

Quantum criticality: quantum matter without quasiparticles

34th Jerusalem Winter School in Theoretical Physics
New Horizons in Quantum Matter
December 27, 2016 - January 5, 2017

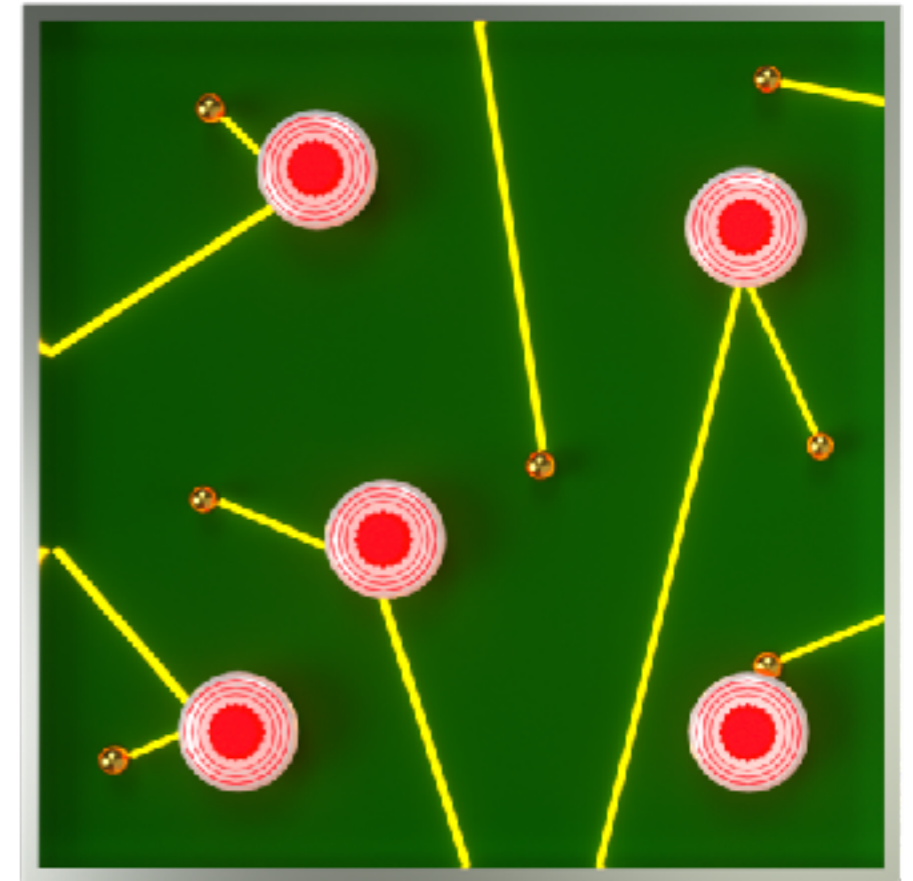
Subir Sachdev

Talk online: sachdev.physics.harvard.edu



Quantum matter with quasiparticles:

- Landau quasi-particles & holes
- Phonon
- Magnon
- Roton
- Plasmon
- Polaron
- Exciton
- Laughlin quasiparticle
- Bogoliubovon
- Anderson-Higgs mode
- Massless Dirac Fermions
- Weyl fermions
-



Quantum matter with quasiparticles:

Most generally, a quasiparticle is
an “additive” excitation:

Quasiparticles can be combined to yield additional excitations, with energy determined by the energies and densities of the constituents. Such a procedure yields all the low-lying excitations. Then we can apply the Boltzmann-Landau theory to make predictions for dynamics.

Quantum matter without quasiparticles:

No quasiparticle structure to excitations.

But how can we be sure that no quasiparticles exist in a given system?
Perhaps there are some exotic quasiparticles inaccessible to current experiments.....

Consider how rapidly the system loses “phase coherence”, reaches local thermal equilibrium, or becomes “chaotic”

Local thermal equilibration or phase coherence time, τ_φ :

- There is an *lower bound* on τ_φ in all many-body quantum systems as $T \rightarrow 0$,

$$\tau_\varphi > C \frac{\hbar}{k_B T},$$

where C is a T -independent constant. Systems *without* quasiparticles have $\tau_\varphi \sim \hbar/(k_B T)$.

- In systems *with* quasiparticles, τ_φ is parametrically larger at low T ;
e.g. in Fermi liquids $\tau_\varphi \sim 1/T^2$,
and in gapped insulators $\tau_\varphi \sim e^{\Delta/(k_B T)}$ where Δ is the energy gap.

K. Damle and S. Sachdev, PRB **56**, 8714 (1997)

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

A bound on quantum chaos:

- In classical chaos, we measure the sensitivity of the position at time t , $q(t)$, to variations in the initial position, $q(0)$, *i.e.* we measure

$$\left(\frac{\partial q(t)}{\partial q(0)} \right)^2 = (\{q(t), p(0)\}_{\text{P.B.}})^2$$

- By analogy, we define τ_L as the LYAPUNOV TIME over which the wavefunction of a quantum system is scrambled by an initial perturbation. This scrambling can be measured by

$$\left\langle \left| [\hat{A}(t), \hat{B}(0)] \right|^2 \right\rangle \sim e^{t/\tau_L}$$

This quantum time was argued to obey lower bound

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}.$$

There is no analogous bound in classical mechanics.

A. I. Larkin and Y. N. Ovchinnikov, JETP **28**, 6 (1969)

J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

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Quantum matter without quasiparticles
 \approx fastest possible many-body quantum chaos

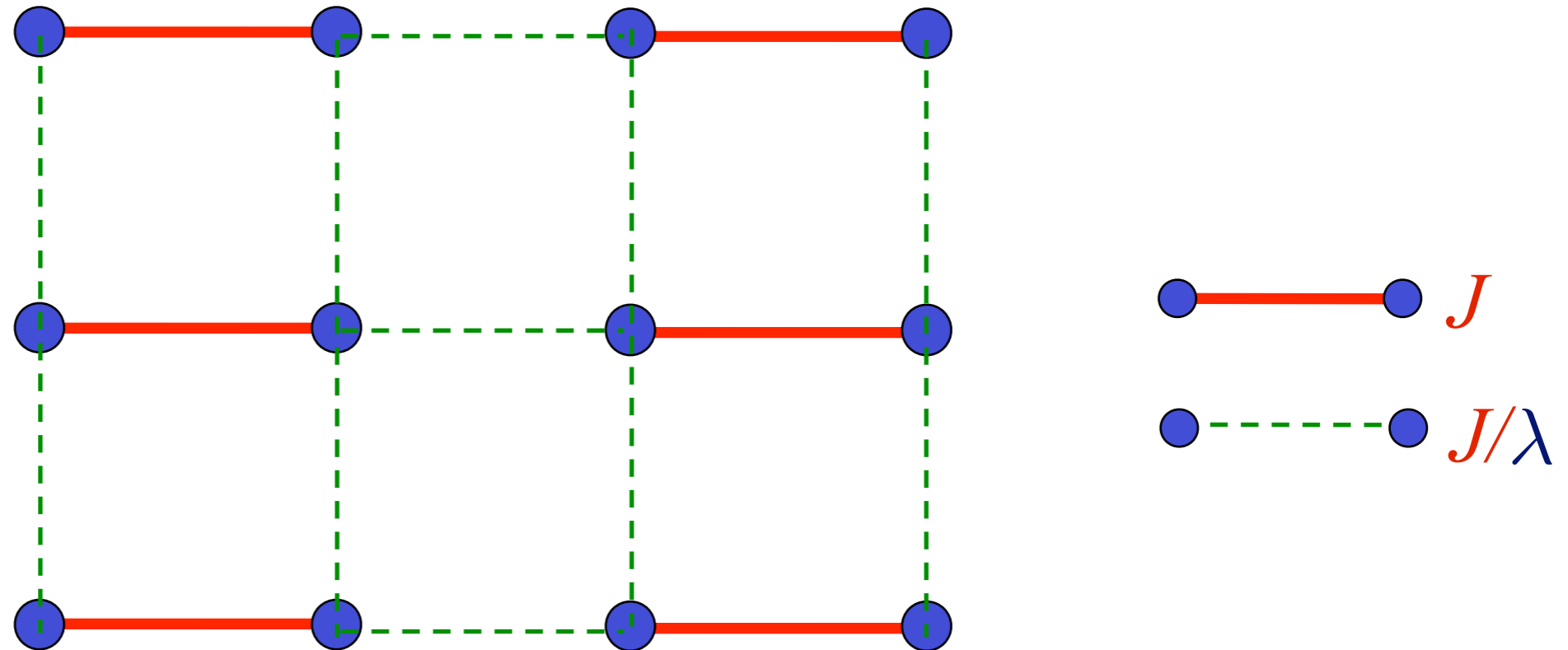
1. Coupled ladder antiferromagnet

2. Superfluid-insulator transition of bosons

3. Ising-nematic criticality in metals

Square lattice antiferromagnet

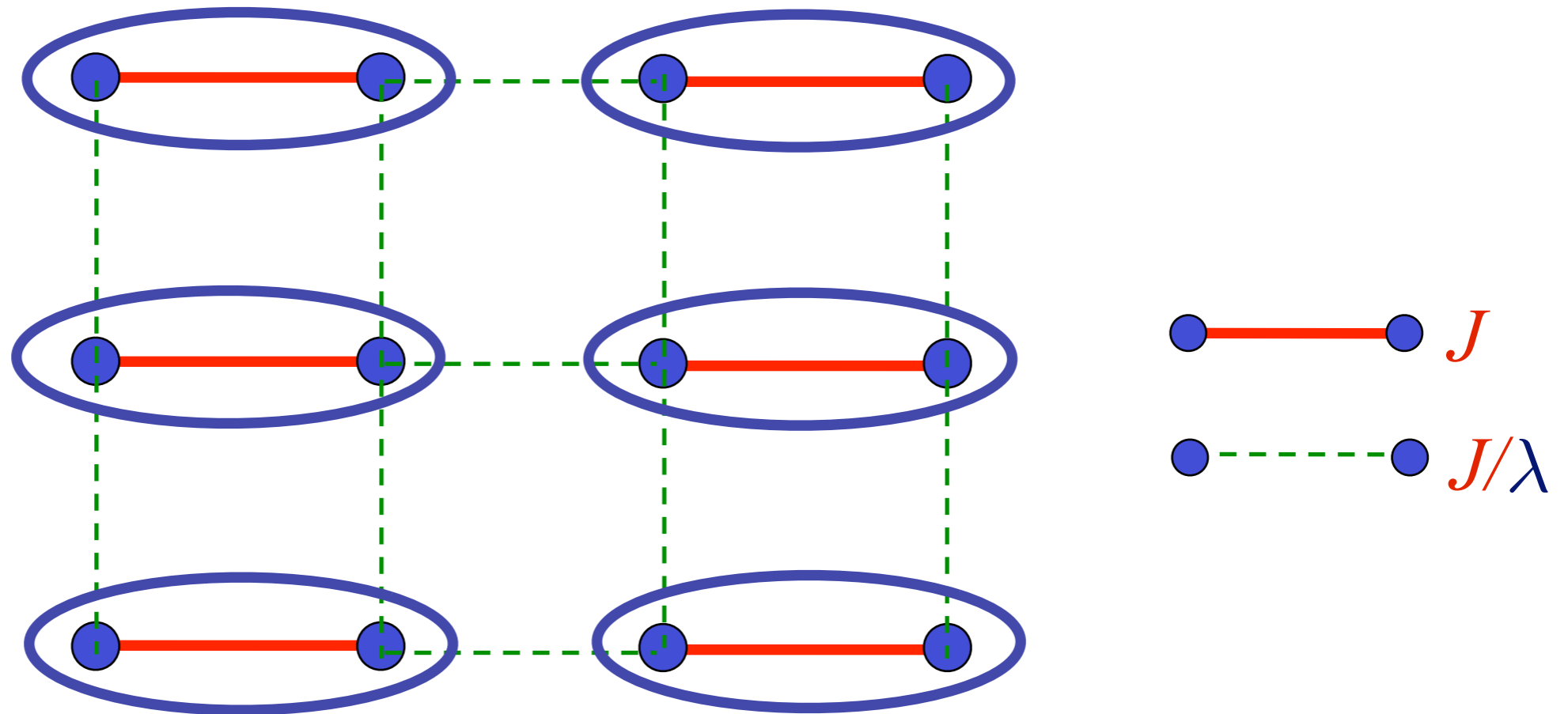
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Weaken some bonds to induce spin entanglement in a new quantum phase

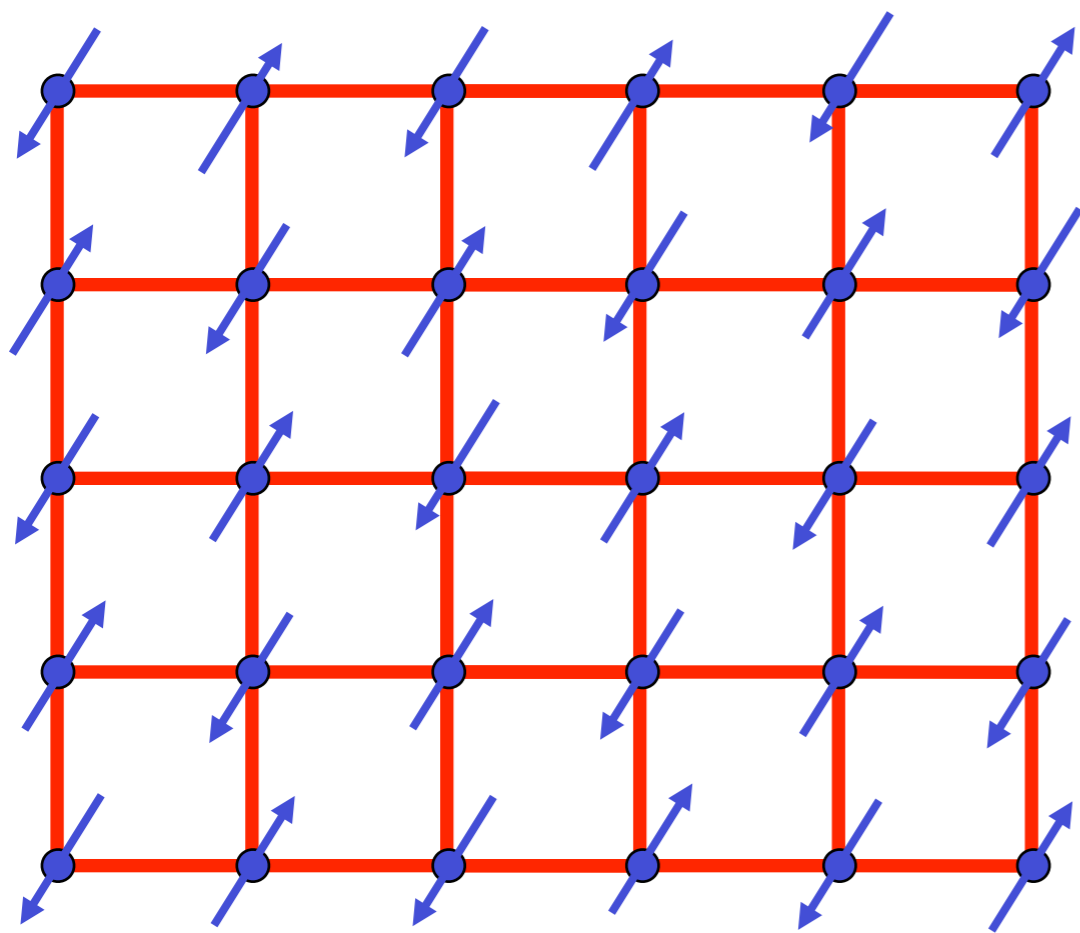
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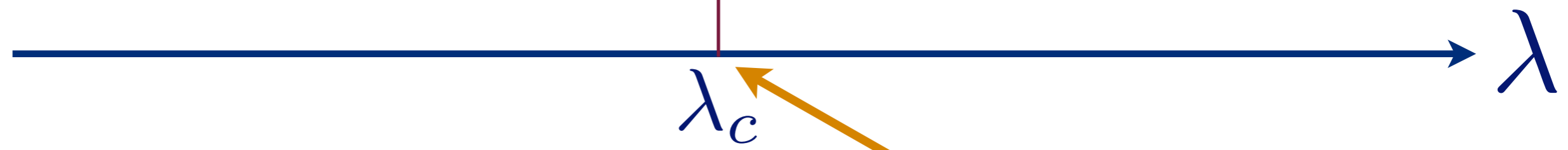
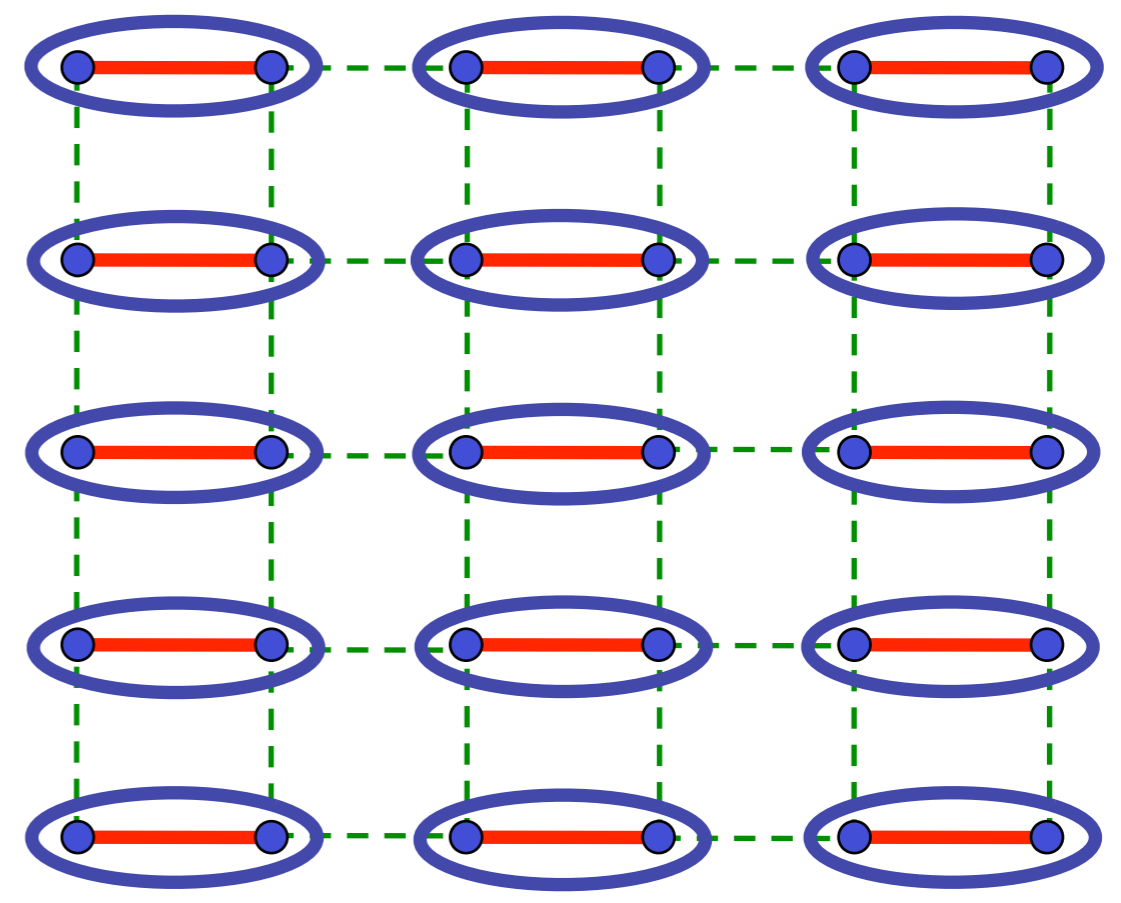


Ground state is a “quantum paramagnet”
with spins locked in valence bond singlets

$$\text{[Diagram of a valence bond singlet]} = \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

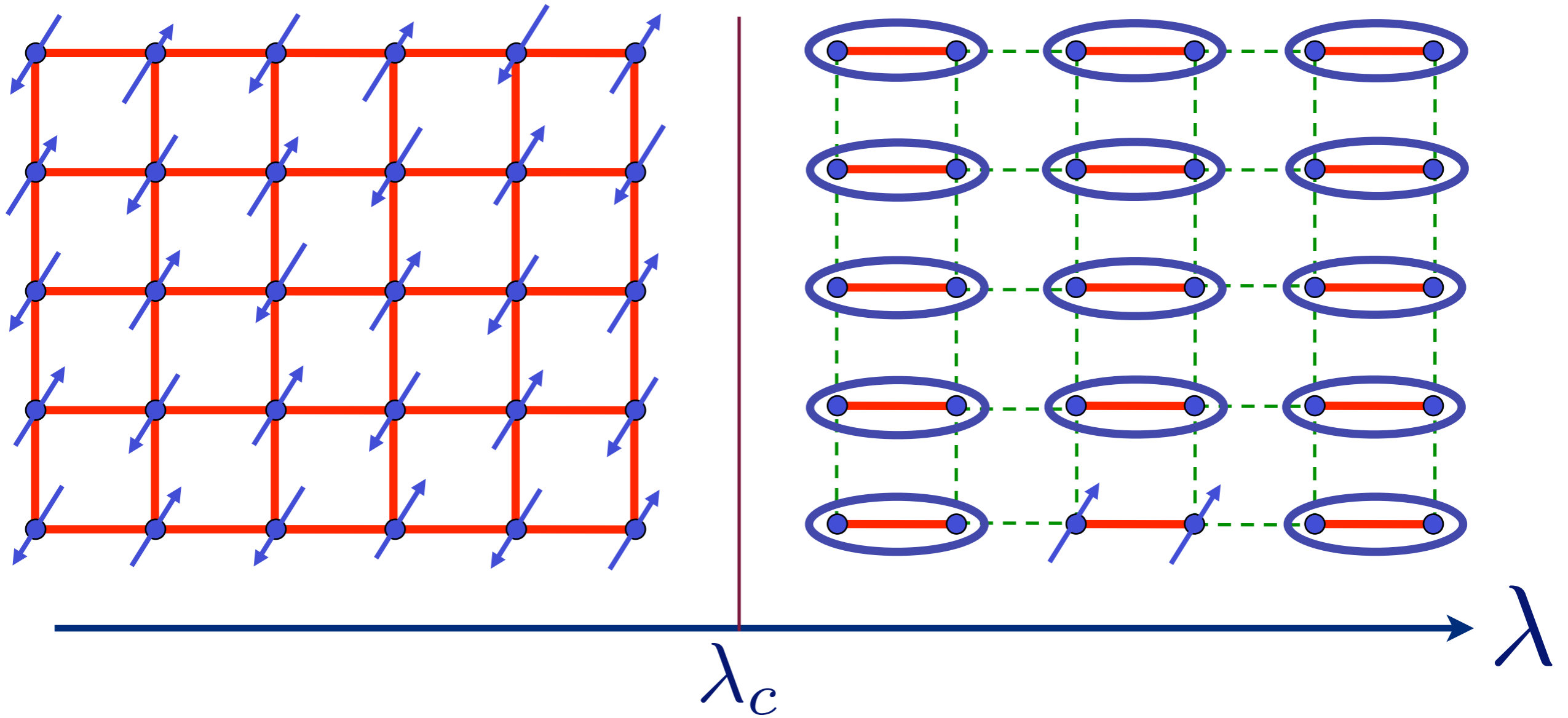


$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

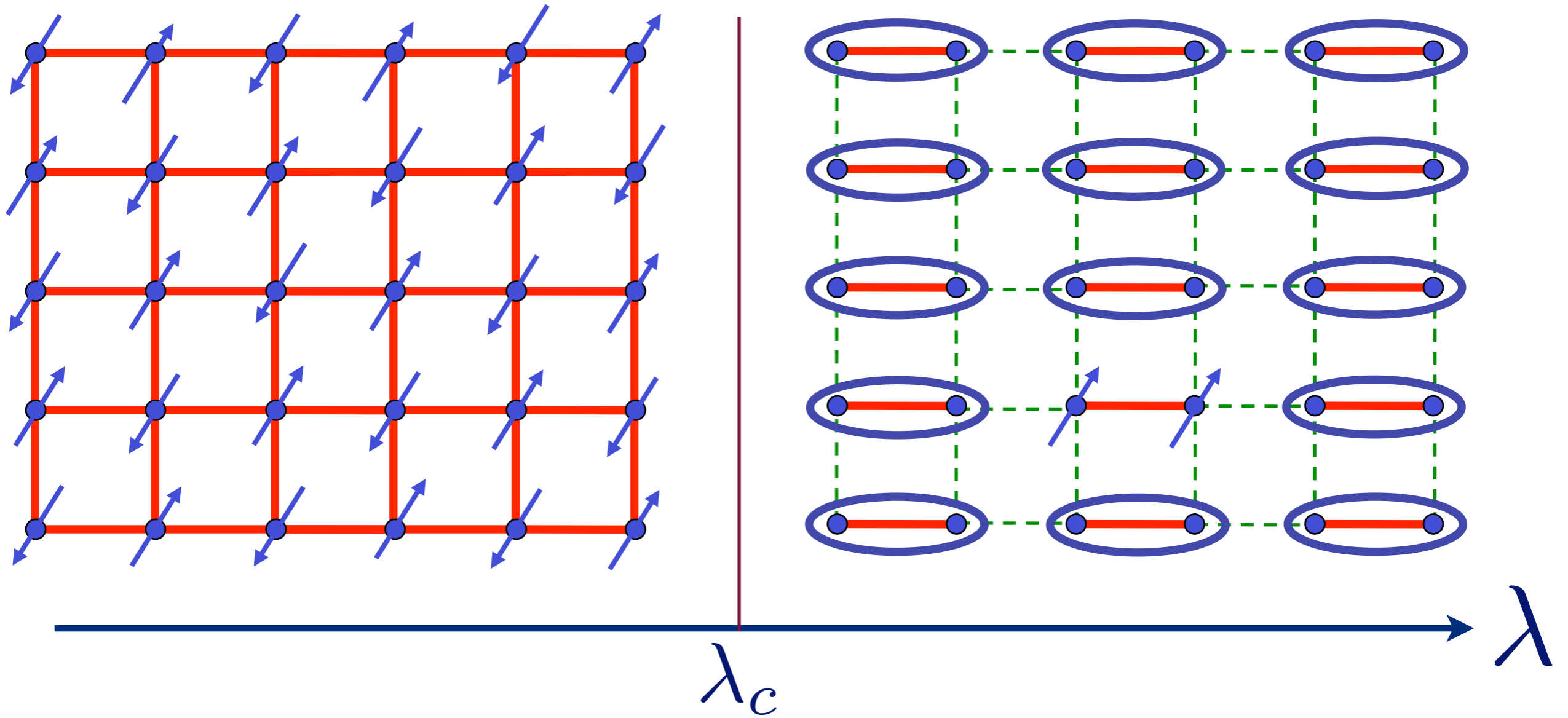


Quantum critical point with non-local entanglement in spin wavefunction

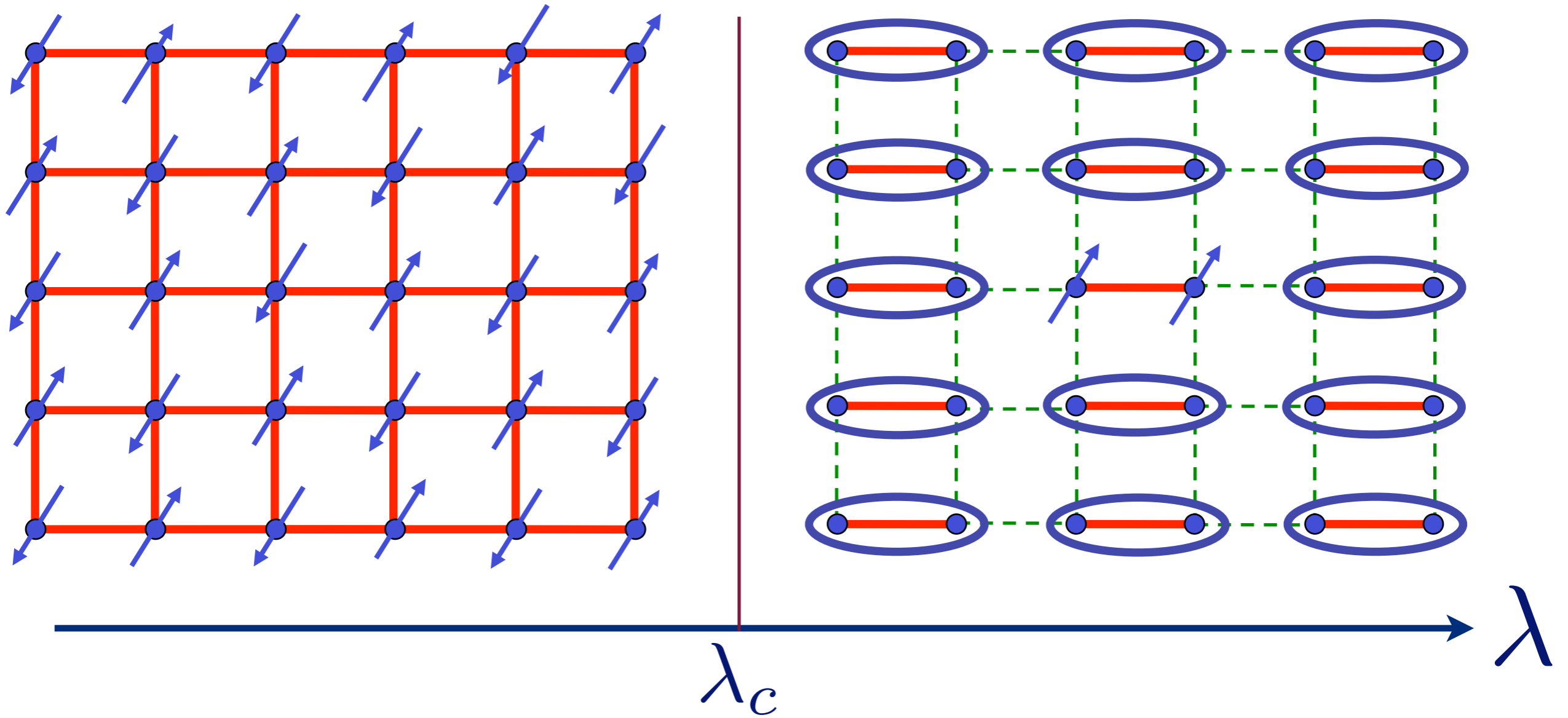
Excitation spectrum in the paramagnetic phase



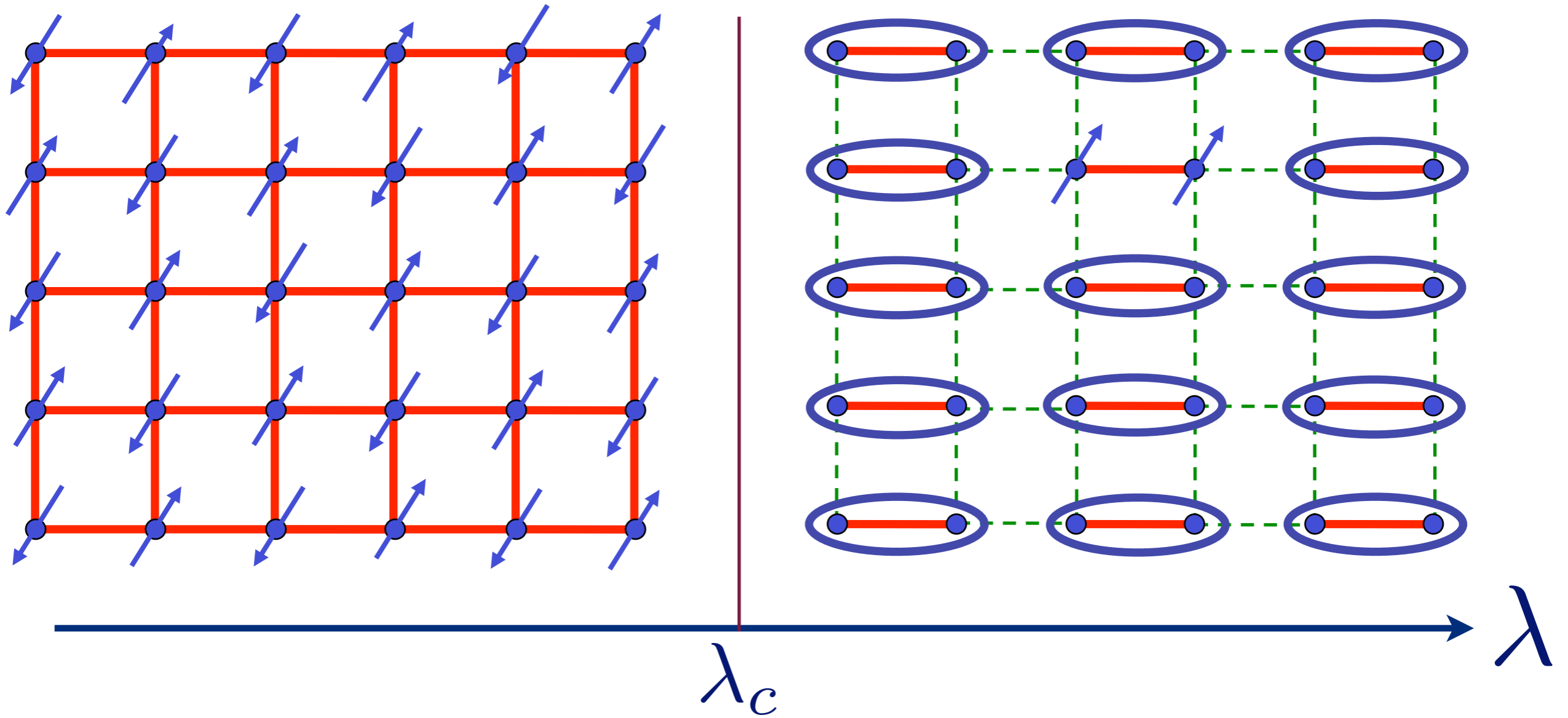
Excitation spectrum in the paramagnetic phase



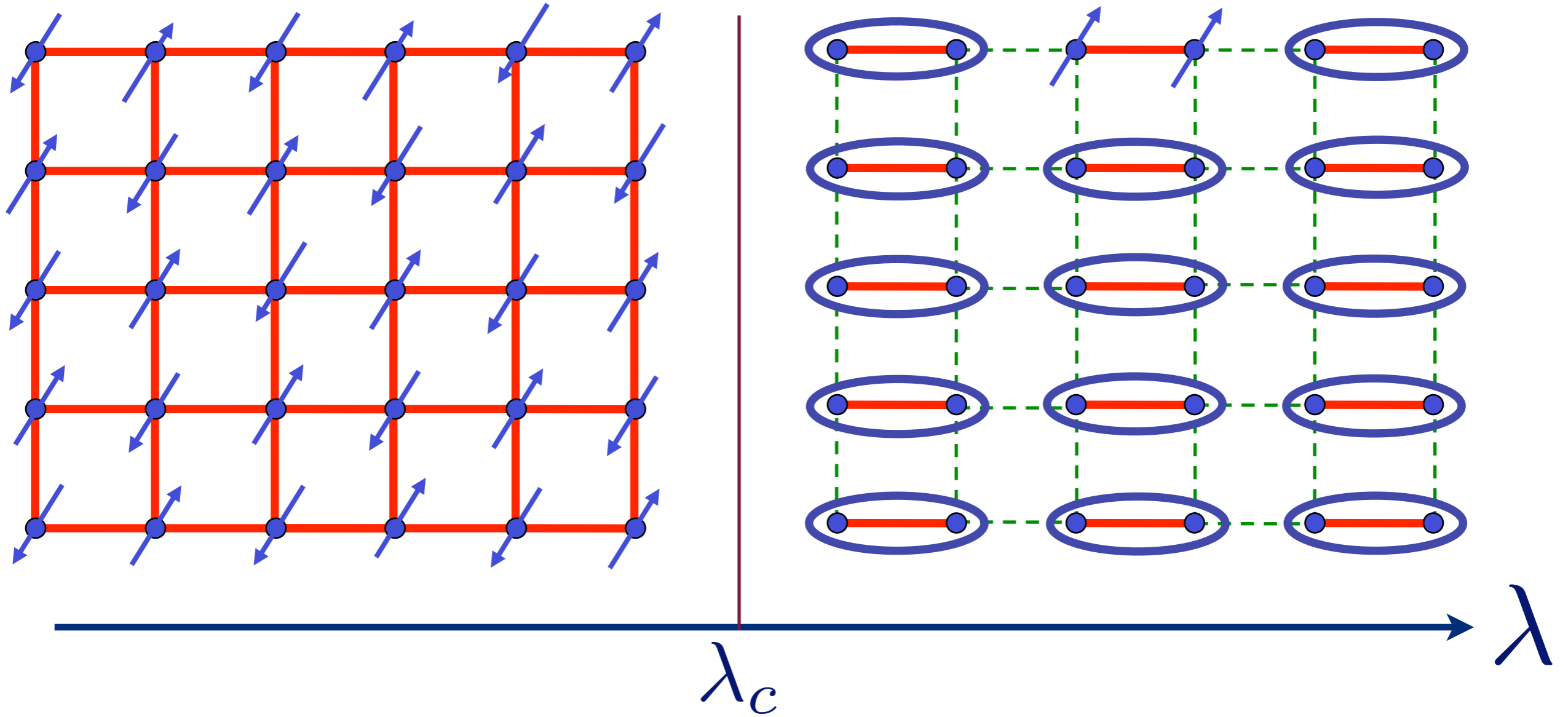
Excitation spectrum in the paramagnetic phase



Excitation spectrum in the paramagnetic phase

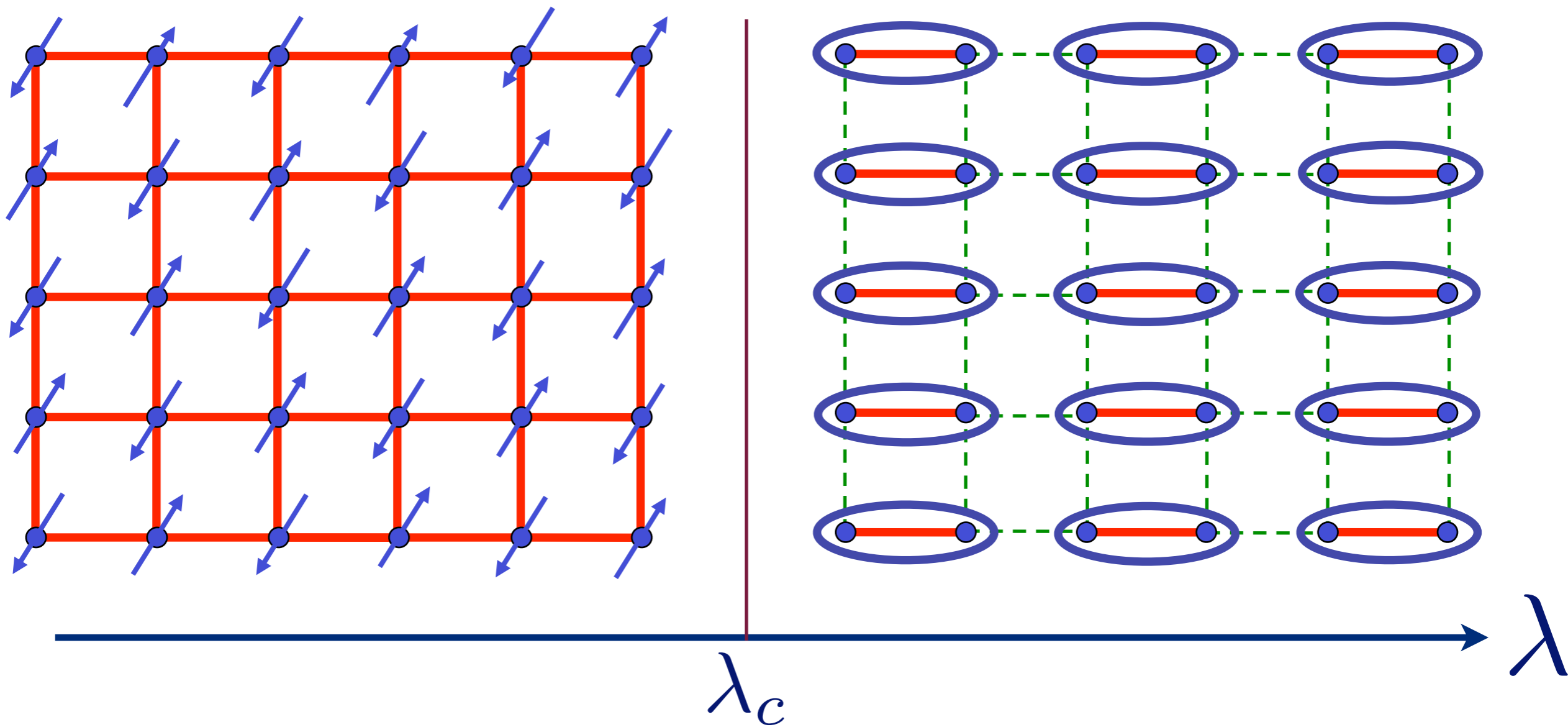


Excitation spectrum in the paramagnetic phase

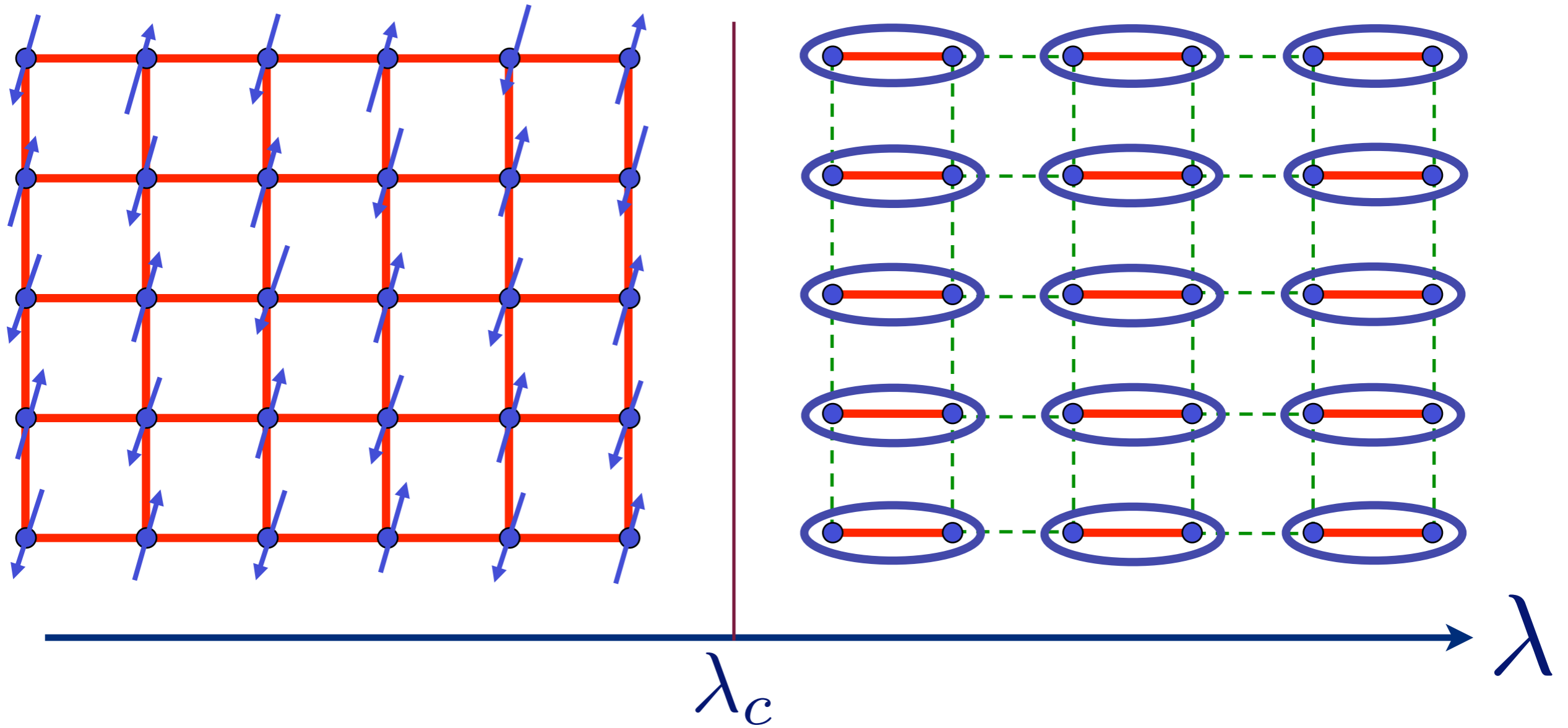


Sharp spin 1
particle excitation
above an energy
gap (spin gap)

