Building strange metals from SYK models

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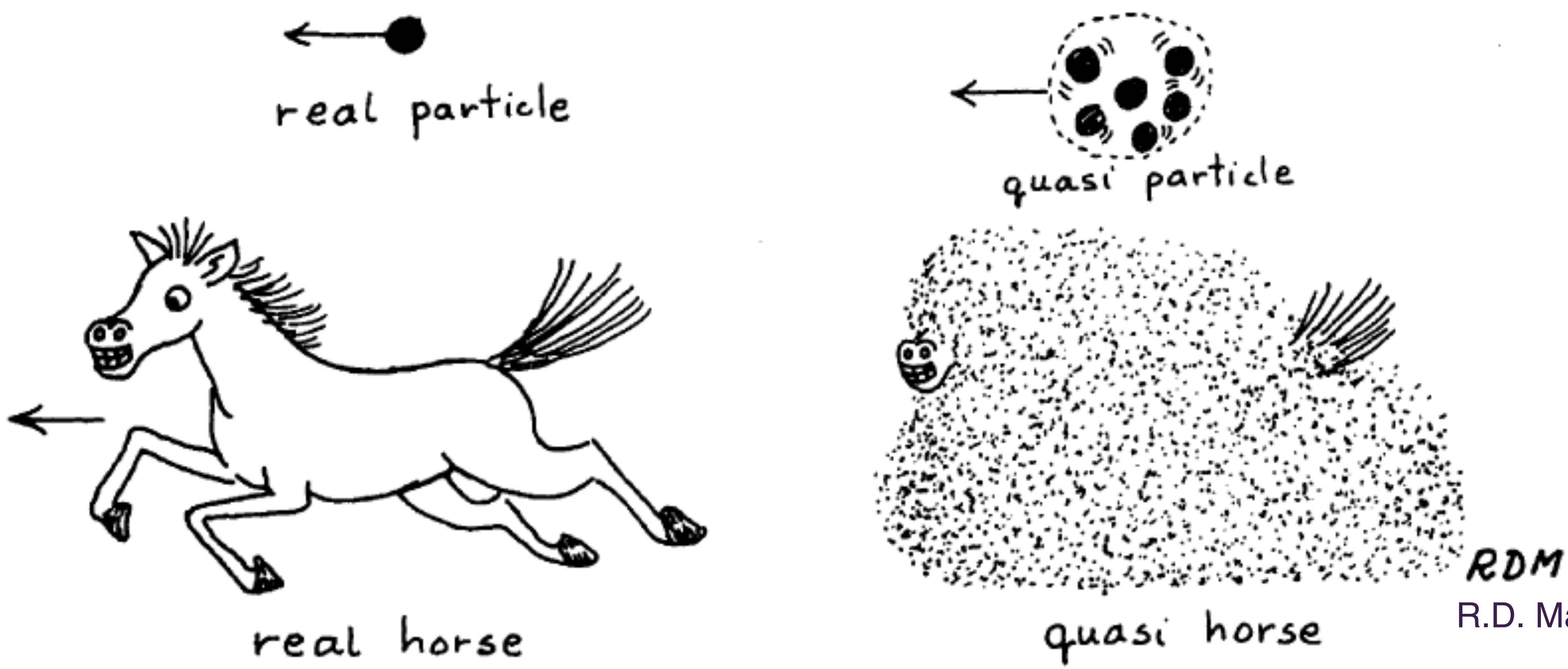
Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.



R.D. Mattuck

Quantum matter with quasiparticles:

The quasiparticle idea is the key reason for the many successes of quantum condensed matter physics:

- Fermi liquid theory of metals, insulators, semiconductors
- Theory of superconductivity (pairing of quasiparticles)
- Theory of disordered metals and insulators (diffusion and localization of quasiparticles)
- Theory of metals in one dimension (collective modes as quasiparticles)
- Theory of the fractional quantum Hall effect (quasiparticles which are `fractions' of an electron)

Quantum matter with quasiparticles:

- **Quasiparticles are additive excitations:**
  The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.

Quantum matter with quasiparticles:

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\rm eq} \sim \frac{\hbar E_F}{(k_B T)^2} \quad , \quad \text{as } T \to 0,$$

where E_F is the Fermi energy.

1. Metal with quasiparticles
   Random matrix model of a `quantum dot'

2. Metal without quasiparticles
   SYK model of a `quantum dot'

3. High temperature superconductors
   and strange metals.

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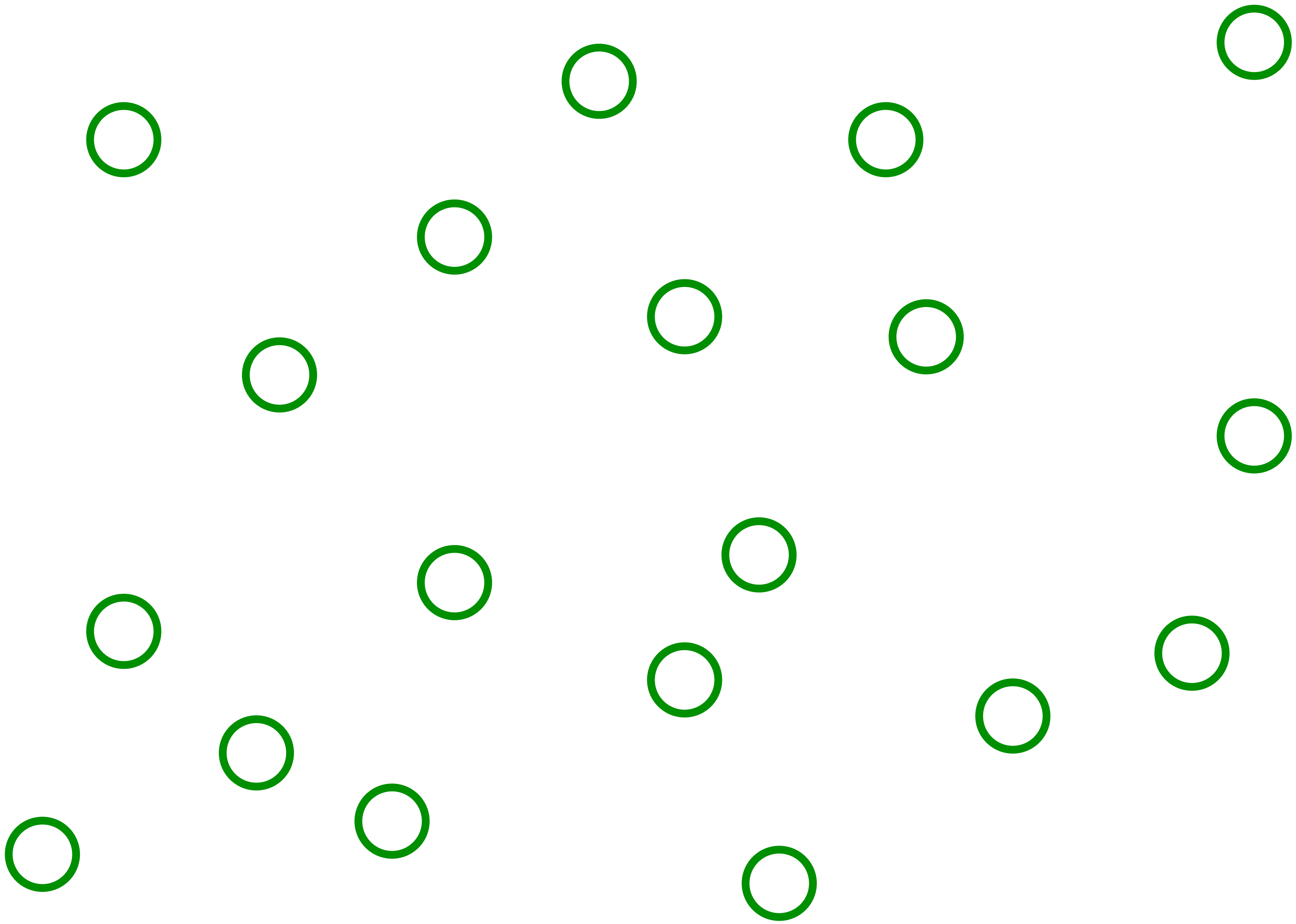
A simple model of a metal with quasiparticles



Pick a set of random positions

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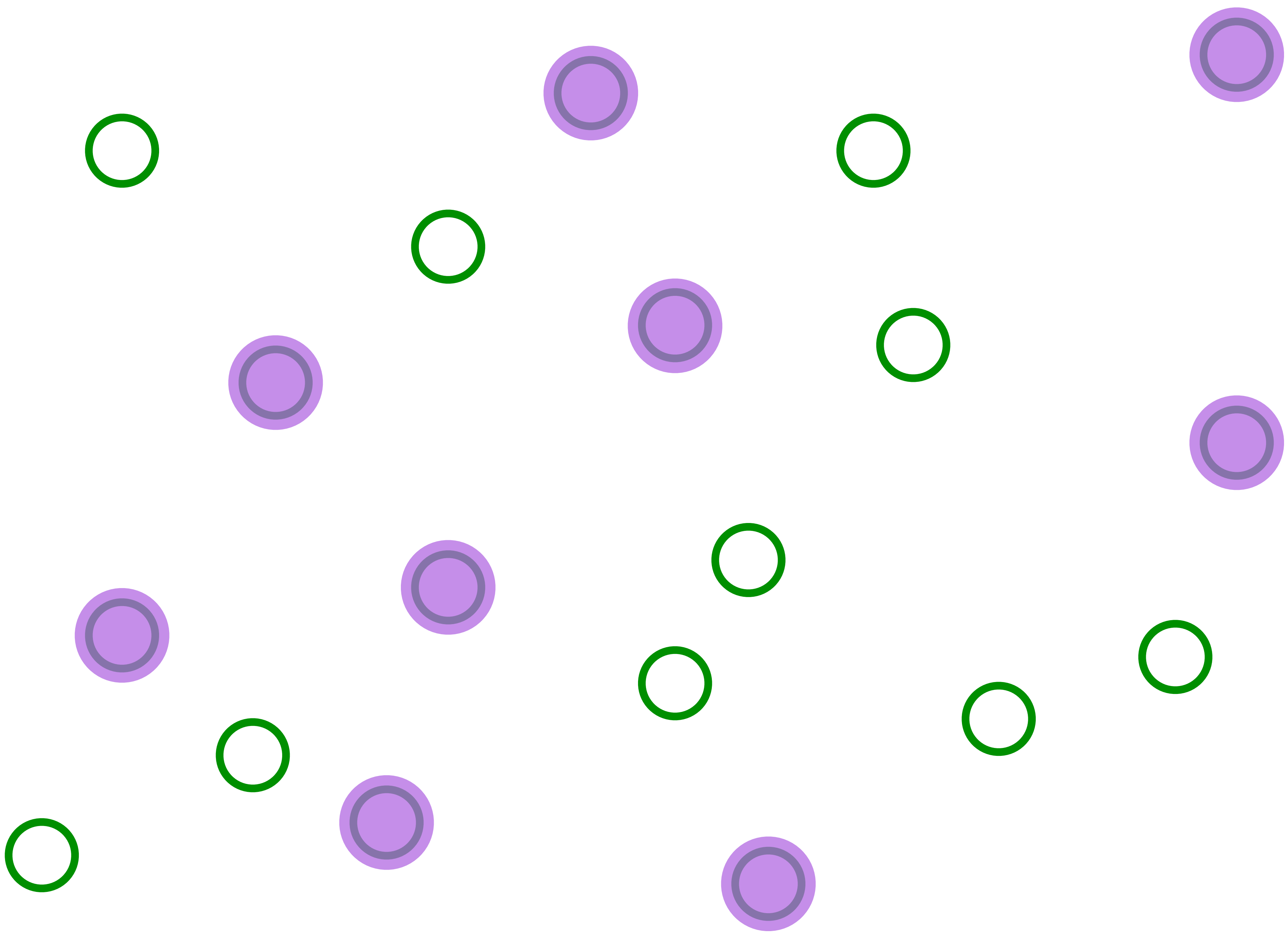

Place electrons randomly on some sites

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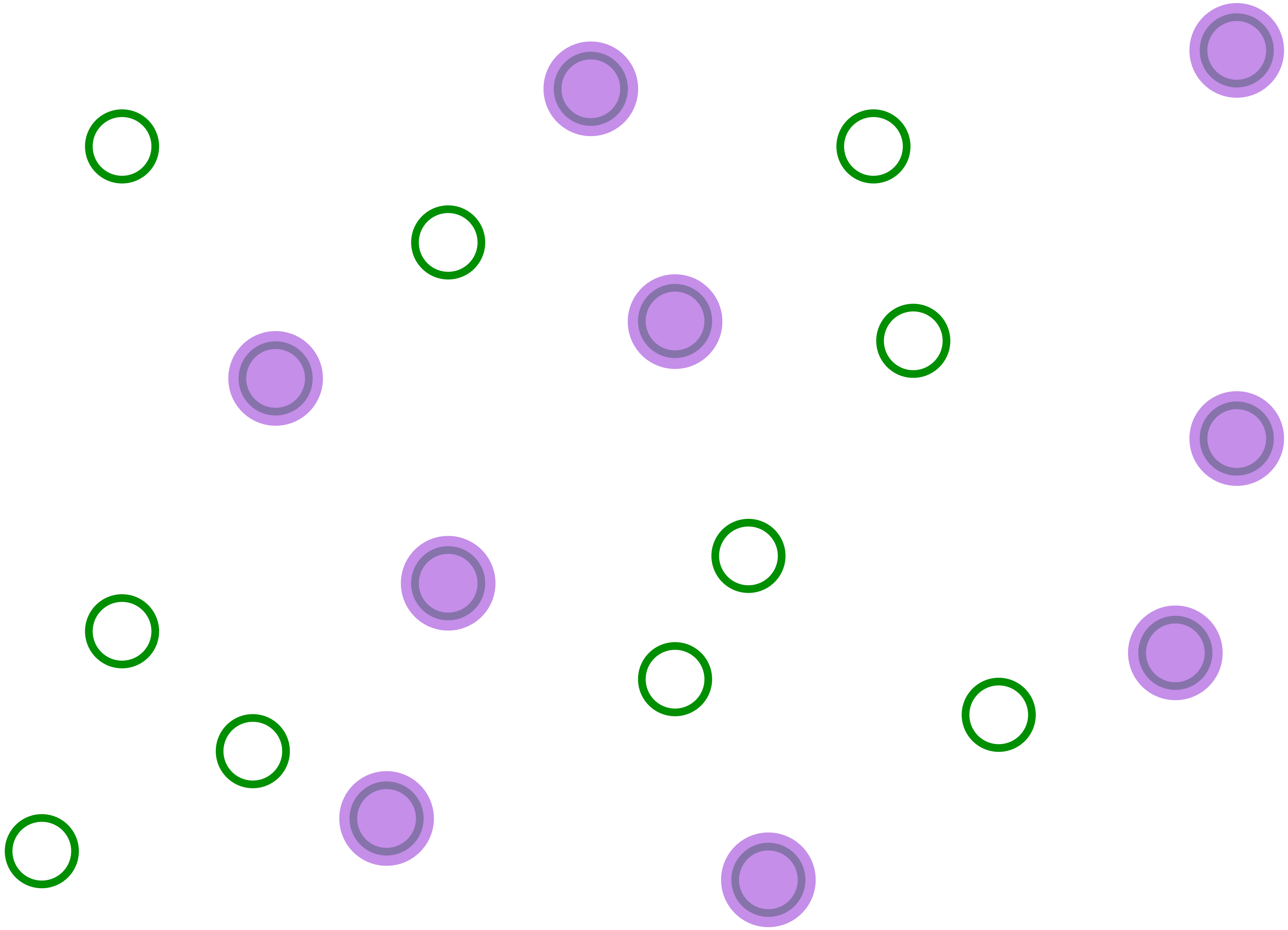

Electrons move one-by-one randomly

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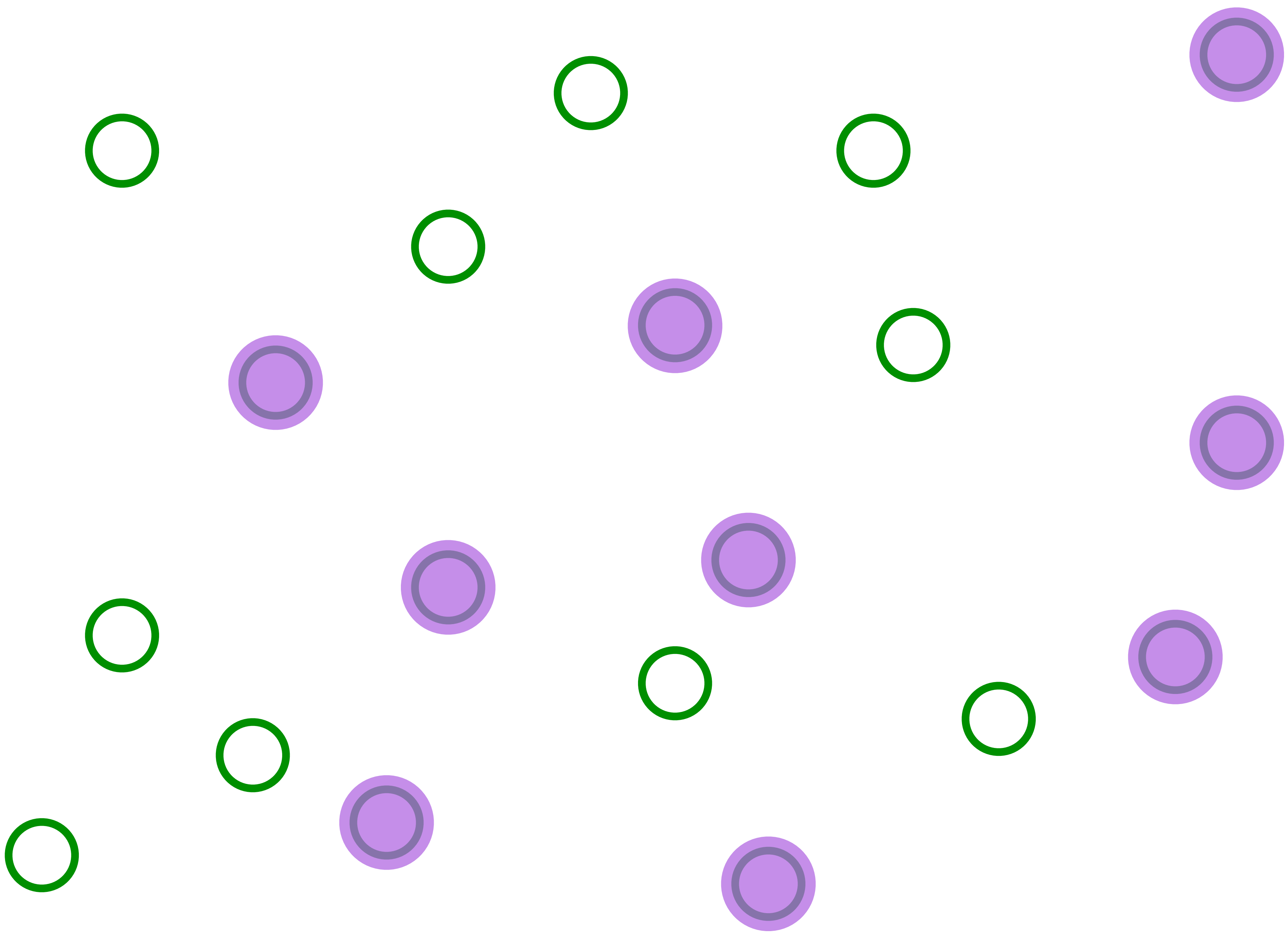

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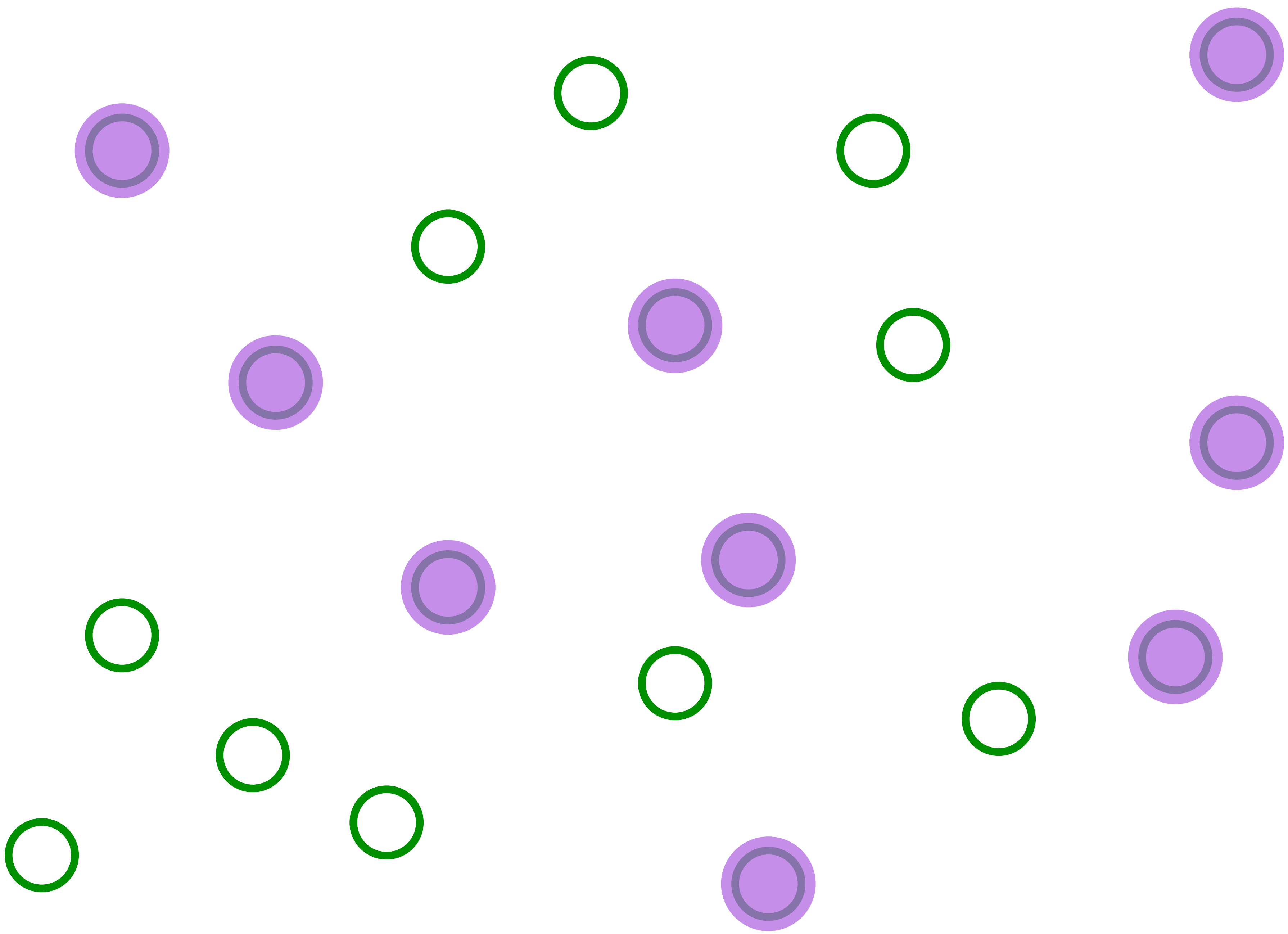

