hardy, D.A. Bonn, Y. Yoshida, H. Eisaki, J. E. Hoffman, B. Keimer, G.A. Sawatzky, and A. Damascelli, arXiv:1402.5415



In addition, by adopting a special experimental geometry, we also resolve the *intra-unit-cell* symmetry of the charge ordered state, which is revealed to be a *d*-wave bond-order. These results represent a fundamental advancement in our microscopic description of charge order in cuprates, and provide crucial insights for the understanding of its origin and interplay with superconductivity and magnetism.

This type of *d*-wave bond order at $\mathbf{Q} = (Q_0, 0)$ was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

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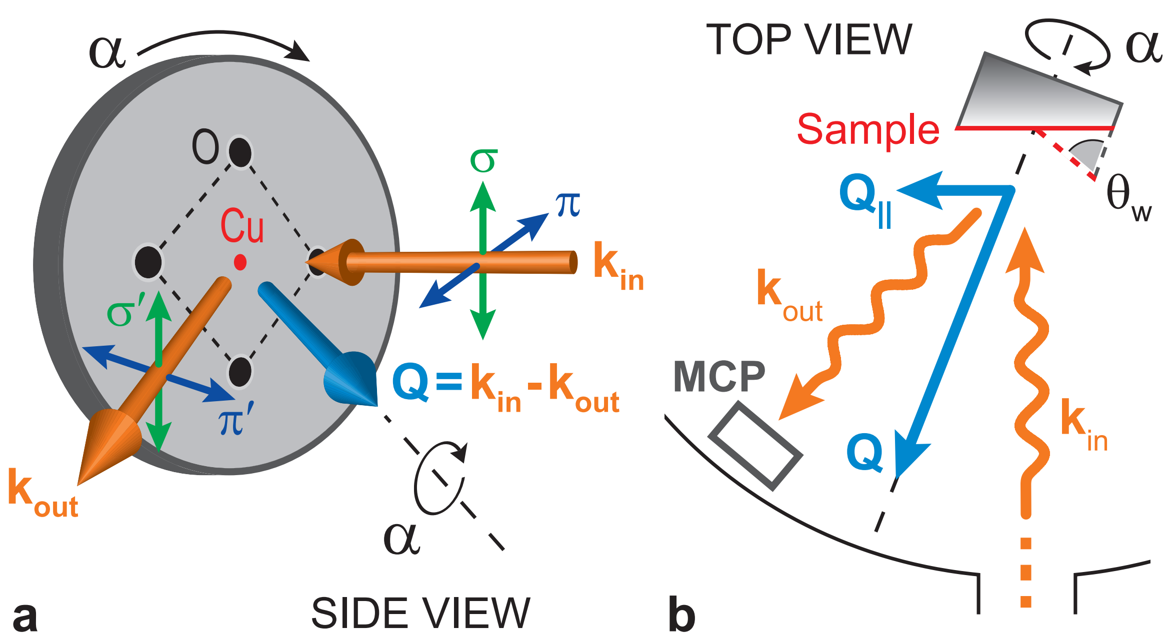

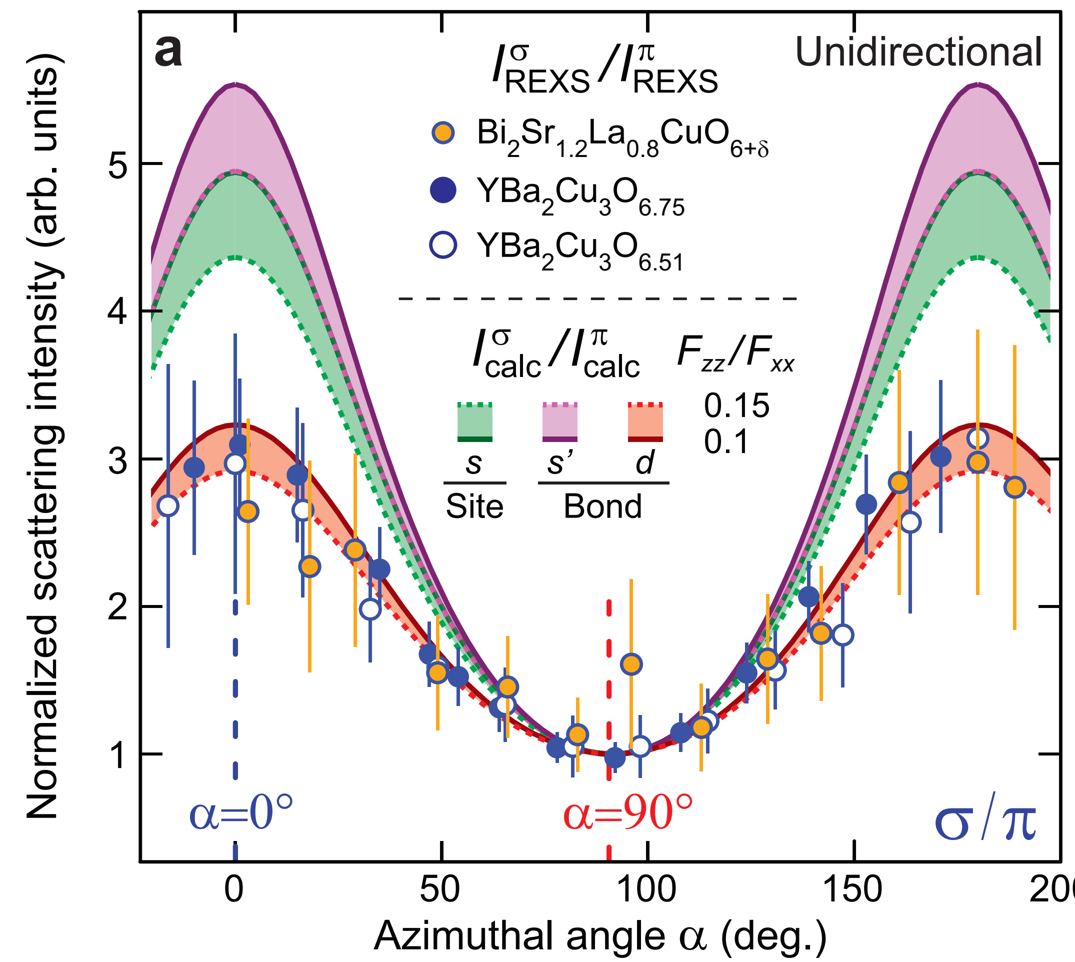

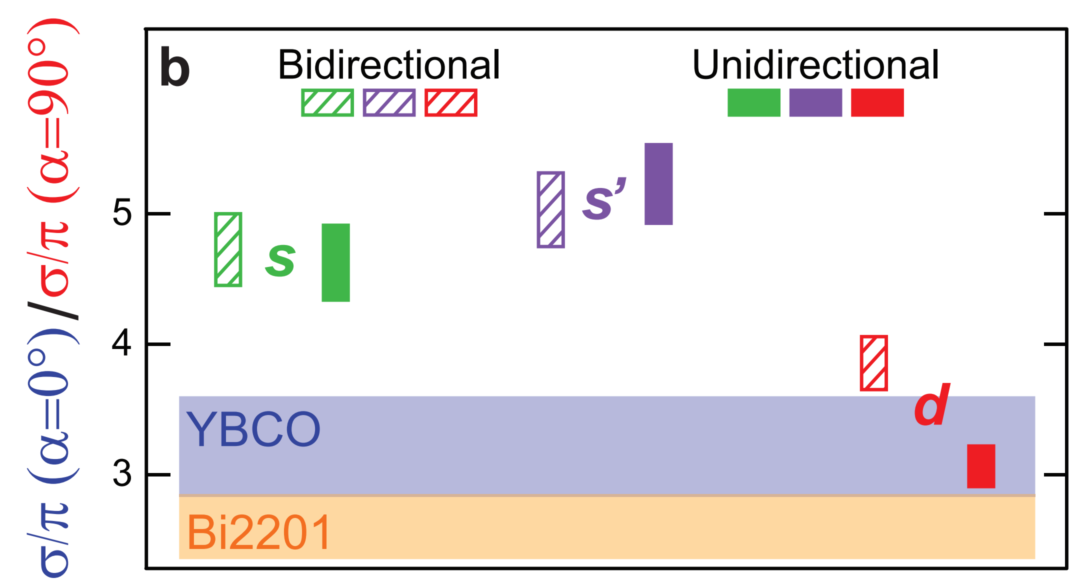

Δ_{CDW}	Probability levels P (%)	
	Bidirectional	Unidirectional
Δ_s	30.3	38.8
$\Delta_{s'}(\cos k_x + \cos k_y)$	12.0	6.0
$\Delta_d\ (\cos k_x - \cos k_y)$	**81.8**	**87.6**

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Outline

1. Antiferromagnetism in metals and *d*-wave superconductivity

2. Competing order: *d*-wave bond order

3. Nematic quantum criticality and the strange metal

4. The pseudogap regime of the hole-doped cuprate superconductors
   Angular fluctuations of a multicomponent order

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Multi-component order parameter for the pseudogap

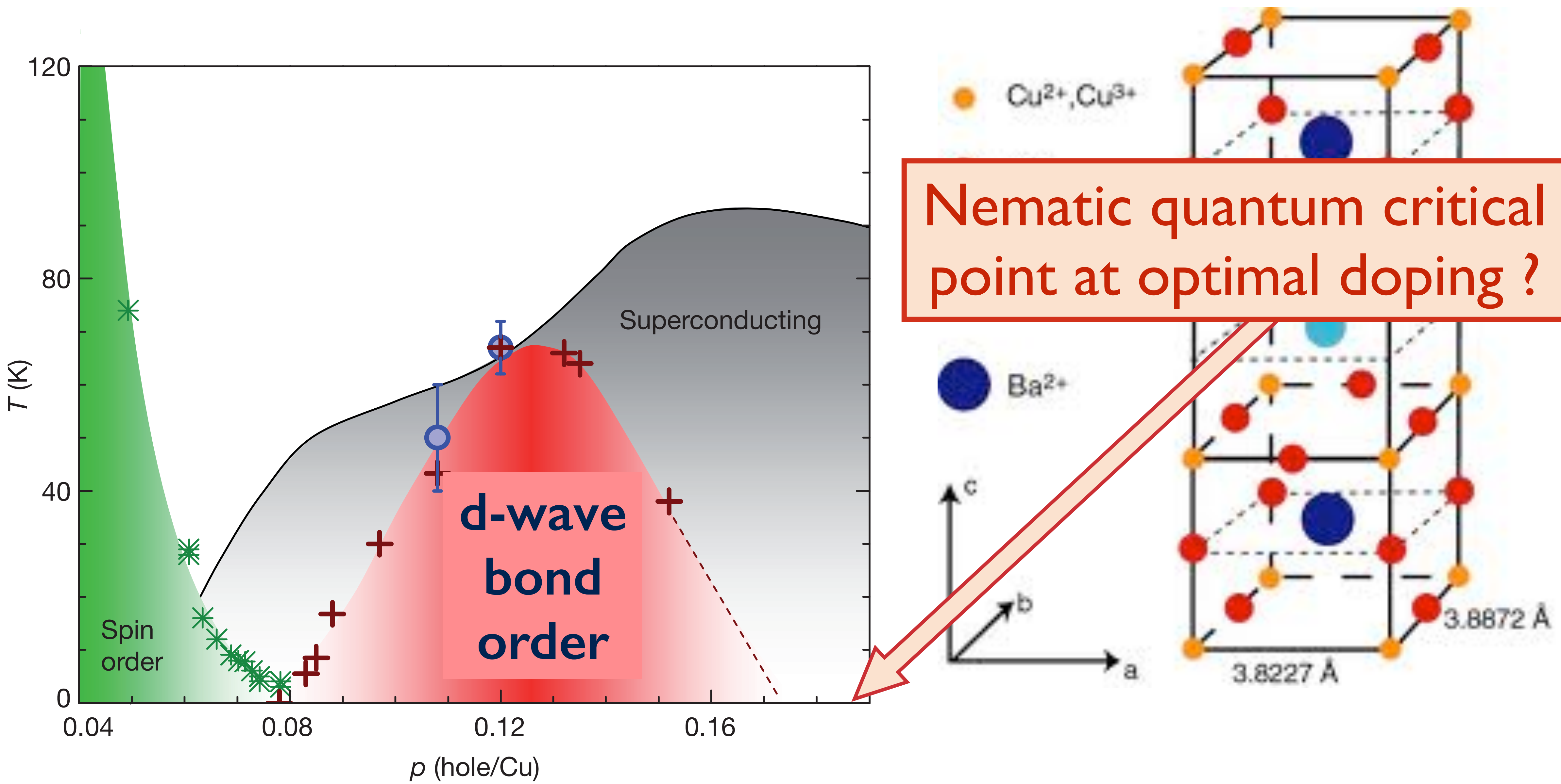
Superconducting order $\Psi(\mathbf{r})$:

$$\left\langle c_{i\alpha}^{\dagger} c_{j\beta}^{\dagger} \right\rangle = \varepsilon_{\alpha\beta} \left[\sum_{\mathbf{k}} \Delta_S(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] \Psi\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right)$$

Charge/bond order $\Phi_{x,y}(\mathbf{r})$ at wavevectors $\mathbf{Q}_{x,y}$:

$$\begin{aligned} \left\langle c_{i\alpha}^{\dagger} c_{j\beta} \right\rangle &= \delta_{\alpha\beta} \left[\sum_{\mathbf{k}} P_{\mathbf{Q}_x}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_x\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_x\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right) \\ &+ \delta_{\alpha\beta} \left[\sum_{\mathbf{k}} P_{\mathbf{Q}_y}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_y\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_y\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right) \end{aligned}$$

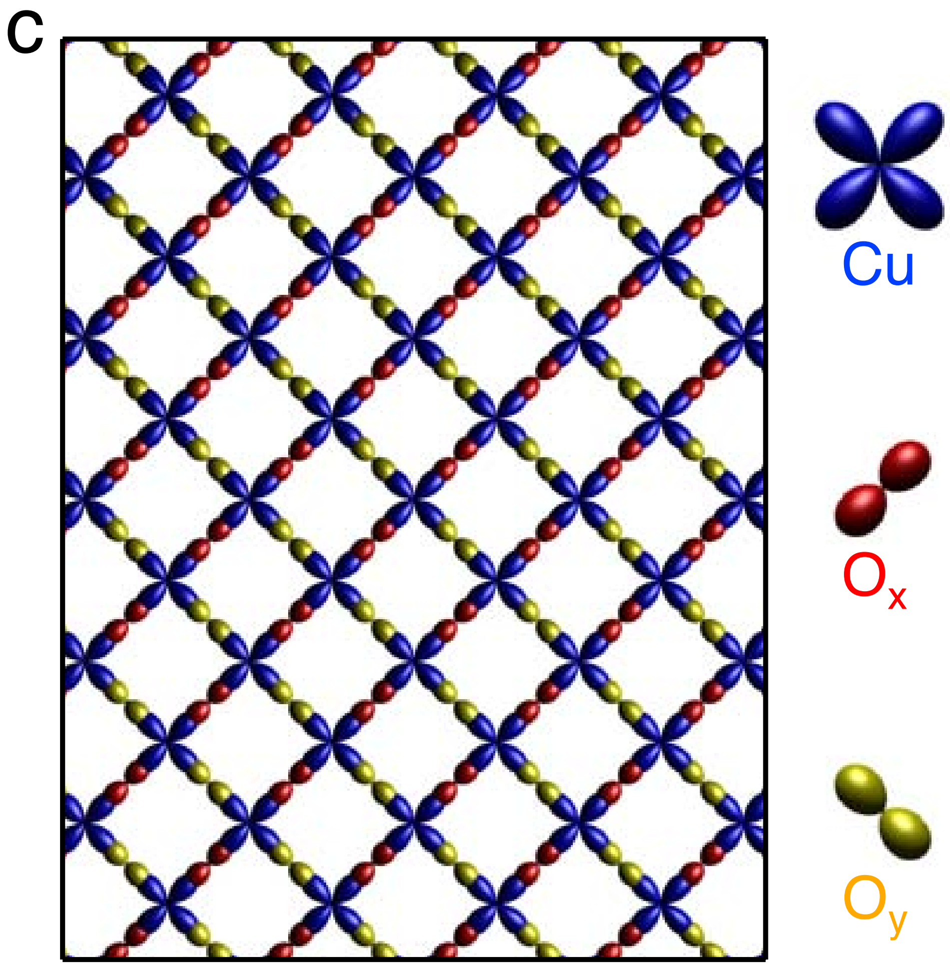
Associated with the bond orders $\Phi_x(\mathbf{r})$ and $\Phi_y(\mathbf{r})$, we can define the Ising-nematic order parameter $\phi(\mathbf{r}) = |\Phi_x(\mathbf{r})|^2 - |\Phi_y(\mathbf{r})|^2$. We can imagine a state with only Ising-nematic order $\langle \phi \rangle \neq 0$, but no bond order $\langle \Phi_x \rangle = \langle \Phi_y \rangle = 0$.

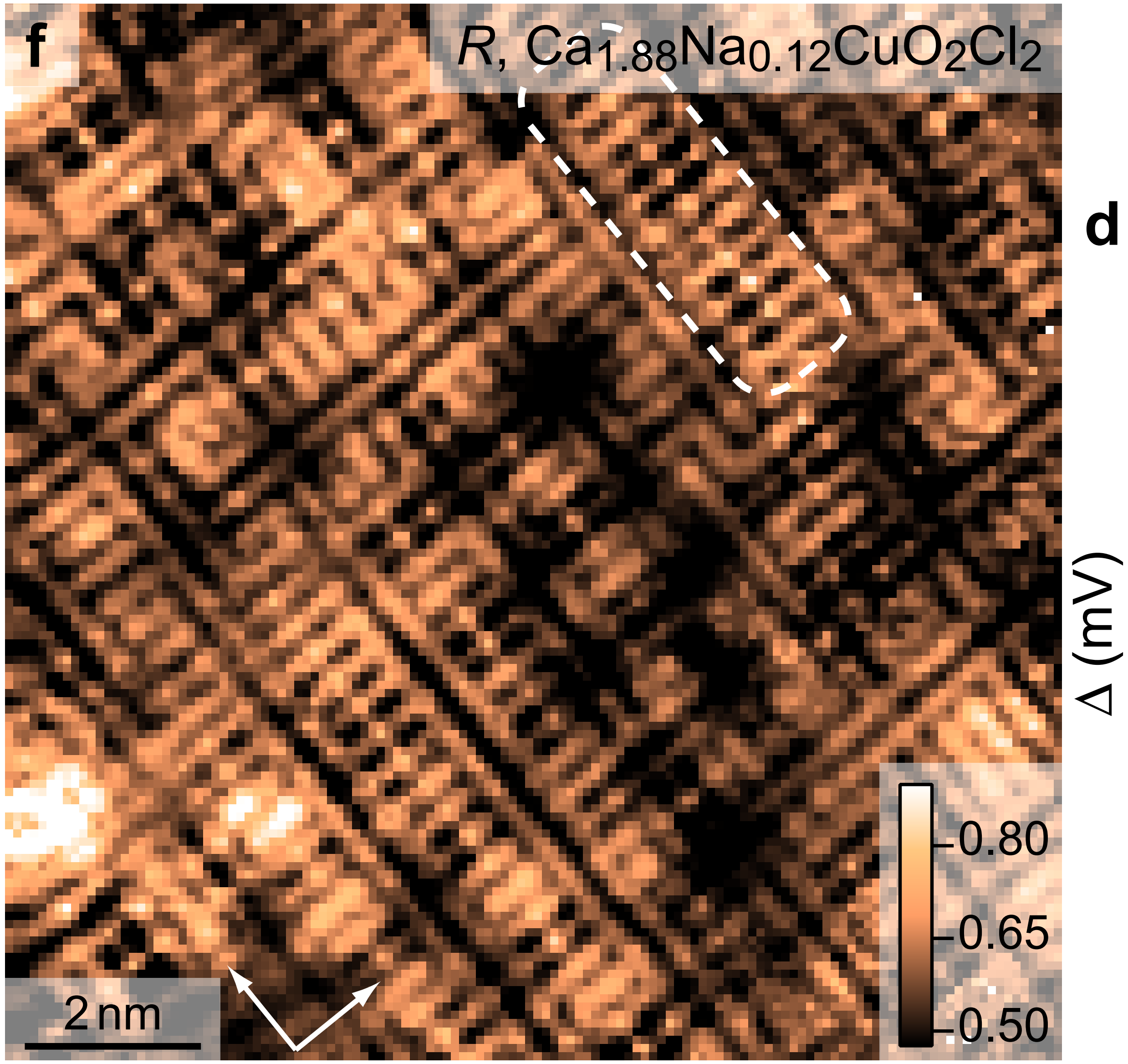


Visualization of the emergence of the pseudogap state and the evolution to superconductivity in a lightly hole-doped Mott insulator

Y. Kohsaka, T. Hanaguri, M. Azuma, M. Takano, J. C. Davis, and H. Takagi

Nature Physics, **8**, 534 (2012).





Evidence for "nematic" order
(*i.e.* breaking of 90° rotation symmetry) in $Ca_{1.88}Na_{0.12}CuO_2Cl_2$.

