Planckian metals and SYK criticality

KAIST, Korea
February 19, 2021

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PHYSICS
VE RI TAS
HARVARD

1. Key puzzle in the cuprates

2. Numerical solution of the SU(2) random t-J model

3. Large-M SYK solution of the SU(M) random t-J model

Fractionalization and deconfined criticality

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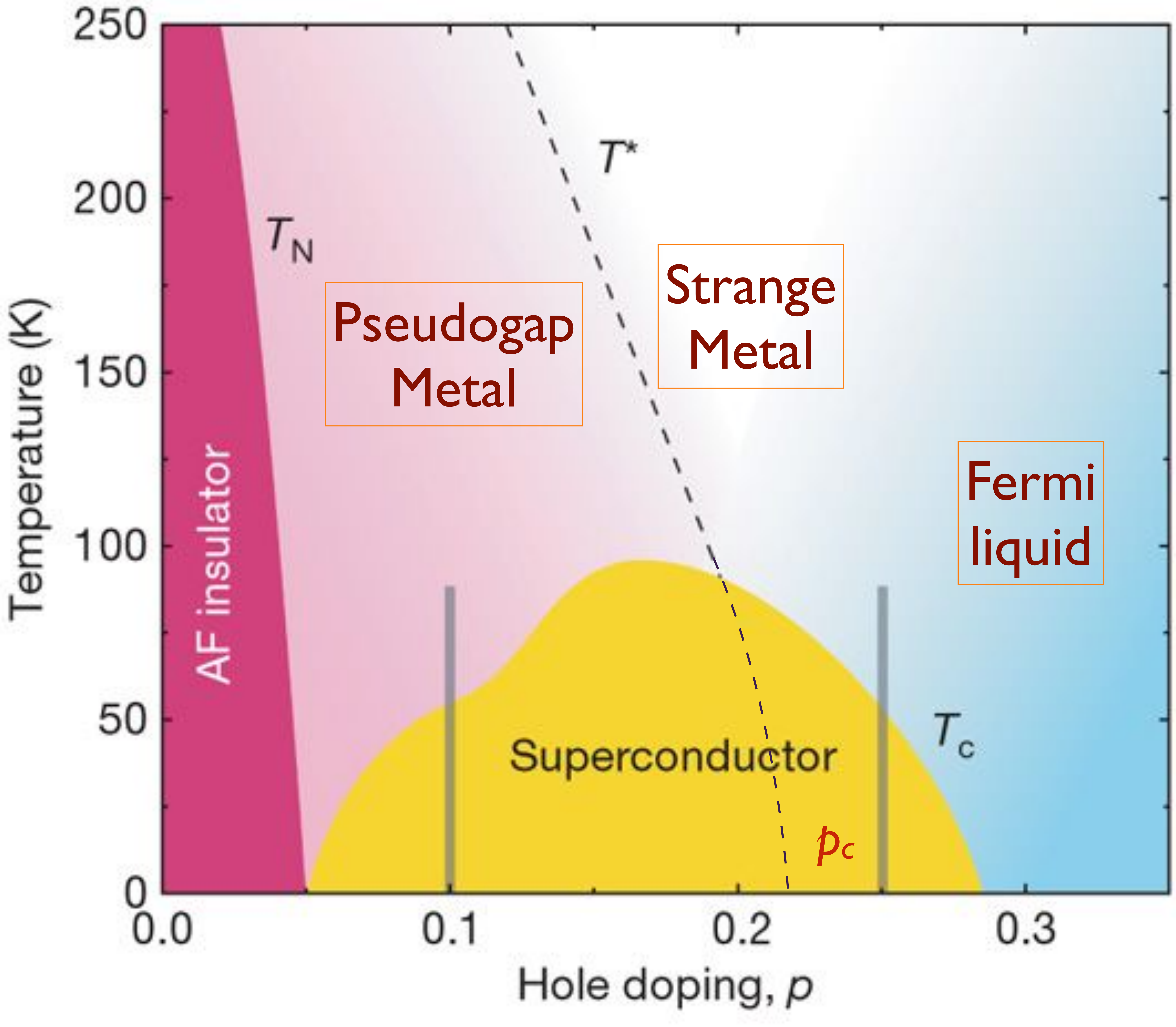
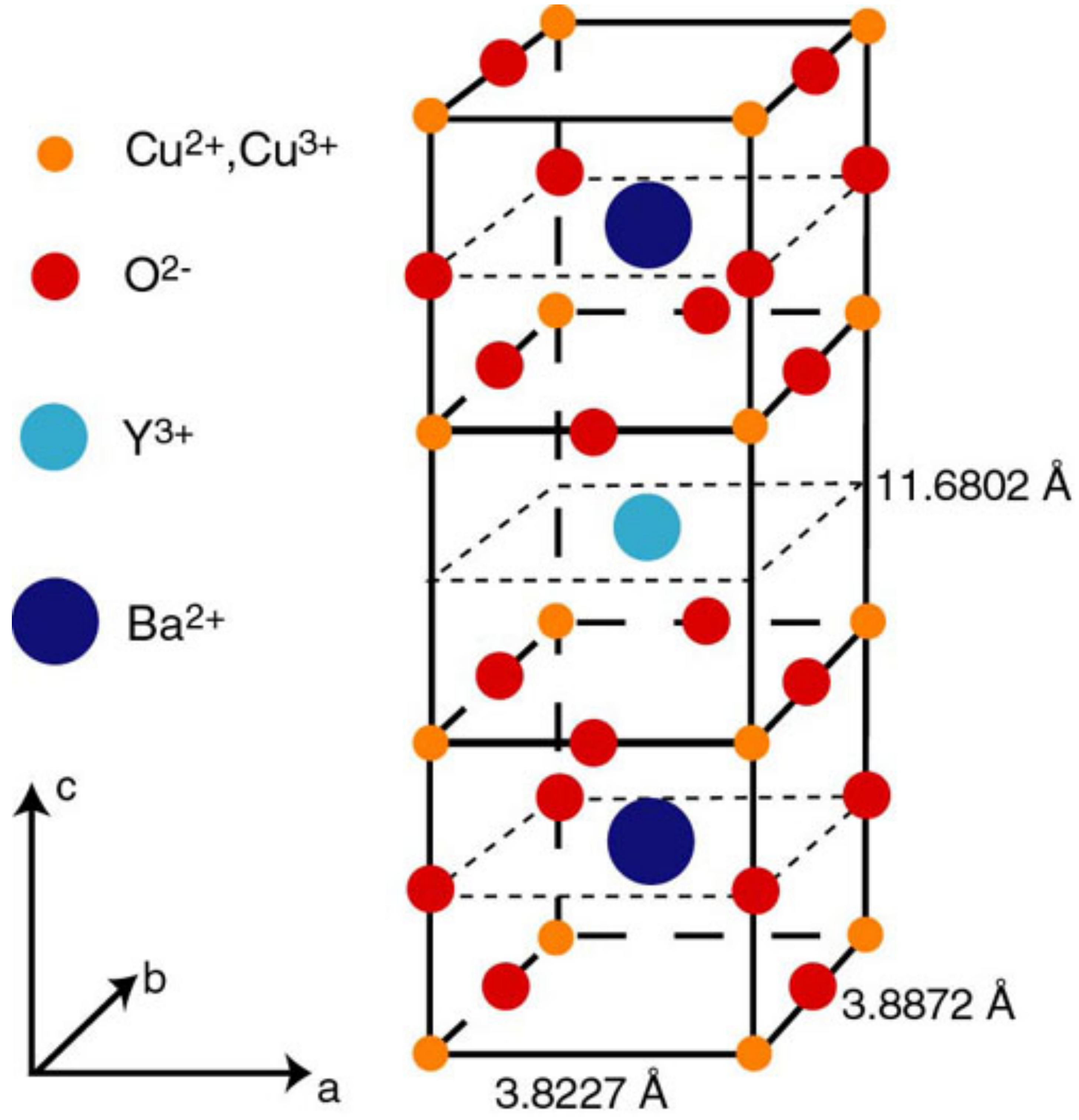
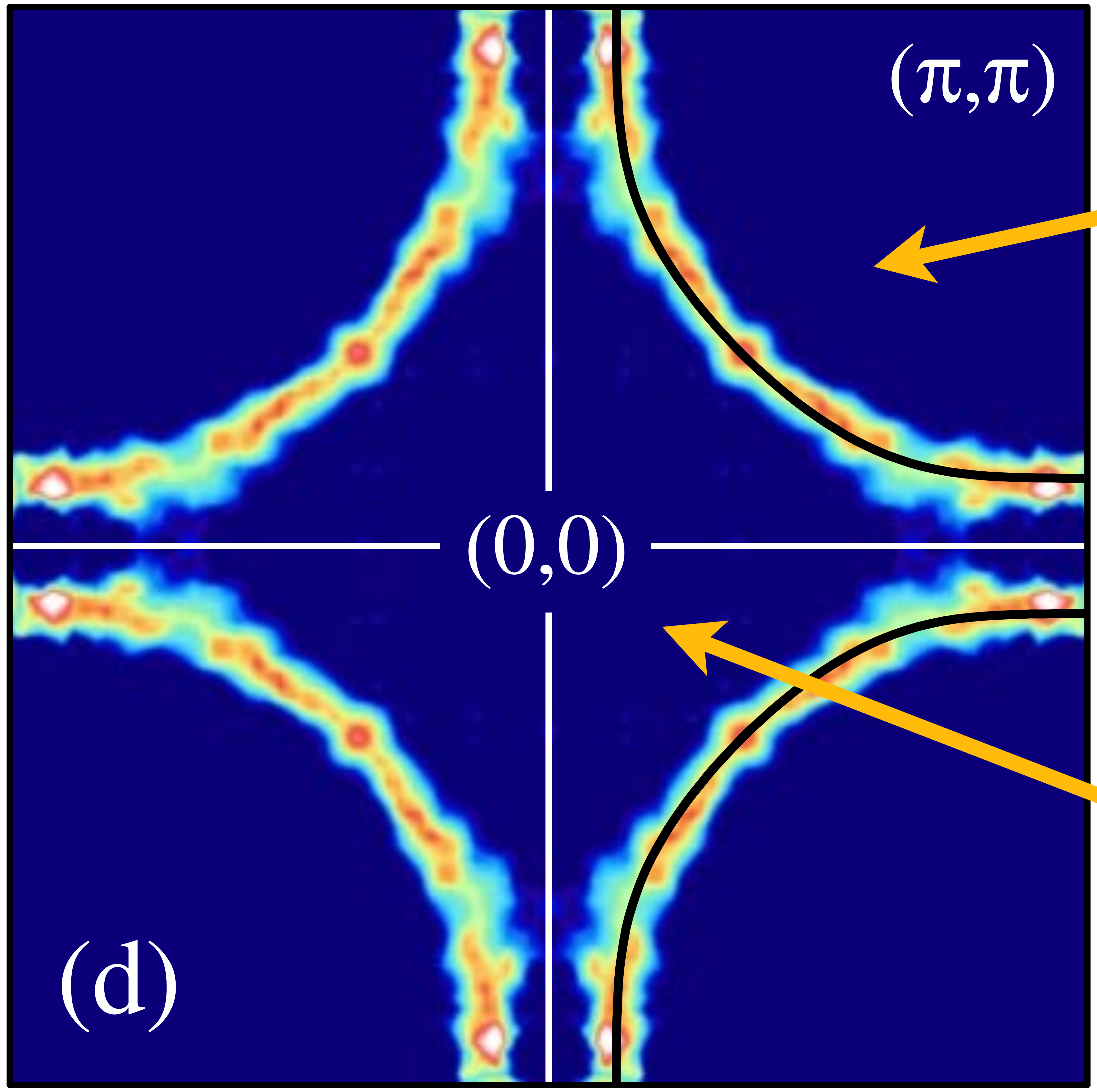
Fractionalization and deconfined criticality


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Pseudogap Metal
Strange Metal
Fermi liquid
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Momentum-space view at large p



$1+p$ holes

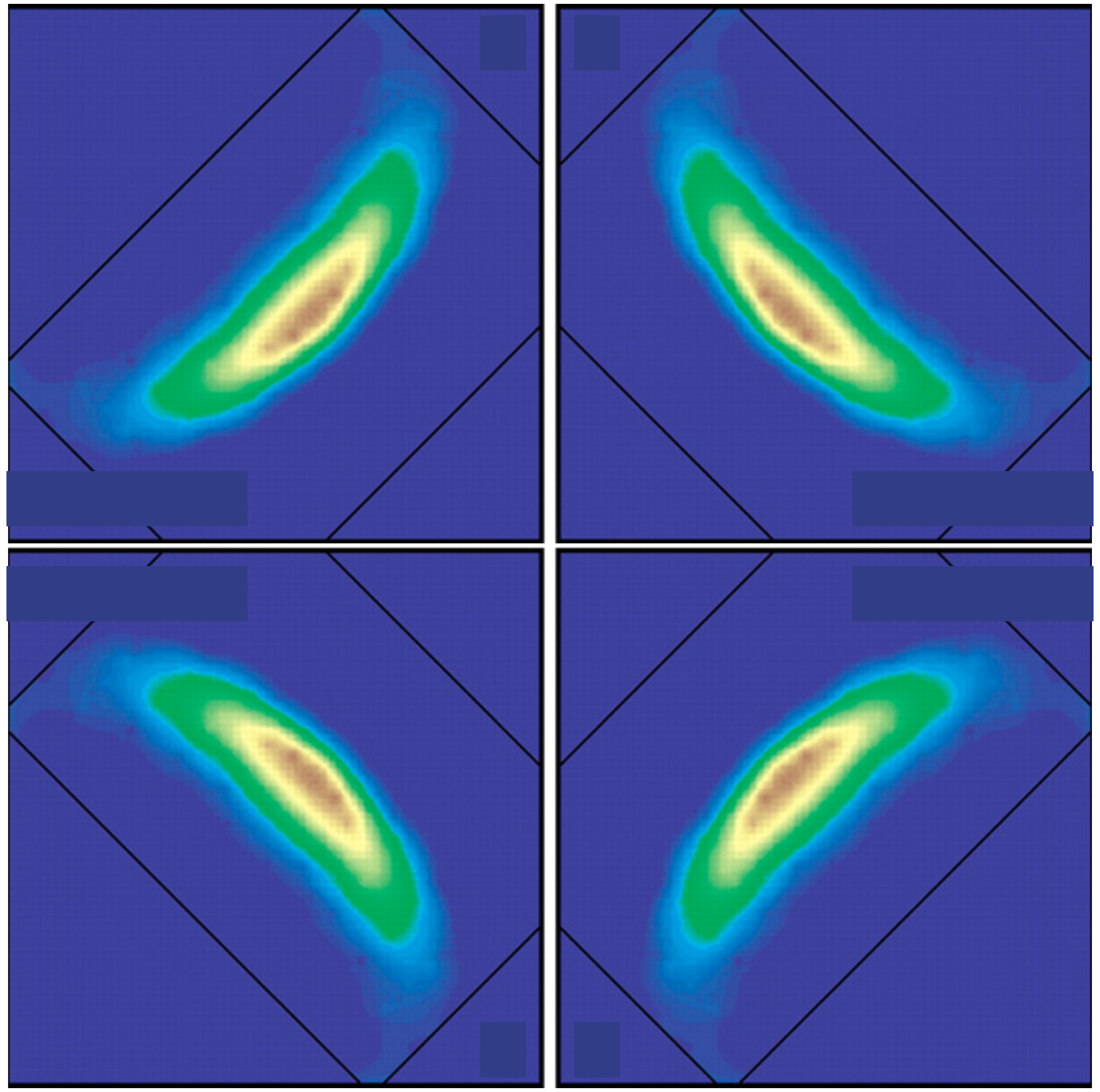
1-p electrons

Overdoped $Tl_2Ba_2CuO_{6+\delta}$
$T_c = 30K$

$1+p$ mobile holes in a filled band

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

Momentum-space view at small p



$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

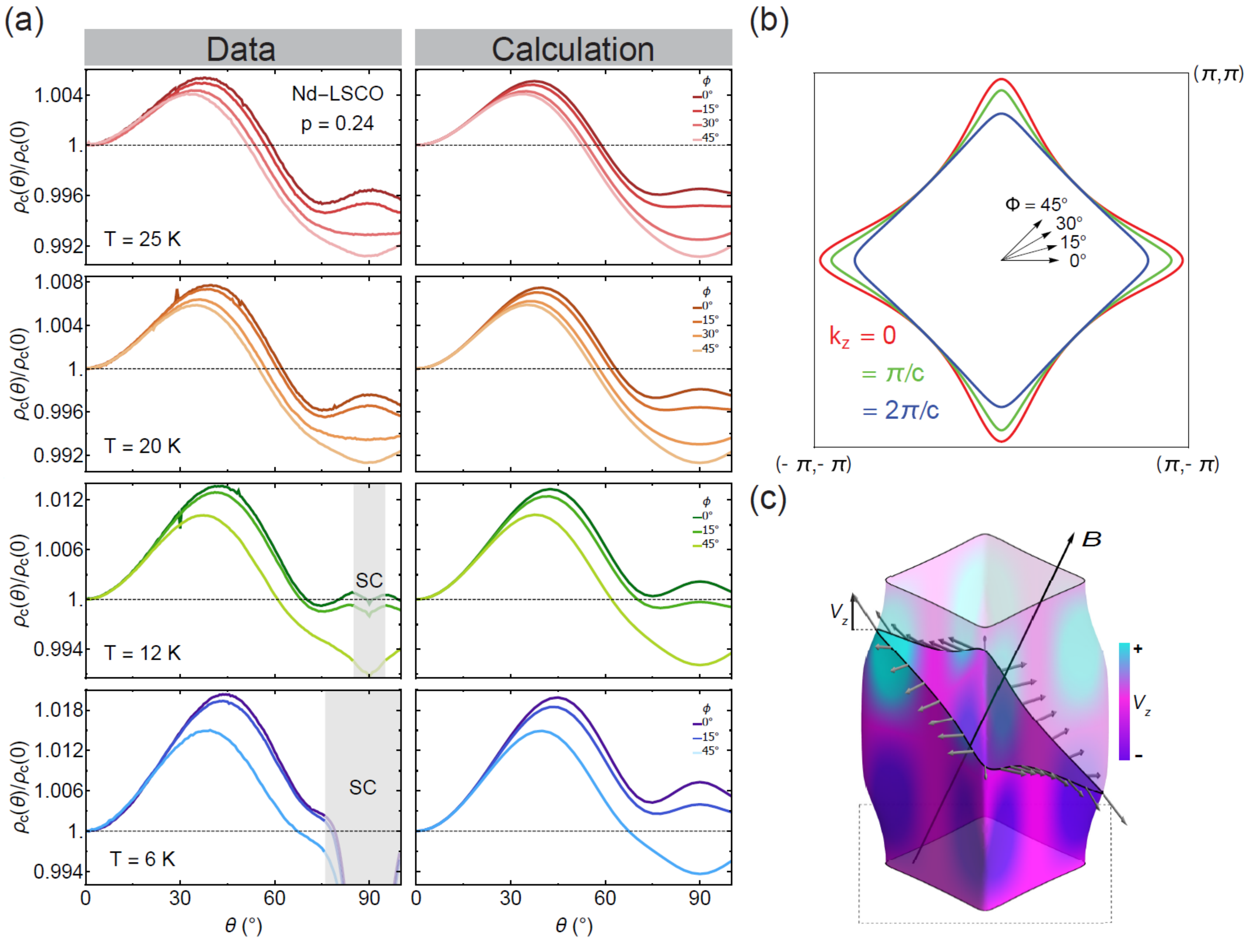
"Fermi arcs"

