The phase diagrams of the high temperature superconductors

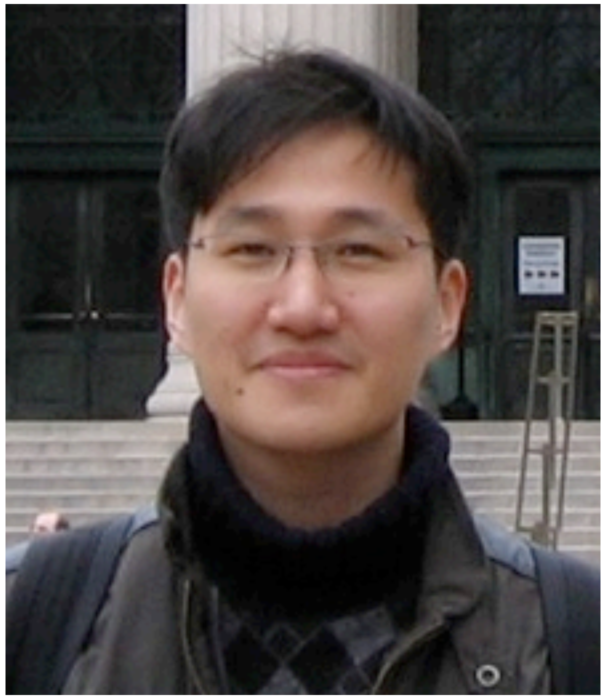
Talk online: sachdev.physics.harvard.edu

PHYSICS

VE RI TAS

HARVARD

Max Metlitski, Harvard



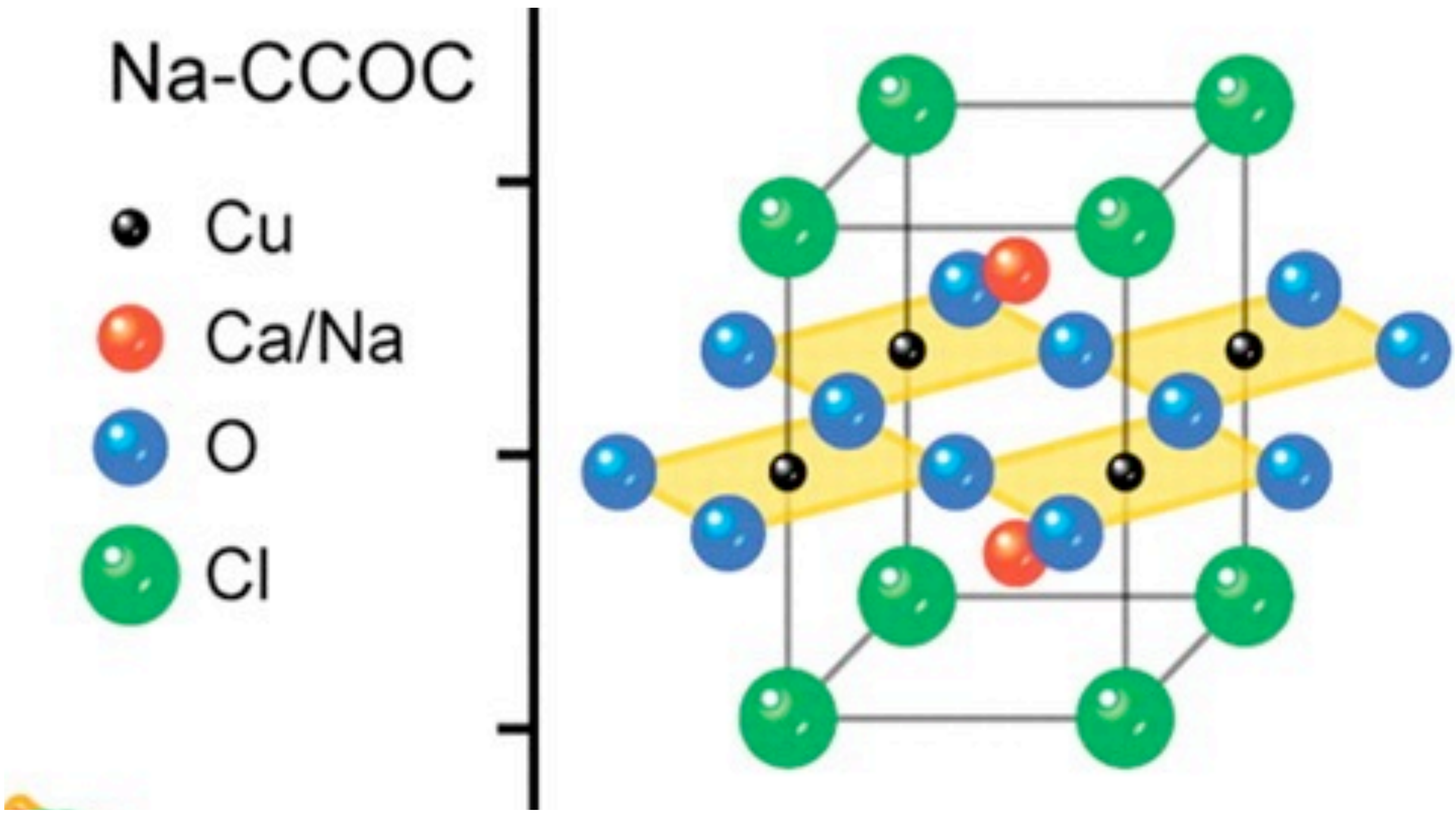
Eun Gook Moon, Harvard

PHYSICS

VE RI TAS

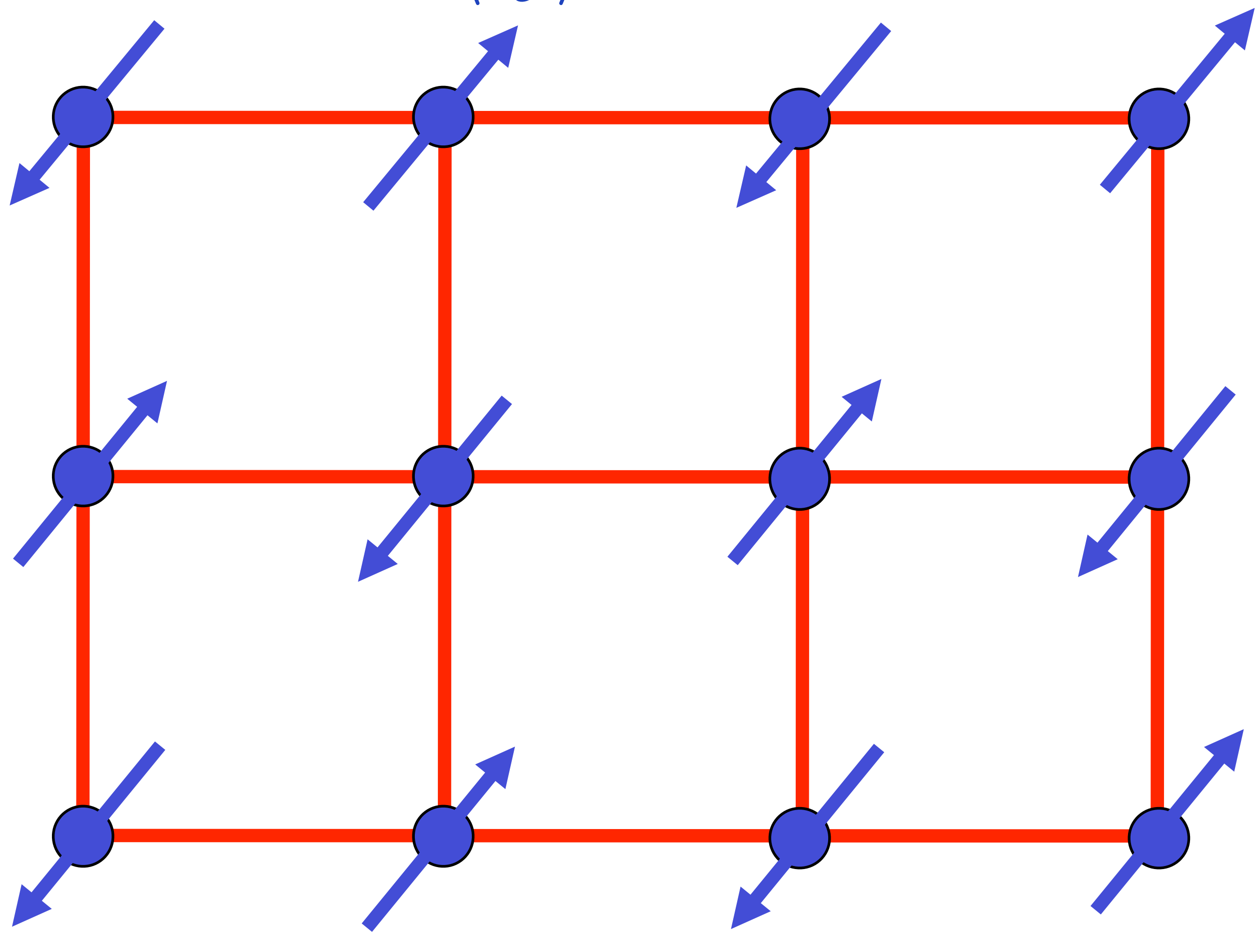
HARVARD

The cuprate superconductors



Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



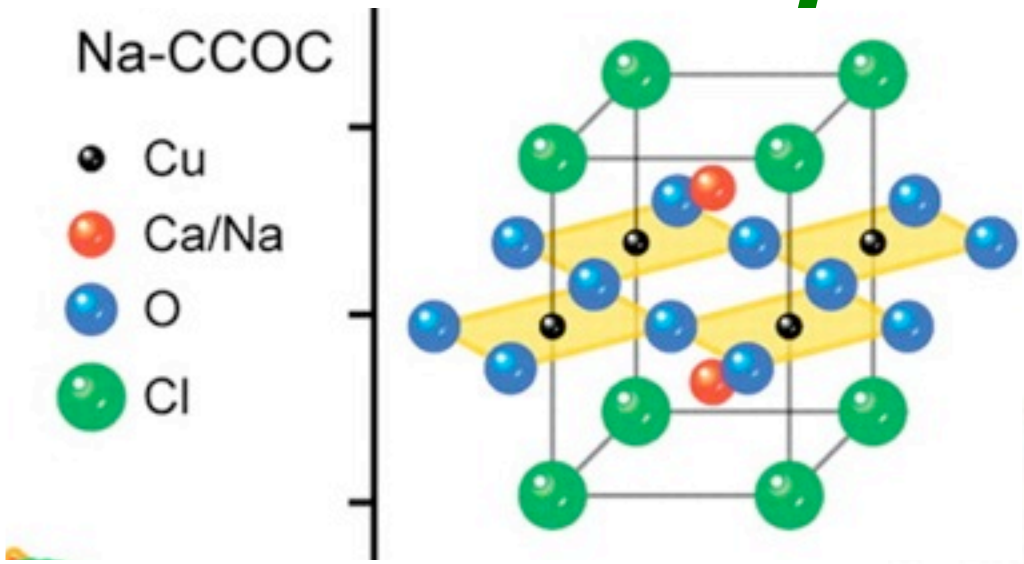
Ground state has long-range Néel order

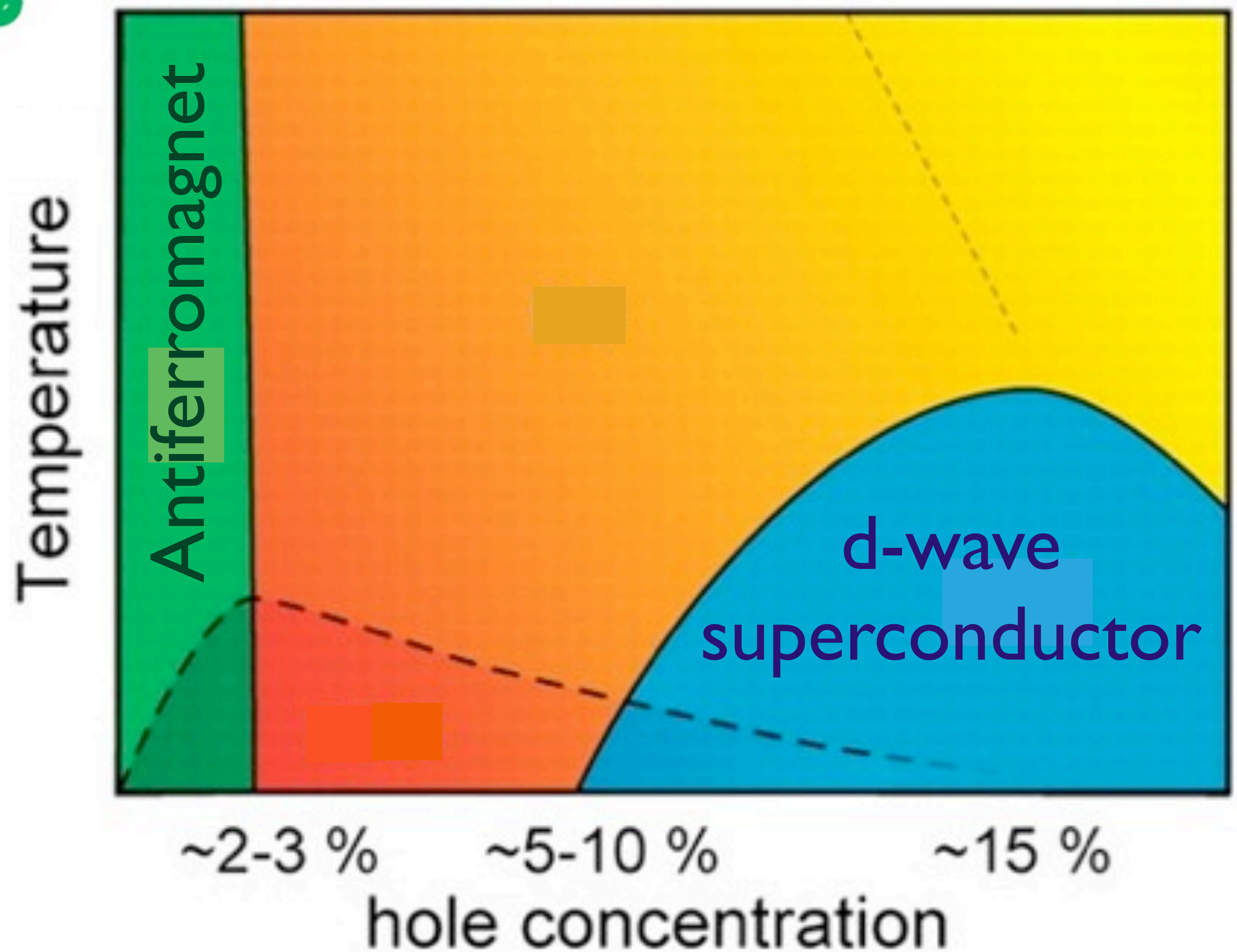
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

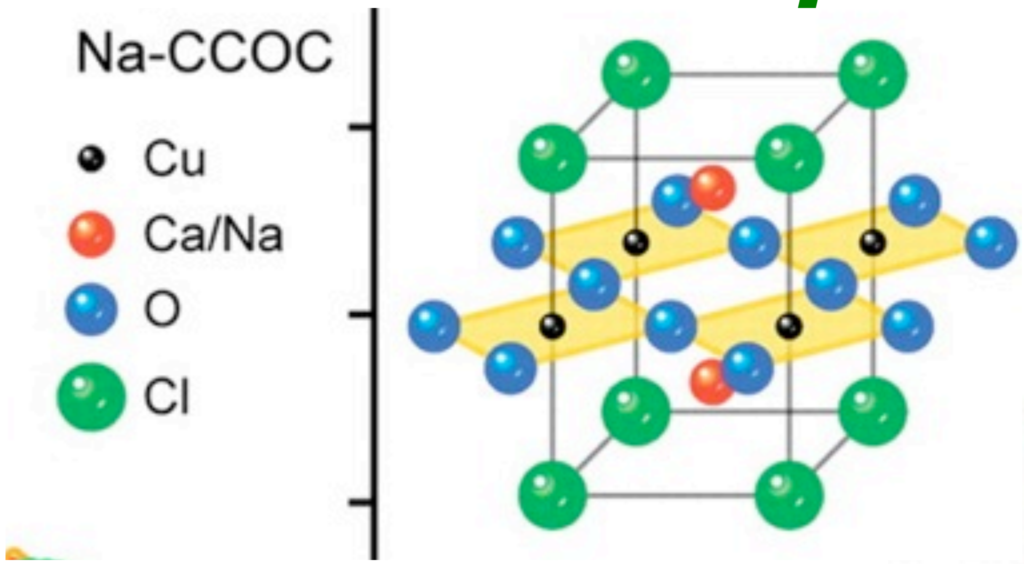
$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

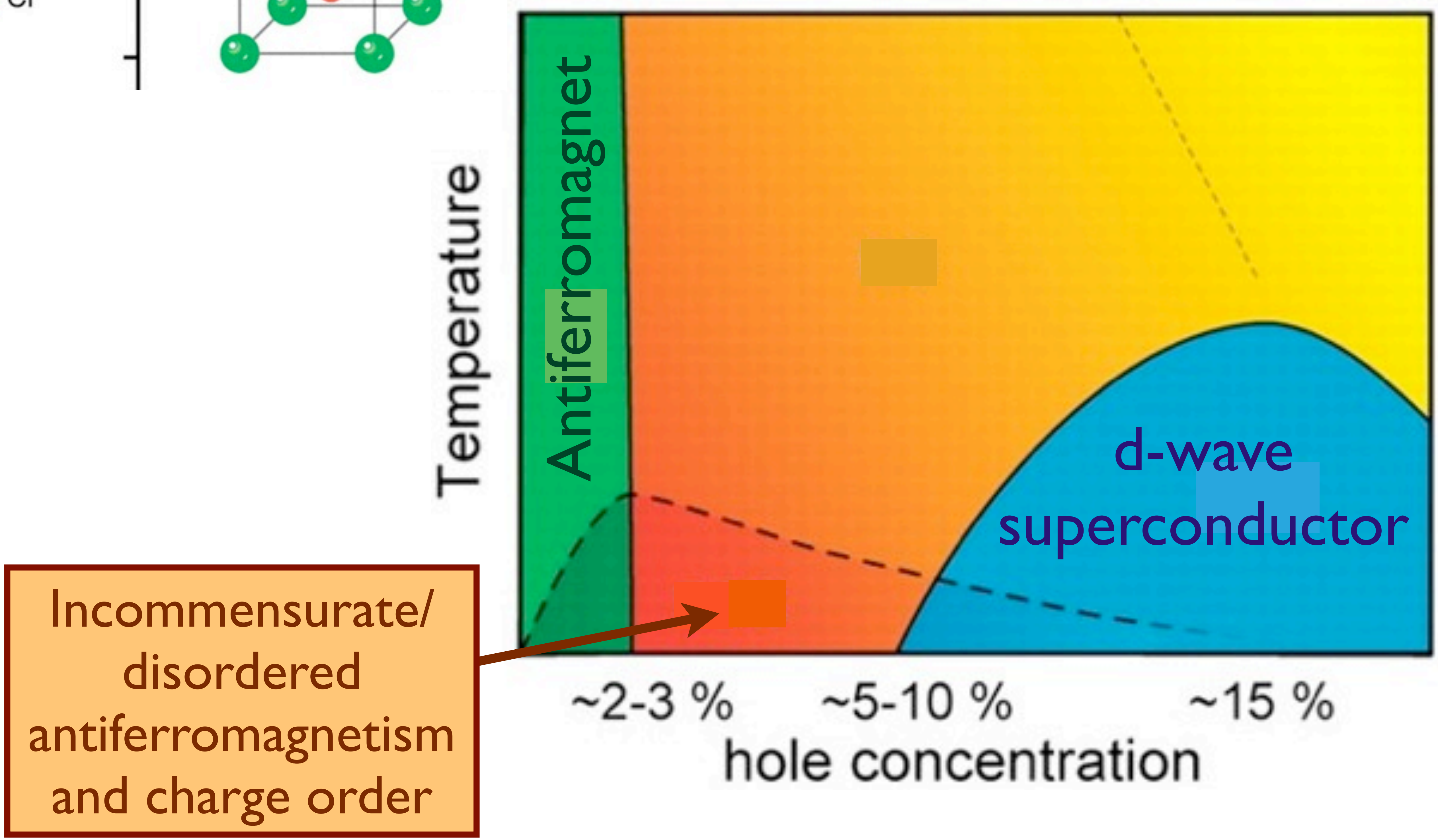
The cuprate superconductors



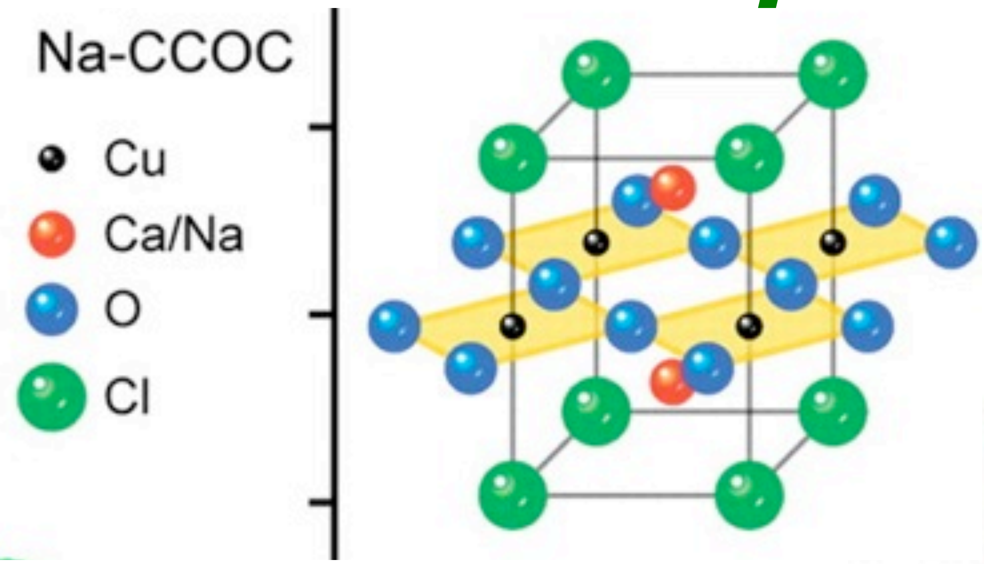


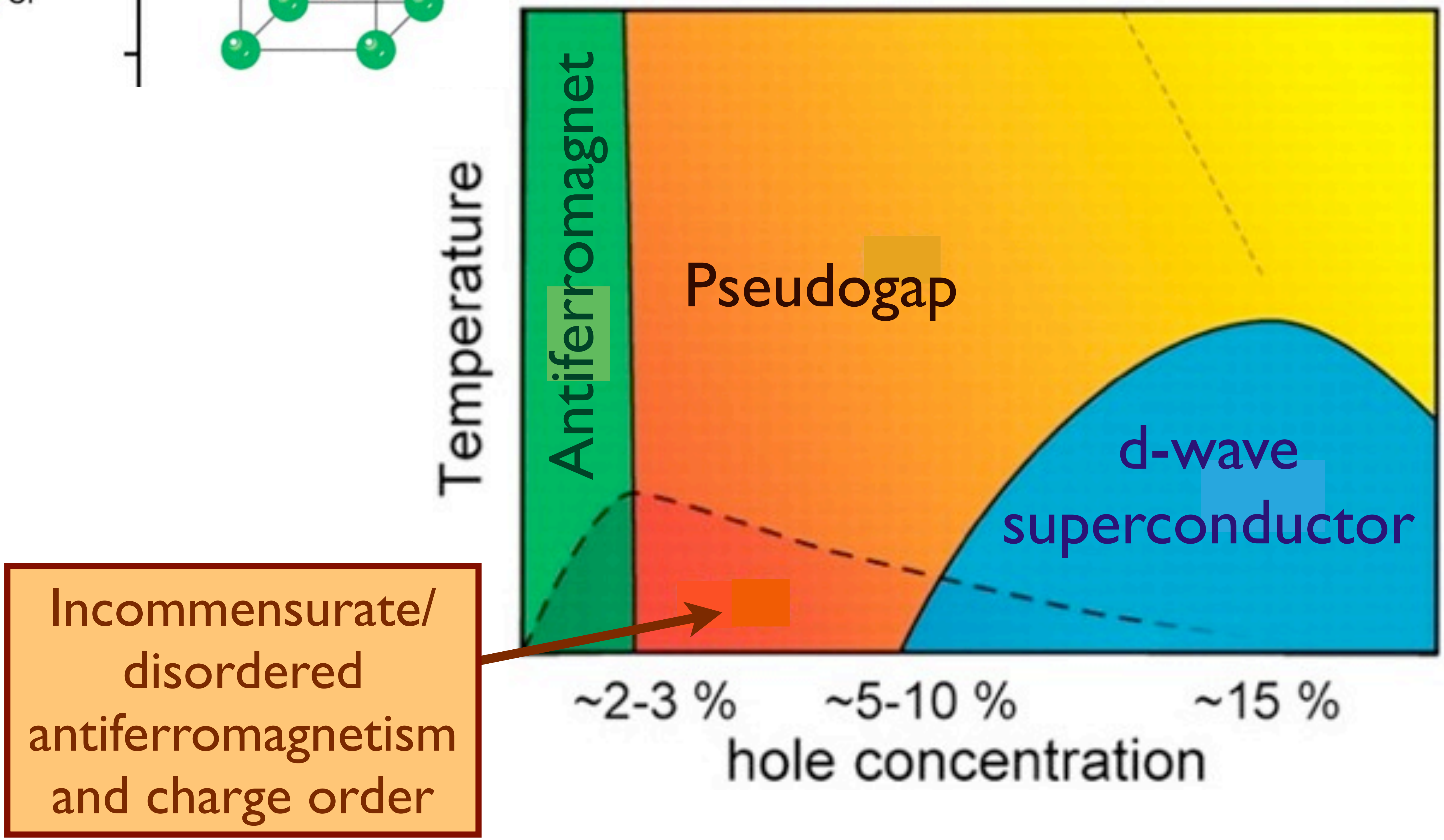
The cuprate superconductors



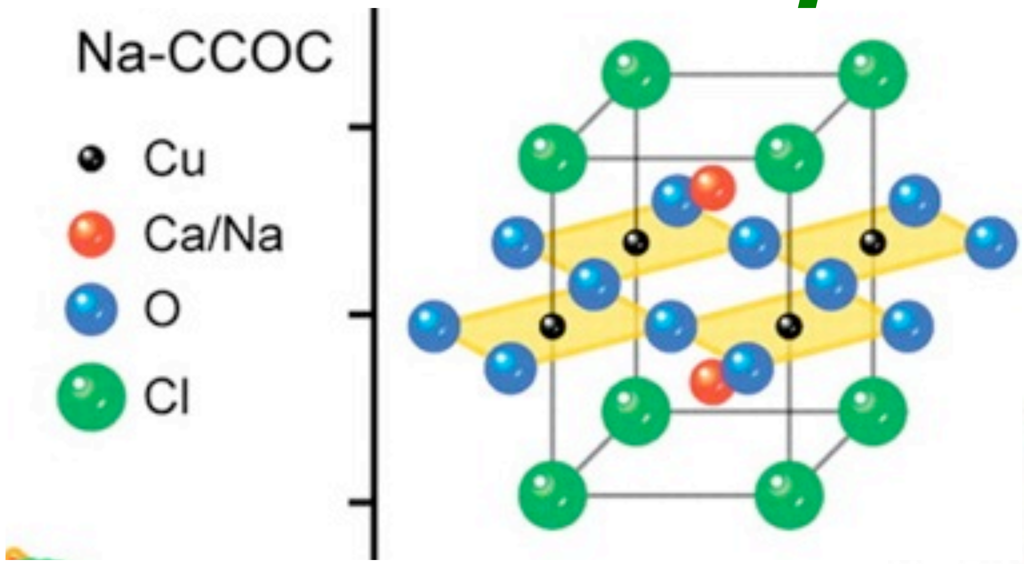


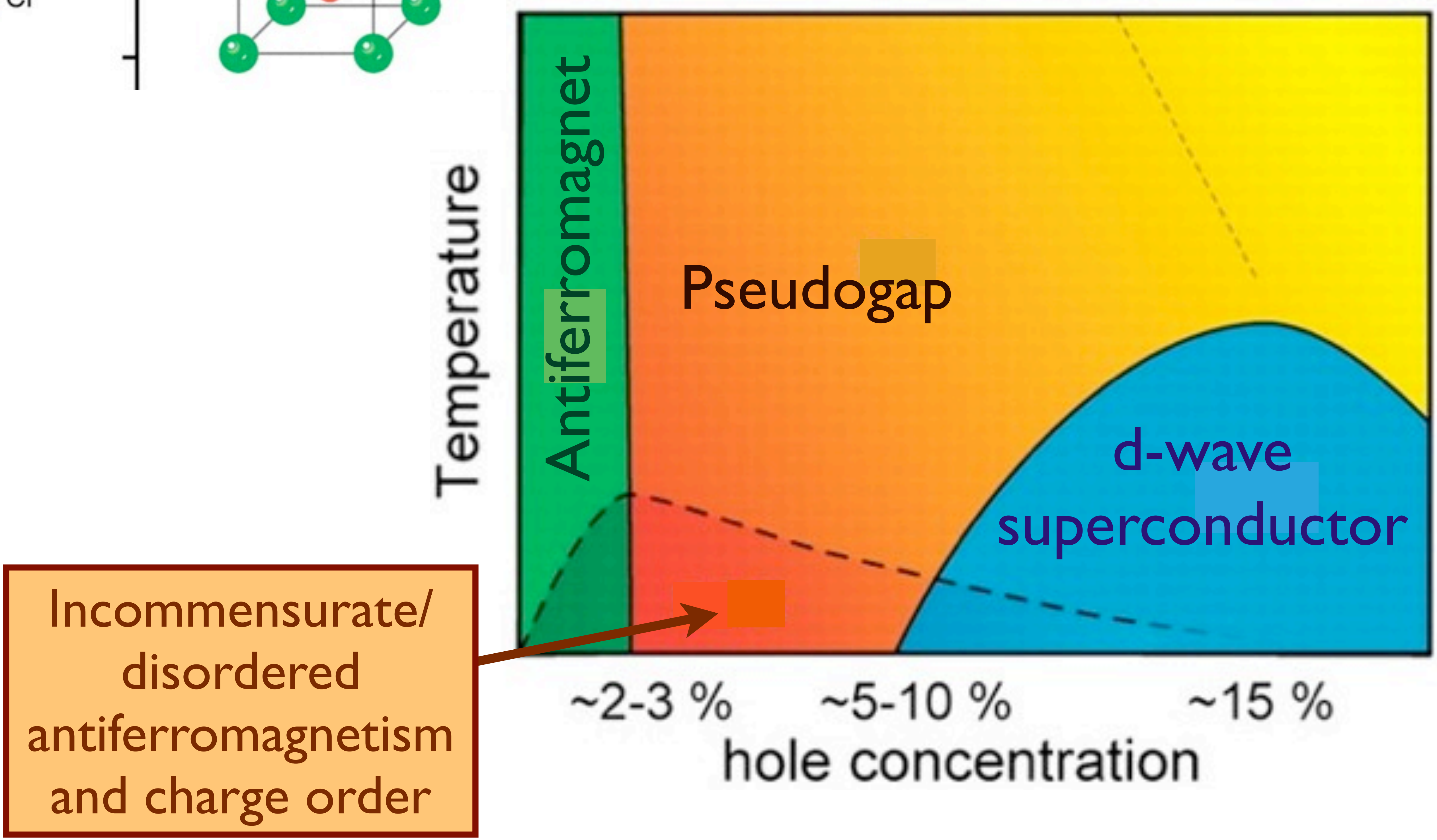
The cuprate superconductors



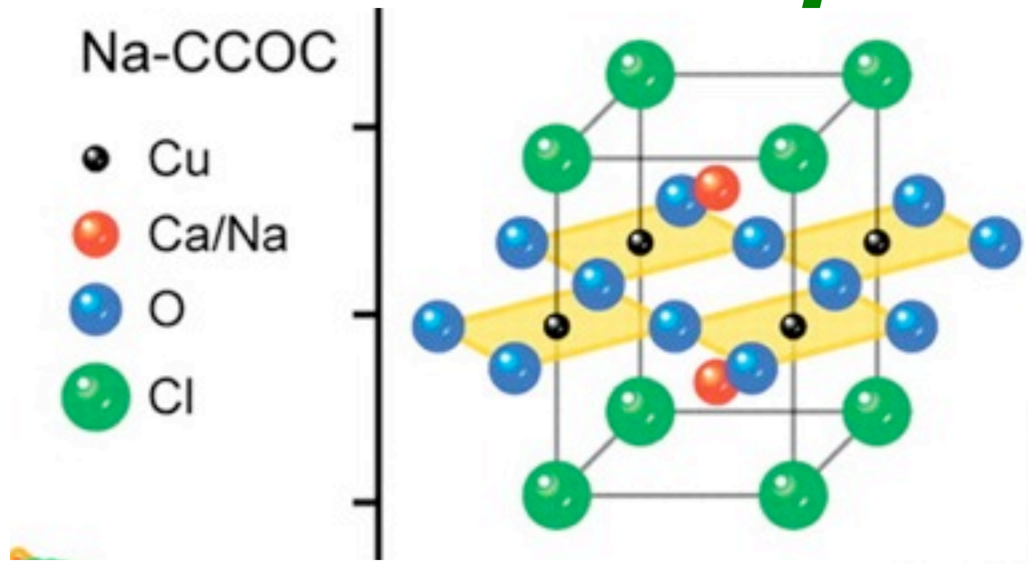


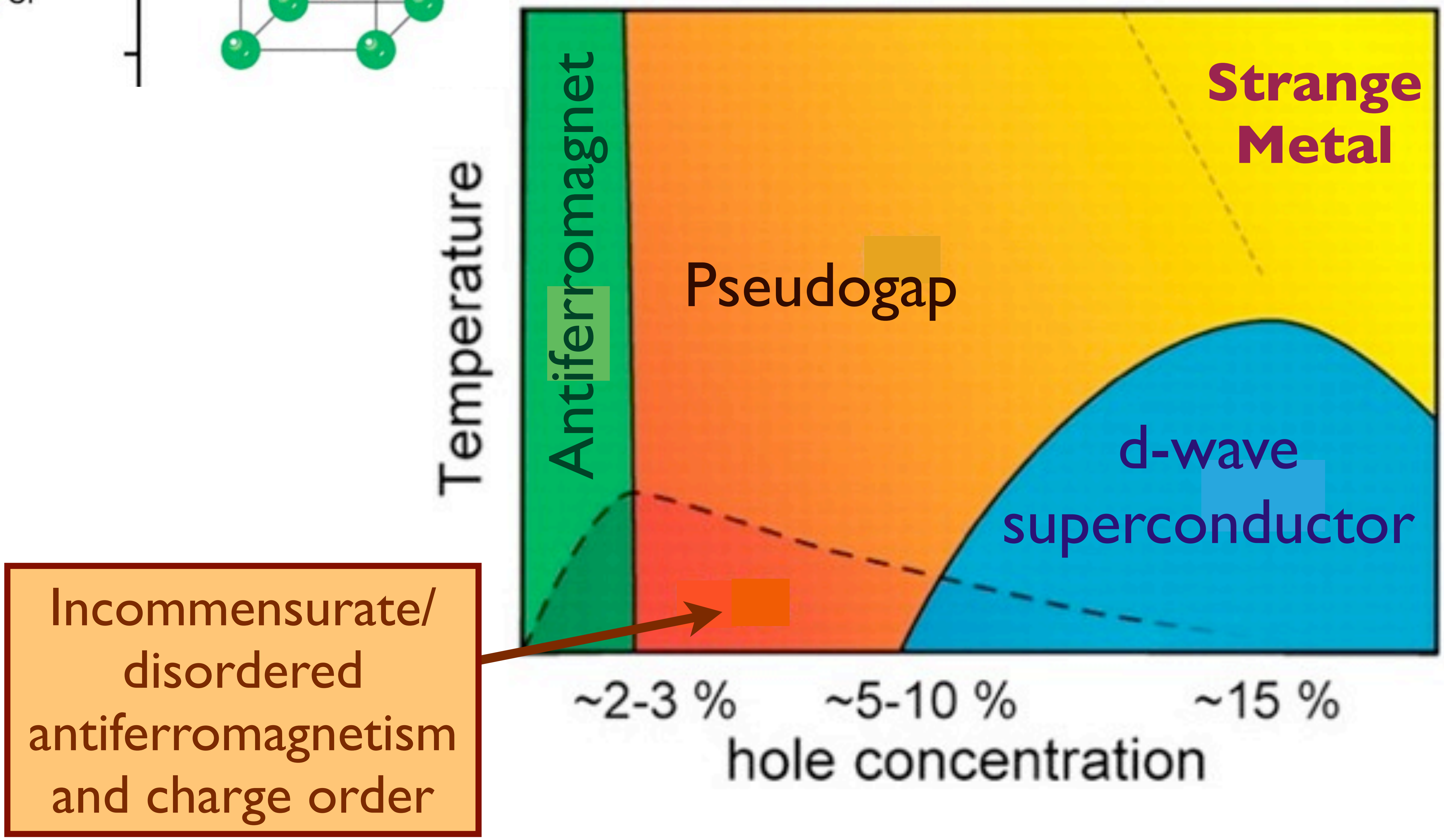
The cuprate superconductors

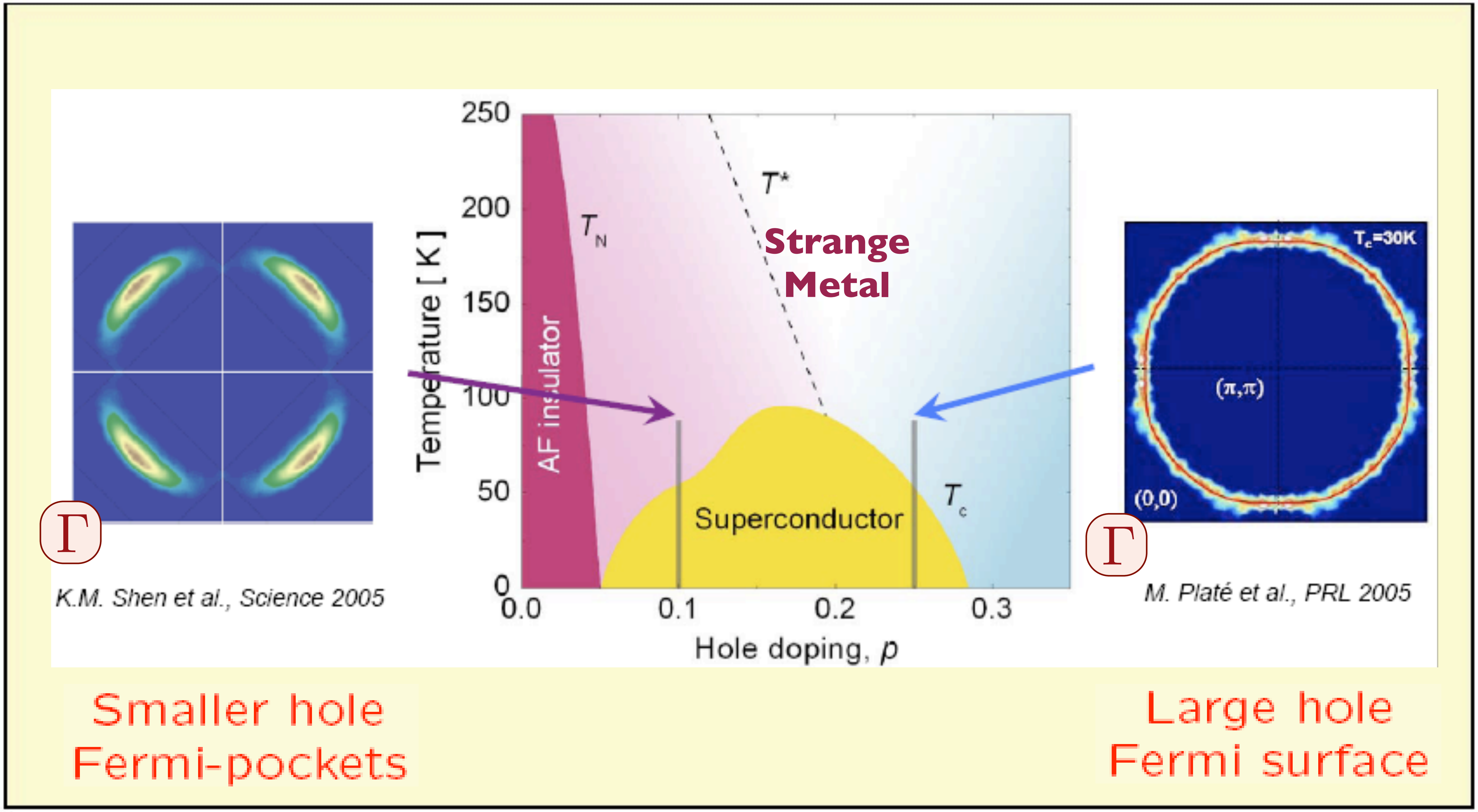




The cuprate superconductors

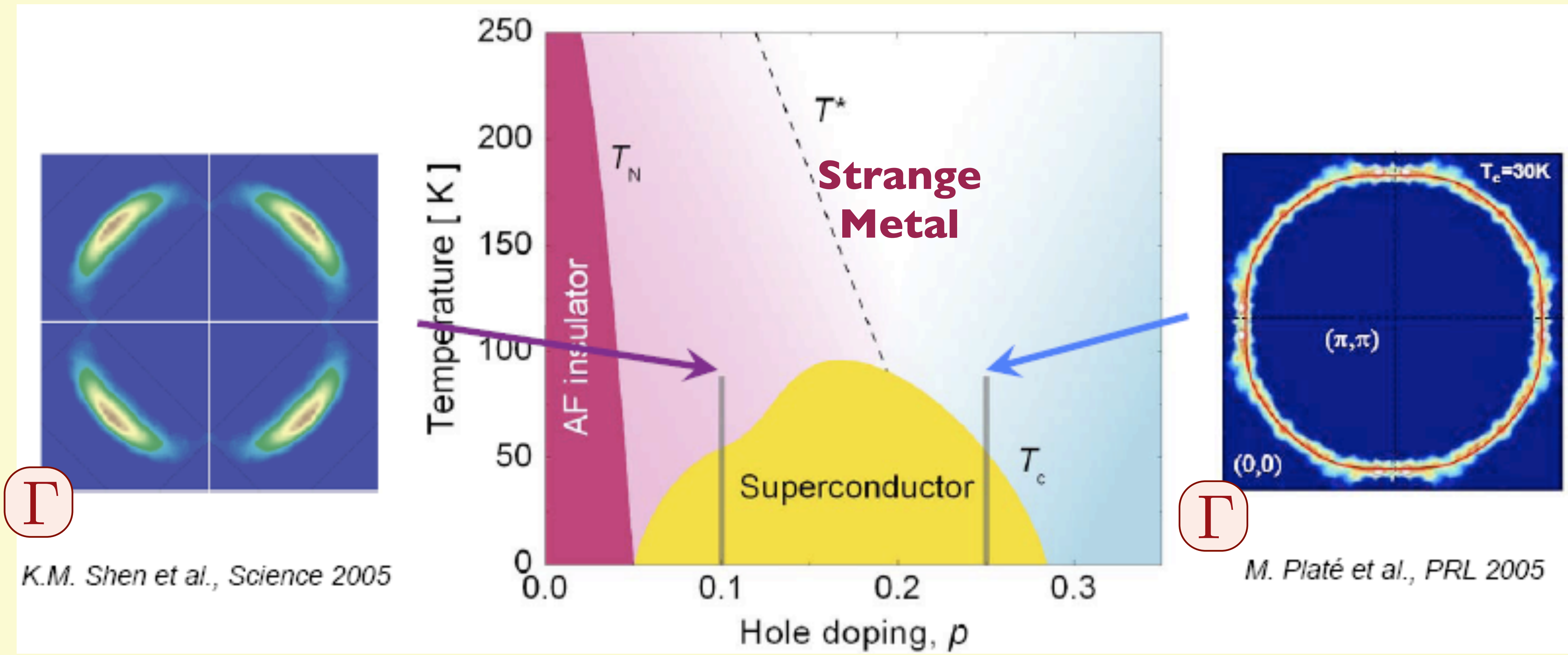






Γ
K.M. Shen et al., Science 2005
250
200
150
100
50
0
Temperature [K]
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Strange
Metal
Superconductor
T_c
T_c=30K
(π,π)
(0,0)
Γ
M. Platé et al., PRL 2005
Smaller hole
Fermi-pockets
Large hole
Fermi surface

Key Ingredients:

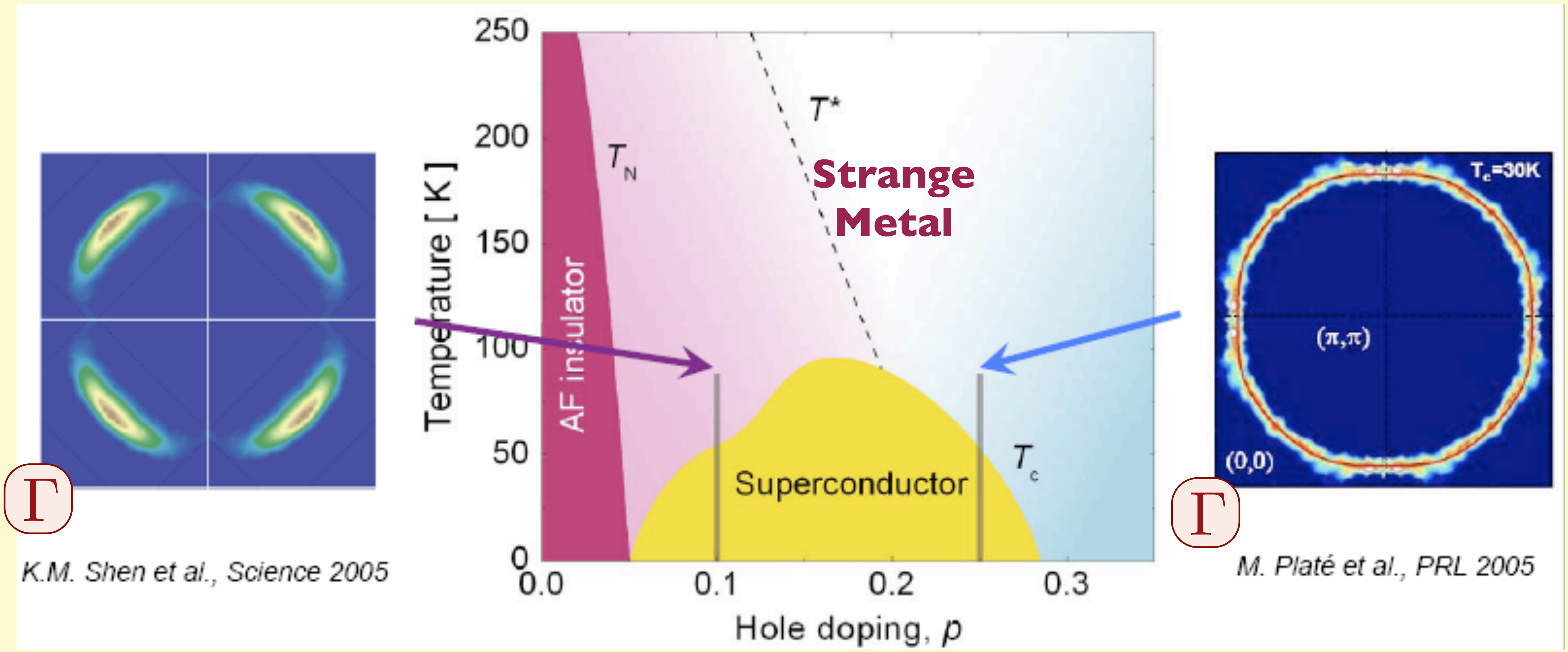


Smaller hole Fermi-pockets

Large hole Fermi surface

Key Ingredients:

Antiferromagnetism (AF)
Spin density wave (SDW)



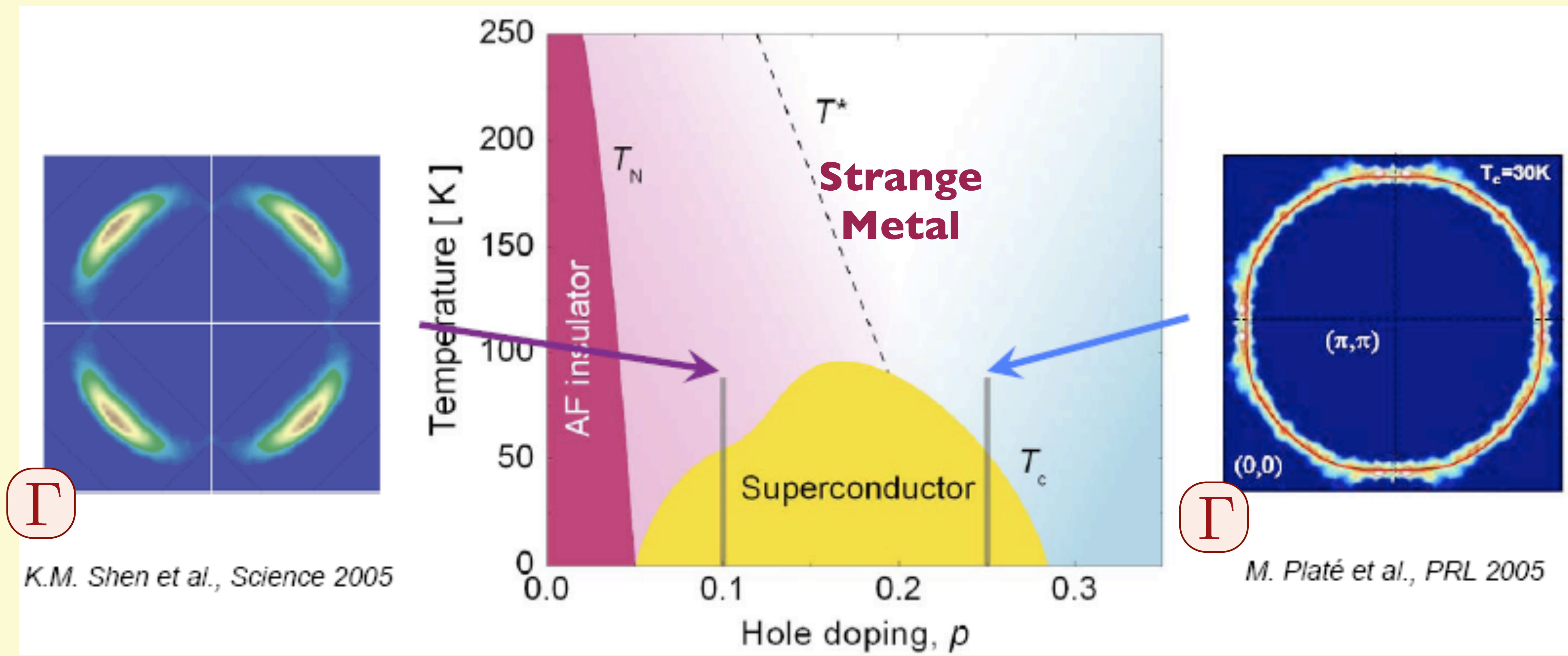
Smaller hole Fermi-pockets

Large hole Fermi surface

Key Ingredients:

d-wave superconductivity

Antiferromagnetism (AF)
Spin density wave (SDW)



