

# Unveiling the order of the high temperature superconductors

University of Innsbruck  
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PERIMETER INSTITUTE  
FOR THEORETICAL PHYSICS



JOHN TEMPLETON  
FOUNDATION



HARVARD

Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)

# Theorists at Harvard



Max Metlitski  
(KITP, UCSB)



Andrea Allais



Matthias Punk  
(Innsbruck)



Debanjan  
Chowdhury



Alexandra  
Thomson

# Cornell



Kazuhiro Fujita  
Cornell/ BNL



Mohammad Hamidian  
Cornell / BNL



Stephen Edkins  
Cornell / St Andrews



Michael Lawler

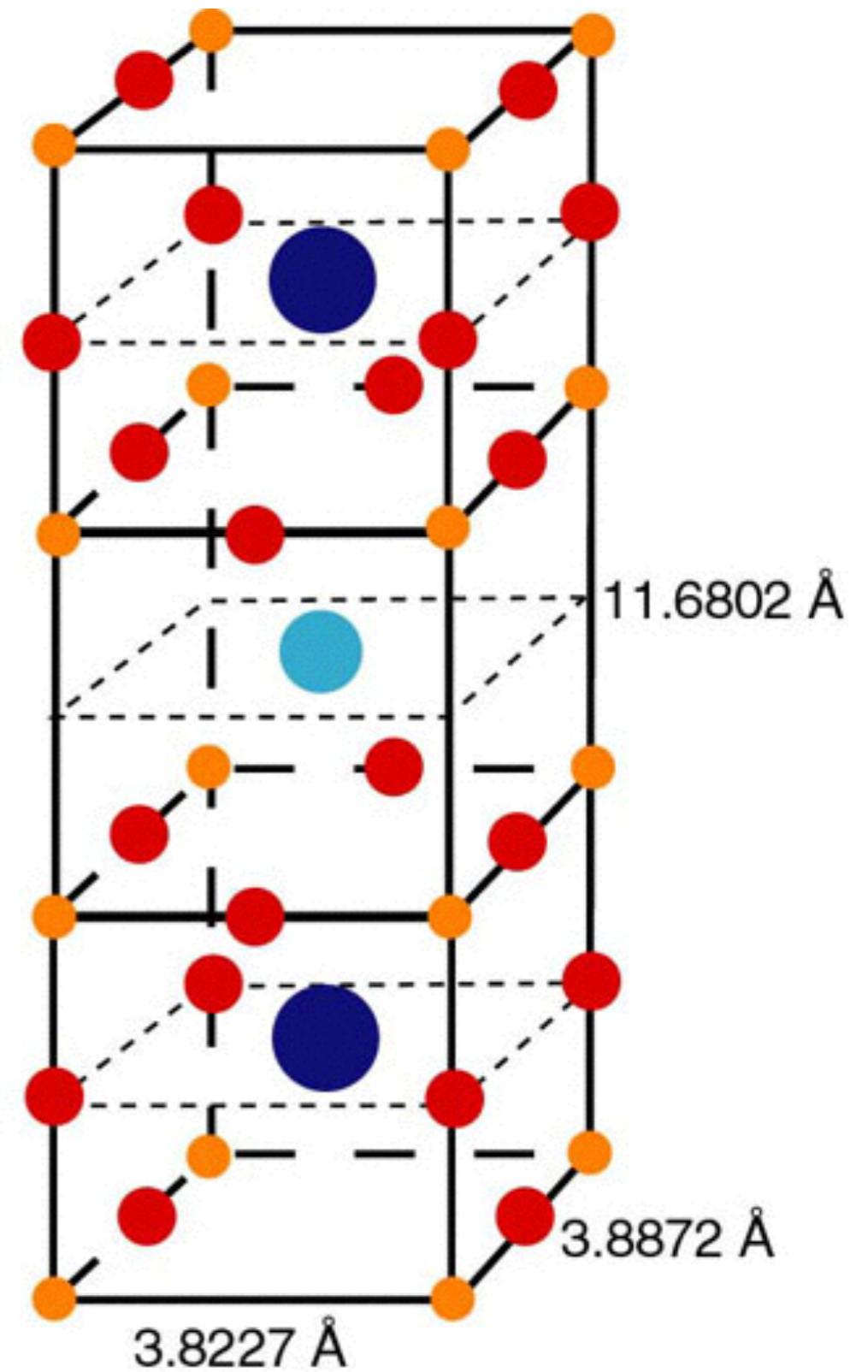
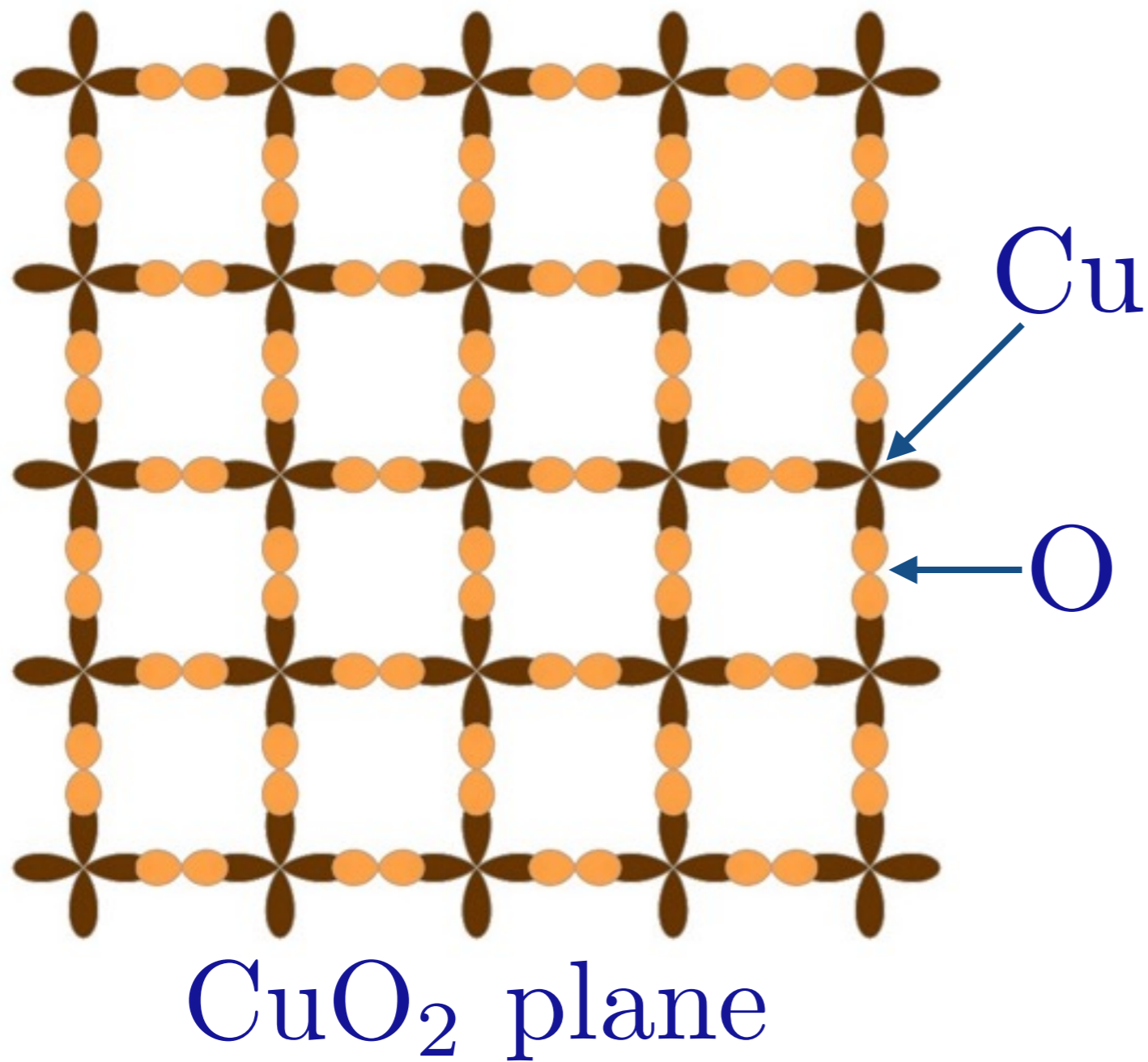


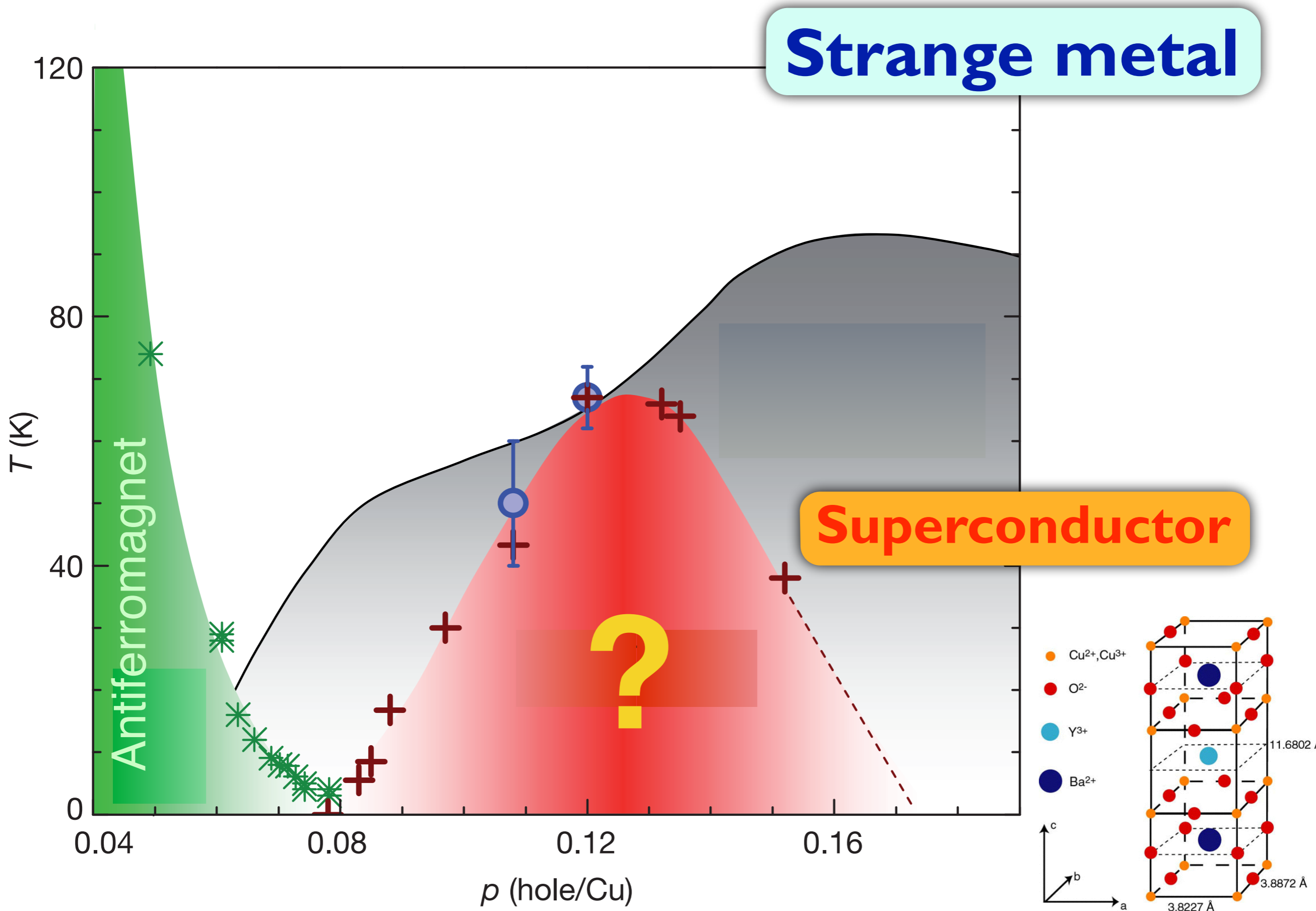
J. C. Seamus Davis



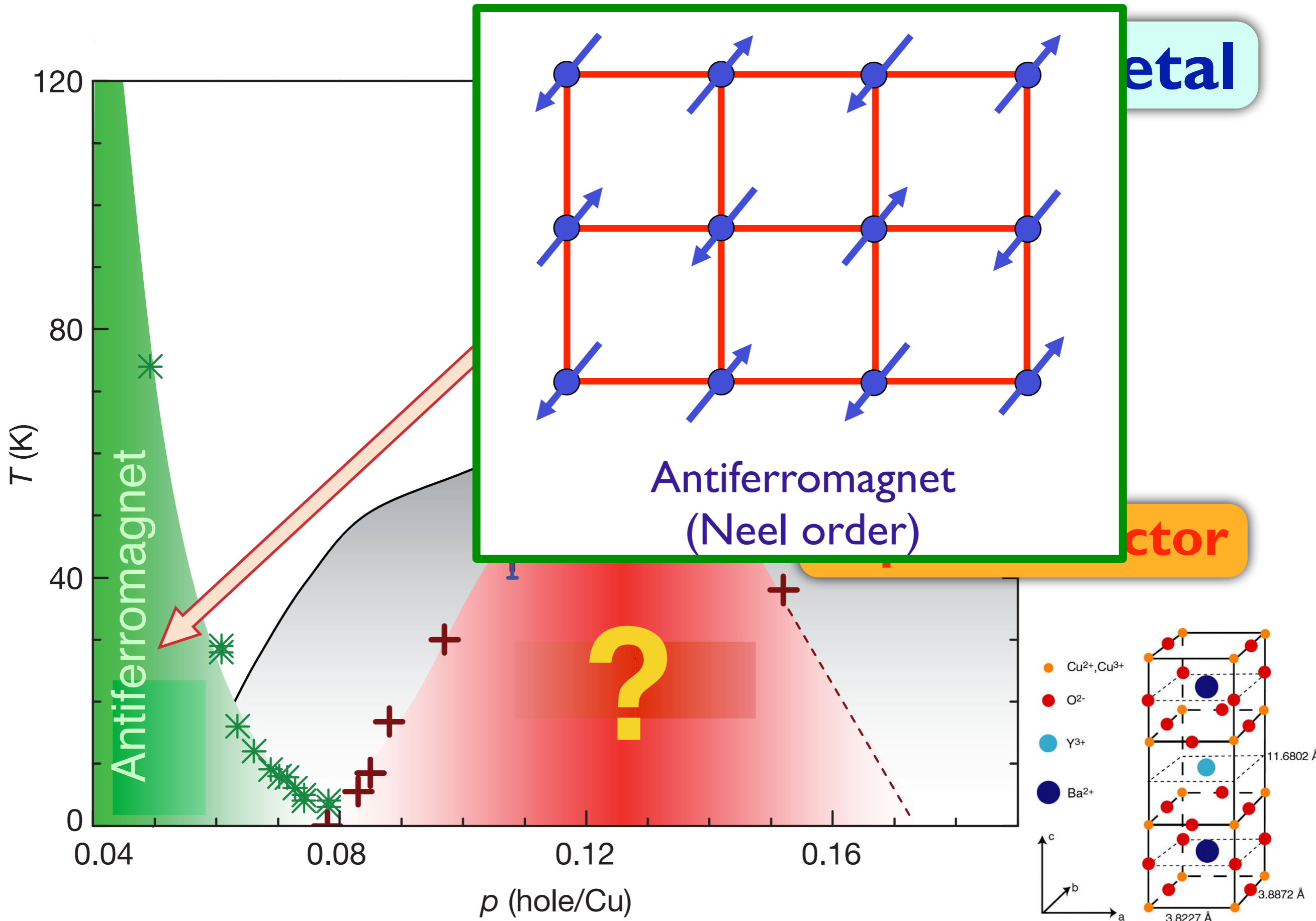
Eun-Ah Kim

# High temperature superconductors

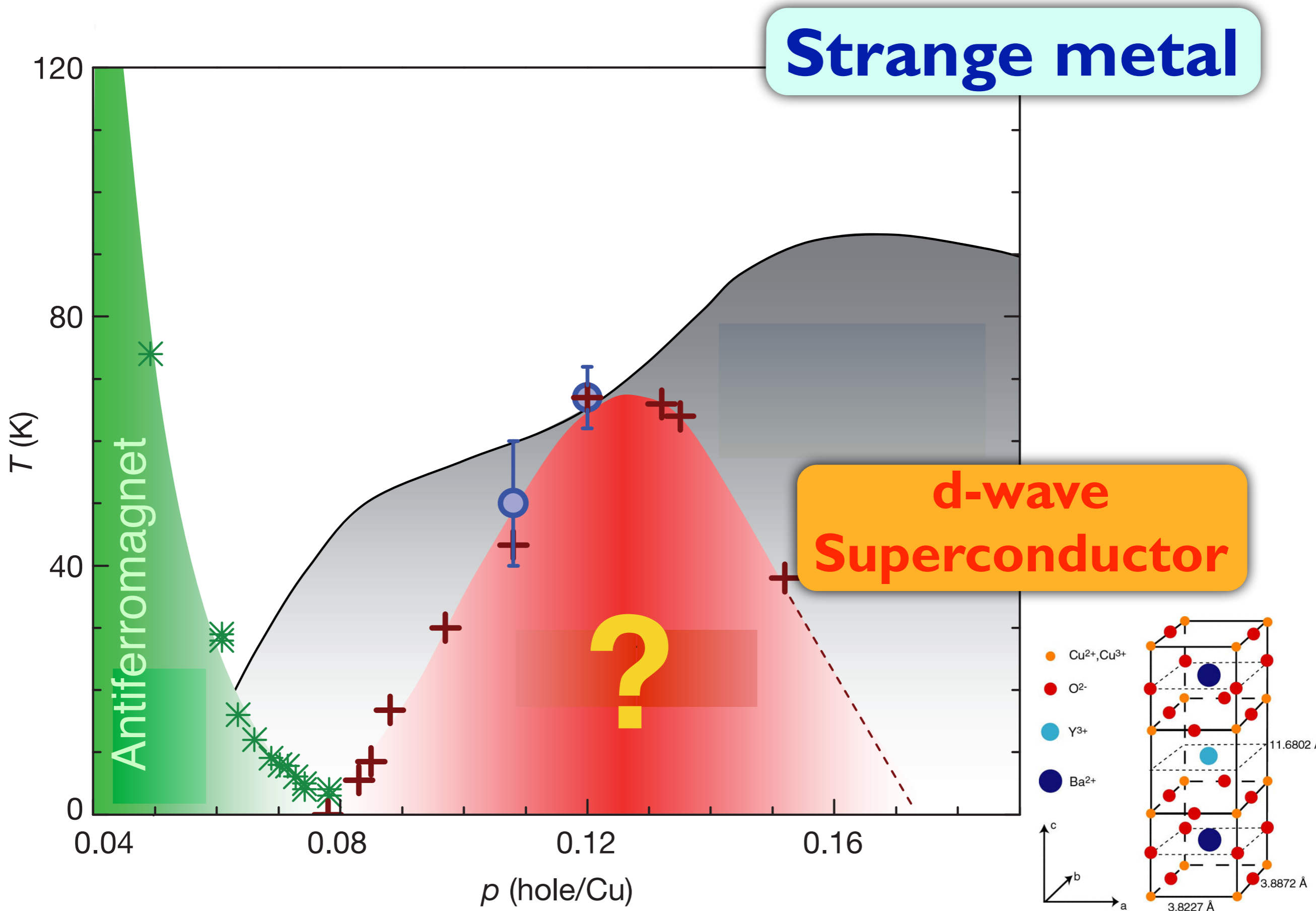




T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).



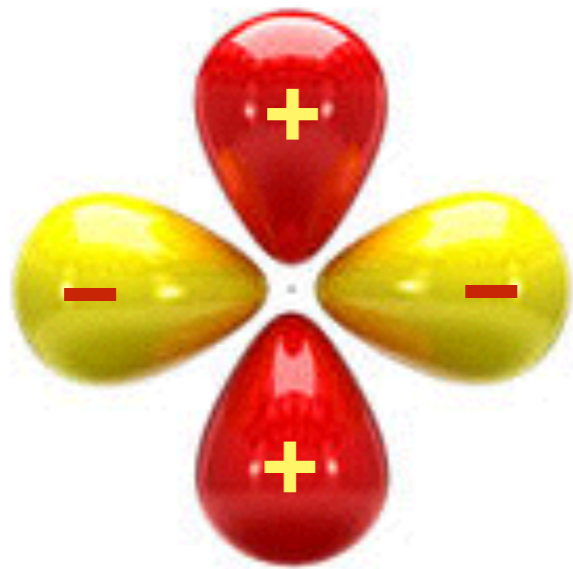
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# Superconductivity: Bose condensation of Cooper pairs of electrons

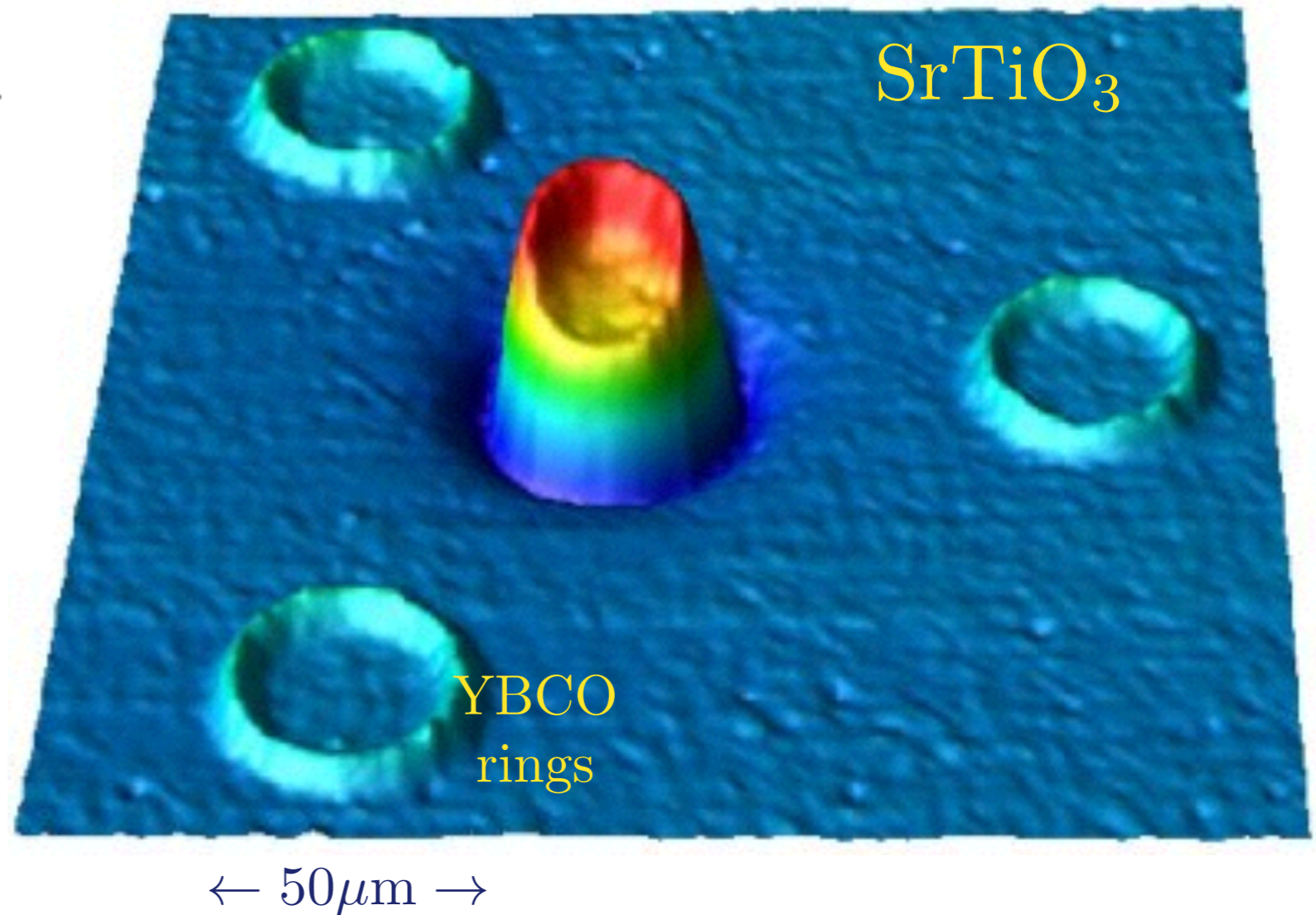
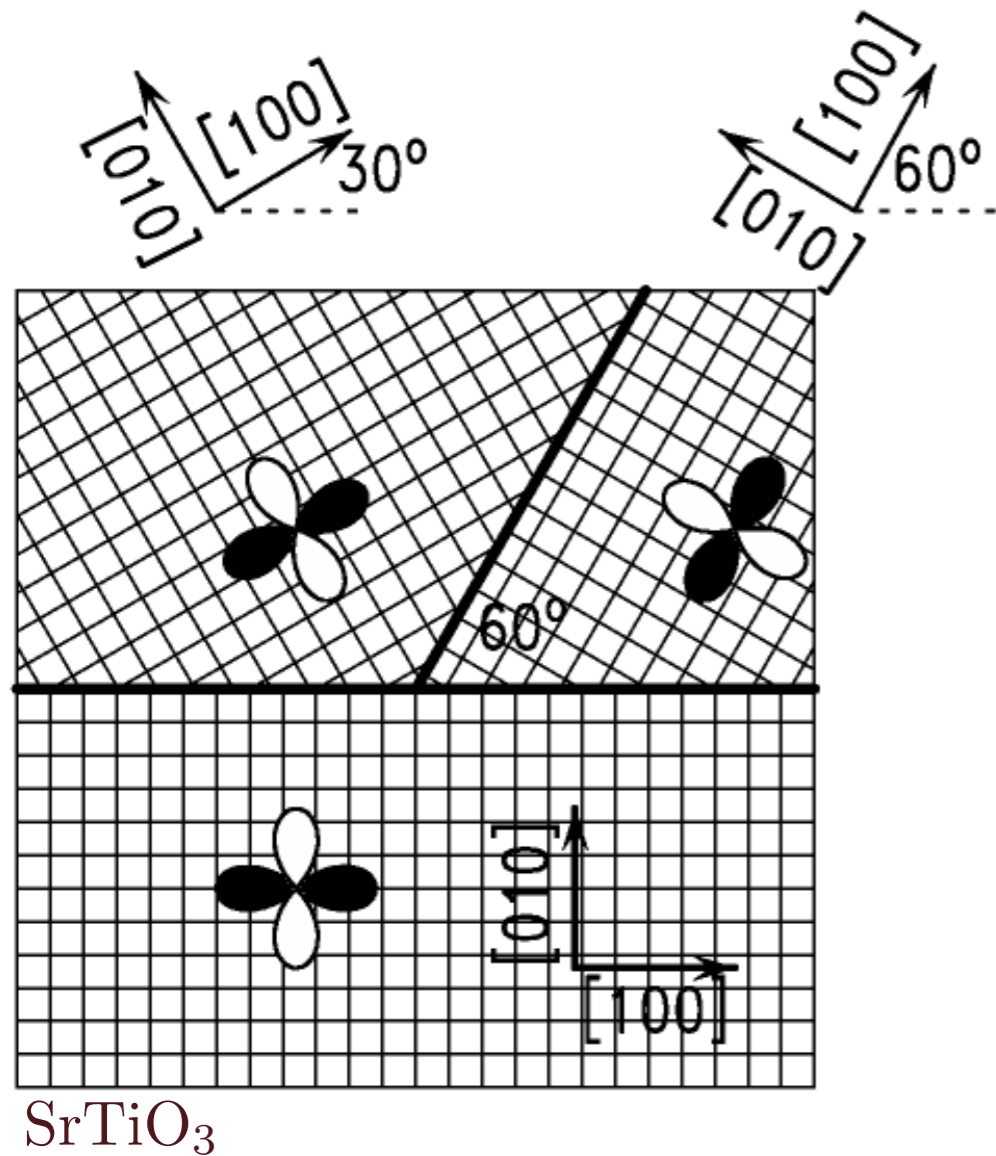
$$\varepsilon^{\alpha\beta} \left\langle c_{\alpha}^{\dagger}(\mathbf{r}_1) c_{\beta}^{\dagger}(\mathbf{r}_2) \right\rangle = \left[ P(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{SC} \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right)$$



Internal Cooper-pair wavefunction.  
Has *d*-wave form in cuprates

$$\alpha, \beta = \uparrow, \downarrow \quad ; \quad \varepsilon^{\uparrow\downarrow} = -\varepsilon^{\downarrow\uparrow} = 1; \quad \varepsilon^{\uparrow\uparrow} = \varepsilon^{\downarrow\downarrow} = 0$$

# Phase-sensitive measurement of the $d$ -wave symmetry of Cooper pairs

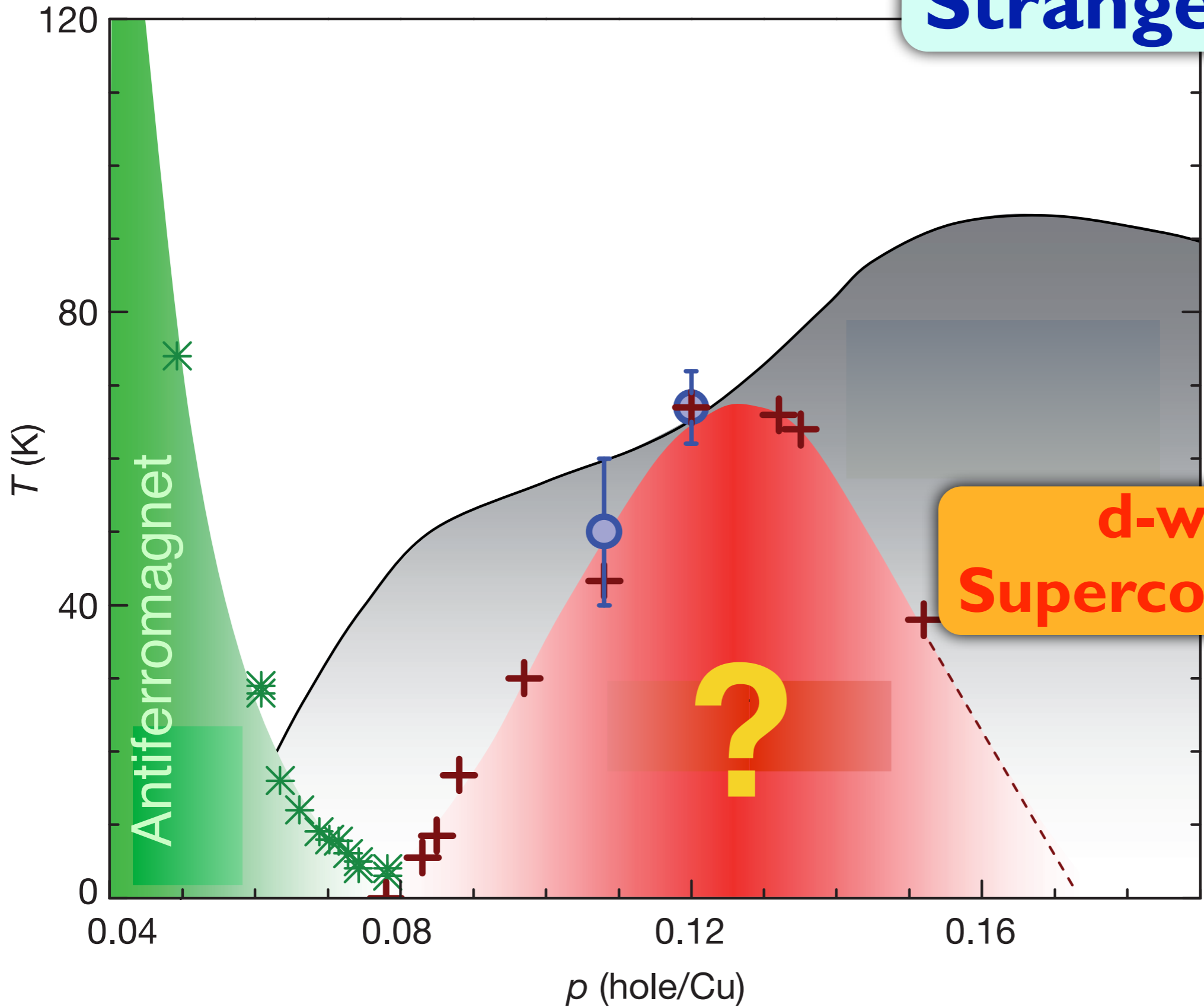


## Pairing Symmetry and Flux Quantization in a Tricrystal Superconducting Ring of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$

C. C. Tsuei, J. R. Kirtley, C. C. Chi,\* Lock See Yu-Jahnes, A. Gupta, T. Shaw, J. Z. Sun, and M. B. Ketchen  
*IBM Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598*

Phys. Rev. Lett. **73**, 593 (1994)

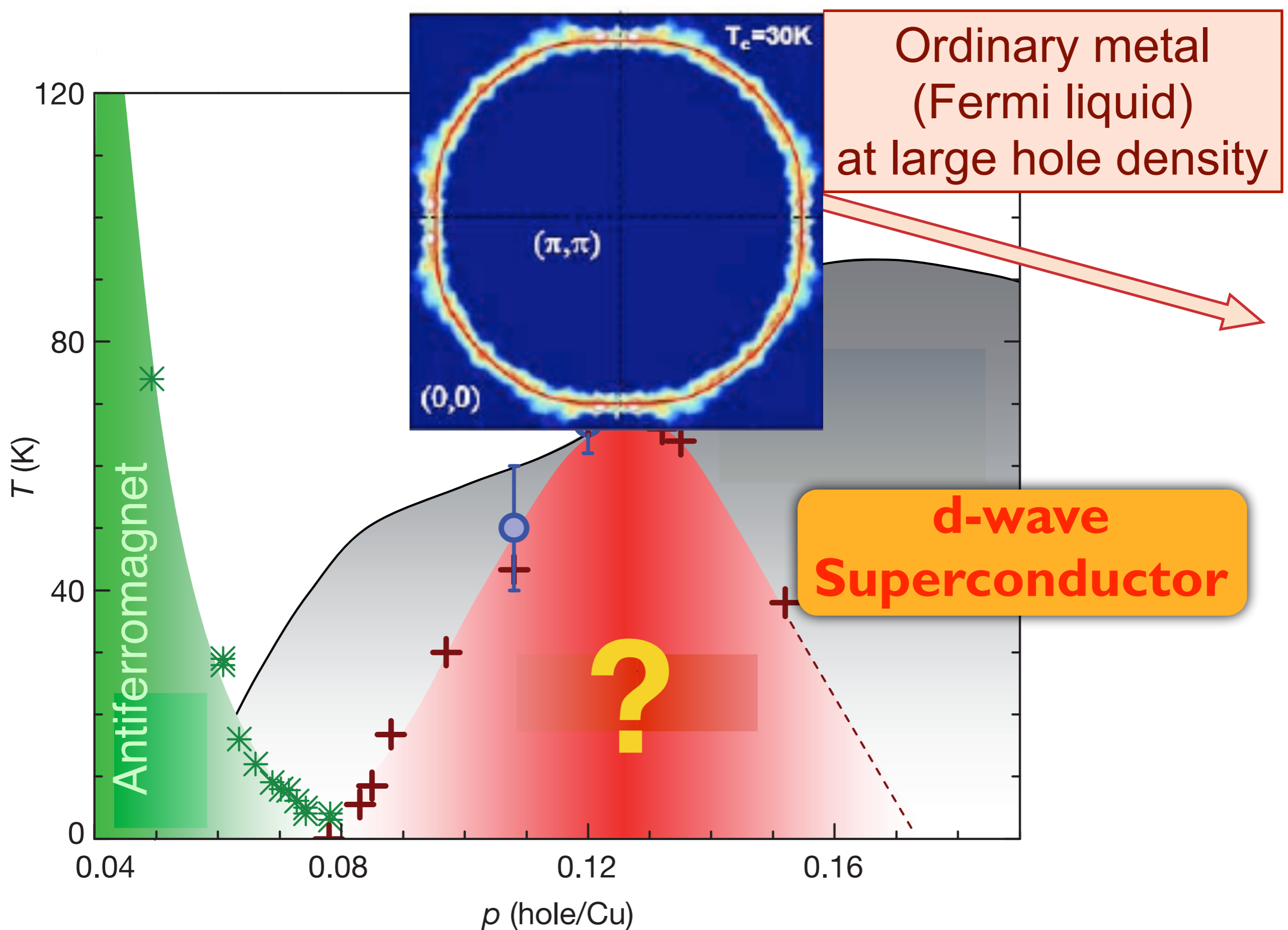
**Strange metal**



Antiferromagnet

d-wave  
Superconductor

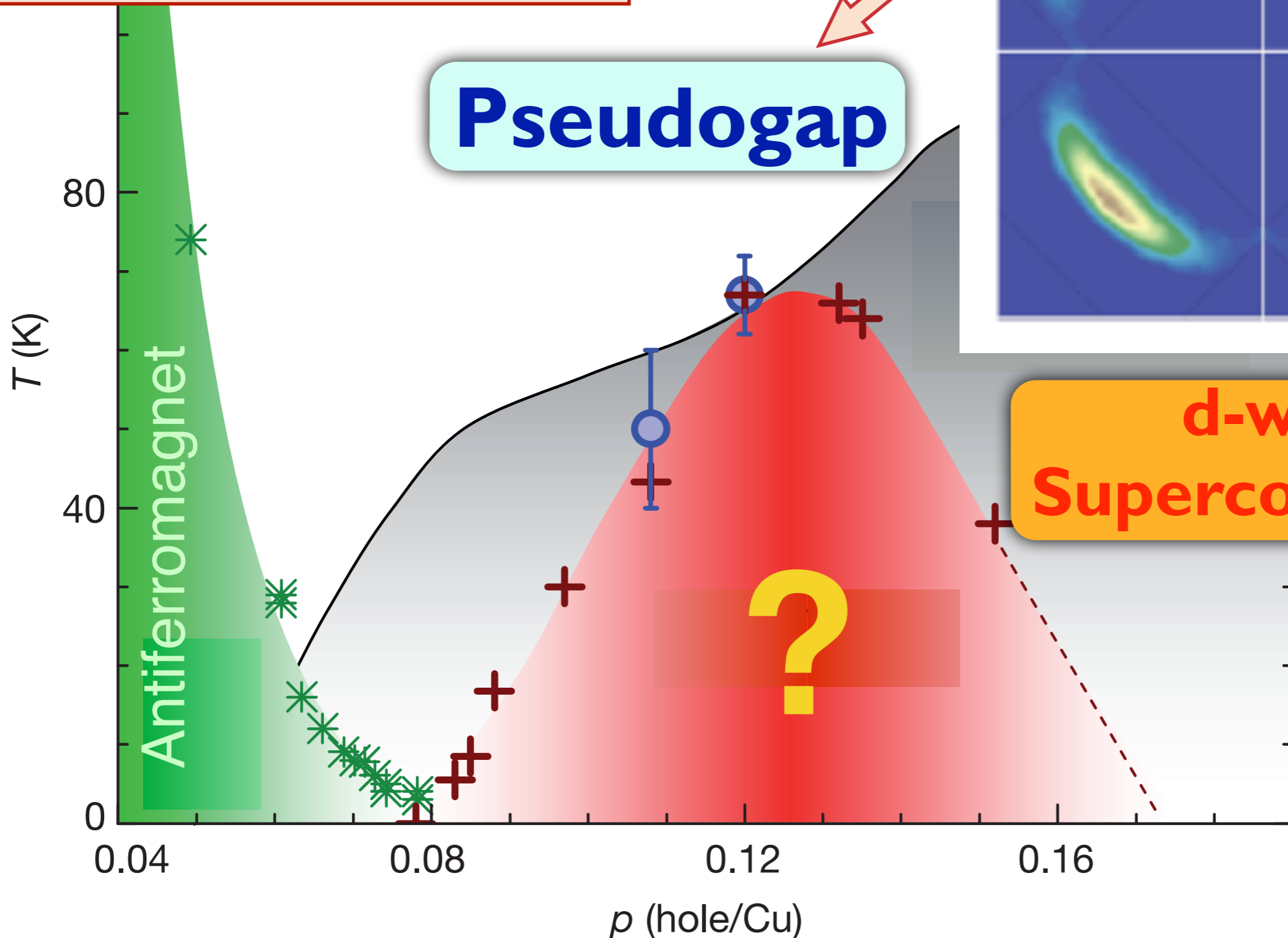
?