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

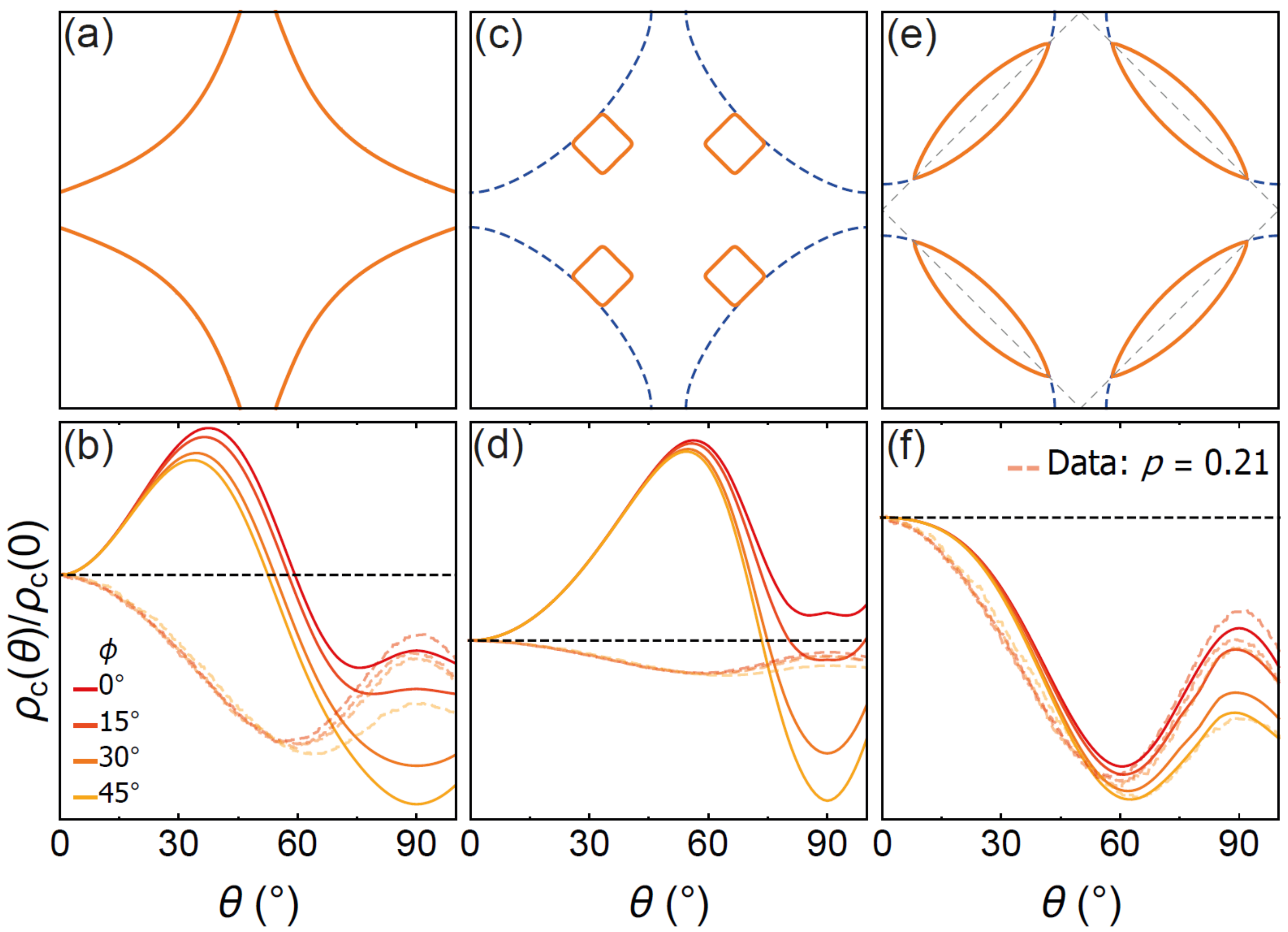
Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, arXiv:2004.01725

We use angle-dependent magnetoresistance (ADMR) to measure the Fermi surface of the cuprate $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$. Above the critical doping p*—outside of the pseudogap phase—we find a Fermi surface that is in quantitative agreement with angle-resolved photoemission. Below p*, however, the ADMR is qualitatively different, revealing a clear change in Fermi surface topology. We find that our data is most consistent with a Fermi surface that has been reconstructed by a Q = (π,π) wavevector. While static Q = (π,π) antiferromagnetism is not found at these dopings, our results suggest that this wavevector is a fundamental organizing principle of the pseudogap phase.



$p > p_c$ Large Fermi surface

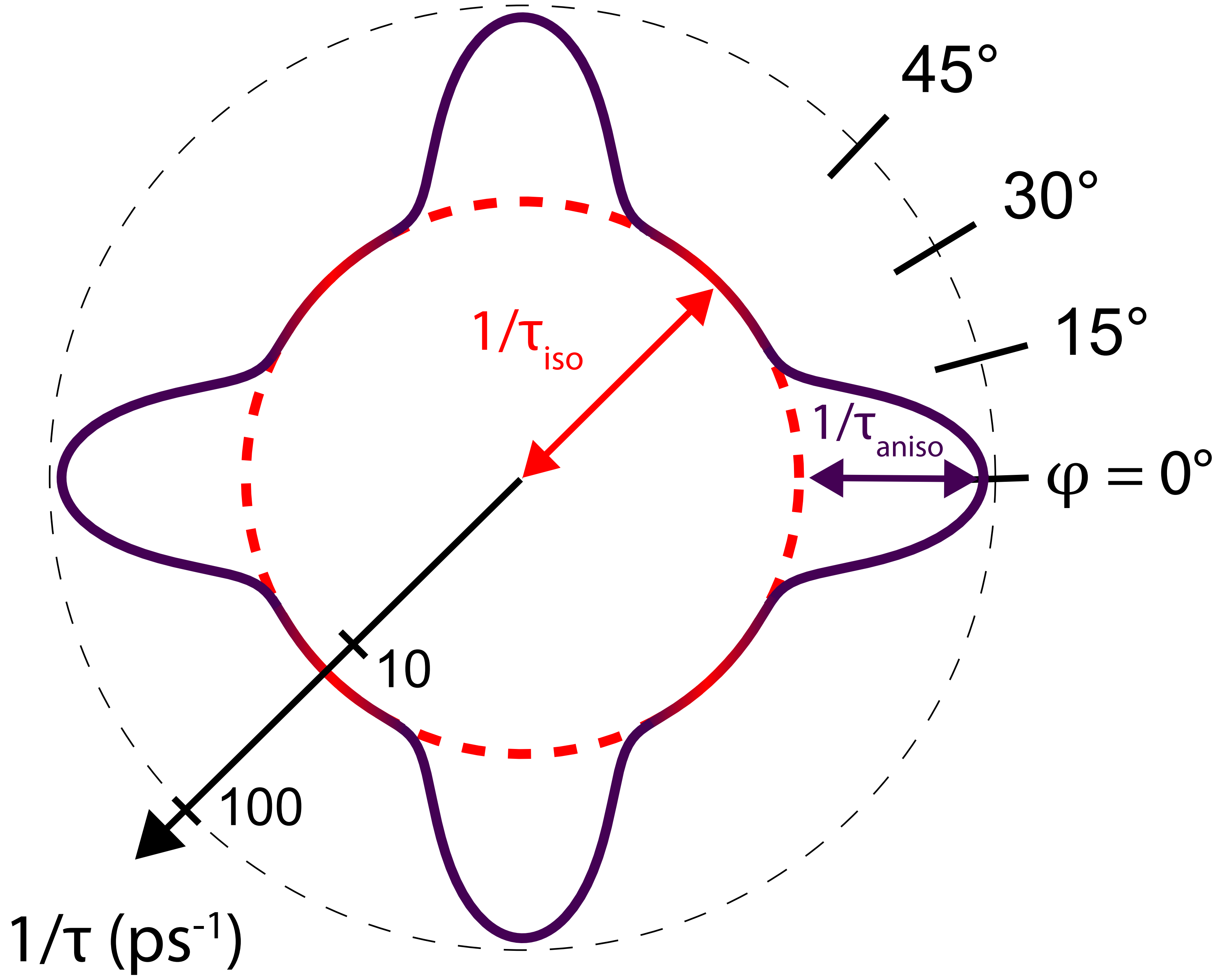


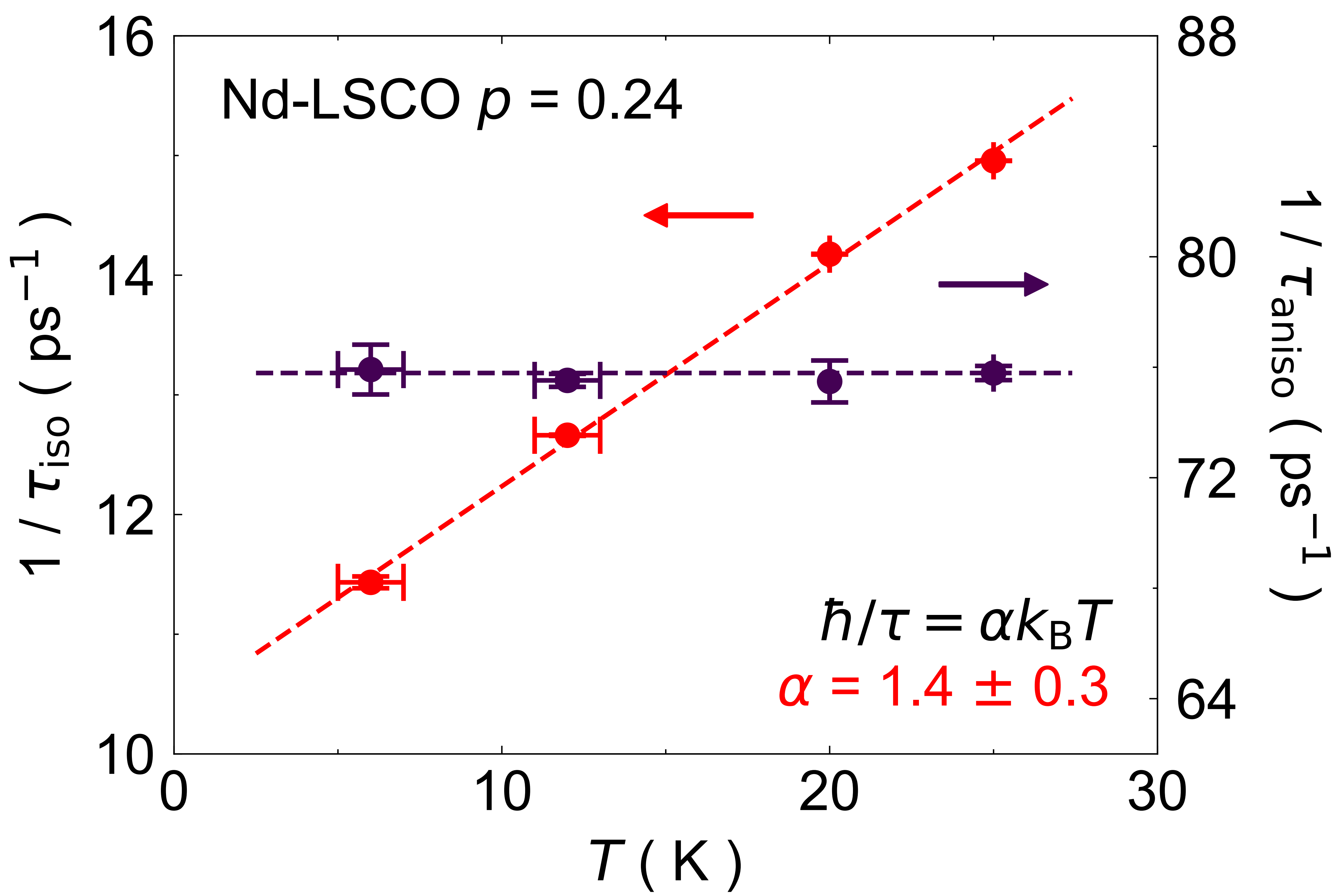
$p < p_c$ Reconstructed Fermi surface

Measurement of the Planckian Scattering Rate

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou,
D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw, arXiv:2011.13054

Angle-dependent magnetoresistance in Nd-LSCO near $p = p_c \approx 0.23$.



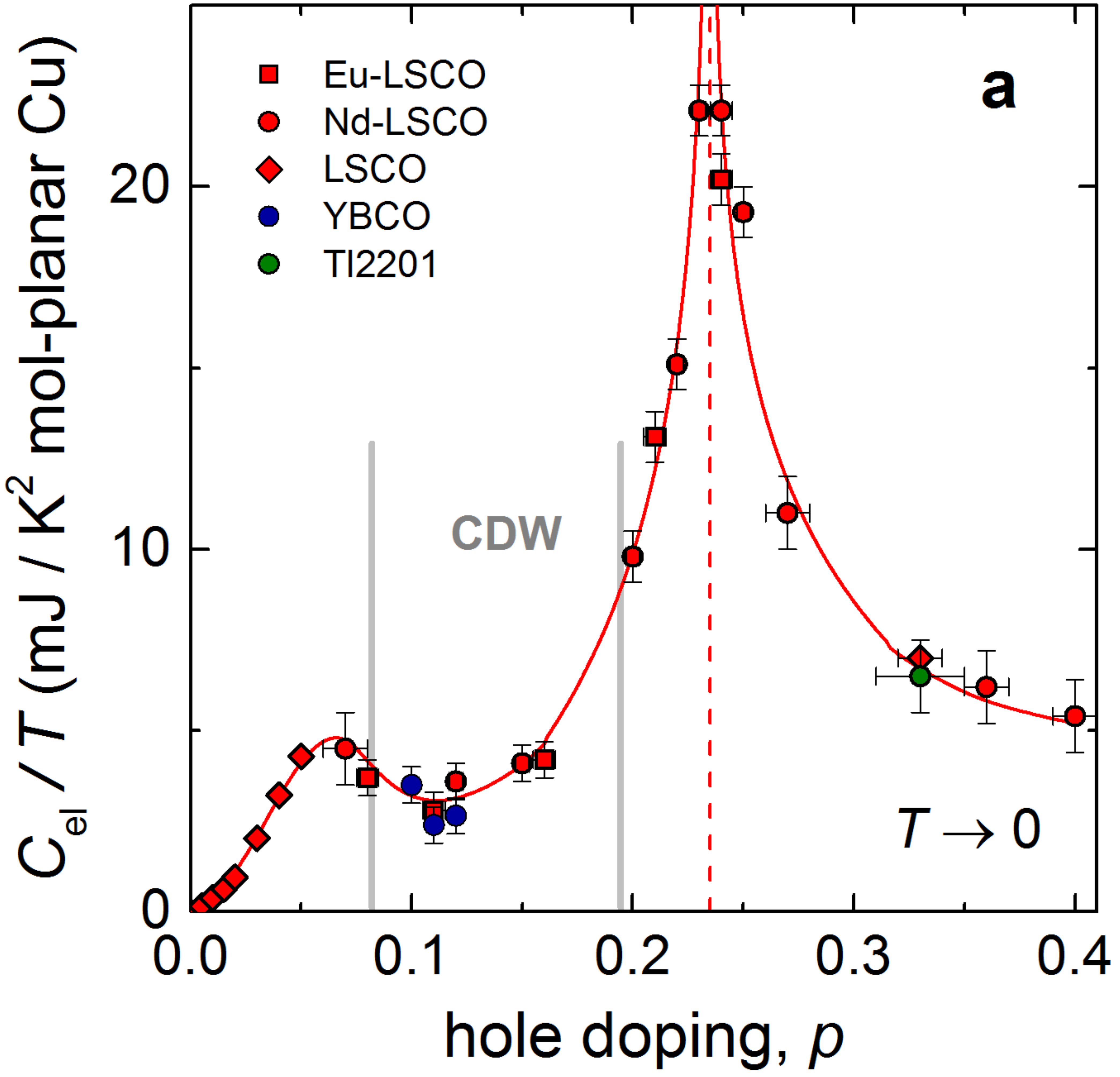


$$\frac{1}{\tau} = \frac{1}{\tau_{\mathrm{aniso}}(\vec{k})} + \frac{\alpha}{\hbar} k_B T$$

Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

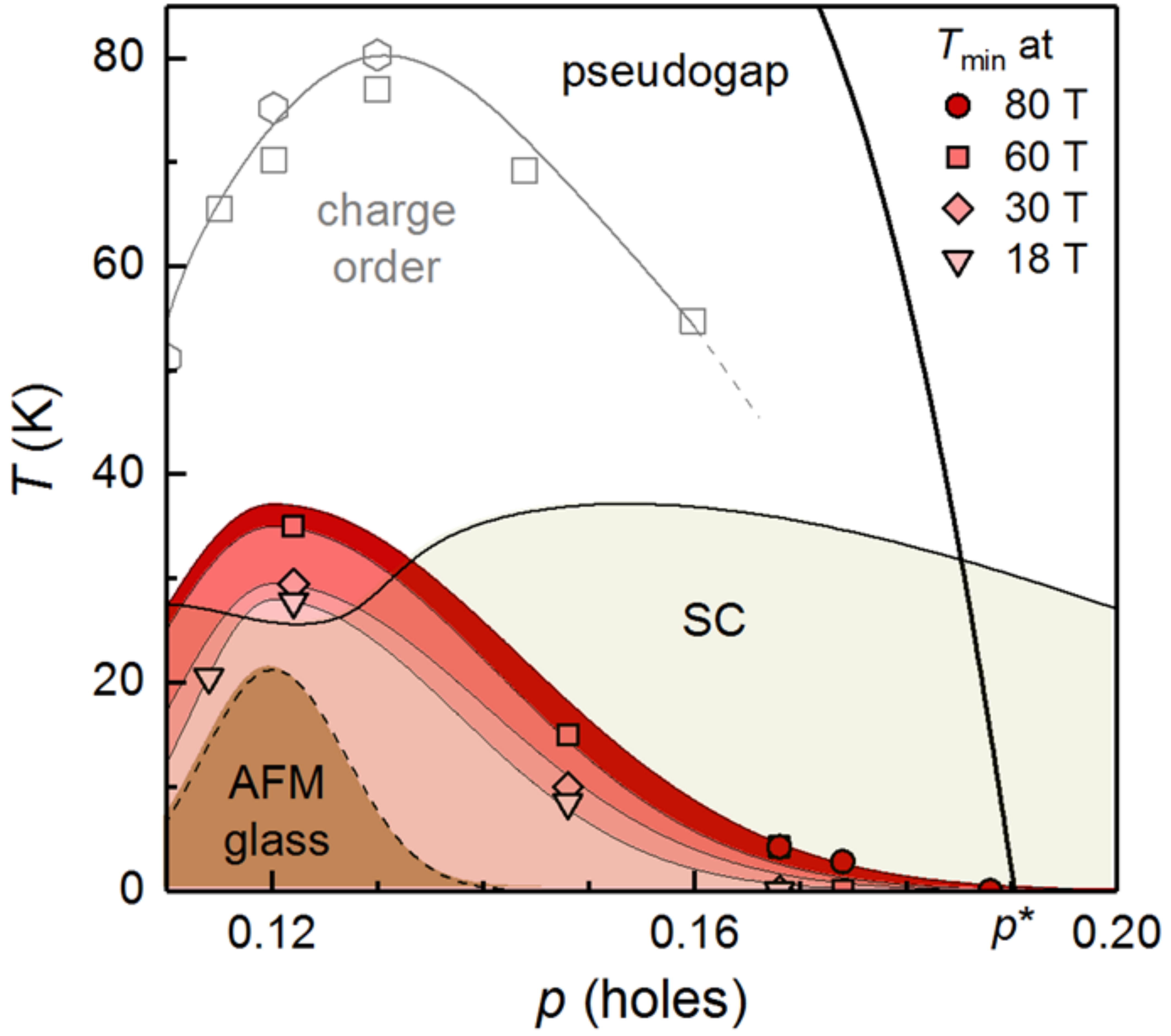
Cyril Proust and Louis Taillefer, Annual Review Condensed Matter Physics **10,** 409 (2019)



Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Nature Physics doi: 10.1038/s41567-020-0950-5

Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1]*, Marc-Henri Julien[1]*



Quasi-static magnetism in the pseudogap state of La2-xSrxCuO4.

Temperature – doping phase diagram representing *T*min, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of *T*min in zero-field, the dashed line (brown area) represents the extrapolated *T*min(*B*=0). While not exactly equal to the freezing temperature *T*f (see Fig. 2), *T*min is closely tied to *T*f and so is expected to have the same doping dependence, including a peak around *p* = 0.12 in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).

