Detecting quantum duality in experiments: how superfluids become solids in two dimensions

Physical Review B **71**, 144508 and 144509 (2005),
cond-mat/0502002

Leon Balents (UCSB)
Lorenz Bartosch (Harvard)
Anton Burkov (Harvard)
Subir Sachdev (Harvard)
Krishnendu Sengupta (HRI, India)

Talk online at http://sachdev.physics.harvard.edu

Outline

I. The superfluid-Mott insulator quantum phase transition

II. Dual theory: vortices and their wavefunctions

III. Vortices in superfluids near the superfluid-insulator quantum phase transition

Vortex wavefunction lives in a dual flavor space

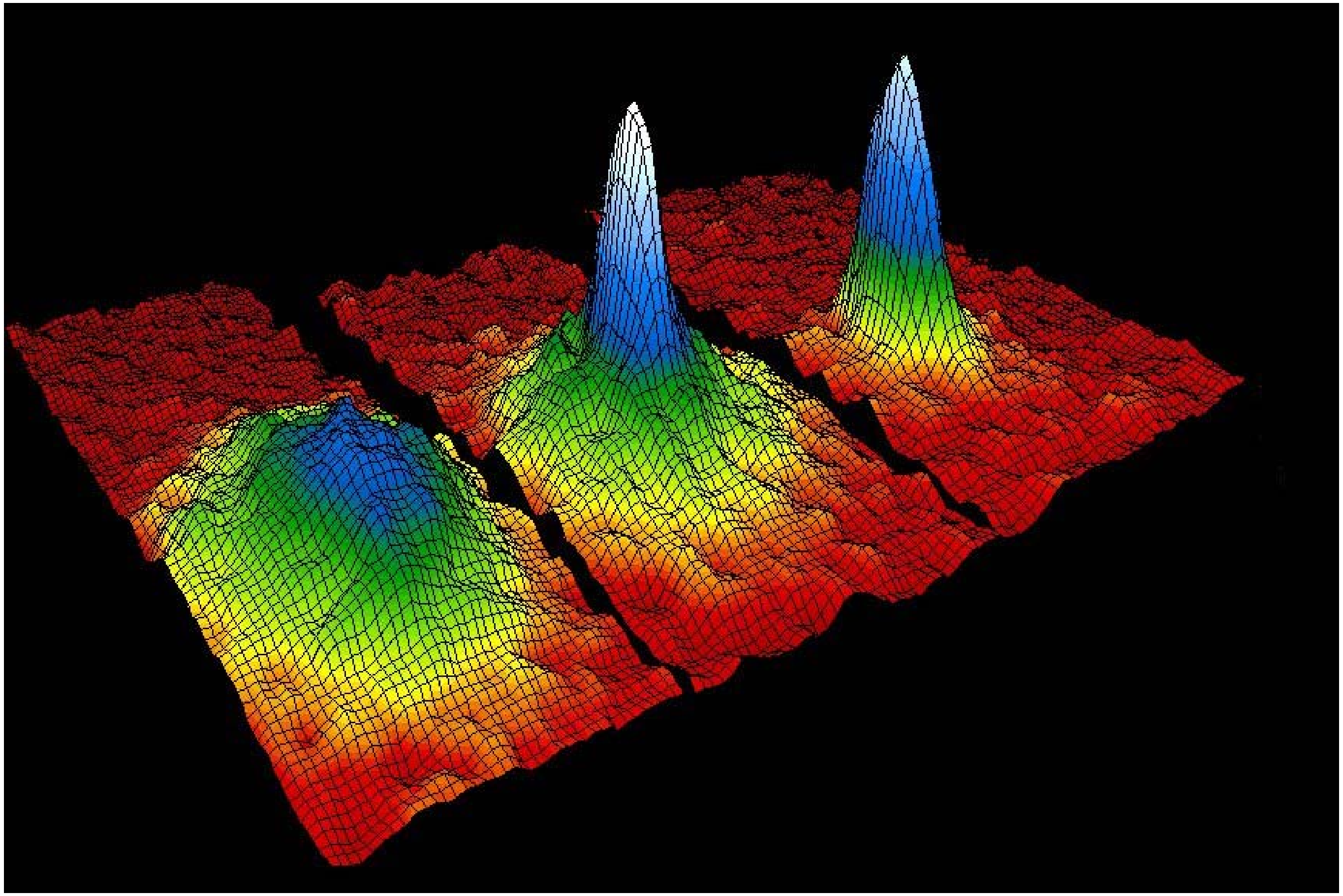
IV. The cuprate superconductors

Detection of vortex flavors ?

I. The superfluid-Mott insulator quantum phase transition

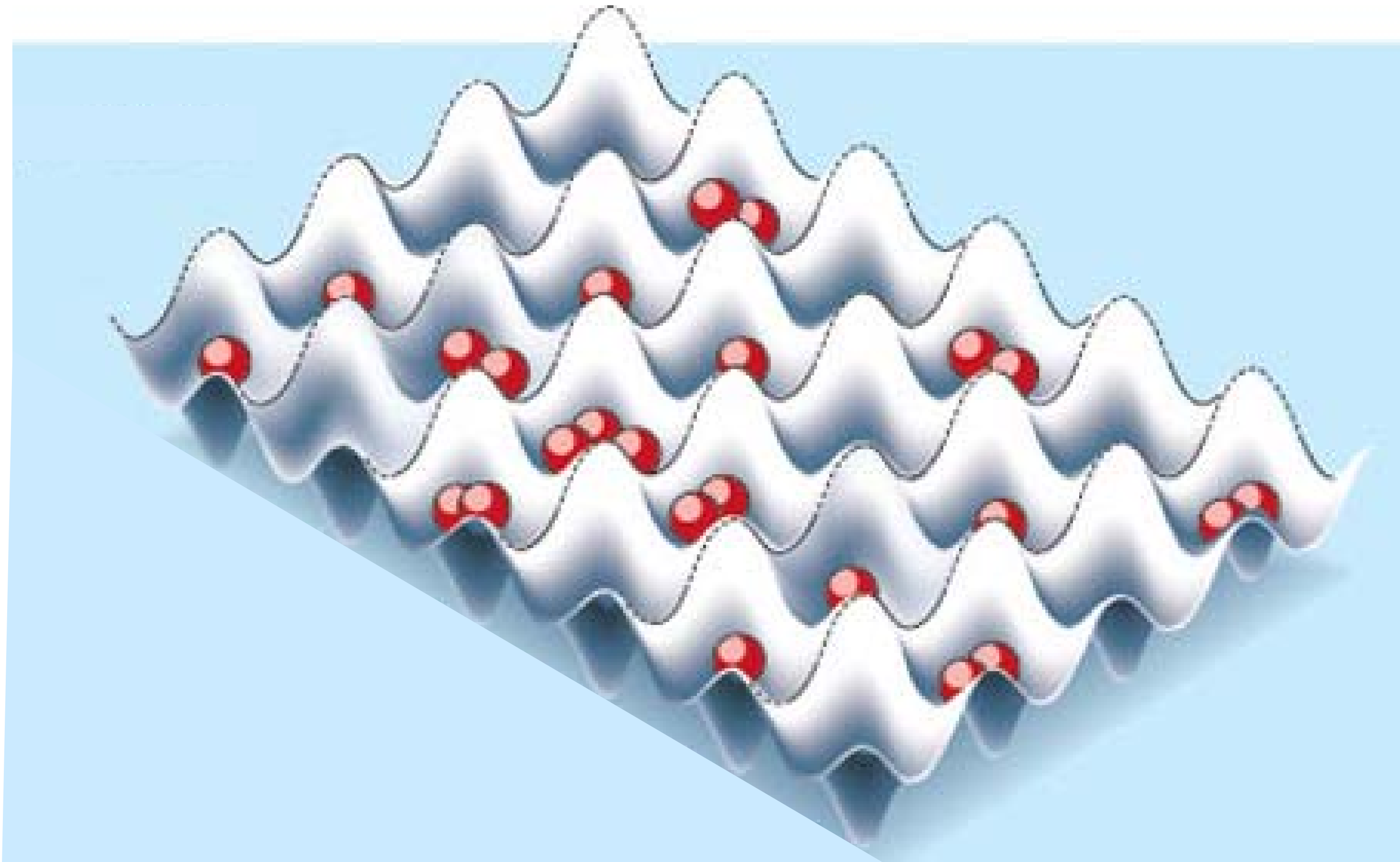
Bose condensation

Velocity distribution function of ultracold ^{87}Rb atoms

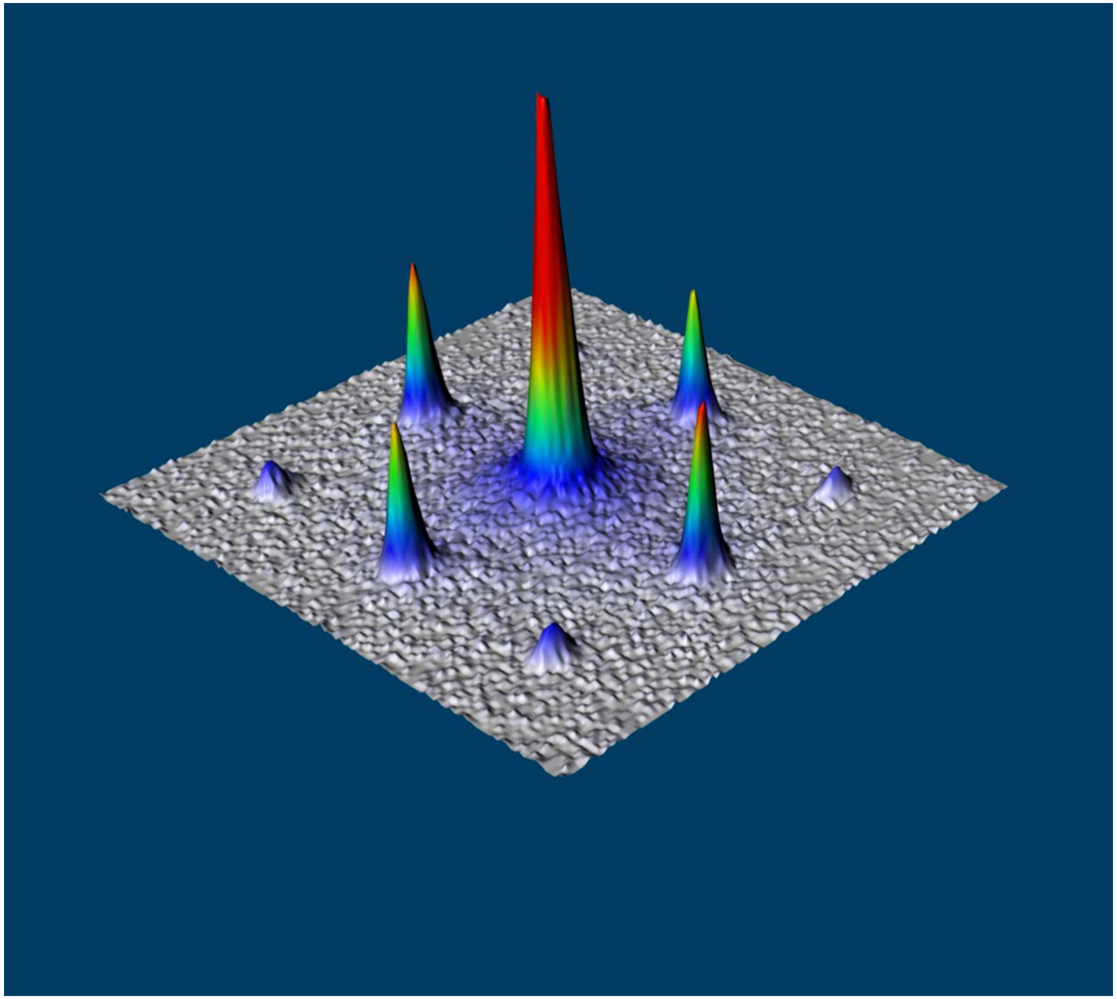


M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman and E. A. Cornell, *Science* **269**, 198 (1995)

Apply a periodic potential (standing laser beams) to trapped ultracold bosons (^{87}Rb)



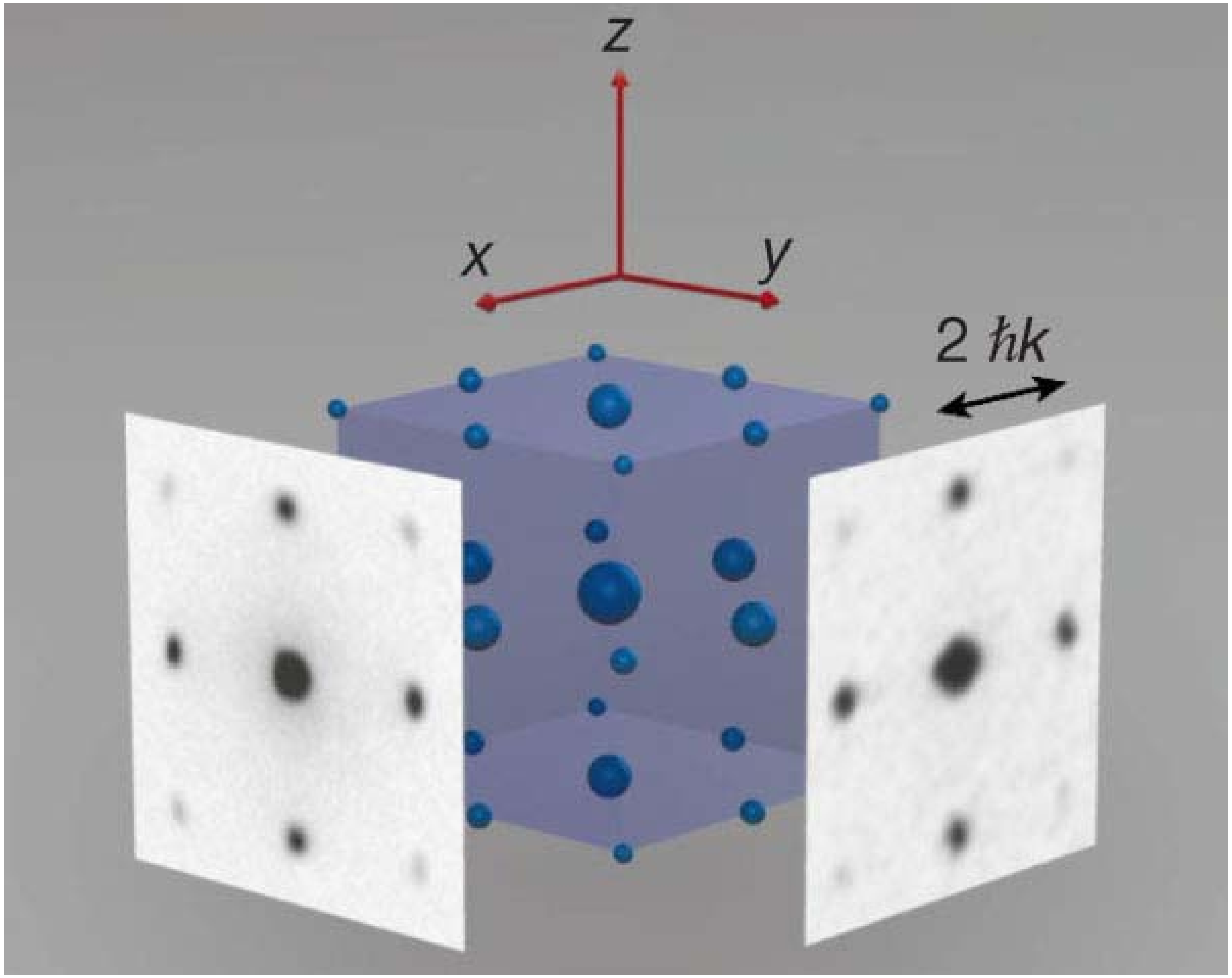
Momentum distribution function of bosons



Bragg reflections of condensate at reciprocal lattice vectors

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

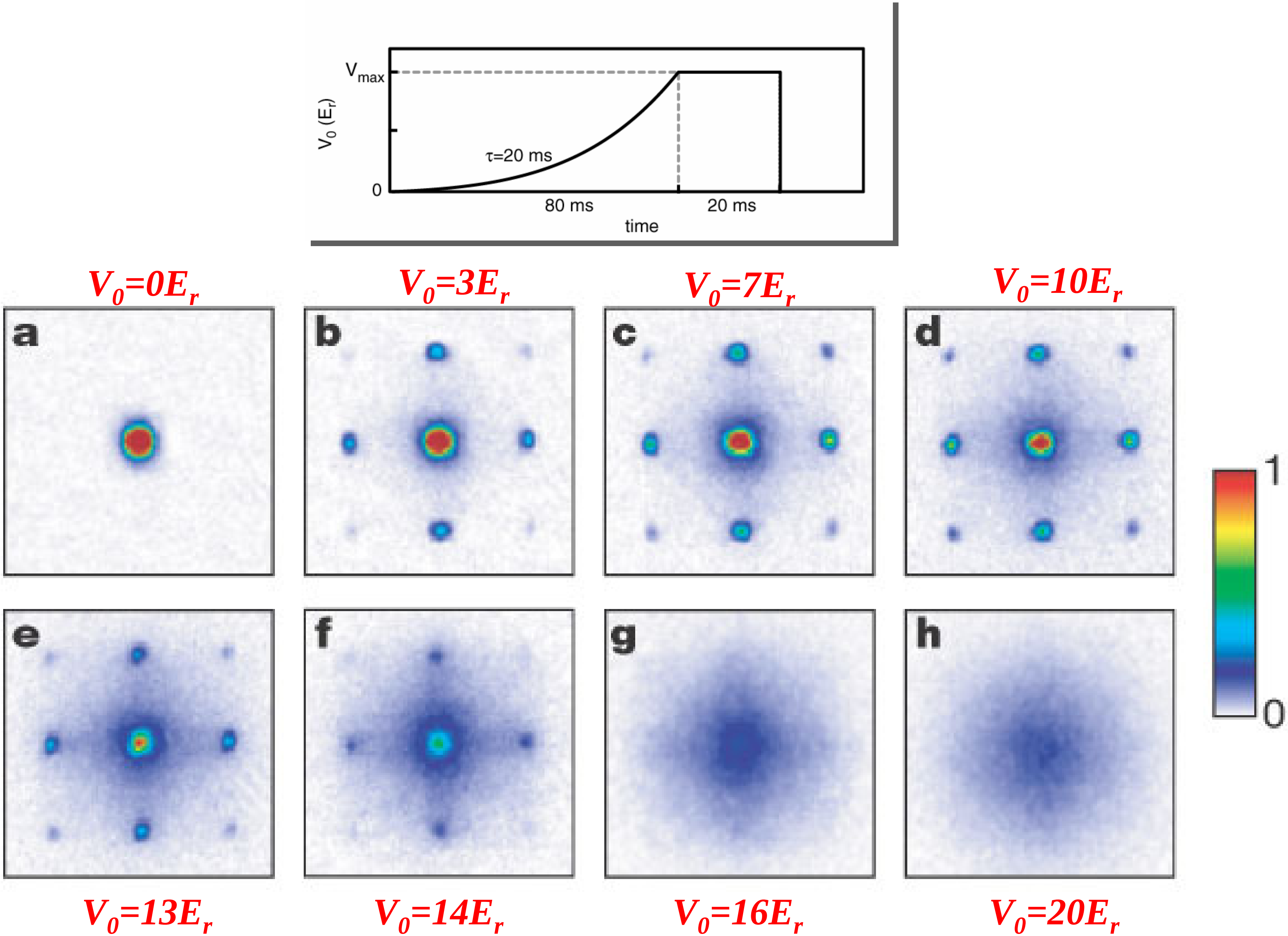
Momentum distribution function of bosons



