Gauge theory for the cuprates near optimal doping

Subir Sachdev

Jerusalem Winter School,
January 6, 2019

Talk online: sachdev.physics.harvard.edu

PHYSICS
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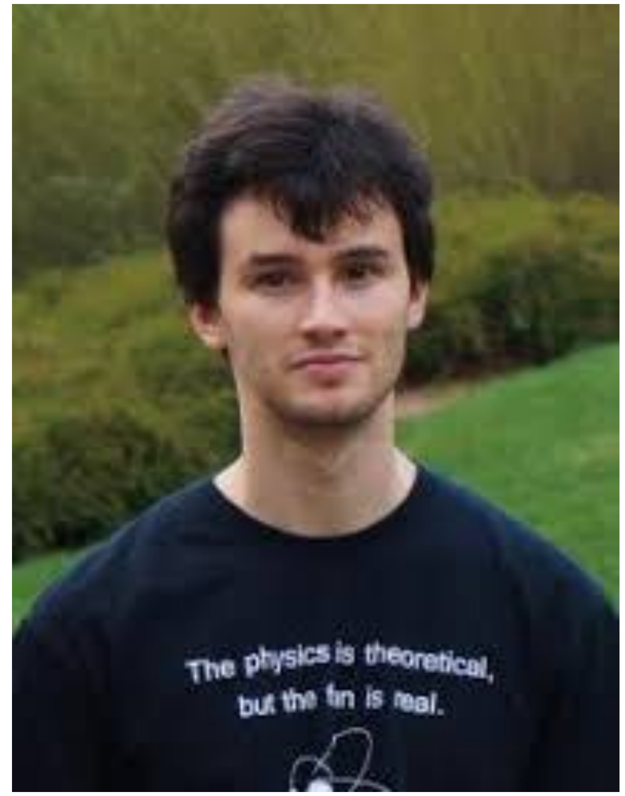
Mathias Scheurer



Grigory Tarnopolsky

Harley Scammell

arXiv:1811.04930

PHYSICS

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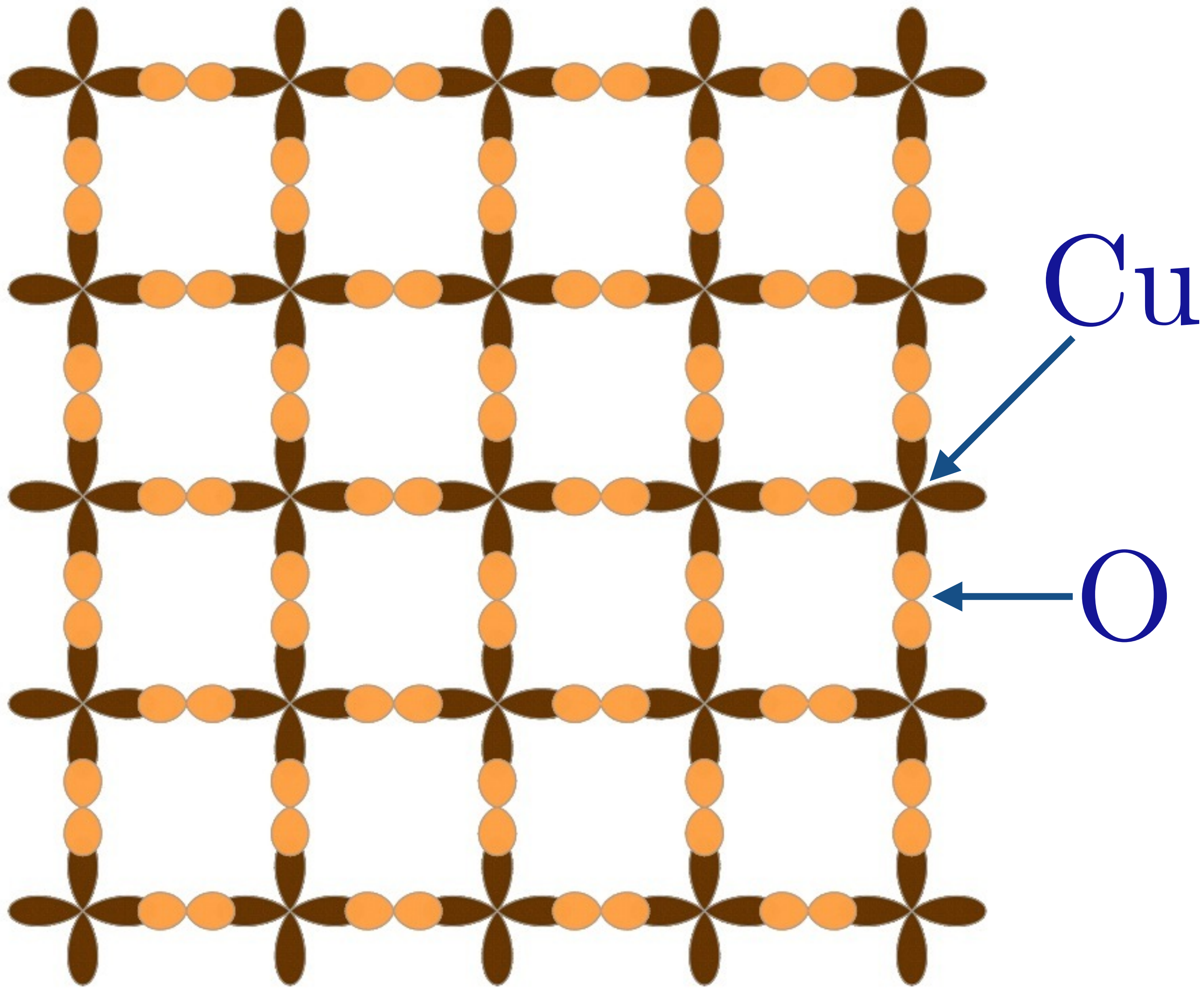
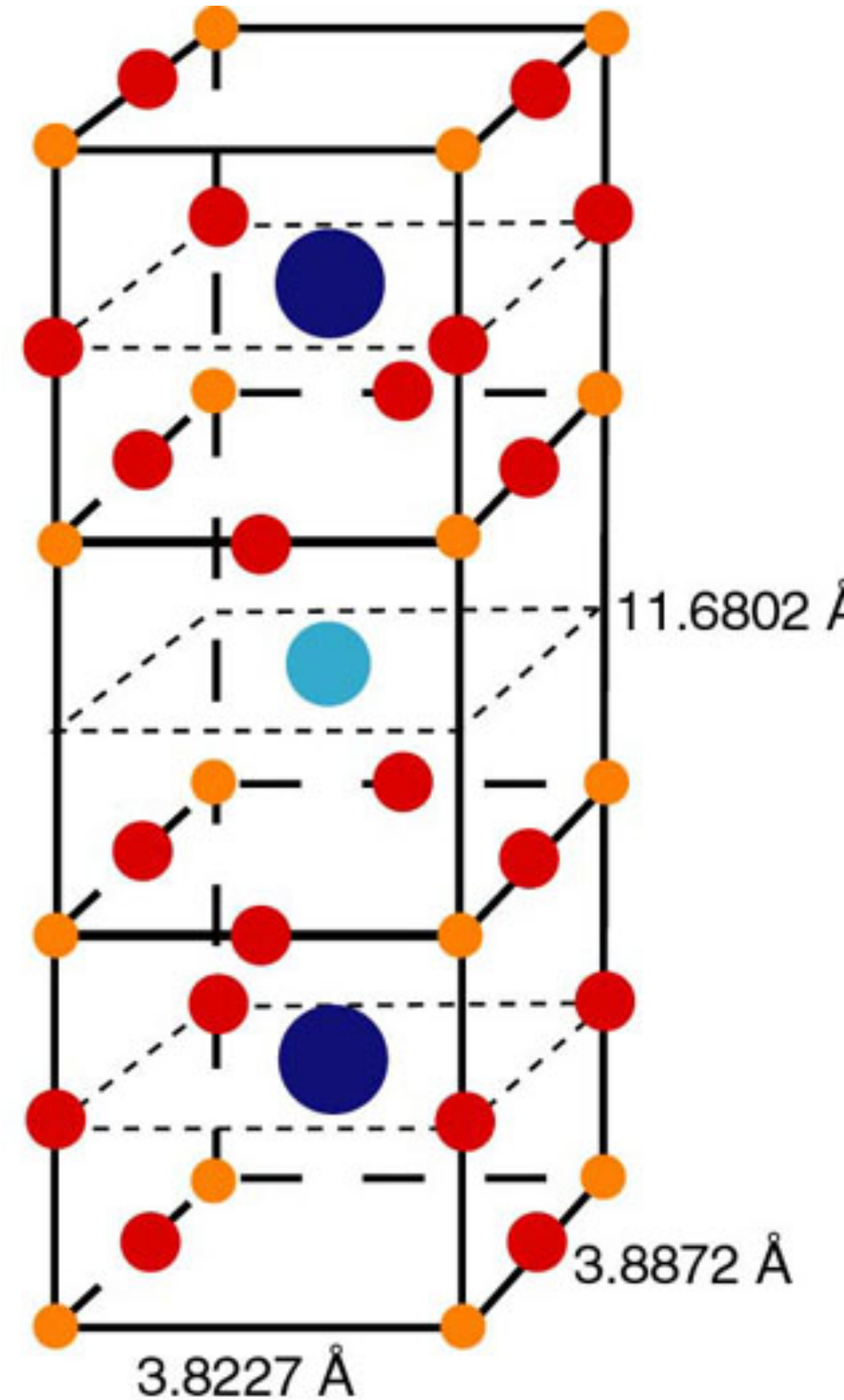
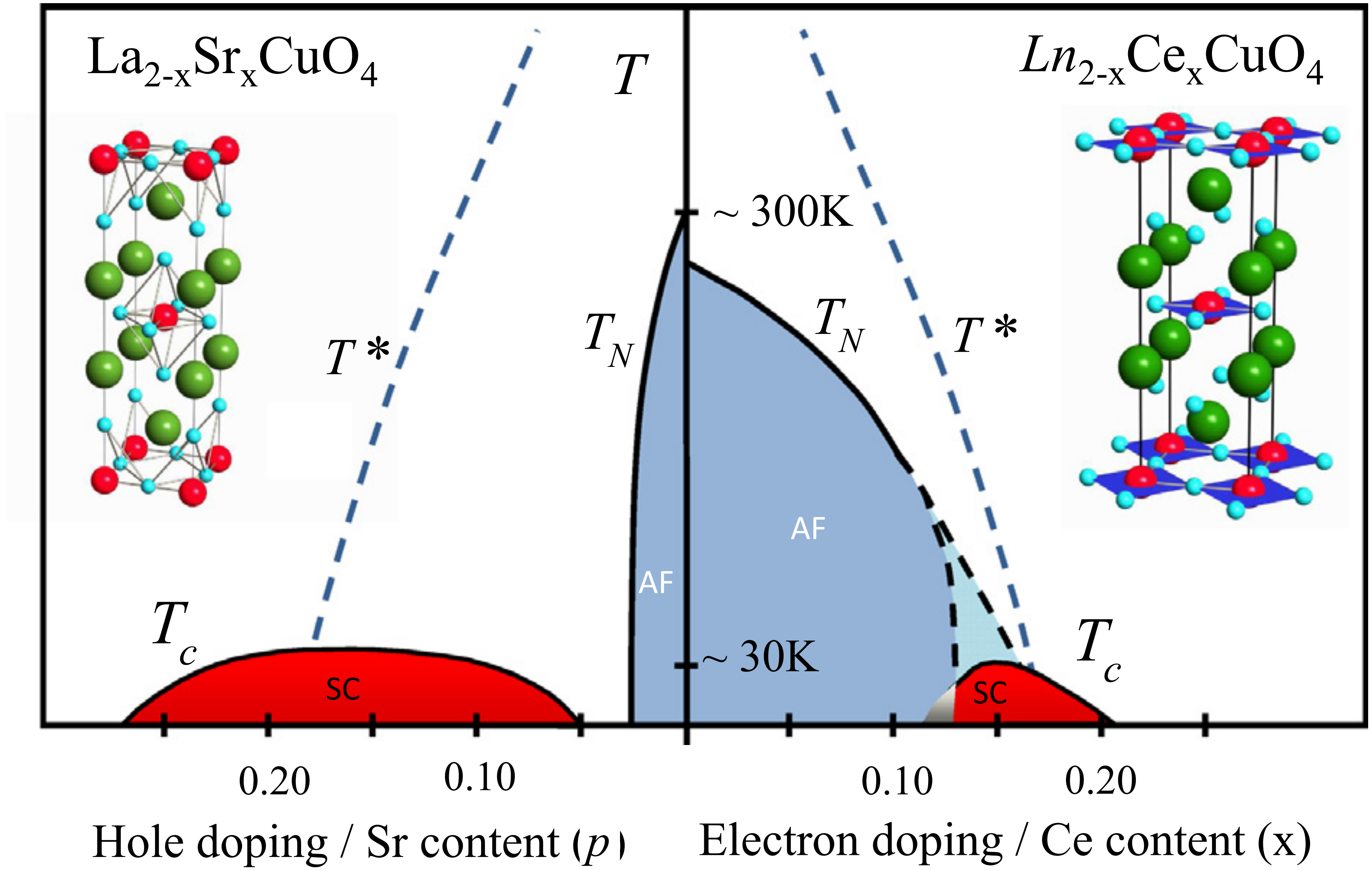
High temperature superconductors



