

Disordered quantum matter without quasiparticles: the SYK models

34th Jerusalem Winter School in Theoretical Physics
New Horizons in Quantum Matter
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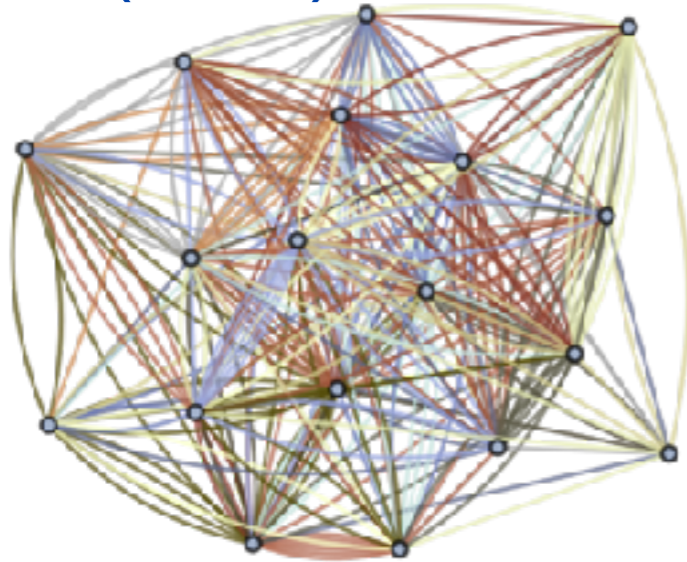
Subir Sachdev

Talk online: sachdev.physics.harvard.edu

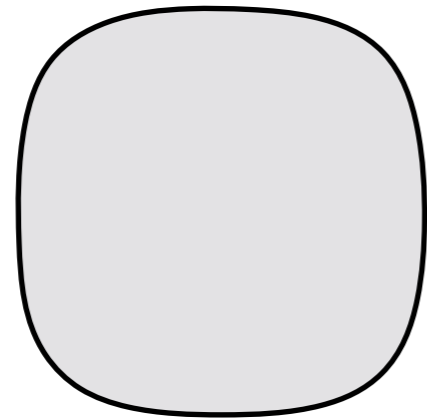
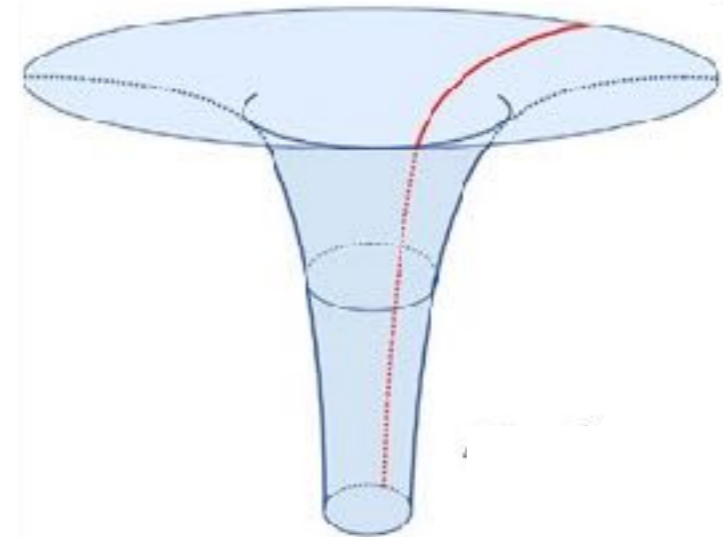


Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



Black holes with AdS₂ horizons

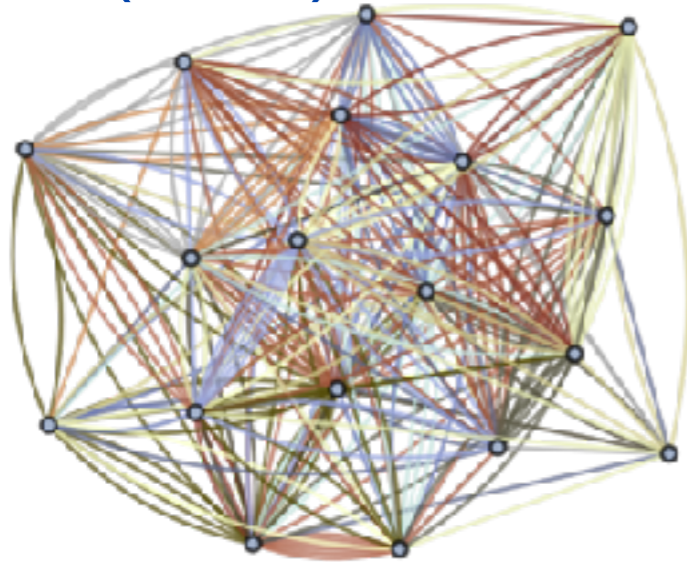


Fermi surface coupled
to a gauge field

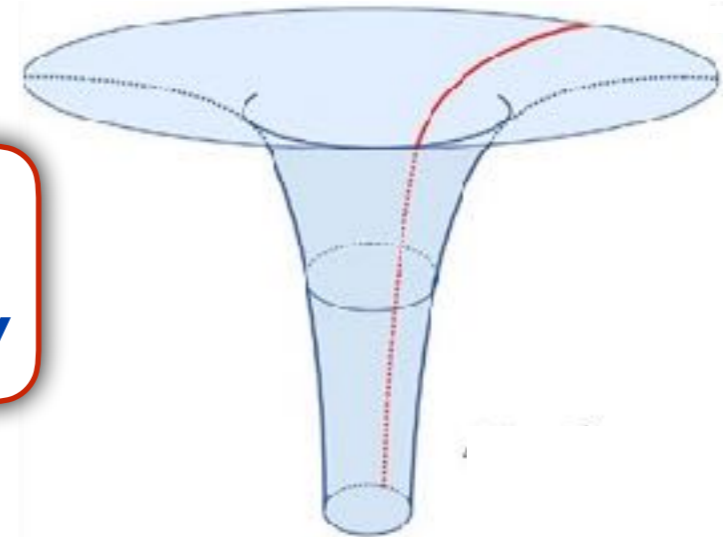
$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - ia_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

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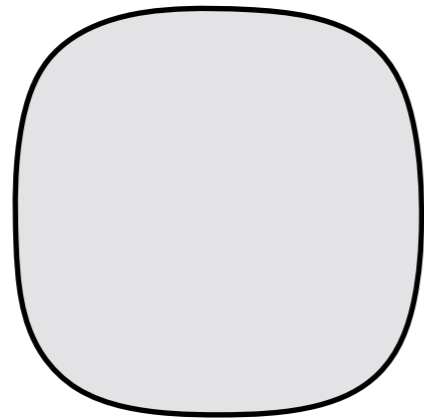
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Same low energy theory

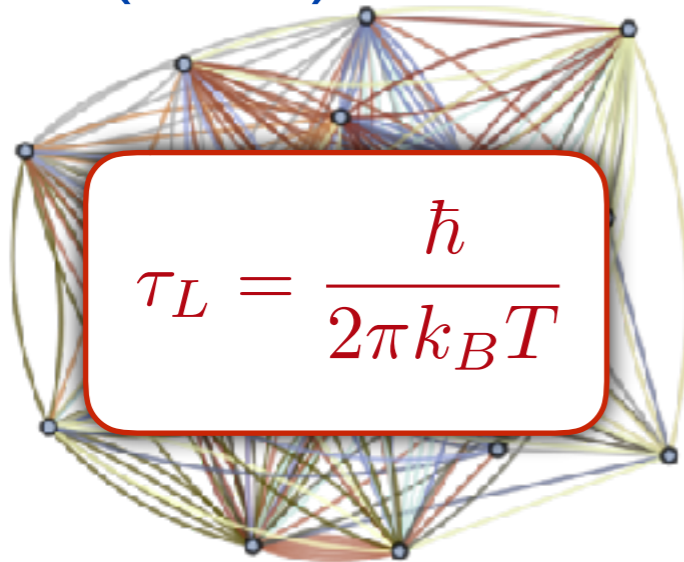


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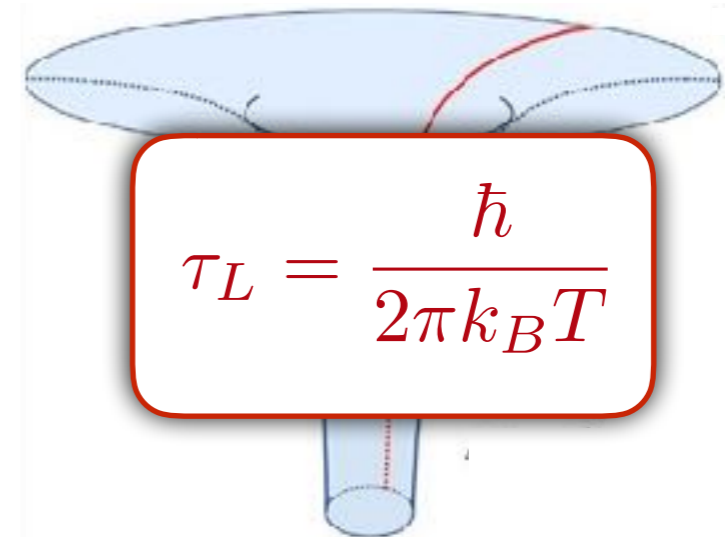
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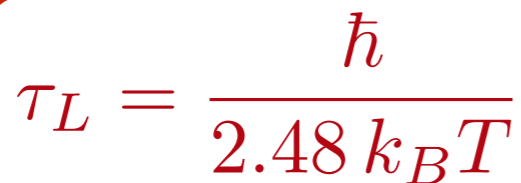


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

Black holes with AdS₂ horizons



$$\tau_L = \frac{\hbar}{2\pi k_B T}$$



A diagram illustrating a Fermi surface coupled to a gauge field. It shows a grey, semi-circular shape representing the Fermi surface. A white box with a red border is overlaid on the diagram, containing the equation for the Lyapunov time τ_L .