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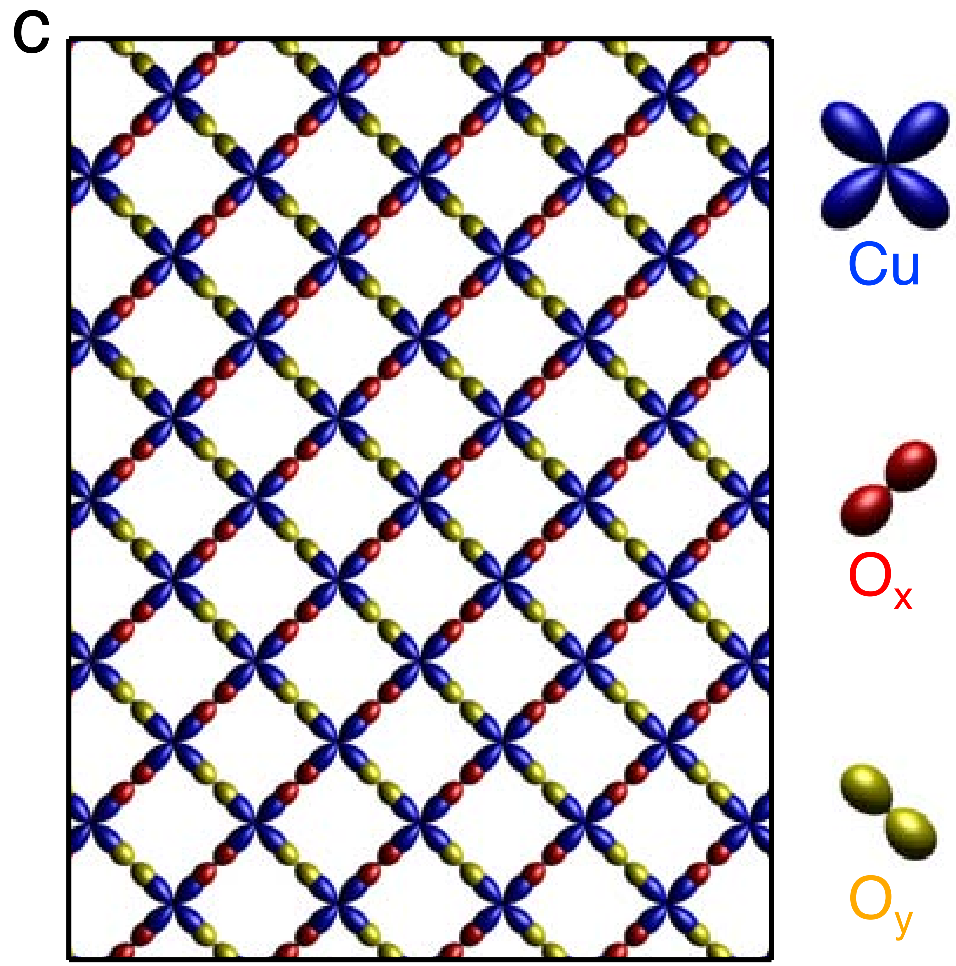

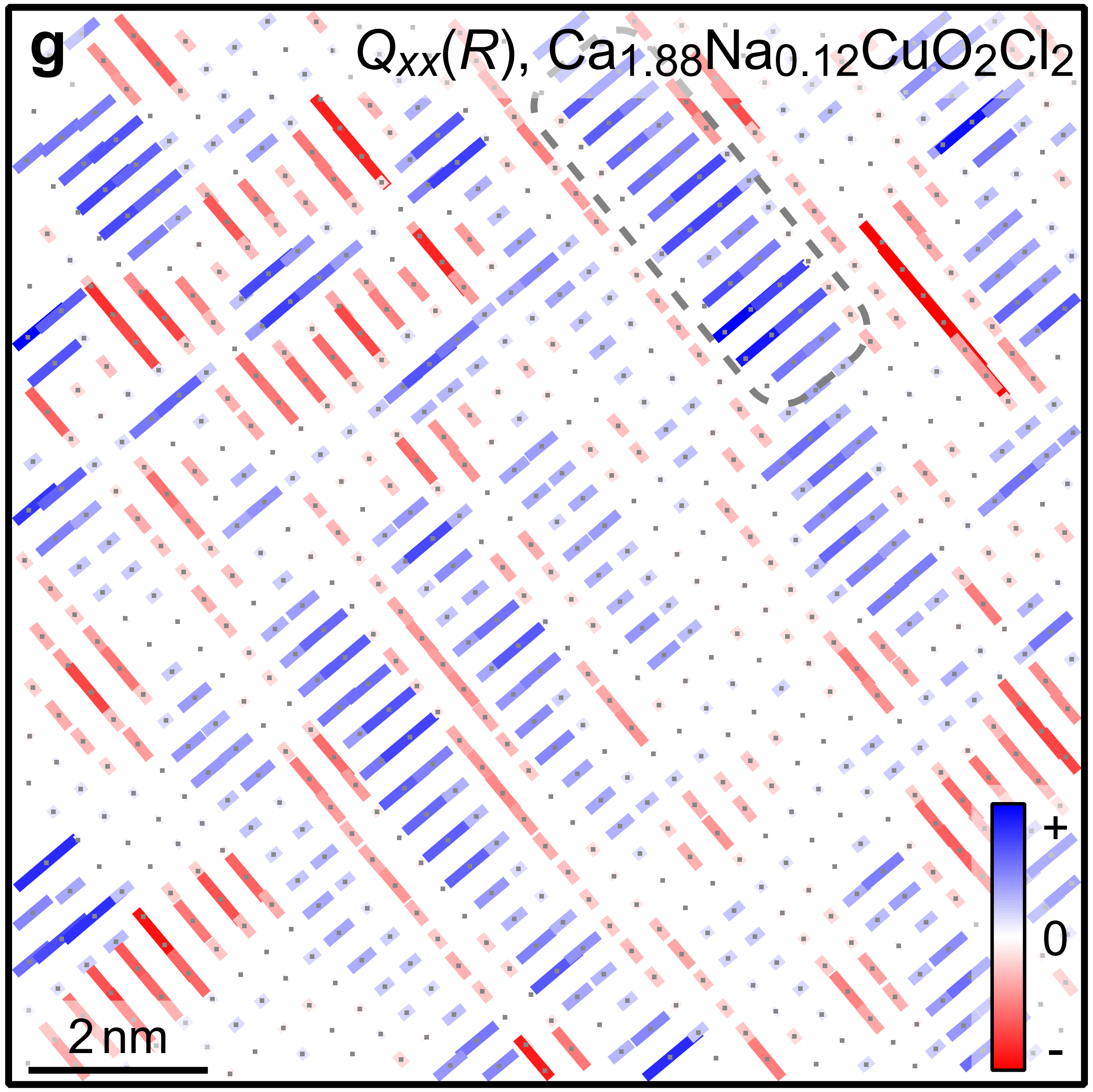

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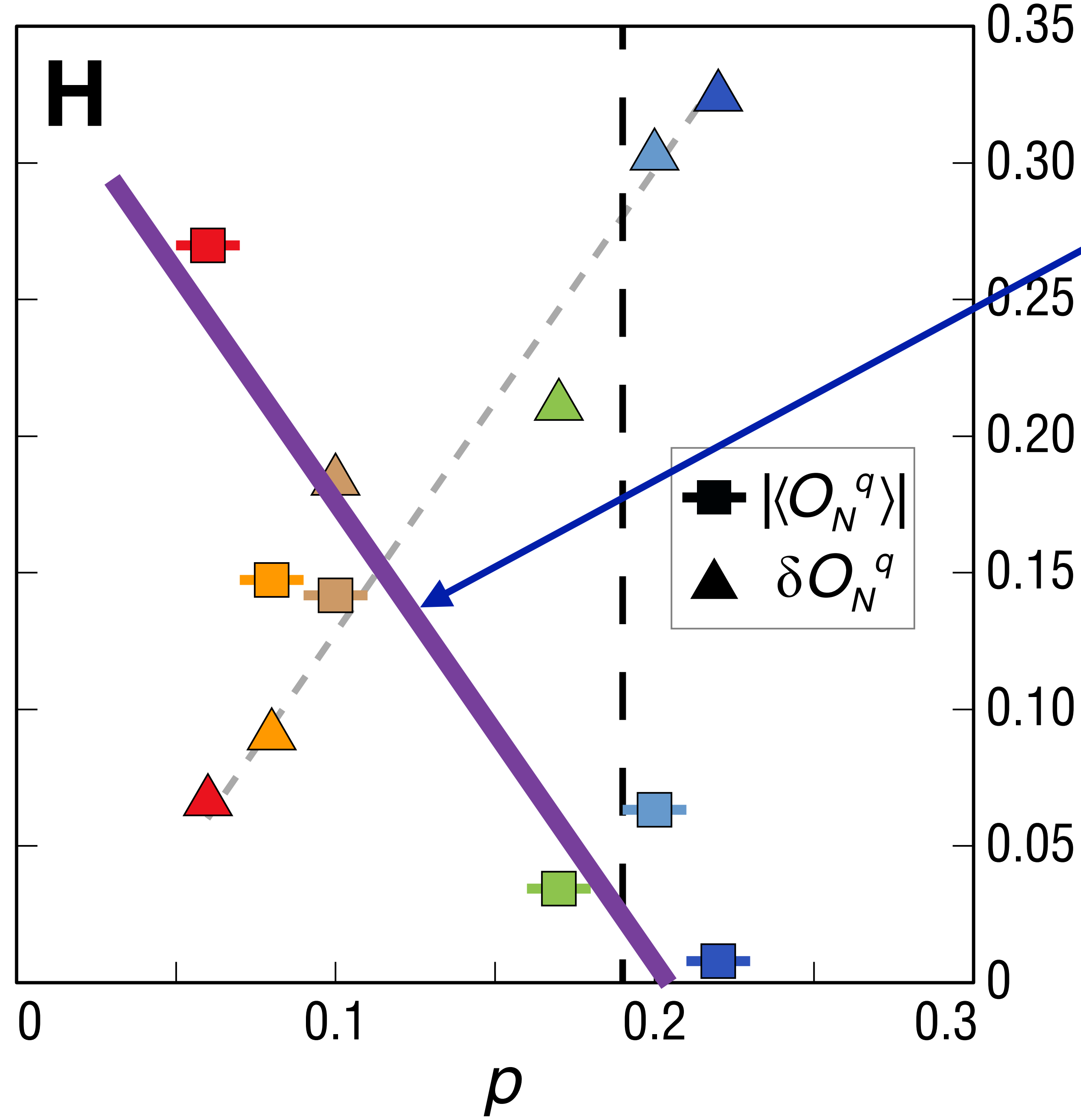
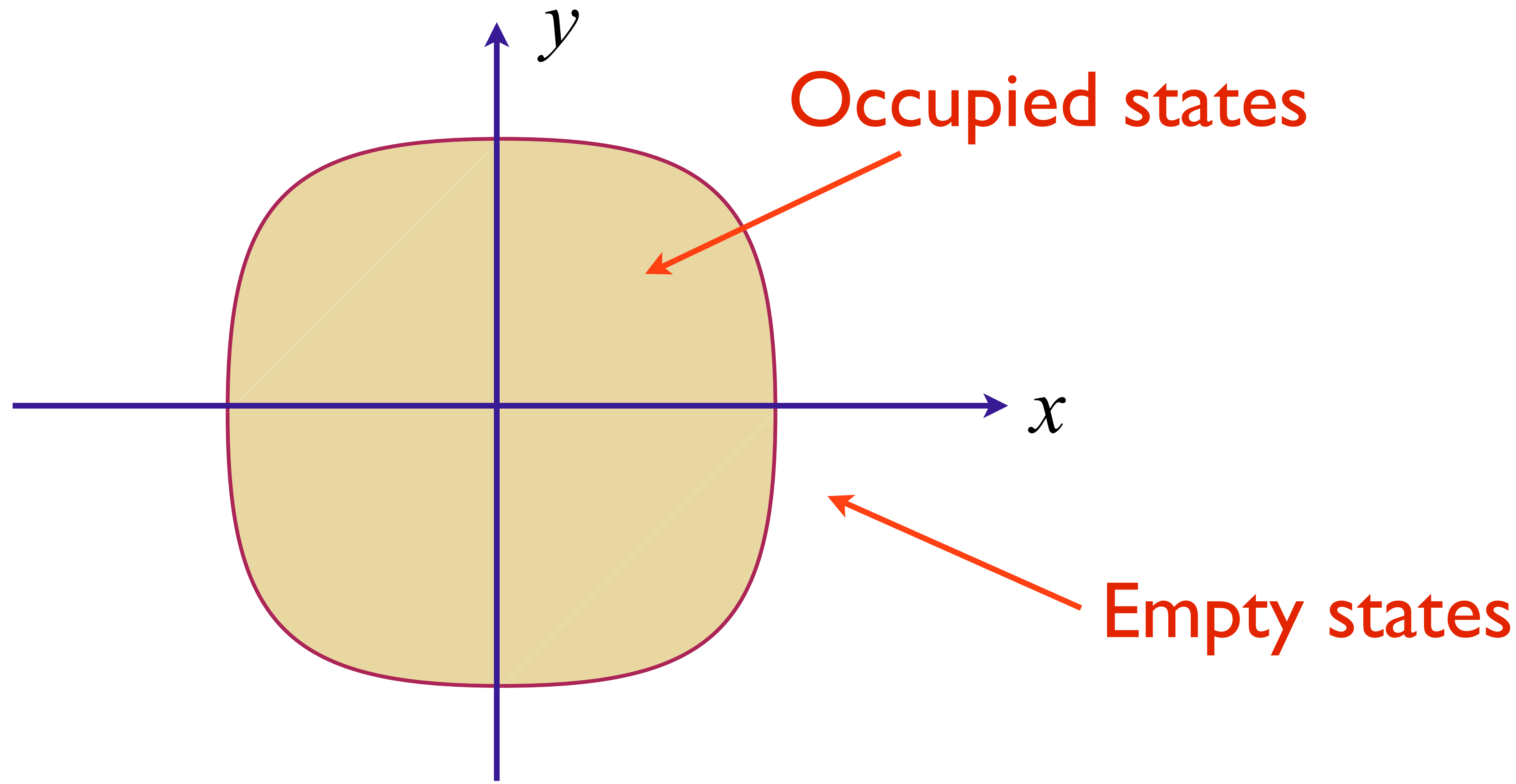


Nematic order as a function of hole density, p

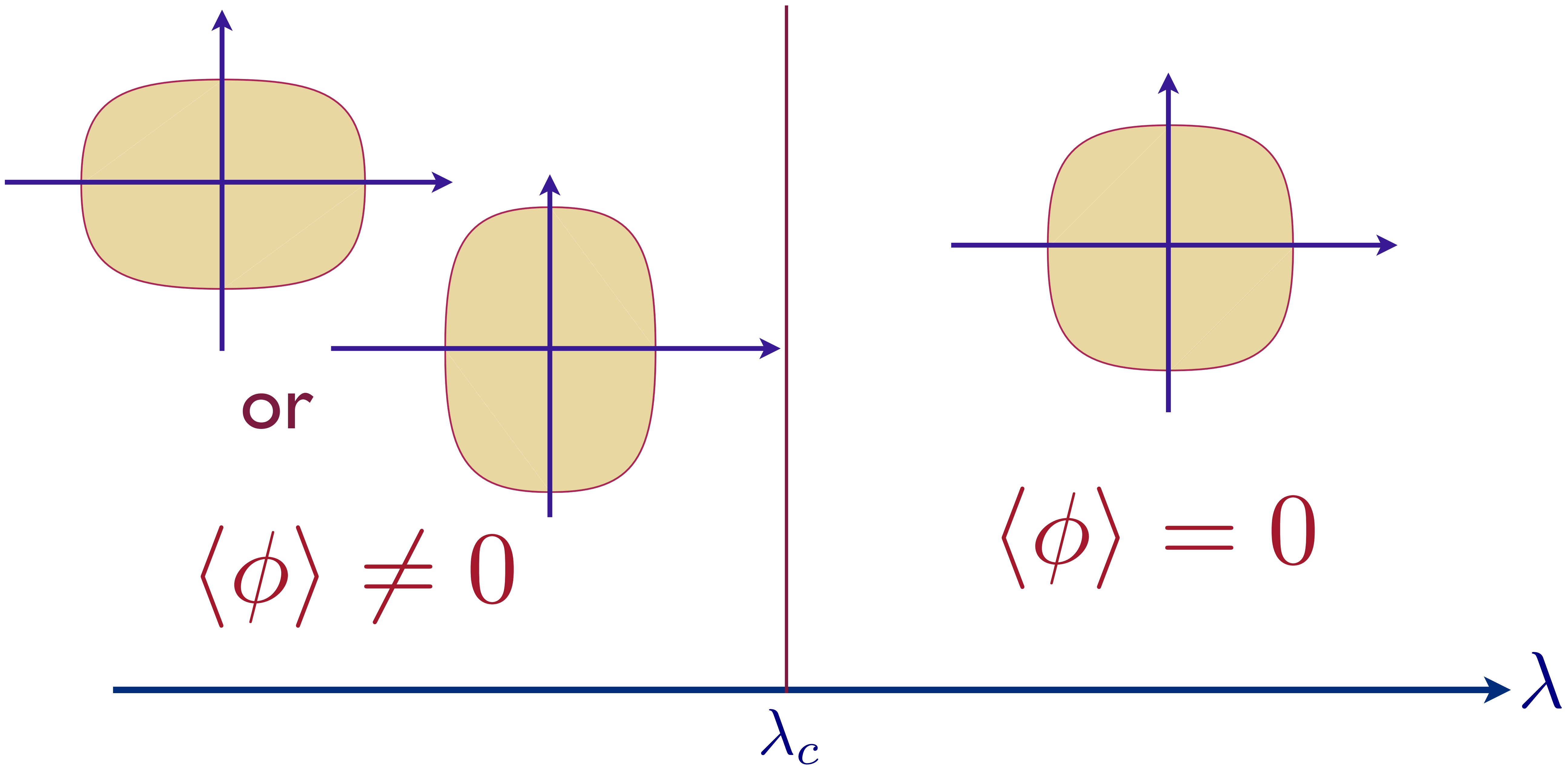
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki , S. Uchida , M. J. Lawler, Eun-Ah Kim, and J. C. Davis, preprint

Quantum criticality of Ising-nematic ordering in a metal



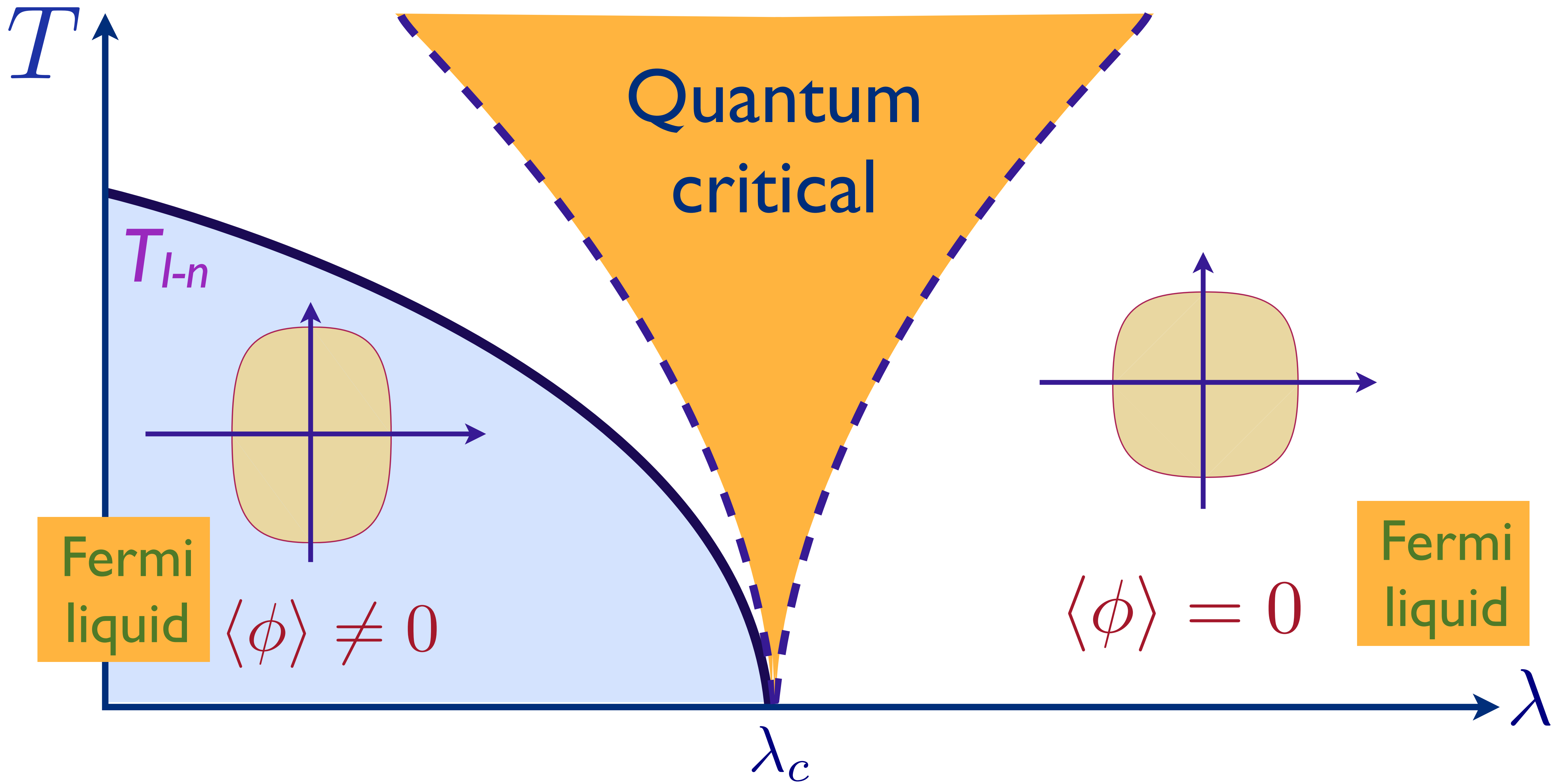
A metal with a Fermi surface
with full square lattice symmetry

Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

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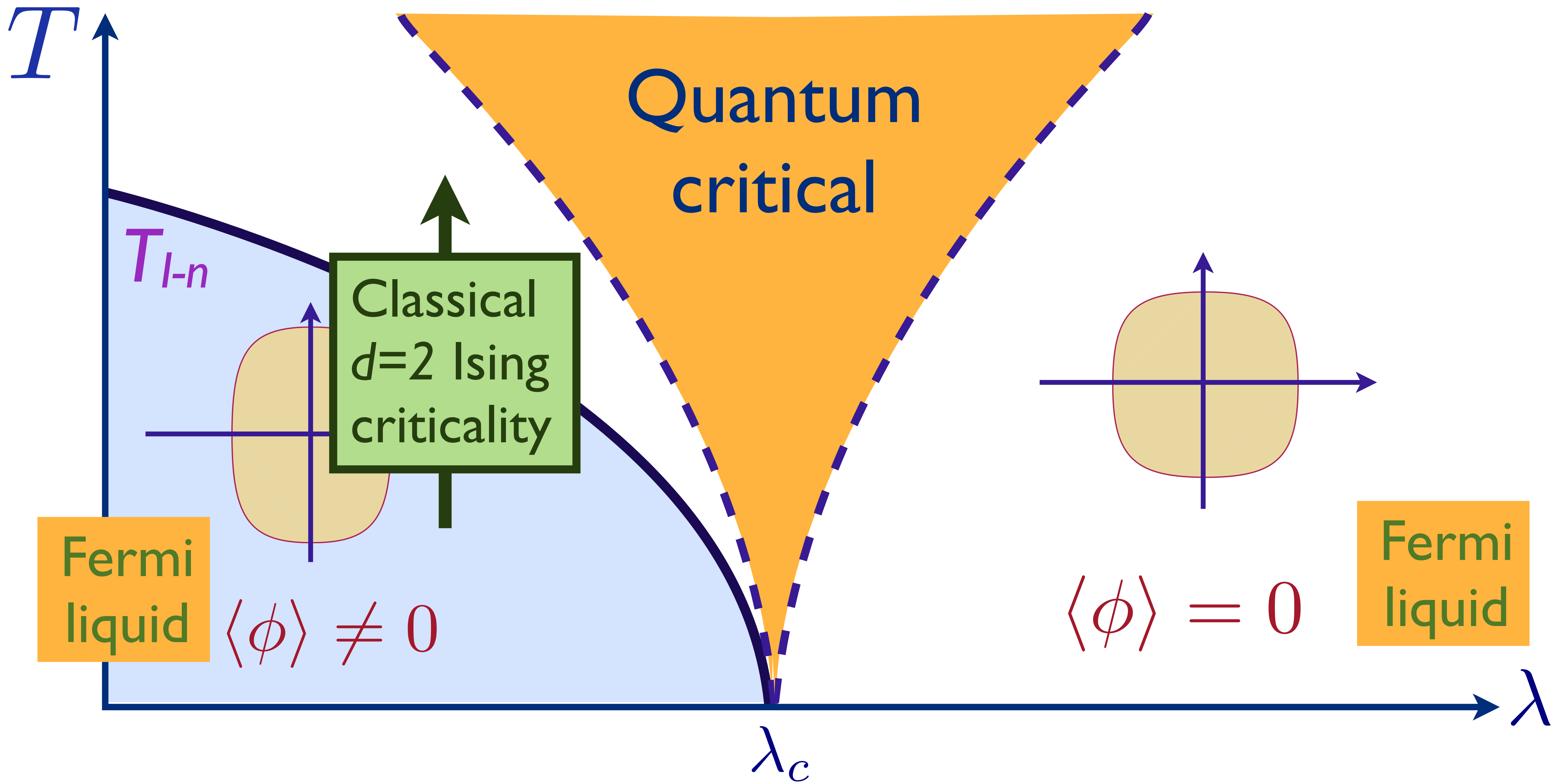

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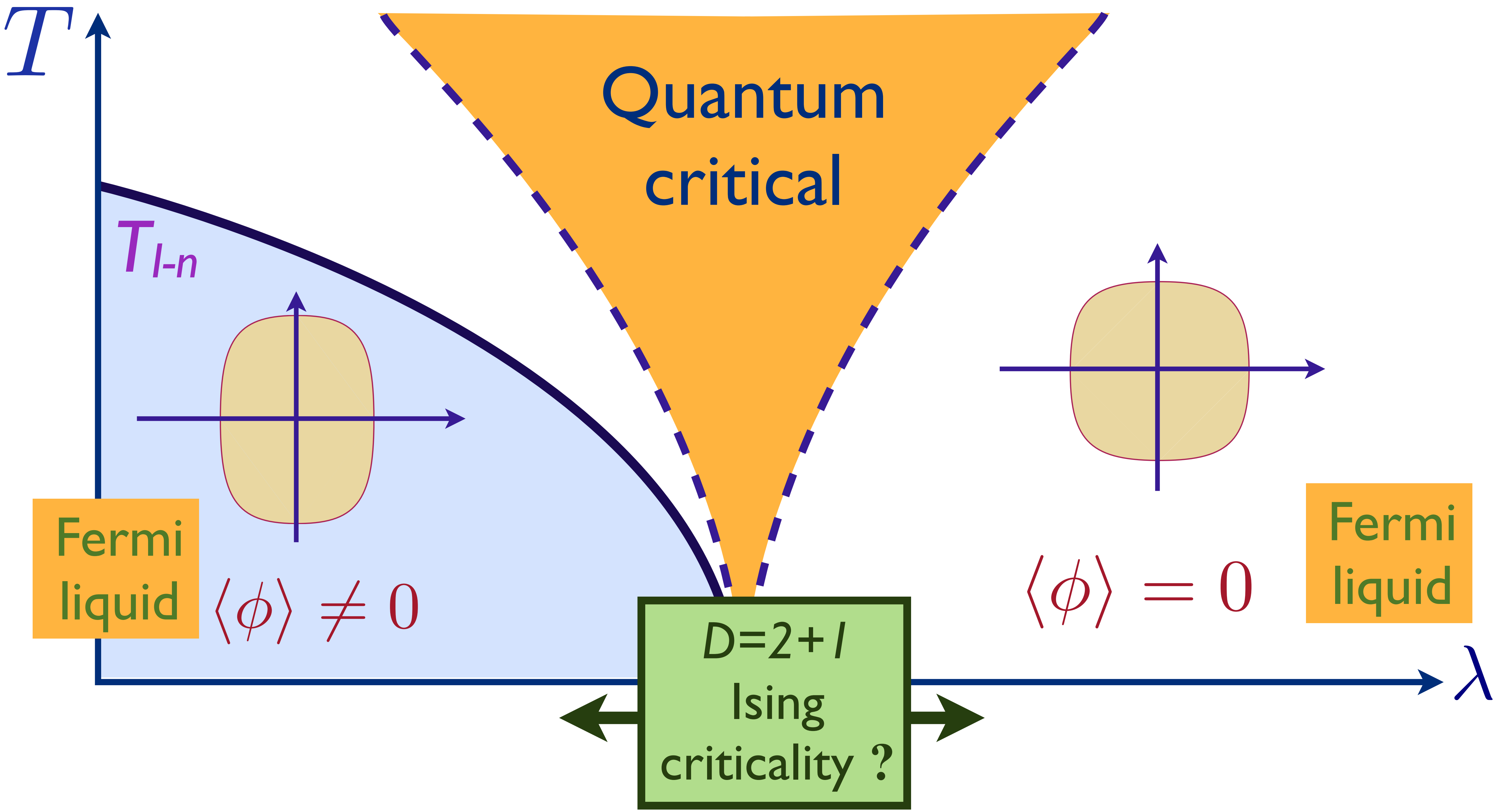

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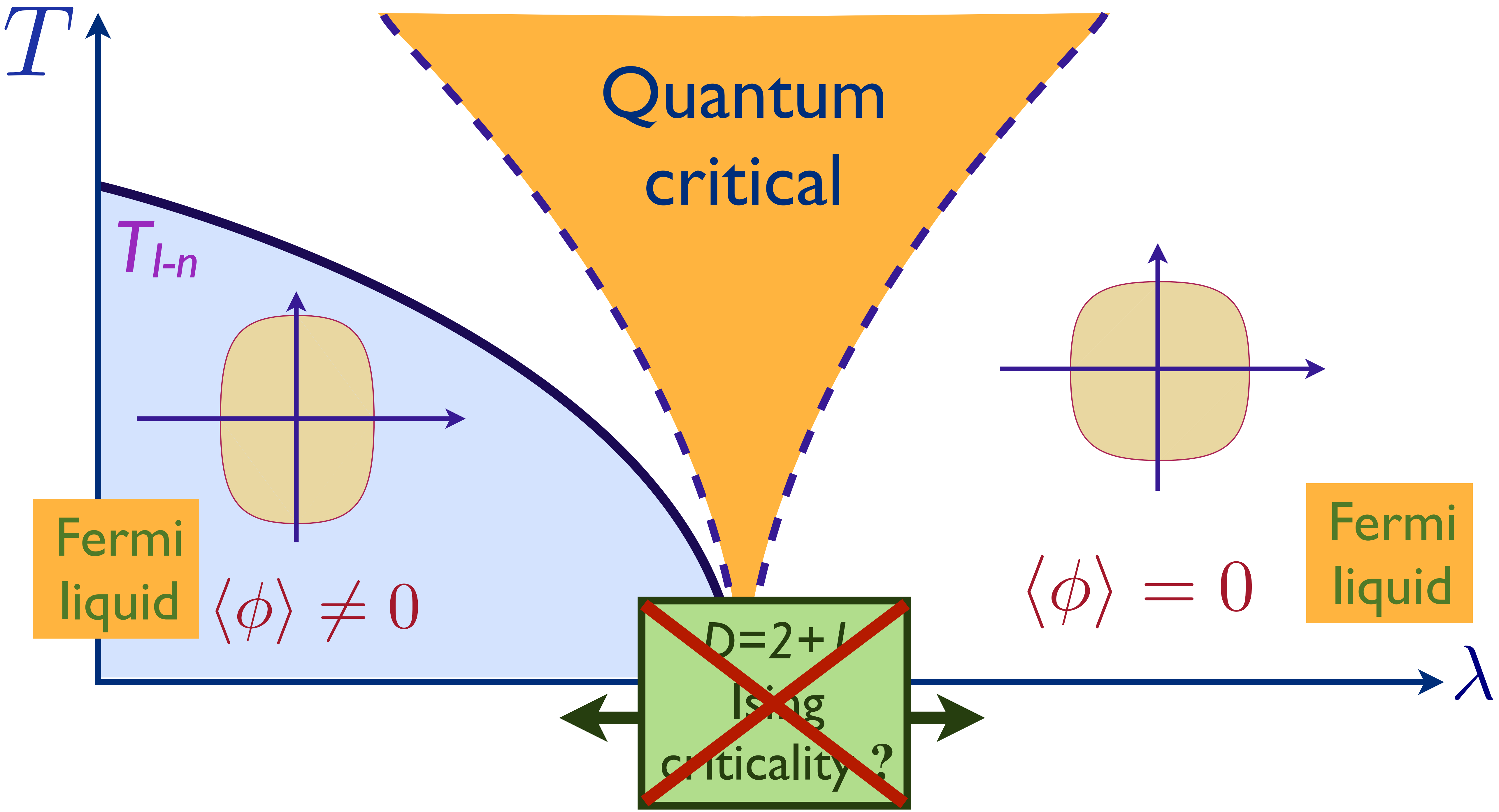

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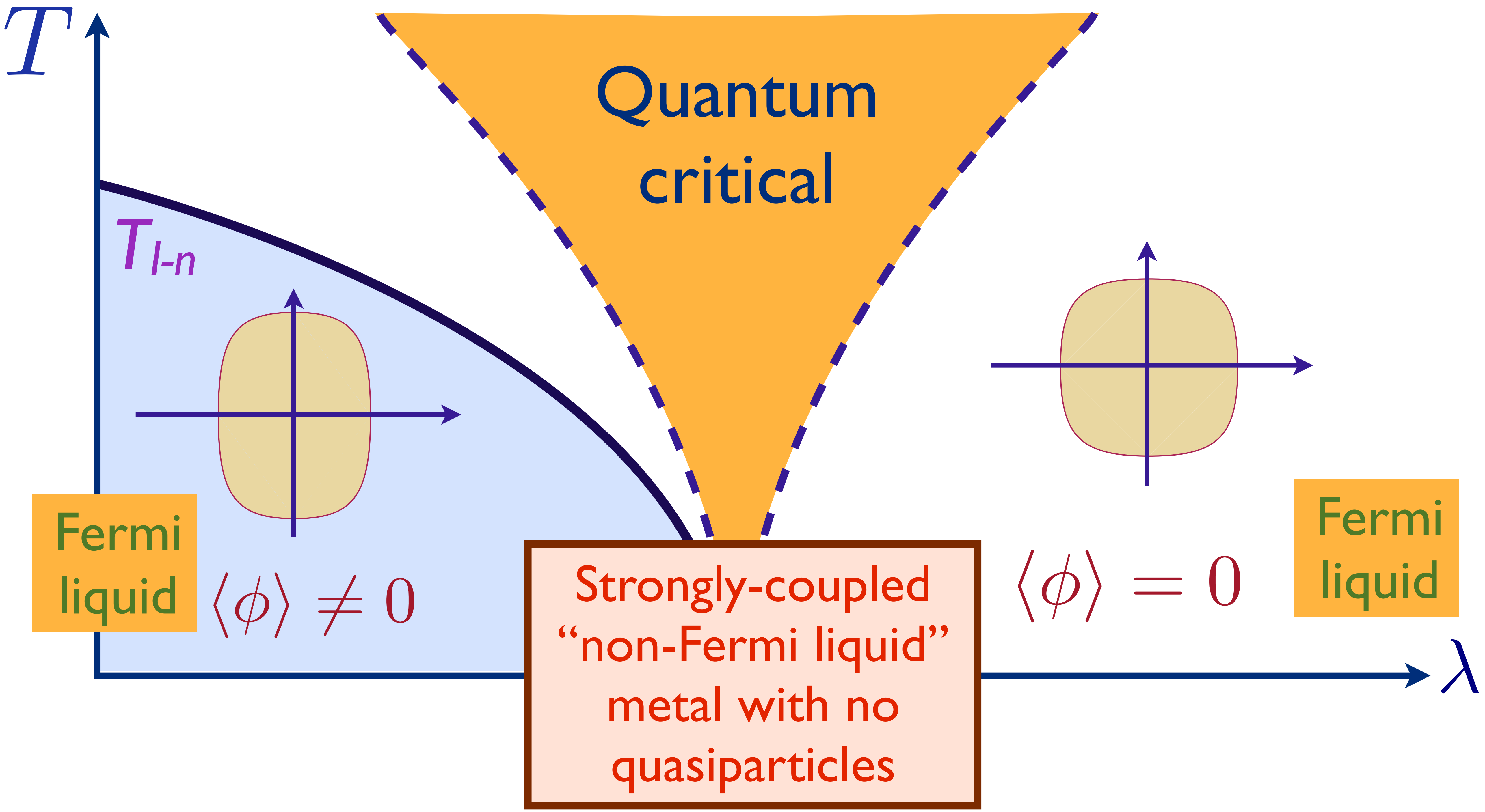

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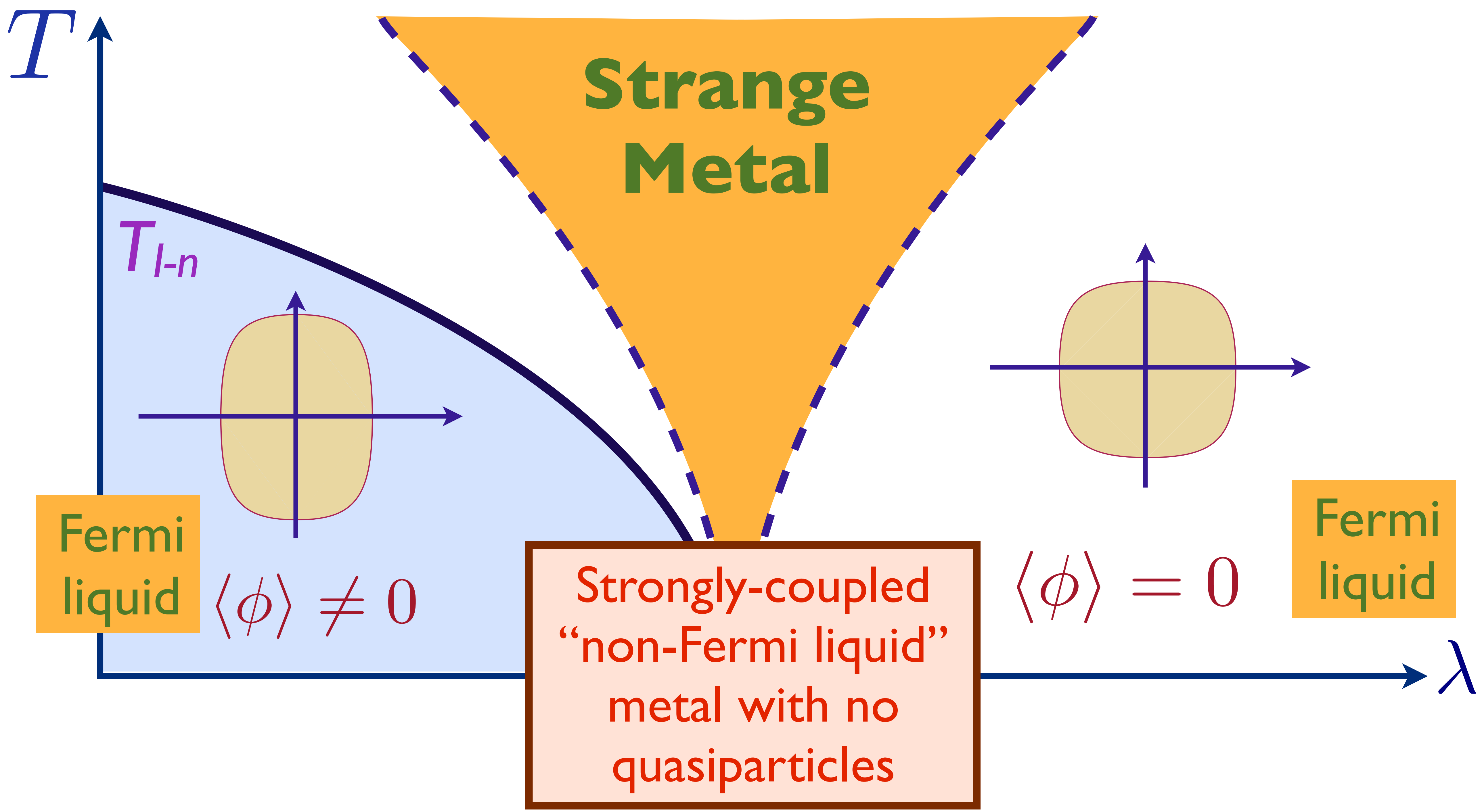

Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4 \right]$$

Quantum criticality of Ising-nematic ordering in a metal

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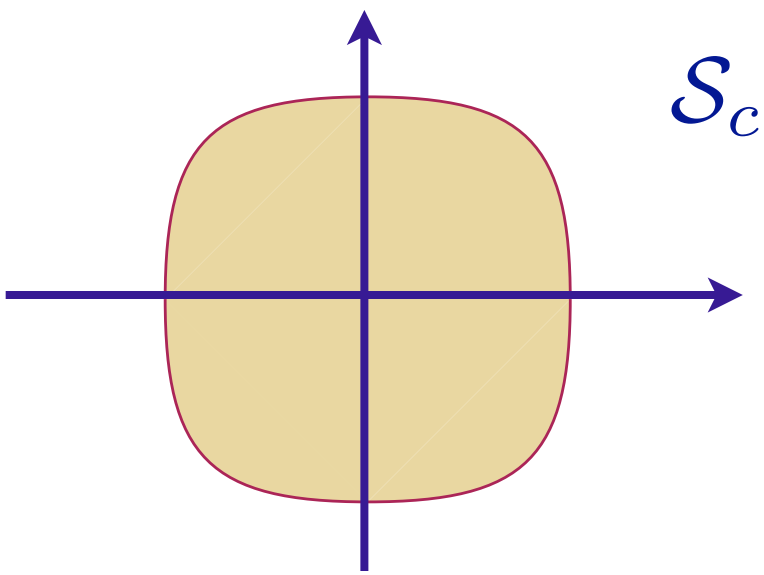
Effective action for electrons:



$$\mathcal{S}_c = \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \right]$$

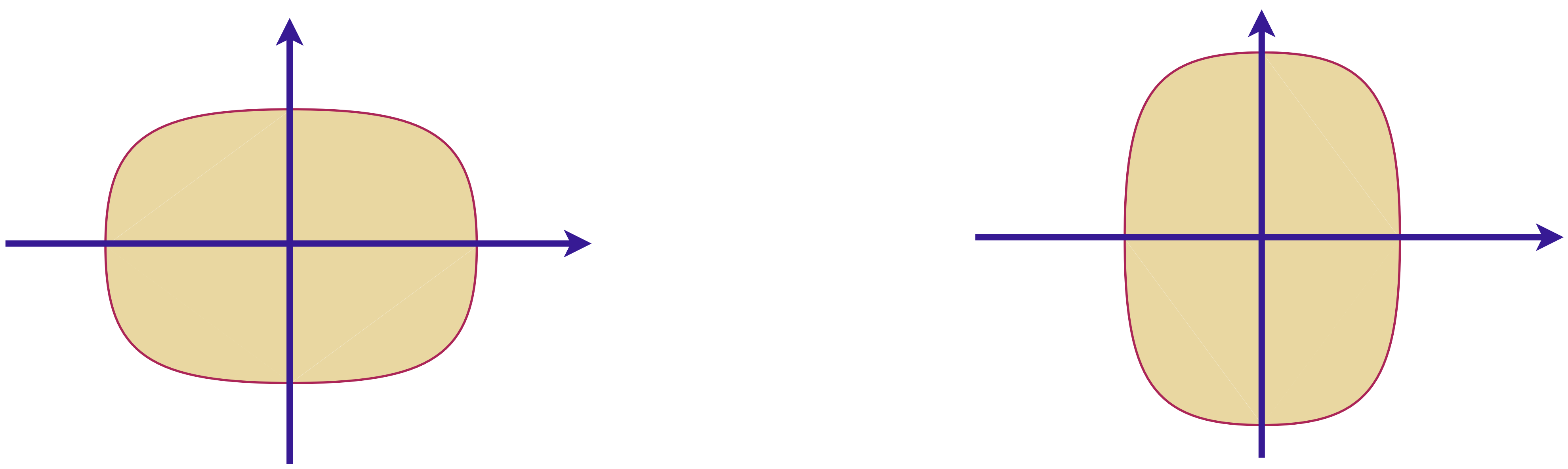
$$\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

