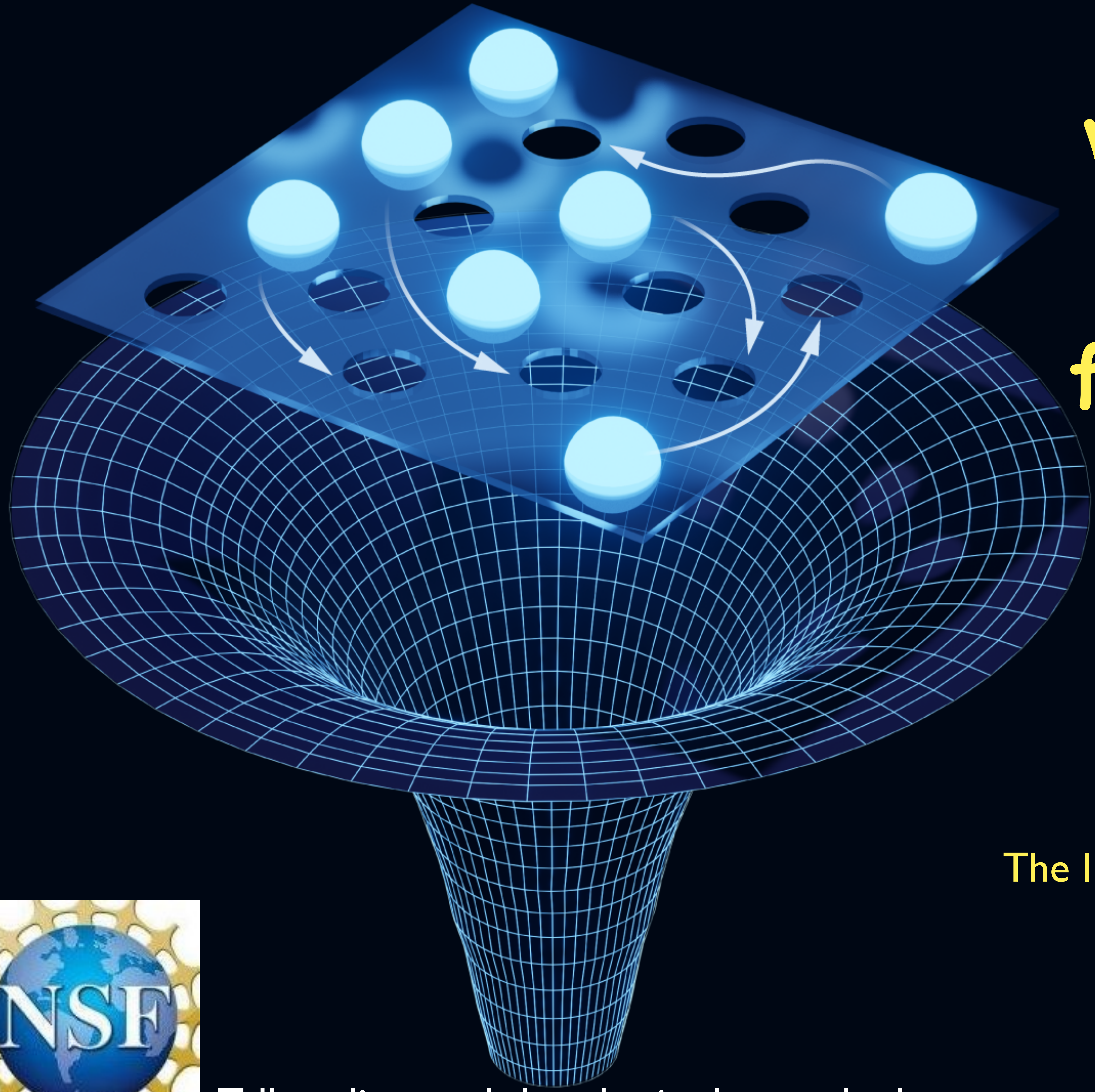


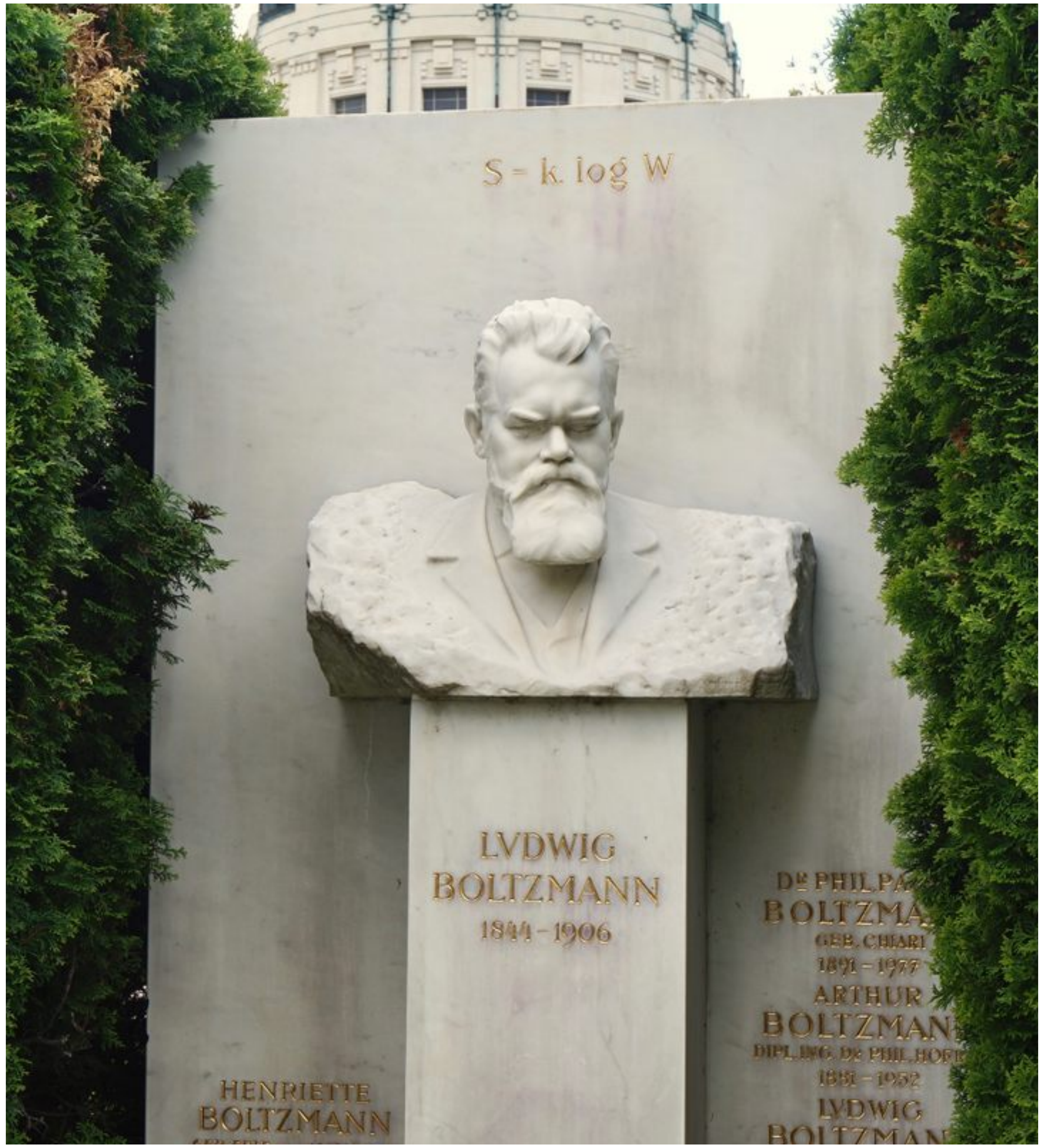
When nature entangles millions of particles: from quantum materials to black holes

Colloquium
The Institute for Solid State Physics
The University of Tokyo
December 9, 2022

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

Foundations
by
Boltzmann



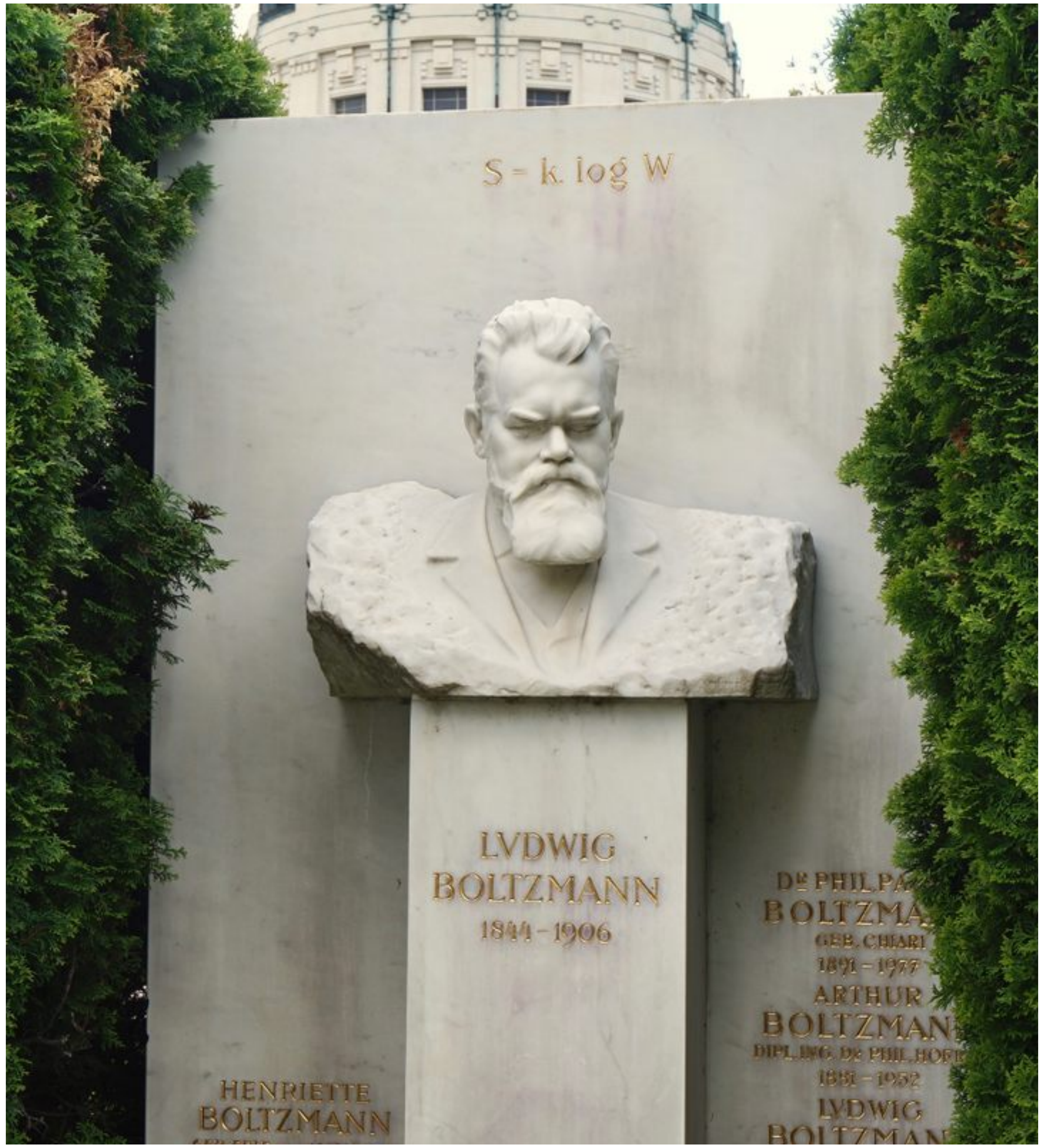
Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states $D(E) = \exp(S(E)/k_B)$

Boltzmann equation (1872)
Dilute classical gas



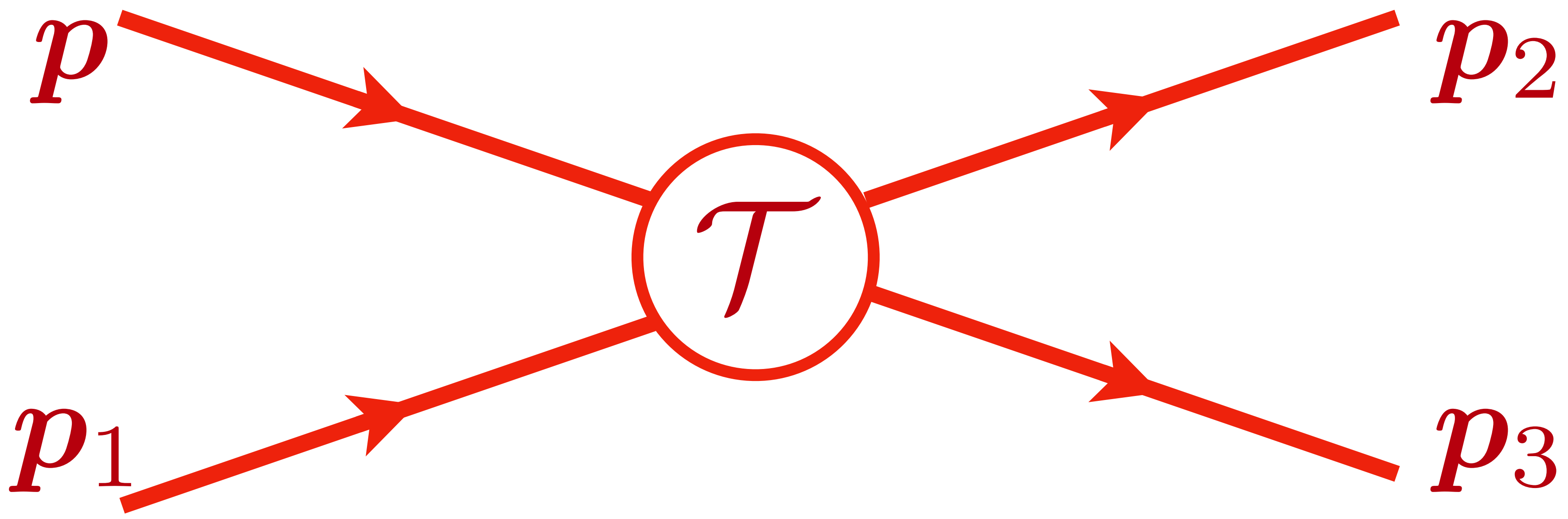
Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

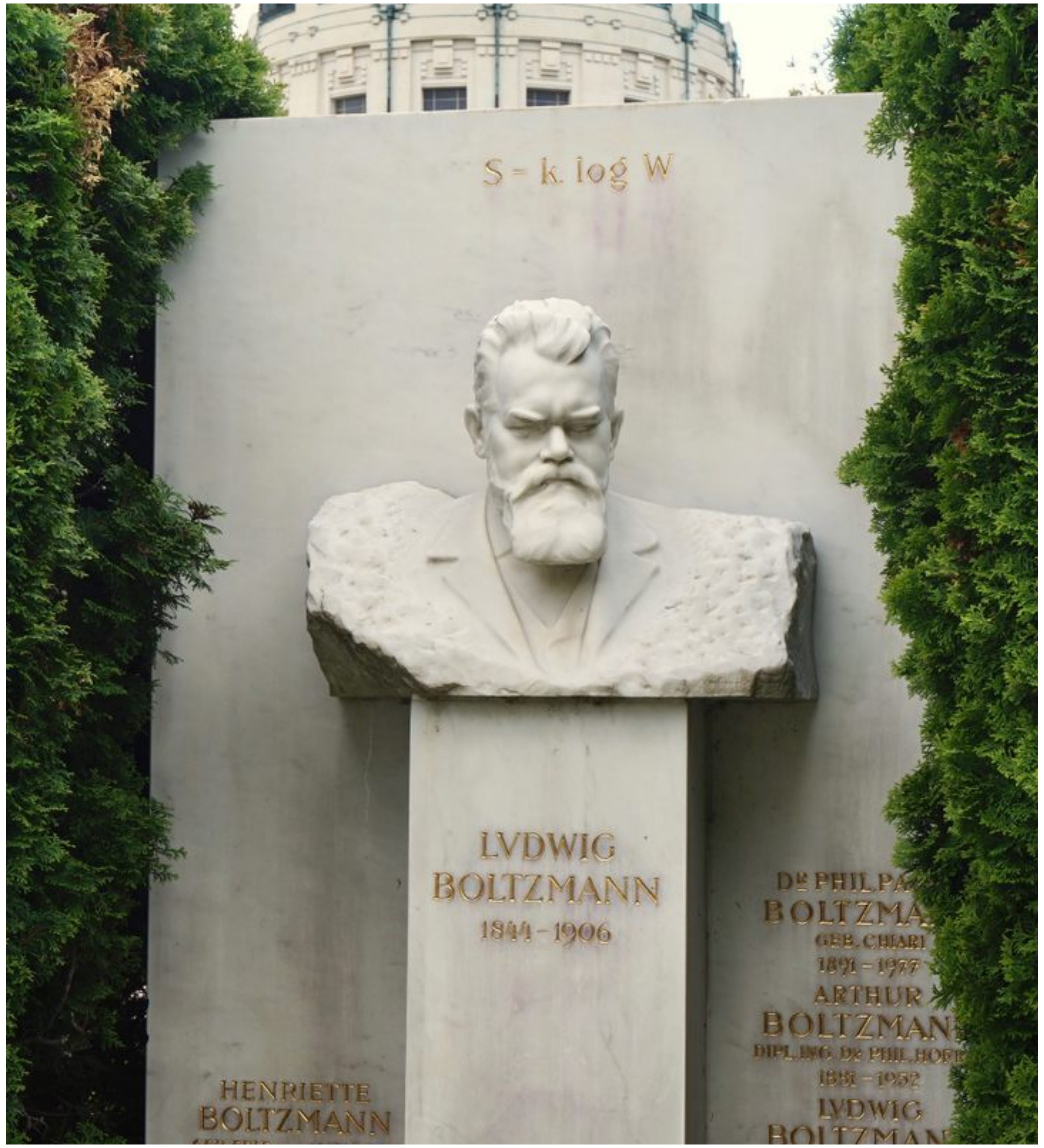
Molecular chaos: successive collisions are statistically independent

$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$

$$-2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$

$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} \right]$$



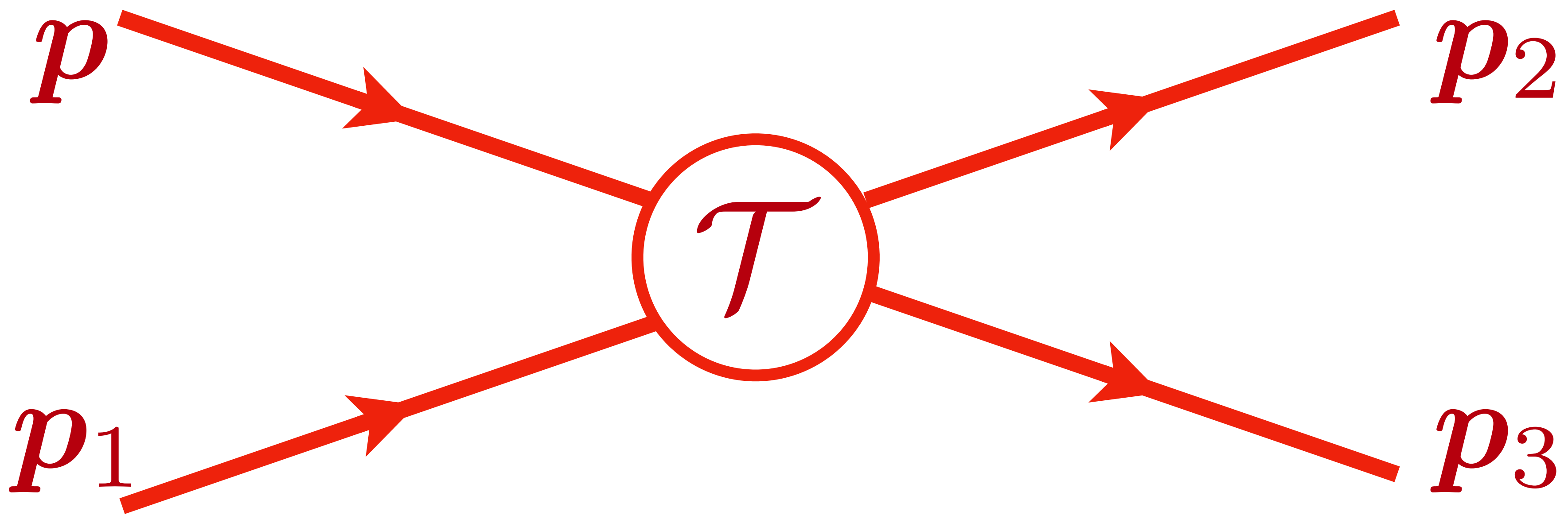


Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

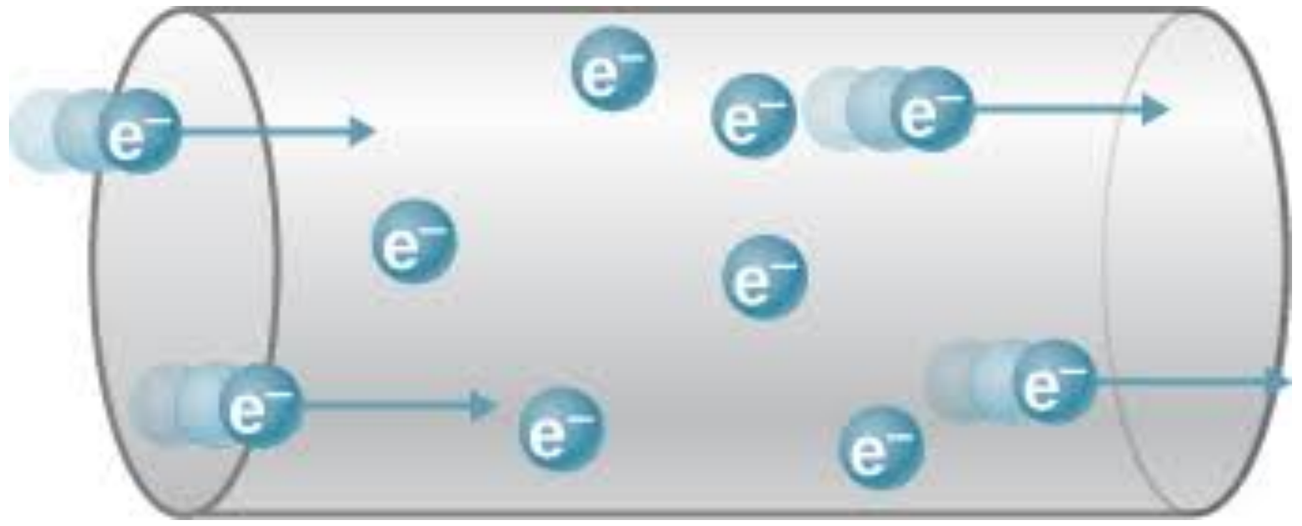
Quantum Boltzmann equation (Landau)
Dense gas of electrons

Neglects quantum interference (entanglement)
between successive collisions

$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$
$$- 2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$
$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} (1 - f_{\boldsymbol{p}_2})(1 - f_{\boldsymbol{p}_3}) - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} (1 - f_{\boldsymbol{p}})(1 - f_{\boldsymbol{p}_1}) \right]$$



Current flow with electrons in Copper

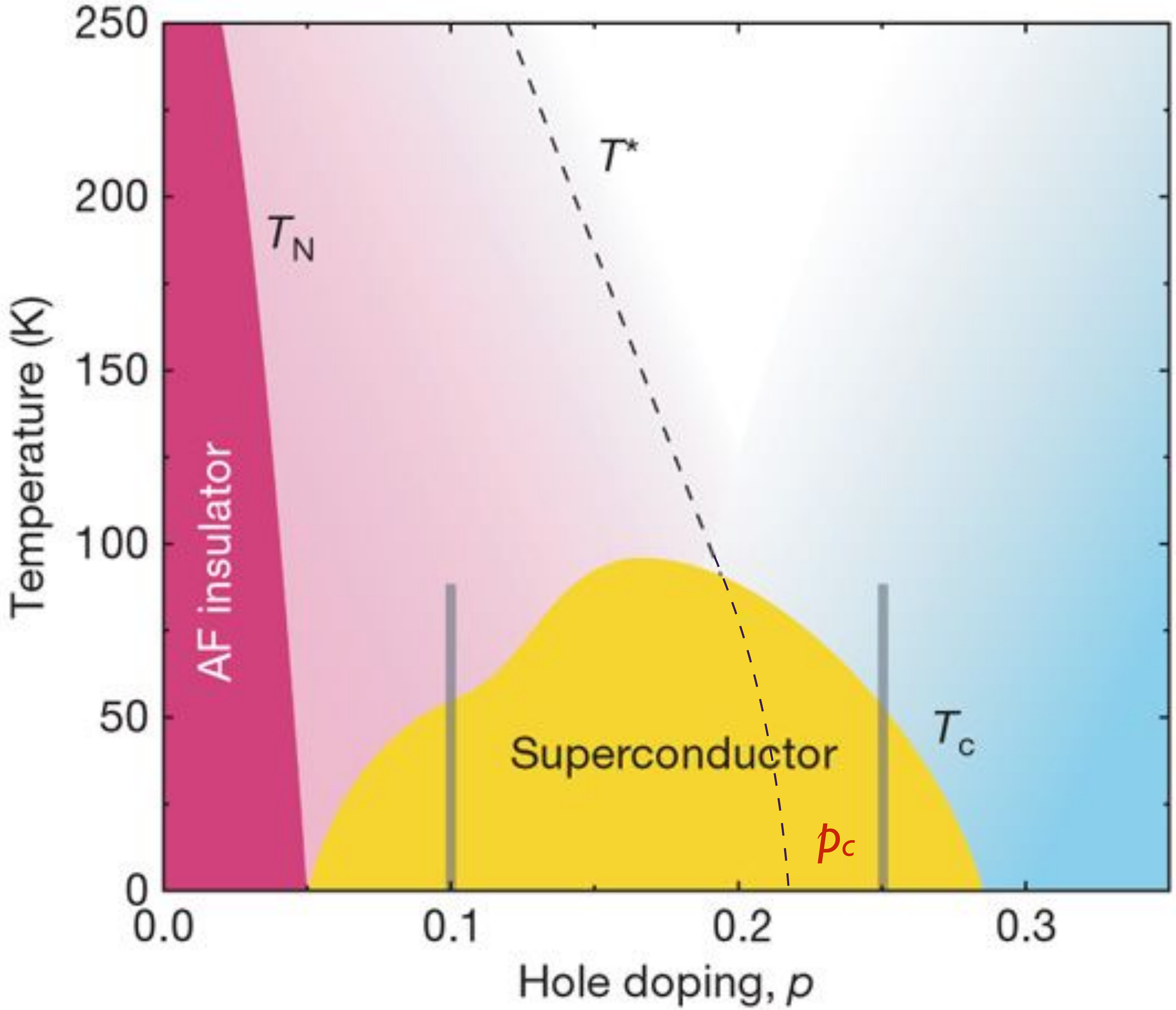


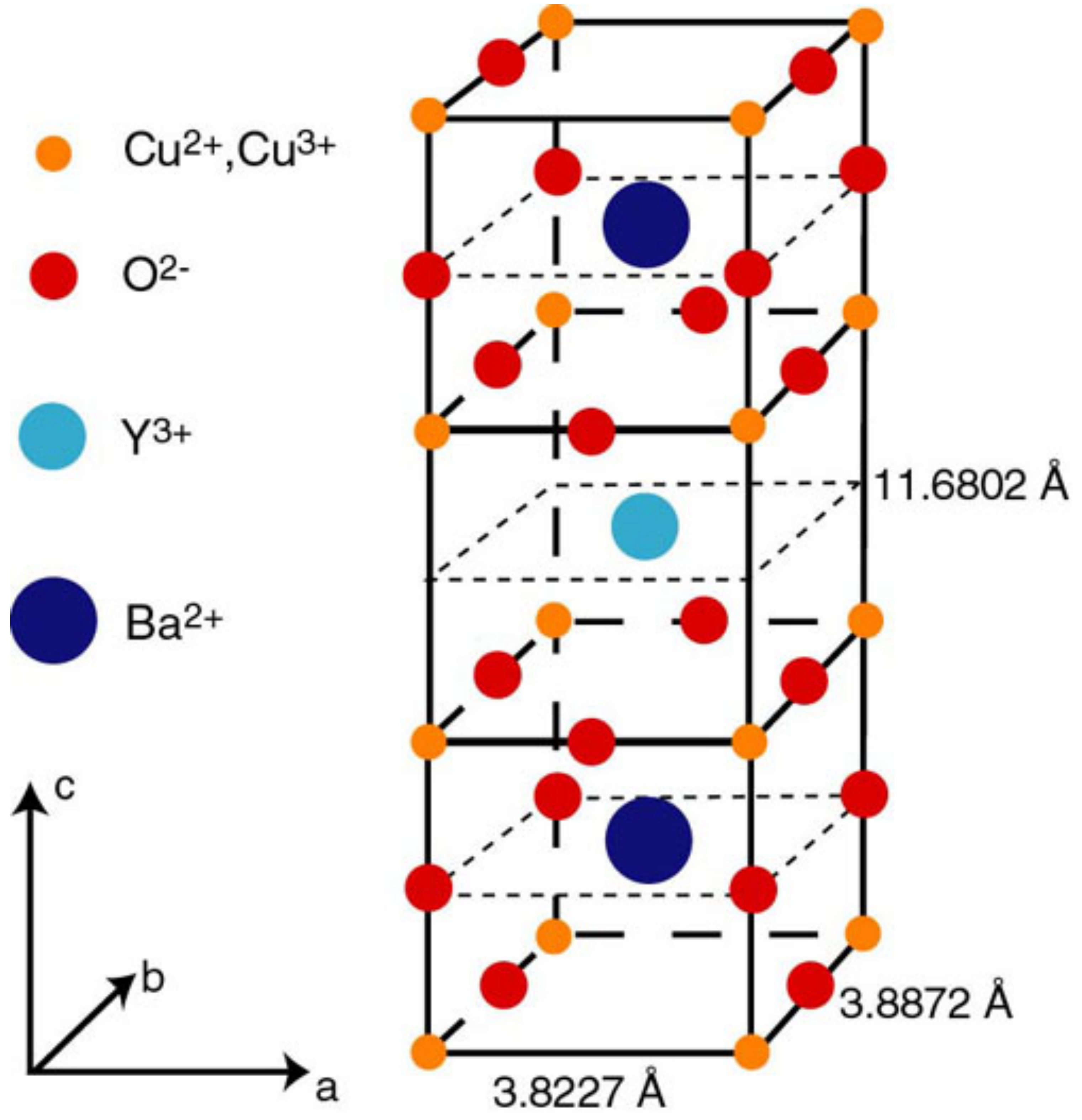
Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

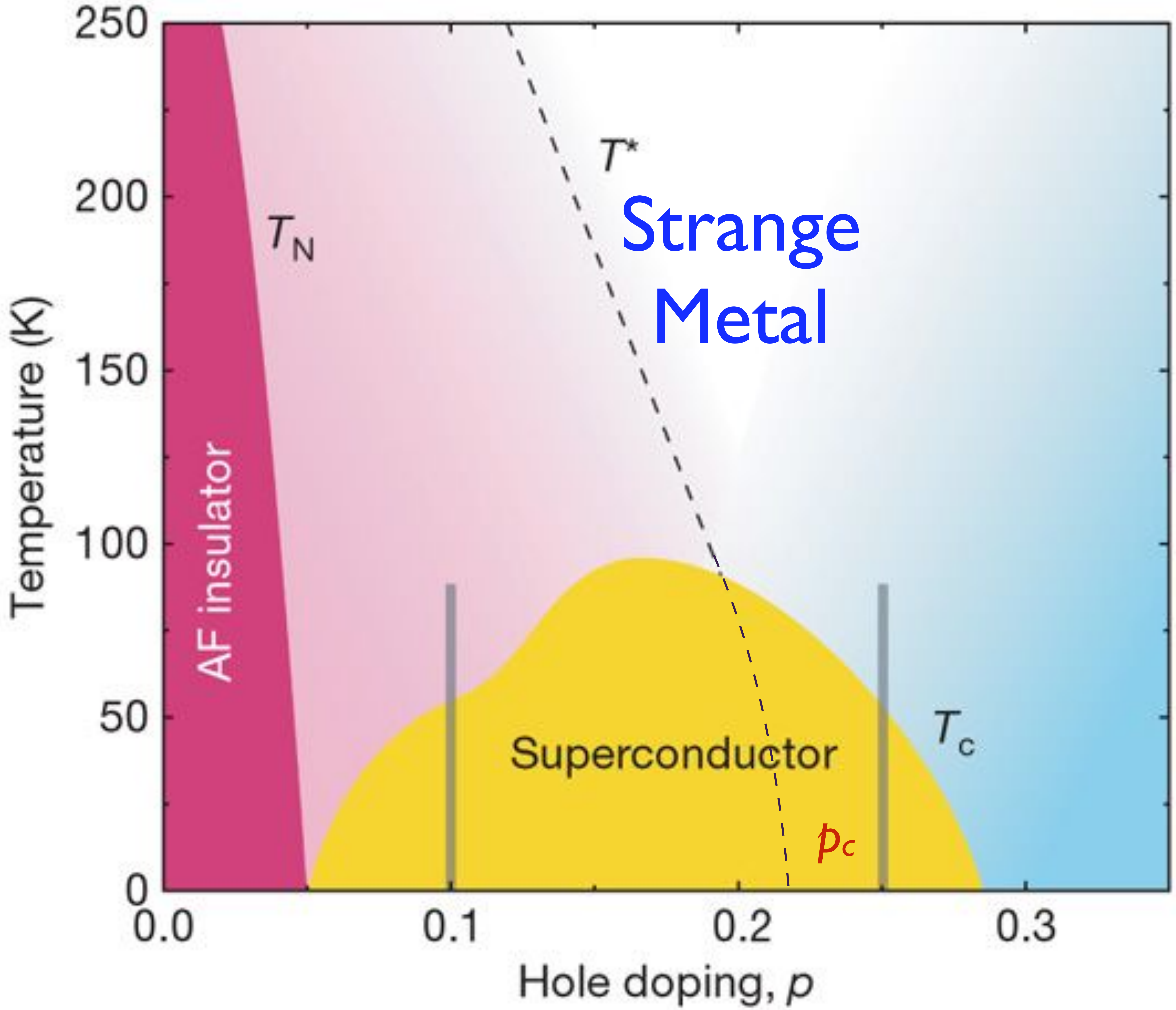
The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

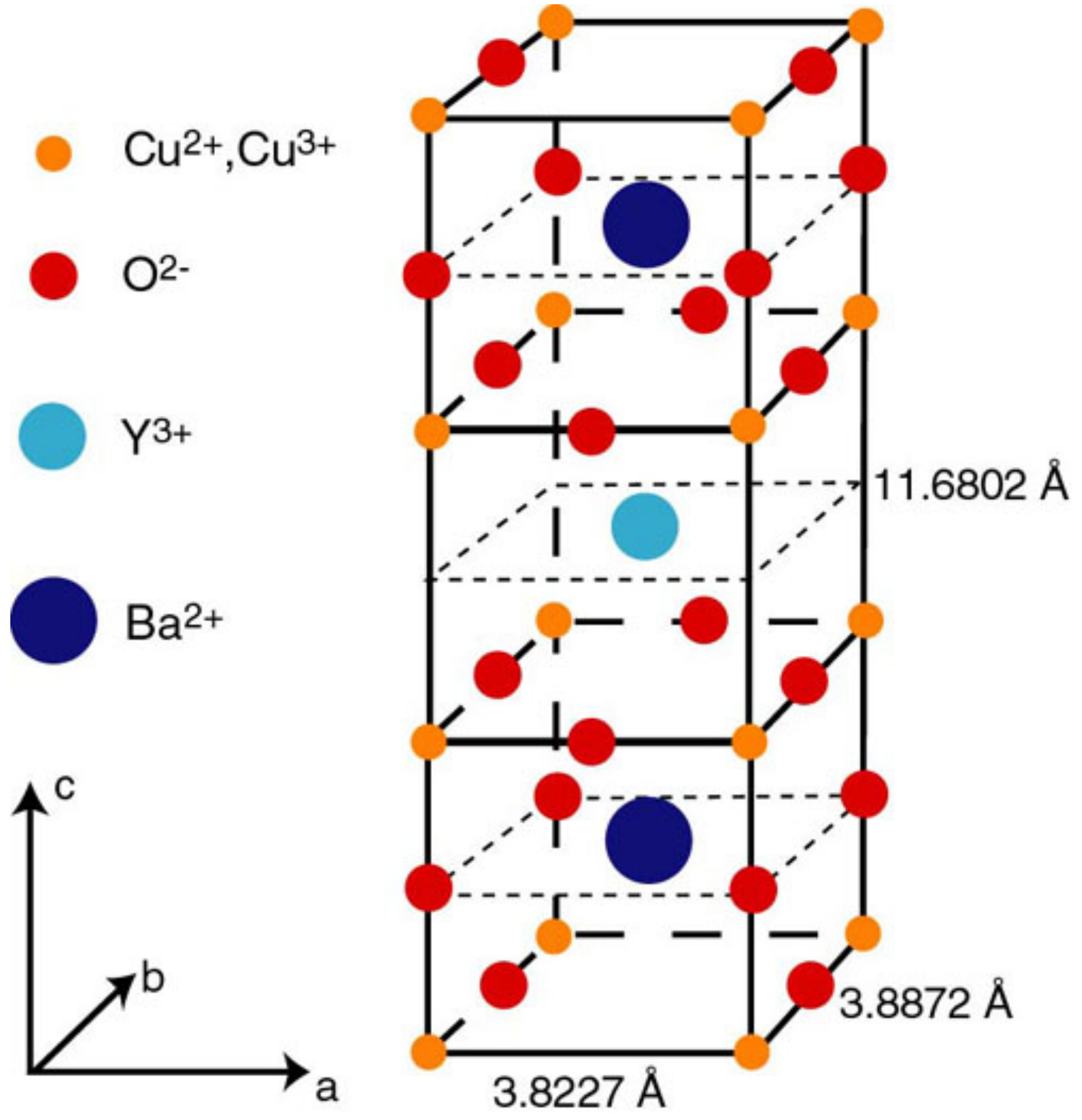
The long scattering time implies that individual electrons are well-defined.