$$\tau_L = \frac{\hbar}{2.48 k_B T}$$

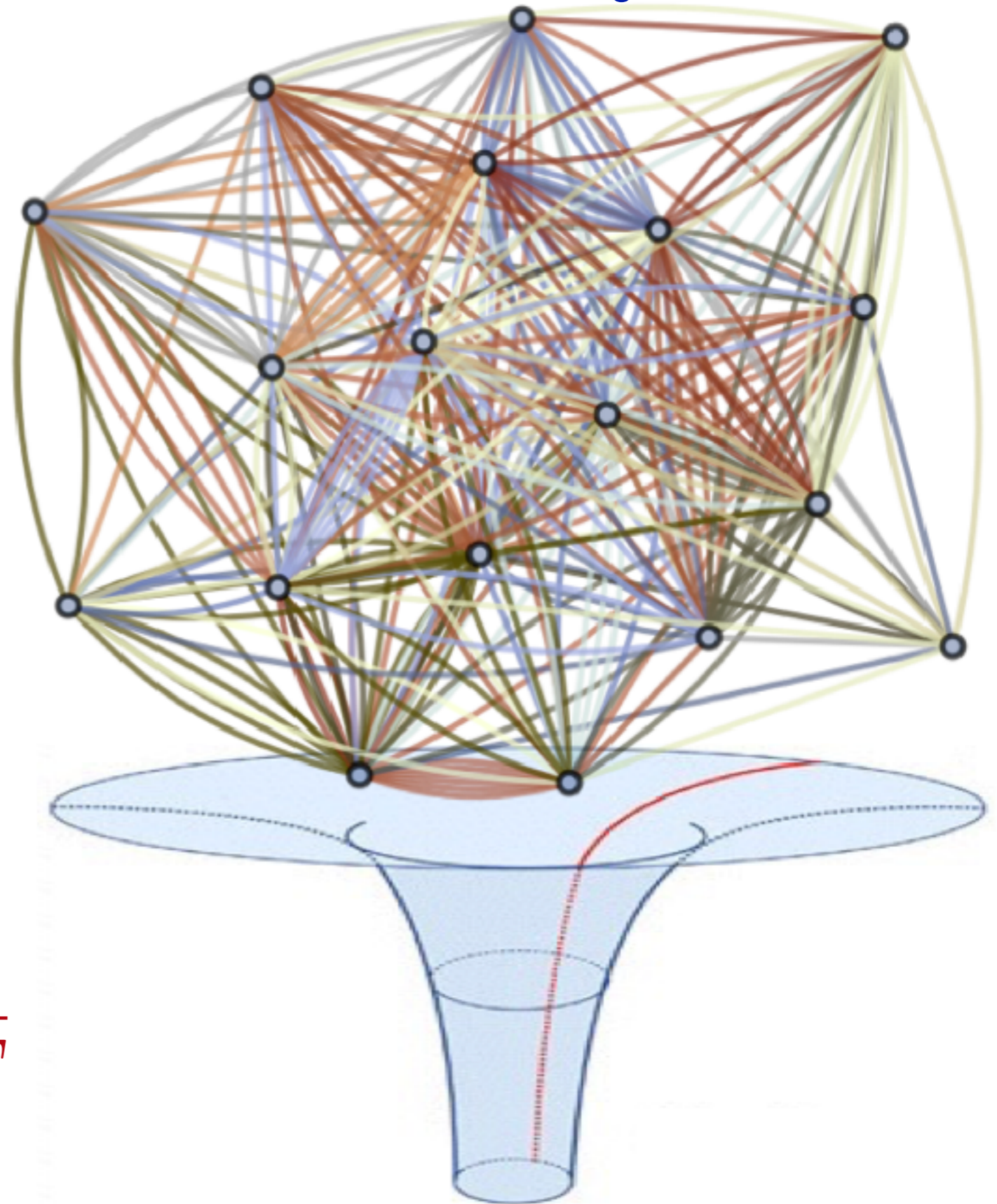
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τ_L : the Lyapunov time to reach quantum chaos

The Sachdev-Ye-Kitaev (SYK) model:

- A theory of a strange metal
- Dual theory of gravity on AdS_2
- Fastest possible quantum chaos with $\tau_L = \frac{\hbar}{2\pi k_B T}$



Infinite-range model with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^N t_{ij} c_i^\dagger c_j + \dots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = Q$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

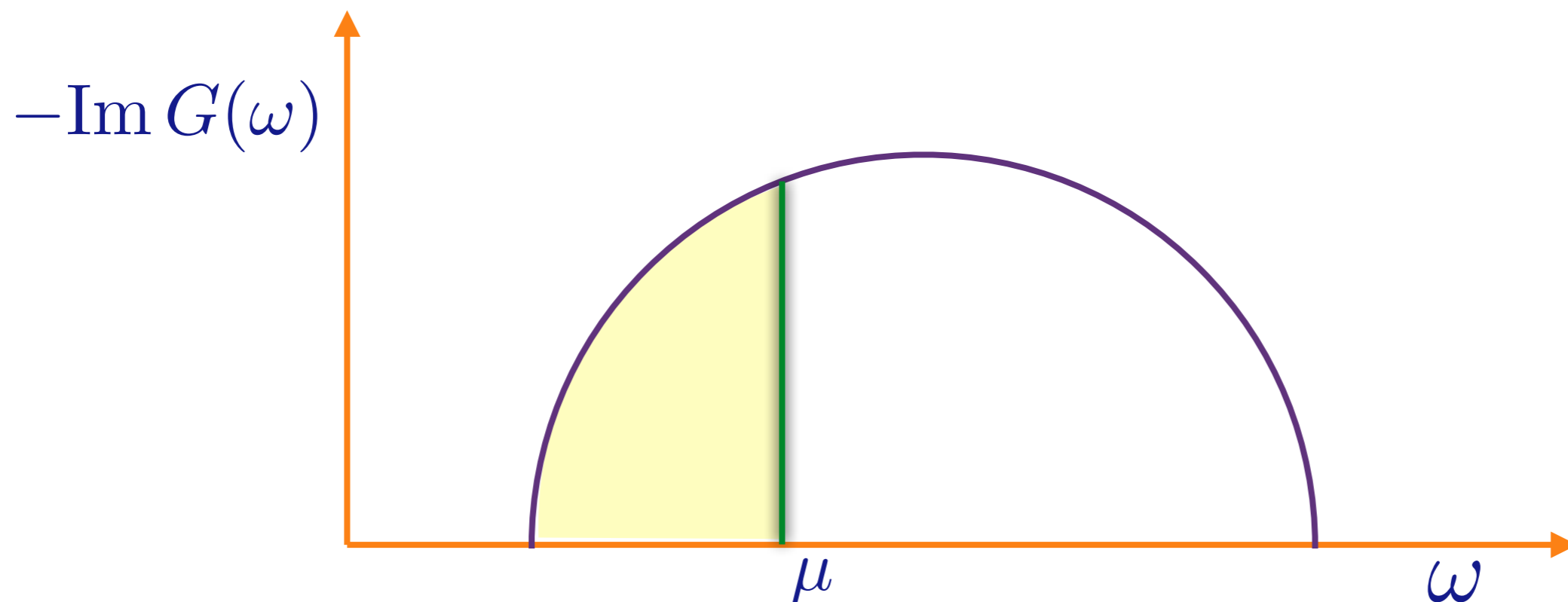
**Fermions occupying the eigenstates of a
 $N \times N$ random matrix**

Infinite-range model with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$
$$G(\tau = 0^-) = Q.$$

$G(\omega)$ can be determined by solving a quadratic equation.



Infinite-range model with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^N t_{ij} c_i^\dagger c_j + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,l=1}^N J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_l$$

$J_{ij;kl}$ are independent random variables with $\overline{J_{ij;kl}} = 0$ and $|\overline{J_{ij;kl}}|^2 = J^2$. We compute the lifetime of a quasiparticle, τ_α , in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamiltonian with energy E_α . By Fermi's Golden rule, for E_α at the Fermi energy

$$\begin{aligned} \frac{1}{\tau_\alpha} &= \pi J^2 \rho_0^2 \int dE_\beta dE_\gamma dE_\delta f(E_\beta)(1 - f(E_\gamma))(1 - f(E_\delta))\delta(E_\alpha + E_\beta - E_\gamma - E_\delta) \\ &= \frac{\pi^3 J^2 \rho_0^2}{4} T^2 \end{aligned}$$

where ρ_0 is the density of states at the Fermi energy.

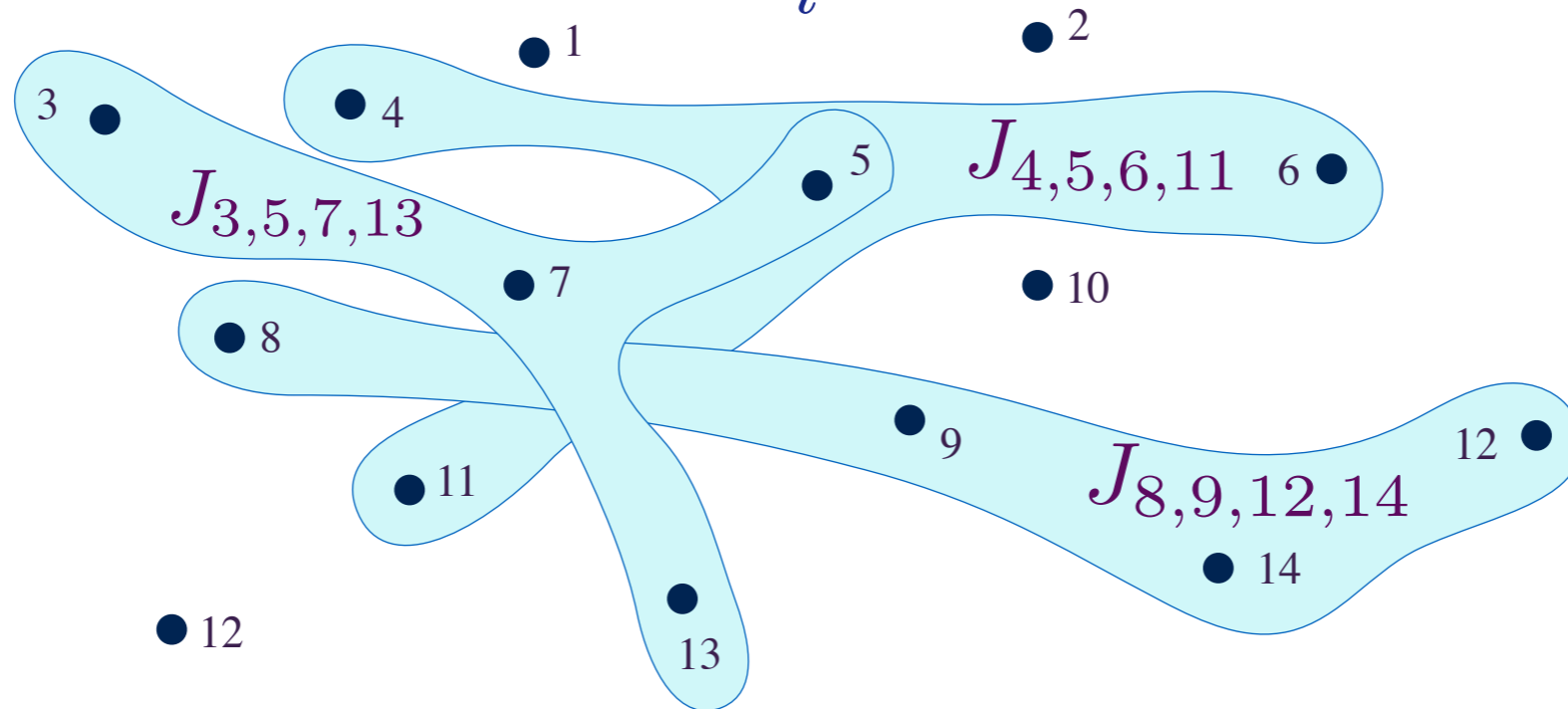
Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^N J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;kl}$ are independent random variables with $\overline{J_{ij;kl}} = 0$ and $\overline{|J_{ij;kl}|^2} = J^2$
 $N \rightarrow \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = Q.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A . The ground state is a non-Fermi liquid, with a continuously variable density Q .

SYK and AdS₂

- Non-zero GPS entropy as $T \rightarrow 0$, $S(T \rightarrow 0) = NS_0 + \dots$
Not a ground state degeneracy: due to an exponentially small (in N) many-body level spacing at all energies down to the ground state energy.