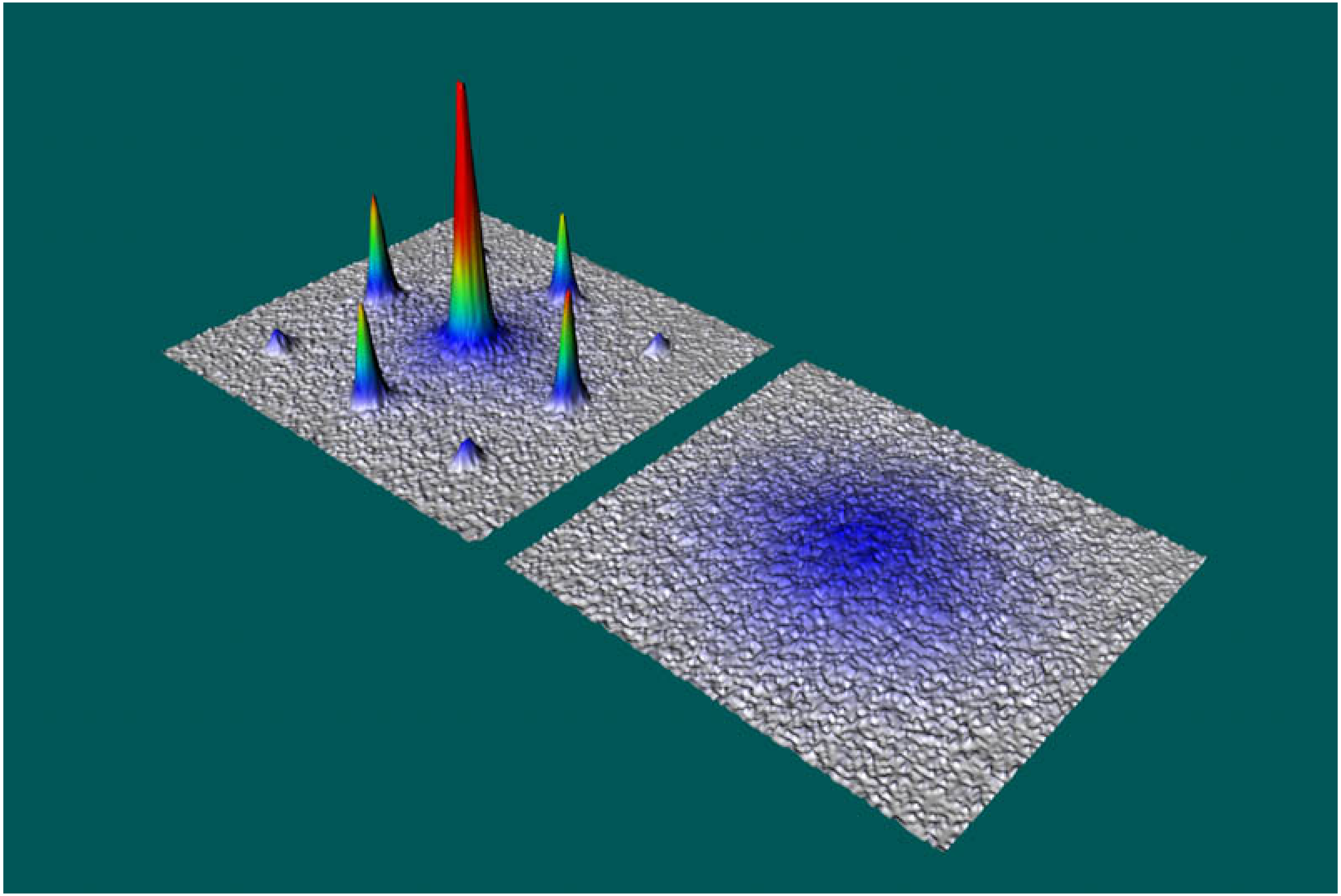
Bragg reflections of condensate at reciprocal lattice vectors

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Superfluid-insulator quantum phase transition at T=0

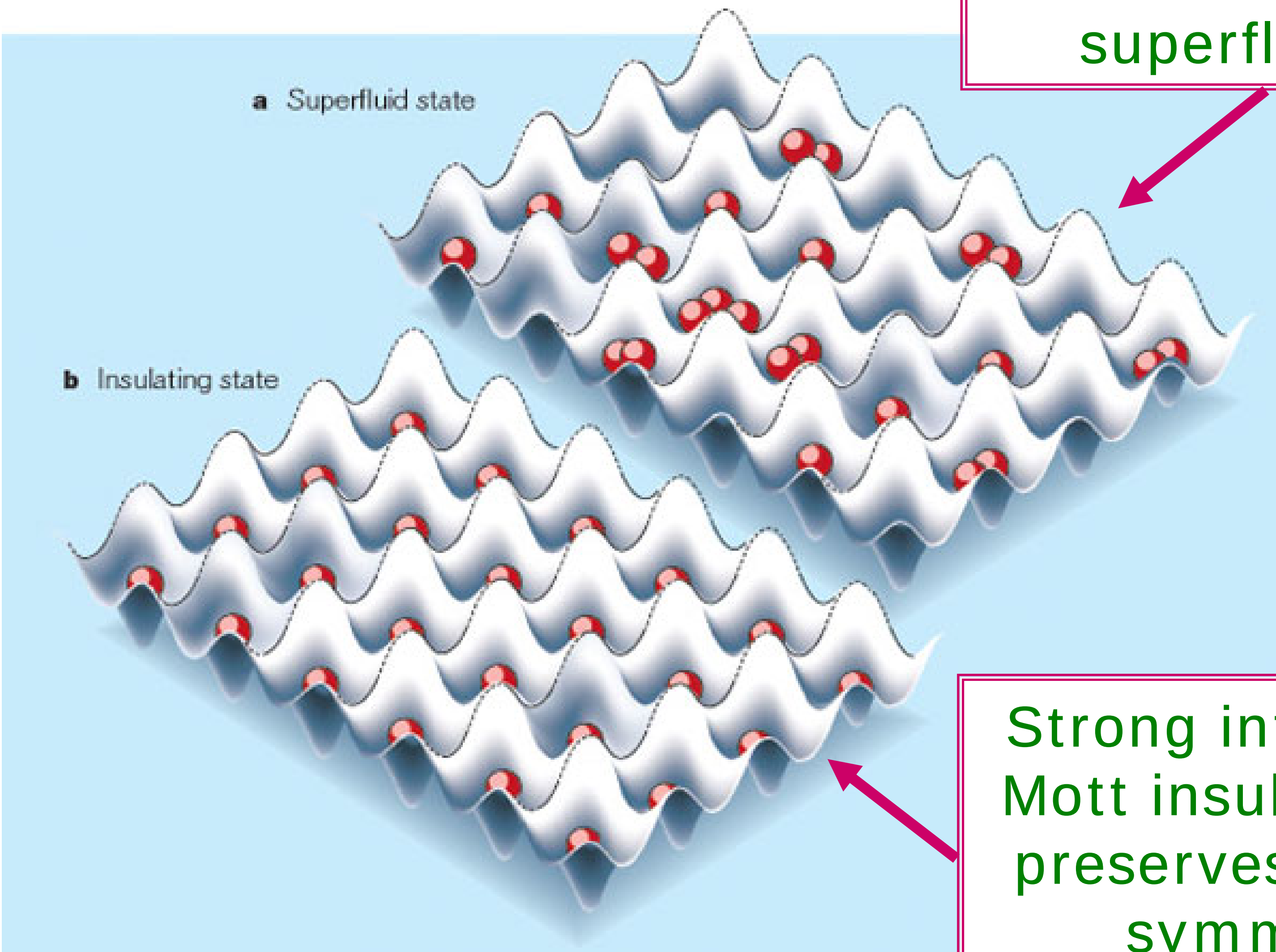


Superfluid-insulator quantum phase transition at T=0



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Bosons at filling fraction $f = 1$