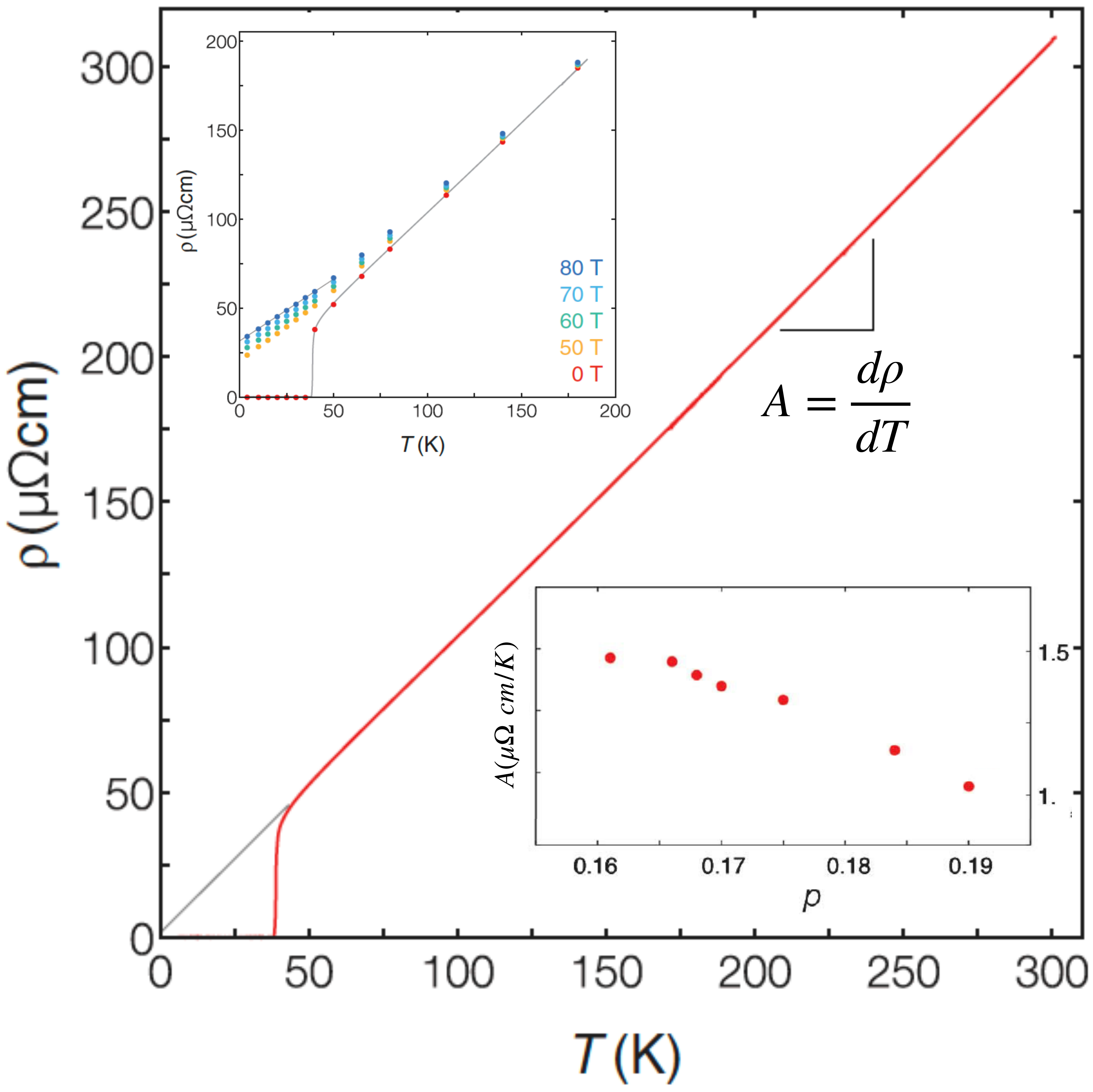
The motion of electrons is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
T^*
T_N
AF insulator
Superconductor
T_c
p_c

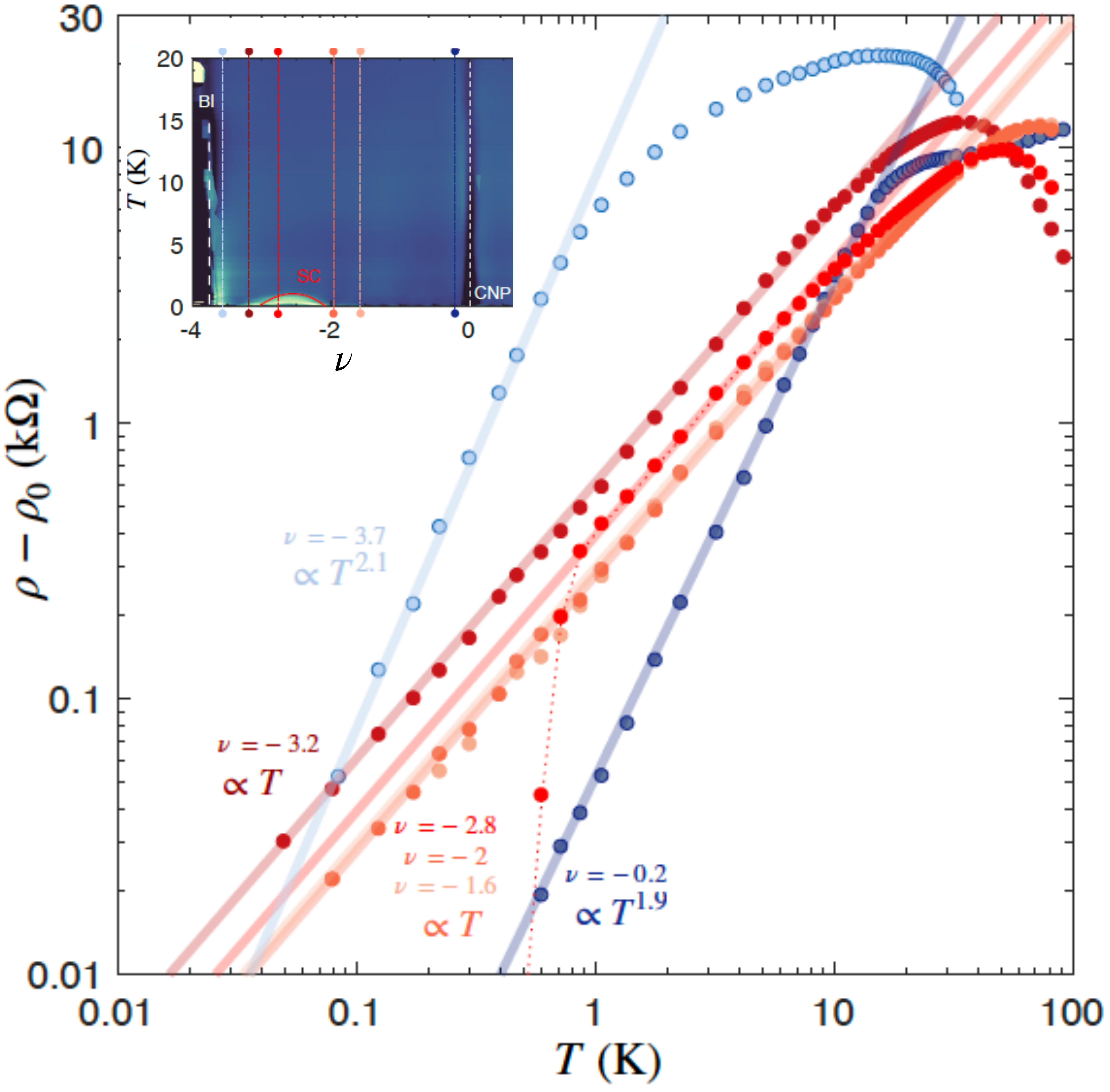

Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Strange Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å



LSCO: Giraldo-Gallo et al. 2018

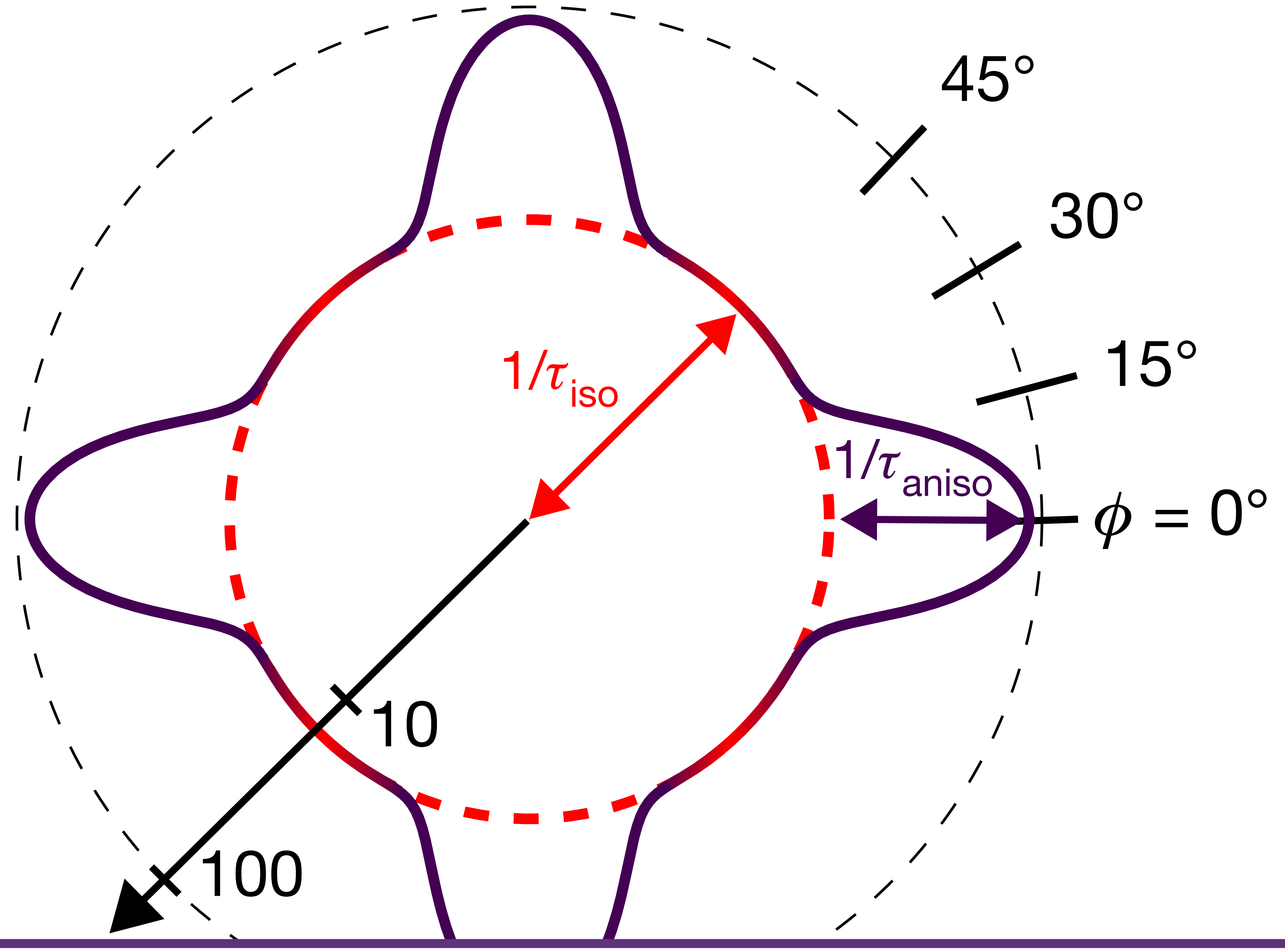


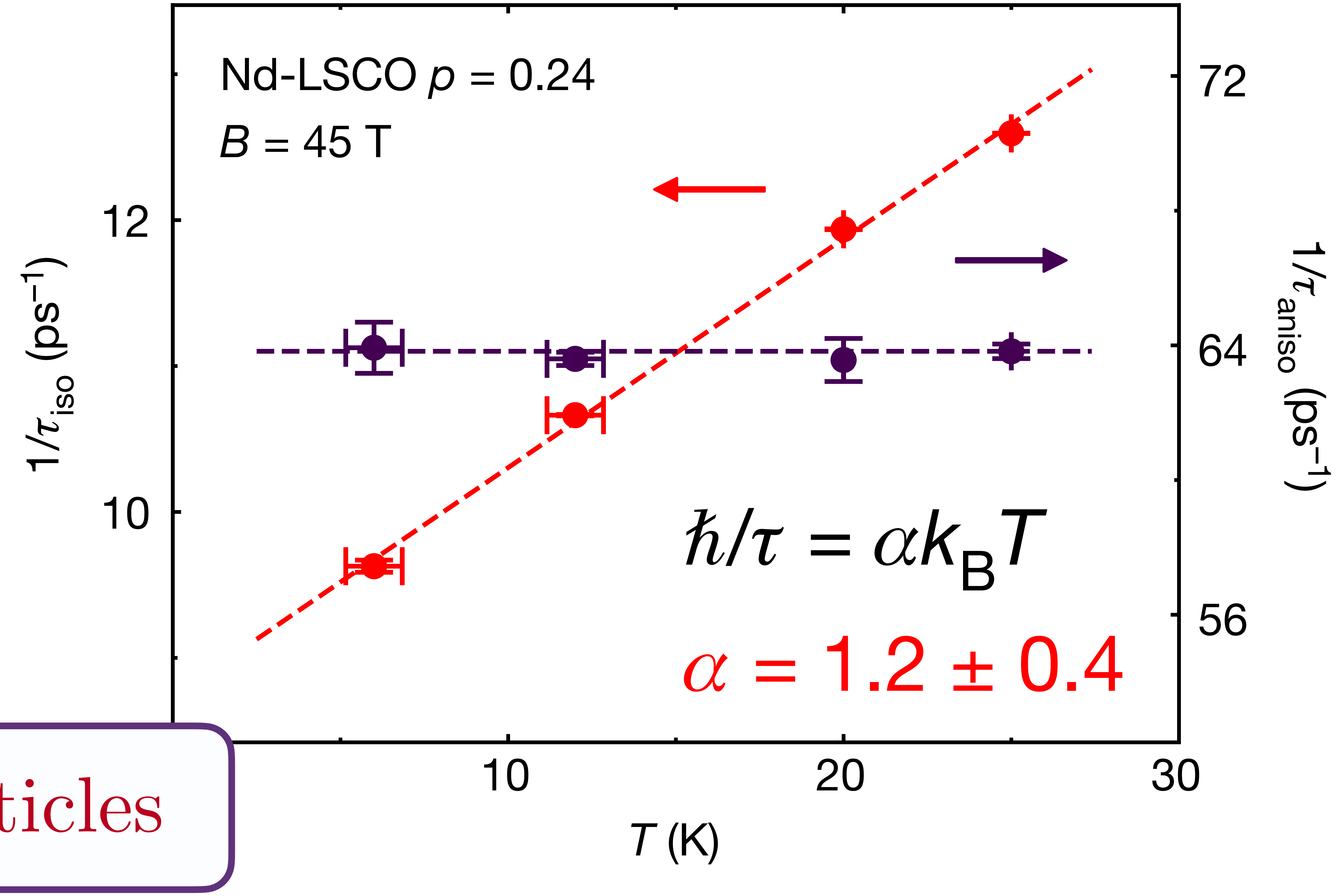
MATBG: Jaoui et al. 2021

Linear-in temperature resistivity from an isotropic Planckian scattering rate

Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw





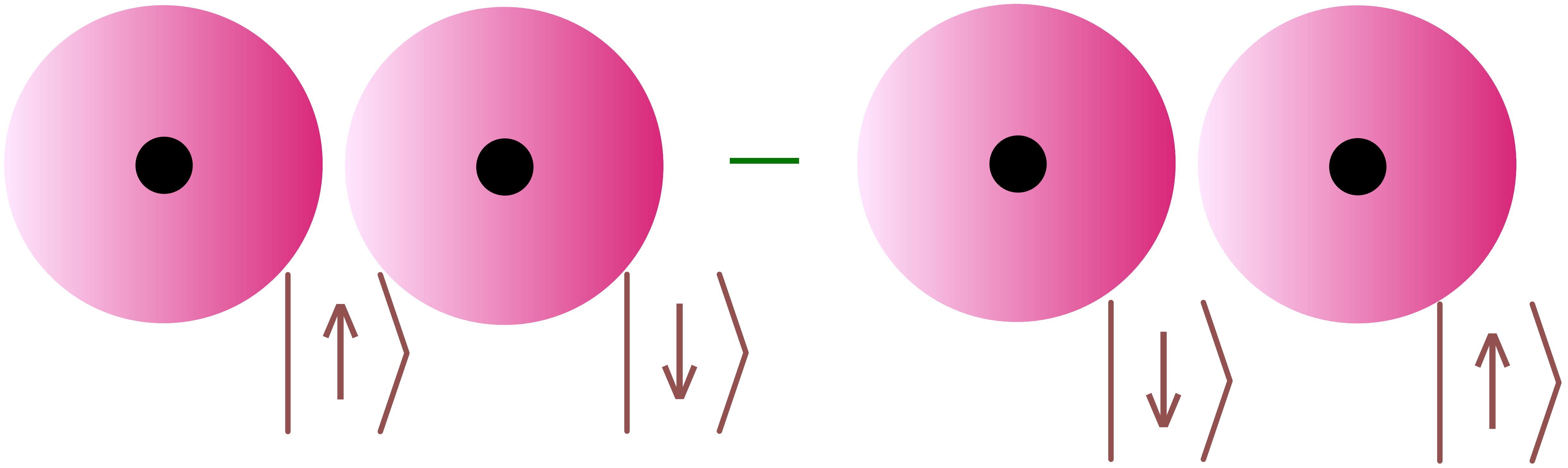
Current flow without quasiparticles

No Boltzmann-Landau quasiparticle description ⇒
Many particle quantum entanglement
from quantum interference between “collisions”

Sachdev-Ye-Kitaev Model

Quantum Entanglement

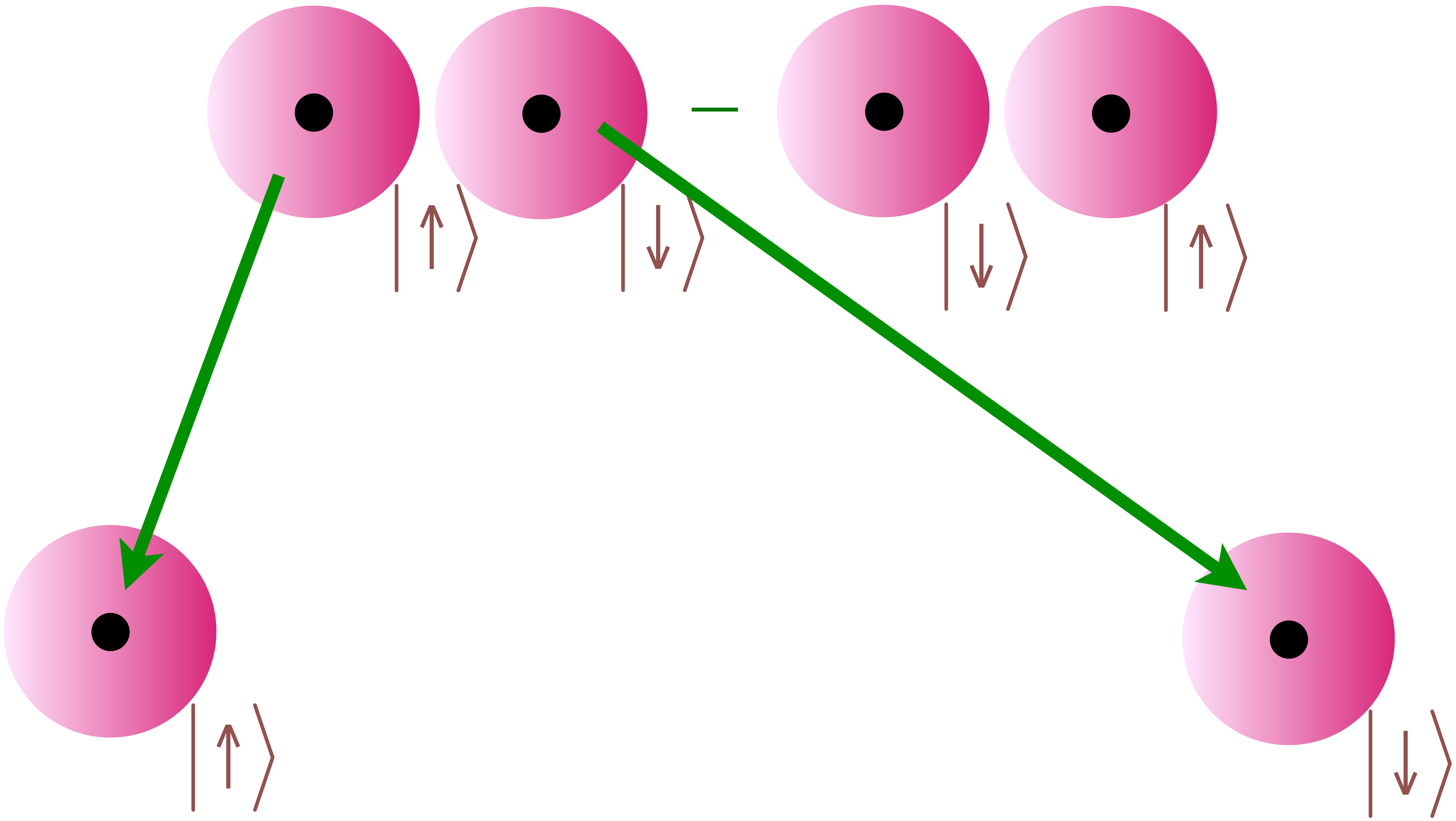
Einstein, Podolsky, Rosen (1935)



$$|\uparrow\rangle\,|\downarrow\rangle - |\downarrow\rangle\,|\uparrow\rangle$$

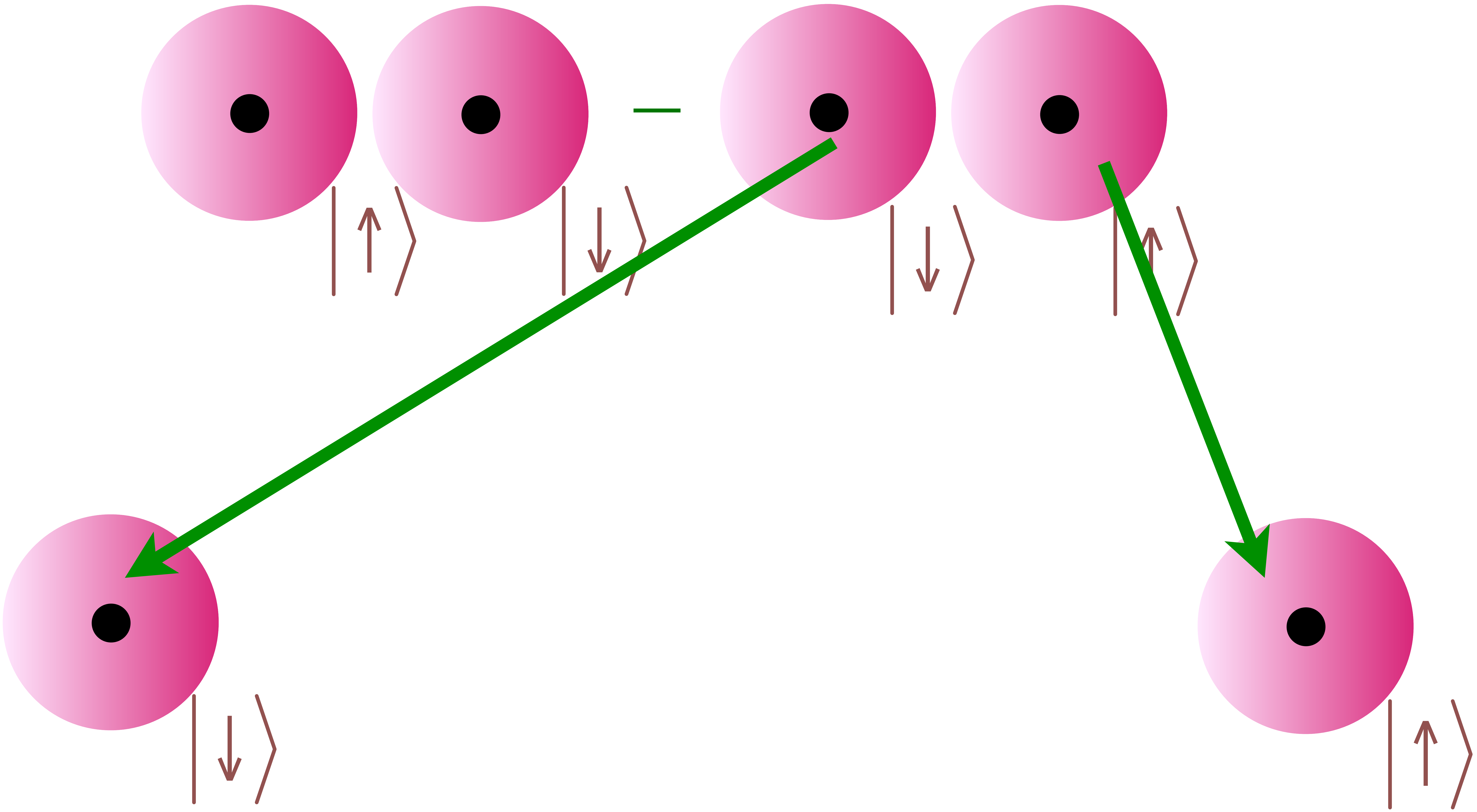
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



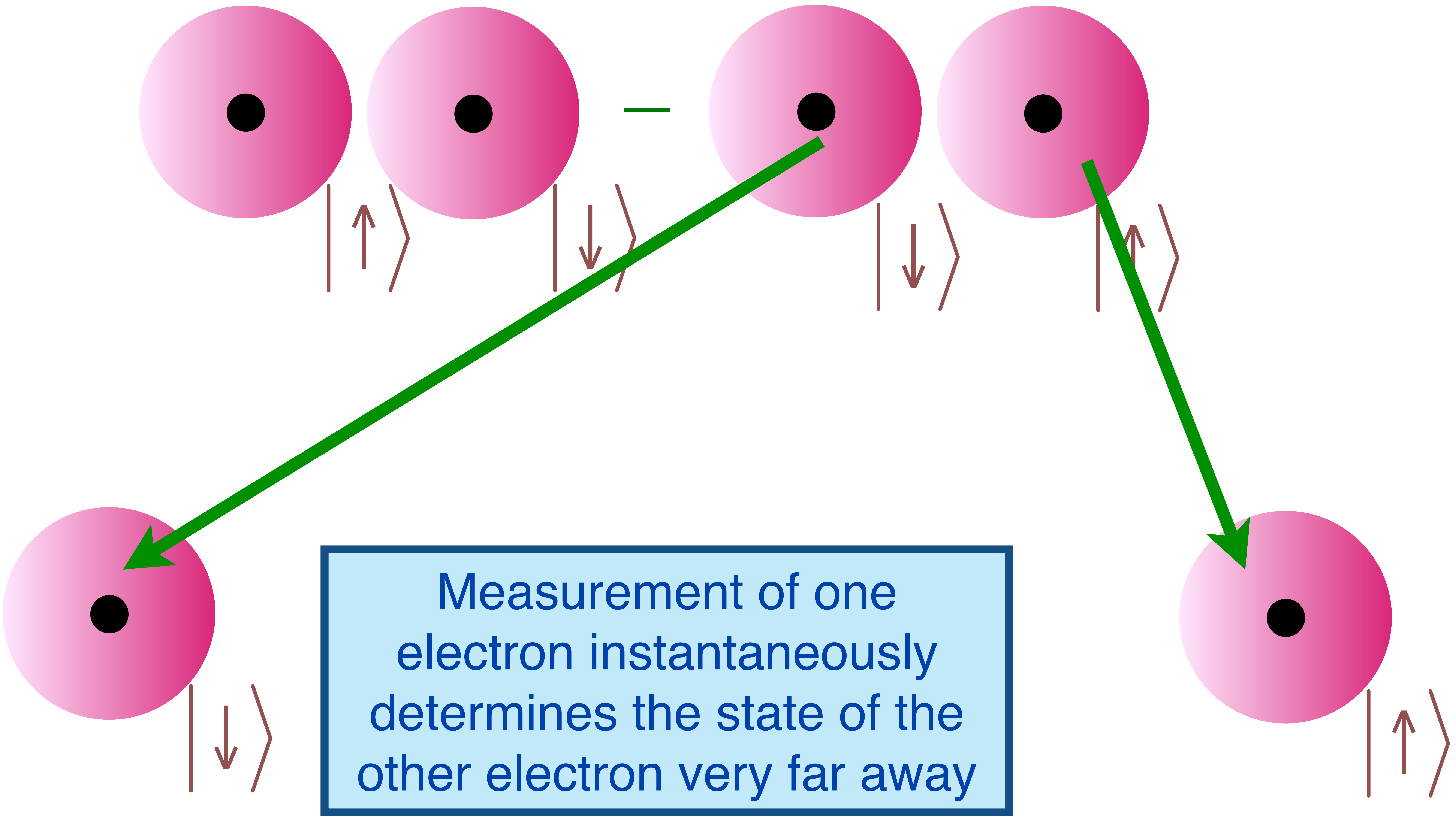
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



Quantum Entanglement

Einstein, Podolsky, Rosen (1935)

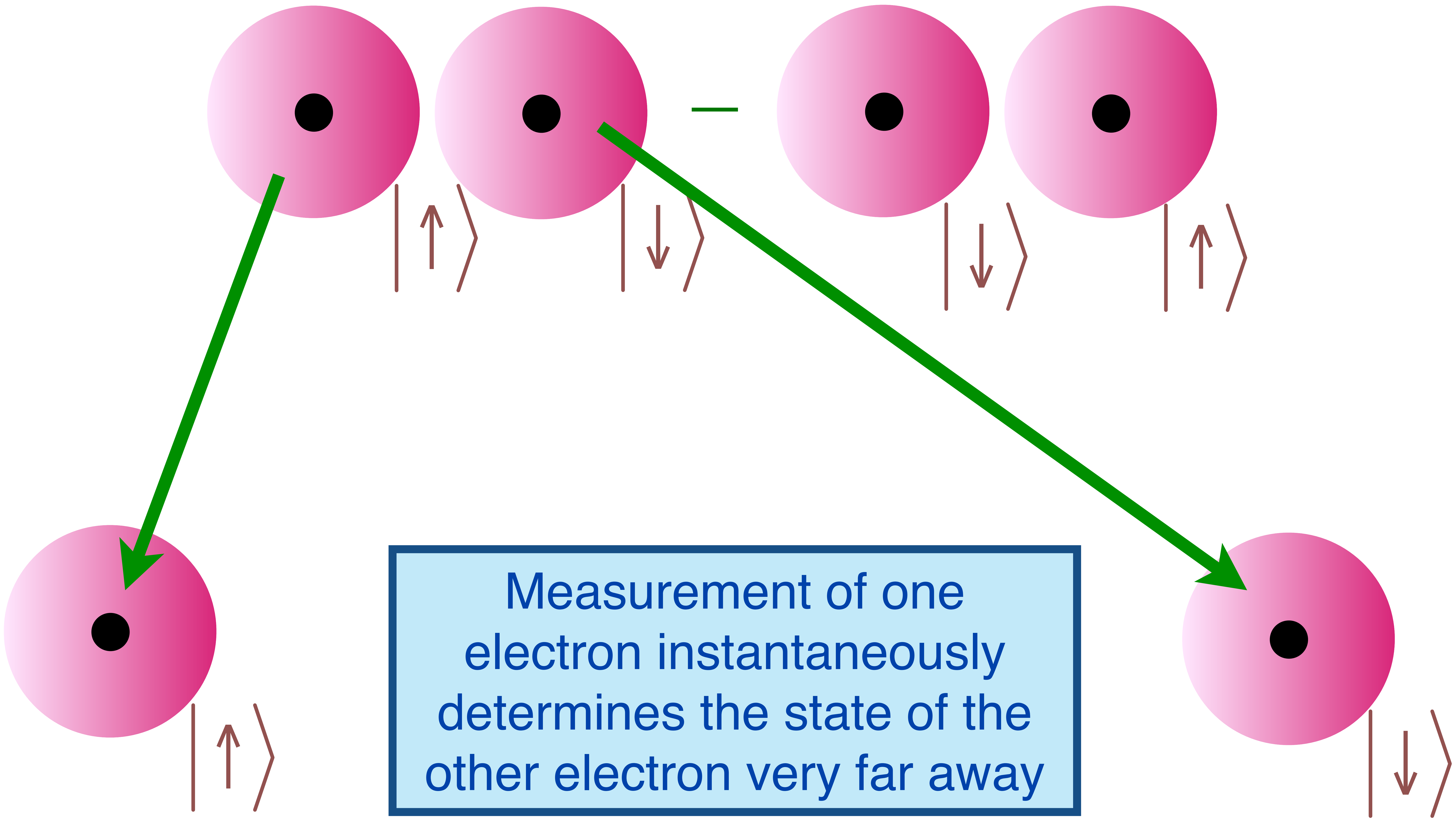


Measurement of one electron instantaneously determines the state of the other electron very far away

Spooky action at a distance! (Einstein, 1947)

Quantum Entanglement

Einstein, Podolsky, Rosen (1935)

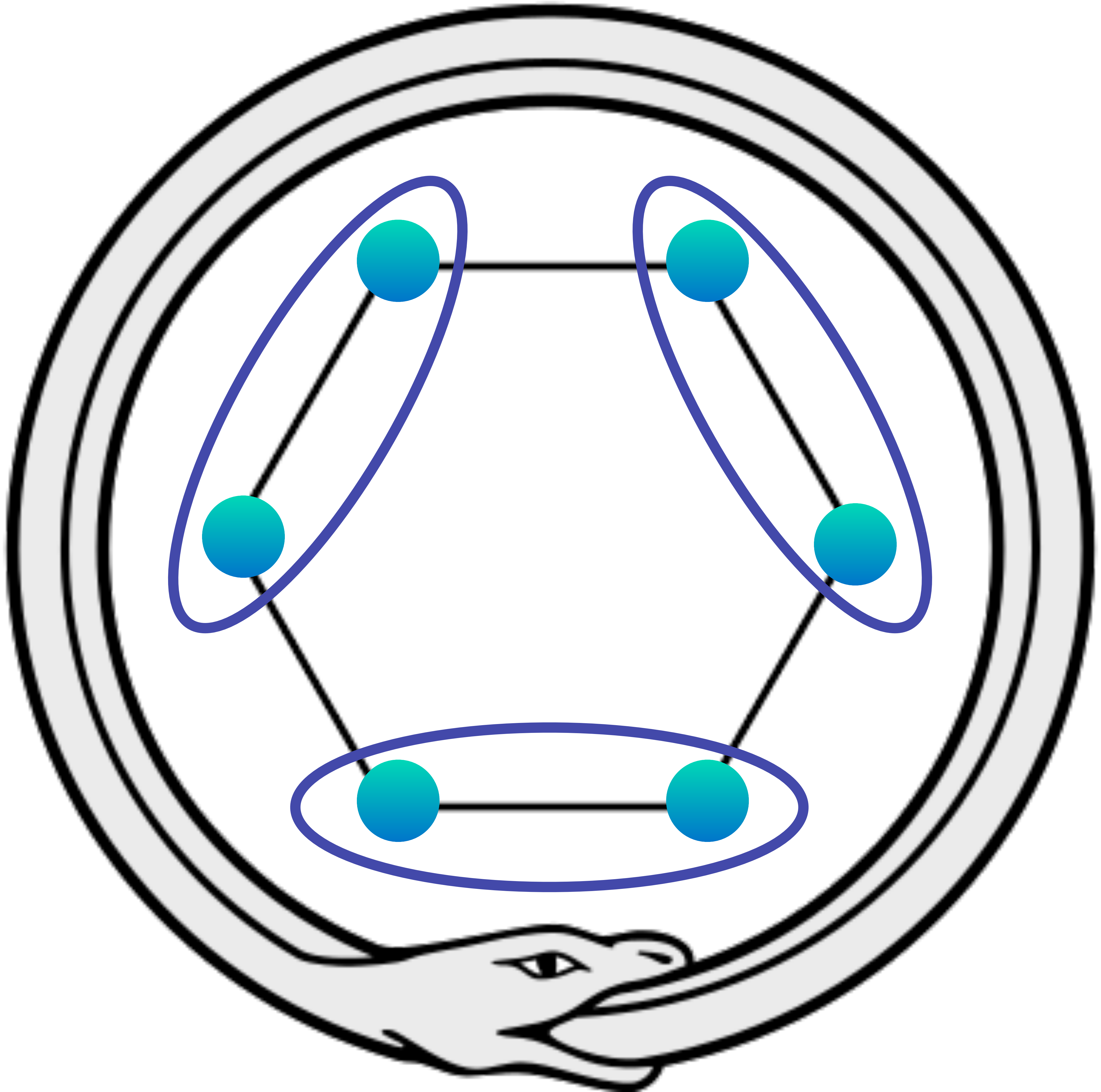


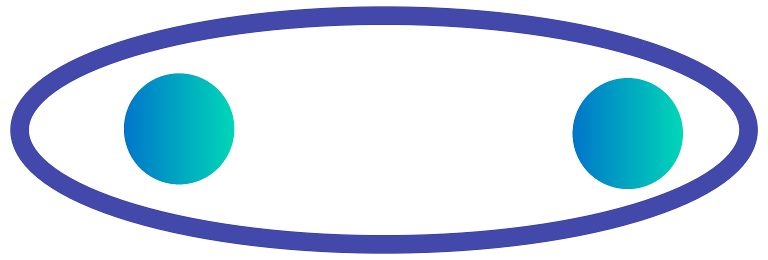
Measurement of one electron instantaneously determines the state of the other electron very far away

Spooky action at a distance! (Einstein, 1947)

Kekule's spooky dream

Here Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail[*]





$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

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[*] Wikipedia

Kekule's spooky dream

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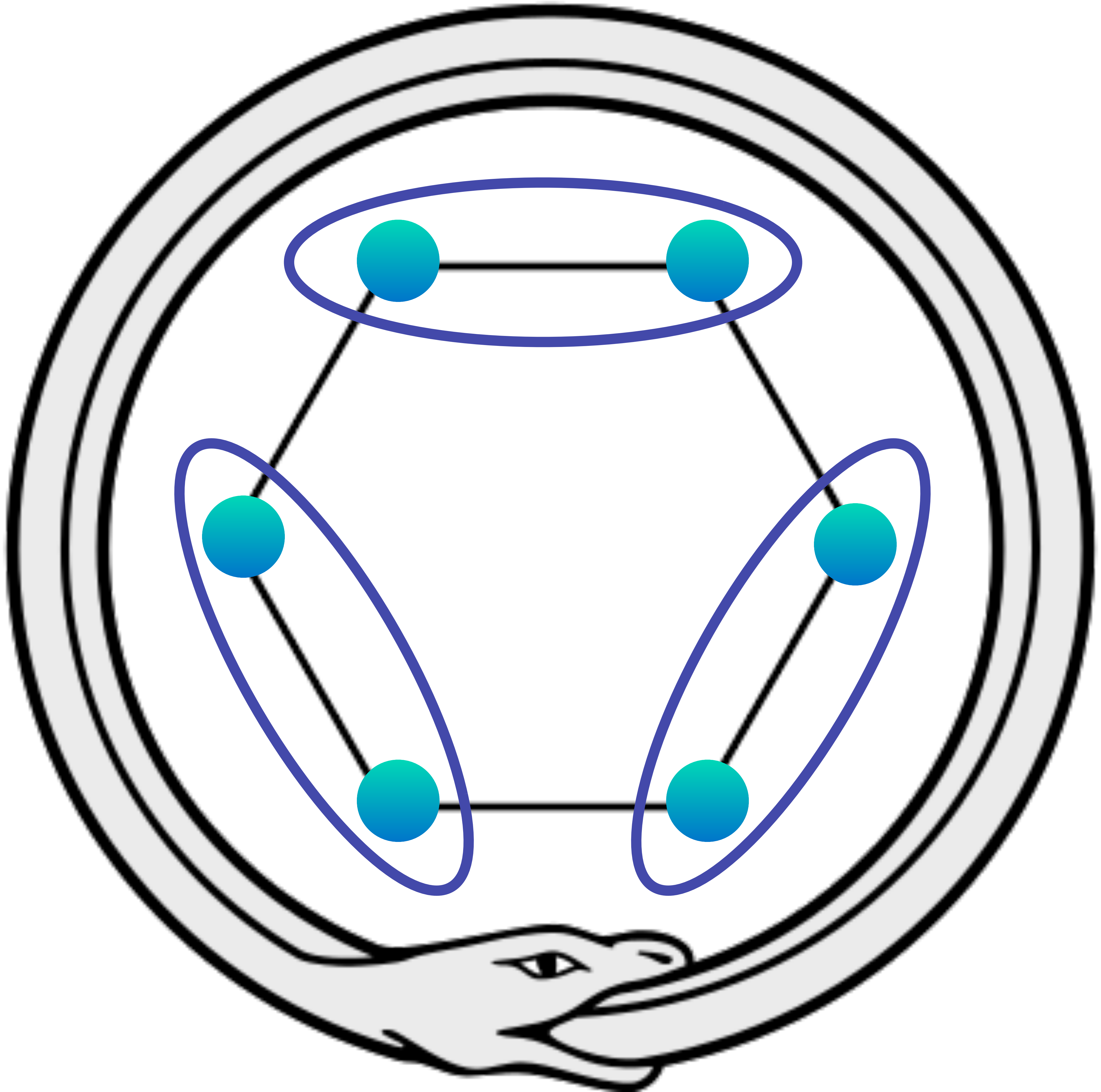

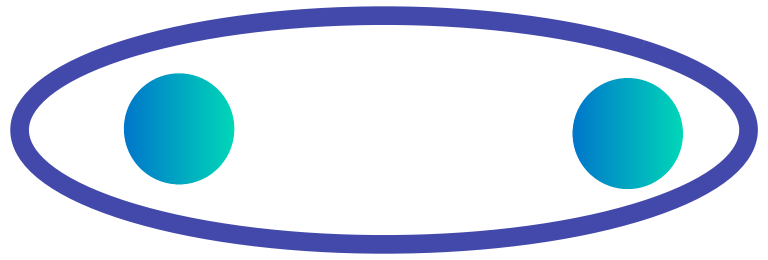

$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

*Wikipedia

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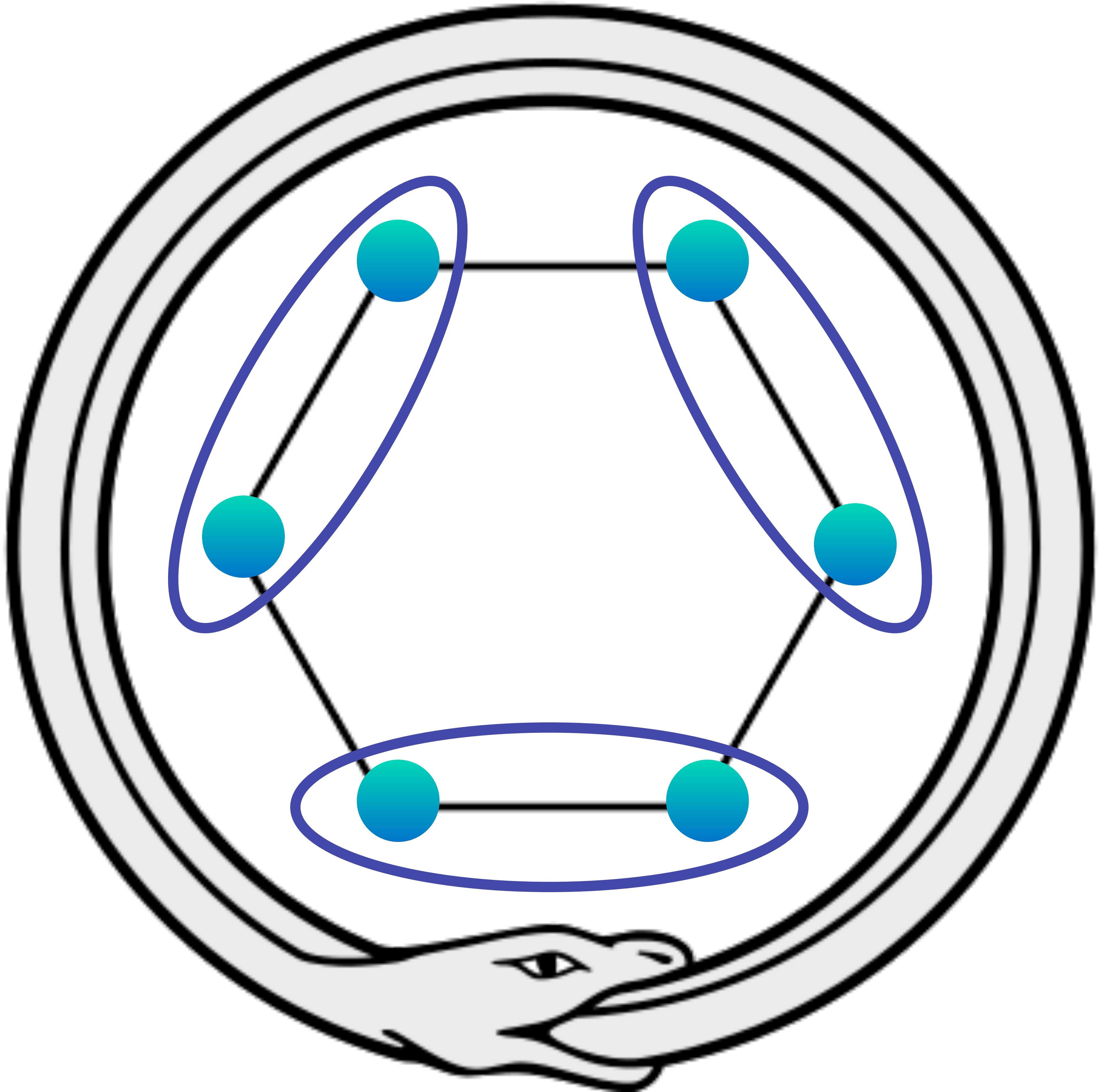
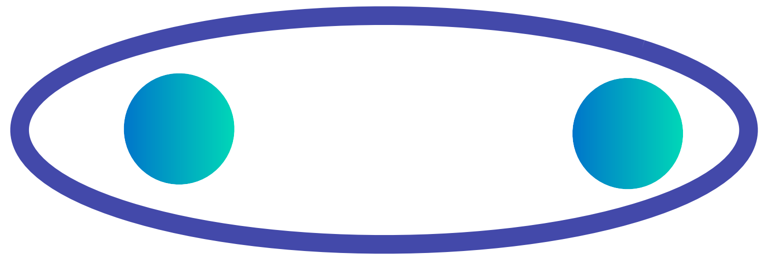
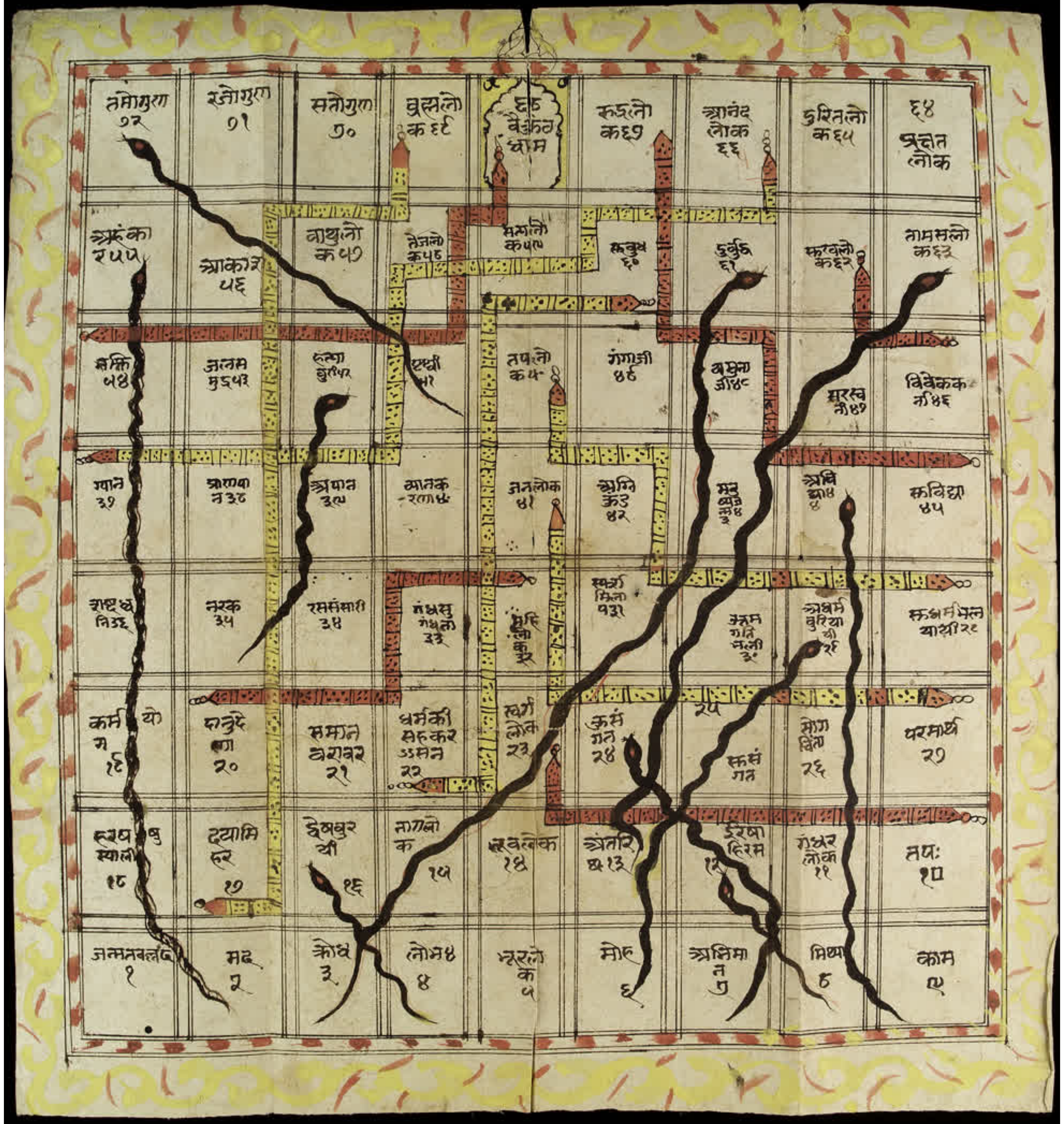




