Quantum spin liquids: from Rydberg atoms to the high temperature superconductors

JOINT QUANTUM INSTITUTE

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INSTITUTE FOR ADVANCED STUDY

PHYSICS HARVARD

1. Spin liquids and Z_2 gauge theory

2. Rydberg atoms as a Z_2 gauge theory

   Probing topological spin liquids

3. Paramagnon fractionalization theory of the pseudogap metal of the Hubbard model

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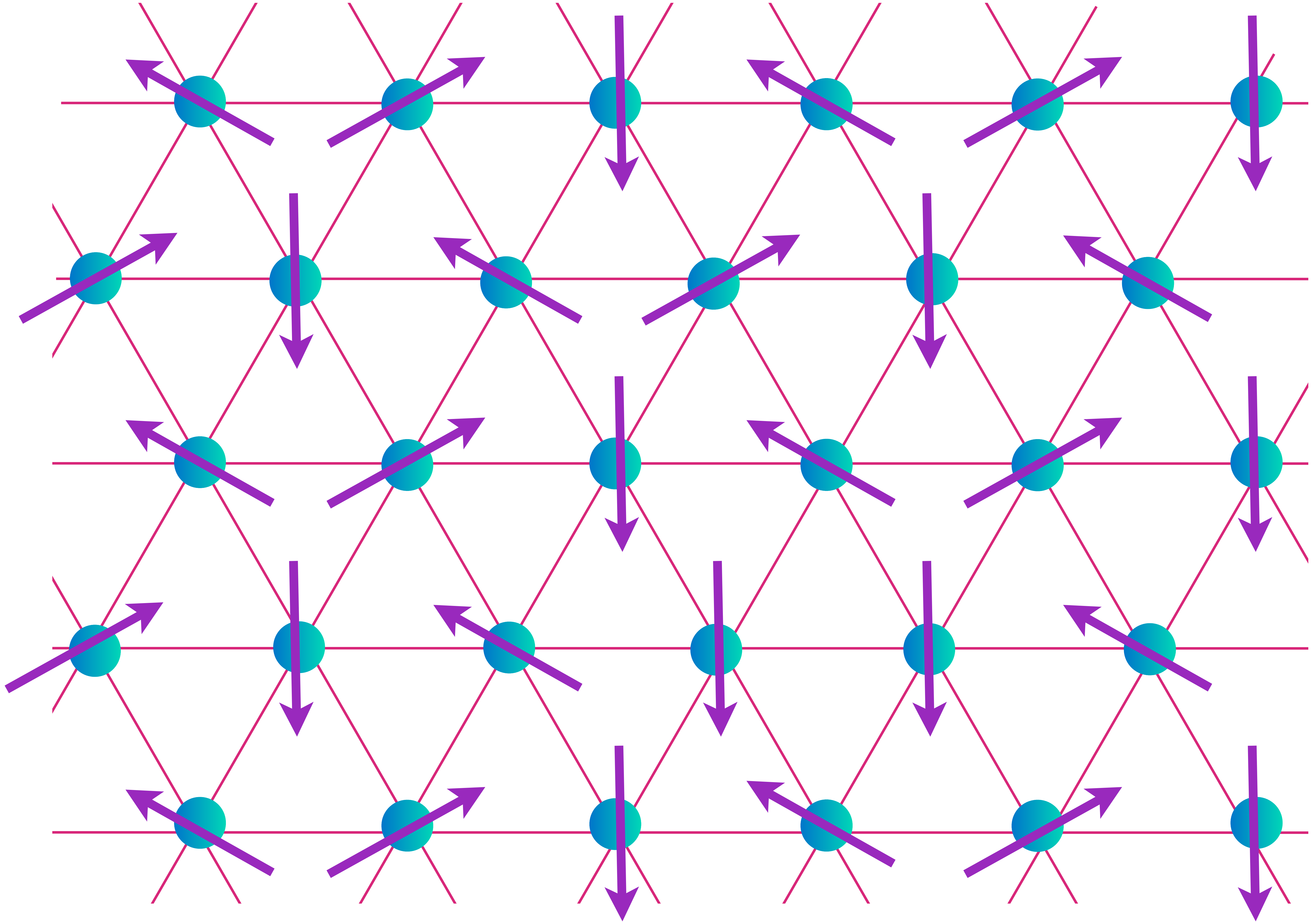
Triangular lattice antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Nearest-neighbor model has non-collinear Neel order

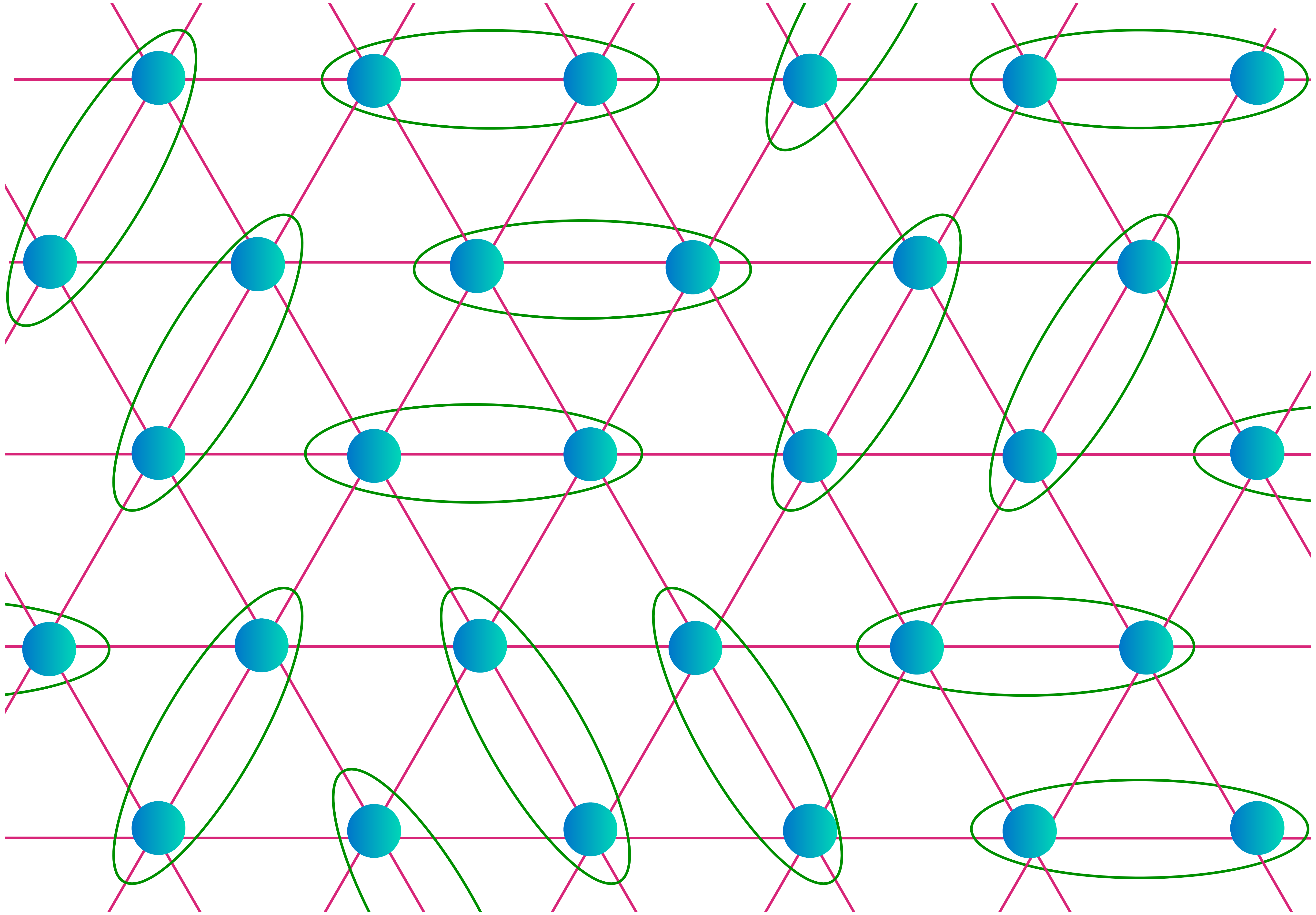
Mapping of bosons and spins

Boson B	Spin $\vec{S}$
$B^\dagger B \leq 1$	$\vec{S}^2 = 3/4$
States $\|0\rangle$, $B^\dagger\|0\rangle$	States $\|\uparrow\rangle$ $\|\downarrow\rangle$
Operators	Operators
$B^\dagger$ $\longleftrightarrow$	$S_+ = S_x + iS_y$
B $\longleftrightarrow$	$S_- = S_x - iS_y$
$B^\dagger B - \frac{1}{2}$ $\longleftrightarrow$	S_z

Spin liquid: resonating valence bonds

Bosons at half-filling,
or a spin model with S=1/2 per unit cell

$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) = \frac{1}{\sqrt{2}} \left(B_1^\dagger - B_2^\dagger \right) |0\rangle$$



$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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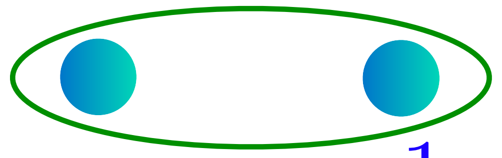

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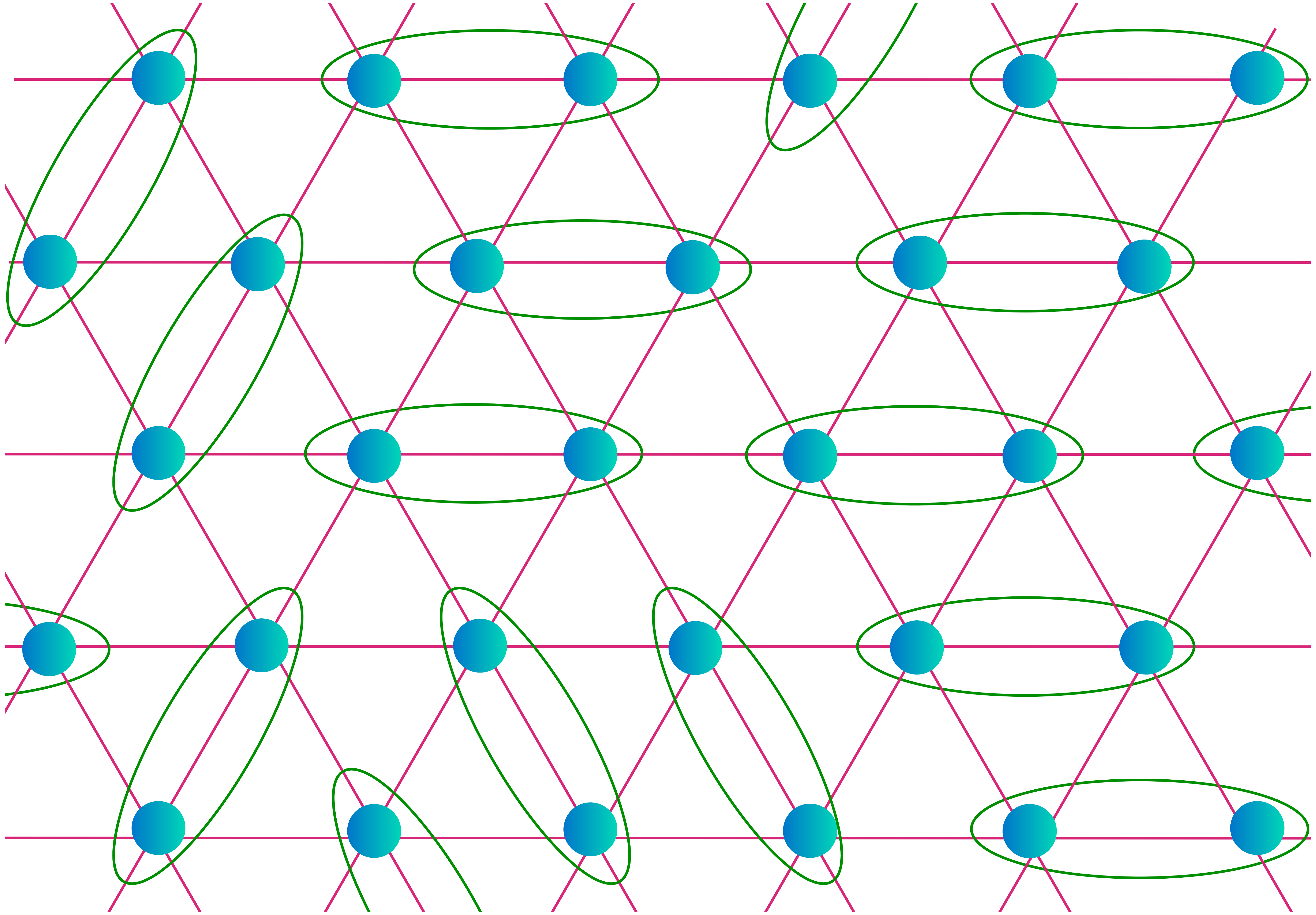

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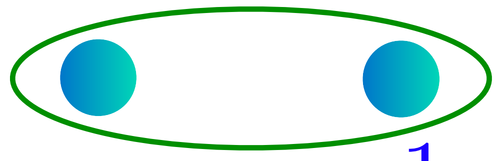

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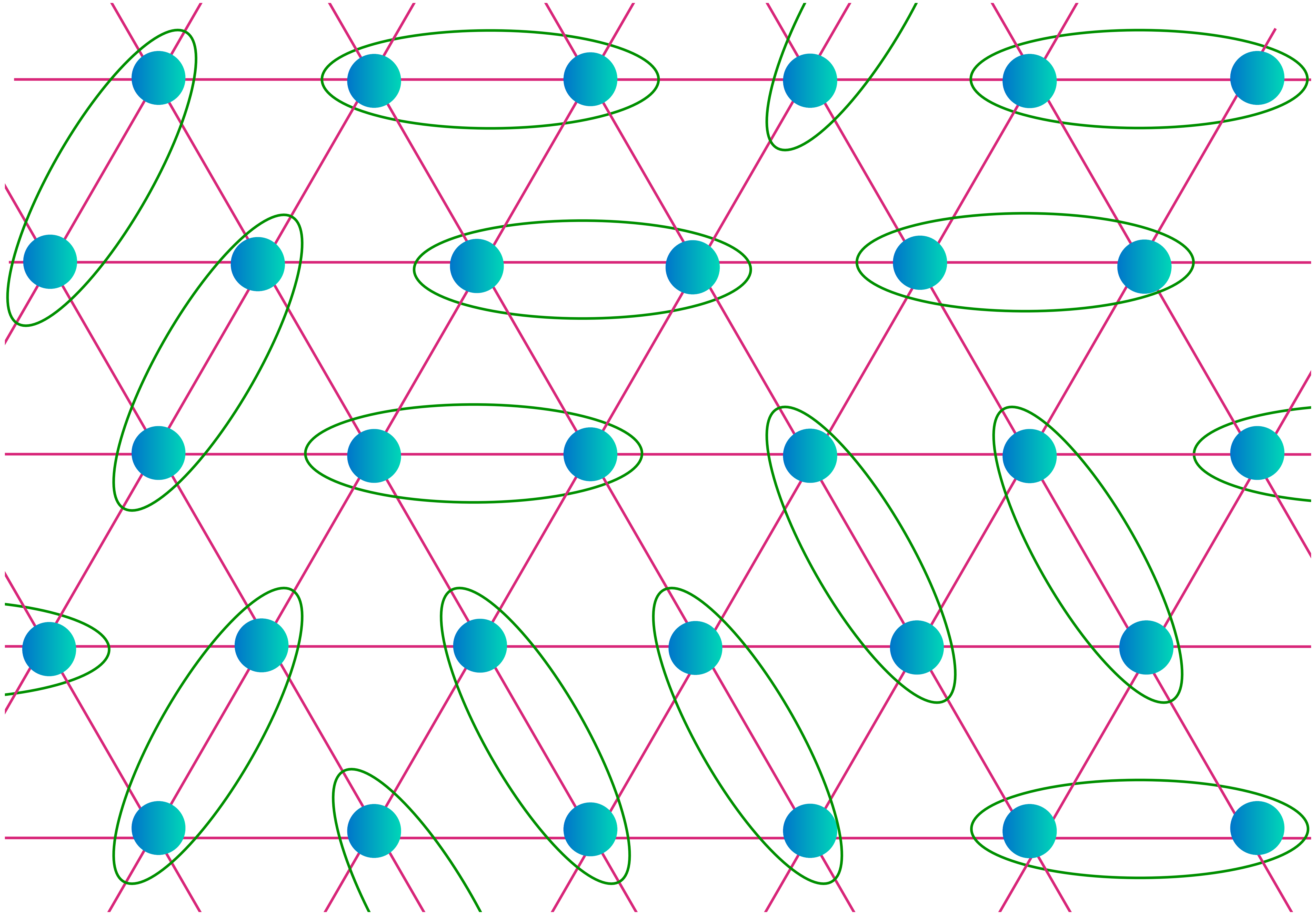

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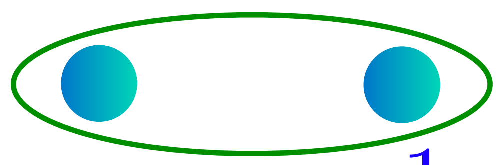

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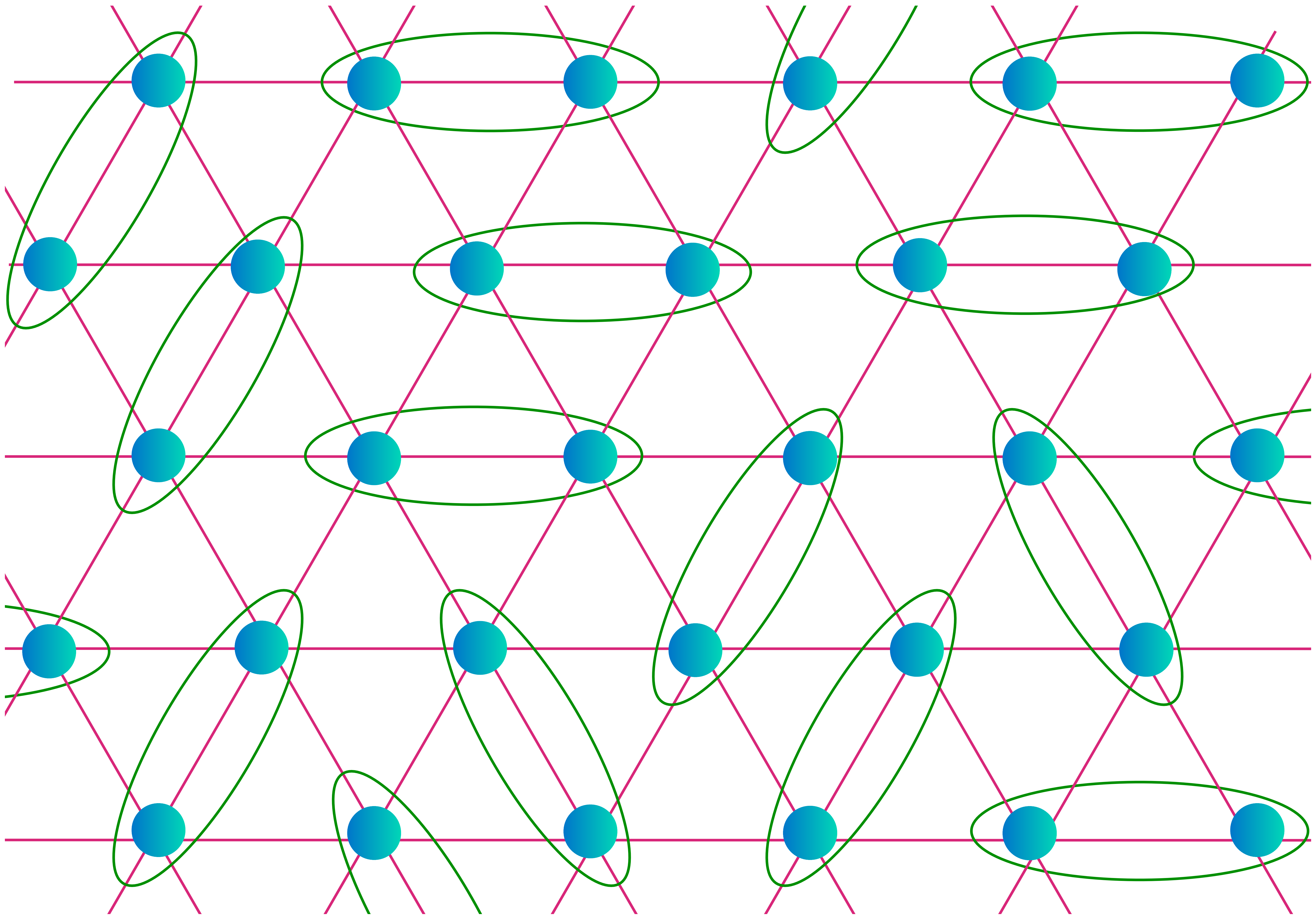

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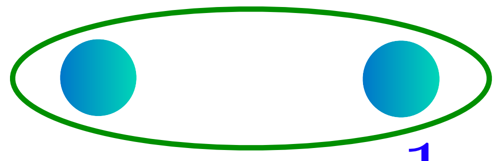

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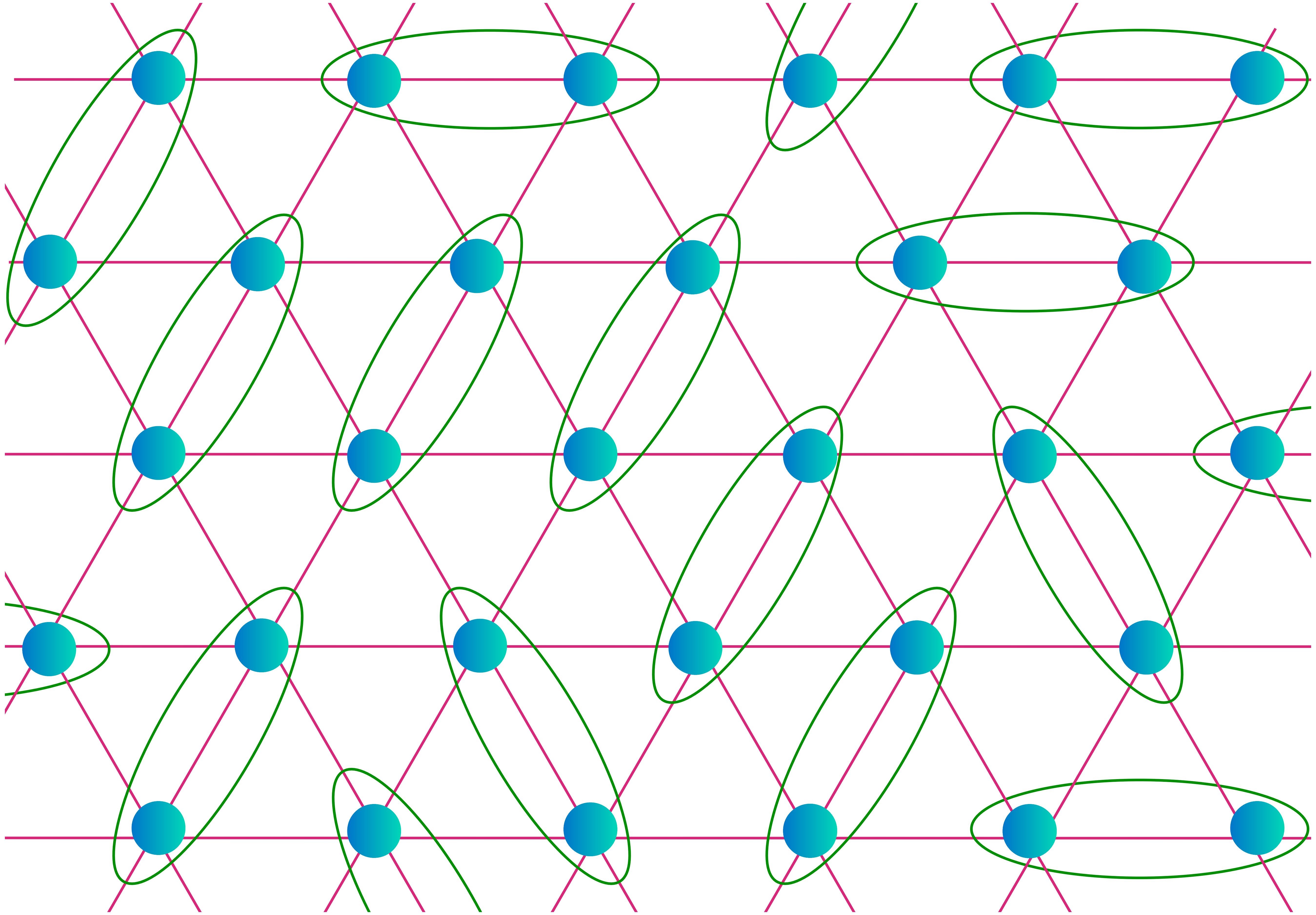

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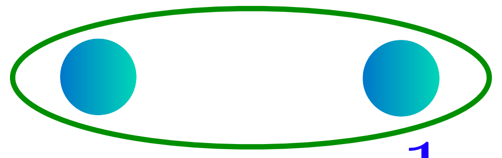

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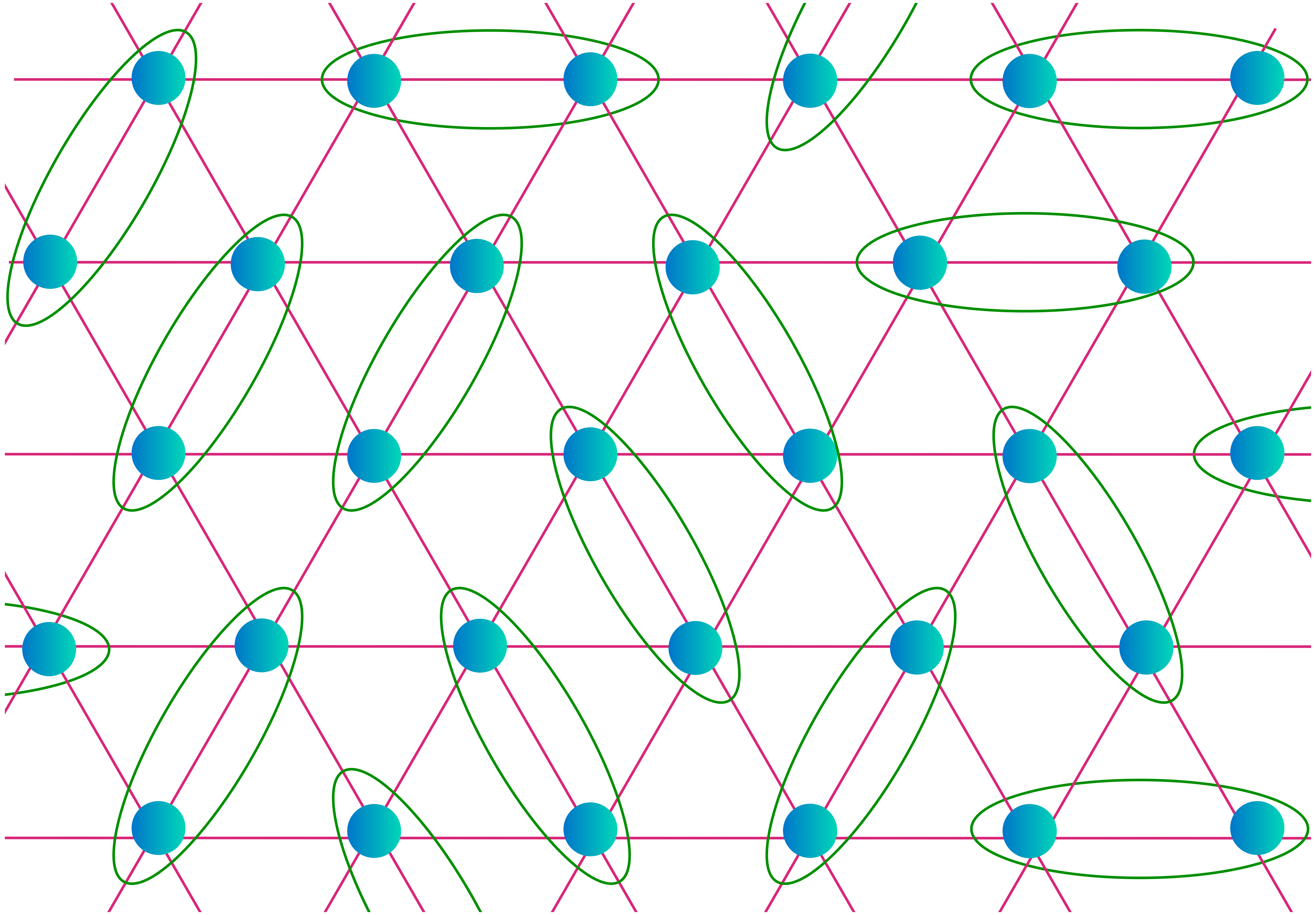

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RVB: Z_2 spin liquid

Read and Sachdev (1990); Wen (1991)

The simplest stable spin liquid (which need not break time-reversal) is the deconfined phase of a $\mathbb{Z}_2$ gauge theory. There are 'spinon' excitations which carry unit $\mathbb{Z}_2$ electric charges, and 'vison' excitations which carry π $\mathbb{Z}_2$ magnetic flux.

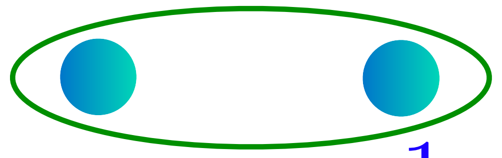
Anyon	e (spinon)	ϵ (spinon)	m (vison)
Boson number	1/2	1/2	0
Self-statistics	boson	fermion	boson

Any pair of e, ϵ, m are mutual semions.

These anyons are 'topological': they cannot be created individually by any local operator, and their existence implies a four-fold ground state degeneracy on a large torus.

RVB: Z_2 spin liquid

Excitations with boson number 1/2



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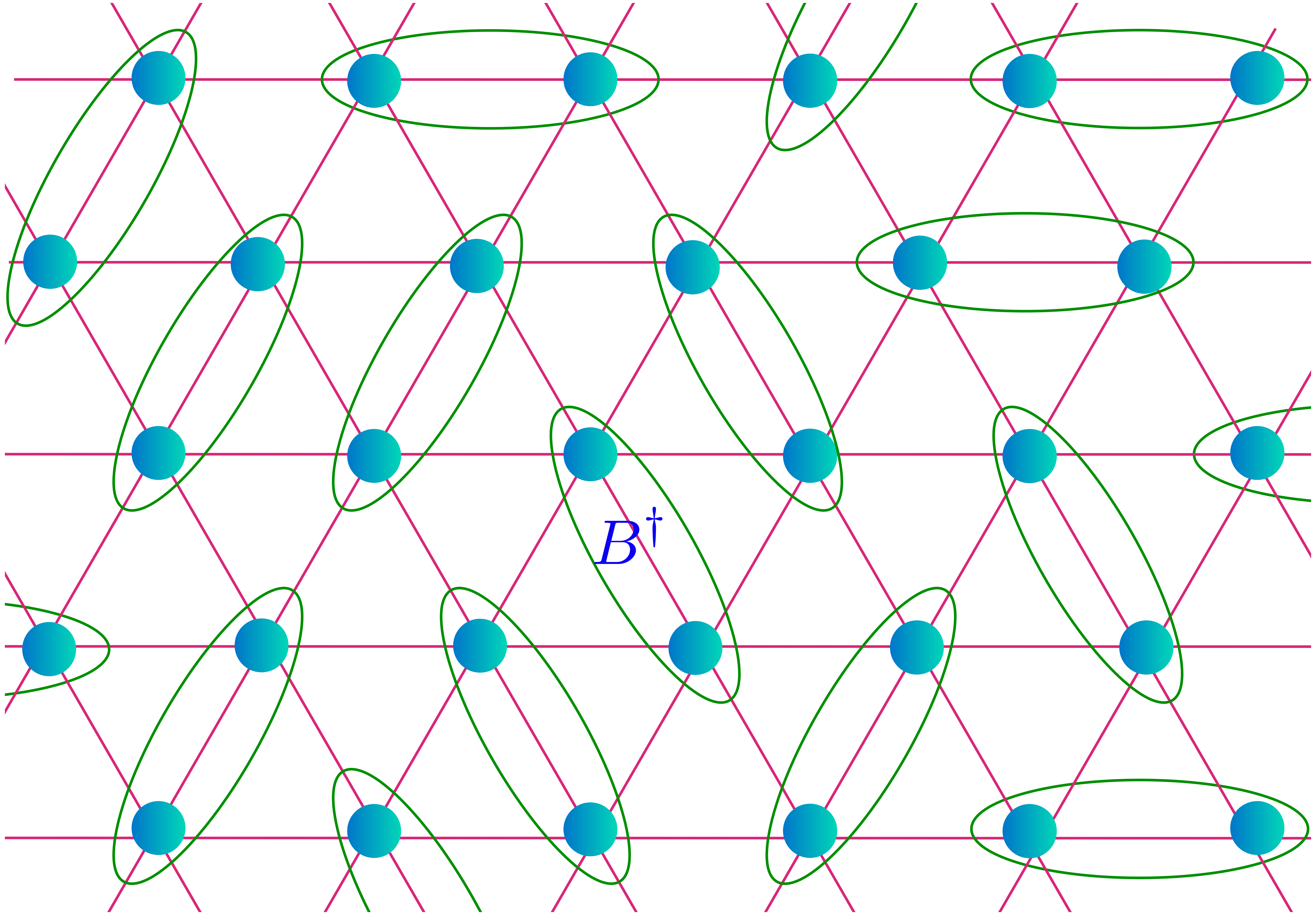
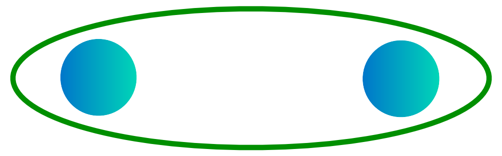
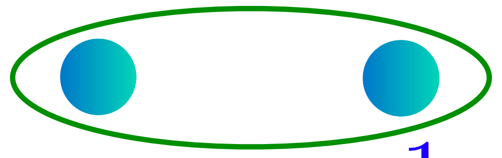


$B_2^\dagger$ 

$$= \frac{1}{\sqrt{2}} B_1^\dagger B_2^\dagger |0\rangle = \frac{1}{\sqrt{2}} | \uparrow\uparrow\rangle$$

RVB: Z_2 spin liquid

Excitations with boson number 1/2
a "spinon"



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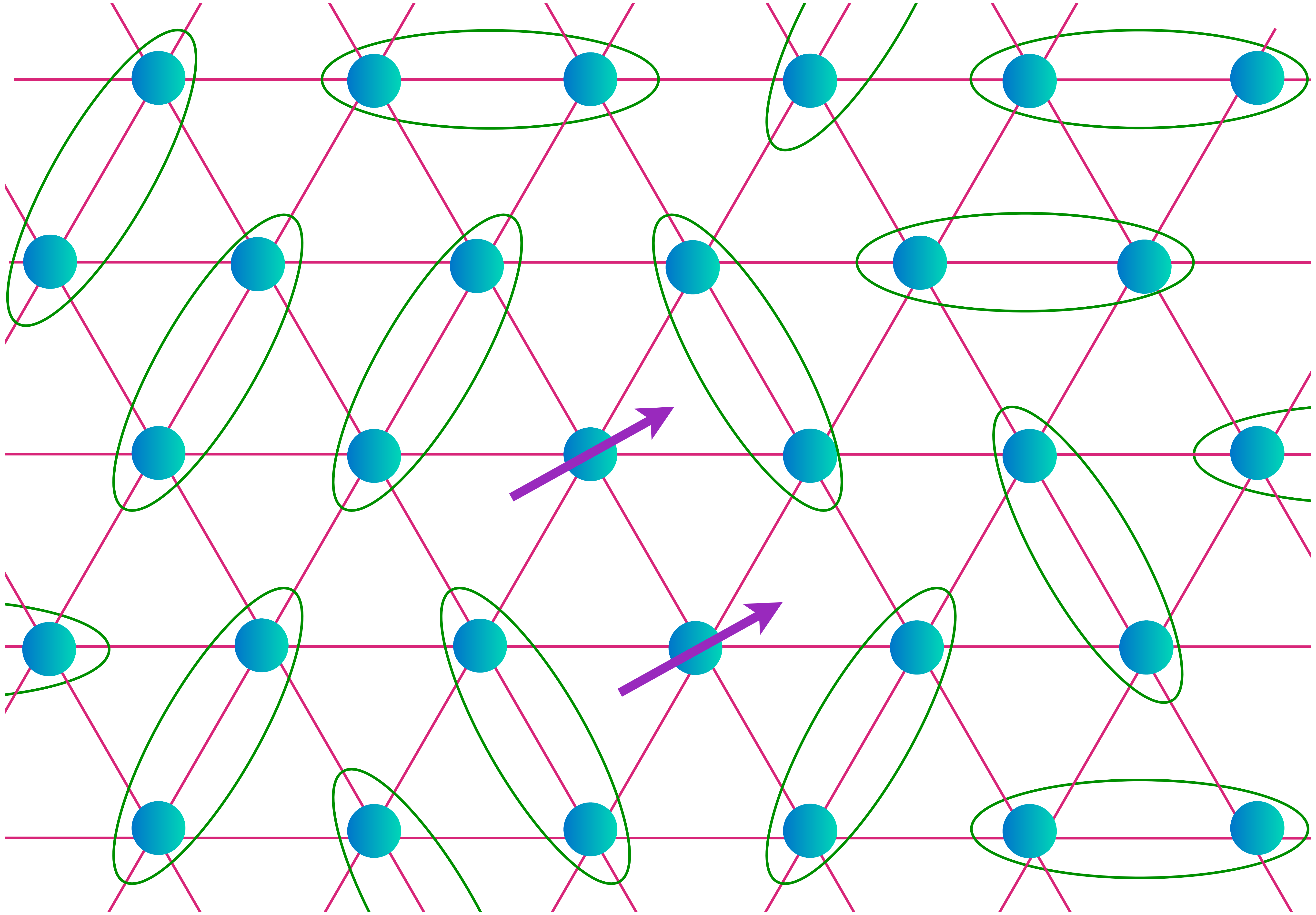

- The boson creation operator $B^\dagger$ creates a *pair* of spinons.
- A single spinon carries boson number $B^\dagger B = 1/2$: fractionalization!