Excitation spectrum in the Néel phase

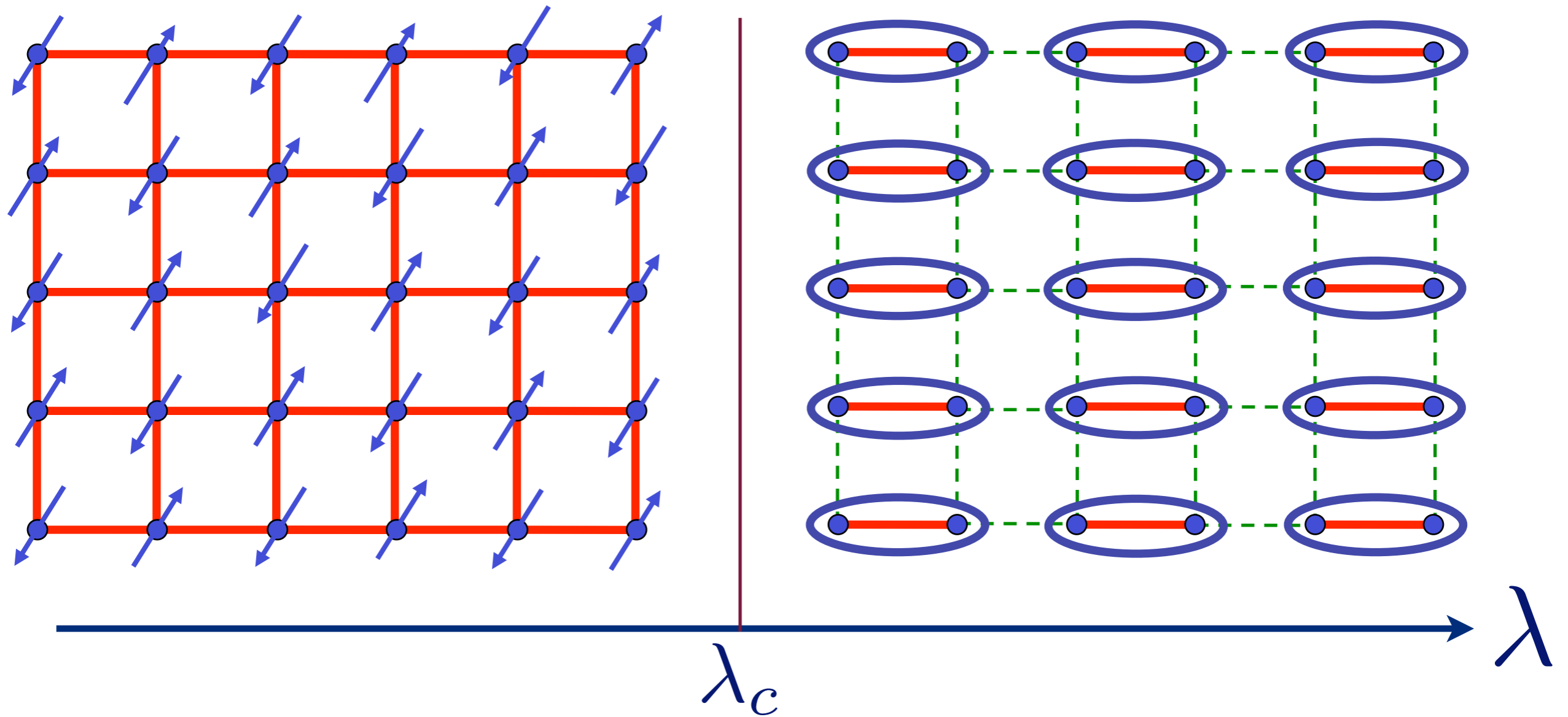


Excitation spectrum in the Néel phase



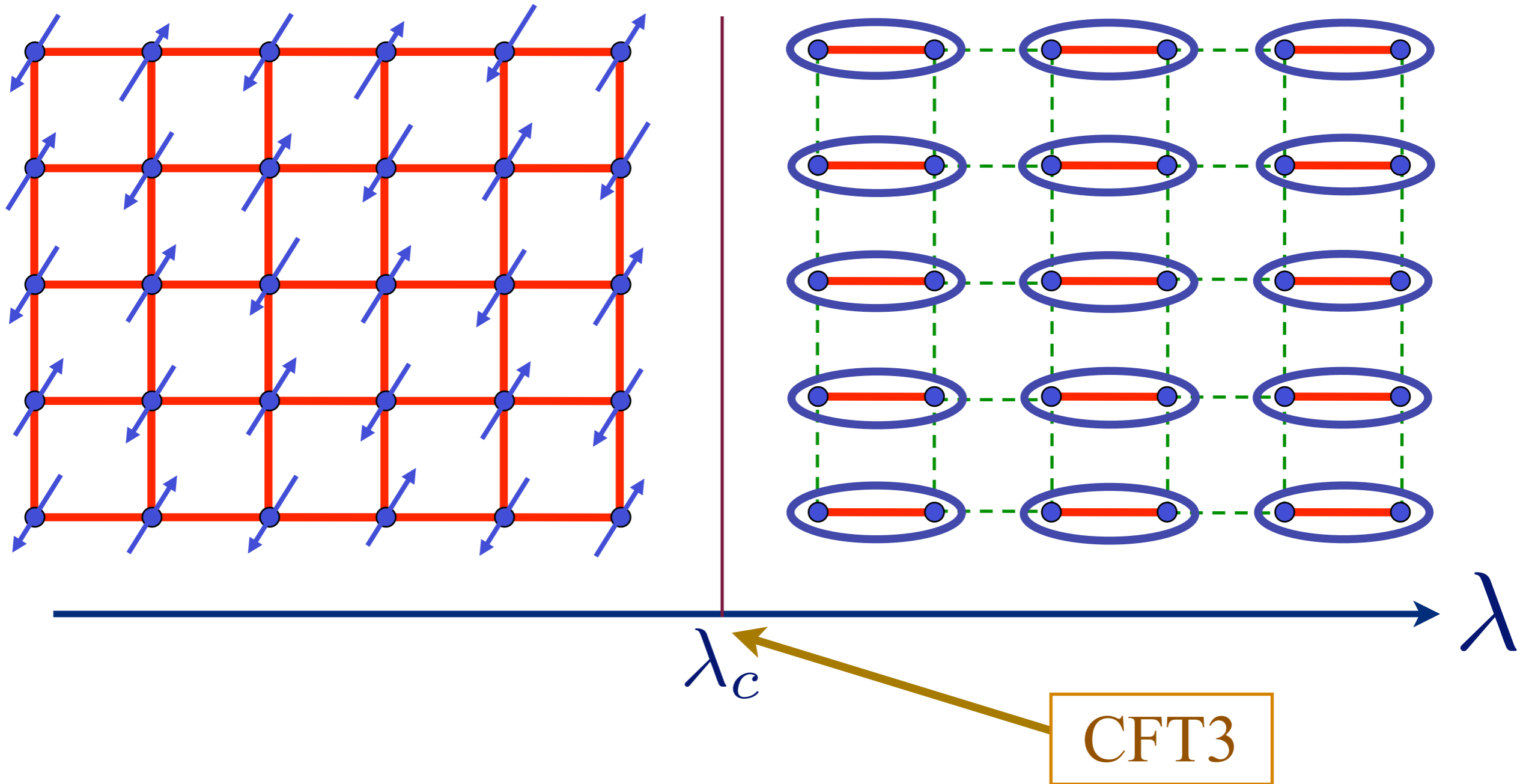
Spin waves

Excitation spectrum in the Néel phase



Spin waves

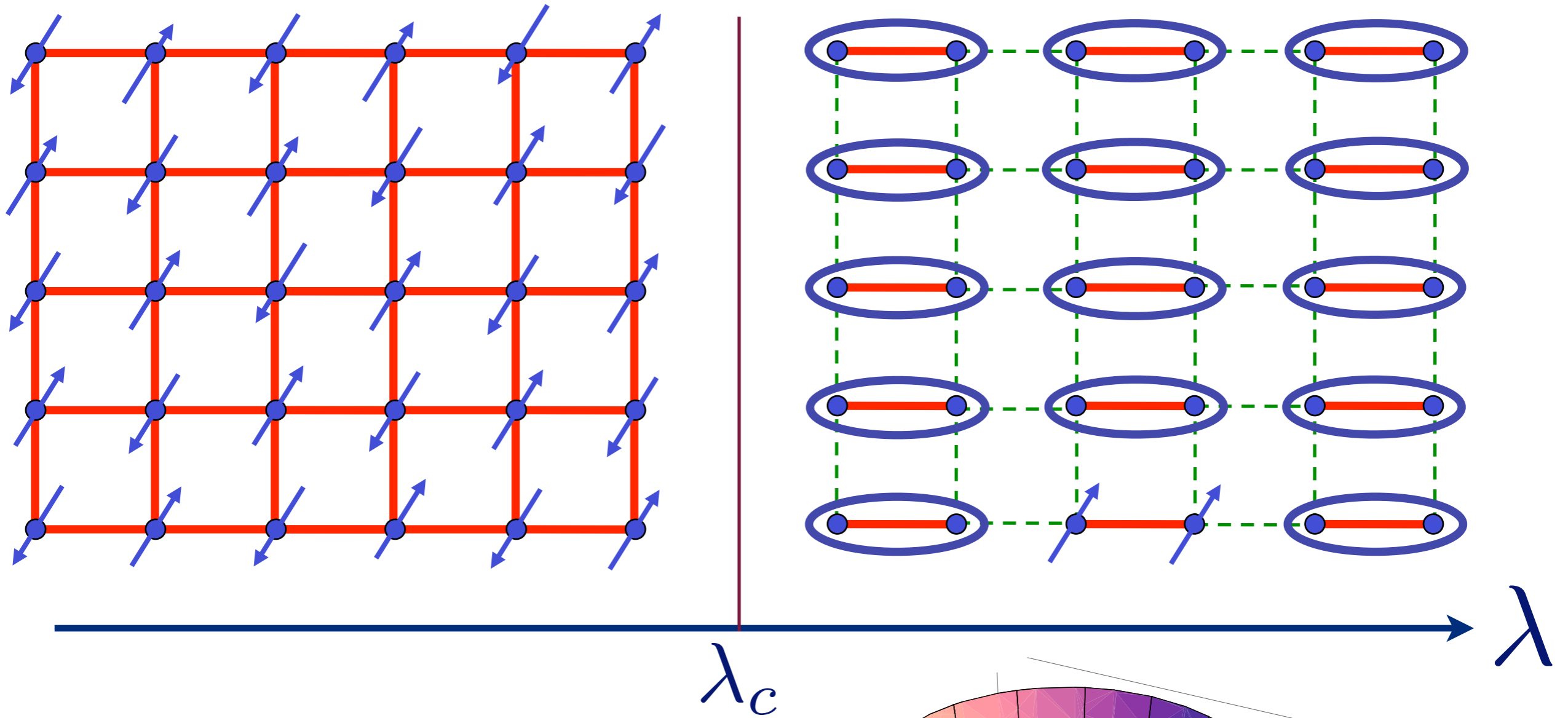
Description using Landau-Ginzburg field theory



$O(3)$ order parameter $\vec{\varphi}$

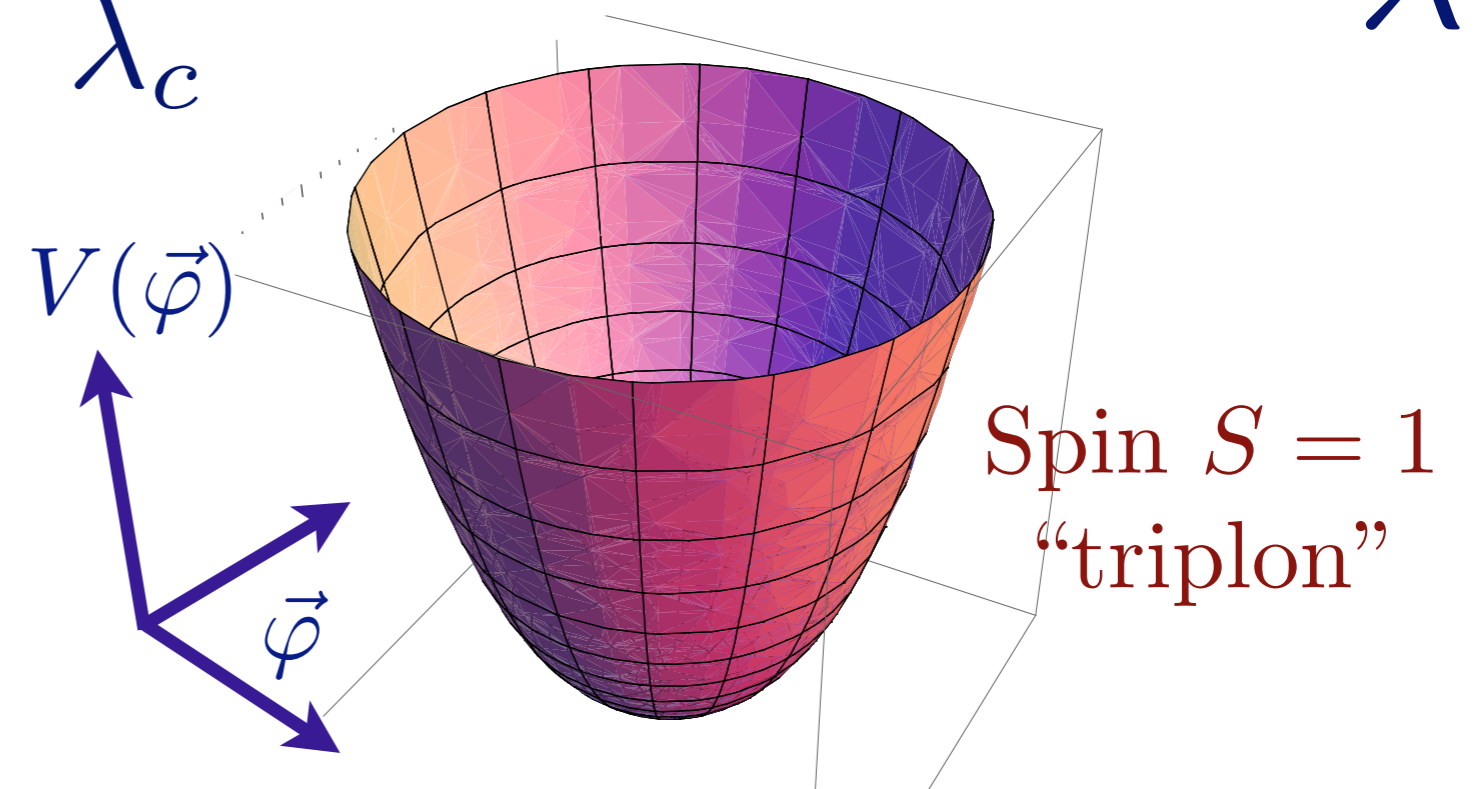
$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \right]$$

Excitation spectrum in the paramagnetic phase

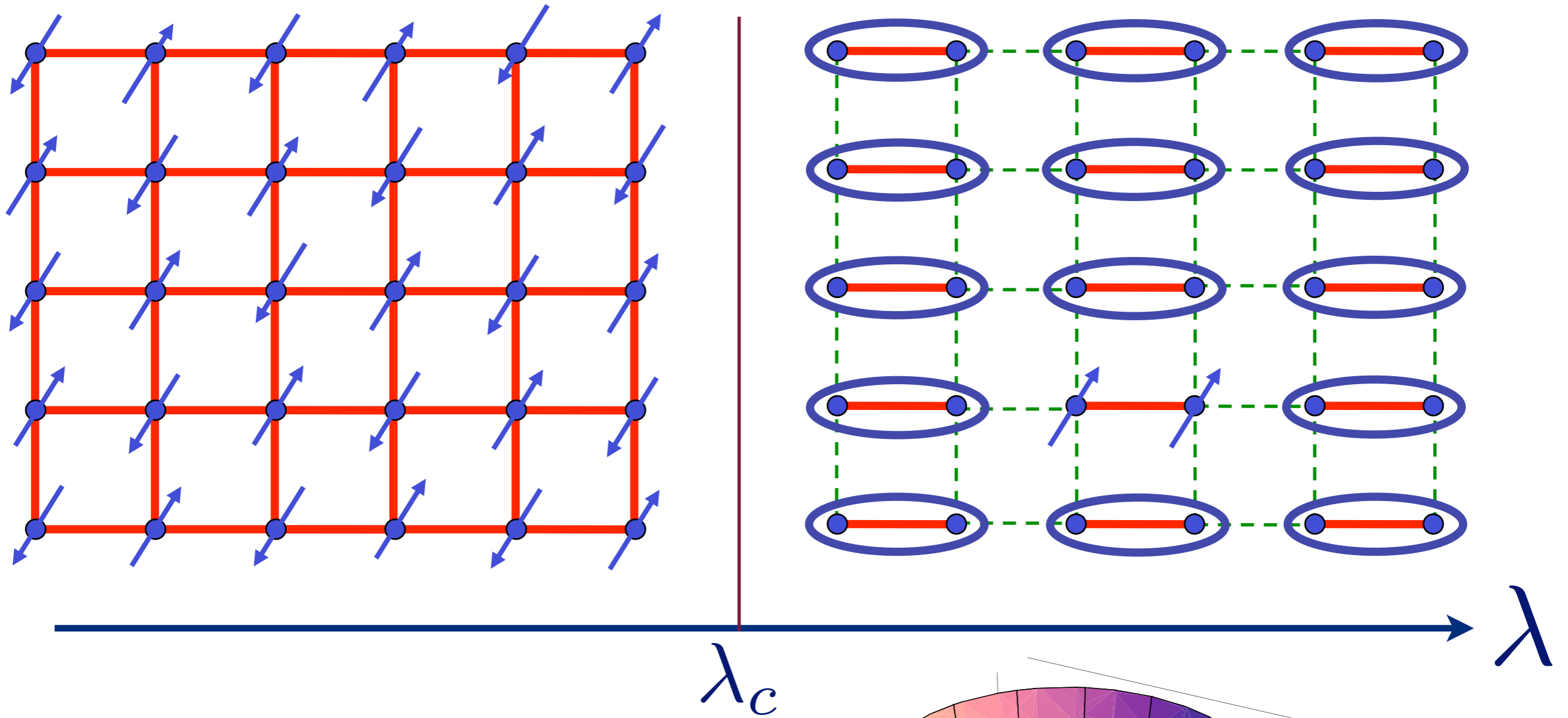


$$V(\vec{\varphi}) = (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2$$

$\lambda > \lambda_c$

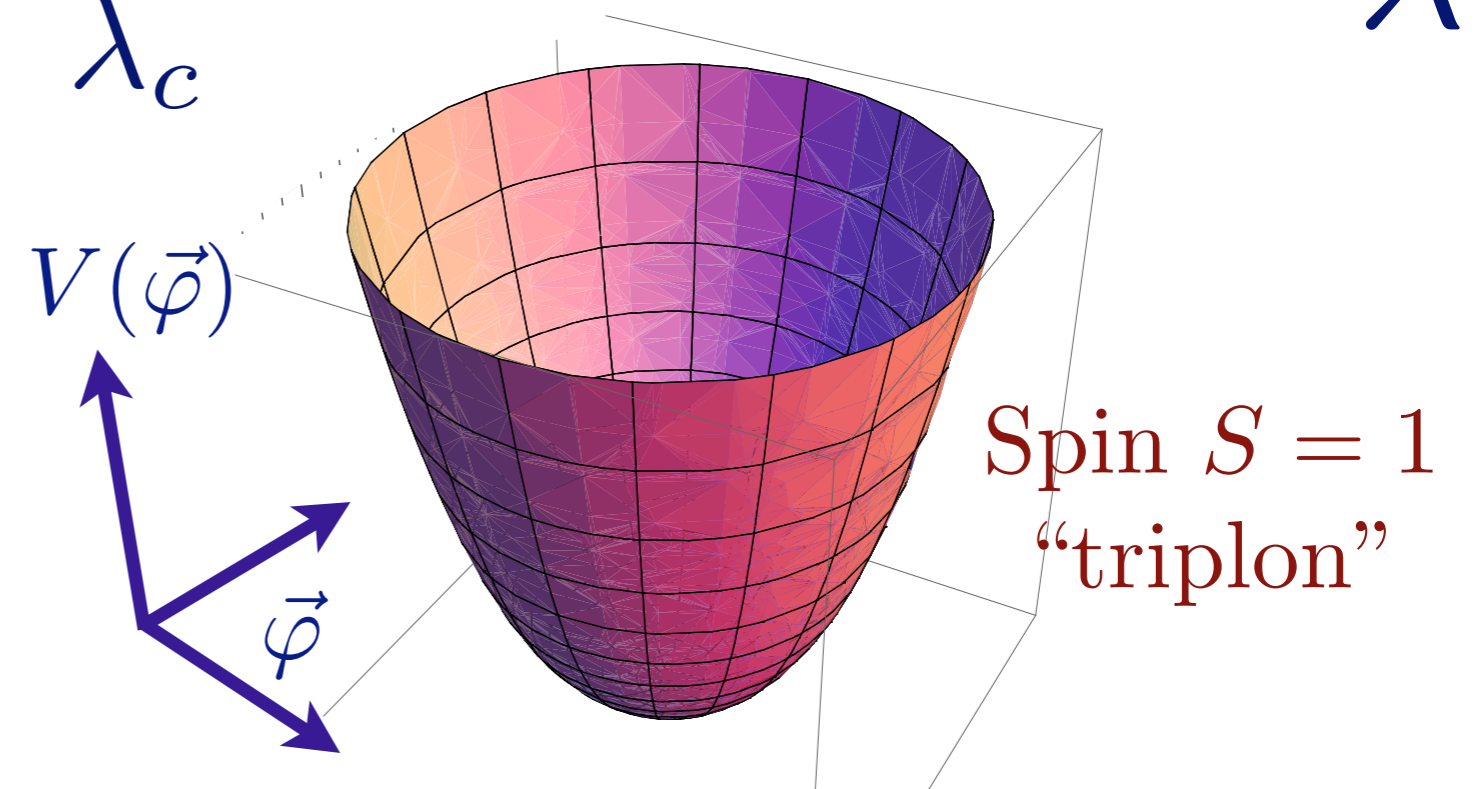


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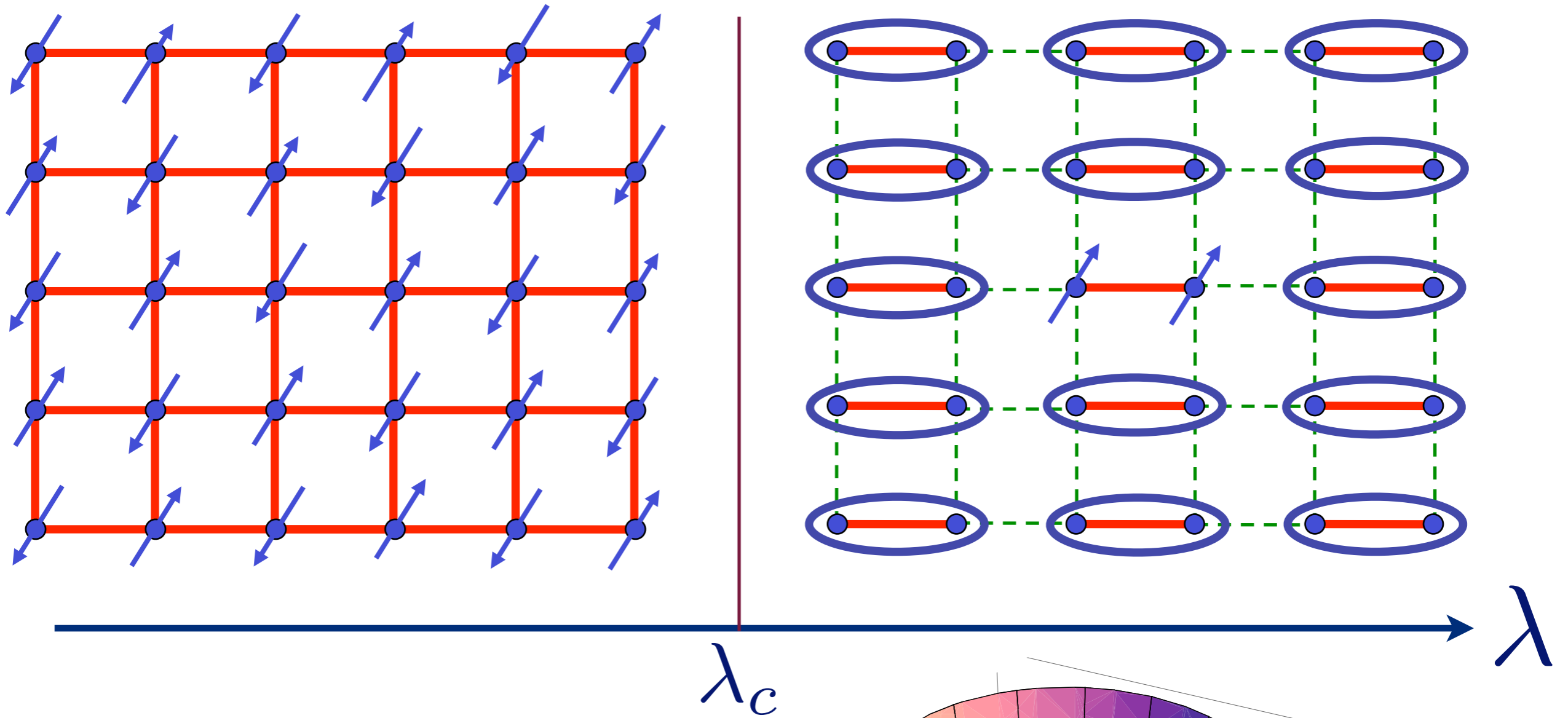


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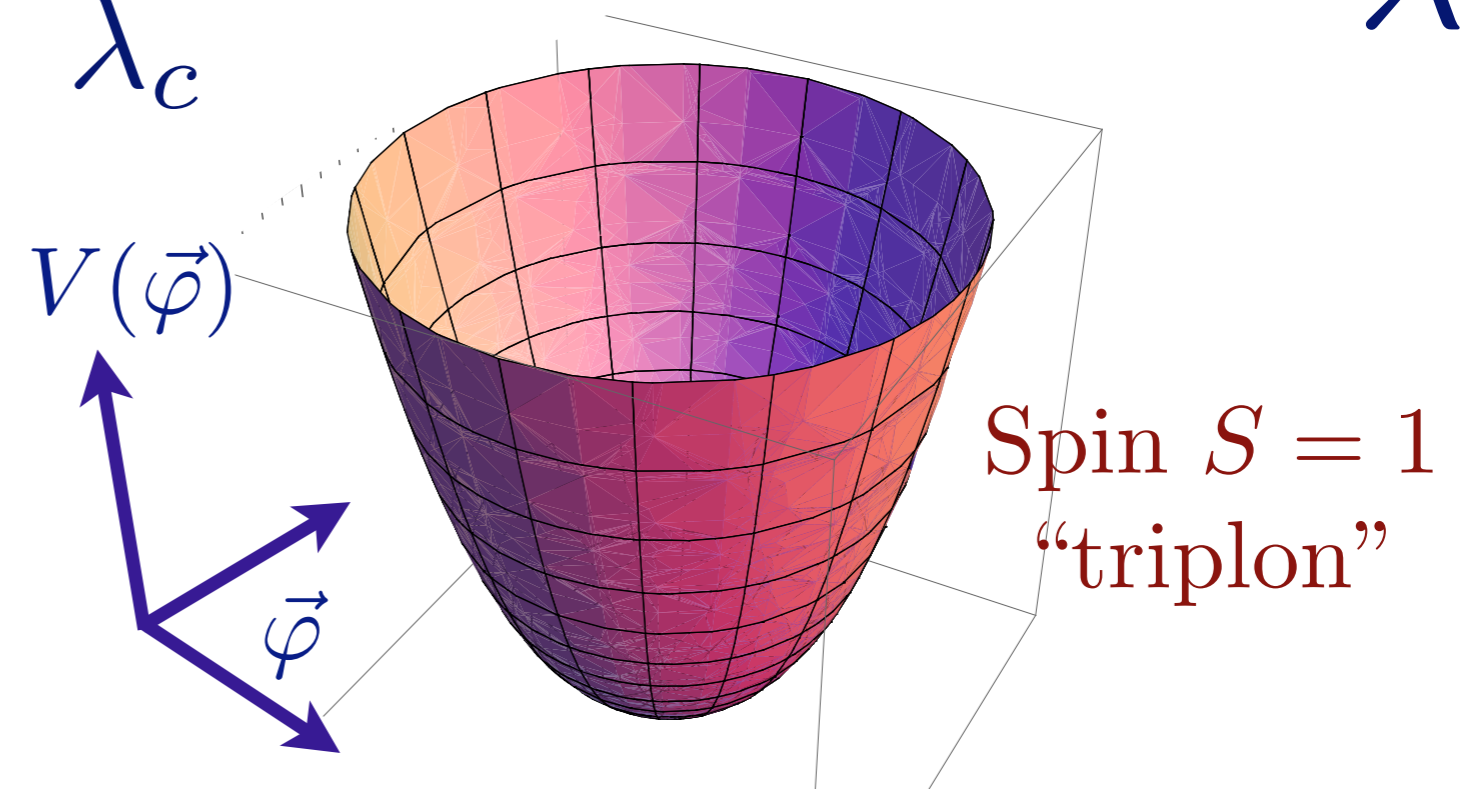


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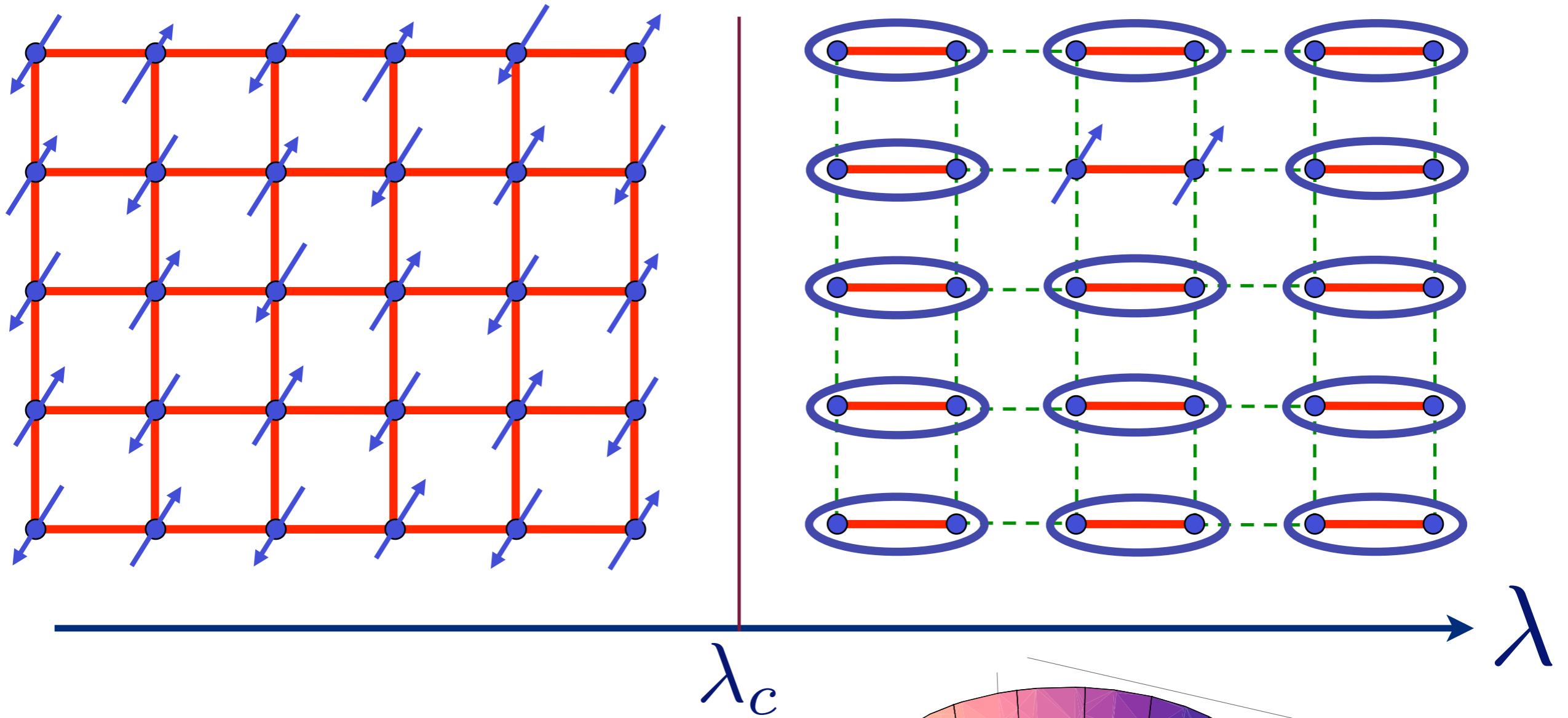


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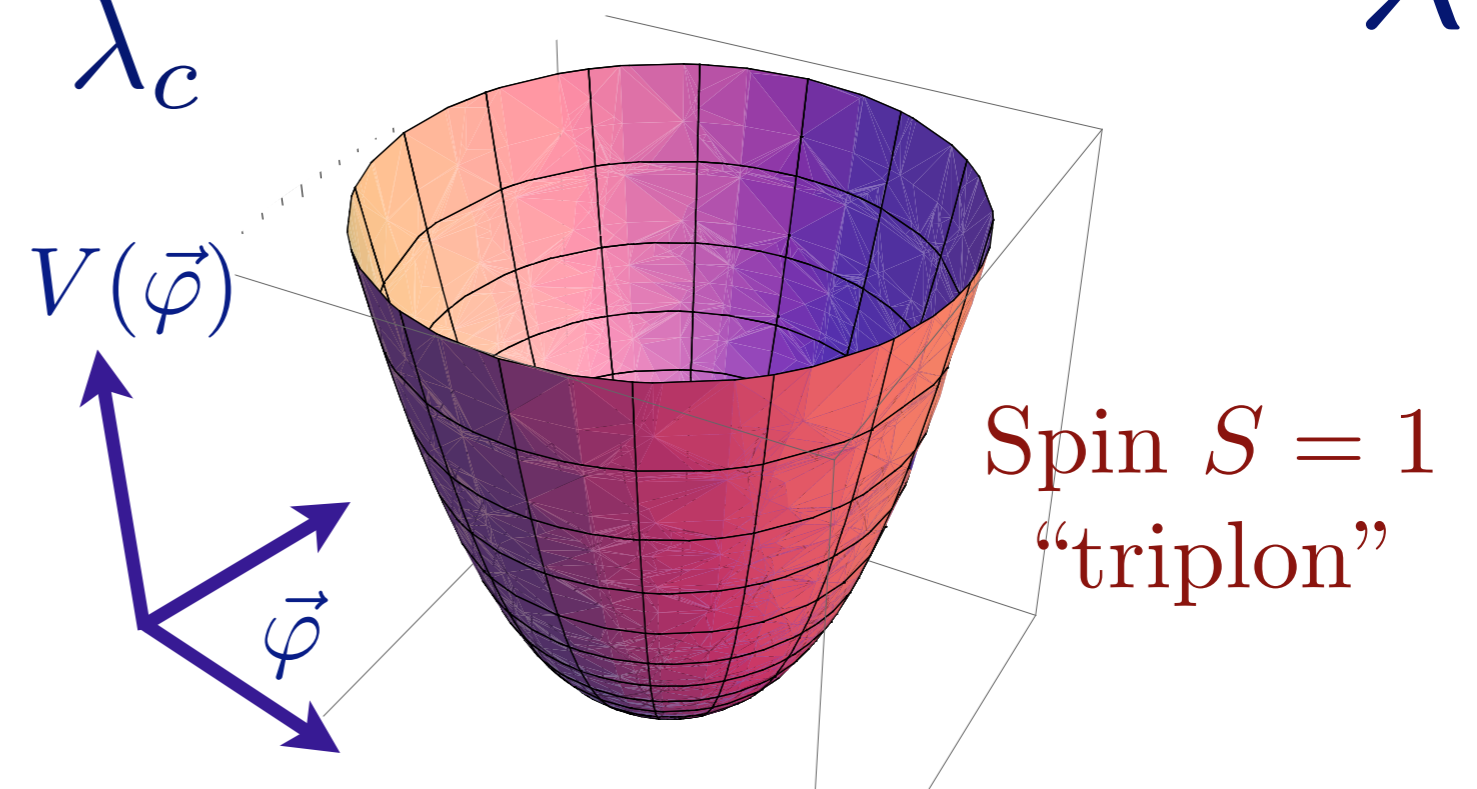


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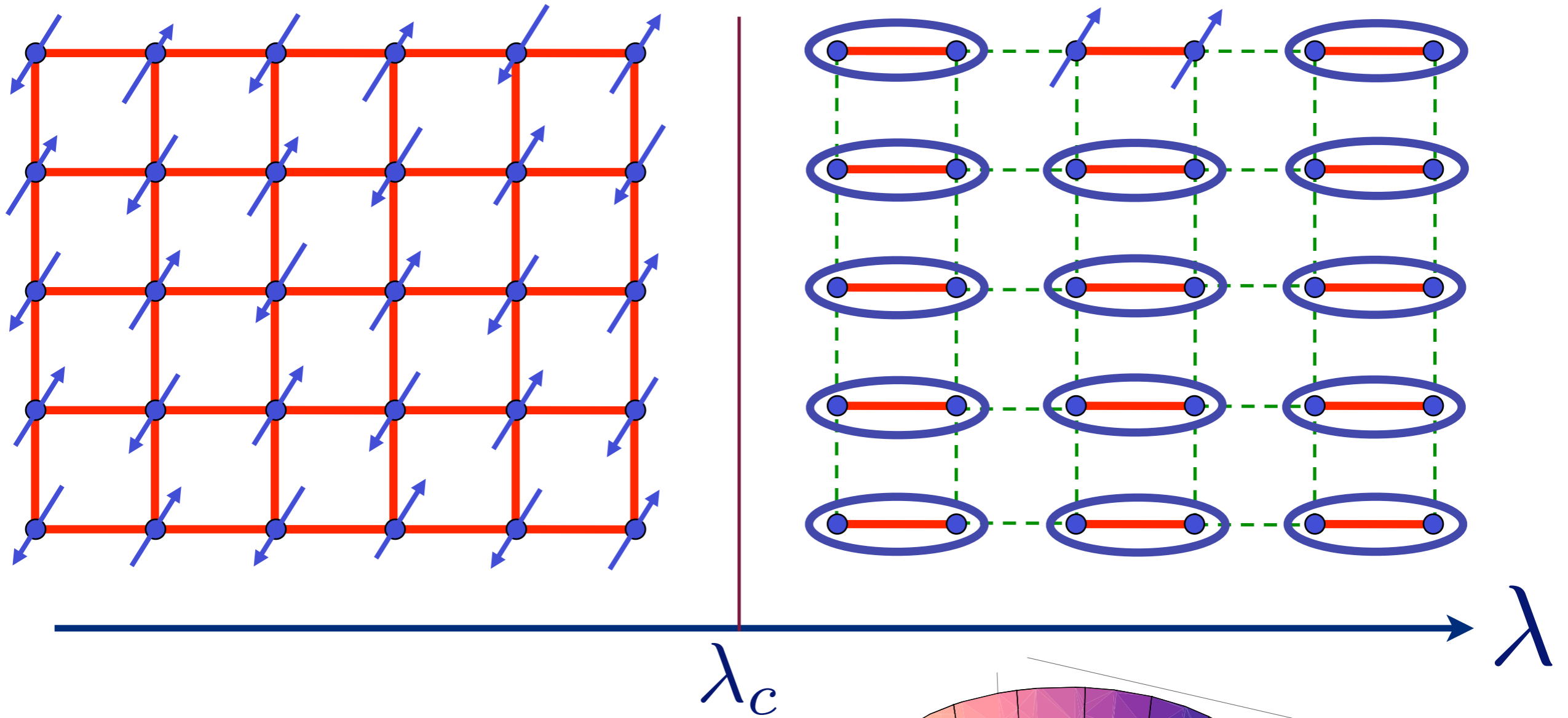


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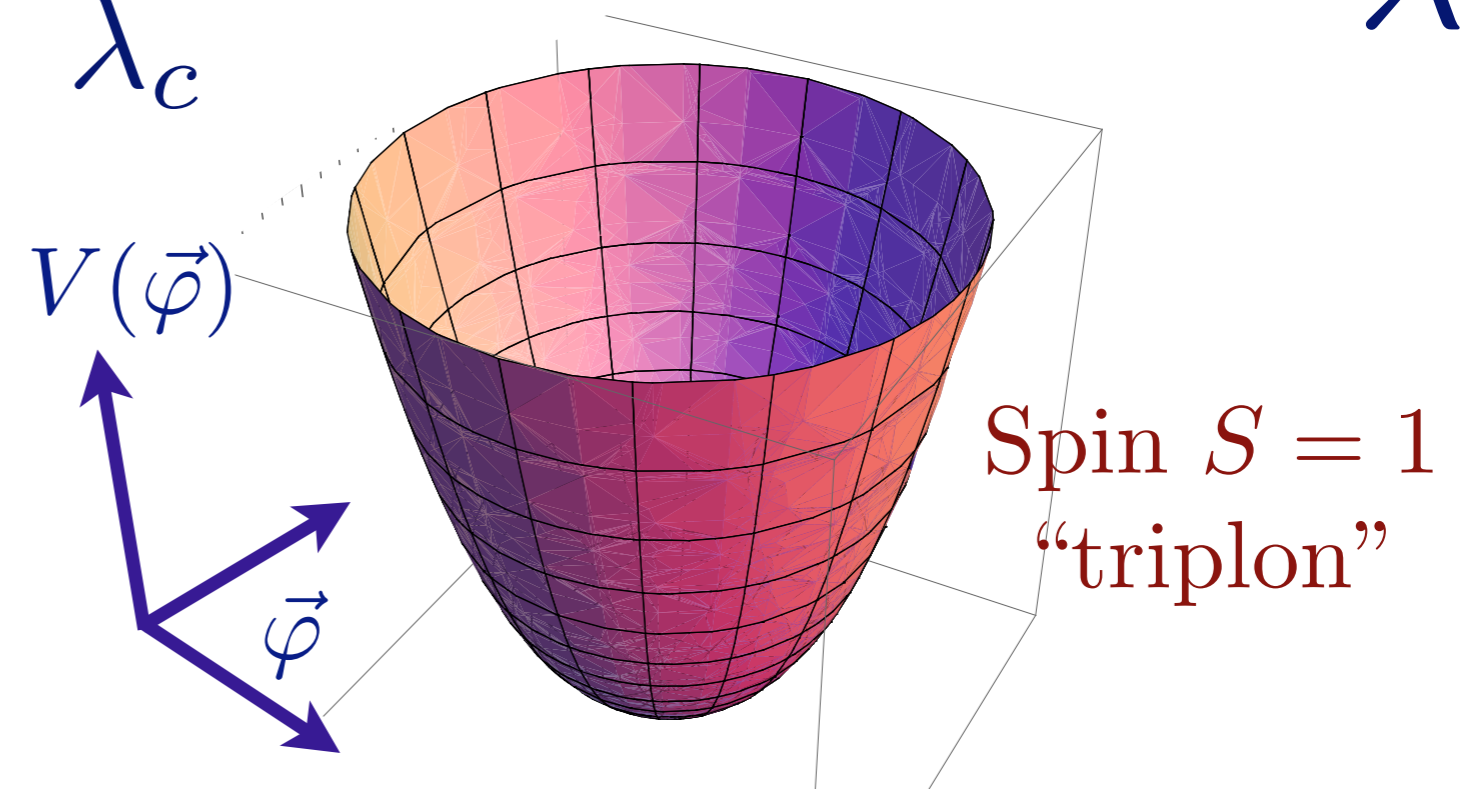