Quantum criticality of Ising-nematic ordering in a metal

"Yukawa" coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y\right) c^{\dagger}_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

for spatially dependent ϕ



$$\langle \phi \rangle > 0 \qquad\qquad \langle \phi \rangle < 0$$

Quantum criticality of Ising-nematic ordering in a metal

The "standard model":

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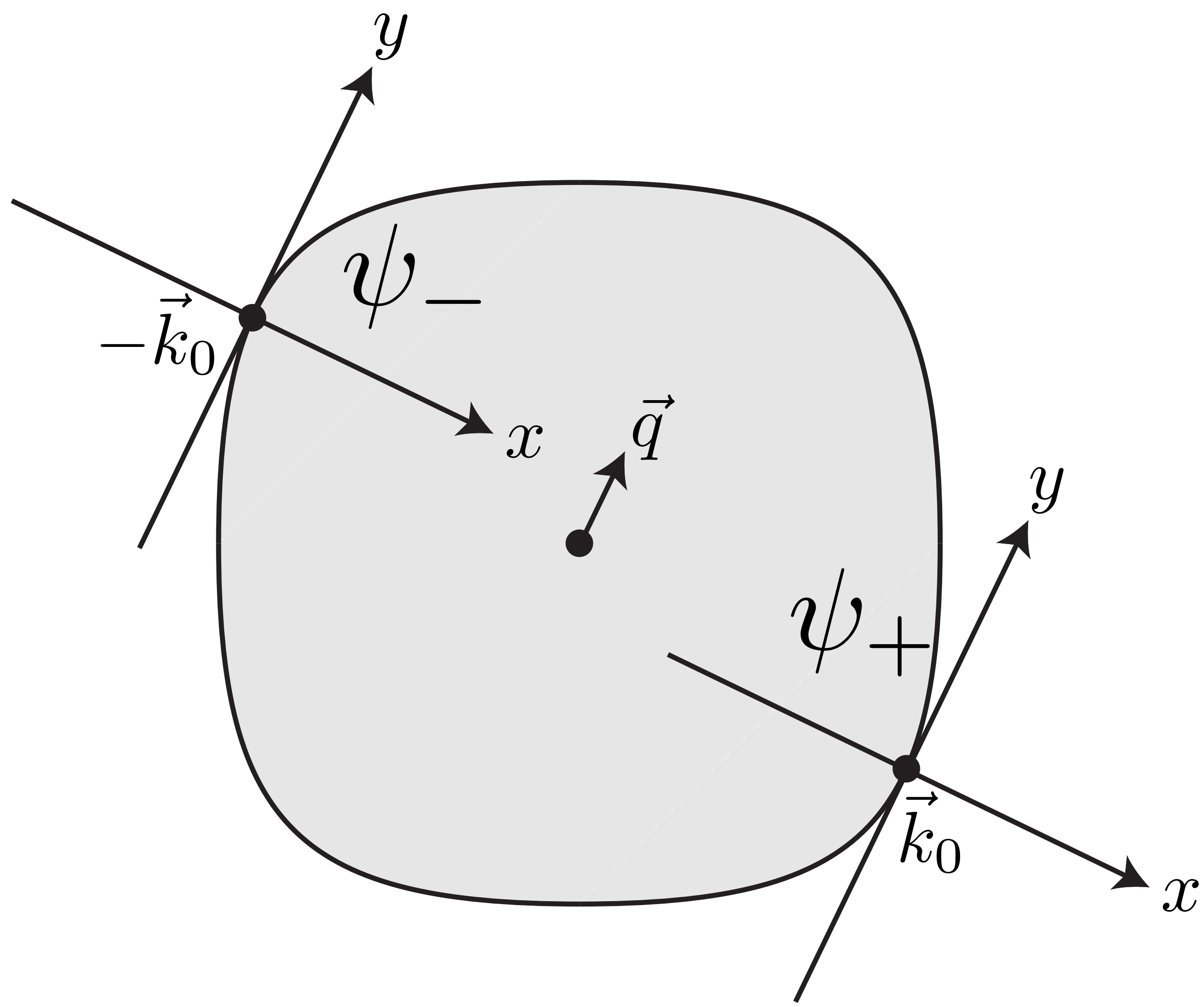
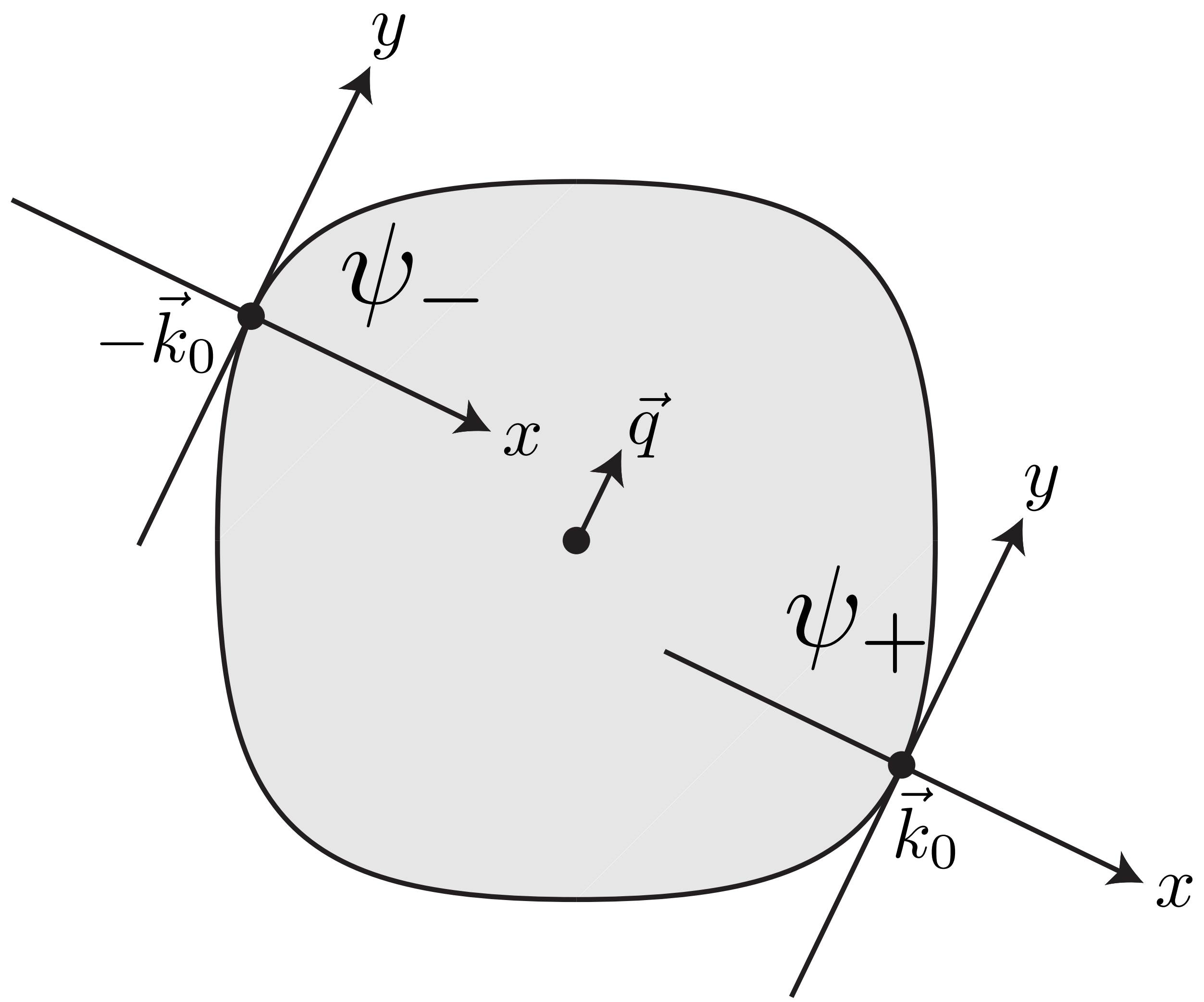
Quantum criticality of Ising-nematic ordering in a metal



- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

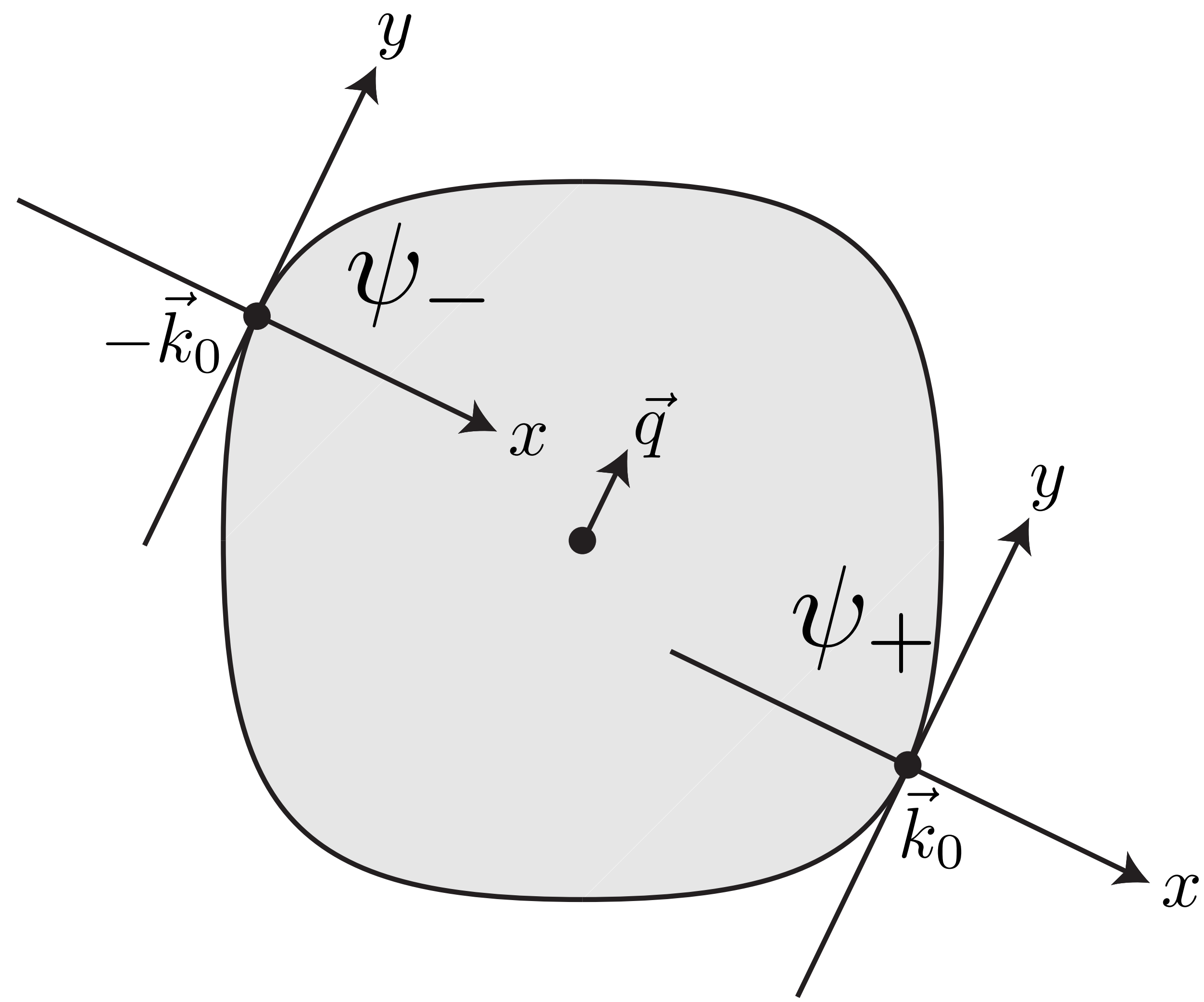
Quantum criticality of Ising-nematic ordering in a metal



- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson (ϕ) kinetic energy about $\vec{q} = 0$.

Quantum criticality of Ising-nematic ordering in a metal



$$\mathcal{L}[\psi_\pm, \phi] =$$

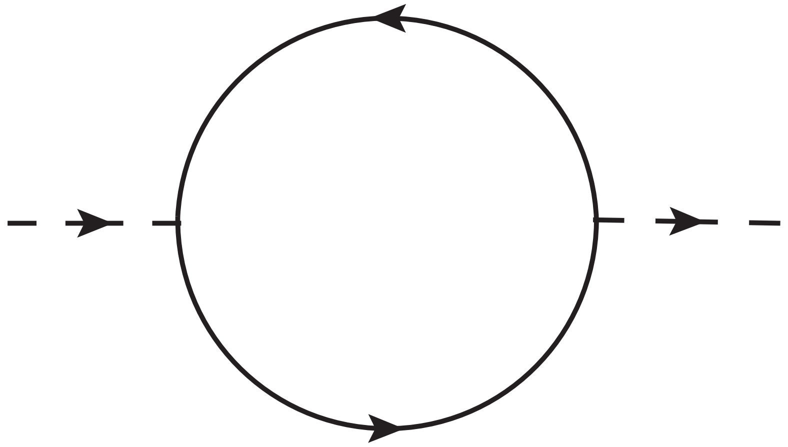
$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-\phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$



One loop ϕ self-energy with N_f fermion flavors:

$$\Sigma_\phi(\vec{q},\omega) = N_f \int \frac{d^2k}{4\pi^2} \frac{d\Omega}{2\pi} \frac{1}{\left[-i(\Omega+\omega) + k_x + q_x + (k_y+q_y)^2\right]\left[-i\Omega - k_x + k_y^2\right]}$$
$$= \frac{N_f}{4\pi} \frac{|\omega|}{|q_y|}$$

Landau-damping

Quantum criticality of Ising-nematic ordering in a metal

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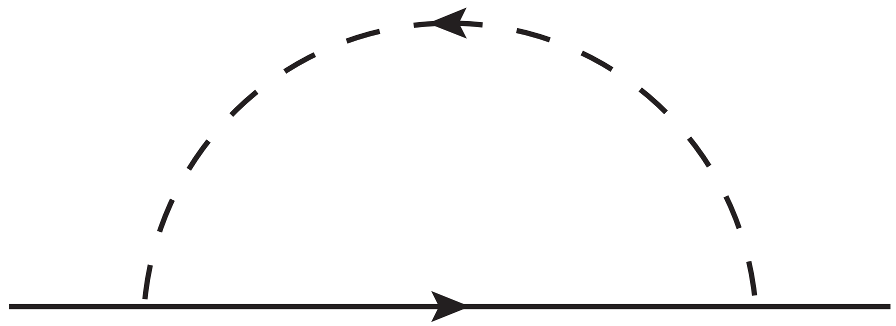

Electron self-energy at order $1/N_f$:

$$\Sigma(\vec{k},\Omega) = -\frac{1}{N_f}\int \frac{d^2q}{4\pi^2}\frac{d\omega}{2\pi}\frac{1}{\left[-i(\omega+\Omega)+k_x+q_x+(k_y+q_y)^2\right]\left[\frac{q_y^2}{g^2}+\frac{|\omega|}{|q_y|}\right]}$$

$$= -i\frac{2}{\sqrt{3}N_f}\left(\frac{g^2}{4\pi}\right)^{2/3}\mathrm{sgn}(\Omega)|\Omega|^{2/3}$$

$\sim |\Omega|^{d/3}$ in dimension d.

Quantum criticality of Ising-nematic ordering in a metal

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Schematic form of ϕ and fermion Green's functions in d dimensions

$$D(\vec{q},\omega) = \frac{1/N_f}{q_y^2 + \dfrac{|\omega|}{|q_y|}} \quad , \quad G_f(\vec{q},\omega) = \frac{1}{q_x + q_y^2 - i\,\mathrm{sgn}(\omega)|\omega|^{2/3}/N_f}$$

In the boson case, $q_y^2 \sim \omega^{1/z_b}$ with $z_b = 3/2$.
In the fermion case, $q_x \sim q_y^2 \sim \omega^{1/z_f}$ with $z_f = 3/d$.

Note $z_f < z_b$ for $d > 2 \Rightarrow$ Fermions have *higher* energy than bosons, and perturbation theory in g is OK.

Strongly-coupled theory in $d = 2$ without quasiparticles.

Quantum criticality of Ising-nematic ordering in a metal

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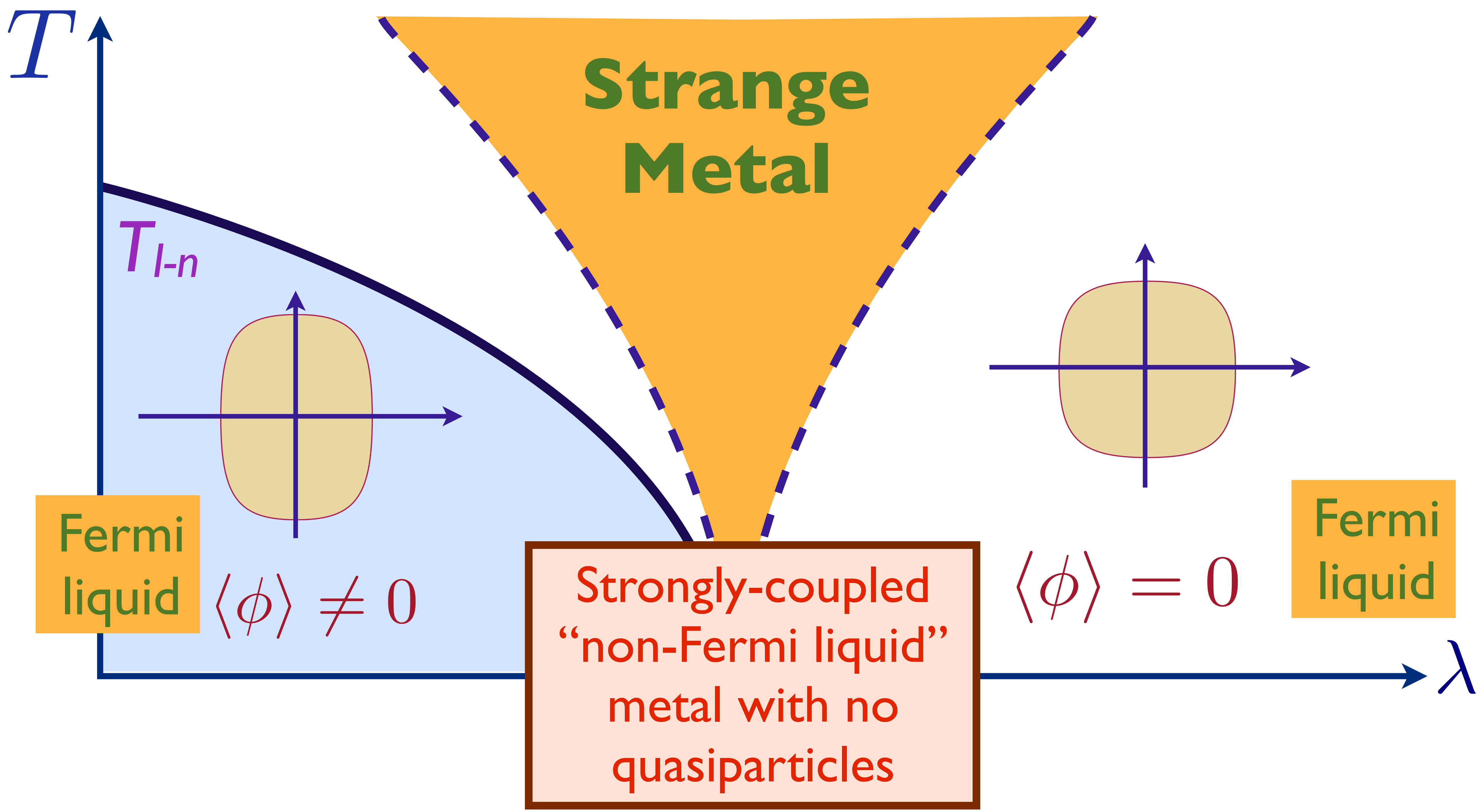
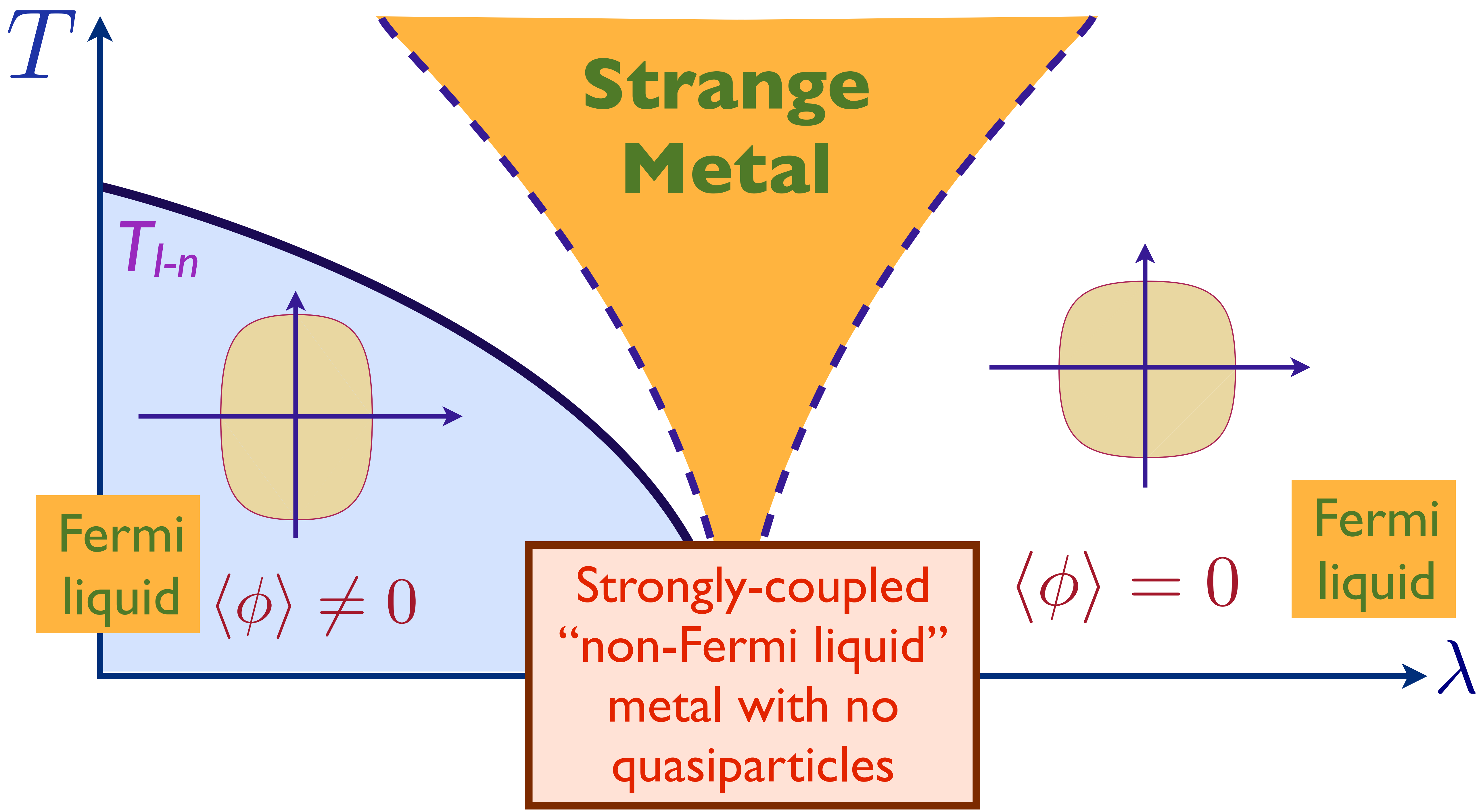


Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



Common theoretical belief from an analysis of scattering of charged electronic quasiparticles off bosonic ϕ fluctuations: resistivity of strange metal $\rho(T) \sim T^{4/3}$.

