Quantum Criticality

S. Sachdev and B. Keimer,
Physics Today, February 2011

Talk online: sachdev.physics.harvard.edu

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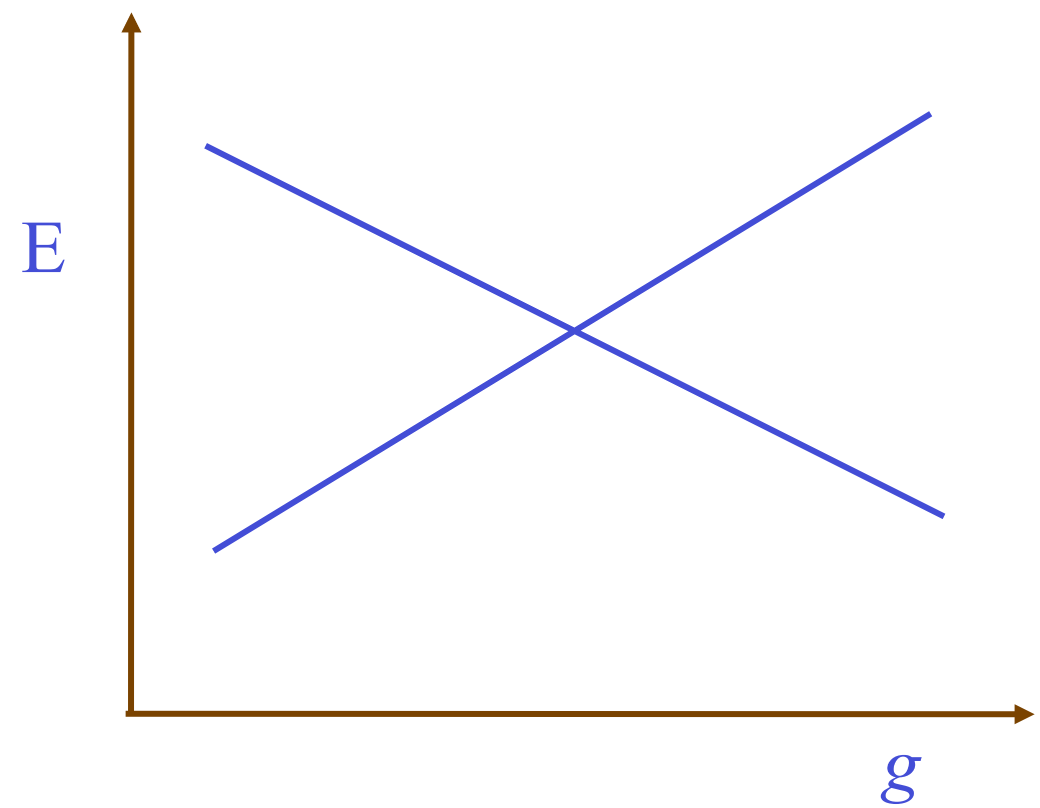
What is a quantum phase transition ?

Non-analyticity in ground state properties as a function of some control parameter g

What is a quantum phase transition ?

Non-analyticity in ground state properties as a function of some control parameter *g*

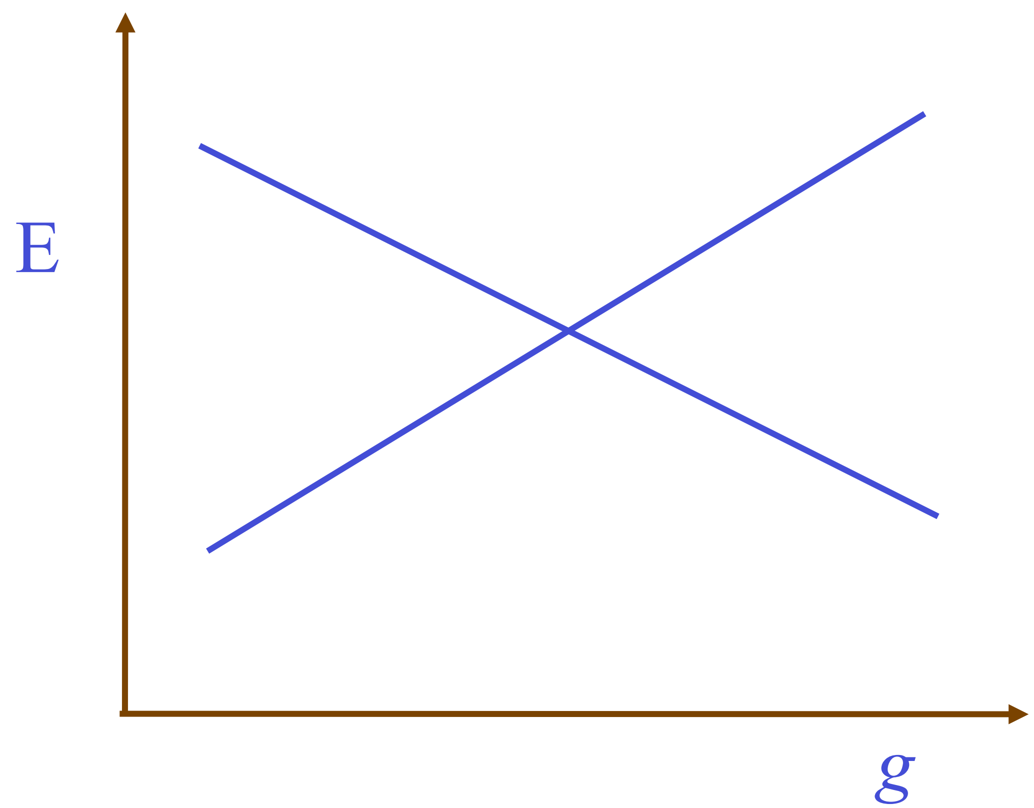


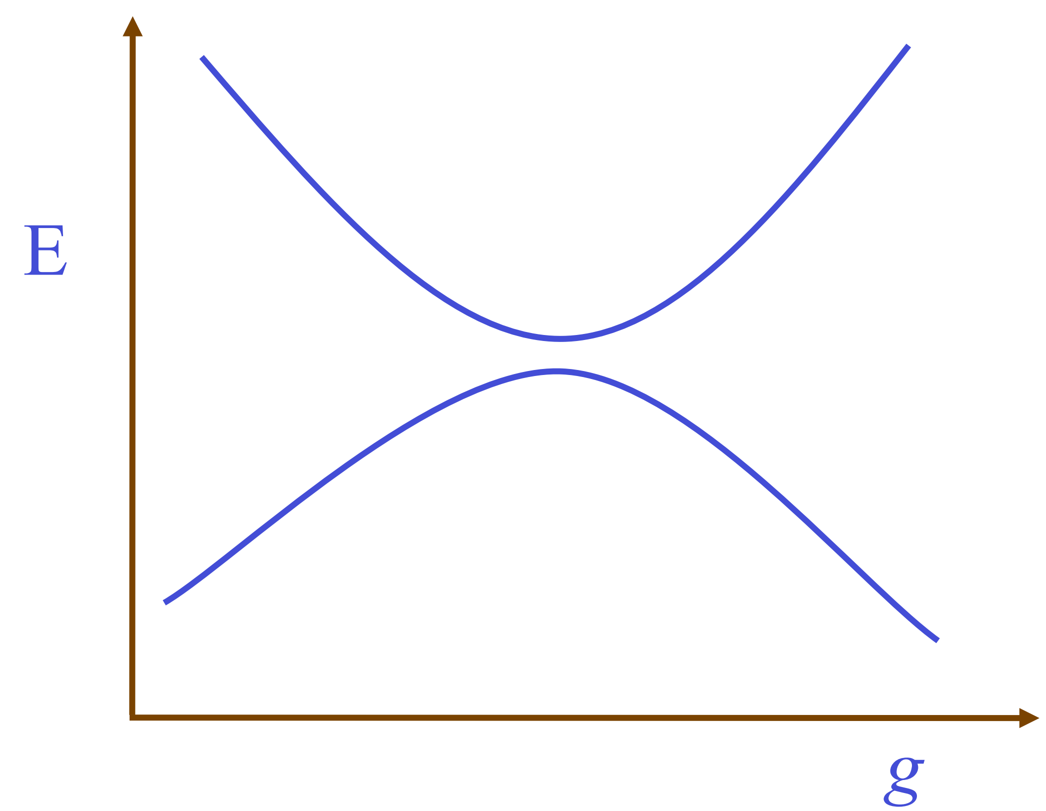
True level crossing:

Usually a *first*-order transition

What is a quantum phase transition ?

Non-analyticity in ground state properties as a function of some control parameter g



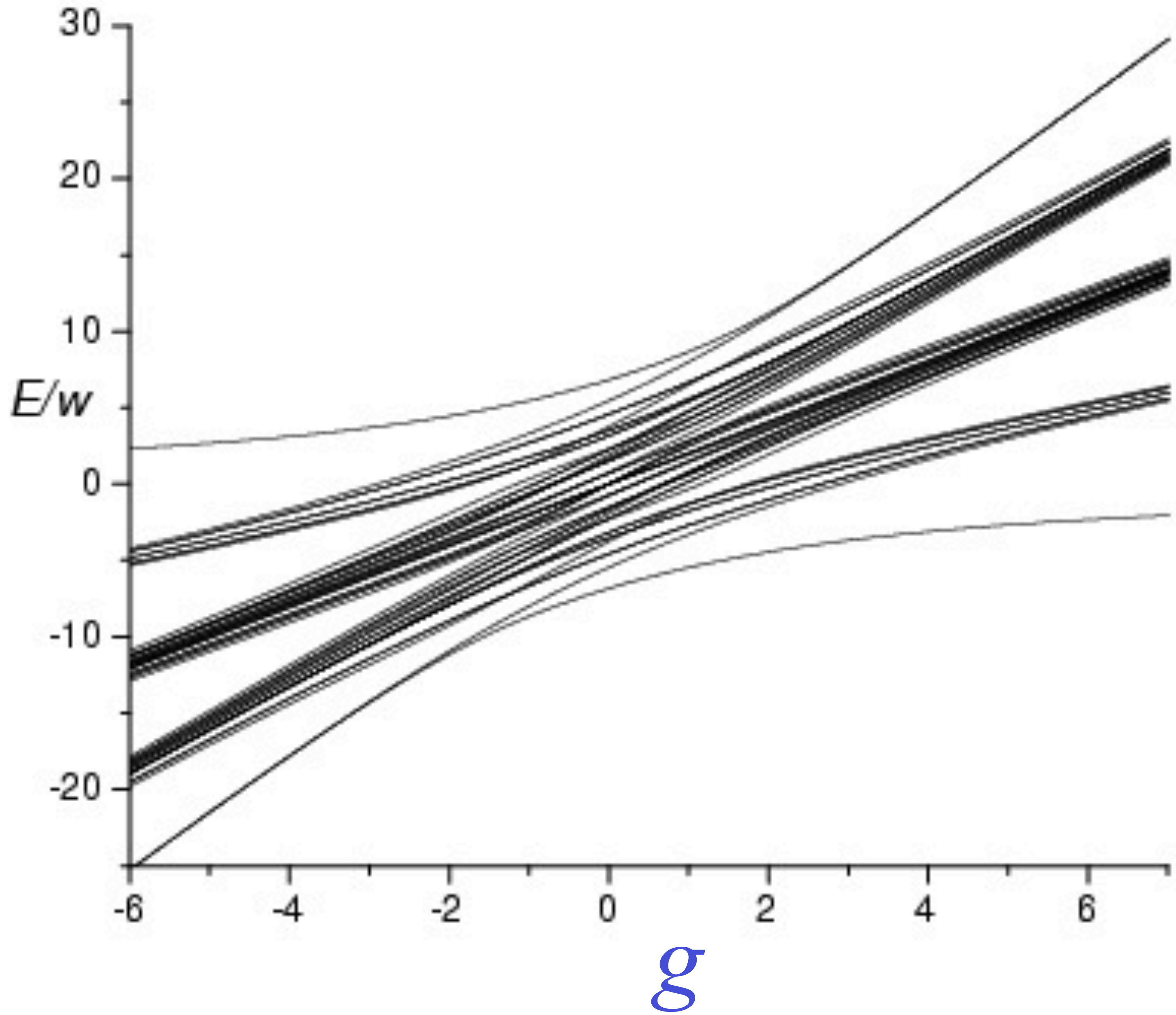


True level crossing:

Usually a *first*-order transition

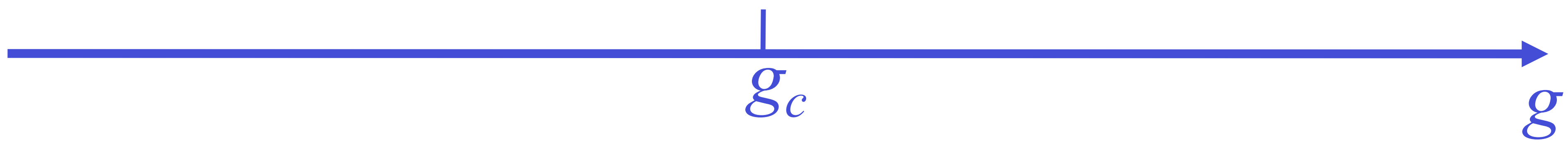
Avoided level crossing which becomes sharp in the infinite volume limit:

second-order transition



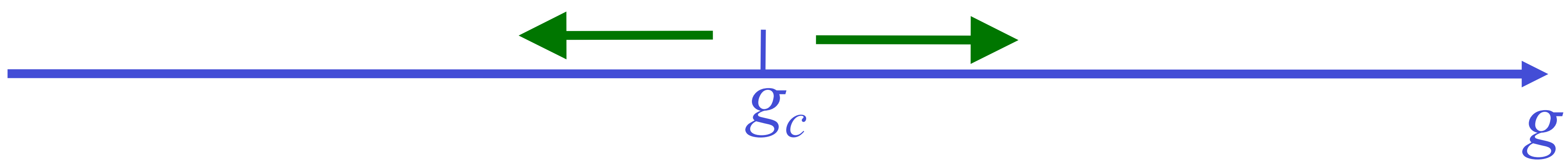
Many levels are important near a second-order quantum phase transition

Why study quantum phase transitions ?



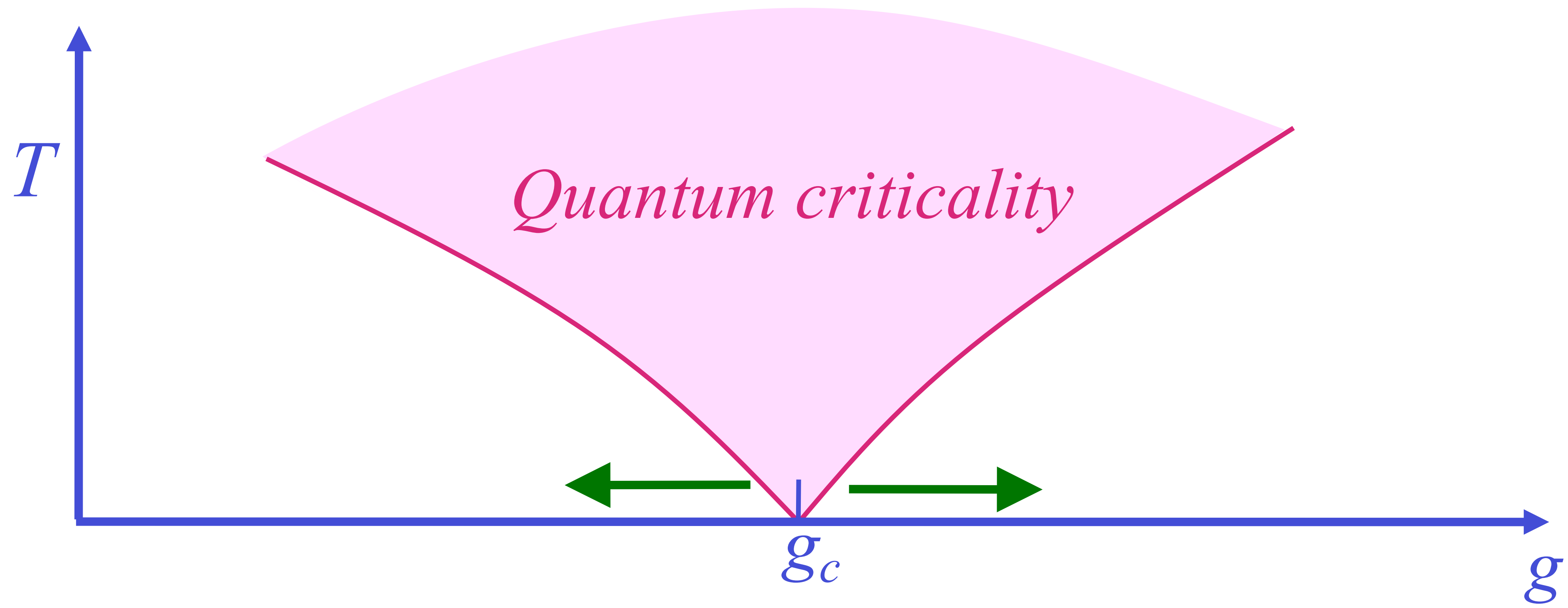
• The ground states at large and small g have wavefunctions which can usually be written as products of wavefunctions of local degrees of freedom *i.e.* they have negligible *quantum entanglement*. The quantum critical state at g_c often has long-range *quantum entanglement* : the "spooky" non-local quantum correlations pointed out by Einstein, Podolsky, and Rosen survive in a macrosopic system at the longest distances.

Why study quantum phase transitions ?



• We are often able to describe the quantum state at g_c by methods drawn from quantum field theory; expansion in g-g_c then allows for a controlled theory in an intermediate coupling regime important for many experimental systems

Why study quantum phase transitions ?



• The quantum critical point controls properties over a wide regime of "quantum criticality" at non-zero temperatures. I will argue that this regime is the key to understanding the physical properties of a variety of modern electronic materials.

Outline

1. The quantum Ising chain

   A. The magnetic insulator $CoNb_2O_6$

   B. Ultracold Rb atoms in an optical lattice

2. Nonzero temperatures and quantum criticality

   Antiferromagnetic insulators

3. Higher temperature superconductors and "strange metals"

   Quantum criticality of fermions and Fermi surfaces

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Degrees of freedom: $j = 1 \ldots N$ qubits, N "large"

$$|\uparrow\rangle_j\,,\ |\downarrow\rangle_j$$

or $$|\rightarrow\rangle_j = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_j + |\downarrow\rangle_j\right)\,,\quad |\leftarrow\rangle_j = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_j - |\downarrow\rangle_j\right)$$

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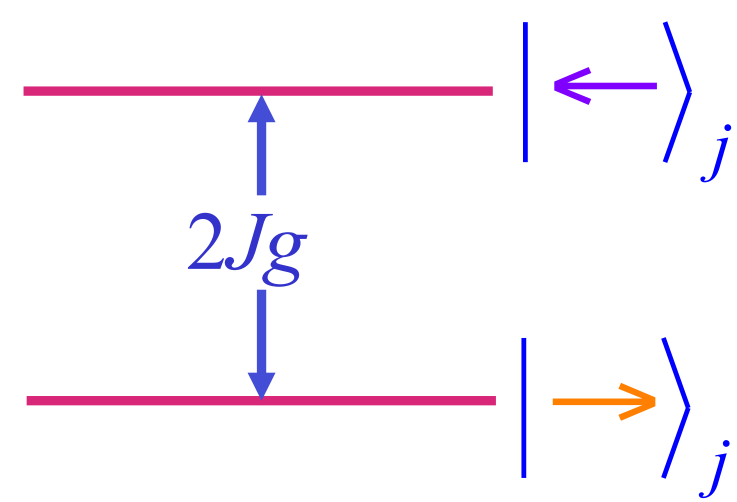
Hamiltonian of decoupled qubits:

$$H_0 = -Jg\sum_j \sigma_j^x$$



Coupling between qubits:

$$H_1 = -J \sum_j \sigma_j^z \sigma_{j+1}^z$$

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$$H_1 = -J\sum_j \sigma_j^z \sigma_{j+1}^z$$

$$\Big(|\rightarrow\rangle_j\langle\leftarrow| + |\leftarrow\rangle_j\langle\rightarrow|\Big)\Big(|\rightarrow\rangle_{j+1}\langle\leftarrow| + |\leftarrow\rangle_{j+1}\langle\rightarrow|\Big)$$

Coupling between qubits:

$$H_1 = -J \sum_j \sigma_j^z \sigma_{j+1}^z$$

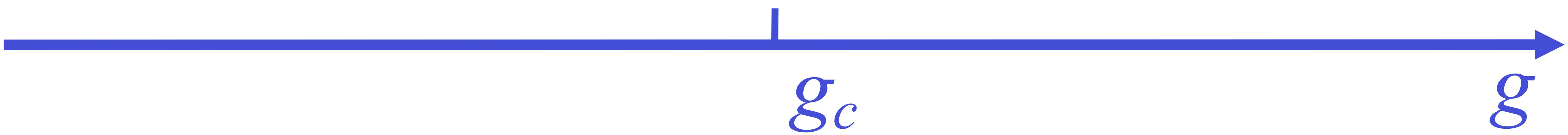
Prefers neighboring qubits

are *either* $|\uparrow\rangle_j |\uparrow\rangle_{j+1}$ *or* $|\downarrow\rangle_j |\downarrow\rangle_{j+1}$

(not entangled)

Full Hamiltonian

$$H = H_0 + H_1 = -J \sum_j \left(g \sigma_j^x + \sigma_j^z \sigma_{j+1}^z \right)$$



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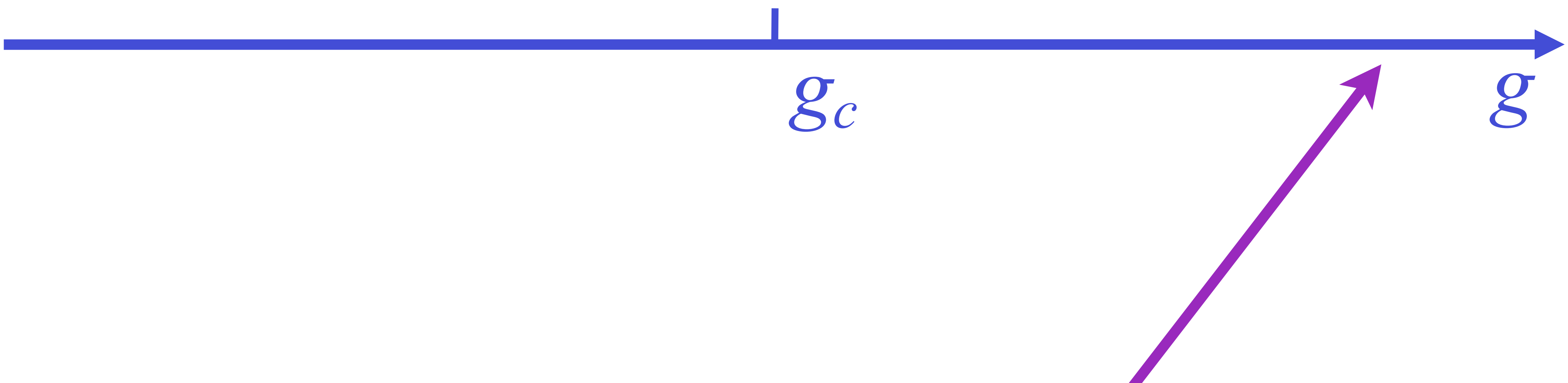

$$|\rightarrow\rangle_1 \; |\rightarrow\rangle_2 \ldots |\rightarrow\rangle_j \ldots |\rightarrow\rangle_{N-1} \; |\rightarrow\rangle_N$$

Product state for large g

Full Hamiltonian

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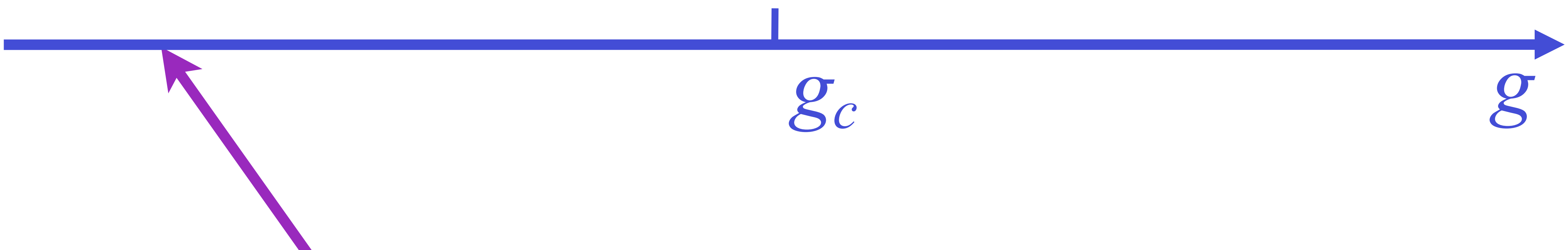


$$|\uparrow\rangle_1 \ |\uparrow\rangle_2 \ \cdots |\uparrow\rangle_j \ \cdots |\uparrow\rangle_{N-1} \ |\uparrow\rangle_N$$

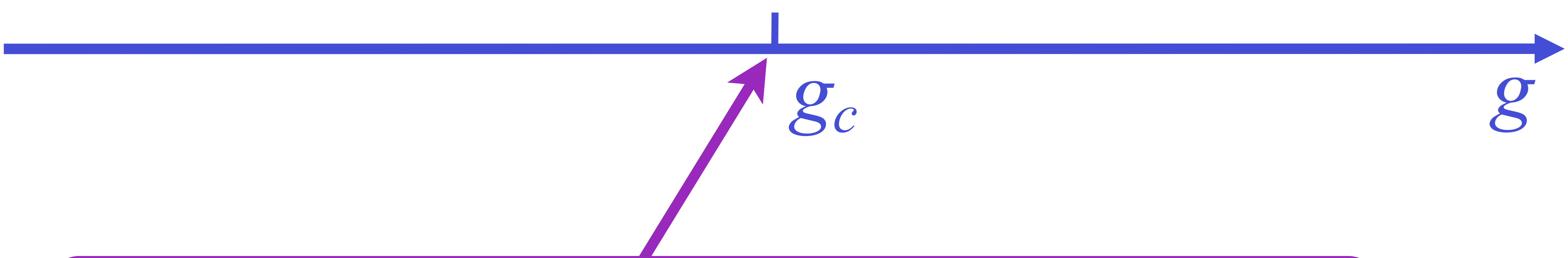
or

$$|\downarrow\rangle_1 \ |\downarrow\rangle_2 \ \cdots |\downarrow\rangle_j \ \cdots |\downarrow\rangle_{N-1} \ |\downarrow\rangle_N$$

Product state for small g

Full Hamiltonian

$$H = H_0 + H_1 = -J \sum_j \left(g\sigma_j^x + \sigma_j^z \sigma_{j+1}^z \right)$$



Entangled state at quantum critical point, involving complicated superposition of 2^N qubit configurations

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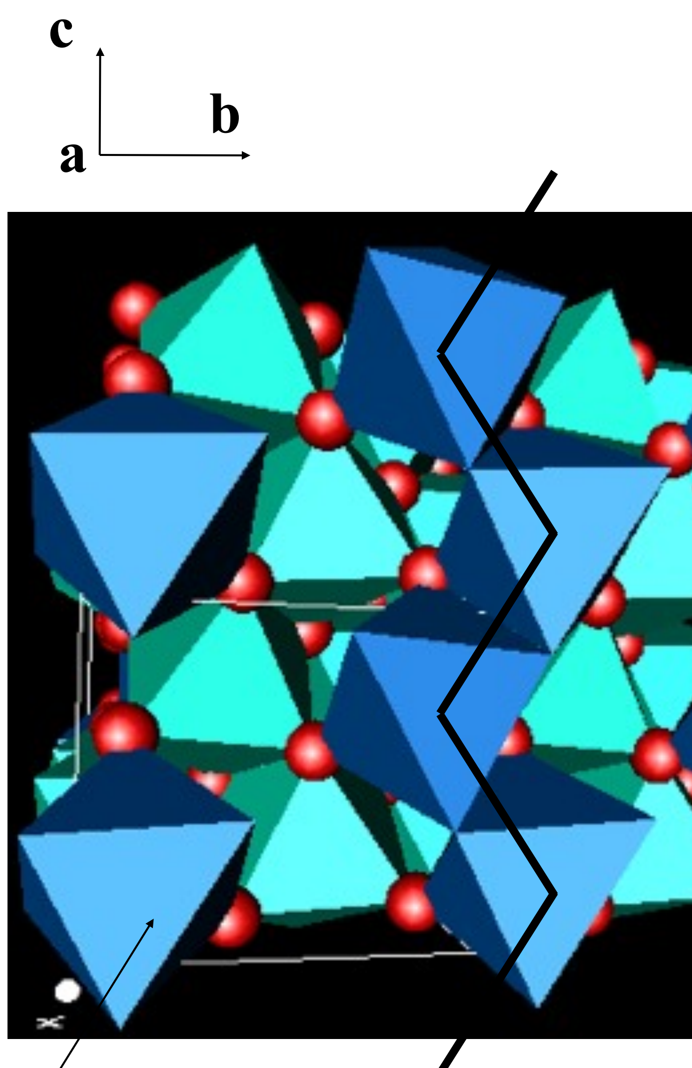
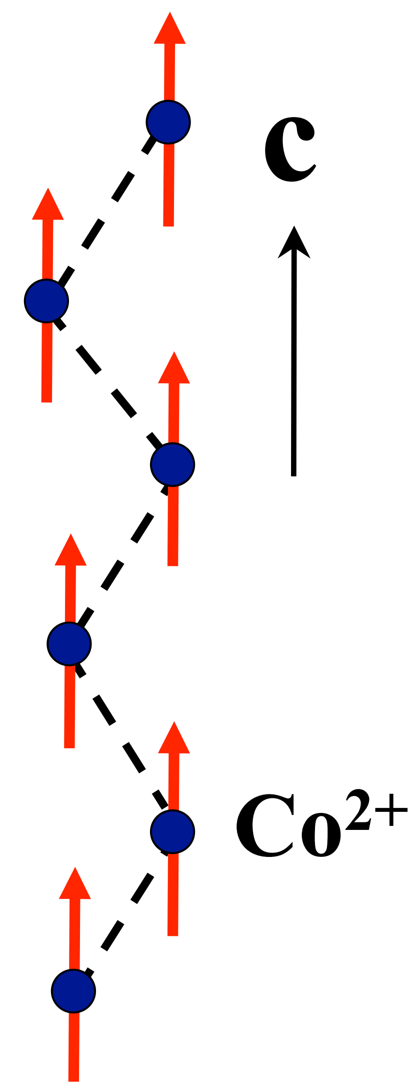
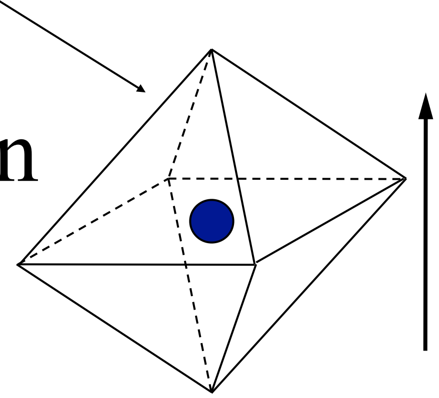
Quasi-1D Ising ferromagnet $CoNb_2O_6$



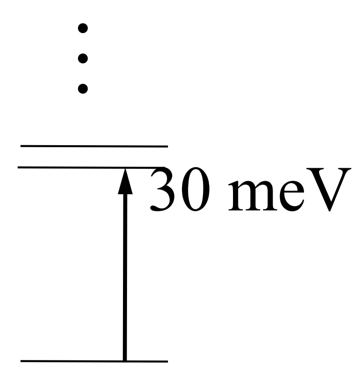
Co^{2+} spin chain along c-axis

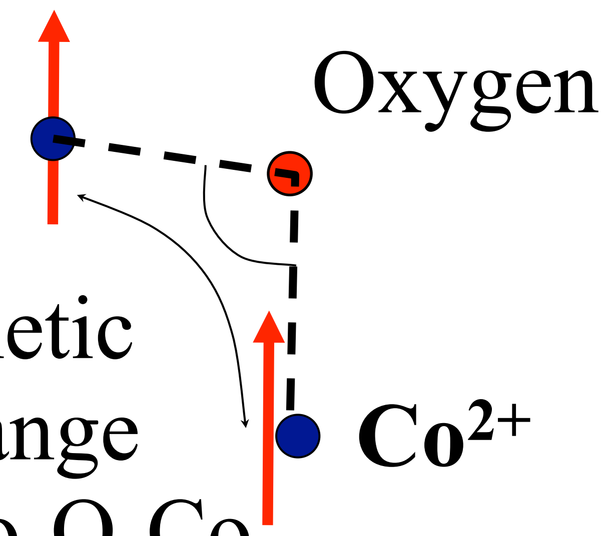


CoO_6 distorted octahedron



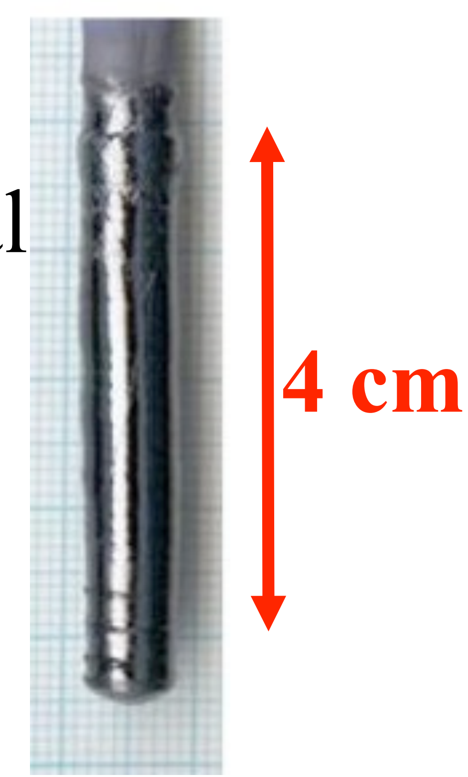
Strong easy-axis (Ising)





Ferromagnetic superexchange
~ 90° bond Co-O-Co
~ 20K ~ 2meV

Single crystal of $CoNb_2O_6$ (Oxford image furnace)



R. Coldea, D. A. Tennant, E. M. Wheeler, E. Wawrzynska, D. Prabhakaran, M. Telling, K. Habicht, P. Smeibidl, and K. Kiefer, Science **327**, 177 (2010).

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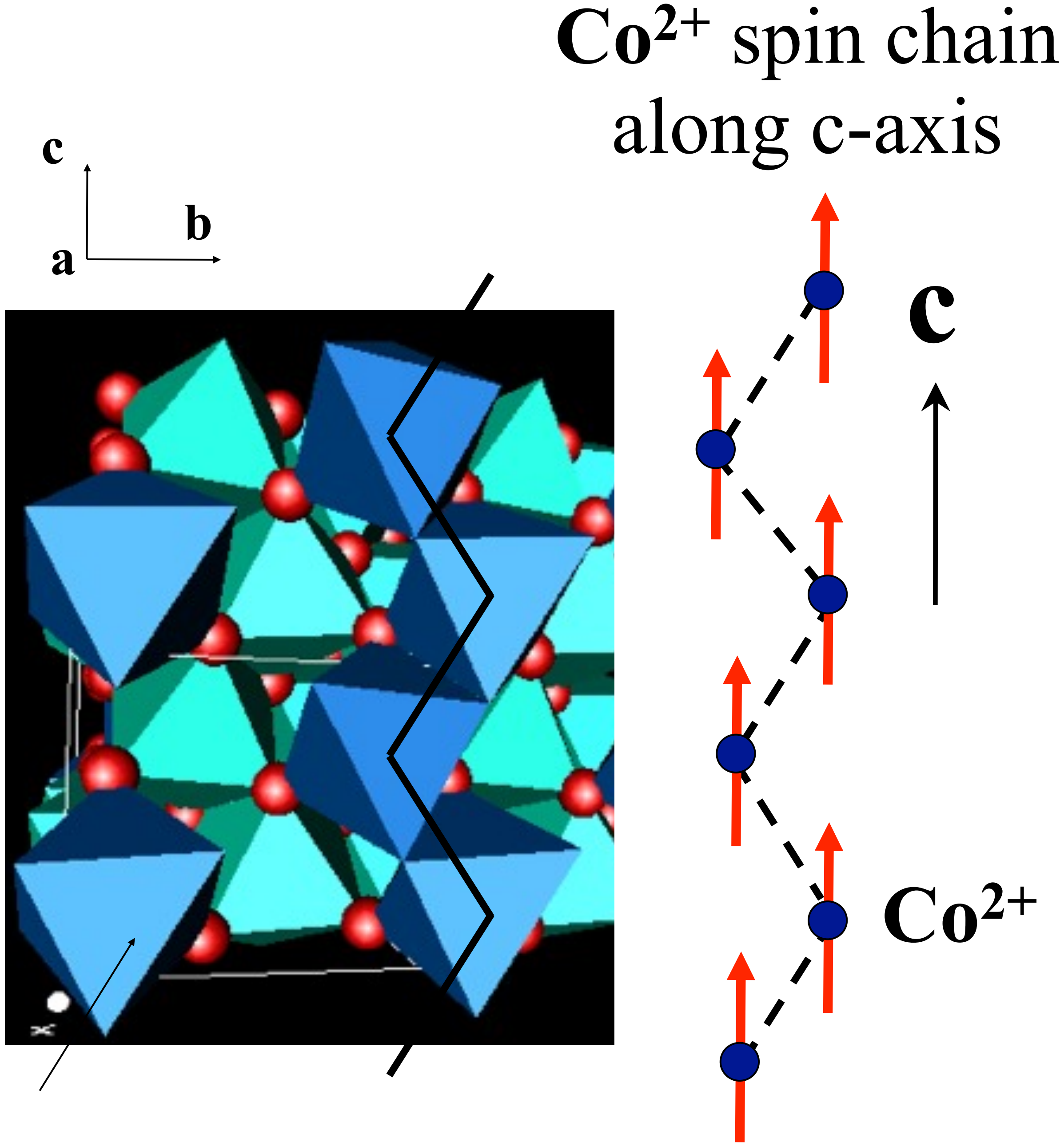

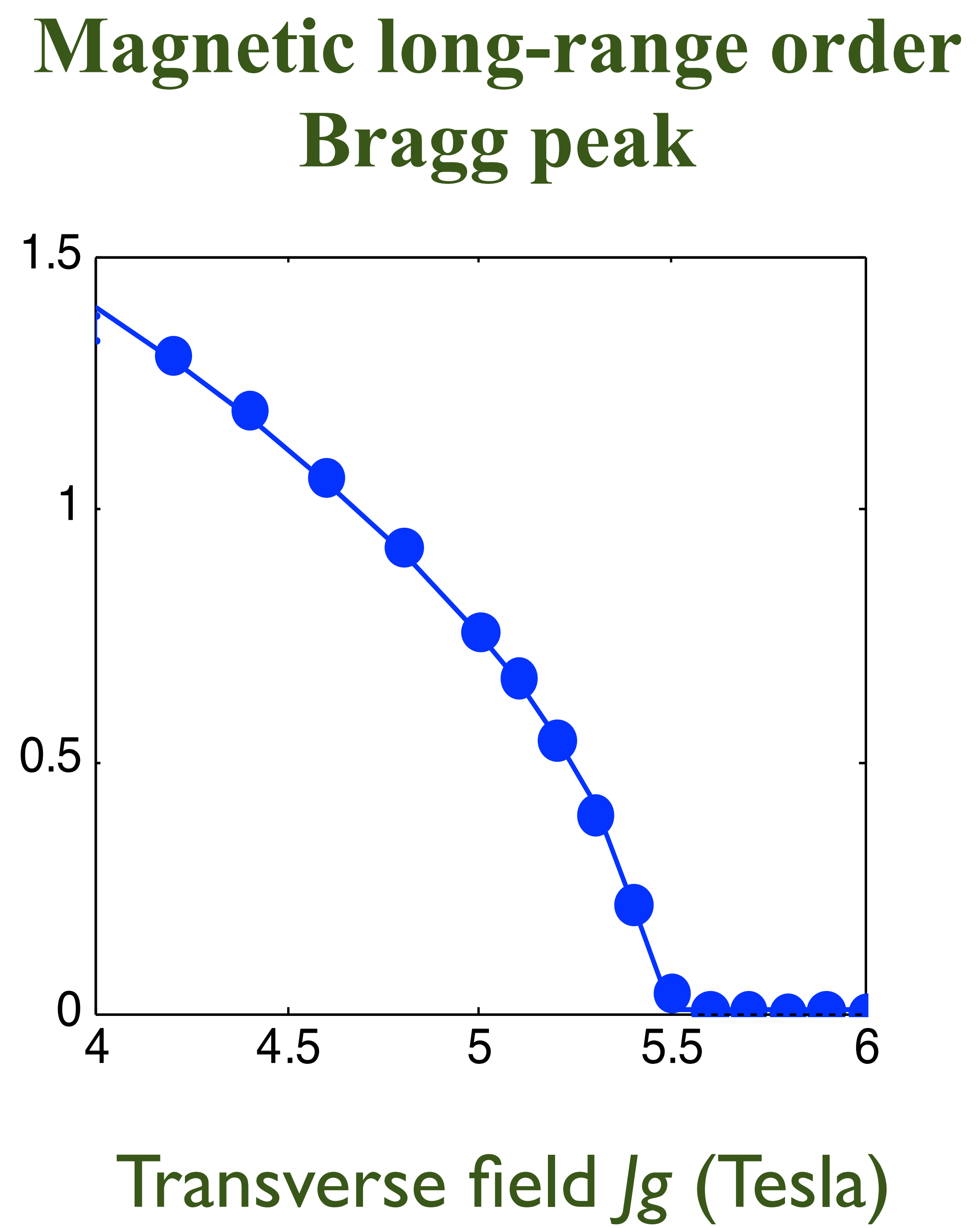

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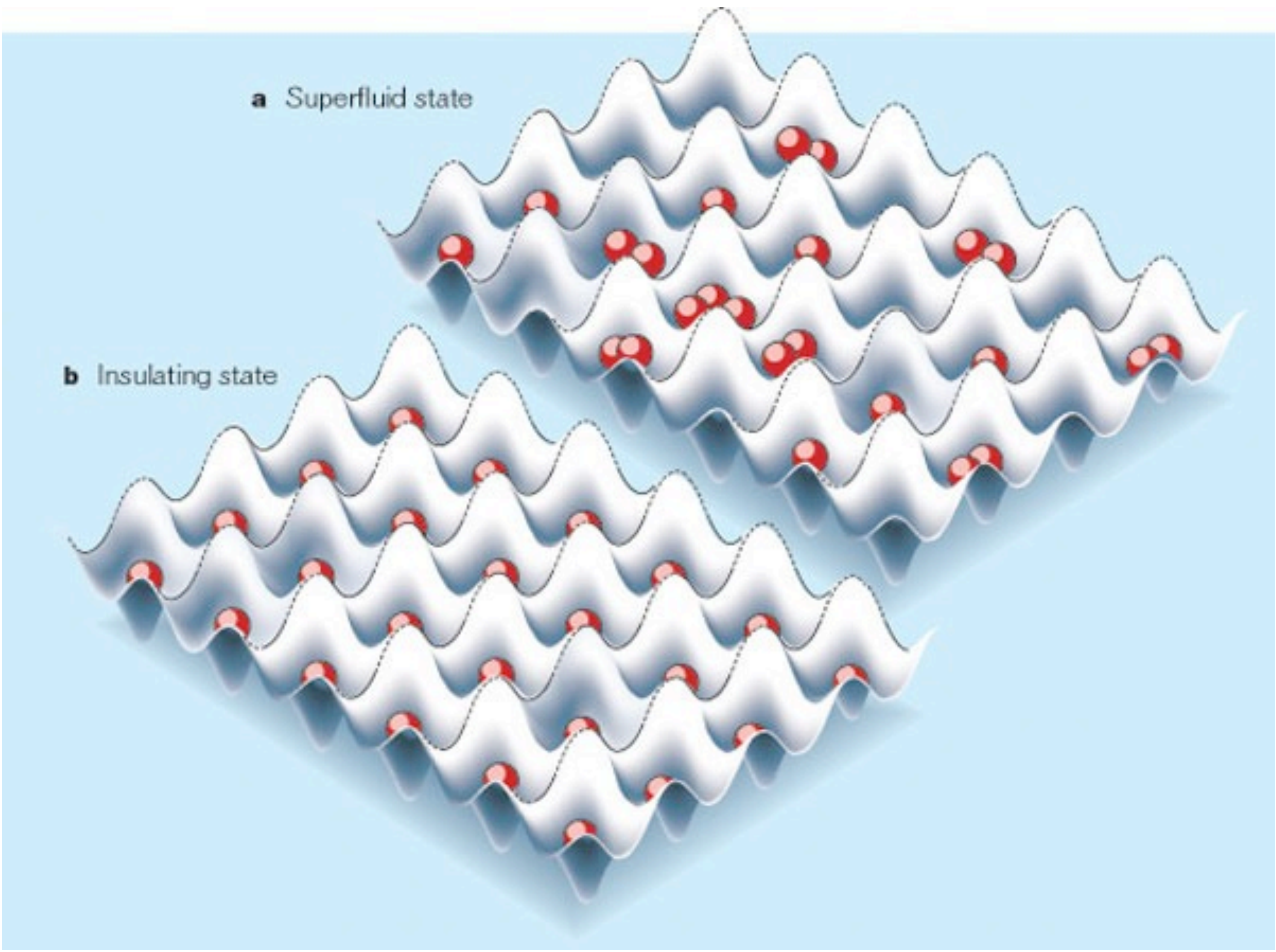
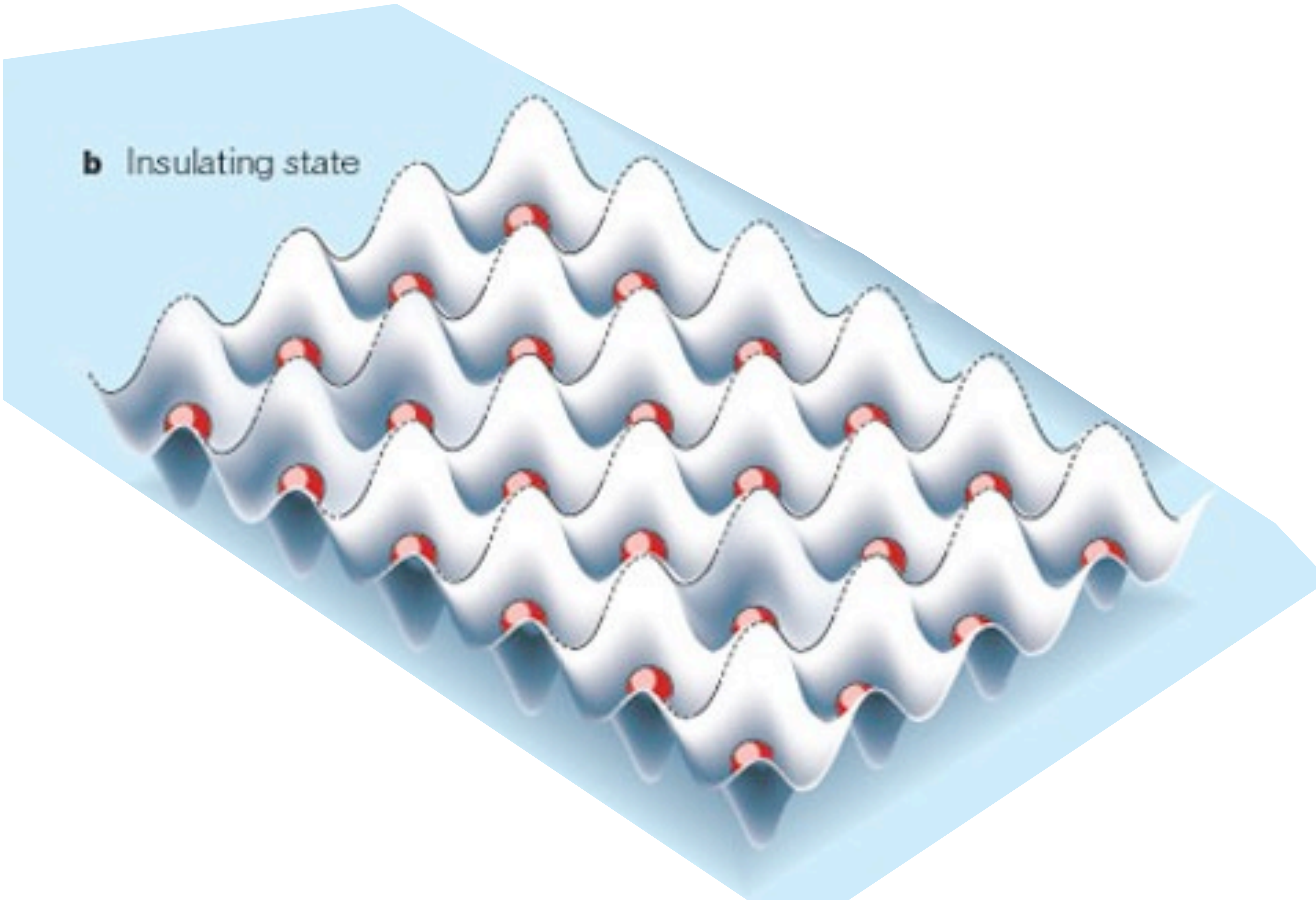
Superfluid-insulator transition of ^{87}Rb atoms in a magnetic trap and an optical lattice potential



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Mott insulator of ^{87}Rb atoms in a magnetic trap and an optical lattice potential



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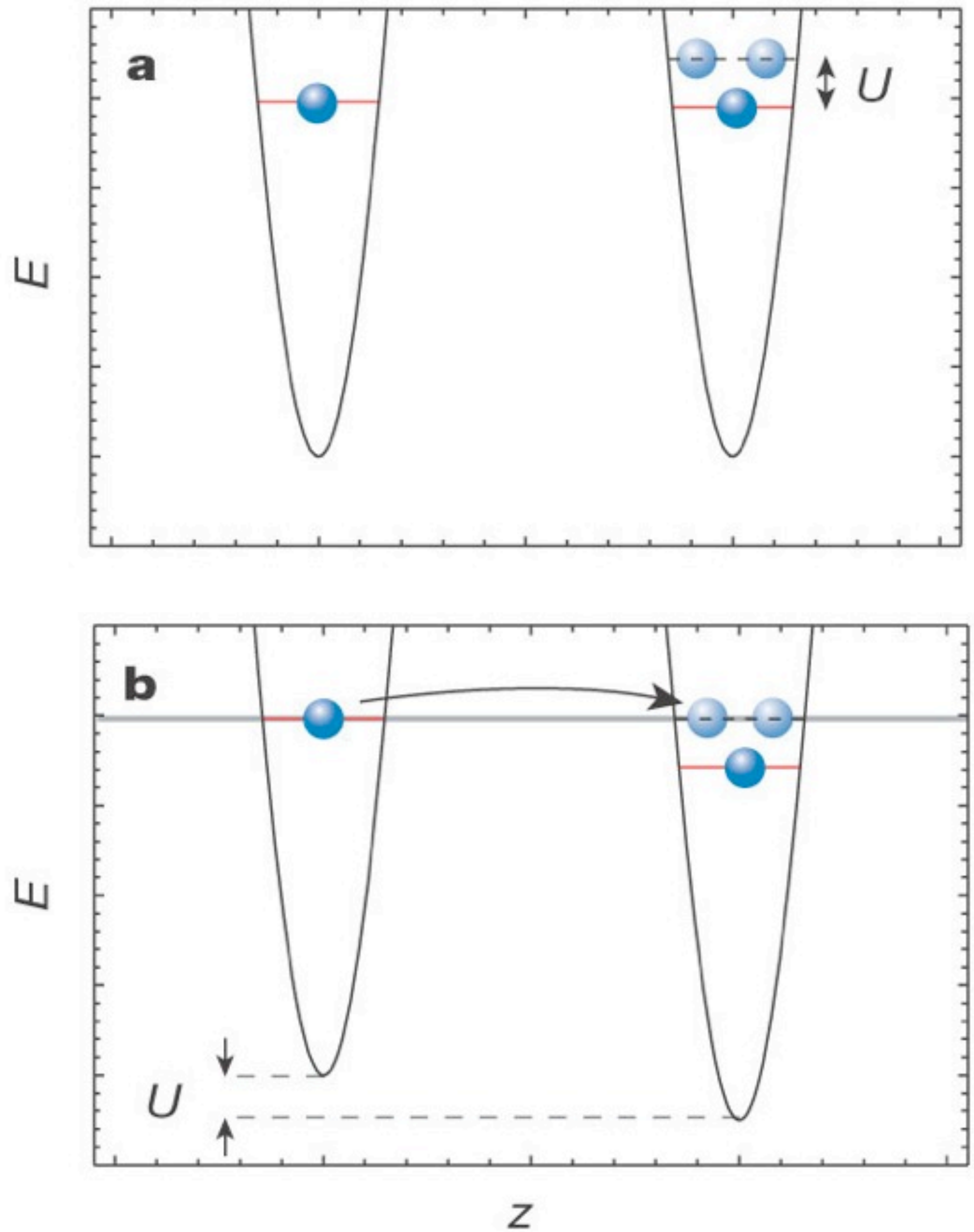
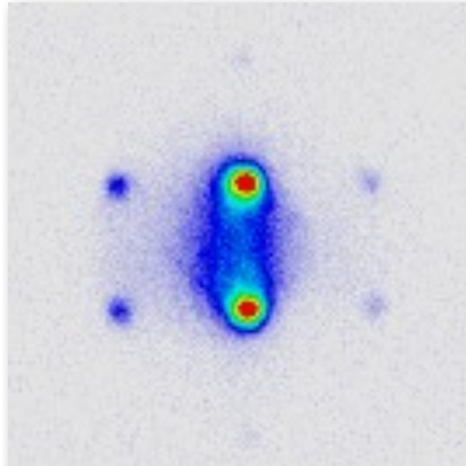
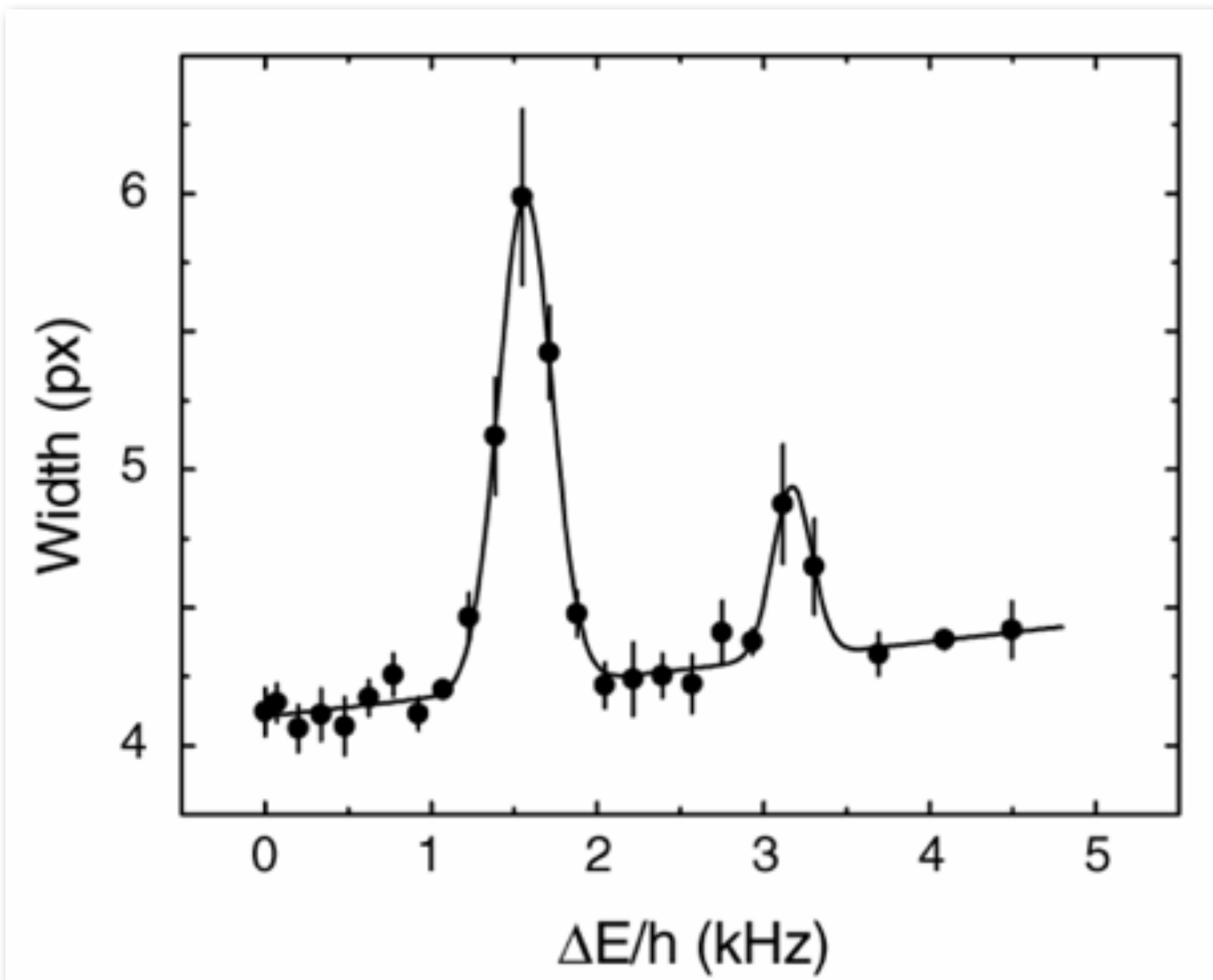
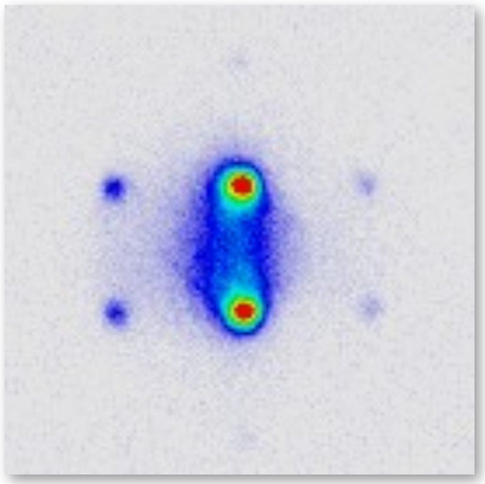
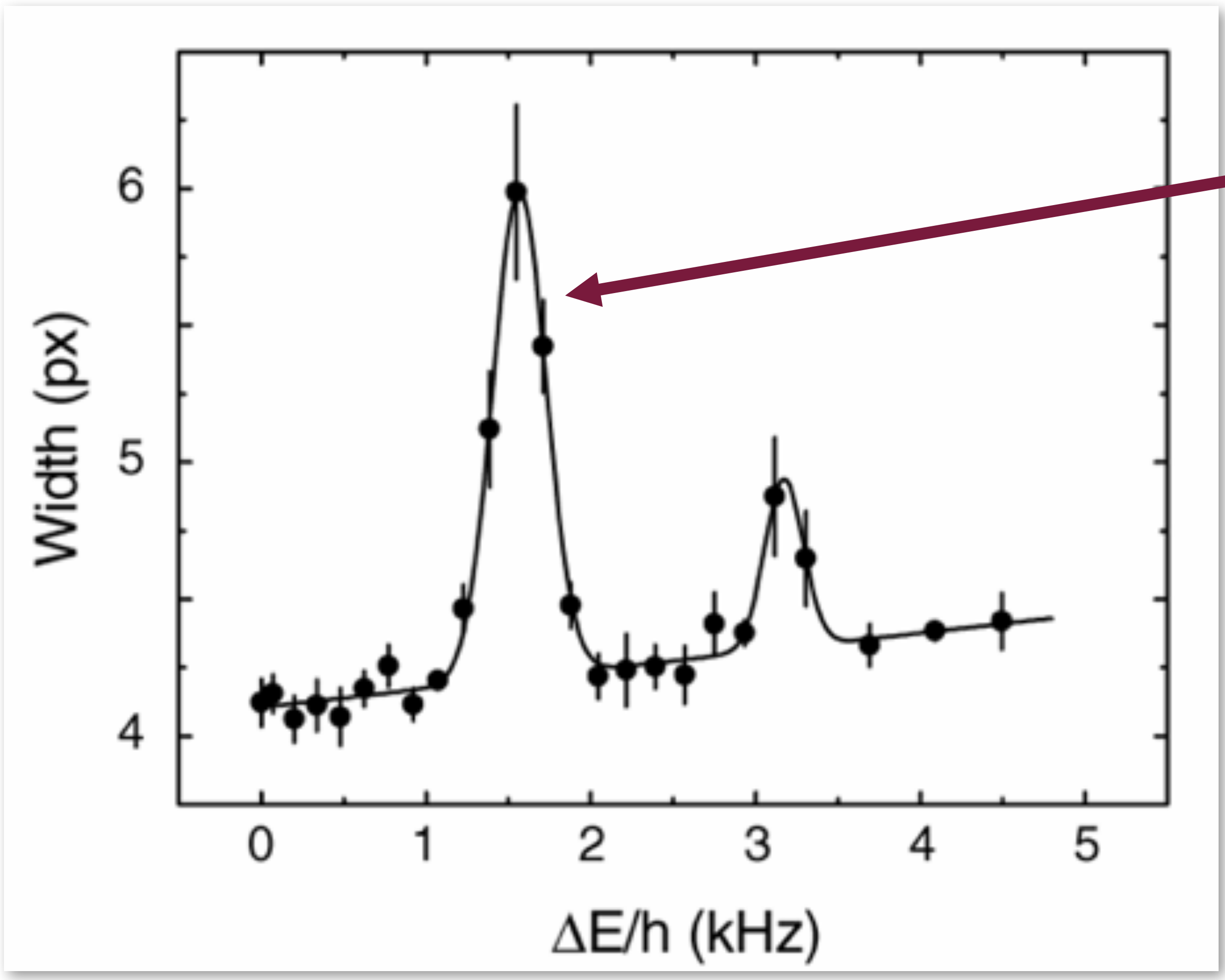
Applying an “electric” field to the Mott insulator



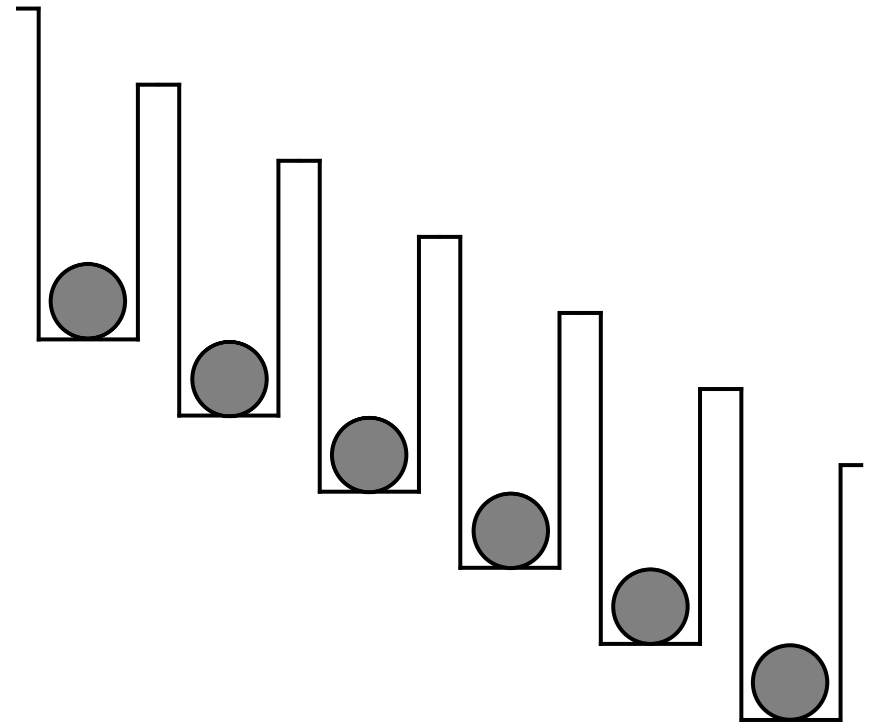



6
5
4
Width (px)
0
1
2
3
4
5
ΔE/h (kHz)





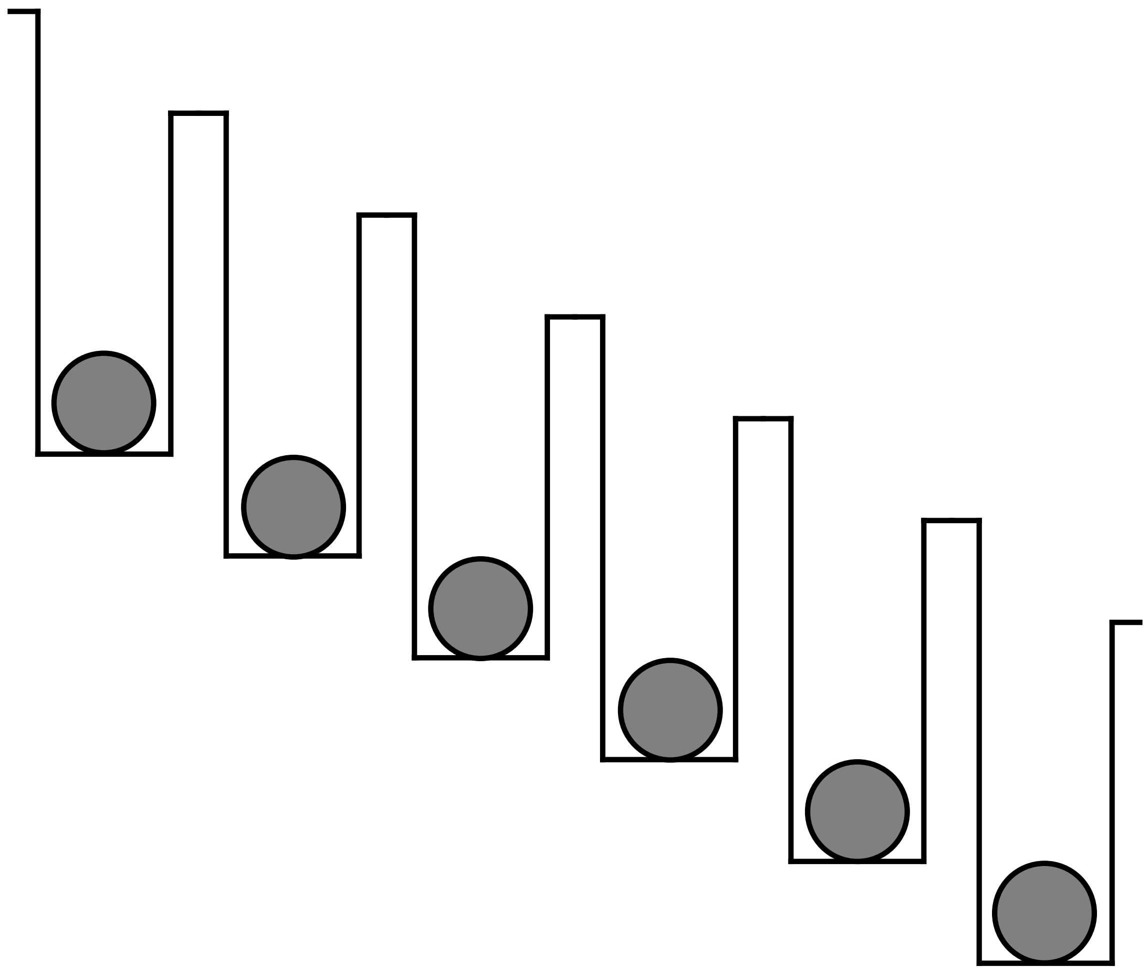
Why is there a peak (and not a threshold) when E = U ?