A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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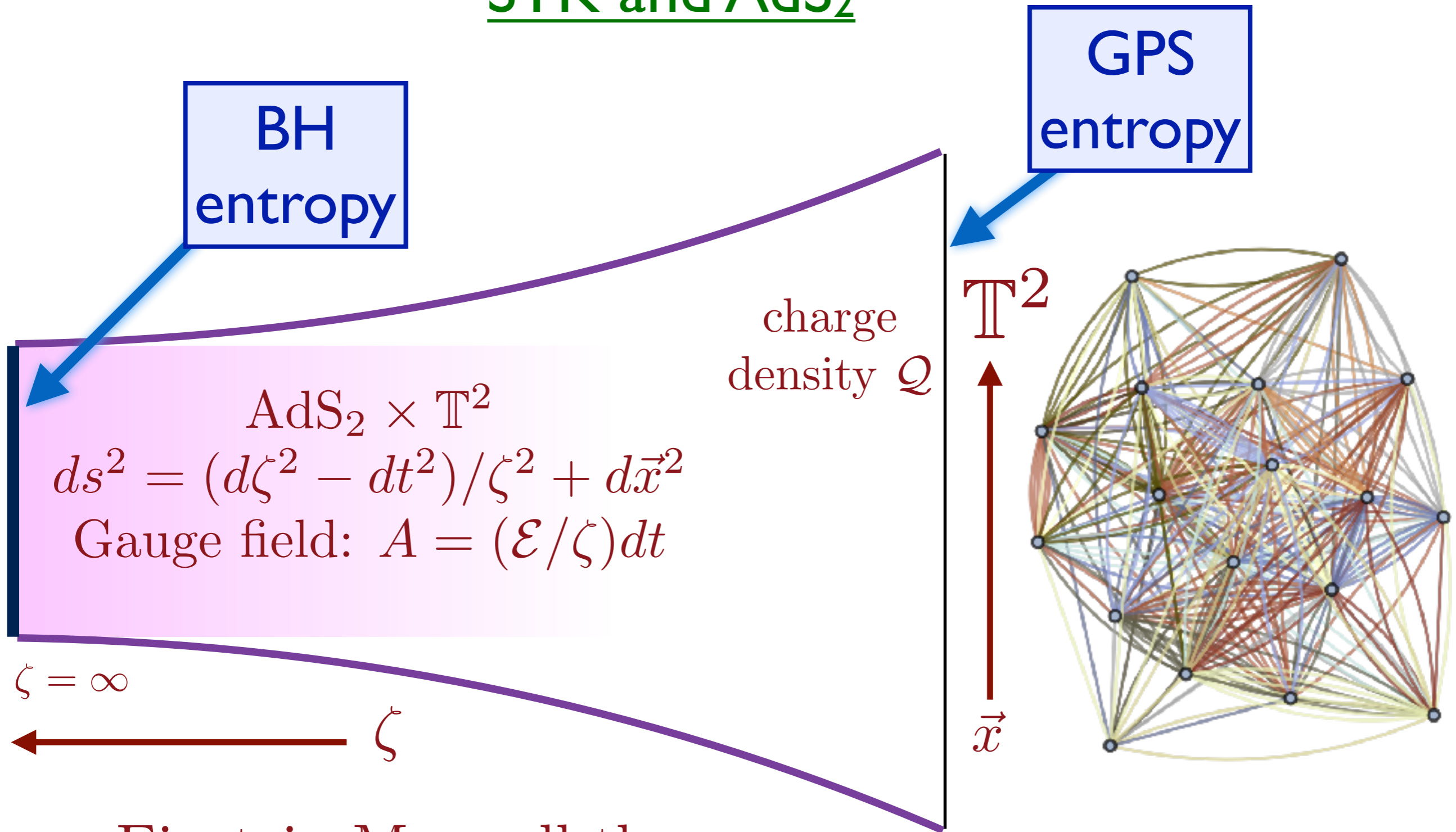
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- This entropy, and other dynamic correlators of the SYK models, imply that the SYK model is holographically dual to black holes with an AdS₂ horizon. The Bekenstein-Hawking entropy of the black hole equals NS_0 :

GPS = BH.

S. Sachdev, PRL **105**, 151602 (2010)

SYK and AdS₂



Mapping to SYK applies when temperature $\ll 1/(\text{size of } T^2)$

SYK and AdS₂

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
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At frequencies $\ll J$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} \tilde{G}(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \tilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

SYK and AdS₂

Let us write the large N saddle point solutions of S as

$$\begin{aligned} G_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-1/2} \\ \Sigma_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-3/2}. \end{aligned}$$

The saddle point will be invariant under a reparamaterization $f(\tau)$ when choosing $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ leads to a transformed $\tilde{G}(\sigma_1, \sigma_2) = G_s(\sigma_1 - \sigma_2)$ (and similarly for Σ). It turns out this is true only for the $\text{SL}(2, \mathbb{R})$ transformations under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d}, \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken down to $\text{SL}(2, \mathbb{R})$ by the saddle point.

SYK and AdS₂

Connections of SYK to gravity and AdS₂ horizons

- Reparameterization and gauge invariance are the ‘symmetries’ of the Einstein-Maxwell theory of gravity and electromagnetism
- SL(2,R) is the isometry group of AdS₂.

$ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$ is invariant under

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d}$$

with $ad - bc = 1$.

SYK and AdS₂

Reparametrization and phase zero modes

We can write the path integral for the SYK model as

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_1) \mathcal{D}\Sigma(\tau_1, \tau_2) e^{-NS[G, \Sigma]}$$

for a known action $S[G, \Sigma]$. We find the saddle point, G_s, Σ_s , and only focus on the “Nambu-Goldstone” modes associated with breaking reparameterization and U(1) gauge symmetries by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2)) e^{i\phi(\tau_1) - i\phi(\tau_2)}$$

(and similarly for Σ). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) e^{-NS_{\text{eff}}[f, \phi]}.$$

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv:1612.00849;
S. Sachdev, PRX **5**, 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

SYK and AdS₂

$$\mathcal{Z} = \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) e^{-N S_{\text{eff}}[f, \phi]}.$$

Symmetry arguments, and explicit computations, show that the effective action is

$$S_{\text{eff}}[f, \phi] = \frac{K}{2} \int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T) \partial_\tau \epsilon)^2 - \frac{\gamma}{4\pi^2} \int_0^{1/T} d\tau \{ \tan(\pi T(\tau + \epsilon(\tau))), \tau \},$$

where $f(\tau) \equiv \tau + \epsilon(\tau)$, the couplings K , γ , and \mathcal{E} can be related to thermodynamic derivatives and we have used the Schwarzian:

$$\{g, \tau\} \equiv \frac{g'''}{g'} - \frac{3}{2} \left(\frac{g''}{g'} \right)^2.$$

Specifically, an argument constraining the effective at $T = 0$ is

$$S_{\text{eff}} \left[f(\tau) = \frac{a\tau + b}{c\tau + d}, \phi(\tau) = 0 \right] = 0,$$

and this is origin of the Schwarzian.

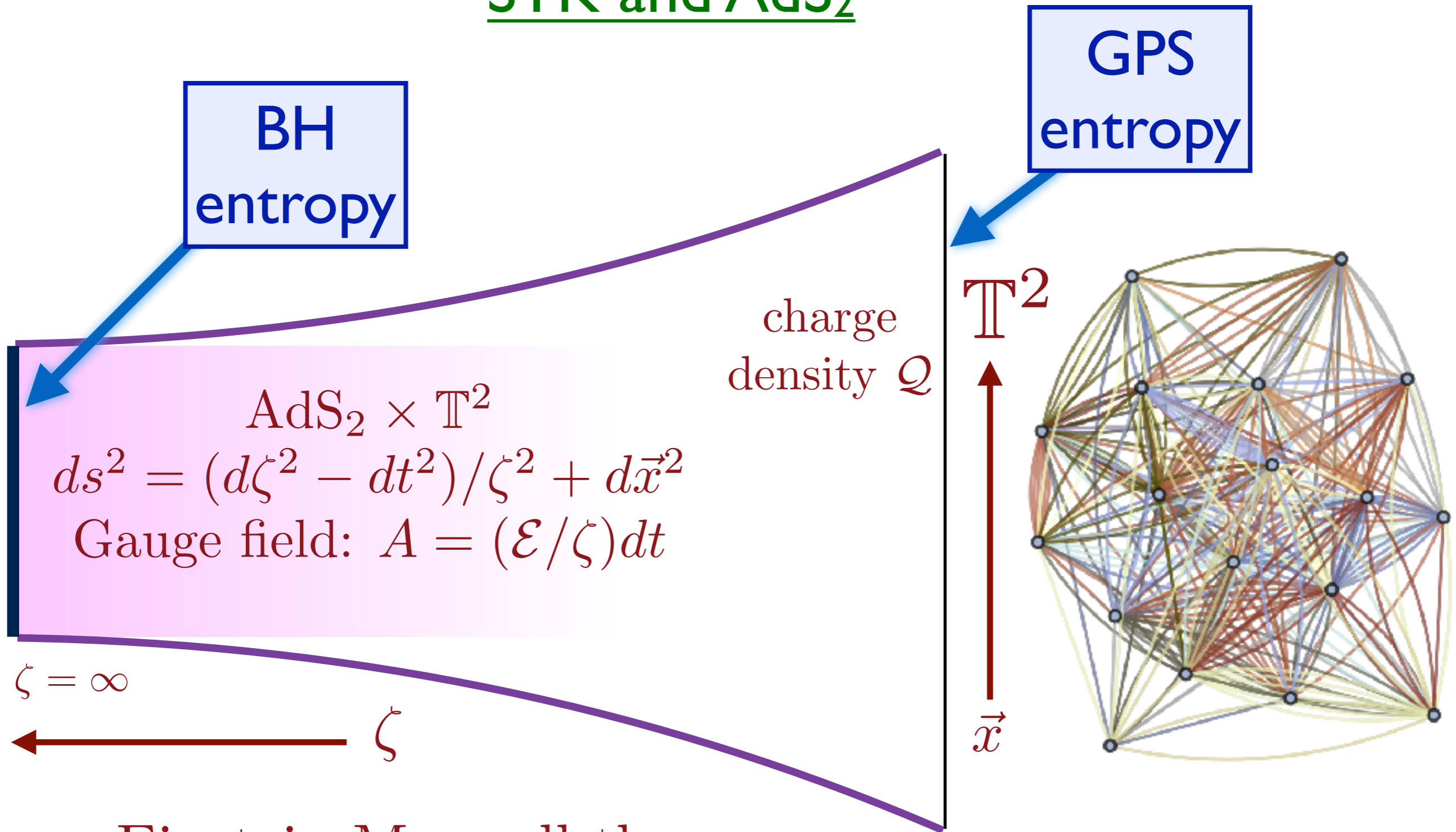
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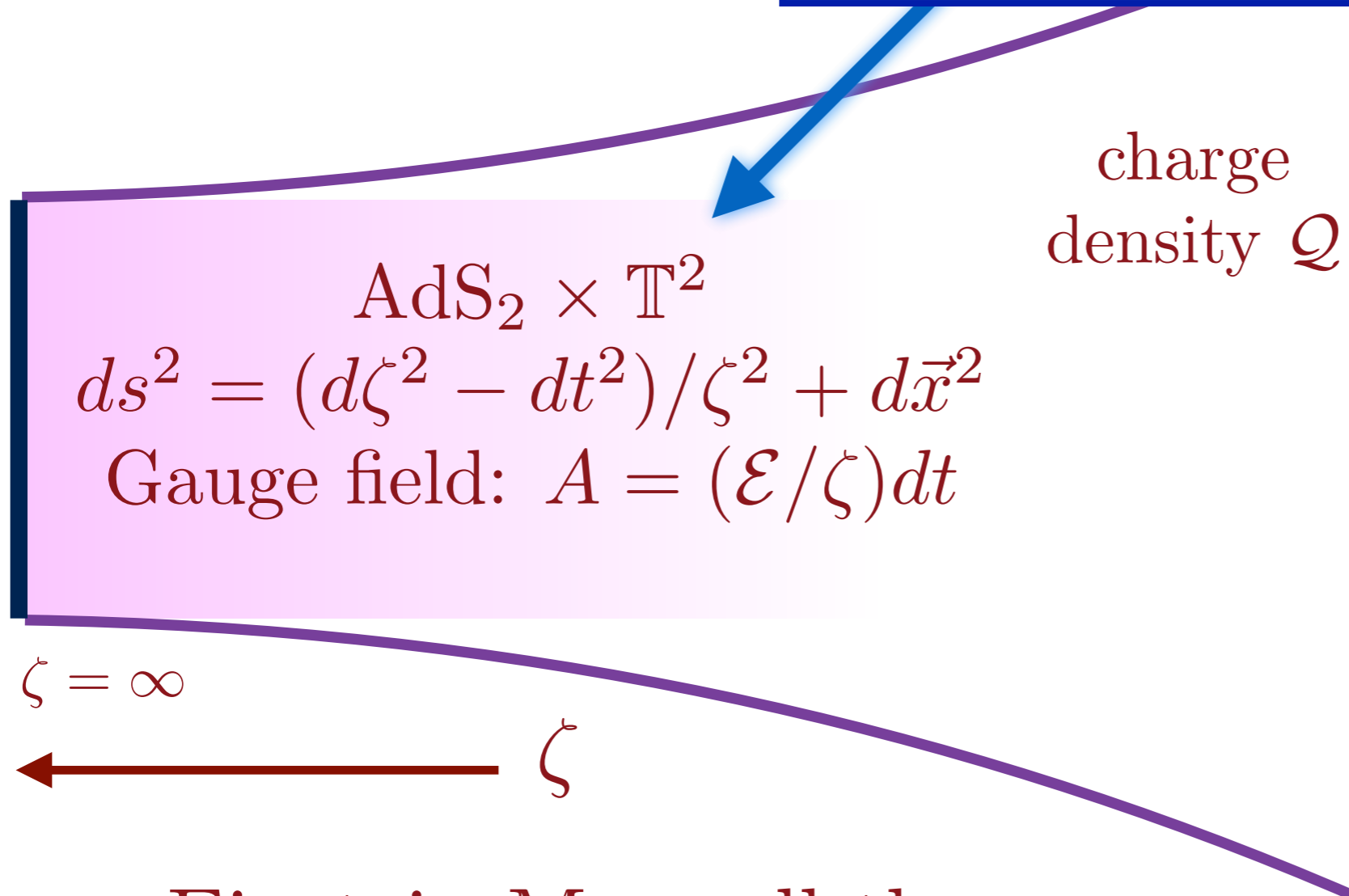
Einstein-Maxwell theory
+ cosmological constant