$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

[*] Wikipedia



My spooky dream*

Ancient Indian game of Snakes and Ladders

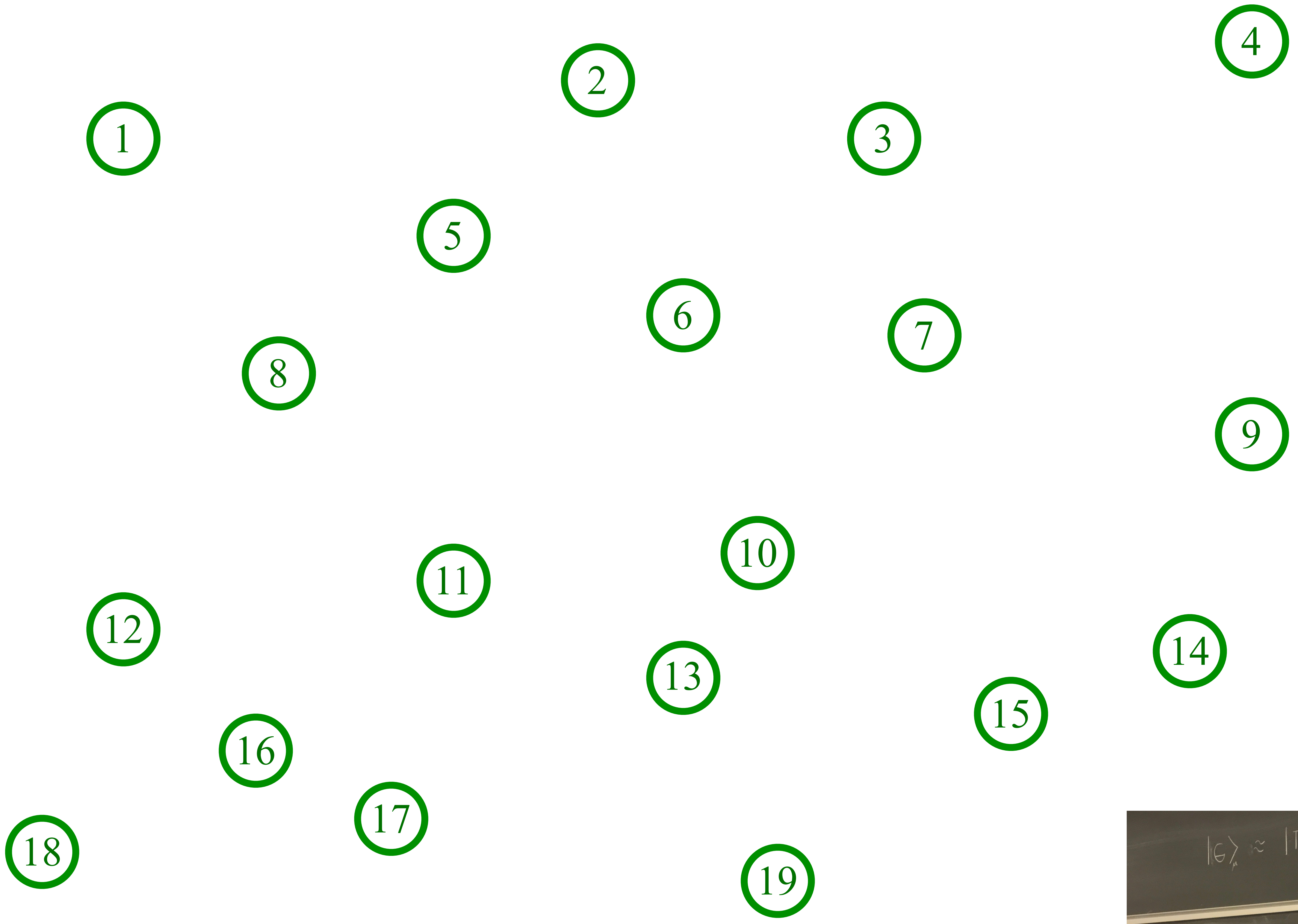
*Not true

A solvable model of multi-particle entanglement which accounts for quantum interference between successive collisions:

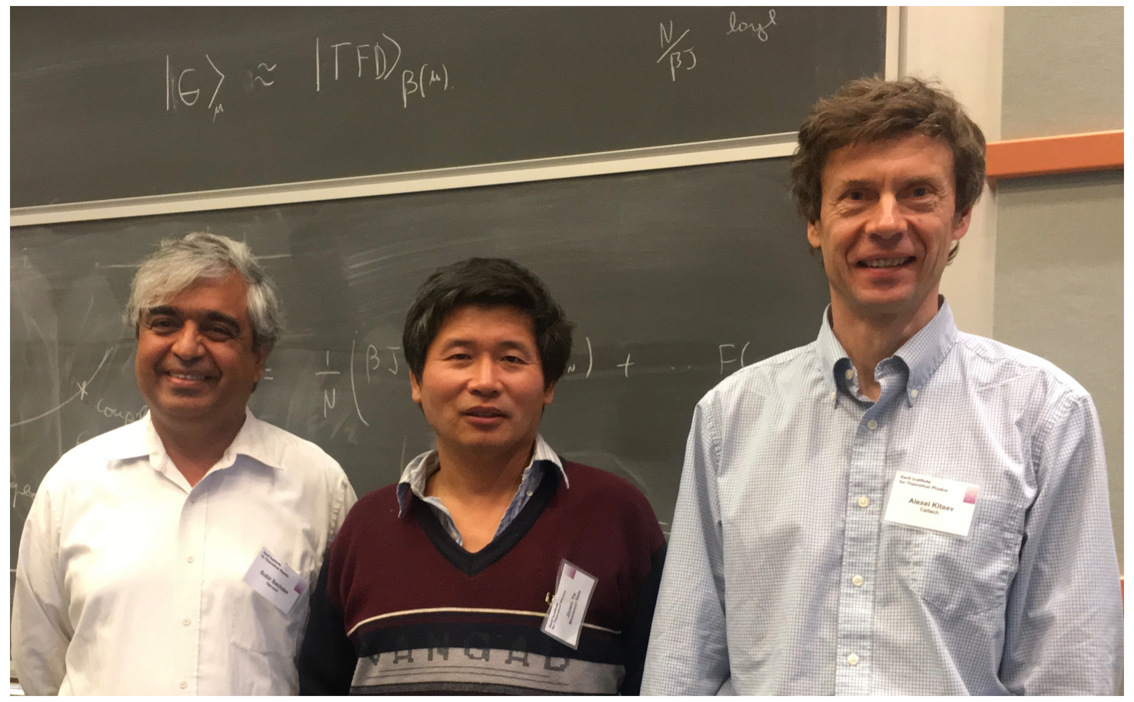
leading to a metal with no particle-like excitations

The SYK model

Sachdev, Ye (1993); Kitaev (2015)

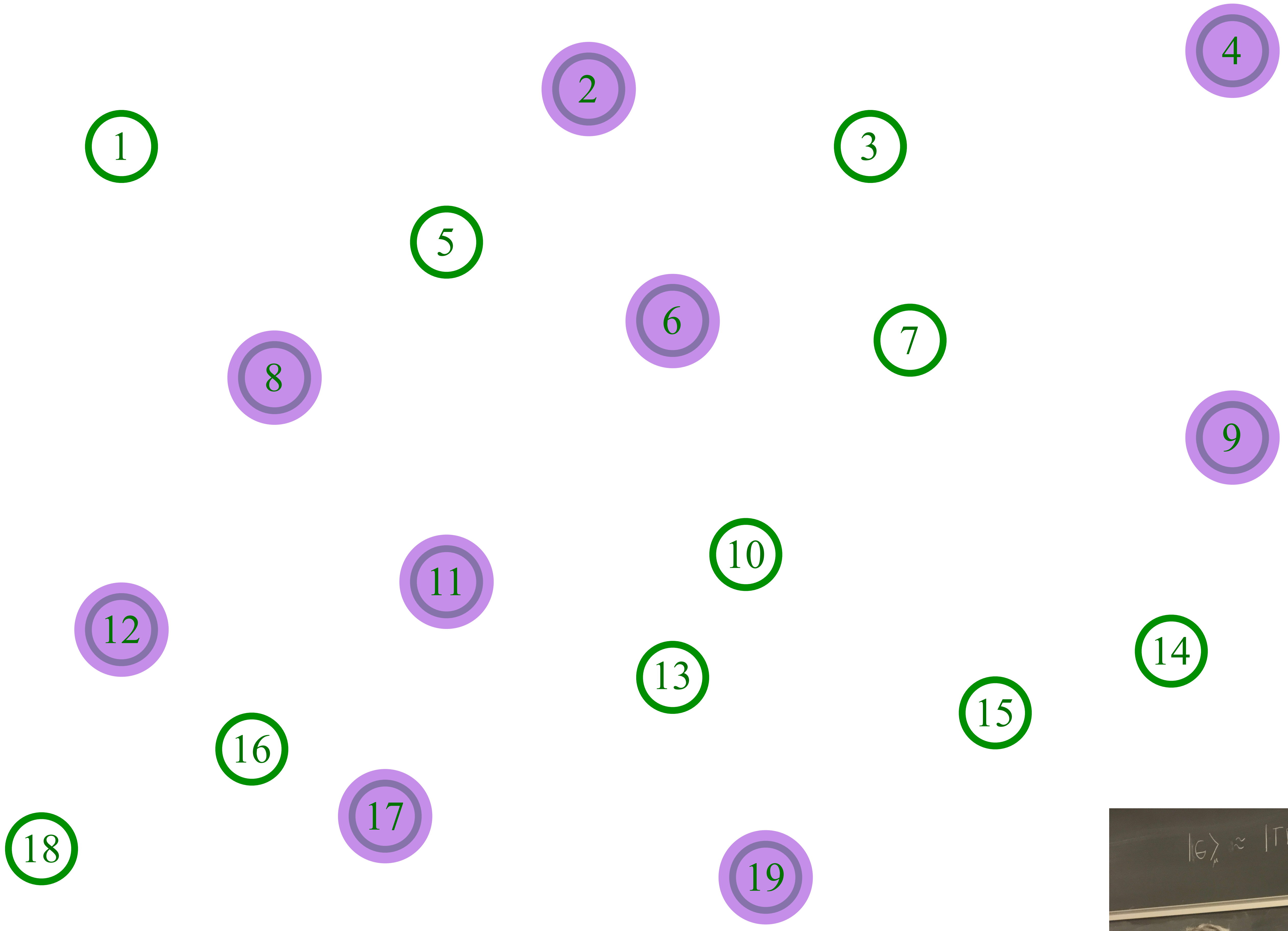


Pick a set of random positions

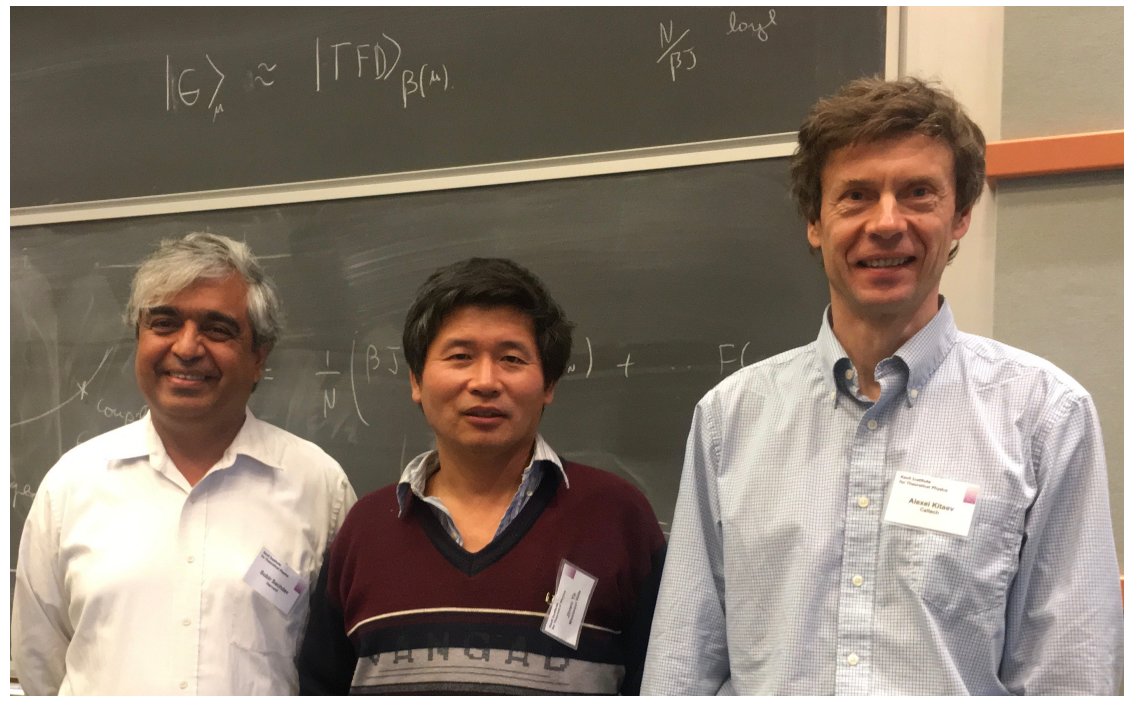


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



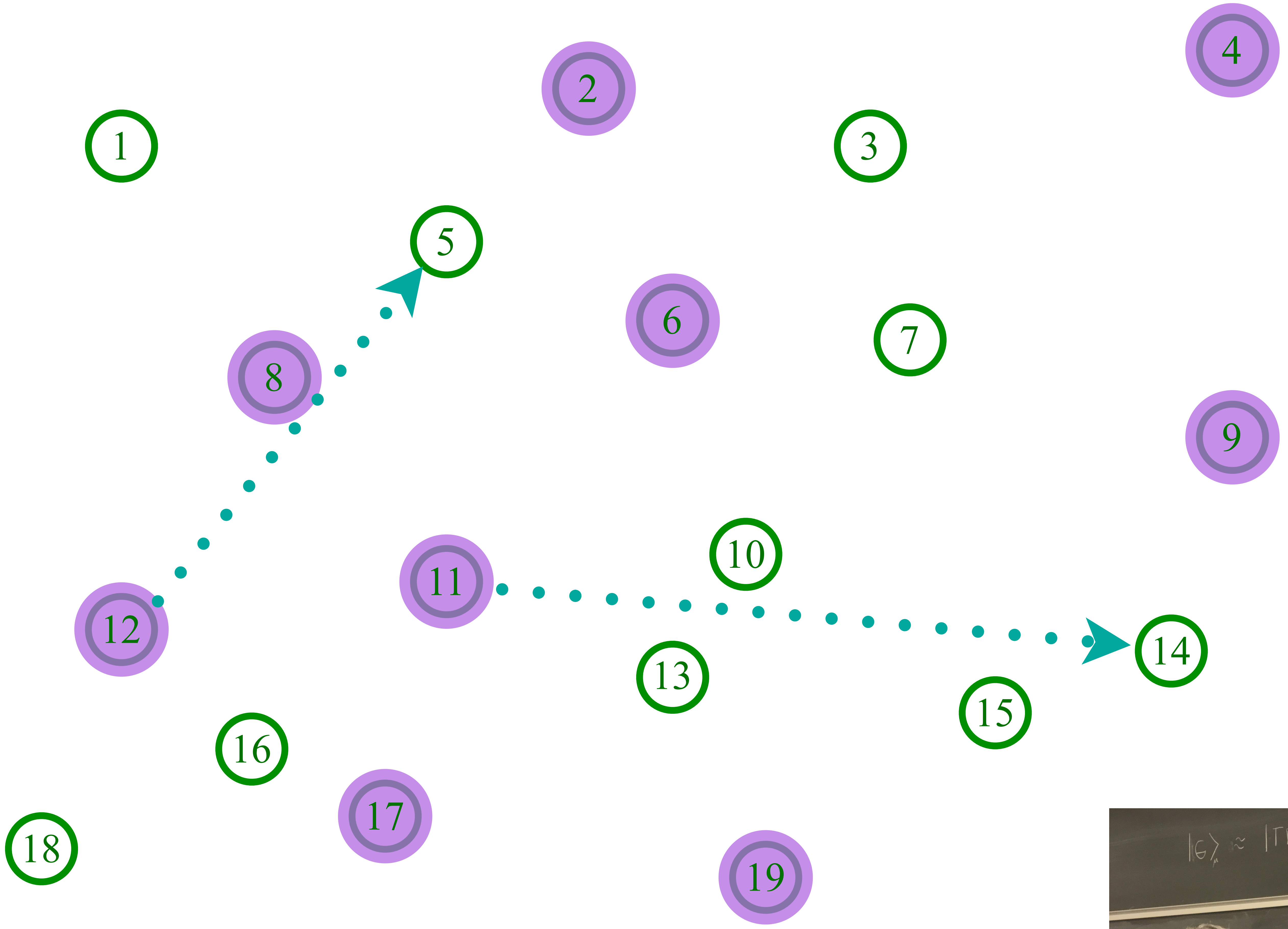
Place electrons randomly on some sites



The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



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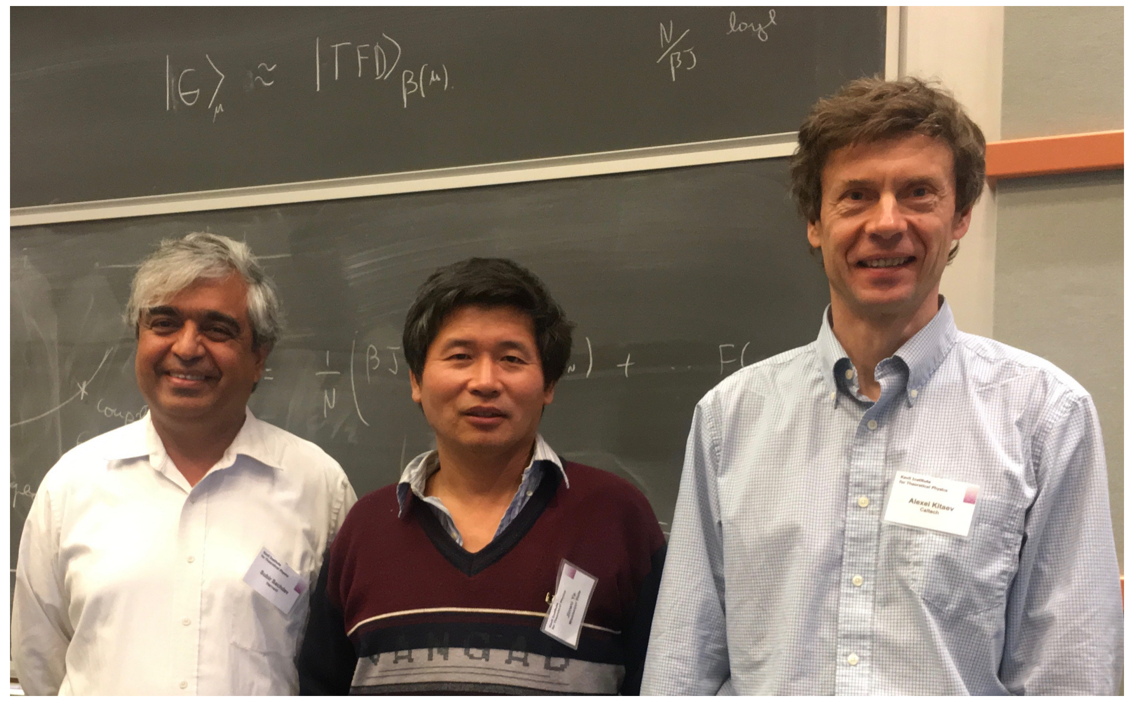

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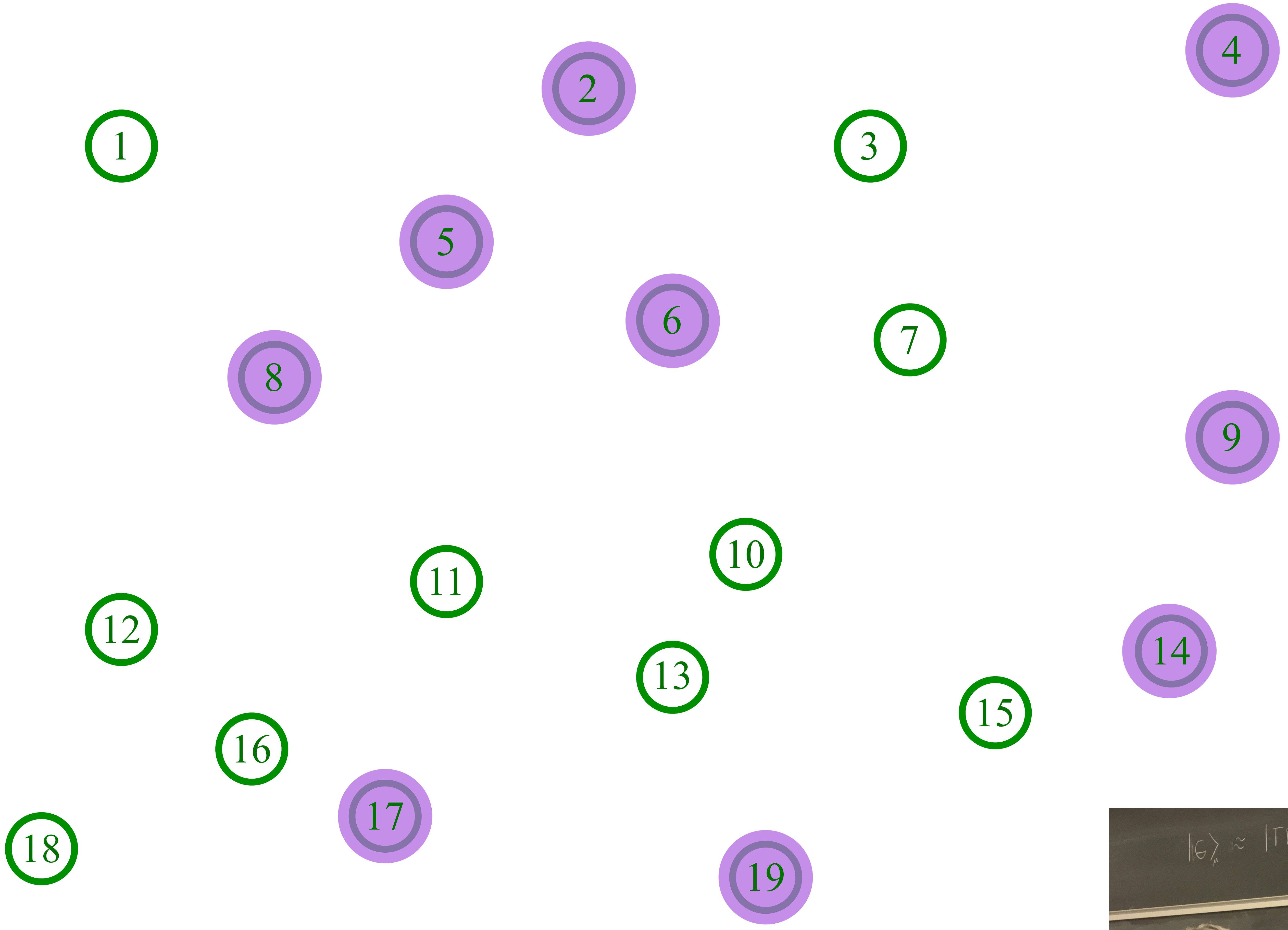
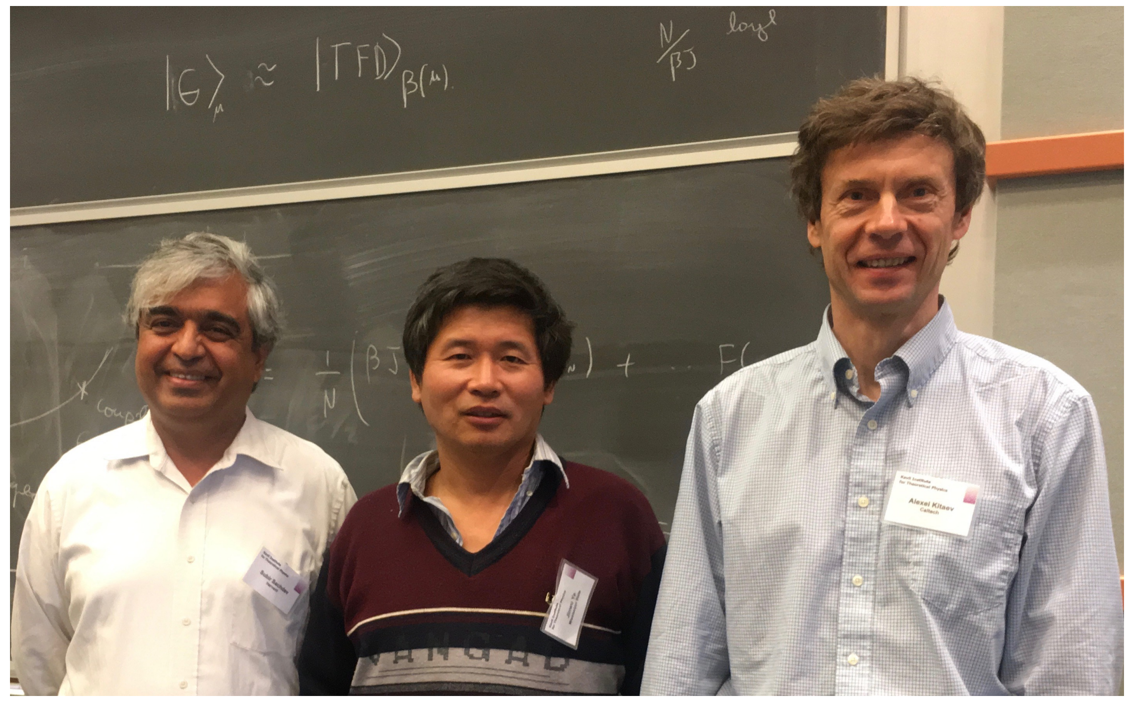


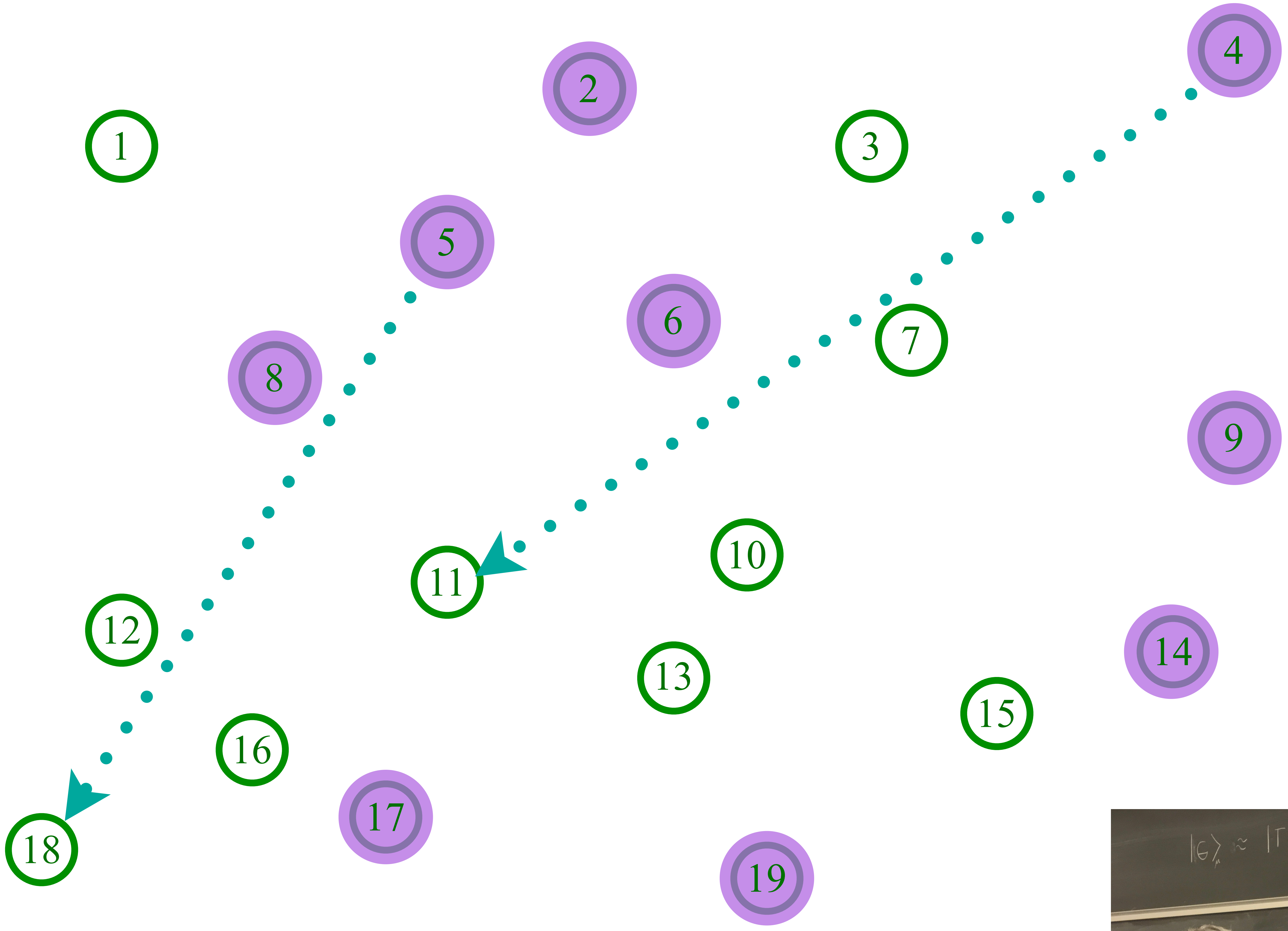
Entangle electrons pairwise randomly



The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



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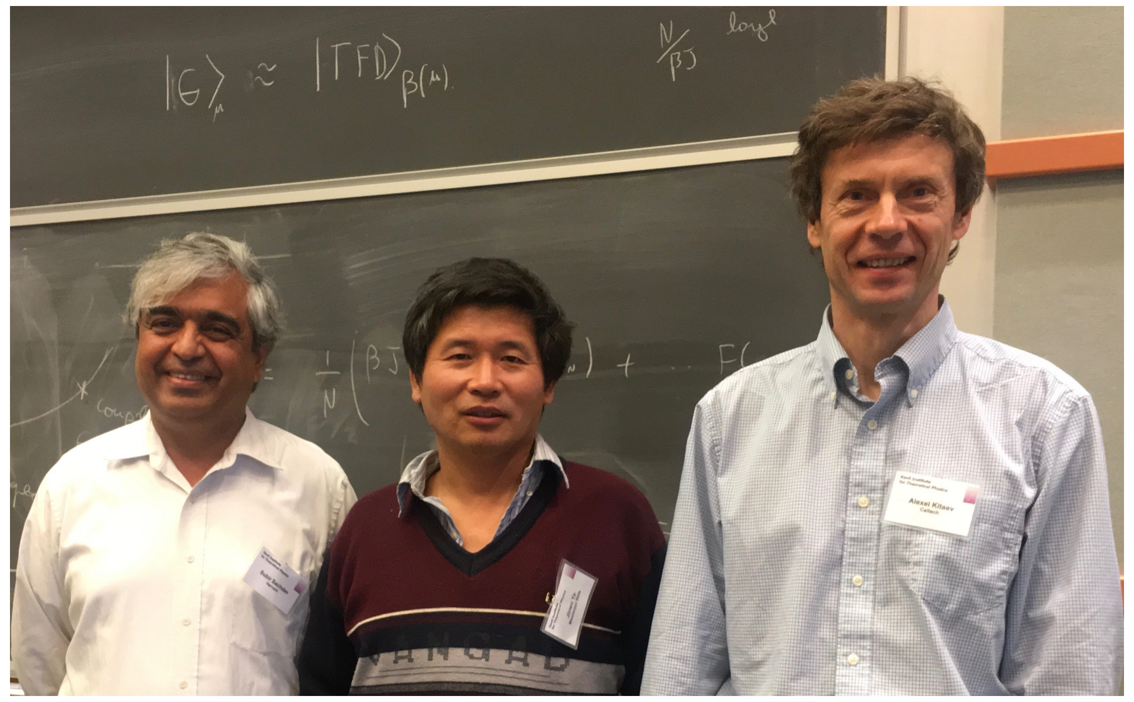
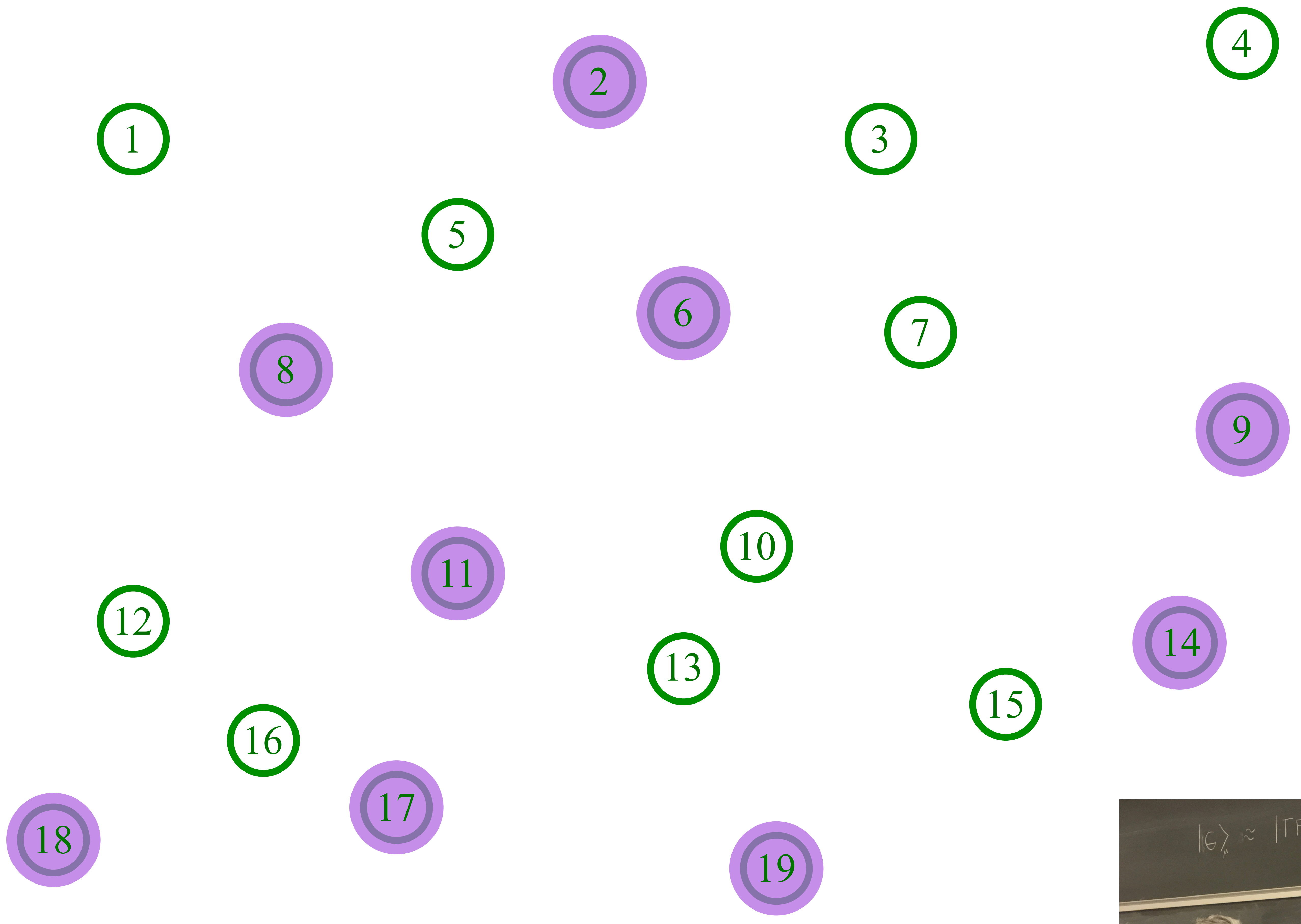


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{4,5;11,18}$



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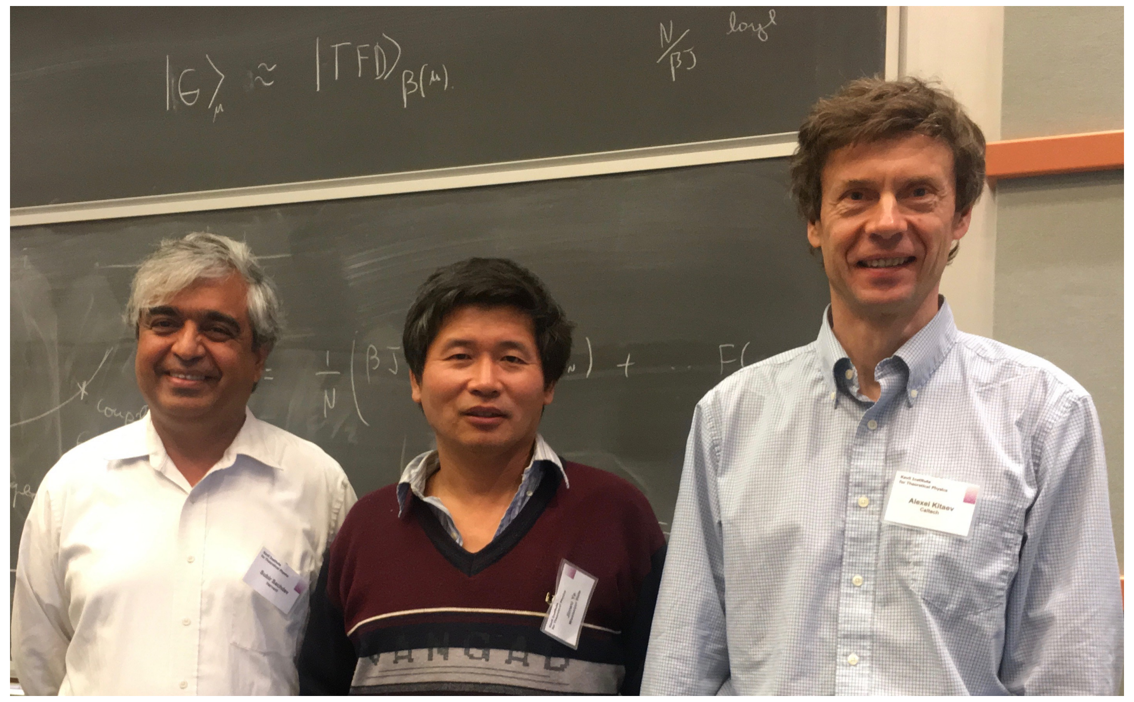
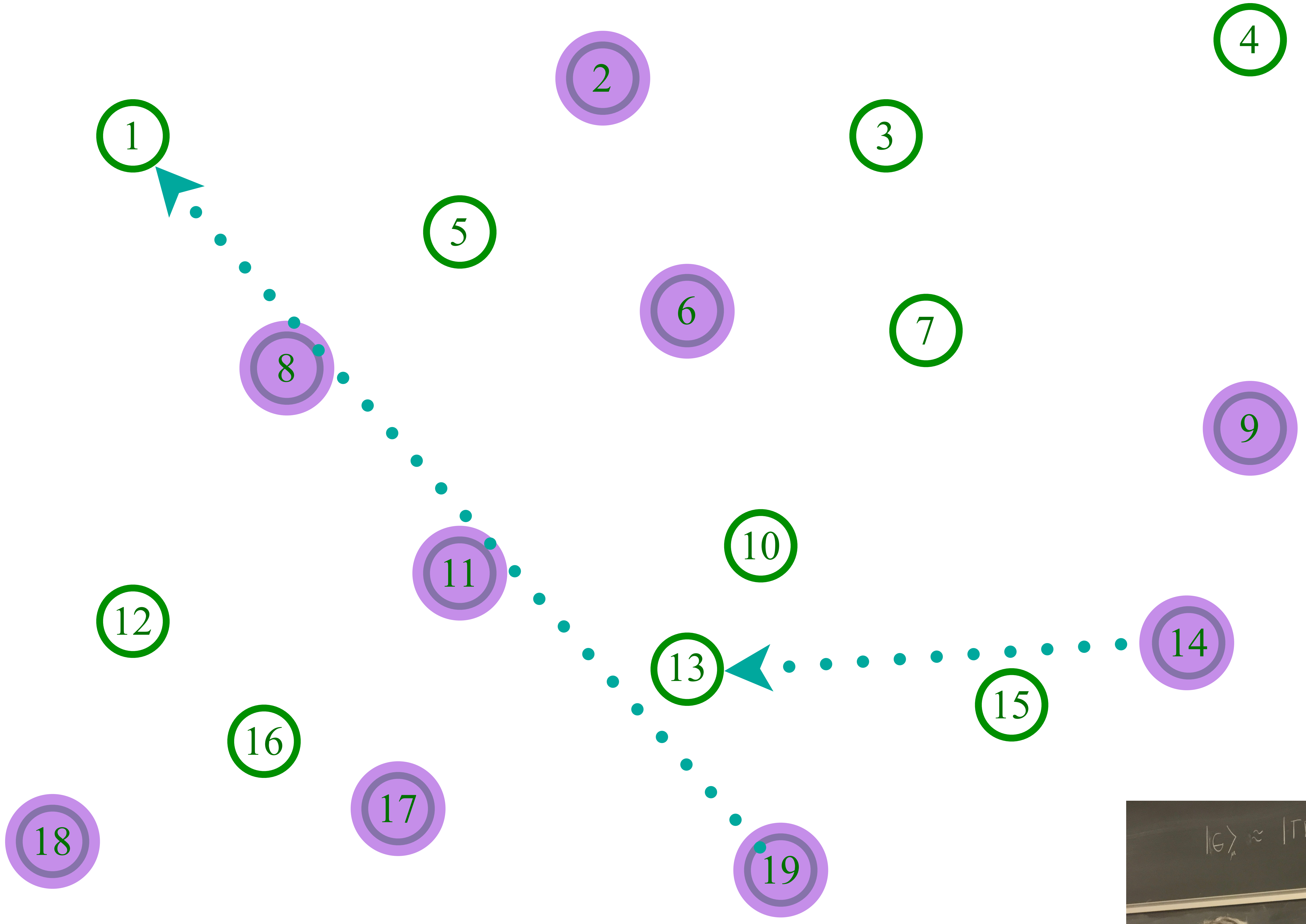