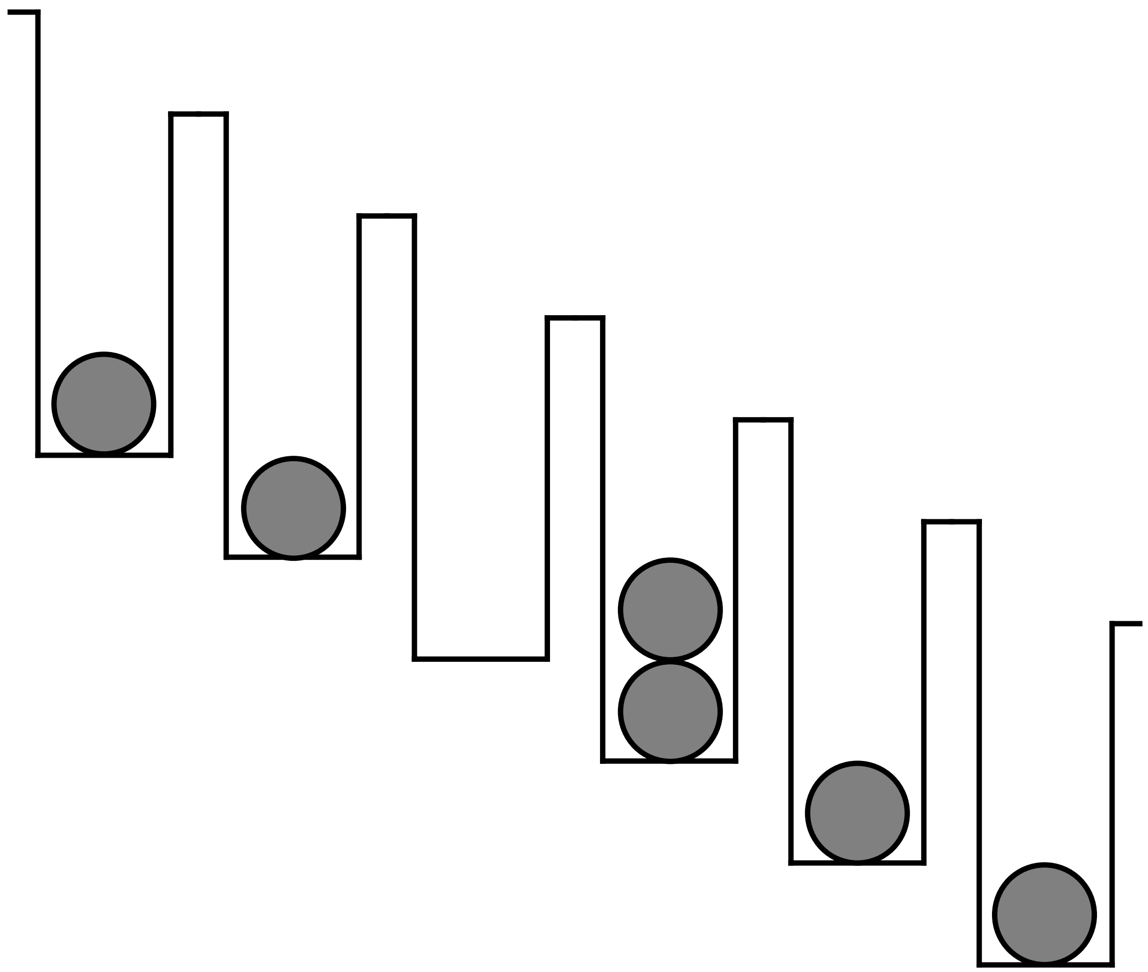


$$H = -t\sum_{\langle ij \rangle}\left(b_i^\dagger b_j + b_j^\dagger b_i\right) + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) - \sum_i \boldsymbol{E}\cdot\boldsymbol{r}_i n_i$$

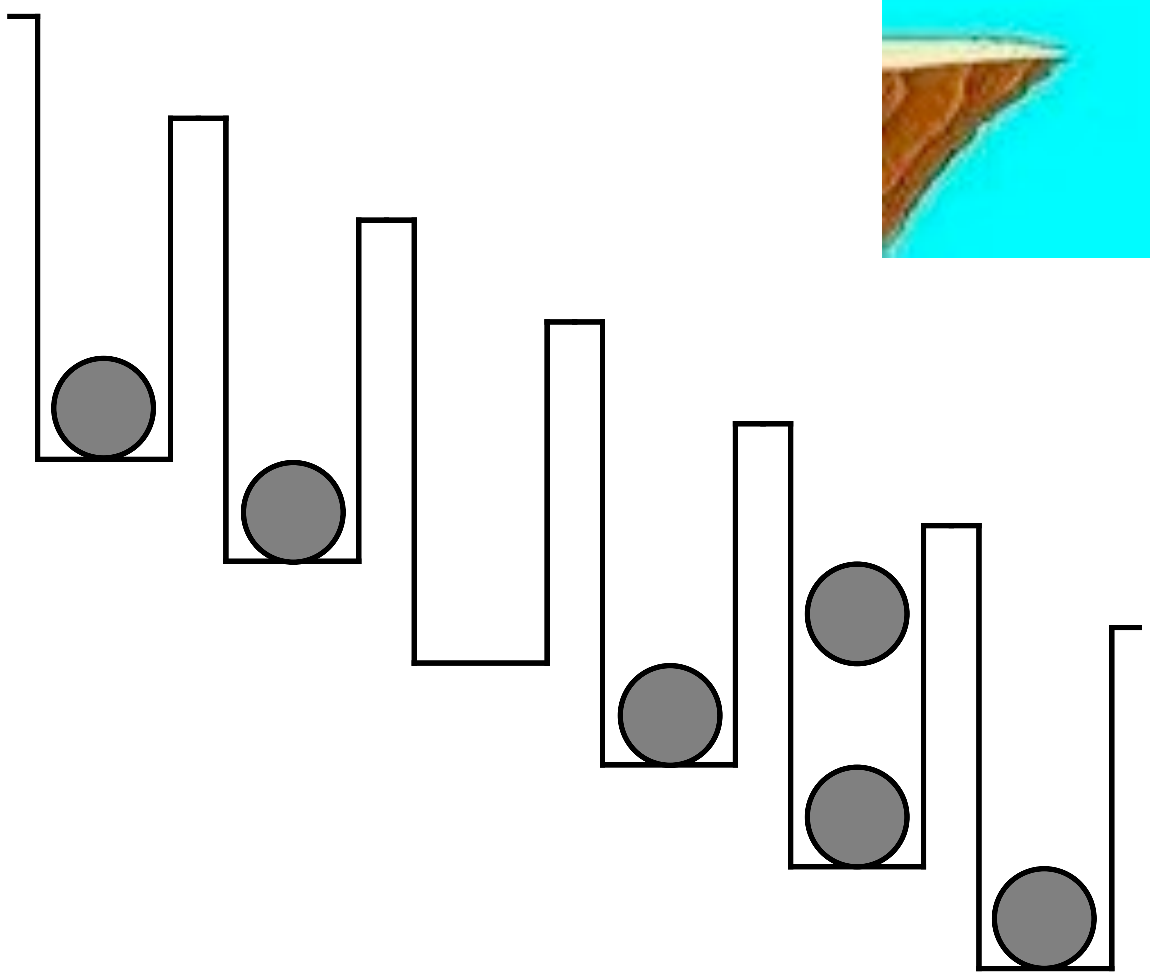
$$n_i = b_i^\dagger b_i$$

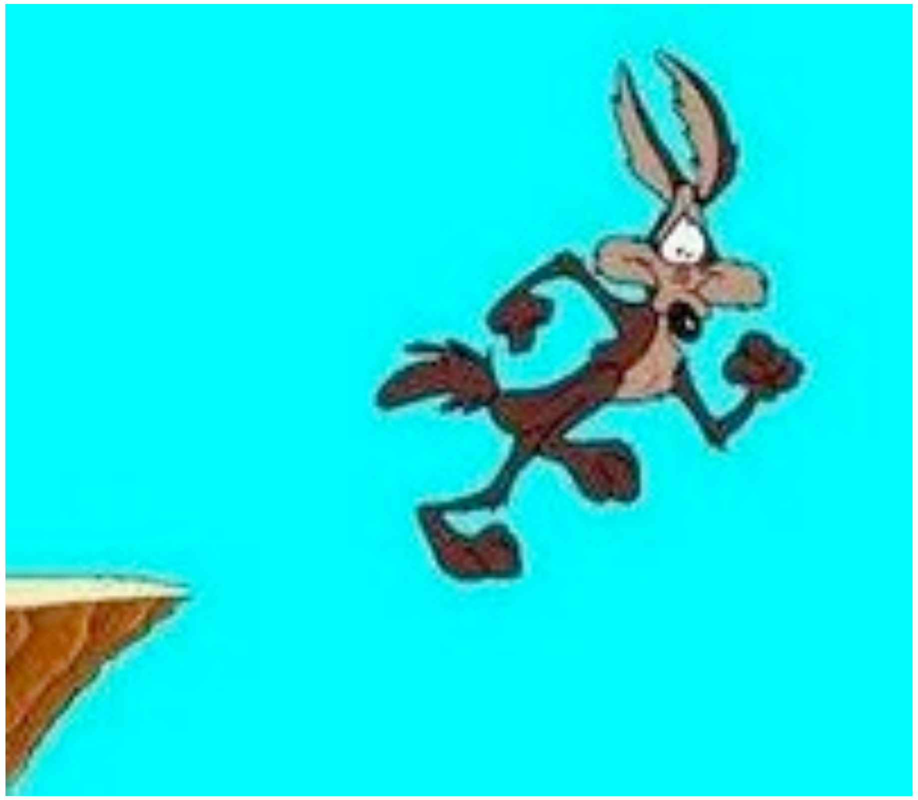
$$\left|U - E\right|, t \ll E, U$$



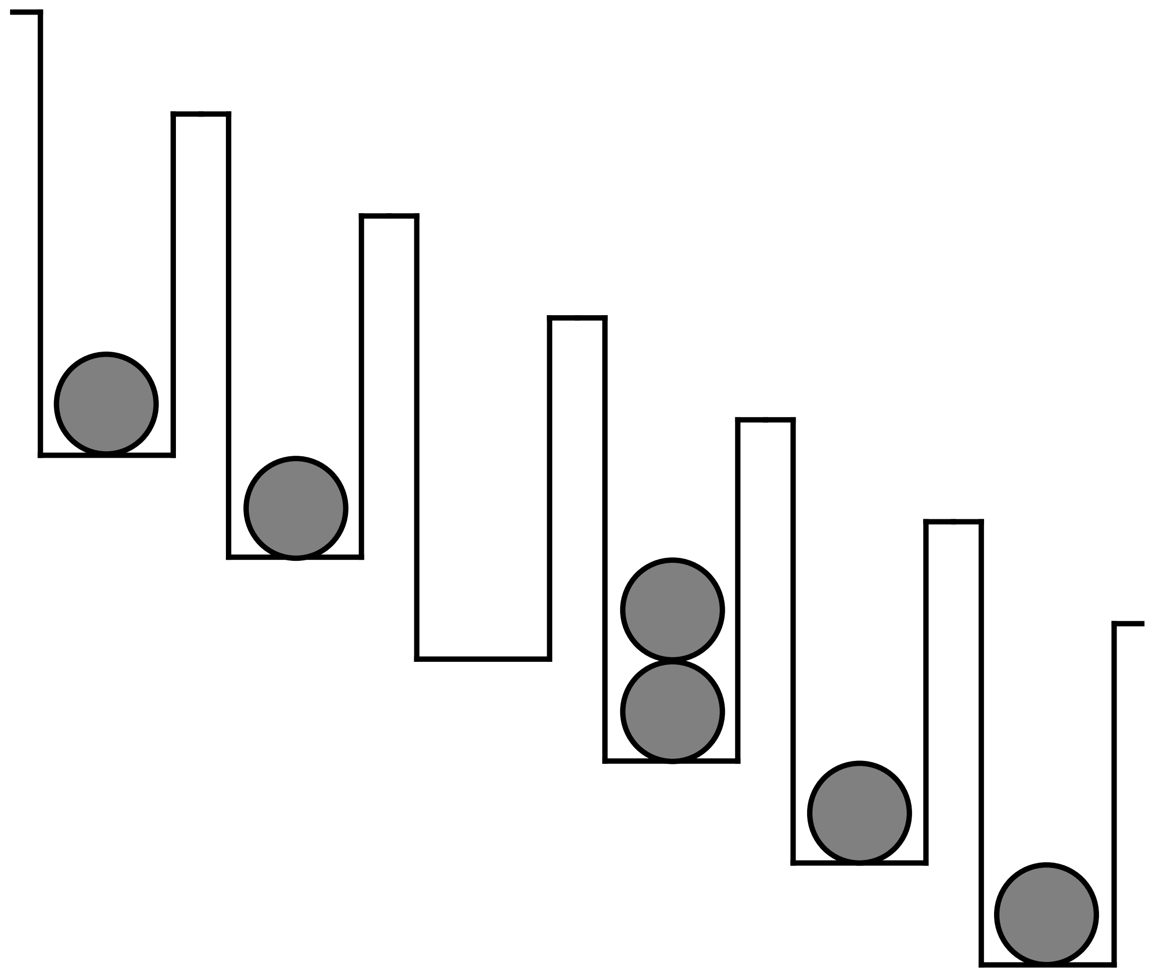


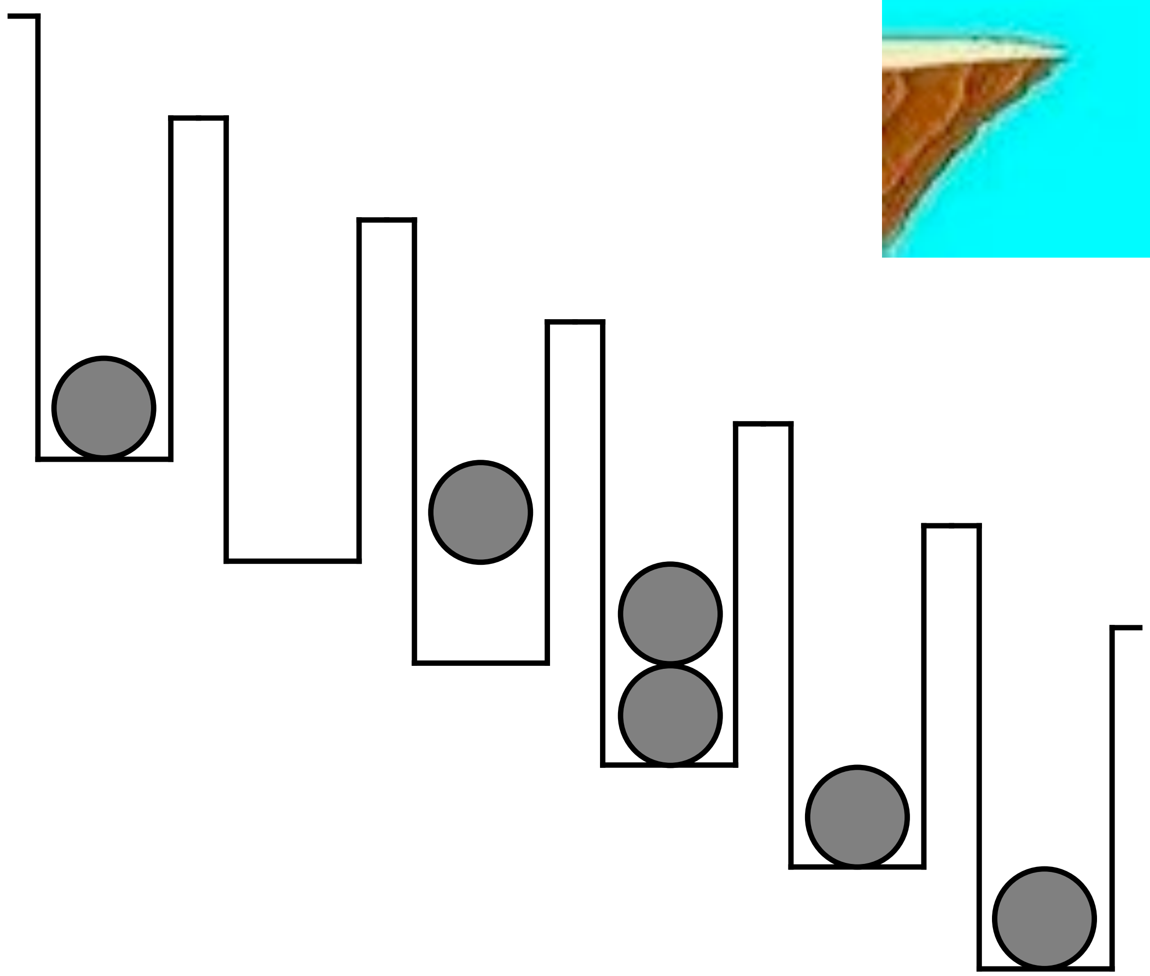
Resonant transition when $E \approx U$

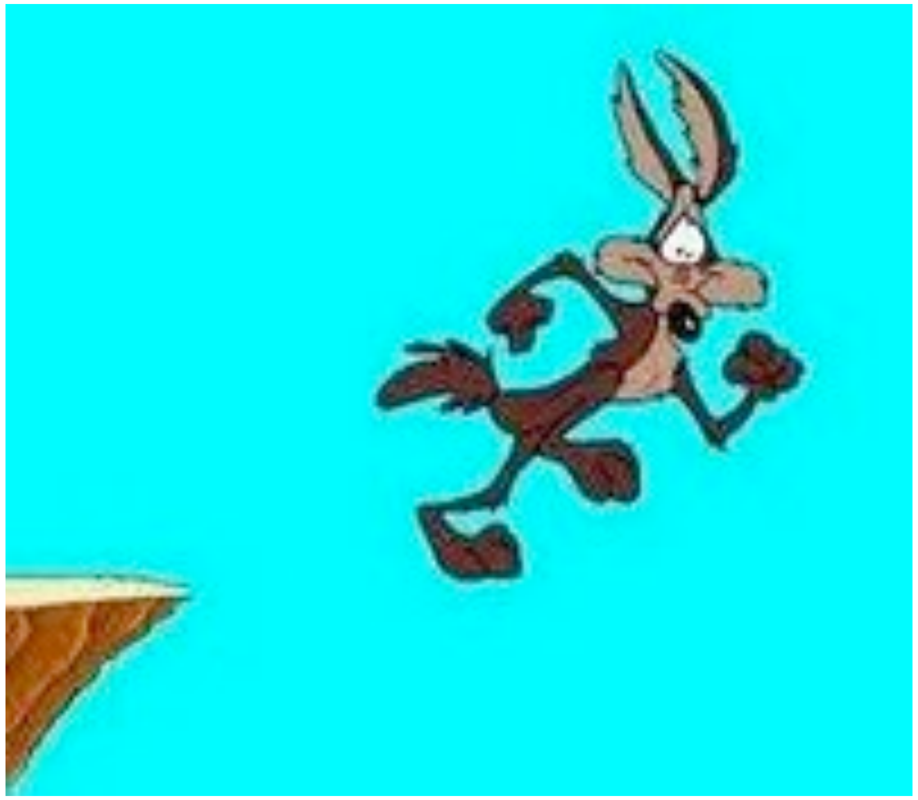




Virtual state







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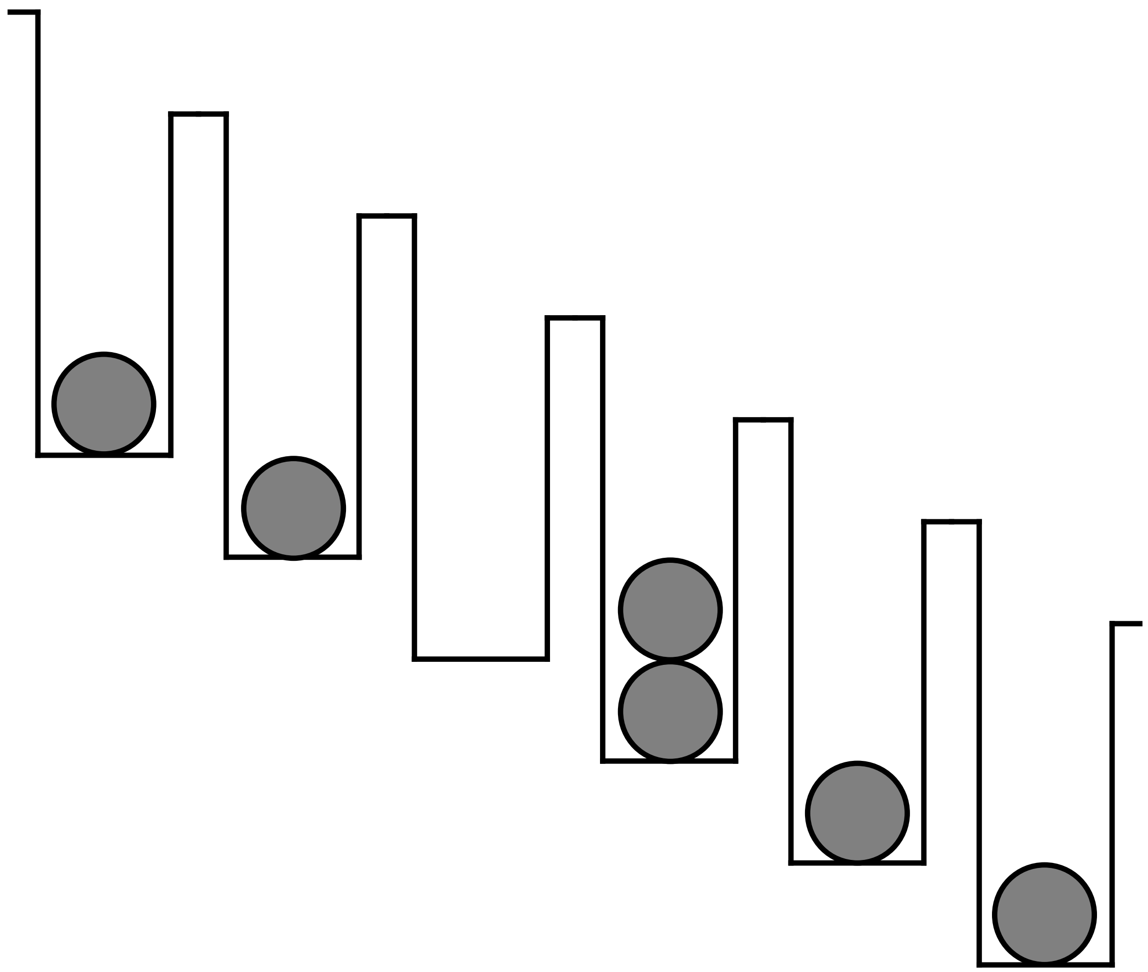

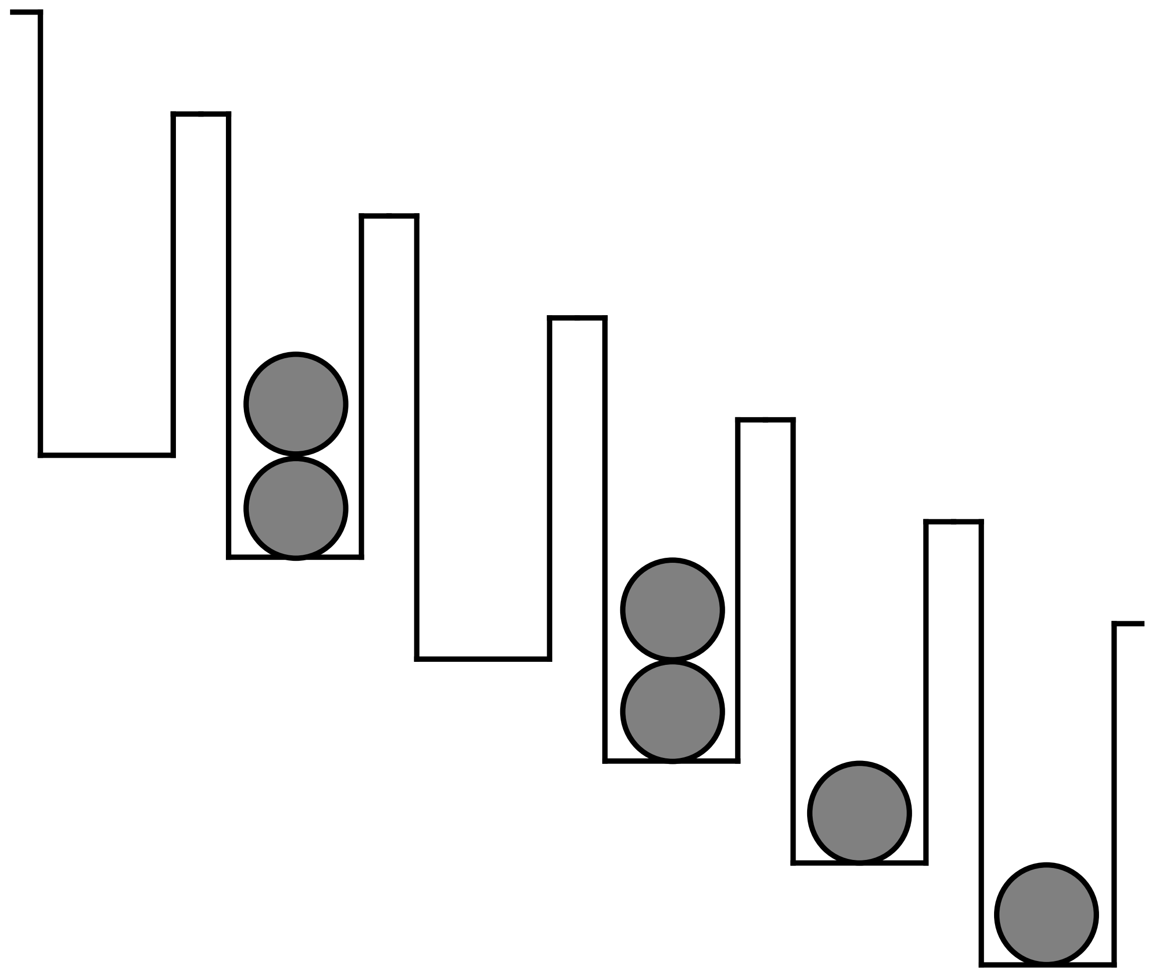

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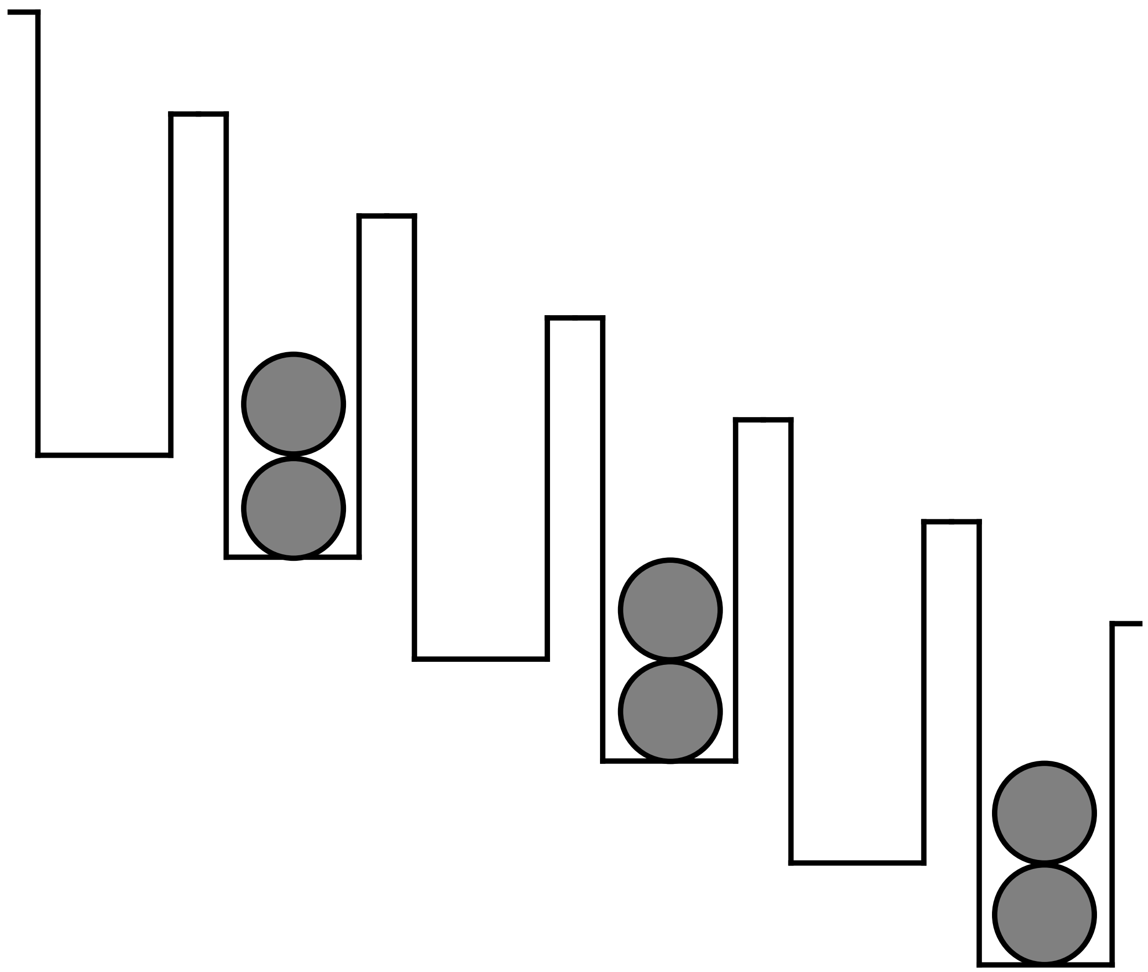

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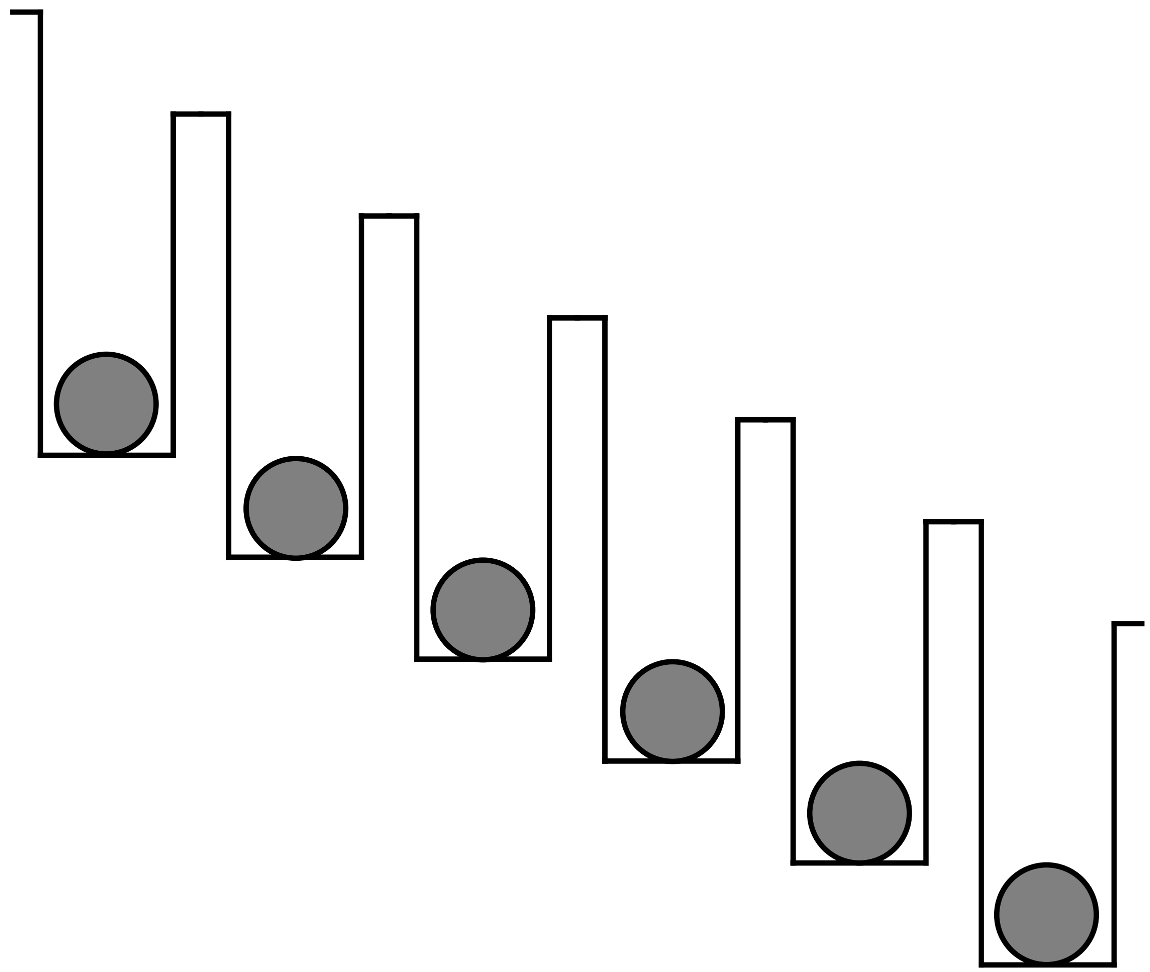
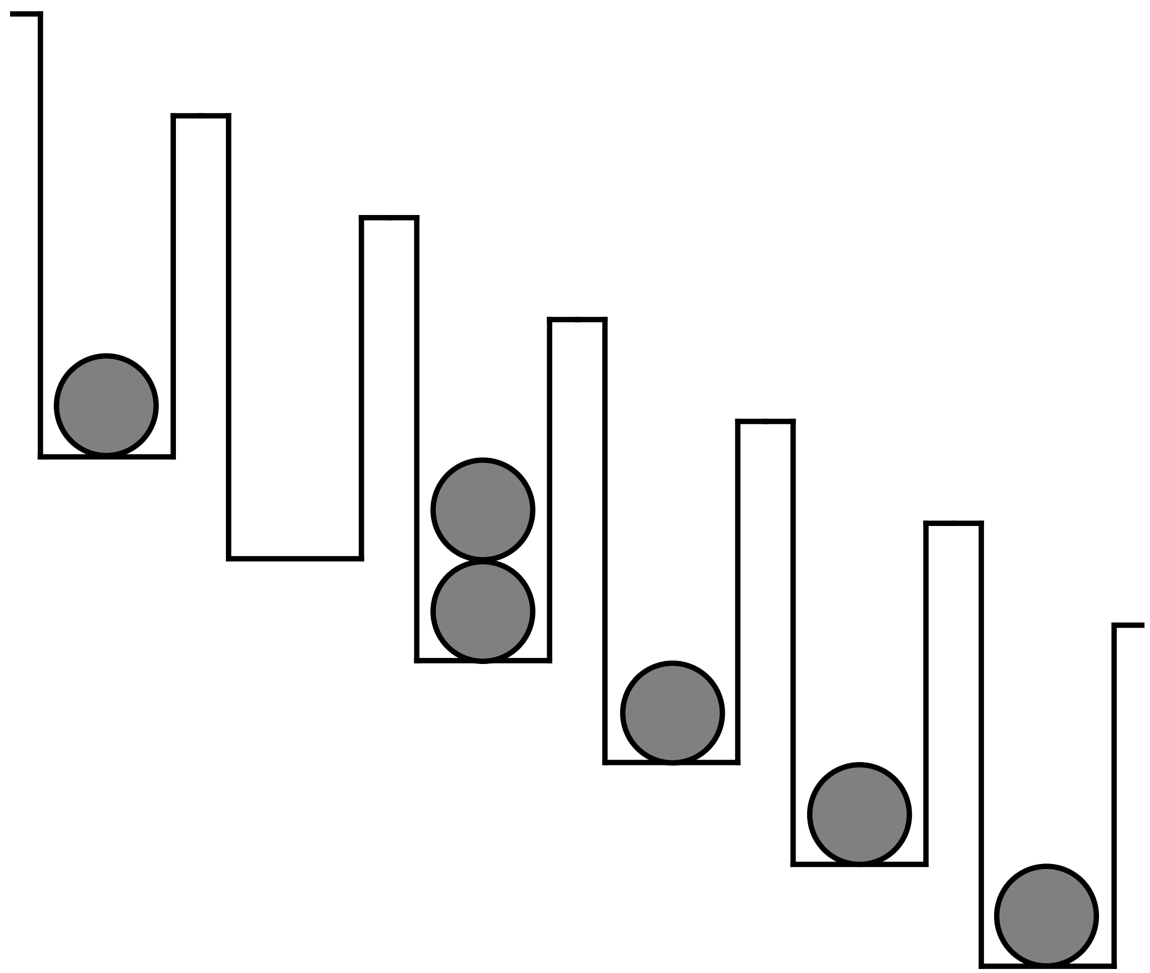
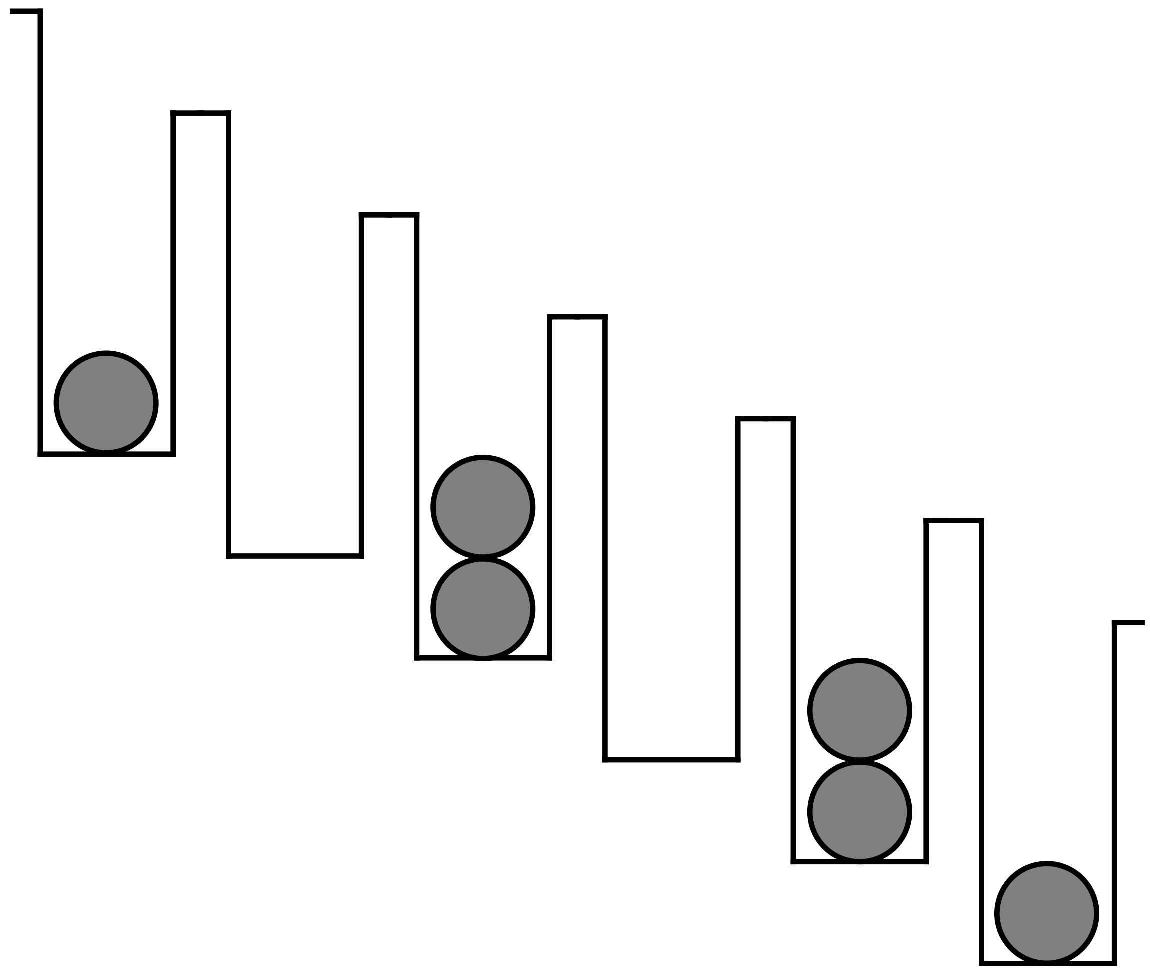
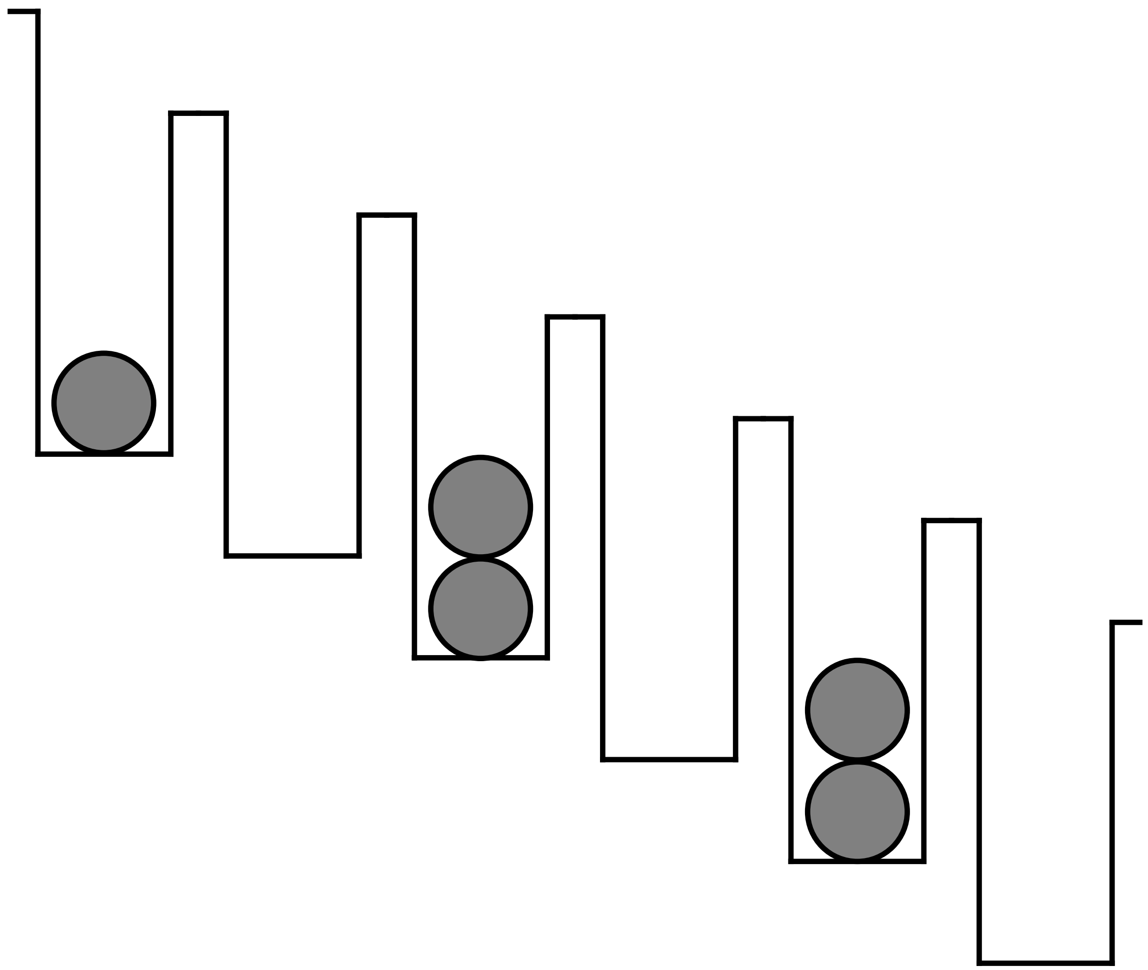
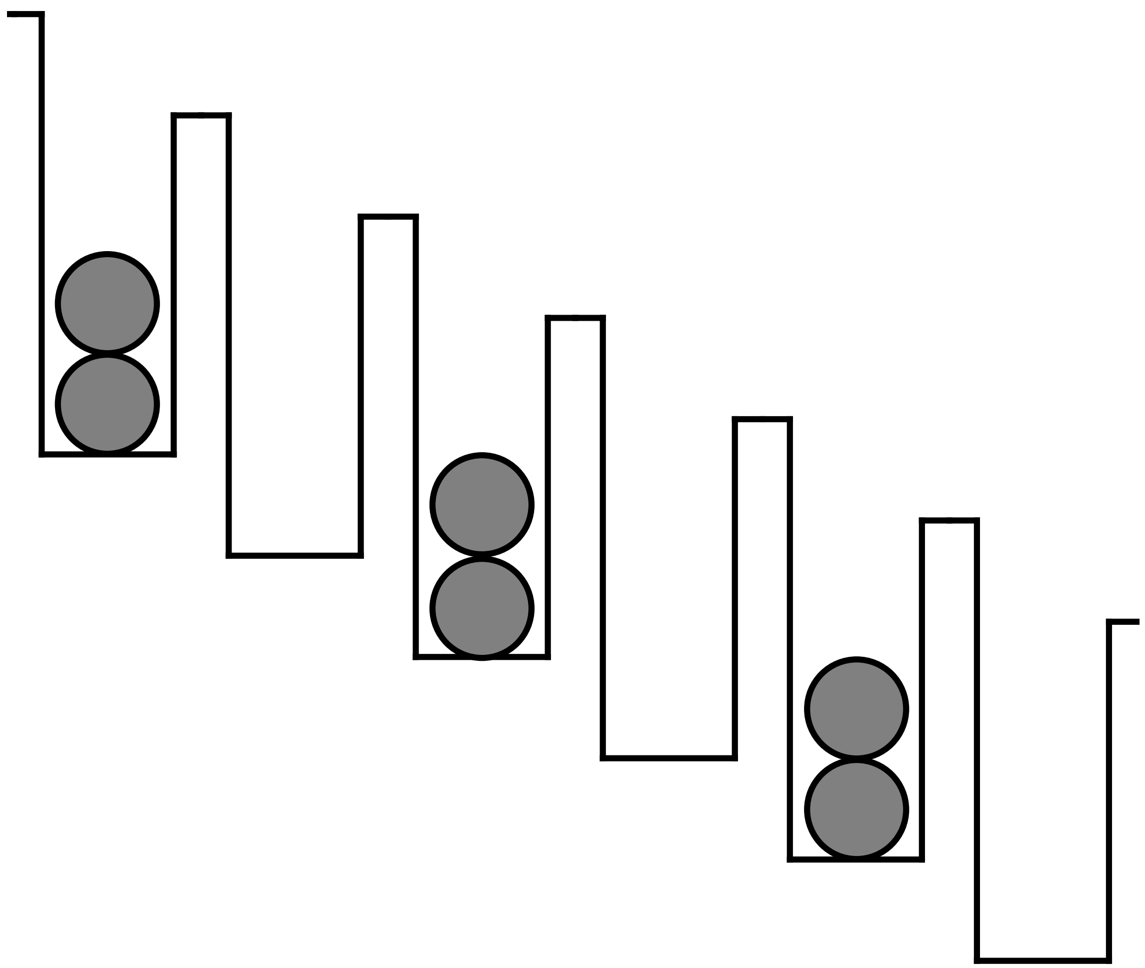


