

Find the lowest eigenvalues $\lambda(\mathbf{Q})$, and corresponding eigenvectors, of the matrix

$$\delta_{\ell m} - \frac{1}{2} \left(\frac{3}{4} \mathcal{J}_\ell + \mathcal{V}_\ell \right) \Pi_{\ell m}(\mathbf{Q}) + \delta_{\ell,0} W(\mathbf{Q}) \Pi_{0m}(\mathbf{Q}), \text{ where}$$

$$\Pi_{\ell m}(\mathbf{Q}) = 2 \sum_{\mathbf{k}} \phi_\ell(\mathbf{k}) \phi_m(\mathbf{k}) \frac{f(\varepsilon(\mathbf{k} - \mathbf{Q}/2)) - f(\varepsilon(\mathbf{k} + \mathbf{Q}/2))}{\varepsilon(\mathbf{k} + \mathbf{Q}/2) - \varepsilon(\mathbf{k} - \mathbf{Q}/2)} \text{ and}$$

$$W(\mathbf{Q}) \equiv \sum_{\ell} \mathcal{V}_\ell \phi_\ell(0) \phi_\ell(\mathbf{Q})$$