

# Fractionalization and emergent gauge fields in metals: application to $CeCoIn_5$ and the cuprates

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February 18, 2021

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Talk online: [sachdev.physics.harvard.edu](https://sachdev.physics.harvard.edu)



PHYSICS



HARVARD



**Maria Tikhanovskaya**



**Yahui Zhang**



**Alexander Nikolaenko**

# Luttinger theorem for Fermi liquids

Consider a general model of interacting electrons  $c_\uparrow, c_\downarrow$  and  $S = 1/2$  spins  $\vec{S} = (1/2)f_\alpha^\dagger \vec{\sigma}_{\alpha\beta} f_\beta$ ,  $\sum_\alpha f_\alpha^\dagger f_\alpha = 1$ .

$$c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}, \quad f_\alpha f_\beta^\dagger + f_\beta^\dagger f_\alpha = \delta_{\alpha\beta}, \quad c_\alpha f_\beta + f_\beta c_\alpha = 0$$

We can also think of the spins as qubits with  $\vec{S} = (1/2)(X, Y, Z)$  the Pauli qubit operators.

$$\begin{aligned} H = & - \sum_{i \neq j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha} + U \sum_i c_{i\uparrow}^\dagger c_{i\uparrow} c_{i\downarrow}^\dagger c_{i\downarrow} \\ & + \sum_{i < j} J_{ij} \vec{S}_i \cdot \vec{S}_j + \sum_i \frac{J_K}{2} \vec{S}_i \cdot c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta} \dots \end{aligned}$$

# Luttinger theorem for Fermi liquids

Luttinger's theorem states that the momentum space volume enclosed by the Fermi surface (the location of a discontinuity in the electron distribution function) equals  $(1/2)(\text{density of all electrons}) \times (2\pi)^d$  modulo the density contained in filled bands (which can accommodate 2 electrons per unit cell).

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The traditional proofs rely upon a perturbative expansion about the free electron limit. In this proof we use the  $f_\alpha$  representation of  $\vec{S}_i$ , and replace the constraint  $\sum_\alpha f_\alpha^\dagger f_\alpha$  by an interaction  $U_f f_\uparrow^\dagger f_\uparrow f_\downarrow^\dagger f_\downarrow$ , and expand about  $U_f = 0$ .

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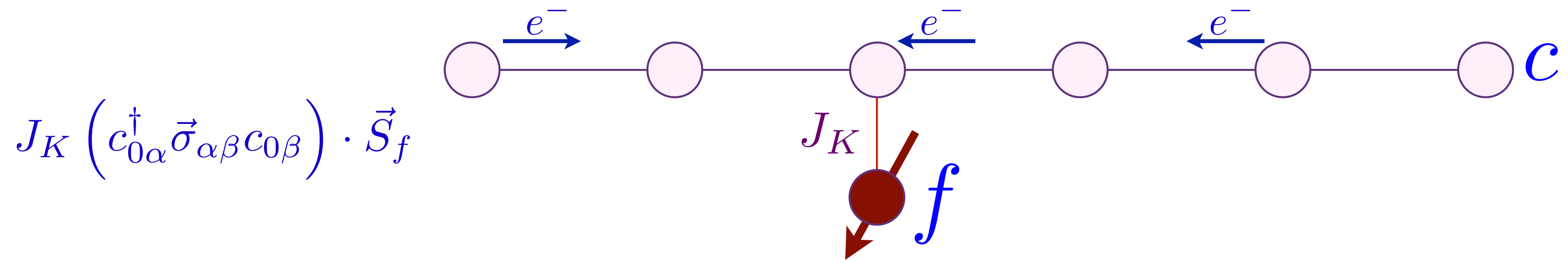
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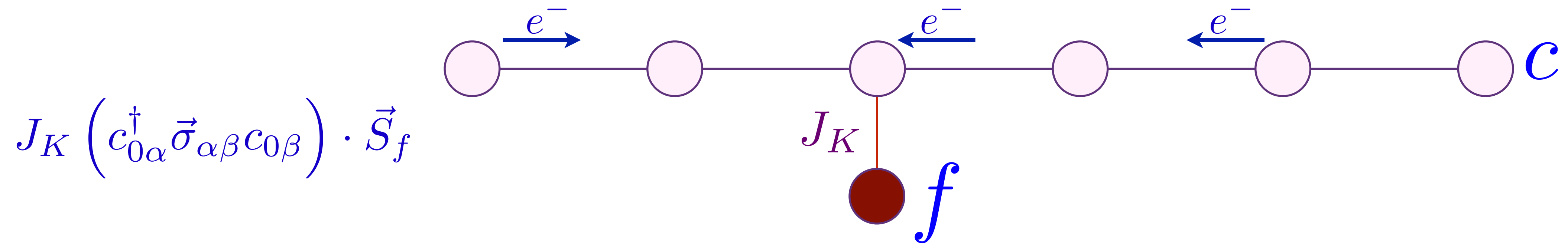
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More recent proofs (Oshikawa; Else, Thorngren, Senthil) use an 'anomaly' associated with the combination of translations and the U(1) symmetry of electron number conservation, and can work directly with spin qubits.

# Kondo model



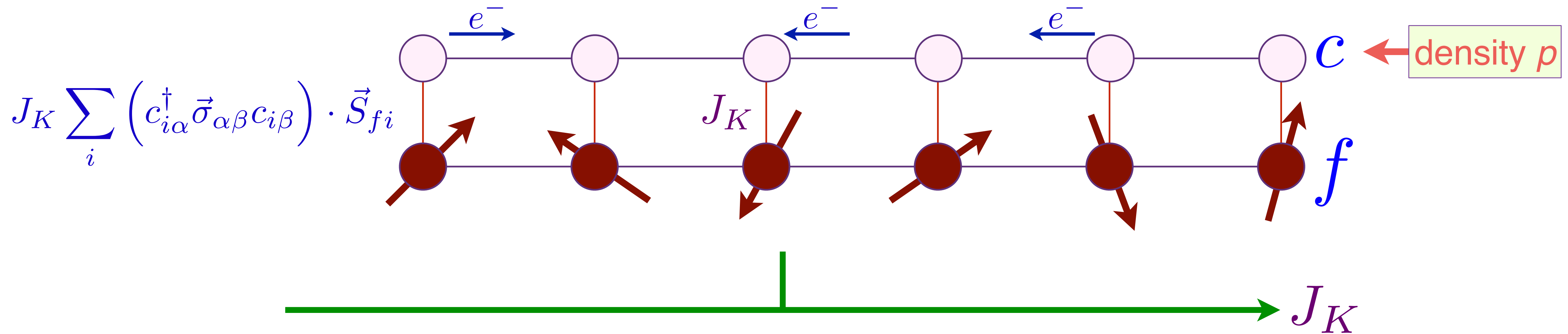
# Kondo model



The  $c$  electrons ‘Kondo screen’ the  $f$  spin at low energies:  
The  $f$  electron ‘dissolves’ into the Fermi sea.

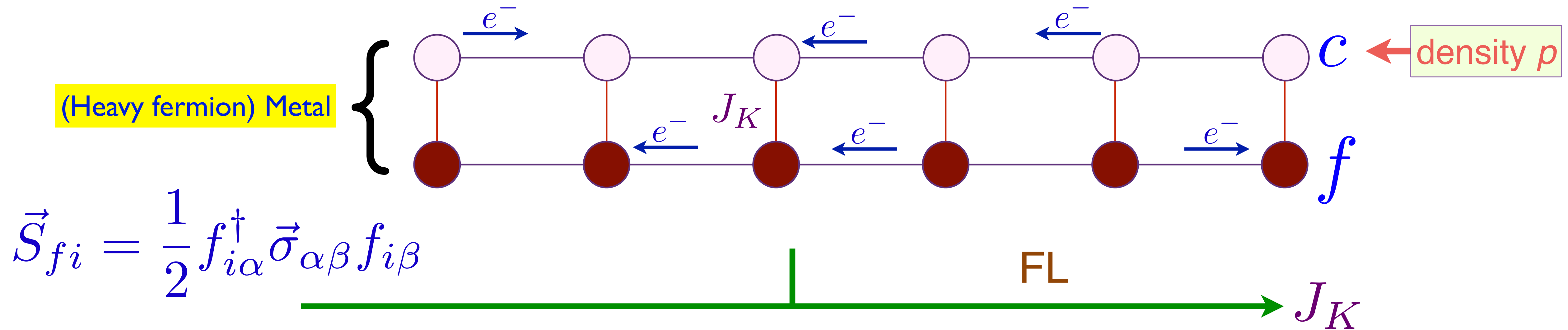
# Luttinger volume in **Kondo lattice** models

Kondo lattice of  $f$  electron spins coupled to a conduction band of  $c$  electrons of density  $p$ .



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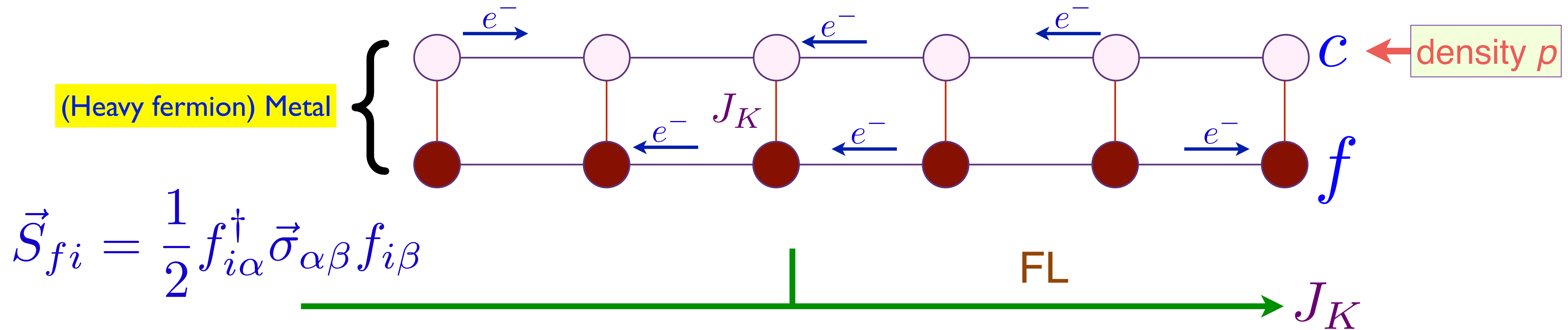
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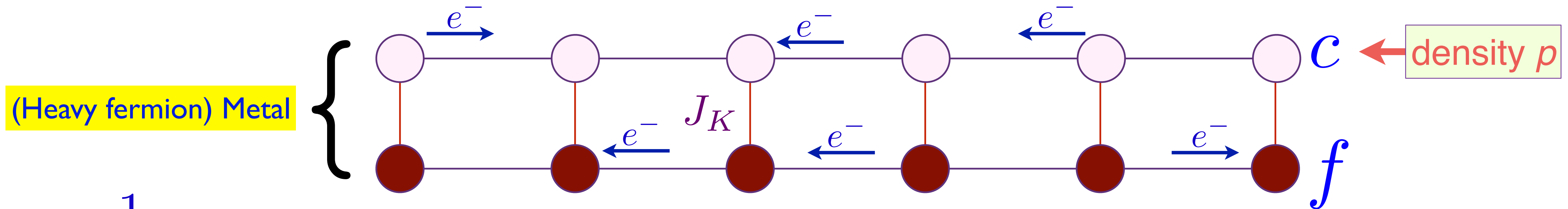


$$\vec{S}_{fi} = \frac{1}{2} f_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} f_{i\beta}$$

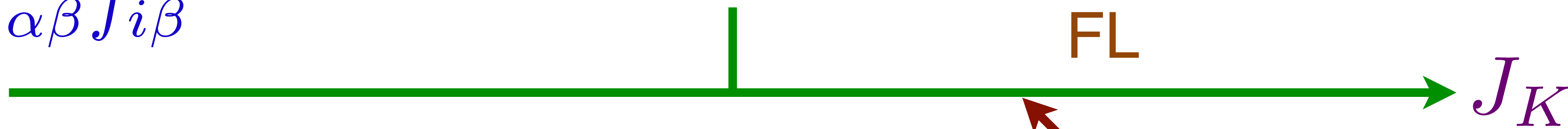
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 The  $f$  electrons ‘dissolve’ into the Fermi sea.  
 The Fermi surface is large: encloses volume of  $1 + p$  electrons.

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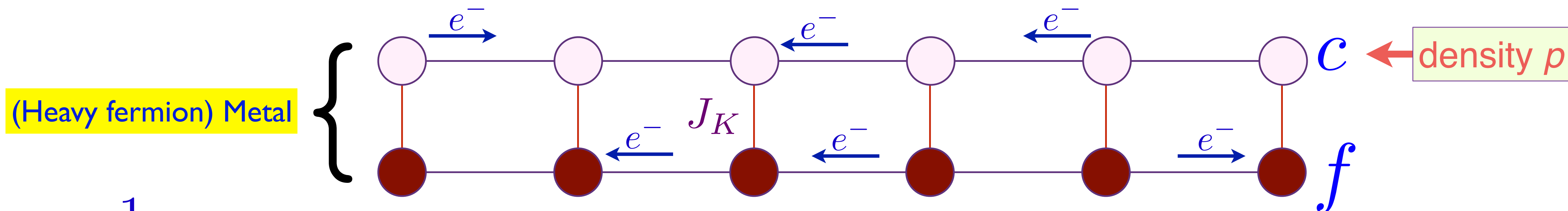


Large Fermi surface of size  $1 + p$

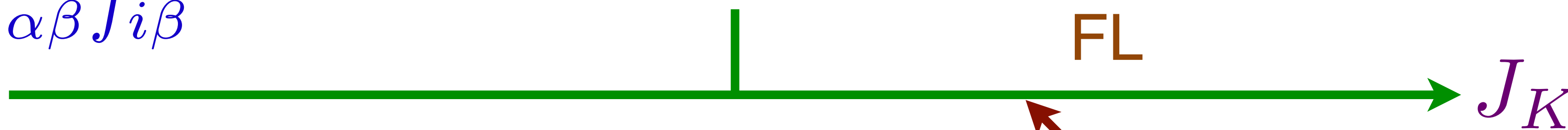
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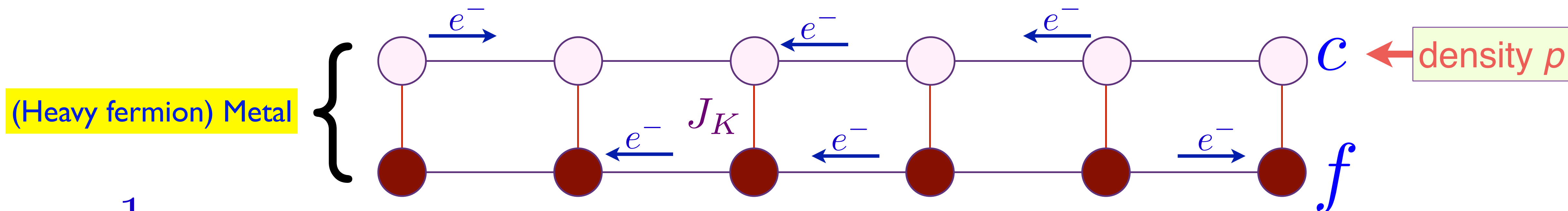
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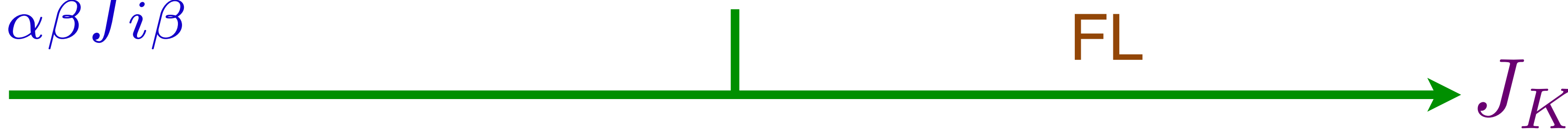
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$$\vec{S}_{fi} = \frac{1}{2} f_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} f_{i\beta}$$

$$\langle c_\alpha^\dagger f_\alpha \rangle \neq 0$$



The Kondo lattice model has a gauge symmetry:  $f_{i\alpha} \rightarrow e^{i\theta_i} f_{i\alpha}$

This gauge symmetry is fully broken by a Higgs condensate  $\langle c_\alpha^\dagger f_\alpha \rangle$  in the FL phase.

1. Luttinger volume violation in Kondo lattice models  
*The FL\* phase and CeCoIn<sub>5</sub>*
2. Introduction to cuprates  
*Small to Large Fermi surface transition*
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# Luttinger volume violation

The Luttinger volume can be violated in a metal when there is 'bulk topological order' *i.e.* excitations associated with fractionalization and emergent gauge fields.

T. Senthil, M.Vojta, and S. Sachdev, PRB **69**, 035111 (2004)

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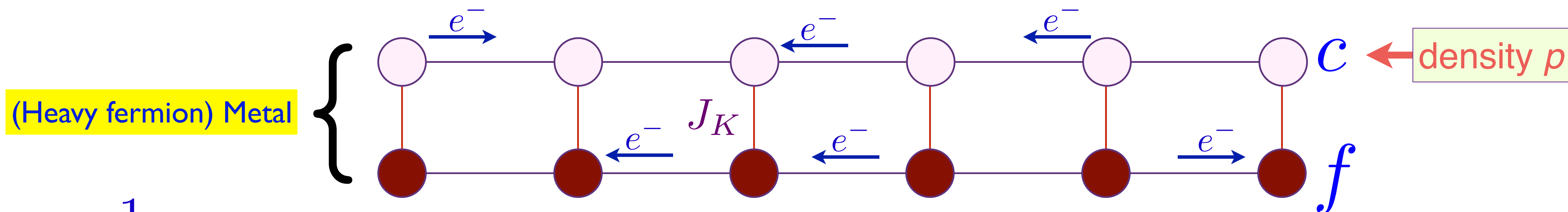
In this manner with obtain the  $FL^*$  phase: a metallic phase with a Fermi surface of Fermi-liquid-like electronic quasiparticles, enclosing a non-Luttinger volume.

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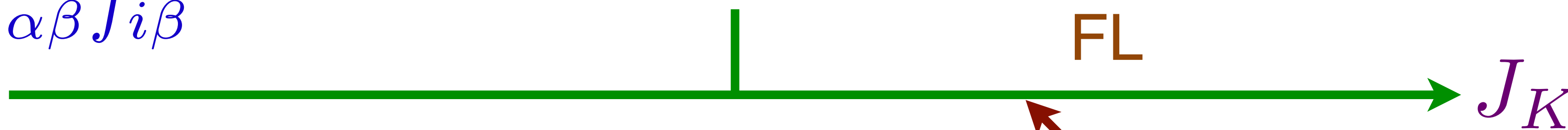
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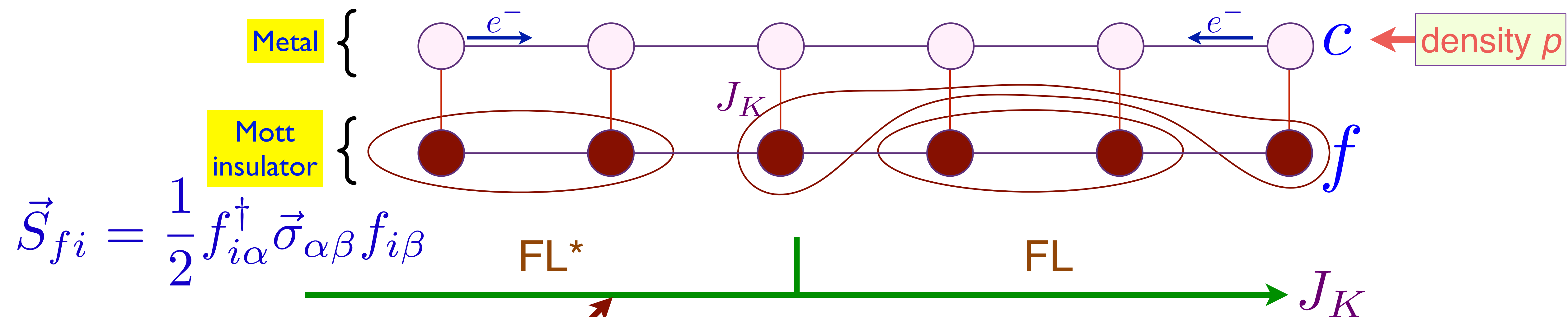
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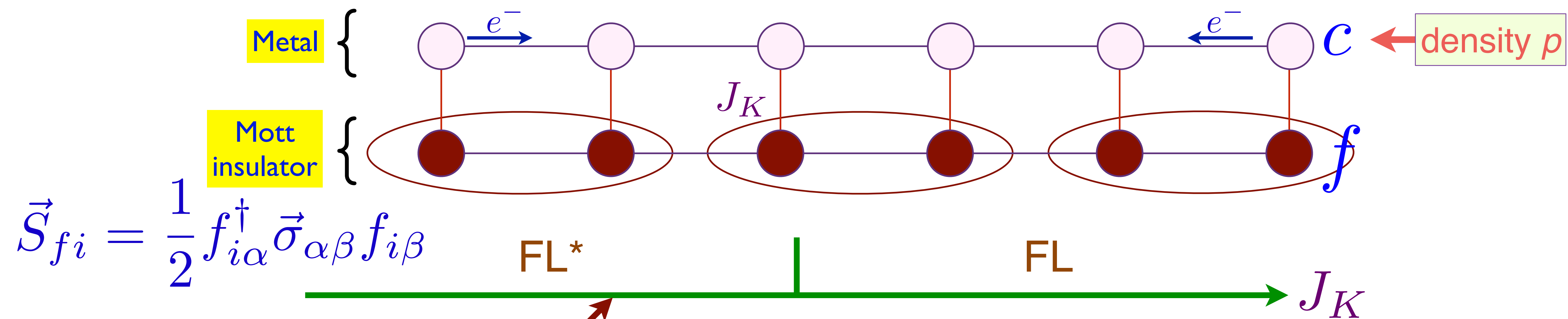
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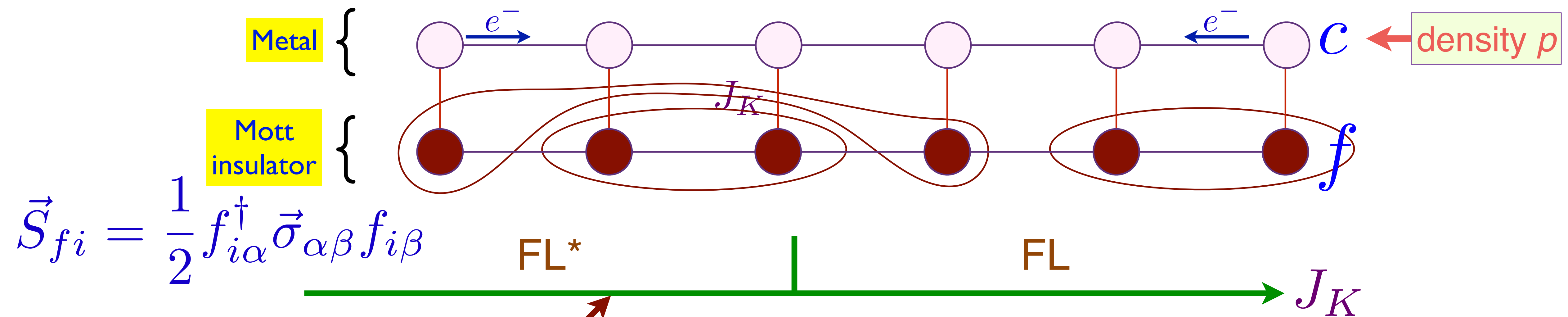
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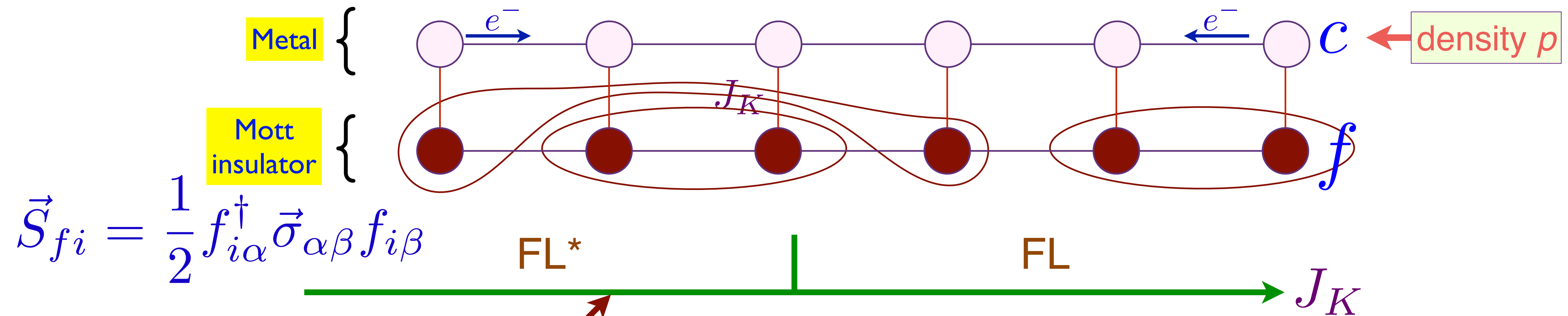
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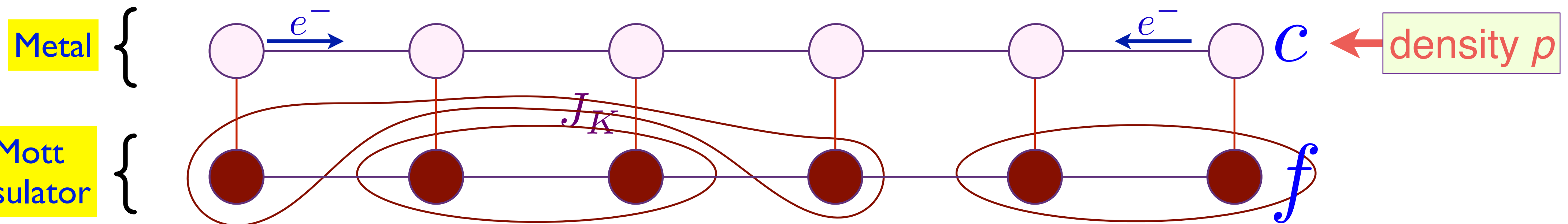
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V.I. Anisimov, I.A. Nekrasov,  
D.E. Kondakov, T.M. Rice & M. Sigrist,  
EPJB **25**, 191 (2002)  
L. de' Medici, A. Georges, S. Biermann,  
PRB **72**, 205124 (2005)

Kondo-breakdown or 'selective Mott' transition



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Kondo-breakdown or ‘selective Mott’ transition

U(1) gauge theory of a ‘hybridization-Higgs’ boson  $b \sim f_{\alpha}^{\dagger} c_{\alpha}$  which condenses on the ‘Large Fermi surface’ side.

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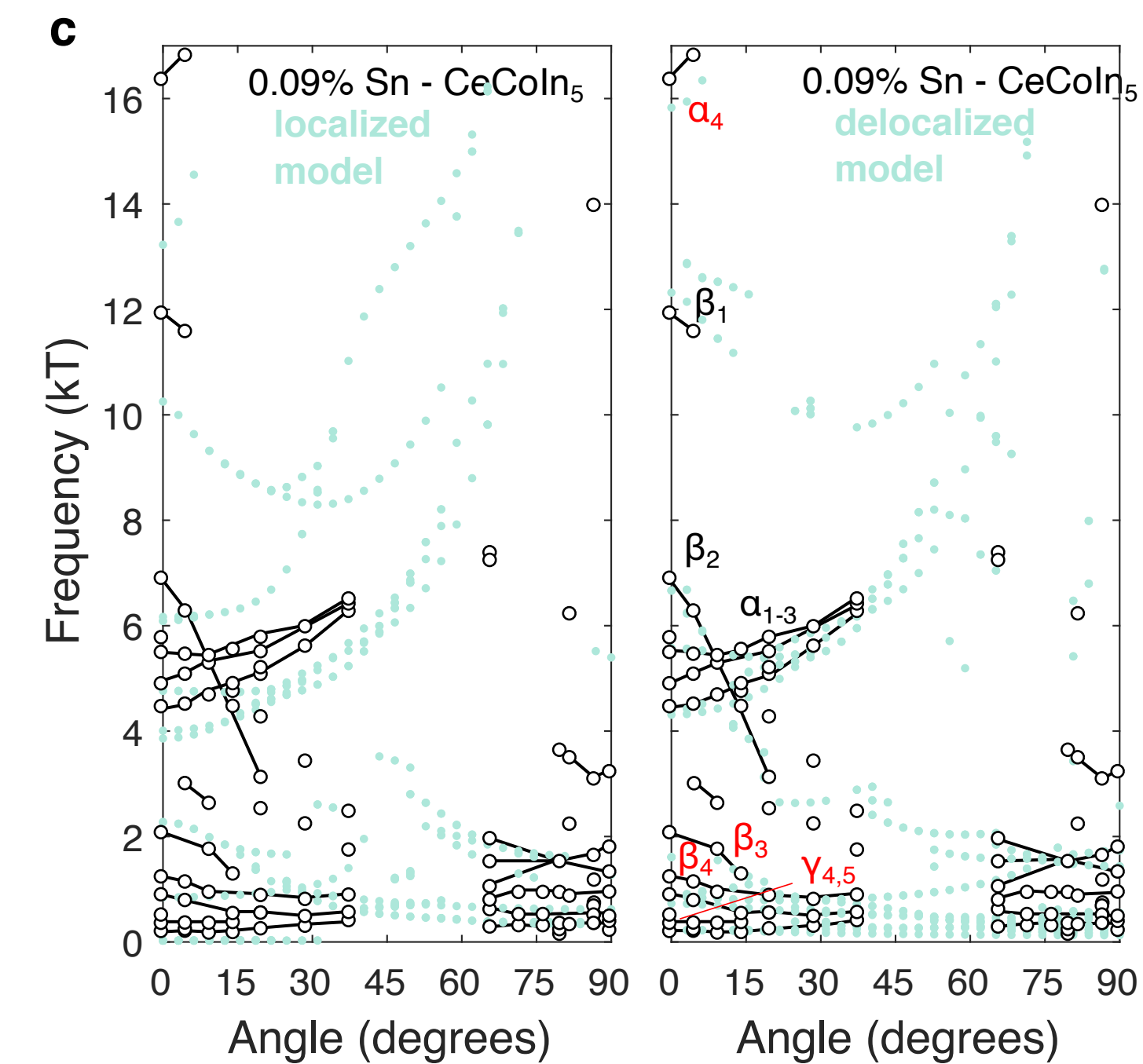
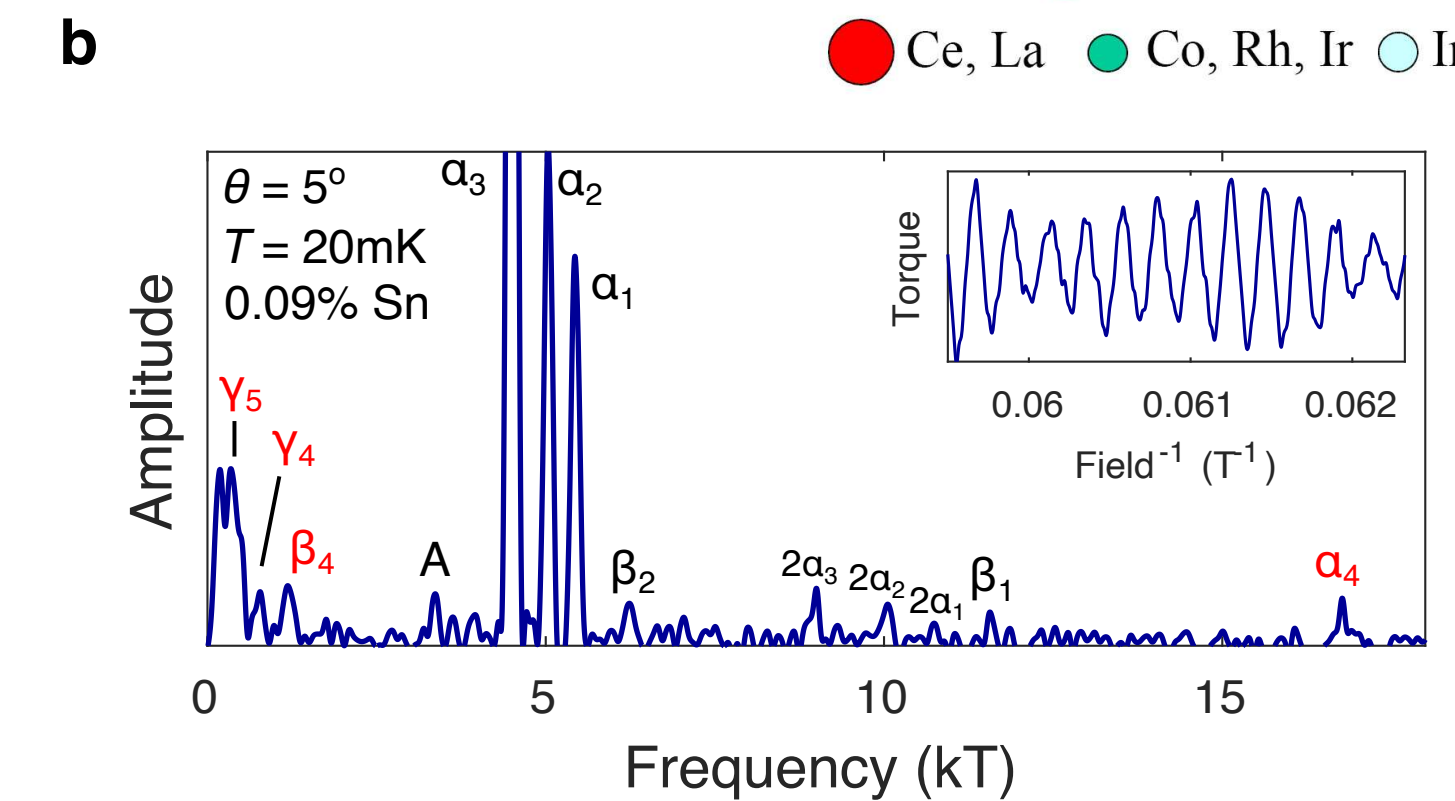
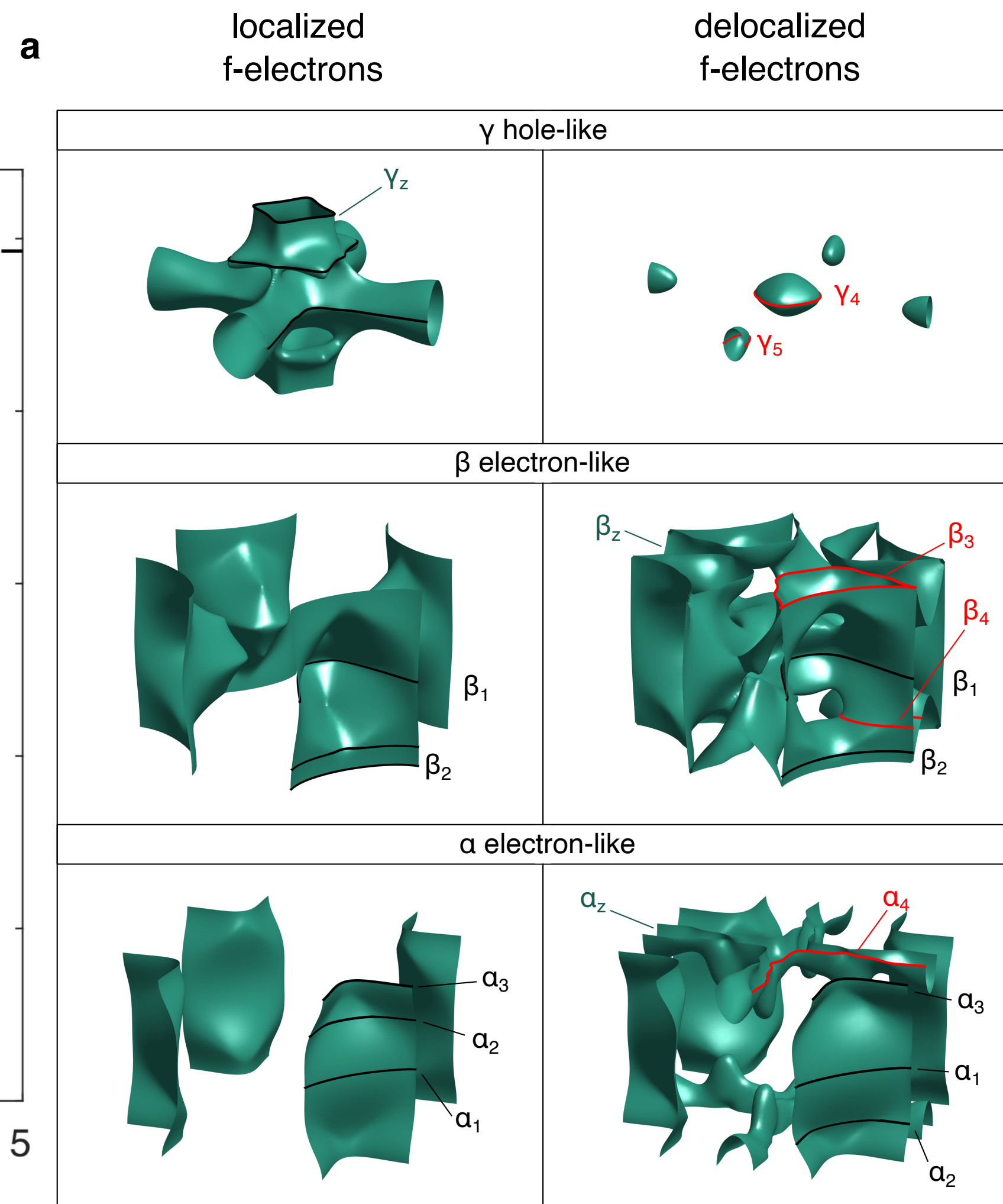
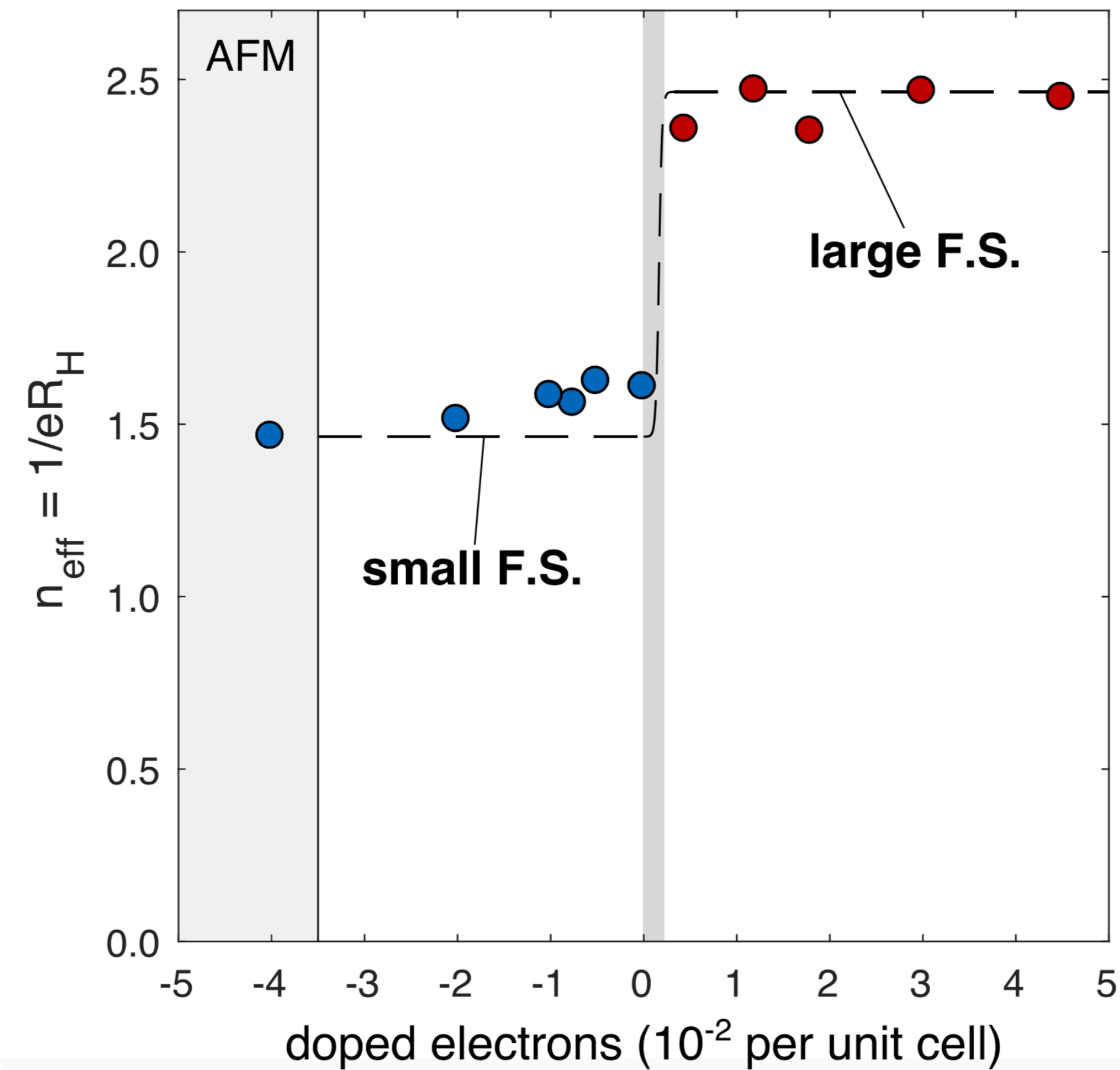
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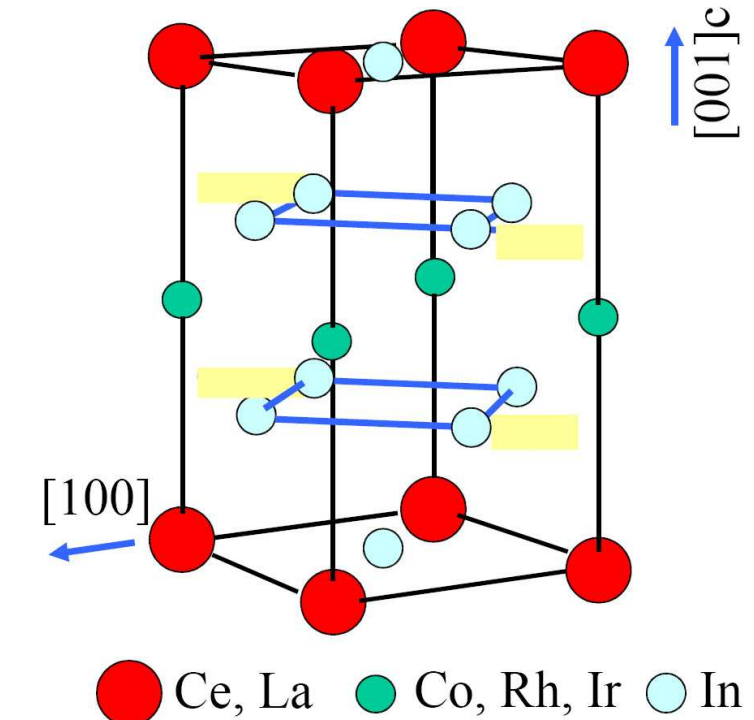