Quantum criticality of Ising-nematic ordering in a metal

The "standard model":

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This continuum theory has a conserved momentum $\mathbf{P}$, and $\chi_{\mathbf{J},\mathbf{P}} \neq 0$, and so the resistivity $\rho(T) = 0$.

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The resistivity of the strange metal is *not* determined by the scattering rate of charged excitations near the Fermi surface, but by the dominant rate of momentum loss by *any* excitation, whether neutral or charged, or fermionic or bosonic

Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

$$\mathcal{S}_{\rm dis} = \int d^2r d\tau \left[V(\mathbf{r})\, c^\dagger c + h(\mathbf{r})\, \phi \right] ,$$

$$\overline{V(\mathbf{r})} = 0 \quad ; \quad \overline{V(\mathbf{r})V(\mathbf{r}')} = V_0^2\, \delta(\mathbf{r}-\mathbf{r}')\,,$$

$$\overline{h(\mathbf{r})} = 0 \quad ; \quad \overline{h(\mathbf{r})h(\mathbf{r}')} = h_0^2\, \delta(\mathbf{r}-\mathbf{r}')\,,$$

we use the memory-function approach to obtain the *resistivity* for current along angle ϑ

$$\rho(T) = \frac{1}{\chi_{\mathbf{J},\mathbf{P}}^2} \lim_{\omega \to 0} \int \frac{d^2k}{(2\pi)^2}\, k^2 \cos^2(\theta_{\mathbf{k}} - \vartheta) \left(V_0^2\, \frac{\mathrm{Im}\, \Pi^R_{c^\dagger c}(\omega, \mathbf{k})}{\omega} + h_0^2\, \frac{\mathrm{Im}\, D^R_\phi(\omega, \mathbf{k})}{\omega} \right) .$$

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`.

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Fermi surface term: Obtain T-dependent corrections to residual resistivity similar to earlier work

G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001)
I. Paul, C. Pépin, B. N. Narozhny, and D. L. Maslov, Phys. Rev. Lett. **95**, 017206 (2005).

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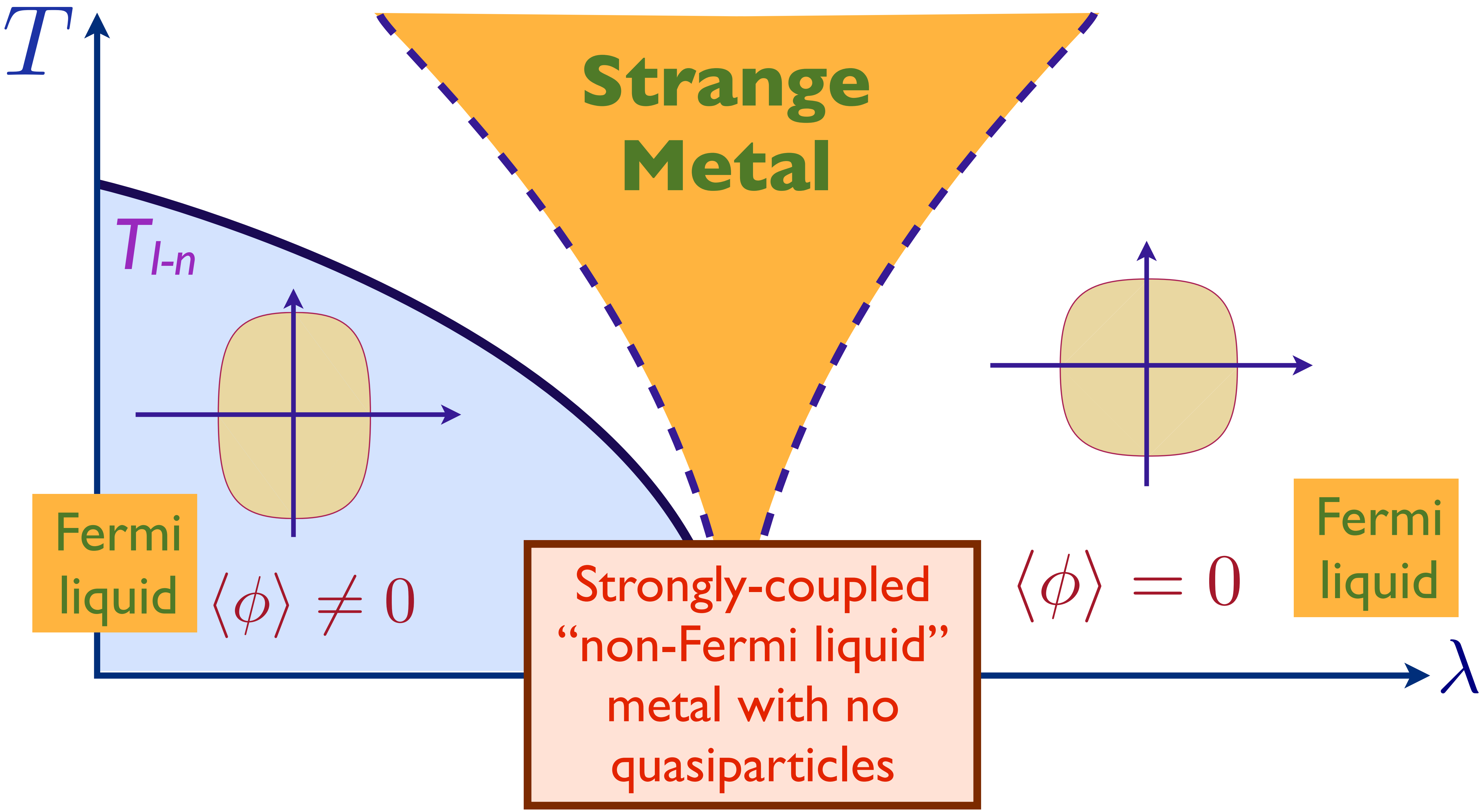
Bosonic term: Dominant contribution:

$$\rho(T) \sim T^{(d-z+\eta)/z}$$

Crosses over from the "relativistic" form ($z = 1$, $\eta \approx 0$) with $\rho(T) \sim T$ at higher T,
to the "Landau-damped" form ($z = 3$, $\eta = 0$) with $\rho(T) \sim (T \ln(1/T))^{-1/2}$ at lower T (subtle corrections to scaling specific to this field theory).

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, arXiv:1401.7012.

Quantum criticality of Ising-nematic ordering in a metal



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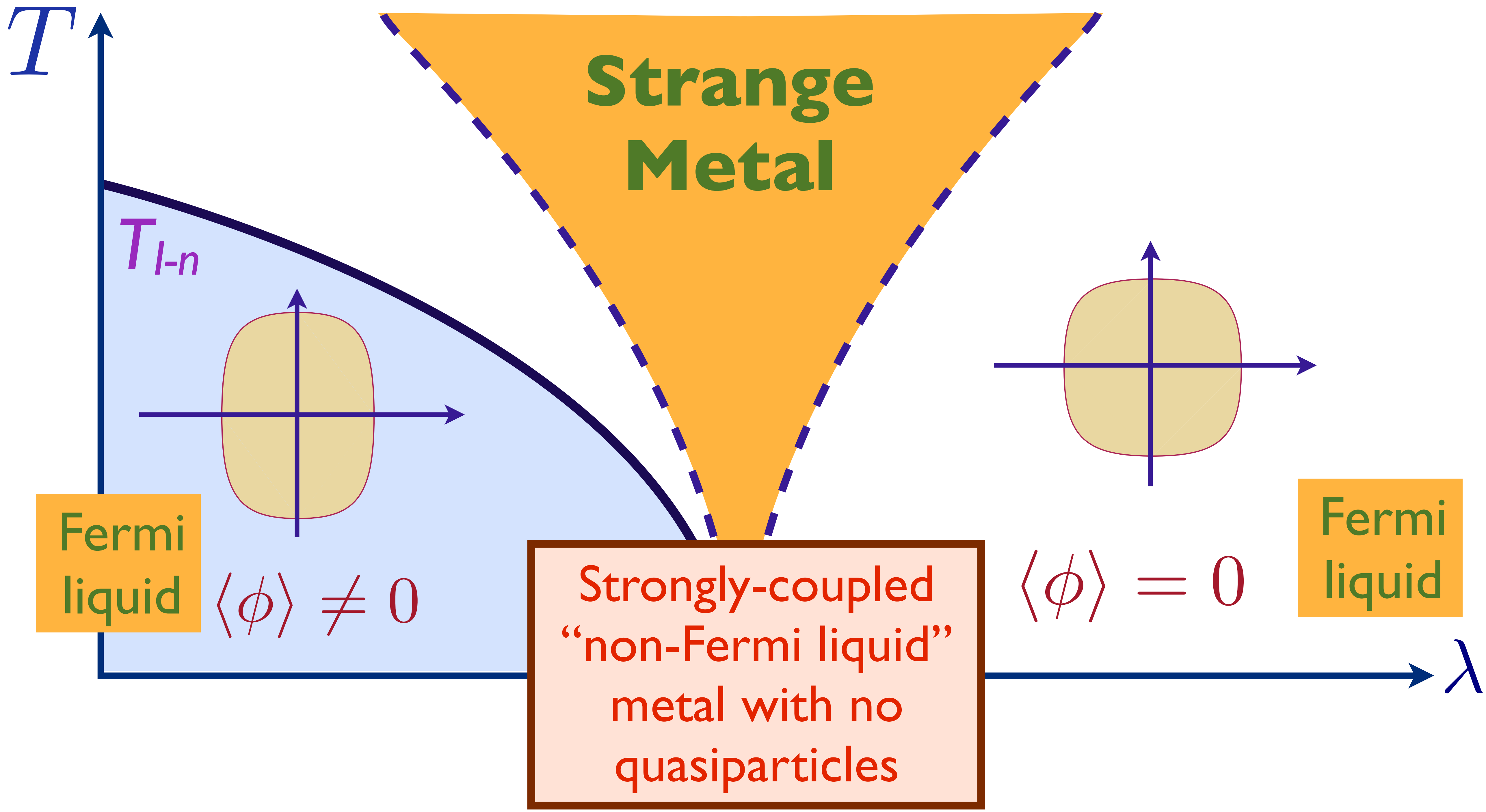

Strange metal regime with $\rho(T) \sim T$
due to scattering of $z = 1$ neutral bosons
off a *random field*.

Outline

1. Antiferromagnetism in metals and *d*-wave superconductivity

2. Competing order: *d*-wave bond order

3. Nematic quantum criticality and the strange metal

4. The pseudogap regime of the hole-doped cuprate superconductors
   Angular fluctuations of a multicomponent order

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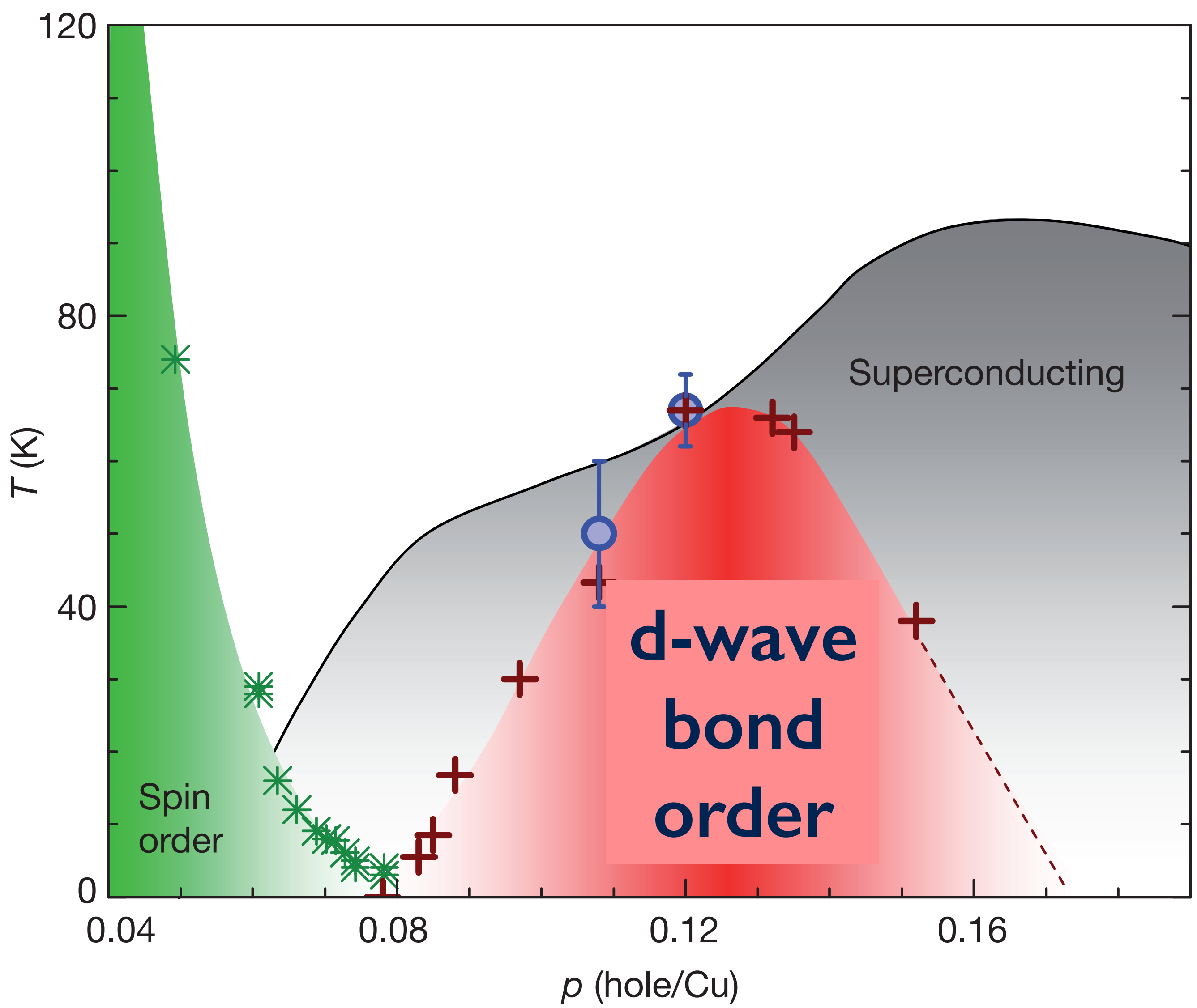

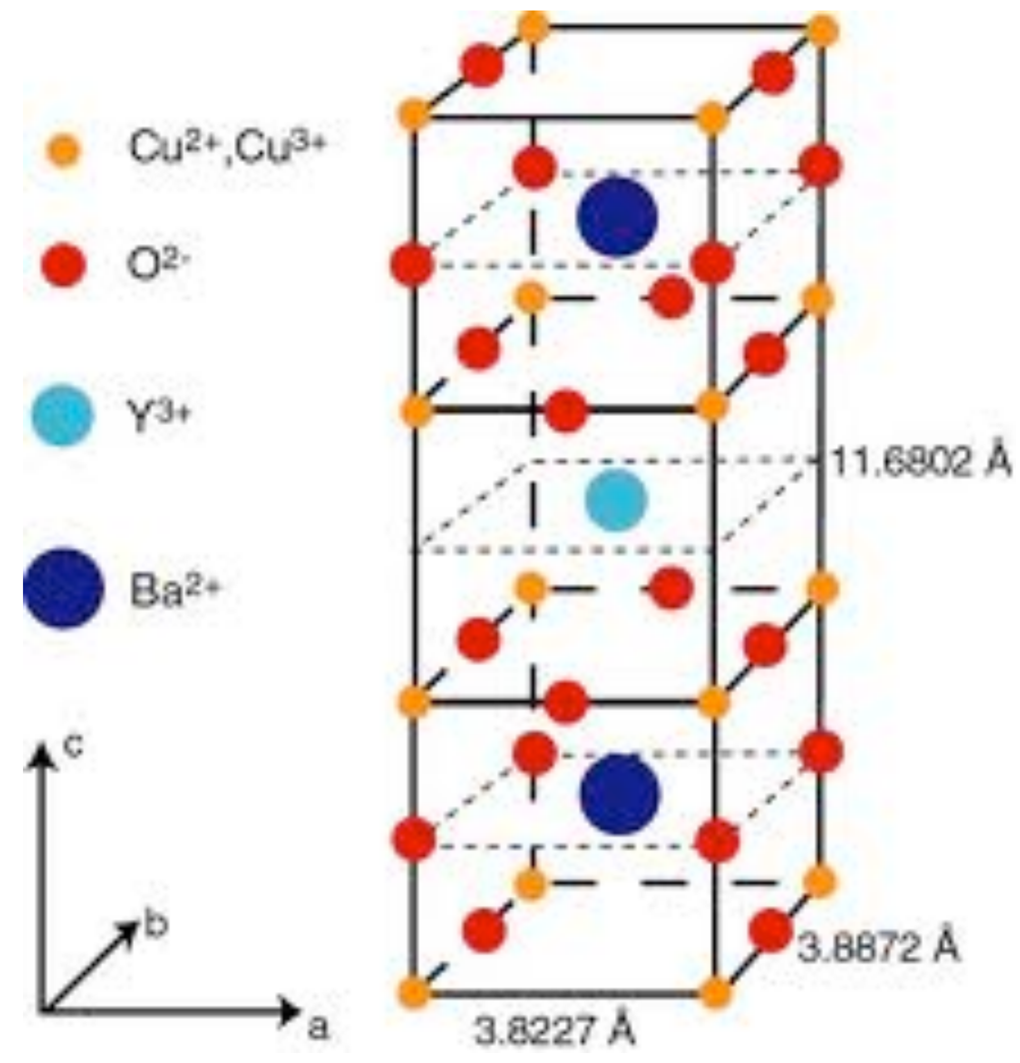

Direct observation of competition between superconductivity and charge density wave order in $YBa_2Cu_3O_{6.67}$

NATURE PHYSICS | VOL 8 | DECEMBER 2012 |

J. Chang[1,2]★, E. Blackburn[3], A. T. Holmes[3], N. B. Christensen[4], J. Larsen[4,5], J. Mesot[1,2], Ruixing Liang[6,7], D. A. Bonn[6,7], W. N. Hardy[6,7], A. Watenphul[8], M. v. Zimmermann[8], E. M. Forgan[3] and S. M. Hayden[9]