RVB: Z_2 spin liquid

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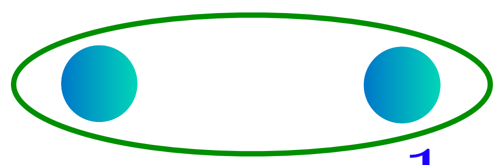

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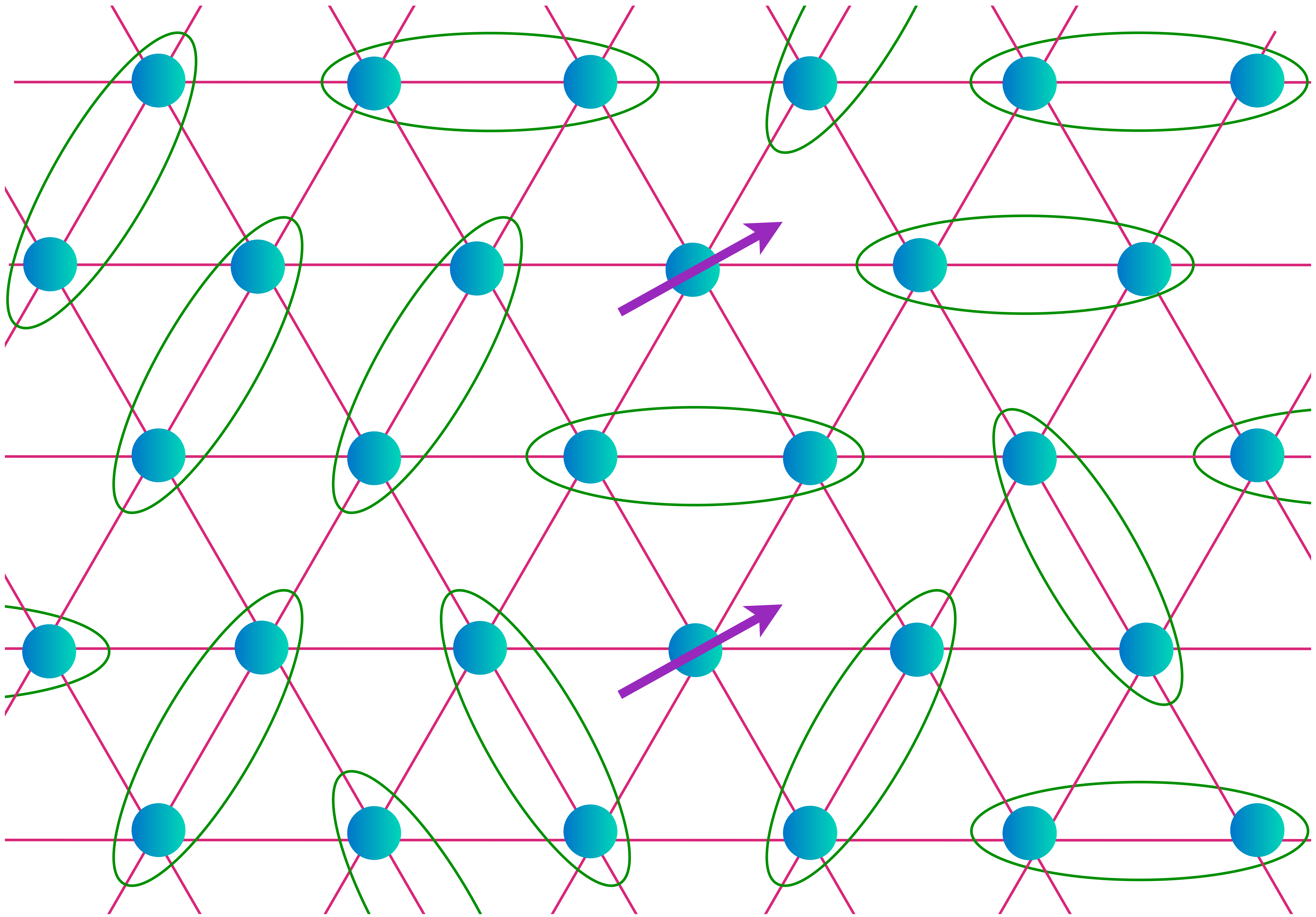

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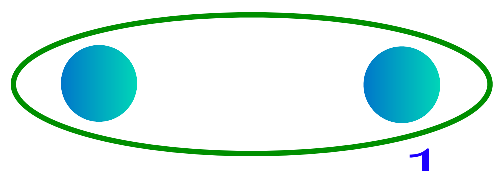

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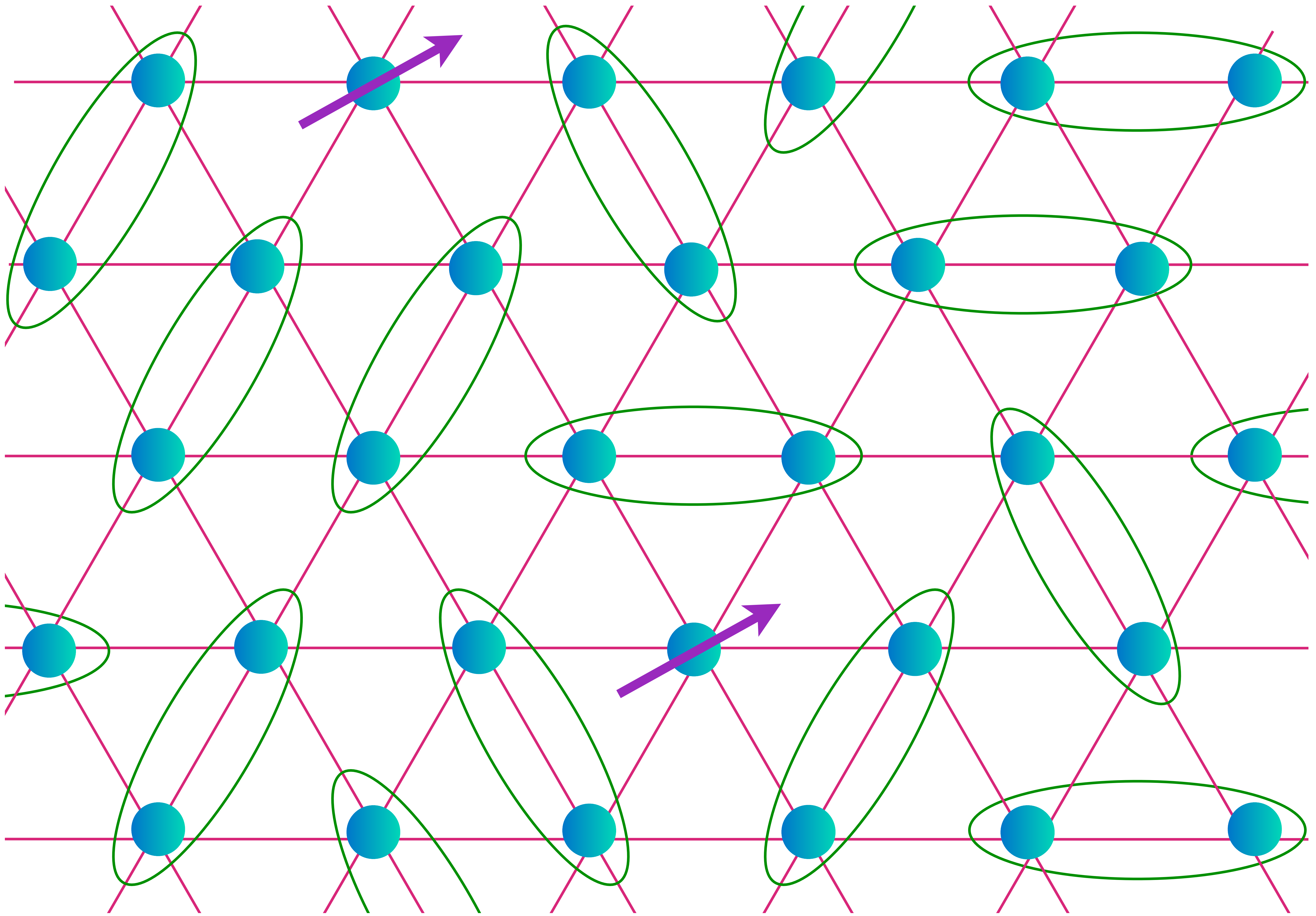

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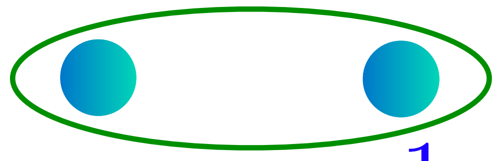

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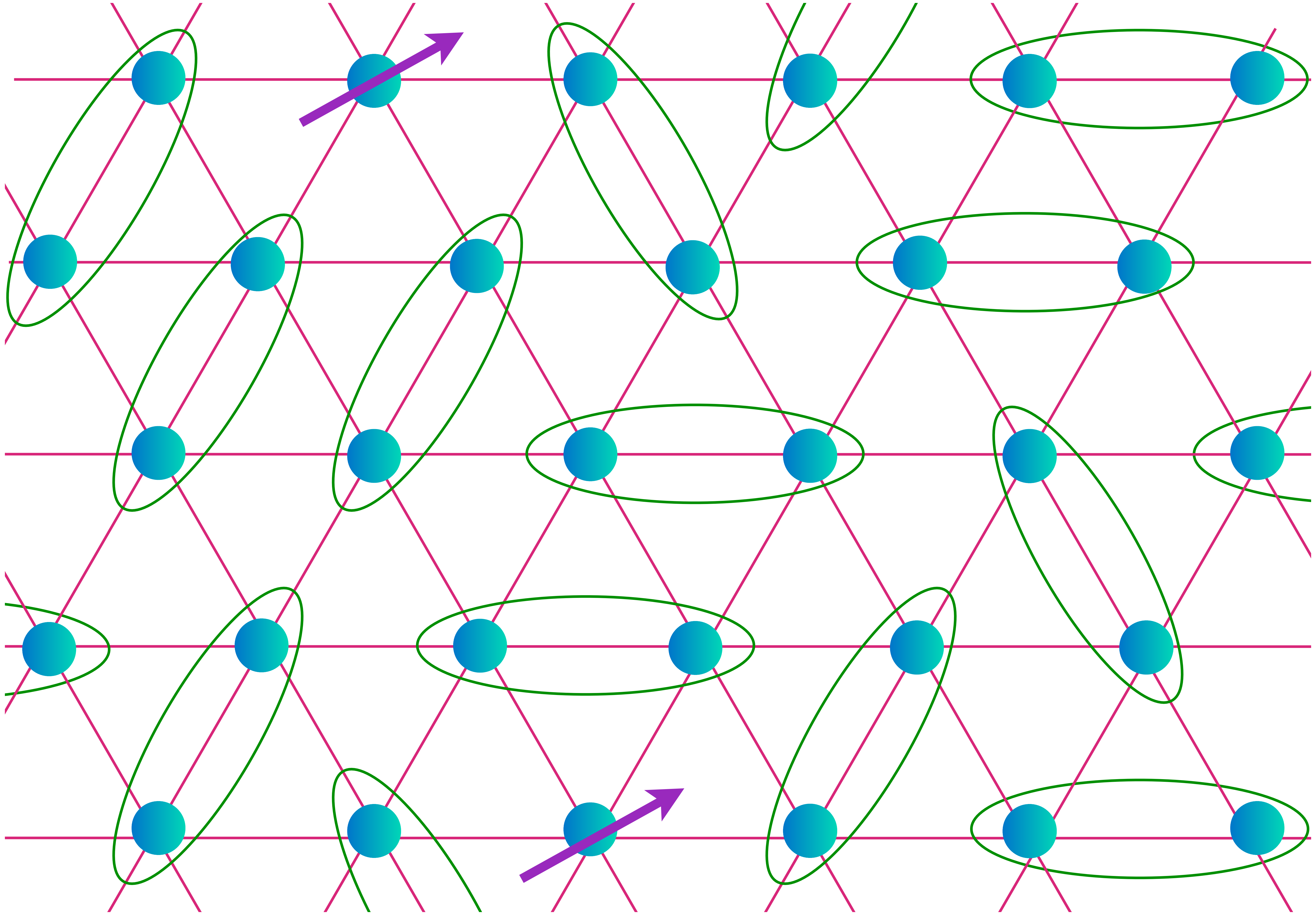

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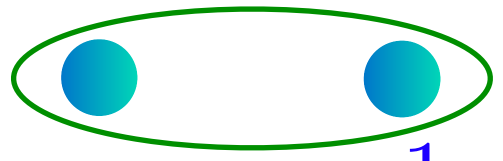
RVB: Z_2 spin liquid

Excitations with boson number 0
a vison (*m* particle)



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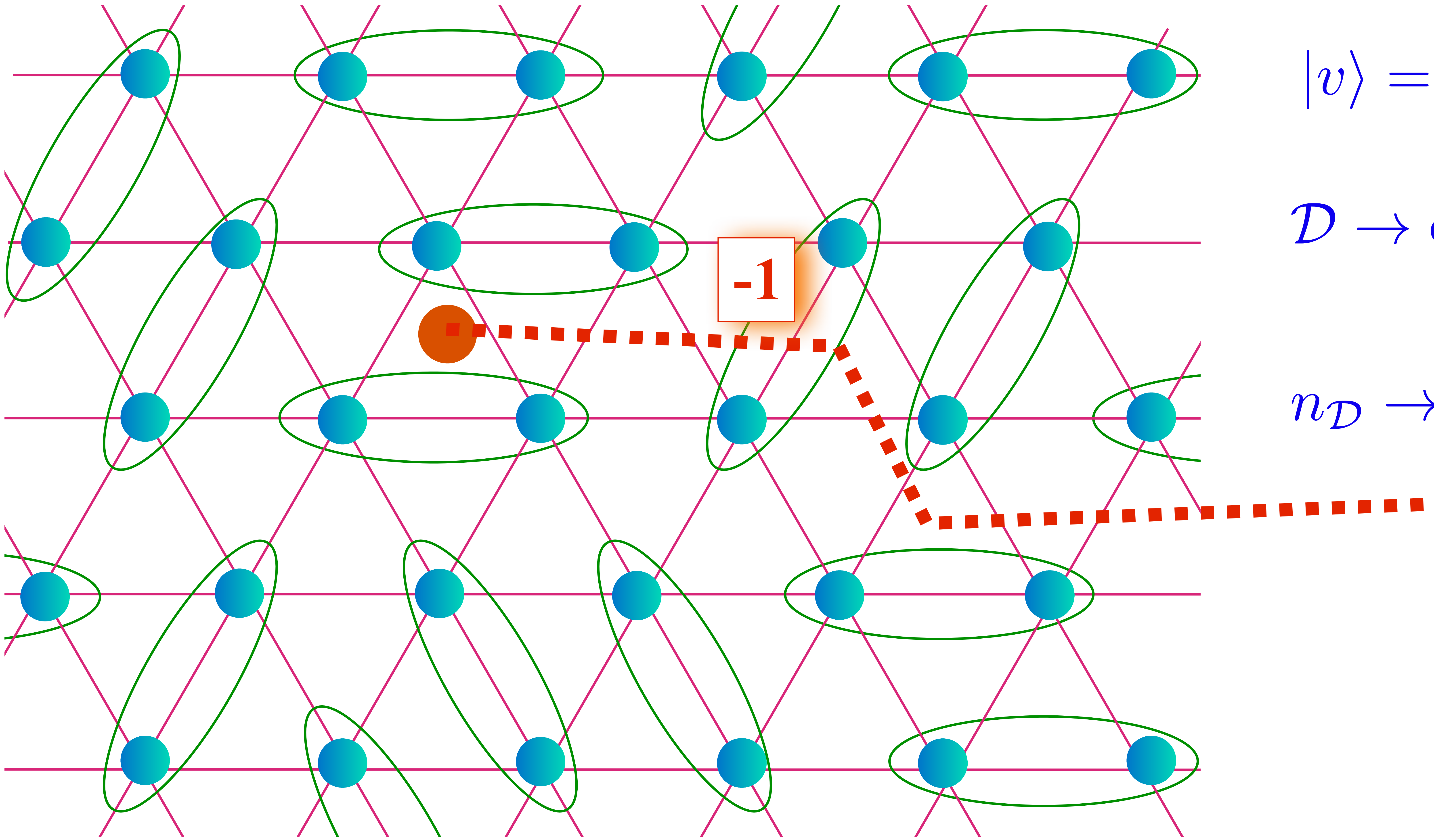

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$\mathcal{D} \rightarrow$ dimer covering of lattice

$n_{\mathcal{D}} \rightarrow$ number of dimers crossing red line

N. Read and B. Chakraborty, Phys. Rev. B 40, 7133 (1989)

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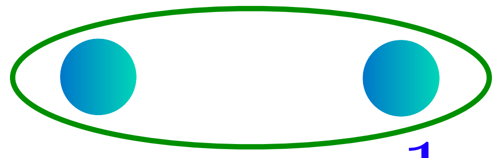

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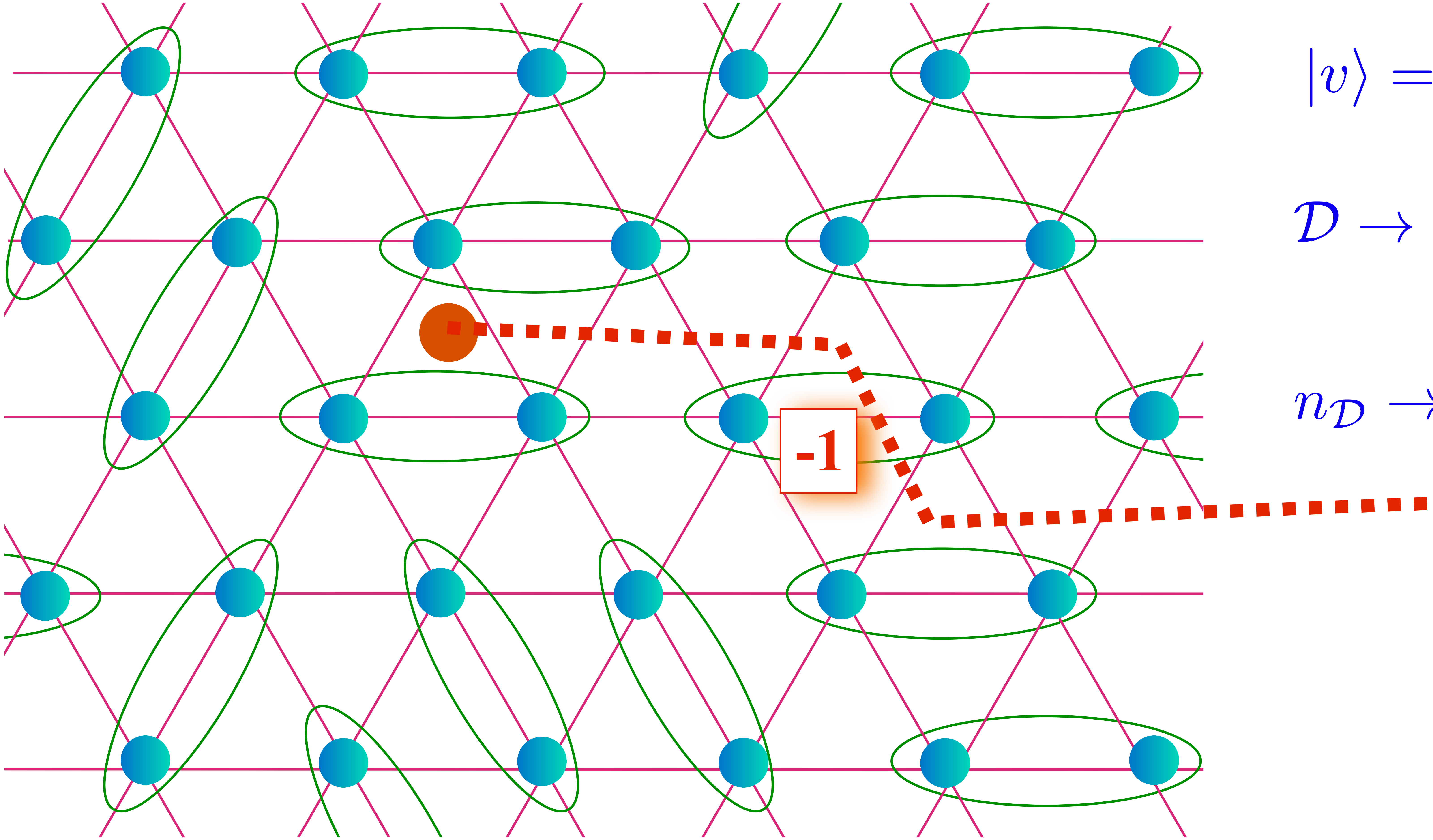

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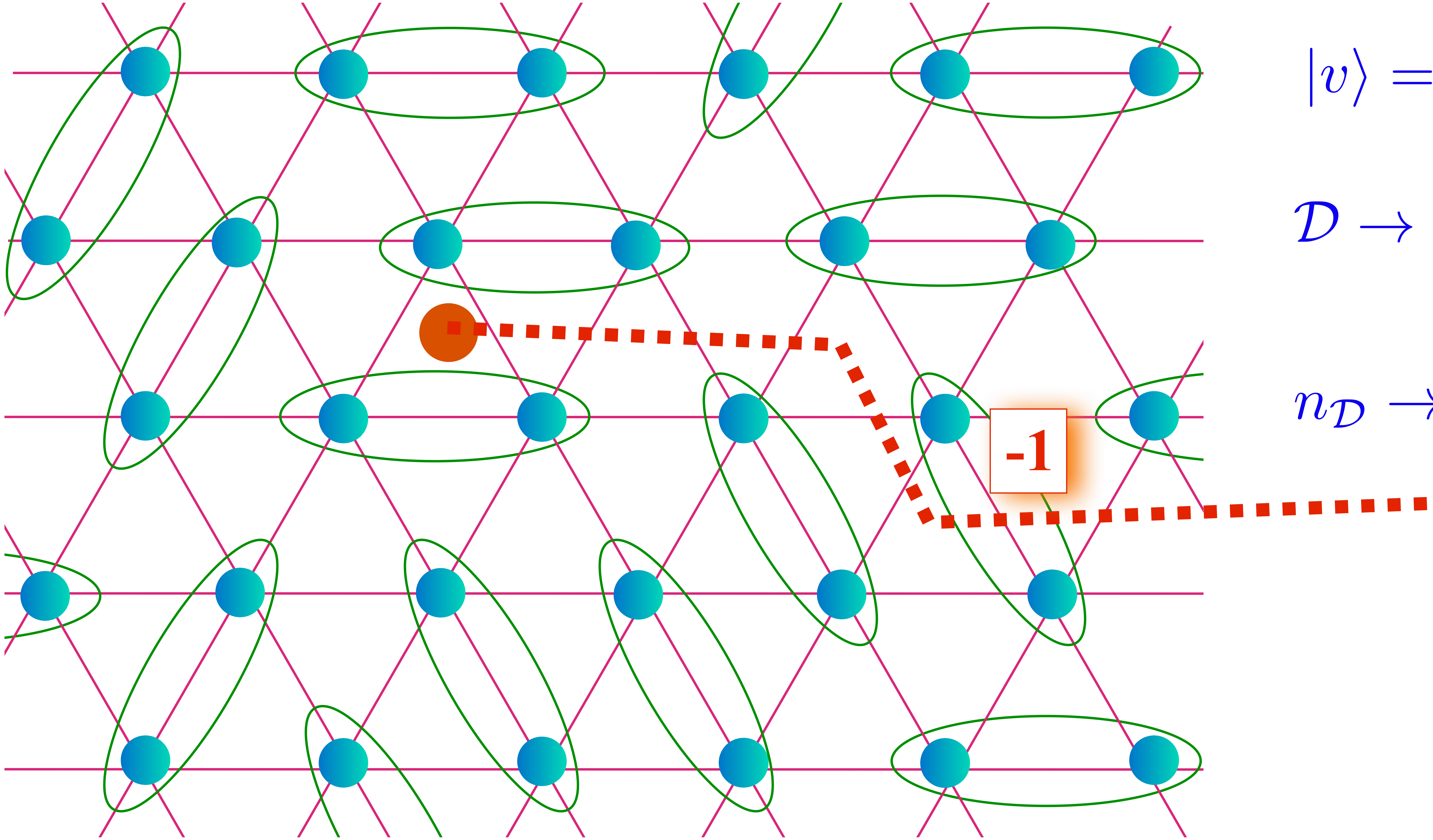

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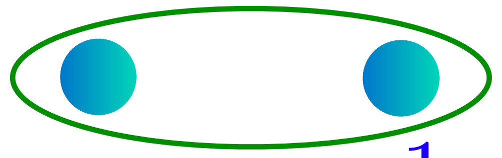

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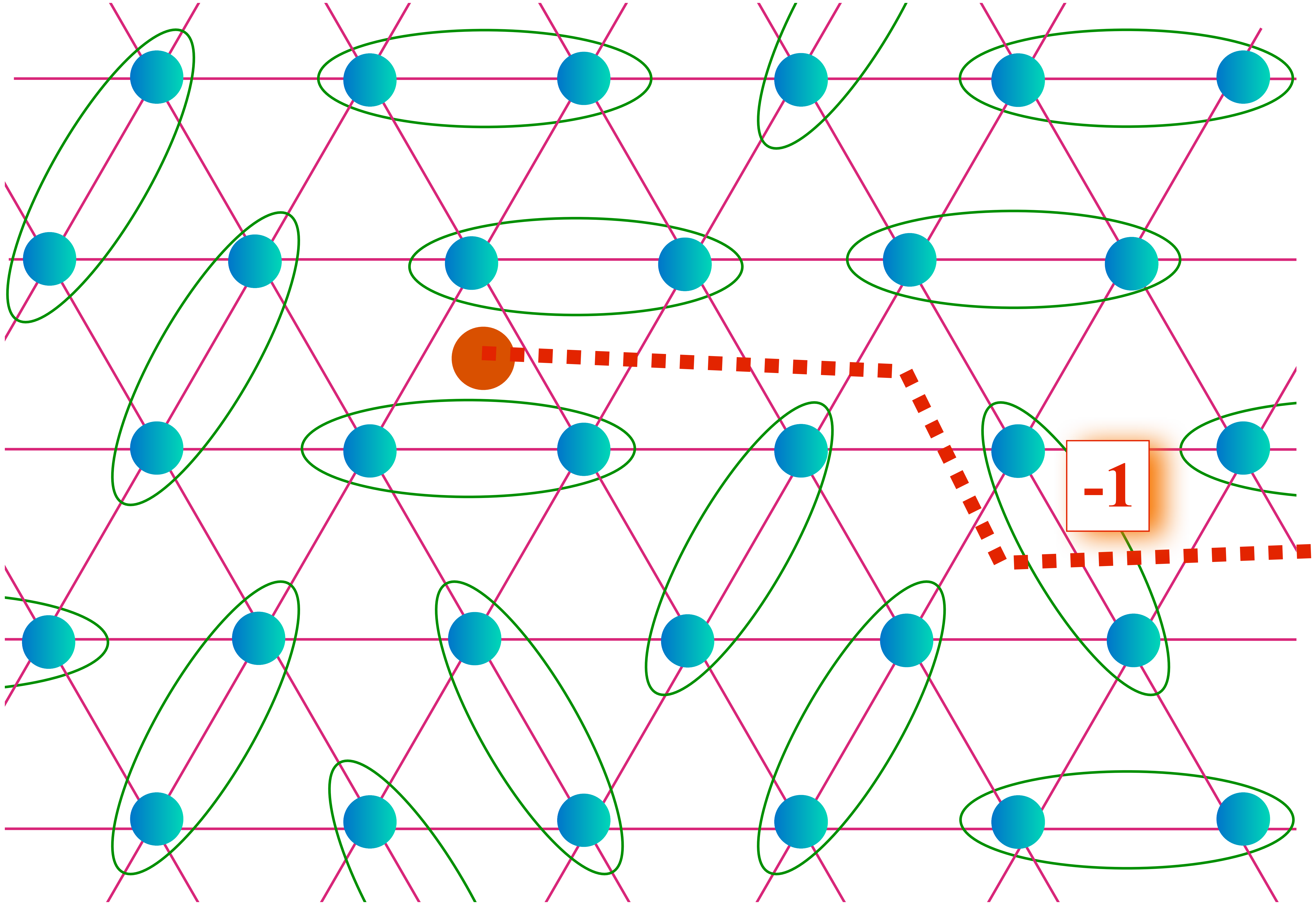

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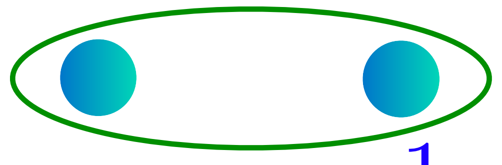