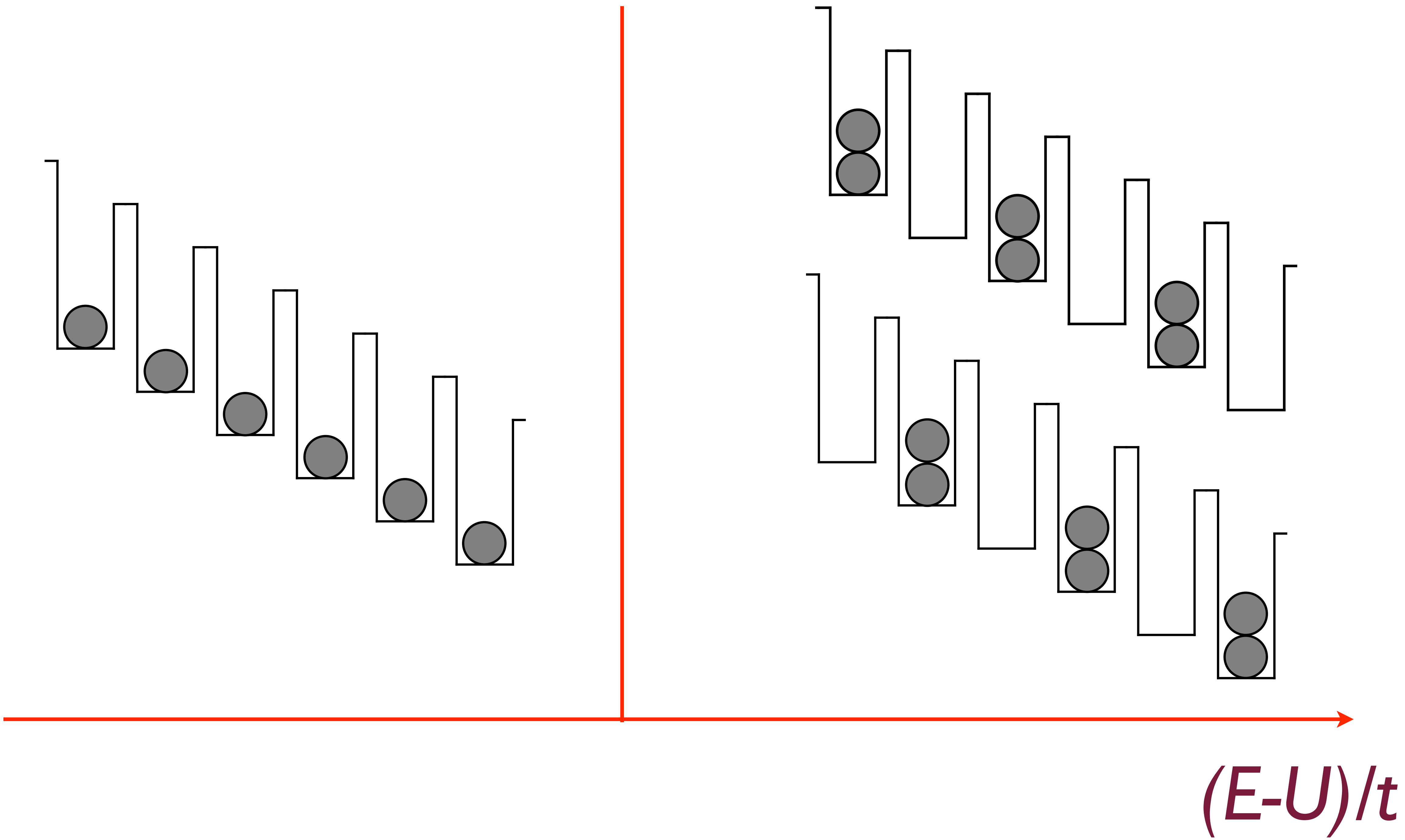








Phase diagram



S. Sachdev, K. Sengupta, and S.M. Girvin, Phys. Rev. B 66, 075128 (2002)

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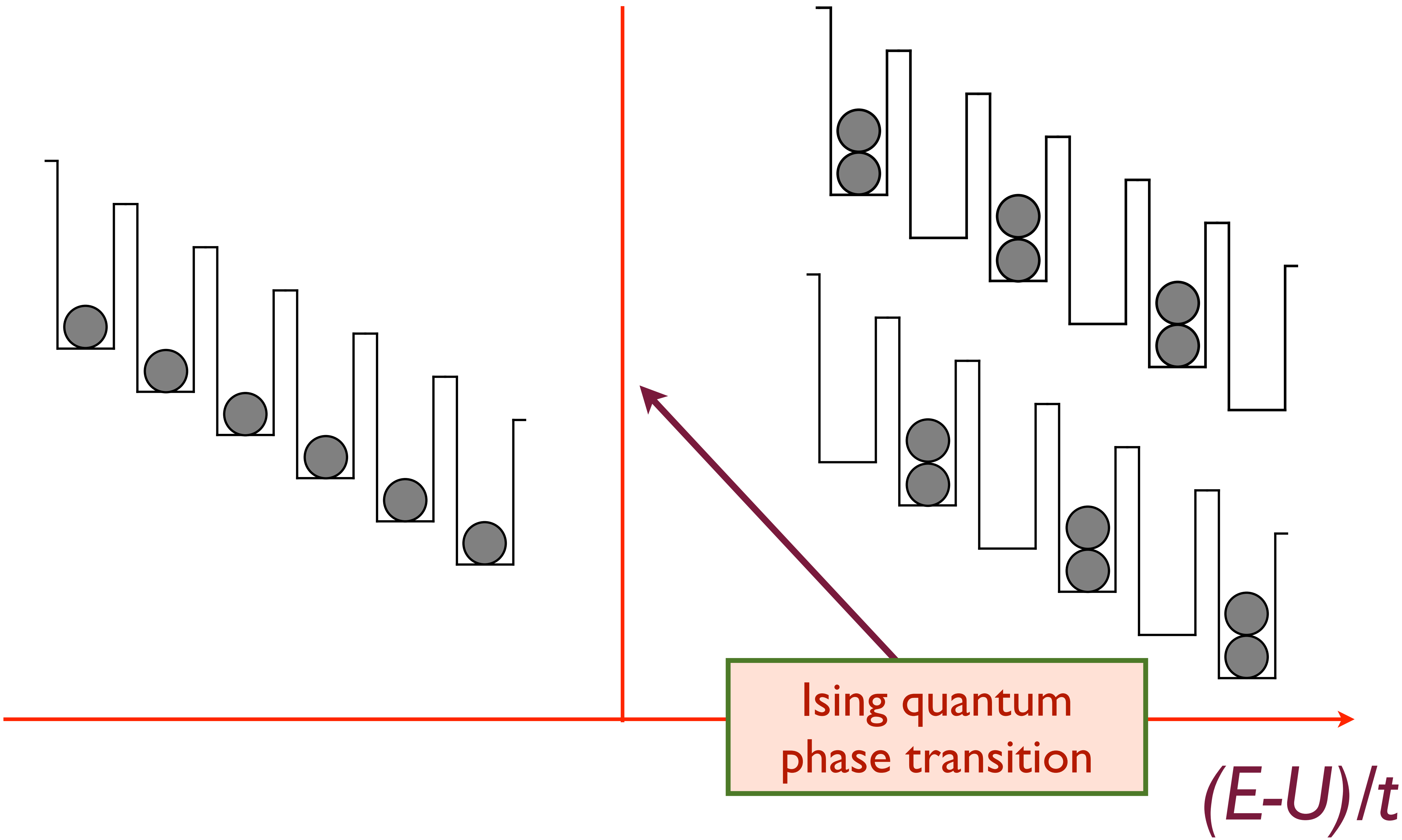

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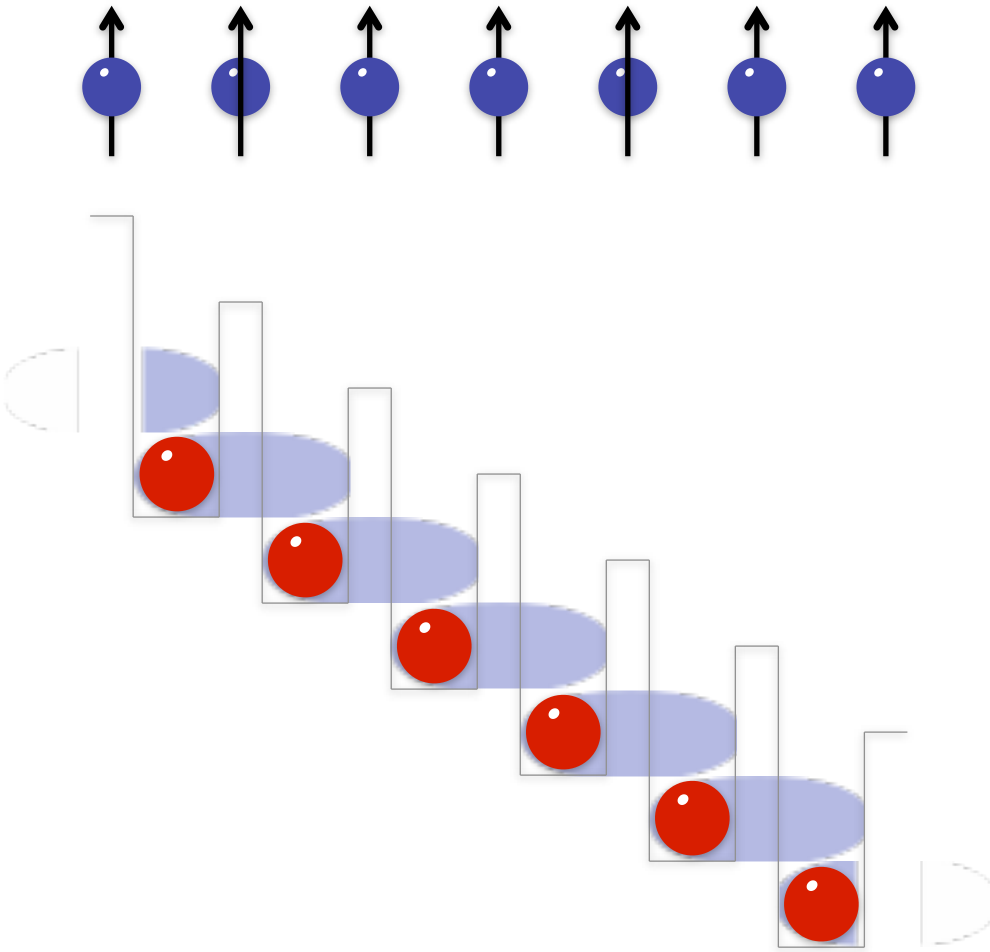
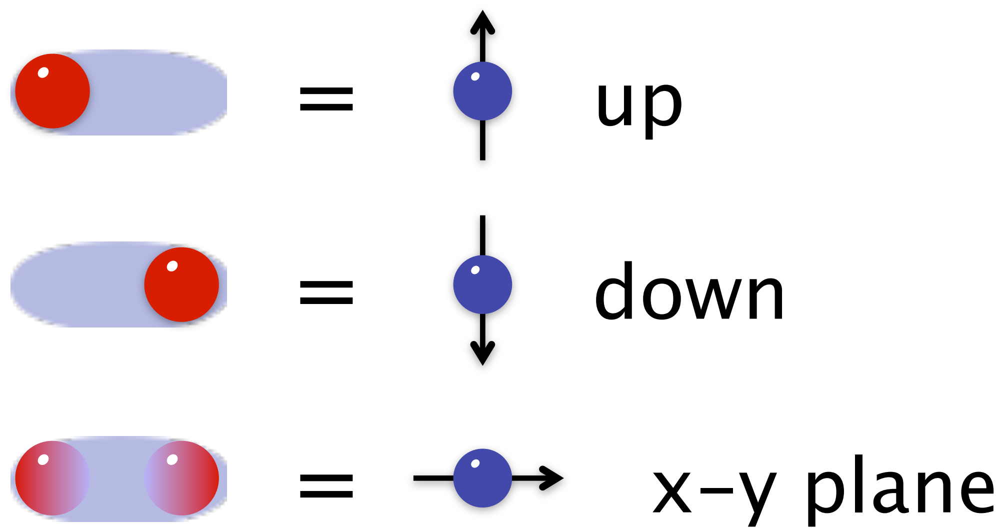
Hamiltonian of resonant subspace



Effective Hamiltonian can be written as spin model



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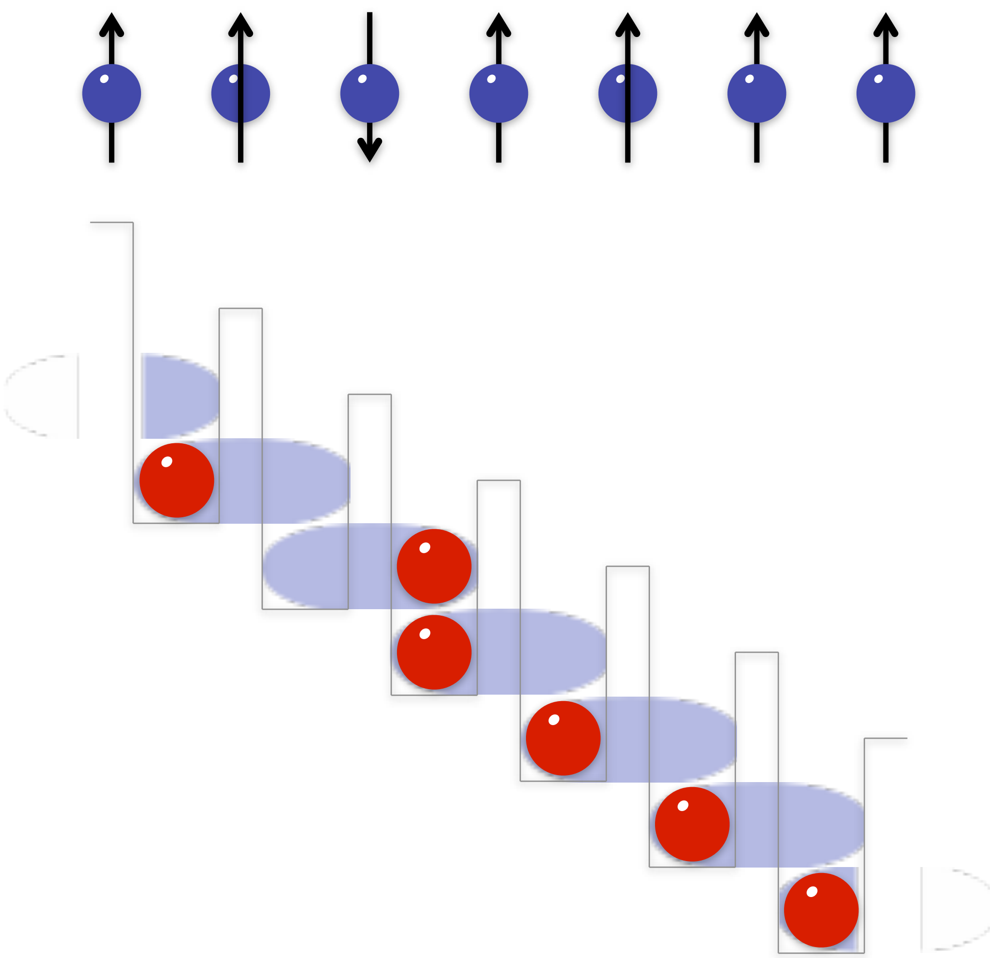

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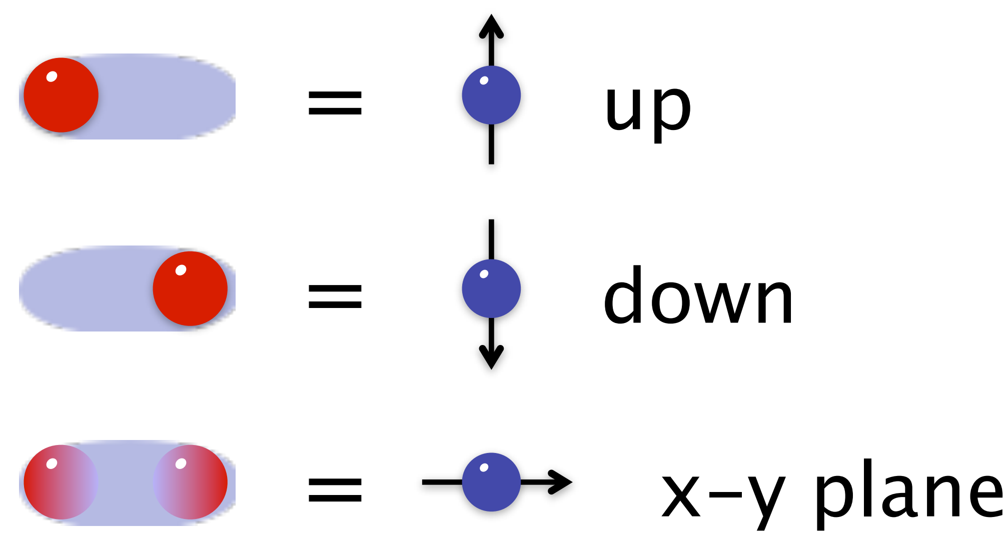

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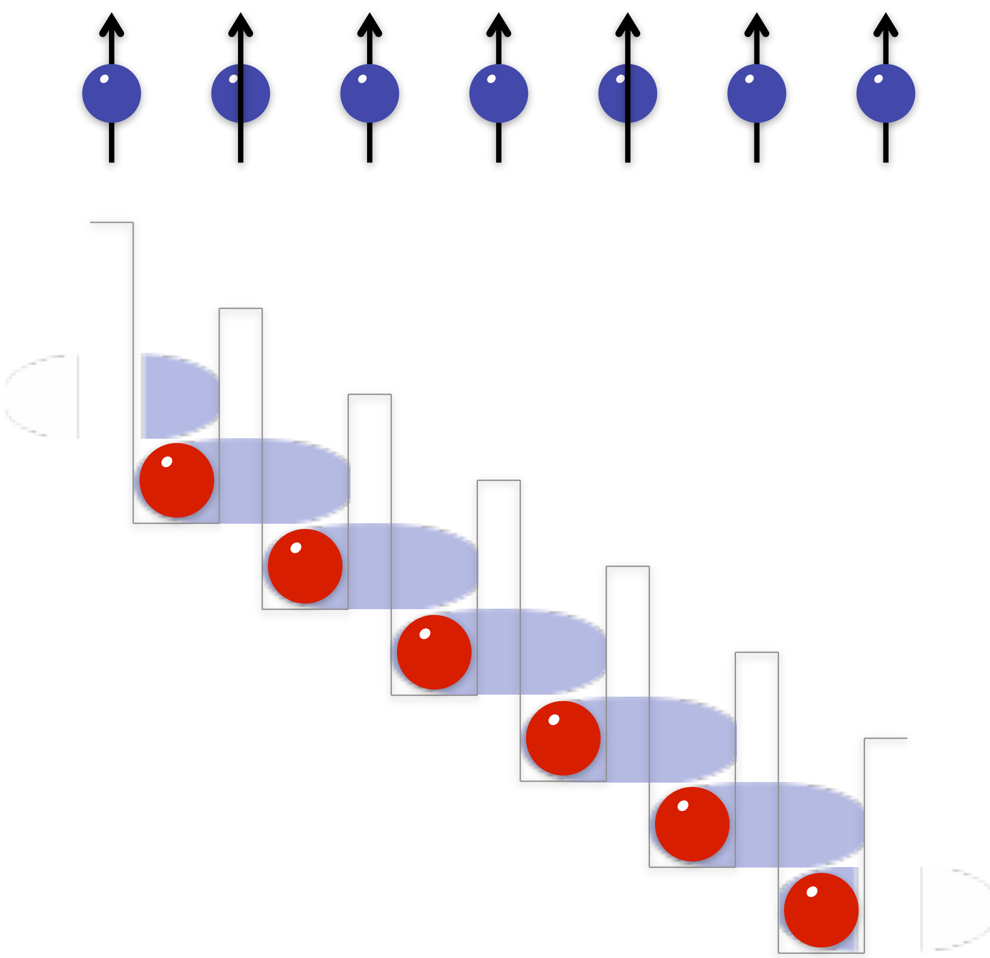

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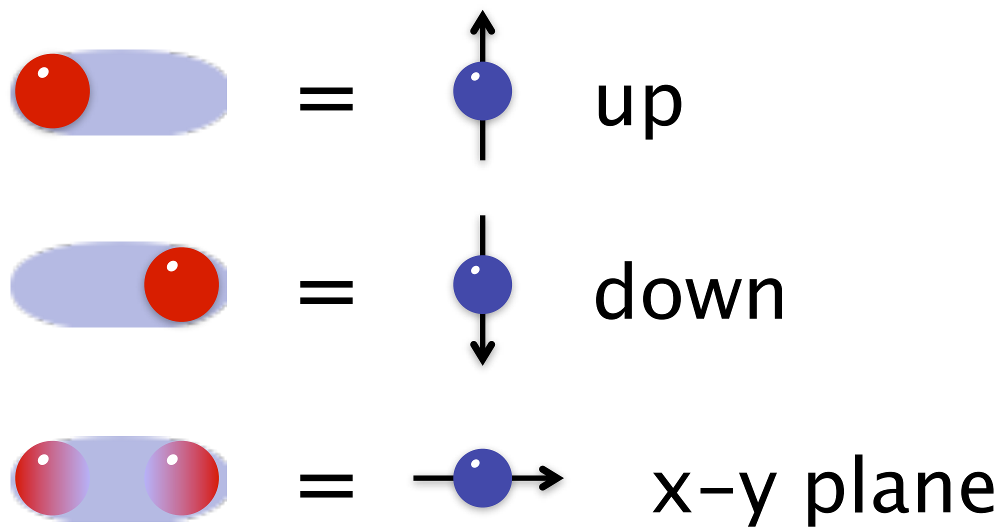

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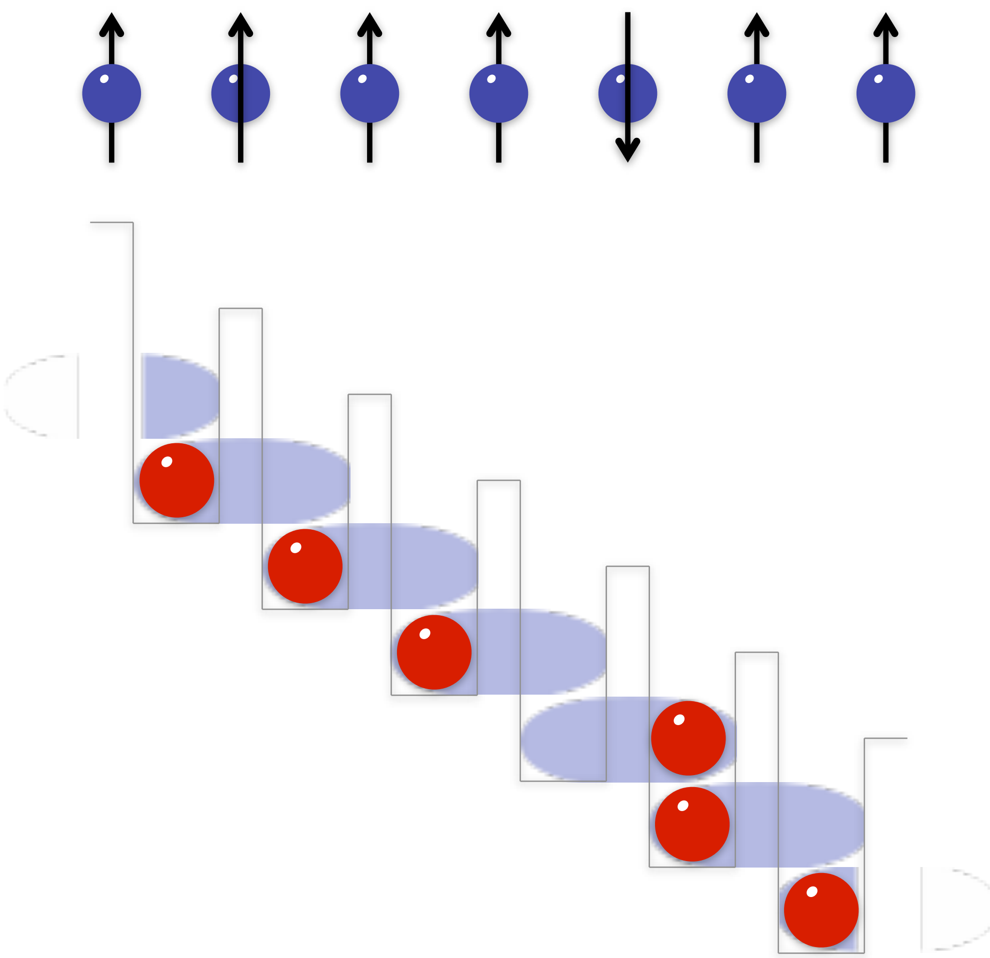

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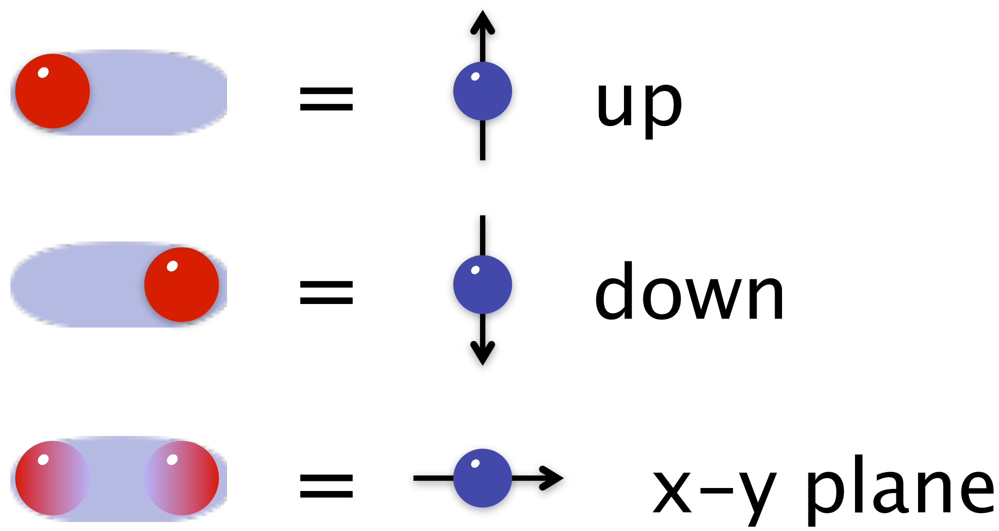

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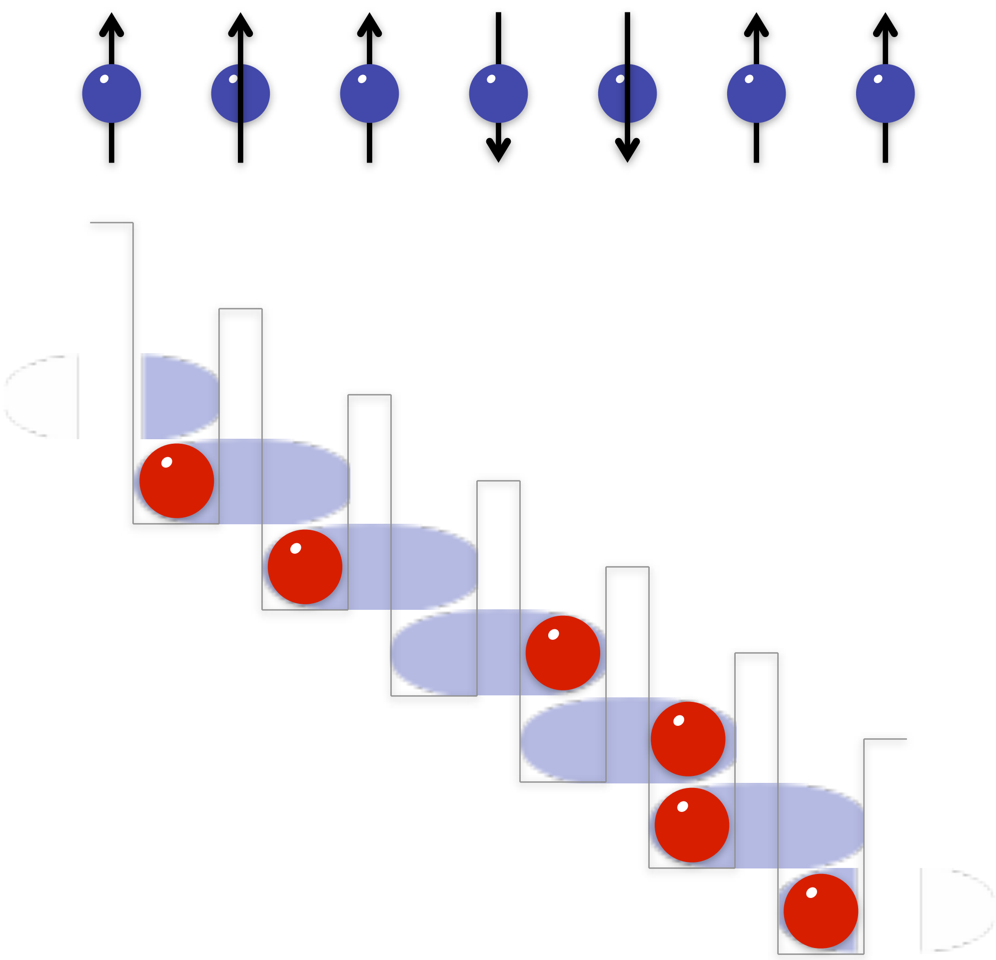

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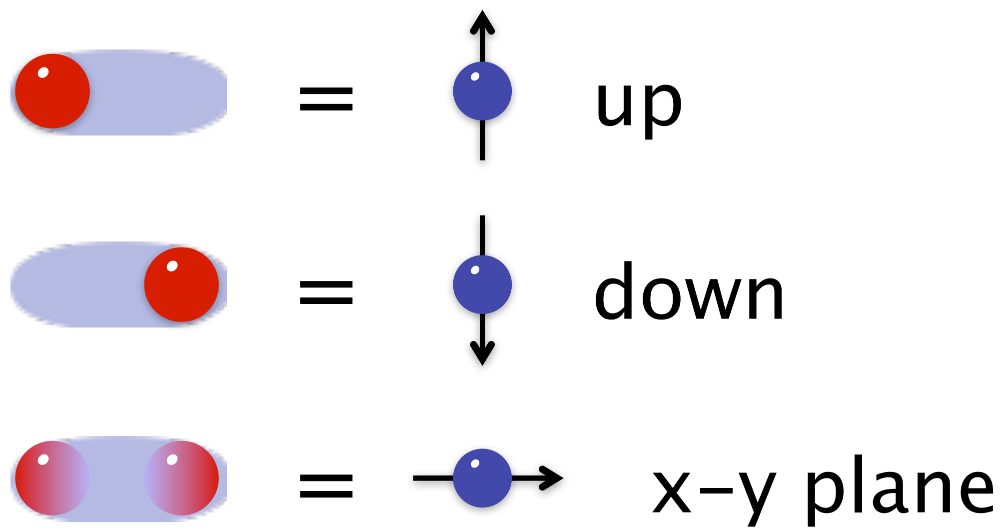
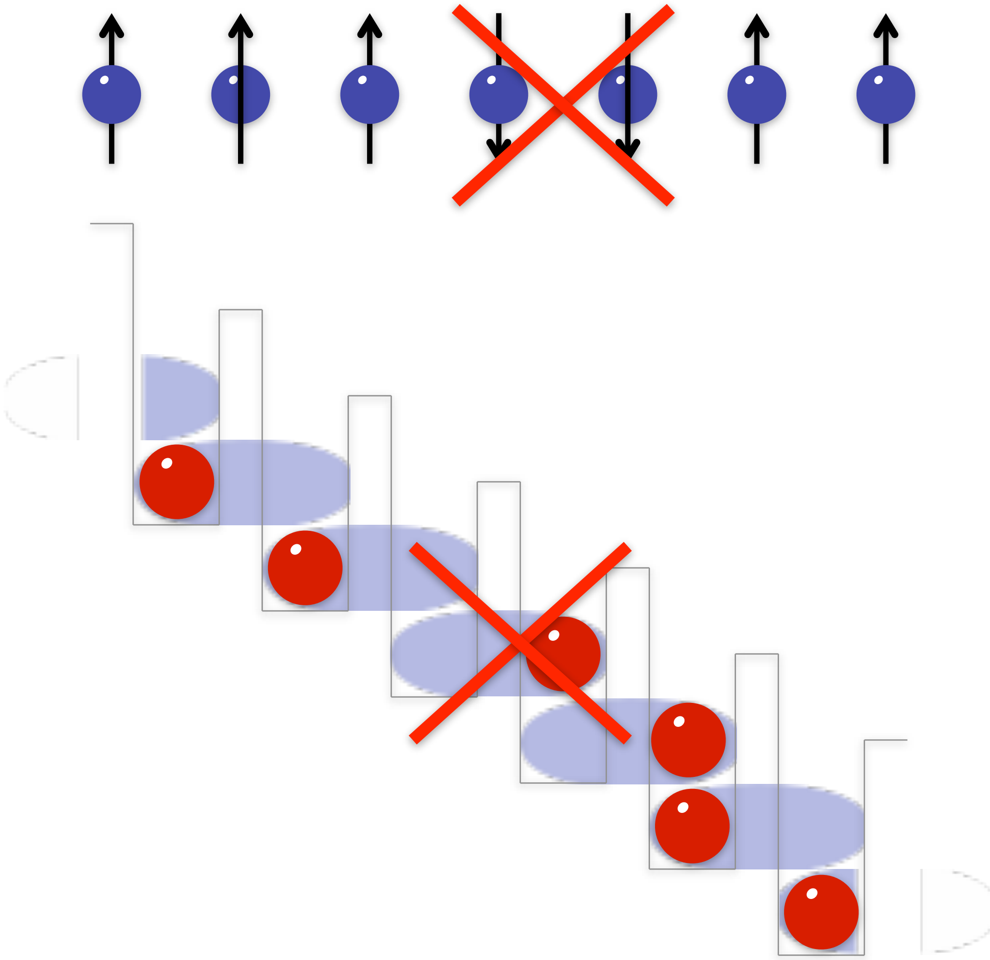
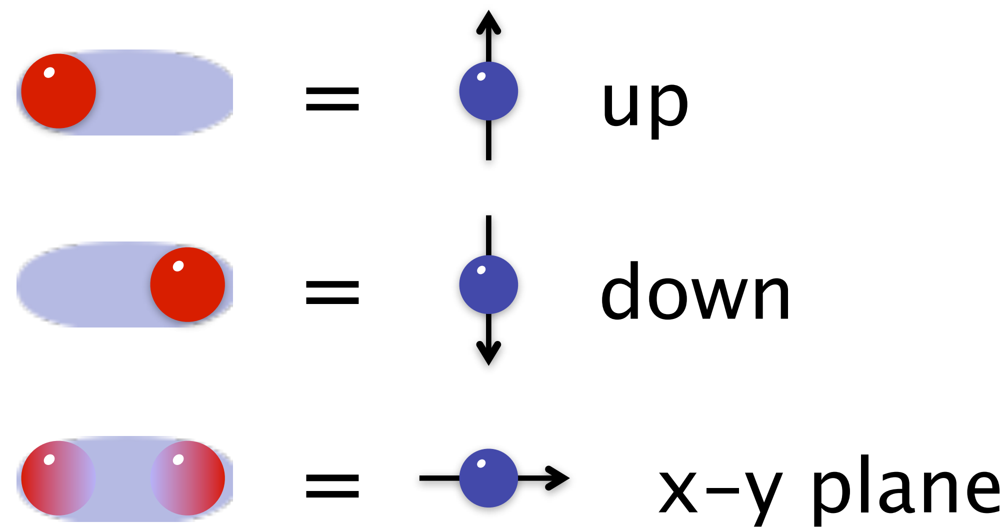


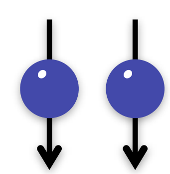
Hamiltonian of resonant subspace



Effective Hamiltonian can be written as spin model

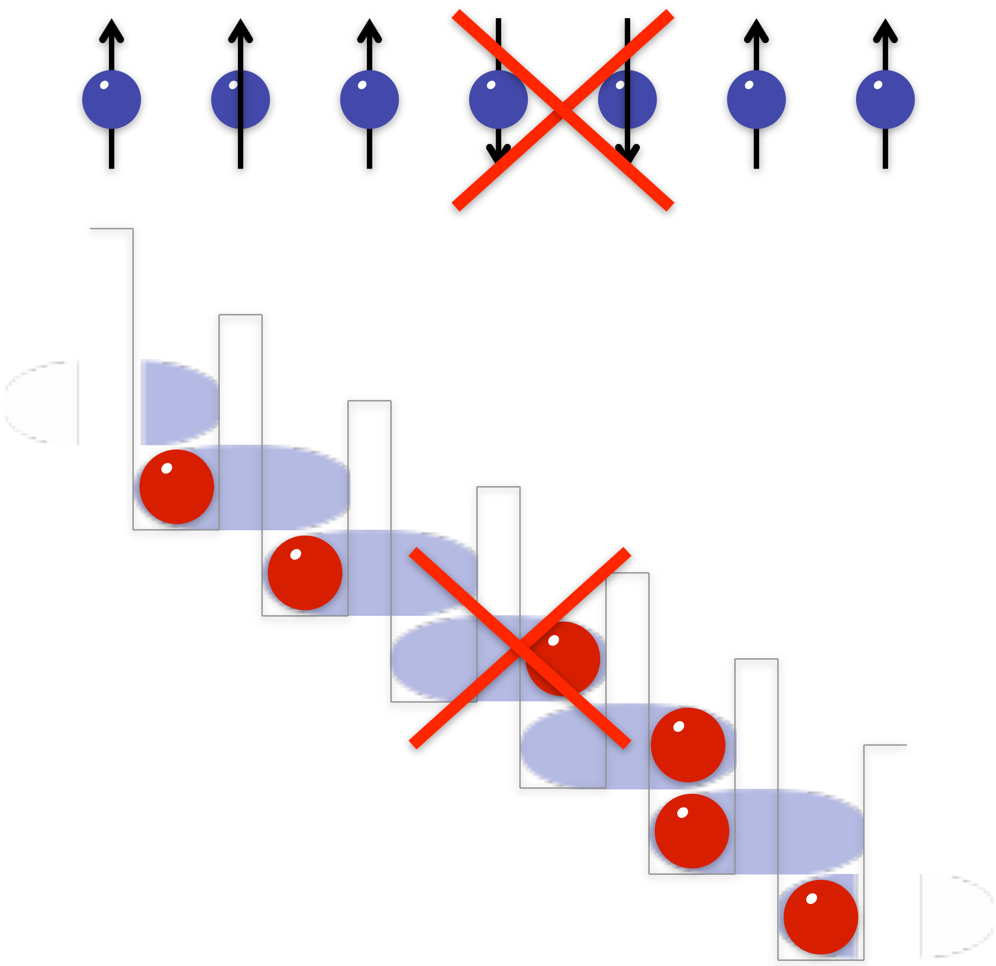


Constraint:

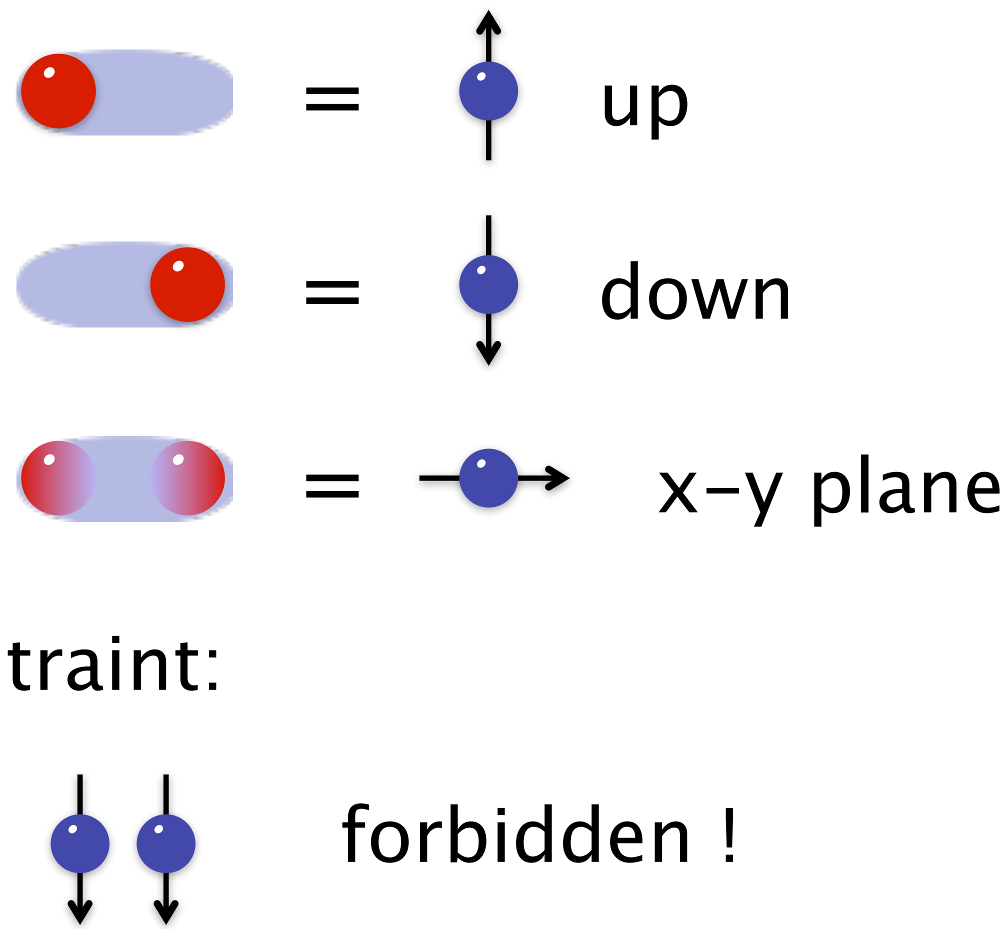


forbidden !

Hamiltonian of resonant subspace

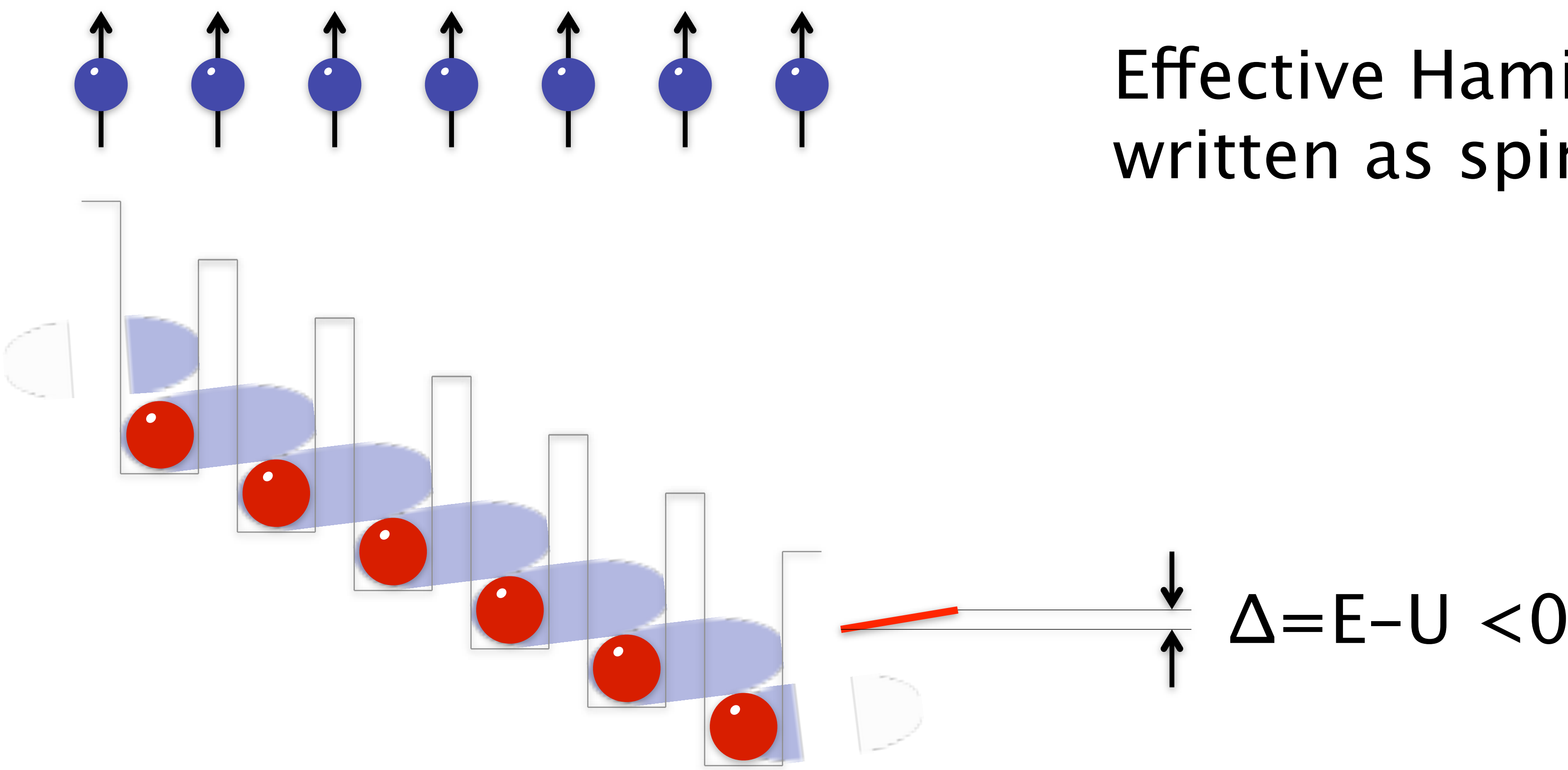


Effective Hamiltonian can be written as spin model



Include a term $(J/4)\sum_i (\sigma_i^z - 1)(\sigma_{i+1}^z - 1)$ and send $J \to \infty$. Infinite exchange interaction !

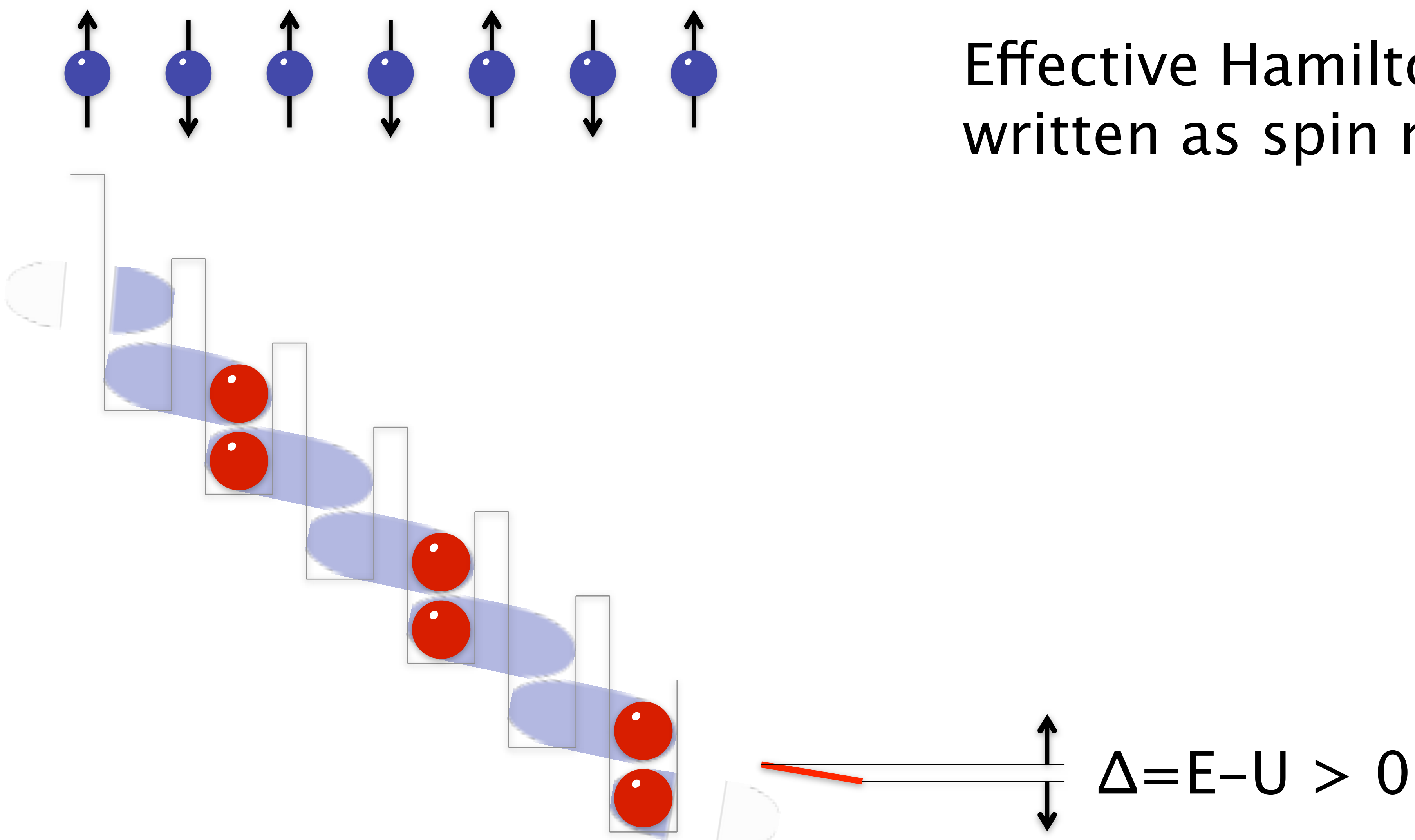
Hamiltonian of resonant subspace



Effective Hamiltonian can be written as spin model

Paramagnetic state

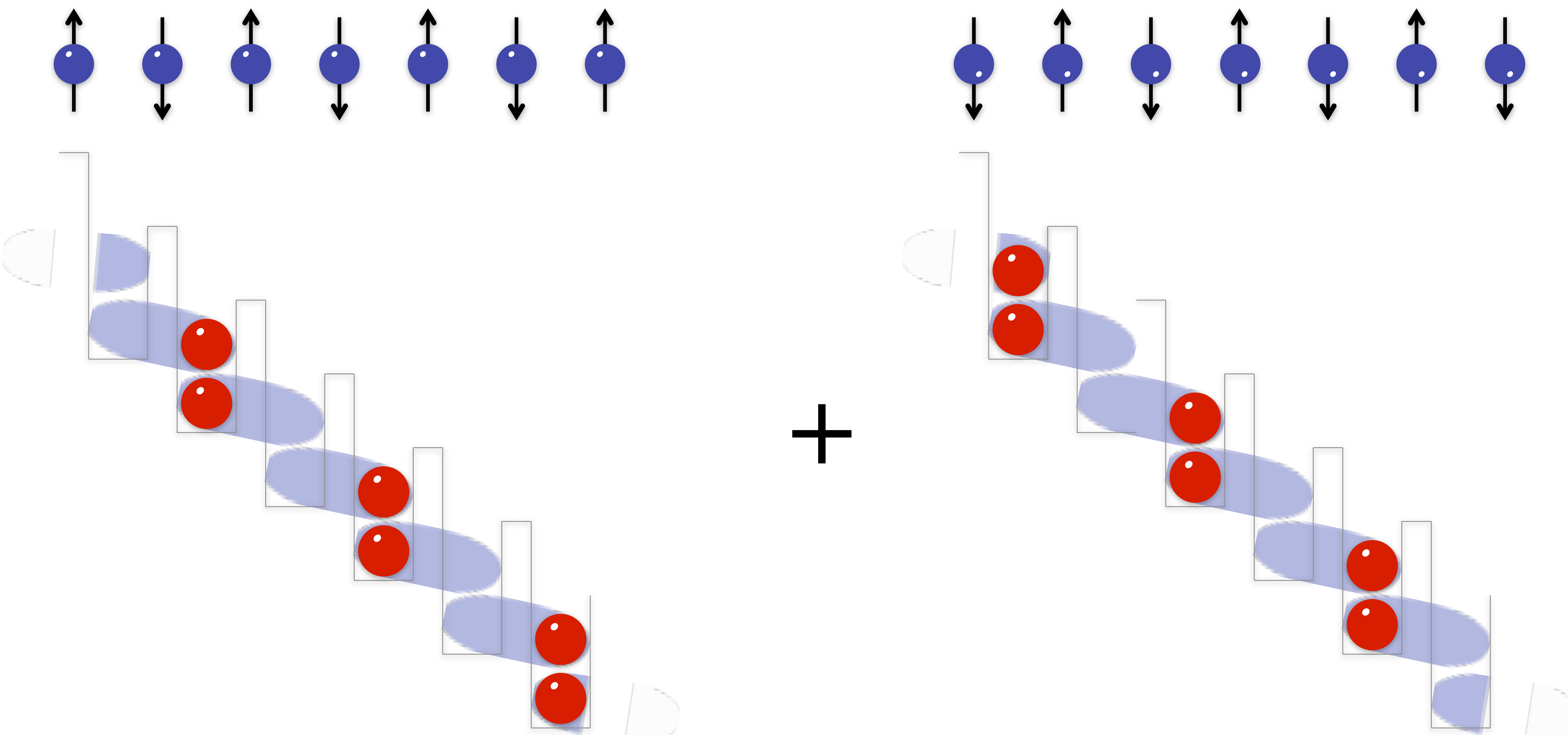
Hamiltonian of resonant subspace



Effective Hamiltonian can be written as spin model

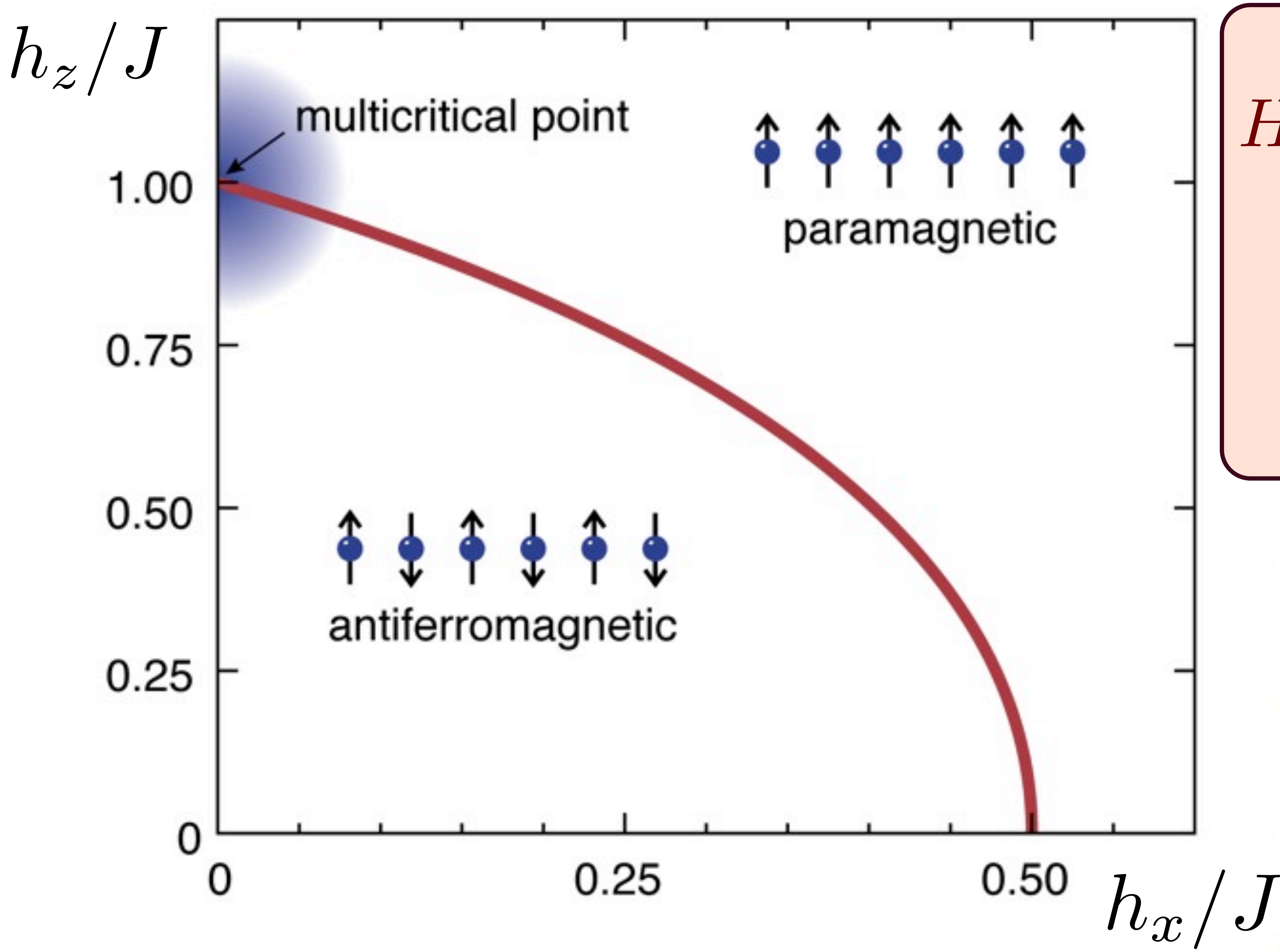
Antiferromagnetic state, two fold degenerate

Hamiltonian of resonant subspace



Antiferromagnetic state, two fold degenerate

Phase diagram of spin model



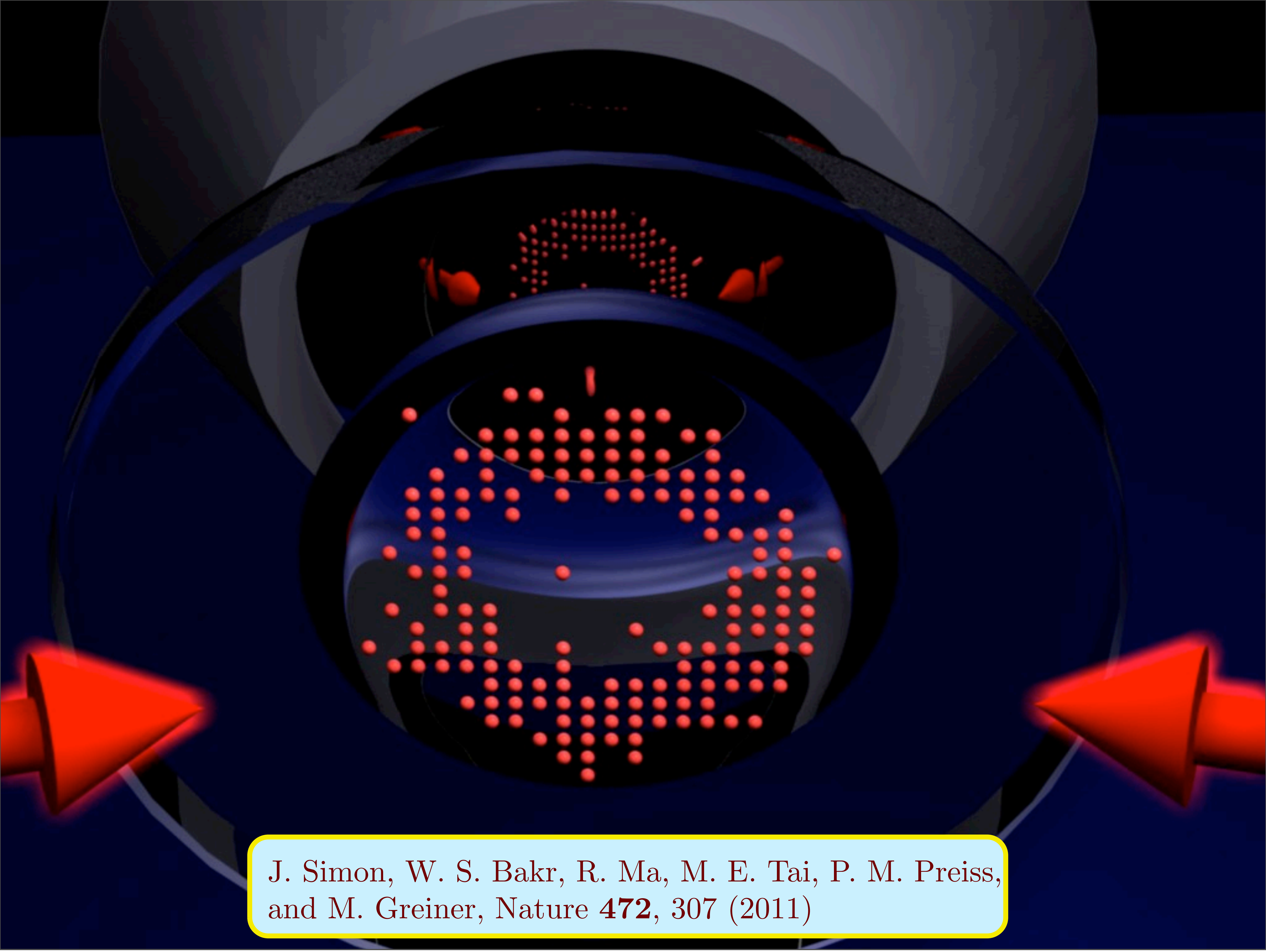
$$H = \sum_i \left[\frac{J}{4} \sigma_i^z \sigma_{i+1}^z - \frac{h_z}{2} \sigma_i^z - \frac{h_x}{2} \sigma_i^x \right]$$

$$J \to \infty,$$

$$h_z = J + (U - E),$$

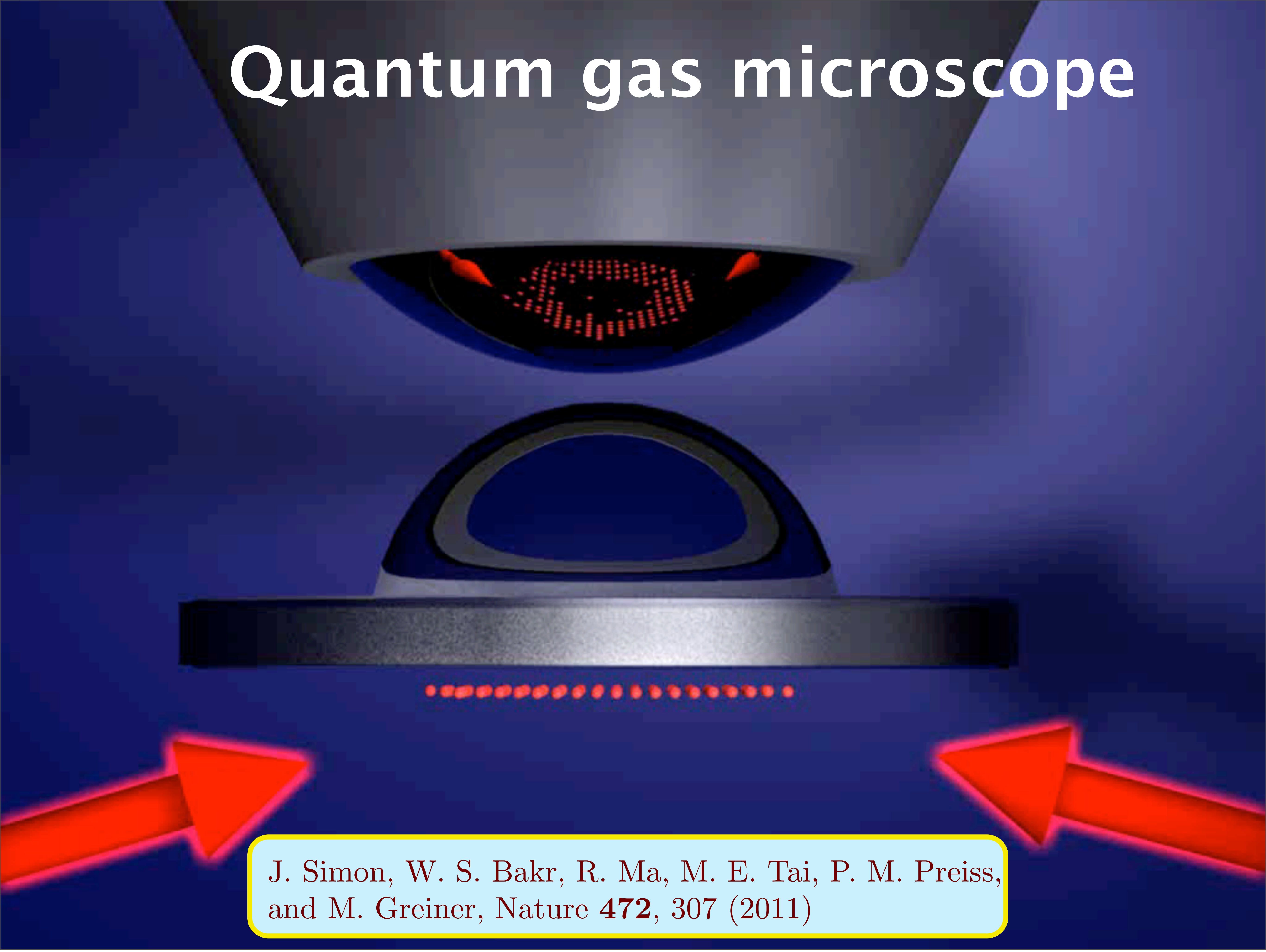
$$h_x = 2\sqrt{2}t$$

hx =0: classical first order phase transition
Finite hx: quantum phase transition, second order