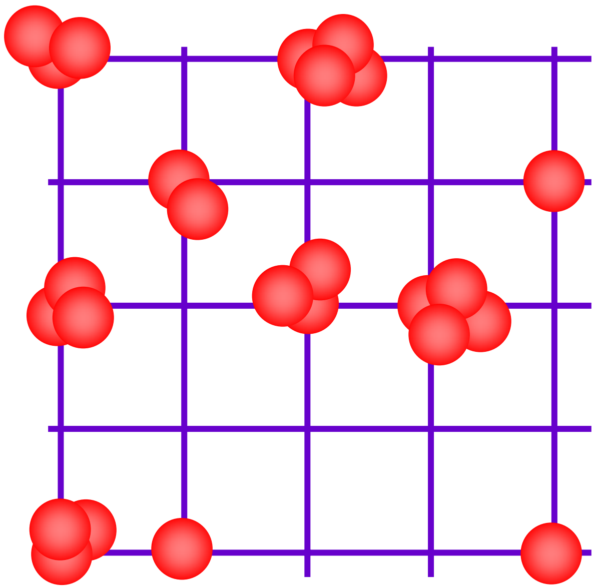


Weak interactions: superfluidity

Strong interactions: Mott insulator which preserves all lattice symmetries

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

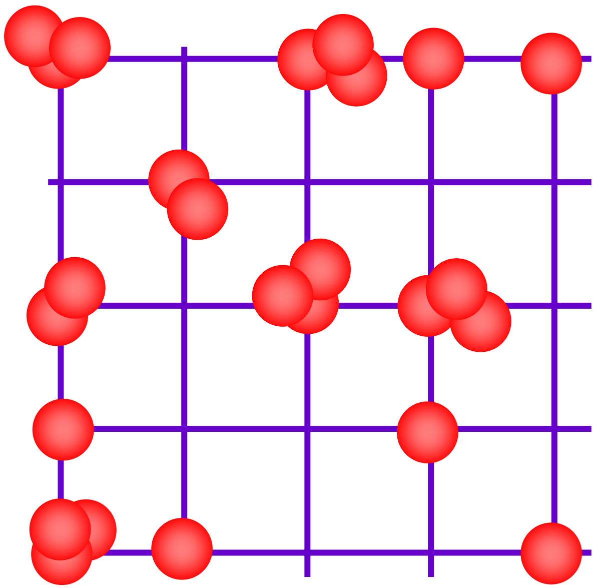
Bosons at filling fraction $f = 1$



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

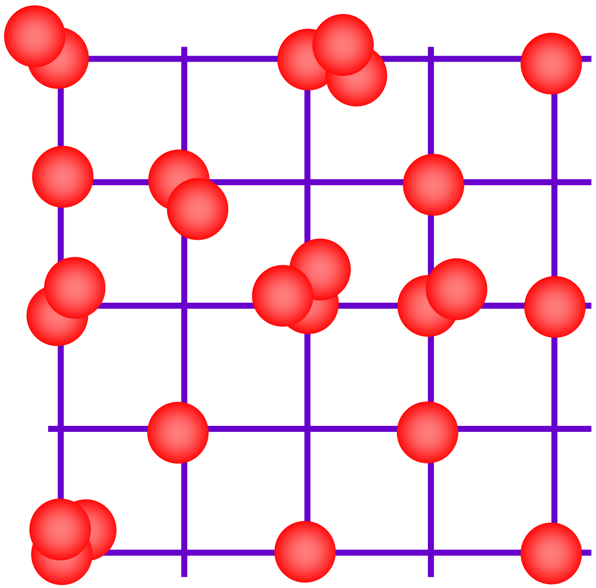
Bosons at filling fraction $f = 1$



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

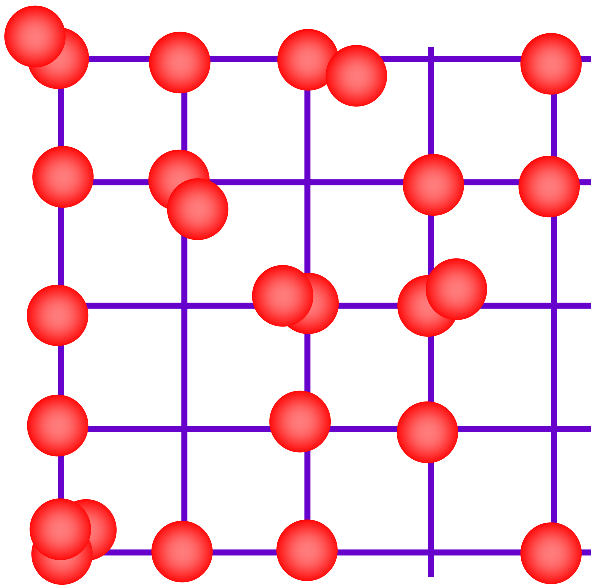
Bosons at filling fraction $f = 1$



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

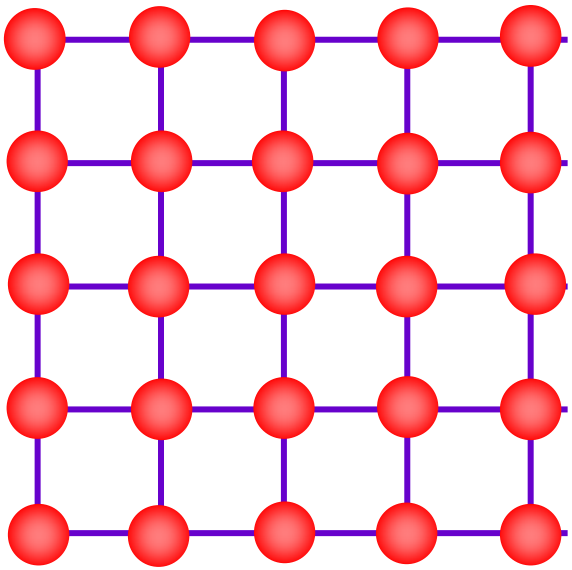
Bosons at filling fraction $f = 1$



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

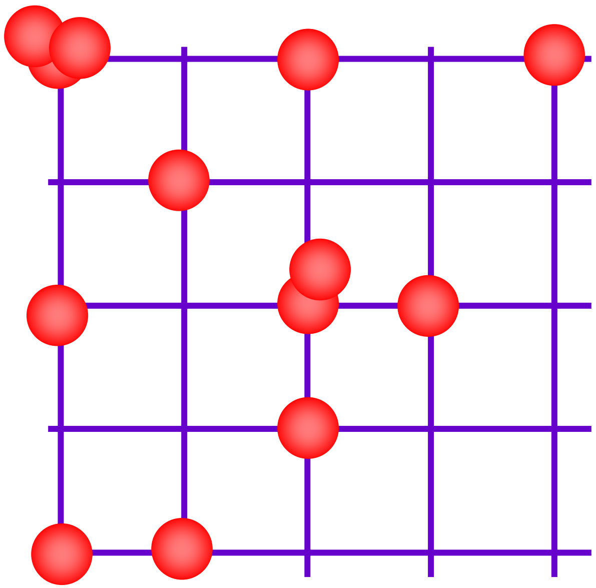
Bosons at filling fraction $f = 1$



$$\langle \Psi \rangle = 0$$

Strong interactions: insulator

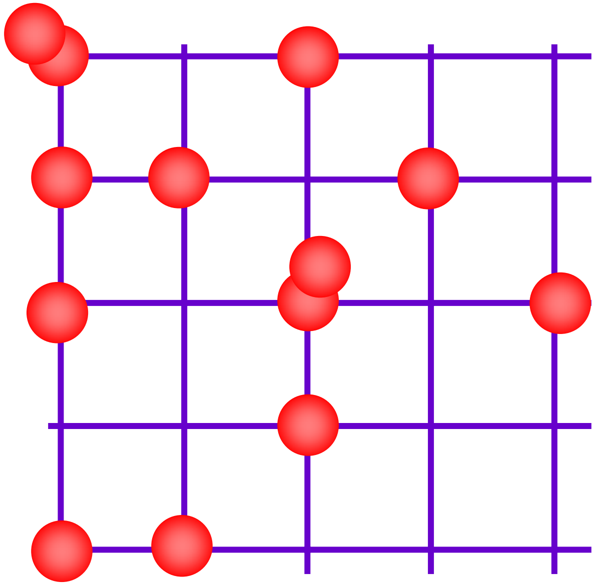
Bosons at filling fraction $f = 1/2$



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

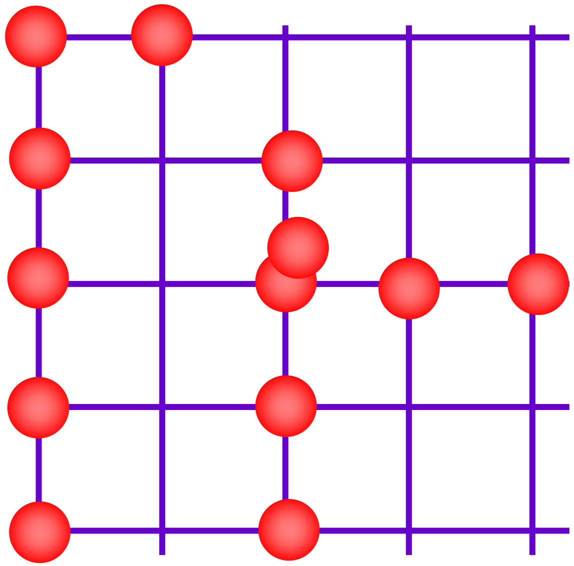
Bosons at filling fraction $f = 1/2$



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

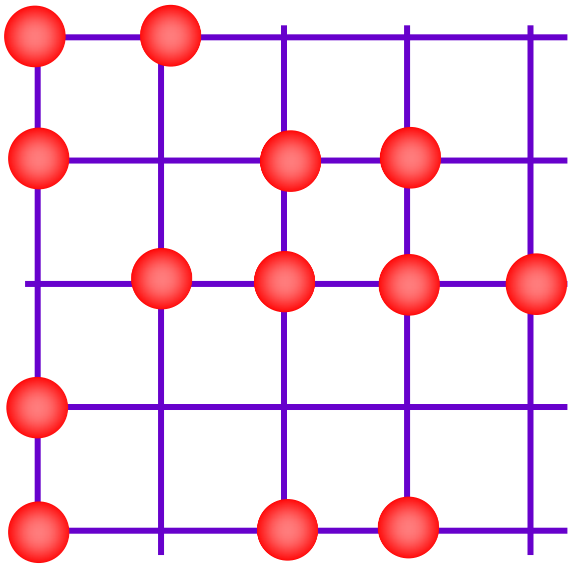
Bosons at filling fraction $f = 1/2$



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

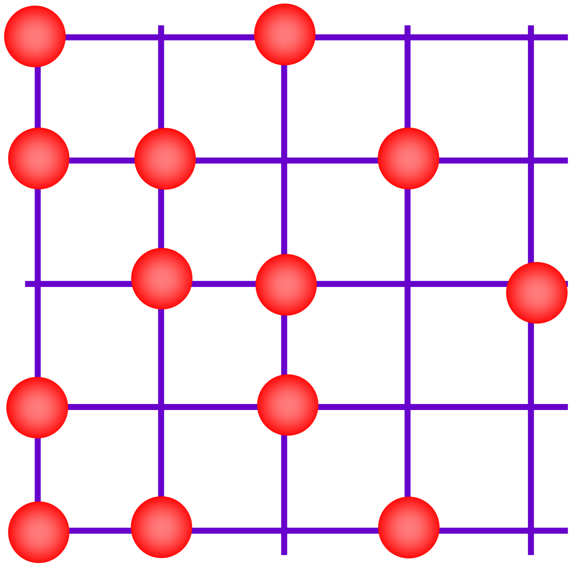
Bosons at filling fraction $f = 1/2$



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

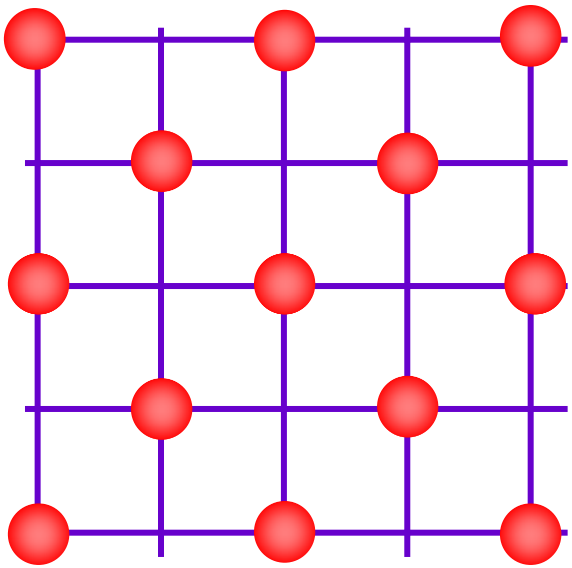
Bosons at filling fraction $f = 1/2$



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

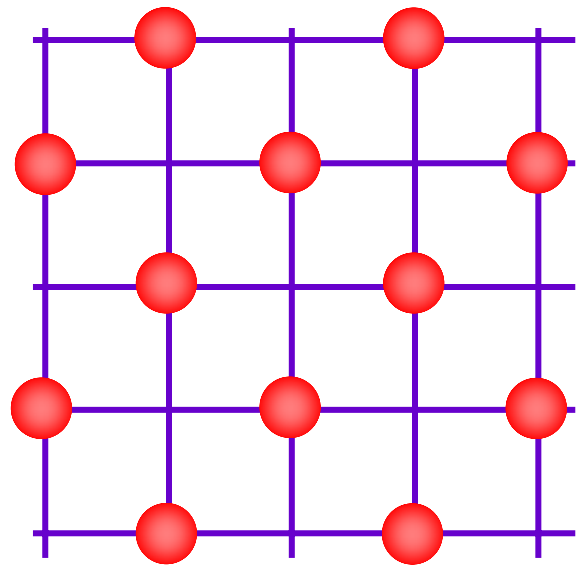
Bosons at filling fraction $f = 1/2$



$$\langle \Psi \rangle = 0$$

Strong interactions: insulator

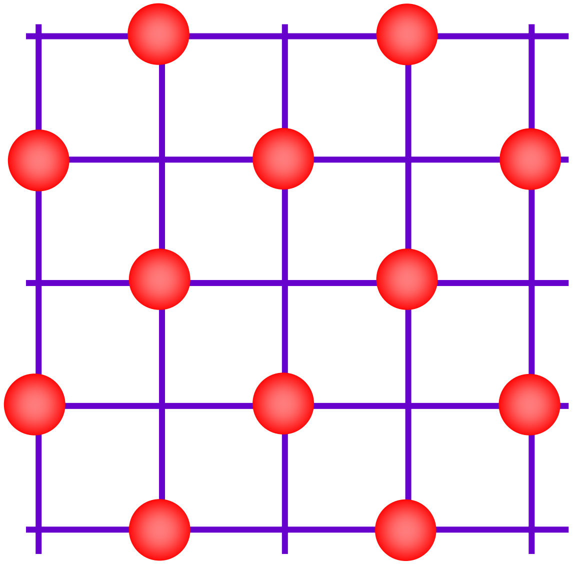
Bosons at filling fraction $f = 1/2$



$$\langle \Psi \rangle = 0$$

Strong interactions: insulator

Bosons at filling fraction $f = 1/2$



$$\langle \Psi \rangle = 0$$

Strong interactions: insulator

Insulator has "density wave" order

Superfluid-insulator transition of bosons at generic filling fraction f

The transition is characterized by multiple distinct order parameters (boson condensate, density-wave order……..)

Traditional (Landau-Ginzburg-Wilson) view:

Such a transition is first order, and there are no precursor fluctuations of the order of the insulator in the superfluid.

Superfluid-insulator transition of bosons at generic filling fraction f

The transition is characterized by multiple distinct order parameters (boson condensate, density-wave order..........)

Traditional (Landau-Ginzburg-Wilson) view:

Such a transition is first order, and there are no precursor fluctuations of the order of the insulator in the superfluid.

Recent theories:

Quantum interference effects can render such transitions second order, and the superfluid does contain precursor CDW fluctuations.

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Outline

I. The superfluid-Mott insulator quantum phase transition

II. Dual theory: vortices and their wavefunctions

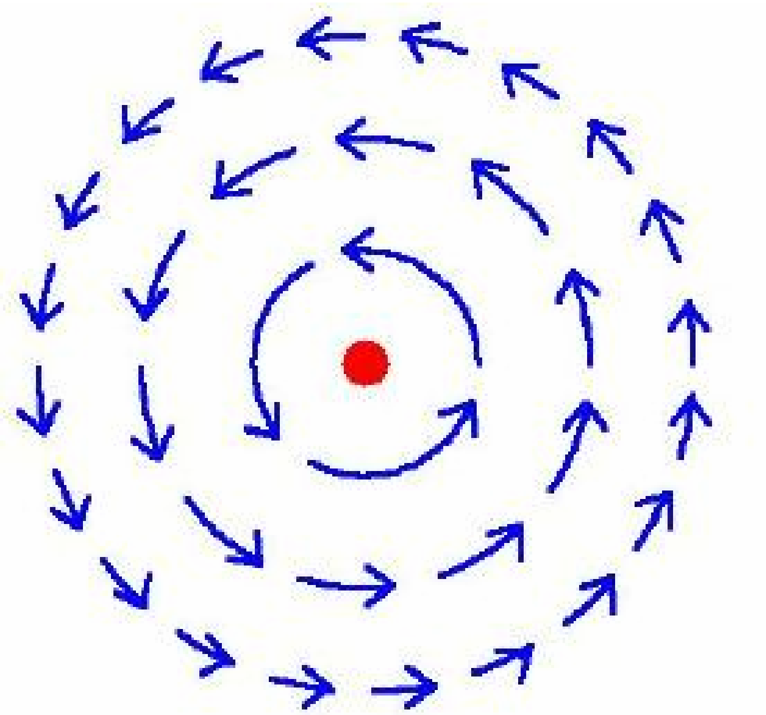
III. Vortices in superfluids near the superfluid-insulator quantum phase transition
Vortex wavefunction lives in a dual flavor space

IV. The cuprate superconductors
Detection of vortex flavors ?

II. The quantum mechanics of vortices

Magnus forces, duality, and point vortices as dual "electric" charges

Excitations of the superfluid: **Vortices**

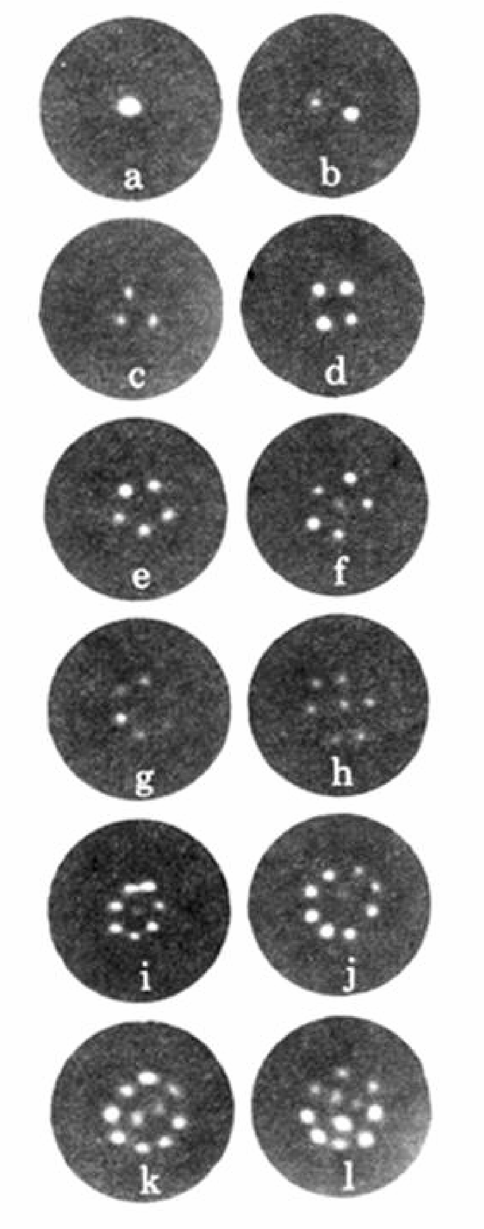


The circulation of a vortex is quantized:

$$\oint \mathbf{v}_s \cdot d\mathbf{r} = \frac{\hbar}{m} \oint \nabla\theta \cdot d\mathbf{r} = n\frac{h}{m}$$

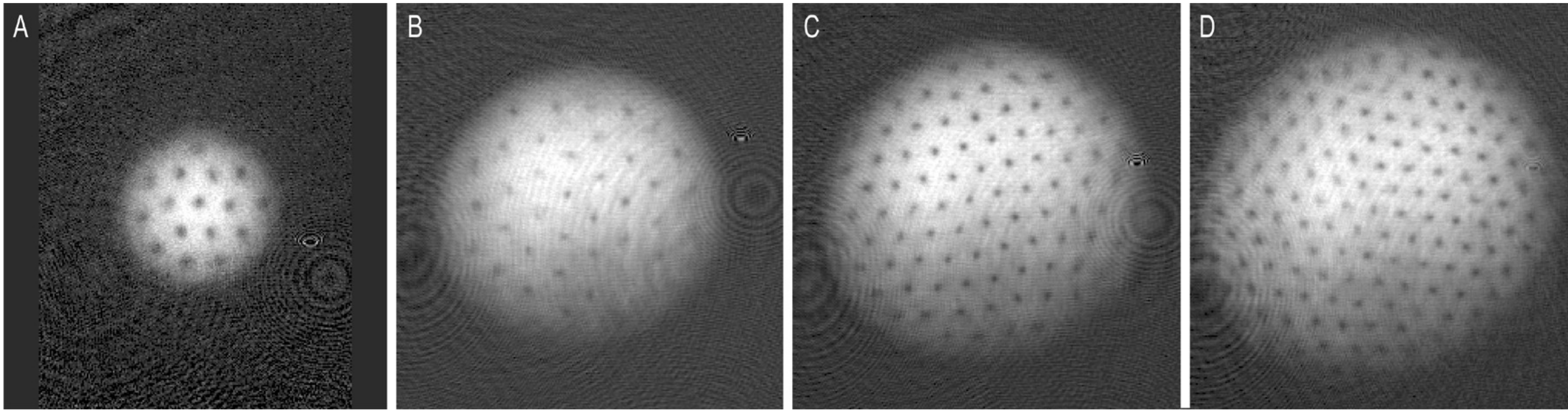
where n is an integer.

Observation of quantized vortices in rotating ^{4}He



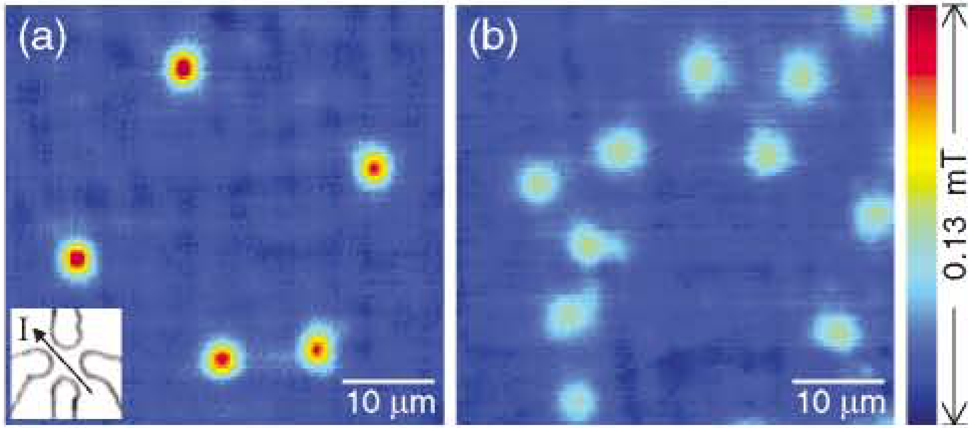
E.J. Yarmchuk, M.J.V. Gordon, and R.E. Packard,
Observation of Stationary Vortex Arrays in Rotating Superfluid Helium,
Phys. Rev. Lett. **43**, 214 (1979).

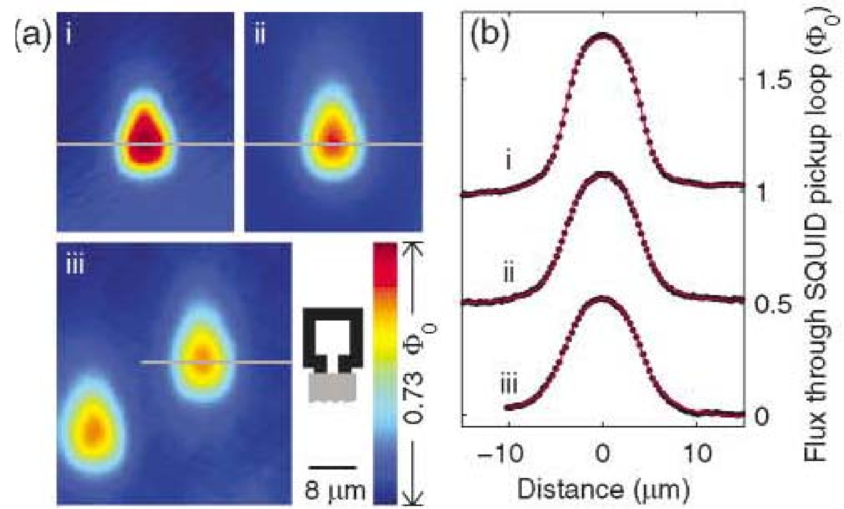
Observation of quantized vortices in rotating ultracold Na



J. R. Abo-Shaeer, C. Raman, J. M. Vogels, and W. Ketterle,
Observation of Vortex Lattices in Bose-Einstein Condensates,
Science **292**, 476 (2001).

Quantized fluxoids in $YBa_2Cu_3O_{6+y}$



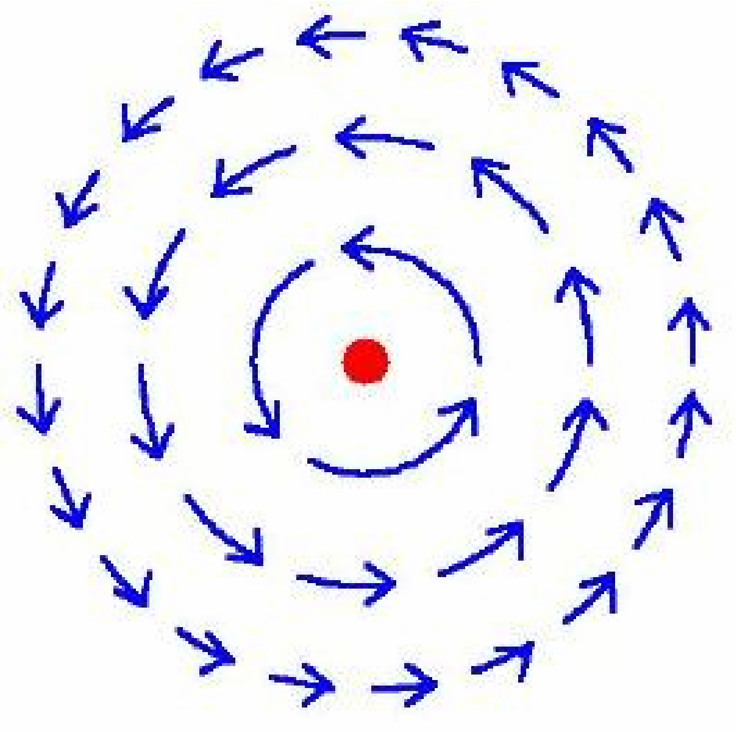


J. C. Wynn, D. A. Bonn, B.W. Gardner, Yu-Ju Lin, Ruixing Liang, W. N. Hardy, J. R. Kirtley, and K. A. Moler, *Phys. Rev. Lett.* **87**, 197002 (2001).