arXiv:2011.12951

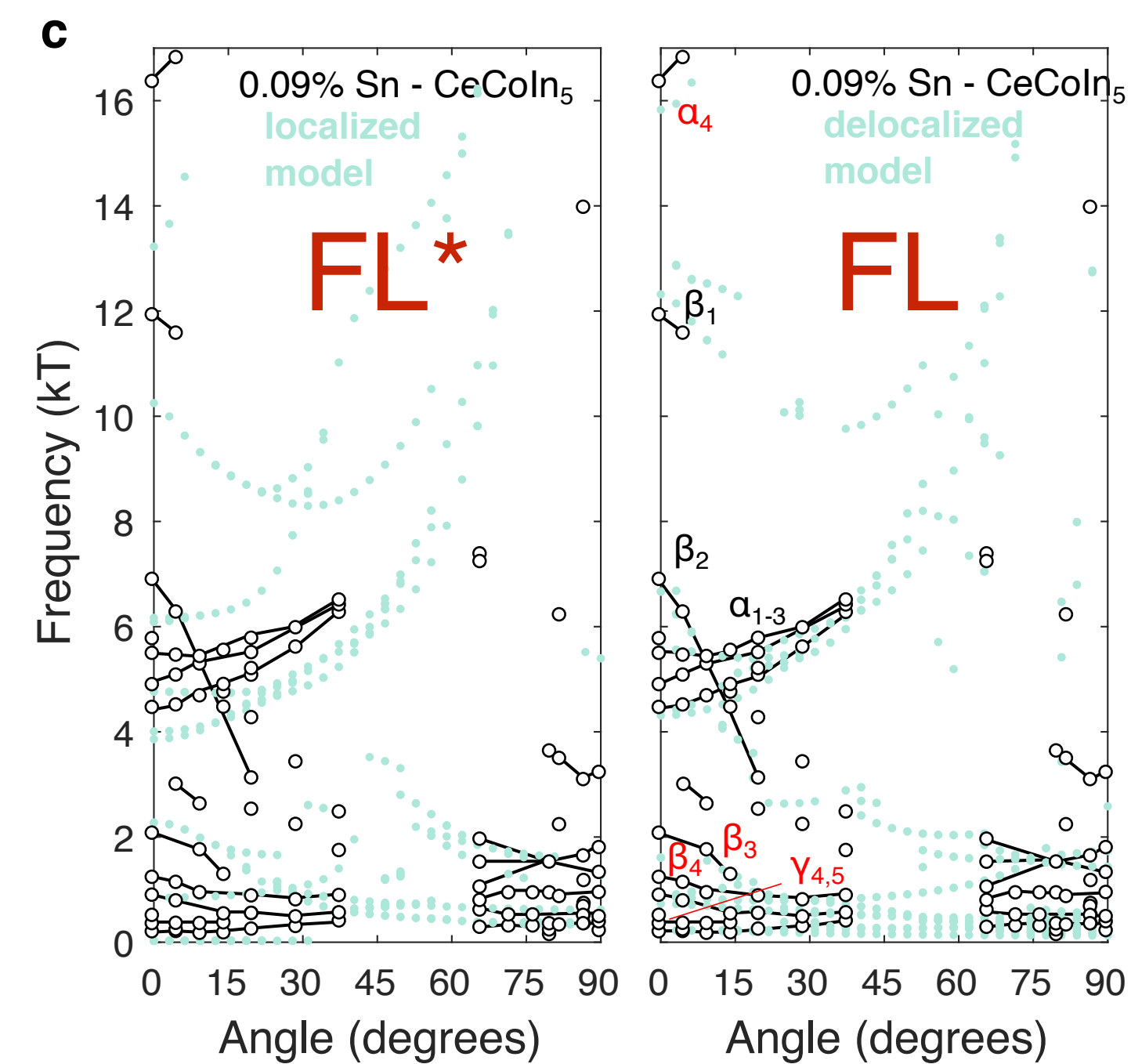
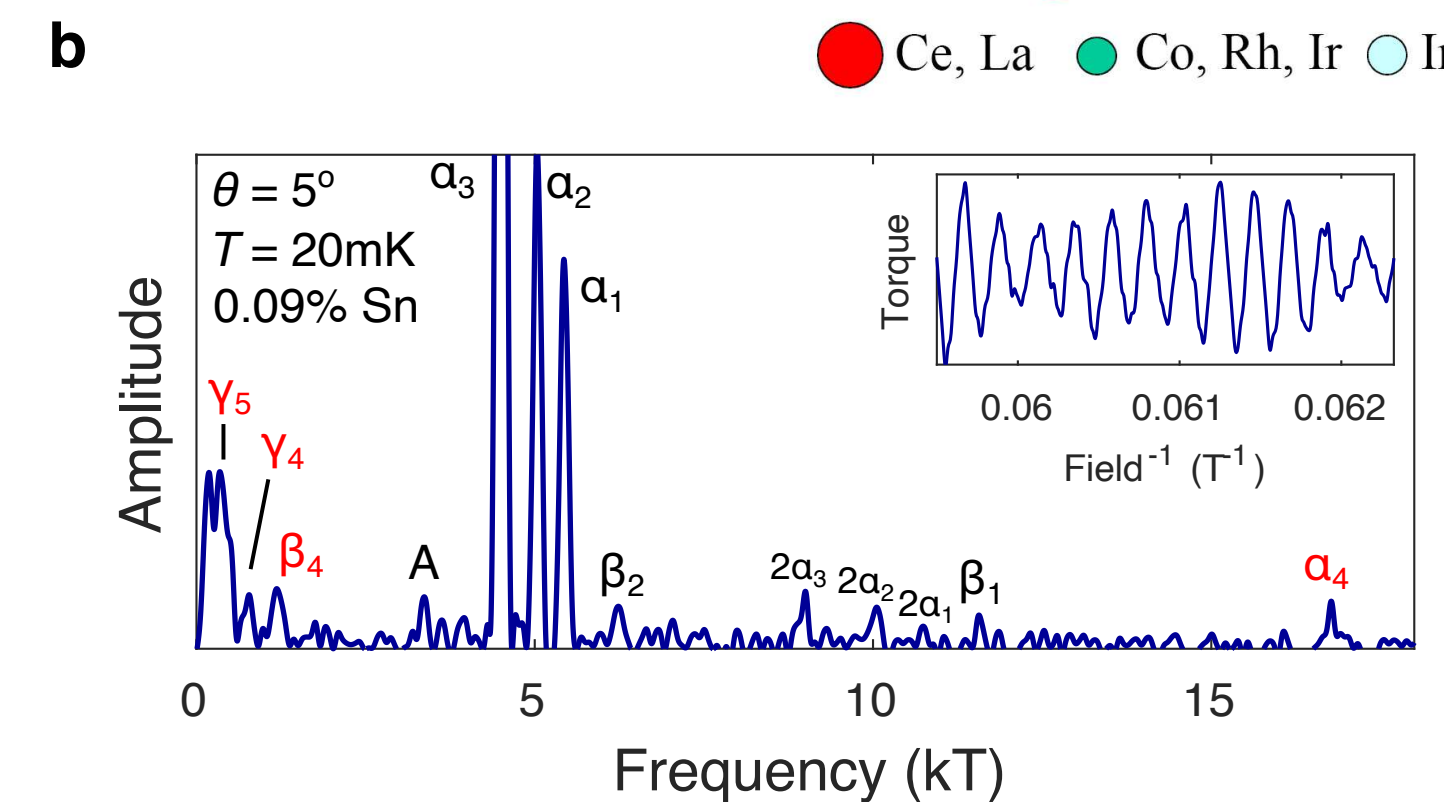
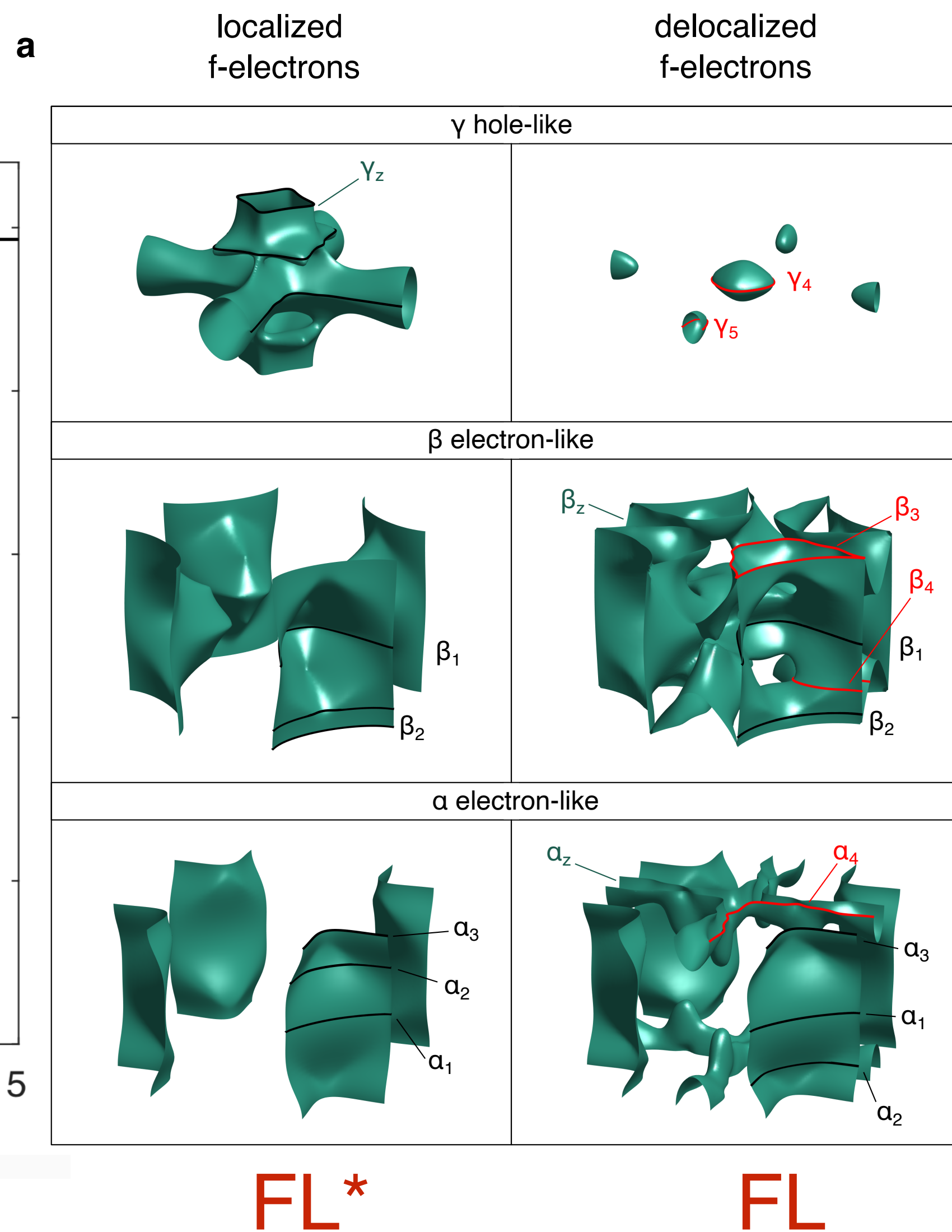
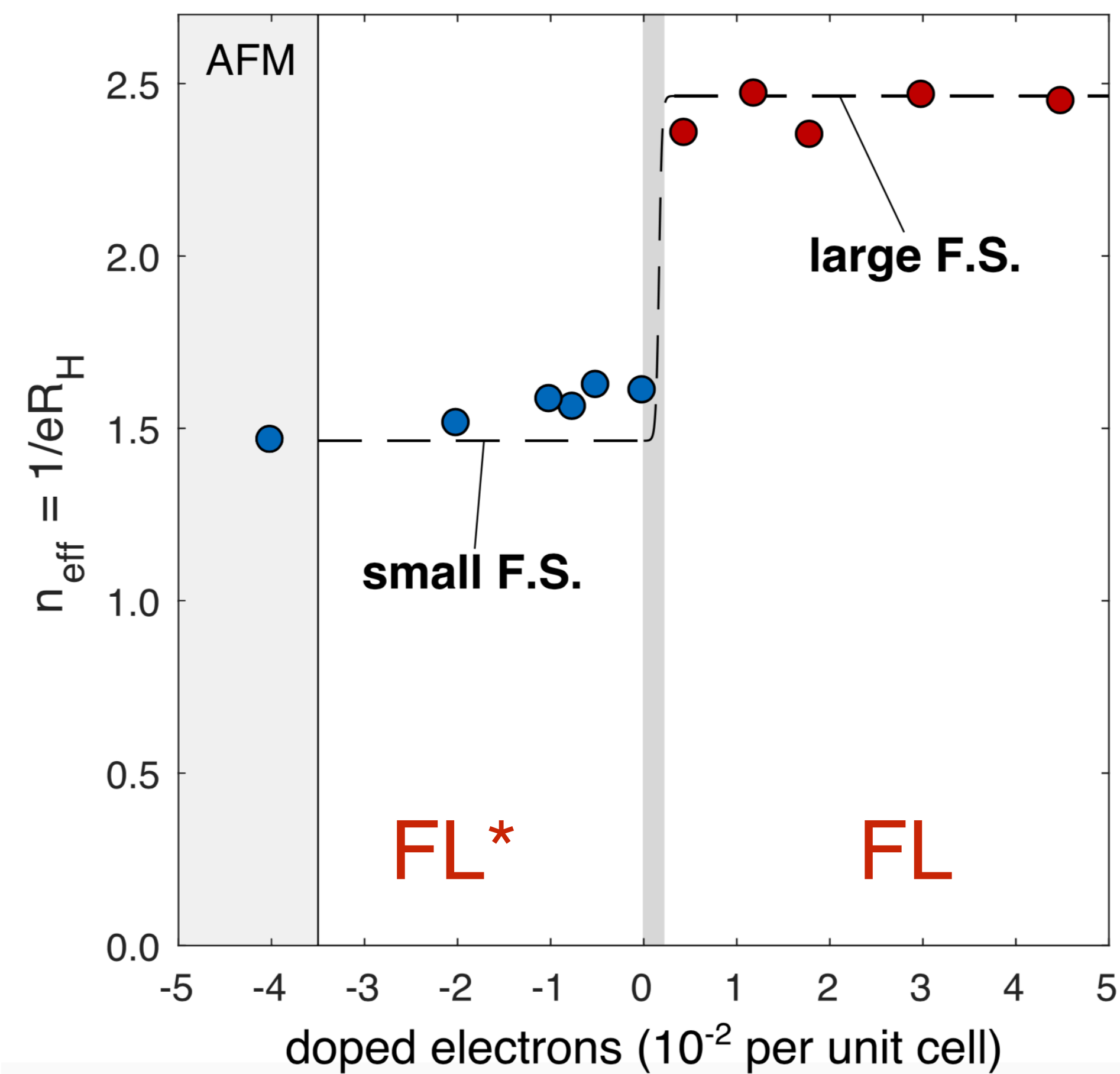


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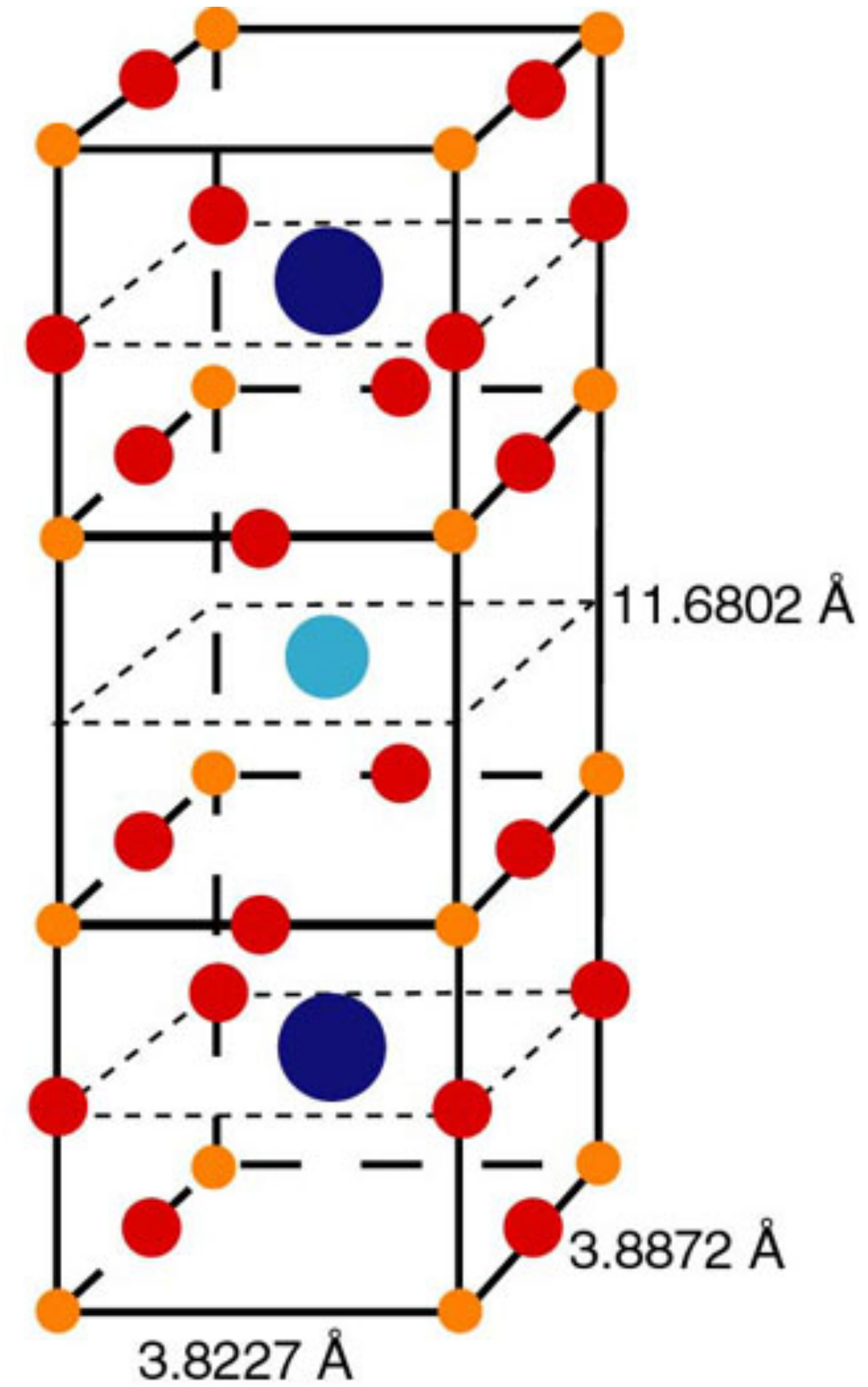
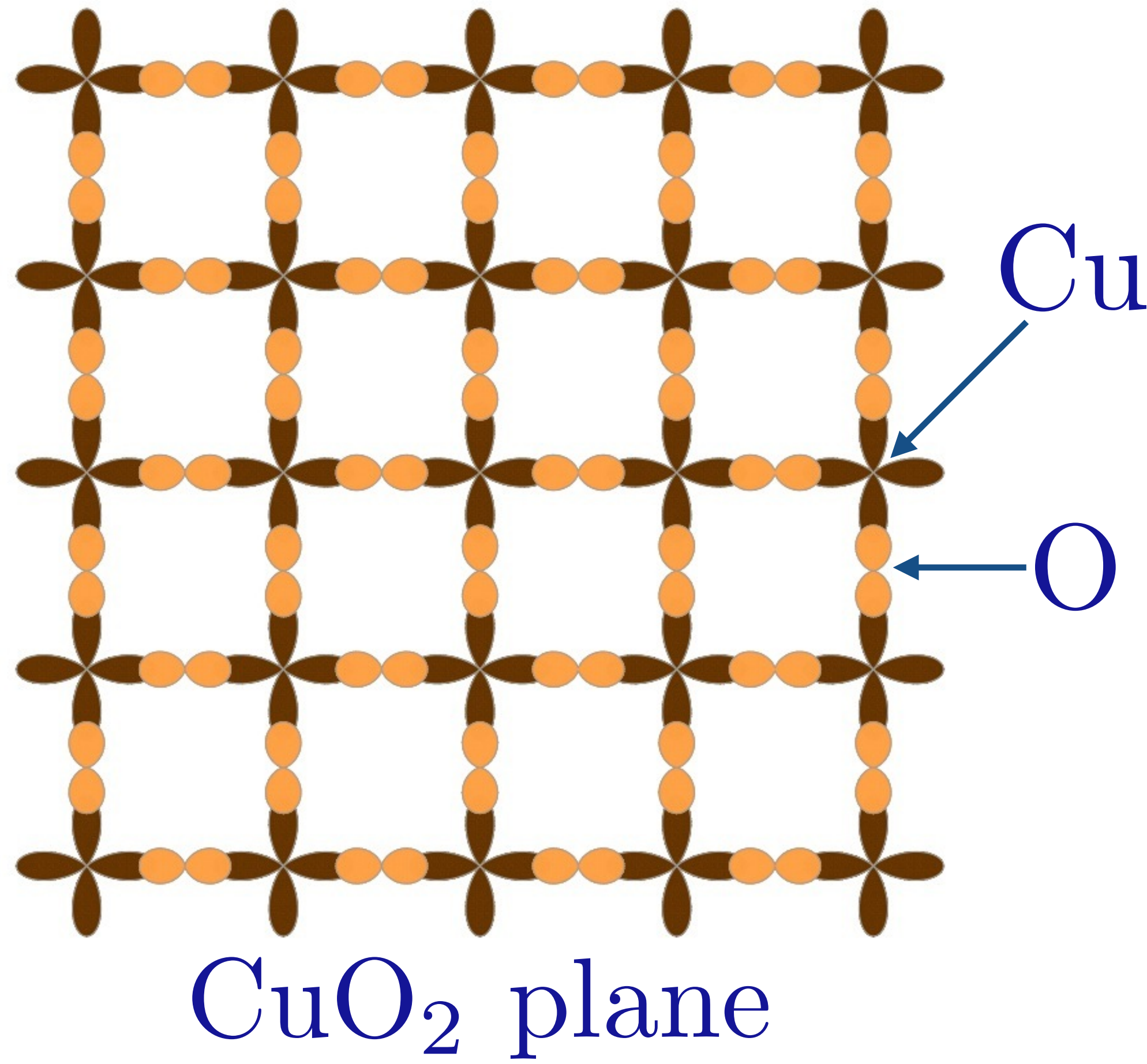
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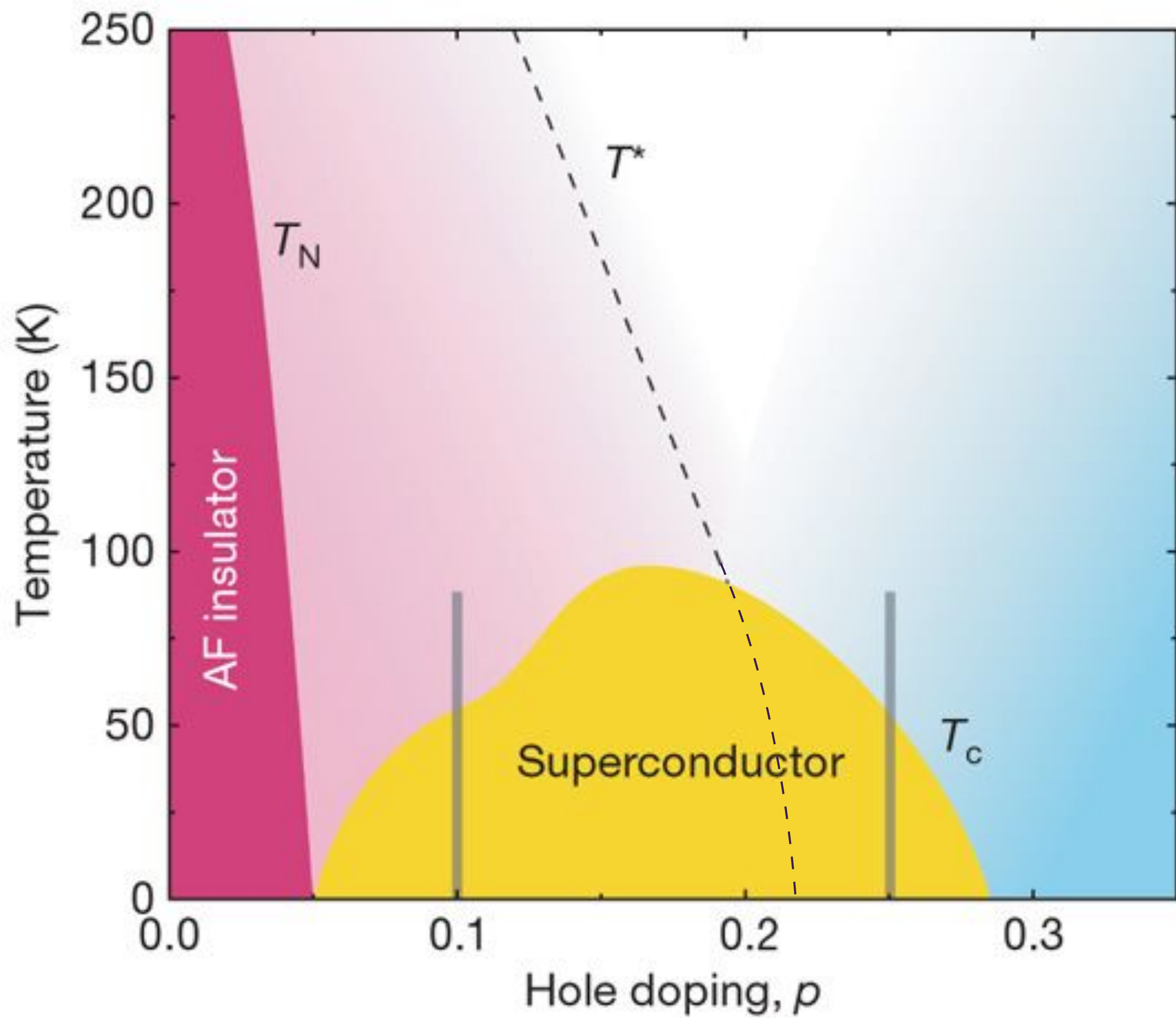
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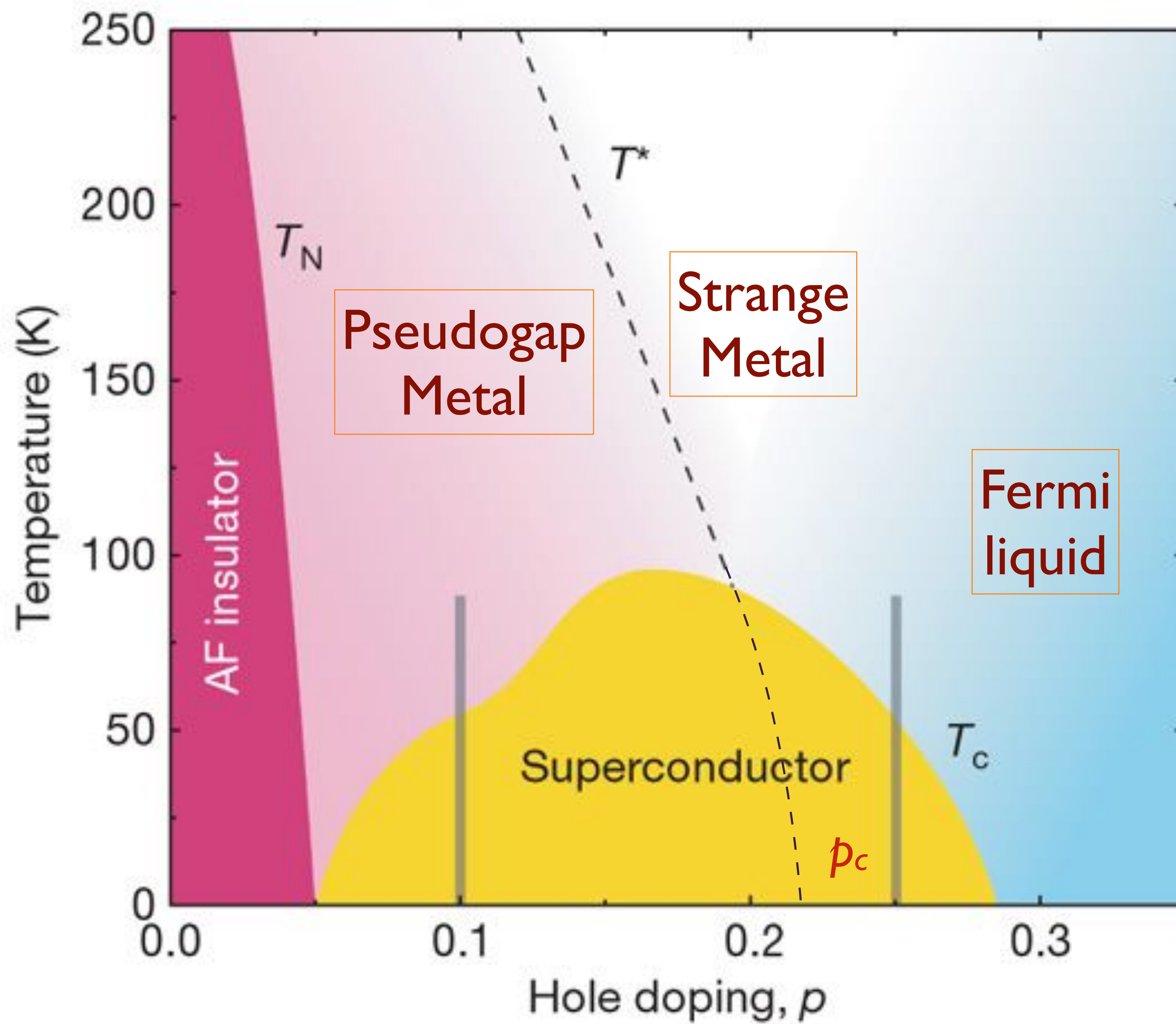
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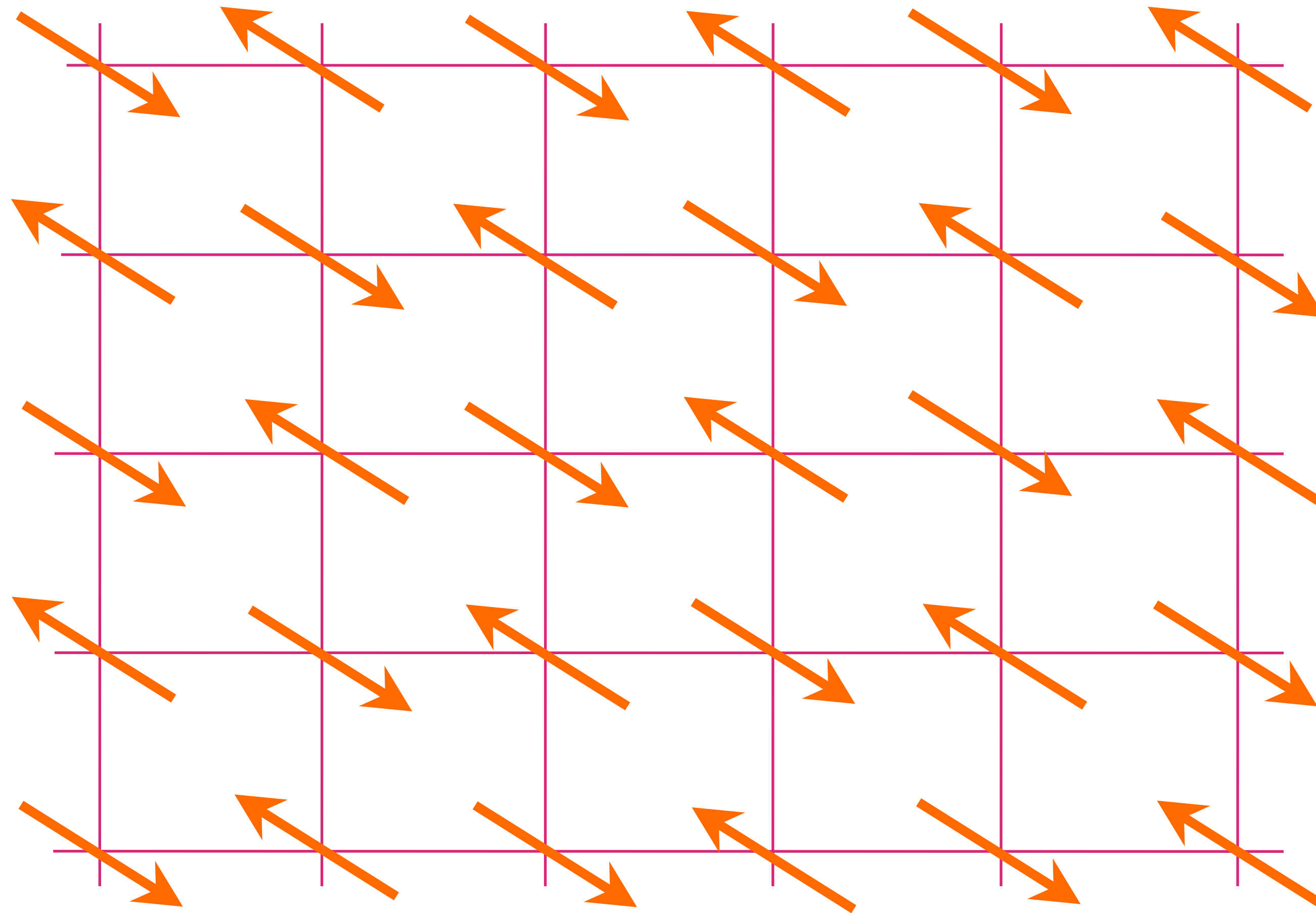
# High temperature superconductors