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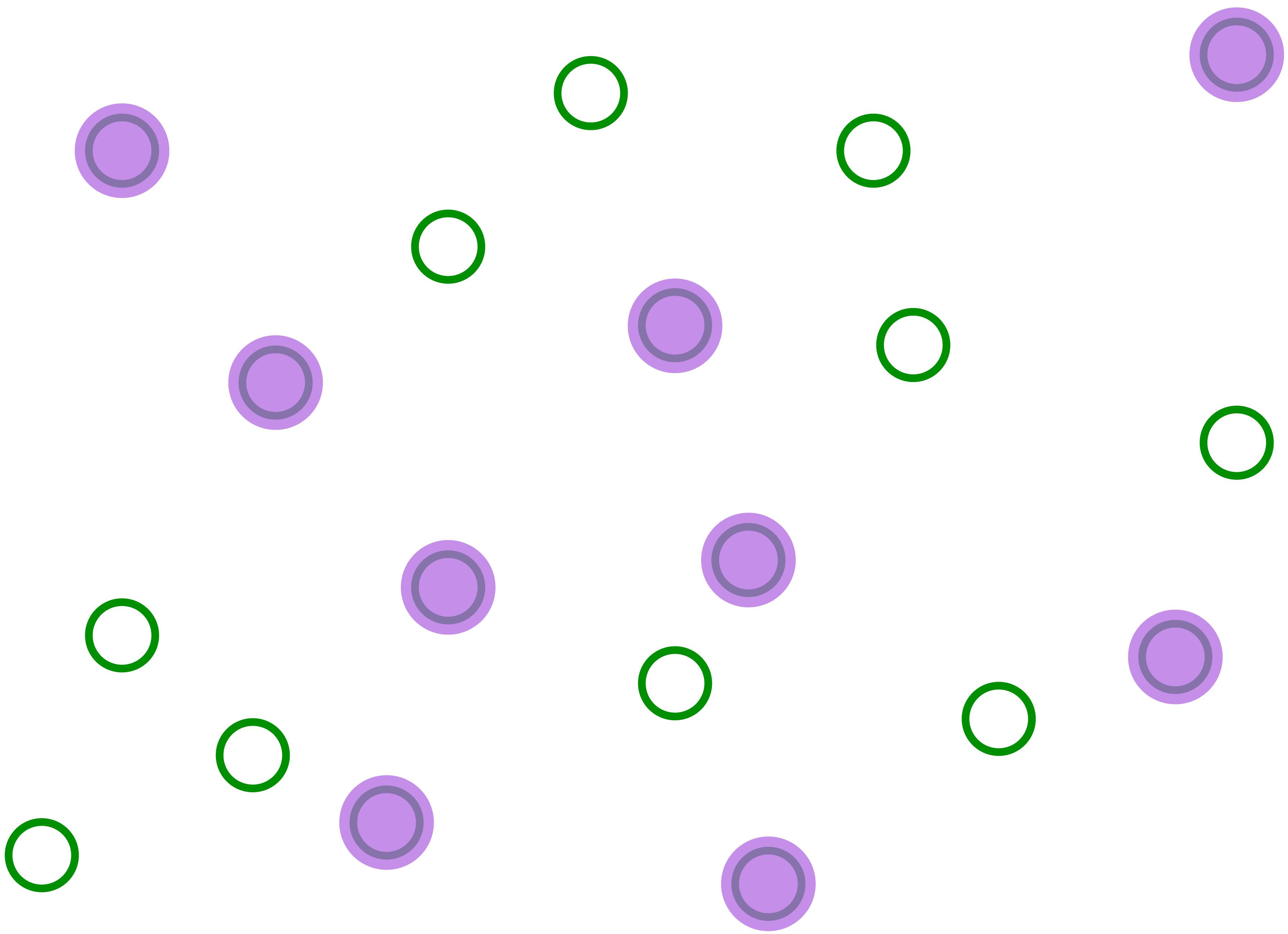

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A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

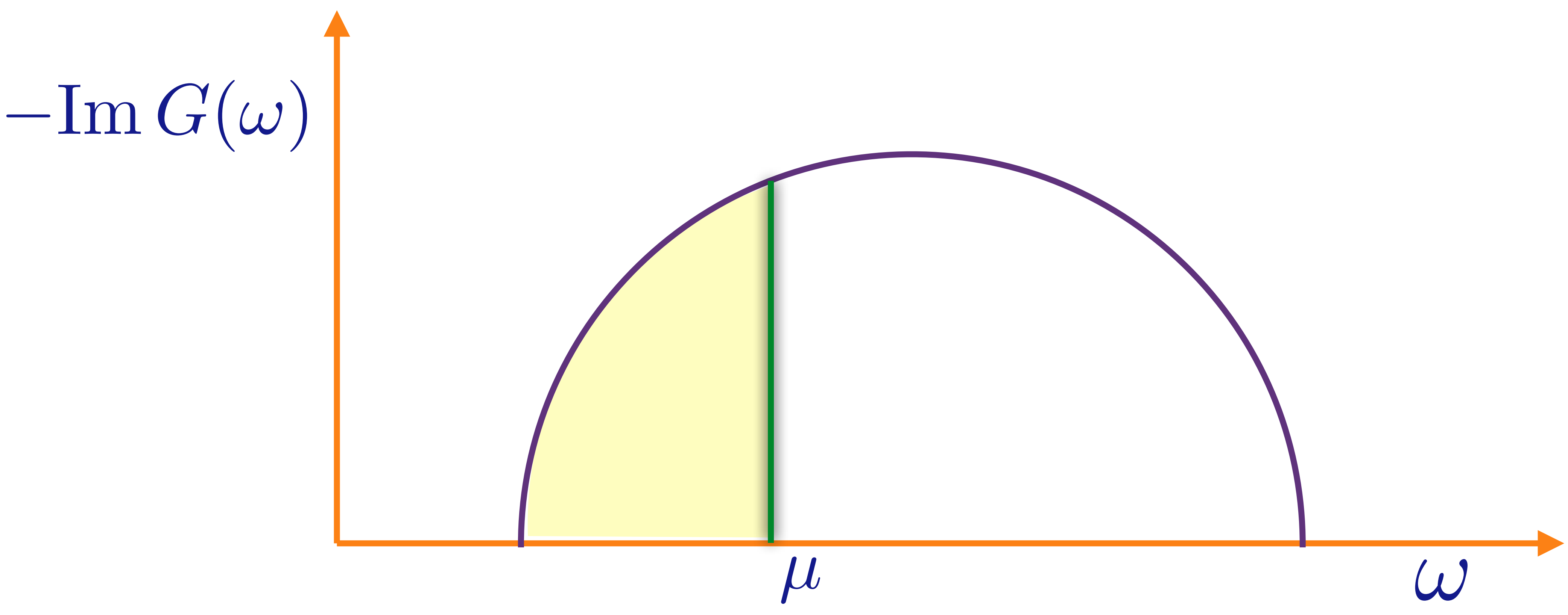
Fermions occupying the eigenstates of a *N* x *N* random matrix

Infinite-range model with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$G(\omega)$ can be determined by solving a quadratic equation.



Infinite-range model with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

$J_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$. We compute the lifetime of a quasiparticle, τ_α, in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamitonian with energy ε_α. By Fermi's Golden rule, for ε_α at the Fermi energy

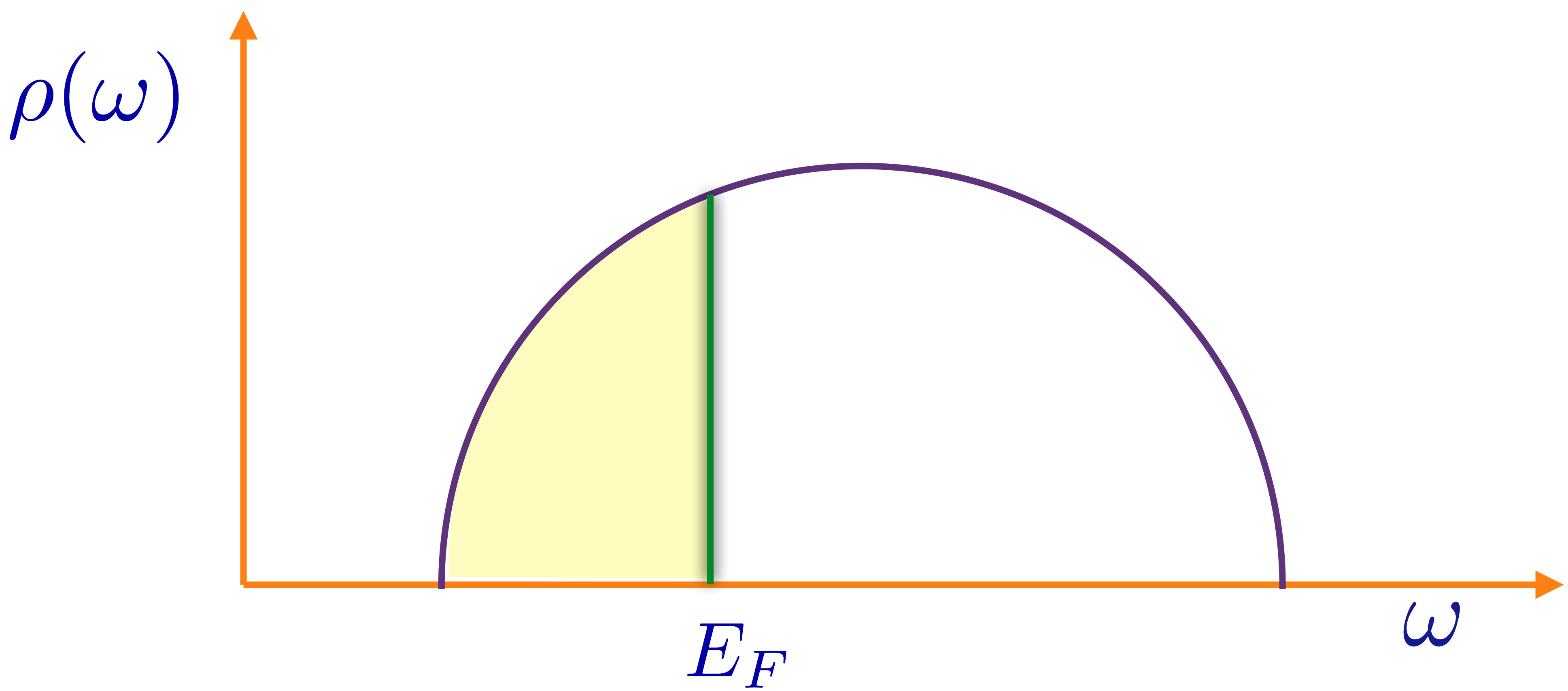
$$\begin{aligned}\frac{1}{\tau_\alpha} &= \pi U^2 \rho_0^2 \int d\varepsilon_\beta d\varepsilon_\gamma d\varepsilon_\delta f(\varepsilon_\beta)(1 - f(\varepsilon_\gamma))(1 - f(\varepsilon_\delta))\delta(\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_\gamma - \varepsilon_\delta) \\ &= \frac{\pi^3 U^2 \rho_0^2}{4} T^2\end{aligned}$$

where ρ_0 is the density of states at the Fermi energy.

Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The density of states is $\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$.



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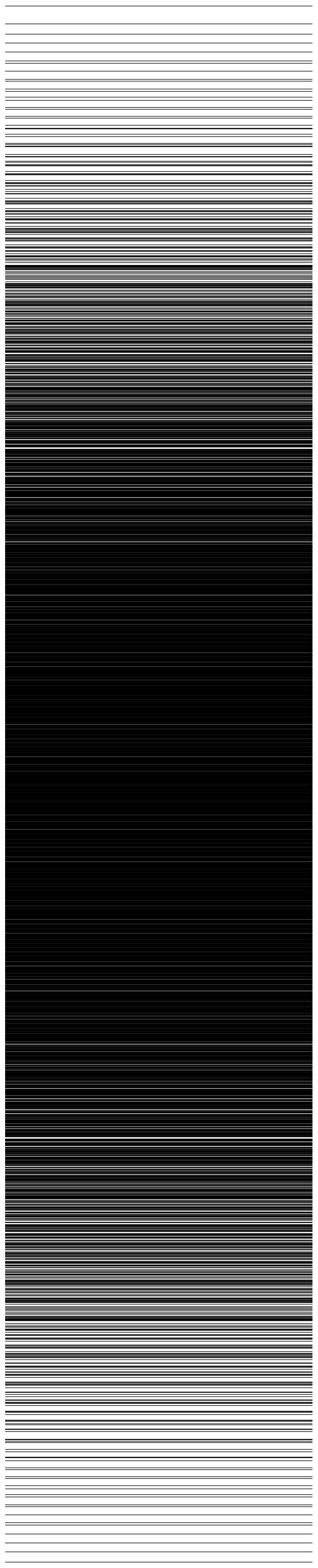

Many-body level spacing $\sim 2^{-N}$

Quasiparticle excitations with spacing $\sim 1/N$

There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

where $n_\alpha = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_α have a level spacing $\sim 1/N$.

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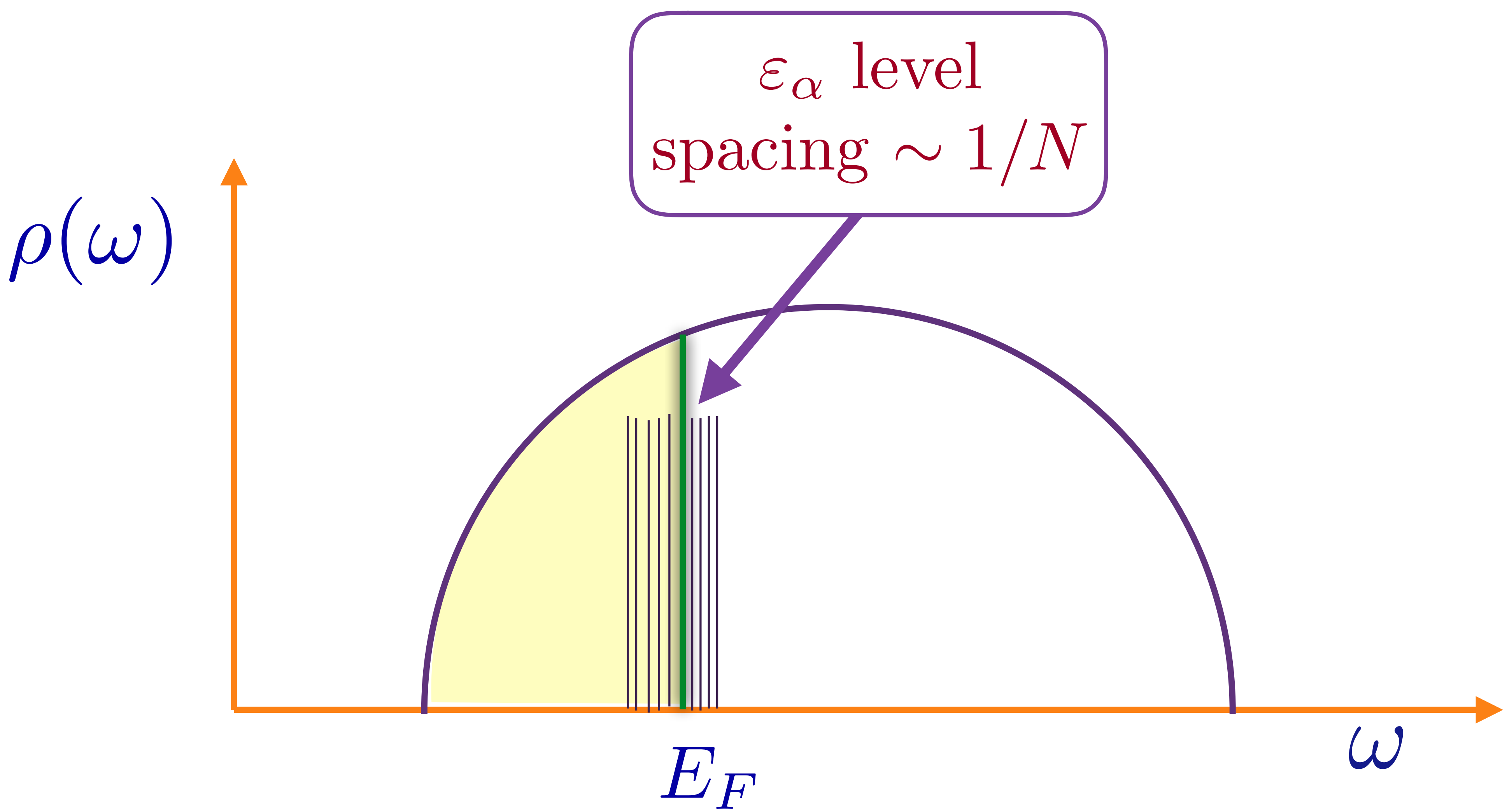

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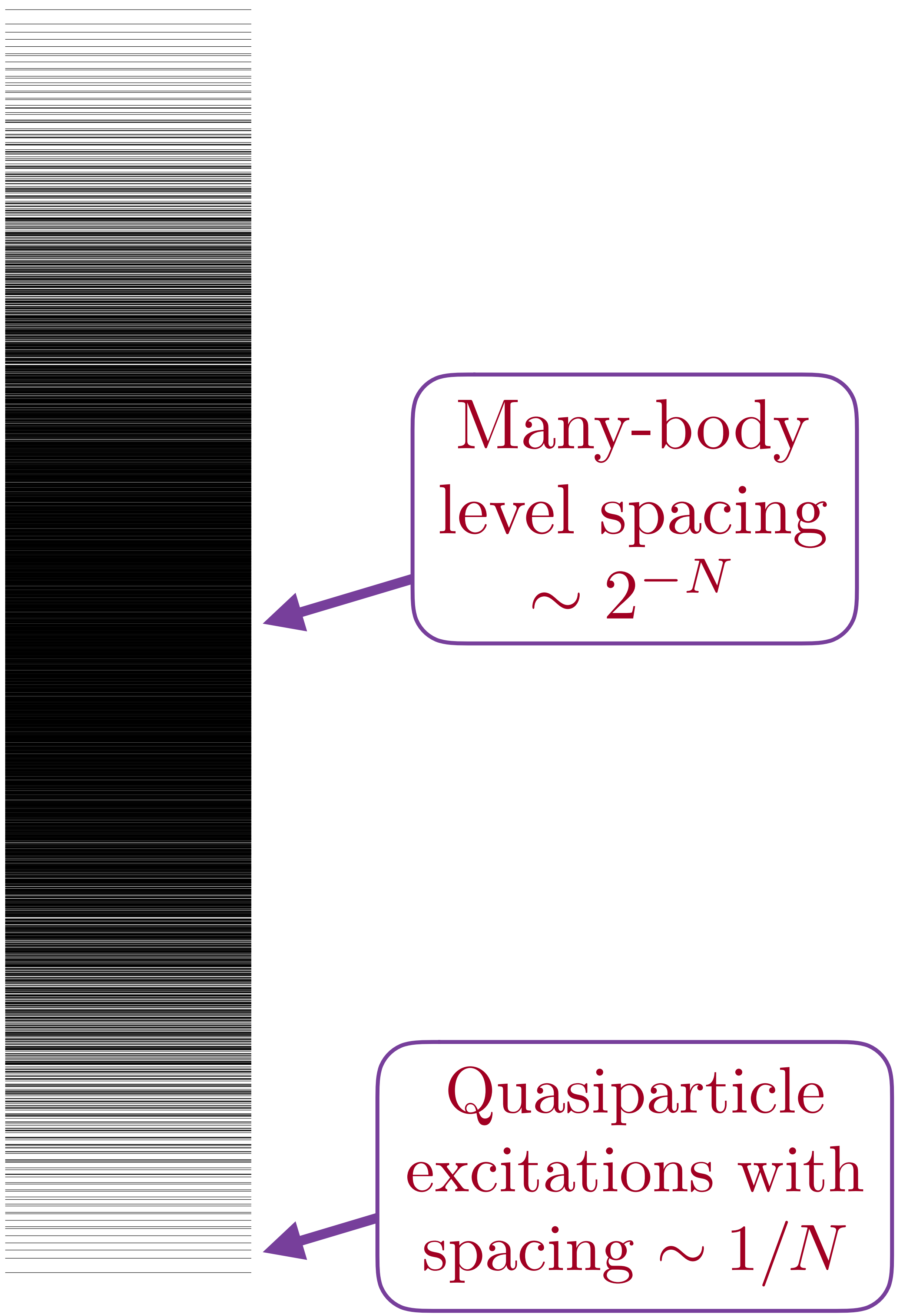