In superconductors, vortices carry quantized magnetic flux:

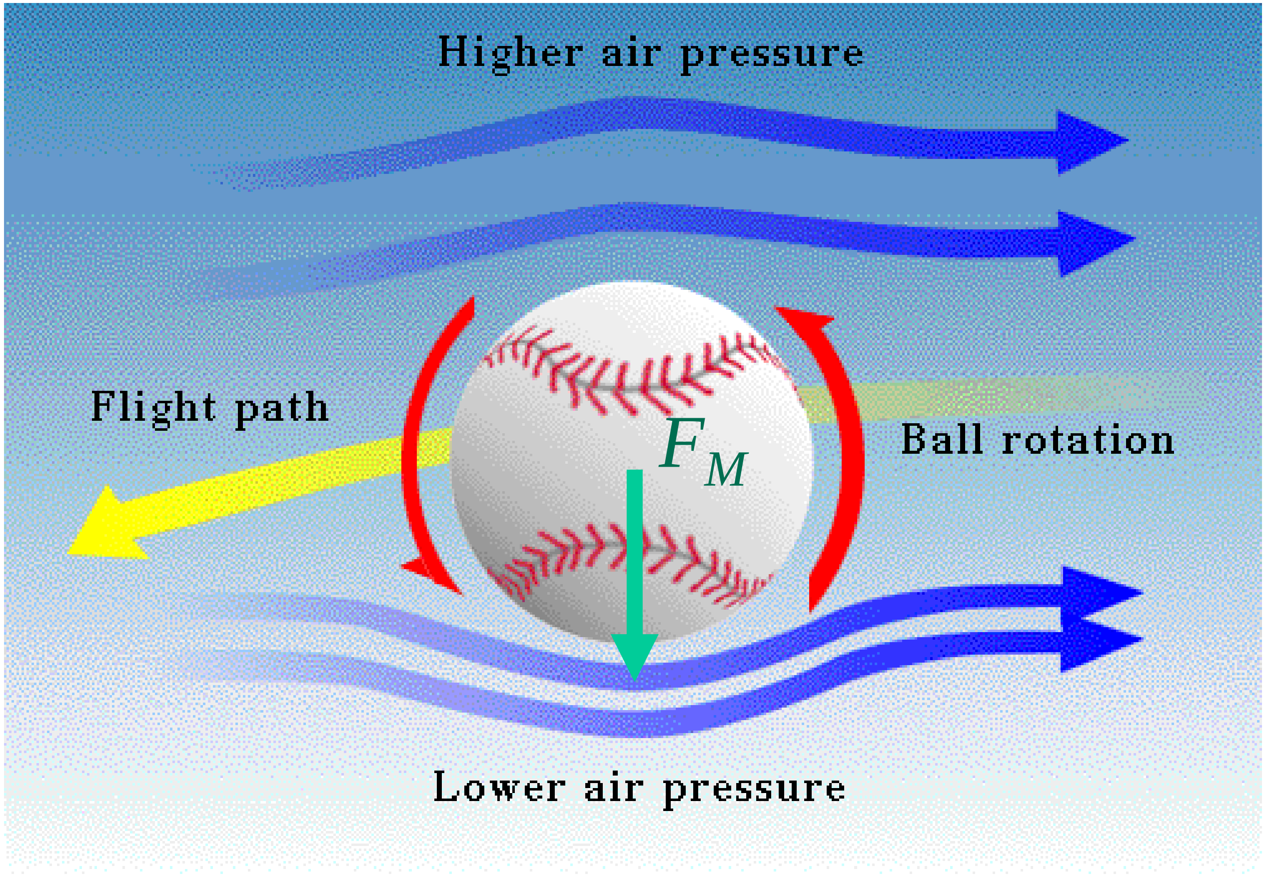
$$\int \mathbf{B} \cdot d\mathbf{S} = n\frac{hc}{2e}$$

Excitations of the superfluid: **Vortices**



<u>Central question</u>:
In two dimensions, we can view the vortices as point particle excitations of the superfluid. What is the quantum mechanics of these "particles" ?

In ordinary fluids, vortices experience the Magnus Force



$$F_M = (\text{mass density of air}) \cdot (\text{velocity of ball}) \cdot (\text{circulation})$$

For a vortex in a superfluid, this is

$$\mathbf{F}_M = (m\rho)\left(\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}}\right)\left(\oint \mathbf{v}_s \cdot d\mathbf{r}\right)$$
$$= nh\rho\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}}$$

where ρ = number density of bosons

$\mathbf{v}_s$ = local velocity of superfluid

$\mathbf{r}_v$ = position of vortex

For a vortex in a superfluid, this is

$$\mathbf{F}_M = (m\rho)\left(\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}}\right)\left(\oint \mathbf{v}_s \cdot d\mathbf{r}\right)$$

$$= nh\rho\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}}$$

$$= n\left(\mathbf{E} + \frac{d\mathbf{r}_v}{dt} \times \mathbf{B}\right)$$

where $\mathbf{E} = \rho\mathbf{v}_s \times \hat{\mathbf{z}}$ and $\mathbf{B} = -h\rho\hat{\mathbf{z}}$

Dual picture:

The vortex is a quantum particle with dual "electric" charge *n*, moving in a dual "magnetic" field of strength = *h*×(number density of Bose particles)

Outline

I. The superfluid-Mott insulator quantum phase transition

II. Dual theory: vortices and their wavefunctions

III. Vortices in superfluids near the superfluid-insulator quantum phase transition

Vortex wavefunction lives in a dual flavor space

IV. The cuprate superconductors

Detection of vortex flavors ?

III. Vortices in superfluids near the superfluid-insulator quantum phase transition

Vortices carry a "flavor" index which encodes the density-wave order of the proximate insulator

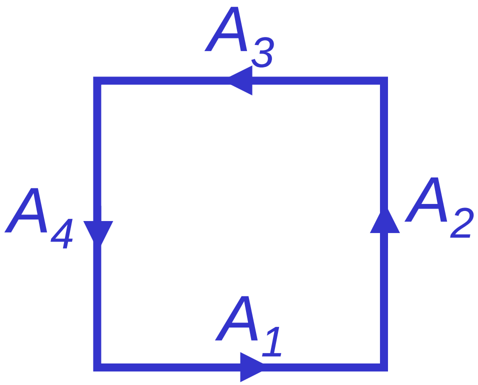
See also work by Z. Tesanovic, M. Franz, A. Melikyan
Phys. Rev. Lett. **93**, 217004; *Phys. Rev*. B **71**, 214511.

- The vortices are quantum particles moving in a periodic potential with the symmetry of the square lattice, and in the presence of a dual "magnetic" field of strength $= h\rho$, where ρ is the number density of bosons per unit cell.

- The vortex motion can be described by the effective Hofstadter Hamiltonian:

$$\mathcal{H}_v = -t \sum_{\langle ij \rangle} \left(e^{iA_{ij}} \varphi_i^* \varphi_j + \text{c.c.} \right)$$

where φ_i is an operator which annihilates a vortex particle at site i of a square lattice.



$$A_1 + A_2 + A_3 + A_4 = 2\pi f$$

where f is the boson filling fraction.

Bosons at filling fraction $f = 1$

• At f=1, the “magnetic” flux per unit cell is 2π, and the vortex does not pick up any phase from the boson density.

• The effective dual “magnetic” field acting on the vortex is zero, and the corresponding component of the Magnus force vanishes.

Bosons at rational filling fraction f=p/q

Quantum mechanics of the vortex "particle" in a periodic potential with *f* flux quanta per unit cell

Space group symmetries of Hofstadter Hamiltonian:

T_x, T_y : Translations by a lattice spacing in the x, y directions

R : Rotation by 90 degrees.

Magnetic space group:

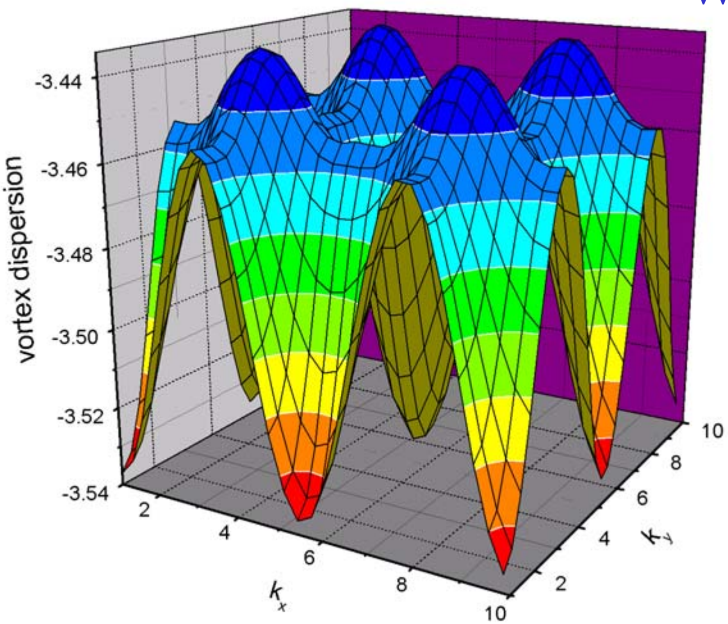
$$T_x T_y = e^{2\pi i f} T_y T_x \quad ;$$

$$R^{-1} T_y R = T_x \quad ; \quad R^{-1} T_x R = T_y^{-1} \quad ; \quad R^4 = 1$$

The low energy vortex states must form a representation of this algebra

Vortices in a superfluid near a Mott insulator at filling $f=p/q$

Hofstadter spectrum of the quantum vortex "particle" with field operator φ



At filling $f=p/q$, there are q species of vortices, φ_ℓ (with ℓ=1…q), associated with q degenerate minima in the vortex spectrum. These vortices realize the smallest, q-dimensional, representation of the magnetic algebra.

$$T_x : \varphi_\ell \rightarrow \varphi_{\ell+1} \quad ; \quad T_y : \varphi_\ell \rightarrow e^{2\pi i \ell f} \varphi_\ell$$

$$R : \varphi_\ell \rightarrow \frac{1}{\sqrt{q}} \sum_{m=1}^{q} \varphi_m e^{2\pi i \ell m f}$$

Vortices in a superfluid near a Mott insulator at filling $f=p/q$

The wavefunction of the φ_ℓ vortices in flavor space characterizes the density-wave order

Density-wave order:

Status of space group symmetry determined by density operators $\rho_{\mathbf{Q}}$ at wavevectors $\mathbf{Q}_{mn} = \frac{2\pi p}{q}(m,n)$

$$\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell mf}$$

$$T_x : \rho_{\mathbf{Q}} \to \rho_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\hat{x}} \quad ; \quad T_y : \rho_{\mathbf{Q}} \to \rho_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\hat{y}}$$

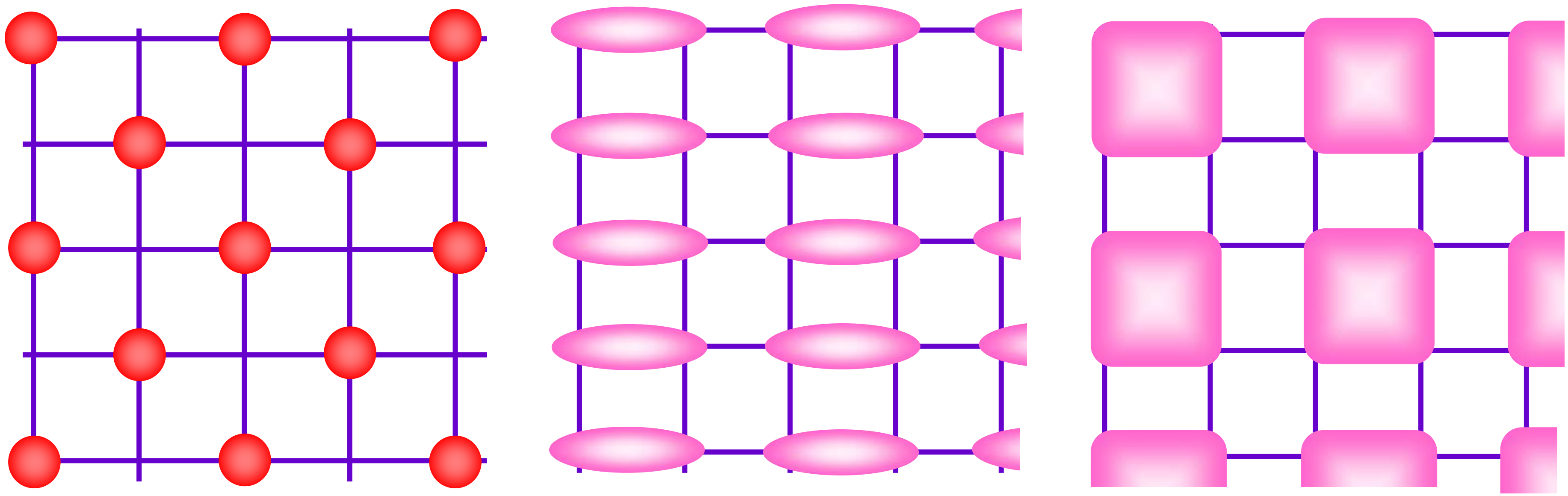
$$R : \rho(\mathbf{Q}) \to \rho(R\mathbf{Q})$$

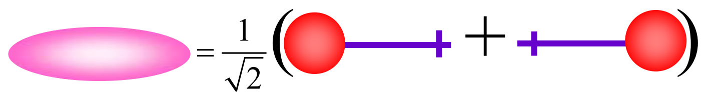
Vortices in a superfluid near a Mott insulator at filling $f=p/q$

- The excitations of the superfluid are described by the quantum mechanics of q flavors of low energy vortices moving in zero dual "magnetic" field.

- The orientation of the vortex in flavor space implies a particular configuration of density-wave order in its vicinity.

Mott insulators obtained by condensing vortices at $f = 1/2$





Charge density wave (CDW) order

Valence bond solid (VBS) order

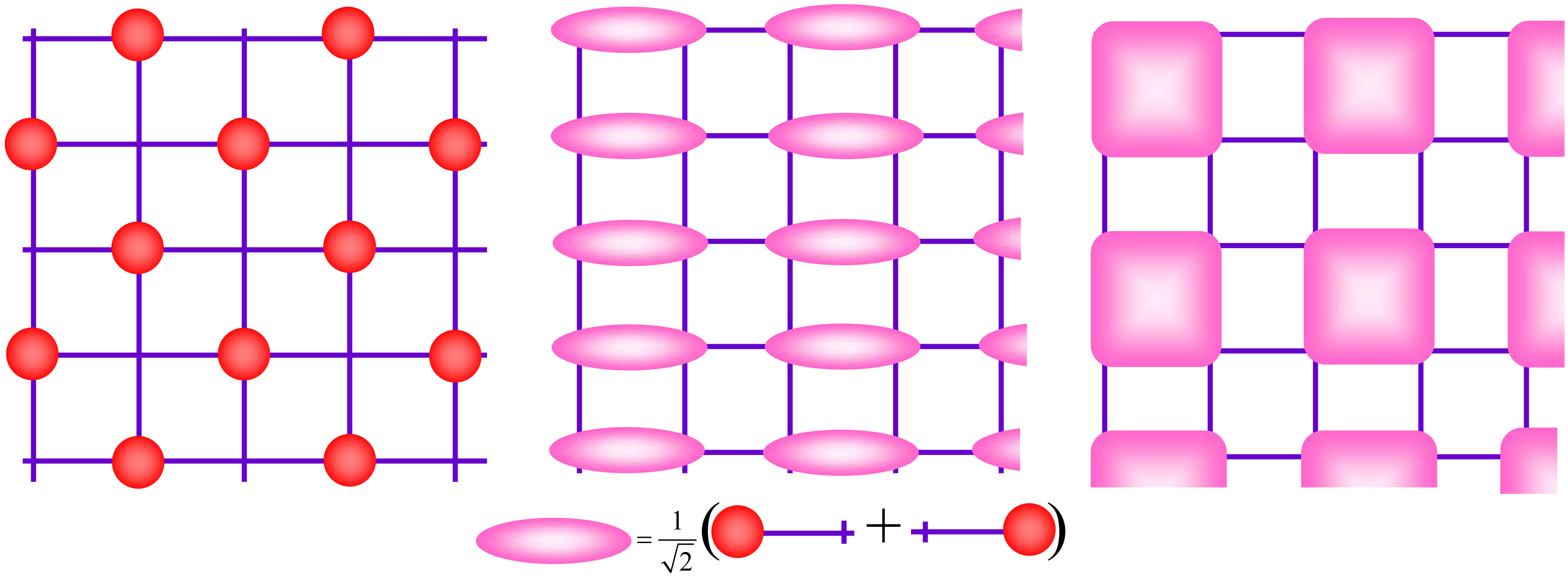
Valence bond solid (VBS) order

Can define a common CDW/VBS order using a generalized "density" $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$

All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\mathbf{Q}} \rangle \neq 0$ for certain $\boldsymbol{Q}$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)
S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Mott insulators obtained by condensing vortices at $f = 1/2$