Smaller hole Fermi-pockets

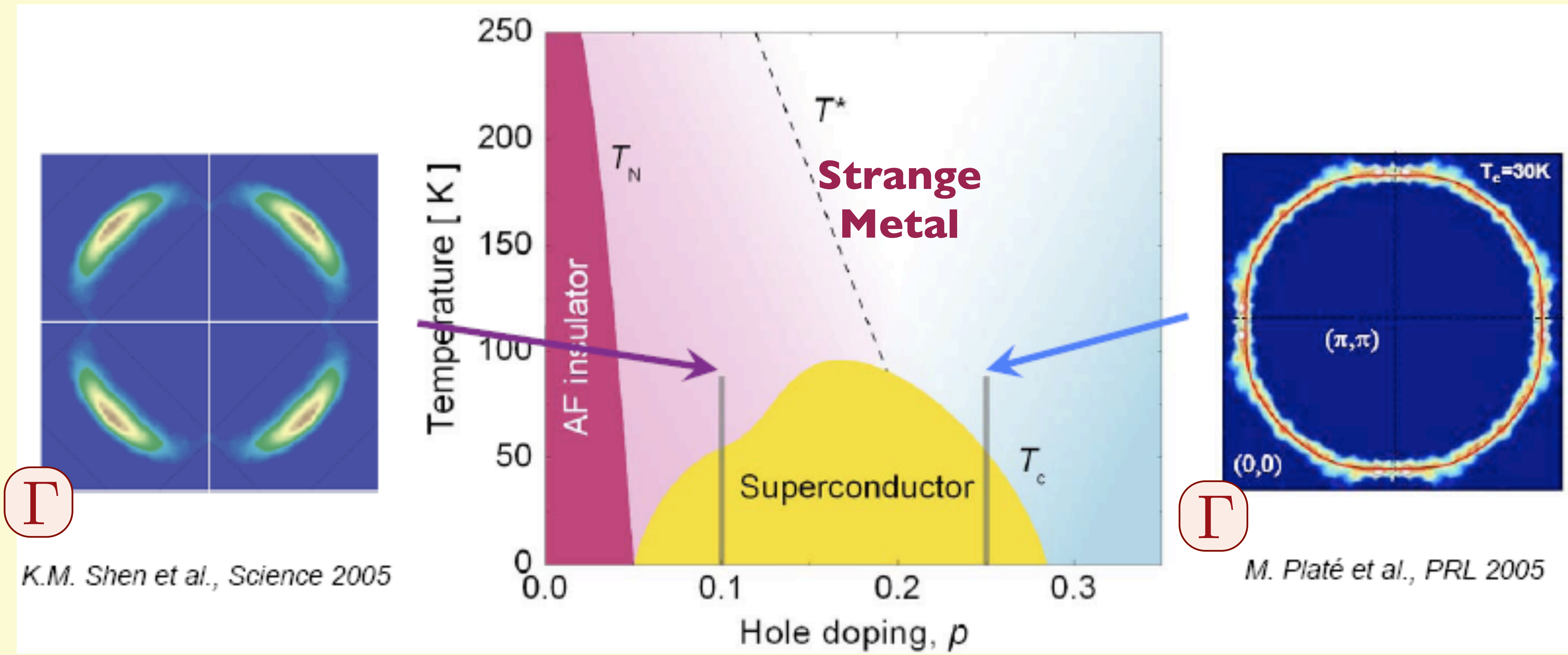
Large hole Fermi surface

Key Ingredients:

d-wave superconductivity

Antiferromagnetism (AF)
Spin density wave (SDW)

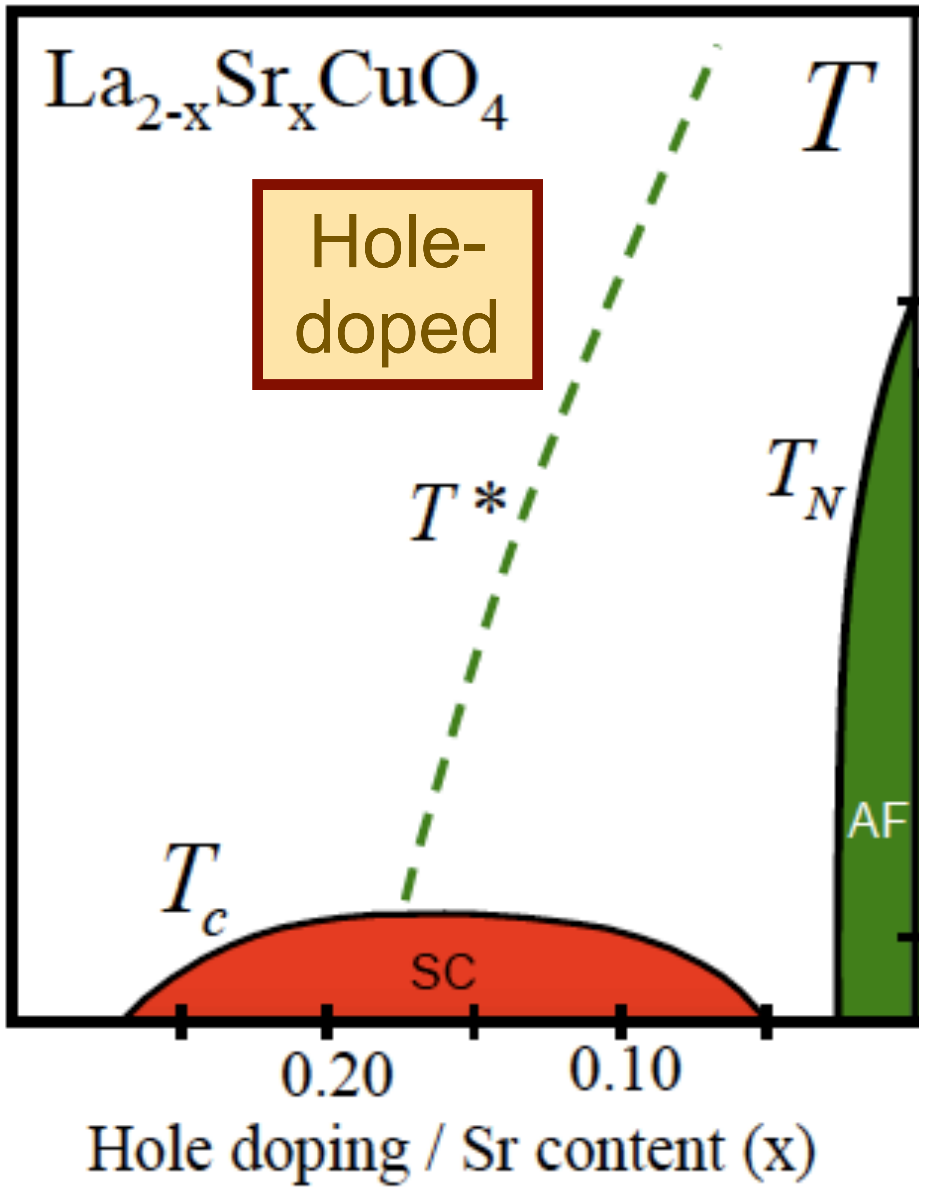
Fermi surface reconstruction



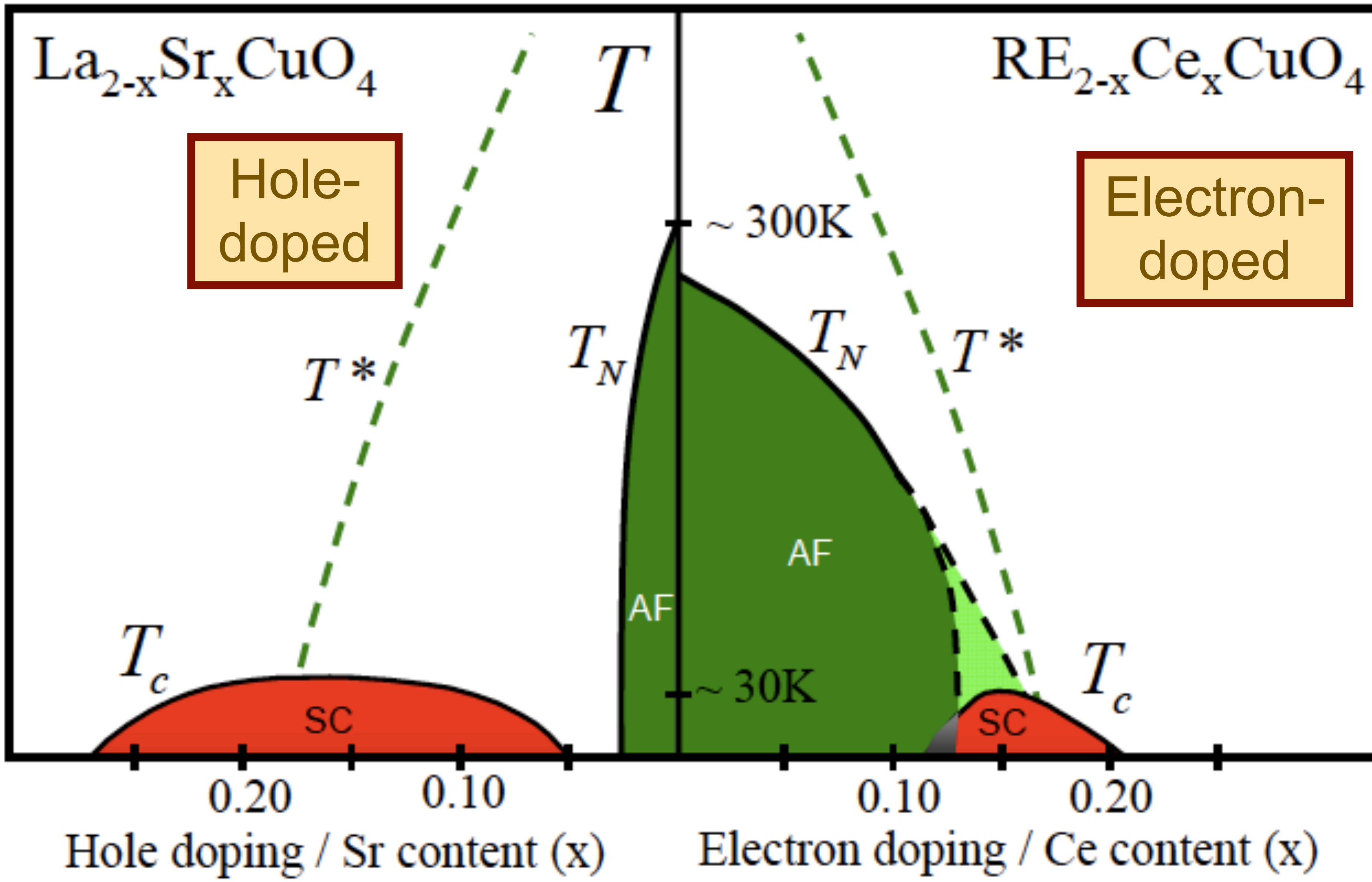
Smaller hole Fermi-pockets

Large hole Fermi surface

Thursday, May 5, 2011


$La_{2-x}Sr_xCuO_4$
T
Hole-doped
T^*
T_N
AF
T_c
SC
0.20
0.10
Hole doping / Sr content (x)

Electron-doped cuprate superconductors



Electron-doped cuprate superconductors

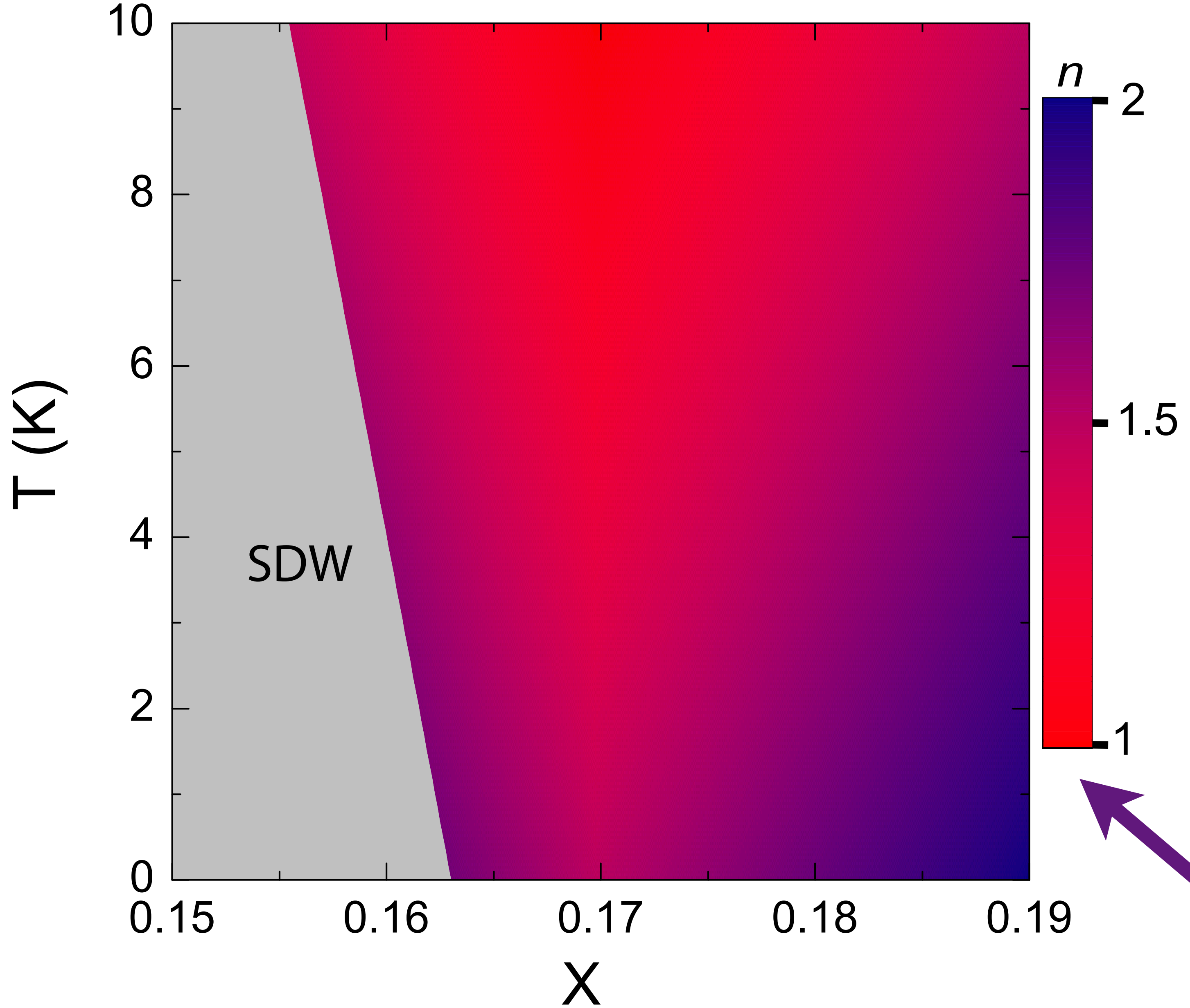
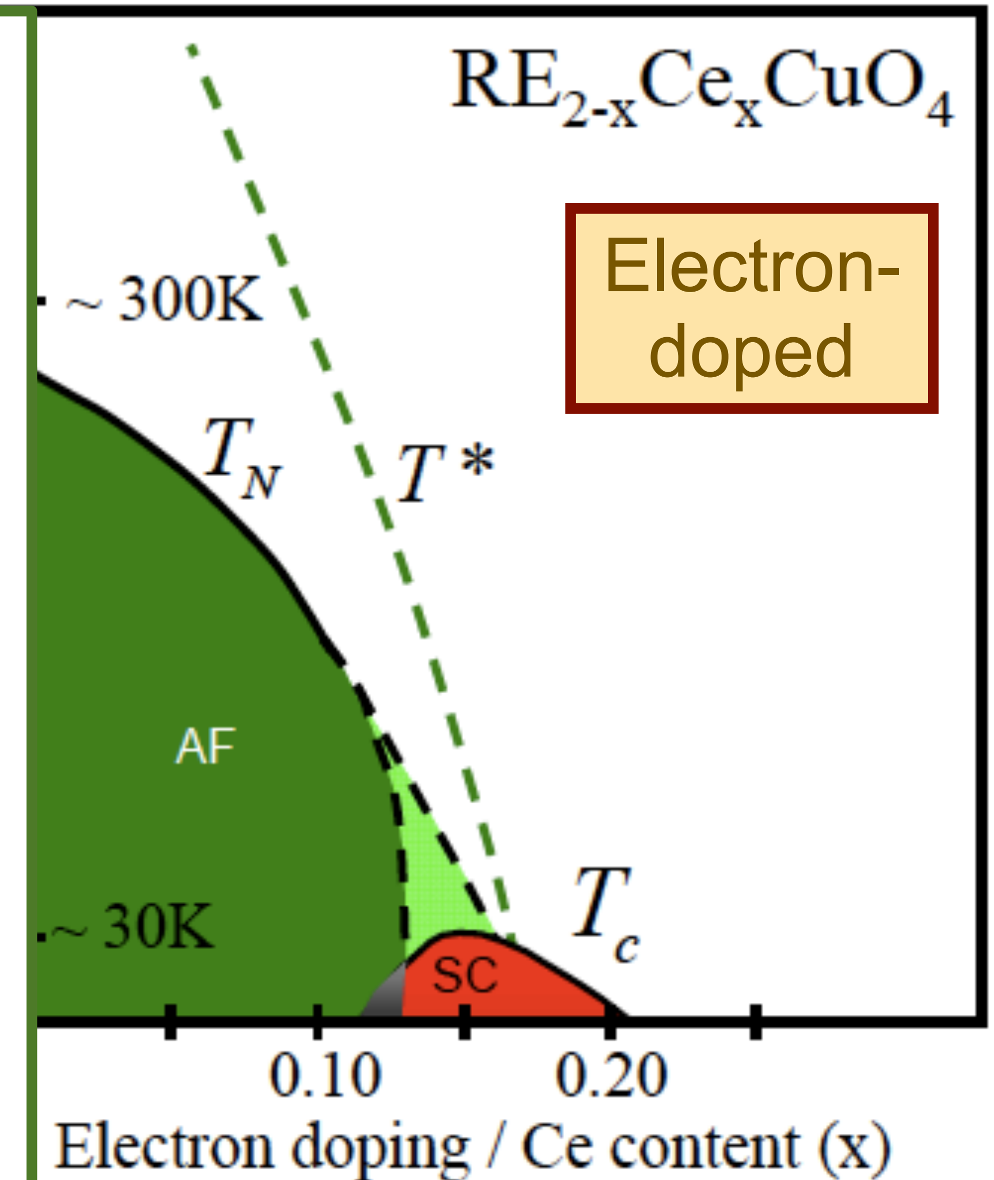


Figure prepared by K. Jin and and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).



Resistivity $\sim \rho_0 + AT^n$

Electron-doped cuprate superconductors

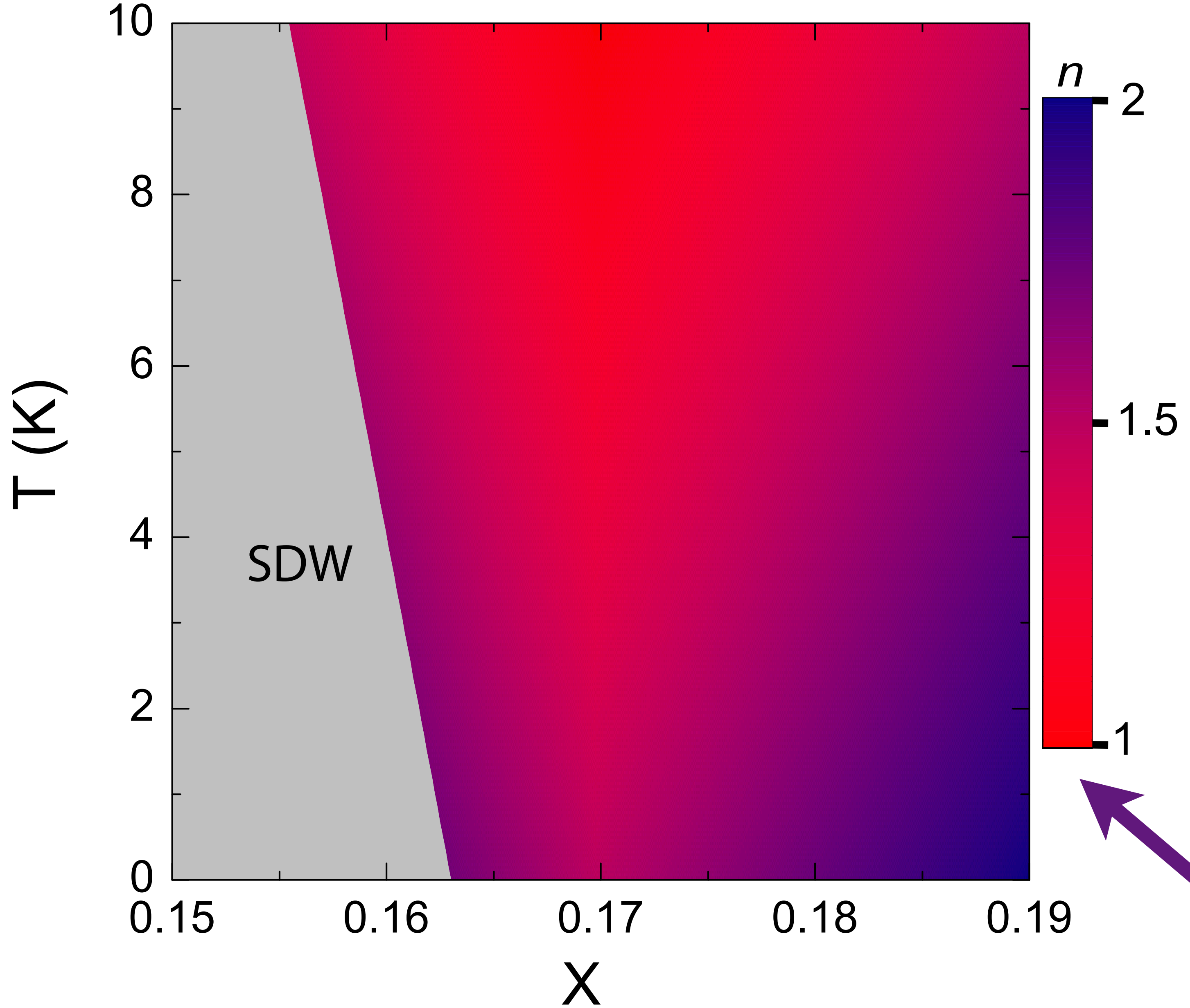
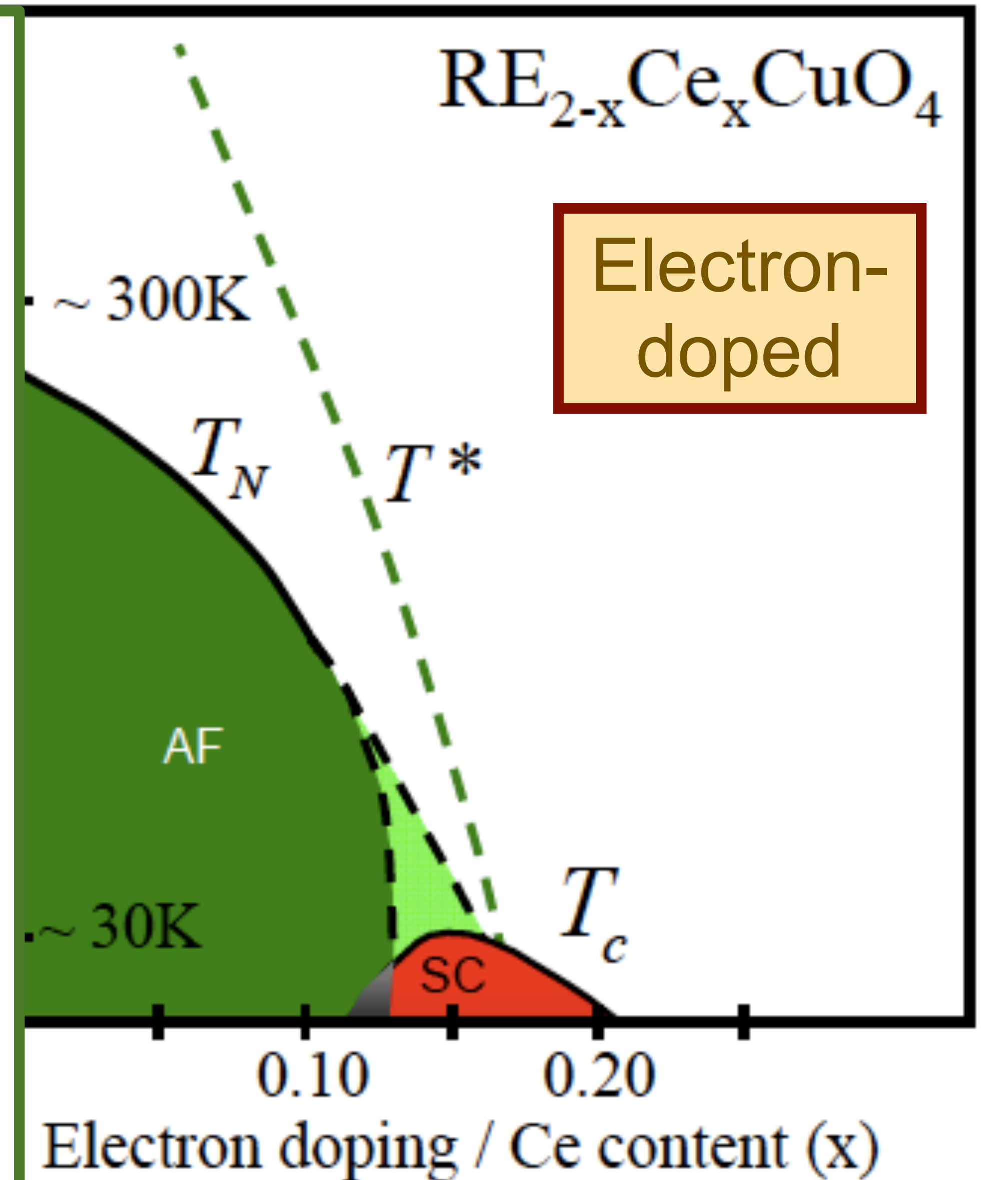


Figure prepared by K. Jin and and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).



Resistivity $\sim \rho_0 + AT^n$

Electron-doped cuprate superconductors

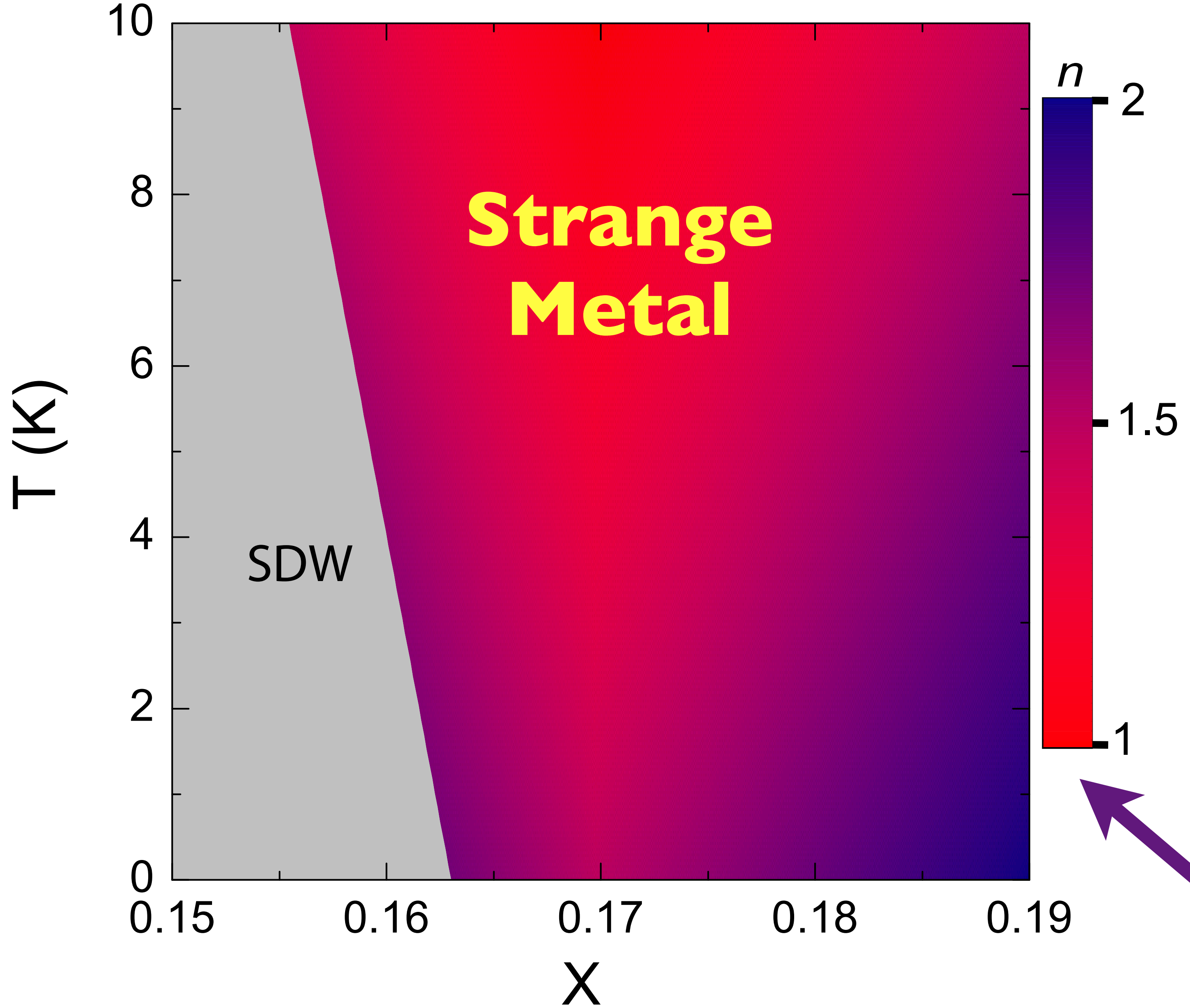
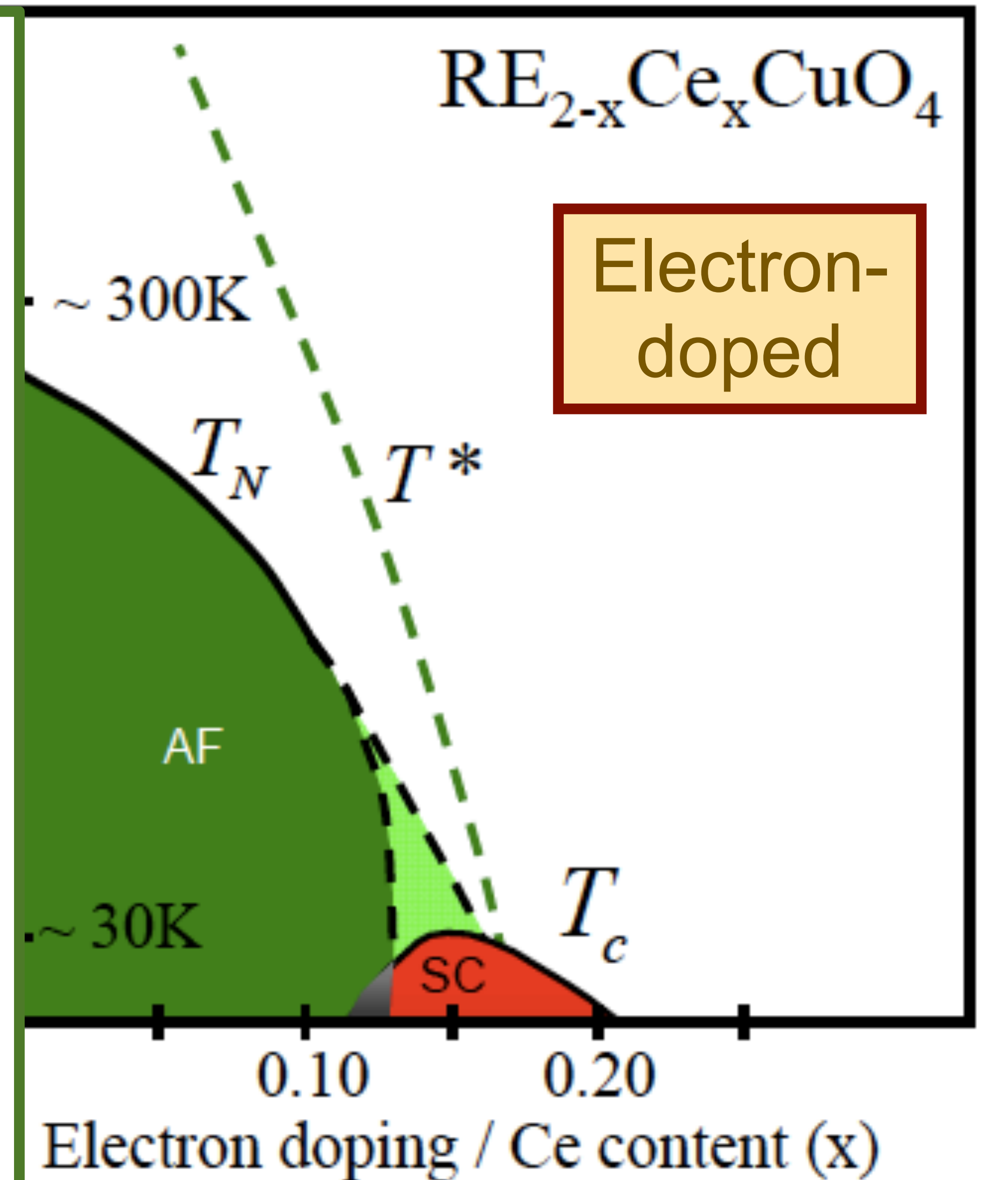


Figure prepared by K. Jin and and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).

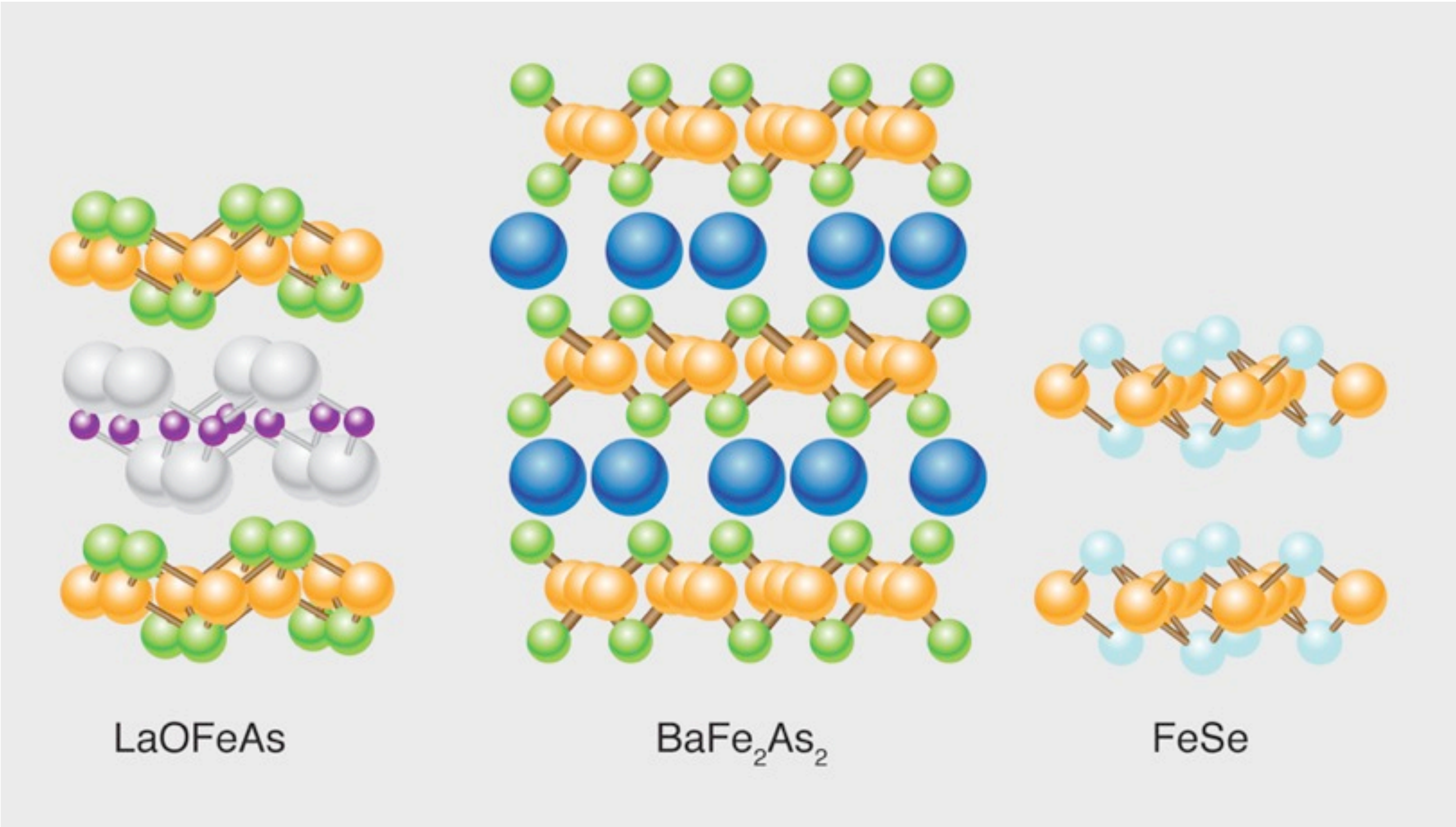


Resistivity

$$\sim \rho_0 + AT^n$$

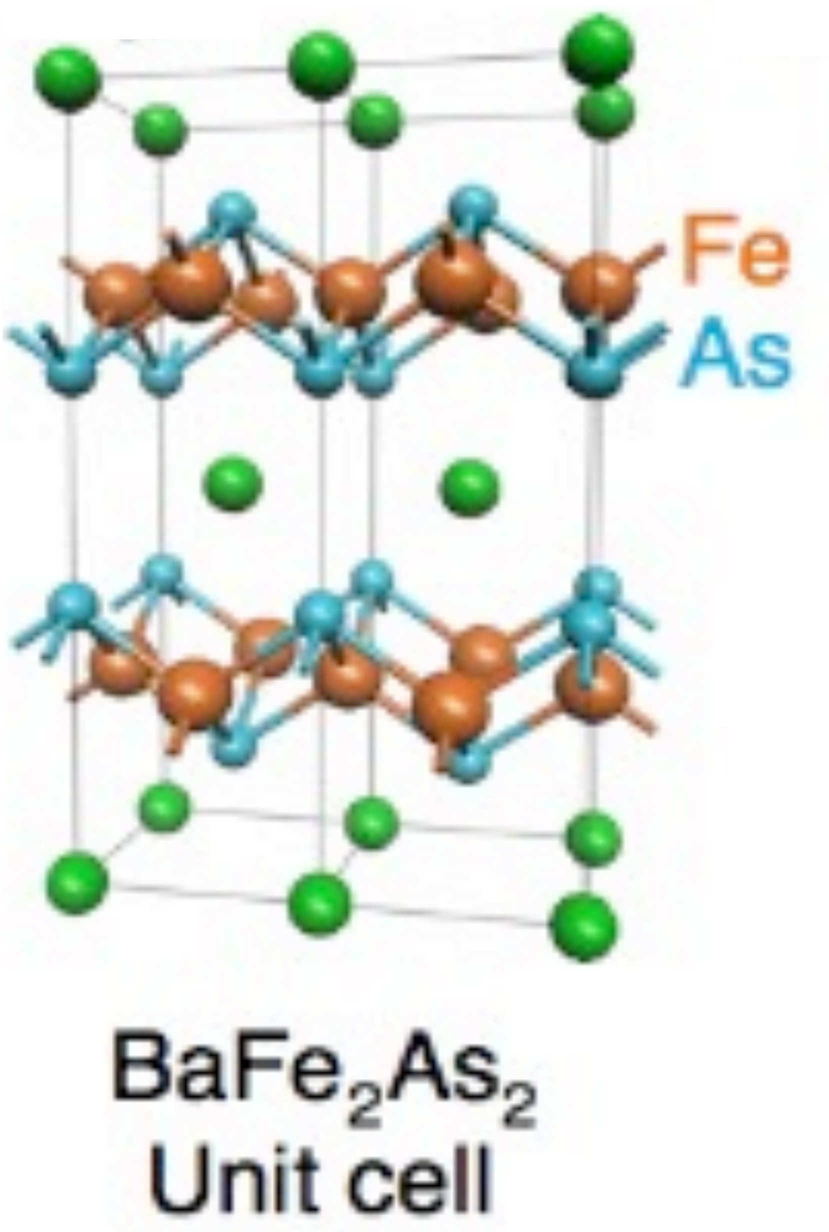
Iron pnictides:

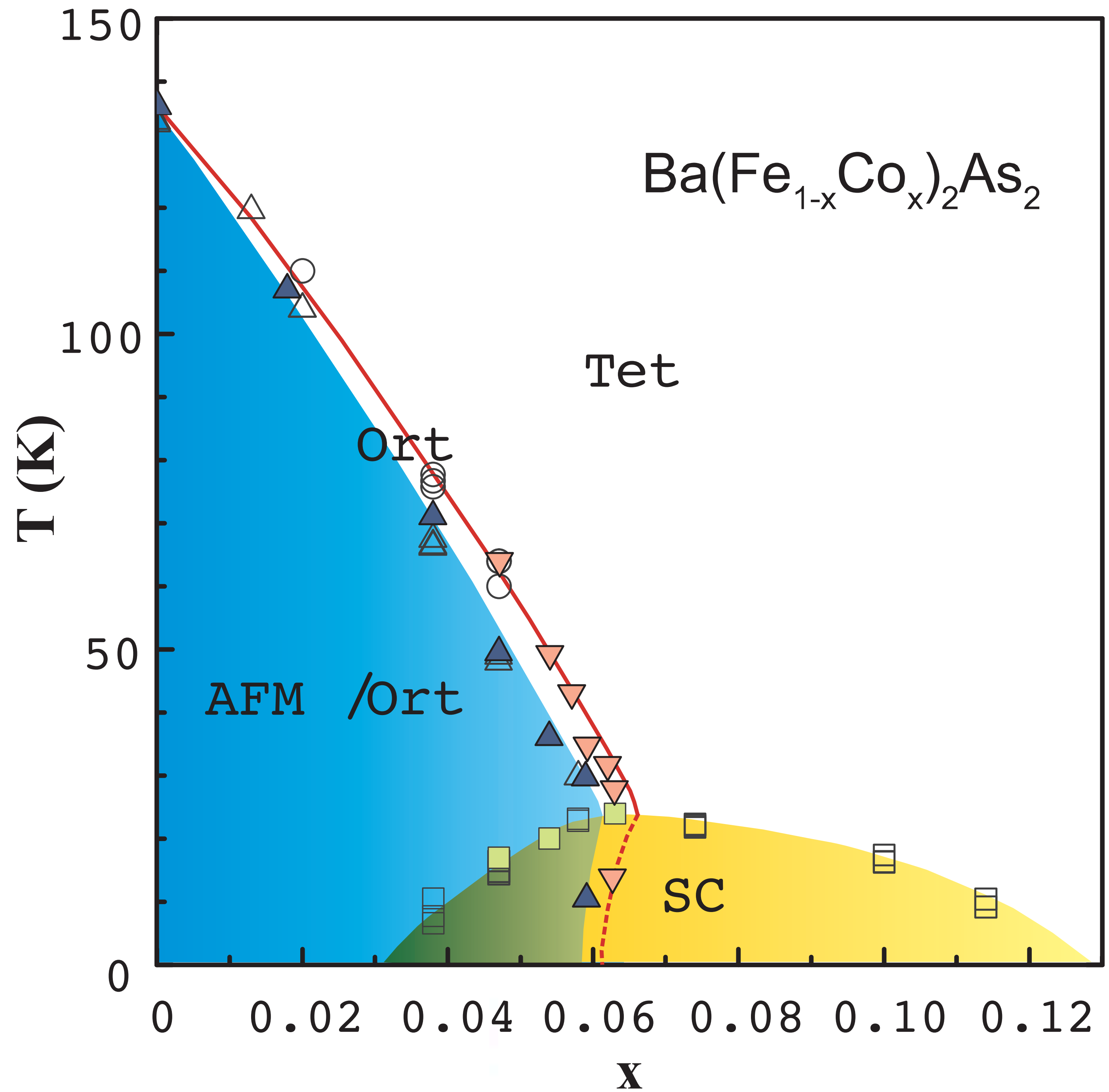
a new class of high temperature superconductors



Iron pnictides:

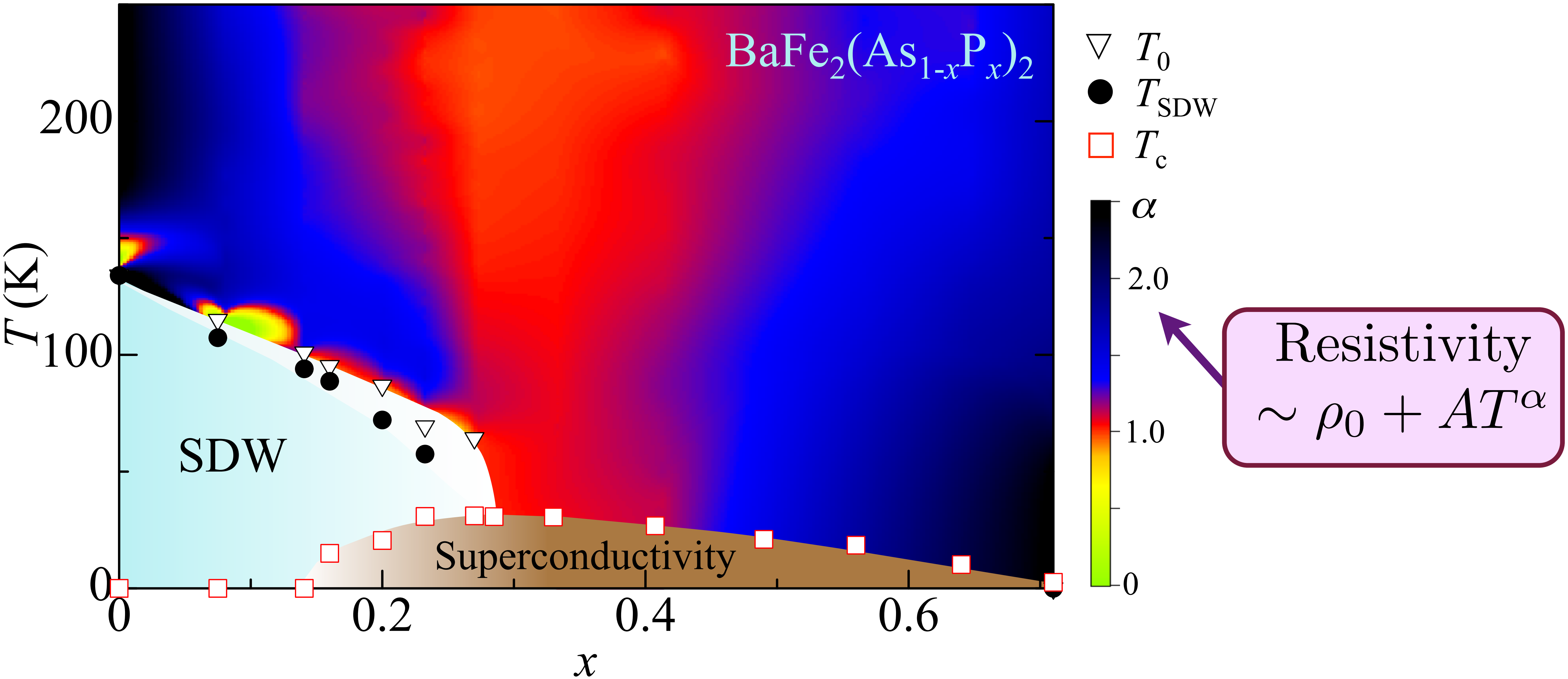
a new class of high temperature superconductors





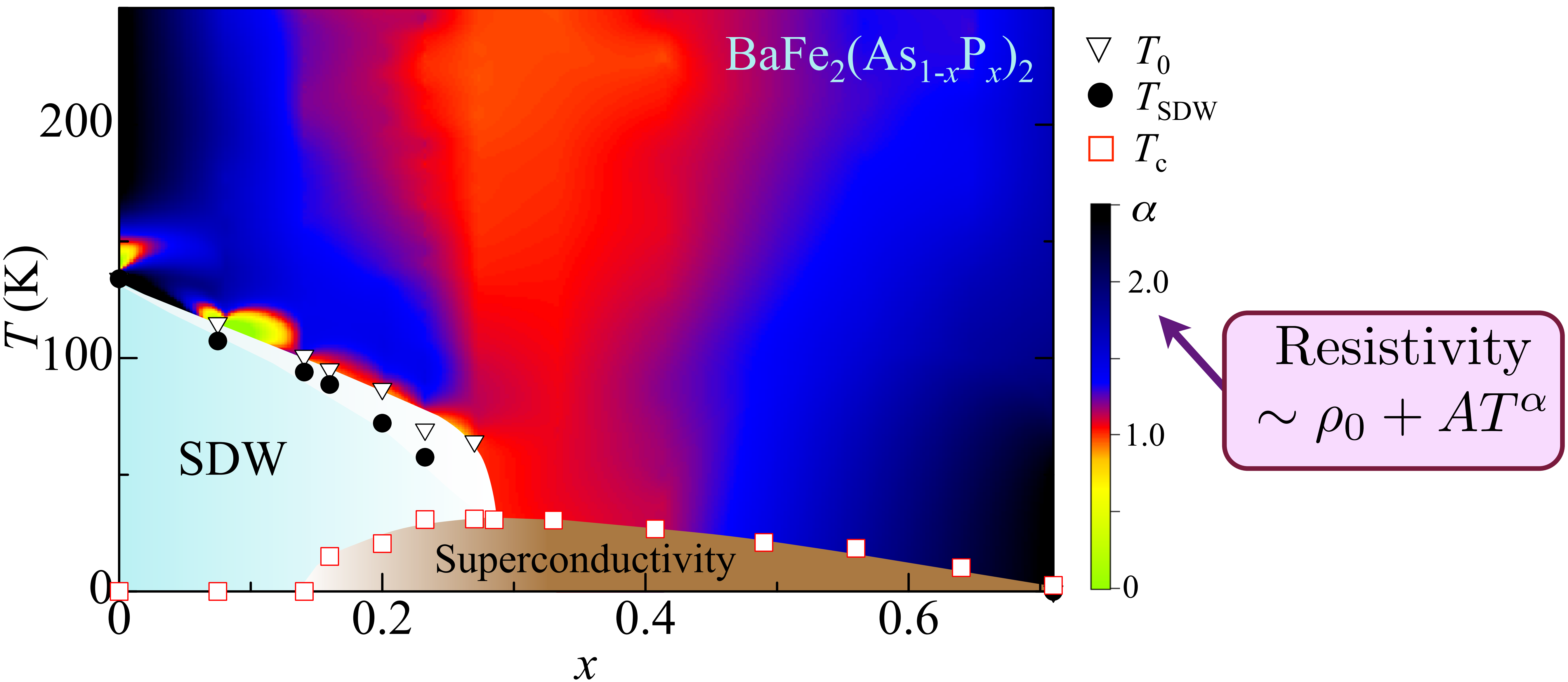
S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, *Physical Review Letters* **104**, 057006 (2010).