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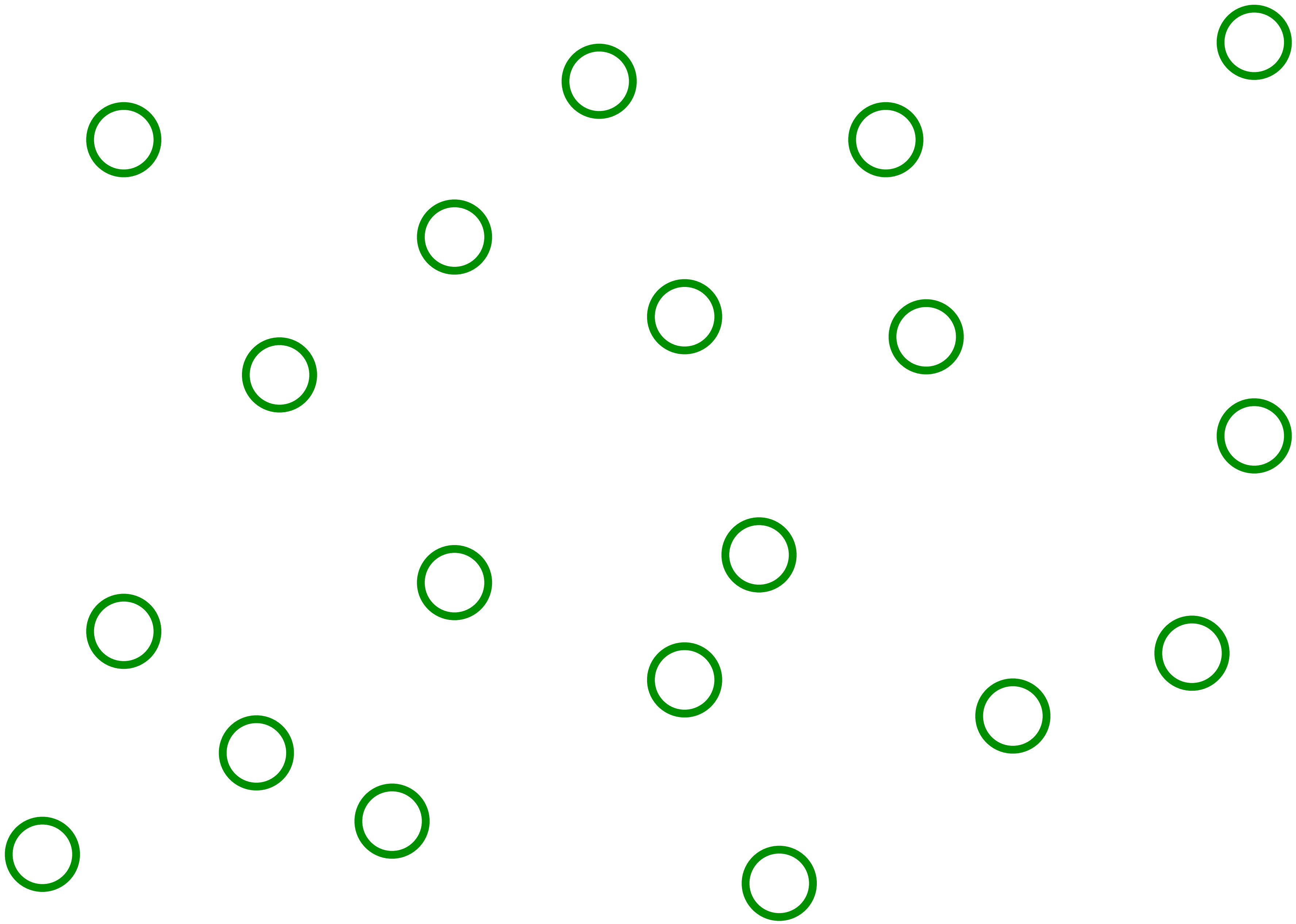
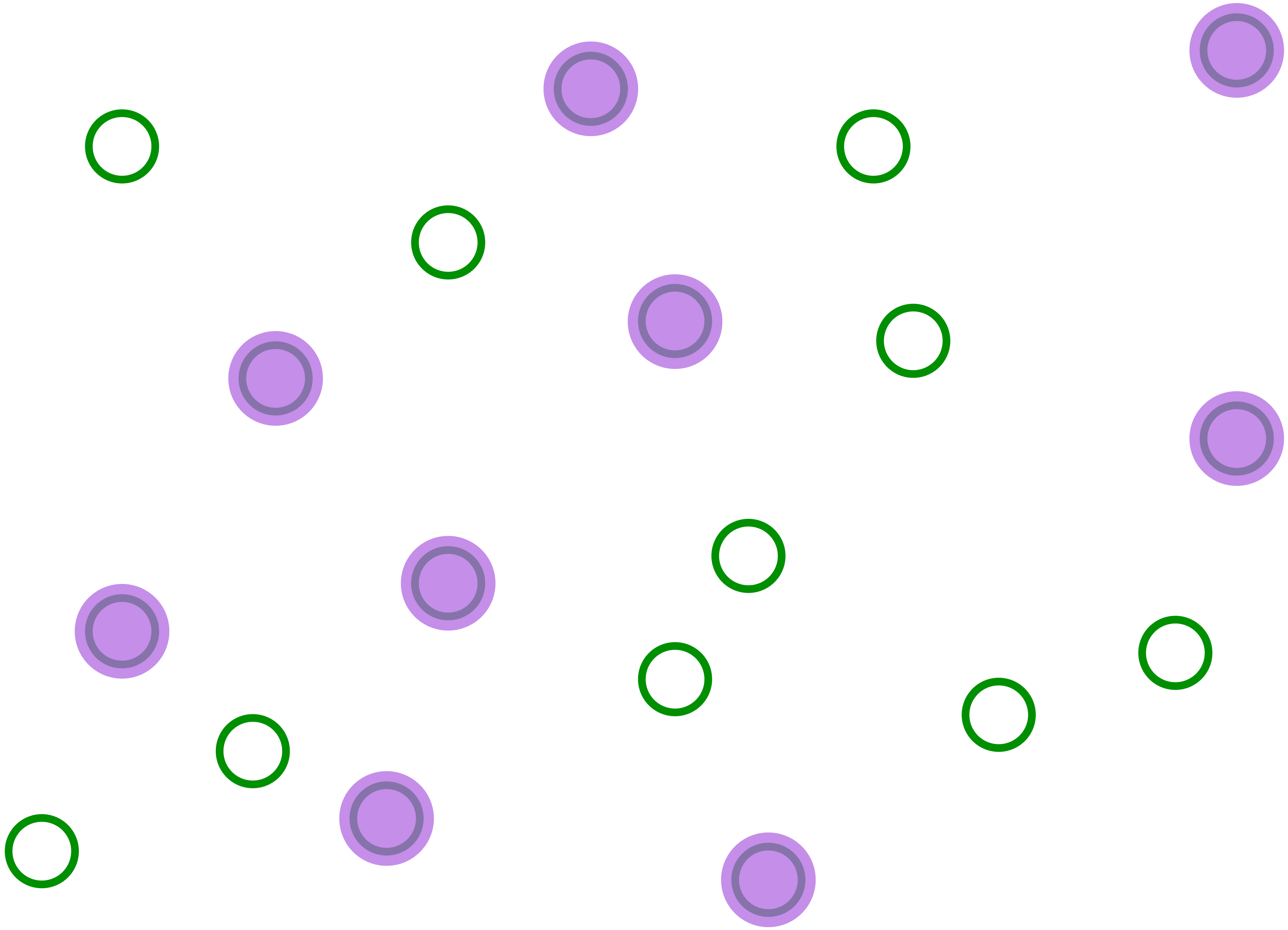
The Sachdev-Ye-Kitaev (SYK) model



Pick a set of random positions

The SYK model



Place electrons randomly on some sites

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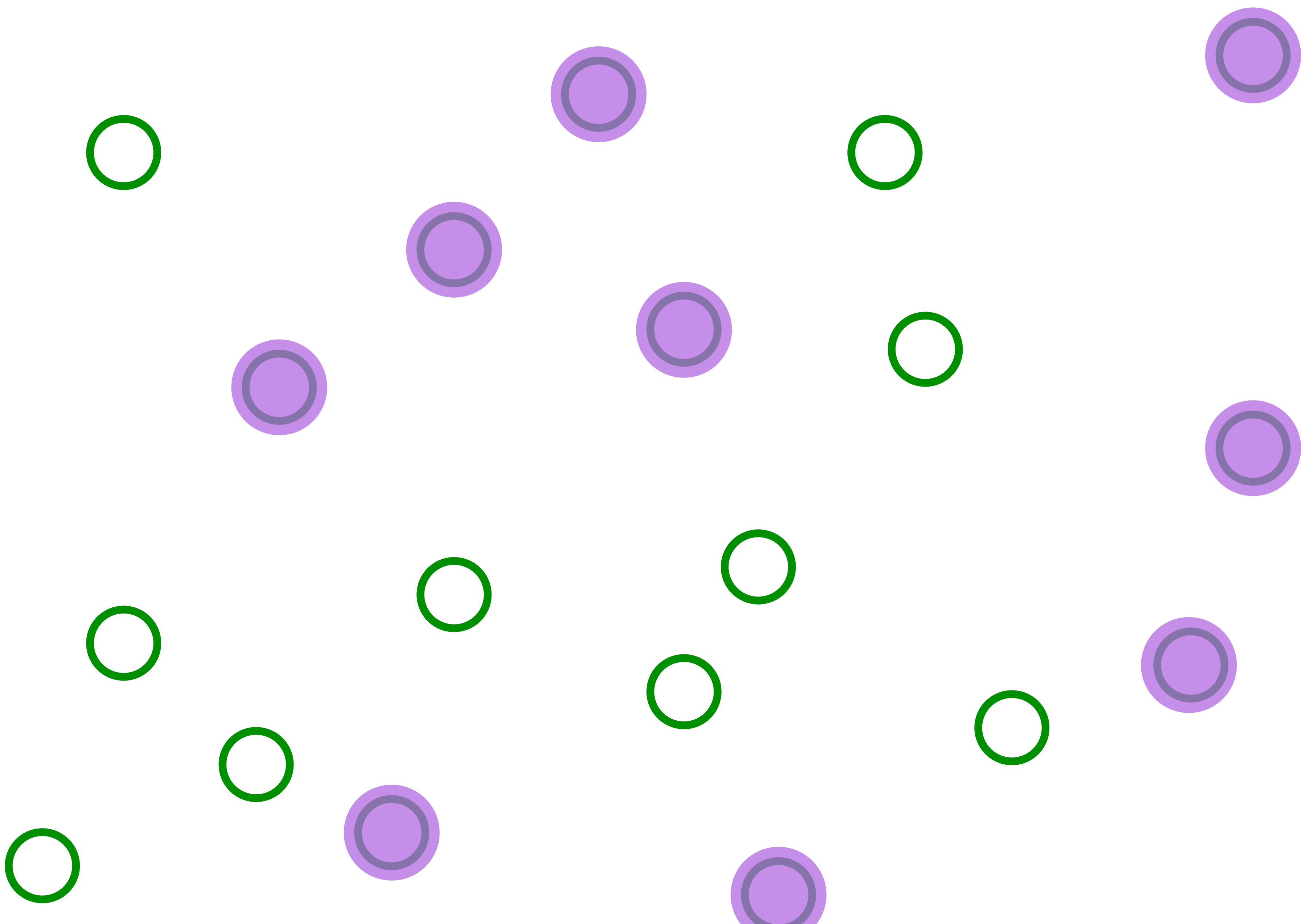
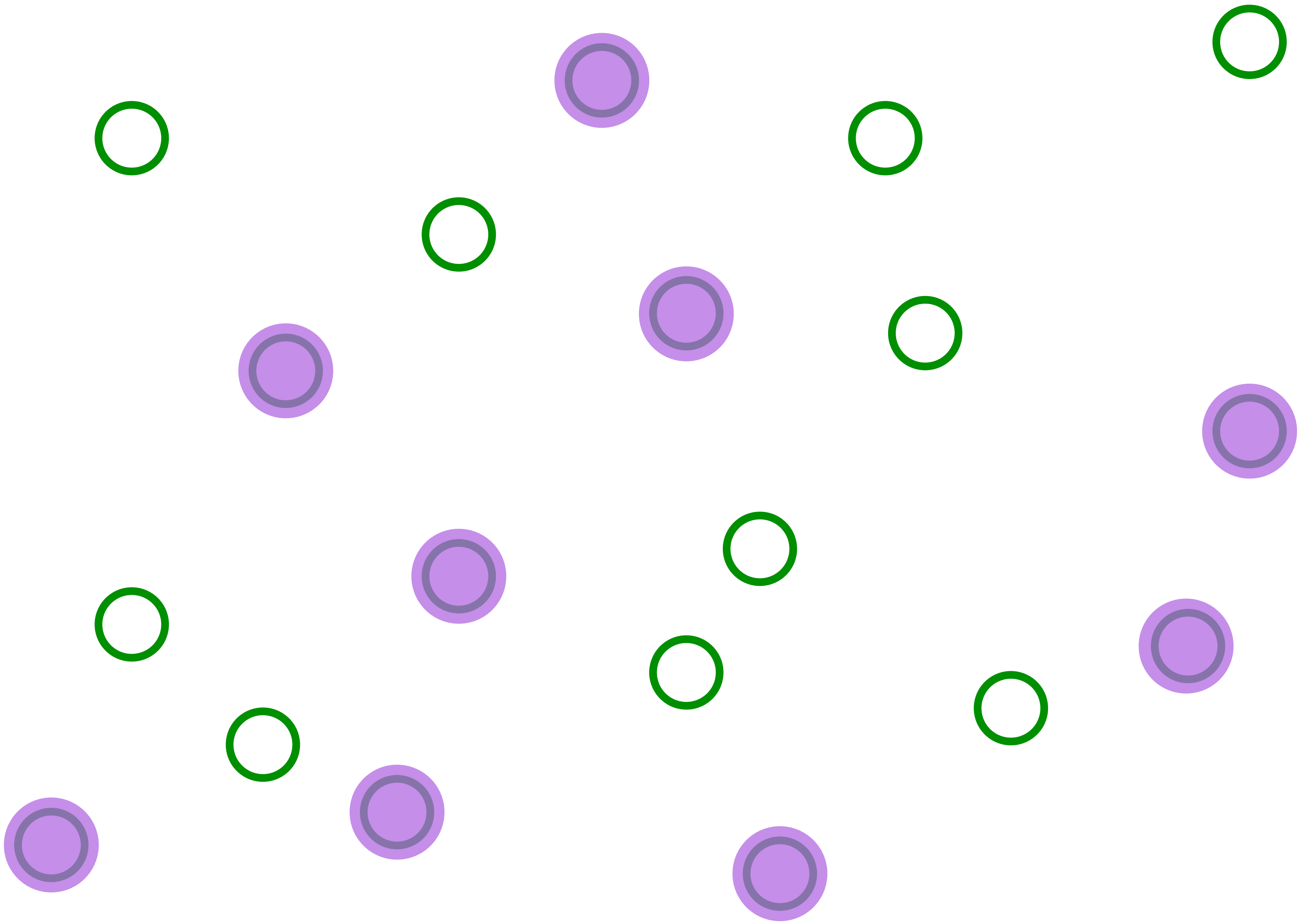


Entangle electrons pairwise randomly

The SYK model



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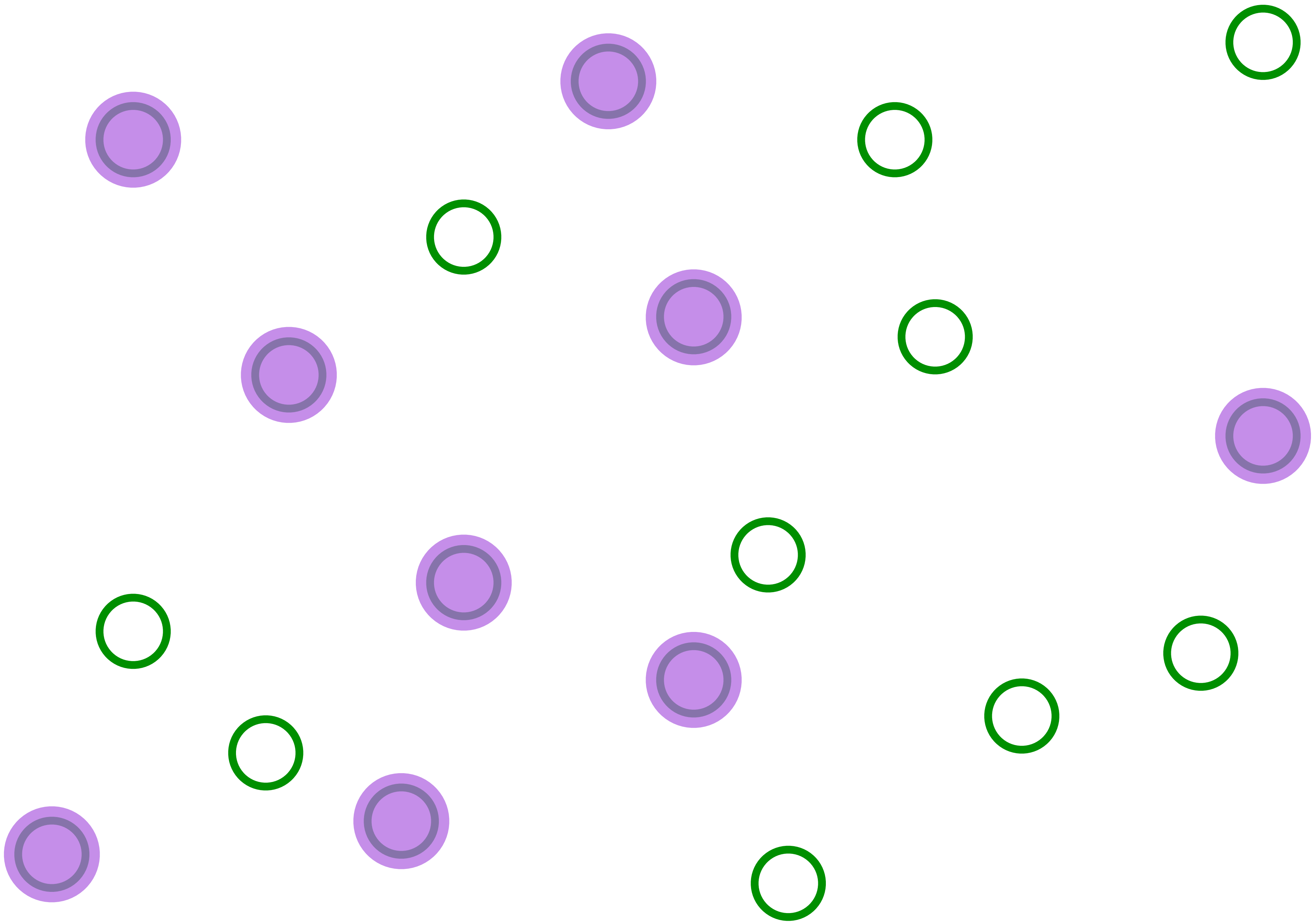

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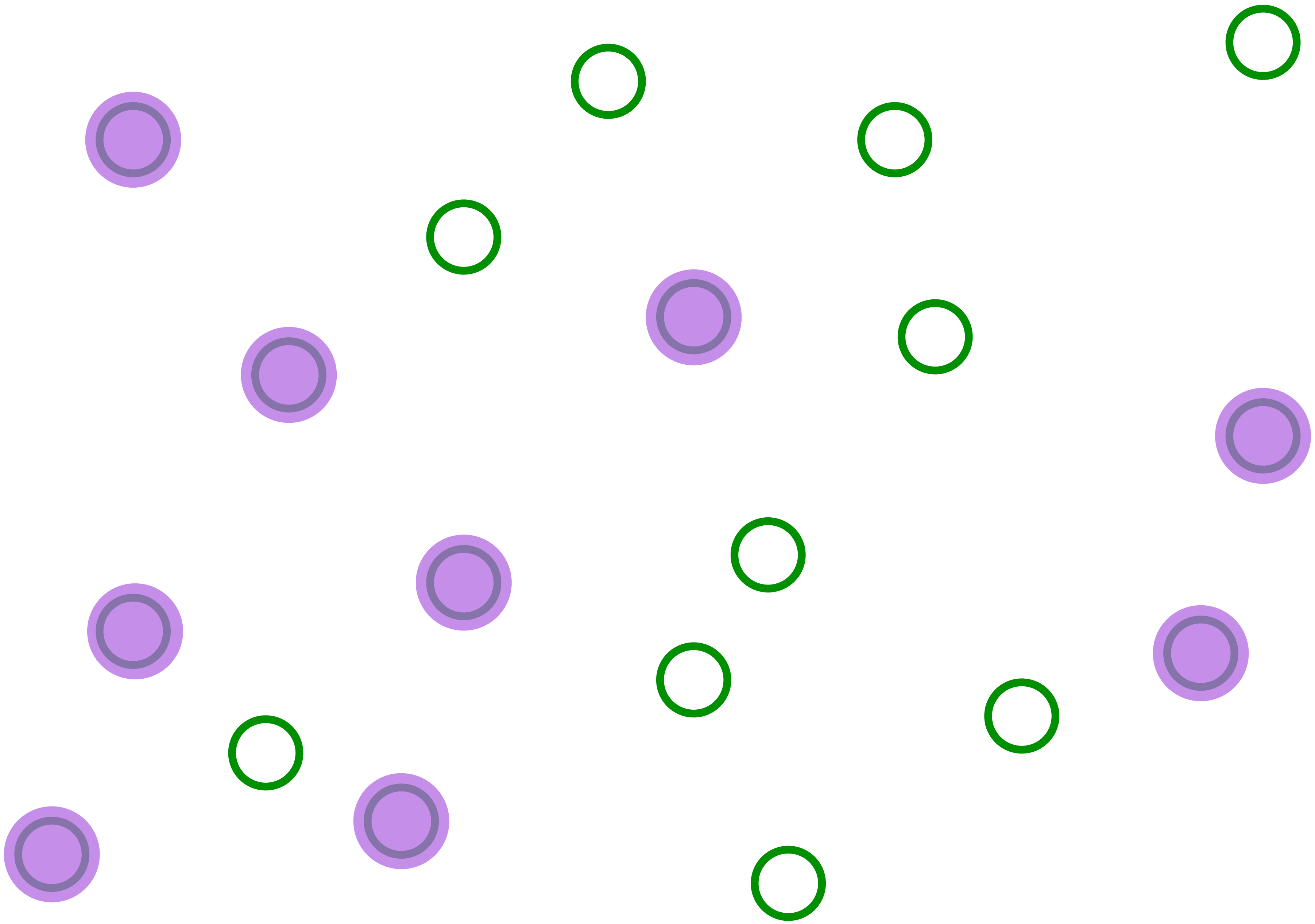