S. Sachdev, PRL **105**, 151602 (2010)

Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$

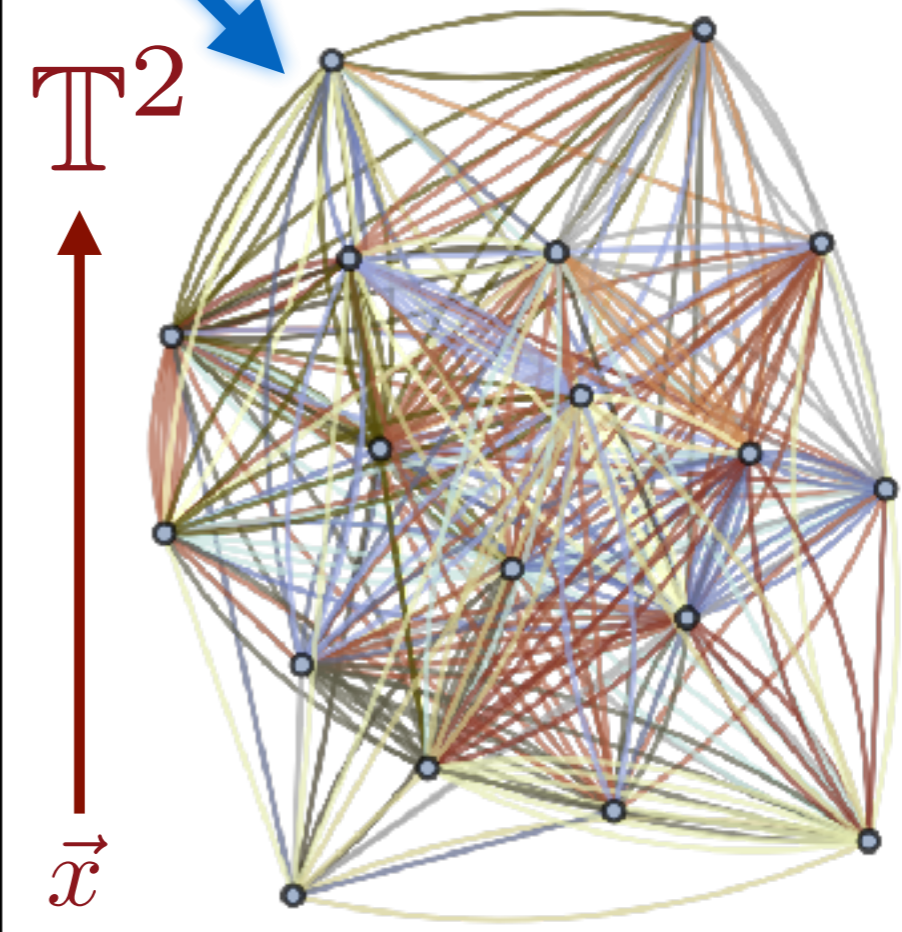
SYK and AdS₂

Same long-time
effective action



$$AdS_2 \times T^2$$
$$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$$

Gauge field: $A = (\mathcal{E}/\zeta)dt$



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Coupled SYK models

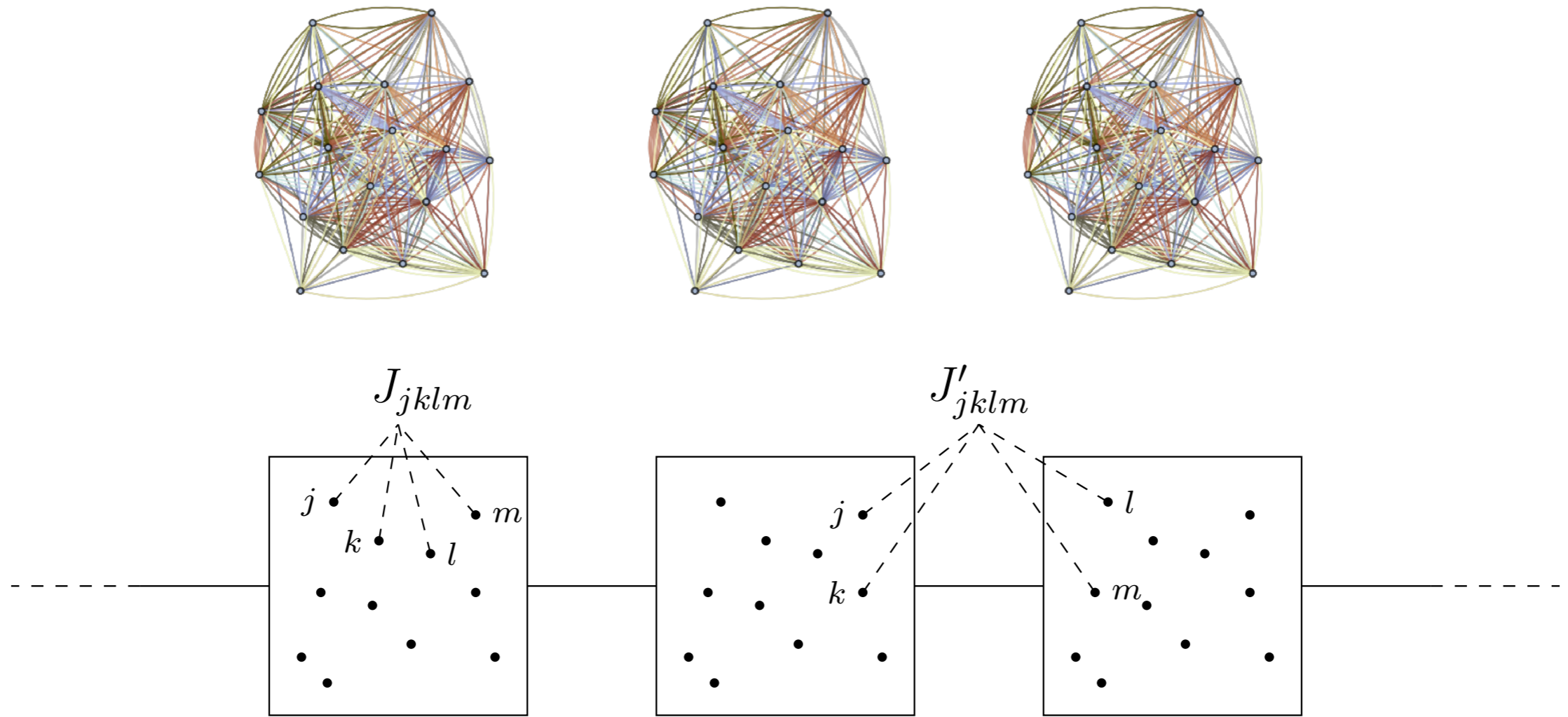
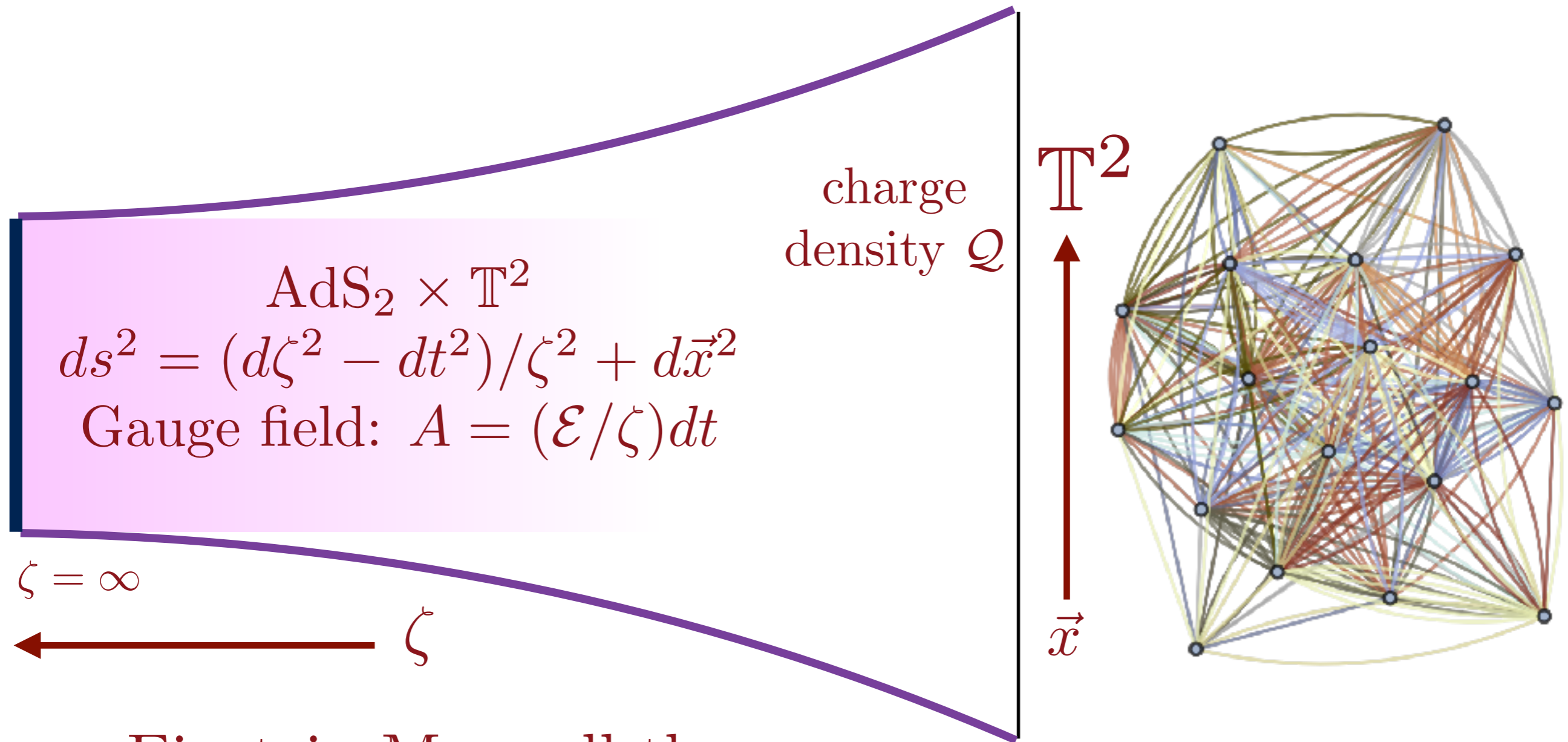