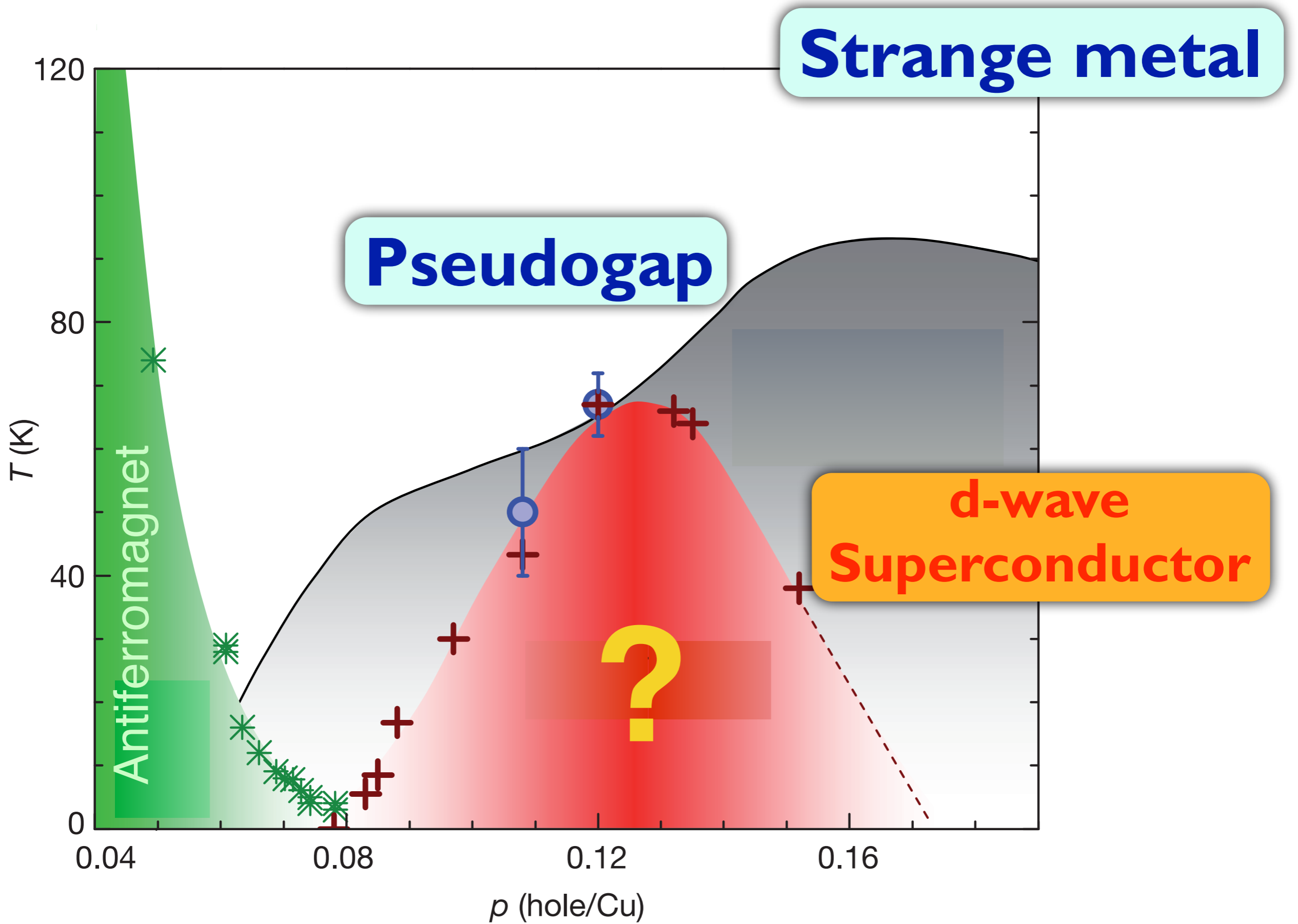


“Fermi arcs” at low doping

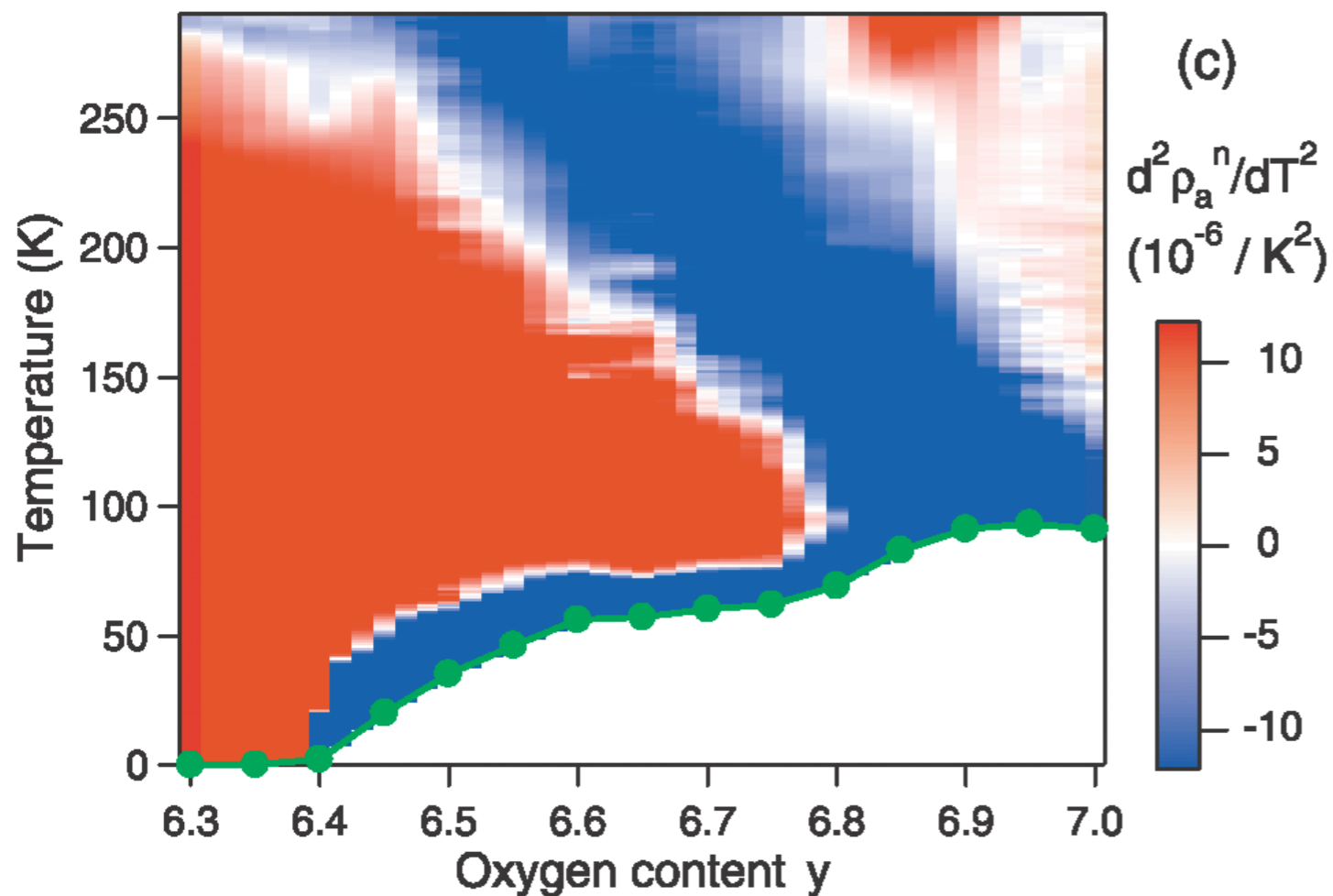
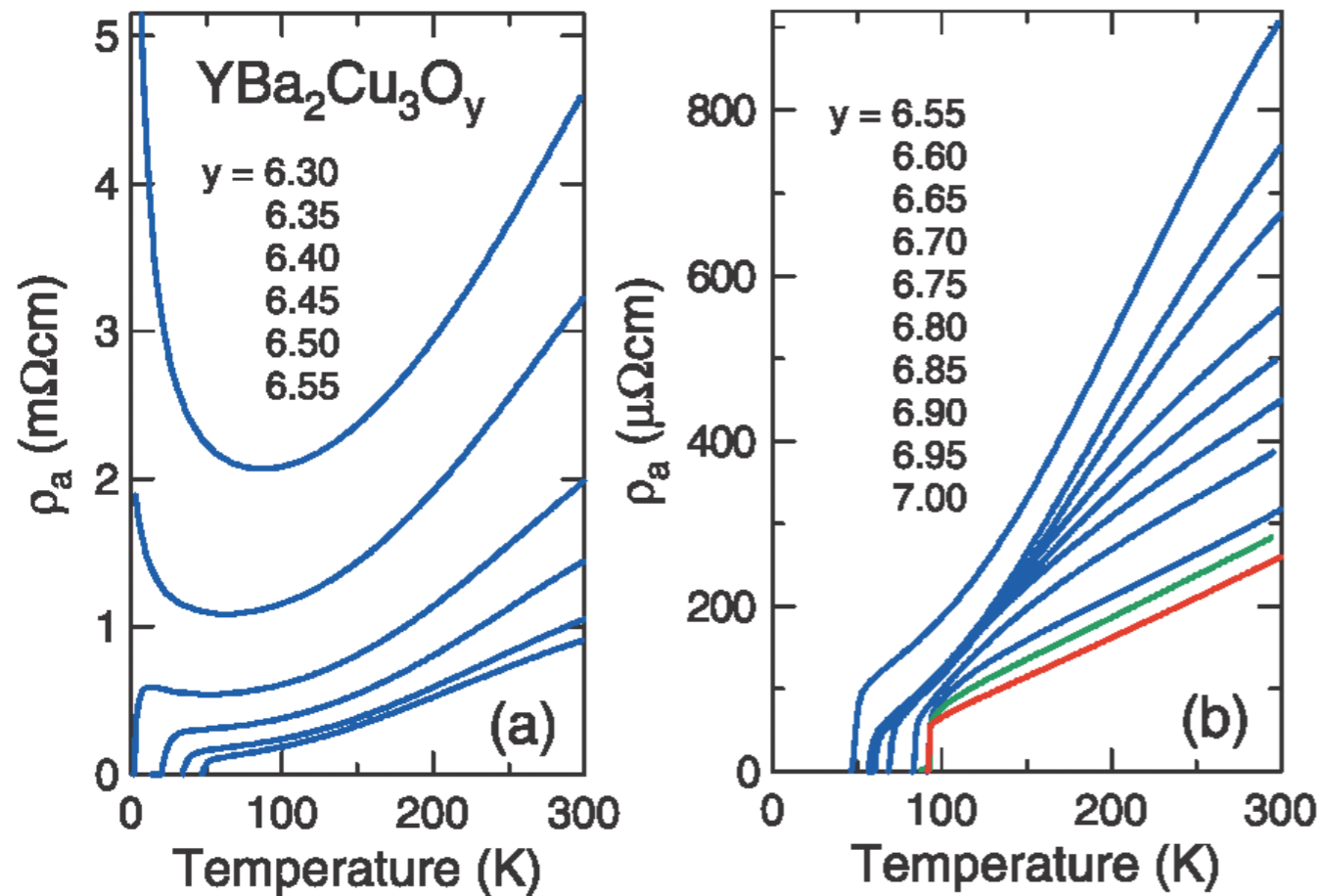
Pseudogap

d-wave Superconductor



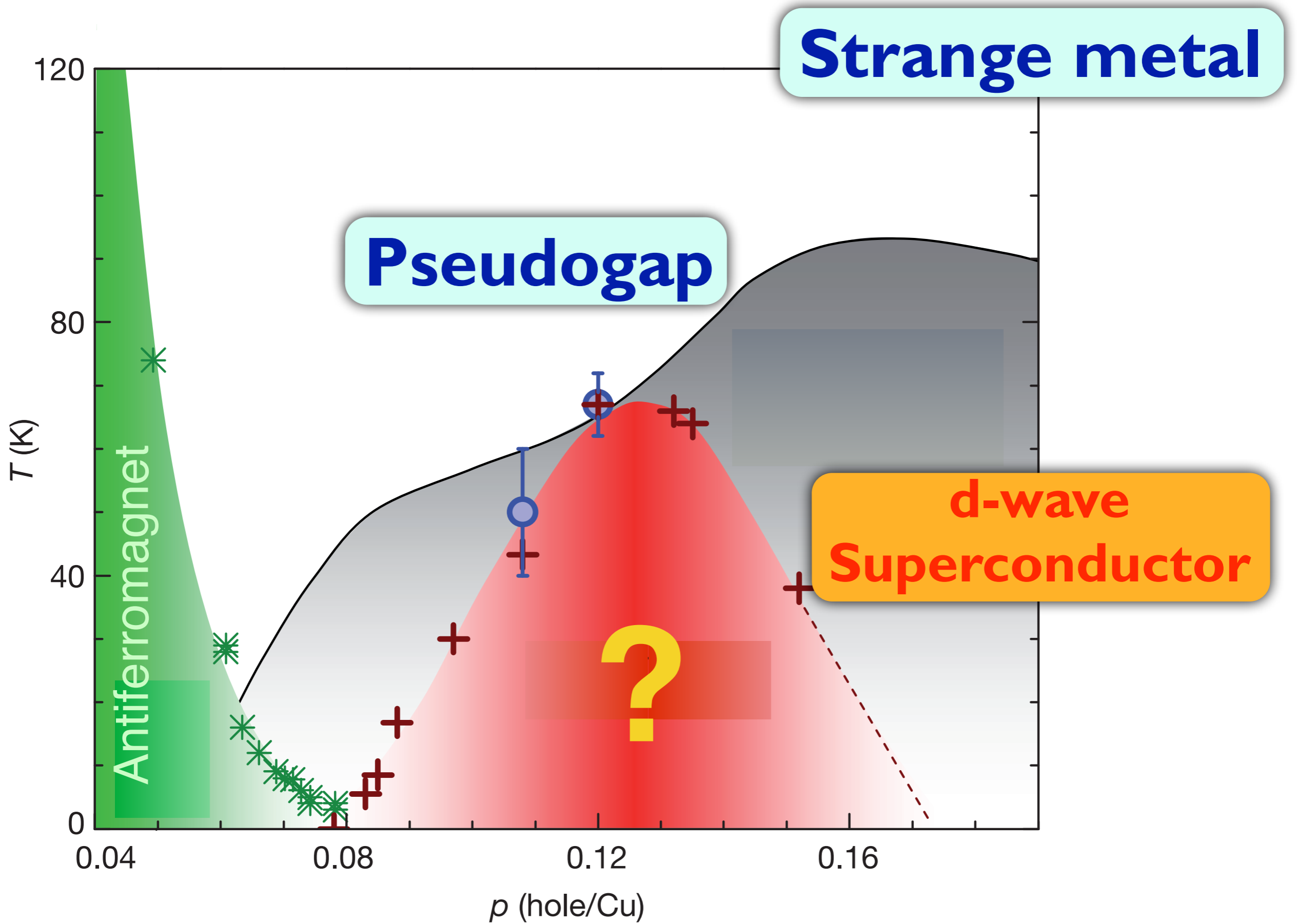


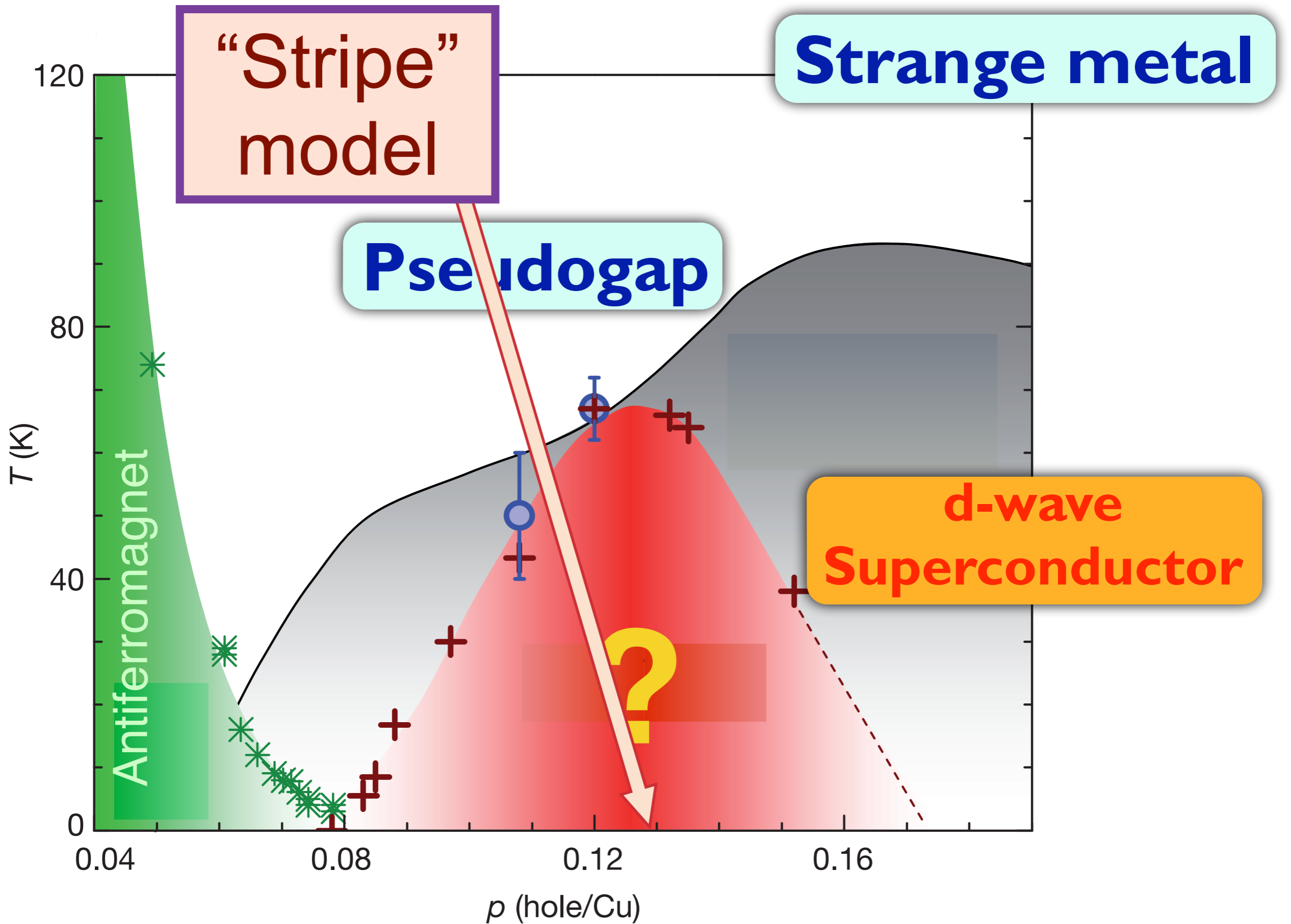
# Strange metal



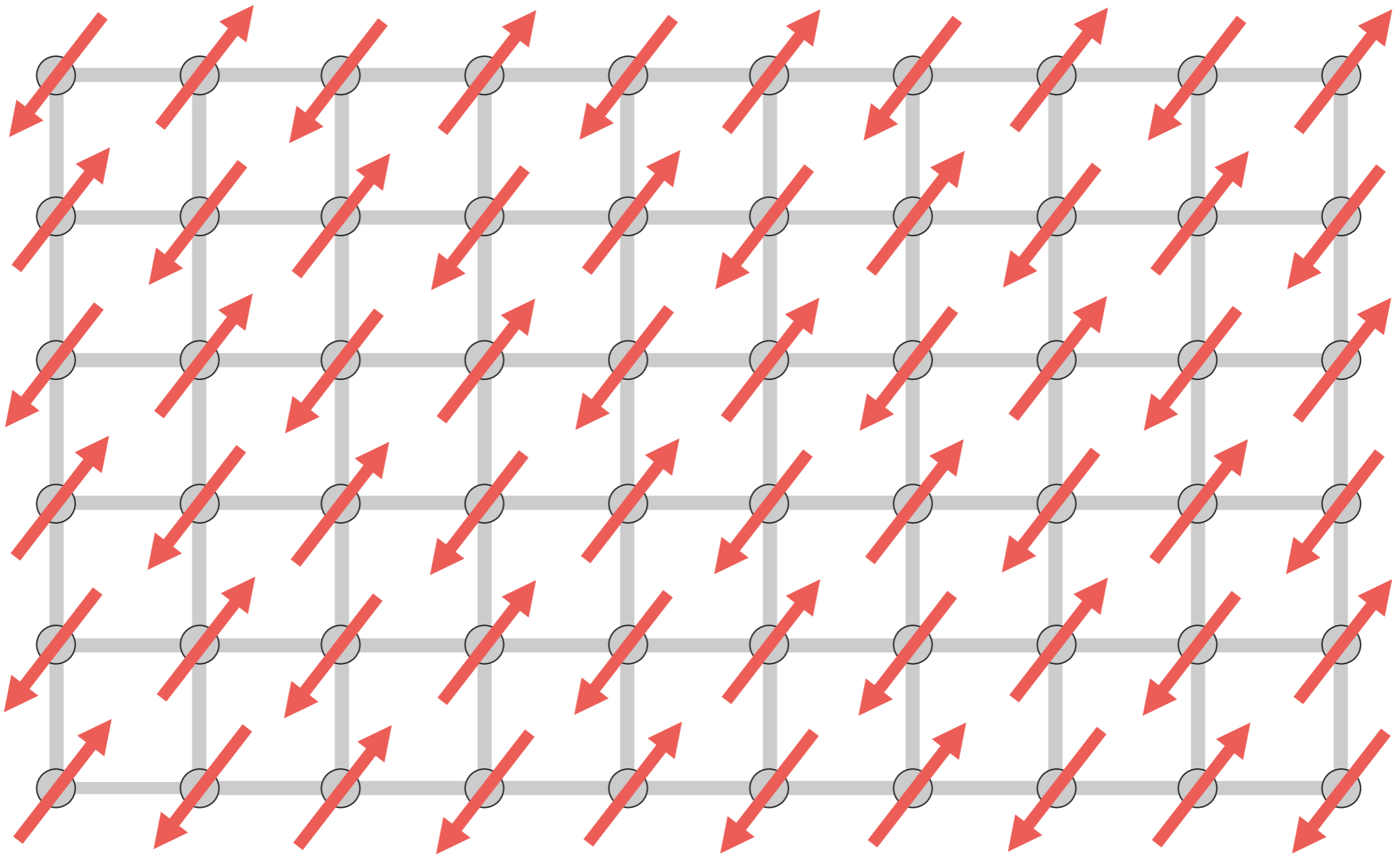
YBCO at optimal doping has resistivity  $\rho(T) \sim T$ .

Yoichi Ando, Seiki Komiya, Kouji Segawa, S. Ono, and Y. Kurita, Phys. Rev. Lett. **93**, 267001 (2004)



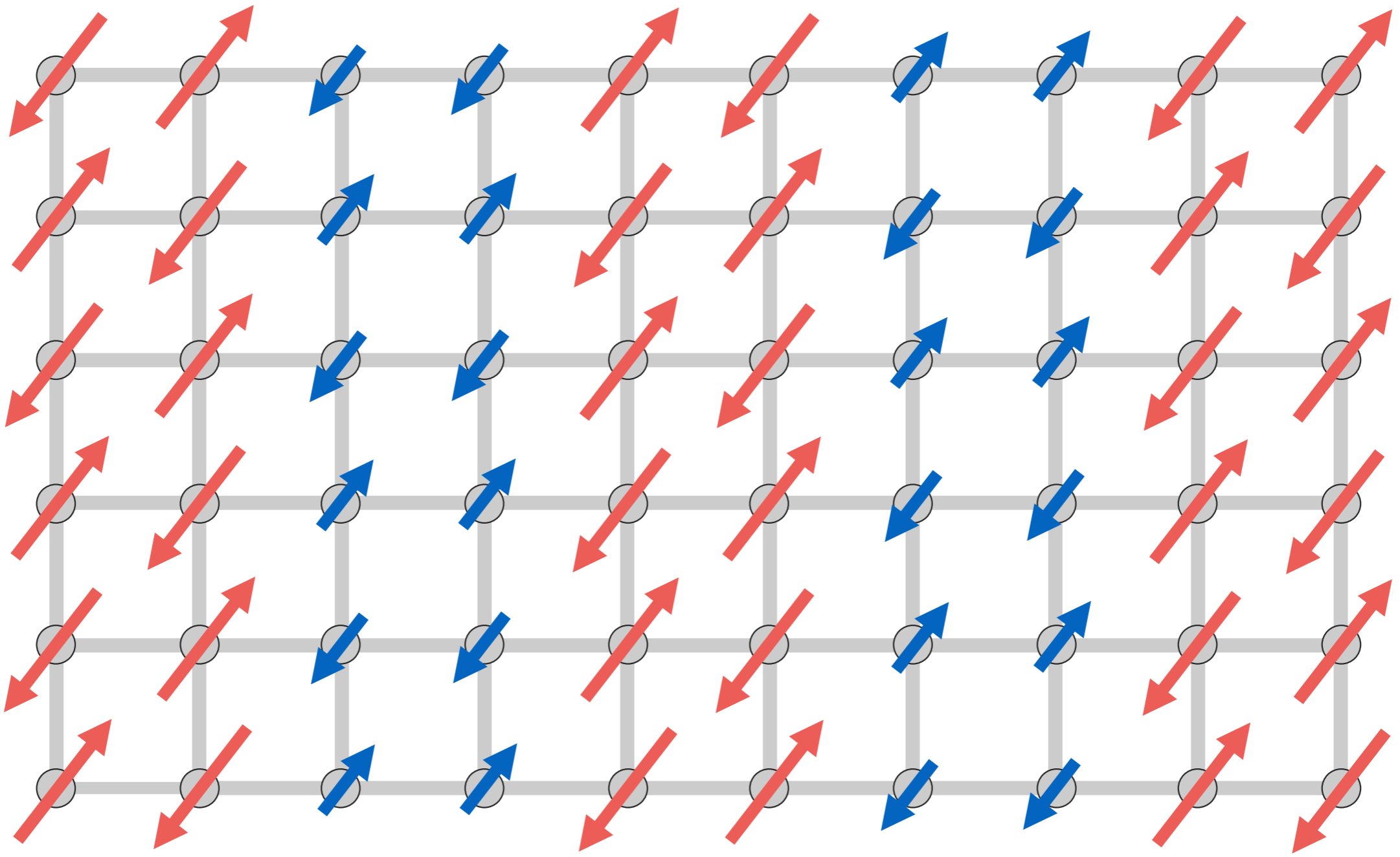


# “Stripe” model



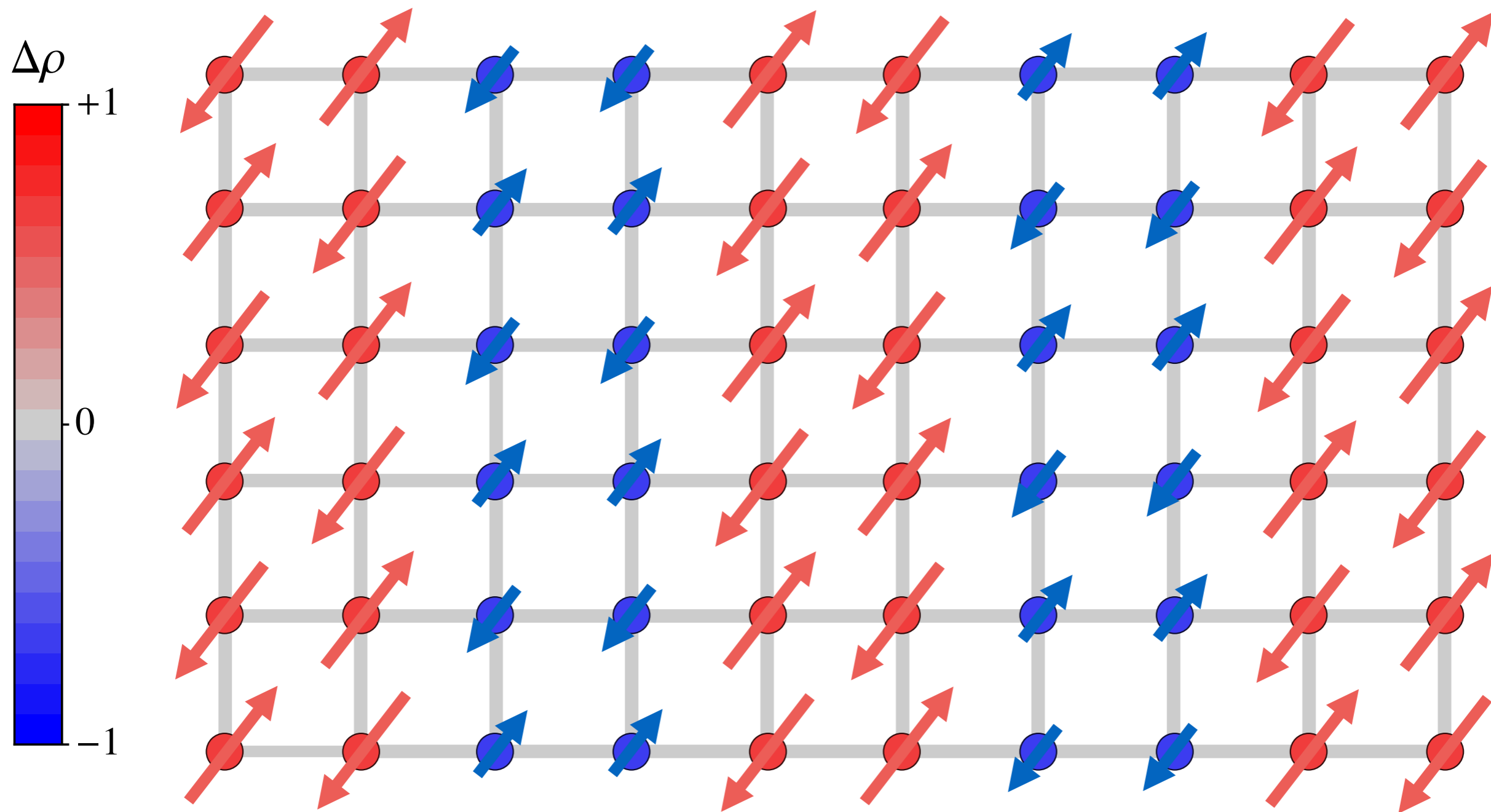
Start with an antiferromagnet

# “Stripe” model



Domain walls 4 lattice spacings apart

“Stripe”  
model

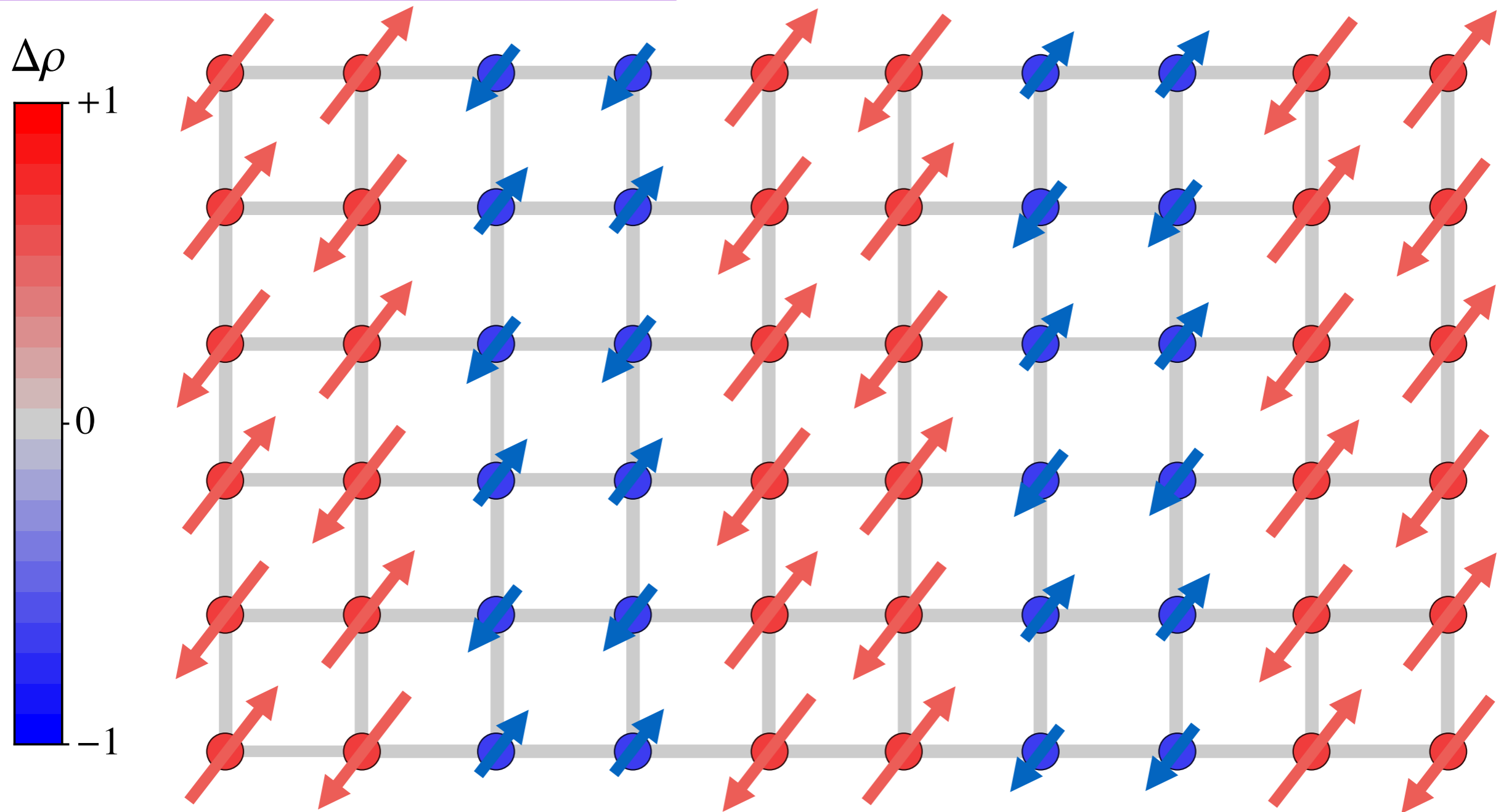


Put the holes in the domain walls

# “Stripe” model

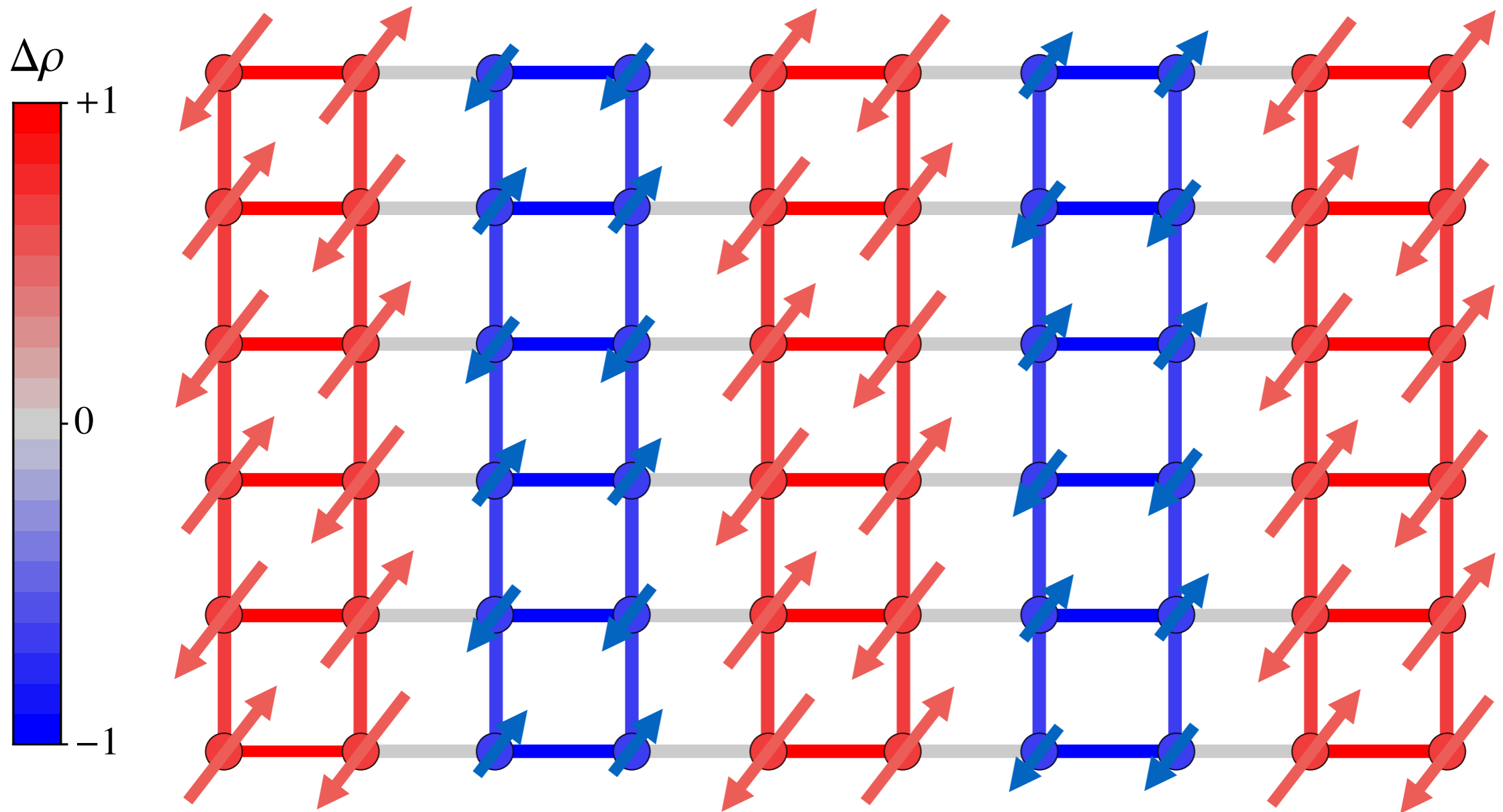
Observed in La-based  
compounds (Tranquada..)

Theory: Zaanen, Kivelson, Fradkin....

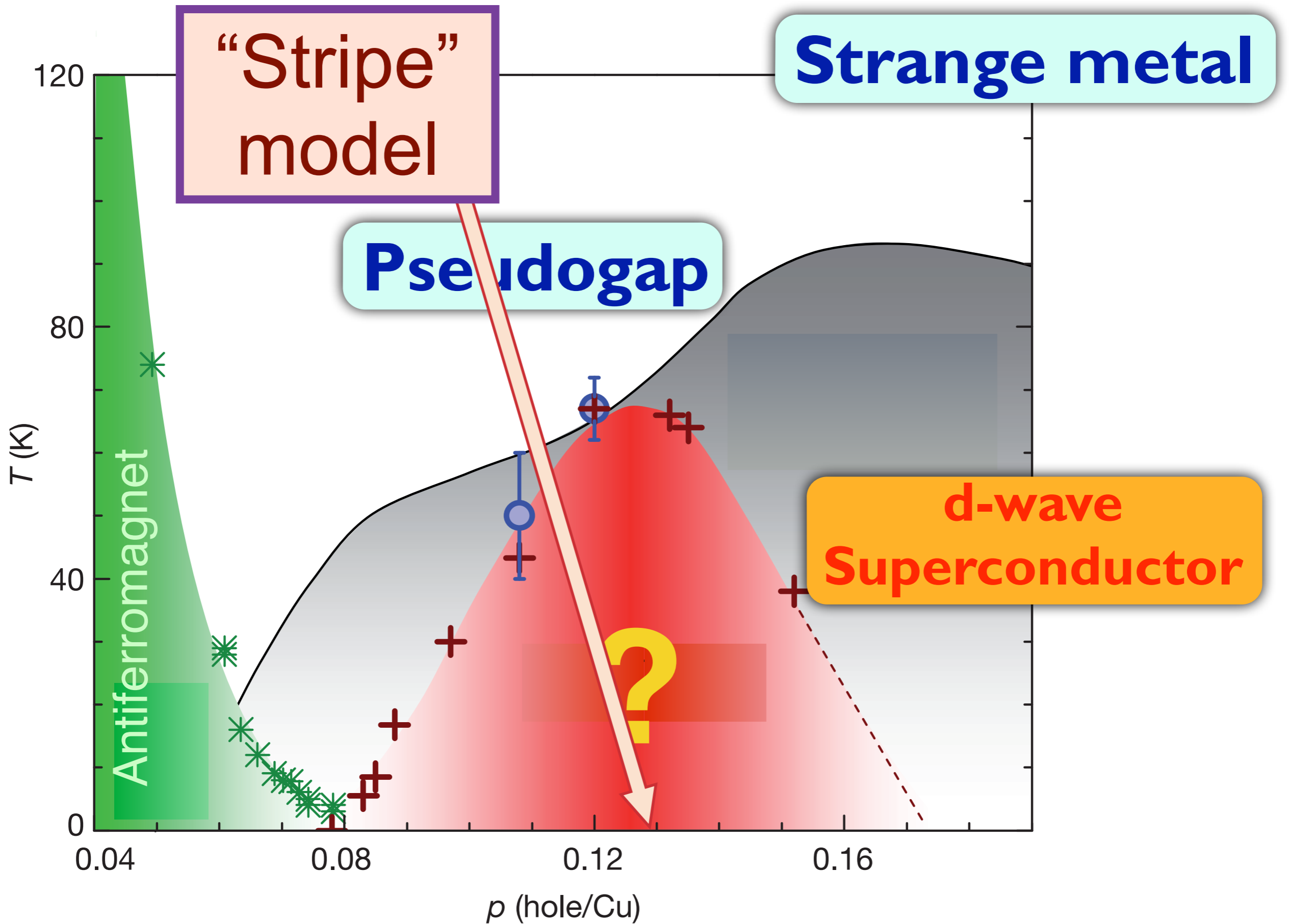


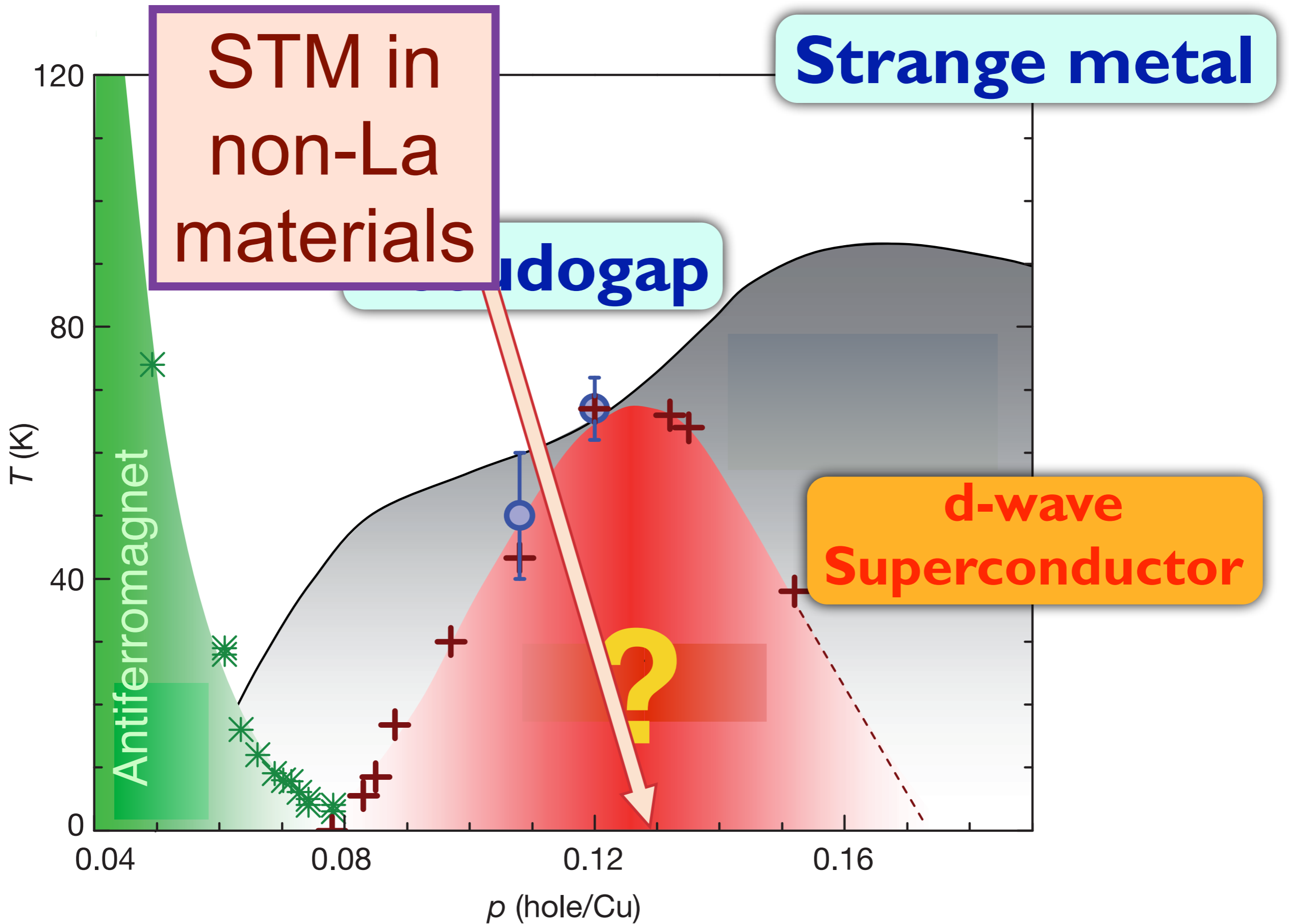
Put the holes in the domain walls

# “Stripe” model



Colors on the bonds map the local exchange energy



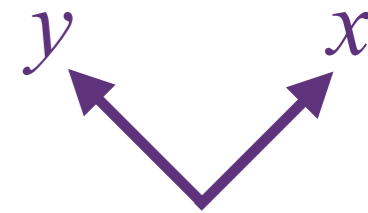
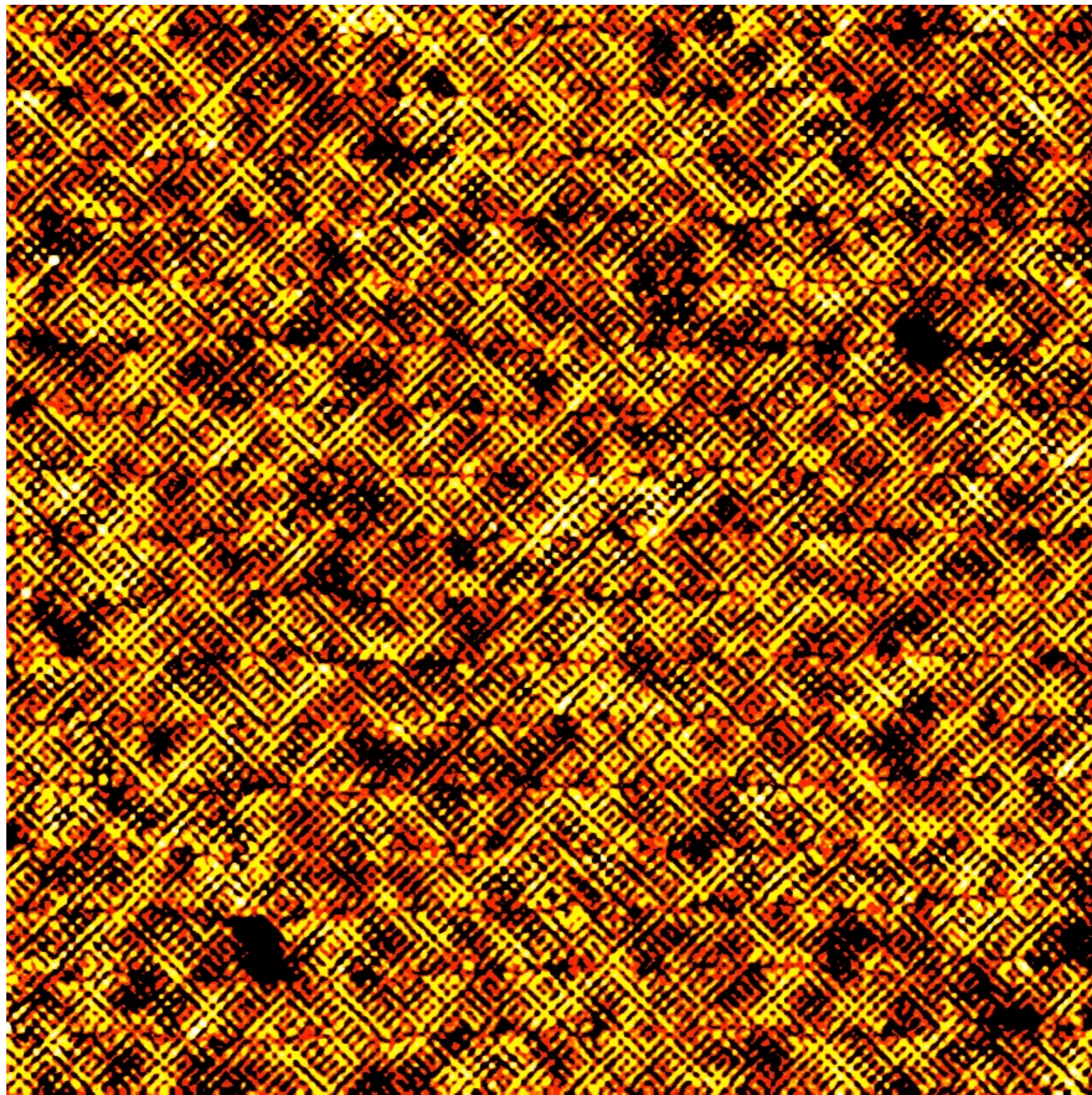


See also

C. Howald, H. Eisaki,  
N. Kaneko, M. Greven,  
and A. Kapitulnik,  
*Phys. Rev. B* **67**,  
014533 (2003);

M. Vershinin, S. Misra,  
S. Ono, Y. Abe, Yoichi  
Ando, and  
A. Yazdani, *Science*  
**303**, 1995 (2004).

W. D. Wise, M. C. Boyer,  
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Y. Wang, and  
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*Nature Phys.* **4**, 696  
(2008).



“R-map” of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007). **Davis group has sub-angstrom resolution capabilities, with lattice drift corrections, which make sublattice phase-resolved STM possible.**

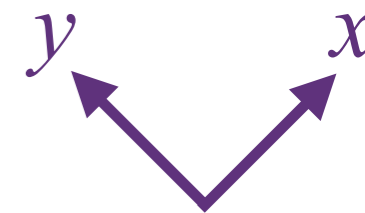
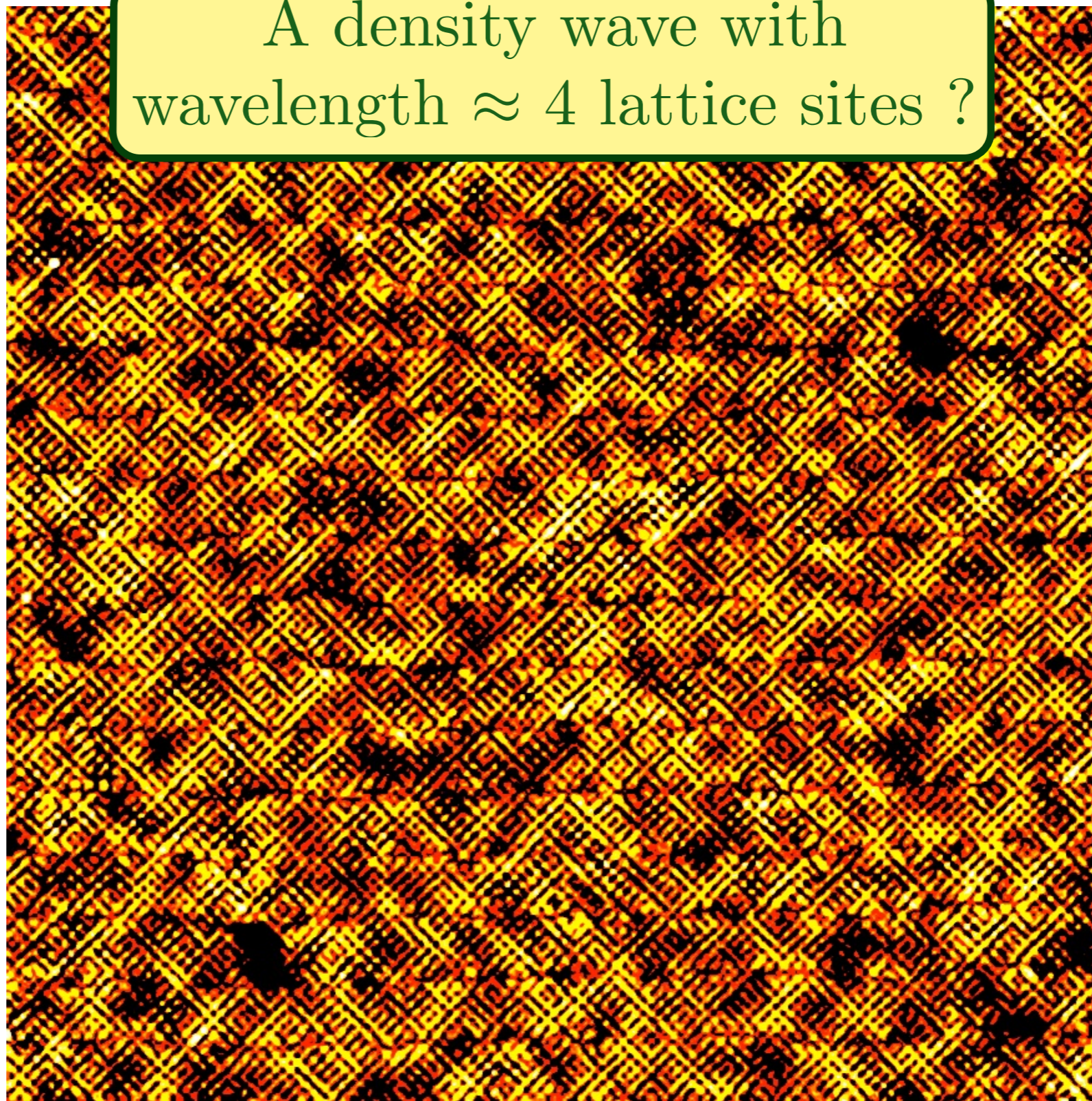
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A density wave with  
wavelength  $\approx 4$  lattice sites ?