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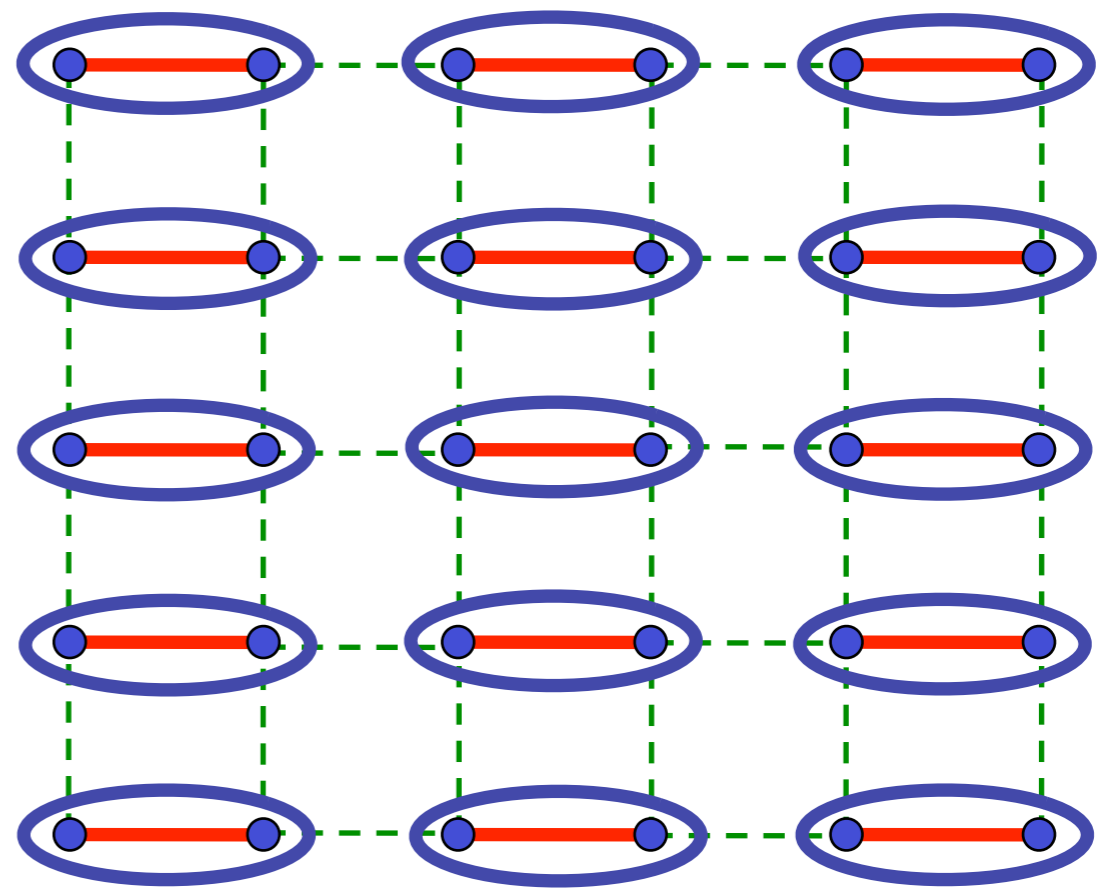
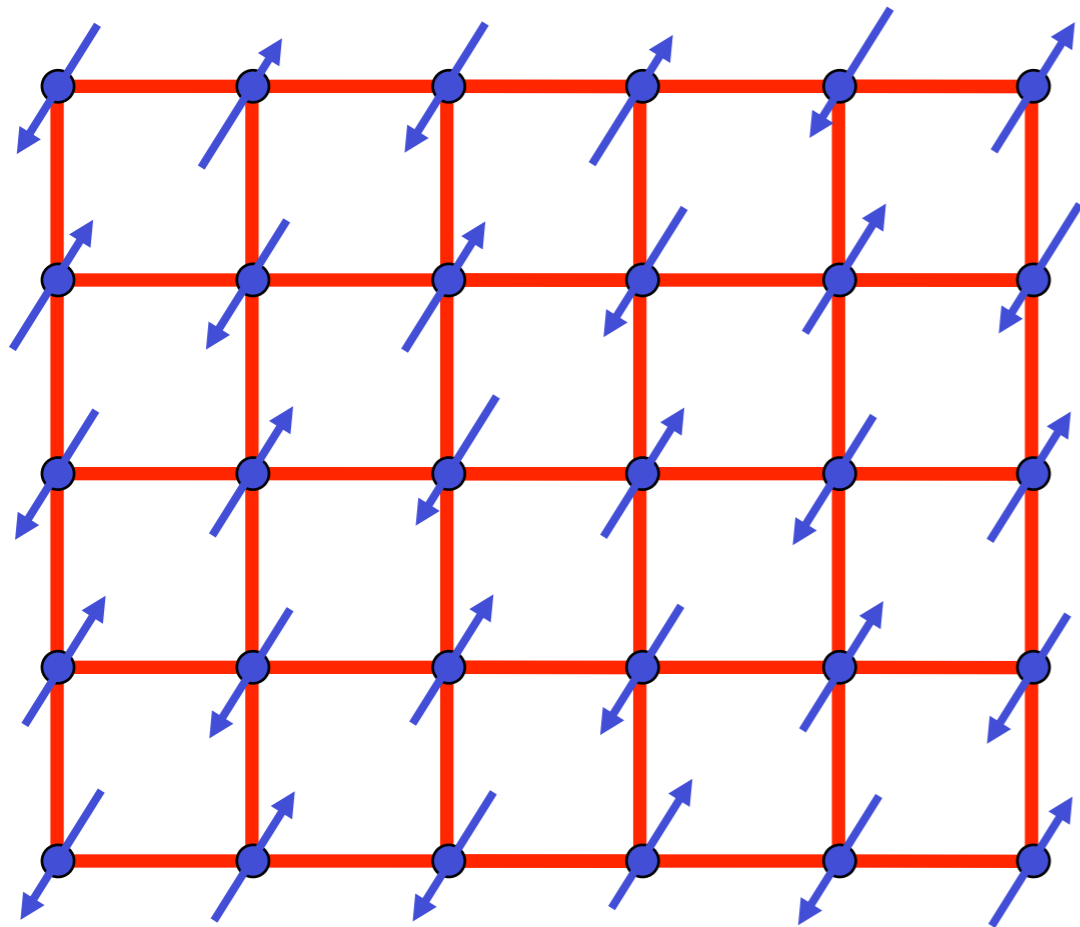


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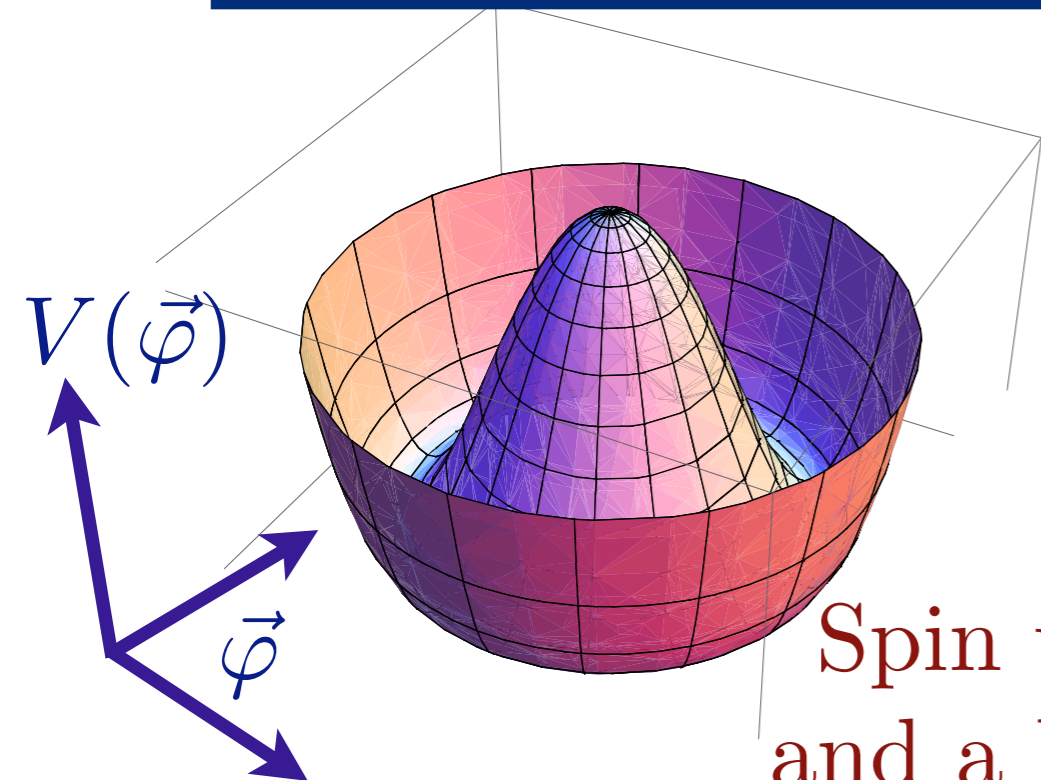


Excitation spectrum in the Néel phase



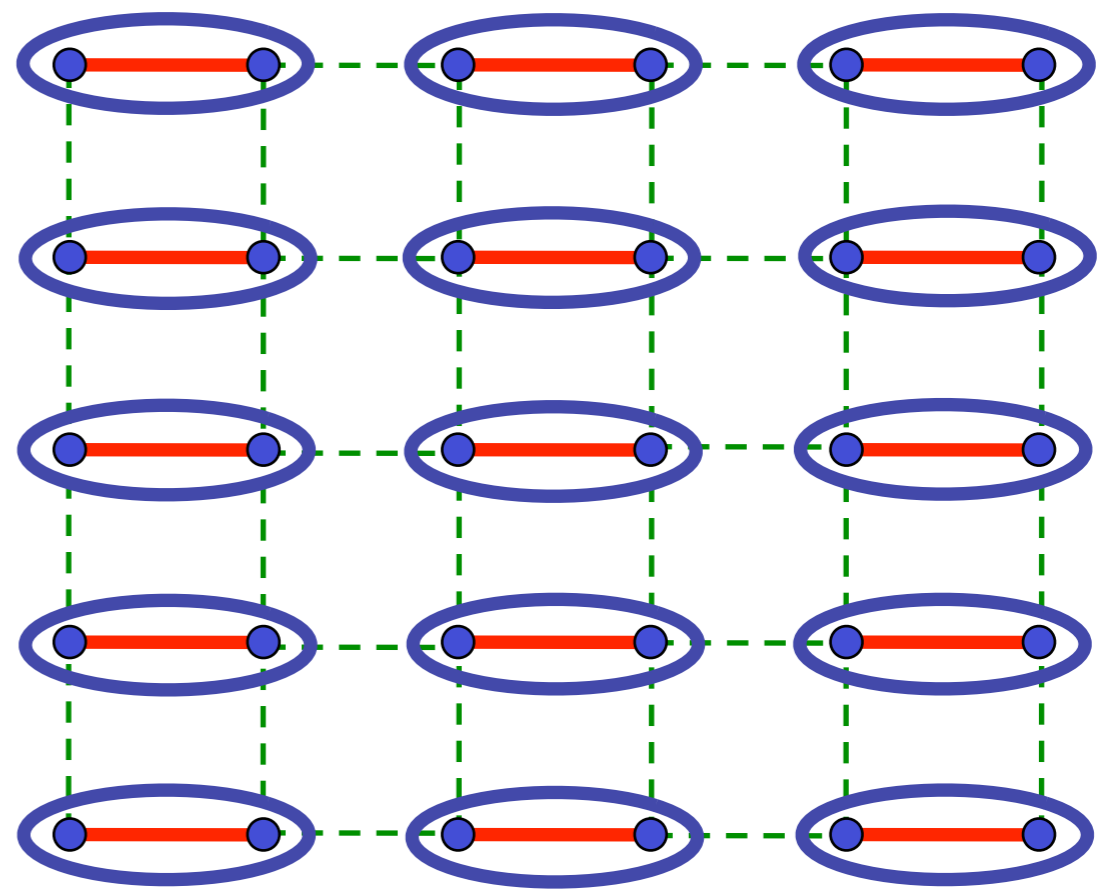
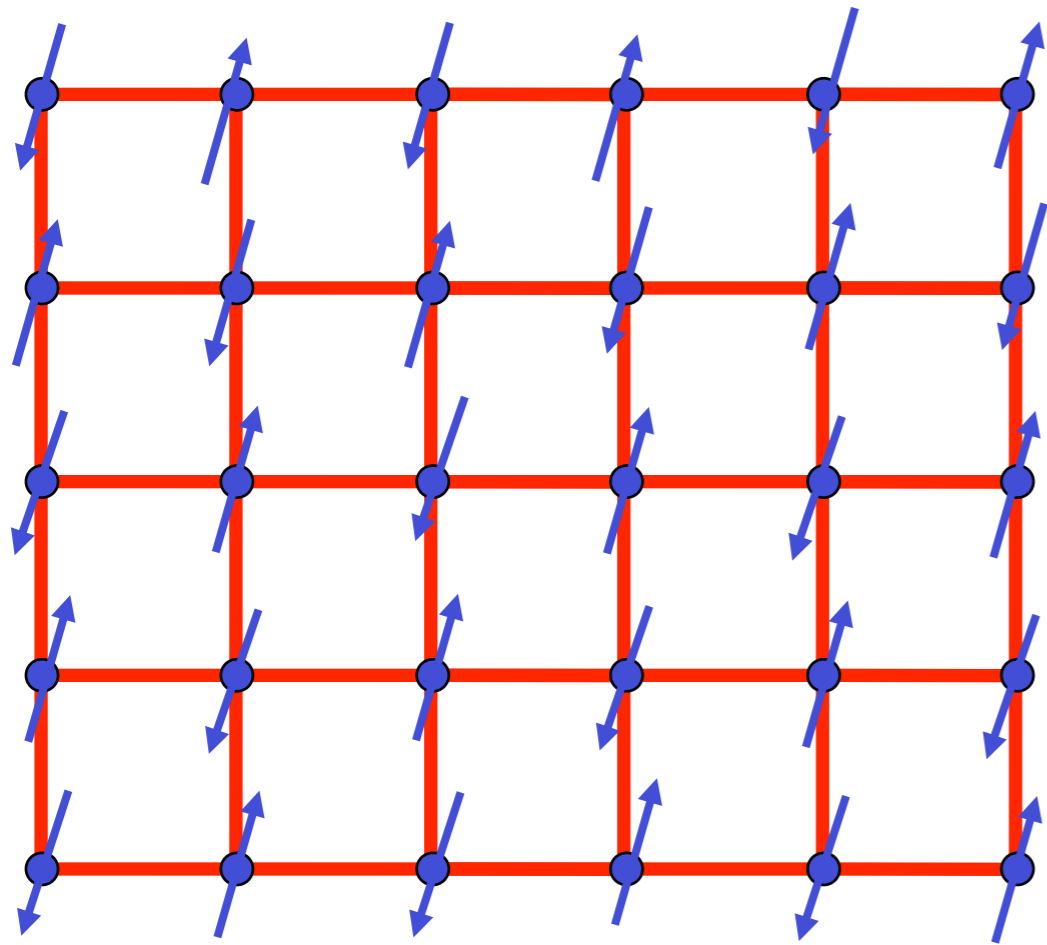
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$$\lambda < \lambda_c$$



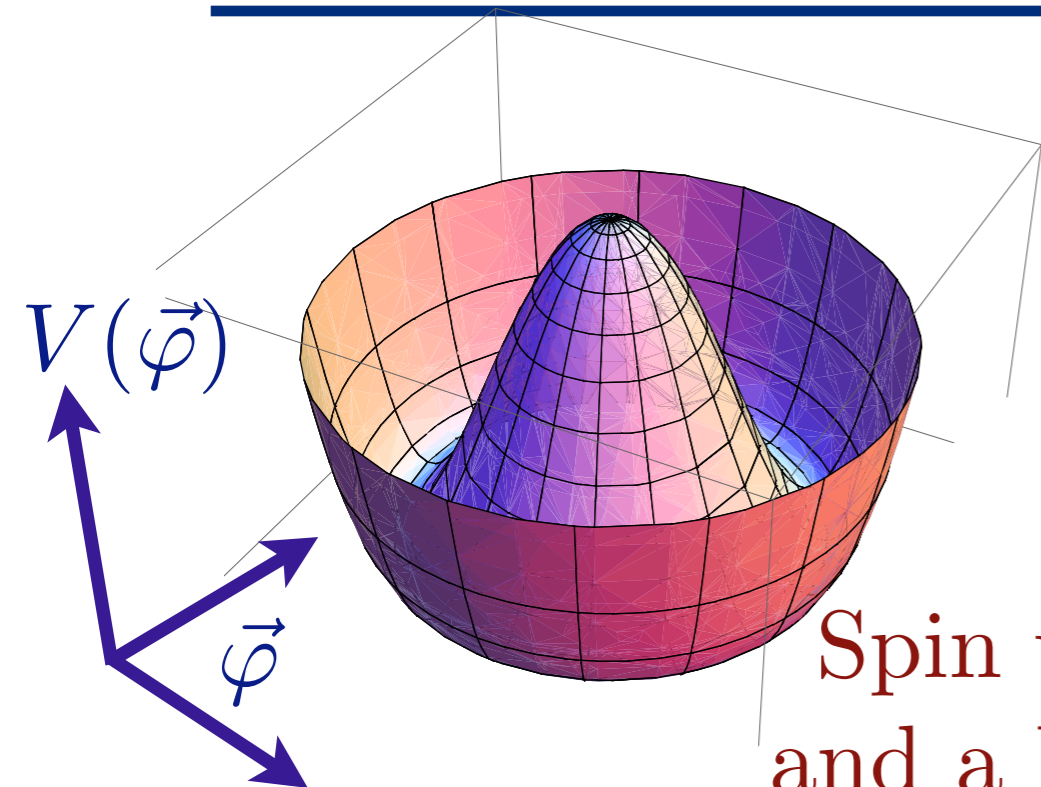
Spin waves (“Goldstone” modes)
and a longitudinal “Higgs” particle

Excitation spectrum in the Néel phase



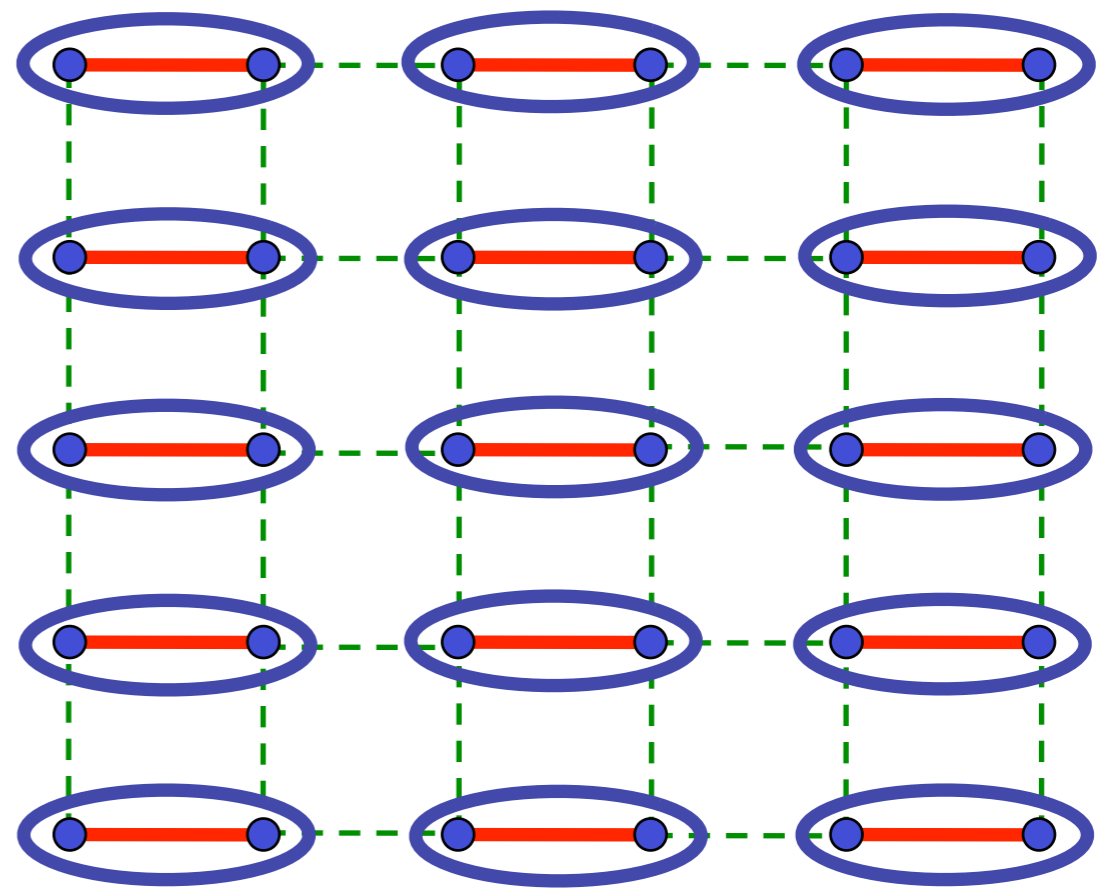
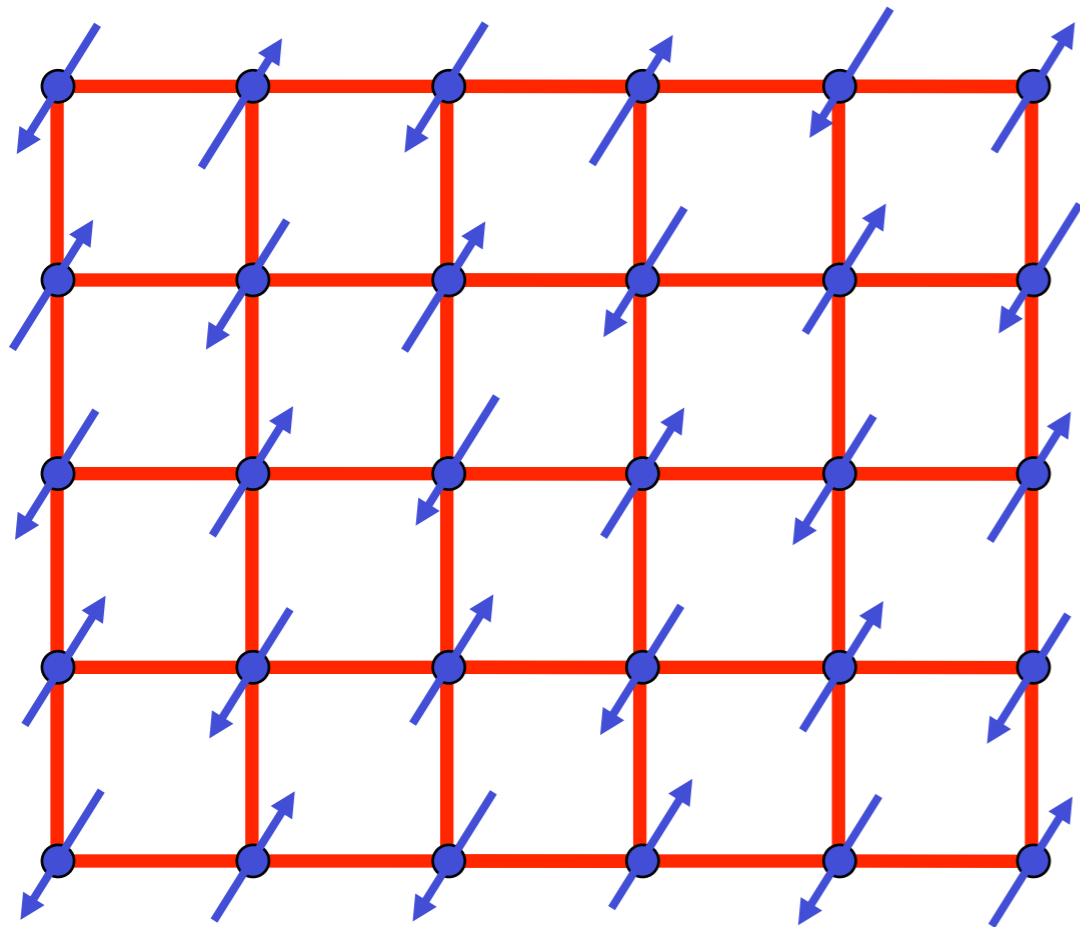
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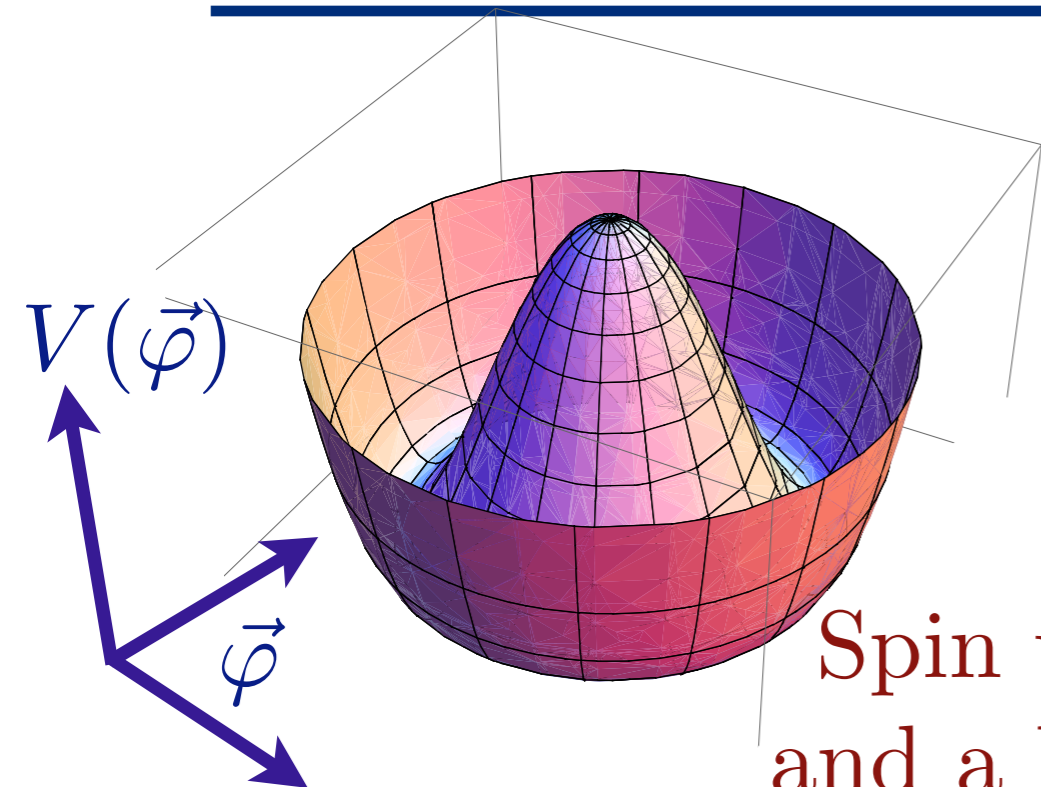
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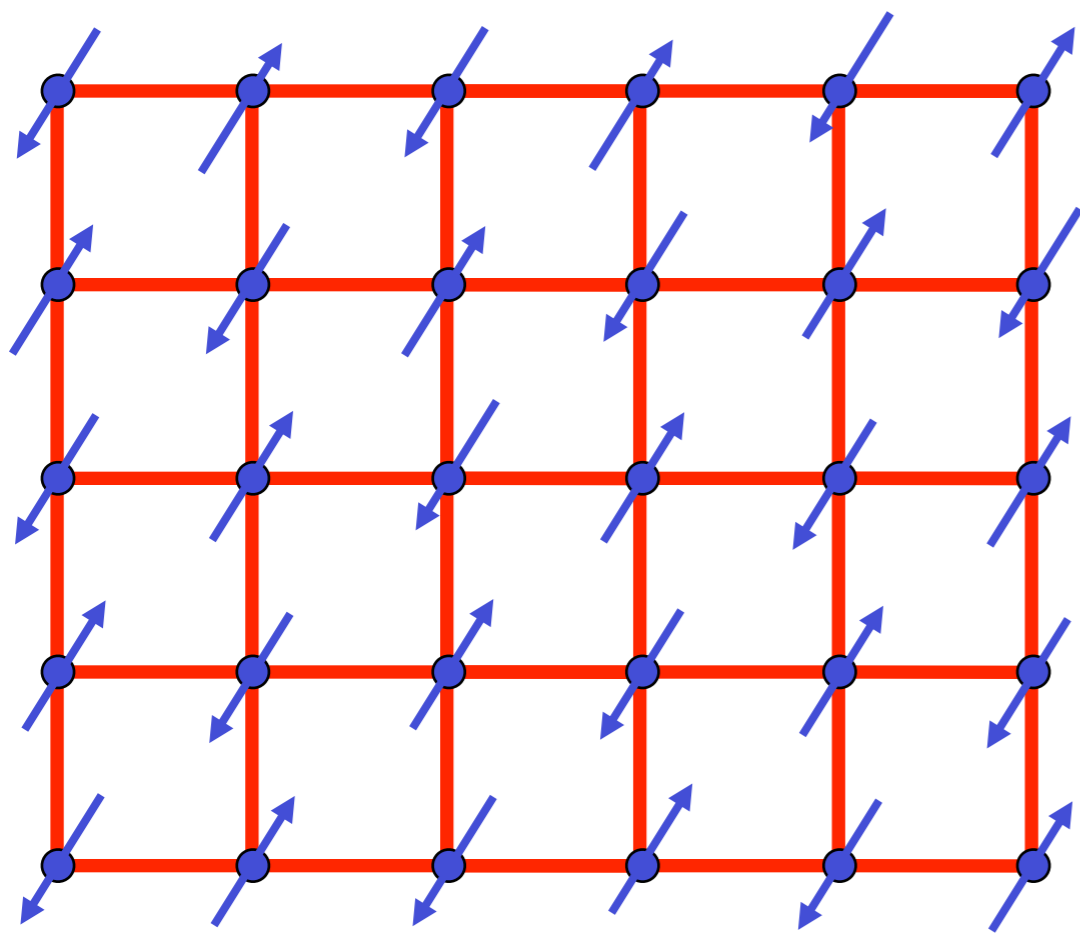


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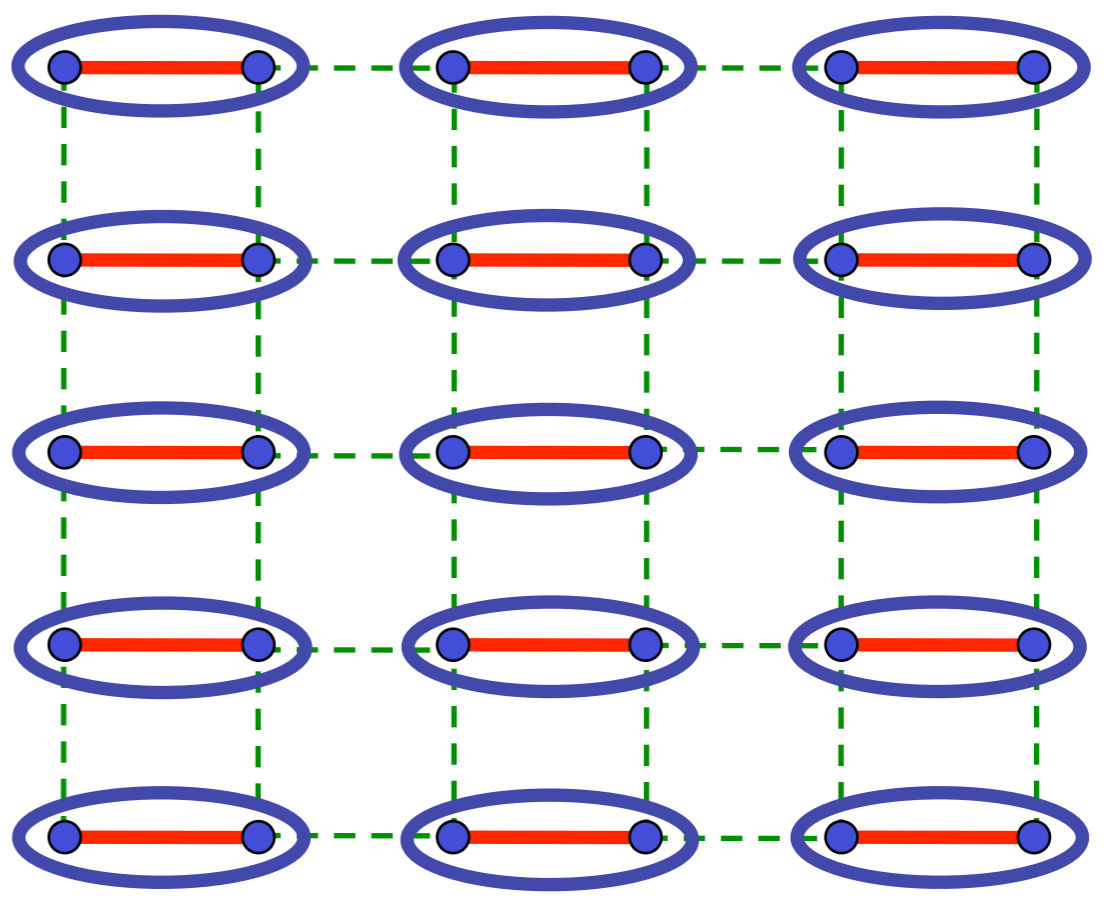
$$\lambda < \lambda_c$$



Spin waves (“Goldstone” modes)
and a longitudinal “Higgs” particle



$$\begin{aligned}
 & \text{Diagram of two blue dots in a blue oval} \\
 & = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)
 \end{aligned}$$



$O(3)$ order parameter $\vec{\varphi}$

CFT3

$$\mathcal{S} = \int d^2r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + s \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \right]$$

Quantum Monte Carlo - critical exponents

Table IV: Fit results for the critical exponents ν , β/ν , and η . We summarize results including a variation of the critical point within its error bar. For the ladder model (top group of values) fit results and quality of fits are also given at the previous best estimate of α_c . The bottom group are results for the plaquette model. Numbers in [...] brackets denote the $\chi^2/\text{d.o.f.}$ For comparison relevant reference values for the 3D $O(3)$ universality class are given in the last line.

α_c	ν^a	β/ν^b	η^c
1.9096 $-\sigma$	0.712(4) [1.8]	0.516(2) [0.5]	0.026(2) [0.2]
1.9096	0.711(4) [1.8]	0.518(2) [1.1]	0.029(5) [0.8]
1.9096 $+\sigma$	0.710(4) [1.8]	0.519(3) [2.5]	0.032(7) [1.4]
1.9107 ^d	0.709(3) [1.7]	0.525(8) [15.3]	0.051(10) [12]
1.8230 $-\sigma$	0.708(4) [0.99]	0.515(2) [0.84]	0.025(4) [0.15]
1.8230	0.706(4) [1.04]	0.516(2) [0.40]	0.028(3) [0.31]
1.8230 $+\sigma$	0.706(4) [1.10]	0.517(2) [1.6]	0.031(5) [0.80]
Ref. 49	0.7112(5)	0.518(1)	0.0375(5)

^a $L > 12$.

^b $L > 16$.

^c $L > 20$.

^dPrevious best estimate of Ref. 19.

S. Wenzel and W. Janke, *Phys. Rev. B* **79**, 014410 (2009)

M. Troyer, M. Imada, and K. Ueda, *J. Phys. Soc. Japan* (1997)

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Field-theoretic
RG of CFT3
E.Vicari *et al.*

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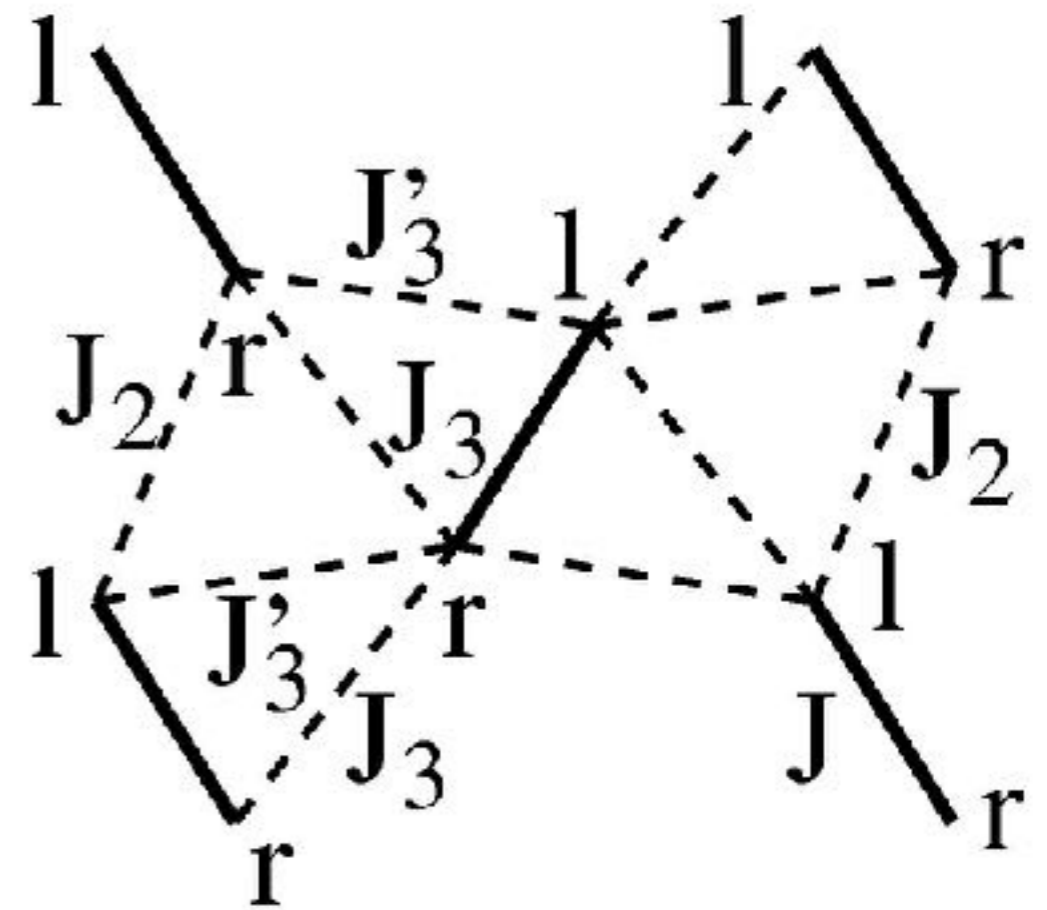
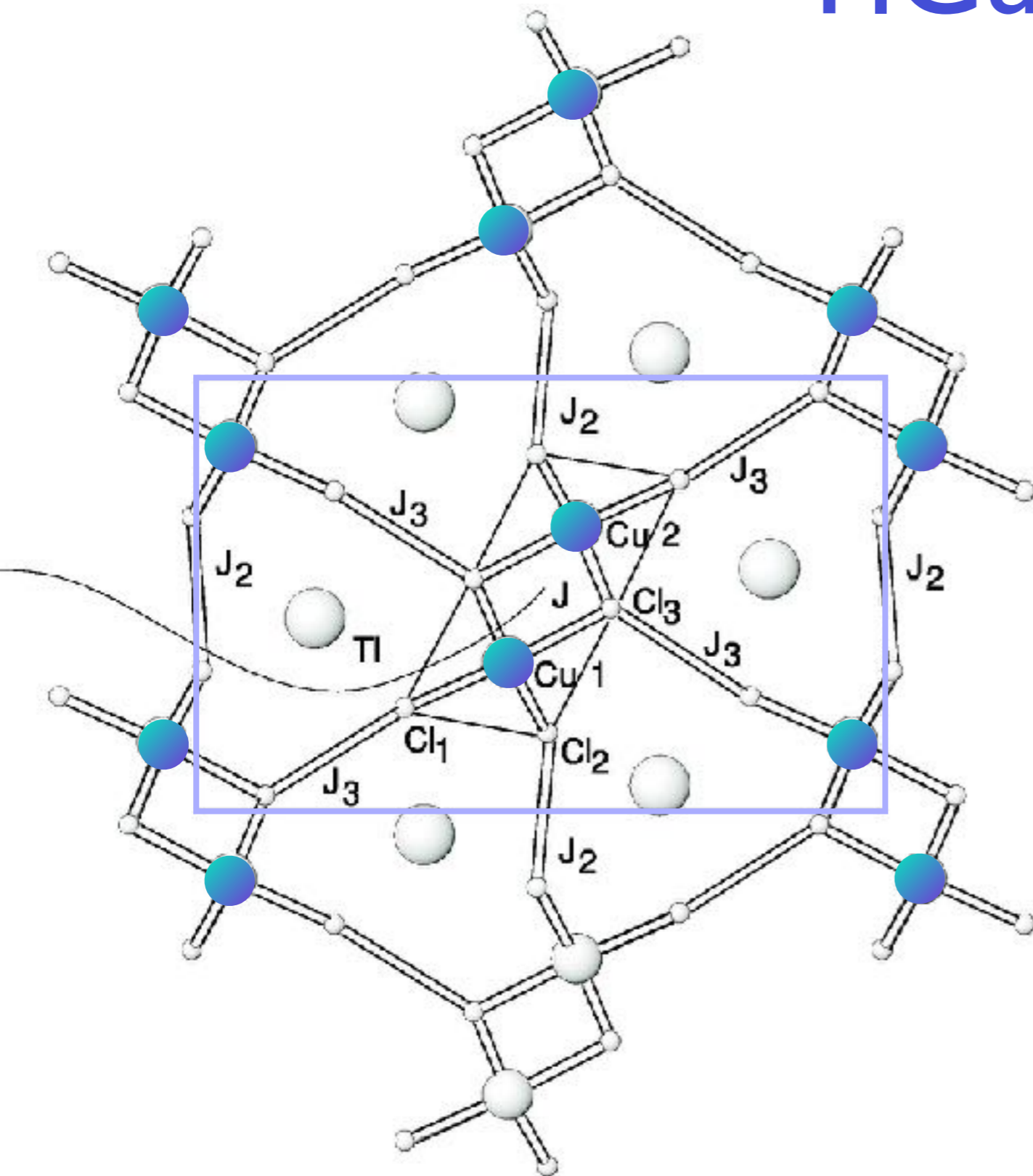
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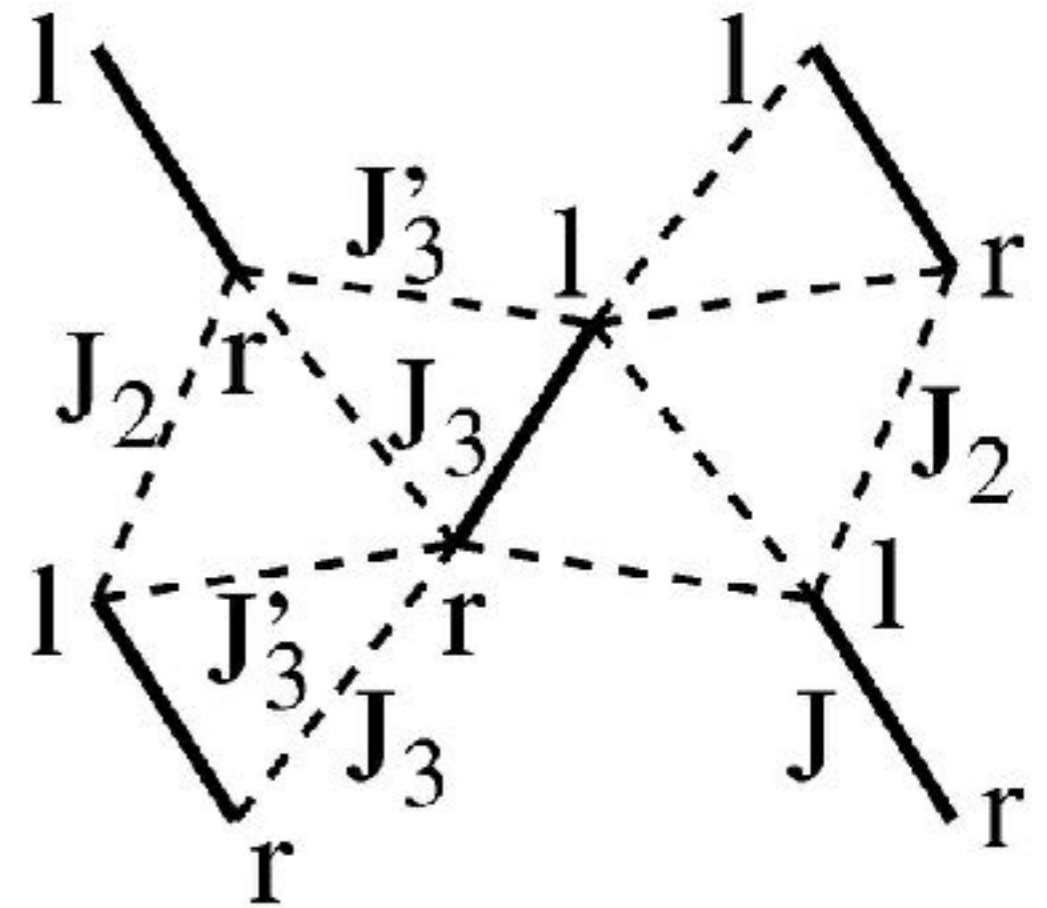
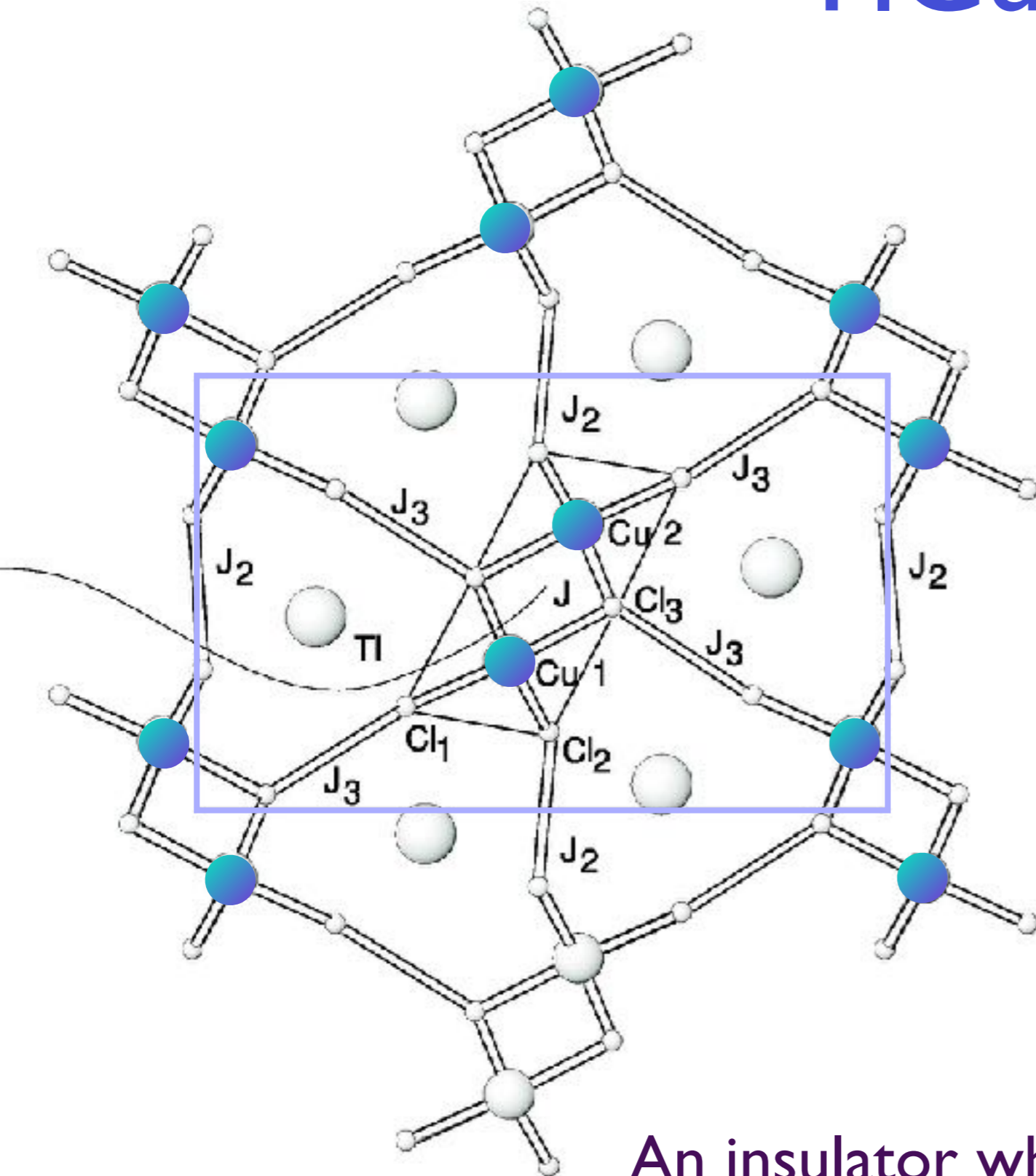
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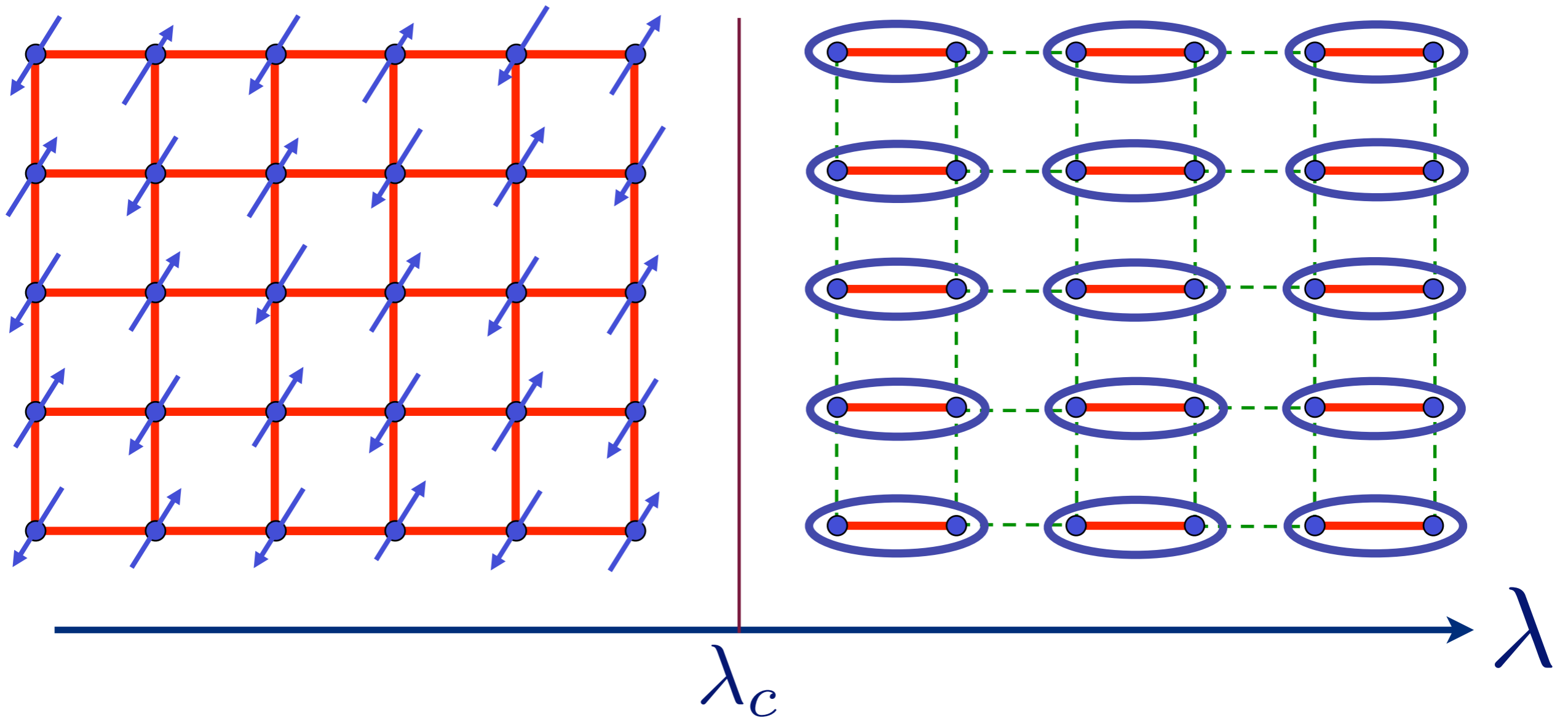
TiCuCl₃



TiCuCl₃



An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.