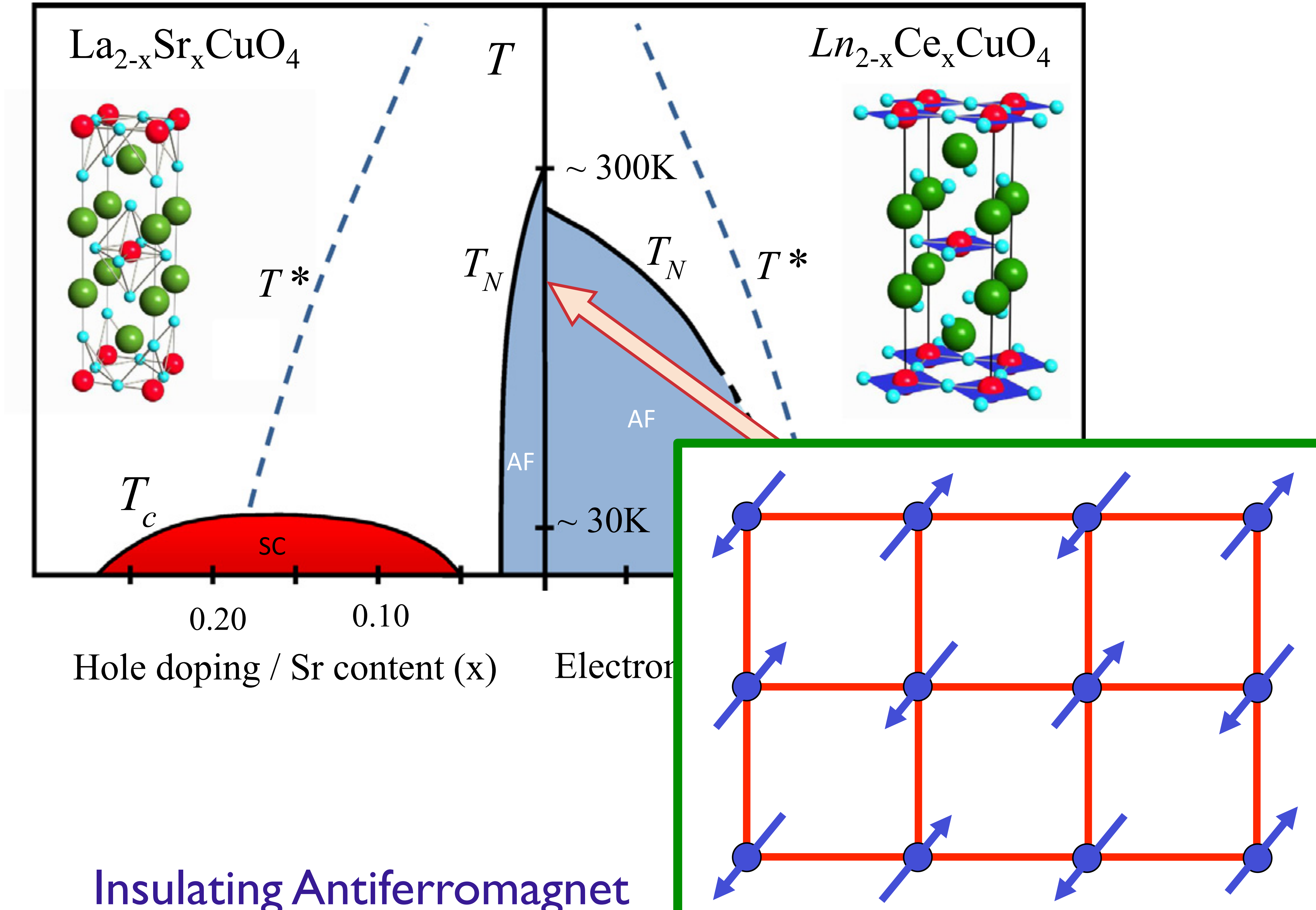
CuO_2 plane



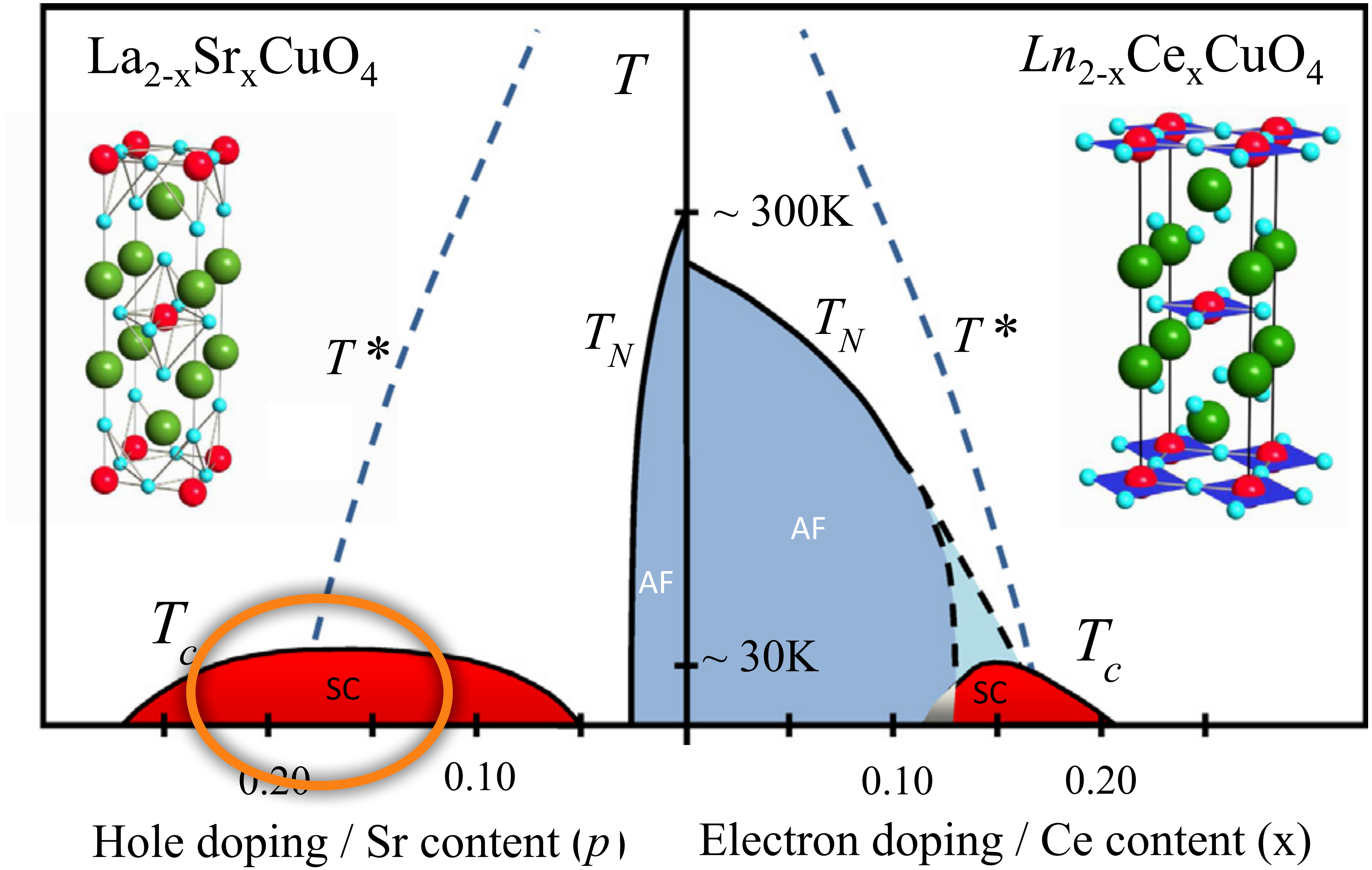
$YBa_2Cu_3O_{6+x}$



$La_{2-x}Sr_xCuO_4$
$Ln_{2-x}Ce_xCuO_4$
T
~ 300K
~ 30K
T^*
T^*
T_N
T_N
AF
AF
T_c
T_c
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)



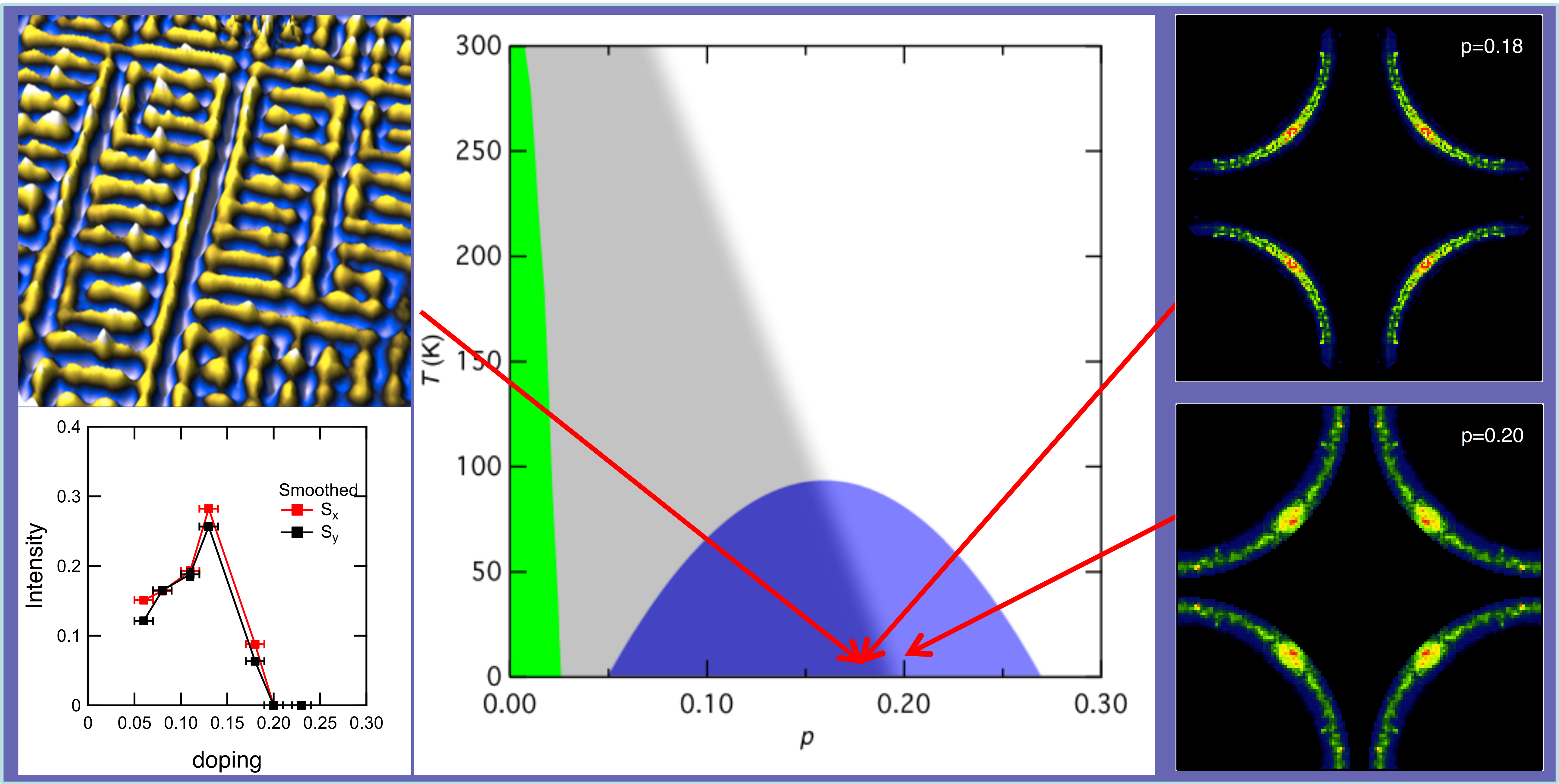
Insulating Antiferromagnet


La2-xSrxCuO4
Ln2-xCexCuO4
T
~ 300K
~ 30K
T*
TN
AF
SC
Tc
0.20
0.10
Hole doping / Sr content (p)
Electron doping / Ce content (x)

Hole doped cuprates

Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, Science **344**, 608 (2014)

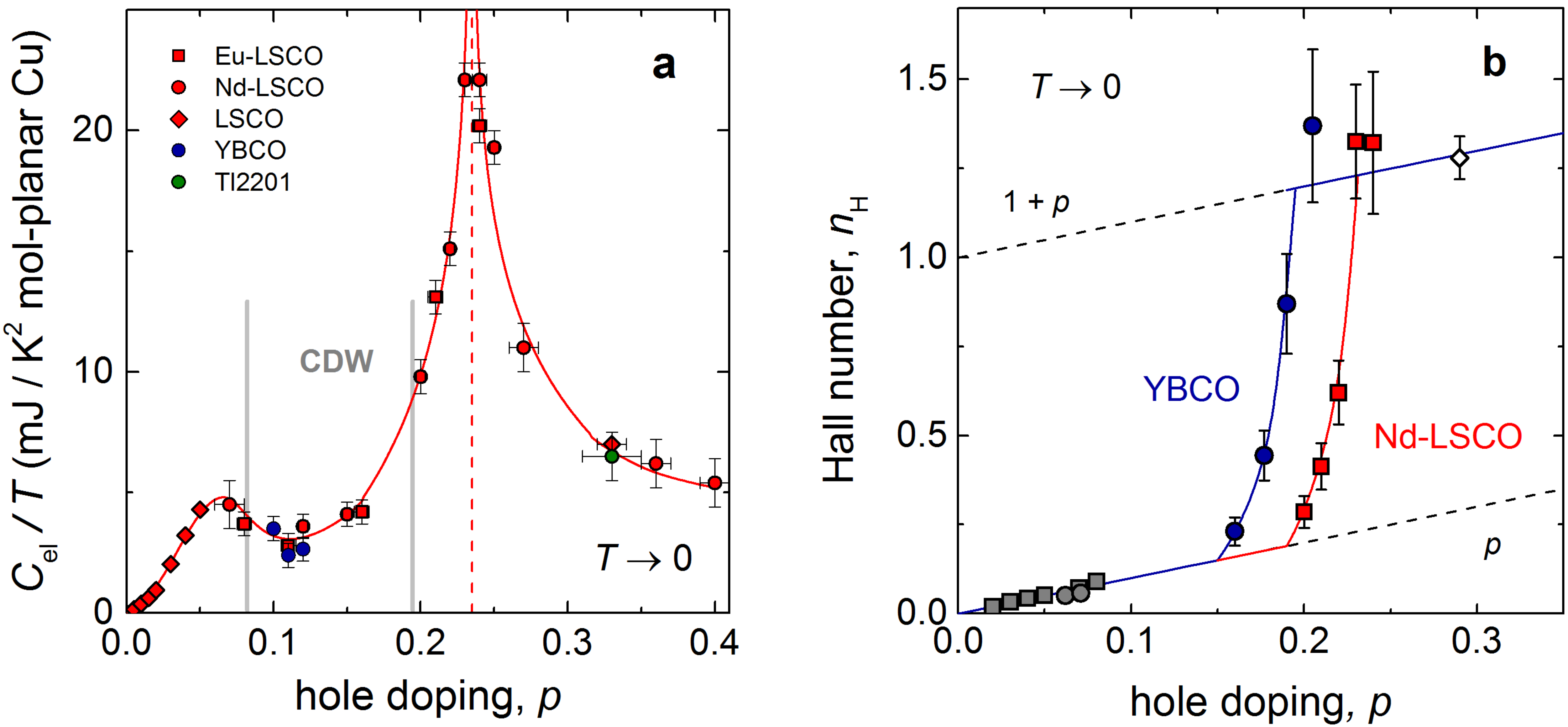
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, Science **344**, 612 (2014)