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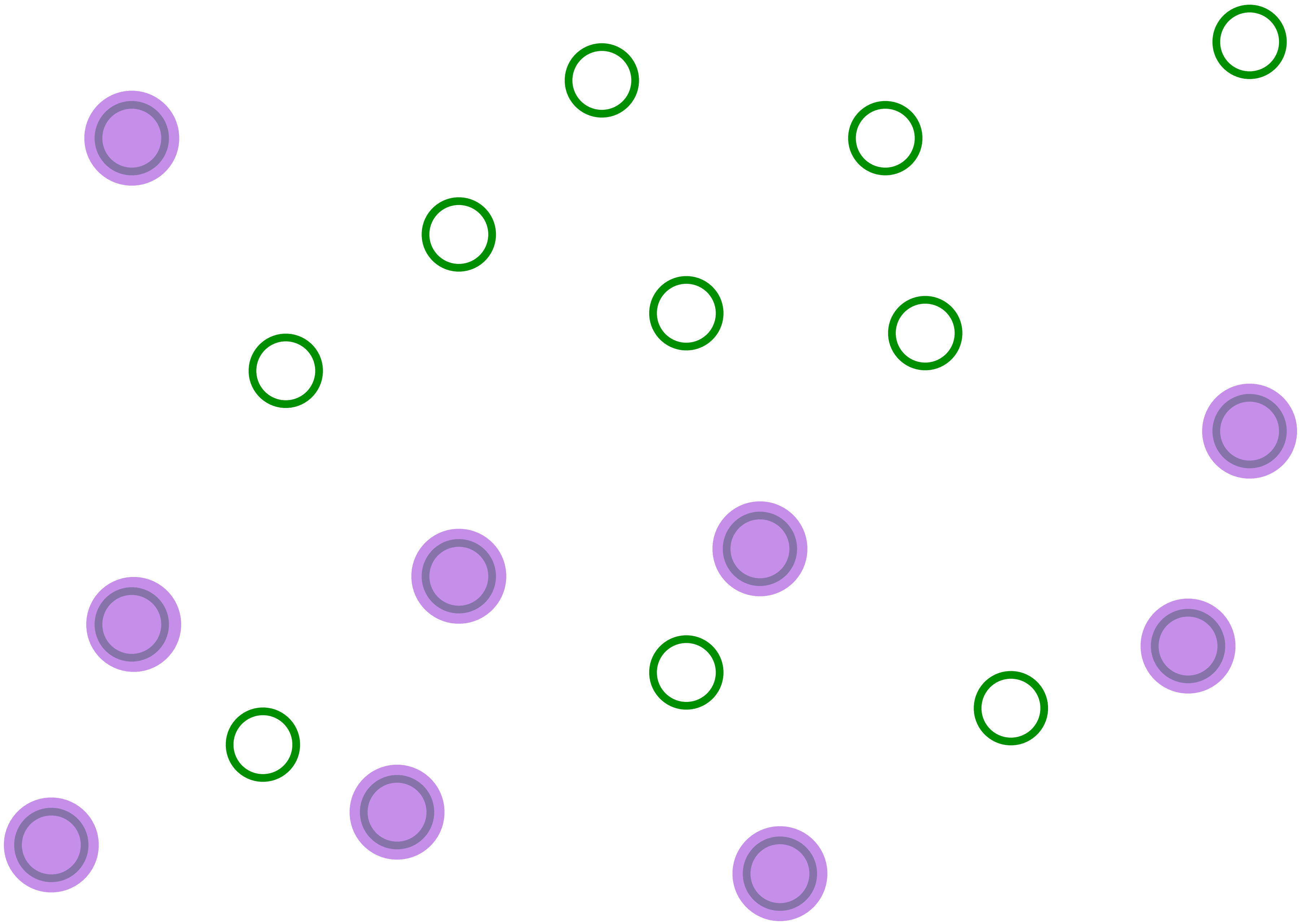

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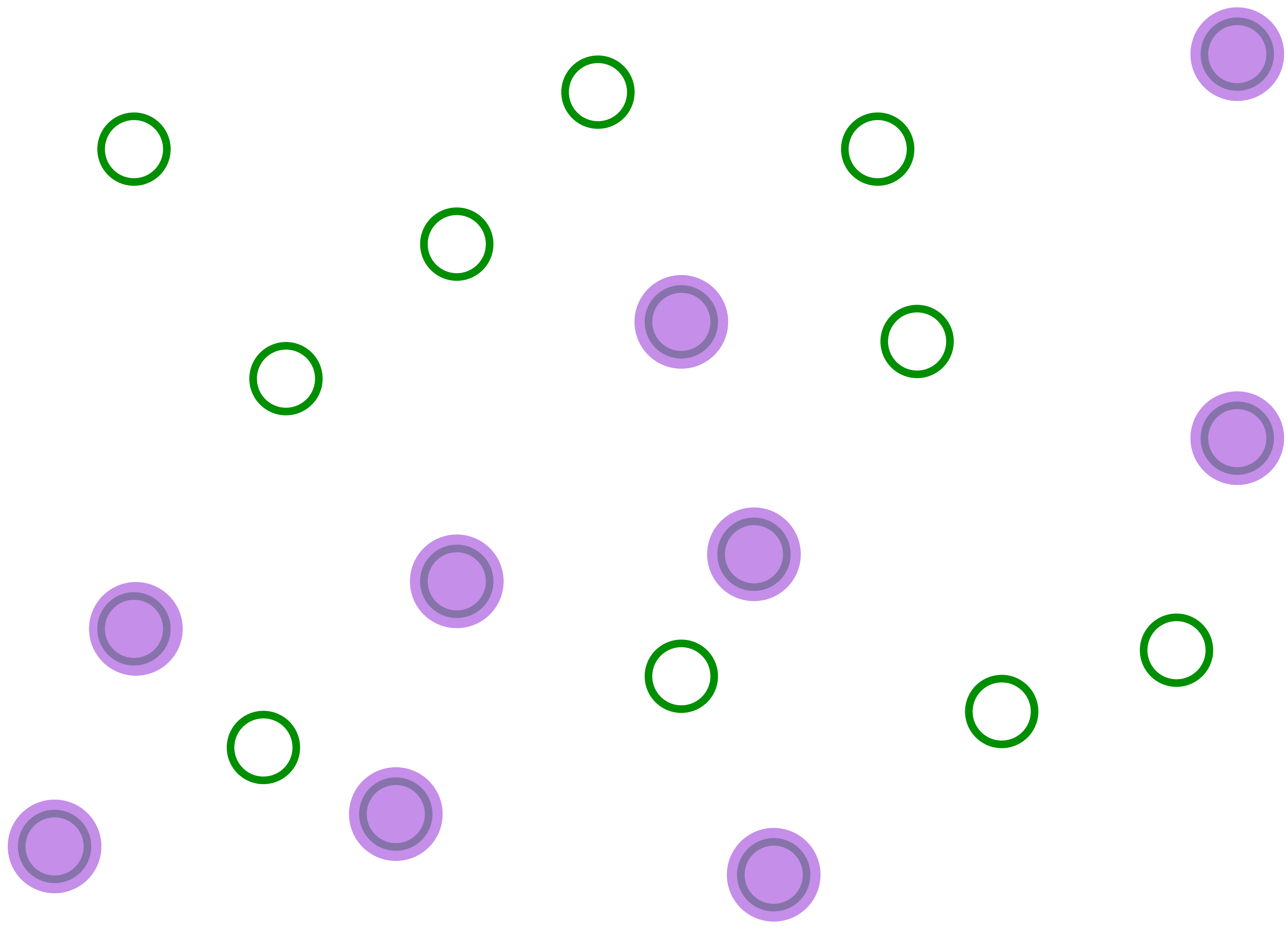

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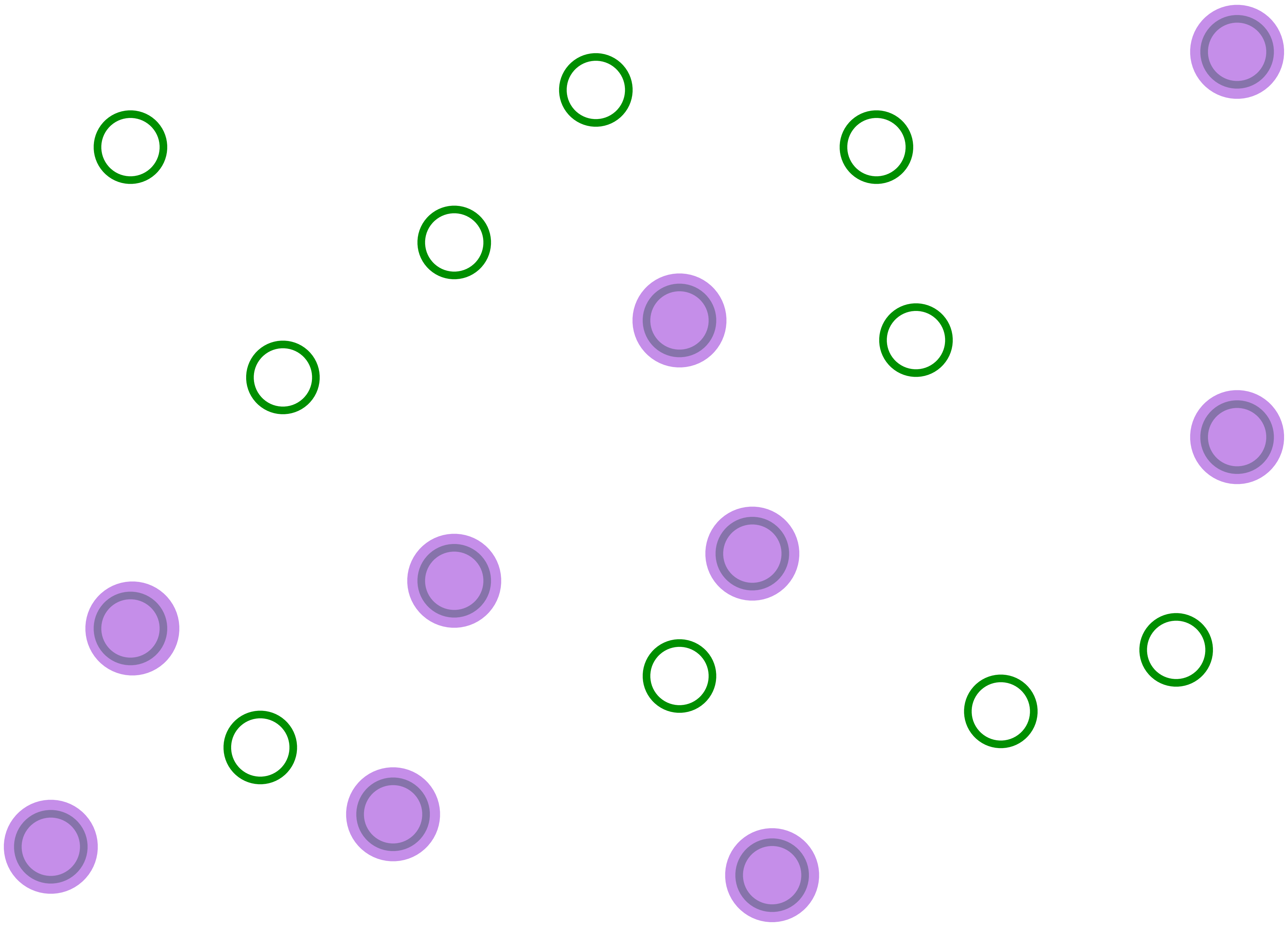


This describes both a strange metal and a black hole!

The SYK model

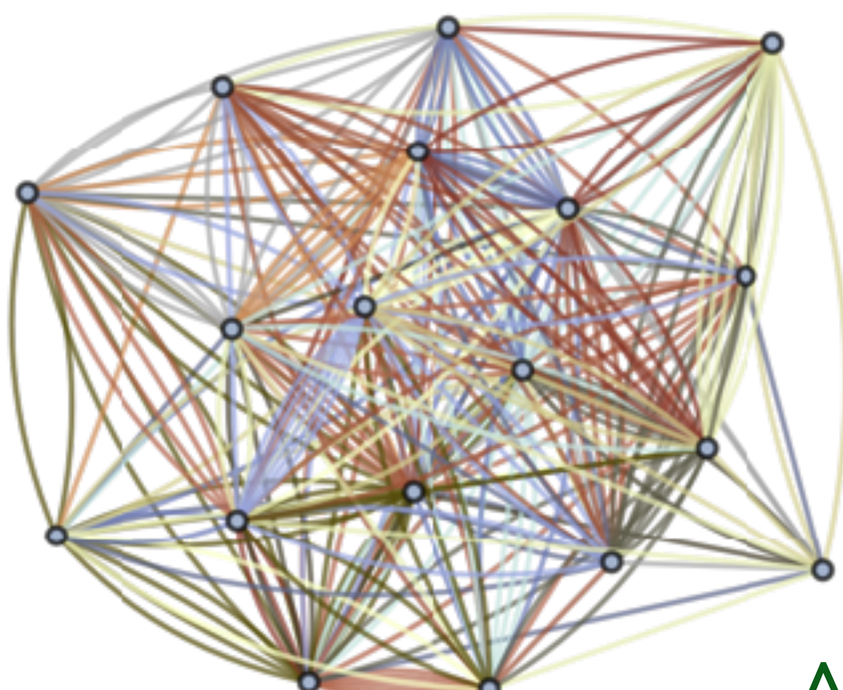
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

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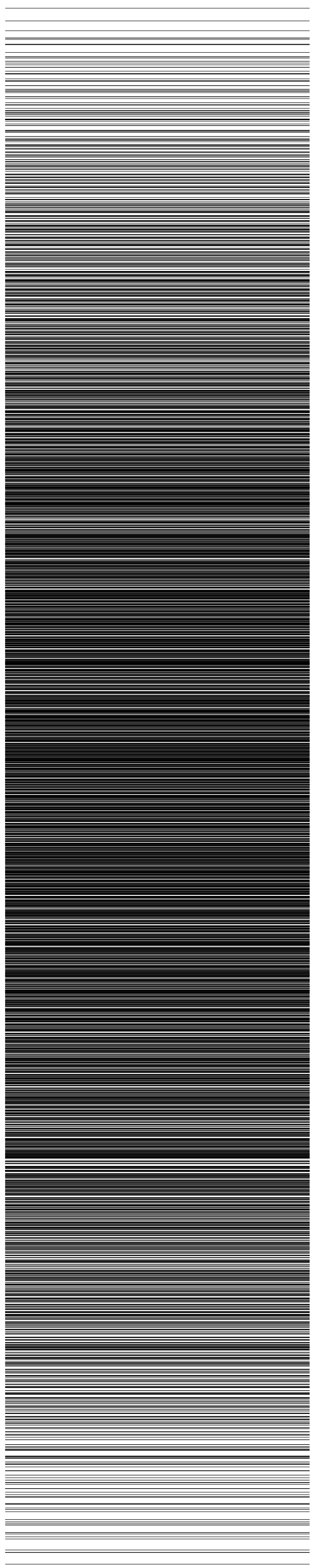

Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$ with

$$\begin{aligned} s_0 &= \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots \\ &< \ln 2 \end{aligned}$$

where G is Catalan's constant, for the half-filled case $Q = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

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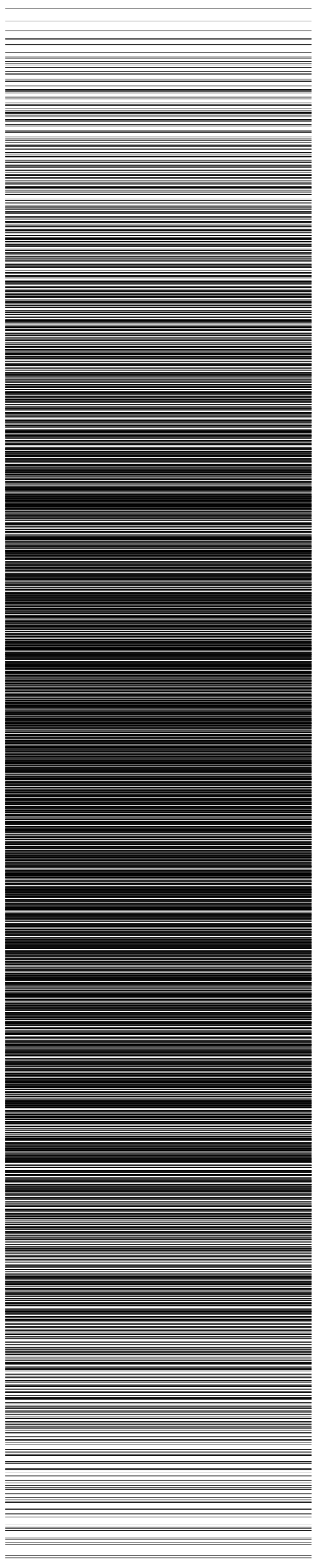

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Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

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No quasiparticles !
$E \neq \sum_\alpha n_\alpha \varepsilon_\alpha$
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W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The SYK model

Feynman graph expansion in $U_{ijk\ell}$, and graph-by-graph average, yields exact equations in the large N limit:

$$\boxed{\begin{gathered} G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau) \\ G(\tau = 0^-) = \mathcal{Q}. \end{gathered}}$$

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$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

where $A = e^{-i\pi/4}(\pi/U^2)^{1/4}$ at half-filling. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

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At frequencies $\ll U$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations. The singular part of the self-energy and the Green's function obey

$$\int_0^\beta d\tau_2\, \Sigma_{\rm sing}(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\Sigma_{\rm sing}(\tau_1, \tau_2) = -U^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)$$

A. Kitaev, 2015
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$$\int_0^\beta d\tau_2 \, \Sigma_(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\Sigma(\tau_1, \tau_2) = -U^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)$$

These equations are invariant under

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{G}(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.
By using $f(\sigma) = \tan(\pi T \sigma)/(\pi T)$ we can
now obtain the $T > 0$ solution from the $T = 0$ solution.

A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

The SYK model

Let us write the large N saddle point solutions of S as

$$\begin{aligned} G_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-1/2} \\ \Sigma_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-3/2}. \end{aligned}$$

The saddle point will be invariant under a reperamaterization $f(\tau)$ when choosing $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ leads to a transformed $\widetilde{G}(\sigma_1, \sigma_2) = G_s(\sigma_1 - \sigma_2)$ (and similarly for Σ). It turns out this is true only for the SL(2, R) transformations under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken down to SL(2, R) by the saddle point.

A. Kitaev

The SYK model

- $T = 0$ fermion Green's function is incoherent: $G(\tau) \sim \tau^{-1/2}$ at large τ. (Fermi liquids with quasiparticles have the coherent: $G(\tau) \sim 1/\tau$)

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- The model exhibits eigenstate thermalization. Each eigenstate scrambles quantum information (as measured in the out-of-time-order correlation) in the fastest possible time of $\hbar/(2\pi k_B T(E))$.

J. Sonner and M. Vielma, arXiv:1707.08013

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It is remarkable to have a solvable model with such properties.

Quantum matter without quasiparticles:

- If there are no quasiparticles, then

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$$\tau_{\rm eq} = \# \frac{\hbar}{k_B T} \quad , \quad \text{the `Planckian time'}.$$

S. Sachdev,
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$$\tau_{\rm eq} = \# \frac{\hbar}{k_B T} \quad , \quad \text{the 'Planckian time'}.$$

- Systems without quasiparticles are the fastest possible in reaching local equilibrium, and all many-body quantum systems obey, as $T \to 0$

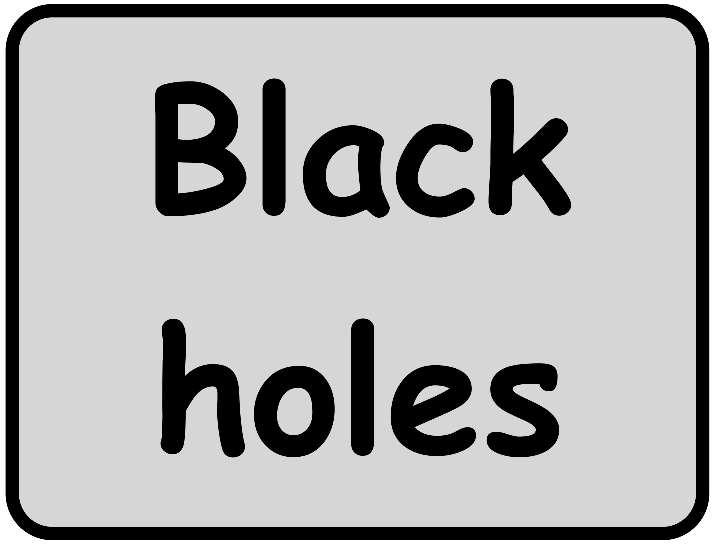
$$\tau_{\rm eq} > C \frac{\hbar}{k_B T} .$$

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

  - In Fermi liquids $\tau_{\rm eq} \sim 1/T^2$, and so the bound is obeyed as $T \to 0$.
  - This bound rules out quantum systems with *e.g.* $\tau_{\rm eq} \sim \hbar/(J k_B T)^{1/2}$.
  - There is no bound in classical mechanics ($\hbar \to 0$). By cranking up frequencies, we can attain equilibrium as quickly as we desire.

SYK models and black holes

- Black holes have an entropy proportional to their surface area, and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- Black holes relax to thermal equilibrium in a time $\sim \hbar/(k_B T_H) = 8\pi G M/c^3$.



