# Quantum spin liquids in antiferromagnets and Rydberg atoms

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1. Introduction to spin liquids

   *The $Z_2$ spin liquid*

2. Rydberg atoms

3. Gapless spin liquids on the square lattice

   *Gauge theories of partons*

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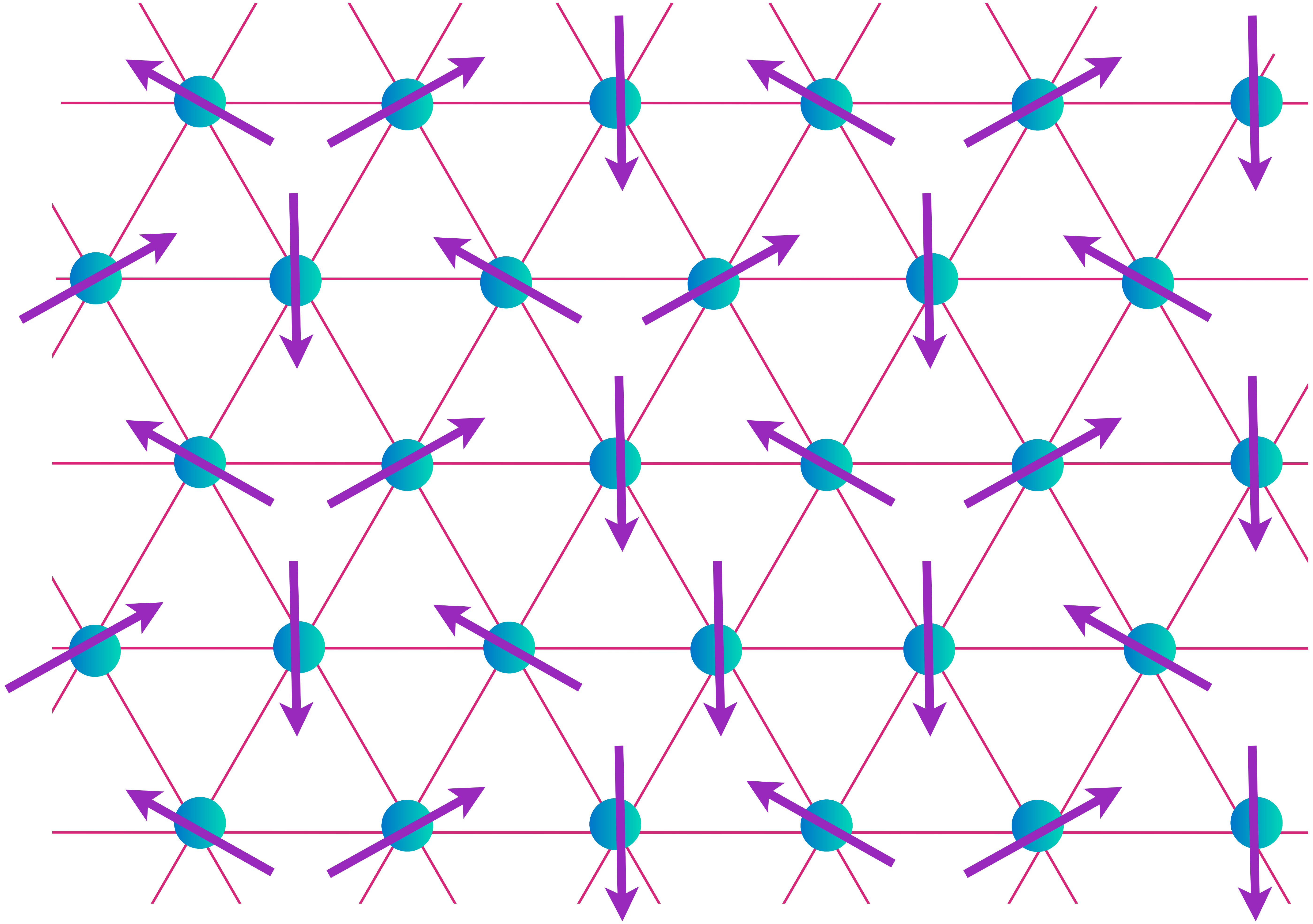

Nearest-neighbor model has non-collinear Neel order

# Mapping of bosons and spins

| Boson $B$ | Spin $\vec{S}$ |
| --- | --- |
| $B^\dagger B \leq 1$ | $\vec{S}^2 = 3/4$ |
| States $\|0\rangle$, $B^\dagger\|0\rangle$ | States $\|\uparrow\rangle$ $\|\downarrow\rangle$ |
| Operators | Operators |
| $B^\dagger$ $\longleftrightarrow$ | $S_+ = S_x + iS_y$ |
| $B$ $\longleftrightarrow$ | $S_- = S_x - iS_y$ |
| $B^\dagger B - \frac{1}{2}$ $\longleftrightarrow$ | $S_z$ |

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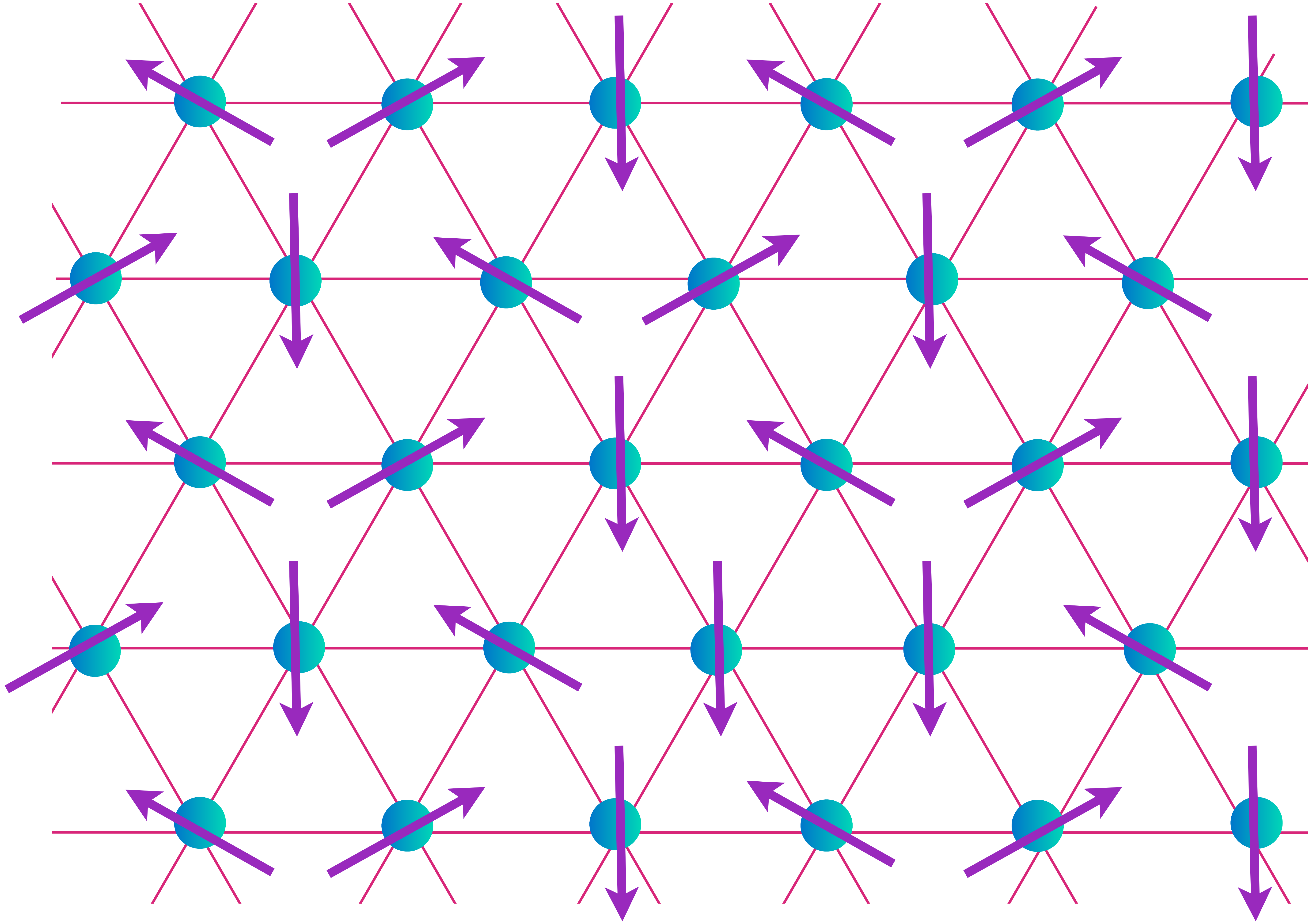

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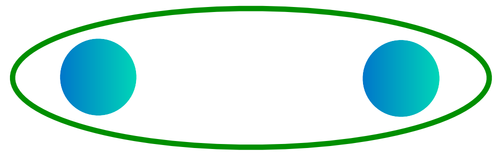

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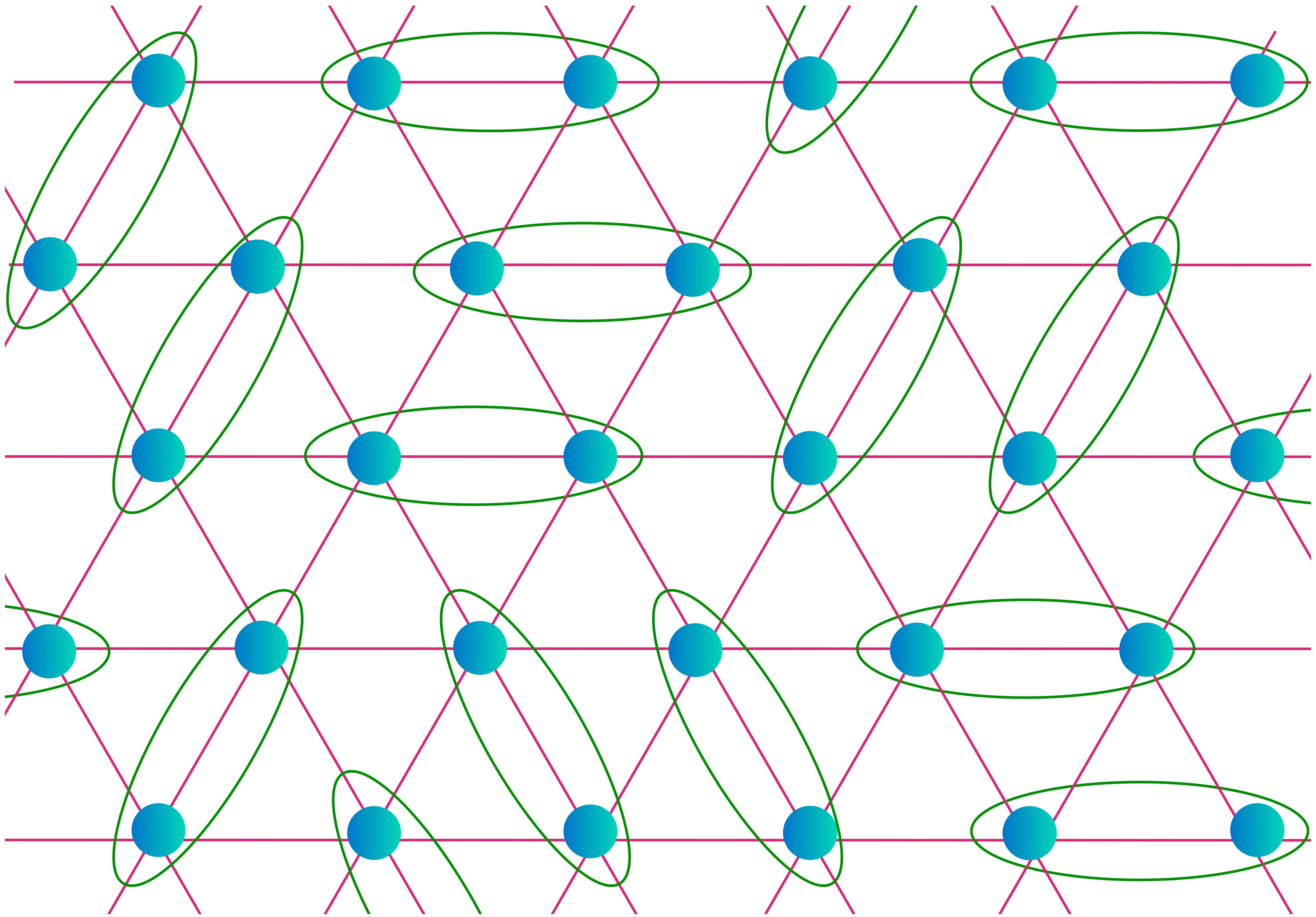

$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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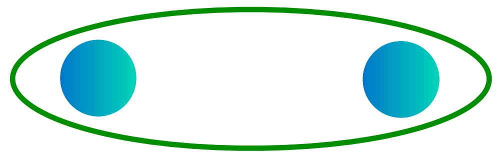

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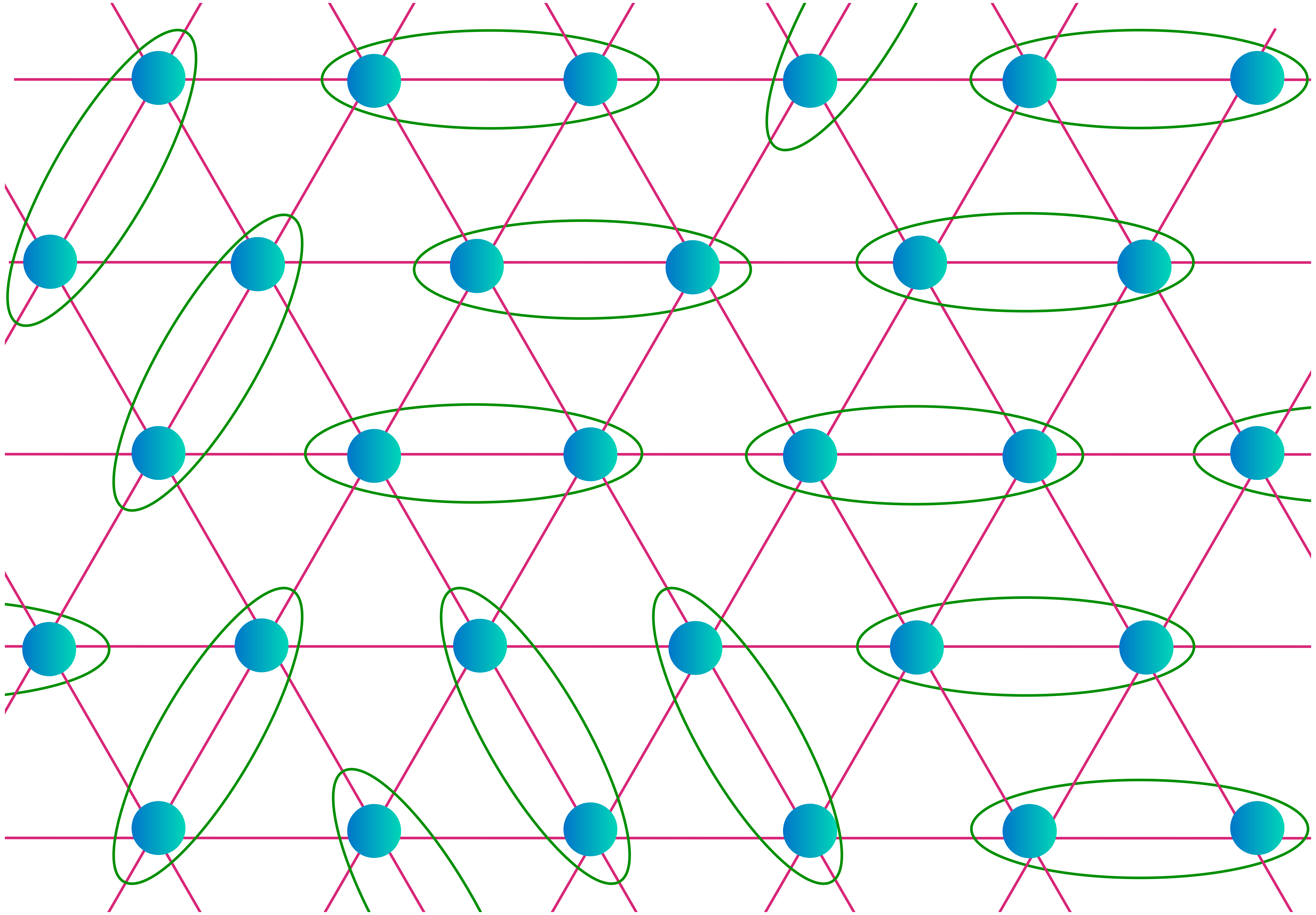

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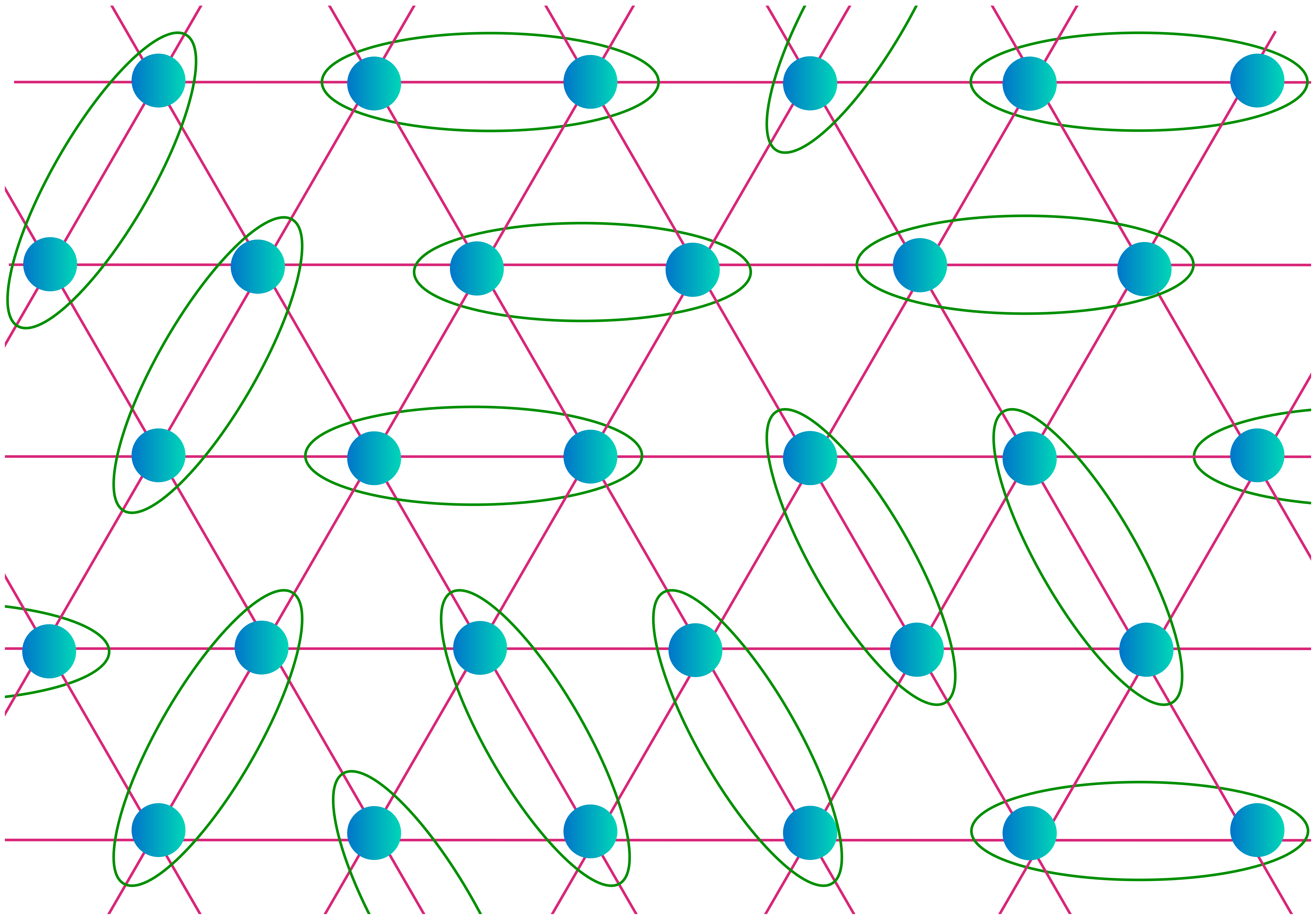

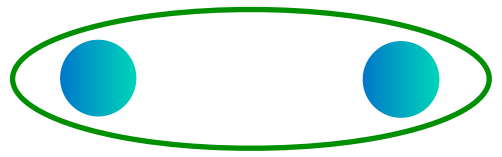

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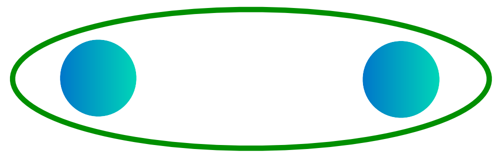

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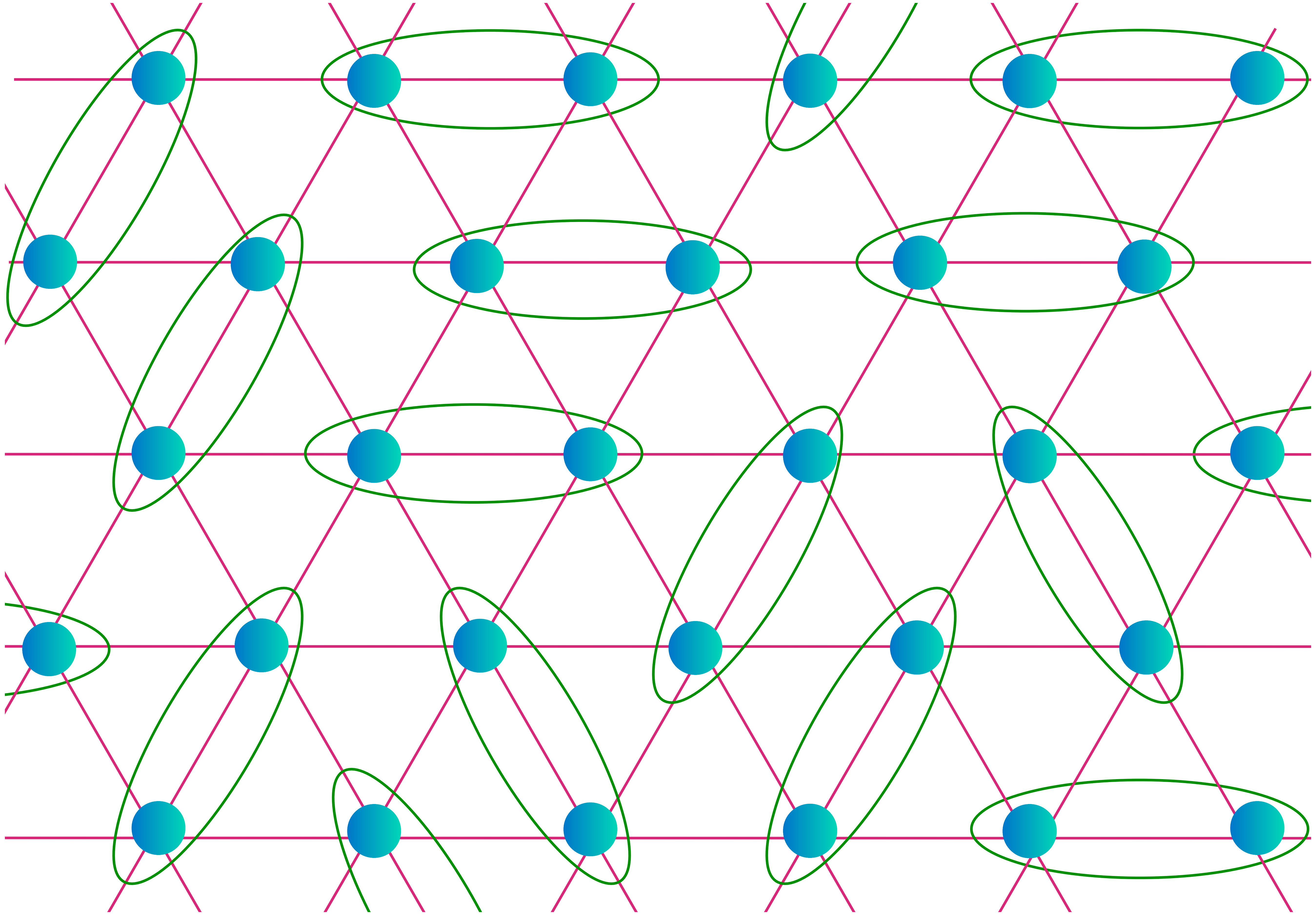

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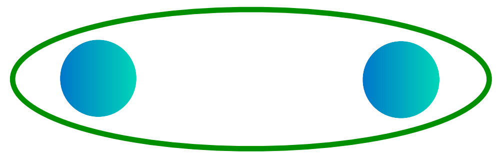

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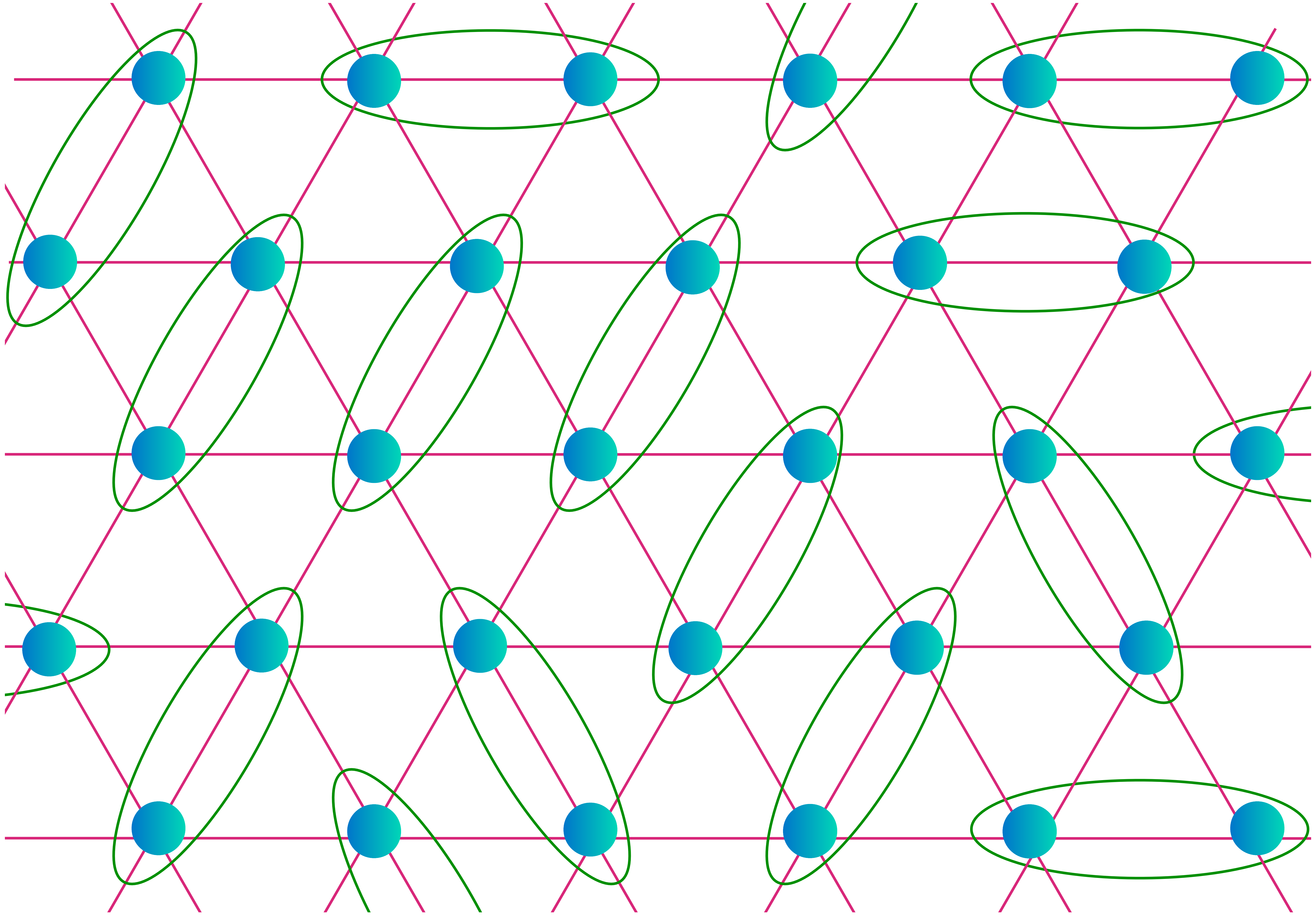