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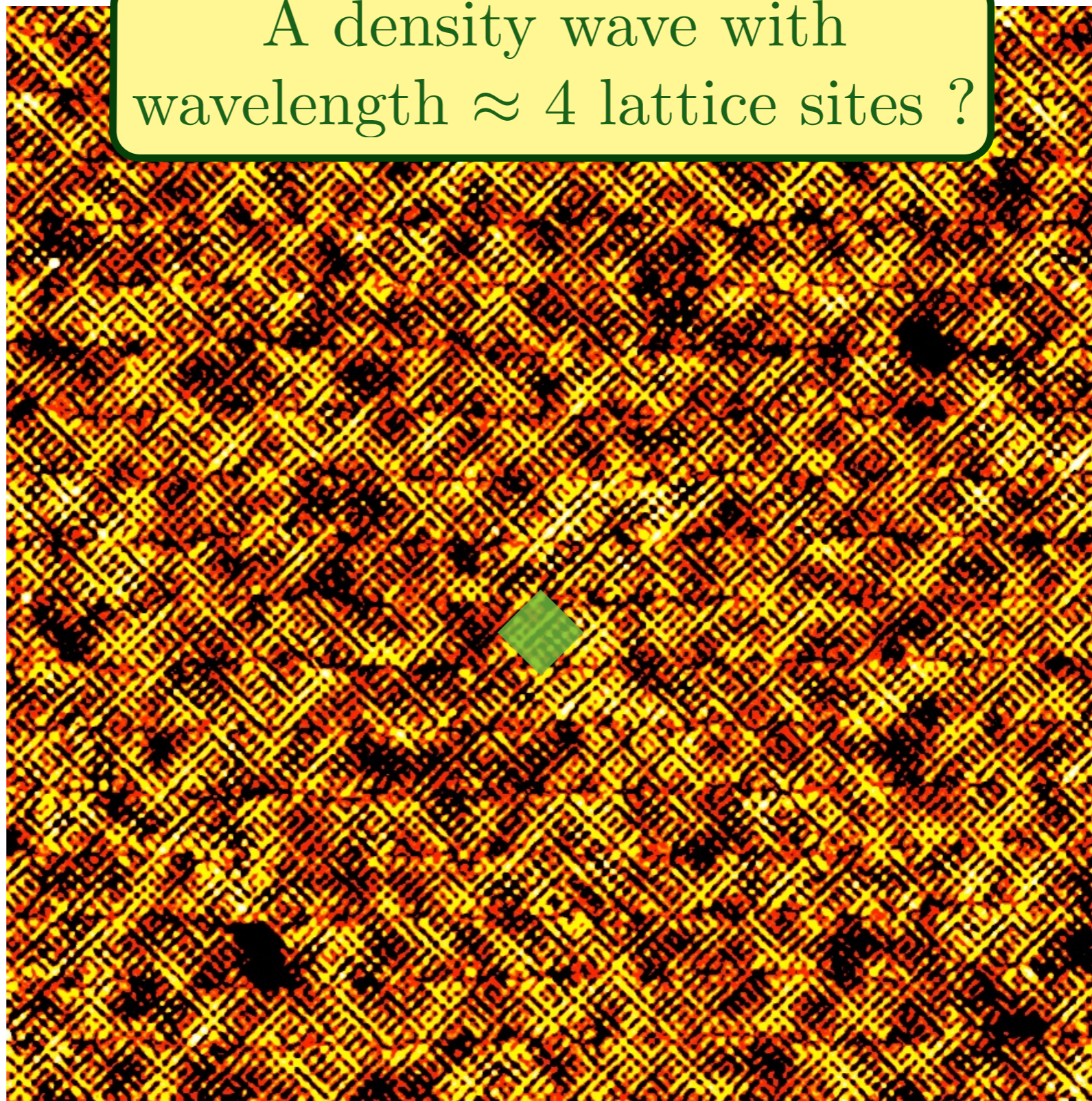
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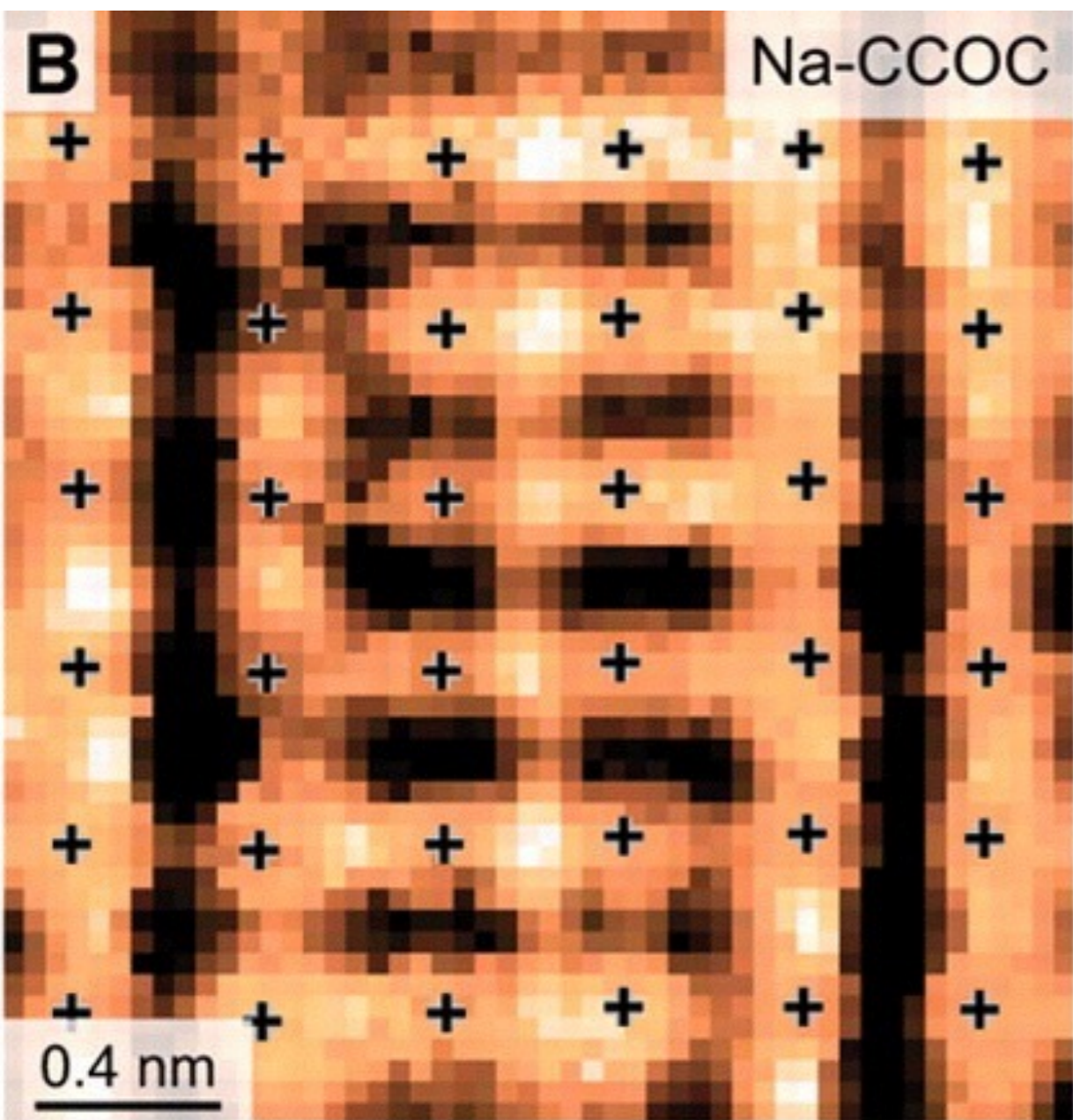
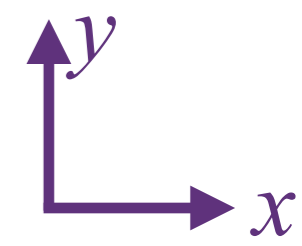
M. Vershinin, S. Misra,  
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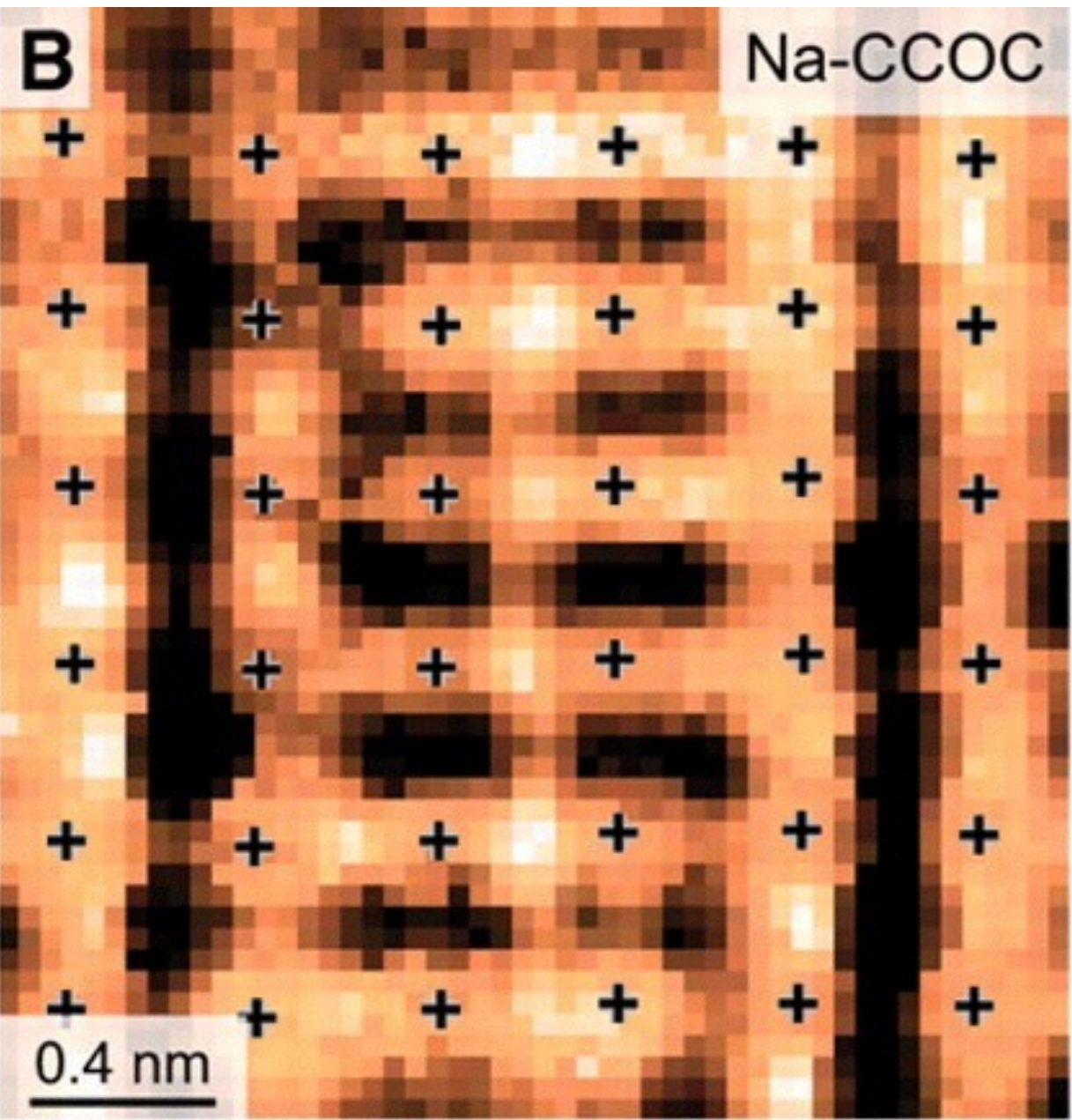
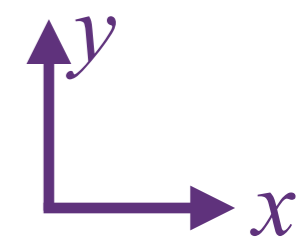
A density wave with  
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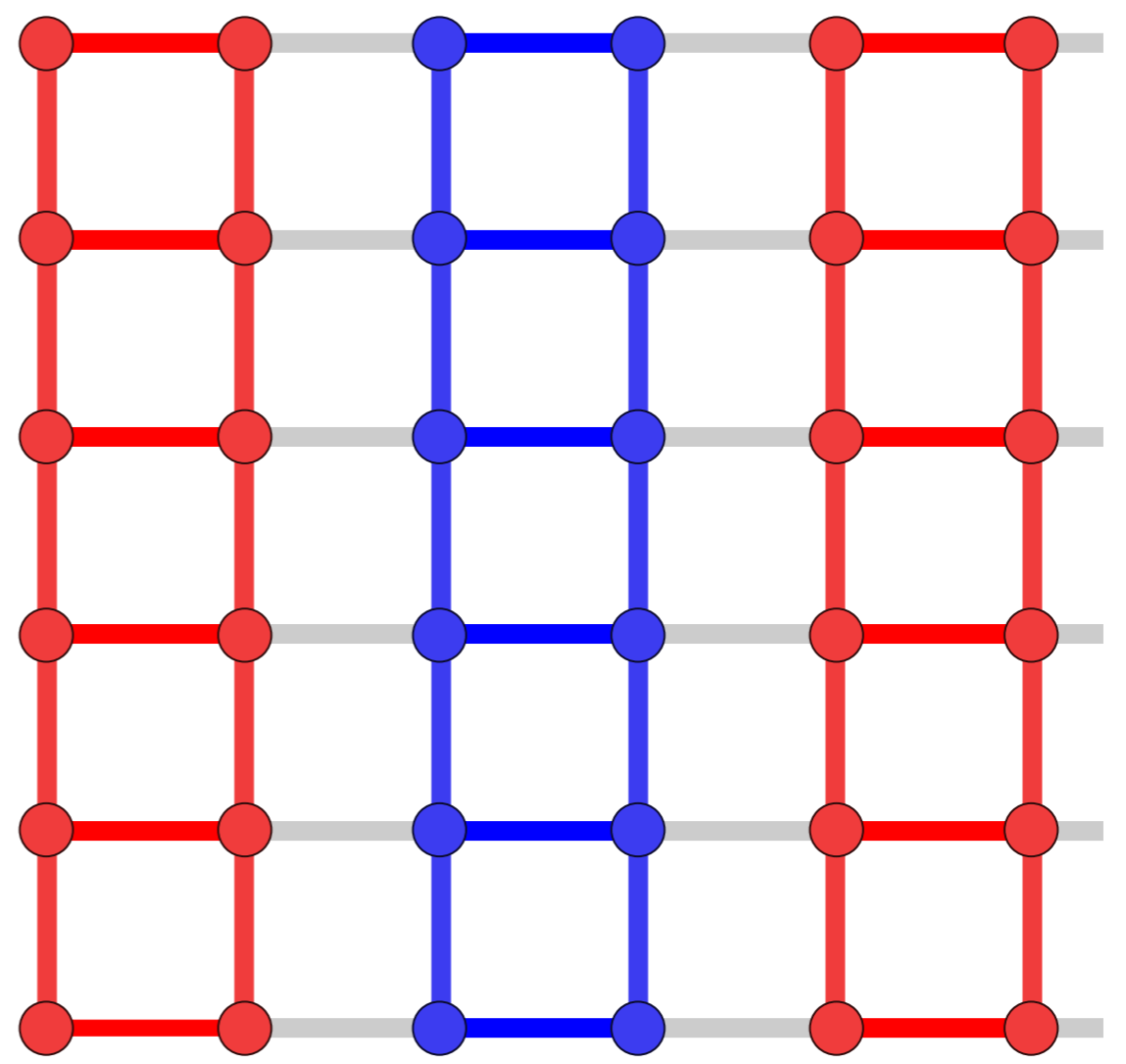
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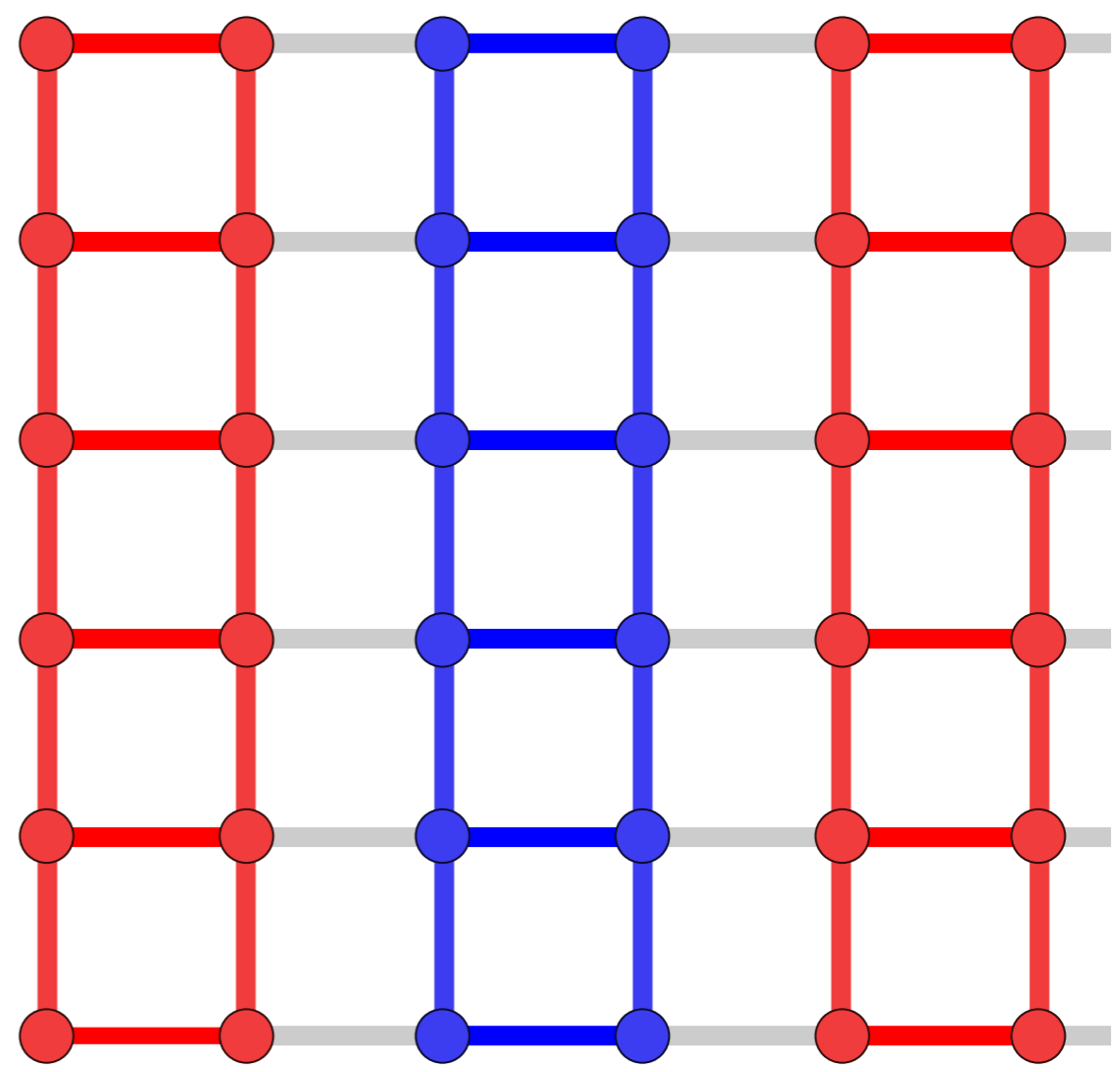
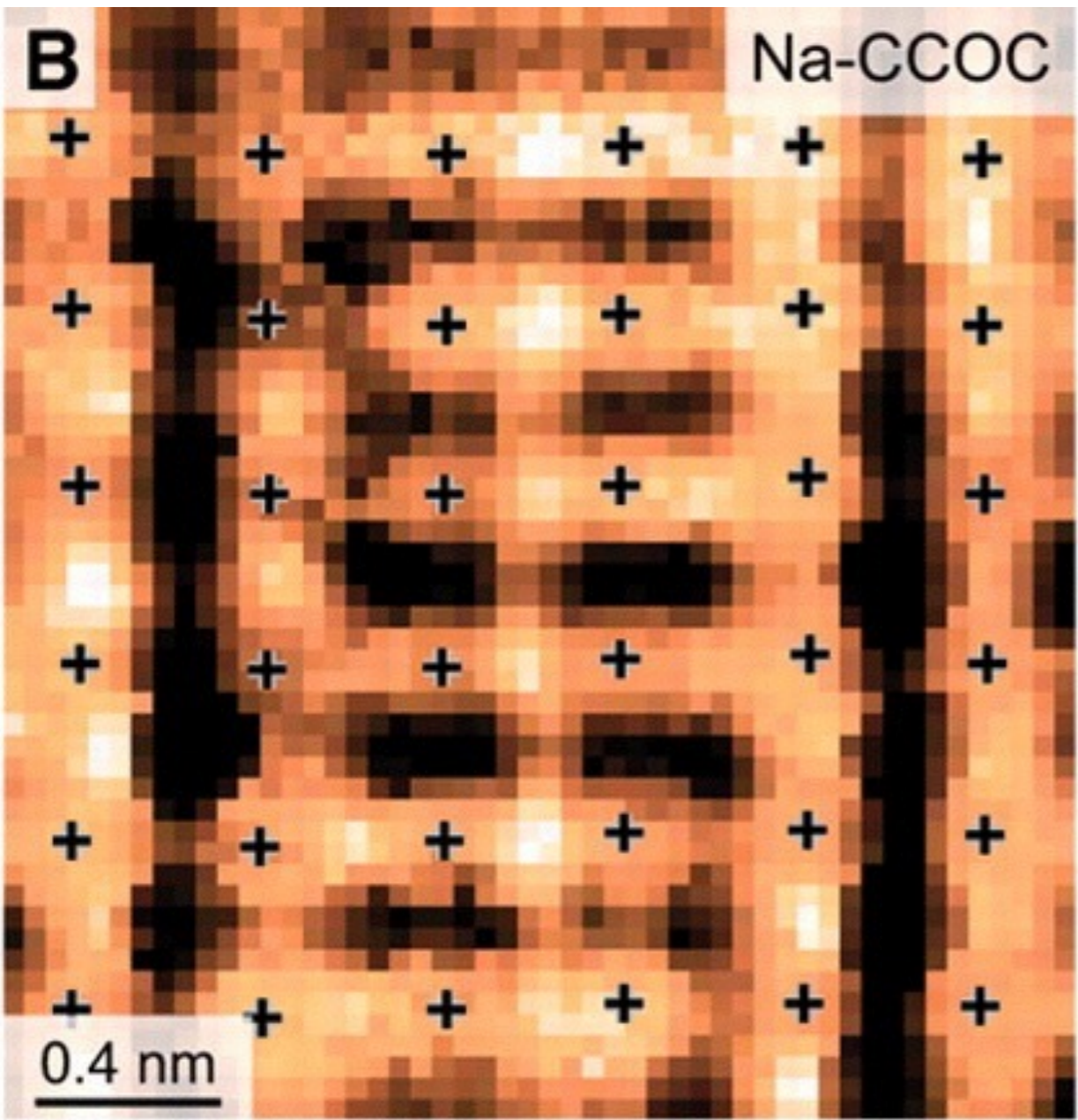
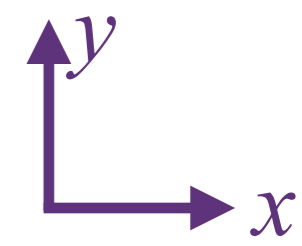
Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



**“Stripe” model**



“Stripe” model

Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

Microstructure of STM picture does not match “stripe” model

**Unconventional density wave (DW) :**  
**Bose condensation of particle-hole pairs**

$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1)c_{\alpha}(\mathbf{r}_2) \rangle$$
$$= \left[ \mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2}$$

Unconventional density wave (DW) :  
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Crucial “center-of-mass” co-ordinate.  
(Not used in previous work)  
Simplifies action of time-reversal

Unconventional density wave (DW) :  
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$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1)c_{\alpha}(\mathbf{r}_2) \rangle = \left[ \mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2}$$

Density wave form factor (internal particle-hole pair wavefunction)

$$\mathcal{P}(\mathbf{r}) = \int \frac{d^2k}{4\pi^2} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}}$$

Time-reversal symmetry requires  $\mathcal{P}(\mathbf{k}) = \mathcal{P}(-\mathbf{k})$ .

We expand (using reflection symmetry for  $\mathbf{Q}$  along axes or diagonals)

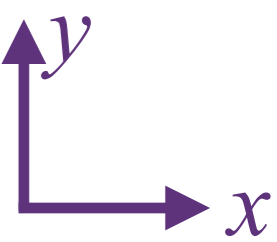
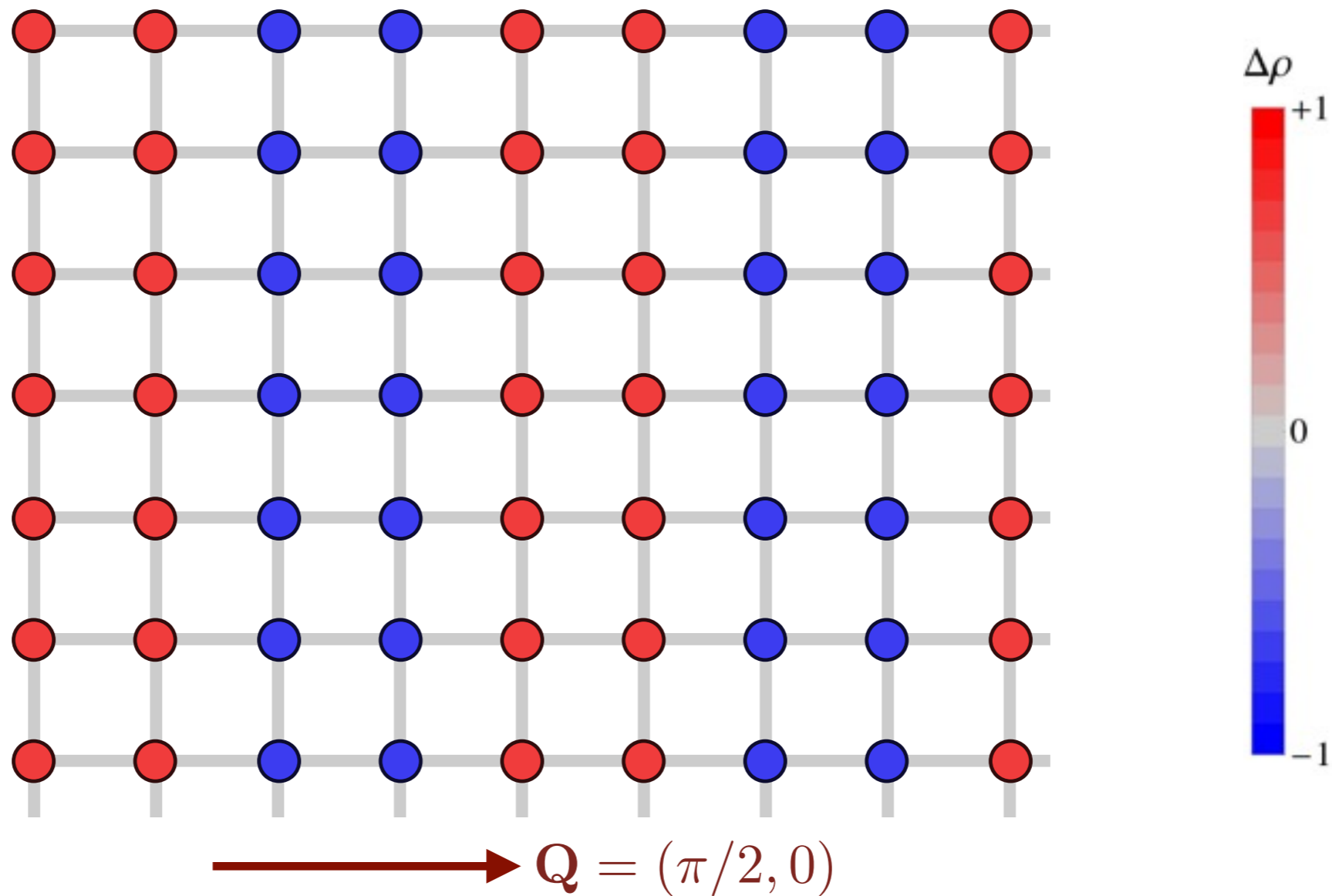
$$\mathcal{P}(\mathbf{k}) = \mathcal{P}_s + \mathcal{P}_{s'}(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y)$$

# Conventional CDW order: $s$ -form factor

Plot of  $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$  for  $i = j$ , and  $i, j$  nearest neighbors.

$$P_{ij} = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = 1 \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

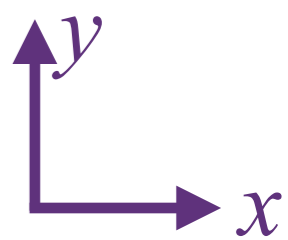
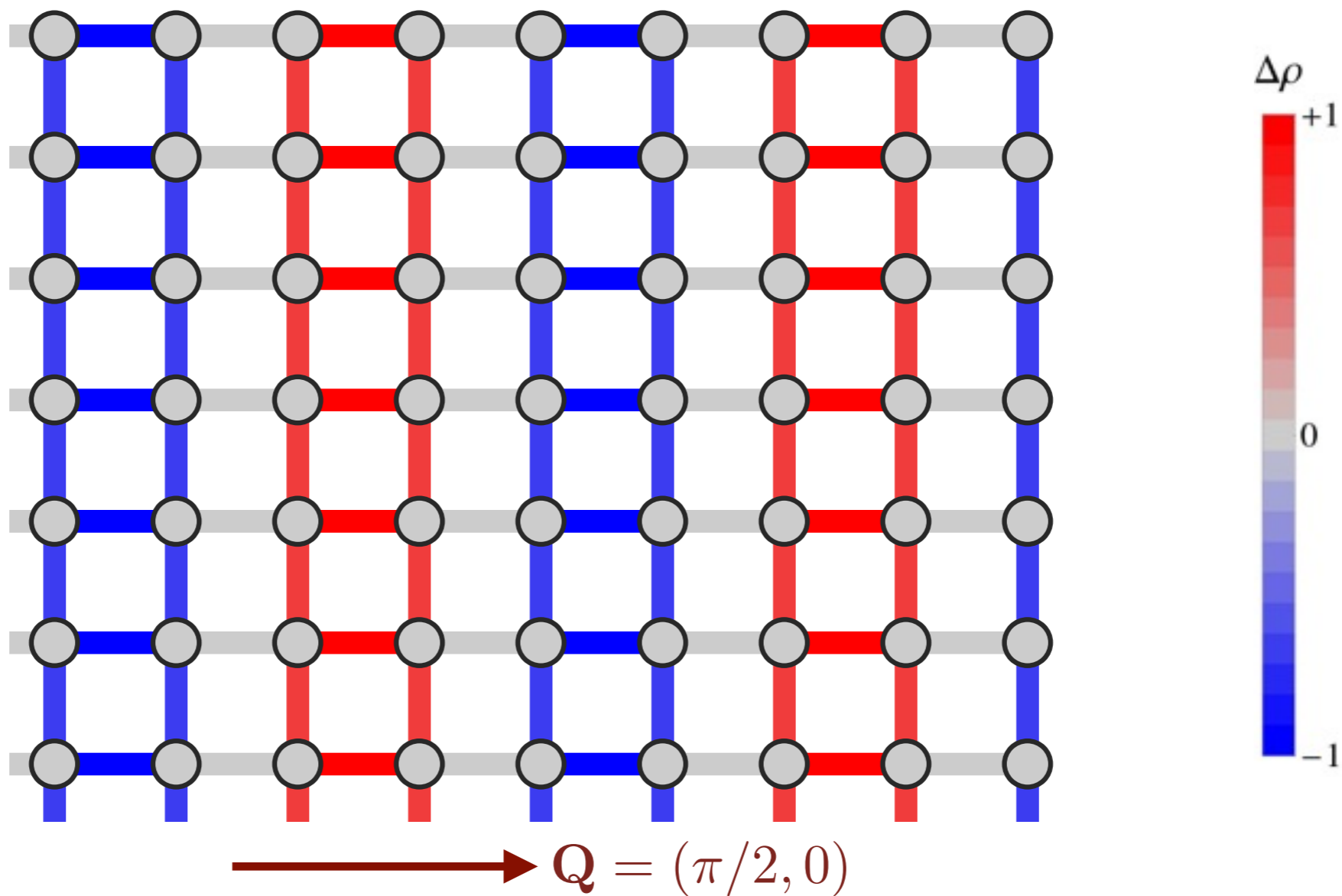


# Unconventional DW order: $s'$ -form factor

Plot of  $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$  for  $i = j$ , and  $i, j$  nearest neighbors.

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$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) + \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

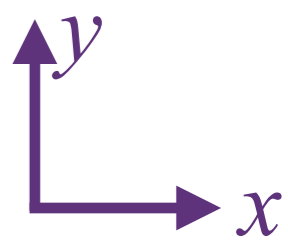


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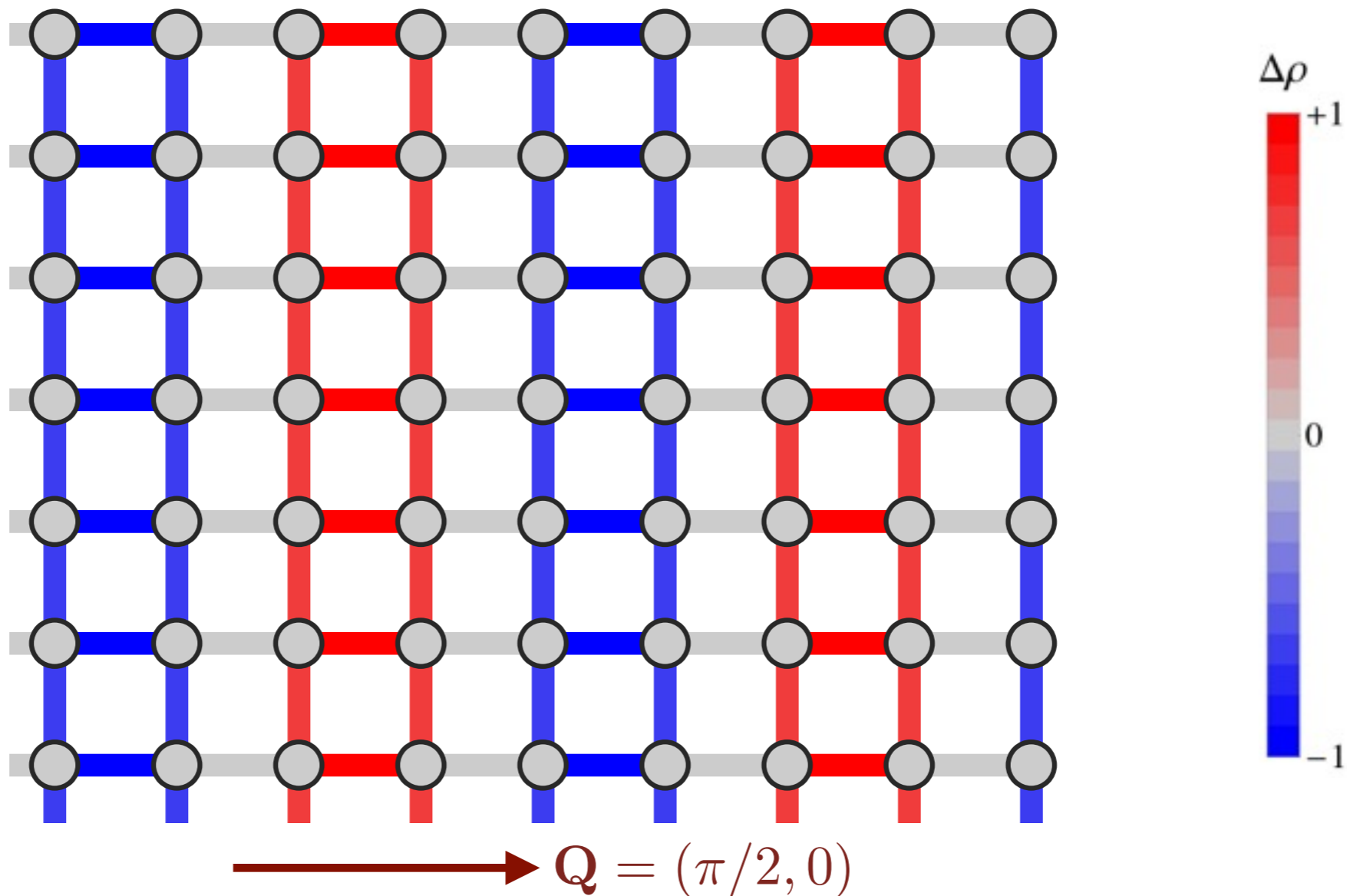
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“Stripe”  
model !

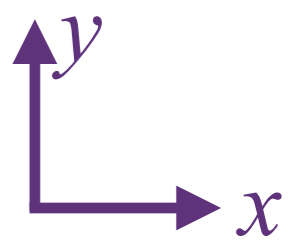


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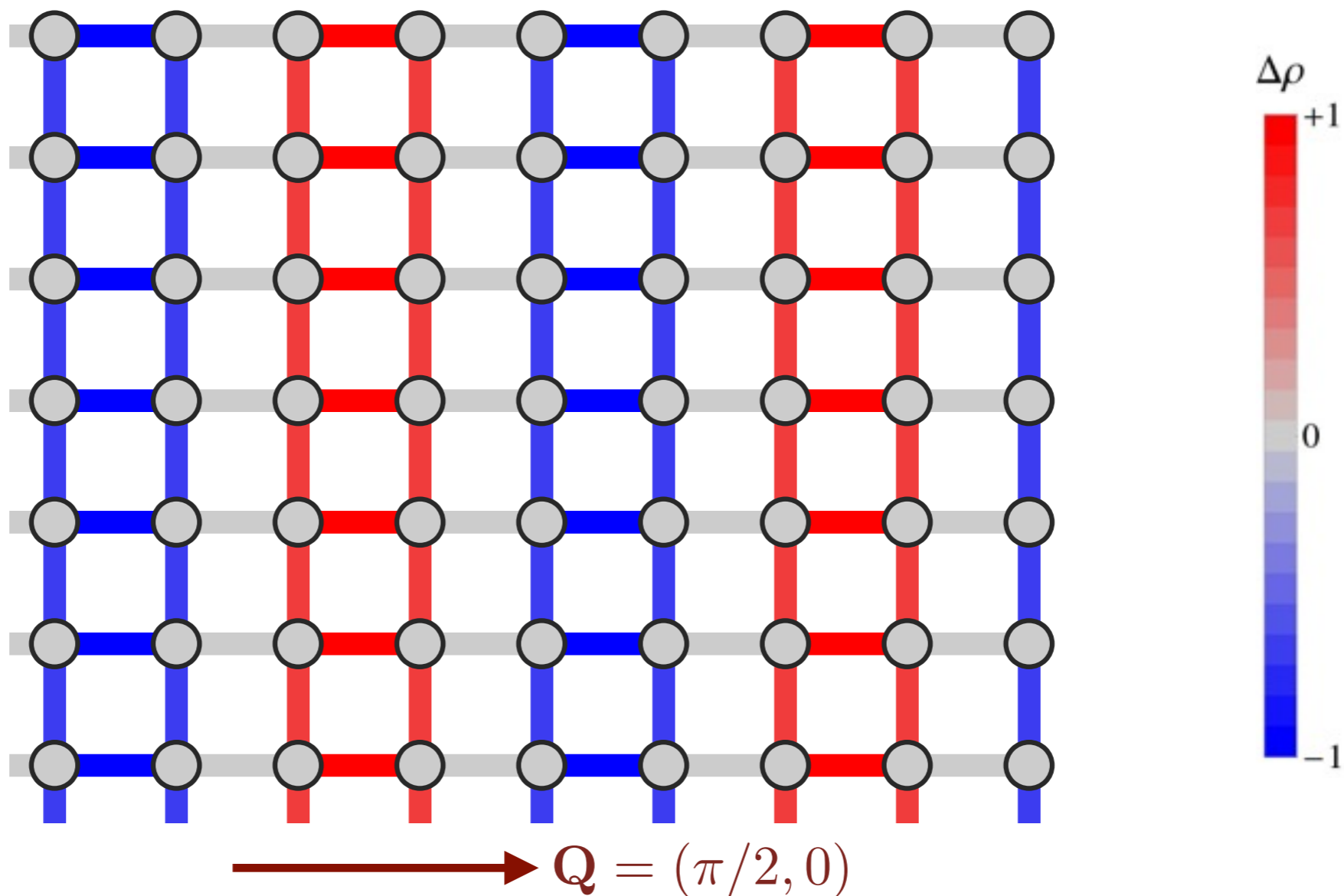
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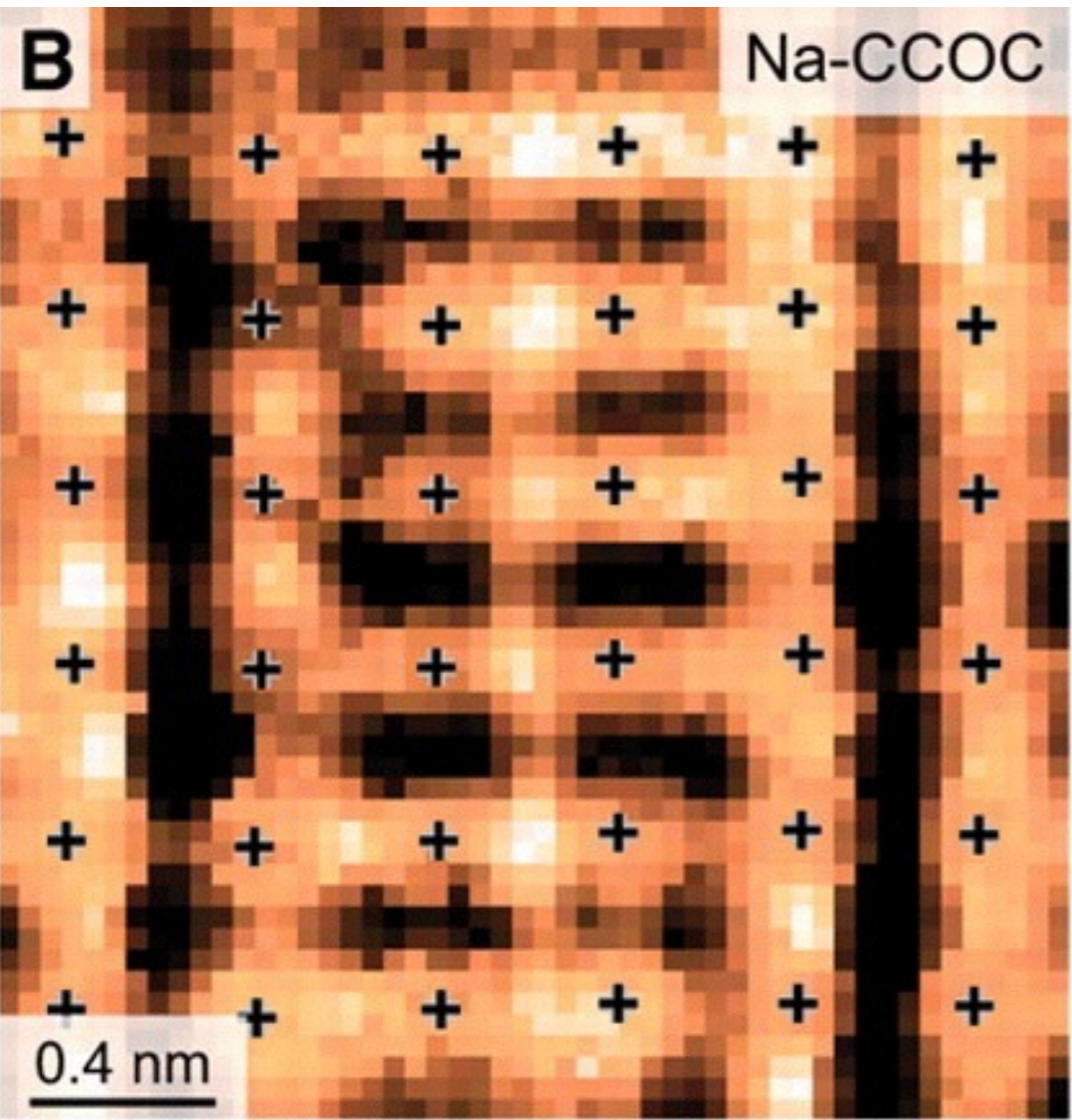
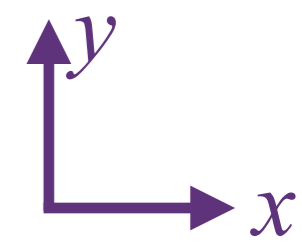
$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) + \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$



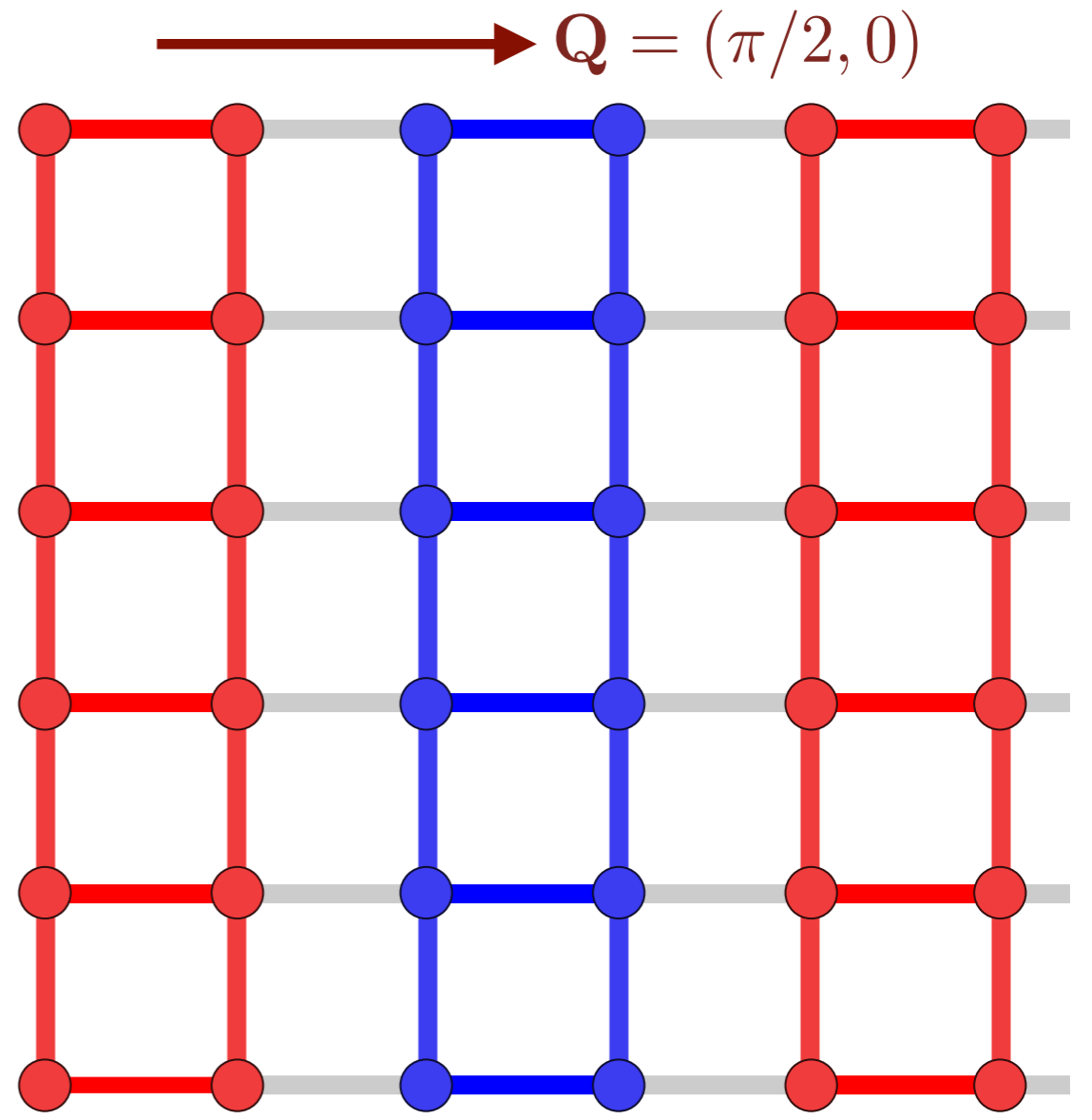
“Stripe”  
model !