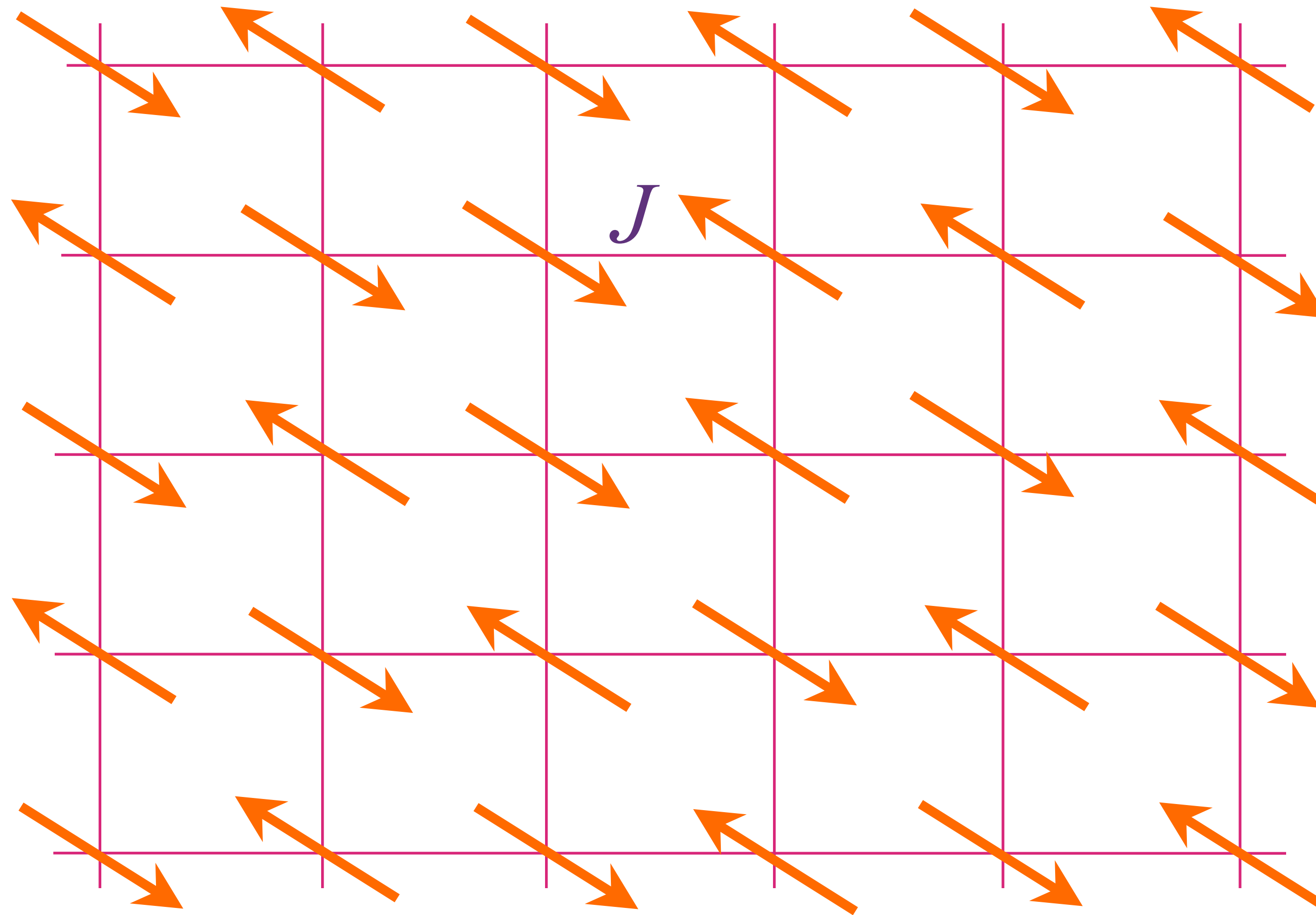




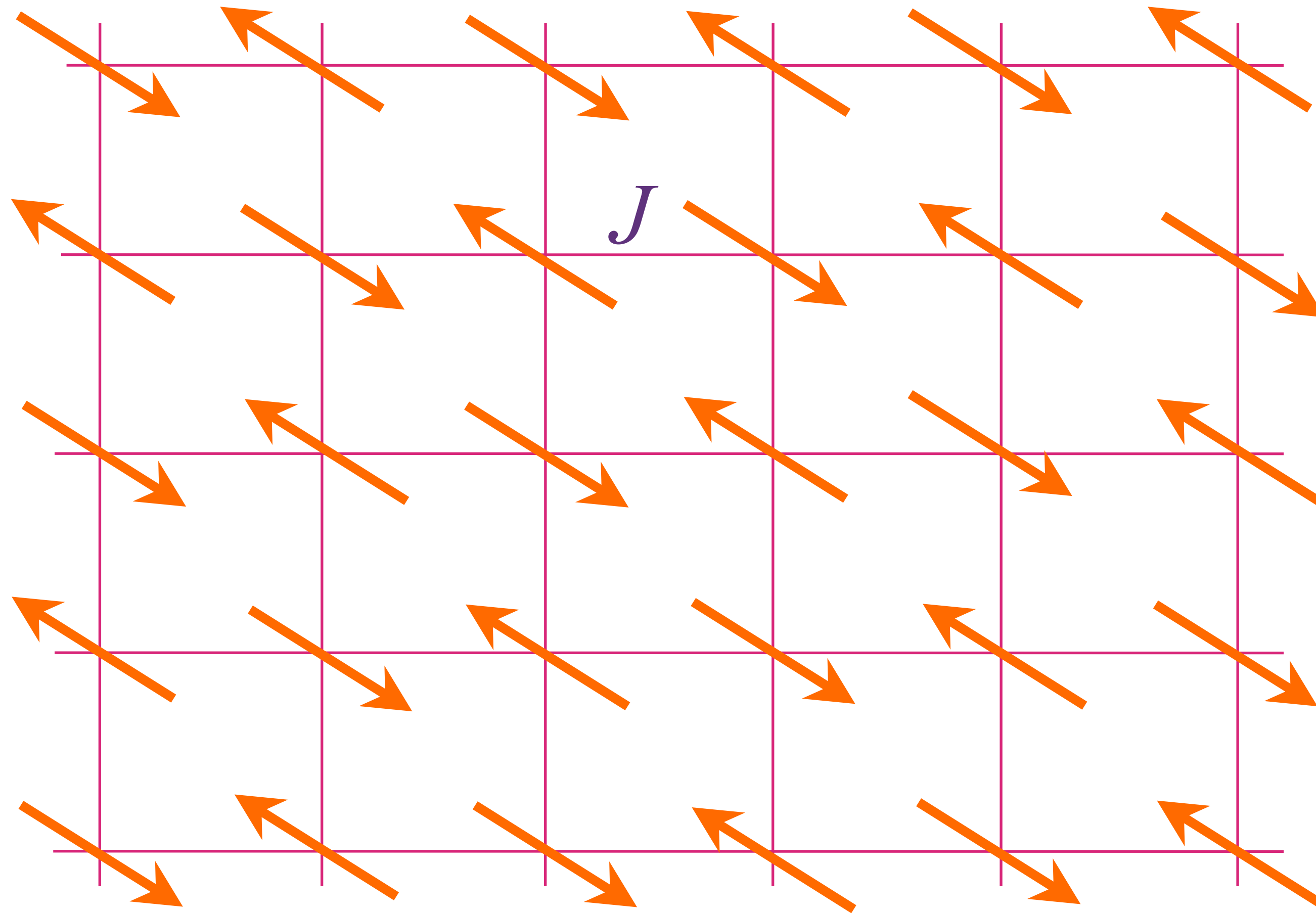
# Insulating antiferromagnet



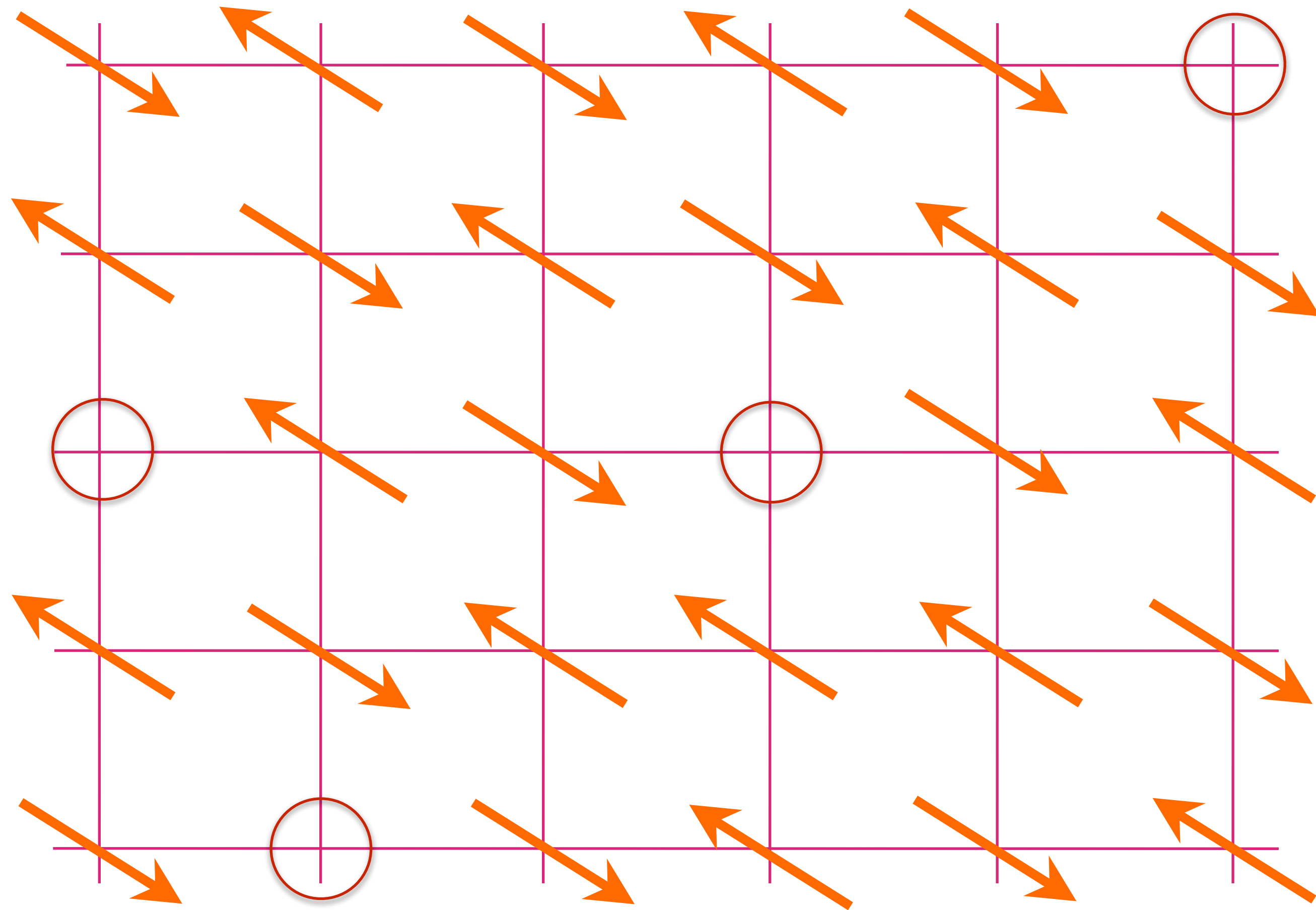
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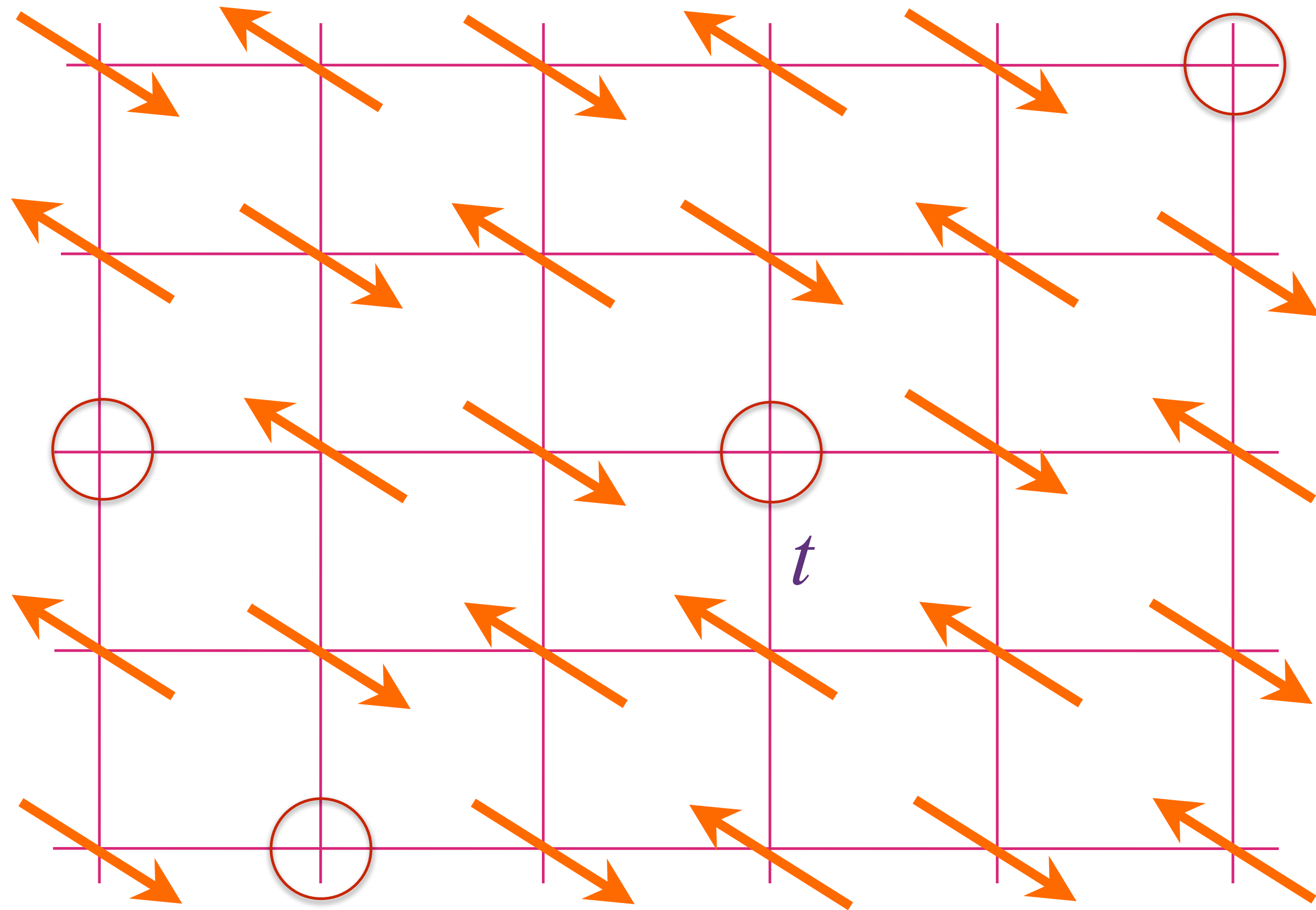
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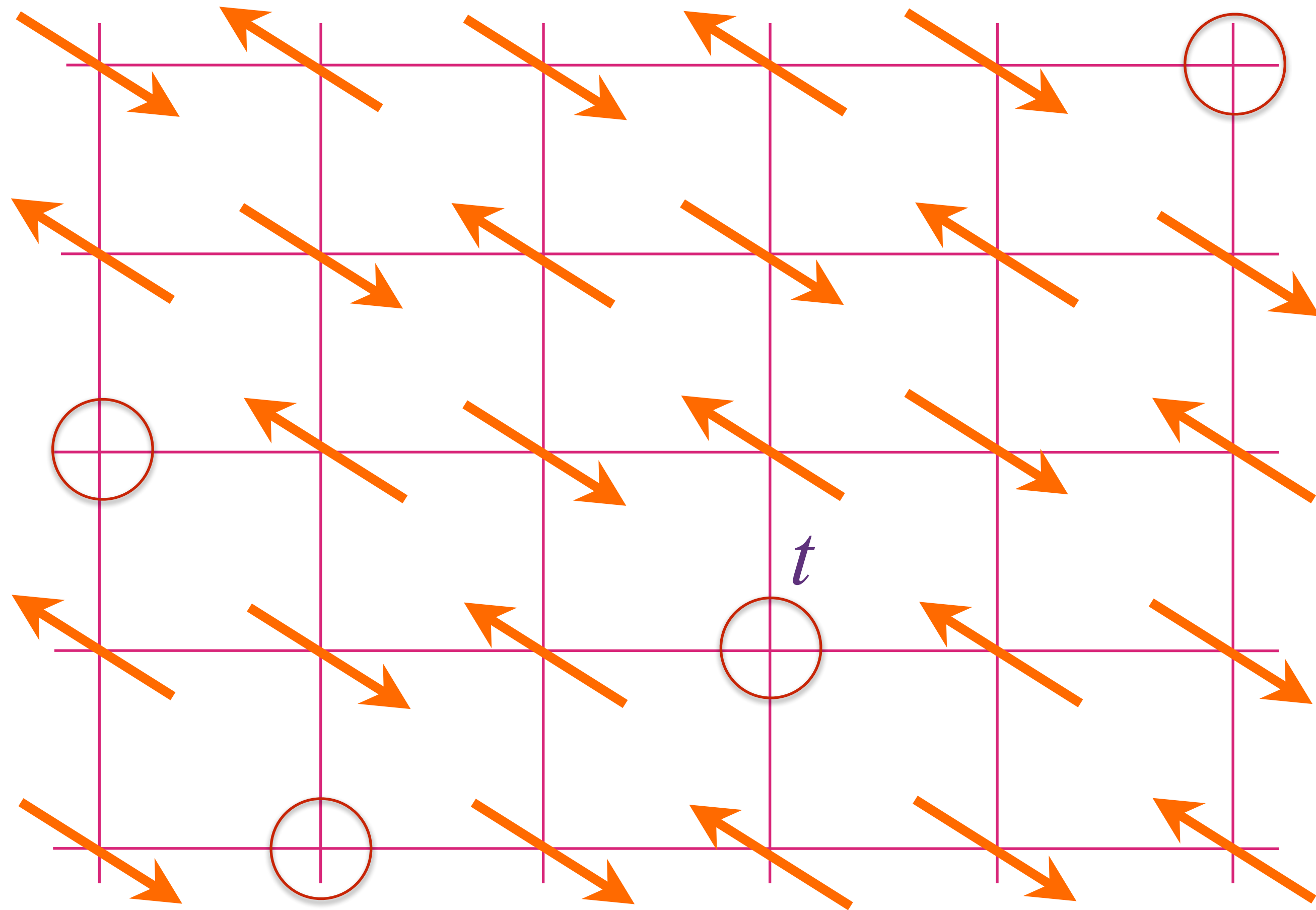
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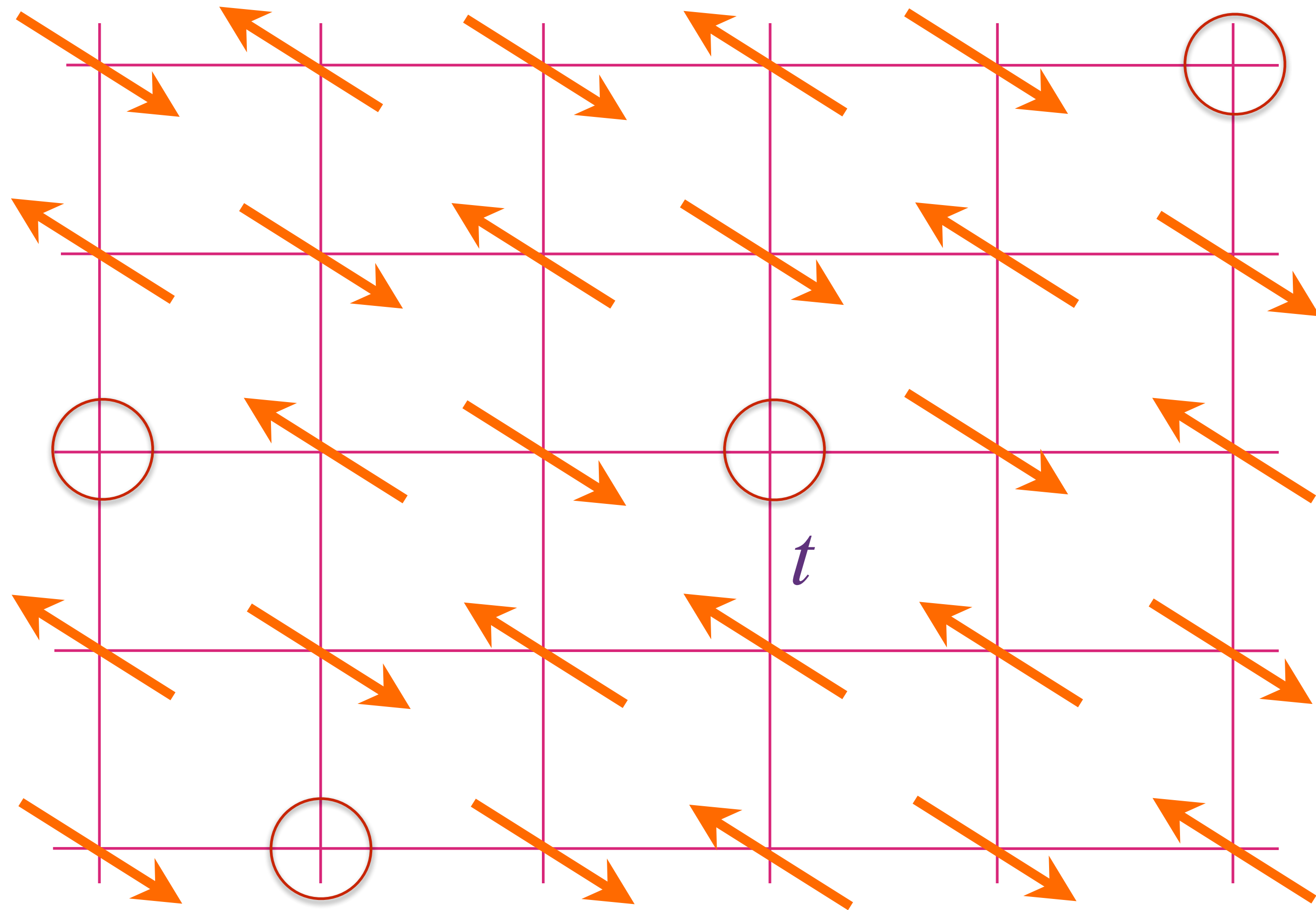
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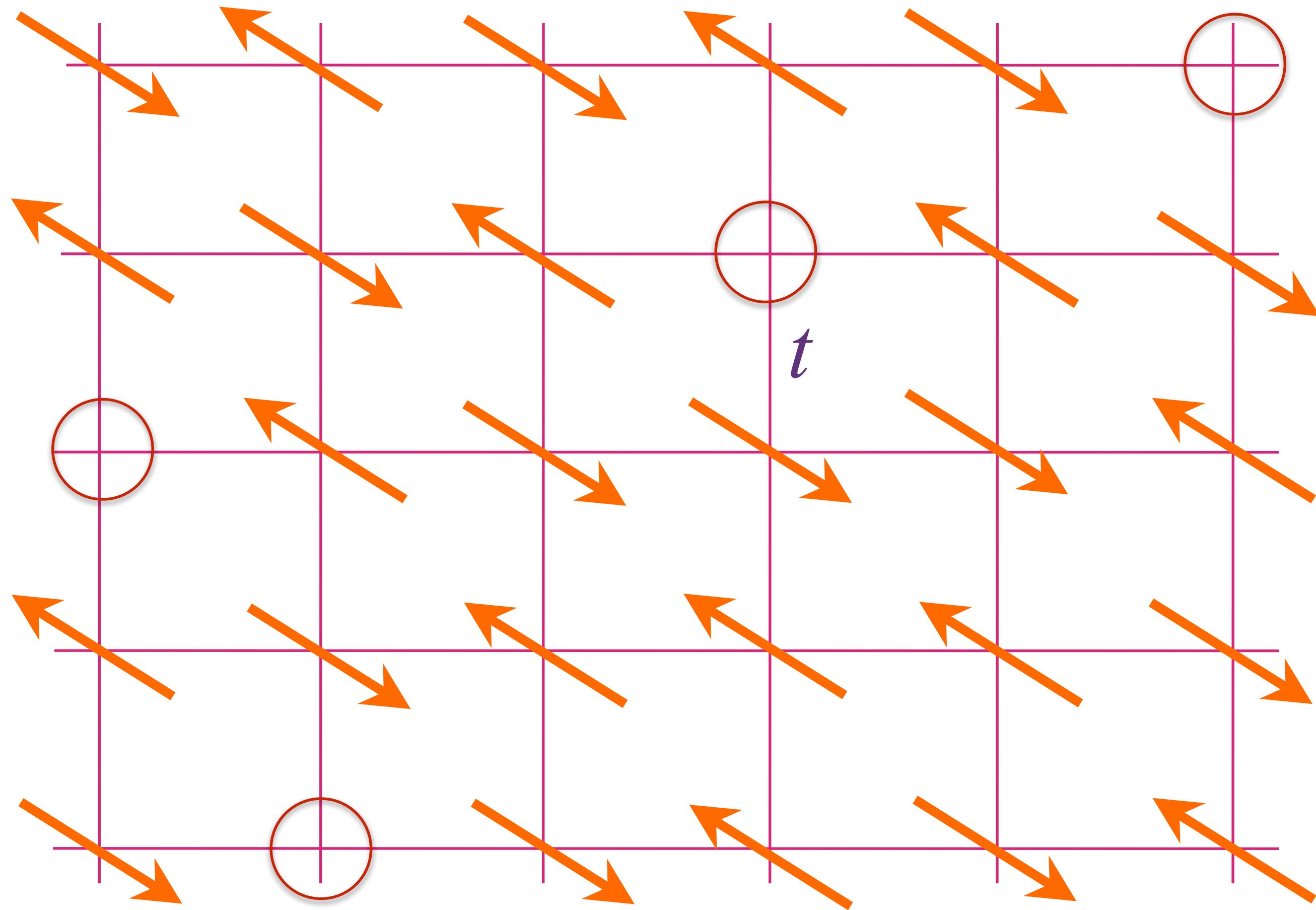


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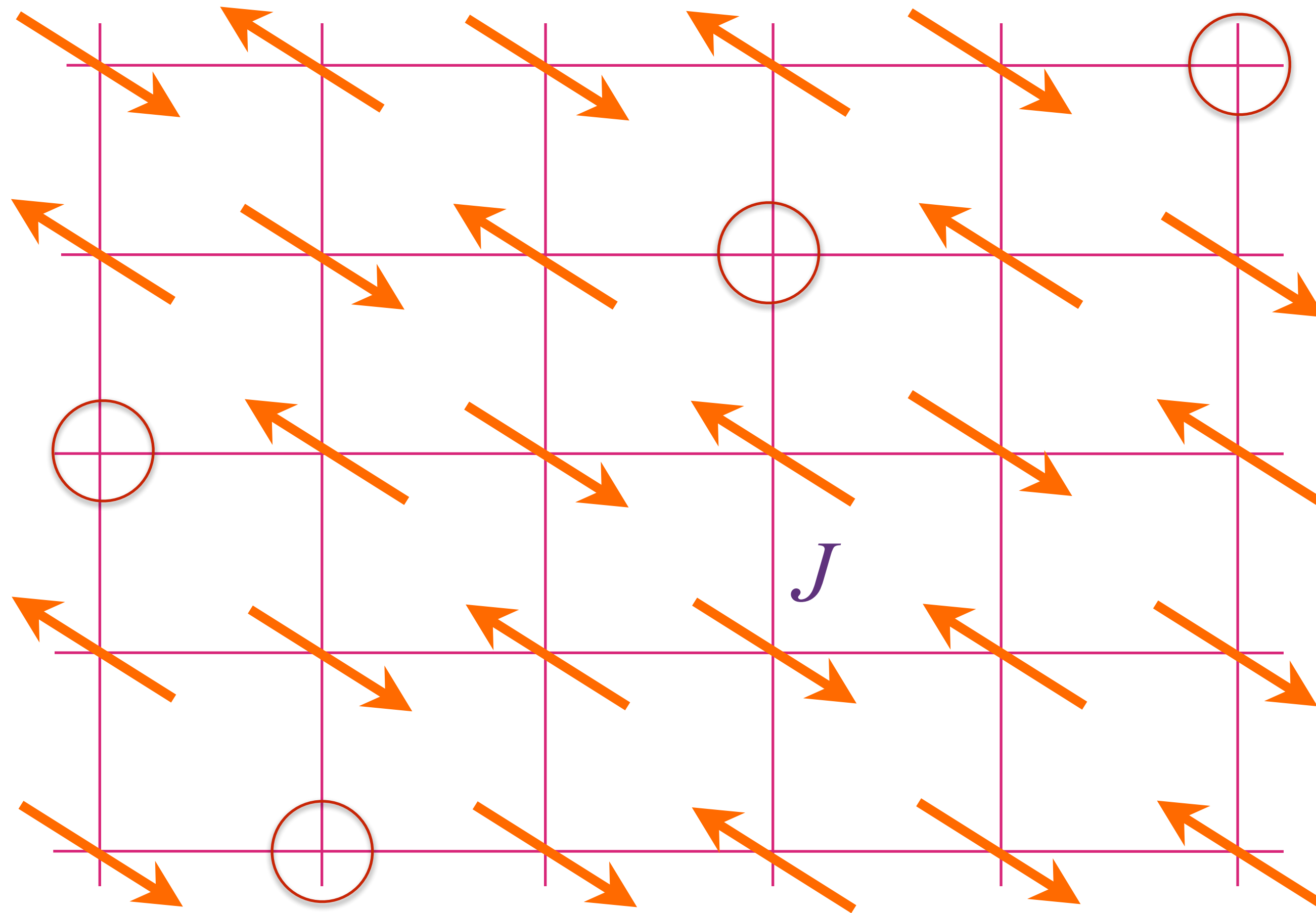
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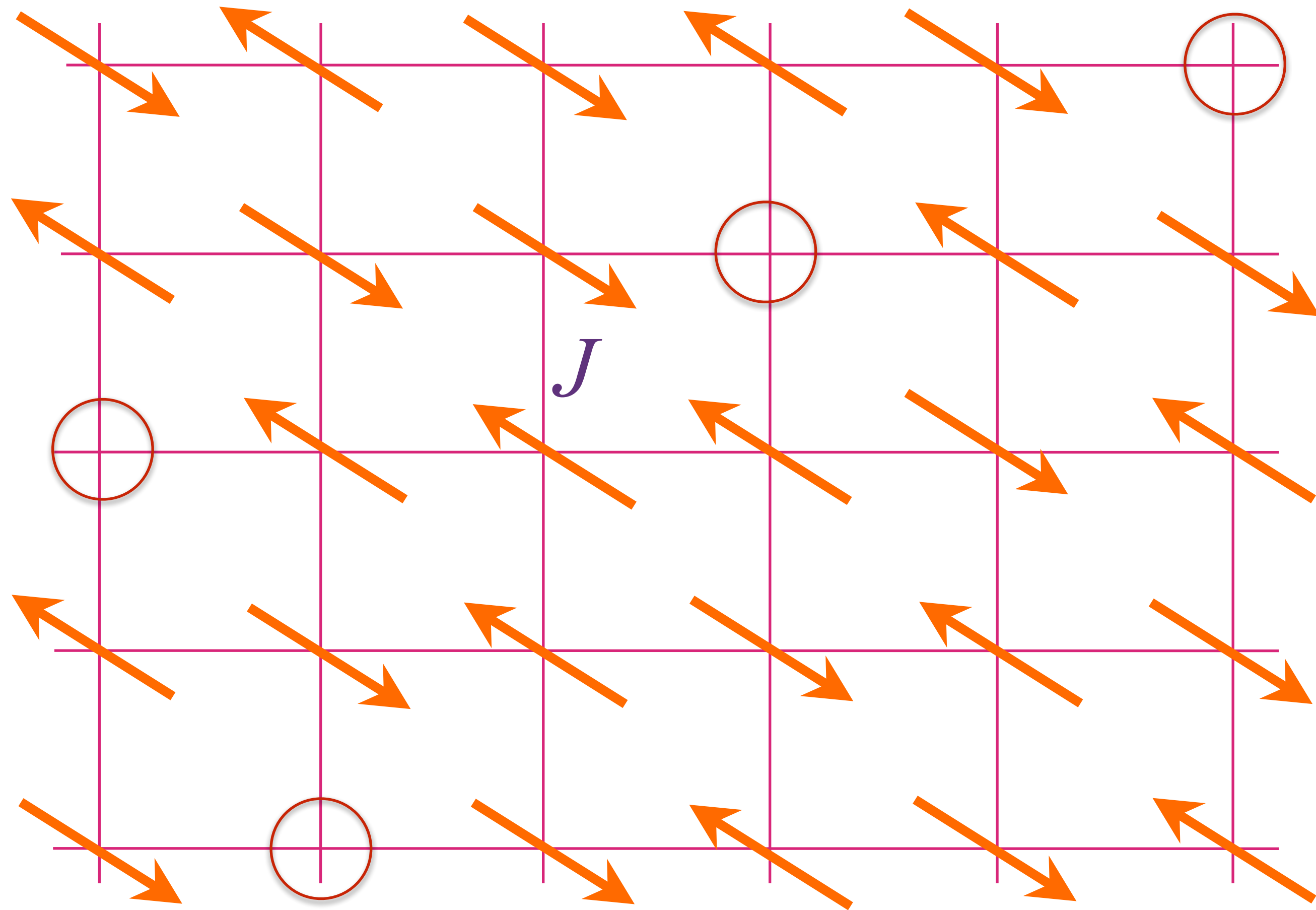
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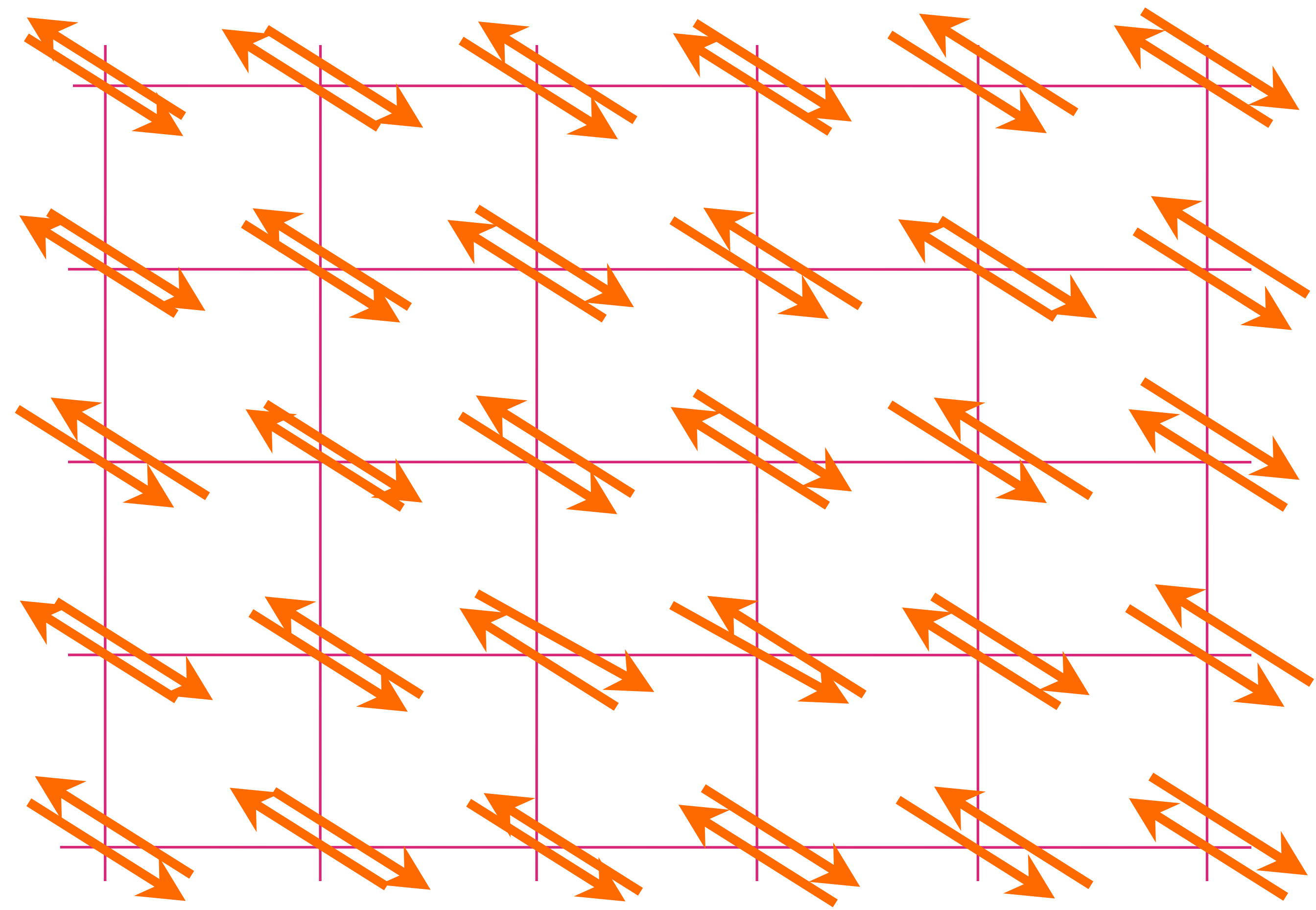
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fluctuating spins

