

$$Z = \int \mathcal{D}\vec{n}(x) \delta(\vec{n}^2(x) - 1) \exp\left(-\frac{\rho_s}{2T} \int d^2x (\nabla_x \vec{n})^2\right)$$

Generalize  $\vec{n}$  to a  $N$ -component vector  $n_\alpha$ ,  $\alpha = 1 \dots N$ , and take the  $N \rightarrow \infty$  limit while taking  $\rho_s \propto N$ . This is implemented by a Lagrange multiplier  $\lambda$

$$Z = \int \mathcal{D}\lambda(x) \mathcal{D}n_\alpha(x) \exp\left(-\frac{\rho_s}{2T} \int d^2x \left[(\nabla_x n_\alpha)^2 + i\lambda(n_\alpha^2 - 1)\right]\right)$$

We can now perform the Gaussian integral over  $n_\alpha$

$$Z = \int \mathcal{D}\lambda(x) \exp\left(-\frac{N}{2} \text{Tr} \ln(-\nabla_x^2 + i\lambda) + \frac{\rho_s}{2T} \int d^2x i\lambda\right)$$

Because  $\rho_s \propto N$ , in the  $N \rightarrow \infty$  limit the partition function is dominated by the saddle point.