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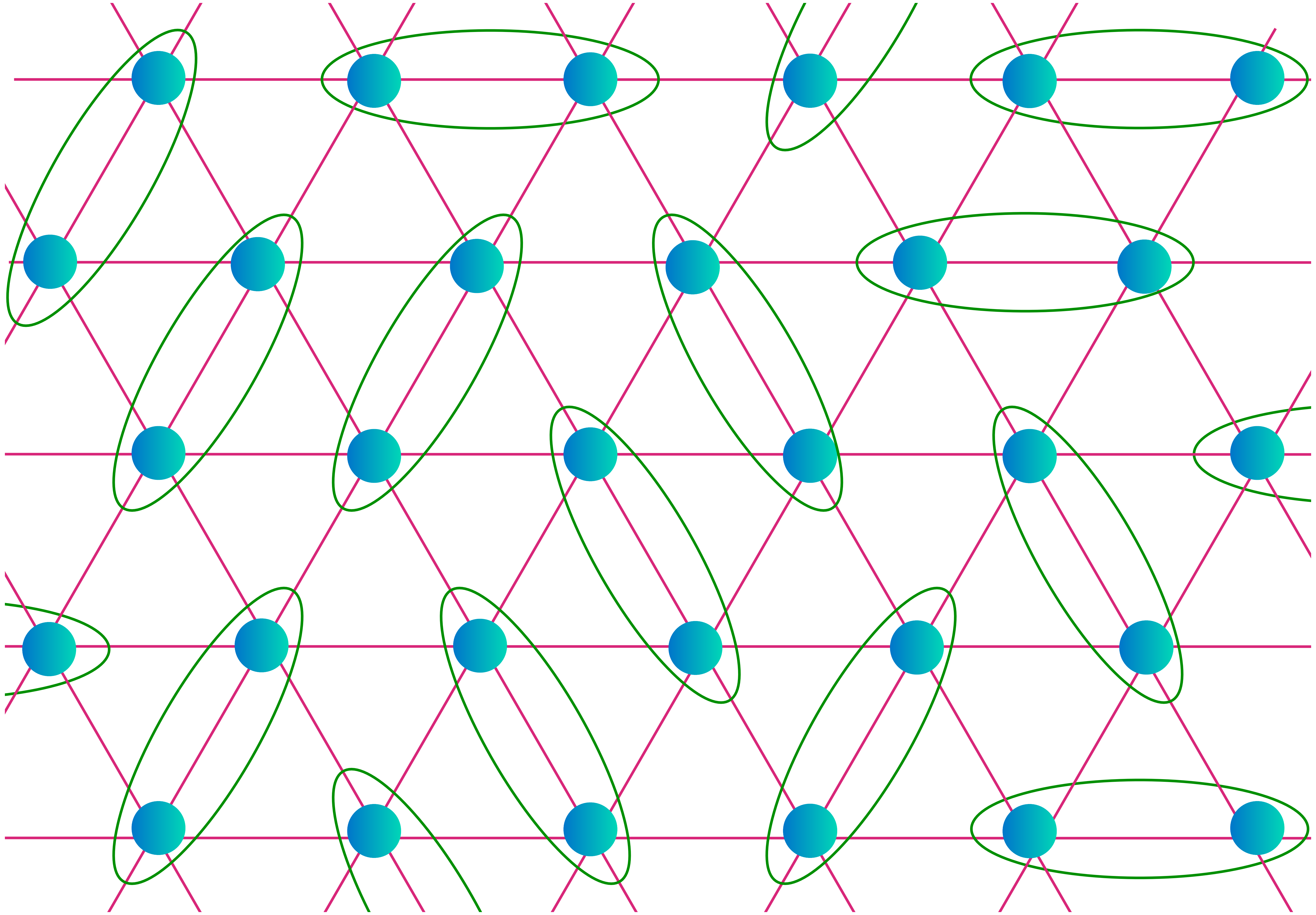

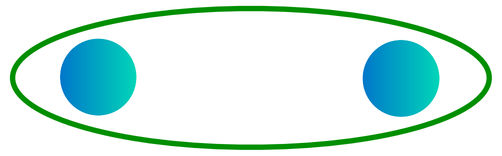

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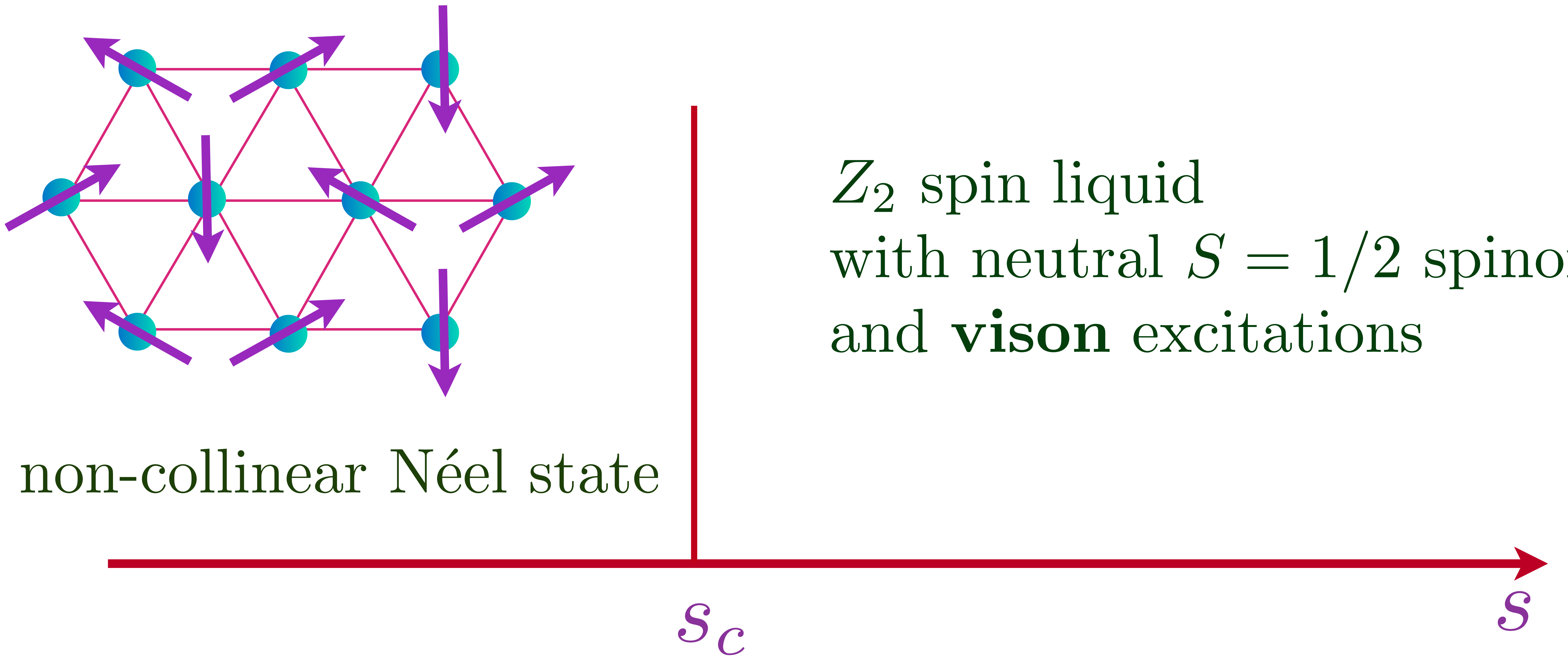


$Z_2$ spin liquid
with neutral $S = 1/2$ spinons
and **vison** excitations

N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

# Excitations of the $Z_2$ Spin liquid

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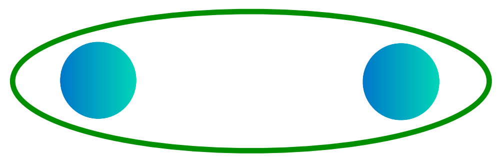
$e$ (boson) or $\epsilon$ (fermion) particle



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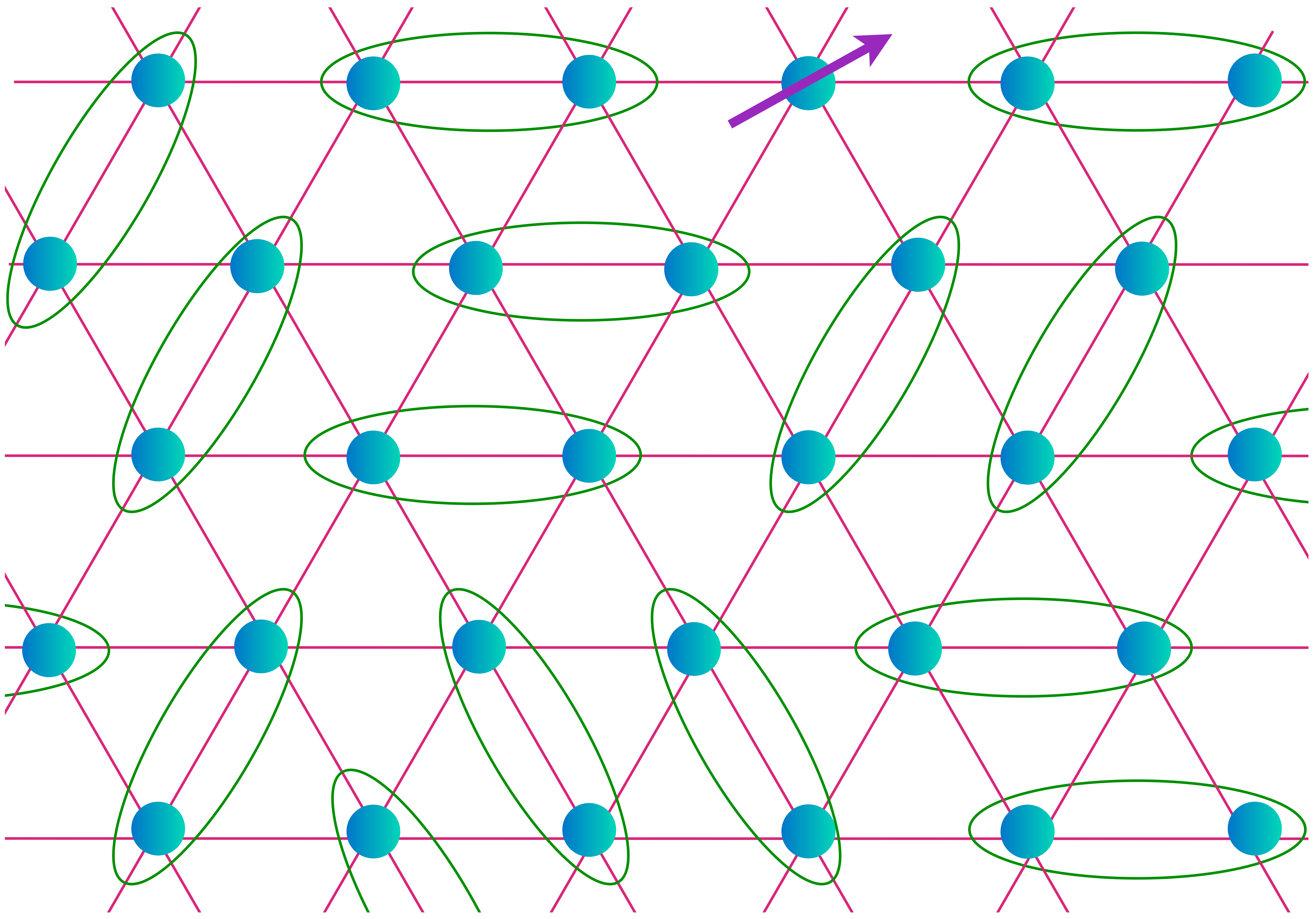

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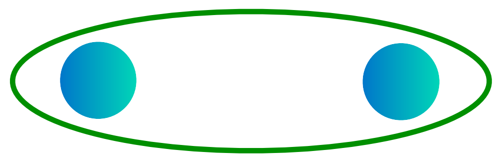

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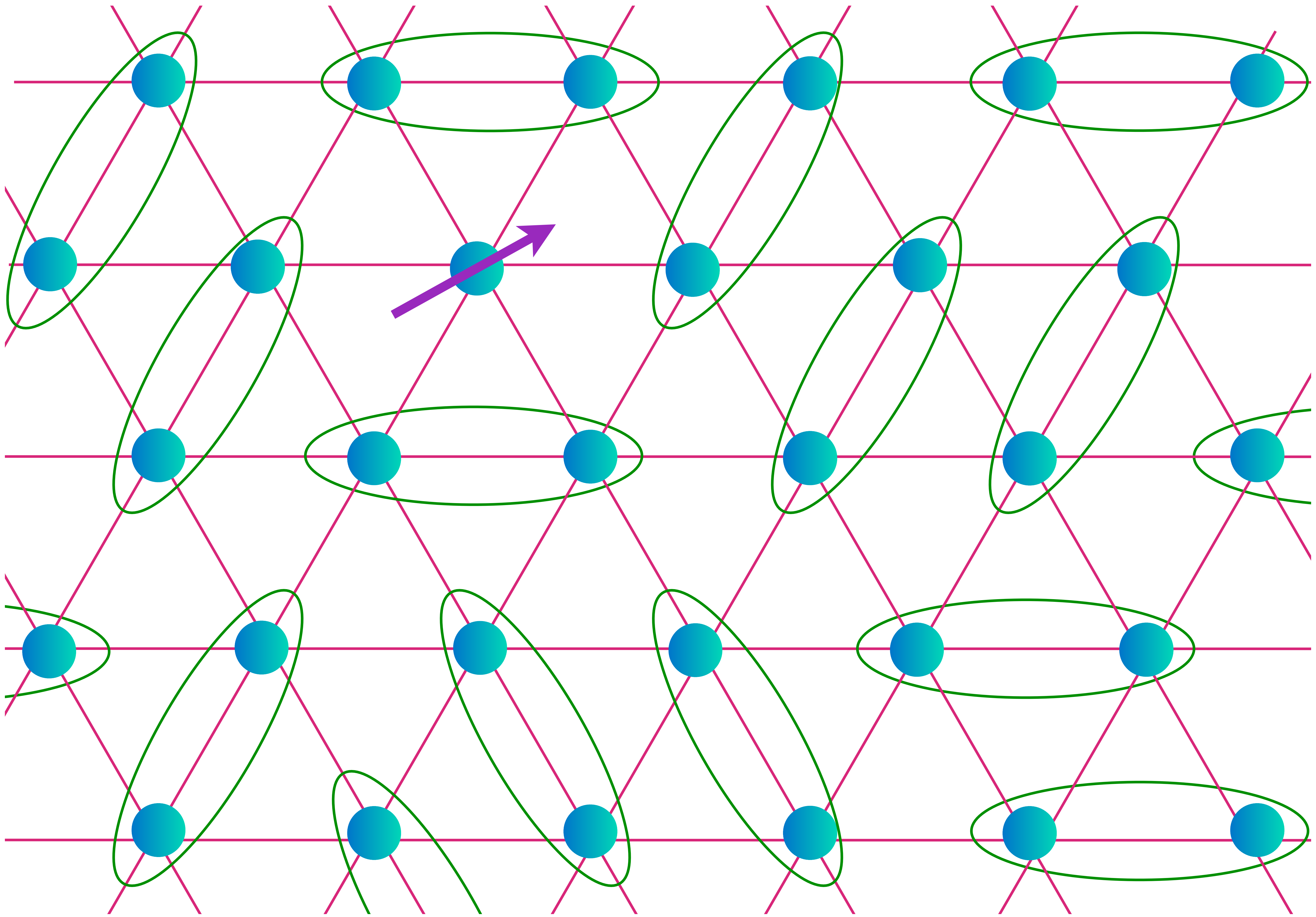

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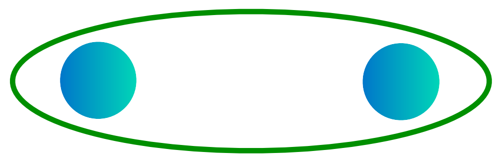

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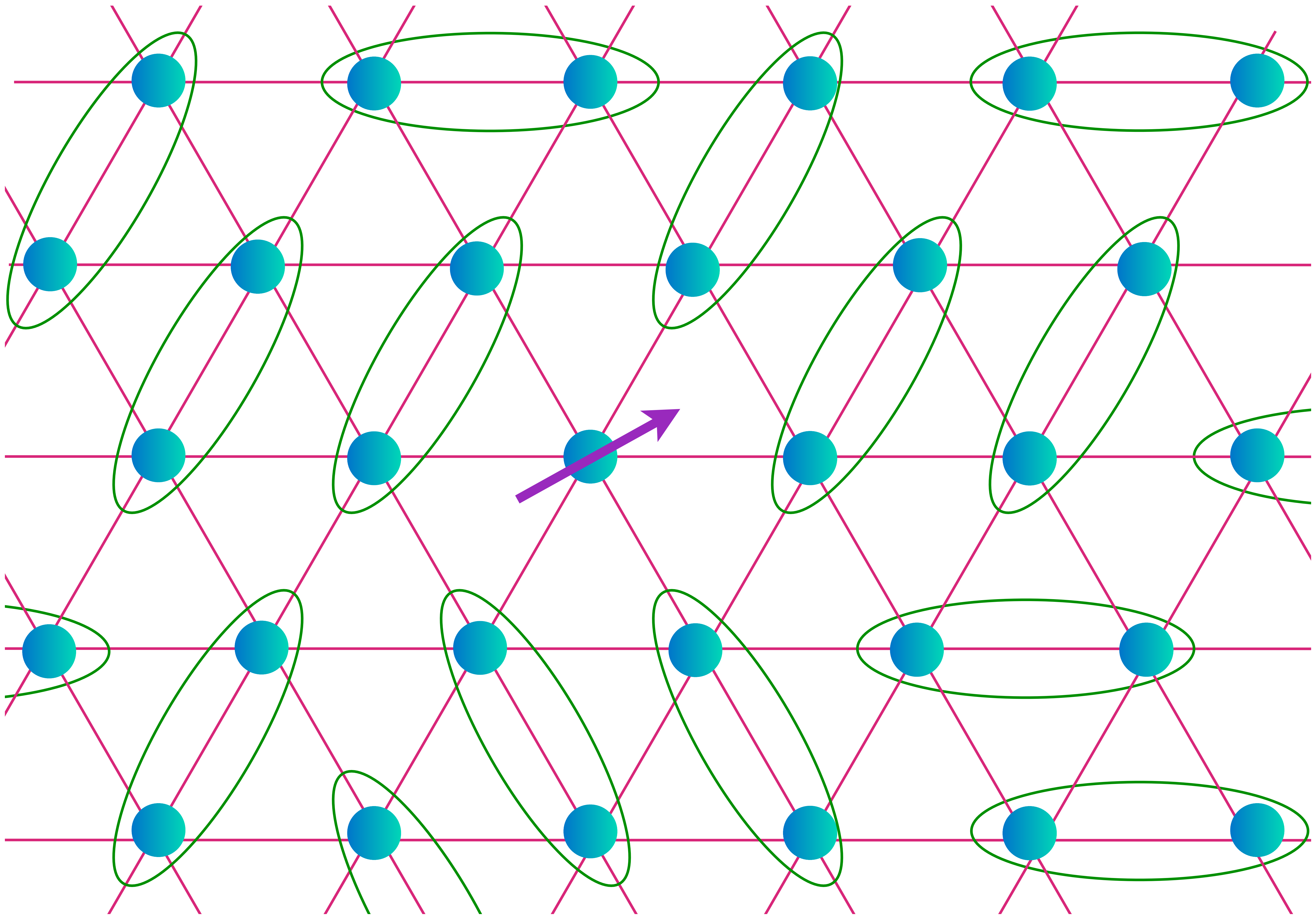

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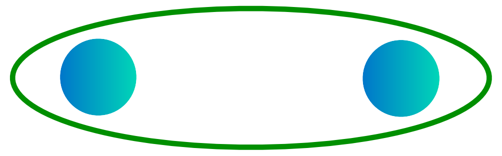

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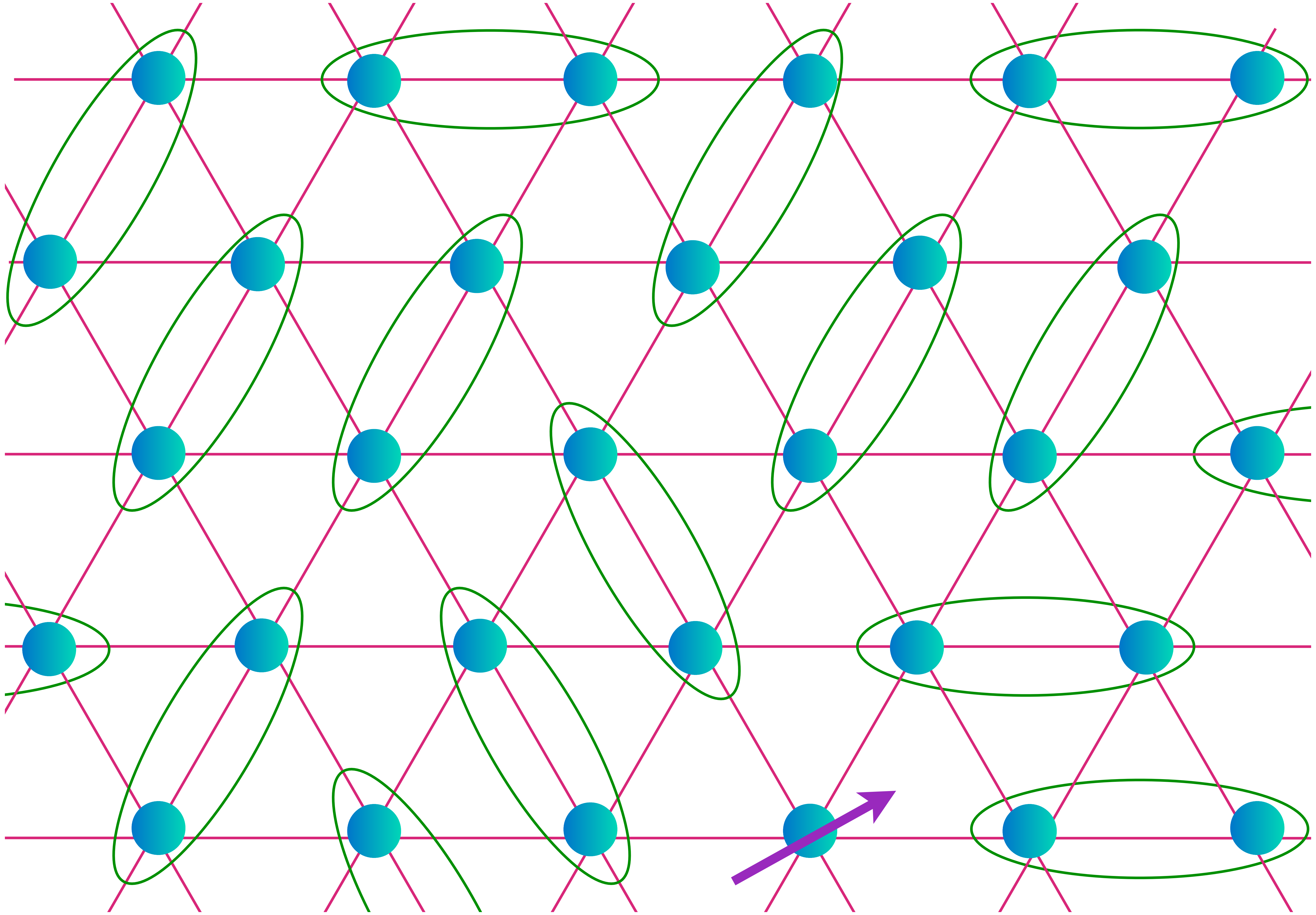

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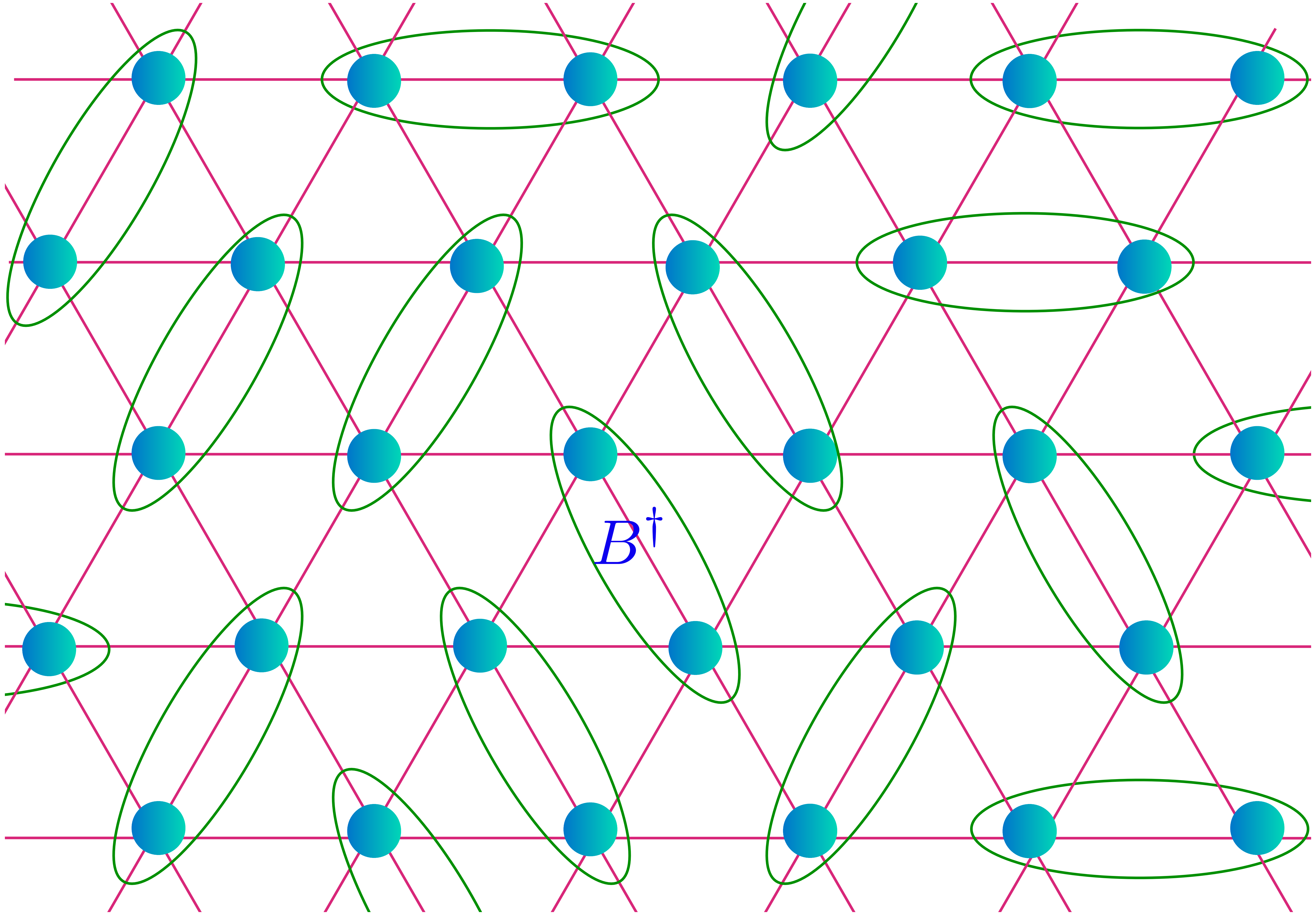


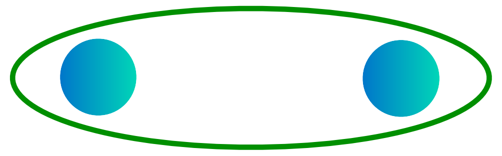

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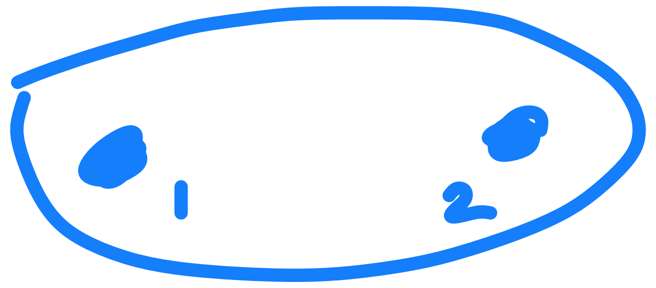
$B_2^\dagger$ 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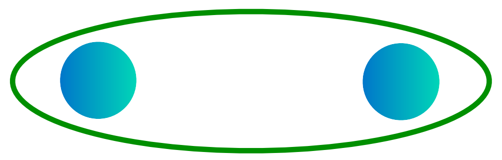