Needed:

A small Fermi surface
to
large Fermi surface
transition in
a single band model
with
Planckian criticality

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Fractionalization and deconfined criticality

Henry Shackleton

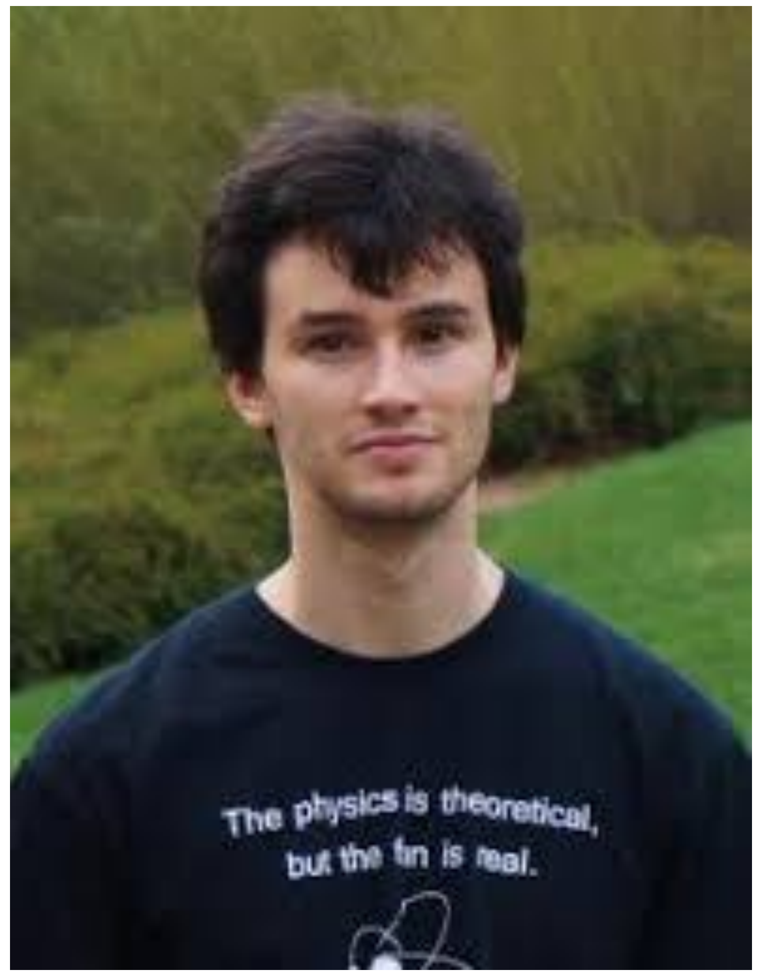
arXiv:2012.06589

Alexander Wietek

Antoine Georges

Maria Tikhanovskaya

Haoyu Guo



Grigory Tarnopolsky

arXiv:2010.09742
arXiv:2012.14449

Random t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^{\dagger}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = 1 - p$$

$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J_{ij}^2} = J^2$$
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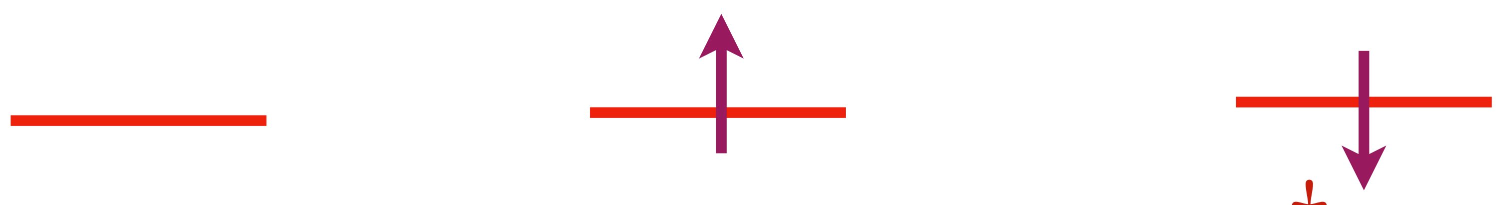

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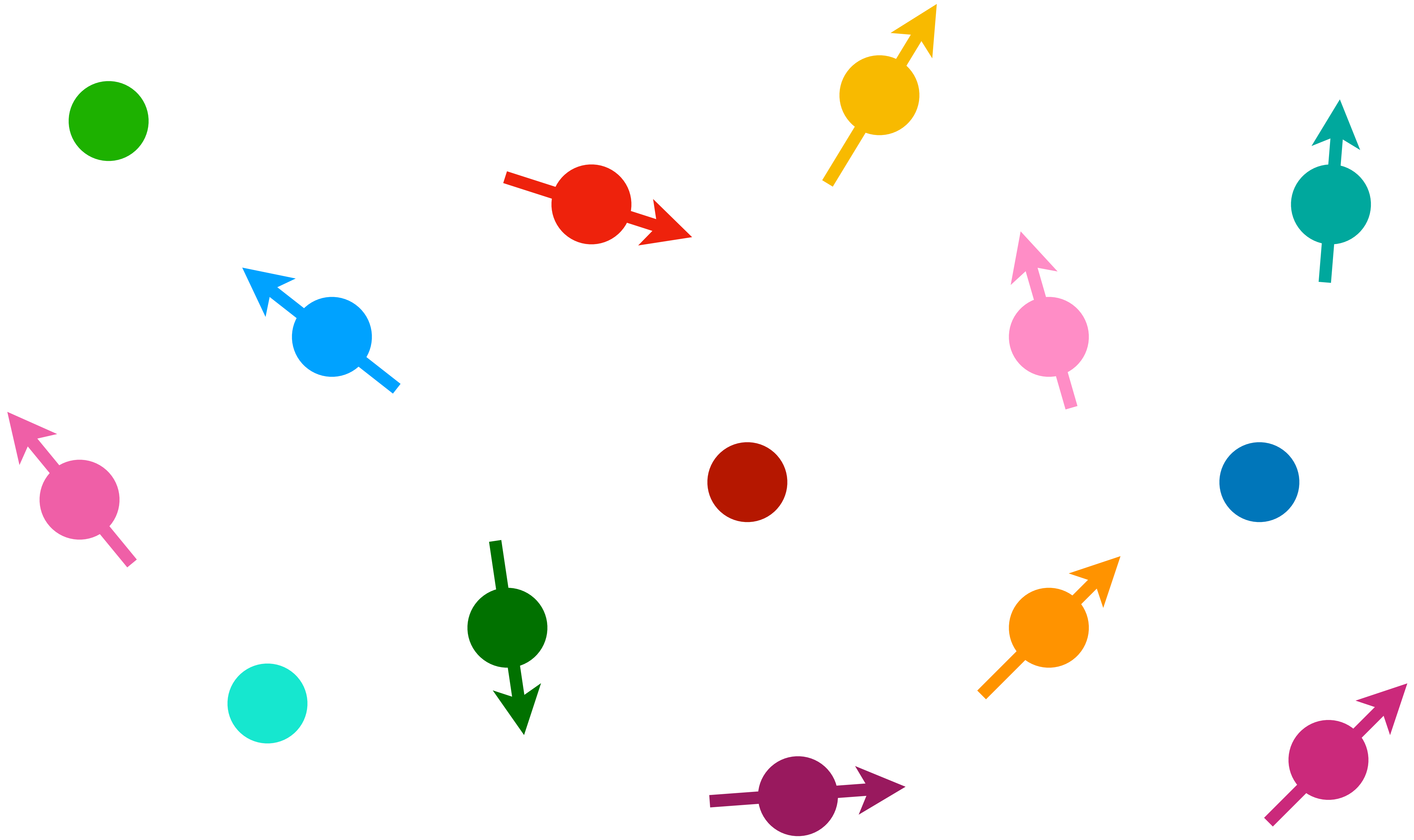

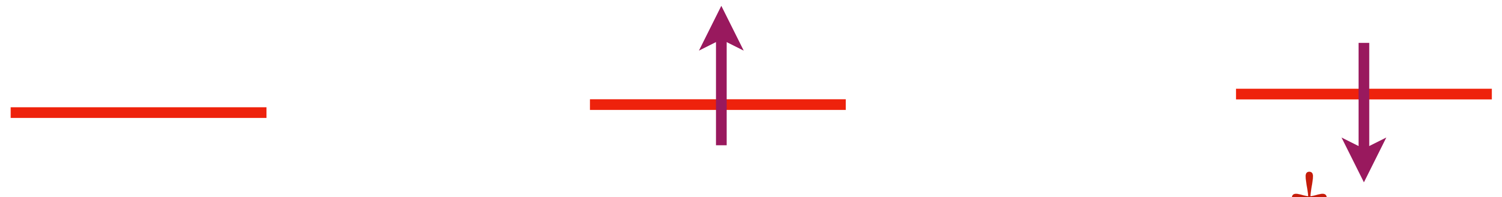

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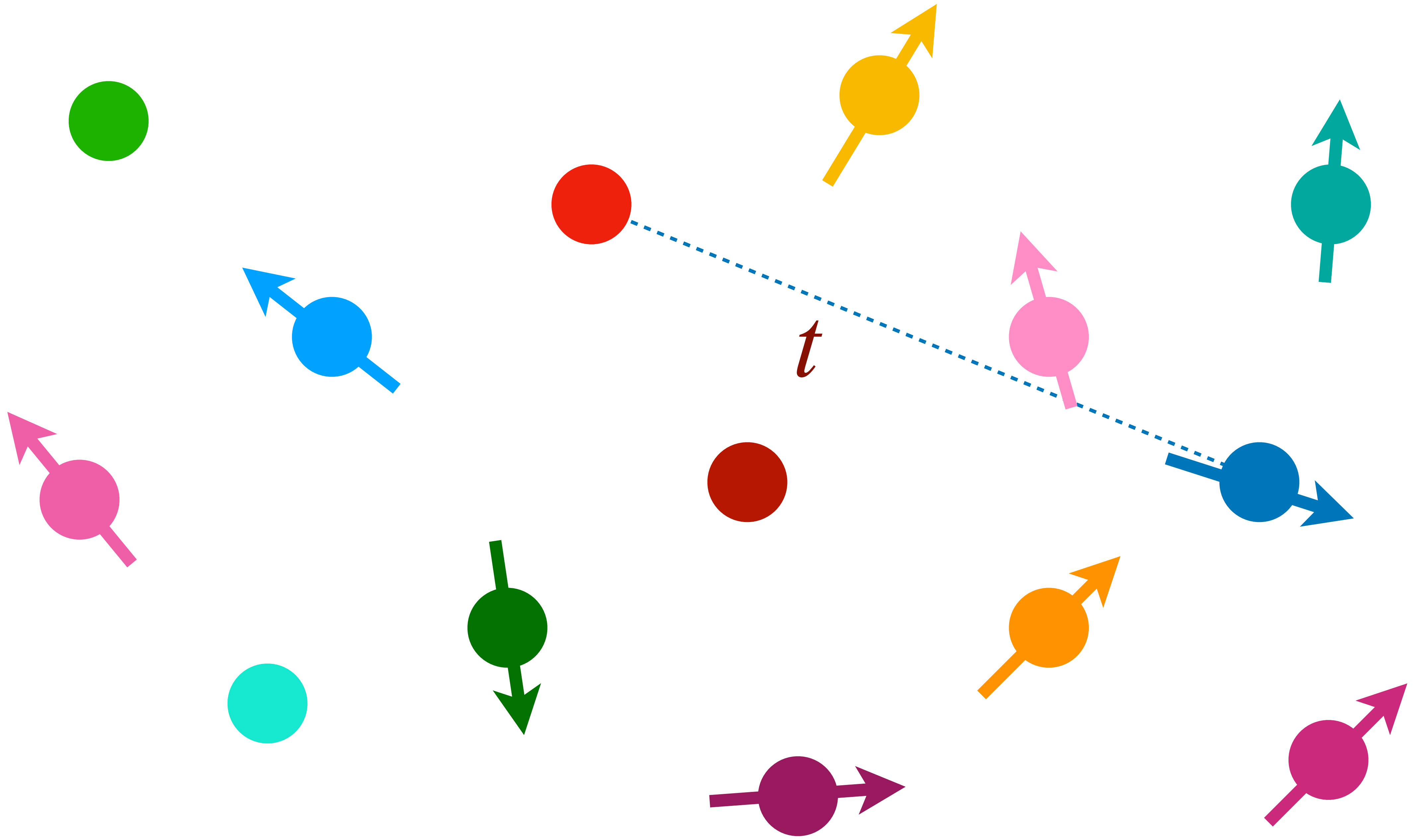

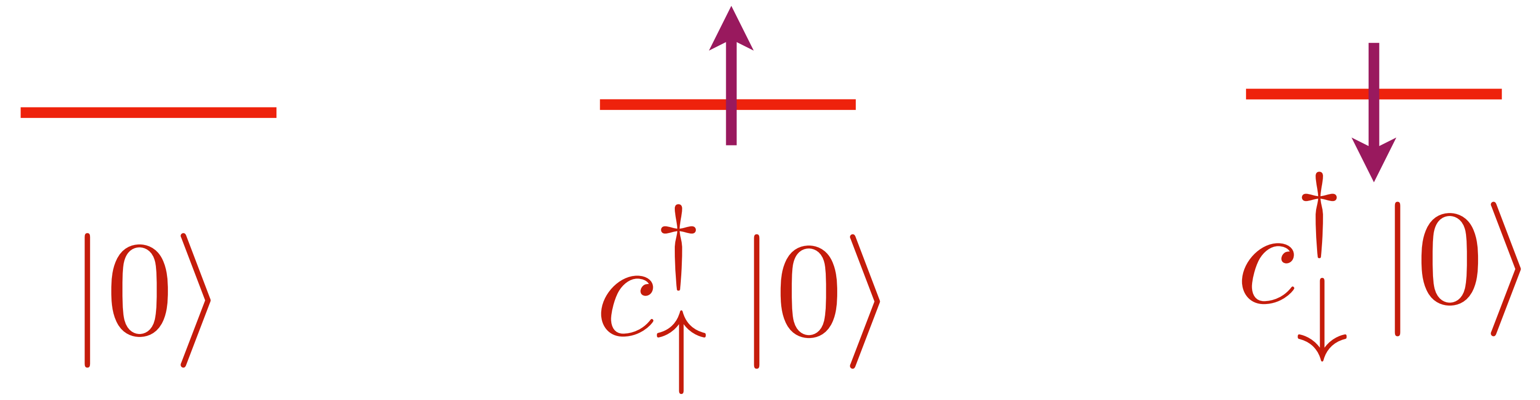

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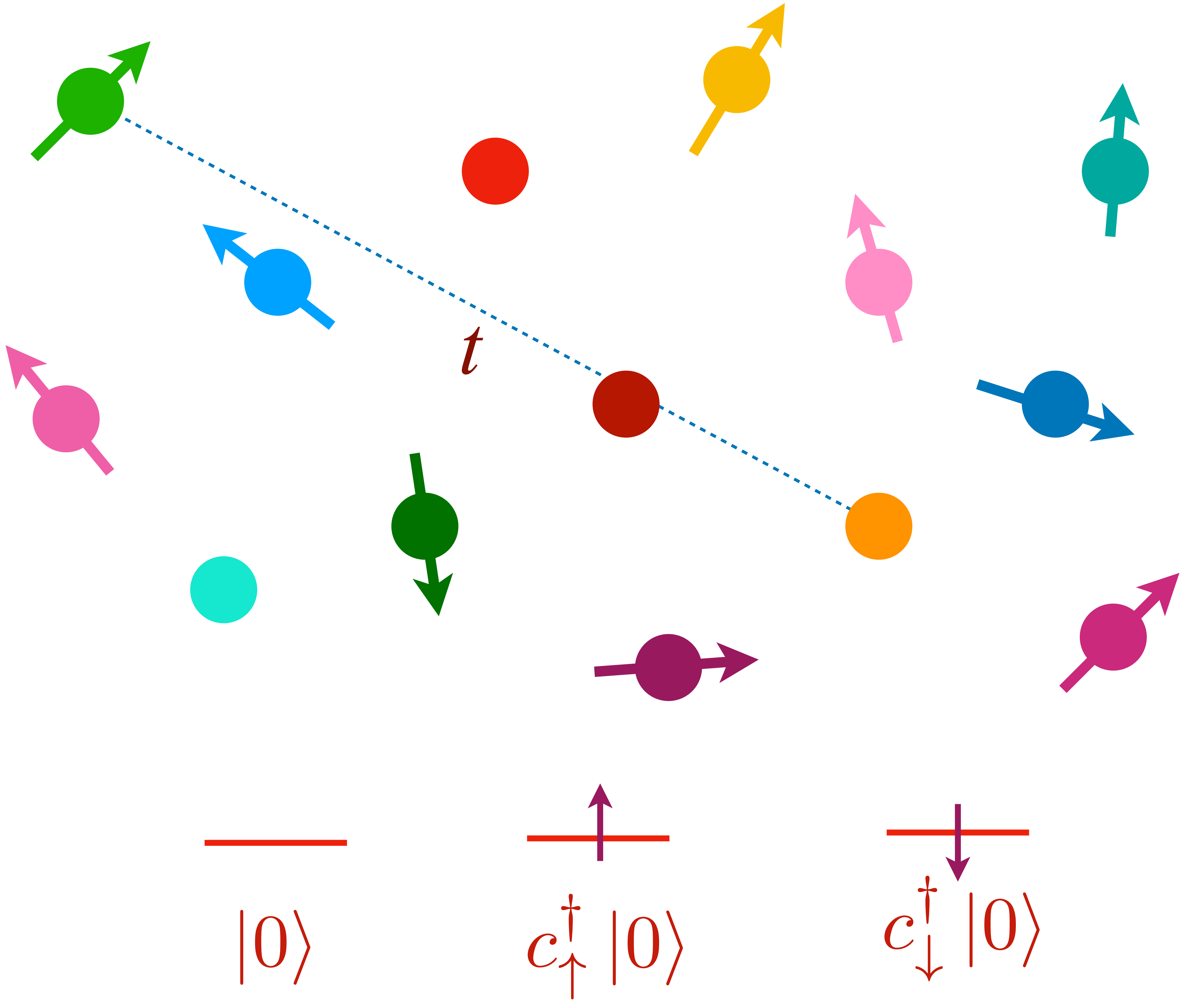

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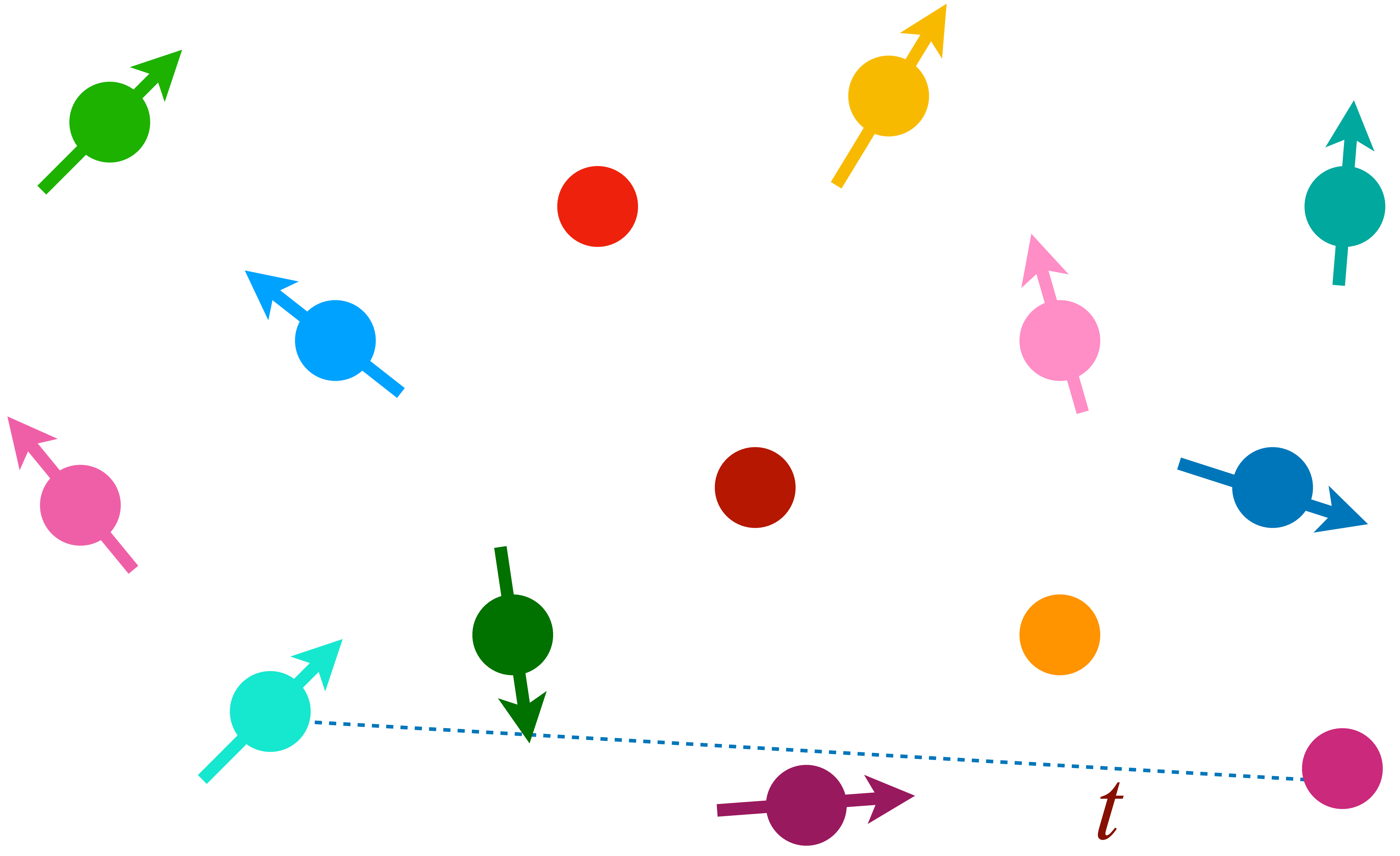