Temperature-doping phase diagram of the iron pnictides:



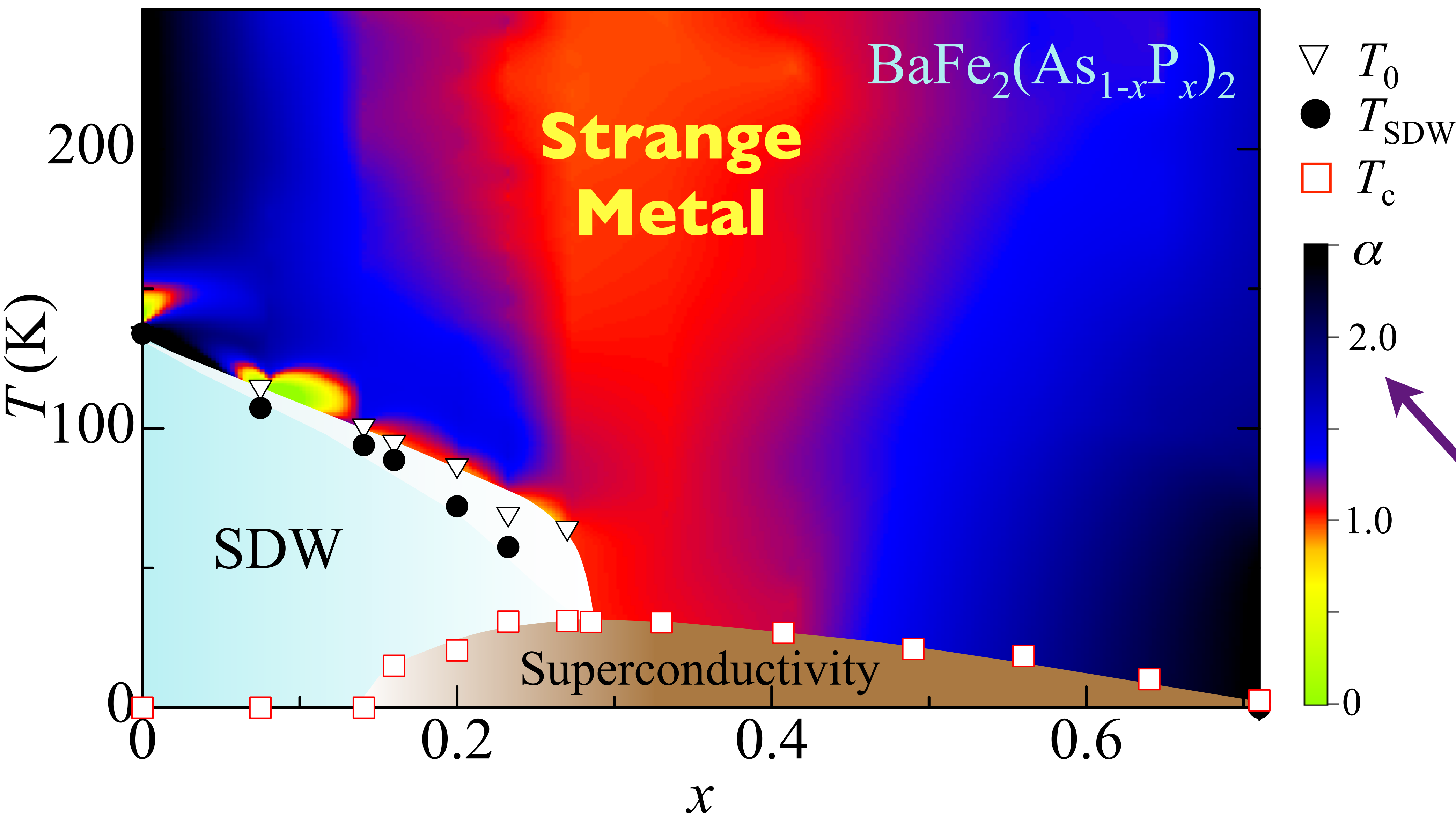
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Temperature-doping phase diagram of the iron pnictides:



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Temperature-doping phase diagram of the iron pnictides:



Resistivity $\sim \rho_0 + AT^\alpha$

S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Questions

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

Questions

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

- *If so, why is there no antiferromagnetism in the hole-doped cuprates near the point where the superconductivity is strongest ?*

Questions

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

- *If so, why is there no antiferromagnetism in the hole-doped cuprates near the point where the superconductivity is strongest ?*

- *What is the physics of the strange metal ?*

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

   Quantum criticality of Fermi surface reconstruction, and the phase diagram in a magnetic field

2. Strongly-coupled quantum criticality in metals

   Fermi surfaces and gapless bosons

3. Instability to unconventional superconductivity

   "Mechanism" of higher temperature superconductivity

4. Theory of the competition between superconductivity and antiferromagnetism

Outline

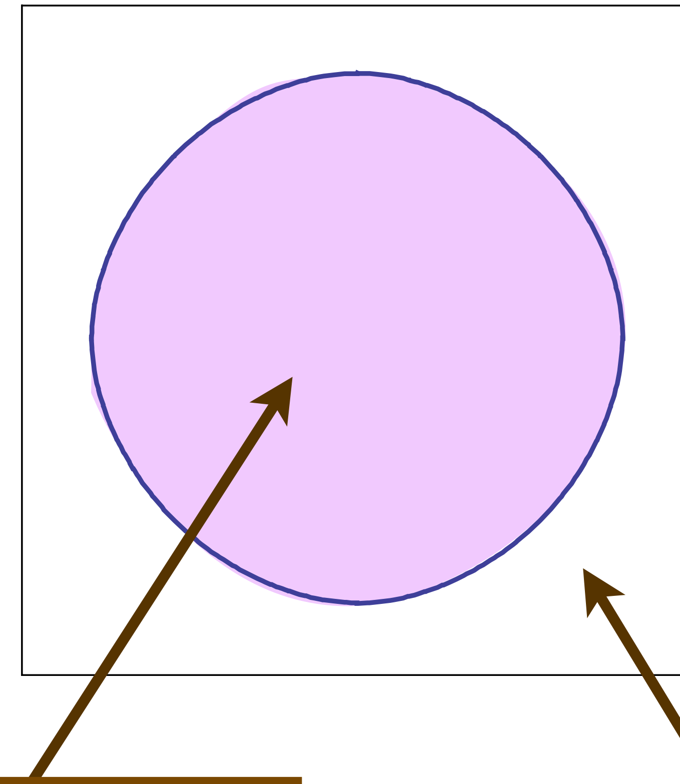
1. Phenomenology of the onset of antiferromagnetism in a metal
   Quantum criticality of Fermi surface reconstruction, and the phase diagram in a magnetic field

2. Strongly-coupled quantum criticality in metals
   Fermi surfaces and gapless bosons

3. Instability to unconventional superconductivity
   "Mechanism" of higher temperature superconductivity

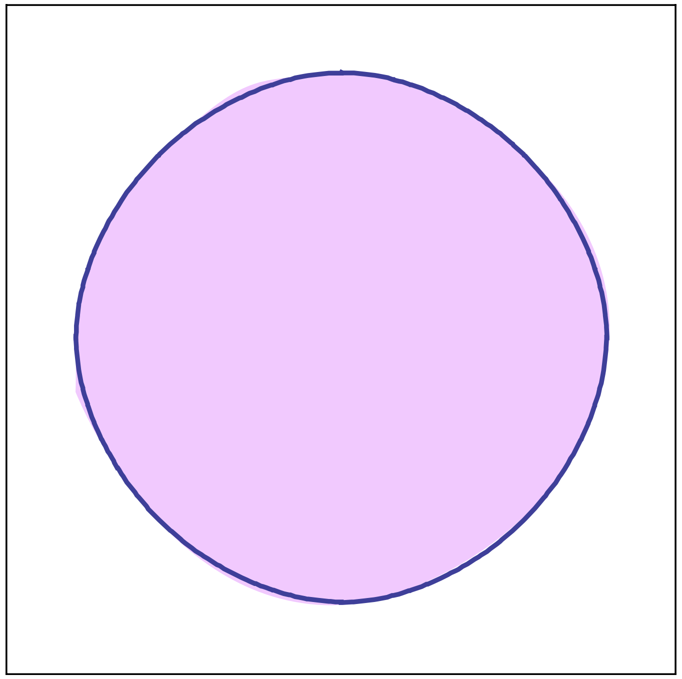
4. Theory of the competition between superconductivity and antiferromagnetism

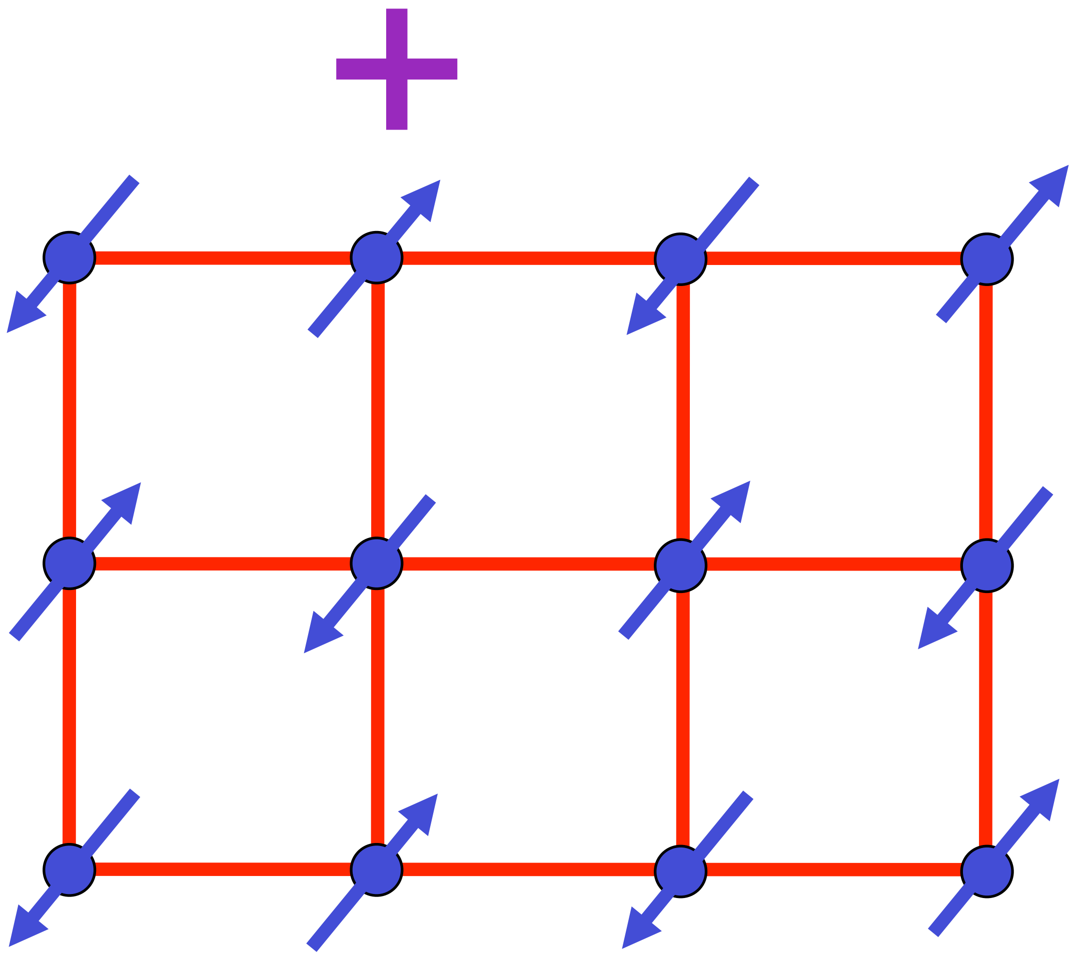
Fermi surface



Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface

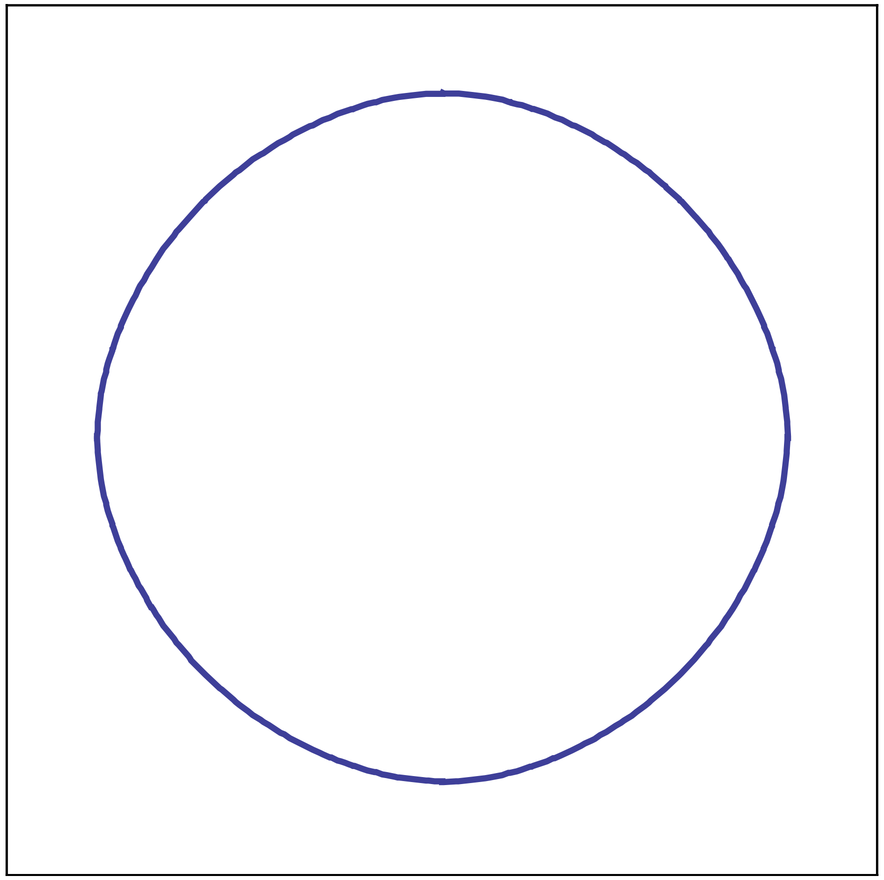




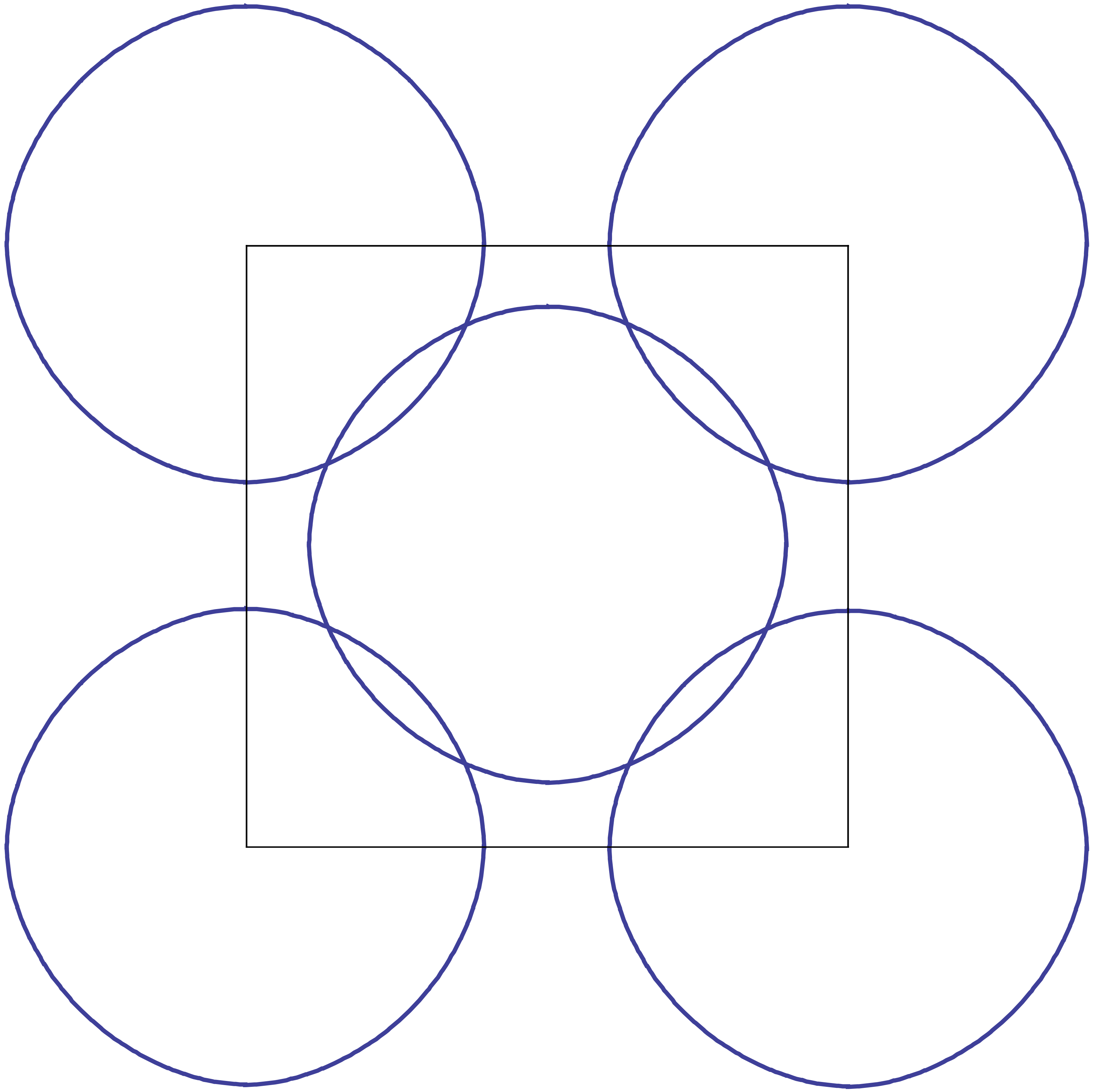
The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

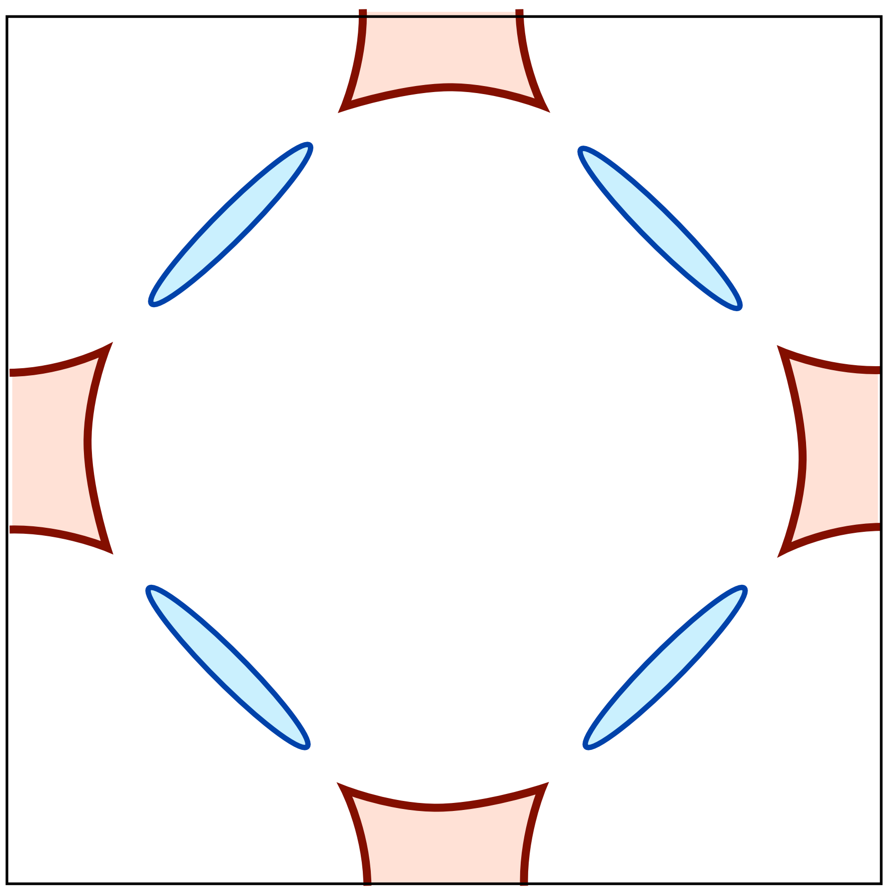
where $\mathbf{K}$ is the ordering wavevector.



Metal with “large” Fermi surface

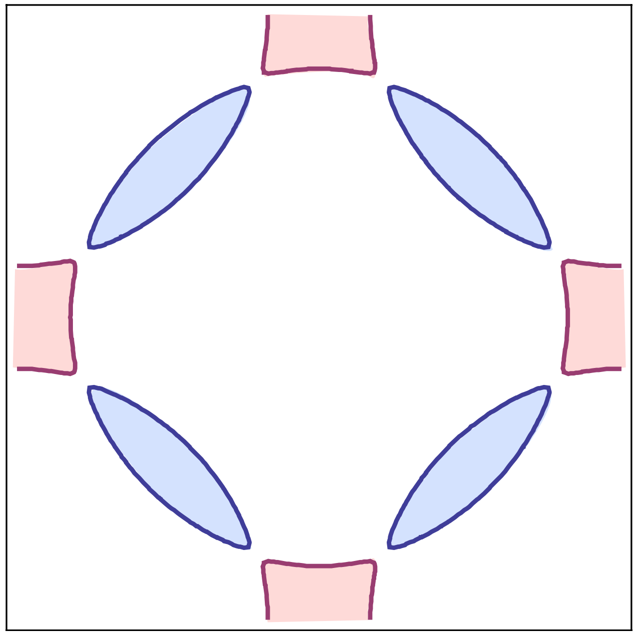


Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.



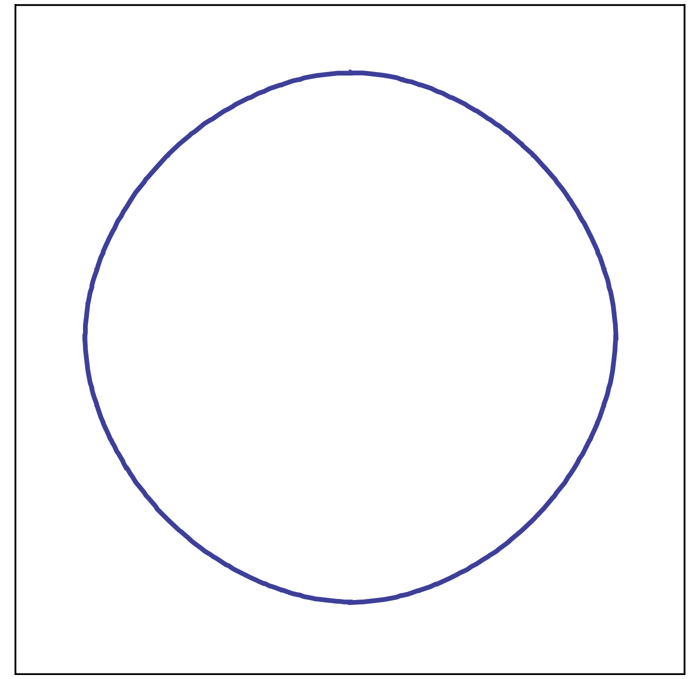
Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$

Fermi surface+antiferromagnetism



$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets



$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

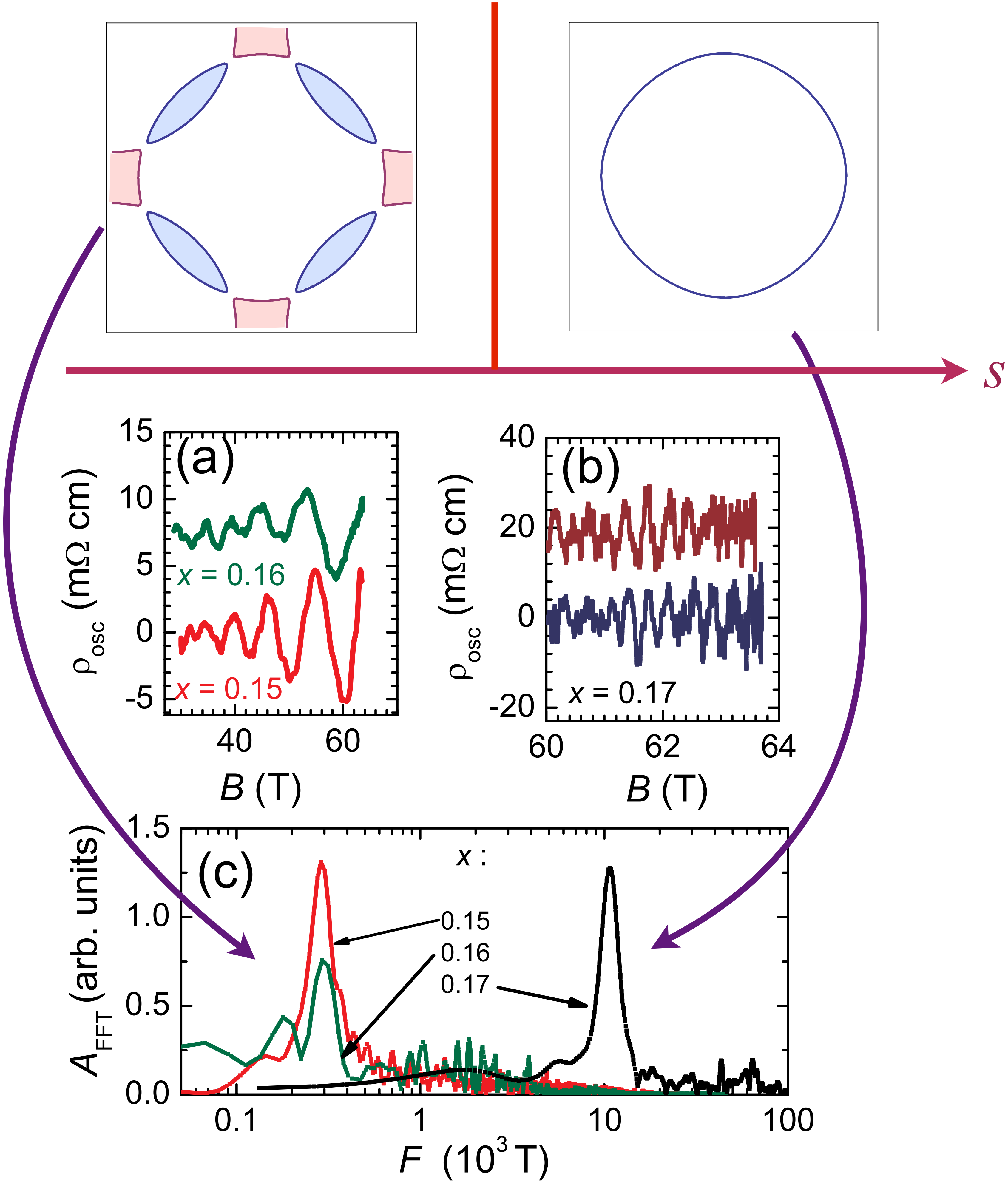
← Increasing interaction

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

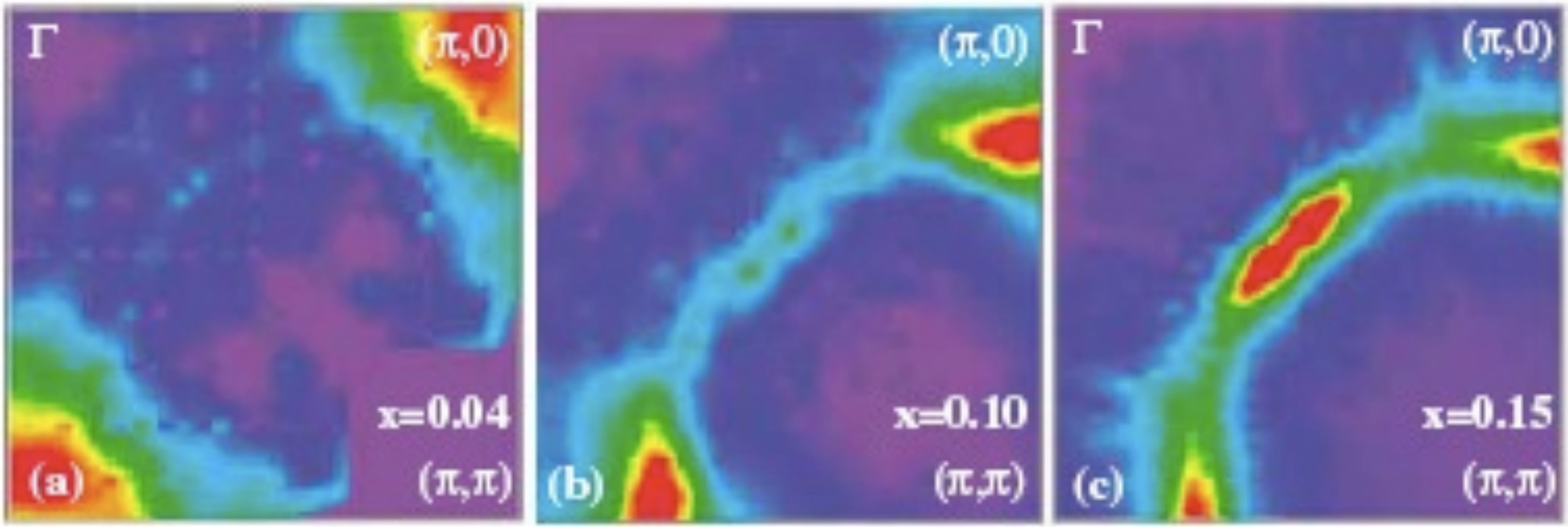
Quantum oscillations

$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross, *Phys. Rev. Lett.* **103**, 157002 (2009).

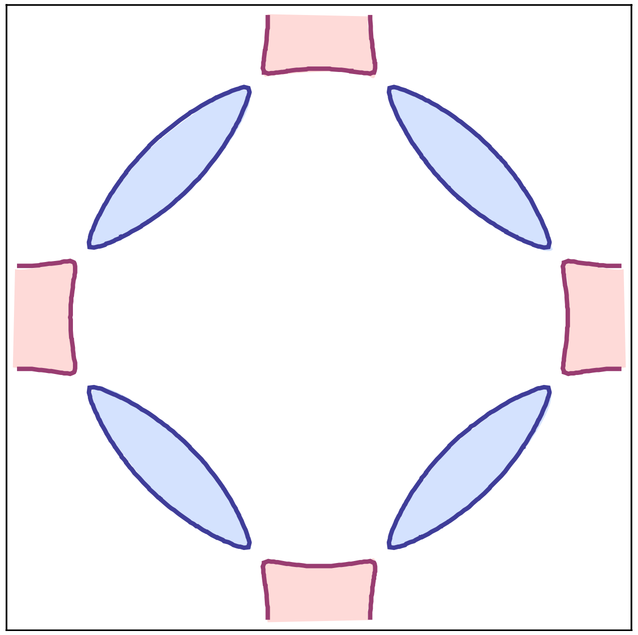


Photoemission in $Nd_{2-x}Ce_xCuO_4$



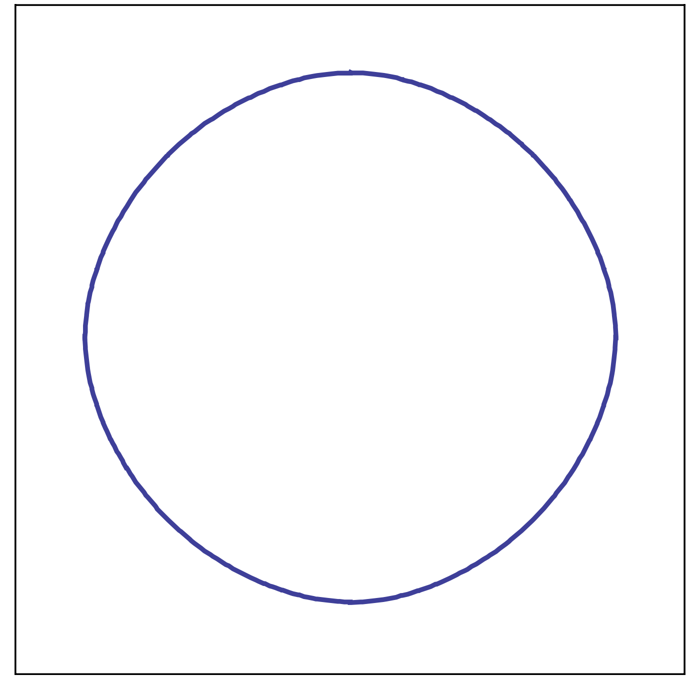
N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets



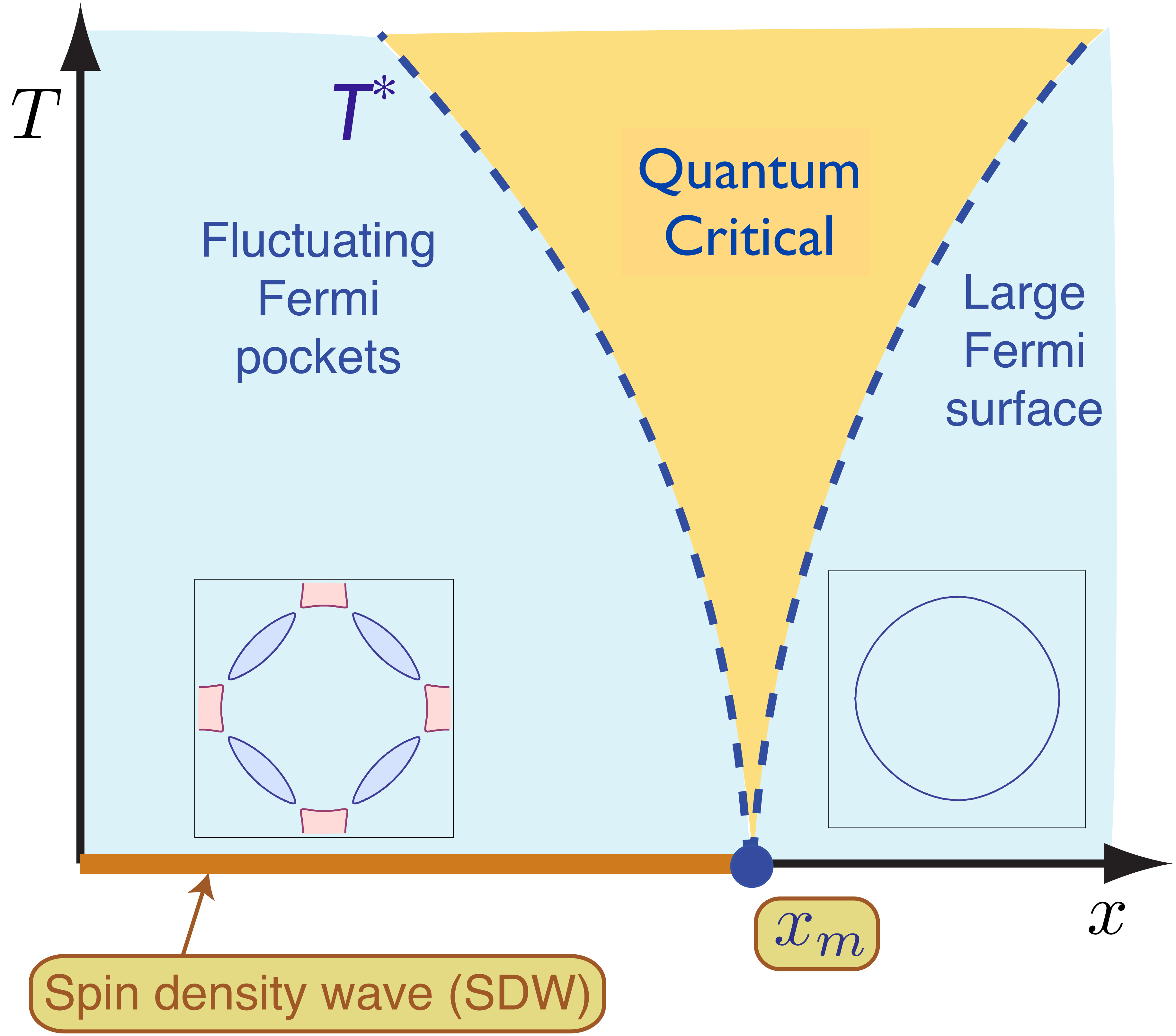
$$\langle \vec{\varphi} \rangle = 0$$

Metal with "large"
Fermi surface

s

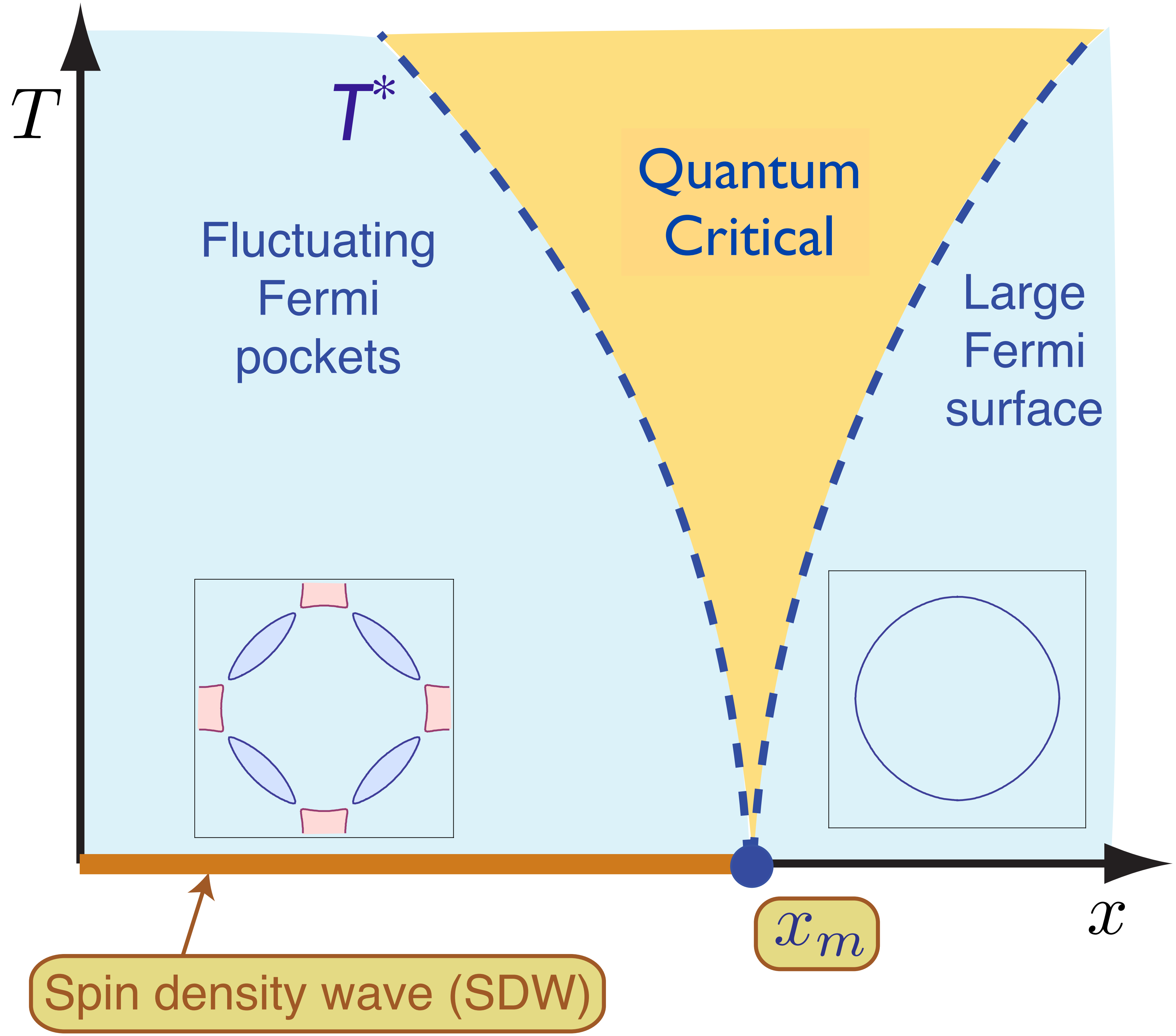
S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Theory of quantum criticality in the cuprates



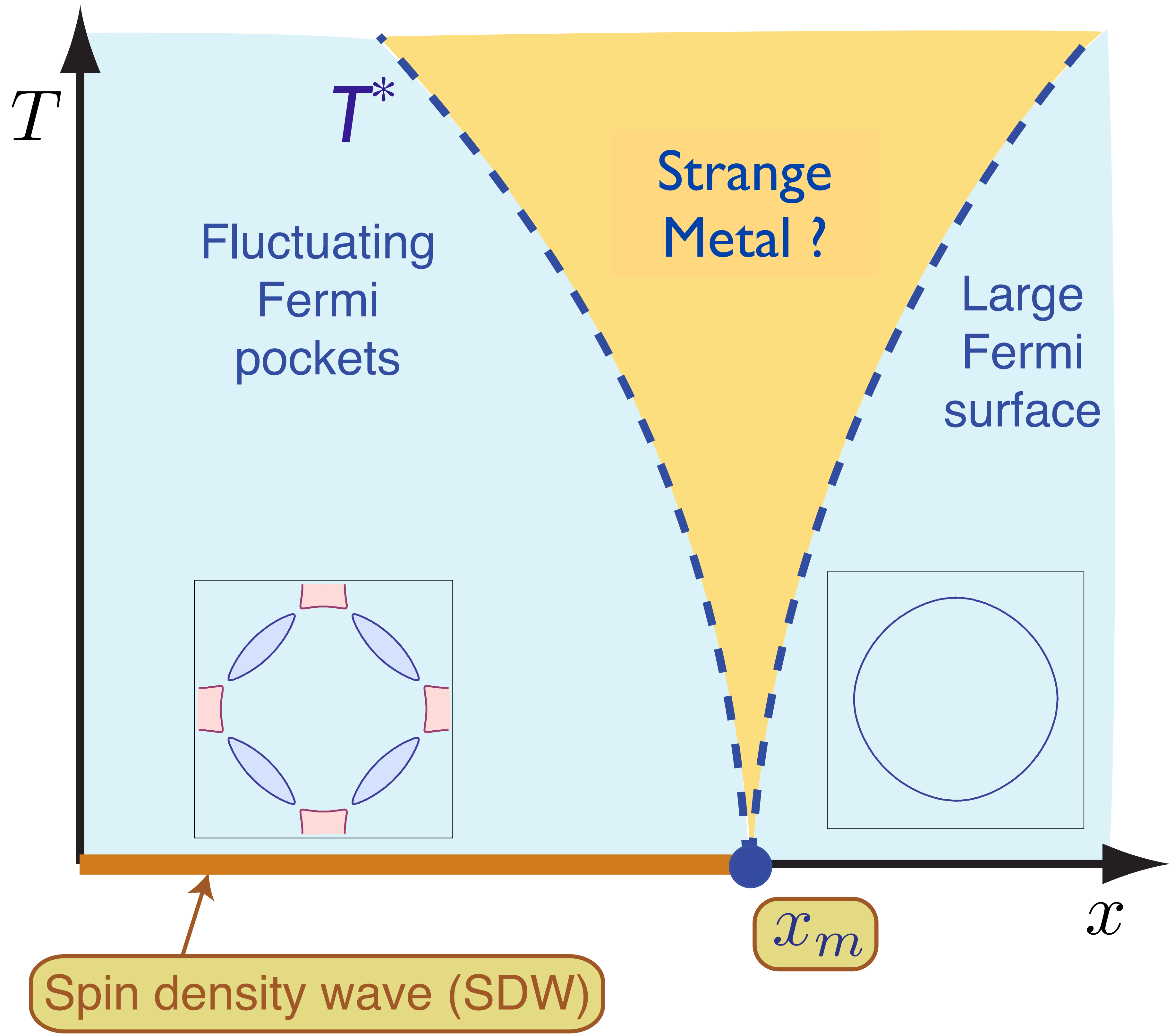
Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Theory of quantum criticality in the cuprates



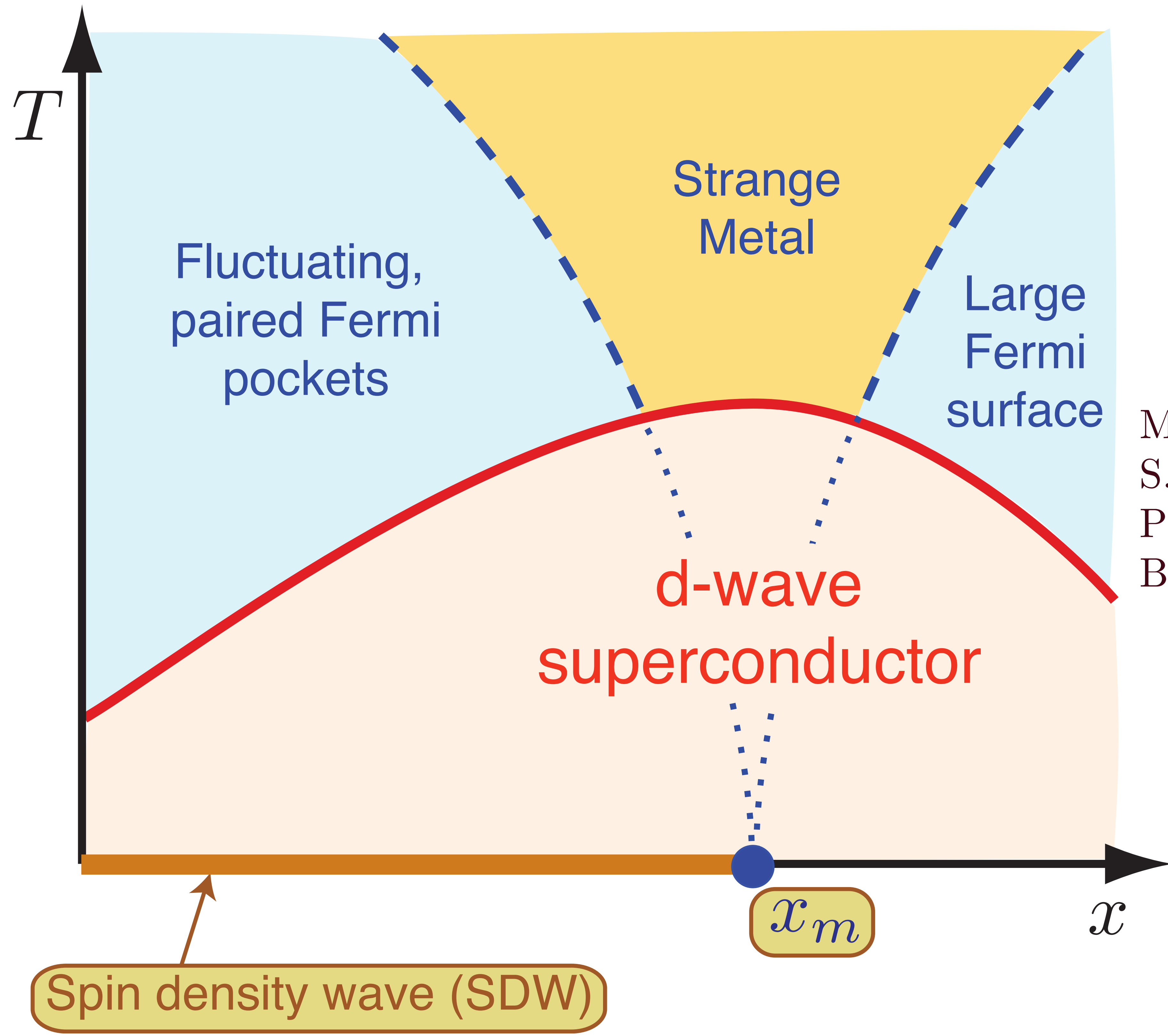
Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