X-ray  
observations  
indicate  
strong  $s'$   
component in  
LBCO





Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



$s + s'$ -form factor density wave

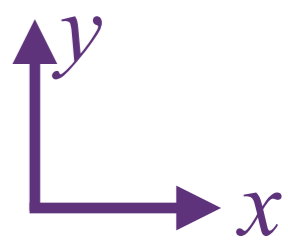
$s + s'$  form factor does not match STM measurements on BSCCO, Na-CCOC.

# Unconventional DW order: $d$ -form factor

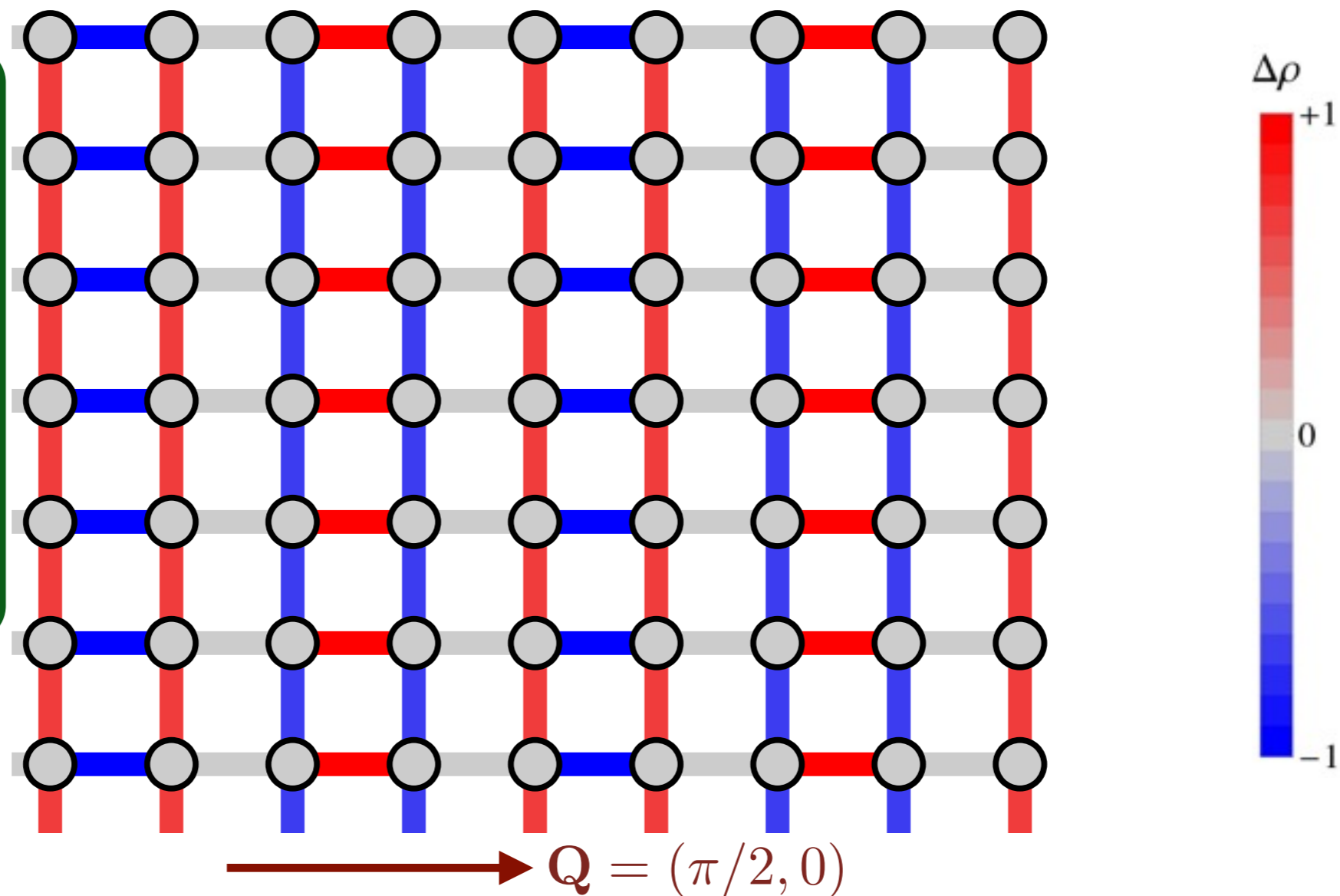
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$$P_{ij} = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

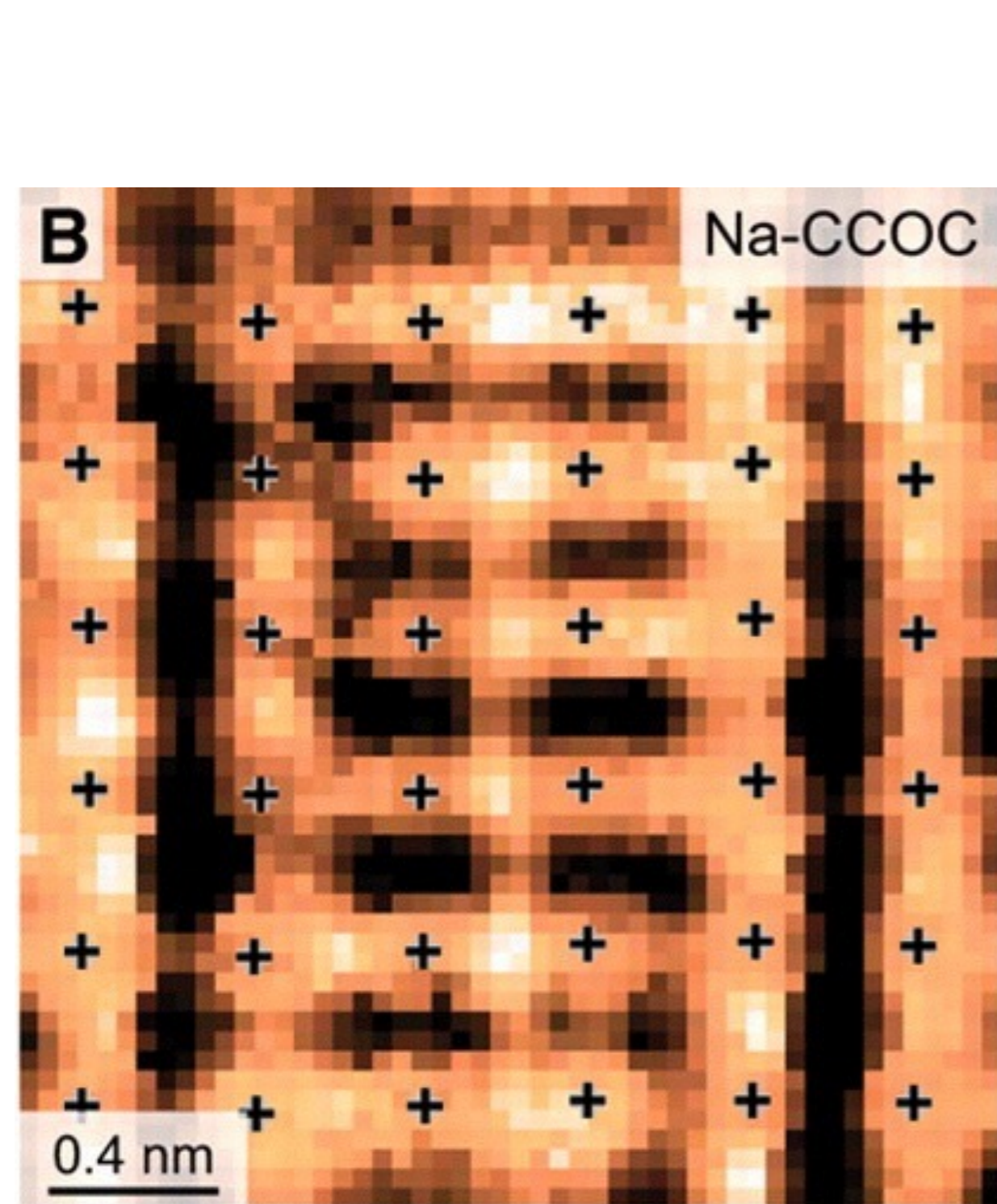
$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$



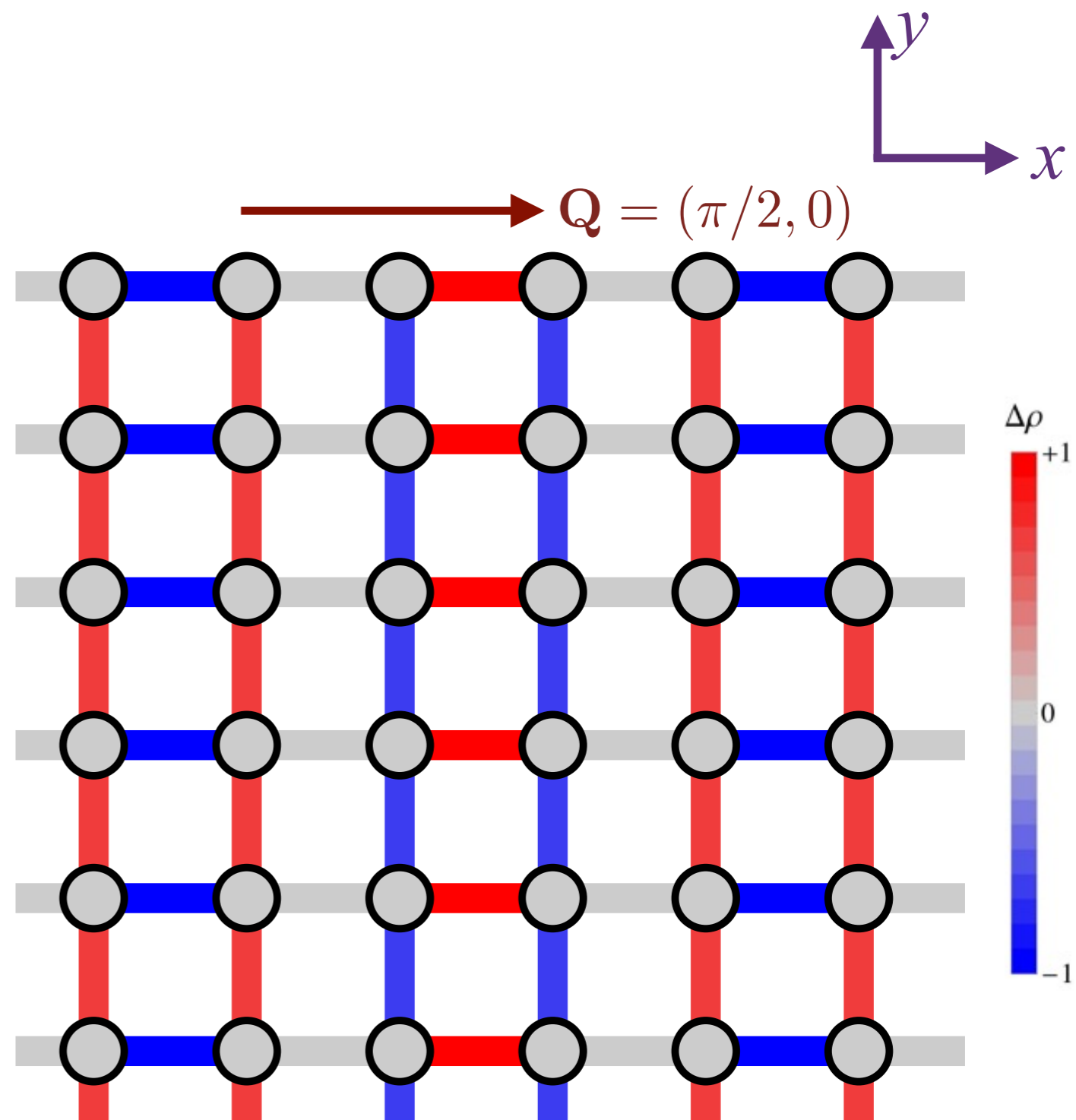
Our prediction:  
Density wave on horizontal bonds has a phase-shift of  $\pi$  relative to the wave on vertical bonds



M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).  
S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

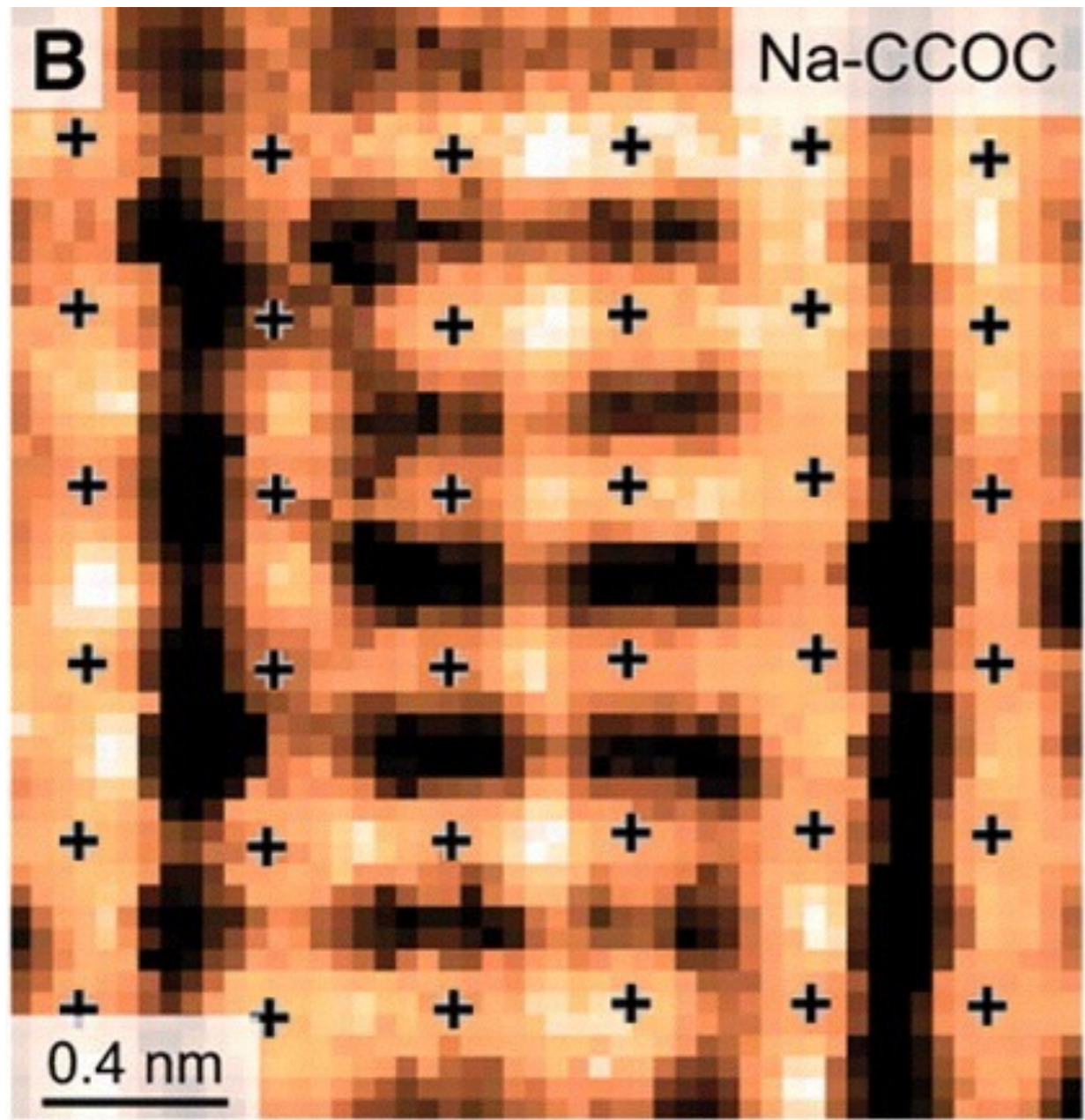
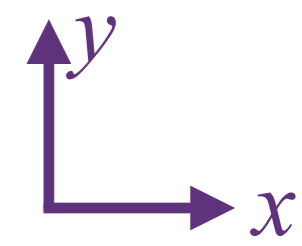


Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

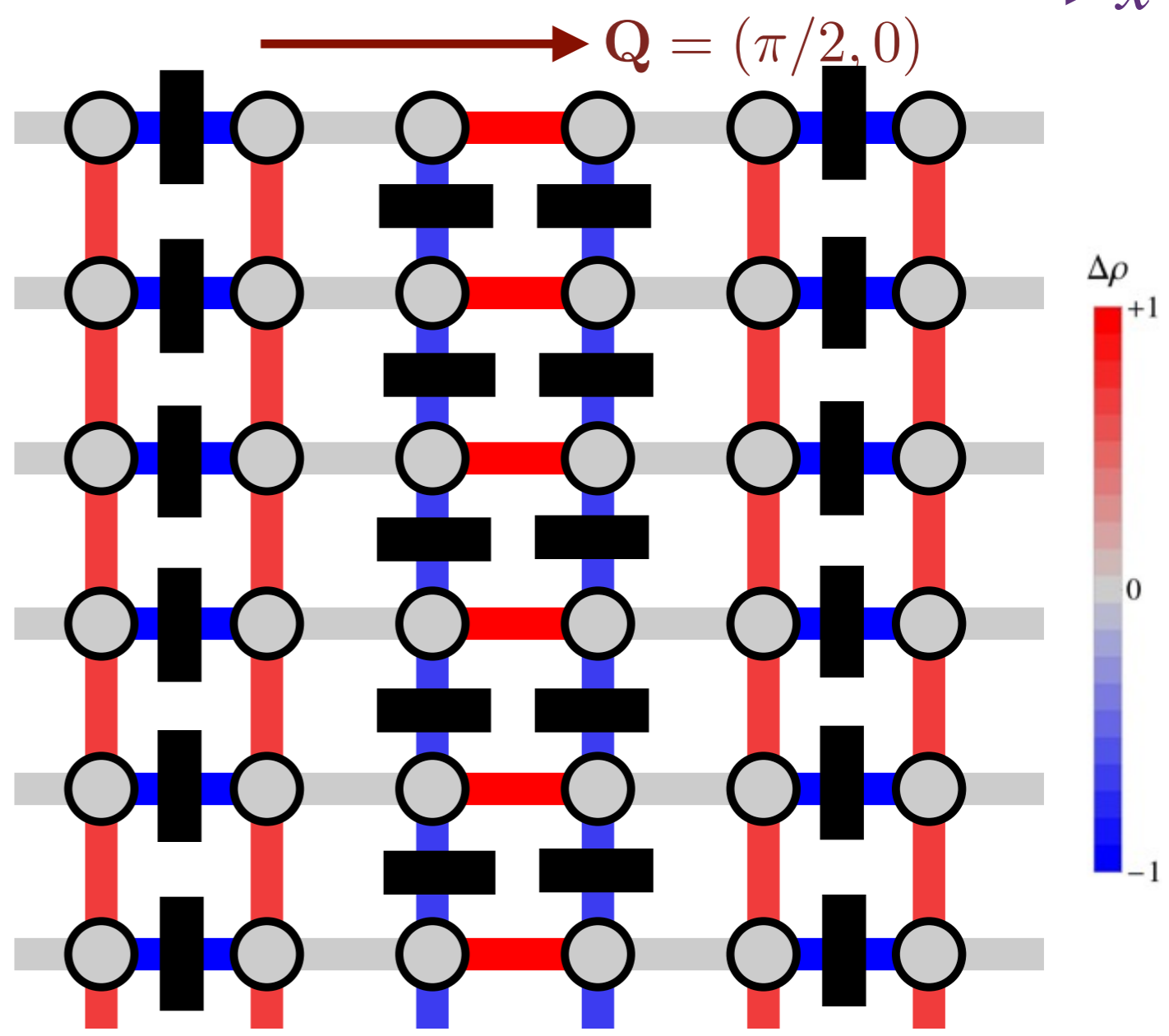


*d*-form factor density wave order

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).  
 S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).



Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



*d*-form factor density wave order

*d* form factor is compatible with STM measurements on BSCCO, Na-CCOC !

# Direct phase-sensitive identification of a $d$ -form factor density wave in underdoped cuprates

Kazuhiro Fujita<sup>a,b,c,1</sup>, Mohammad H. Hamidian<sup>a,b,1</sup>, Stephen D. Edkins<sup>b,d</sup>, Chung Koo Kim<sup>a</sup>, Yuhki Kohsaka<sup>e</sup>, Masaki Azuma<sup>f</sup>, Mikio Takano<sup>g</sup>, Hidenori Takagi<sup>c,h,i</sup>, Hiroshi Eisaki<sup>j</sup>, Shin-ichi Uchida<sup>c</sup>, Andrea Allais<sup>k</sup>, Michael J. Lawler<sup>b,l</sup>, Eun-Ah Kim<sup>b</sup>, Subir Sachdev<sup>k,m</sup>, and J. C. Séamus Davis<sup>a,b,d,2</sup>

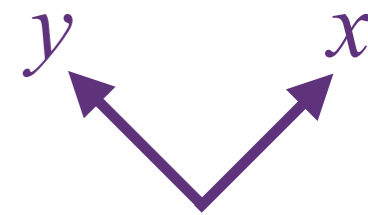
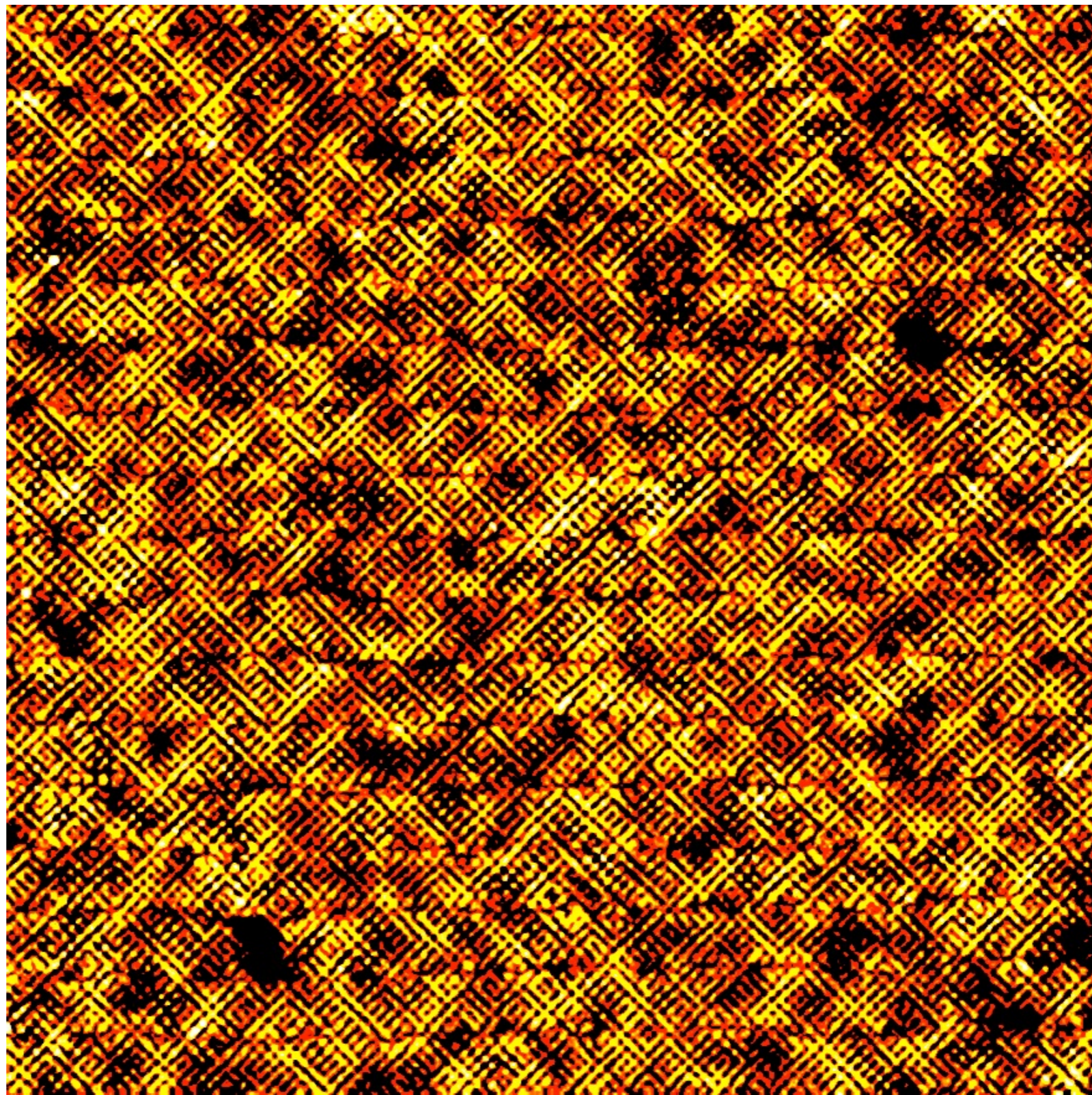
The identity of the fundamental broken symmetry (if any) in the underdoped cuprates is unresolved. However, evidence has been accumulating that this state may be an unconventional density wave. Here we carry out site-specific measurements within each  $\text{CuO}_2$  unit cell, segregating the results into three separate electronic structure images containing only the Cu sites [ $\text{Cu}(r)$ ] and only the  $x/y$  axis O sites [ $\text{O}_x(r)$  and  $\text{O}_y(r)$ ]. Phase-resolved Fourier analysis reveals directly that the modulations in the  $\text{O}_x(r)$  and  $\text{O}_y(r)$  sublattice images consistently exhibit a relative phase of  $\pi$ . We confirm this discovery on two highly distinct cuprate compounds, ruling out tunnel matrix-element and materials-specific systematics. These observations demonstrate by direct sublattice phase-resolved visualization that the density wave found in underdoped cuprates consists of modulations of the intraunit-cell states that exhibit a predominantly  $d$ -symmetry form factor.

See also

C. Howald, H. Eisaki,  
N. Kaneko, M. Greven,  
and A. Kapitulnik,  
*Phys. Rev. B* **67**,  
014533 (2003);

M. Vershinin, S. Misra,  
S. Ono, Y. Abe, Yoichi  
Ando, and  
A. Yazdani, *Science*  
**303**, 1995 (2004).

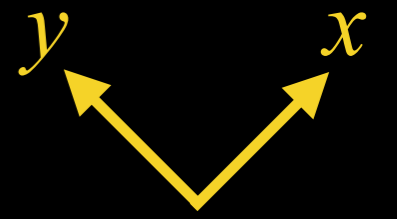
W. D. Wise, M. C. Boyer,  
K. Chatterjee, T. Kondo,  
T. Takeuchi, H. Ikuta,  
Y. Wang, and  
E. W. Hudson,  
*Nature Phys.* **4**, 696  
(2008).



“R-map” of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007). **Davis group has sub-angstrom resolution capabilities, with lattice drift corrections, which make sublattice phase-resolved STM possible.**

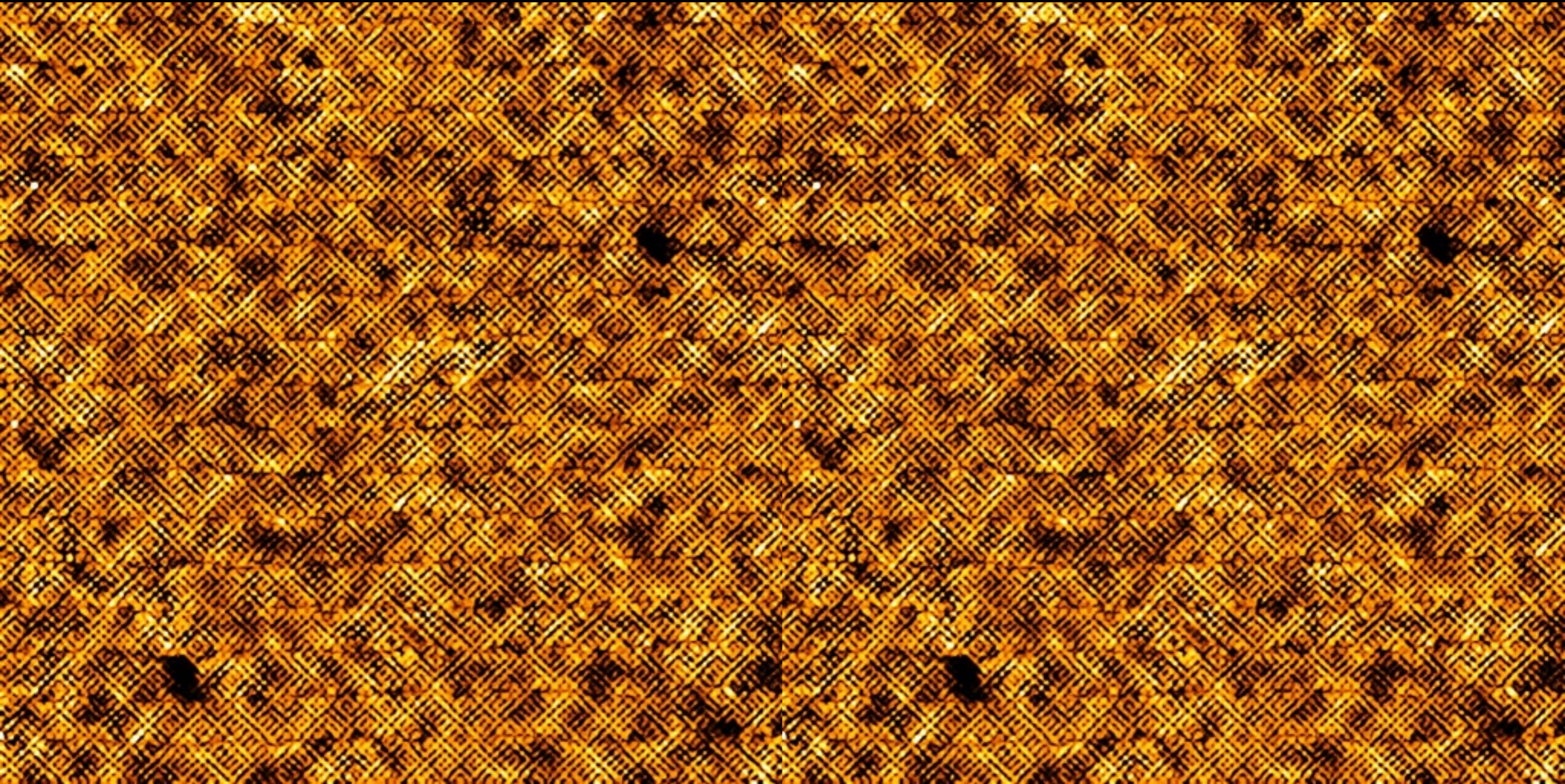
UD45K  
BSCCO

$R(r, 150\text{mV})$



$R(r, 150\text{mV})$

$R(r, 150\text{mV})$



Note that these are identical images.

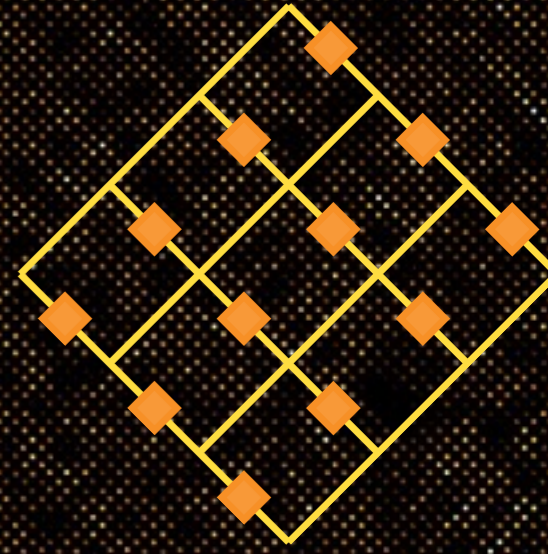
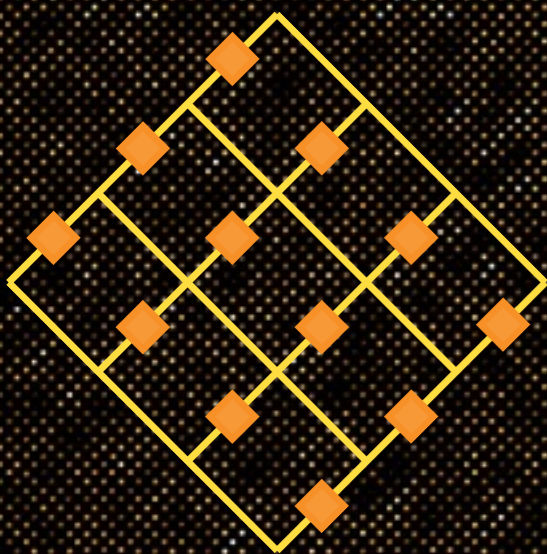
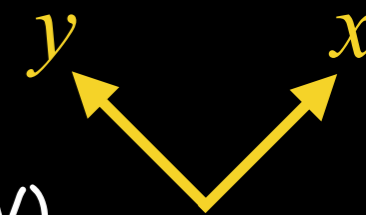
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)

UD45K

$R(r=0, 150\text{mV})$

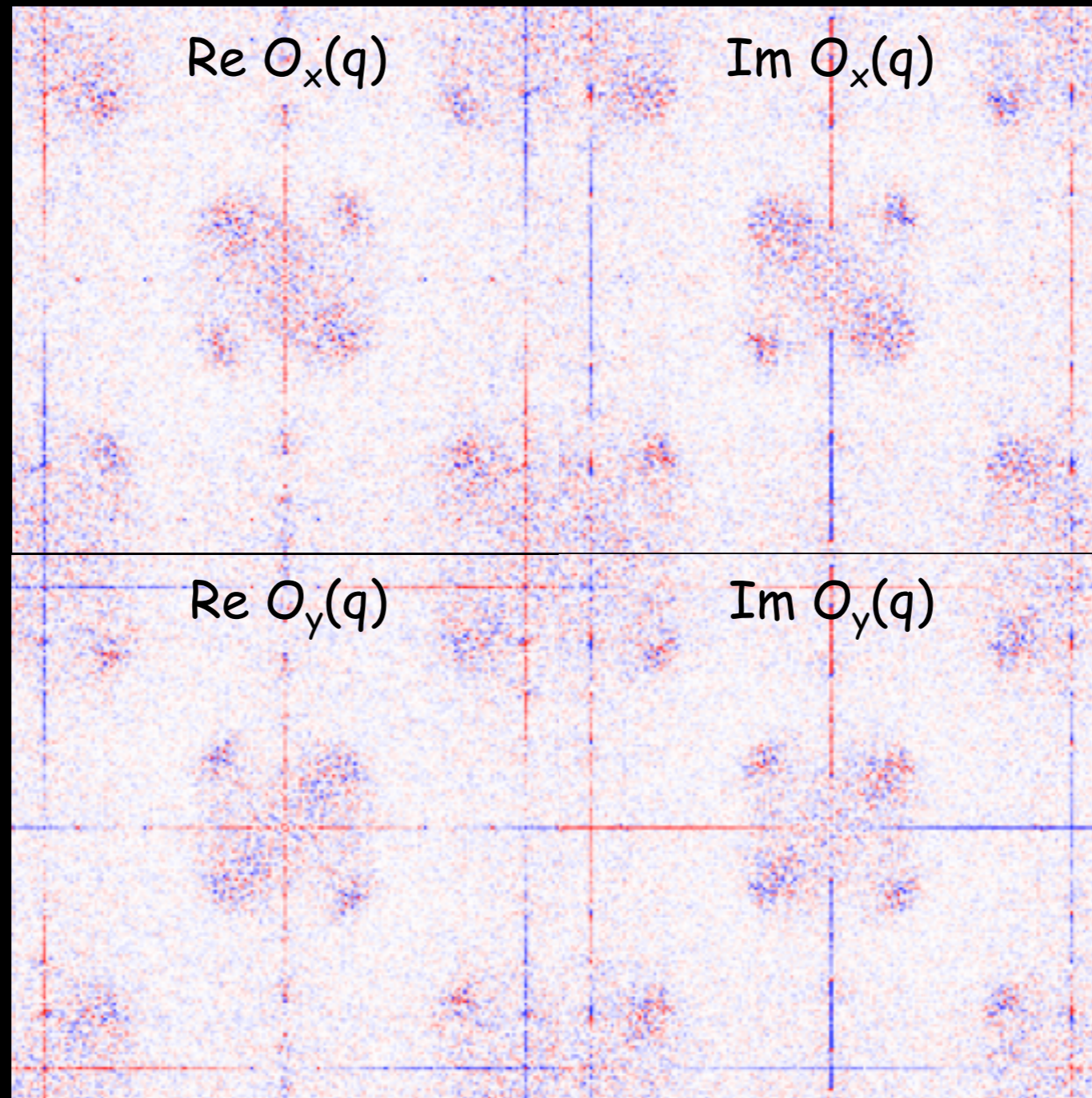
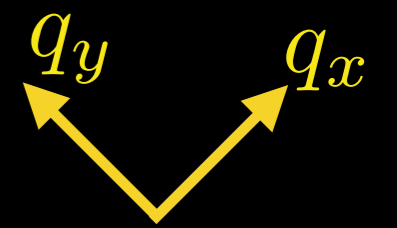
$R(r=O_x, 150\text{mV})$

$R(r=O_y, 150\text{mV})$

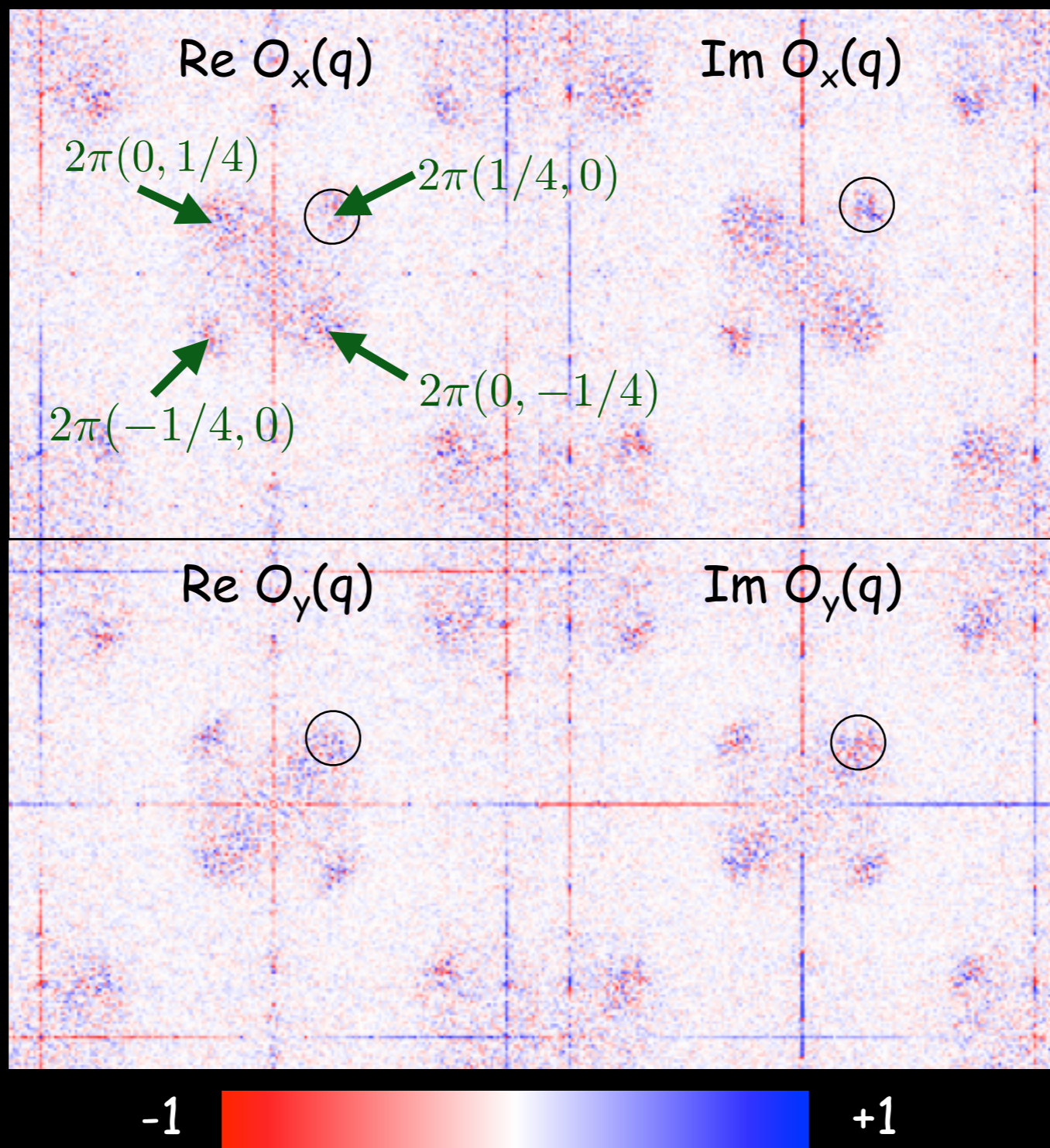
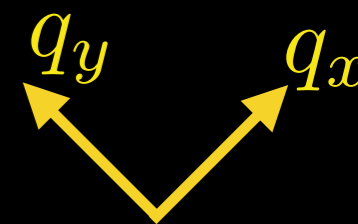


UD45K

# Broad (0,Q) and (Q,0) DW Features

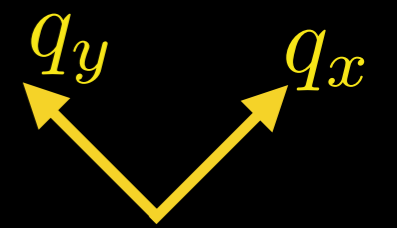


## Broad (0,Q) and (Q,0) DW Features

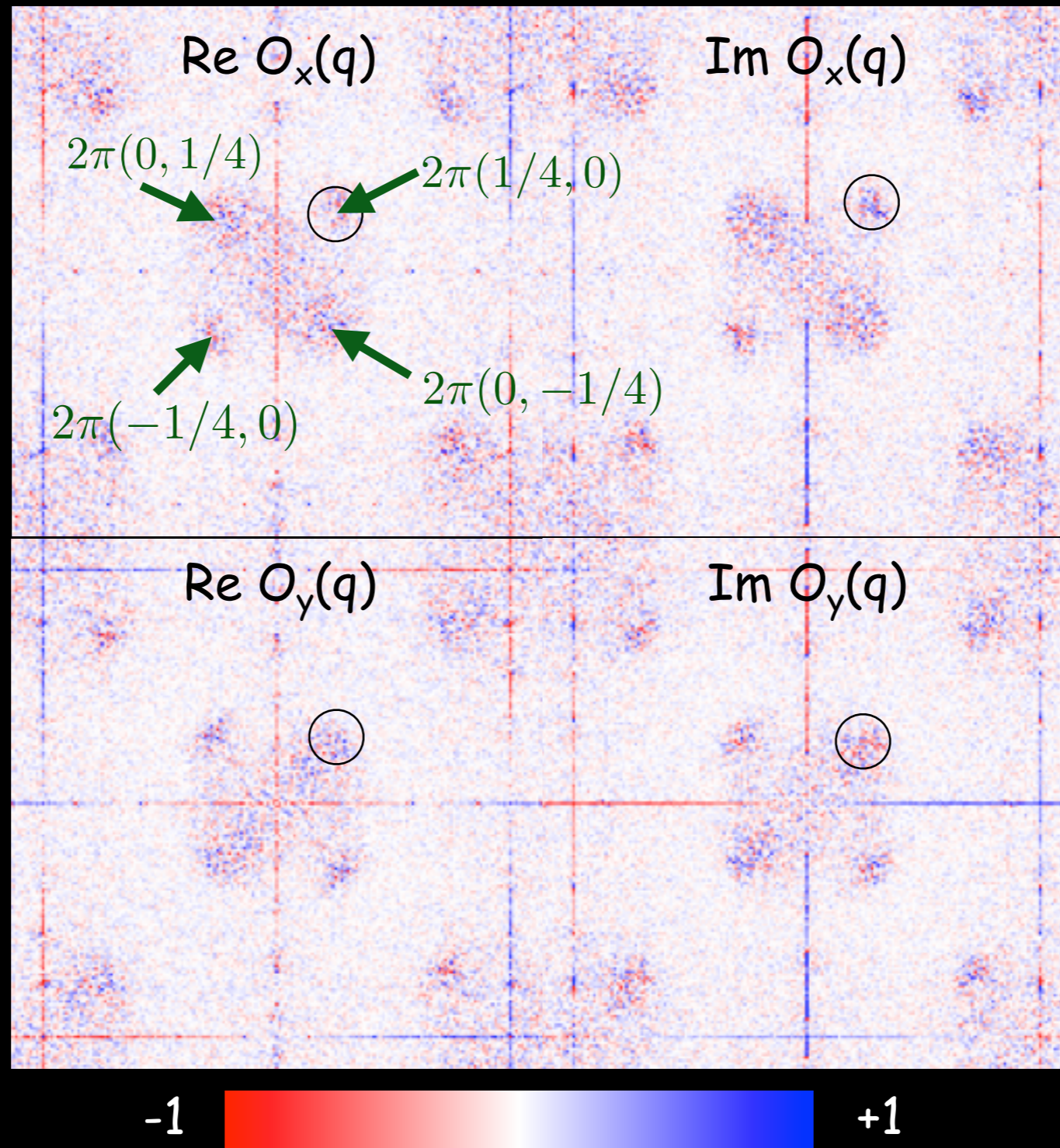


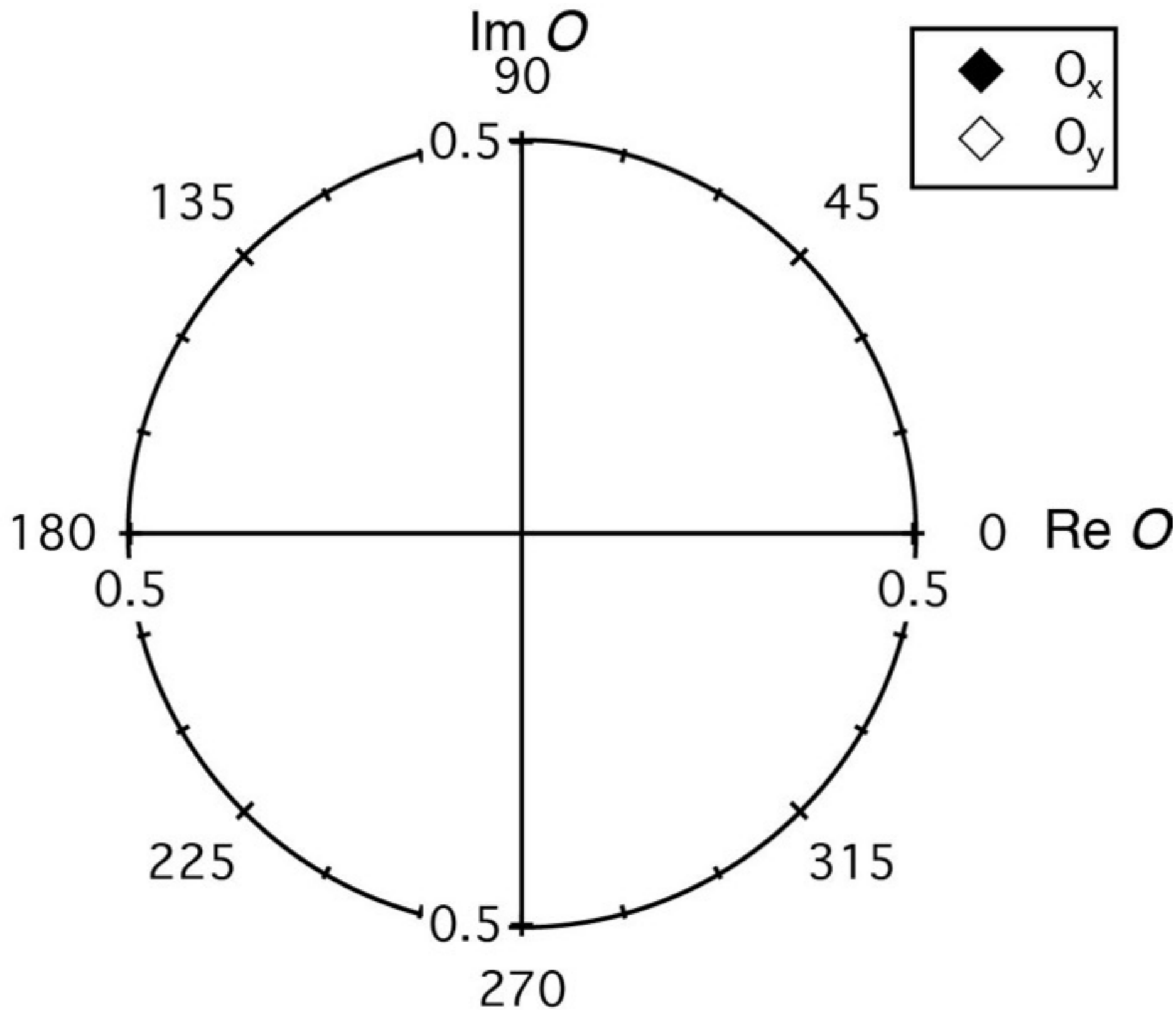
UD45K

## Broad (0,Q) and (Q,0) DW Features

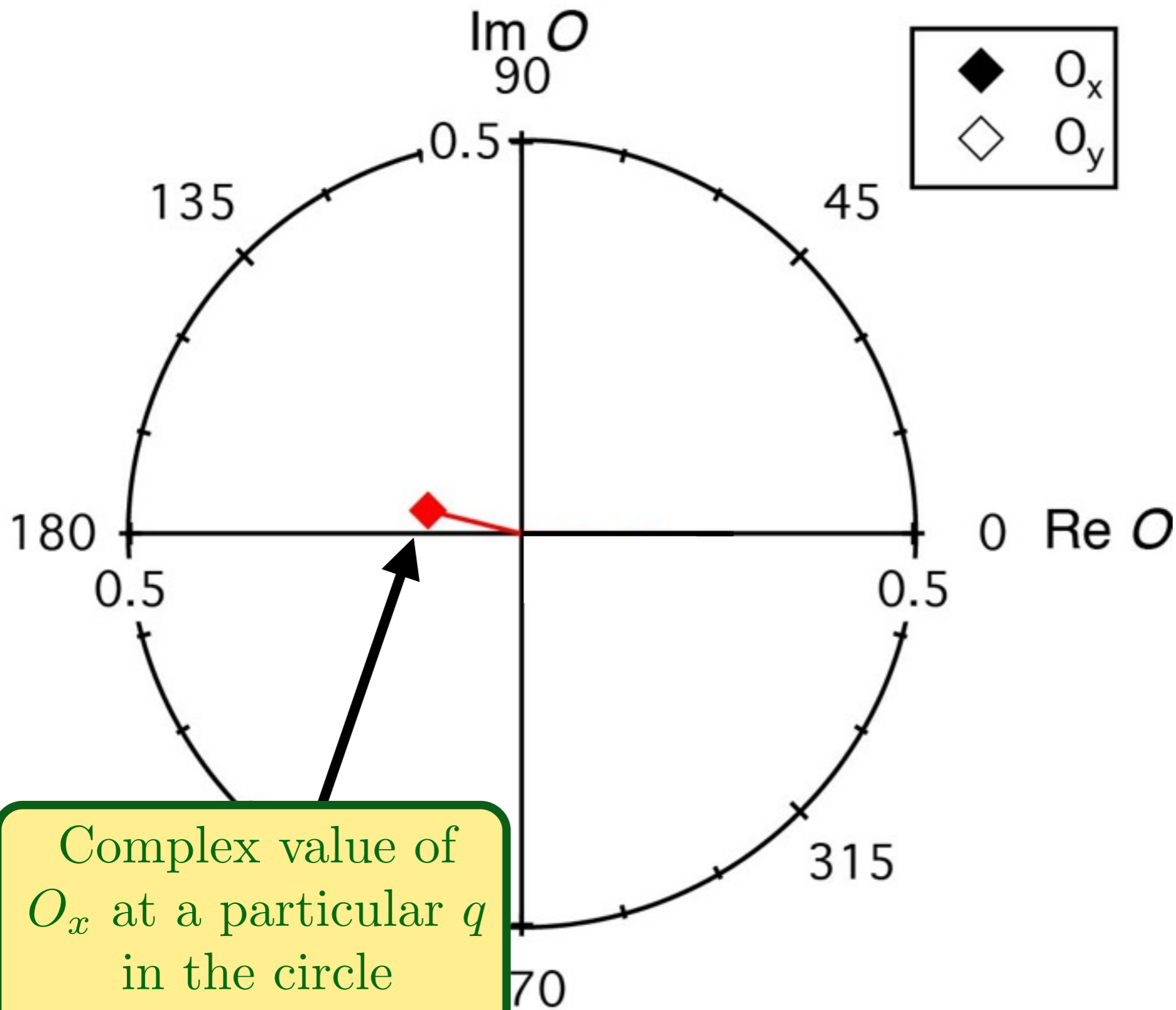


For each pixel in the circles, we obtain 2 complex numbers,  $O_x(q)$  and  $O_y(q)$ .