Charge density wave (CDW) order

Valence bond solid (VBS) order

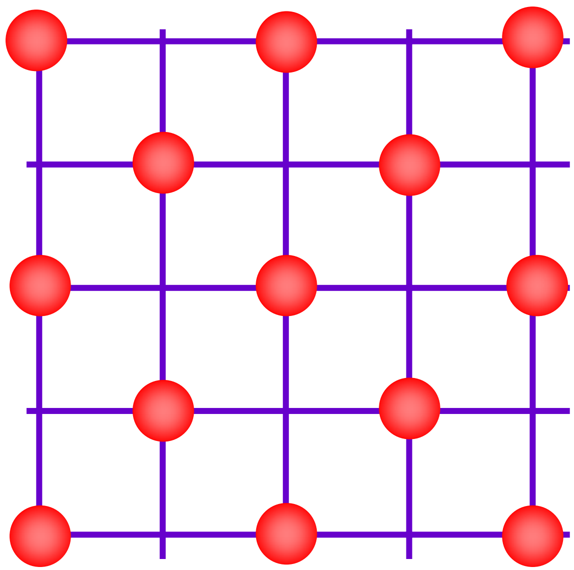
Valence bond solid (VBS) order

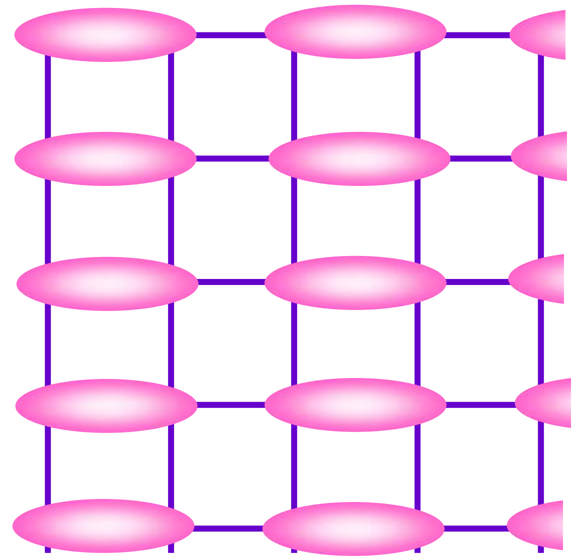
Can define a common CDW/VBS order using a generalized "density" $\rho(\boldsymbol{r}) = \sum_{\boldsymbol{Q}} \rho_{\boldsymbol{Q}} e^{i\boldsymbol{Q}.\boldsymbol{r}}$

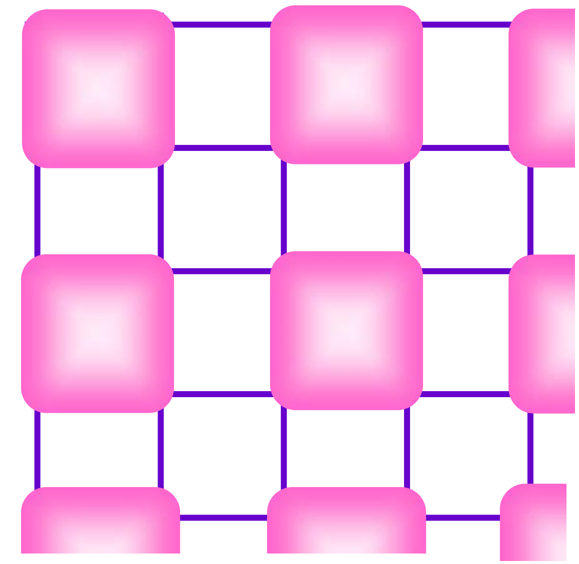
All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\boldsymbol{Q}} \rangle \neq 0$ for certain $\boldsymbol{Q}$

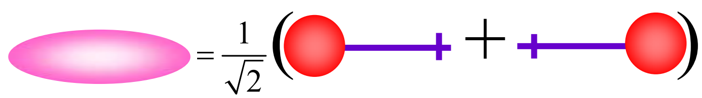
C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)
S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Mott insulators obtained by condensing vortices at $f = 1/2$









Charge density wave (CDW) order

Valence bond solid (VBS) order

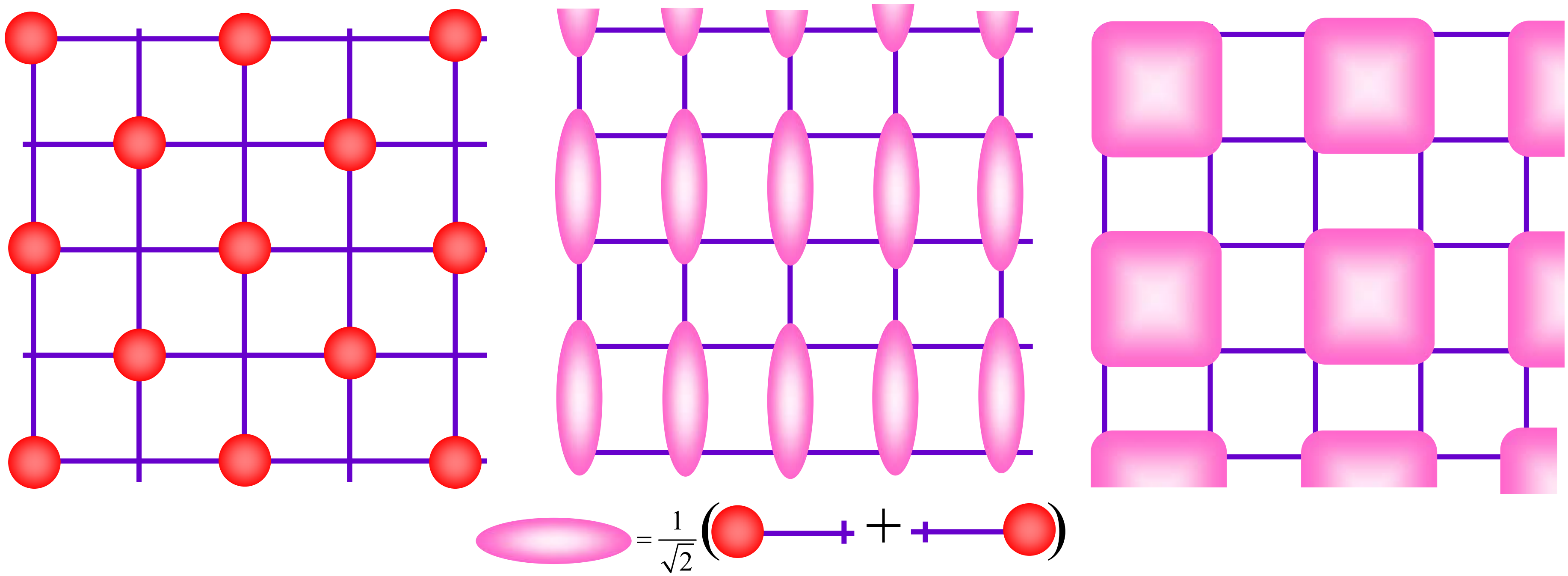
Valence bond solid (VBS) order

Can define a common CDW/VBS order using a generalized "density" $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$

All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\mathbf{Q}} \rangle \neq 0$ for certain $\mathbf{Q}$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)
S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Mott insulators obtained by condensing vortices at $f = 1/2$



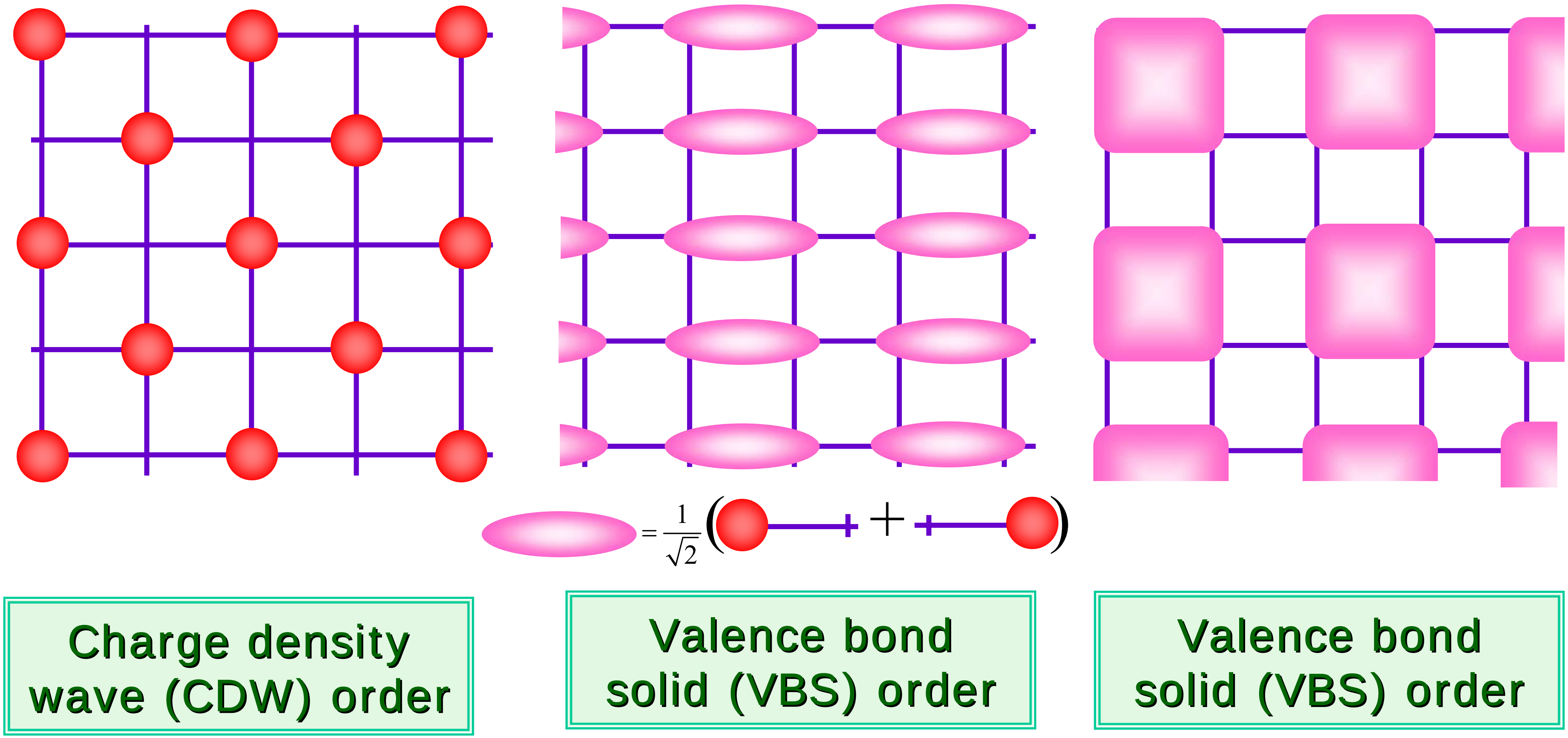
Charge density wave (CDW) order	Valence bond solid (VBS) order	Valence bond solid (VBS) order

Can define a common CDW/VBS order using a generalized "density" $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$

All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\mathbf{Q}} \rangle \neq 0$ for certain $\mathbf{Q}$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)
S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Mott insulators obtained by condensing vortices at $f = 1/2$

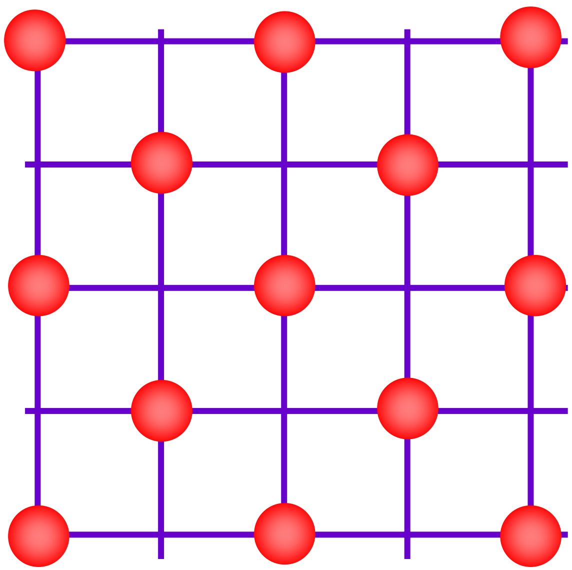


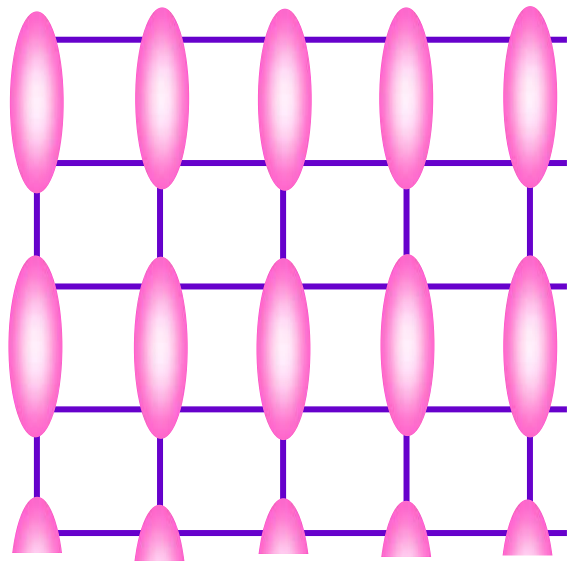
Can define a common CDW/VBS order using a generalized "density" $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$

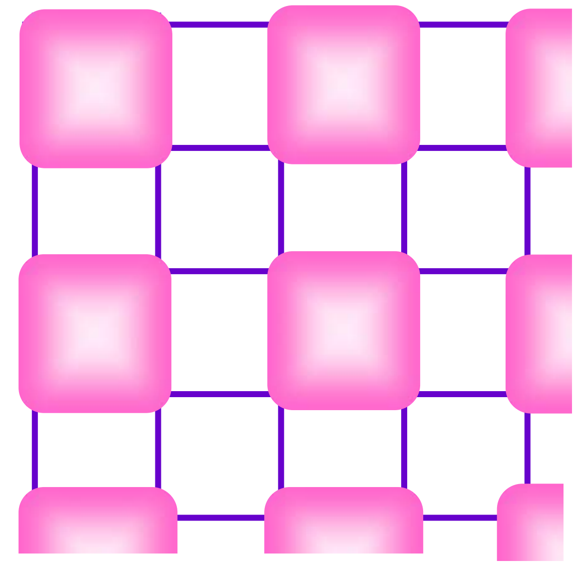
All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\mathbf{Q}} \rangle \neq 0$ for certain $\mathbf{Q}$

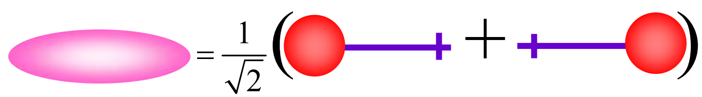
C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)
S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Mott insulators obtained by condensing vortices at $f = 1/2$









Charge density wave (CDW) order

Valence bond solid (VBS) order

Valence bond solid (VBS) order

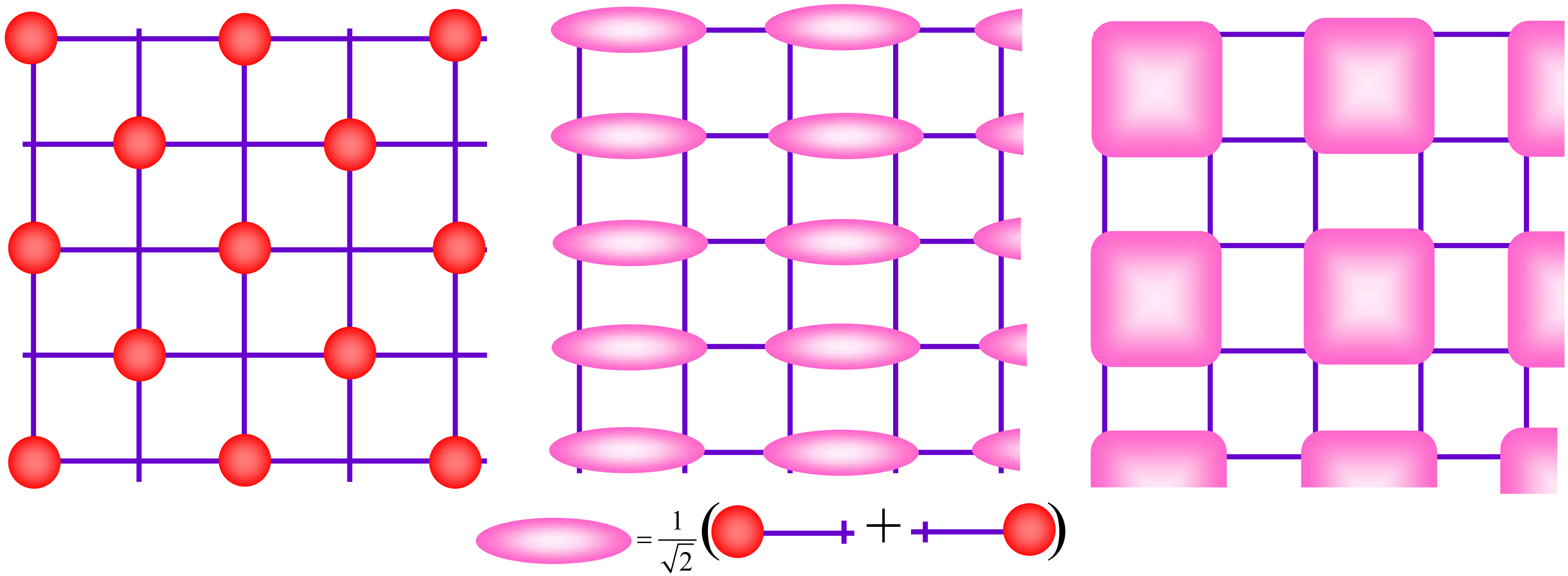
Can define a common CDW/VBS order using a generalized "density" $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$

All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\mathbf{Q}} \rangle \neq 0$ for certain $\mathbf{Q}$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Mott insulators obtained by condensing vortices at $f = 1/2$



Charge density wave (CDW) order

Valence bond solid (VBS) order

Valence bond solid (VBS) order

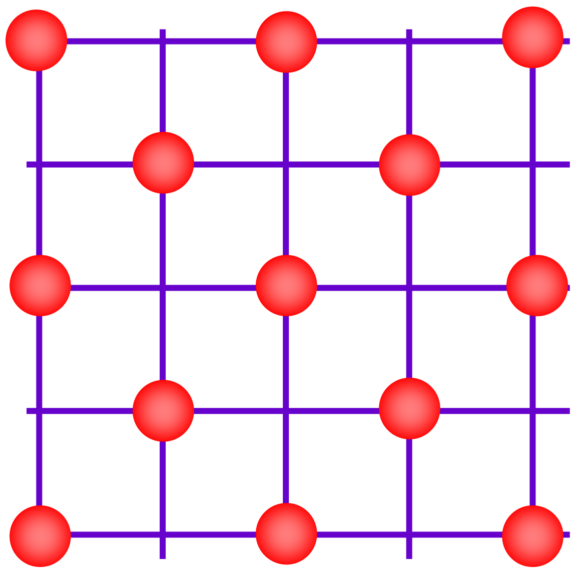
Can define a common CDW/VBS order using a generalized "density" $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$