Theory of quantum criticality in the cuprates



Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

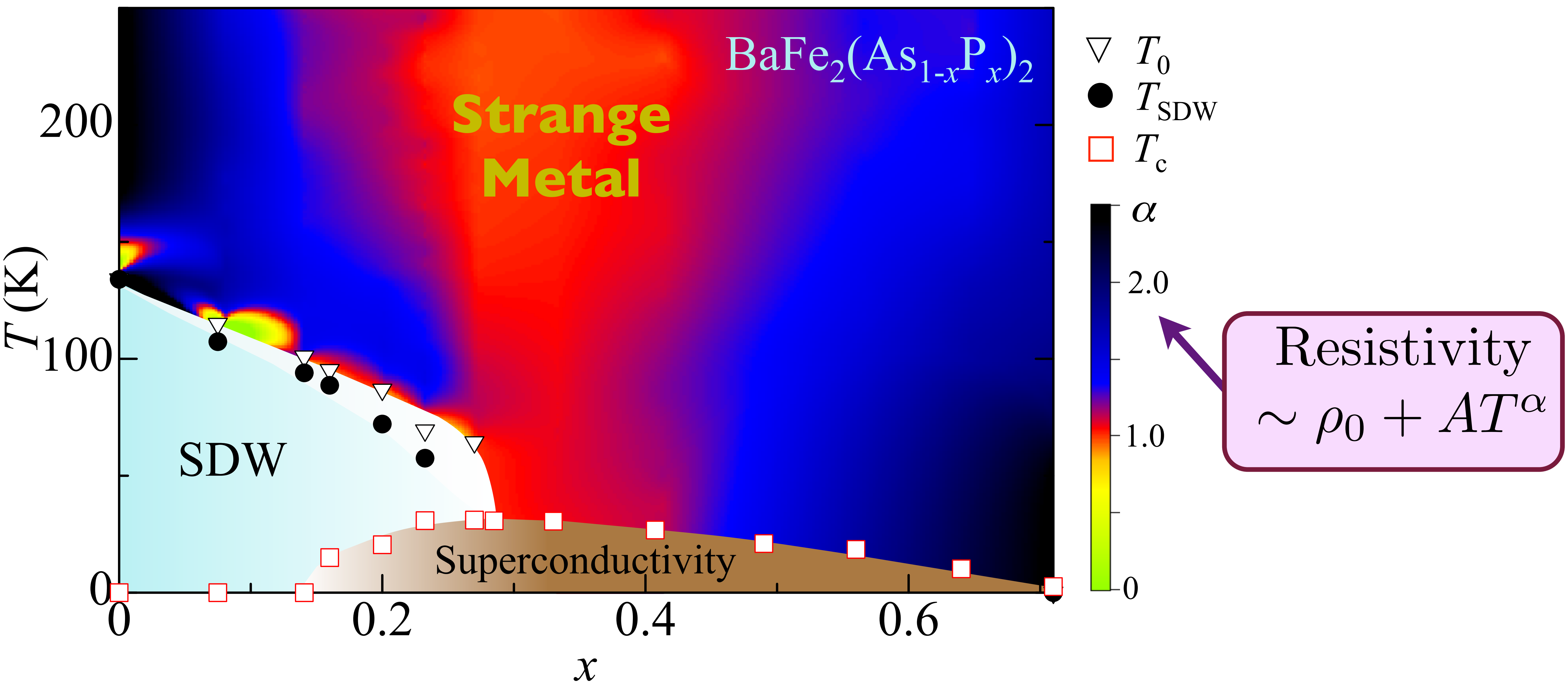
Theory of quantum criticality in the cuprates



M. A. Metlitski and S. Sachdev, Physical Review B **82**, 075128 (2010)

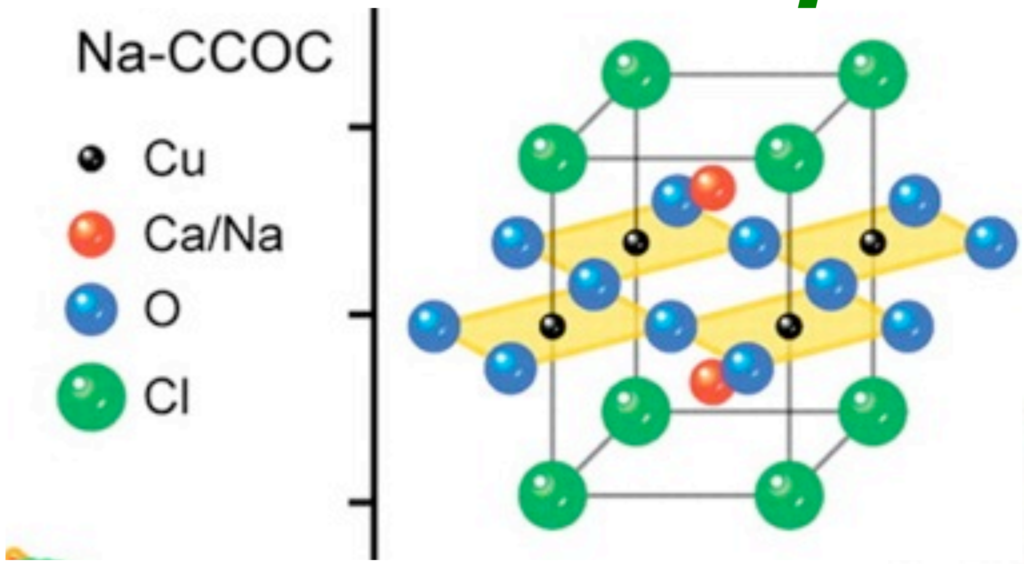
SDW quantum critical point is unstable to *d*-wave superconductivity
This instability is stronger than that in the BCS theory

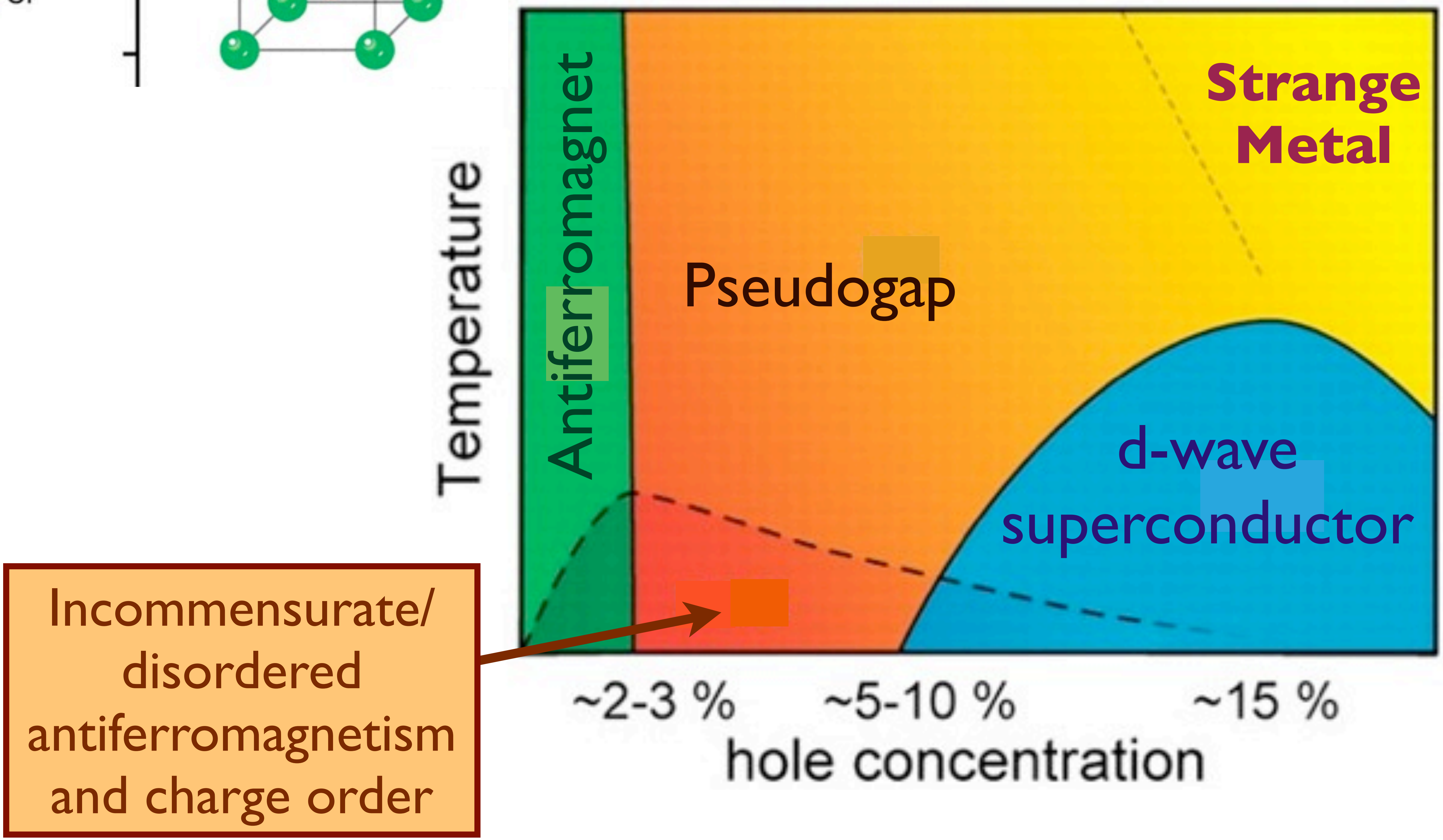
Temperature-doping phase diagram of the iron pnictides:



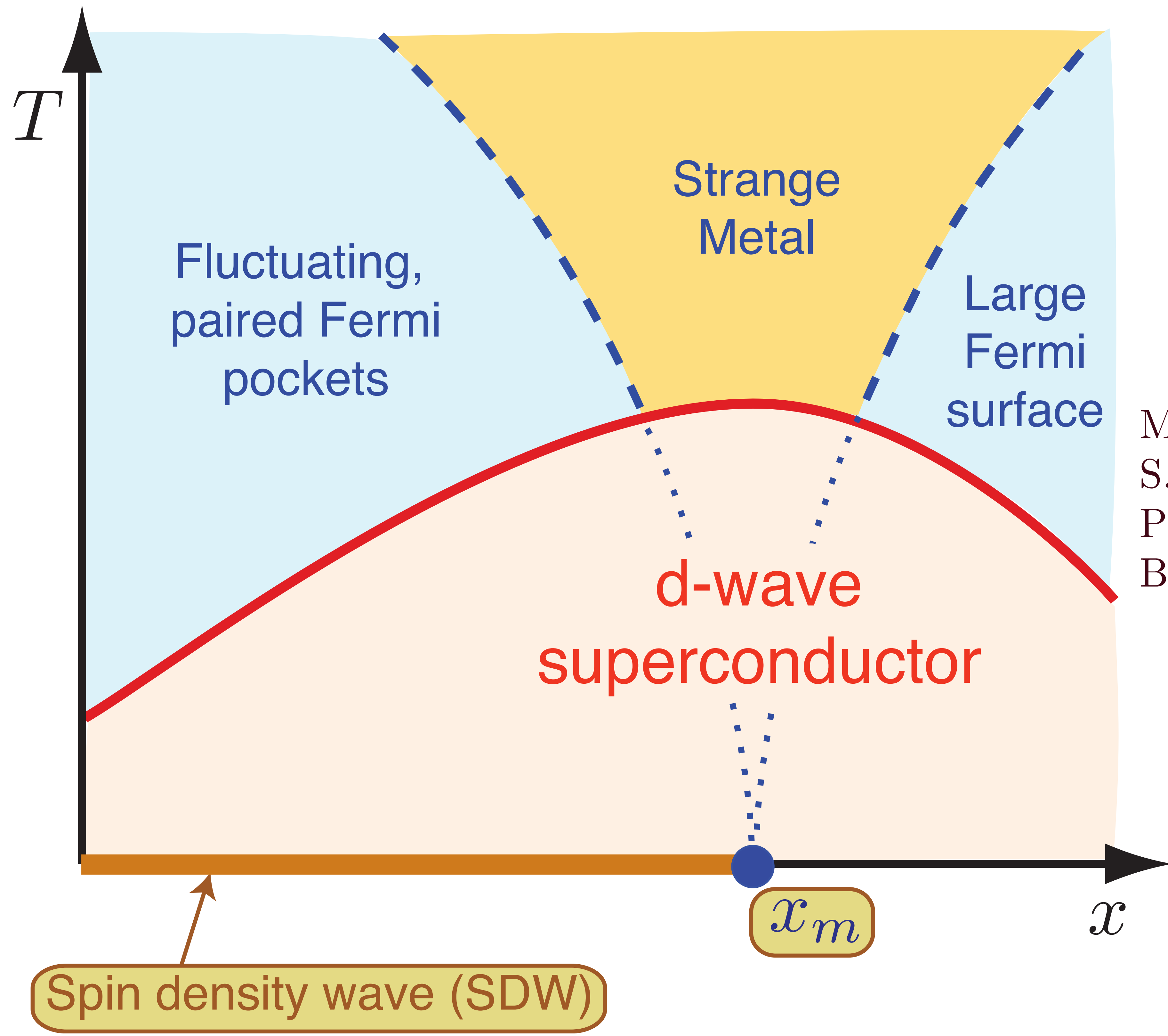
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

The cuprate superconductors





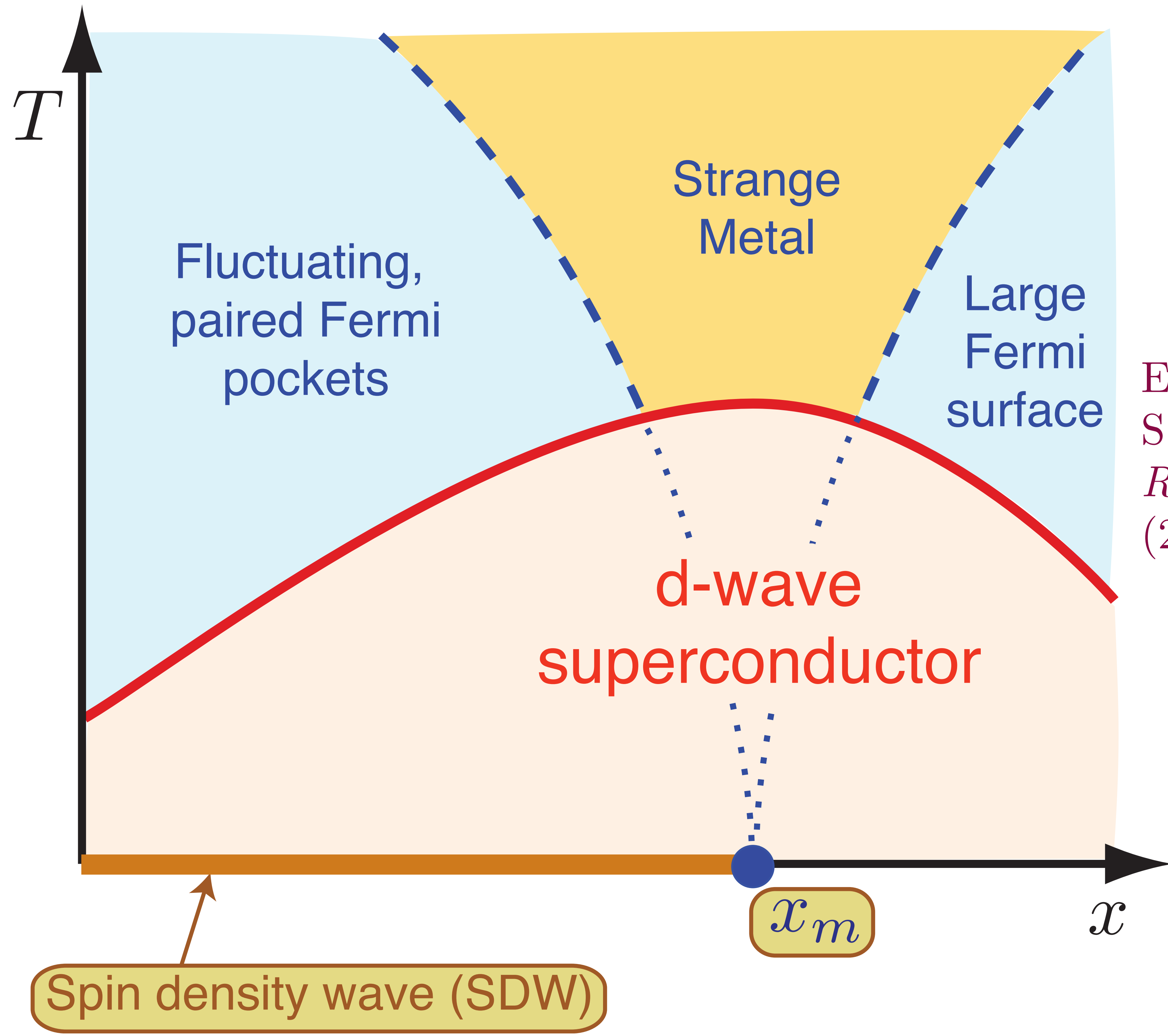
Theory of quantum criticality in the cuprates



M. A. Metlitski and S. Sachdev, Physical Review B **82**, 075128 (2010)

SDW quantum critical point is unstable to d-wave superconductivity
This instability is stronger than that in the BCS theory

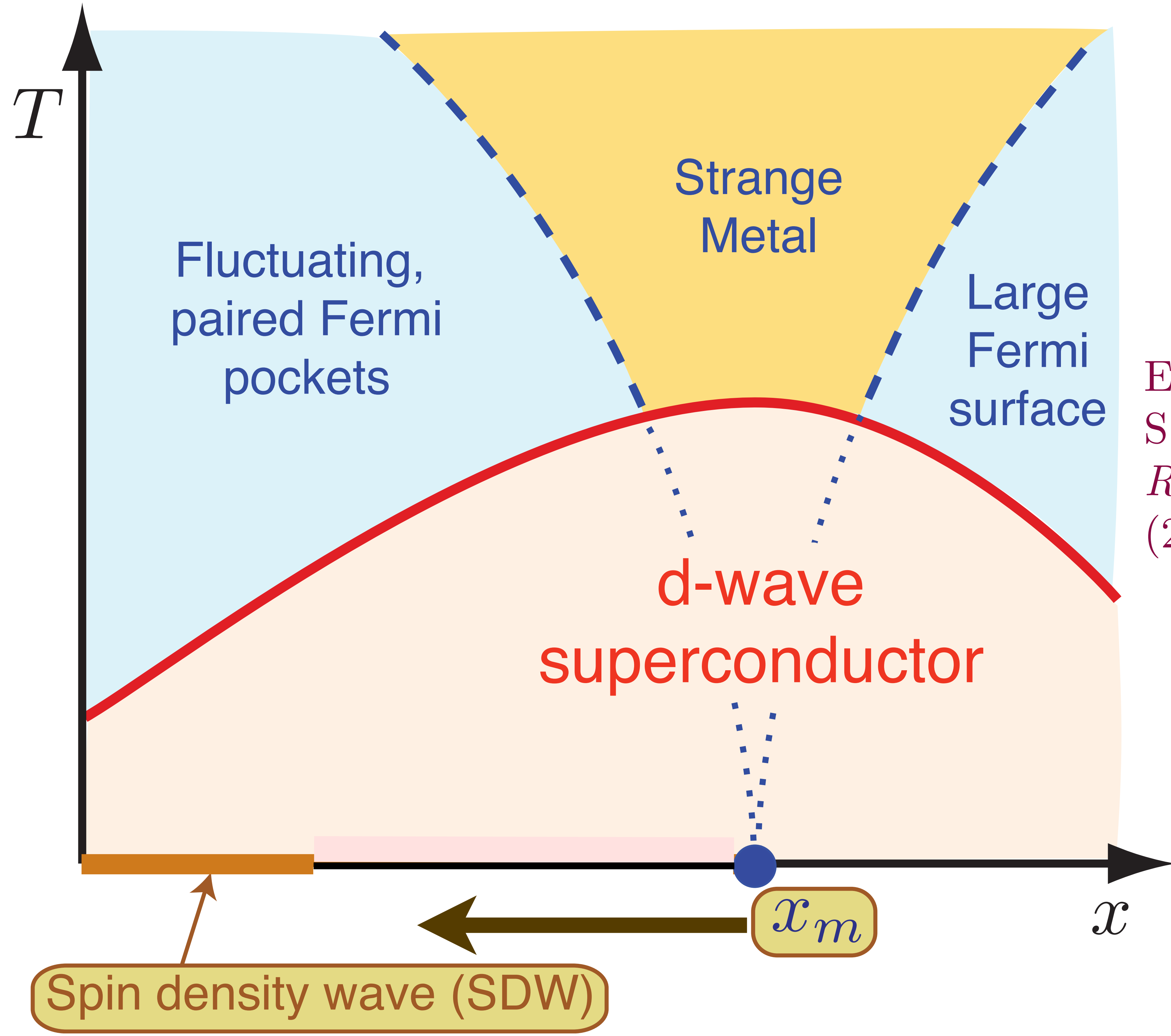
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

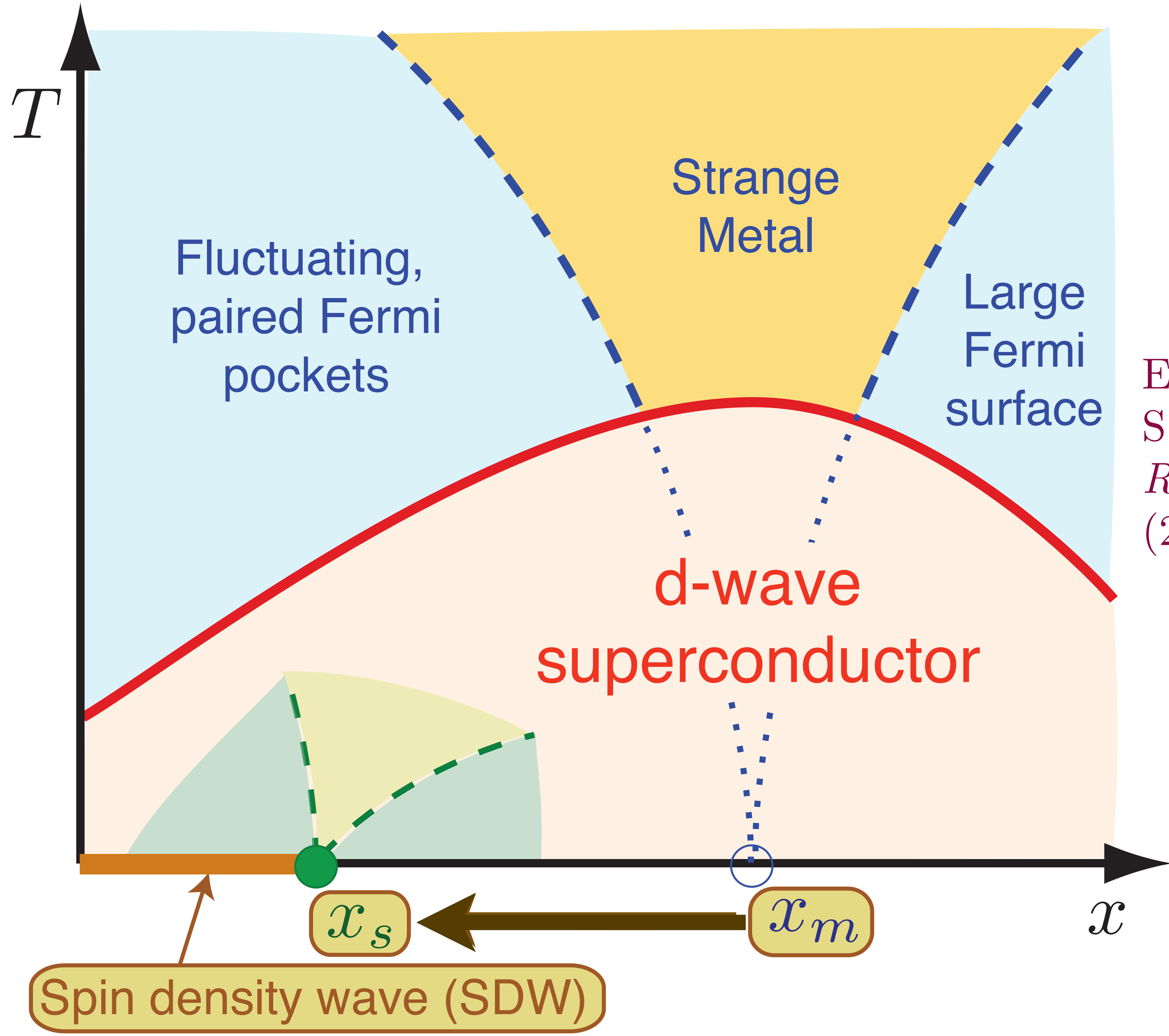
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

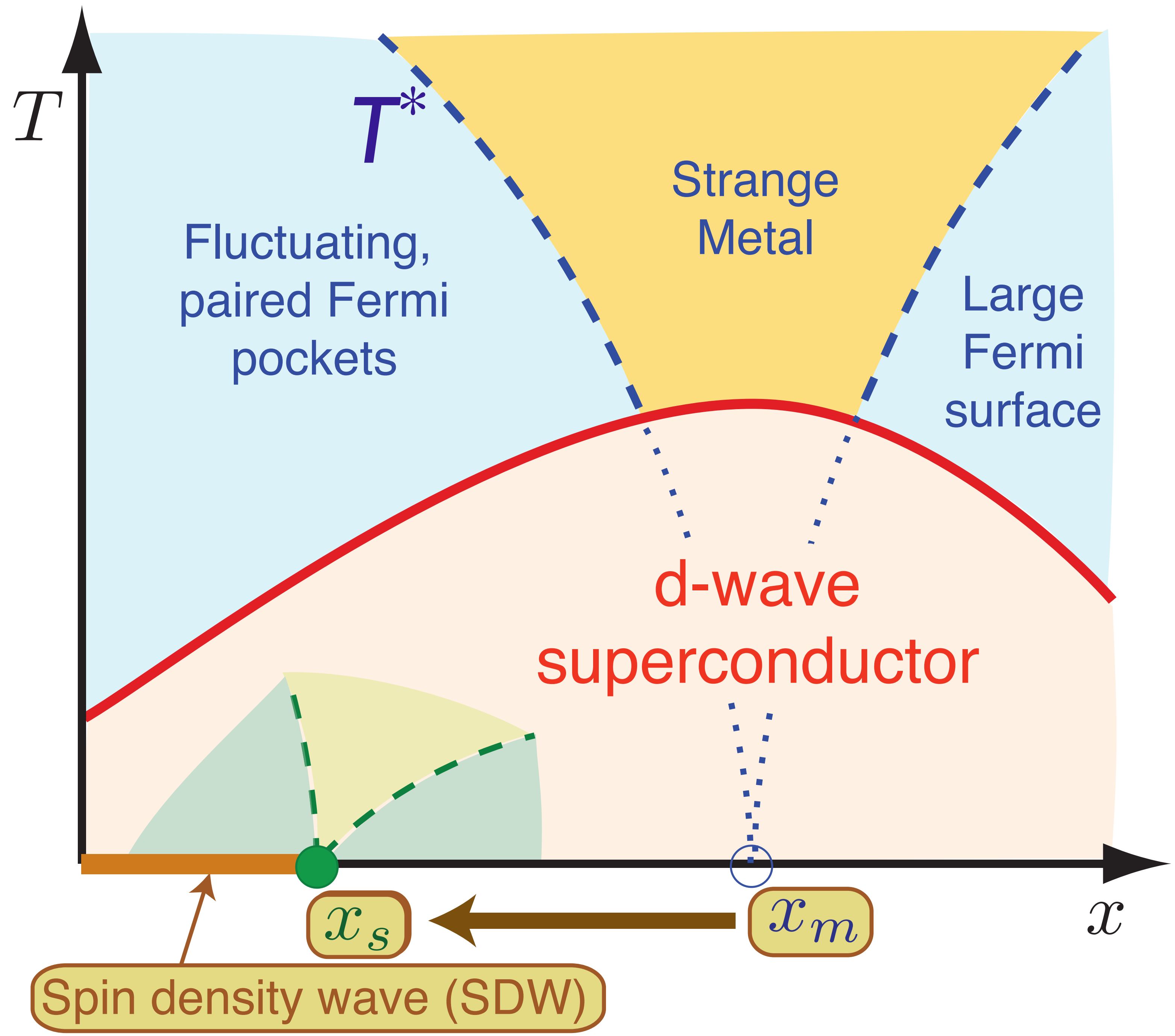
Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



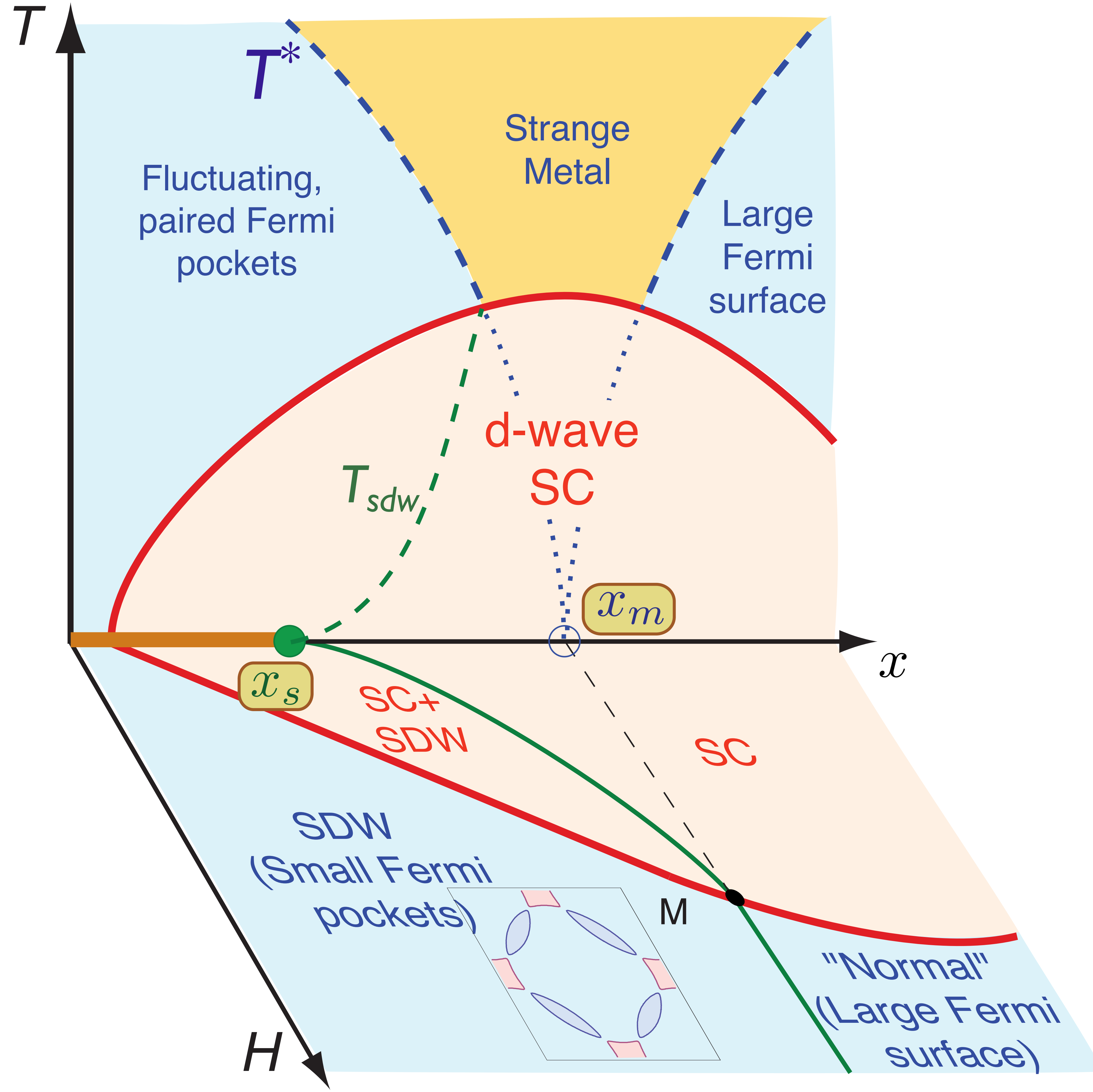
E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

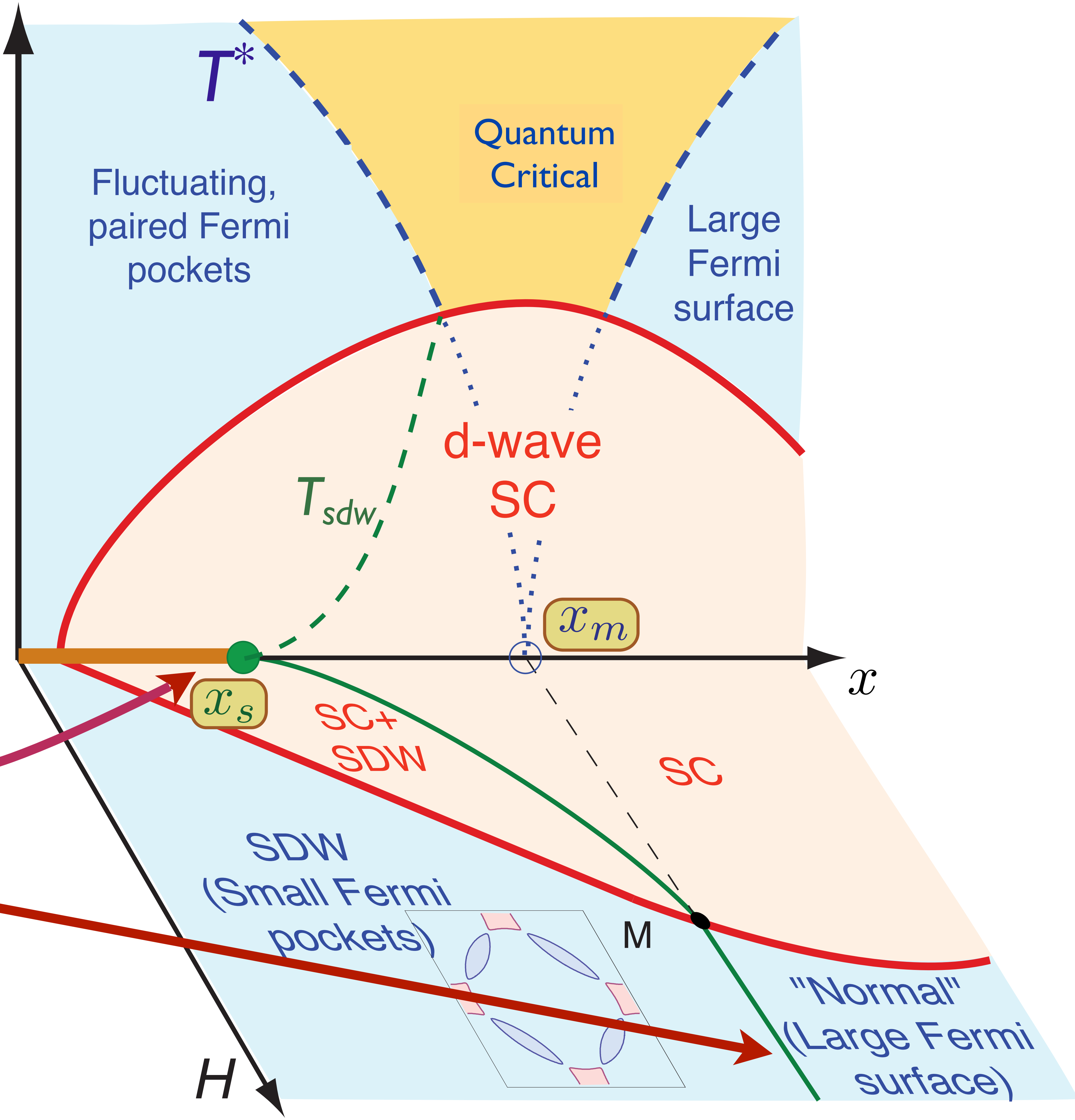


T
T^*
Fluctuating,
paired Fermi
pockets
Strange
Metal
Large
Fermi
surface
d-wave
superconductor
x_s
x_m
x
Spin density wave (SDW)

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



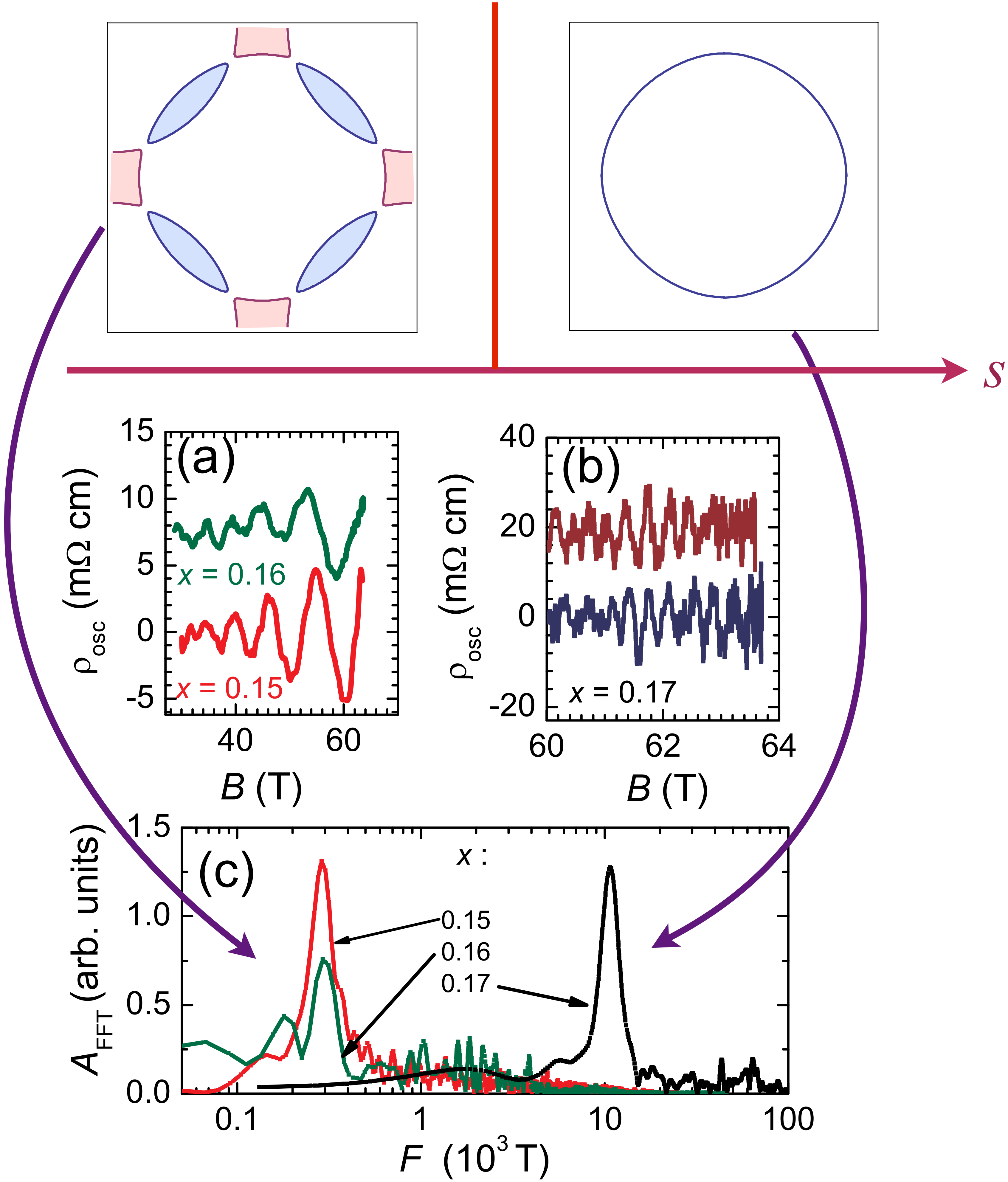
E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



Neutron scattering experiments on $Nd_{2-x}Ce_xCuO_4$ show that at low fields $x_s = 0.14$, while quantum oscillations at high fields show that $x_m = 0.165$.

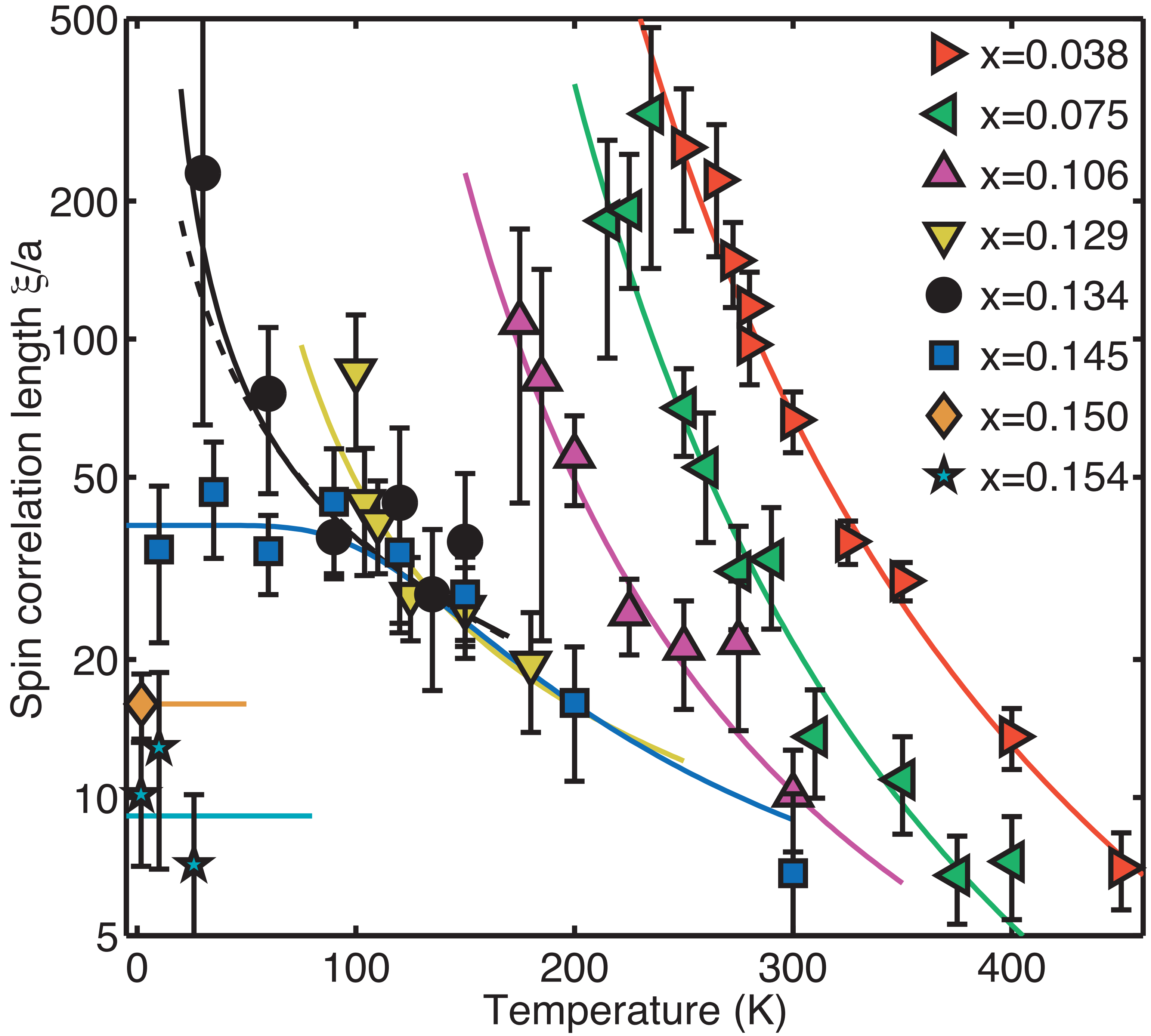
Quantum oscillations

$Nd_{2-x}Ce_xCuO_4$



T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross, *Phys. Rev. Lett.* **103**, 157002 (2009).

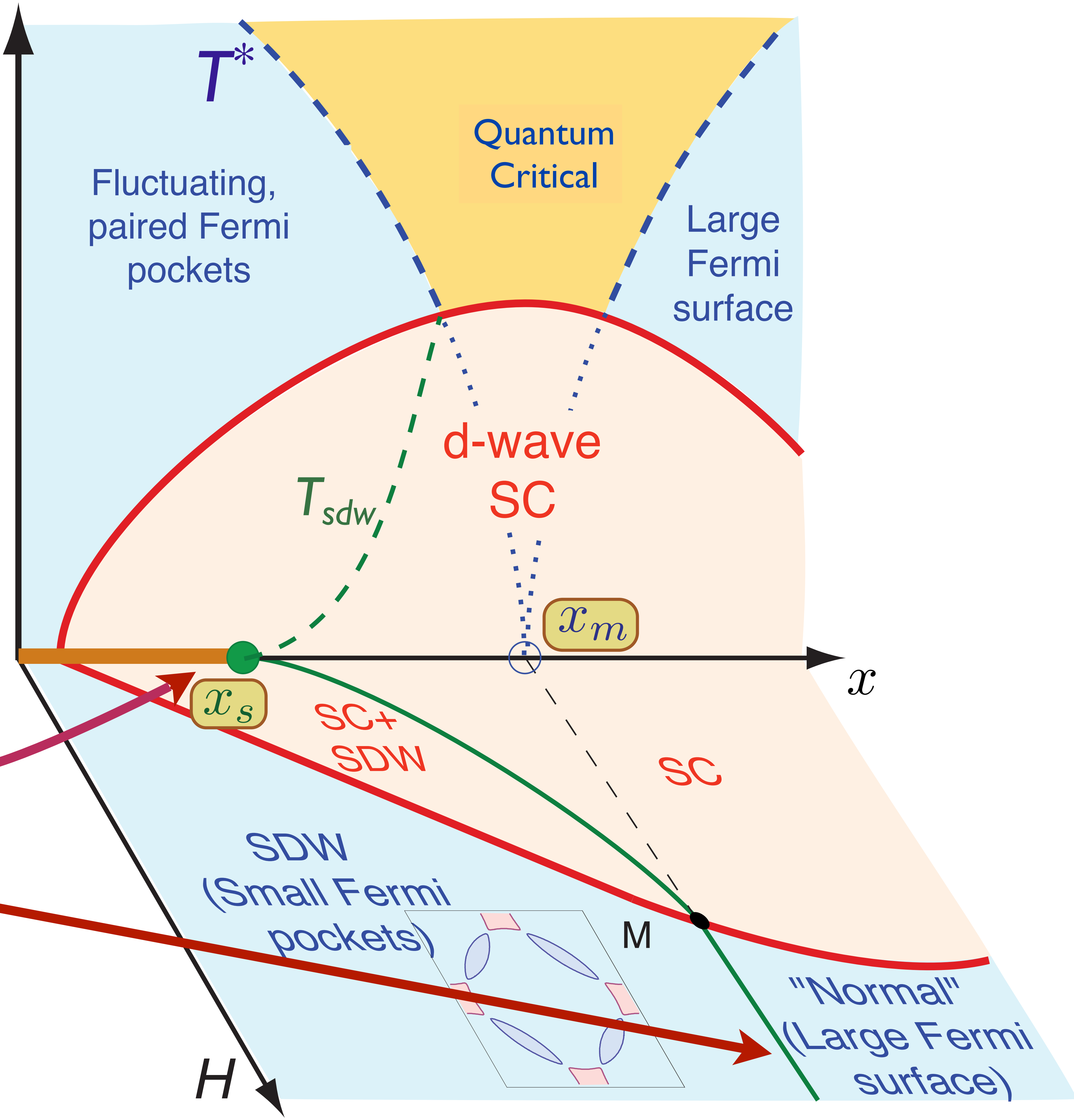
$Nd_{2-x}Ce_xCuO_4$



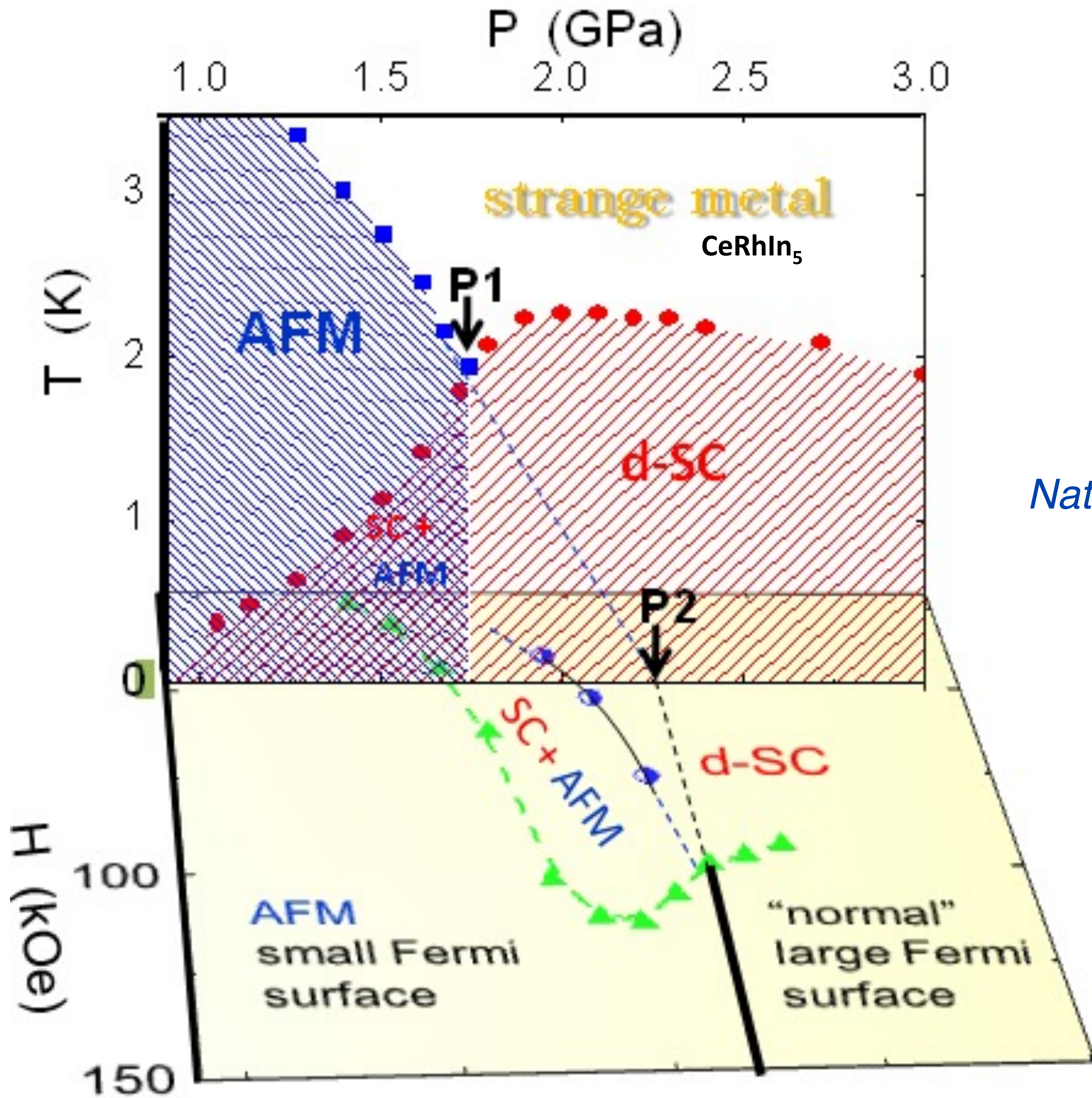
E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven, *Nature* **445**, 186 (2007).