Figure 1: A chain of coupled SYK sites: each site contains $N \gg 1$ fermion with SYK interaction. The coupling between nearest neighbor sites are four fermion interaction with two from each site.

Yingfei Gu, Xiao-Liang Qi, and D. Stanford, arXiv:1609.07832
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv:1612.00849

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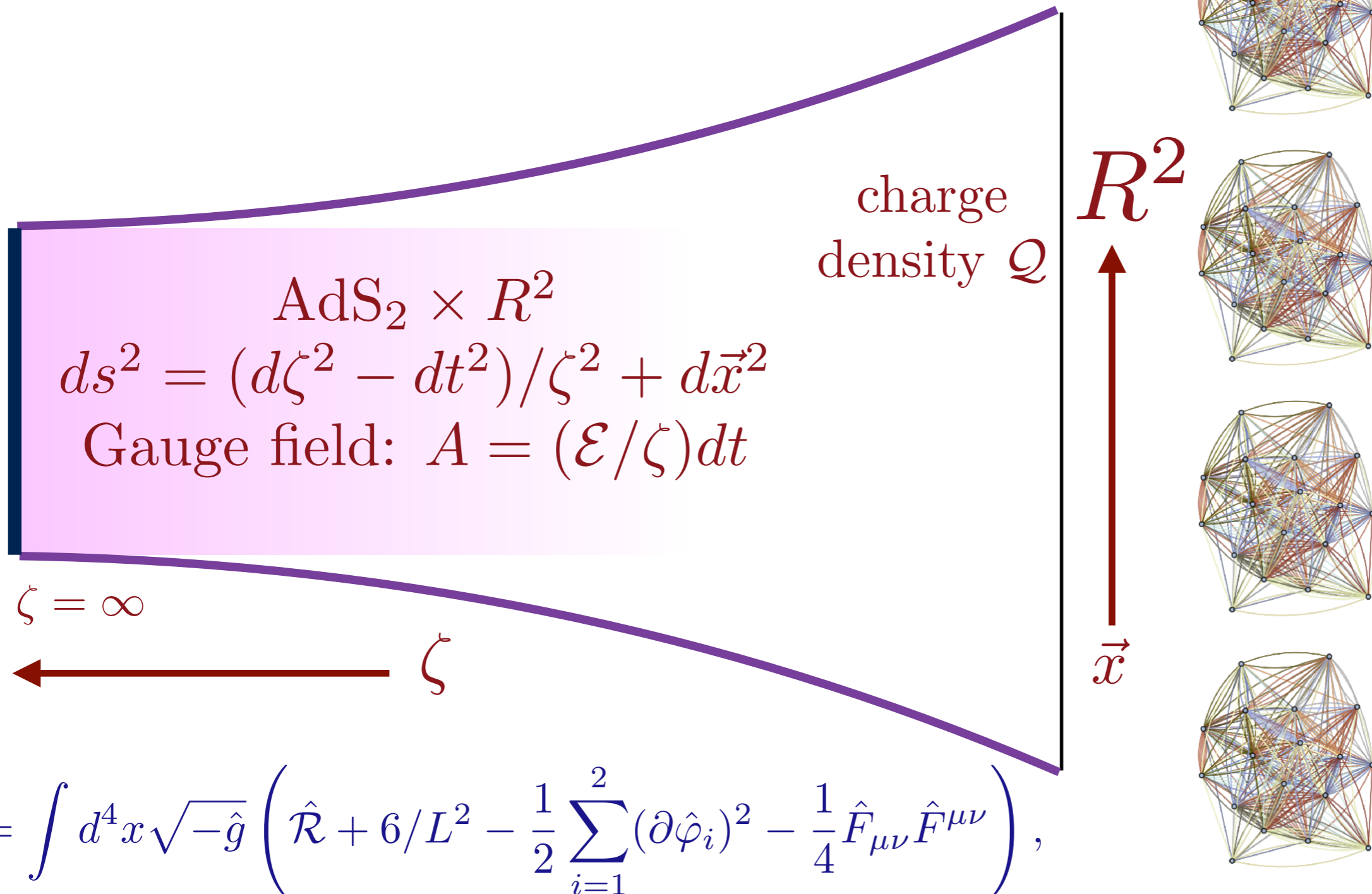


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Coupled SYK and AdS₄



$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{2} \sum_{i=1}^2 (\partial \hat{\varphi}_i)^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),$$

Einstein-Maxwell-axion theory with saddle point $\hat{\varphi}_i = kx_i$ leading to momentum dissipation

Coupled SYK and AdS₄

The response functions of the density, Q , and the energy, E exhibit diffusion

$$\begin{pmatrix} \langle Q; Q \rangle_{k,\omega} & \langle E - \mu Q; Q \rangle_{k,\omega} / T \\ \langle E - \mu Q; Q \rangle_{k,\omega} & \langle E - \mu Q; E - \mu Q \rangle_{k,\omega} / T \end{pmatrix} = [i\omega(-i\omega + Dk^2)^{-1} + 1] \chi_s$$

where the diffusivities are related to the thermoelectric conductivities by the Einstein relations

$$D = \begin{pmatrix} \sigma & \alpha \\ \alpha T & \bar{\kappa} \end{pmatrix} \chi_s^{-1}.$$

The Seebeck co-efficient (thermopower), α/σ , is given exactly by a thermodynamic derivative

$$\frac{\alpha}{\sigma} = \frac{\partial S_0}{\partial Q}$$

The coupled-SYK and AdS₄ models realize a disordered metal with no quasiparticle excitations.
(a “strange metal”)

Quantum chaos:

- In both the SYK and holographic models, the growth of chaos is characterized by

$$\left\langle \left| \{c(x, t), c^\dagger(0, 0)\} \right|^2 \right\rangle \sim \exp \left(\frac{1}{\tau_L} \left(t - \frac{|x|}{v_B} \right) \right)$$

where the Lyapunov time saturates the lower bound $\hbar/(2\pi k_B T)$ and the BUTTERFLY VELOCITY $v_B \sim T^{1/2}$.

- The thermal diffusivity, D_E is given exactly by

$$D_E = v_B^2 \tau_L.$$

There is no universal relationship between the charge diffusivity, D_c , and v_B .

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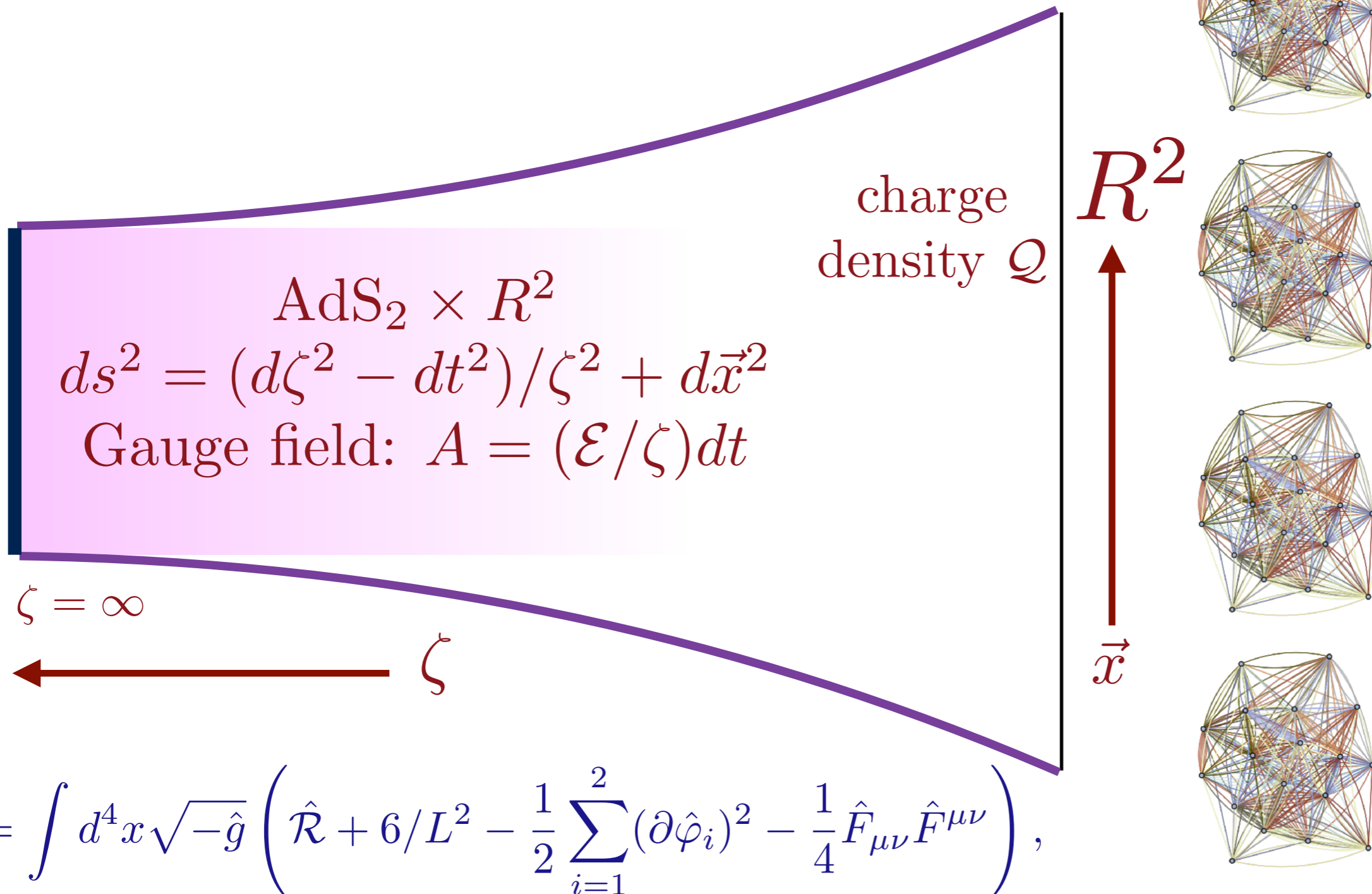
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- Quantum chaos is intimately linked to the loss of phase coherence from electron-electron interactions. As the time derivative of the local phase is determined by the local energy, phase fluctuations and chaos are linked to interaction-induced energy fluctuations, and hence thermal diffusivity.