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



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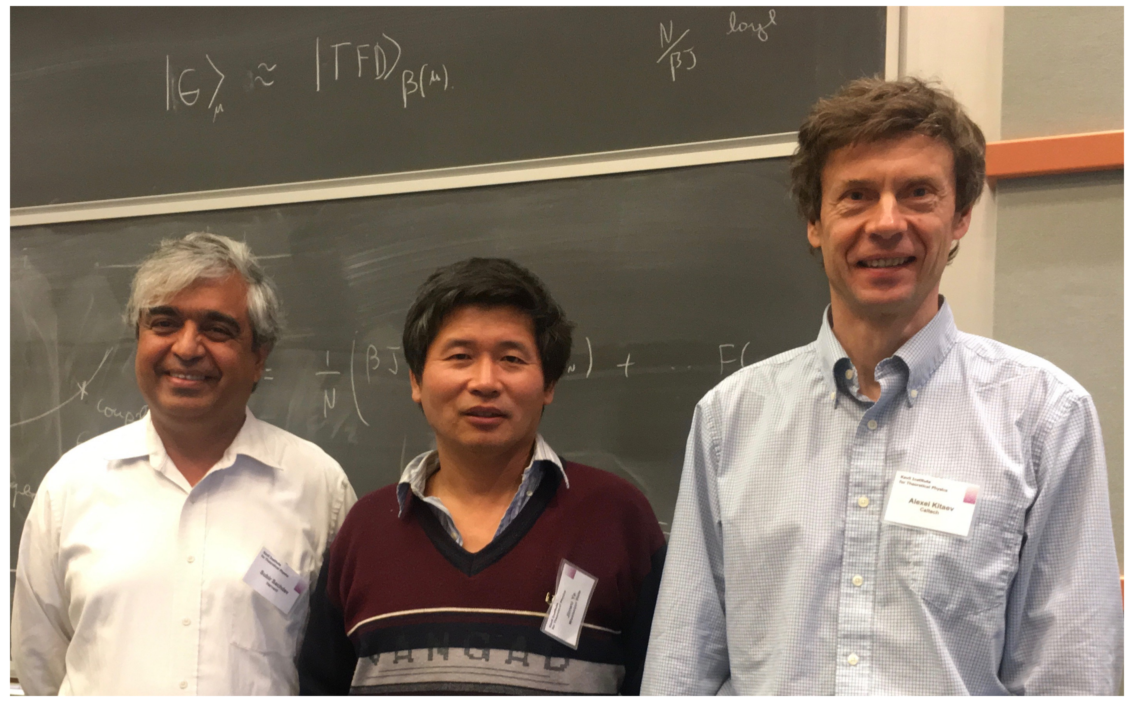

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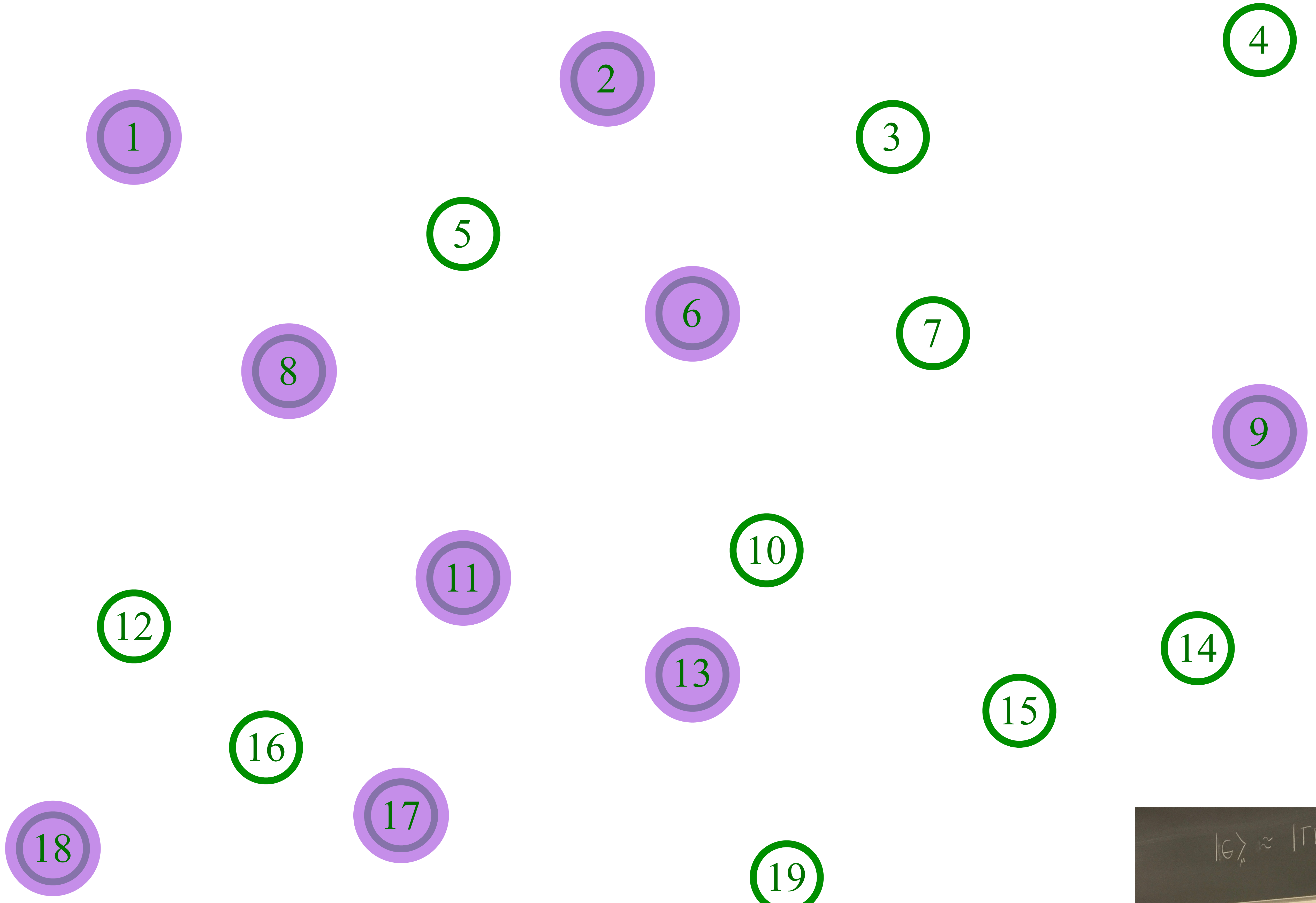
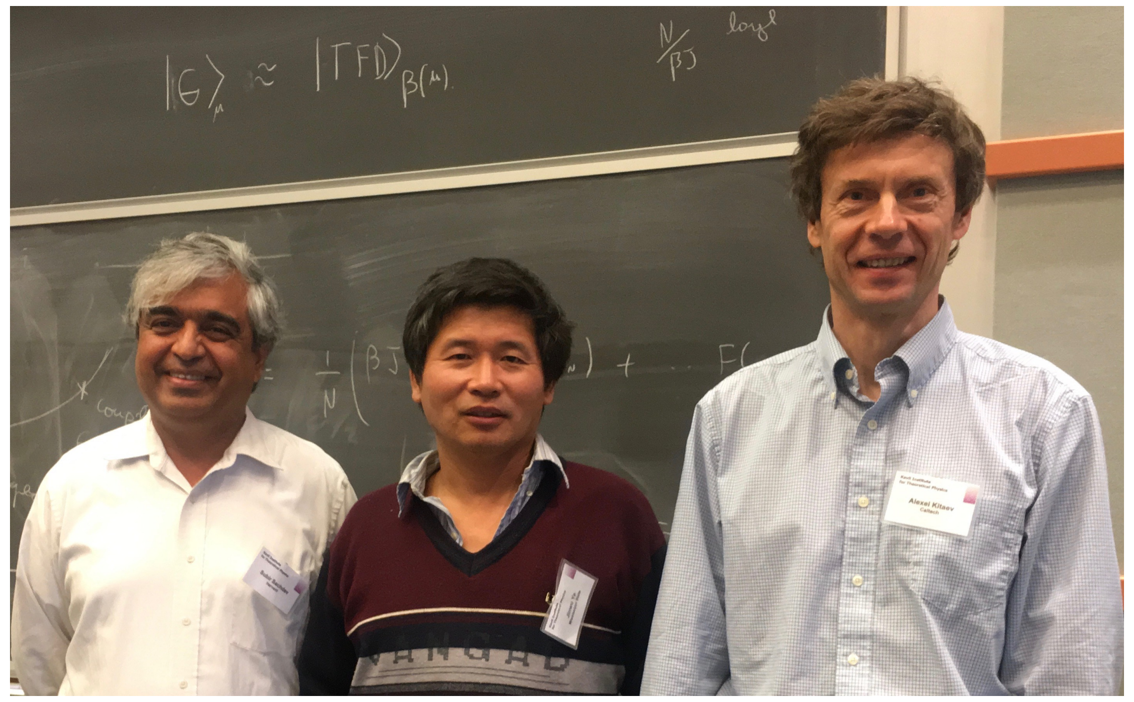


$$U_{14,19;1,13}$$

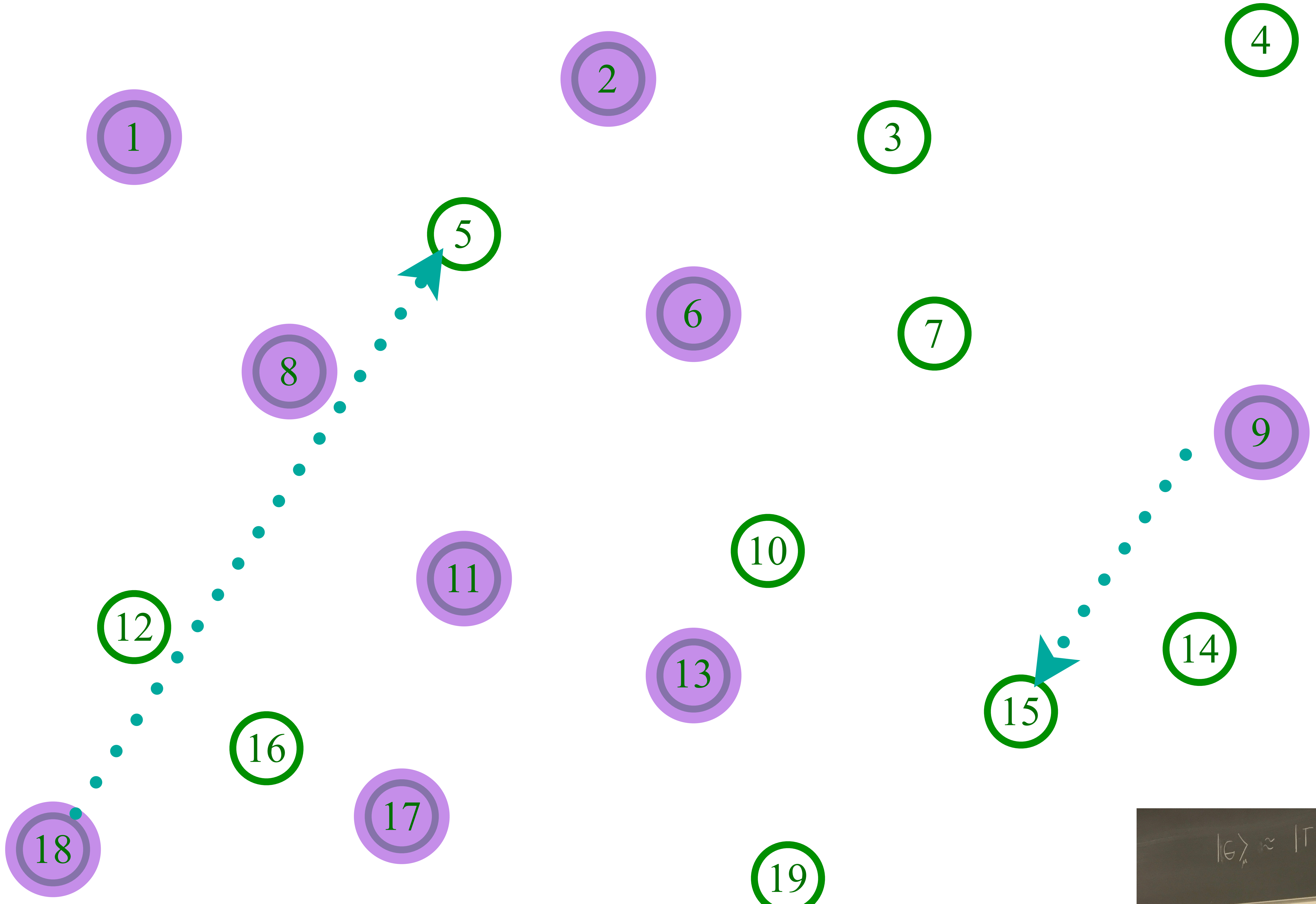
Entangle electrons pairwise randomly



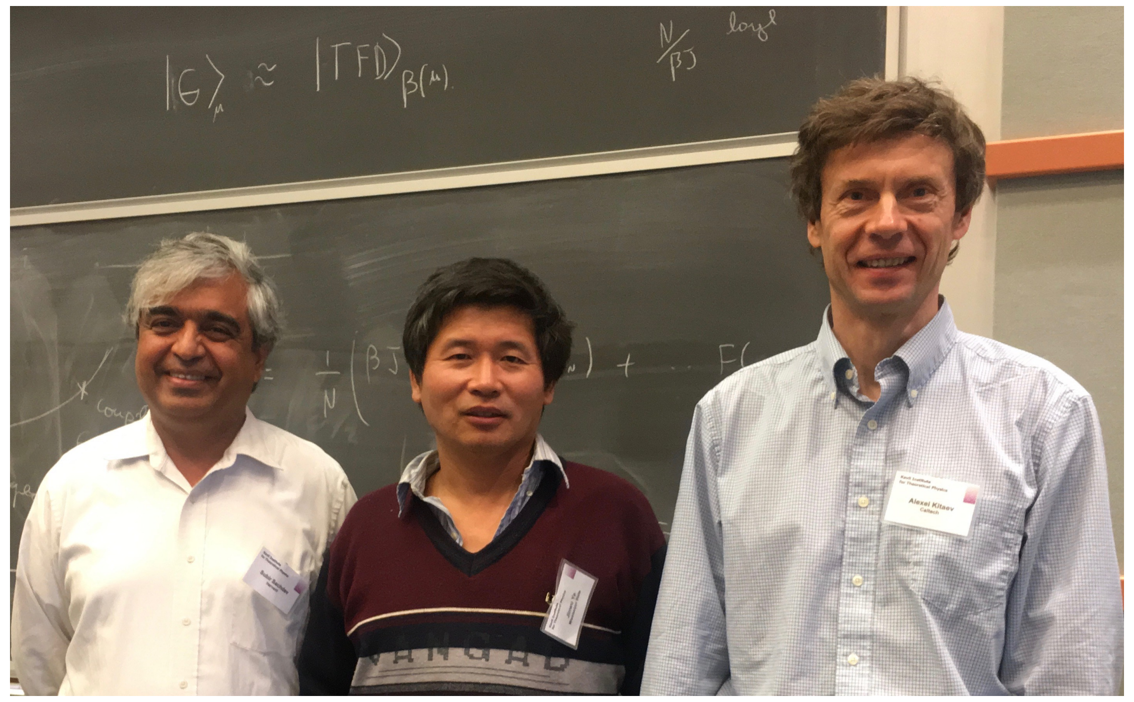
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



Entangle electrons pairwise randomly



The SYK model

Sachdev, Ye (1993); Kitaev (2015)

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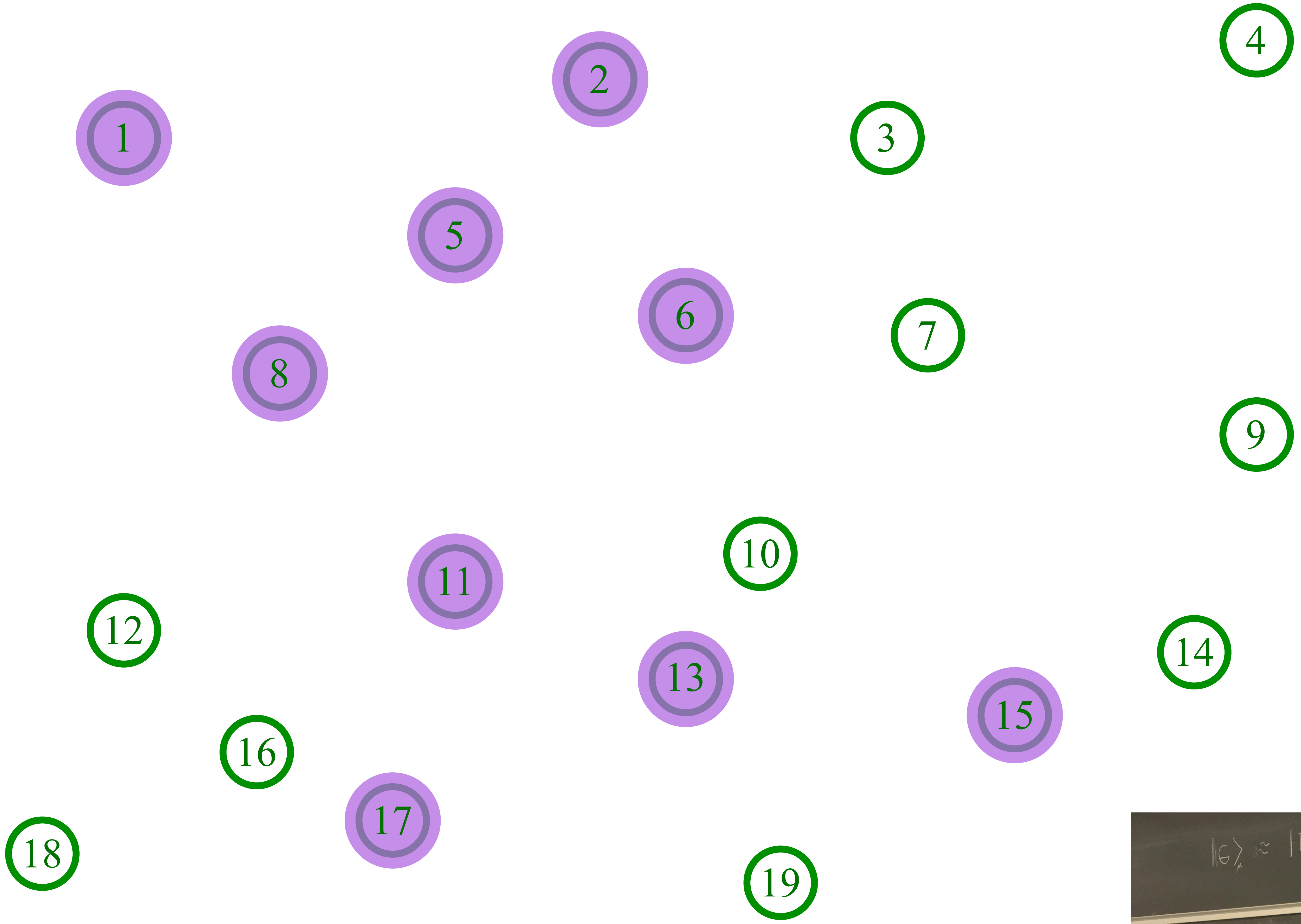
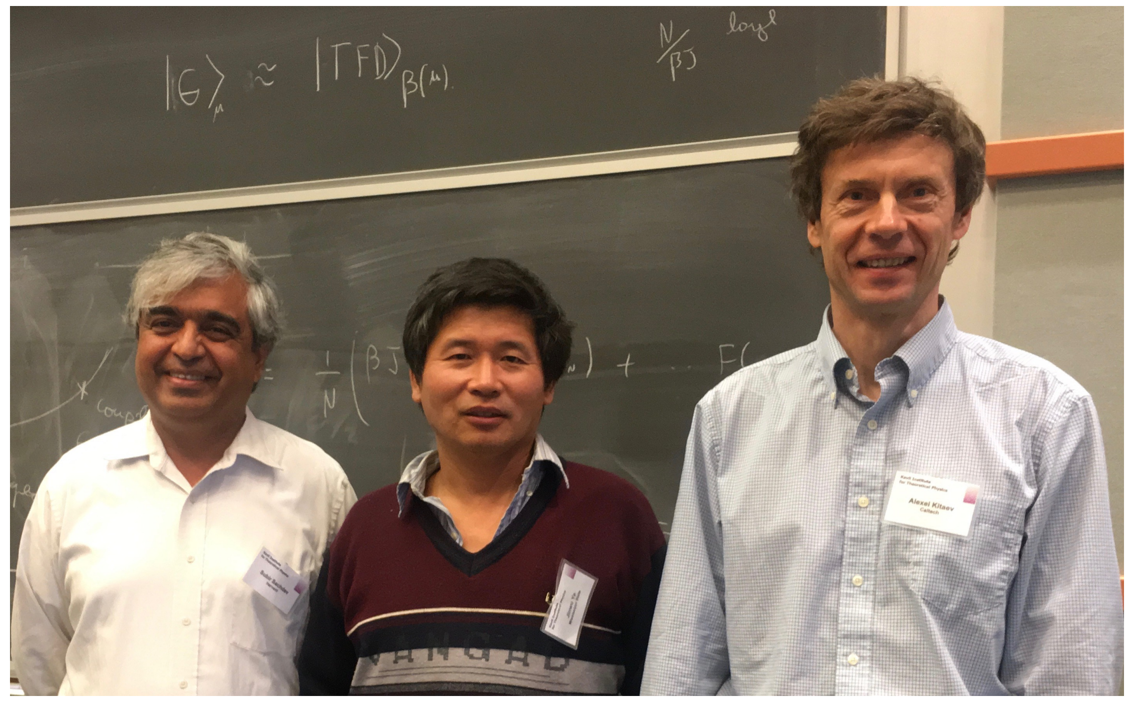


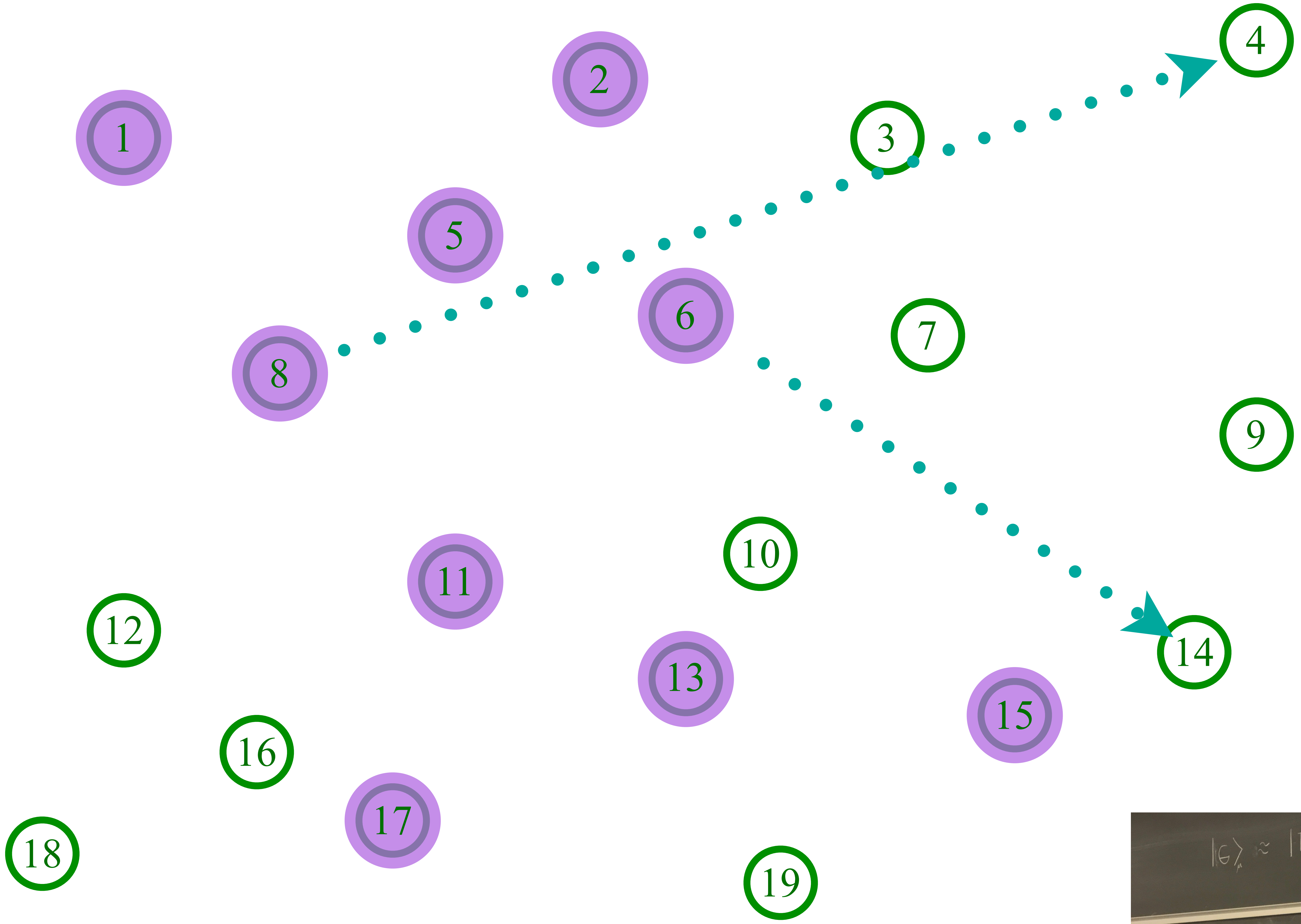
Entangle electrons pairwise randomly



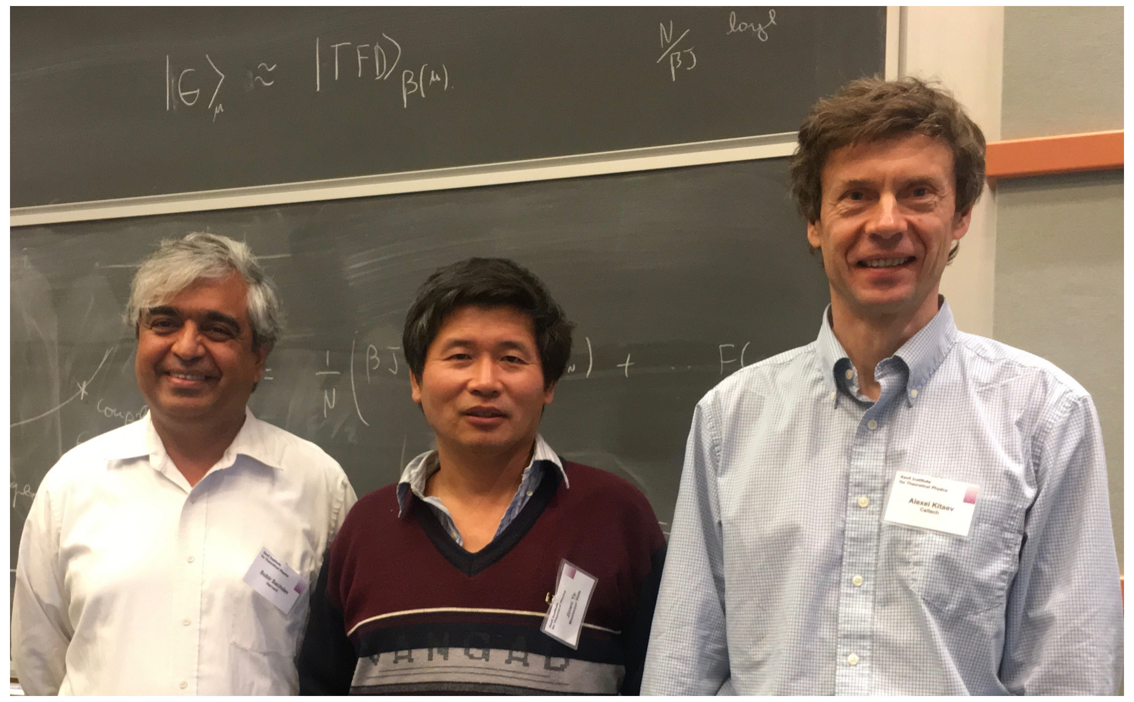
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



Entangle electrons pairwise randomly



The SYK model

Sachdev, Ye (1993); Kitaev (2015)

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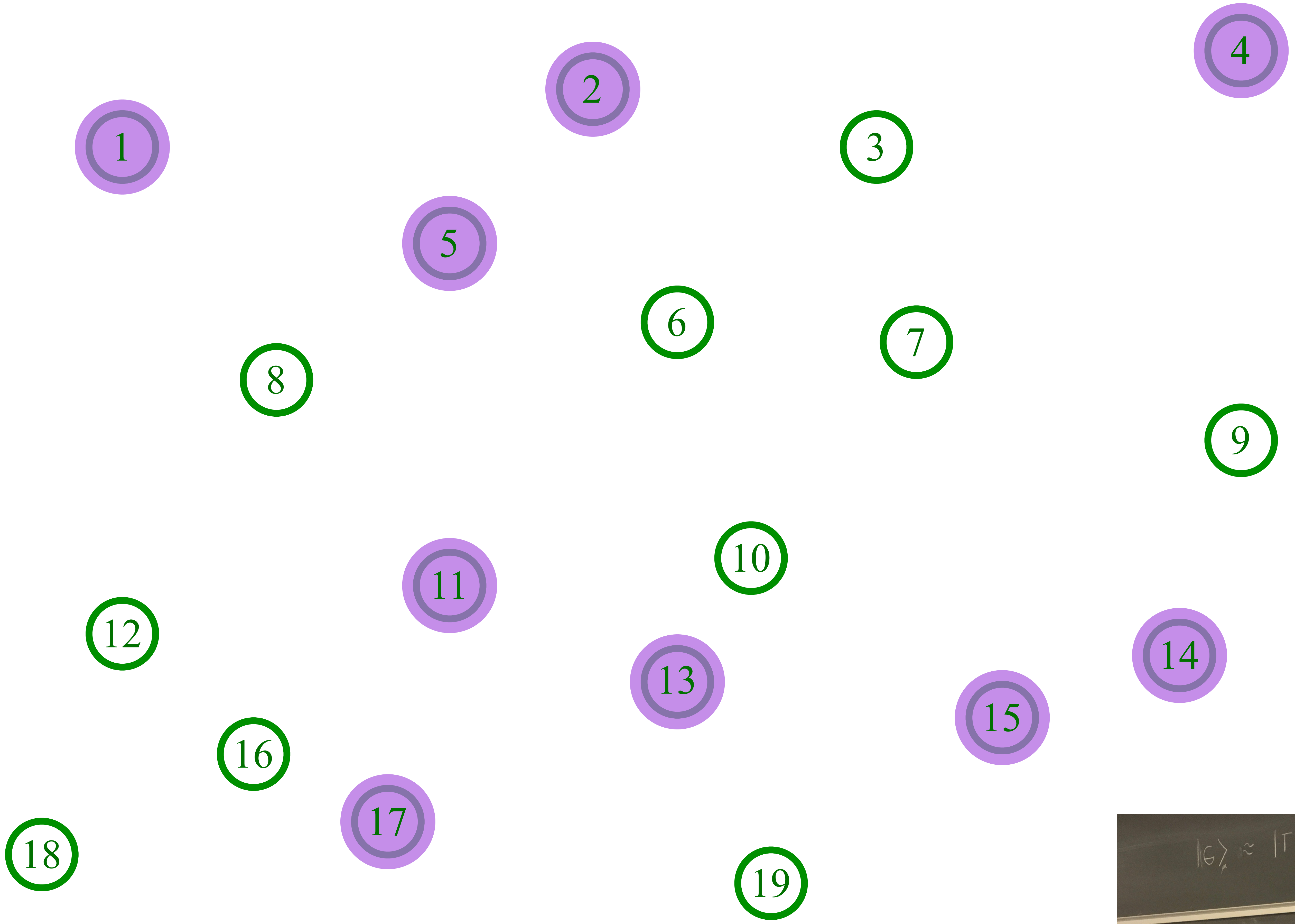
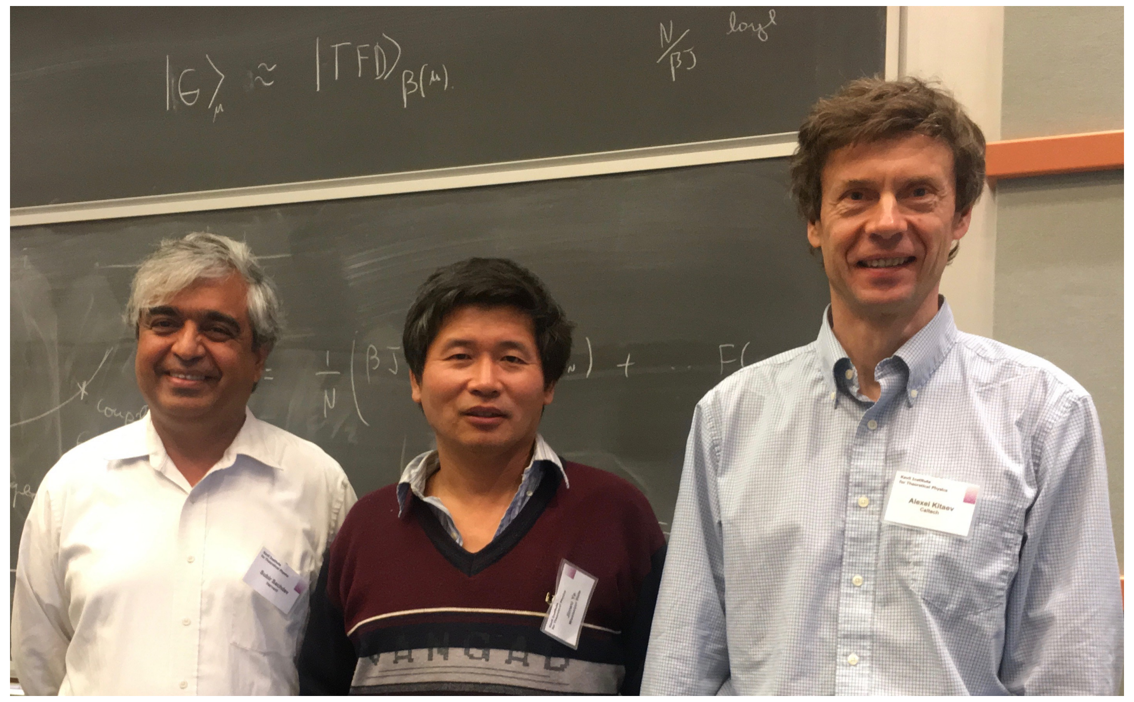


Entangle electrons pairwise randomly



The SYK model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

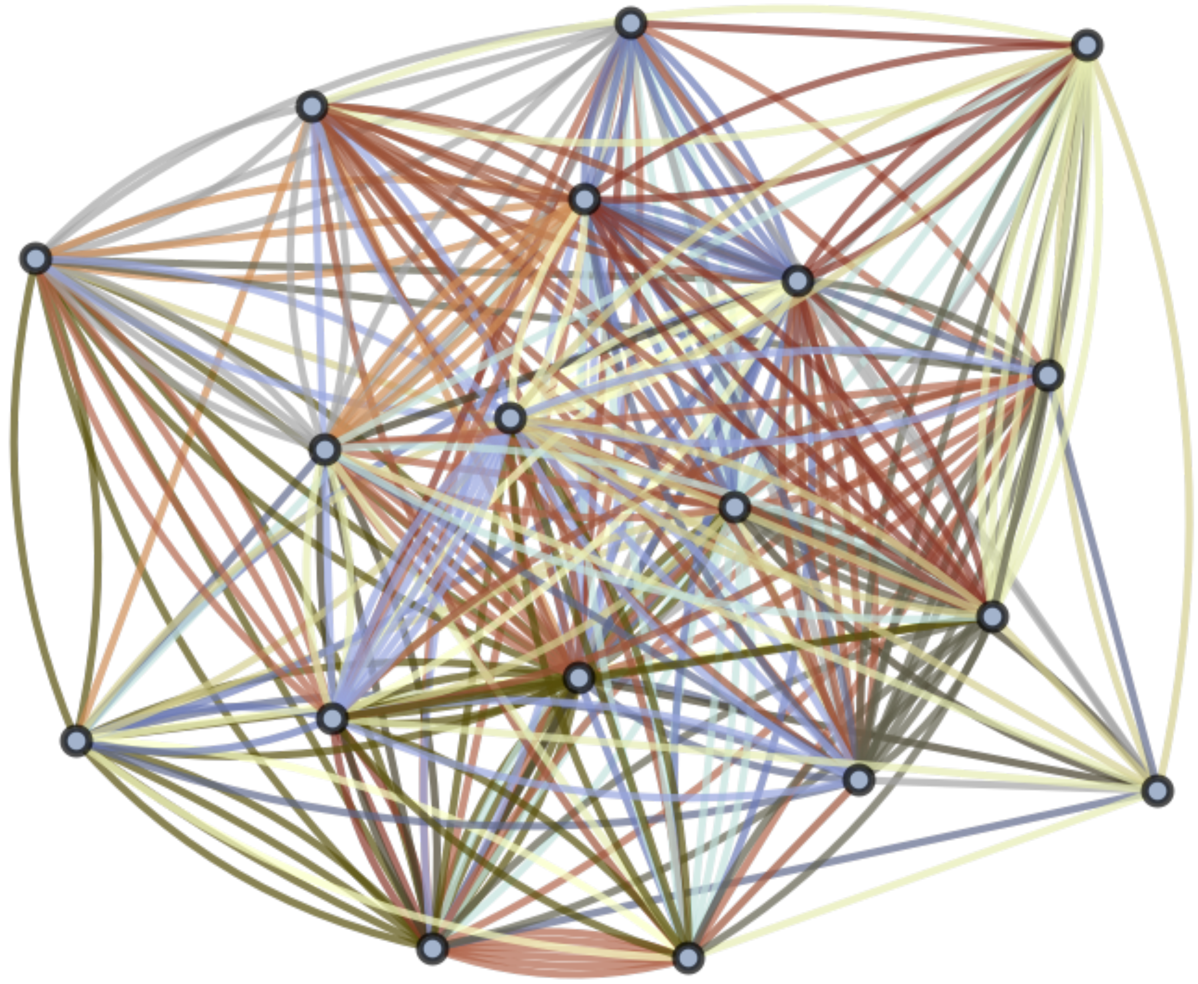
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