# Antiferromagnet doped with hole density $p$



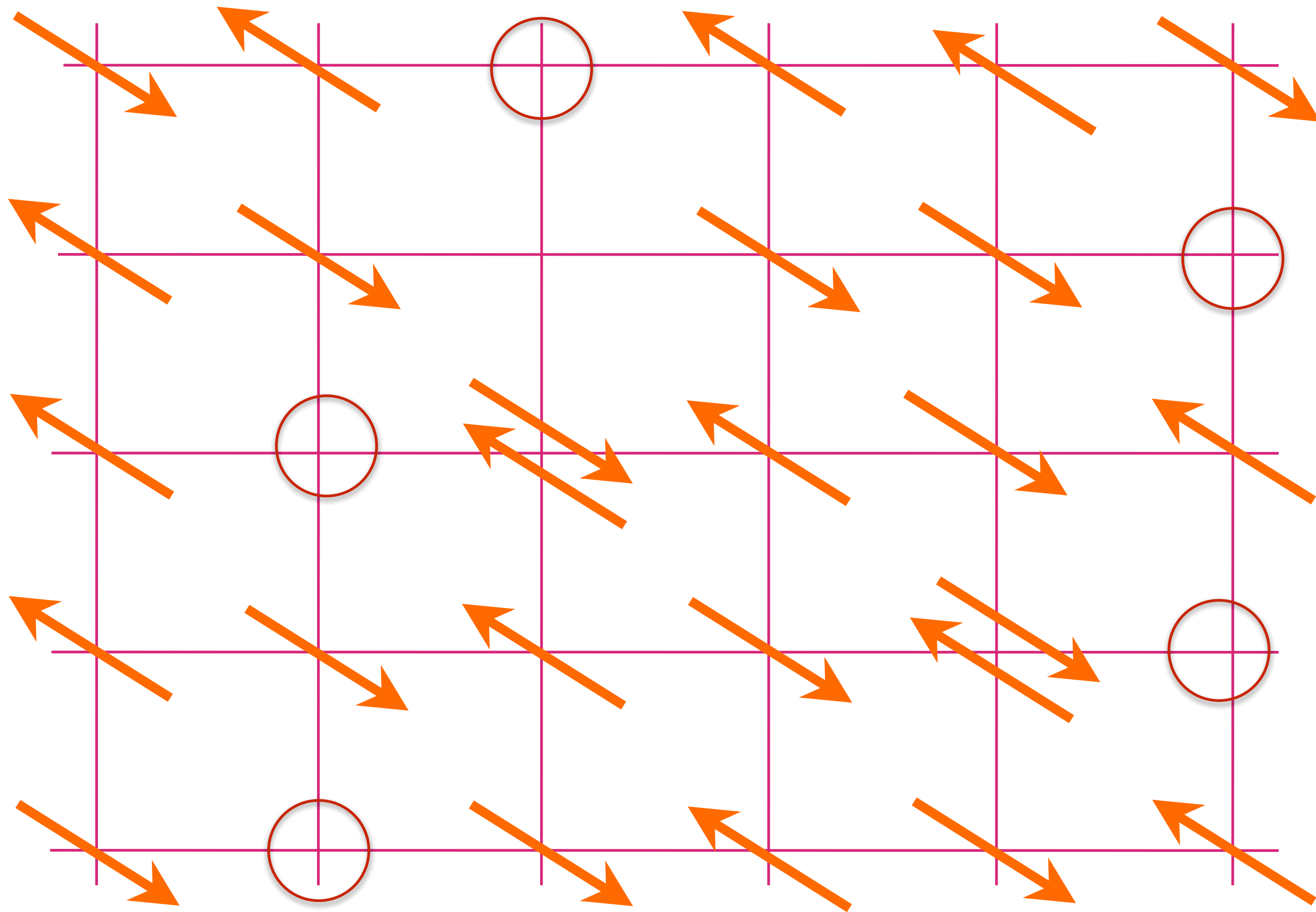
$p$  mobile holes in a background of  
fluctuating spins

# Momentum-space view at large $p$



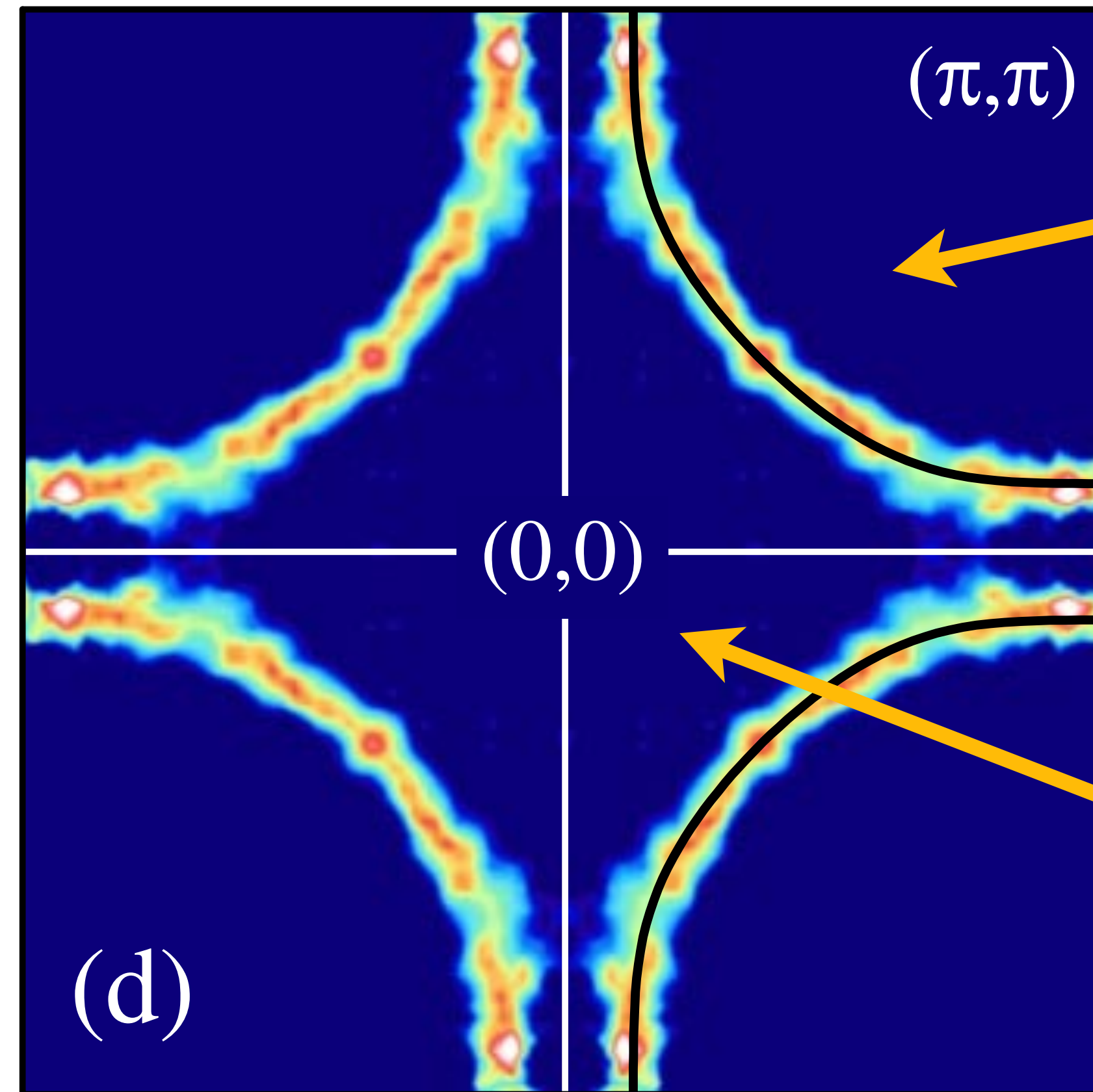
Filled  
Band

# Momentum-space view at large $p$



$1-p$  mobile electrons =  
 $1+p$  mobile holes in a filled band

# Momentum-space view at large $p$



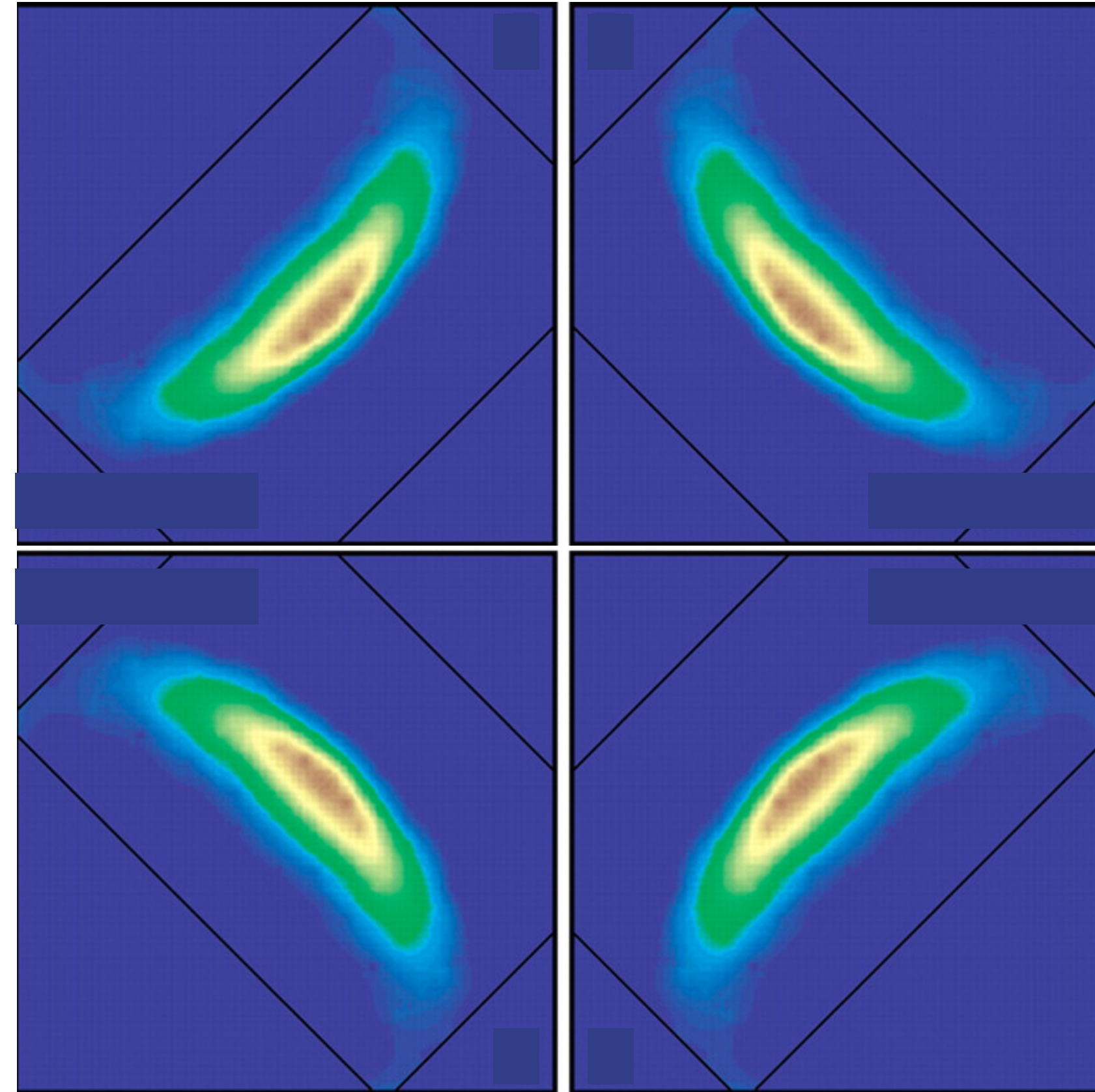
$l+p$  holes

Overdoped  $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+\delta}$   
 $T_c = 30\text{K}$

$l-p$  electrons

$l+p$  mobile holes in a filled band

# Momentum-space view at small $p$



$\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$   
at  $x = 0.10$

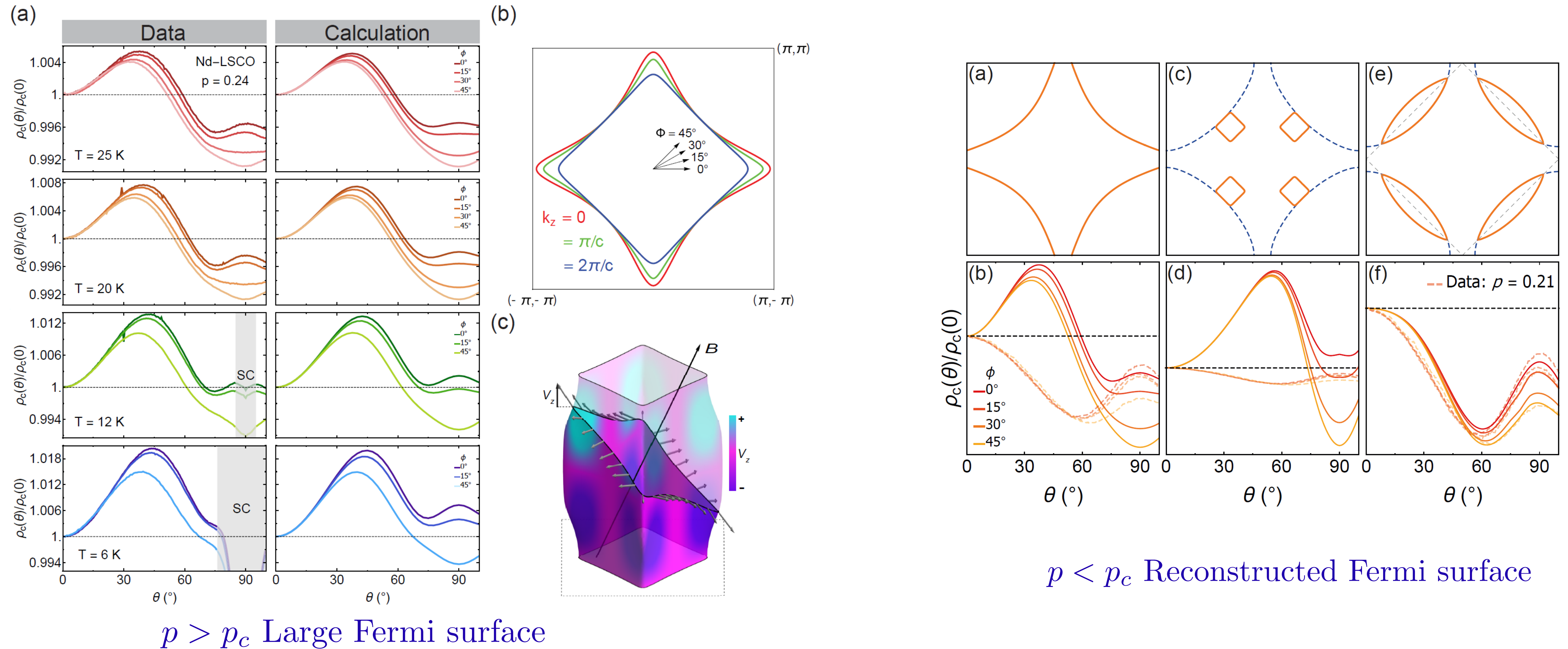
*“Fermi arcs”*

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, *Science* **307**, 901 (2005)

# Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, arXiv:2004.01725

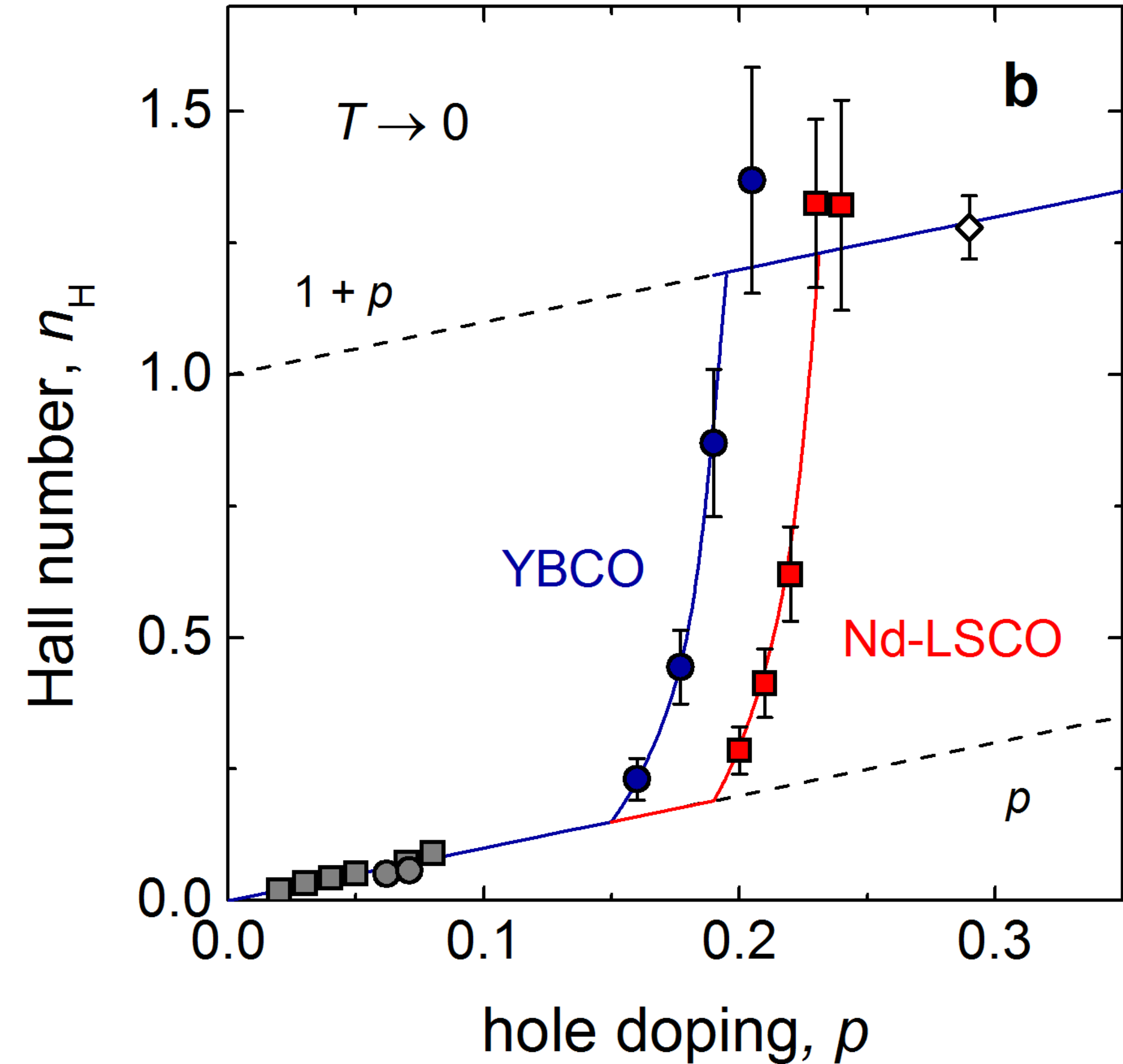
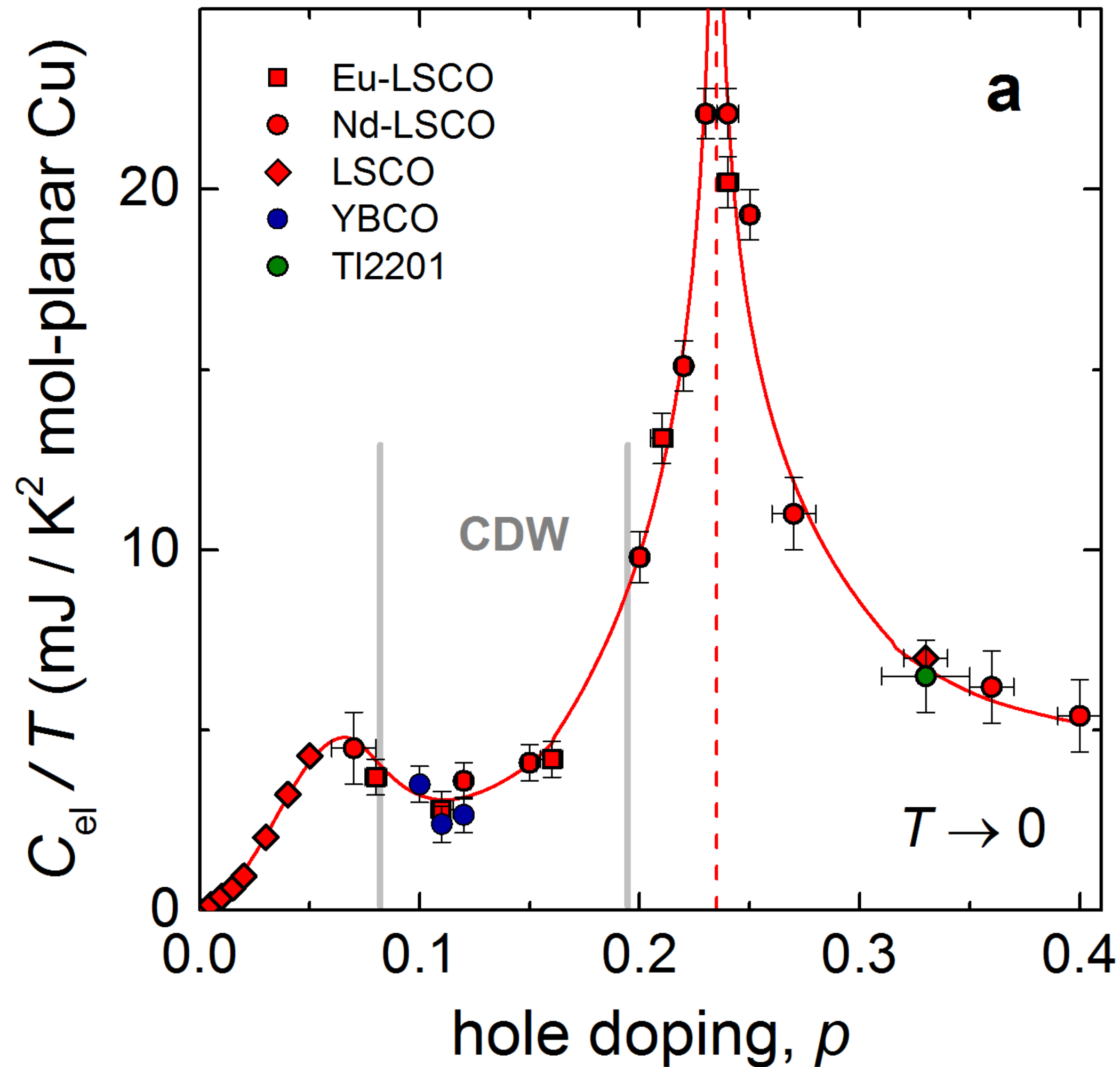
We use angle-dependent magnetoresistance (ADMR) to measure the Fermi surface of the cuprate  $\text{La}_{1.6-x}\text{Nd}_{0.4}\text{Sr}_x\text{CuO}_4$ . Above the critical doping  $p^*$  — outside of the pseudogap phase — we find a Fermi surface that is in quantitative agreement with angle-resolved photoemission. Below  $p^*$ , however, the ADMR is qualitatively different, revealing a clear change in Fermi surface topology. We find that our data is most consistent with a Fermi surface that has been reconstructed by a  $Q = (\pi, \pi)$  wavevector. While static  $Q = (\pi, \pi)$  antiferromagnetism is not found at these dopings, our results suggest that this wavevector is a fundamental organizing principle of the pseudogap phase.

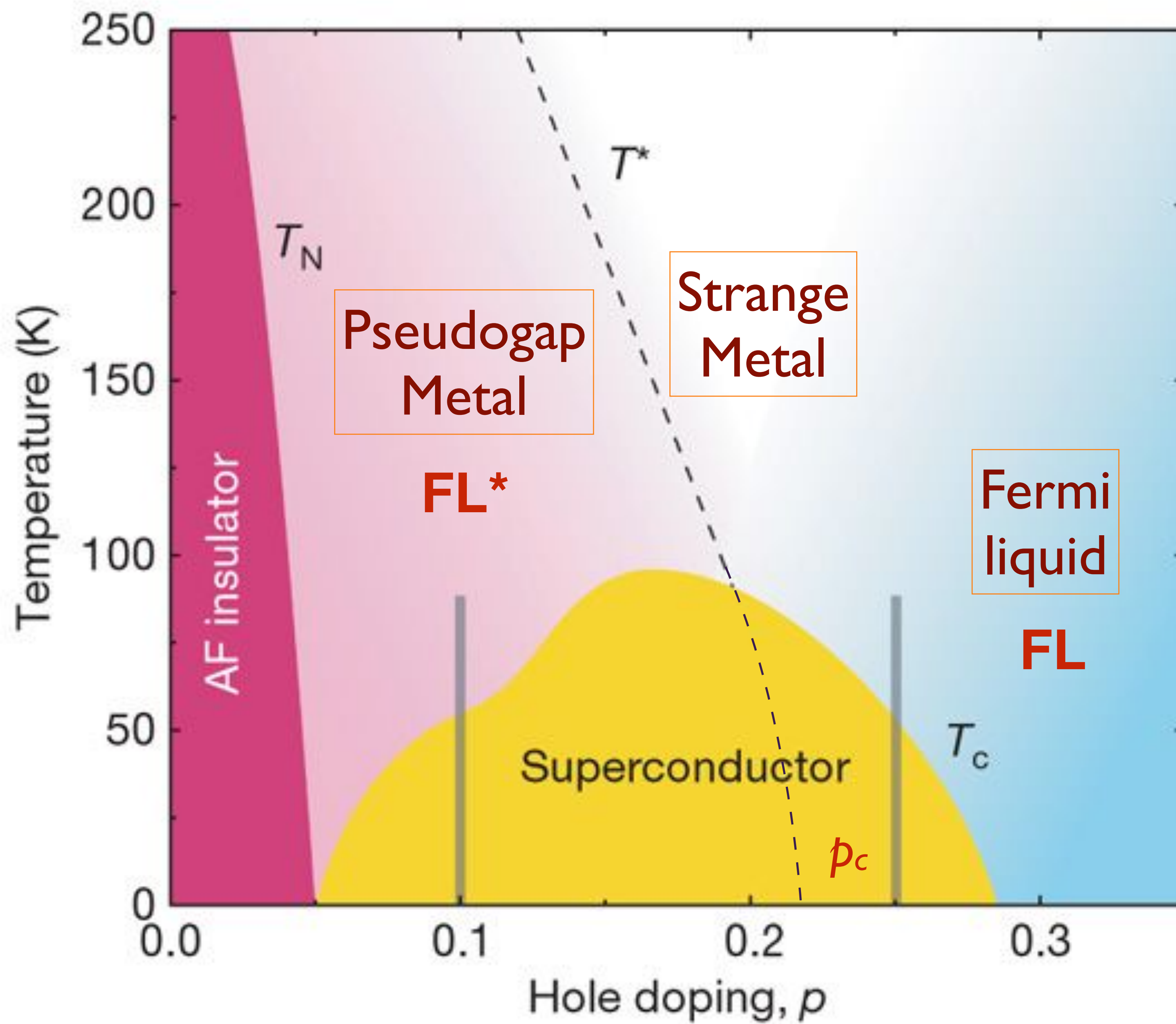


# Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, Annual Review Condensed Matter Physics **10**, 409 (2019)





1. Luttinger volume violation in  
Kondo lattice models

*The FL\* phase and CeCoIn<sub>5</sub>*

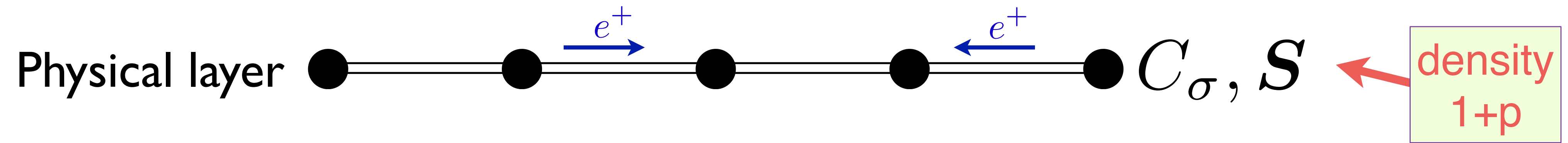
2. Introduction to cuprates

*Small to Large Fermi surface transition*

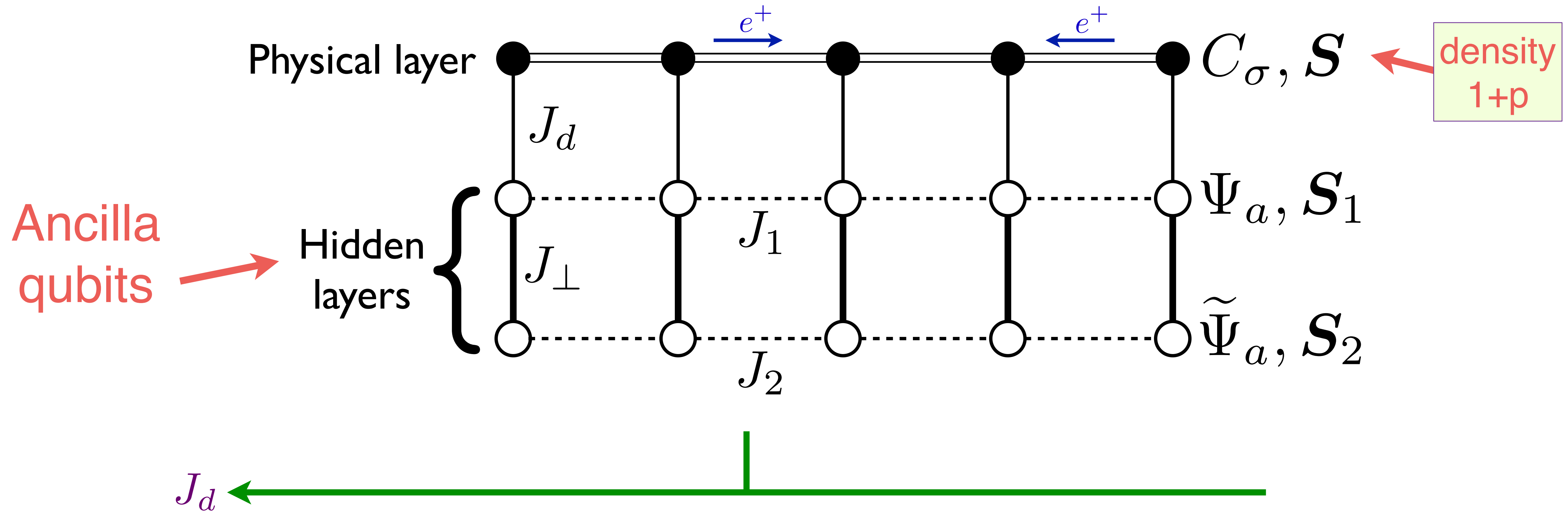
3. Luttinger volume violation a one-band model

*Ancilla qubits and ghost Fermi surfaces*

# FL\* and FL in a **one-band** model



# FL\* and FL in a **one-band** model



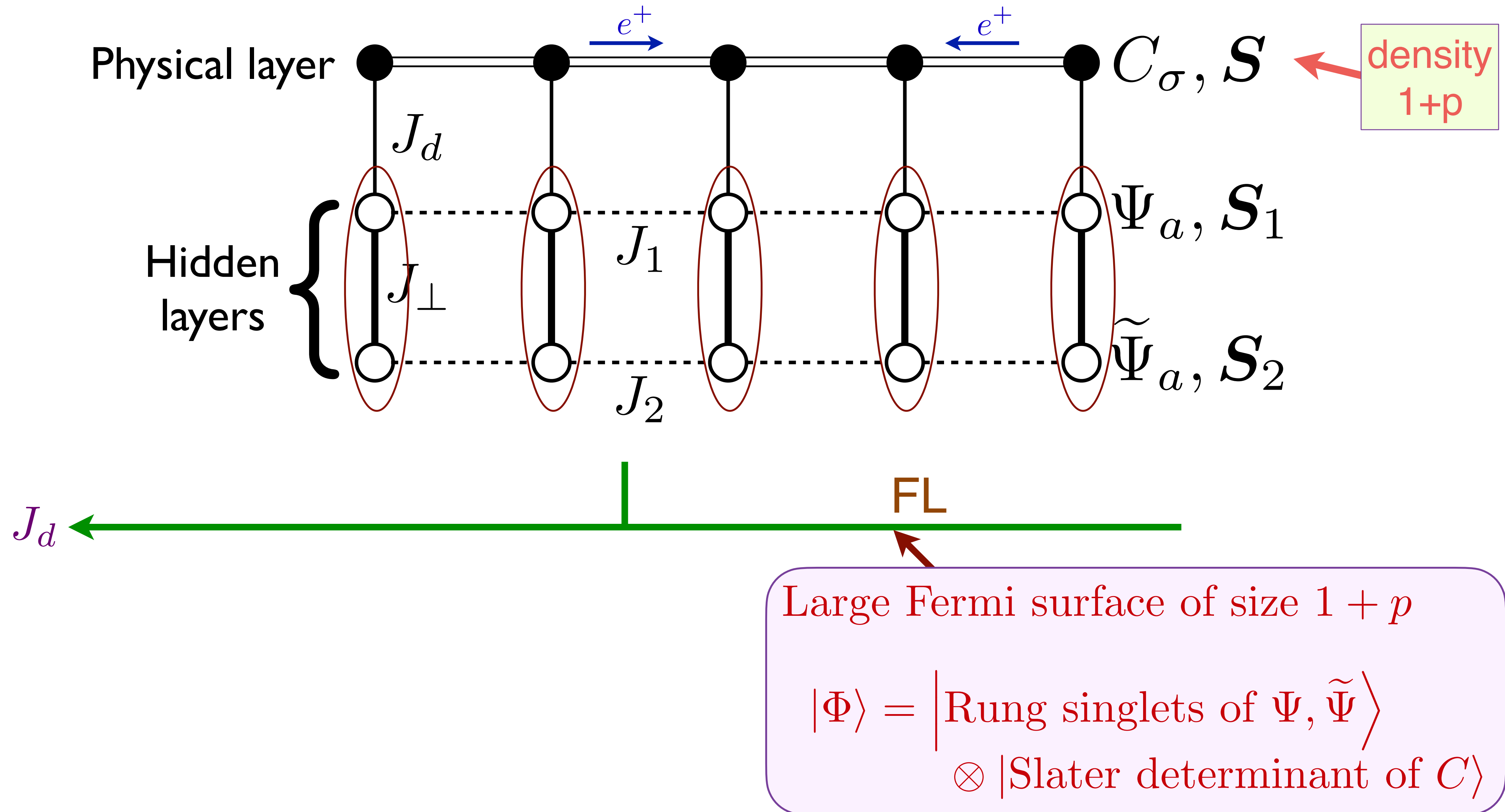
Ya-Hui Zhang

Precursor with one extra band of fermions in FL\* phase:

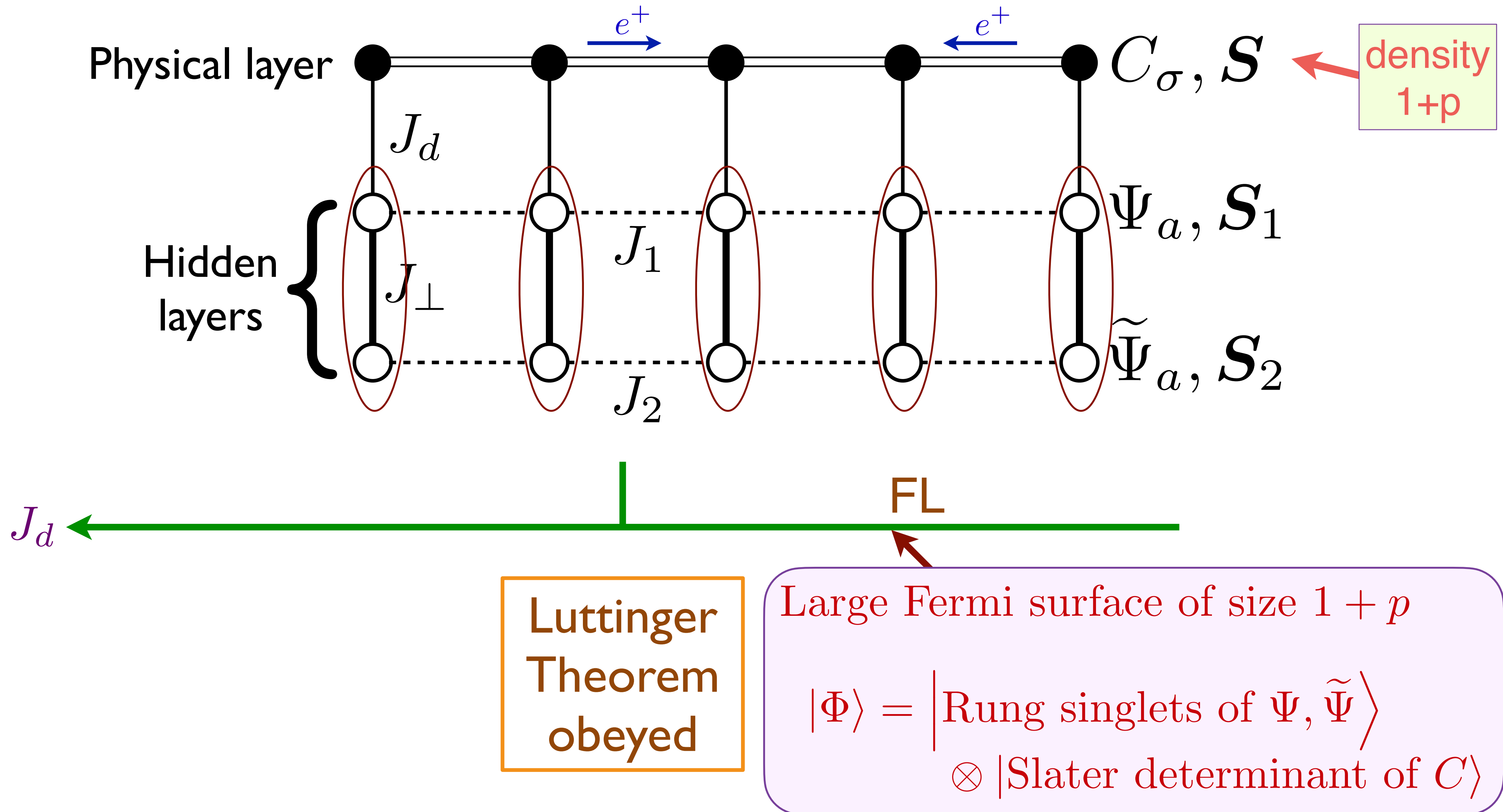
Y. Qi and S. Sachdev, PRB **81**, 115129 (2010)

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011)