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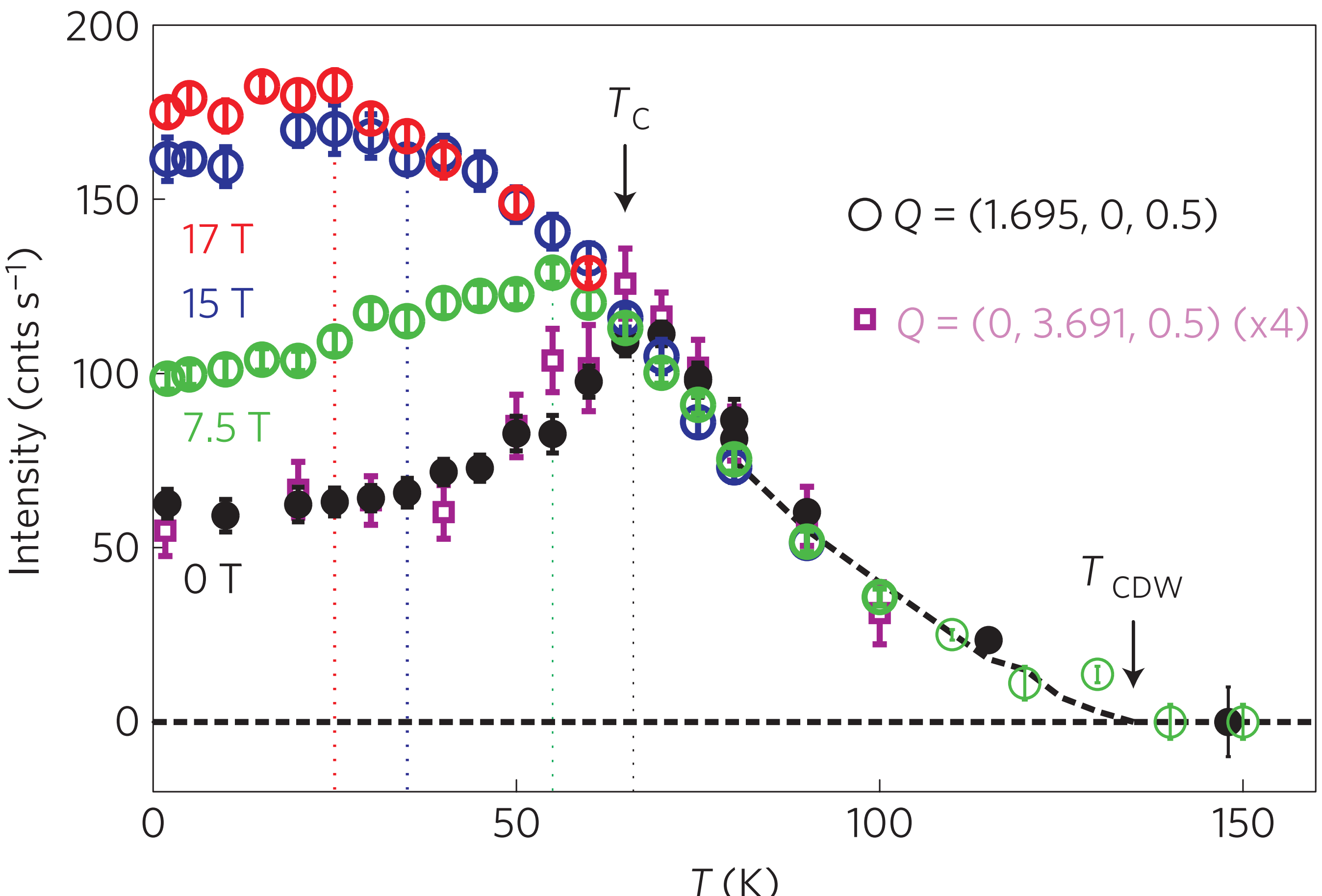

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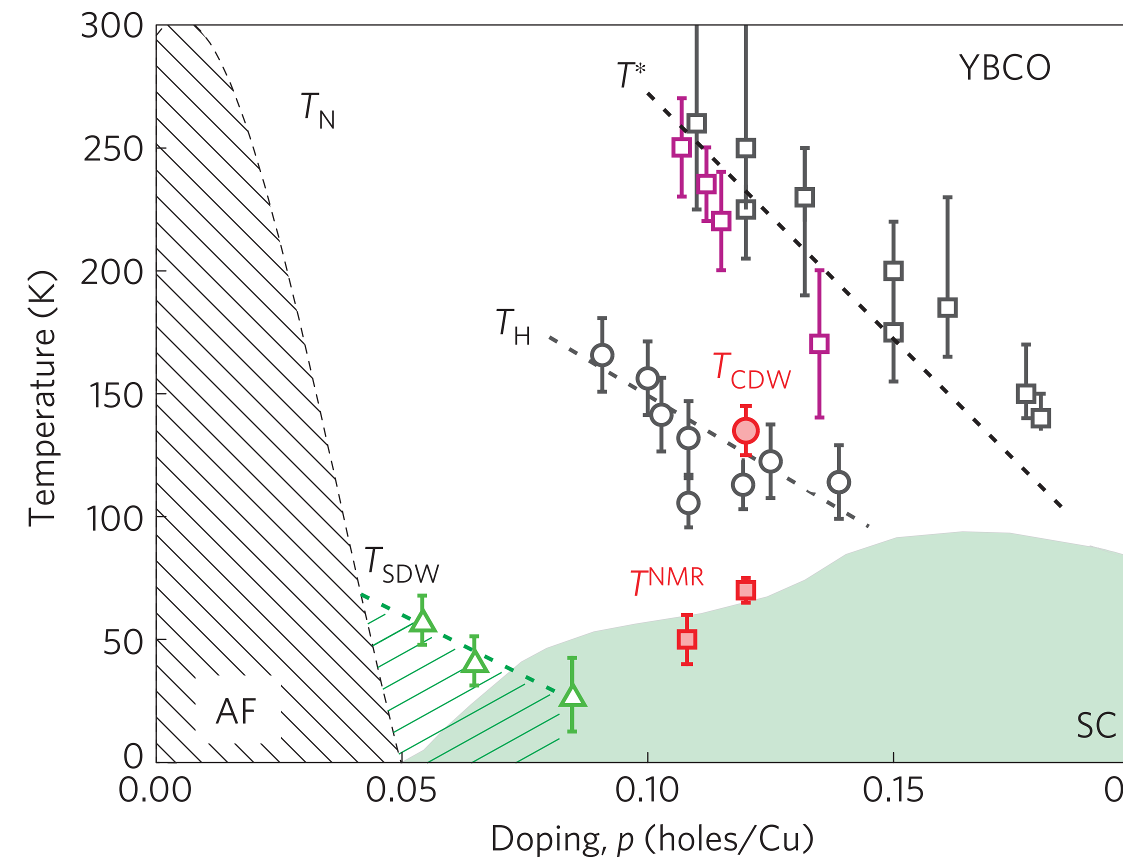
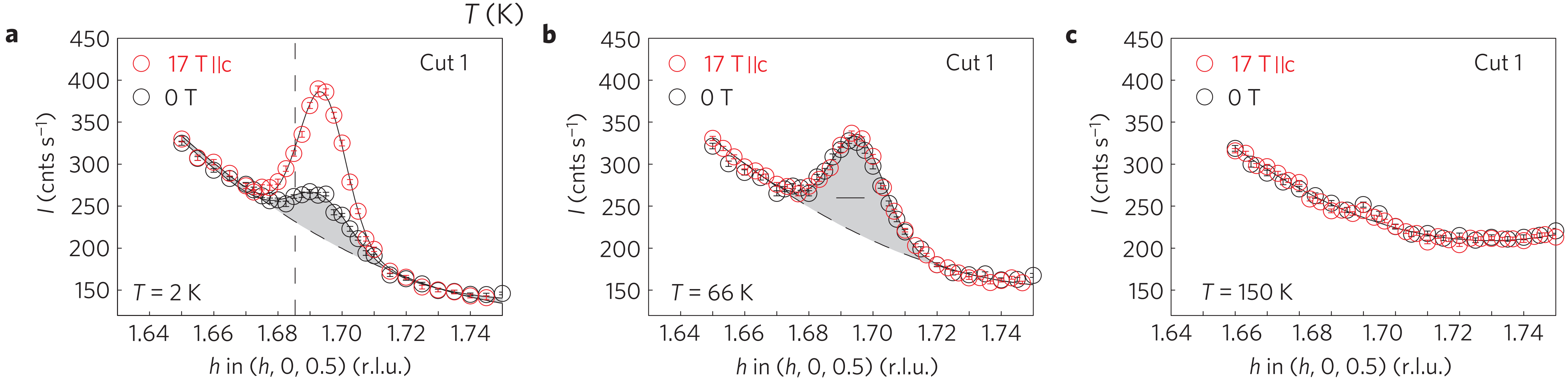
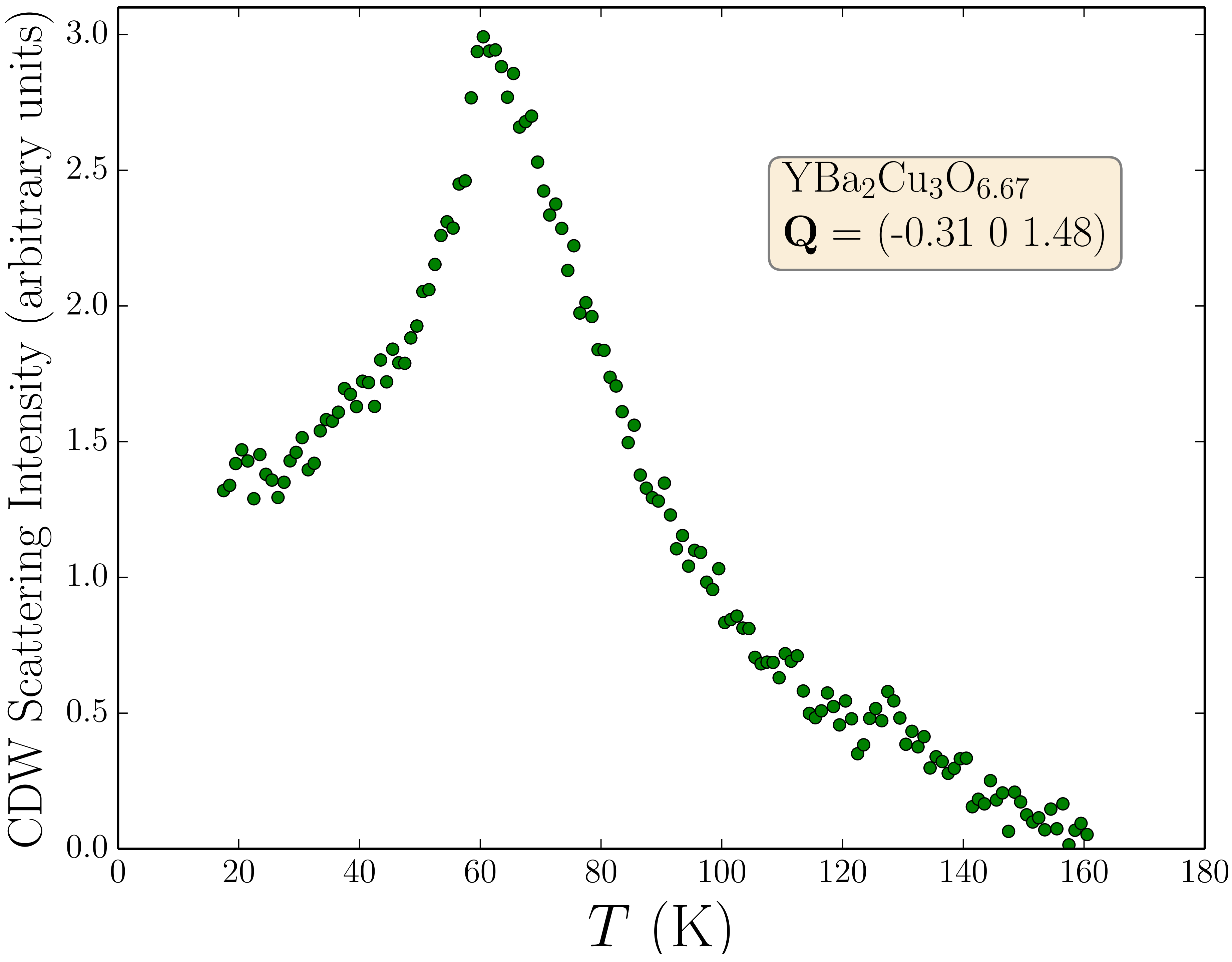
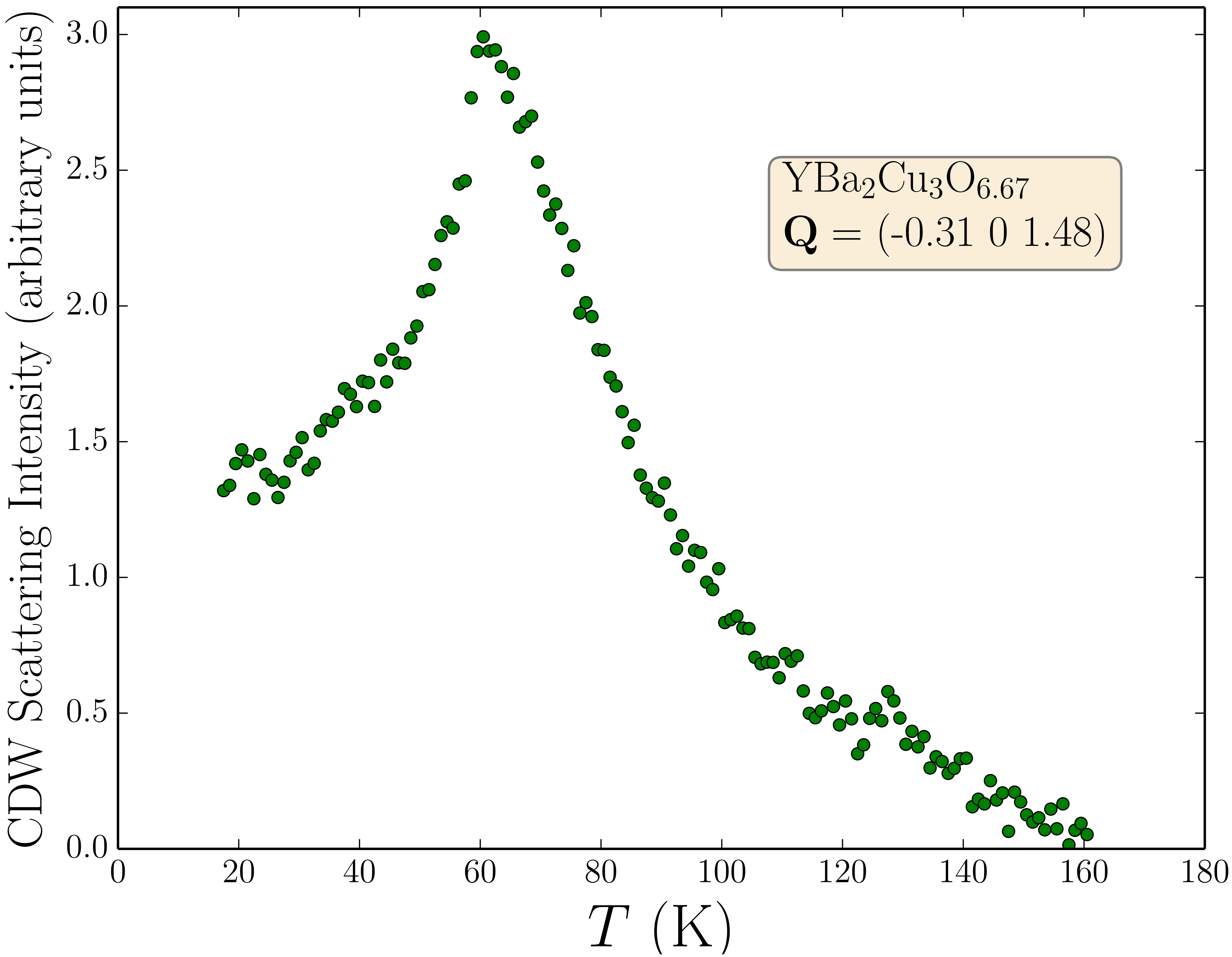




A. J. Achkar, X. Mao, Christopher McMahon, R. Sutarto, F. He, Ruixing Liang, D. A. Bonn, W. N. Hardy, D. G. Hawthorn, arXiv:1312.6630.



A. J. Achkar, X. Mao, Christopher McMahon, R. Sutarto, F. He, Ruixing Liang, D. A. Bonn, W. N. Hardy, D. G. Hawthorn, arXiv:1312.6630.



Onset is unlike an arrested ordering transition, or precursor critical fluctuations

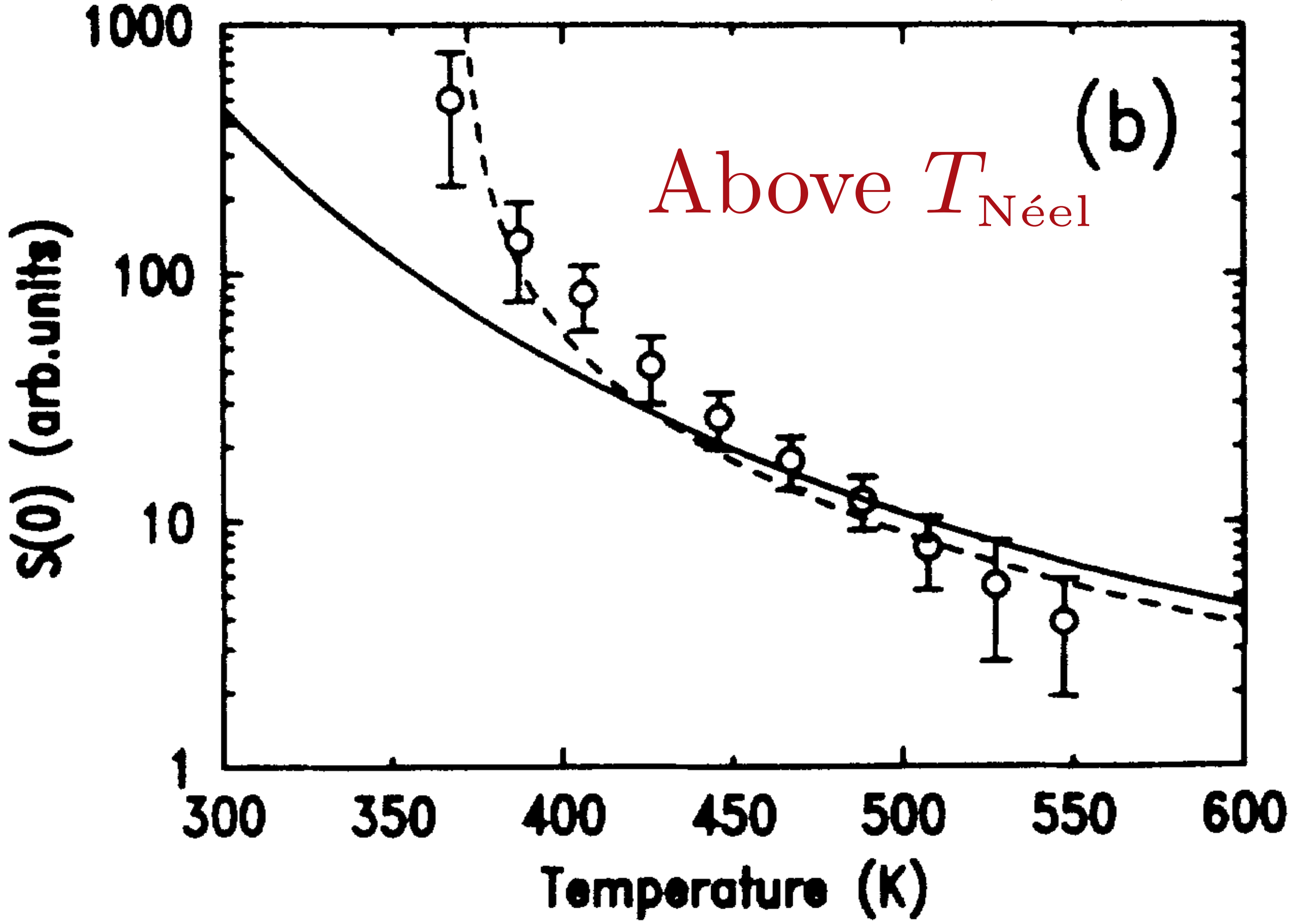
Key idea: analogy with the onset of antiferromagnetism in the *insulator* La_2CuO_4

$$T_{\text{Néel}} = 325\text{K}$$

Gradual onset of intensity over a wide range of T is a consequence of *angular thermal* fluctuations of an order parameter with 3 or more components in 2 spatial dimensions

Polyakov, 1975
Chakravarty, Halperin, Nelson 1989

B. Keimer *et al.*,
Phys. Rev. B **46**, 14034 (1992).



O(3) non-linear sigma model

$$Z = \int \mathcal{D}\vec{n}(x)\delta\left(\vec{n}^2(x) - 1\right)\exp\left(-\frac{\rho_s}{2T}\int d^2x\left(\nabla_x\vec{n}\right)^2\right)$$

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Generalize $\vec{n}$ to a N-component vector n_α, $\alpha = 1 \ldots N$, and take the $N \to \infty$ limit while taking $\rho_s \propto N$. This is implemented by a Lagrange multiplier λ

$$Z = \int \mathcal{D}\lambda(x)\mathcal{D}n_\alpha(x)\exp\left(-\frac{\rho_s}{2T}\int d^2x\left[\left(\nabla_x n_\alpha\right)^2 + i\lambda(n_\alpha^2 - 1)\right]\right)$$

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We can now perform the Gaussian integral over n_α

$$Z = \int \mathcal{D}\lambda(x)\exp\left(-\frac{N}{2}\mathrm{Tr}\ln\left(-\nabla_x^2 + i\lambda\right) + \frac{\rho_s}{2T}\int d^2x\, i\lambda\right)$$

Because $\rho_s \propto N$, in the $N \to \infty$ limit the partition function is dominated by the saddle point.

O(3) non-linear sigma model

At the saddle point, we set $i\lambda(x) = \xi^{-1}$, and then the "structure factor" $S(k)$ of the order parameter is

$$S(k) = \int d^2x \, \langle n_\alpha(x) n_\alpha(0) \rangle \, e^{ikx} = \frac{NT}{\rho_s} \frac{1}{(k^2 + \xi^{-2})}$$

This identifies ξ as the correlation length.

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This identifies ξ as the correlation length. The value of ξ is determined by the saddle-point equation, which simply enforces the constraint $n_\alpha^2(x) = 1$. So we have

$$\frac{NT}{\rho_s} \int \frac{d^2k}{4\pi^2} \frac{1}{k^2 + \xi^{-2}} = 1$$

Performing the k integral with a momentum cutoff Λ we obtain

$$\frac{NT}{4\pi\rho_s} \ln\left(1 + \Lambda^2\xi^2\right) = 1 \quad \Rightarrow \quad \xi = \Lambda^{-1} \exp\left(\frac{2\pi\rho_s}{NT}\right)$$

So ξ is *finite* at all non-zero T (no LRO), and diverges exponentially as $T \to 0$ (consistent with Mermin-Wagner theorem).