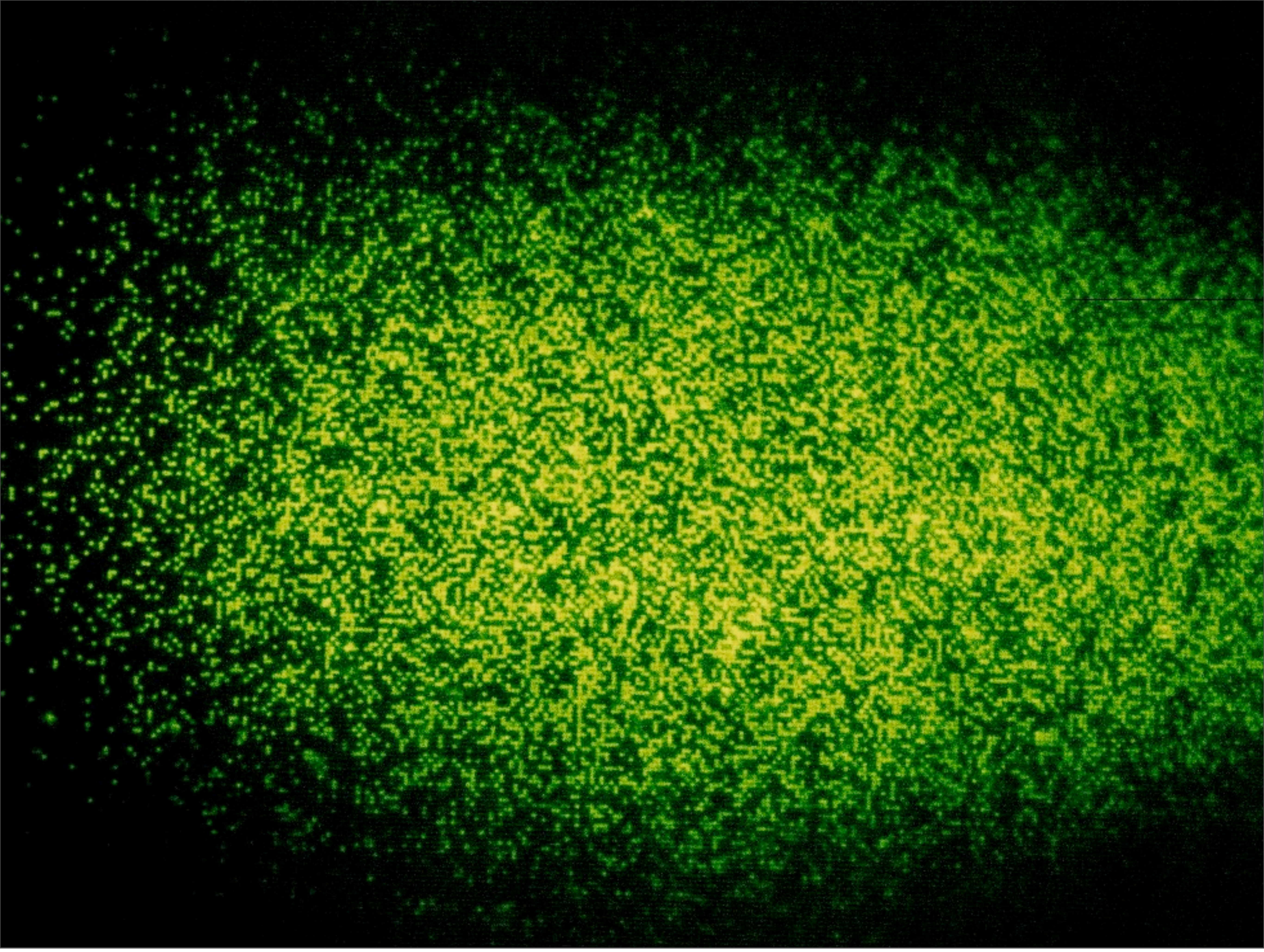


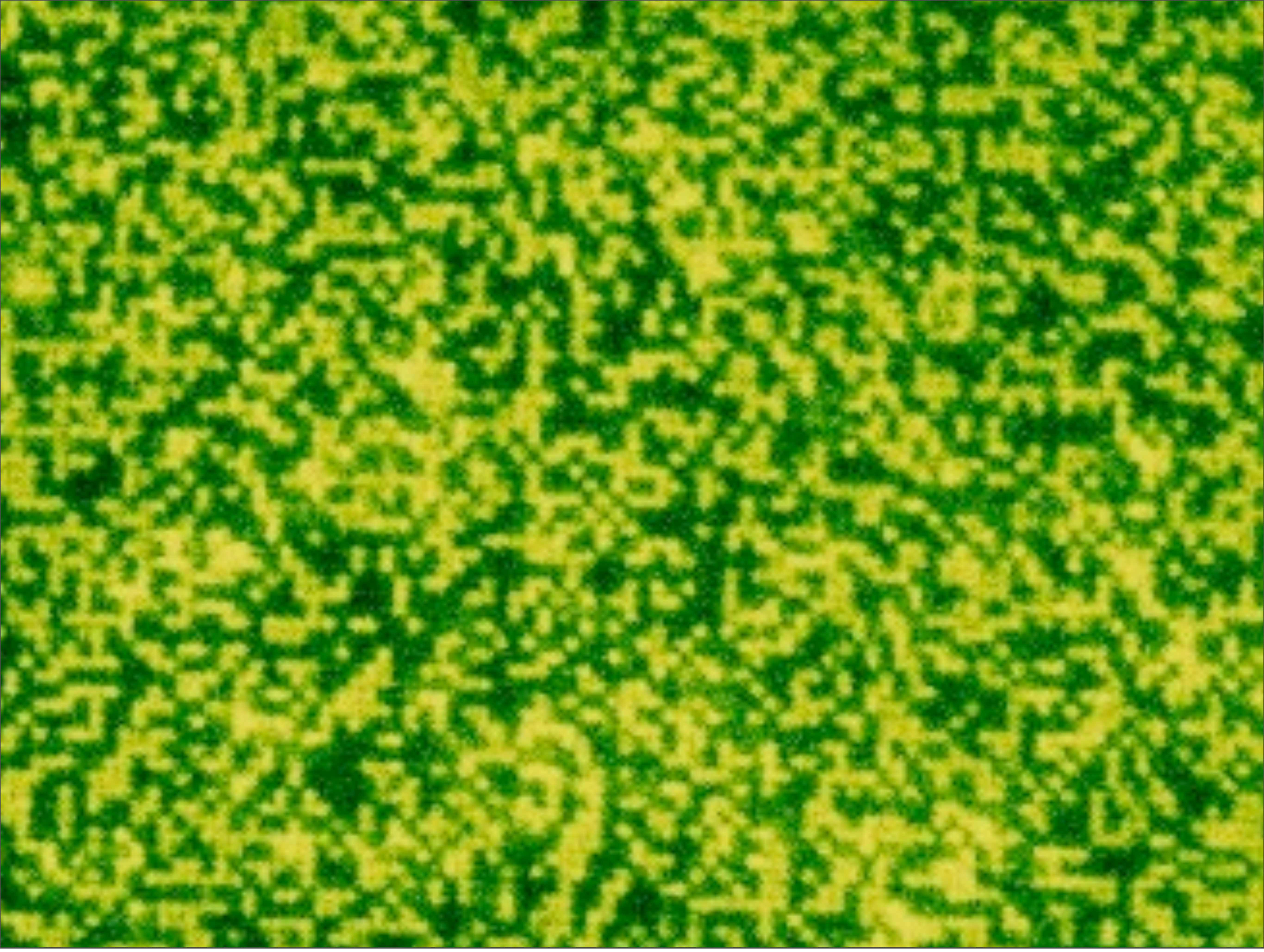
J. Simon, W. S. Bakr, R. Ma, M. E. Tai, P. M. Preiss, and M. Greiner, Nature **472**, 307 (2011)

Quantum gas microscope

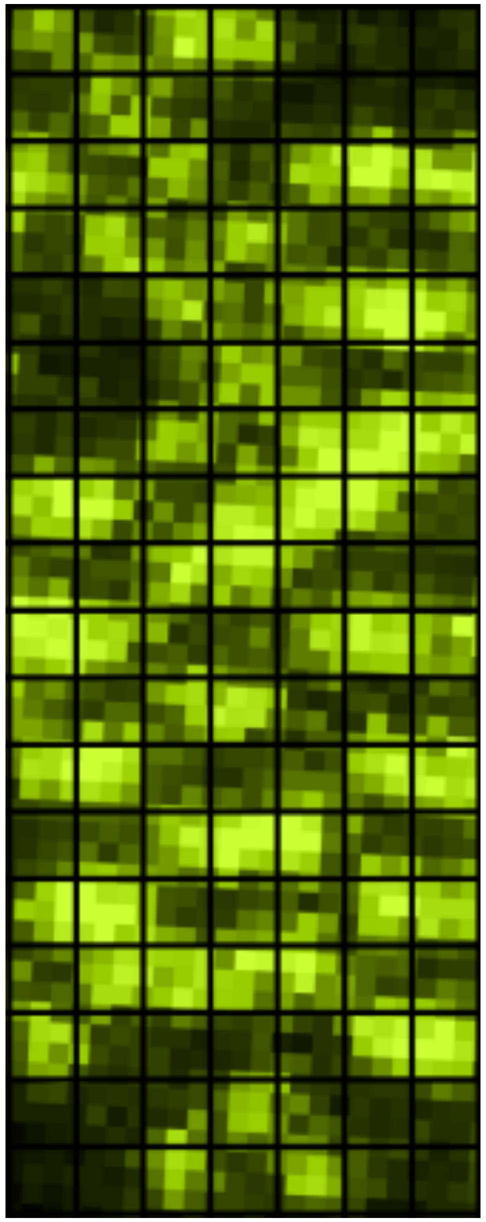


J. Simon, W. S. Bakr, R. Ma, M. E. Tai, P. M. Preiss, and M. Greiner, Nature **472**, 307 (2011)





In–situ imaging of antiferromagnetic chains



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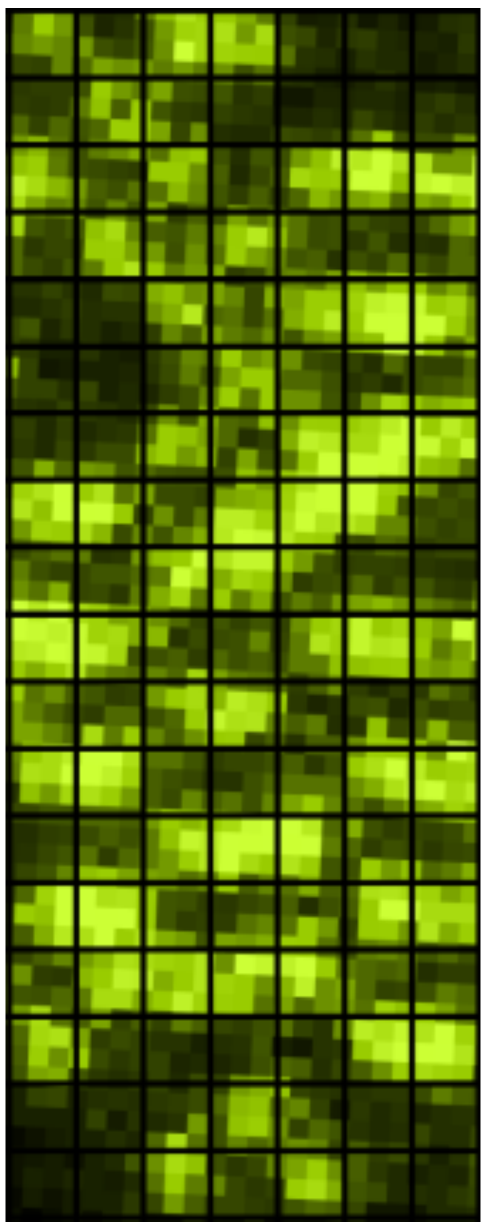
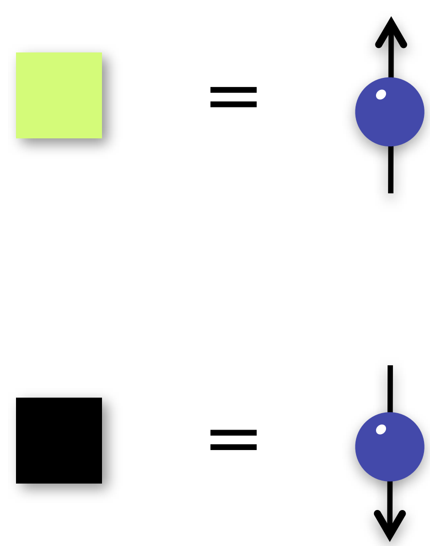
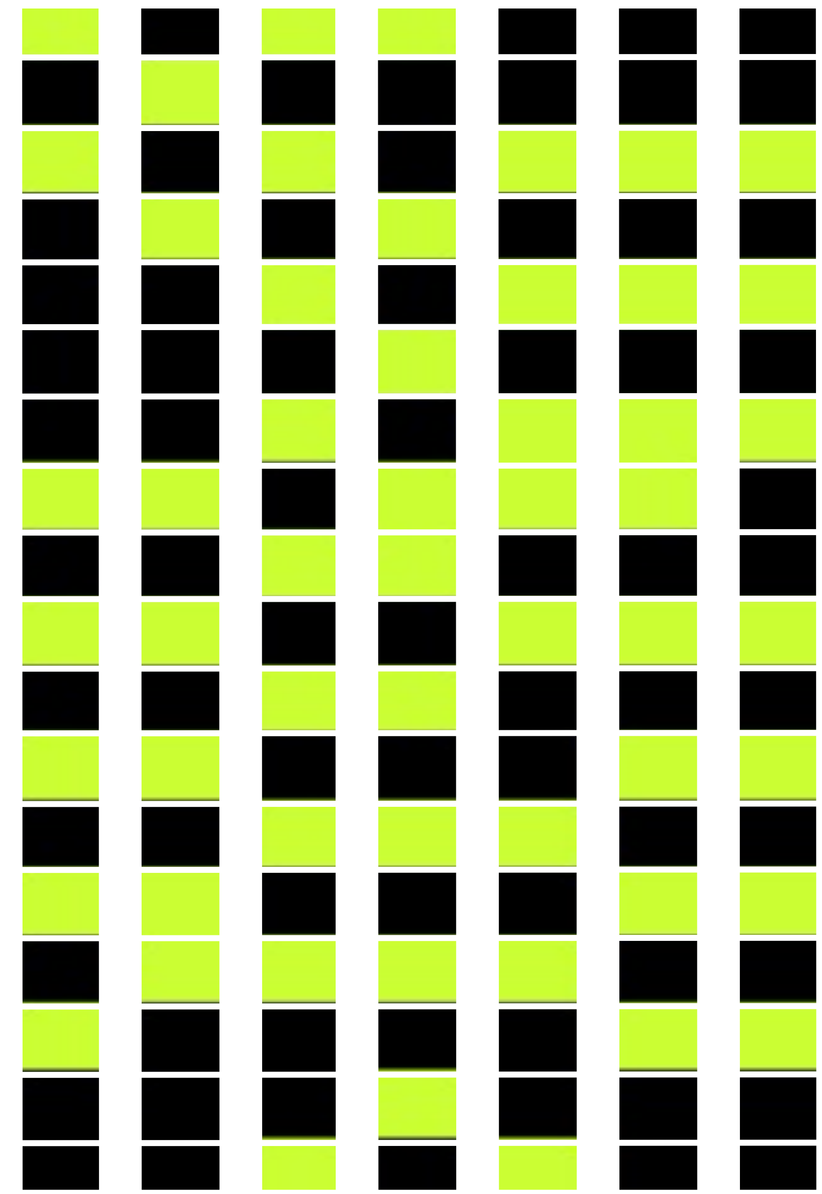
In-situ imaging of antiferromagnetic chains







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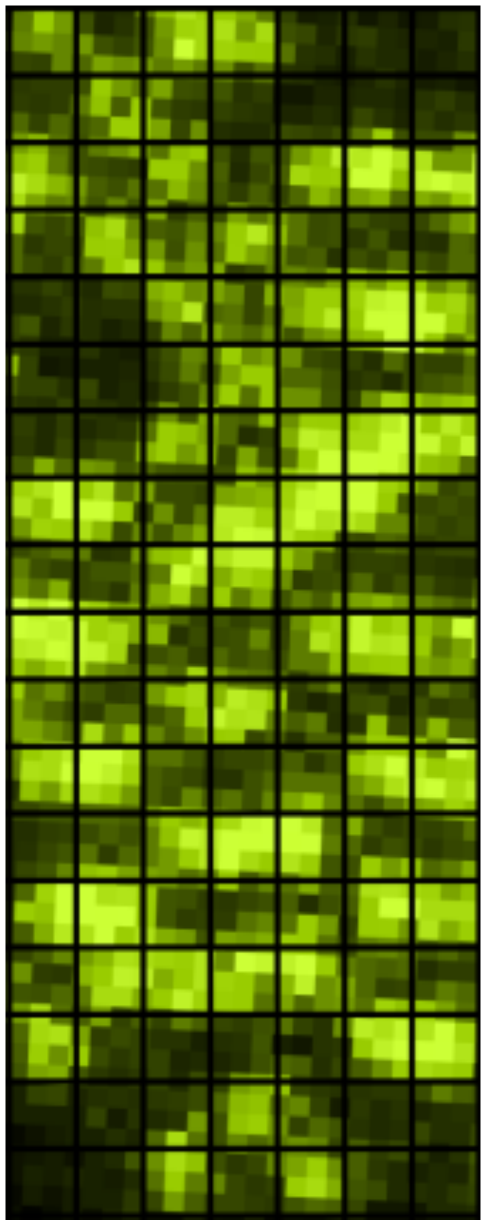
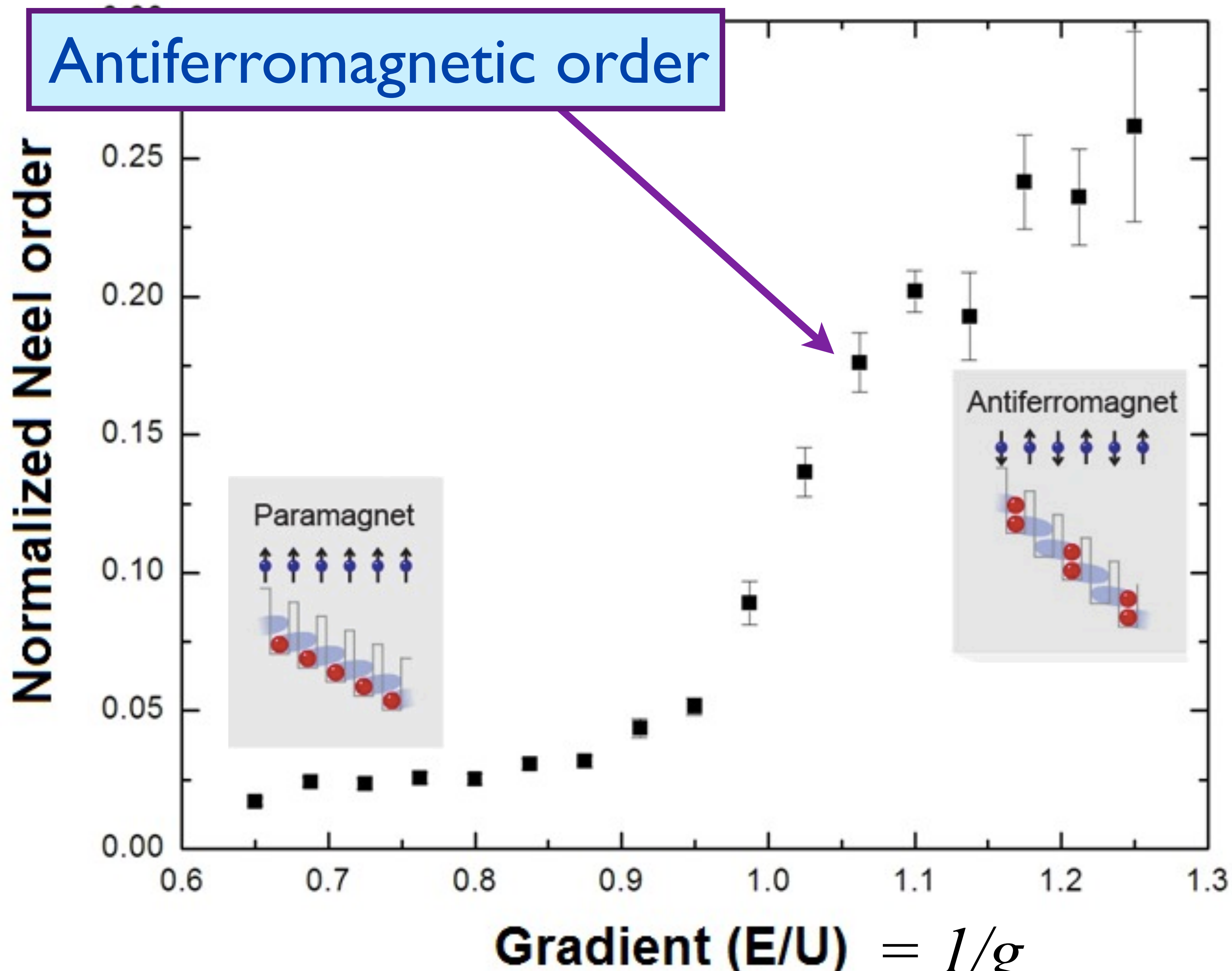
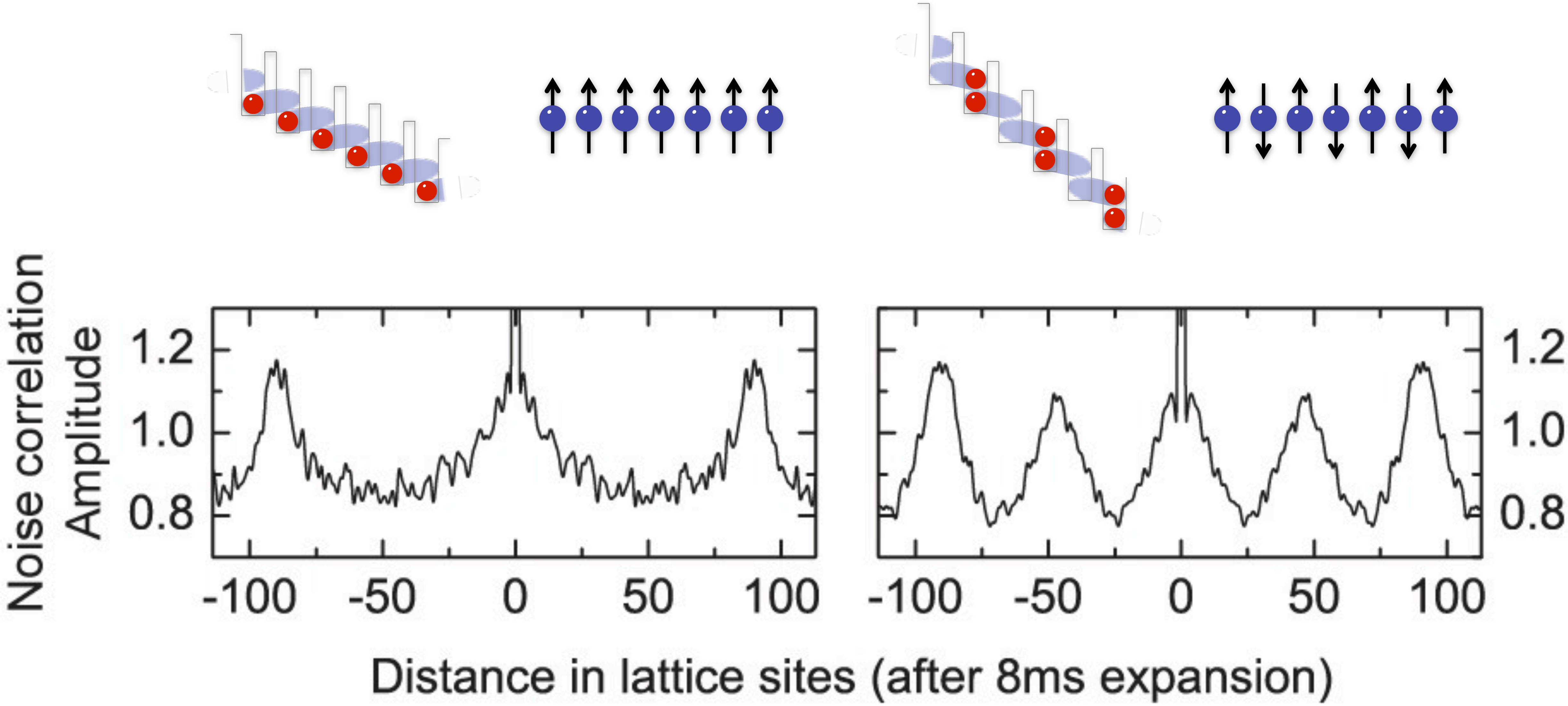




$= 1/g$

J. Simon, W. S. Bakr, R. Ma, M. E. Tai, P. M. Preiss, and M. Greiner, Nature **472**, 307 (2011)

Hanbury-Brown-Twiss noise correlations measure Fourier transform of boson density



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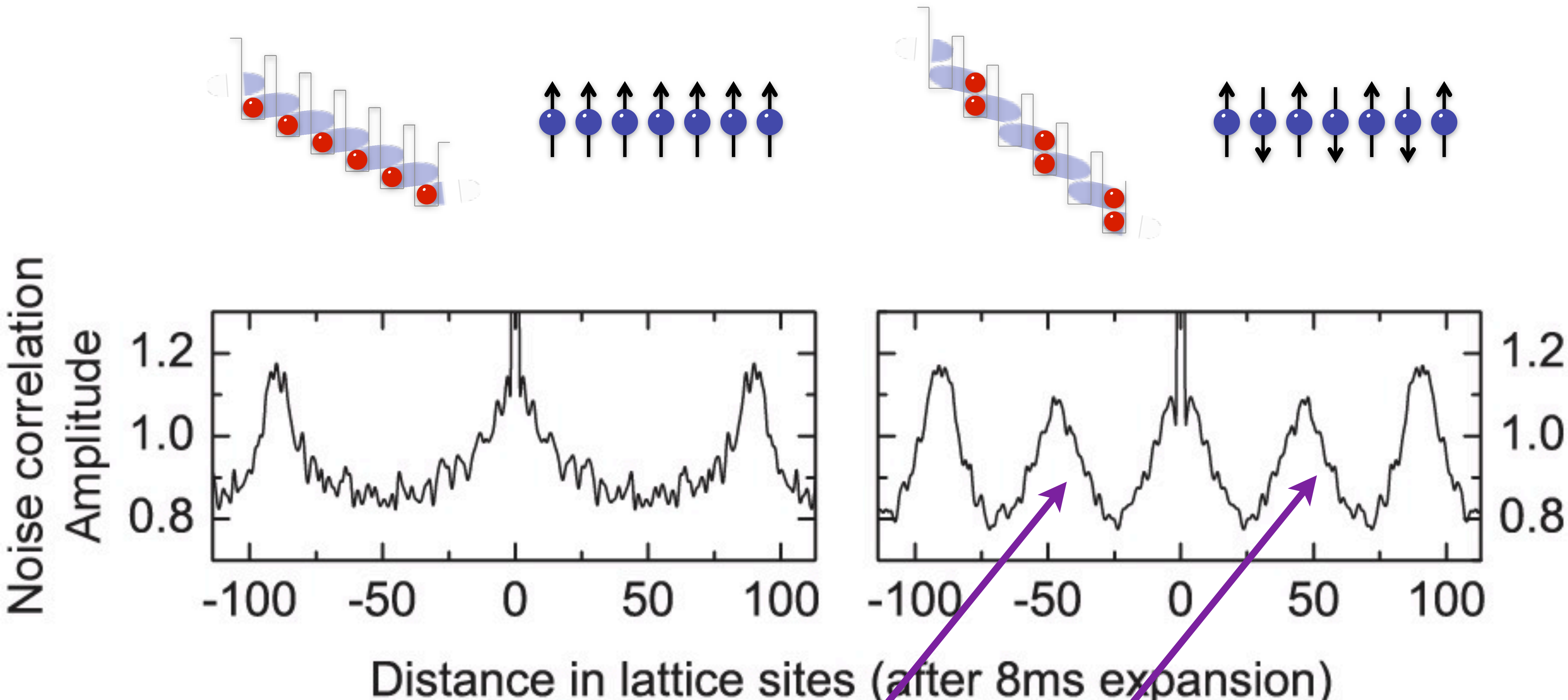

Peaks from antiferromagnetic order

J. Simon, W. S. Bakr, R. Ma, M. E. Tai, P. M. Preiss, and M. Greiner, Nature **472**, 307 (2011)

Outline

1. The quantum Ising chain

    A. The magnetic insulator $CoNb_2O_6$

    B. Ultracold Rb atoms in an optical lattice

2. Nonzero temperatures and quantum criticality

    Antiferromagnetic insulators

3. Higher temperature superconductors and “strange metals”

    Quantum criticality of fermions and Fermi surfaces

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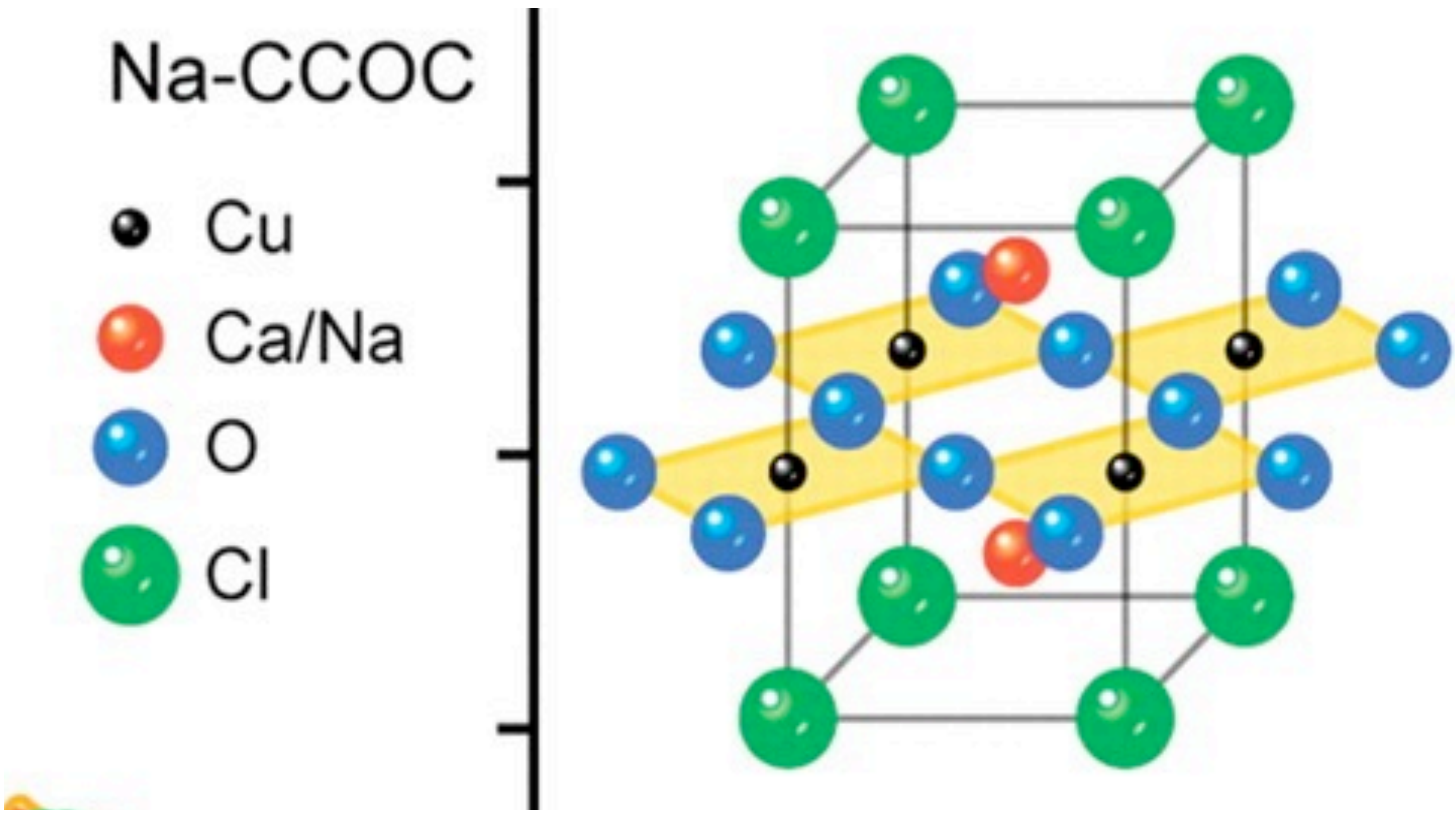
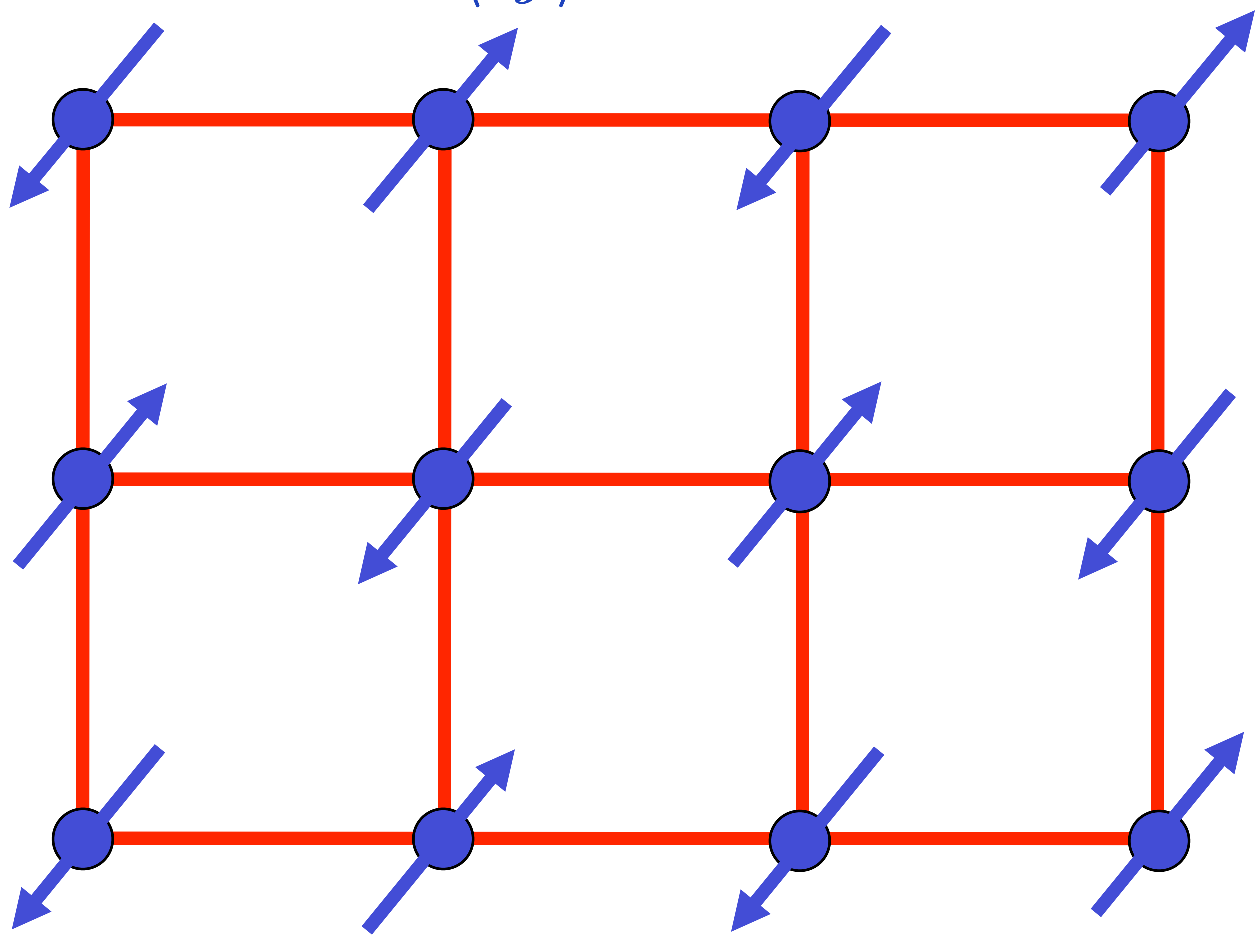
The cuprate superconductors



Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

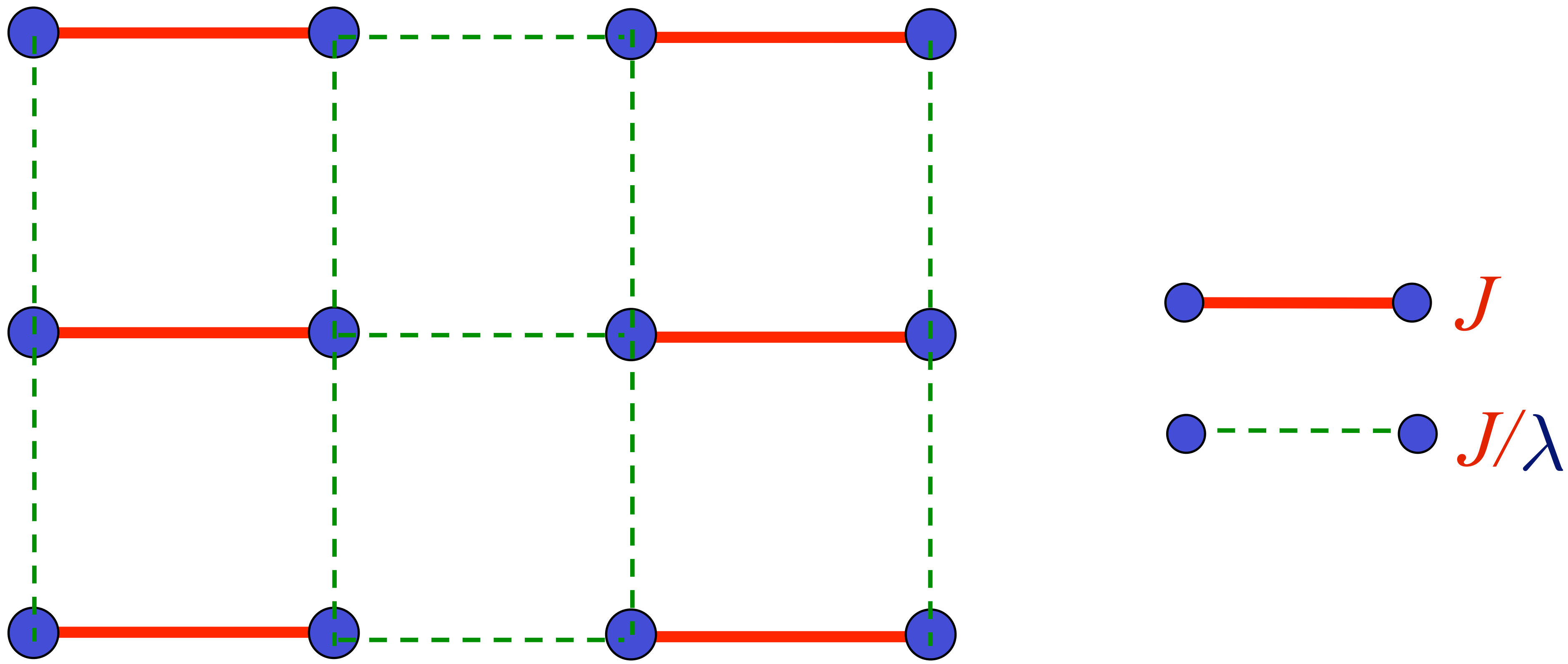
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

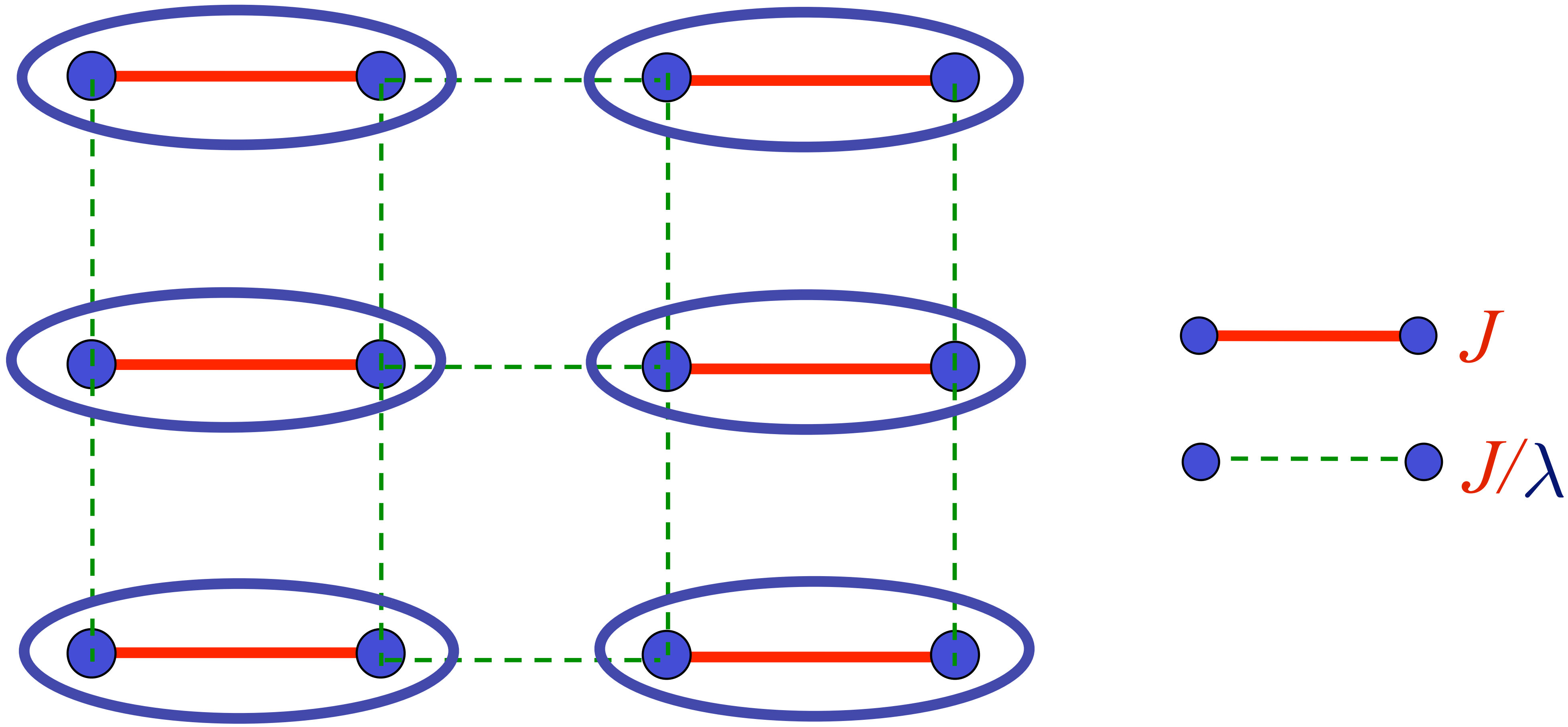
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



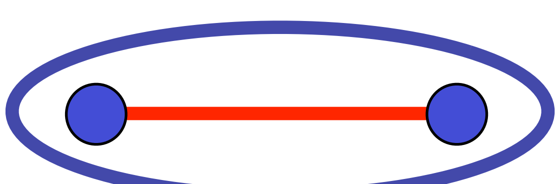
Weaken some bonds to induce spin entanglement in a new quantum phase

Square lattice antiferromagnet

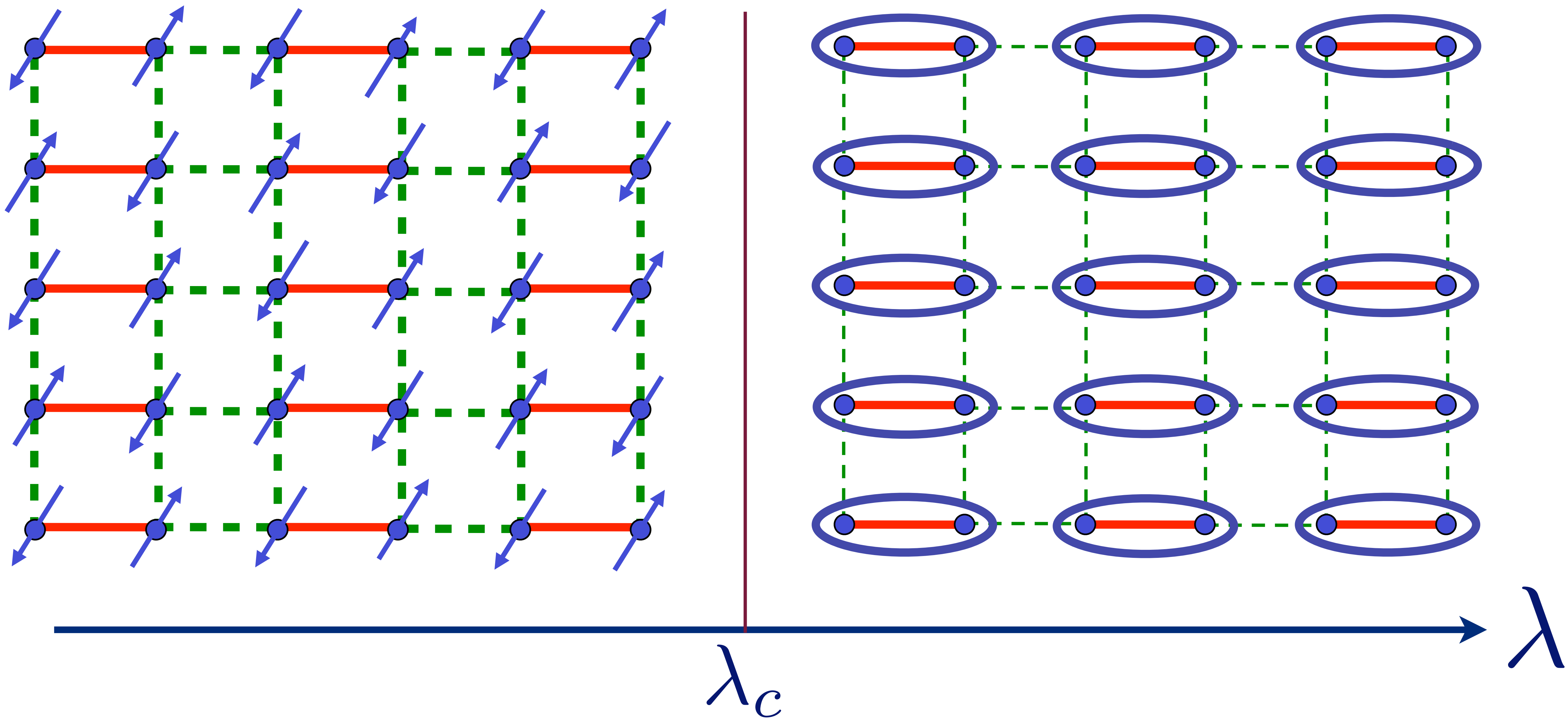
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

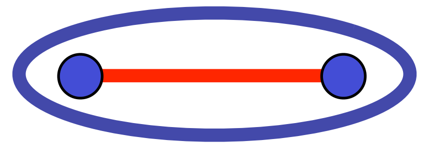


Ground state is a "quantum paramagnet" with spins locked in valence bond singlets

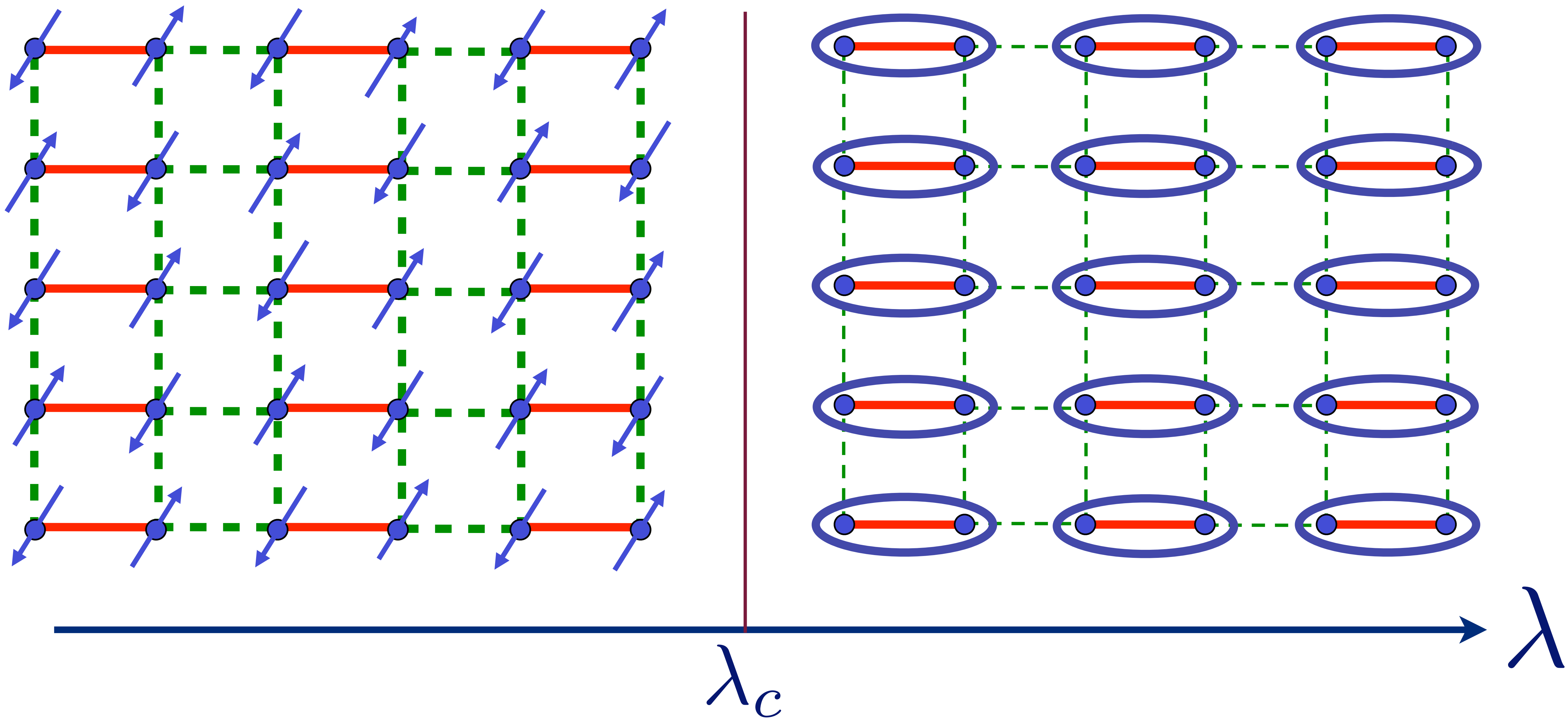
 $= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$

$$= \frac{1}{\sqrt{2}}\Big(\big|\uparrow\downarrow\big\rangle - \big|\downarrow\uparrow\big\rangle\Big)$$





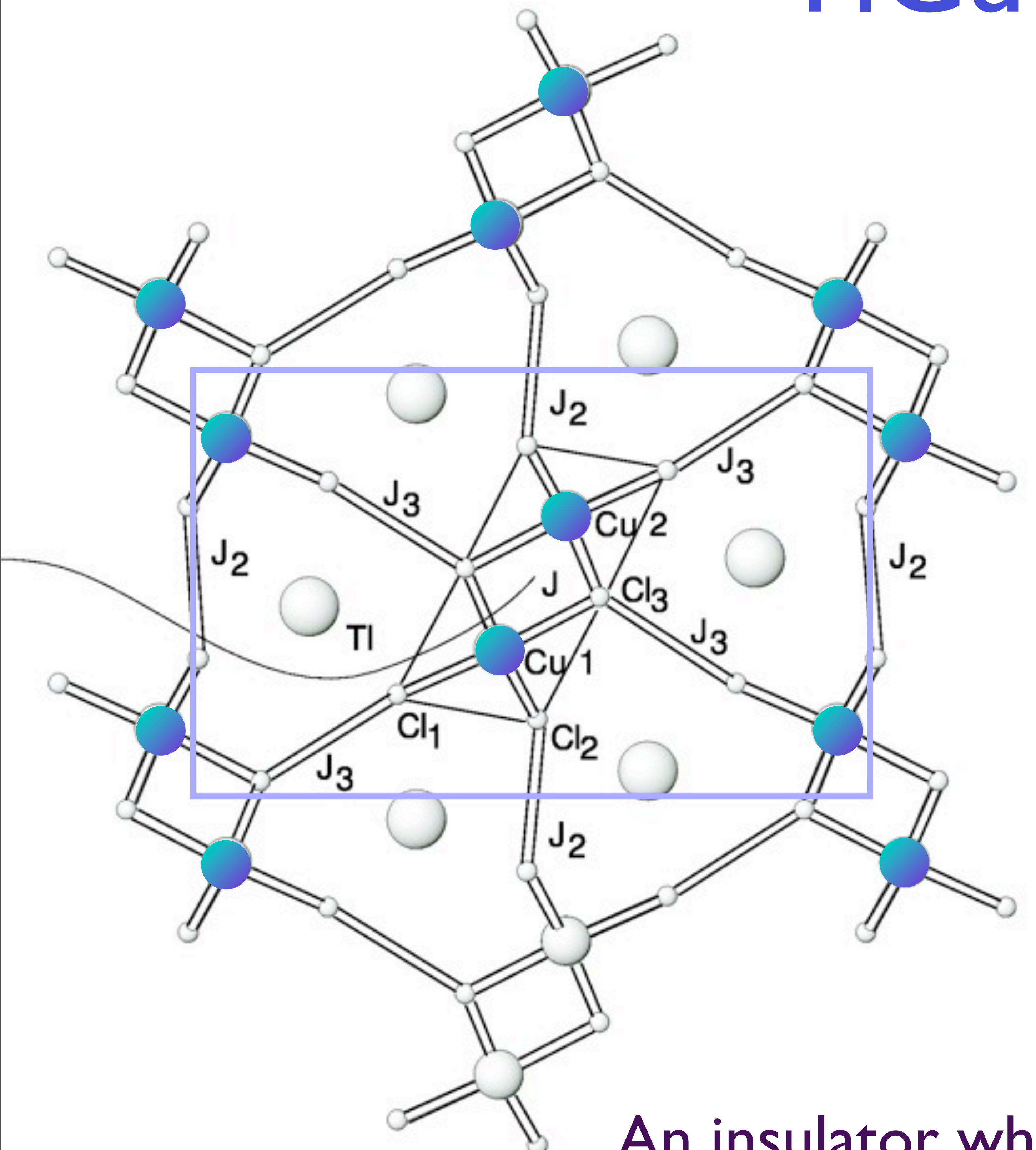
$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