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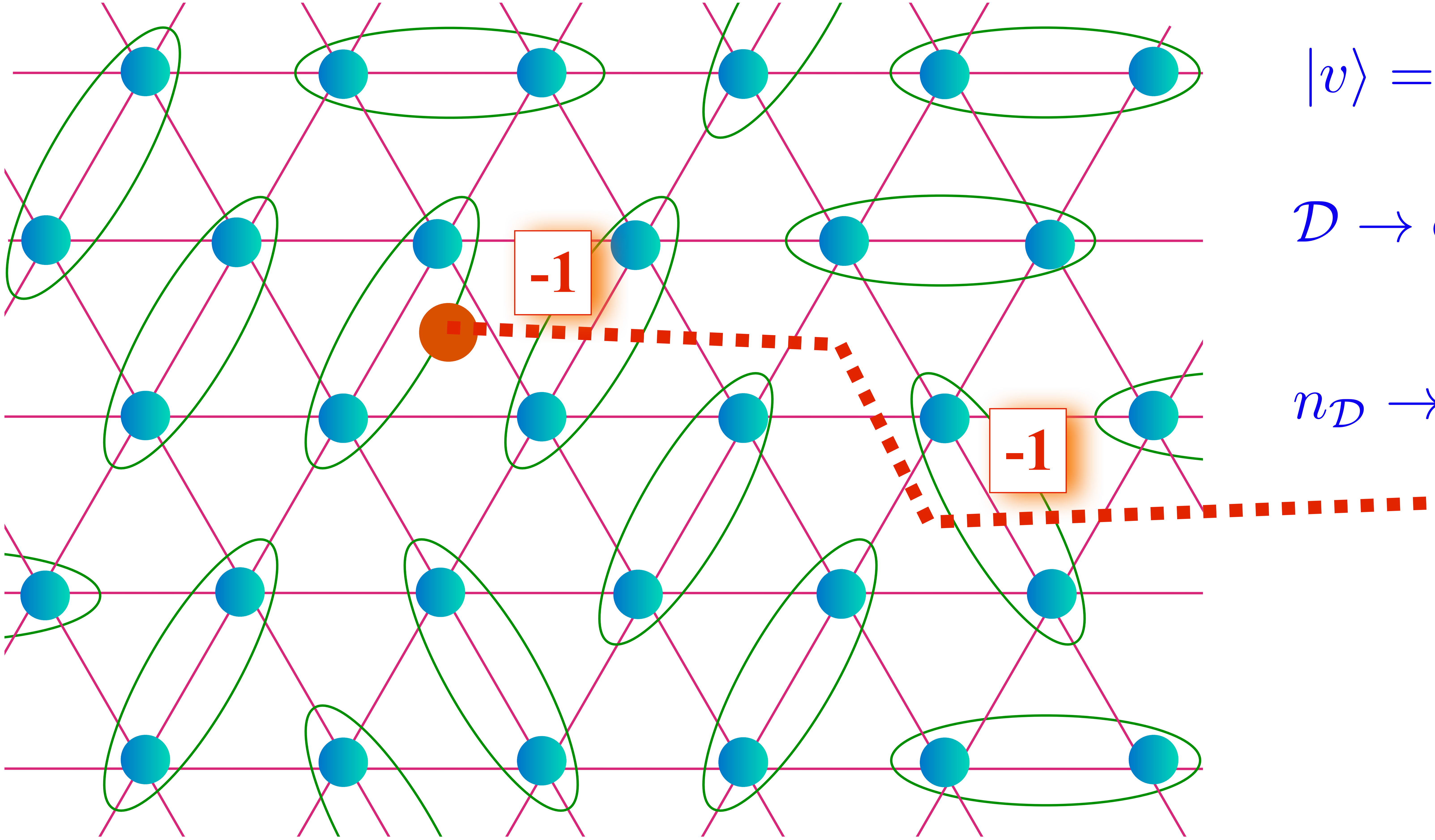

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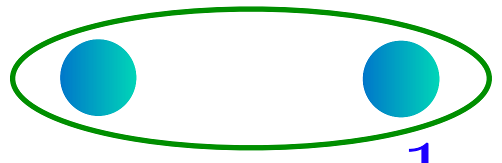

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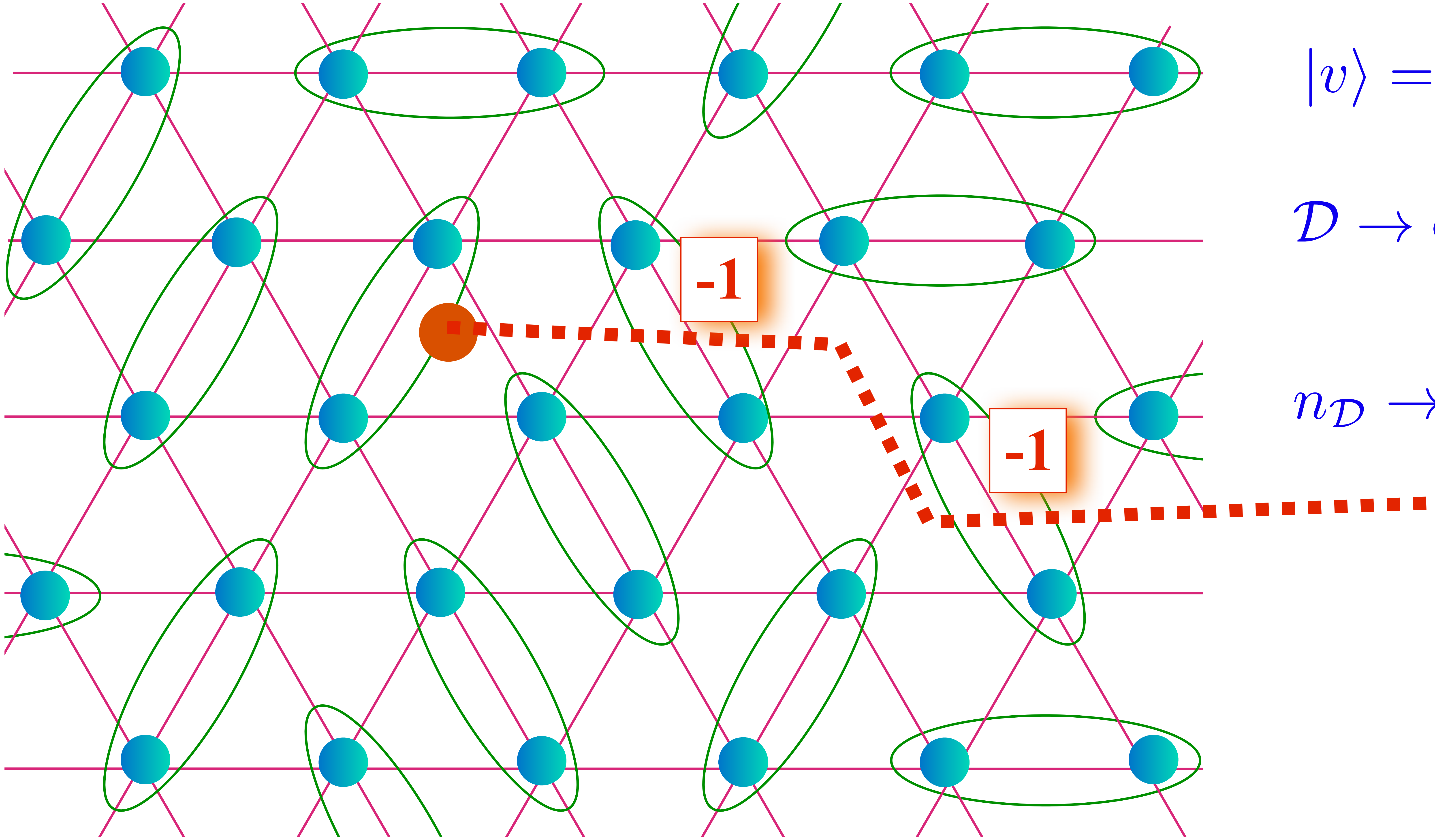

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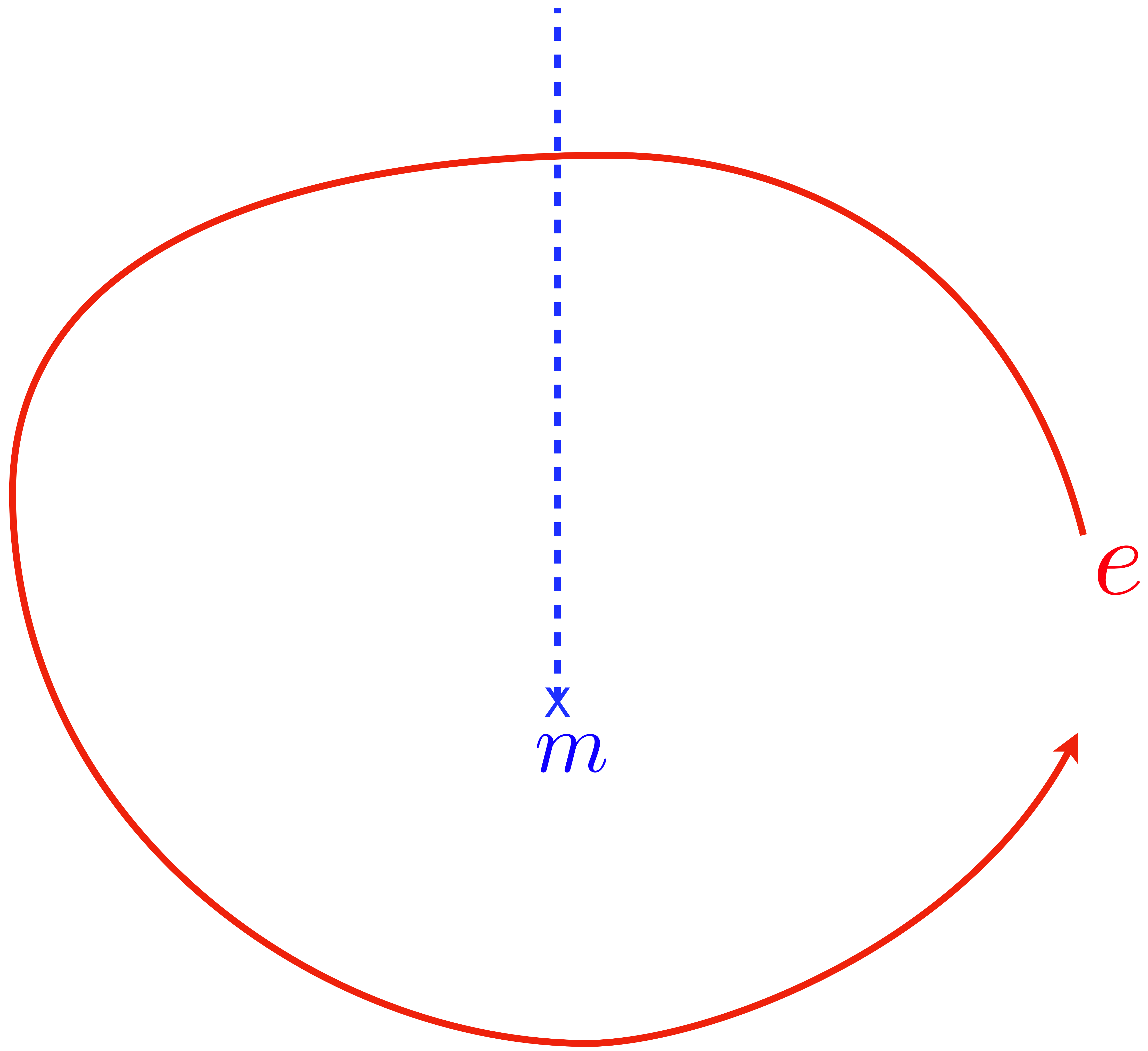
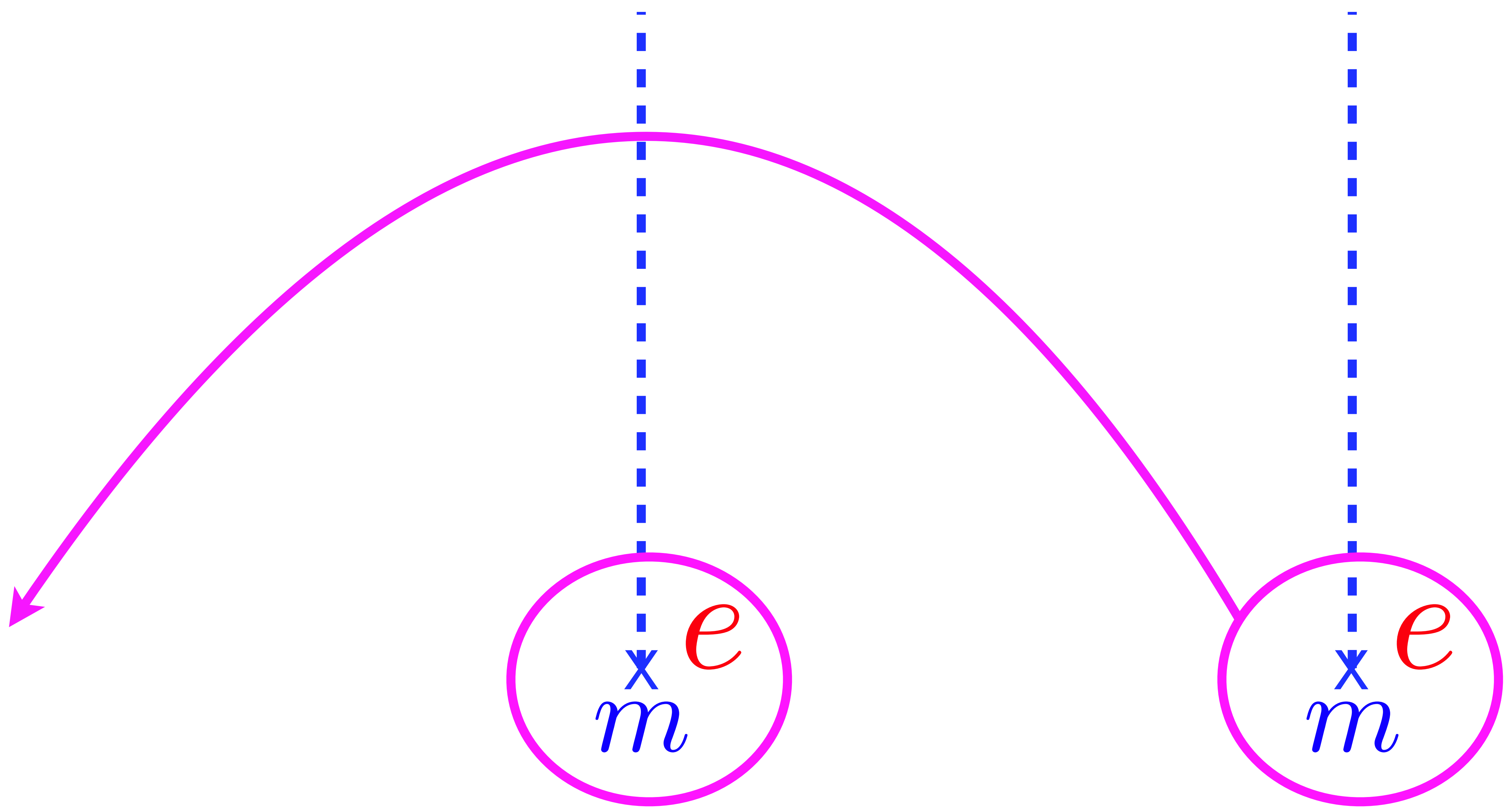


The e spinon and the m vison are *mutual* semions

RVB: Z_2 spin liquid



The ϵ spinon is a fermion.

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Quantum phases of Rydberg atoms on a kagome lattice,
Rhine Samajdar, Wen Wei Ho, Hannes Pichler, M. D. Lukin, and S. S.,
Proceedings of the National Academy of Sciences **118**, e2015785118 (2021); arXiv:2011.12295

Emergent Z_2 gauge theories and topological excitations in Rydberg atom arrays,
Rhine Samajdar, Darshan G. Joshi, Yanting Teng, and S. S., arXiv:2204.00632

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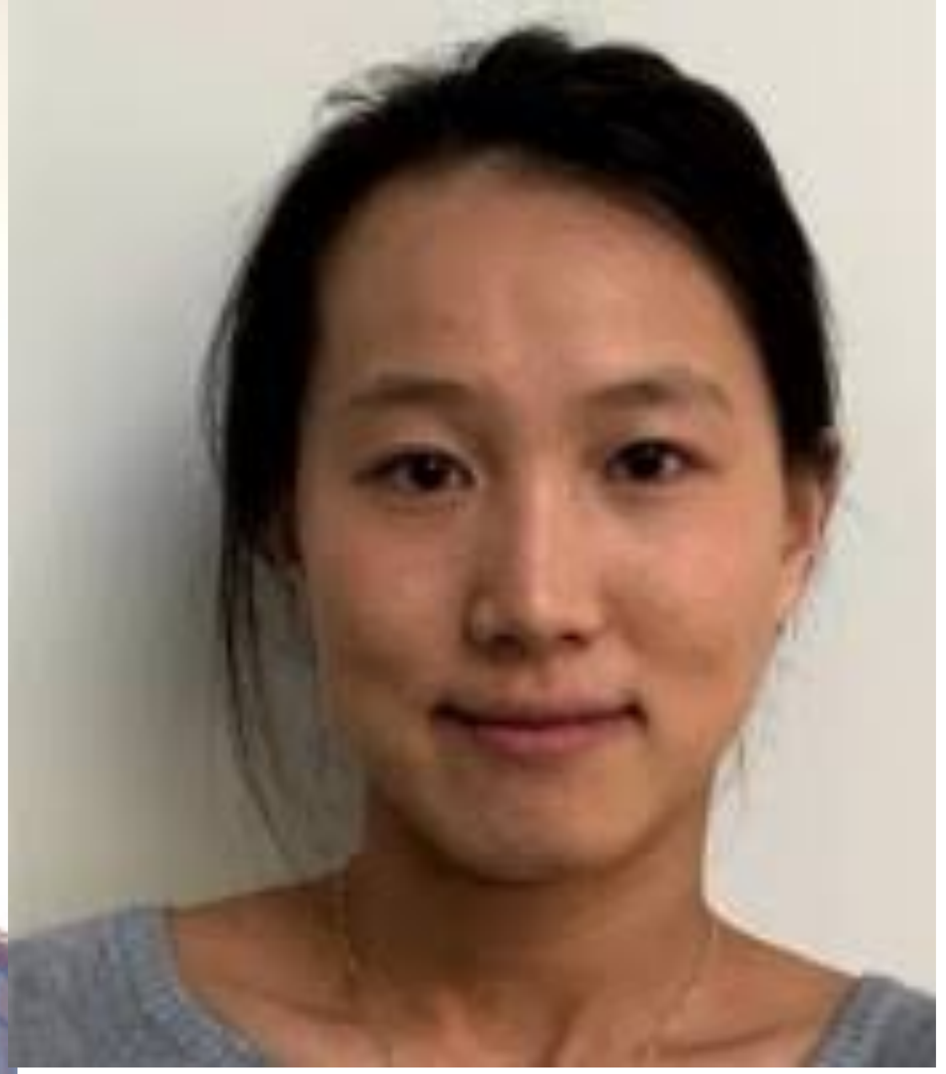
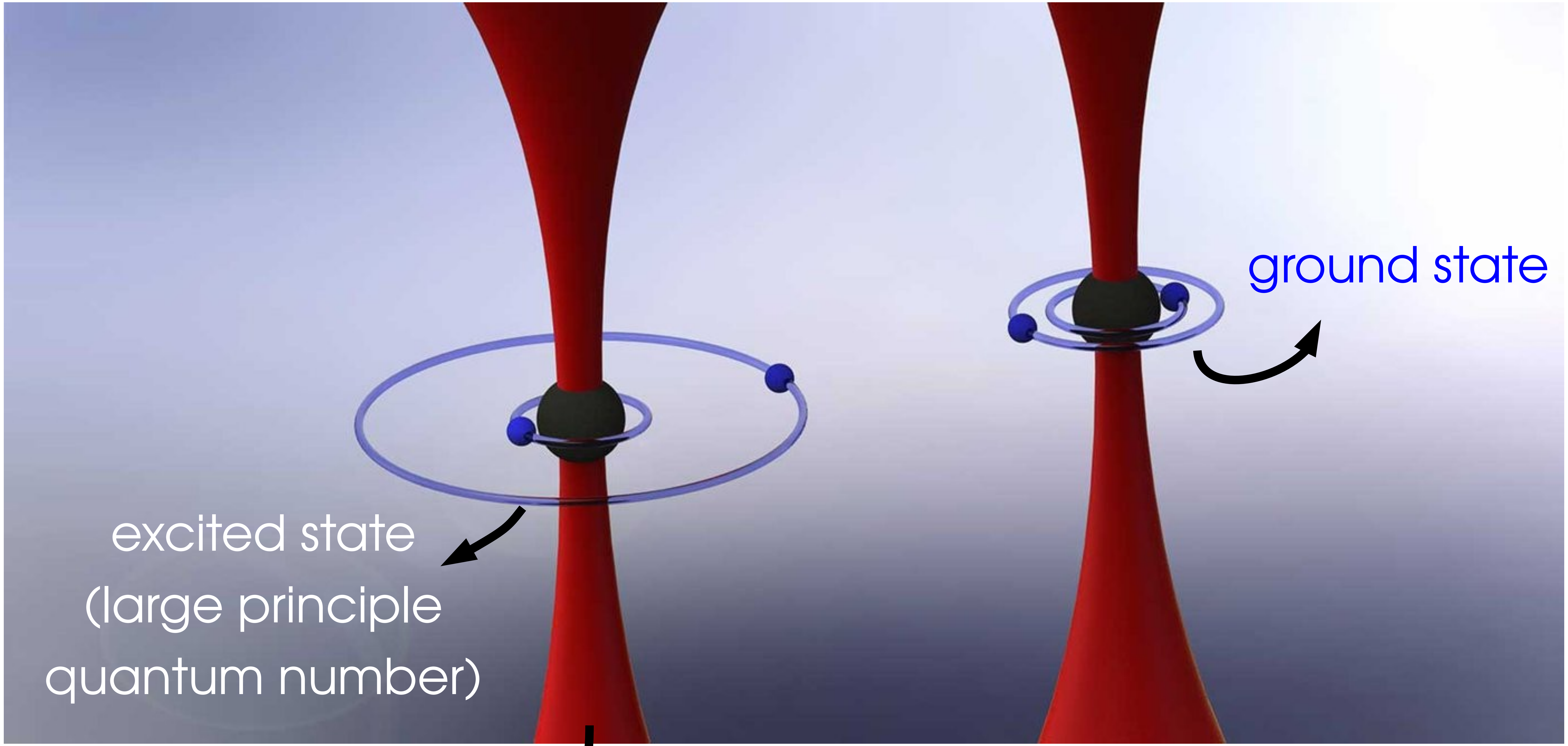


Yanting Teng



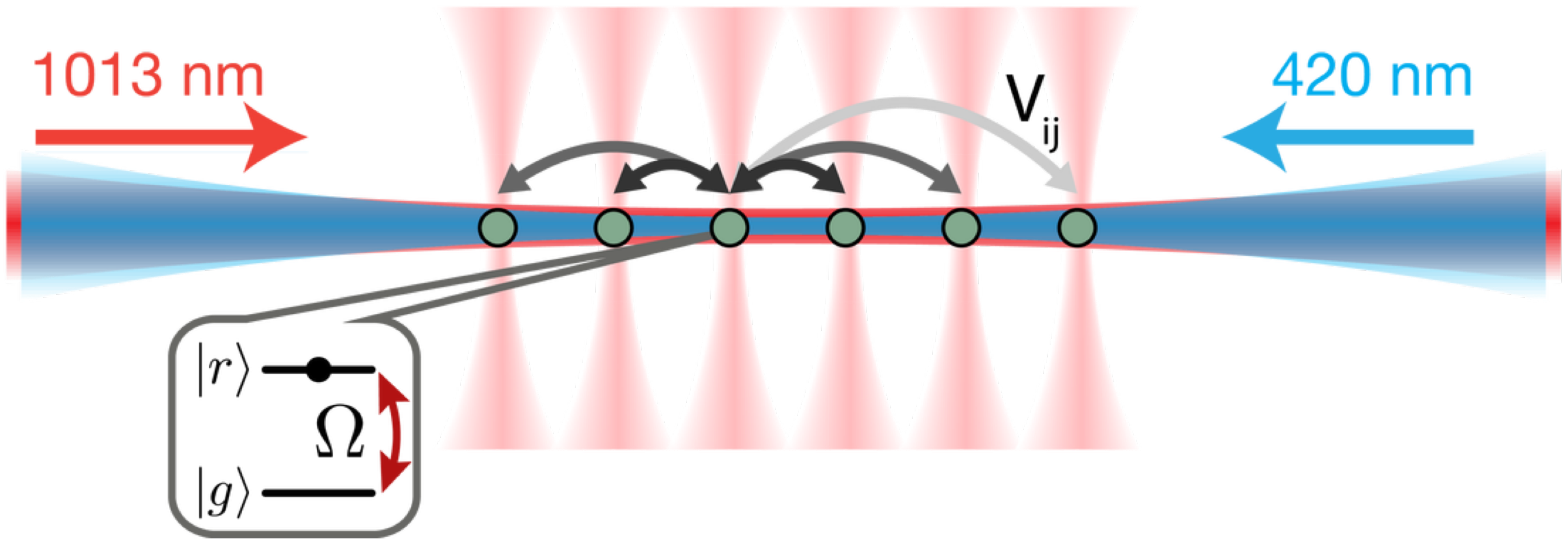
$$V_{|\ell-\ell'|} \sim \frac{1}{|\ell-\ell'|^6}$$

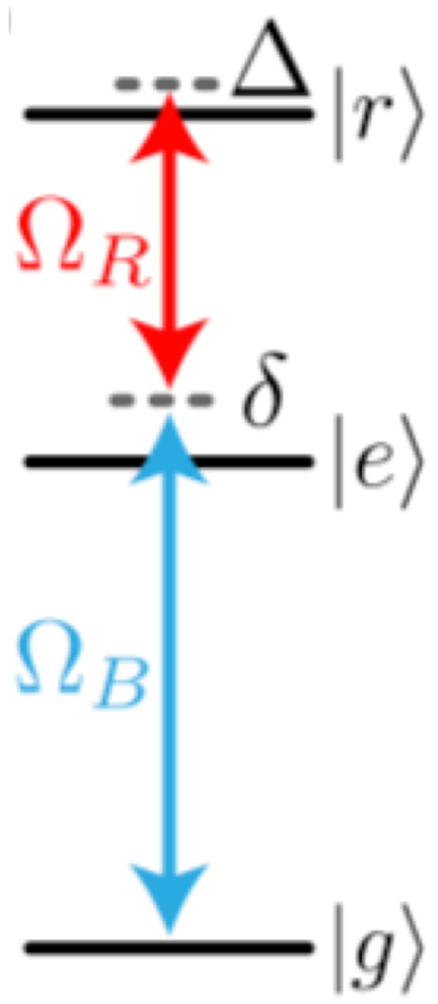
optical tweezer (traps atom)

Fig: https://www.caltech.edu/about/news/quantum-innovations-achieved-using-alkaline-earth-atoms

$$H_{\mathrm{Ryd}} = \sum_{\ell}\left[\frac{\Omega}{2}\left(|g\rangle\langle r| + |r\rangle\langle g|\right)_{\ell} - \Delta|r\rangle\langle r|\right] + \sum_{(\ell,\ell')} V_{|\ell-\ell'|}\Big(|r\rangle\langle r|_{\ell}\otimes|r\rangle\langle r|_{\ell'}\Big)$$

QPTs in a Rydberg quantum simulator





$$|g\rangle \equiv |0\rangle$$

$$|r\rangle \equiv B^\dagger |0\rangle$$

$$\mathcal{H} = \sum_\ell \left[\frac{\Omega}{2} \left(B_\ell + B_\ell^\dagger \right) - \Delta n_\ell \right] + \sum_{\ell < \ell'} V_{|\ell - \ell'|} n_\ell n_{\ell'}$$

$$V_{|\ell-\ell'|} \sim \frac{1}{|\ell - \ell'|^6}$$

$$n_\ell \equiv B_\ell^\dagger B_\ell$$

$n_\ell = 0, 1$ 'hard core' bosons

FSS model

S. Sachdev, K. Sengupta, and S.M. Girvin, PRB **66**, 075128 (2002)
P. Fendley, K. Sengupta, S. Sachdev, PRB **69**, 075106 (2004)

From the FSS model to an emergent $\mathbb{Z}_2$ gauge theory

$$\mathcal{H} = \sum_{\ell} \left[\frac{\Omega}{2} \left(B_\ell + B_\ell^\dagger \right) - \Delta n_\ell \right] + \sum_{\ell < \ell'} V_{|\ell - \ell'|} n_\ell n_{\ell'}$$

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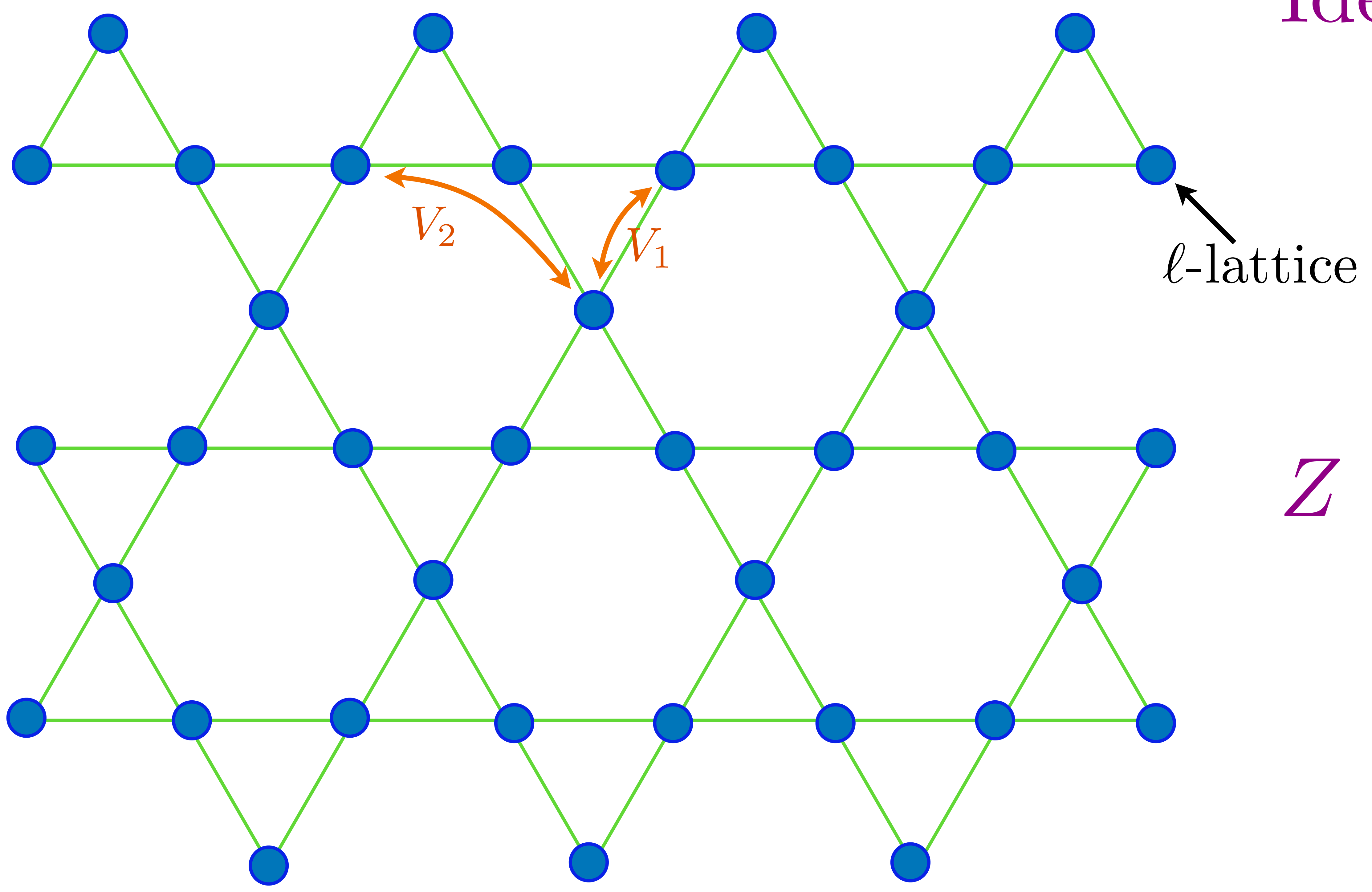

Identify hard core bosons with a qubit X, Y, Z

$$B_\ell + B_\ell^\dagger \Leftrightarrow Z_\ell$$

$$n_\ell \Leftrightarrow (1 - X_\ell)/2$$

Z will become the $\mathbb{Z}_2$ gauge field

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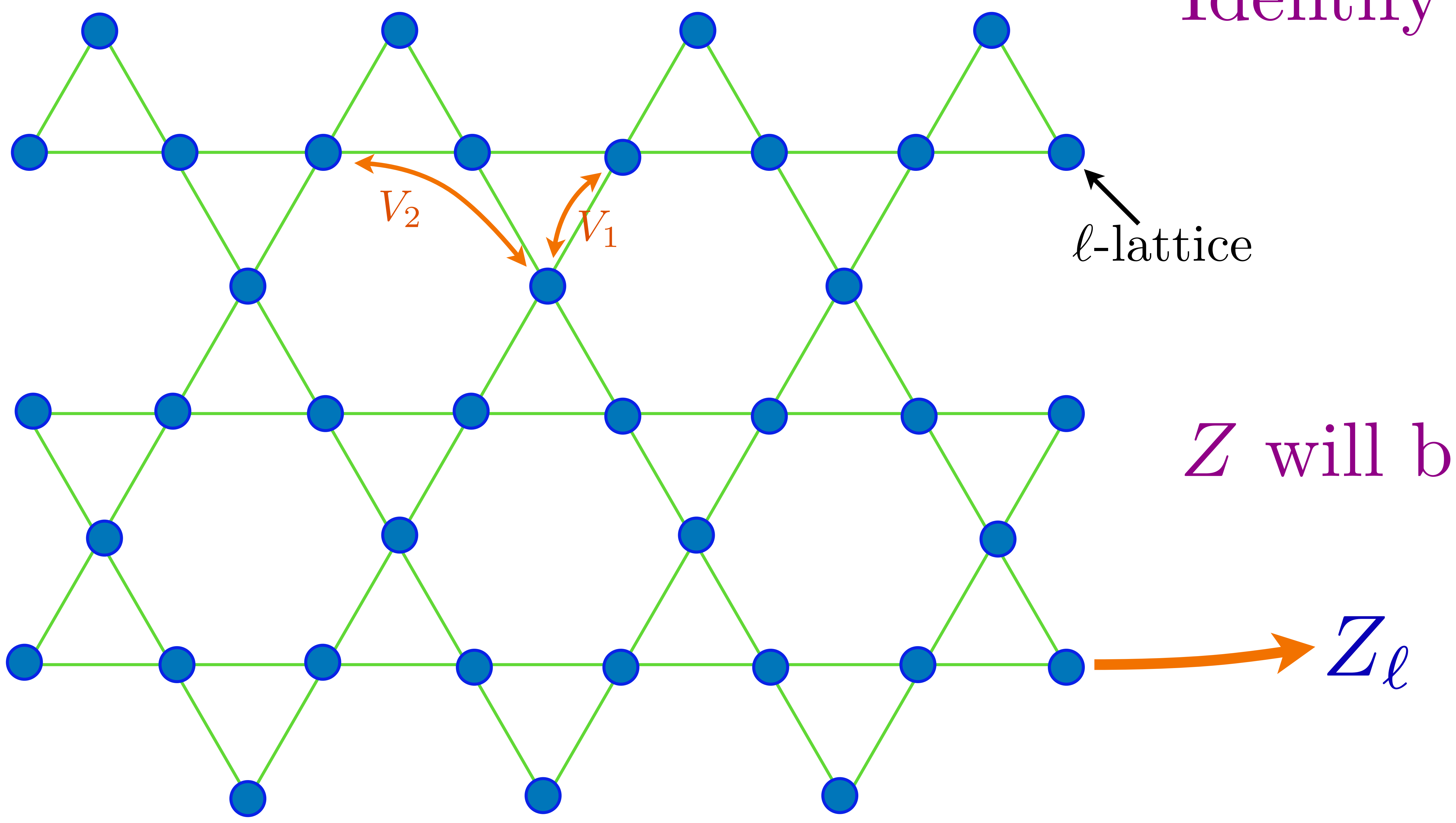
$$\mathcal{H} = \sum_{\ell} \left[\frac{\Omega}{2} Z_\ell + \frac{\Delta}{2} X_\ell \right] + \sum_{\ell < \ell'} \frac{V_{|\ell - \ell'|}}{4} (1 - X_\ell)(1 - X_{\ell'})$$