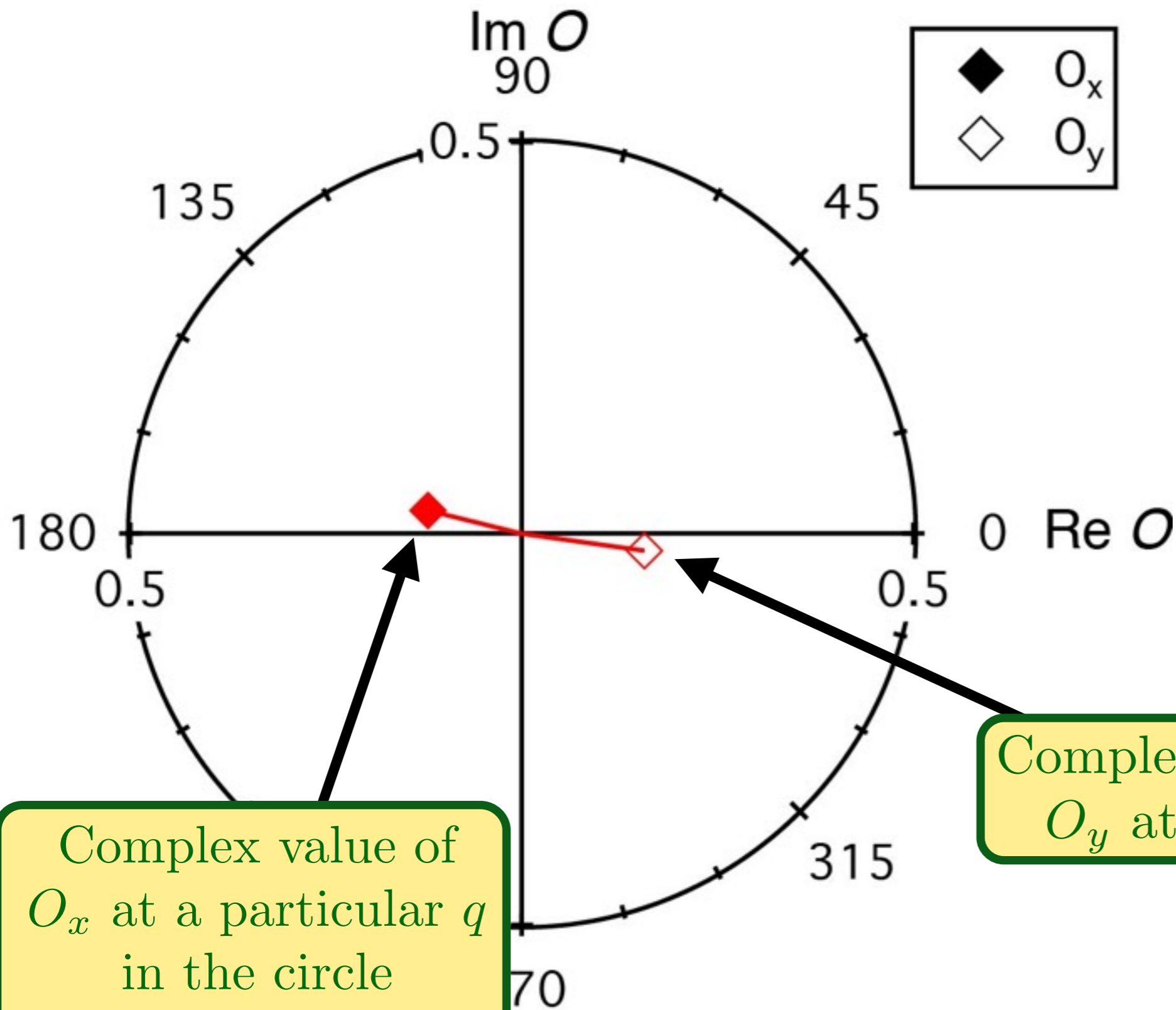


**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



**Phase-sensitive measurement of the  $d$ -form factor of density wave order**

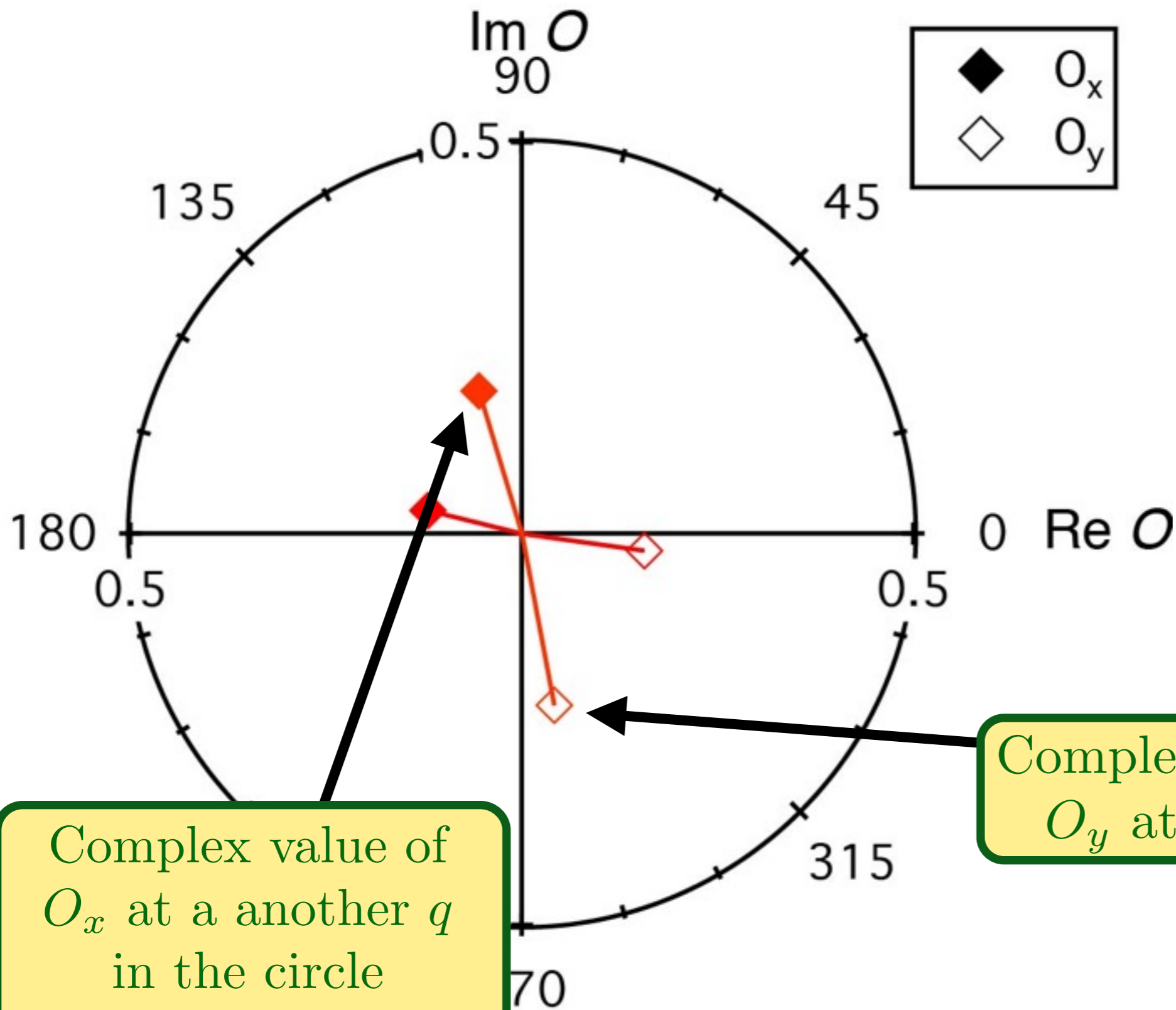
Complex value of  $O_x$  at a particular  $q$  in the circle around  $2\pi(1/4, 0)$ .



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Complex value of  $O_x$  at a particular  $q$  in the circle around  $2\pi(1/4, 0)$ .

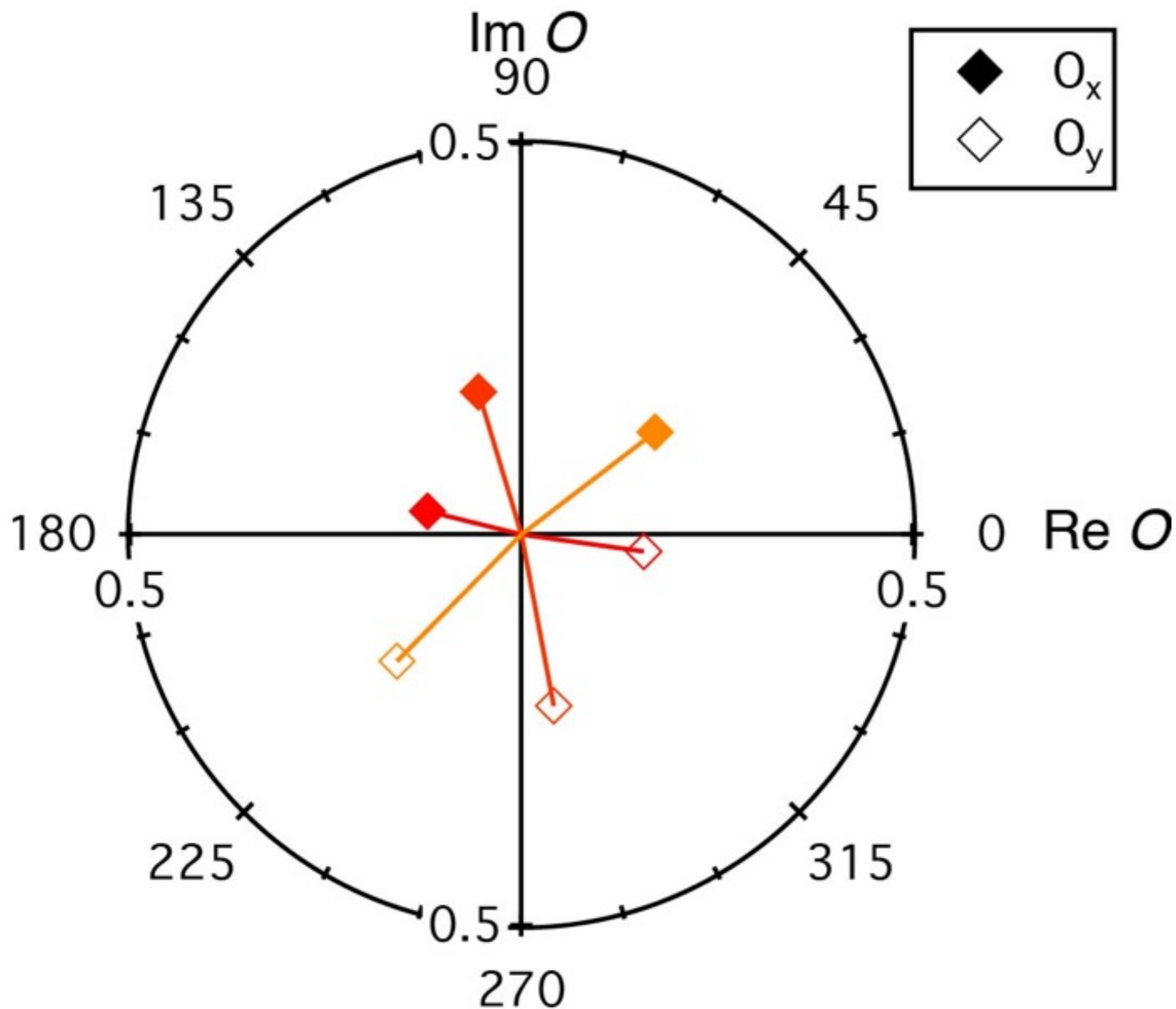
Complex value of  $O_y$  at same  $q$



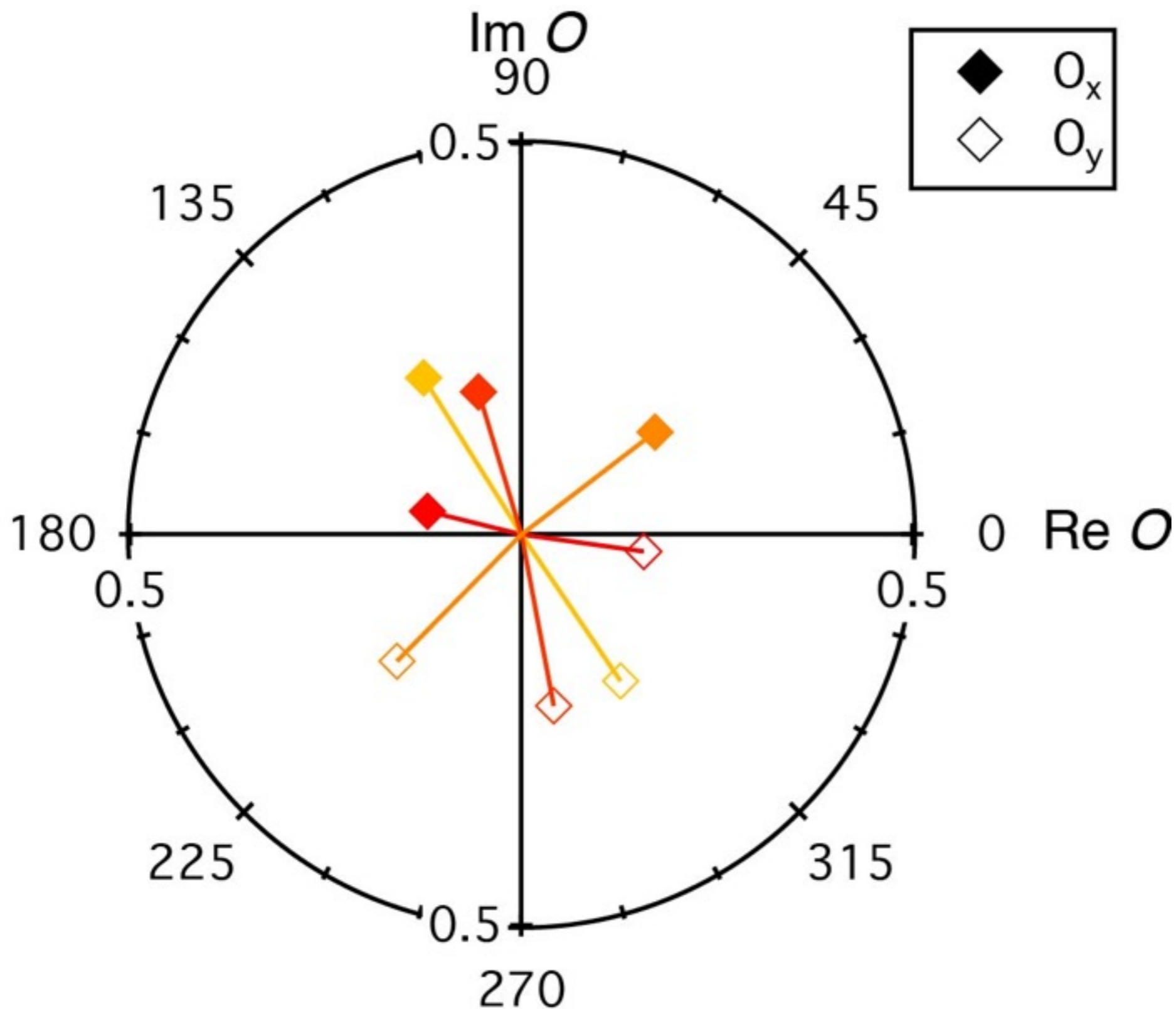
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**

Complex value of  $O_x$  at a another  $q$  in the circle around  $2\pi(1/4, 0)$ .

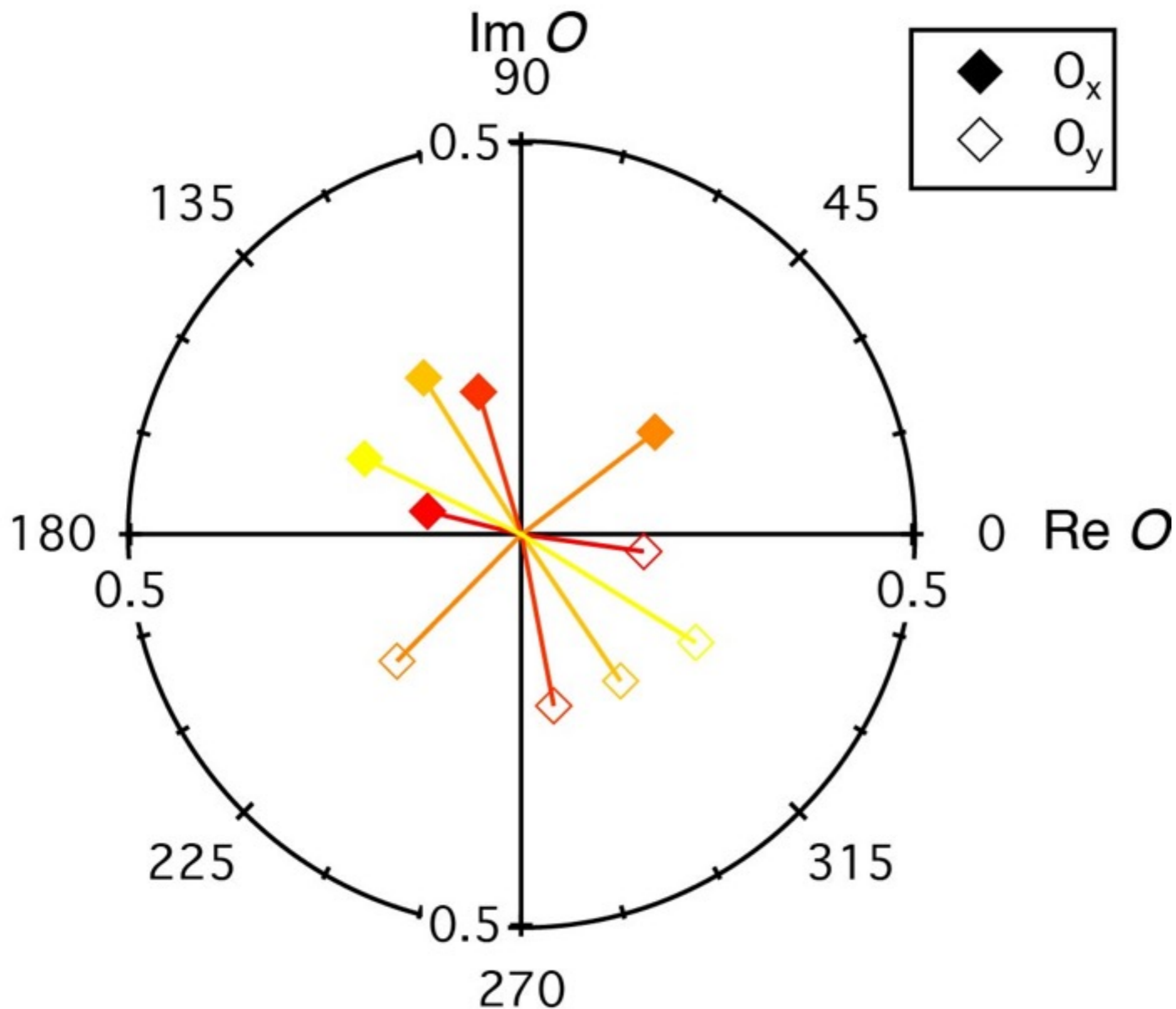
Complex value of  $O_y$  at same  $q$



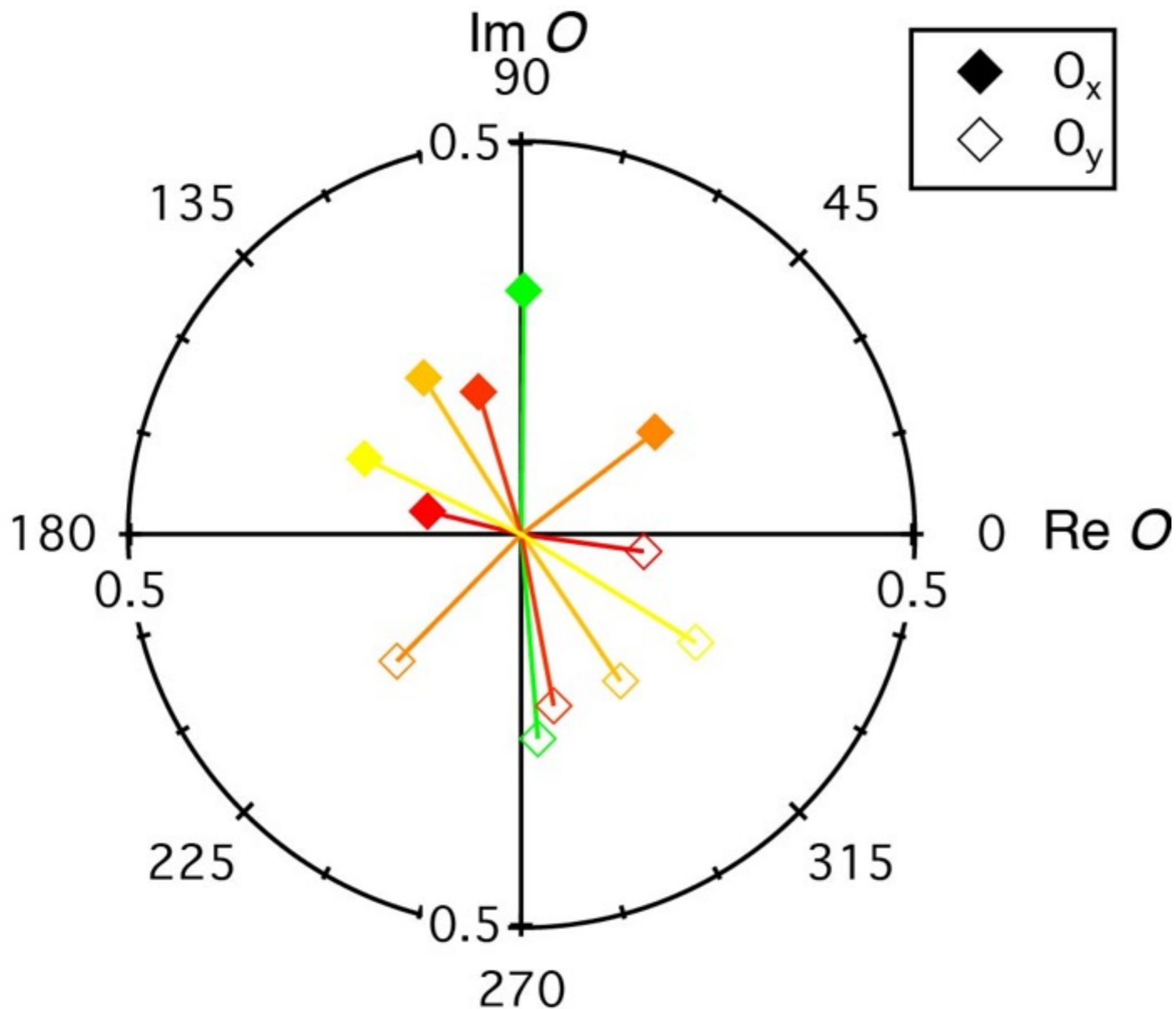
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



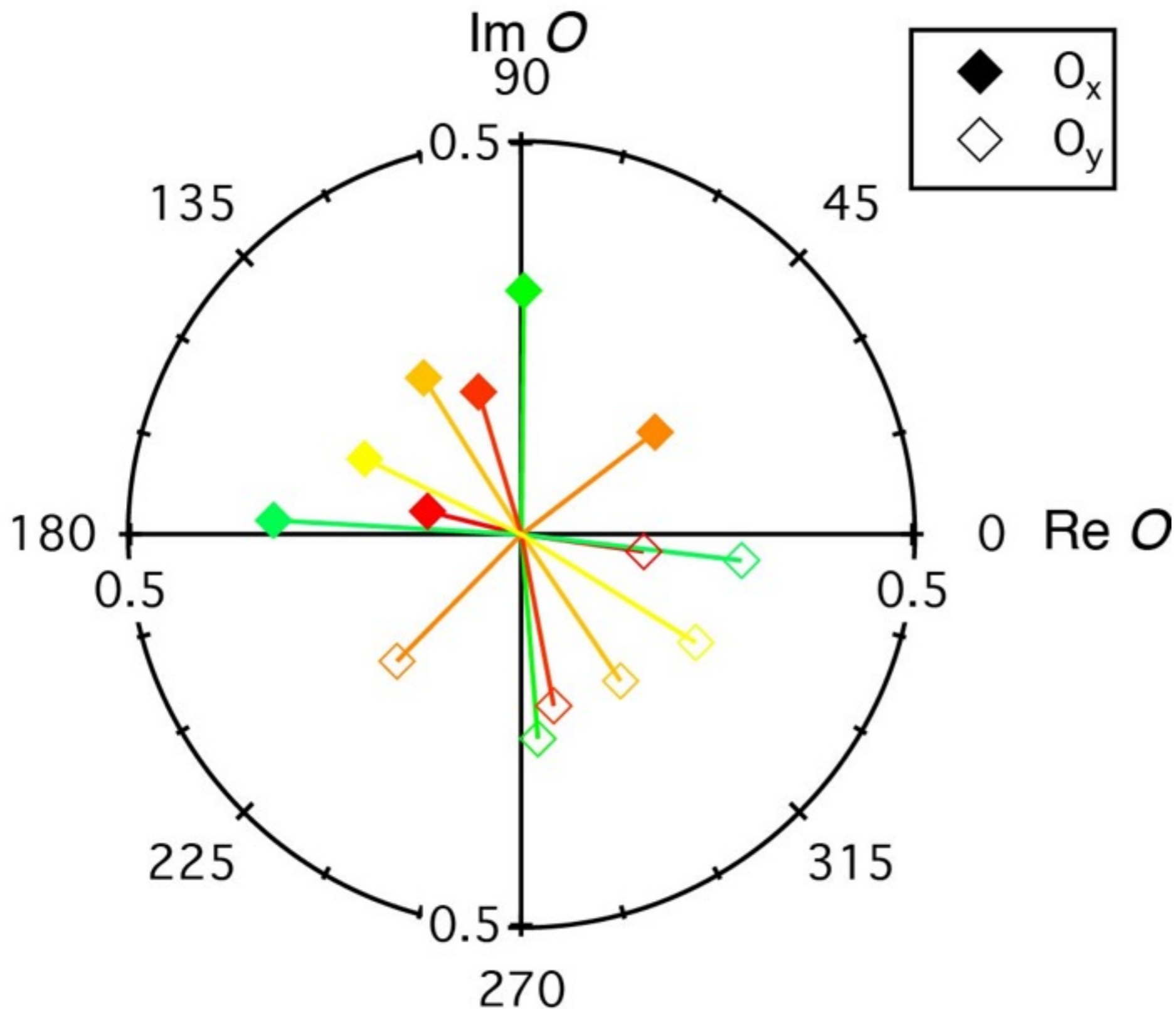
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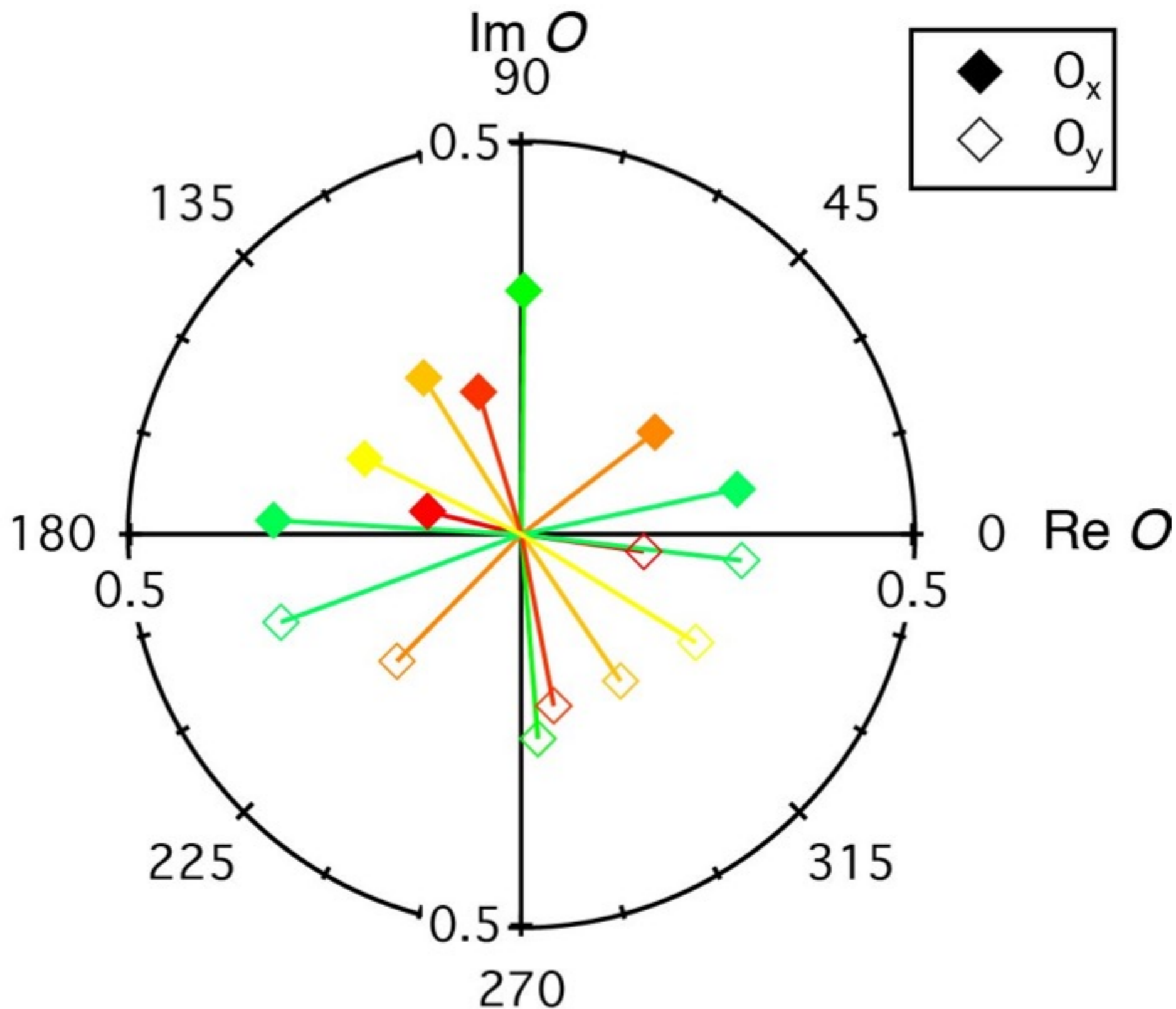
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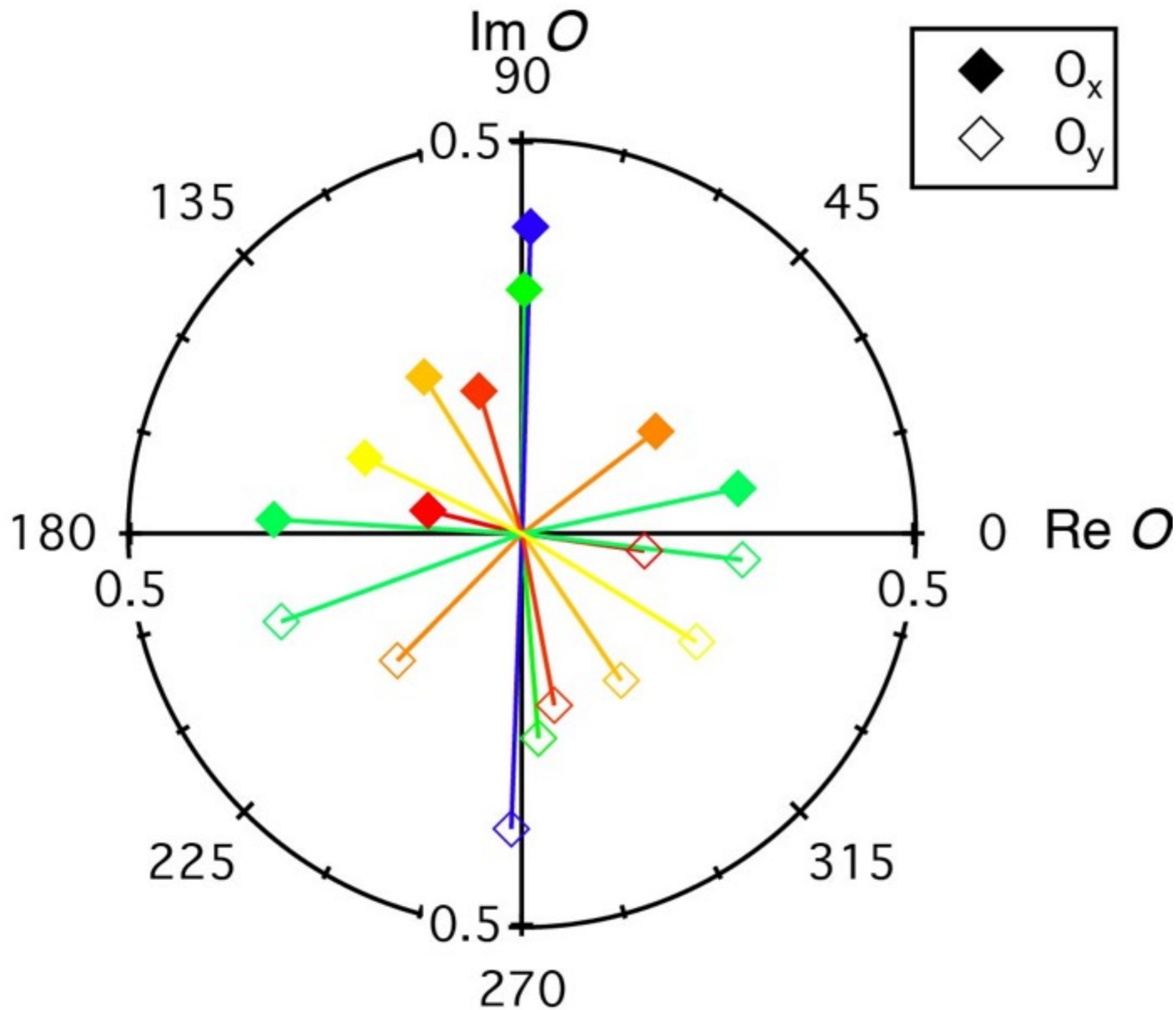
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



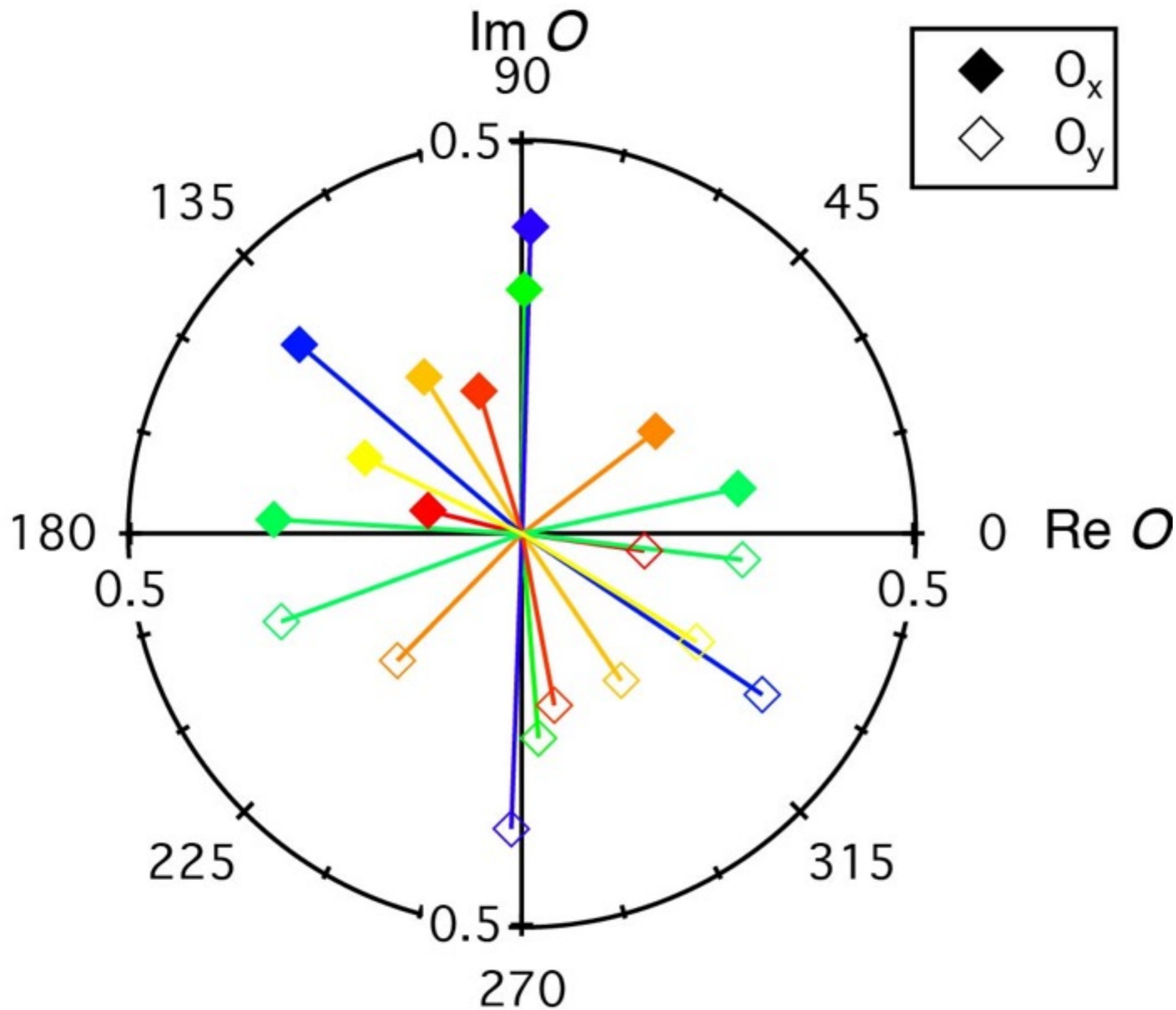
**Phase-sensitive measurement of the *d*-form factor of density wave order**



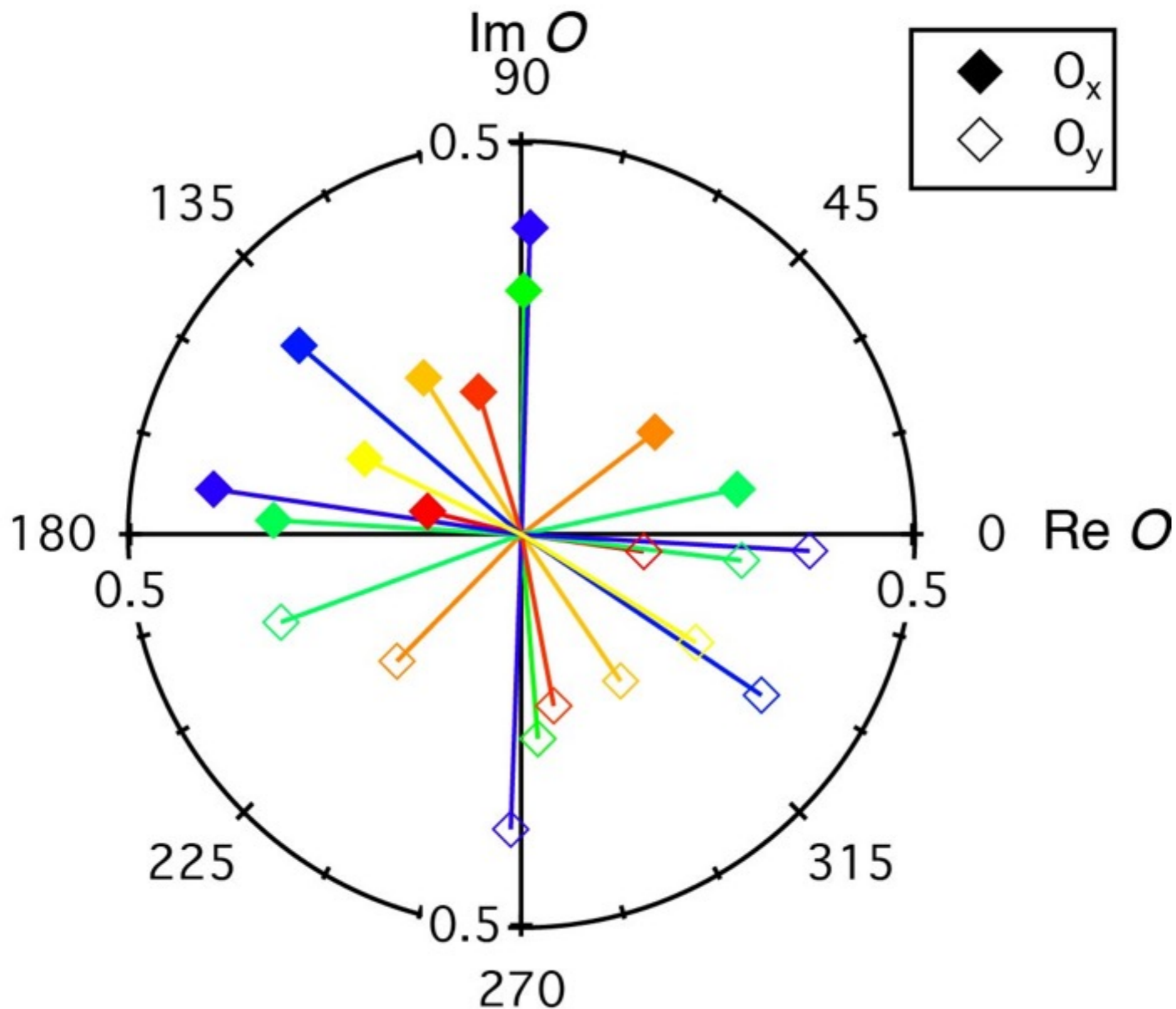
**Phase-sensitive  
measurement of  
the  $d$ -form factor  
of density wave  
order**