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Neutron scattering experiments on $Nd_{2-x}Ce_xCuO_4$ show that at low fields $x_s = 0.14$, while quantum oscillations at high fields show that $x_m = 0.165$.

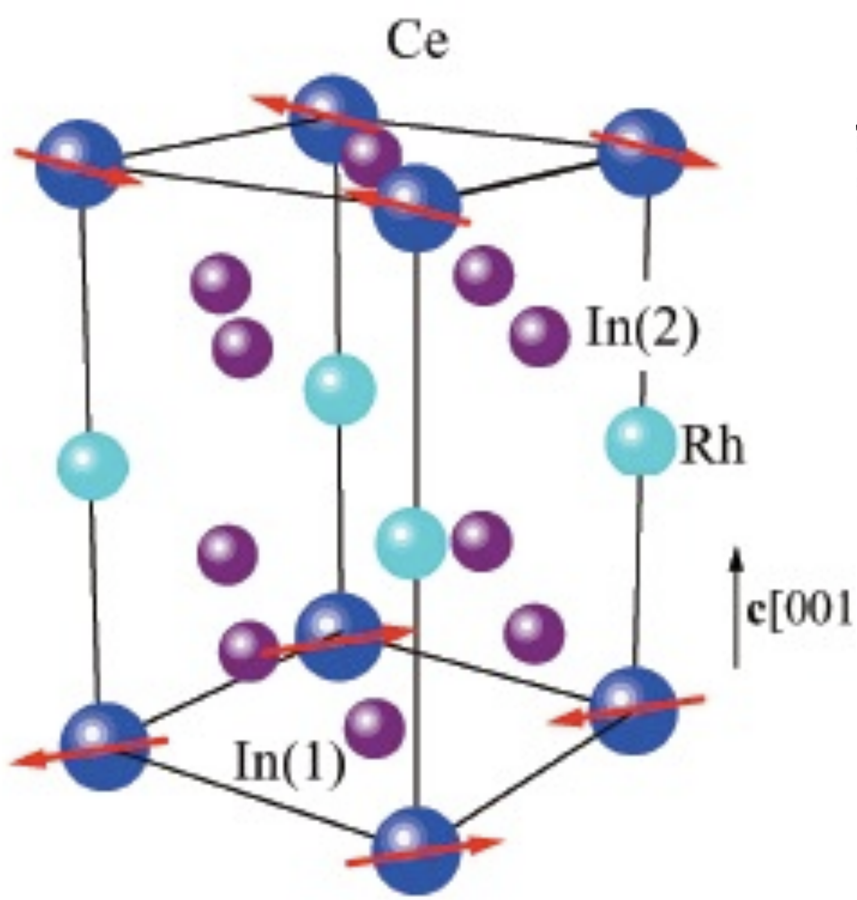


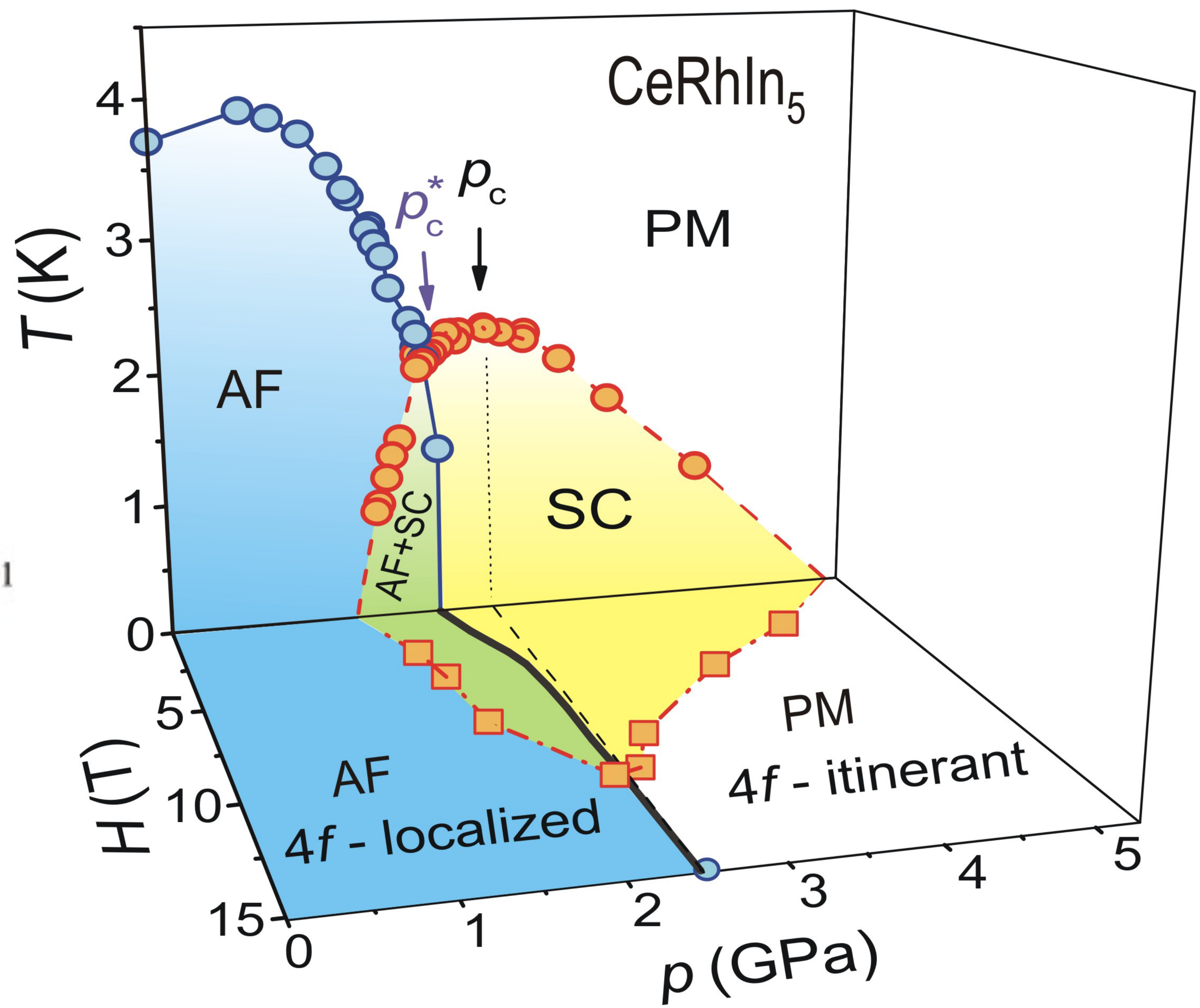
Similar phase diagram for $CeRhIn_5$



T. Park et al.,
Nature **440**, 65 (2006)

Similar phase diagram for $CeRhIn_5$

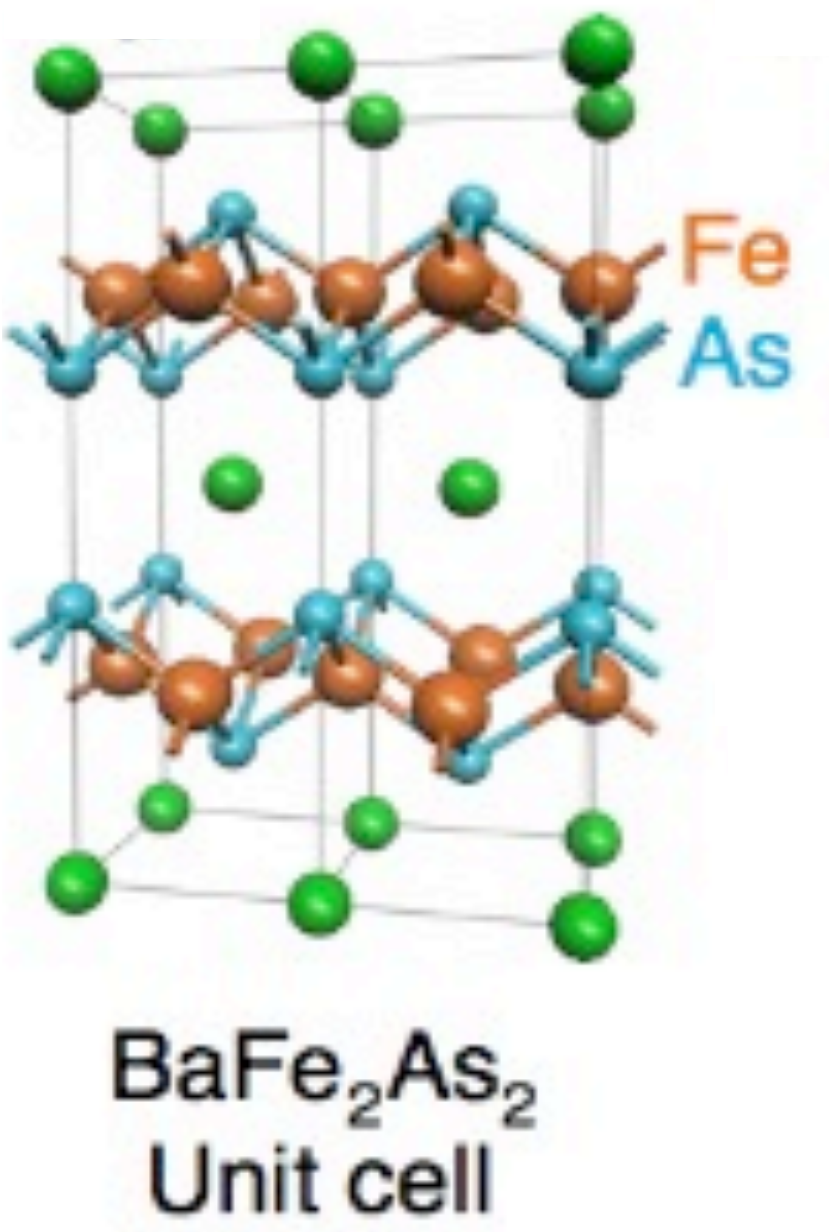


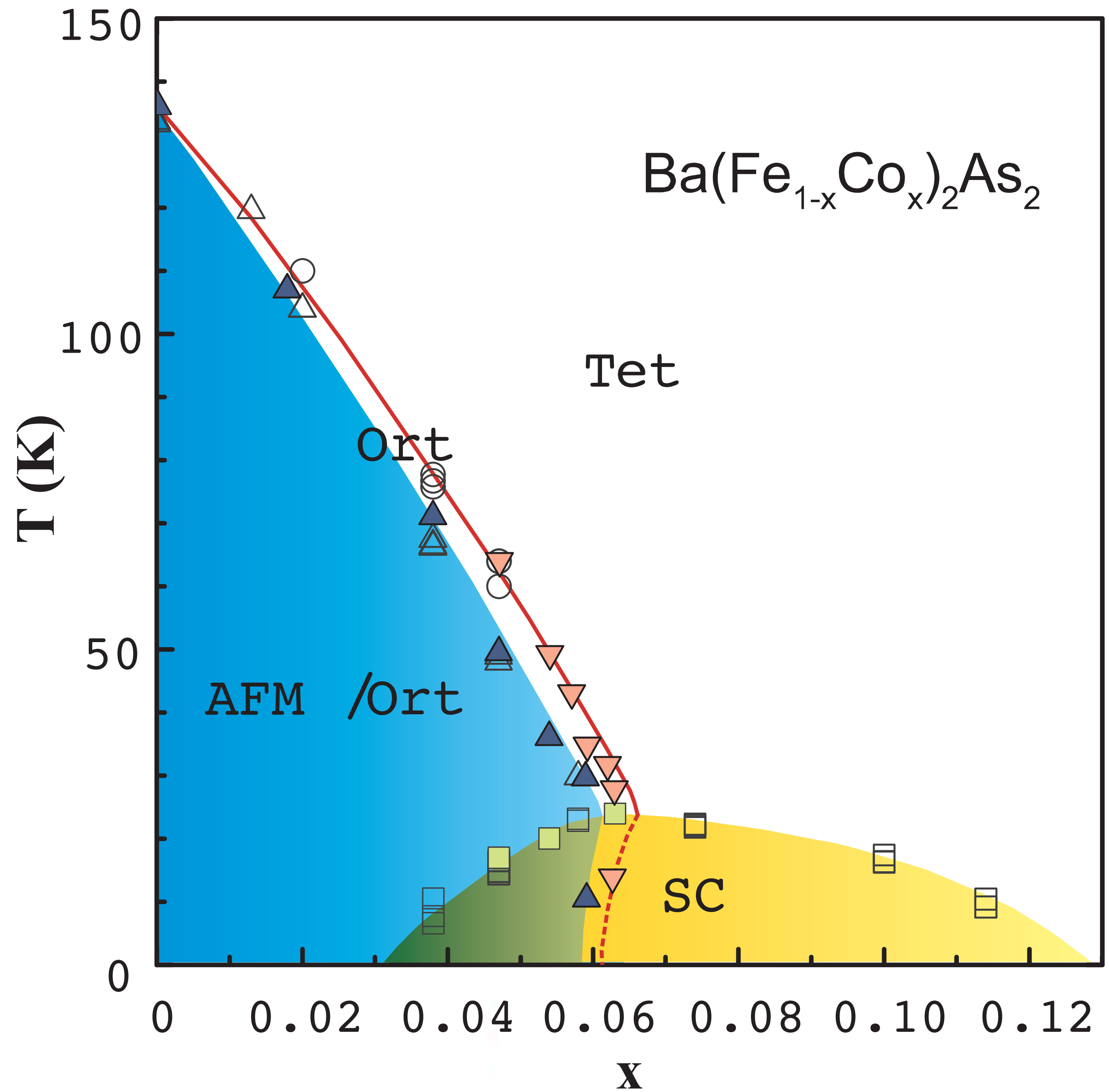


G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

Iron pnictides:

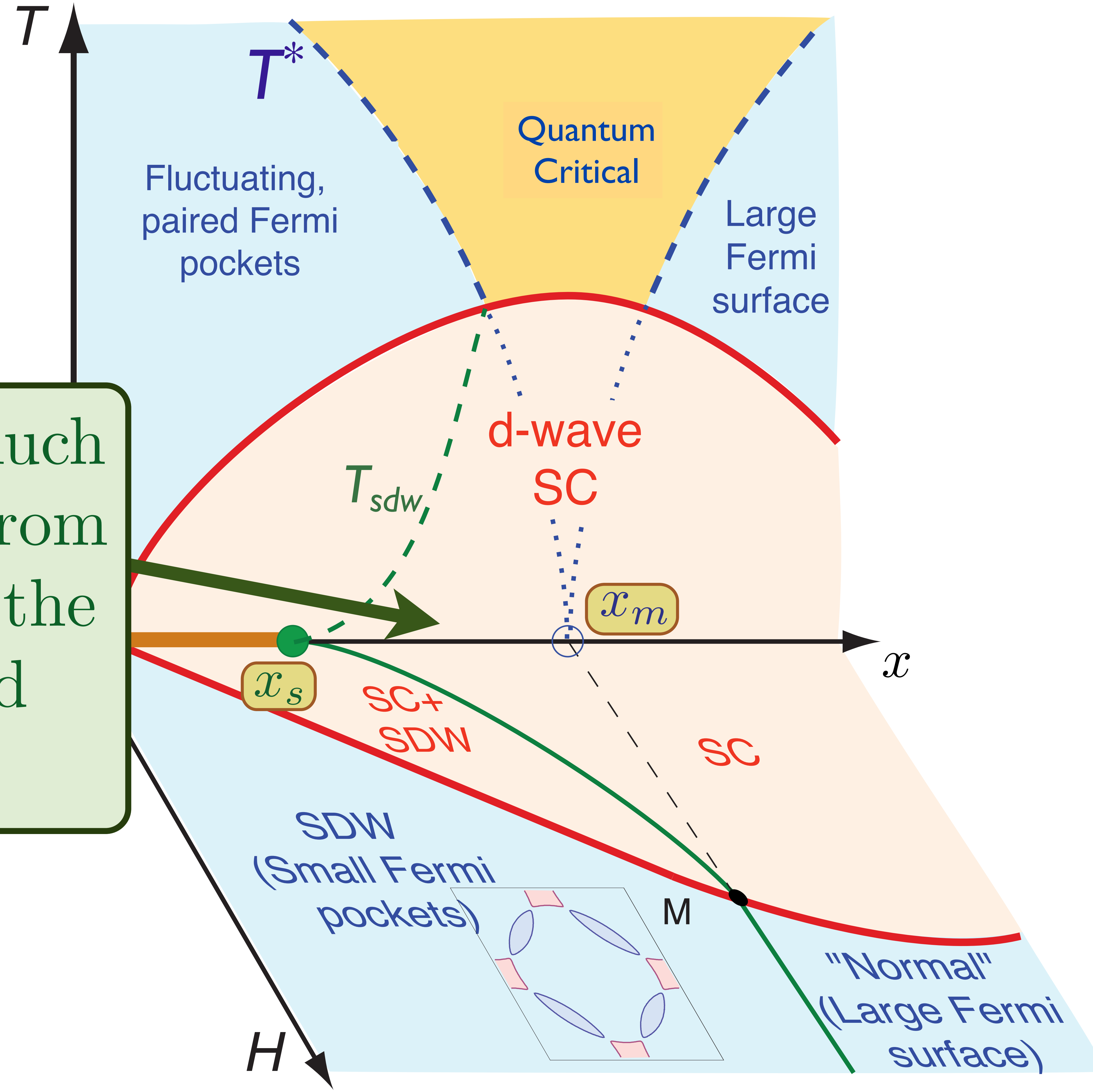
a new class of high temperature superconductors



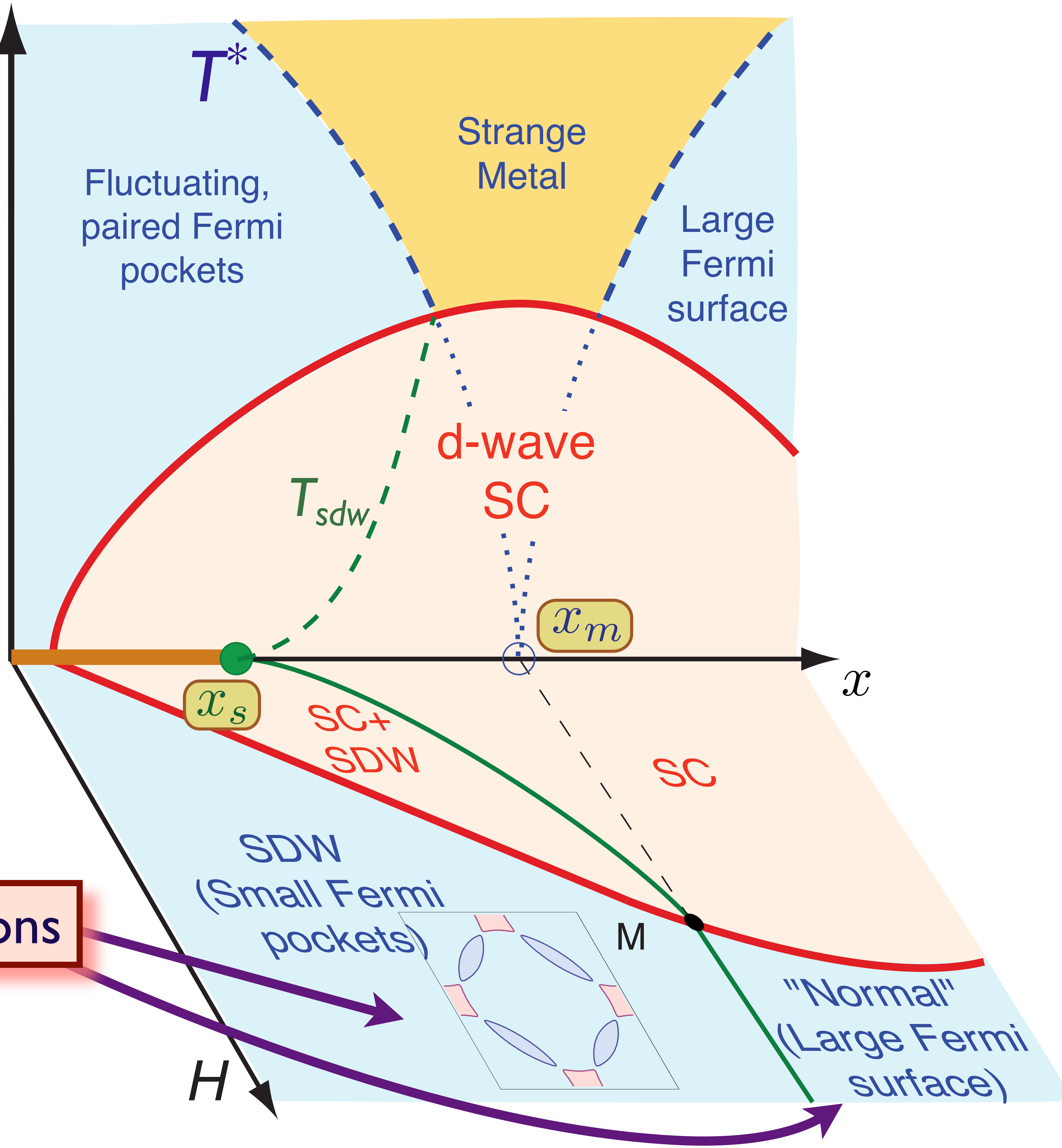


S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, *Physical Review Letters* **104**, 057006 (2010).

There is a much larger shift from x_m to x_s in the hole-doped cuprates.



E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



Quantum oscillations

Evidence for small Fermi pockets in hole-doped cuprates

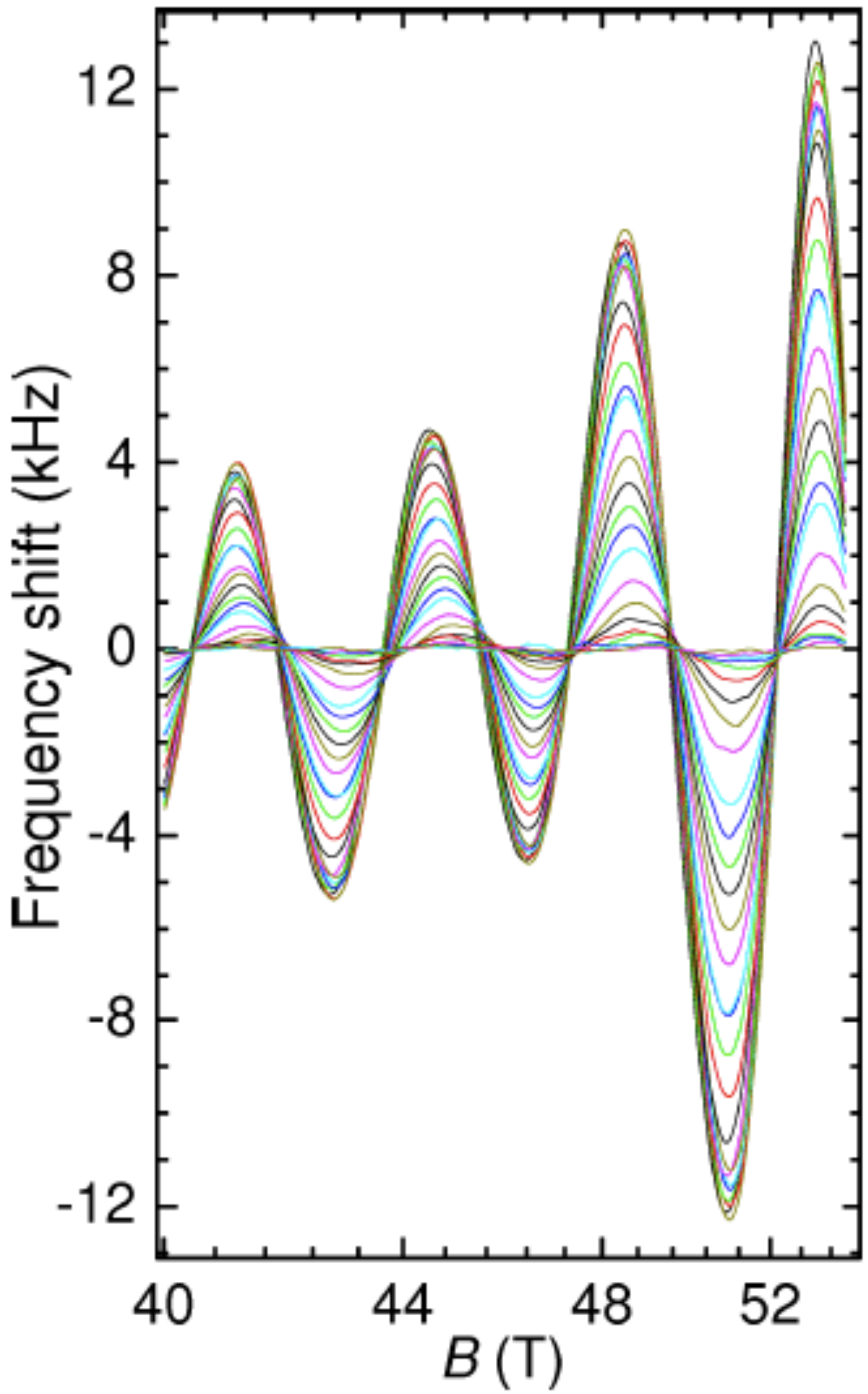
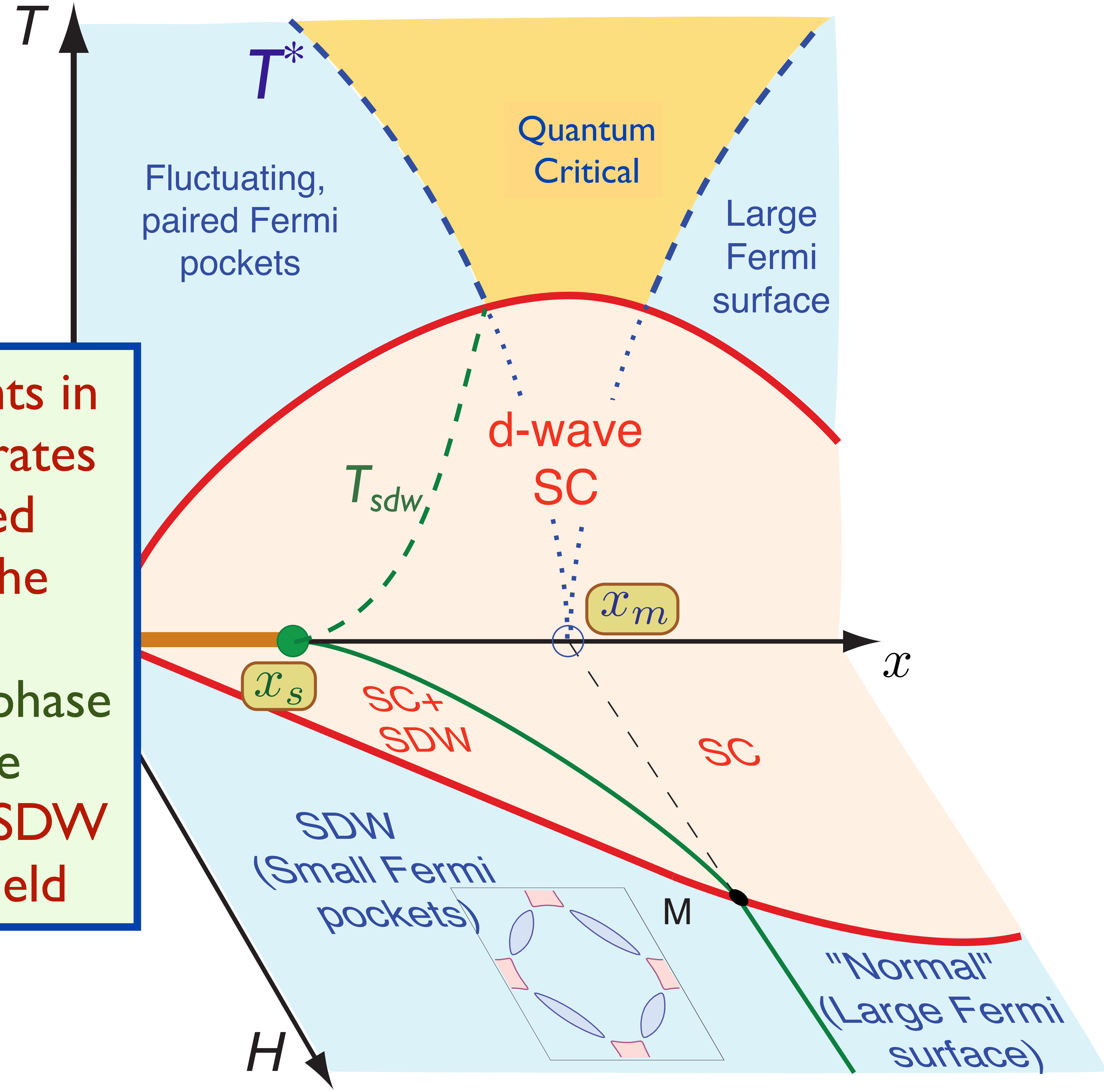


FIG. 2: Magnetic quantum oscillations measured in $YBa_2Cu_3O_{6+x}$ with $x \approx 0.56$ (after background polynomial subtraction). This restricted interval in $B = |\mathbf{B}|$ furnishes a dynamic range of ~ 50 dB between $T = 1$ and 18 K. The actual T values are provided in Fig. 3.

Suchitra E. Sebastian, N. Harrison,
M. M. Altarawneh, Ruixing Liang, D. A. Bonn,
W. N. Hardy, and G. G. Lonzarich
Physical Review B **81**, 140505(R) (2010)

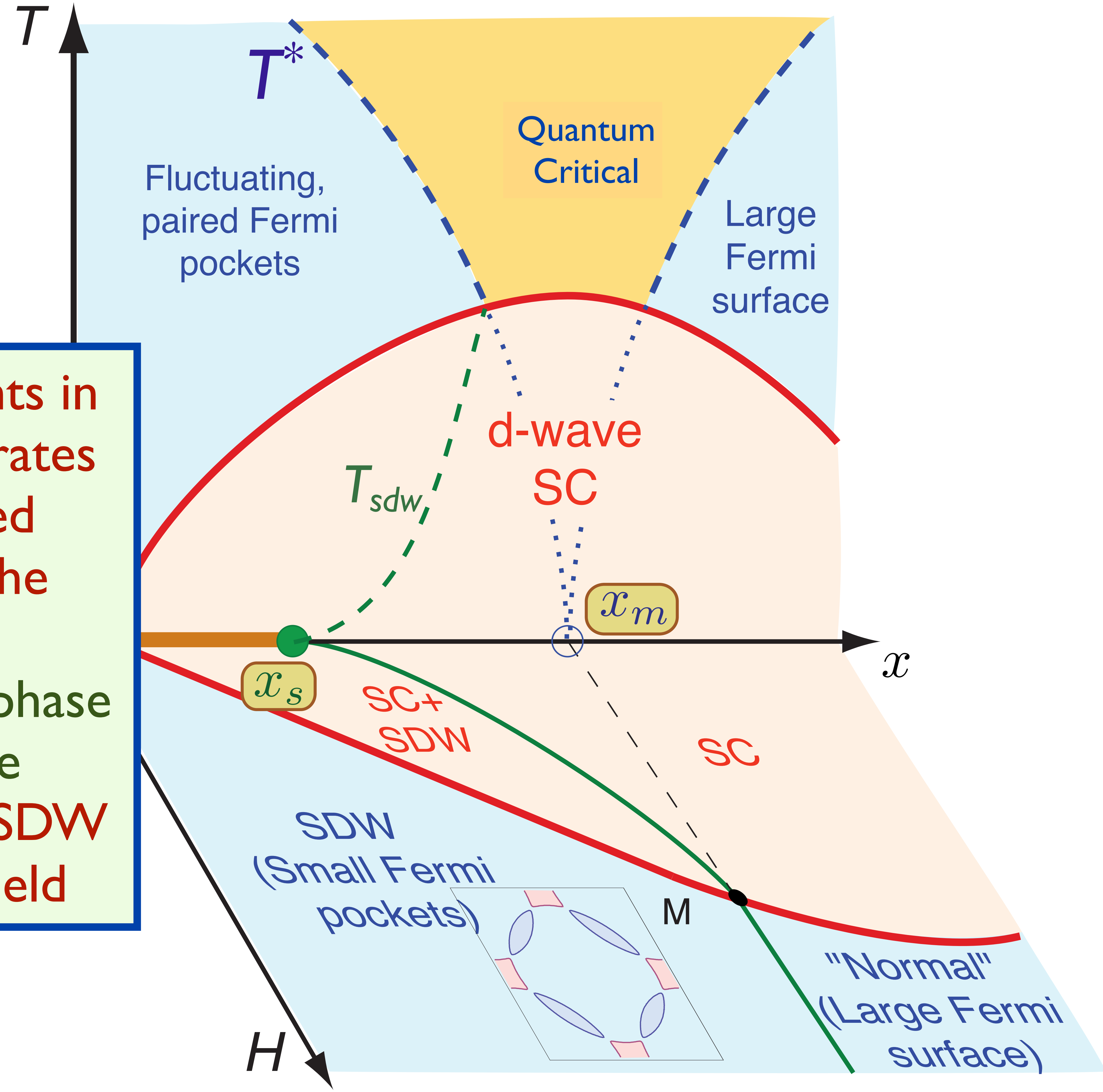
Original observation:
N. Doiron-Leyraud, C. Proust,
D. LeBoeuf, J. Levallois,
J.-B. Bonnemaison, R. Liang,
D. A. Bonn, W. N. Hardy,
and L. Taillefer,
Nature **447**, 565 (2007)

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



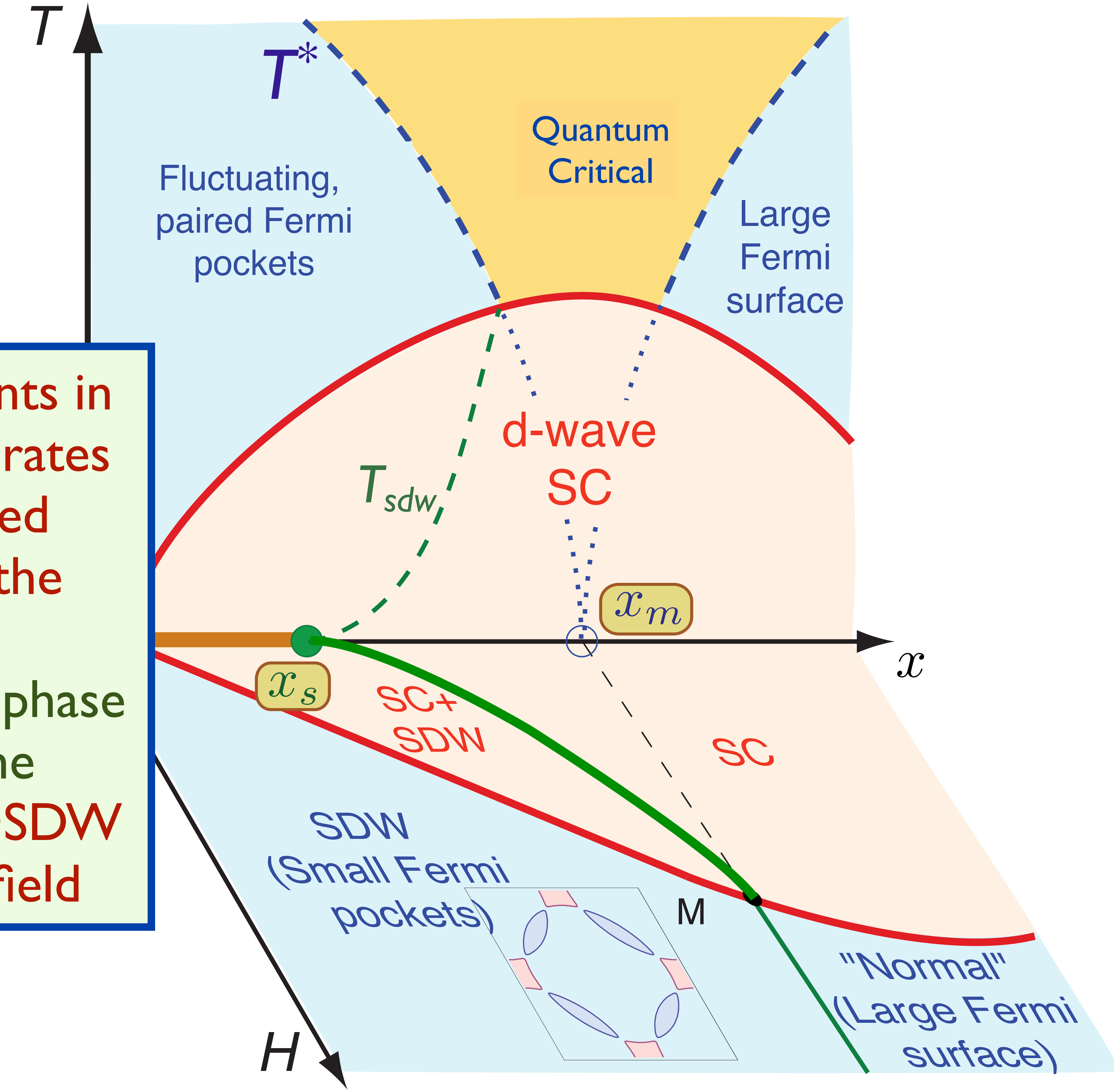
Many experiments in hole-doped cuprates have presented evidence for the predicted green quantum phase transition line from SC to SC+SDW in a magnetic field

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

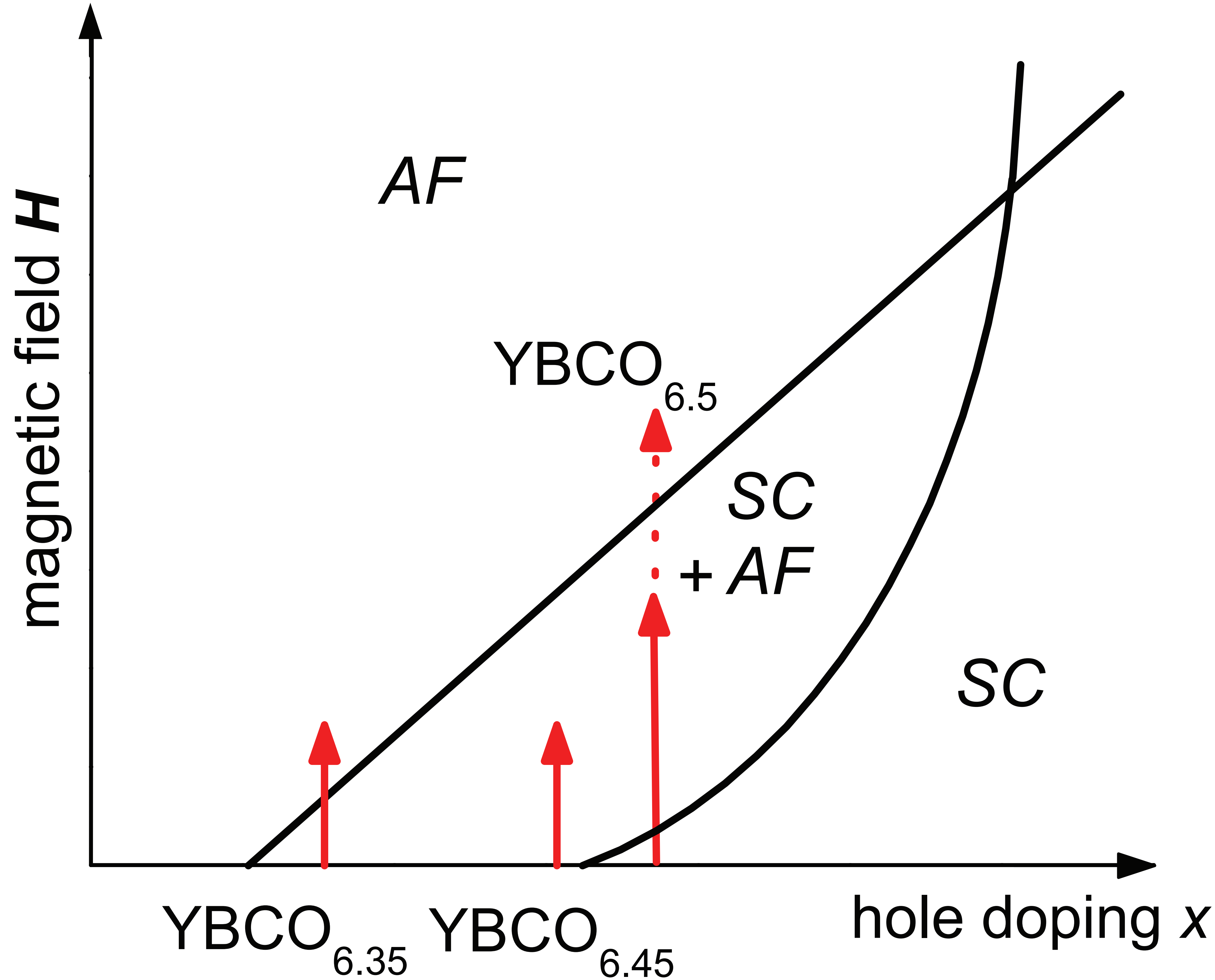


Many experiments in hole-doped cuprates have presented evidence for the predicted green quantum phase transition line from SC to SC+SDW in a magnetic field

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

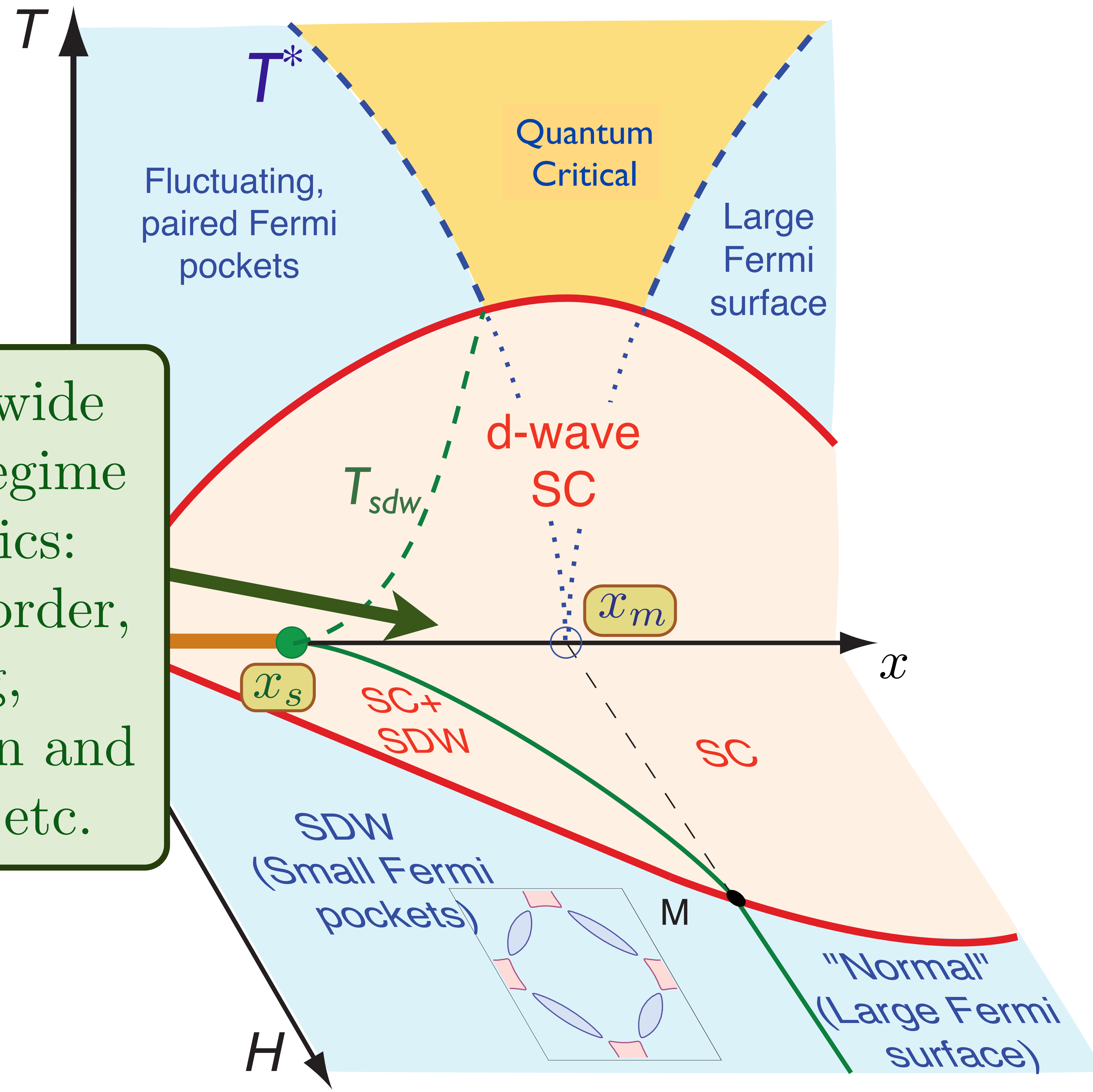


Many experiments in hole-doped cuprates have presented evidence for the predicted green quantum phase transition line from SC to SC+SDW in a magnetic field



D. Haug, V. Hinkov, Y. Sidis, P. Bourges, N. B. Christensen, A. Ivanov, T. Keller, C. T. Lin, and B. Keimer, *New J. Phys.* **12**, 105006 (2010)

This opens a wide intermediate regime for new physics: bond-nematic order, T-breaking, fractionalization and Mott physics etc.



Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

   Quantum criticality of Fermi surface reconstruction, and the phase diagram in a magnetic field

2. Strongly-coupled quantum criticality in metals

   Fermi surfaces and gapless bosons

3. Instability to unconventional superconductivity

   "Mechanism" of higher temperature superconductivity

4. Theory of the competition between superconductivity and antiferromagnetism

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

   Quantum criticality of Fermi surface reconstruction, and the phase diagram in a magnetic field

2. Strongly-coupled quantum criticality in metals

   Fermi surfaces and gapless bosons

3. Instability to unconventional superconductivity

   "Mechanism" of higher temperature superconductivity

4. Theory of the competition between superconductivity and antiferromagnetism

Spin-fermion model: Elections with dispersion $\varepsilon_{\mathbf{k}}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$+ \int d\tau d^2 r \left[\frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi} \right)^2 + \frac{s}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{r}_i} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

Spin-fermion model: Elections with dispersion $\varepsilon_{\mathbf{k}}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$+ \int d\tau d^2 r \left[\frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi} \right)^2 + \frac{s}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{r}_i} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

Coupling between fermions and antiferromagnetic order: $\lambda^2 \sim U$, the Hubbard repulsion

A technical aside......