← Pressure in TlCuCl_3

TlCuCl₃ at ambient pressure

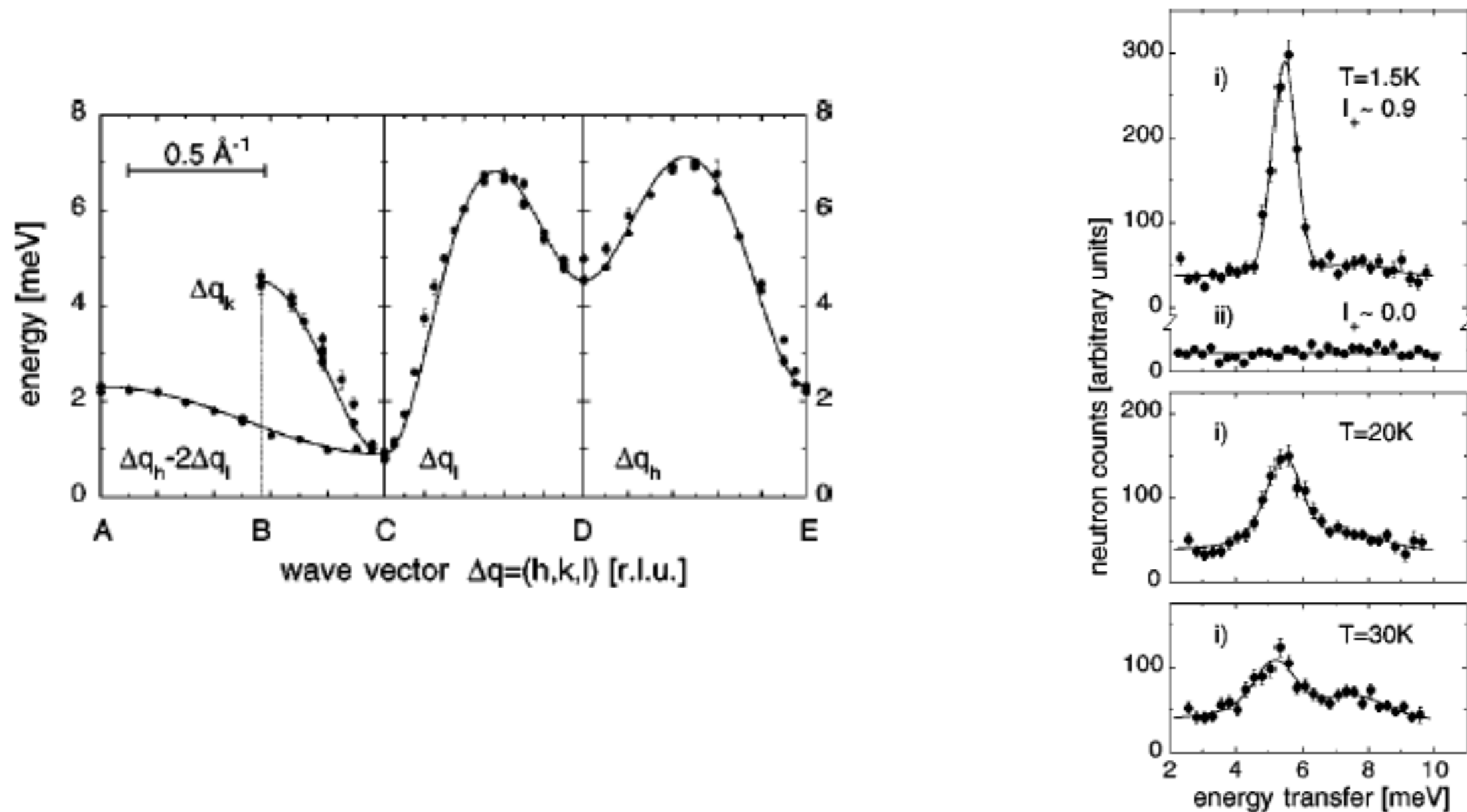
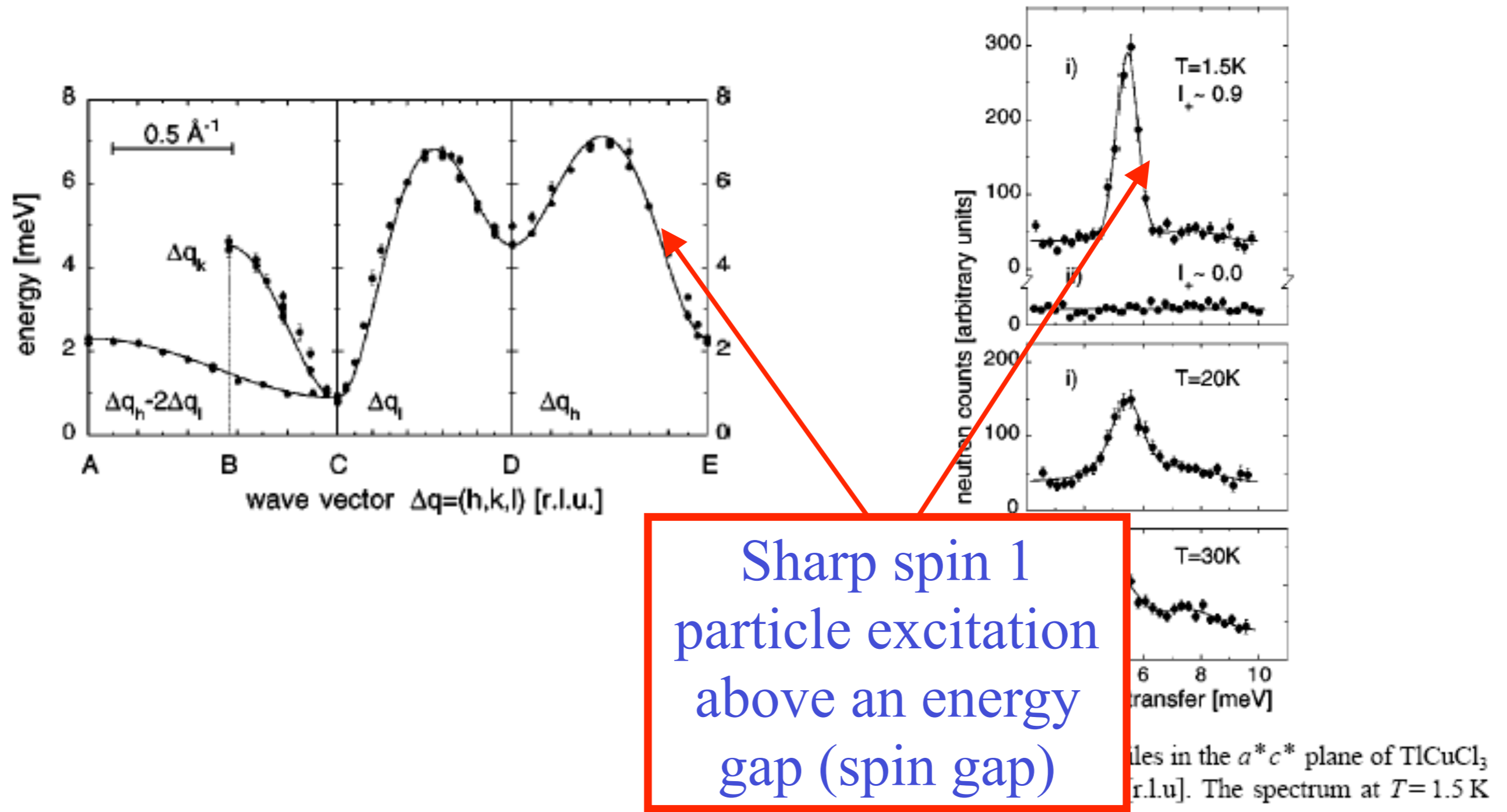


FIG. 1. Measured neutron profiles in the a^*c^* plane of TlCuCl₃ for $i = (1.35, 0, 0)$, $ii = (0, 0, 3.15)$ [r.l.u.]. The spectrum at $T = 1.5 \text{ K}$

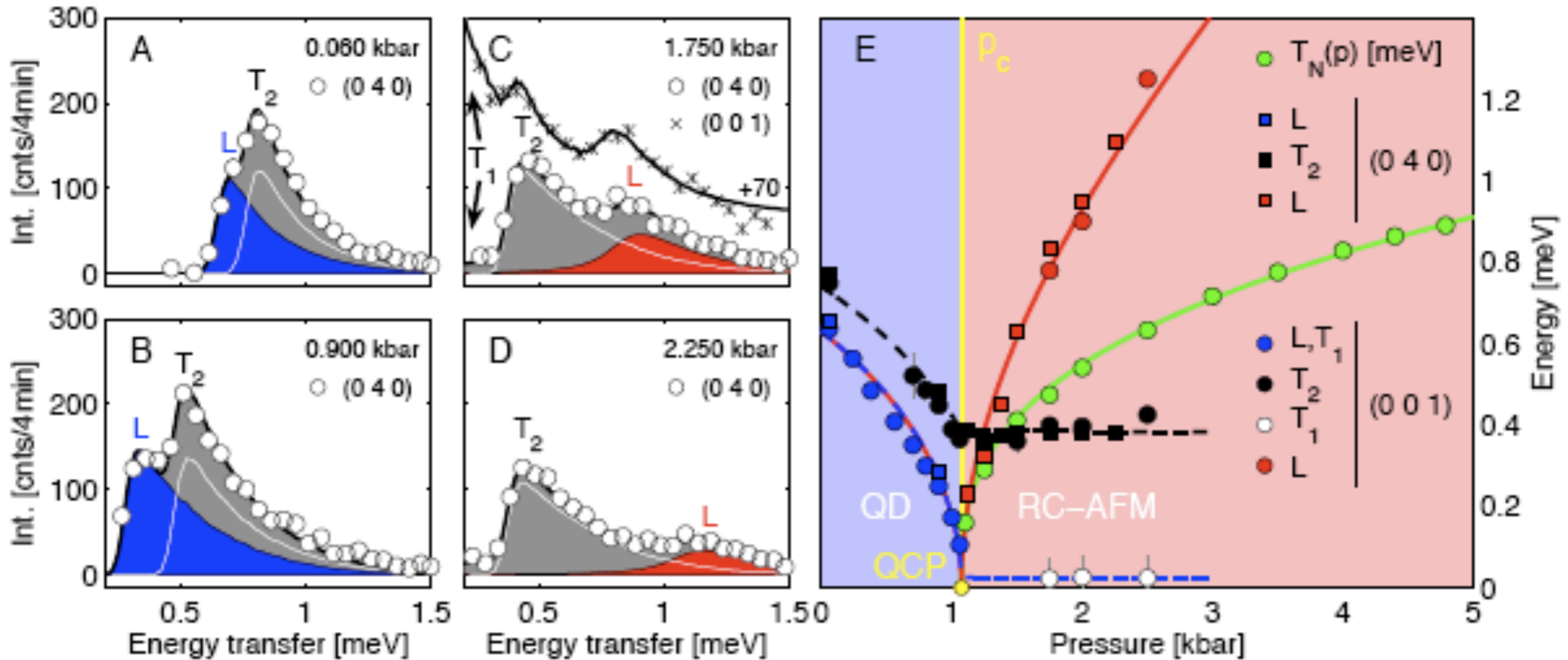
N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev. B* 63 172414 (2001).

TlCuCl₃ at ambient pressure



N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev. B* 63 172414 (2001).

TiCuCl₃ with varying pressure



Observation of 3 → 2 low energy modes, emergence of new longitudinal mode (the “Higgs boson”) in Néel phase, and vanishing of Néel temperature at quantum critical point

Christian Ruedg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorro, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Prediction of quantum field theory

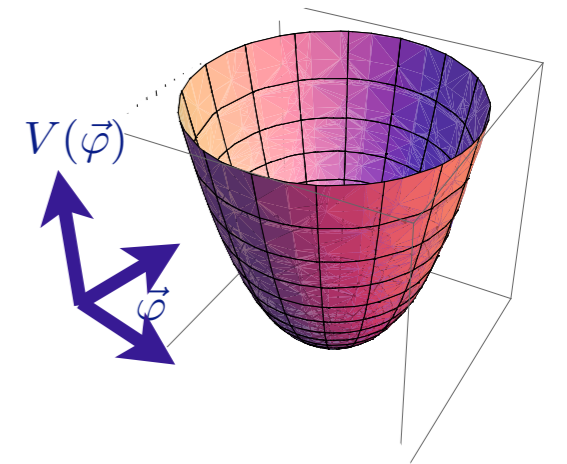
Potential for $\vec{\varphi}$ fluctuations: $V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u(\vec{\varphi}^2)^2$

Paramagnetic phase, $\lambda > \lambda_c$

Expand about $\vec{\varphi} = 0$:

$$V(\vec{\varphi}) \approx (\lambda - \lambda_c)\vec{\varphi}^2$$

Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$



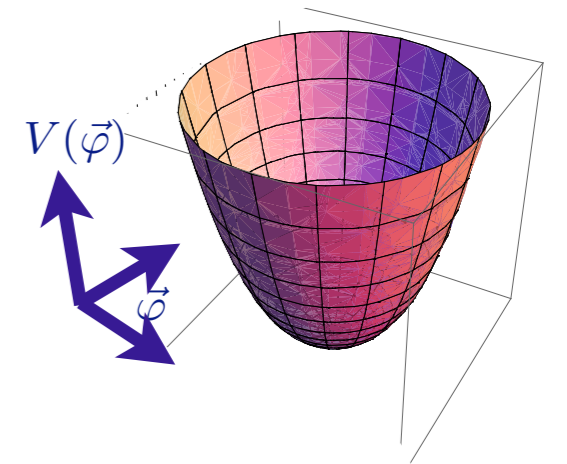
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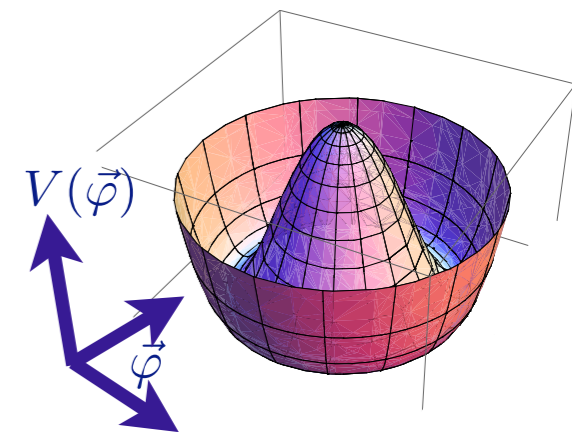


Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$

Néel phase, $\lambda < \lambda_c$

Expand $\vec{\varphi} = (0, 0, \sqrt{(\lambda_c - \lambda)/(2u)}) + \vec{\varphi}_1$:

$$V(\vec{\varphi}) \approx 2(\lambda_c - \lambda)\varphi_{1z}^2$$

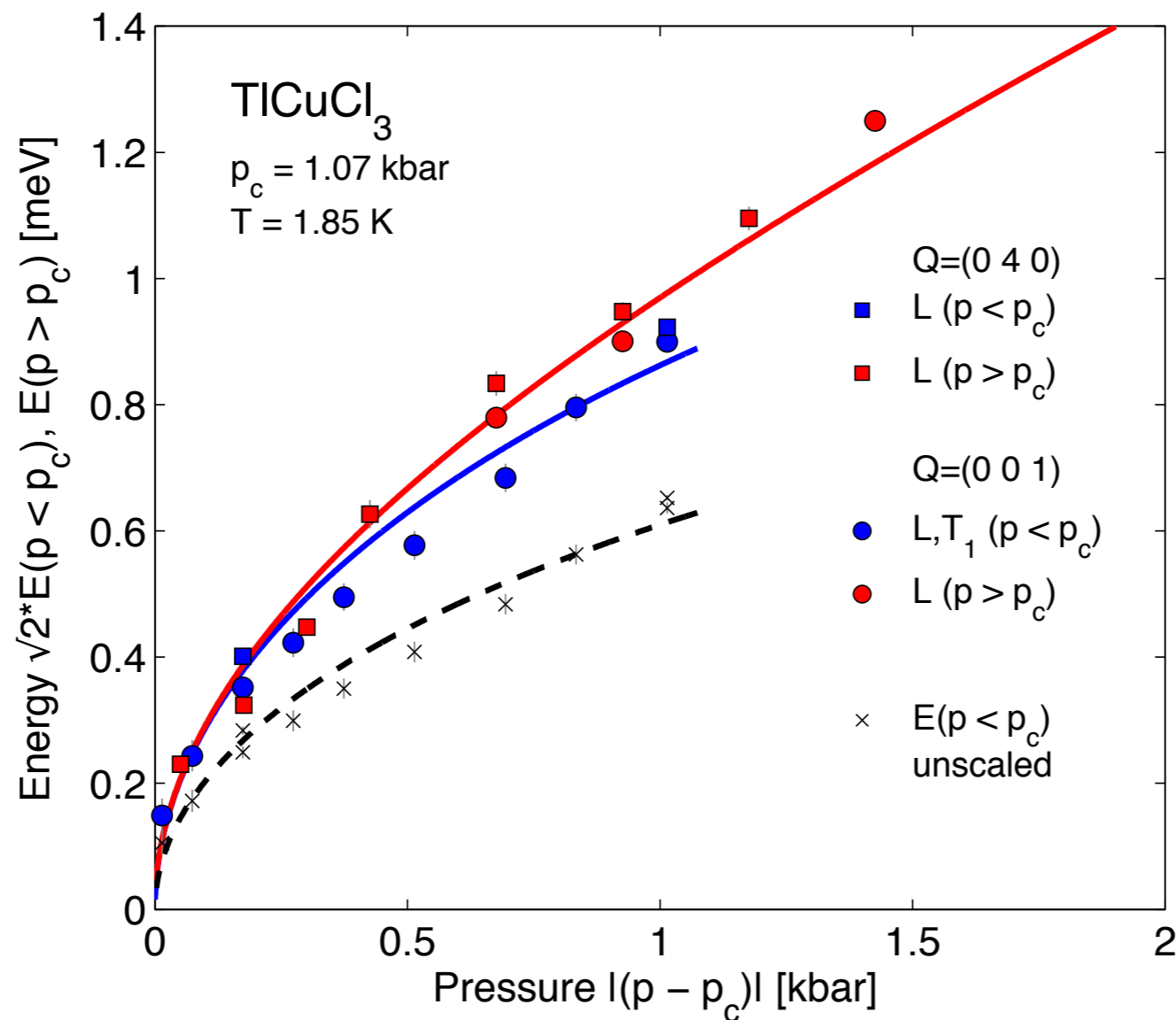


Yields 2 gapless spin waves and one Higgs particle with energy gap $\sim \sqrt{2(\lambda_c - \lambda)}$

Prediction of quantum field theory

$$\frac{\text{Energy of Higgs particle}}{\text{Energy of triplon}} = \sqrt{2}$$

$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u(\vec{\varphi}^2)^2$$

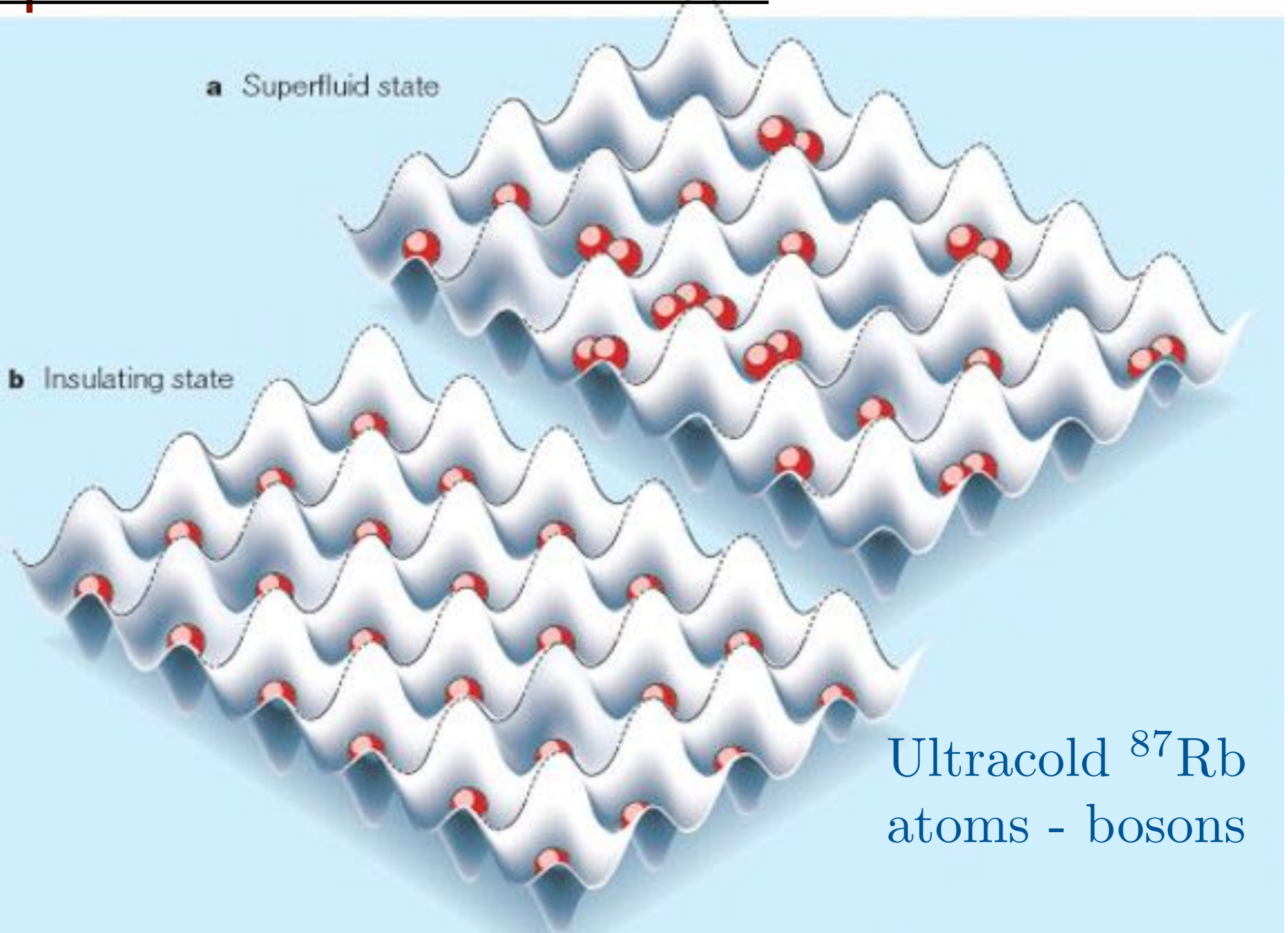


1. Coupled ladder antiferromagnet

2. Superfluid-insulator transition of bosons

3. Ising-nematic criticality in metals

Superfluid-insulator transition



Ultracold ^{87}Rb
atoms - bosons

The Superfluid-Insulator transition

Boson Hubbard model

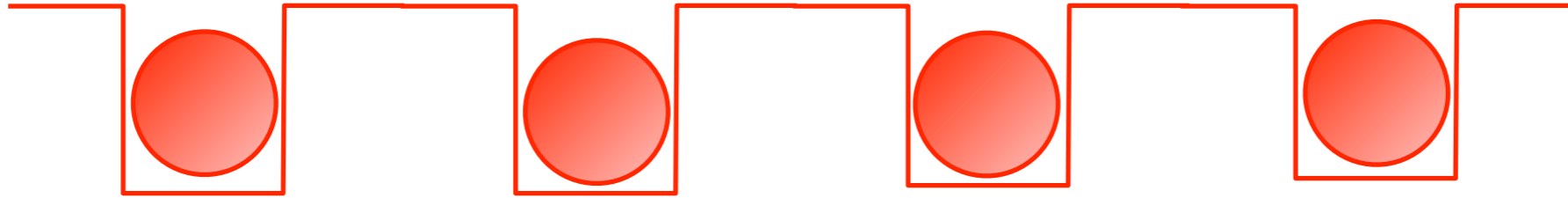
Degrees of freedom: Bosons, b_j^\dagger , hopping between the sites, j , of a lattice, with short-range repulsive interactions.

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2} \sum_j n_j (n_j - 1) + \dots$$

$$n_j \equiv b_j^\dagger b_j$$

$$[b_j, b_k^\dagger] = \delta_{jk}$$

$$\underline{U \gg t}$$

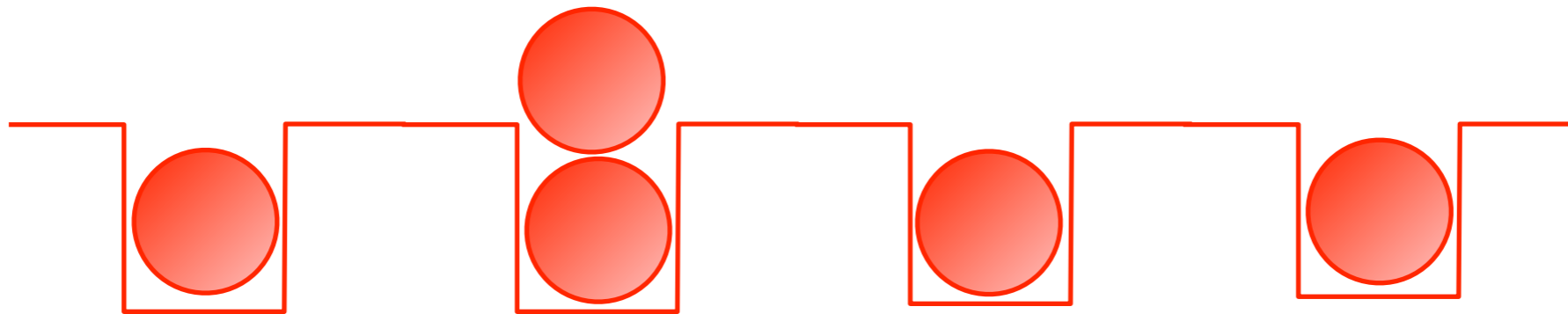


Insulator (the vacuum)
at large repulsion between bosons

$$|\text{Ground state}\rangle = \prod_i b_i^\dagger |0\rangle$$

$$\underline{U \gg t}$$

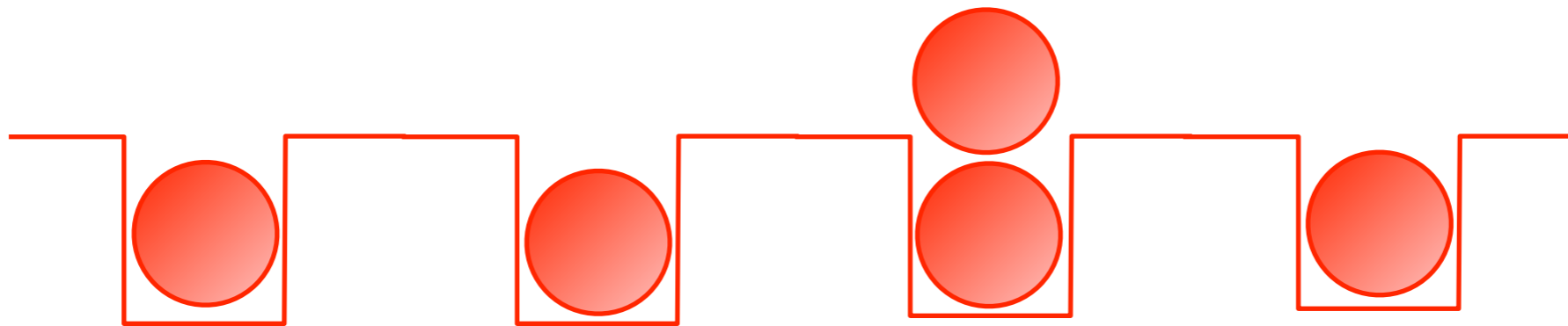
Excitations of the insulator:



Particles $\sim \psi^\dagger$

$$\underline{U \gg t}$$

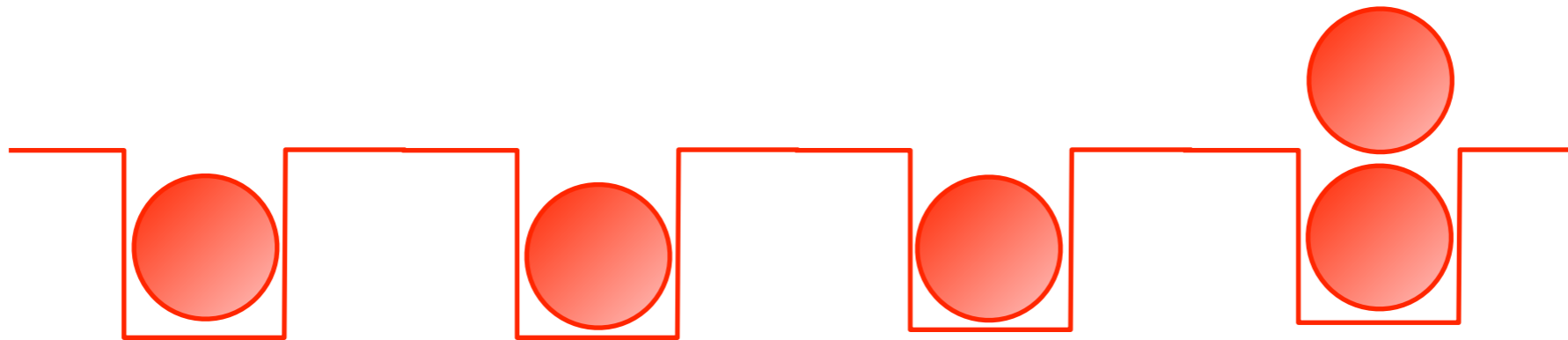
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Excitations of the insulator:



Particles $\sim \psi^\dagger$

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Excitations of the insulator:



Holes $\sim \psi$

$$\underline{U \gg t}$$

Excitations of the insulator:



Holes $\sim \psi$

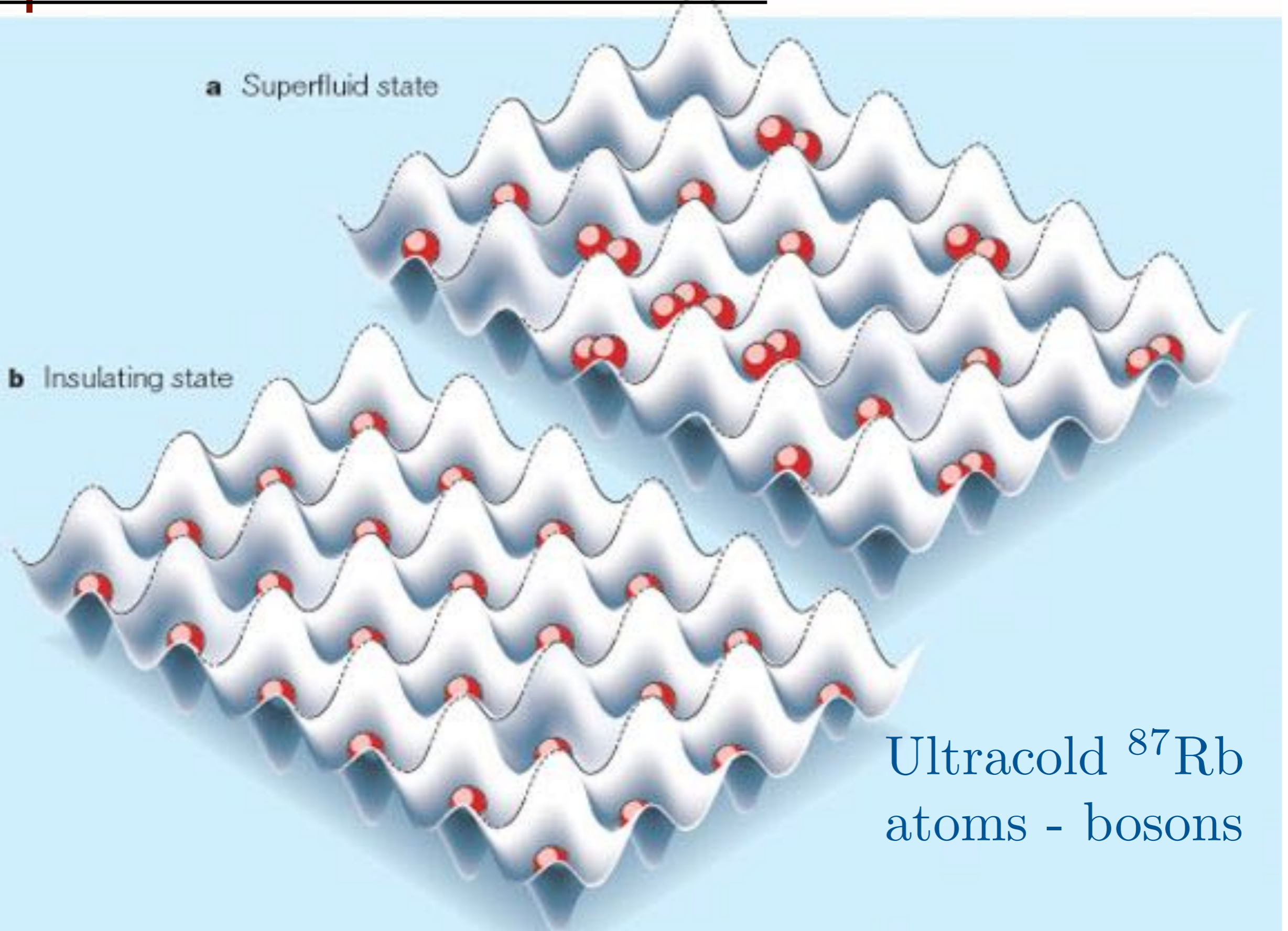
$$\underline{U \gg t}$$

Excitations of the insulator:



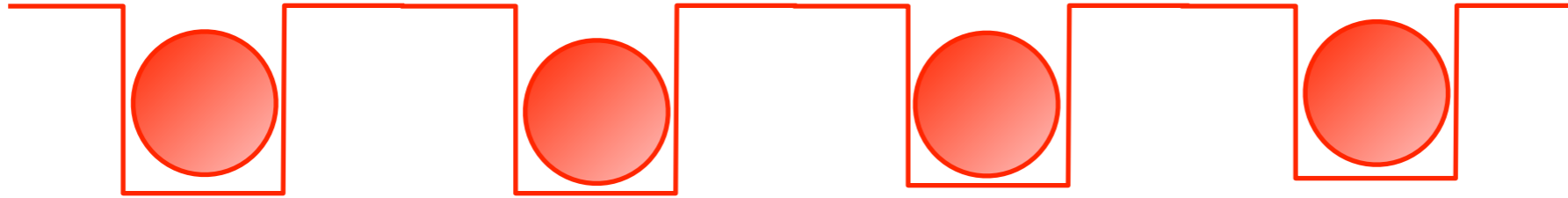
Holes $\sim \psi$

Superfluid-insulator transition



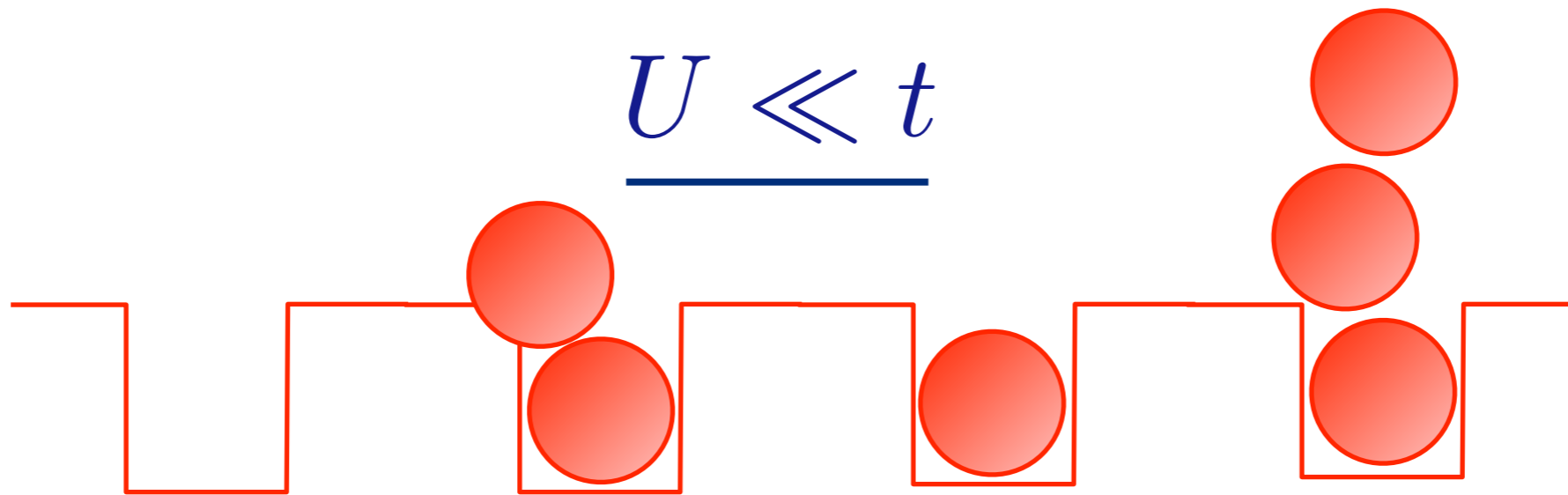
Ultracold ^{87}Rb
atoms - bosons

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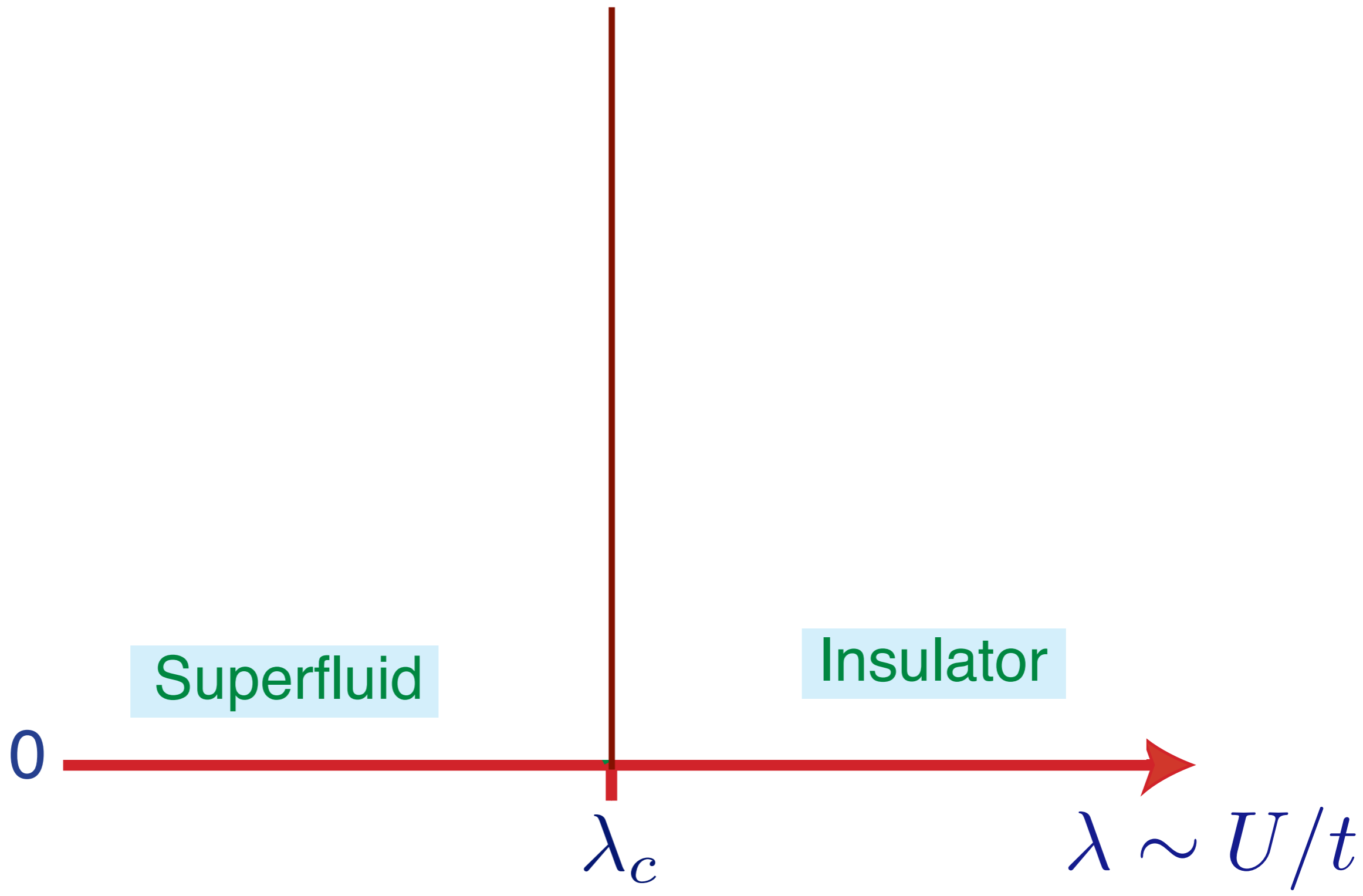
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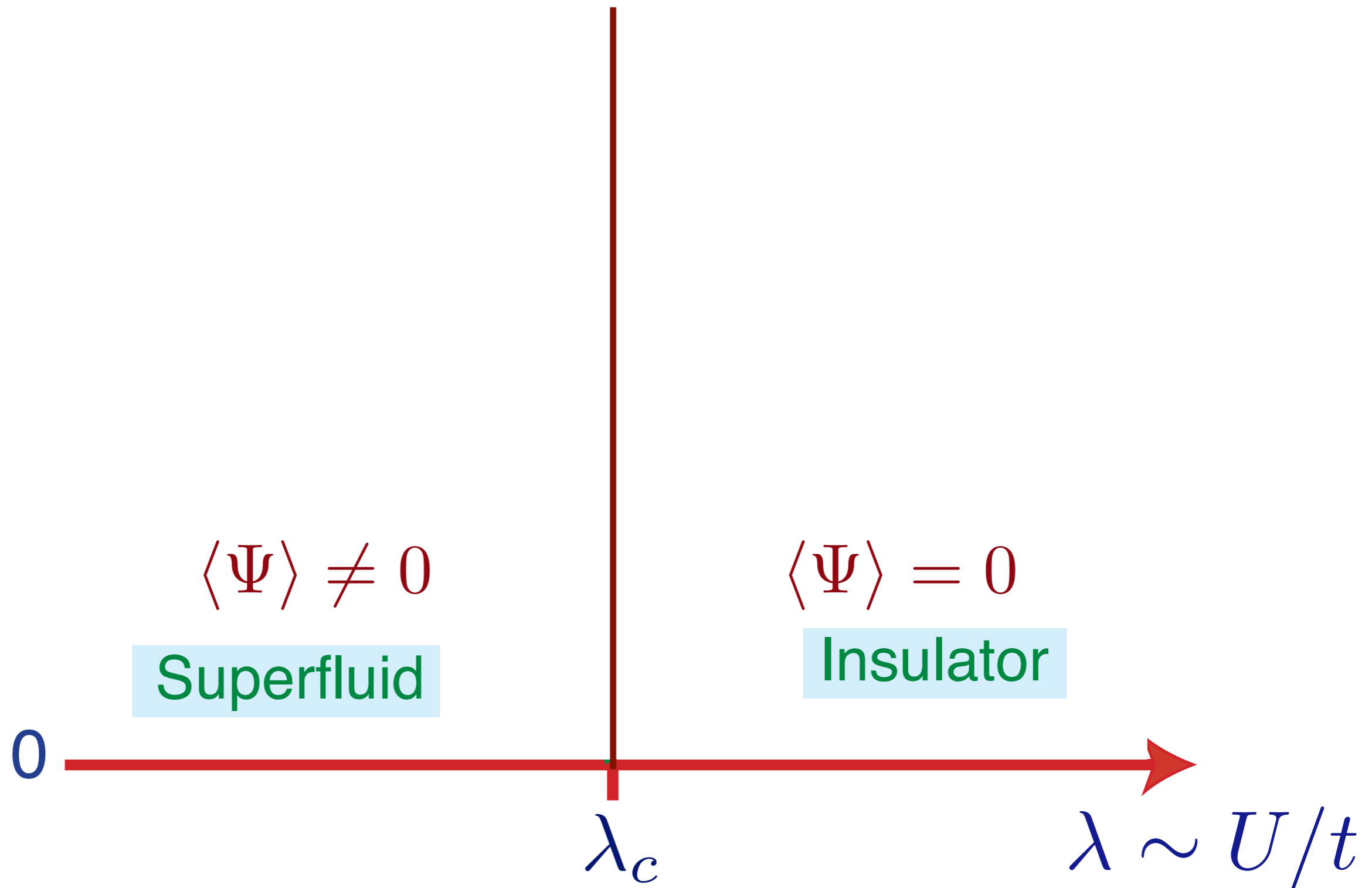