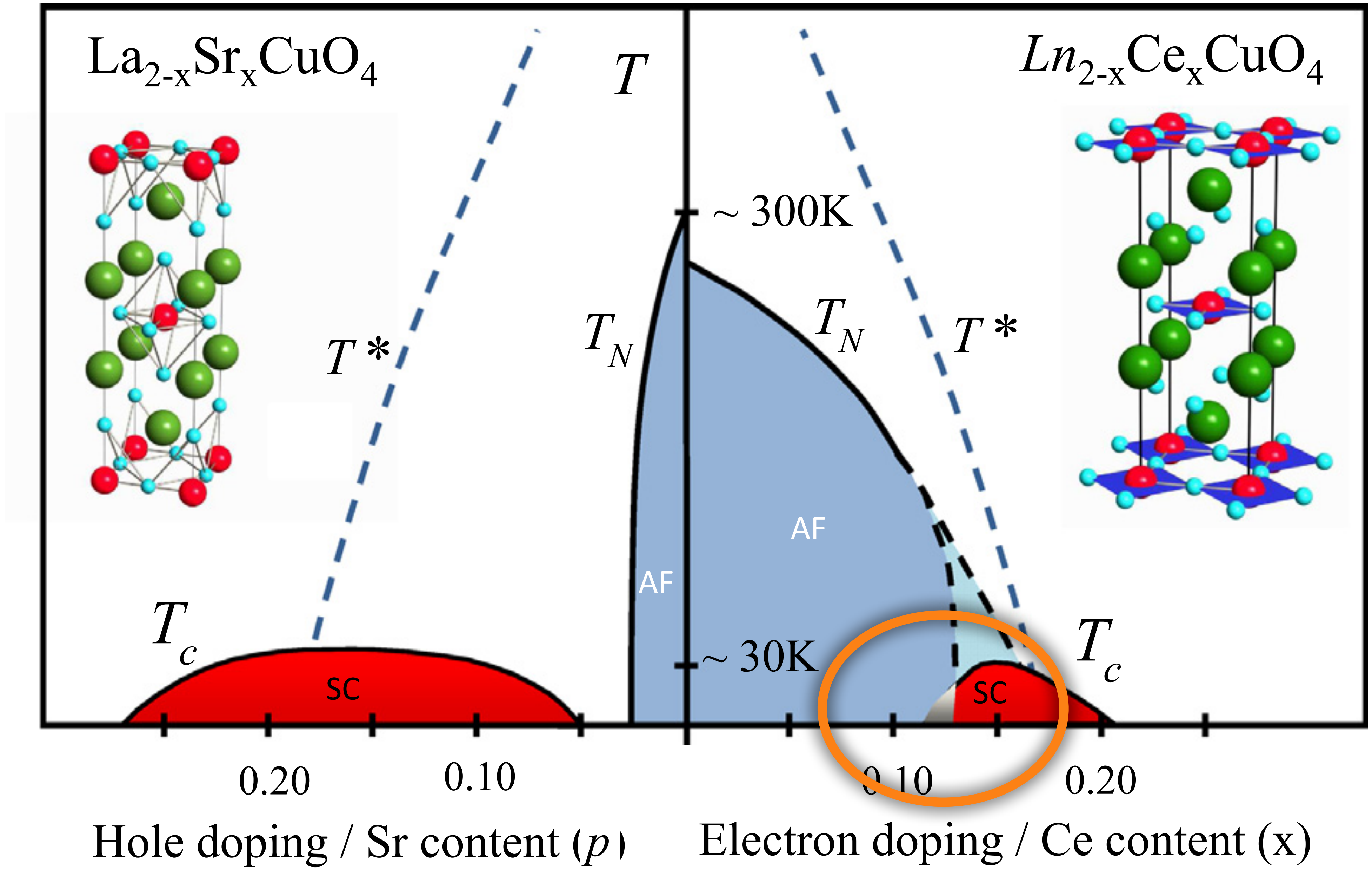


Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507




La2-xSrxCuO4
Ln2-xCexCuO4
T
~ 300K
~ 30K
T*
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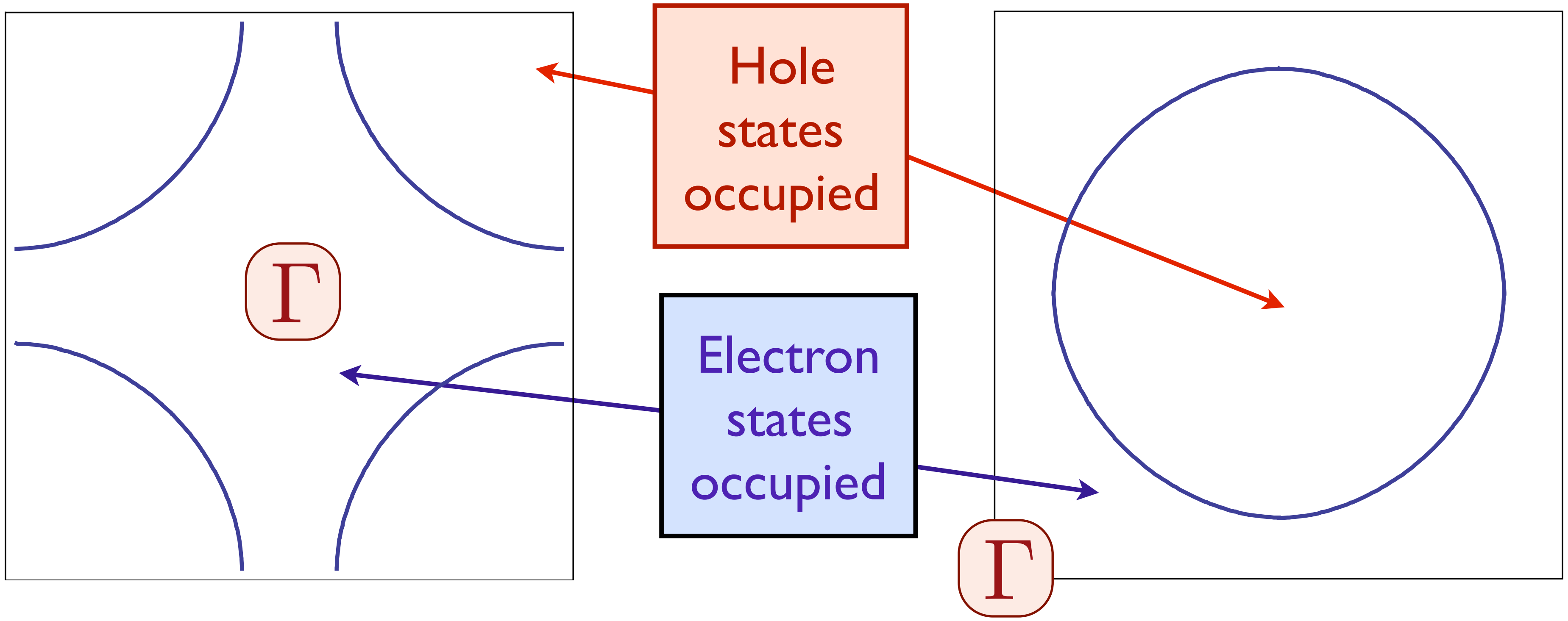
TN
AF
AF
Tc
Tc
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)

Fermi surface+antiferromagnetism



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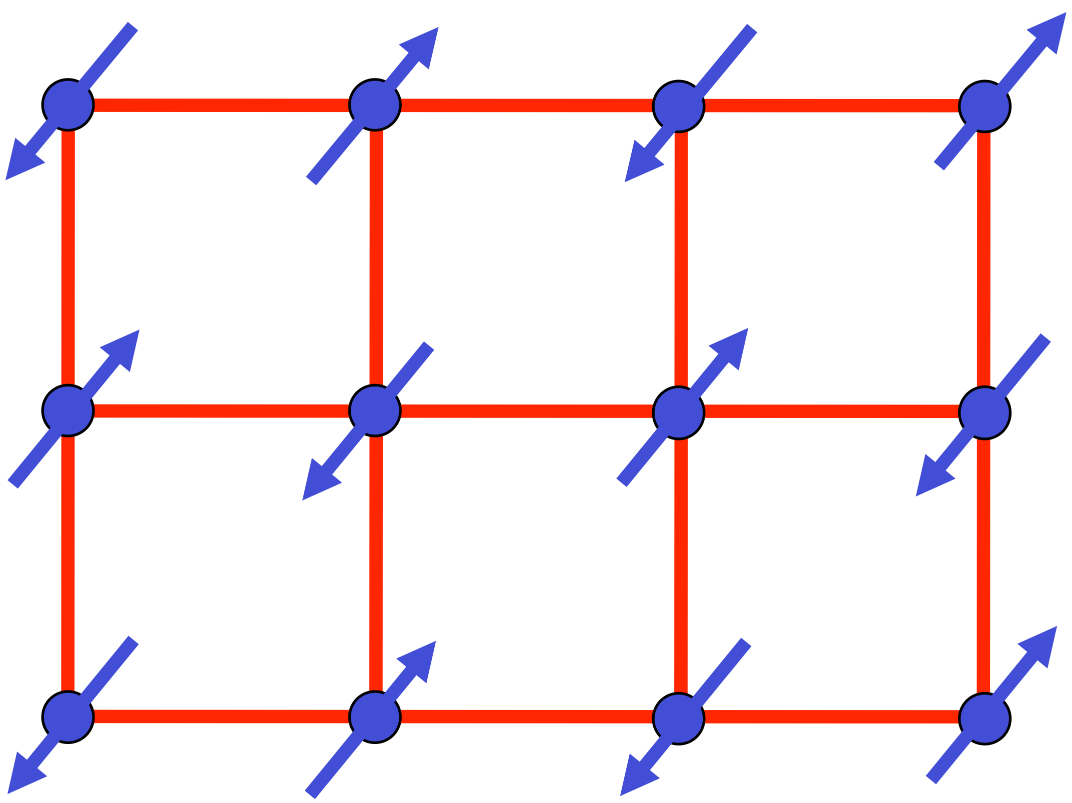
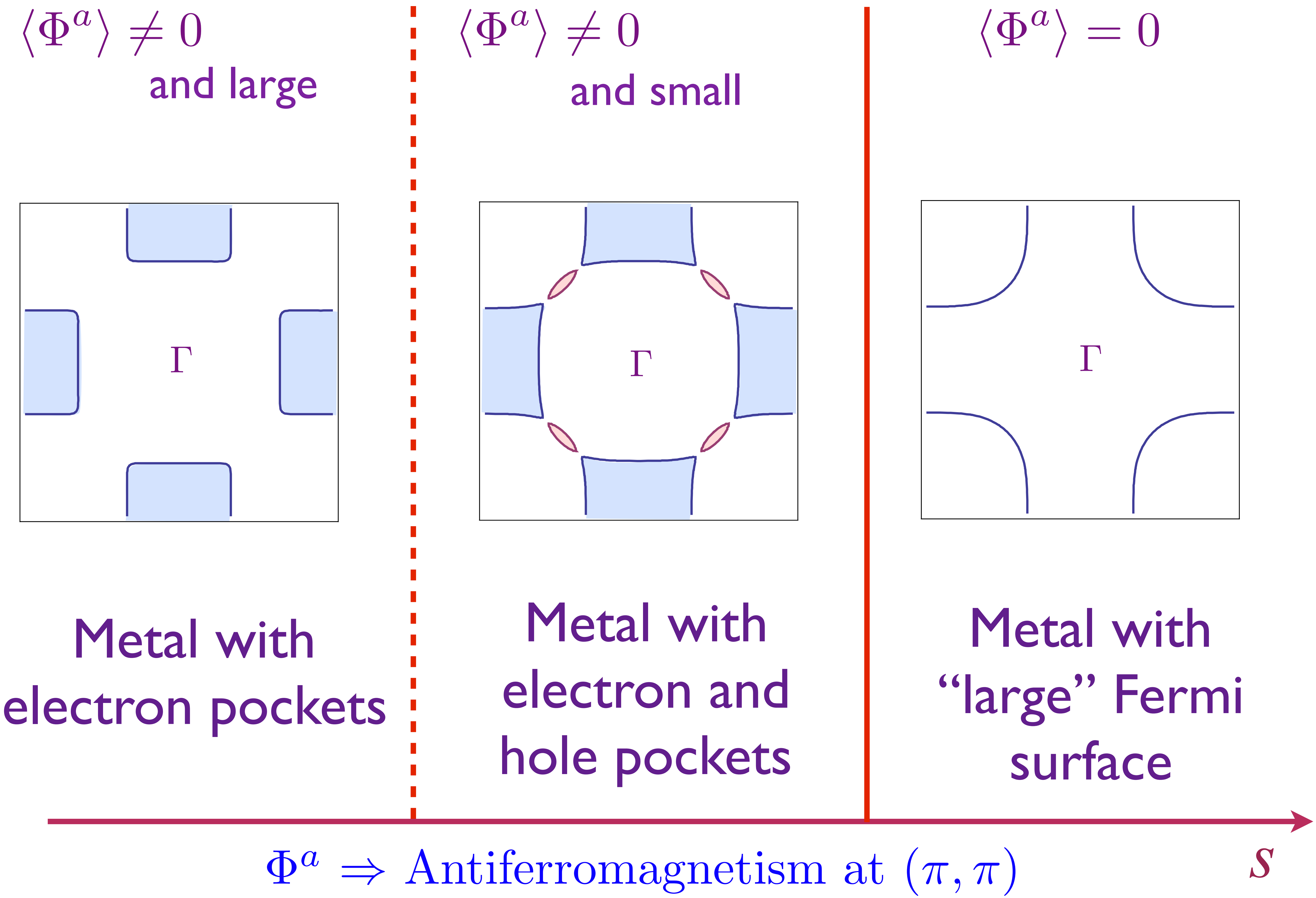


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\Phi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

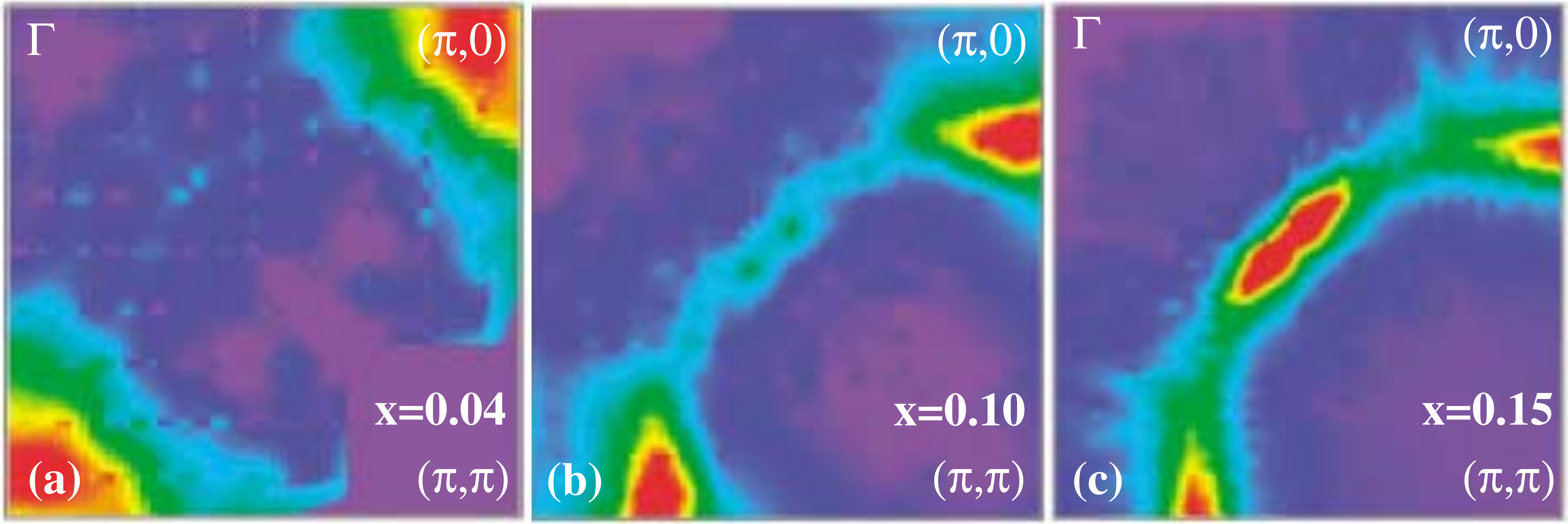
where $\mathbf{K}$ is the ordering wavevector.