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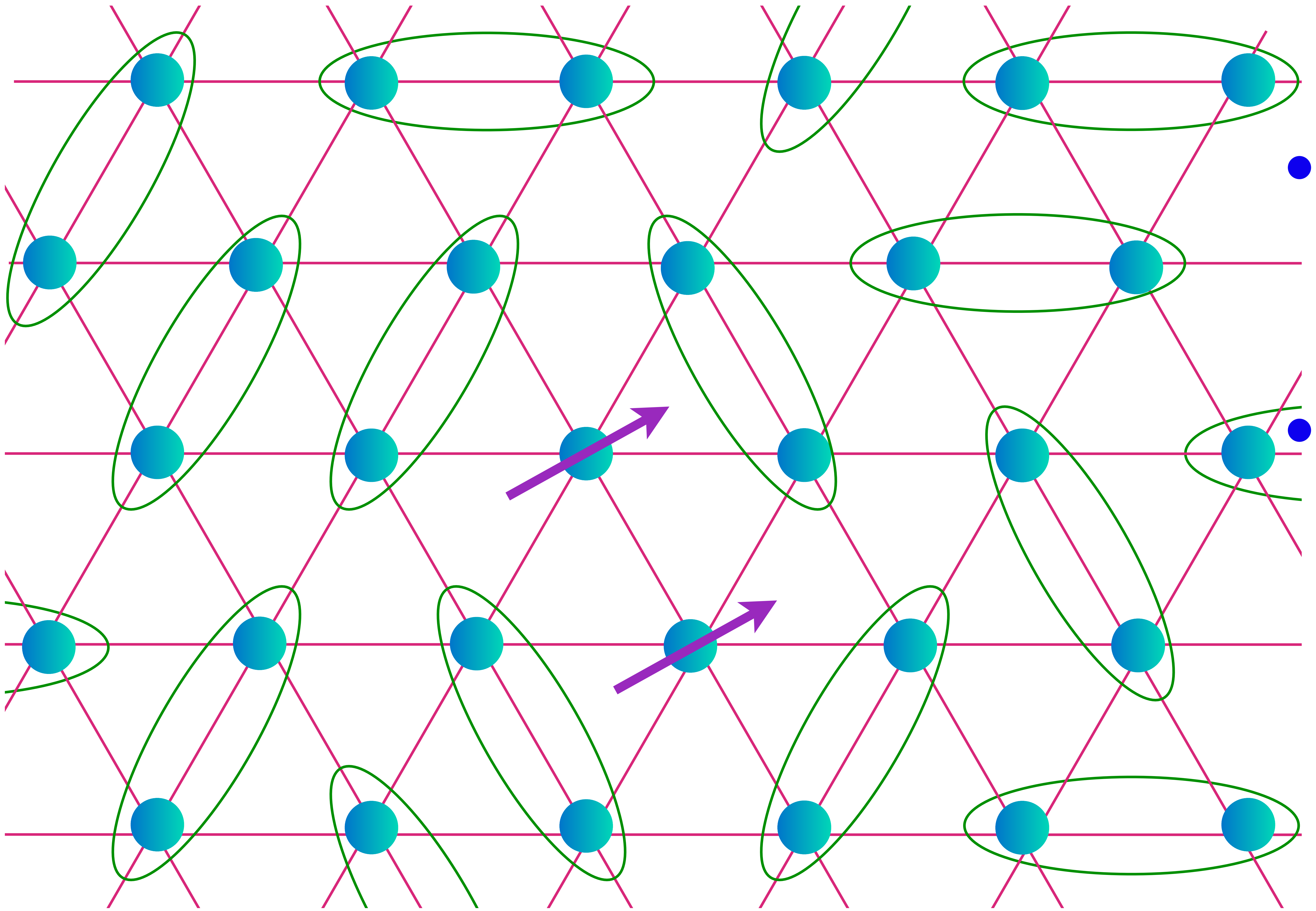

- Spinons can only be created in pairs by a local operator ($e.g.$ $B^\dagger$)
- A single spinon carries boson number $B^\dagger B = 1/2$: fractionalization!

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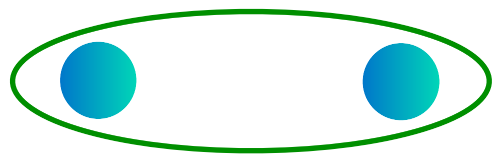

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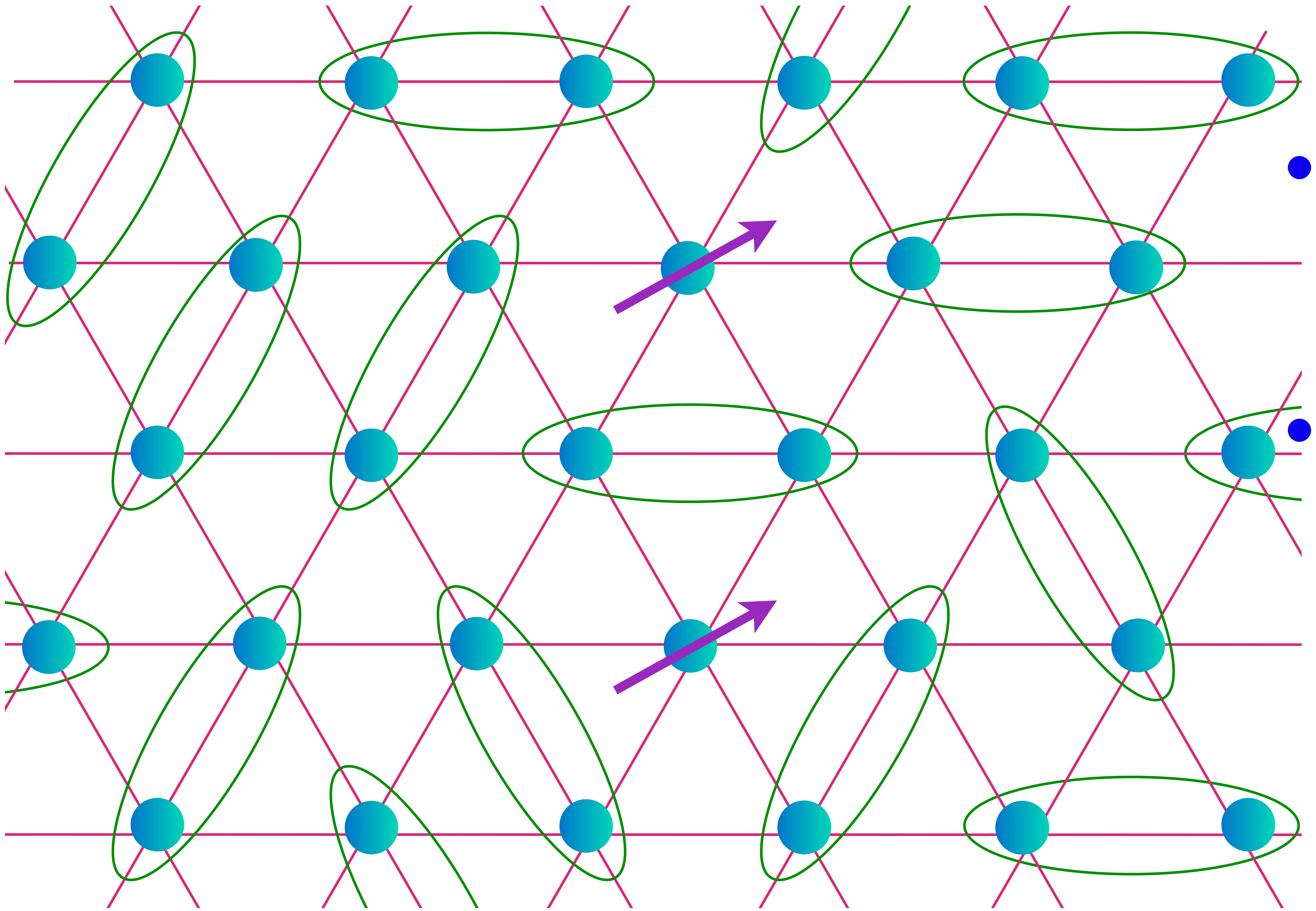

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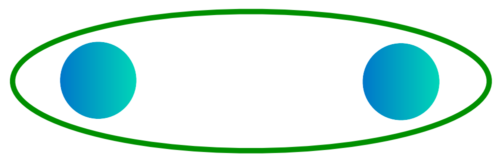

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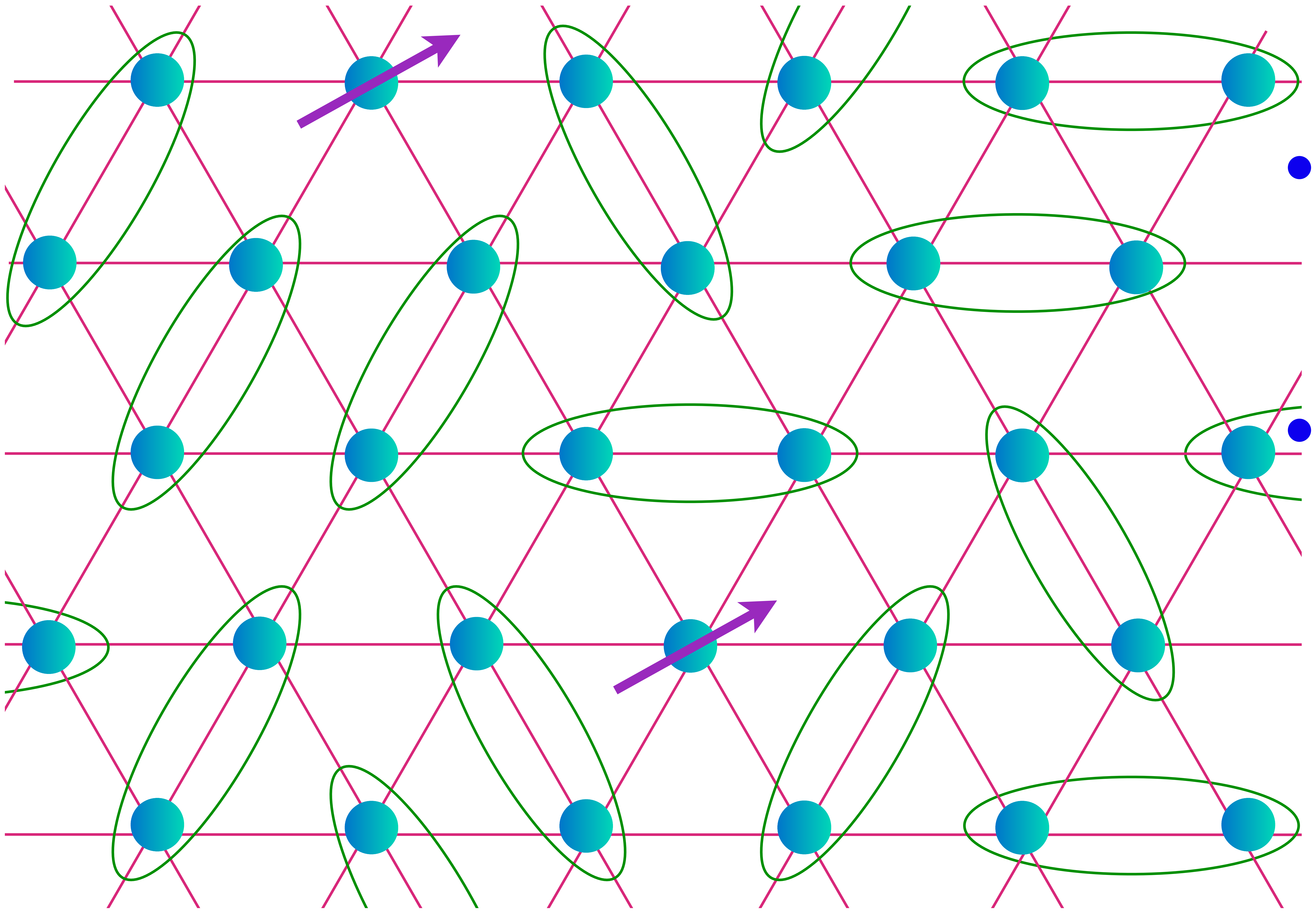

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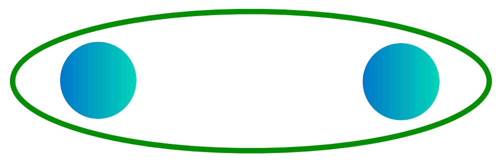

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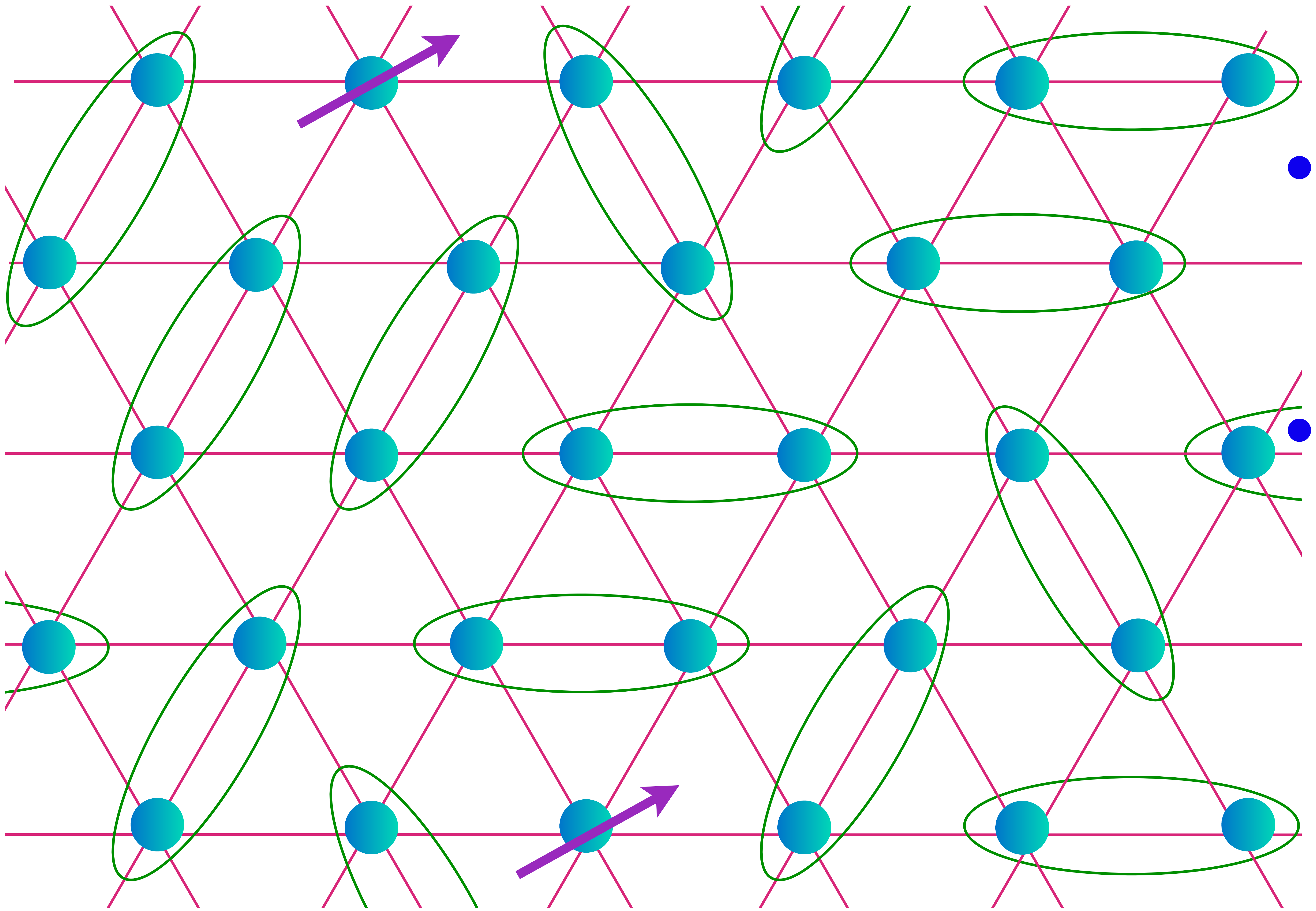

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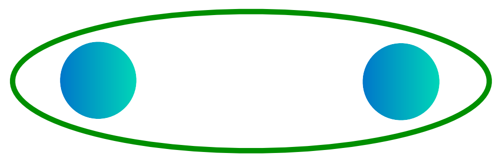

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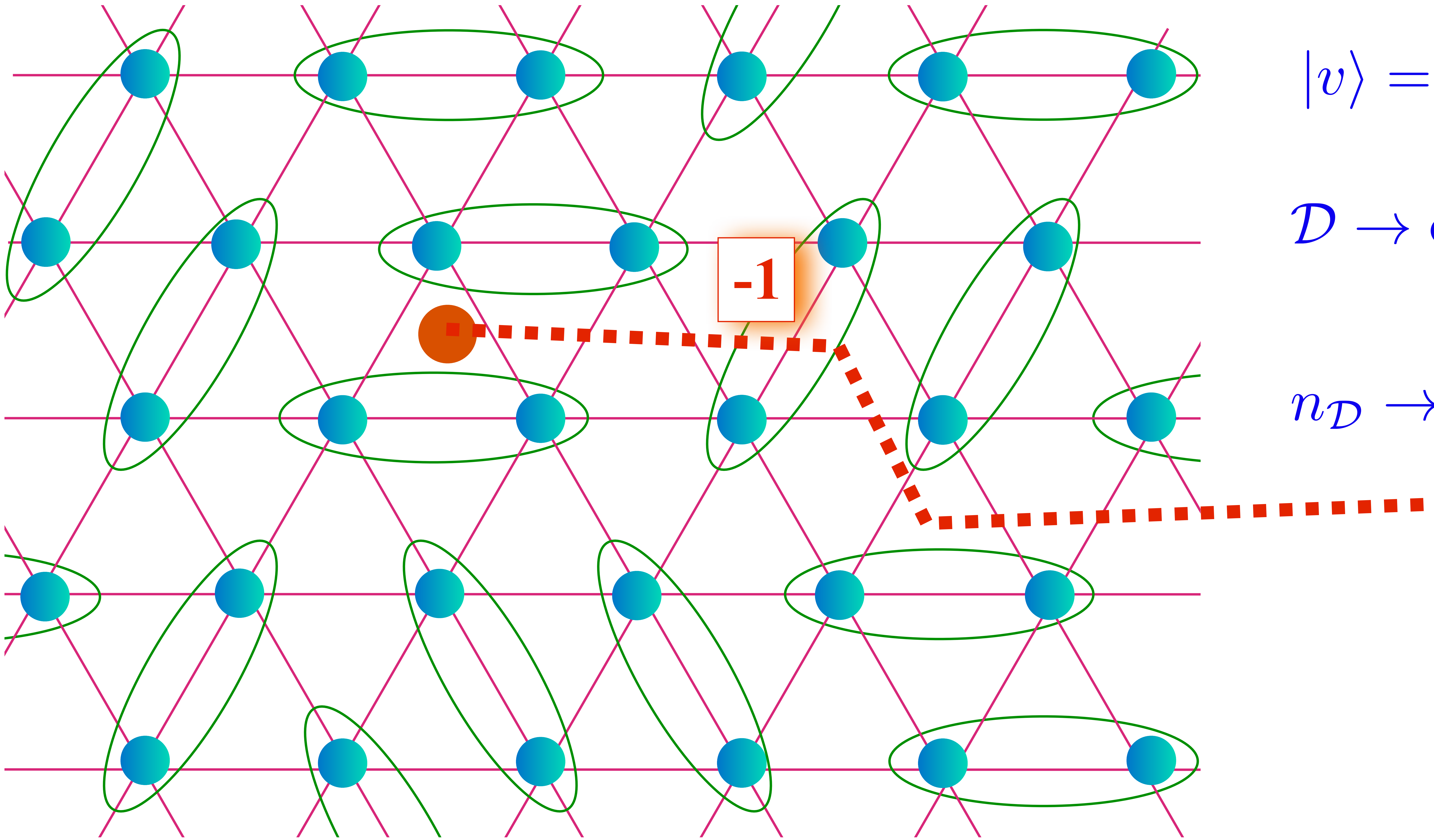


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$\mathcal{D} \to$ dimer covering of lattice

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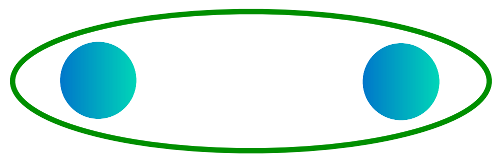

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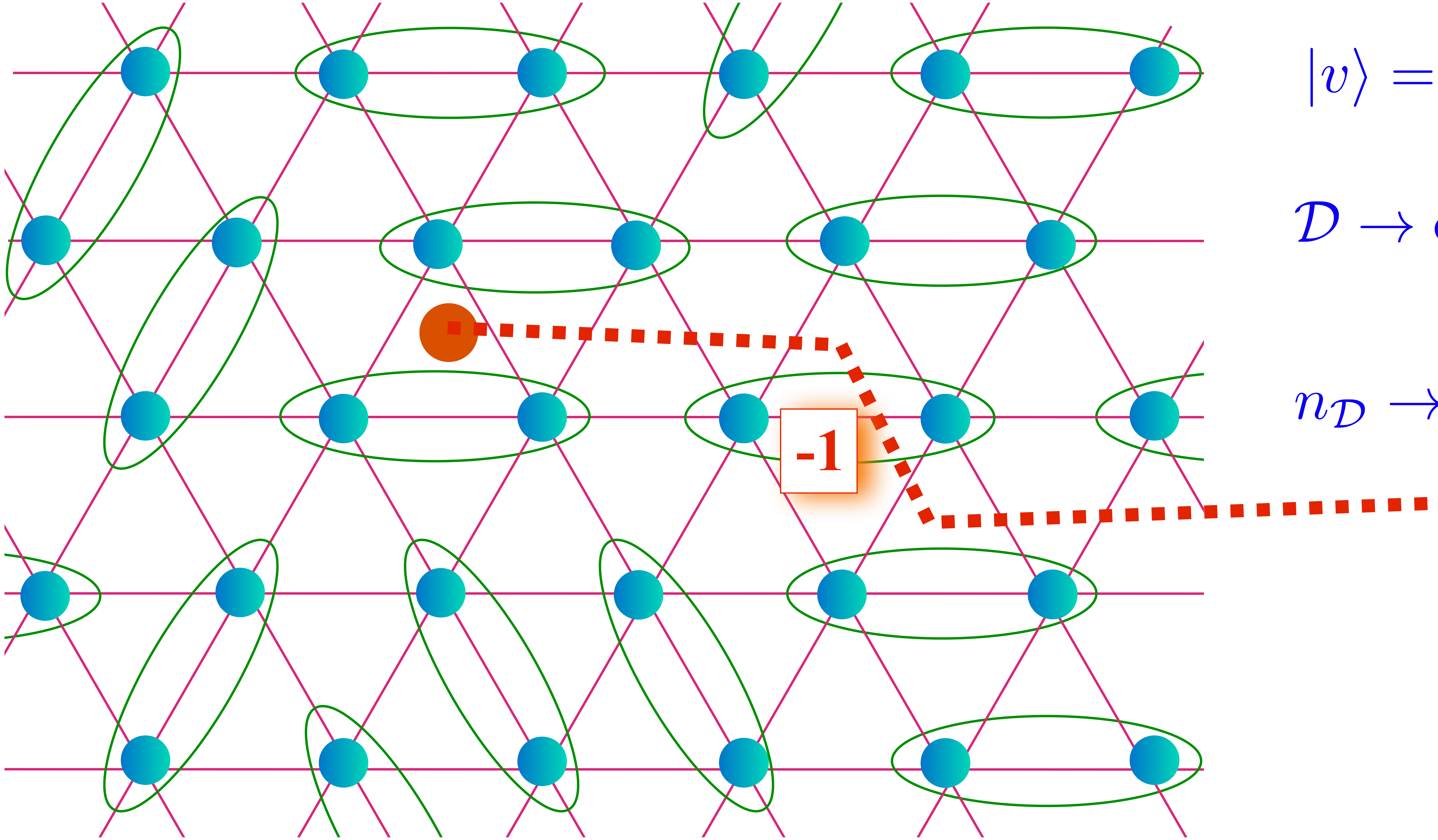


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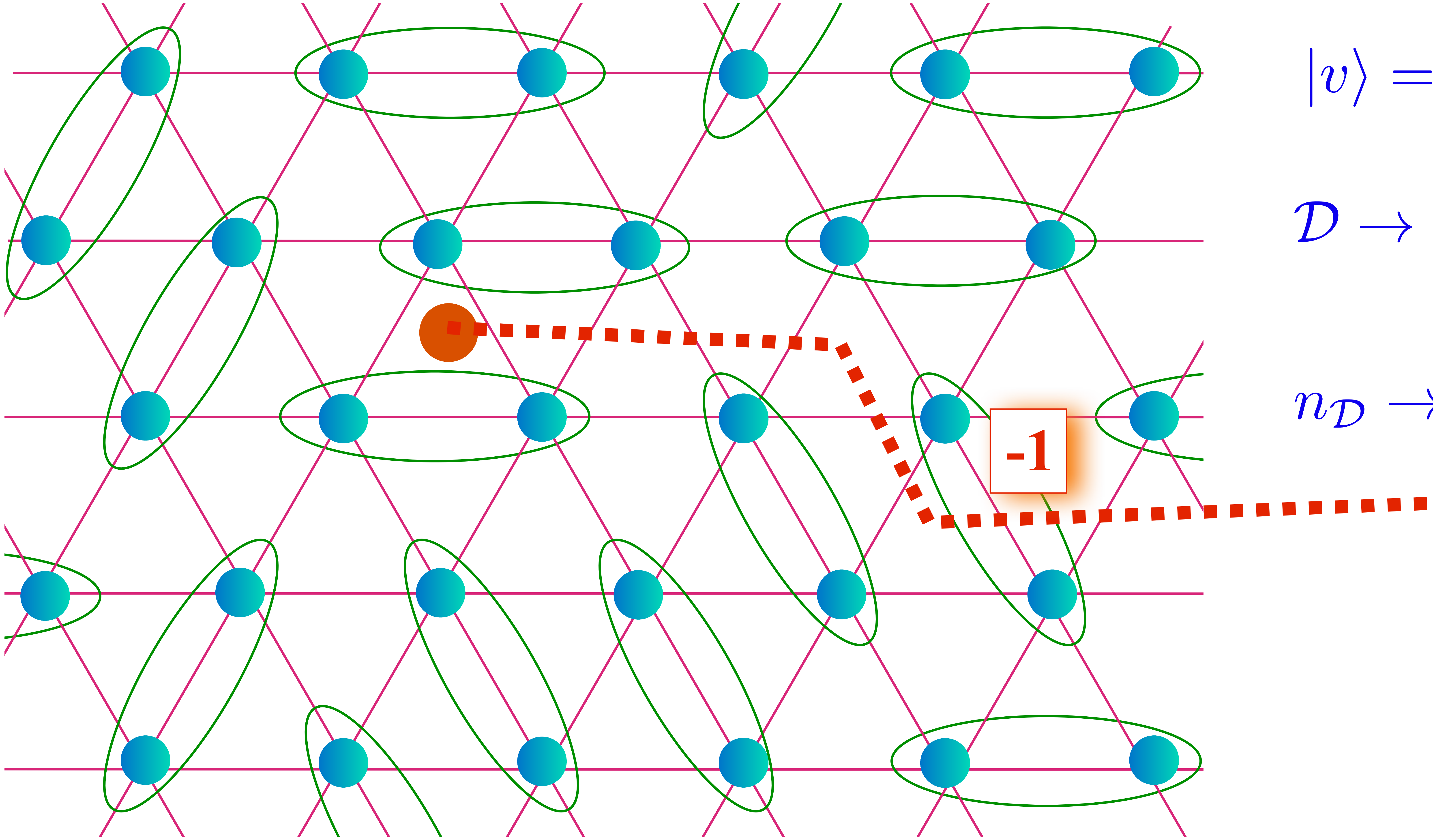