Superfluid
at small repulsion between bosons

$$|\text{Ground state}\rangle = \left[\sum_i b_i^\dagger \right]^N |0\rangle$$

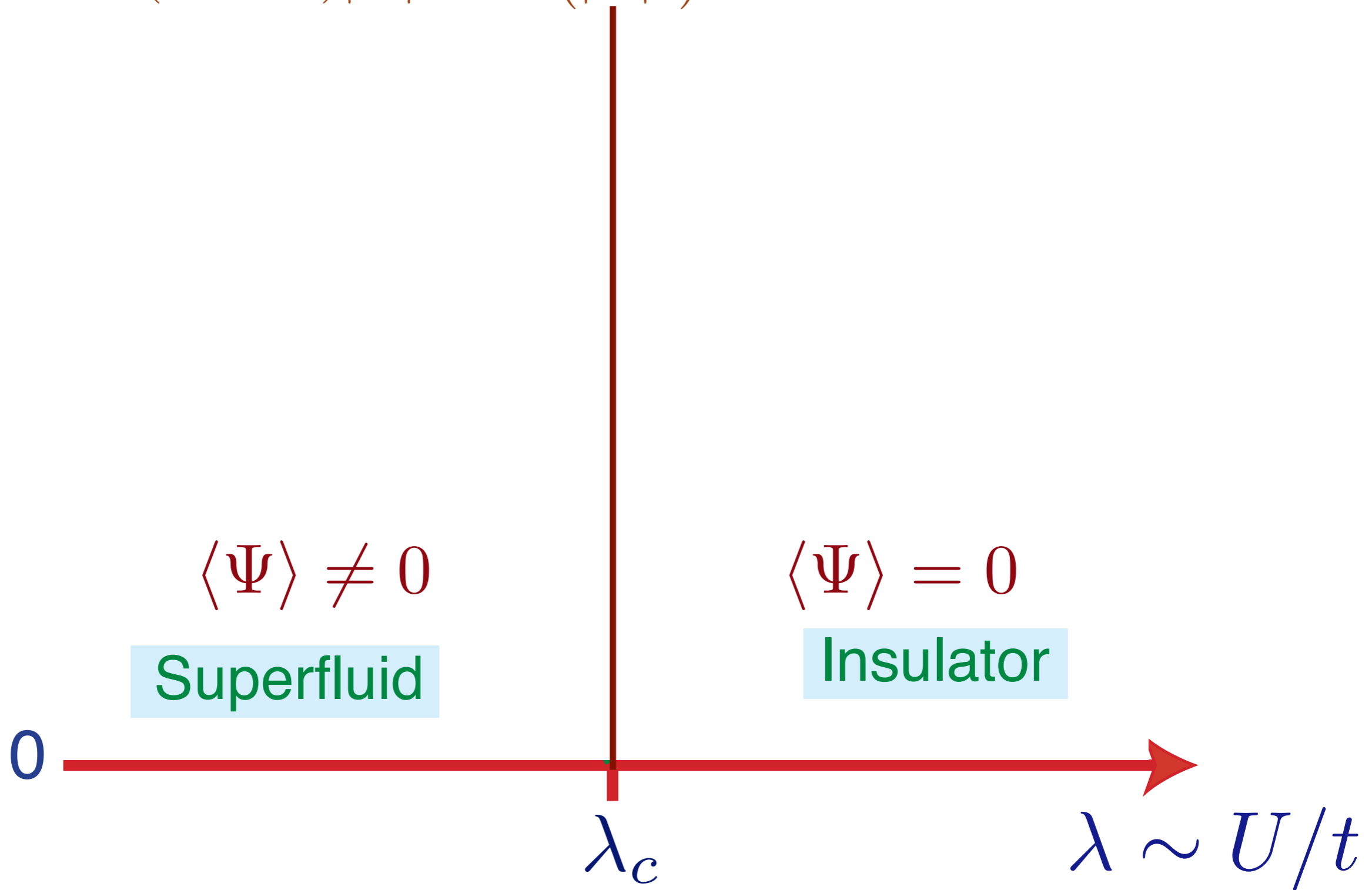


$\Psi \rightarrow$ a complex field representing the Bose-Einstein condensate of the superfluid



$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

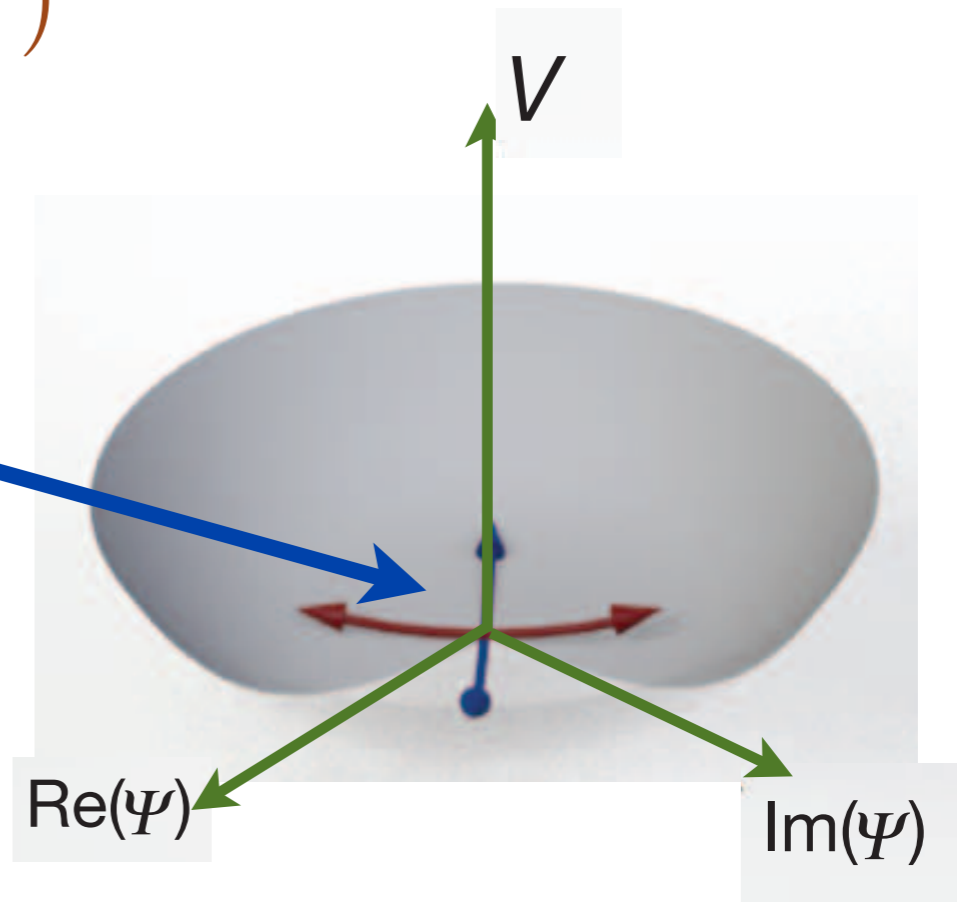
$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$



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Particles and holes correspond to the 2 normal modes in the oscillation of Ψ about $\Psi = 0$.



$$\langle \Psi \rangle \neq 0$$

Superfluid

$$\langle \Psi \rangle = 0$$

Insulator

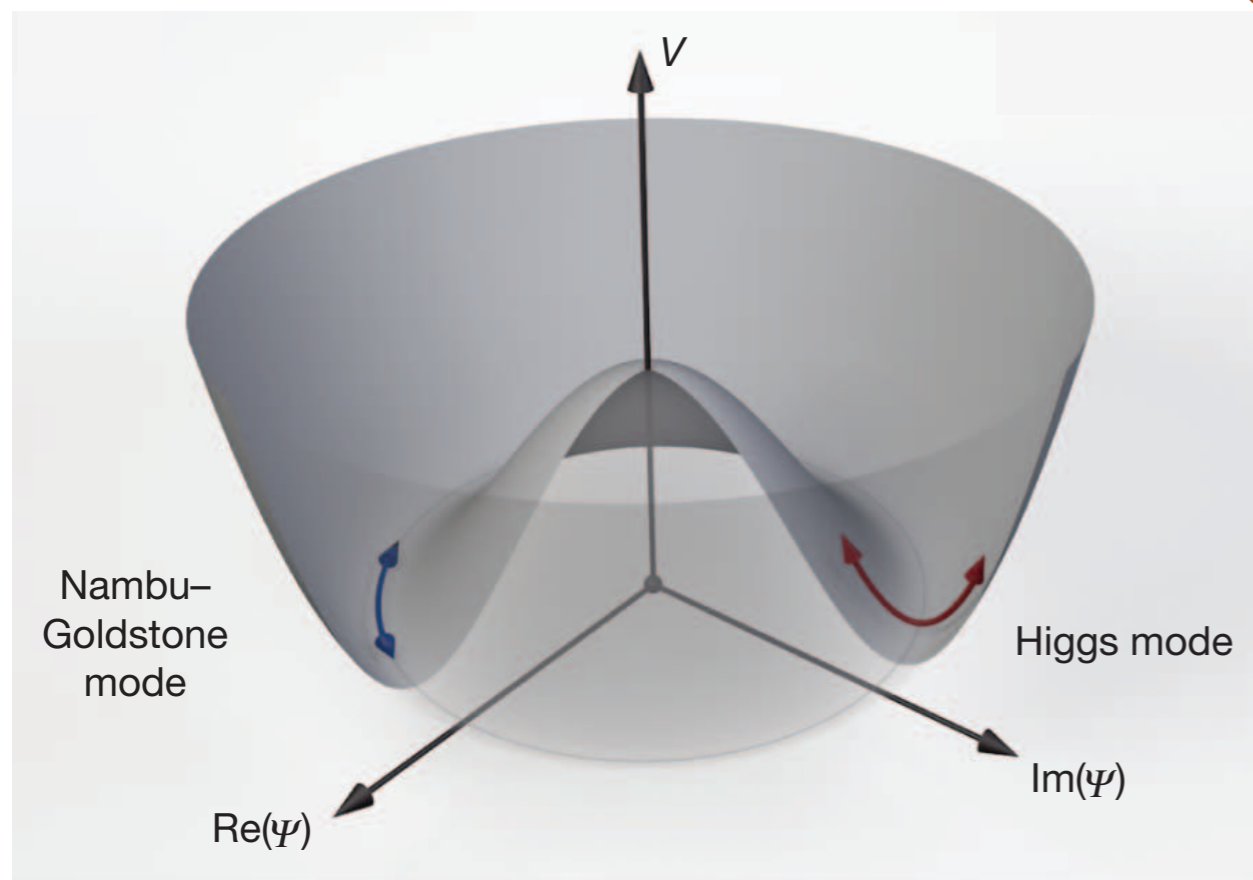
0

λ_c

$\lambda \sim U/t$

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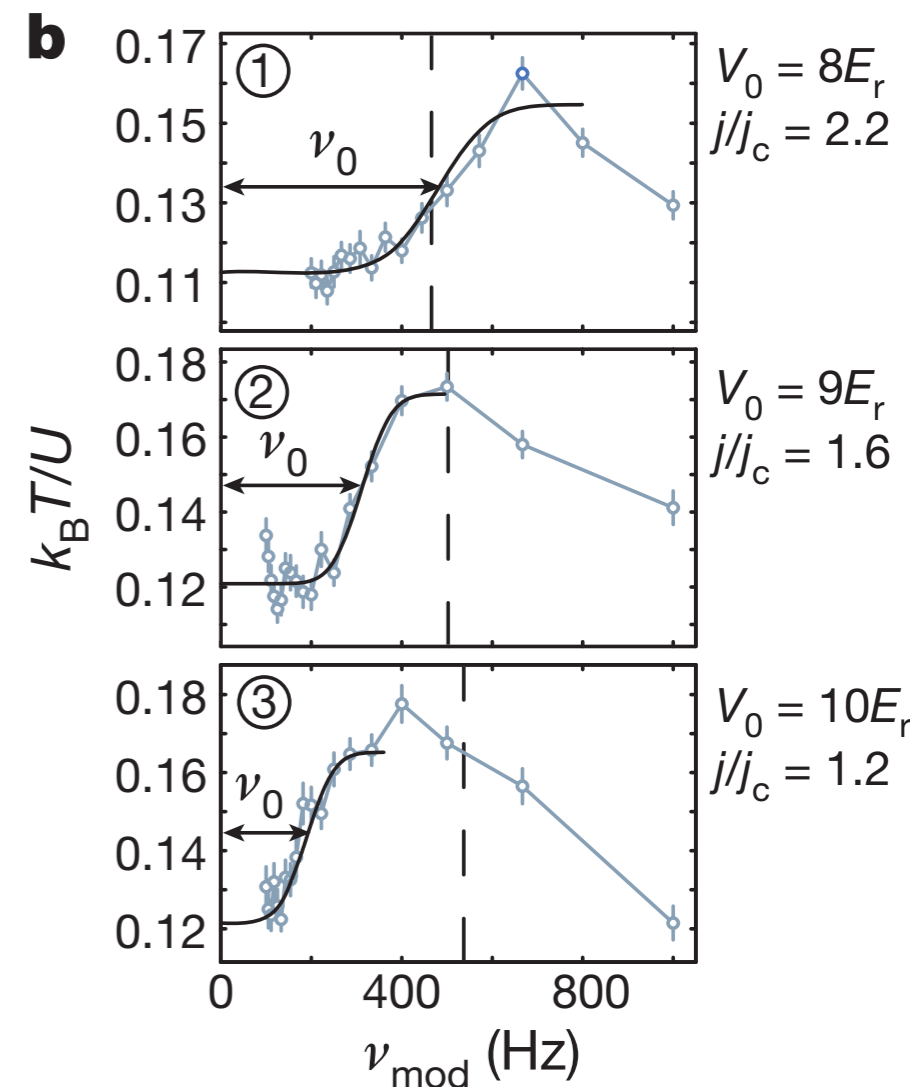
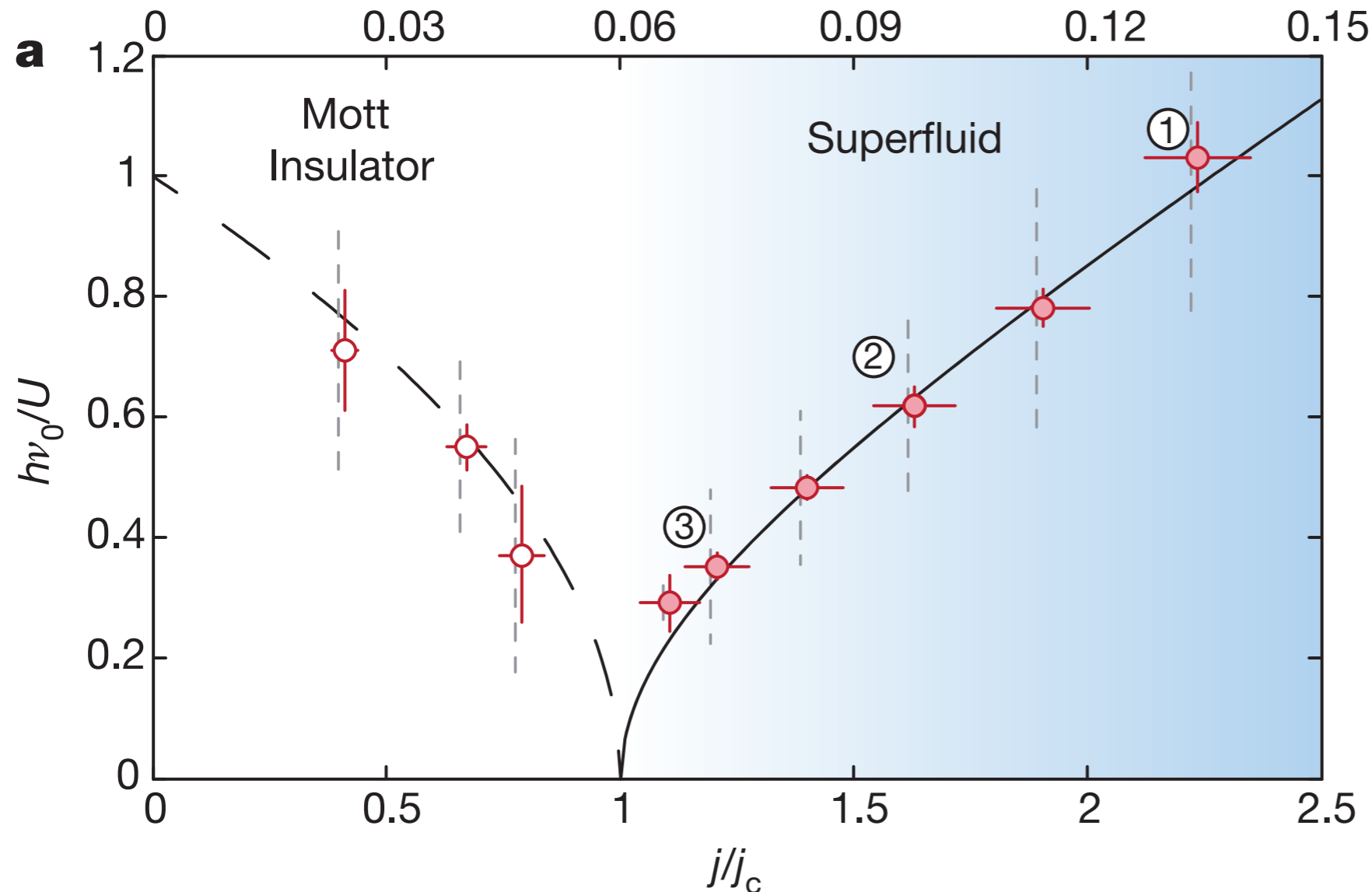
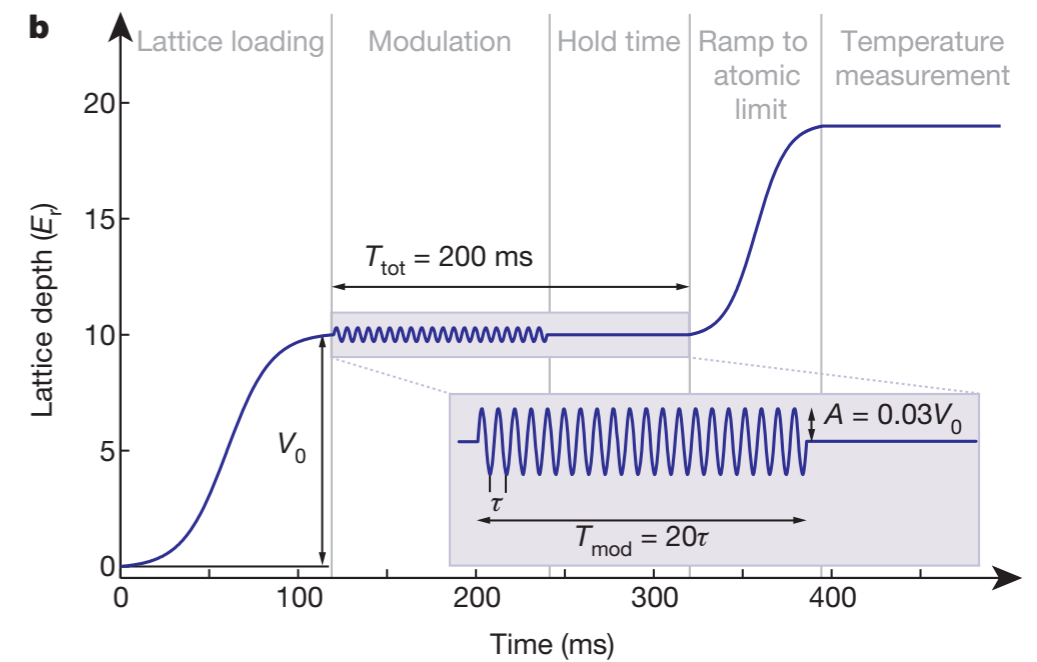
0

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$\lambda \sim U/t$

Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole

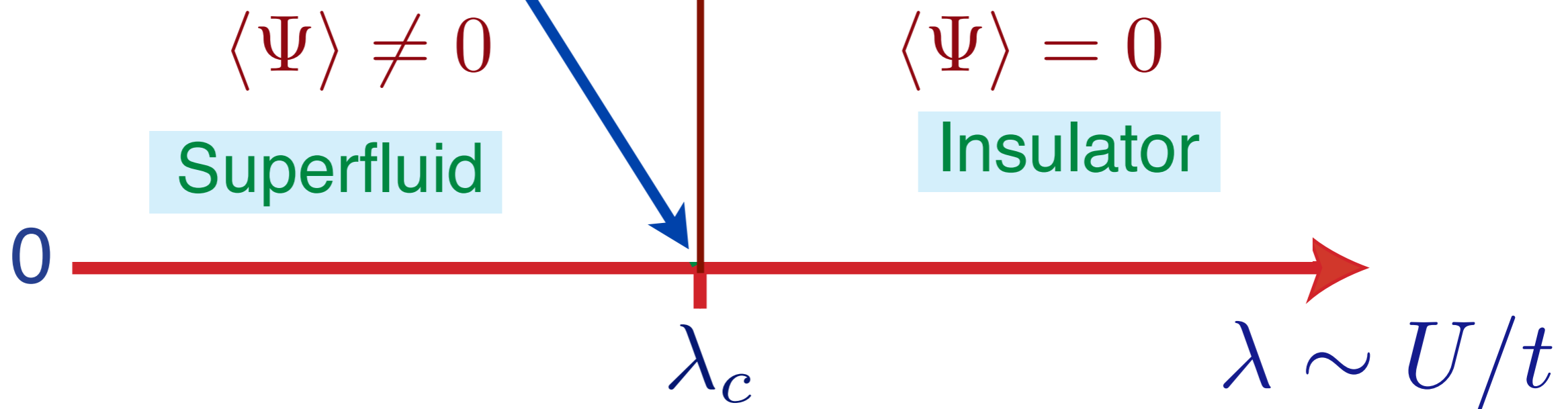


Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

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$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$

Quantum state with
complex, many-body,
“long-range” quantum entanglement



Characteristics of quantum critical point

- Long-range entanglement

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- No quasiparticles - no simple description of excitations.

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- The low energy excitations are described by a theory which has the same structure as Einstein's theory of special relativity, but with the spin-wave velocity playing the role of the velocity of light.

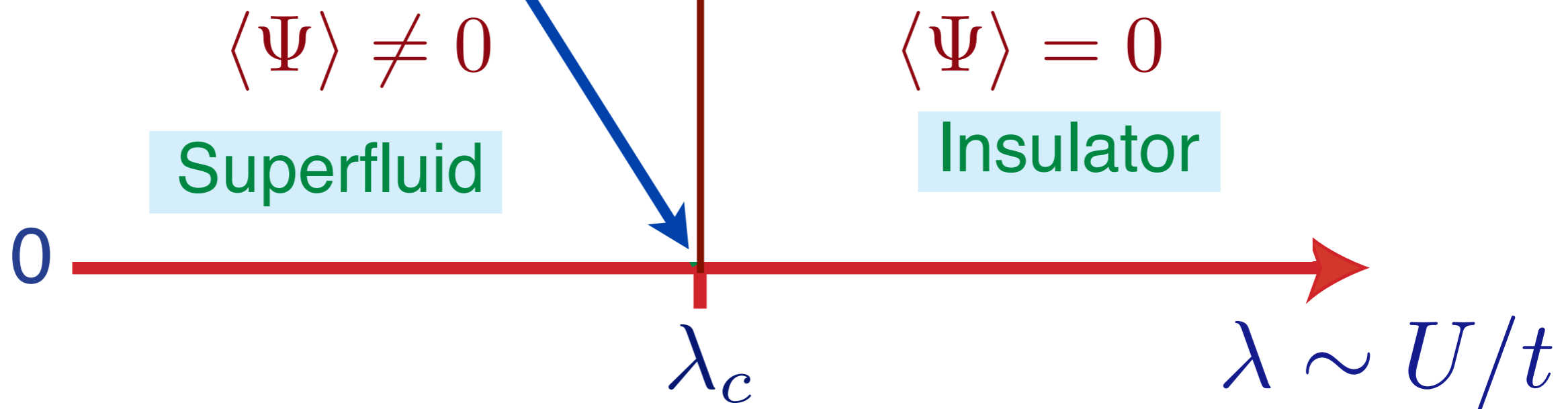
Characteristics of quantum critical point

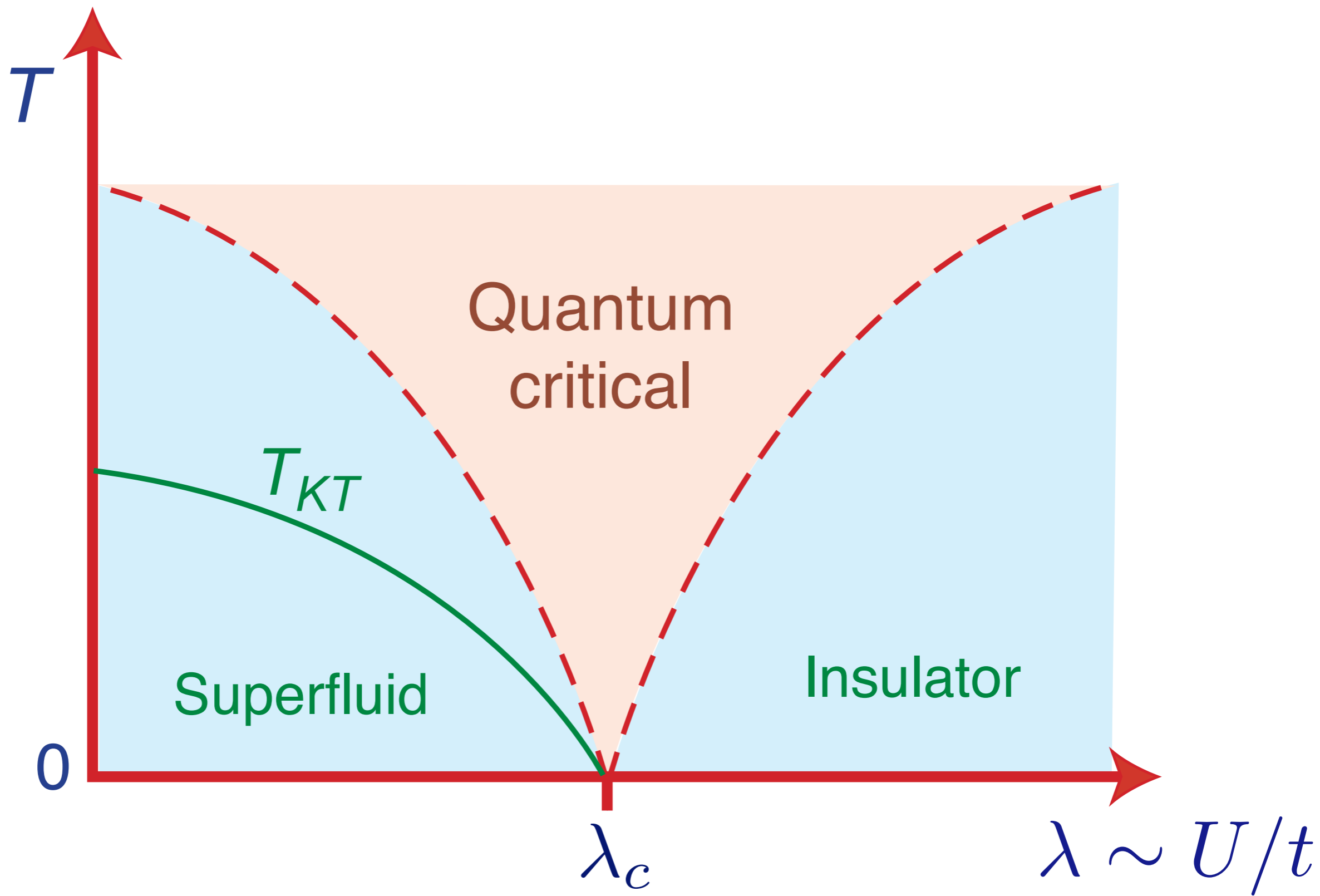
- Long-range entanglement
- No quasiparticles - no simple description of excitations.
- The low energy excitations are described by a theory which has the same structure as Einstein's theory of special relativity, but with the spin-wave velocity playing the role of the velocity of light.
- The theory of the critical point is strongly-coupled because the quartic-coupling u flows to a renormalization group fixed point (the Wilson-Fisher fixed point). This fixed point has an even larger symmetry corresponding to conformal transformations of spacetime: we refer to such a theory as a **CFT₃**

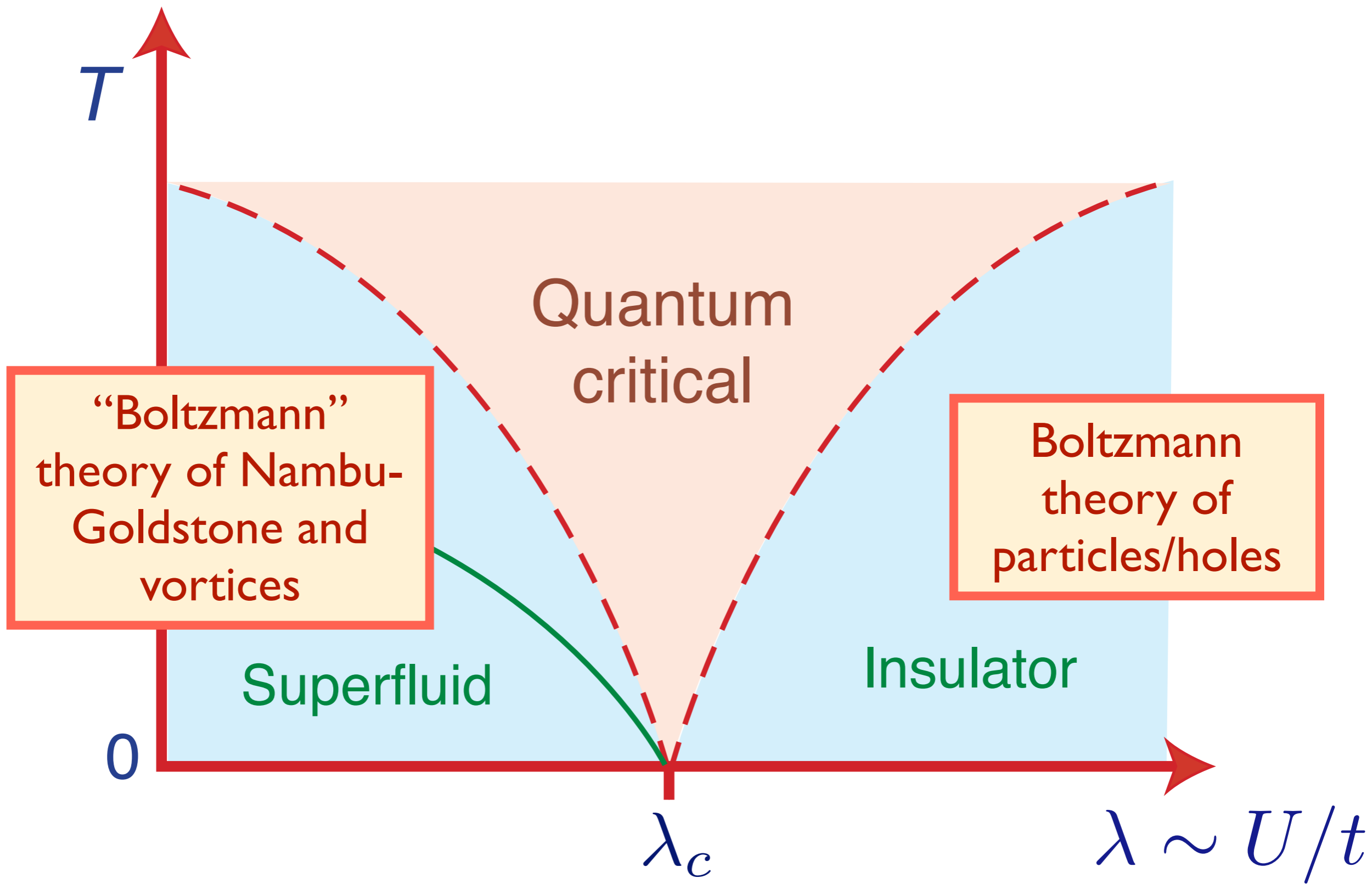
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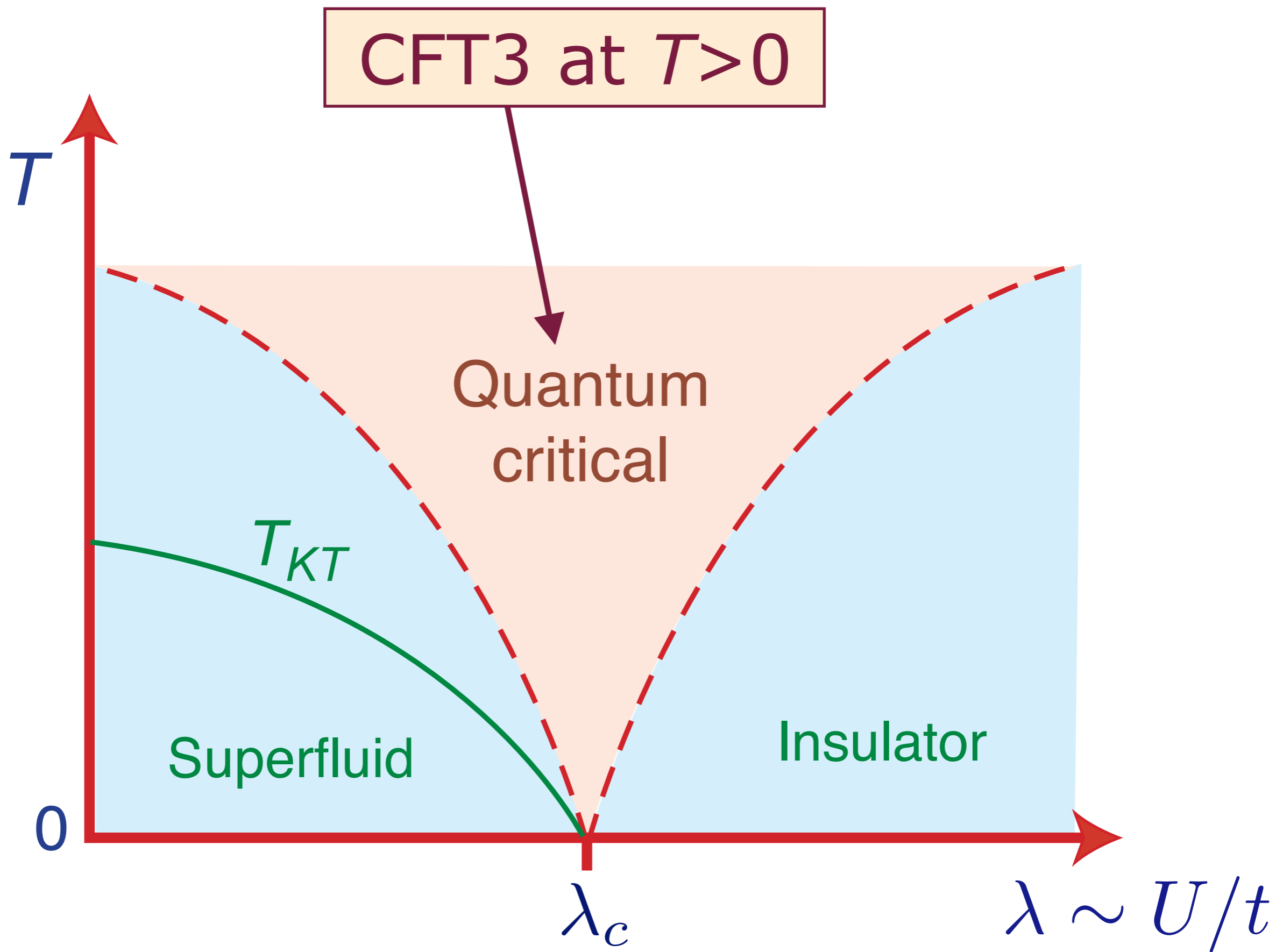
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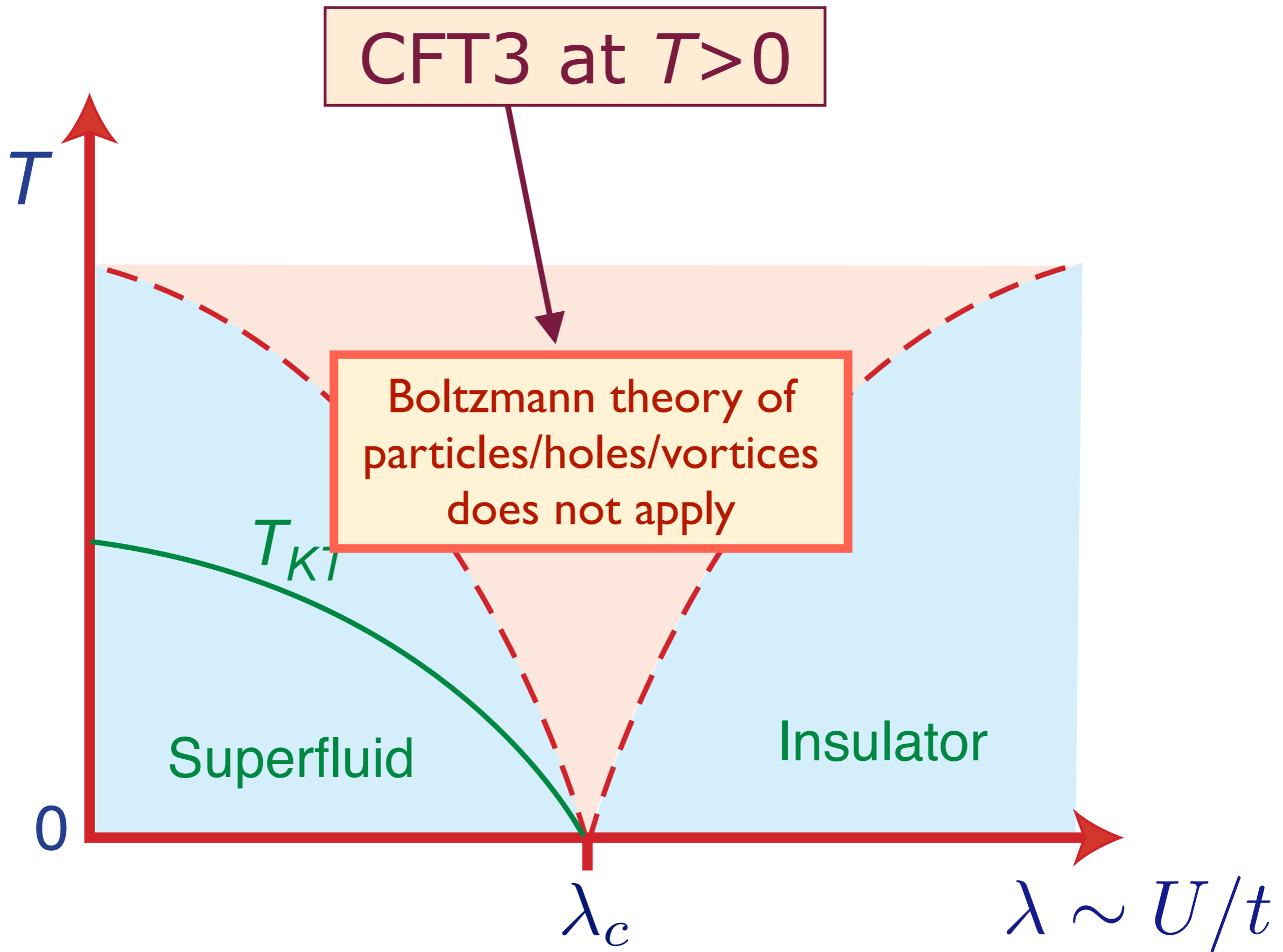
A conformal field theory
in 2+1 spacetime dimensions:
a CFT3