Square lattice Hubbard model with electron doping



Electron doped cuprates

$$\mathrm{Nd}_{2-x}\mathrm{Ce}_x\mathrm{CuO}_{4\pm\delta}$$



Doping Dependence of an n-Type Cuprate Superconductor Investigated by Angle-Resolved Photoemission Spectroscopy

N. P. Armitage, F. Ronning, D. H. Lu, C. Kim, A. Damascelli, K. M. Shen, D. L. Feng, H. Eisaki, Z.-X. Shen, P. K. Mang, N. Kaneko, M. Greven, Y. Onose, Y. Taguchi, and Y. Tokura

Phys. Rev. Lett. **88**, 257001 (2002)

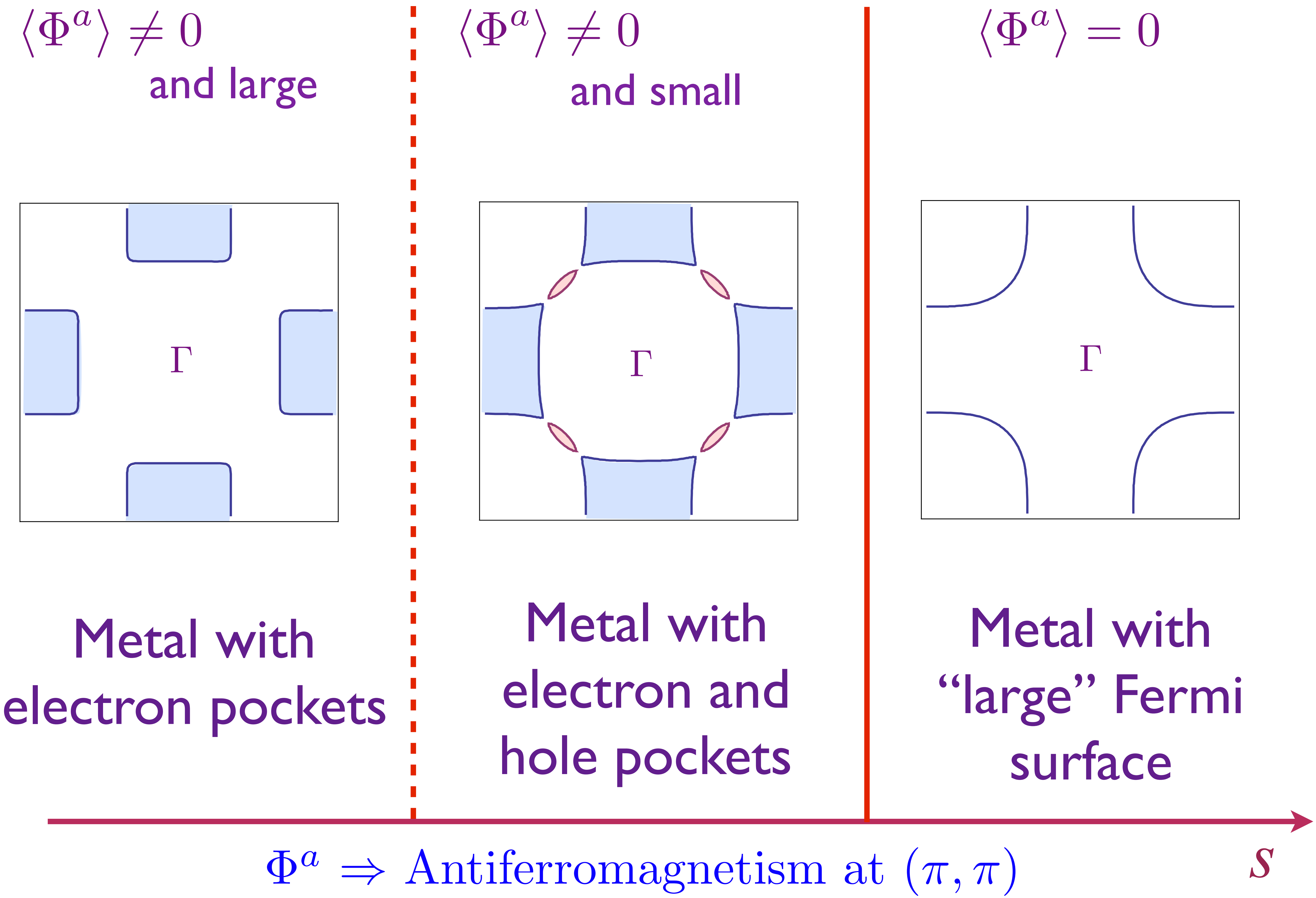
arXiv:1811.04992

Fermi surface reconstruction in electron-doped cuprates without antiferromagnetic long-range order

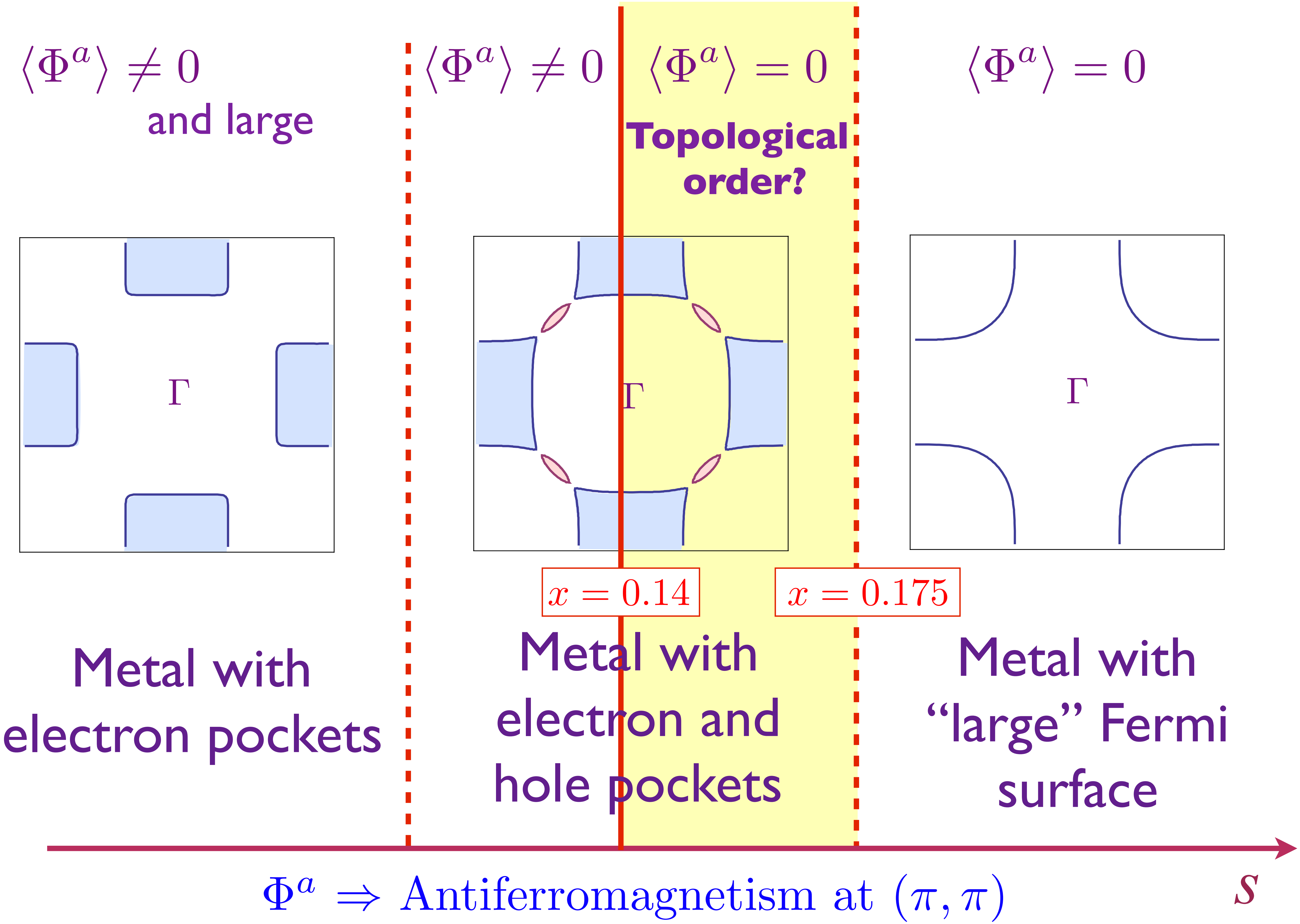
Junfeng He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto, K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu, Y. S. Lee, T. P. Devereaux and Z.-X. Shen

- New photoemission measurements at zero magnetic field show Fermi surfaces in quantitative agreement with quantum oscillation measurements.
- The energy gap between the electron and hole pockets collapses near $x = 0.17$ like an order parameter.
- "The totality of the data points to a mysterious order between $x = 0.14$ and $x = 0.17$, whose appearance favors the FS reconstruction and disappearance defines the quantum critical doping. A recent topological proposal provides an ansatz for its origin."

Square lattice Hubbard model with electron doping



Square lattice Hubbard model with electron doping



1. SU(2) gauge theory of fluctuating antiferromagnetism

2. Electron-doped cuprates

$N_h = 1$ adjoint Higgs field

3. Hole-doped cuprates

$N_h = 4$ adjoint Higgs field

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Gauge theory of fluctuating antiferromagnetism

We can (exactly) transform the Hubbard model to the "spin-fermion" model: **electrons** $c_{i\alpha}$ on the square lattice with dispersion

$$\begin{aligned}\mathcal{H}_c &= -\sum_{i,\rho} t_\rho \left(c_{i,\alpha}^\dagger c_{i+\boldsymbol{v}_\rho,\alpha} + c_{i+\boldsymbol{v}_\rho,\alpha}^\dagger c_{i,\alpha} \right) \\ &\quad -\mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} + \mathcal{H}_{\text{int}}\end{aligned}$$

are coupled to a magnetic moment order parameter $\Phi^p(i)$, $p = x, y, z$

$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \Phi^p(i) c_{i,\alpha}^\dagger \sigma_{\alpha\beta}^p c_{i,\beta} + V_\Phi$$

Gauge theory of fluctuating antiferromagnetism

For fluctuating antiferromagnetism (spin density waves (SDW)), we transform to a rotating reference frame using the SU(2) rotation R_i

$$\left(\begin{array}{c} c_{i\uparrow} \\ c_{i\downarrow} \end{array} \right) = R_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right),$$

in terms of fermionic "chargons" ψ_s and a **Higgs field** $H^a(i)$

$$\sigma^p \Phi^p(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

The Higgs field is the SDW order in the rotating reference frame.

Fluctuating antiferromagnetism

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the AFM order replaced by the Higgs field.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi_{i,s}^\dagger \psi_{i+\boldsymbol{v}_\rho,s} + \psi_{i+\boldsymbol{v}_\rho,s}^\dagger \psi_{i,s} \right) - \mu \sum_i \psi_{i,s}^\dagger \psi_{i,s} + \mathcal{H}_{\rm int}$$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i H^a(i) \psi_{i,s}^\dagger \sigma_{ss'}^a \psi_{i,s'} + V_H$$

IF we can transform to a rotating reference frame in which $H^a(i) =$ a constant independent of i and time, **THEN** the ψ fermions in the presence of fluctuating AFM will inherit the small Fermi surfaces of the electrons in the presence of static AFM.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Field	Symbol	Statistics	$\mathrm{SU(2)_{gauge}}$	$\mathrm{SU(2)_{spin}}$	$\mathrm{U(1)_{e.m.charge}}$
Electron	c	fermion	$\mathbf{1}$	$\mathbf{2}$	-1
AF order	Φ	boson	$\mathbf{1}$	$\mathbf{3}$	0
Chargon	ψ	fermion	$\mathbf{2}$	$\mathbf{1}$	-1
Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

Note that this representation is ambiguous up to a SU(2) gauge transformation, V_i

$$\begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix} \rightarrow V_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix}$$
$$R_i \rightarrow R_i V_i^\dagger$$
$$\sigma^a H^a(i) \rightarrow V_i \, \sigma^b H^b(i) \, V_i^\dagger .$$

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

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Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

Describe the (potential) "topological" transition near optimal doping by a theory for the Higgs field alone

Gauge theory of fluctuating antiferromagnetism

We obtain different numbers of adjoint Higgs scalars, N_h, depending upon the spatial dependence of the local spin correlations:

Neel correlations (electron doped cuprates):
$N_h = 1$,
$\boldsymbol{K} = (\pi, \pi)$,
$H^a(i) = H_1^a(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}_i}$

Bidirectional incommensurate correlations (hole doped cuprates):
$N_h = 4$,
$\boldsymbol{K}_y = (\pi, \pi - \delta)$, $\boldsymbol{K}_x = (\pi - \delta, \pi)$,
$H^a(i) = \mathrm{Re}\left\{\left[H_1^a(\boldsymbol{r}) + iH_2^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_x\cdot\boldsymbol{r}_i} + \left[H_3^a(\boldsymbol{r}) + iH_4^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_y\cdot\boldsymbol{r}_i}\right\}$

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SU(2) gauge theory

SU(2) gauge theory with N_h adjoint Higgs fields H_ℓ^a ($a = 1, 2, 3$, $\ell = 1 \ldots N_h$), with potential $V(H_\ell^a)$ and SU(2) gauge field A_μ^a. There is O(N_h) global flavor symmetry.

$$\mathcal{L} = \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2} \left(\partial_\mu H_\ell^a - \epsilon_{abc} A_\mu^b H_\ell^c \right)^2 + V(H_\ell^a)$$

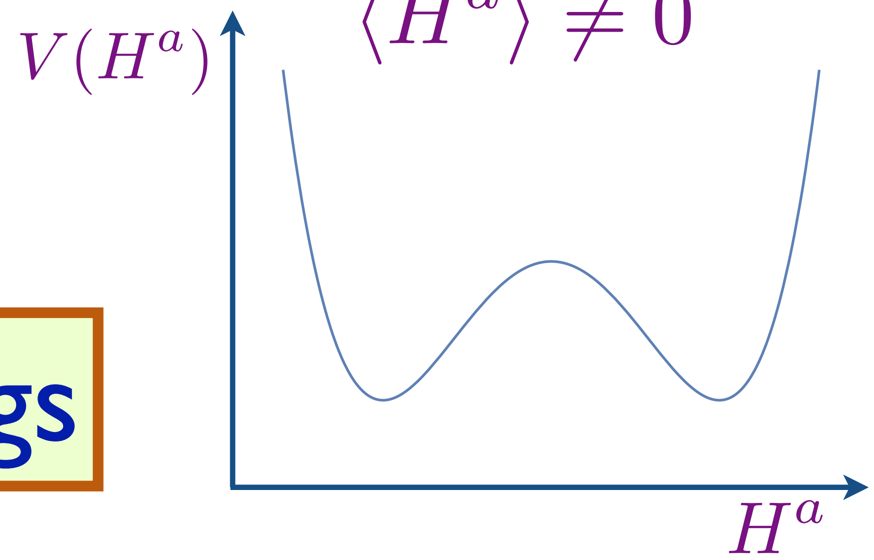
$$\begin{aligned} F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \epsilon_{abc} A_\mu^b A_\nu^c \\ V(H_\ell^a) &= s\, H_\ell^a H_\ell^a + u_0\, H_\ell^a H_\ell^a H_m^b H_m^b \\ &\quad + u_1 \left(H_\ell^a H_m^a H_\ell^b H_m^b - \frac{1}{N_h} H_\ell^a H_\ell^a H_m^b H_m^b \right) \end{aligned}$$