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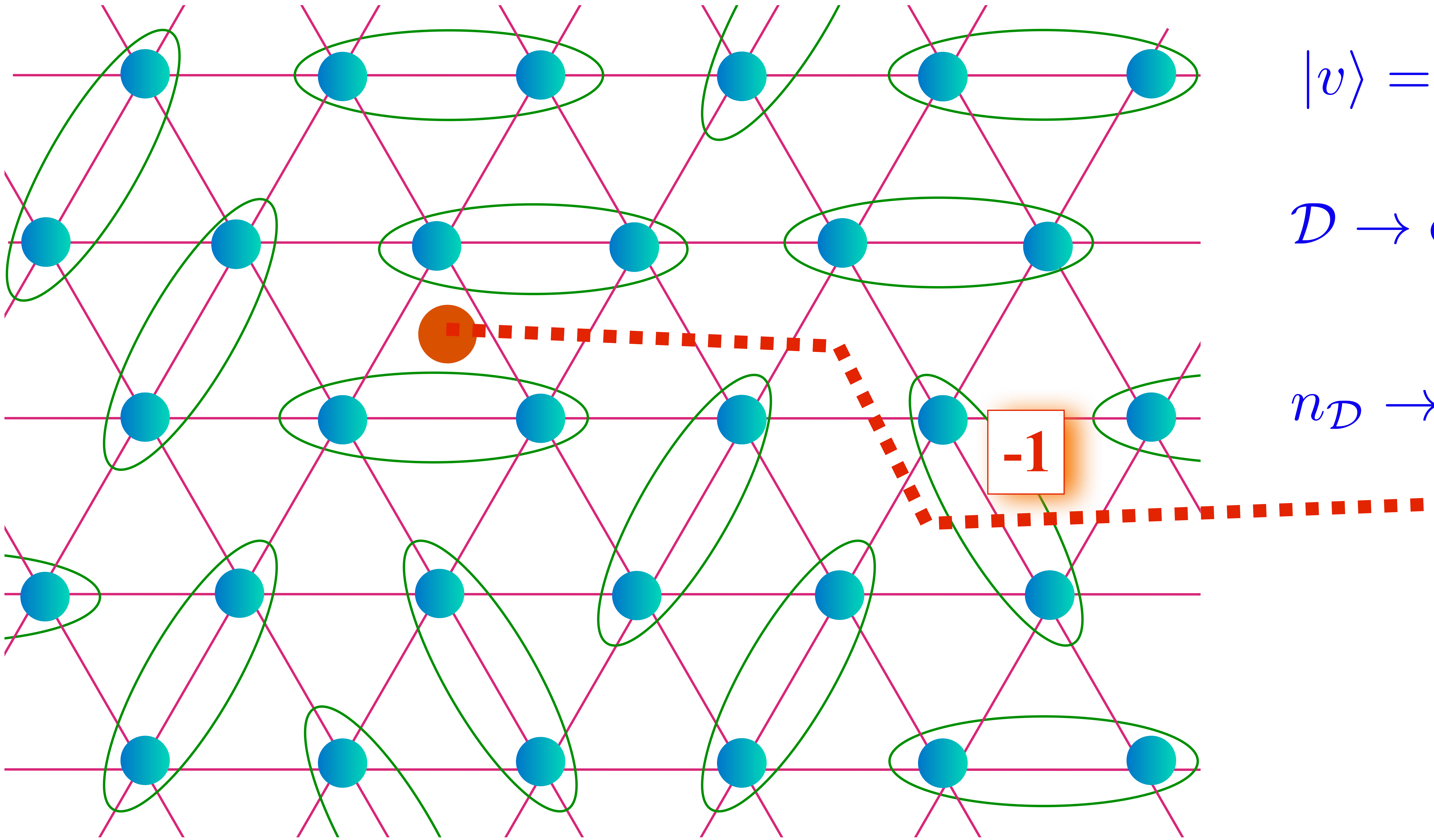


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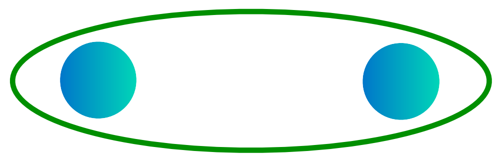

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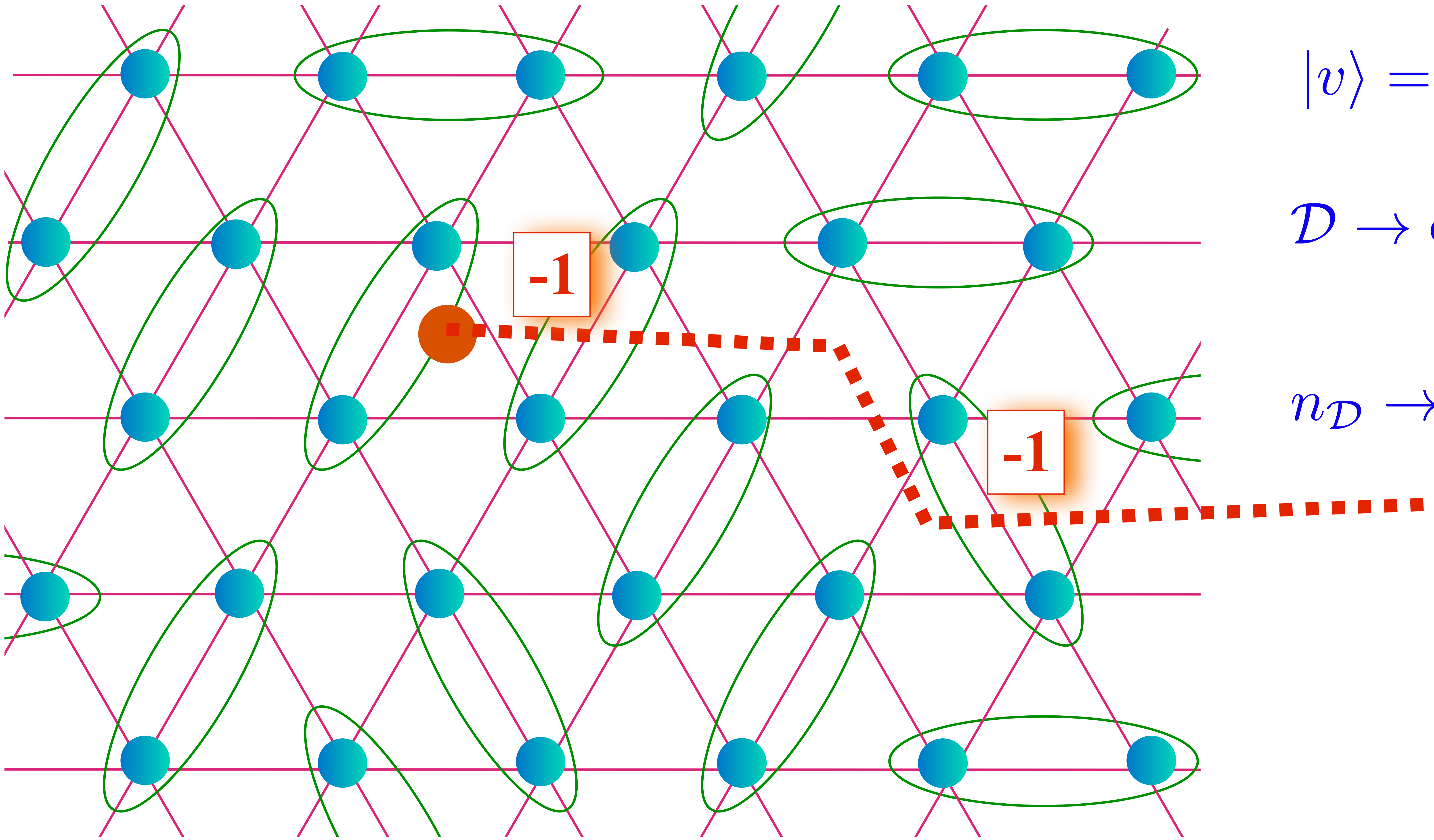


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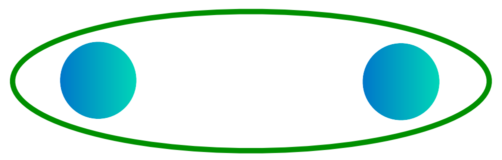

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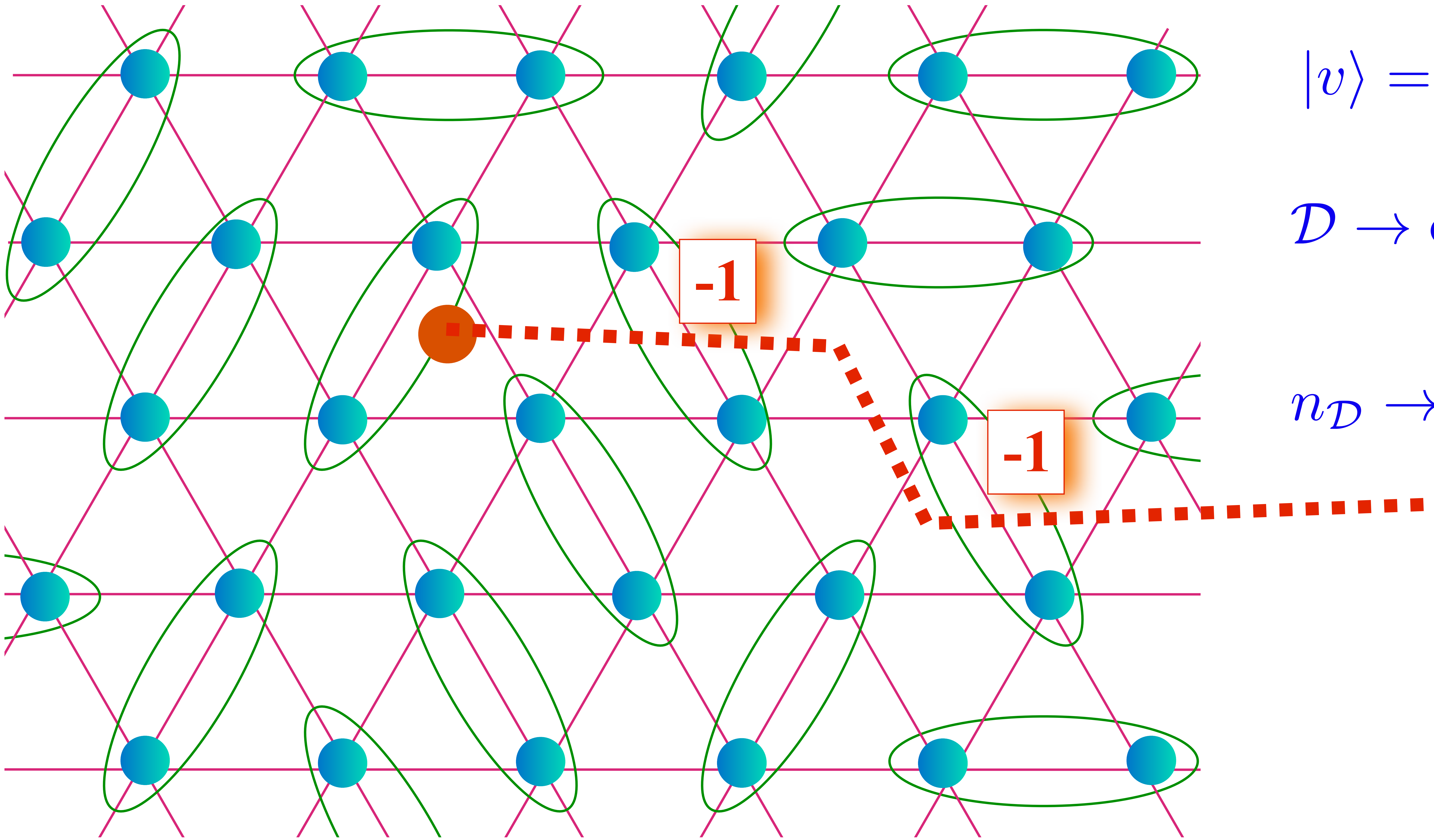


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# Excitations of the $Z_2$ Spin liquid

- A spinon adiabatically transported around a vison picks up a phase factor of $-1$: spinons and visons are mutual semions.
- A bound state of a spinon and a vison picks up a phase factor of $-1$ when exchanged with another bound state of a spinon and a vison:
  - The $\epsilon$ spinon (fermion) is a bound state of the $e$ spinon (boson) and a vison ($\epsilon = e \times m$).
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# Ground state degeneracy on the torus

Place insulator on a torus:

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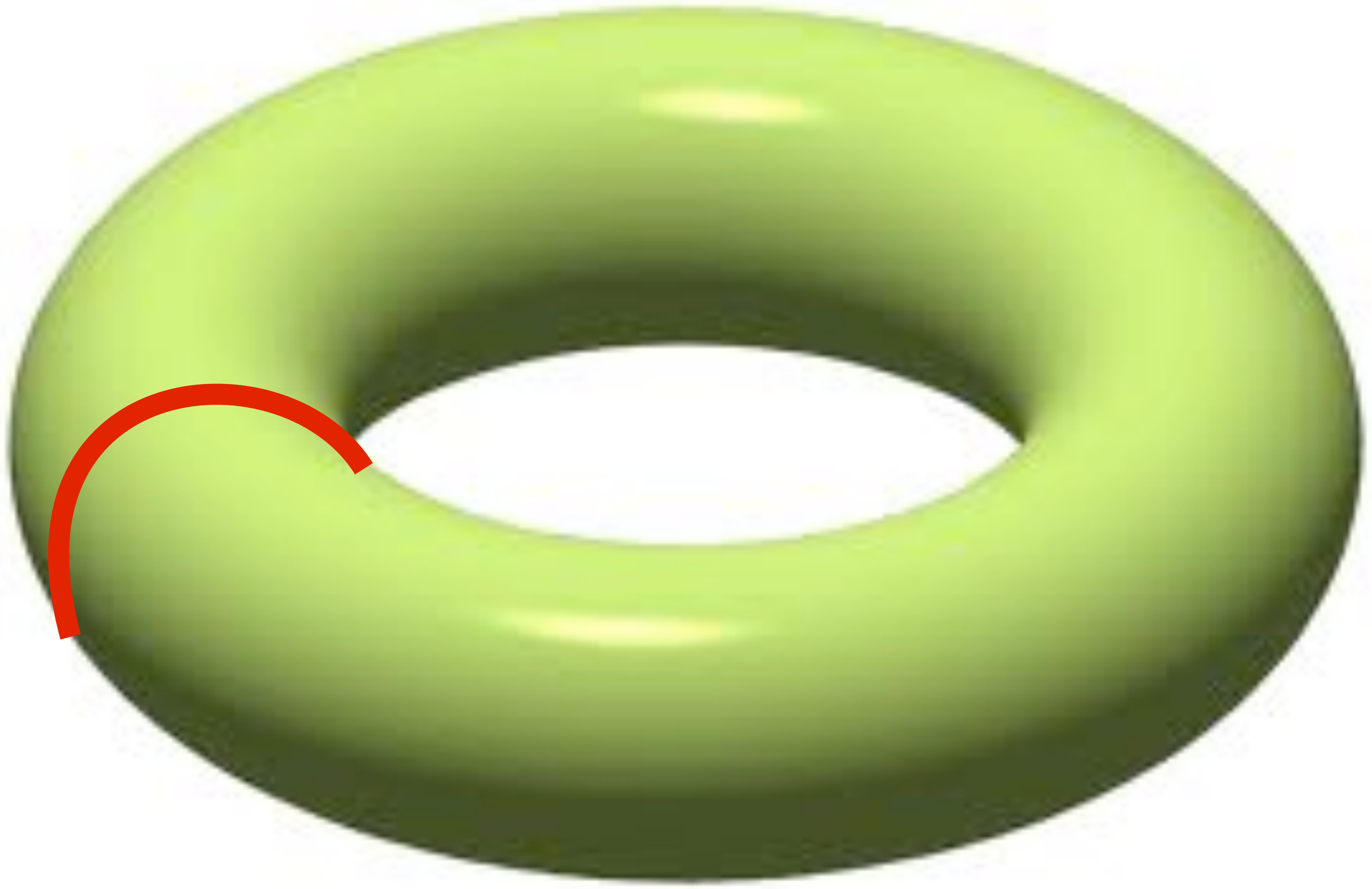
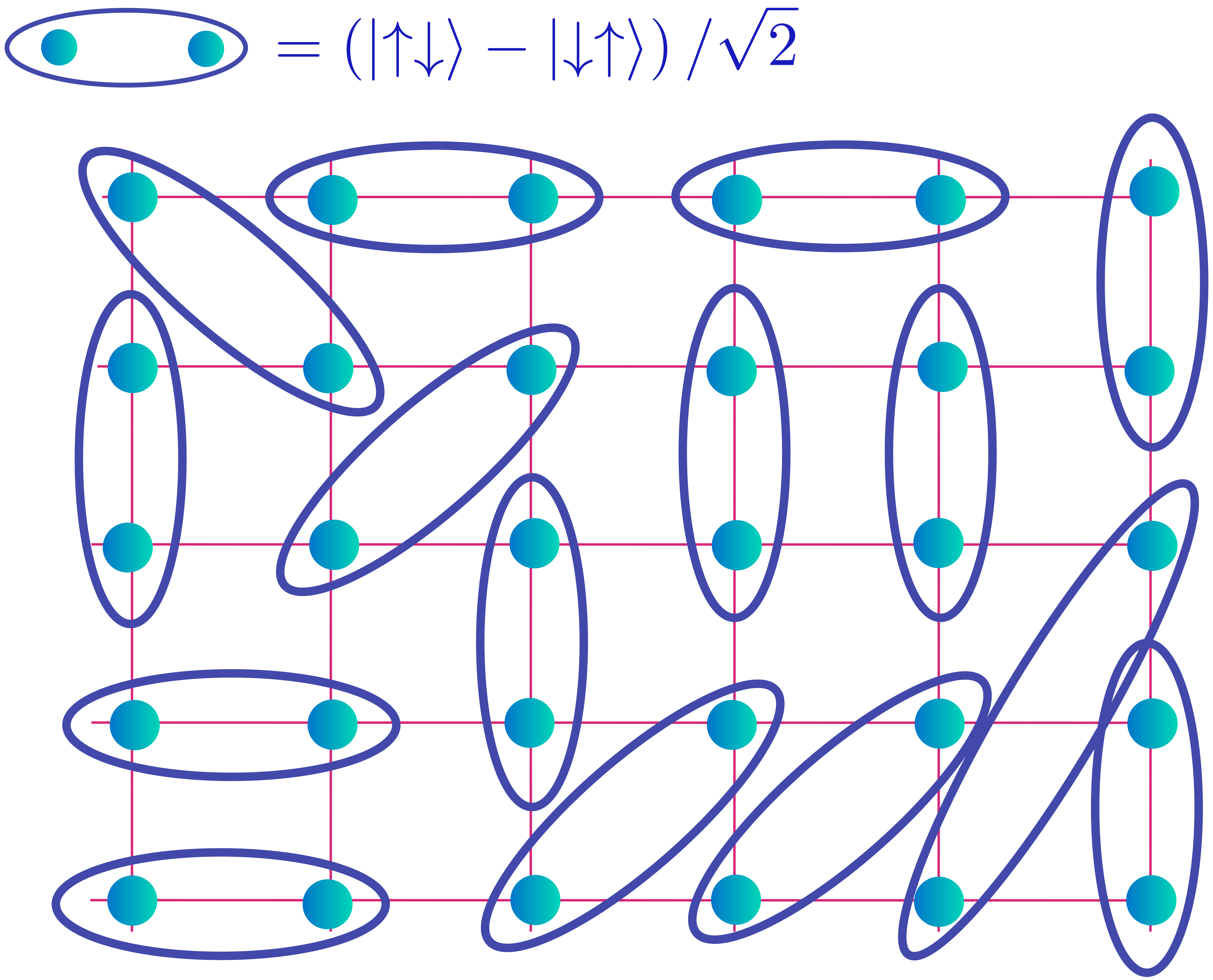


**Place insulator on a torus:**

Obtain a degenerate orthogonal state by modifying the wavefunction on a "branch-cut" encircling the torus.

# Ground state degeneracy on the torus



Place insulator on a torus:

Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

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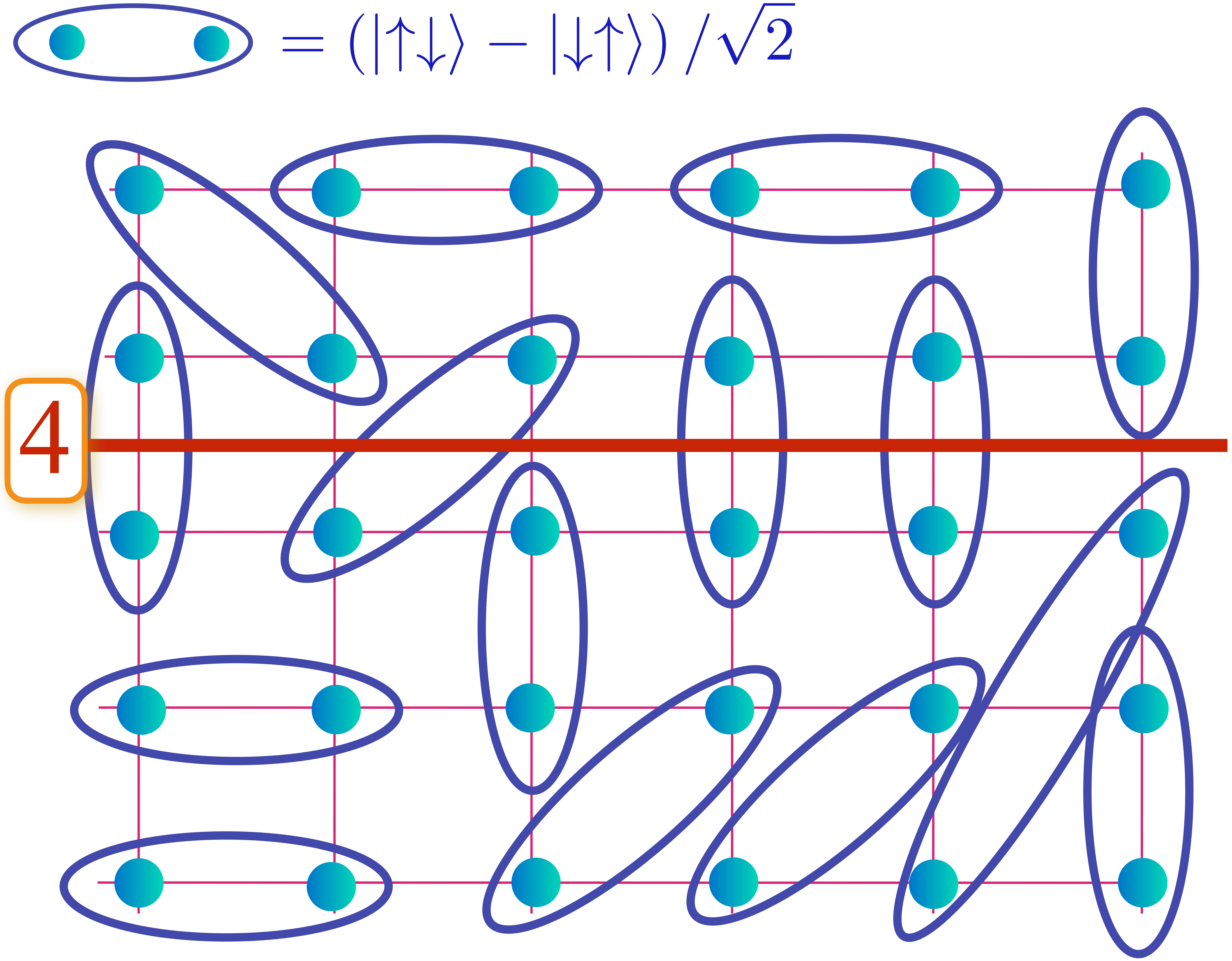


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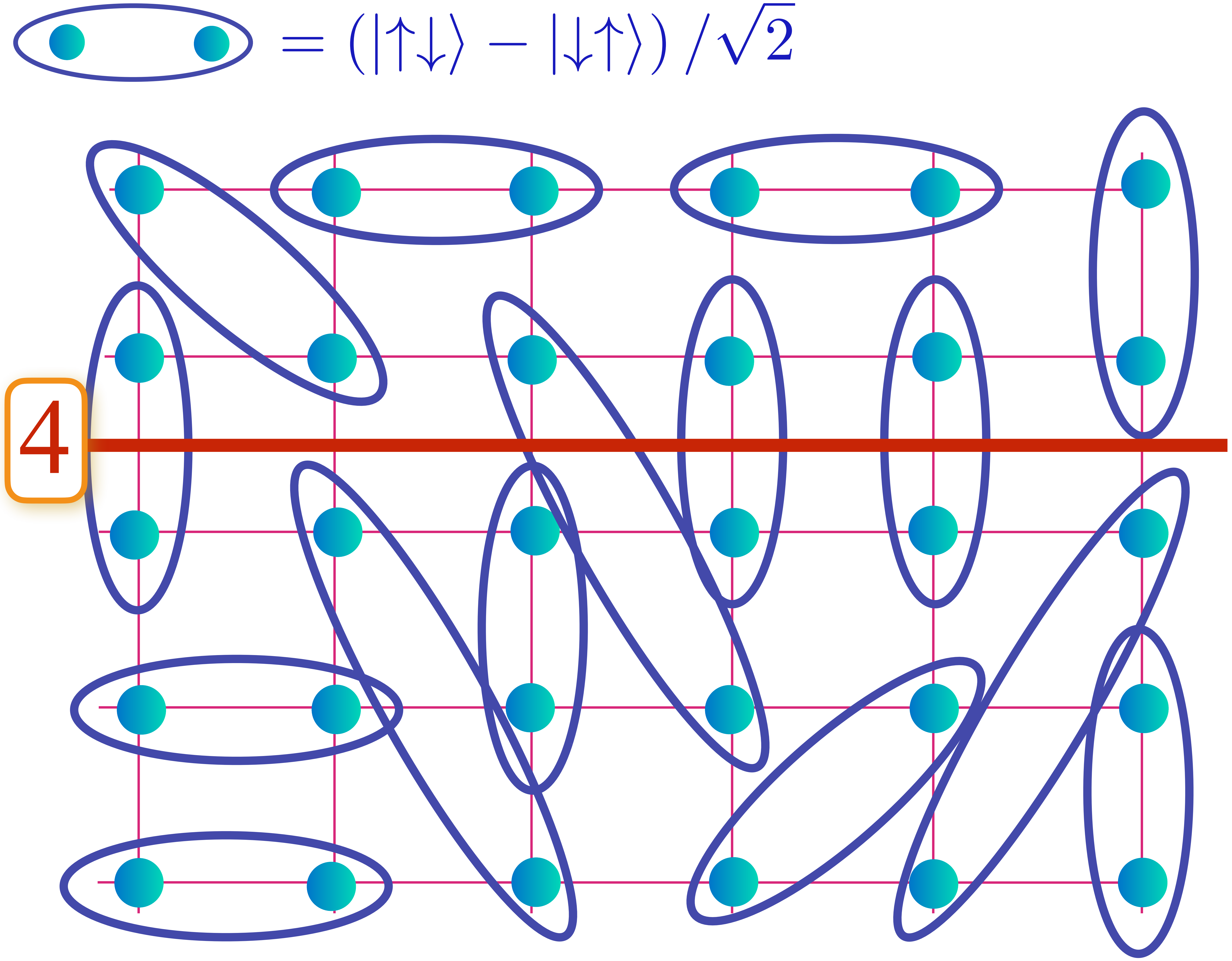