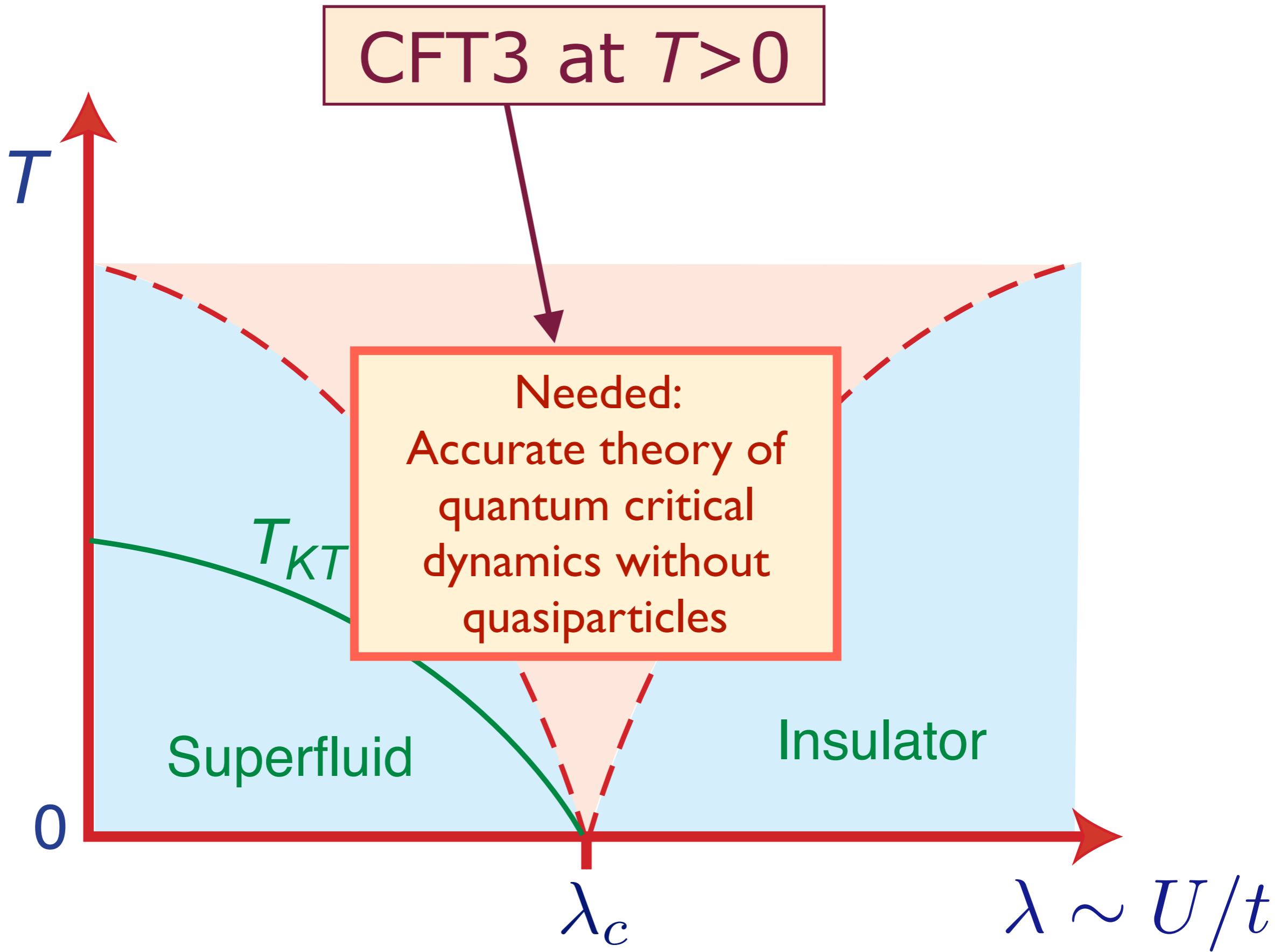




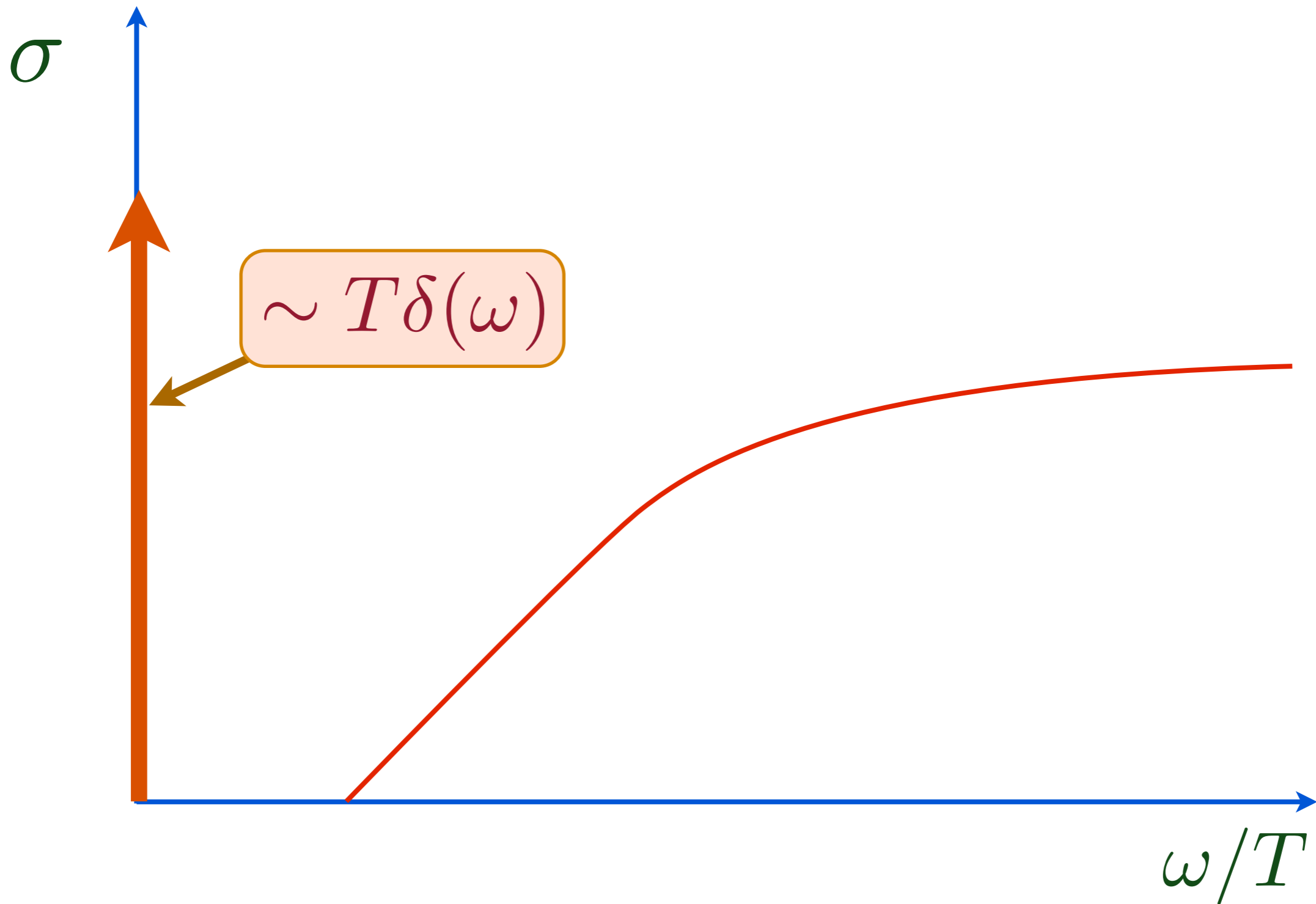




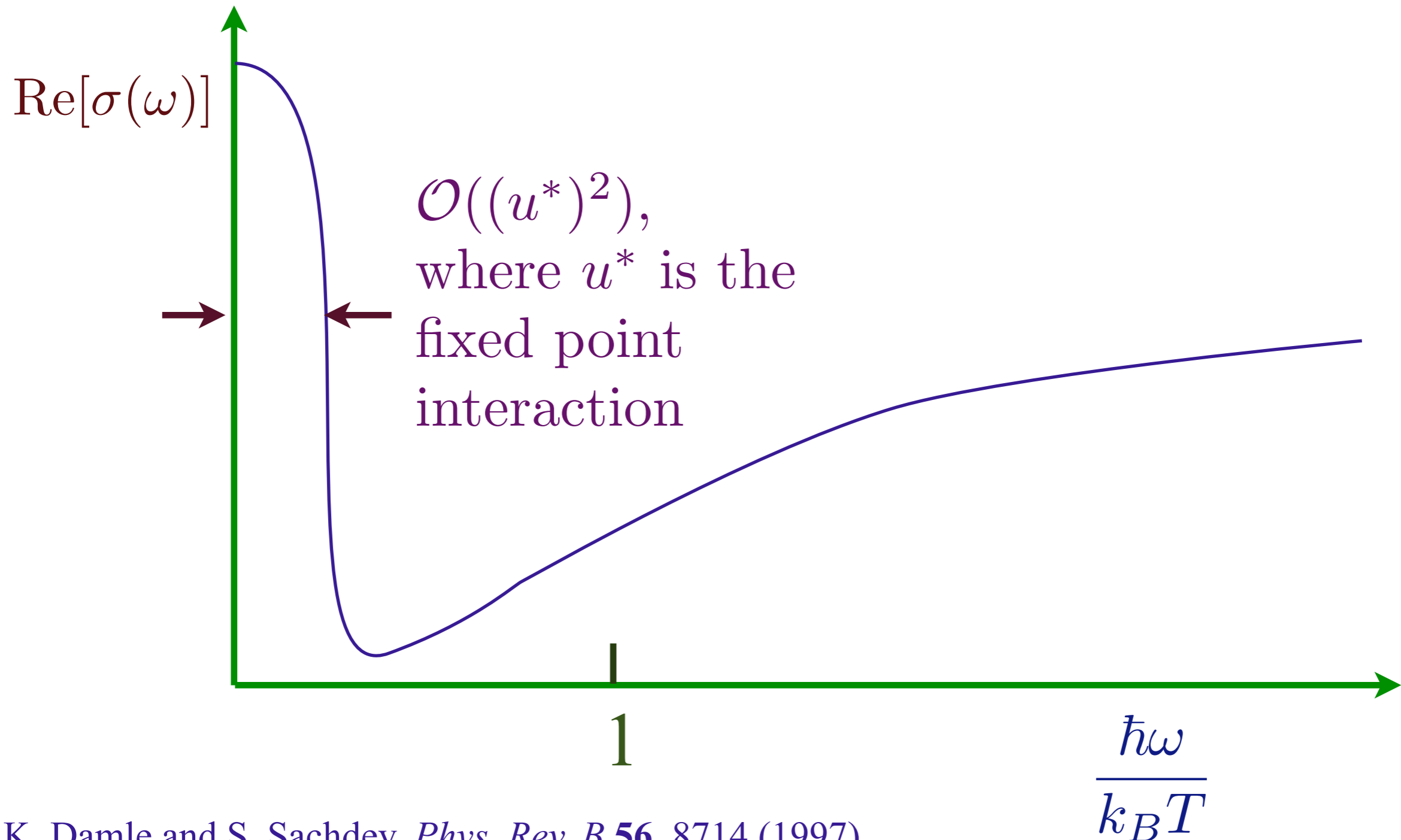




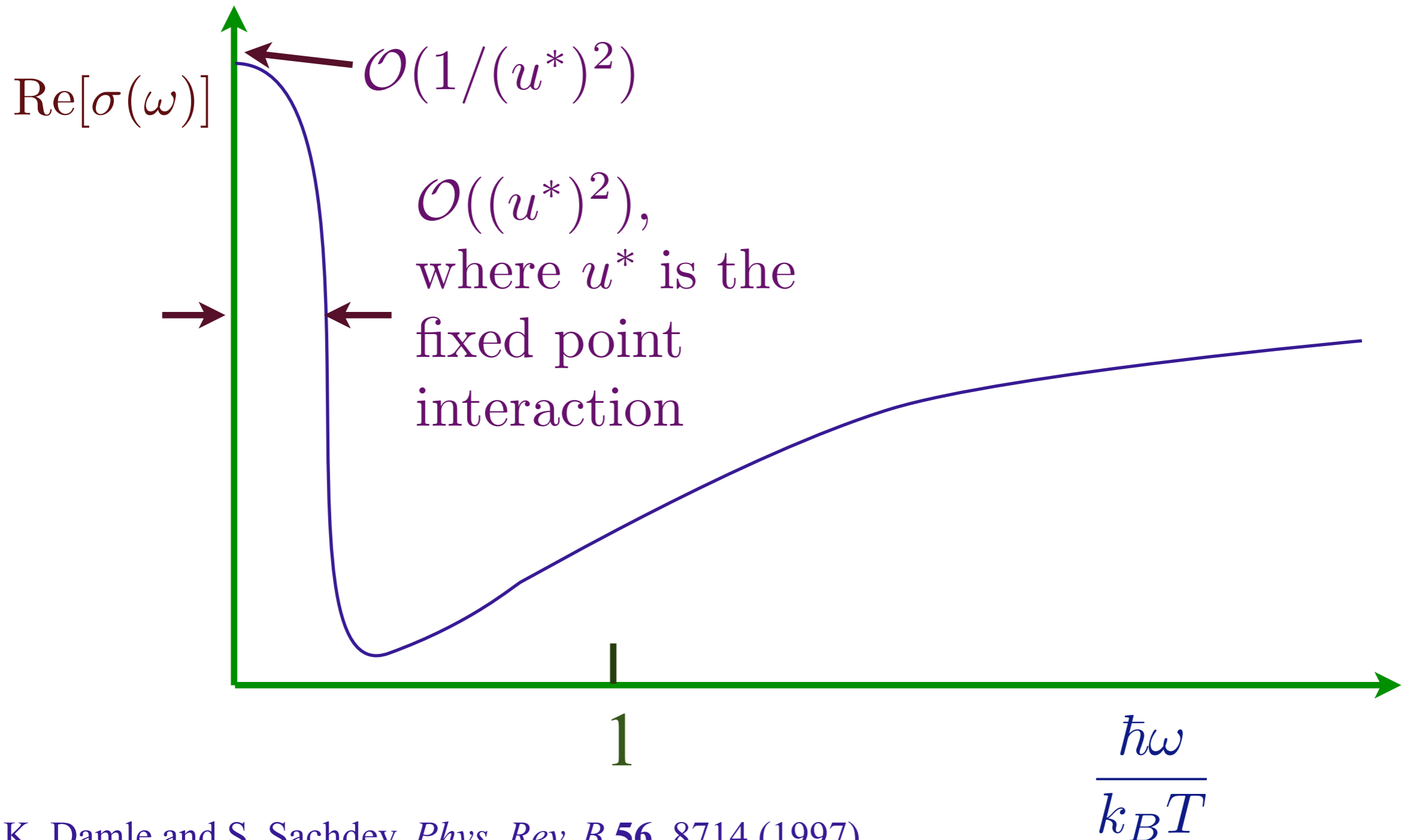
Electrical transport in a free quasiparticle CFT3 for $T > 0$



Quasiparticle view of quantum criticality (Boltzmann equation):
Electrical transport for a (weakly) interacting CFT3



Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3

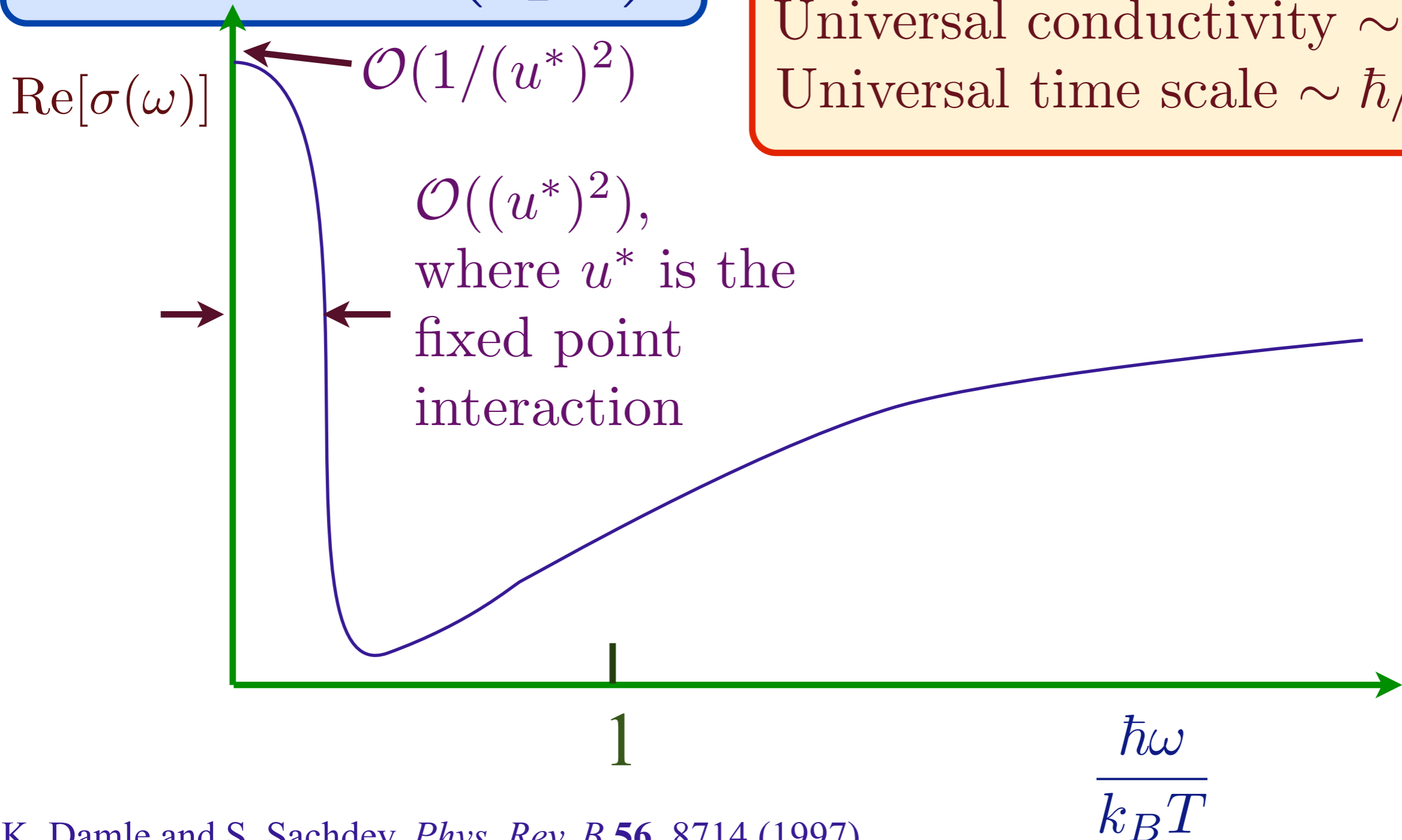


Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h} \Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

$\Sigma \rightarrow$ a universal function

Universal conductivity $\sim e^2/h$
Universal time scale $\sim \hbar/k_B T$



Quasiparticle view of quantum criticality (Boltzmann equation):
Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h} \Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

$\Sigma \rightarrow$ a universal function
 Universal conductivity $\sim e^2/h$
 Universal time scale $\sim \hbar/k_B T$

Re[$\sigma(\omega)$]

$\mathcal{O}(1/(u^*)^2)$

$\mathcal{O}((u^*)^2),$

v
f
i

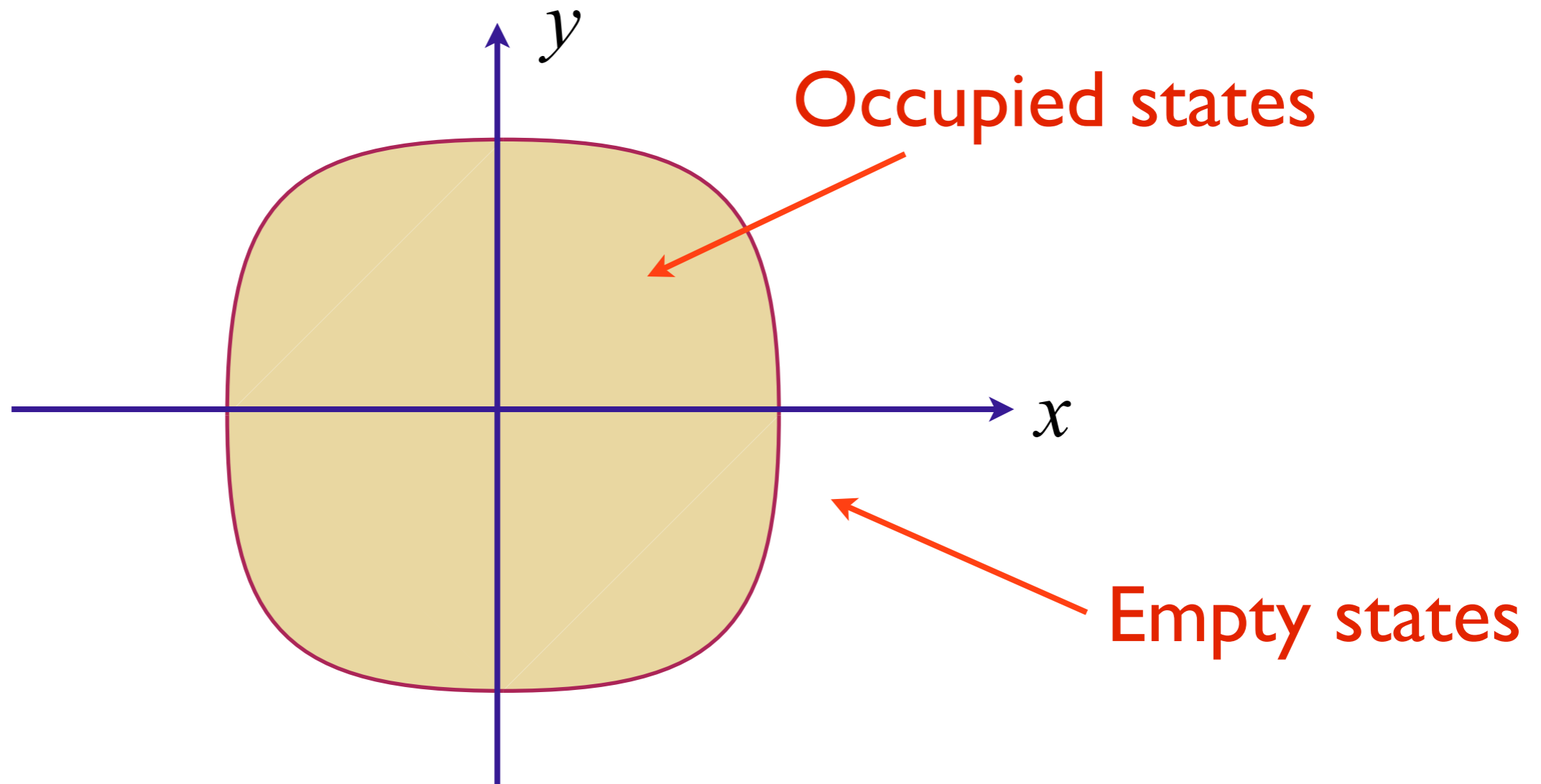
Needed:
 a method for computing
 the universal conductivity
 of strongly interacting CFT3s

1

$\frac{\hbar\omega}{k_B T}$

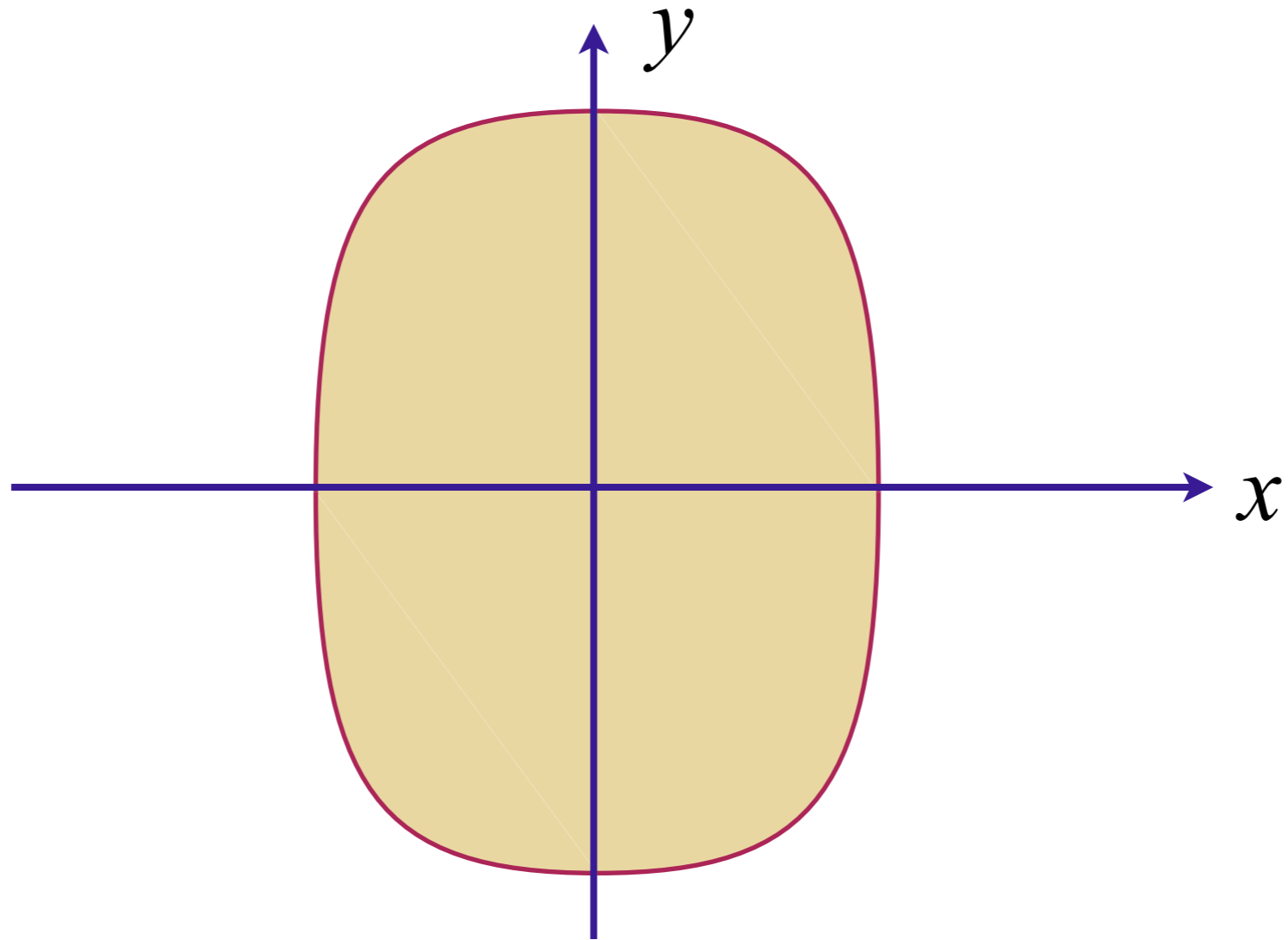
1. Coupled ladder antiferromagnet
2. Superfluid-insulator transition of bosons
3. Ising-nematic criticality in metals

Quantum criticality of Ising-nematic ordering in a metal



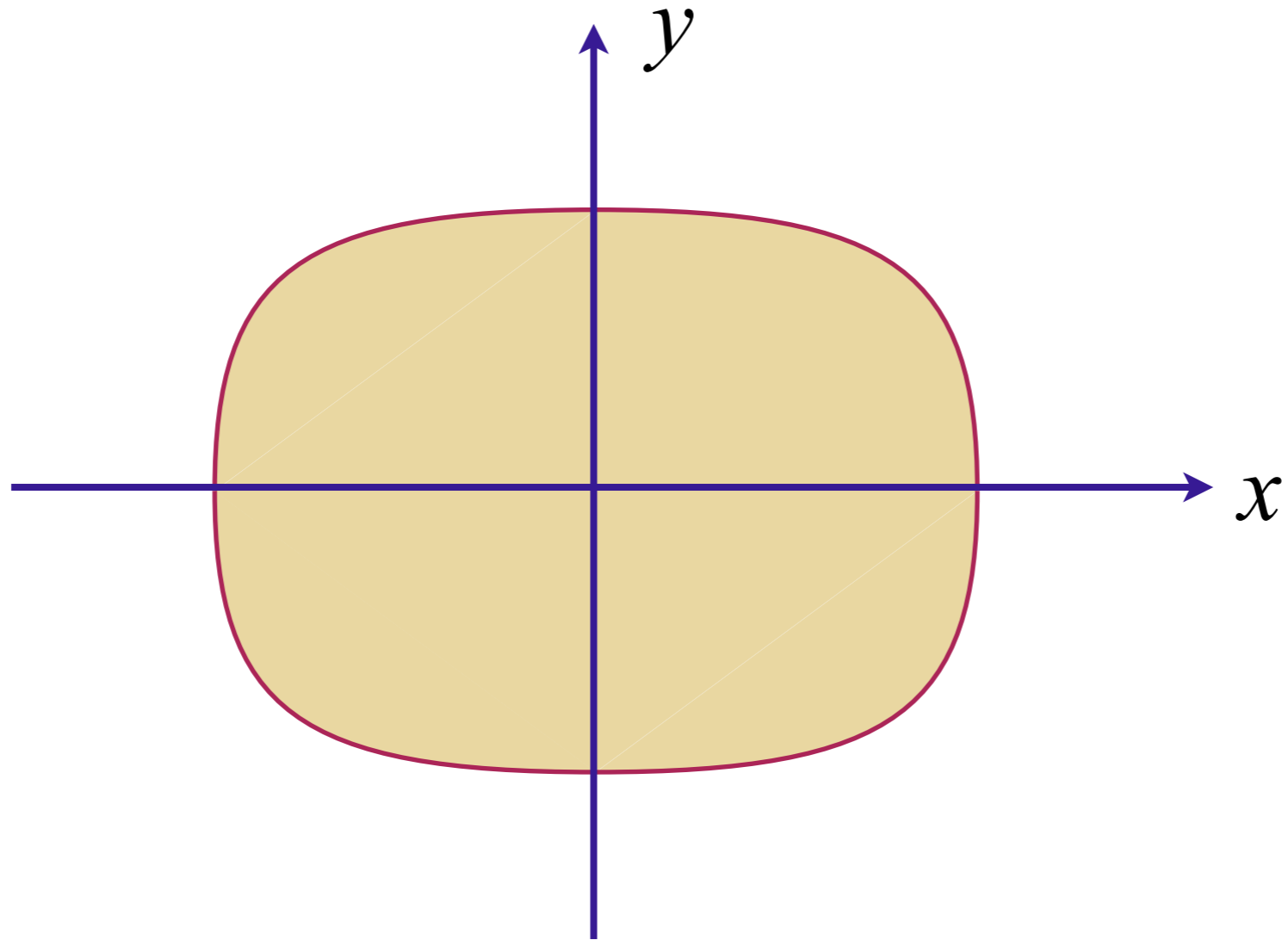
A metal with a Fermi surface
with full square lattice symmetry

Quantum criticality of Ising-nematic ordering in a metal



Spontaneous elongation along y direction:

Quantum criticality of Ising-nematic ordering in a metal



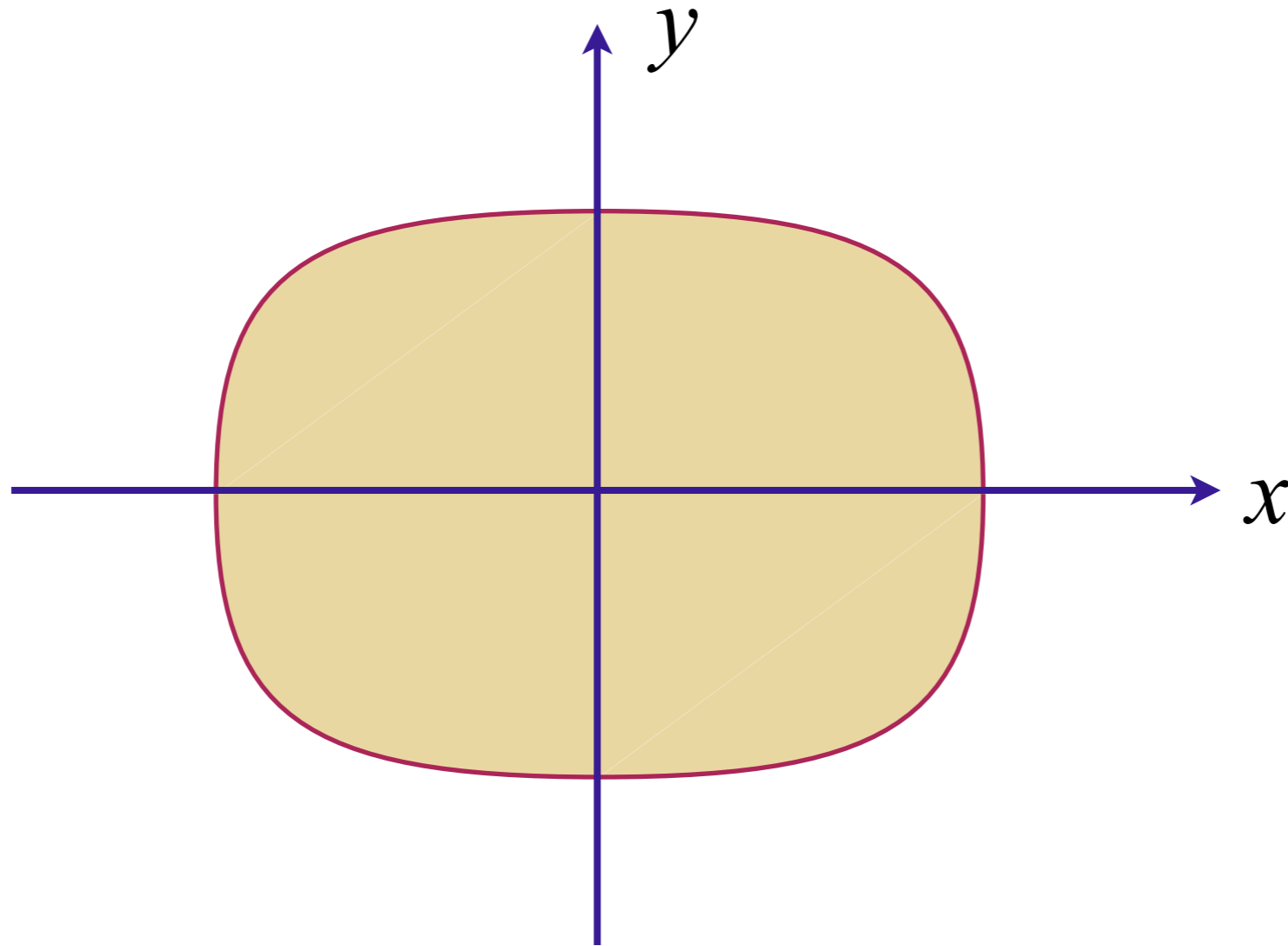
Spontaneous elongation along x direction:

Ising-nematic order parameter

$$\phi \sim \int d^2 k (\cos k_x - \cos k_y) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}$$

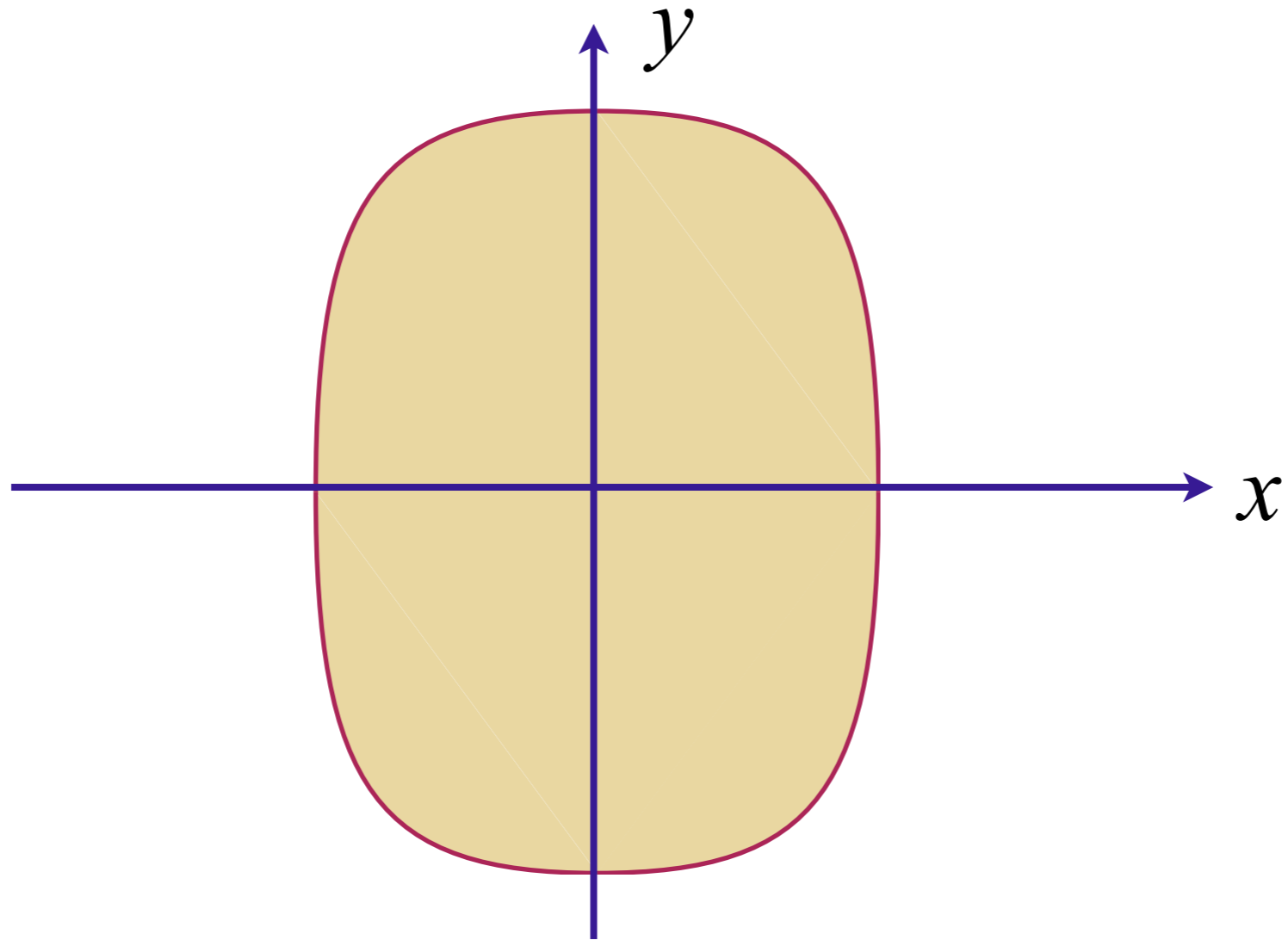
Measures spontaneous breaking of square lattice point-group symmetry of underlying Hamiltonian

Quantum criticality of Ising-nematic ordering in a metal



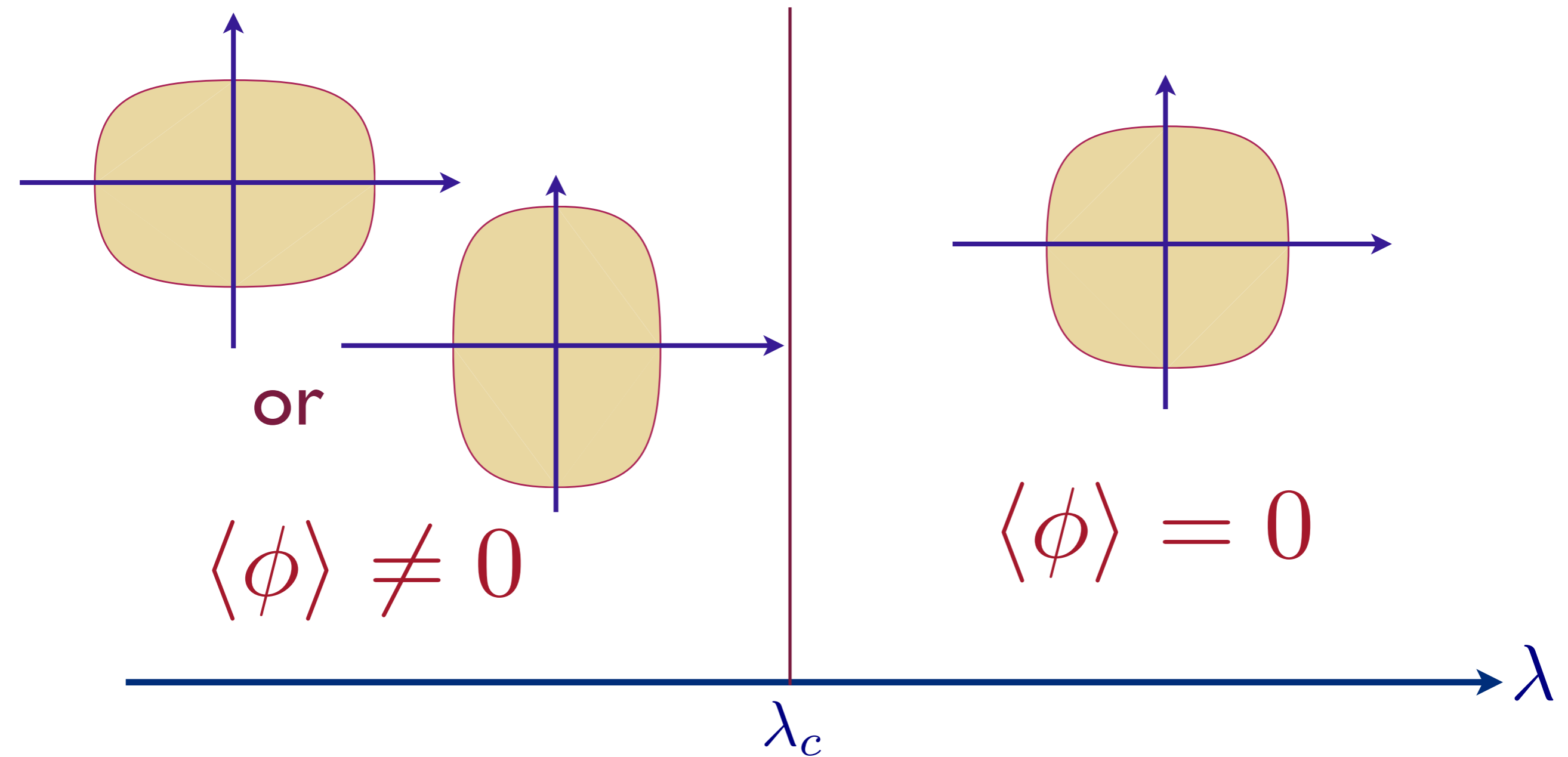
Spontaneous elongation along x direction:
Ising order parameter $\phi > 0$.