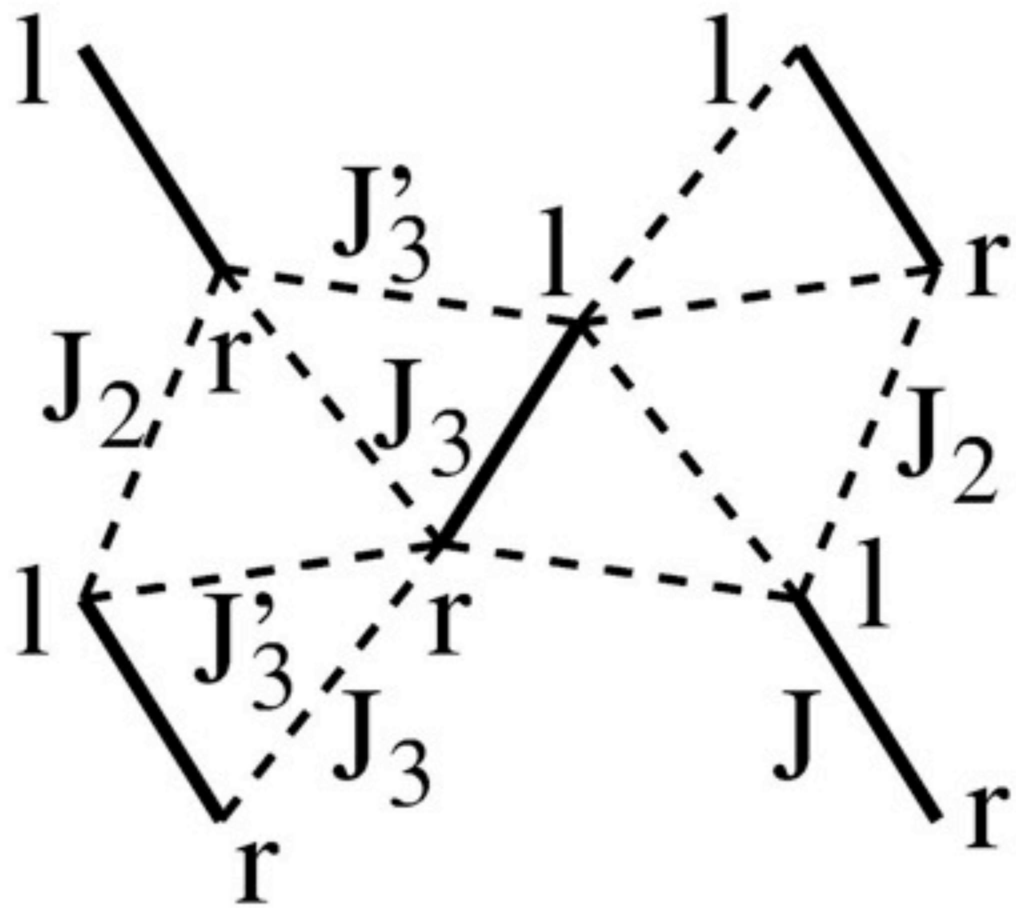


Pressure in $TlCuCl_3$

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

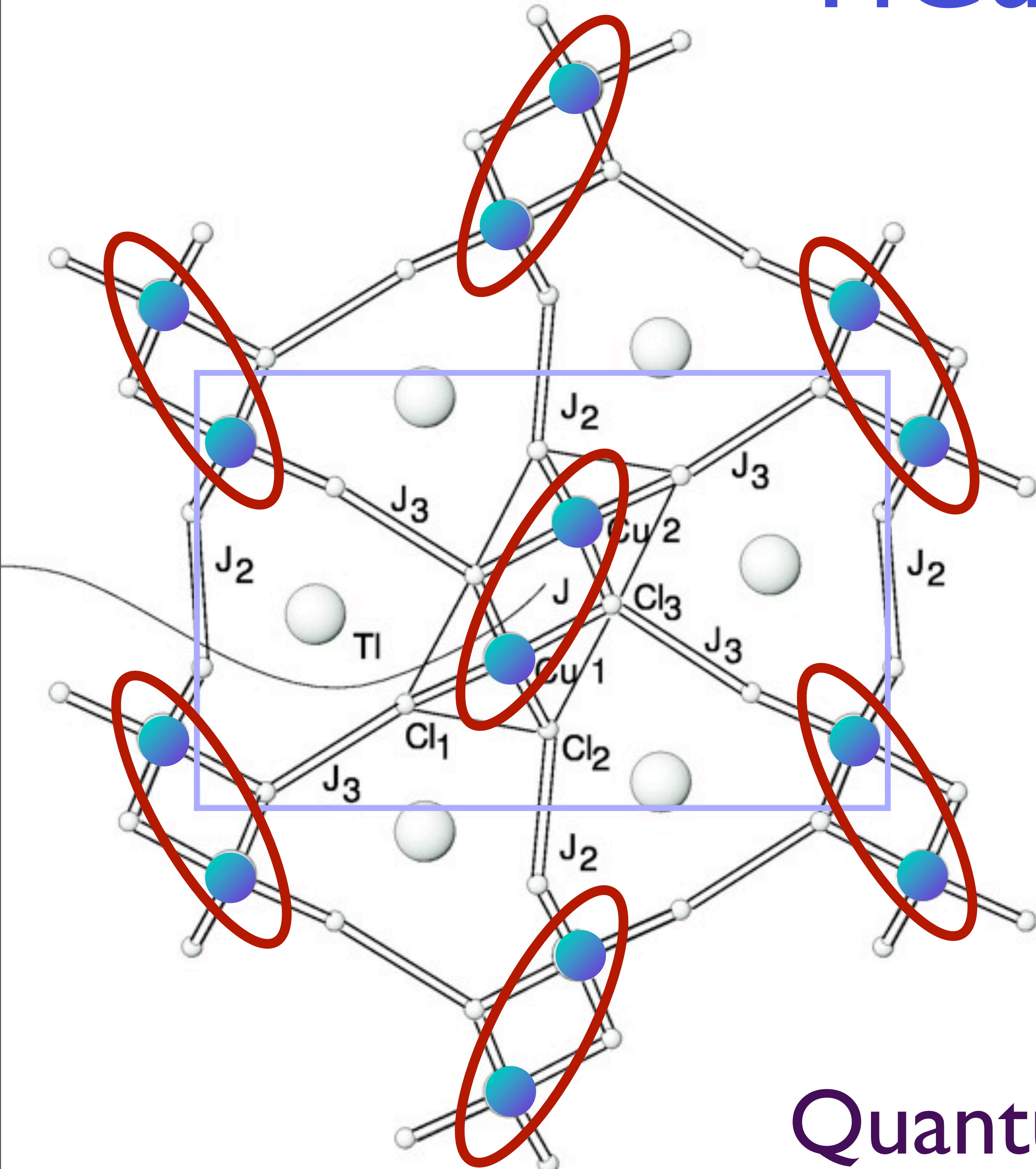
$TlCuCl_3$

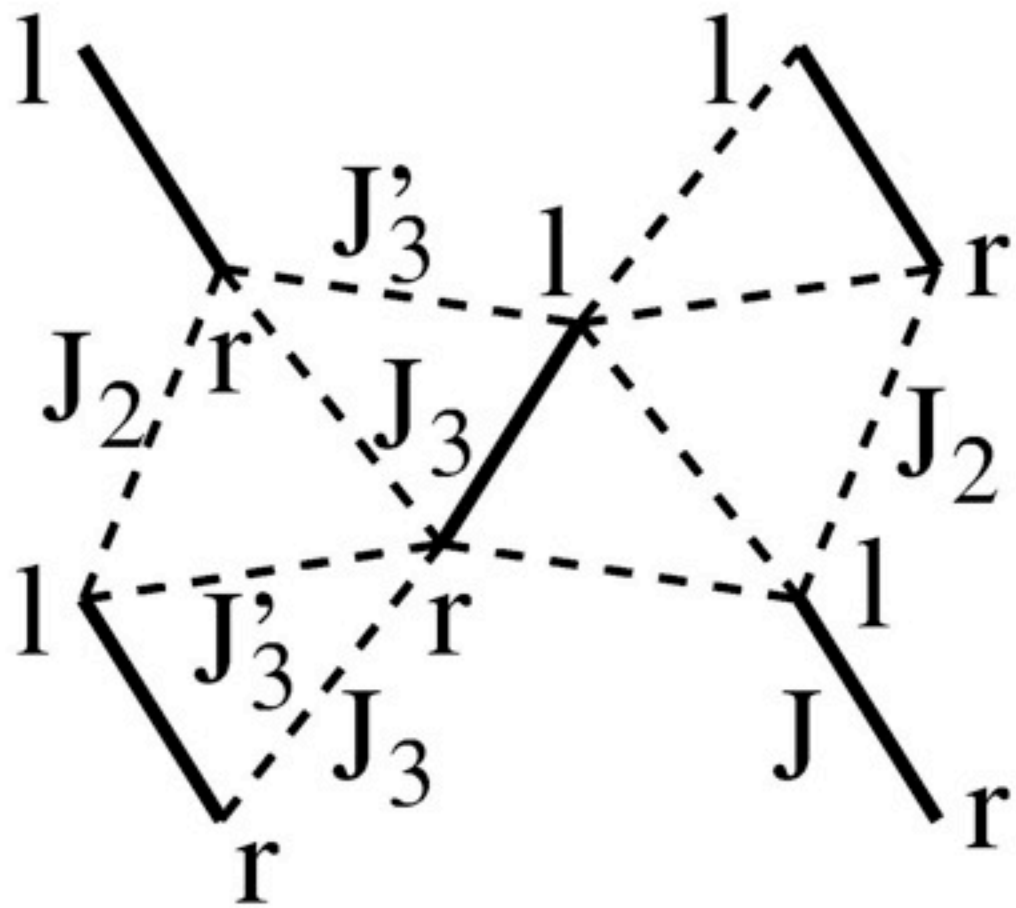




An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

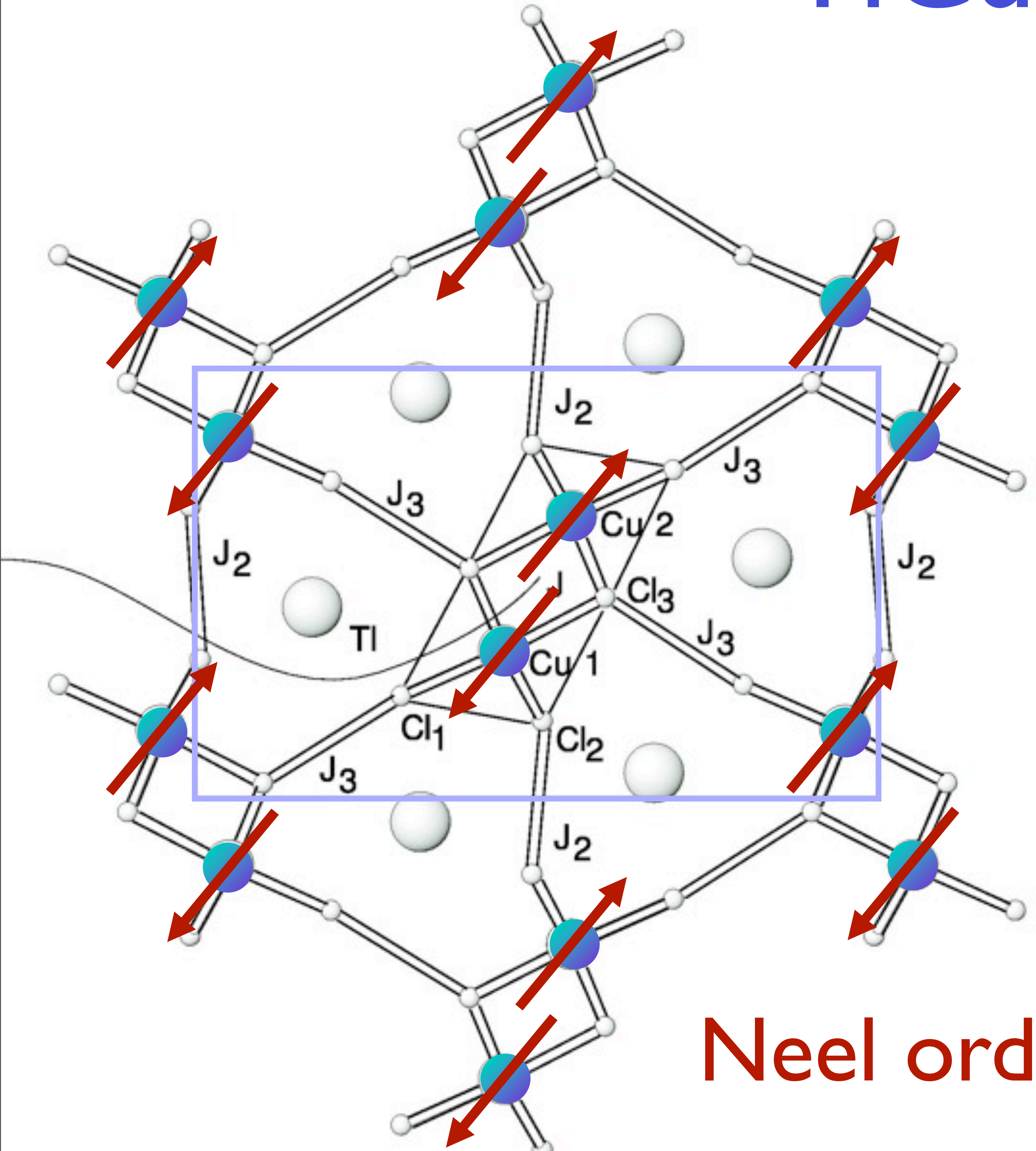
TlCuCl$_3$

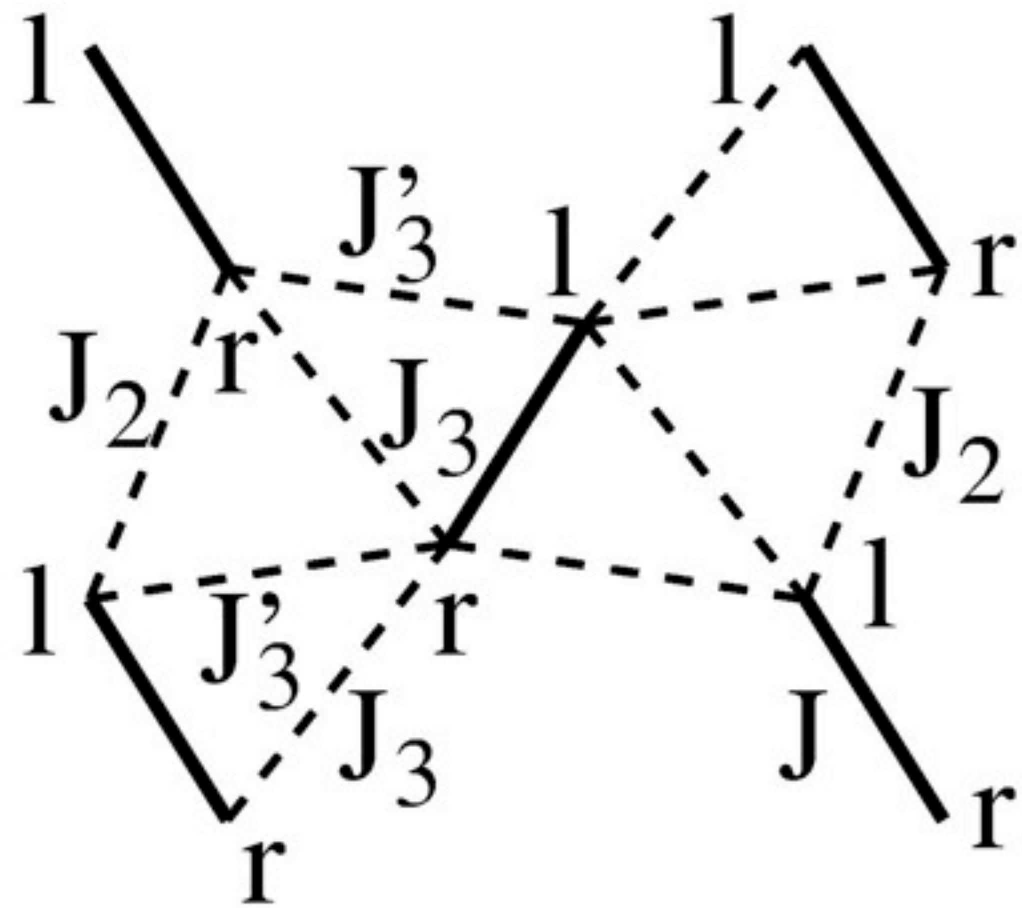




Quantum paramagnet at ambient pressure

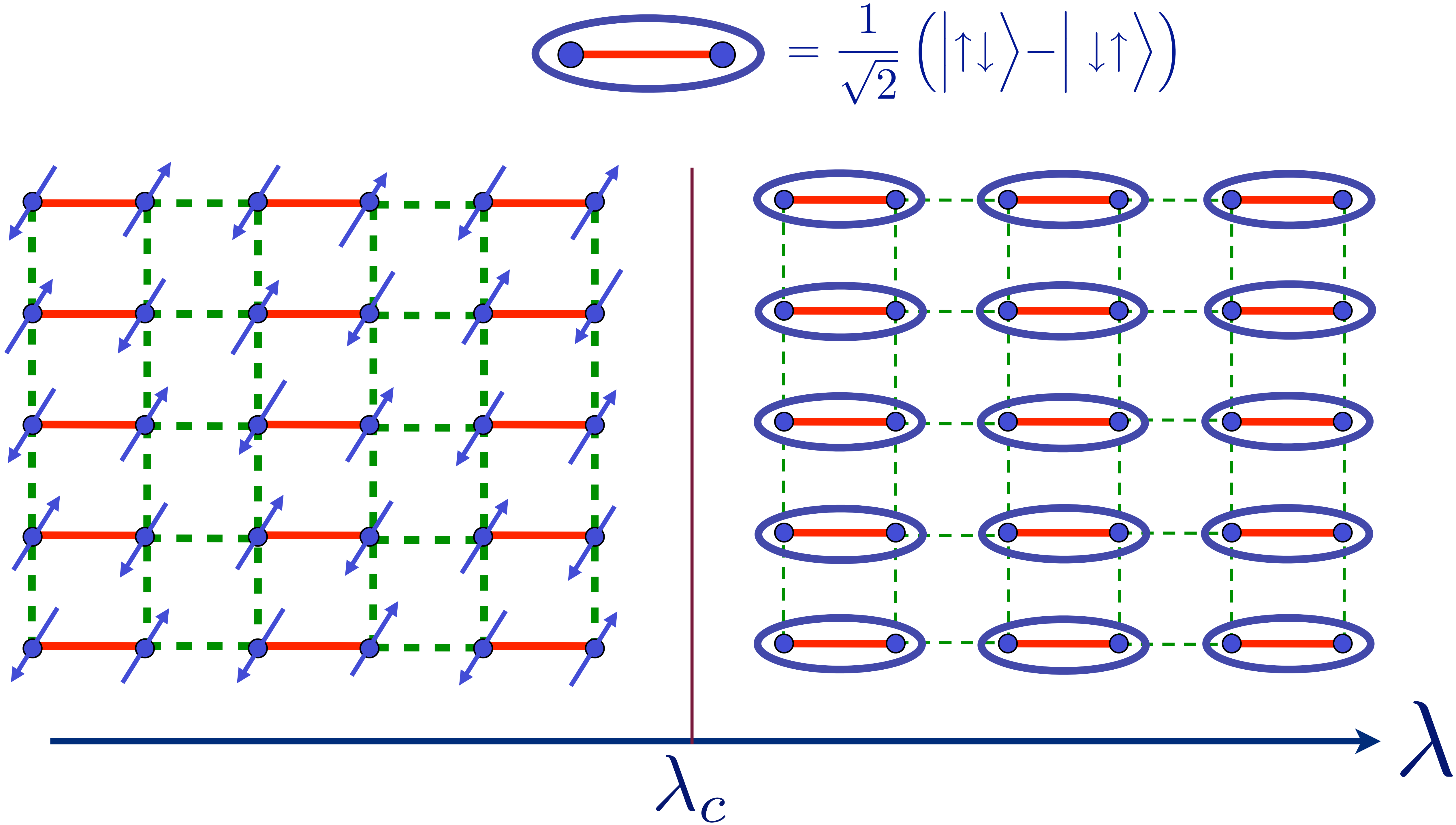
$TlCuCl_3$





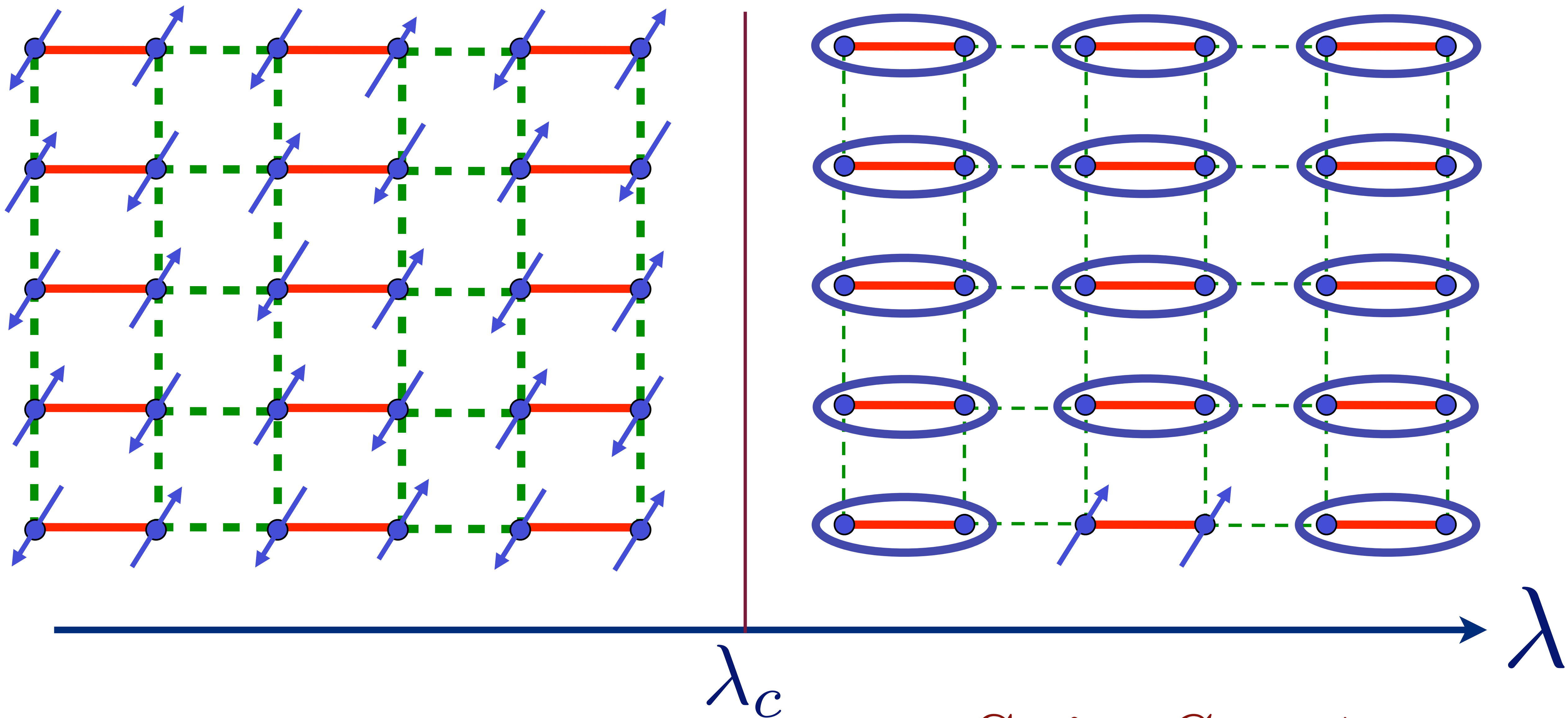
Neel order under pressure

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).



$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$
λ_c
λ

Excitation spectrum in the paramagnetic phase



Spin $S = 1$
"triplon"

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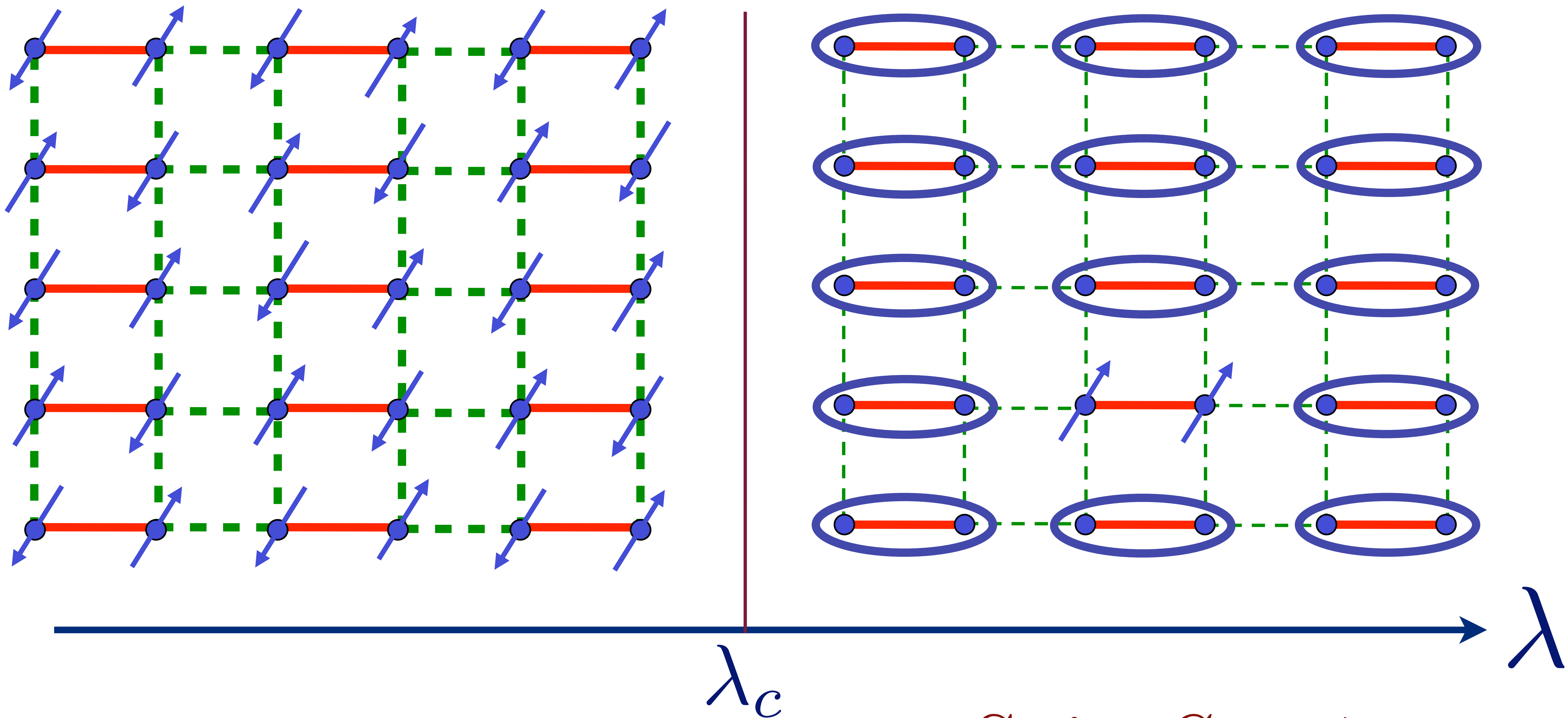

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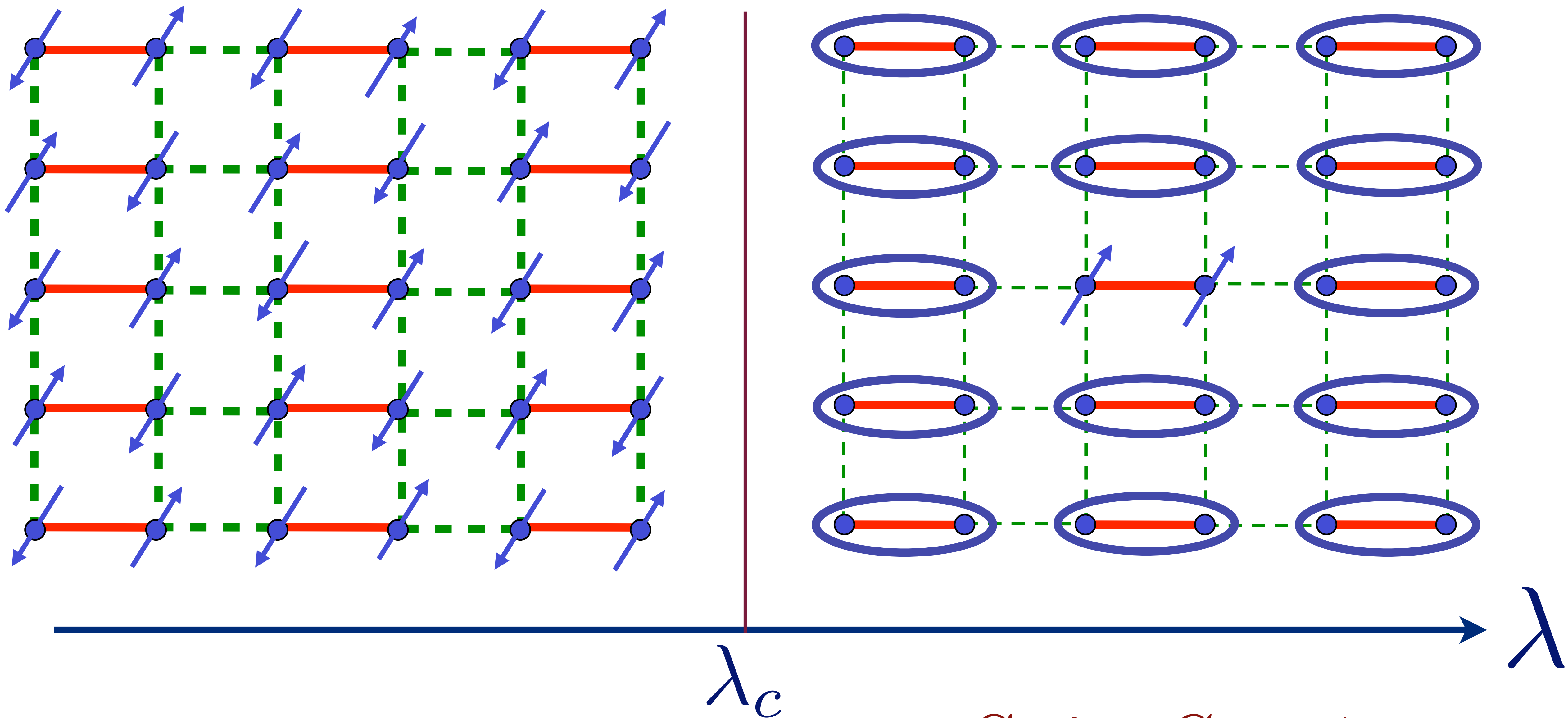

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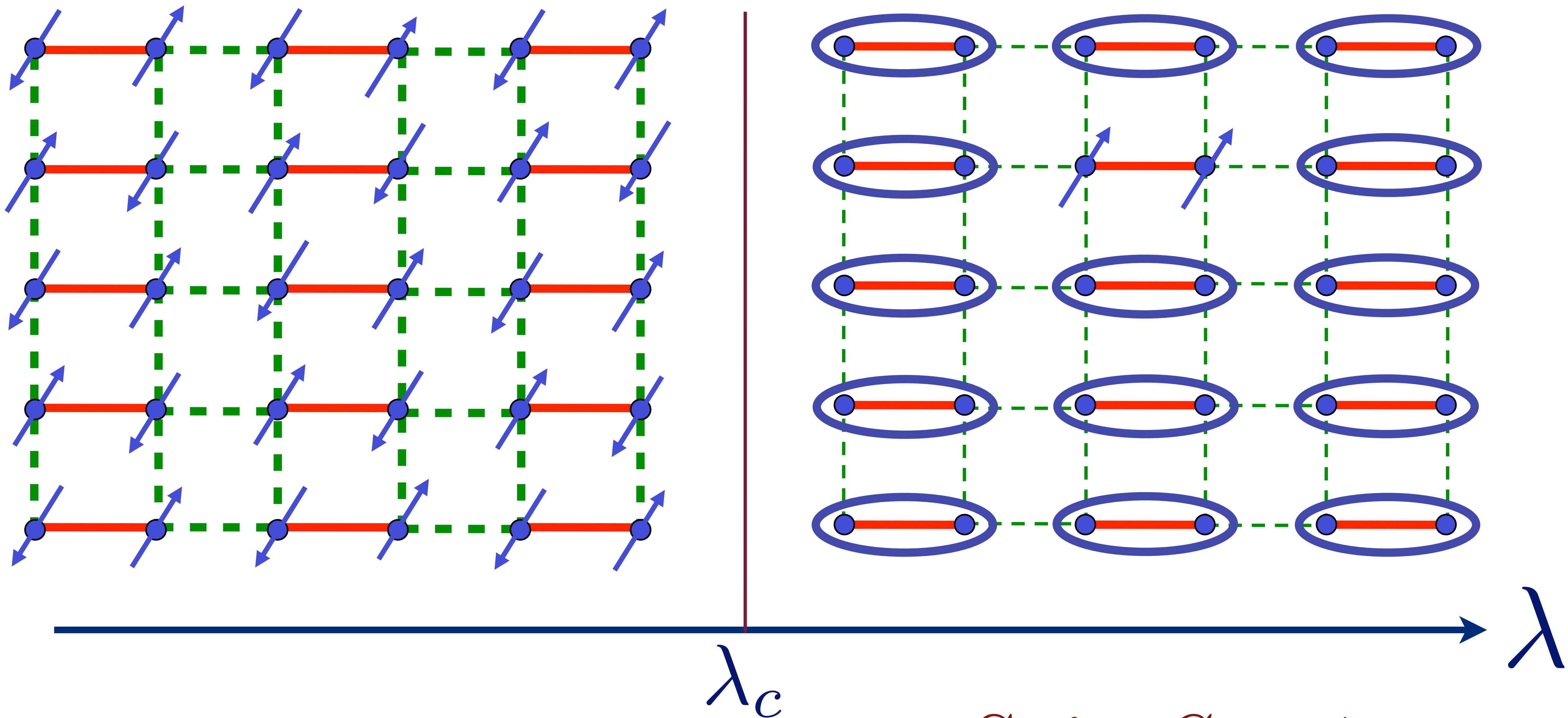
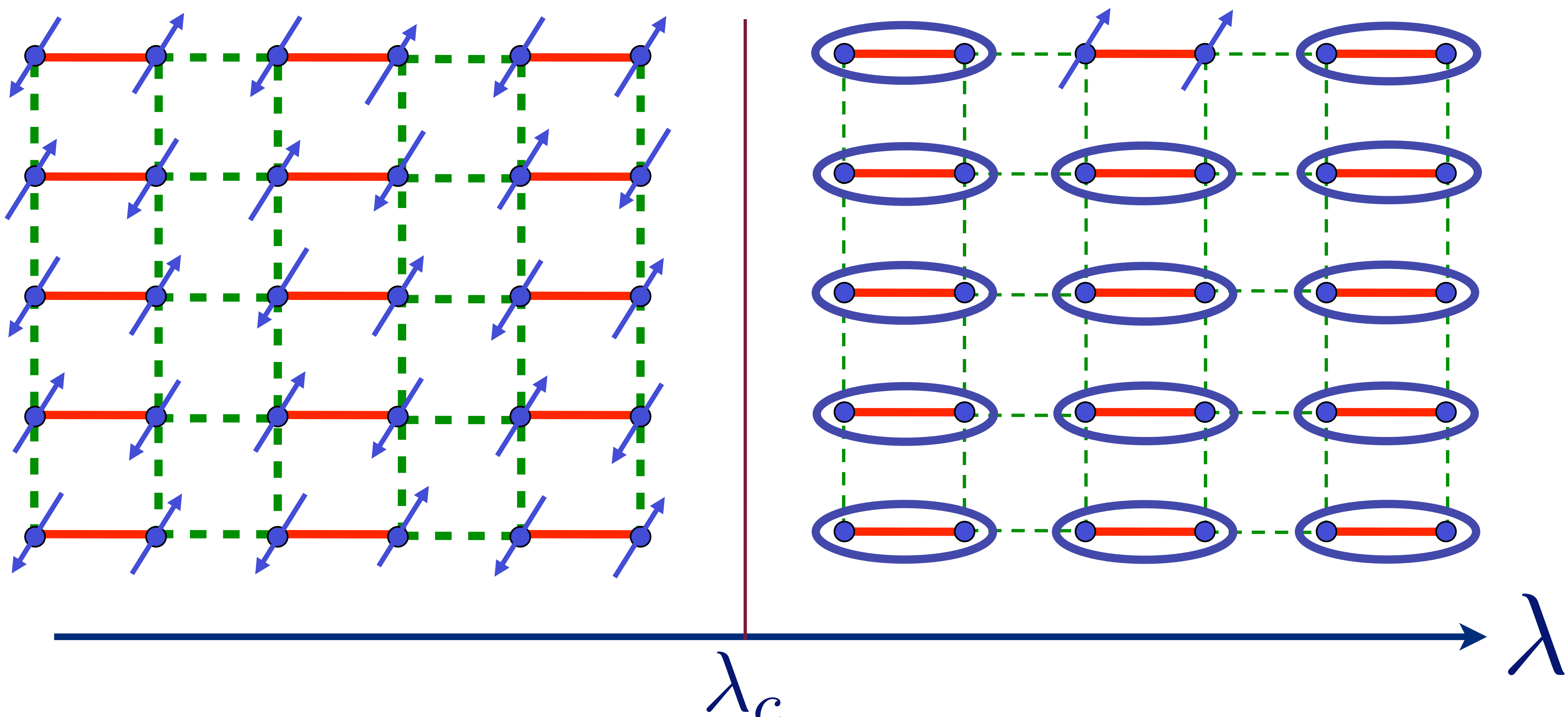


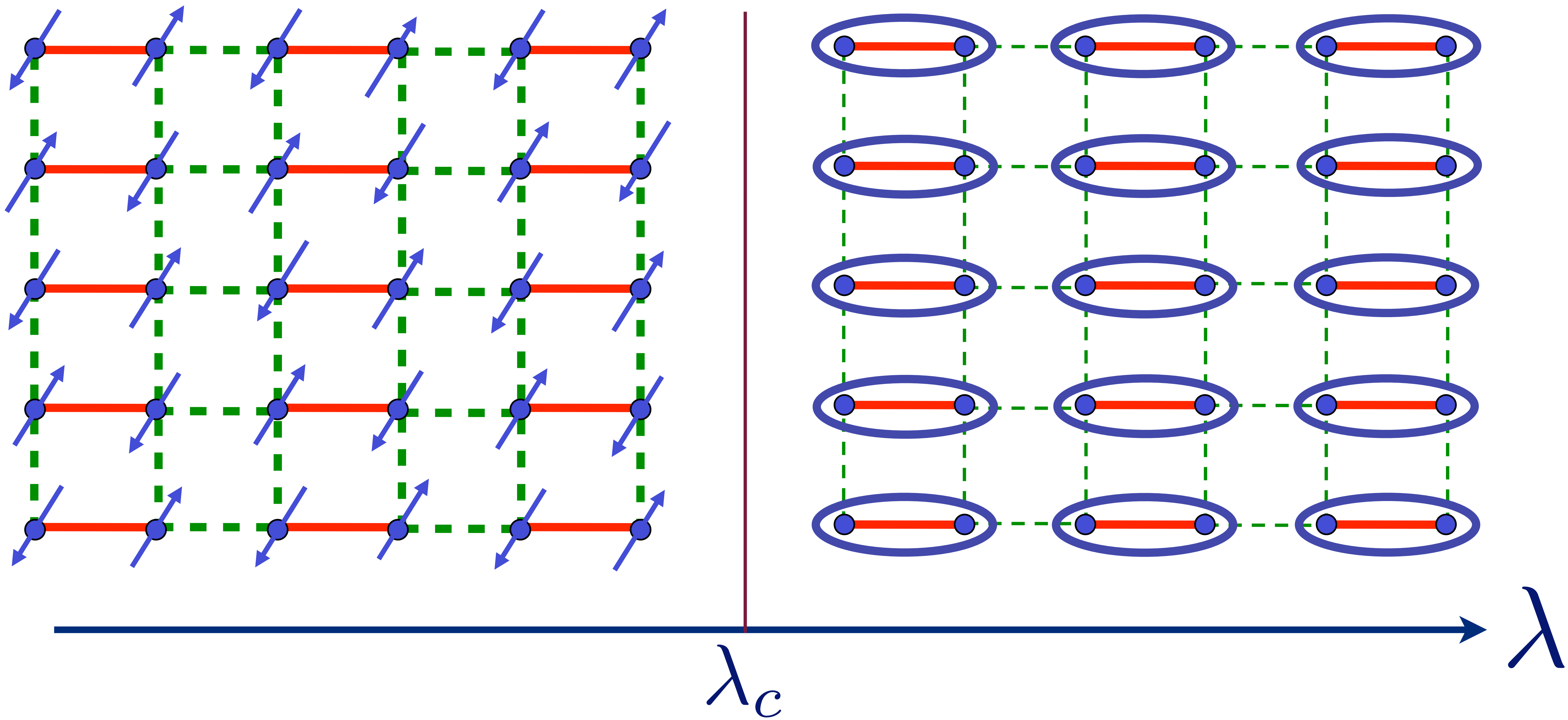
Spin $S = 1$
"triplon"

Excitation spectrum in the paramagnetic phase



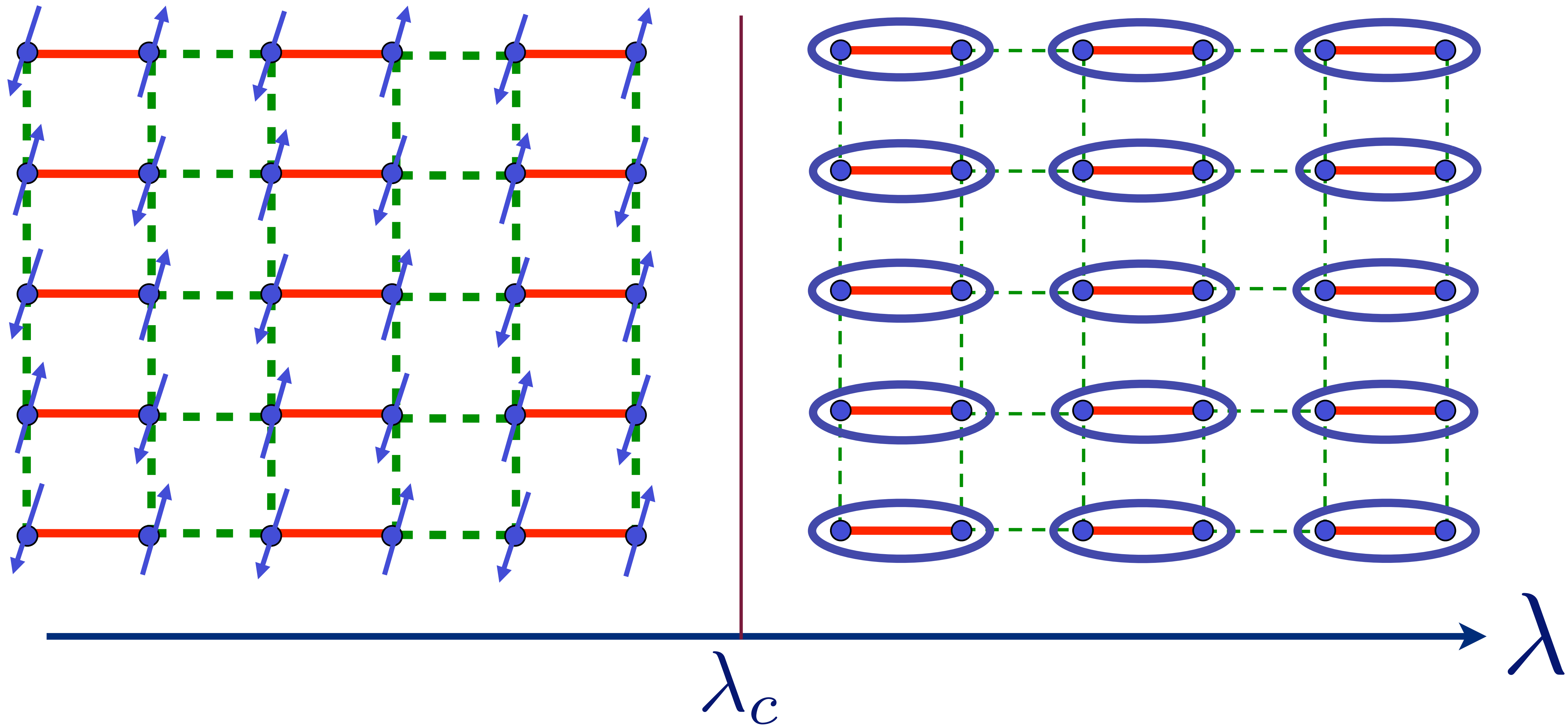
Spin $S = 1$
"triplon"

Excitation spectrum in the Néel phase



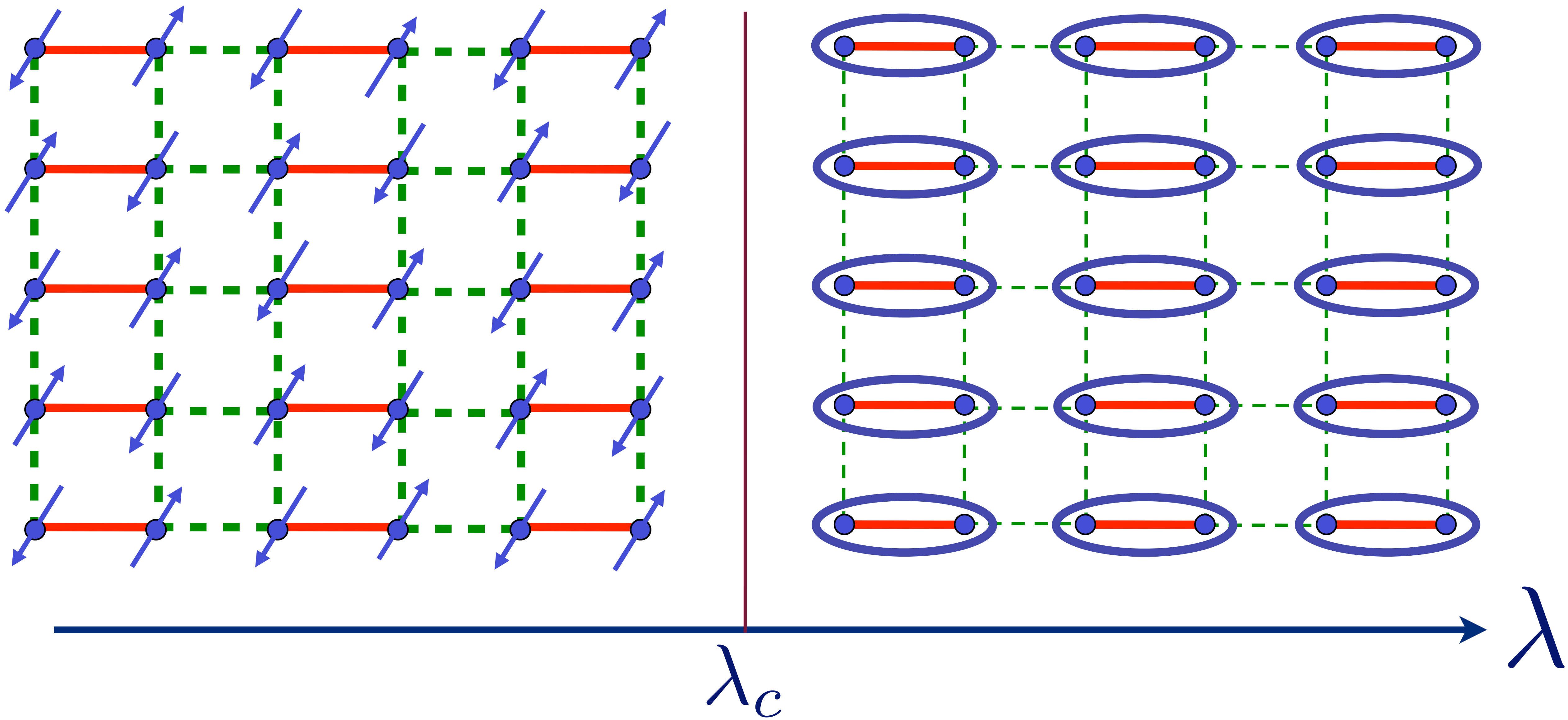
Spin waves

Excitation spectrum in the Néel phase



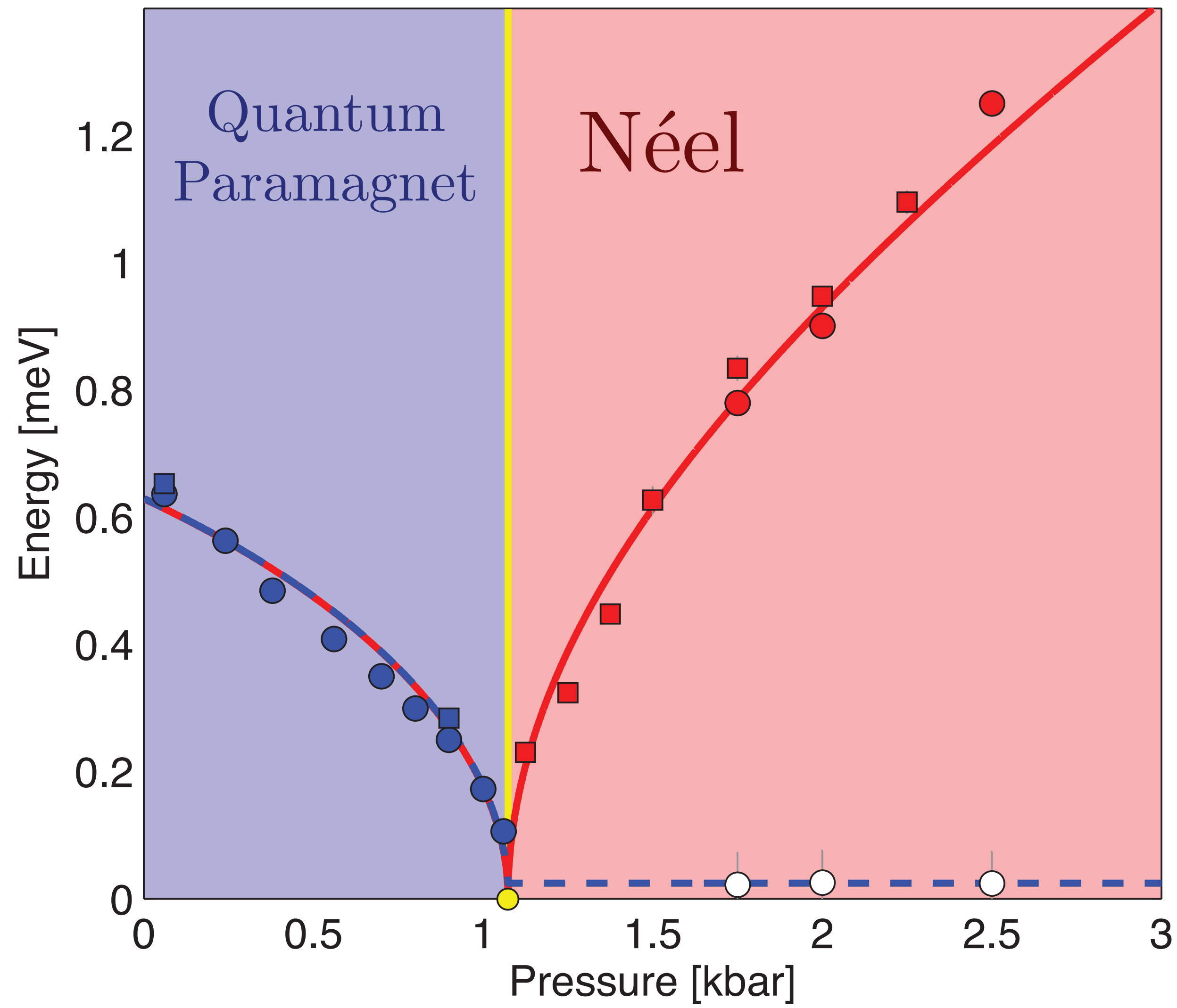
Spin waves

Excitation spectrum in the Néel phase



Spin waves

$TlCuCl_3$ with varying pressure



Triplon in the quantum paramagnet.
Spin waves and a new "Higgs" particle in the Néel phase: the latter represents longitudinal oscillations in the magnitude of the Néel order.