Hertz-Moriya-Millis theory

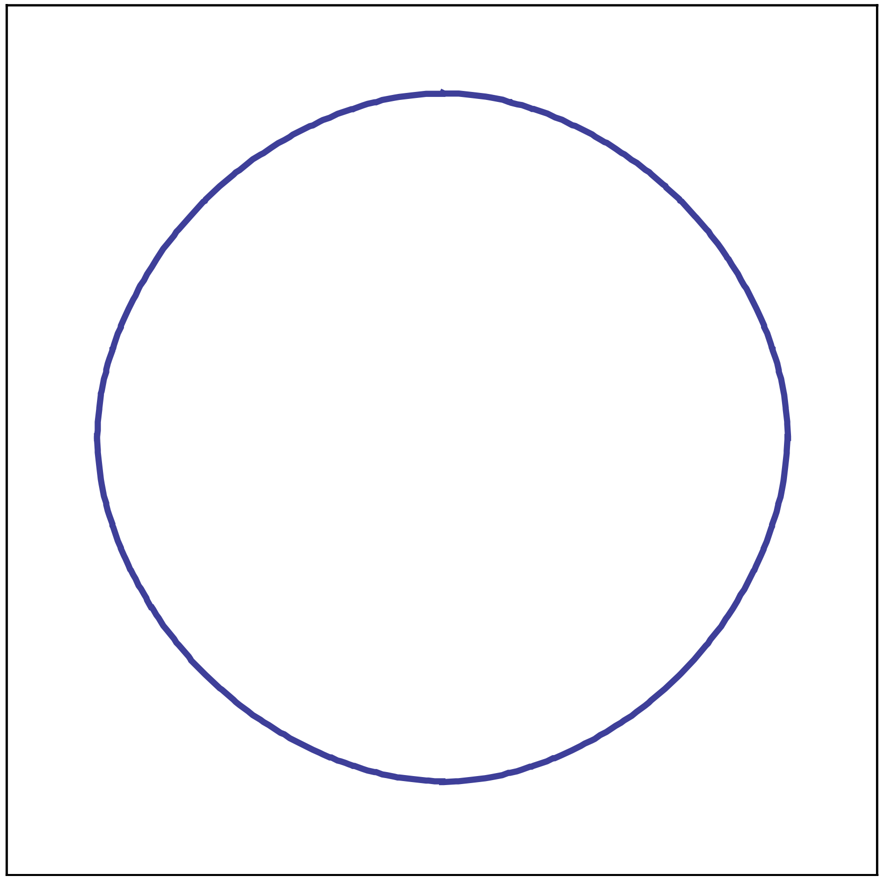
Integrate out fermions and obtain an effective action for the boson field $\vec{\varphi}$ alone. Because the fermions are gapless, this is potentially dangerous, and will lead to non-local terms in the $\vec{\varphi}$ effective action. Hertz focused on only the simplest such non-local term. However, there are an infinite number of non-local terms at higher order, and these lead to a breakdown of the Hertz theory in $d = 2$.

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

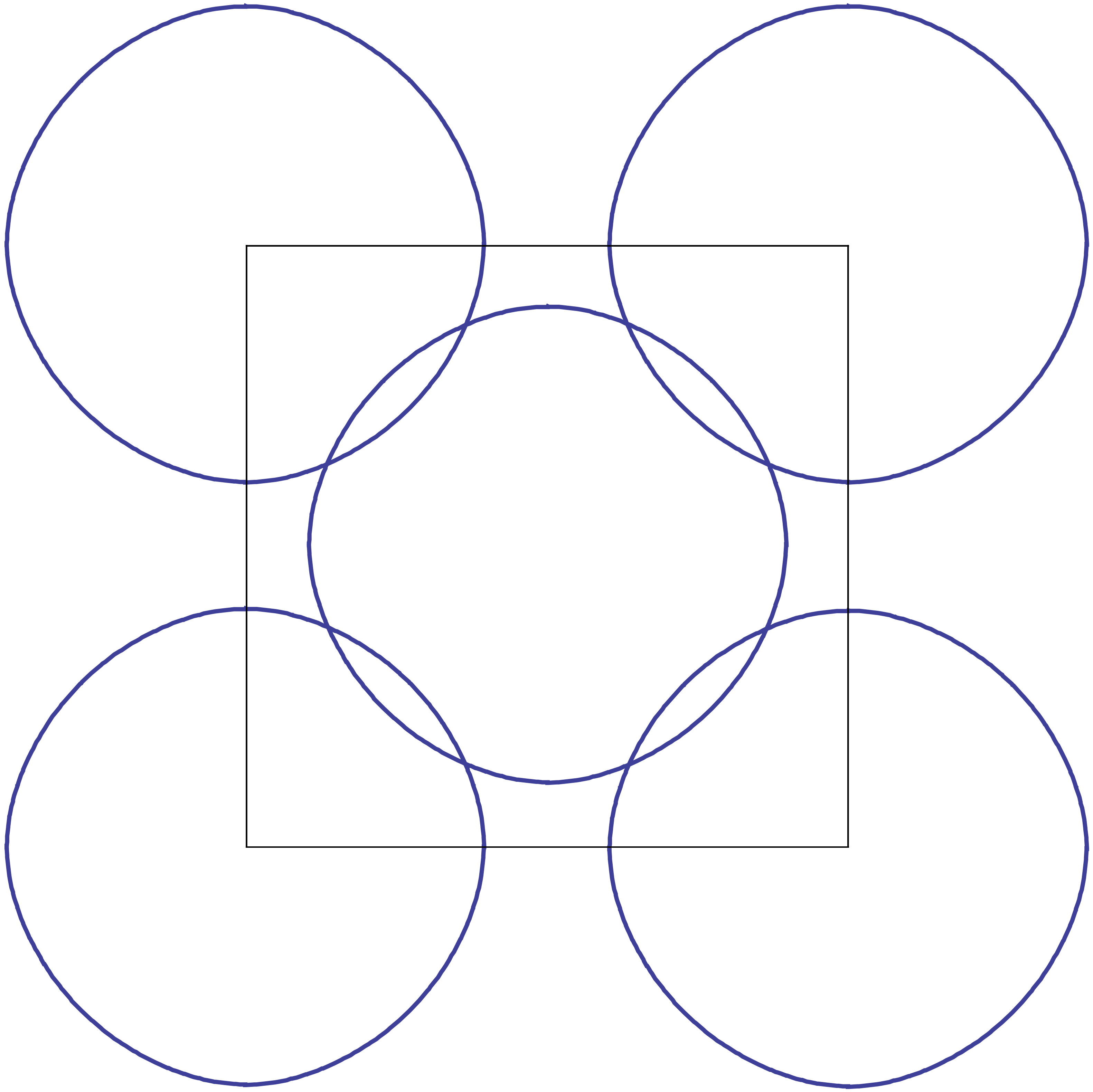
A technical aside......

We need to perform an RG analysis on a local theory of both the fermions and the $\vec{\varphi}$. It appears that such a theory can be analyzed using a $1/N$ expansion, where N is the number of fermion flavors. At two-loop order, the $1/N$ expansion is well-behaved, and we can determine consistent RG flow equations. However, at higher loops we find corrections to the renormalizations which require summation of all planar graphs even at the leading order in $1/N$, and the $1/N$ expansion appears to be organized as a genus expansion of random surfaces. But even this genus expansion breaks down in the renormalization of a quartic coupling of $\vec{\varphi}$. In the following, I will describe some of the two loop results.

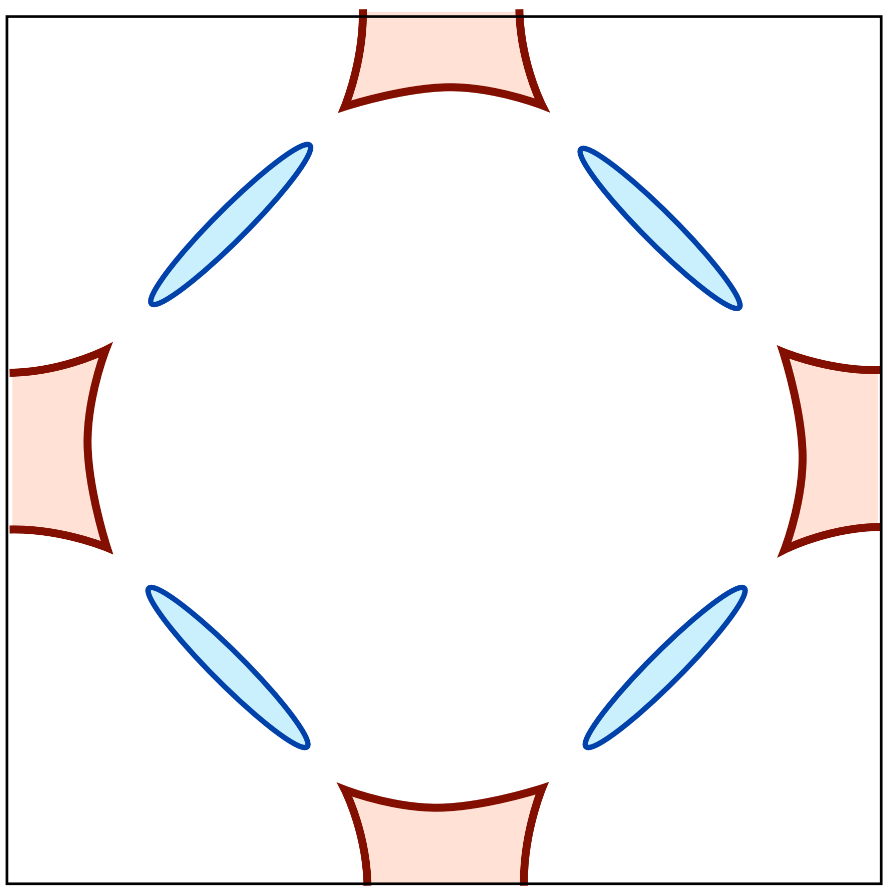
M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



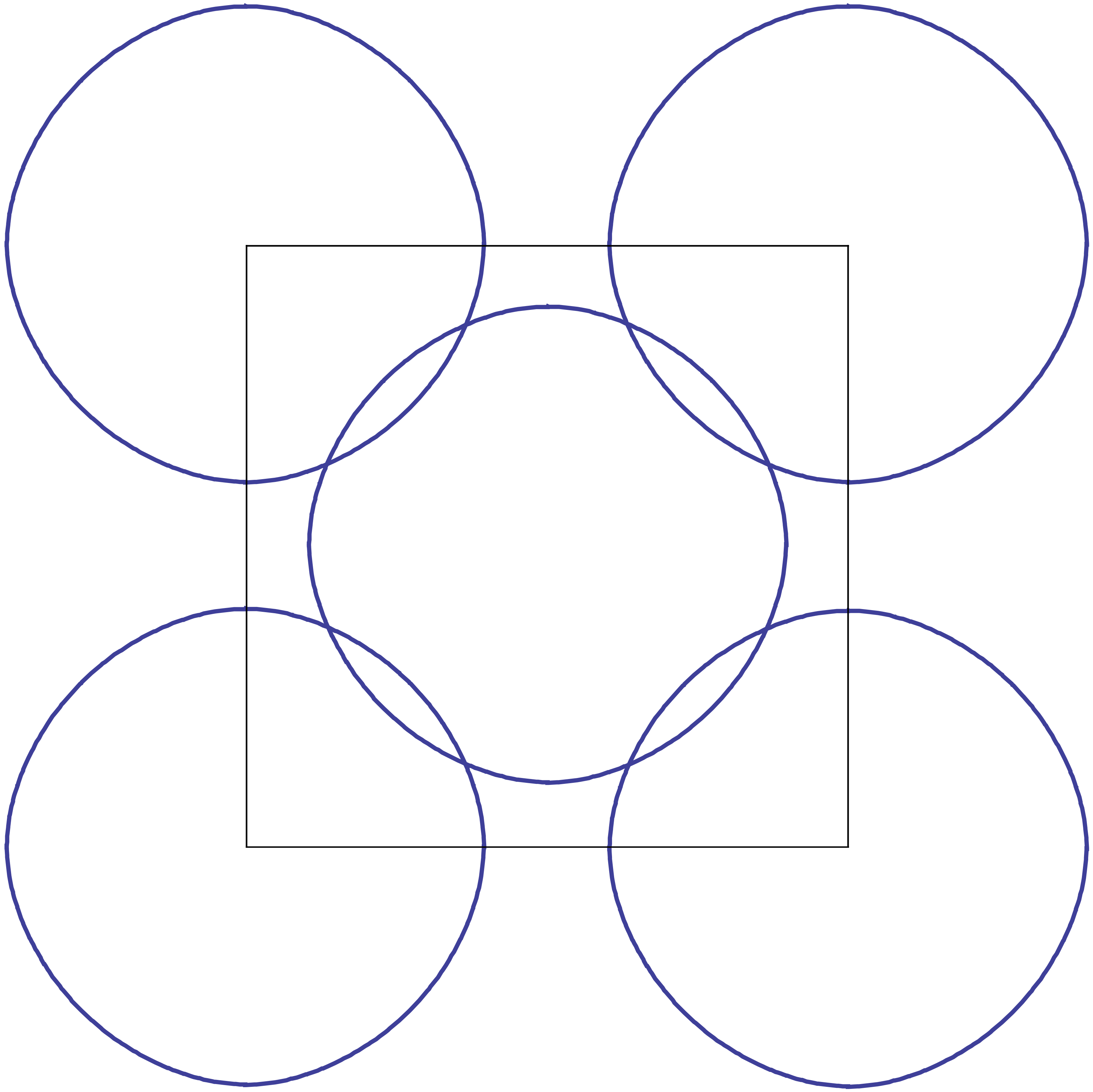
Metal with “large” Fermi surface



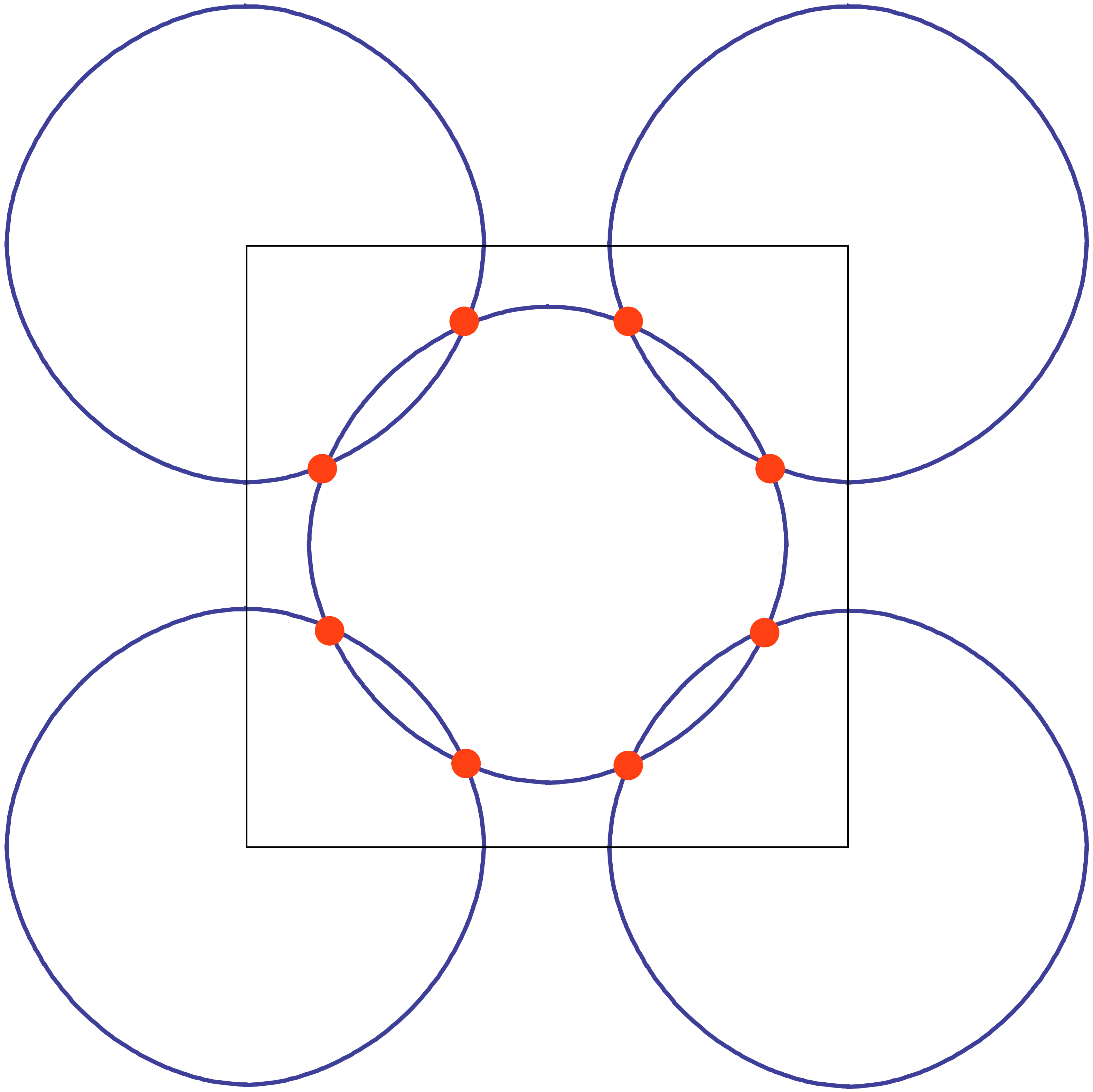
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.



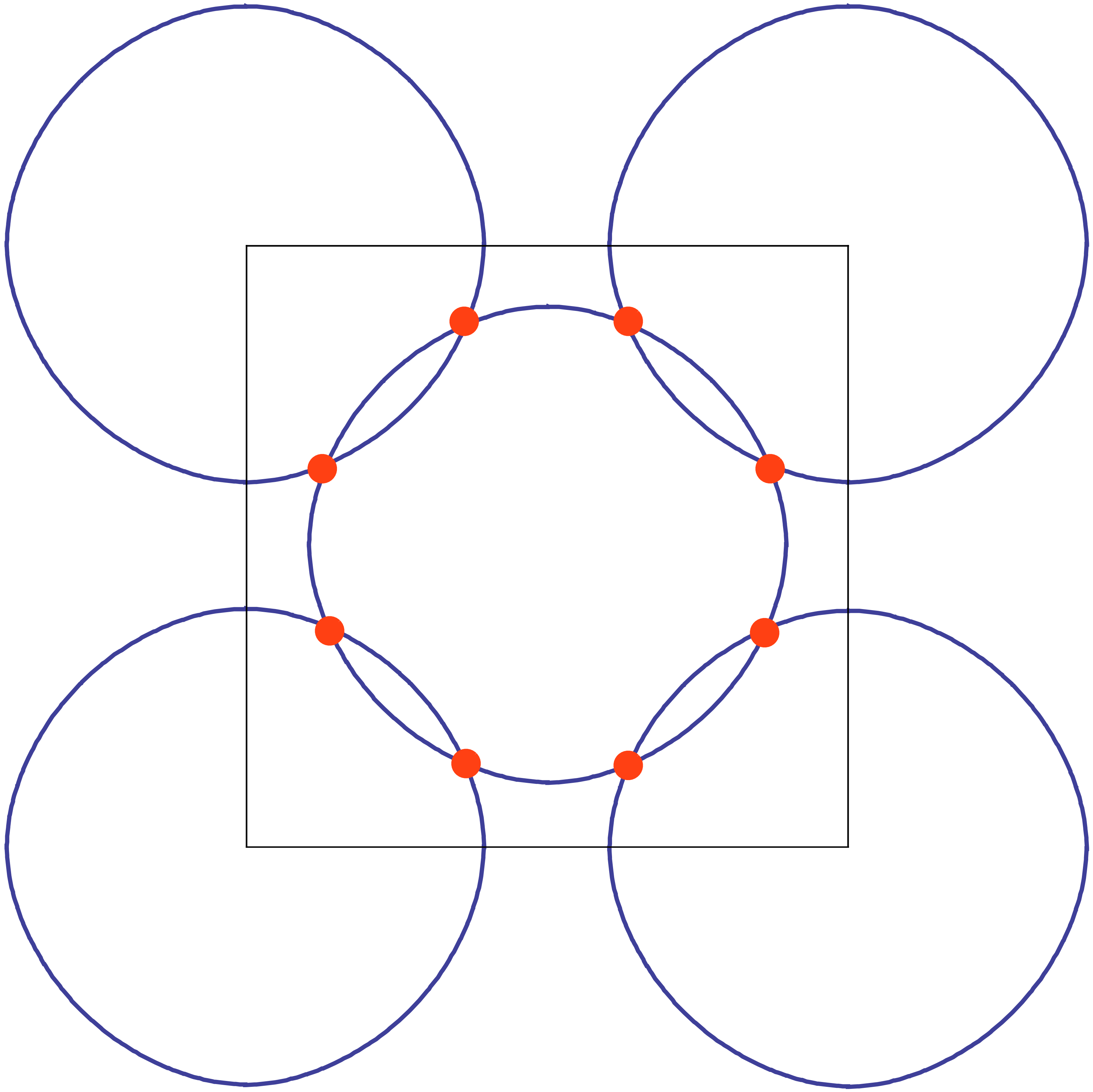
Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$



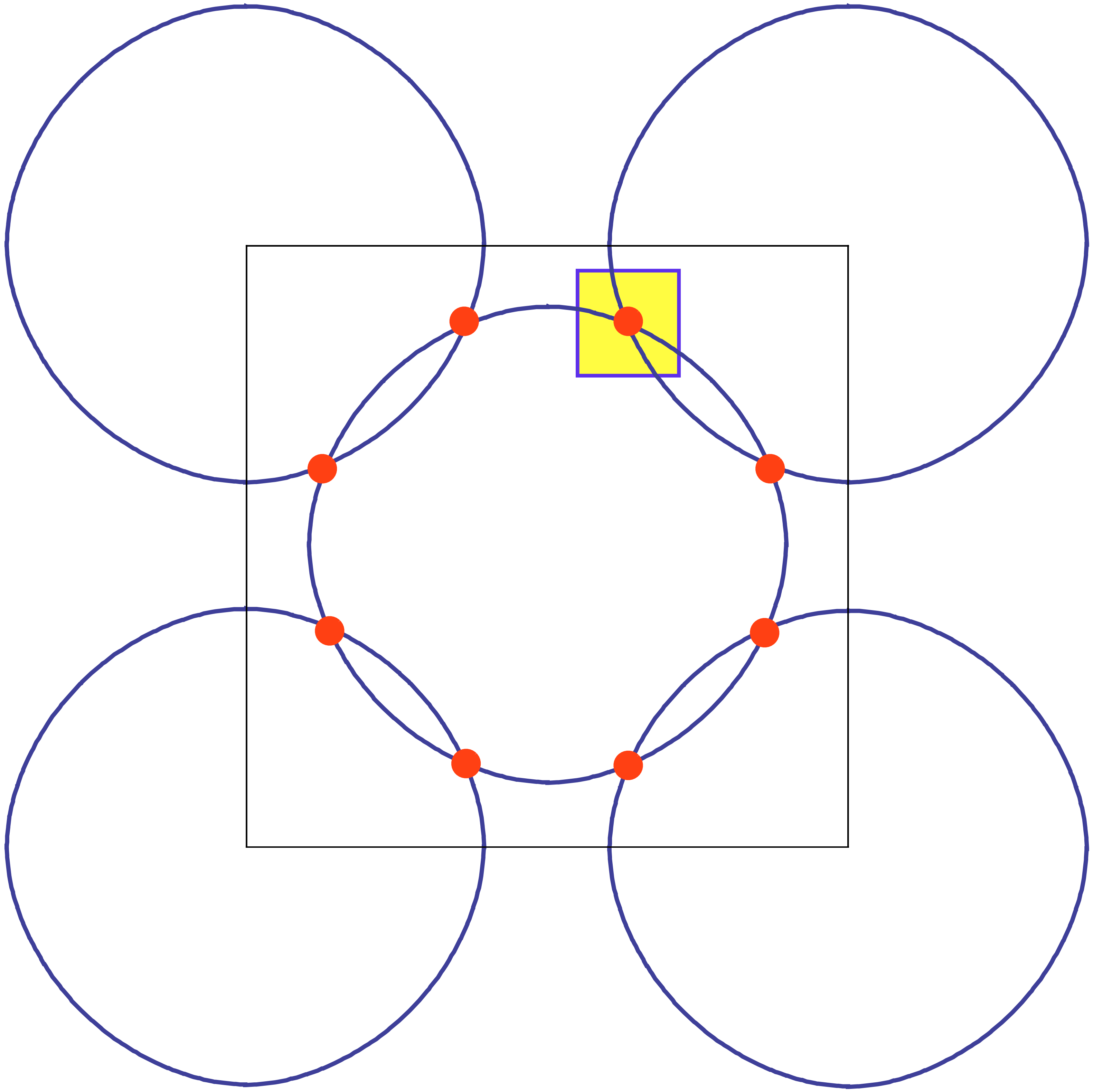
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.



“Hot” spots

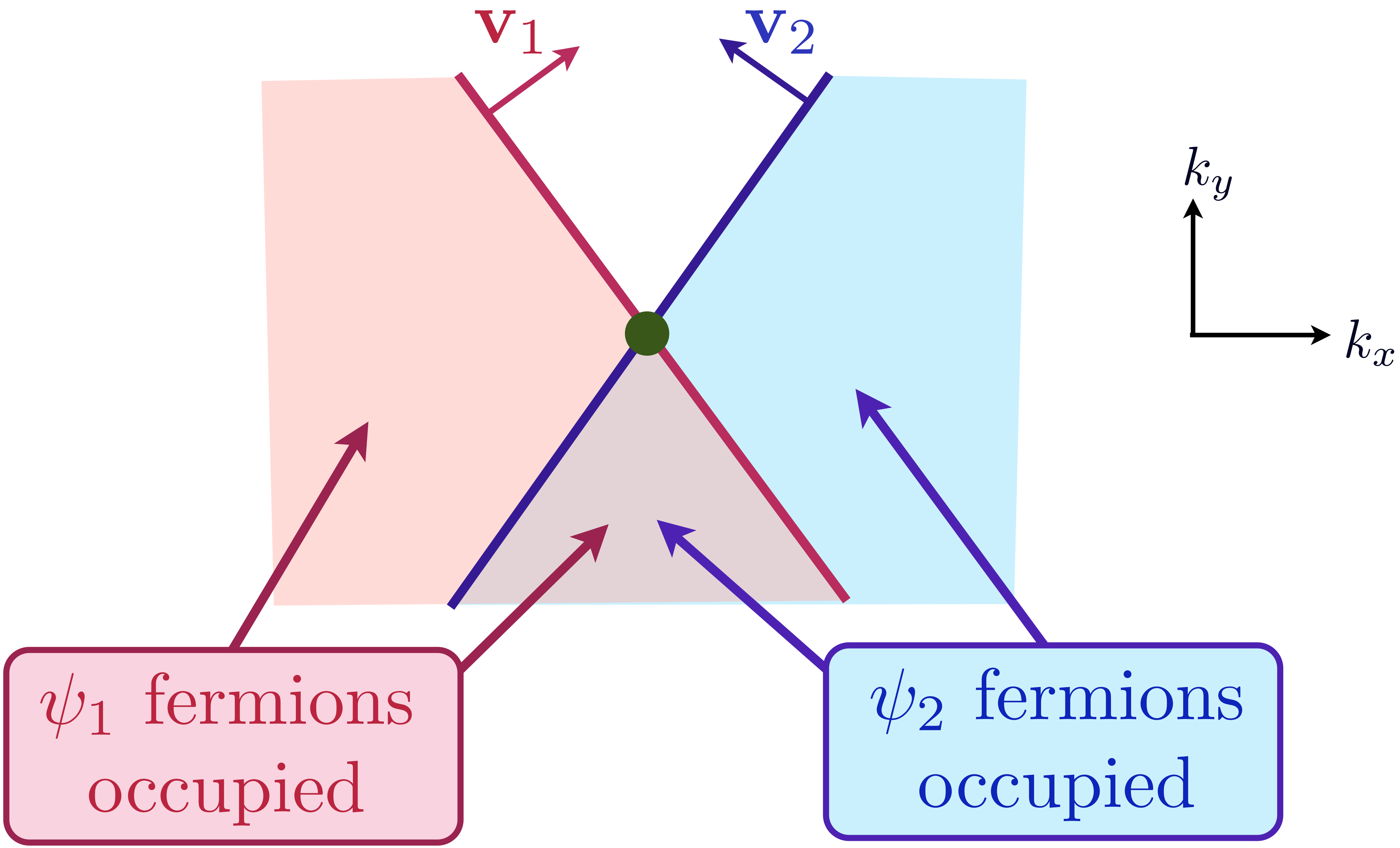


Low energy theory for critical point near hot spots

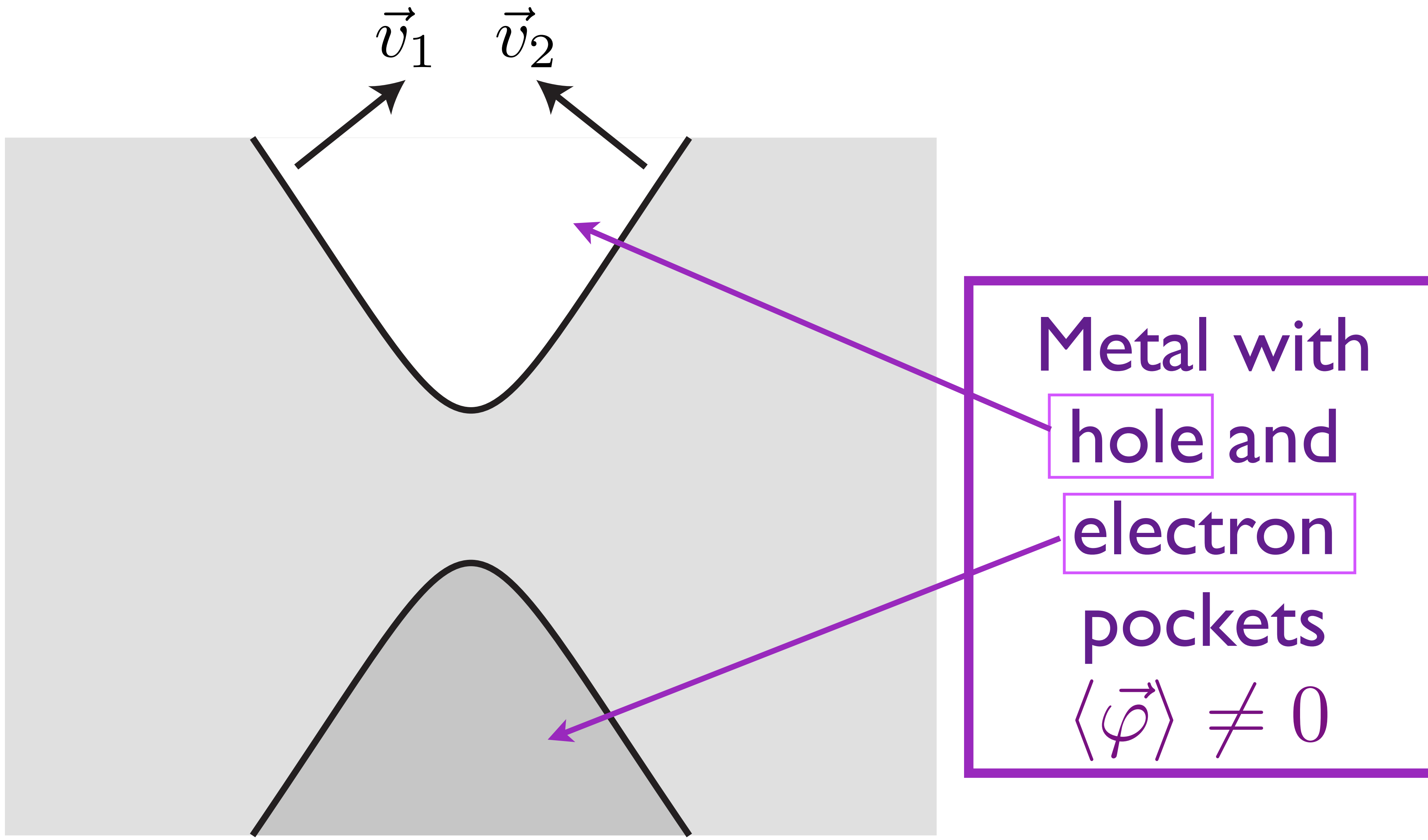


Low energy theory for critical point near hot spots

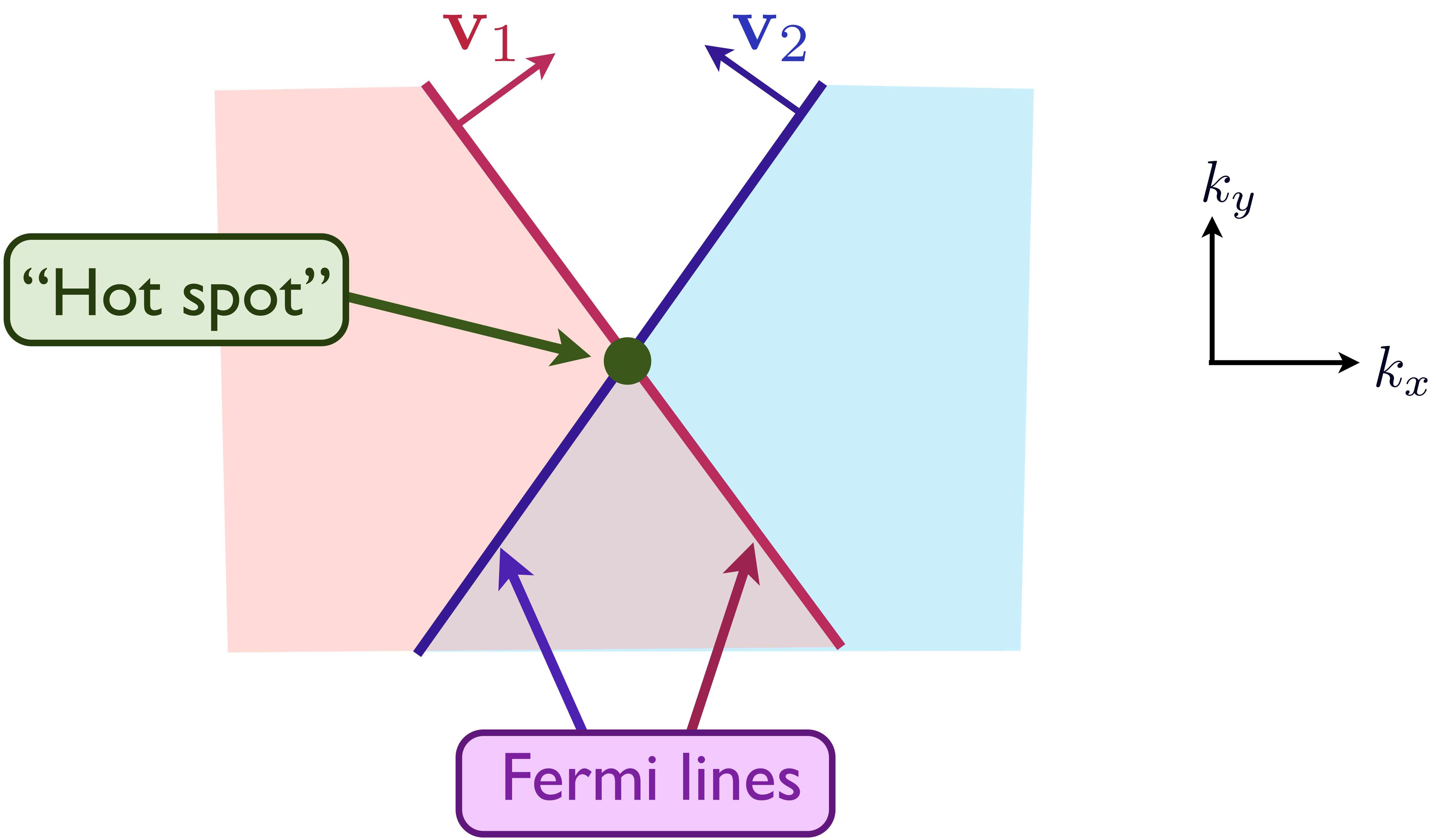
Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ



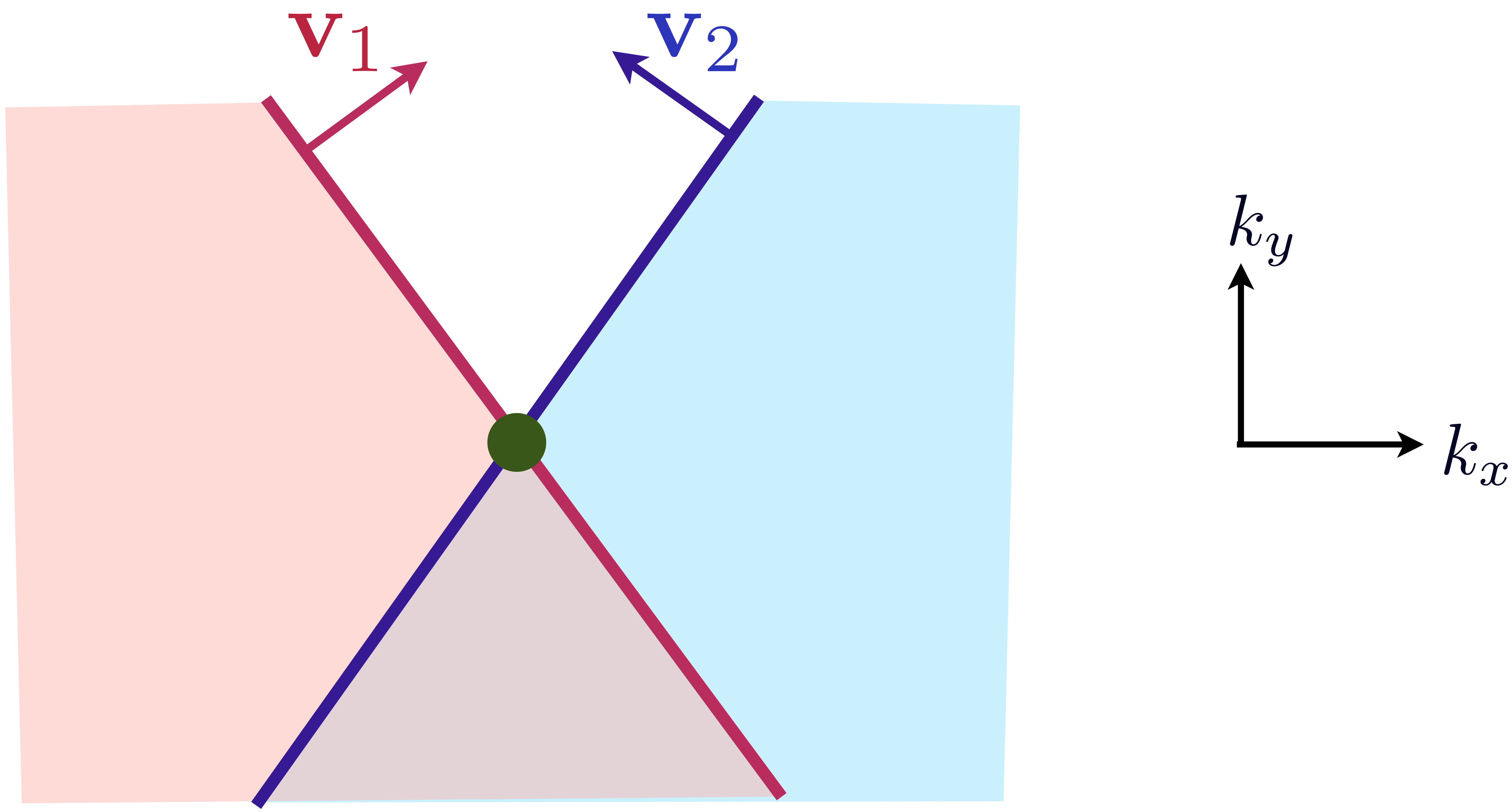
Fermi lines reconnect in antiferromagnetic phase



Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ

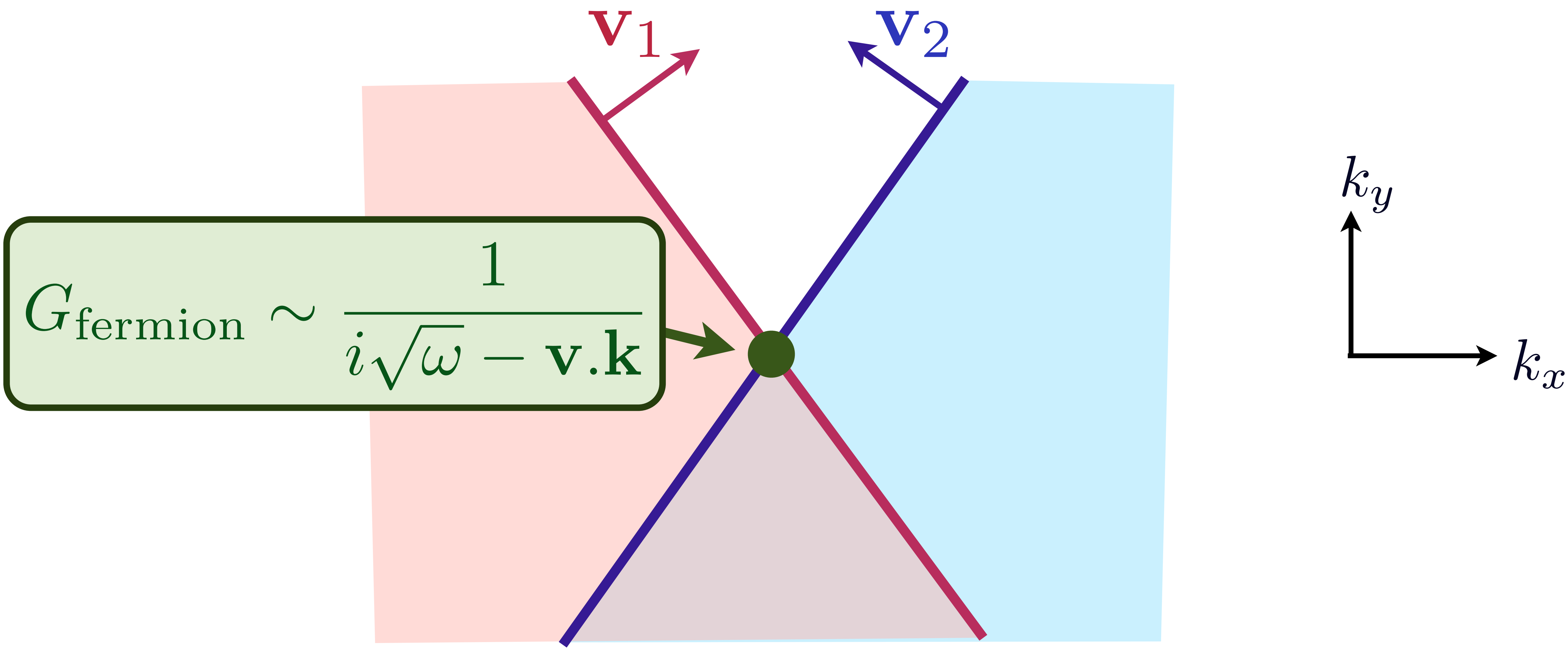


Critical point theory is strongly coupled in $d = 2$
Results are *independent* of coupling λ



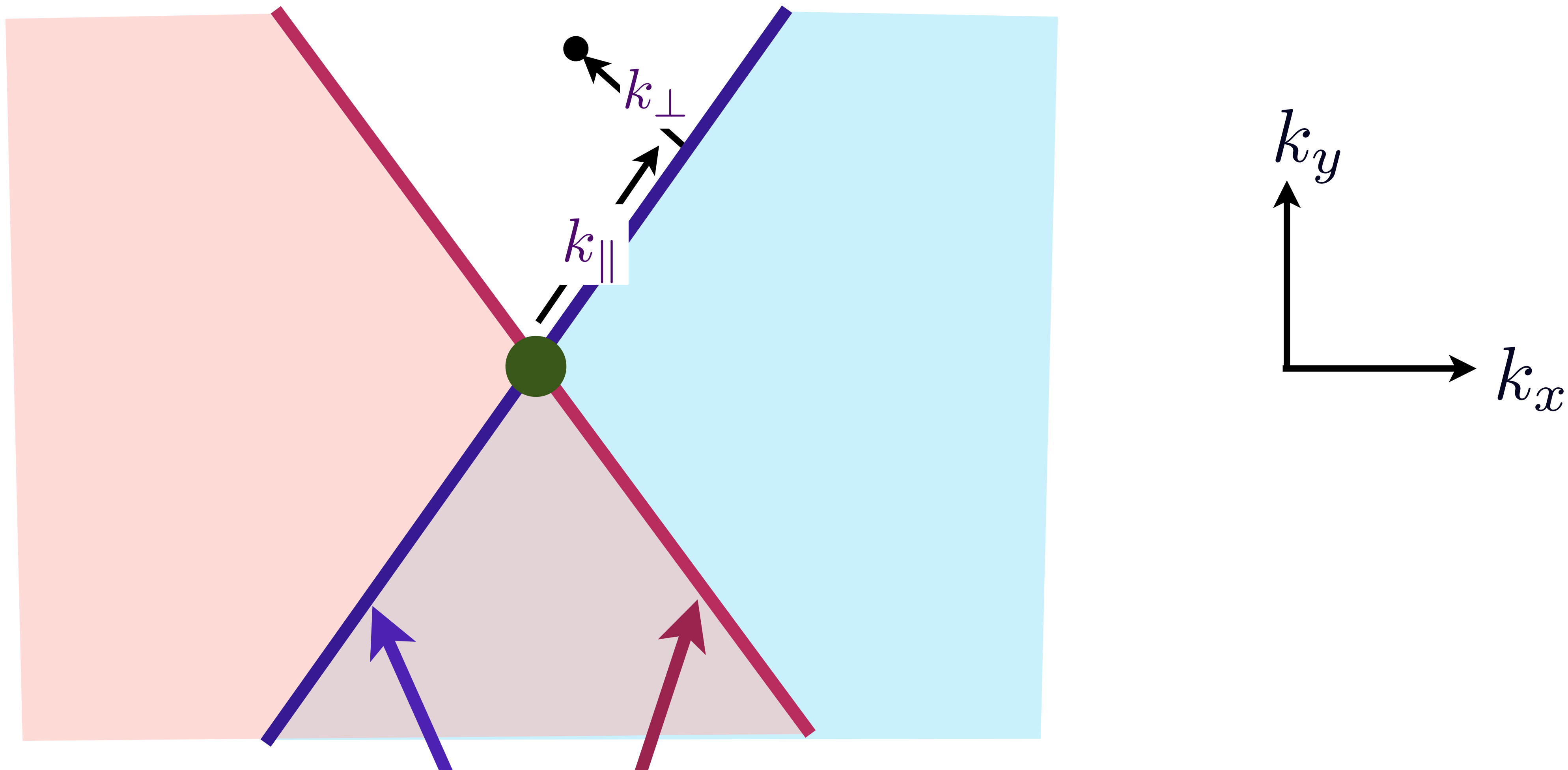
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Critical point theory is strongly coupled in $d = 2$
Results are *independent* of coupling λ



A. J. Millis, *Phys. Rev. B* **45**, 13047 (1992)
Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004)

Critical point theory is strongly coupled in $d = 2$
Results are *independent* of coupling λ



$$G_{\text{fermion}} = \frac{Z(k_\parallel)}{i\omega - v_F(k_\parallel)k_\perp}, \quad Z(k_\parallel) \sim v_F(k_\parallel) \sim k_\parallel$$

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

   Quantum criticality of Fermi surface reconstruction, and the phase diagram in a magnetic field

2. Strongly-coupled quantum criticality in metals

   Fermi surfaces and gapless bosons

3. Instability to unconventional superconductivity

   "Mechanism" of higher temperature superconductivity

4. Theory of the competition between superconductivity and antiferromagnetism

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

   Quantum criticality of Fermi surface reconstruction, and the phase diagram in a magnetic field

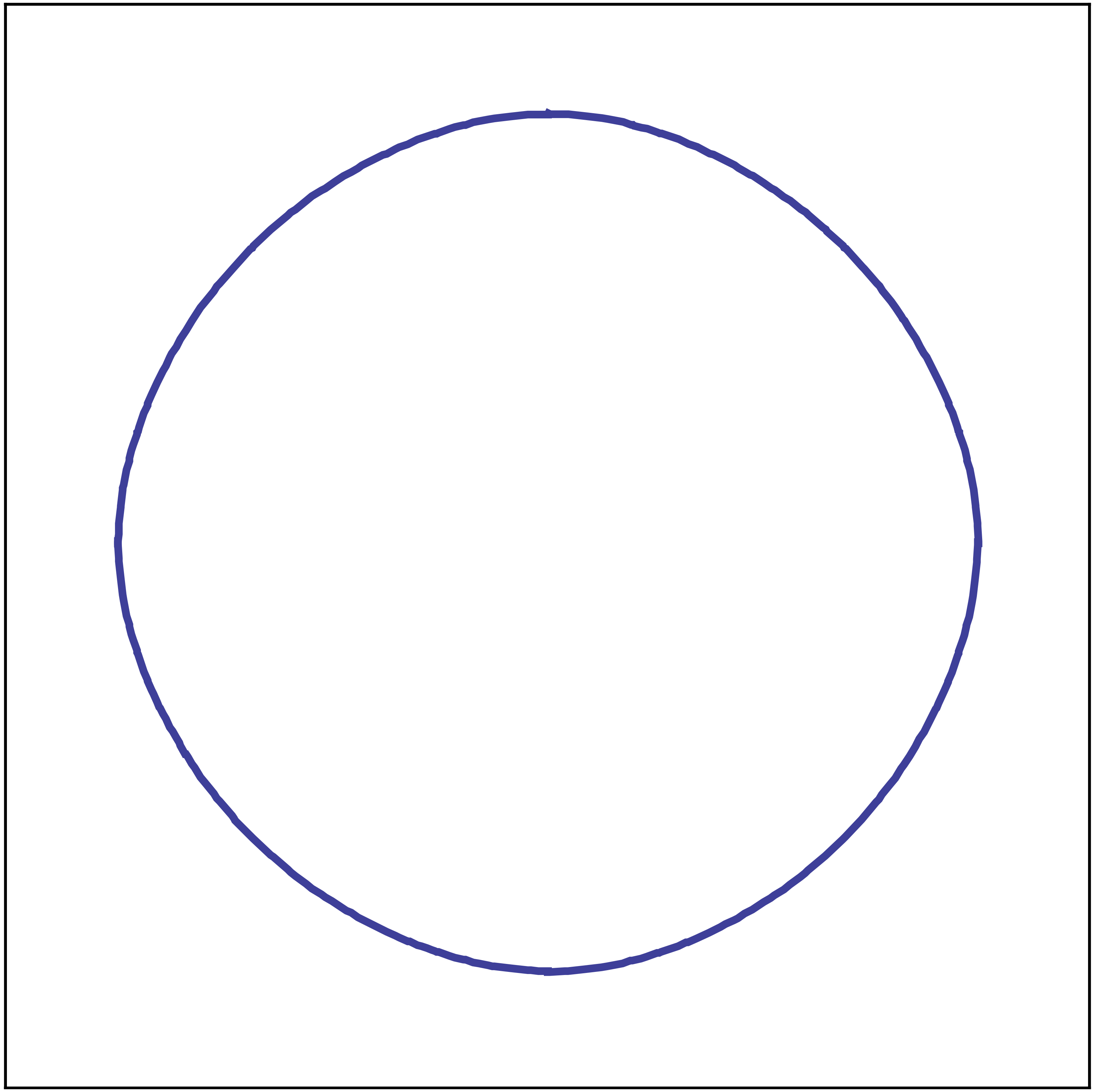
2. Strongly-coupled quantum criticality in metals