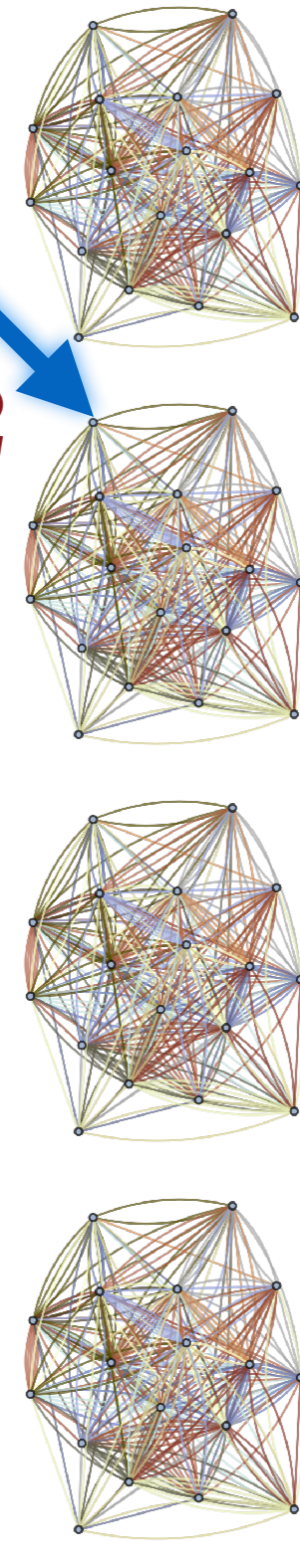
Coupled SYK and AdS₄



Einstein-Maxwell-axion theory with saddle point $\hat{\varphi}_i = kx_i$
 leading to momentum dissipation

Coupled SYK and AdS₄

Matching correlators for thermoelectric diffusion, and quantum chaos



charge density \mathcal{Q}

R^2

\vec{x}

$\text{AdS}_2 \times R^2$

$$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$$

$$\text{Gauge field: } A = (\mathcal{E}/\zeta)dt$$

$\zeta = \infty$

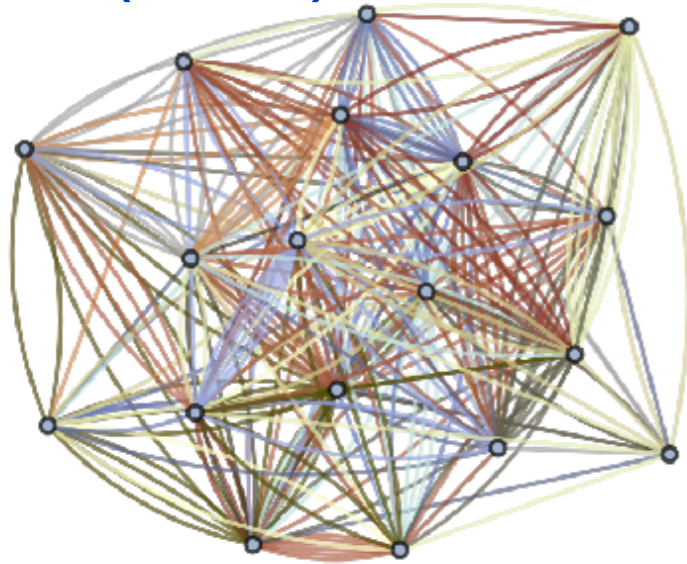
ζ

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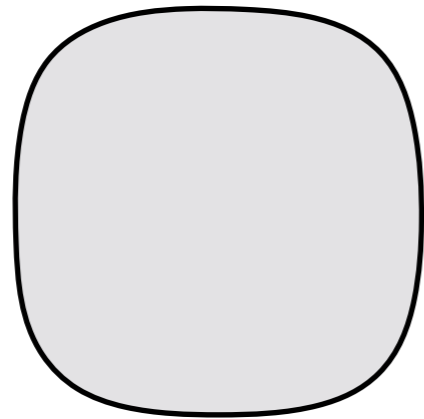
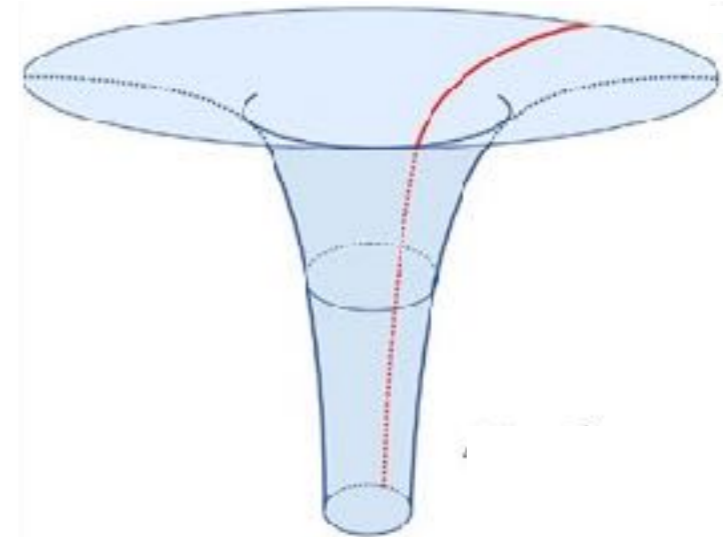
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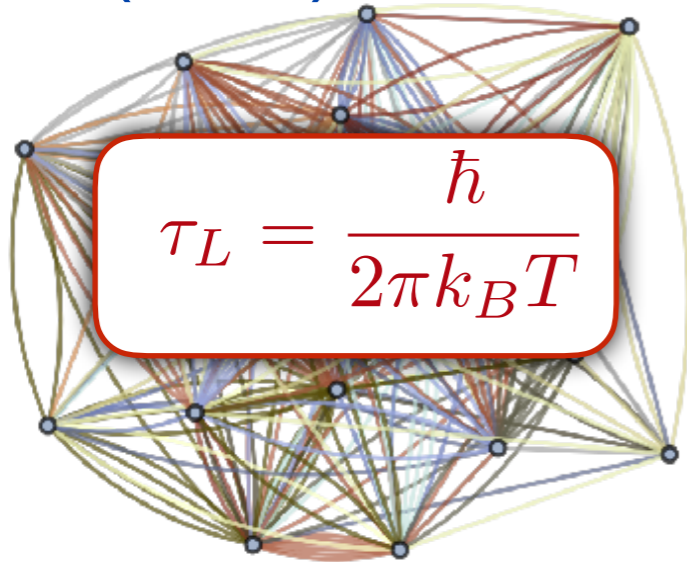


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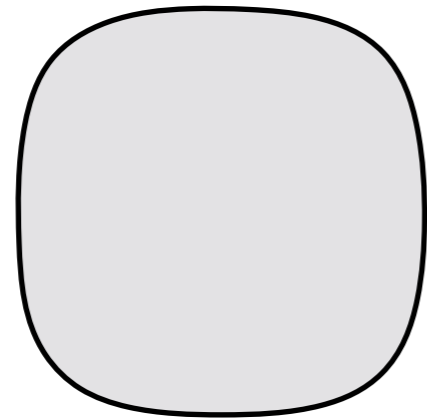
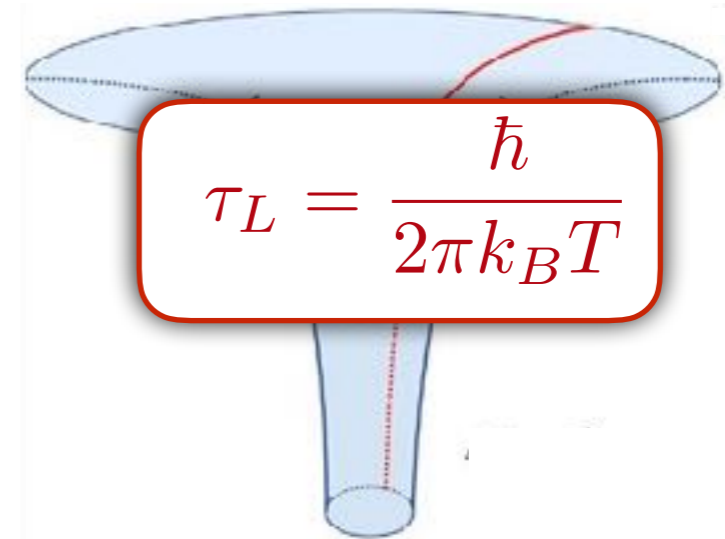
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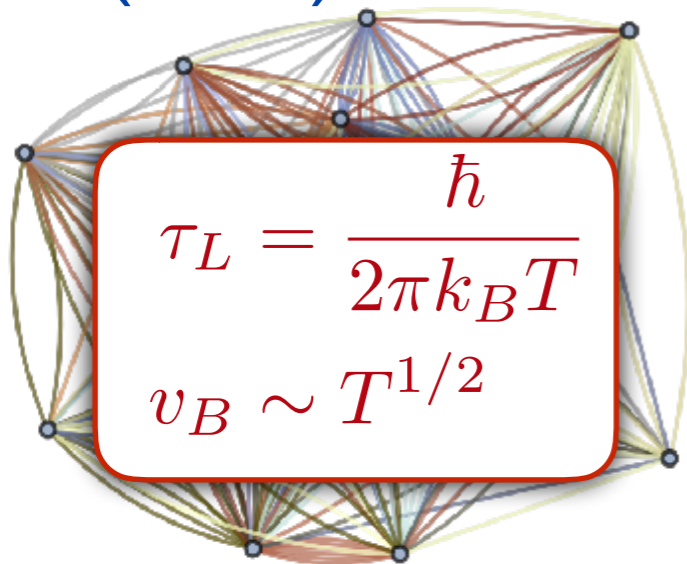
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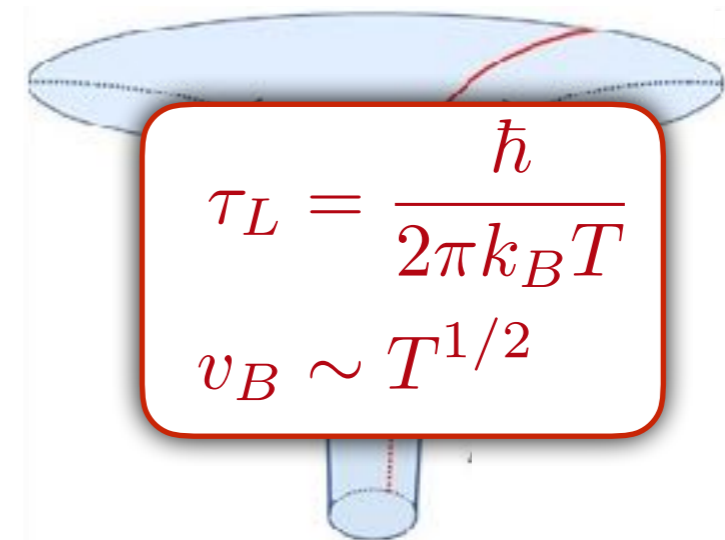
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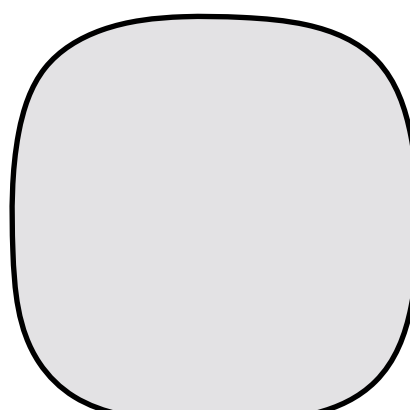
The Sachdev-Ye-Kitaev (SYK) models



Black holes with AdS₂ horizons



Fermi surface coupled to a gauge field



A diagram of a Fermi surface coupled to a gauge field, showing a gray, rounded rectangular shape representing the Fermi surface.