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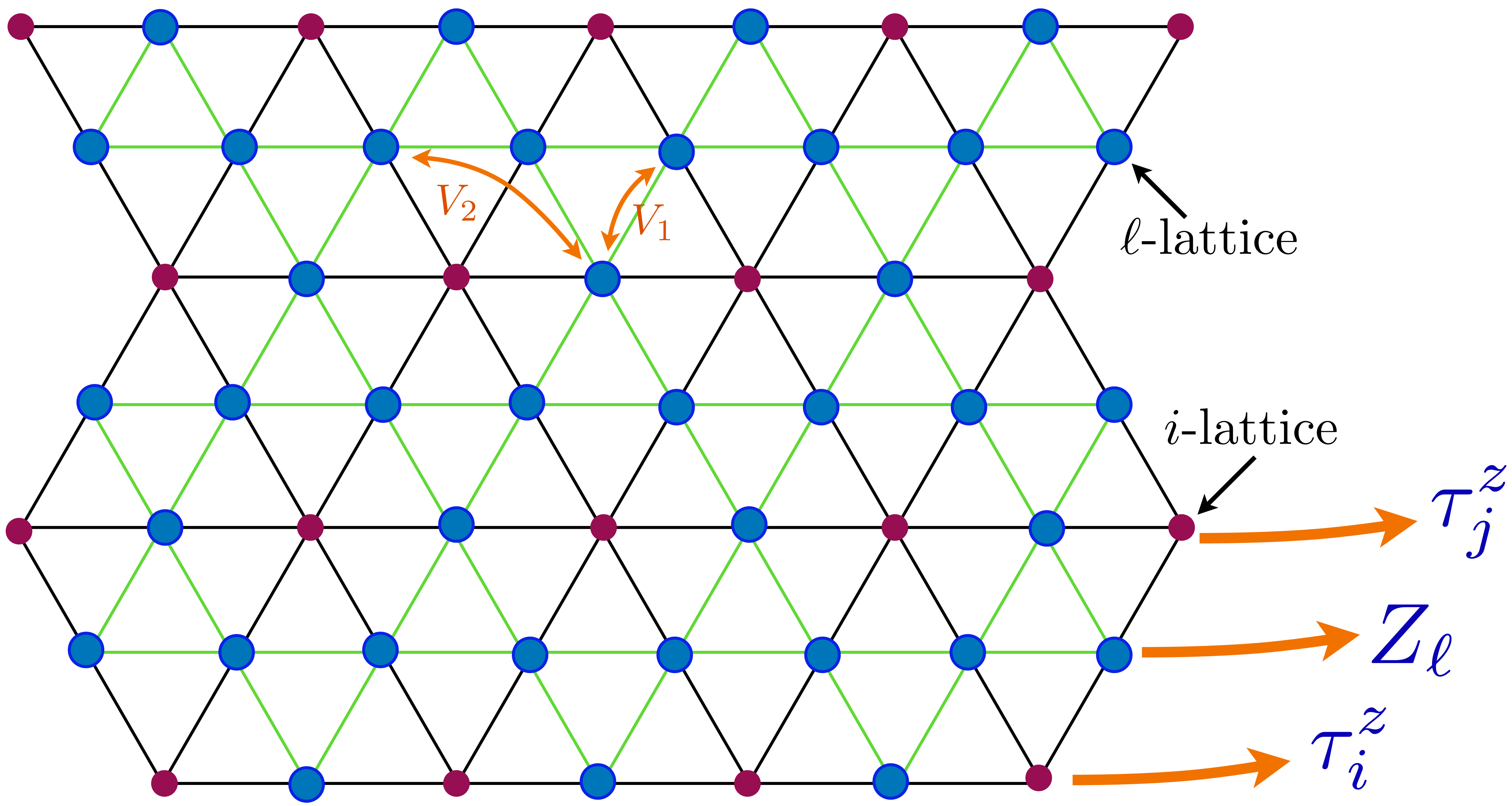

R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, S. Sachdev, PNAS **118**, e2015785118 (2021)

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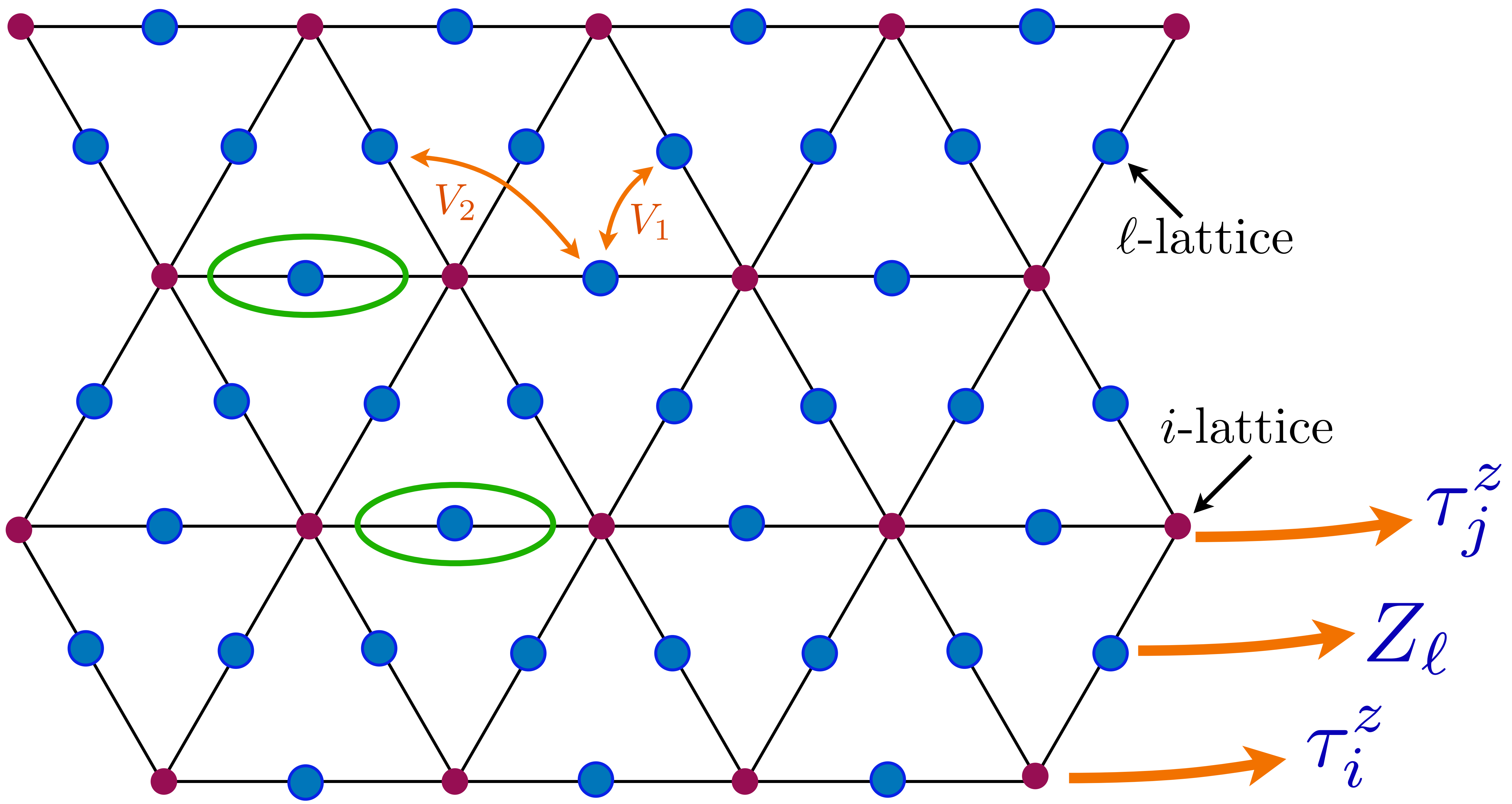

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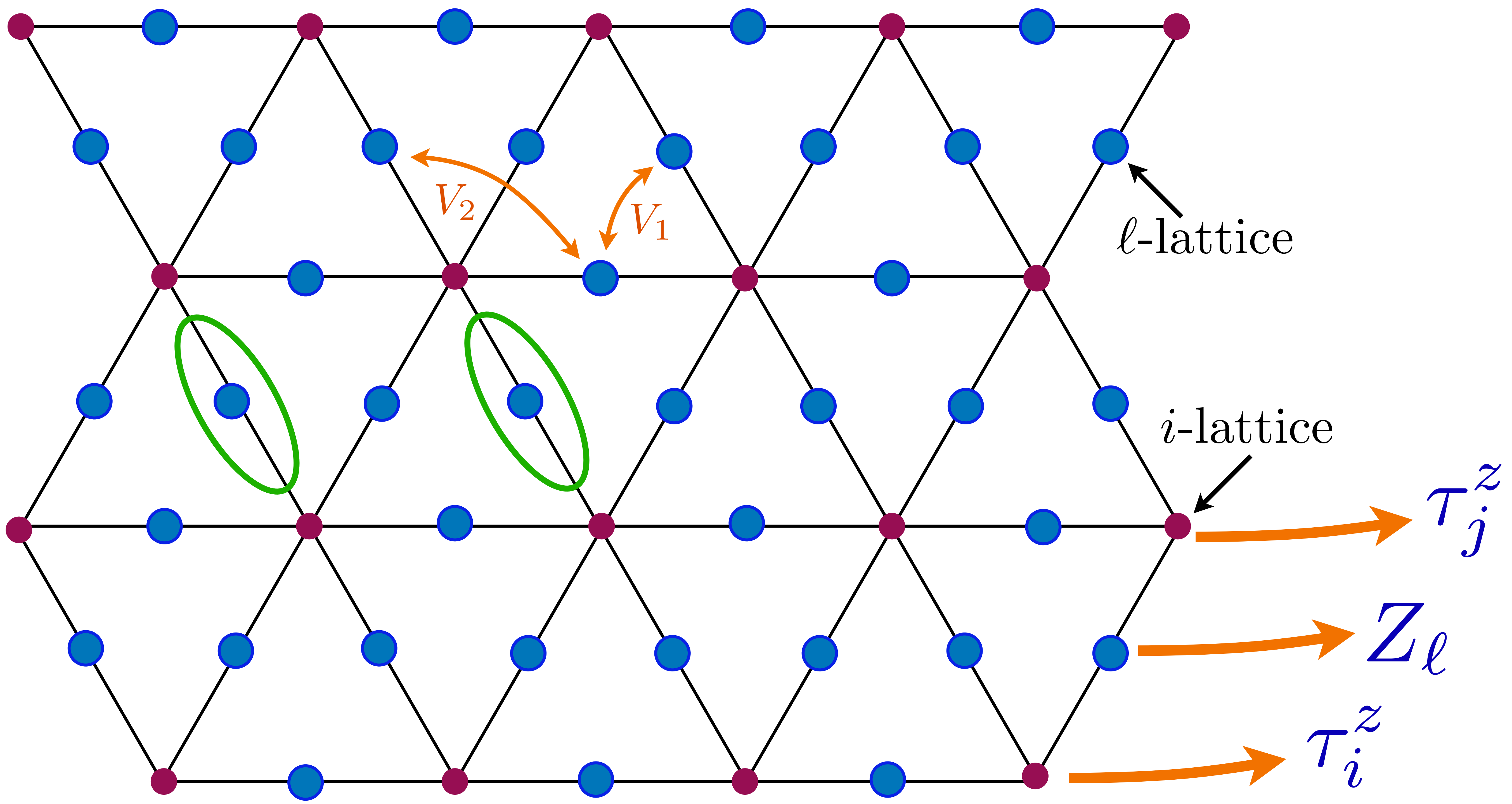

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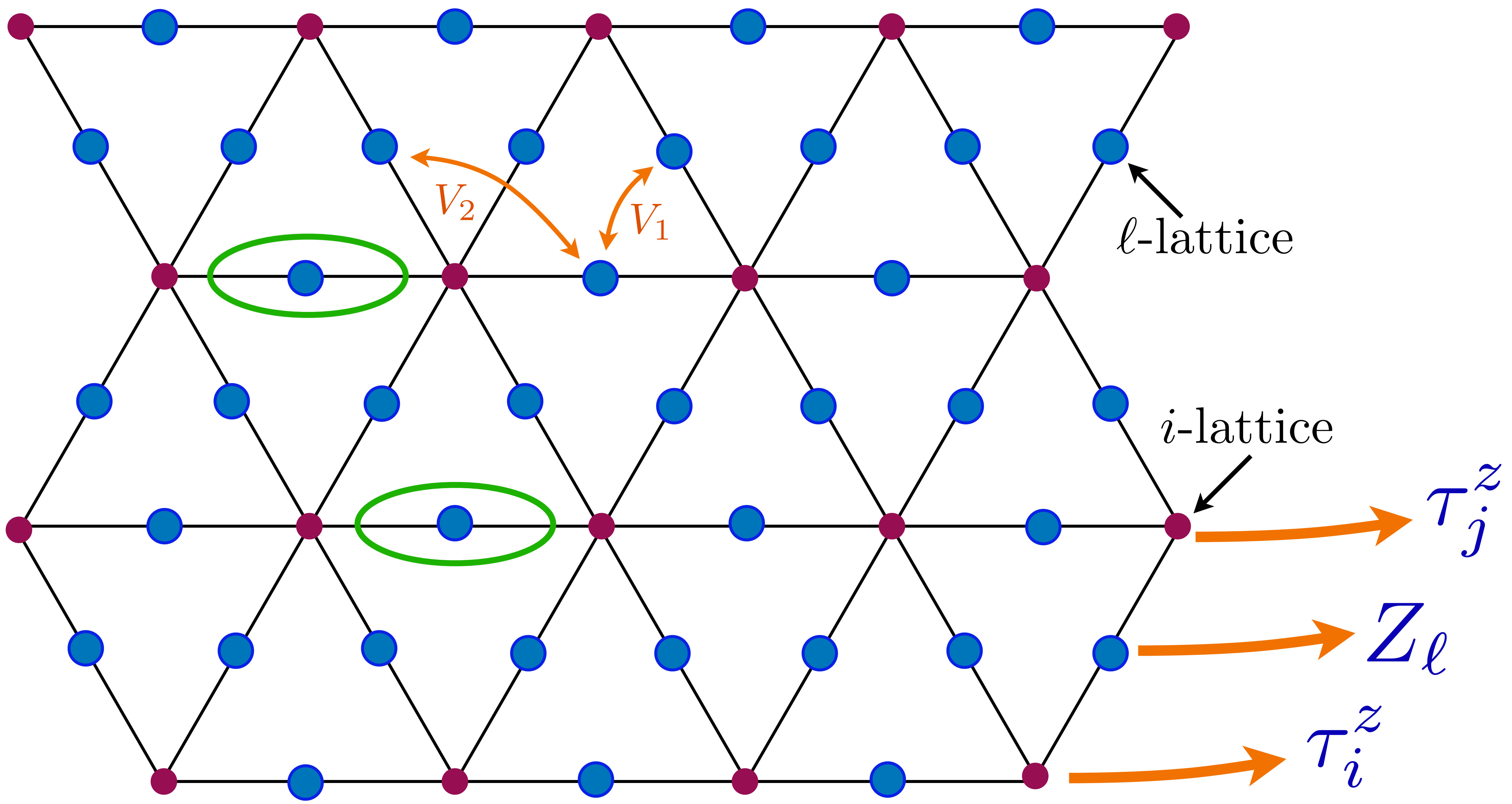

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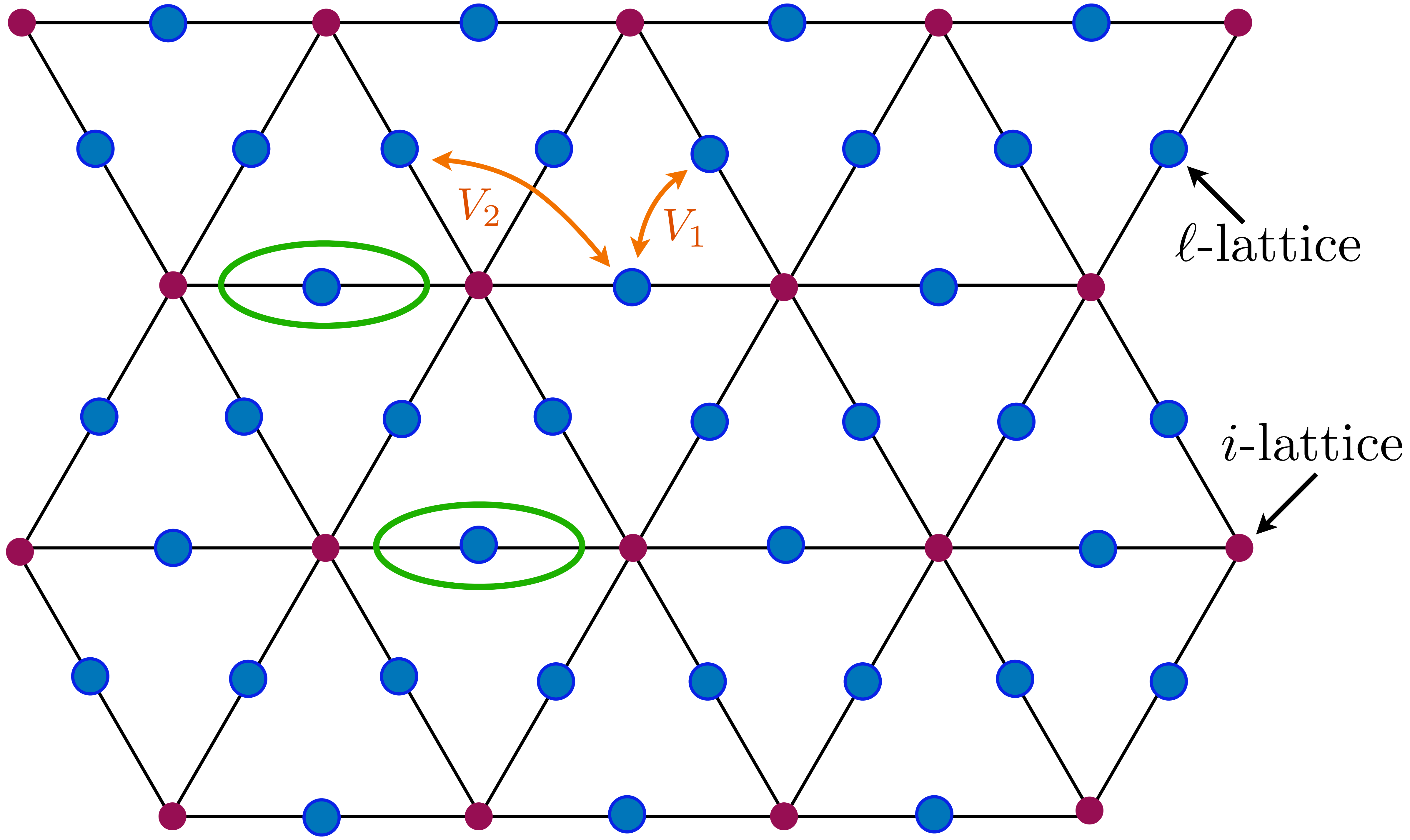

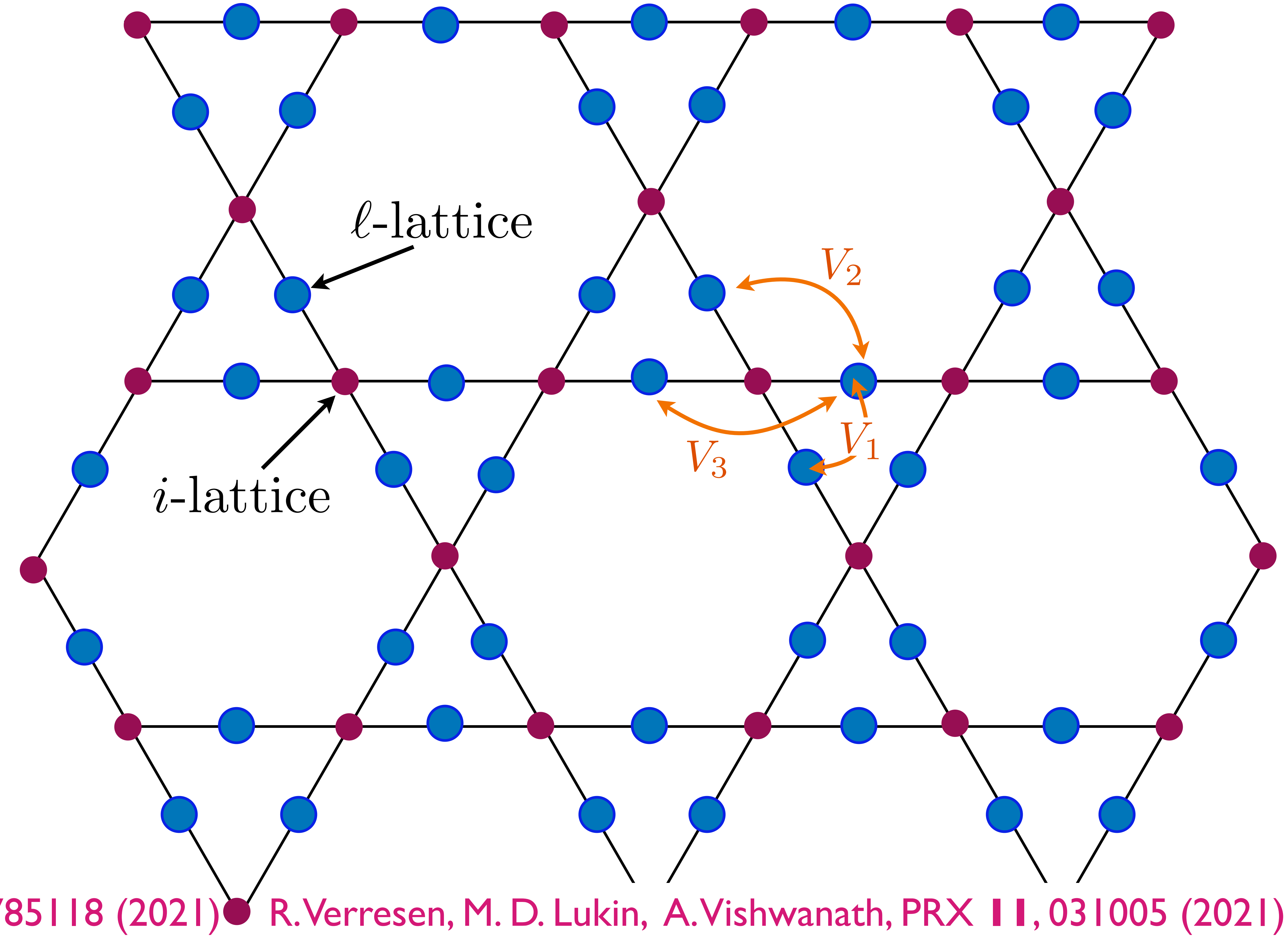

R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, S. Sachdev, PNAS **118**, e2015785118 (2021)
R. Verresen, M. D. Lukin, A. Vishwanath, PRX **11**, 031005 (2021)

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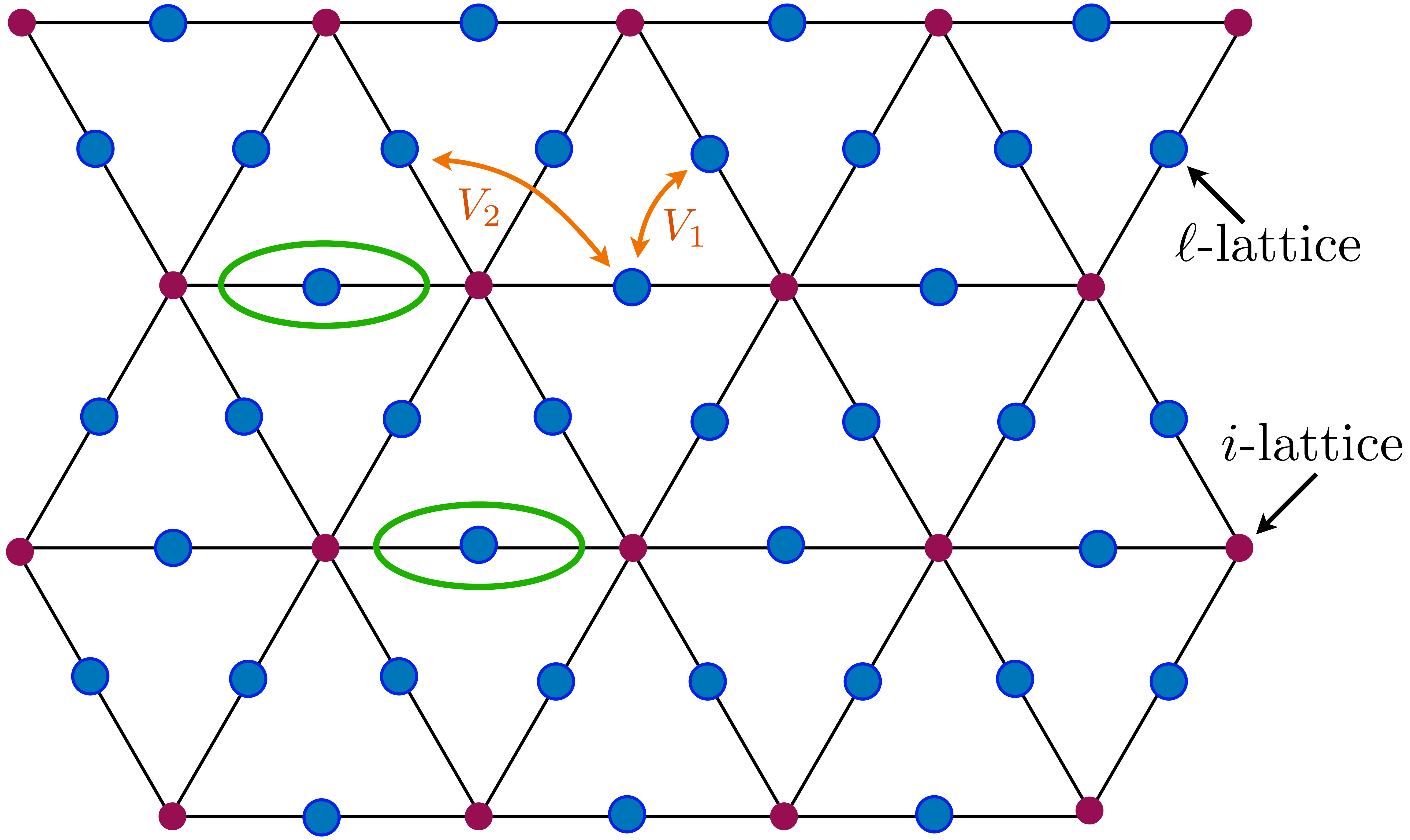

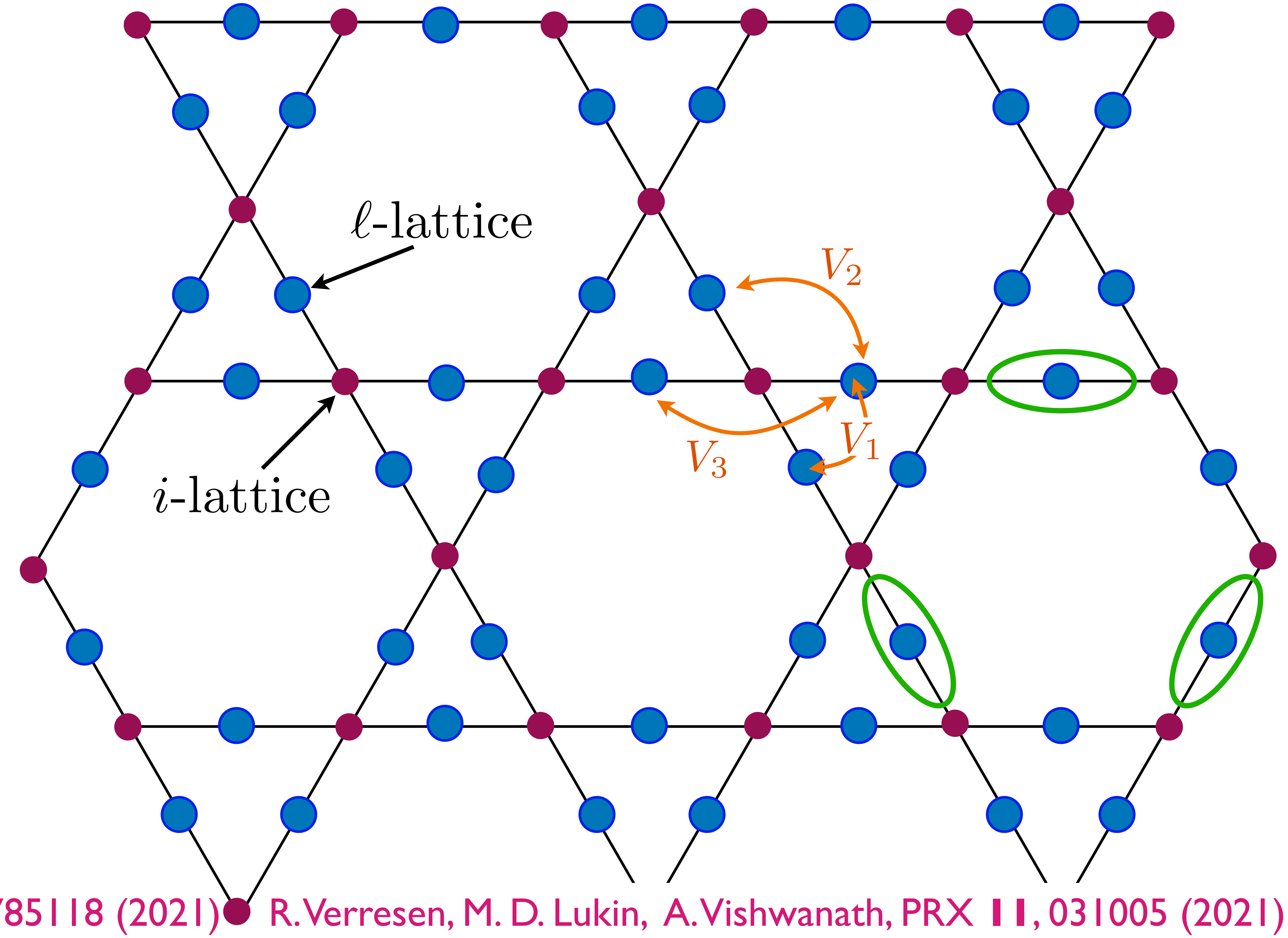

R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, S. Sachdev, PNAS **118**, e2015785118 (2021) R. Verresen, M. D. Lukin, A. Vishwanath, PRX **11**, 031005 (2021)

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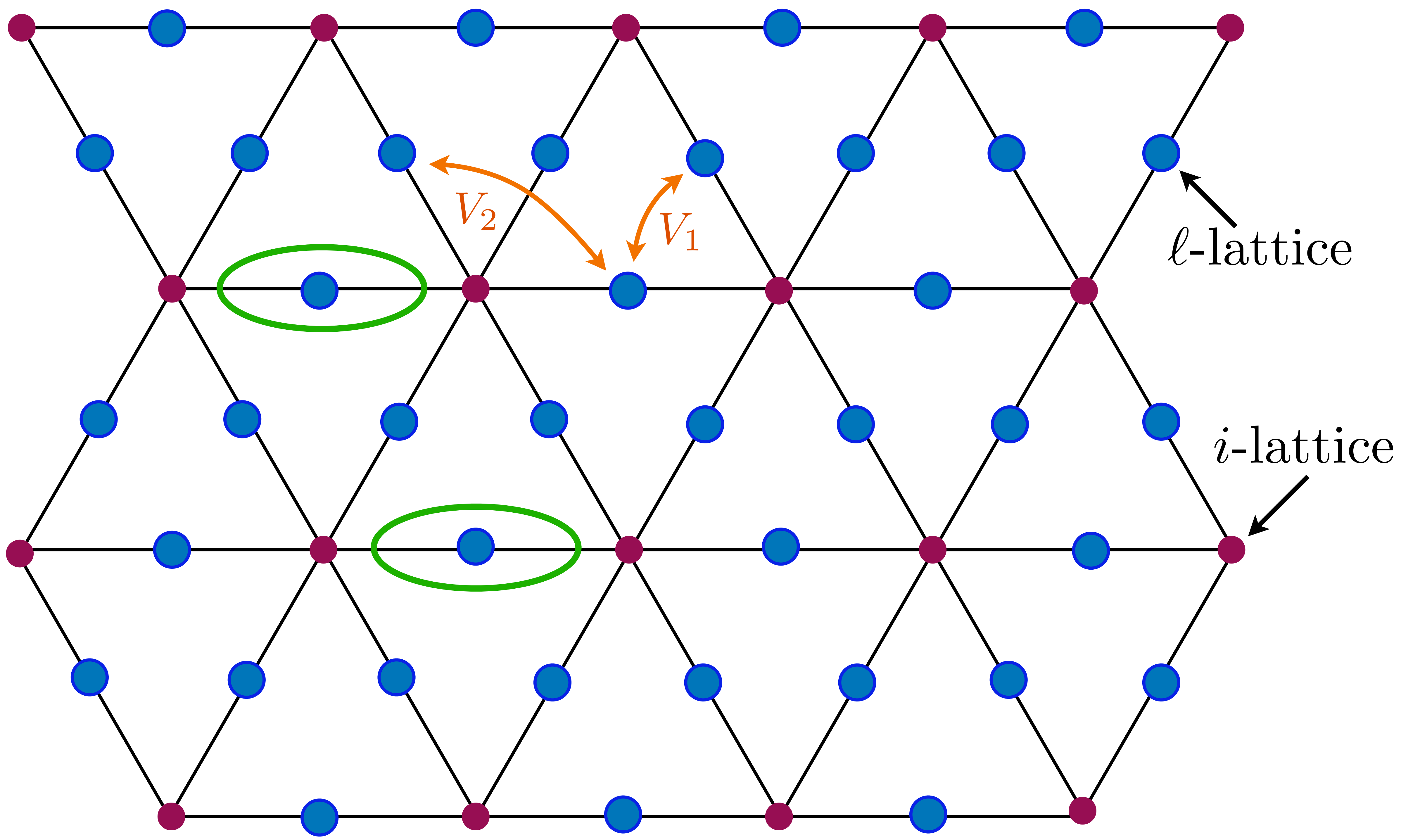

