At the saddle point, we set $i\lambda(x) = \xi^{-1}$, and then the "structure factor" $S(k)$ of the order parameter is

$$S(k) = \int d^2x \, \langle n_\alpha(x) n_\alpha(0) \rangle \, e^{ikx} = \frac{NT}{\rho_s} \frac{1}{(k^2 + \xi^{-2})}$$

This identifies ξ as the correlation length. The value of ξ is determined by the saddle-point equation, which simply enforces the constraint $n_\alpha^2(x) = 1$. So we have

$$\frac{NT}{\rho_s} \int \frac{d^2k}{4\pi^2} \frac{1}{k^2 + \xi^{-2}} = 1$$

Performing the k integral with a momentum cutoff Λ we obtain

$$\frac{NT}{4\pi\rho_s} \ln\left(1 + \Lambda^2 \xi^2\right) = 1 \quad \Rightarrow \quad \xi = \Lambda^{-1} \exp\left(\frac{2\pi\rho_s}{NT}\right)$$

So ξ is *finite* at all non-zero T (no LRO), and diverges exponentially as $T \to 0$ (consistent with Mermin-Wagner theorem).