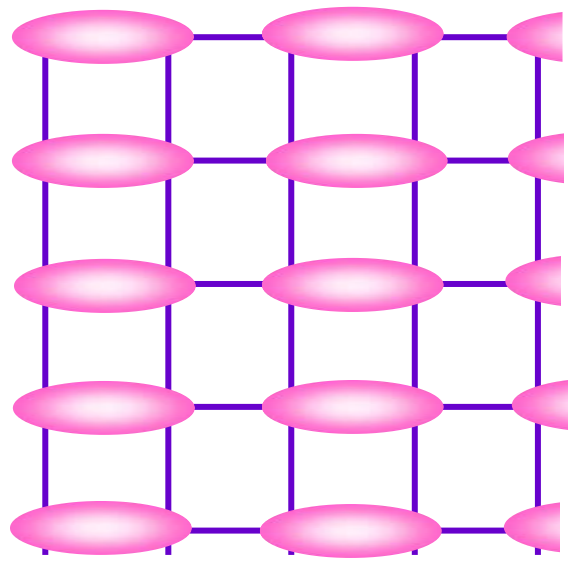
All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\mathbf{Q}} \rangle \neq 0$ for certain $\mathbf{Q}$

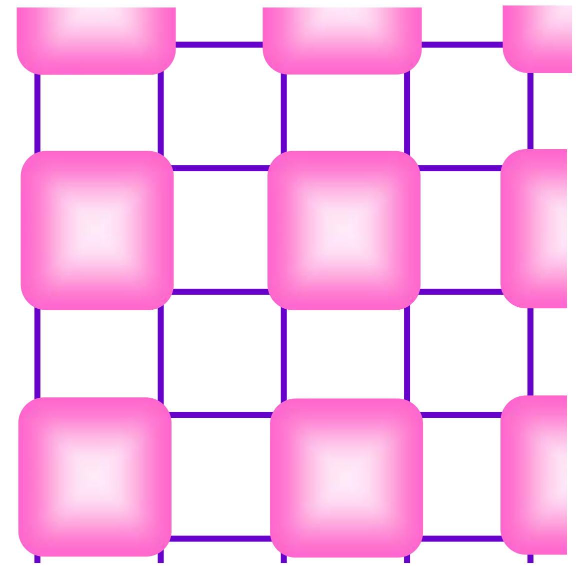
C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)

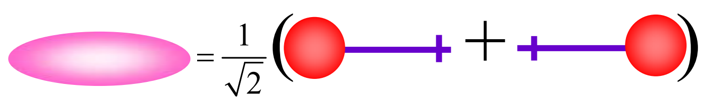
S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Mott insulators obtained by condensing vortices at $f = 1/2$









Charge density wave (CDW) order

Valence bond solid (VBS) order

Valence bond solid (VBS) order

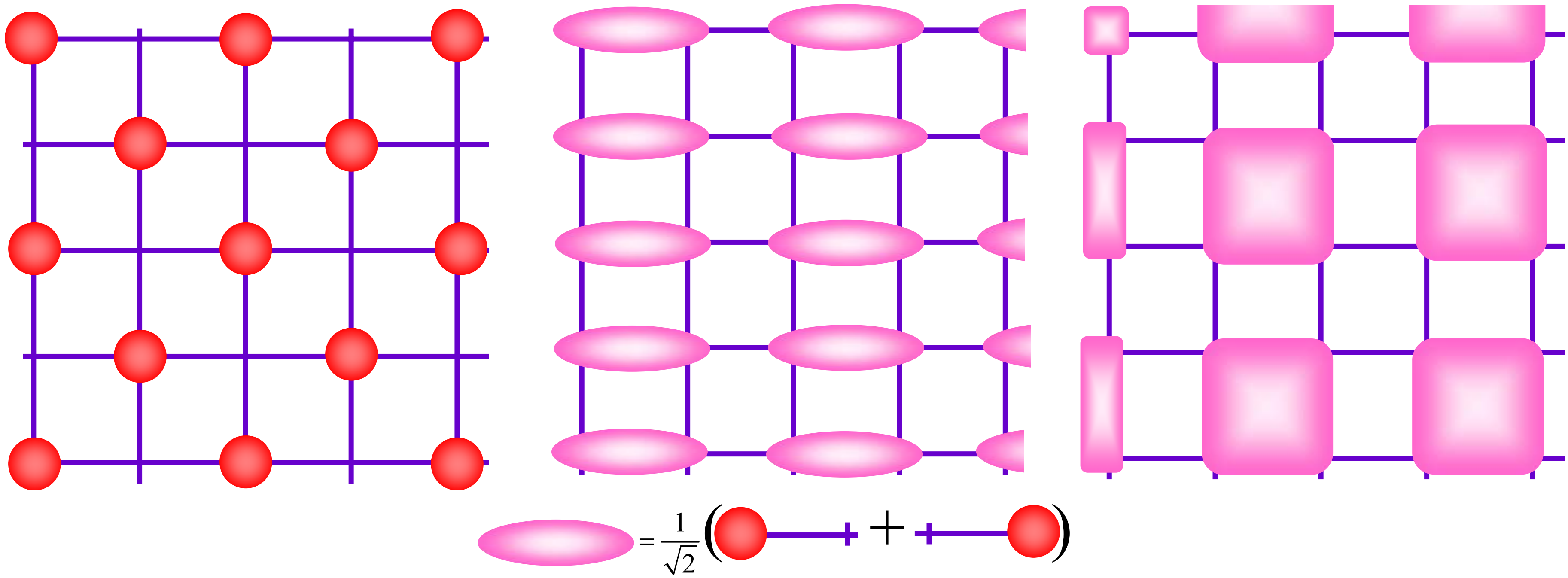
Can define a common CDW/VBS order using a generalized "density" $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$

All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\mathbf{Q}} \rangle \neq 0$ for certain $\mathbf{Q}$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Mott insulators obtained by condensing vortices at $f = 1/2$



Charge density wave (CDW) order

Valence bond solid (VBS) order

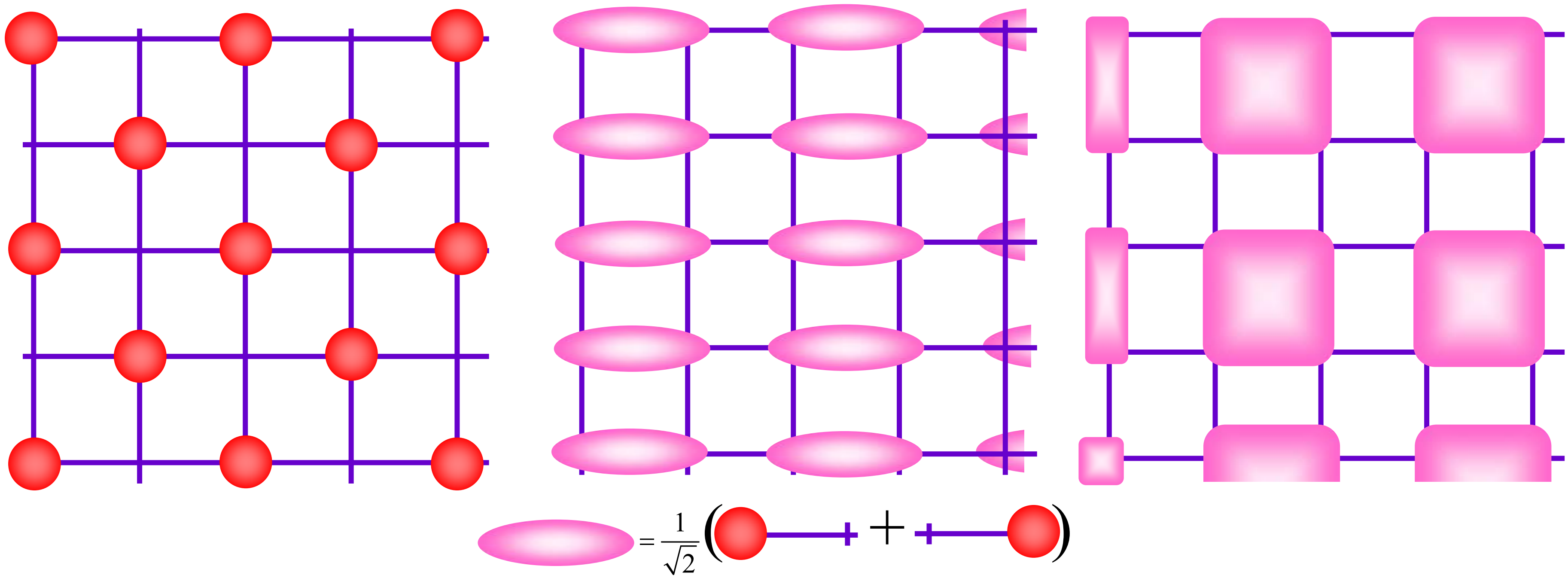
Valence bond solid (VBS) order

Can define a common CDW/VBS order using a generalized "density" $\rho(\boldsymbol{r}) = \sum_{\boldsymbol{Q}} \rho_{\boldsymbol{Q}} e^{i\boldsymbol{Q}.\boldsymbol{r}}$

All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\boldsymbol{Q}} \rangle \neq 0$ for certain $\boldsymbol{Q}$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)
S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Mott insulators obtained by condensing vortices at $f = 1/2$



Charge density wave (CDW) order

Valence bond solid (VBS) order

Valence bond solid (VBS) order

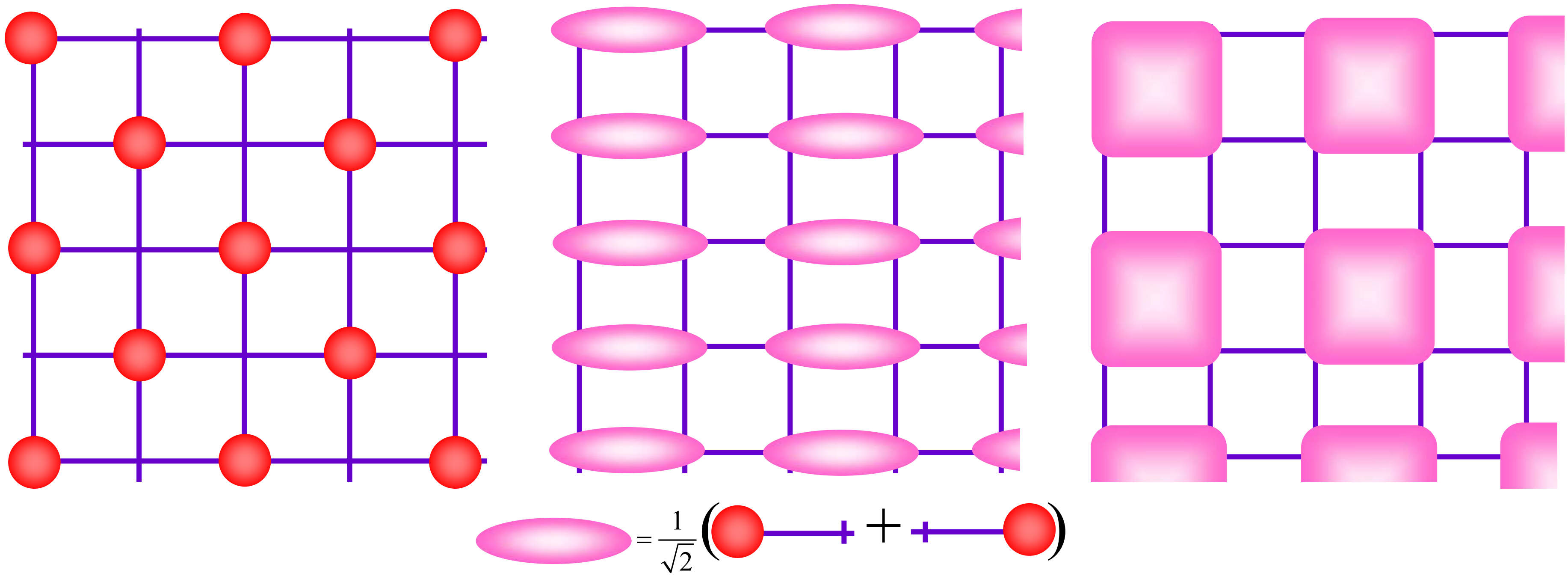
Can define a common CDW/VBS order using a generalized "density" $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$

All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\mathbf{Q}} \rangle \neq 0$ for certain $\mathbf{Q}$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Mott insulators obtained by condensing vortices at $f = 1/2$



Charge density wave (CDW) order

Valence bond solid (VBS) order

Valence bond solid (VBS) order

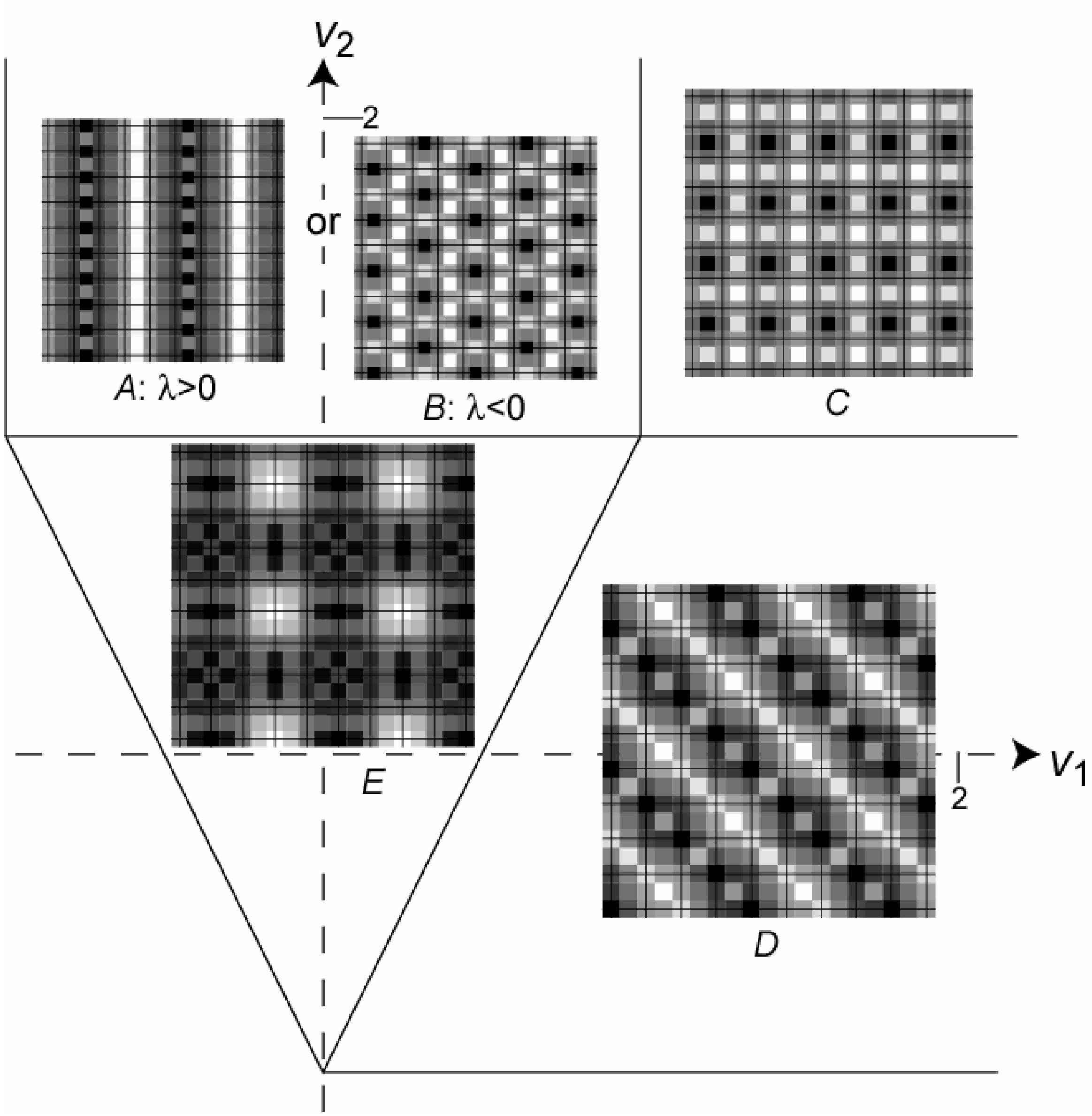
Can define a common CDW/VBS order using a generalized "density" $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$

All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\mathbf{Q}} \rangle \neq 0$ for certain $\mathbf{Q}$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Mott insulators obtained by condensing vortices at $f = 1/4, 3/4$



$a \times b$ unit cells;

q/a, q/b, ab/q,

all integers

Vortices in a superfluid near a Mott insulator at filling $f=p/q$

- The excitations of the superfluid are described by the quantum mechanics of q flavors of low energy vortices moving in zero dual "magnetic" field.

- The orientation of the vortex in flavor space implies a particular configuration of VBS order in its vicinity.

Vortices in a superfluid near a Mott insulator at filling $f=p/q$

- The excitations of the superfluid are described by the quantum mechanics of q flavors of low energy vortices moving in zero dual "magnetic" field.

- The orientation of the vortex in flavor space implies a particular configuration of VBS order in its vicinity.