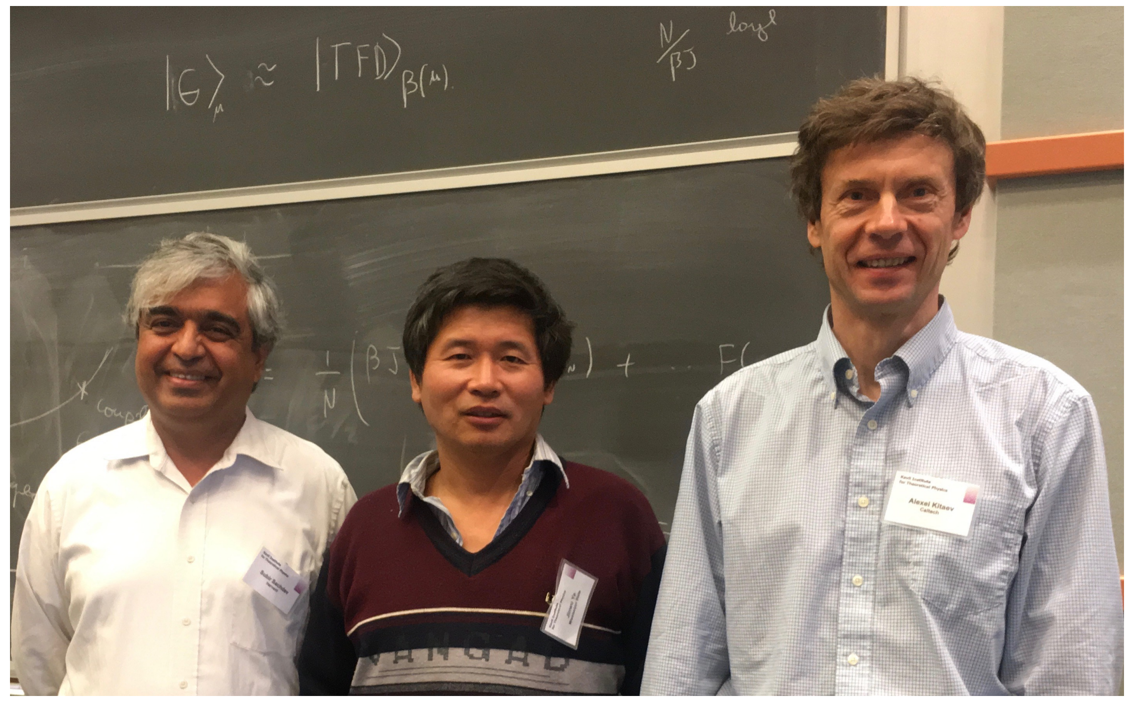
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha \, ; \quad [\mathcal{H}, \mathcal{Q}] = 0 \, ; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





The SYK model

$$\begin{aligned}\mathcal{Z}(\mathcal{Q},T) &= \mathrm{Tr}_{\mathcal{Q}} \exp\left(-\frac{\mathcal{H}}{T}\right) = \exp(-F/T)\,;\ \text{Entropy } S = -\frac{\partial F}{\partial T}. \\ &\equiv \int_{E_0^-}^{\infty} D(E) e^{-E/T}\,; \quad D(E) = \sum_i \delta(E - E_i)\,; \quad \mathcal{H}\left|\Psi_i\right\rangle = E_i \left|\Psi_i\right\rangle\end{aligned}$$

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$$\lim_{T\rightarrow 0}\lim_{N\rightarrow\infty}\frac{S}{N} = s_0$$

A. Georges, O. Parcollet, and S. Sachdev,
PRB **63**, 134406 (2001)

The constant s_0 is a known universal number. At $\mathcal{Q} = 1/2$, we have

$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4} = 0.464847699170805107492692486833\ldots\ .$$

The SYK model

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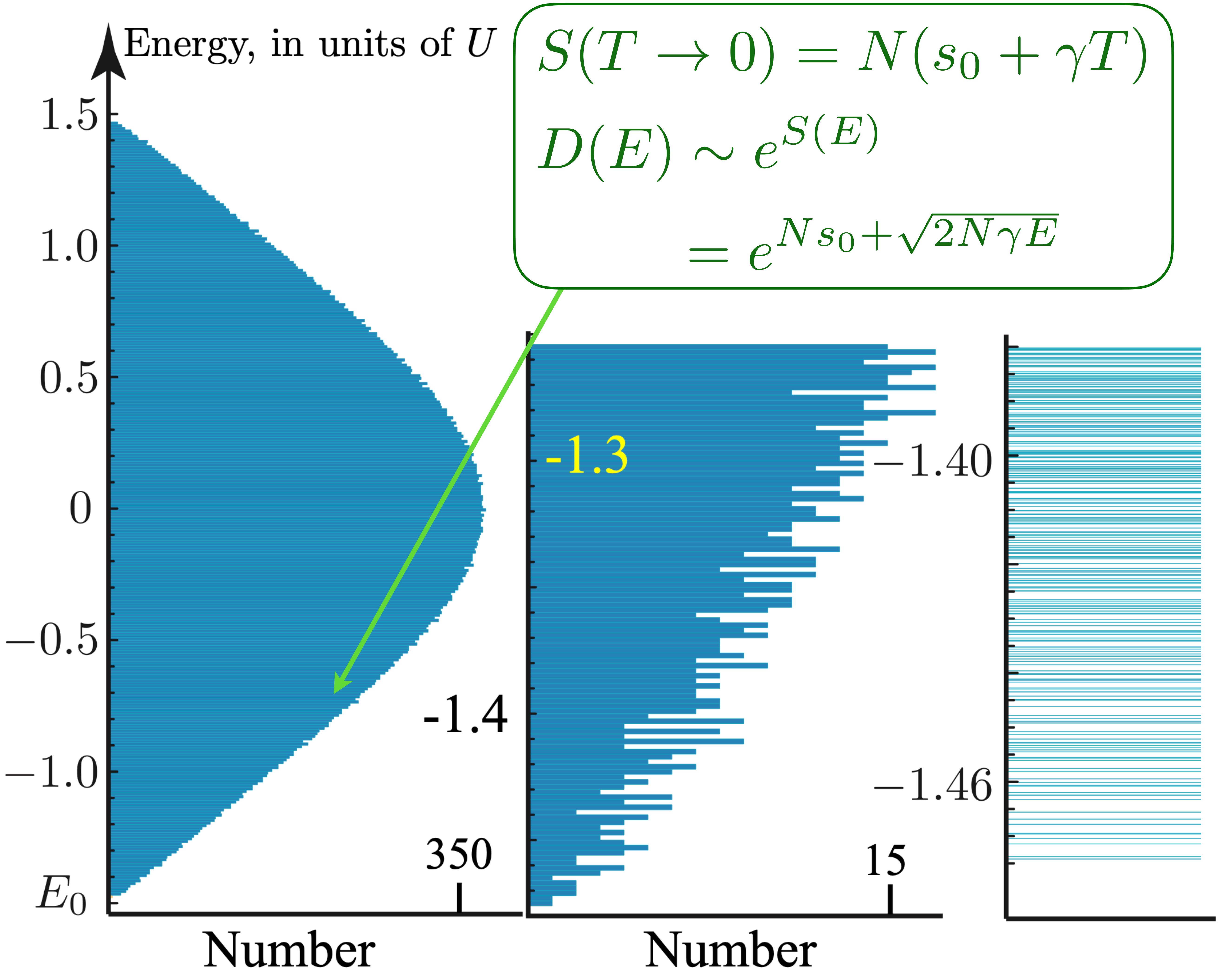
Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



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Complex SYK model

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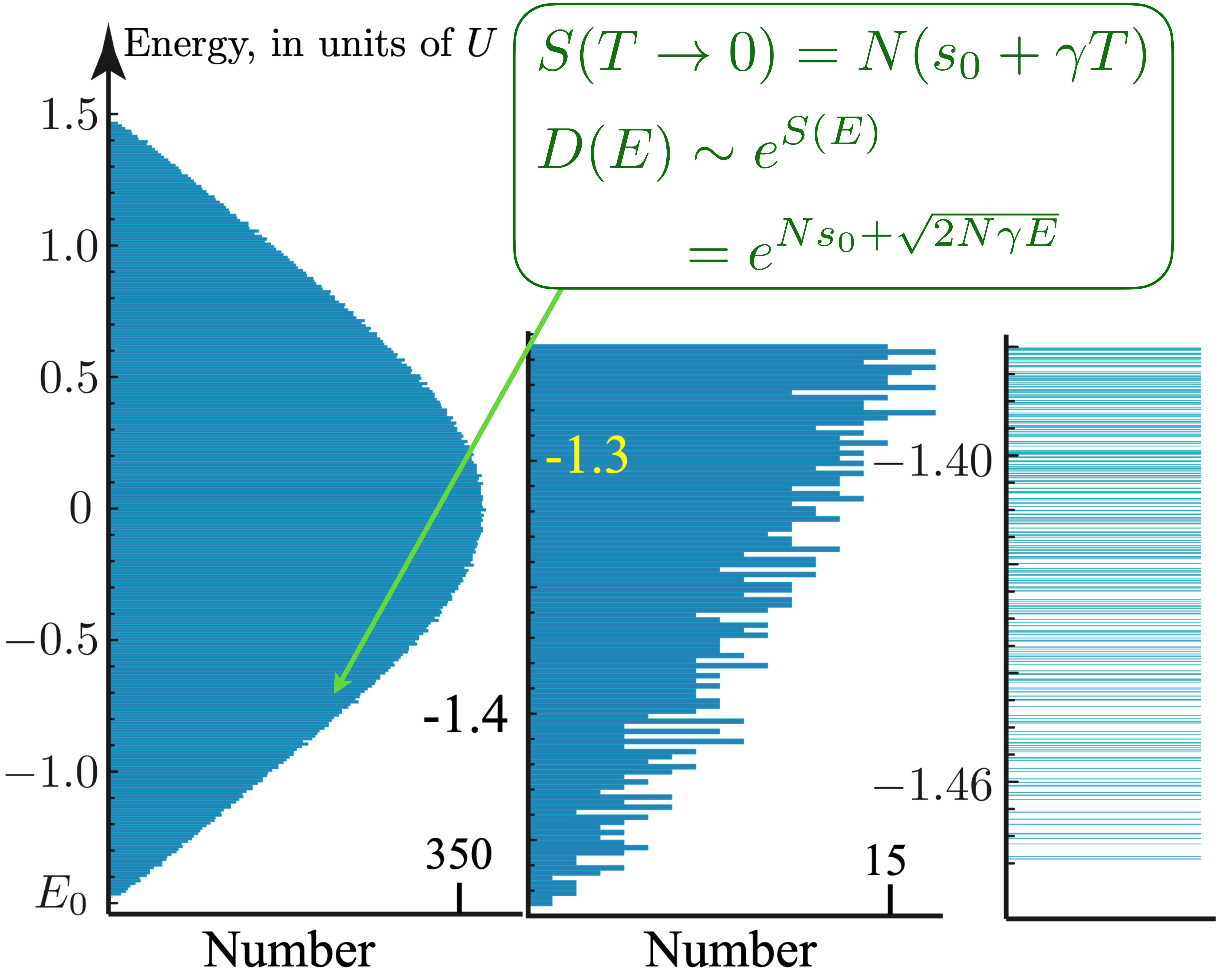

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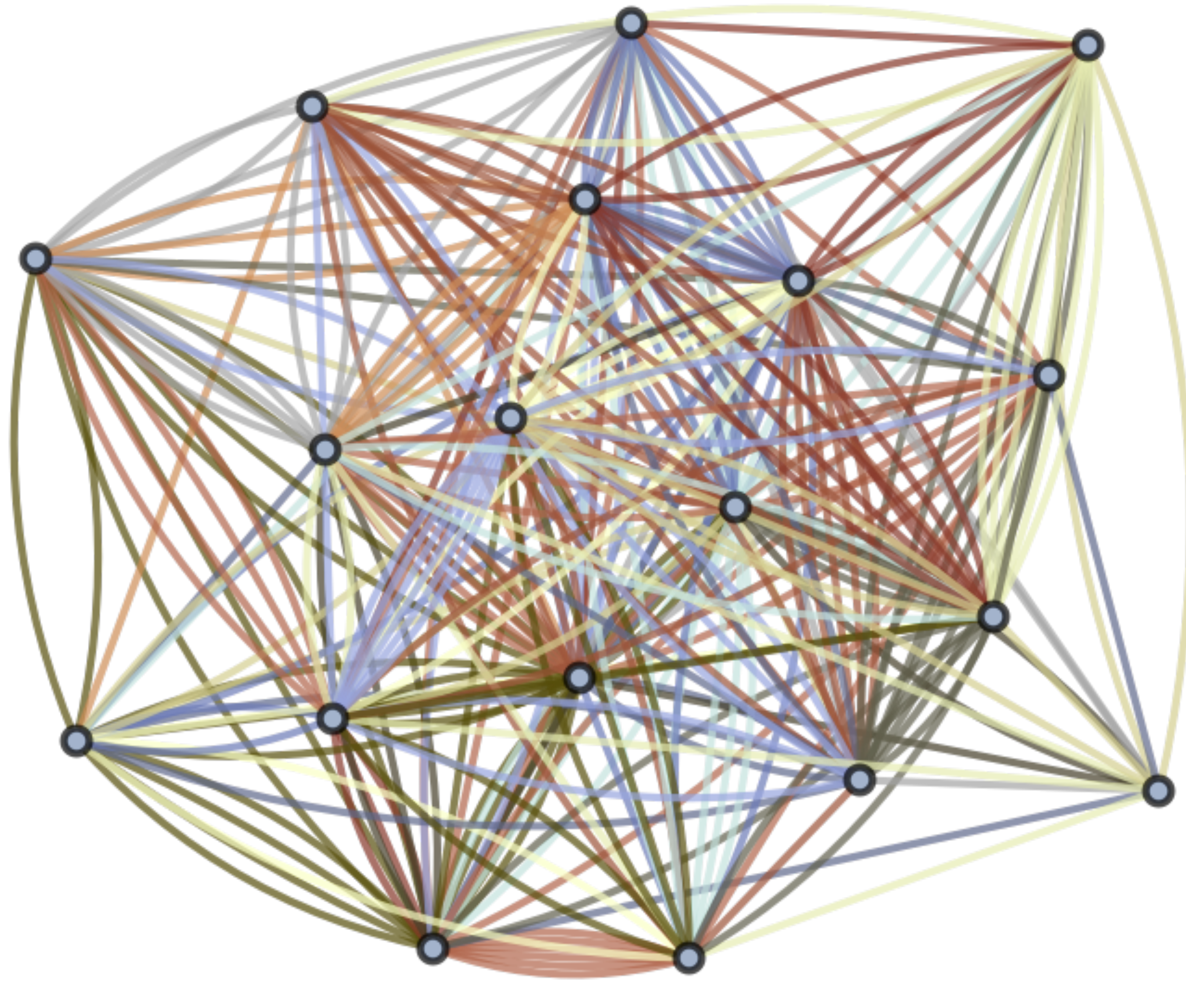
Energy level spacing $\sim e^{-N s_0}$!

Complex SYK model

The Sachdev-Ye-Kitaev (SYK) model

The disorder-averaged partition is exactly this G-Σ theory:



$$\mathcal{Z} = \int \mathcal{D}G(\tau_1,\tau_2)\mathcal{D}\Sigma(\tau_1,\tau_2)\exp(-NI)$$

$$I = \ln\det\left[\delta(\tau_1-\tau_2)(\partial_{\tau_1}+\mu) - \Sigma(\tau_1,\tau_2)\right] + \int d\tau_1 d\tau_2 \left[\Sigma(\tau_1,\tau_2)G(\tau_2,\tau_1) + (U^2/2)G^2(\tau_2,\tau_1)G^2(\tau_1,\tau_2)\right]$$

Saddle-point equations for $G(\tau_1-\tau_2)$ and $\Sigma(\tau_1-\tau_2)$:

$$G(i\omega) = \frac{1}{i\omega+\mu-\Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2G^2(\tau)G(-\tau)$$

$$G(\tau=0^-) = \mathcal{Q}.$$

G-Σ path integral

At frequencies $\ll U$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \, e^{-i\phi(\sigma_1) + i\phi(\sigma_2)} \, \widetilde{G}(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \, e^{-i\phi(\sigma_1) + i\phi(\sigma_2)} \, \widetilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $\phi(\sigma)$ are arbitrary functions.

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

Time reparametrization and phase soft modes

The dominant fluctuations of the bilocal Green's function $G(\tau_1, \tau_2)$ involve a deformation of the conformal Green's function by a time reparameterization and a gauge transformation.

$$G(\tau_1, \tau_2) = [f'(\tau_1) f'(\tau_2)]^{1/4} G_*(f(\tau_1) - f(\tau_2)) e^{i\phi(\tau_1) - i\phi(\tau_2)}$$

Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) e^{-E_0/T + N s_0 - N I_{\text{eff}}[f,\phi]} ,$$

where $E_0 \propto N$ is the ground state energy.

A. Kitaev (2015); J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5,** 041025 (2015)

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Exact path integral over time reparameterizations:

$$\boxed{S = N(s_0 + \gamma T) - \frac{3}{2}\ln\left(\frac{U}{T}\right) - \frac{\ln N}{2} \quad \Rightarrow \quad D(E) \sim N^{-1} \exp\left(N s_0\right) \sinh(\sqrt{2N\gamma E})}$$

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A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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J. S. Cotler et al., JHEP 05 (2017) 118

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Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, JHEP 02 (2020) 157

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

J. S. Cotler et al., JHEP 05 (2017) 118

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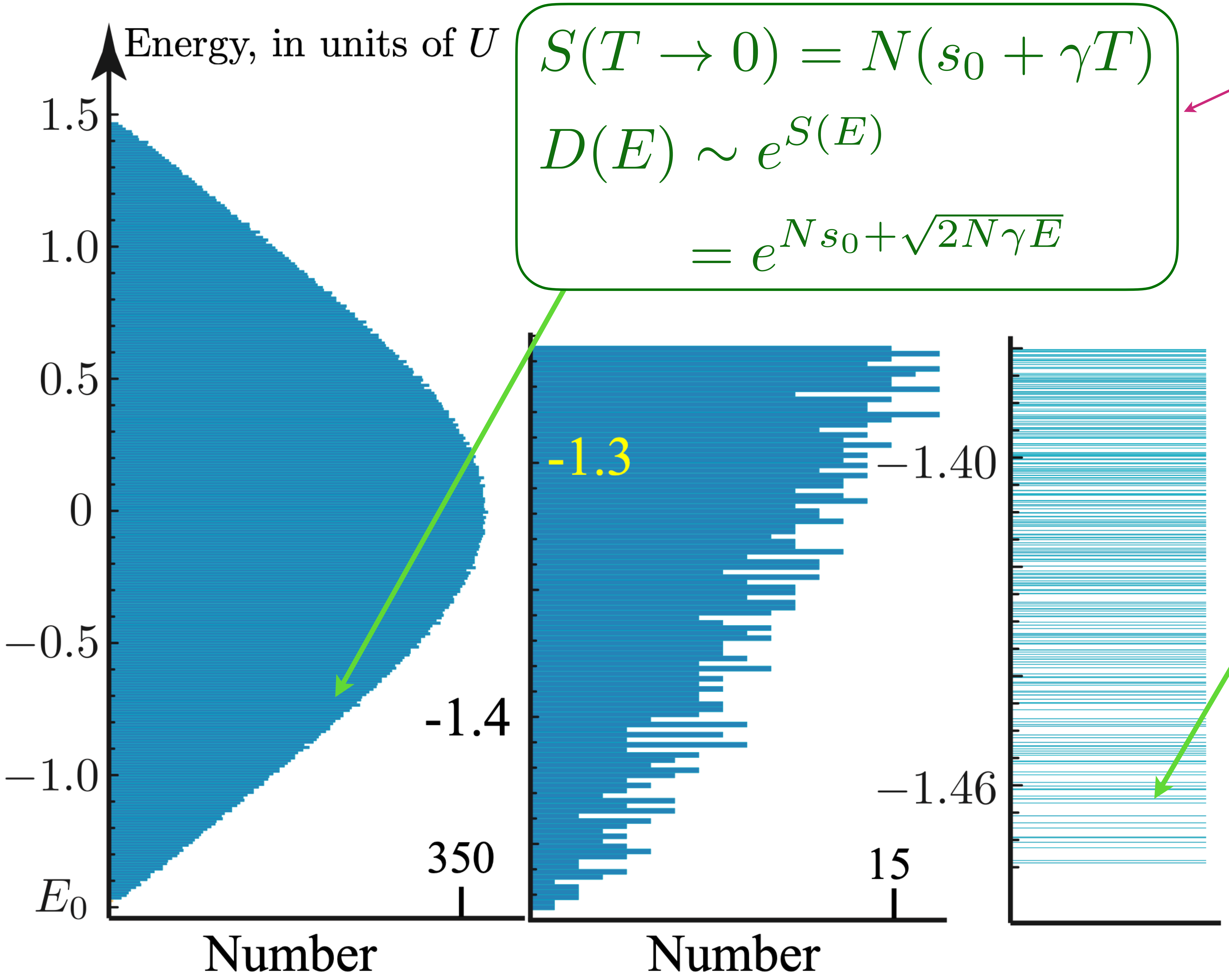

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$$D(E) \sim e^{N s_0} \sqrt{2\gamma E/N}$$

Complex SYK model

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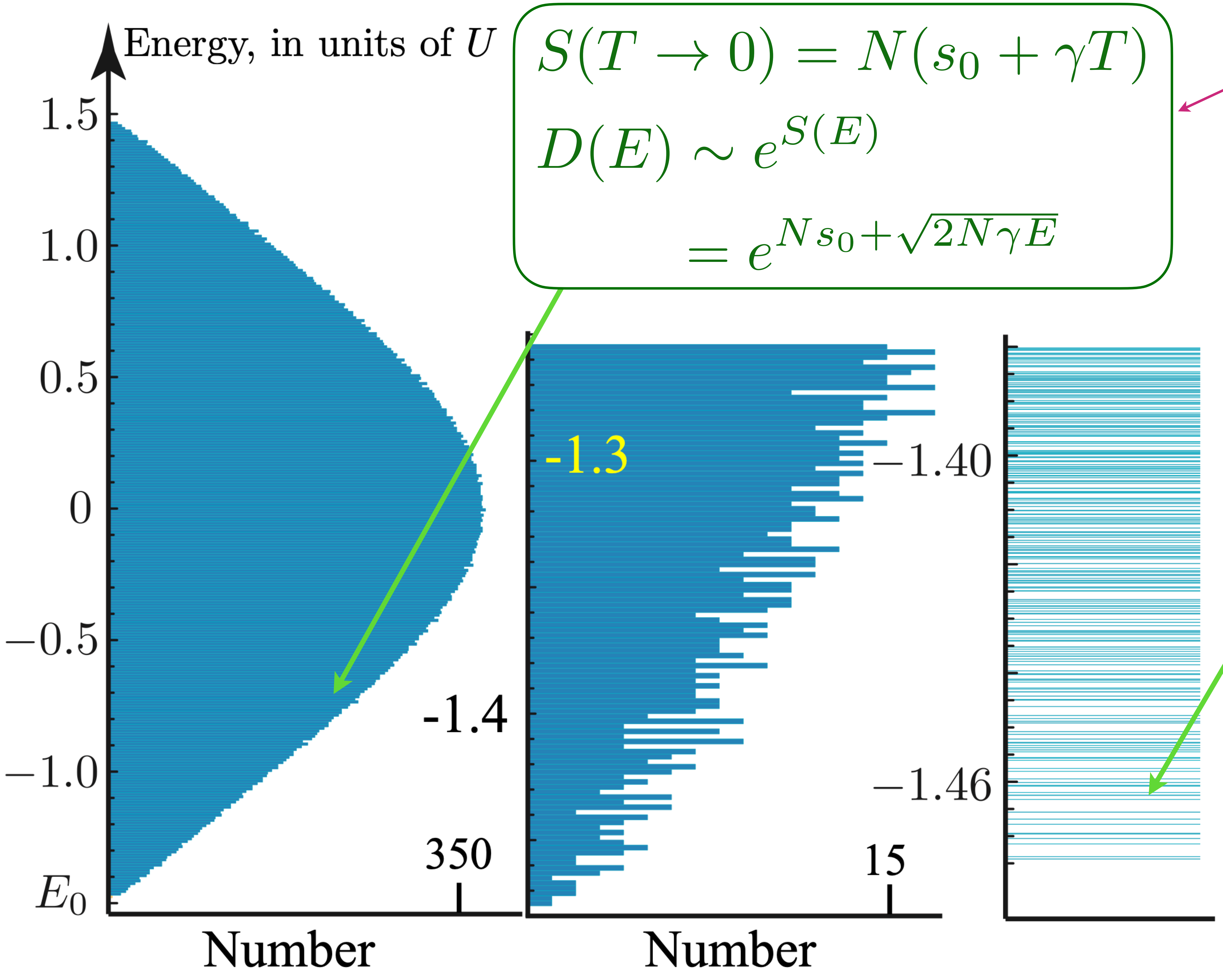

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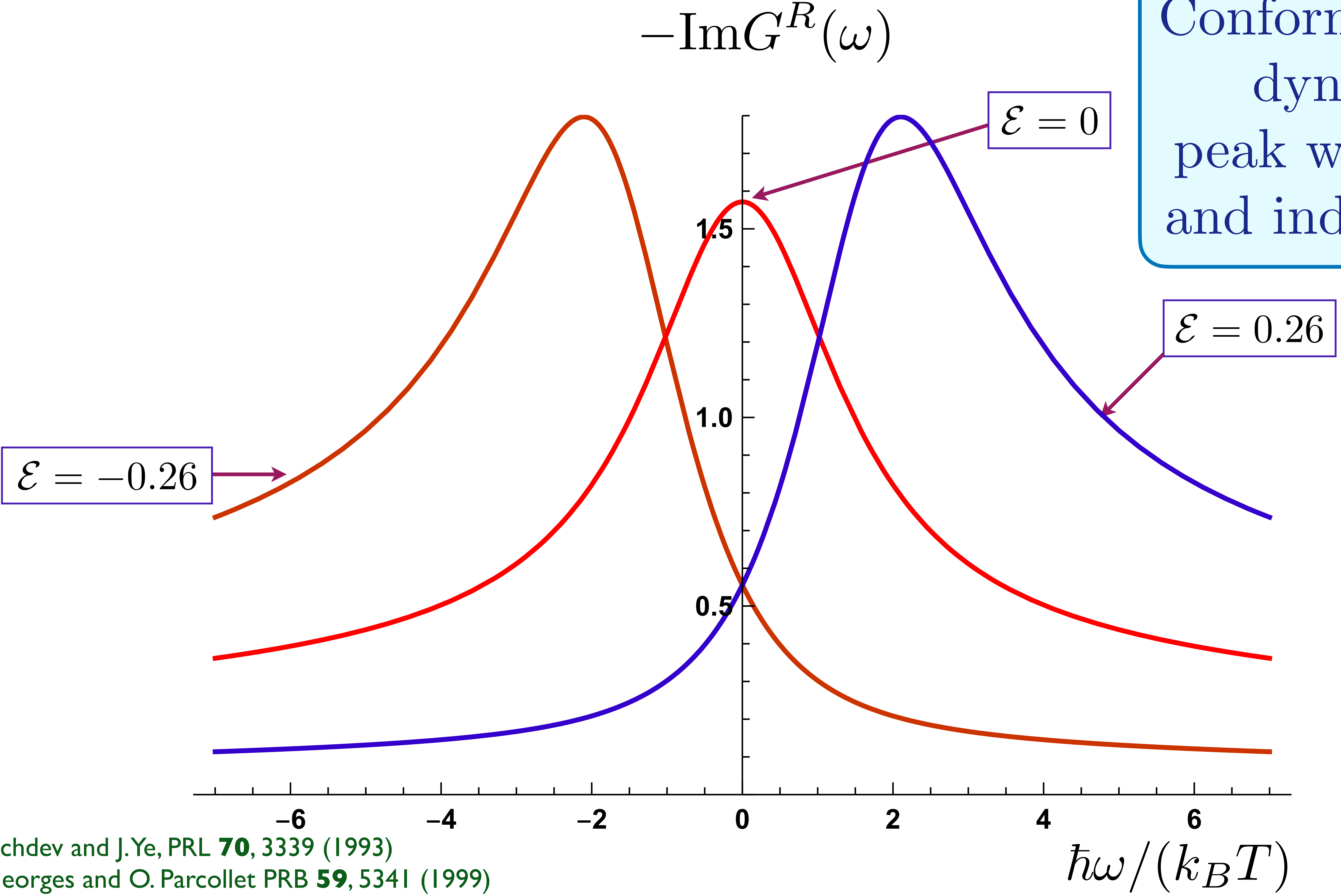
$$D(E) \sim e^{N s_0} \sqrt{2\gamma E/N}$$

No exponentially large degeneracy, but exponentially small level spacing!
No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

Complex SYK model

The SYK model



Conformal 'Planckian' dynamics with peak width $\sim k_B T/\hbar$ and independent of U

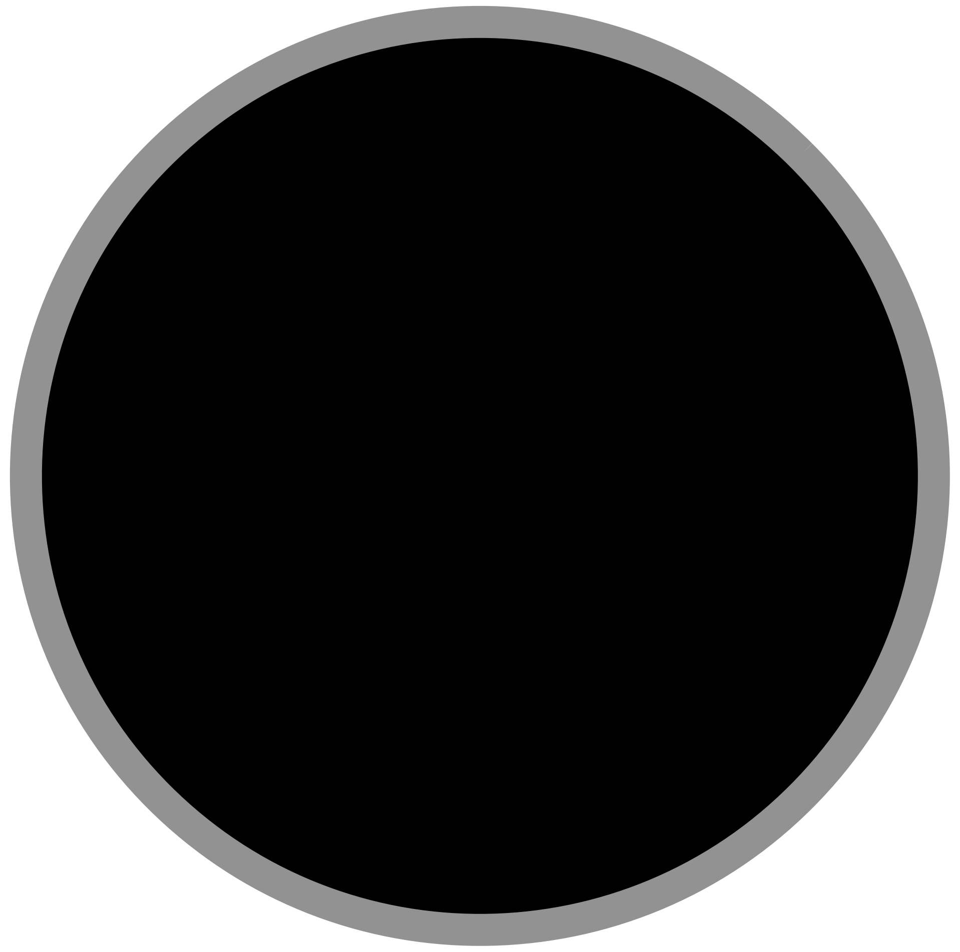
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

Quantum black holes

Black Holes

Objects so dense that light is gravitationally bound to them.

$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

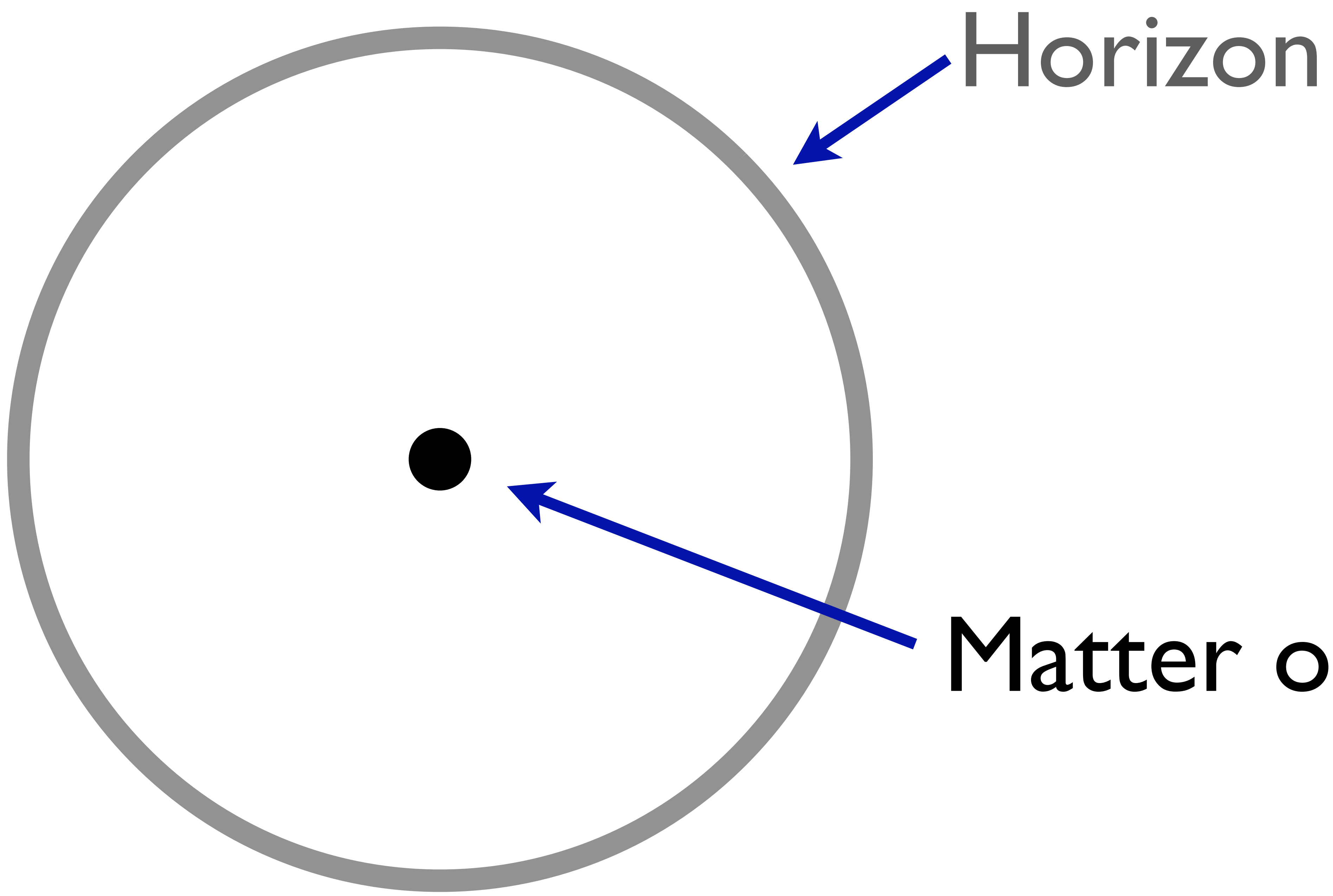


G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



Horizon

Matter of infinite density!

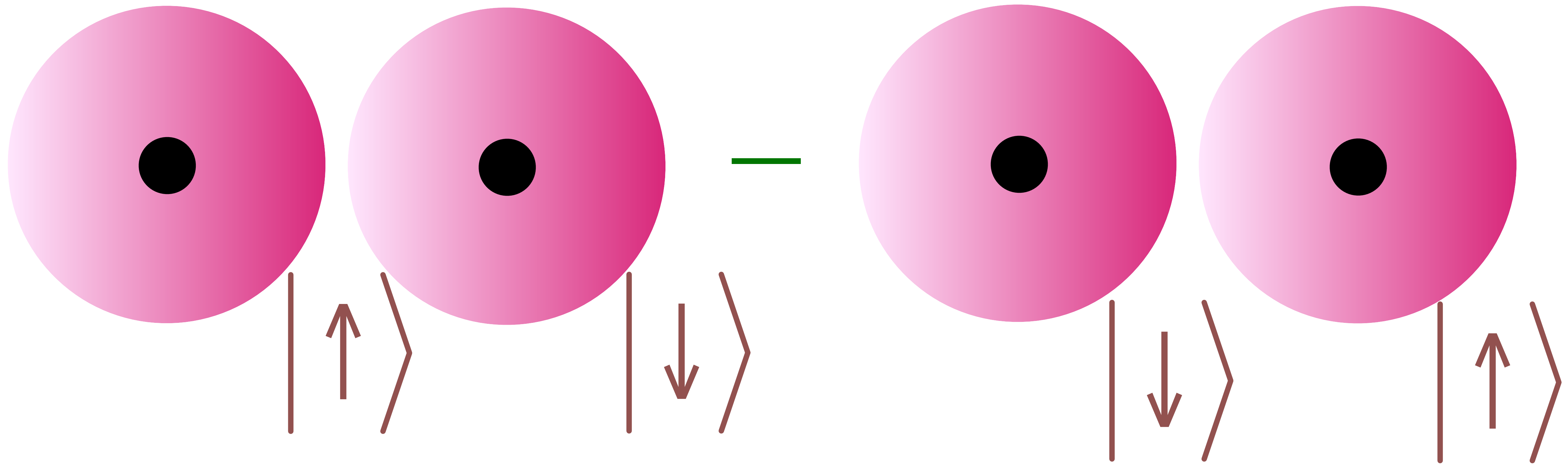
What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.

This singularity convinced many early on that black holes were unphysical solutions of Einstein's equations, and did not exist in our universe.

In any case, it was clear that quantum theory should be applied to the collapsed matter, but no one knew how to.