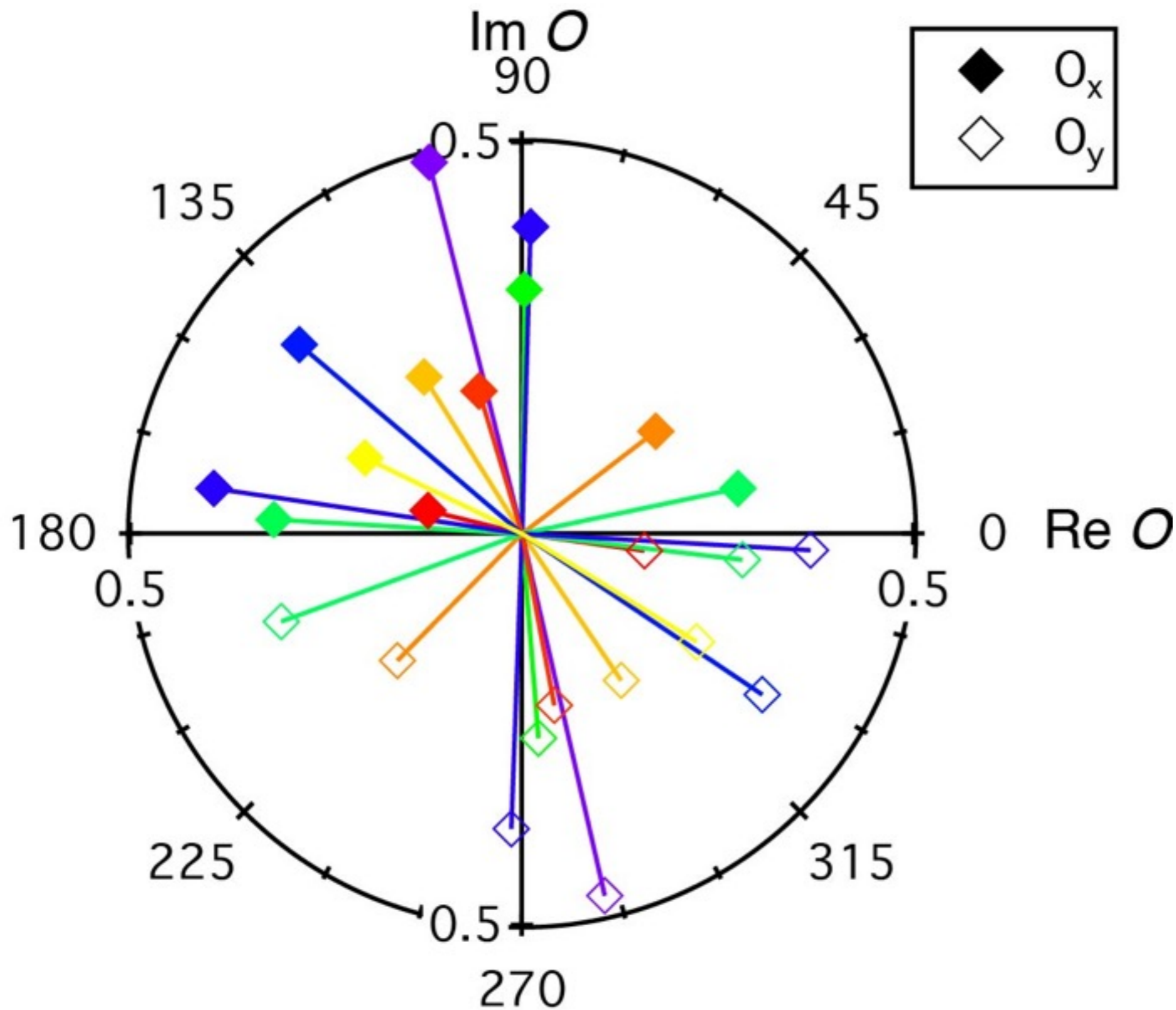
**Phase-sensitive measurement of the *d*-form factor of density wave order**



**Phase-sensitive measurement of the  $d$ -form factor of density wave order**

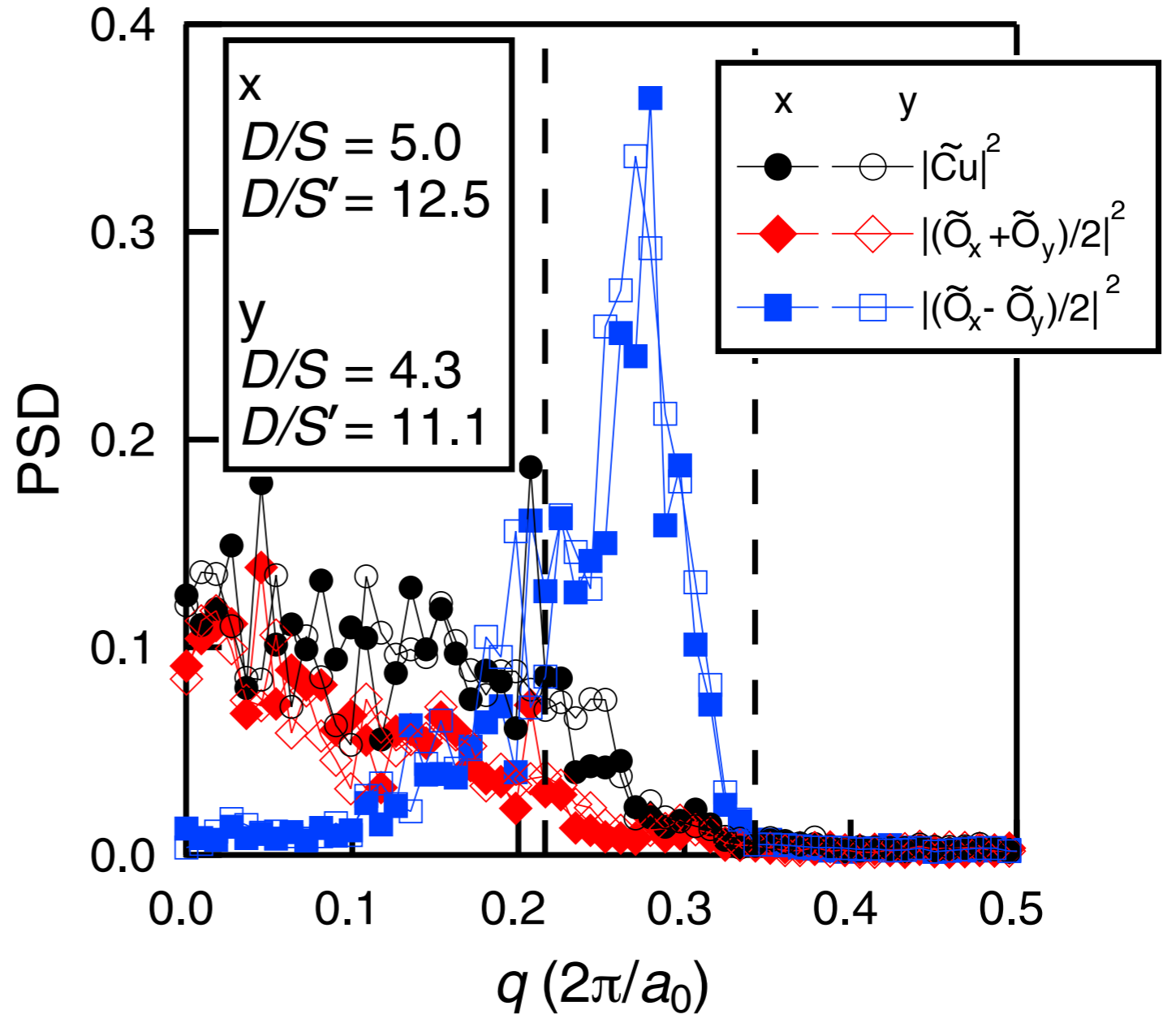
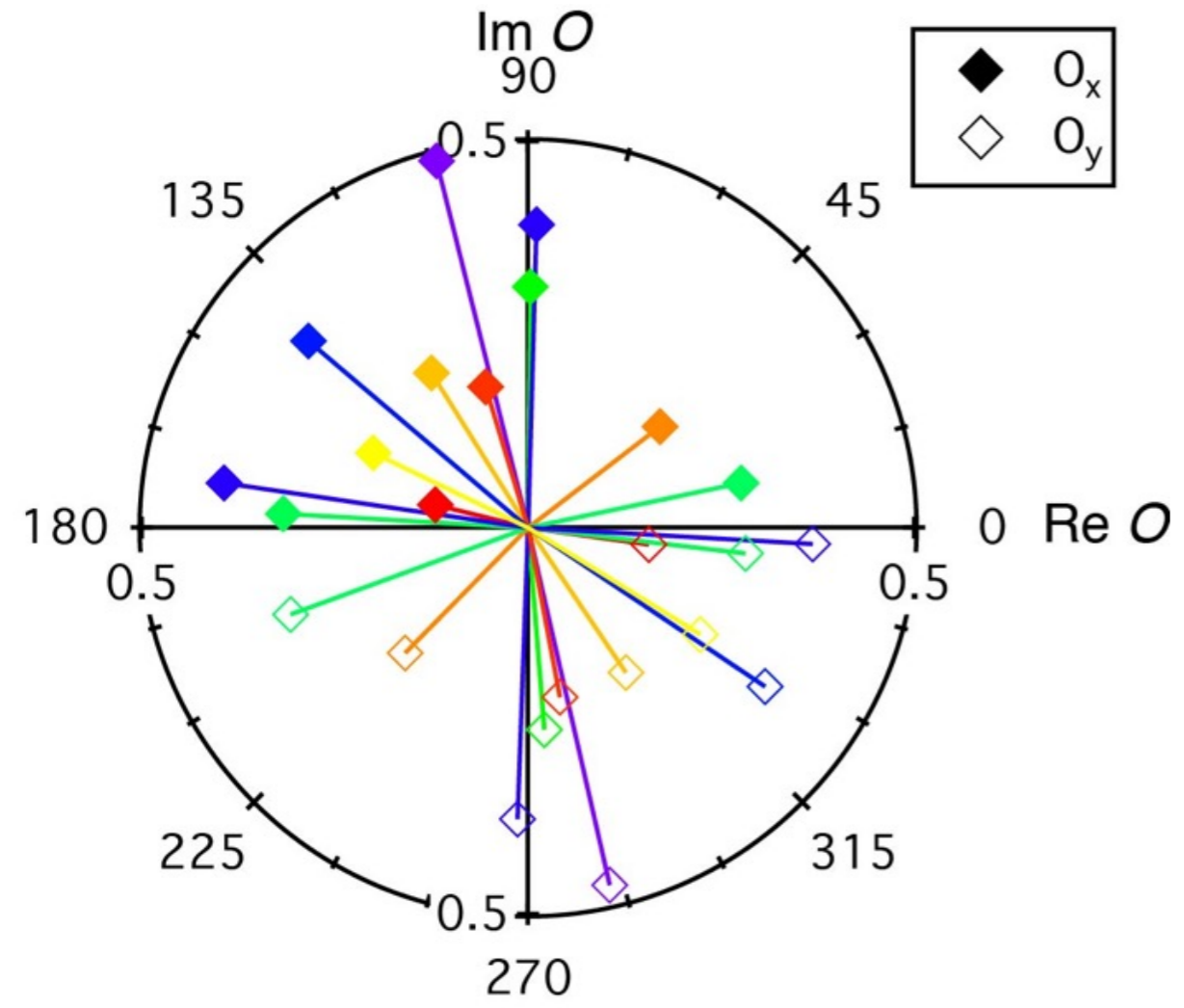


**Phase-sensitive  
measurement of  
the  $d$ -form factor  
of density wave  
order**

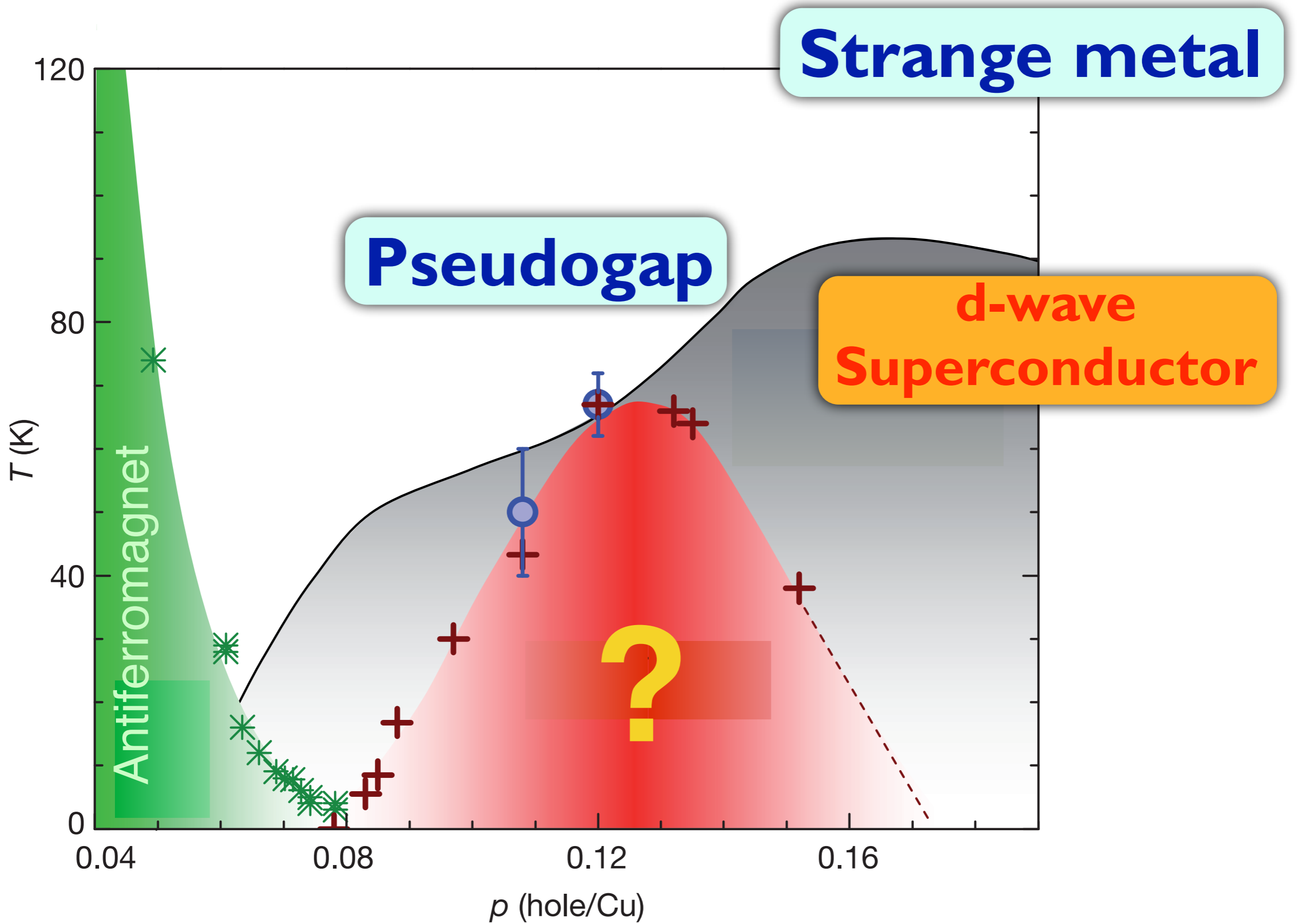


**Phase-sensitive measurement of the *d*-form factor of density wave order**

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K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS 111, E3026 (2014)

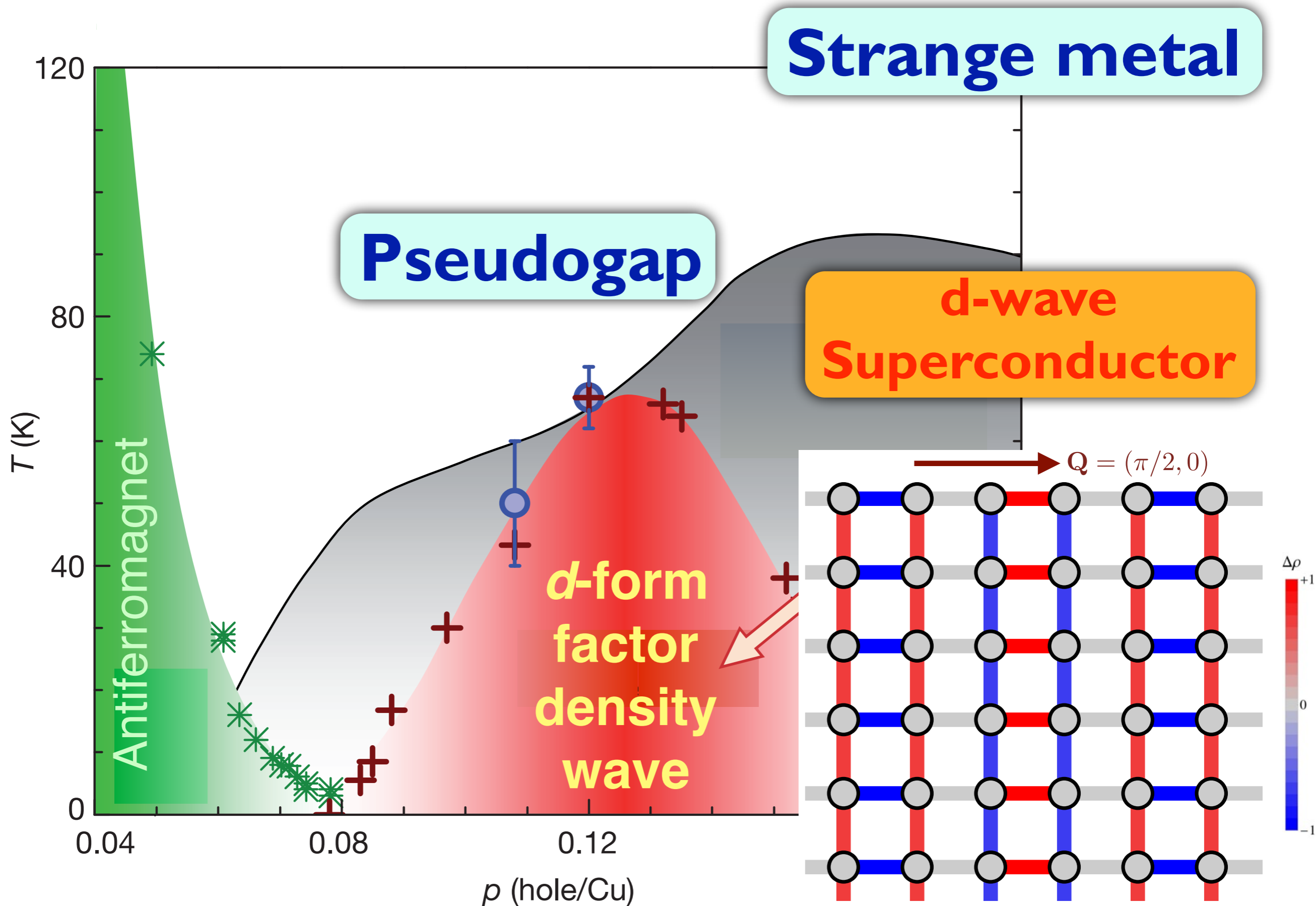


**Strange metal**

**Pseudogap**

**d-wave  
Superconductor**

?



K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS 111, E3026 (2014)

# Outline

## 1. Density waves in the underdoped cuprates

*STM observation of d-form factor density wave*

## 2. RPA theory of density waves

## 3. Fractionalized Fermi liquids (FL\*) in doped square lattice antiferromagnets

*d-form factor density waves with the correct wavevector*

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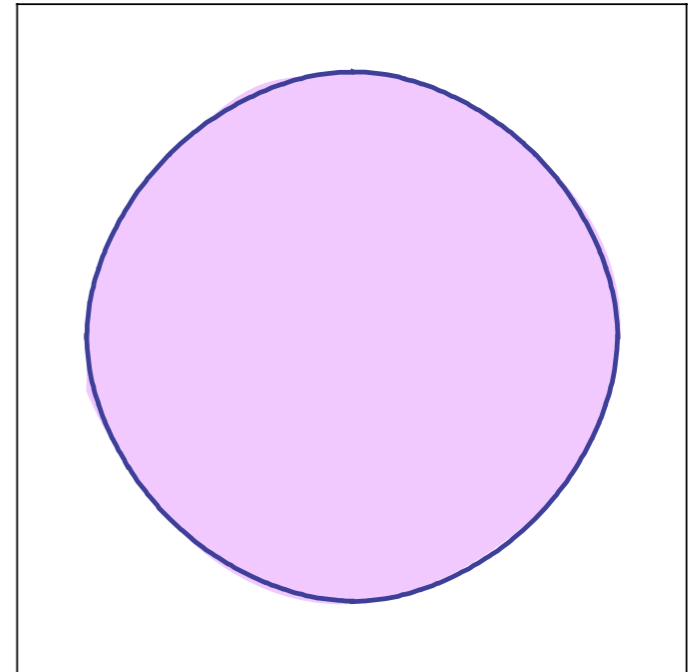
## 2. RPA theory of density waves

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*d-form factor density waves with the correct wavevector*

# Fermi surface+antiferromagnetism

Metal with “large”  
Fermi surface

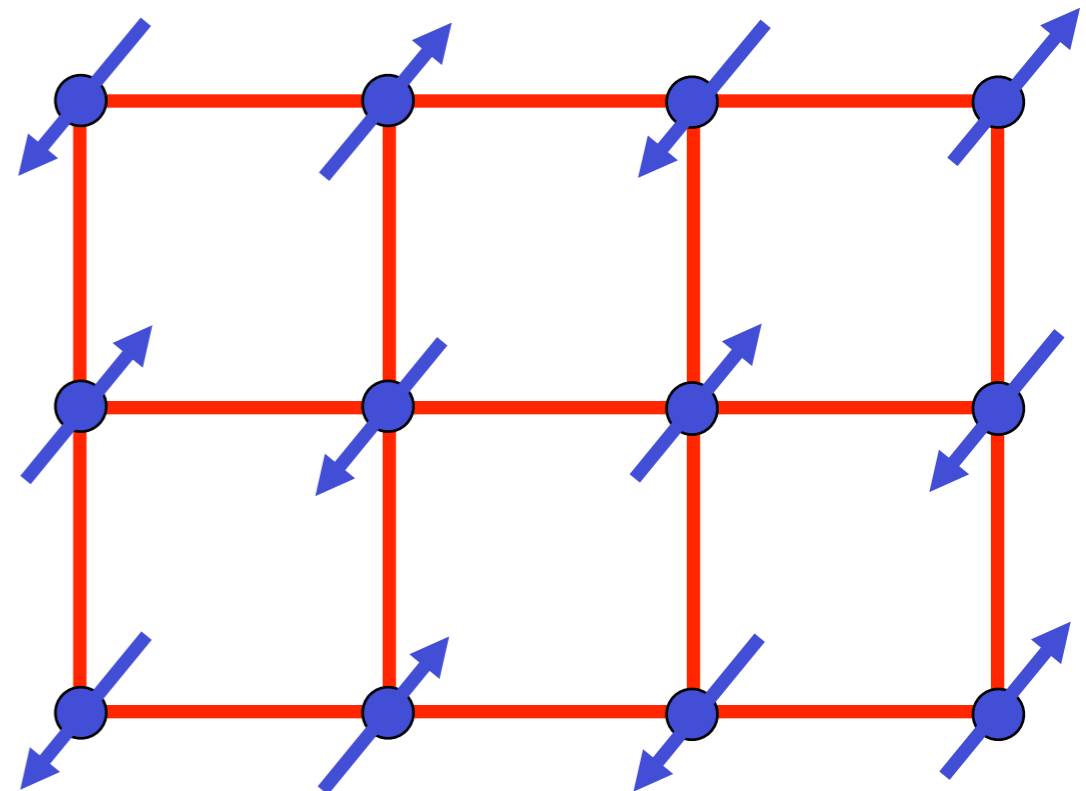


+

The electron spin polarization obeys

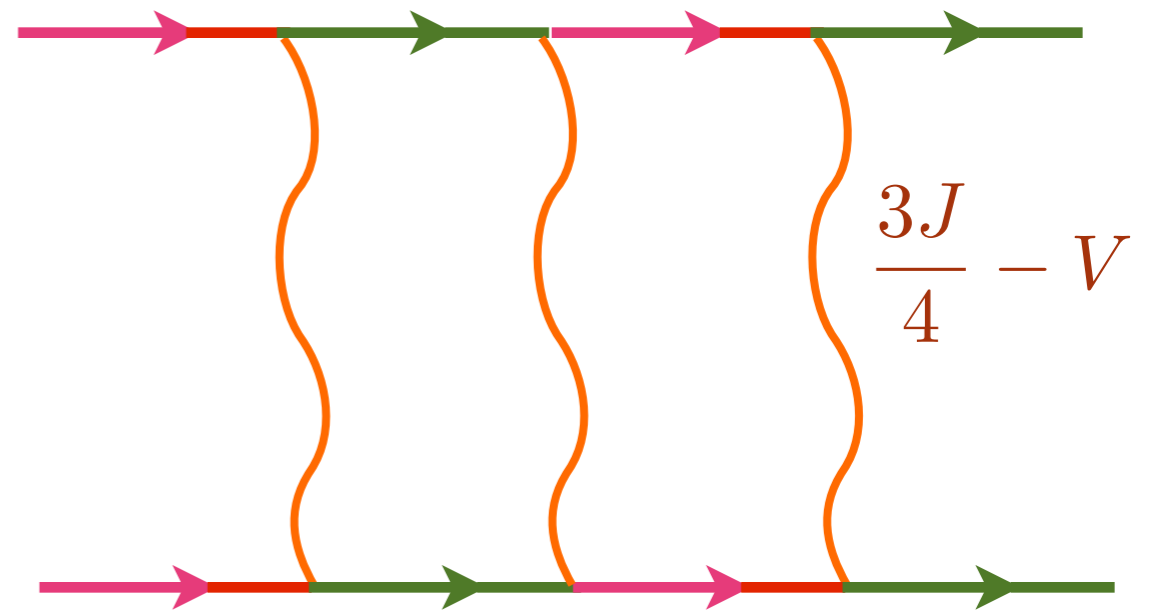
$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

where  $\mathbf{K} = (\pi, \pi)$  is the ordering  
wavevector.

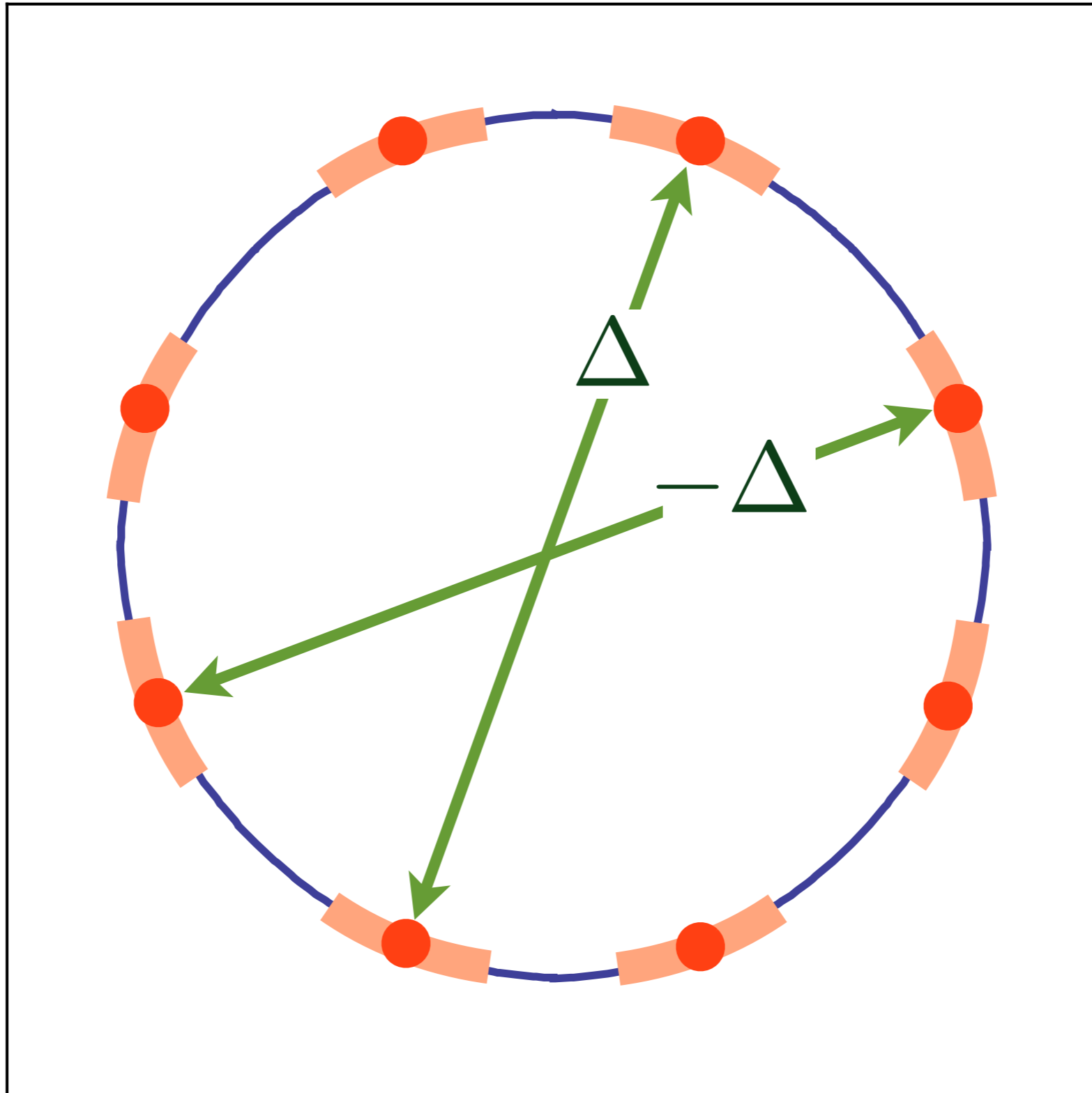


# Pairing “glue” from antiferromagnetic fluctuations

$$H = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + V \sum_{\langle ij \rangle} n_i n_j + \dots$$



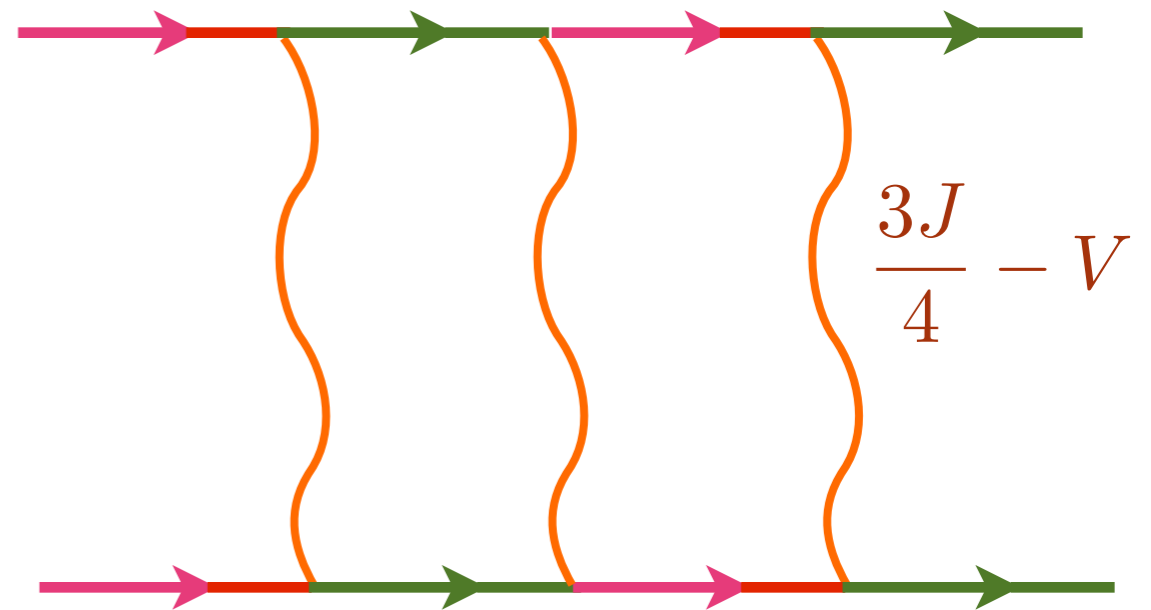
$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



Unconventional pairing at and near hot spots

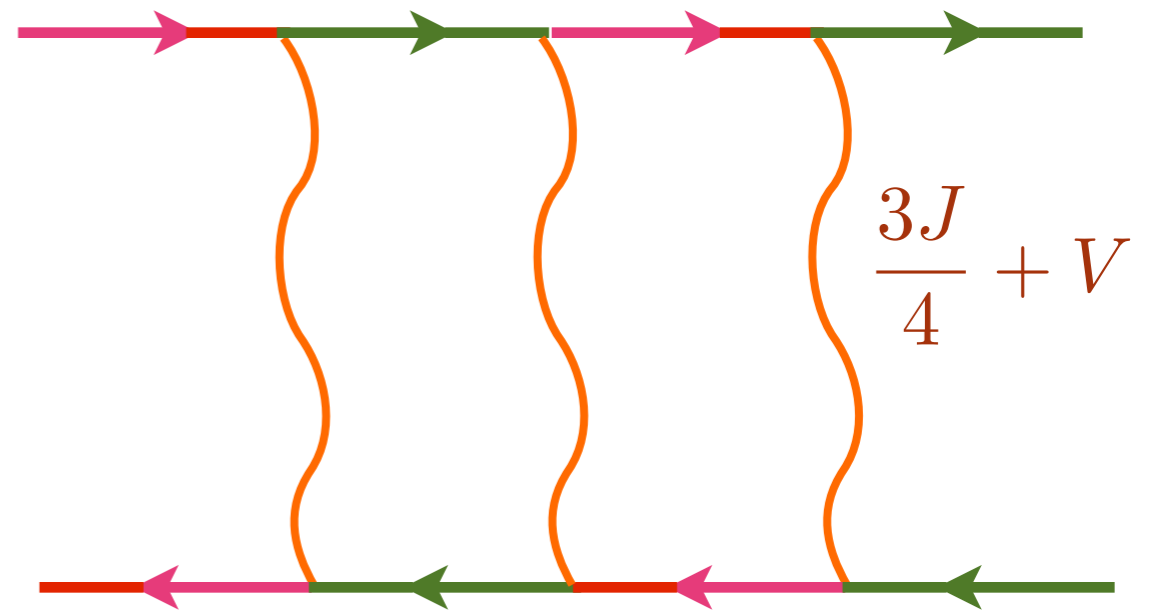
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$$H = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + V \sum_{\langle ij \rangle} n_i n_j + \dots$$

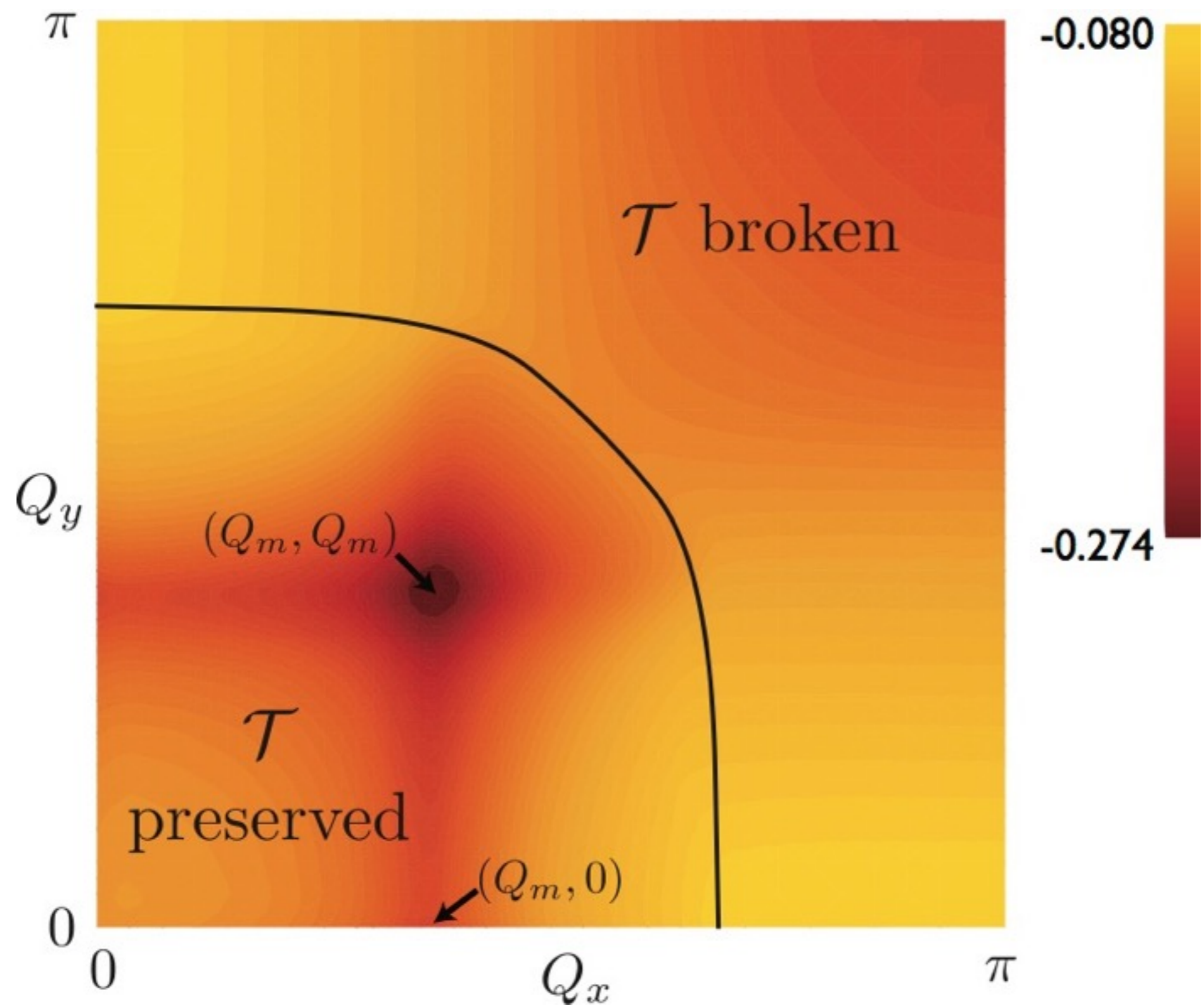


# Same “glue” can lead to particle-hole pairing

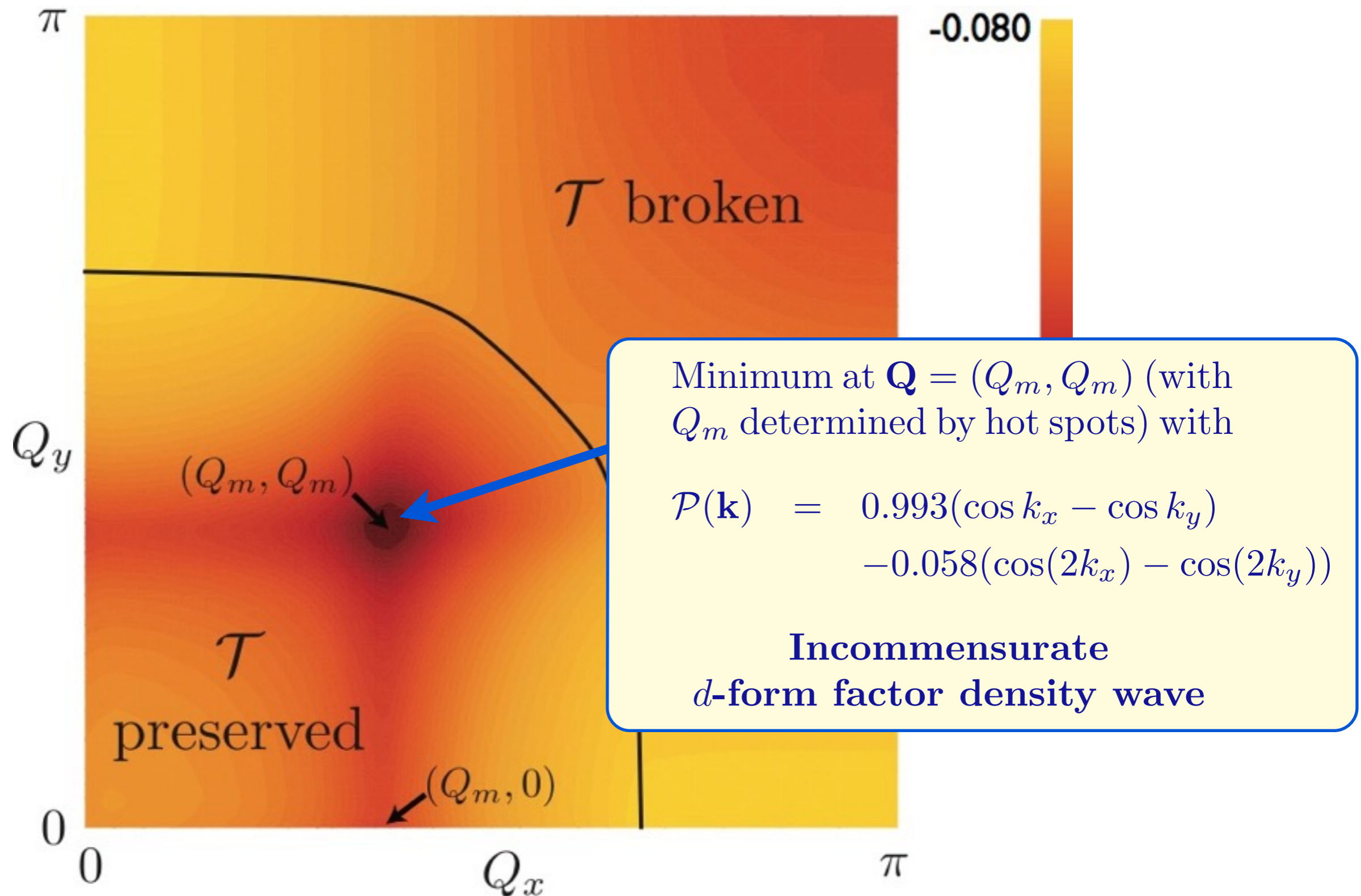
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M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)  
S. Sachdev and R. La Placa, *Phys. Rev. Lett.* **111**, 027202 (2013)



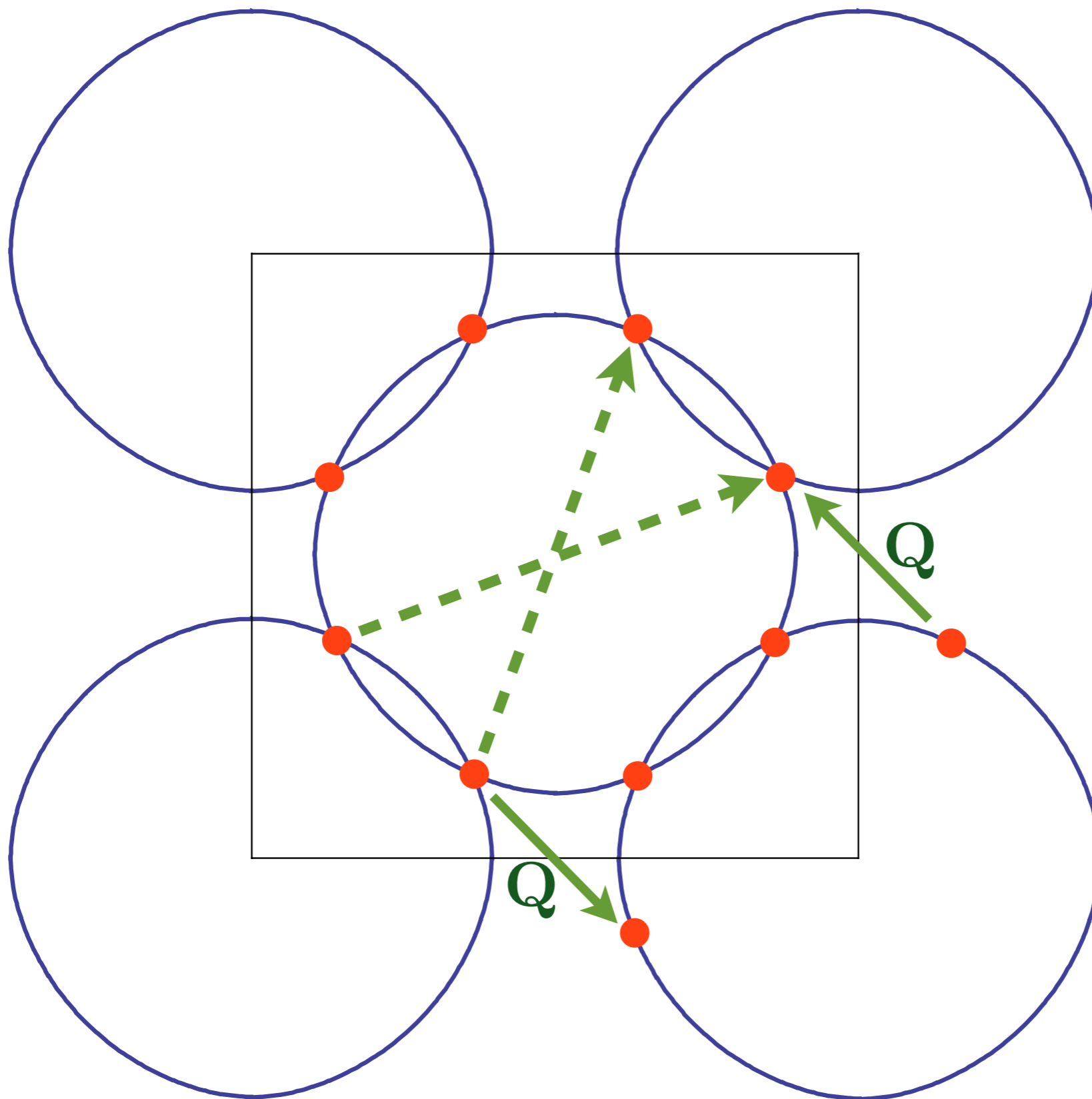
Eigenvalues,  $\lambda(\mathbf{Q})$ , of the spin-singlet, particle-hole propagator. The corresponding eigenvector is  $\mathcal{P}(\mathbf{k})$  and this leads to the order  $\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = [\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$