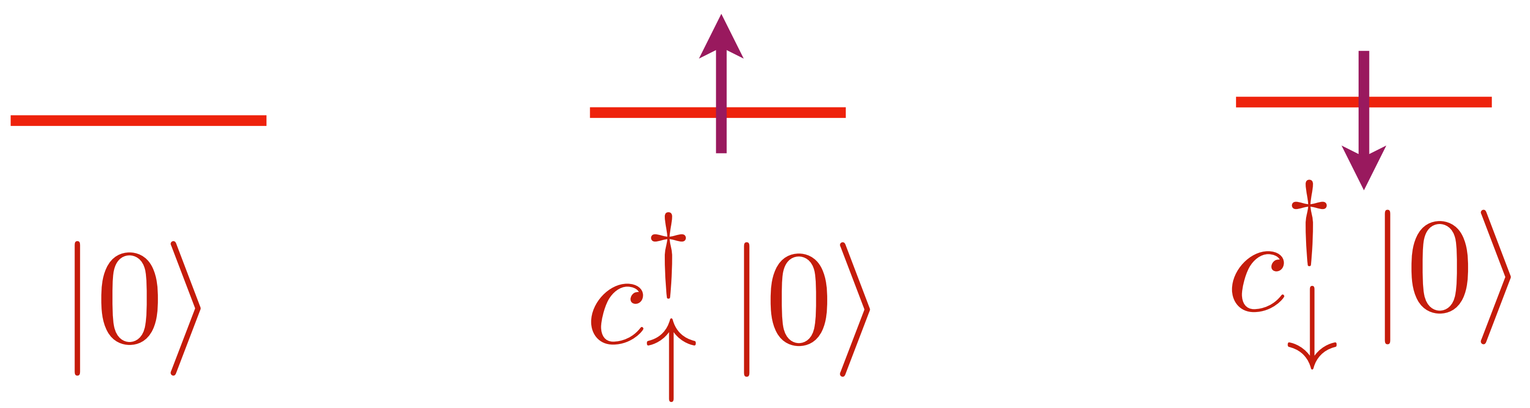

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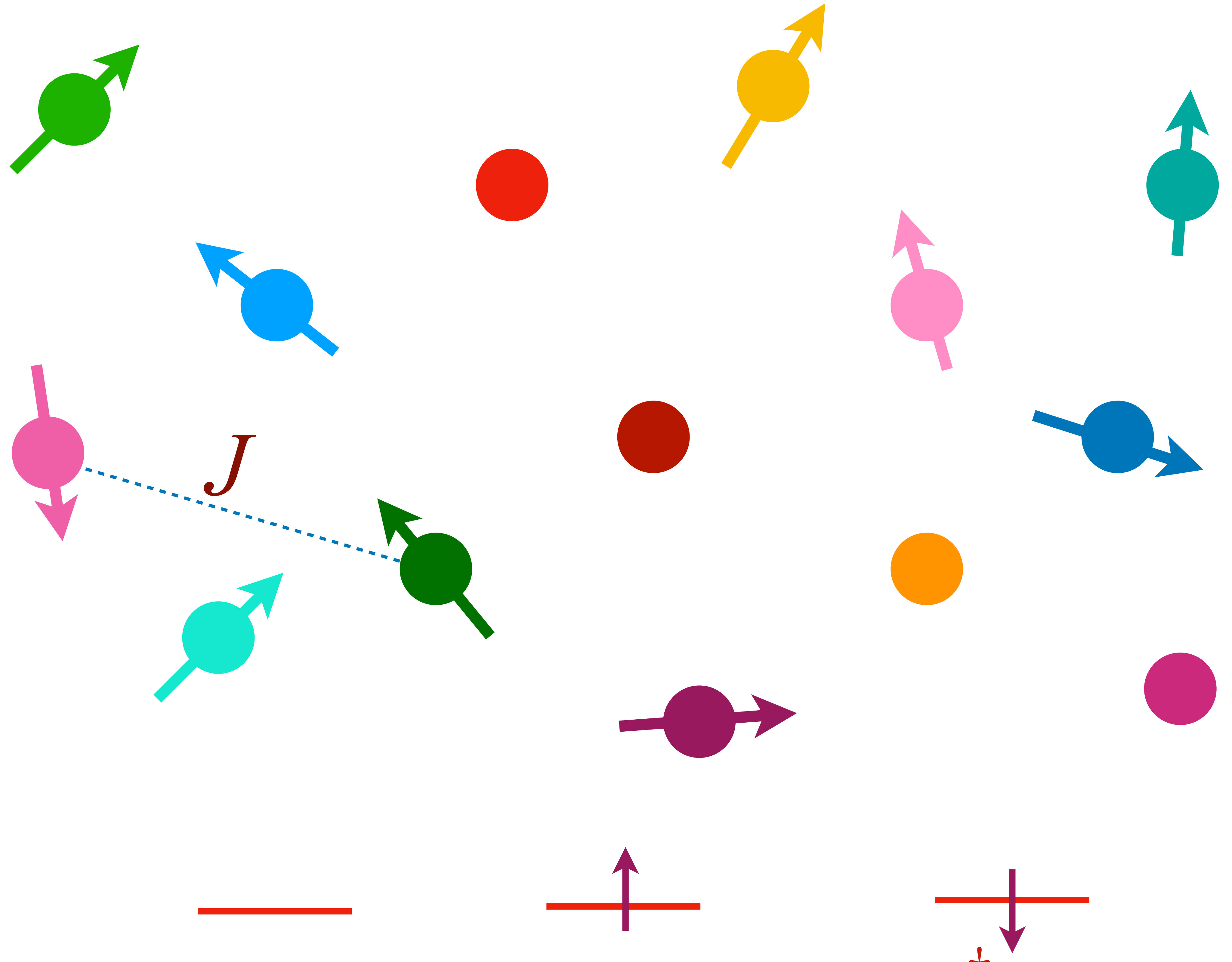

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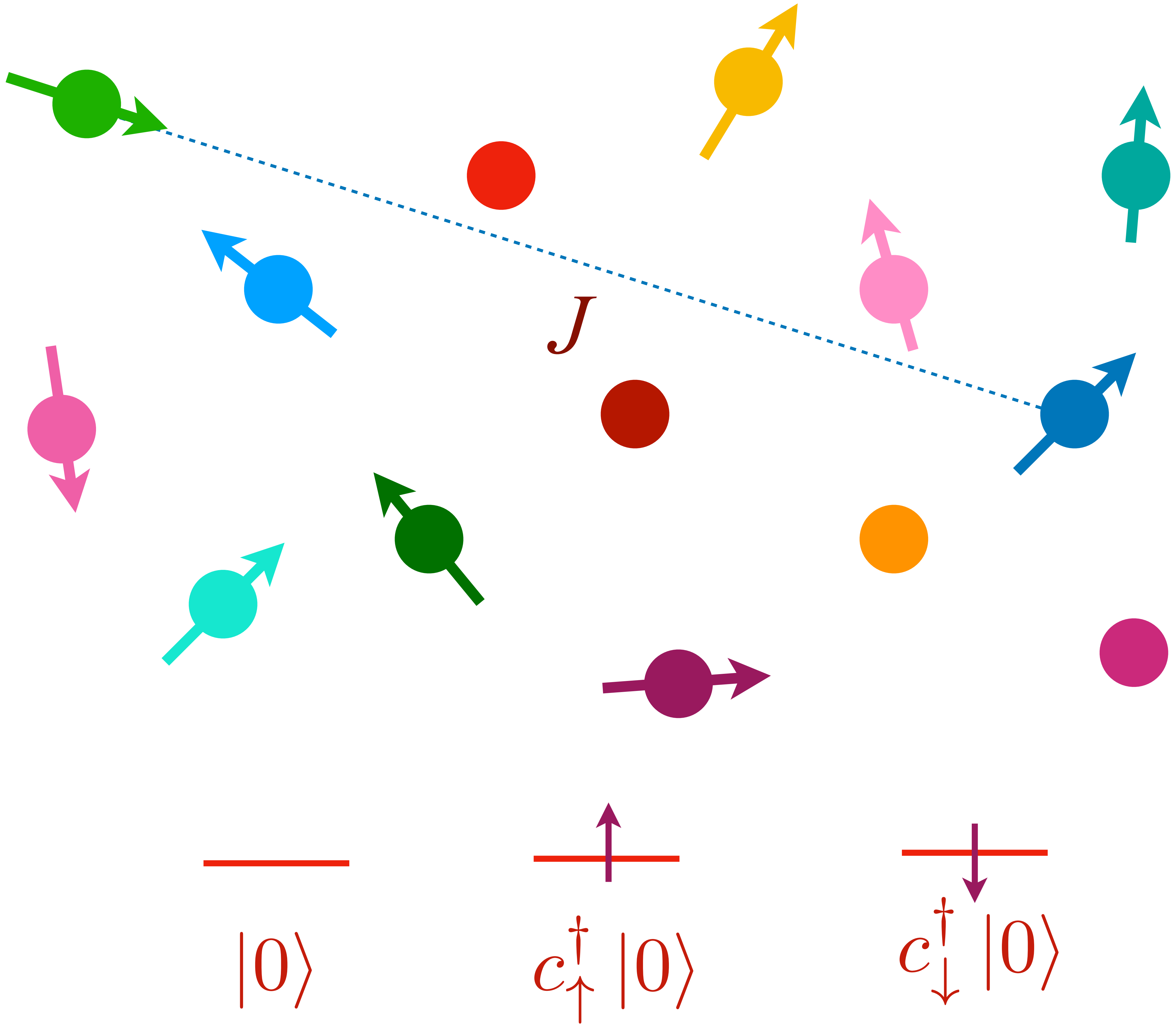

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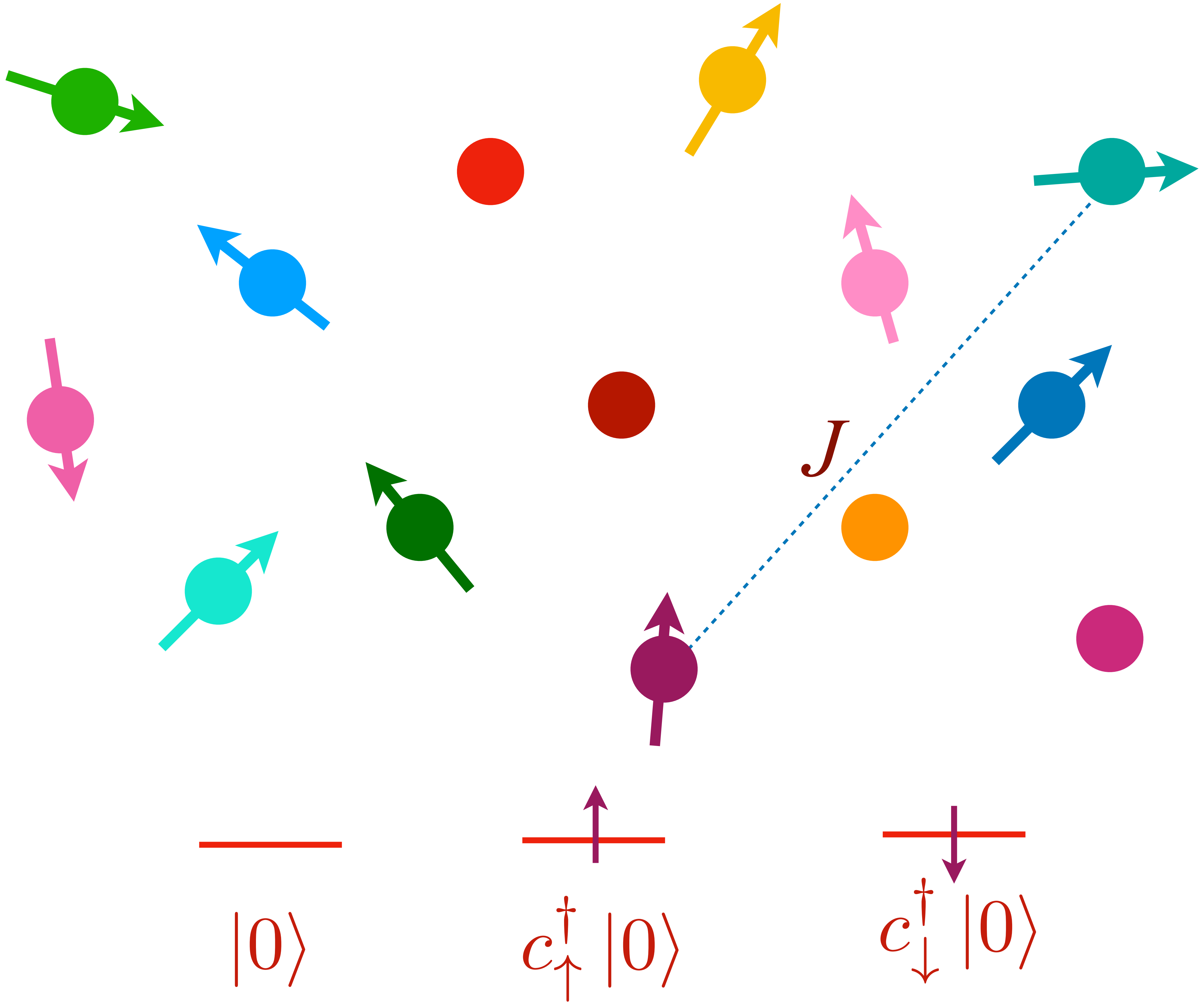

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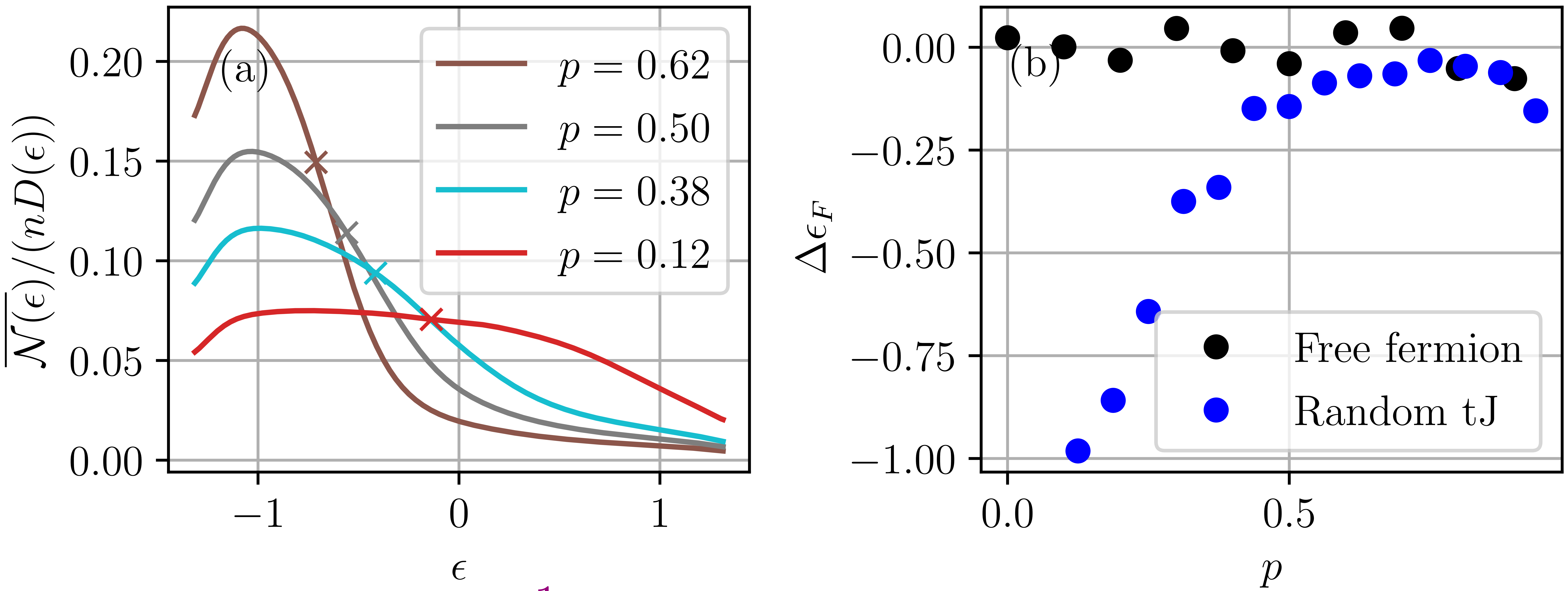
- Introducing randomness removes the "distractions" of multiple competing orders
- Averaging over many samples allows smoother and faster approach to the thermodynamic limit from finite size studies.

One particle energy distribution function



$$\mathcal{N}(\epsilon) = \frac{1}{N} \sum_{\lambda} \delta(\epsilon - \epsilon_\lambda) \sum_{ij\sigma} \langle \lambda | i \rangle \langle c^\dagger_{i\sigma} c_{j\sigma} \rangle \langle j | \lambda \rangle$$

where $|\lambda\rangle$ are one-particle eigenstates of the t_{ij}. In a Fermi liquid, the Luttinger identity implies that $\mathcal{N}(\epsilon)$ has a discontinuity at the free particle Fermi energy ϵ_F. ($D(\epsilon)$ is the Wigner semi-circle density of states.)

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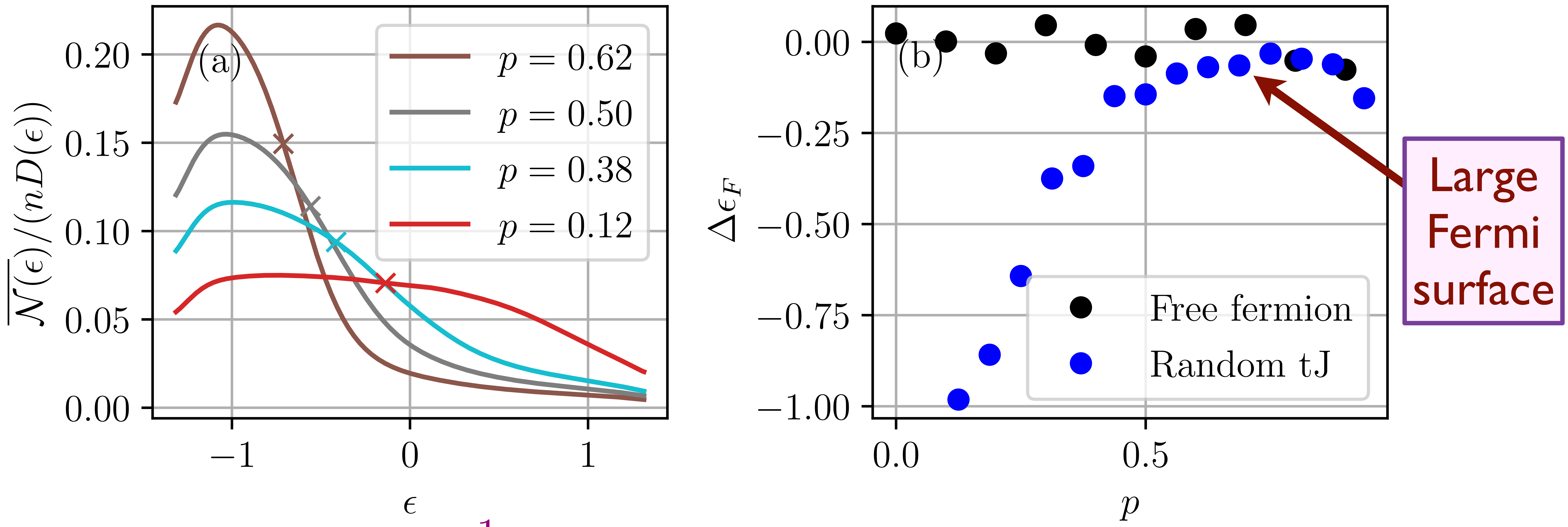