Quantum Entanglement across a black hole horizon



$$|\uparrow\rangle \; |\downarrow\rangle - |\downarrow\rangle \; |\uparrow\rangle$$

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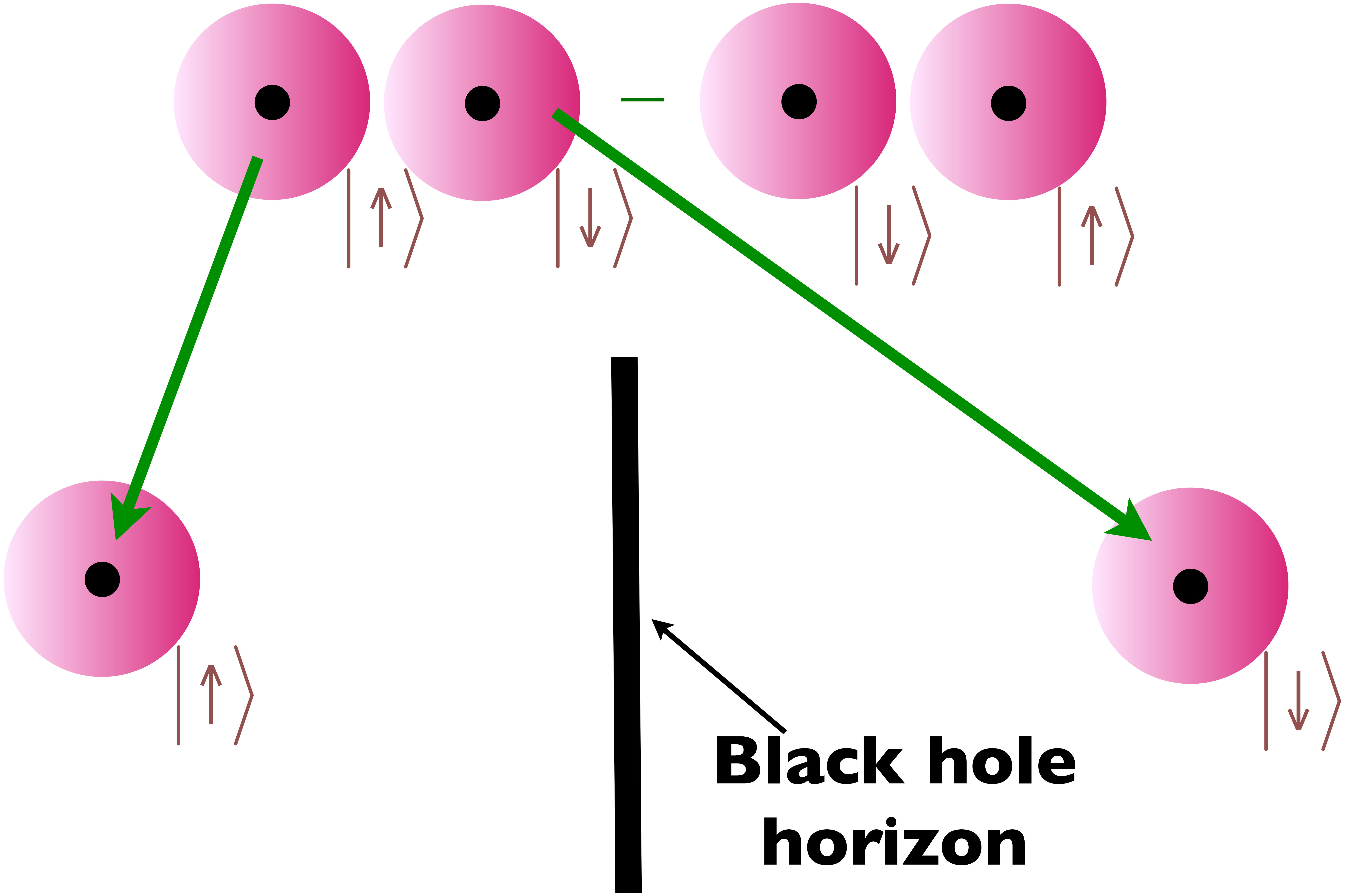

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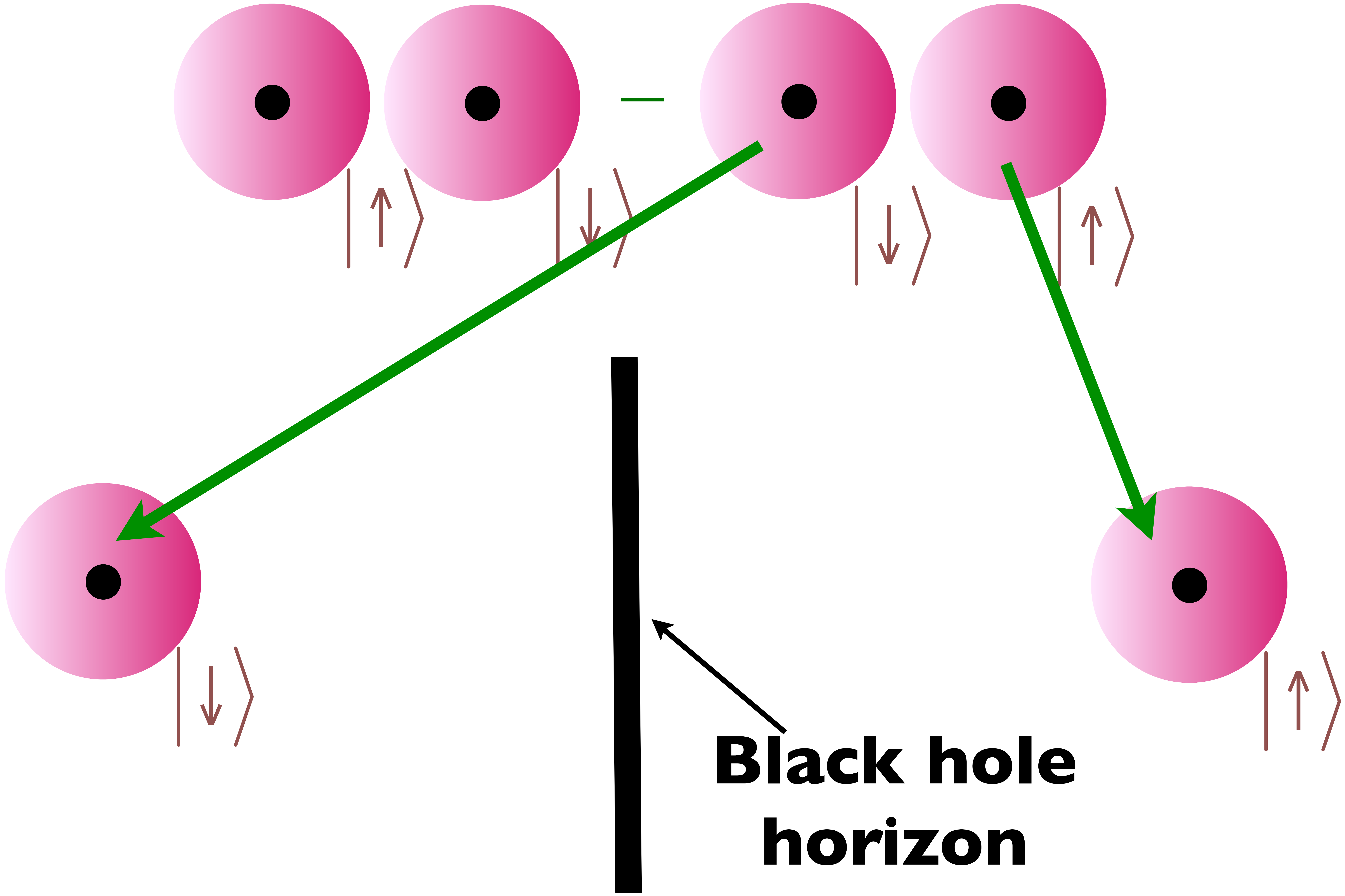
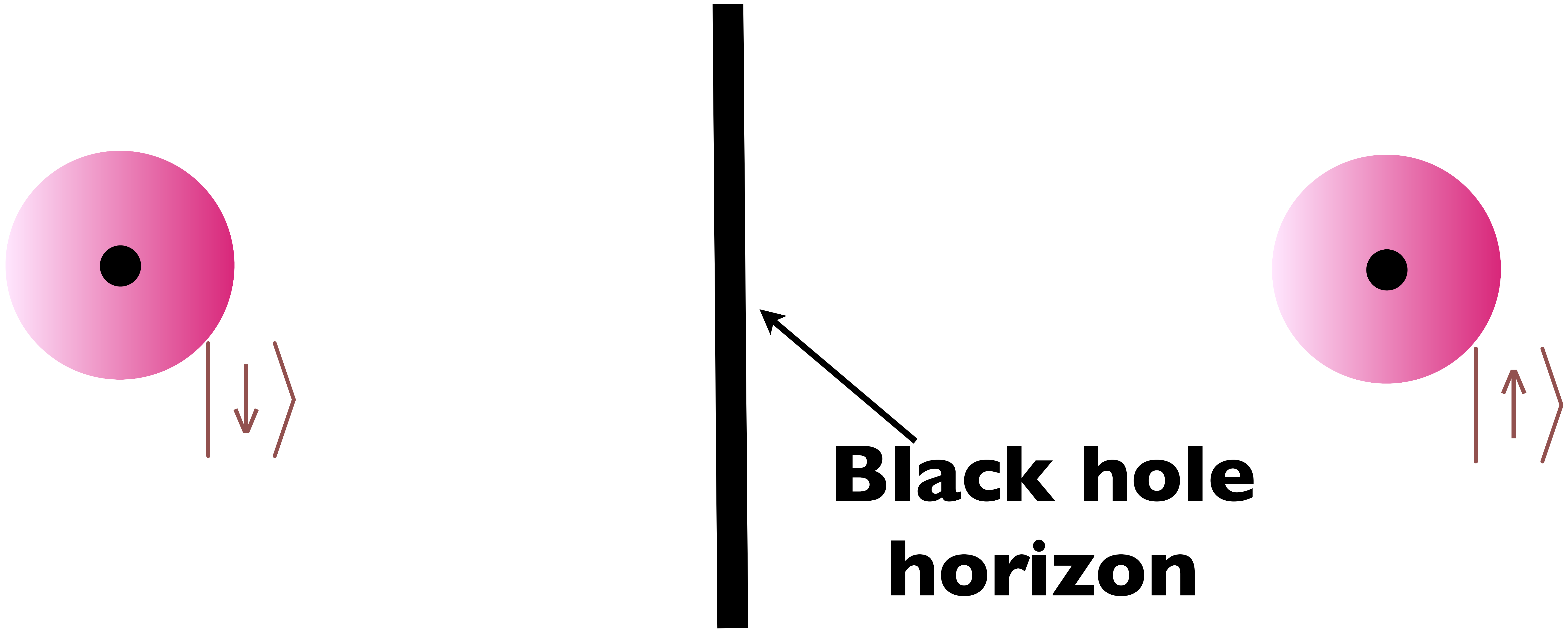


Black hole horizon

Quantum Entanglement across a black hole horizon

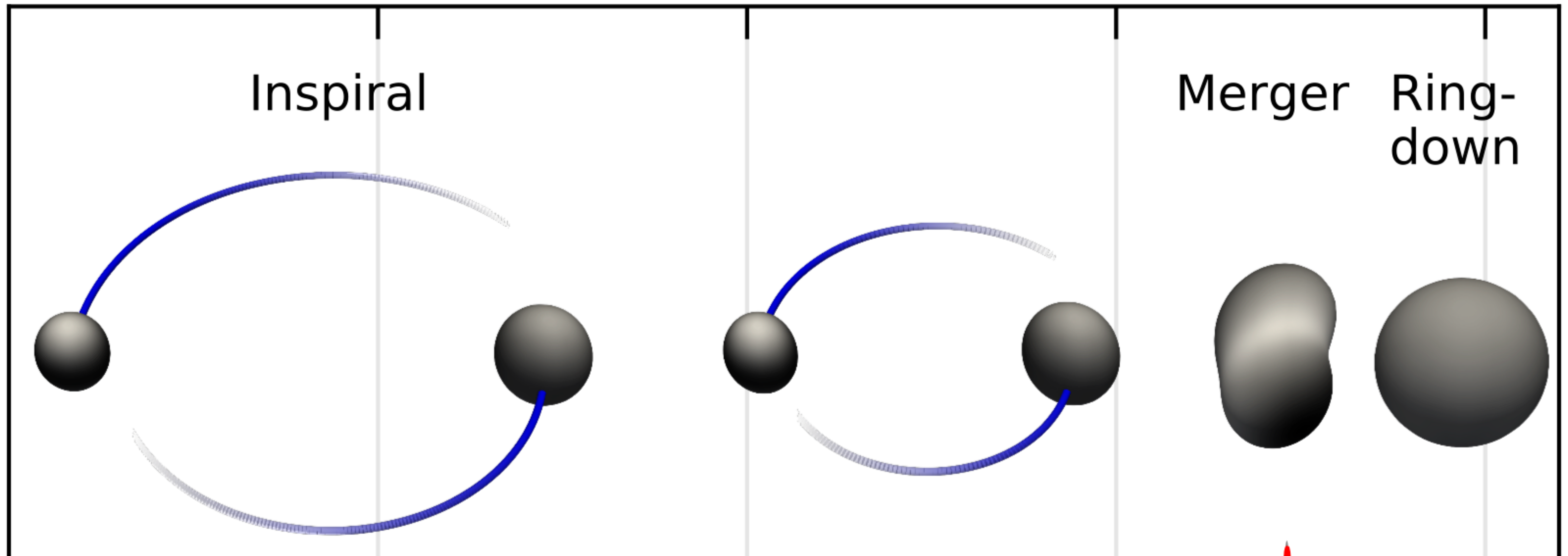
Hawking (1975) used other arguments to show that black hole horizons have a temperature
(The entanglement reasoning: to an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.)



Black hole horizon

Quantum black holes

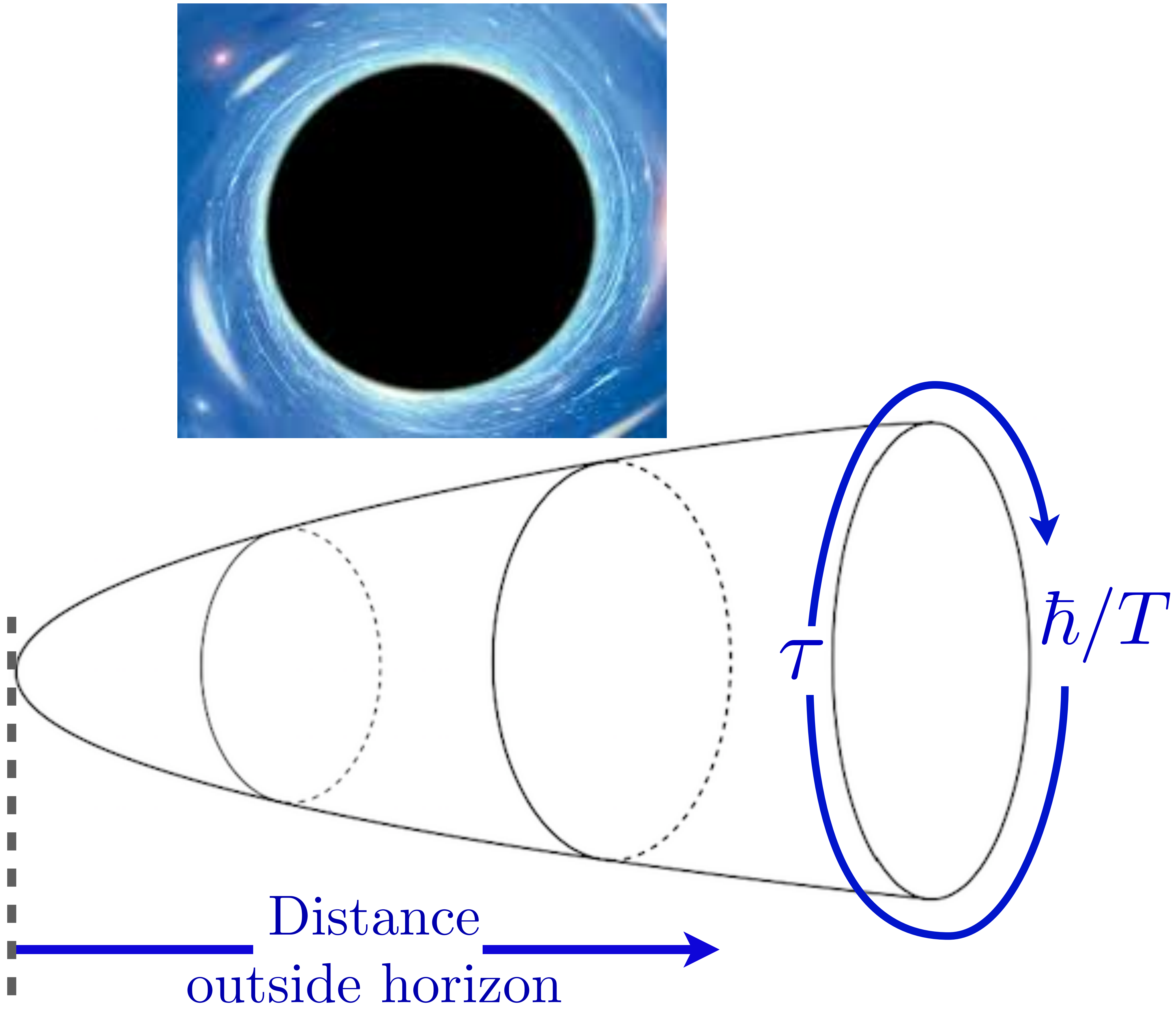
- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- The entropy is proportional to their surface area. $S = A k_B c^3/(4G\hbar)$.
- They relax to thermal equilibrium in a time $\sim 8\pi G M/c^3 = \hbar/(k_B T_H)$ which is Planckian!



J. D. Bekenstein, PRD **7**, 2333 (1973)
S. W. Hawking, Nature **248**, 30 (1974)
C.V. Vishveshwara, Nature **227**, 936 (1970)

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\mathcal{Z}(\mathcal{Q},T) = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu},A_\mu]\right)$$



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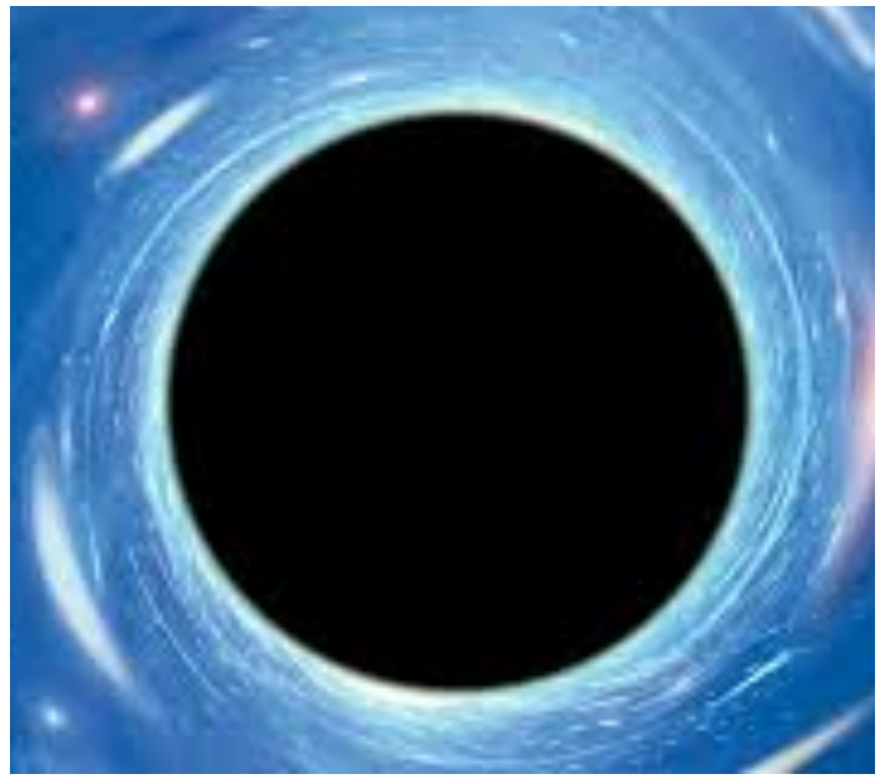
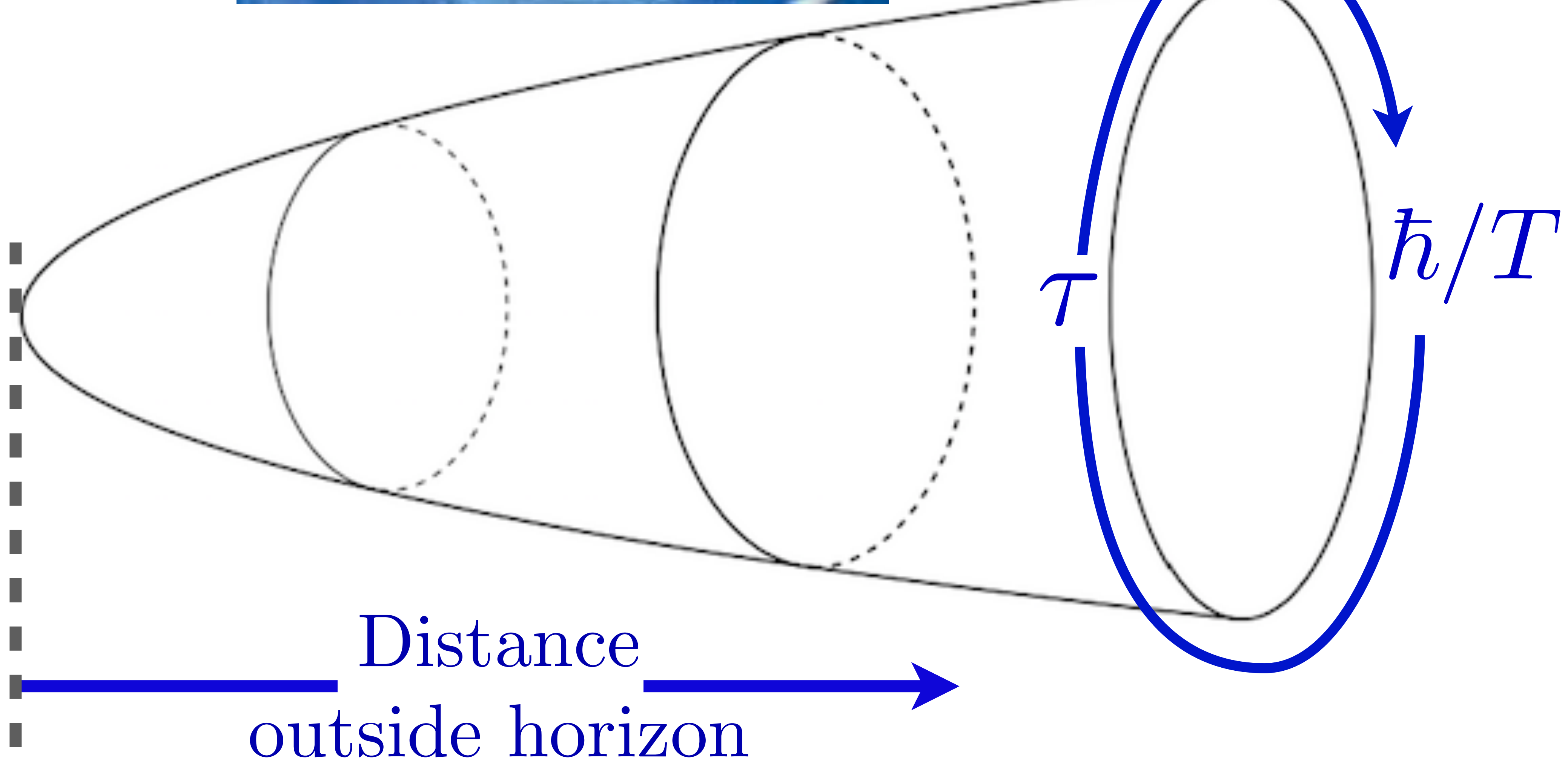
Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)



$$S_{BH}(T\to 0,\mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0c^3}{4G\hbar}\left(1+\frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T=0$.

Obtained from the saddle-point of the gravity path integral in the imaginary time spacetime outside the black hole.



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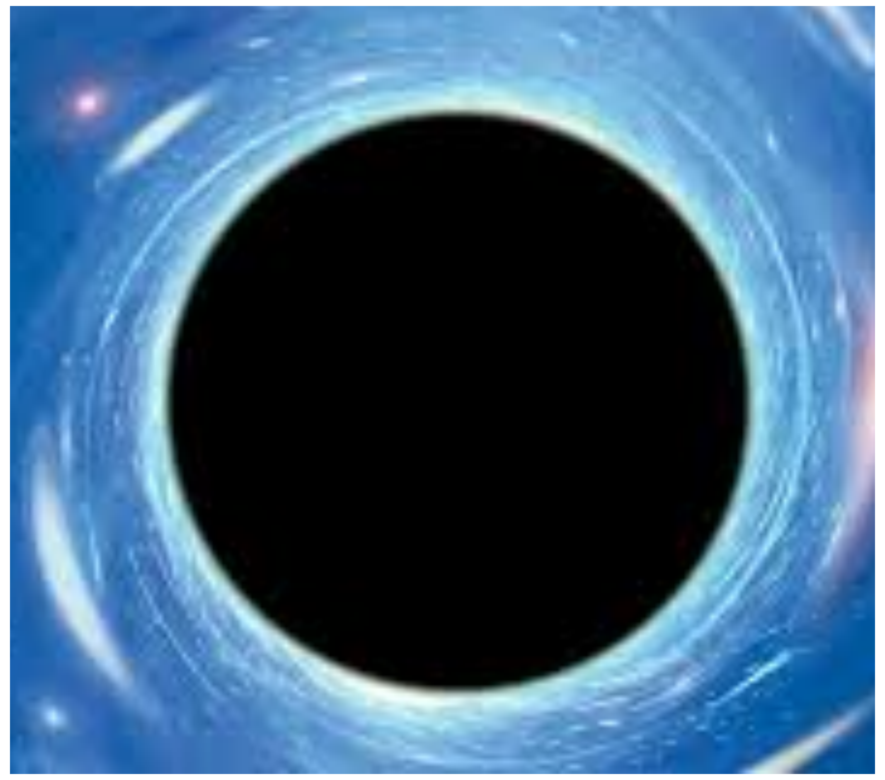

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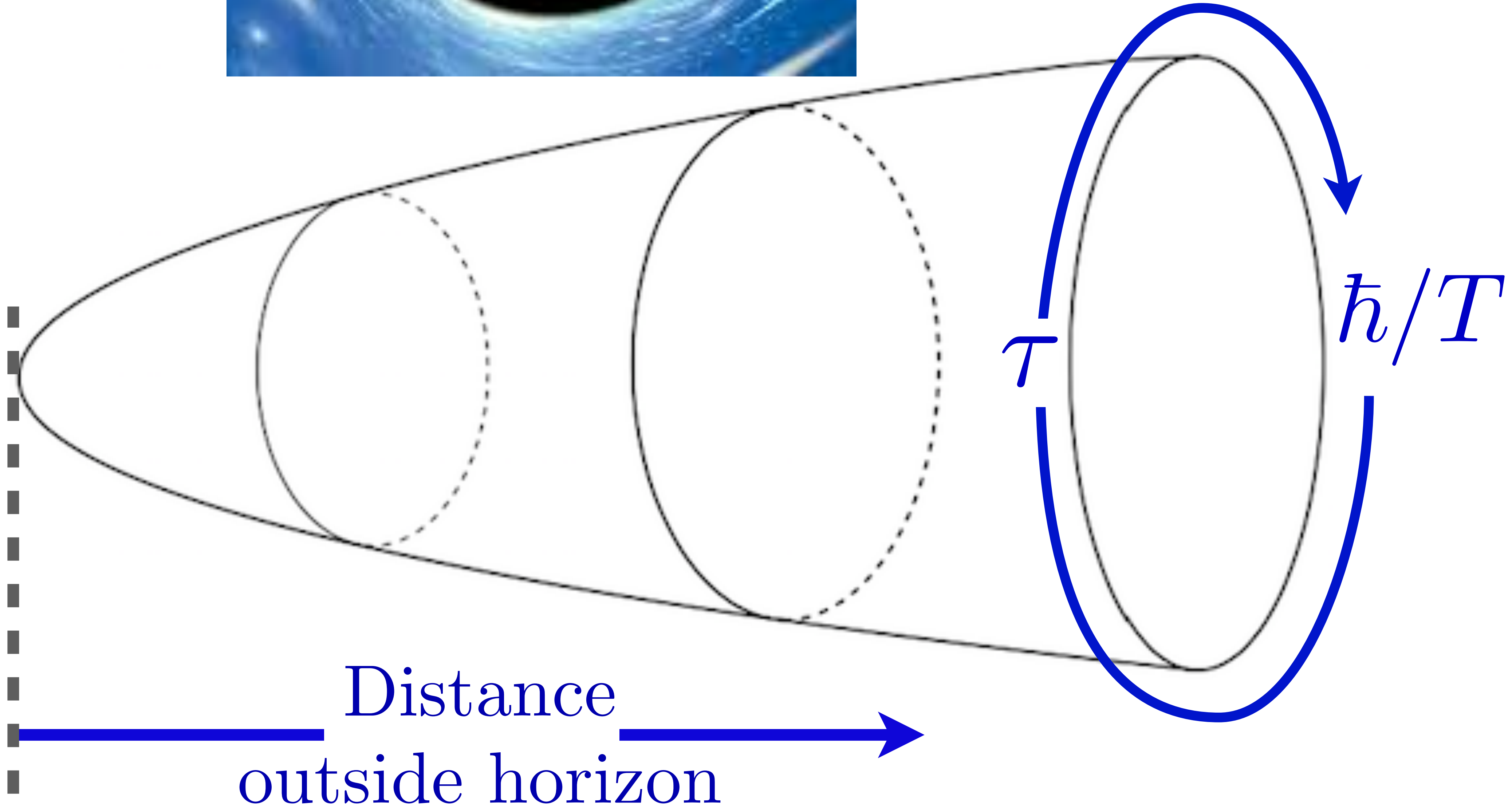


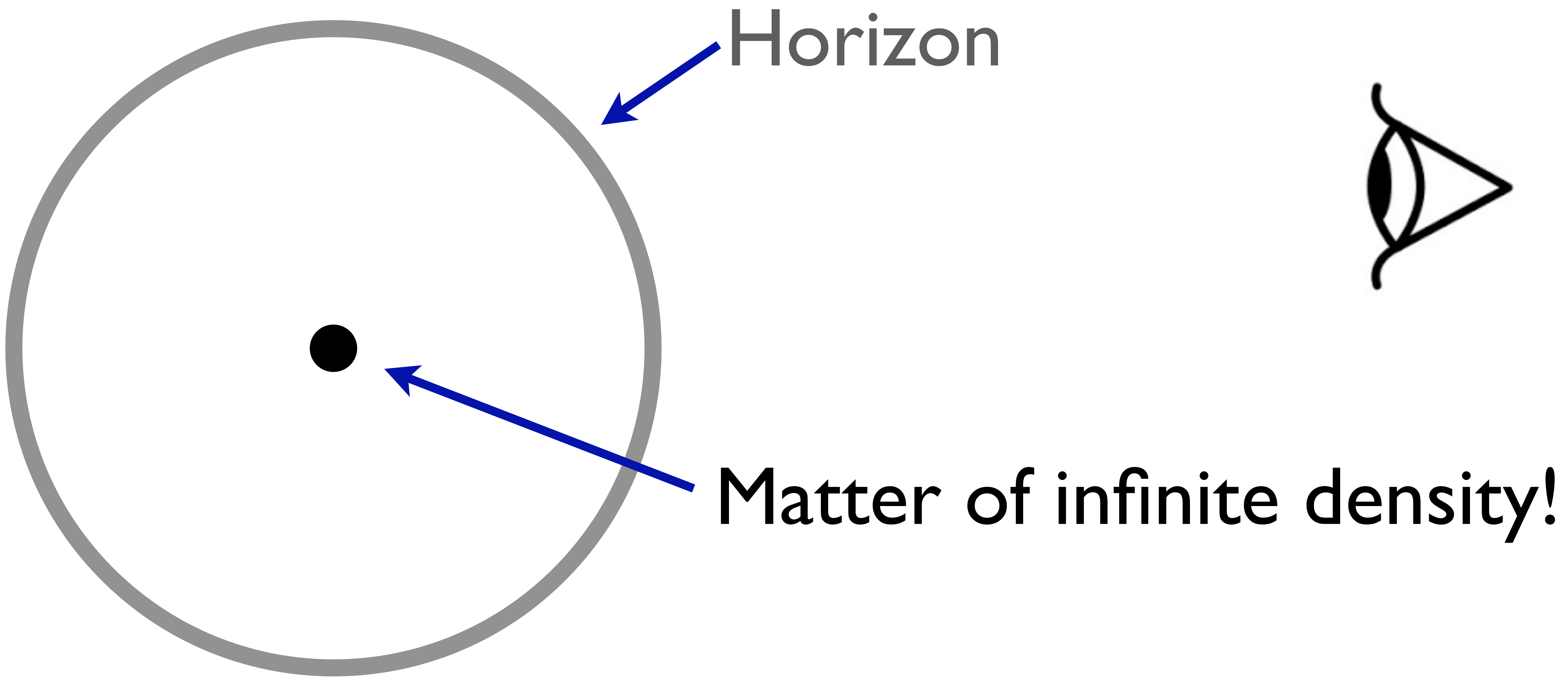
Note the similarity to the large *N* entropy of the SYK model !
(along with other similarities)

Sachdev PRL 2010

Quantum Black Holes

Hawking obtained the black hole entropy by semiclassical computations for an observer *outside* the black hole horizon.

This allowed Hawking to avoid the contradictions associated with the singularity at the center of the black hole.



Horizon

Matter of infinite density!

Quantum Black Holes

Hawking obtained the black hole entropy by semiclassical computations for an observer *outside* the black hole horizon.

Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* matches the Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics ?

From the SYK model to a quantum theory of charged black holes

Reissner-Nordstrom black hole of Einstein-Maxwell theory

Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

The isometry group of AdS_2 is the 0+1 dimensional conformal group SL(2,R).

Faulkner, Liu, McGreevy, Vegh 2009; Sachdev 2010

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$$\mathcal{Z}(\mathcal{Q},T) = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu},A_\mu]\right)$$

Saddle-point:

$$S_{BH}(T\to 0,\mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0c^3}{4G\hbar}\left(1+\frac{2(\pi A_0)^{1/2}T}{\hbar c}+\ldots\right)$$

$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T=0$.

Maldacena, Stanford, Yang (2016)

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

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$$\approx \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\; I^{(1+1)}_{\text{JT gravity of AdS}_2\text{+boundary graviton}}[g_{\mu\nu},A_\mu]\right)$$

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$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)\exp\left(-\frac{1}{\hbar} I_{\text{SYK}}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)]\right)$$

Saddle-point:

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$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T=0$.

Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019); Iliesiu, Turaci (2020)

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$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)\exp\left(-\frac{1}{\hbar} I_{\text{SYK}}[\text{time reparameterizations } f(\tau),\text{ phase rotations } \phi(\tau)]\right)$$

$$S(T\to 0,\mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} - \frac{3}{2}\ln\left(\frac{(\hbar c^5/G)^{1/2}}{T}\right)$$

The $\ln T$ term is the SYK/boundary-graviton correction to Bekenstein-Hawking.

There is also a

$$-\frac{559}{180}\ln\left(\frac{A_0 c^3}{\hbar G}\right)$$

term from other massless modes; Sen (2011)
Iliesiu, Murthy, Turiaci (2022)

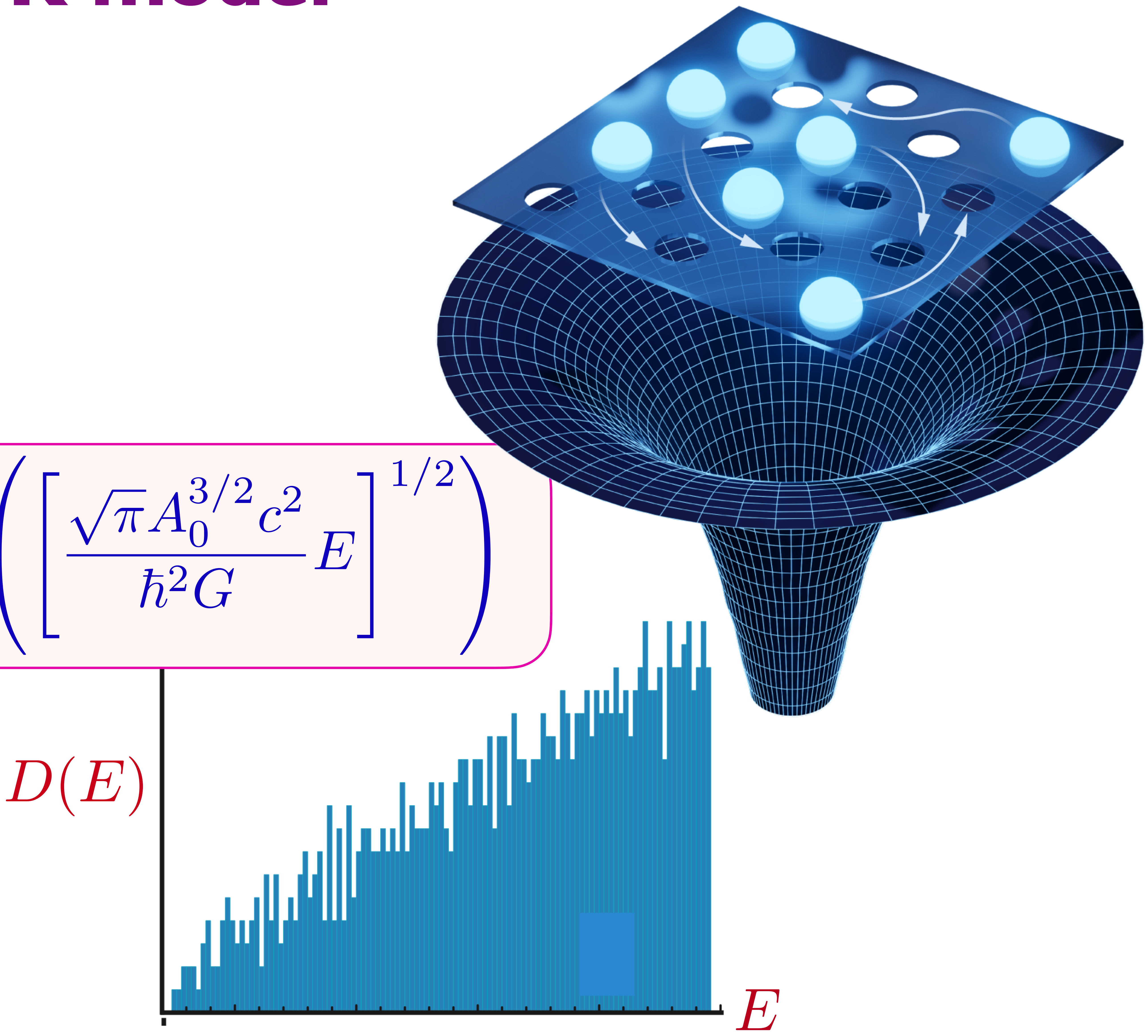
Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019); Iliesiu, Turaci (2020)

Quantum simulation of charged black holes by the SYK model

- For generic charged black holes in 3+1 dimensions, the SYK model yields, in terms of $A_0 = 2GQ^2/c^4$ the horizon area at $T = 0$:

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

There is no degeneracy, but an exponentially small level spacing down to the ground state.



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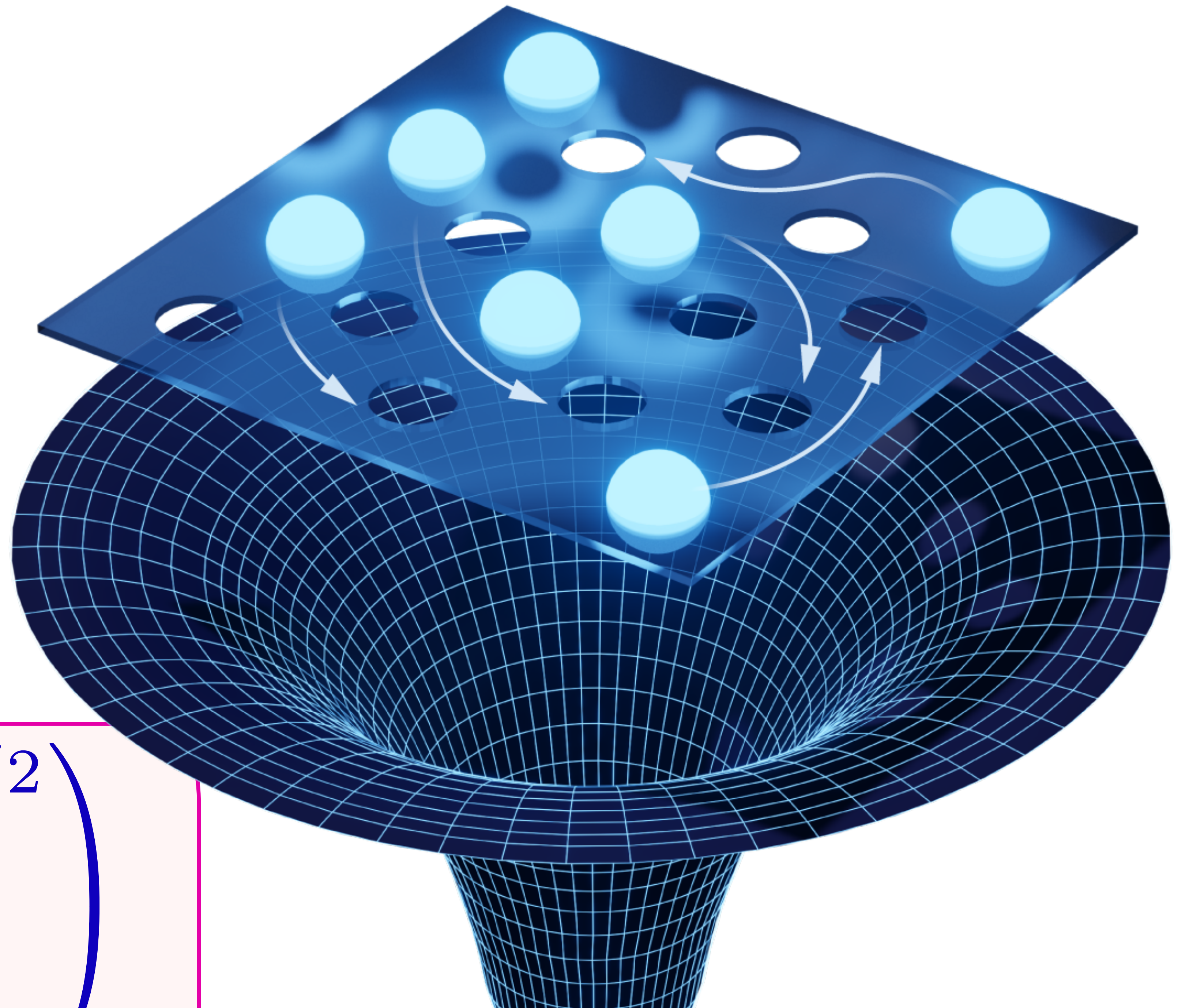

Bekenstein-Hawking

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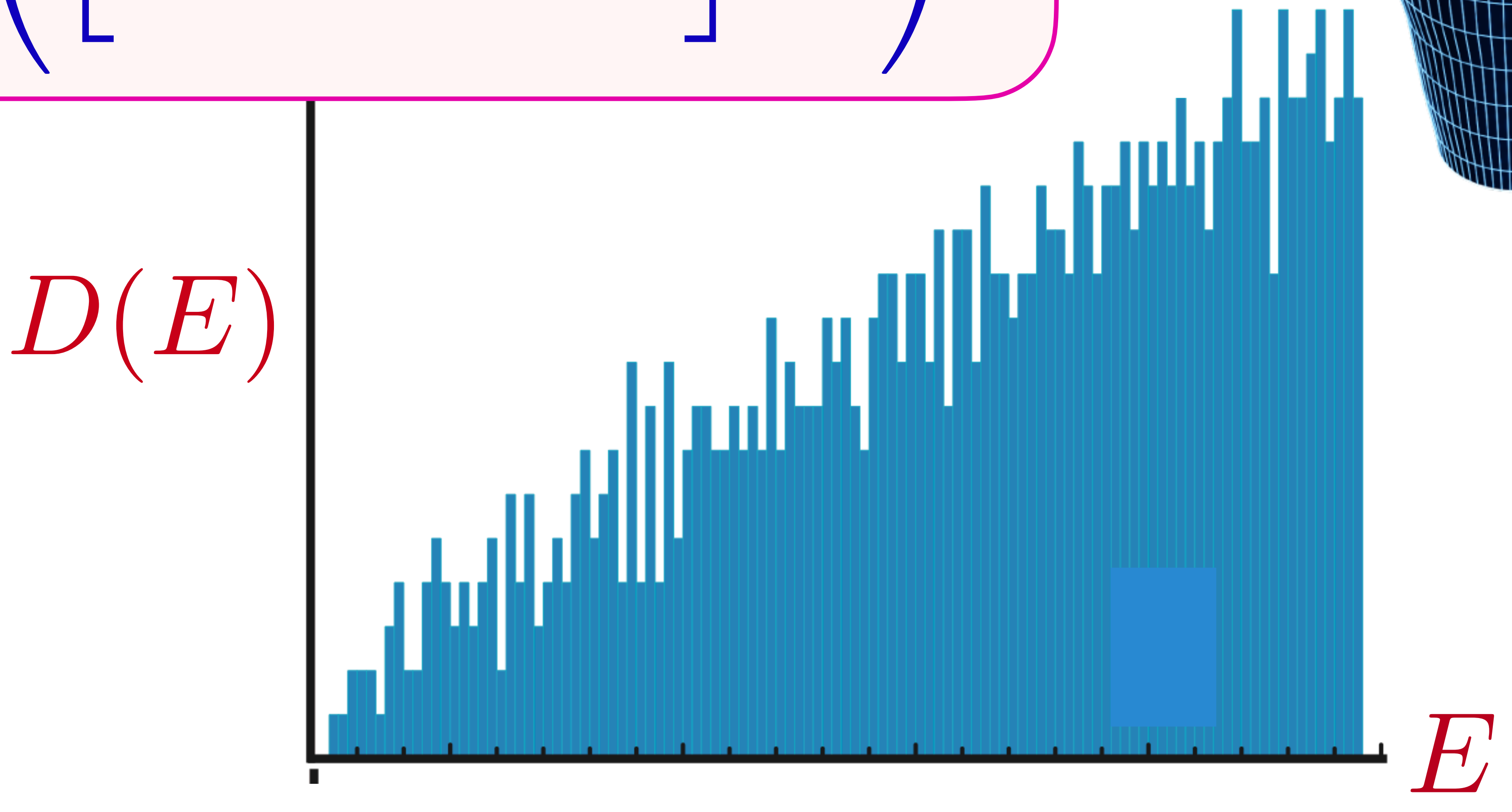

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Bekenstein-Hawking

Developments from the SYK model

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi}\, A_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