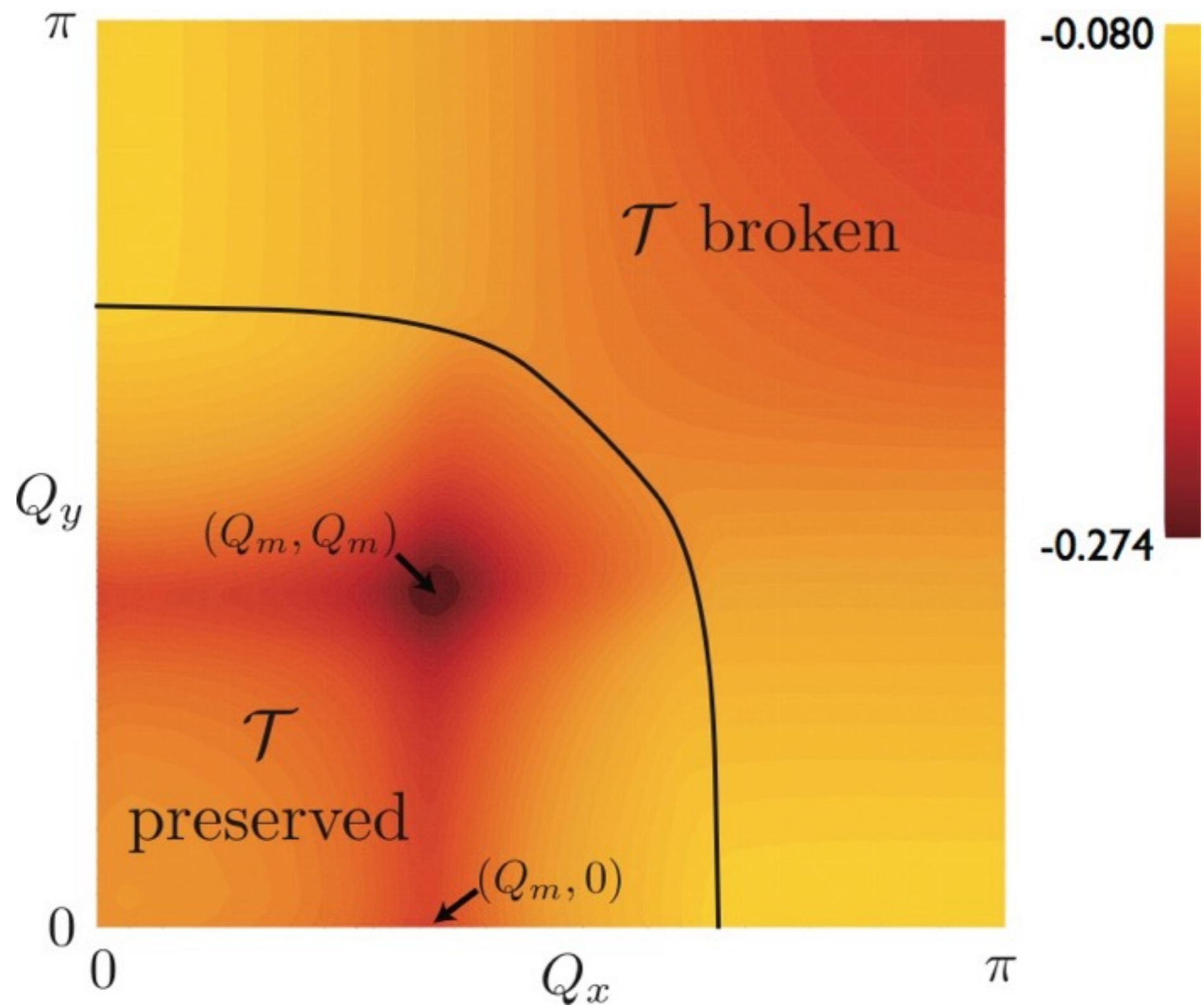
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$$\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2}$$

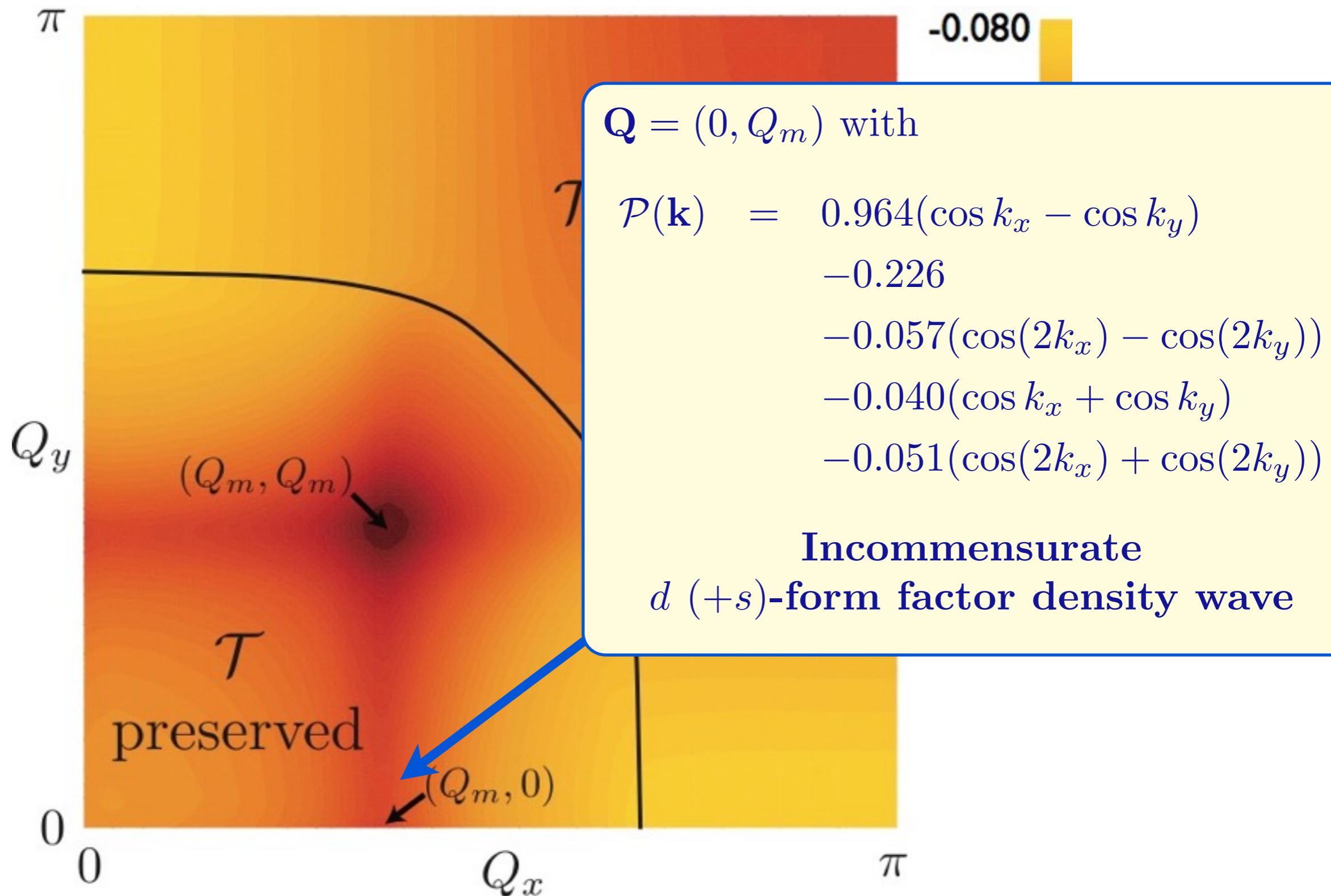
# $d$ -form factor density wave



$$\left\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \mathcal{P}_d(\cos k_x - \cos k_y)$$

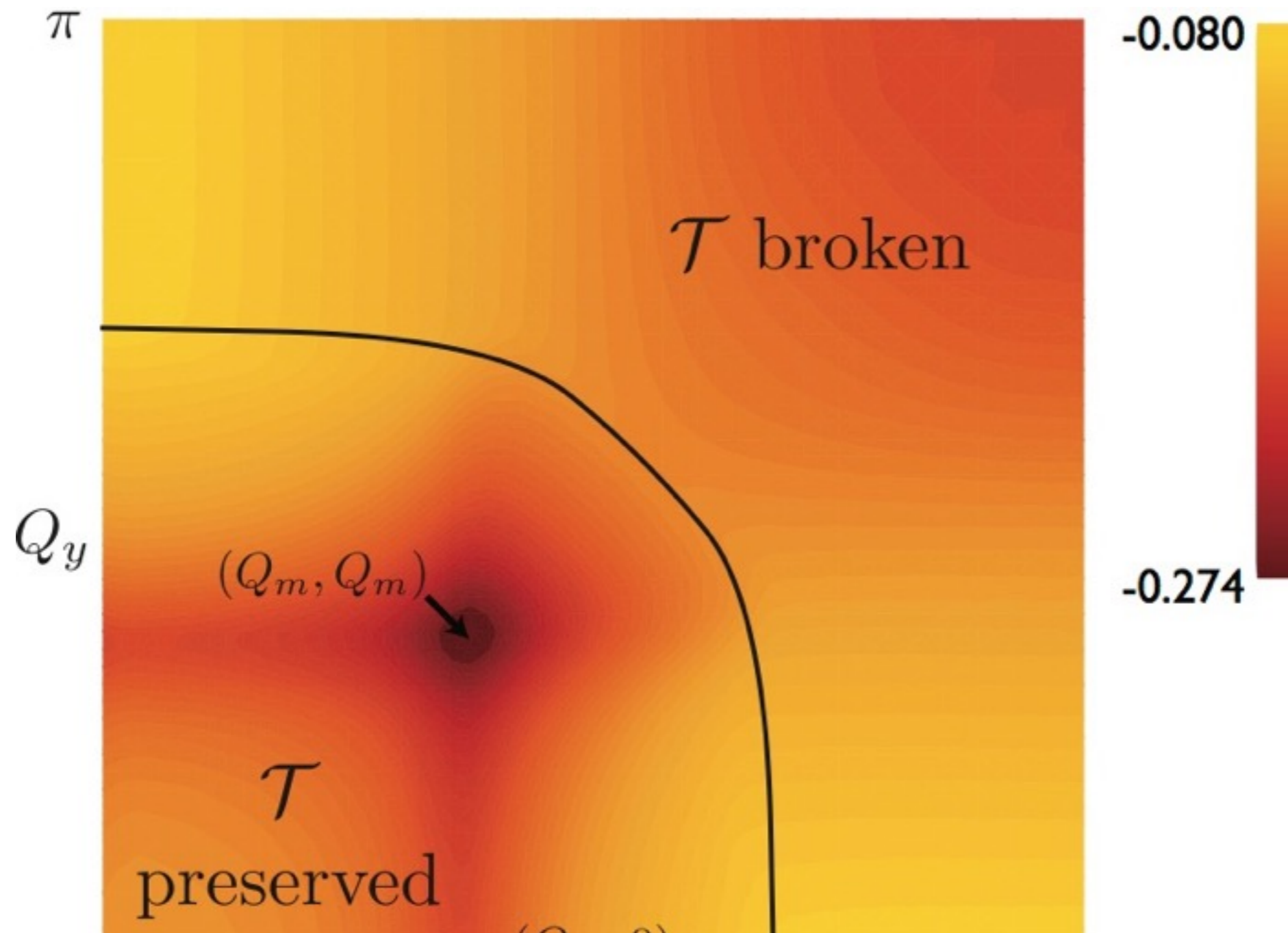


Eigenvalues,  $\lambda(\mathbf{Q})$ , of the spin-singlet, particle-hole propagator. The corresponding eigenvector is  $\mathcal{P}(\mathbf{k})$  and this leads to the order  $\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = [\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$

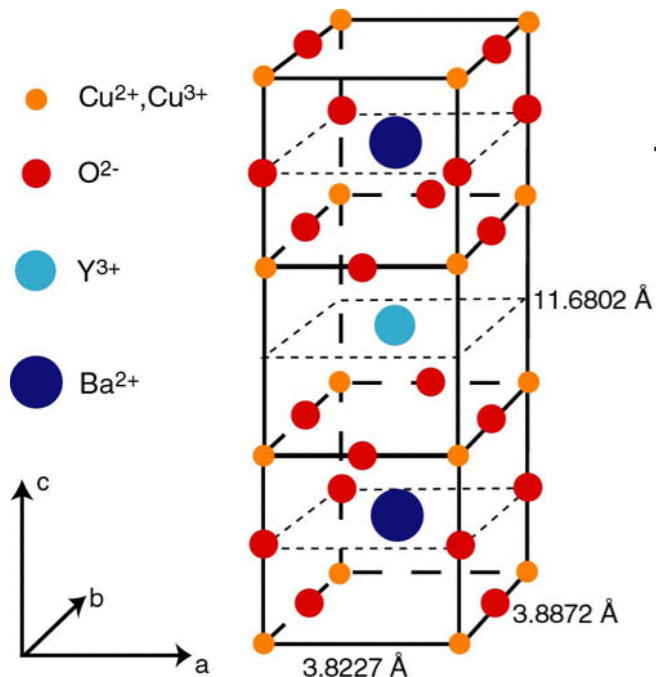


Eigenvalues,  $\lambda(\mathbf{Q})$ , of the spin-singlet, particle-hole propagator. The corresponding eigenvector is  $\mathcal{P}(\mathbf{k})$  and this leads to the order

$$\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2}$$



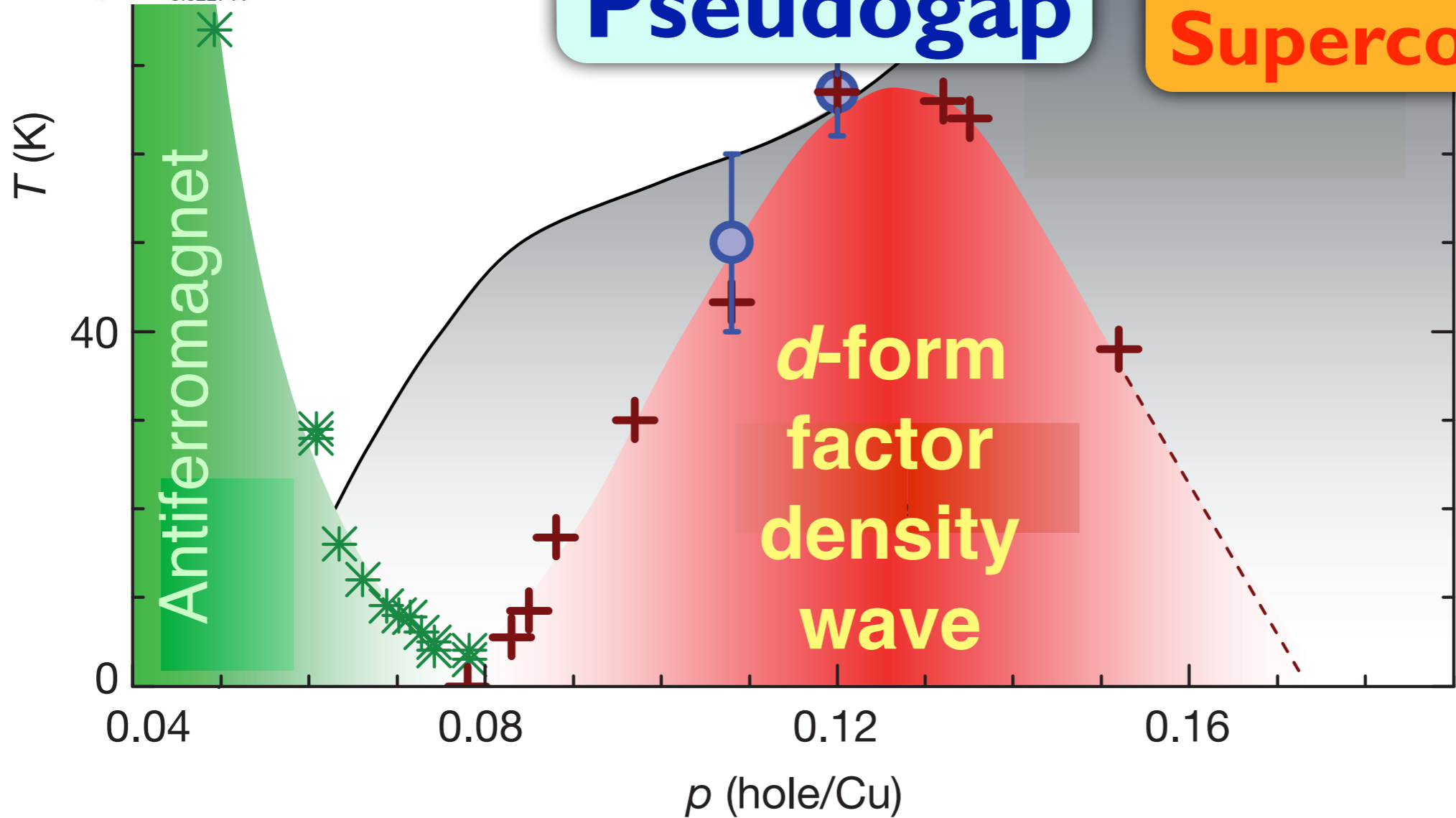
This theory yields the correct form factor, but the incorrect  $\mathbf{Q}$ . We have found the same features in all theories (one- and three-band) with a large Fermi surface

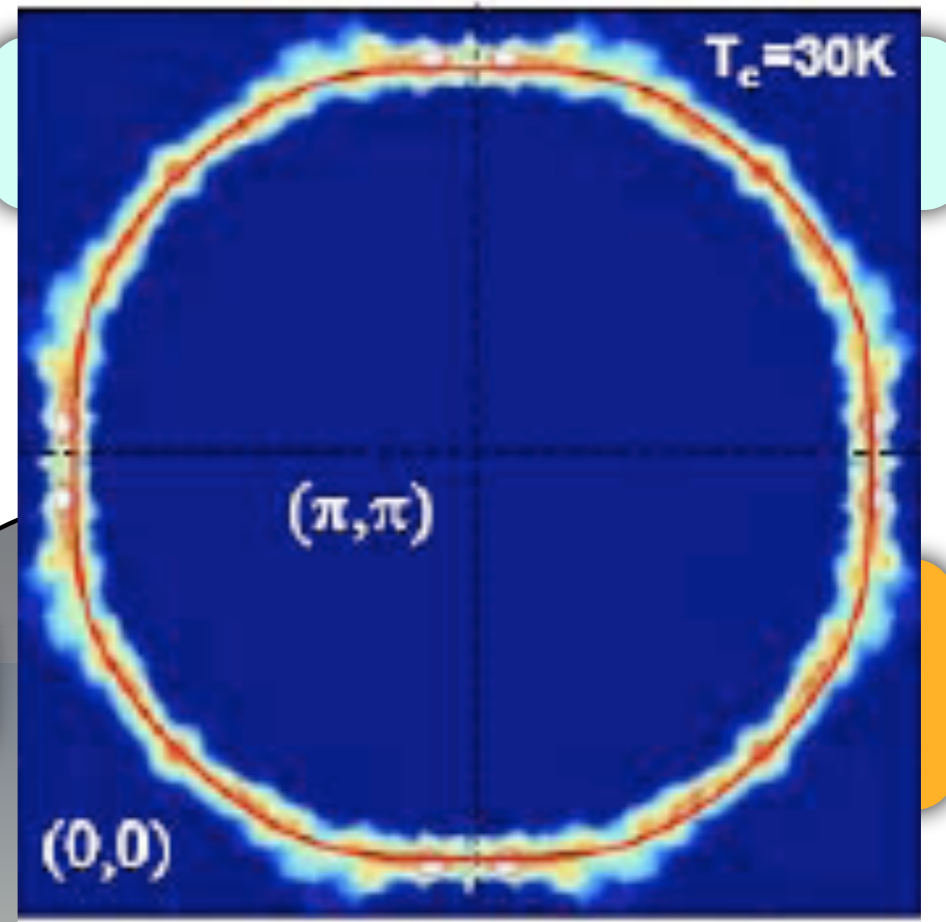
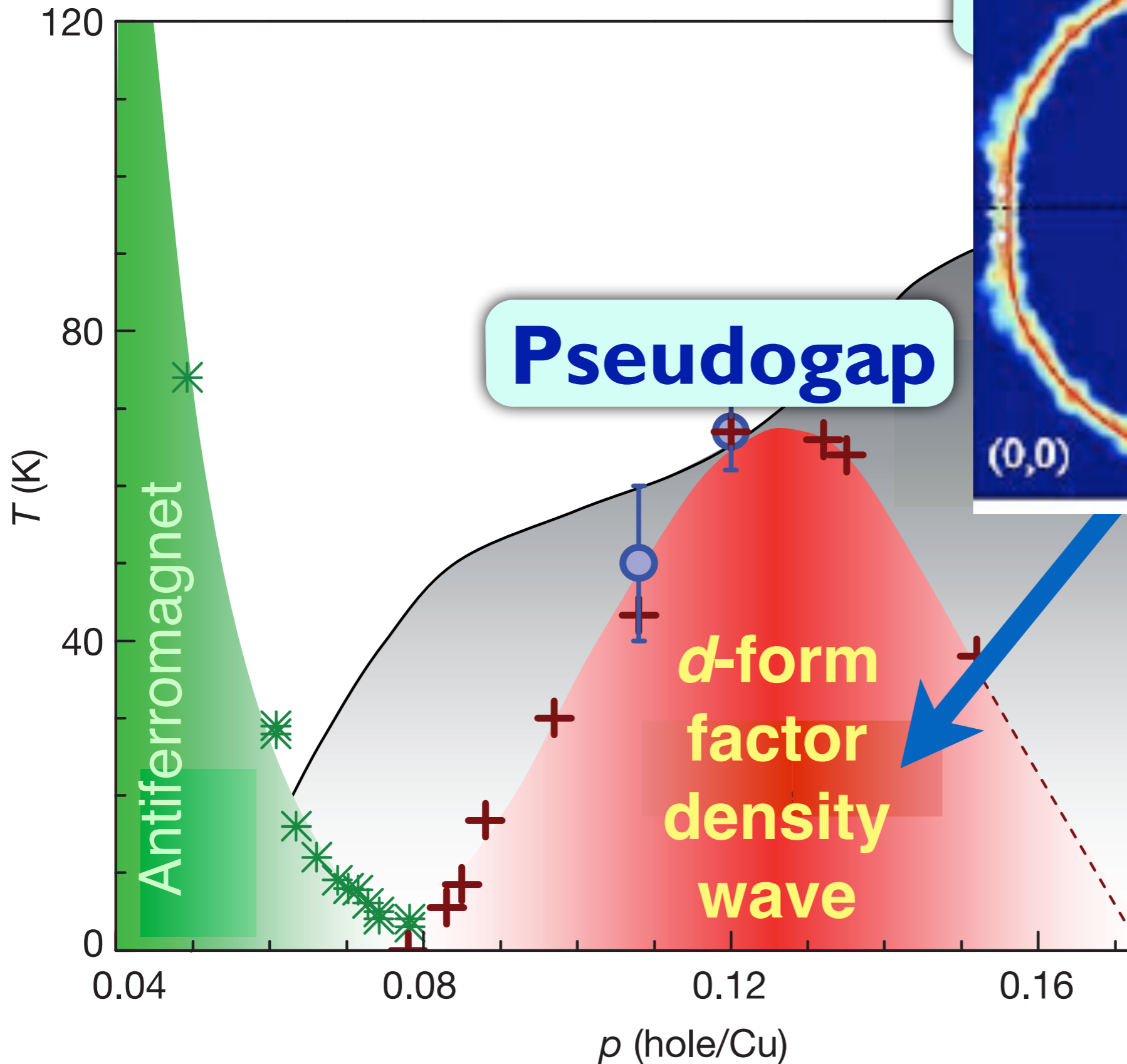


**Strange metal**

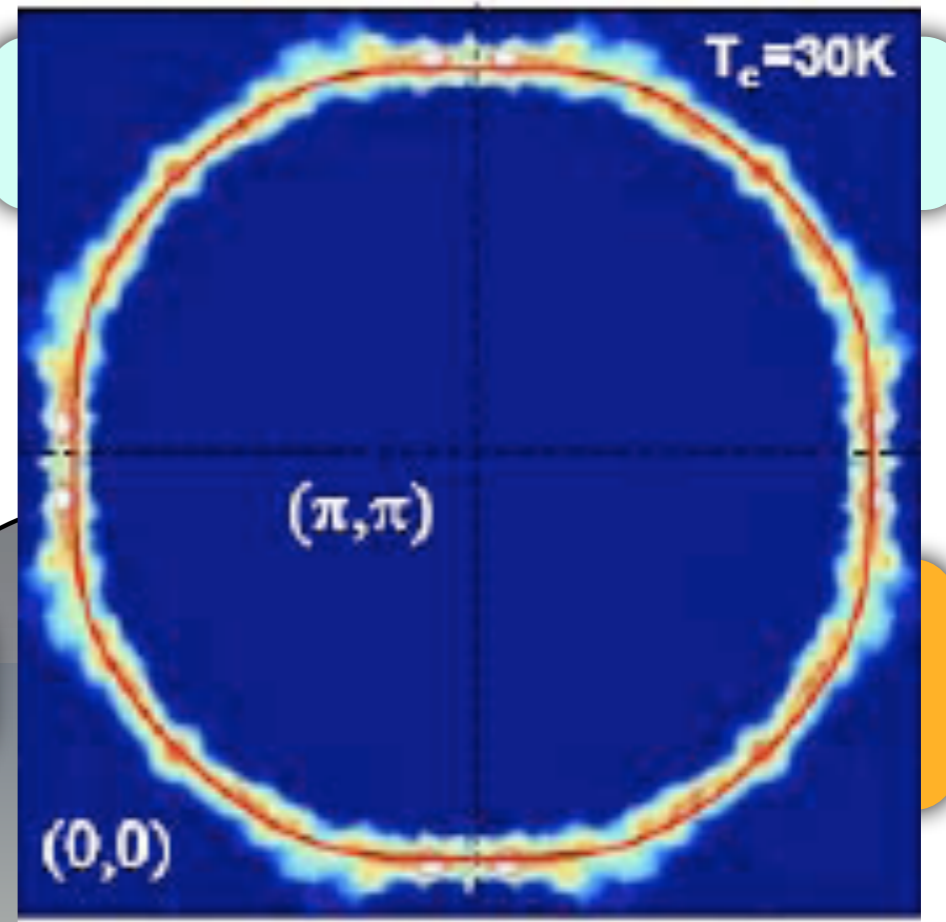
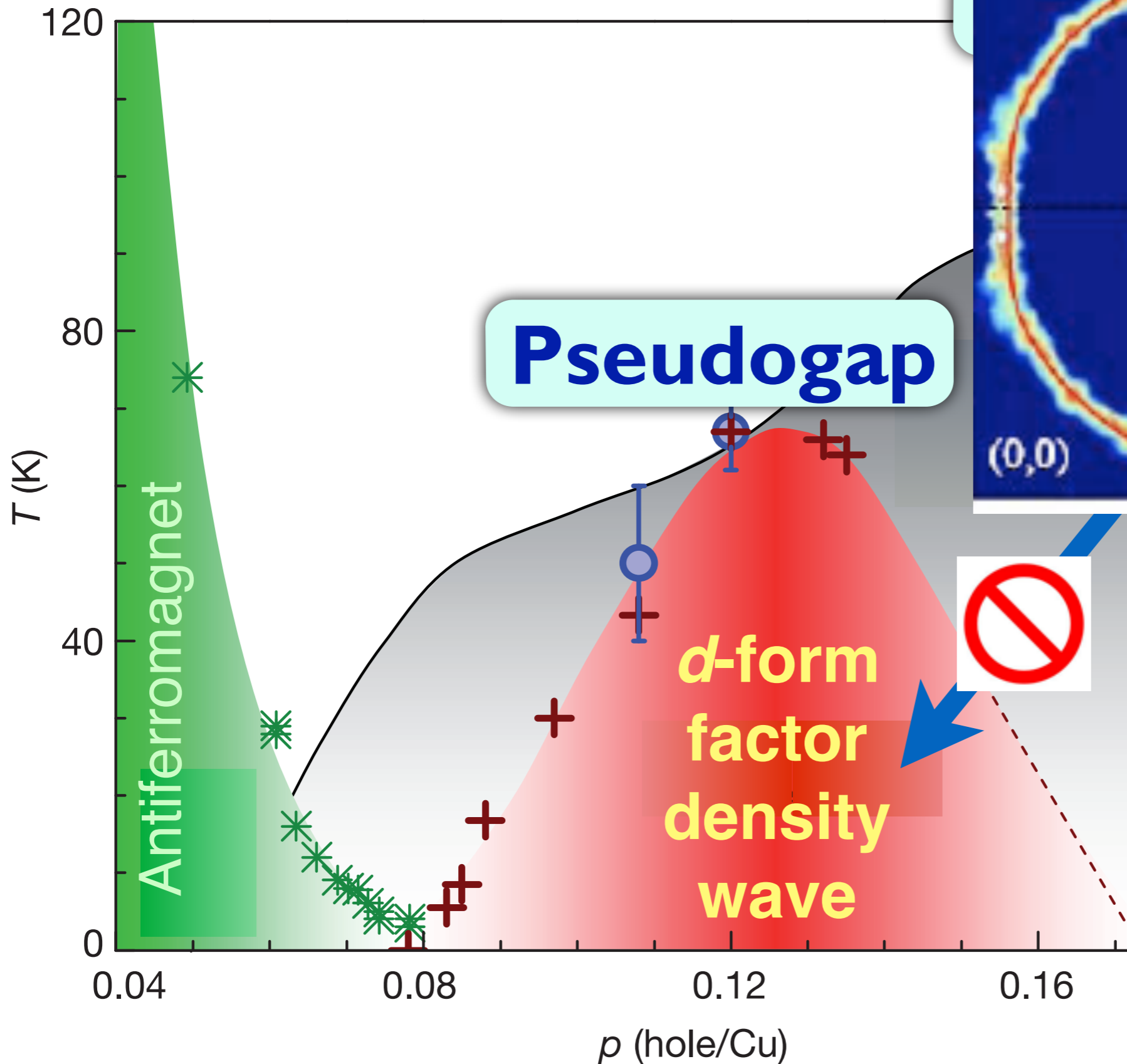
**Pseudogap**

**d-wave Superconductor**

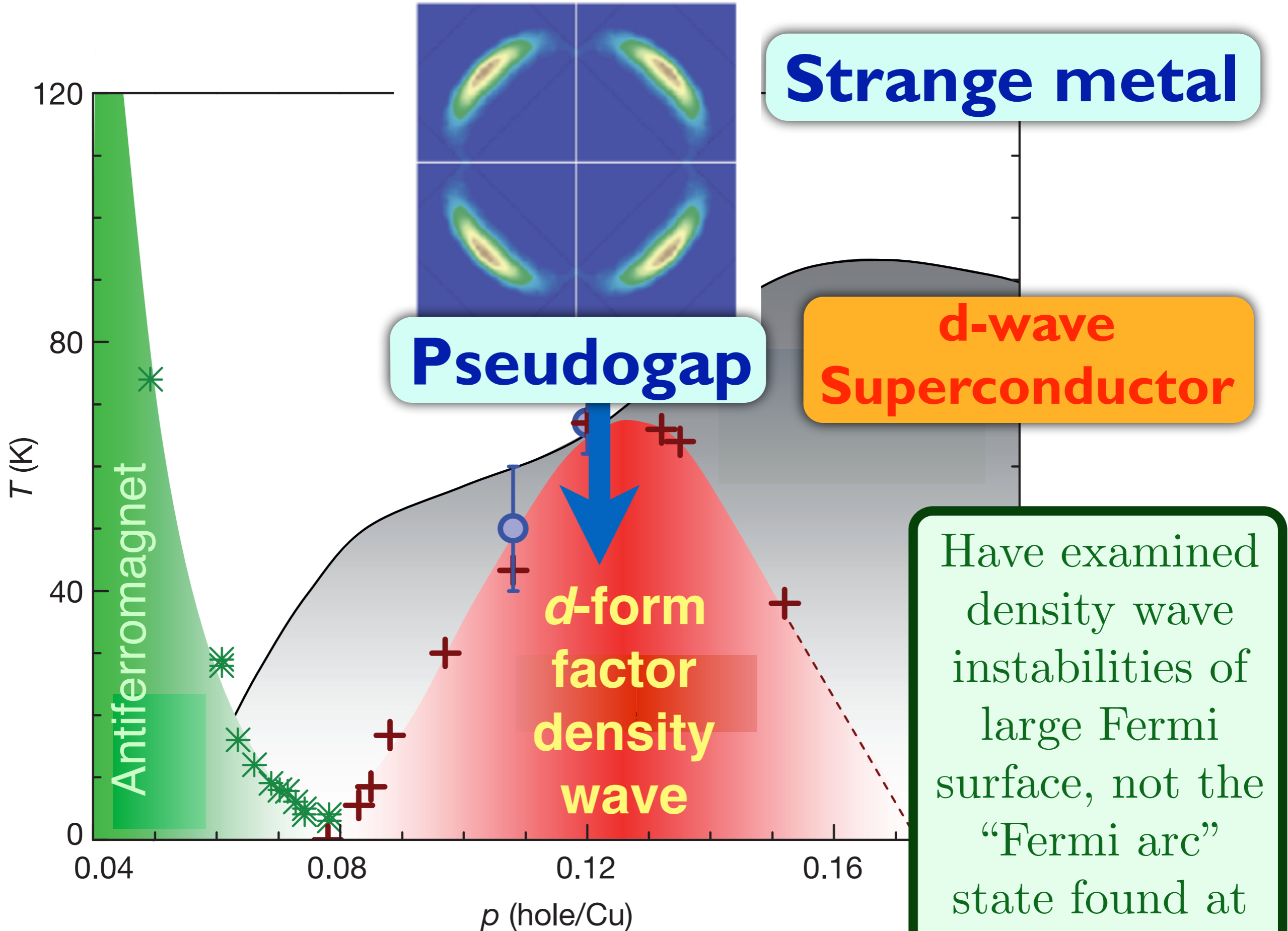




Have examined density wave instabilities of large Fermi surface, not the “Fermi arc” state found at low doping.



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# Outline

## 1. Density waves in the underdoped cuprates

*STM observation of d-form factor density wave*

## 2. RPA theory of density waves

## 3. Fractionalized Fermi liquids (FL\*) in doped square lattice antiferromagnets

*d-form factor density waves with the correct wavevector*

# Outline

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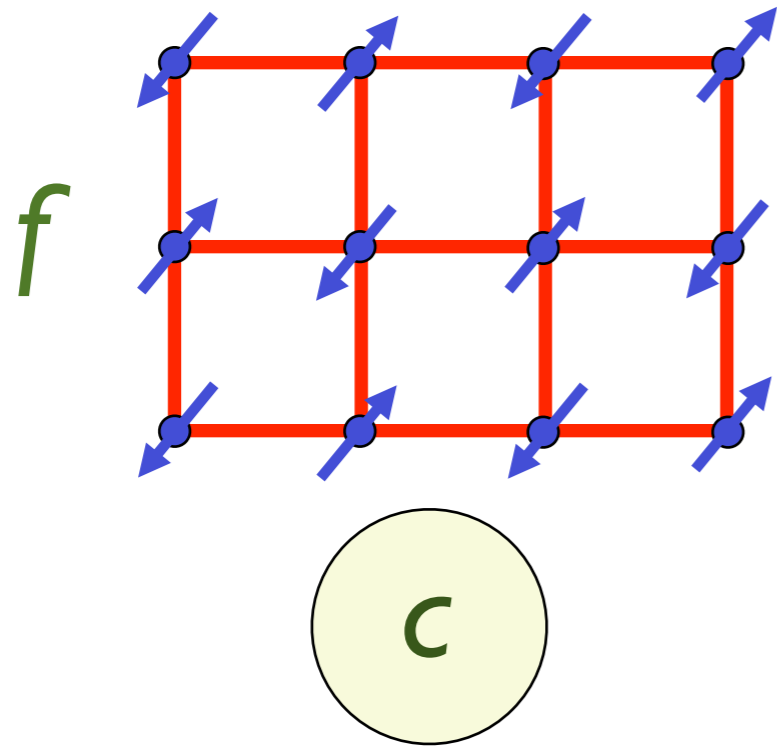
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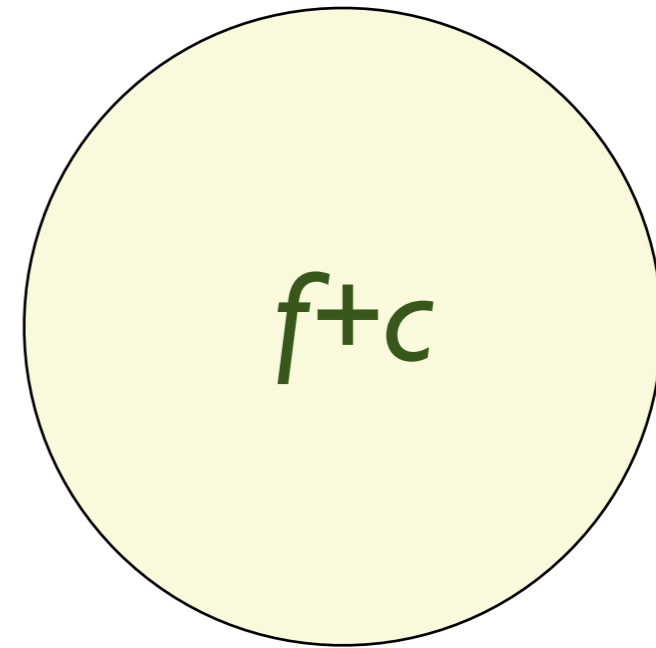
*d-form factor density waves with the correct wavevector*

# Magnetic order and the heavy Fermi liquid in the Kondo lattice



$$\langle \vec{\varphi} \rangle \neq 0$$

Magnetic Metal:  
f-electron moments  
and  
c-conduction electron  
Fermi surface

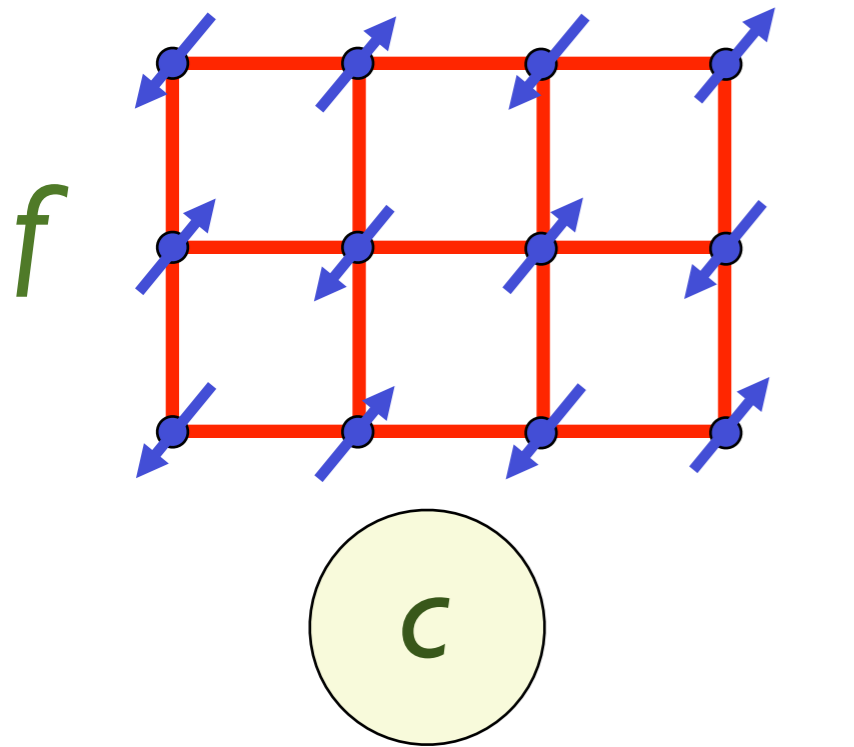


$$\langle \vec{\varphi} \rangle = 0$$

Heavy Fermi liquid  
with “large” Fermi  
surface of  
hybridized f and  
c-conduction  
electrons

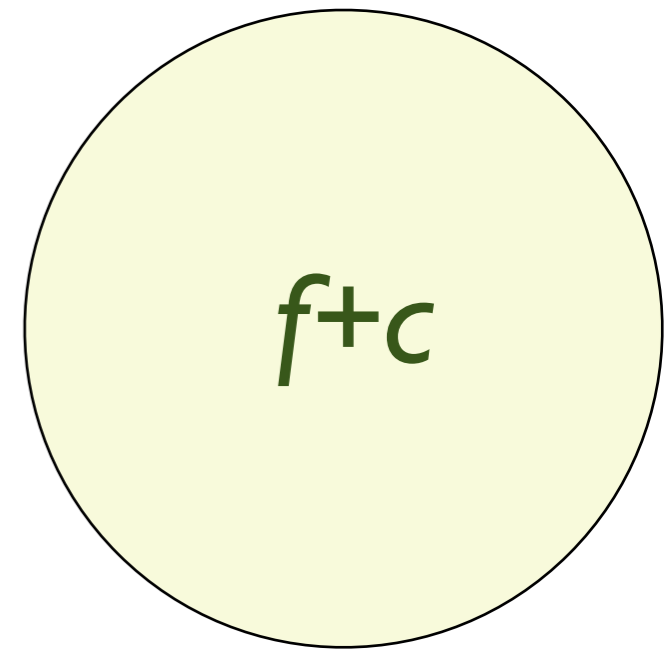


# Separating onset of SDW order and the heavy Fermi liquid in the Kondo lattice



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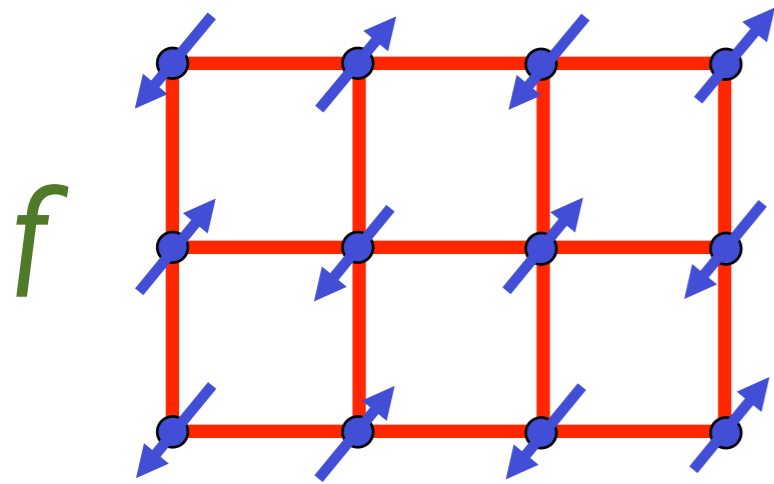


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Heavy Fermi liquid  
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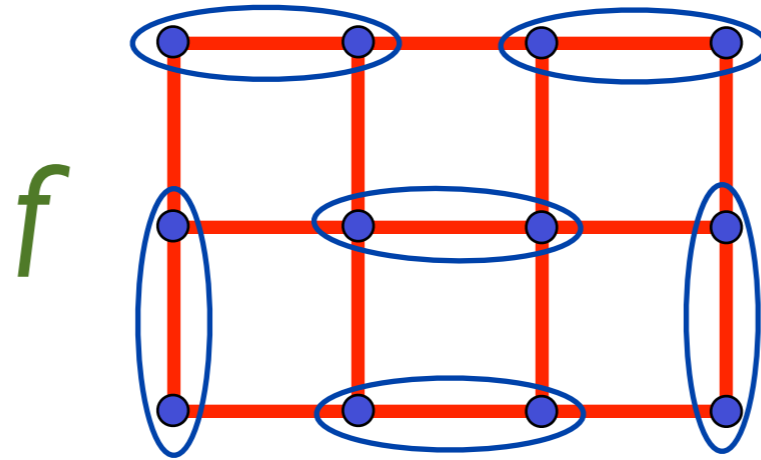


# Separating onset of SDW order and the heavy Fermi liquid in the Kondo lattice



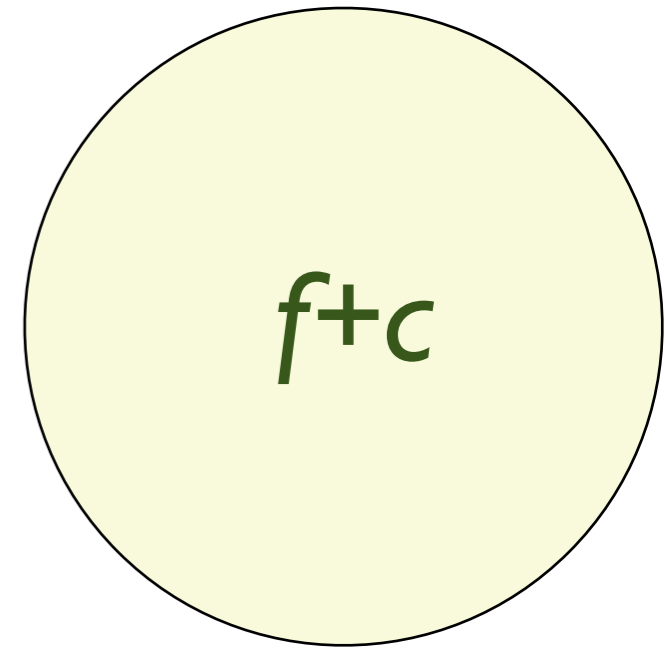
$$\langle \vec{\varphi} \rangle \neq 0$$