O(3) non-linear sigma model

The *exact* result (for the exponential) at finite N is

$$\xi = \Lambda^{-1} \exp\left(\frac{2\pi\rho_s}{(N-2)T}\right)$$

O(3) non-linear sigma model

The *exact* result (for the exponential) at finite N is

$$\xi = \Lambda^{-1} \exp\left(\frac{2\pi\rho_s}{(N-2)T} \right)$$

Neutron scattering measures the structure factor, and the peak value is $S(0)$

$$S(0) = \frac{NT}{\rho_s}\xi^2 = \frac{NT}{\Lambda^2\rho_s} \exp\left(\frac{4\pi\rho_s}{(N-2)T} \right)$$

So there is no Bragg peak at the ordering wavevector for any two-dimensional antiferromagnet.

La_2CuO_4 has a non-zero ordering temperature $T_N =$ 325K, and this arises solely from the *inter-layer* coupling.

<u>Key idea</u>: analogy with the onset of antiferromagnetism in the *insulator* La_2CuO_4

Gradual onset of intensity over a wide range of T is a consequence of <u>*angular thermal*</u> fluctuations of an order parameter with <u>3 or more</u> components in <u>2</u> spatial dimensions

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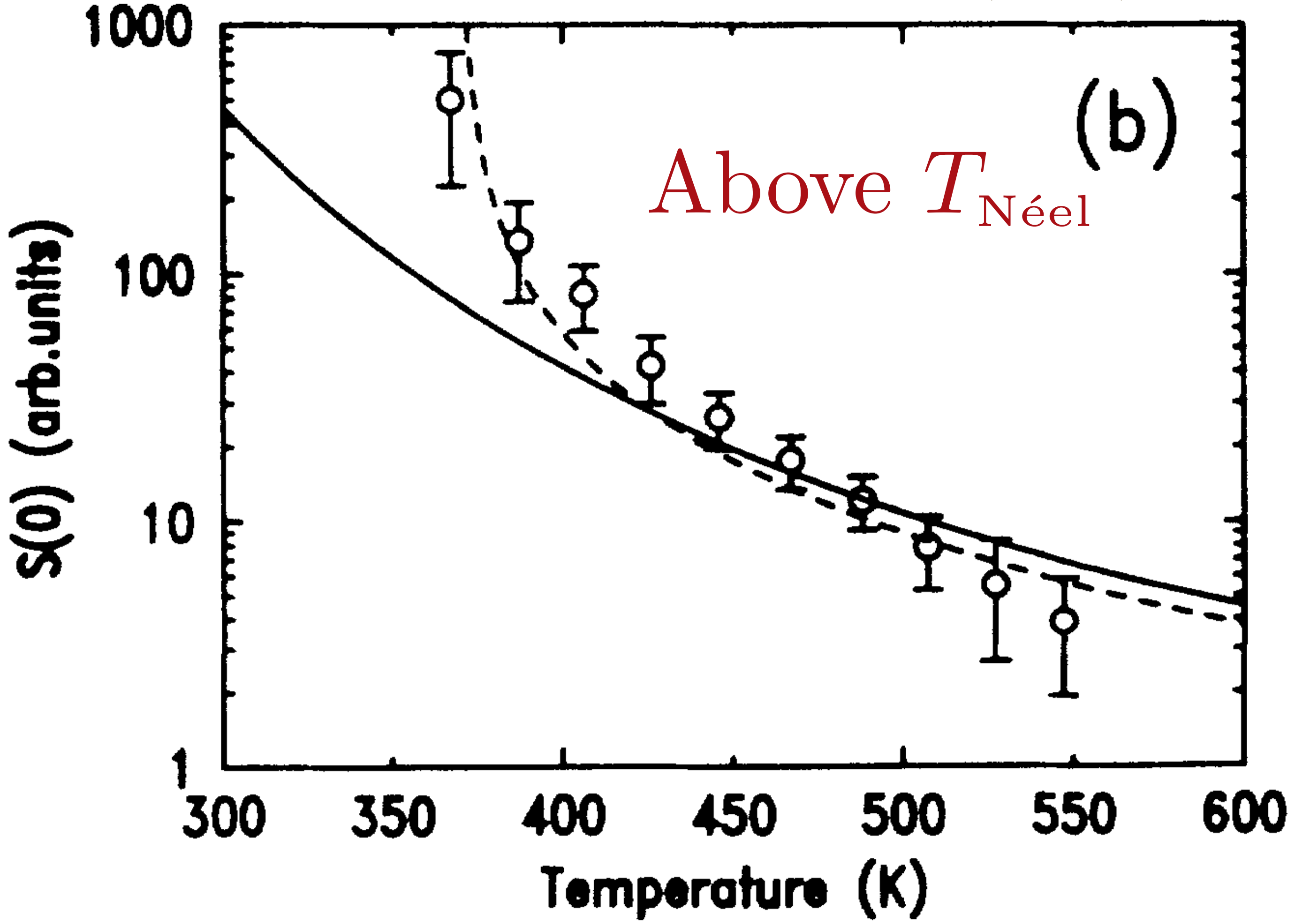


Multi-component order parameter for the pseudogap

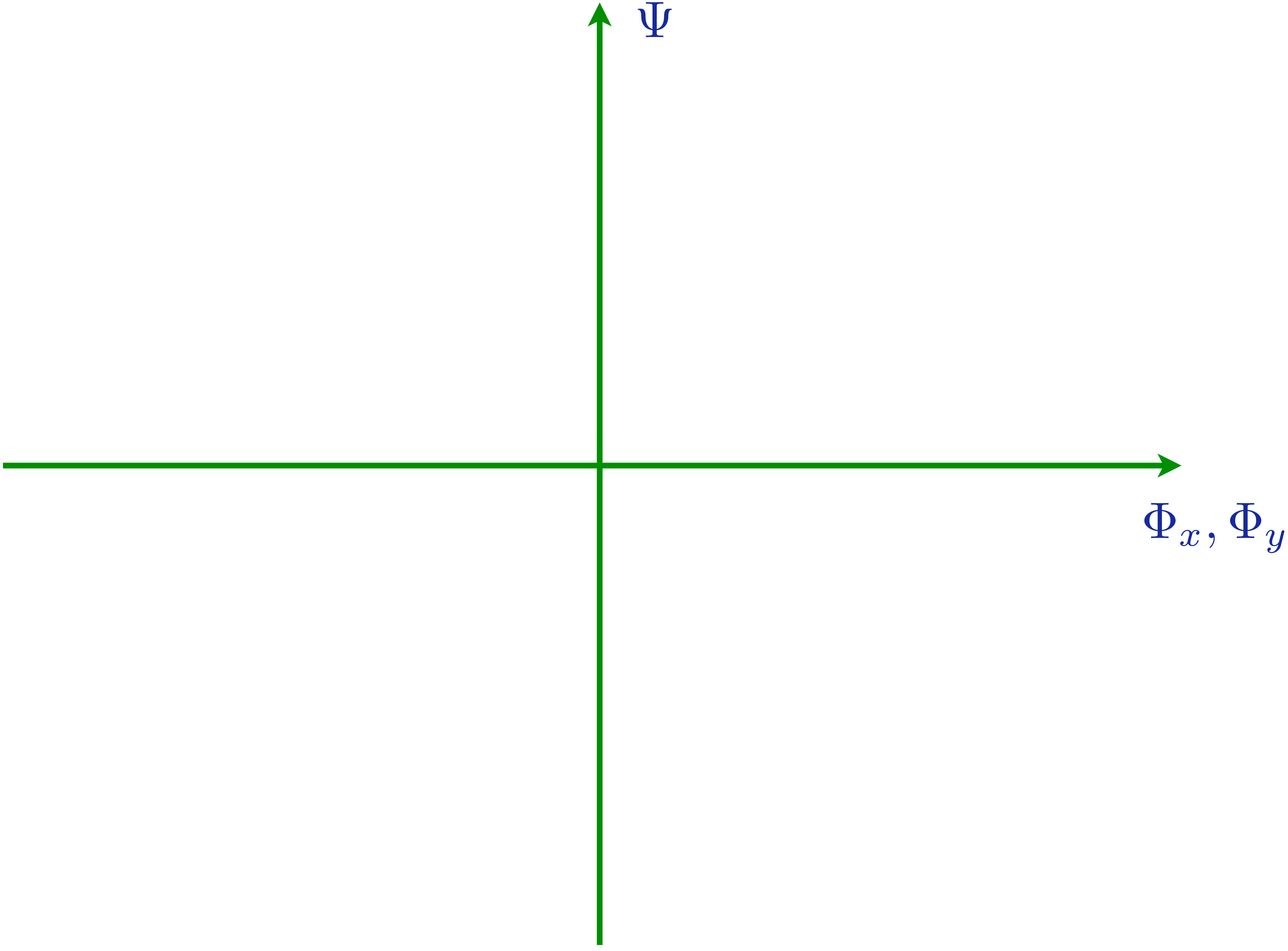
Superconducting order $\Psi(\mathbf{r})$:

$$\left\langle c_{i\alpha}^{\dagger} c_{j\beta}^{\dagger} \right\rangle = \varepsilon_{\alpha\beta} \left[\sum_{\mathbf{k}} \Delta_S(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] \Psi\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right)$$

Charge/bond order $\Phi_{x,y}(\mathbf{r})$ at wavevectors $\mathbf{Q}_{x,y}$:

$$\begin{aligned} \left\langle c_{i\alpha}^{\dagger} c_{j\beta} \right\rangle = & \, \delta_{\alpha\beta} \left[\sum_{\mathbf{k}} P_{\mathbf{Q}_x}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_x\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_x\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right) \\ & + \delta_{\alpha\beta} \left[\sum_{\mathbf{k}} P_{\mathbf{Q}_y}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_y\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_y\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right) \end{aligned}$$

Multi-component order parameter



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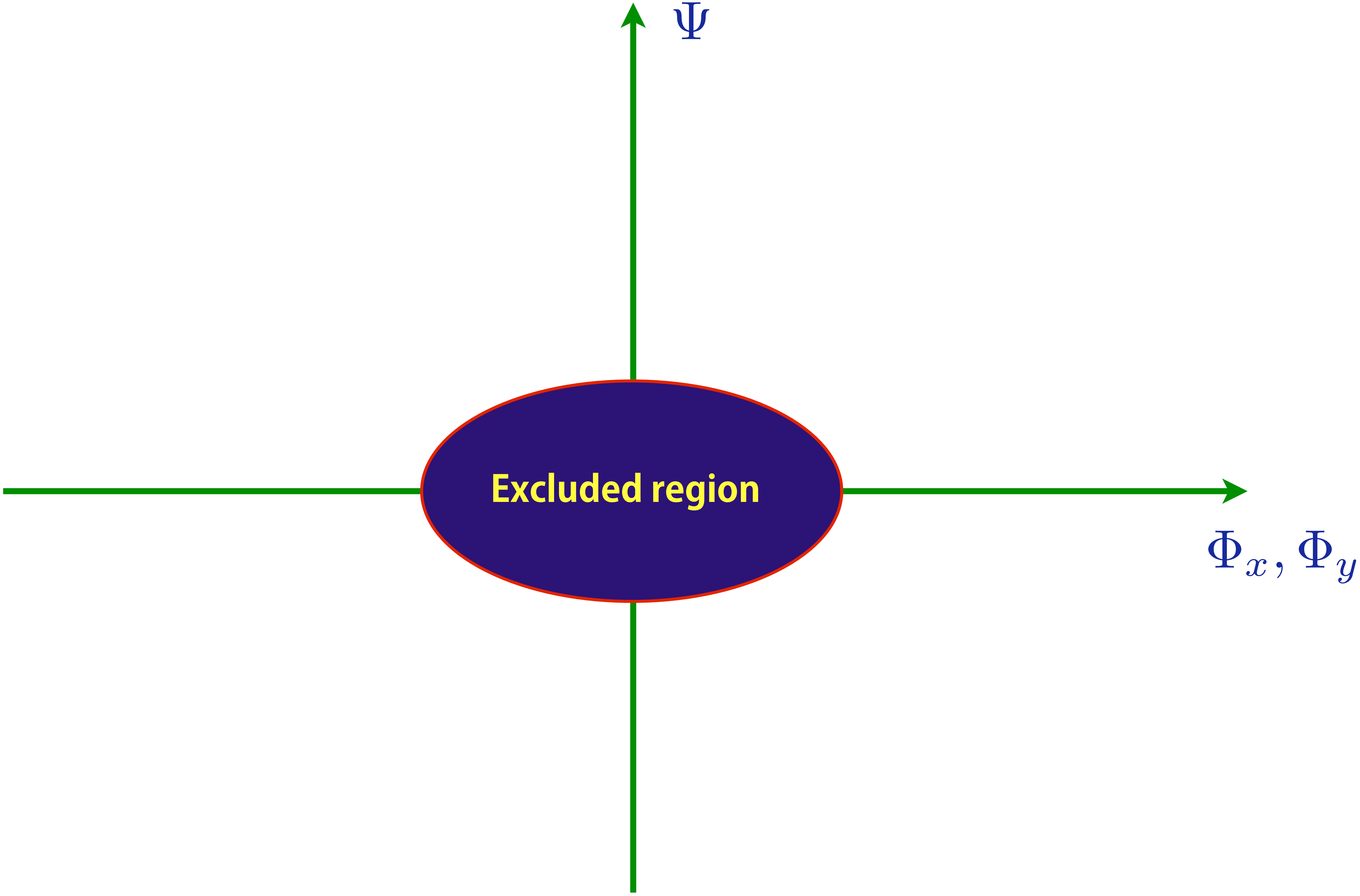

Support from theory of antiferromagnetic quantum criticality

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

K. B. Efetov, H. Meier, and C. Pepin, *Nature Physics* **9**, 442 (2013)

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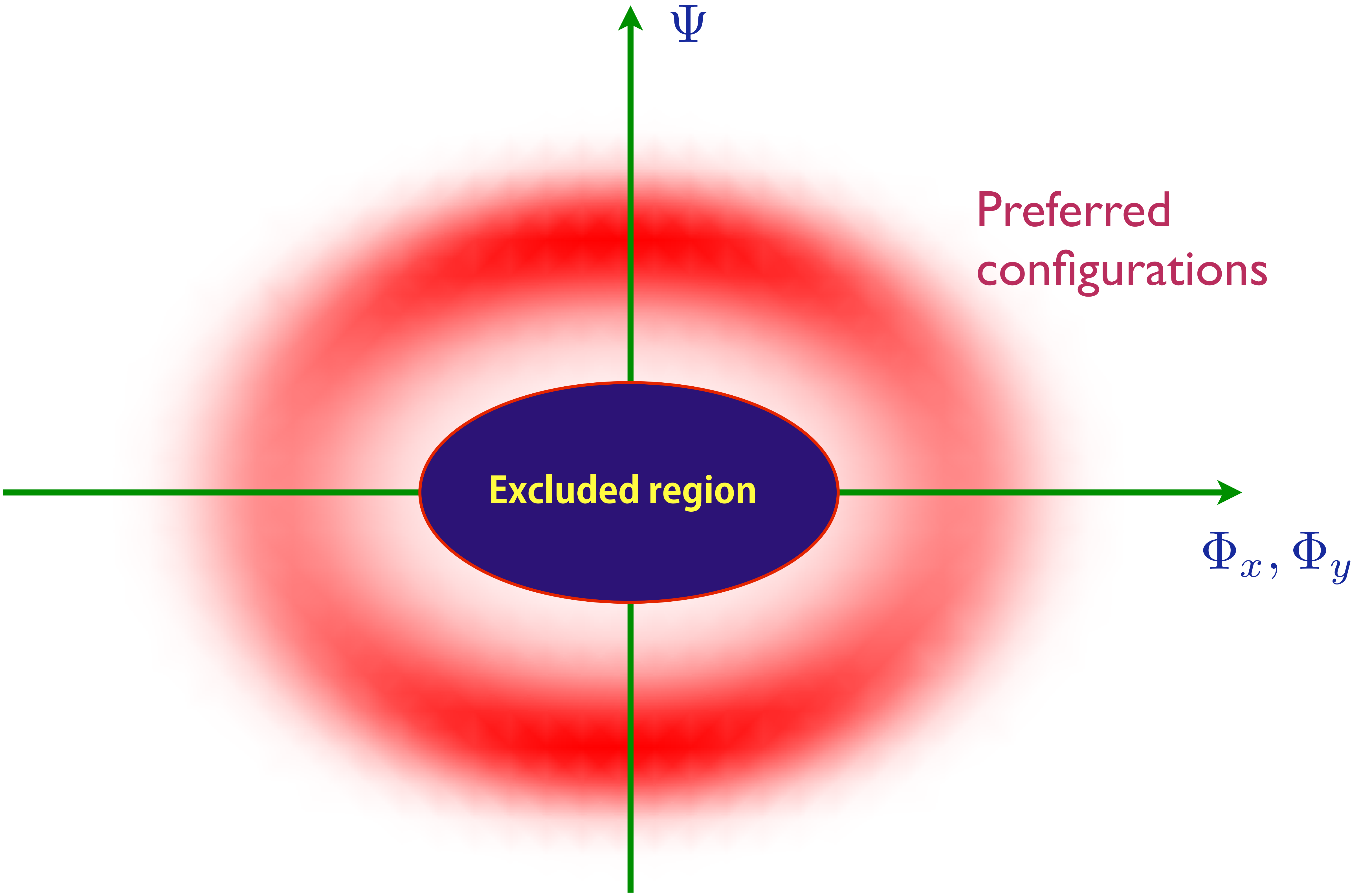

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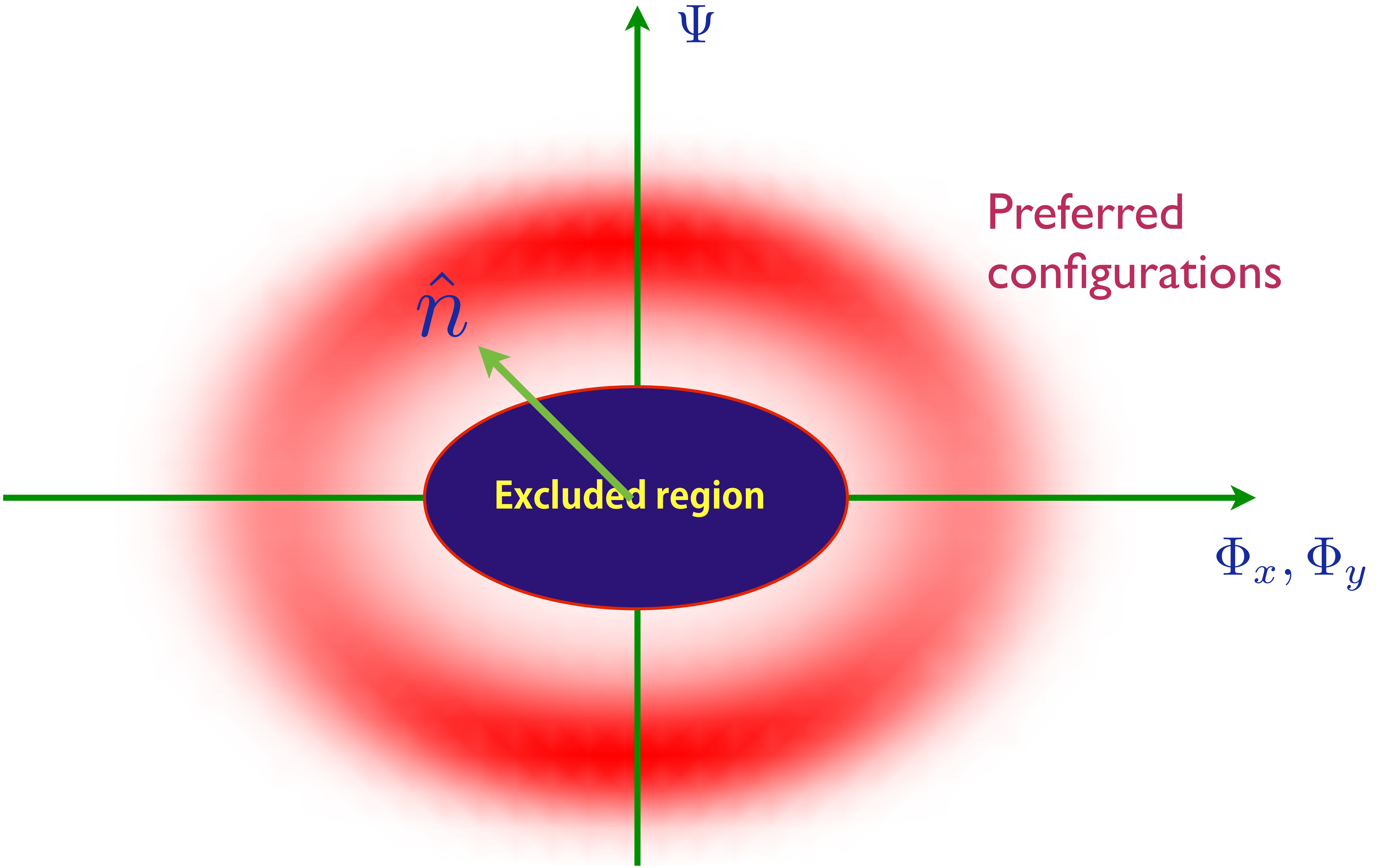


Label order parameter by a 6-component unit vector n_α with $\sum_\alpha n_\alpha^2 = 1$

O(6) non-linear sigma model

$$\mathcal{Z} = \int \mathcal{D}n_\alpha(\mathbf{r})\,\delta\left(\sum_{\alpha=1}^{6} n_\alpha^2(\mathbf{r}) - 1\right)\exp\Bigg(-\frac{\rho_s}{2T}\int d^2r\Bigg[\sum_{\alpha=1}^{2}(\nabla n_\alpha)^2 + \lambda\sum_{\alpha=3}^{6}(\nabla n_\alpha)^2 + g\sum_{\alpha=3}^{6} n_\alpha^2 + w\left[\left(n_3^2+n_4^2\right)^2 + \left(n_5^2+n_6^2\right)^2\right]\Bigg]\Bigg).$$

where $\Psi \propto n_1 + in_2$, $\Phi_x \propto n_3 + in_4$, $\Phi_y \propto n_5 + in_6$.