# FL\* and FL in a **one-band** model

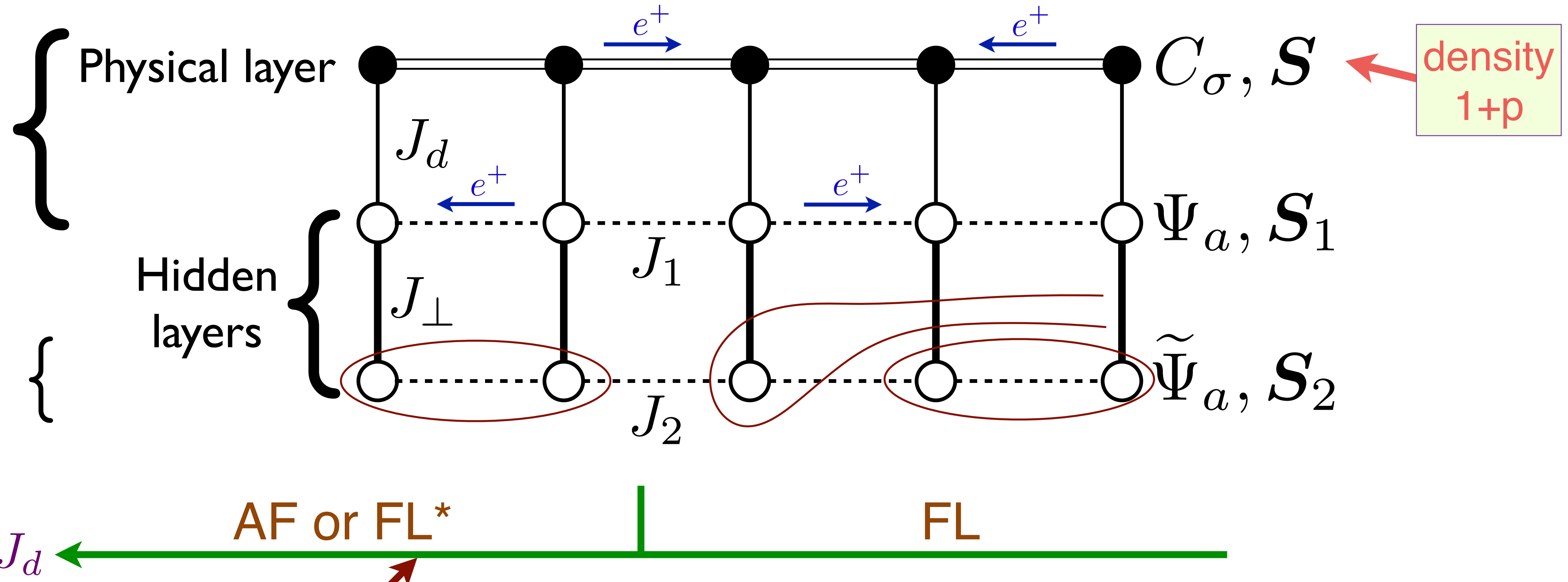


# FL\* and FL in a **one-band** model



# FL\* and FL in a **one-band** model

Metal.  
Density  
 $2 + p \cong p$



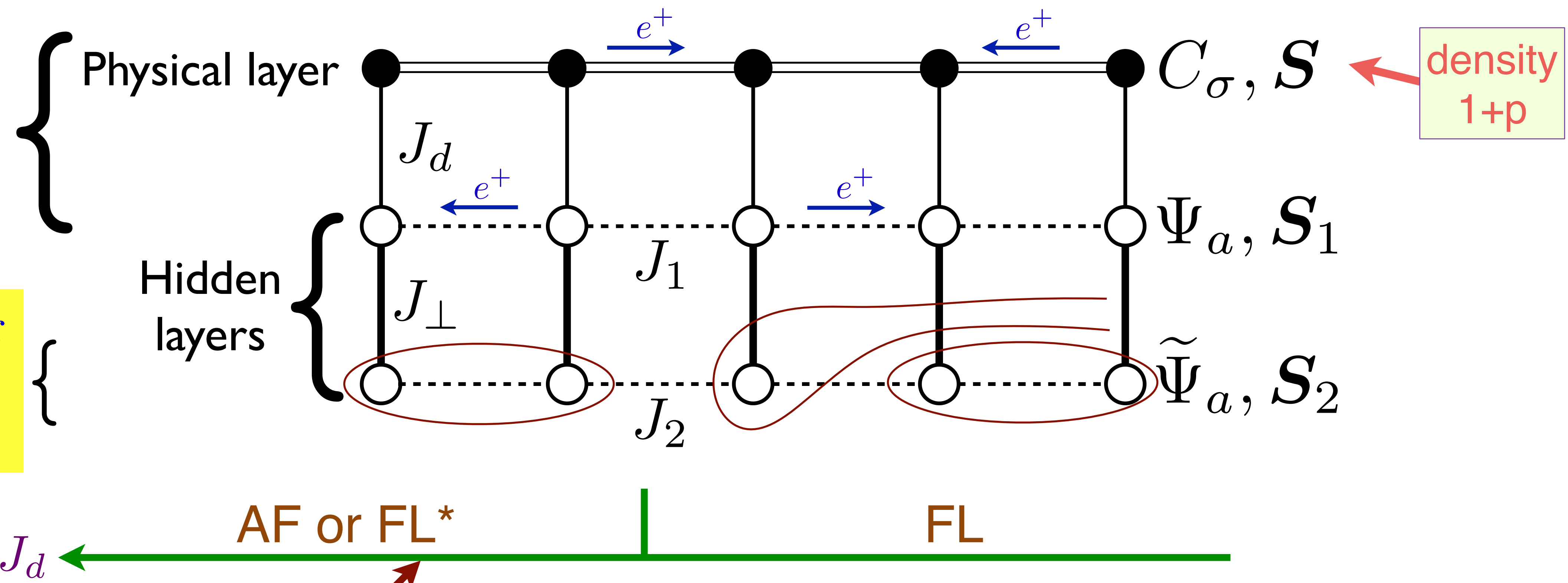
Small Fermi surface of size  $p$

$$|\Phi\rangle = \left[ \text{Projection onto rung singlets of } \Psi, \tilde{\Psi} \right] \otimes |\text{Slater determinant of } (C, \Psi)\rangle \otimes |\text{Slater determinant of } \tilde{\Psi}\rangle$$

# FL\* and FL in a **one-band** model

Metal.  
Density  
 $2 + p \cong p$

Mott insulator  
Spin liquid  
or AF order



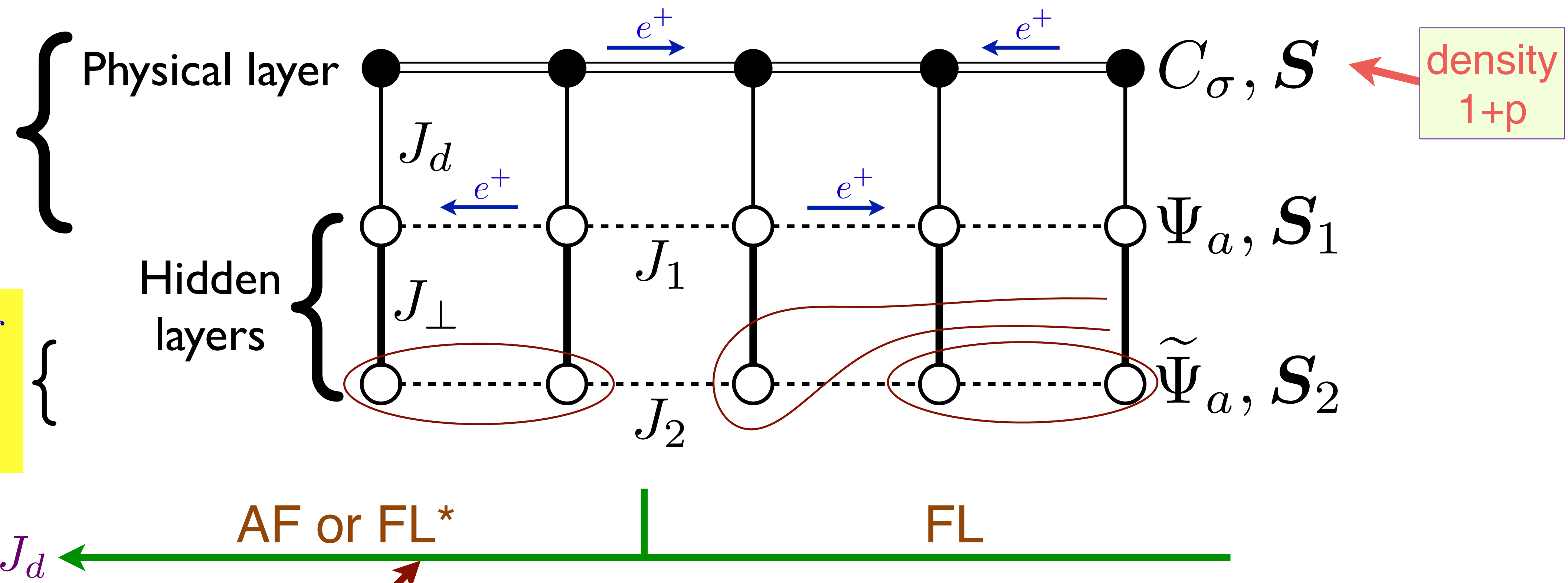
Small Fermi surface of size  $p$

$$|\Phi\rangle = \left[ \text{Projection onto rung singlets of } \Psi, \tilde{\Psi} \right] \otimes |\text{Slater determinant of } (C, \Psi)\rangle \otimes |\text{Slater determinant of } \tilde{\Psi}\rangle$$

# FL\* and FL in a **one-band** model

Metal.  
Density  
 $2 + p \cong p$

Mott insulator  
Spin liquid  
or AF order



Small Fermi surface of size  $p$

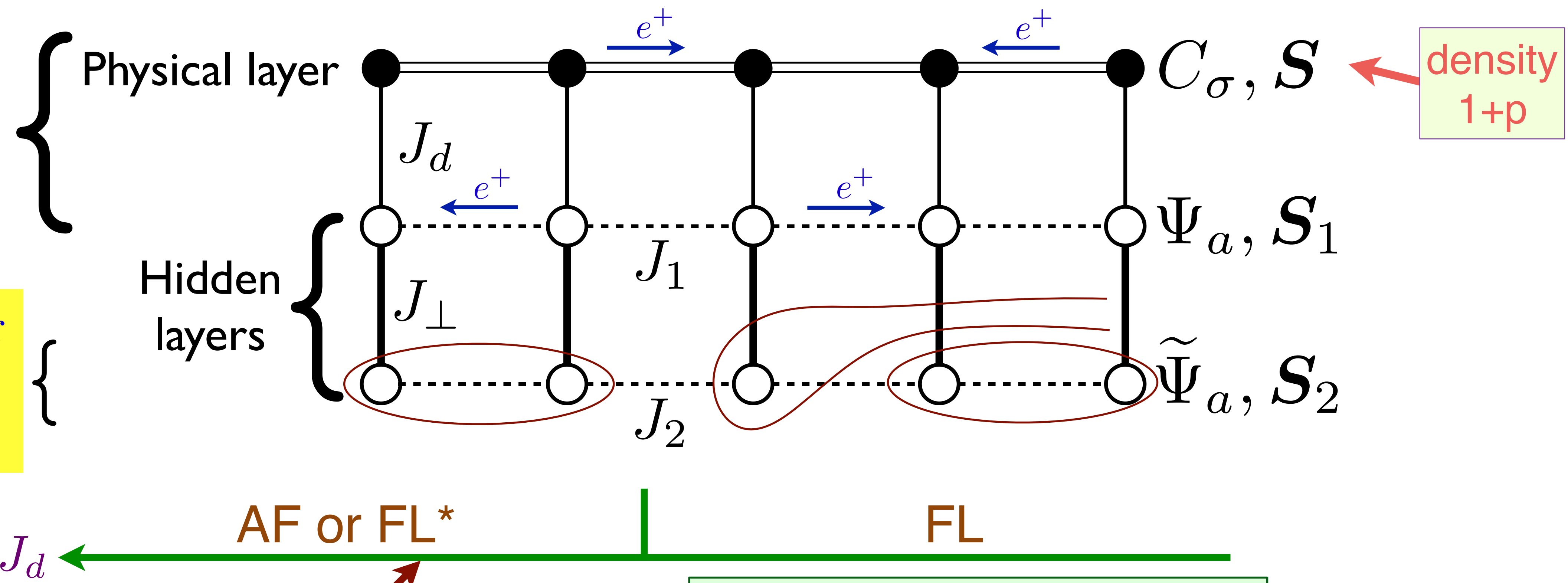
$$|\Phi\rangle = \left[ \text{Projection onto rung singlets of } \Psi, \tilde{\Psi} \right] \otimes |\text{Slater determinant of } (C, \Psi)\rangle \otimes |\text{Slater determinant of } \tilde{\Psi}\rangle$$

Luttinger Theorem violated;  
OK, because of topological order of  $\tilde{\Psi}$

# FL\* and FL in a **one-band** model

Metal.  
Density  
 $2 + p \cong p$

Mott insulator  
Spin liquid  
or AF order

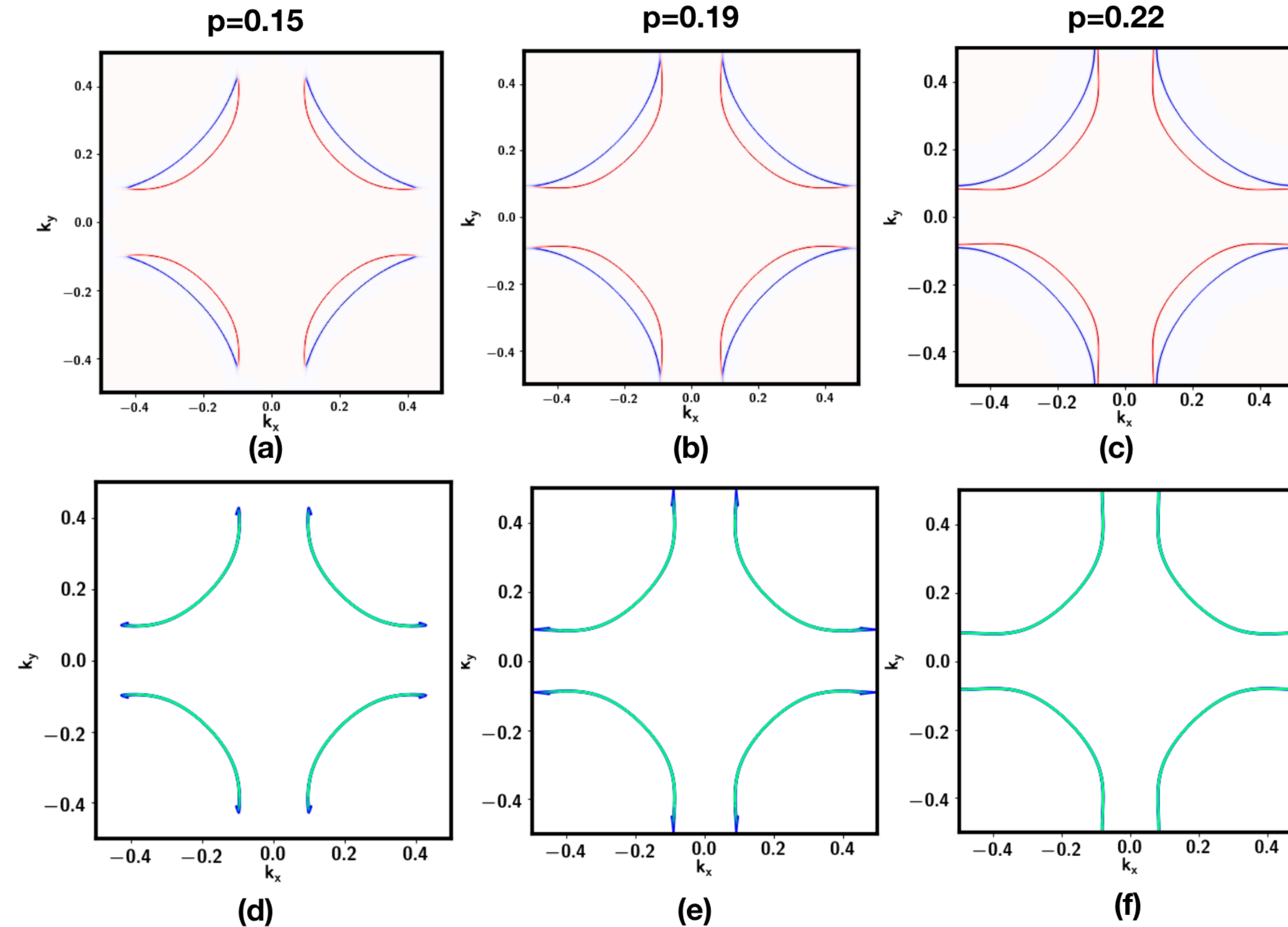


Small Fermi surface of size  $p$

$$|\Phi\rangle = \left[ \text{Projection onto rung singlets of } \Psi, \tilde{\Psi} \right] \otimes |\text{Slater determinant of } (C, \Psi)\rangle \otimes |\text{Slater determinant of } \tilde{\Psi}\rangle$$

Similar to a selective Mott transition in hidden layer 1:  $\Psi$  fermions are insulating in FL phase, and metallic in FL\* phase.

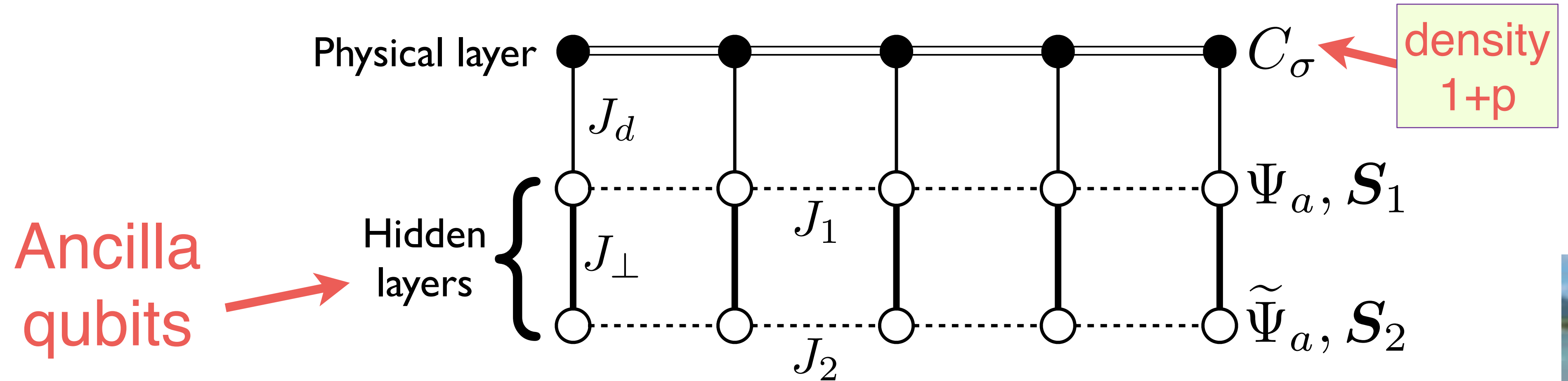
# FL\* in a **one-band** model



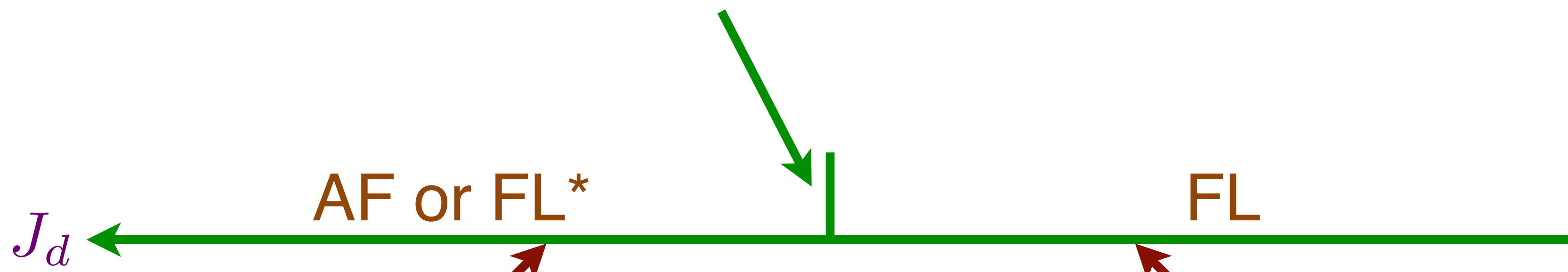
“Fermi arc”  
spectral functions  
in the FL\* phase

Zero frequency spectral density of electrons (red) and ghosts (blue)

# FL\* and FL in a **one-band** model



Yahui Zhang



Small Fermi surface of size  $p$

$$|\Phi\rangle = \left[ \text{Projection onto rung singlets of } \Psi, \tilde{\Psi} \right]$$

$$\propto |\text{Slater determinant of } (C, \Psi)\rangle$$

$$\otimes |\text{Slater determinant of } \tilde{\Psi}\rangle$$

Large Fermi surface of size  $1 + p$

$$|\Phi\rangle = |\text{Rung singlets of } \Psi, \tilde{\Psi}\rangle$$

$$\otimes |\text{Slater determinant of } C\rangle$$

# FL\* and FL in a **one-band** model

Write fermion operators as  $2 \times 2$  matrices

$$\Psi = \begin{pmatrix} \Psi_{\uparrow} & -\Psi_{\downarrow}^{\dagger} \\ \Psi_{\downarrow} & \Psi_{\uparrow}^{\dagger} \end{pmatrix}, \quad \tilde{\Psi} = \begin{pmatrix} \tilde{\Psi}_{\uparrow} & -\tilde{\Psi}_{\downarrow}^{\dagger} \\ \tilde{\Psi}_{\downarrow} & \tilde{\Psi}_{\uparrow}^{\dagger} \end{pmatrix}$$

Single occupancy constraints of  $\Psi$ ,  $\tilde{\Psi}$  leads to  $SU(2)_1 \times SU(2)_2$  gauge symmetry:

$$\begin{aligned} SU(2)_1 : & \quad \Psi \rightarrow \Psi U_1, & \tilde{\Psi} & \rightarrow \tilde{\Psi} \\ SU(2)_2 : & \quad \Psi \rightarrow \Psi, & \tilde{\Psi} & \rightarrow \tilde{\Psi} U_2 \end{aligned}$$

P.A. Lee, N. Nagaosa, and X.-G. Wen, RMP **78**, 17 (2006)

Local singlet formation ('antiferromagnetism')  $\mathcal{S}_1 + \mathcal{S}_2 \approx 0$  leads to  $SU(2)_S$  gauge symmetry:

$$SU(2)_S : \quad \Psi \rightarrow U_S \Psi, \quad \tilde{\Psi} \rightarrow U_S \tilde{\Psi}$$

S. Sachdev, M.A. Metlitski, Yang Qi, and Cenke Xu, PRB **80**, 155129 (2009)

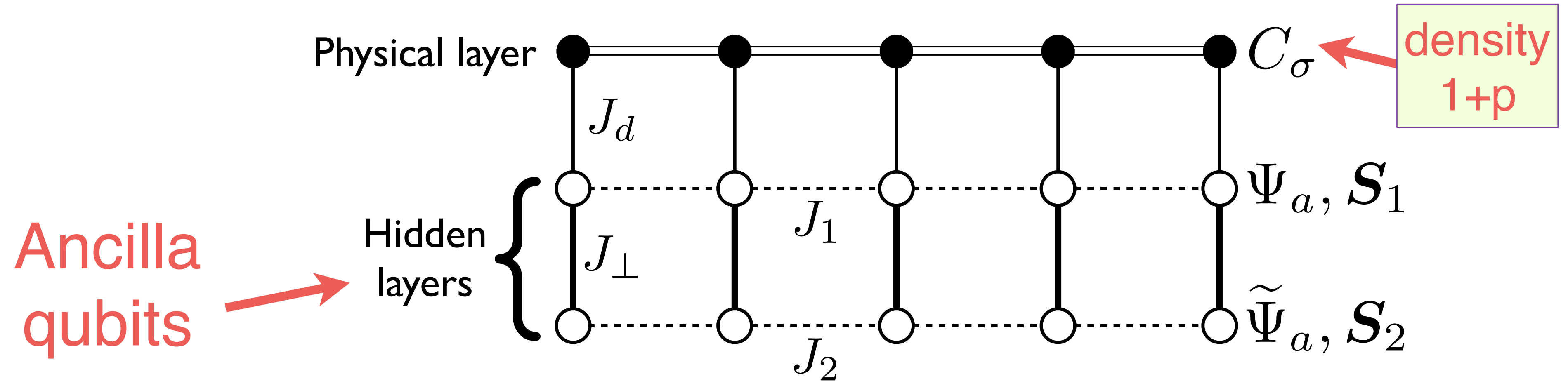
S. Sachdev, H. D. Scammell, M. S. Scheurer, and G. Tarnopolsky, PRB **99**, 054516 (2019)



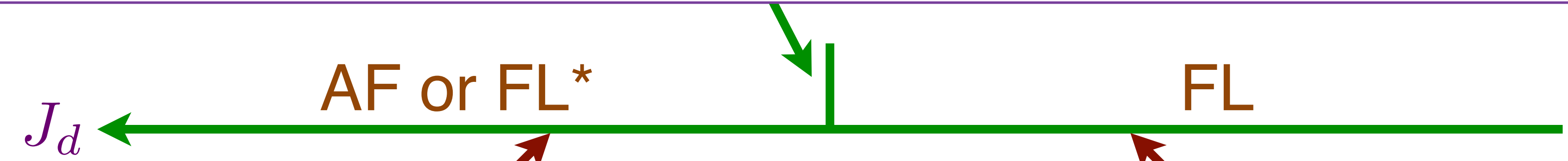
Yahui Zhang

Ya-Hui Zhang, S. Sachdev, PRR **2**, 023172; PRB **102**, 155124 (2020)

# Metal-metal transitions in a **one-band** model



$(U(1)_S \times U(1)_1)/Z_2$  or  $(SU(2)_S \times U(1)_1)/Z_2$  gauge theory of a  $\Psi$  ghost Fermi surface and a ‘hybridization-Higgs’ boson  $\sim C_\sigma^\dagger \Psi_a$  which condenses on the ‘Small Fermi surface’ side.



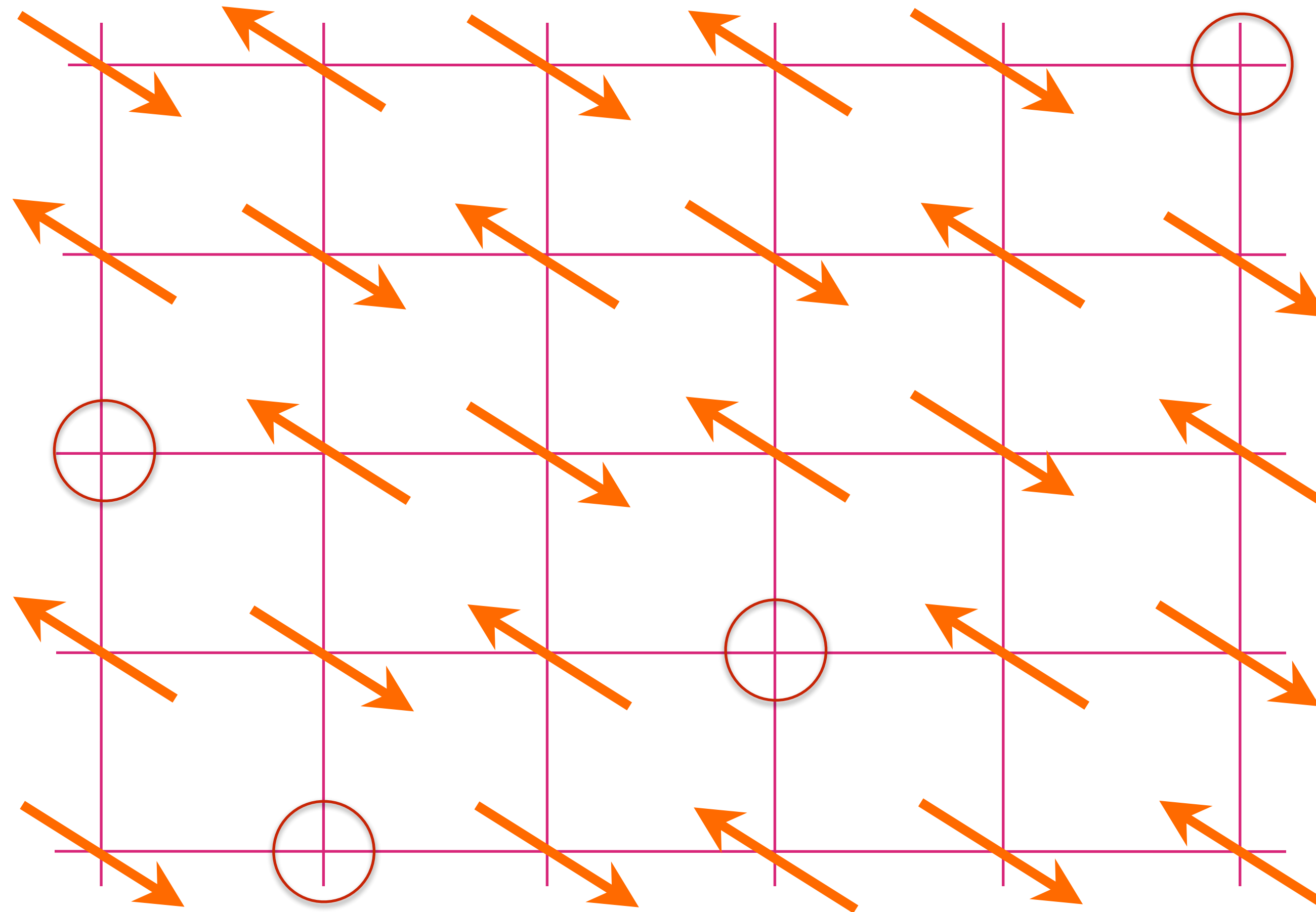
Small Fermi surface of size  $p$

$|\Phi\rangle = \left[ \text{Projection onto rung singlets of } \Psi, \tilde{\Psi} \right]$   
 $\otimes |\text{Slater determinant of } (C, \Psi)\rangle$   
 $\otimes |\text{Slater determinant of } \tilde{\Psi}\rangle$

Large Fermi surface of size  $1+p$

$|\Phi\rangle = \left| \text{Rung singlets of } \Psi, \tilde{\Psi} \right\rangle$   
 $\otimes |\text{Slater determinant of } C\rangle$

# Earlier approach to FL\* in a **one-band** model

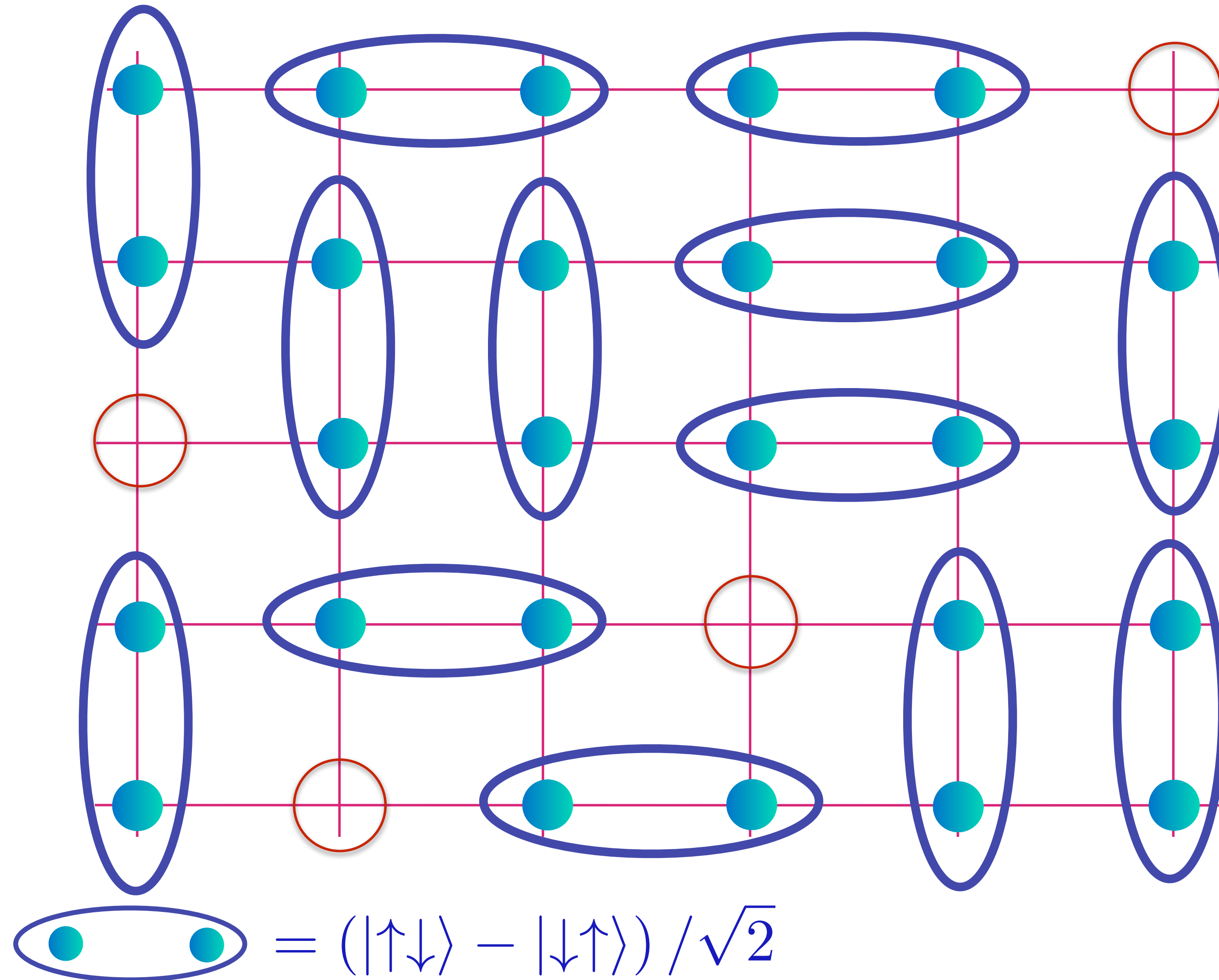


Anti-ferromagnet  
with  $p$  holes  
per square

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

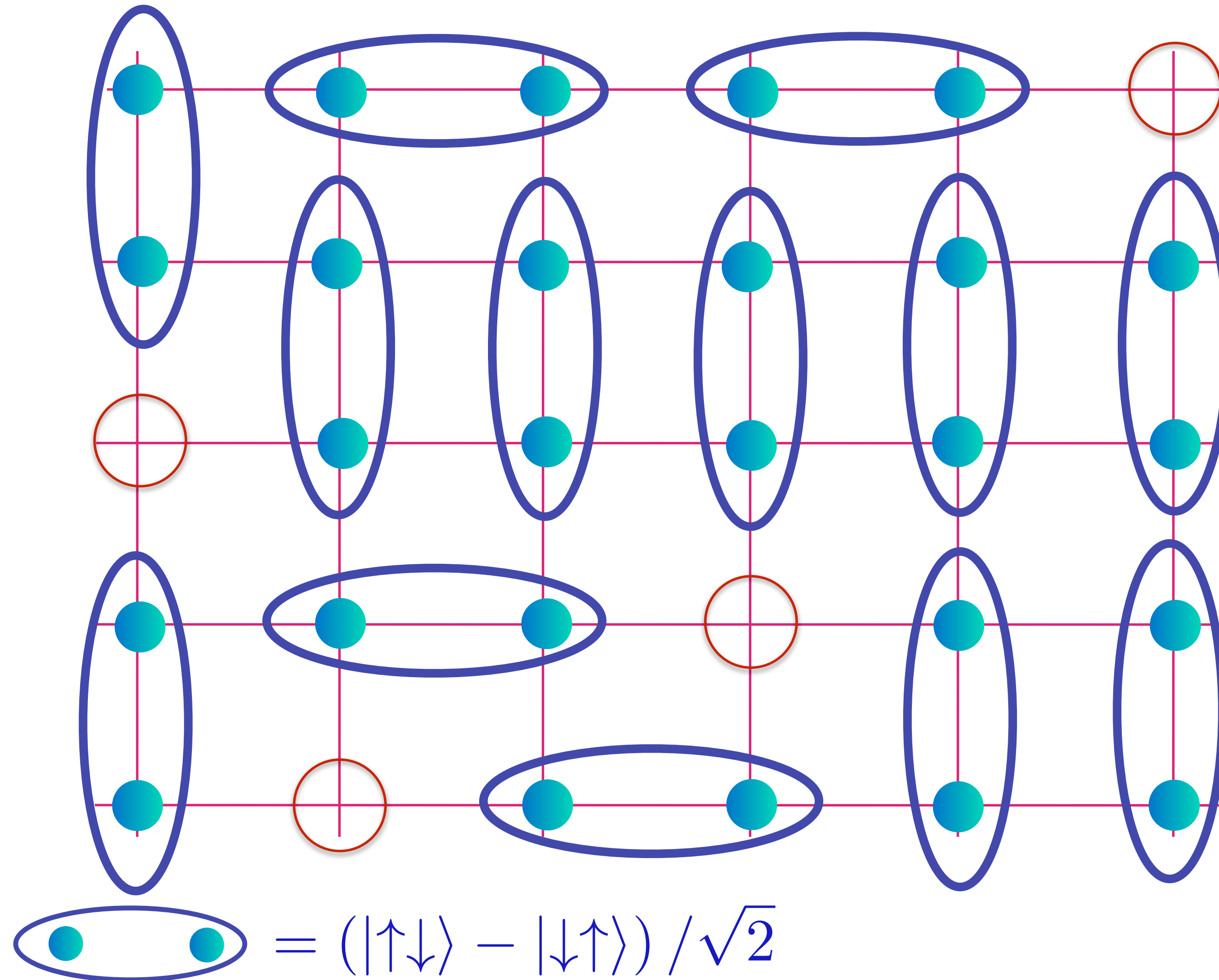


Spin liquid  
with density  
 $\rho$  of spinless,  
charge  $+e$   
“holons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