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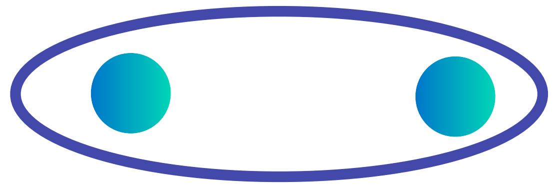
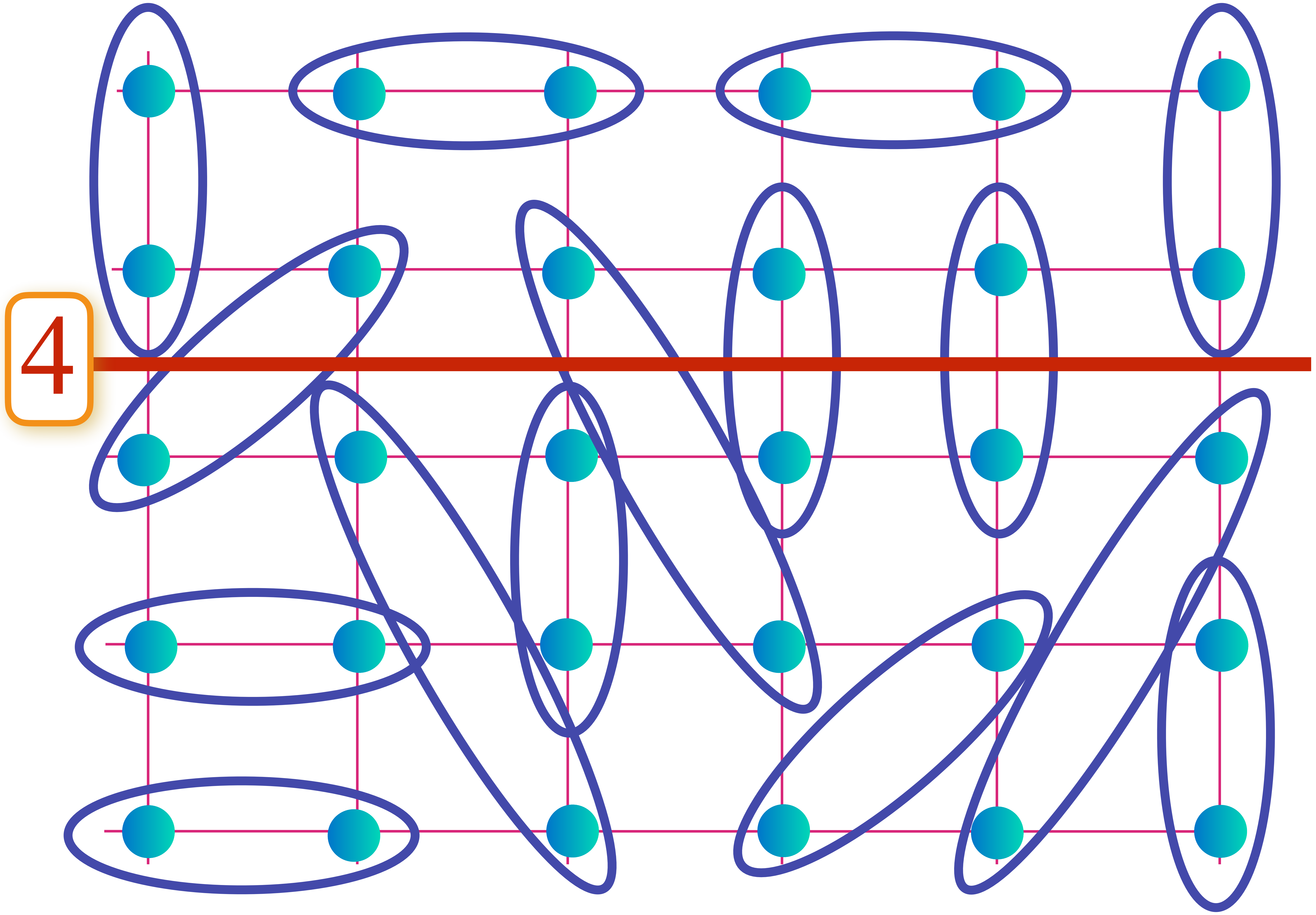

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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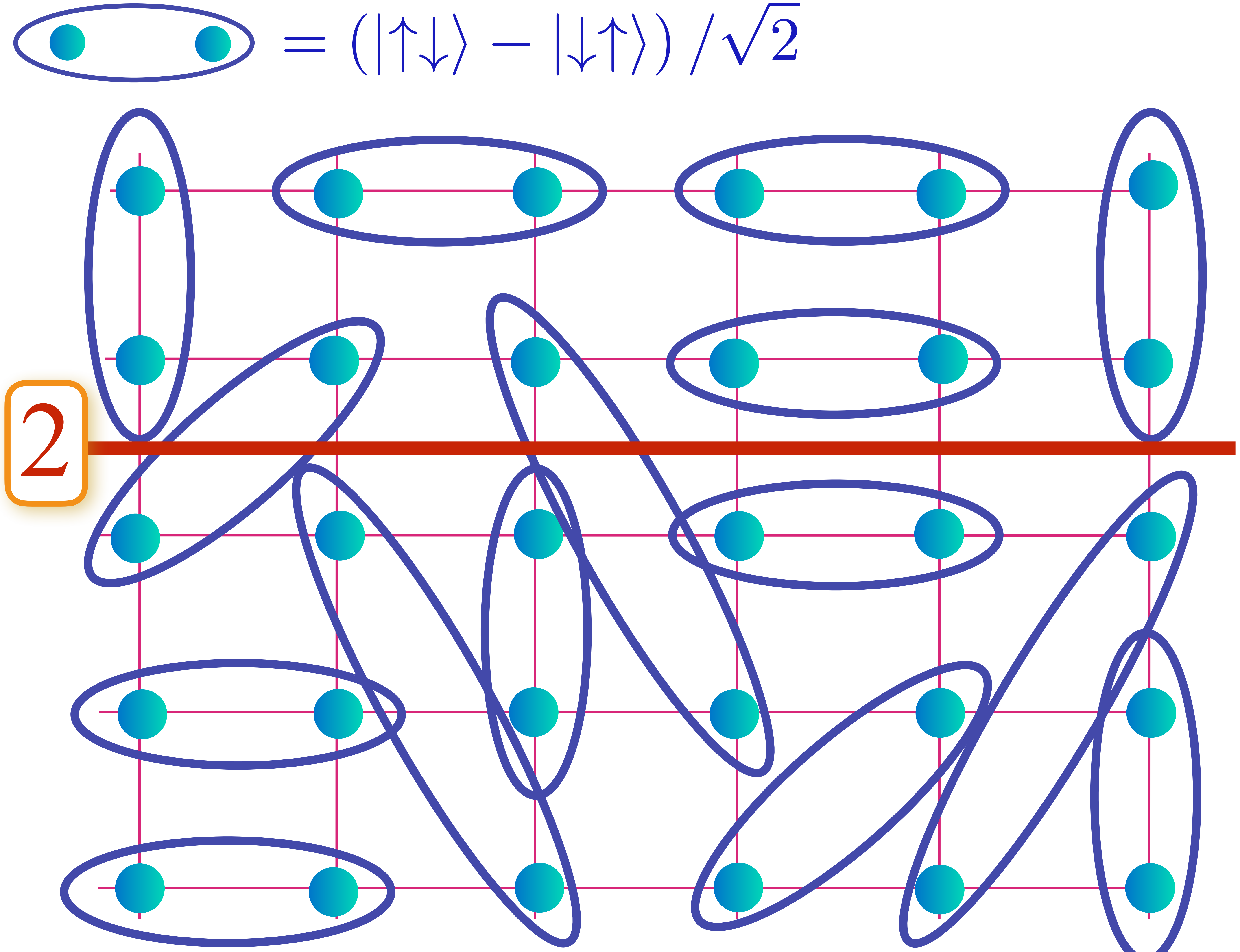


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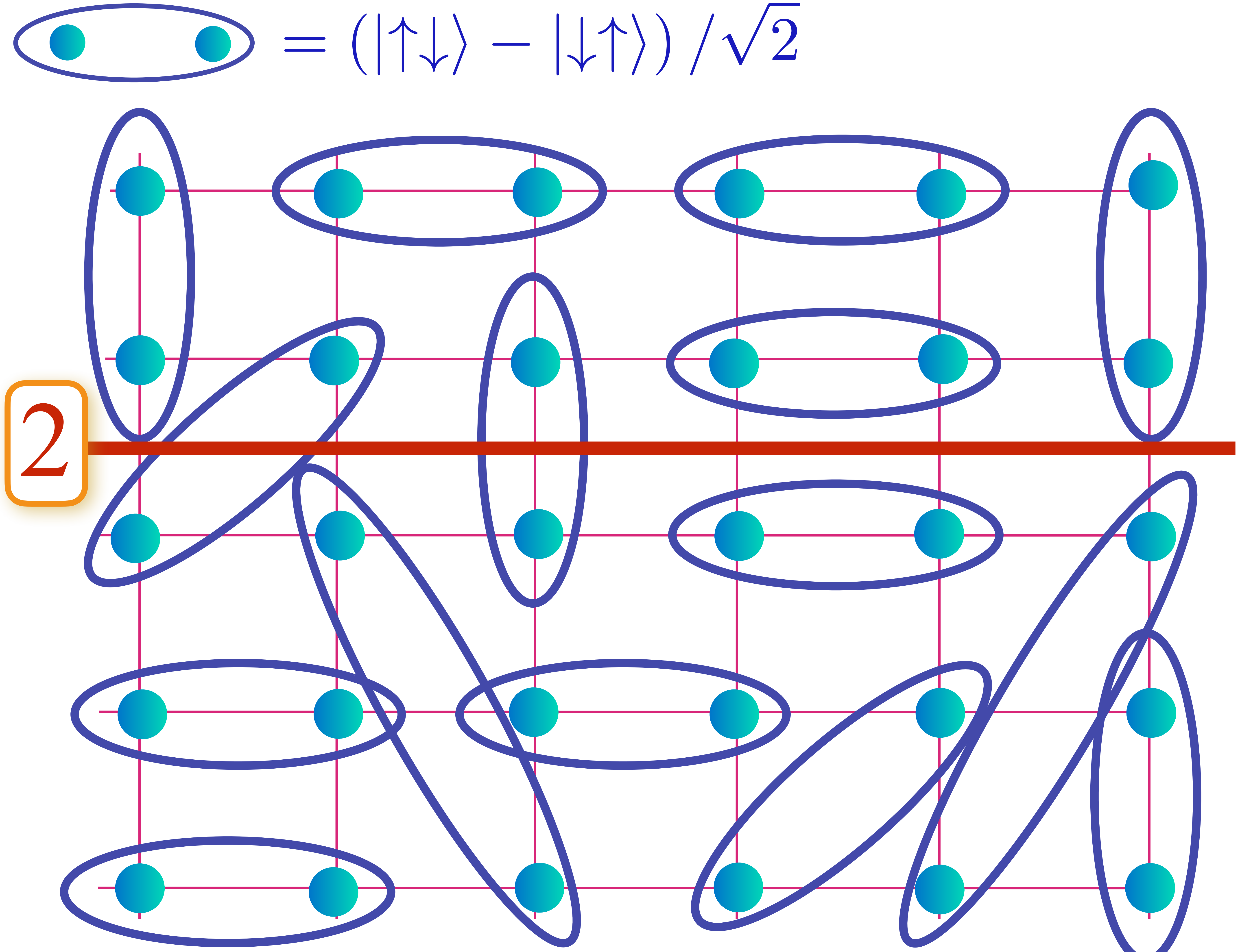


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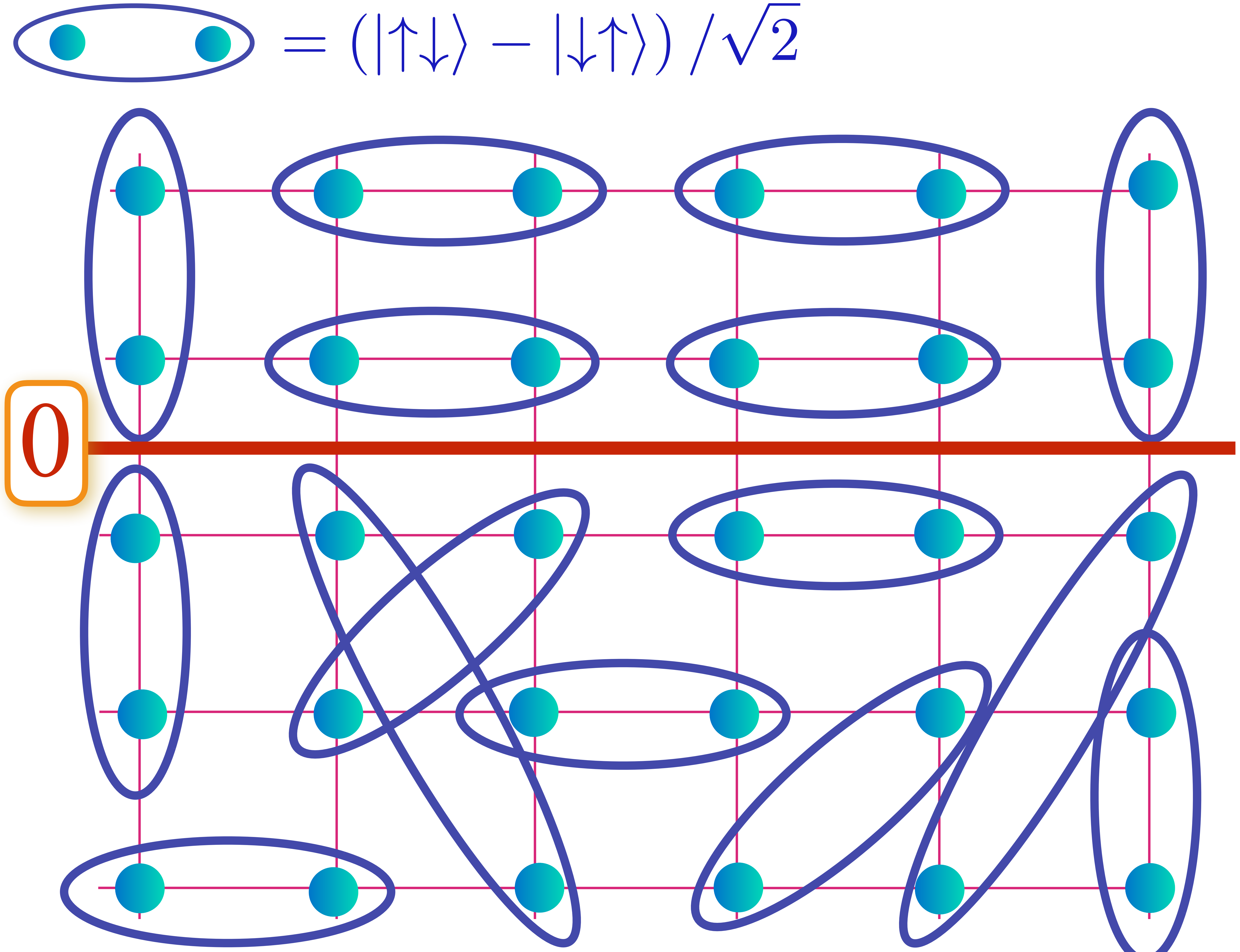


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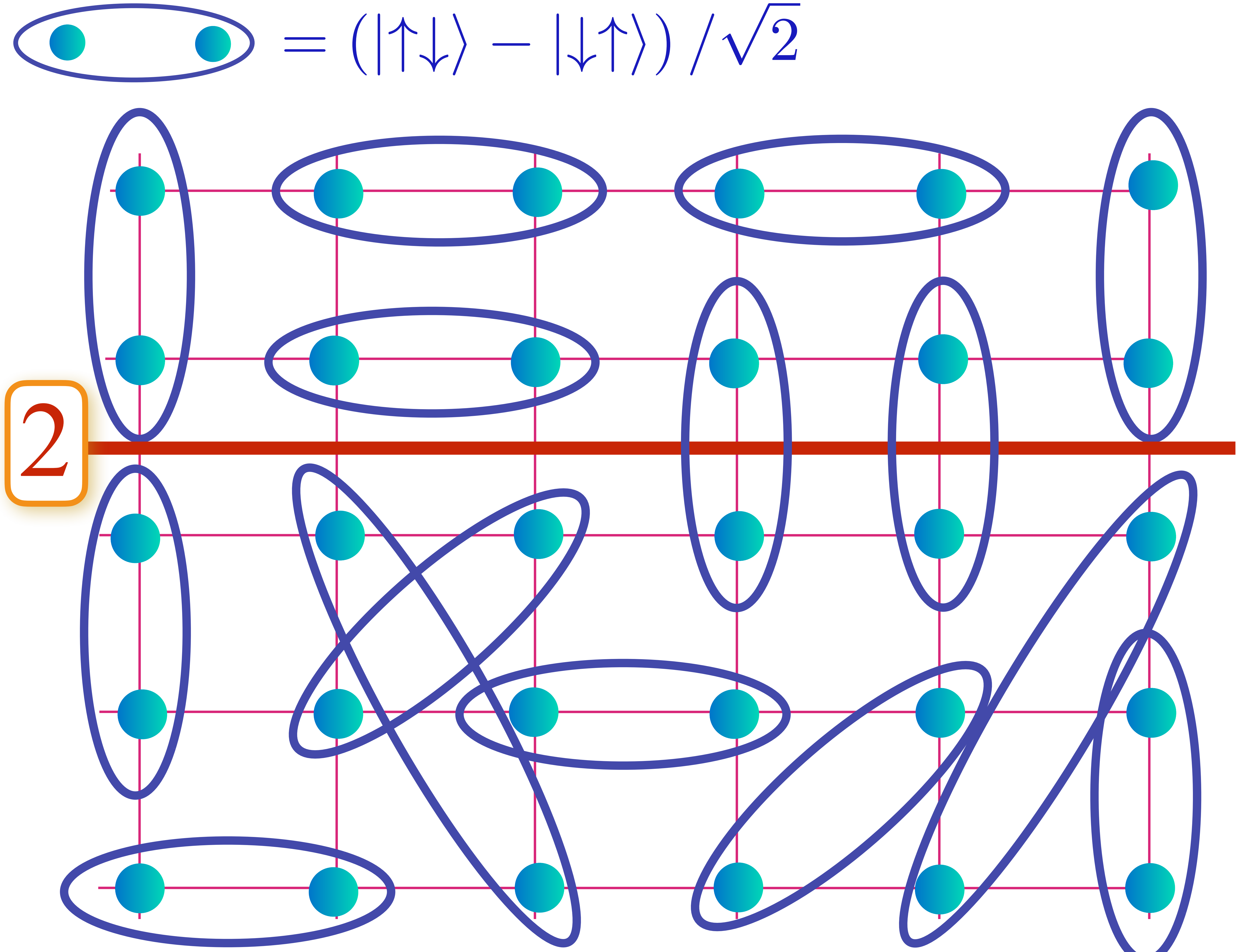


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# $Z_2$ spin liquid for interacting bosons at filling $n$

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  - On a square lattice, the single vison state exhibit ‘translational symmetry fractionalization’ with
  $$T_x T_y = T_y T_x e^{2\pi i n},$$
  with $n$ integer or half-integer.
  - For antiferromagnets of spin $S$, the translational symmetry fractionalization is
  $$T_x T_y = T_y T_x e^{2\pi i S}.$$

R. A. Jalabert and S. Sachdev, PRB **44**, 686 (1991); S. Sachdev and M Vojta, J. Phys. Soc. Jpn. Suppl. B **69**, (2000); T. Senthil and M.P.A. Fisher, PRB **62**, 7850 (2000).

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*The $Z_2$ spin liquid*

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*Gauge theories of partons*

Rhine Samajdar


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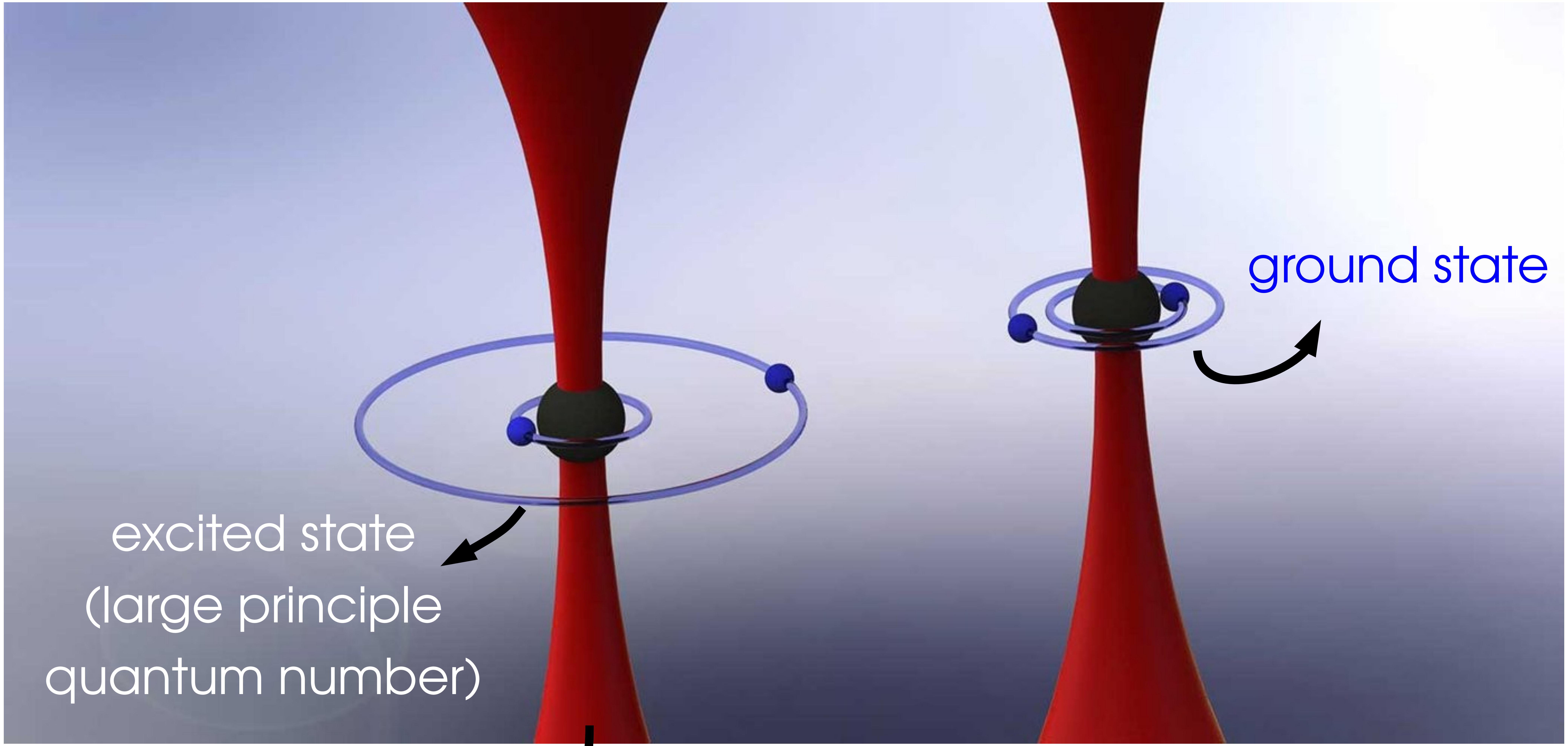



$$V_{|i-j|} \sim |i-j|^{-6}$$

optical tweezer (traps atom)

Fig: https://www.caltech.edu/about/news/quantum-innovations-achieved-using-alkaline-earth-atoms

$$H_{\mathrm{Ryd}} = \sum_i \left[ \frac{\Omega}{2} \left( |g\rangle\langle r| + |r\rangle\langle g| \right)_i - \delta |r\rangle\langle r| \right] + \sum_{(i,j)} V_{|i-j|} \Big( |r\rangle\langle r|_i \otimes |r\rangle\langle r|_j \Big)$$

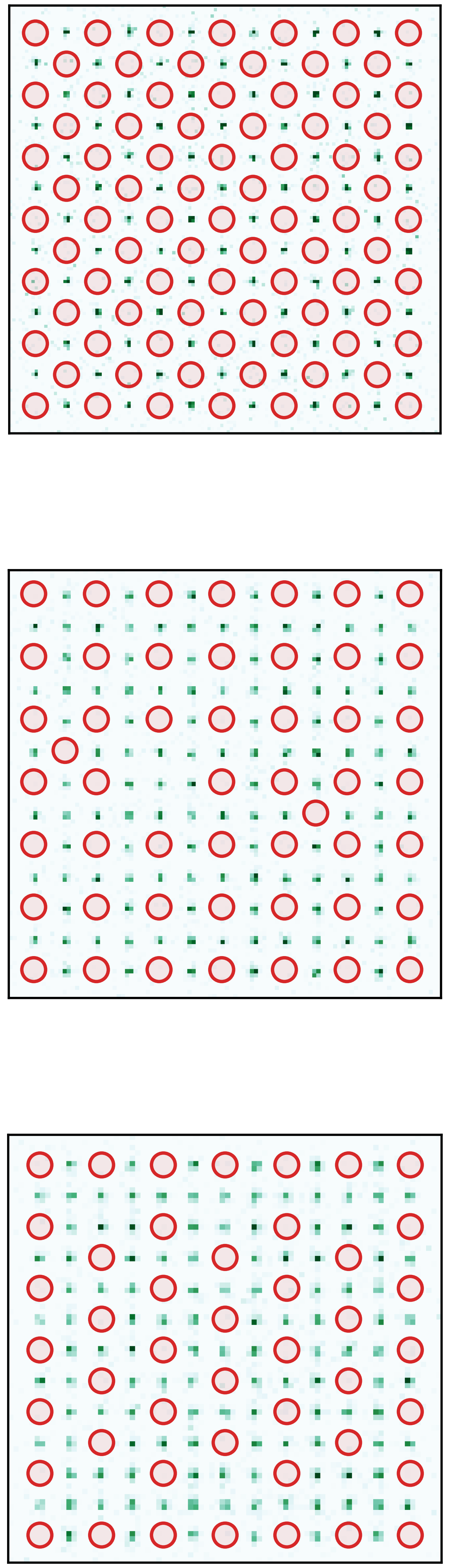
# *Rydberg atoms on the square lattice: theory*

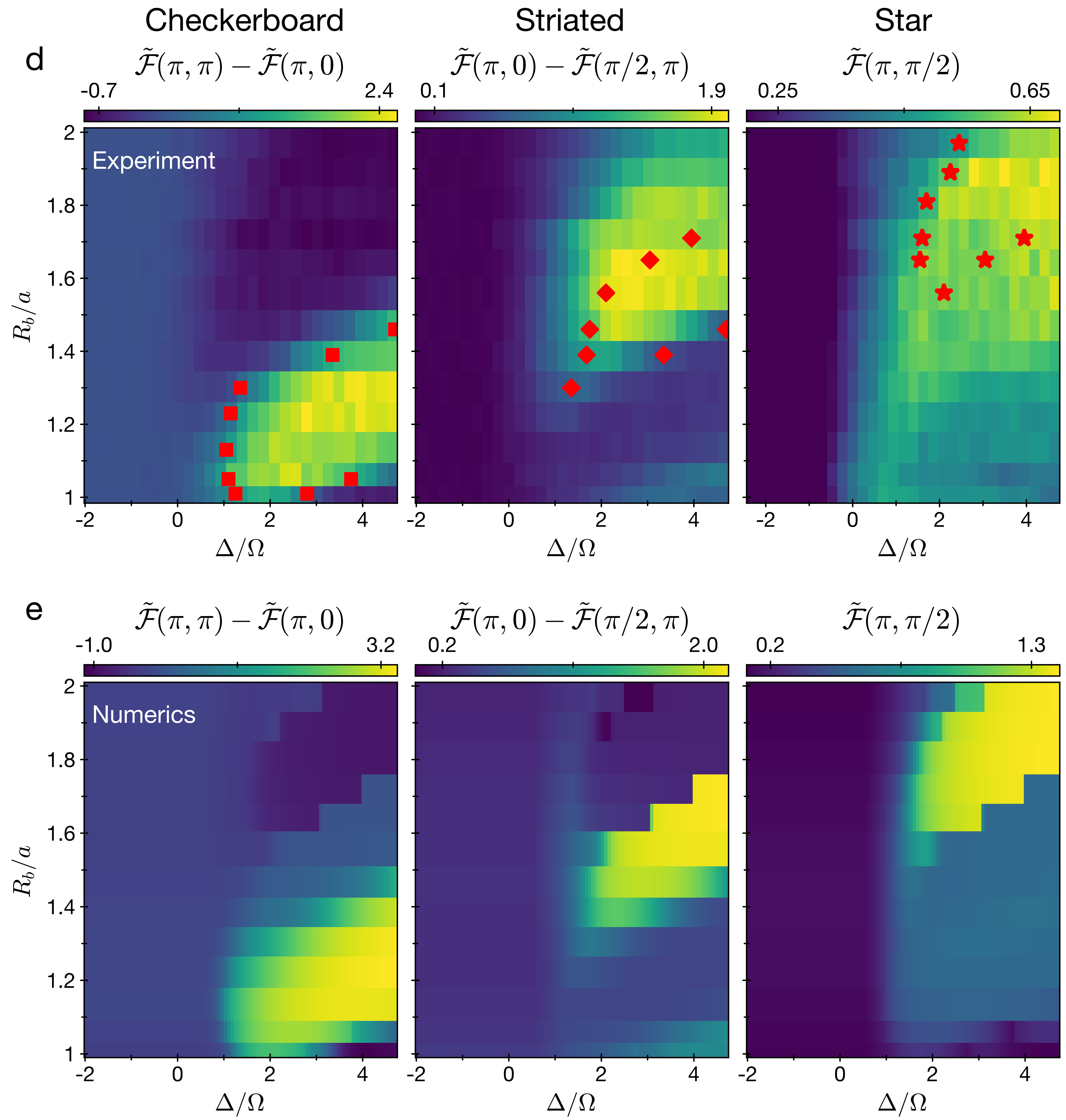

R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and S. Sachdev, Physical Review Letters **124**, 103601 (2020)