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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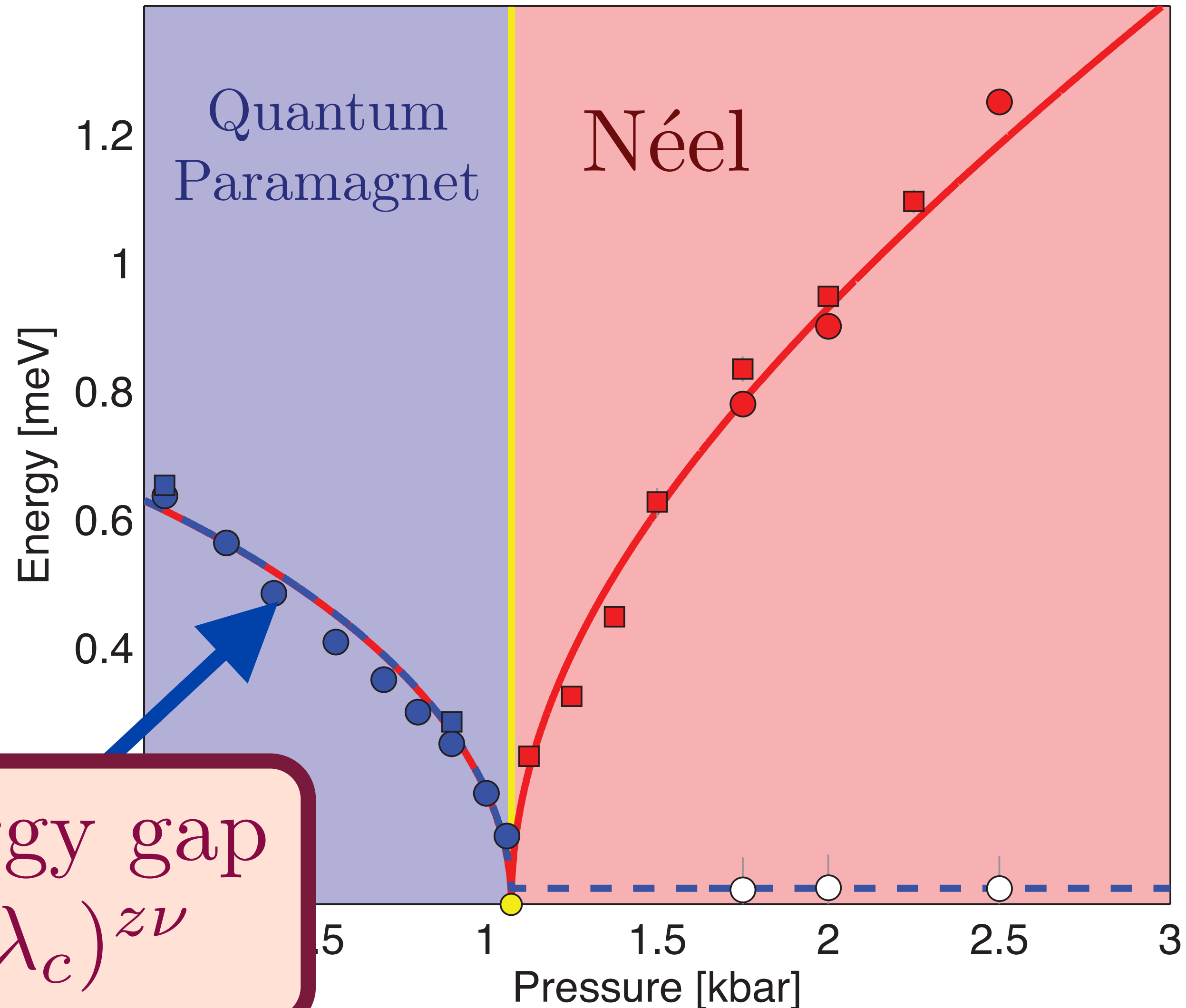

Triplon energy gap

$$\Delta \sim (\lambda - \lambda_c)^{z\nu}$$

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Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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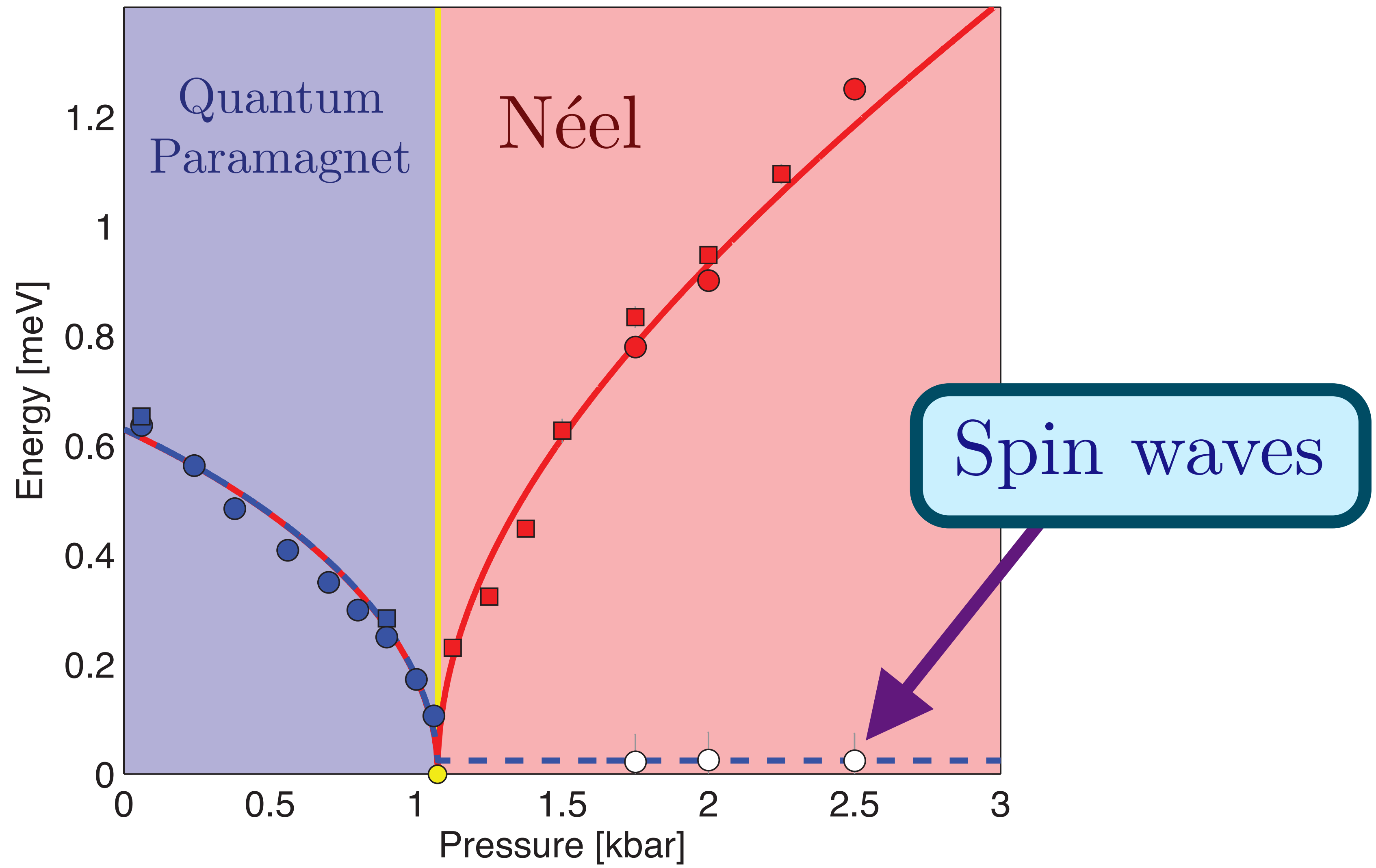

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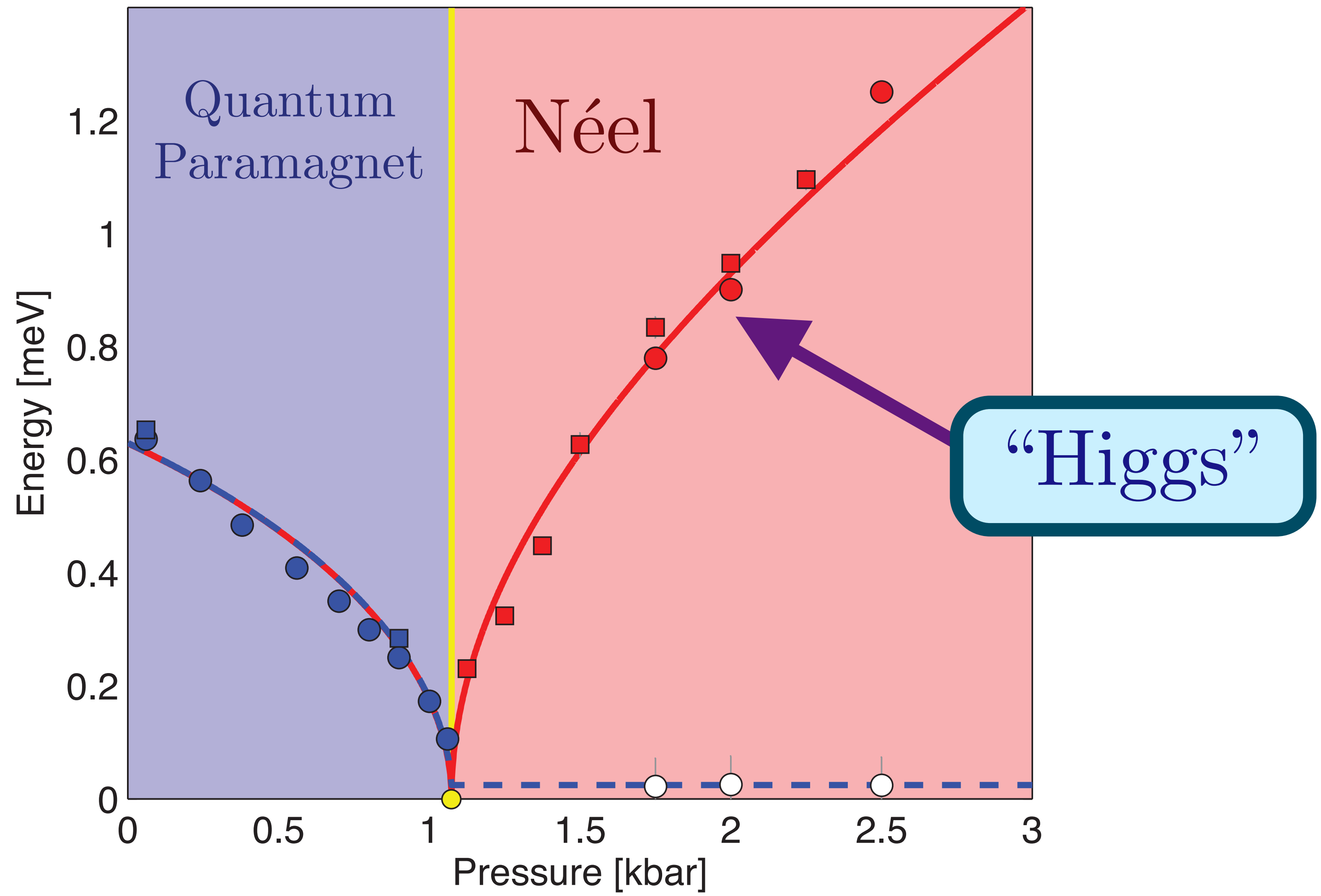

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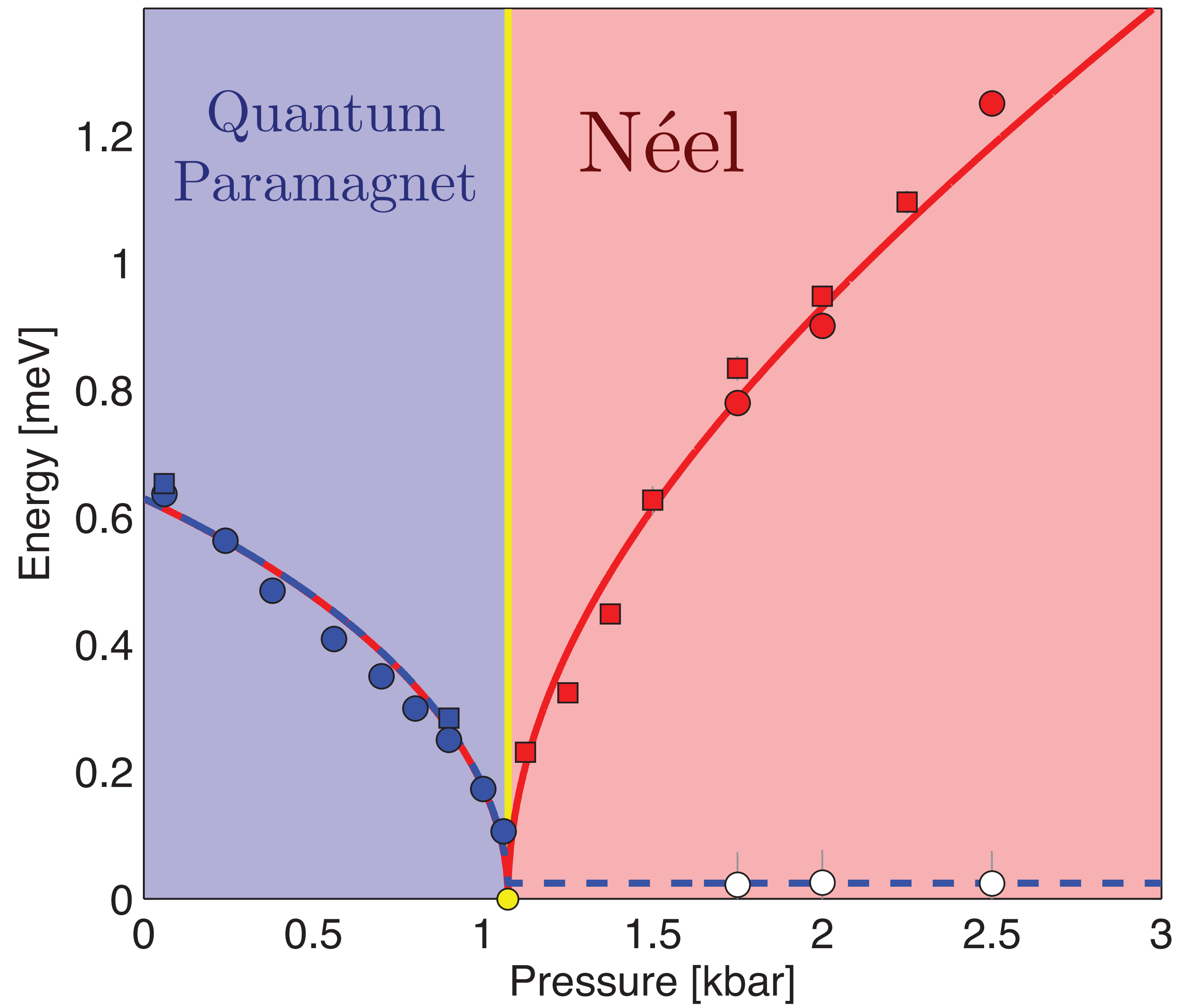

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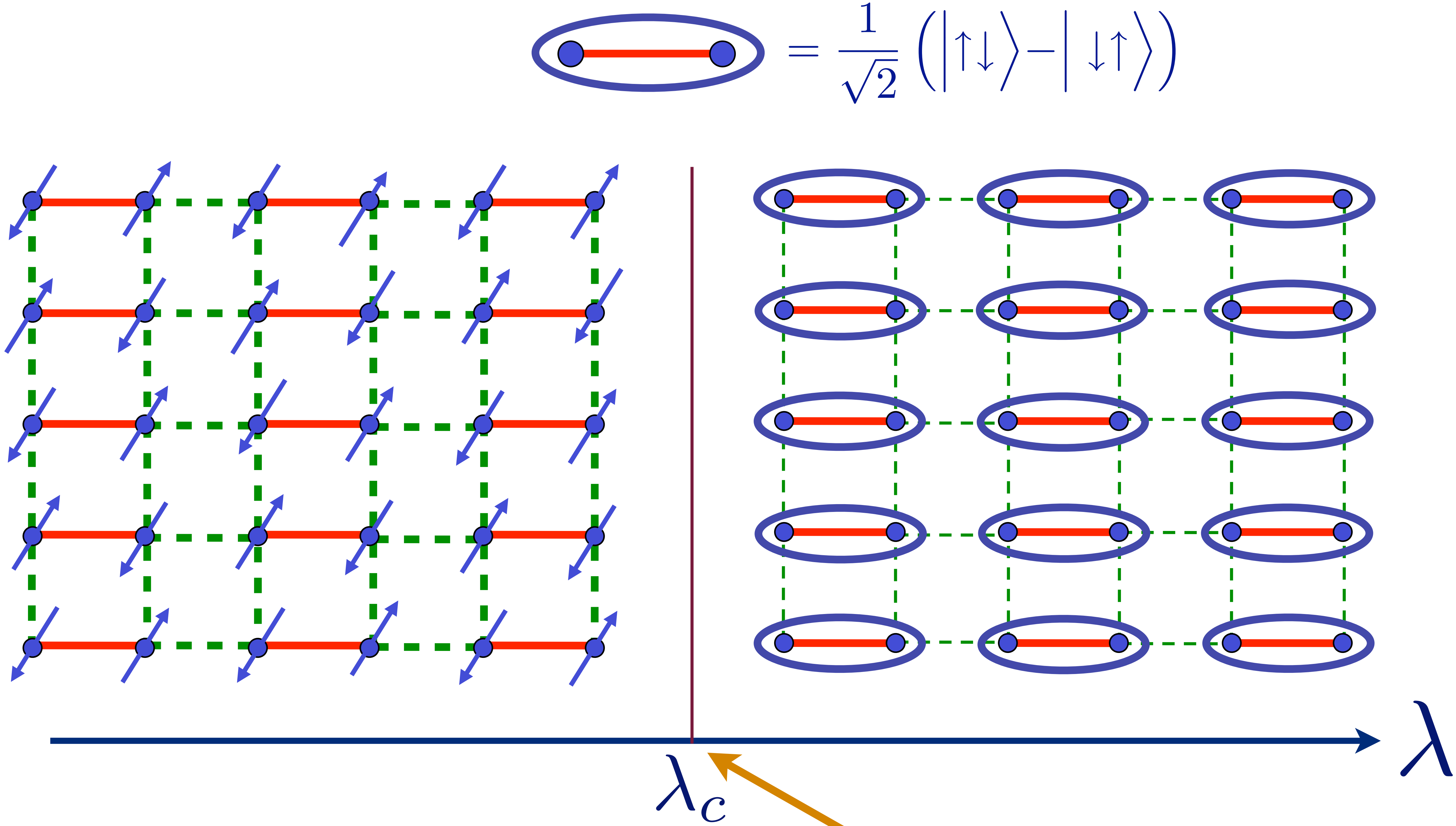
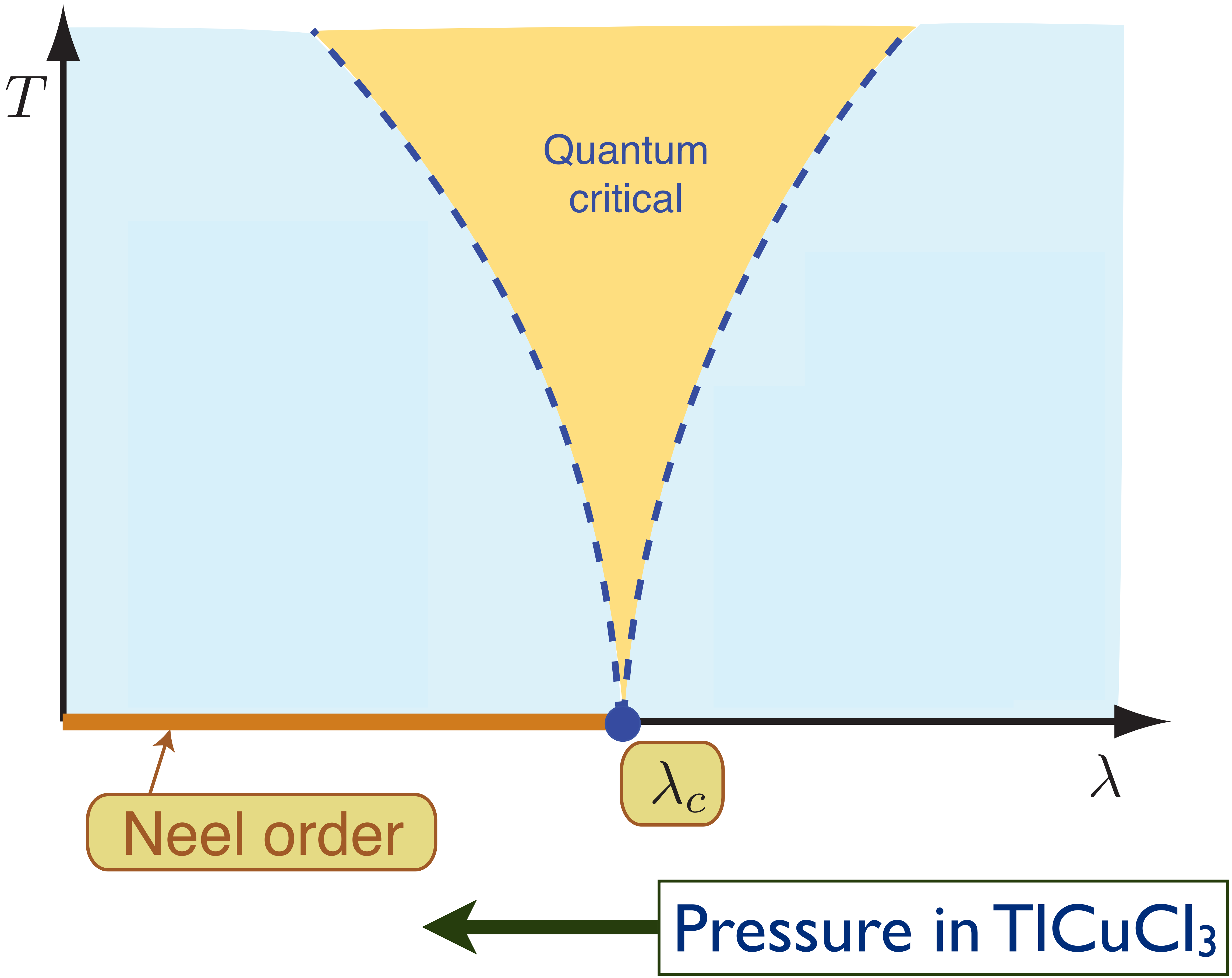
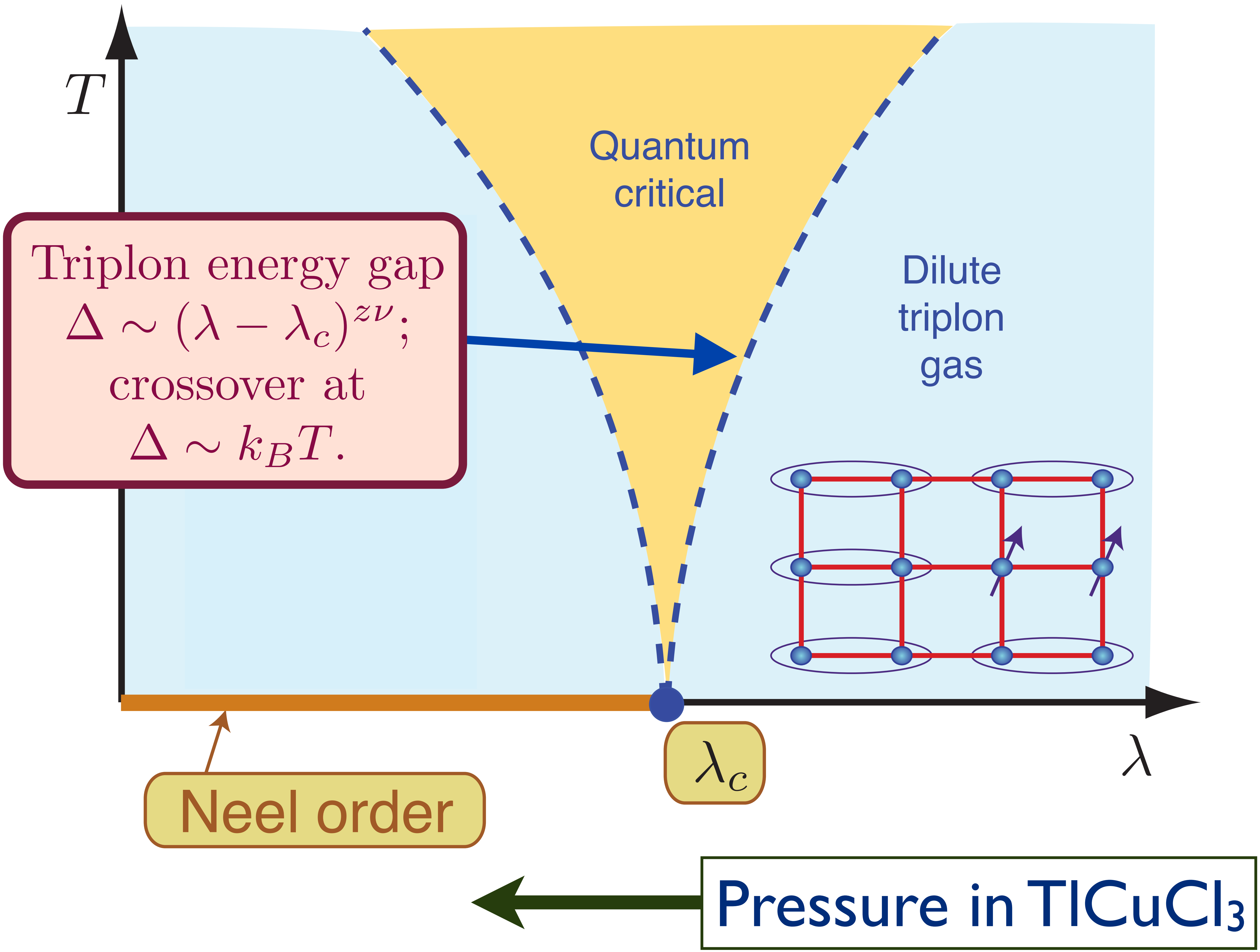


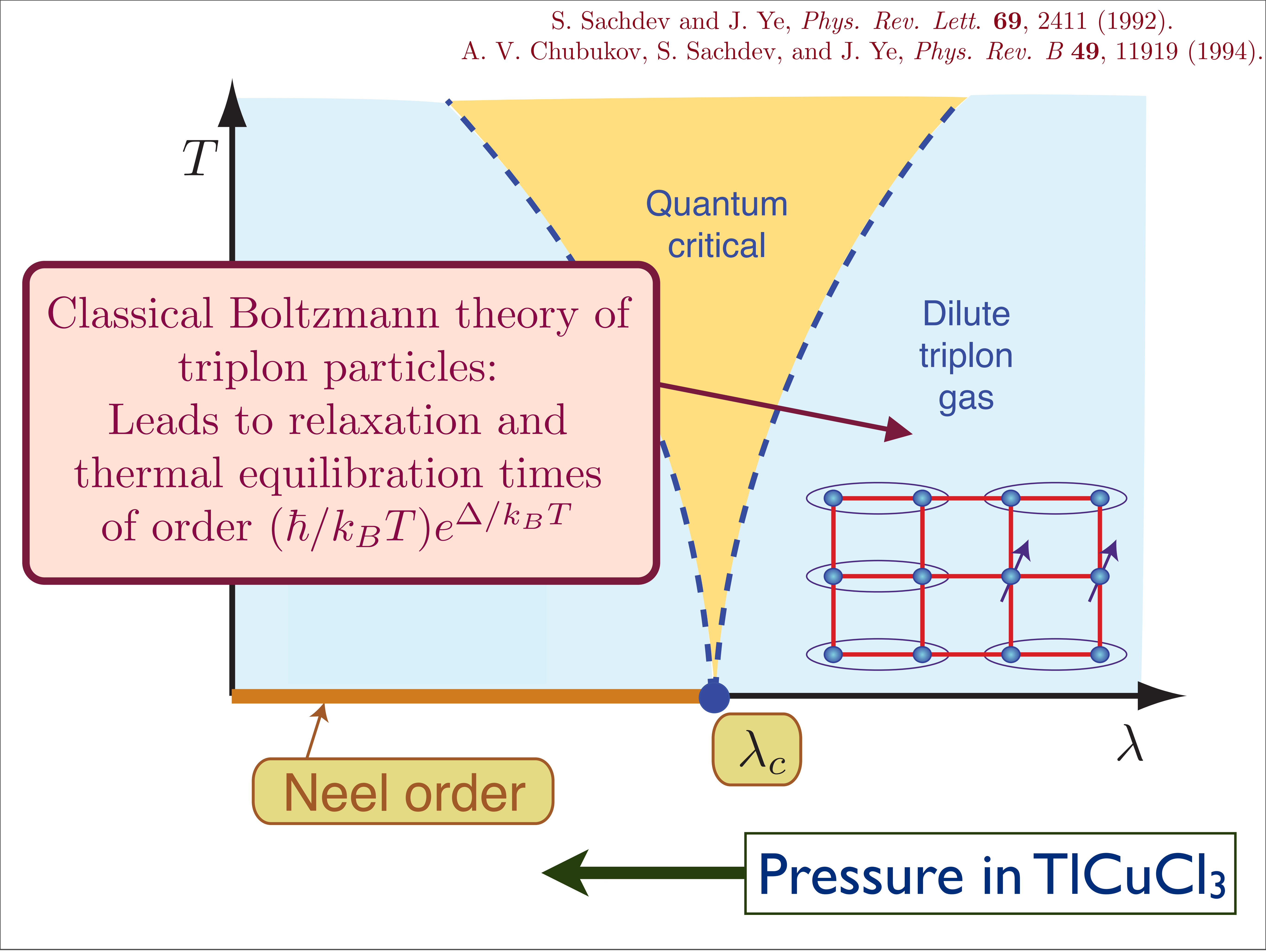
Quantum critical point with non-local entanglement in spin wavefunction

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).



S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).

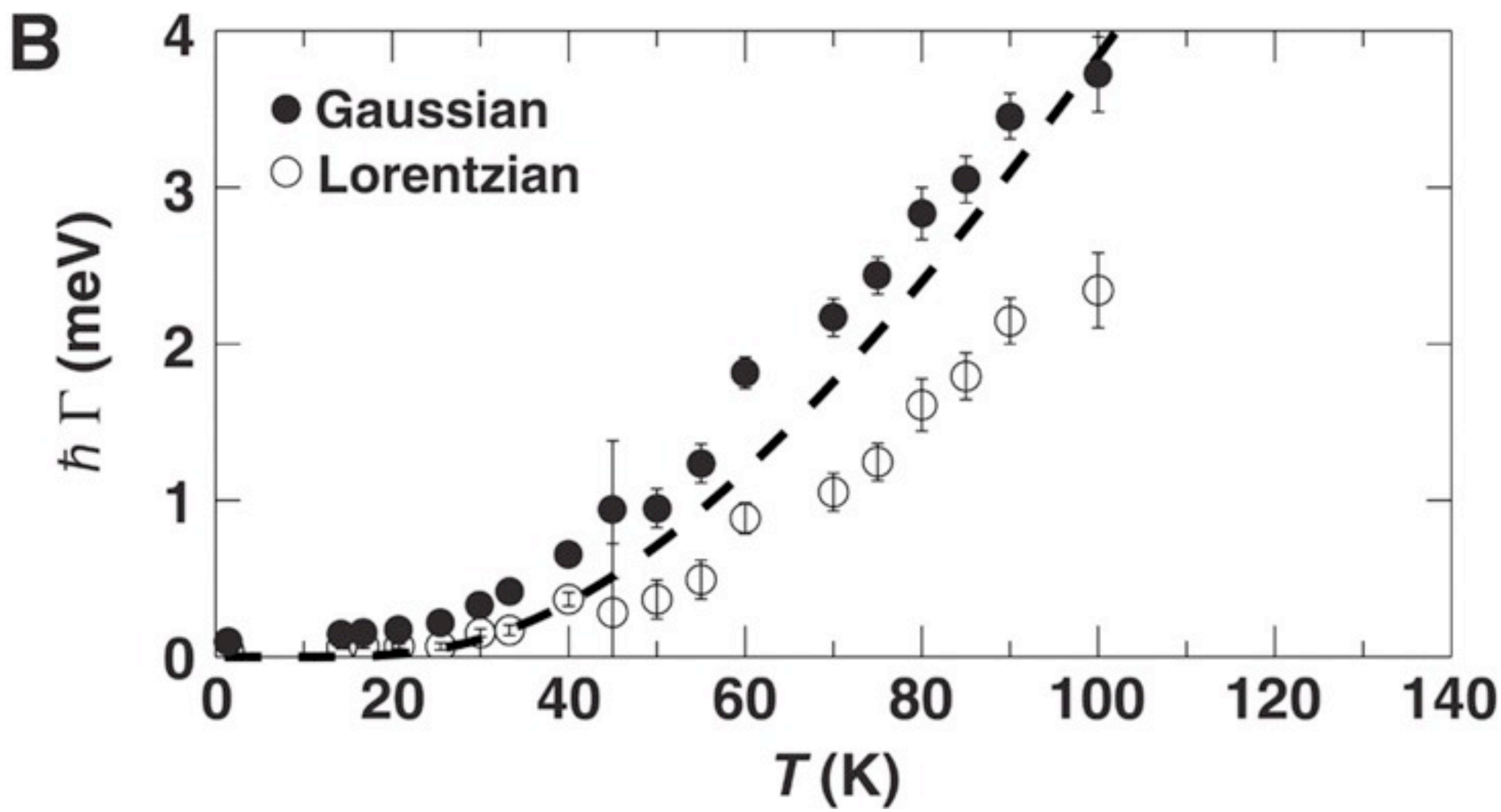


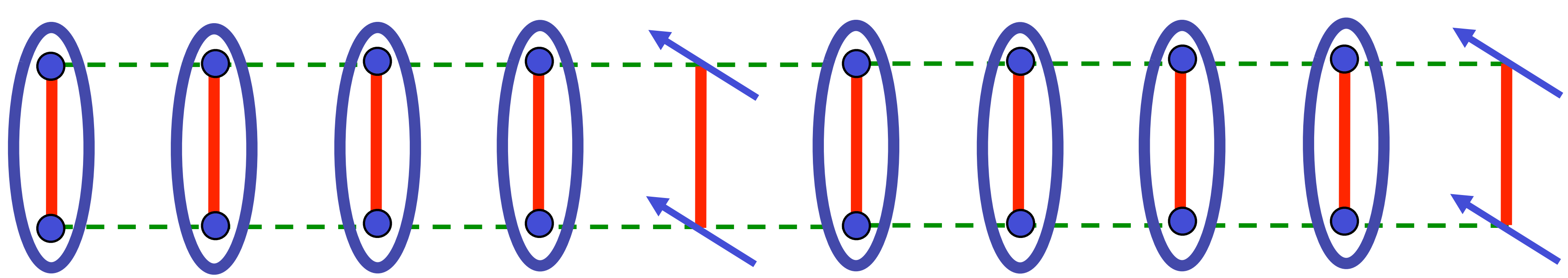


Thursday, May 5, 2011

Neutron scattering measurements of the collisions between triplons in Y_2BaNiO_5

Guangyong Xu, C. Broholm, Yeong-Ah Soh, G. Aeppli, J. F. DiTusa, Ying Chen, M. Kenzelmann, C. D. Frost, T. Ito, K. Oka, H. Takagi, Science **317**, 1049 (2007)





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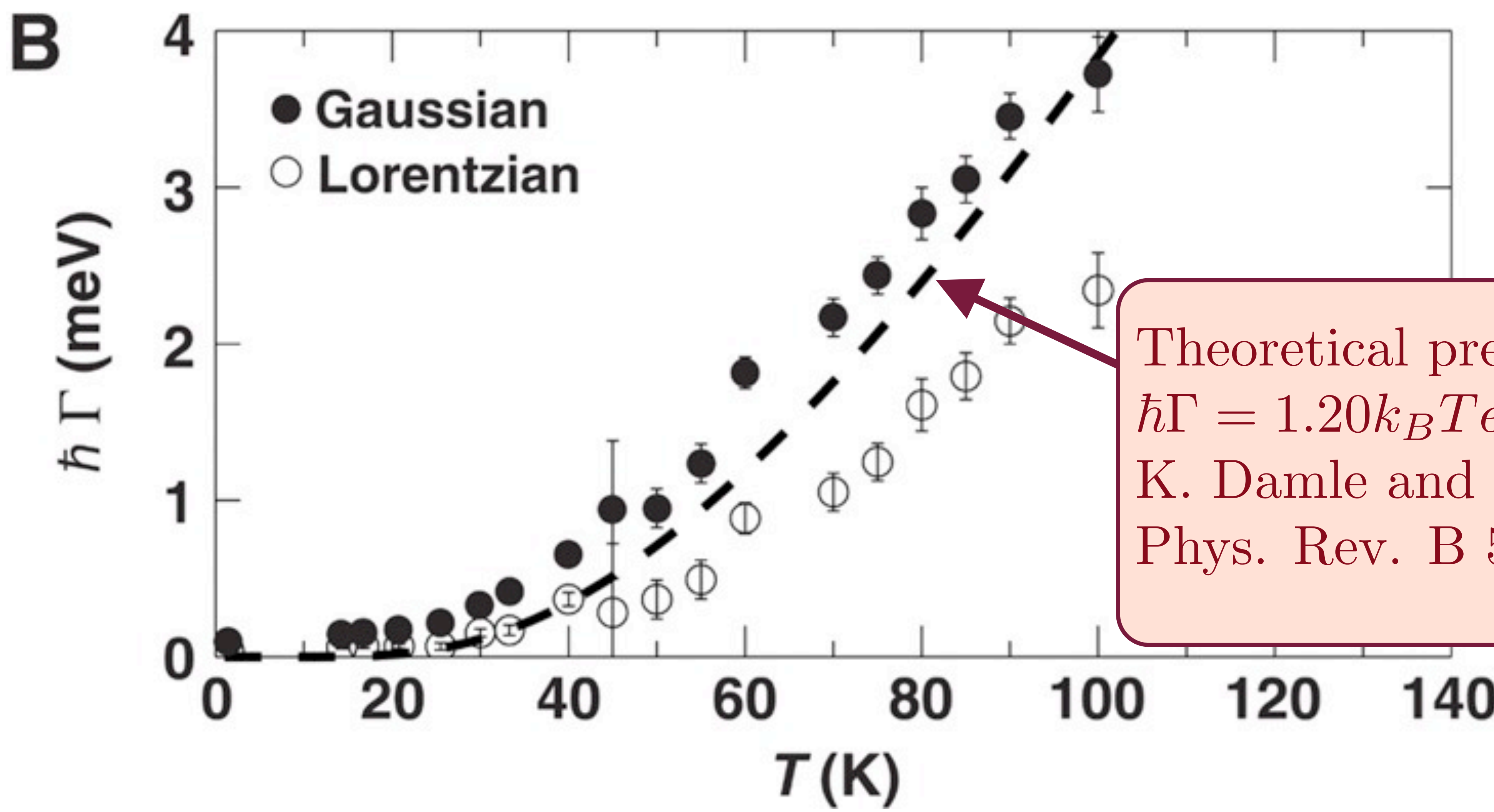
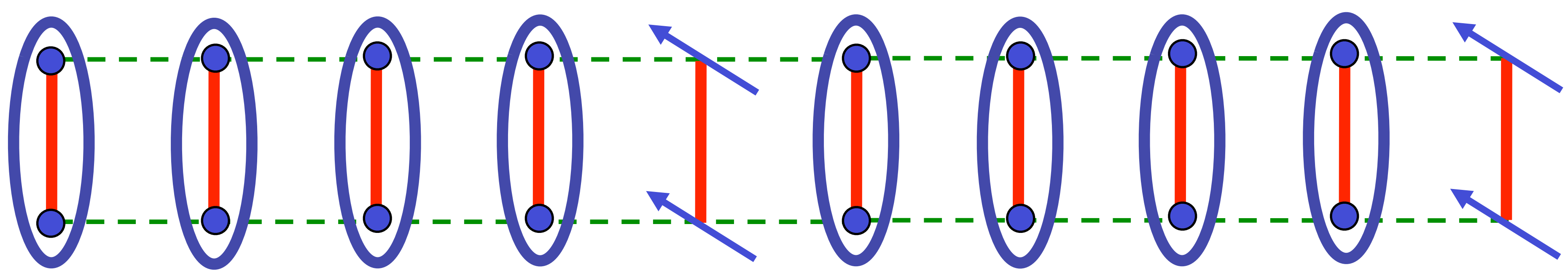


Theoretical prediction
$\hbar\Gamma = 1.20 k_B T e^{\Delta/k_B T}$
K. Damle and S. Sachdev, Phys. Rev. B 57, 8307 (1998)



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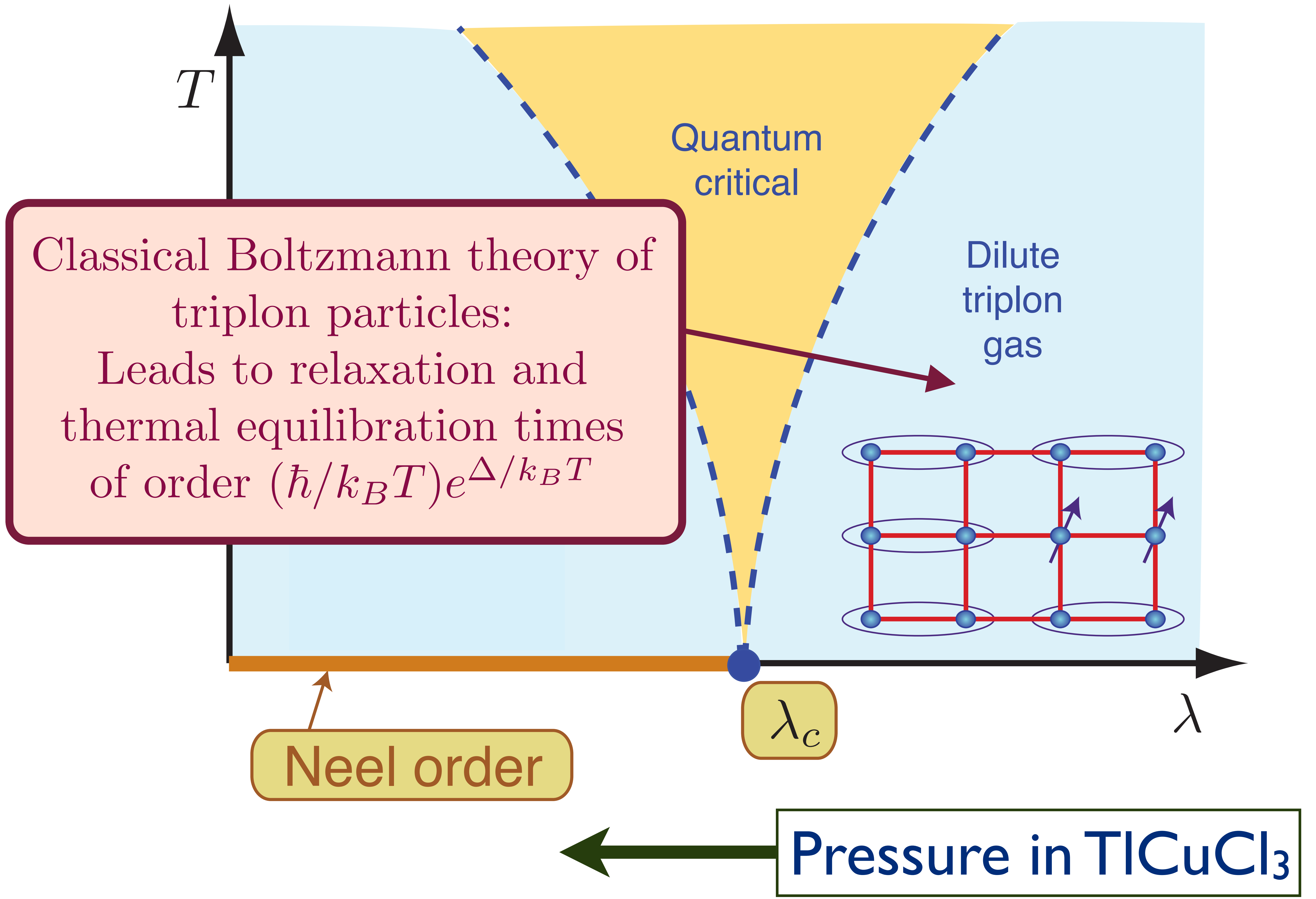

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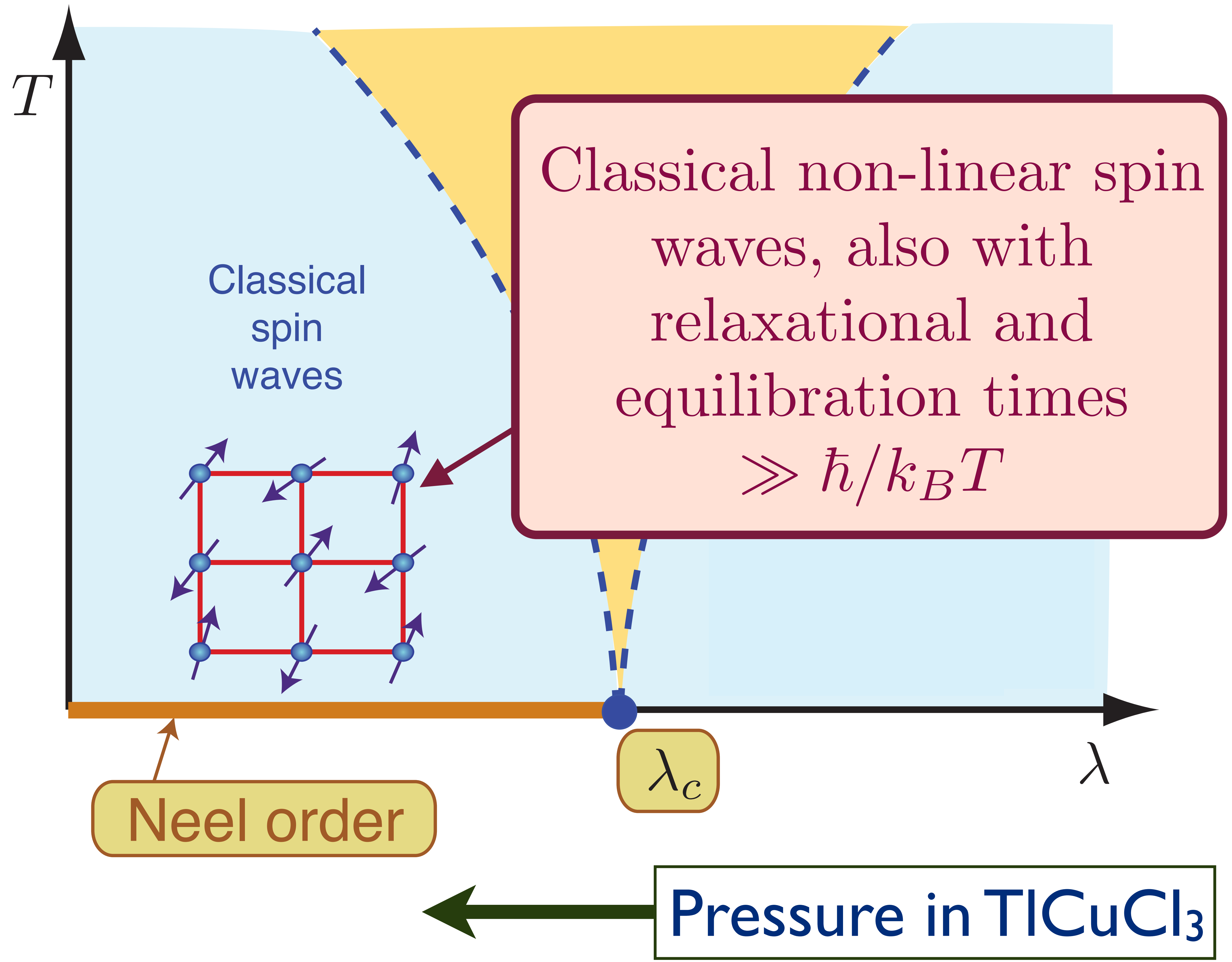

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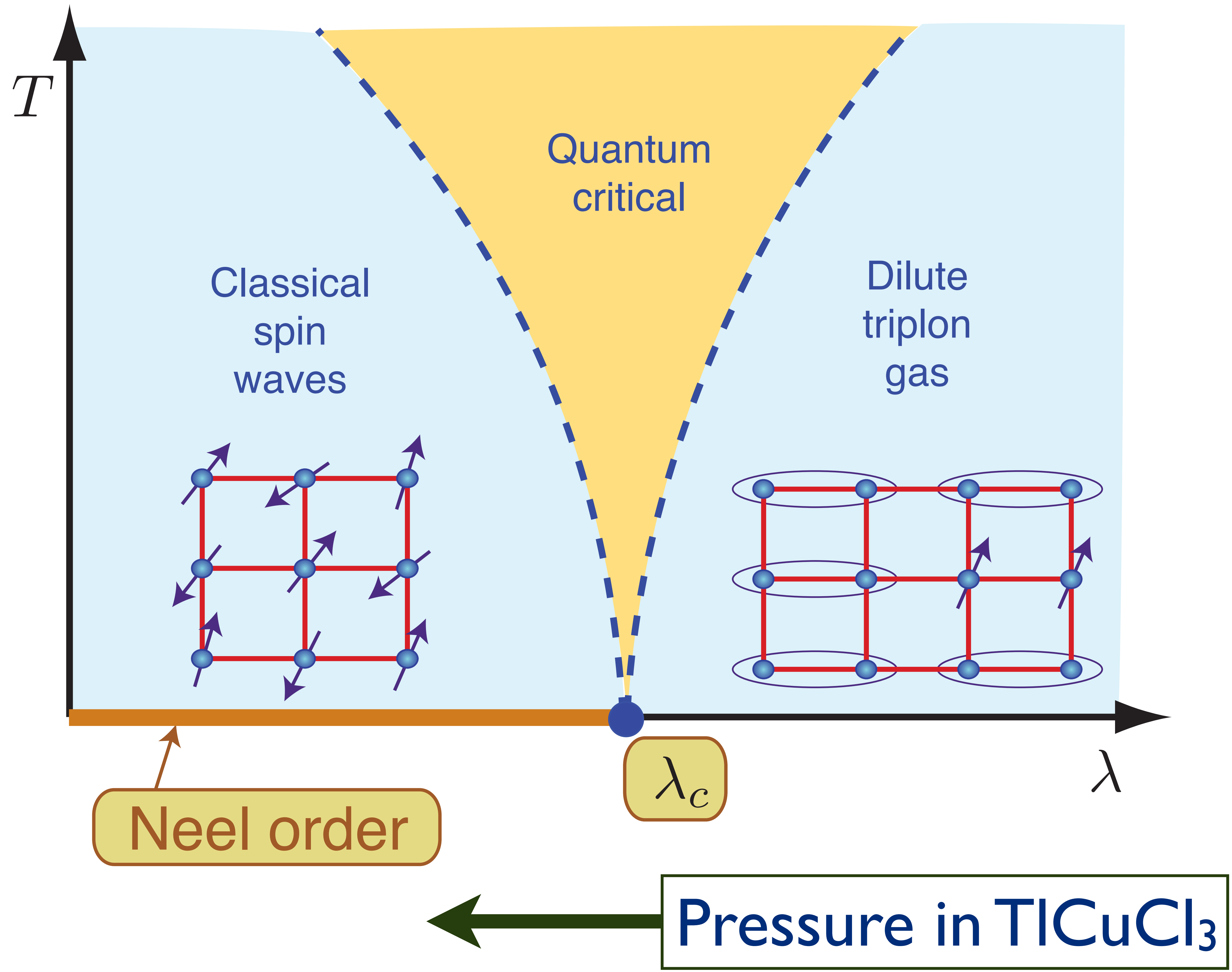

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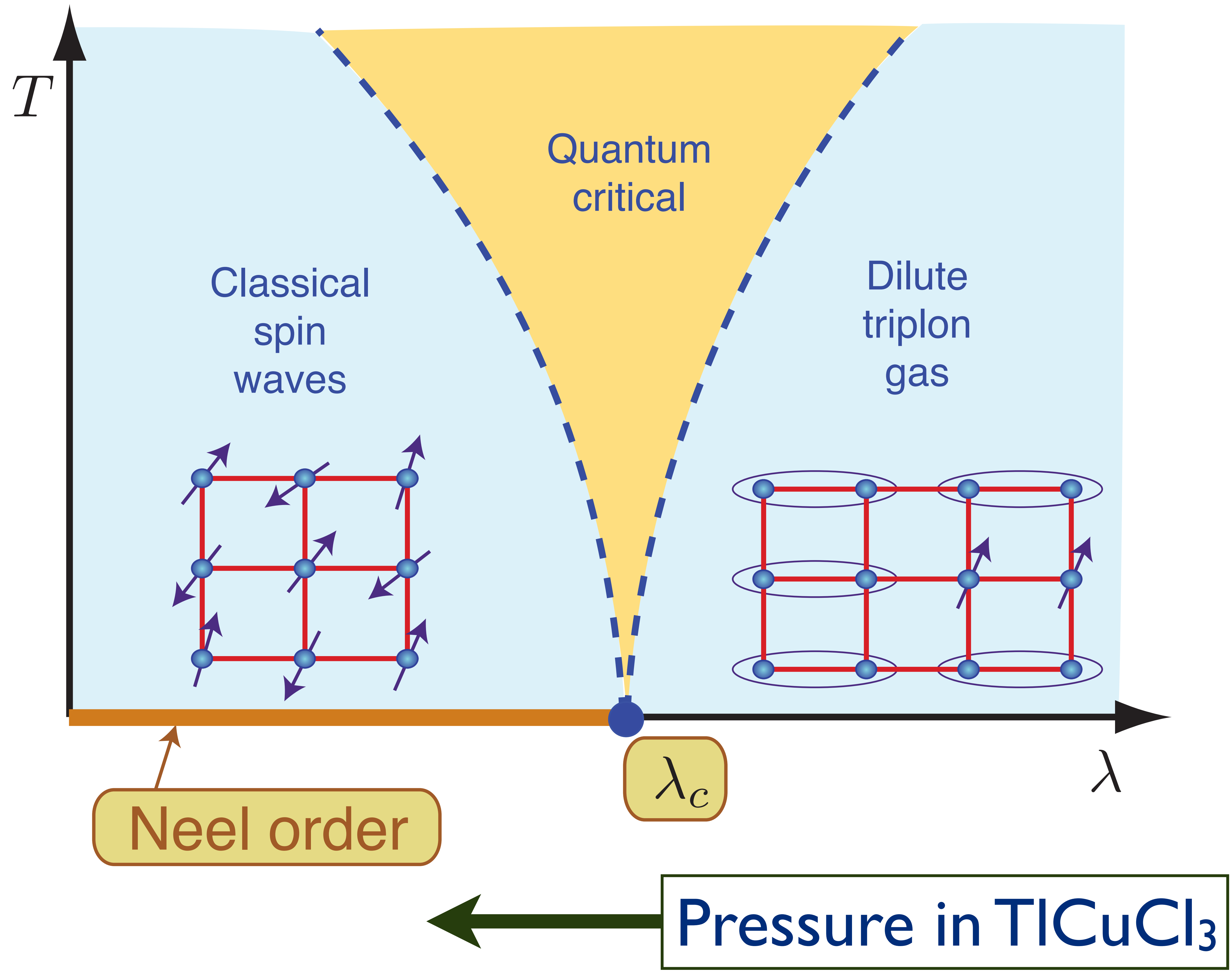

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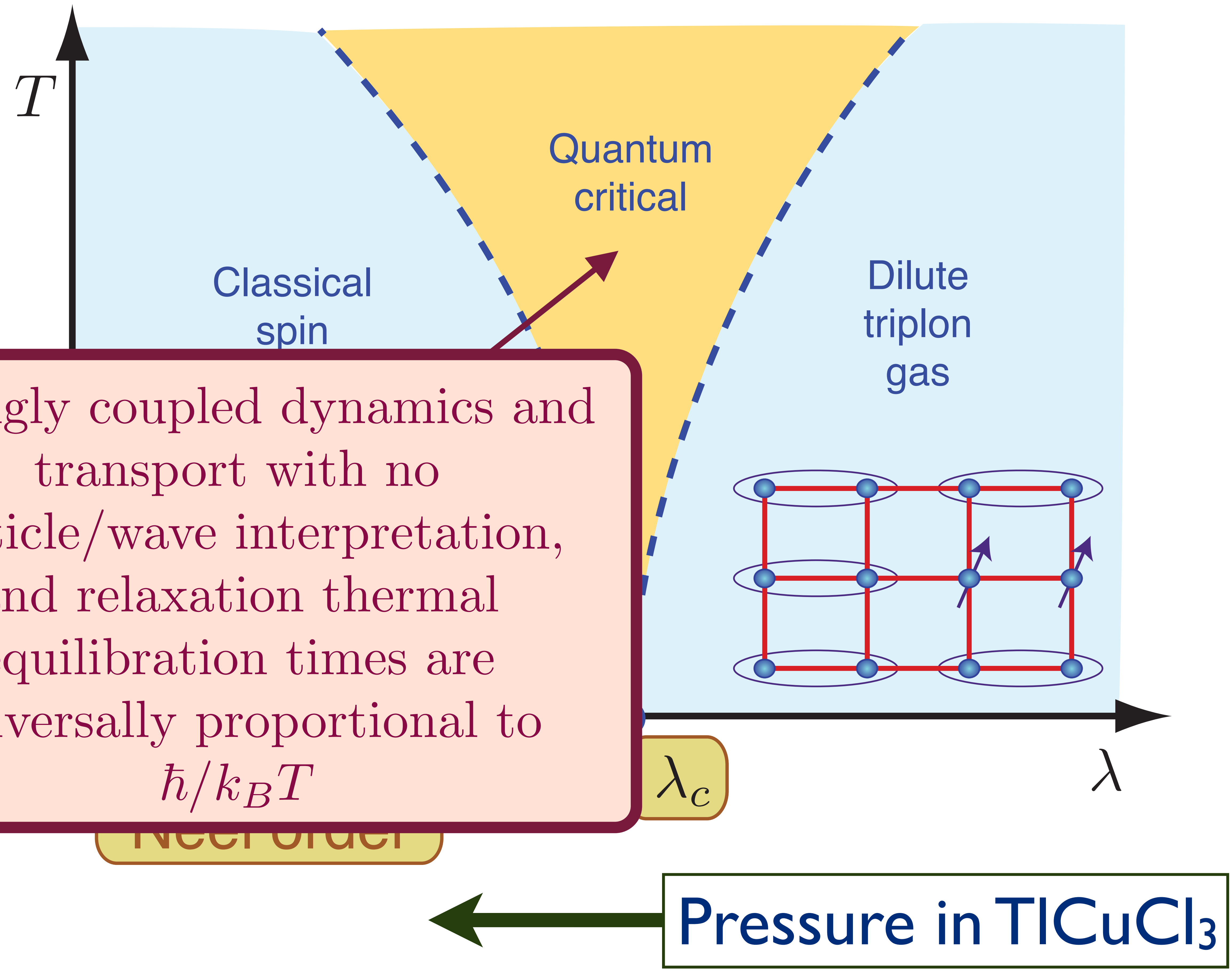


Strongly coupled dynamics and transport with no particle/wave interpretation, and relaxation thermal equilibration times are universally proportional to $\hbar/k_B T$

Quantum critical transport

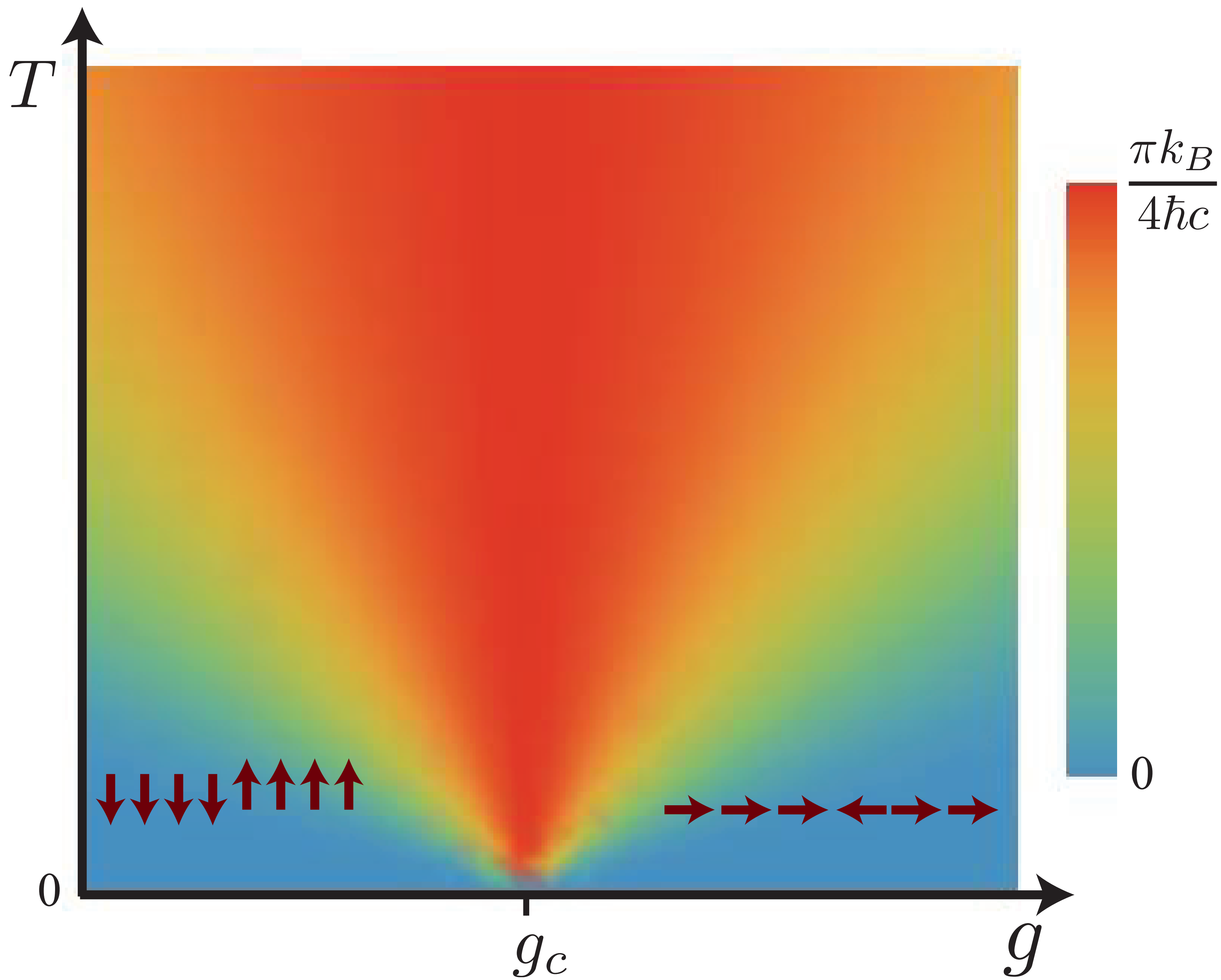
Quantum *"nearly perfect fluid"*
with shortest possible
relaxation time, τ_R

$$\tau_R = \mathcal{C} \frac{\hbar}{k_B T}$$

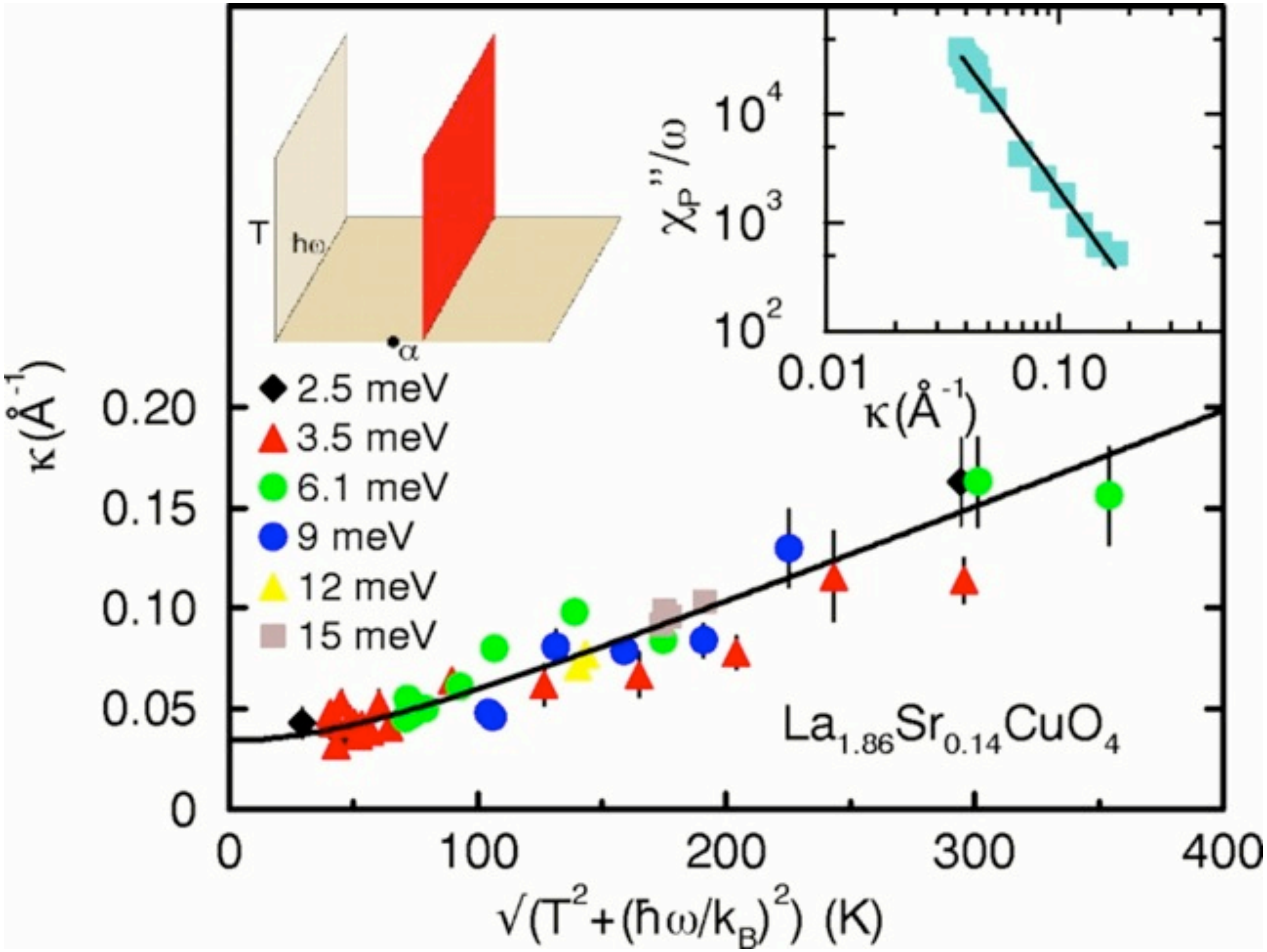
where $\mathcal{C}$ is a *universal* constant

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Non-zero temperature crossovers for the quantum Ising chain



Color density plot of $d\xi^{-1}/dT$, where ξ is the spin correlation length. This quantity measures the strength of the interactions between the thermal excitations.