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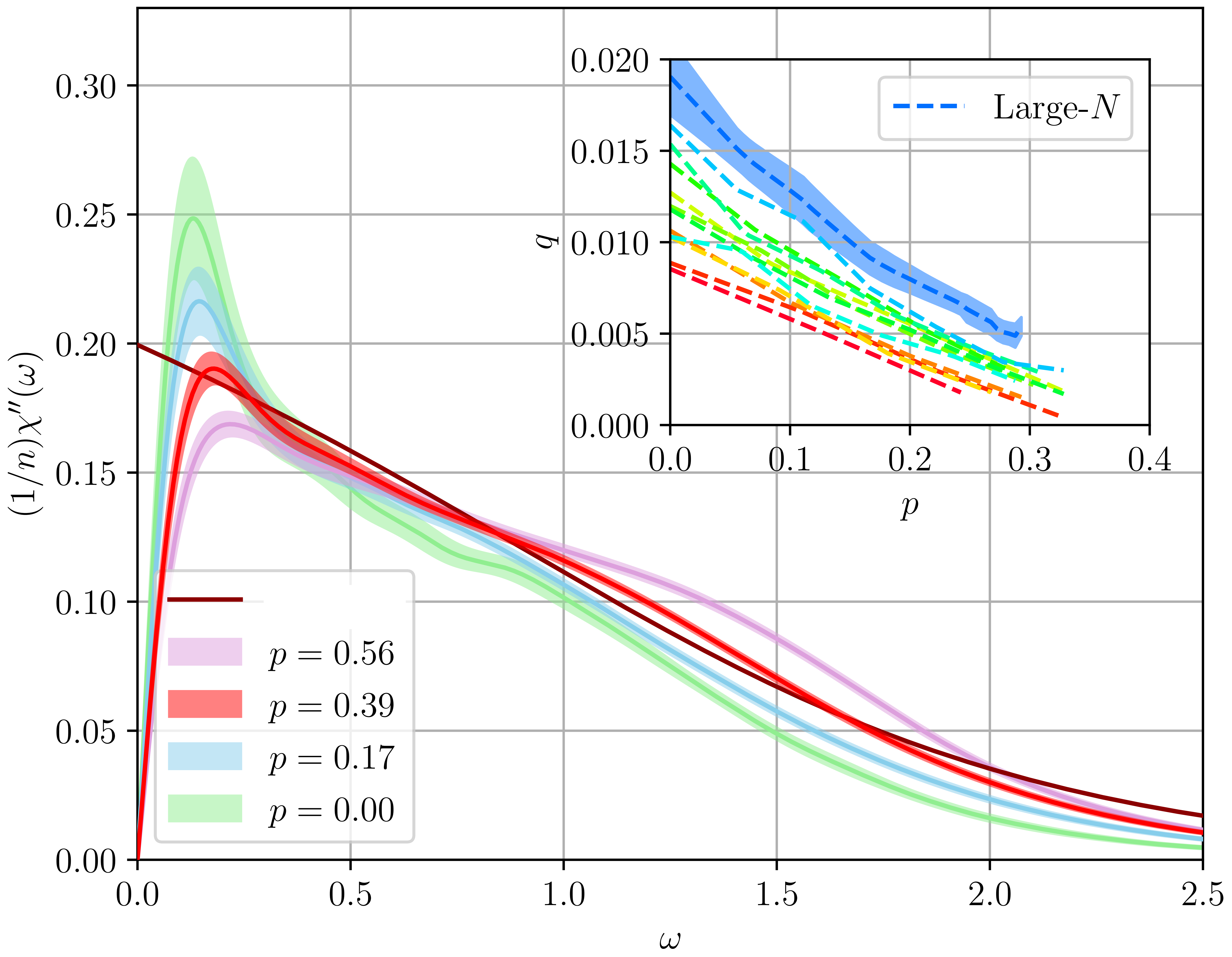
Dynamic spin susceptibility



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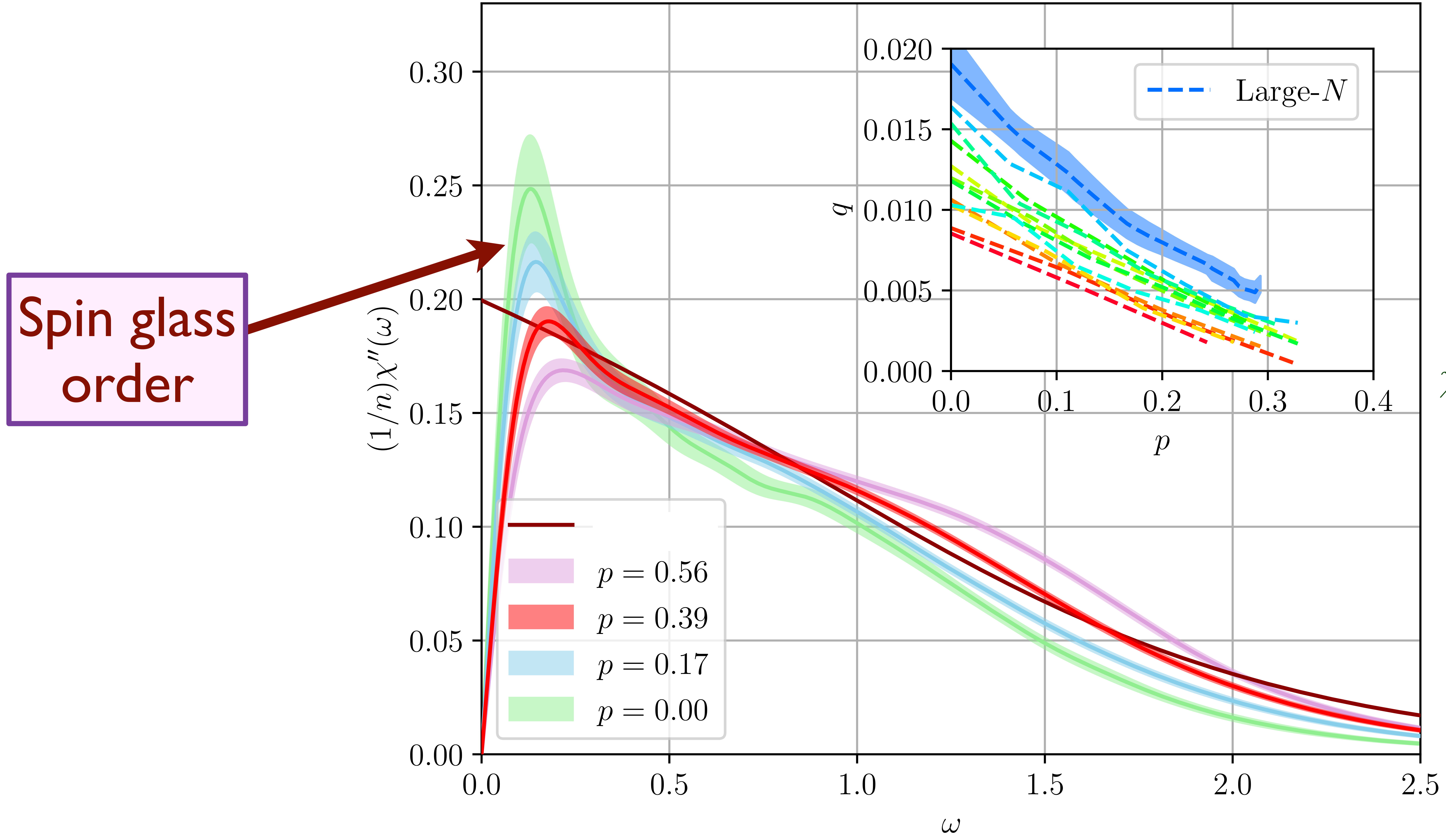
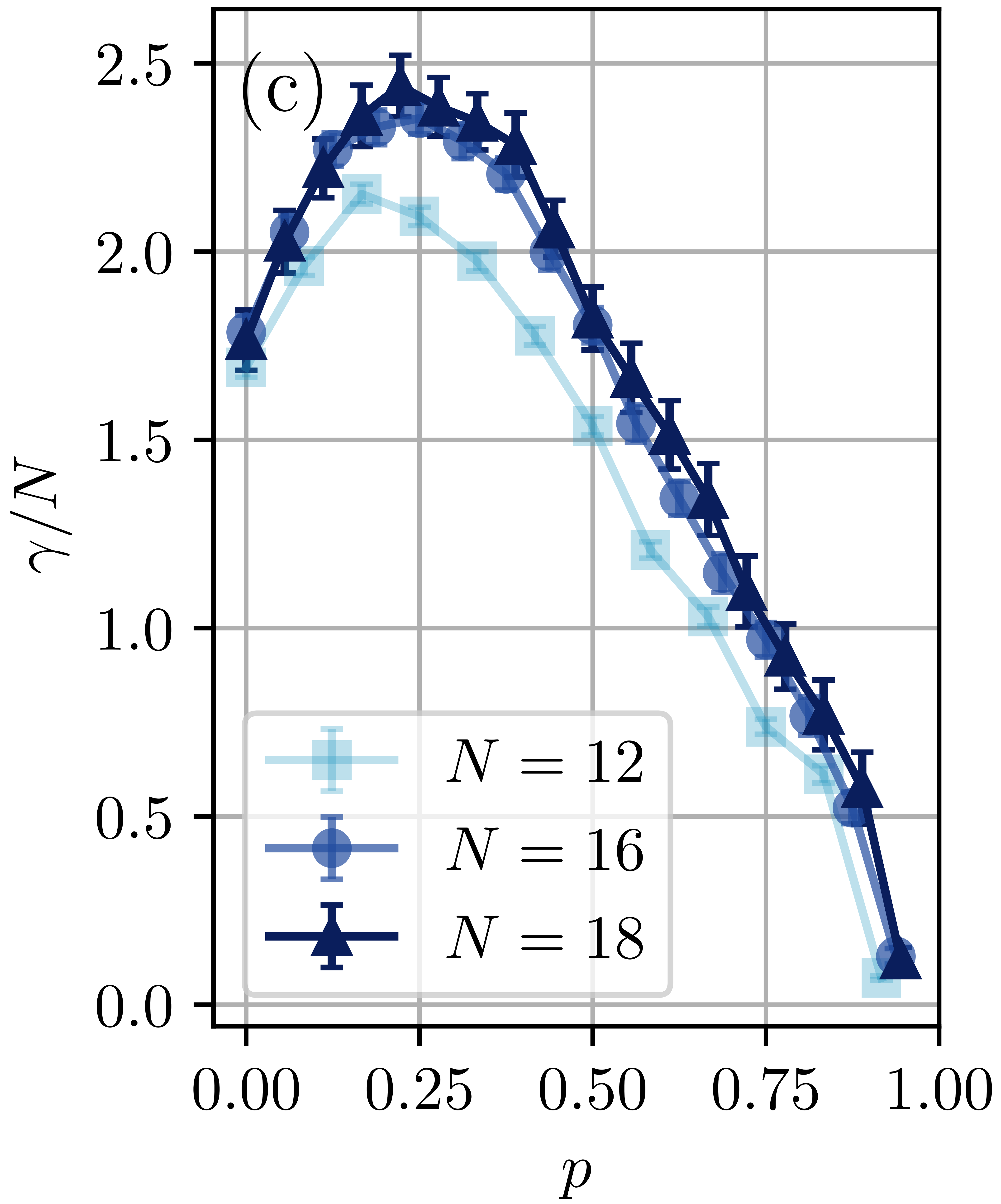


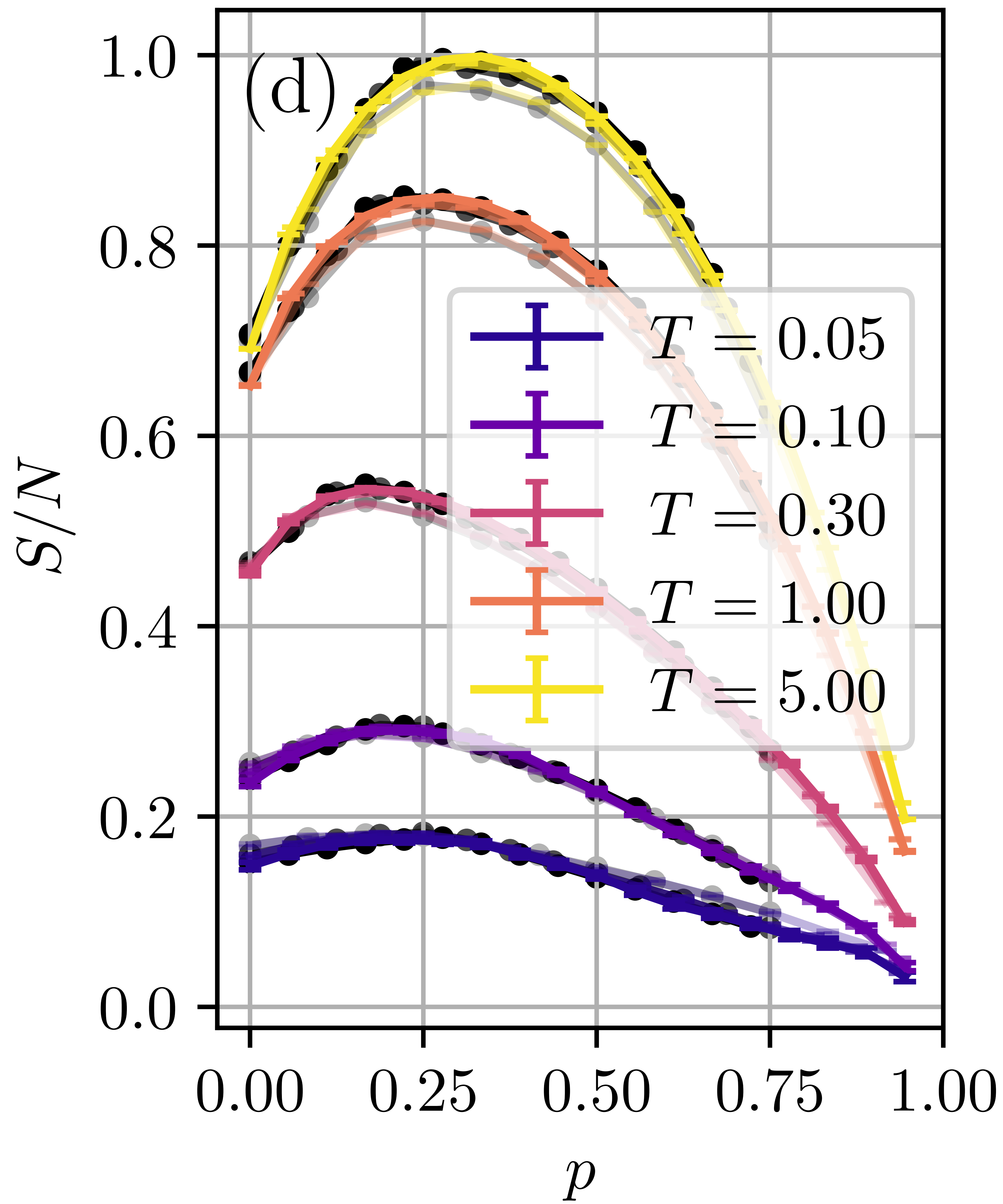
$$\chi''(\omega) = \sum_n |\langle 0| S_{+i} |n\rangle|^2 \times \delta(\hbar\omega - E_n + E_0),$$
(at $T = 0$)

Evidence for a quantum critical point at $p = p_c \approx 0.3$.
Spin glass order q non-zero for $p < p_c$

Specific heat

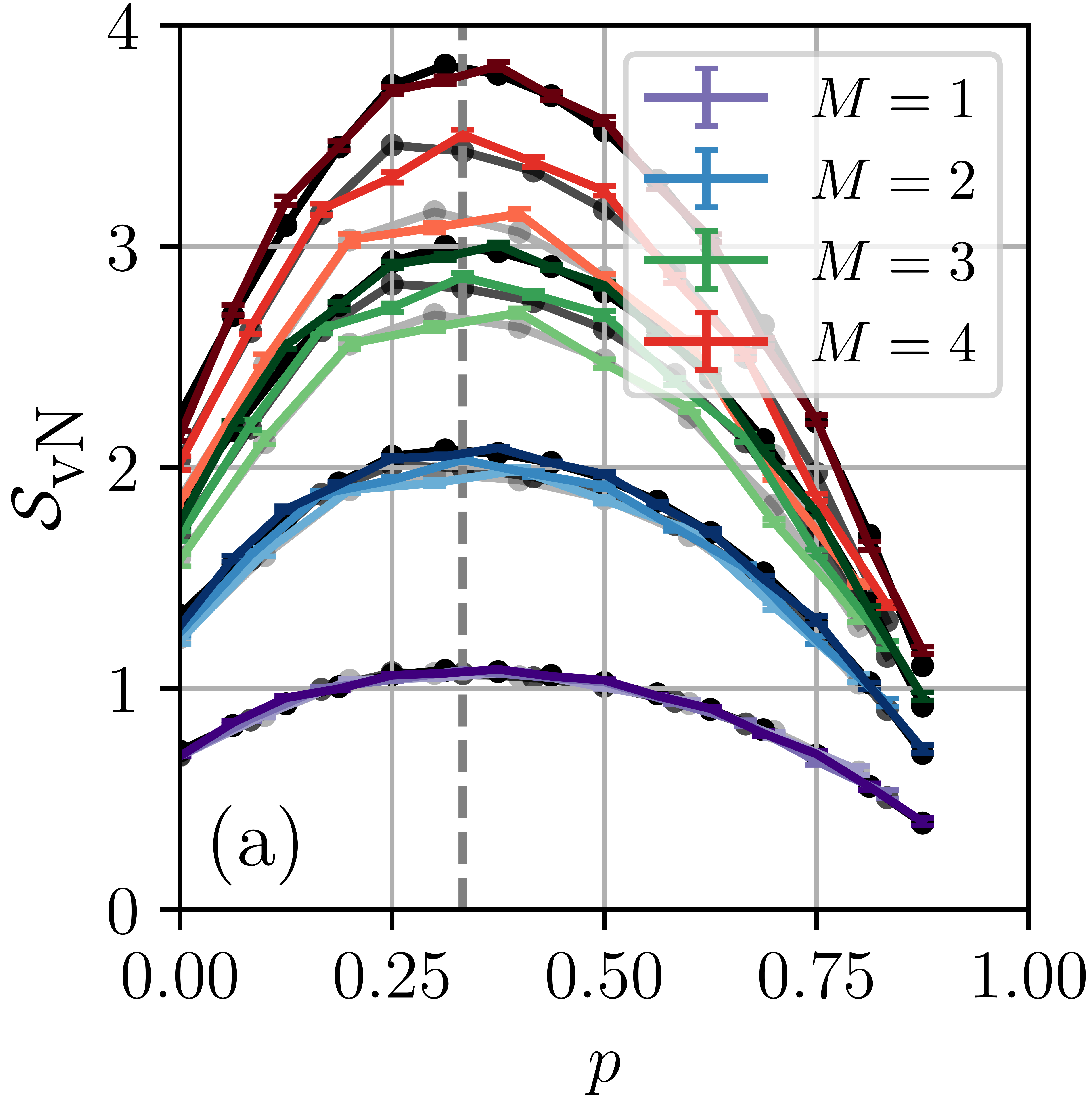


Entropy



(d)
S/N
p
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
T = 0.05
T = 0.10
T = 0.30
T = 1.00
T = 5.00

Entanglement Entropy



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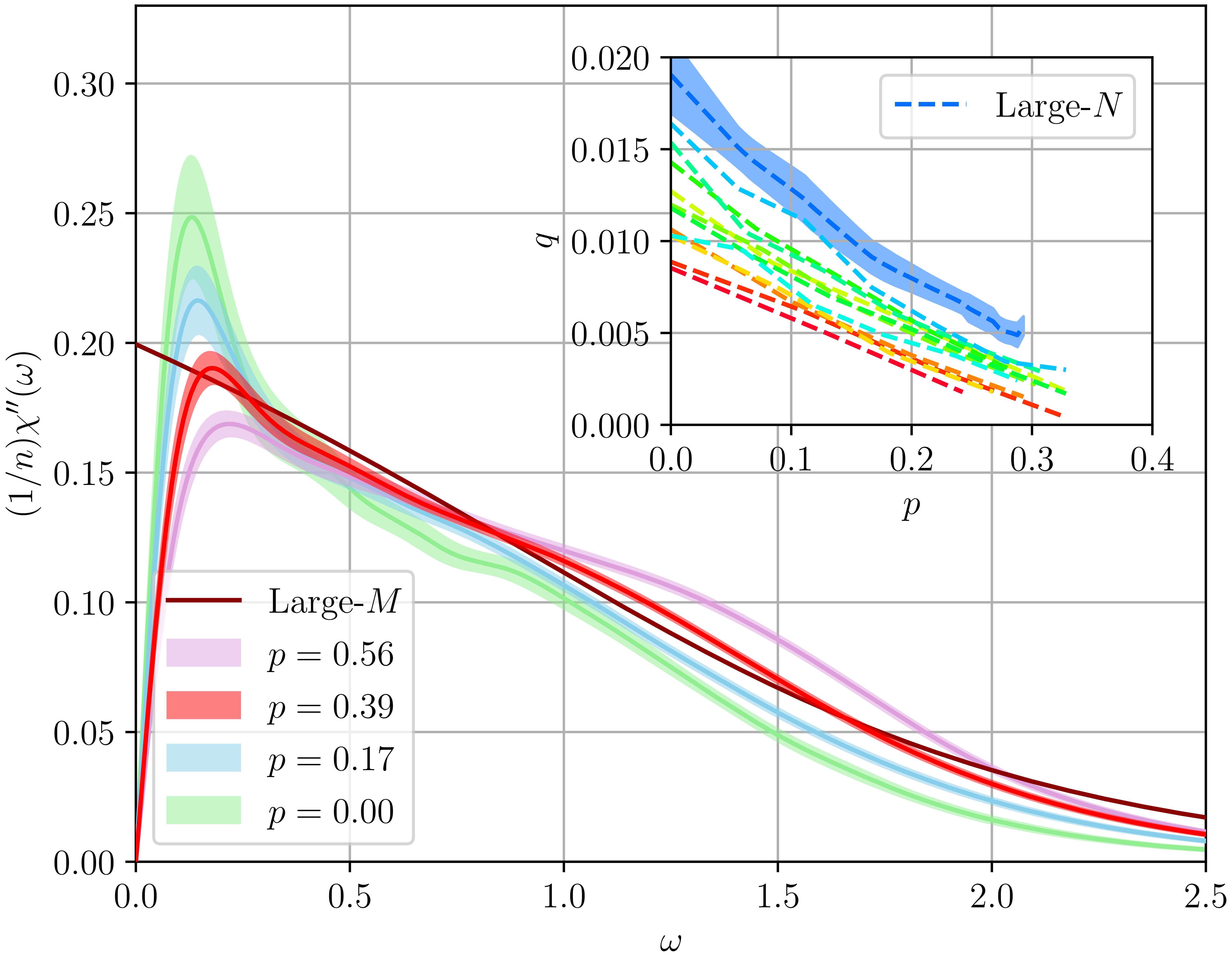