Quantum criticality of Ising-nematic ordering in a metal



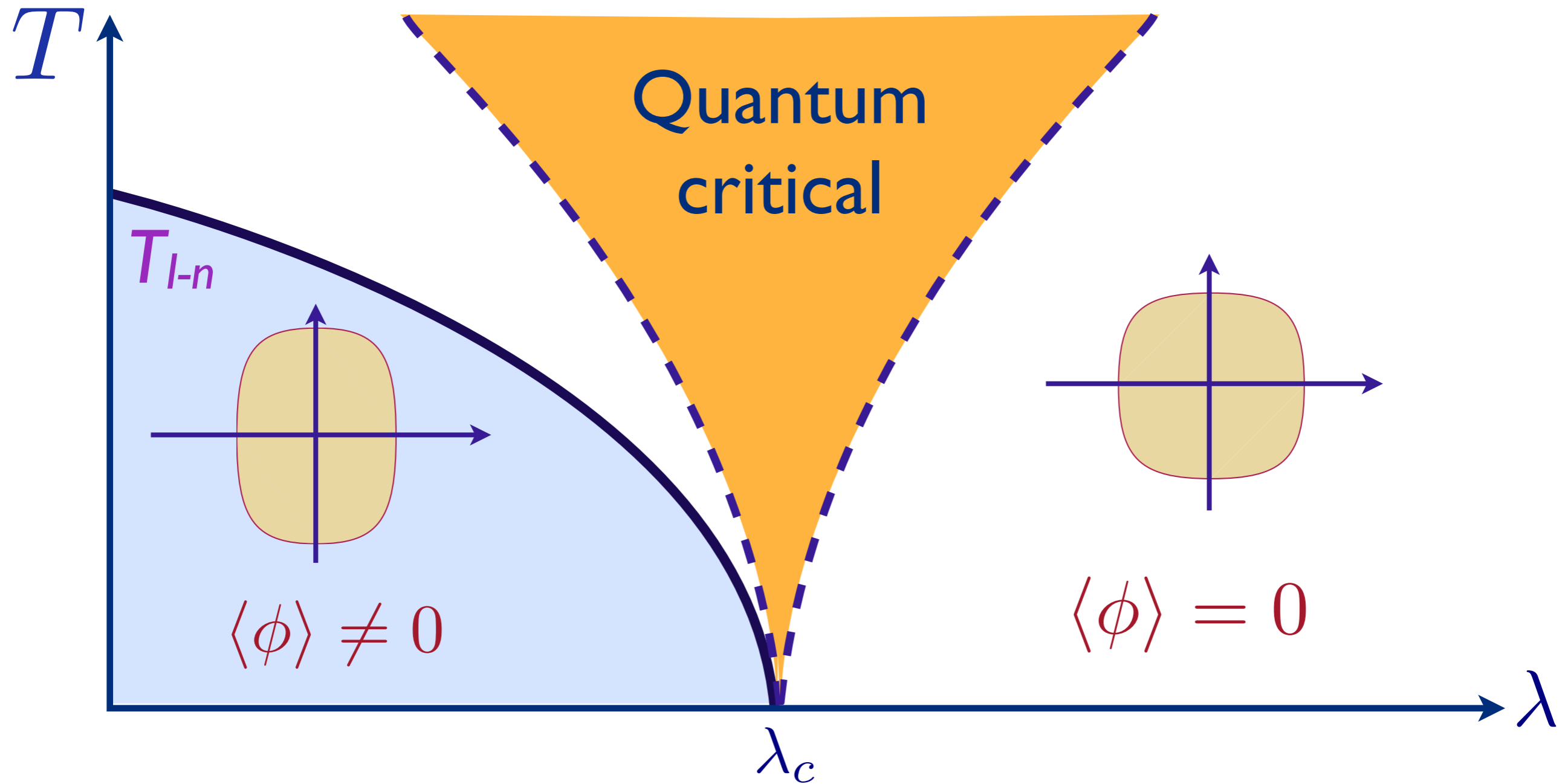
Spontaneous elongation along y direction:
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Quantum criticality of Ising-nematic ordering in a metal



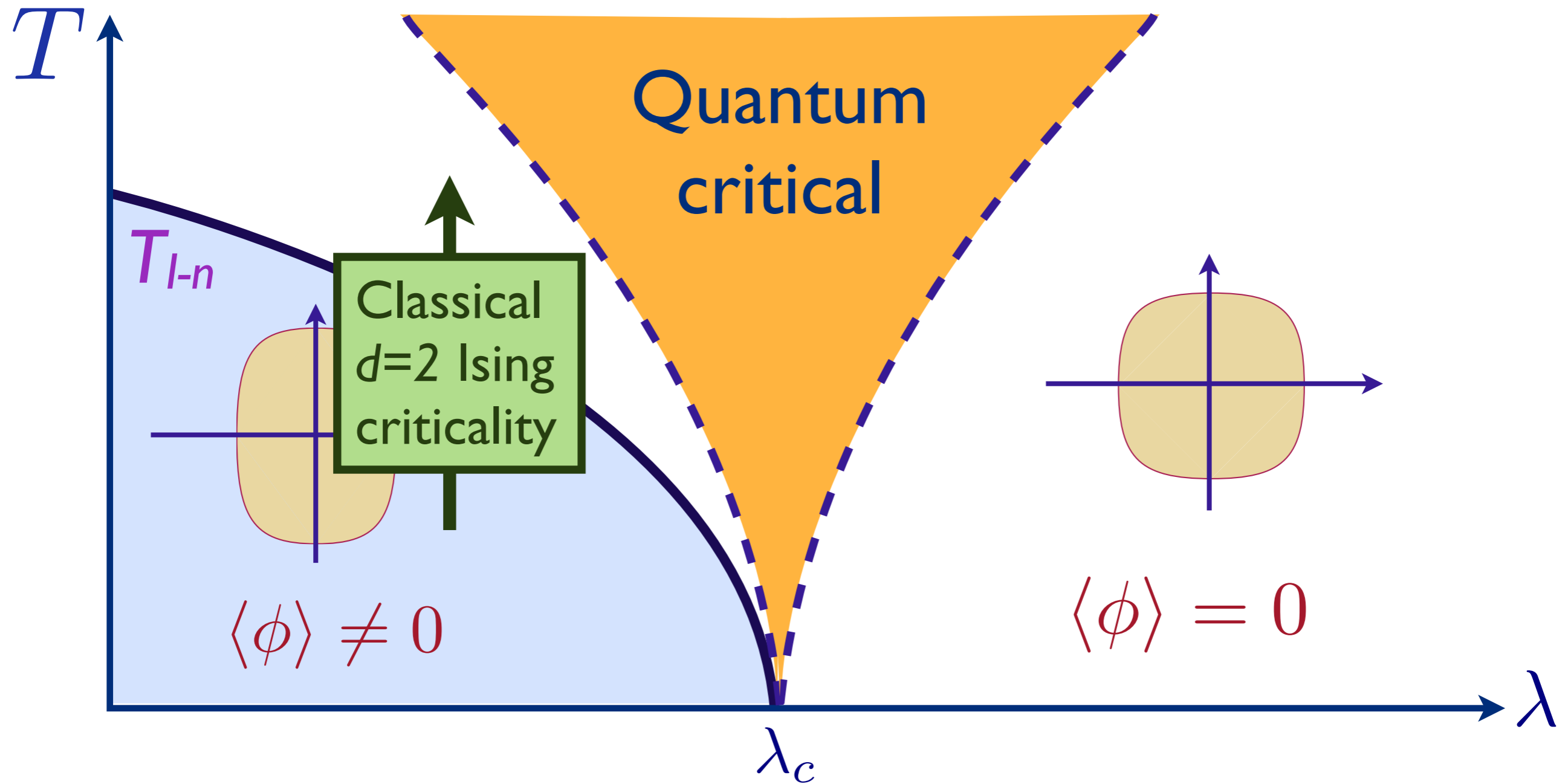
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal



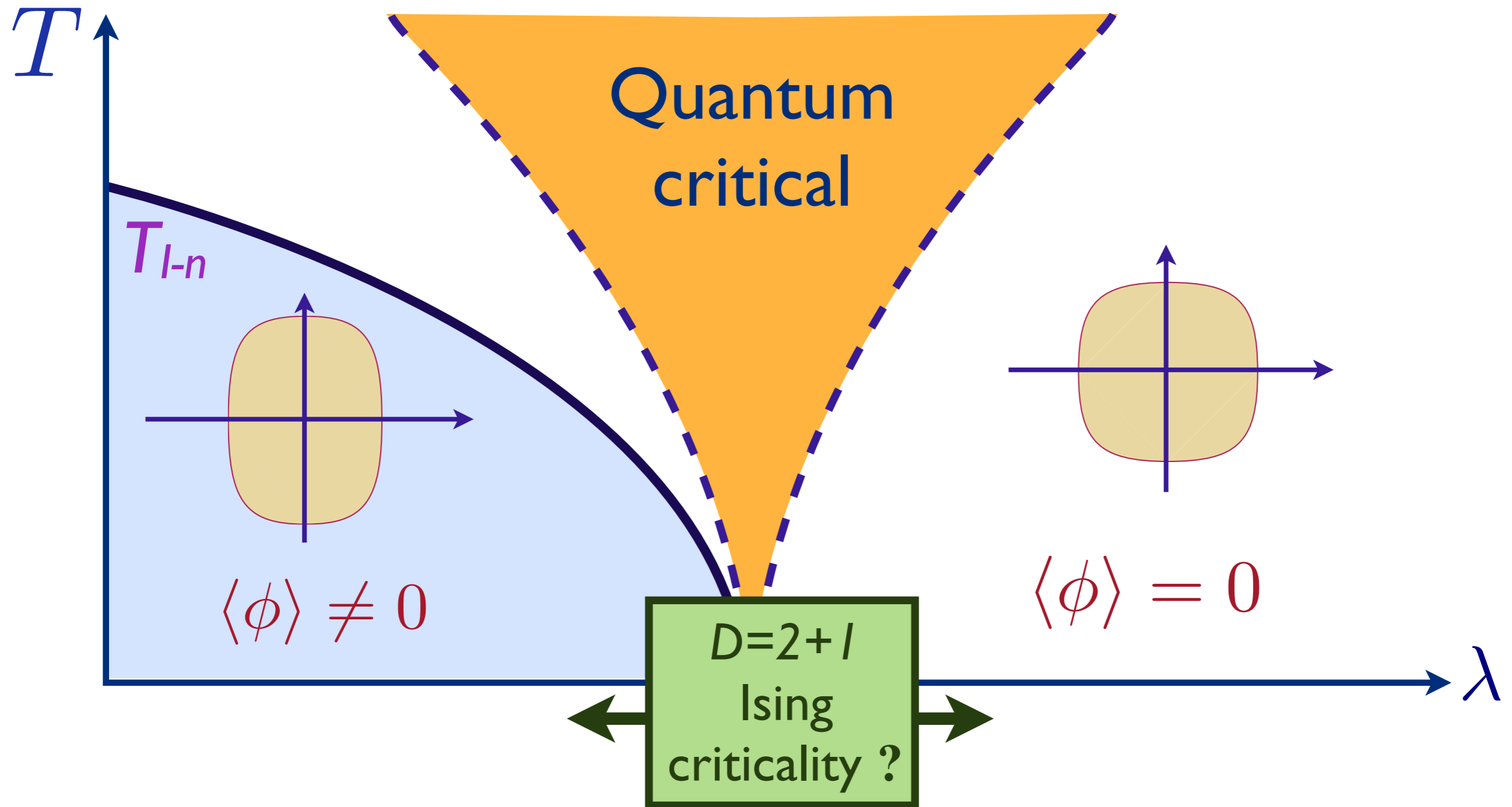
Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



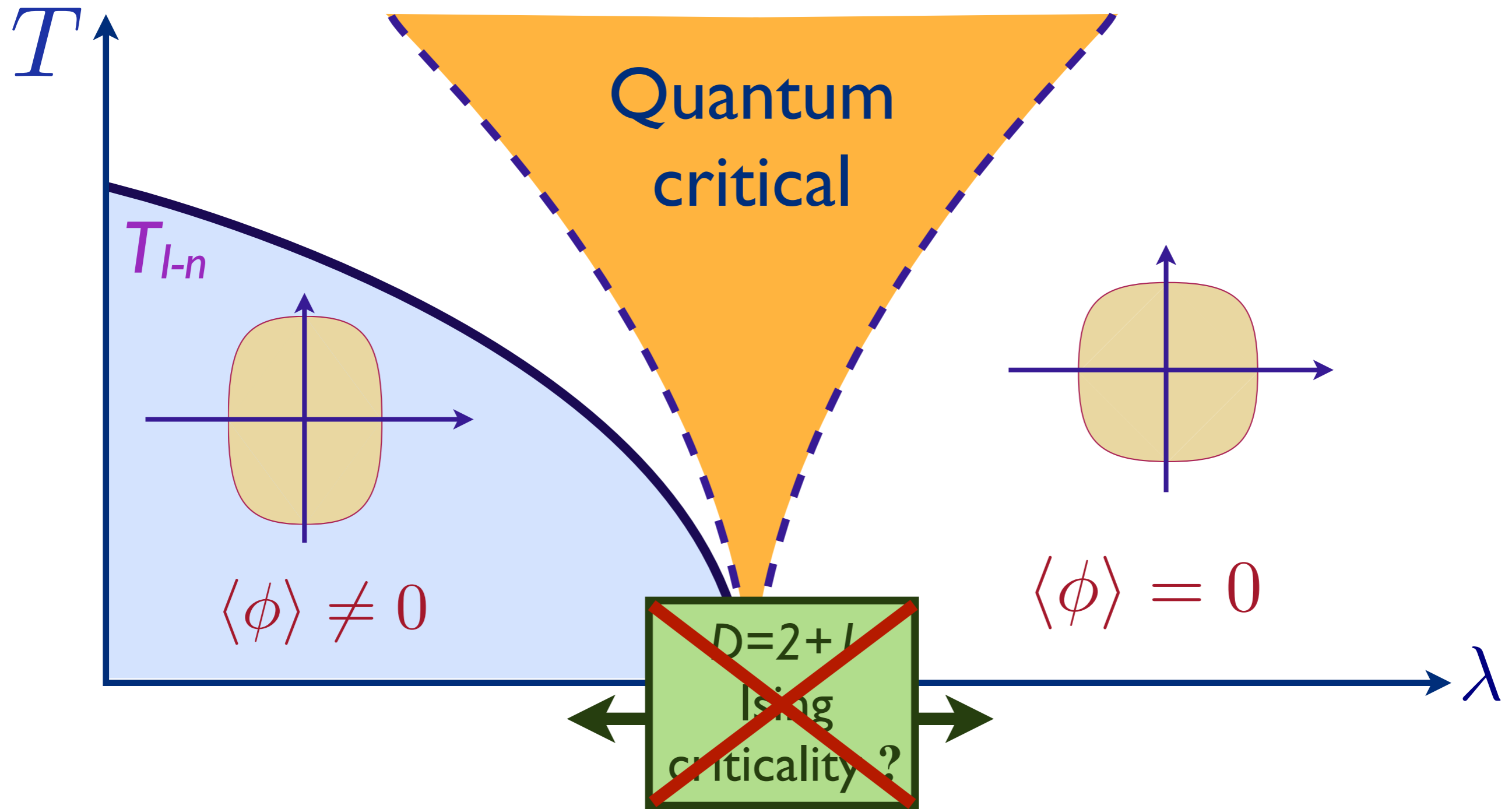
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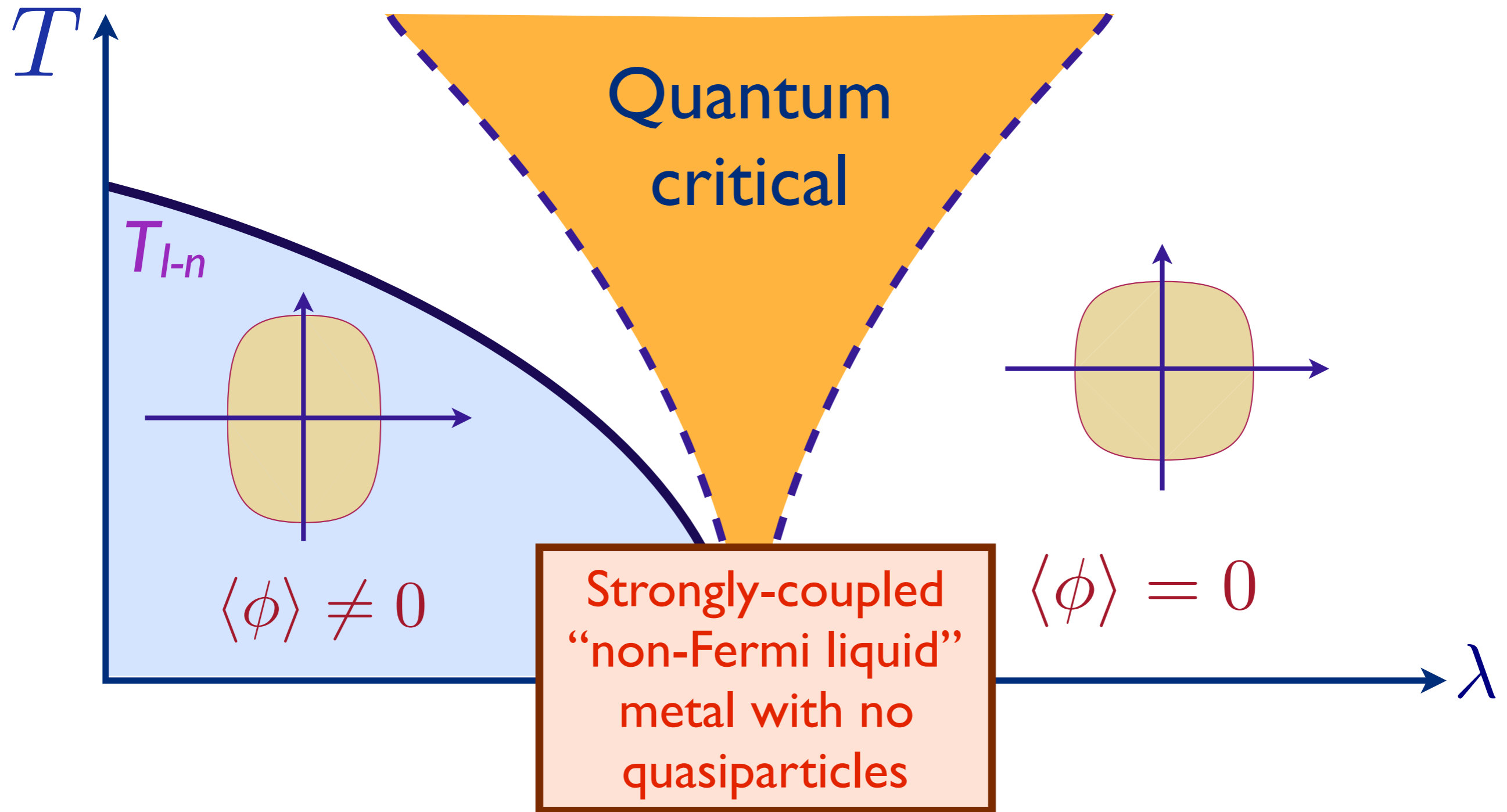
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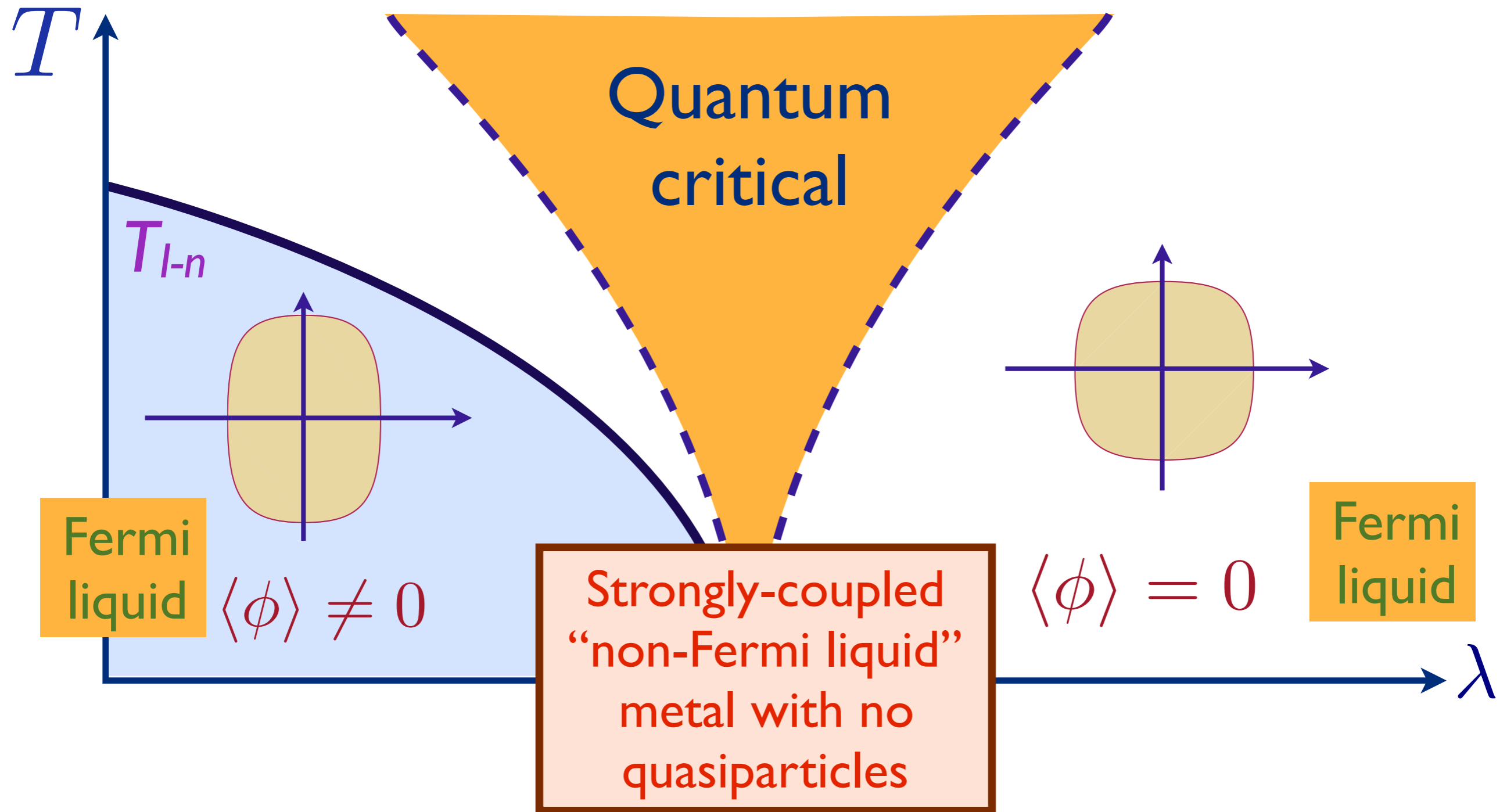
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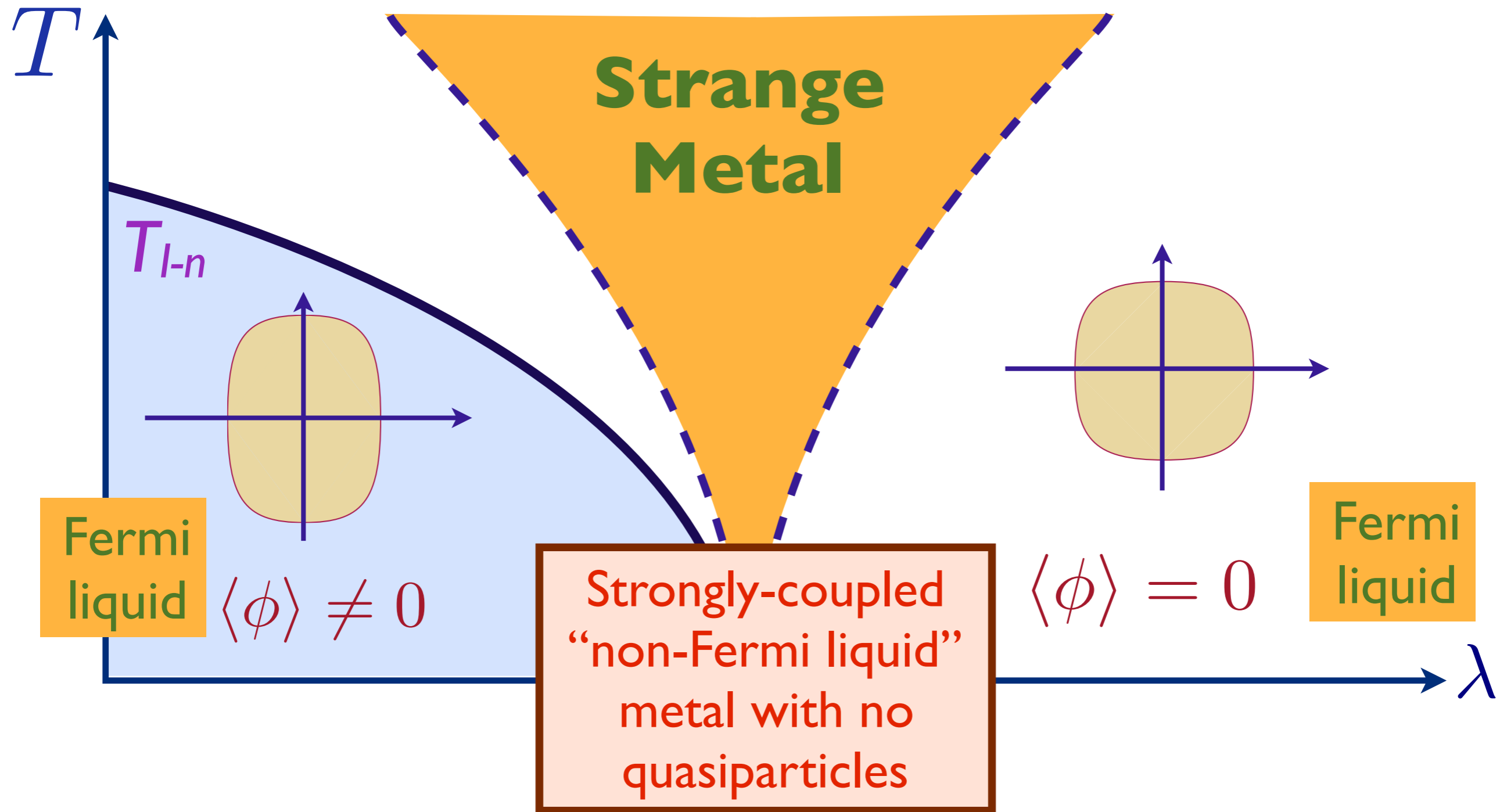
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Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

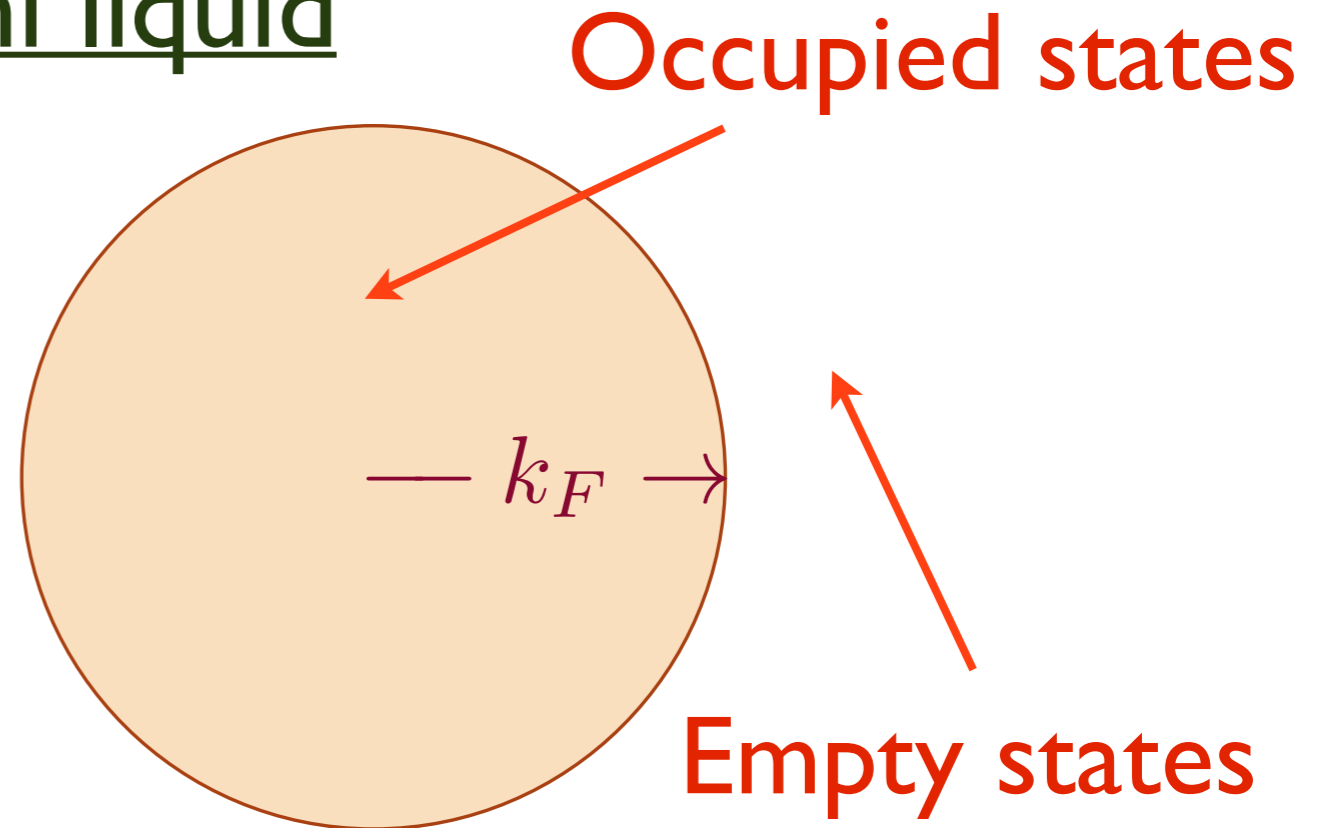
Quantum criticality of Ising-nematic ordering in a metal



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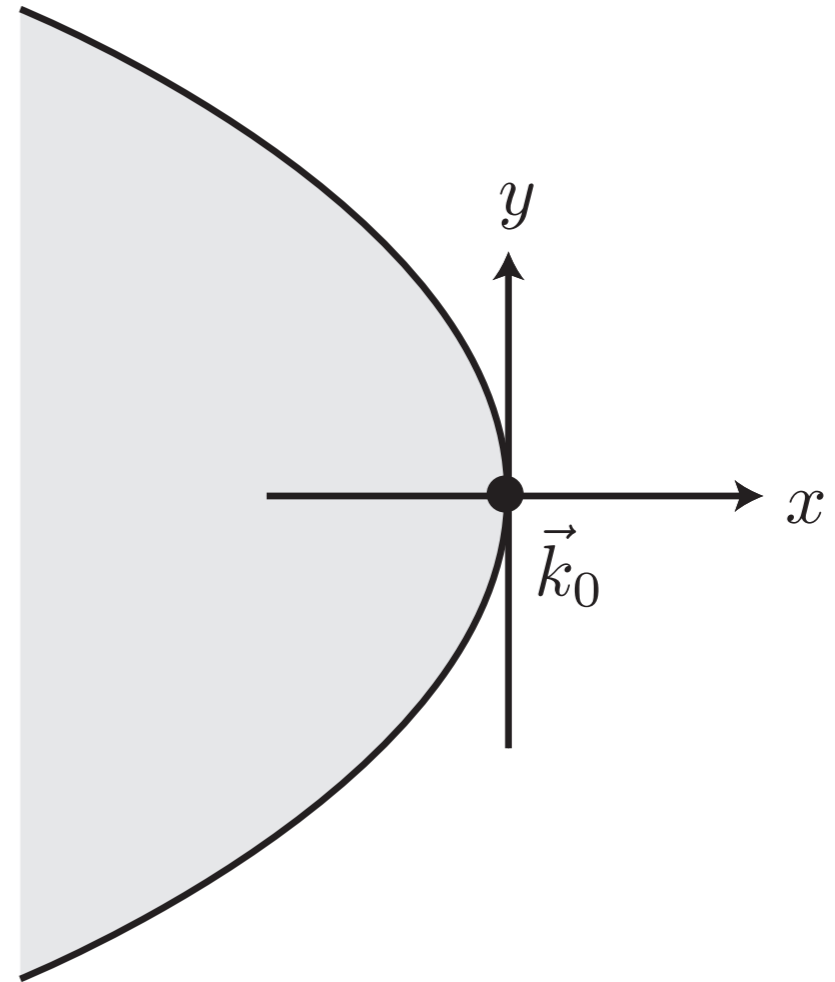
The Fermi liquid

$$\mathcal{L} = f_{\alpha}^{\dagger} \left(\partial_{\tau} - \frac{\nabla^2}{2m} - \mu \right) f_{\alpha} + u f_{\alpha}^{\dagger} f_{\beta}^{\dagger} f_{\beta} f_{\alpha}$$



The Fermi liquid: RG

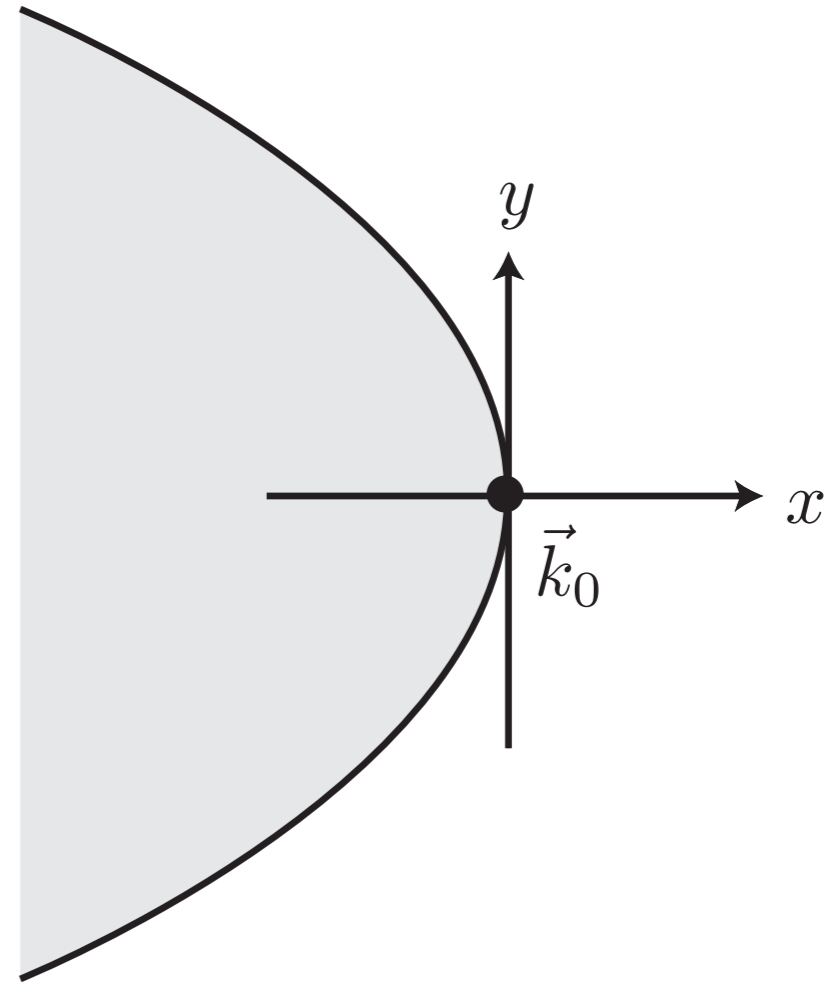
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- Expand fermion kinetic energy at wavevectors about \vec{k}_0 , by writing $f_{\alpha}(\vec{k}_0 + \vec{q}) = \psi_{\alpha}(\vec{q})$

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$$\mathcal{L}[\psi_\alpha] = \psi_\alpha^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2 \right) \psi_\alpha + u \psi_\alpha^\dagger \psi_\beta^\dagger \psi_\beta \psi_\alpha$$

The Fermi liquid: RG

$$\mathcal{S}[\psi_\alpha] = \int d^{d-1}y dx d\tau \left[\psi_\alpha^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_\alpha + u \psi_\alpha^\dagger \psi_\beta^\dagger \psi_\beta \psi_\alpha \right]$$

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The kinetic energy is invariant under the rescaling $x \rightarrow x/s$, $y \rightarrow y/s^{1/2}$, and $\tau \rightarrow \tau/s^z$, provided $z = 1$ and

$$\psi \rightarrow \psi s^{(d+1)/4}.$$

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Then we find $u \rightarrow u s^{(1-d)/2}$, and so we have the RG flow

$$\frac{du}{d\ell} = \frac{(1-d)}{2} u$$

Interactions are *irrelevant* in $d = 2$!

The Fermi liquid: RG

$$\mathcal{S}[\psi_\alpha] = \int d^{d-1}y dx d\tau \left[\psi_\alpha^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_\alpha + u \psi_\alpha^\dagger \psi_\beta^\dagger \psi_\beta \psi_\alpha \right]$$

The fermion Green's function to order u^2 has the form (upto logs)

$$G(\vec{q}, \omega) = \frac{\mathcal{A}}{\omega - q_x - q_y^2 + ic\omega^2}$$

So the quasiparticle pole is sharp.

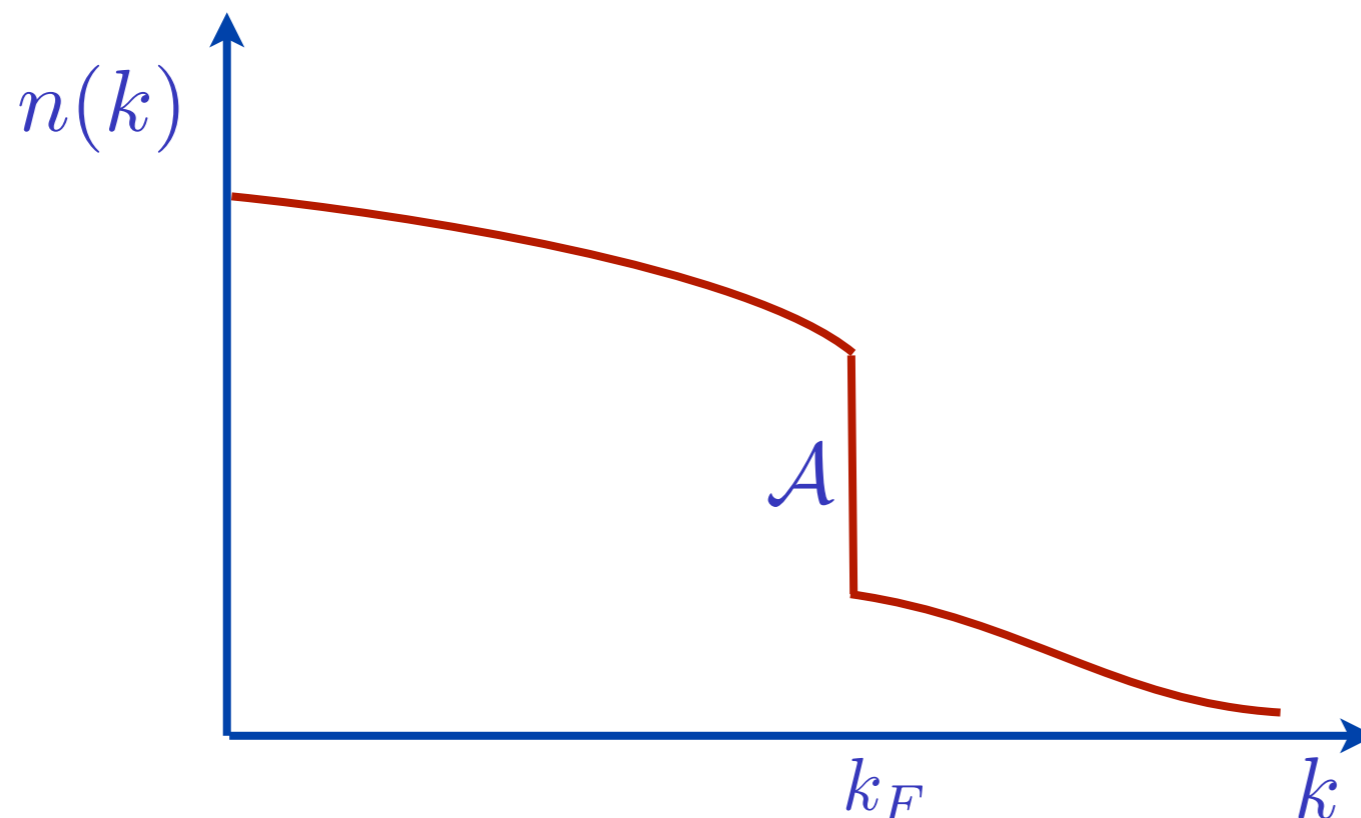
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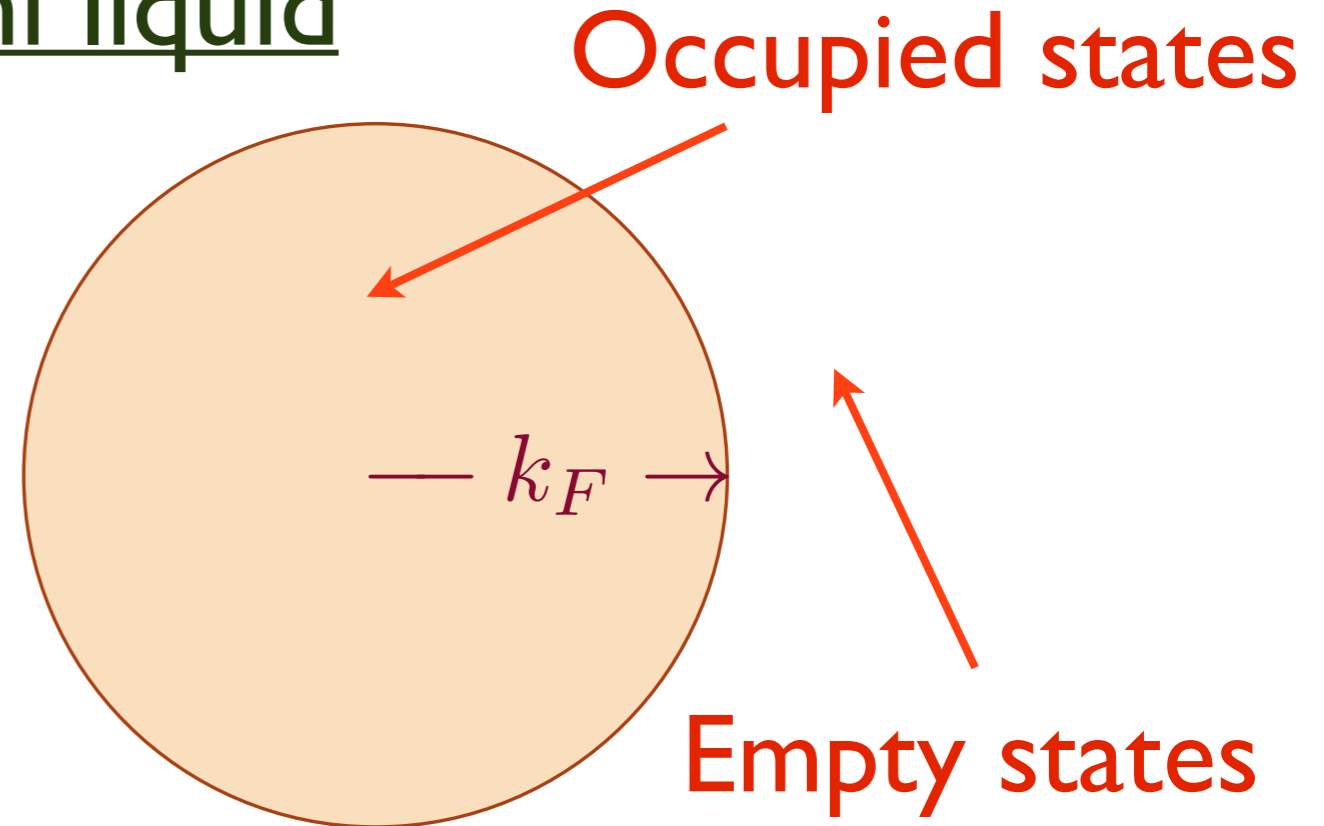
So the quasiparticle pole is sharp. And fermion momentum distribution function $n(\vec{k}) = \langle f_\alpha^\dagger(\vec{k}) f_\alpha(\vec{k}) \rangle$ had the following form:



The Fermi liquid

$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f$$

+ 4 Fermi terms

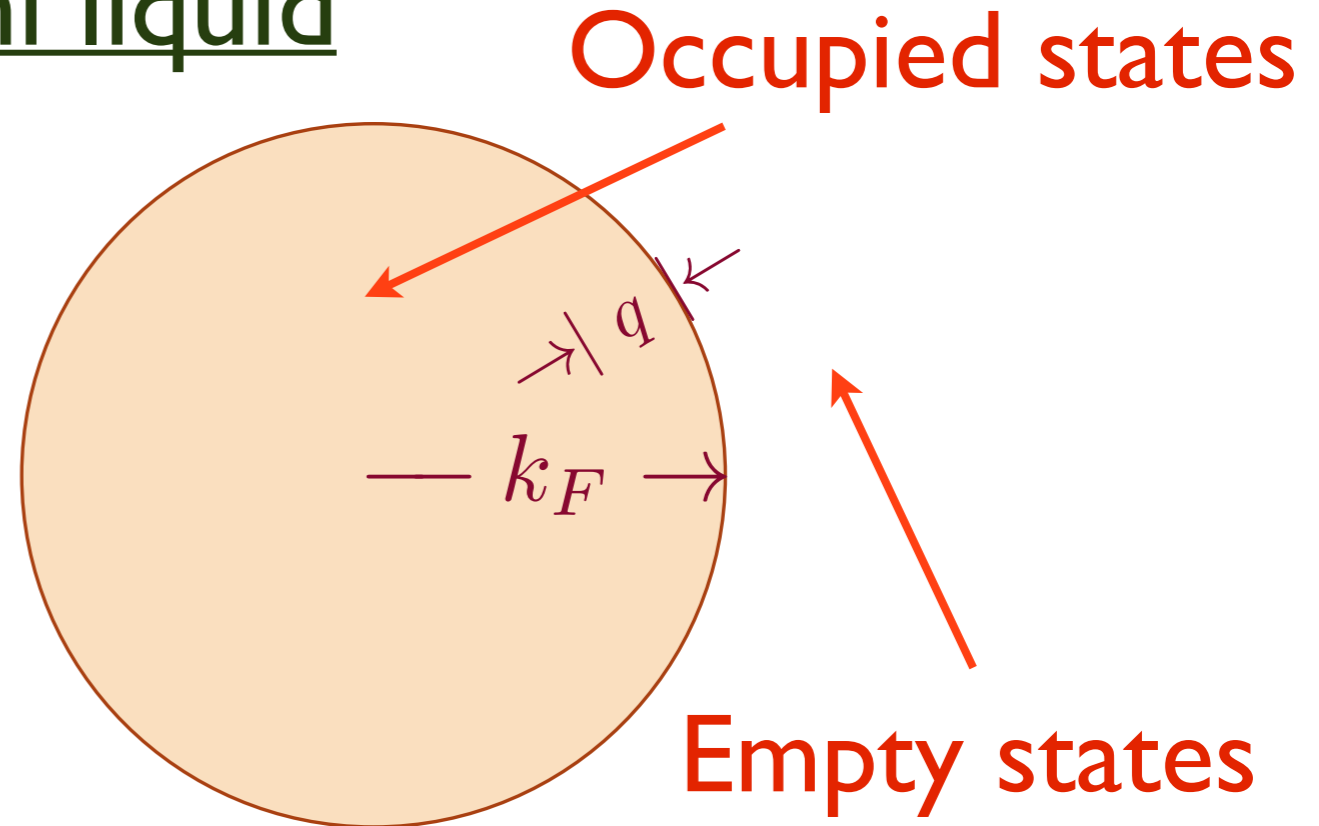


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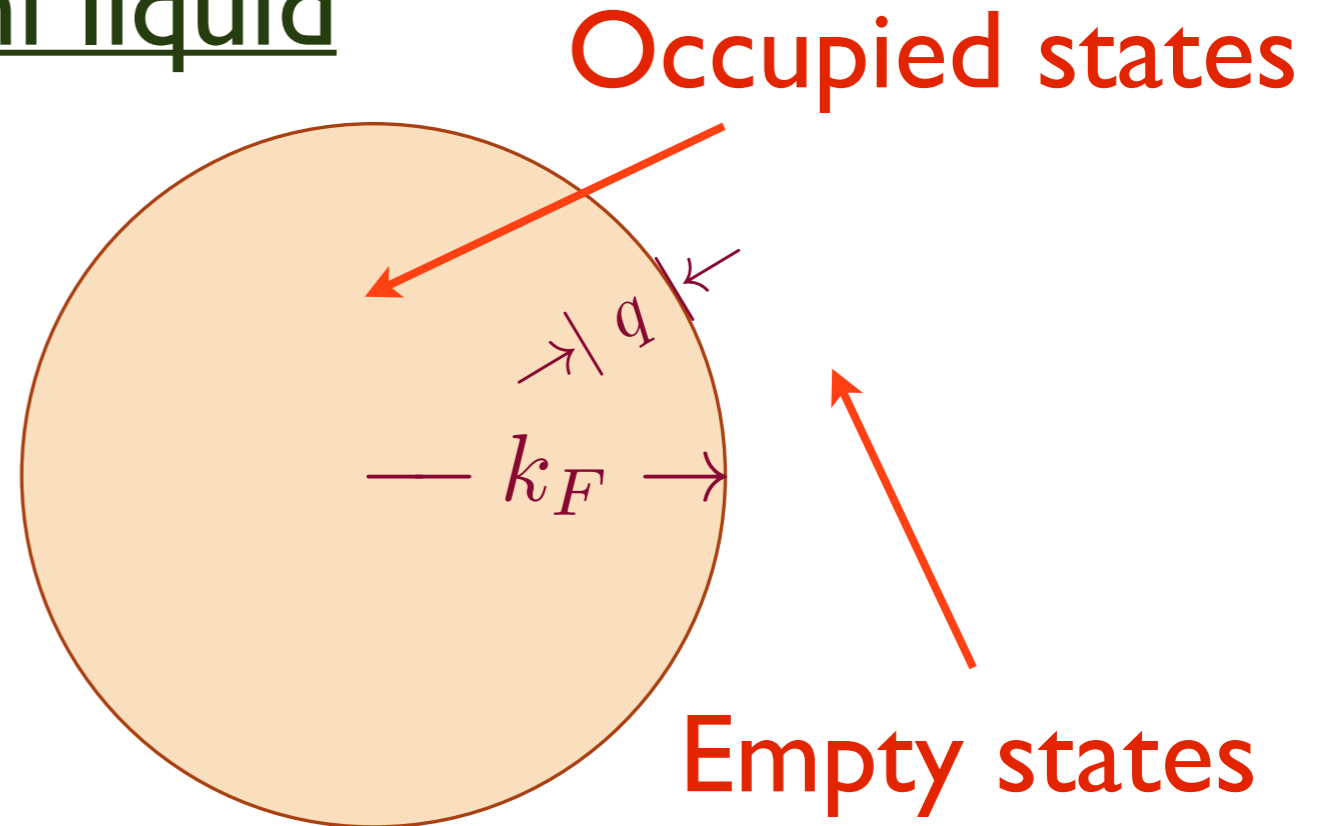


- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$, with dynamic exponent $z = 1$.