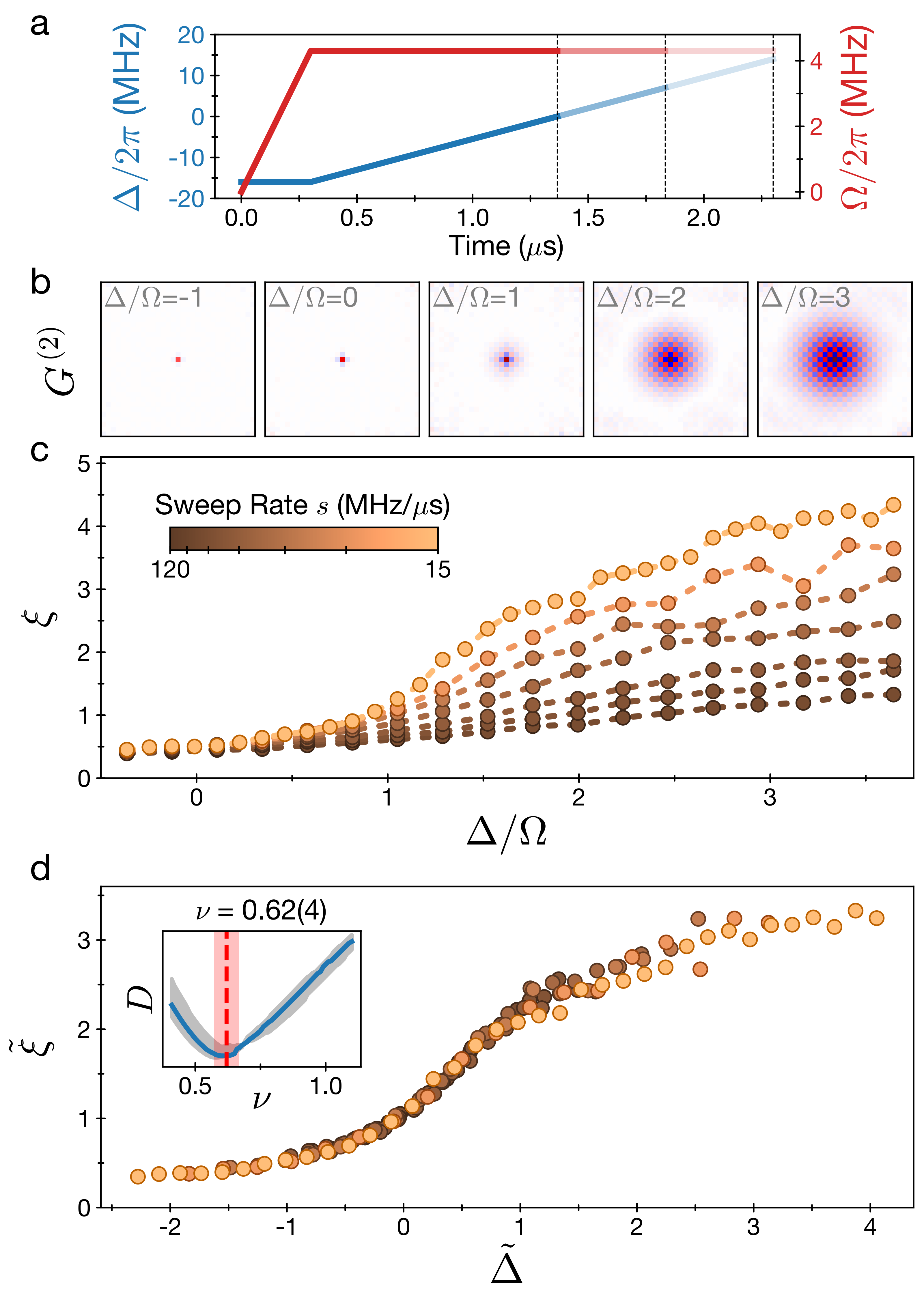
# Rydberg atoms on the square lattice: experiment

$$\tilde{\xi} = \xi (s/s_0)^{\mu}$$

$$\tilde{\Delta} = (\Delta - \Delta_c)(s/s_0)^{\kappa}$$






Sepehr Ebadi, Tout T. Wang, Harry Levine, Alexander Keesling, Giulia Semeghini, Ahmed Omran, Dolev Bluvstein, Rhine Samajdar, Hannes Pichler, Wen Wei Ho, Soonwon Choi, Subir Sachdev, Markus Greiner, Vladan Vuletic, and Mikhail D. Lukin, arXiv:2012.12281
Pascal Scholl et al. arXiv:2012.12268

First observation of Ising quantum phase transition in 2+1 dimensions

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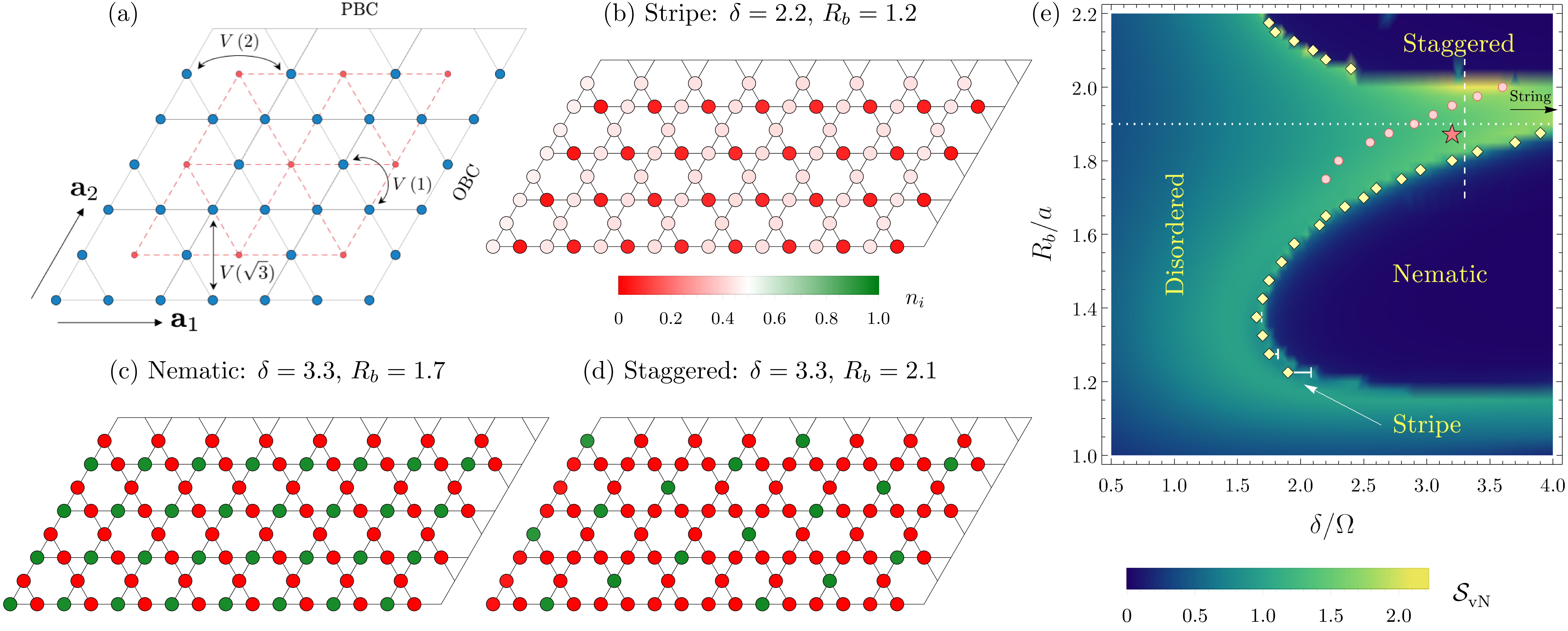


R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and S. Sachdev, PNAS **118**, e2015785118 (2021)

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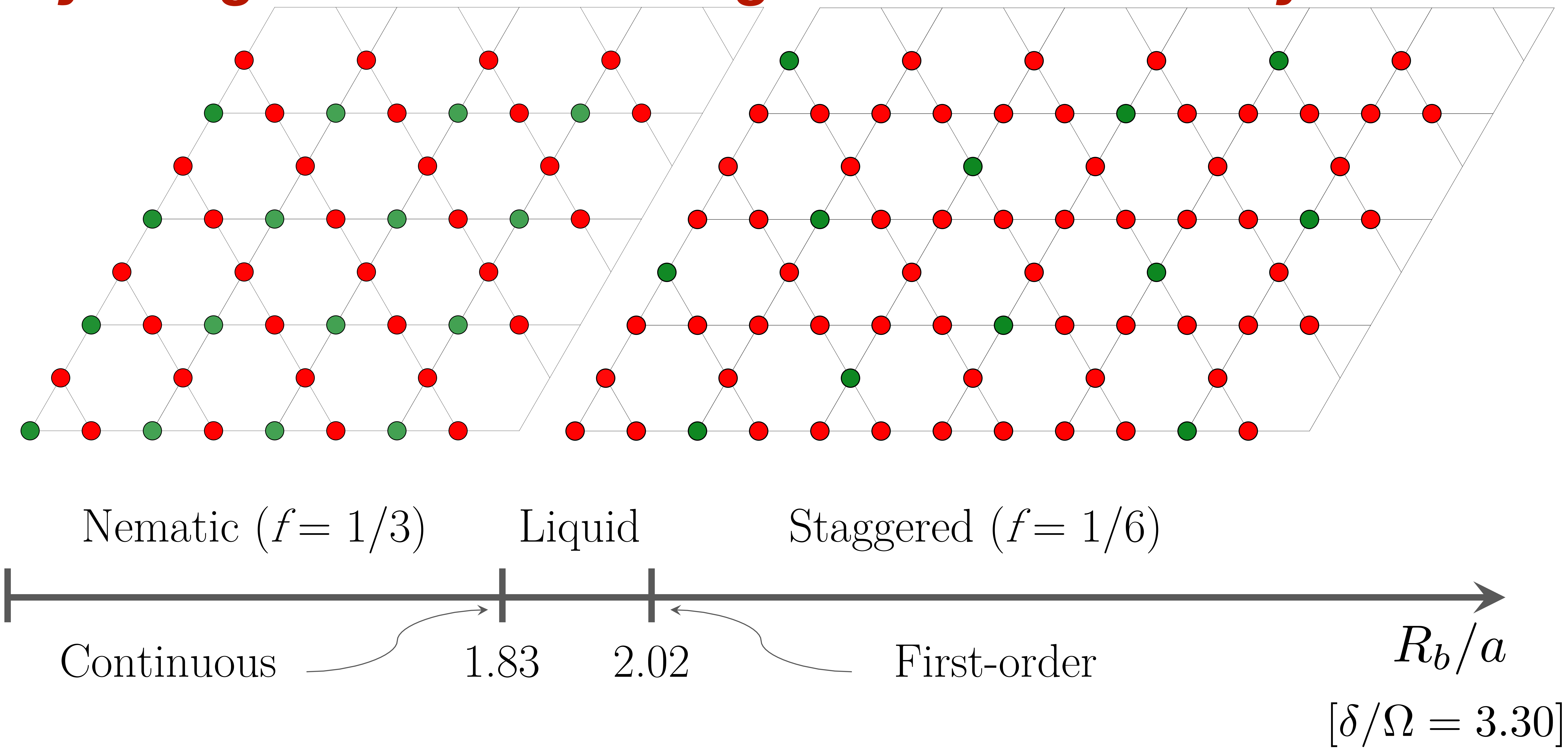


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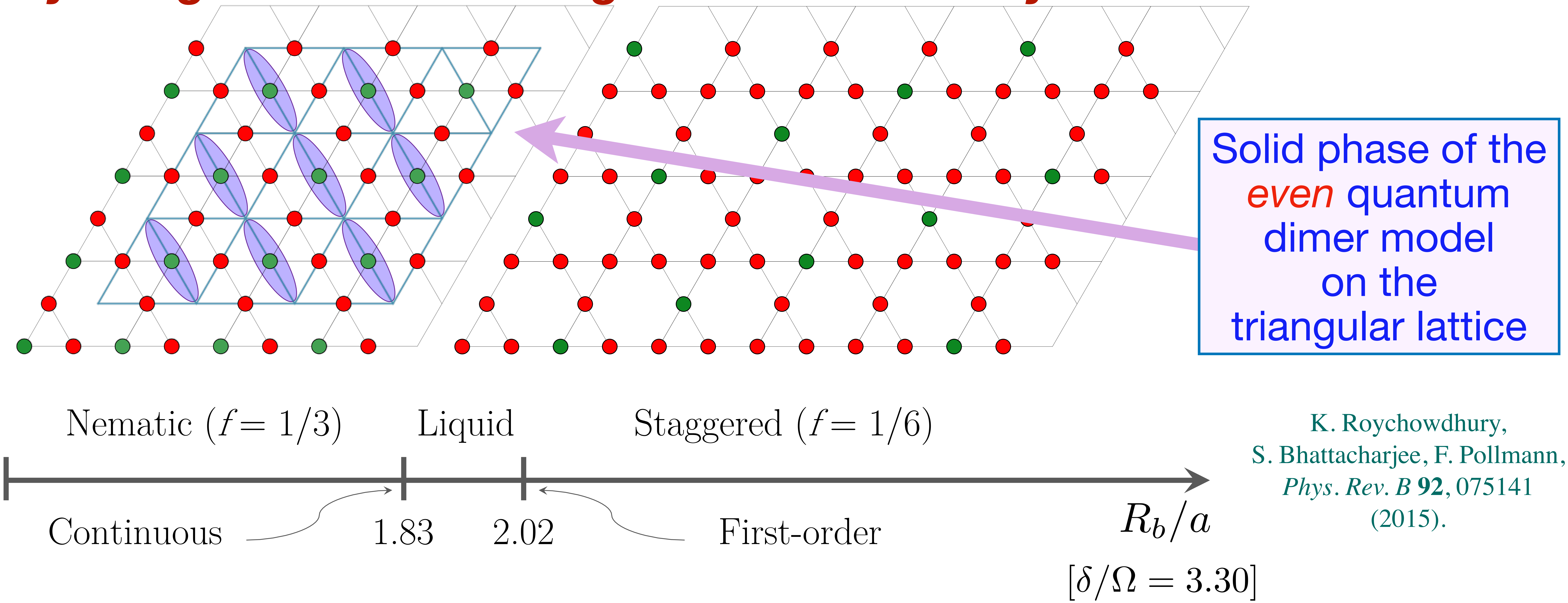


K. Roychowdhury, S. Bhattacharjee, F. Pollmann, *Phys. Rev. B* **92**, 075141 (2015).

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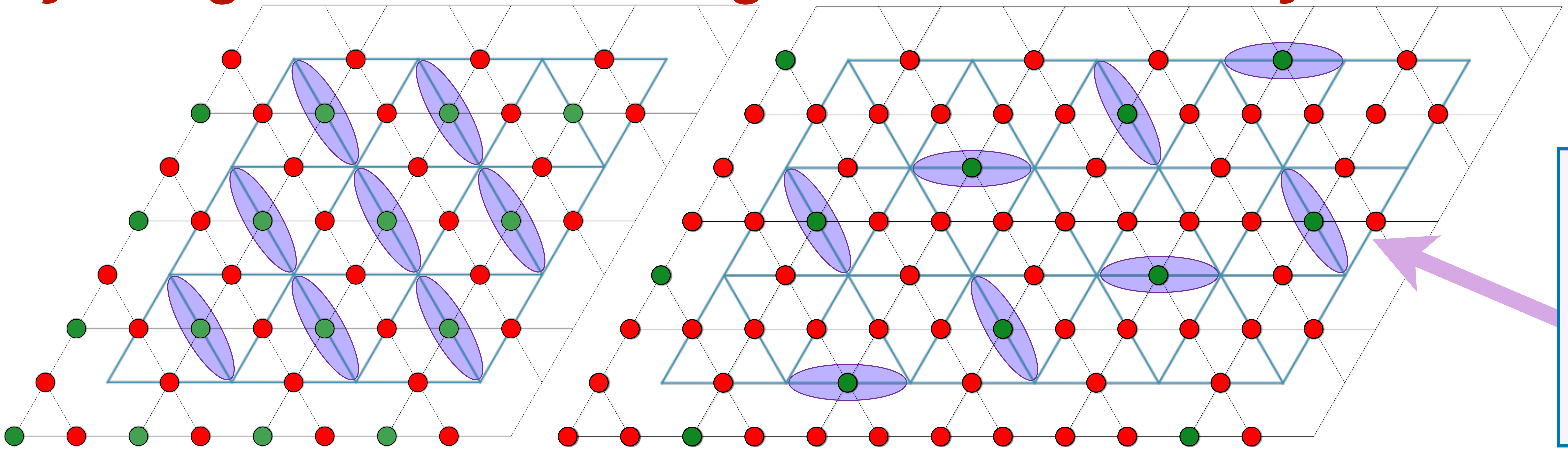
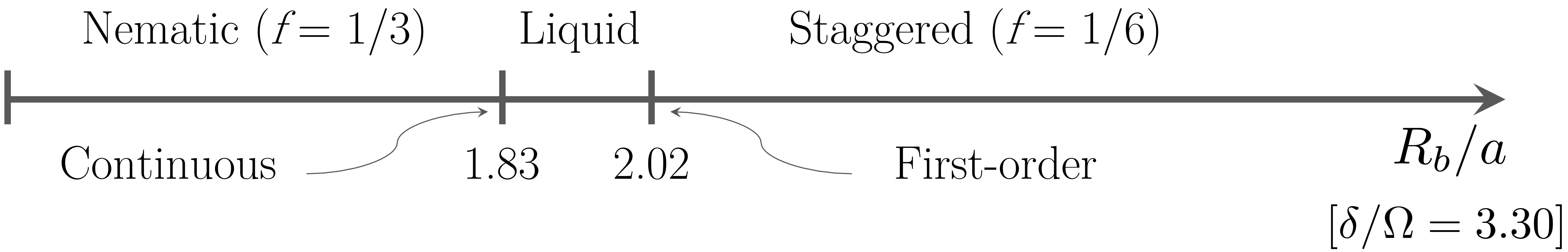


Solid phase of the *odd* quantum dimer model on the triangular lattice




R. Moessner, S. L. Sondhi, *Phys. Rev. Lett.* **86**, 1881 (2001).

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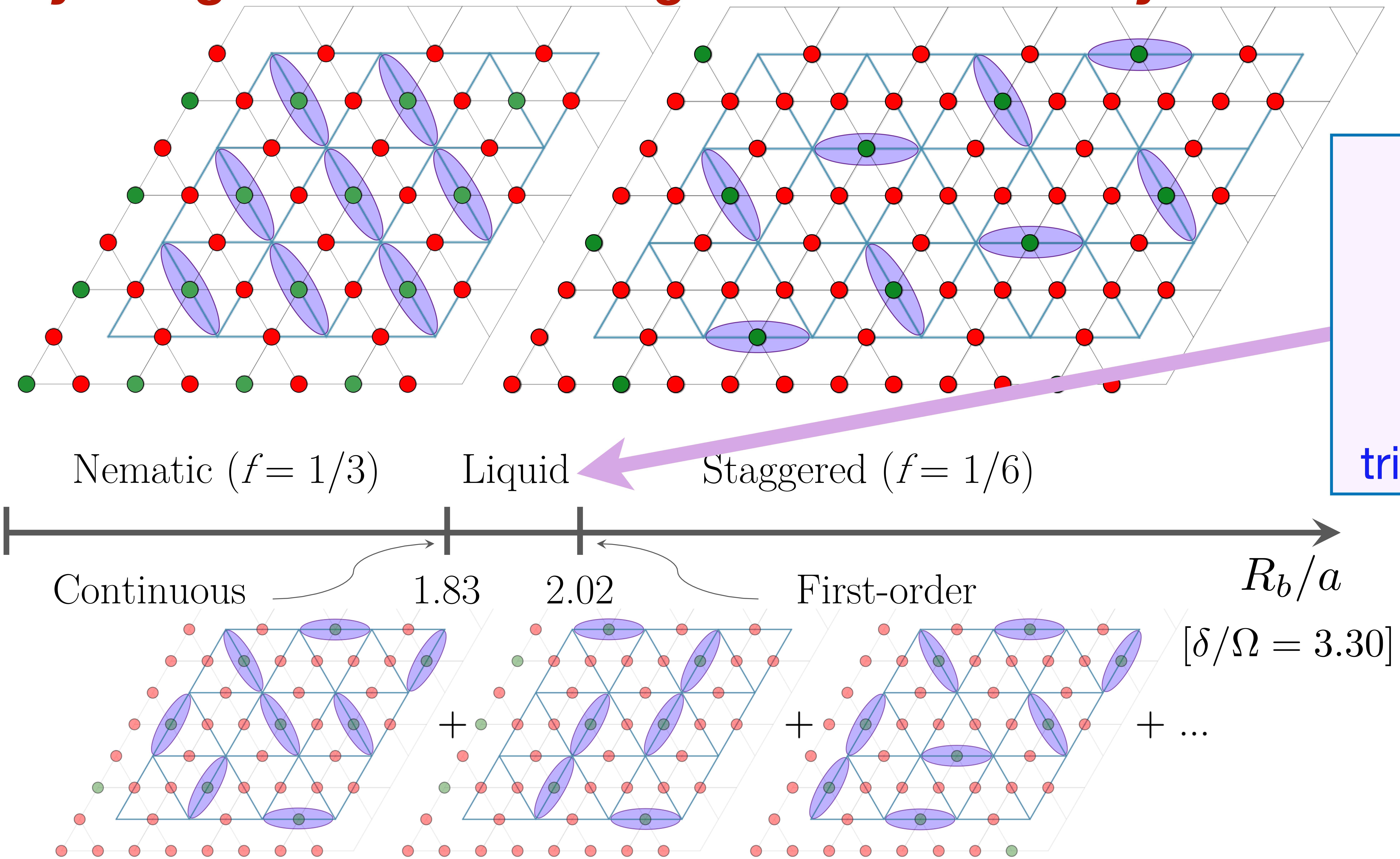


Quantum liquid phase of the *odd/even* quantum dimer model on the triangular lattice ??

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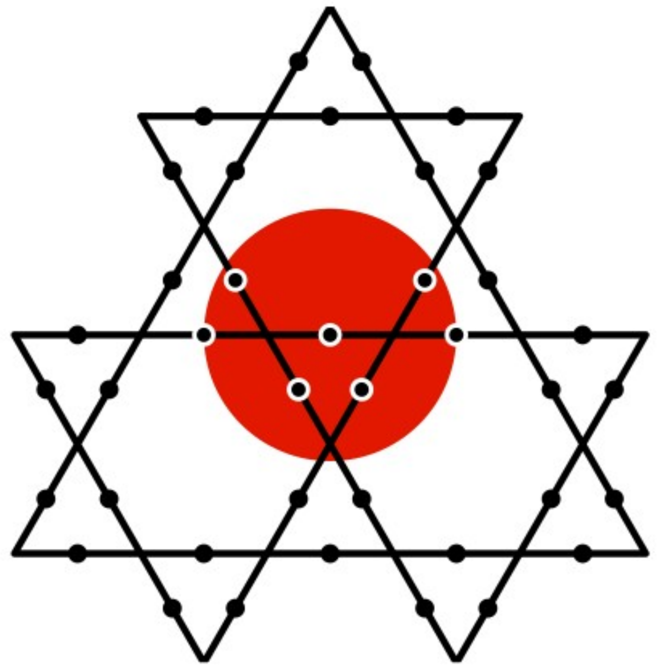
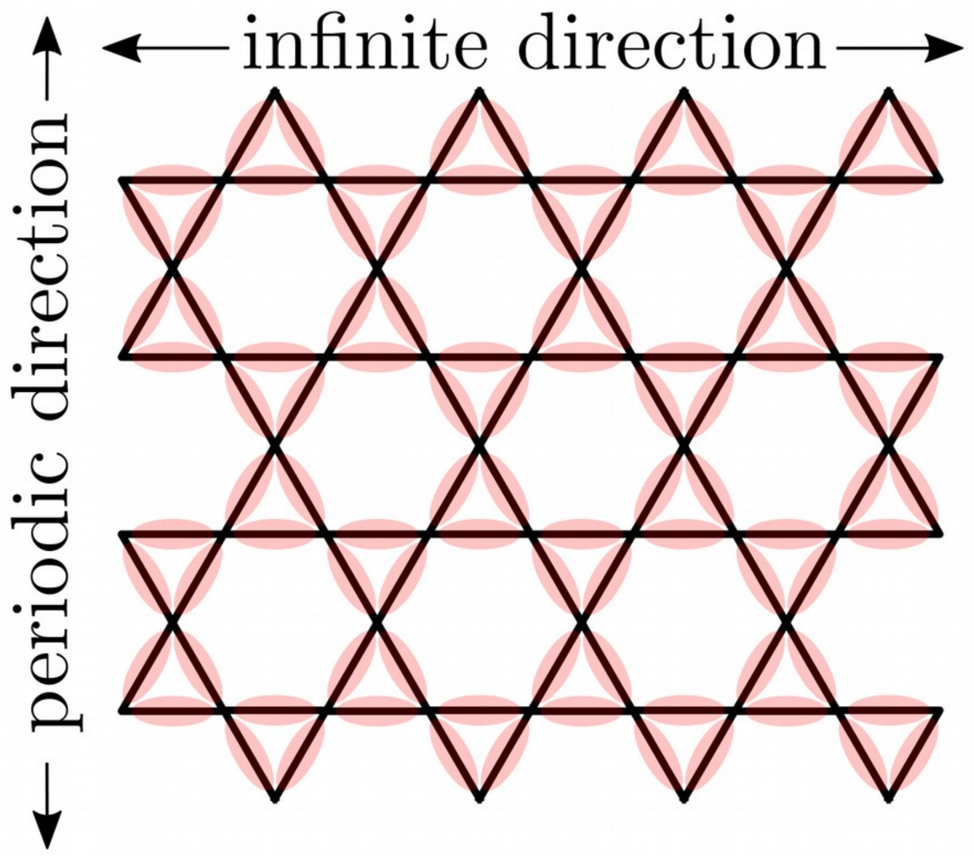
$$H = \frac{\Omega}{2} \sum_i P \sigma_i^x P - \delta \sum_i n_i$$