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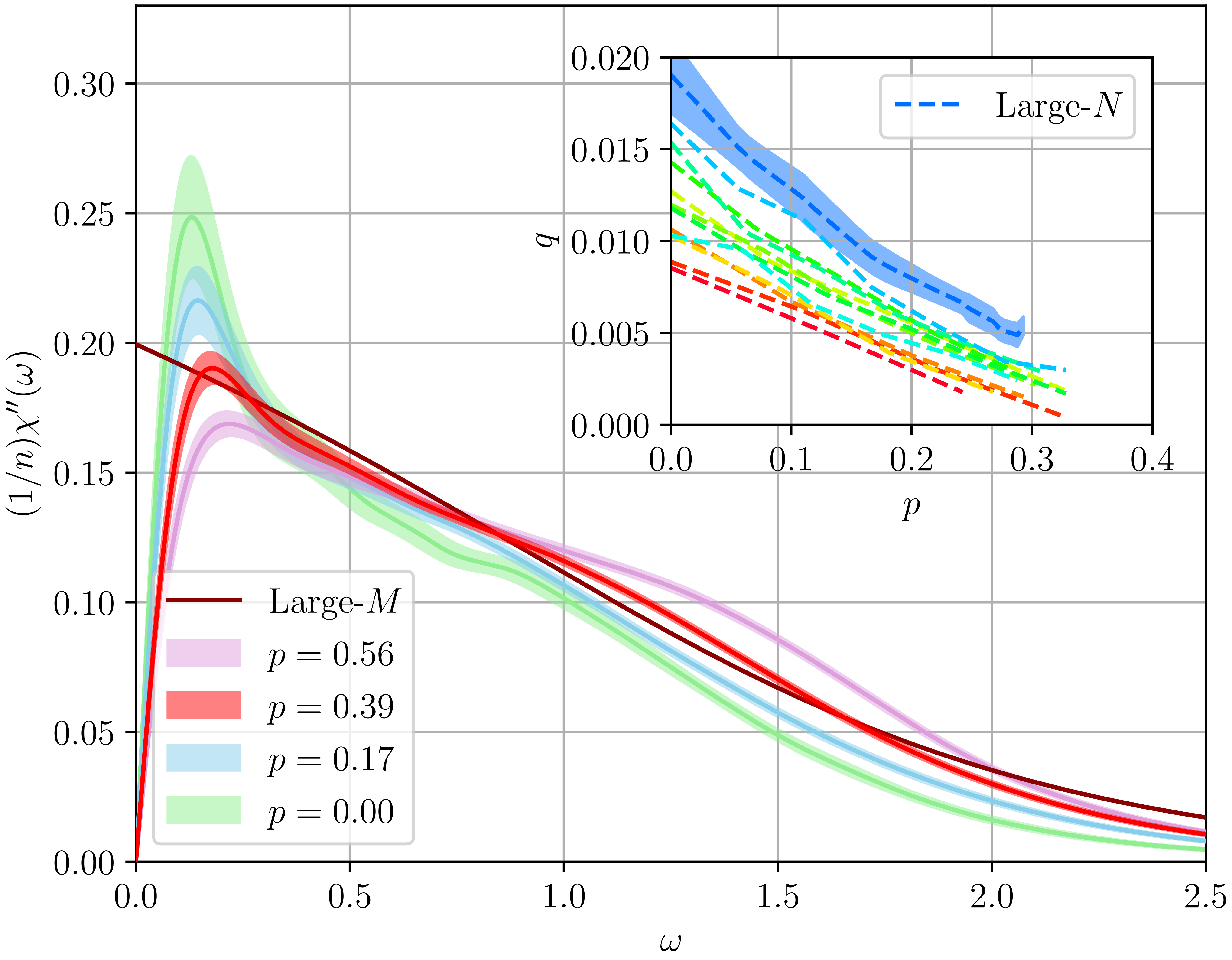


Critical spin susceptibility matches the large M SU(M) SYK model.

$\chi''(\omega) \sim \text{sgn}(\omega) \left[1 - \mathcal{C}\gamma|\omega| + \ldots\right]$ has the 'marginal' sgn(ω) form, with a linear ω correction.

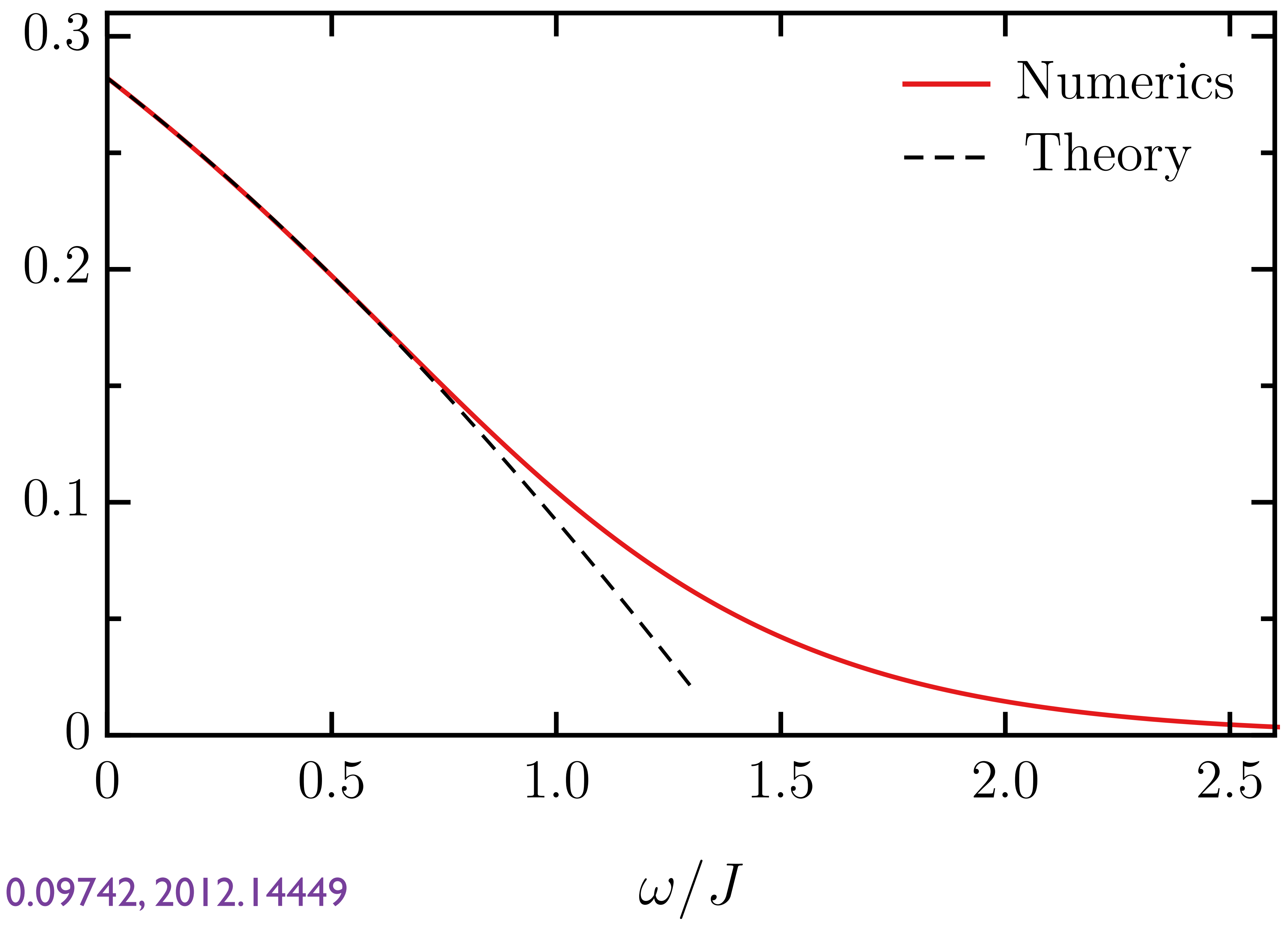
Shown is the numerical solution of SYK equations (SY, PRL 1993), after rescaling J.

Consequences of 2D-gravity for the dynamic spin susceptibility of SYK model

$$\chi_L(\omega) = \sum_n |\langle 0| X_i |n\rangle|^2 \, \delta(\hbar\omega - E_n + E_0), \text{ (at } T = 0)$$

$$\mathrm{Im}\chi_L(\omega) \sim \mathrm{sgn}(\omega)\left[1 - \mathcal{C}\gamma|\omega| - \frac{7}{16}(\mathcal{C}\gamma)^2|\omega|^2 - \mathcal{C}'|\omega|^{2.77354\ldots} + \frac{37}{48}(\mathcal{C}\gamma)^3|\omega|^3 - \ldots\right]$$

Numerical solution of SYK equations (SY, PRL 1993), compared with conformal perturbation theory. $\mathcal{C}$ is the co-efficient of the action for the 'boundary graviton' in holographic dual.



Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

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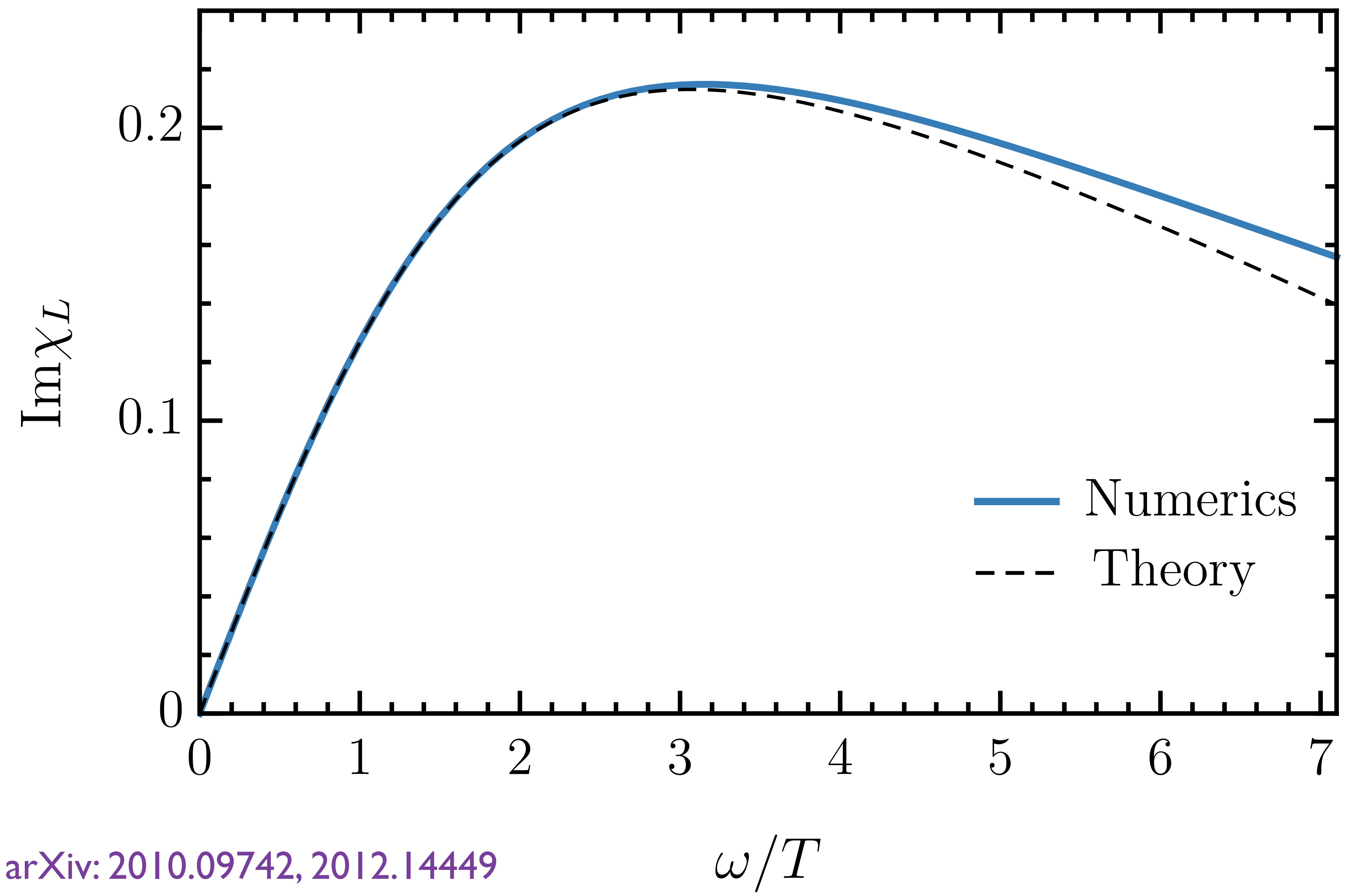

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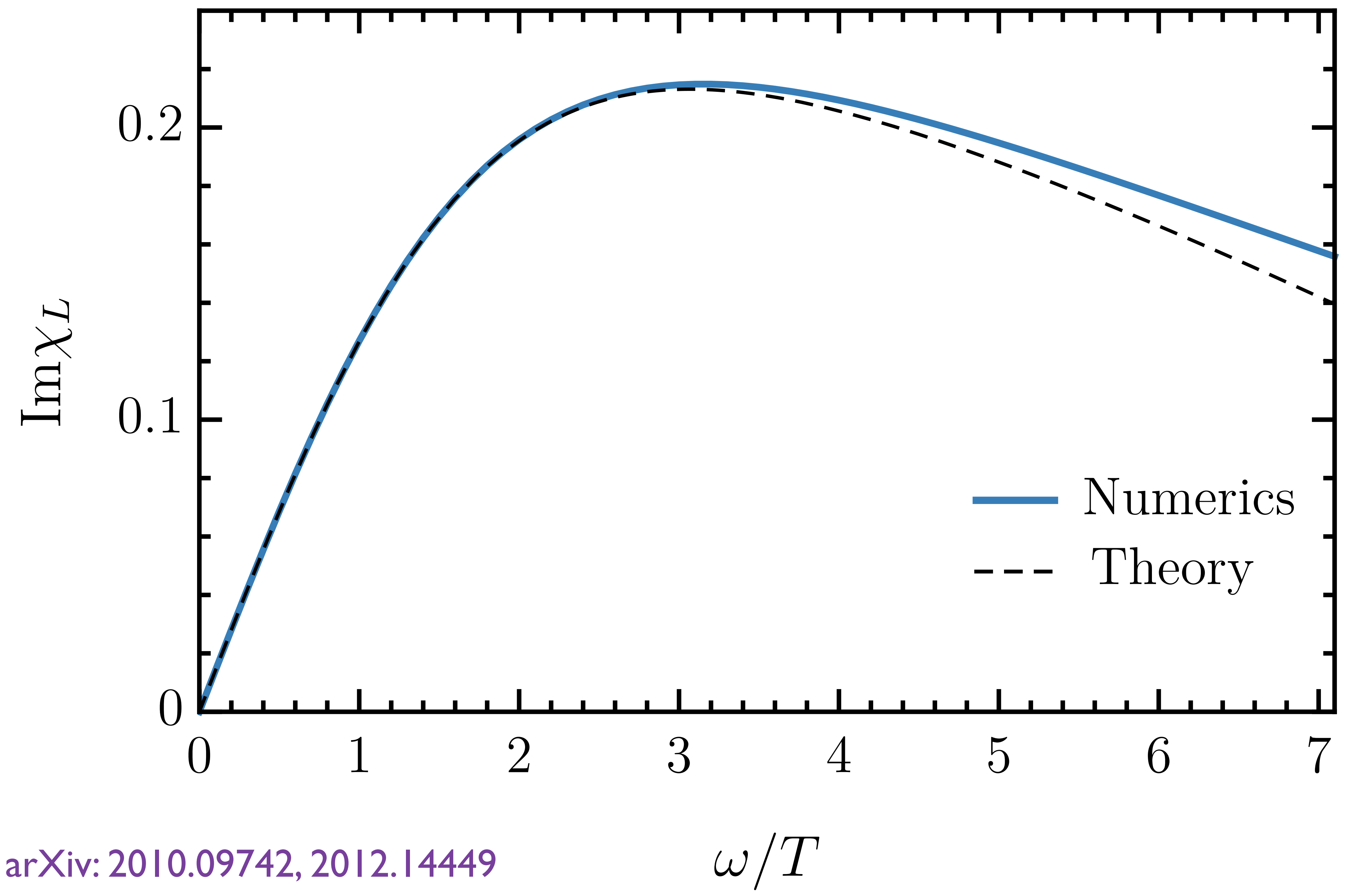
Conformally (SL(2,R)) invariant result with characteristic dissipative time $\sim \hbar/(k_B T)$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)



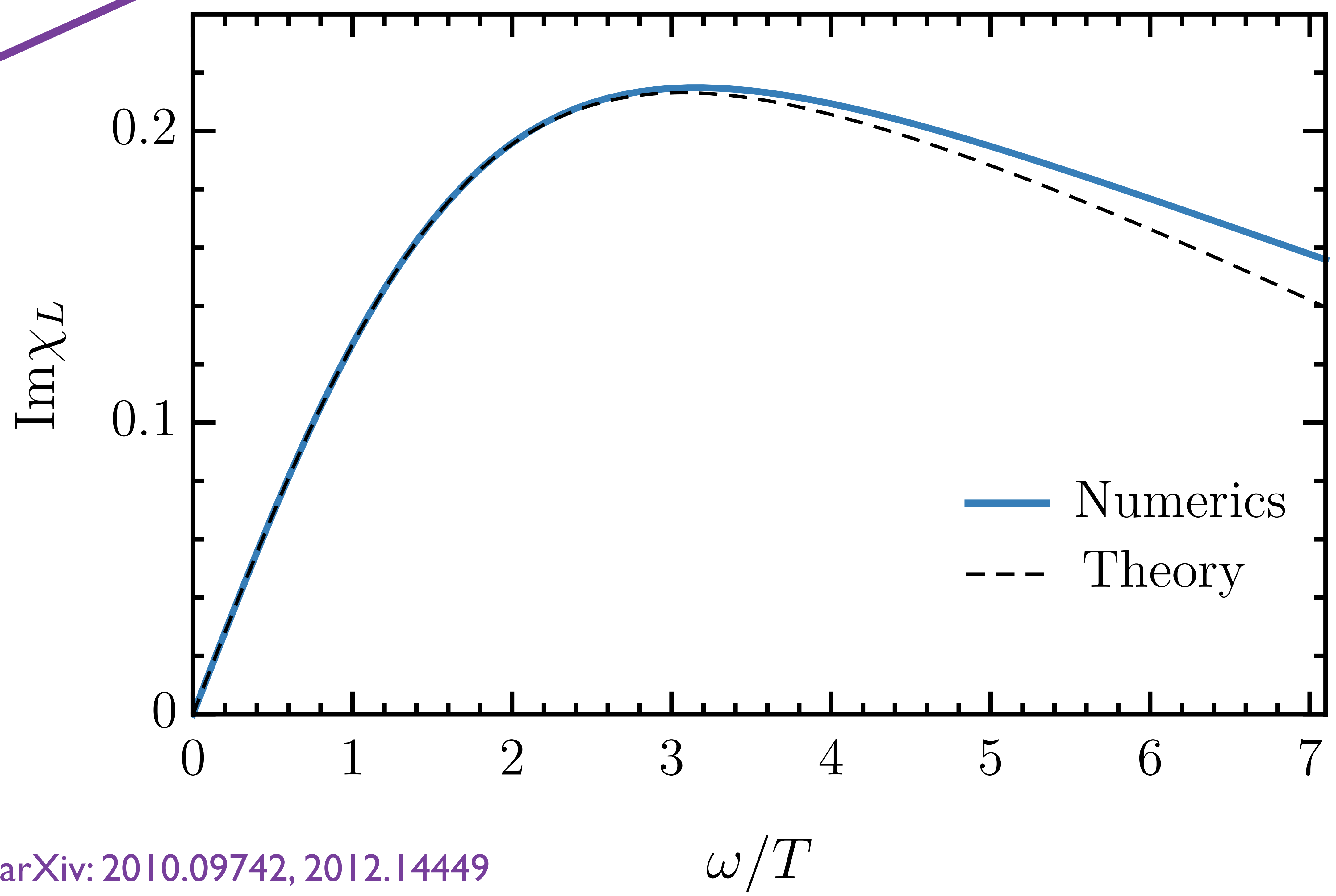
Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