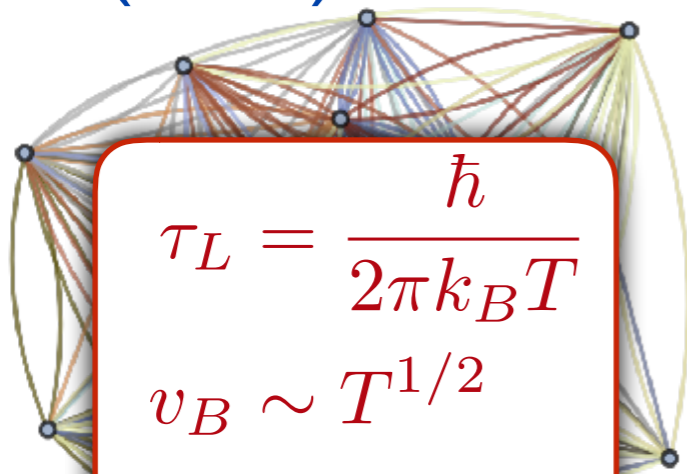
$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - ia_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L : the Lyapunov time to reach quantum chaos

v_B : the “butterfly velocity” for the spatial propagation of chaos

Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

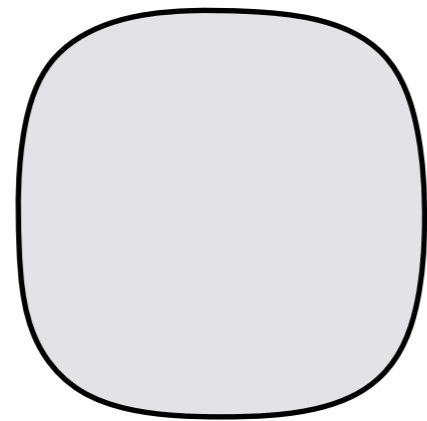


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$
$$v_B \sim T^{1/2}$$
$$D_E = v_B^2 \tau_L$$

Black holes with AdS₂ horizons



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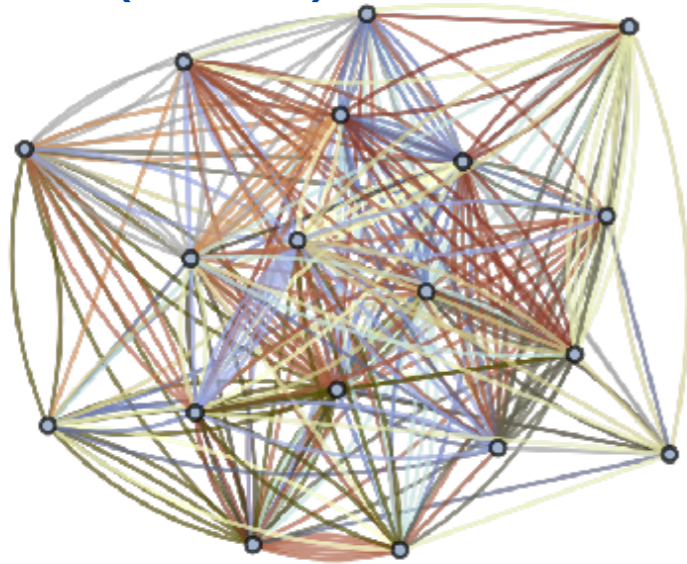
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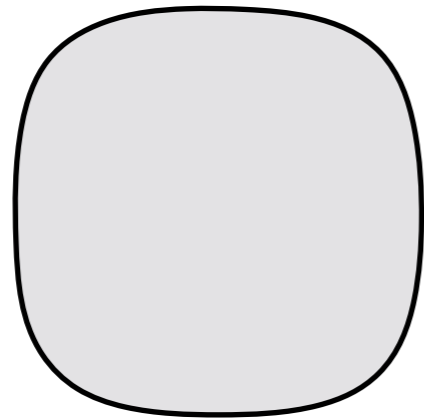
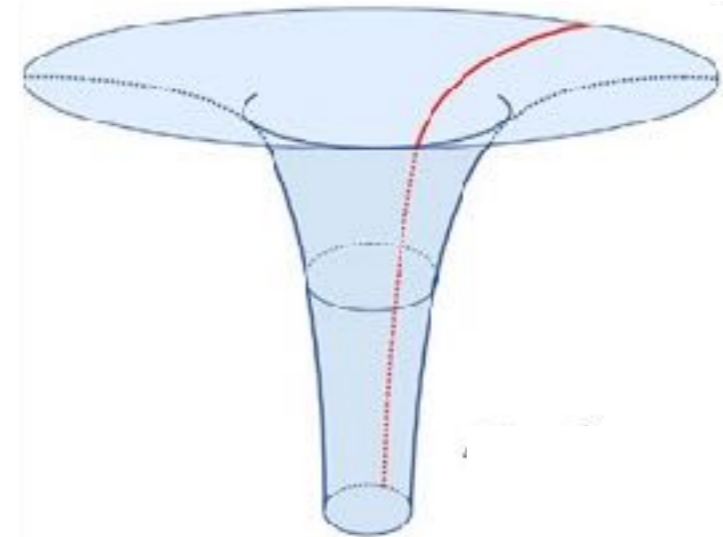
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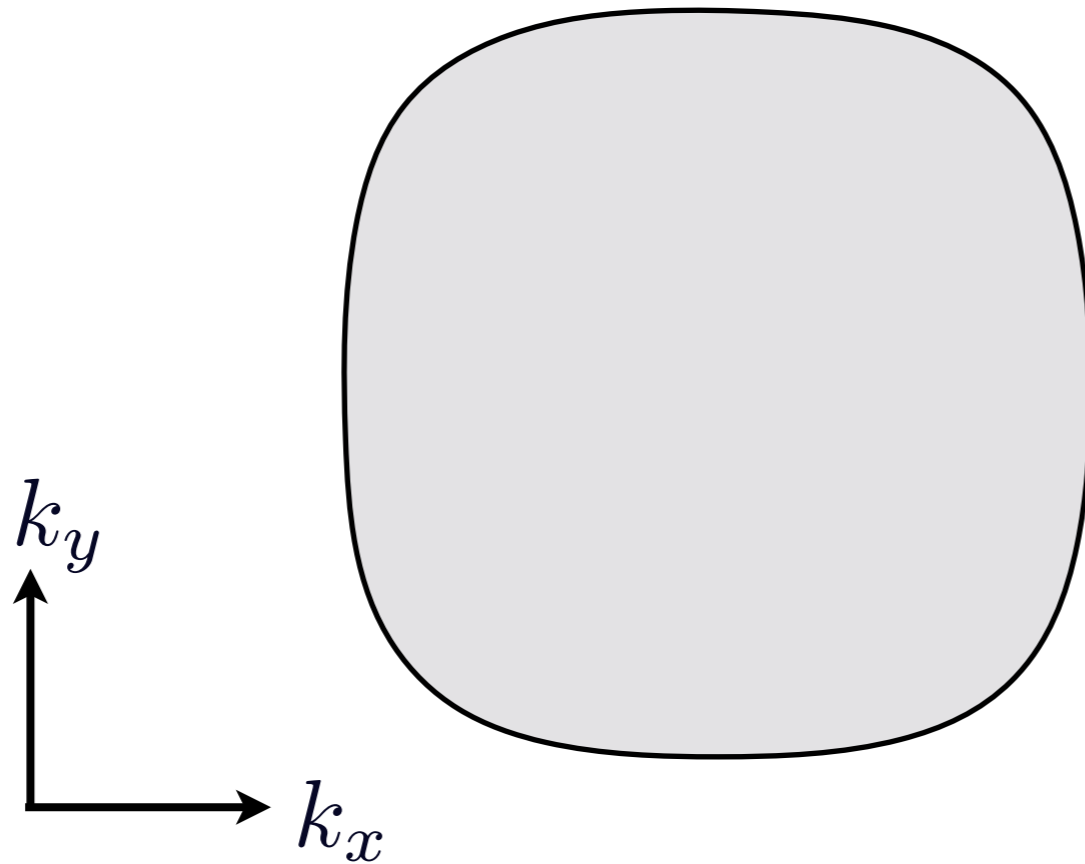
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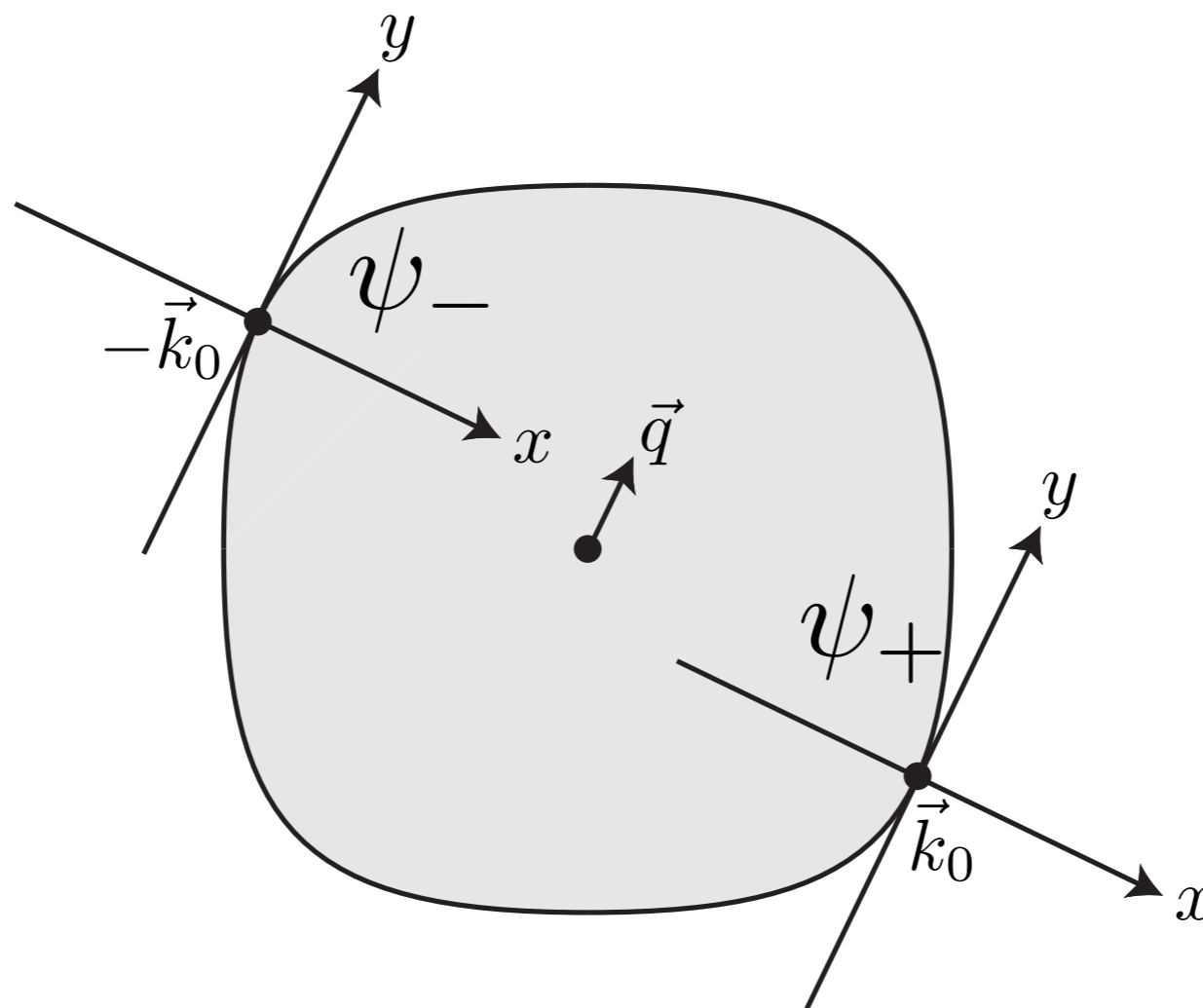
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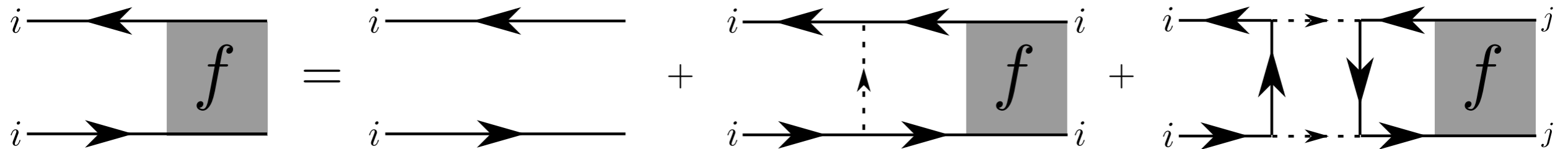
Fermi surface coupled to a gauge field