Neutron scattering measurements on $La_{1.86}$ $Sr_{0.14}$ Cu O_4, showing scaling of the dynamic spin susceptibility at an incommensurate wavevector:

$$\chi''(\omega, T) = \frac{A}{T^{2-\eta}} \Phi\left(\frac{\hbar\omega}{k_B T}\right)$$

G. Aeppli, T. E. Mason, S. M. Hayden, H. A. Mook and J. Kulda, *Science*, **278**, 1432 (1997).

Outline

1. The quantum Ising chain

    A. The magnetic insulator $CoNb_2O_6$

    B. Ultracold Rb atoms in an optical lattice

2. Nonzero temperatures and quantum criticality

    Antiferromagnetic insulators

3. Higher temperature superconductors
    and "strange metals"

    Quantum criticality of fermions and Fermi surfaces

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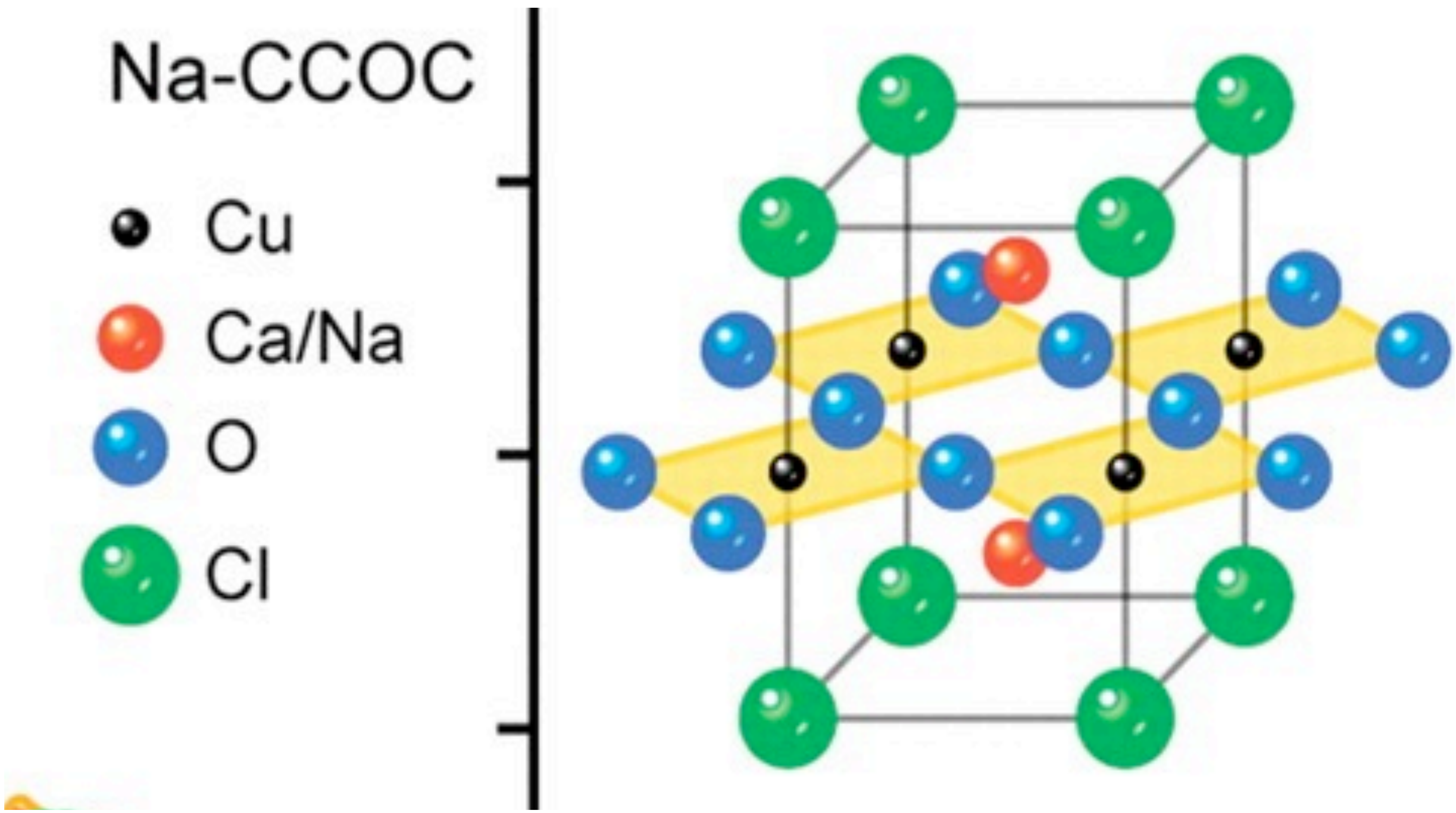
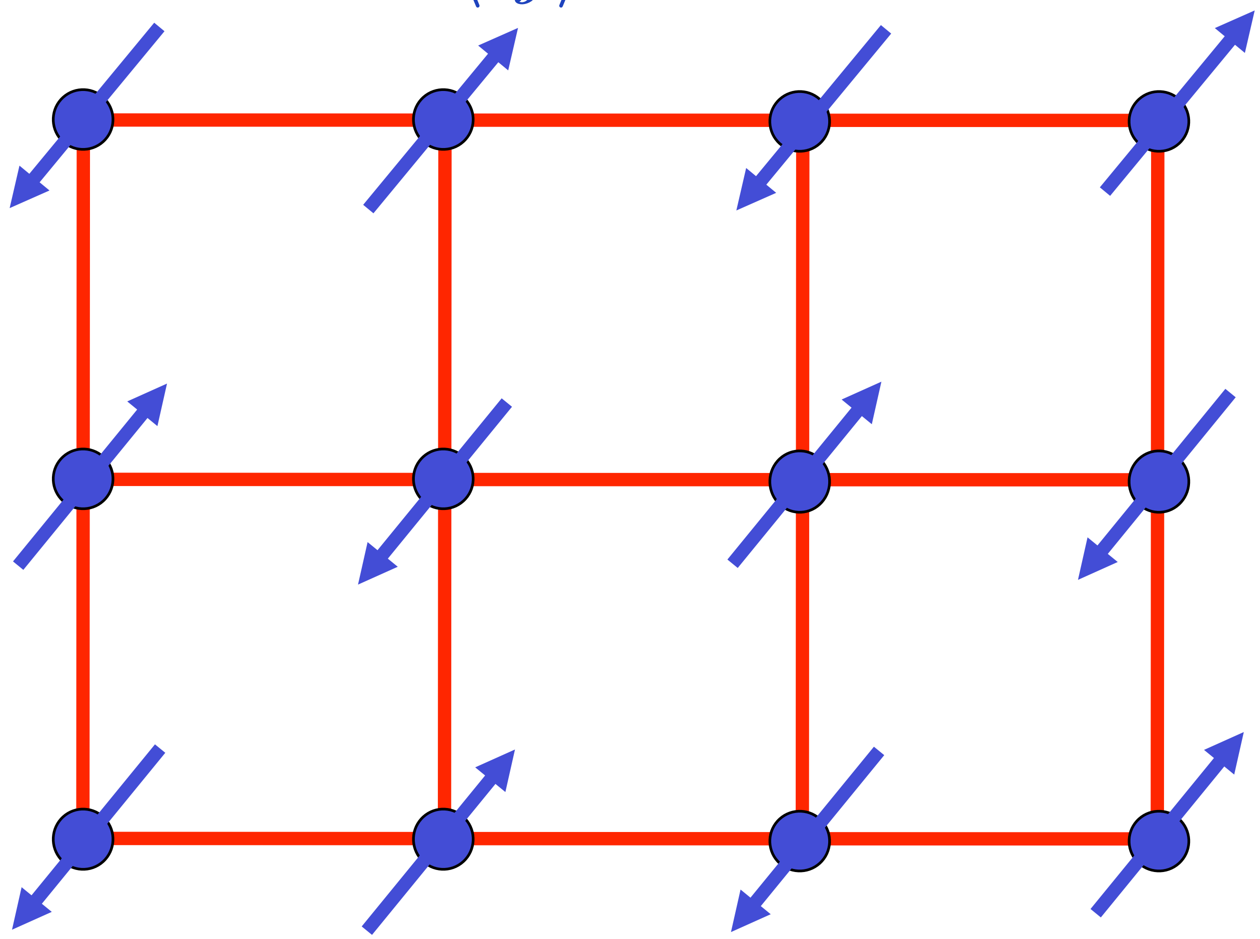
The cuprate superconductors



Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

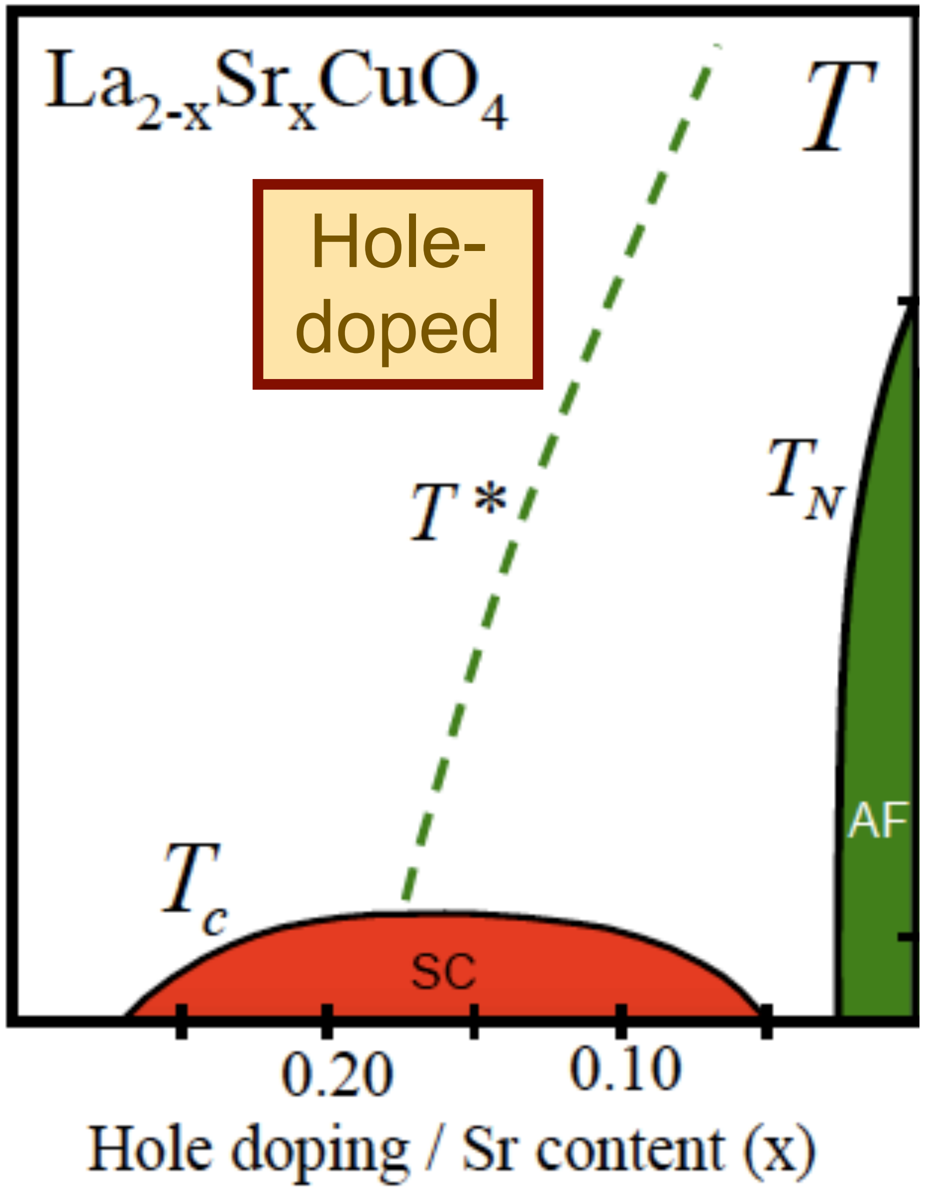


Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

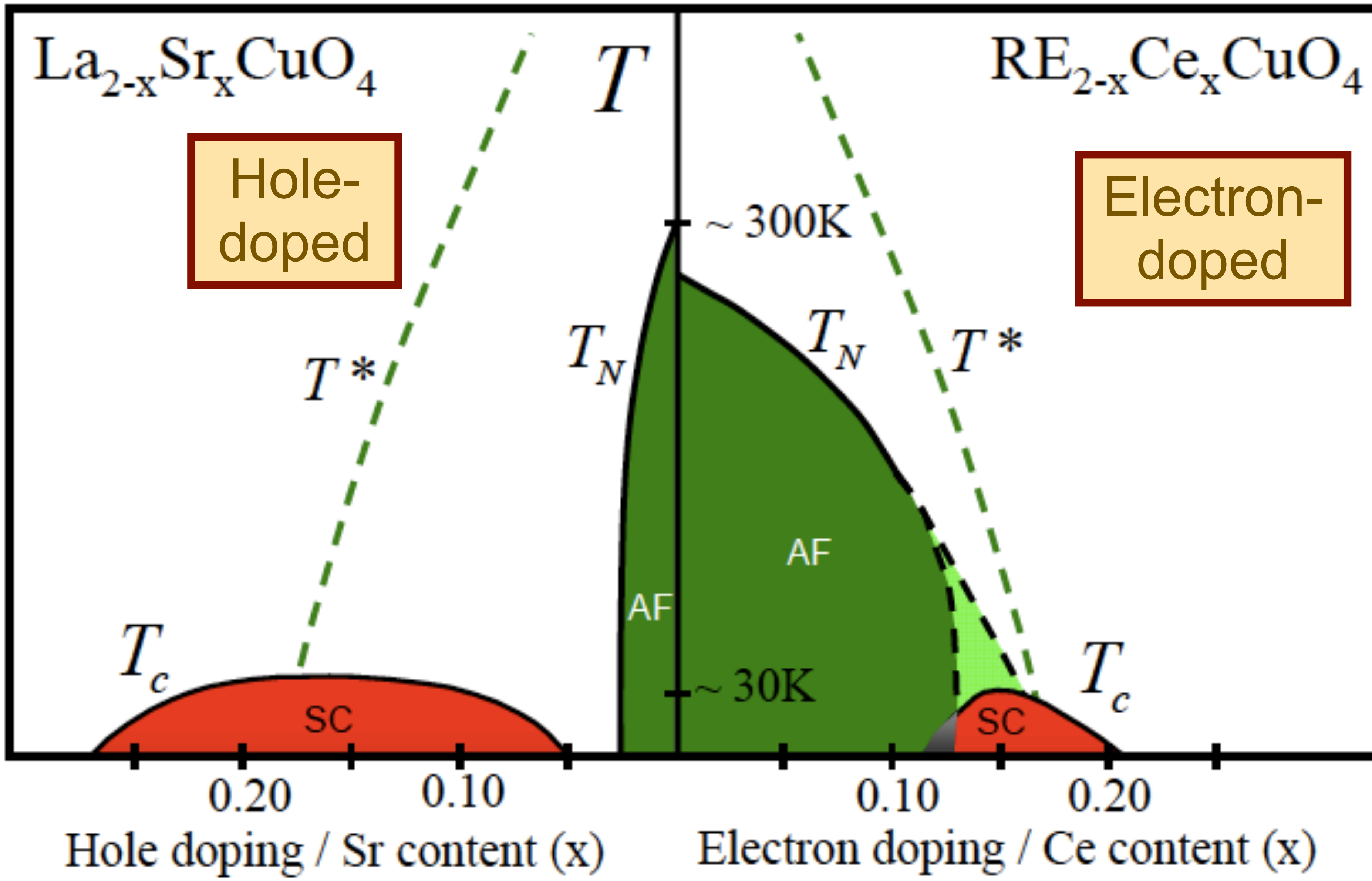
$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.



$La_{2-x}Sr_xCuO_4$
Hole-doped
T
T^*
T_N
AF
T_c
SC
0.20
0.10
Hole doping / Sr content (x)

Electron-doped cuprate superconductors



Electron-doped cuprate superconductors

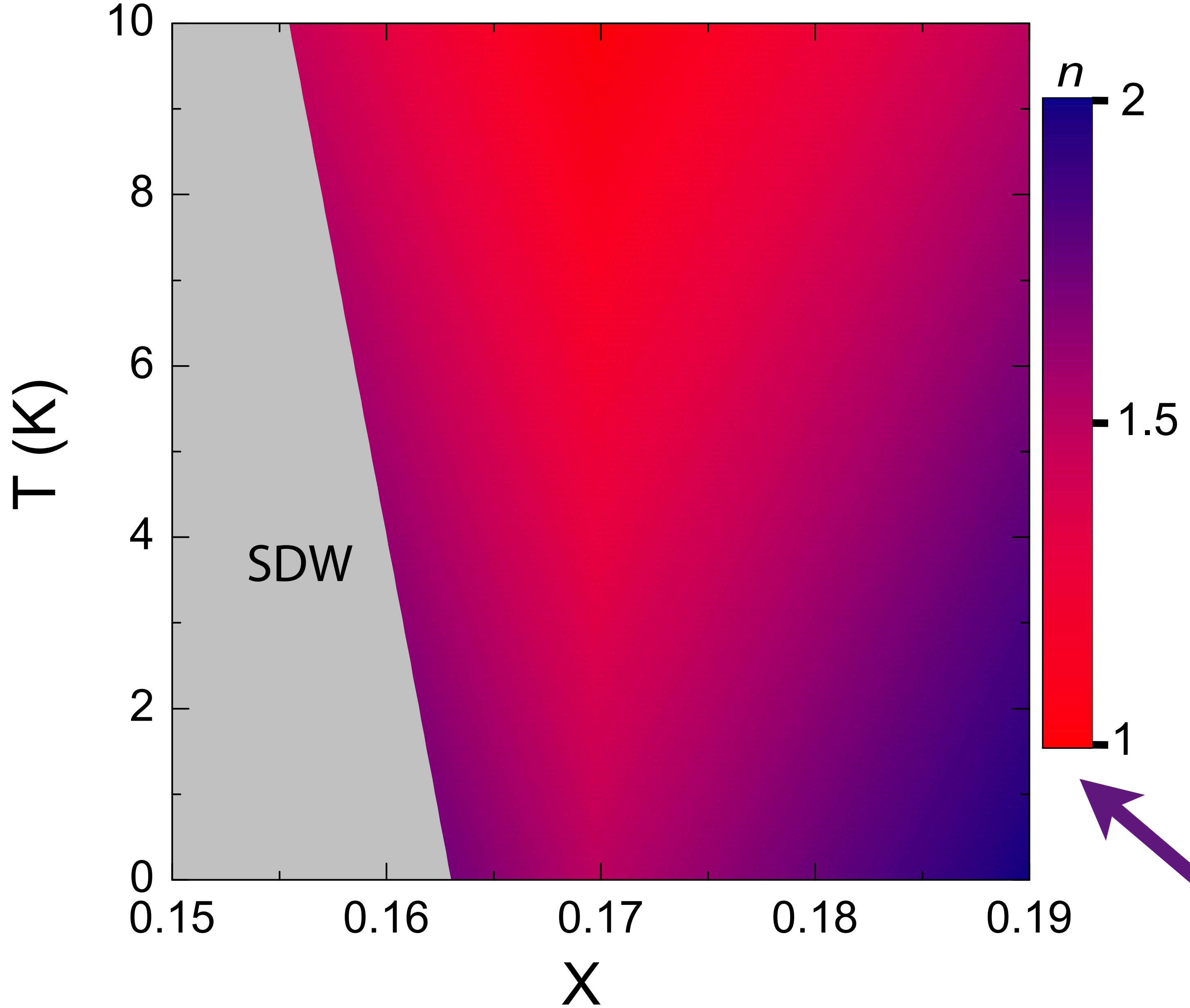
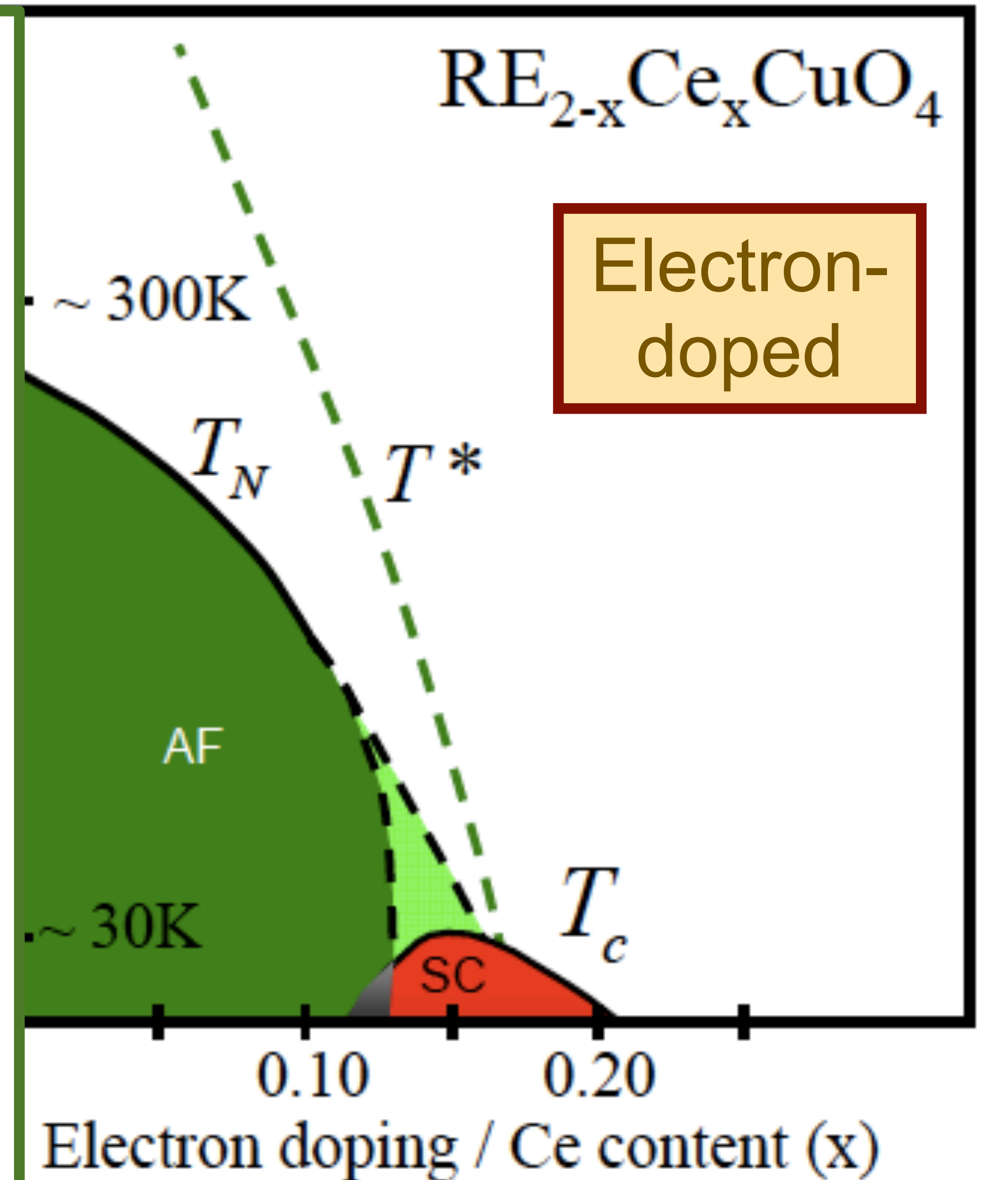
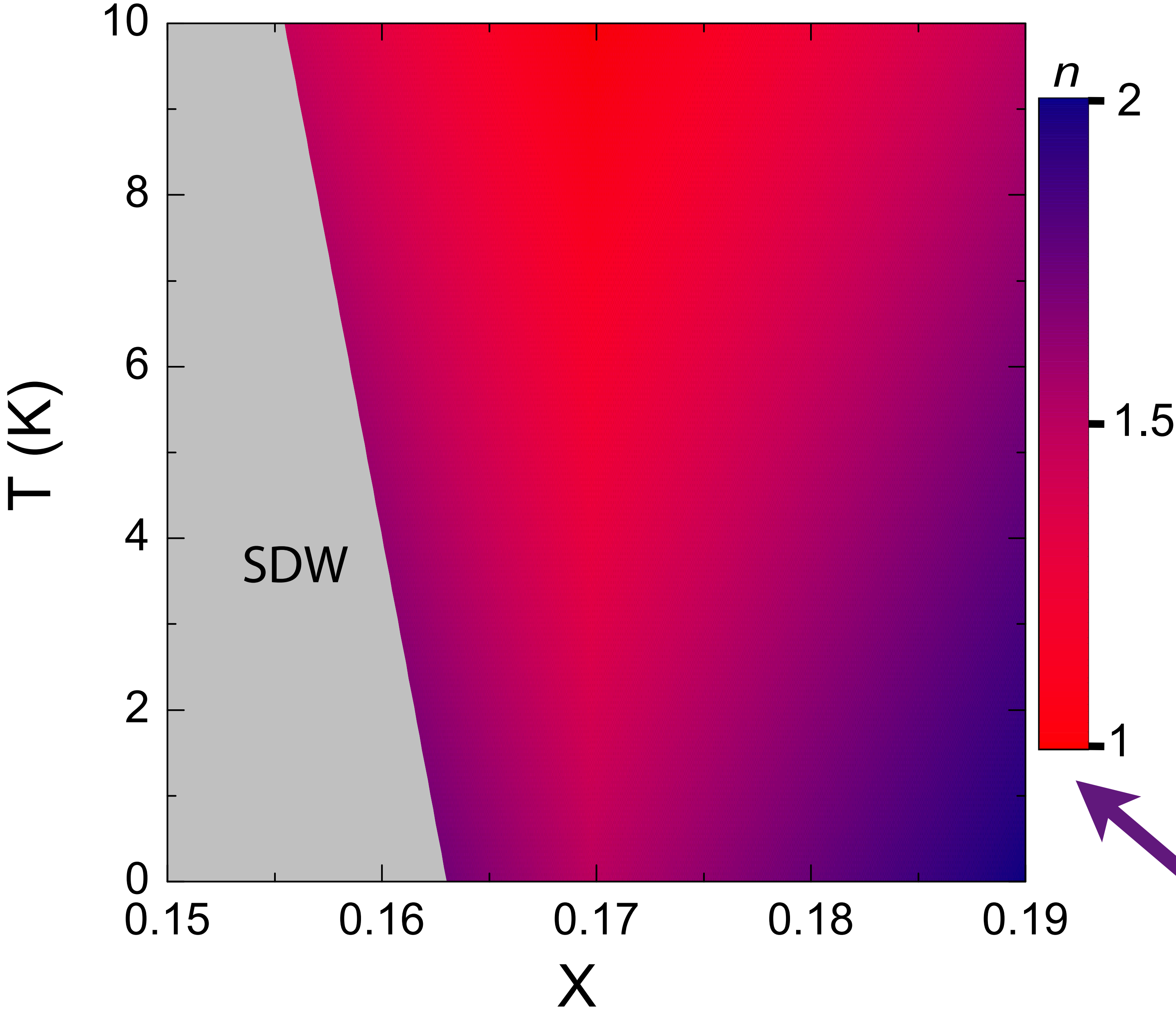


Figure prepared by K. Jin and and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).



Resistivity
$\sim \rho_0 + AT^n$

Electron-doped cuprate superconductors



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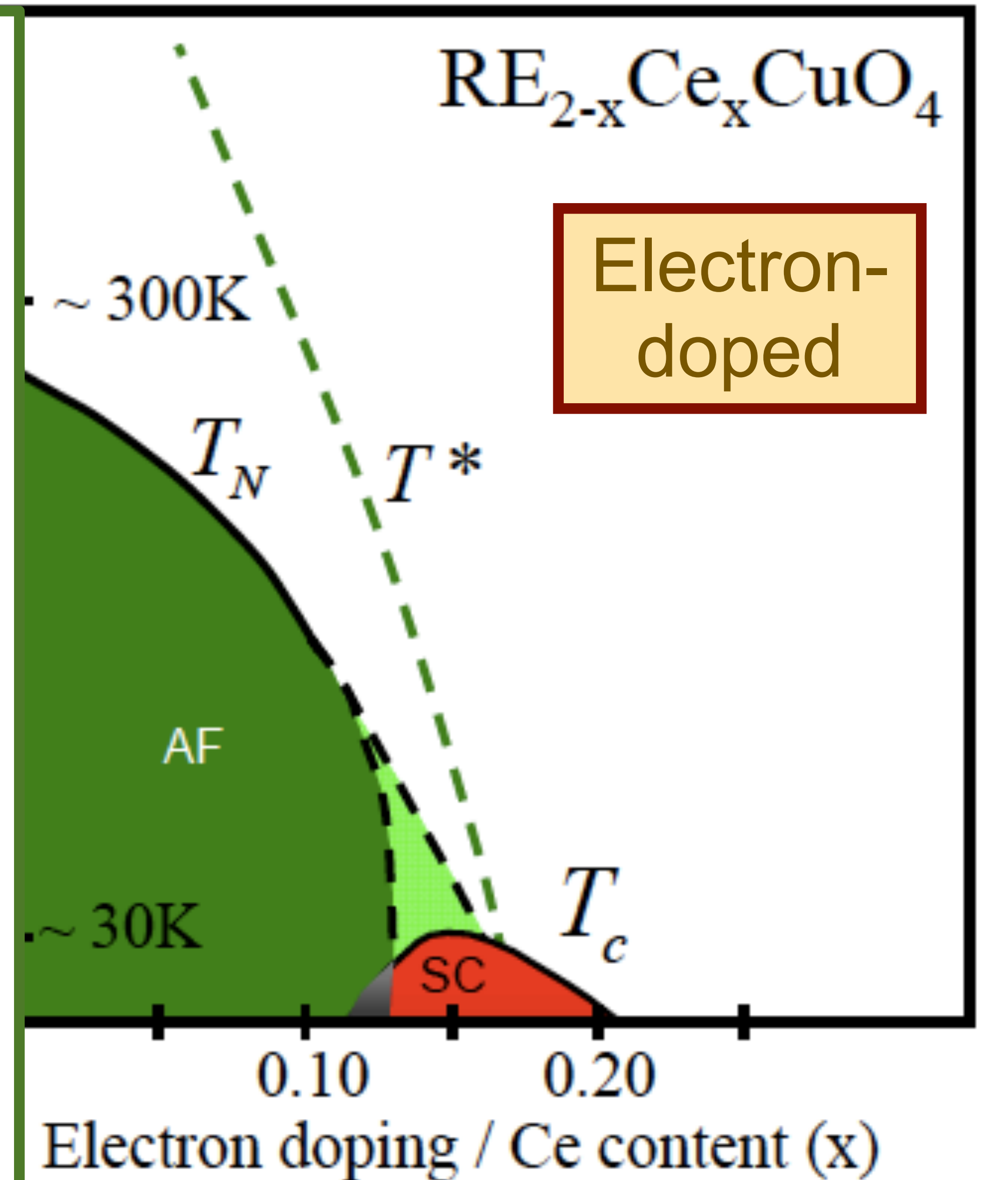

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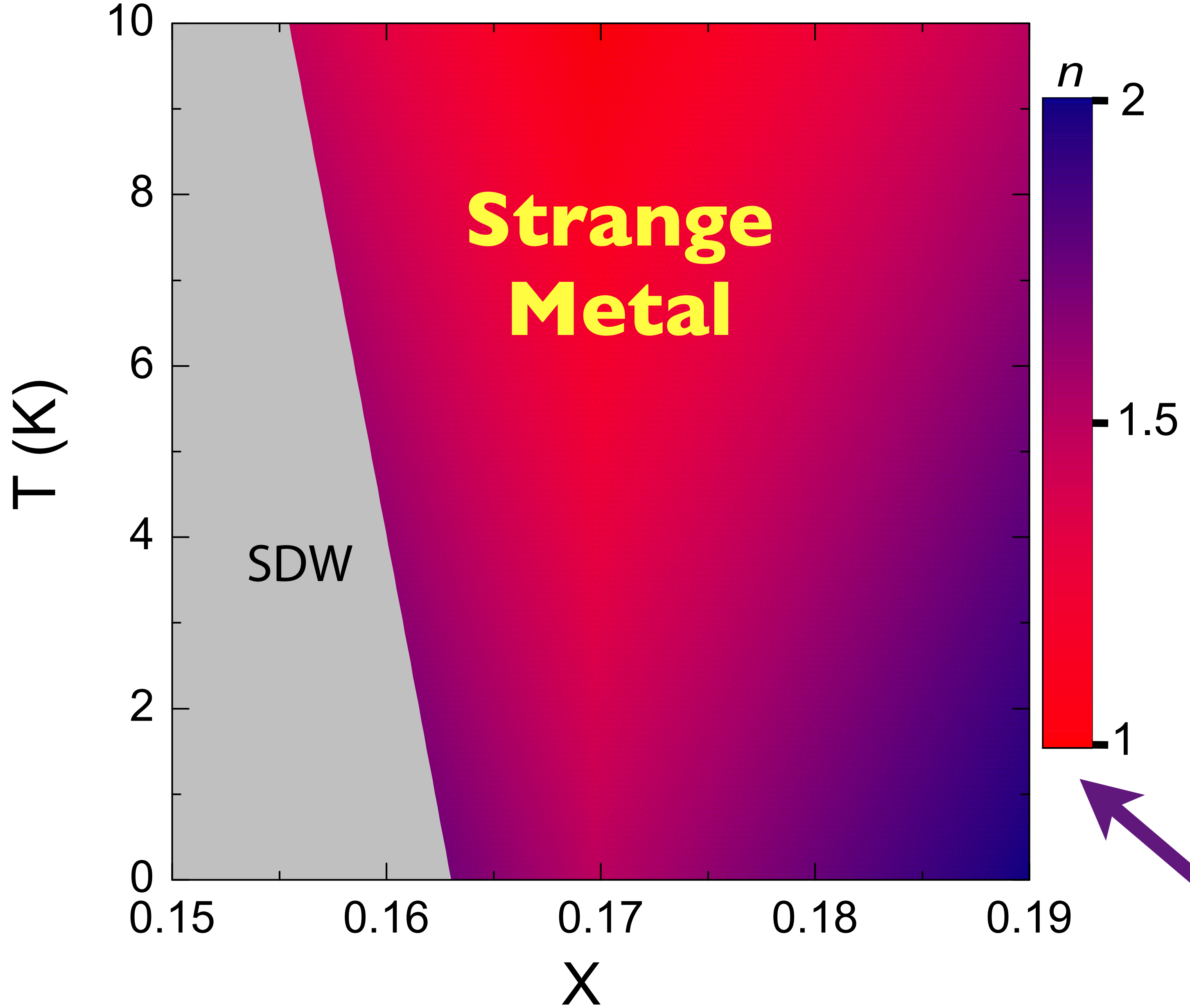

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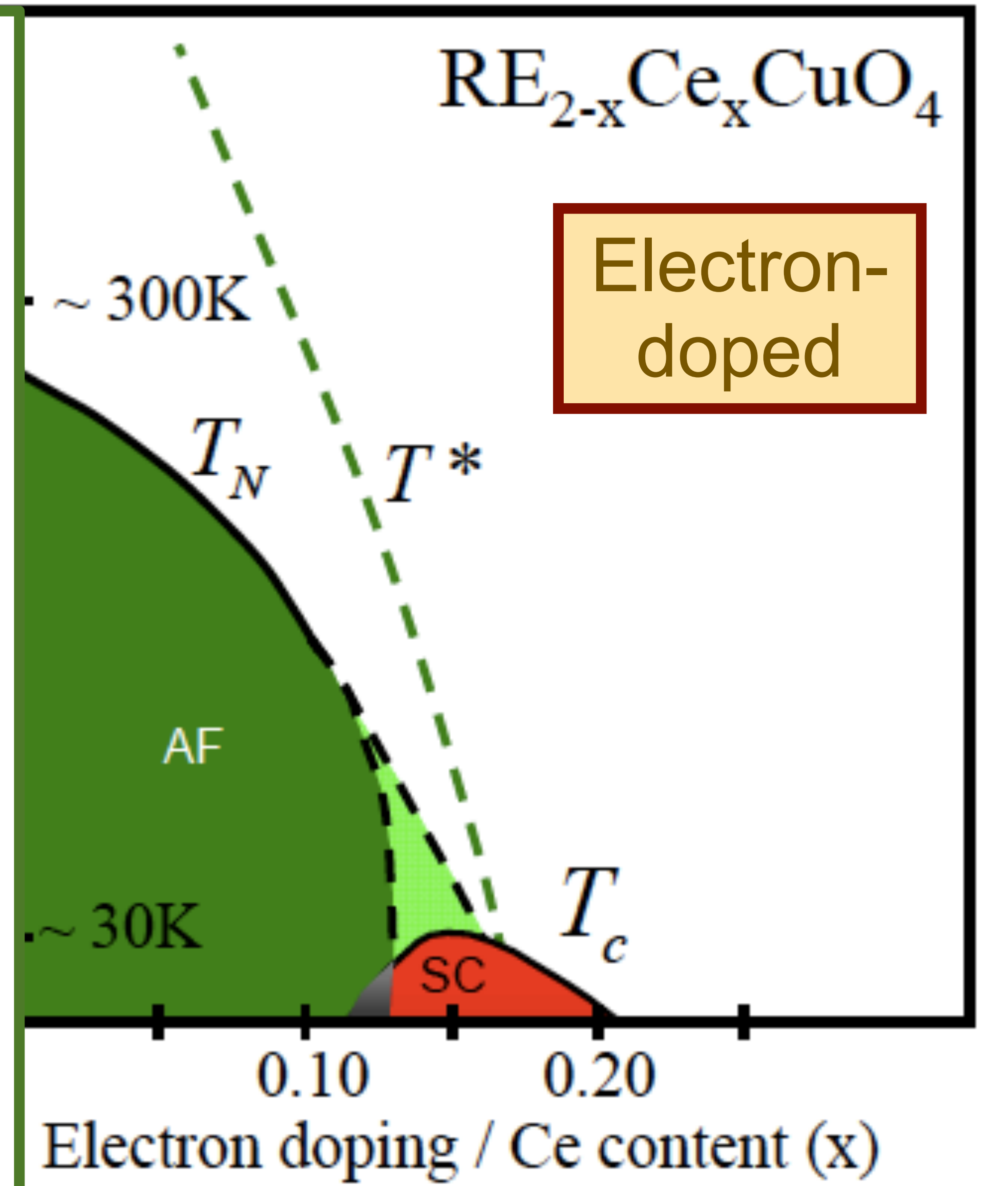
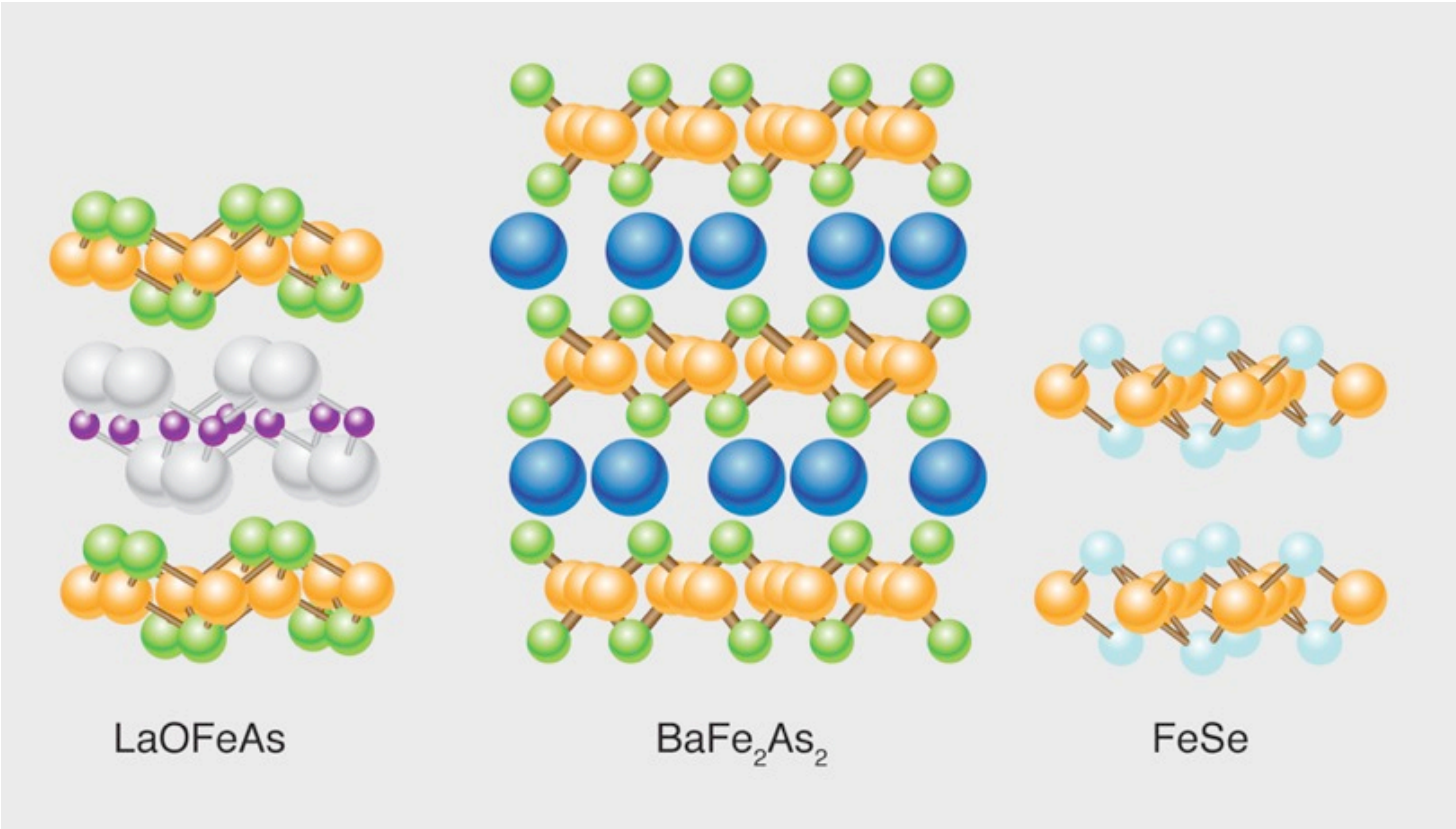


Resistivity $\sim \rho_0 + AT^n$

Iron pnictides:

a new class of high temperature superconductors



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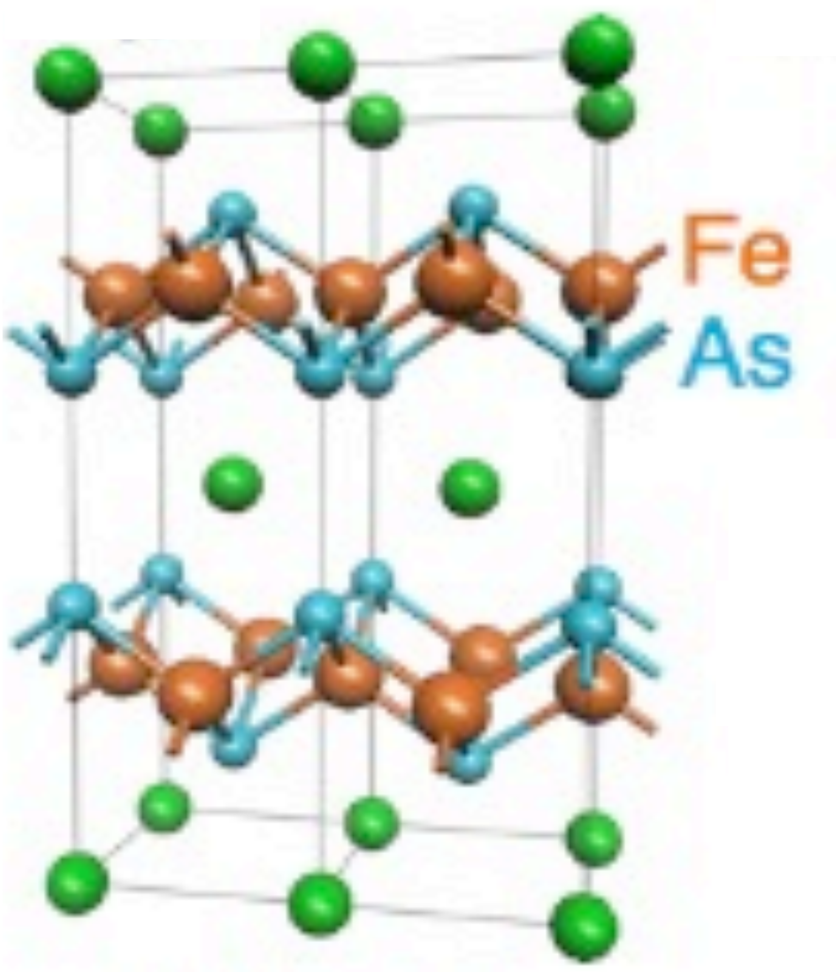
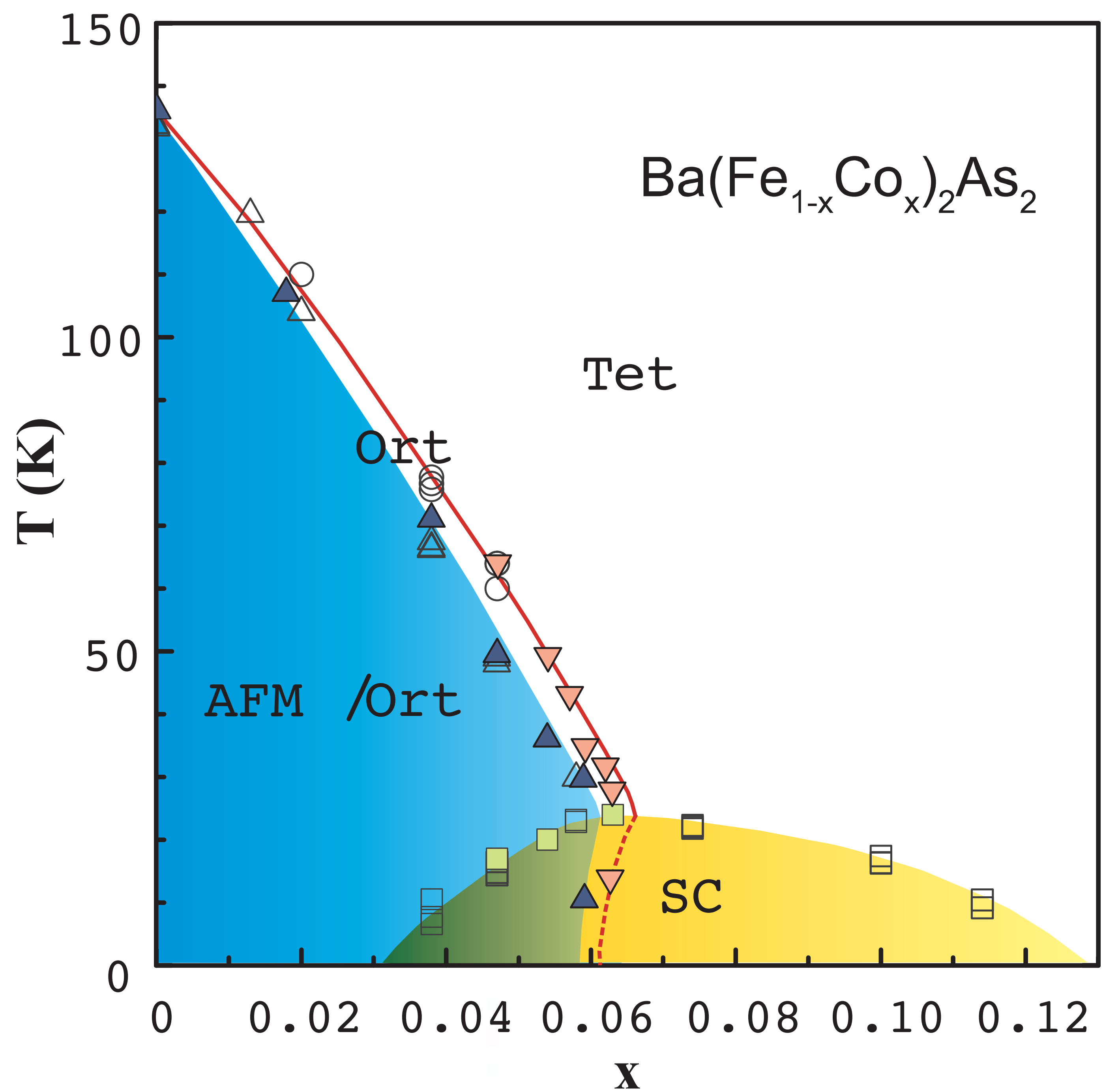
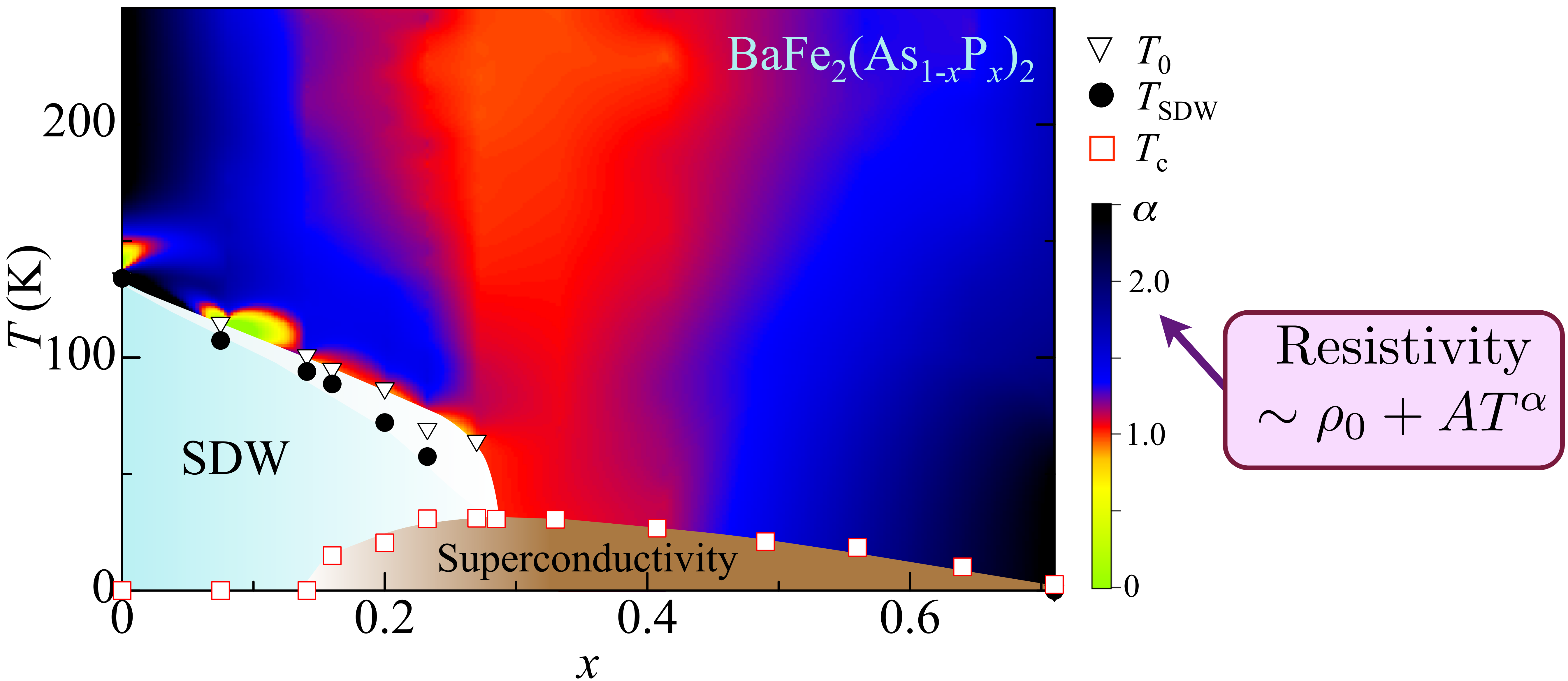


$BaFe_2As_2$
Unit cell



S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, *Physical Review Letters* **104**, 057006 (2010).