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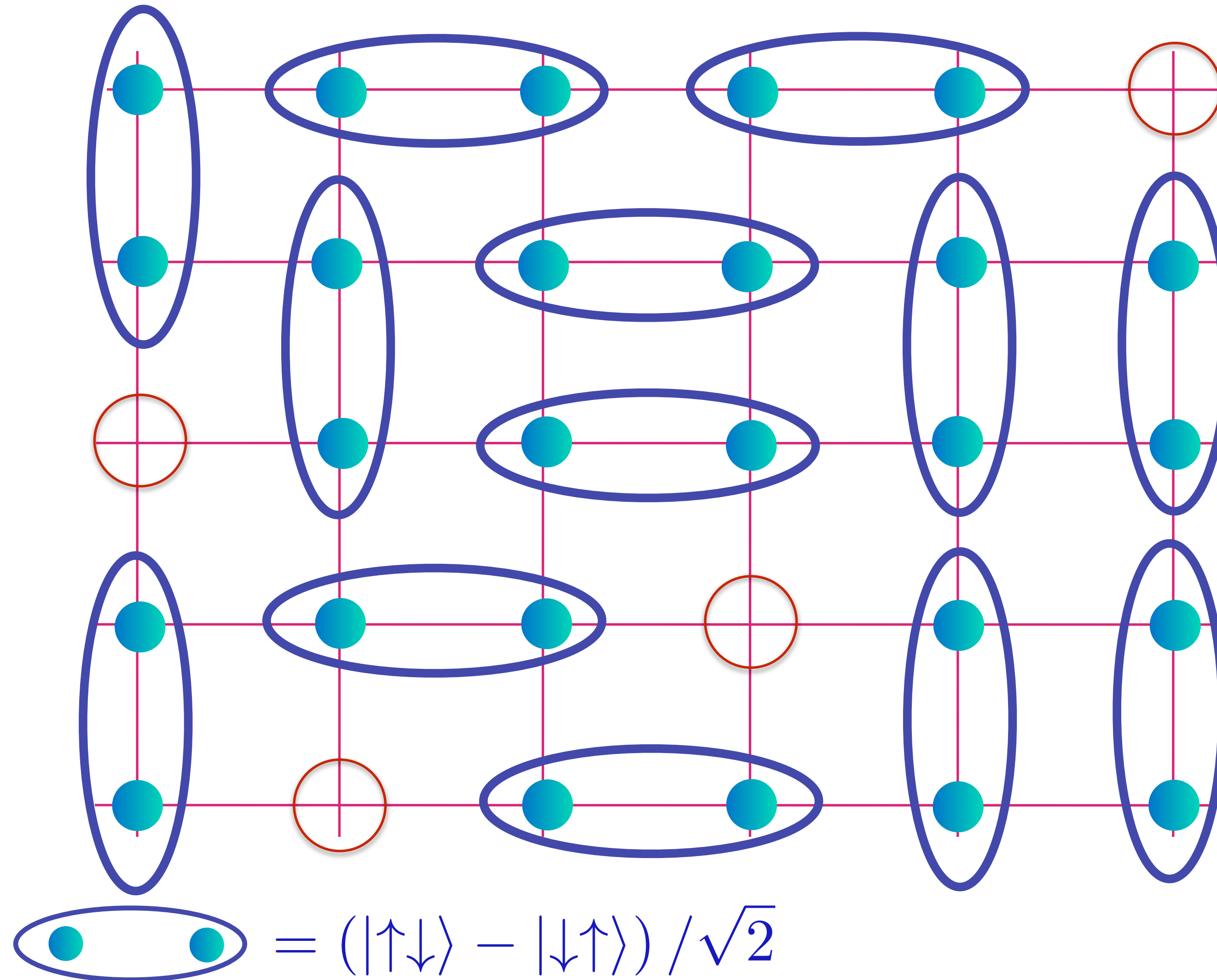


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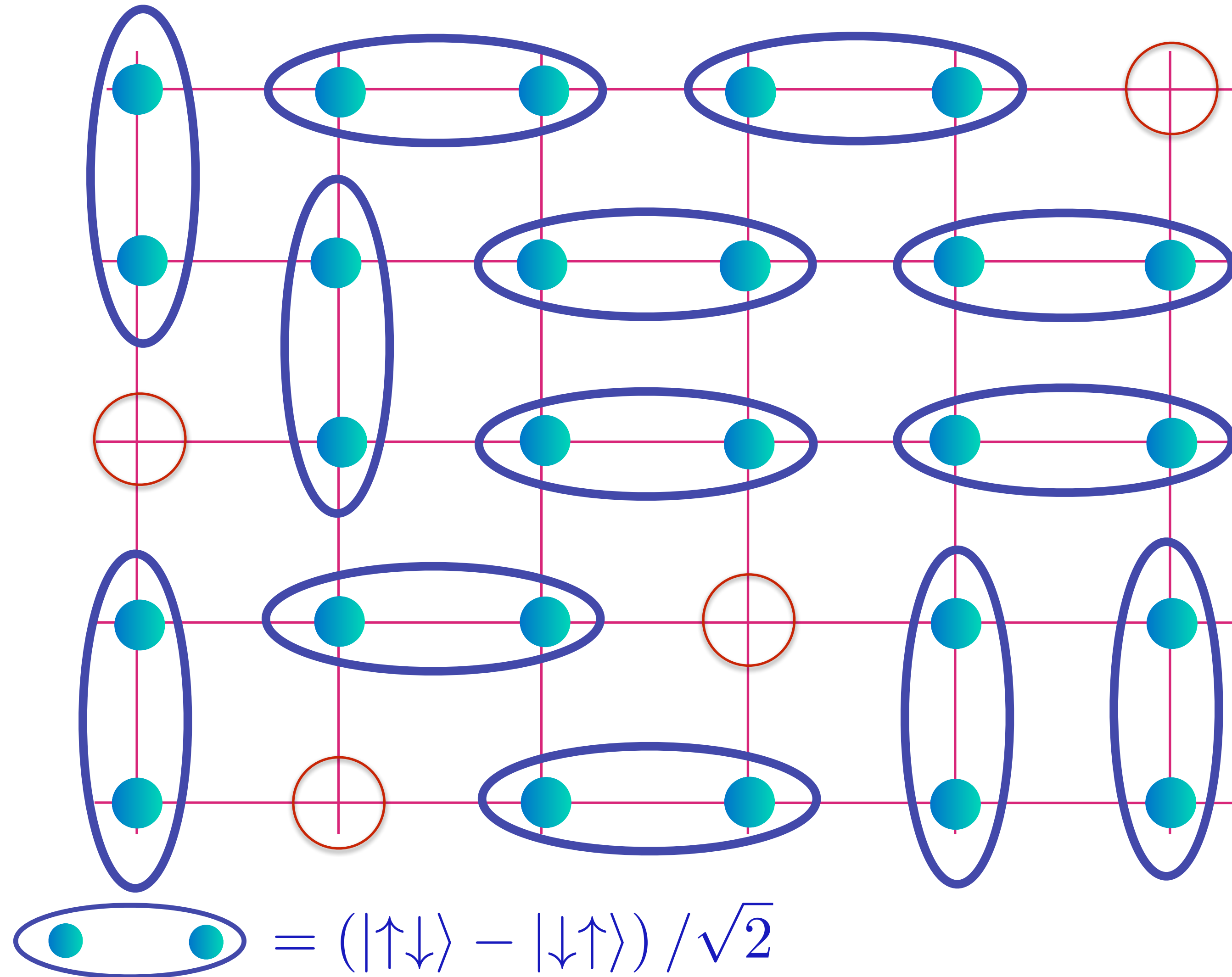


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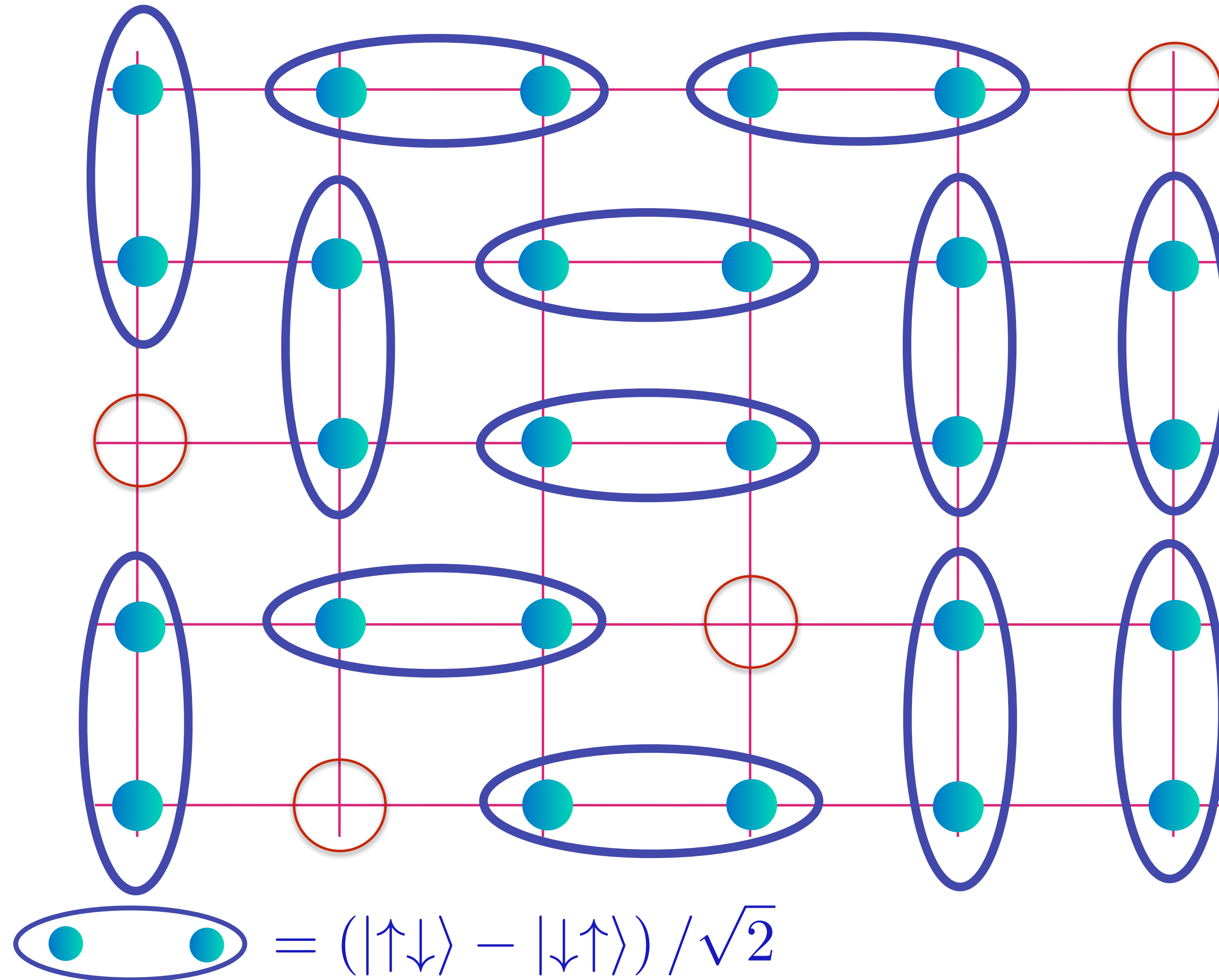


Spin liquid  
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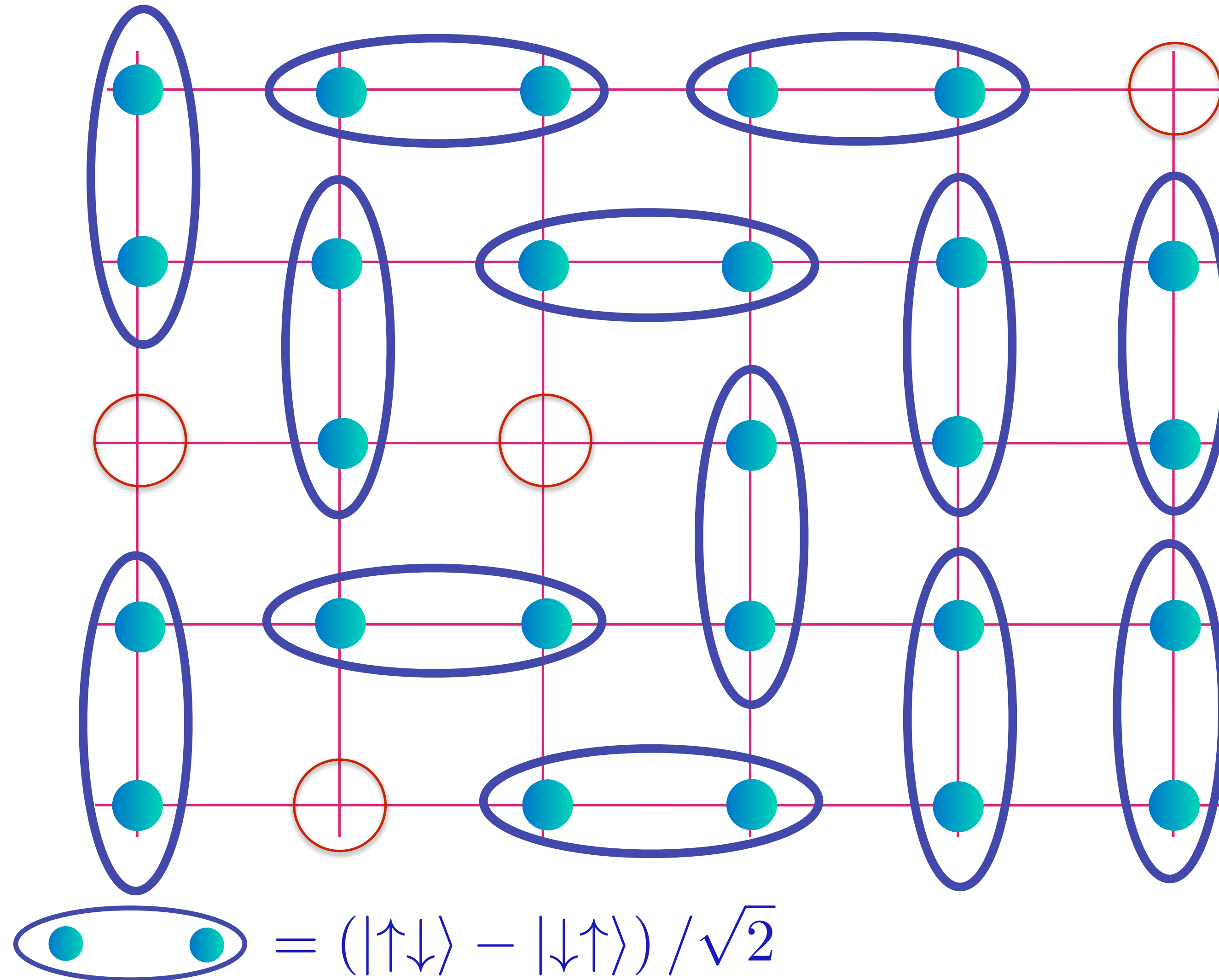


Spin liquid  
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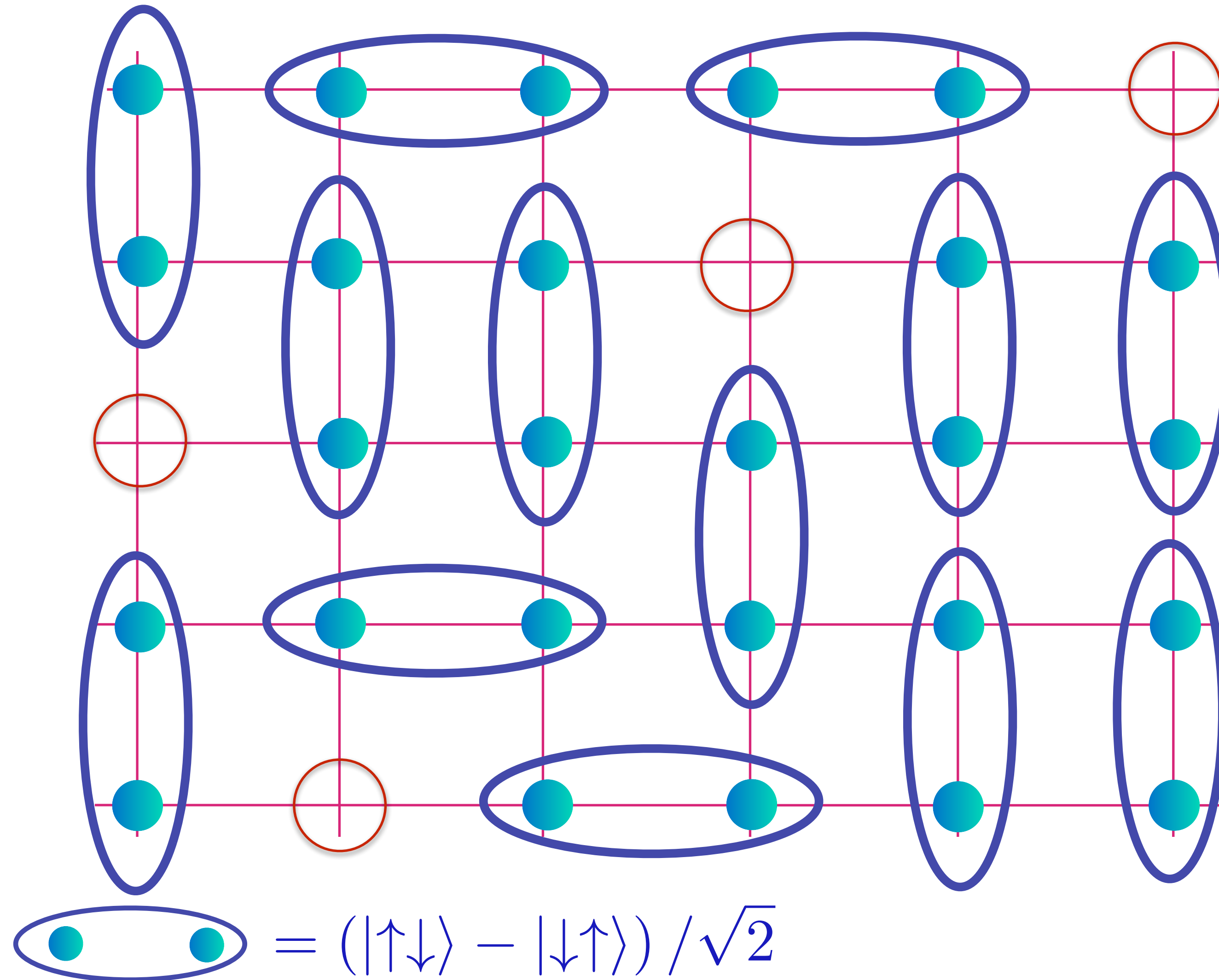


Spin liquid  
with density  
 $\rho$  of spinless,  
charge  $+e$   
“holons”.

# Holon metal

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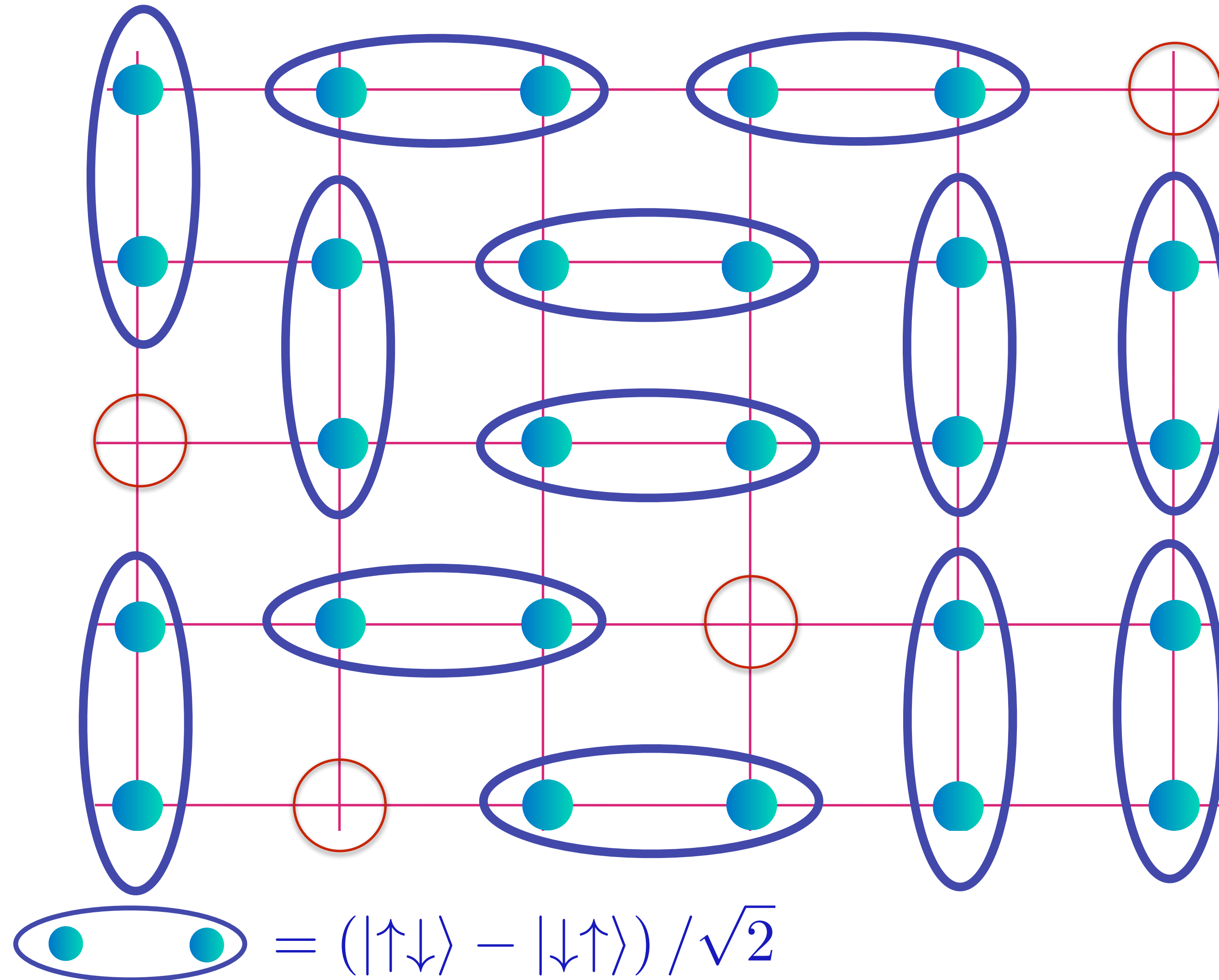


Spin liquid  
with density  
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“holons”.

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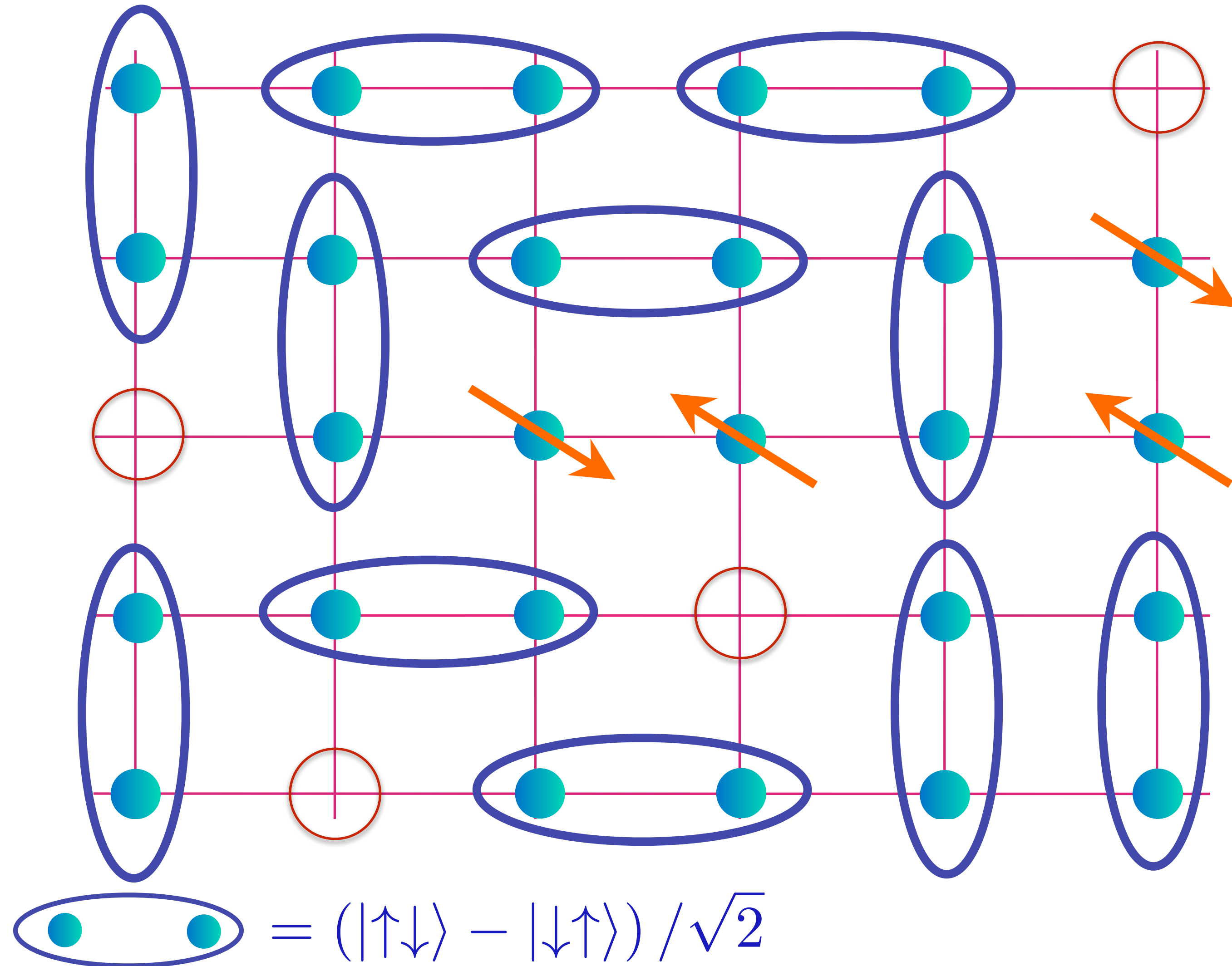


Spin liquid  
with density  
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charge  $+e$   
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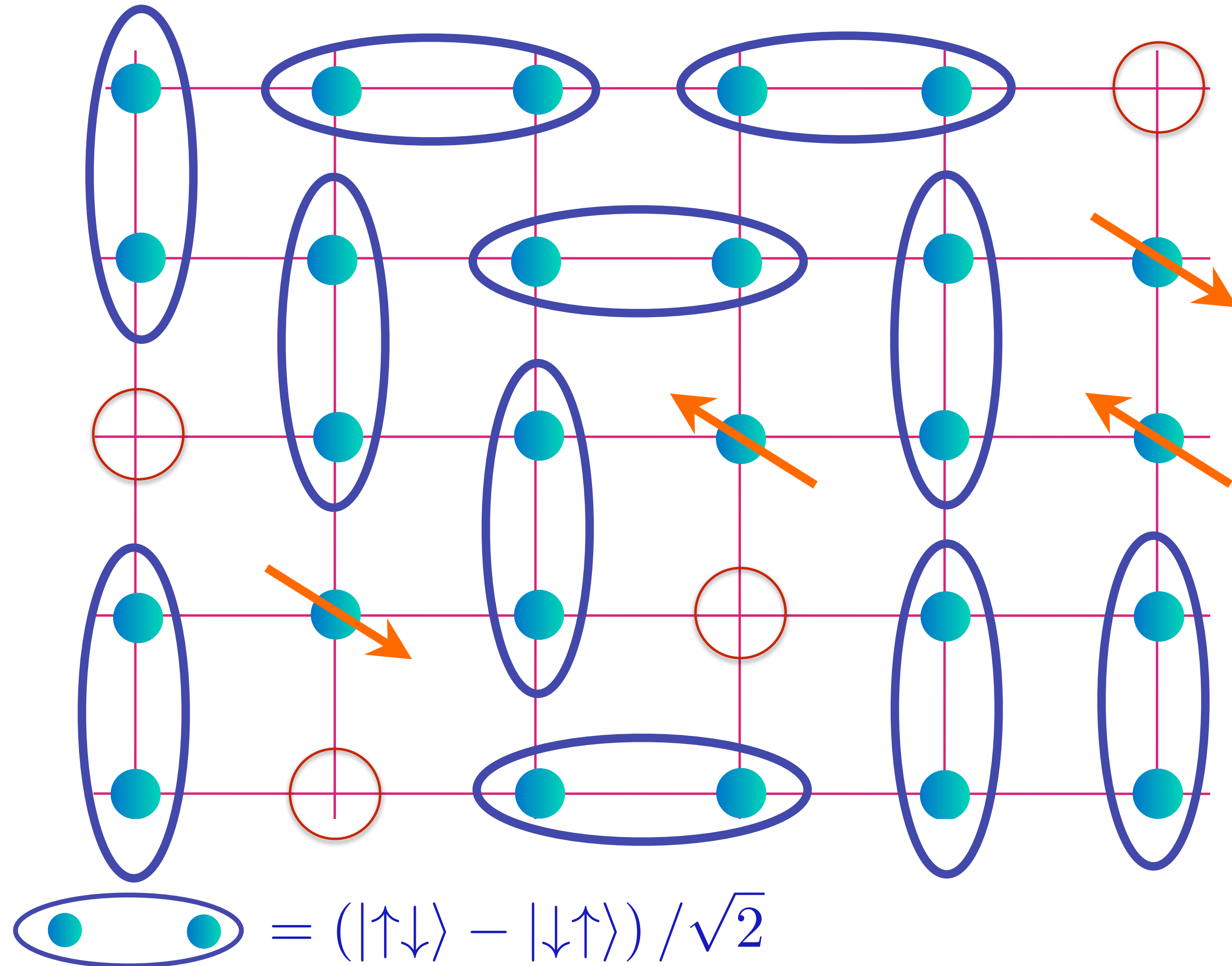


Spin liquid  
with density  
 $\rho$  of spinless,  
charge  $+e$   
“holons” and  
charge 0, spin-1/2  
“spinons”.

# Holon metal

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D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

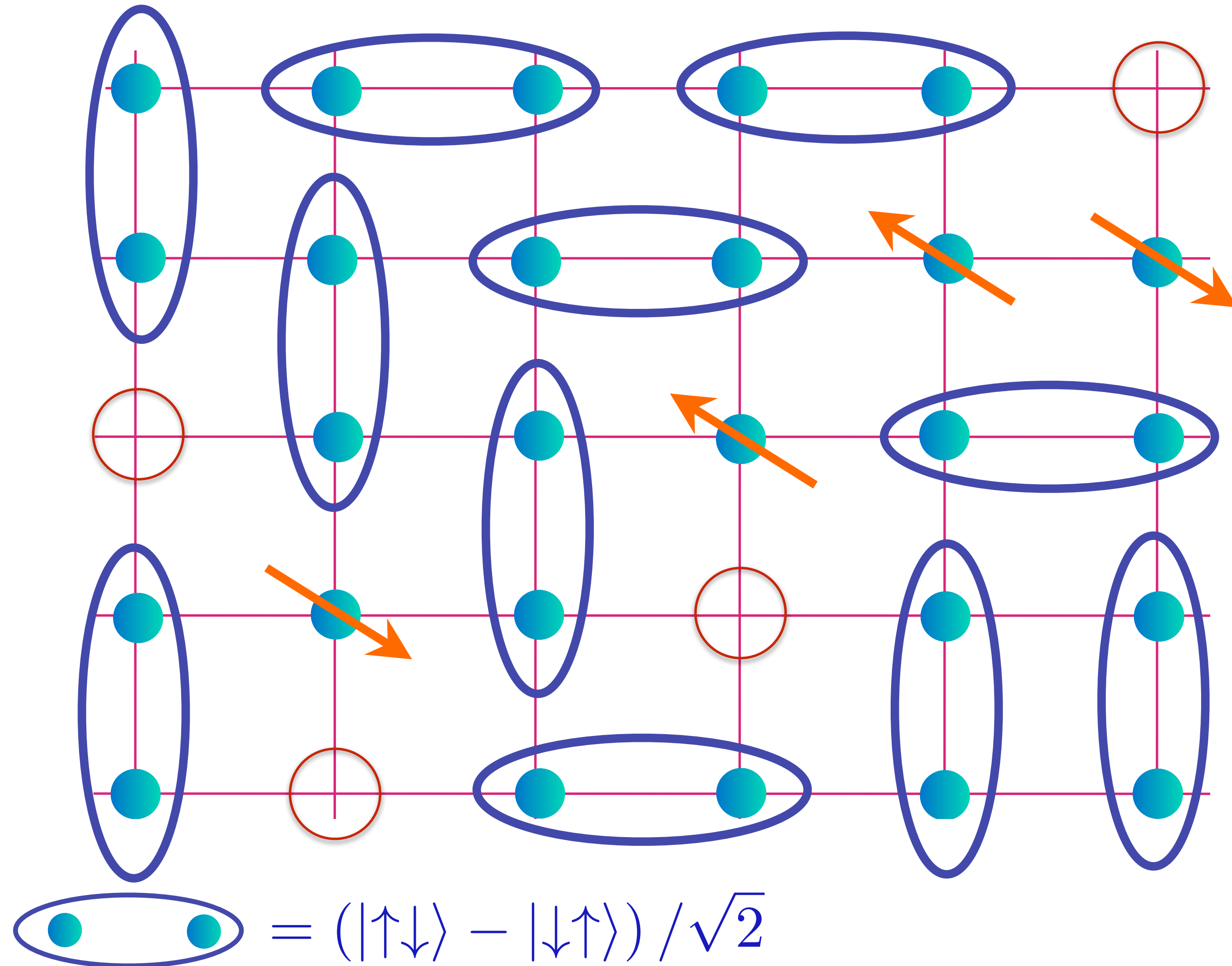


Spin liquid  
with density  $\rho$  of spinless,  
charge  $+e$   
“holons” and  
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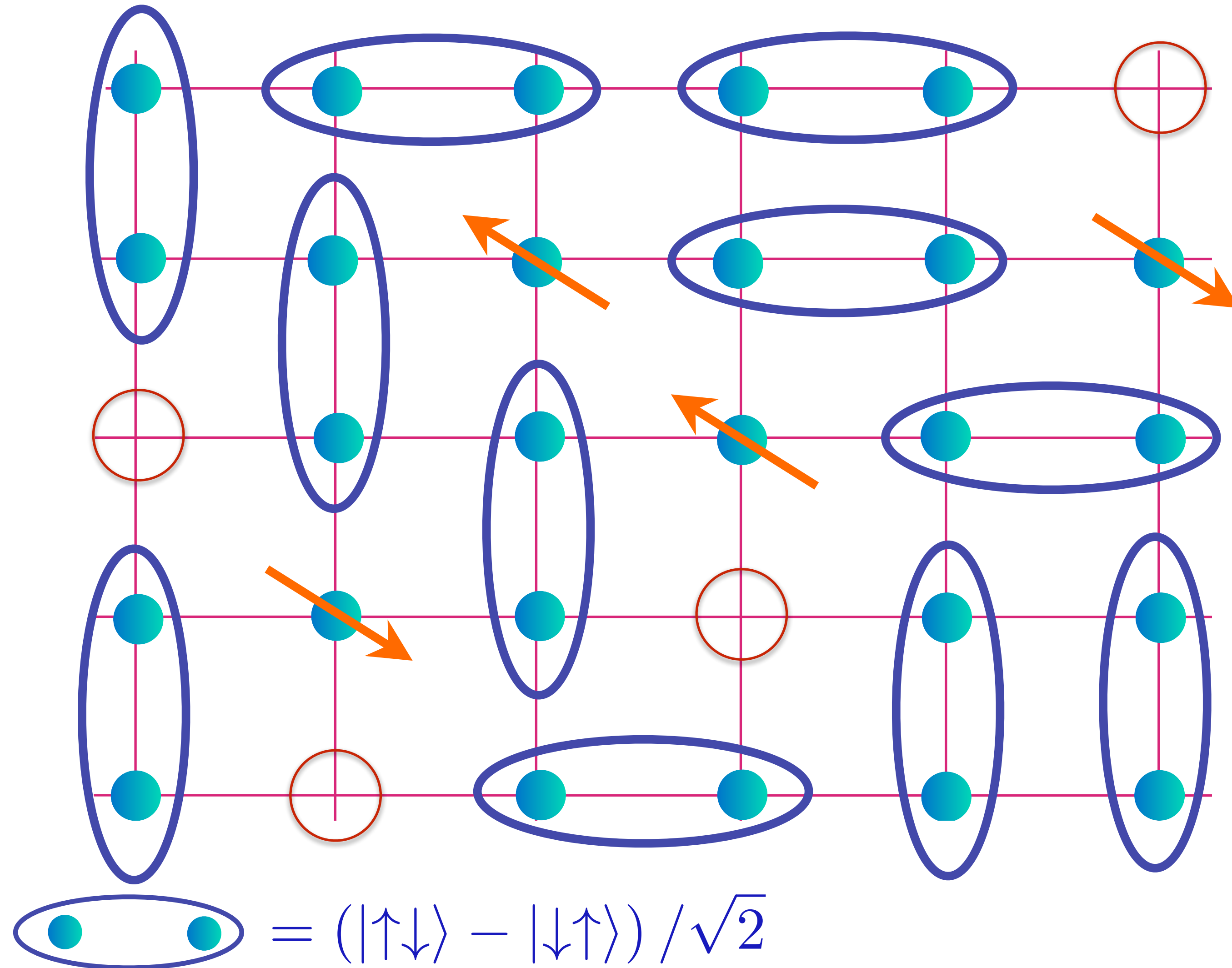


Spin liquid  
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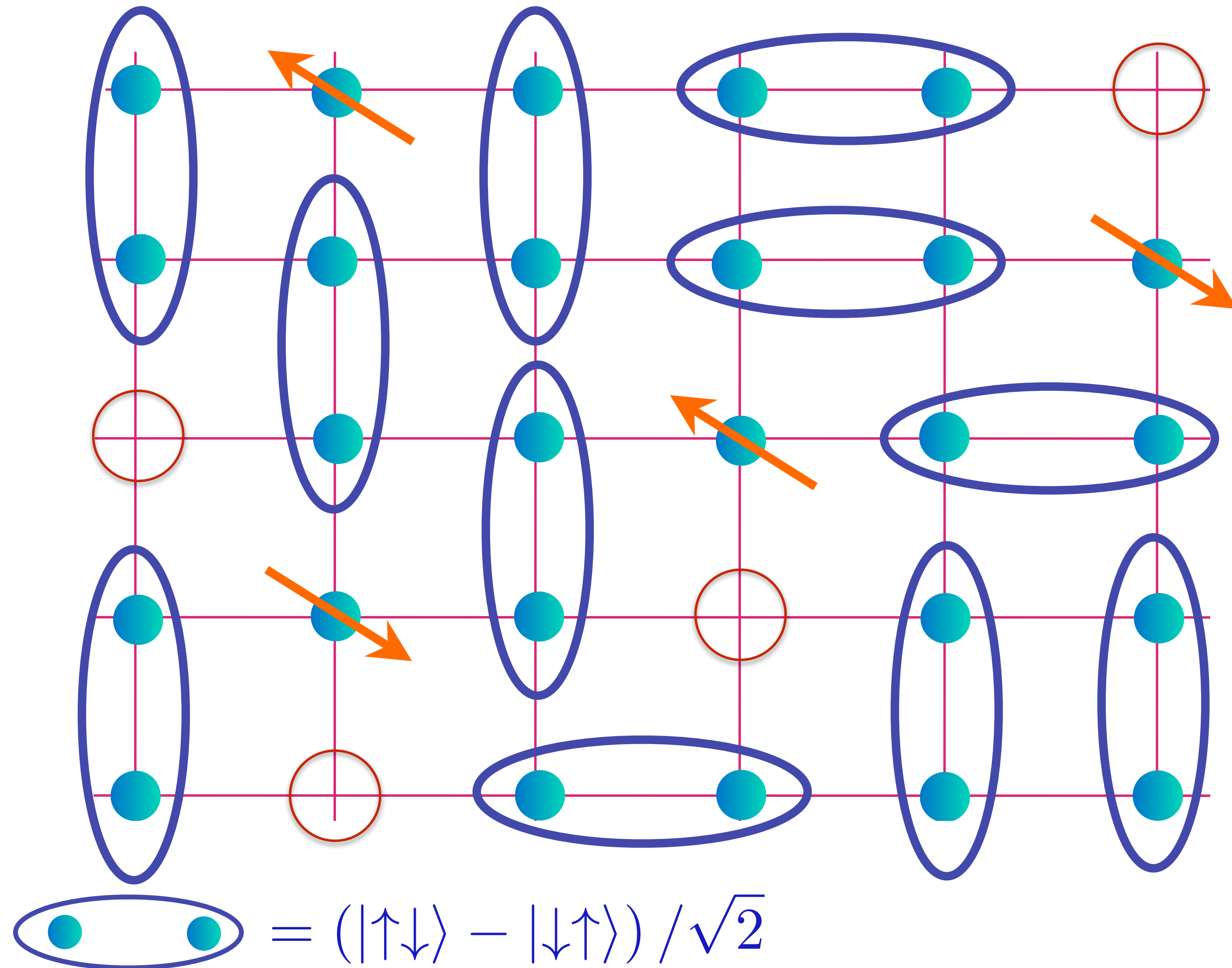