The Fermi liquid

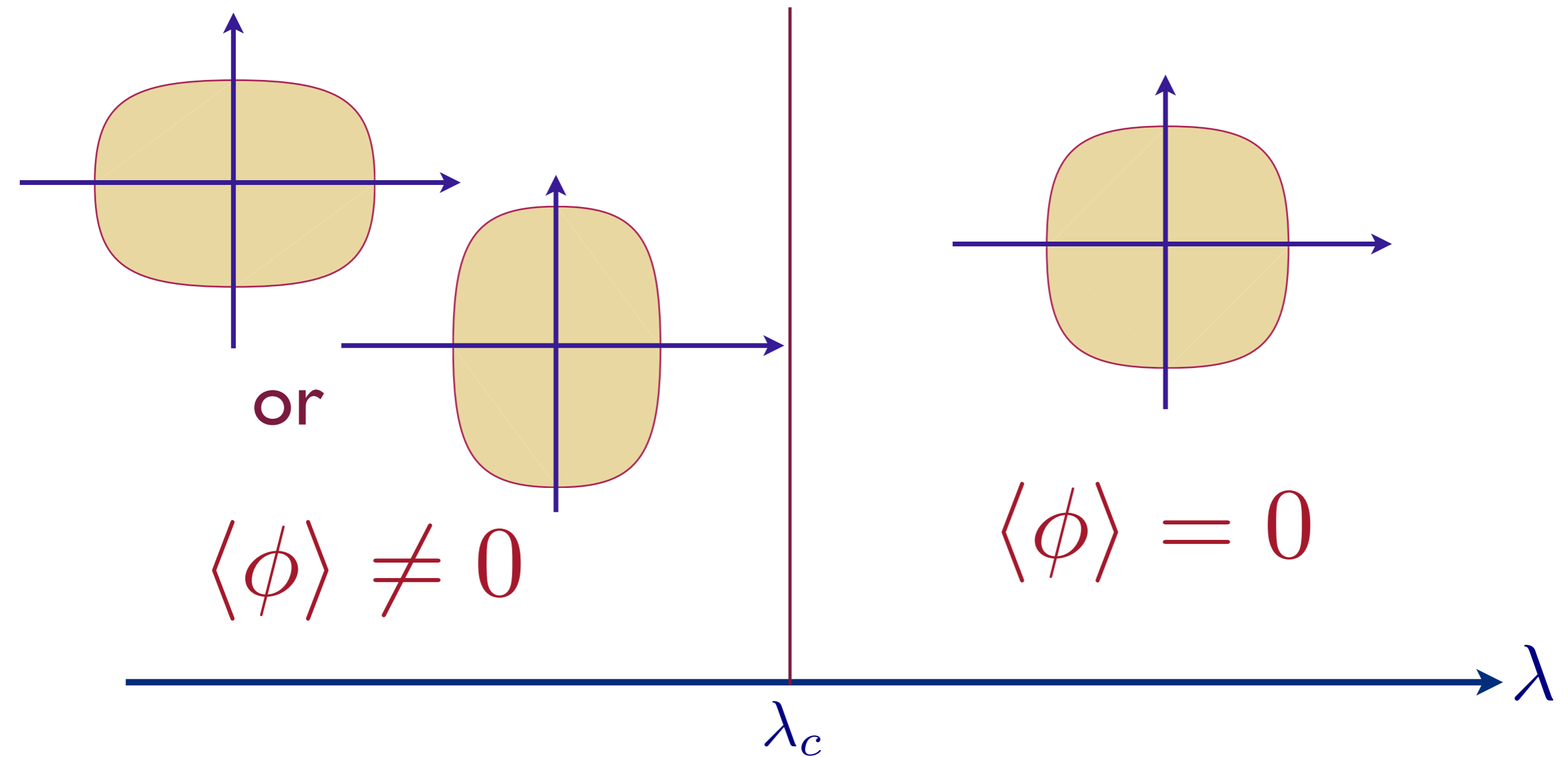
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- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$, with dynamic exponent $z = 1$.
- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T$. It is useful to write this as $S \sim T^{(d-\theta)/z}$, with violation of hyperscaling exponent $\theta = d - 1$.

Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal

Effective action for Ising order parameter

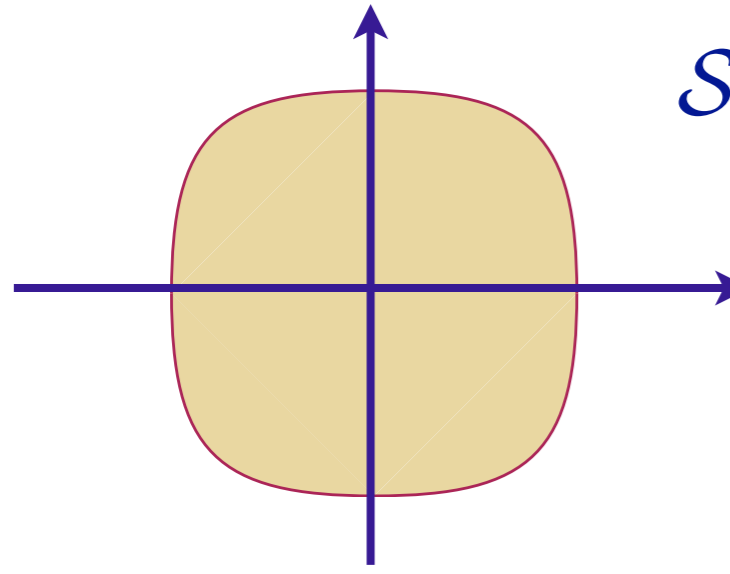
$$\mathcal{S}_\phi = \int d^2r d\tau [(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4]$$

Quantum criticality of Ising-nematic ordering in a metal

Effective action for Ising order parameter

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Effective action for electrons:

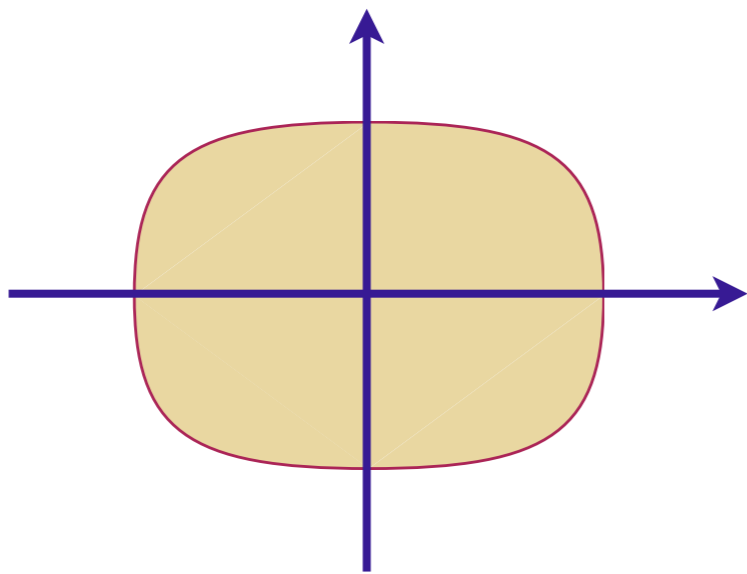

$$\begin{aligned} \mathcal{S}_c &= \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \right] \\ &\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger (\partial_\tau + \varepsilon_{\mathbf{k}}) c_{\mathbf{k}\alpha} \end{aligned}$$

Quantum criticality of Ising-nematic ordering in a metal

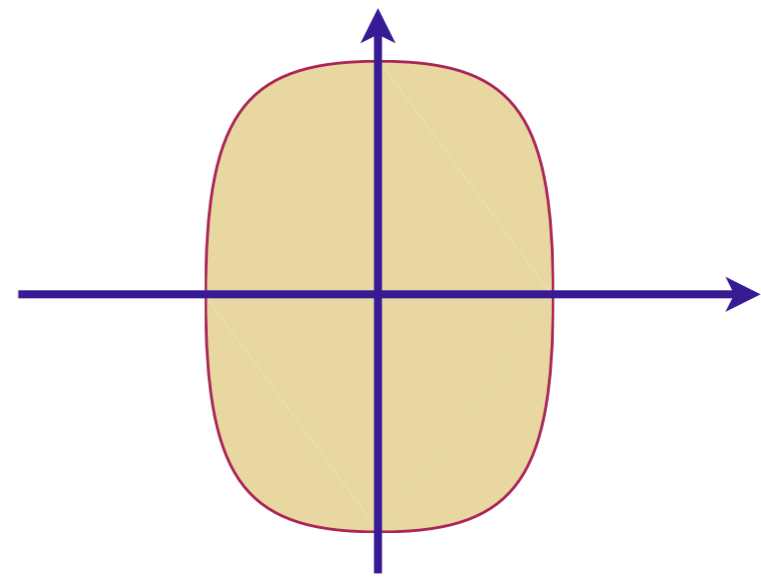
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}, \mathbf{q}} \phi_{\mathbf{q}} (\cos k_x - \cos k_y) c_{\mathbf{k}+\mathbf{q}/2, \alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2, \alpha}$$

for spatially dependent ϕ



$$\langle \phi \rangle > 0$$



$$\langle \phi \rangle < 0$$

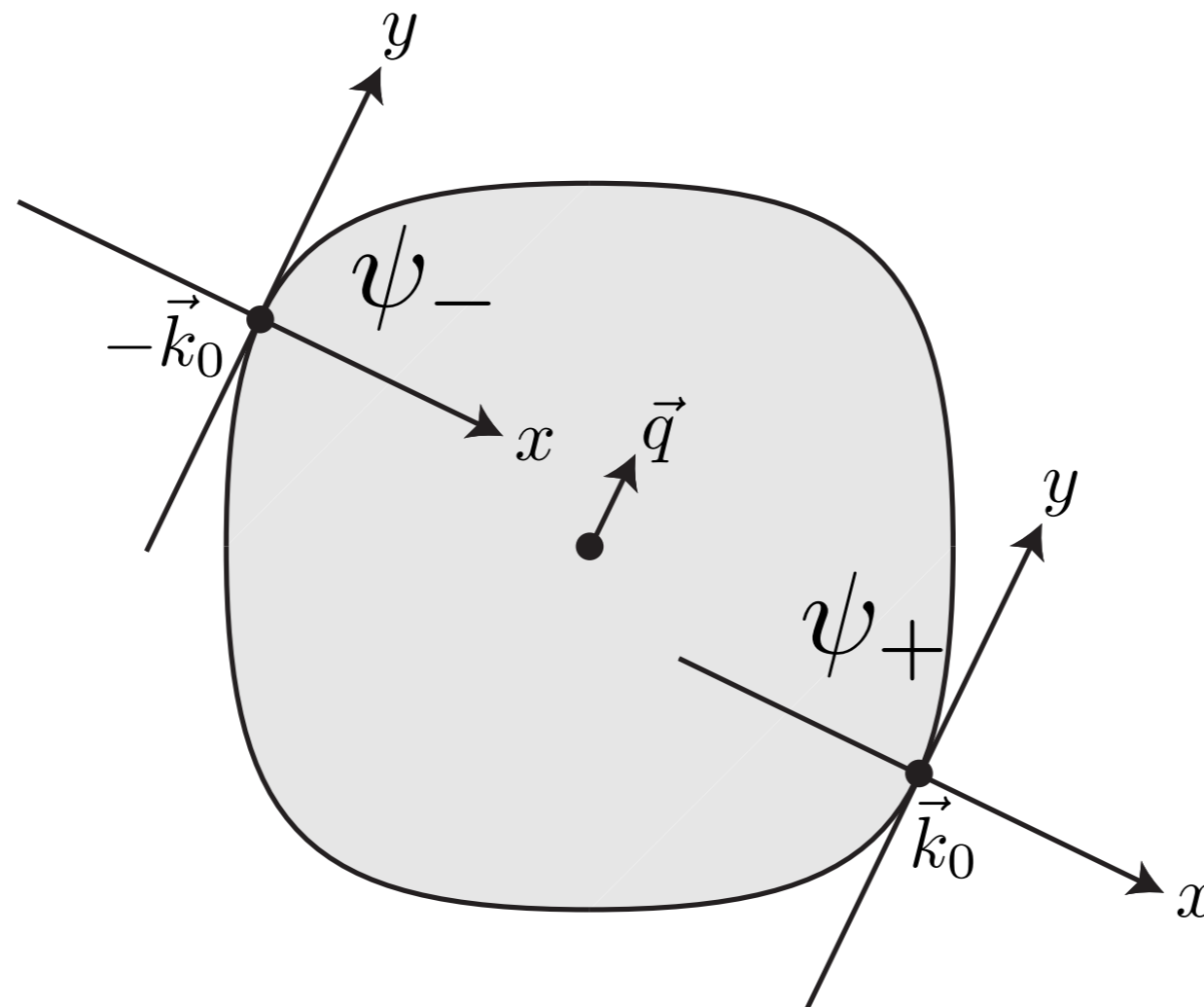
Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{S}_\phi = \int d^2r d\tau [(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4]$$

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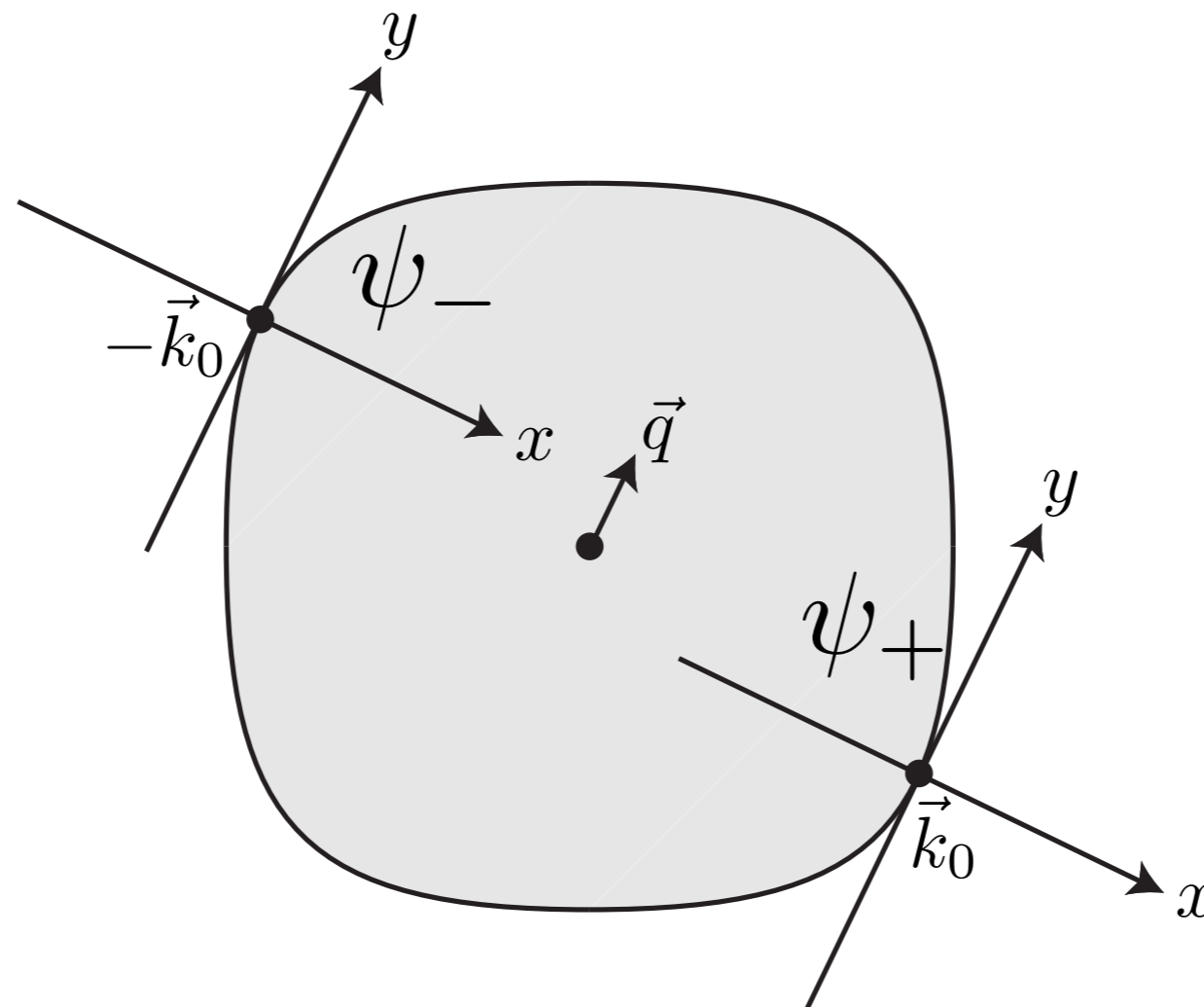
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Quantum criticality of Ising-nematic ordering in a metal



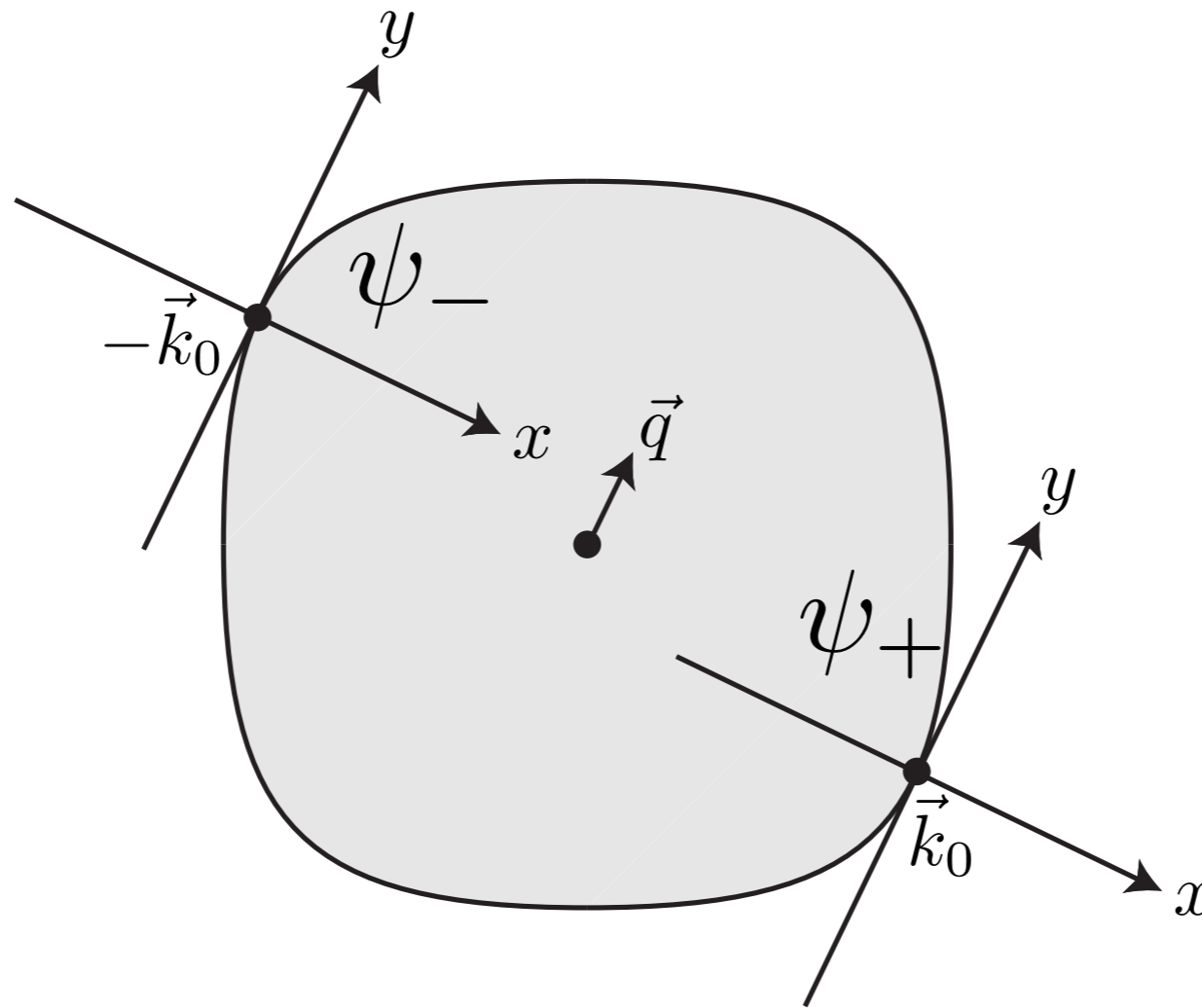
- ϕ fluctuation at wavevector \vec{q} couples most efficiently to fermions near $\pm\vec{k}_0$.

Quantum criticality of Ising-nematic ordering in a metal



- ϕ fluctuation at wavevector \vec{q} couples most efficiently to fermions near $\pm\vec{k}_0$.
- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson (ϕ) kinetic energy about $\vec{q} = 0$.

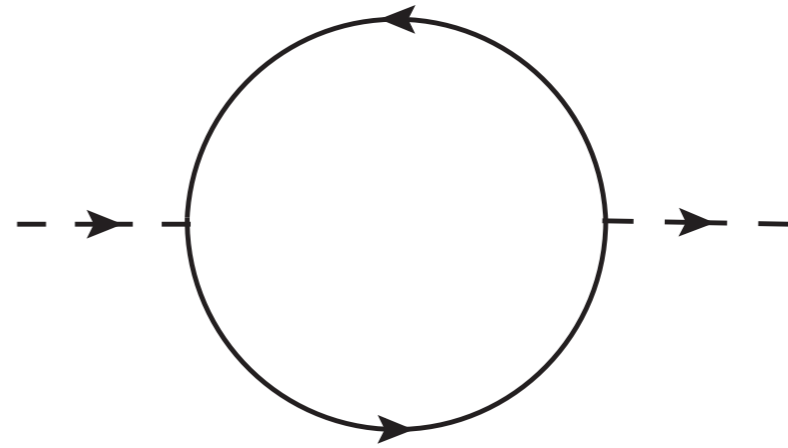
Quantum criticality of Ising-nematic ordering in a metal



$$\begin{aligned} \mathcal{L}[\psi_{\pm}, \phi] = & \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- \\ & - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2 \end{aligned}$$

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2$$



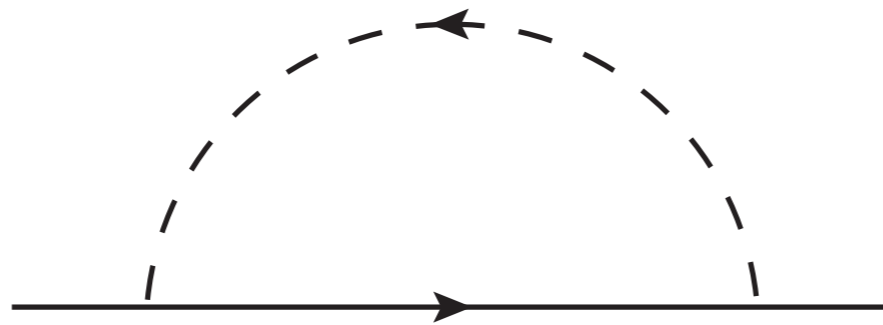
One loop ϕ self-energy with N_f fermion flavors:

$$\begin{aligned} \Sigma_\phi(\vec{q}, \omega) &= N_f \int \frac{d^2 k}{4\pi^2} \frac{d\Omega}{2\pi} \frac{1}{[-i(\Omega + \omega) + k_x + q_x + (k_y + q_y)^2] [-i\Omega - k_x + k_y^2]} \\ &= \frac{N_f}{4\pi} \frac{|\omega|}{|q_y|} \end{aligned}$$

Landau-damping

Quantum criticality of Ising-nematic ordering in a metal

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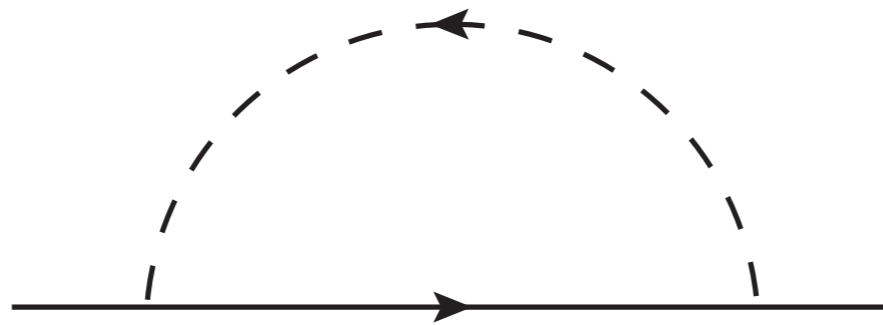


Electron self-energy at order $1/N_f$:

$$\begin{aligned} \Sigma(\vec{k}, \Omega) &= -\frac{1}{N_f} \int \frac{d^2 q}{4\pi^2} \frac{d\omega}{2\pi} \frac{1}{[-i(\omega + \Omega) + k_x + q_x + (k_y + q_y)^2] \left[\frac{q_y^2}{g^2} + \frac{|\omega|}{|q_y|} \right]} \\ &= -i \frac{2}{\sqrt{3} N_f} \left(\frac{g^2}{4\pi} \right)^{2/3} \text{sgn}(\Omega) |\Omega|^{2/3} \end{aligned}$$

Quantum criticality of Ising-nematic ordering in a metal

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Quantum criticality of Ising-nematic ordering in a metal

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Schematic form of ϕ and fermion Green's functions in d dimensions

$$D(\vec{q}, \omega) = \frac{1/N_f}{q_\perp^2 + \frac{|\omega|}{|q_\perp|}}, \quad G_f(\vec{q}, \omega) = \frac{1}{q_x + q_\perp^2 - i \text{sgn}(\omega) |\omega|^{d/3} / N_f}$$

In the boson case, $q_\perp^2 \sim \omega^{1/z_b}$ with $z_b = 3/2$.

In the fermion case, $q_x \sim q_\perp^2 \sim \omega^{1/z_f}$ with $z_f = 3/d$.

Note $z_f < z_b$ for $d > 2 \Rightarrow$ Fermions have *higher* energy than bosons, and perturbation theory in g is OK.

Strongly-coupled theory in $d = 2$.

Quantum criticality of Ising-nematic ordering in a metal

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In *both* cases $q_x \sim q_y^2 \sim \omega^{1/z}$, with $z = 3/2$. Note that the bare term $\sim \omega$ in G_f^{-1} is irrelevant.

Strongly-coupled theory without quasiparticles.

Quantum criticality of Ising-nematic ordering in a metal

$$\begin{aligned} \mathcal{L} = & \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- \\ & - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2 \end{aligned}$$

Simple scaling argument for $z = 3/2$.

Quantum criticality of Ising-nematic ordering in a metal

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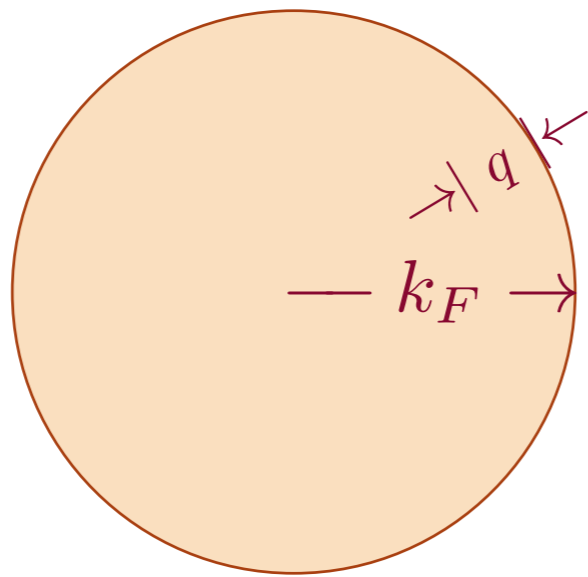
Simple scaling argument for $z = 3/2$.

Under the rescaling $x \rightarrow x/s$, $y \rightarrow y/s^{1/2}$, and $\tau \rightarrow \tau/s^z$, we find invariance provided

$$\begin{aligned} \phi &\rightarrow \phi s \\ \psi &\rightarrow \psi s^{(2z+1)/4} \\ g &\rightarrow g s^{(3-2z)/4} \end{aligned}$$

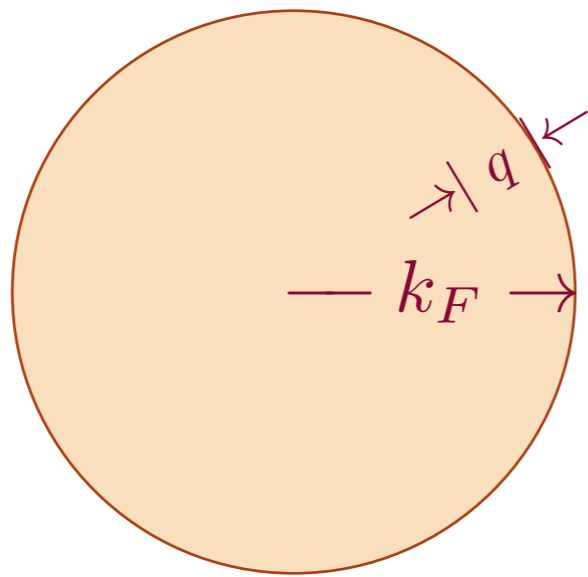
So the action is invariant provided $z = 3/2$.

FL Fermi liquid



- $k_F^d \sim Q$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

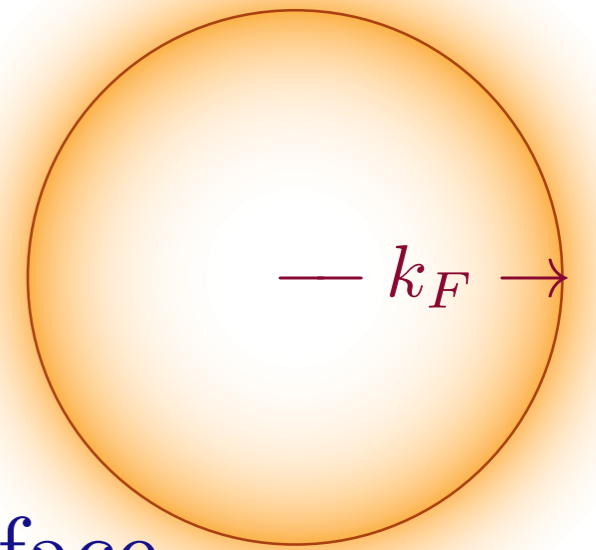
FL
Fermi
liquid



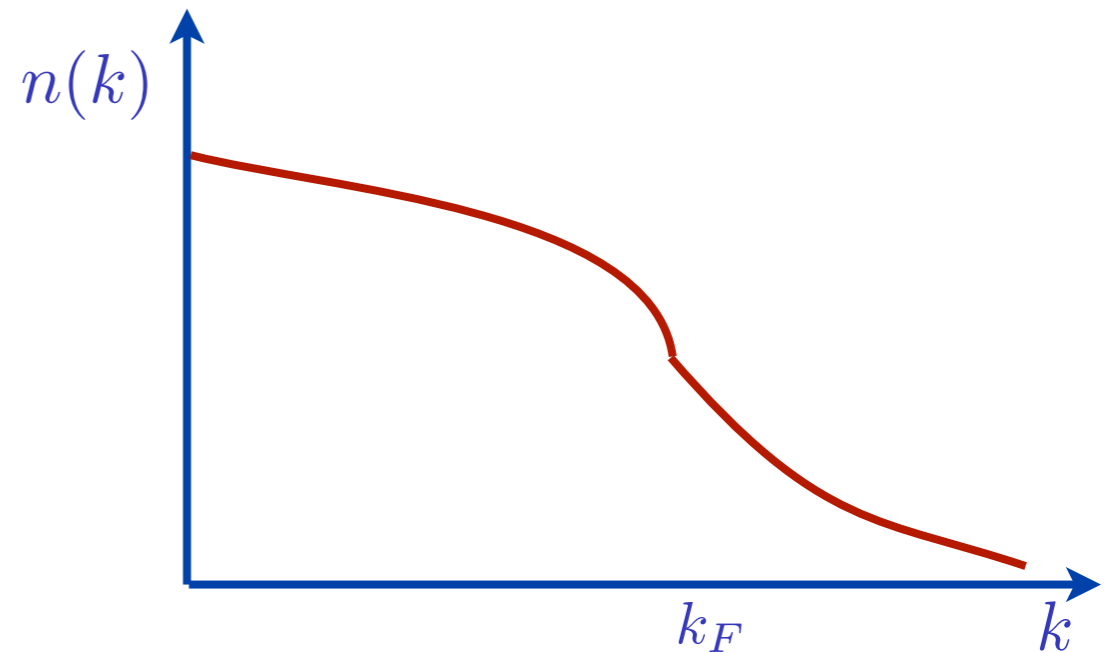
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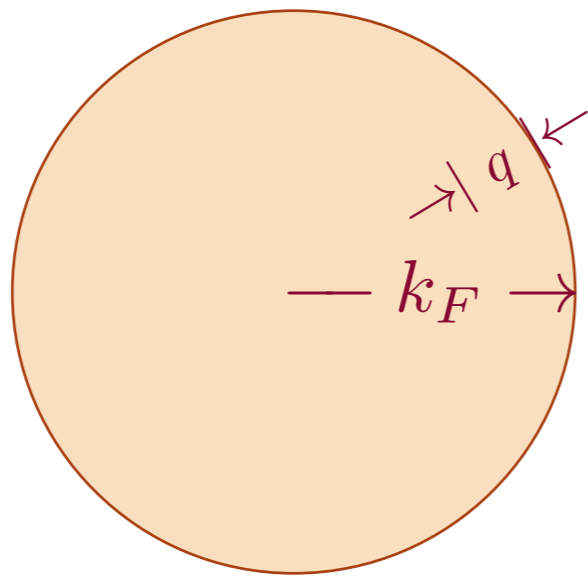
NFL
Nematic
QCP



- Fermi surface with $k_F^d \sim Q$.



FL Fermi liquid

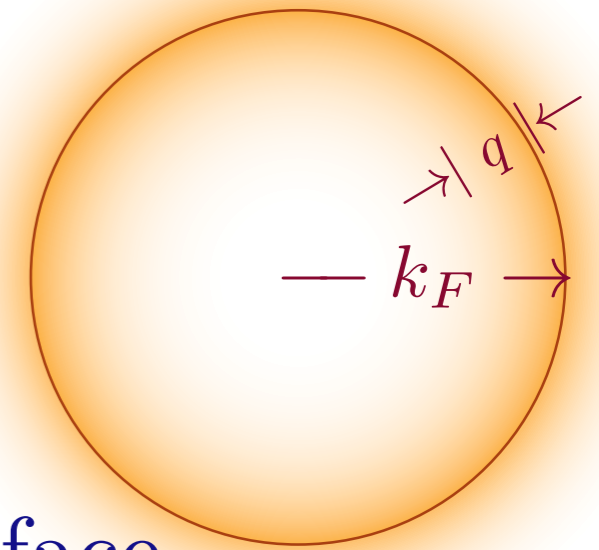


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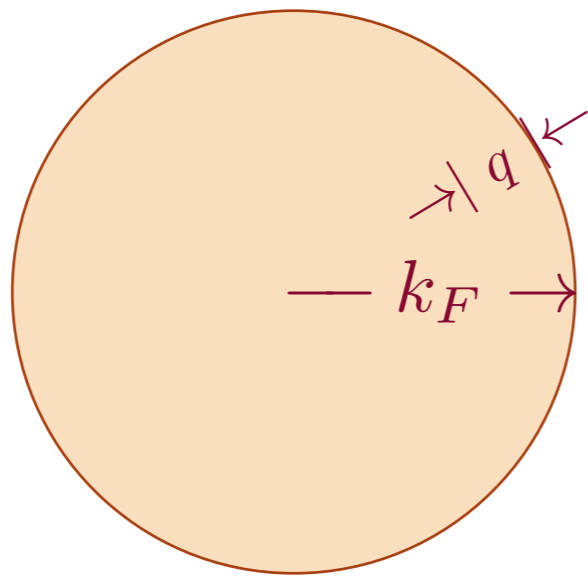
NFL Nematic QCP



- Fermi surface with $k_F^d \sim Q$.

- Diffuse fermionic excitations with $z = 3/2$ to three loops.

FL Fermi liquid

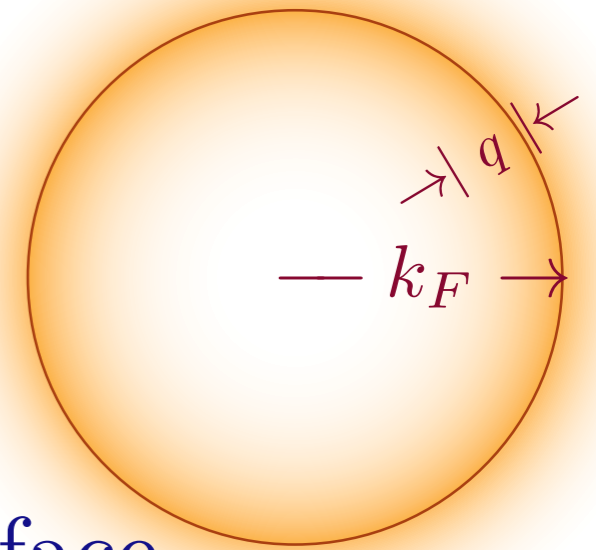


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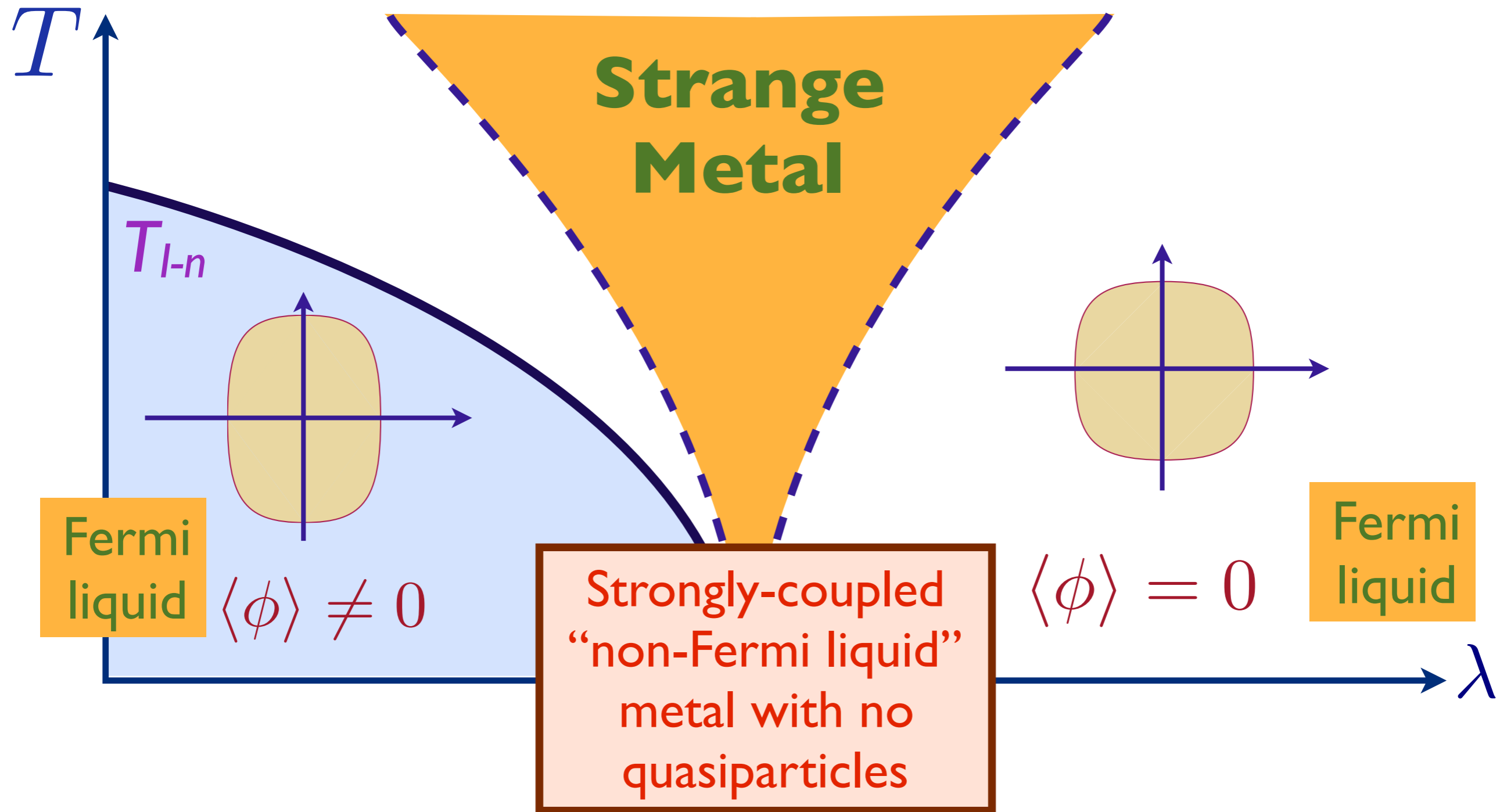


- Fermi surface with $k_F^d \sim Q$.

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- $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.

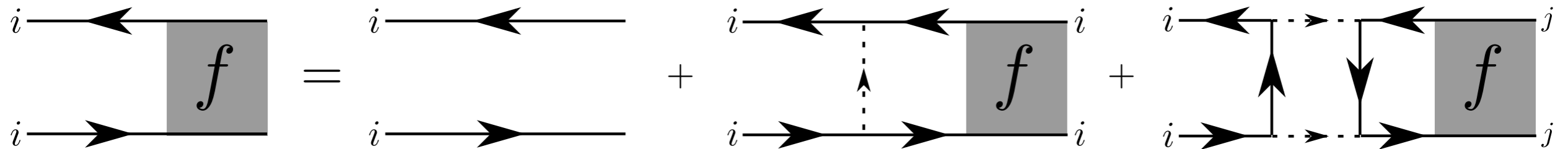
Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

Fermi surface coupled to a gauge field

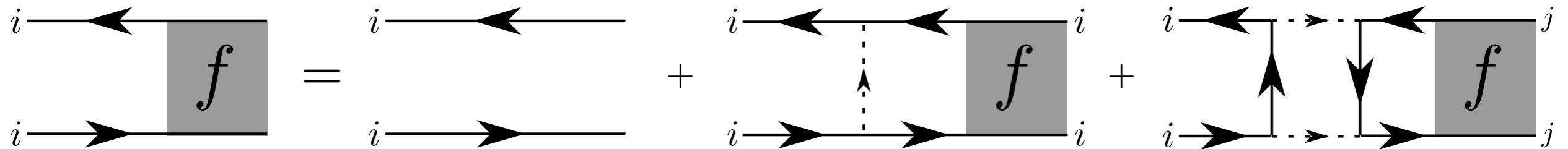
Compute out-of-time-order correlator to
diagnose quantum chaos



$$f(t) = \frac{1}{N^2} \theta(t) \sum_{i,j=1}^N \int d^2x \operatorname{Tr} \left[e^{-\beta H/2} \{ \psi_i(x, t), \psi_j^\dagger(0) \} \right. \\ \left. \times e^{-\beta H/2} \{ \psi_i(x, t), \psi_j^\dagger(0) \}^\dagger \right] \\ \sim \exp\left((t - x/v_B)/\tau_L \right)$$

Fermi surface coupled to a gauge field

Compute out-of-time-order correlator to diagnose quantum chaos



Strongly-coupled theory with no quasiparticles and fast scrambling:

$$\begin{aligned}\tau_L &\approx \frac{\hbar}{2.48 k_B T} \\ v_B &\approx 4.1 \frac{NT^{1/3}}{e^{4/3}} \frac{v_F^{5/3}}{\gamma^{1/3}} \\ D_E &\approx 0.42 v_B^2 \tau_L\end{aligned}$$



N is the number of fermion flavors, v_F is the Fermi velocity, γ is the Fermi surface curvature, e is the gauge coupling constant.