   Fermi surfaces and gapless bosons

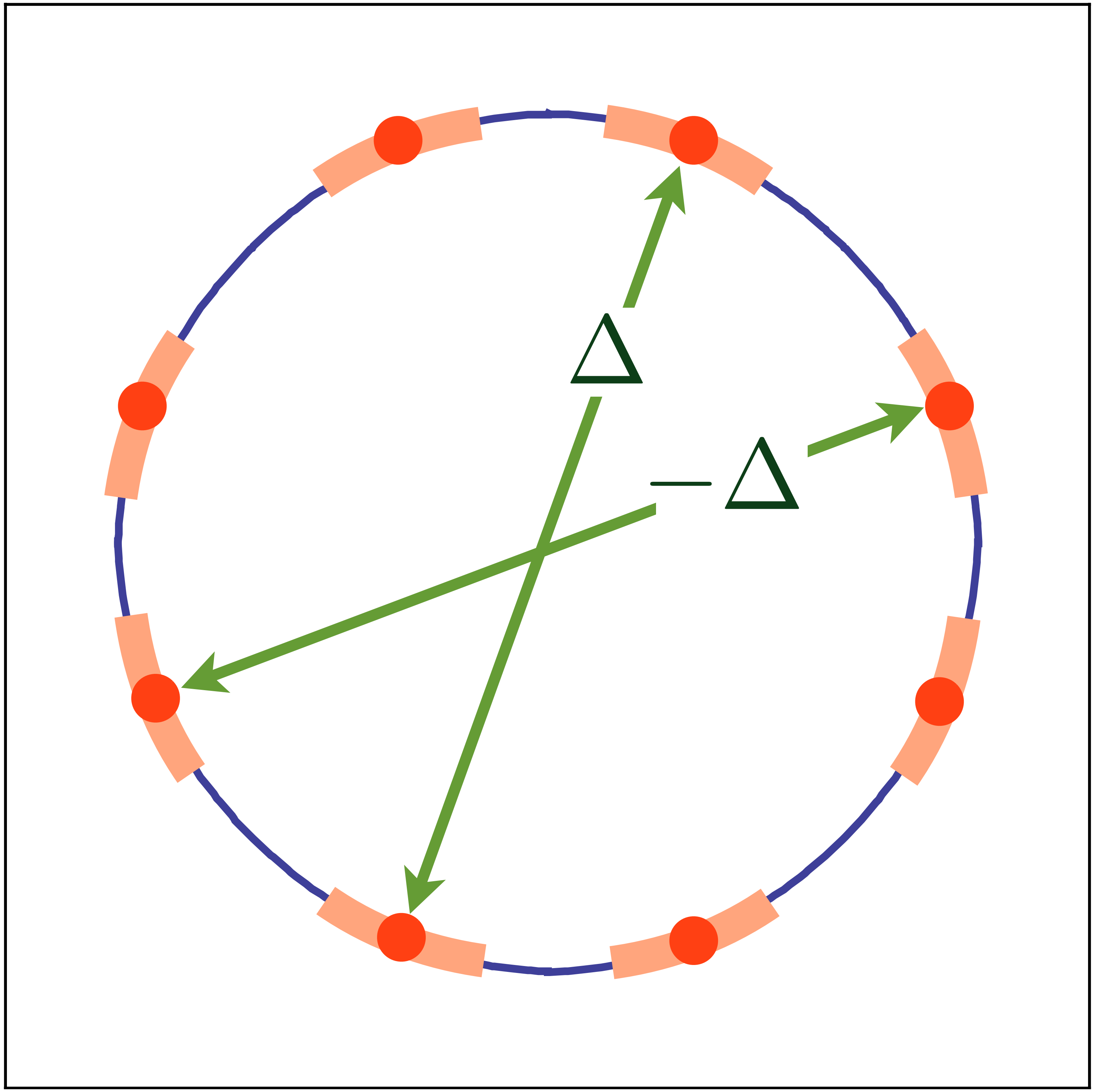
3. Instability to unconventional superconductivity

   "Mechanism" of higher temperature superconductivity

4. Theory of the competition between superconductivity and antiferromagnetism

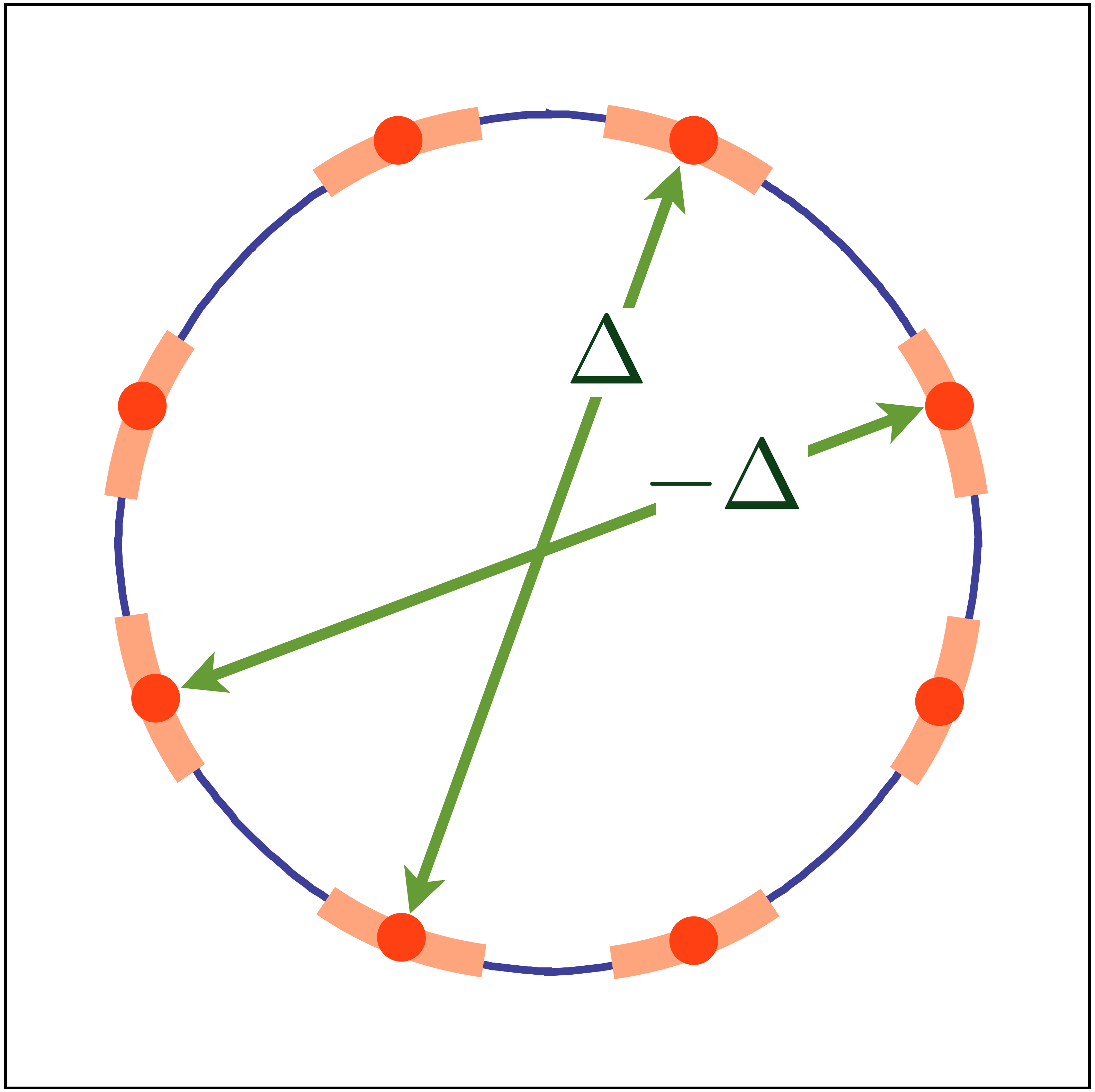


Metal with "large" Fermi surface



Unconventional pairing at *and near* hot spots

$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$

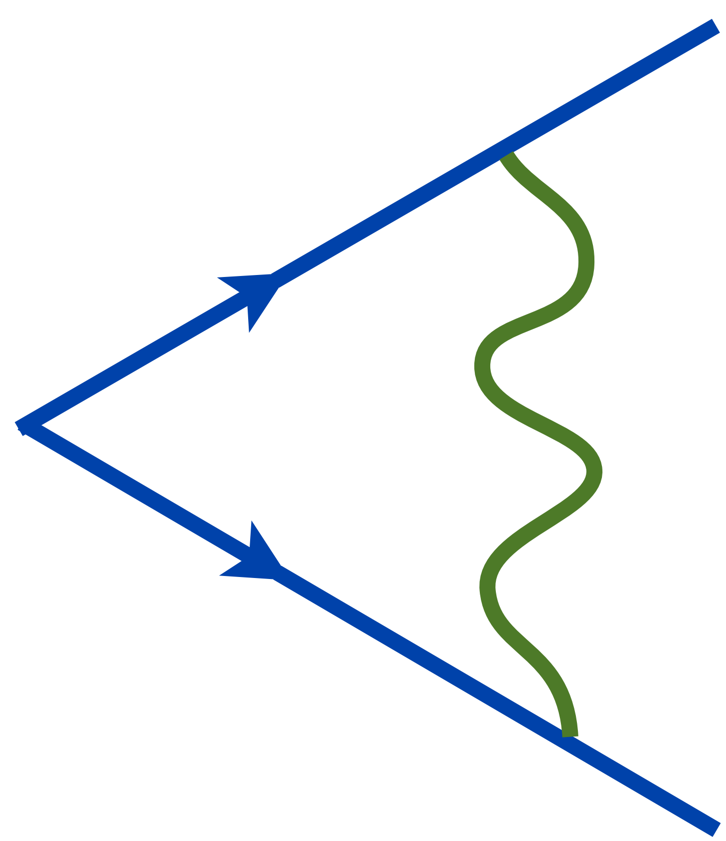


Unconventional pairing at *and near* hot spots

BCS theory

$$1 + \lambda_{\text{e-ph}} \log\left(\frac{\omega_D}{\omega}\right)$$

Cooper logarithm



BCS theory

$$1 + \lambda_{\text{e-ph}} \log\left(\frac{\omega_D}{\omega}\right)$$

Electron-phonon coupling

BCS theory

$$1 + \lambda_{\text{e-ph}} \log\left(\frac{\omega_D}{\omega}\right)$$

Electron-phonon coupling

Debye frequency

BCS theory

$$1 + \lambda_{\text{e-ph}} \log\left(\frac{\omega_D}{\omega}\right)$$

Electron-phonon coupling

Debye frequency

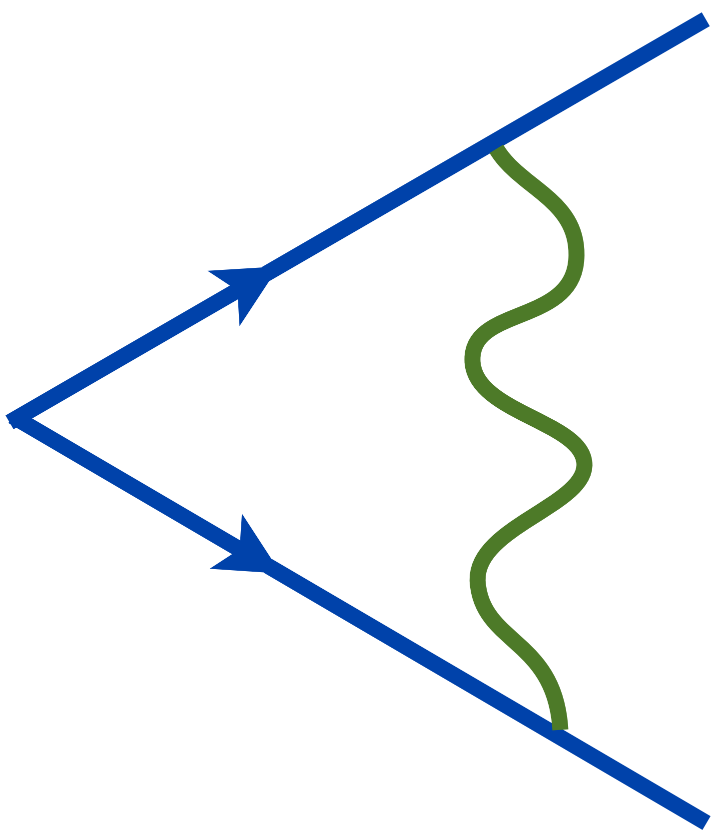
Implies
$T_c \sim \omega_D \exp\left(-1/\lambda\right)$

Enhancement of pairing susceptibility by interactions

Antiferromagnetic fluctuations: weak-coupling

$$1 + \left(\frac{U}{t}\right)^2 \underbrace{\log\left(\frac{E_F}{\omega}\right)}_{\text{Cooper logarithm}}$$

Cooper logarithm



V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M.Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

Enhancement of pairing susceptibility by interactions

Antiferromagnetic fluctuations: weak-coupling

$$1+\left(\frac{U}{t}\right)^2\log\left(\frac{E_F}{\omega}\right)$$

Fermi energy

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

Enhancement of pairing susceptibility by interactions

Antiferromagnetic fluctuations: weak-coupling

$$1 + \left(\frac{U}{t}\right)^2 \log\left(\frac{E_F}{\omega}\right)$$

Applies in a Fermi liquid as repulsive interaction $U \to 0$.

Fermi energy

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M.Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

Enhancement of pairing susceptibility by interactions

Antiferromagnetic fluctuations: weak-coupling

$$1 + \left(\frac{U}{t}\right)^2 \log\left(\frac{E_F}{\omega}\right)$$

Applies in a Fermi liquid as repulsive interaction $U \to 0$.

Fermi energy

Implies

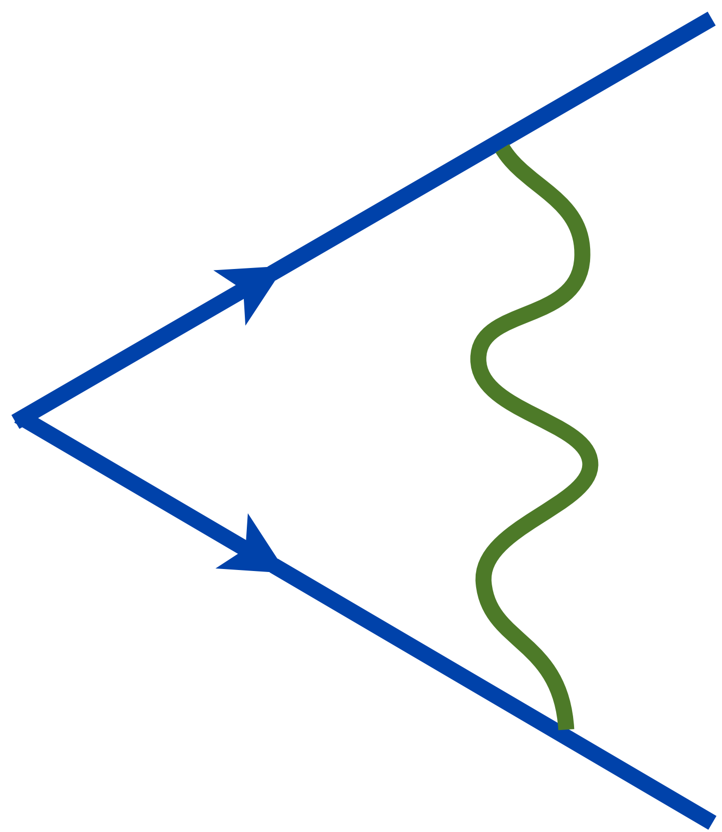
$$T_c \sim E_F \exp\left(-(t/U)^2\right)$$

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M.Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2\left(\frac{E_F}{\omega}\right)$$



M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2\left(\frac{E_F}{\omega}\right)$$

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$

Fermi energy

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Enhancement of pairing susceptibility by interactions

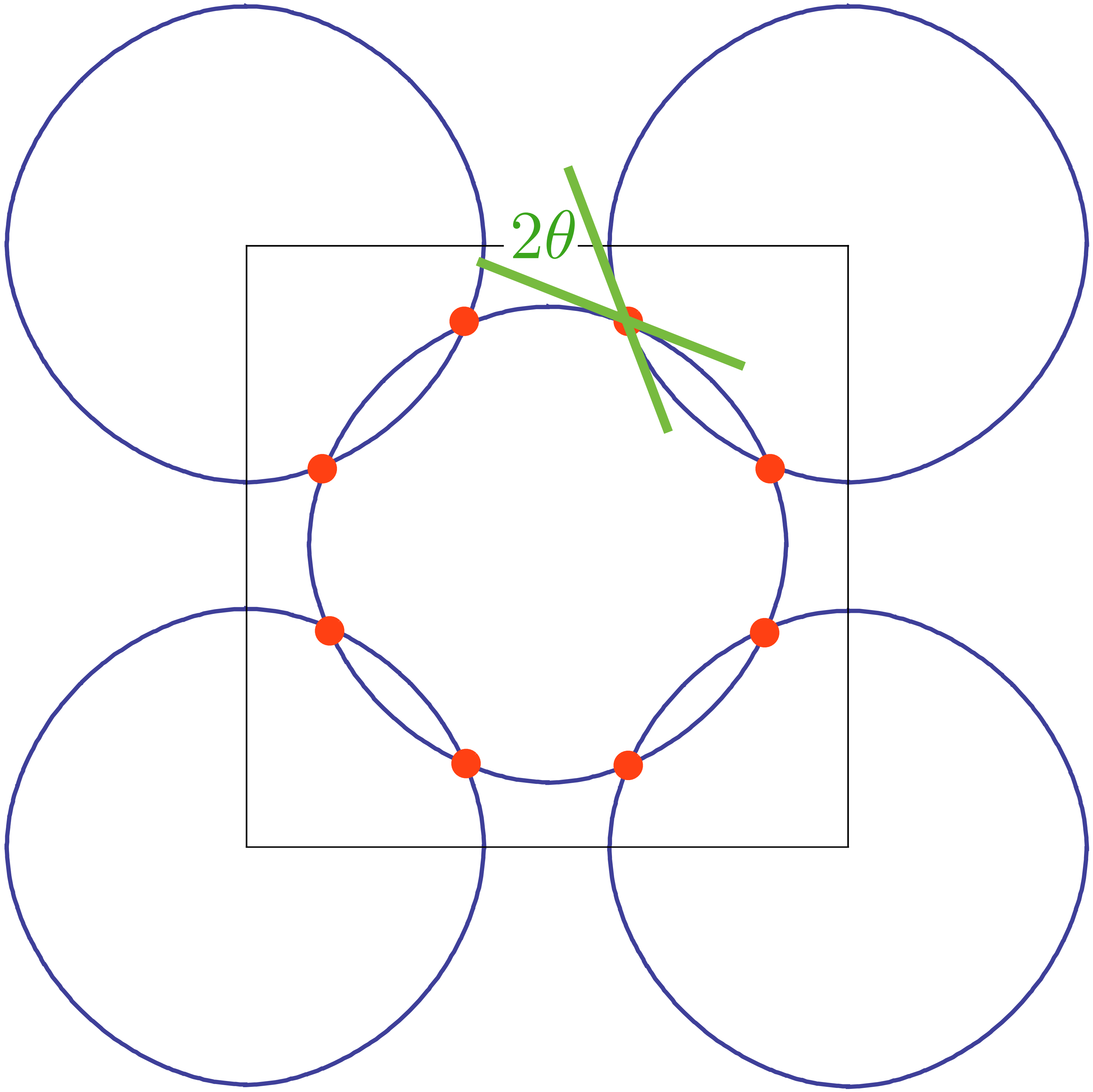
Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$

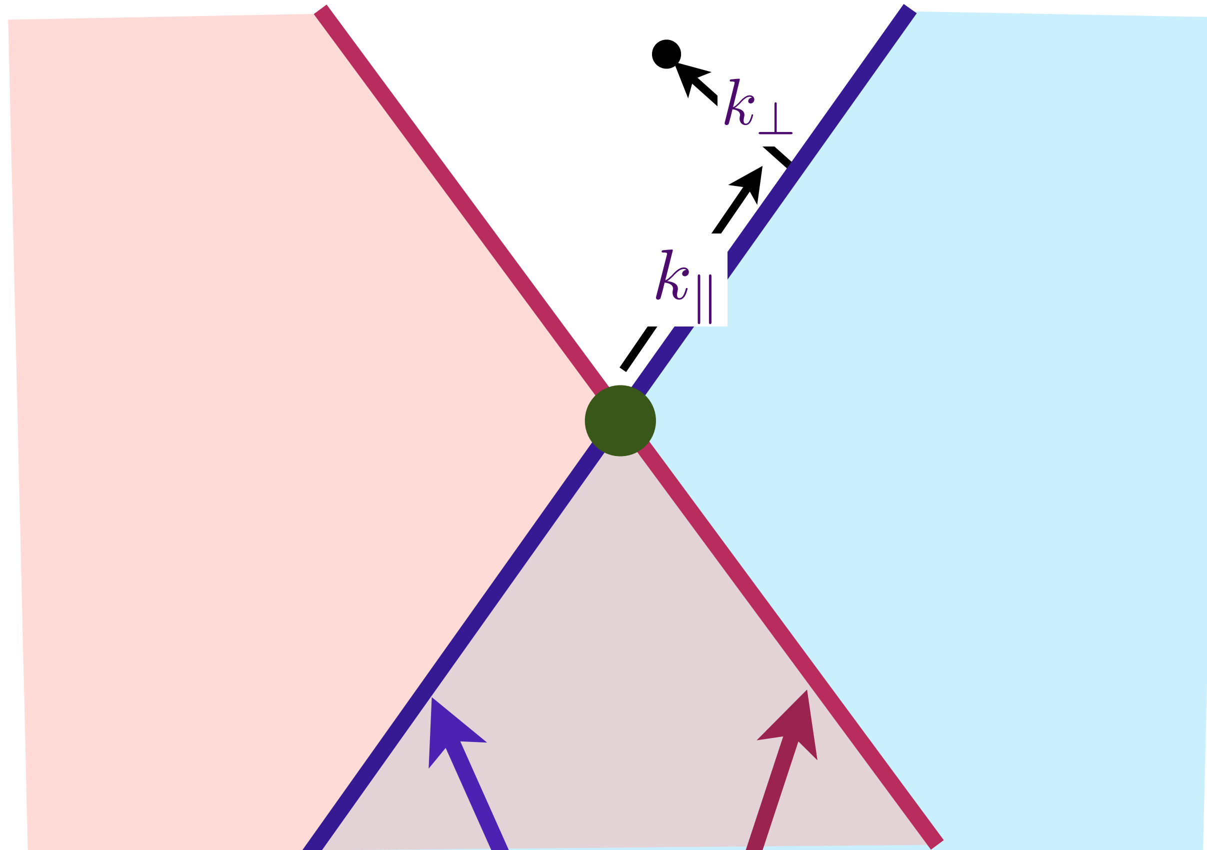
Fermi energy

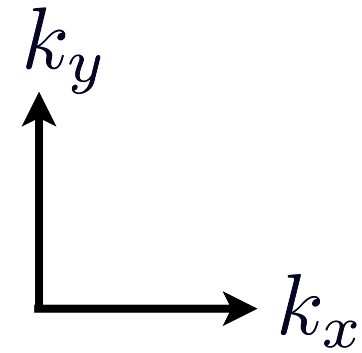
$\alpha = \tan\theta$, where 2θ is the angle between Fermi lines. *Independent* of interaction strength U in 2 dimensions.

(see also Ar. Abanov, A.V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001))
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

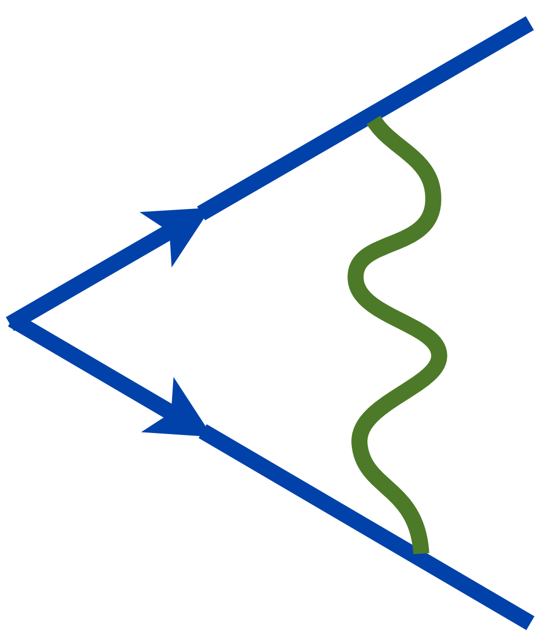

2θ

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



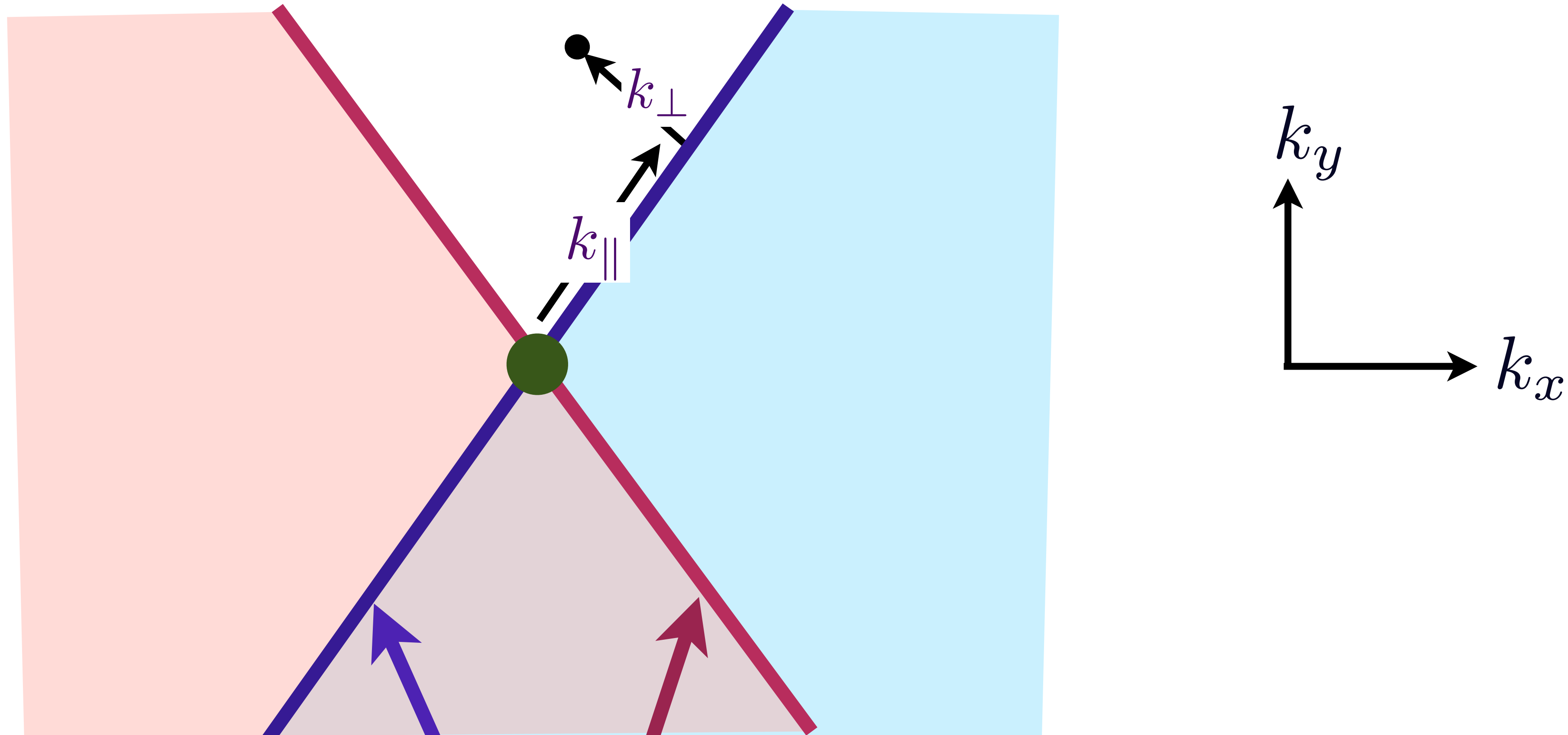


$$G_{\text{fermion}} = \frac{Z(k_\parallel)}{i\omega - v_F(k_\parallel) k_\perp}, \quad Z(k_\parallel) \sim v_F(k_\parallel) \sim k_\parallel$$

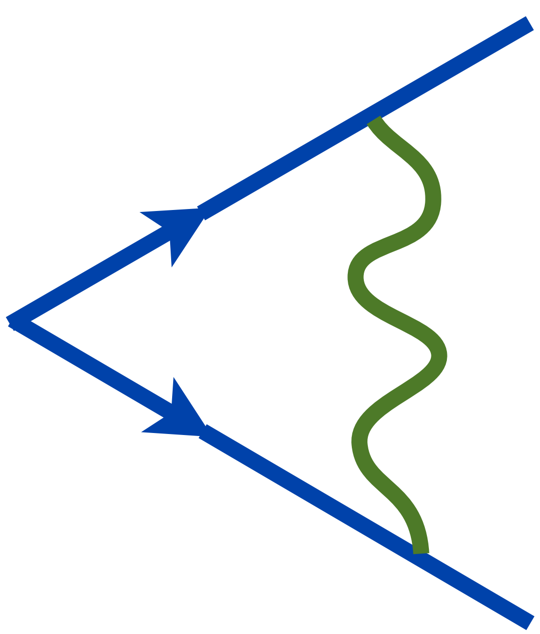


$$\int dk_\parallel \, \frac{1}{k_\parallel^2} \left(\frac{Z^2(k_\parallel)}{v_F(k_\parallel)} \right) \log \frac{k_\parallel^2}{\omega}$$

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



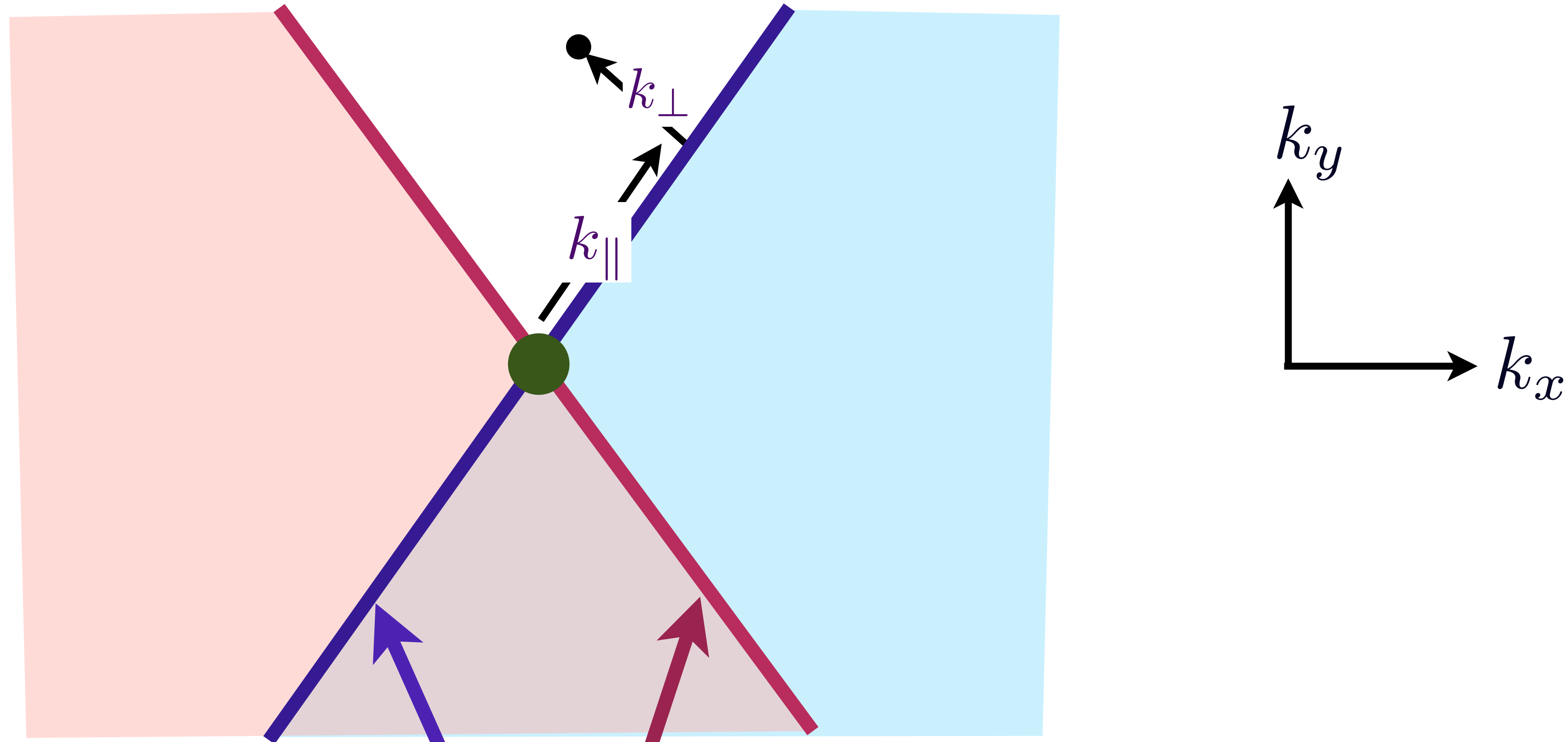
$$G_{\text{fermion}} = \frac{Z(k_\parallel)}{i\omega - v_F(k_\parallel)k_\perp}, \quad Z(k_\parallel) \sim v_F(k_\parallel) \sim k_\parallel$$



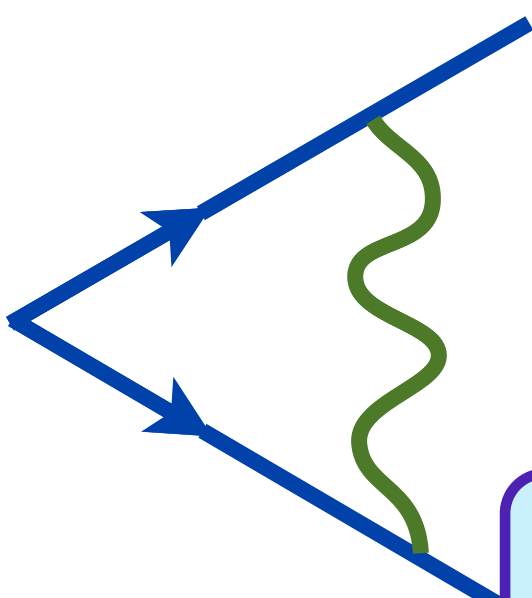
$$\int dk_\parallel \, \frac{1}{k_\parallel^2} \left(\frac{Z^2(k_\parallel)}{v_F(k_\parallel)} \right) \log \frac{k_\parallel^2}{\omega}$$

Cooper logarithm

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



$$G_{\text{fermion}} = \frac{Z(k_\parallel)}{i\omega - v_F(k_\parallel)k_\perp}, \quad Z(k_\parallel) \sim v_F(k_\parallel) \sim k_\parallel$$



$$\int dk_\parallel \frac{1}{k_\parallel^2} \left(\frac{Z^2(k_\parallel)}{v_F(k_\parallel)} \right) \log \frac{k_\parallel^2}{\omega}$$

Spin fluctuation propagator

Cooper logarithm

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2\left(\frac{E_F}{\omega}\right)$$

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$

- $\log^2$ singularity arises from Fermi lines; singularity *at* hot spots is weaker.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2\left(\frac{E_F}{\omega}\right)$$

- $\log^2$ singularity arises from Fermi lines; singularity *at* hot spots is weaker.
- Interference between BCS and quantum-critical logs.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$

- $\log^2$ singularity arises from Fermi lines; singularity *at* hot spots is weaker.
- Interference between BCS and quantum-critical logs.
- Momentum dependence of self-energy is crucial.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2\left(\frac{E_F}{\omega}\right)$$

- $\log^2$ singularity arises from Fermi lines; singularity *at* hot spots is weaker.
- Interference between BCS and quantum-critical logs.
- Momentum dependence of self-energy is crucial.
- Not suppressed by $1/N$ factor in $1/N$ expansion.

Ar. Abanov, A.V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001)
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

   Quantum criticality of Fermi surface reconstruction, and the phase diagram in a magnetic field

2. Strongly-coupled quantum criticality in metals

   Fermi surfaces and gapless bosons

3. Instability to unconventional superconductivity

   "Mechanism" of higher temperature superconductivity

4. Theory of the competition between superconductivity and antiferromagnetism

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

   Quantum criticality of Fermi surface reconstruction, and the phase diagram in a magnetic field

2. Strongly-coupled quantum criticality in metals

   Fermi surfaces and gapless bosons

3. Instability to unconventional superconductivity

   "Mechanism" of higher temperature superconductivity

4. Theory of the competition between superconductivity and antiferromagnetism

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (Δ):

$$\mathcal{S} = \int d^2r d\tau \left[\frac{1}{2}(\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2}(\nabla_x \vec{\varphi})^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\left(\vec{\varphi}^2\right)^2 + \kappa\, \vec{\varphi}^2 \left|\Delta\right|^2 \right] + \int d^2r \left[\left|(\nabla_x - i(2e/\hbar c)\mathcal{A})\Delta\right|^2 - |\Delta|^2 + \frac{|\Delta|^4}{2} \right]$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (Δ):

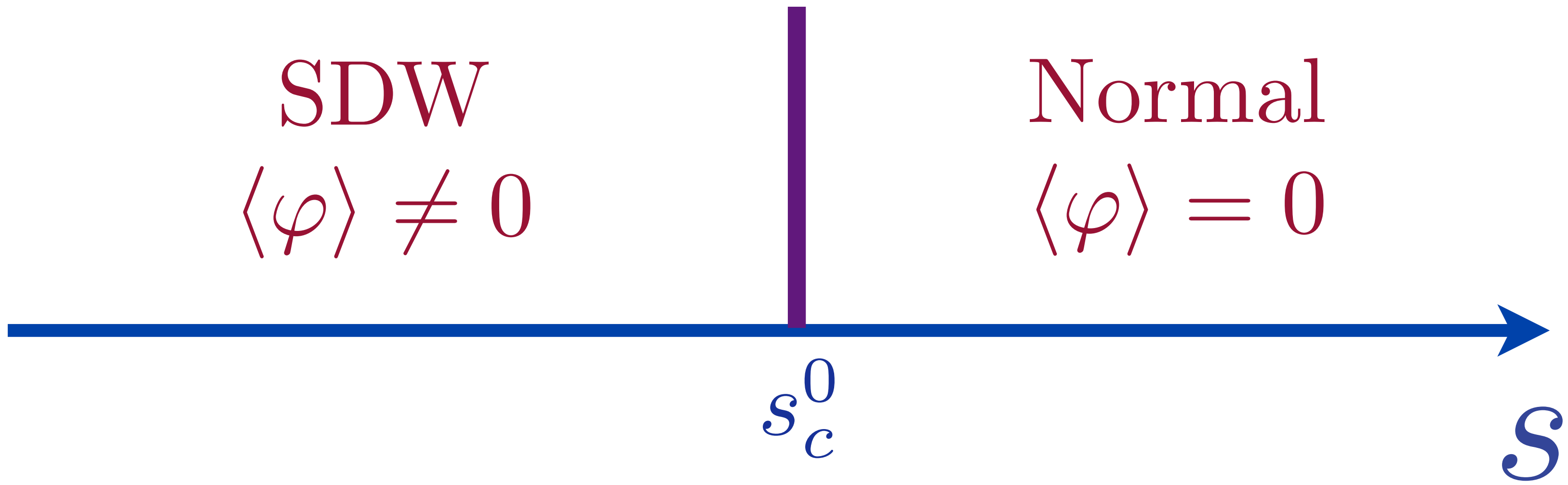
$$\mathcal{S} = \int d^2r d\tau \left[\frac{1}{2}(\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2}(\nabla_x \vec{\varphi})^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\left(\vec{\varphi}^2\right)^2 + \kappa\, \vec{\varphi}^2 \left|\Delta\right|^2 \right] + \int d^2 r \left[\left|(\nabla_x - i(2e/\hbar c)\mathcal{A})\Delta\right|^2 - \left|\Delta\right|^2 + \frac{\left|\Delta\right|^4}{2} \right]$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

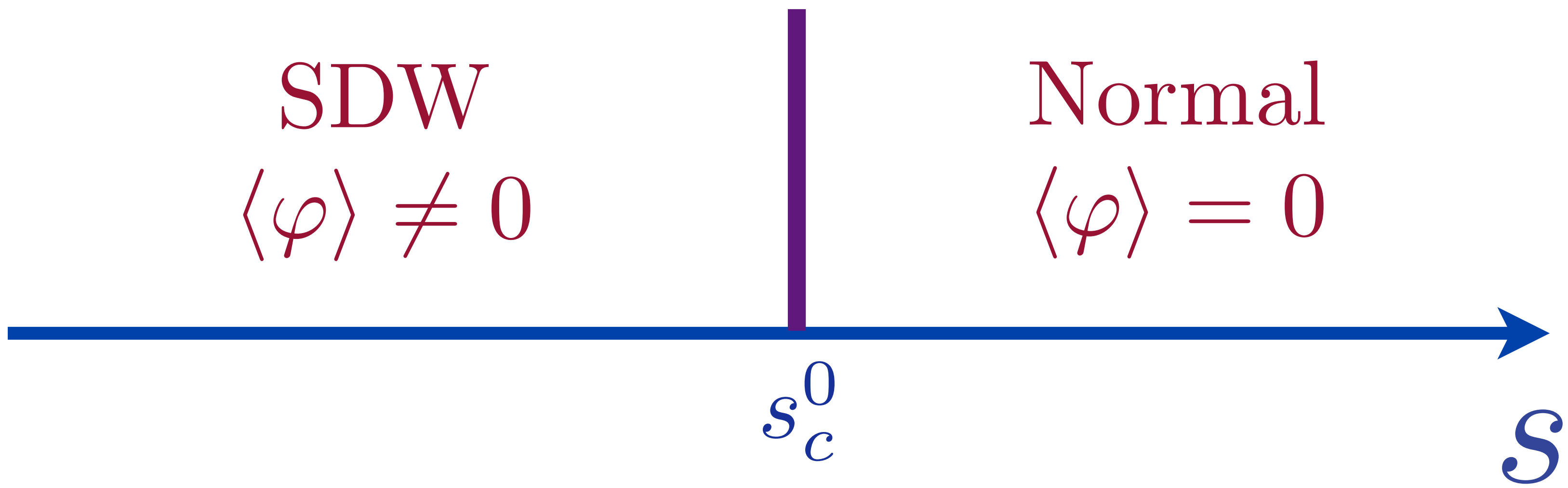


No SC

$\Delta = 0$

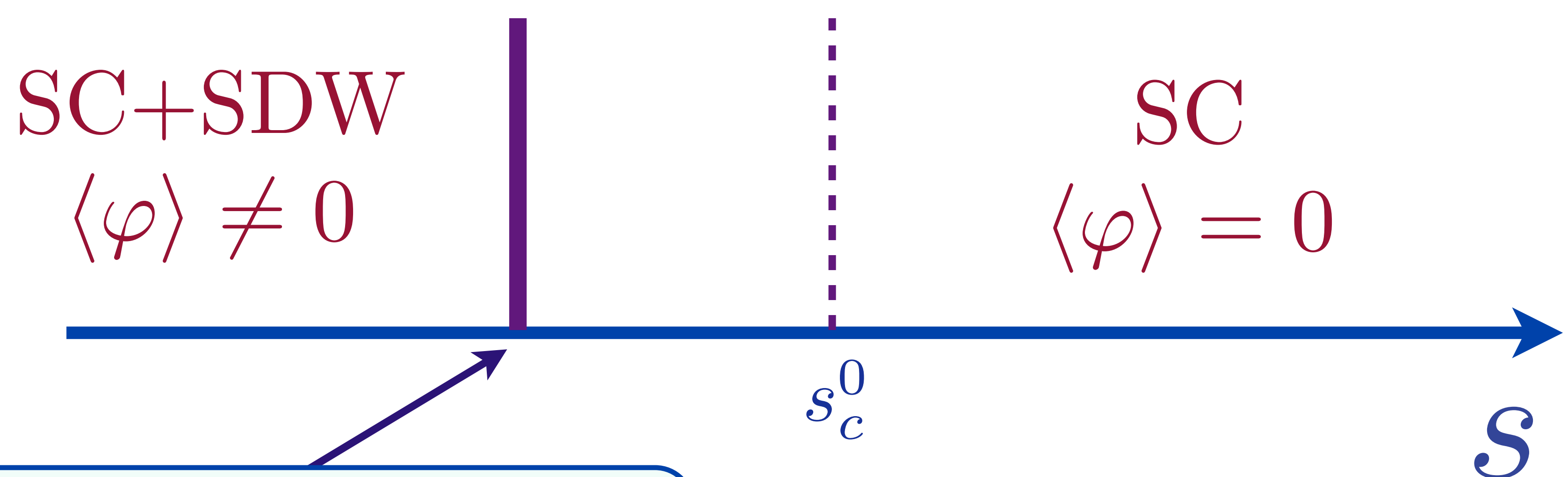
Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order



No SC

$\Delta = 0$



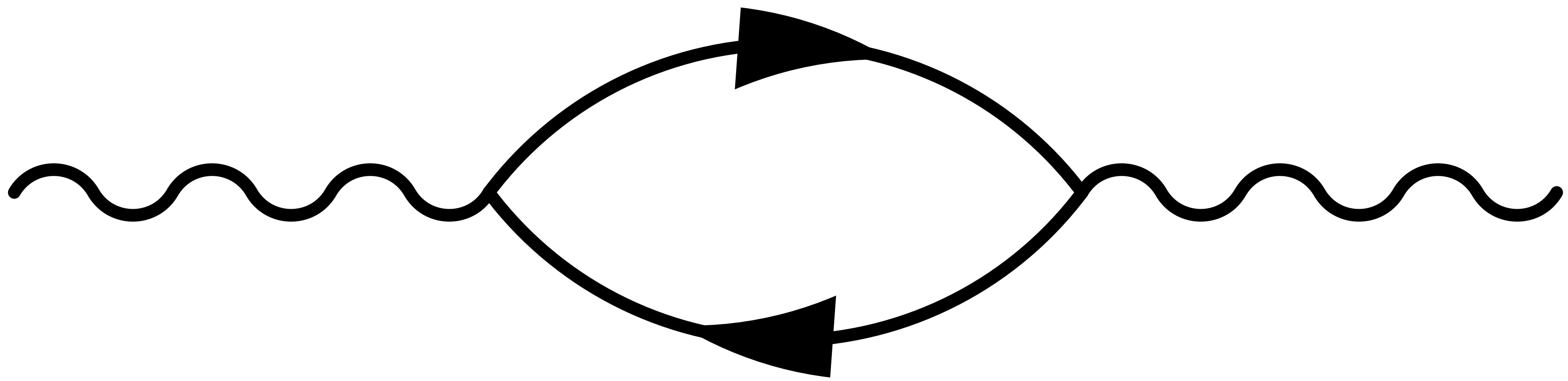
SC

$\Delta \neq 0$

$$s_c = s_c^0 - \kappa |\Delta|^2$$

Fermi surface theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order



Compute the SDW susceptibility, χ, in the superconducting state. As $\Delta \to 0$, we find

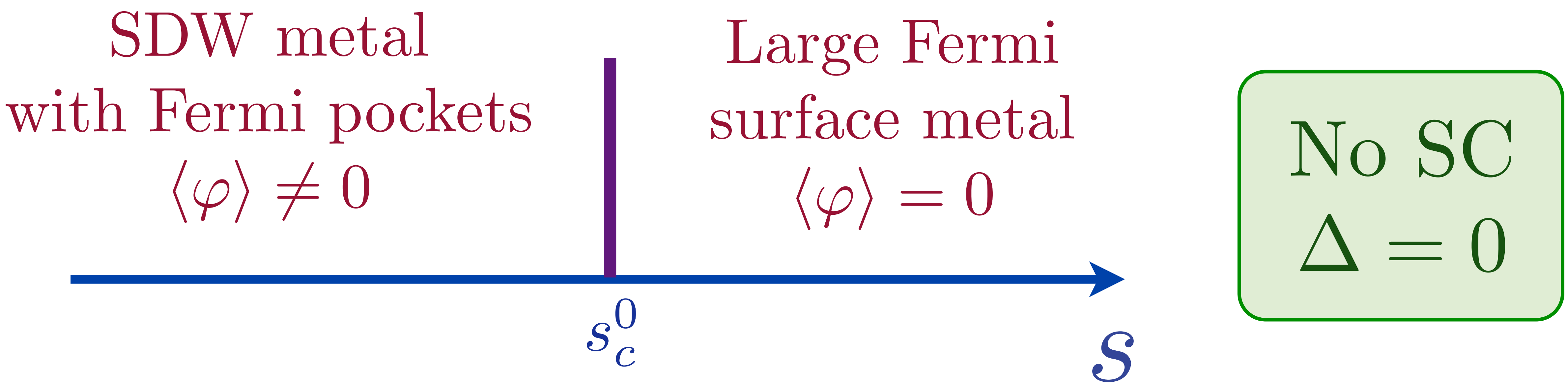
$$\chi(\Delta) = \chi(0) - C|\Delta|$$

where C is a universal constant dominated by the vicinity of the hot spots.