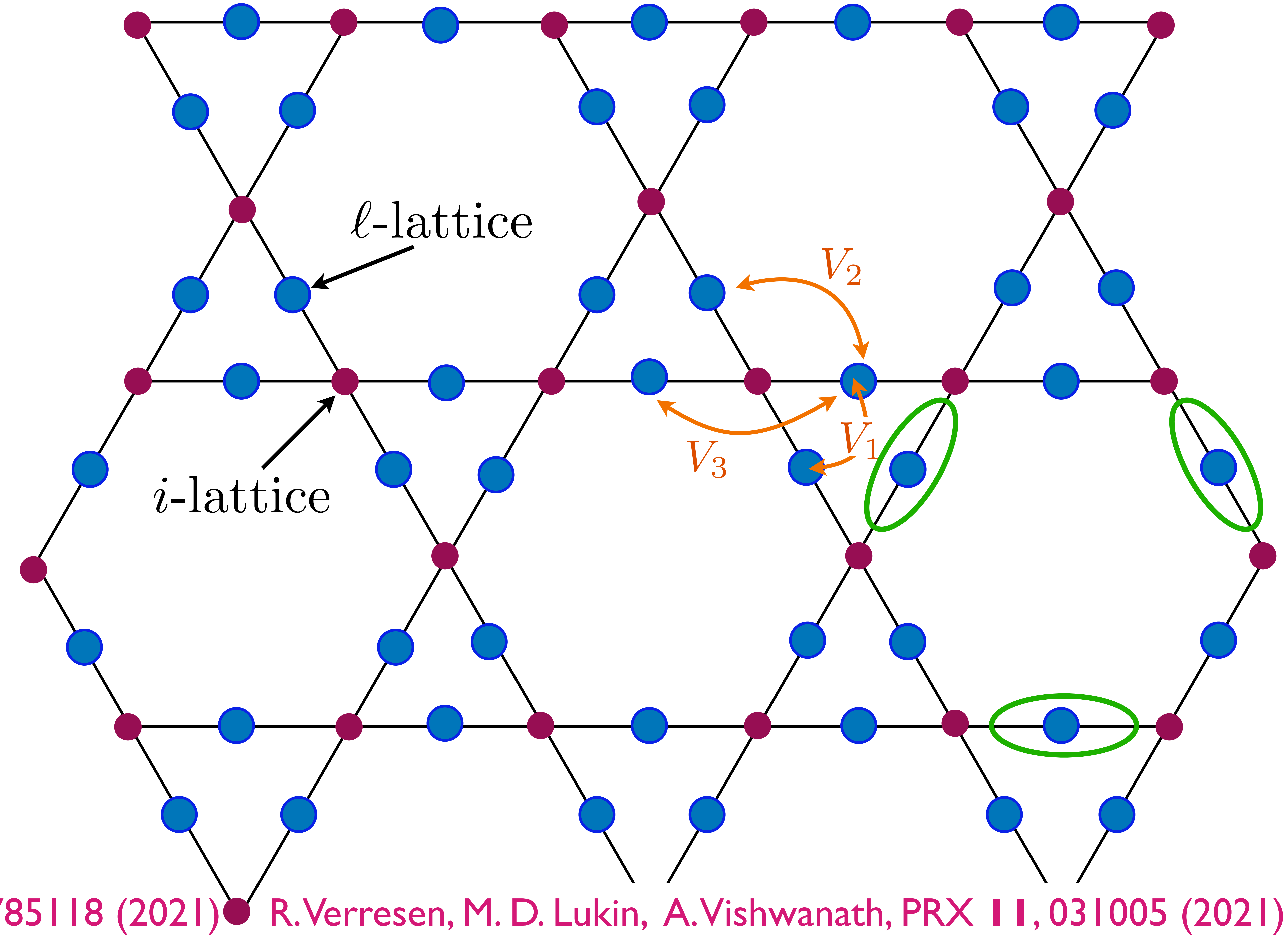

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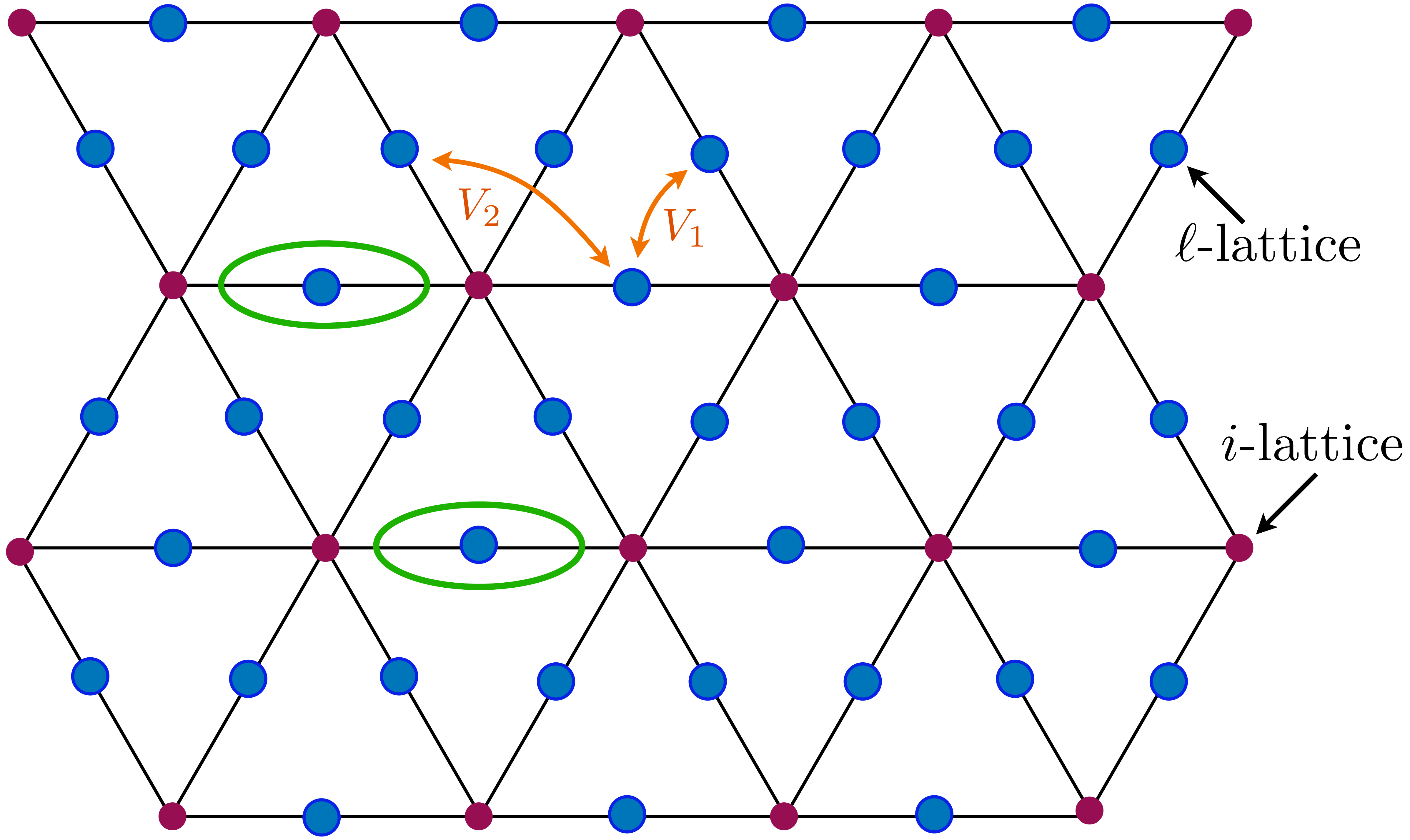

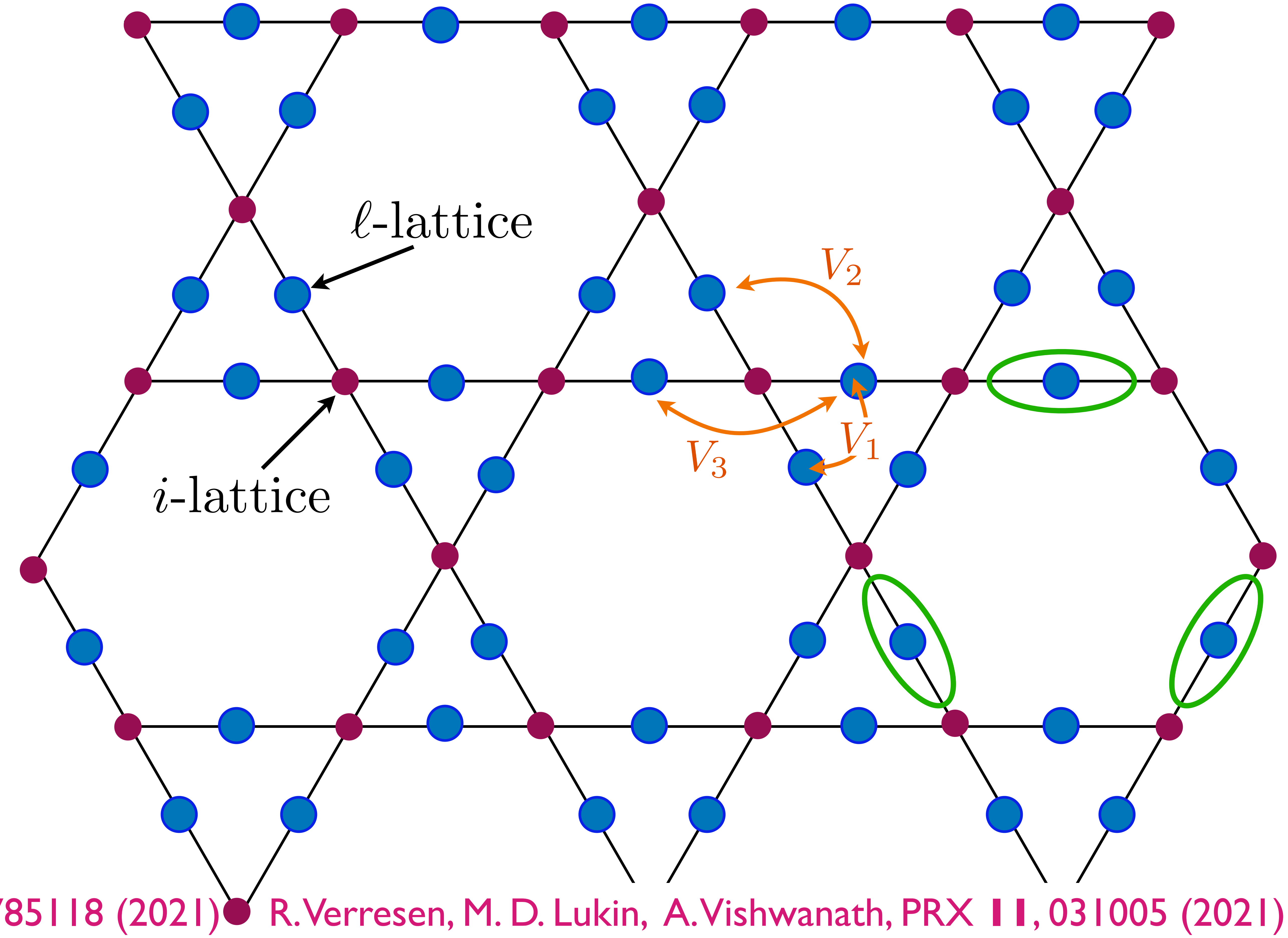

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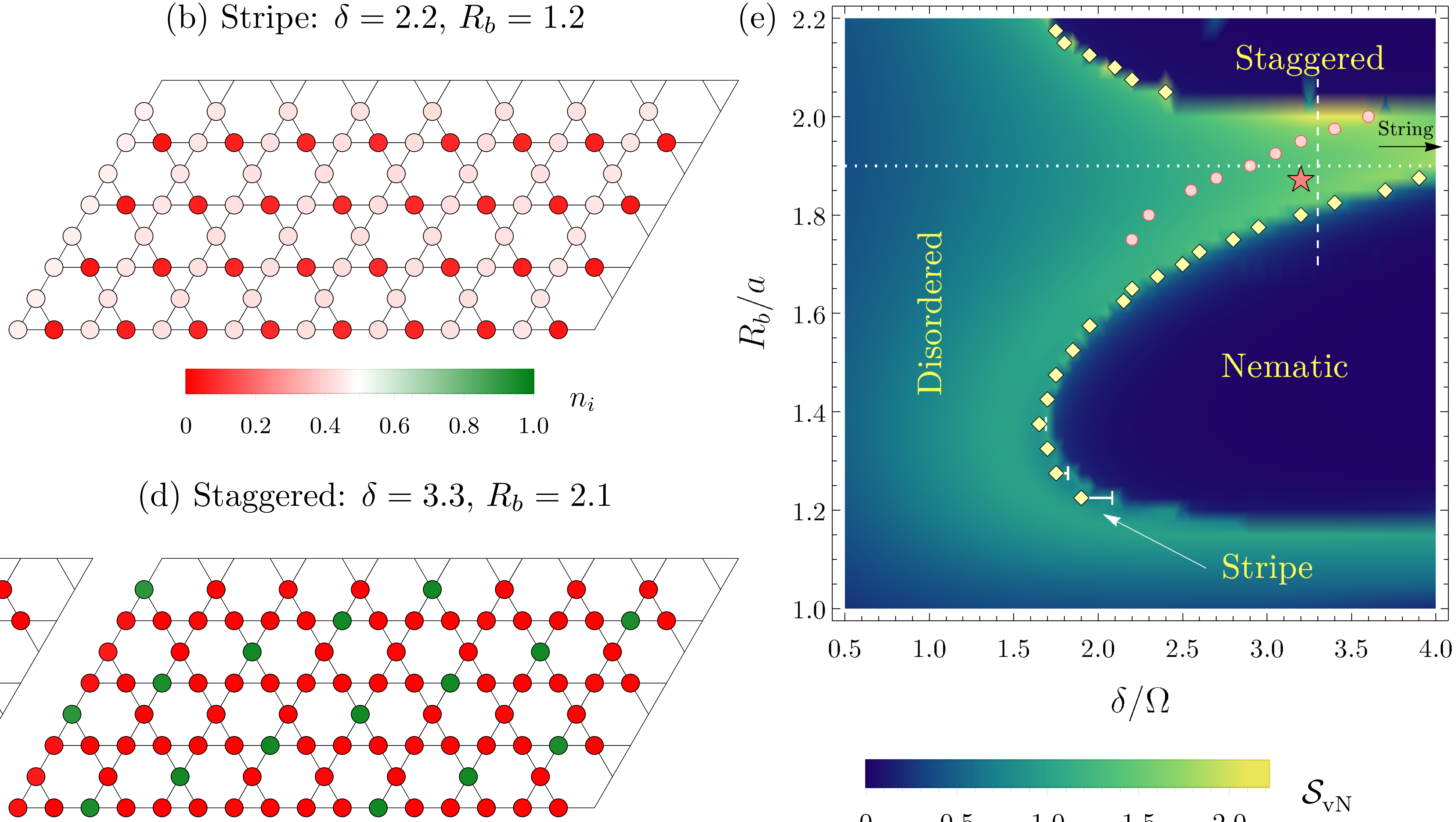
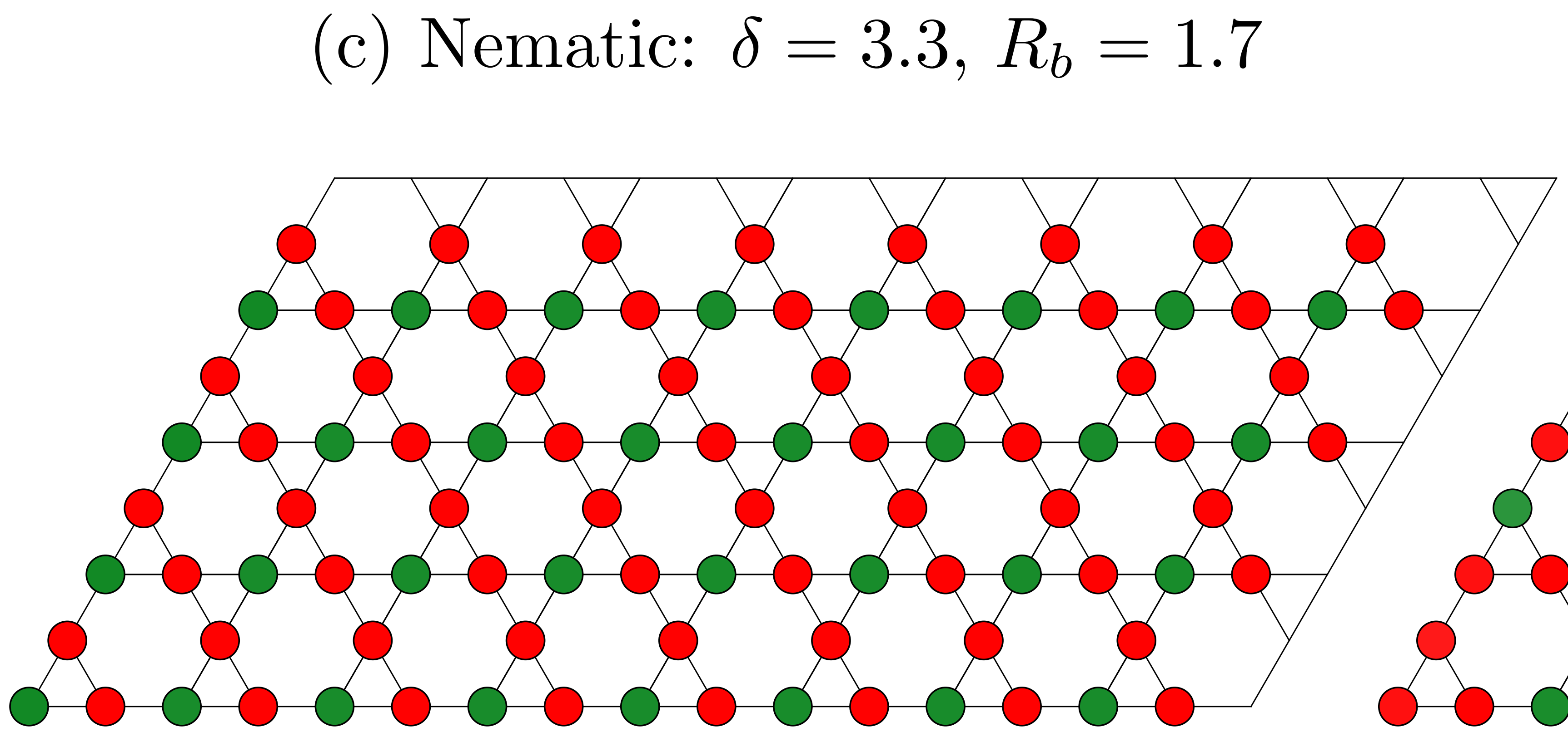
Rydberg atoms on site-kagome lattice: theory





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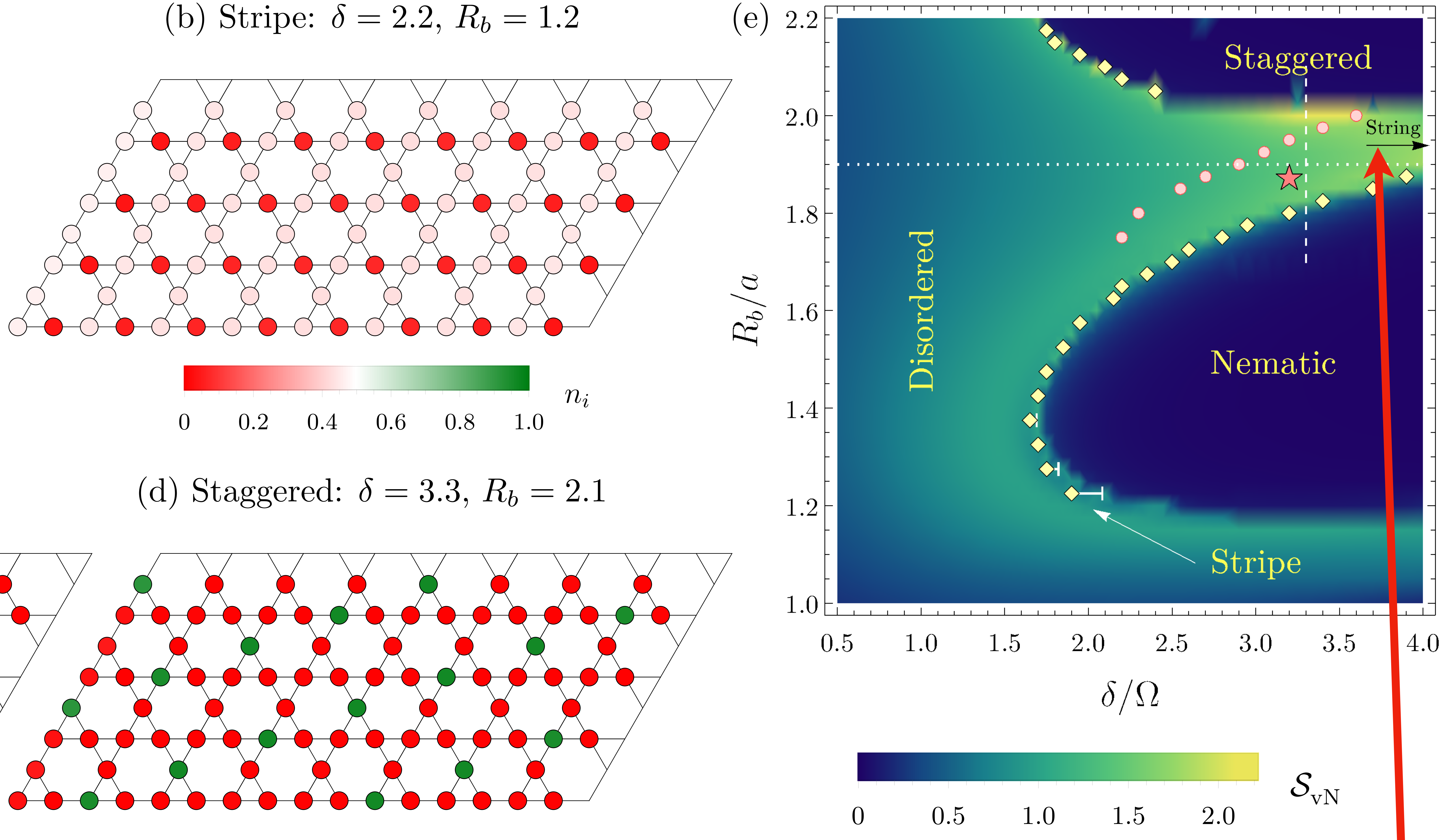
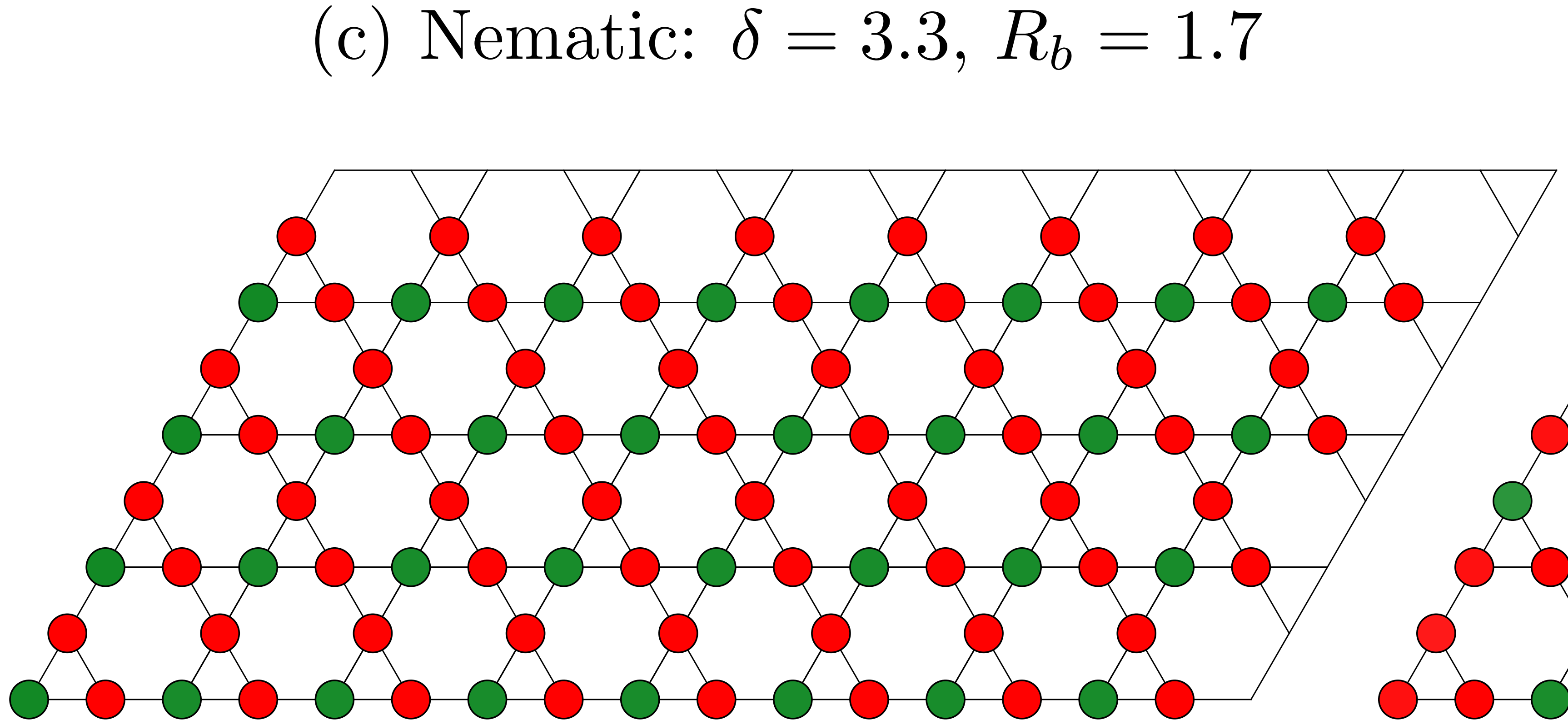




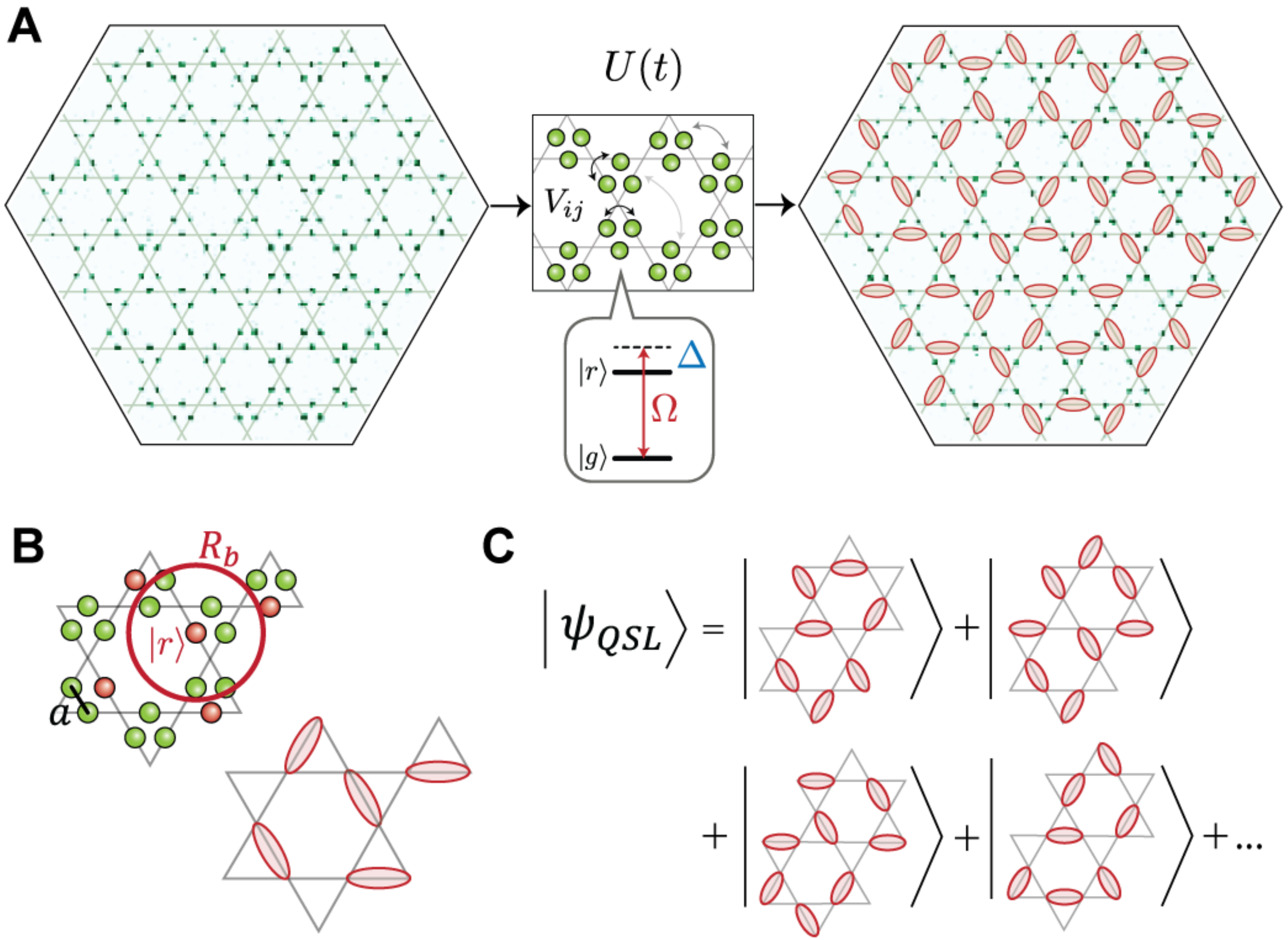
Topological spin liquid described by emergent $\mathbb{Z}_2$ gauge theory **?**

R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and S. Sachdev, PNAS **118**, e2015785118 (2021)

Probing Topological Spin Liquids on a Programmable Quantum Simulator

G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T. T. Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletic, M. D. Lukin, Science **374**, 1242 (2021).