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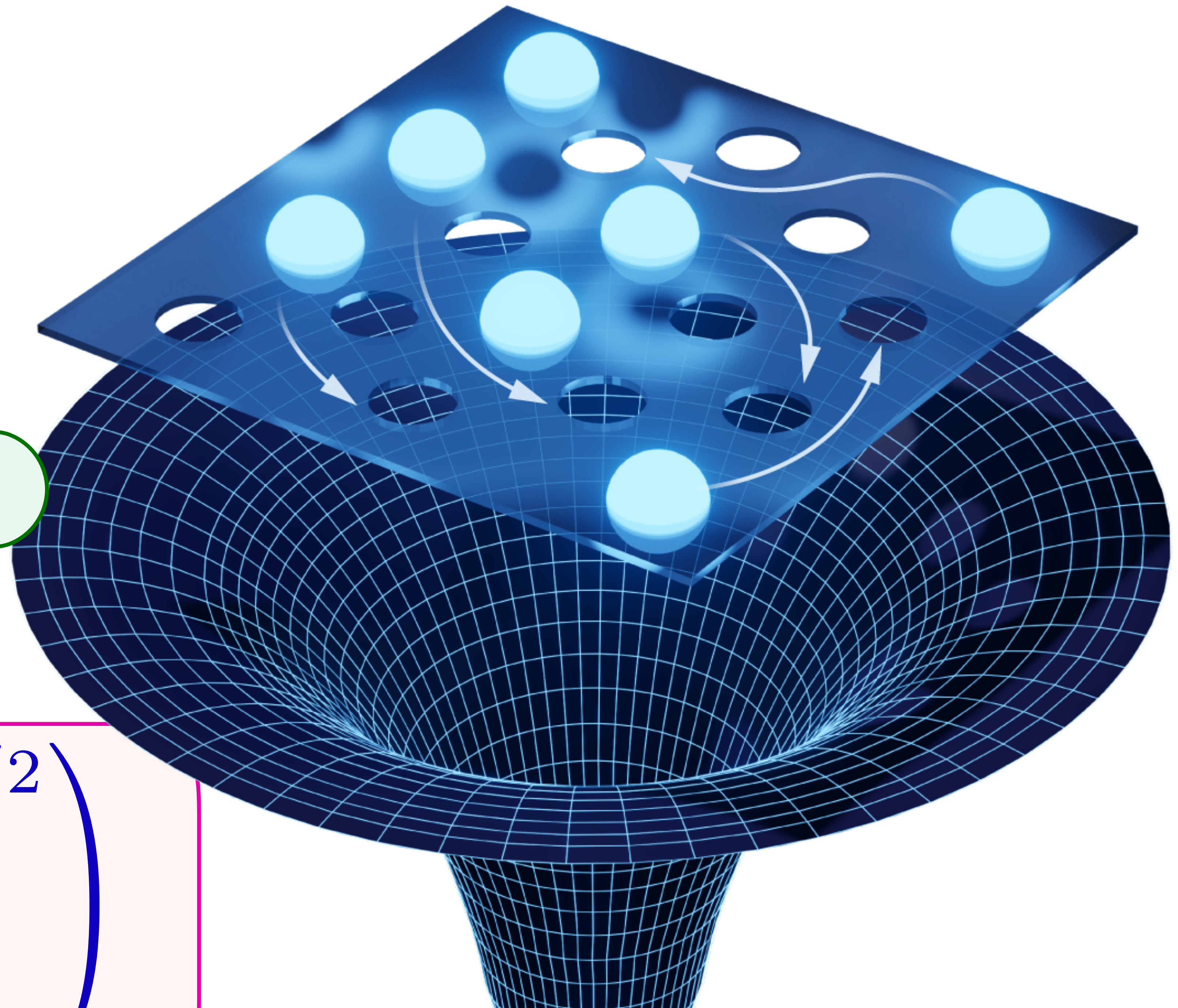
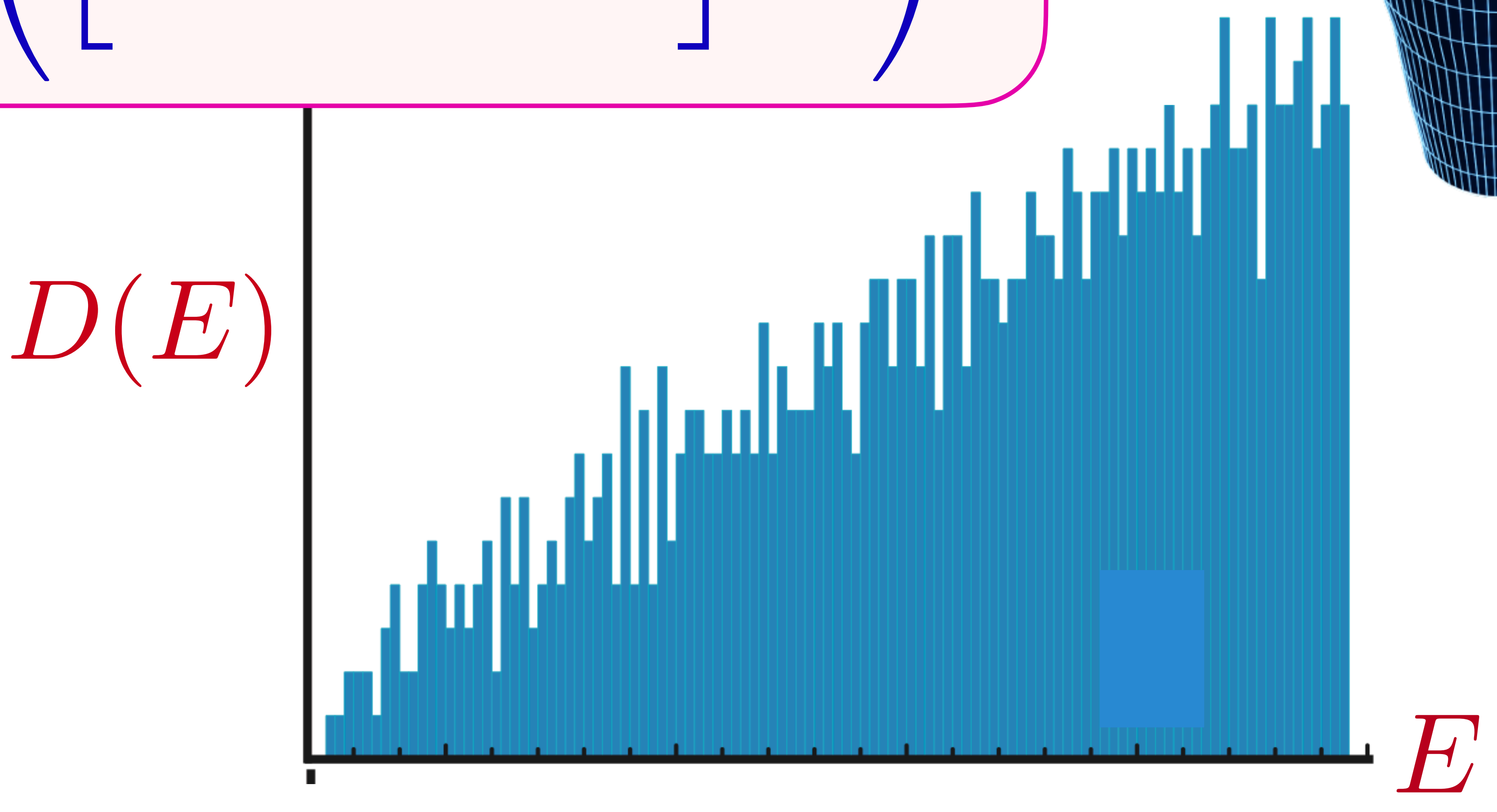
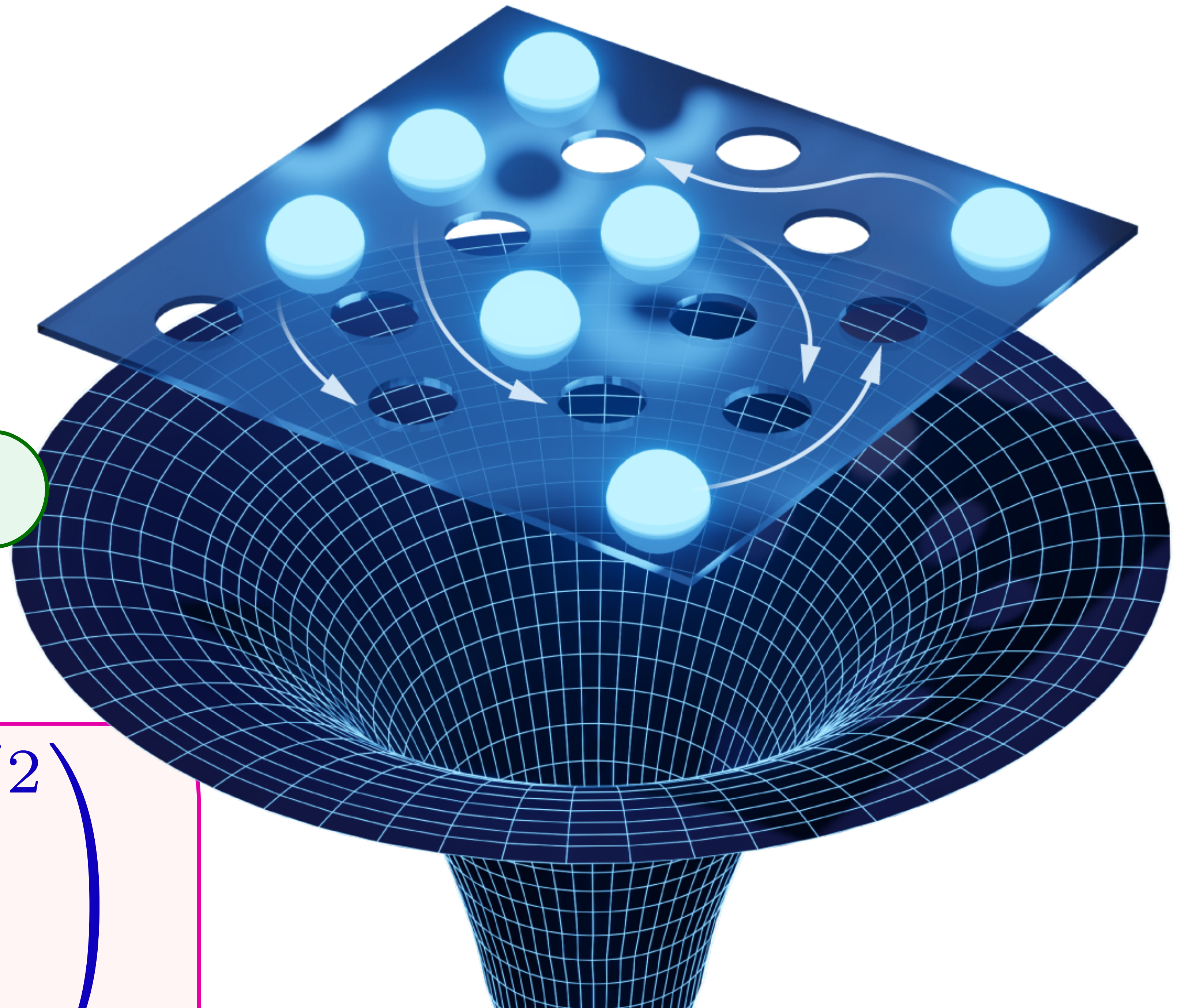




Quantum simulation of charged black holes by the SYK model



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Iliesiu, Murthy, Turiaci (2022)

Bekenstein-Hawking

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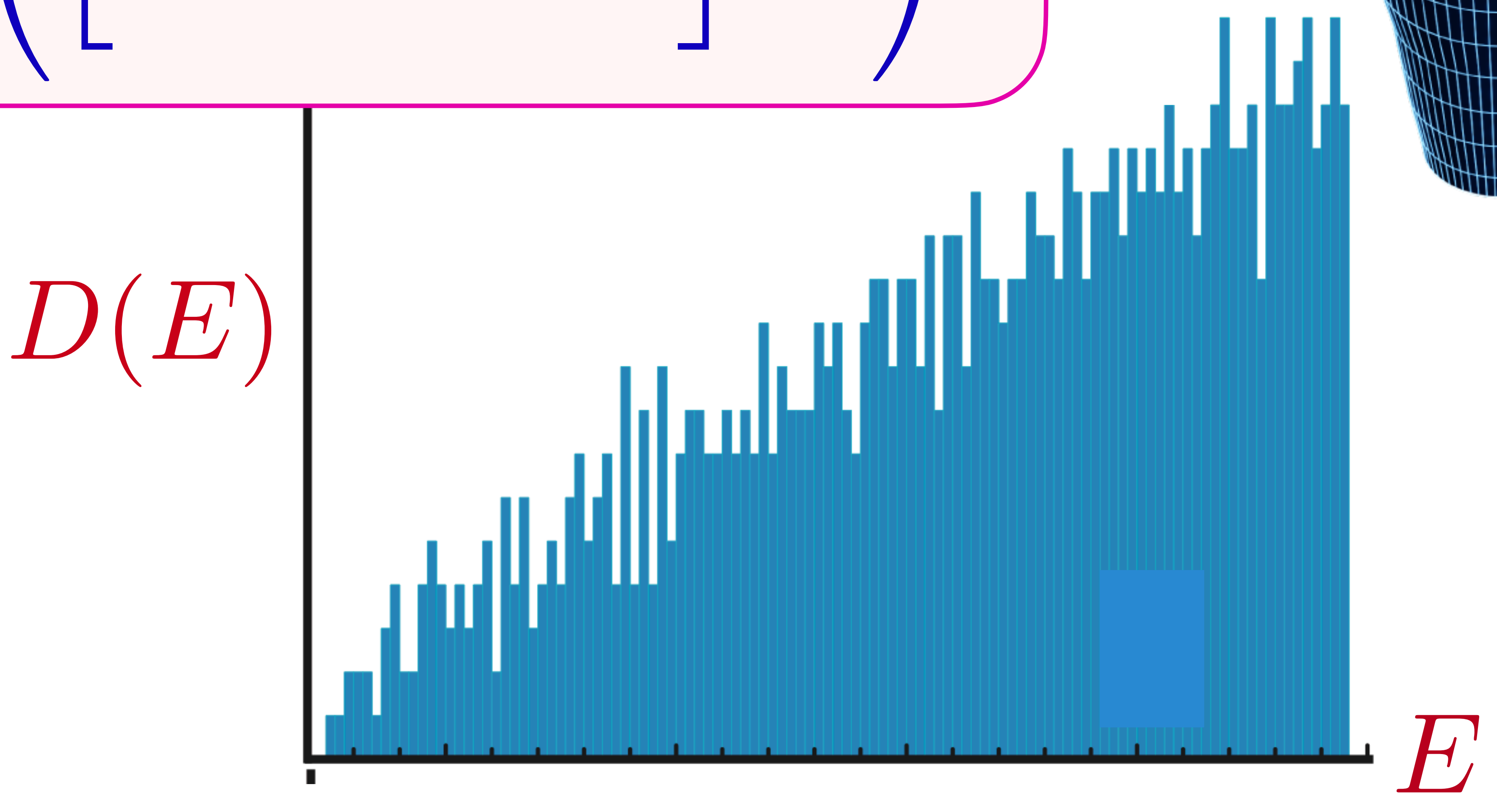


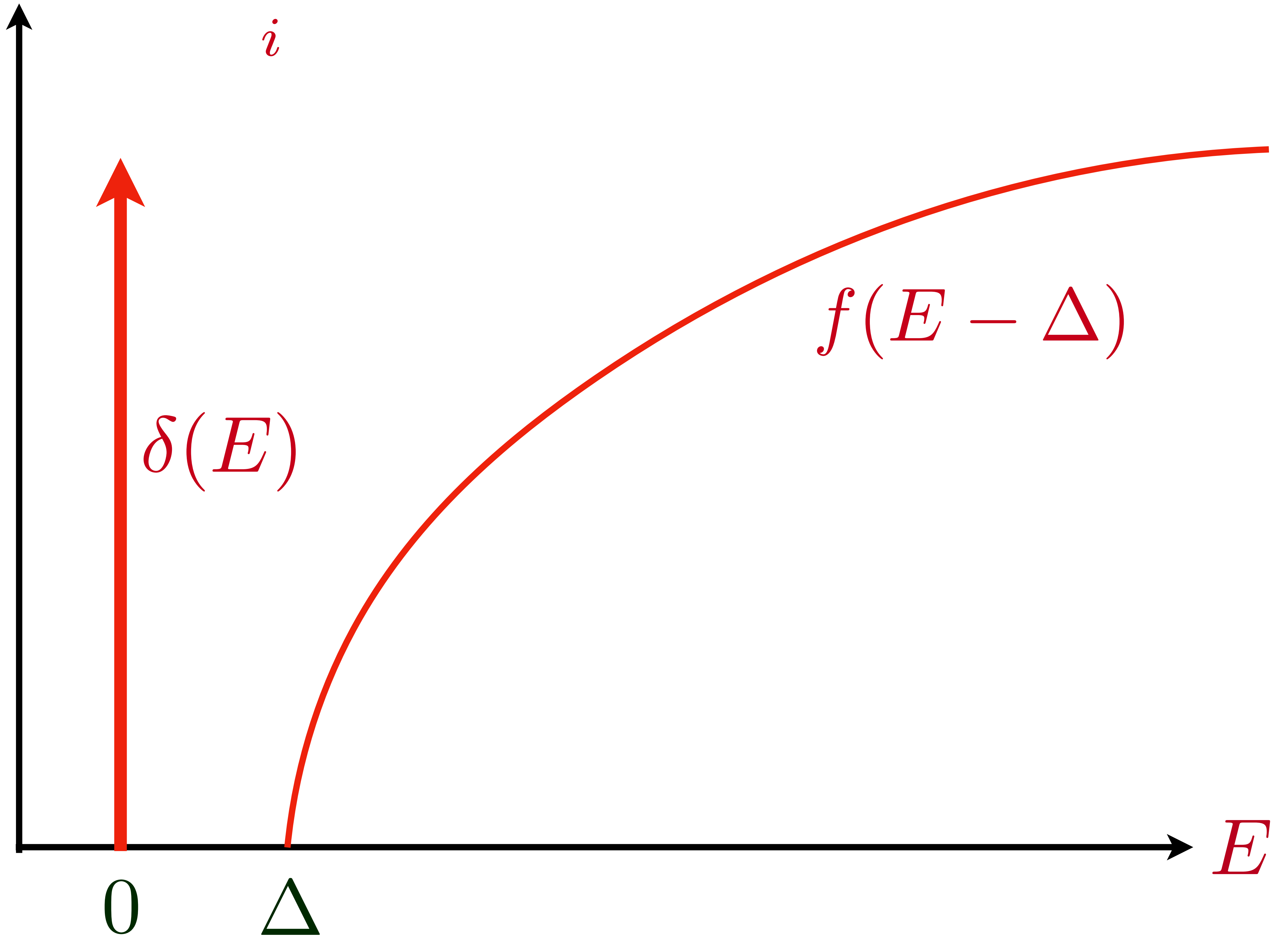
Strong theory of charged black holes

- With sufficient low energy supersymmetry, string theory yields:

$$D(E) = \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\delta(E) + \theta(E-\Delta) f(E-\Delta) + \ldots$$

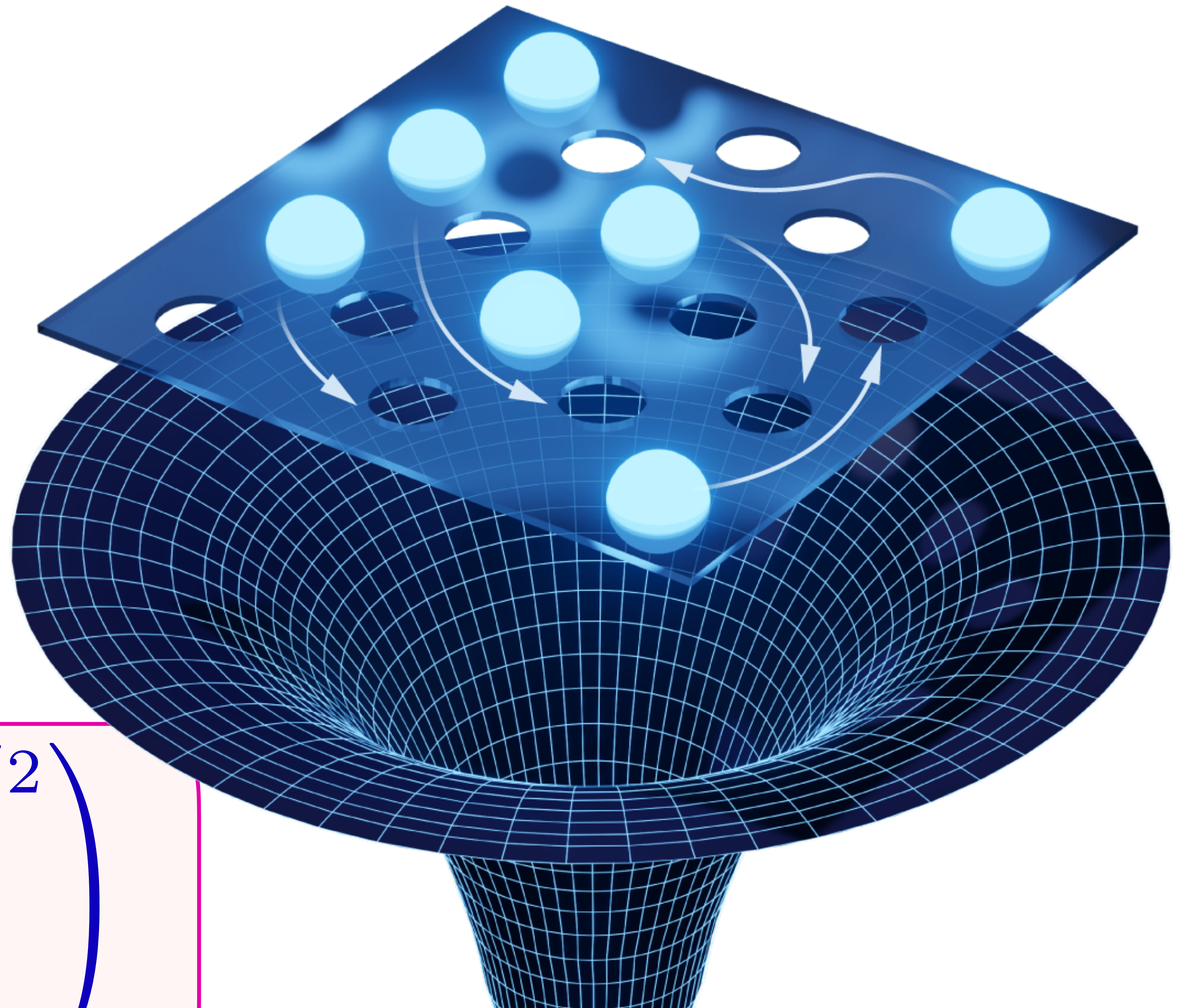
There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state.

$$D(E) = \sum_i \delta(E - E_i)$$



M. Heydeman, L.V. Iliesiu, G. J. Turiaci, and W. Zhao, 2020
L.V. Iliesiu, S. Murthy, G. J. Turiaci, 2022

Quantum simulation of charged black holes by the SYK model



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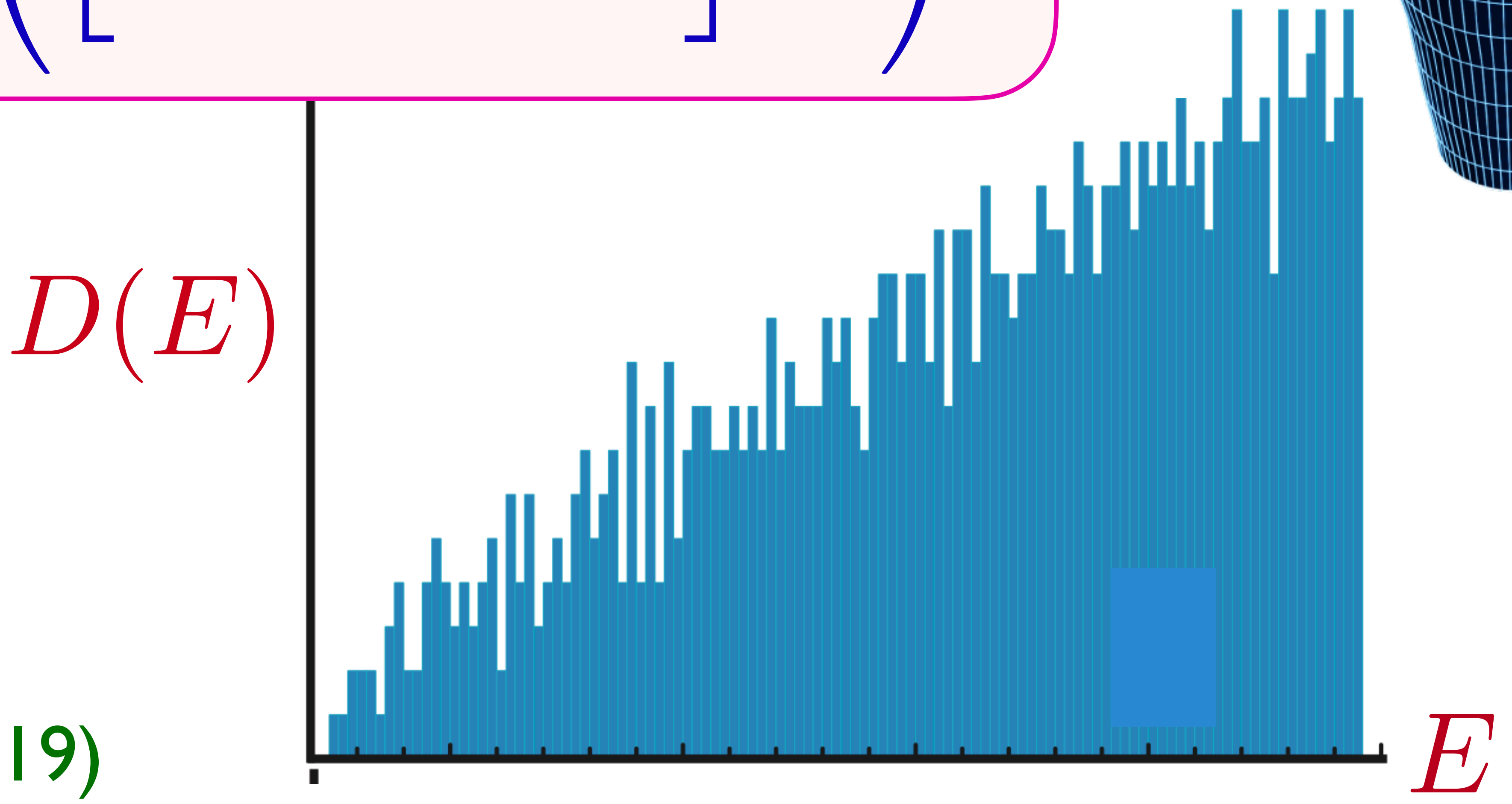
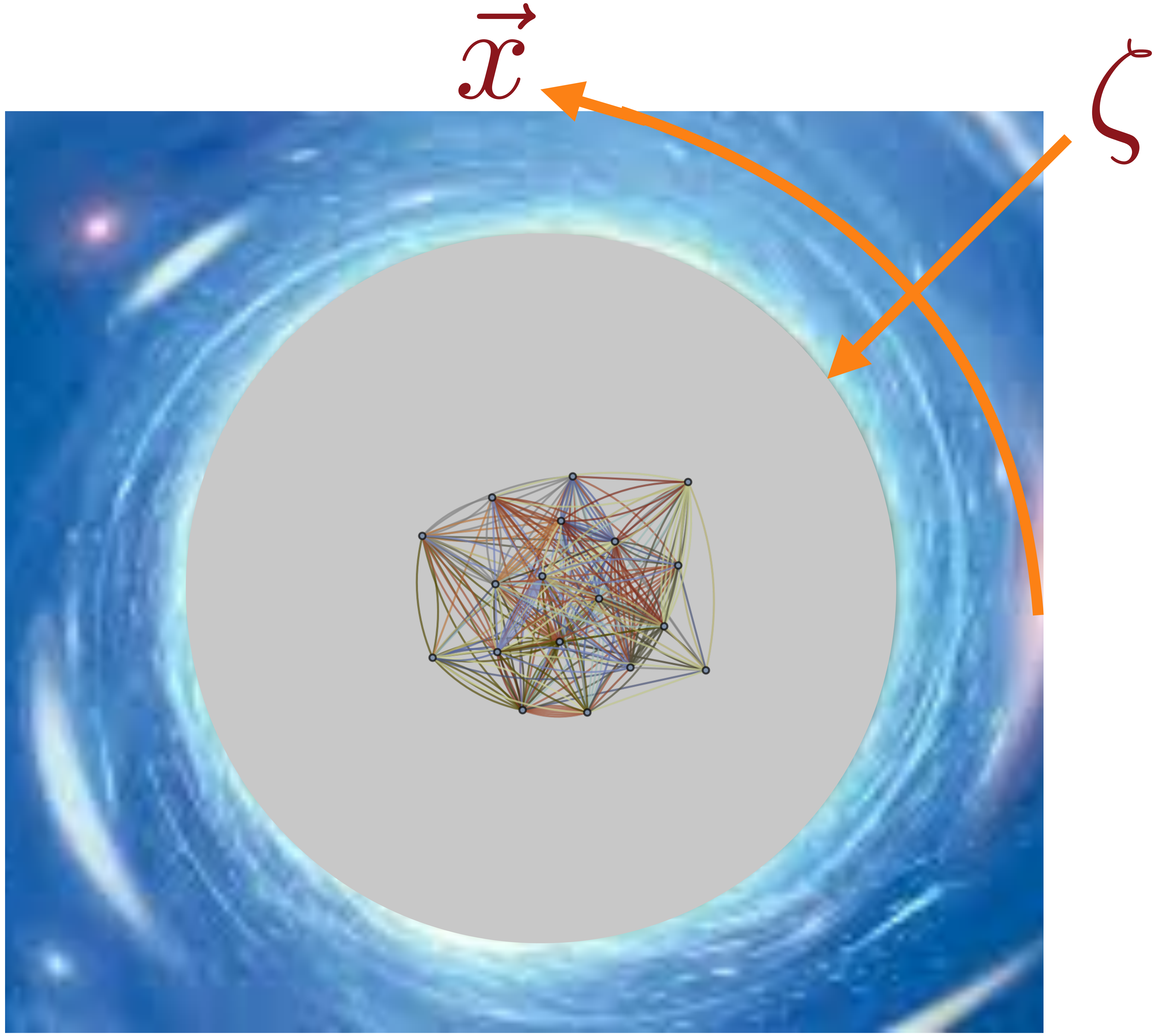
- ‘Wormhole’ contributions to this quantum simulation have led to an understanding of the Page curve of entanglement entropy of evaporating black holes.

Saad, Shenker, Stanford (2019)



Quantum simulation of charged black holes by the SYK model



The SYK provides the needed realization of the black hole interior, and its density of quantum states matches gravitational entropy computations for charged black holes !

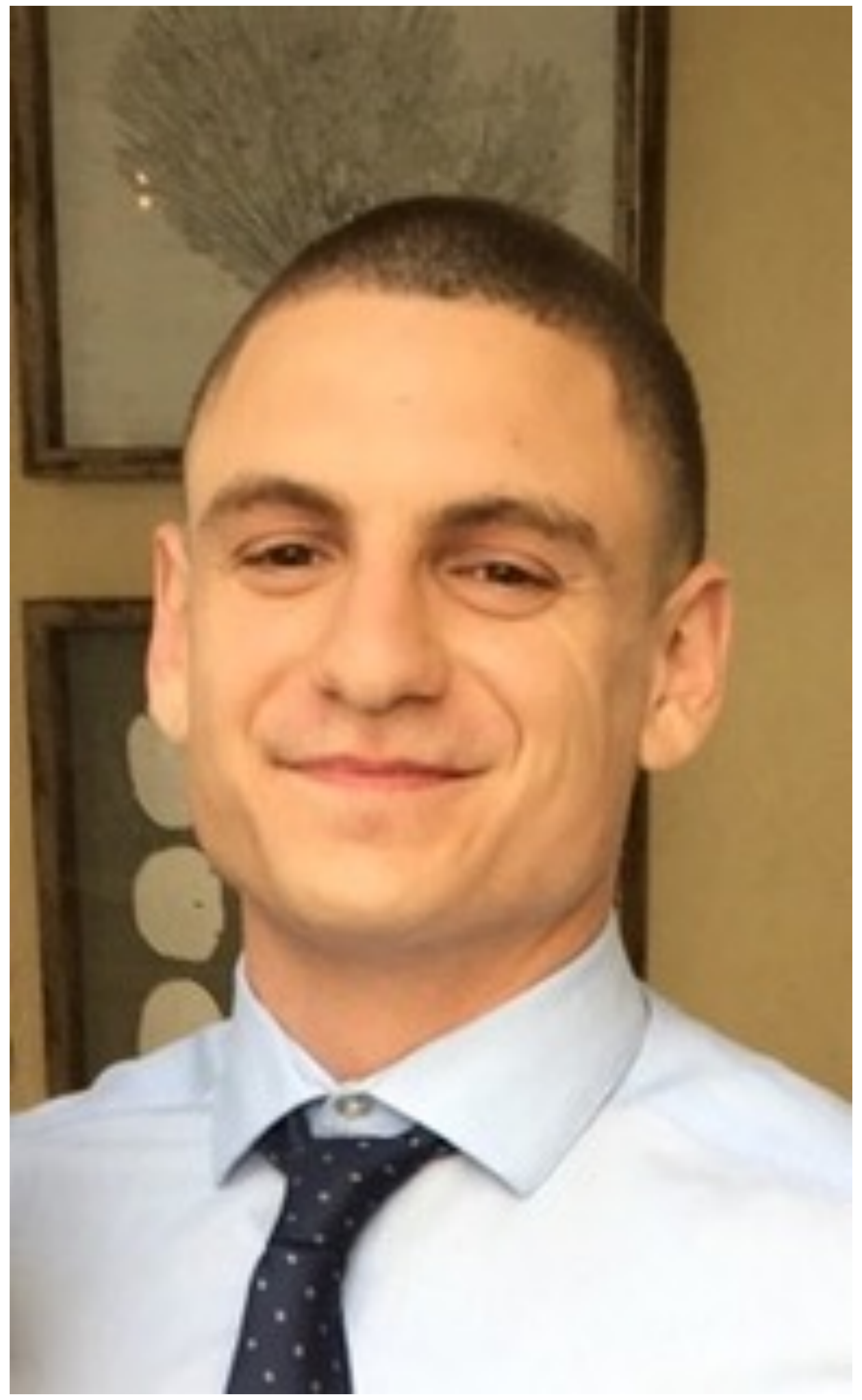
From the SYK model to a universal theory of strange metals

Aavishkar Patel

Flatiron Institute, NYC

Haoyu Guo

Harvard



Ilya Esterlis

Harvard ➔ Wisconsin

arXiv: 2103.08615, 2203.04990, 2207.08841

E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, arXiv:2012.00763

Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

3. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon……A. Georges, arXiv:2205.04030

4. Photoemission: nearly “marginal Fermi liquid” electron spectral density:

$$\mathrm{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau(\omega)} \sim |\omega| \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

T.J. Reber….D. Dessau, Nature Communications **10**, 5737 (2019)

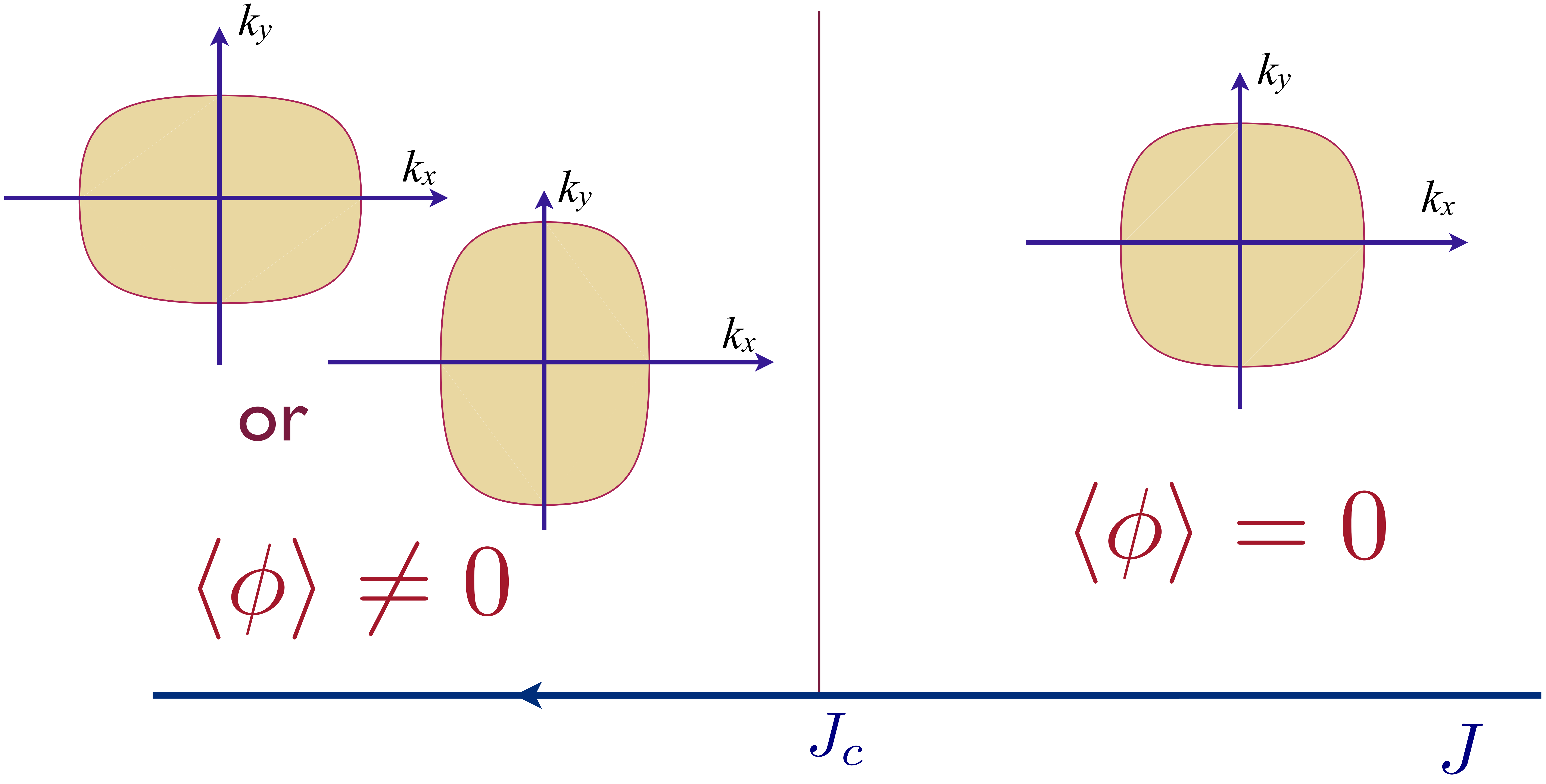
Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell}\psi_i^\dagger \psi_j \phi_\ell \,,$$

$g_{ij\ell}$ independent random numbers with zero mean. Large N limit leads to Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$.