E. G. Moon and S. Sachdev, *Phy. Rev. B* **82**, 104516 (2010)

Fermi surface theory of competing orders

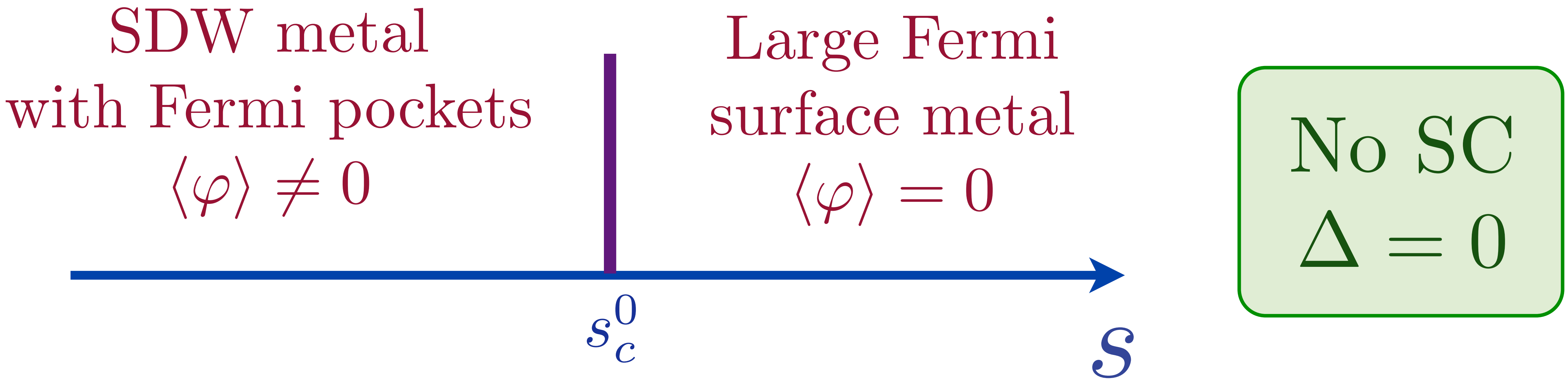
Competition between superconductivity (SC) and spin-density wave (SDW) order

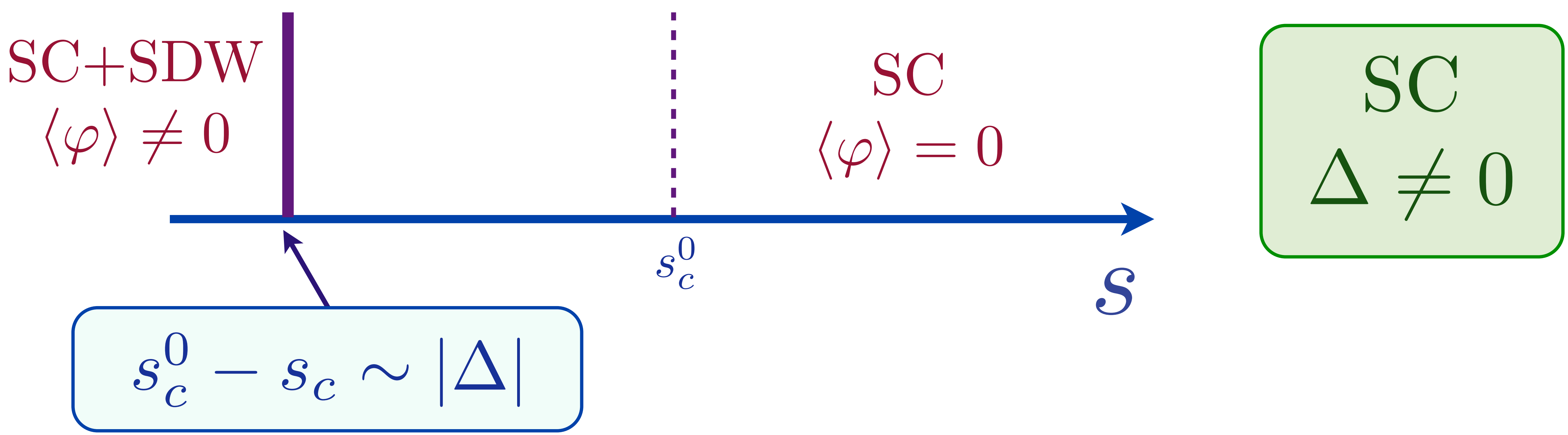


E. G. Moon and S. Sachdev, *Phy. Rev. B* **82**, 104516 (2010)

Fermi surface theory of competing orders

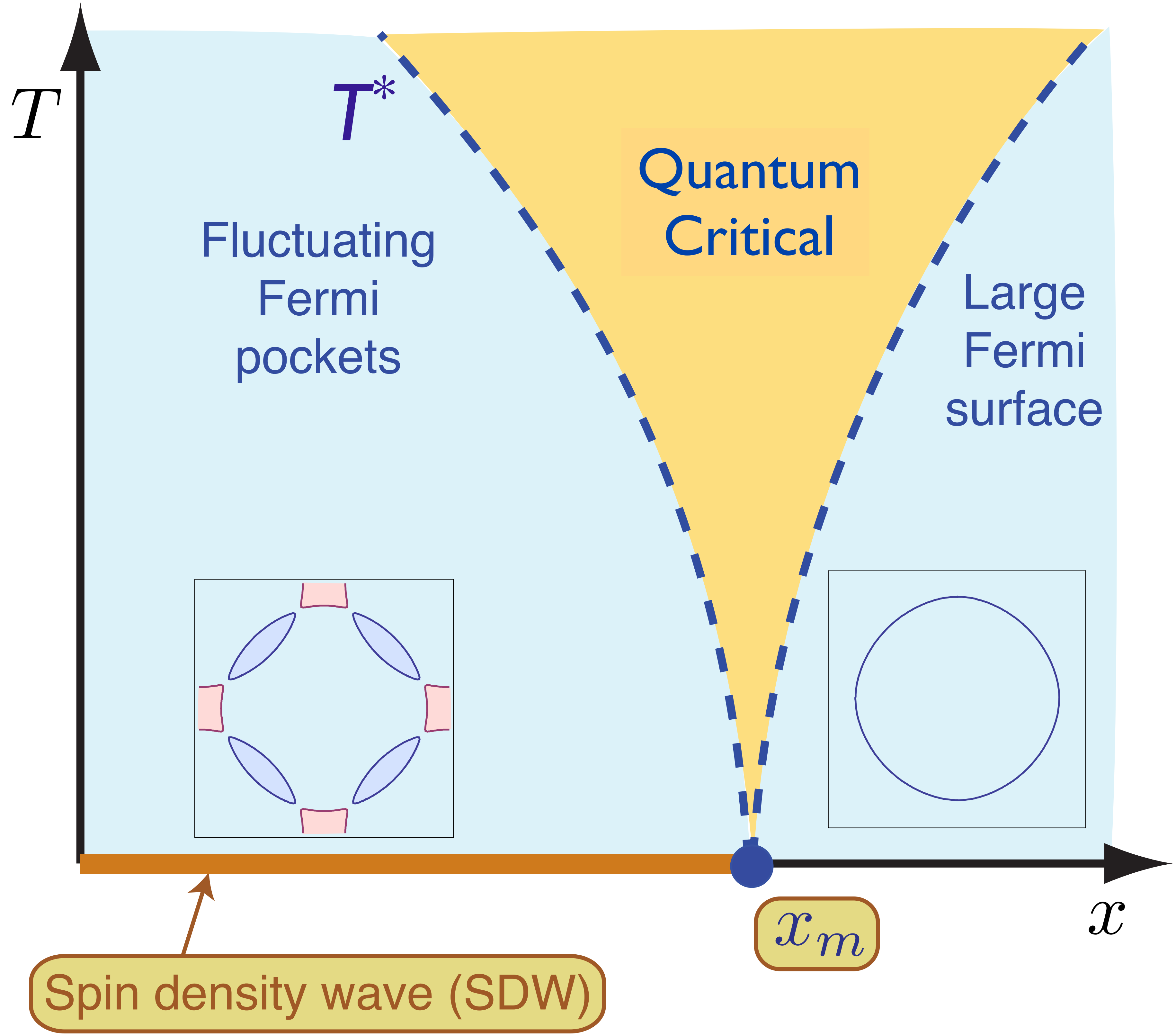
Competition between superconductivity (SC) and spin-density wave (SDW) order





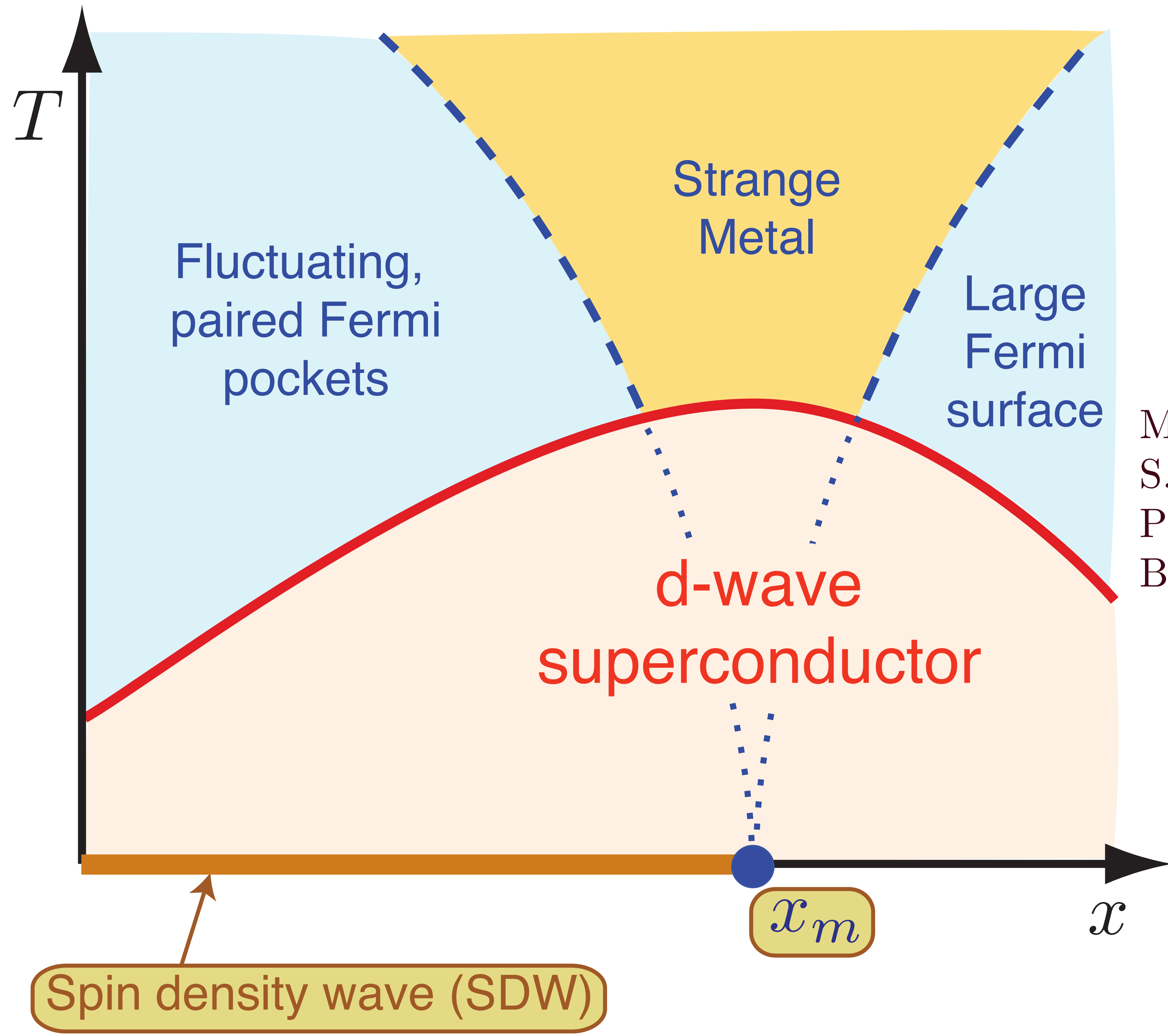
E. G. Moon and S. Sachdev, *Phy. Rev. B* **82**, 104516 (2010)

Theory of quantum criticality in the cuprates



Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

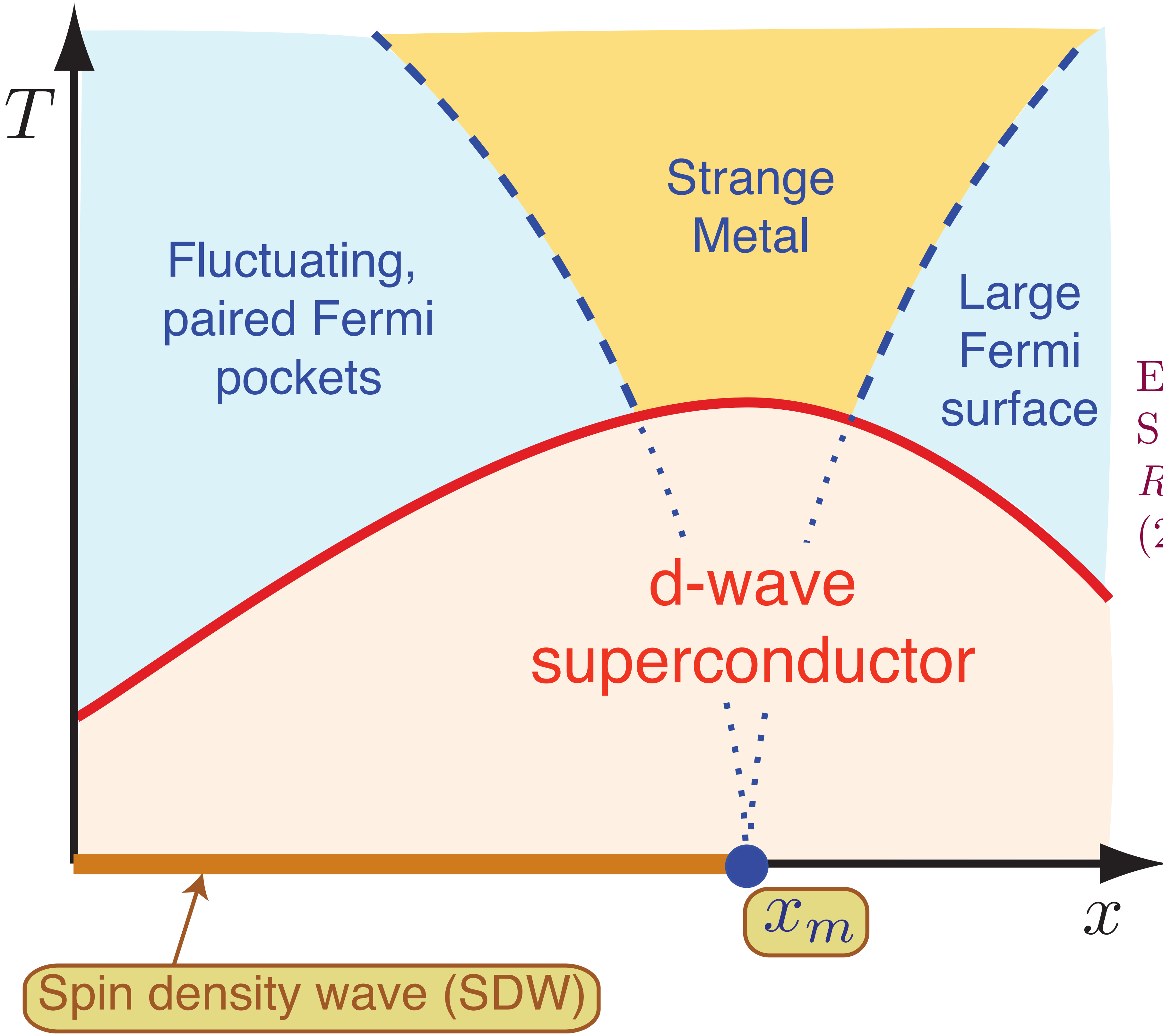
Theory of quantum criticality in the cuprates



M. A. Metlitski and S. Sachdev, Physical Review B **82**, 075128 (2010)

SDW quantum critical point is unstable to *d*-wave superconductivity
This instability is stronger than that in the BCS theory

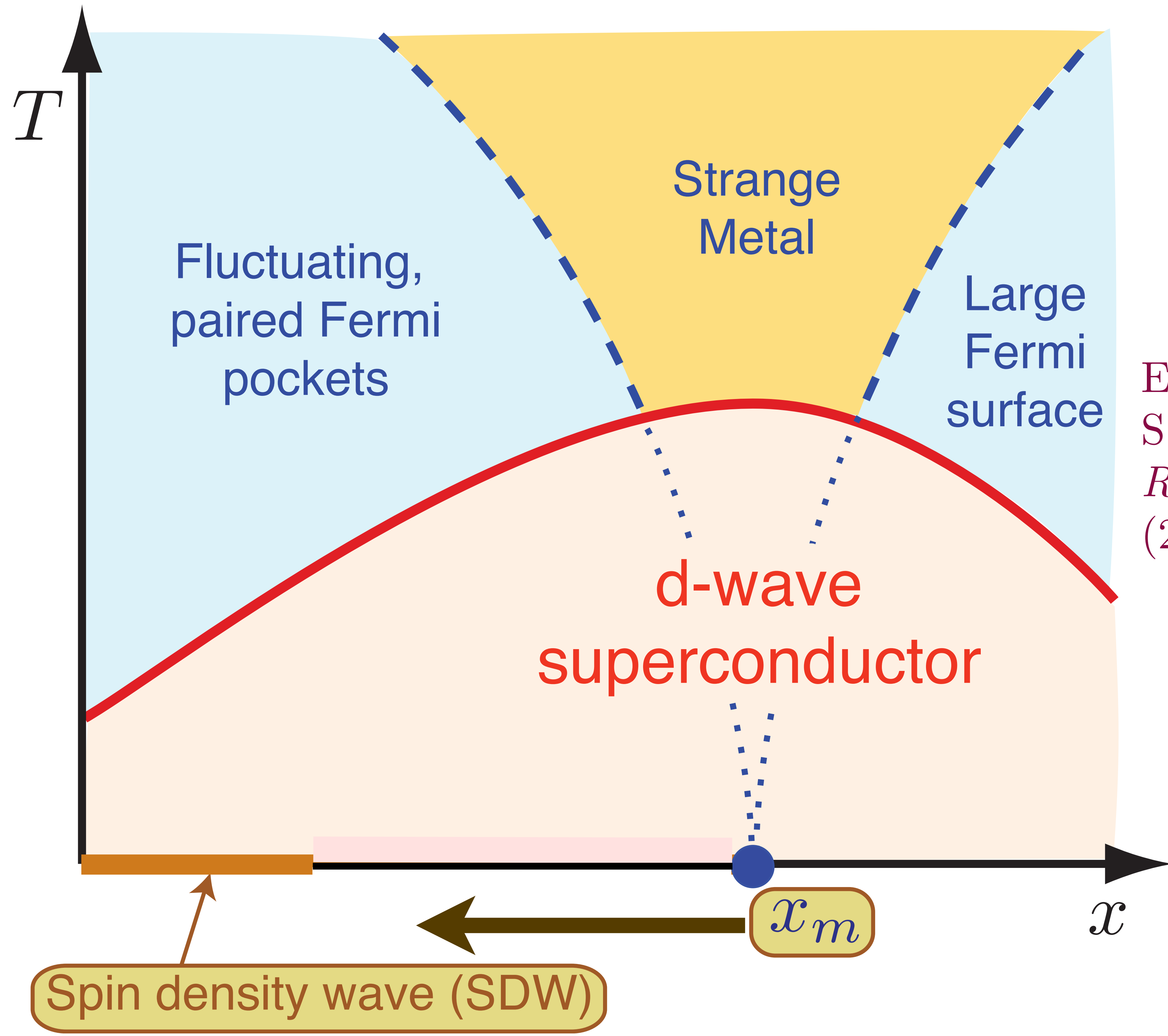
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

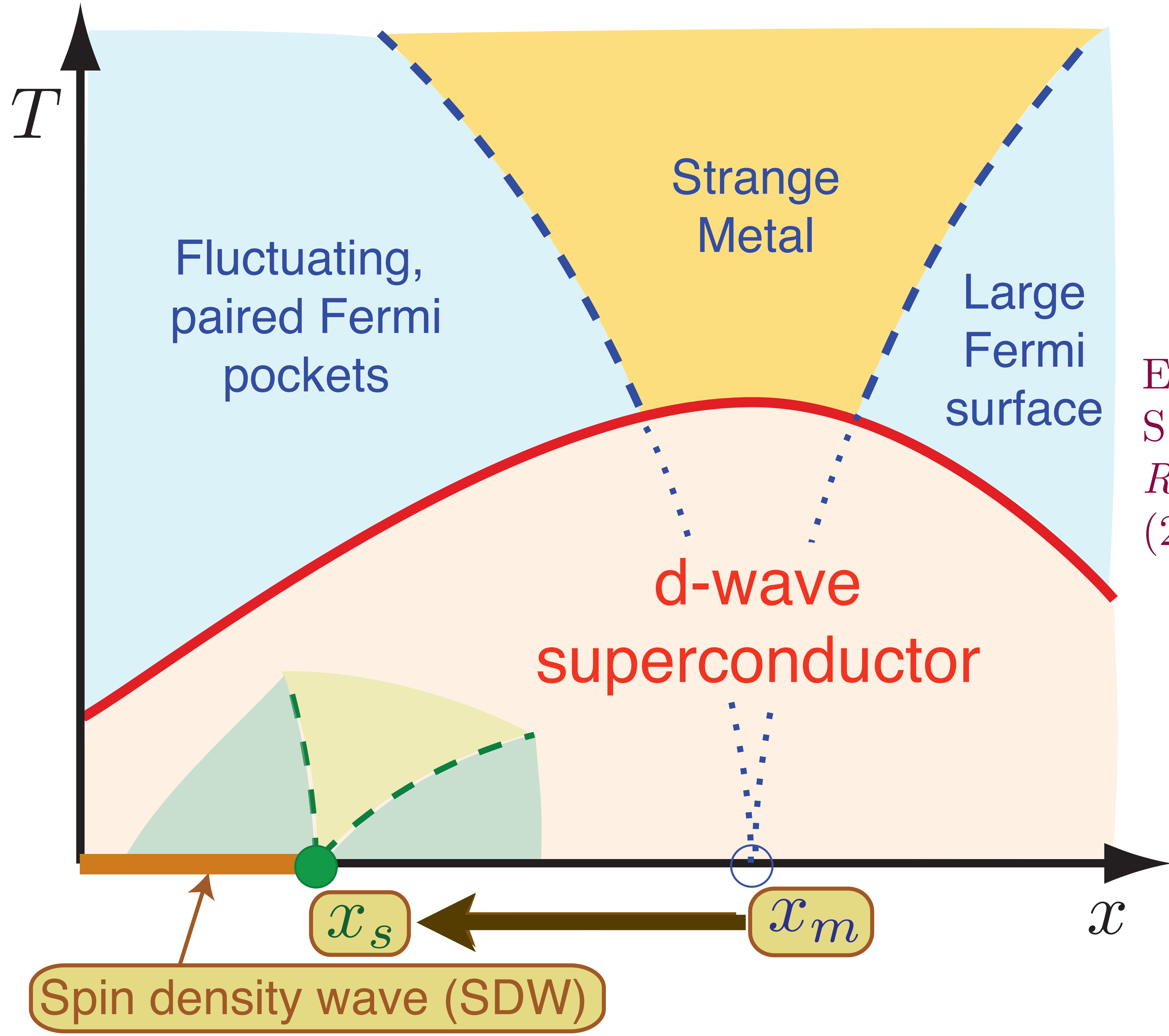
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Questions

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

- *If so, why is there no antiferromagnetism in the hole-doped cuprates near the point where the superconductivity is strongest ?*

- *What is the physics of the strange metal ?*

Questions and answers

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

  Yes

- *If so, why is there no antiferromagnetism in the hole-doped cuprates near the point where the superconductivity is strongest ?*

- *What is the physics of the strange metal ?*

Questions and answers

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

    Yes

- *If so, why is there no antiferromagnetism in the hole-doped cuprates near the point where the superconductivity is strongest ?*

    Competition between antiferromagnetism and superconductivity has shifted the antiferromagnetic quantum-critical point (QCP), and shrunk the region of antiferromagnetism. This QCP shift is largest in the cuprates

- *What is the physics of the strange metal ?*

Questions and answers

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

  Yes

- *If so, why is there no antiferromagnetism in the hole-doped cuprates near the point where the superconductivity is strongest ?*

  Competition between antiferromagnetism and superconductivity has shifted the antiferromagnetic quantum-critical point (QCP), and shrunk the region of antiferromagnetism. This QCP shift is largest in the cuprates

- *What is the physics of the strange metal ?*

  Proposal: strongly-coupled quantum criticality of Fermi surface change in a metal

Possible exotic intermediate phases

Fermi pockets without spin density wave order

Transform electrons to a
"rotating reference frame",
quantizing spins in the direction of the
local antiferromagnetic order

Possible exotic intermediate phases

Fermi pockets without spin density wave order

Transform electrons to a
"rotating reference frame",
quantizing spins in the direction of the
local antiferromagnetic order

This is facilitated by writing the
vector antiferromagnetic order parameter $\vec{\varphi}$
in terms of a bosonic spinor z_α,
with $\alpha = \uparrow, \downarrow$ and

$$\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta.$$

Possible exotic intermediate phases

Fermi pockets without spin density wave order

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

Possible exotic intermediate phases

Fermi pockets without spin density wave order

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

Spinless fermions

Possible exotic intermediate phases

Fermi pockets without spin density wave order

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \bullet \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$\mathrm{SU}(2)_{\mathrm{spin}}$

Possible exotic intermediate phases

Fermi pockets without spin density wave order

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \bullet \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$\mathrm{U(1)_{charge}}$

Possible exotic intermediate phases

Fermi pockets without spin density wave order

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \bullet \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$U \times U^{-1}$

$\mathrm{SU(2)}_{\mathrm{s;gauge}}$

Possible exotic intermediate phases

Fermi pockets without spin density wave order

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z^*_\downarrow \\ z_\downarrow & z^*_\uparrow \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

The Hubbard model can be written as a lattice gauge theory with a

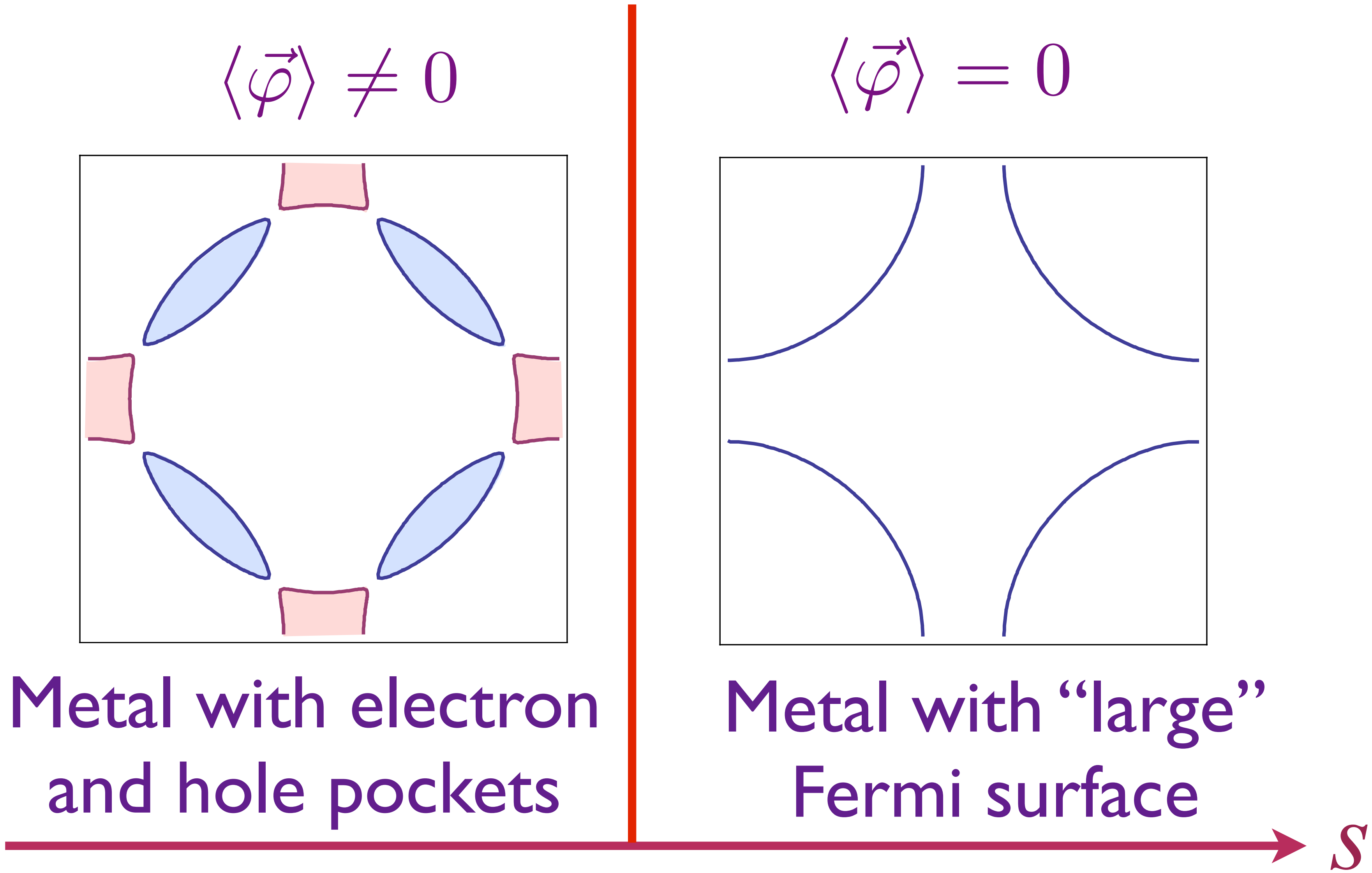
$$SU(2)_{s;g} \times SU(2)_{\rm spin} \times U(1)_{\rm charge}$$

invariance.

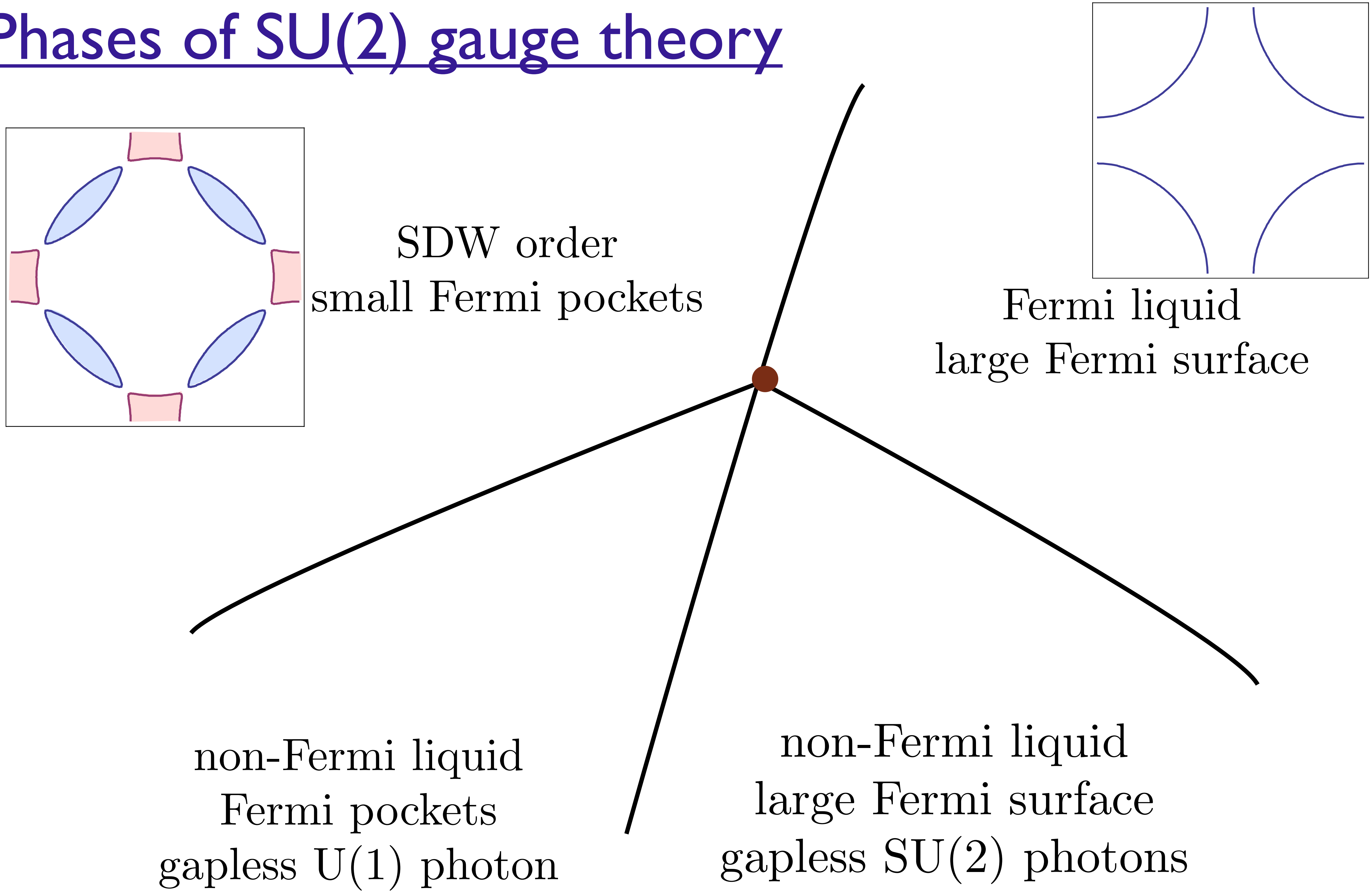
The $SU(2)_{s;g}$ is a gauge invariance, while $SU(2)_{\rm spin} \times U(1)_{\rm charge}$ is a global symmetry

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Conventional phases considered so far

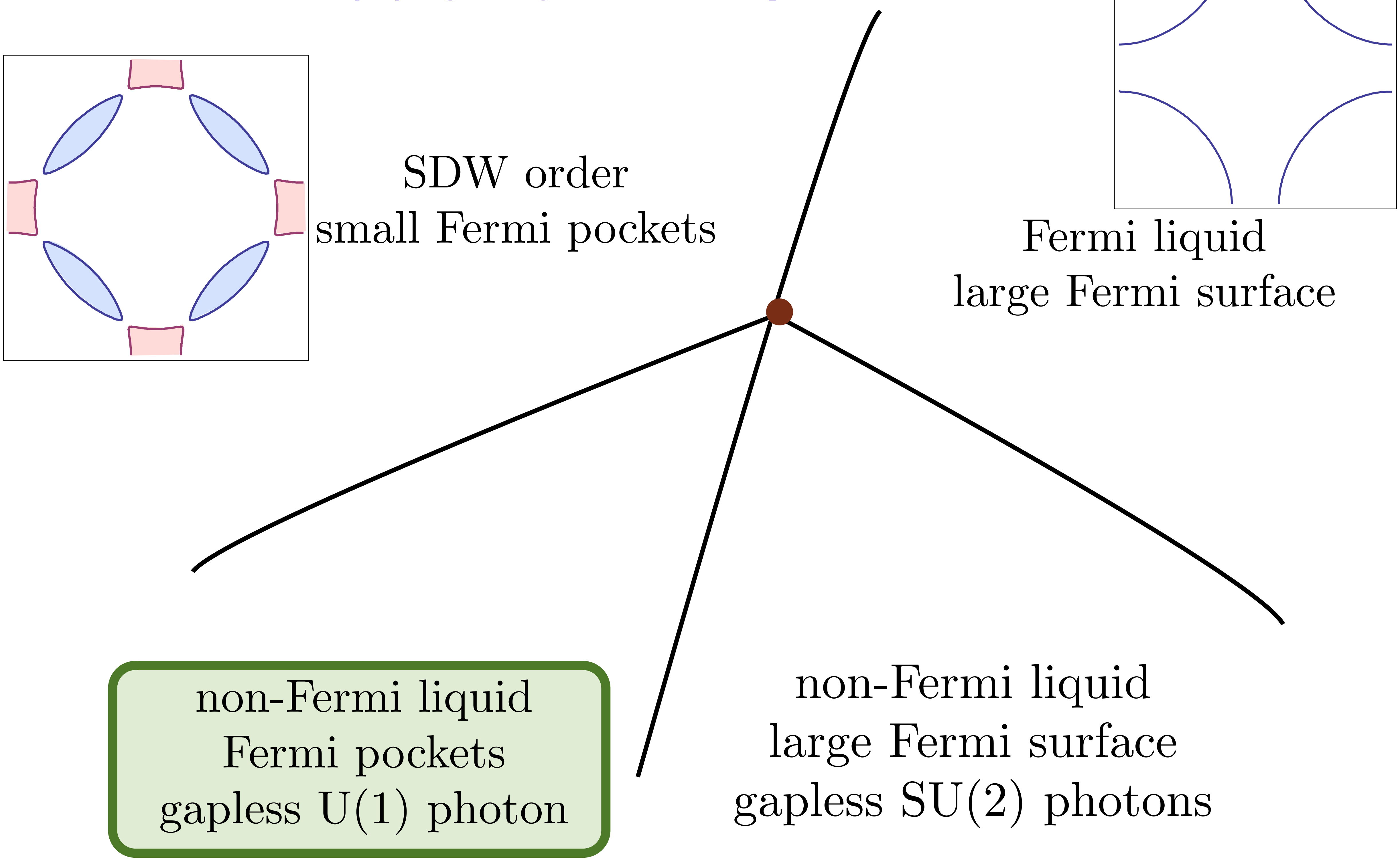


Phases of SU(2) gauge theory



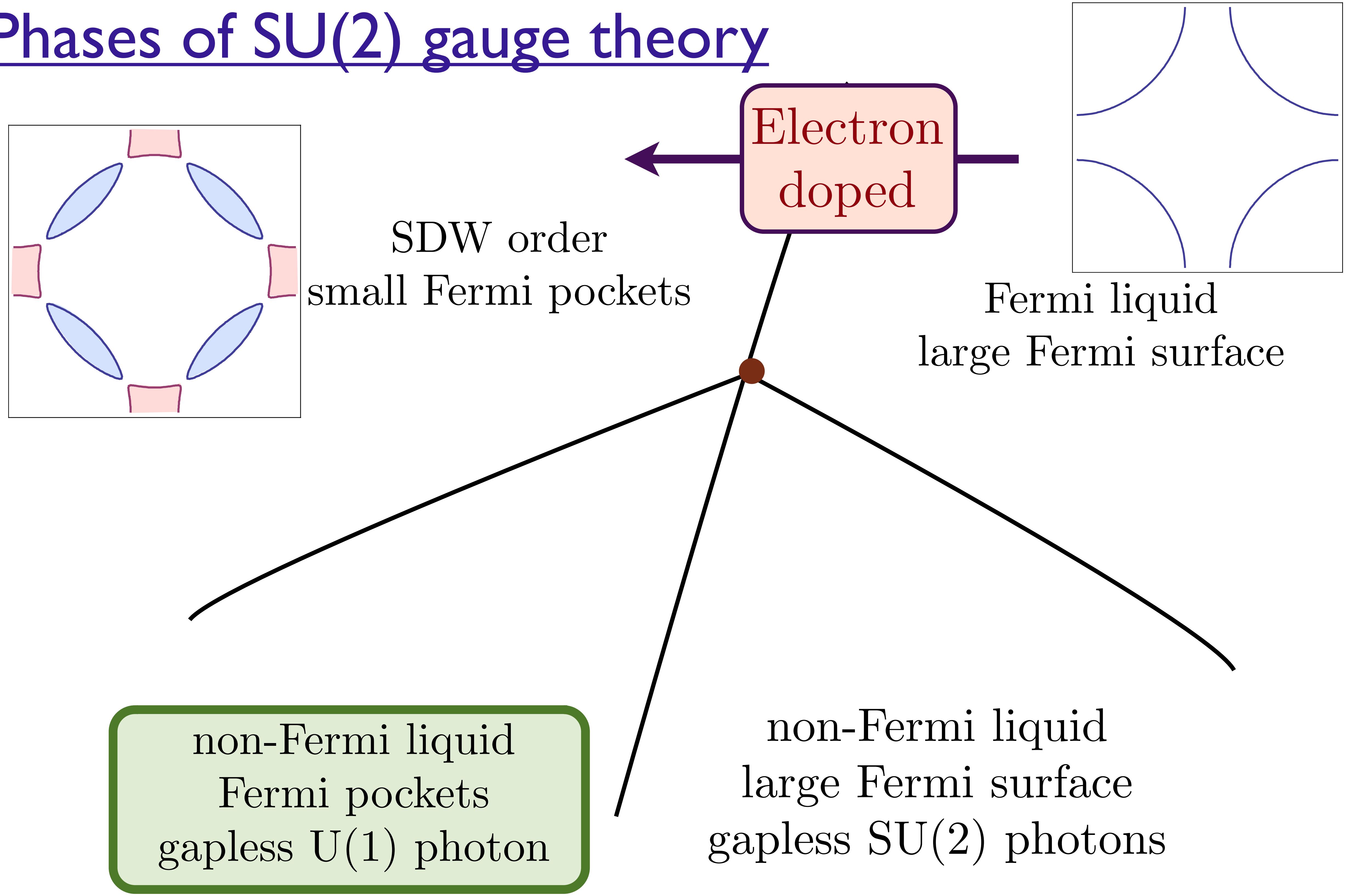
S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Phases of SU(2) gauge theory



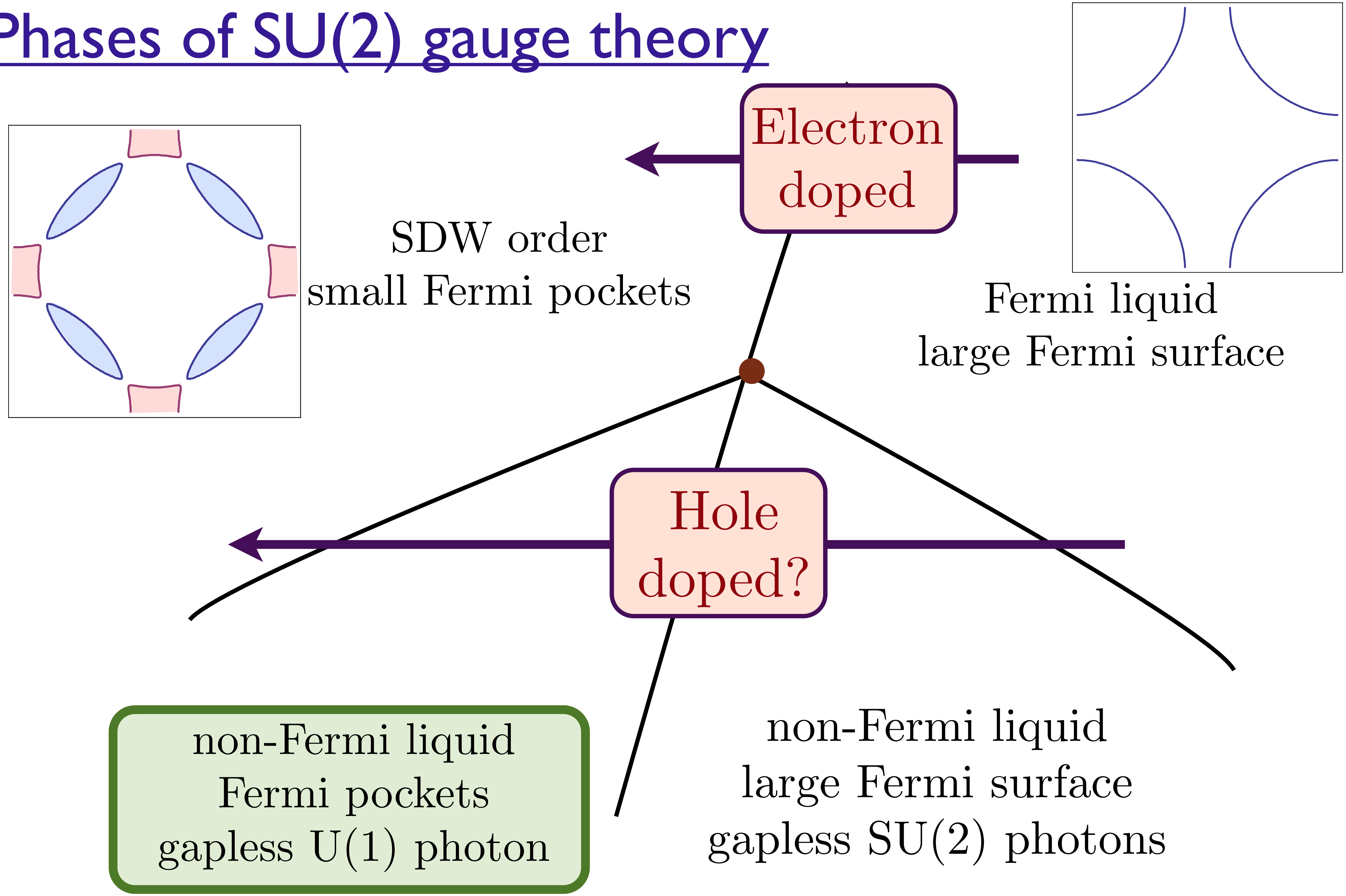
S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Phases of SU(2) gauge theory



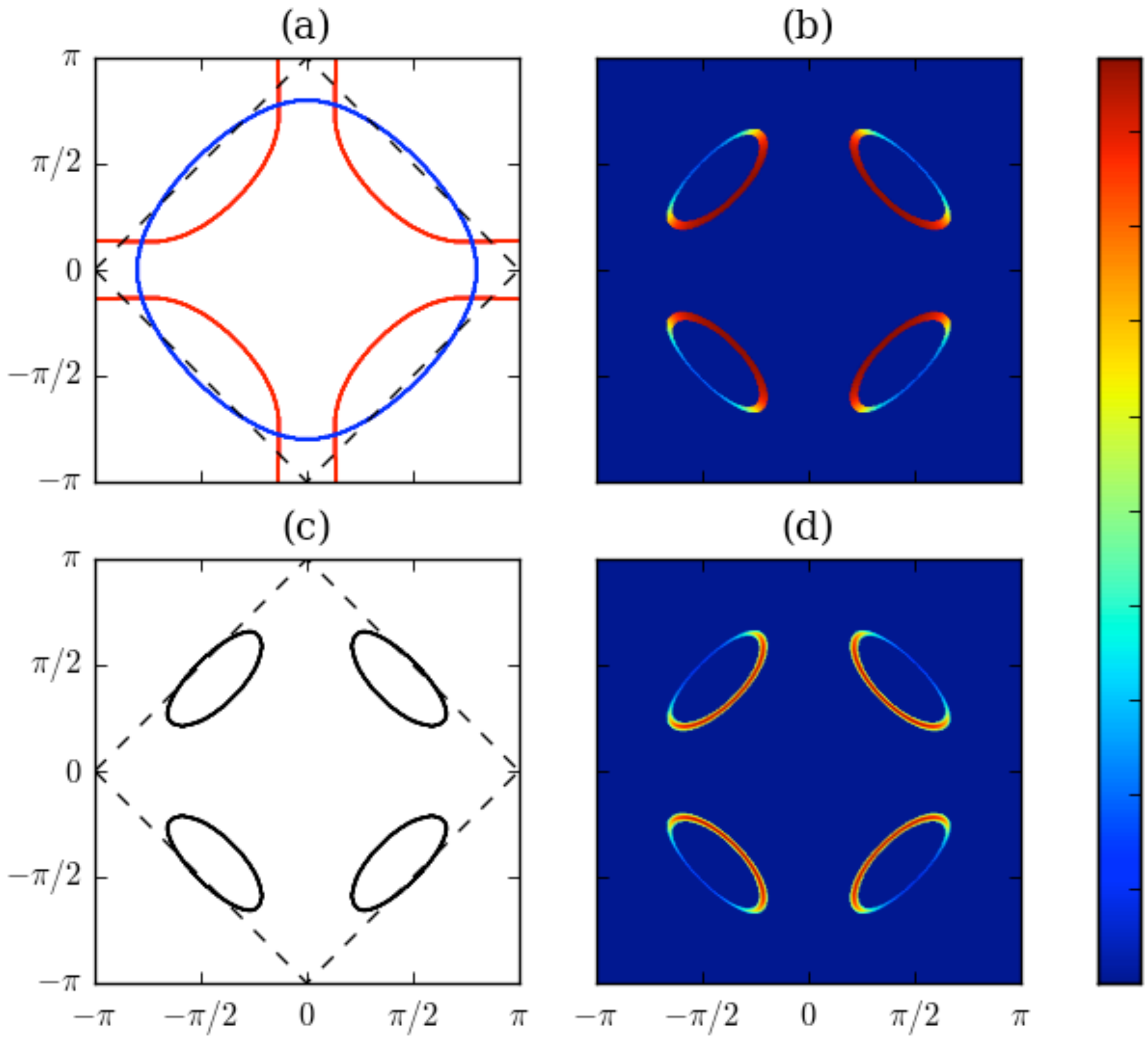
S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Phases of SU(2) gauge theory



S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Exotic non-Fermi liquid has Fermi pockets without long-range antiferromagnetism, along with emergent gauge excitations



Y. Qi and S. Sachdev, *Physical Review B* **81**, 115129 (2010)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, *Physical Review B* **75** , 235122 (2007)

R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, *Nature Physics* **4**, 28 (2008)