$N_h = 1$

Phase diagrams of SU(2) gauge theory

Higgs

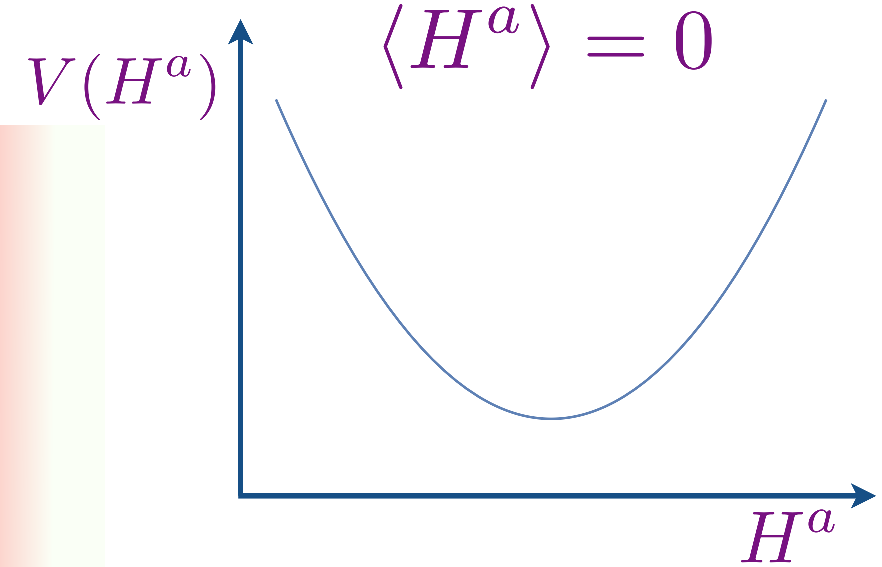
$\langle H^a \rangle \neq 0$



- Condensation of H^a breaks SU(2) to U(1)

Crossover

$\langle H^a \rangle = 0$



Confinement

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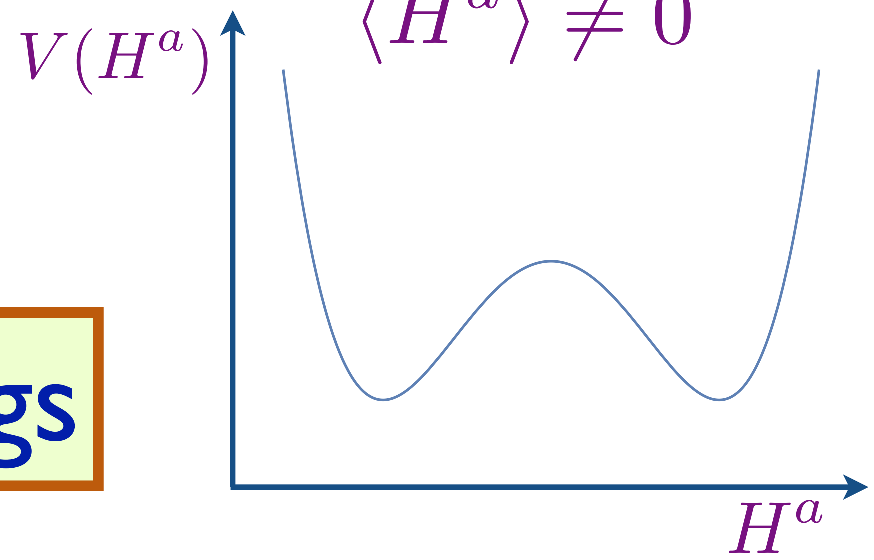
A. C. Davis, A. Hart, T.W. B. Kibble, and A. Rajantie, Phys. Rev. D 65, 125008 (2002)

$N_h = 1$

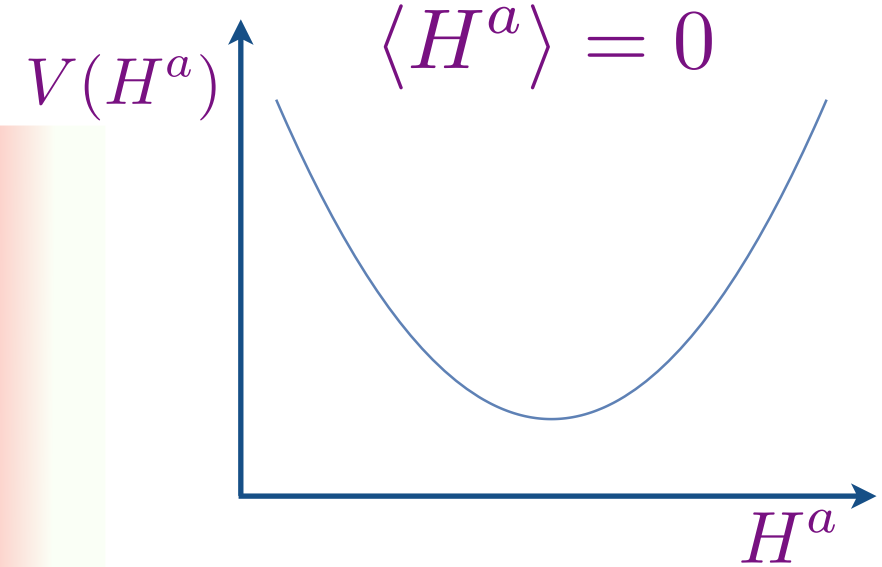
Phase diagrams of SU(2) gauge theory



Higgs

- Condensation of H^a breaks SU(2) to U(1)
- U(1) confines because of proliferation of 'tHooft-Polyakov monopoles

Crossover



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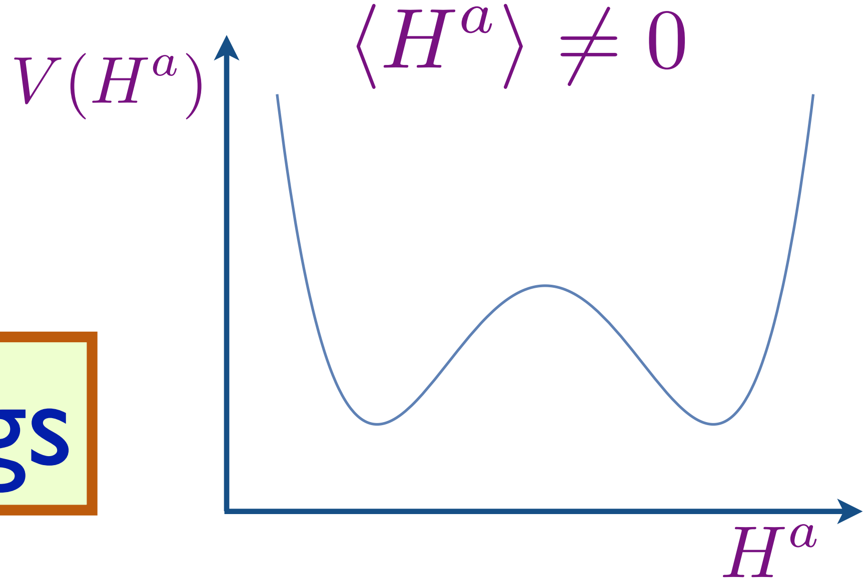
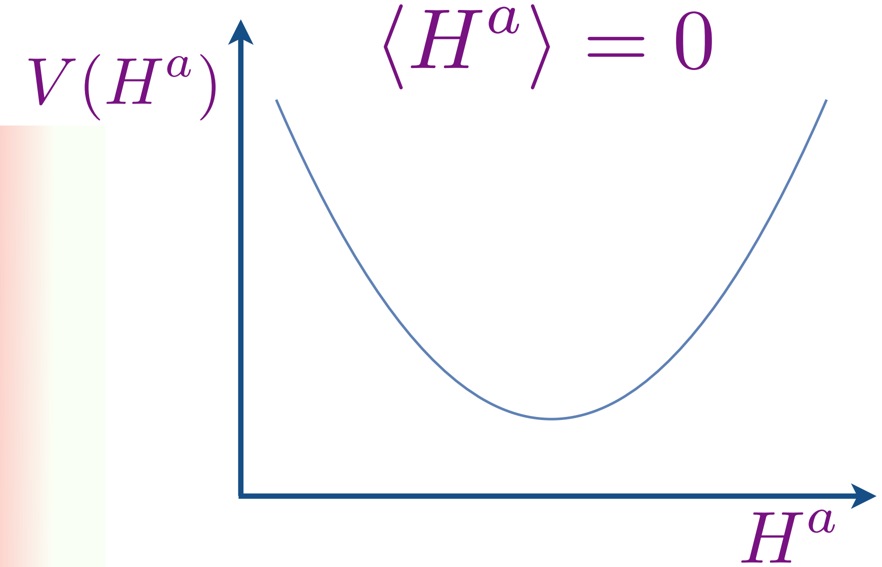


- Condensation of H^a breaks SU(2) to U(1)
- U(1) confines because of proliferation of 'tHooft-Polyakov monopoles
- Monopole action $\sim \sqrt{-s}$, leading to an exponentially large confinement scale

Crossover



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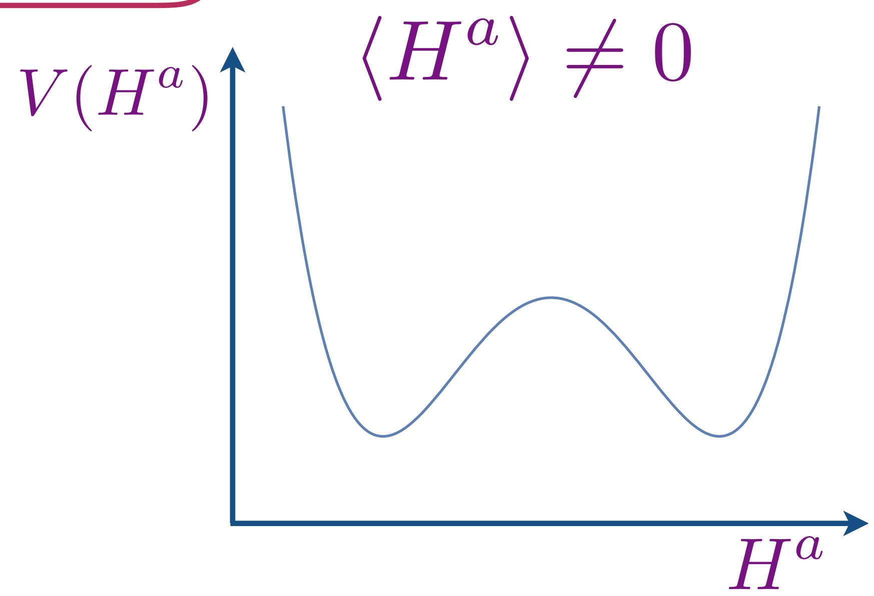

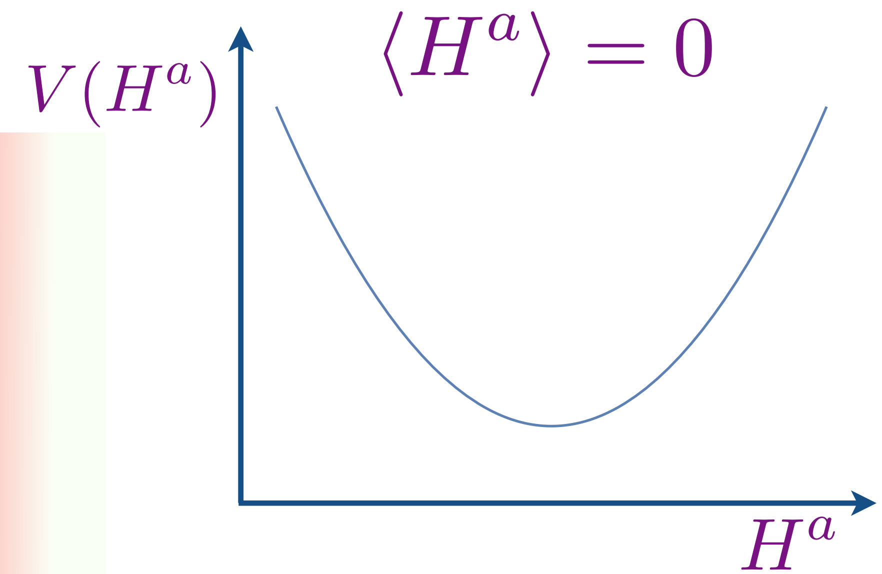

Higgs/U(1) confinement

Reconstructed (FL*) Fermi surfaces, with large length scale confinement in a U(1) gauge theory, leading to re-emergence of large Fermi surface

Crossover

Confinement

Fermi liquid with large Fermi surface

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A. C. Davis, A. Hart, T.W. B. Kibble, and A. Rajantie, Phys. Rev. D 65, 125008 (2002)