Consequences of 2D-gravity for the dynamic spin susceptibility of SYK model

$$\chi_L(\omega) = \sum_n |\langle 0| X_i |n\rangle|^2 \, \delta(\hbar\omega - E_n + E_0), \text{ (at } T = 0)$$

$$\chi_L(\omega) \sim \tanh\left(\frac{\hbar\omega}{2k_B T}\right)\left[1 - \mathcal{C}\gamma\,\omega \tanh\left(\frac{\hbar\omega}{2k_B T}\right) - \ldots\right]$$

Correction from the boundary graviton



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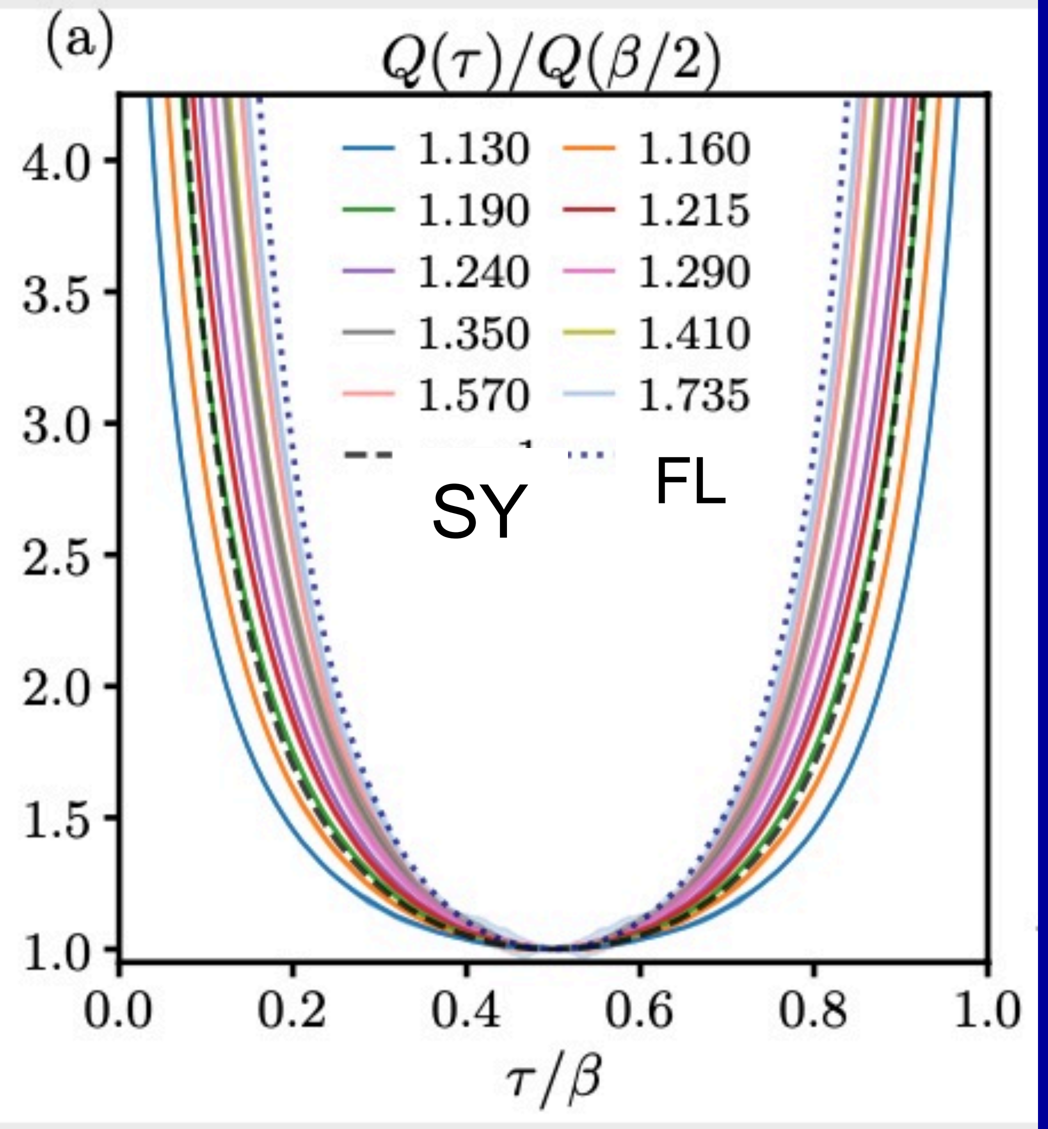
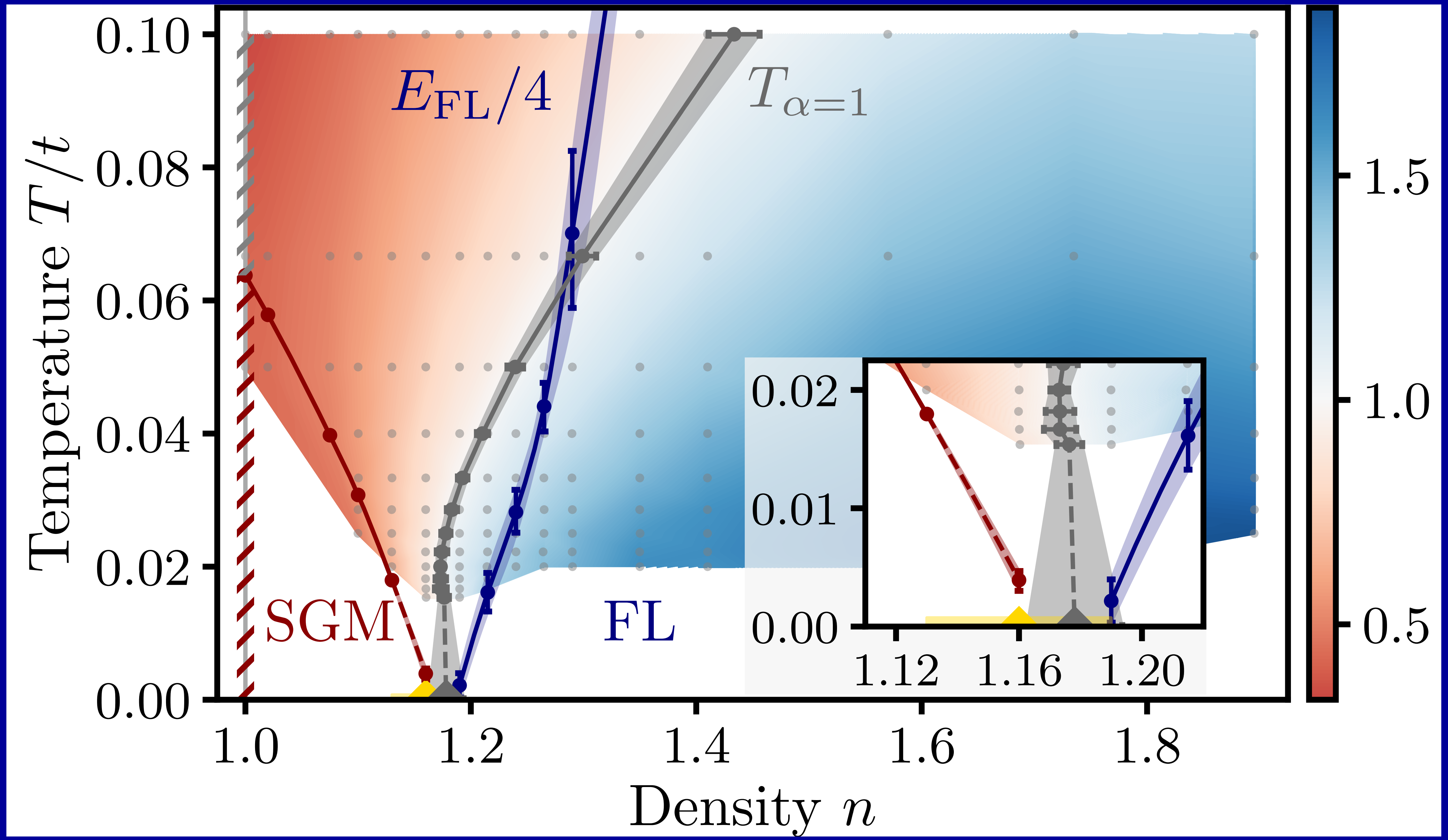
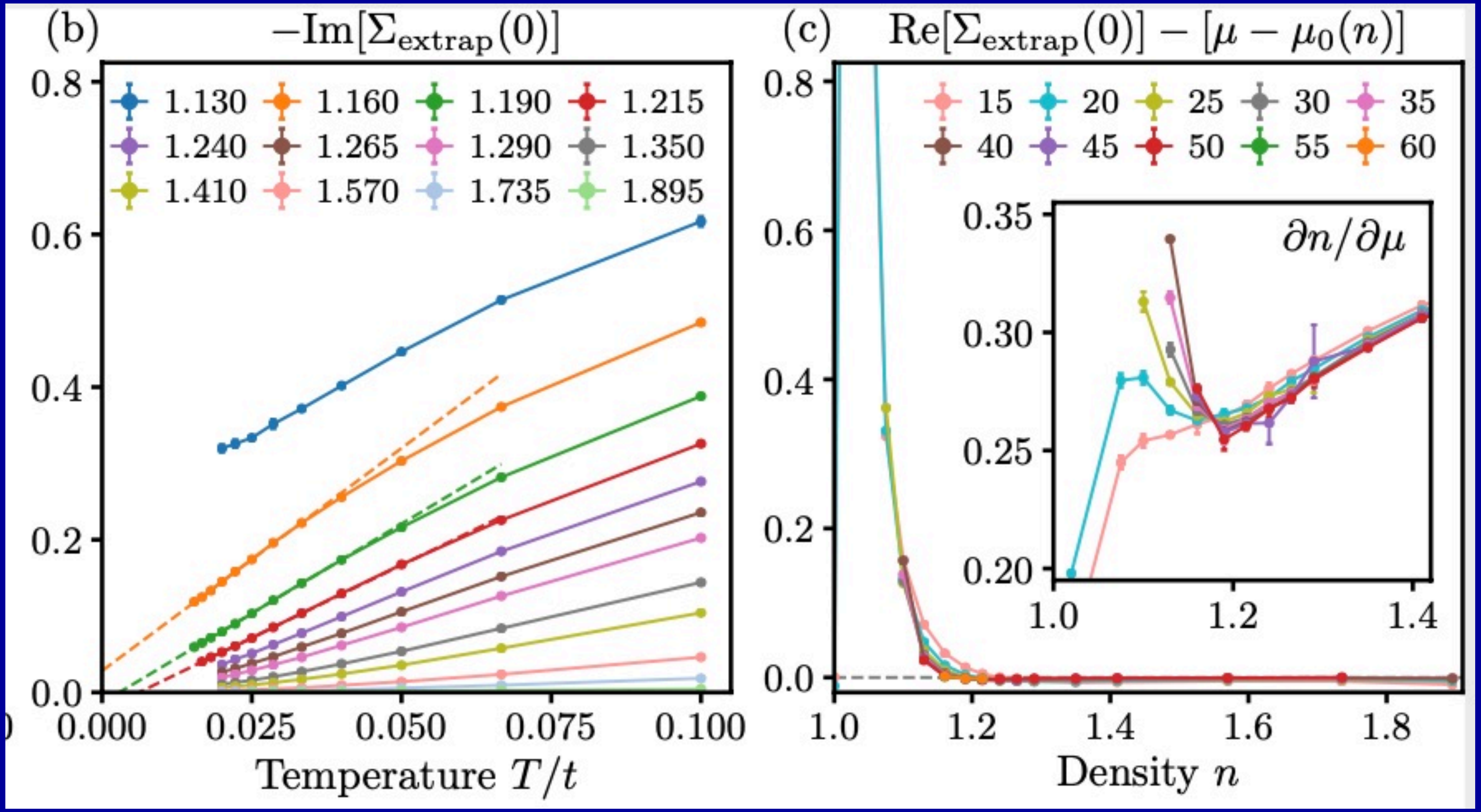
Doping the SU(2) t-J-U SY Model : An Intringuing Quantum Critical Point Solving the EDMFT Equations

Dumitrescu, Wentzell, AG, Parcollet
Soon on arXiv…







Scattering rate
Linear in T
at QCP

Luttinger volume of FS
Breaks down at QCP

Dumitrescu, Wentzell, AG, Parcollet, Soon on arXiv…

The random t-J model has

- Spin glass order for $p < p_c$.
- Fermi liquid with Luttinger volume Fermi surface for $p > p_c$
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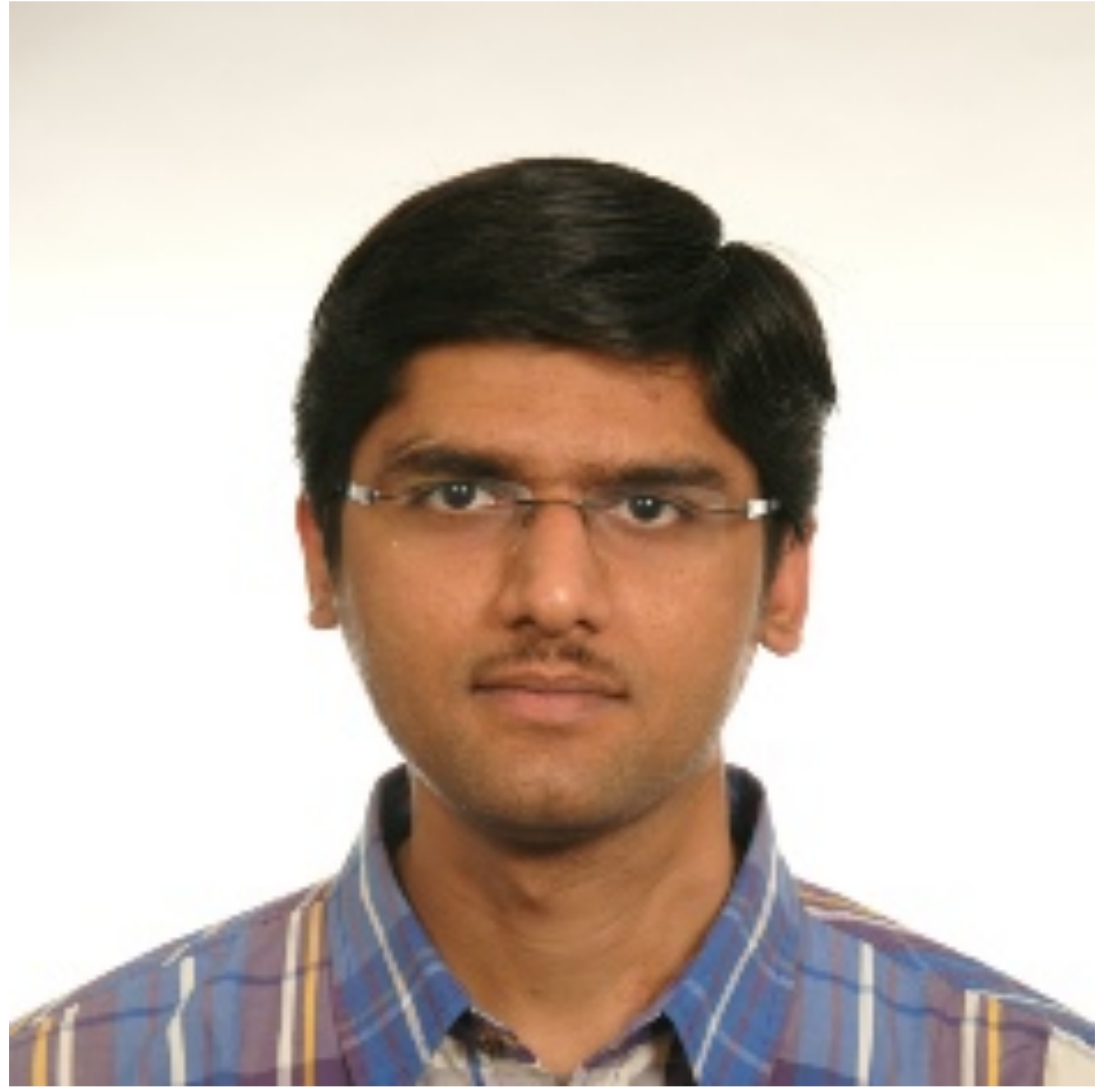
1. Key puzzle in the cuprates

2. Numerical solution of the SU(2) random *t*-*J* model

3. Large-*M* SYK solution of the SU(*M*) random *t*-*J* model

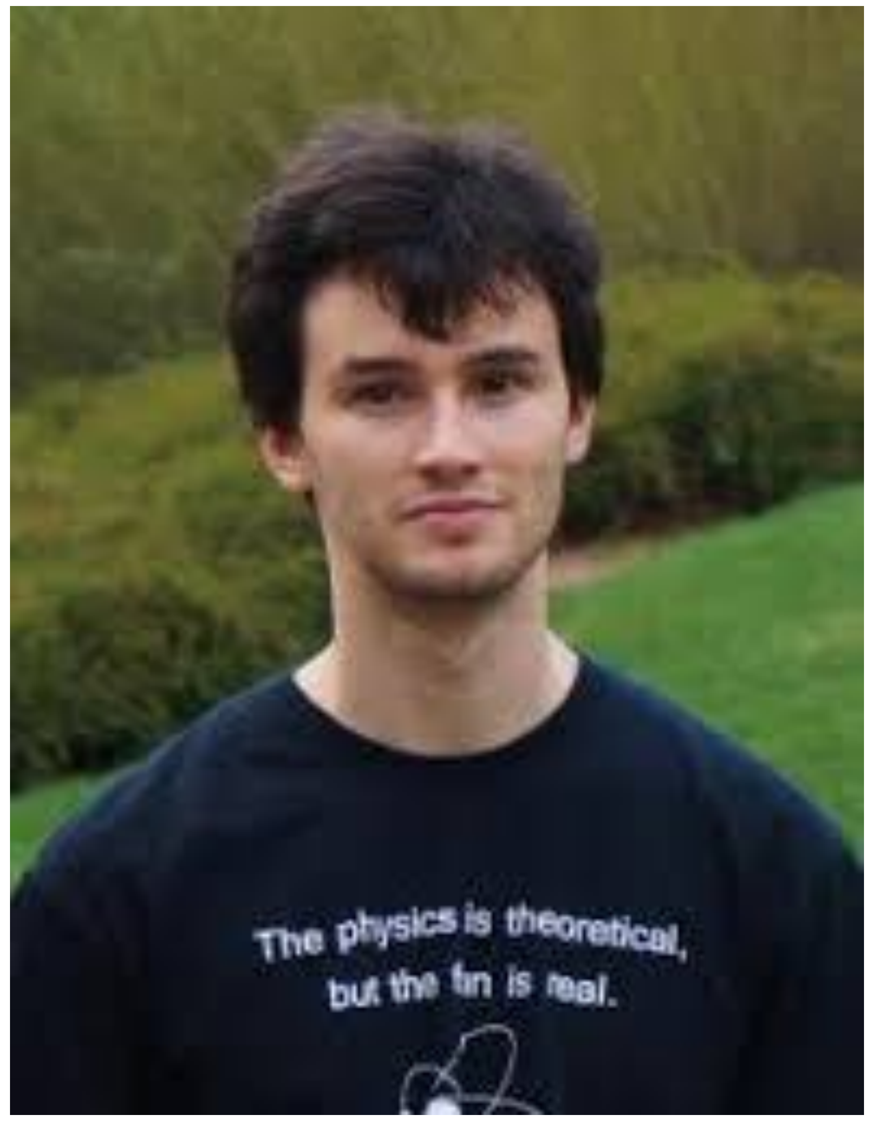
   Fractionalization and deconfined criticality