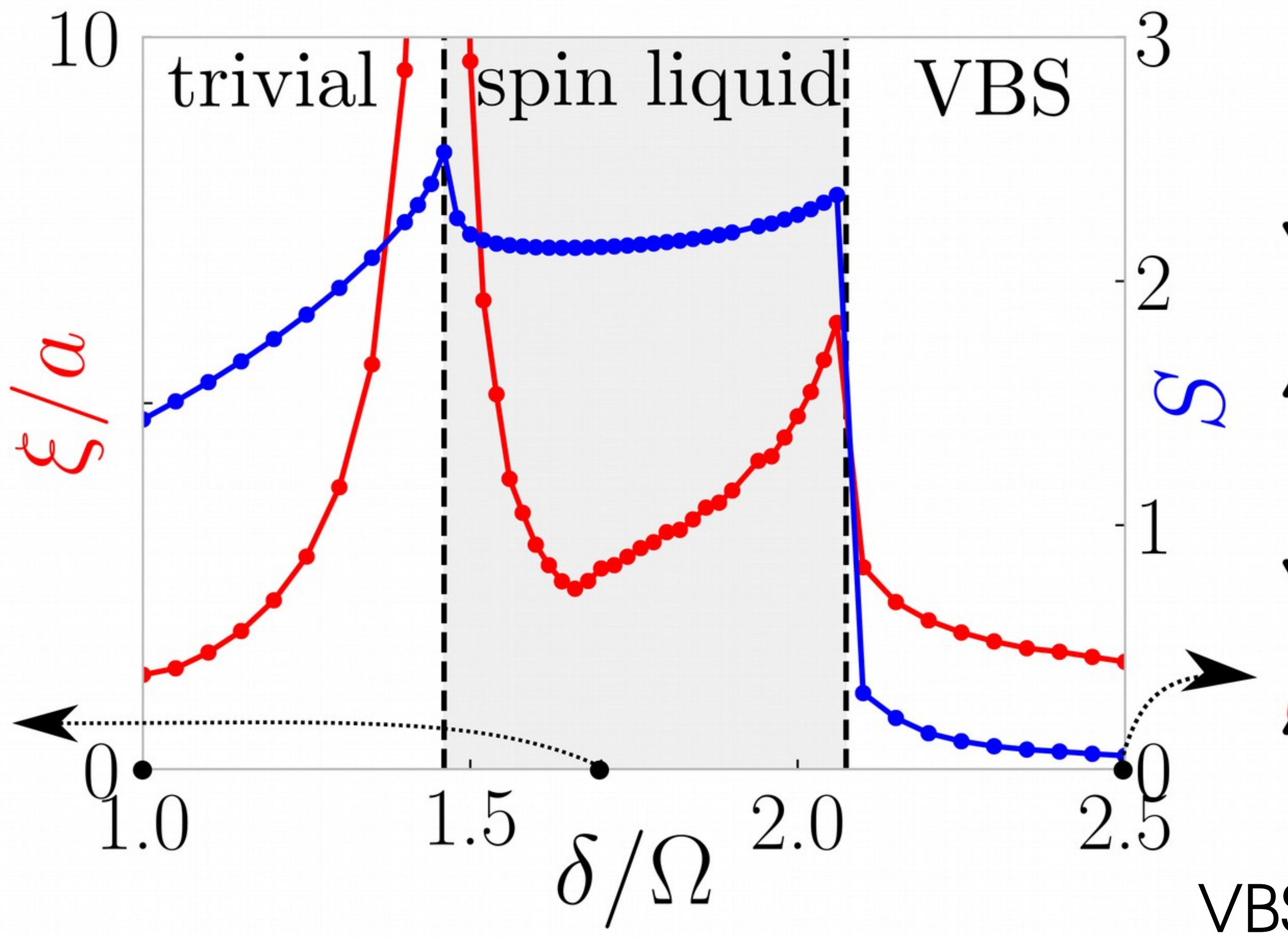
we put the model on an infinitely-long cylinder

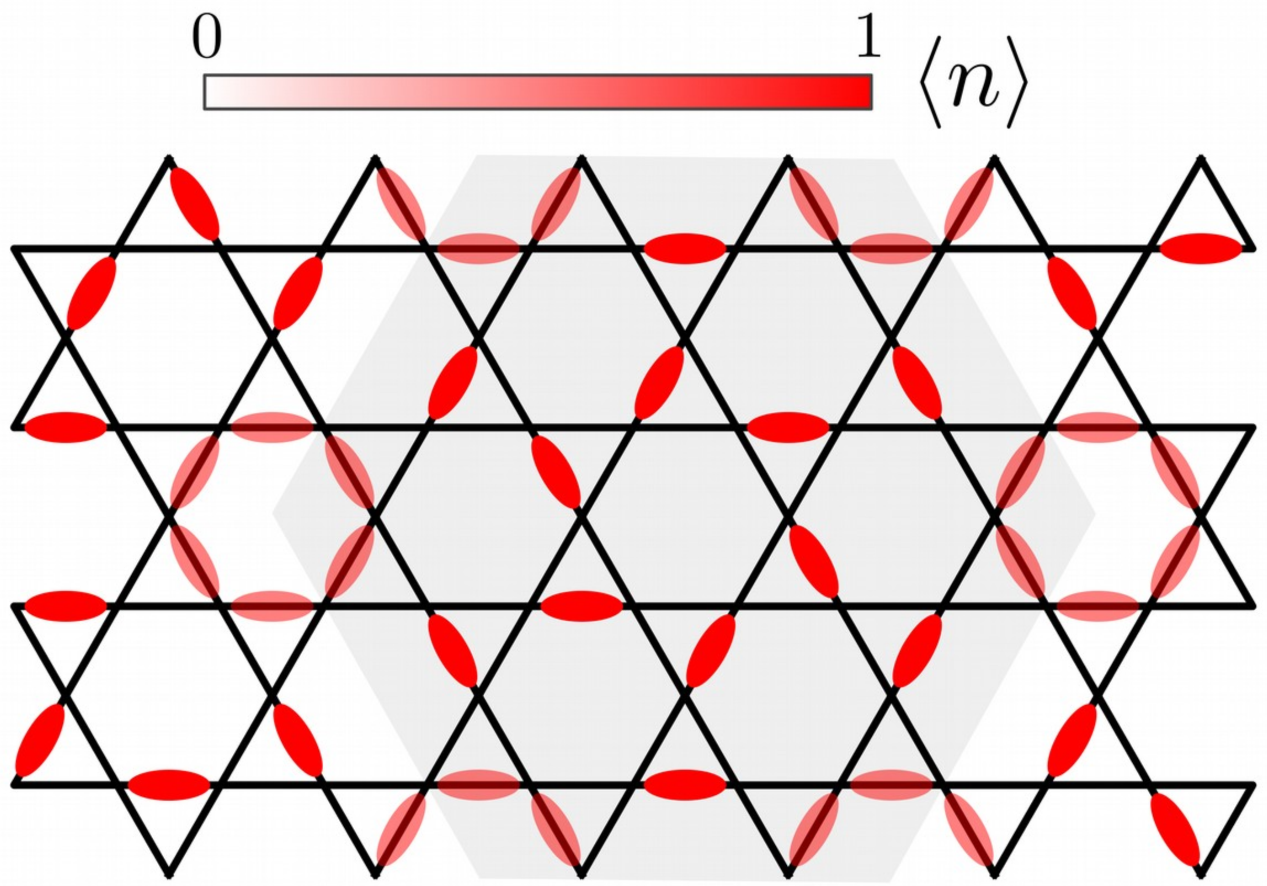
→ use density matrix renormalization group (DMRG)

(White '92, Stoudenmire '13, Hauschild '18)




intermediate featureless phase




VBS pattern studied before in context of Heisenberg model on kagome lattice (Nikolic and Senthil, Singh and Huse, ...)

Ruben Verresen, Mikhail D. Lukin, Ashvin Vishwanath, arXiv: 2011.12310

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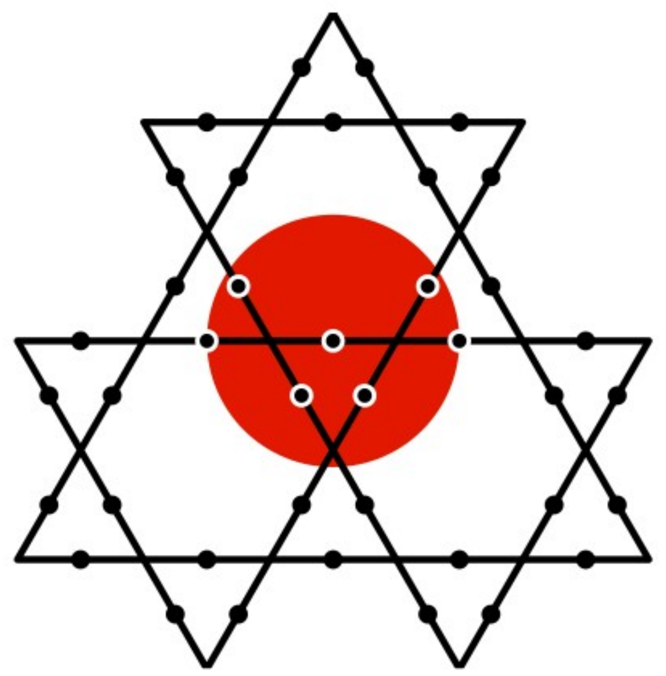

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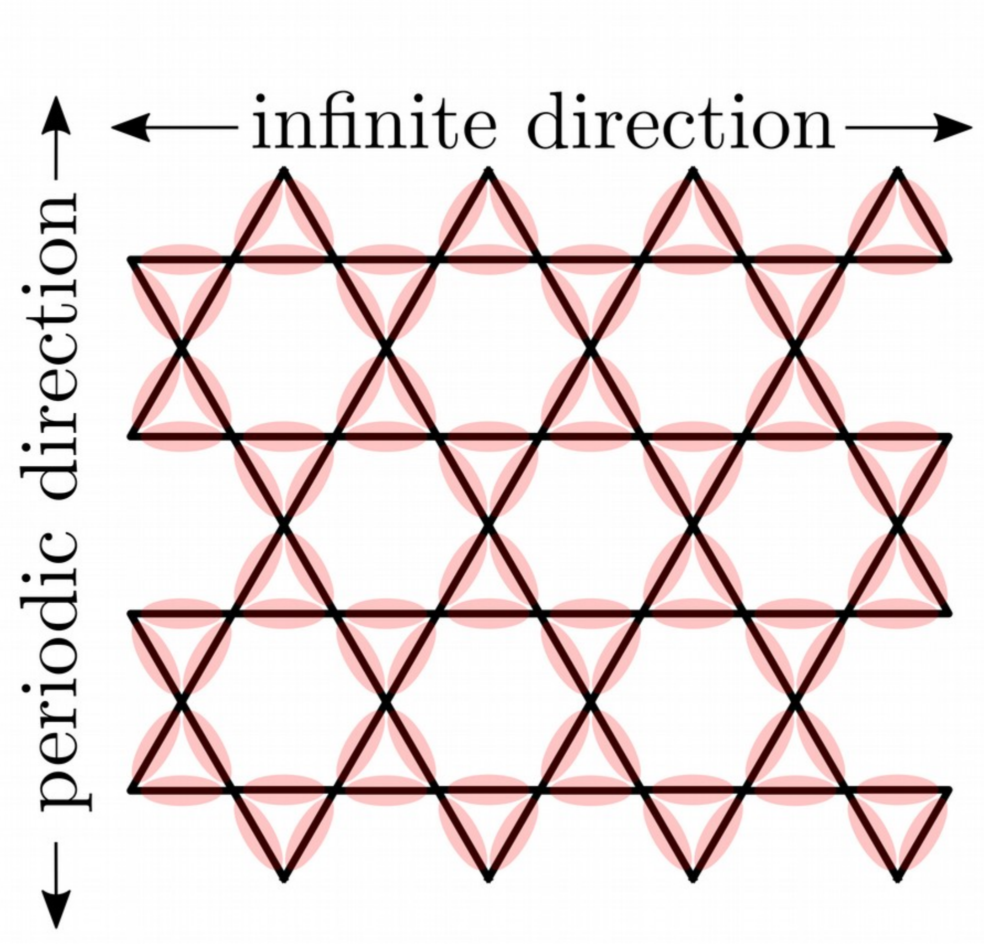


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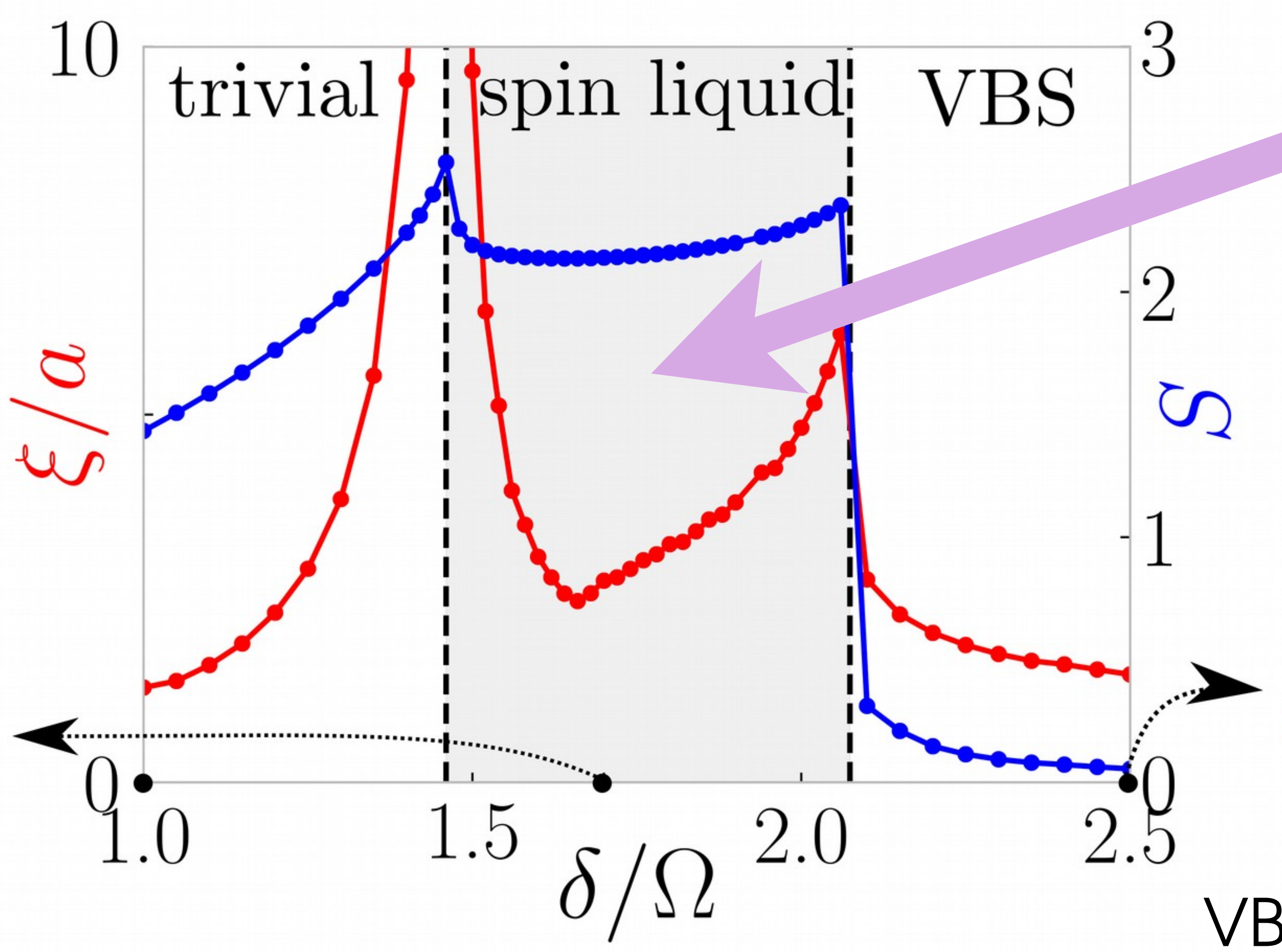


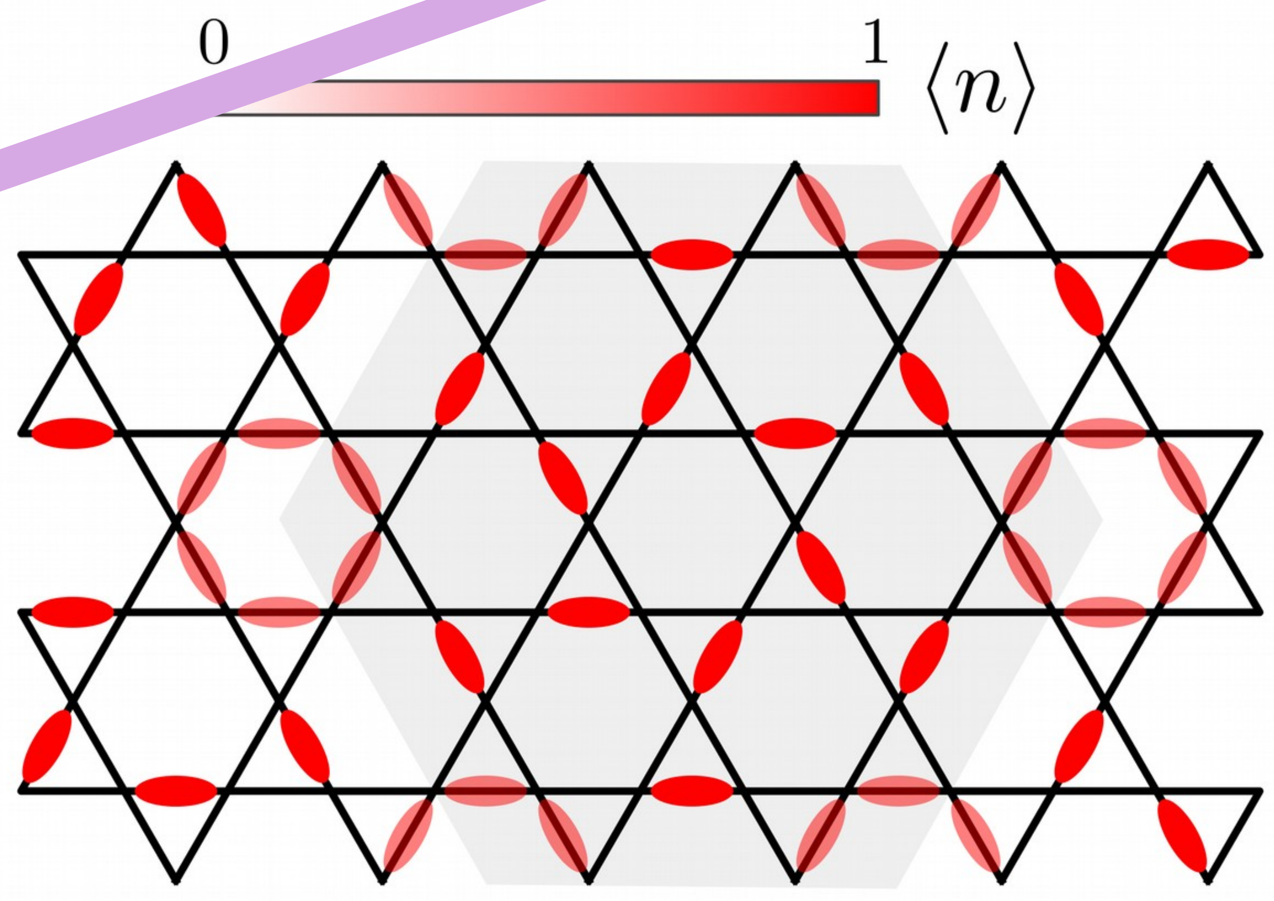


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# Parton construction of spin liquids

Fermionic partons

$$\vec{S} = f_\alpha^\dagger \frac{\vec{\sigma}_{\alpha\beta}}{2} f_\beta$$

Constraint $\sum_\alpha f_\alpha^\dagger f_\alpha = 1$

SU(2) gauge invariance

$$\begin{pmatrix} f_\uparrow \\ f_\downarrow^\dagger \end{pmatrix} \to U \begin{pmatrix} f_\uparrow \\ f_\downarrow^\dagger \end{pmatrix}$$

Mean-field Hamiltonian for free spinons

$$H_f = \sum_{i,j} \left( t_{ij} f_{i\alpha}^\dagger f_{j\alpha} + Q_{ij} \varepsilon_{\alpha\beta} f_{i\alpha} f_{j\beta} + \text{H.c.} \right)$$

$H_f$ can be invariant under SU(2), U(1), or $\mathbb{Z}_2$ for fixed $t_{ij}$, $Q_{ij}$.

Bosonic partons

$$\vec{S} = b_\alpha^\dagger \frac{\vec{\sigma}_{\alpha\beta}}{2} b_\beta$$

Constraint $\sum_\alpha b_\alpha^\dagger b_\alpha = 1$

U(1) gauge invariance

$$b_\alpha \to e^{i\theta} b_\alpha$$

Mean-field Hamiltonian for free spinons

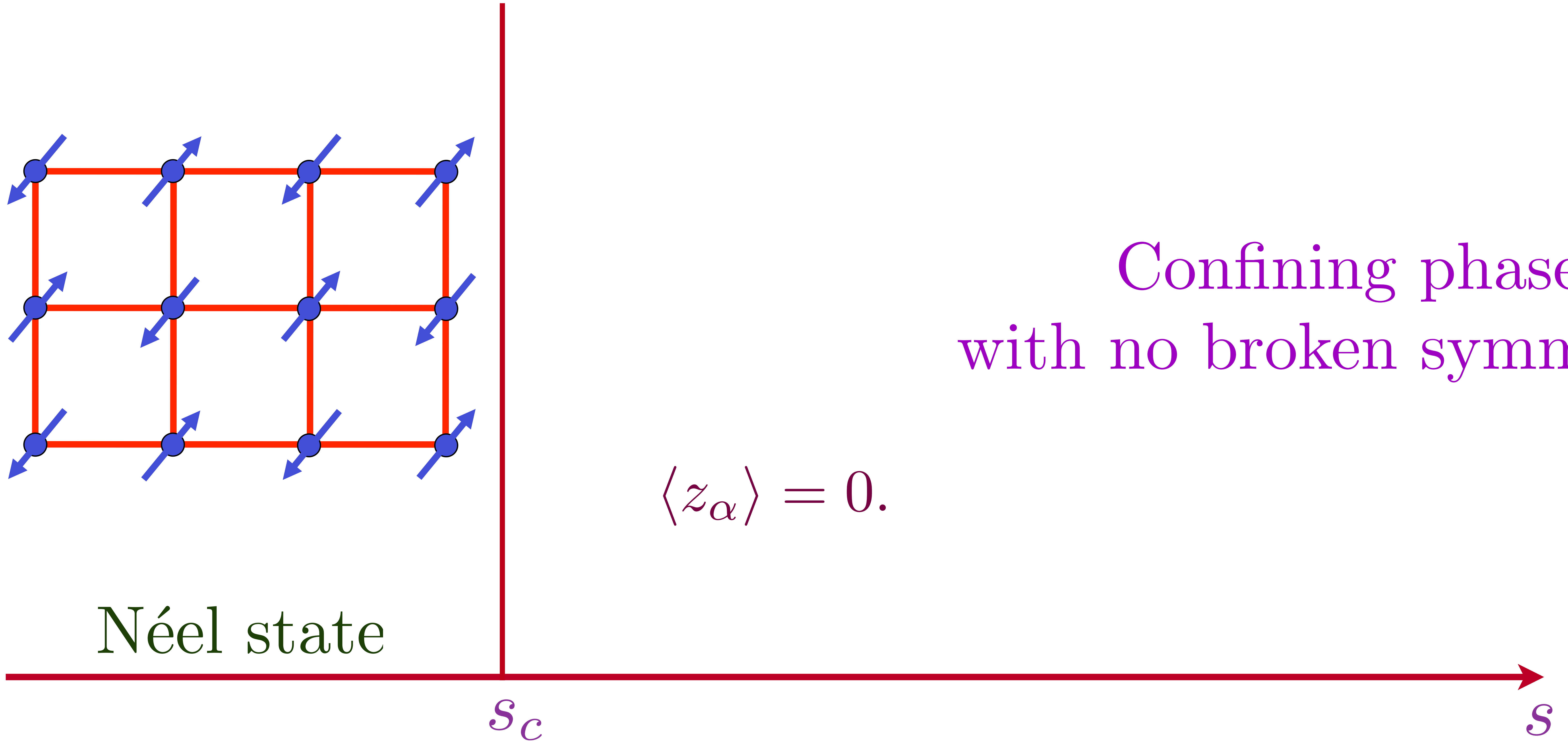
$$H_b = \sum_{i,j} \left( t_{ij} b_{i\alpha}^\dagger b_{j\alpha} + Q_{ij} \varepsilon_{\alpha\beta} b_{i\alpha} b_{j\beta} + \text{H.c.} \right)$$

$H_b$ can be invariant under U(1) or $\mathcal{Z}_2$ for fixed $t_{ij}$, $Q_{ij}$.

# Low energy theory of bosonic partons on the square lattice

$$\mathcal{S} = \int d^2x d\tau \left[ |(\partial_\mu - iA_\tau) z_\alpha|^2 + c^2 |(\vec{\nabla}_x - i\vec{A}) z_\alpha|^2 + s|z_\alpha|^2 + \frac{u}{2} \left( |z_\alpha|^2 \right)^2 \right]$$

The $\mathrm{CP}^1$ field theory of relativistic bosonic spinons $z_\alpha$
coupled to a U(1) gauge field $A_\mu$.
The antiferromagnetic (Néel) order parameter is $(-1)^i \vec{S}_i \sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$.