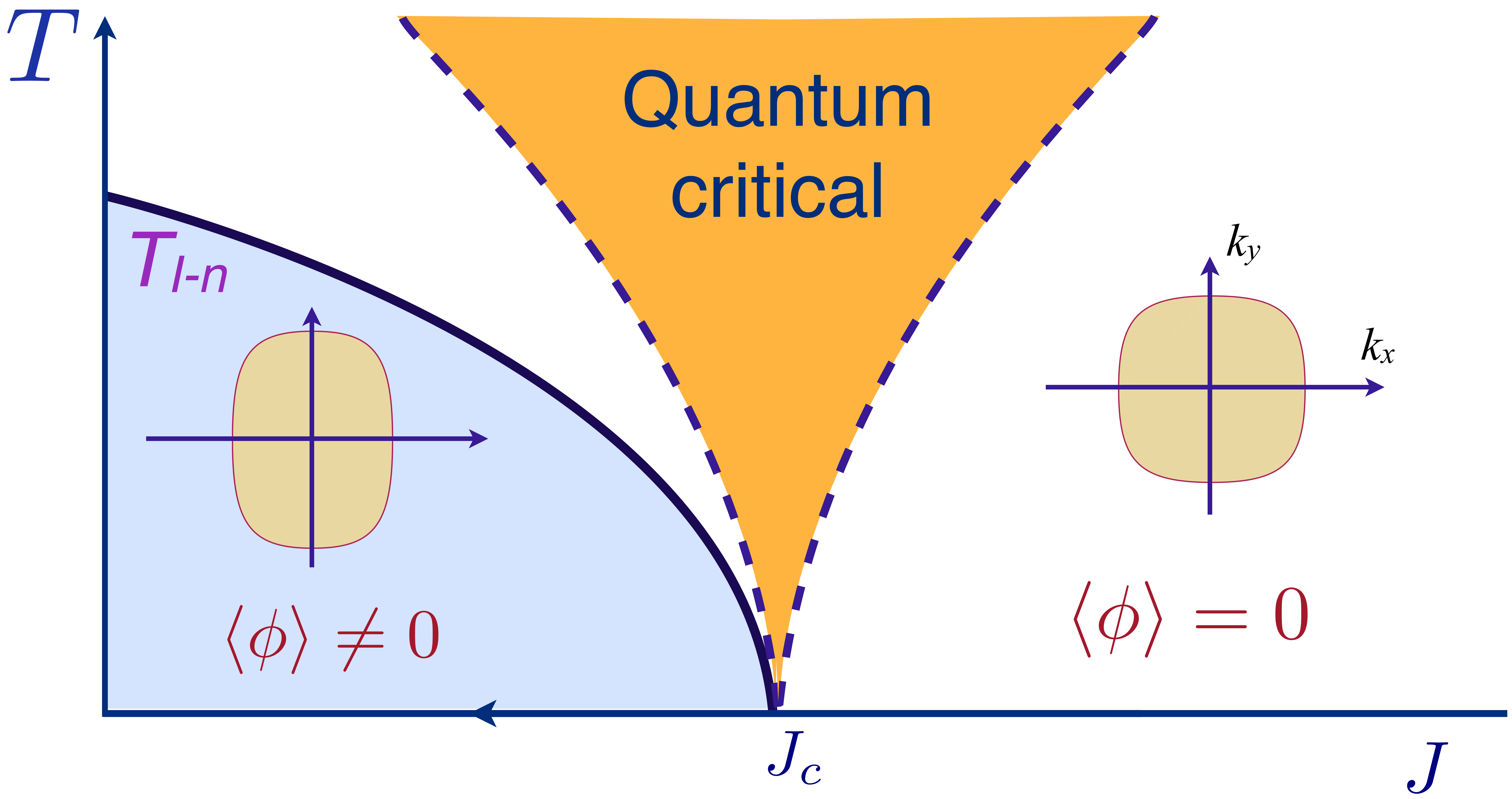
W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Quantum criticality of Ising-nematic ordering in a metal

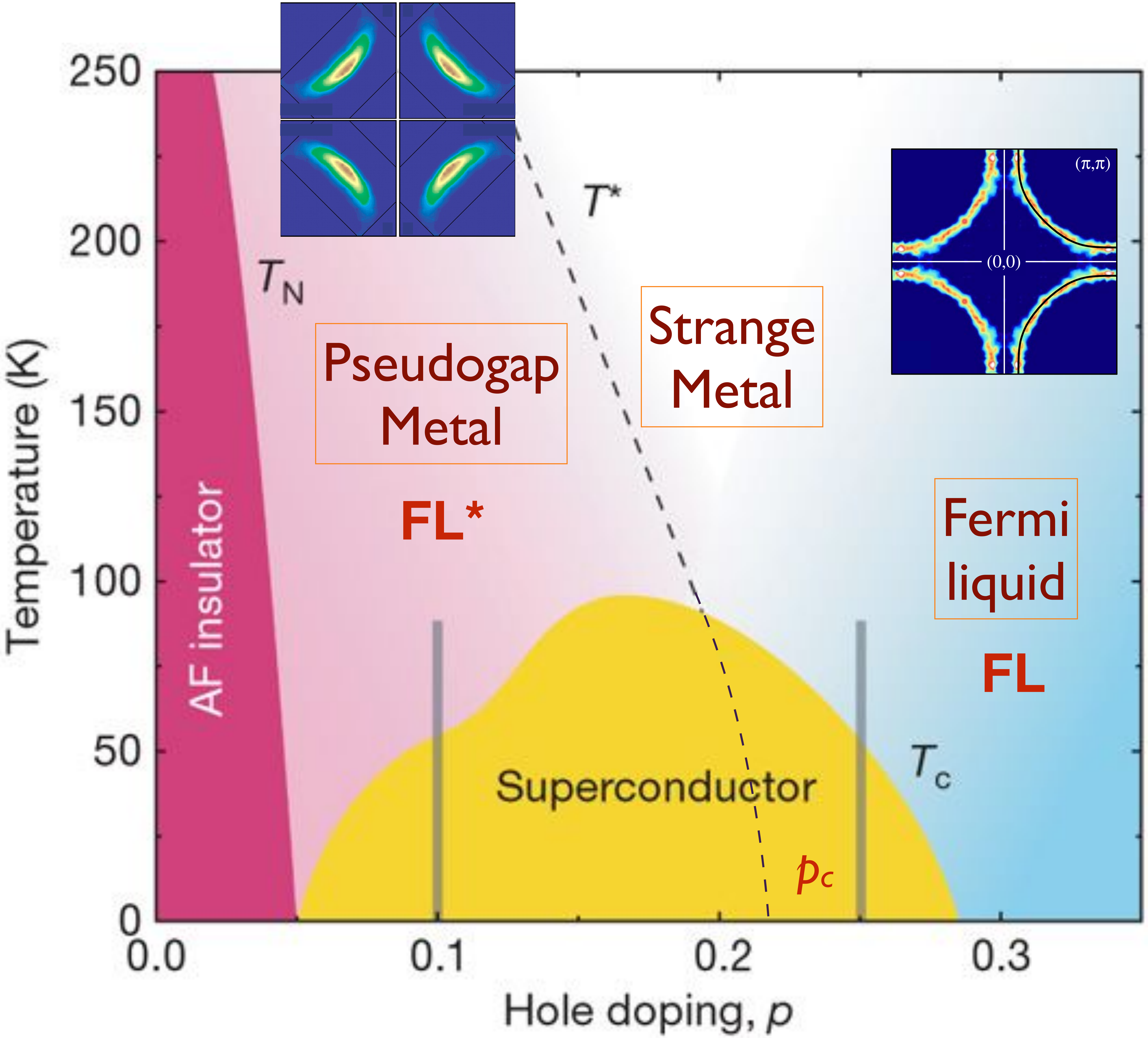


Pomeranchuk instability as a function of coupling J

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and J



Our results also apply
to the transition from a

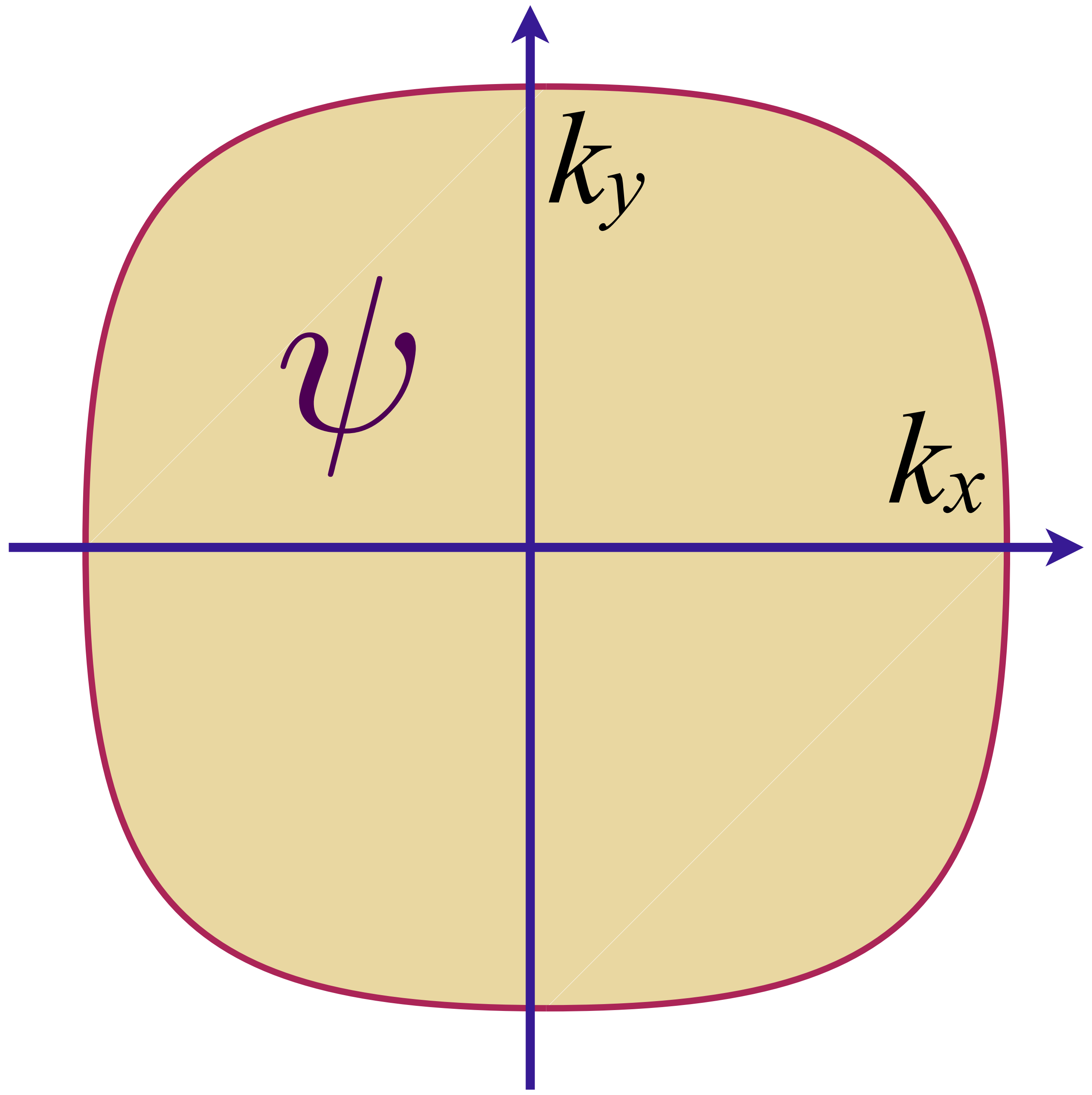
fractionalized Fermi liquid (FL*)
to a Fermi liquid (FL)

in a single-band Hubbard model,
or in a Kondo lattice model;

a Higgs field ("slave boson")
takes the place of ϕ.

Fermi surface

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

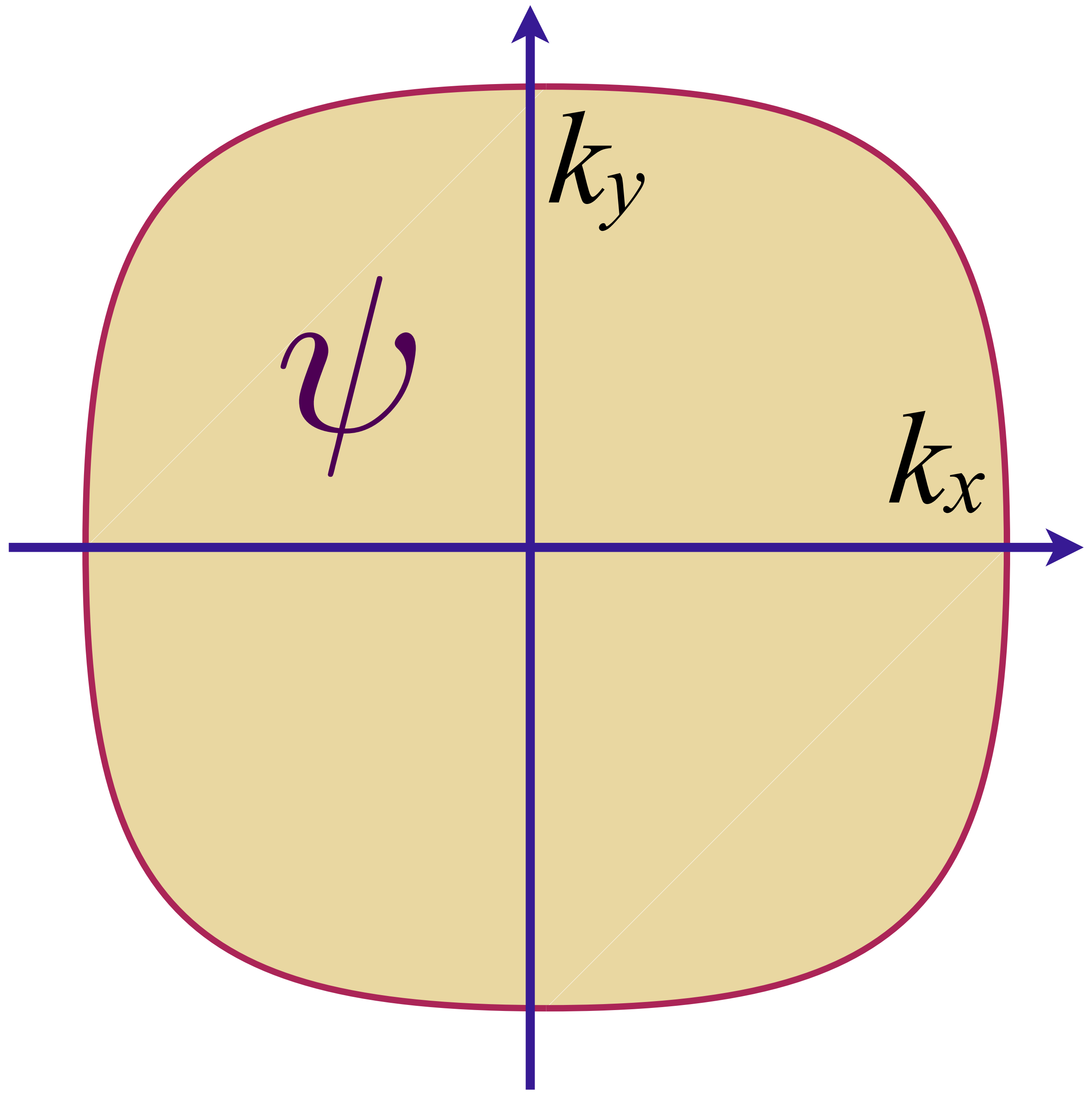


$$- J \, \psi^\dagger(\boldsymbol{r}) \psi^\dagger(\boldsymbol{r}) \psi(\boldsymbol{r}) \psi(\boldsymbol{r})$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order,
Higgs field ...



$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
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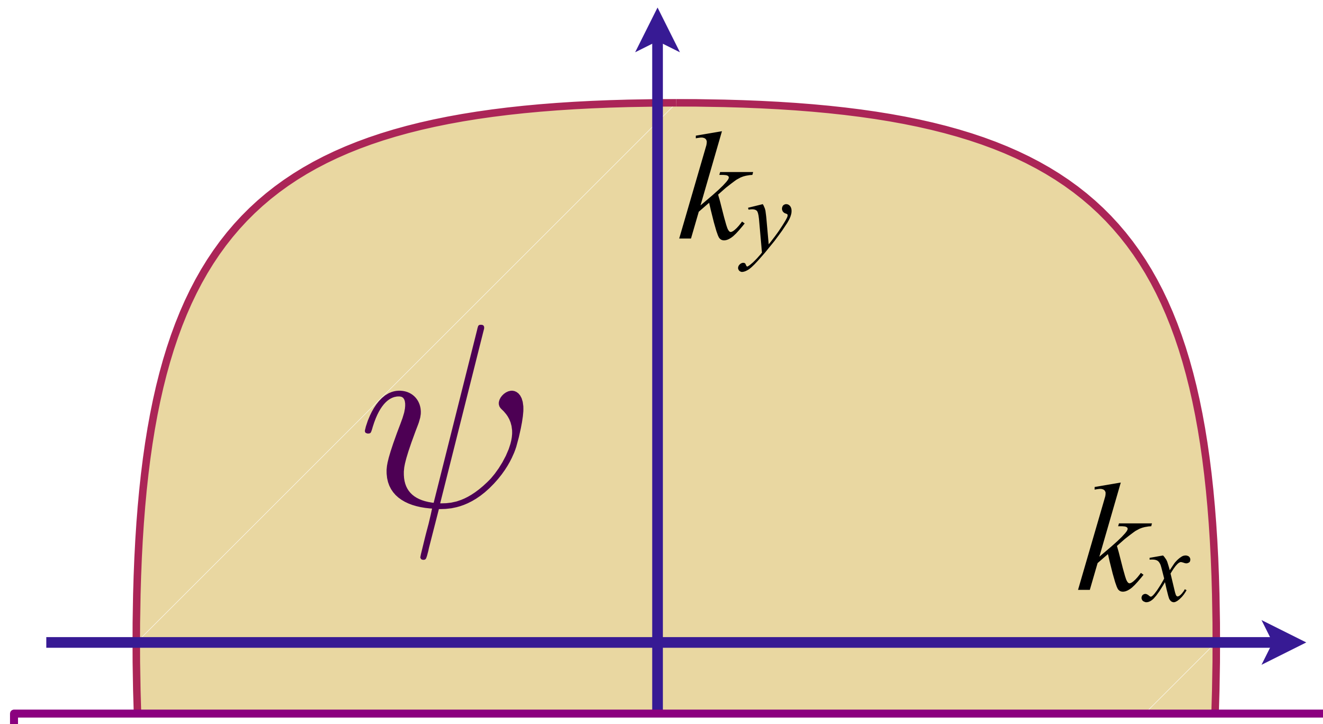


$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Solve in a large N limit with Yukawa coupling

$$\frac{g_{ij\ell}}{N} \int d^2 r d\tau \, \psi^\dagger_i(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau) \quad , \quad \overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

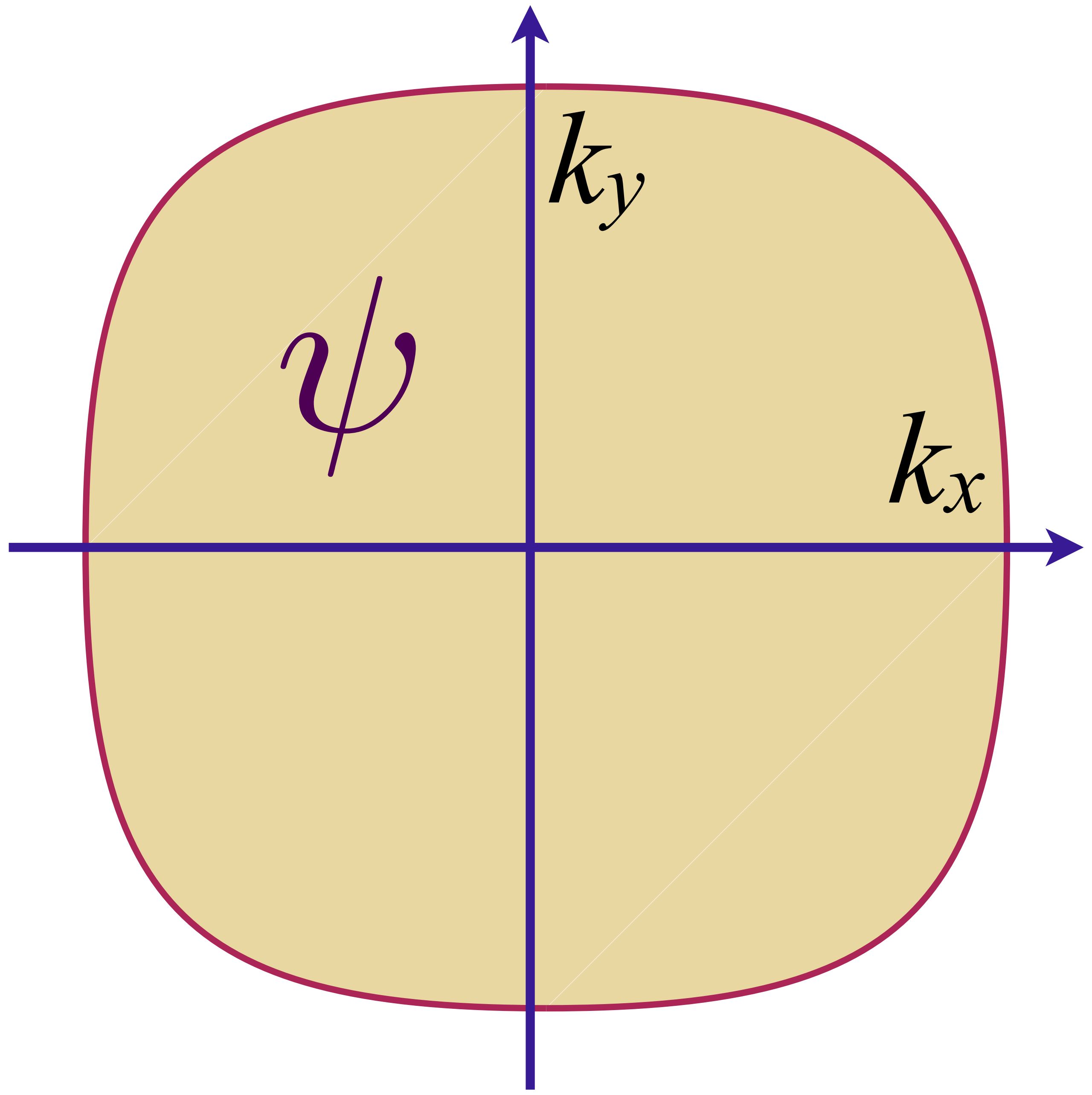
to obtain Eliashberg solution for electron (G) and boson (D) Green's functions at small ω:

$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i \text{sgn}(\omega)|\omega|^{2/3}, \quad G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)}, \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma|\Omega|/q}$$

P.A. Lee (1989)

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order,
Higgs field . . .

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Transport—a perfect metal!

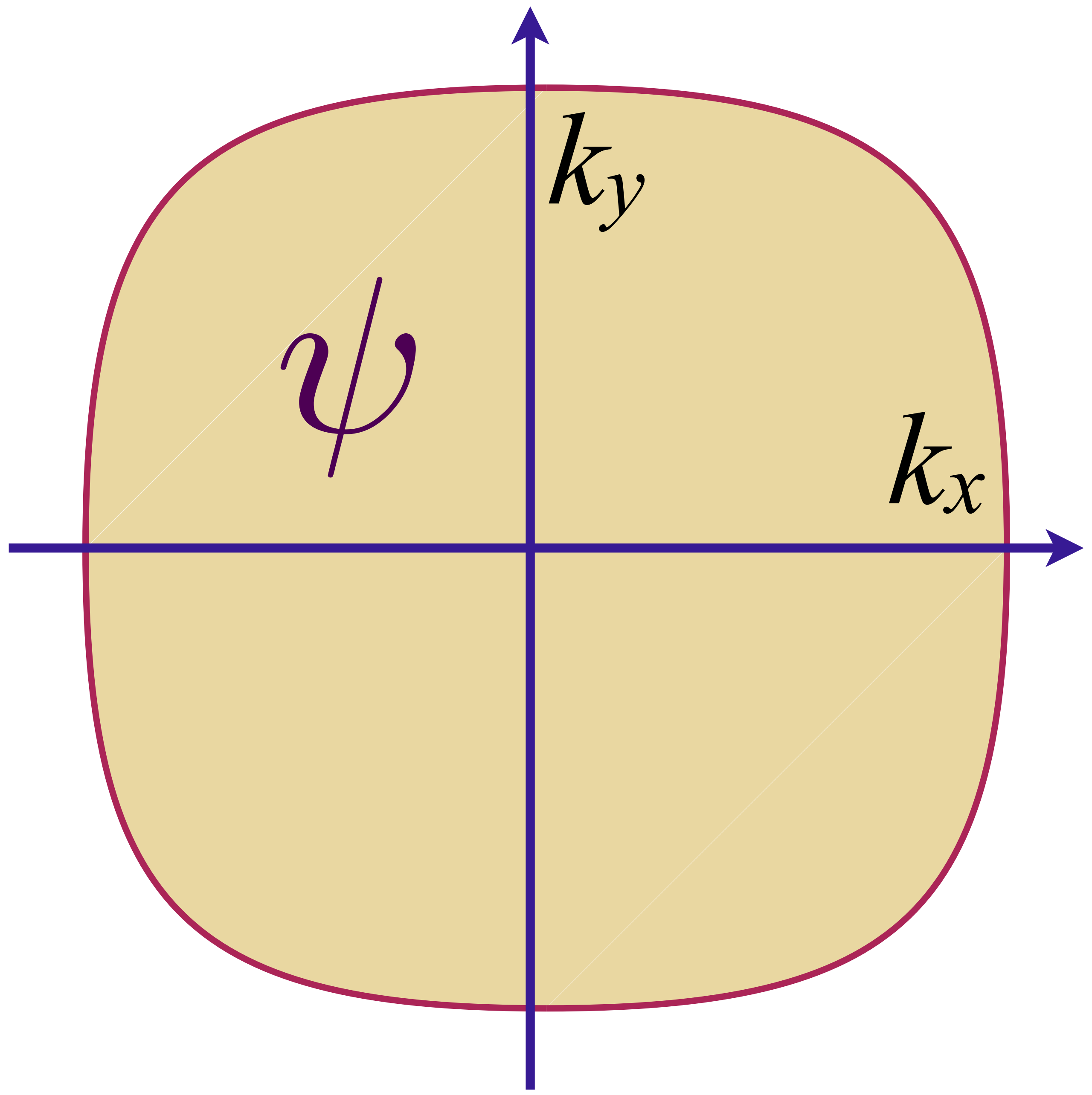
Conservation of momentum and fermion-boson drag imply:

$$\sigma(\omega) \sim \frac{1}{-i\omega} + |\omega|^0 + \cdots \quad (\omega^{-2/3} \text{ term has vanishing co-efficient})$$

Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S. PRB **106**, 115151 (2022)

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order,
Higgs field ...

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

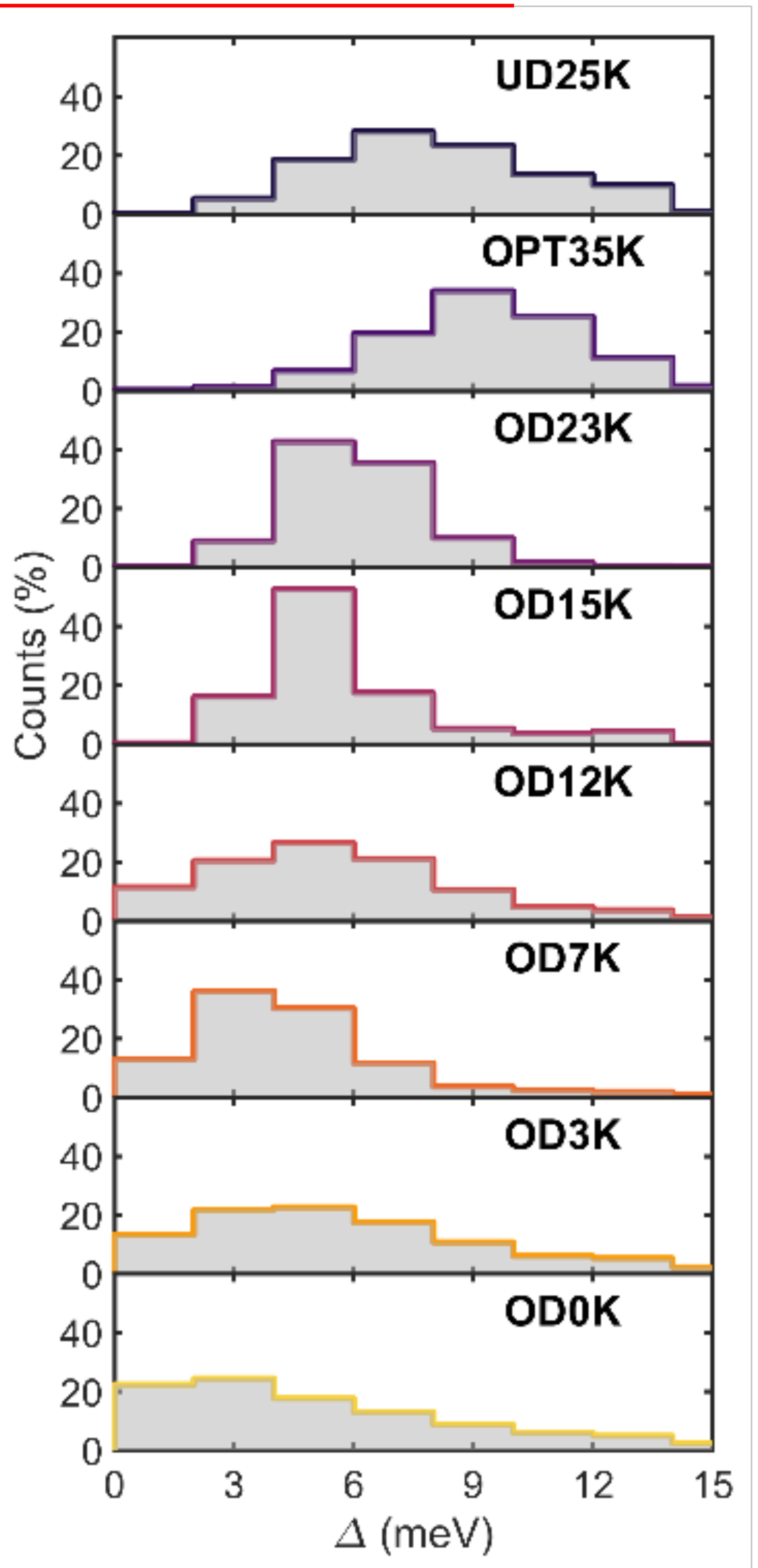
Spatially random interactions!

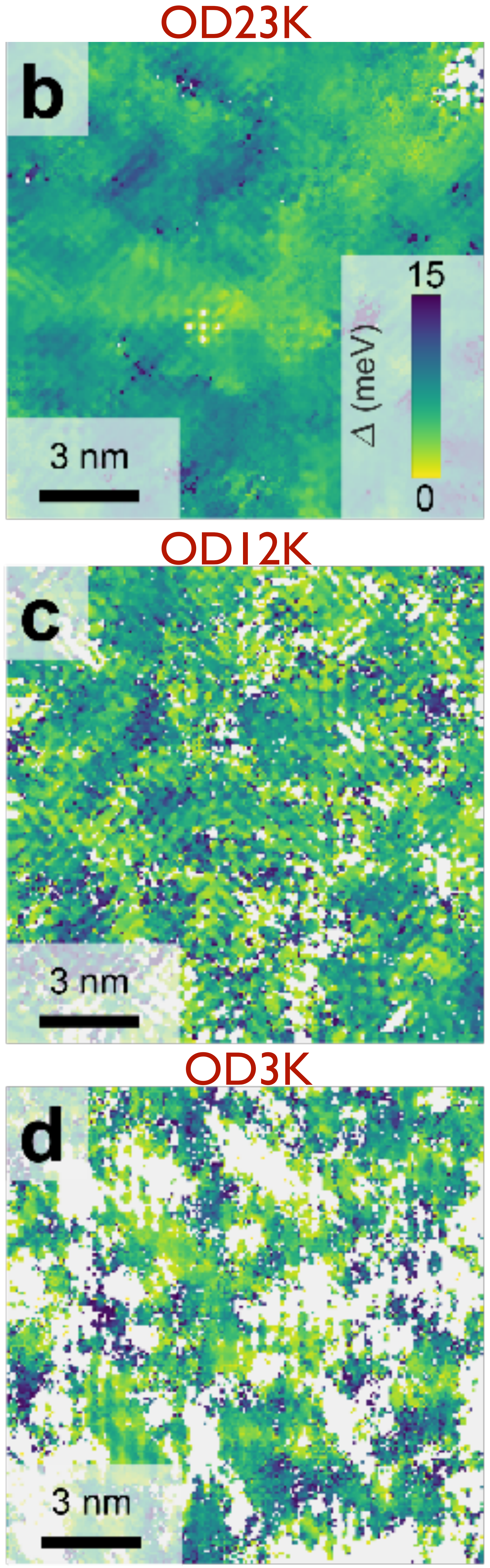
Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped (Pb,Bi)$_2$Sr$_2$CuO$_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge,
Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos,
Amber Vervloet, Steef Smit, Erik van Heumen,
Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin,
Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen,
Milan P. Allan

Our scanning tunneling spectroscopy measurements in the overdoped regime of the (Pb,Bi)$_2$Sr$_2$CuO$_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

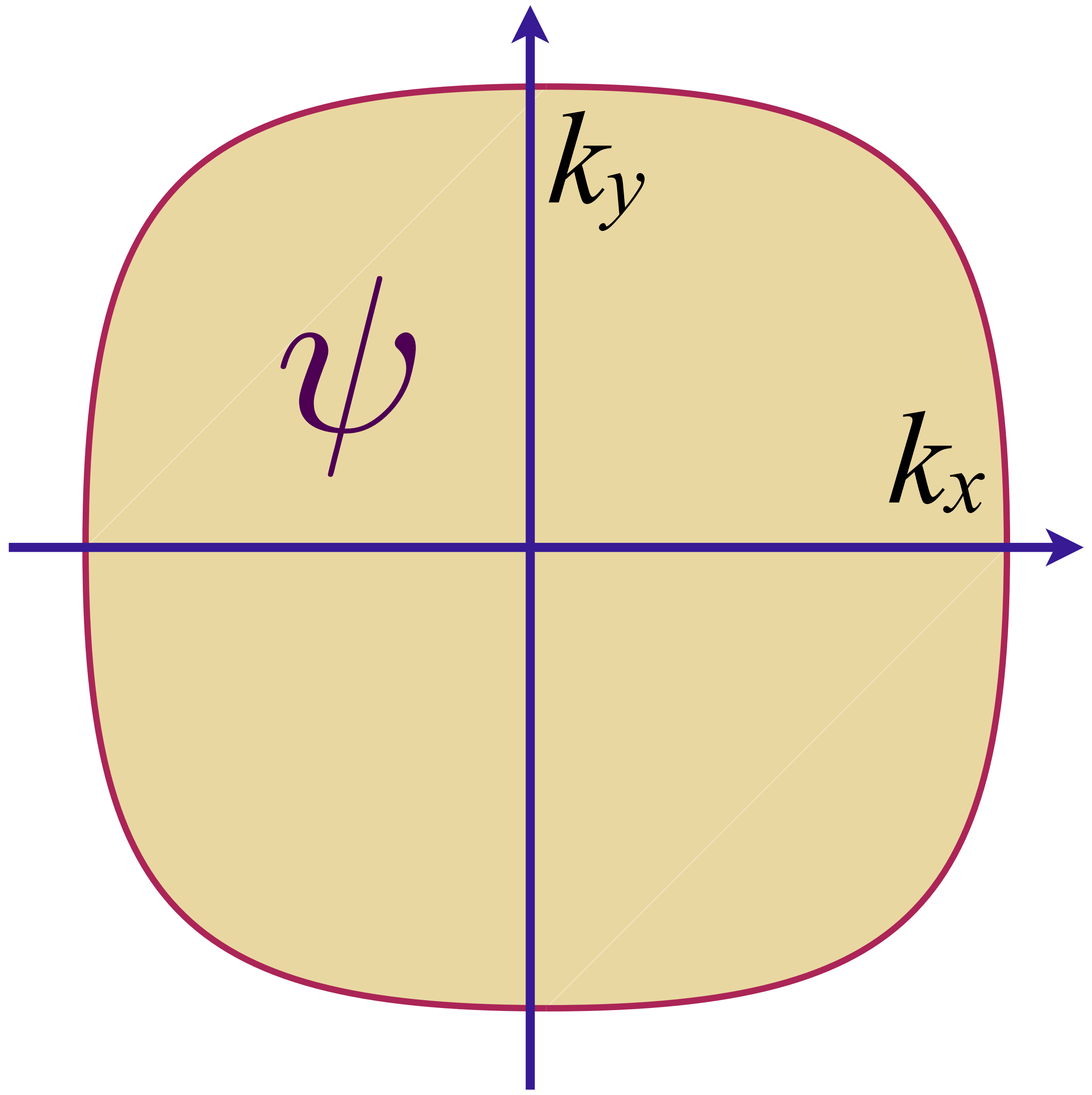
arXiv:2205.09740





Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order,
Higgs field ...

$$\frac{[\phi(\boldsymbol{r})]^2}{J + J'(\boldsymbol{r})} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Fermi surface coupled to a critical boson with disorder

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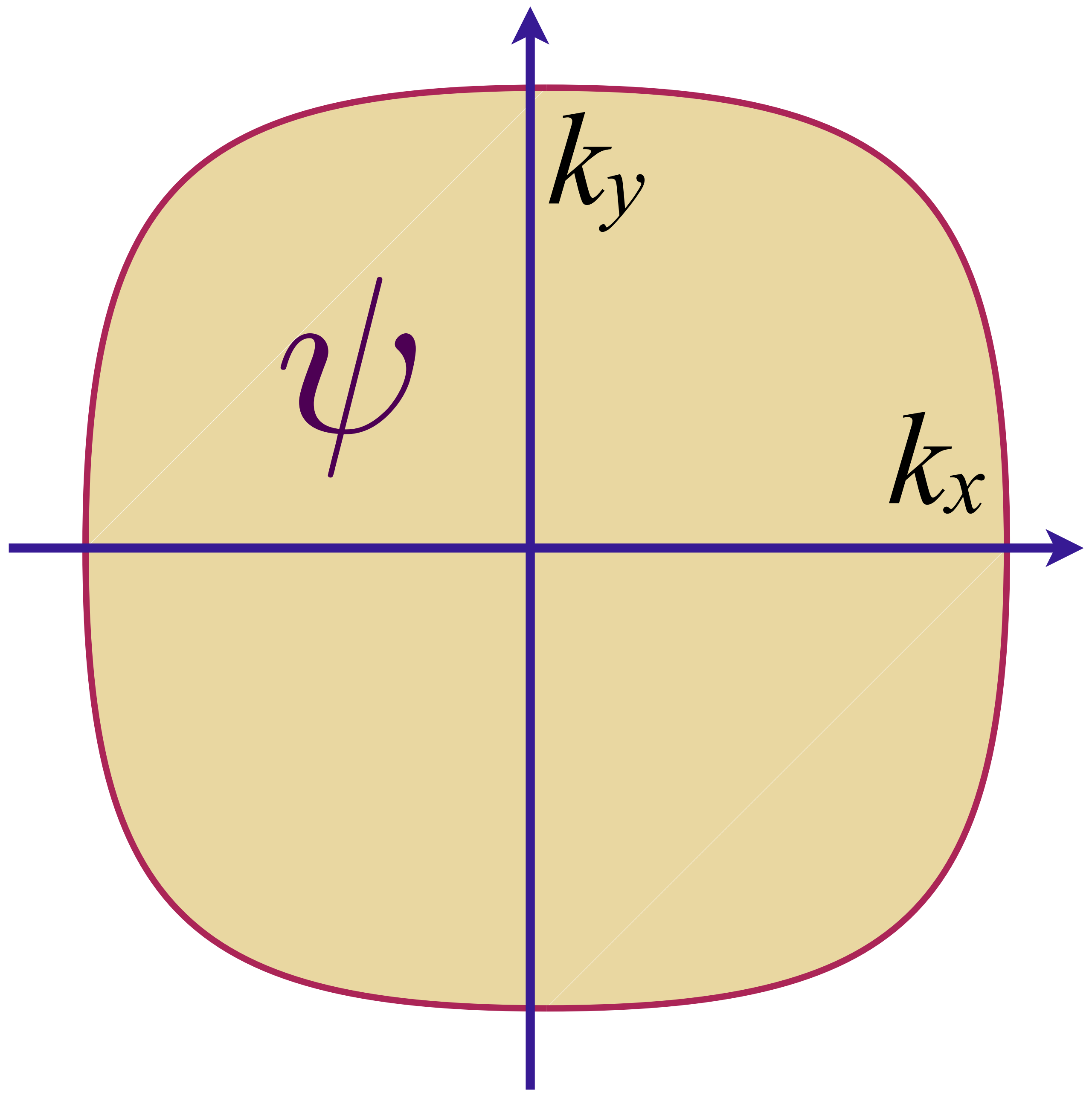


a critical boson ϕ
e.g. Ising-nematic order,
Higgs field ...

$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

ϕ^2 "mass" disorder $J'(\boldsymbol{r})$ is strongly relevant;
rescale ϕ to move disorder to the Yukawa coupling;

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Fermi surface coupled to a critical boson with disorder

$$\text{Conductivity: } \sigma(\omega) \sim \frac{1}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \, \dfrac{m^*_{\rm trans}(\omega)}{m}}$$

$$\frac{1}{\tau_{\rm trans}(\omega)} \sim v^2 + g'^2 |\omega| \quad ; \quad \frac{m^*_{\rm trans}(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2; Transport insensitive to g

Fermi surface coupled to a critical boson with disorder

$$\text{Conductivity: } \sigma(\omega) \sim \frac{1}{\dfrac{1}{\tau_{\text{trans}}(\omega)} - i\omega\,\dfrac{m^*_{\text{trans}}(\omega)}{m}}$$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*_{\text{trans}}(\omega)}{m} \sim \frac{2g'^2}{\pi}\ln(\Lambda/\omega)$$

$$\text{Electron Green's function: } G(\omega) \sim \frac{1}{\omega\,\dfrac{m^*(\omega)}{m} - \varepsilon(\boldsymbol{k}) + i\left(\dfrac{1}{\tau_e} + \dfrac{1}{\tau_{\text{in}}(\omega)}\right)\text{sgn}(\omega)}$$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2\right)|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi}\left(\frac{g^2}{v^2} + g'^2\right)\ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2;
Transport insensitive to g; Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat.

Summary

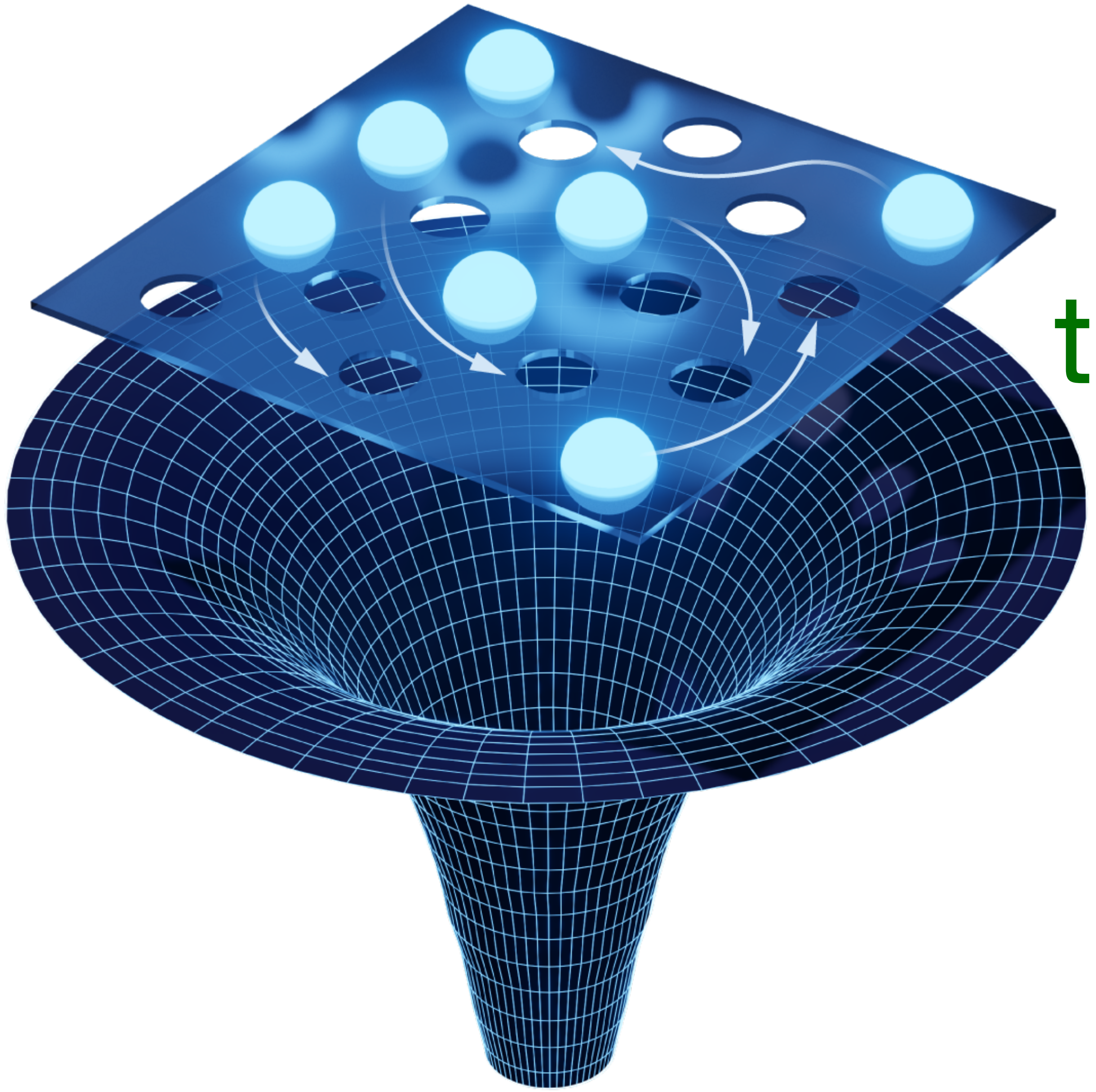
- SYK: a solvable toy model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

- Toy SYK model captures the correct universal low energy quantum theory of charged black holes, and provides a Hamiltonian realization of black hole microstates.

- Linear-T resistivity, $T \ln(1/T)$ specific heat, $\sim 1/\omega$ optical conductivity, and marginal Fermi liquid electron spectrum *all* arise from a SYK-like model with spatially random interactions in a two-dimensional quantum-critical metal.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990

The Sachdev-Ye-Kitaev (SYK) model



The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)

☯

In a *dual* set of variables it describes ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)