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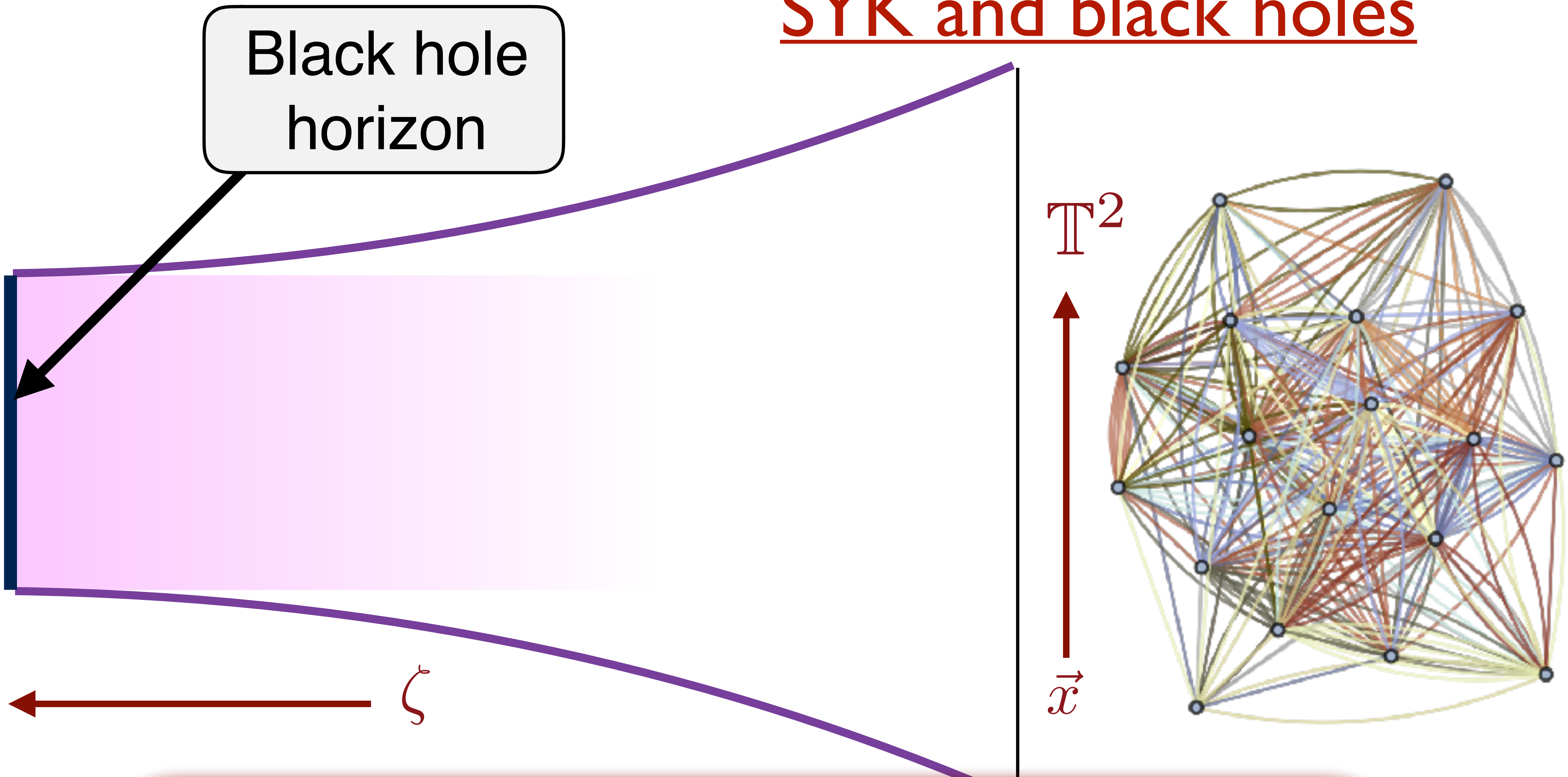
- Black holes in $d+1$ spatial dimensions are similar to a quantum system without quasiparticles in d spatial dimensions.

Black holes

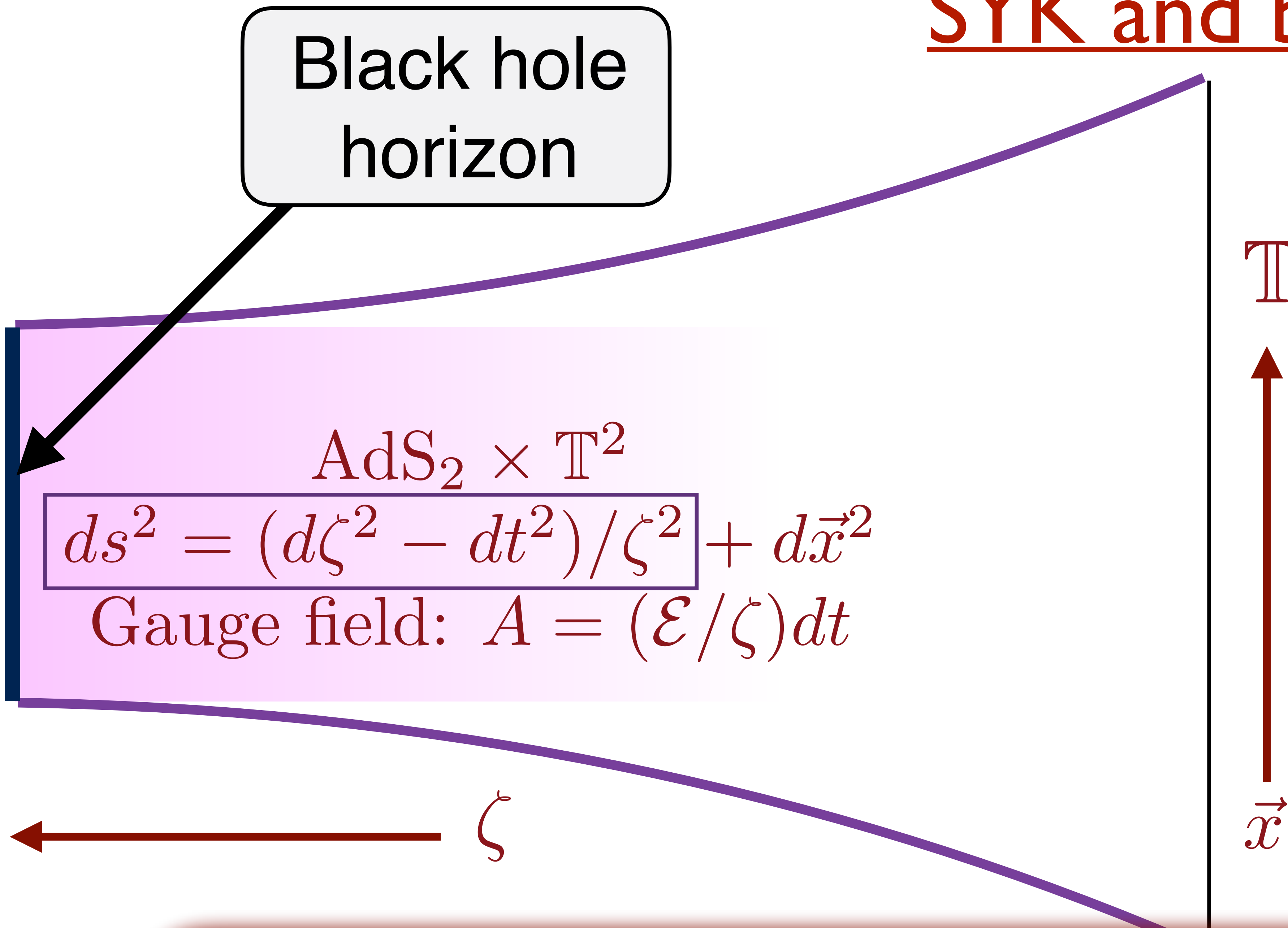
SYK and black holes

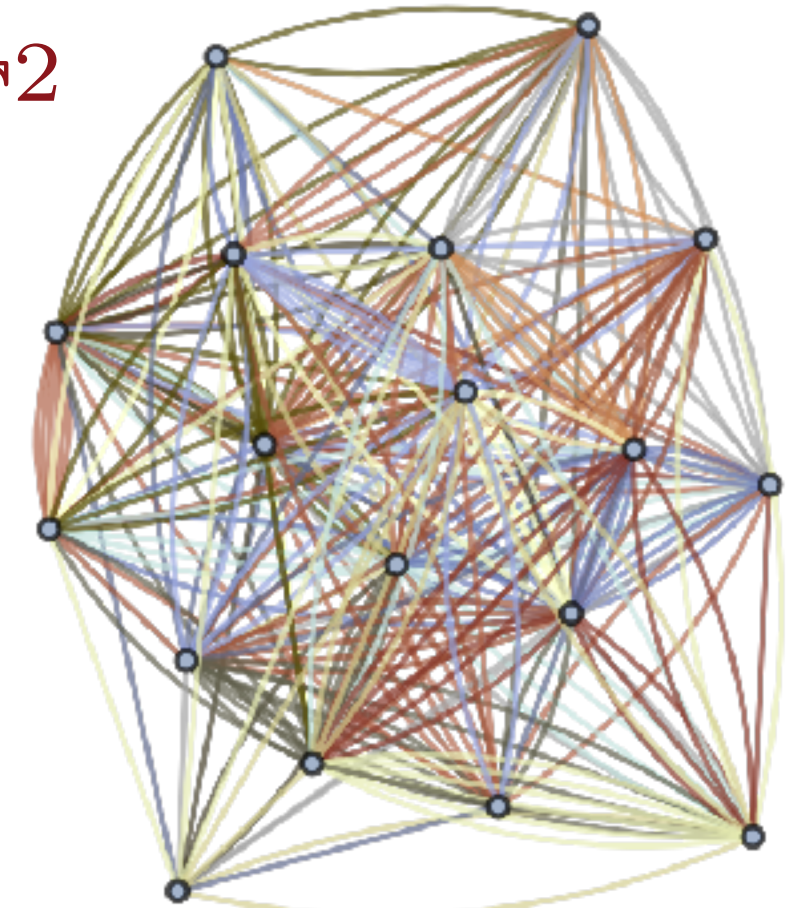


Black holes with a near-horizon AdS_2 geometry (described by quantum gravity in 1+1 spacetime dimensions) match the properties of the 0+1 dimensional SYK model in the previous slide: Ns_0 is the Bekenstein-Hawking entropy

S. Sachdev, PRL **105**, 151602 (2010); A. Kitaev (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857

SYK and black holes





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SYK and AdS_2

Connections of SYK to gravity and AdS_2 horizons

- Reparameterization and gauge invariance are the 'symmetries' of the Einstein-Maxwell theory of gravity and electromagnetism

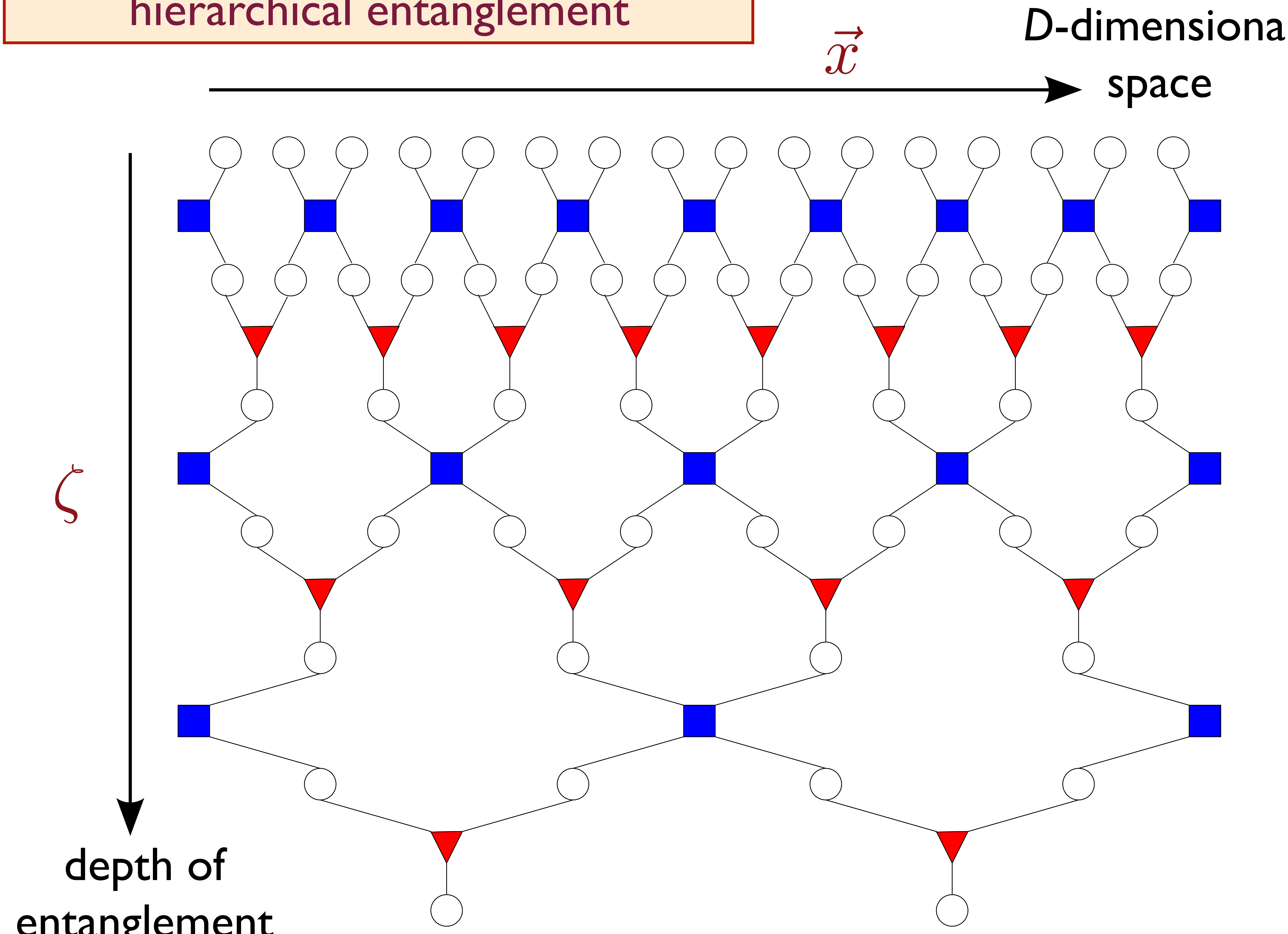
- SL(2,R) is the isometry group of AdS_2.

$ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$ is invariant under

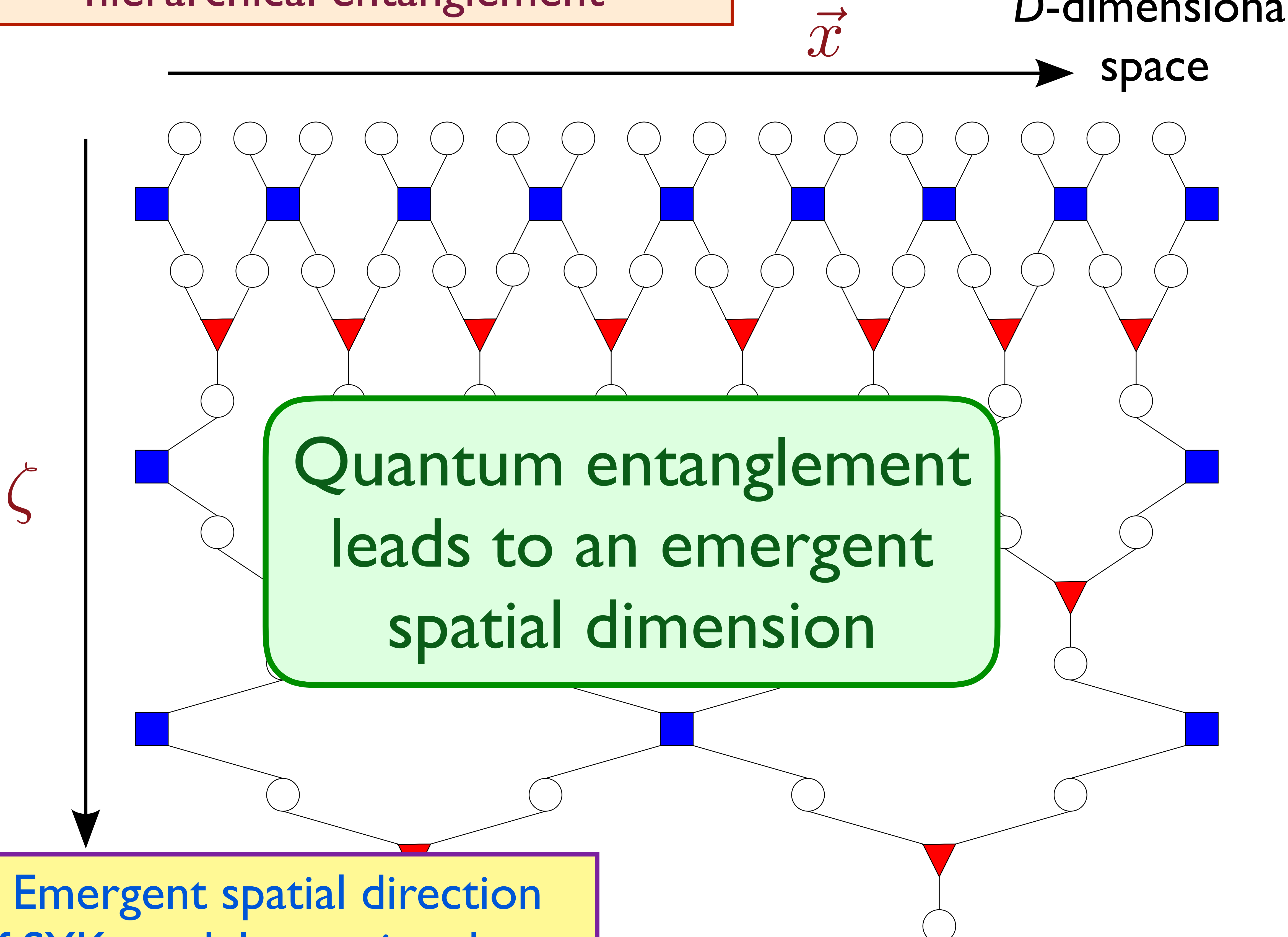
$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d}$$

with $ad - bc = 1$.

Tensor network of hierarchical entanglement



Tensor network of
hierarchical entanglement



Emergent spatial direction
of SYK model or string theory

B. Swingle

Digital Quantum Simulation of Minimal AdS/CFT

L. García-Álvarez,[1] I. L. Egusquiza,[2] L. Lamata,[1] A. del Campo,[3] J. Sonner,[4] and E. Solano[1,5]

We propose the digital quantum simulation of a minimal AdS/CFT model in controllable quantum platforms. We consider the Sachdev-Ye-Kitaev model describing interacting Majorana fermions with randomly distributed all-to-all couplings, encoding nonlocal fermionic operators onto qubits to efficiently implement their dynamics via digital techniques. Moreover, we also give a method for probing nonequilibrium dynamics and the scrambling of information. Finally, our approach serves as a protocol for reproducing a simplified low-dimensional model of quantum gravity in advanced quantum platforms as trapped ions and superconducting circuits.