Rydberg atoms on the link-kagome lattice: experiment



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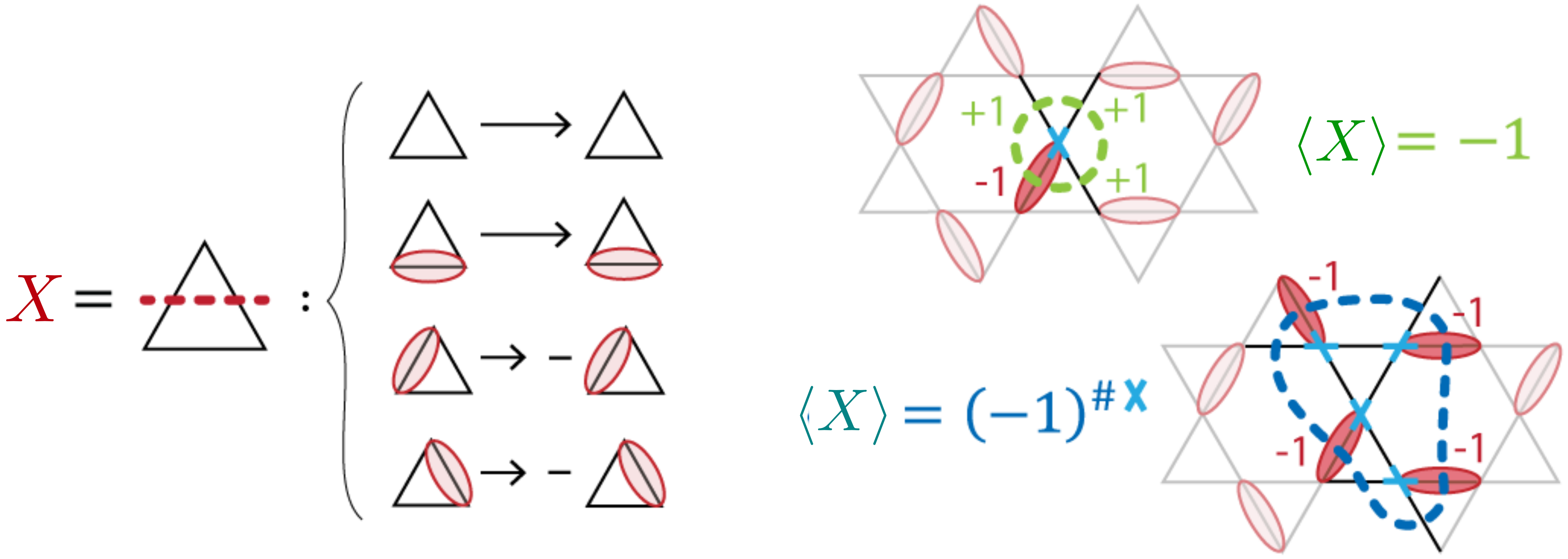
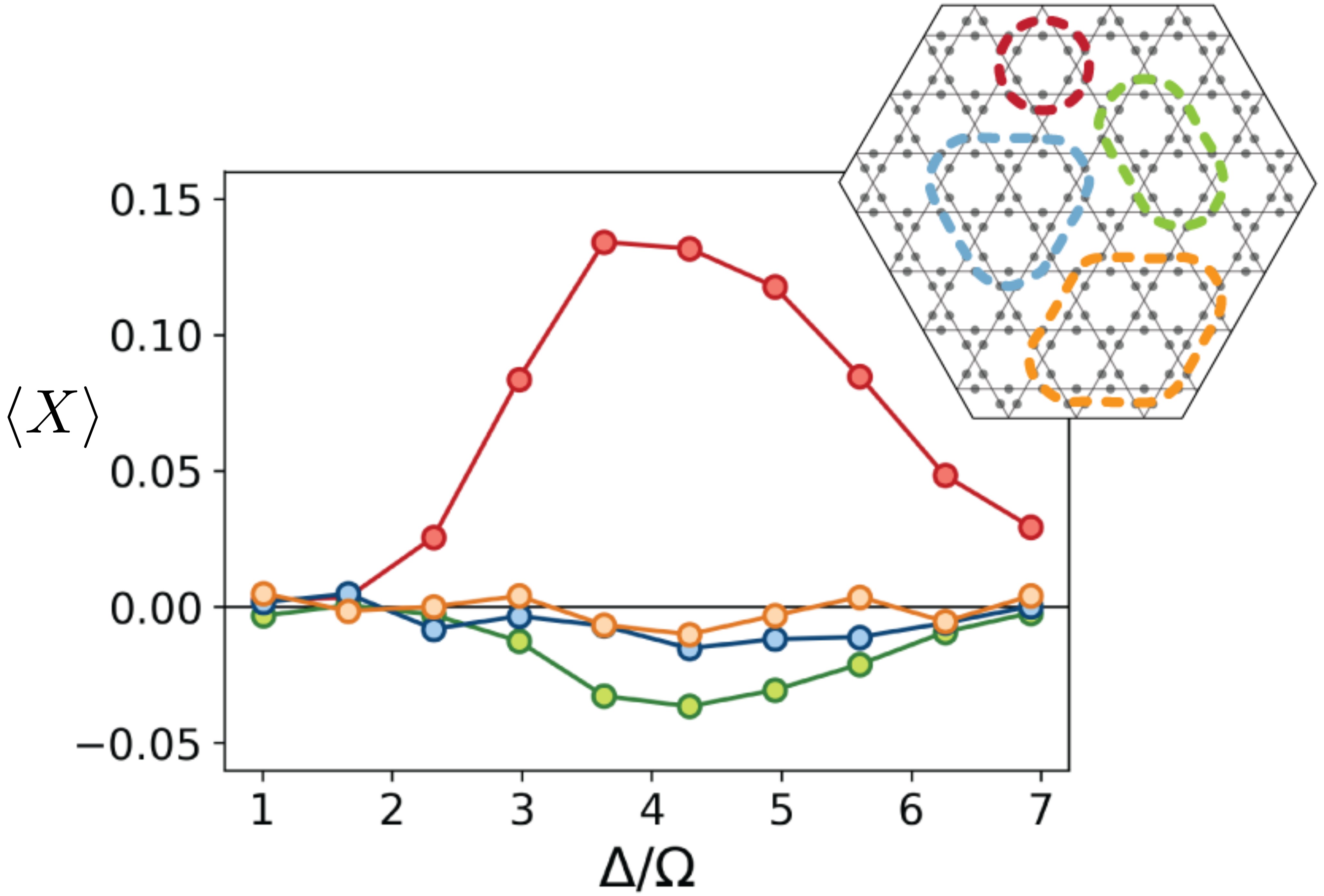


Measurement of
the topological
X operator
$= \prod_{\text{loop}} \sigma^x_\ell$.
Detects close-packed dimers.



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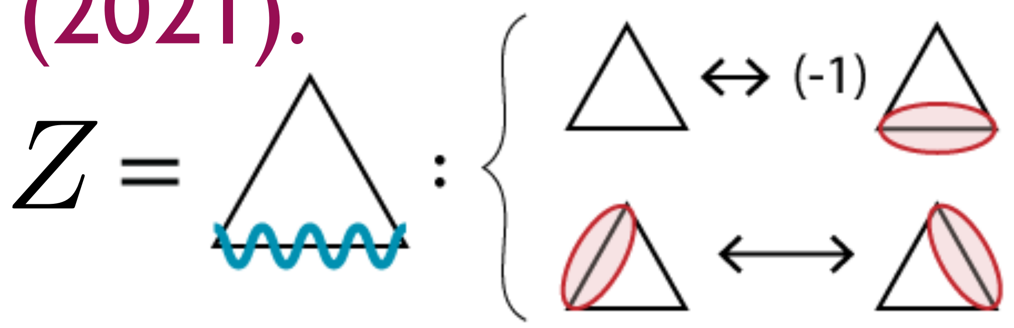

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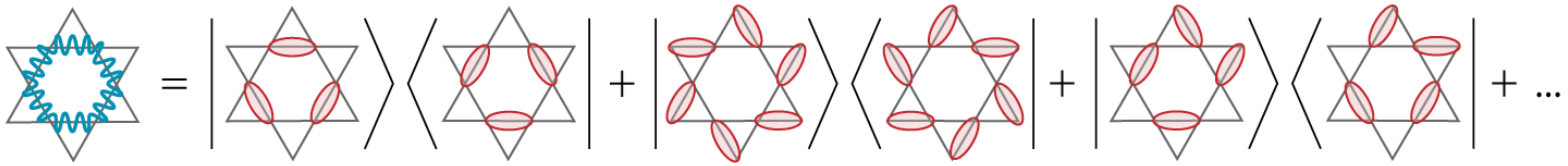
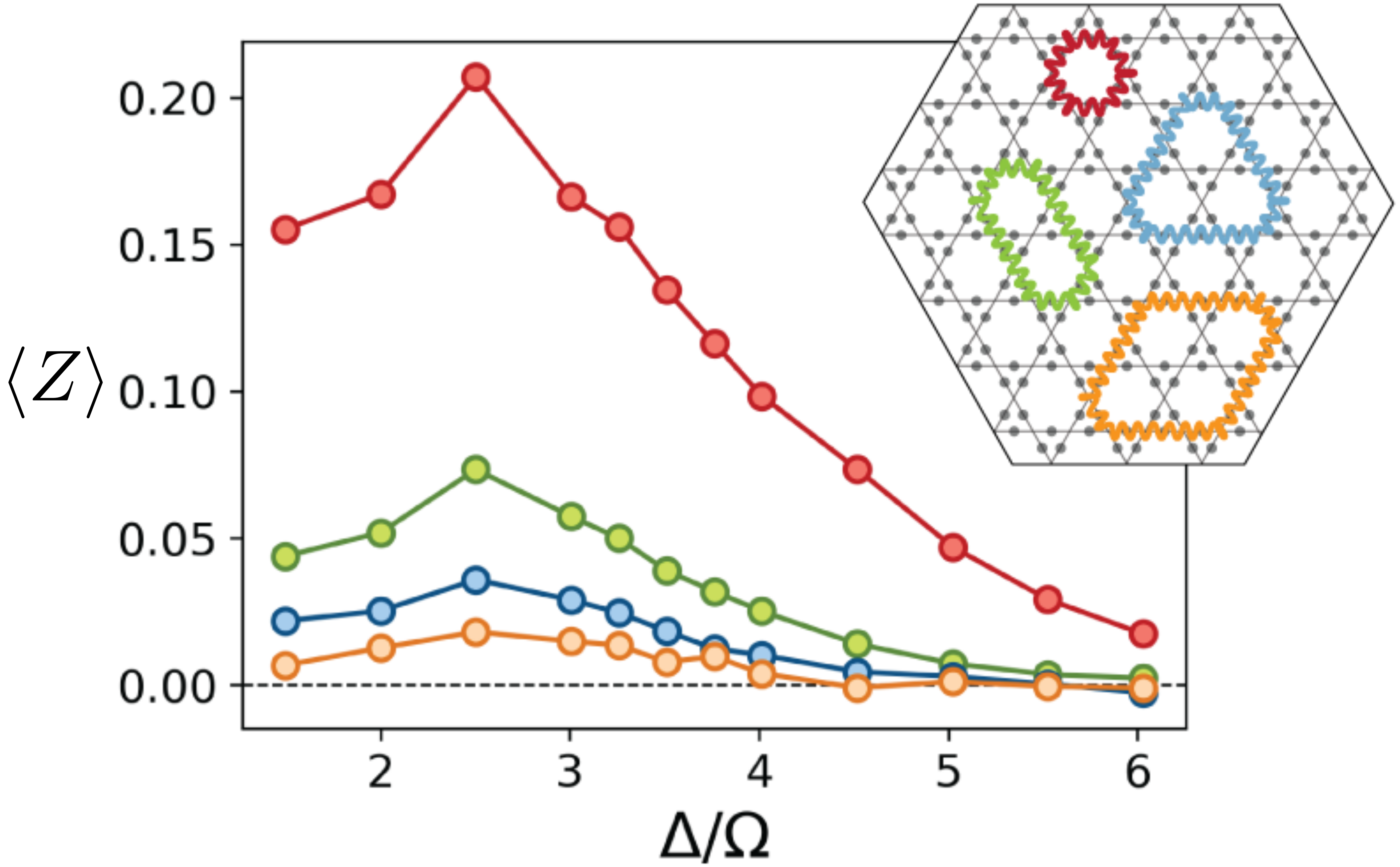


Measurement of the topological Z operator. Detects resonance between dimer loops.



1. Spin liquids and Z_2 gauge theory

2. Rydberg atoms as a Z_2 gauge theory

   Probing topological spin liquids

3. Paramagnon fractionalization theory of the pseudogap metal of the Hubbard model

Yahui Zhang

Alexander
Nikolaenko

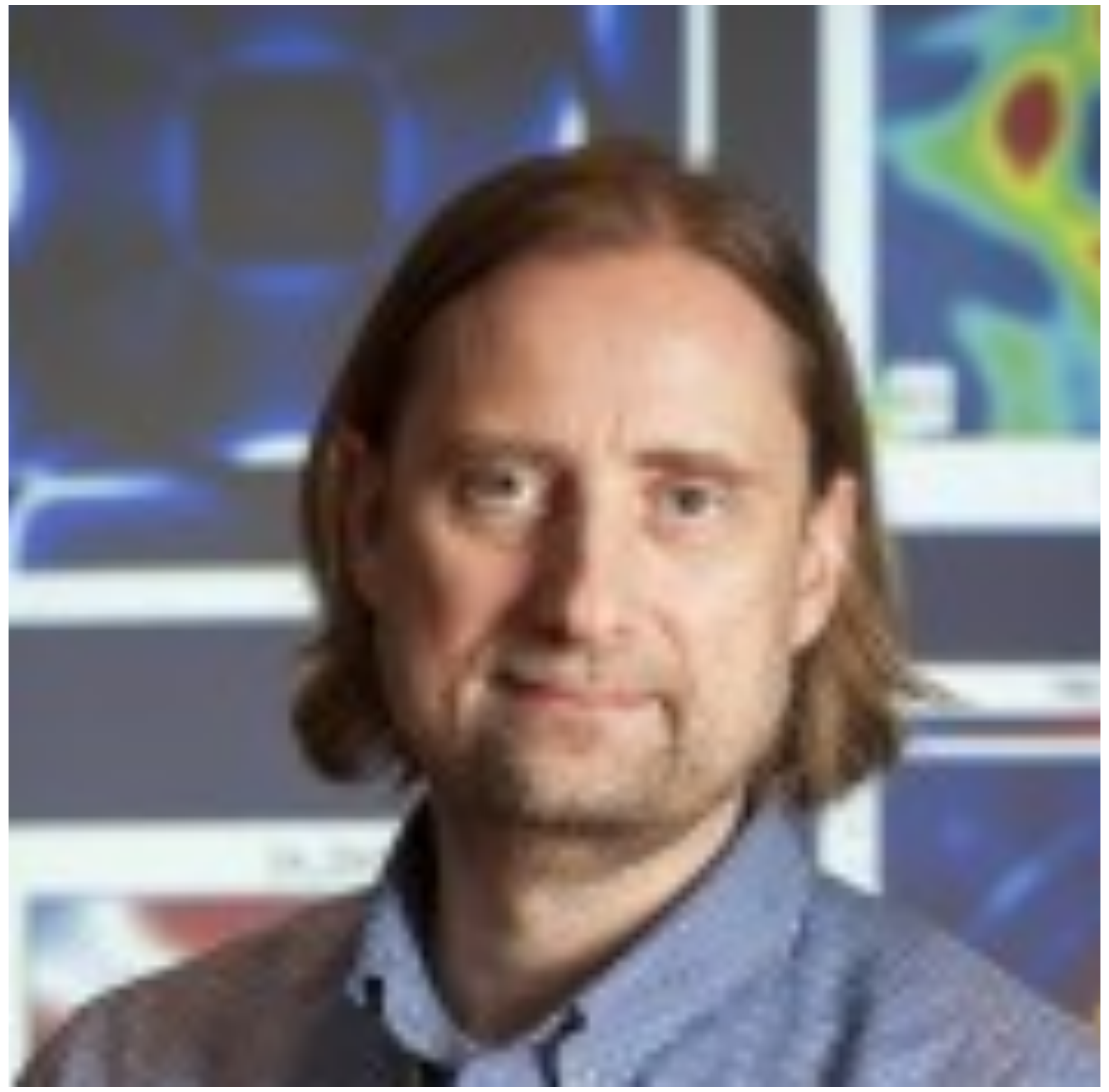
Maria
Tikhanovskaya

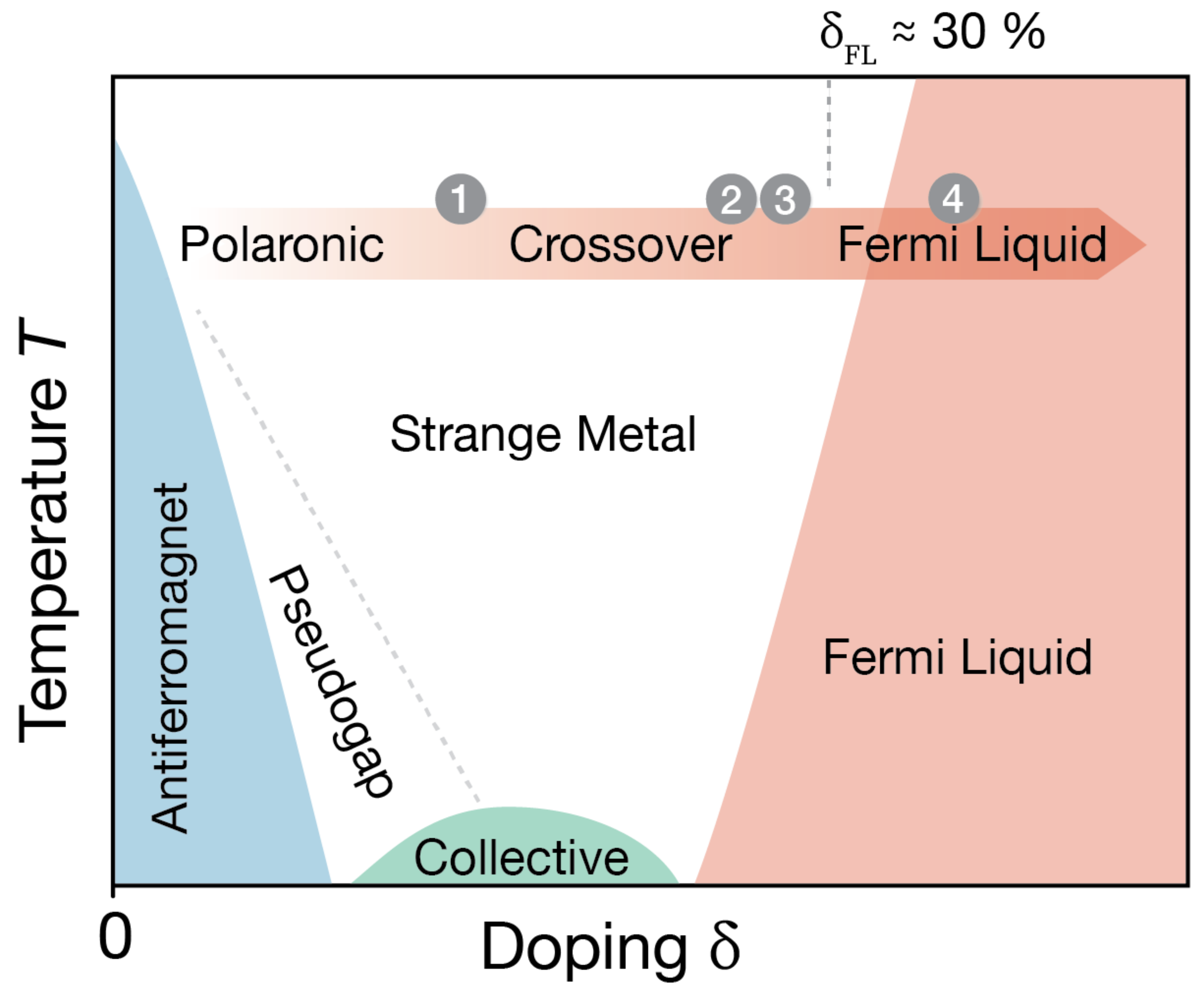
Eric Mascot

arXiv: 2001.09159
arXiv: 2103.05009

arXiv: 2006.01140
arXiv: 2111.13703

Dirk Morr



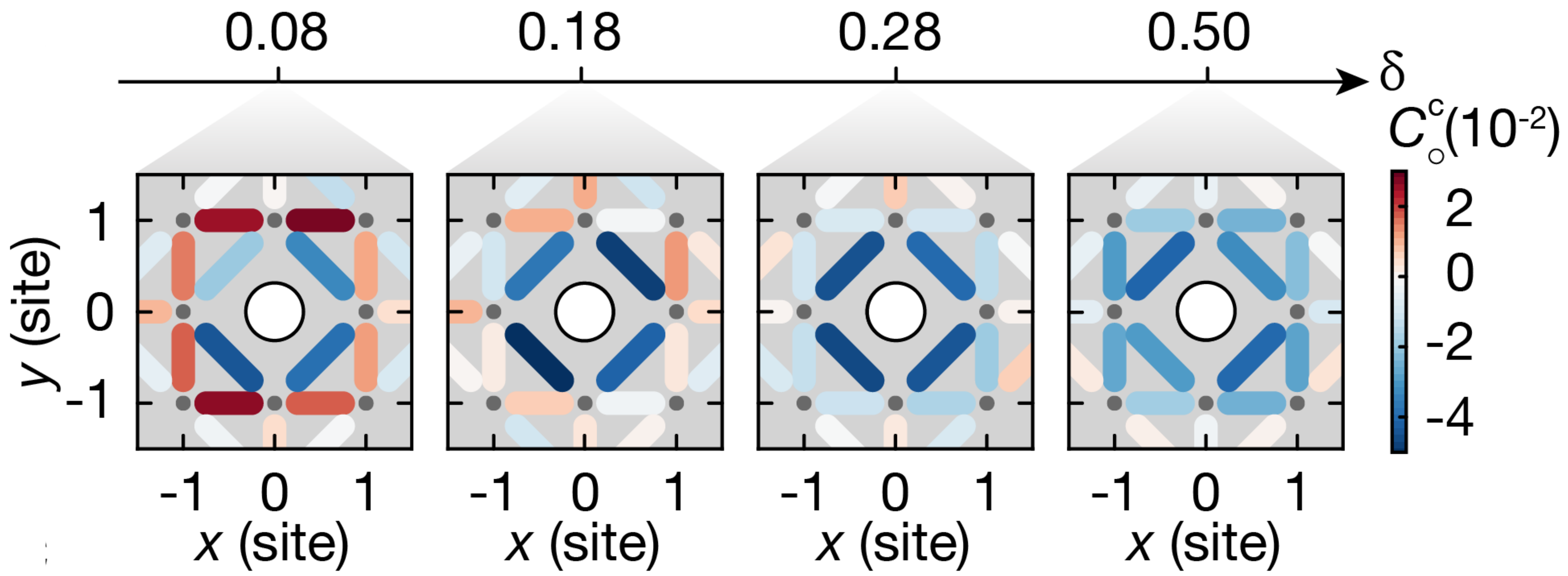


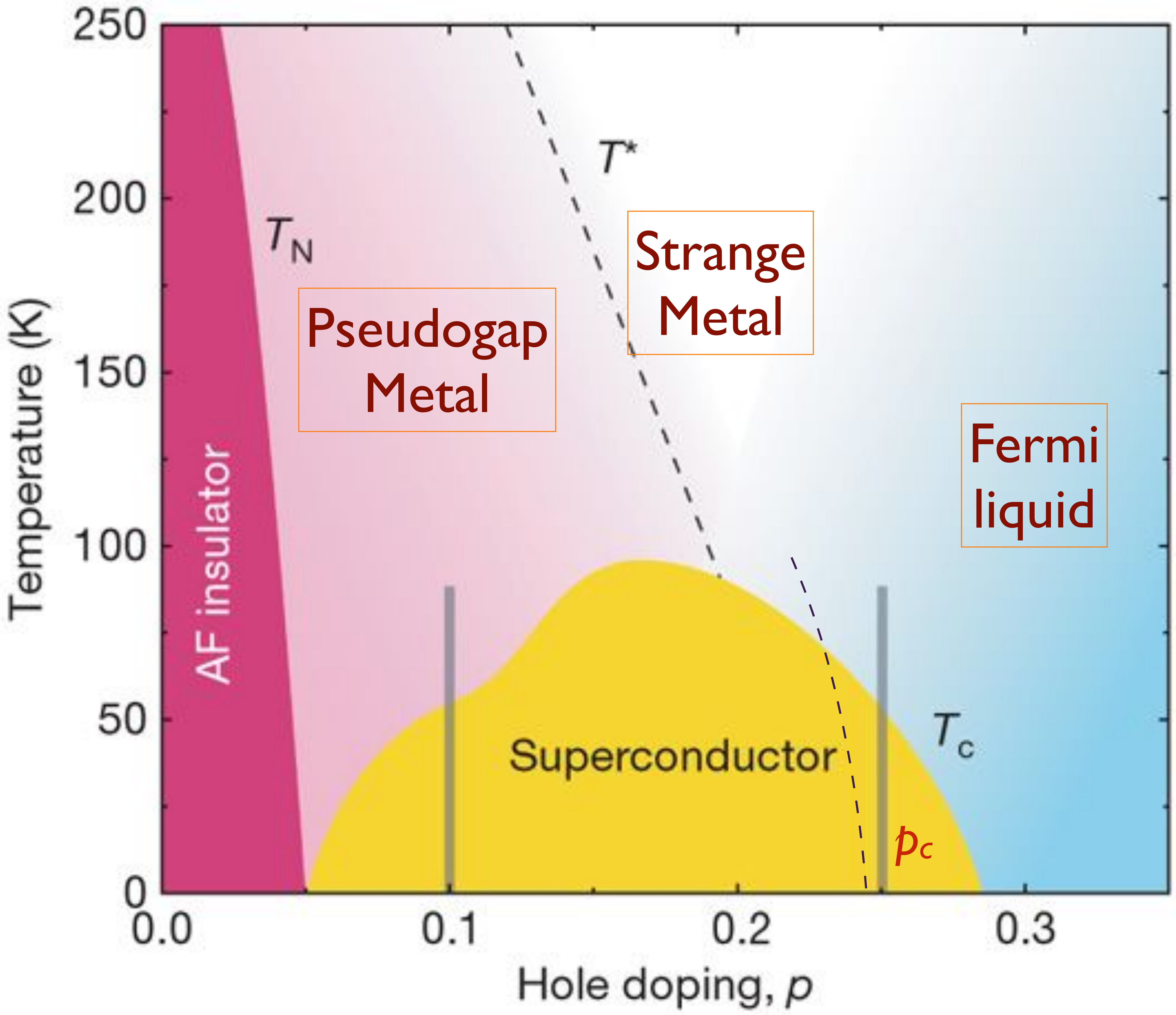
Ultracold fermionic atoms in optical lattices

Microscopic evolution of doped Mott insulators from polaronic metal to Fermi liquid

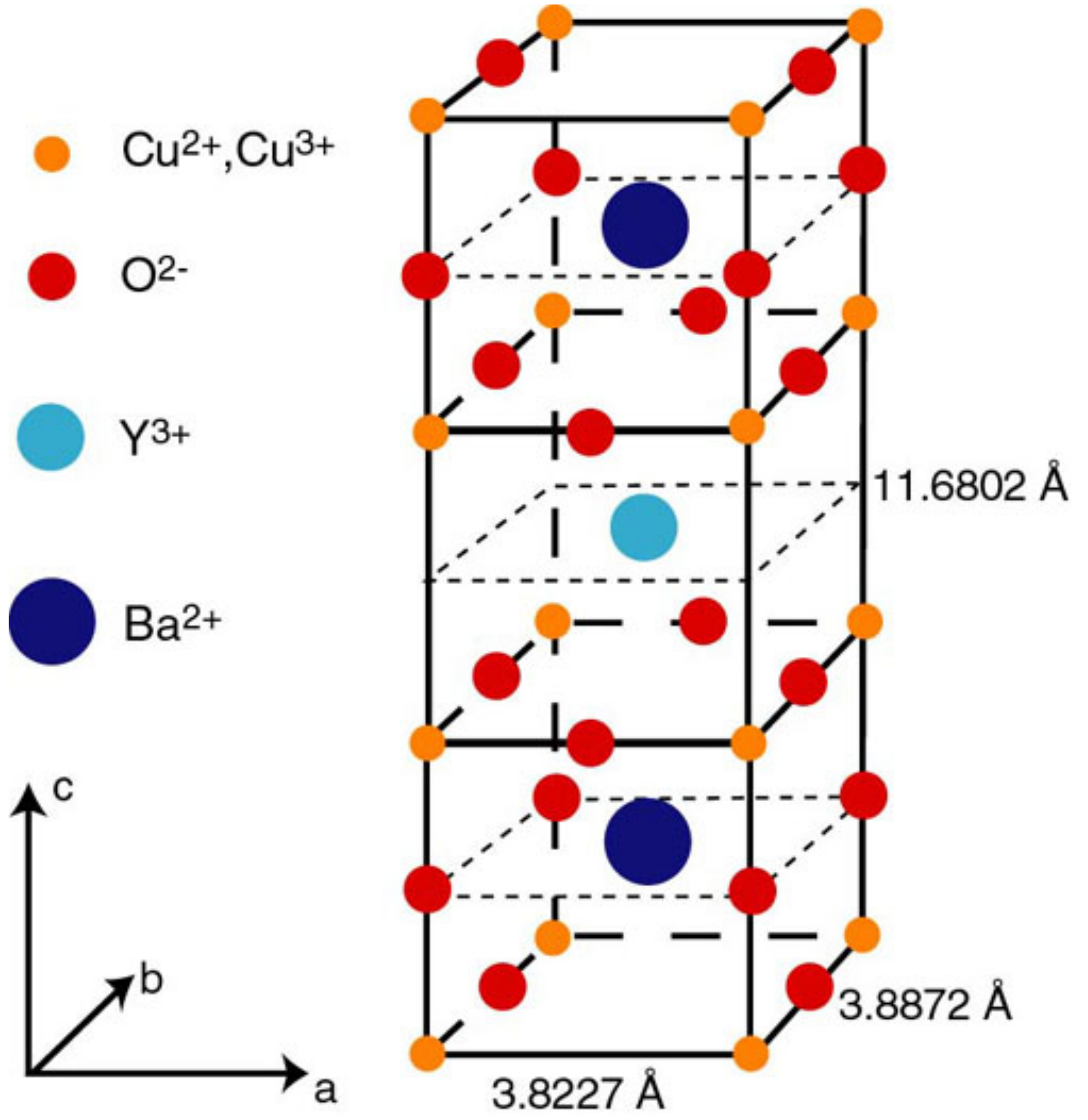
Joannis Koepsell, Dominik Bourgund, Pimonpan Sompet, Sarah Hirthe, Annabelle Bohrdt, Yao Wang, Fabian Grusdt, Eugene Demler, Guillaume Salomon, Christian Gross, Immanuel Bloch

Science **374** (2021) 82

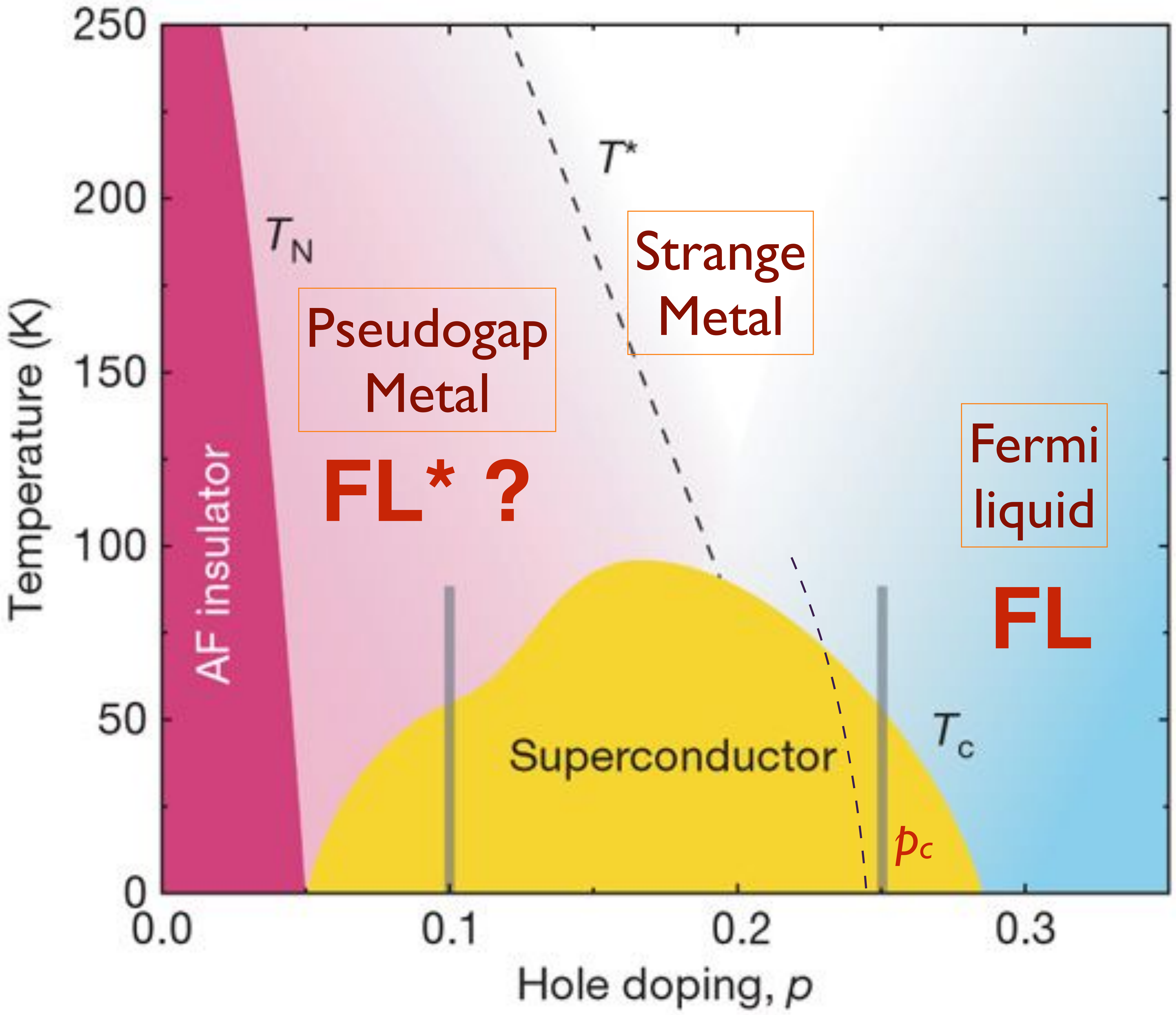


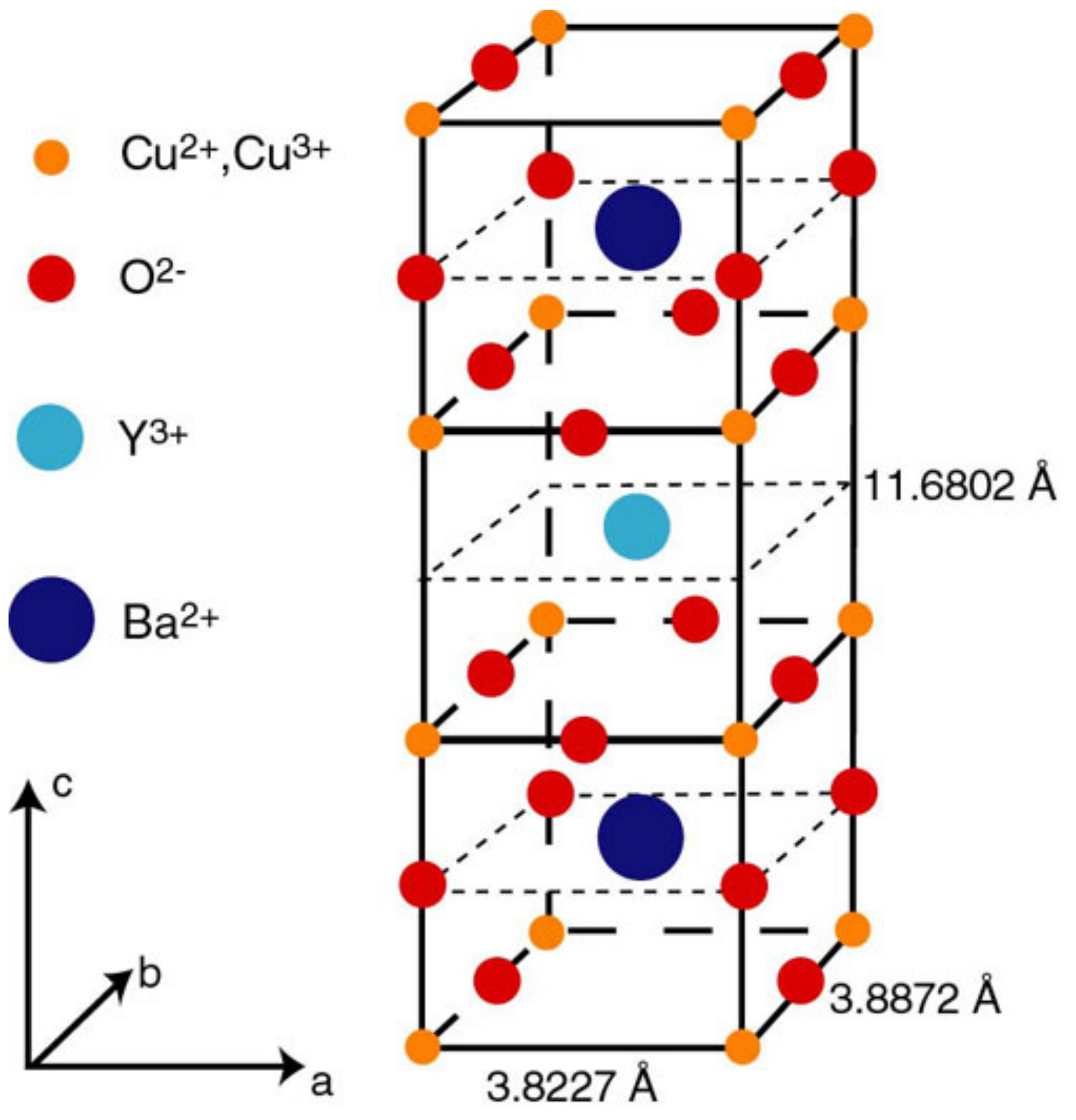


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Pseudogap Metal
Strange Metal
Fermi liquid
Superconductor
T_c
p_c