Spin liquid  
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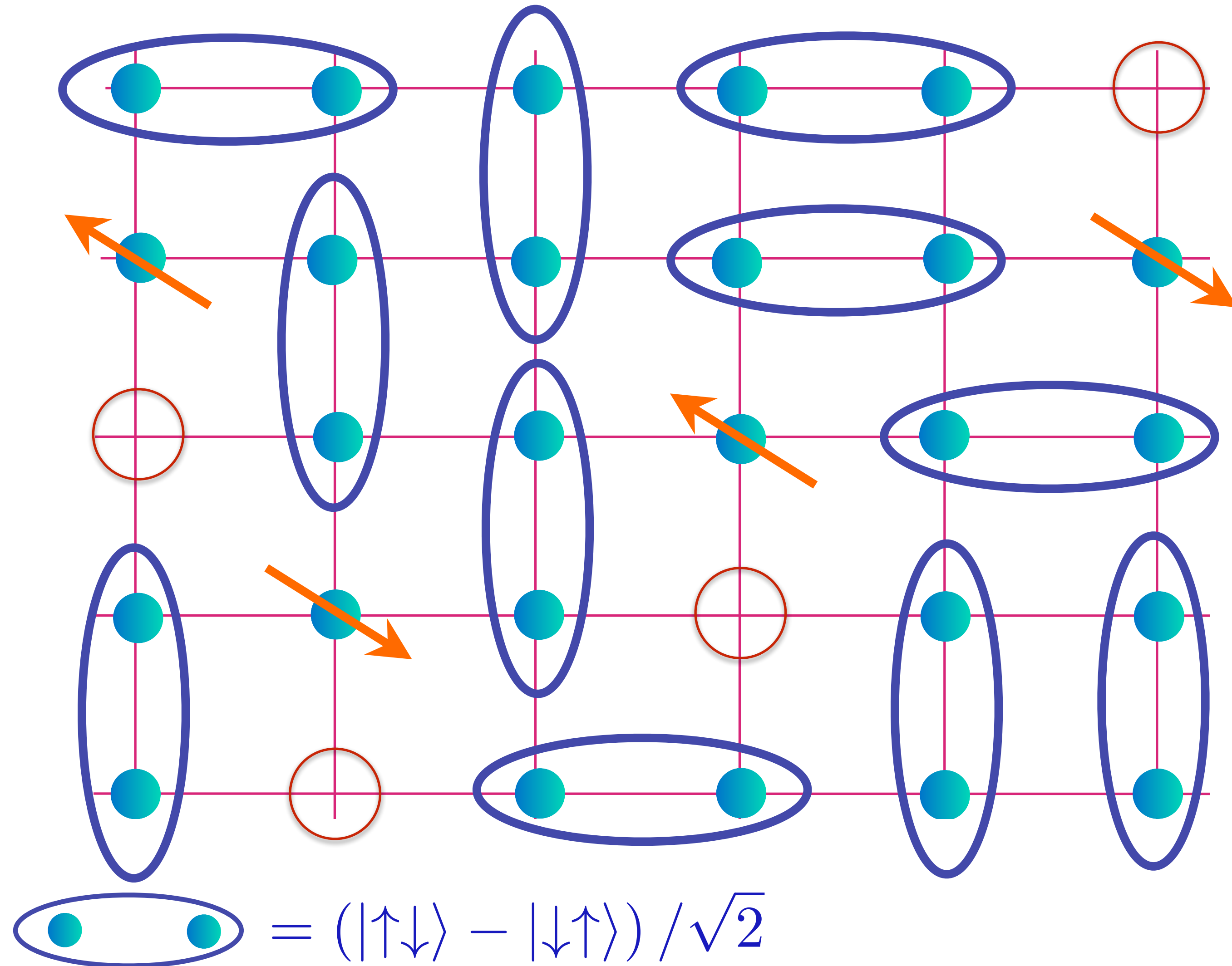


Spin liquid  
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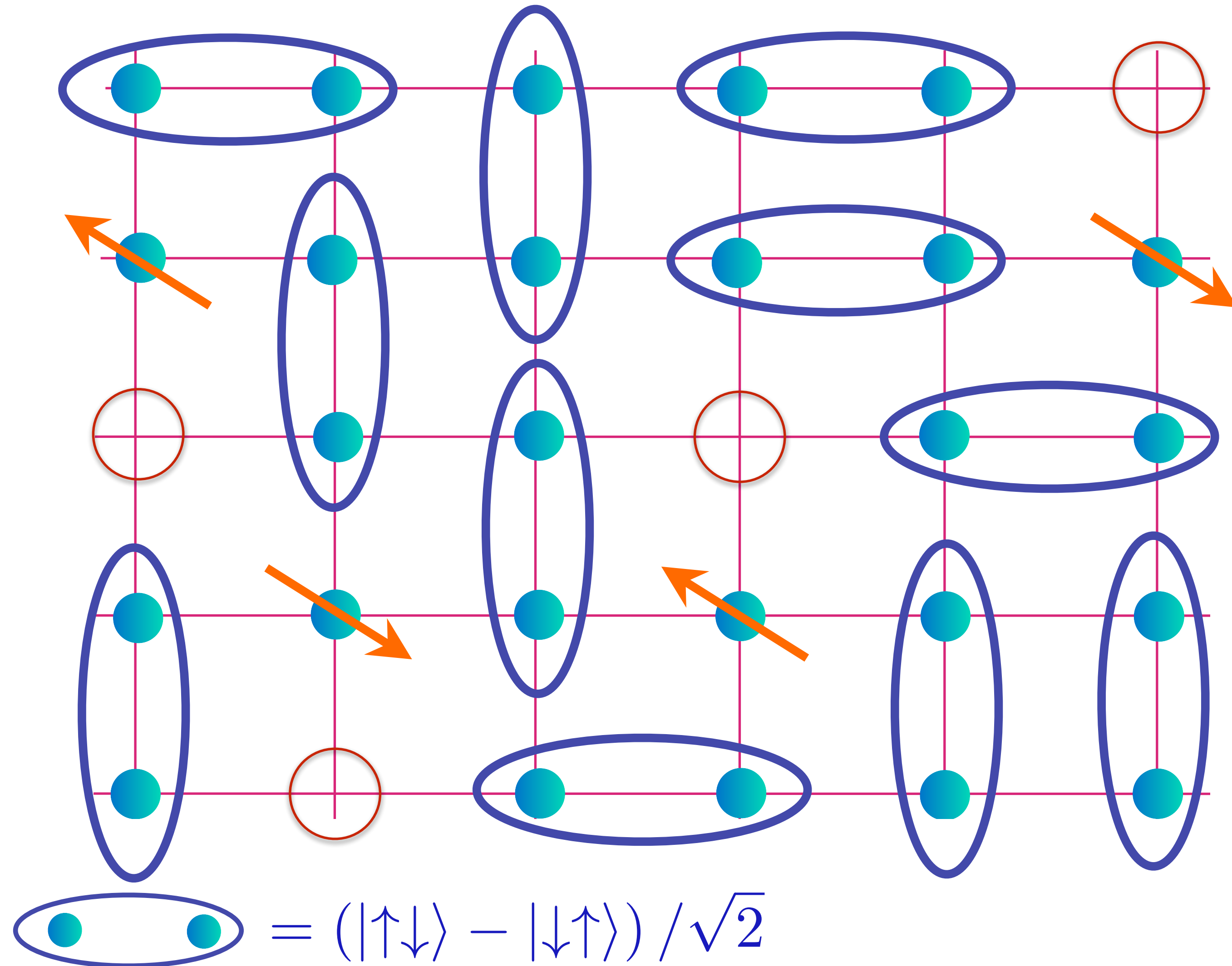


Spin liquid  
with density  
 $\rho$  of spinless,  
charge  $+e$   
“holons” and  
charge 0, spin-1/2  
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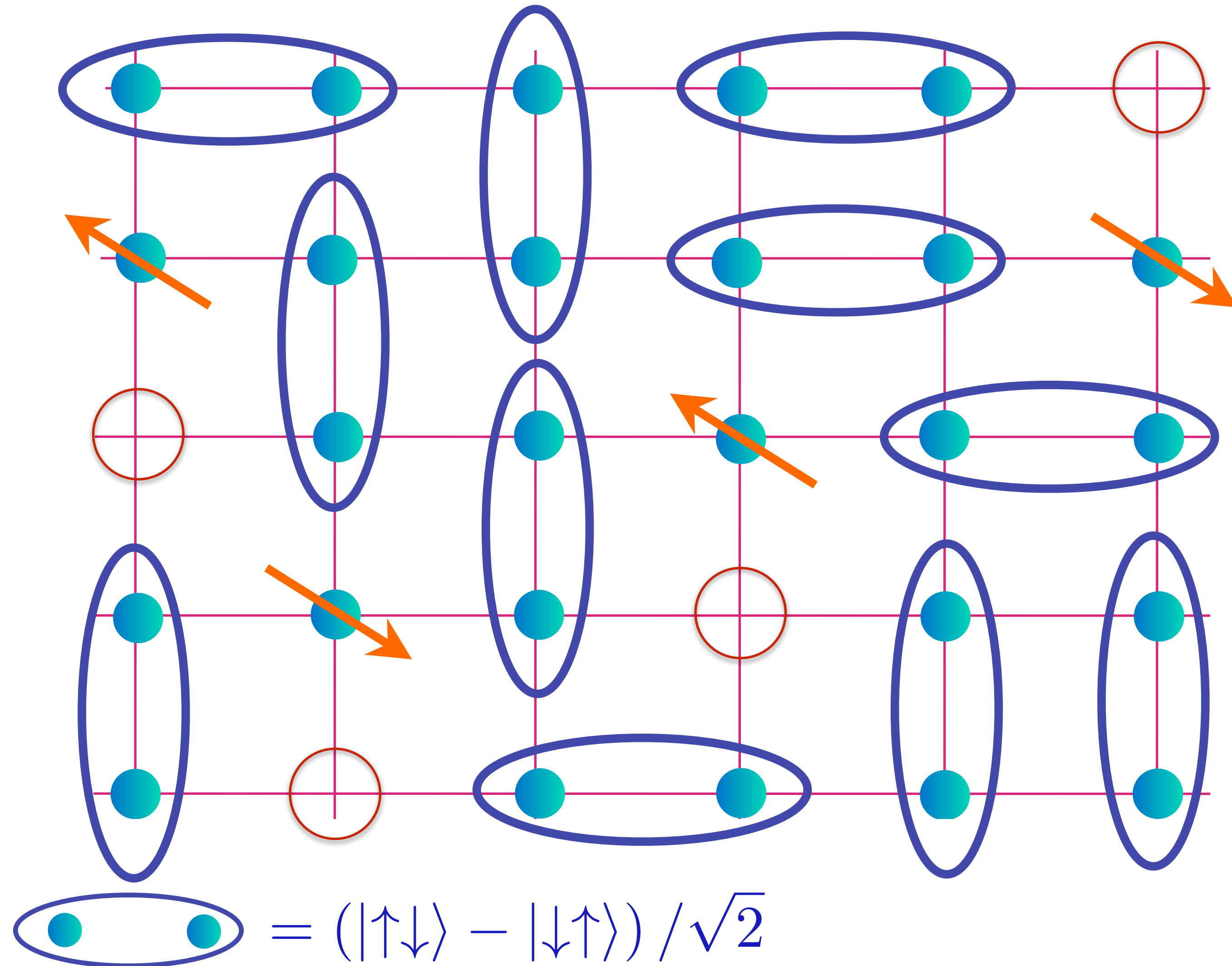


Spin liquid  
with density  $p$  of spinless,  
charge  $+e$   
“holons” and  
charge 0, spin-1/2  
“spinons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

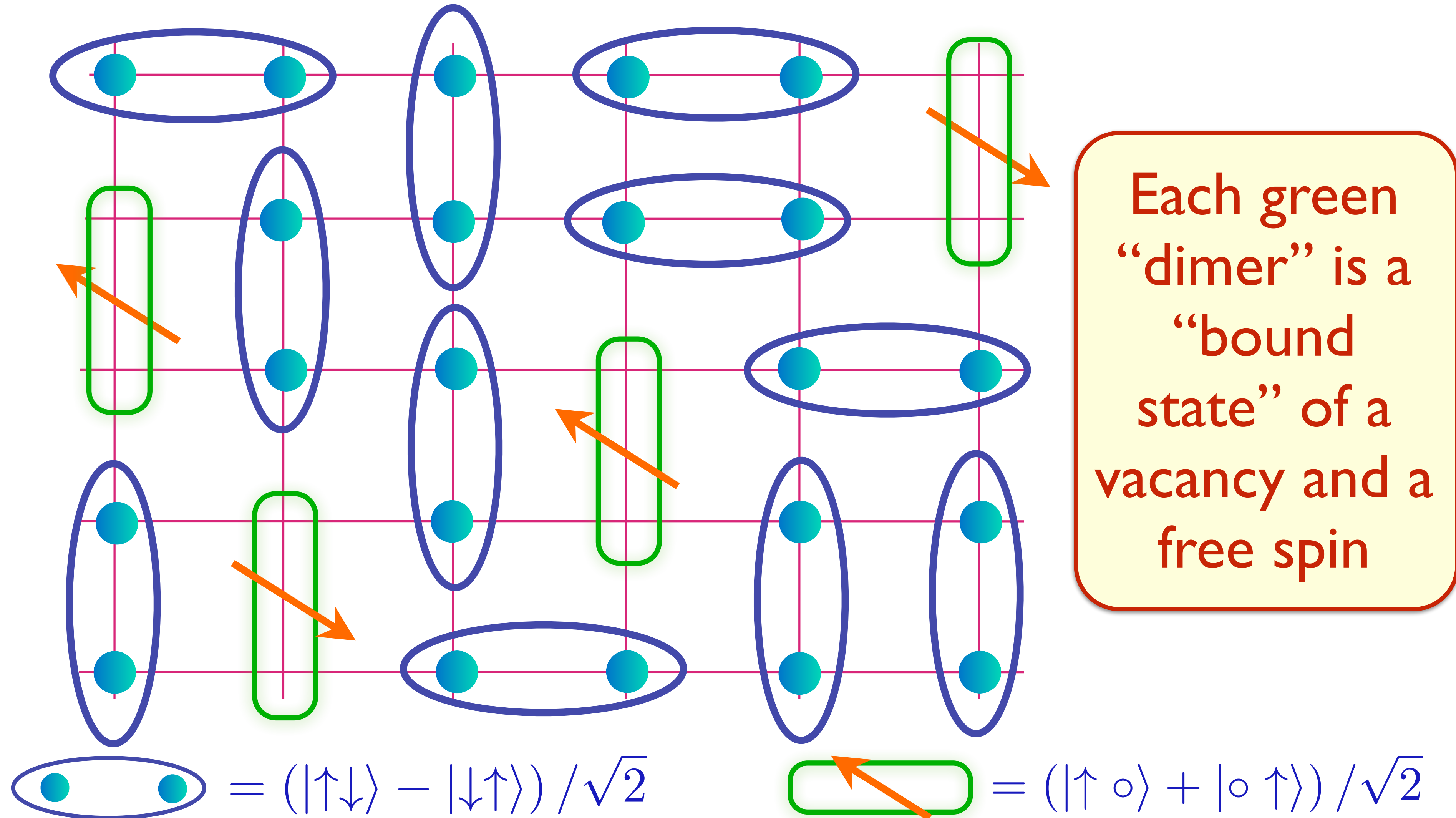


Spin liquid  
with density  
 $\rho$  of spinless,  
charge  $+e$   
“holons” and  
charge 0, spin-1/2  
“spinons”.

# Earlier approach to FL\* in a *one-band* model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)

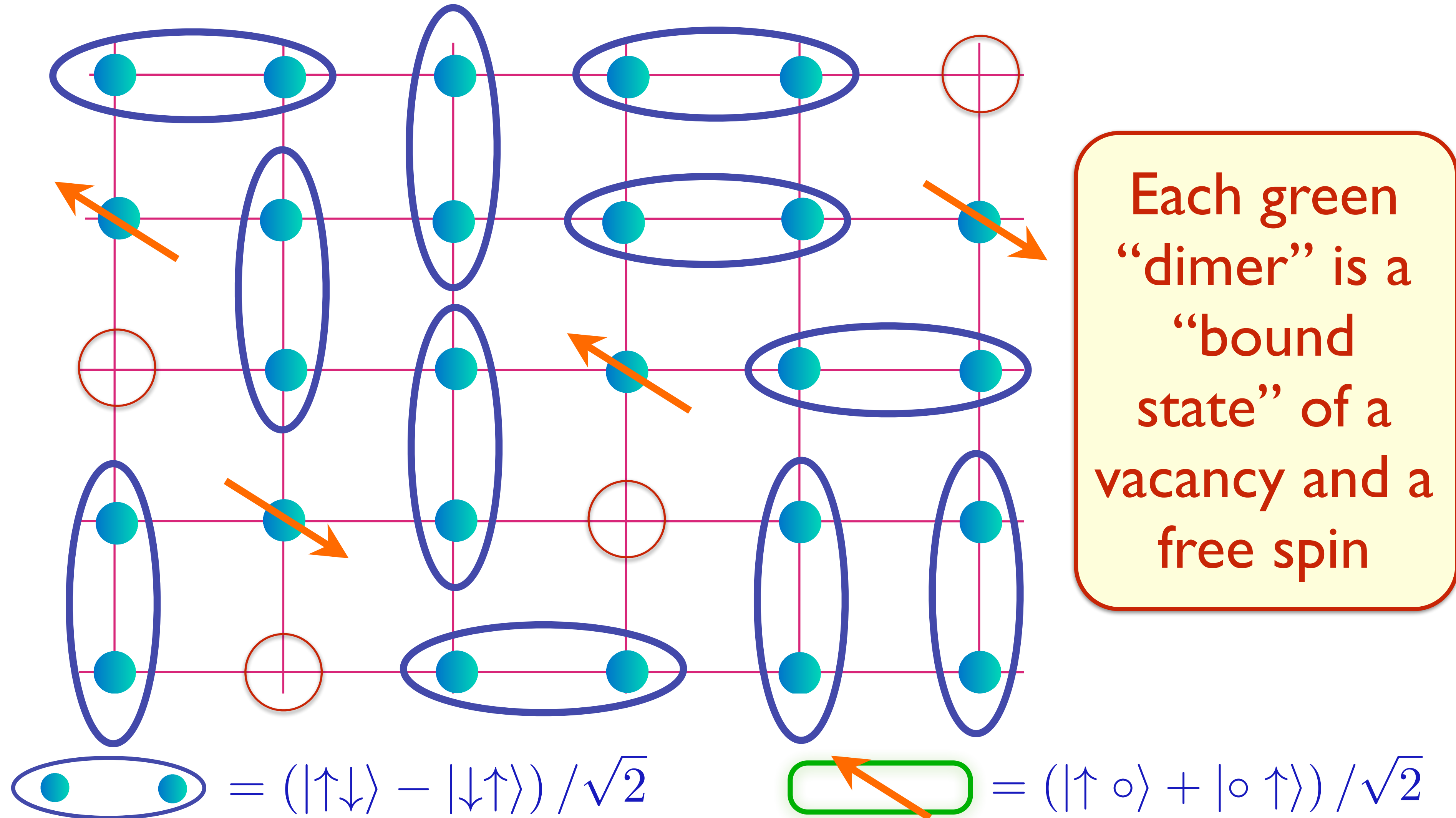


E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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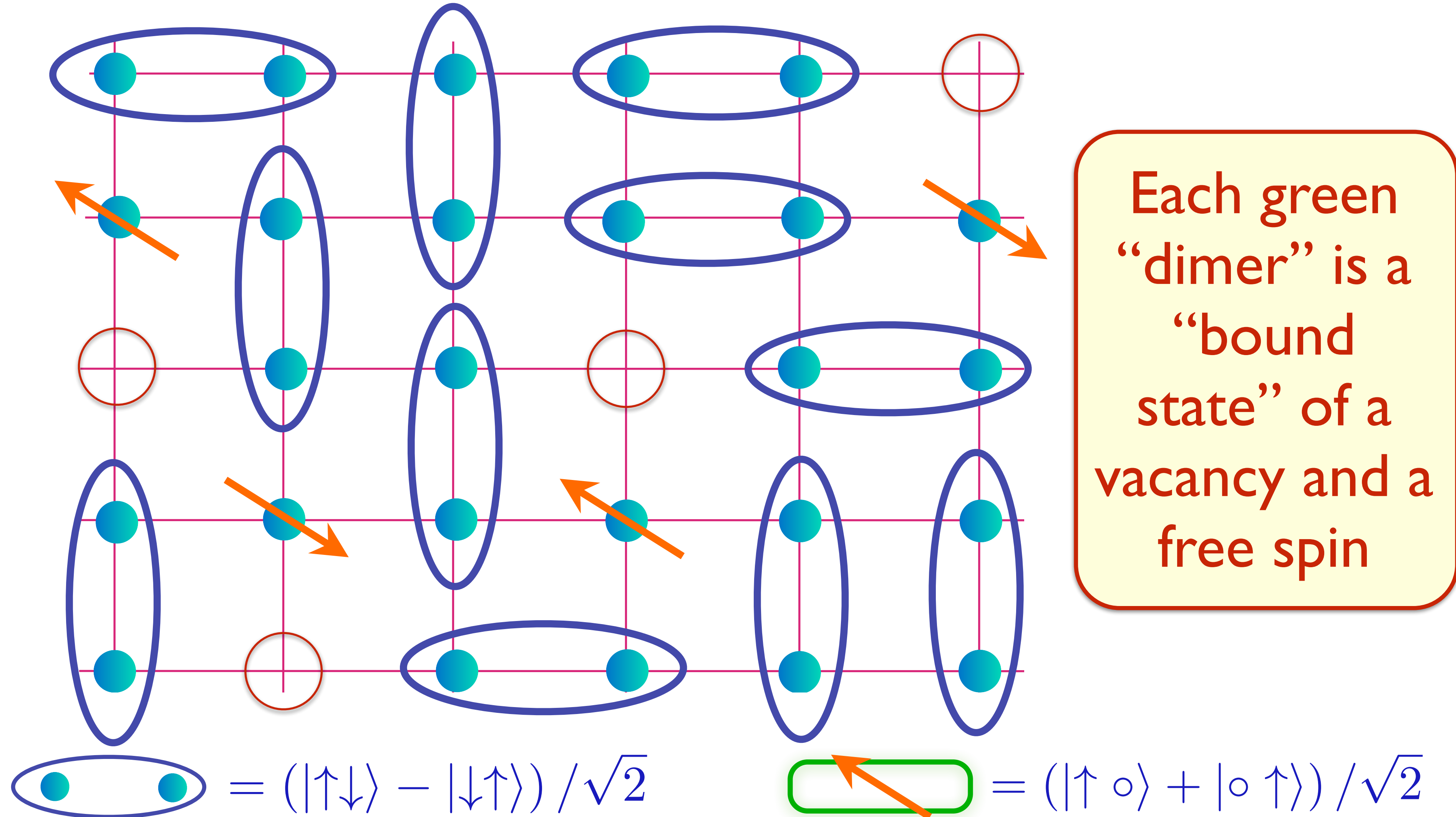


E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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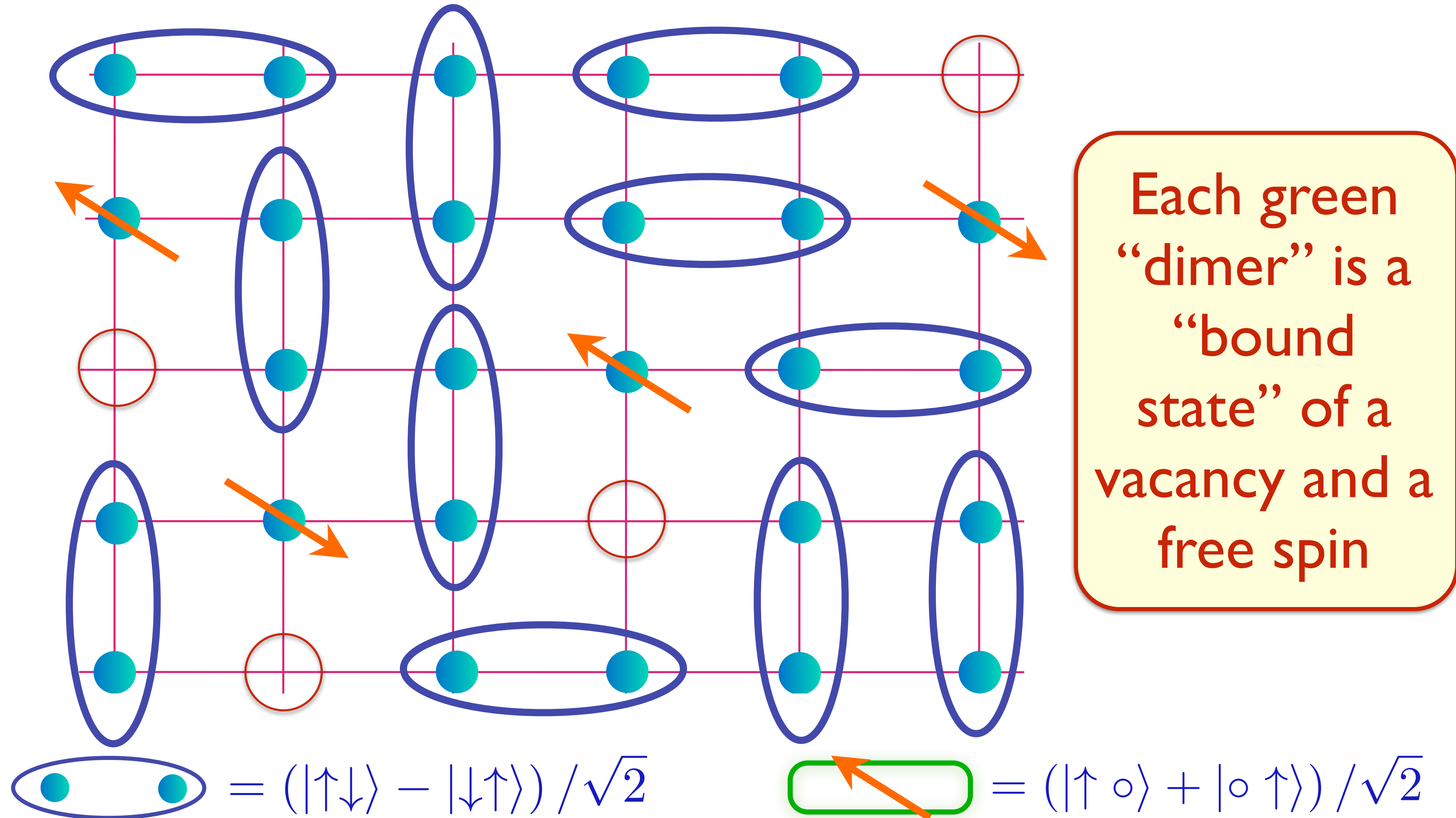
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



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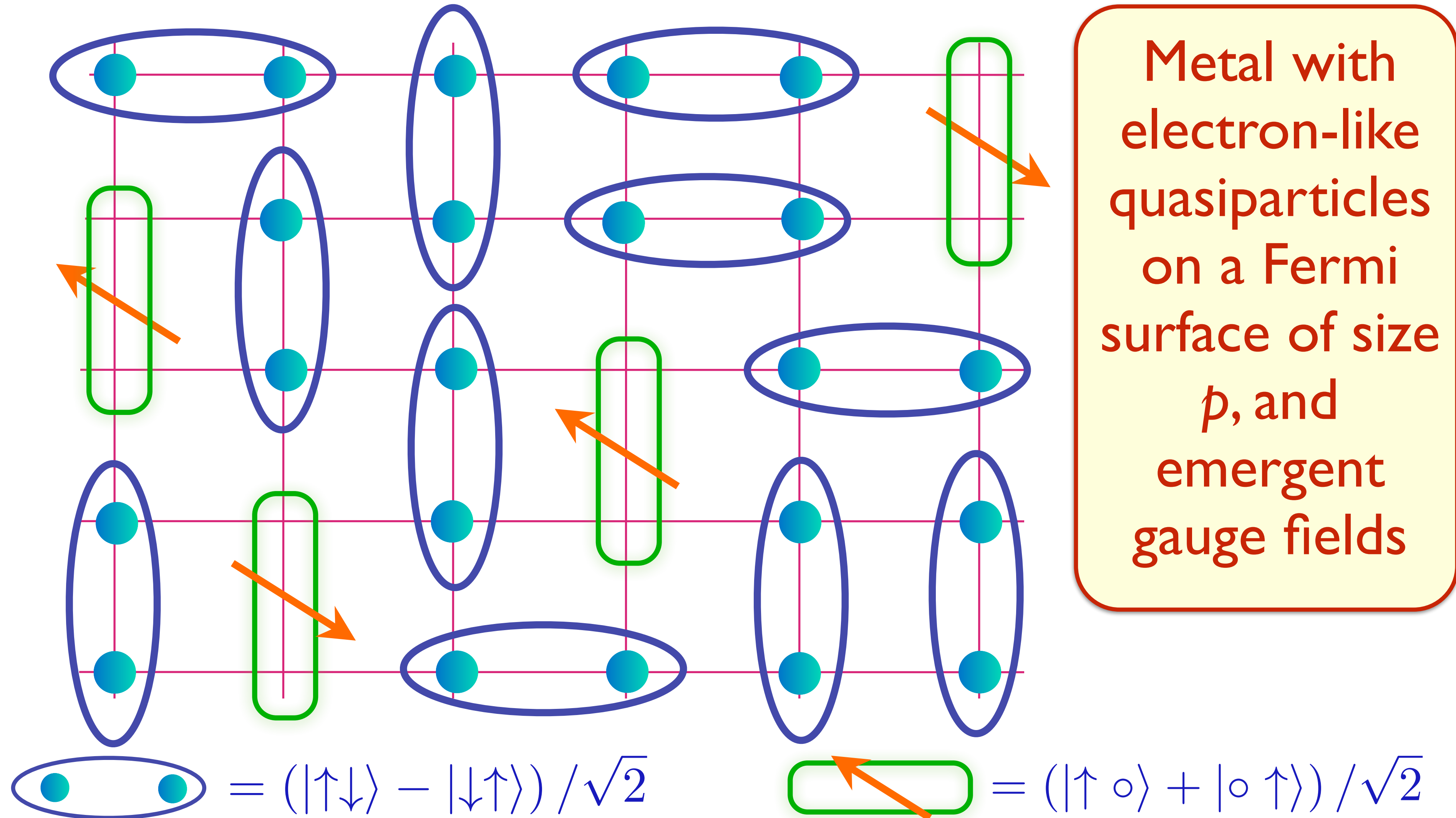


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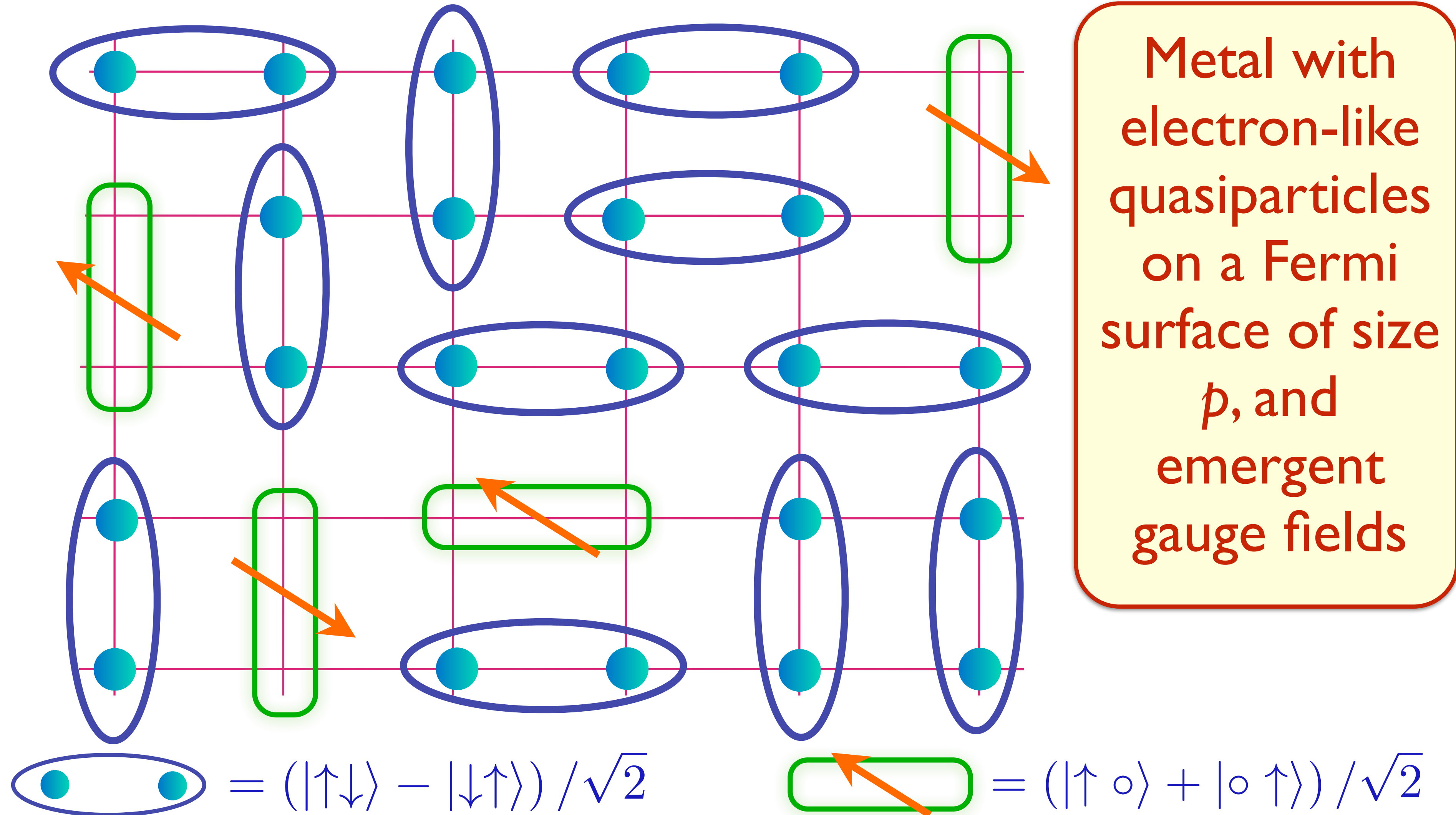


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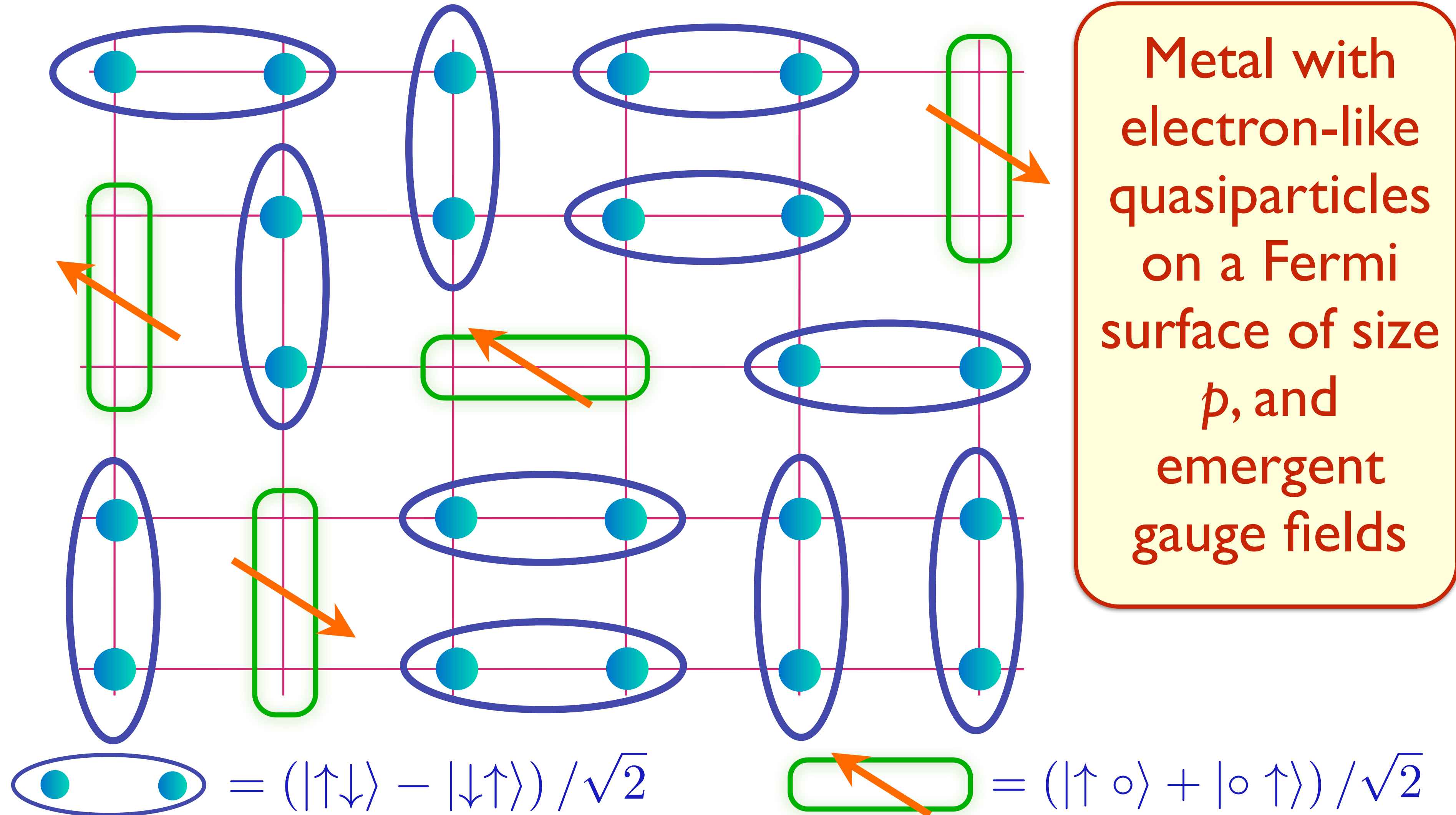