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$N_h = 2$

- There is a global $\mathrm{O}(N_h)$ symmetry, and we can define a gauge-invariant order parameter

$$Q_{\ell m} = H^a_\ell H^a_m - \frac{\delta_{\ell m}}{N_h} H^a_n H^a_n$$

- For $u_1 < 0$, the Higgs condensate is of the form

$$H^a_1 = (H_0, 0, 0) \quad , \quad H^a_2 = (0, 0, 0)$$

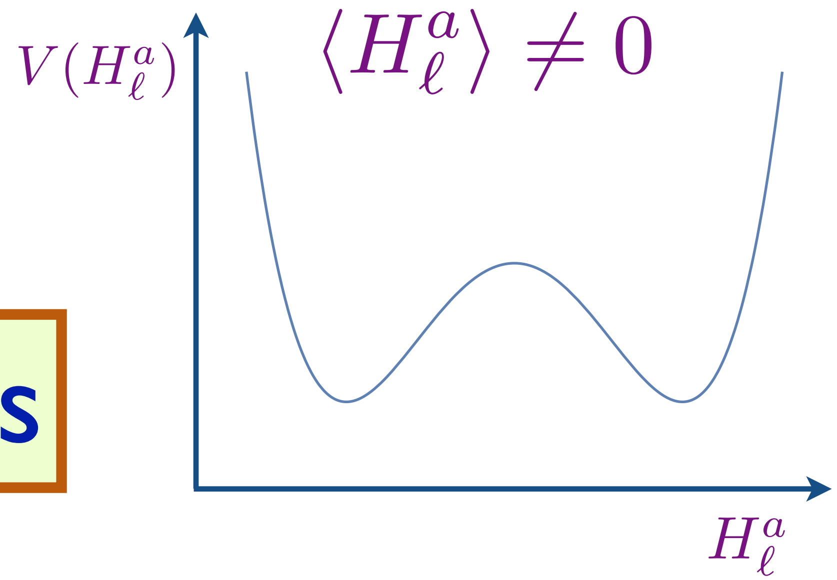
This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1$. But the global O(2) is broken because

$$\langle Q_{\ell m} \rangle \neq 0$$

$N_h = 2$

Phase diagrams of SU(2) gauge theory

Higgs

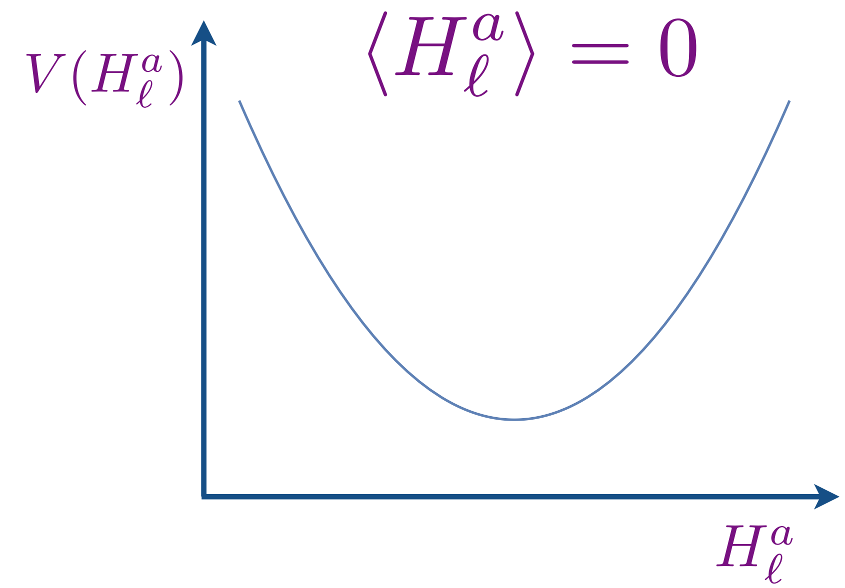


$u_1 < 0$

- Condensation of H^a breaks SU(2) to U(1), which confines
- Broken O(2) symmetry, $\langle Q_{\ell m} \rangle \neq 0$.

Possible Deconfined critical SU(2) gauge theory

Confinement



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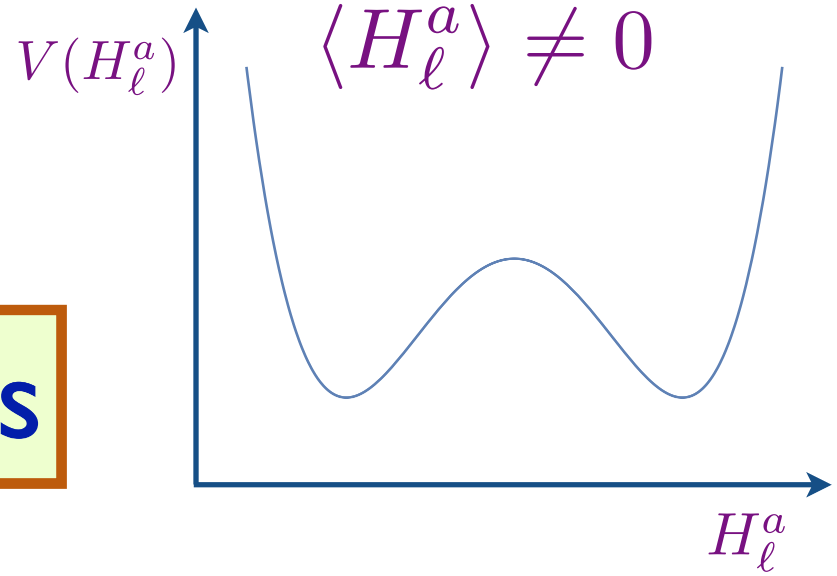
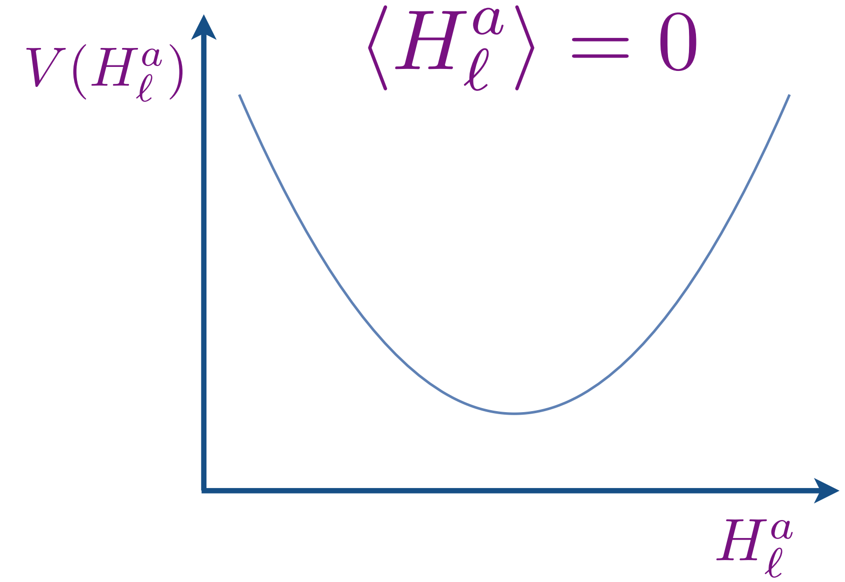
$N_h = 2$

Phase diagrams of SU(2) gauge theory





Higgs

$u_1 < 0$

Confinement

- Condensation of H^a breaks SU(2) to U(1), which confines
- Broken O(2) symmetry, $\langle Q_{\ell m} \rangle \neq 0$.

Possible Deconfined critical SU(2) gauge theory

Multi-universality of a Landau-allowed transition ?

s

$N_h = 2$

- There is a global $\mathrm{O}(N_h)$ symmetry, and we can define a gauge-invariant order parameter

$$Q_{\ell m} = H_\ell^a H_m^a - \frac{\delta_{\ell m}}{N_h} H_n^a H_n^a$$

- For $u_1 < 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, 0, 0)$$

This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1$. But the global O(2) is broken because

$$\langle Q_{\ell m} \rangle \neq 0$$

- For $u_1 > 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, H_0, 0)$$

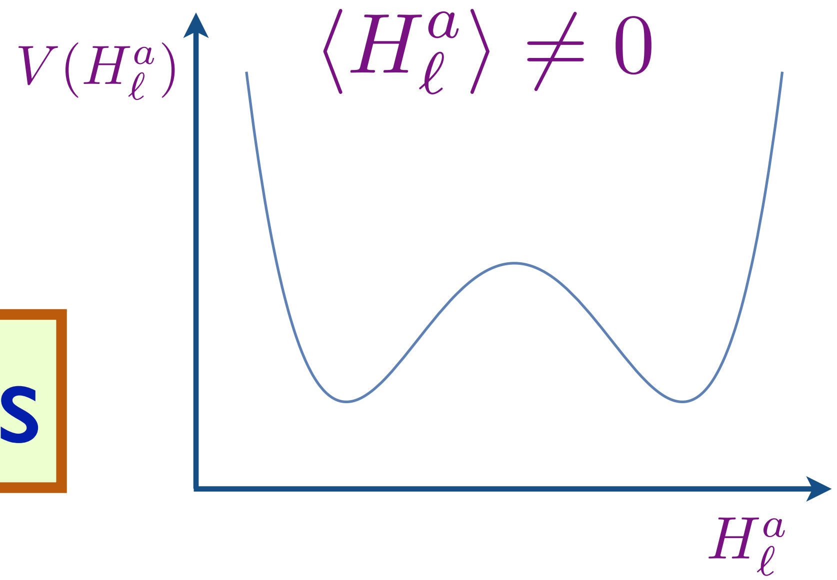
This breaks gauge SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order (as in the 'toric code'). But now the global O(2) is unbroken because

$$\langle Q_{\ell m} \rangle = 0$$

$N_h = 2$

Phase diagrams of SU(2) gauge theory

Higgs

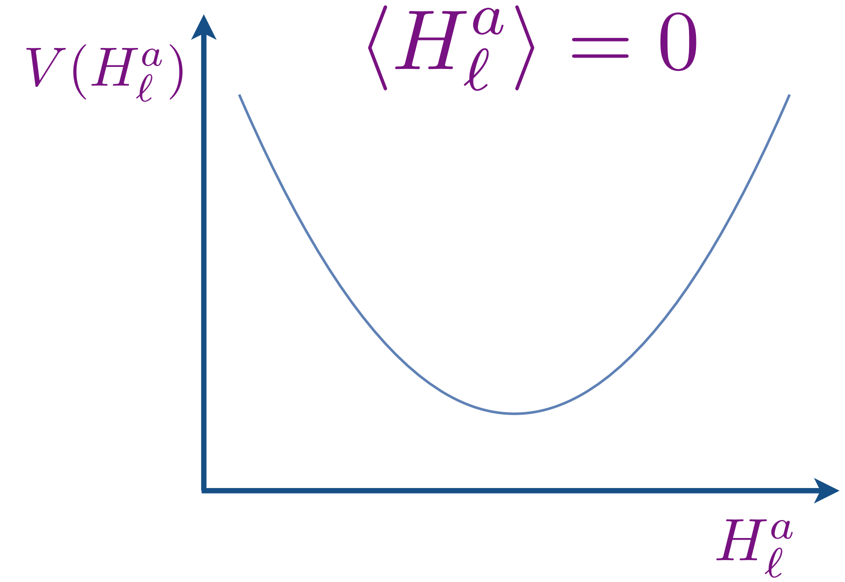


$u_1 > 0$

- Condensation of H^a breaks SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order.
- Unbroken O(2) symmetry, $\langle Q_{\ell m} \rangle = 0$.

Possible Deconfined critical SU(2) gauge theory

Confinement



s

$N_h = 3$

- For $u_1 < 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, 0, 0) \quad , \quad H_3^a = (0, 0, 0)$$

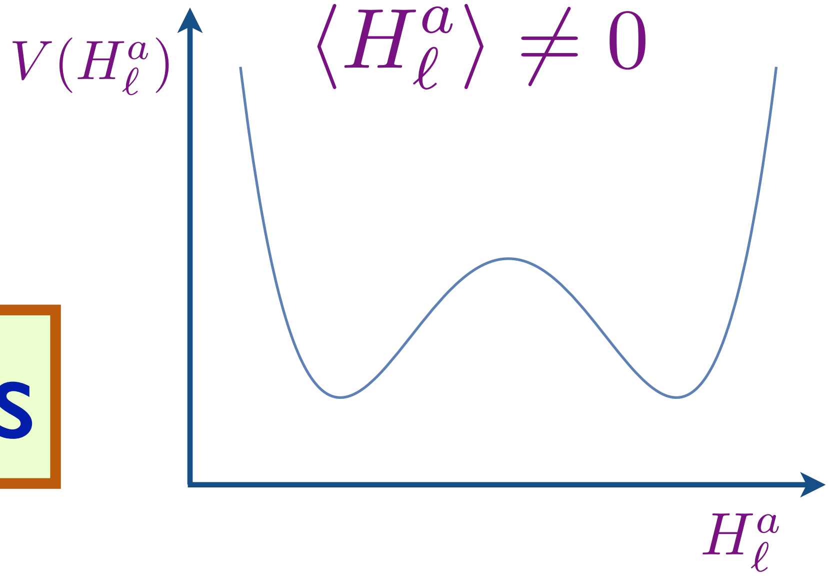
This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1, 2$. The global O(3) is broken to O(2) and

$$\langle Q_{\ell m} \rangle \neq 0$$

$N_h = 3$

Phase diagrams of SU(2) gauge theory

Higgs

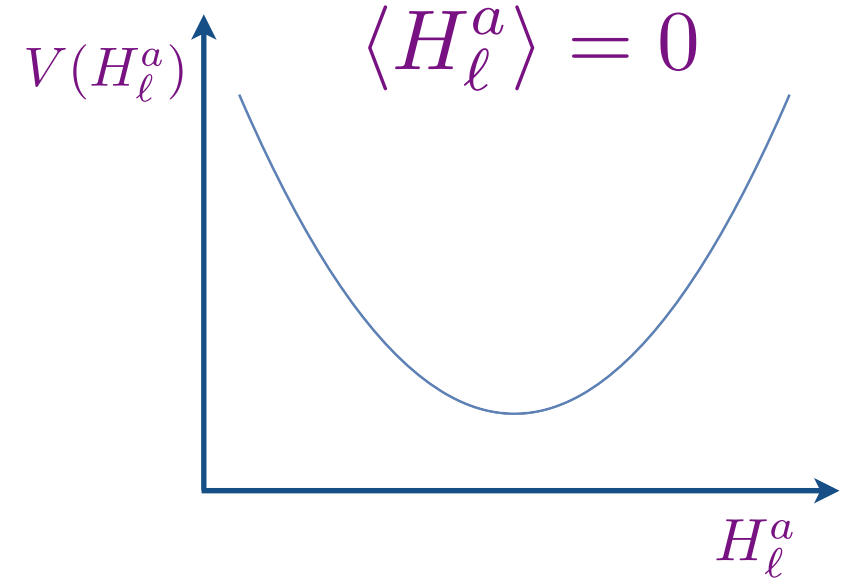


$u_1 < 0$

- Condensation of H^a breaks SU(2) to U(1), which confines
- O(3) symmetry broken to O(2), $\langle Q_{\ell m} \rangle \neq 0$.

Possible Deconfined critical SU(2) gauge theory

Confinement



Multi-universality of a Landau-allowed transition ?