Darshan Joshi

Chenyuan Li

Physical Review X
10, 021033 (2020)



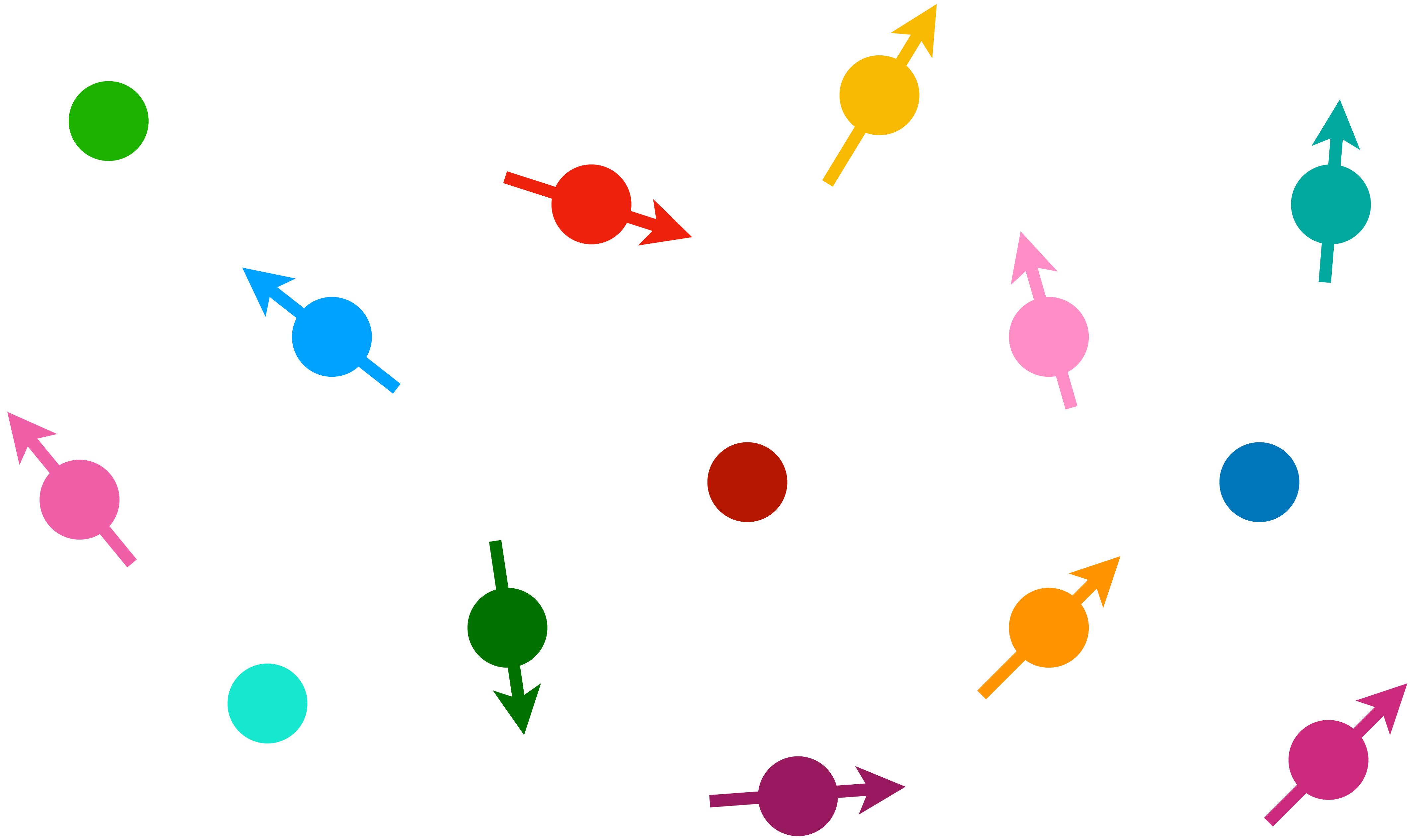
Grigory Tarnopolsky

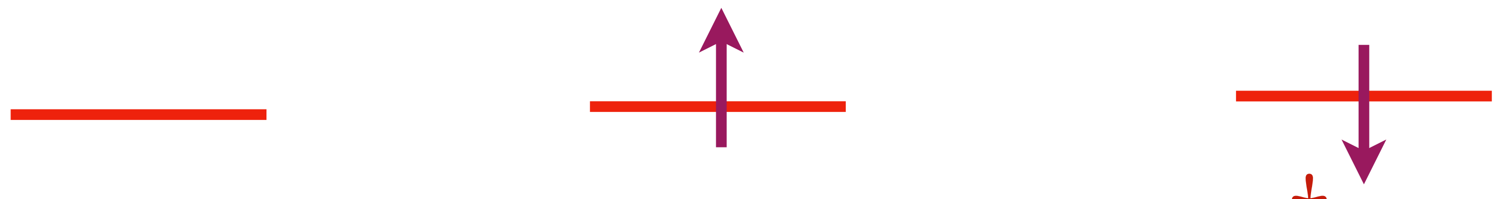
Antoine Georges

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.



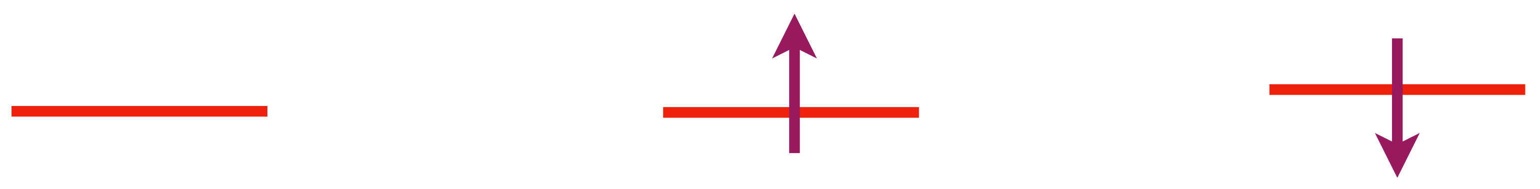


$|0\rangle$ $\quad c_{\uparrow}^{\dagger}|0\rangle$ $\quad c_{\downarrow}^{\dagger}|0\rangle$

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):



$$b^\dagger \left|v\right\rangle \qquad f_\uparrow^\dagger \left|v\right\rangle \qquad f_\downarrow^\dagger \left|v\right\rangle$$

$$\begin{aligned} c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\ f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \end{aligned}$$

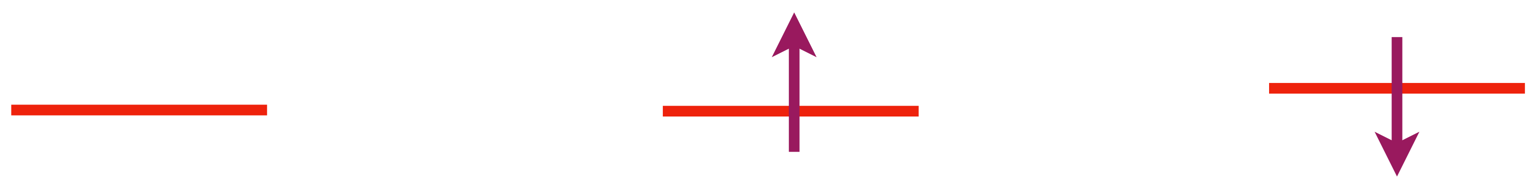
U(1) gauge invariance, $\quad b \rightarrow b e^{i\phi}\,, \quad f_\alpha \rightarrow f_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a fermion $\mathfrak{f}$ (the holon) and a boson $\mathfrak{b}_\alpha$ (the spinon):



$\mathfrak{f}^{\dagger}\,|v\rangle$ $\qquad$ $\mathfrak{b}_{\uparrow}^{\dagger}\,|v\rangle$ $\qquad$ $\mathfrak{b}_{\downarrow}^{\dagger}\,|v\rangle$

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^\dagger \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_\alpha^\dagger \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^\dagger \mathfrak{b}_\alpha + \mathfrak{f}^\dagger \mathfrak{f} &= 1 \end{aligned}$$

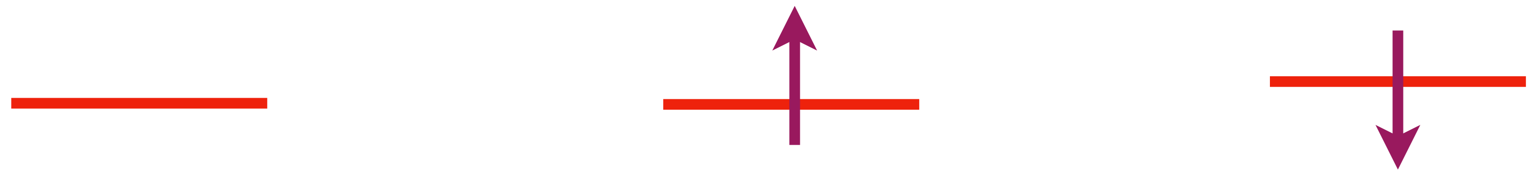
U(1) gauge invariance, $\quad \mathfrak{f} \to \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a fermion $\mathfrak{f}$ (the holon) and a boson $\mathfrak{b}_\alpha$ (the spinon):



$\mathfrak{f}^\dagger |v\rangle$ $\qquad$ $\mathfrak{b}_\uparrow^\dagger |v\rangle$ $\qquad$ $\mathfrak{b}_\downarrow^\dagger |v\rangle$

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$$\mathrm{SU}(1|2) \equiv \mathrm{SU}(2|1)$$

U(1) gauge invariance, $\quad \mathfrak{f} \to \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

Random *t-J* model: large *M* limit

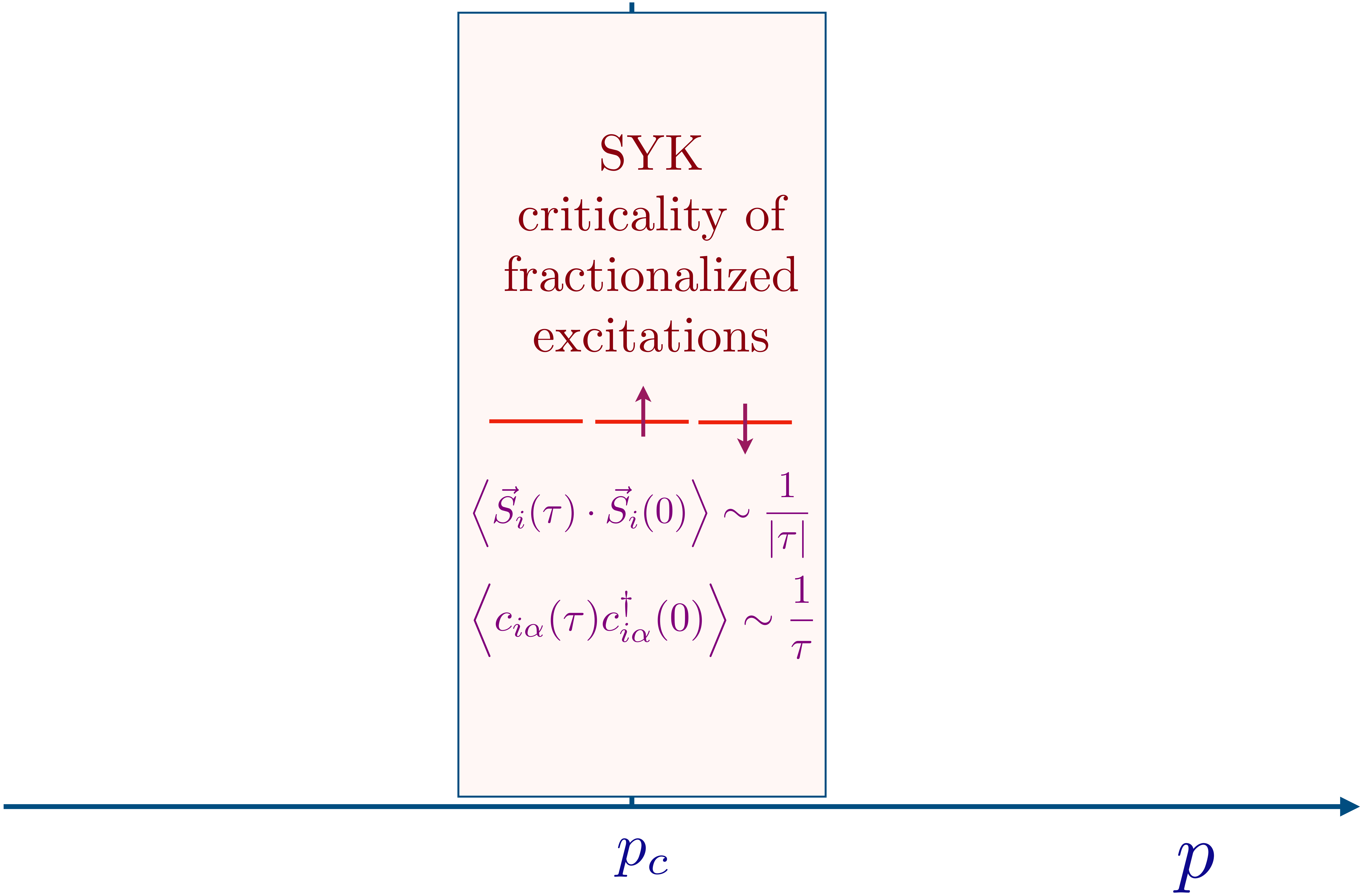
Both the t and the J terms involve <u>four</u> single-particle operators. Consequently, in a large M limit, the saddle-point equations are very similar to the $q = 4$ SYK equations. These equations realize a critical phase with SYK criticality, provided none of the bosons condense.

$$
\begin{aligned}
G_b(i\omega_n) &= \frac{1}{i\omega_n + \mu_b - \Sigma_b(i\omega_n)} \\
\Sigma_b(\tau) &= -t^2 G_f(\tau) G_f(-\tau) G_b(\tau) \\
G_f(i\omega_n) &= \frac{1}{i\omega_n + \mu_f - \Sigma_f(i\omega_n)} \\
\Sigma_f(\tau) &= -J^2 G_f^2(\tau) G_f(-\tau) + k\, t^2 G_f(\tau) G_b(\tau) G_b(-\tau)
\end{aligned}
$$

Here μ_f and μ_b are chemical potentials chosen to satisfy

$$
\langle f^\dagger f \rangle = \frac{1}{2} - k\delta \quad , \quad \langle b^\dagger b \rangle = \delta \,.
$$

Random *t-J* model: large *M* limit



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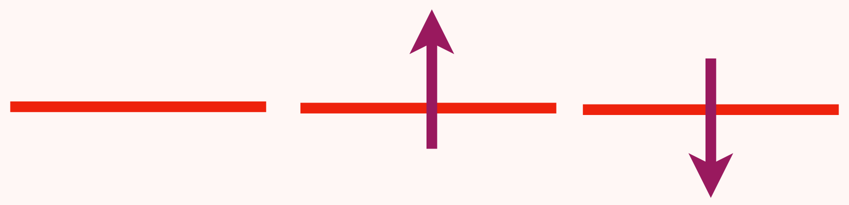
Random *t-J* model: large *M* limit

SYK
criticality of
fractionalized
excitations



$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

$$\left\langle c_{i\alpha}(\tau) c^\dagger_{i\alpha}(0) \right\rangle \sim \frac{1}{\tau}$$

Zeroth order, $p_c = 1/3$ $\qquad p_c \qquad p$

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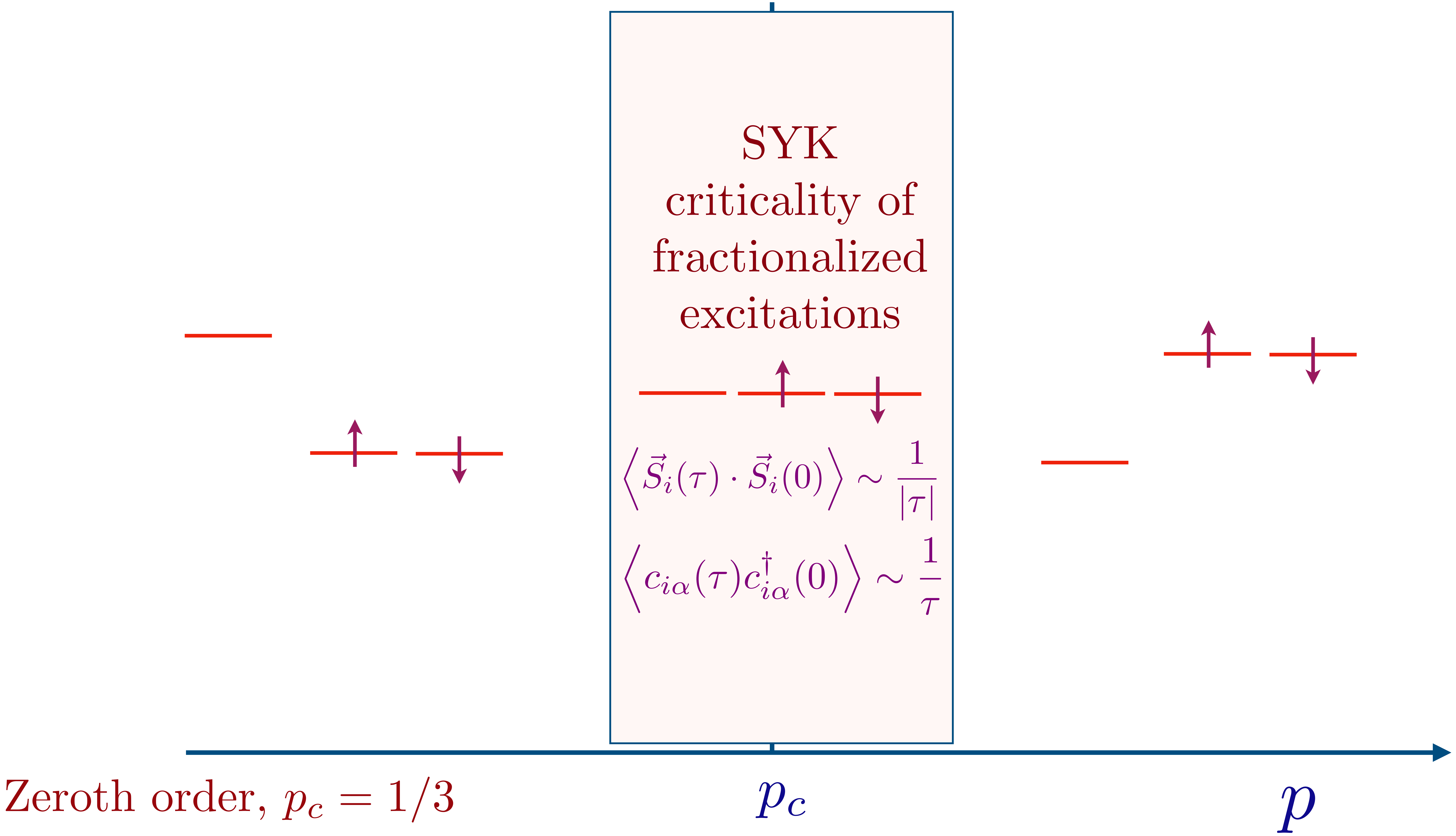

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Random *t-J* model: large *M* limit

SU(1|2) theory

SYK criticality of fractionalized excitations

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Disordered Fermi liquid. Condense holon b, f_α carrier density $1+p$

$f^\dagger_\uparrow |v\rangle$ $f^\dagger_\downarrow |v\rangle$

$b^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

$$\left\langle c_{i\alpha}(\tau) c^\dagger_{i\alpha}(0) \right\rangle \sim \frac{1}{\tau}$$

Zeroth order, $p_c = 1/3$

p_c

p

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

Random *t-J* model: large *M* limit

SU(2|1) theory

Metallic spin glass. Condense spinon $\mathfrak{b}_\alpha$, $\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger |v\rangle$

$\mathfrak{b}_\uparrow^\dagger |v\rangle$ $\mathfrak{b}_\downarrow^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

SYK criticality of fractionalized excitations

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

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Zeroth order, $p_c = 1/3$ $\quad p_c \quad p$

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Partial understanding of these properties is achieved by a theory of SYK criticality of bosonic and fermionic partons