- Any pinned vortex must pick an orientation in flavor space: this induces a halo of VBS order in its vicinity

Outline

I. The superfluid-Mott insulator quantum phase transition

II. Dual theory: vortices and their wavefunctions

III. Vortices in superfluids near the superfluid-insulator quantum phase transition

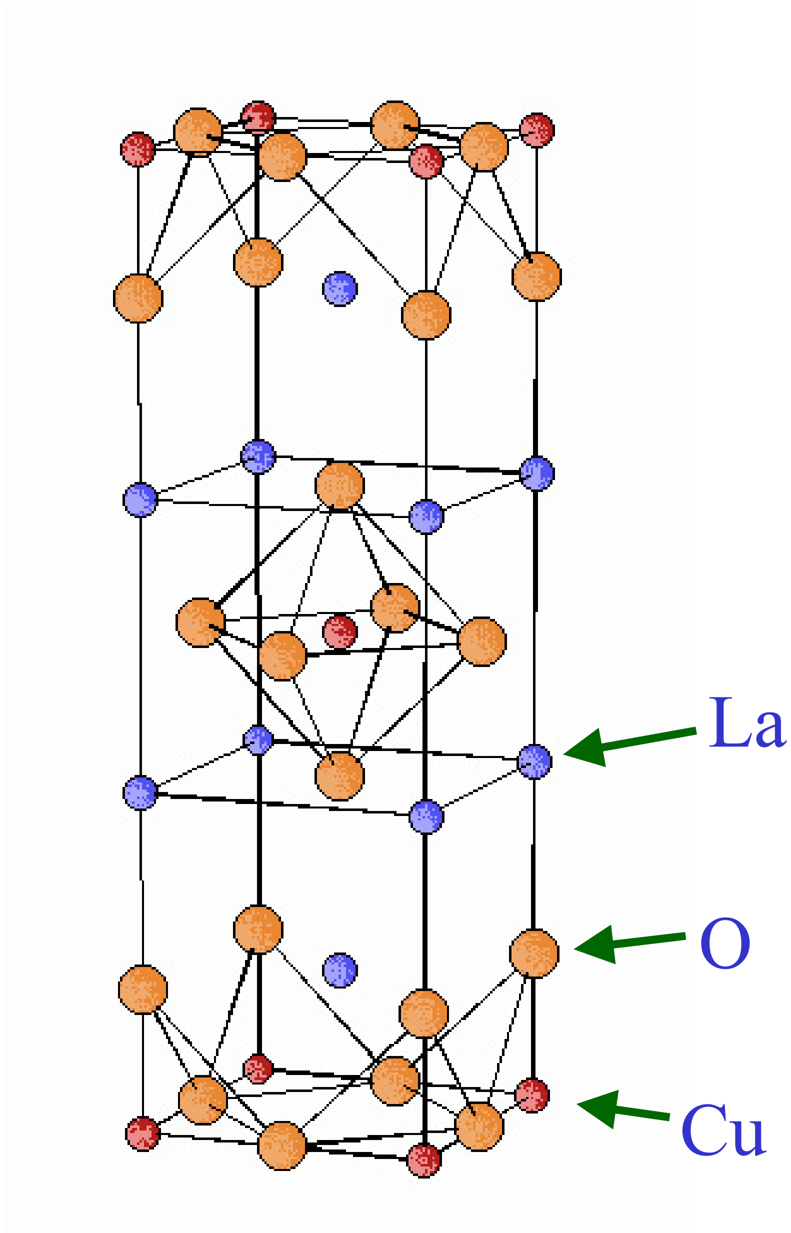
Vortex wavefunction lives in a dual flavor space

IV. The cuprate superconductors

Detection of vortex flavors ?

IV. The cuprate superconductors

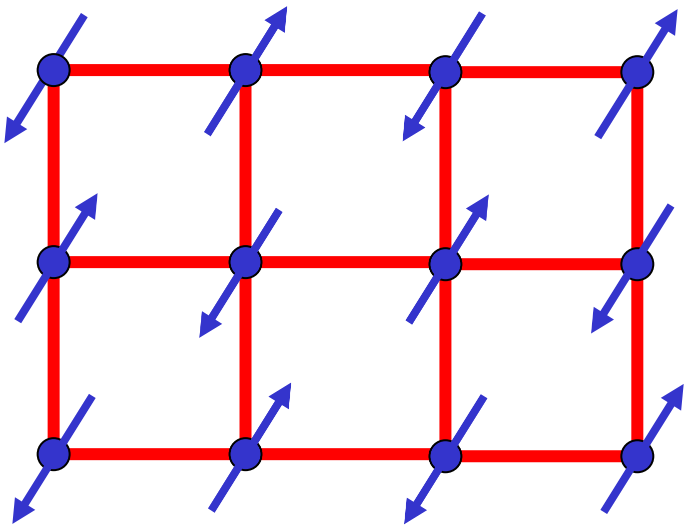
Detection of dual vortex wavefunction in STM experiments ?



La_2CuO_4

La_2CuO_4

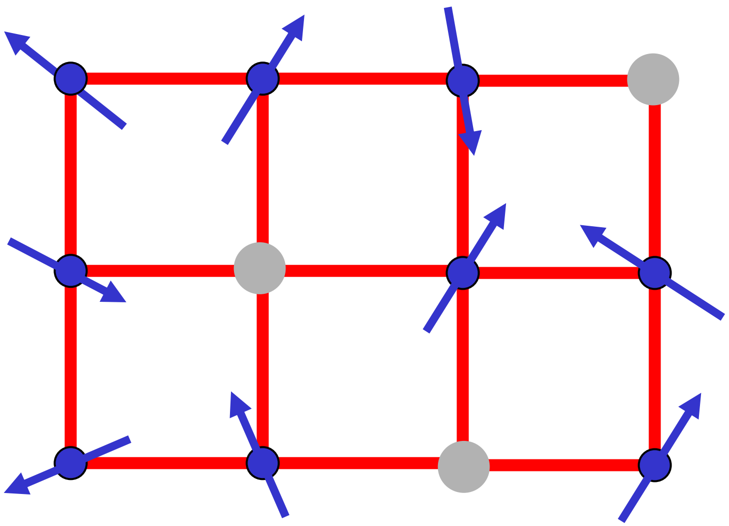
Mott insulator: square lattice antiferromagnet



$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$La_{2-\delta}Sr_{\delta}CuO_4$

Superfluid: condensate of paired holes



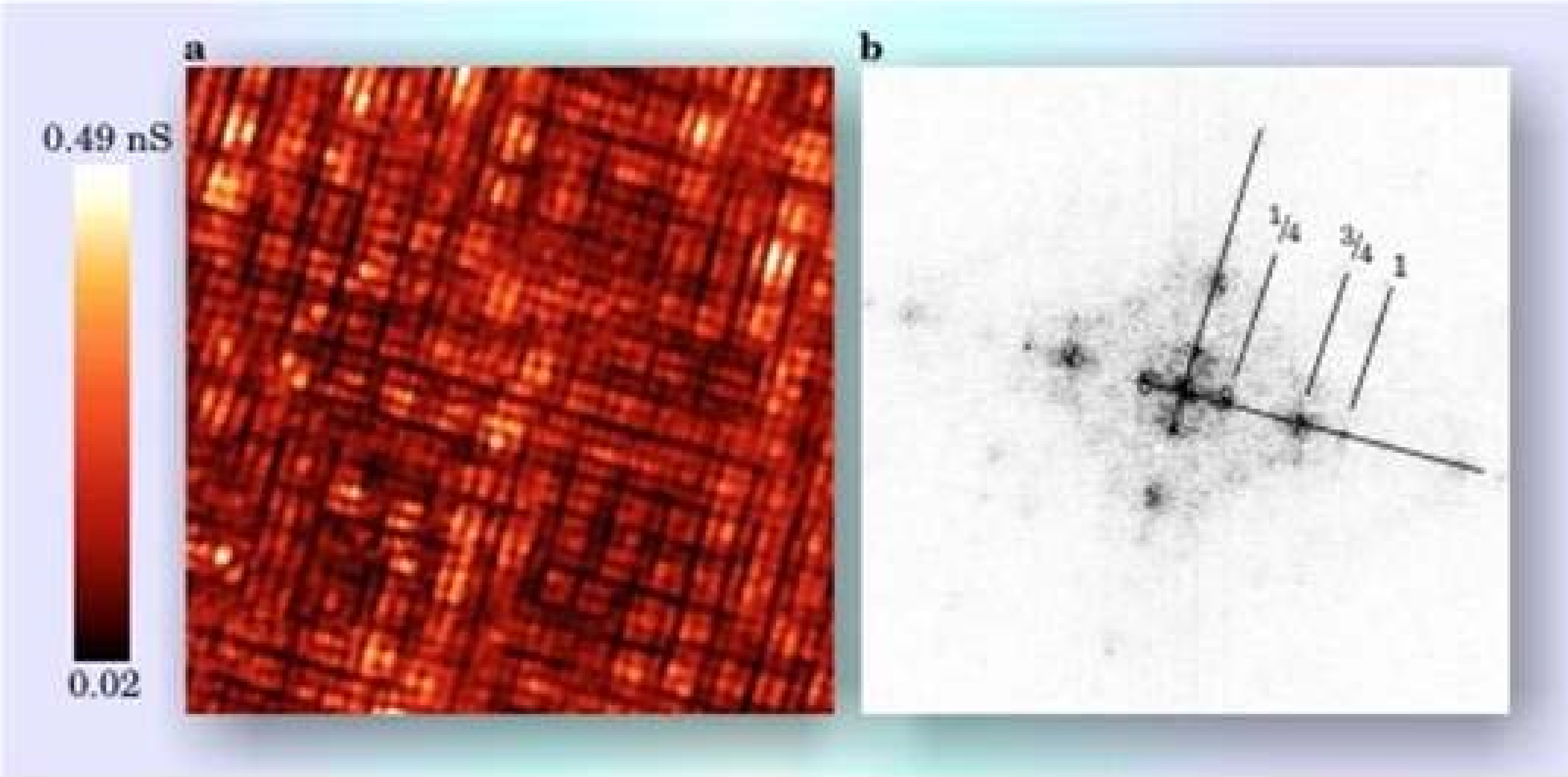
$$\langle \vec{S} \rangle = 0$$

Many experiments on the cuprate superconductors show:

• Tendency to produce modulations in spin singlet observables at wavevectors $(2\pi/a)(1/4,0)$ and $(2\pi/a)(0,1/4)$.

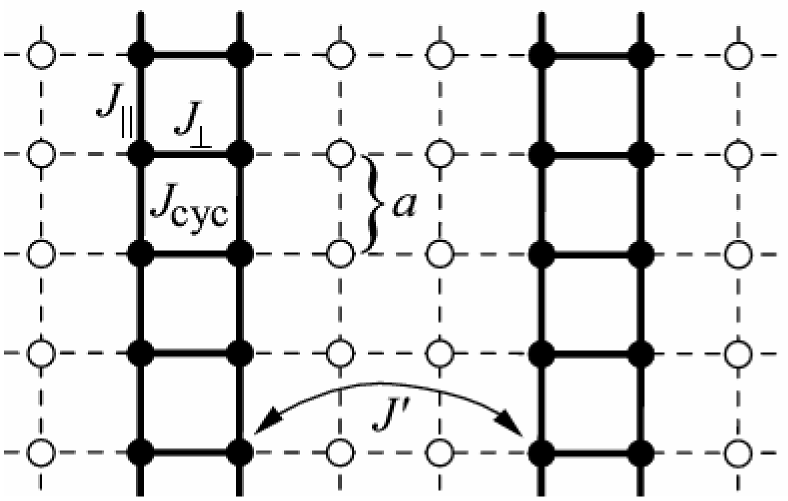
• Proximity to a Mott insulator at hole density $\delta=1/8$ with long-range charge modulations at wavevectors $(2\pi/a)(1/4,0)$ and $(2\pi/a)(0,1/4)$.

The cuprate superconductor $Ca_{2-x}Na_xCuO_2Cl_2$

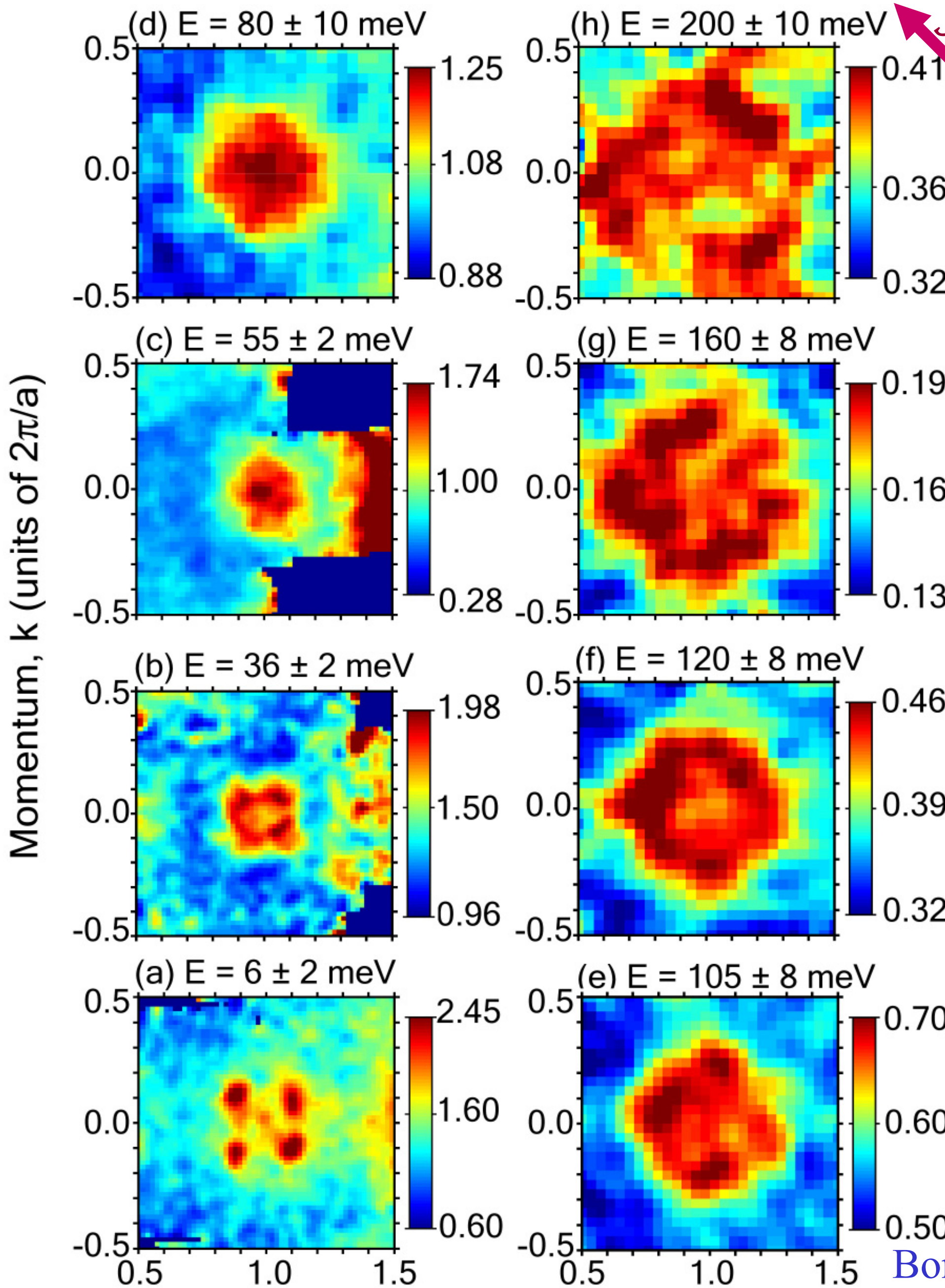


T. Hanaguri, C. Lupien, Y. Kohsaka, D.-H. Lee, M. Azuma, M. Takano, H. Takagi, and J. C. Davis, *Nature* **430**, 1001 (2004).

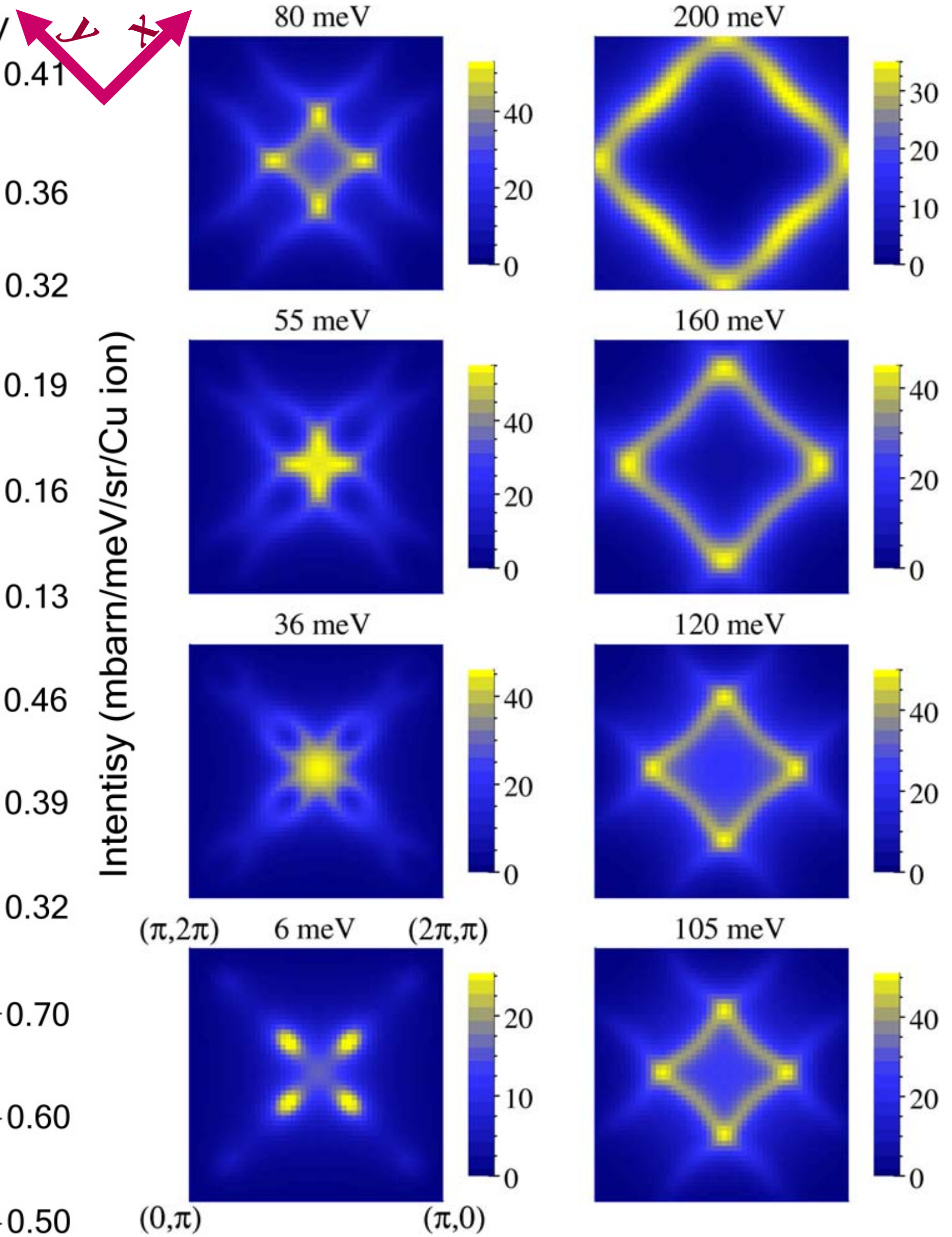
Possible structure of VBS order



This strucuture also explains spin-excitation spectra in neutron scattering experiments



Tranquada *et al., Nature* **429**, 534 (2004)



Bond operator (S. Sachdev and R.N. Bhatt, *Phys. Rev.* B 41, 9323 (1990)) theory of coupled-ladder model, M. Vojta and T. Ulbricht, *Phys. Rev. Lett.* **93**, 127002 (2004)

Many experiments on the cuprate superconductors show:

• Tendency to produce modulations in spin singlet observables at wavevectors $(2\pi/a)(1/4,0)$ and $(2\pi/a)(0,1/4)$.