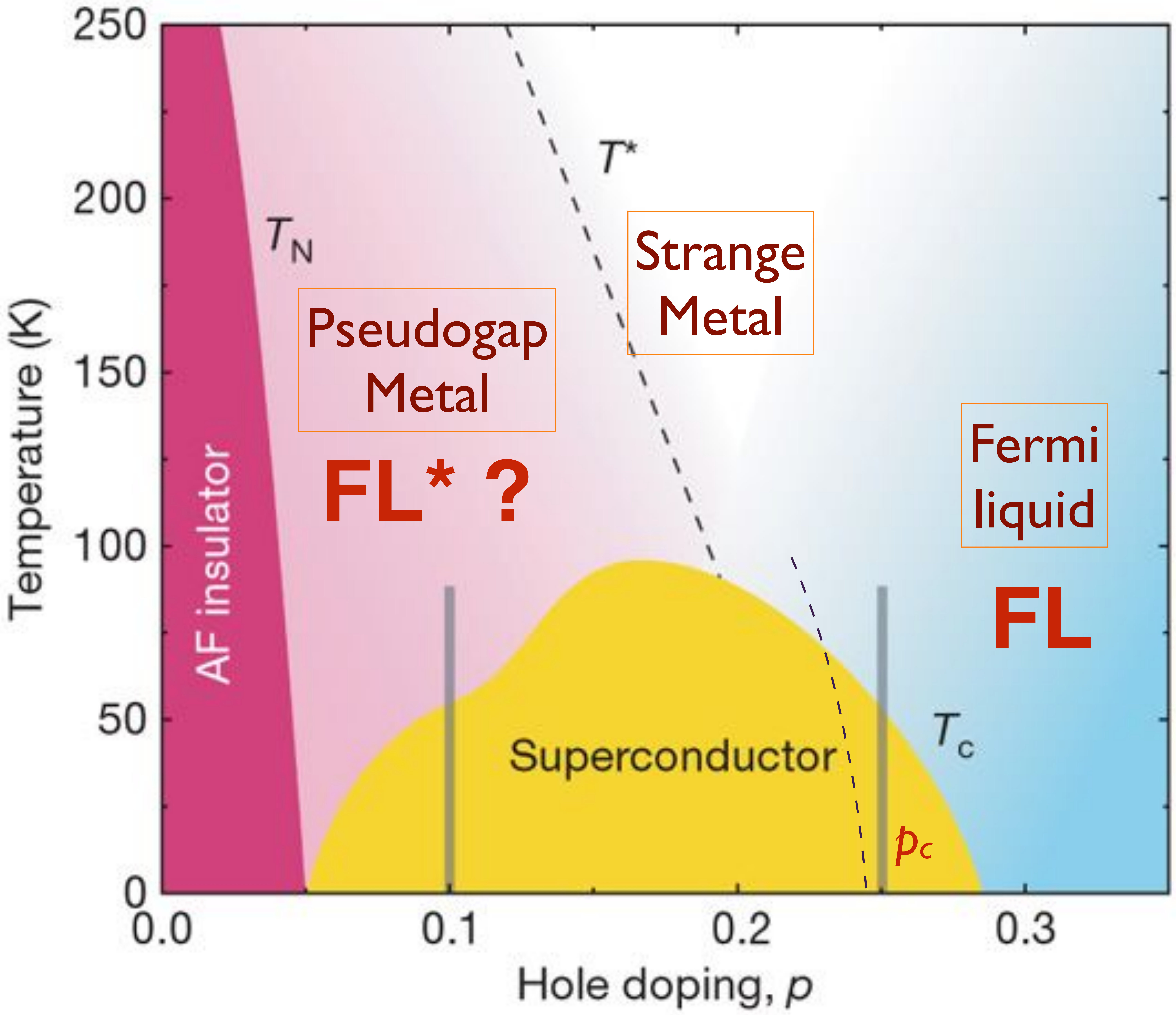


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å





FL*: a metal with a Fermi surface of electron-like quasiparticles, in which the Fermi volume violates the Luttinger relation



Violation of the Luttinger relation requires the presence of a spin liquid with anyons.

In Kondo lattice models, the *f* electrons can form a spin liquid, while the conduction electrons are in a separate band.

However, there is no complete theory for a spin liquid in a metallic phase of the single-band Hubbard model.

Paramagnon theory of the Hubbard model

$$H = -\sum_{i<j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_{i} \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_{i} c_{i\sigma}^{\dagger} c_{i\sigma}$$

We use the operator equation (valid on each site i):

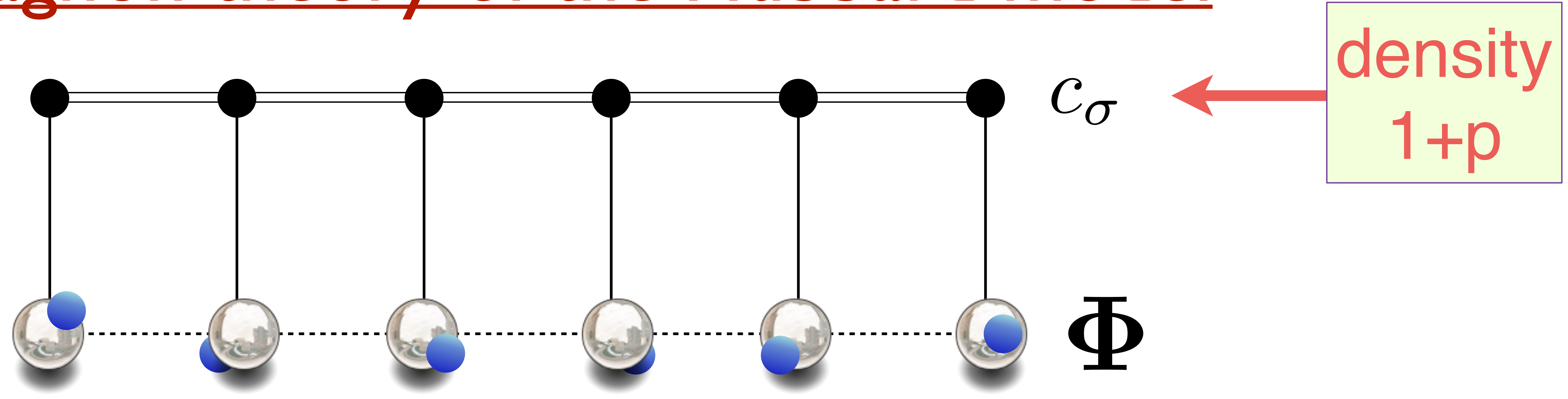
$$U \left(n_{\uparrow} - \frac{1}{2} \right) \left(n_{\downarrow} - \frac{1}{2} \right) = -\frac{2U}{3} \boldsymbol{S}^2 + \frac{U}{4}$$

Then we decouple the interaction via

$$\exp\left(\frac{2U}{3} \sum_i \int d\tau \boldsymbol{S}_i^2 \right) = \int \mathcal{D}\boldsymbol{\Phi}_i(\tau) \exp\left(-\sum_i \int d\tau \left[\frac{3}{8U} \boldsymbol{\Phi}_i^2 - \boldsymbol{\Phi}_i \cdot c_{i\sigma}^{\dagger} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \right] \right)$$

This yields the 'Scalapino-Pines-Chubukov-Schmalian...' theory for a 'paramagnon quantum rotor' $\boldsymbol{\Phi}_i$ coupled to otherwise free fermions $c_{i\sigma}$.

Paramagnon theory of the Hubbard model



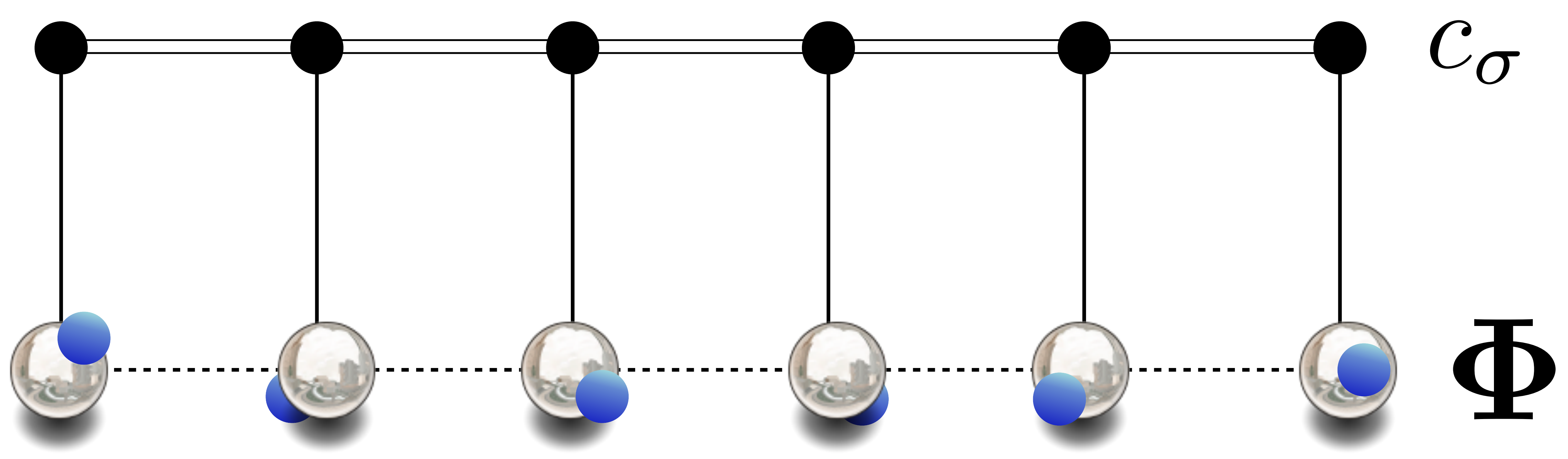
Quantum rotors $|\boldsymbol{\Phi}_i| = 1$

$$\mathcal{H}_{\rm rotor} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_i \frac{g}{2} \boldsymbol{L}_i^2 - \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_i$$

$$[L_{i\alpha}, L_{j\beta}] = i\epsilon_{\alpha\beta\gamma}\delta_{ij}L_{i\gamma}, \quad [L_{i\alpha}, \Phi_{j\beta}] = i\epsilon_{\alpha\beta\gamma}\delta_{ij}\Phi_{i\gamma}, \quad [\Phi_{i\alpha}, \Phi_{j\beta}] = 0$$

Paramagnon theory of the Hubbard model

Quantum rotors $|\boldsymbol{\Phi}_i| = 1$



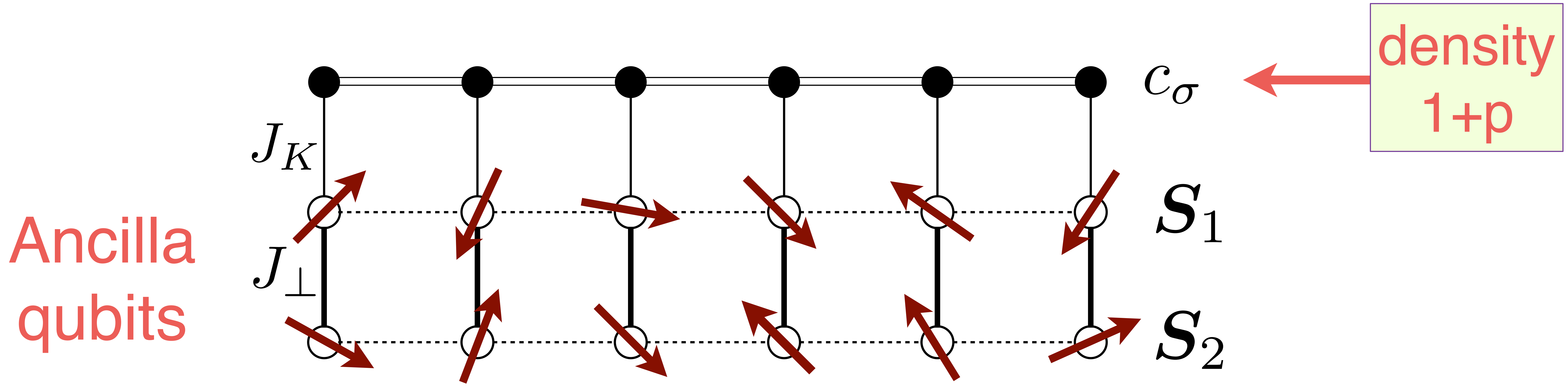
$$\mathcal{H}_{\text{rotor}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + \sum_i \frac{g}{2} \boldsymbol{L}_i^2 - \sum_i c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_i$$

Each rotor has eigenvalues $g\ell(\ell+1)/2$, degeneracy $2\ell+1$, $\ell = 0, 1, 2 \ldots$
Restrict to the $\ell = 0, 1$ states, and represent each rotor by 2 "ancilla qubits", $S = 1/2$ spins $\boldsymbol{S}_{1i}$ and $\boldsymbol{S}_{2i}$, with an antiferromagnetic coupling $J_\perp = g$

$$\boldsymbol{L}_i = \boldsymbol{S}_{1i} + \boldsymbol{S}_{2i}$$

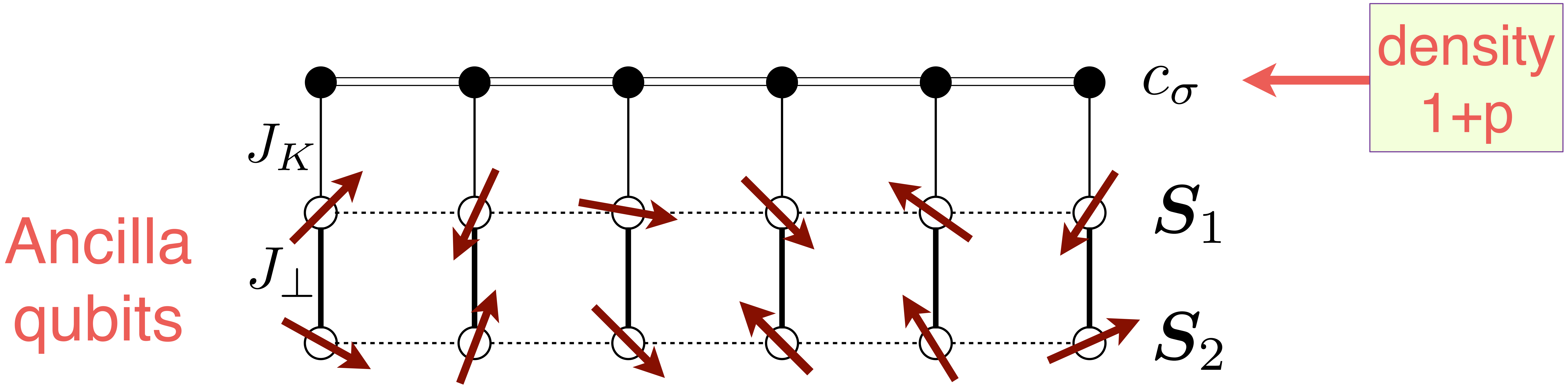
$$\boldsymbol{\Phi}_i = \frac{1}{\sqrt{3}} \left(\boldsymbol{S}_{2i} - \boldsymbol{S}_{1i} \right)$$

Ancilla theory of the Hubbard model



$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_i \left[J_K \, c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{1i} + J_\perp \, \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i} \right] + \sum_{\langle ij \rangle} \left[J_1 \, \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{1j} + J_2 \, \boldsymbol{S}_{2i} \cdot \boldsymbol{S}_{2j} \right]$$

Ancilla theory of the Hubbard model



$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_i \left[J_K \, c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{1i} + J_\perp \, \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i} \right] + \sum_{\langle ij \rangle} \left[J_1 \, \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{1j} + J_2 \, \boldsymbol{S}_{2i} \cdot \boldsymbol{S}_{2j} \right]$$

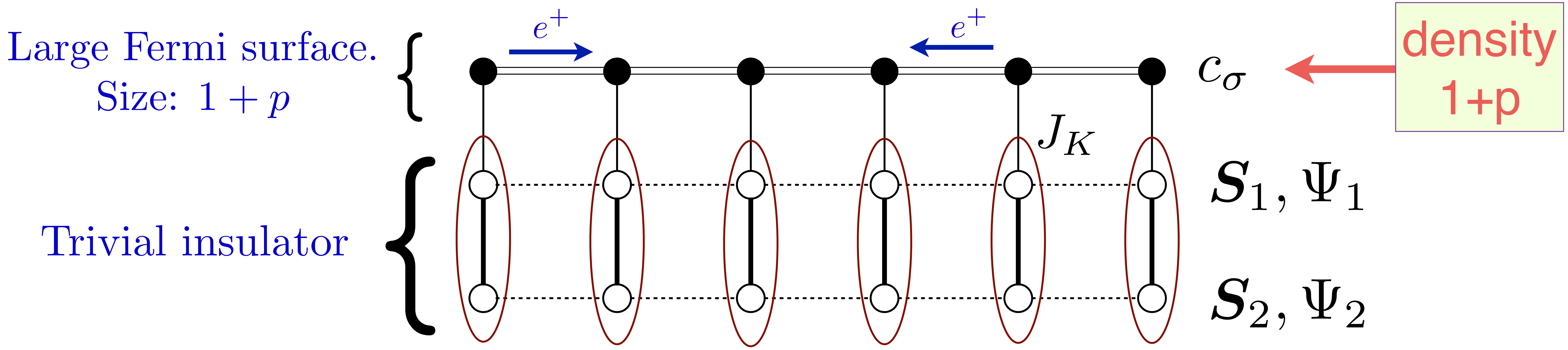
Performing a Schrieffer-Wolff transformation in powers of $1/J_\perp$, we obtain

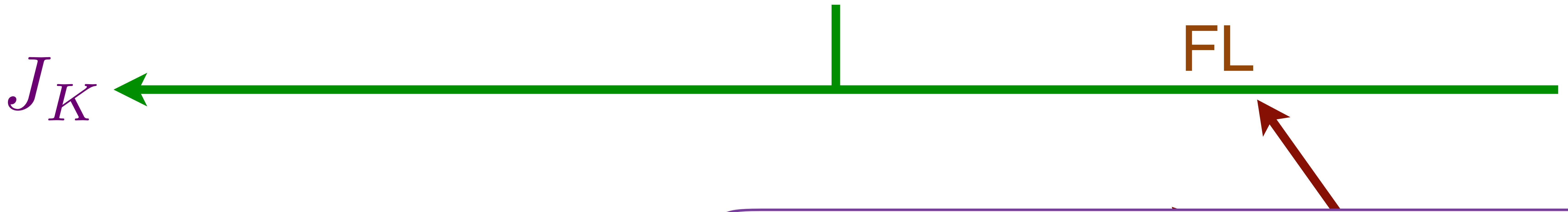
$$\mathcal{H} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + U \sum_i \left[c^\dagger_{i\uparrow} c_{i\uparrow} \right] \left[c^\dagger_{i\downarrow} c_{i\downarrow} \right] + J \sum_{\langle ij \rangle} \left[c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \right] \cdot \left[c^\dagger_{j\rho} \frac{\boldsymbol{\tau}_{\rho\rho'}}{2} c_{j\rho'} \right]$$

i.e. we recover a Hubbard-Heisenberg model with *no ancillas* and

$$U = \frac{3J_K^2}{8J_\perp} + \frac{3J_K^3}{16J_\perp^2} + \ldots \quad , \quad J = \frac{J_K^2 (J_1 + J_2)}{4J_\perp^2}$$

Ancilla theory of the Hubbard model

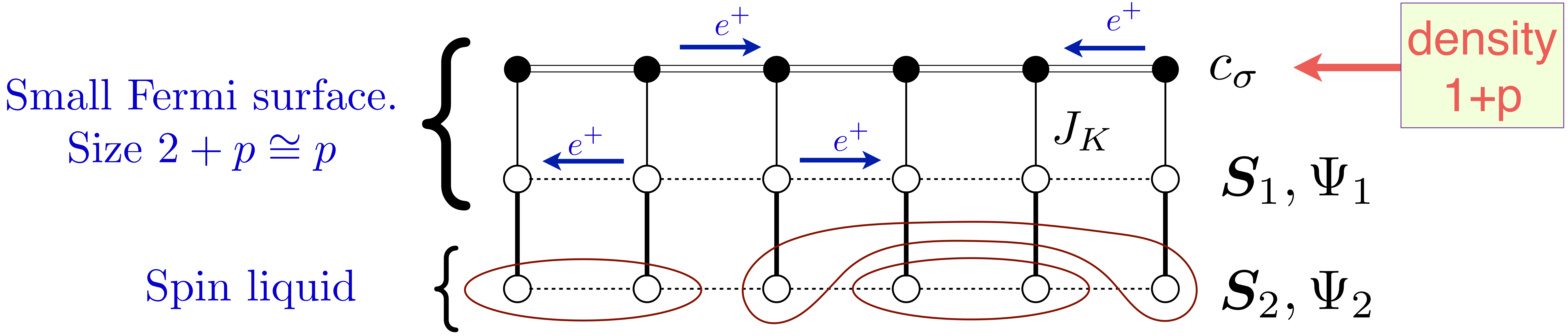


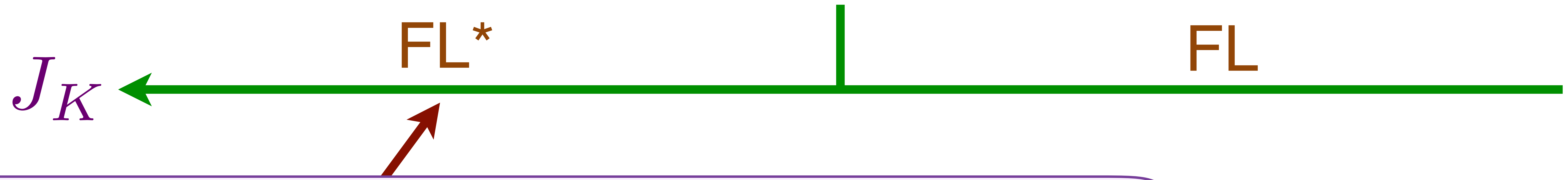


Large Fermi surface of size $1 + p$

$$|\text{FL}\rangle = |\text{Rung singlets of } \Psi_1, \Psi_2\rangle \otimes |\text{Slater determinant of } c\rangle$$

Ancilla theory of the Hubbard model

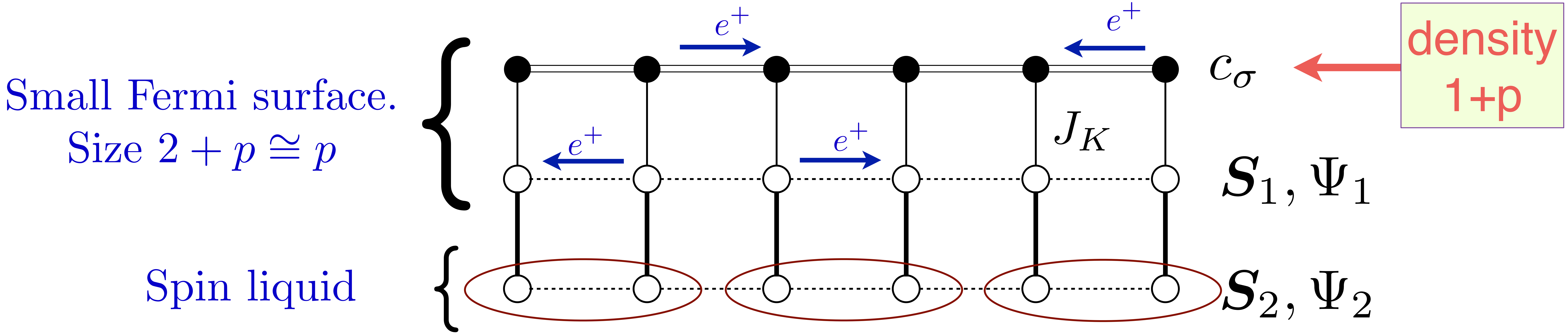


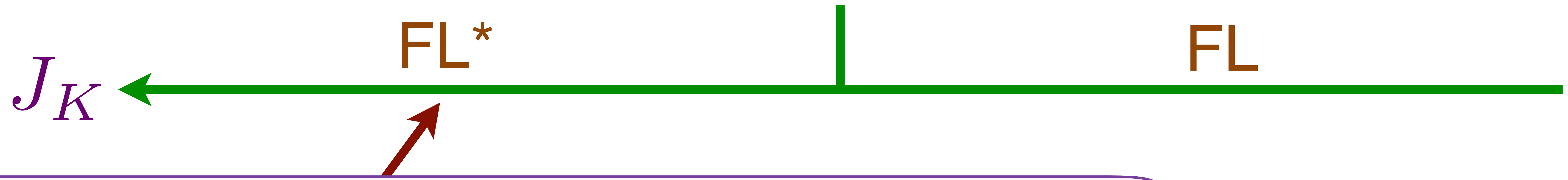


Small Fermi surface of size p

$$|\mathrm{FL}*\rangle = [\text{Projection onto rung singlets of } \Psi_1, \Psi_2]$$
$$\bowtie |\text{Slater determinant of } (c, \Psi_1)\rangle$$
$$\otimes |\text{Slater determinant of } \Psi_2\rangle$$

Ancilla theory of the Hubbard model





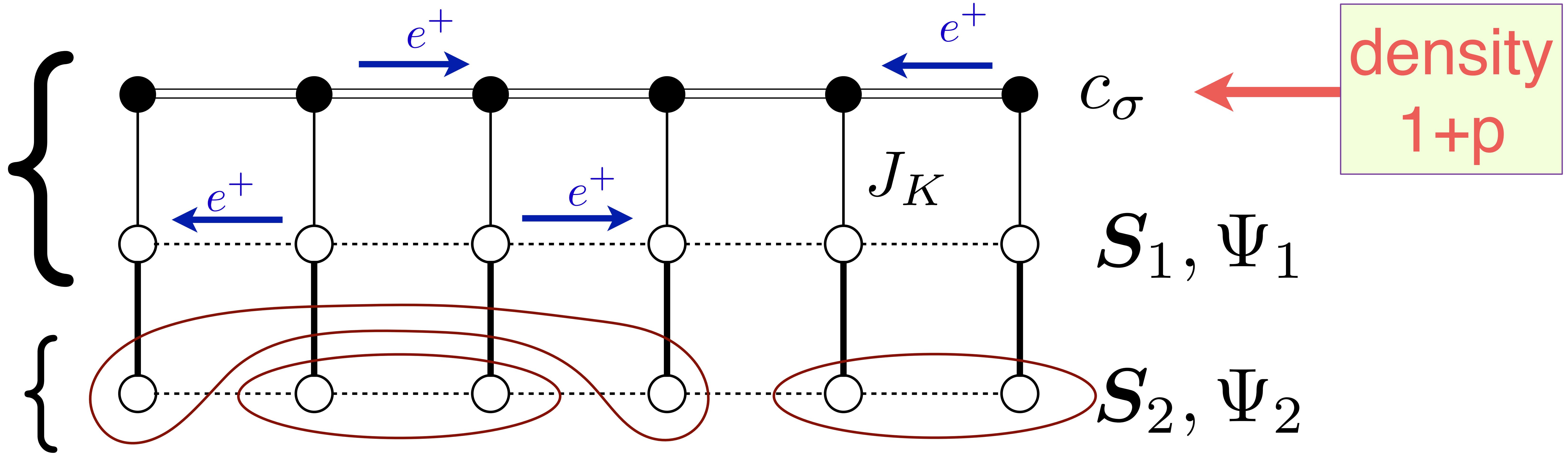
Small Fermi surface of size p

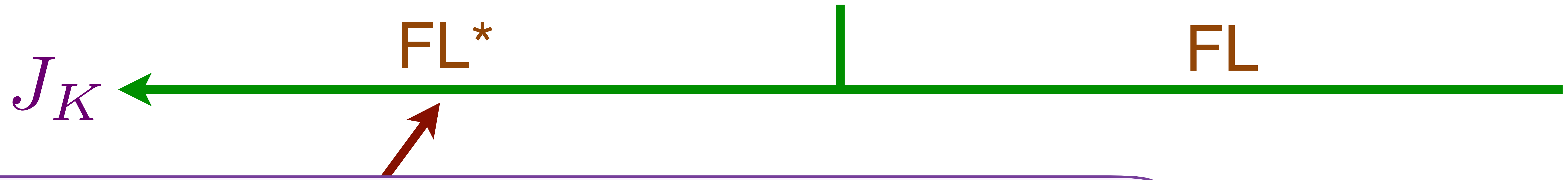
$$
\begin{aligned}
|\mathrm{FL}*\rangle = &\left[\text{Projection onto rung singlets of } \Psi_1, \Psi_2\right] \\
&\bowtie |\text{Slater determinant of } (c, \Psi_1)\rangle \\
&\otimes |\text{Slater determinant of } \Psi_2\rangle
\end{aligned}
$$