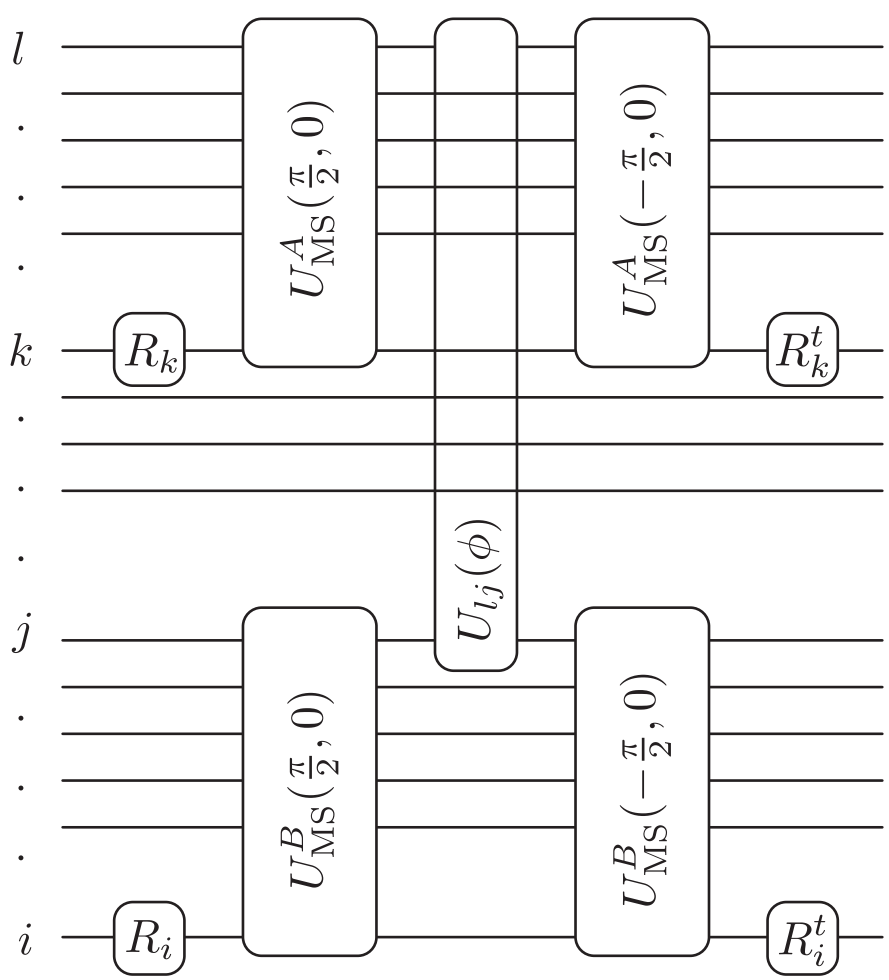


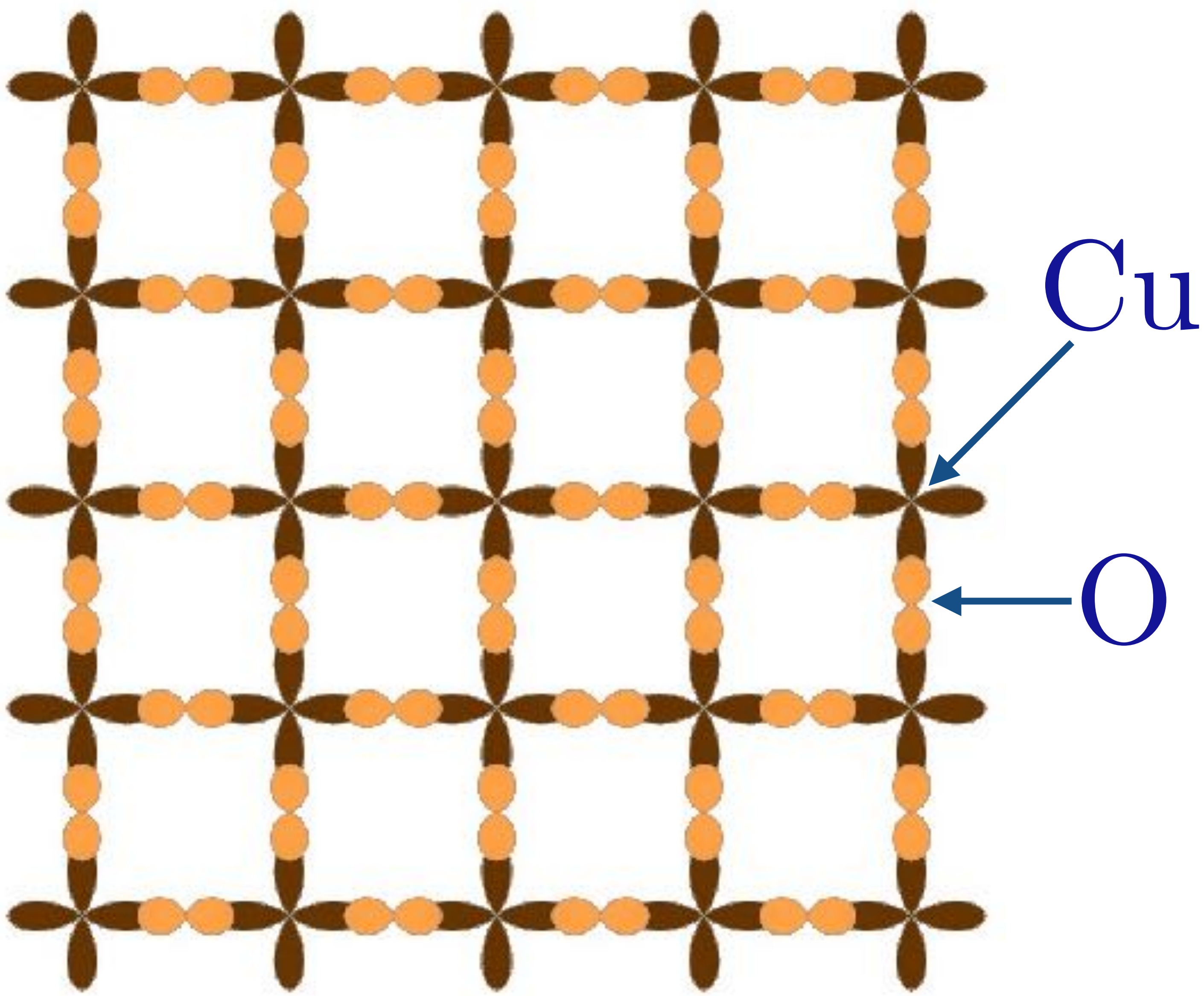
FIG. 1. Engineering many-body interactions in trapped-ion qubits. Operation sequence of single-qubit and multiqubit gates, inside a Trotter step, acting on trapped-ion qubits to generate a generic interaction term (13). The single-qubit rotations R_i and R_k act on qubits i and k, respectively, and the phase ϕ of the two-qubit entangling gate, $U_{ij}(\phi)$, must be chosen adequately in order to produce the desired combination of $\alpha_i\alpha_j\alpha_k\alpha_l$ in the interaction.

1. Metal with quasiparticles
   Random matrix model of a `quantum dot’

2. Metal without quasiparticles
   SYK model of a `quantum dot’

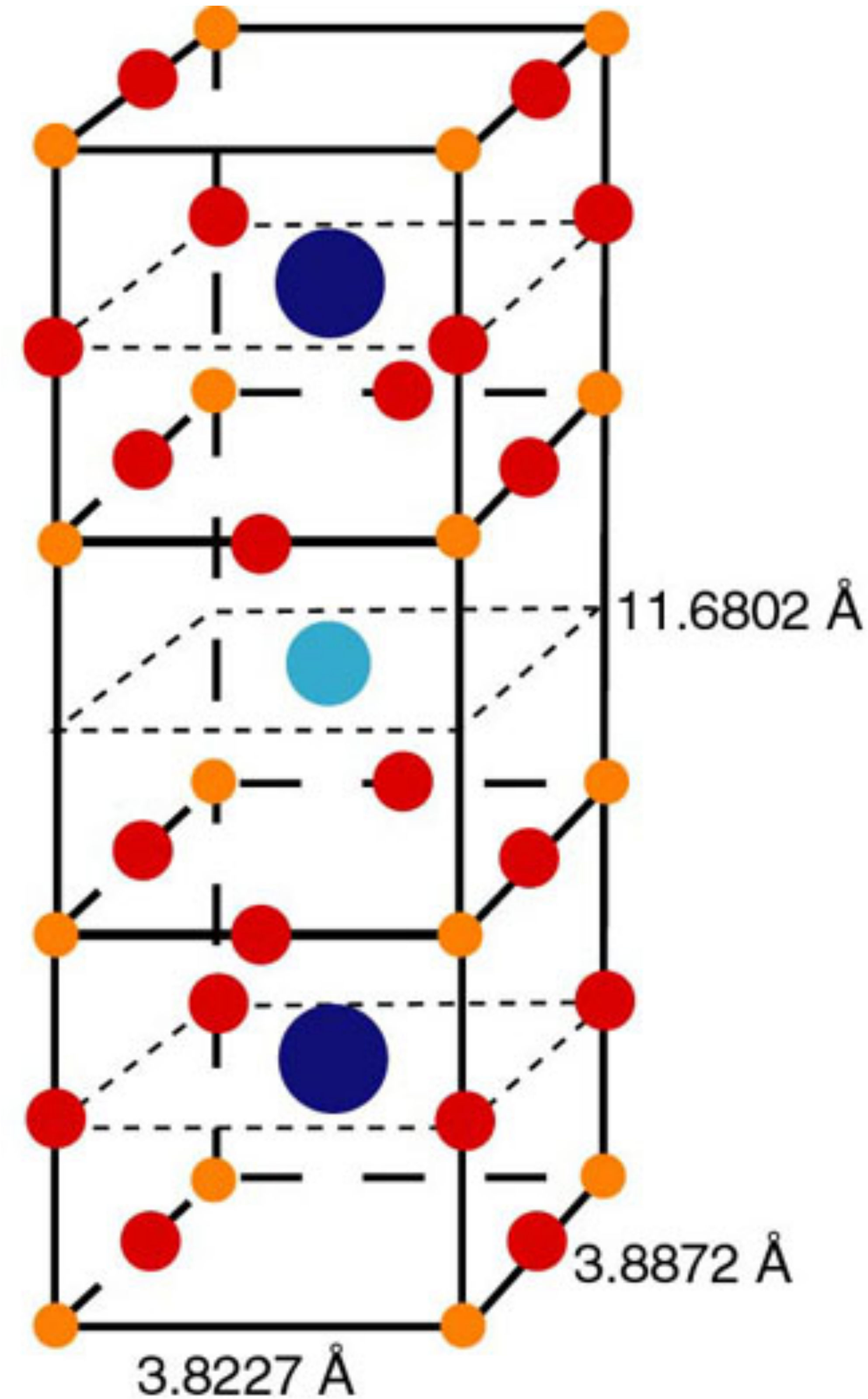
3. High temperature superconductors
   and strange metals.

High temperature superconductors

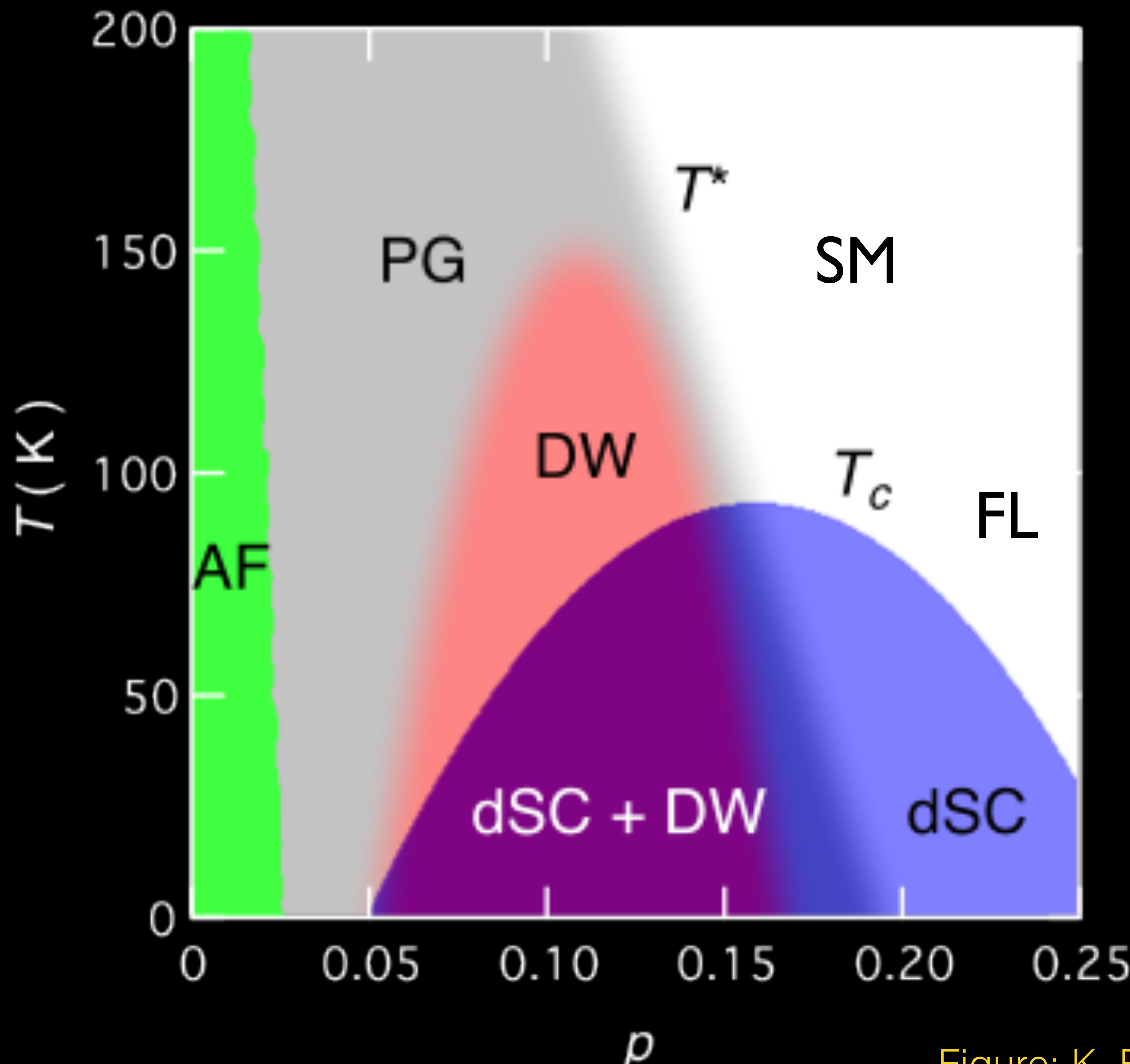


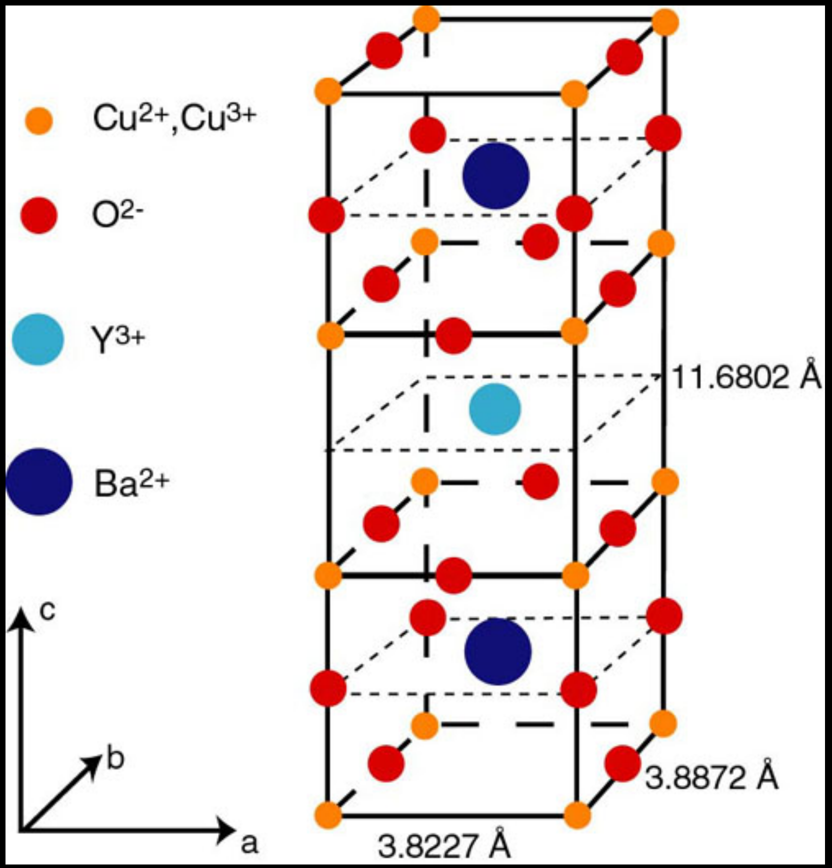
CuO_2 plane

Described by a Hubbard model on the Cu sites



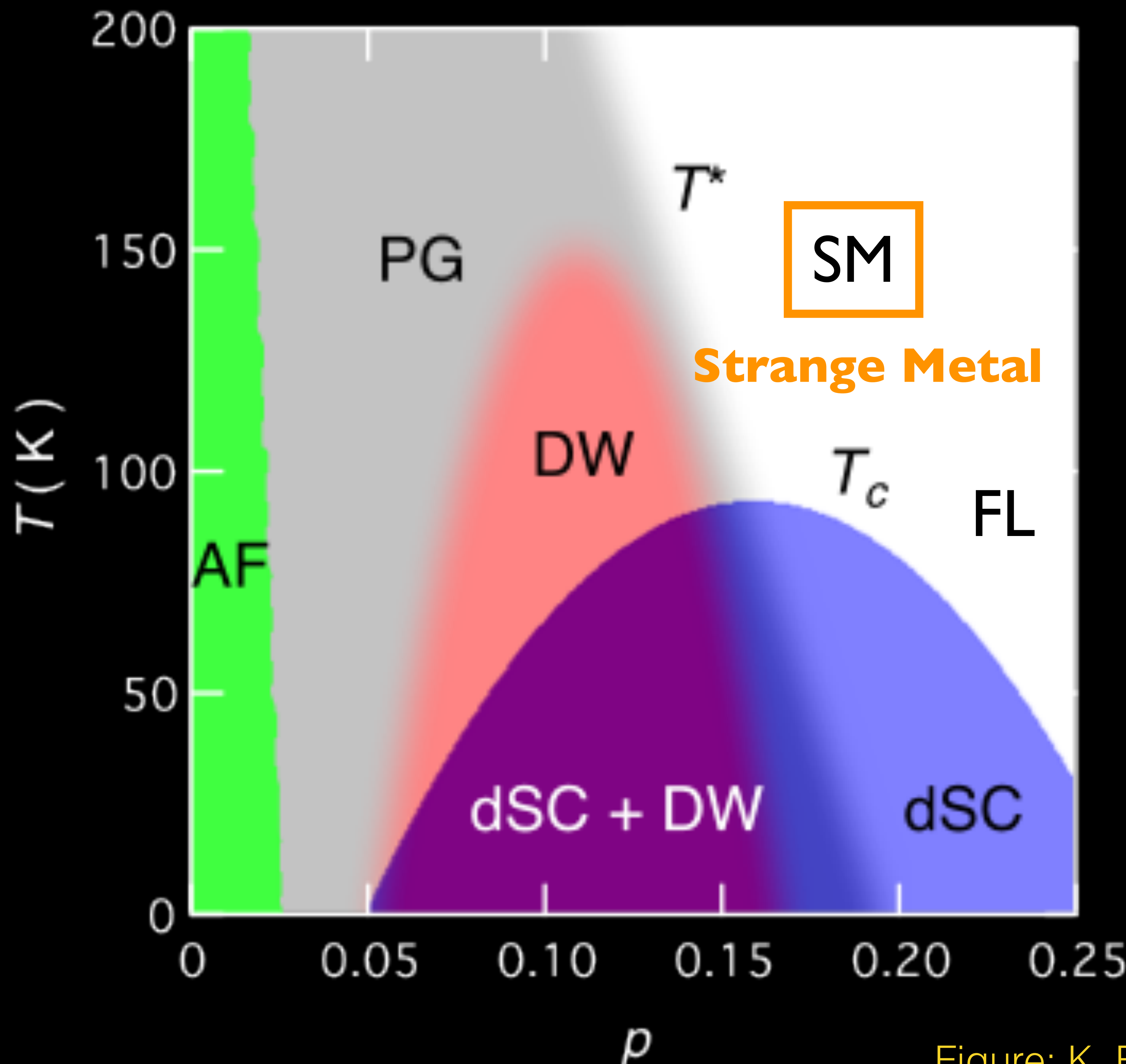
$YBa_2Cu_3O_{6+x}$

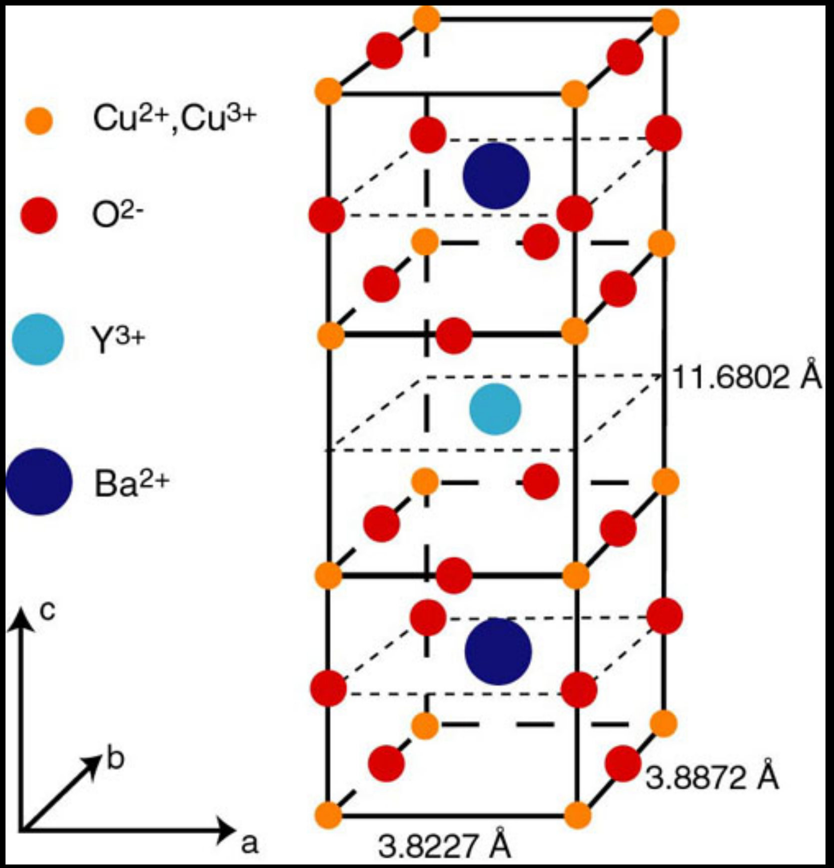




$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis





$\mathrm{YBa_2Cu_3O_{6+x}}$

Figure: K. Fujita and J. C. Seamus Davis

Ubiquitous

"Strange",

"Bad",

"Incoherent",

or "Ultra-quantum"