$$\begin{aligned} \mathcal{L}[\psi_{\pm}, a] = & \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- \\ & - a \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y a)^2 \end{aligned}$$

Fermi surface coupled to a gauge field

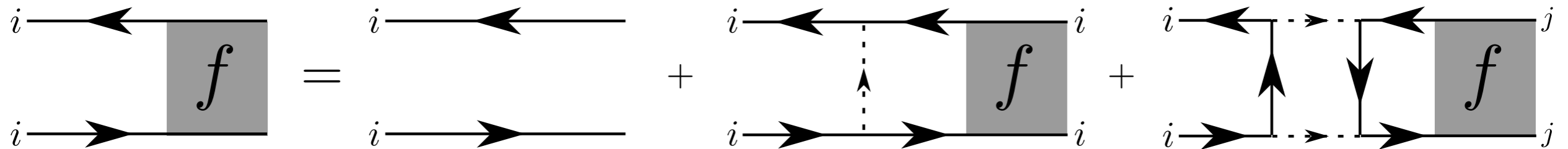
Compute out-of-time-order correlator to
diagnose quantum chaos



$$f(t) = \frac{1}{N^2} \theta(t) \sum_{i,j=1}^N \int d^2x \operatorname{Tr} \left[e^{-\beta H/2} \{ \psi_i(x, t), \psi_j^\dagger(0) \} \right. \\ \left. \times e^{-\beta H/2} \{ \psi_i(x, t), \psi_j^\dagger(0) \}^\dagger \right] \\ \sim \exp\left((t - x/v_B)/\tau_L \right)$$

Fermi surface coupled to a gauge field

Compute out-of-time-order correlator to diagnose quantum chaos



Strongly-coupled theory with no quasiparticles and fast scrambling:

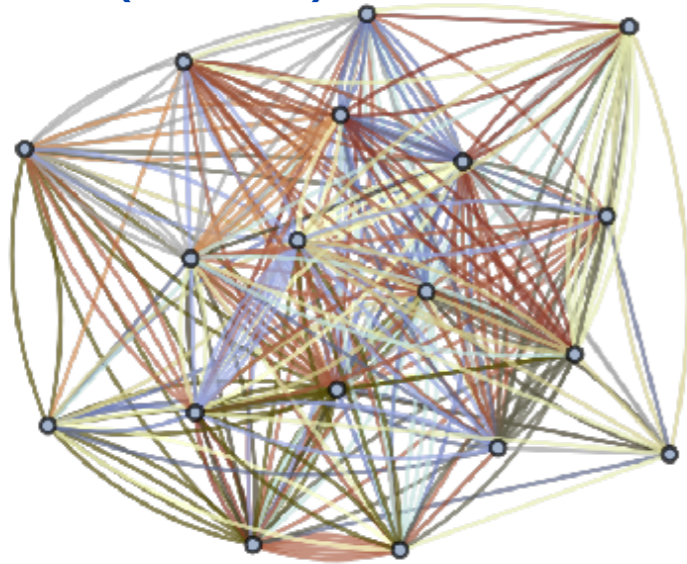
$$\begin{aligned}\tau_L &\approx \frac{\hbar}{2.48 k_B T} \\ v_B &\approx 4.1 \frac{NT^{1/3}}{e^{4/3}} \frac{v_F^{5/3}}{\gamma^{1/3}} \\ D_E &\approx 0.42 v_B^2 \tau_L\end{aligned}$$



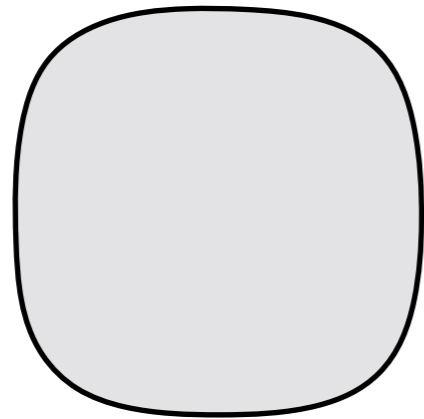
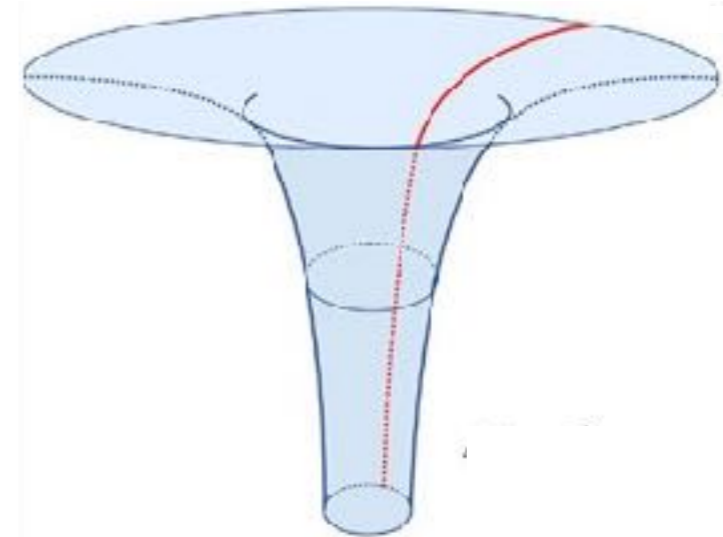
N is the number of fermion flavors, v_F is the Fermi velocity, γ is the Fermi surface curvature, e is the gauge coupling constant.

Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



Black holes with AdS₂ horizons

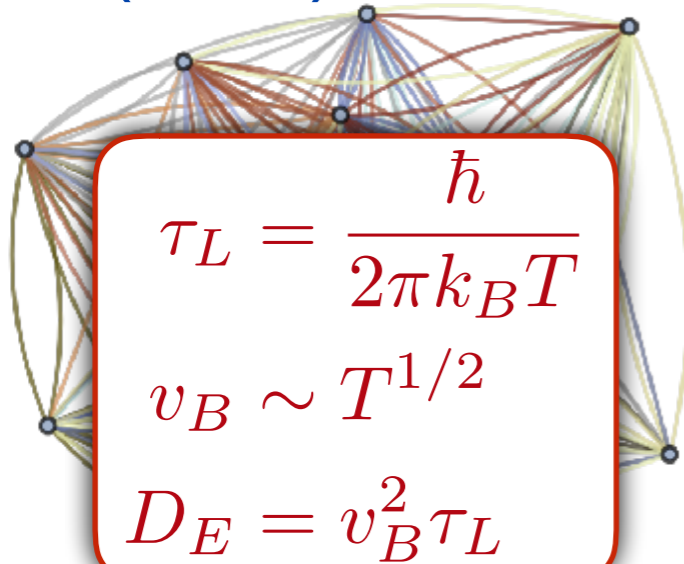


Fermi surface coupled
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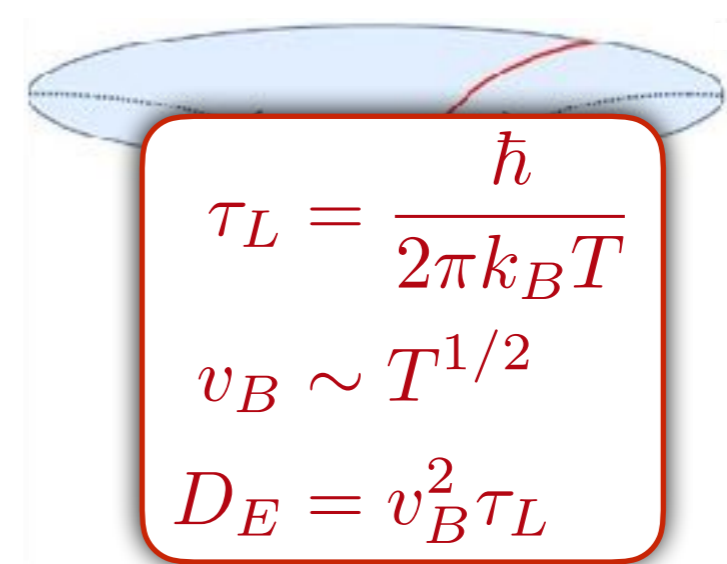


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Black holes with AdS₂ horizons



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$$v_B \sim \frac{N v_F^{5/3}}{e^{4/3} \gamma^{1/3}} T^{1/3}$$

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$\mathcal{L}[\Psi$

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τ_L : the Lyapunov time to reach quantum chaos

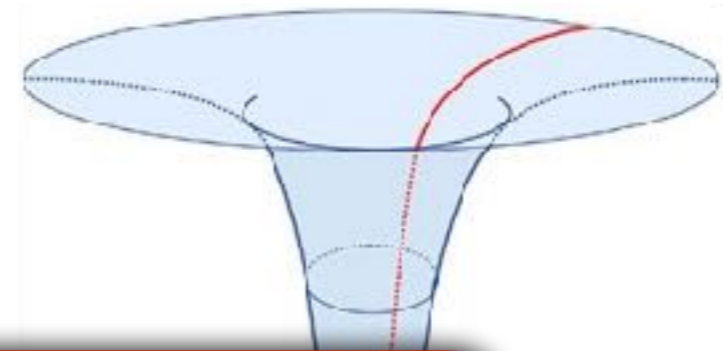
v_B : the “butterfly velocity” for the spatial propagation of chaos

Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



Black holes with AdS₂ horizons



Thermal diffusivity, D_E :

$$D_E = (\text{universal number}) \times v_B^2 \tau_L$$

in all three models

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