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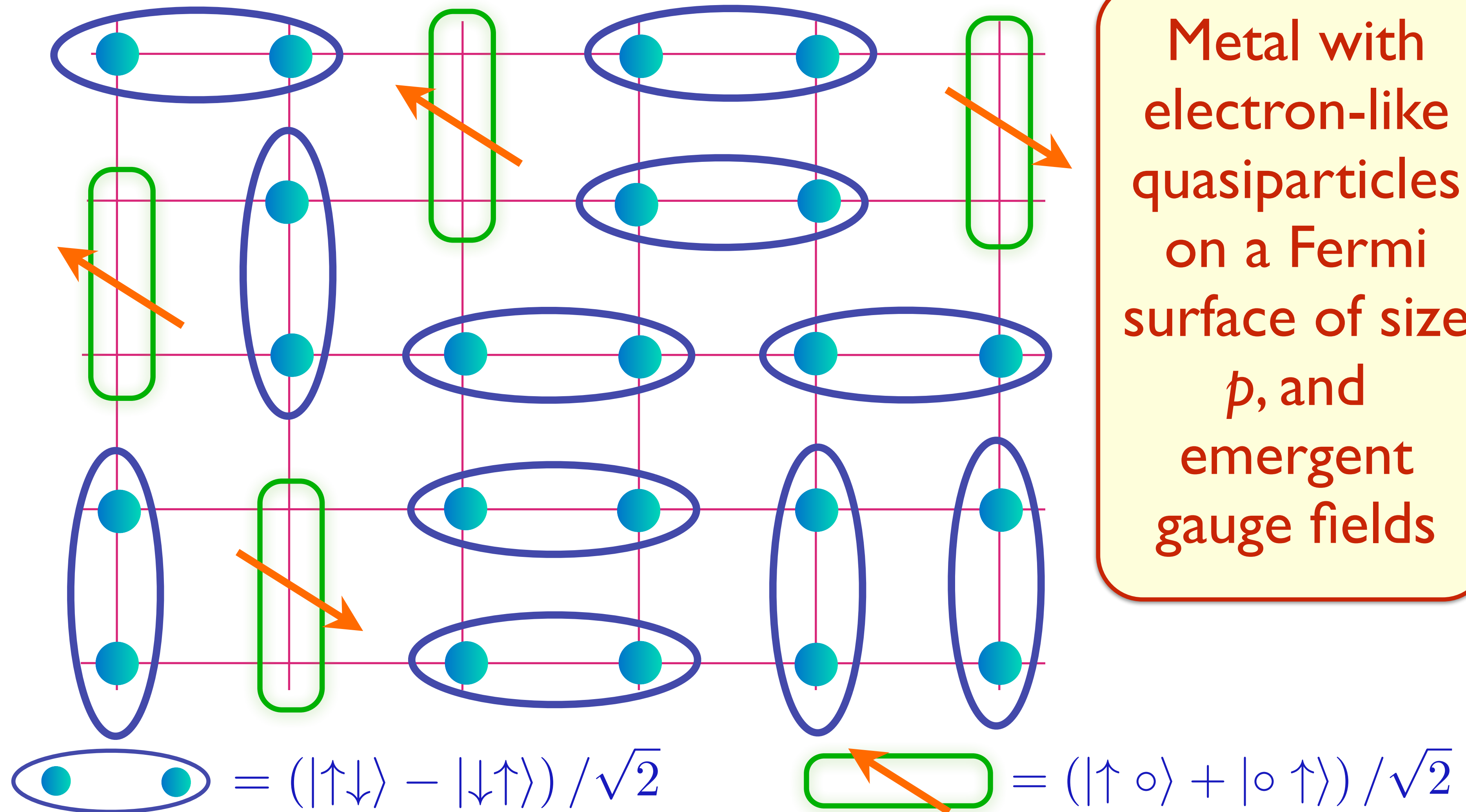


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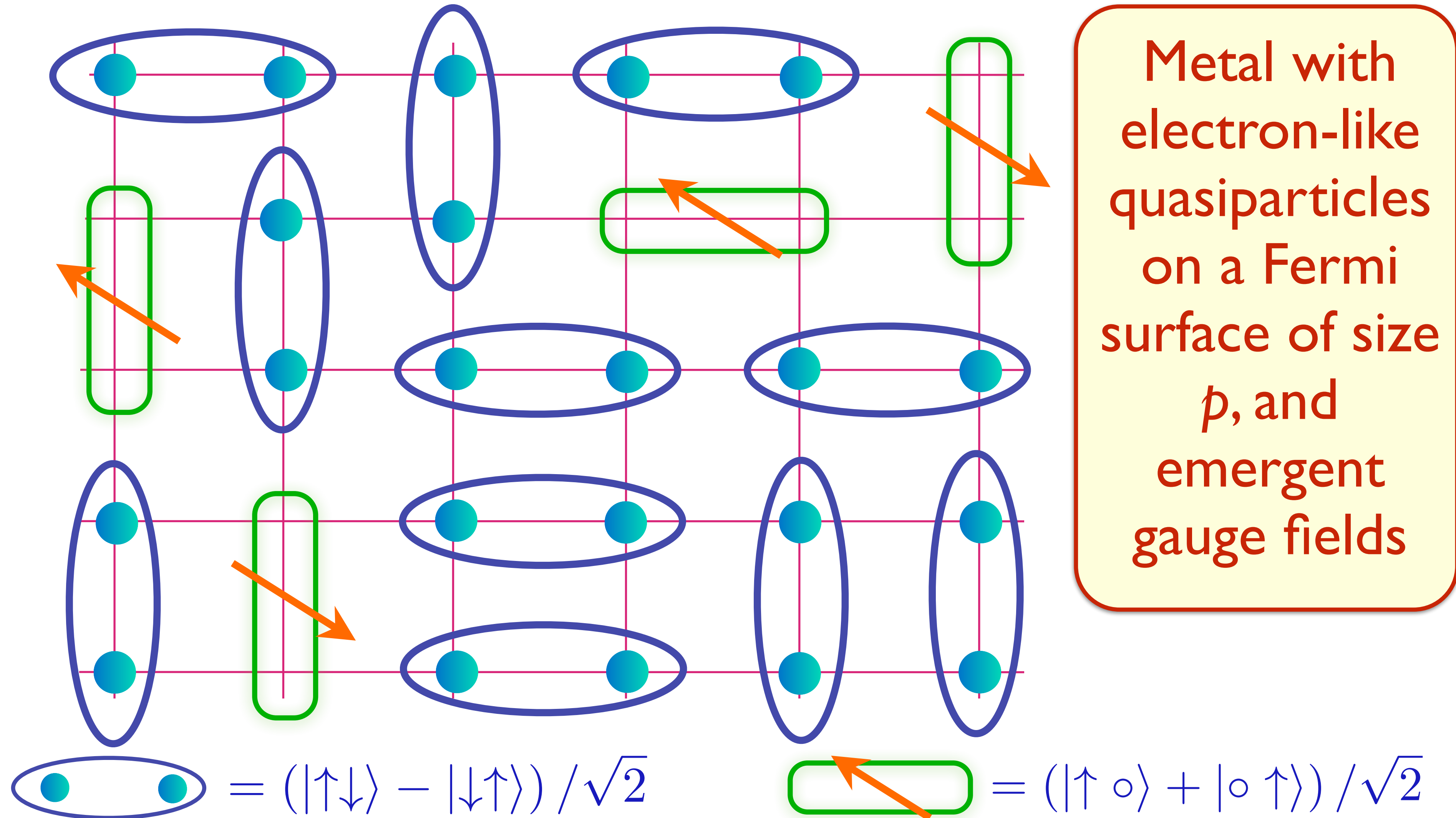


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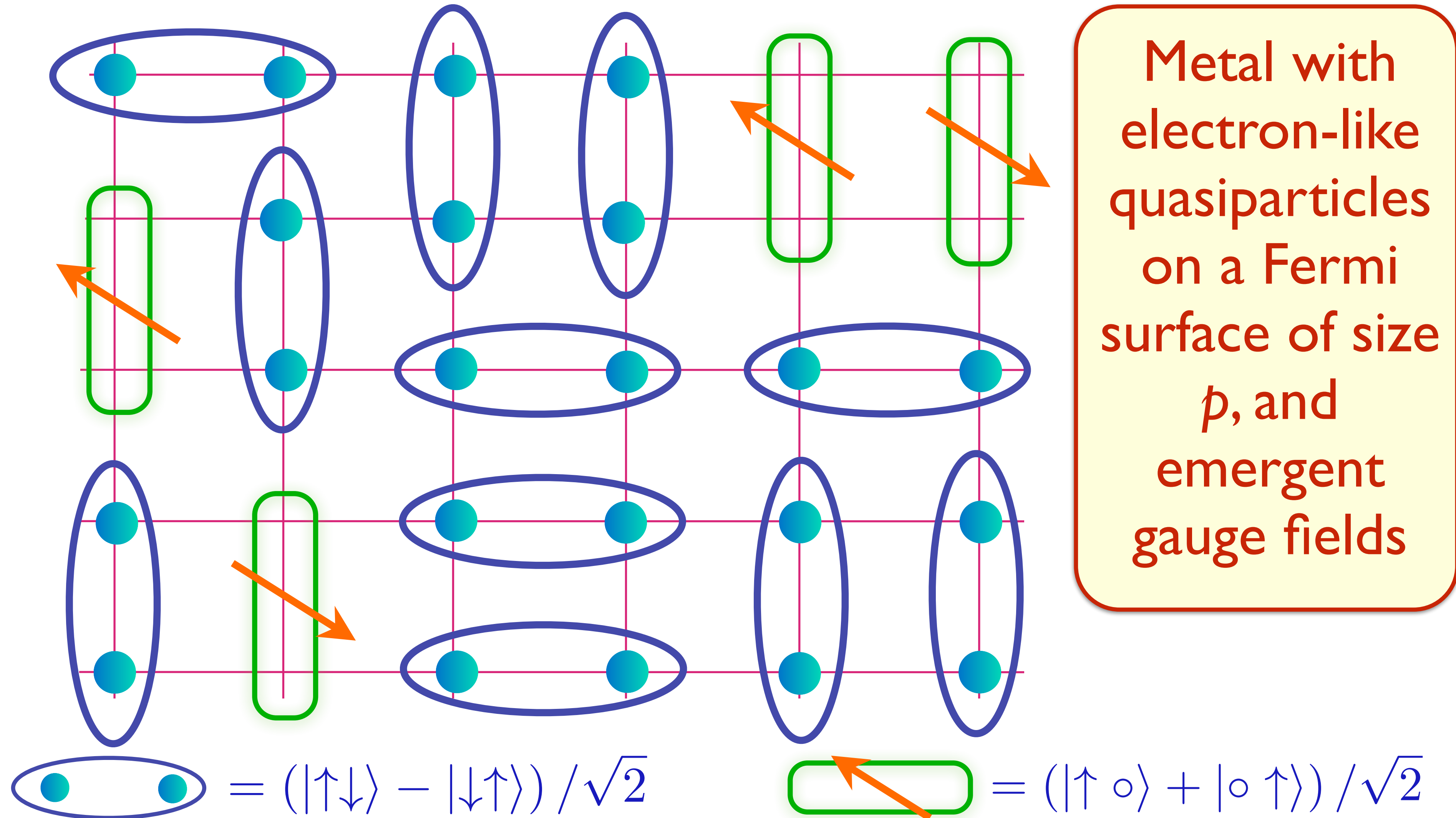


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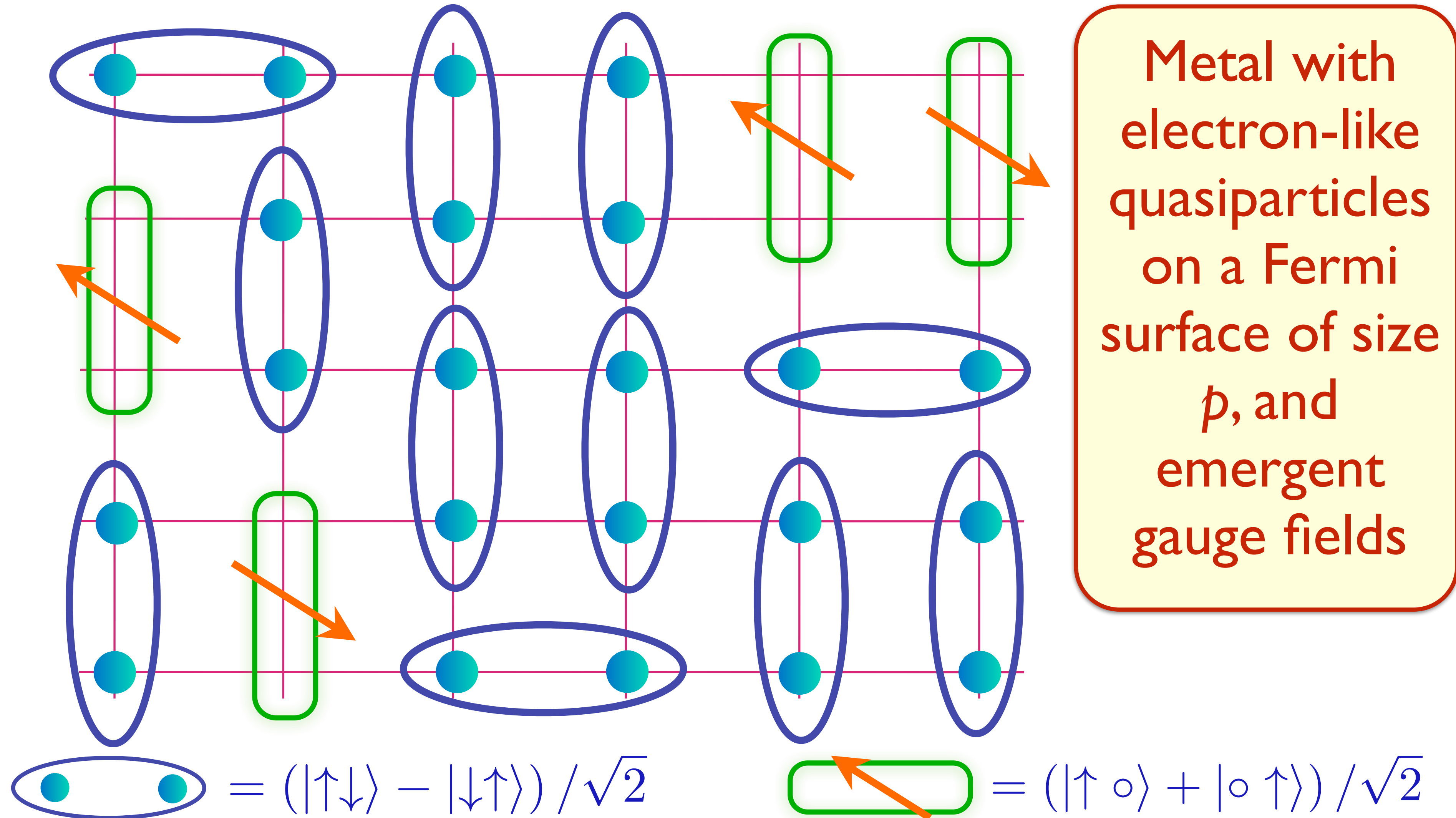


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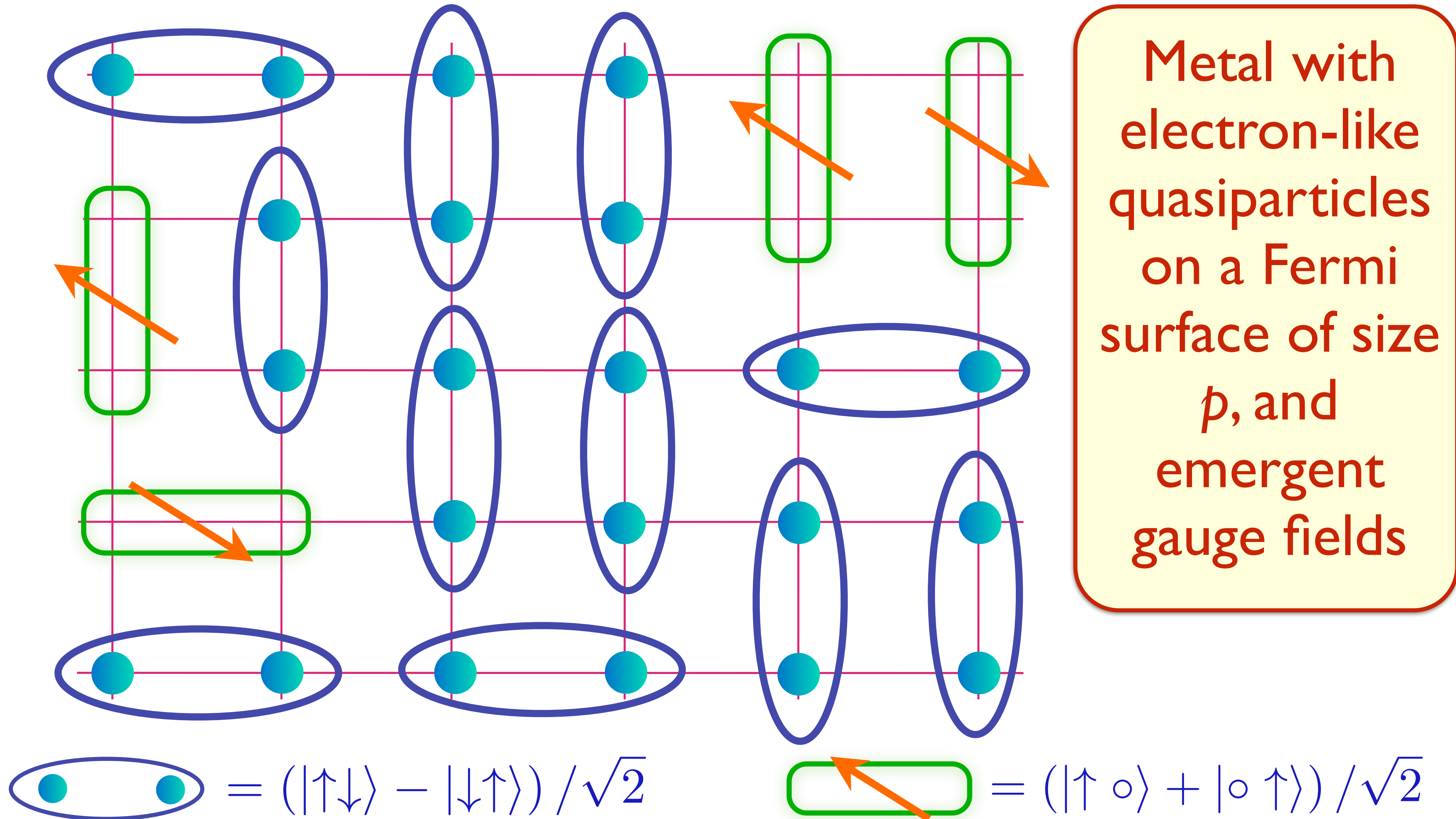


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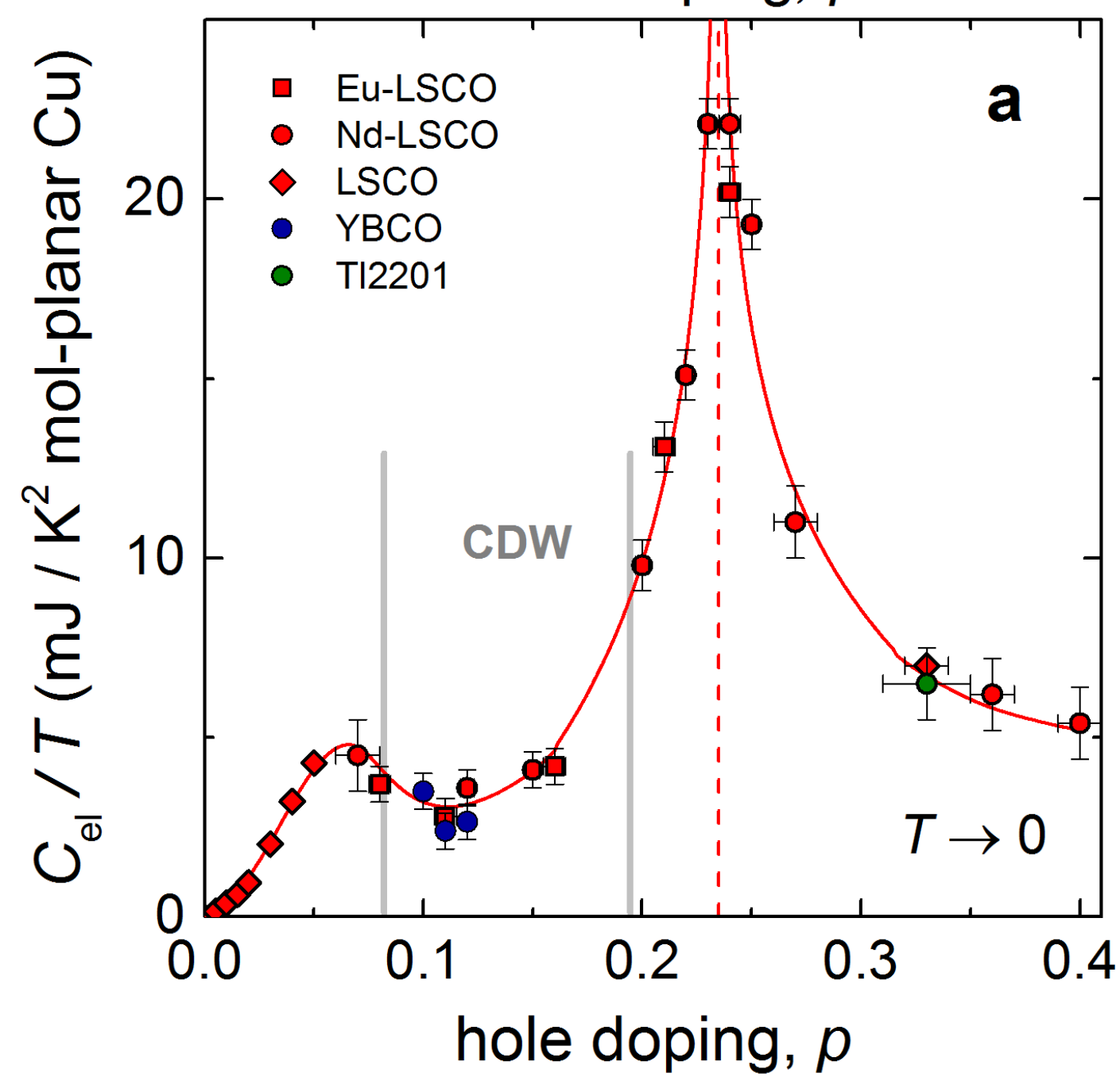
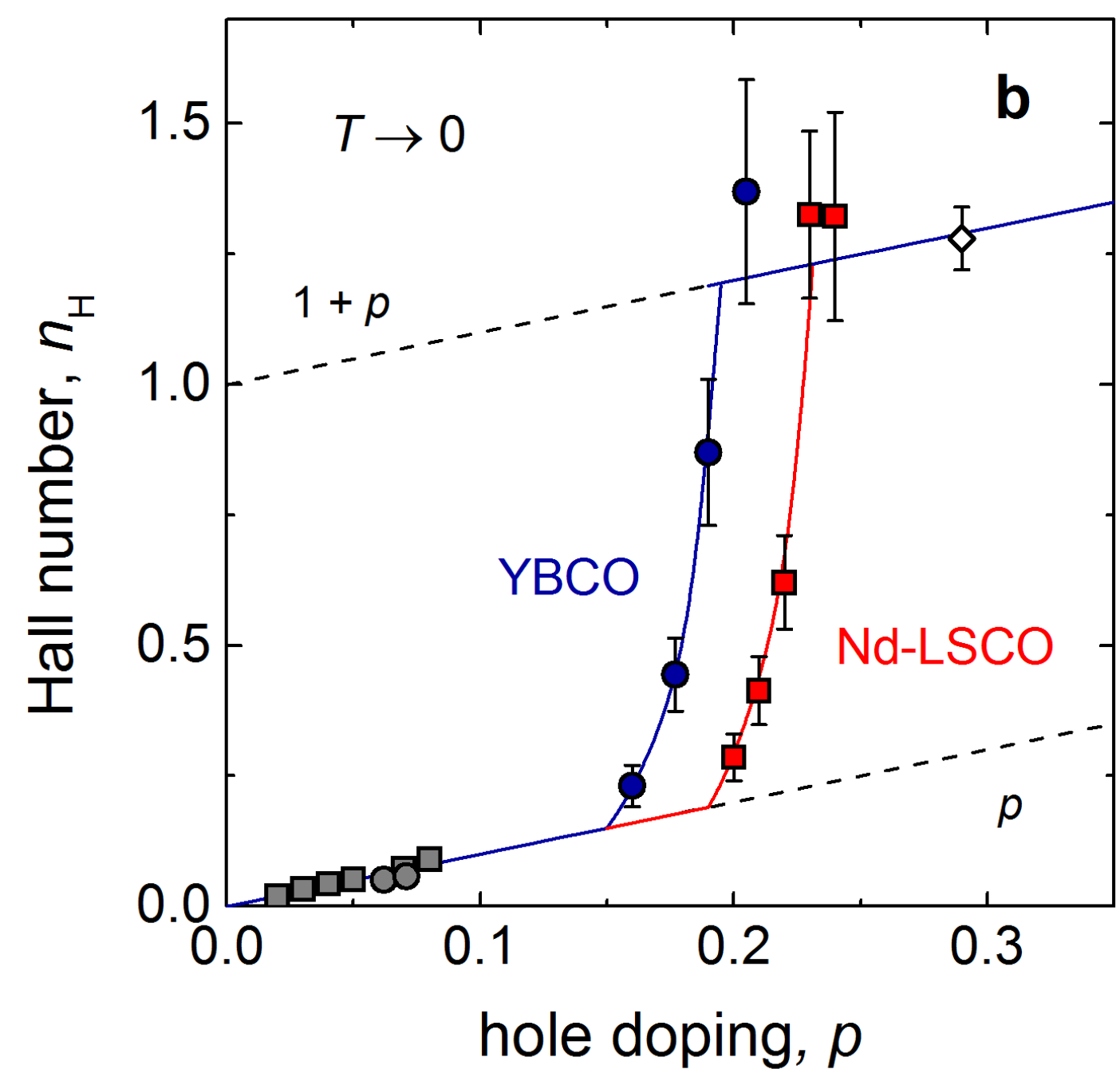
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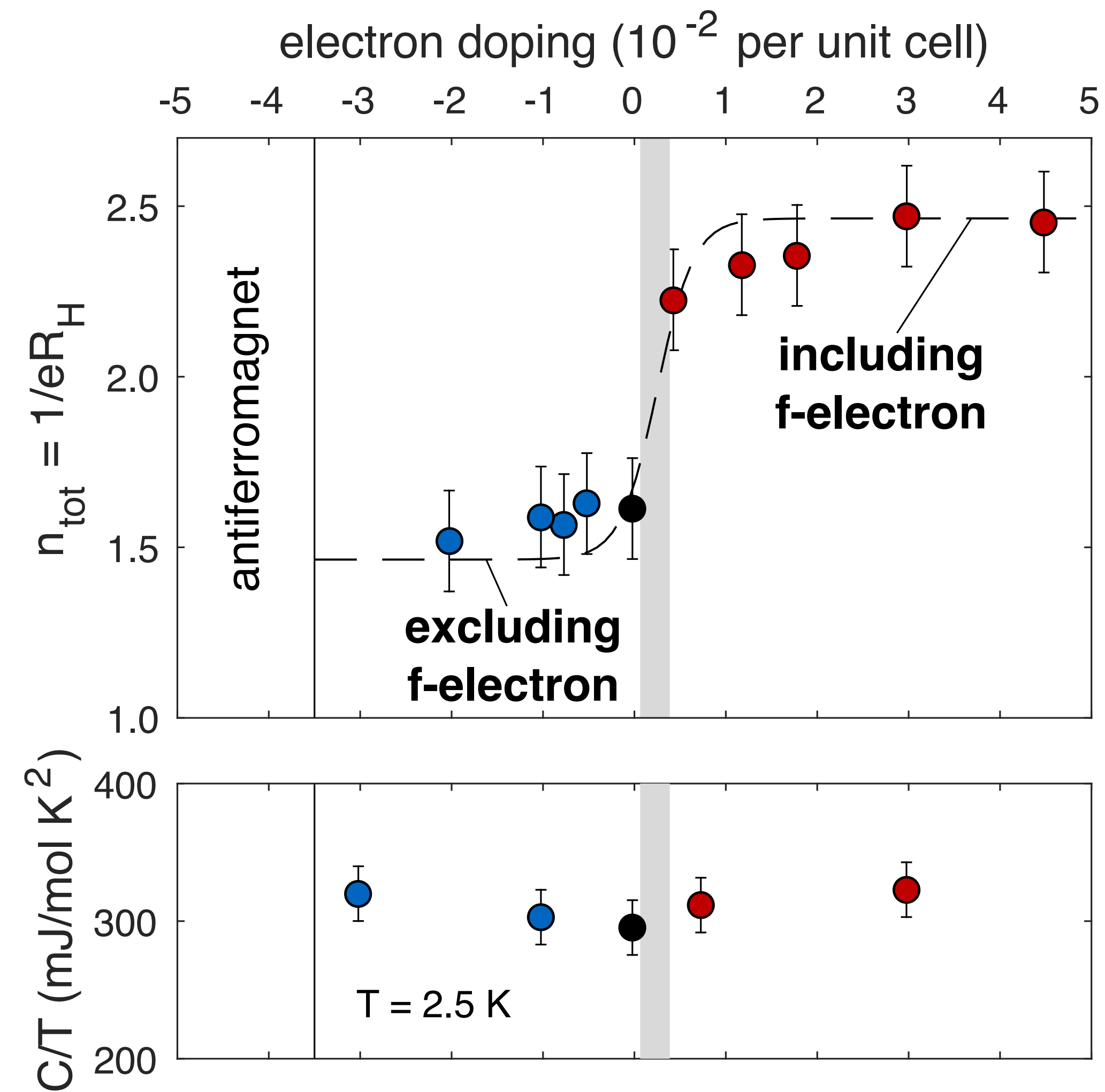
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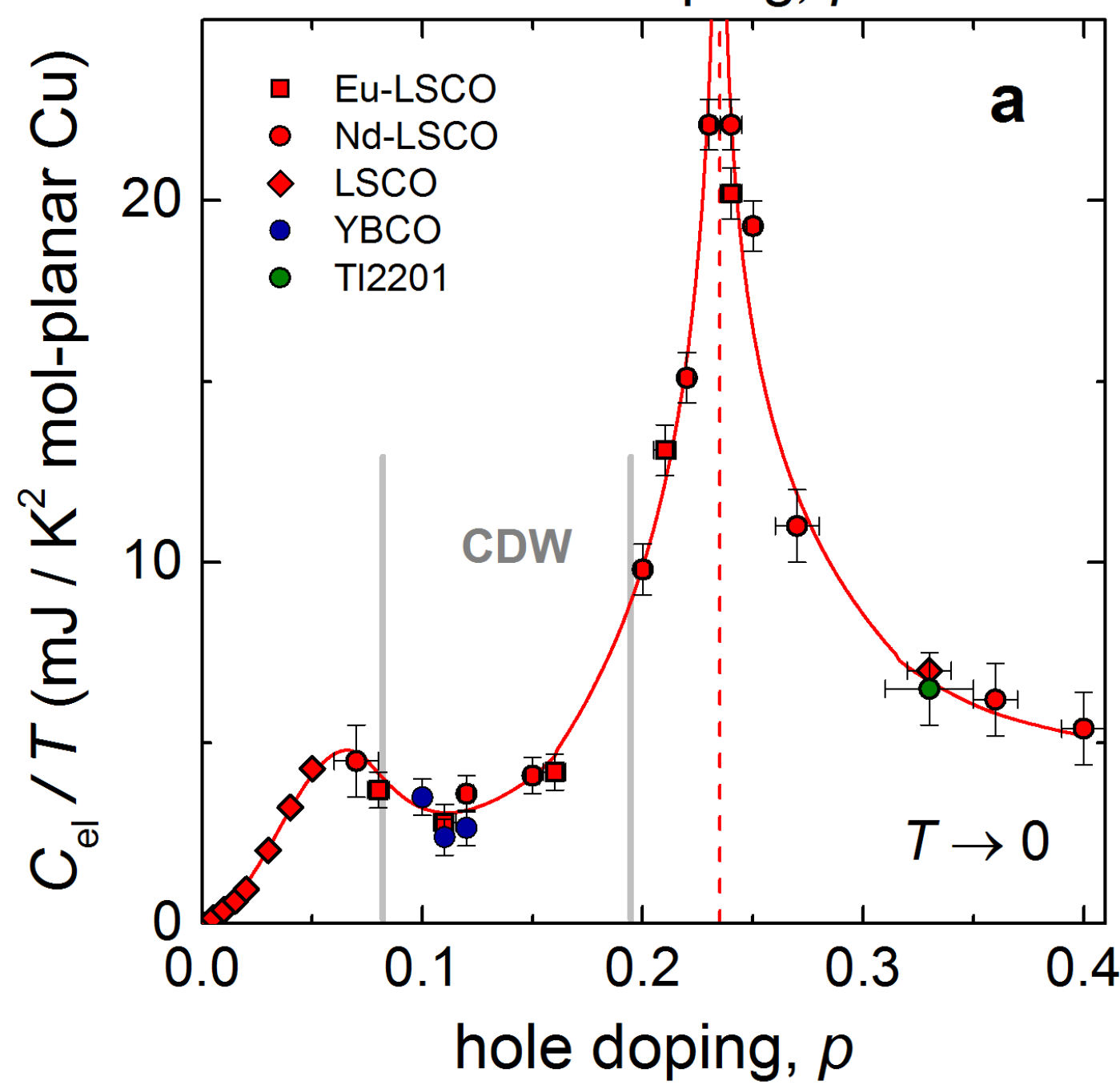
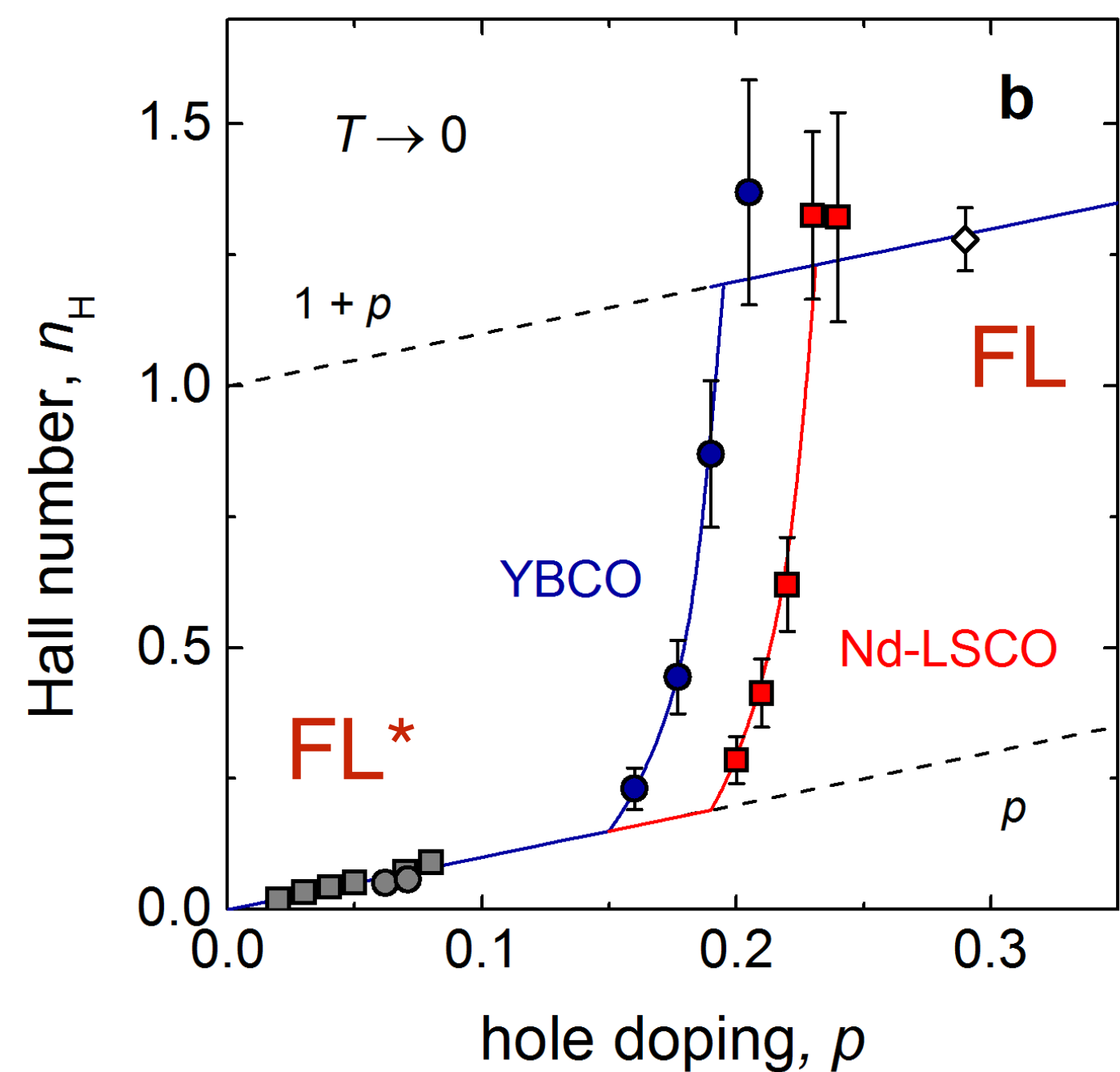
# Cuprates



# CeCoIn<sub>5</sub>



# Cuprates



Evidence for ghost Fermi surfaces in the  $FL^*$ - $FL$  transition in a single-band model ?

# CeCoIn<sub>5</sub>