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$N_h = 3$

- For $u_1 < 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, 0, 0) \quad , \quad H_3^a = (0, 0, 0)$$

This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1, 2$. The global O(3) is broken to O(2) and

$$\langle Q_{\ell m} \rangle \neq 0$$

- For $u_1 > 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, H_0, 0) \quad , \quad H_3^a = (0, 0, H_0)$$

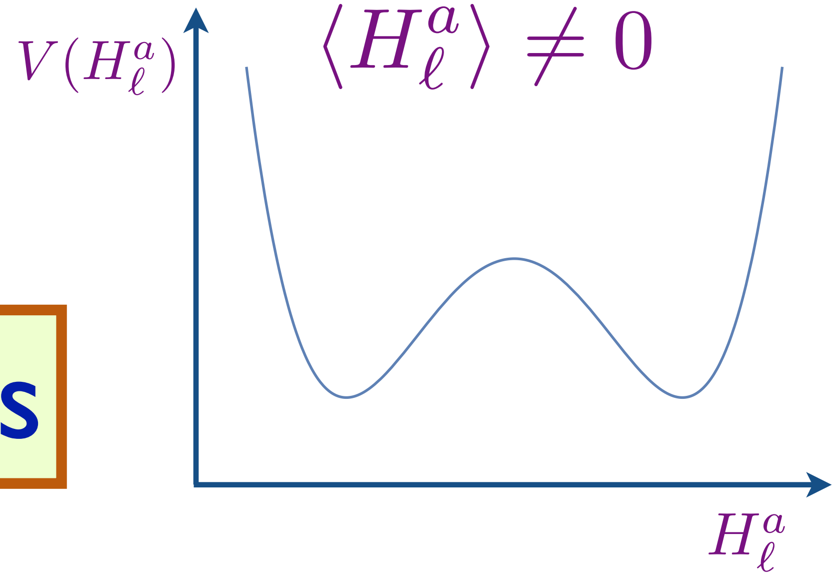
This breaks gauge SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order (as in the 'toric code'). But now the global O(3) is unbroken because

$$\langle Q_{\ell m} \rangle = 0$$

$N_h = 3$

Phase diagrams of SU(2) gauge theory

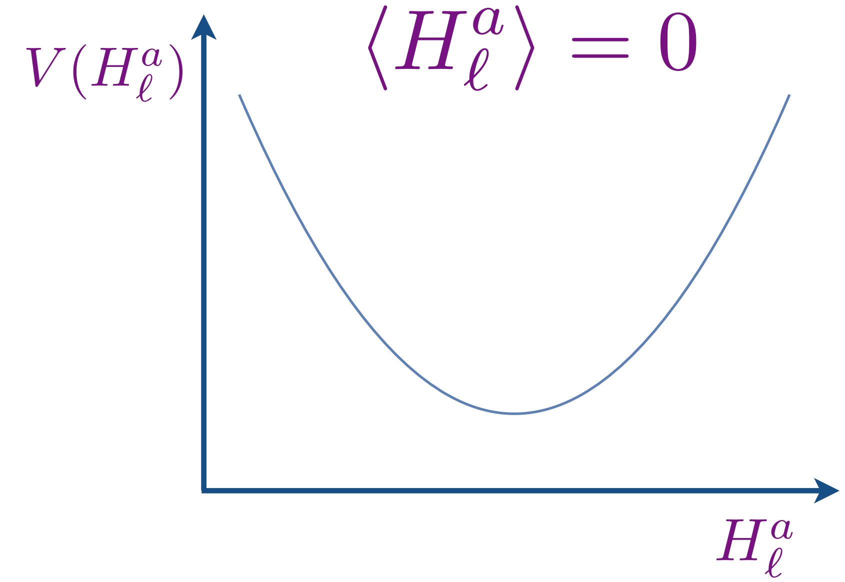
Higgs



$u_1 > 0$

- Condensation of H^a breaks SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order.
- Unbroken O(3) symmetry, $\langle Q_{\ell m} \rangle = 0$.

Possible Deconfined critical SU(2) gauge theory



Confinement

s

$N_h \geq 4$

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- For $u_1 < 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0) \quad , \quad H_{\ell>1}^a = (0,0,0)$$

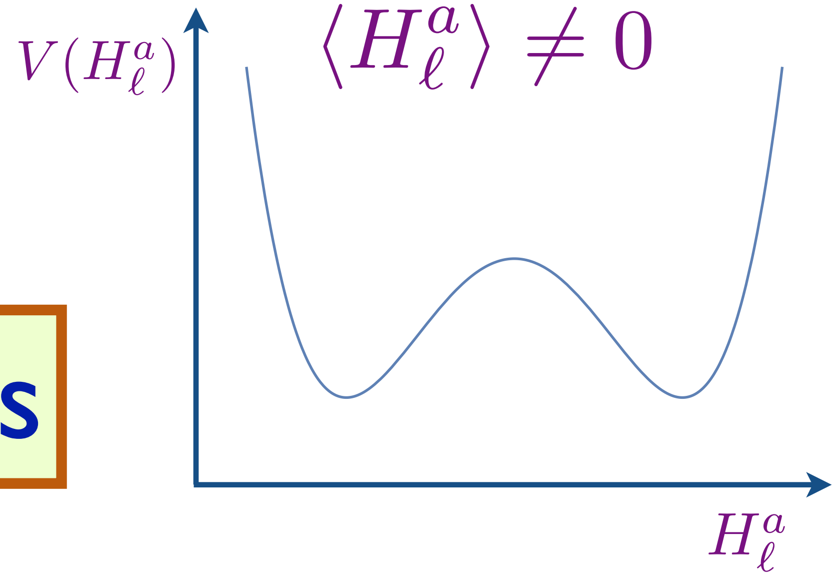
This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1, 2$. The global $\mathrm{O}(N_h)$ is broken to $\mathrm{O}(N_h - 1)$ and

$$\langle Q_{\ell m} \rangle \neq 0$$

$N_h \geq 4$

Phase diagrams of SU(2) gauge theory

Higgs



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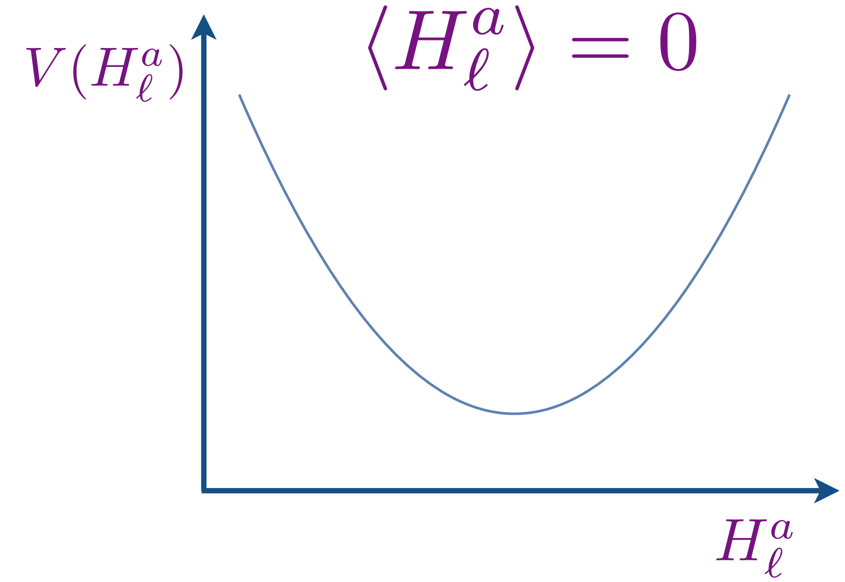
- Condensation of H^a breaks SU(2) to U(1), which confines
- $O(N_h)$ symmetry broken to $O(N_h - 1)$, $\langle Q_{\ell m} \rangle \neq 0$.

Confinement



Possible Deconfined critical SU(2) gauge theory

Multi-universality of a Landau-allowed transition ?

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This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1, 2$. The global O(N_h) is broken to O($N_h - 1$) and

$$\langle Q_{\ell m} \rangle \neq 0$$

- For $u_1 > 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, H_0, 0) \quad , \quad H_3^a = (0, 0, H_0) \quad , \quad H_{\ell>3}^a = (0, 0, 0)$$

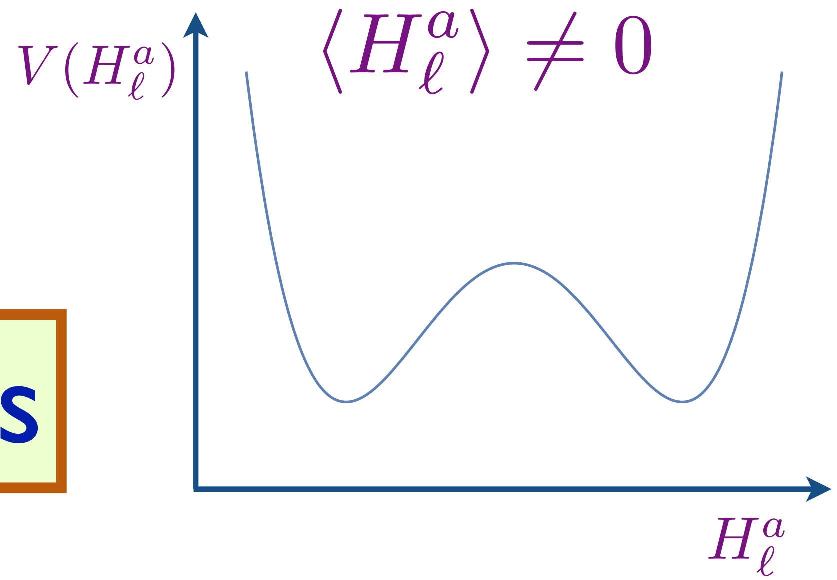
This breaks gauge SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order (as in the 'toric code'). But now the global O(N_h) is broken to O($N_h - 3$)×O(3), and

$$\langle Q_{\ell m} \rangle \neq 0$$

$N_h \geq 4$

Phase diagrams of SU(2) gauge theory

Higgs



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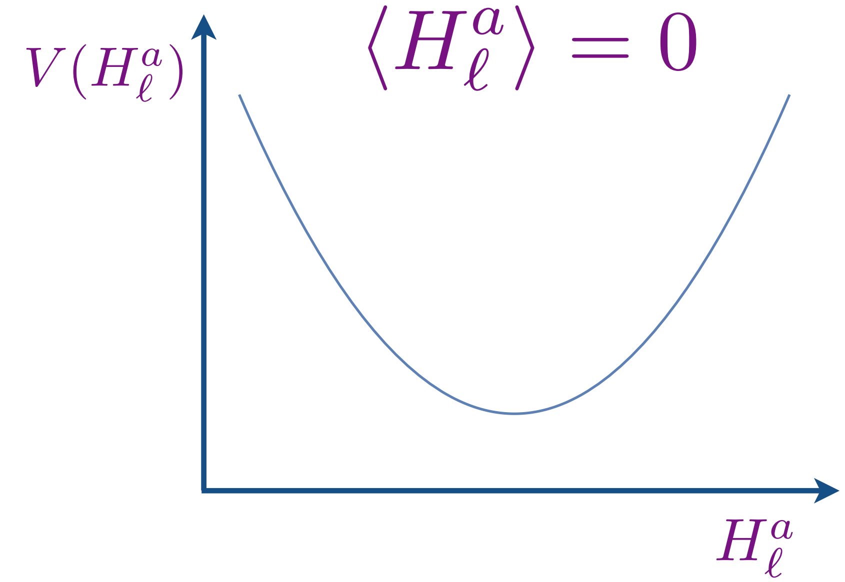
Confinement



- Condensation of H^a breaks SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order.
- $\mathrm{O}(N_h)$ symmetry broken to $\mathrm{O}(N_h - 3) \times \mathrm{O}(3)$, $\langle Q_{\ell m} \rangle \neq 0$.

Possible Deconfined critical SU(2) gauge theory

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Gauge theory of fluctuating antiferromagnetism

For the hole-doped cuprates, $N_h = 4$, we define complex Higgs fields

$$\mathcal{H}_x^a = H_1^a + iH_2^a \quad , \quad \mathcal{H}_y^a = H_3^a + iH_4^a \, .$$

The SU(2) gauge theory is

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}\left|\partial_\mu \mathcal{H}_x^a - \epsilon_{abc} A_\mu^b \mathcal{H}_x^c\right|^2 + \frac{1}{2}\left|\partial_\mu \mathcal{H}_y^a - \epsilon_{abc} A_\mu^b \mathcal{H}_y^c\right|^2 \\
&\quad + \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + V(\mathcal{H}_{x,y}^a) \\
F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \epsilon_{abc} A_\mu^b A_\nu^c \\
V(\mathcal{H}_{x,y}^a) &= s\left(\mathcal{H}_x^{a*}\mathcal{H}_x^a + \mathcal{H}_y^{a*}\mathcal{H}_y^a\right) + u_0\left(\mathcal{H}_x^{a*}\mathcal{H}_x^a + \mathcal{H}_y^{a*}\mathcal{H}_y^a\right)^2 \\
&\quad \frac{u_1}{4}\left(\mathcal{H}_x^{a*}\mathcal{H}_x^a - \mathcal{H}_y^{a*}\mathcal{H}_y^a\right)^2 + \frac{u_2}{2}\left[\left|\mathcal{H}_x^a\mathcal{H}_x^a\right|^2 + \left|\mathcal{H}_y^a\mathcal{H}_y^a\right|^2\right] \\
&\quad + u_3\left(\left|\mathcal{H}_x^a\mathcal{H}_y^a\right|^2 + \left|\mathcal{H}_x^a\mathcal{H}_y^{a*}\right|^2\right) \, .
\end{aligned}$$

Gauge theory of fluctuating antiferromagnetism

There are multiple order parameters for different broken symmetries (Note: spin rotations are preserved and there is no SDW order)

- Ising nematic order

$$\phi = \mathcal{H}_x^{a*} \mathcal{H}_x^a - \mathcal{H}_y^{a*} \mathcal{H}_y^a$$

- Charge density wave (CDW) order at wavevectors $2\boldsymbol{K}_{x,y}$

$$\Phi_x = \mathcal{H}_x^a \mathcal{H}_x^a \quad , \quad \Phi_y = \mathcal{H}_y^a \mathcal{H}_y^a$$

- Charge density wave (CDW) order at wavevectors $\boldsymbol{K}_x \pm \boldsymbol{K}_y$

$$\Phi_+ = \mathcal{H}_x^a \mathcal{H}_y^a \quad , \quad \Phi_- = \mathcal{H}_x^a \mathcal{H}_y^{a*}$$

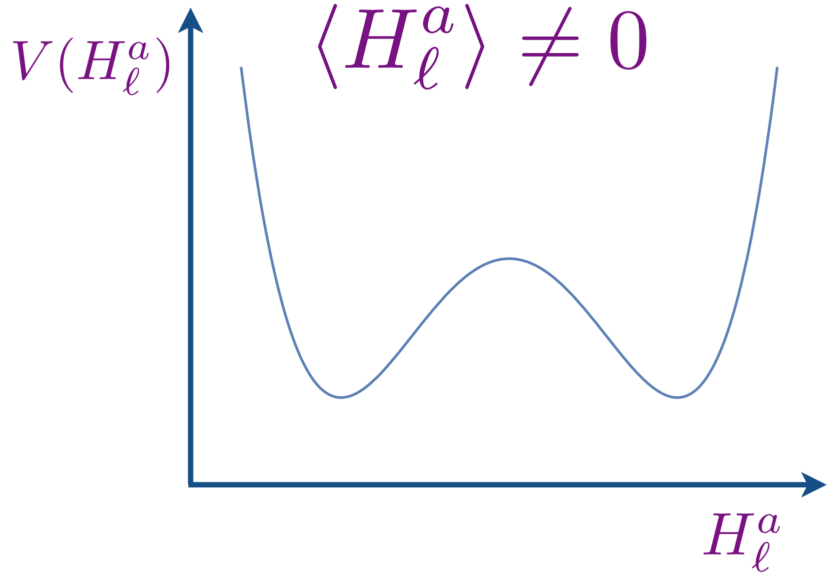
- (Modulated) scalar spin chirality

$$\chi_{ijk} = \epsilon_{abc} H^a(\boldsymbol{r}_i) H^b(\boldsymbol{r}_j) H^c(\boldsymbol{r}_k)$$