Describes $\mathrm{O}(6) \Rightarrow \mathrm{O}(2)\times\mathrm{O}(2)\times\mathrm{O}(2)\rtimes\mathbb{Z}_2$. The coupling g determines the anisotropy between superconductivity and charge order.

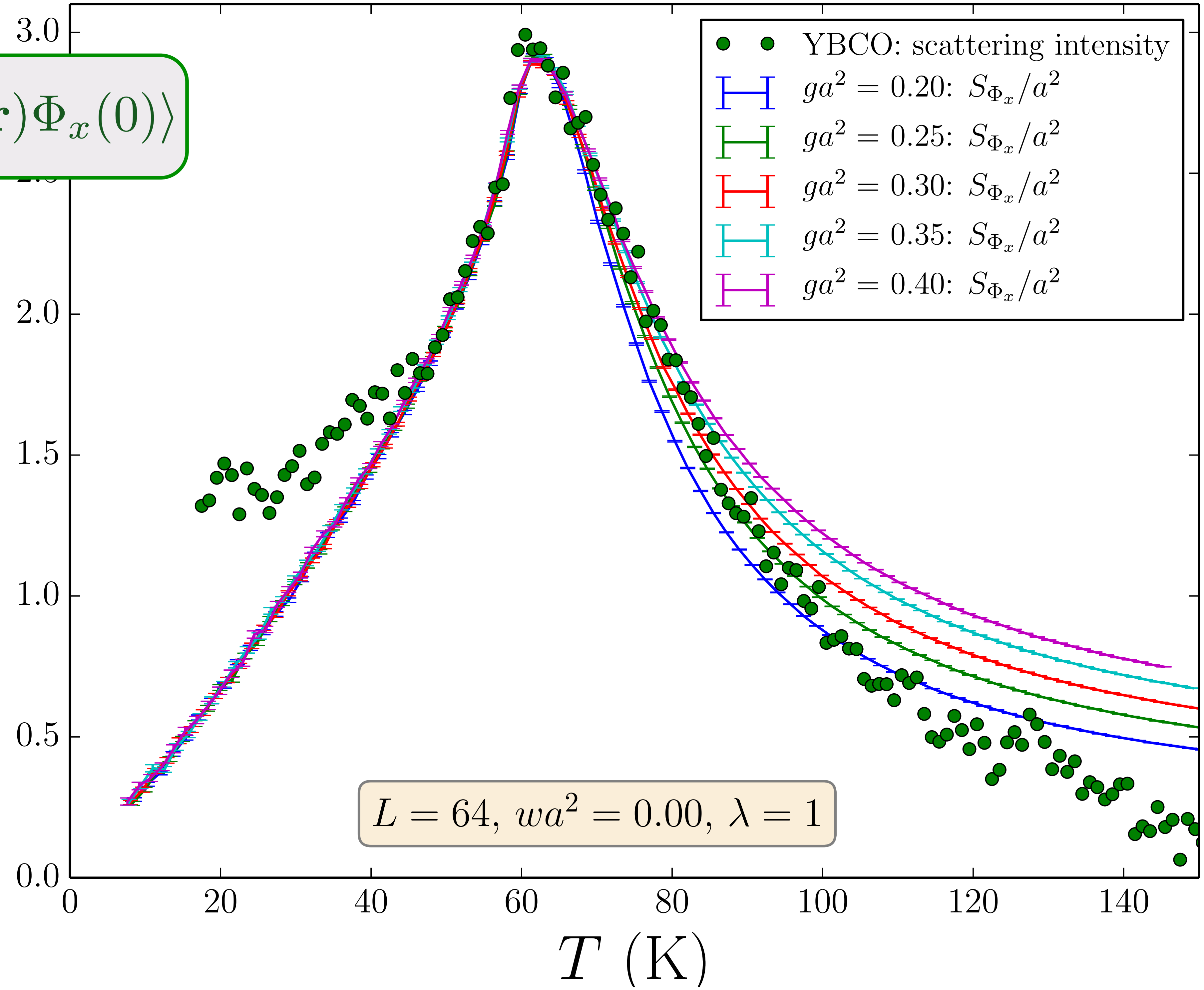
Solve by cluster Monte Carlo and $1/N$ expansion.

L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, *Science*, in press, arXiv:1309.6639

Comparison of Monte Carlo with experiments

$$S_{\Phi_x} = \int d^2 r \, \langle \Phi_x(\mathbf{r}) \Phi_x(0) \rangle$$

Charge order structure factor S_{Φ_x}



For $ga^2 = 0.30$ and $wa^2 = 0.0$ we have $\rho_s = 160\text{K}$.
The height was also rescaled to make the peak heights match.

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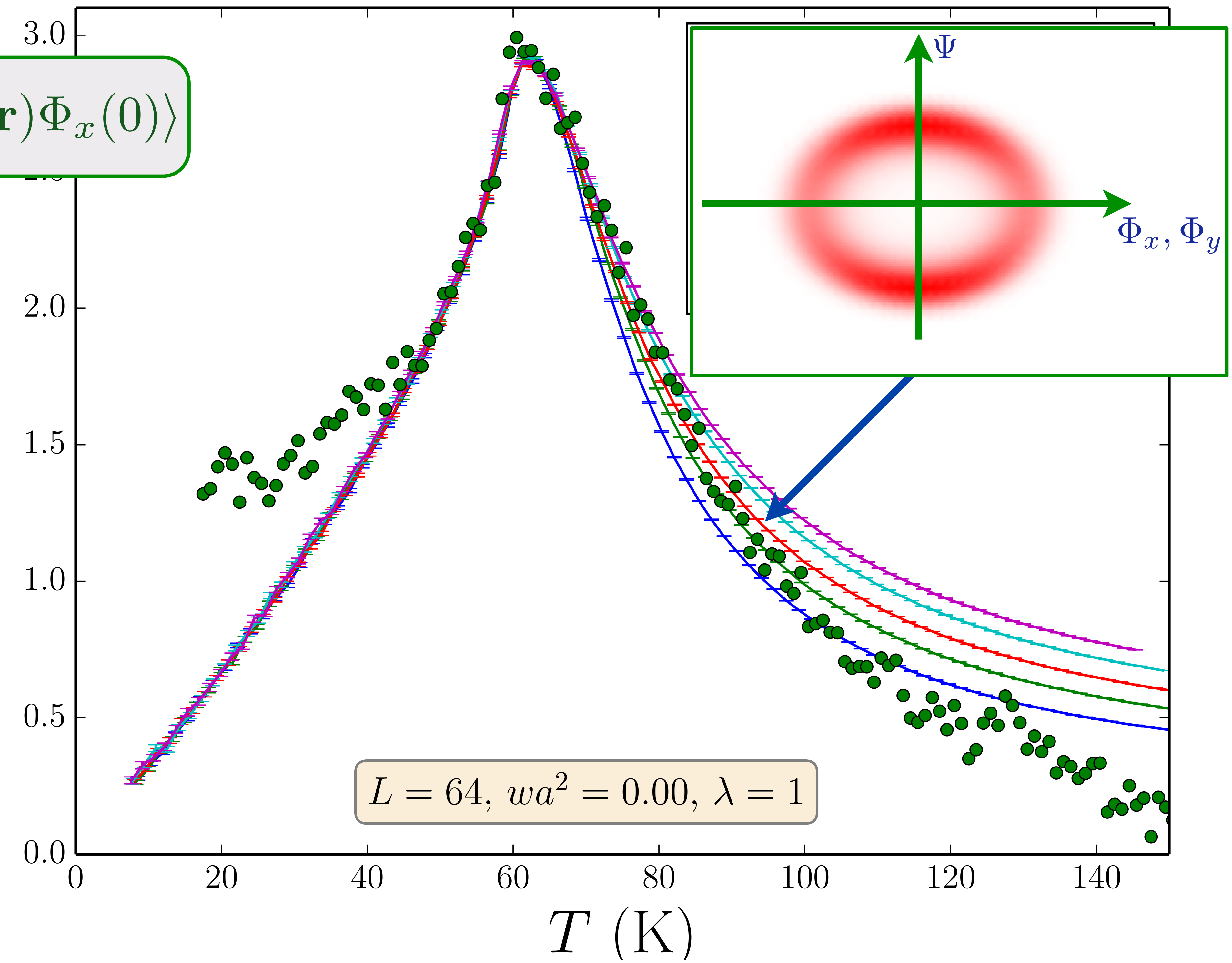

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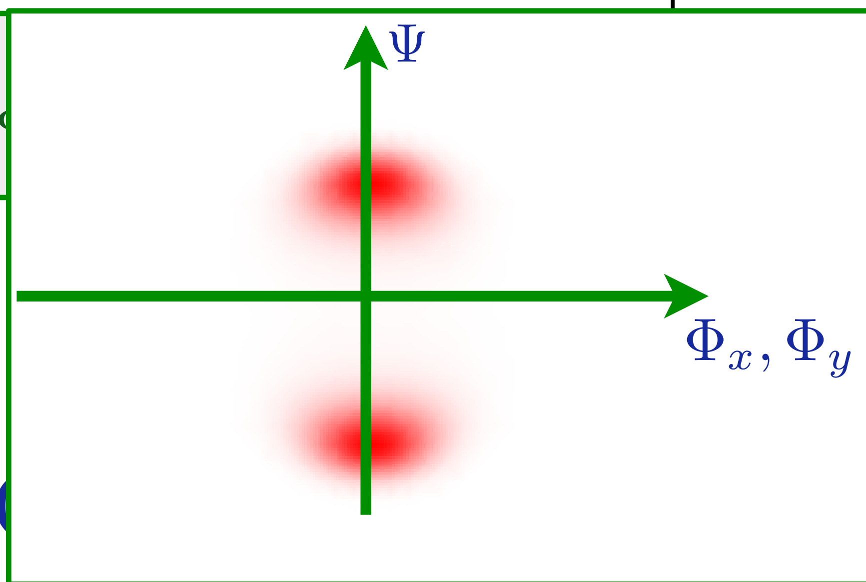
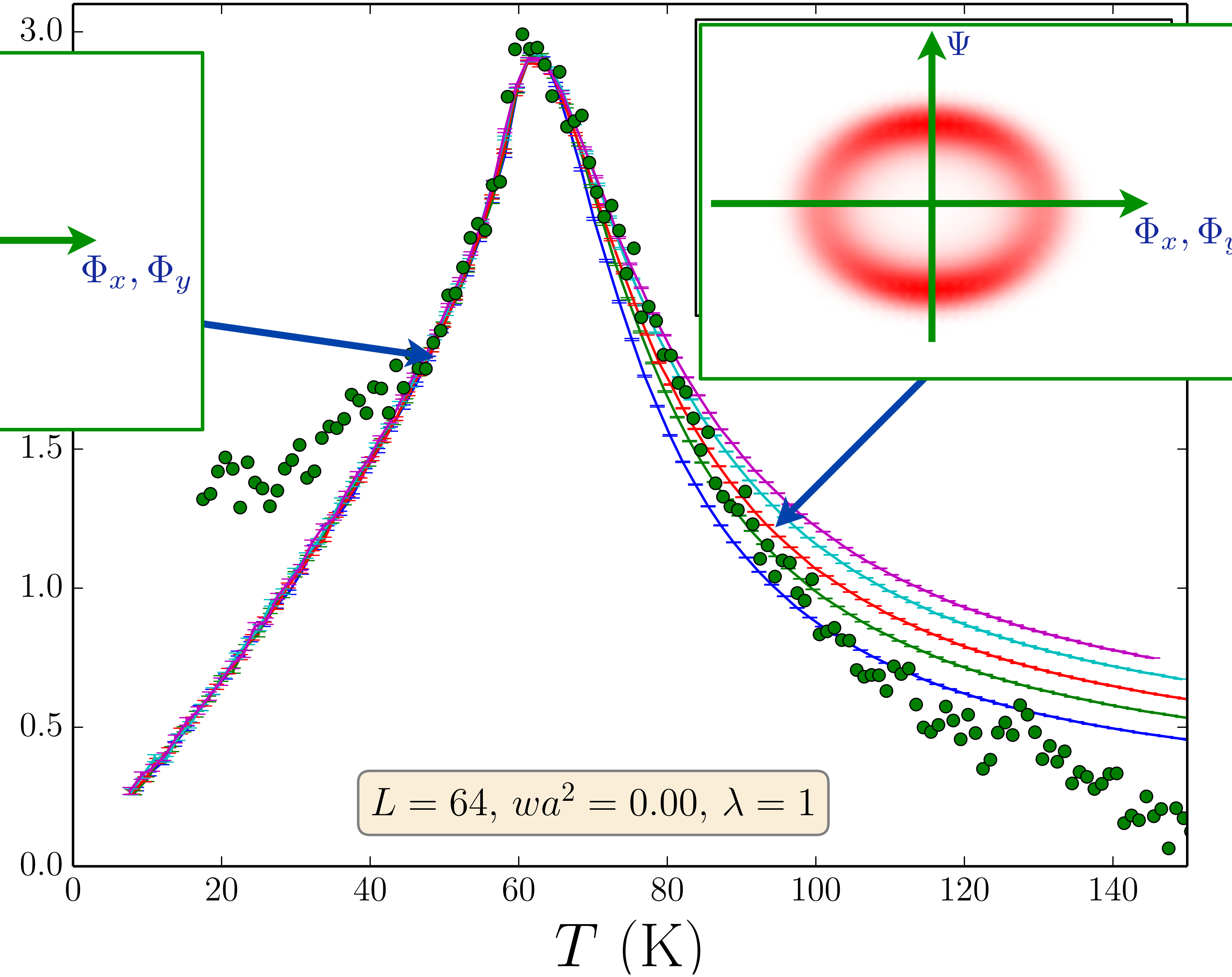


structure factor S_{Φ_x}



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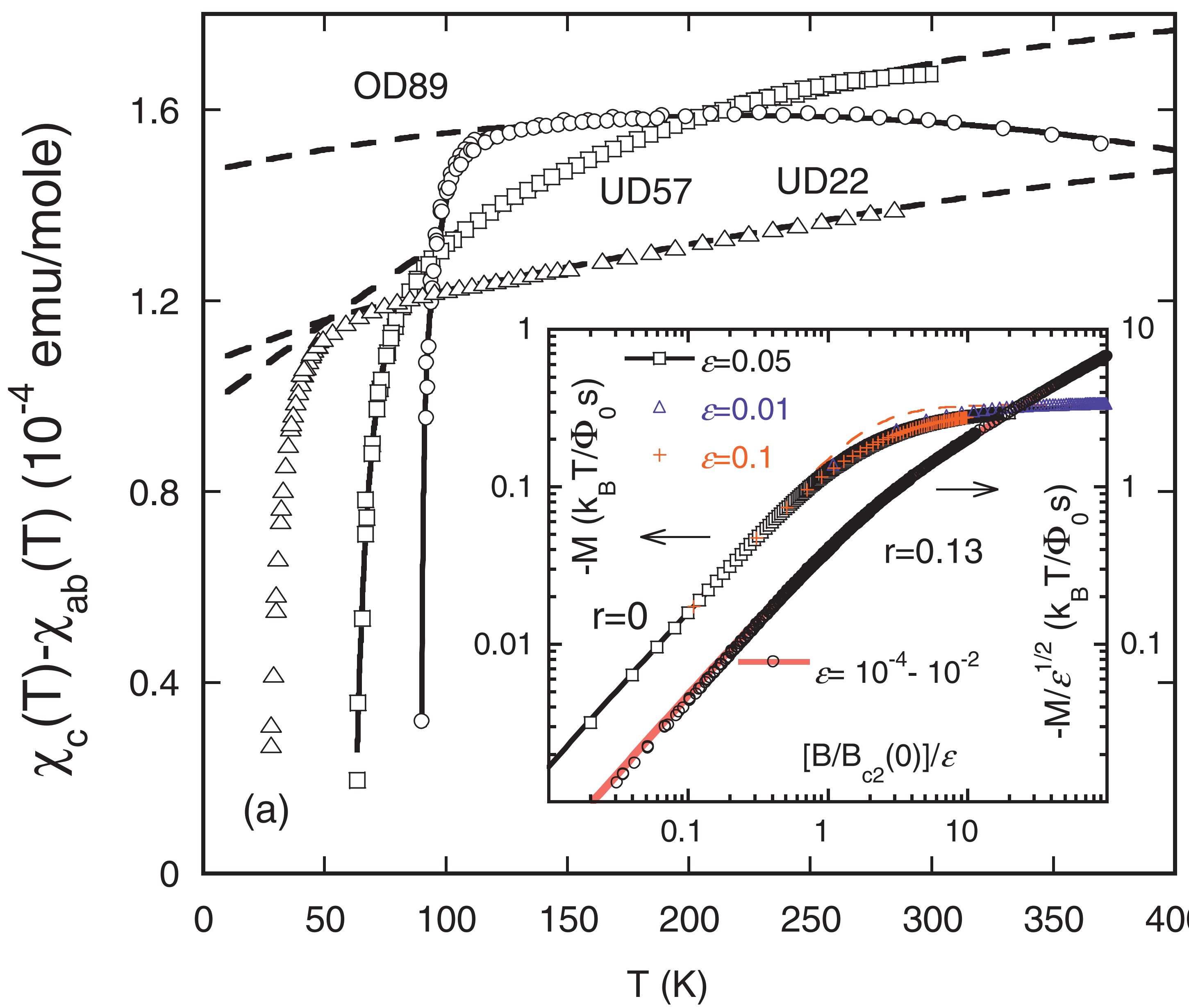
Diamagnetism in the pseudogap

Diamagnetism of $YBa_2Cu_3O_{6+x}$ crystals above T_c: Evidence for Gaussian fluctuations

I. Kokanović,[1,2,*] D. J. Hills,[1] M. L. Sutherland,[1] R. Liang,[3] and J. R. Cooper[1]

PHYSICAL REVIEW B **88**, 060505(R) (2013)



Diamagnetism in the pseudogap

- The *same* set of parameters used to describe X-ray scattering, also predict the strength of superconducting fluctuations above T_c. We characterize the diamagnetism by computing a dimensionless ratio, $R(T)$, between the diamagnetic susceptibility, χ_d, and the charge order correlation length:

$$R(T) \equiv \frac{12\pi \, \chi_d(T)}{k_B T \xi_{\rm cdw}^2}$$

L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, *to appear*

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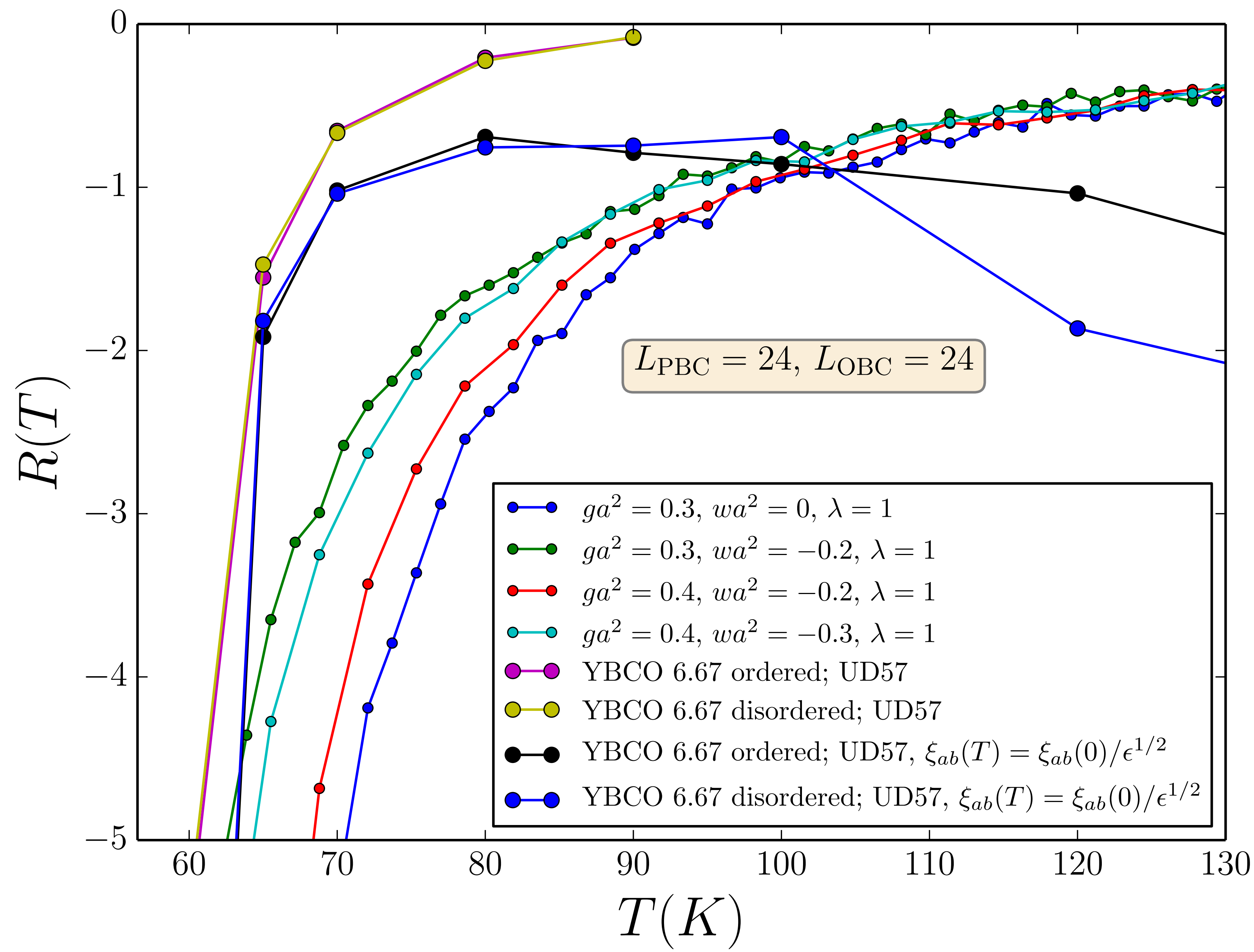

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