Ancilla theory of the Hubbard model

Small Fermi surface.
Size $2+p \cong p$

Spin liquid

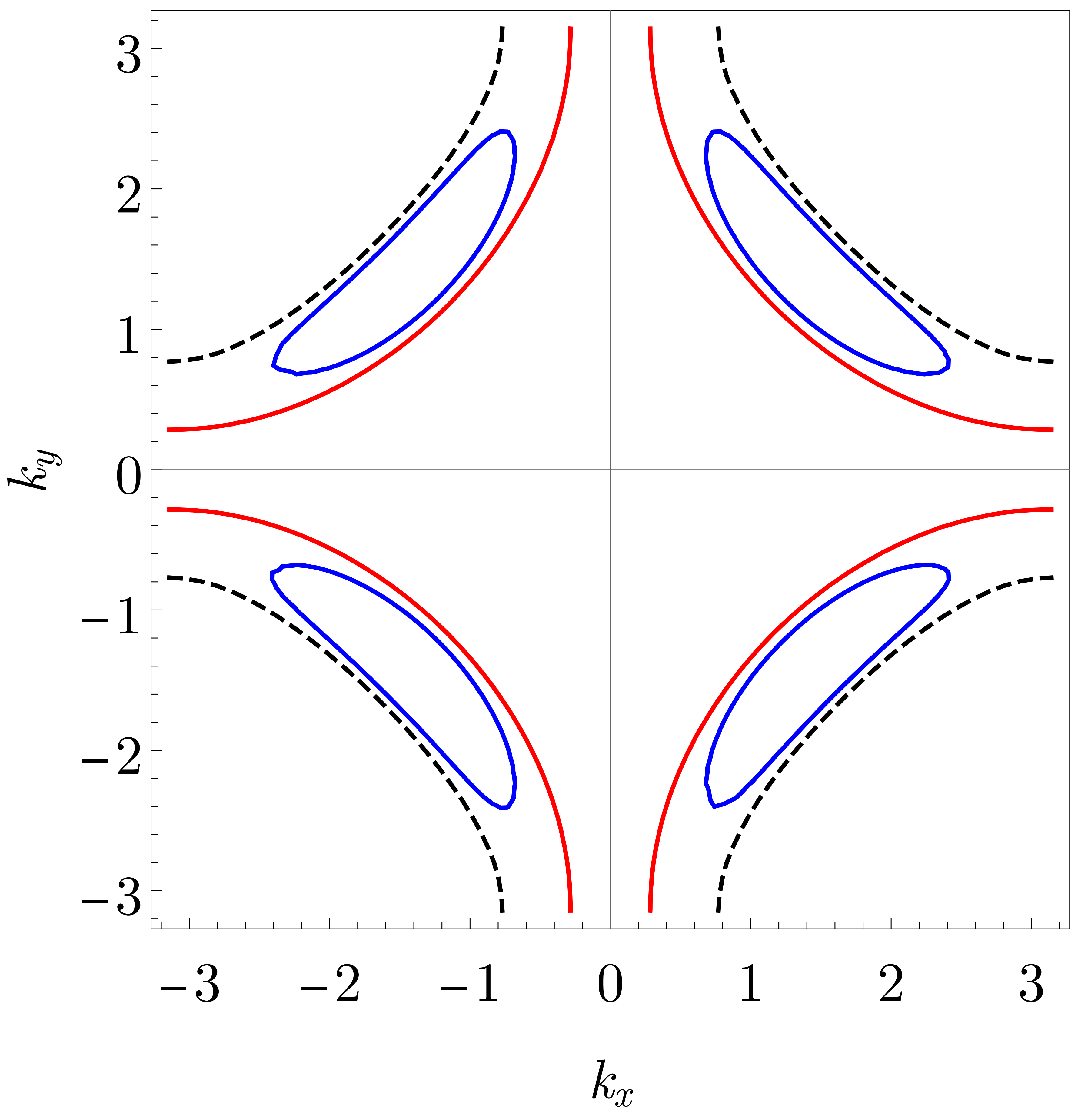




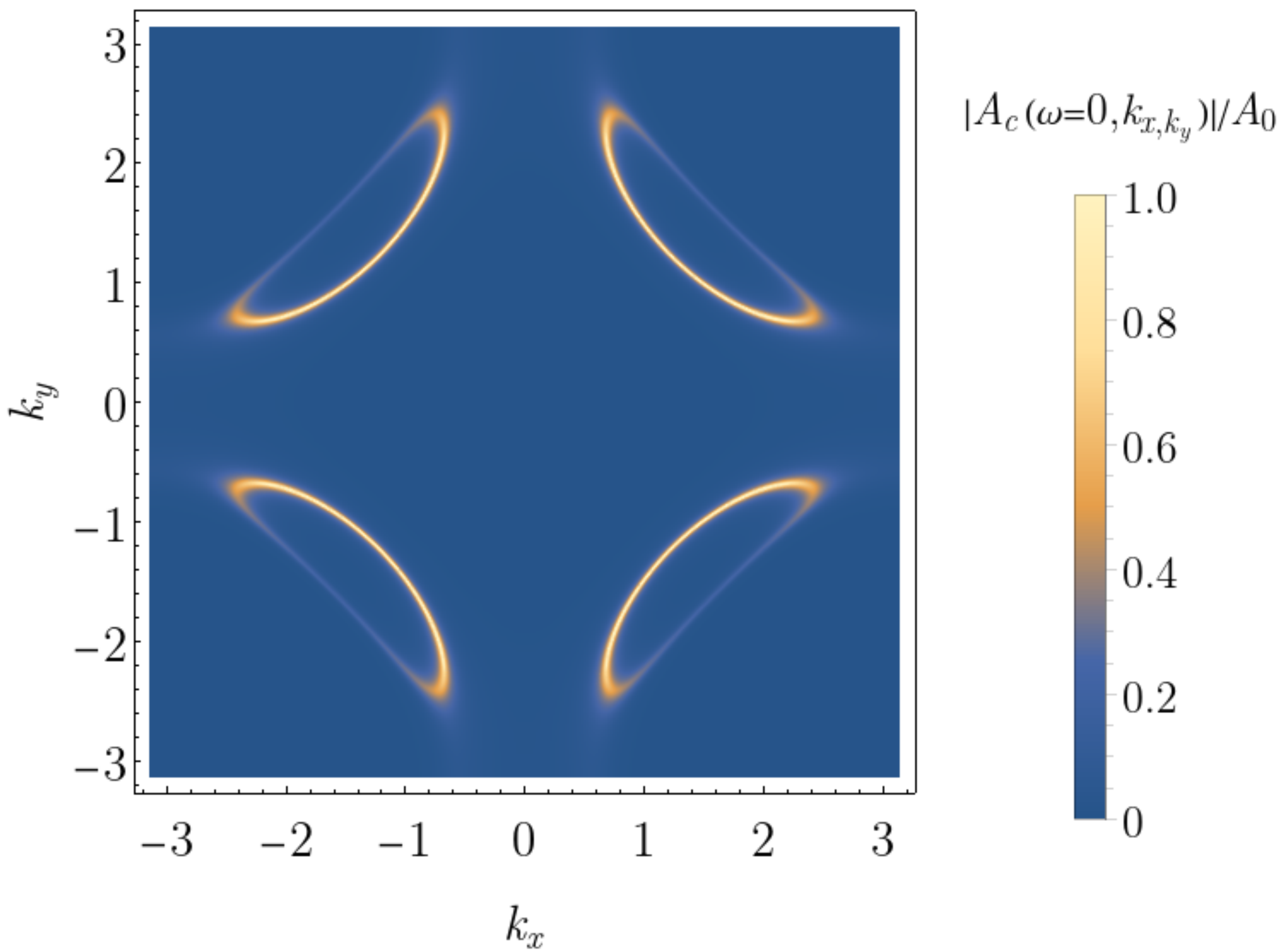
Small Fermi surface of size p

$$|\mathrm{FL}*\rangle = [\text{Projection onto rung singlets of } \Psi_1, \Psi_2]$$
$$\bowtie |\text{Slater determinant of } (c, \Psi_1)\rangle$$
$$\otimes |\text{Slater determinant of } \Psi_2\rangle$$

FL* in a ***one-band*** model



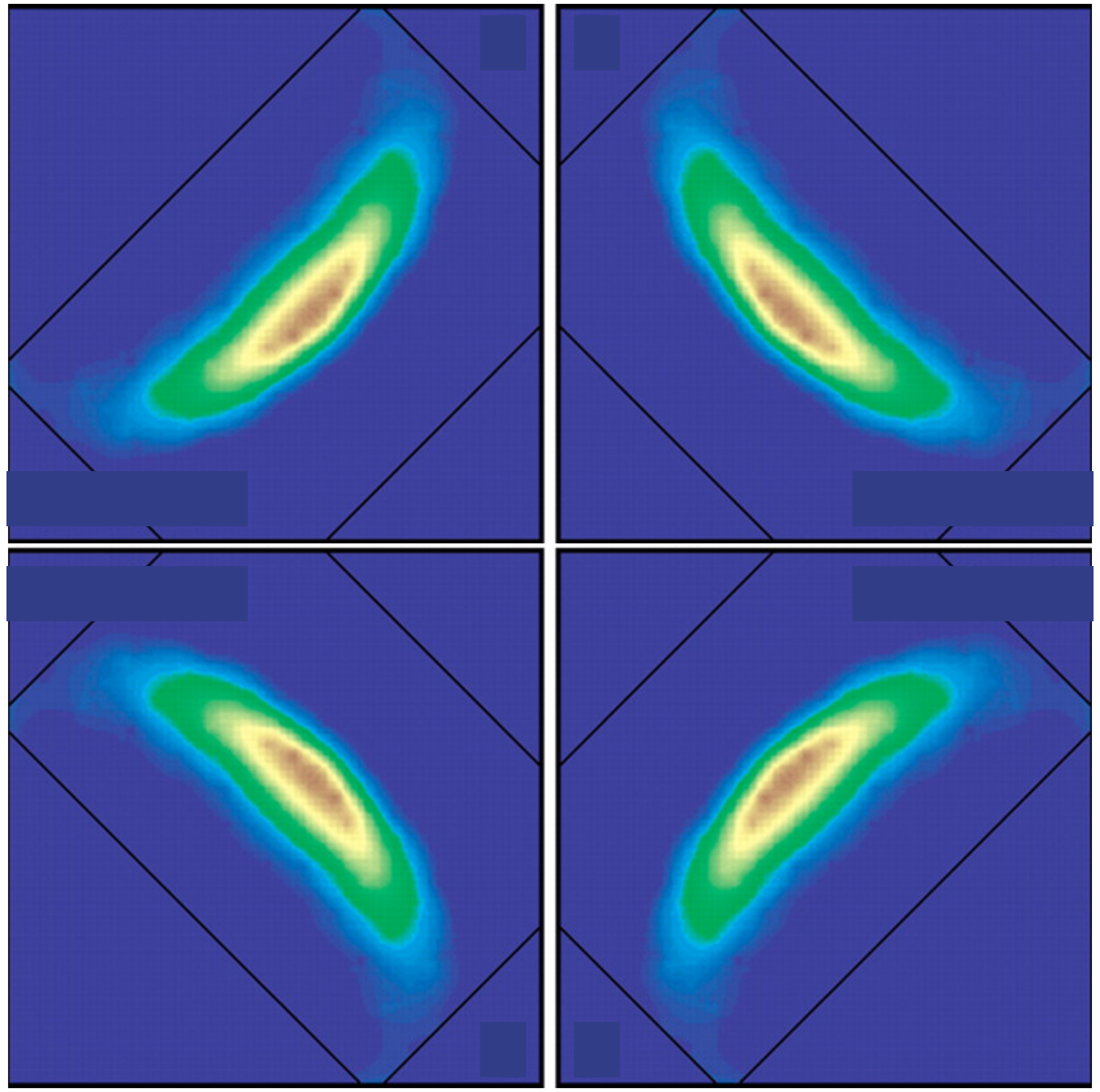
"Fermi arc" spectral functions



FL* Hamiltonian: $[(\mathrm{SU}(2)_1\times\mathrm{SU}(2)_S)/\mathbb{Z}_2]\times\mathrm{U}(1)_{\mathrm{em}}$ is broken to $\mathrm{U}(1)_{\mathrm{diag}}$ by Higgs condensate Φ:

$$H = -\sum_{i,j} t_{ij} c^\dagger_{i;\alpha} c_{j;\alpha} + \sum_{i,j} t_{1,ij} \Psi^\dagger_{1i;\alpha} \Psi_{1j;\alpha} + \sum_i \Phi (c^\dagger_{i;\alpha} \Psi_{1i;\alpha} + \Psi^\dagger_{1i;\alpha} c_{i;\alpha})$$

Photoemission at small p

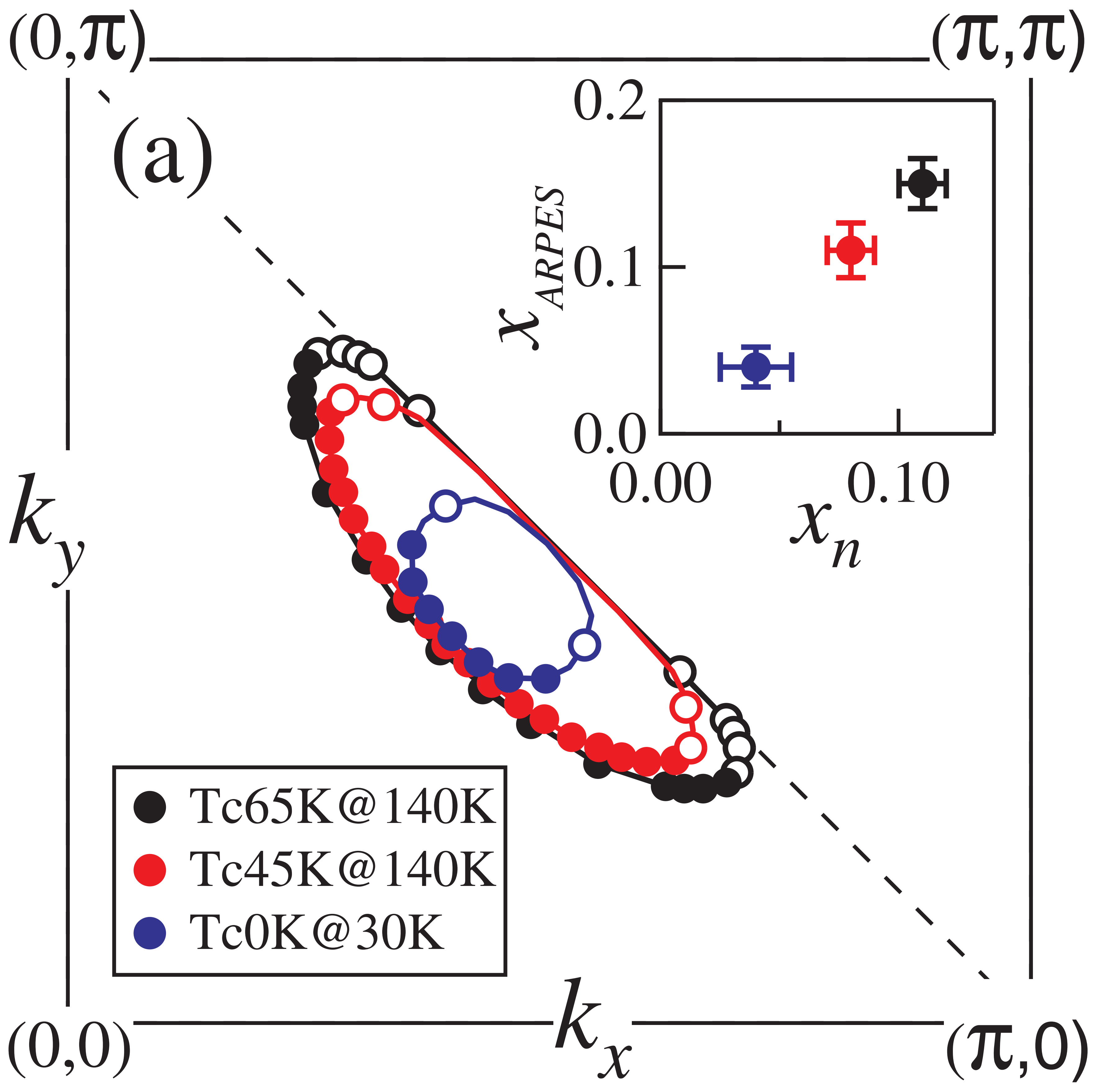


$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

"Fermi arcs"

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

Photoemission at small p



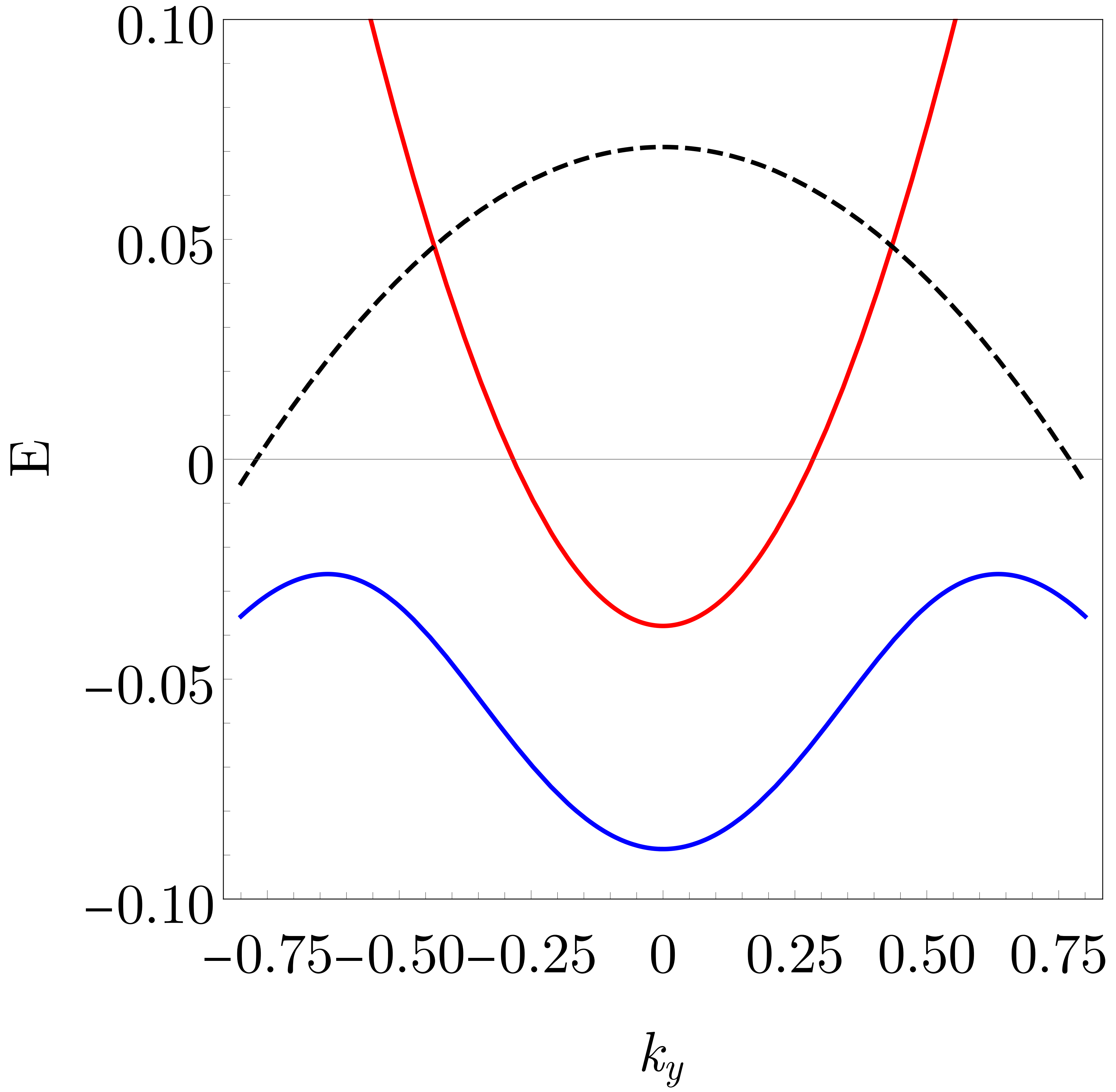
"Fermi pockets"

Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).

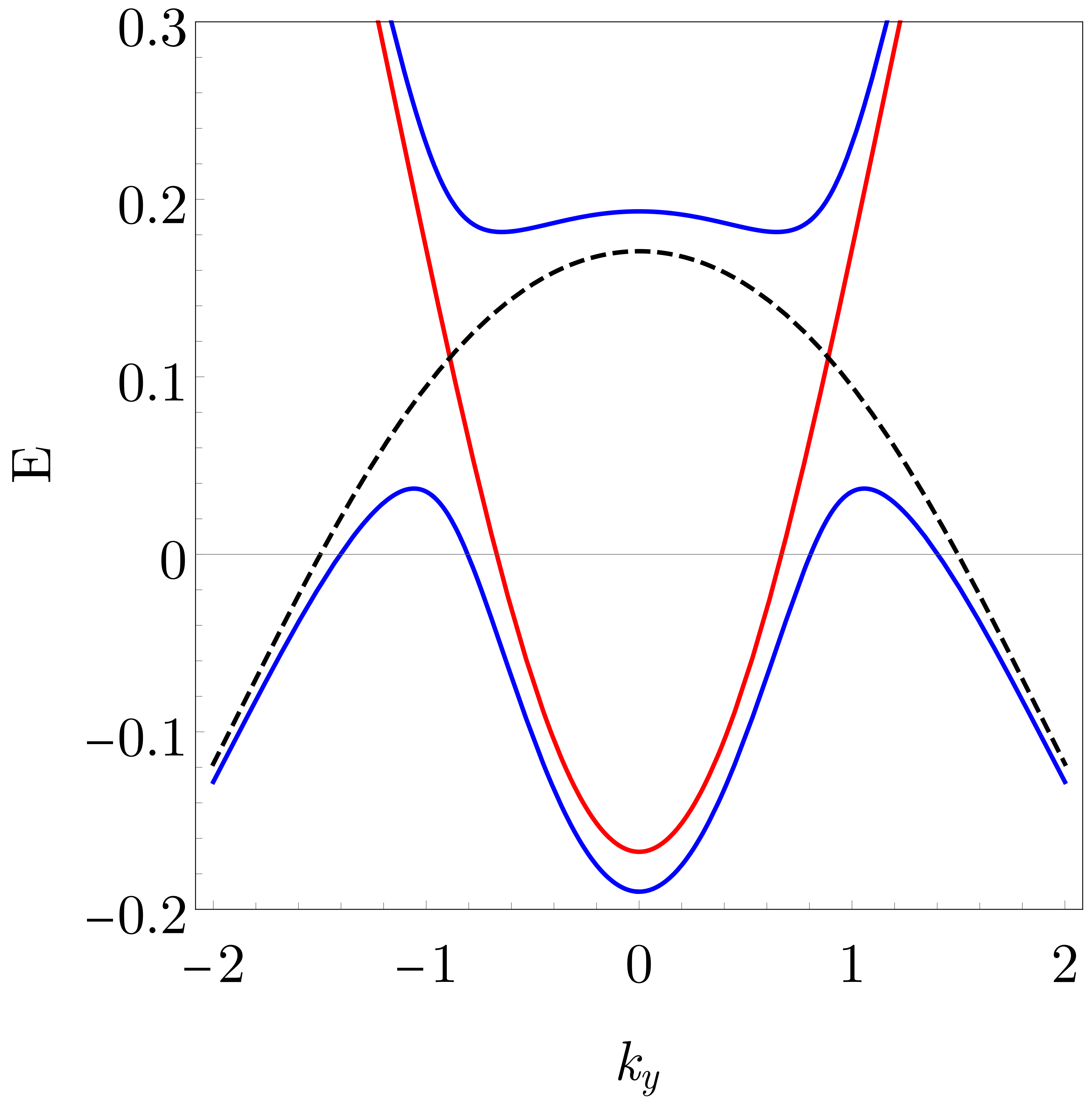
FL* in a ***one-band*** model

Electronic dispersion

Anti-node: $k_x = \pi$



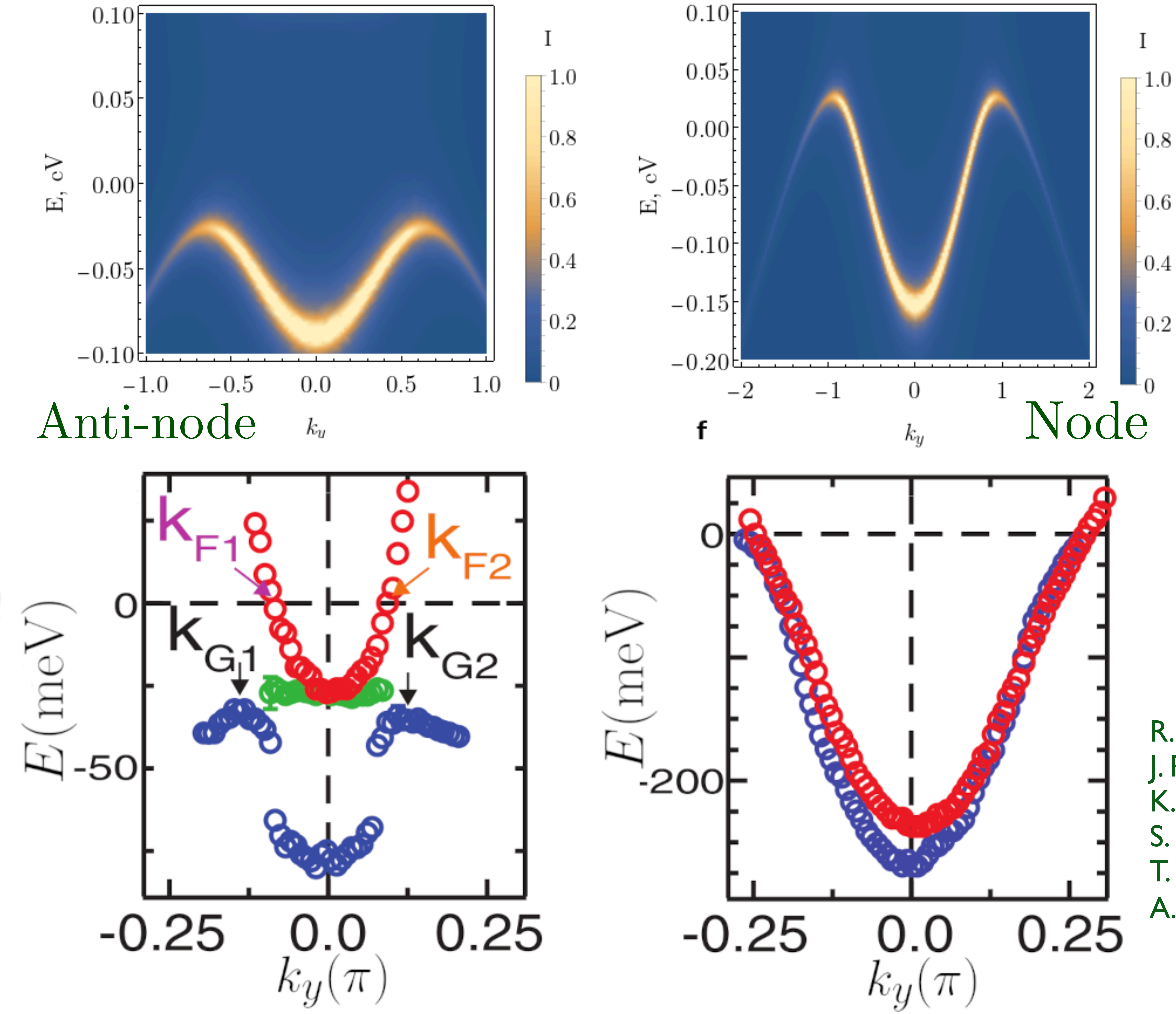
Node: $k_x = 2$



FL* Hamiltonian: $[(\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_S)/\mathbb{Z}_2] \times \mathrm{U}(1)_{\mathrm{em}}$ is broken to $\mathrm{U}(1)_{\mathrm{diag}}$ by Higgs condensate Φ:

$$H = -\sum_{i,j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + \sum_{i,j} t_{1,ij} \Psi^\dagger_{1i\sigma} \Psi_{1j\sigma} + \sum_i \Phi \left(c^\dagger_{i\sigma} \Psi_{1i\sigma} + \Psi^\dagger_{1i\sigma} c_{i\sigma} \right)$$

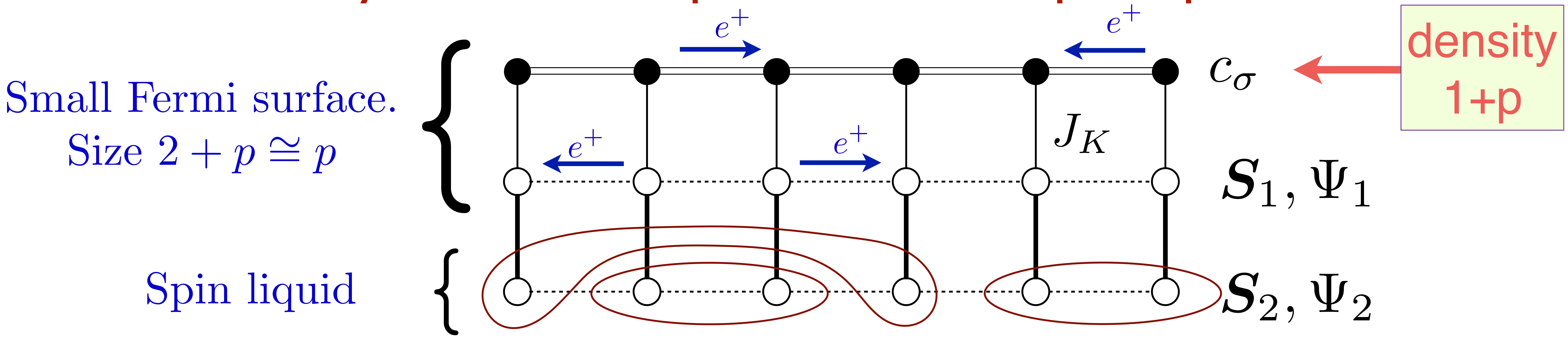
FL* in a ***one-band*** model



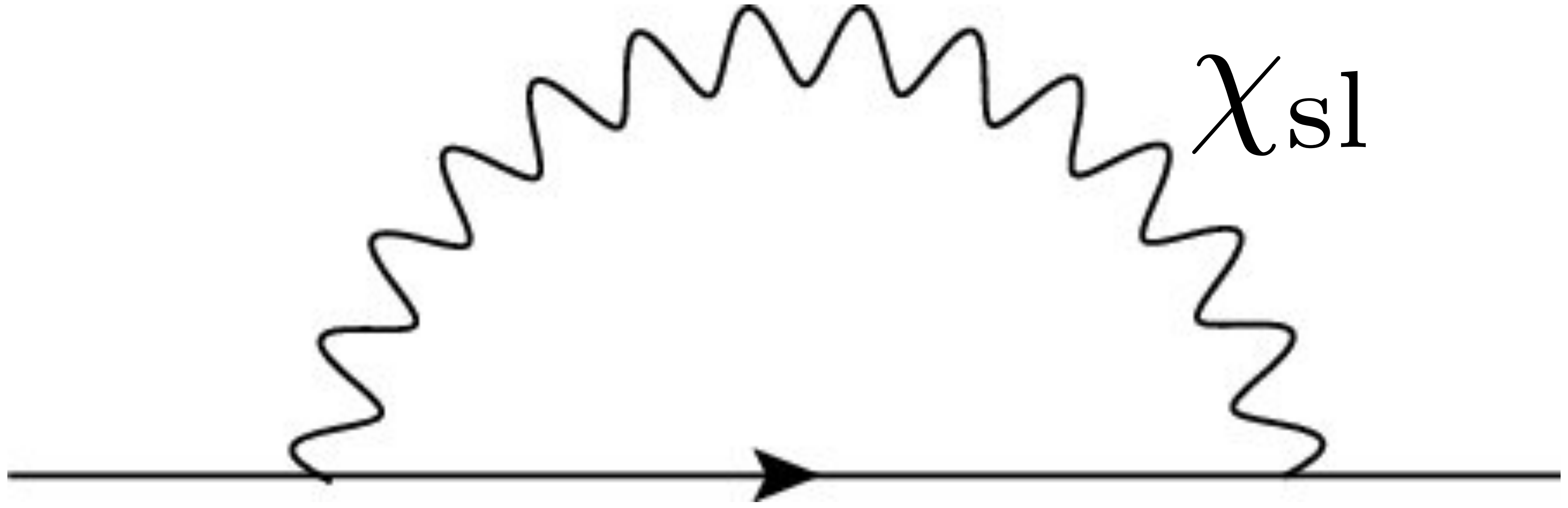
ARPES on Bi2201

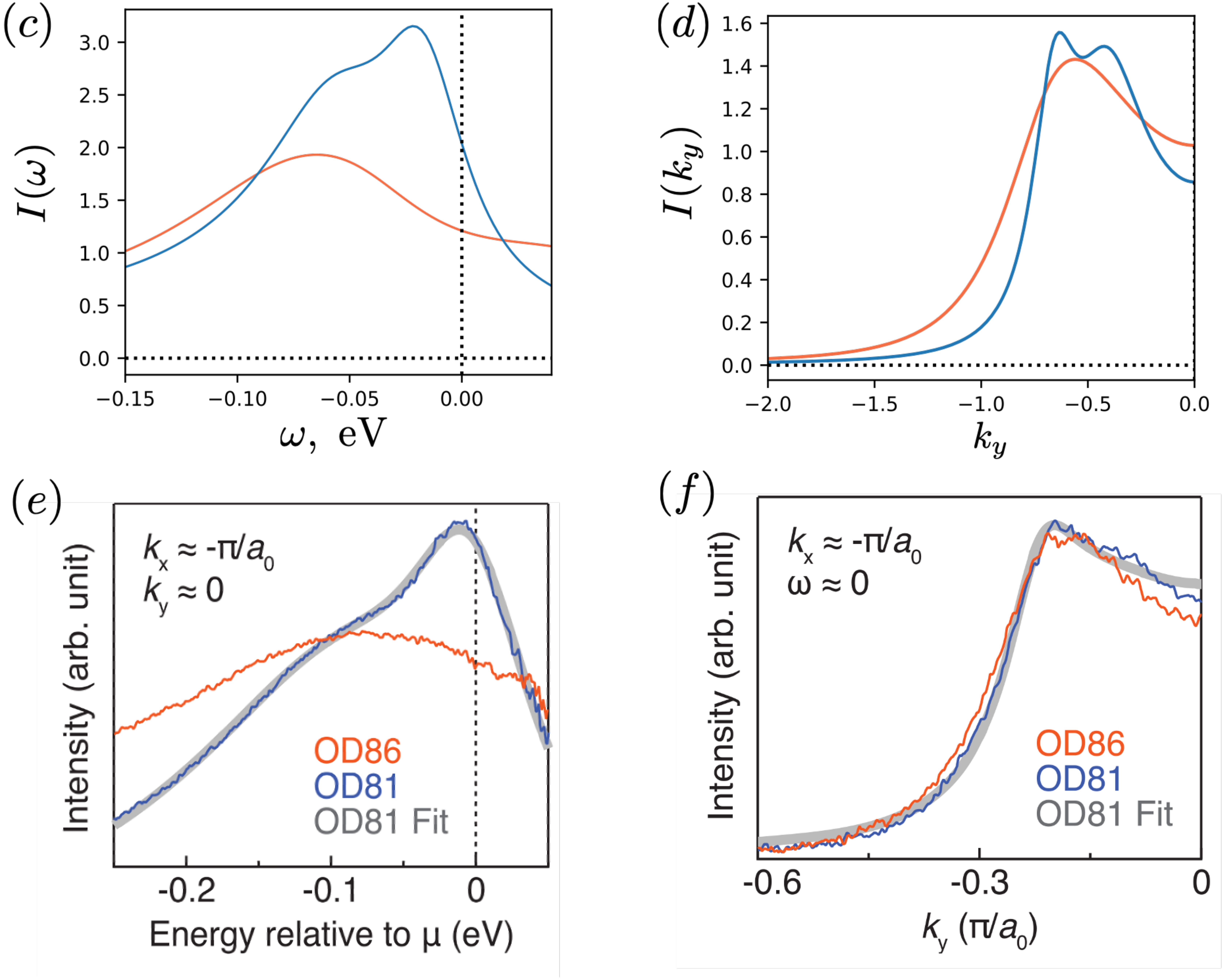
R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

Dynamic consequences of the spin liquid



The only singular gauge fluctuations are those in the spin liquid of the Ψ_2. We can compute their influence on the electronic spectrum perturbatively in the exchange couplings in terms of the dynamic spin susceptibility $\chi_{\rm sl}$.





Antinodal EDC and MDC

(c,d) Theory with SYK spin liquid in Ψ_2 layer. Similar EDC obtained by gapless $\mathbb{Z}_2$ spin liquid

(e,f) Experiments on Bi2212 by S.-D. Chen, M. Hashimoto, Y. He, D. Song, K.-J. Xu, J.-F. He, T. P. Devereaux, H. Eisaki, D.-H. Lu, J. Zaanen, and Z.-X. Shen, Science **366**, 1099 (2019).

Summary

- Probing $\mathbb{Z}_2$ spin liquid with Rydberg atoms:
  Two-state Rydberg atoms on the kagome and ruby lattices can be written *exactly* as a $\mathbb{Z}_2$ gauge theory. Evidence for deconfined phase of $\mathbb{Z}_2$ gauge theory on the ruby lattice.

- Ancilla theory of FL* for the pseudogap metal of the cuprate high temperature superconductors:
  Don't fractionalize the mobile electron, but fractionalize the 'paramagnon rotor' into 'ancilla qubits'.
  Predicts electronic spectra in good agreement with observations in *both* nodal and anti-nodal regions.