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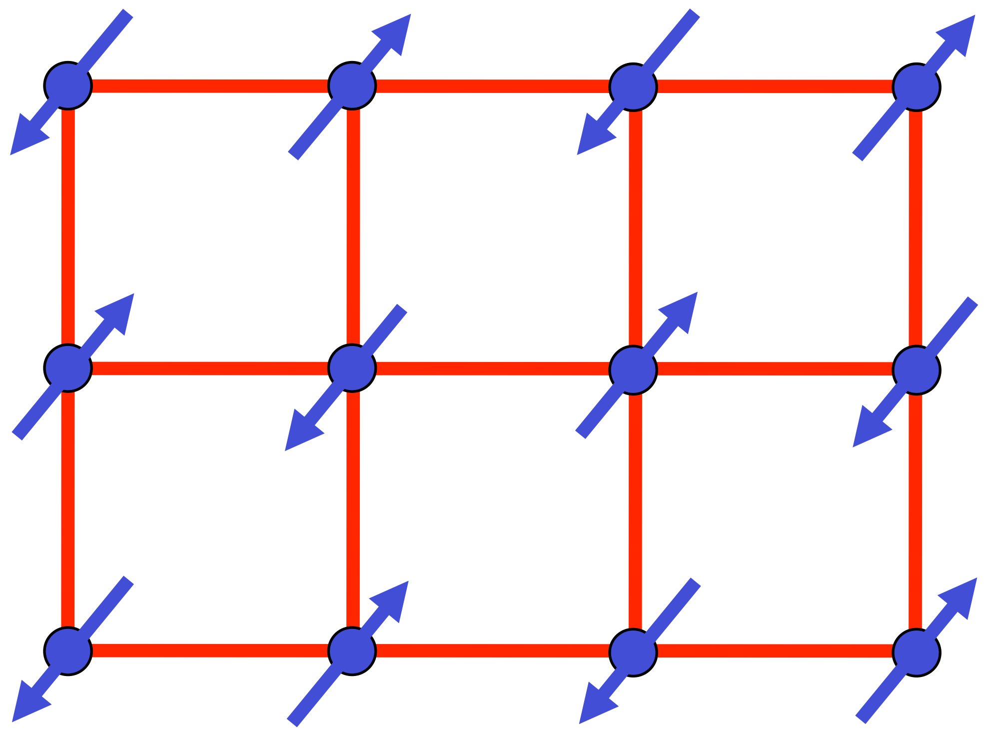
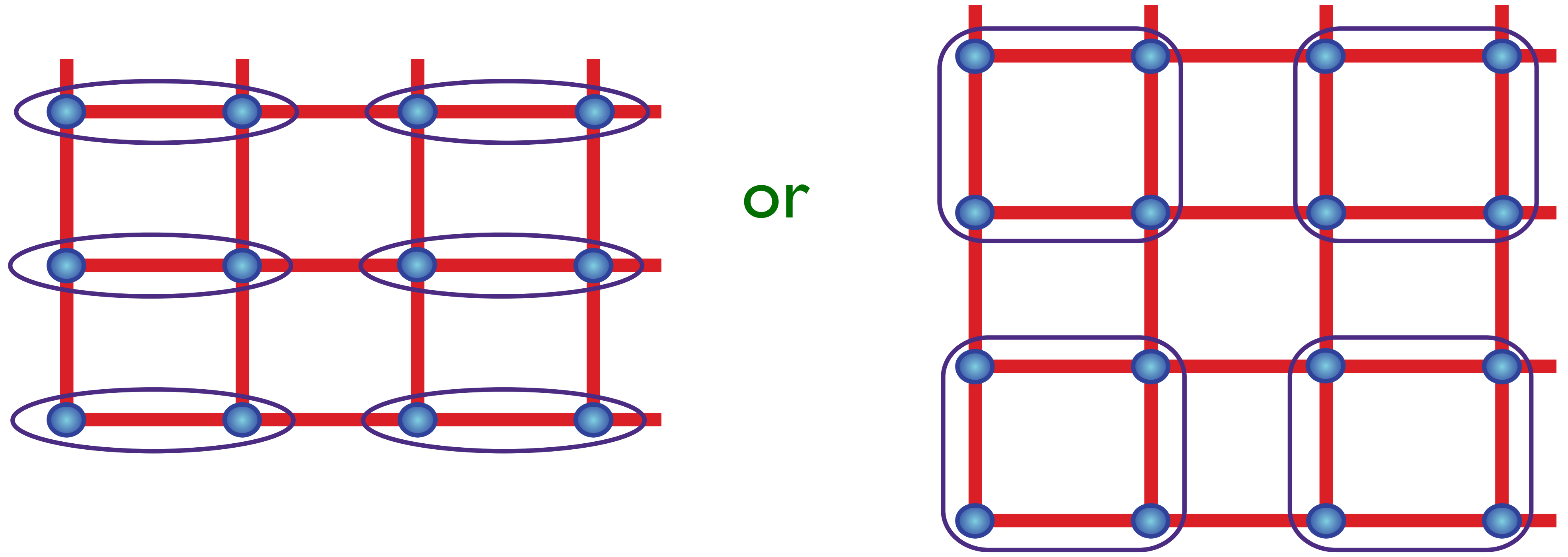


Néel state

Monopoles acquire Berry phases
from background spins
Condensation of monopoles leads
to valence bond solid (VBS) order

$\langle z_\alpha \rangle = 0.$




$s_c$ $\quad$ $s$

N. Read and
S. Sachdev,
*Phys. Rev. Lett.*
**62**, 1694 (1989).

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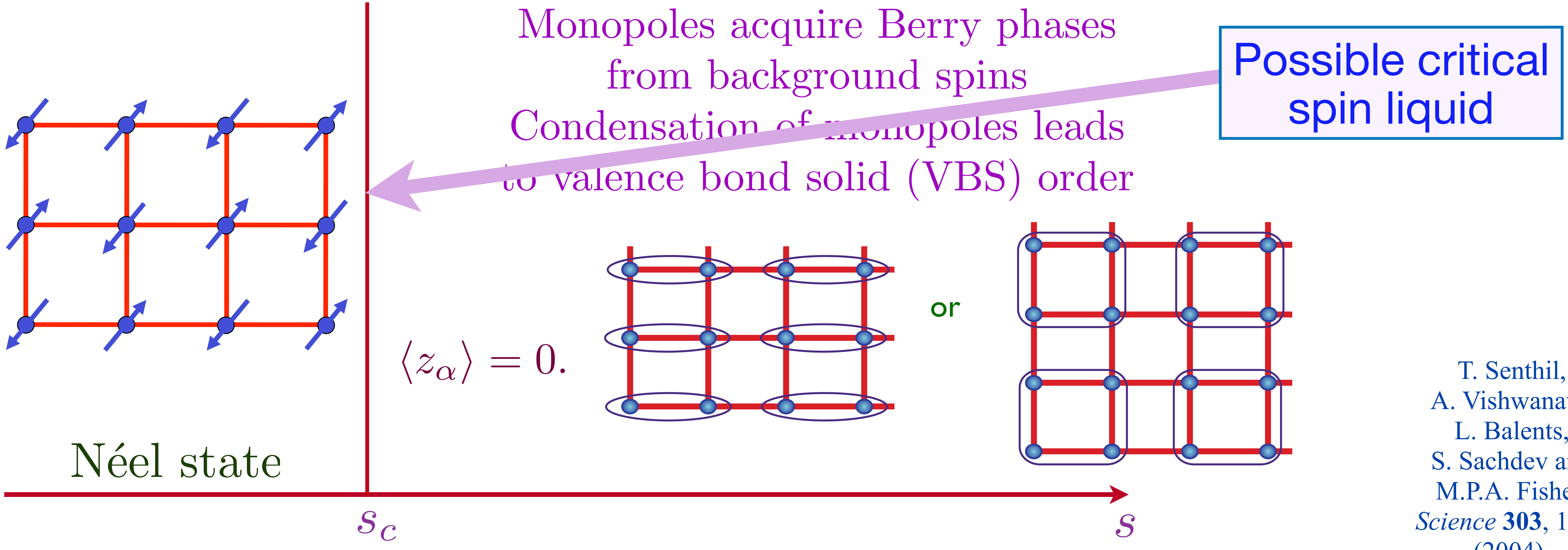


T. Senthil,
A. Vishwanath,
L. Balents,
S. Sachdev and
M.P.A. Fisher,
*Science* **303**, 1490 (2004).

# Low energy theory of fermionic partons on the square lattice

$$\mathcal{S} = \int d^2x d\tau \; \overline{\psi} \gamma_\mu \left( \partial_\mu - i a_\mu \right) \psi + \ldots$$

SU(2) gauge theory with $N_f = 2$ massless (2-component) Dirac fermions *i.e.* QCD$_3$.
This can realize a critical spin liquid, or undergo
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Ying Ran and
Xiao-Gang Wen
cond-mat/
0609620

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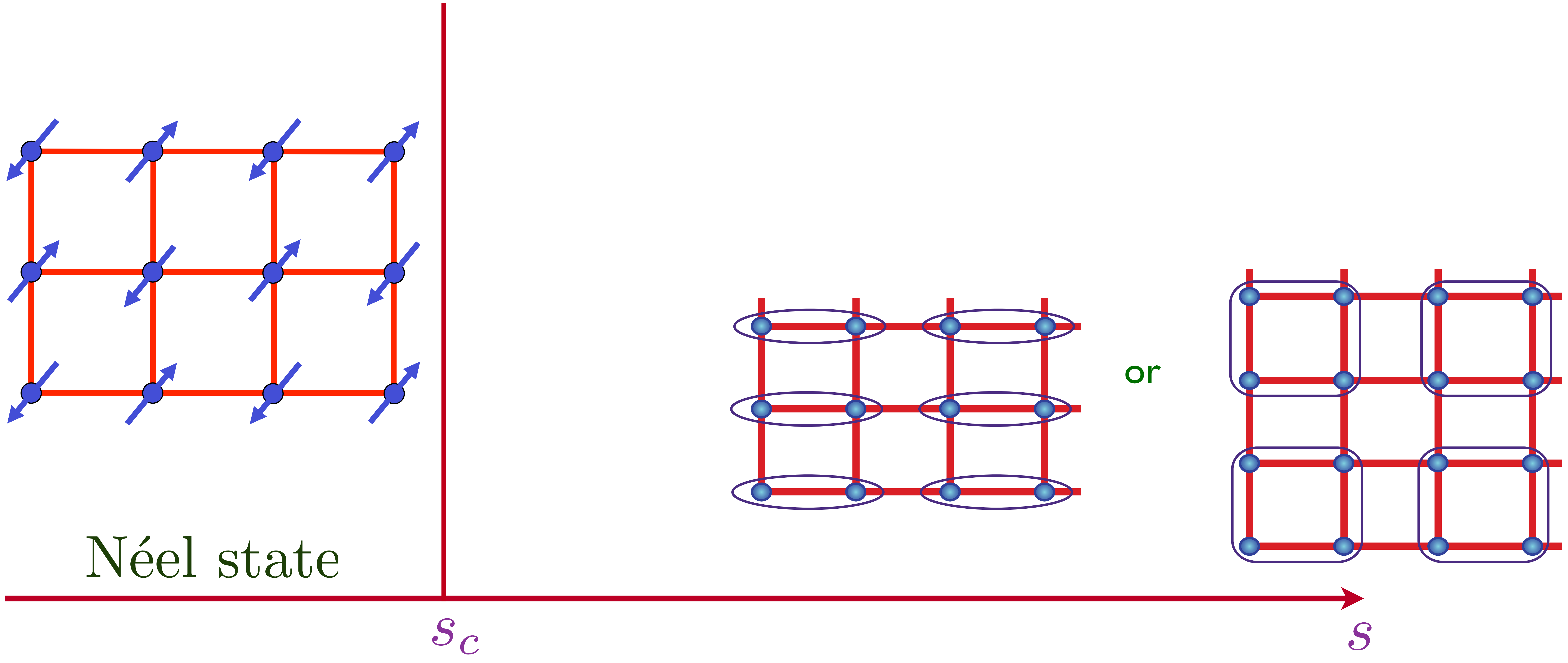


Ying Ran and Xiao-Gang Wen cond-mat/0609620

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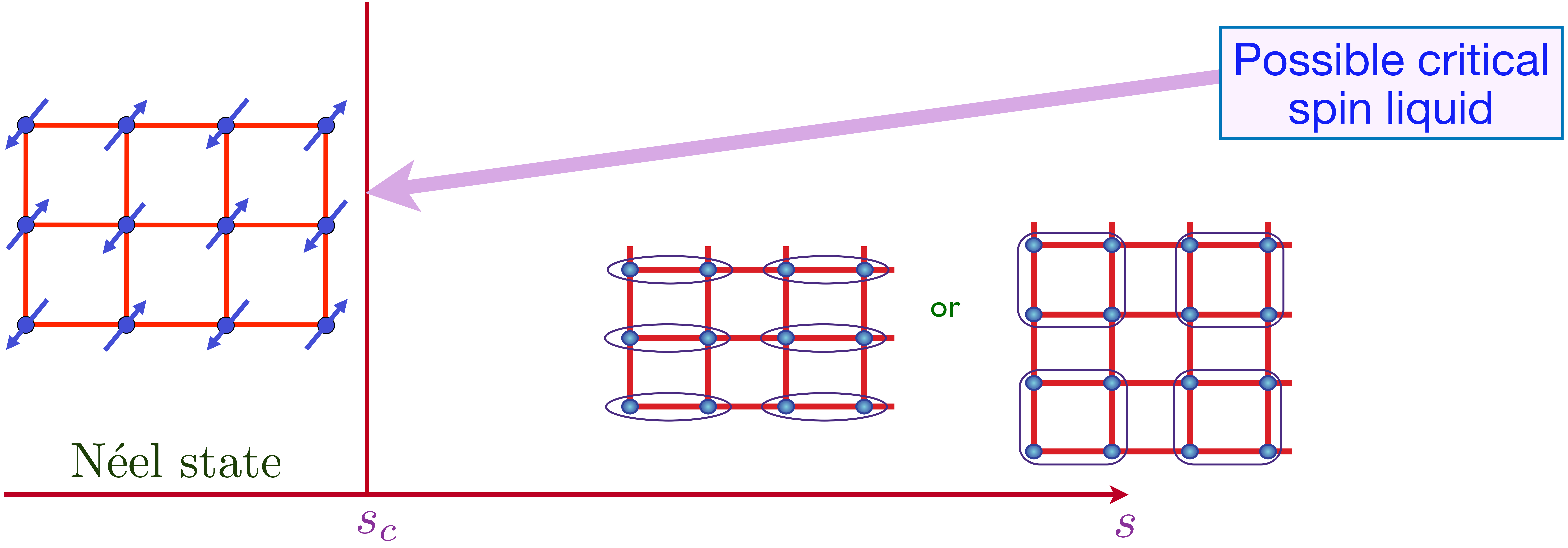

# Critical spin liquids the square lattice

A natural conjecture is that the bosonic parton critical theory (at $s = s_c$)

$$\mathcal{S} = \int d^2x d\tau \left[ |(\partial_\mu - iA_\tau) z_\alpha|^2 + c^2 |(\vec{\nabla}_x - i\vec{A}) z_\alpha|^2 + s|z_\alpha|^2 + \frac{u}{2} \left( |z_\alpha|^2 \right)^2 \right]$$

is the boson-fermion dual of the fermionic parton critical theory

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This is supported by many careful arguments involving monopoles, anomalies, and connections to SPT phases.

Chong Wang, Adam Nahum, Max A. Metlitski, Cenke Xu, and T. Senthil, Phys. Rev. X **7**, 031051 (2017)

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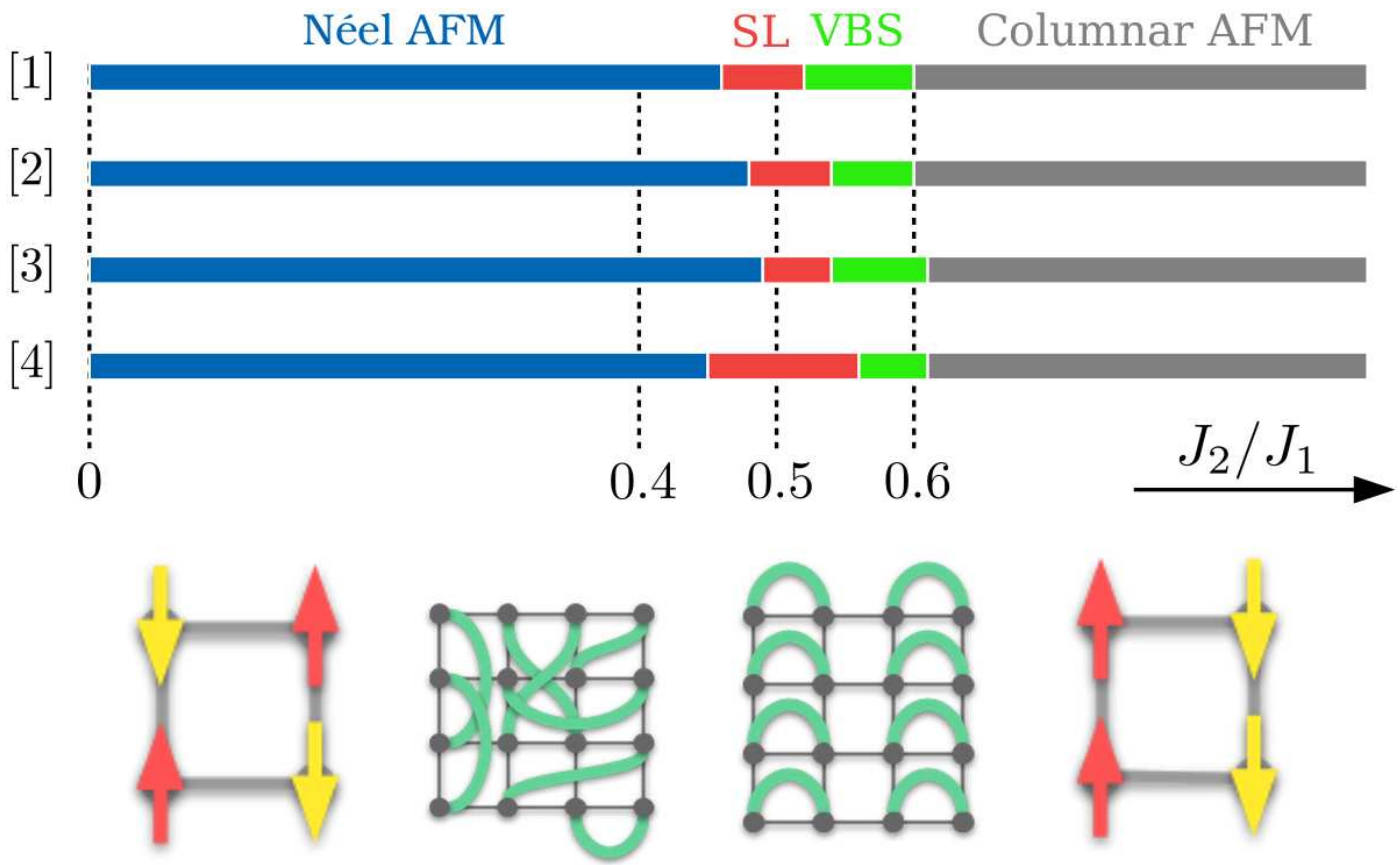
The current numerical evidence supports the existence of such a state up to a large length scale (~ 100 lattice spacings in some models), but a truly critical spin liquid of this type does not exist.

# $J_1$-$J_2$ antiferromagnet the square lattice

Talk by Federico Becca, KITP, 5 Nov, 2020





[1] L. Wang and A.W. Sandvik, Phys. Rev. Lett. **121**, 107202 (2018)

[2] F. Ferrari, F. Becca, Phys. Rev. B **102**, 014417 (2020)

[3] Y. Nomura and M. Imada, arXiv:2005.14142

[4] W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, Z.-C. Gu, arXiv:2009.01821

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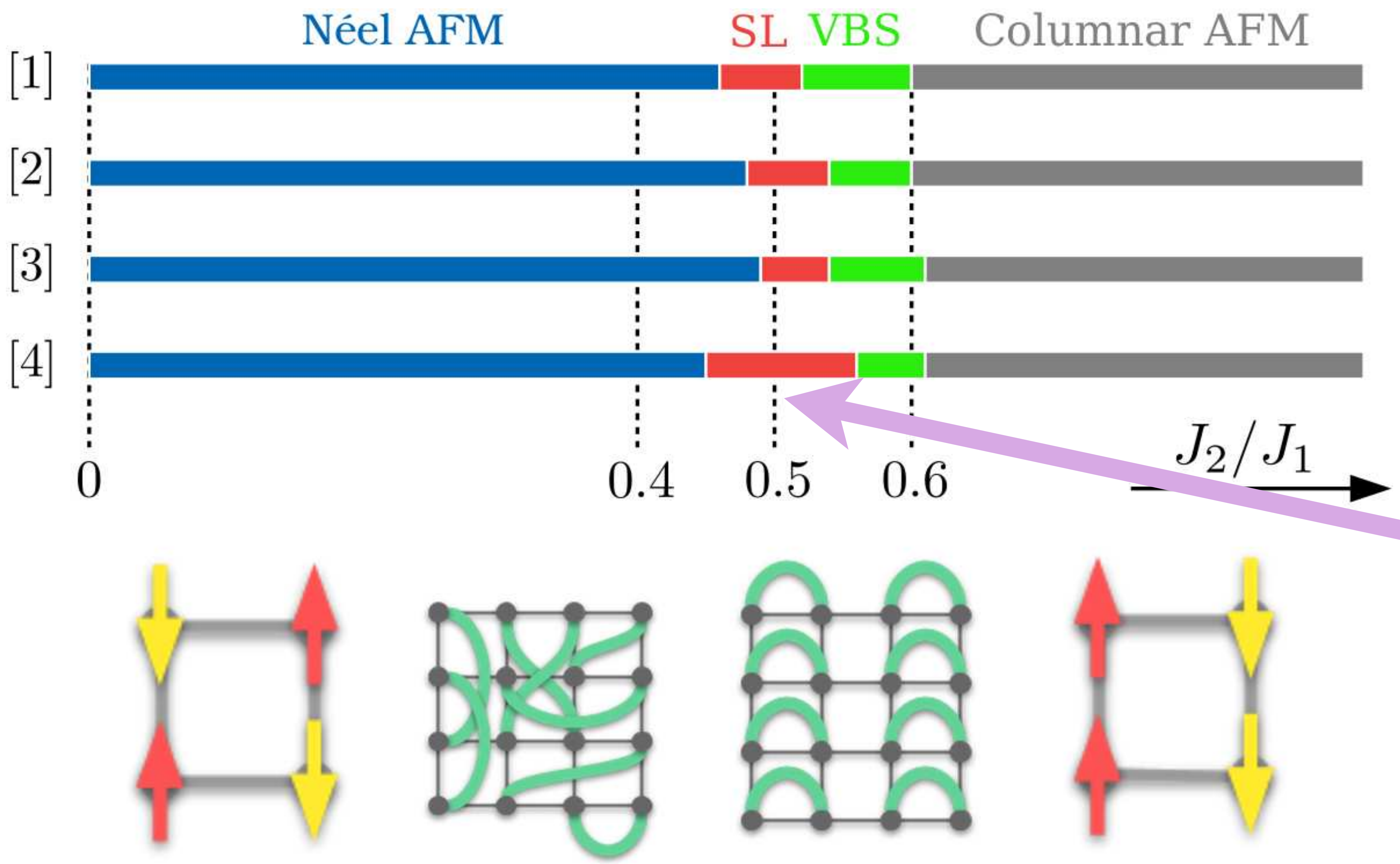


Gapless $Z_2$ spin liquid phase !

Wen-Jun Hu,
Federico Becca,
Alberto Parola,
Sandro Sorella,
PRB **88**, 060402(R) (2013).


[1] L. Wang and A.W. Sandvik, Phys. Rev. Lett. **121**, 107202 (2018)

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**Bosonic partons:**

Condense a charge 2 Higgs scalar, $\Phi$, and ensure that no symmetry is broken.

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S. Sachdev, PRB **45**, 12377 (1992)
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Condense *two* adjoint Higgs scalars, $\Phi_1^a$, $\Phi_2^a$, and ensure no symmetry is broken

$$\mathcal{S} = \int d^2x d\tau \Big[ i\overline{\psi}\gamma_\mu \left(\partial_\mu - i a_\mu^a \sigma^a\right) \psi + \ldots + \lambda \left(\Phi_1^a\, \overline{\psi}\, \mu^z \gamma^x \sigma^a \psi + \Phi_2^a\, \overline{\psi}\, \mu^x \gamma^y \sigma^a \psi\right) + \frac{w}{2}\left(\Phi_1^a \Phi_1^a + \Phi_2^a \Phi_2^a\right) + \ldots \Big]$$

Ying Ran and Xiao-Gang Wen, cond-mat/0609620

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Ying Ran and Xiao-Gang Wen, cond-mat/0609620

These 2 spin liquids are not the same

# $Z_2$ spin liquids the square lattice

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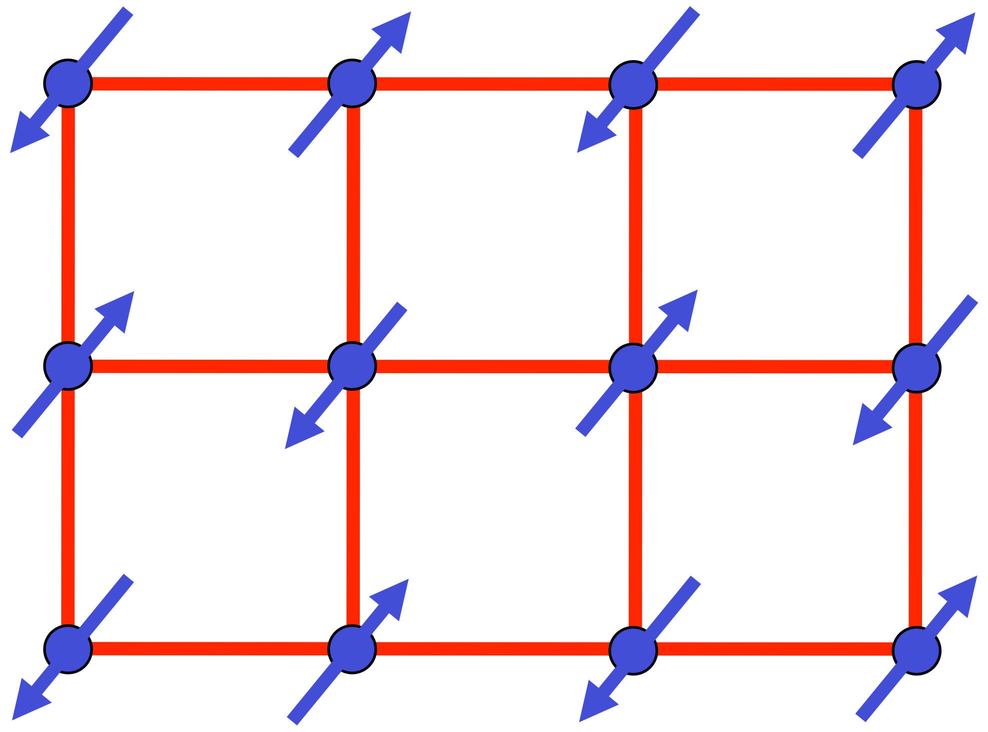
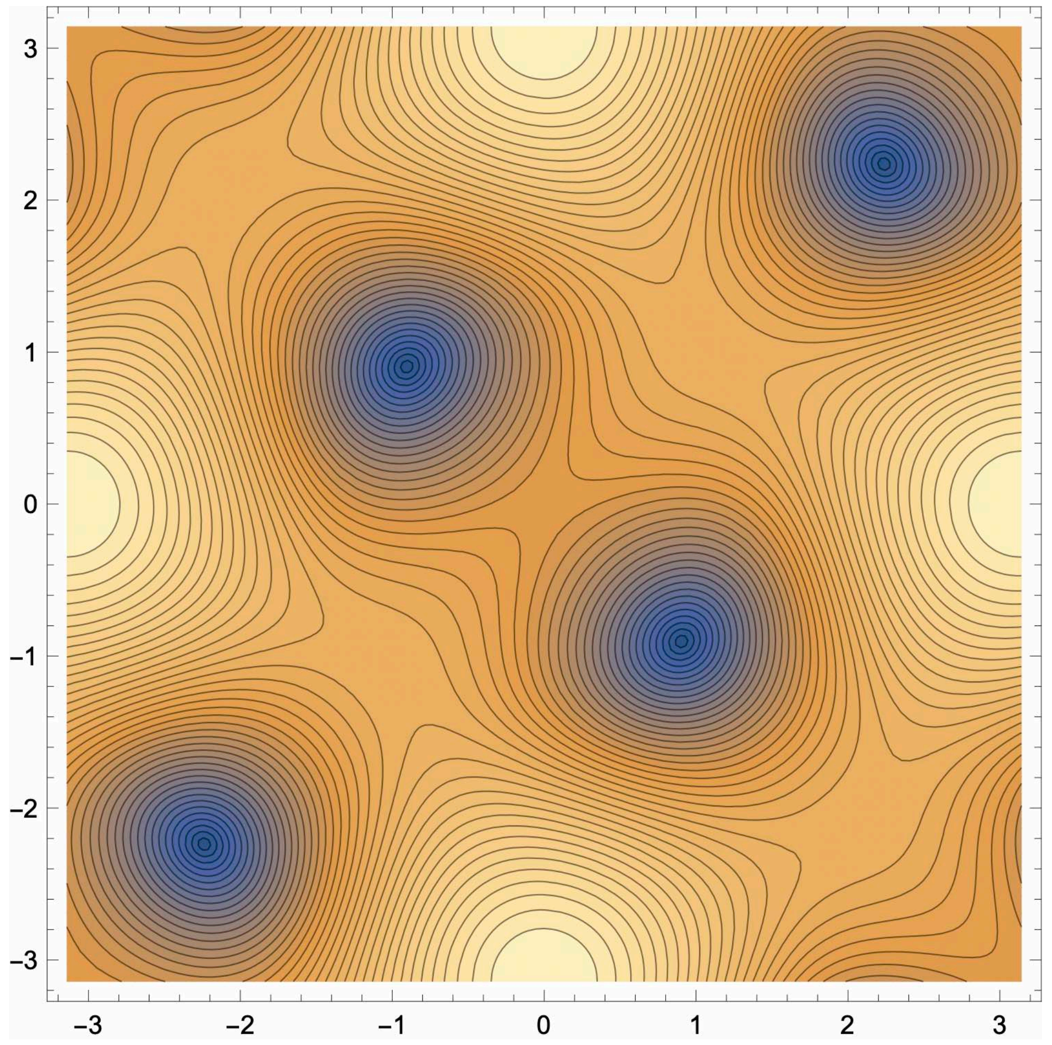
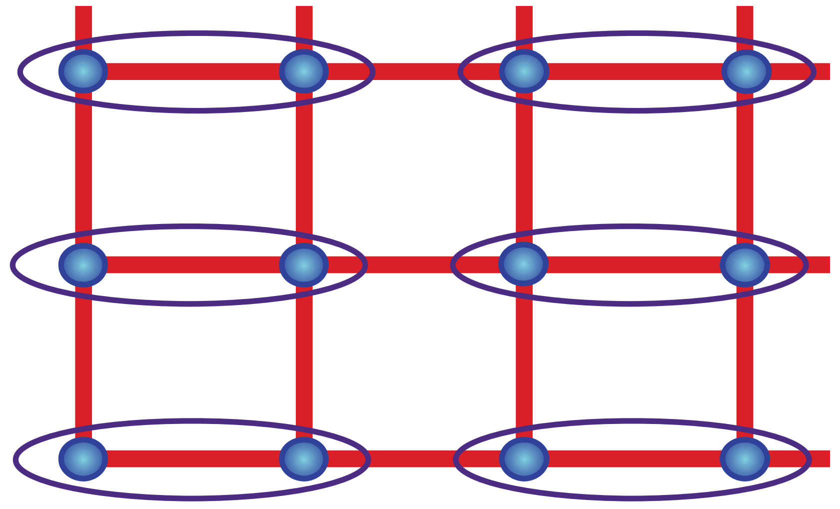


Neel




Gapless $Z_2$ spin liquid



VBS

$w_c$ $w$

Henry Shackleton

Alex Thomson

1. Introduction to spin liquids

*The $Z_2$ spin liquid*

2. Rydberg atoms

3. Gapless spin liquids on the square lattice

*Gauge theories of partons*

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   *The $Z_2$ spin liquid*

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$Z_2$ spin liquids finally found in realistic models! (by numerics)