• Proximity to a Mott insulator at hole density $\delta=1/8$ with long-range charge modulations at wavevectors $(2\pi/a)(1/4,0)$ and $(2\pi/a)(0,1/4)$.

Many experiments on the cuprate superconductors show:

• Tendency to produce modulations in spin singlet observables at wavevectors $(2\pi/a)(1/4,0)$ and $(2\pi/a)(0,1/4)$.

• Proximity to a Mott insulator at hole density $\delta=1/8$ with long-range charge modulations at wavevectors $(2\pi/a)(1/4,0)$ and $(2\pi/a)(0,1/4)$.

Do vortices in the superfluid "know" about these "density" modulations ?

Consequences of our theory:

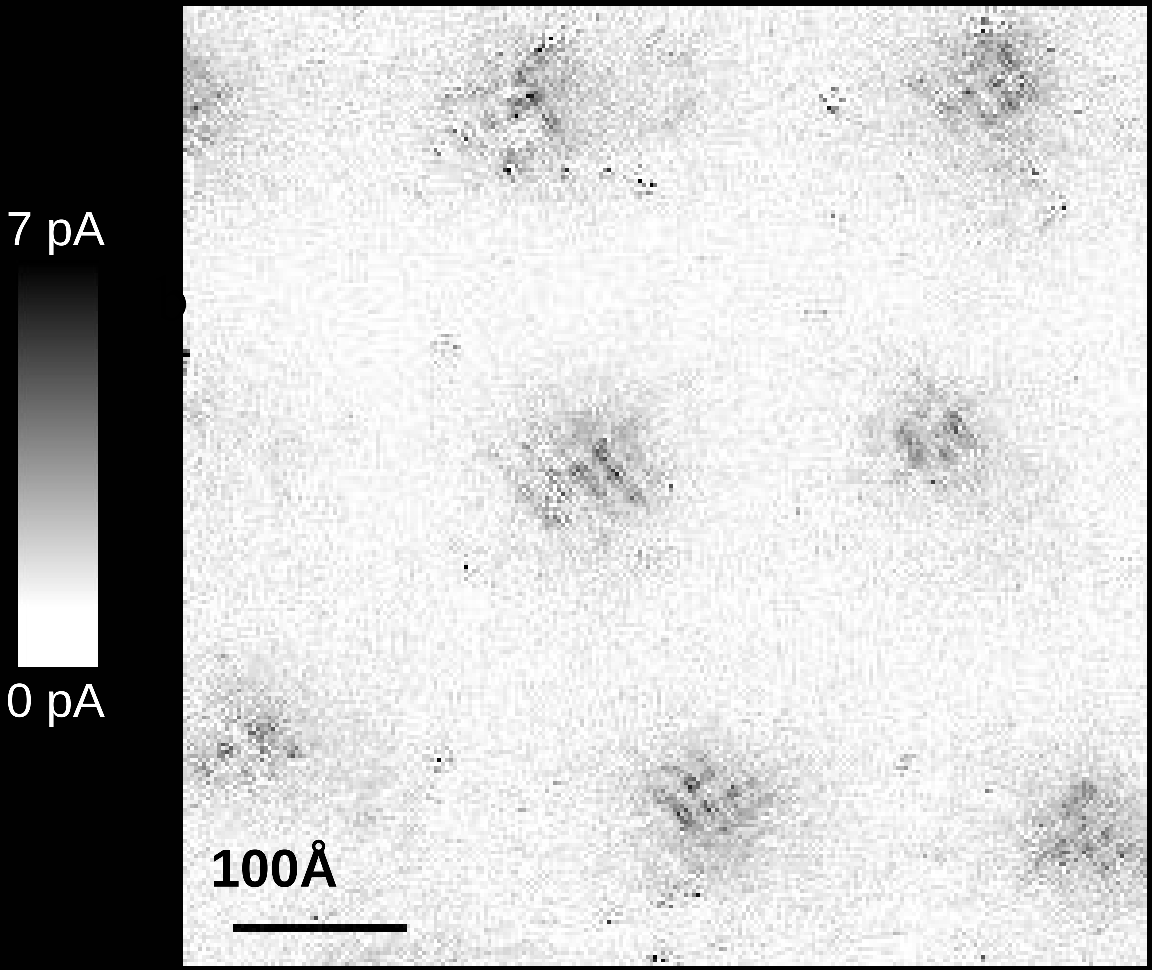
Information on VBS order is contained in the vortex flavor space

Density operators $\rho_{\mathbf{Q}}$ at wavevectors $\mathbf{Q}_{mn} = \frac{2\pi p}{q}(m,n)$

$$\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \varphi_{\ell}^{*} \varphi_{\ell+n} e^{2\pi i \ell mf}$$

Each pinned vortex in the superfluid has a halo of density wave order over a length scale ≈ the zero-point quantum motion of the vortex. This scale diverges upon approaching the insulator

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K



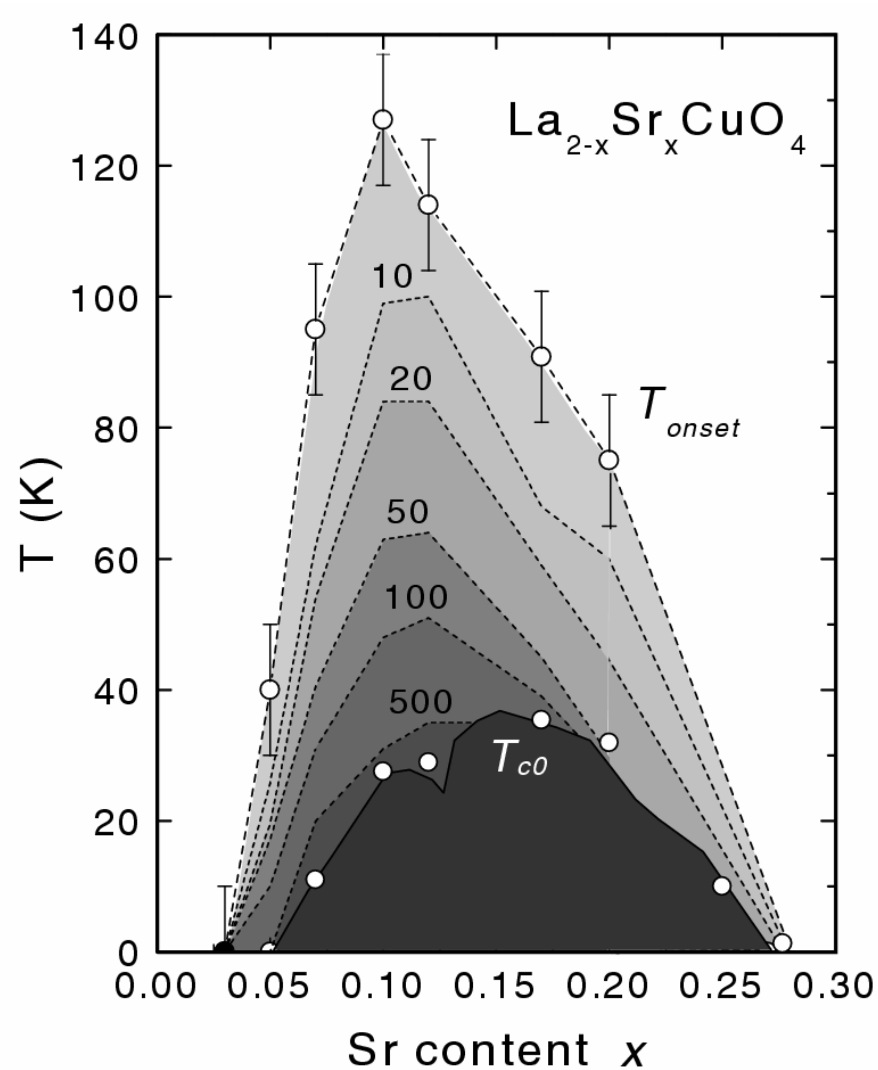
Vortices have halos with LDOS modulations at a period ≈ 4 lattice spacings

J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Prediction of VBS order near vortices: K. Park and S. Sachdev, Phys. Rev. B **64**, 184510 (2001).

Distinct experimental charcteristics of underdoped cuprates at $T > T_c$

Measurements of Nernst effect are well explained by a model of a liquid of vortices and anti-vortices

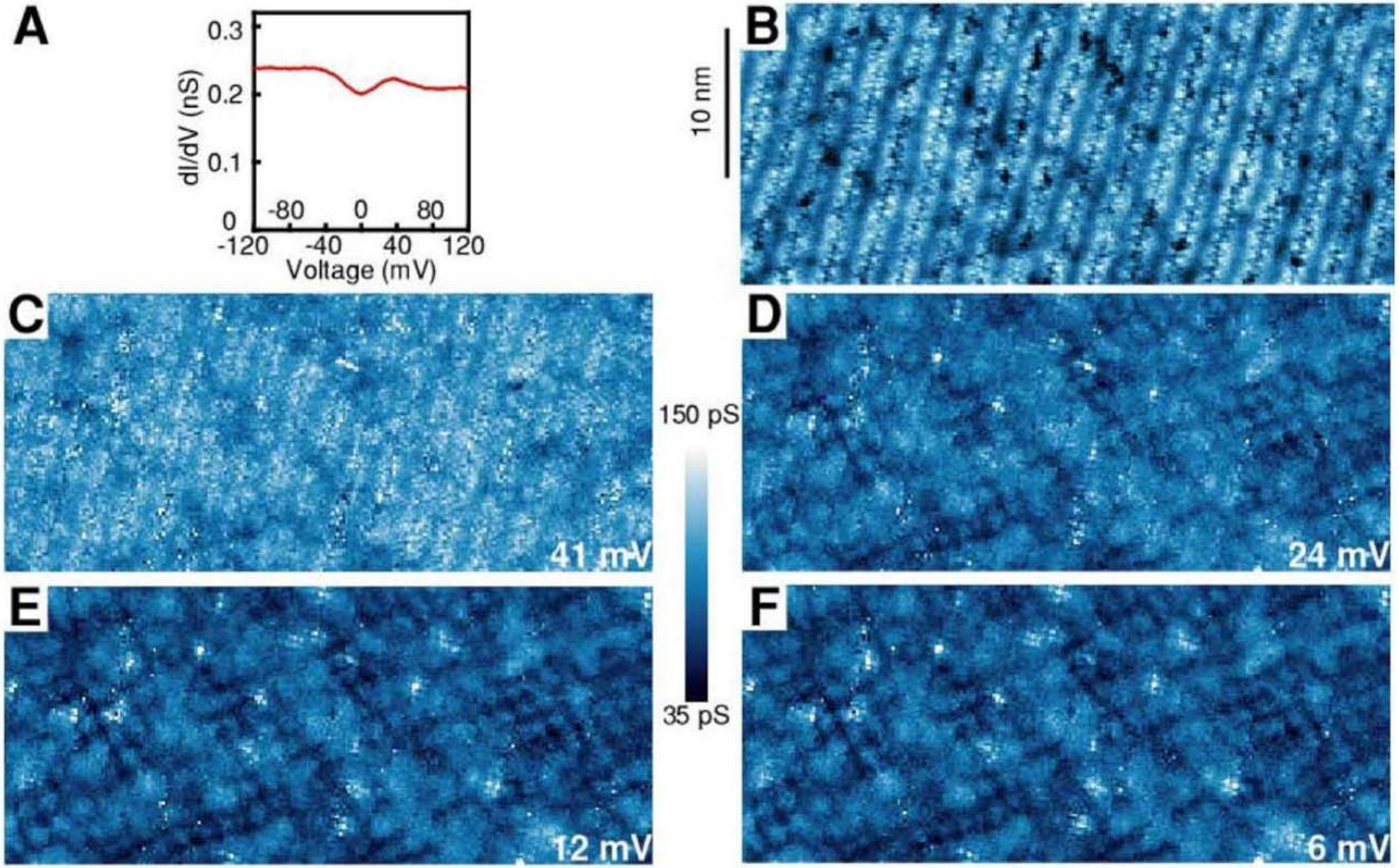


N. P. Ong, Y. Wang, S. Ono, Y. Ando, and S. Uchida, *Annalen der Physik* **13**, 9 (2004).

Y. Wang, S. Ono, Y. Onose, G. Gu, Y. Ando, Y. Tokura, S. Uchida, and N. P. Ong, *Science* **299**, 86 (2003).

Distinct experimental charcteristics of underdoped cuprates at $T > T_c$

STM measurements observe "density" modulations with a period of ≈ 4 lattice spacings

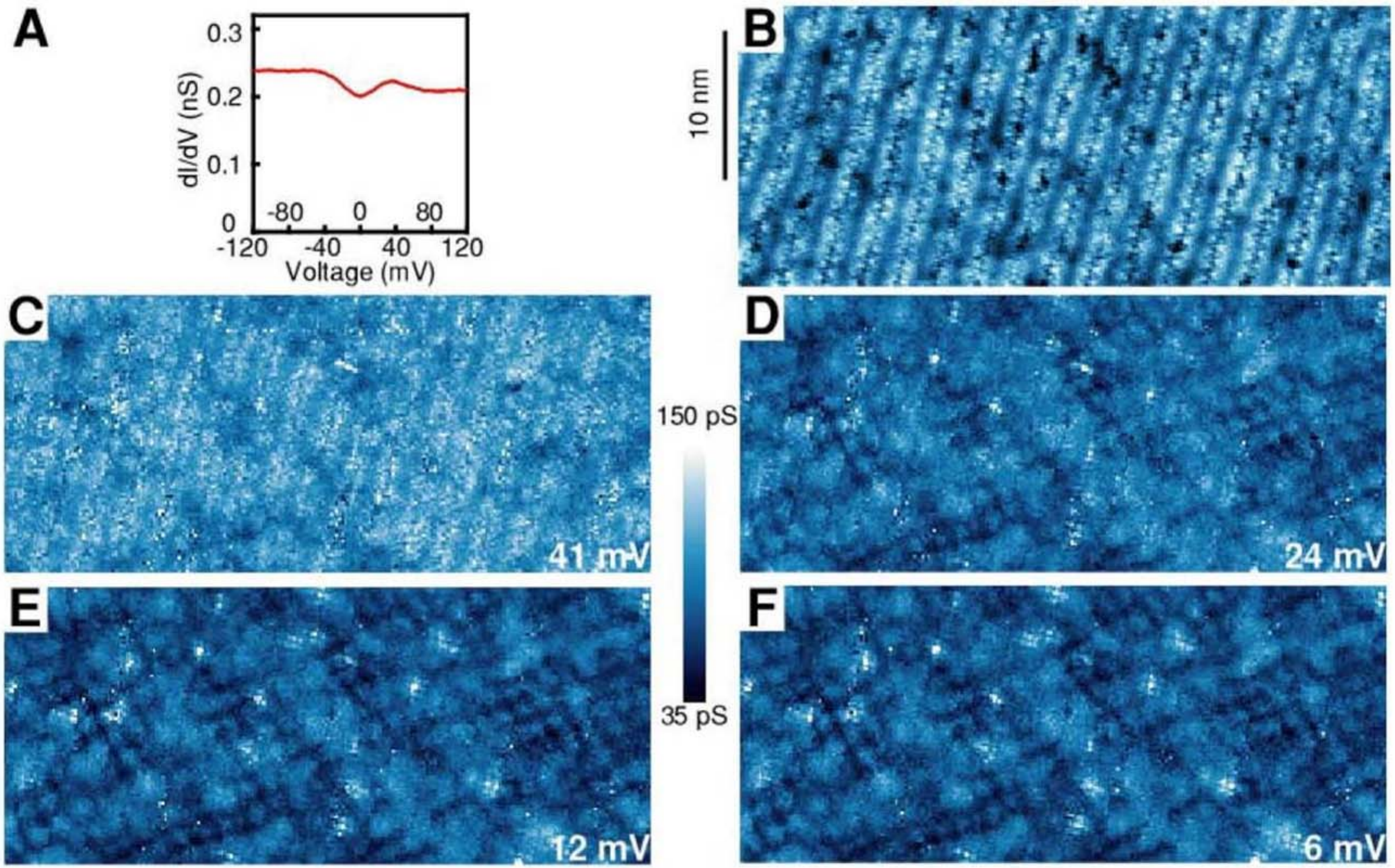


LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ at 100 K.

M. Vershinin, S. Misra, S. Ono, Y. Abe, Y. Ando, and A. Yazdani, *Science*, **303**, 1995 (2004).

Distinct experimental charcteristics of underdoped cuprates at $T > T_c$

Our theory: modulations arise from pinned vortex-anti-vortex pairs – these thermally excited vortices are also responsible for the Nernst effect



LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ at 100 K.

M. Vershinin, S. Misra, S. Ono, Y. Abe, Y. Ando, and A. Yazdani, *Science*, **303**, 1995 (2004).

Superfluids near Mott insulators

The Mott insulator has average Cooper pair density, $f = p/q$ per site, while the density of the superfluid is close (but need not be identical) to this value

- Dual description using vortices with flux *h/(2e)* which come in multiple (usually q) “flavors”
- The lattice space group acts in a projective representation on the vortex flavor space.
- These flavor quantum numbers provide a distinction between superfluids: they constitute a “quantum order”
- Any pinned vortex must chose an orientation in flavor space. This necessarily leads to modulations in the local density of states over the spatial region where the vortex executes its quantum zero point motion.