metal has a resistivity, ρ, which obeys

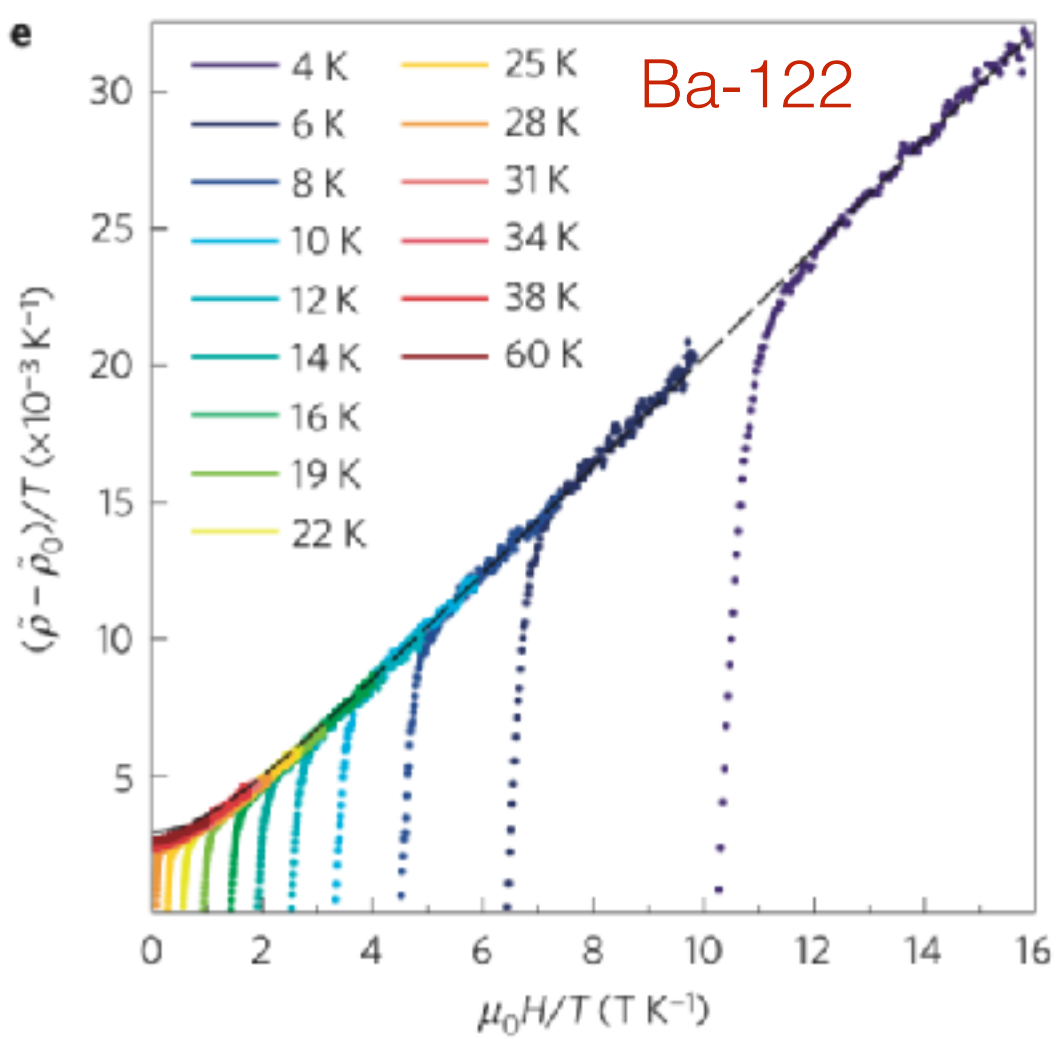
$$\rho \sim T,$$

and

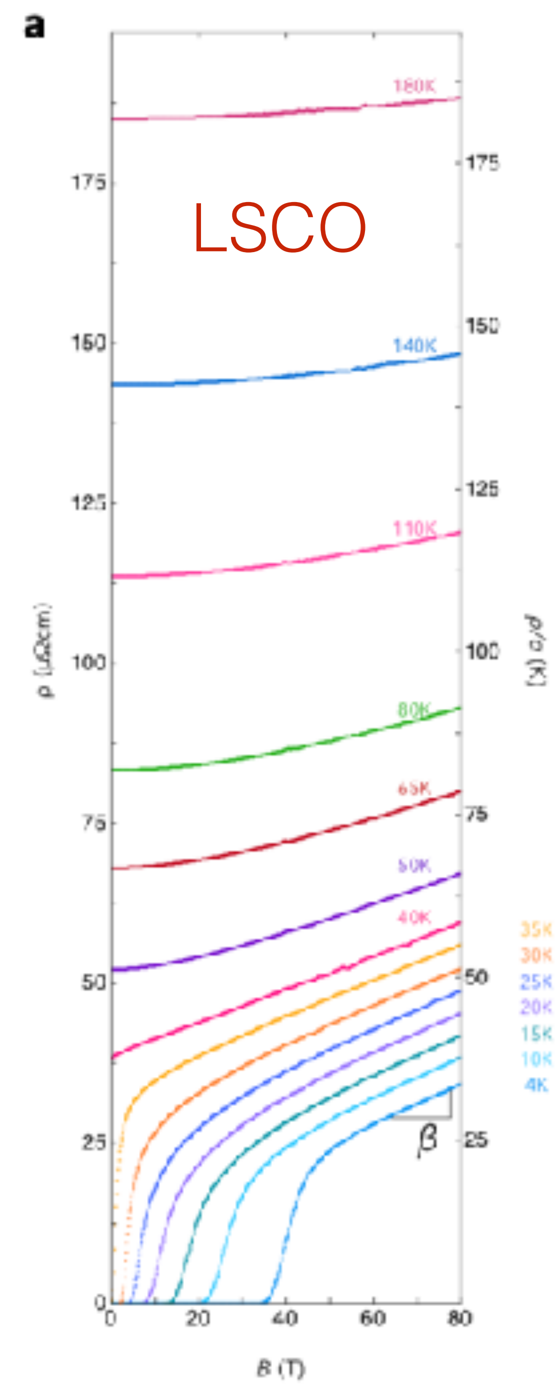
in some cases $\rho \gg h/e^2$
(in two dimensions),
where h/e^2 is the quantum unit of resistance.

Ultra-quantum metals just got stranger…

B-linear magnetoresistance!?



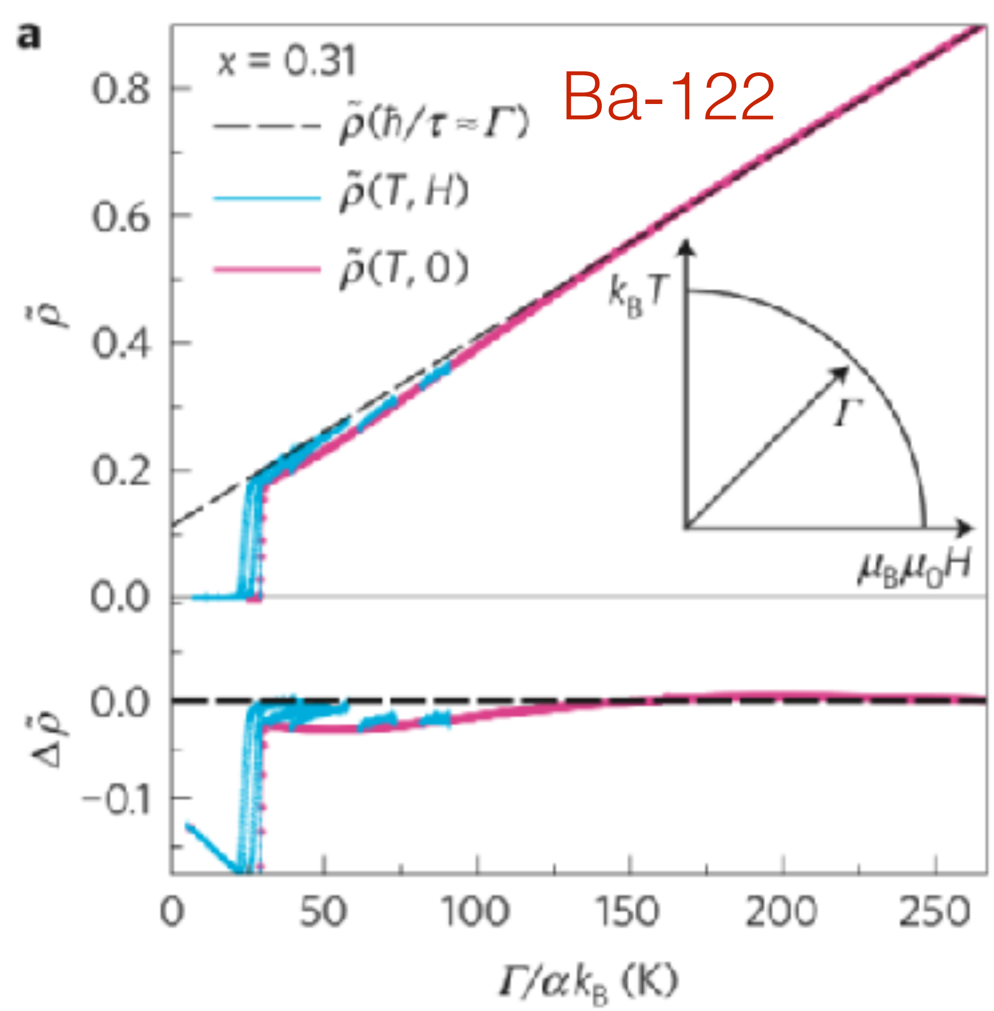
I. M. Hayes et. al., Nat. Phys. 2016



P. Giraldo-Gallo et. al., arXiv:1705.05806

Ultra-quantum metals just got stranger…

Scaling between B and T !?

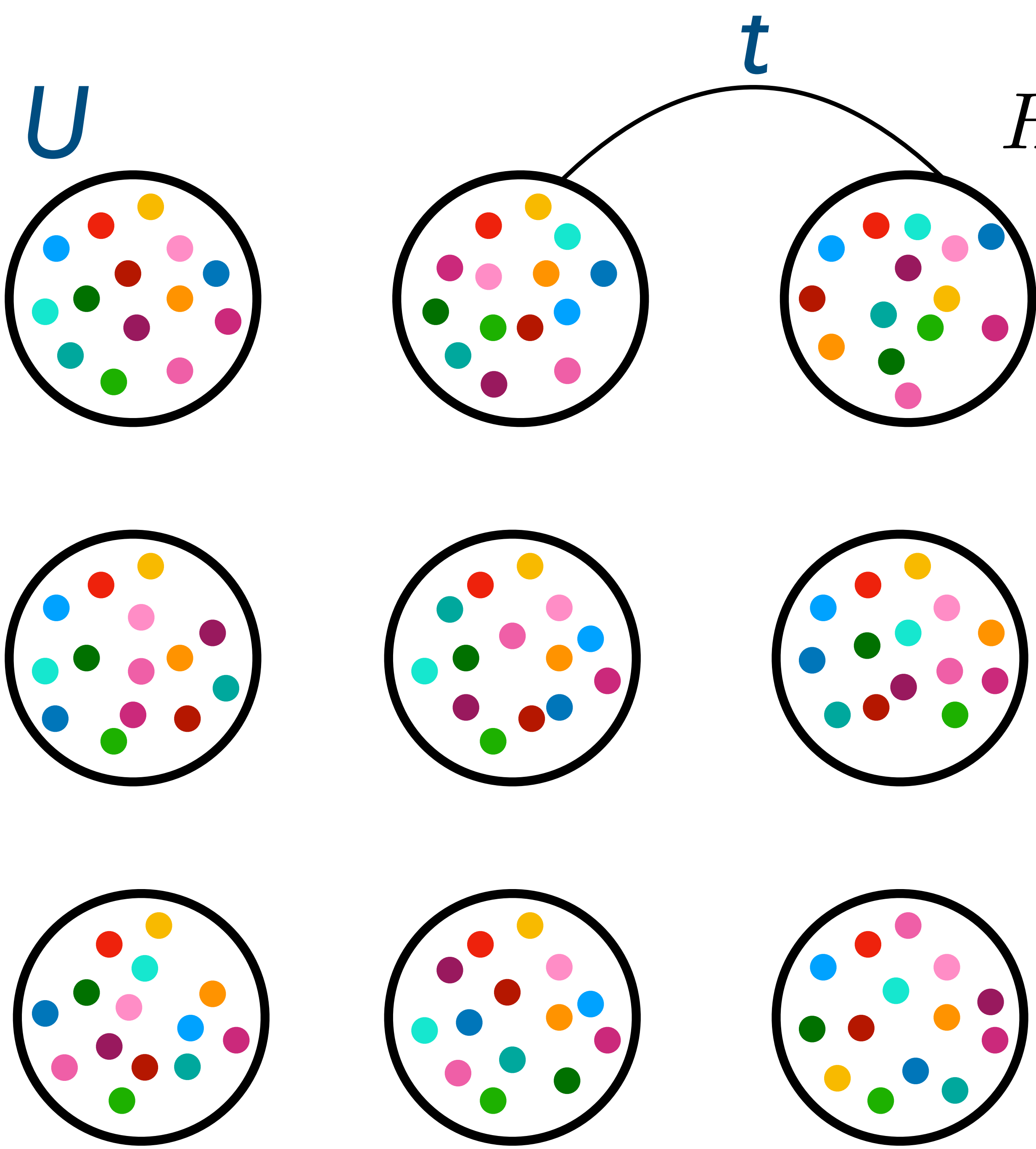


$$\rho(H,T) - \rho(0,0) \propto \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B \mu_0 H)^2} \equiv \Gamma$$

I. M. Hayes et. al., Nat. Phys. 2016

SYK building blocks for a strange metal

SYK quantum islands of electrons with random hopping between them.



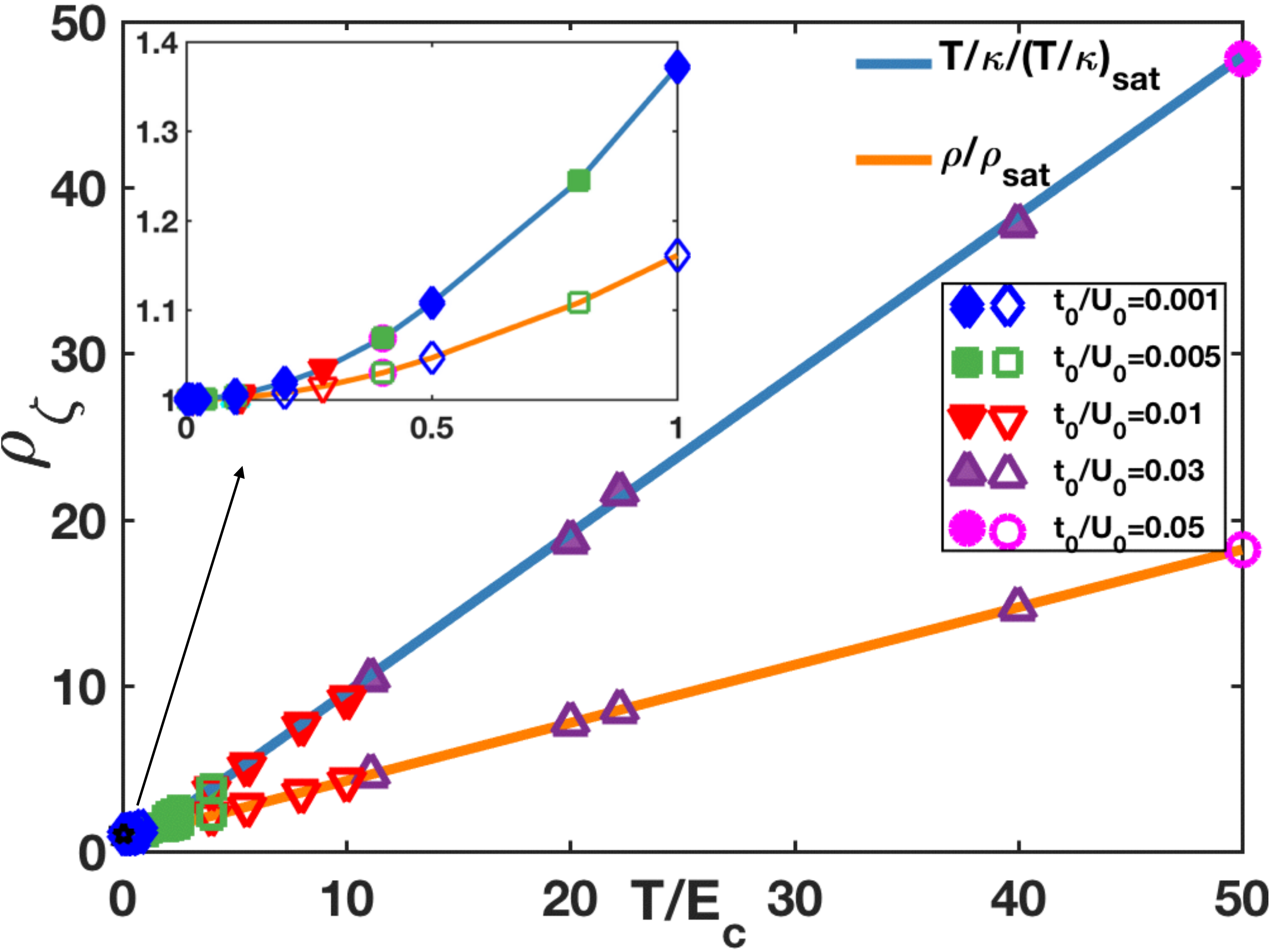
$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx}$$

$$+ \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Low ‘coherence’ scale



$$E_c \sim \frac{t_0^2}{U}$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

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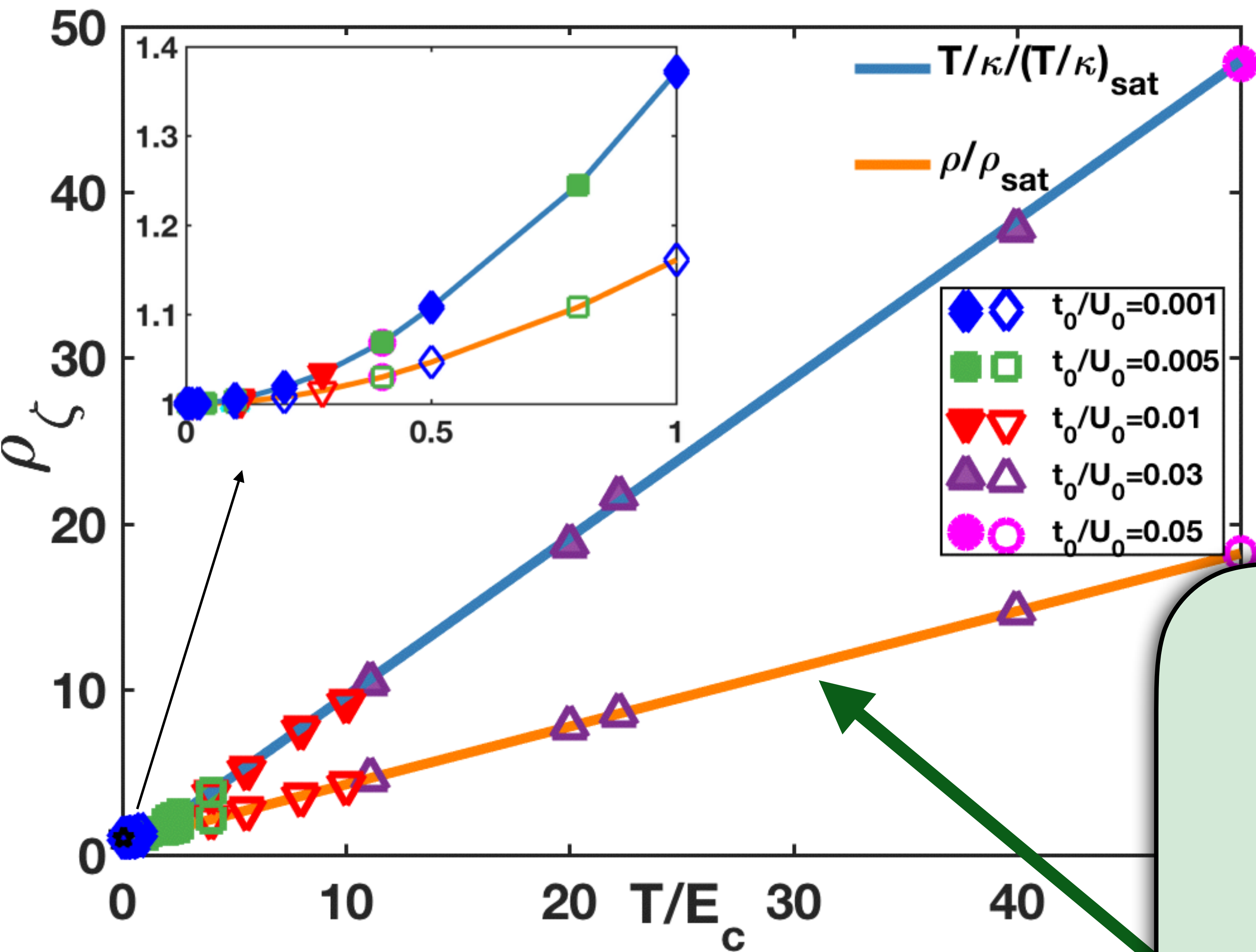

For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right) , \quad s = s_0$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