Temperature-doping phase diagram of the iron pnictides:



SDW= spin density wave = antiferromagnetism in a metal

S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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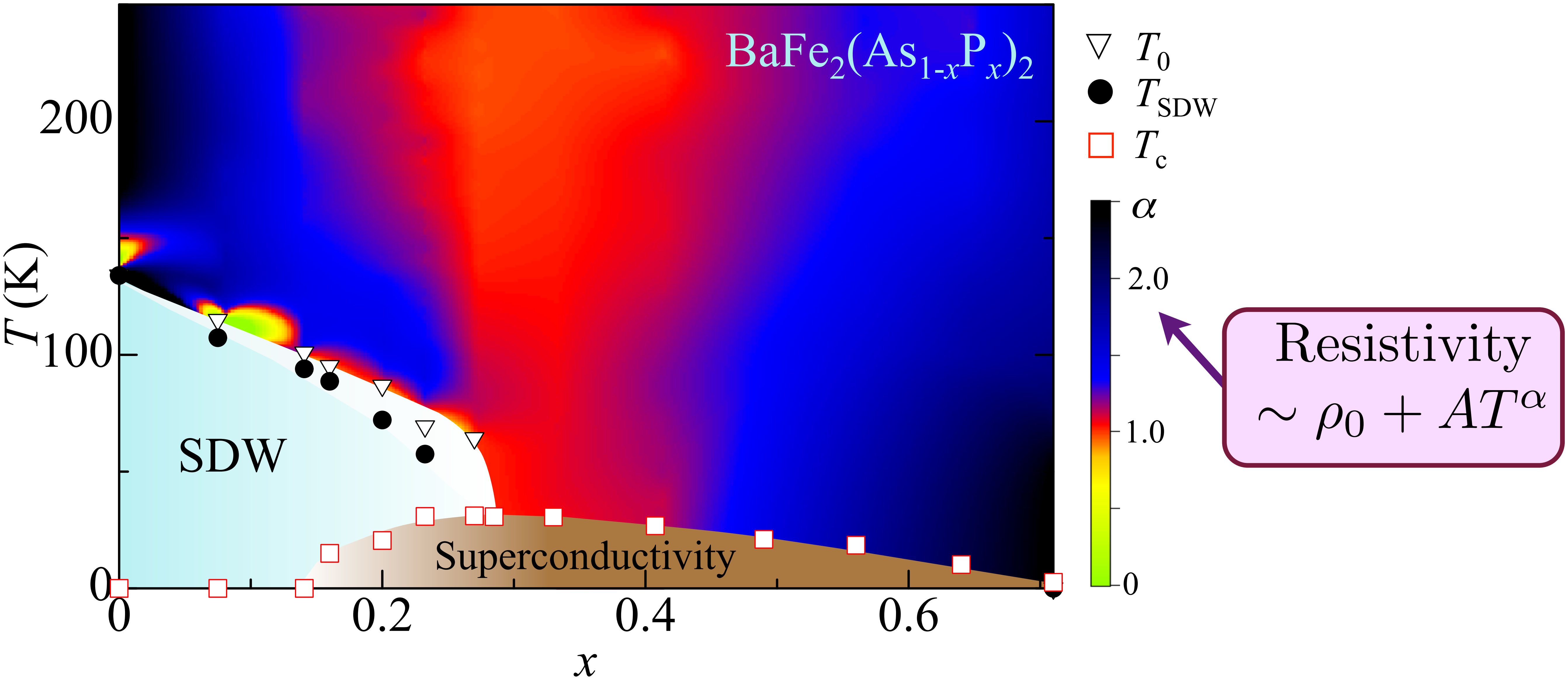

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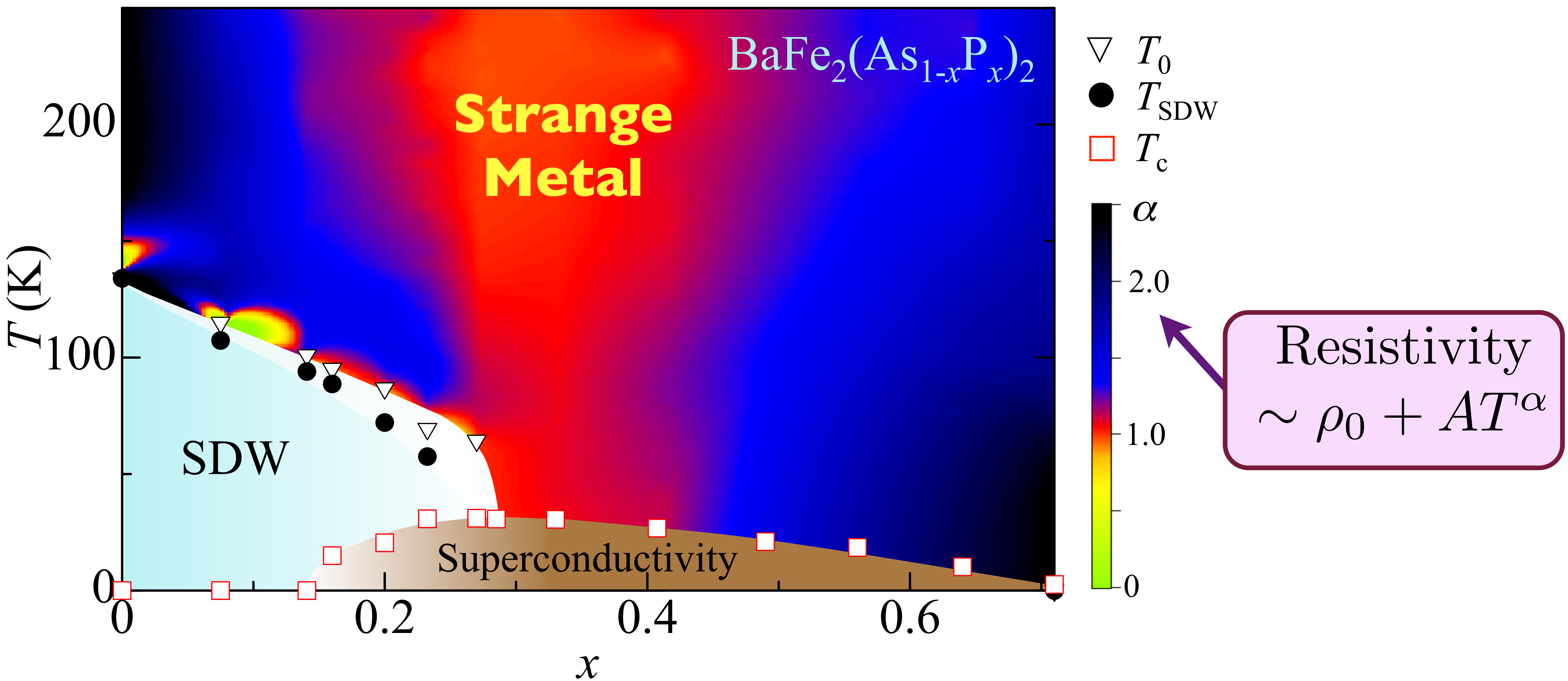

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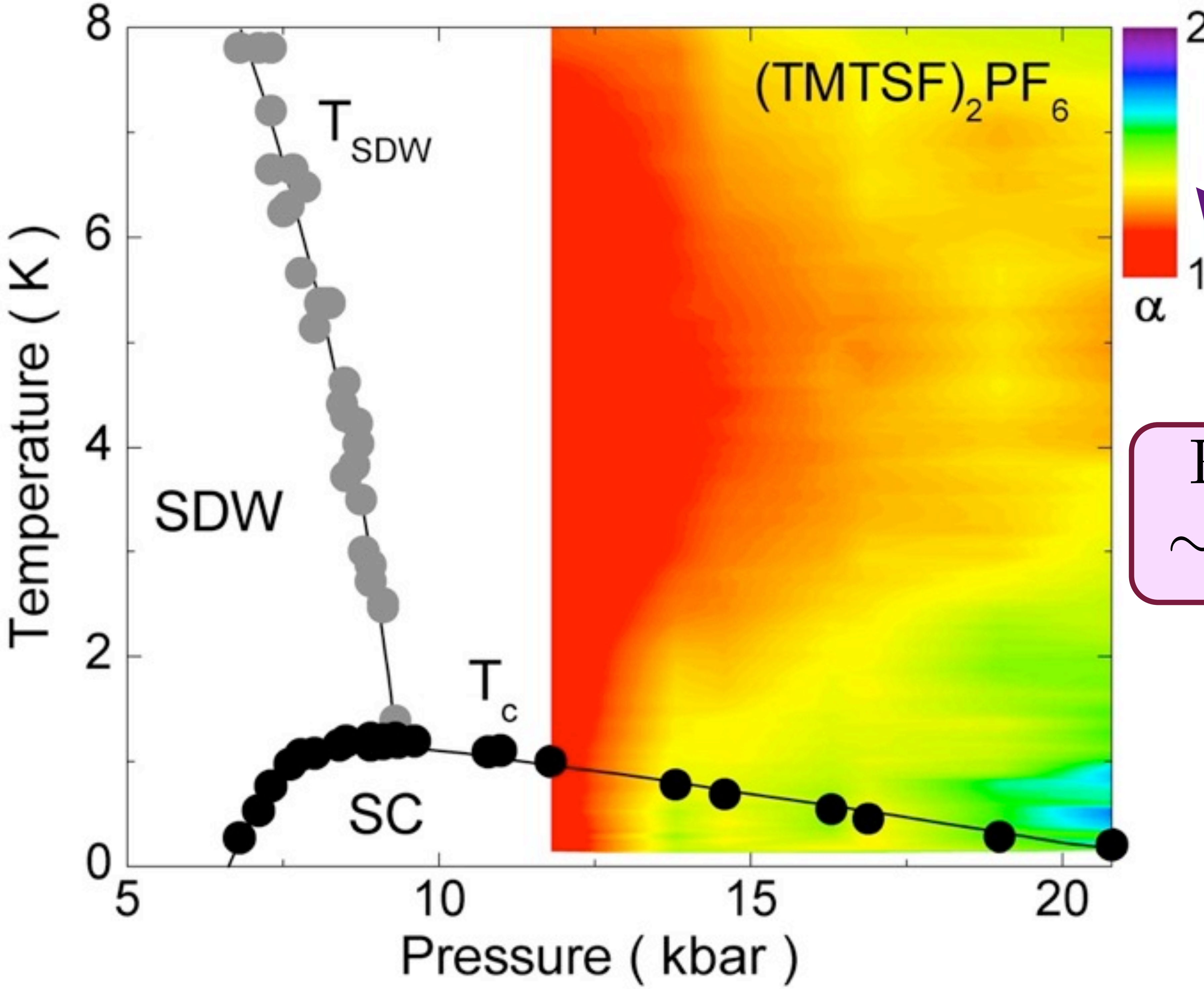
Organic Bechgaard compounds



Resistivity
$\sim \rho_0 + AT^\alpha$

*Doiron-Leyraud et al., PRB **80**, 214531 (2009)*

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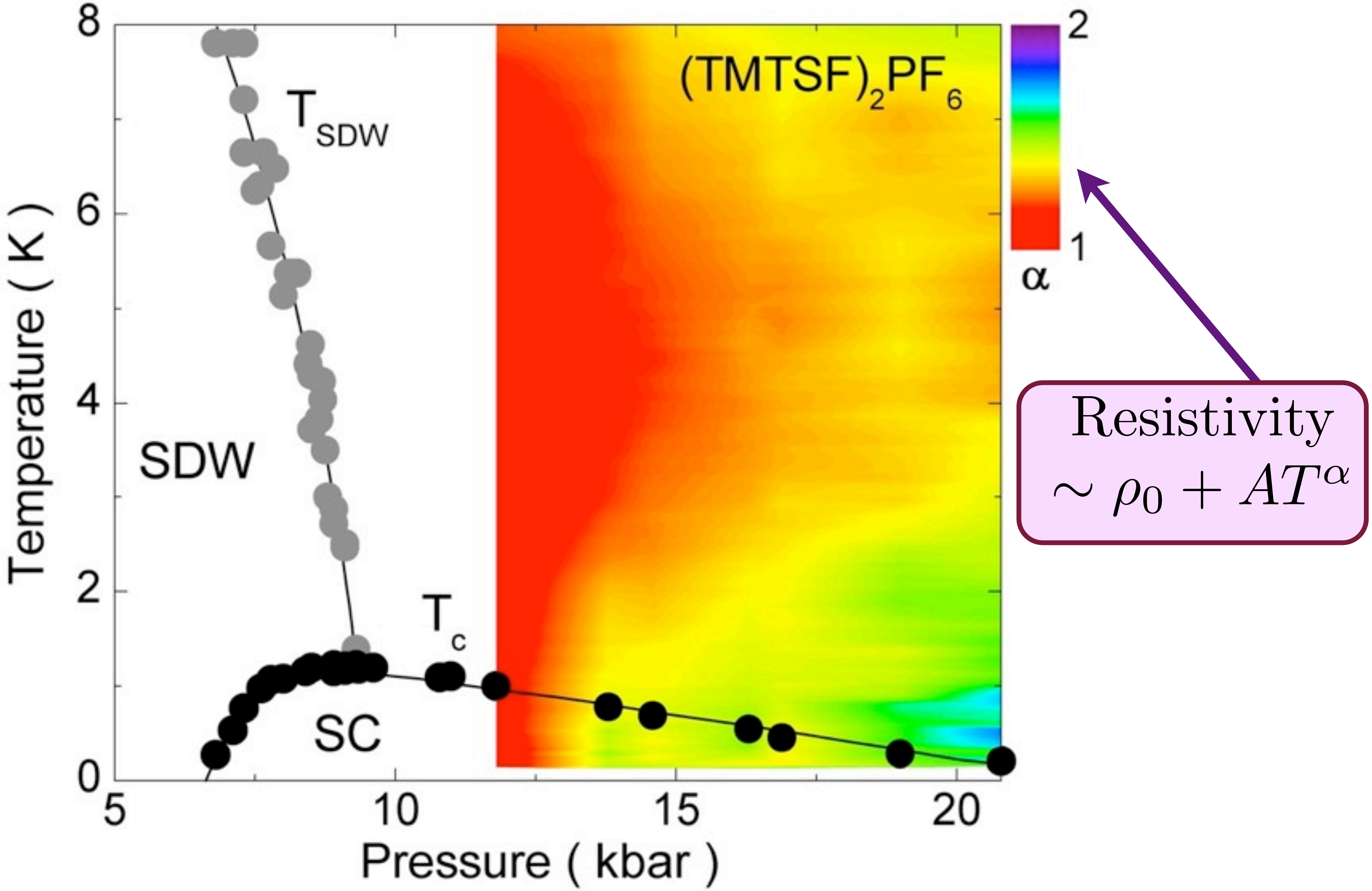

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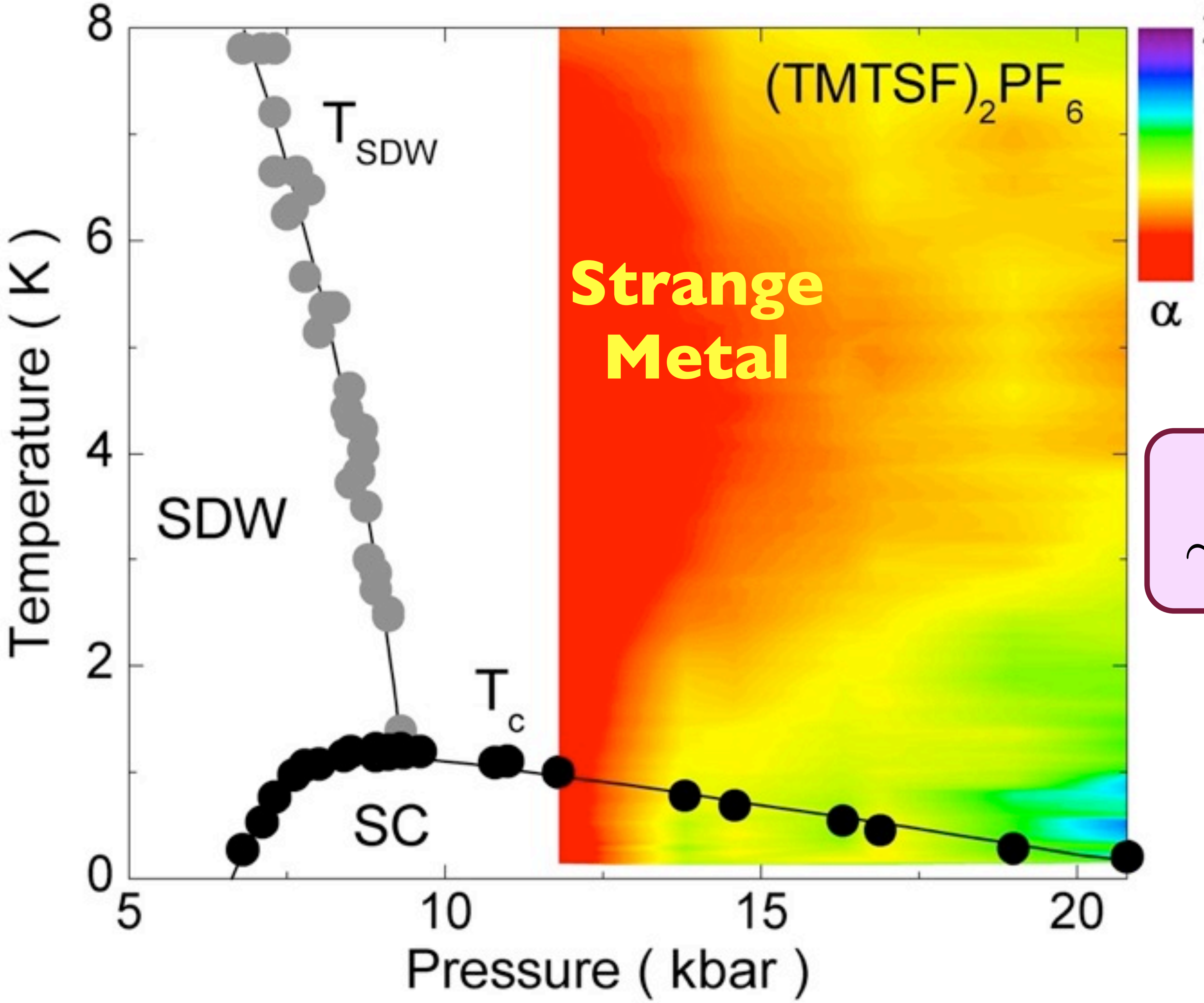

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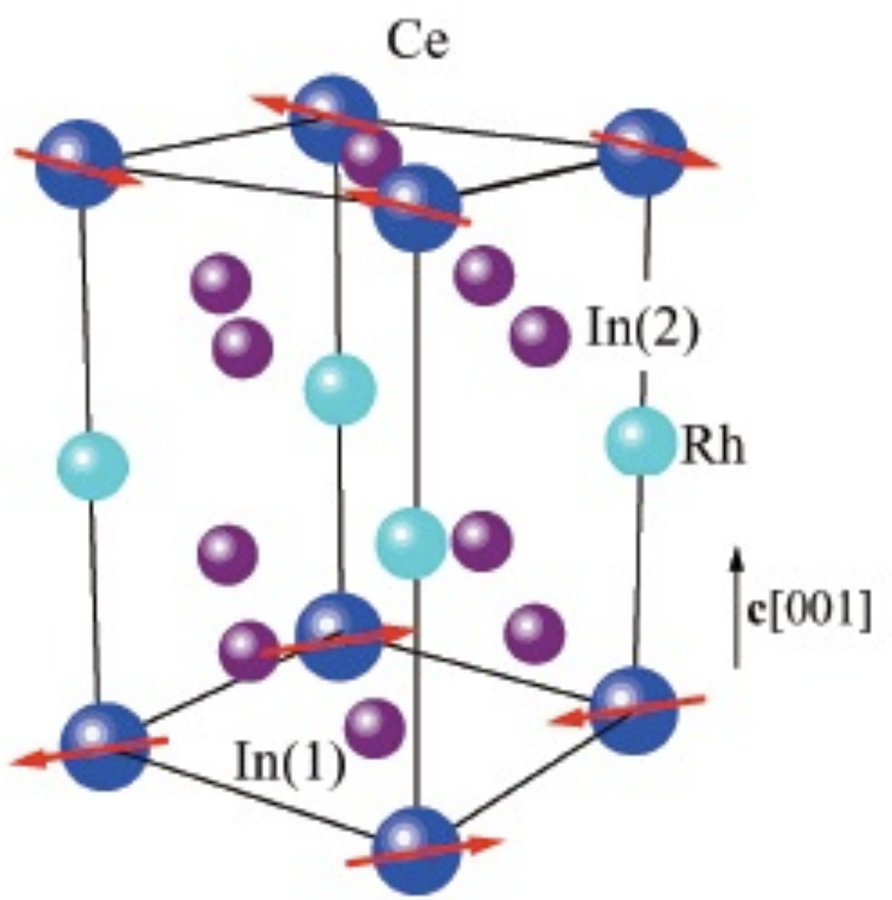
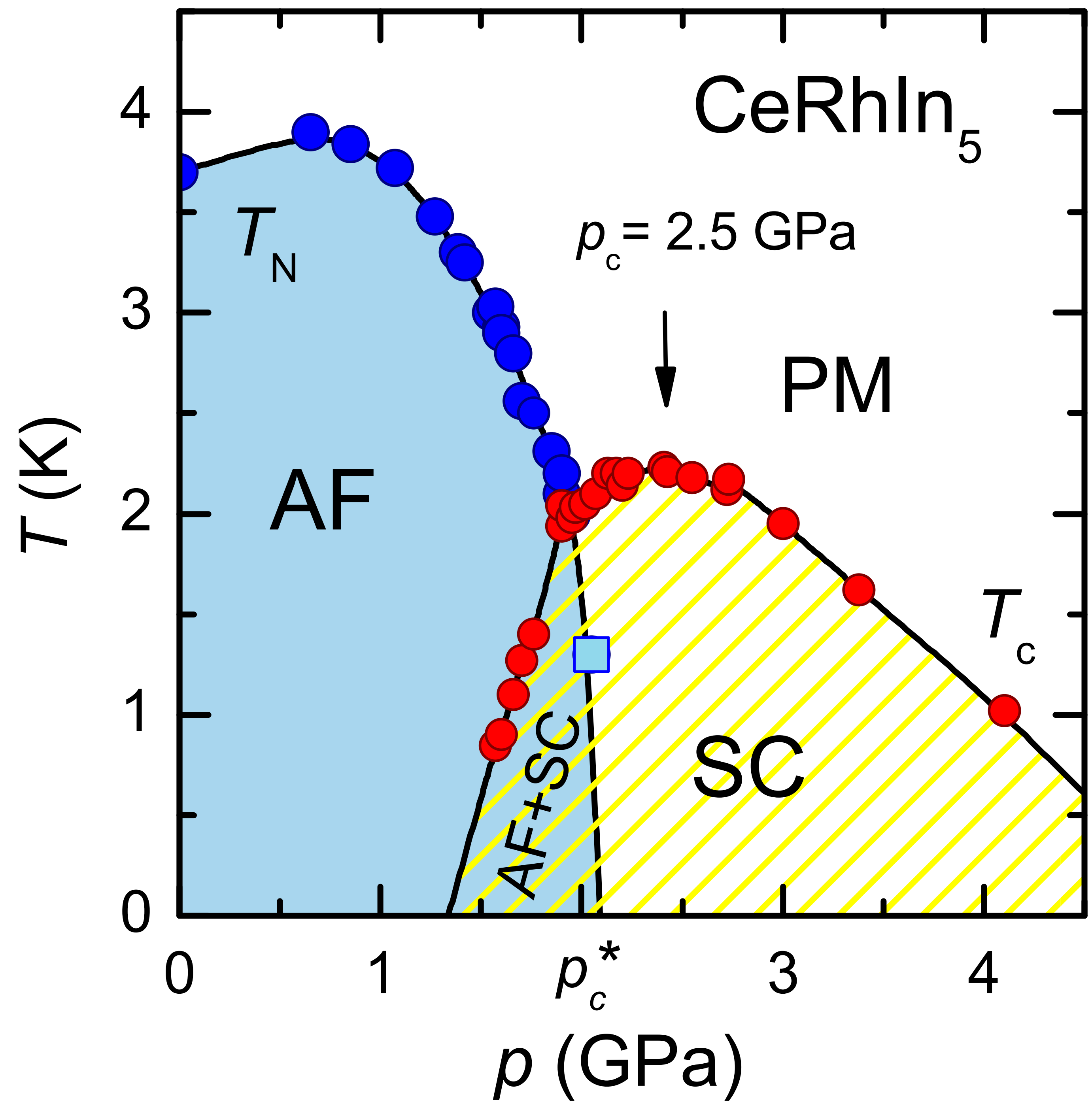
*Doiron-Leyraud et al., PRB **80**, 214531 (2009)*

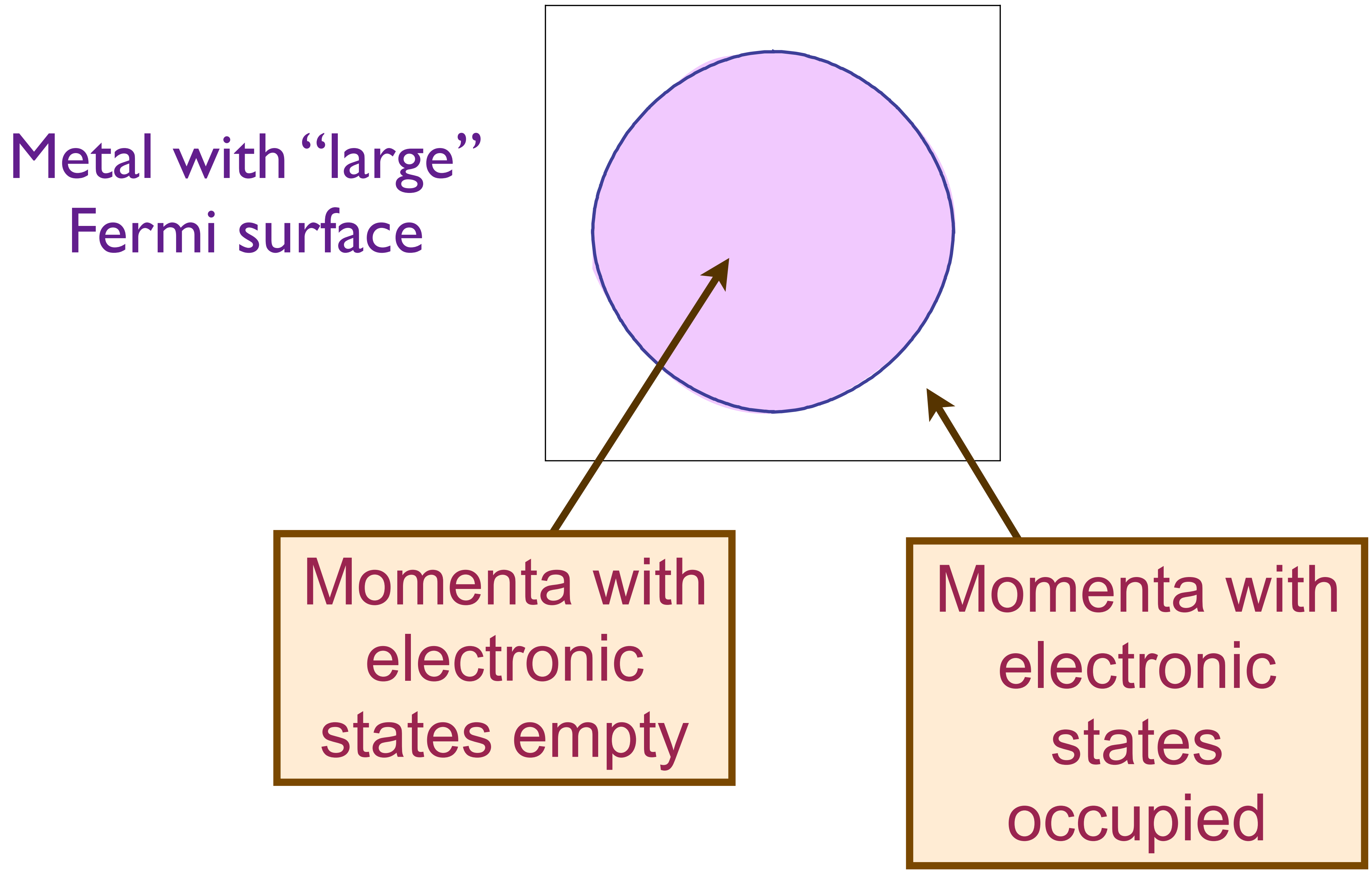
Lower T_c superconductivity in the heavy fermion compounds





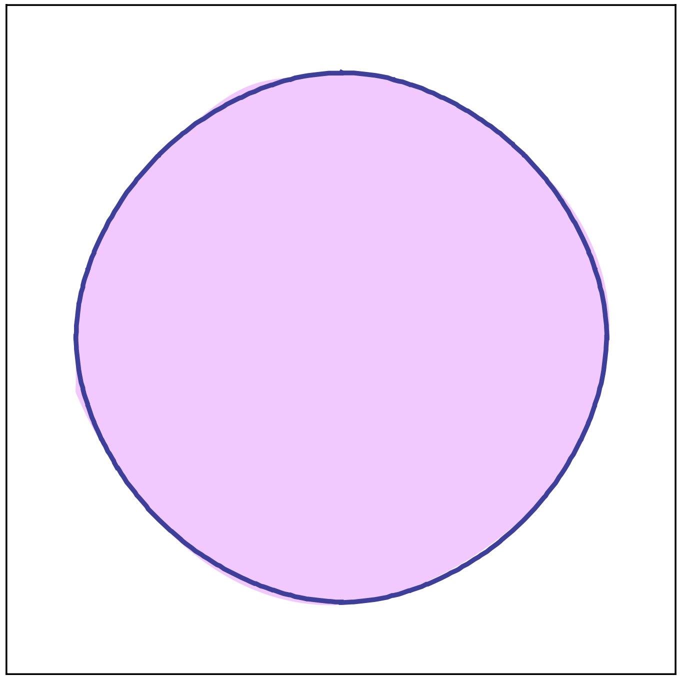
G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

Fermi surface

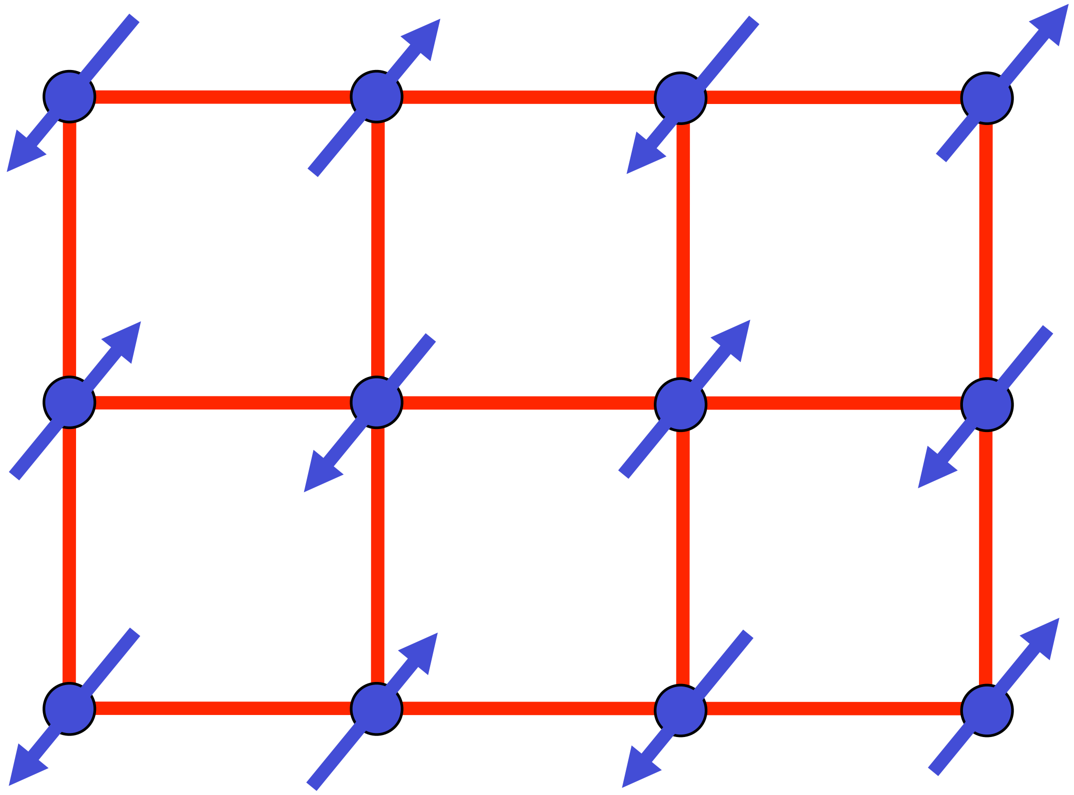


Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface



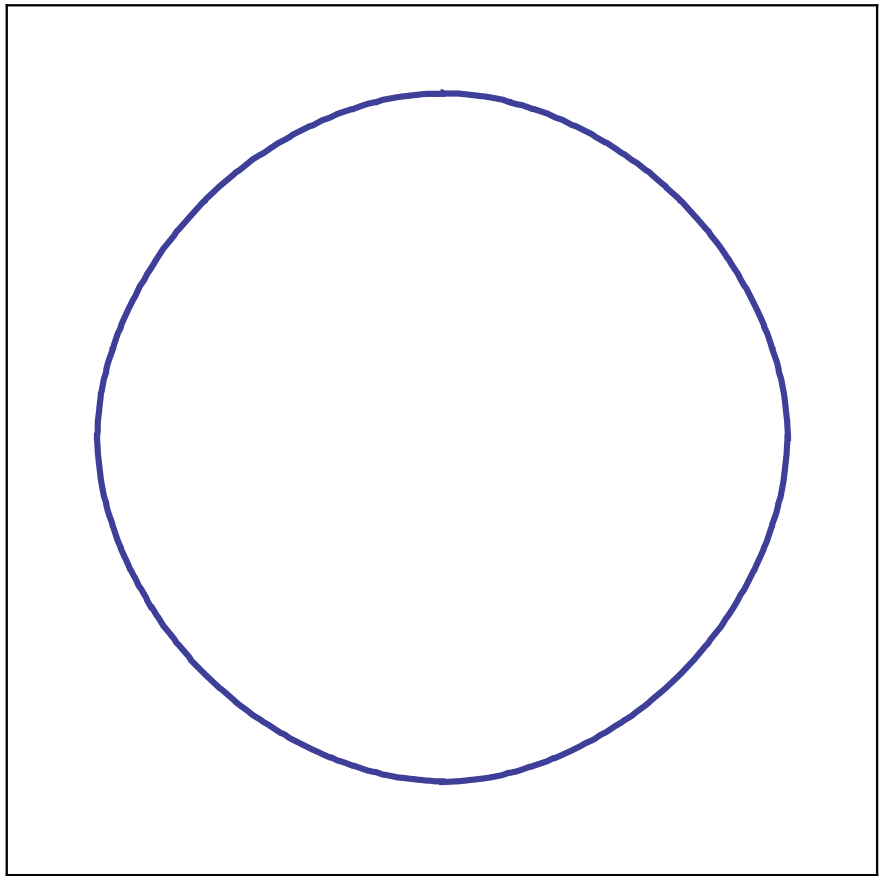
+



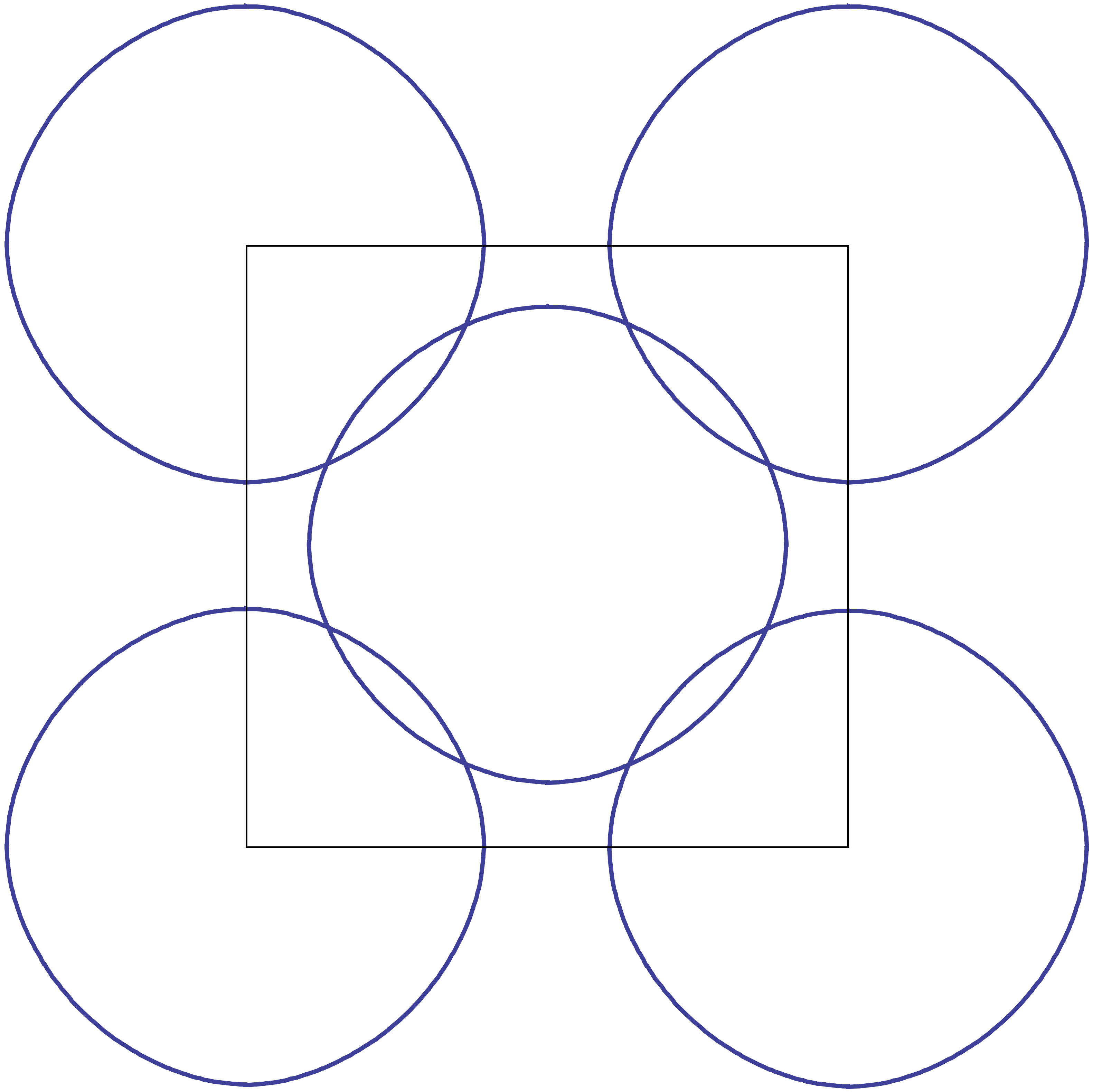
The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

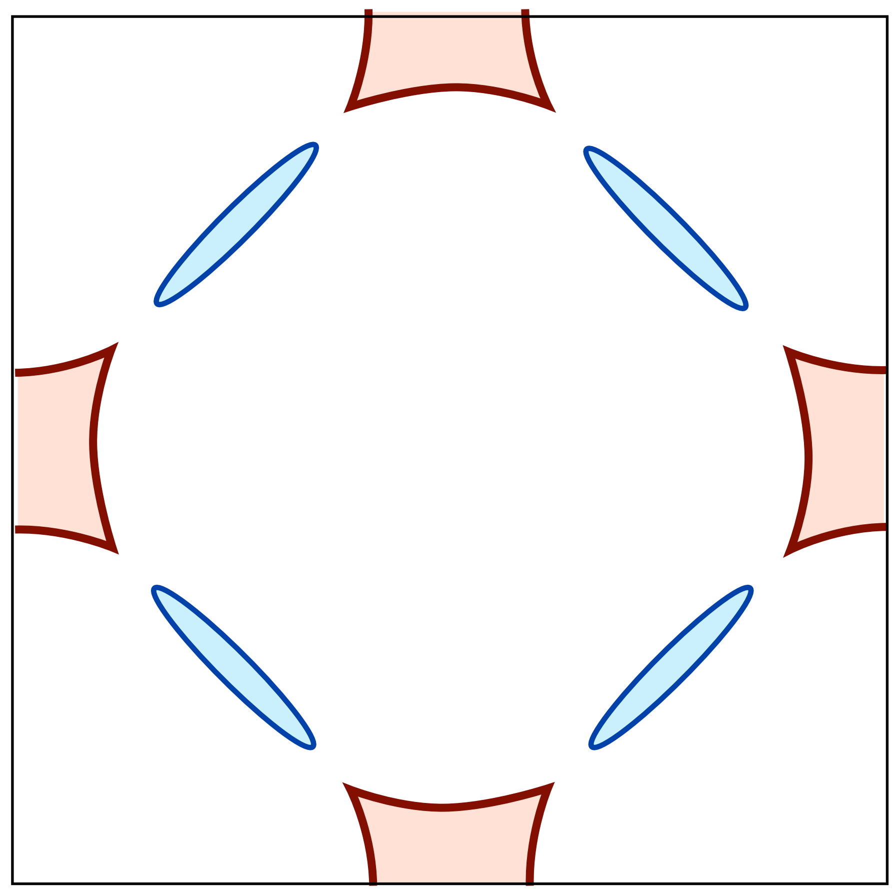
where $\mathbf{K}$ is the ordering wavevector.



Metal with “large” Fermi surface

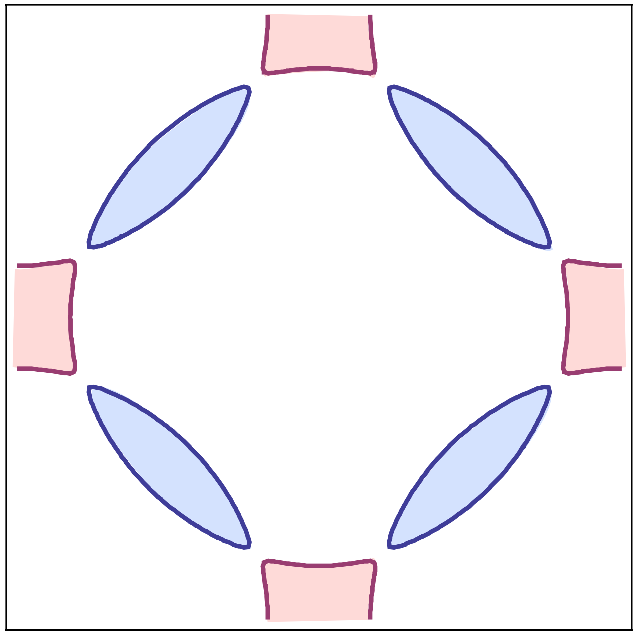


Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.



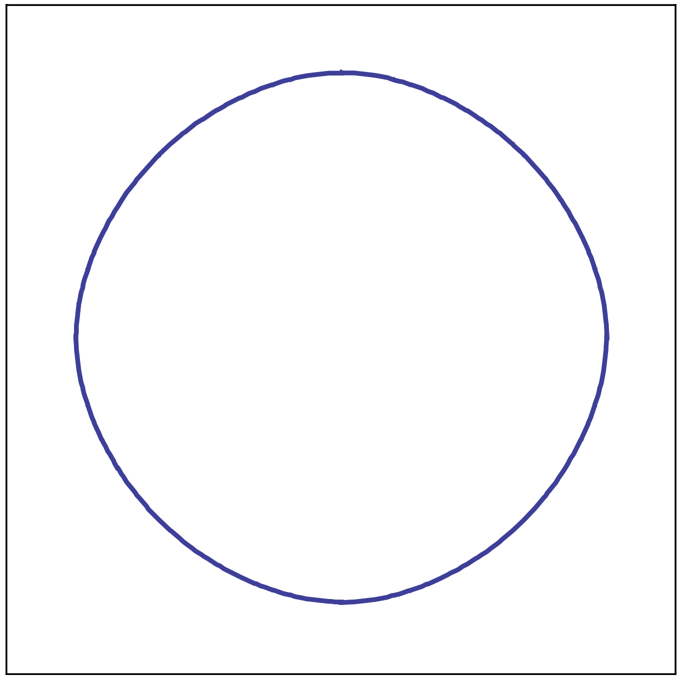
Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$

Fermi surface+antiferromagnetism



$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets



$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

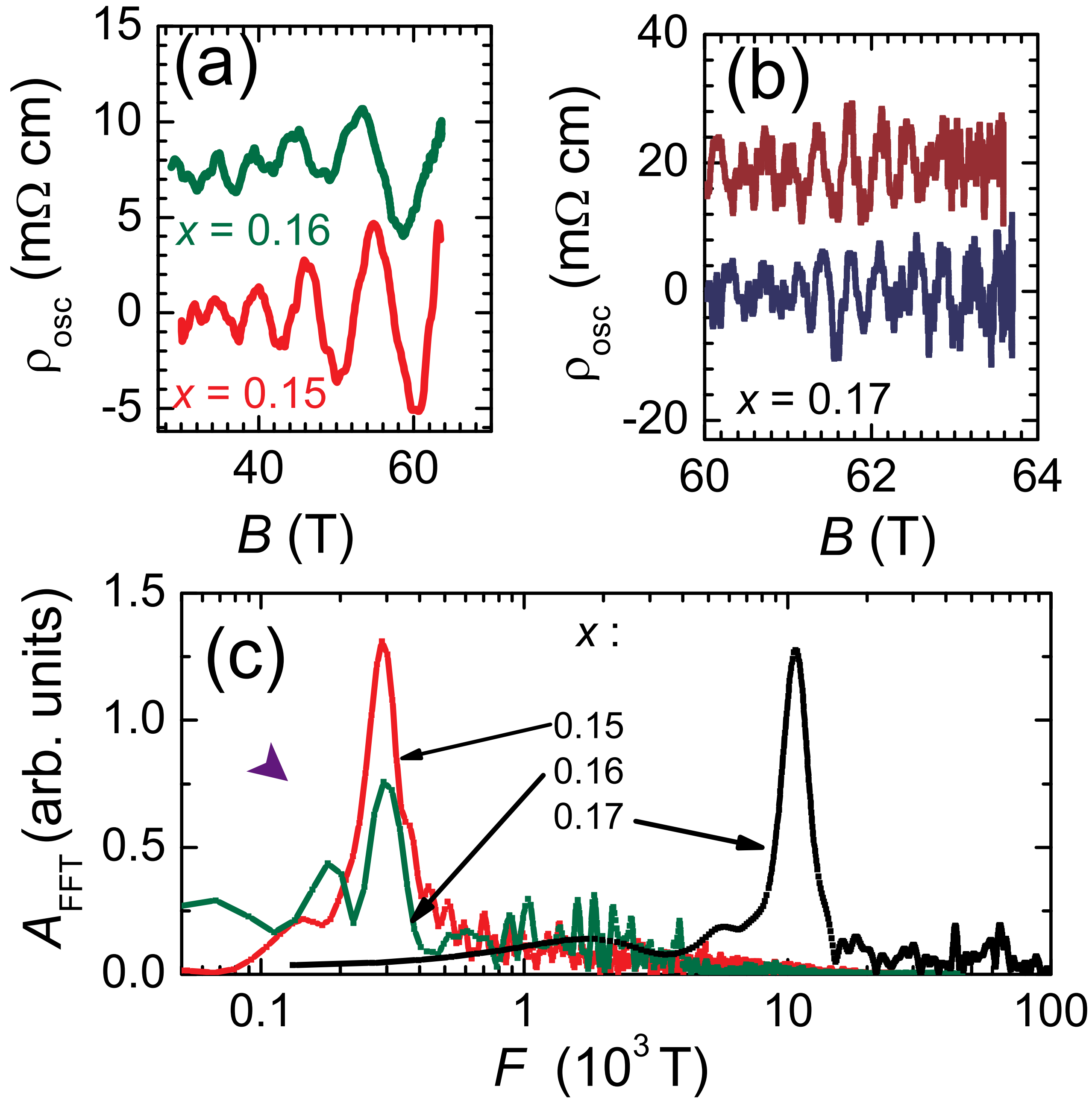
Increasing interaction

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Quantum oscillations

$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross,
Phys. Rev. Lett. **103**, 157002 (2009).



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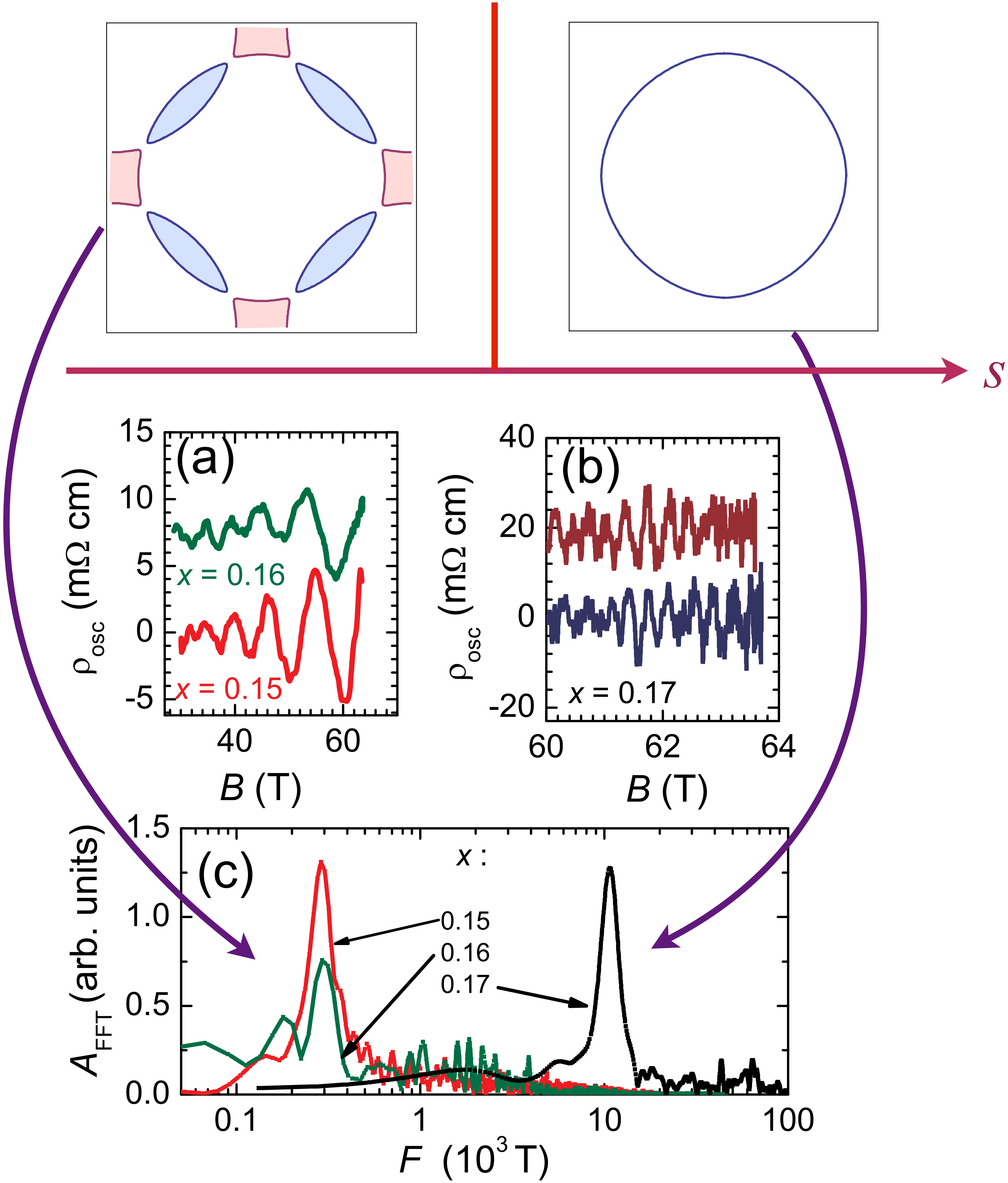
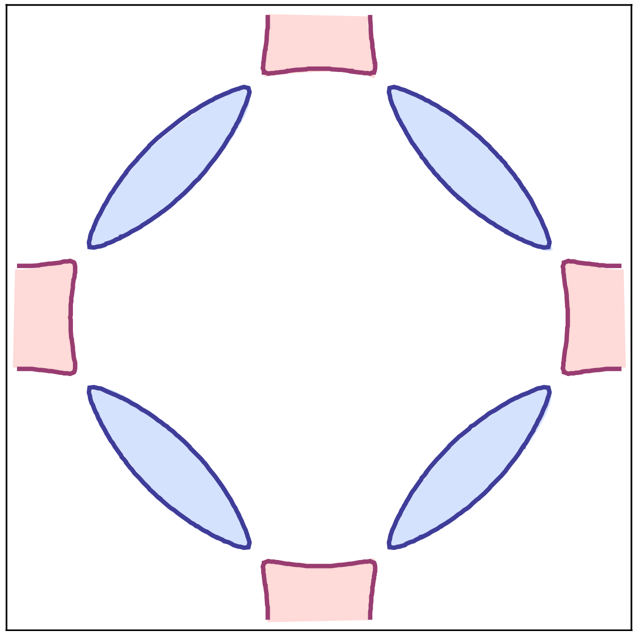
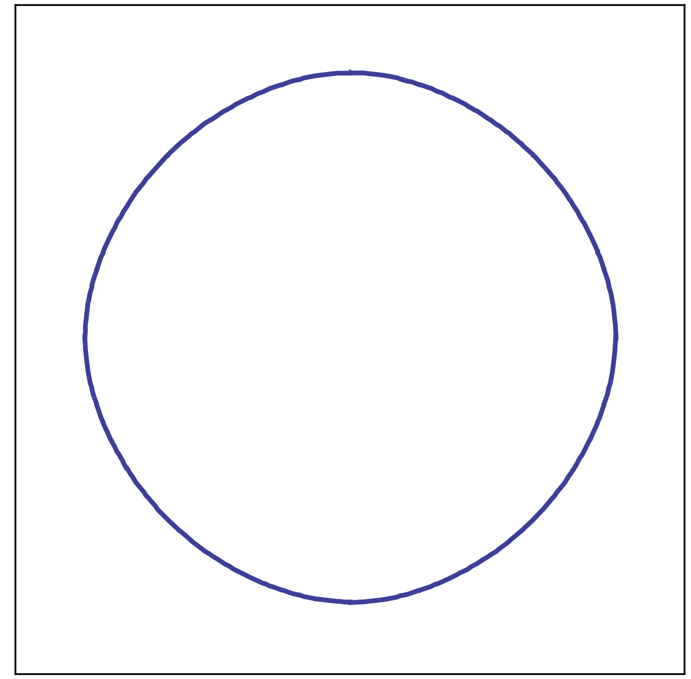


Fermi surface+antiferromagnetism





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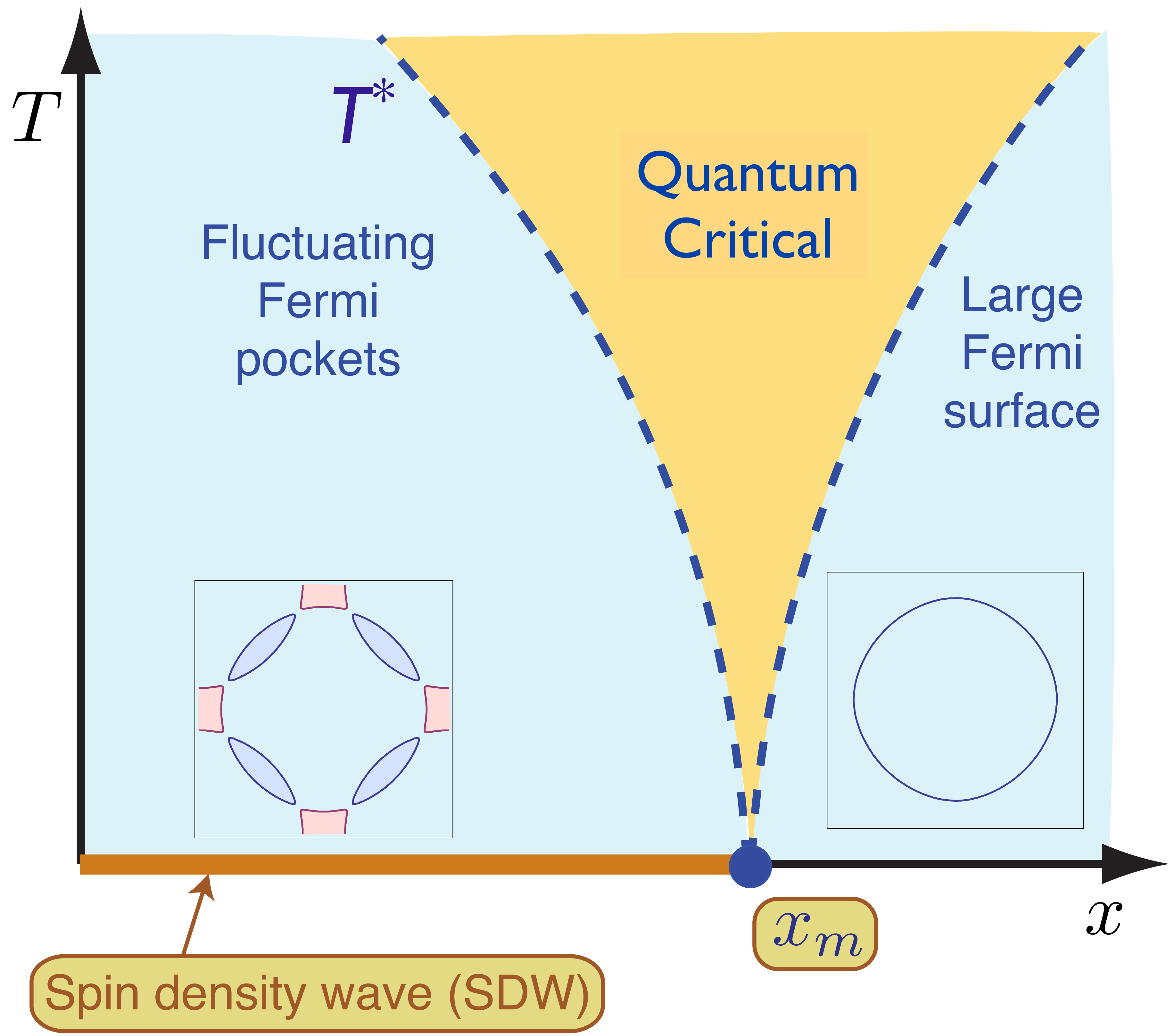
Quantum criticality of antiferromagnetism and Fermi surfaces



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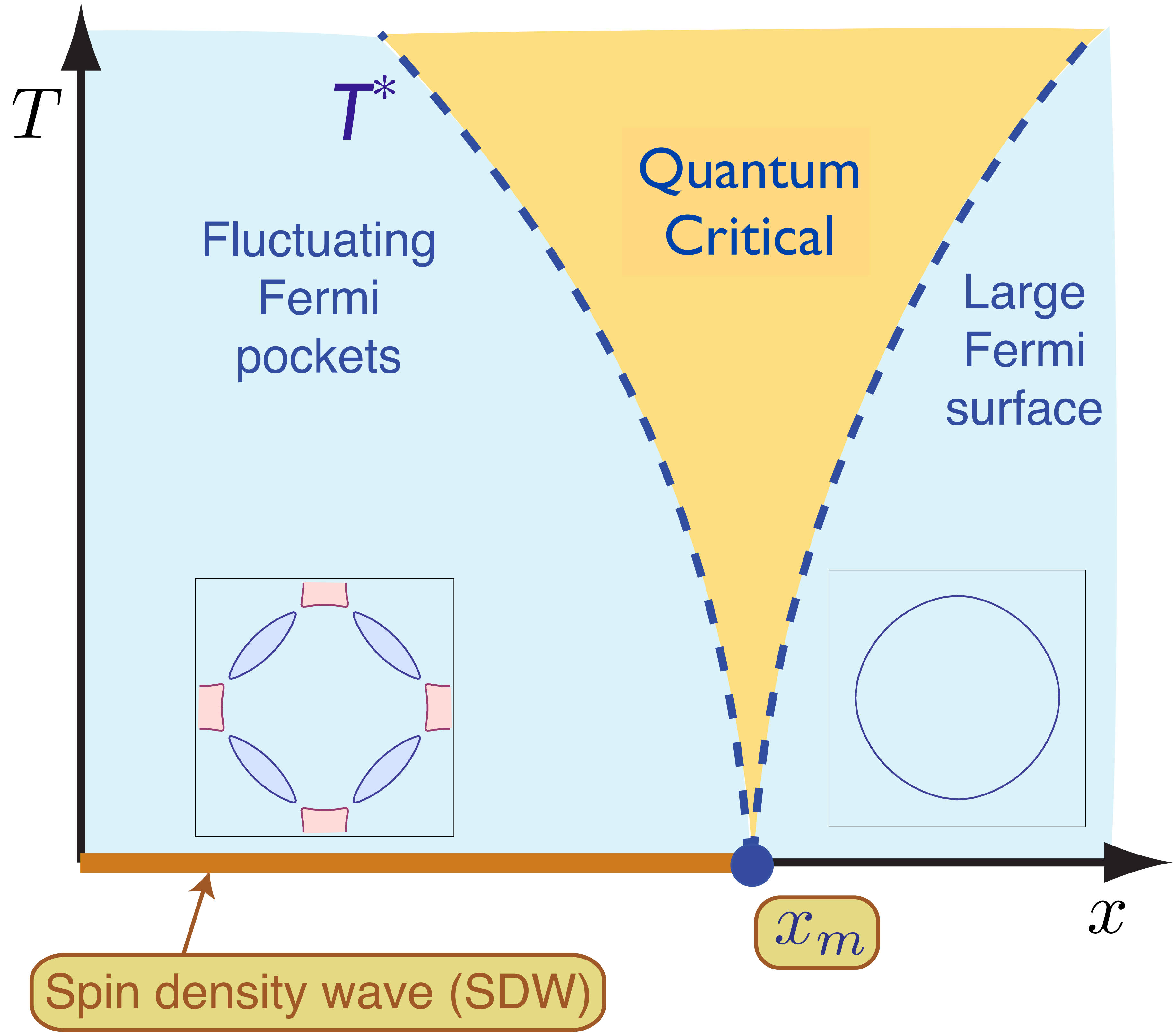

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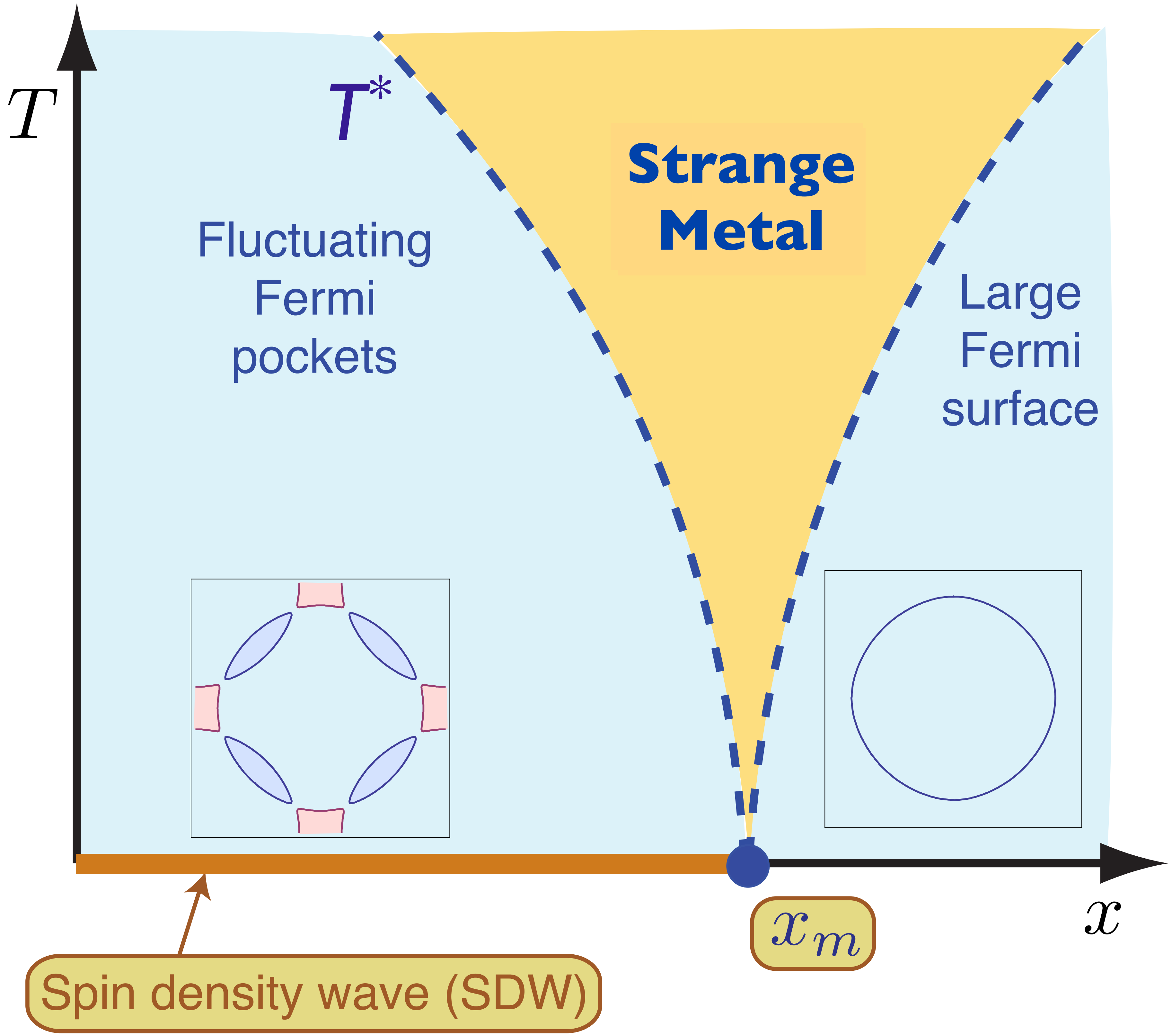

Conclusions

Paradigm of quantum phase transitions:
the quantum Ising chain, realized in
the ferromagnetic insulator $CoNb_2O_6$,
and
ultracold atoms in “tilted” optical lattices

Conclusions

Described excitations spectrum of the coupled-dimer antiferromagnet $TlCuCl_3$.
It has regimes of classical dynamics of triplon particles,
and of non-linear spin waves,
and a regime of quantum-criticality with characteristic equilibration time

$$\hbar/k_B T$$

Conclusions

Quantum criticality of
antiferromagnetism
and Fermi surface reconstruction
controls "strange metal" regime of
quasi-two dimensional
higher temperature superconductors