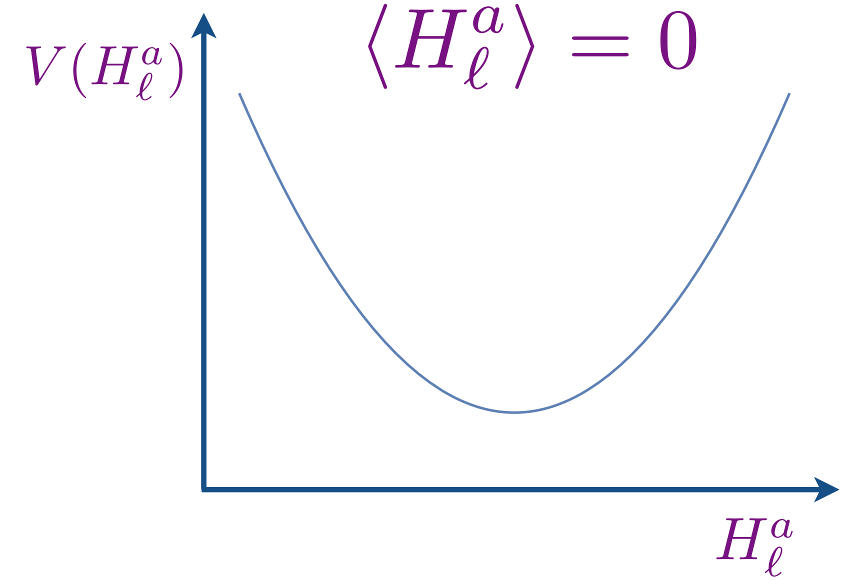
$N_h = 4$

Phase diagrams of SU(2) gauge theory

$\langle H_\ell^a \rangle \neq 0$



$\langle H_\ell^a \rangle = 0$



Higgs/U(1) confinement /Z_2 deconfined

One or more of Ising-nematic, CDW, scalar spin chirality, and $\mathbb{Z}_2$ topological orders

Reconstructed (FL*) Fermi surfaces, with large length scale confinement in the U(1) cases

Possible Deconfined critical SU(2) gauge theory

Confinement

Fermi liquid with large Fermi surface

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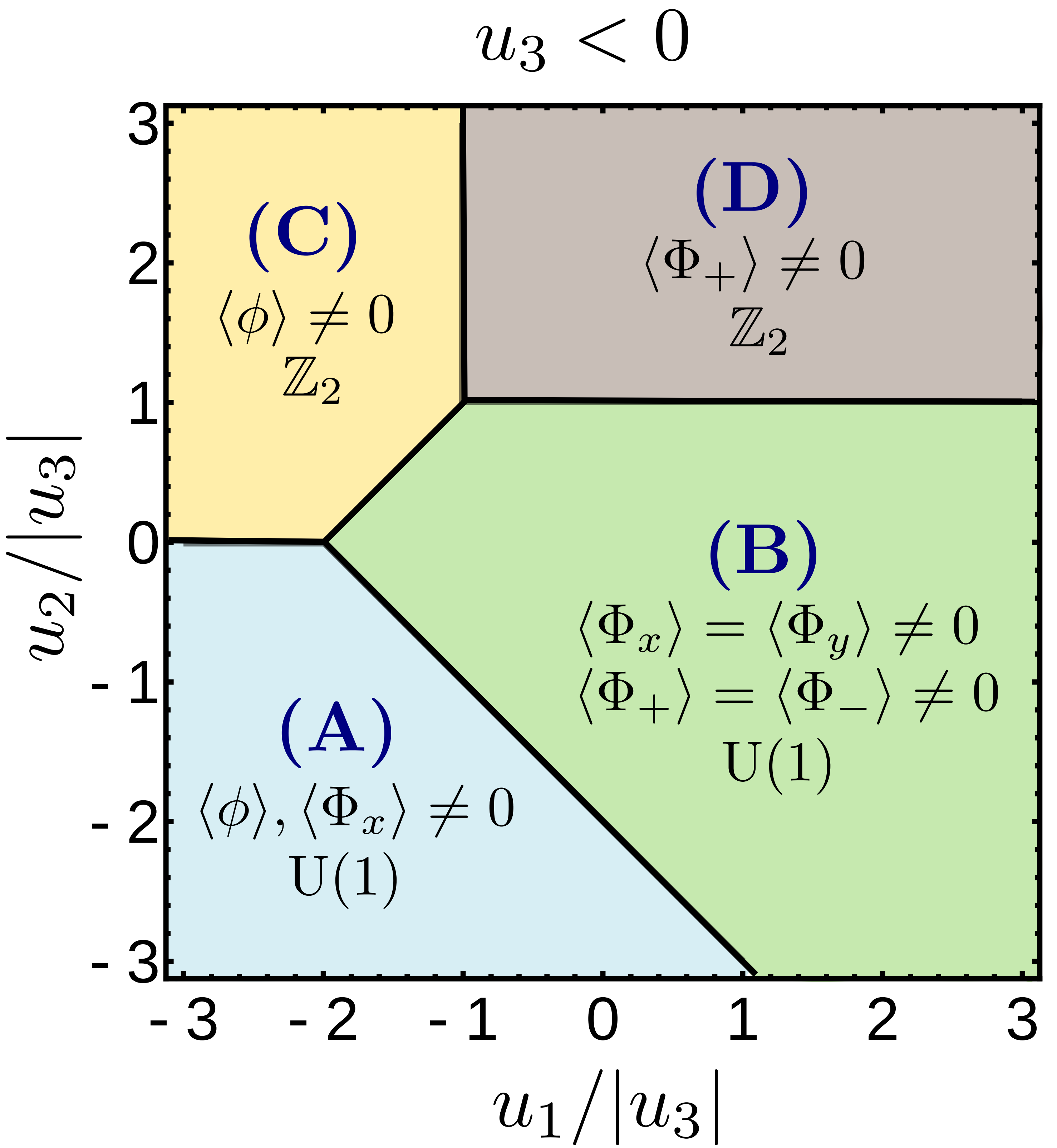
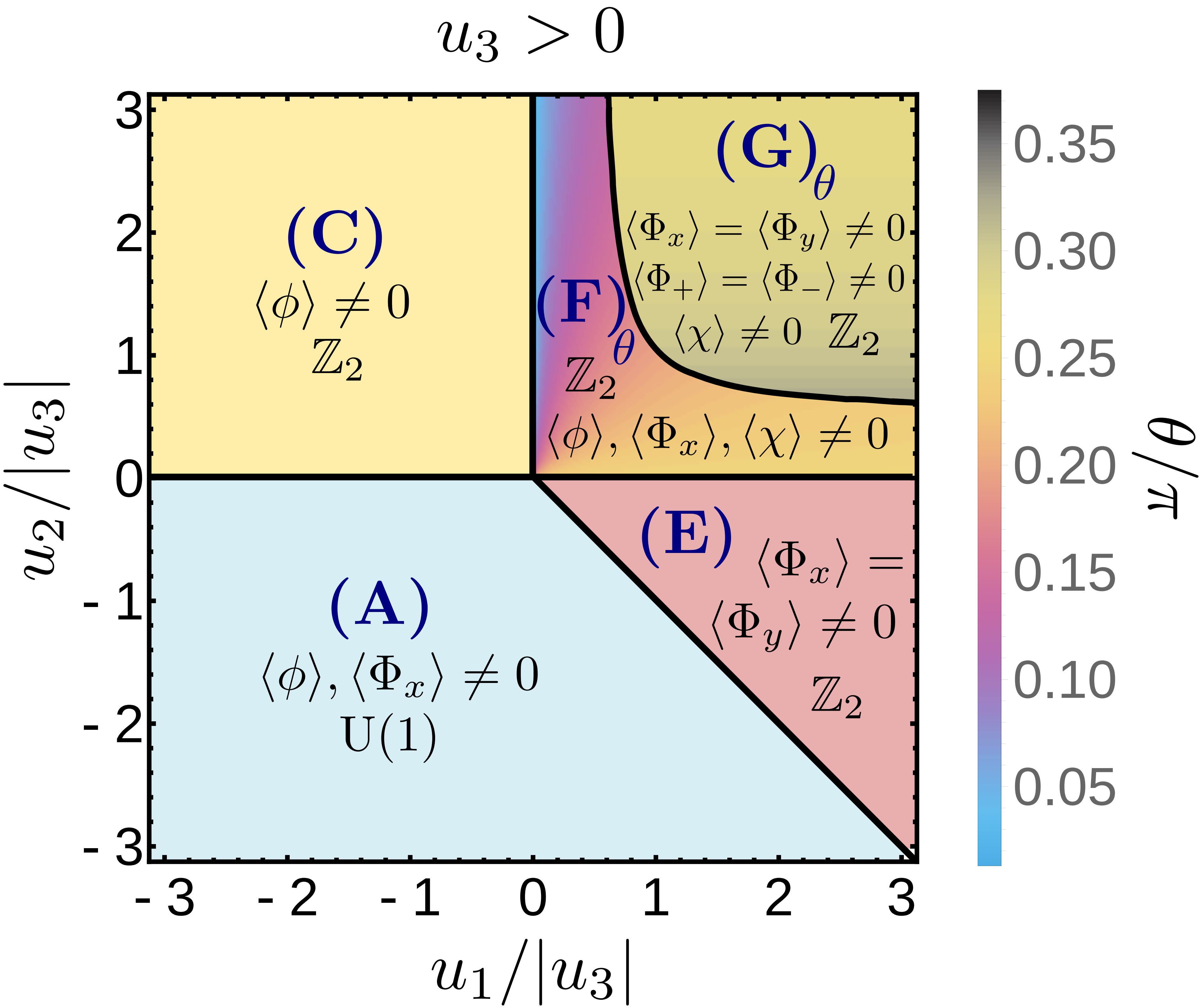
Optimal doping for hole-doped cuprates

Broken symmetries and topological order in the Higgs phase



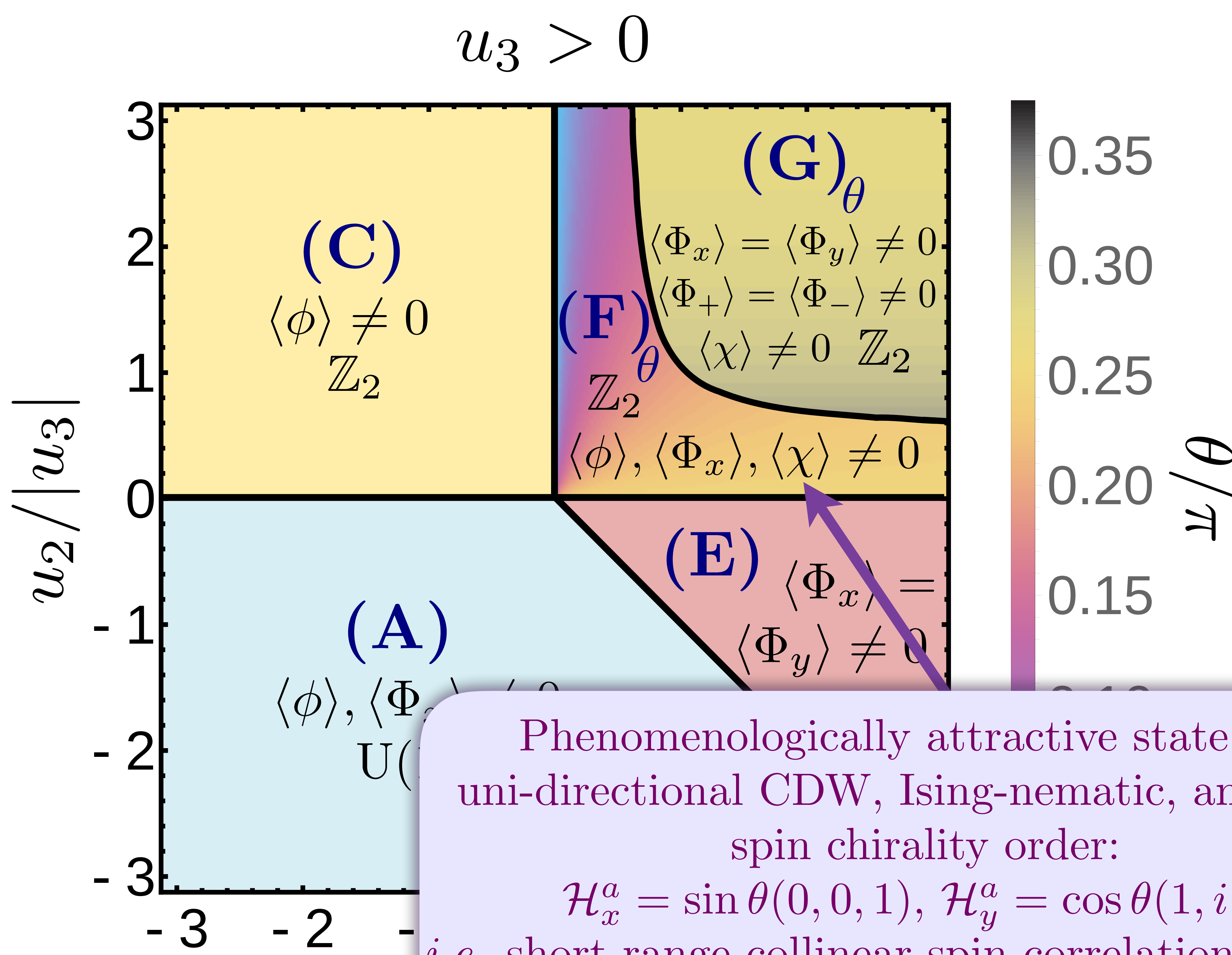
Optimal doping for hole-doped cuprates

Broken symmetries and topological order in the Higgs phase



Optimal doping for hole-doped cuprates

Broken symmetries and topological order in the Higgs phase



Phenomenologically attractive state with uni-directional CDW, Ising-nematic, and scalar spin chirality order:

$\mathcal{H}^a_x = \sin\theta(0,0,1)$, $\mathcal{H}^a_y = \cos\theta(1,i,0)$

i.e. short-range collinear spin correlations along x, and short-range spiral spin correlations along y.

- Cuprates are described across optimal doping by the Higgs-to-confinement crossover/transition of a SU(2) gauge theory with N_h adjoint Higgs fields coupled to a large Fermi surface of gauge-neutral electrons.

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- Electron doped cuprates are described by $N_h = 1$. In this case, there is no phase transition, only a crossover. The underdoped cuprates are described by the Higgs regime, while the overdoped Fermi liquid is the confinement regime.

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- Hole doped cuprates are described by $N_h = 4$. In this case, a phase transition must occur, at least in the absence of disorder. The Higgs phase is characterized by:
  - Stable $\mathbb{Z}_2$ topological order, or U(1) topological up to an exponentially large length scale
  - One or more broken symmetries involving Ising-nematic, CDW, and scalar spin chirality.