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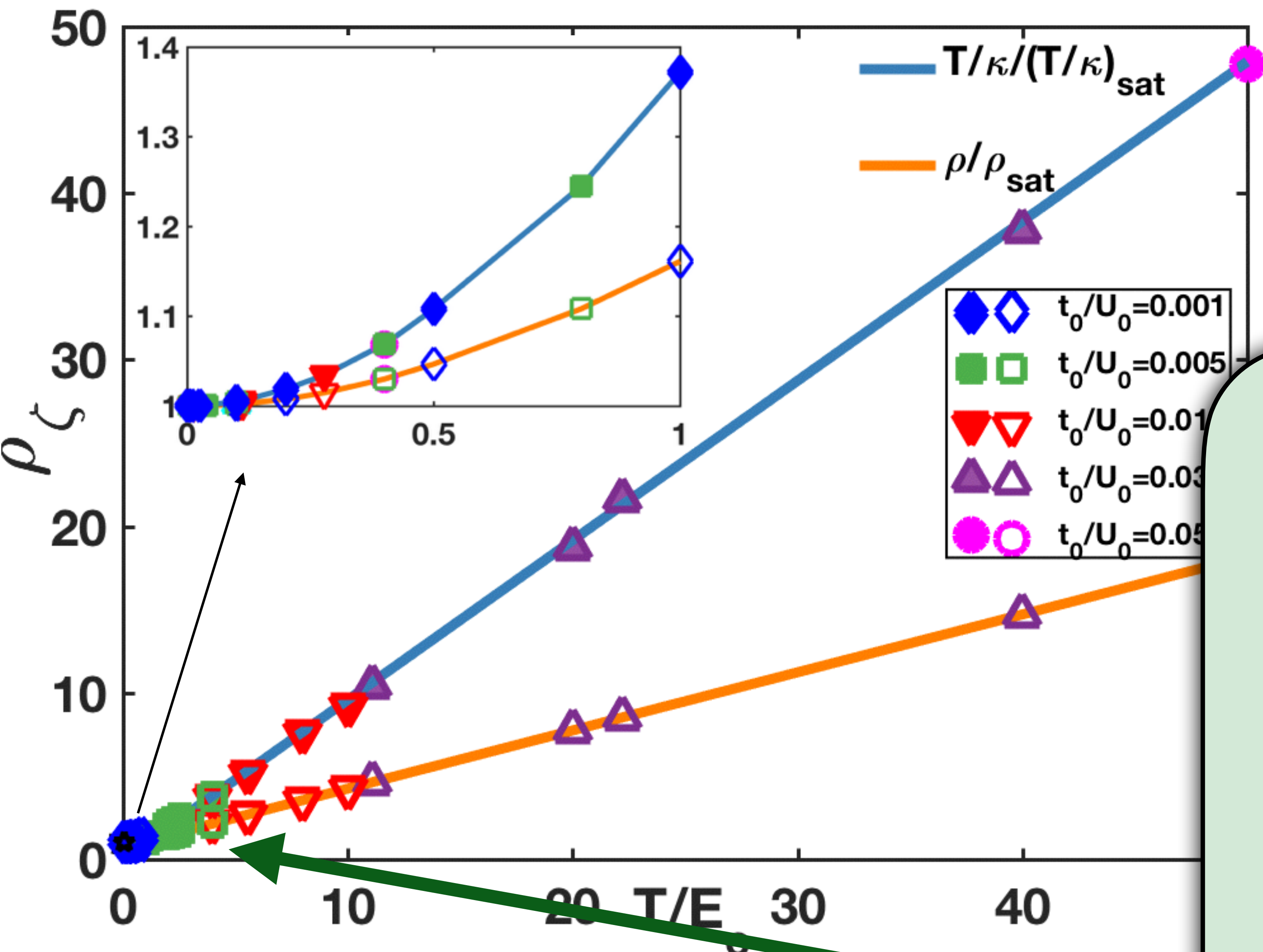


$$E_c \sim \frac{t_0^2}{U}$$

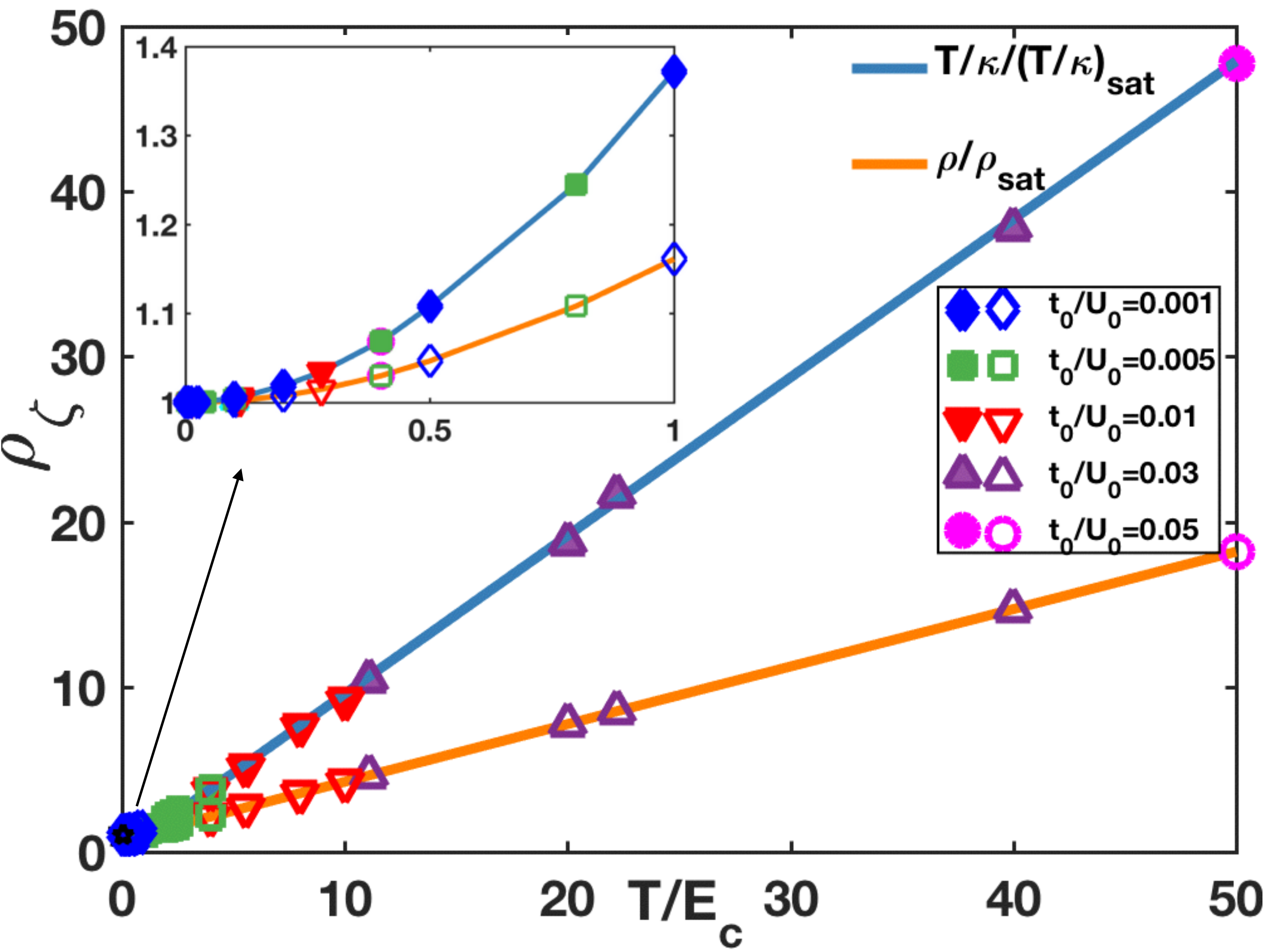
For $T < E_c$, the resistivity, ρ, and entropy density, s, are

$$\rho = \frac{h}{e^2}\left[c_1 + c_2\left(\frac{T}{E_c}\right)^2\right]$$

$$s \sim s_0\left(\frac{T}{E_c}\right)$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Low 'coherence' scale



$$E_c \sim \frac{t_0^2}{U}$$

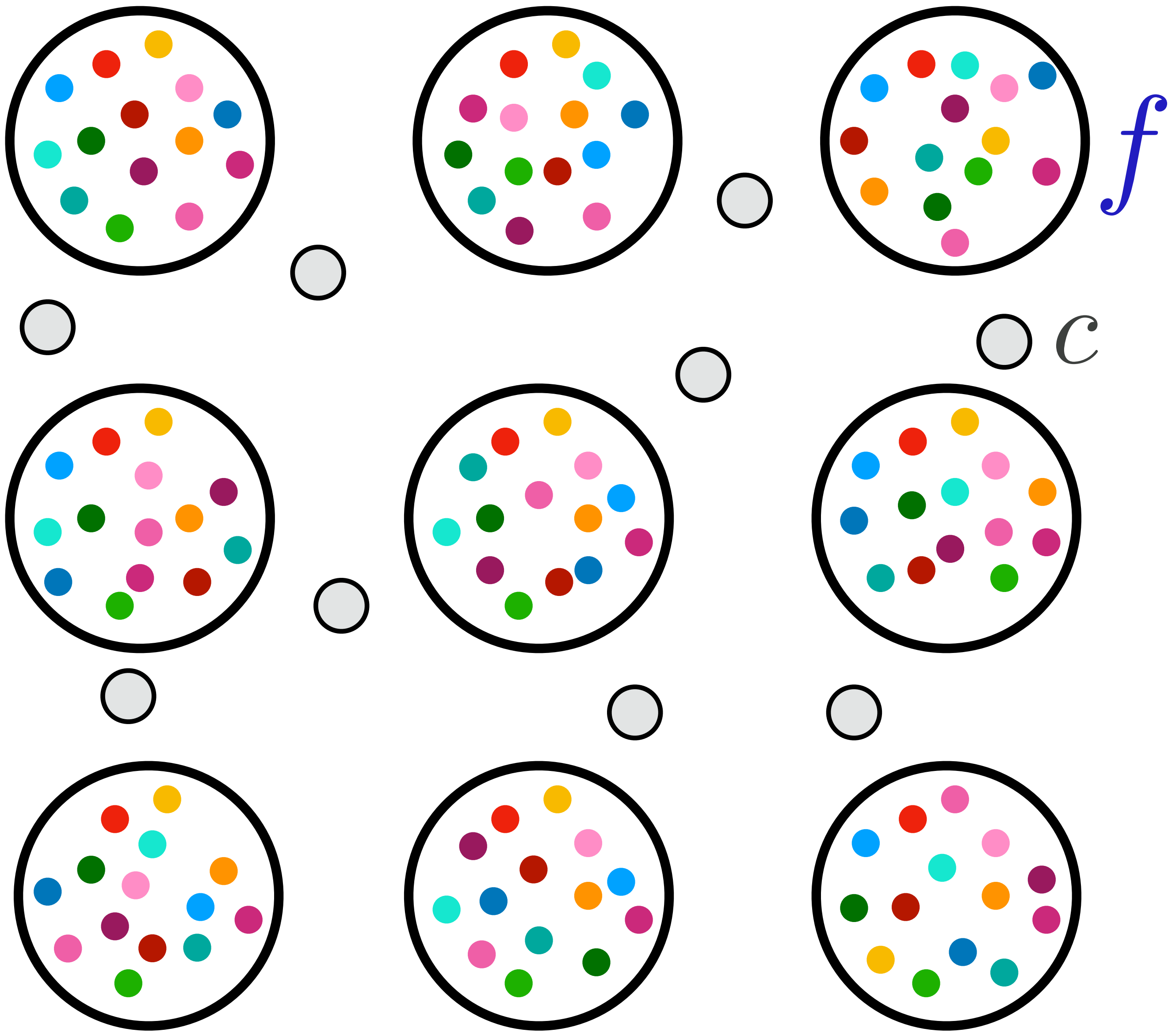
But this model exhibits negligible magnetoresistivity !

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Infecting a Fermi liquid and making it SYK

Mobile electrons (c) interacting with SYK quantum islands (f) with exchange interactions.

This yields the first model agreeing with magnetotransport in strange metals !



Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, arXiv:1712.05026

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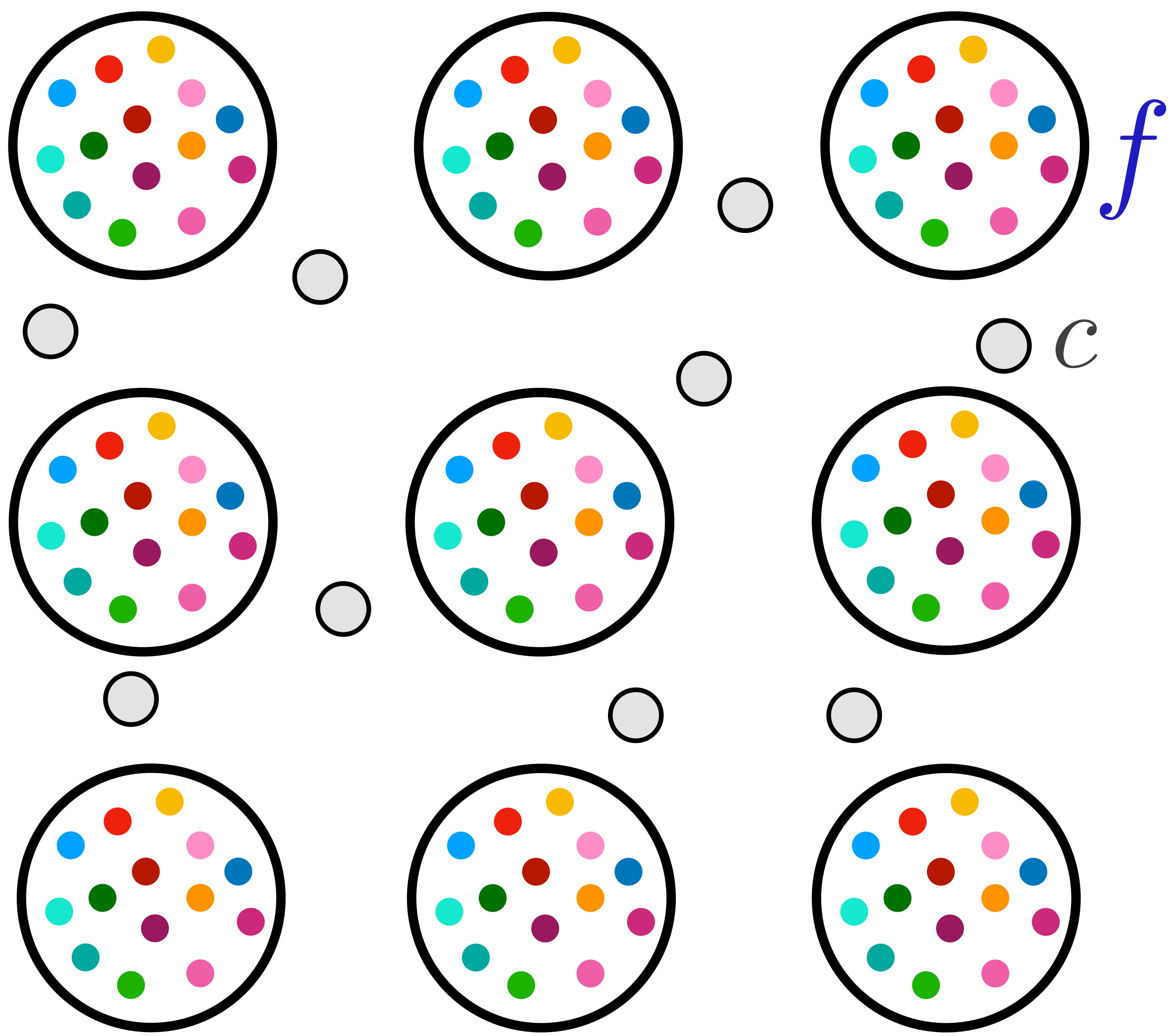


Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, arXiv:1801.06178

Infecting a Fermi liquid and making it SYK

Mobile electrons (c) interacting with SYK quantum islands (f) with exchange interactions.

Large N solution (with or without microscopic disorder) yields a 'marginal Fermi liquid' metal, with conductivities of the form:

$$\begin{aligned}\sigma_{xx}(B,T) &= \frac{1}{T}\Phi_L\left(\frac{B}{T}\right)\\ \sigma_{xy}(B,T) &= \frac{B}{T^2}\Phi_H\left(\frac{B}{T}\right)\end{aligned}$$

where the scaling functions interpolate as

$$\Phi_{L,H}(b\to 0)\sim \text{constant} \quad ; \quad \Phi_{L,H}(b\to\infty)\sim 1/b^2$$

This solution exhibits B/T scaling, but the magnetoresistance ρ_{xx} saturates for $B \gg T$.

Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, arXiv:1712.05026

Infecting a Fermi liquid and making it SYK

Need mesoscopic disorder to obtain linear-in-B magnetoresistance

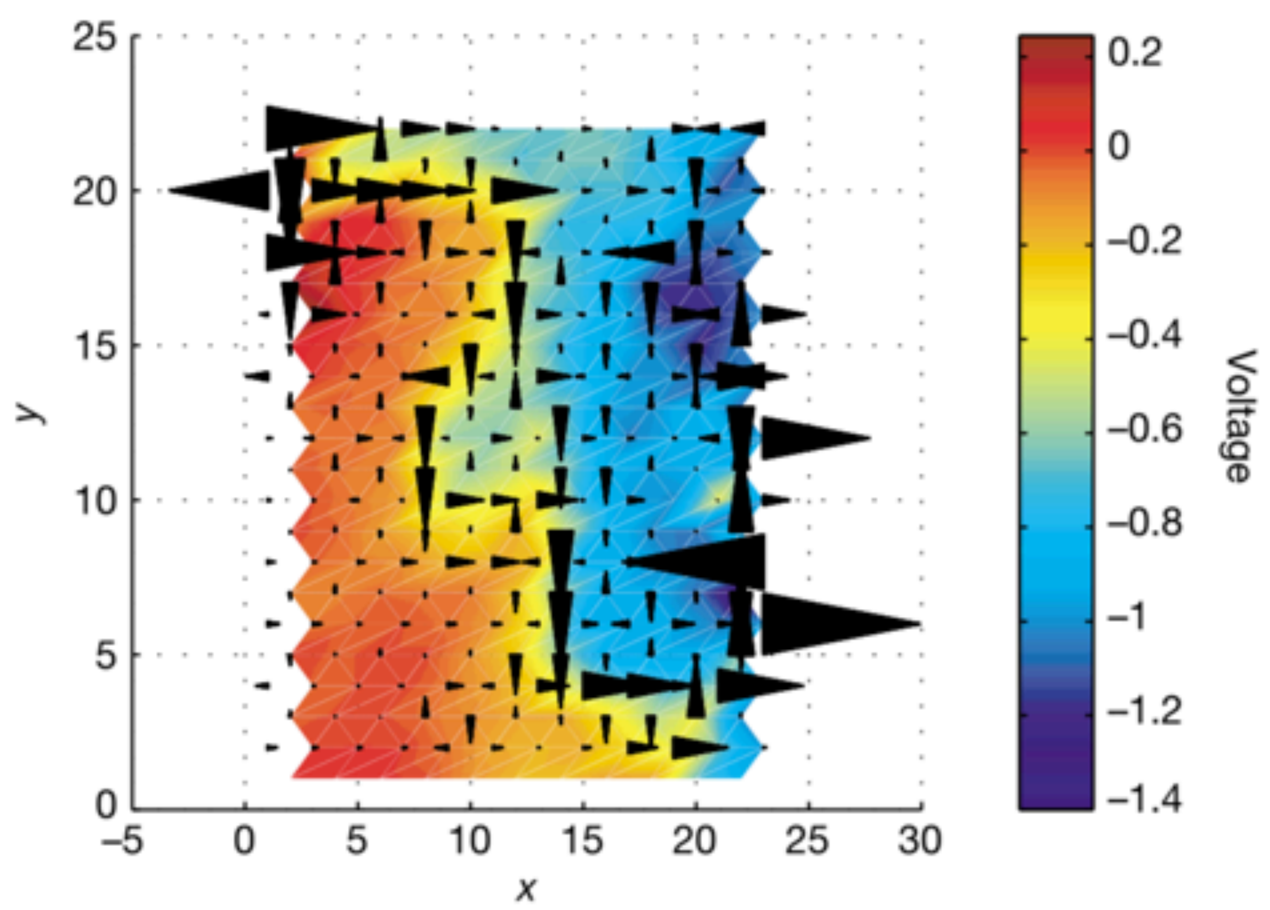
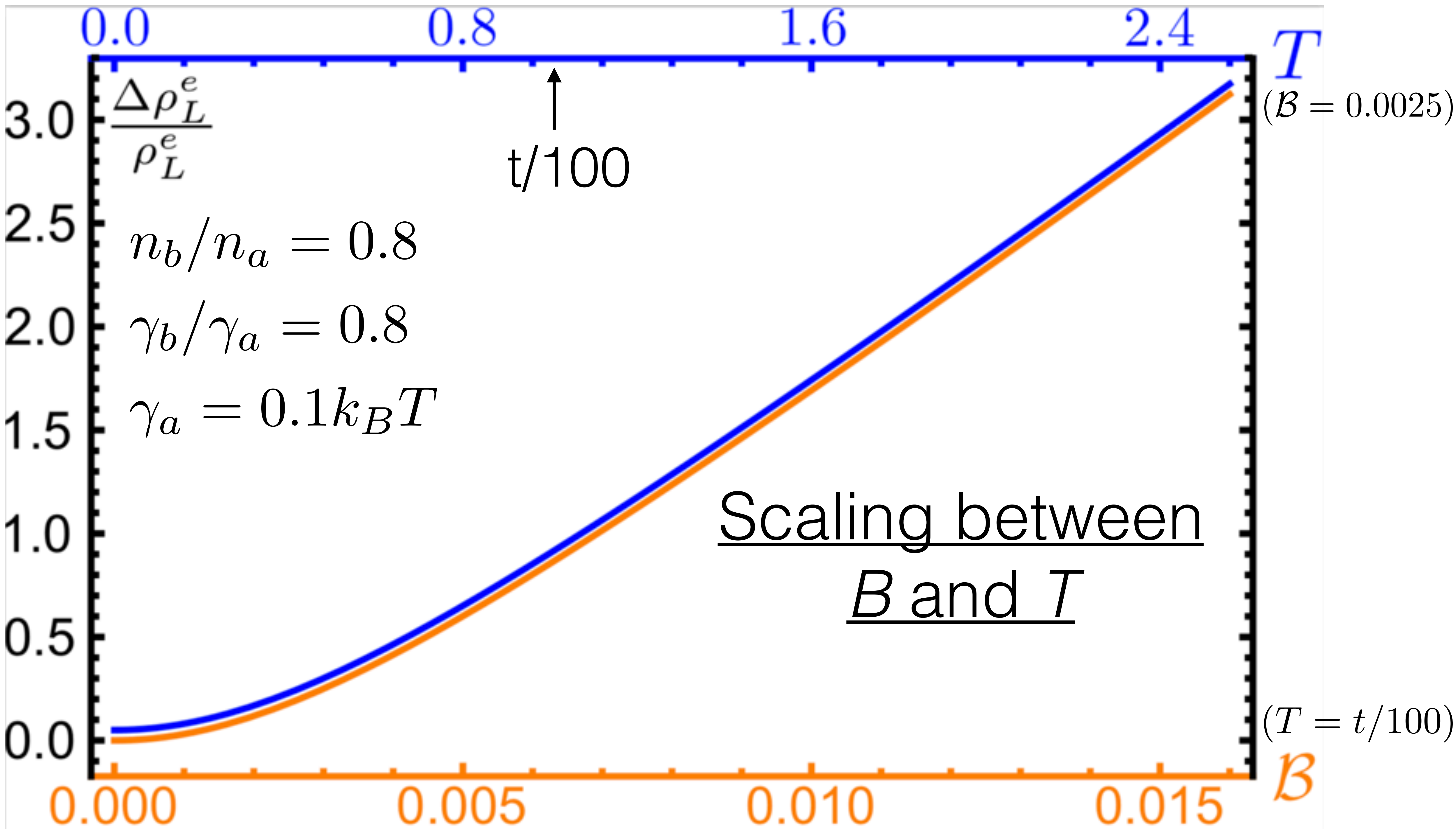


Figure 3 Visualization of currents and voltages at large magnetic field in a 10 × 10 random network of disks with radii 1 (arbitrary units), where the potential difference $U = -1$ V. The black arrows represent the currents, and arrow size depicts the magnitude of the current. The major current path is perpendicular to the applied voltage for a significant proportion of the time, which implies that the magnetoresistance is provided internally by the Hall effect, which is therefore linear in H.

- Current path length increases linearly with B at large B due to local Hall effect, which causes the global resistance to increase linearly with B at large B.

Exact numerical solution of charge-transport equations in a random-resistor network. (M. M. Parish and P. Littlewood, Nature 426, 162 (2003))

Infecting a Fermi liquid and making it SYK



~ 50 T (a = 3.82 A)

Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, arXiv:1712.05026

Quantum matter <u>without</u> quasiparticles:

- No quasiparticle decomposition of low-lying states.
- Thermalization and many-body chaos in the shortest possible time of order $\hbar/(k_B T)$.

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- Thermalization and many-body chaos in the shortest possible time of order $\hbar/(k_B T)$.
- These characteristics are realized in the *solvable* SYK model.
- These are also characteristics of black holes in quantum gravity.