Magnetic Metal:  
f-electron moments  
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Fermi surface



$$\langle \vec{\varphi} \rangle = 0$$

Conduction electron  
Fermi surface  
and  
spin-liquid of  
f-electrons

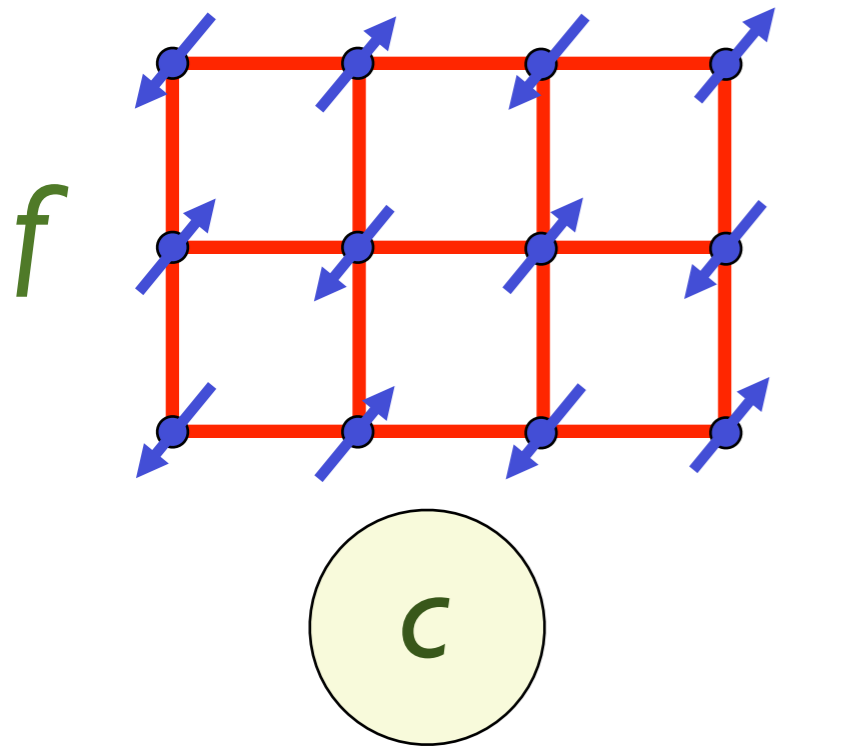


$$\langle \vec{\varphi} \rangle = 0$$

Heavy Fermi liquid  
with "large" Fermi  
surface of  
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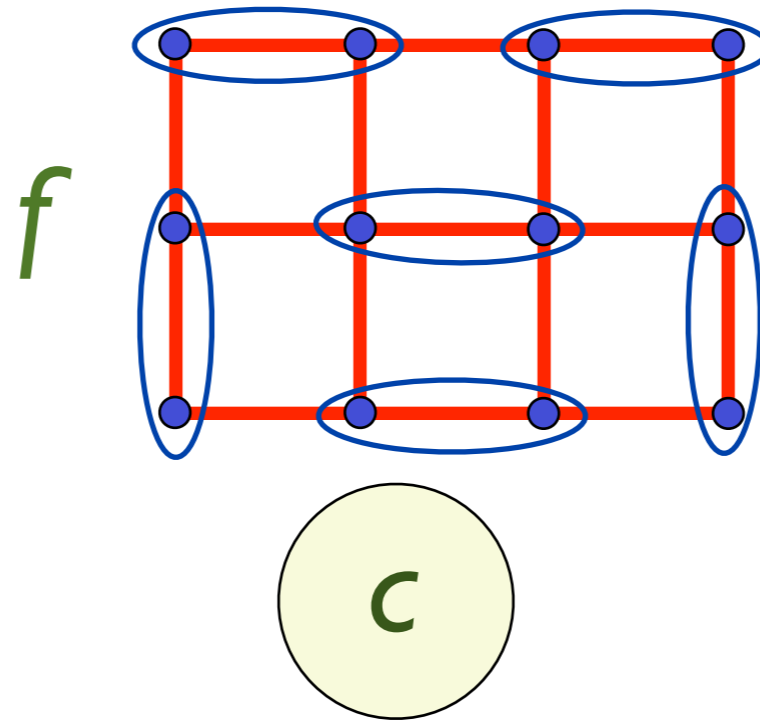


# Separating onset of SDW order and the heavy Fermi liquid in the Kondo lattice



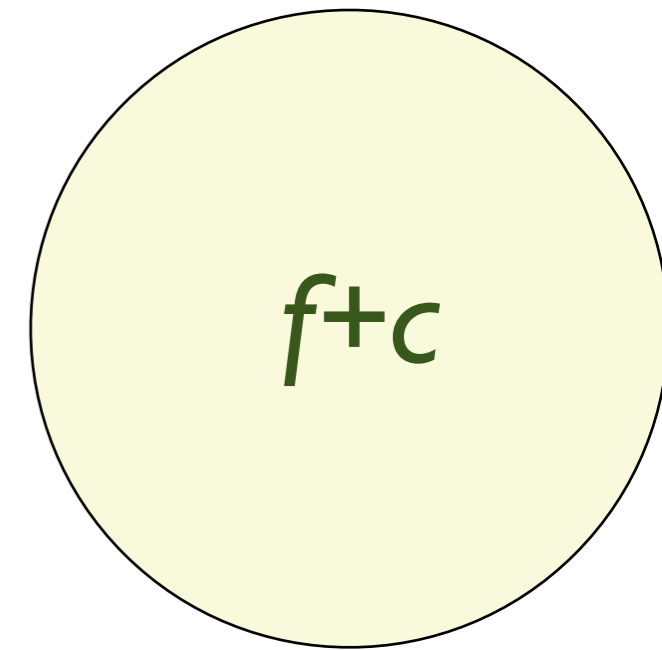
$$\langle \vec{\varphi} \rangle \neq 0$$

Magnetic Metal:  
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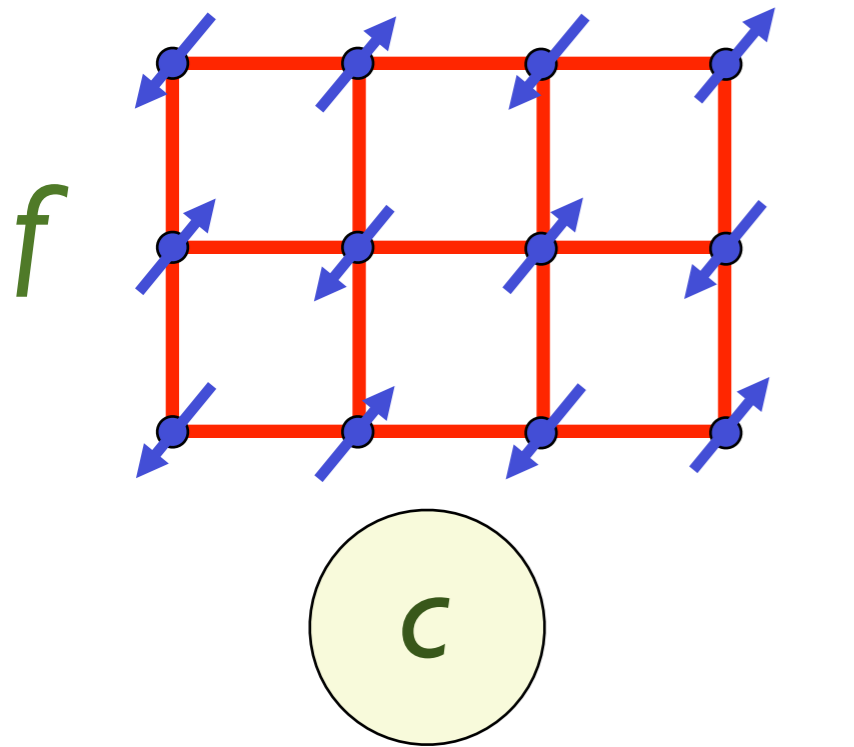
Fractionalized Fermi  
liquid (FL\*) phase  
with no symmetry  
breaking and “small”  
Fermi surface



$$\langle \vec{\varphi} \rangle = 0$$

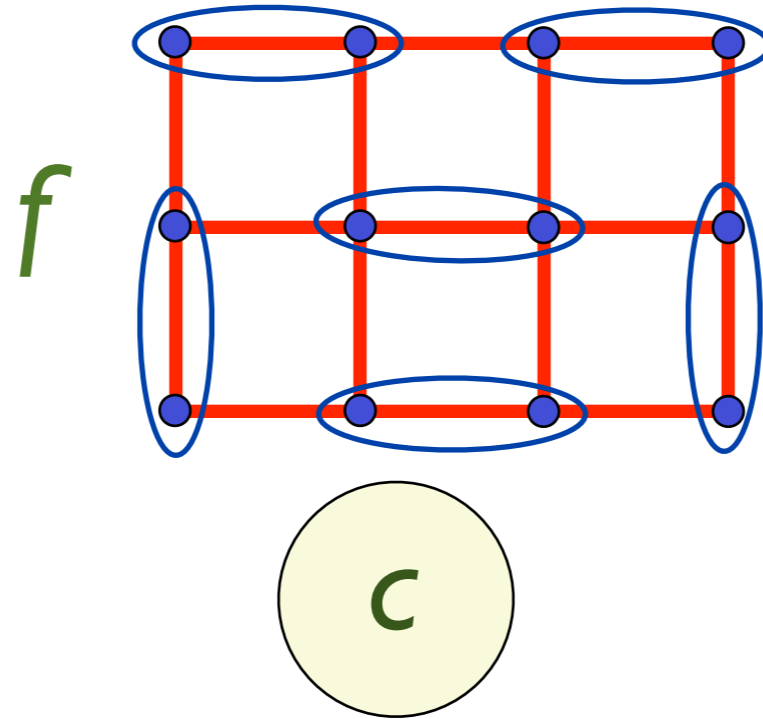
Heavy Fermi liquid  
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# Separating onset of SDW order and the heavy Fermi liquid in the Kondo lattice

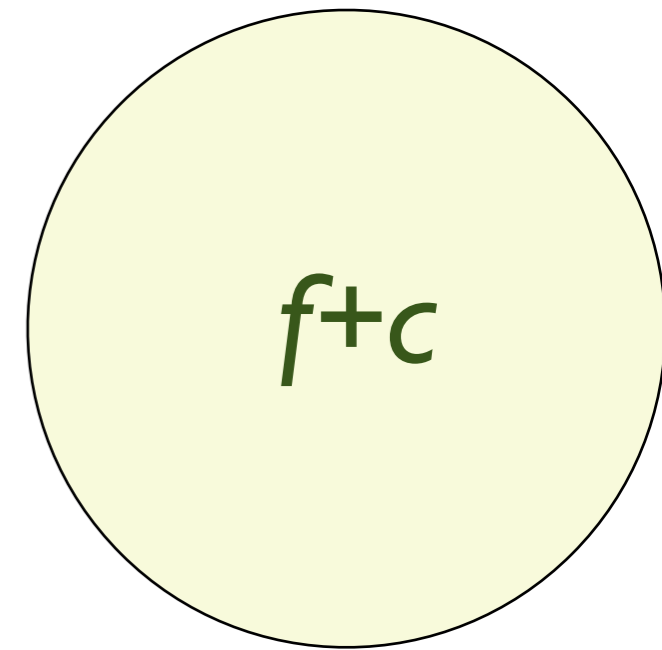


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Magnetic Metal:  
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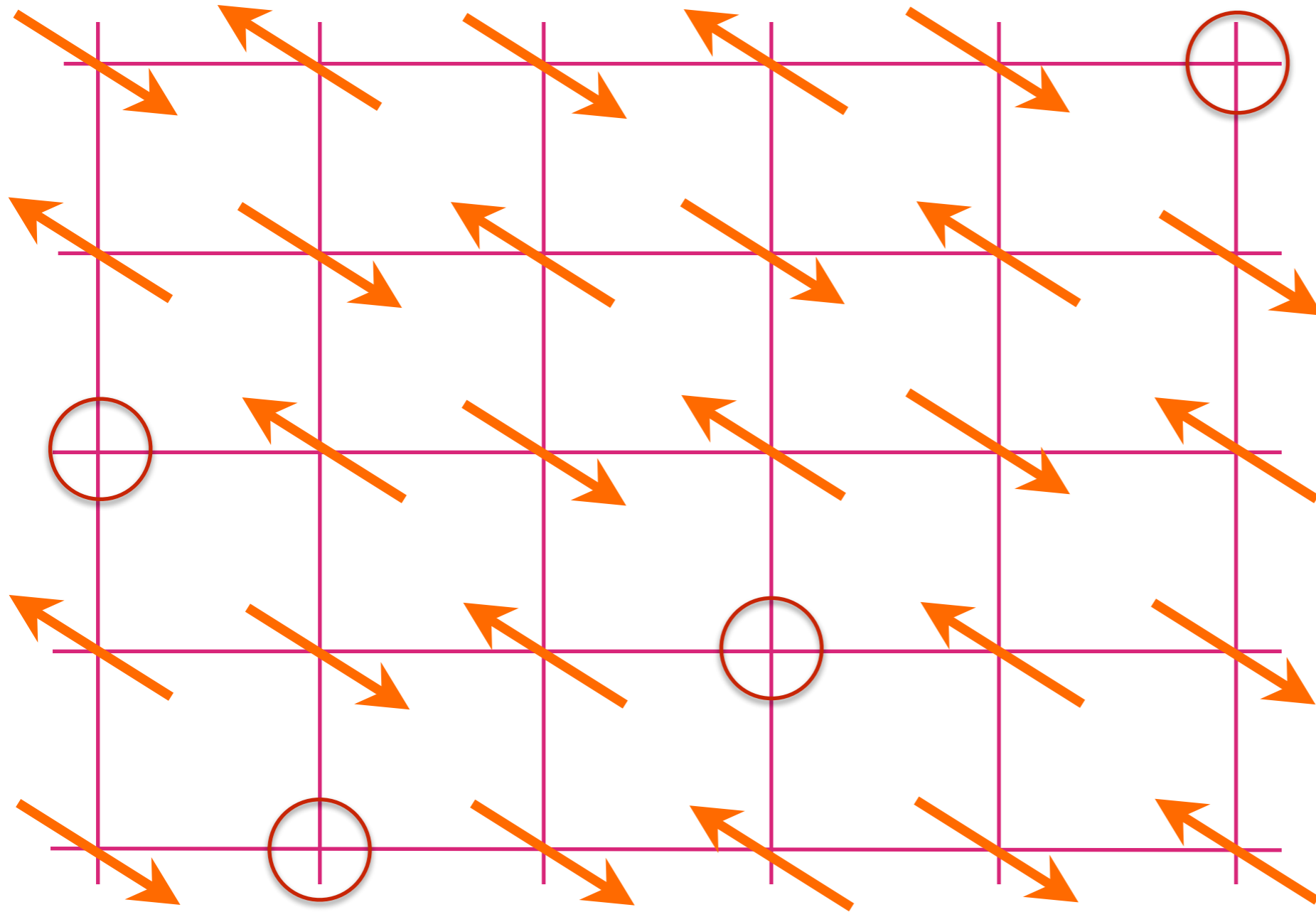
$$\langle \vec{\varphi} \rangle = 0$$

A “topological”  
phase transition  
with no  
conventional  
order parameter

## Characteristics of FL\* phase

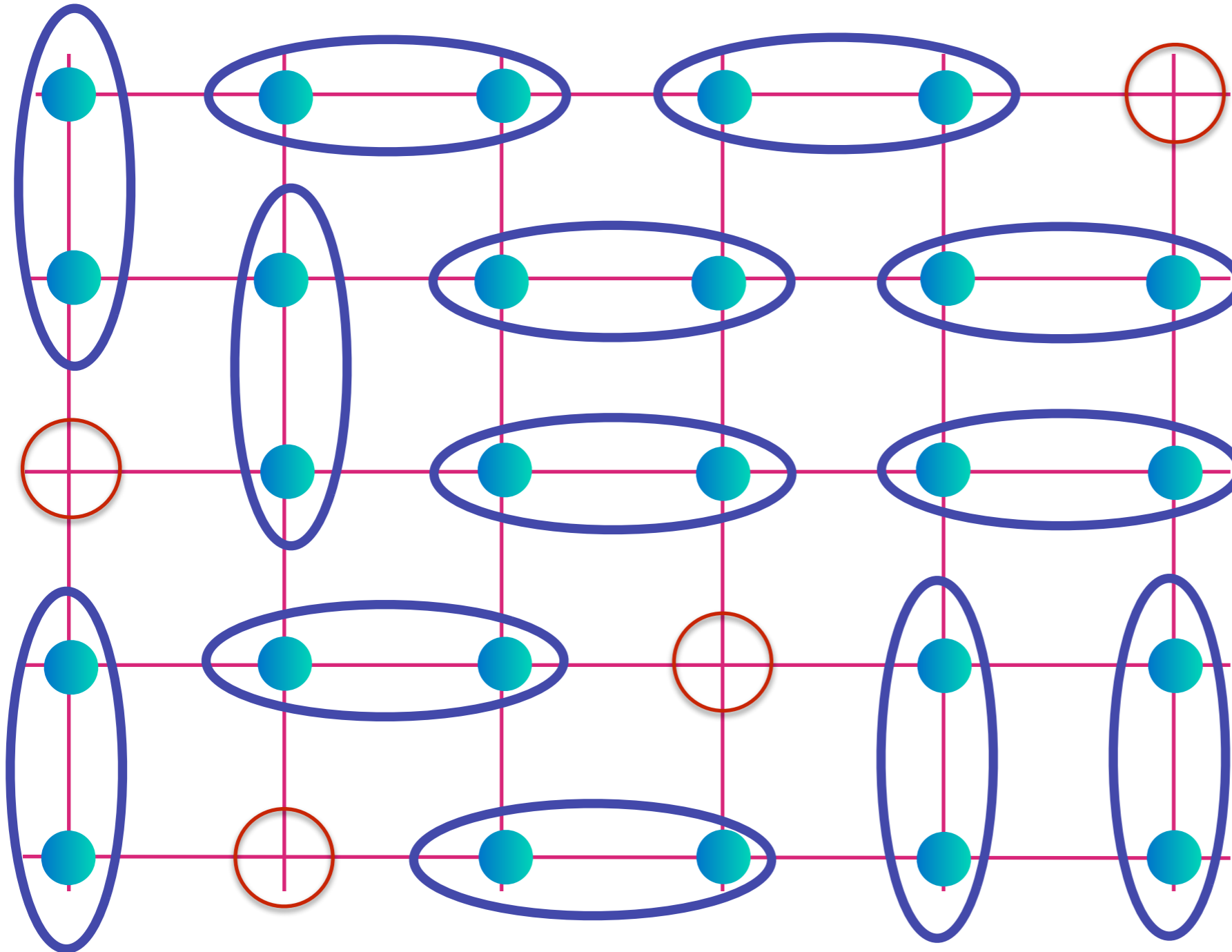
- Fermi surface volume does not count all electrons.
- Such a phase *must* have low energy collective gauge excitations (“topological” order).
- These low energy gauge excitations are needed to account for the deficit in the Fermi surface volume, in M. Oshikawa’s proof of the Luttinger theorem.

# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)



Doped  
anti-  
ferromagnet

# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)



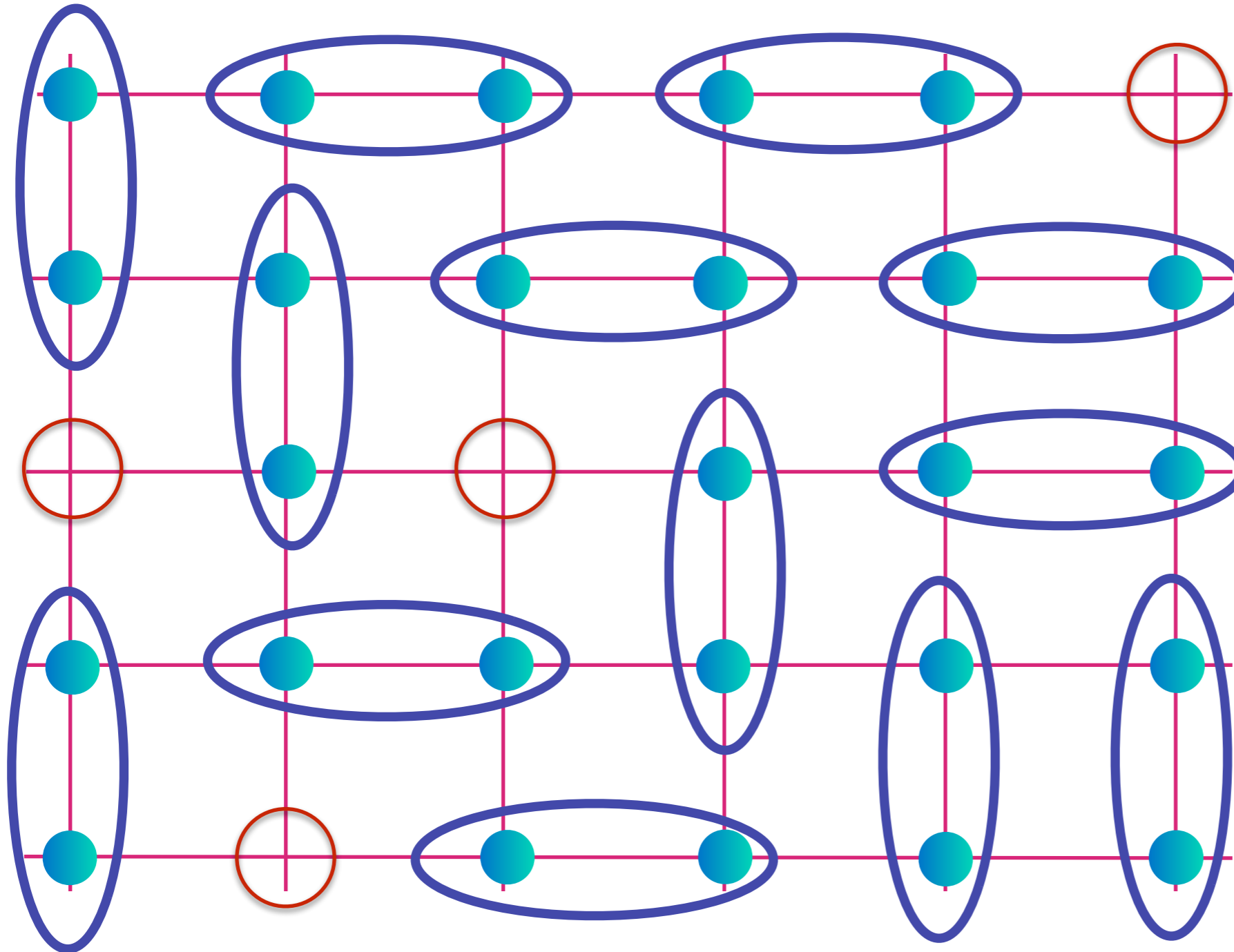
Spin  
liquid

Spinless  
charge  $+e$   
holons

$$\text{[Blue oval with two teal dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)



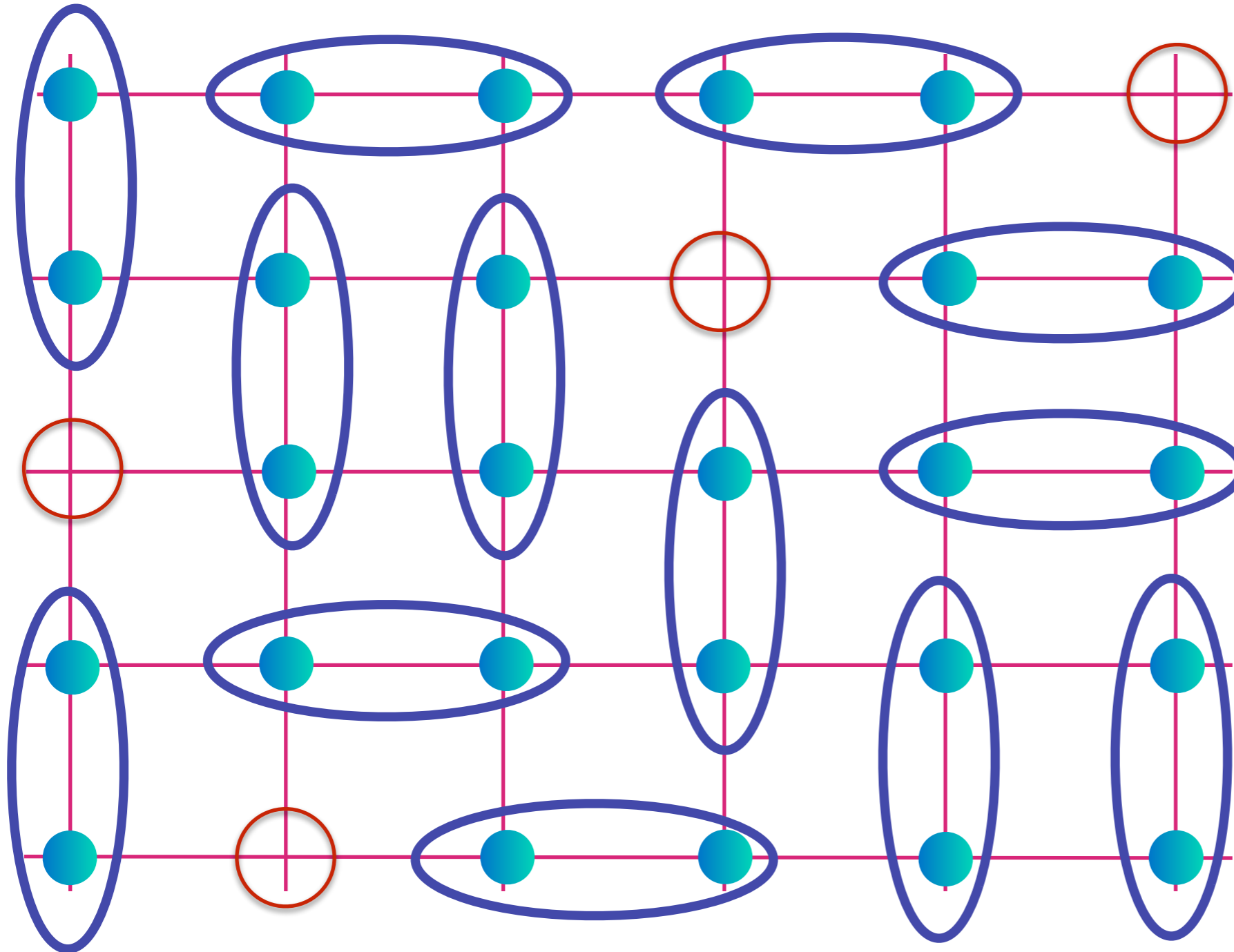
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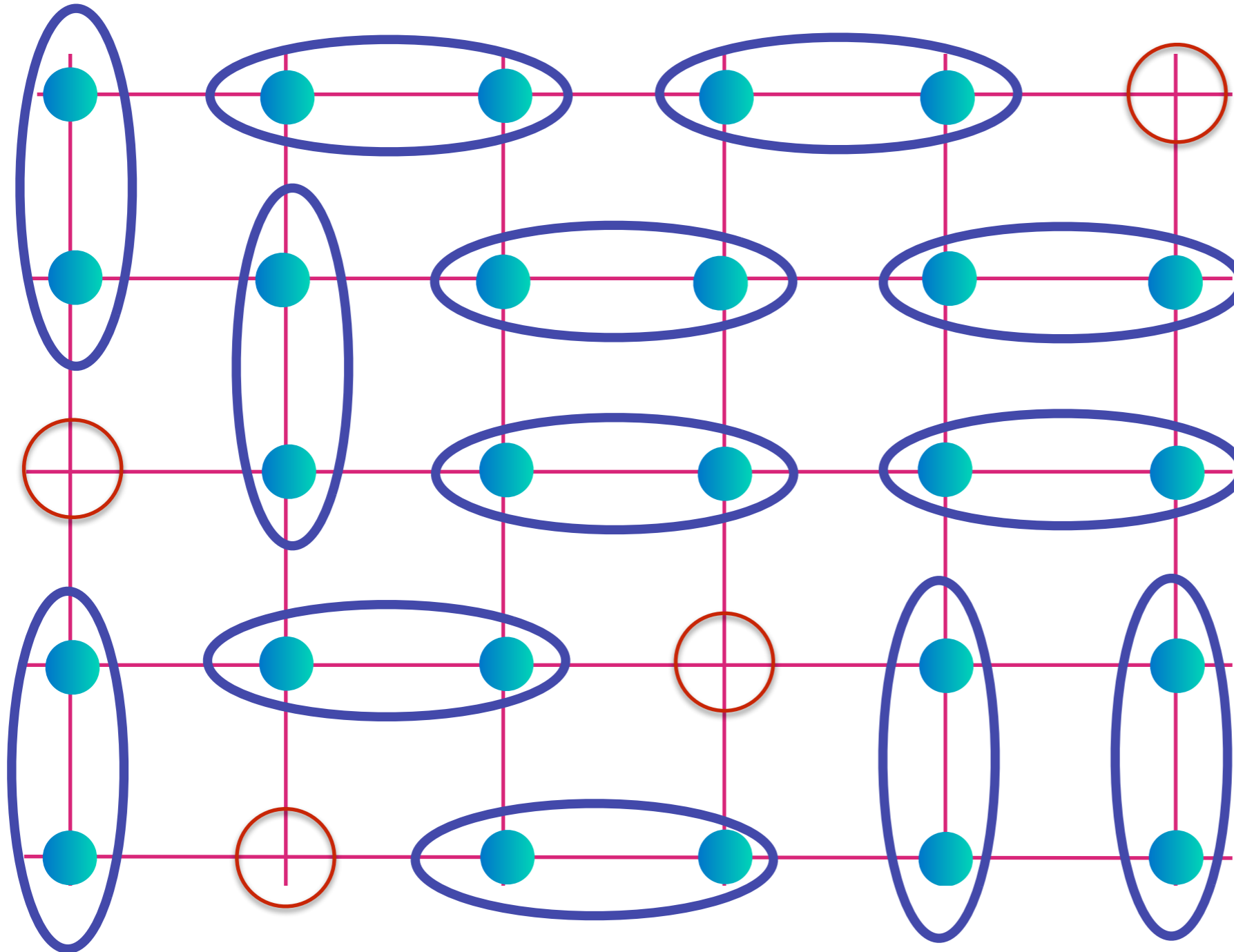
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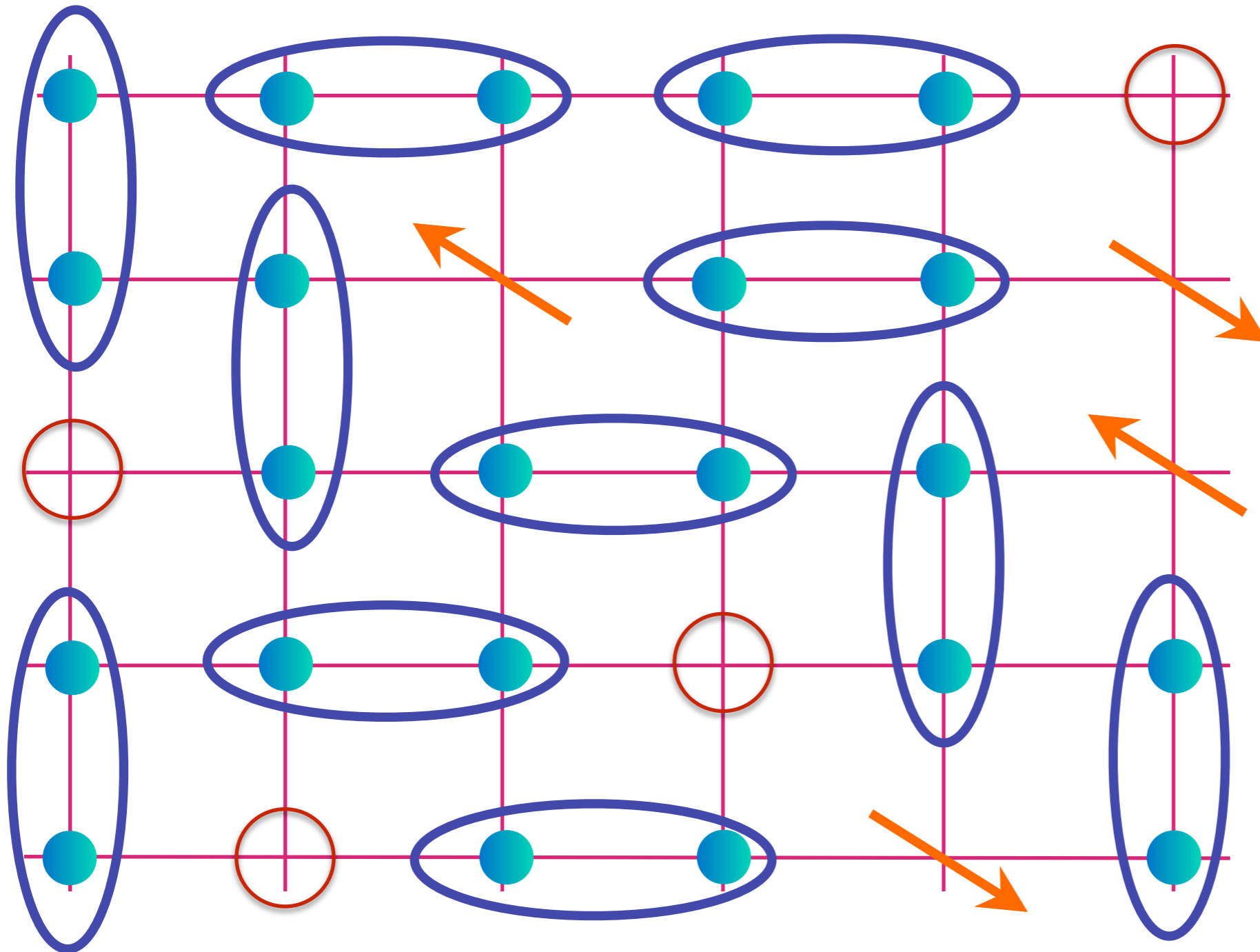
Spin  
liquid

Spinless  
charge  $+e$   
holons

$$\text{[Pair of dots in oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)



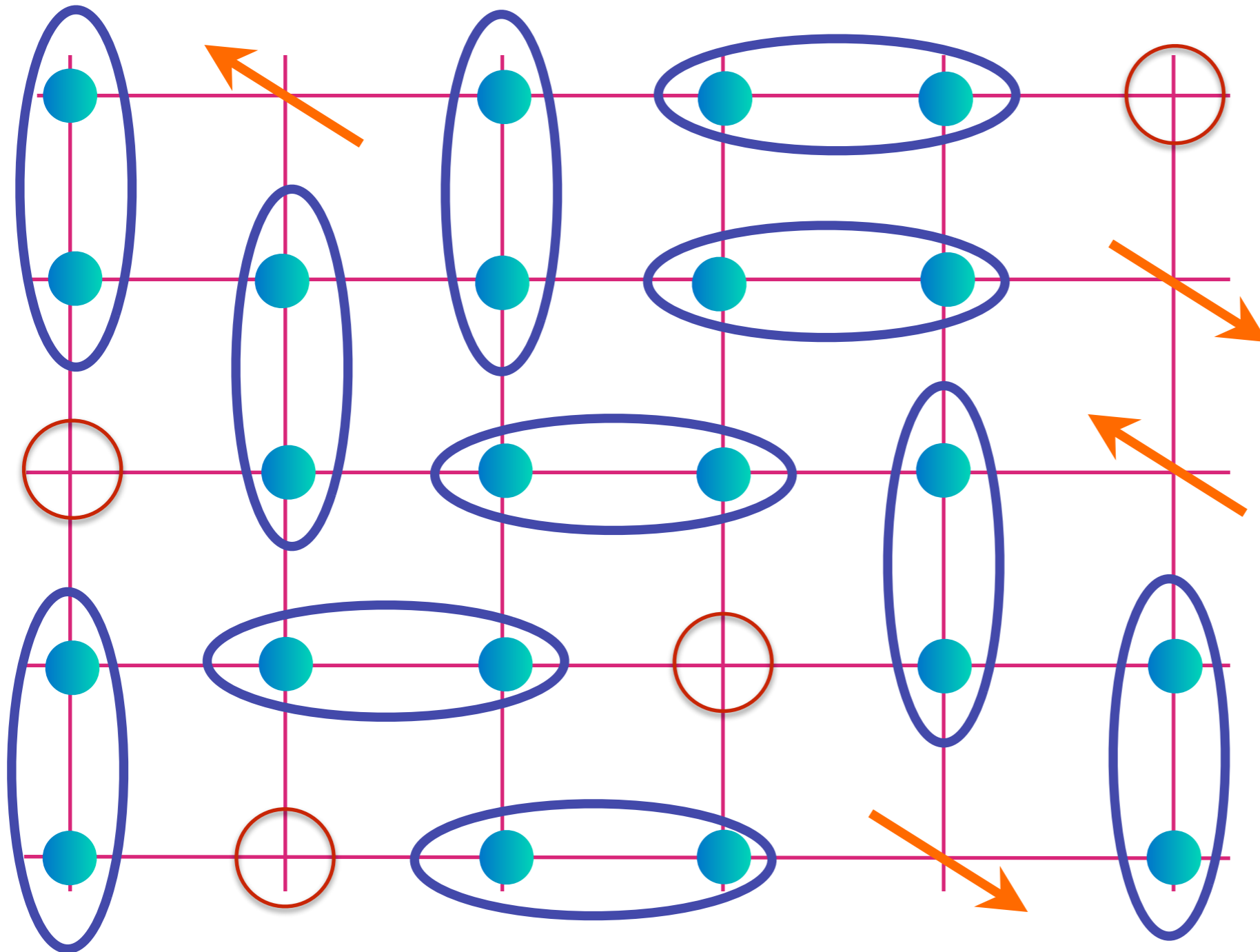
Spin  
liquid

Spinless  
charge  $+e$   
holons  
*and*  
 $S=1/2$   
neutral  
spinons

$$\text{[Blue oval with two teal dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

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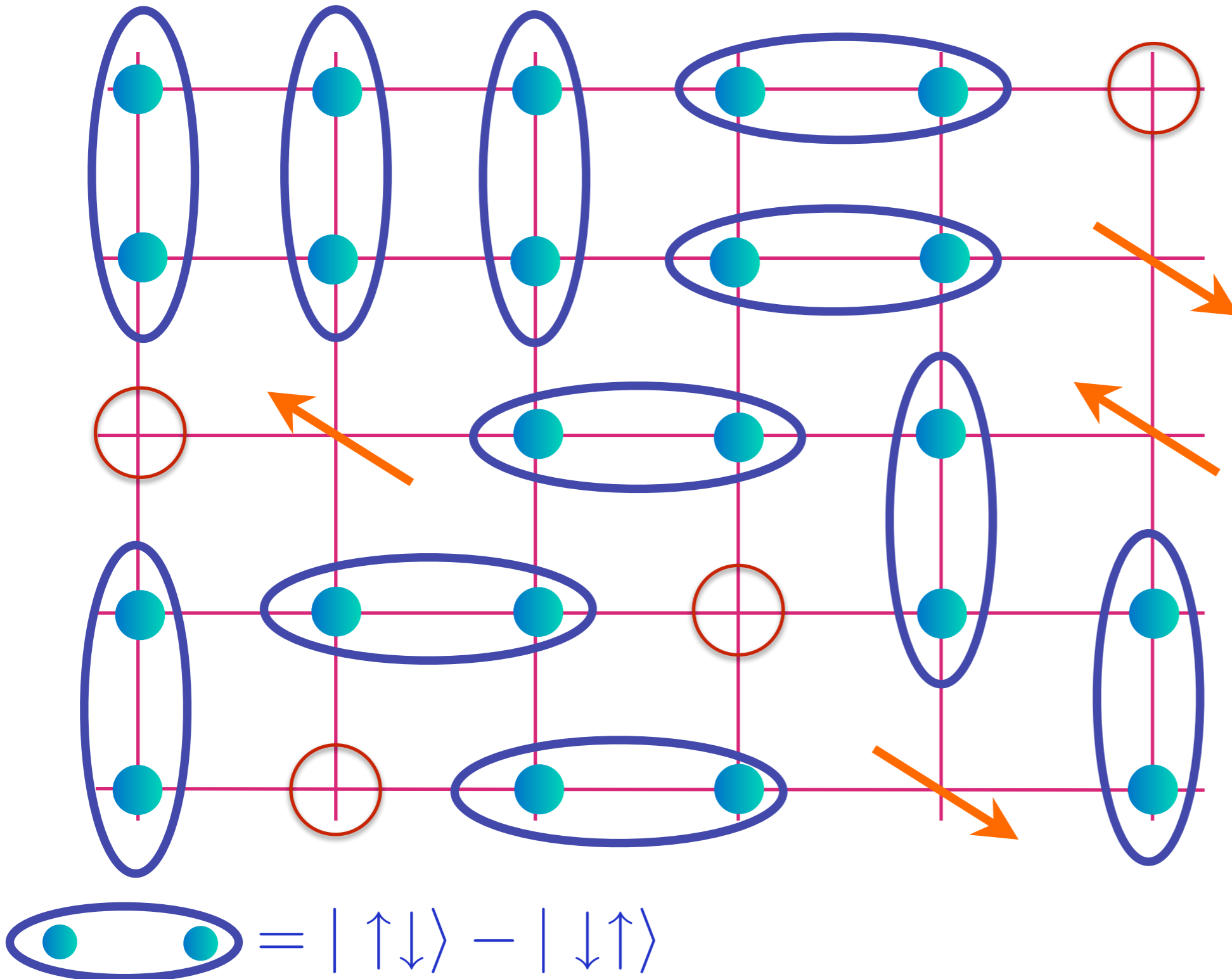
Spin  
liquid

Spinless  
charge  $+e$   
holons  
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neutral  
spinons

 =  $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

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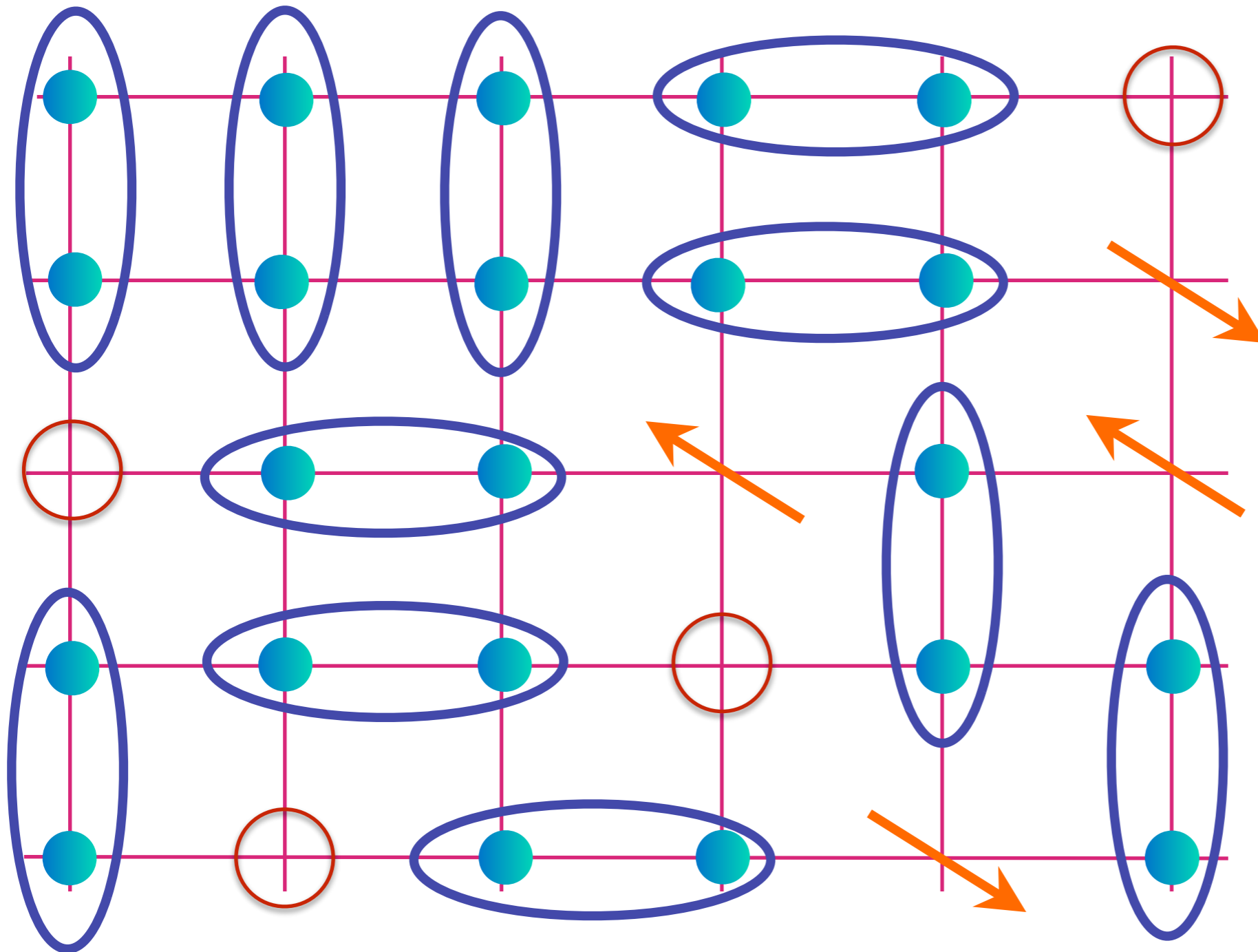


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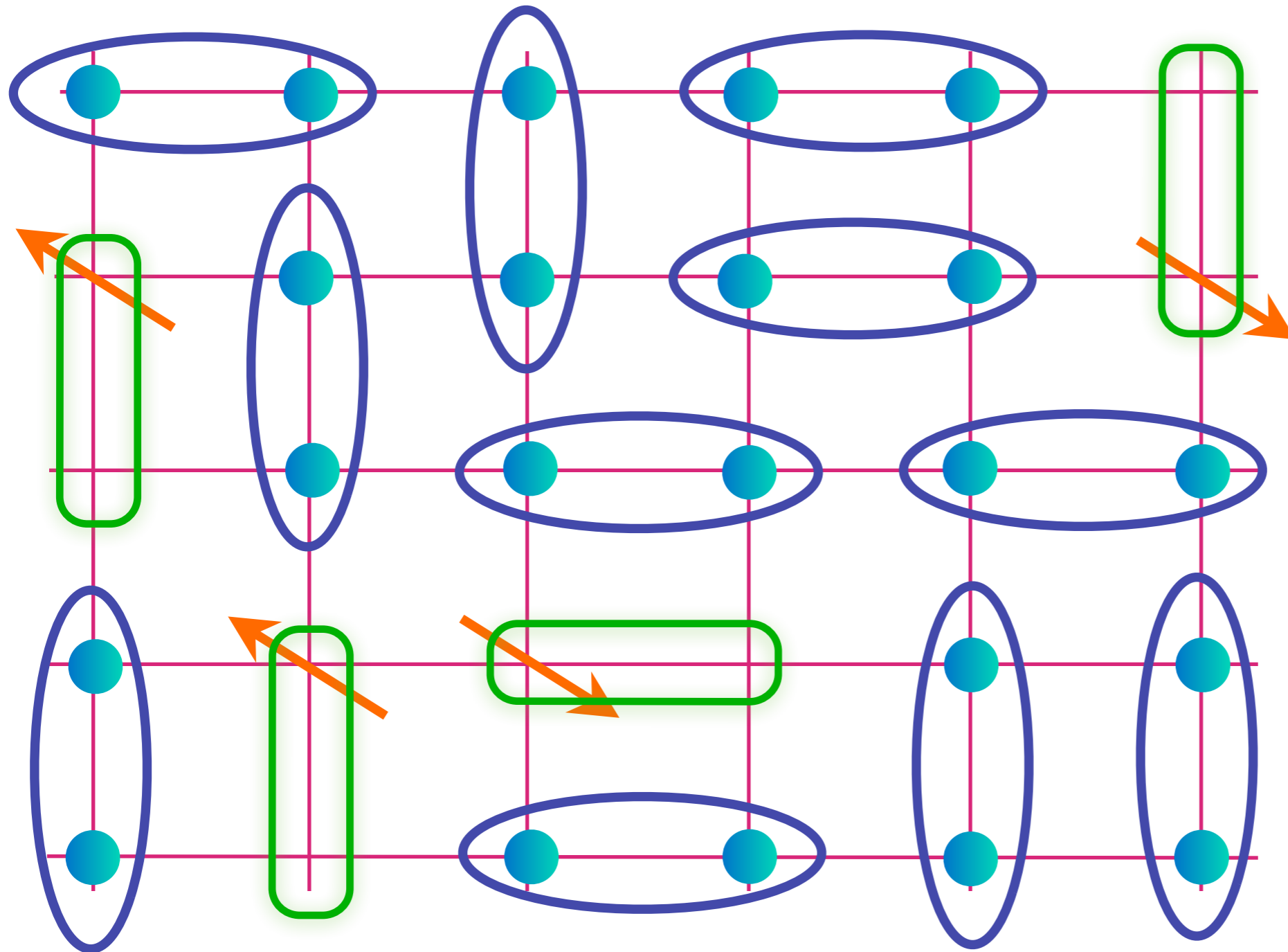


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Baskaran, Zou, Anderson, Fradkin, Kivelson...

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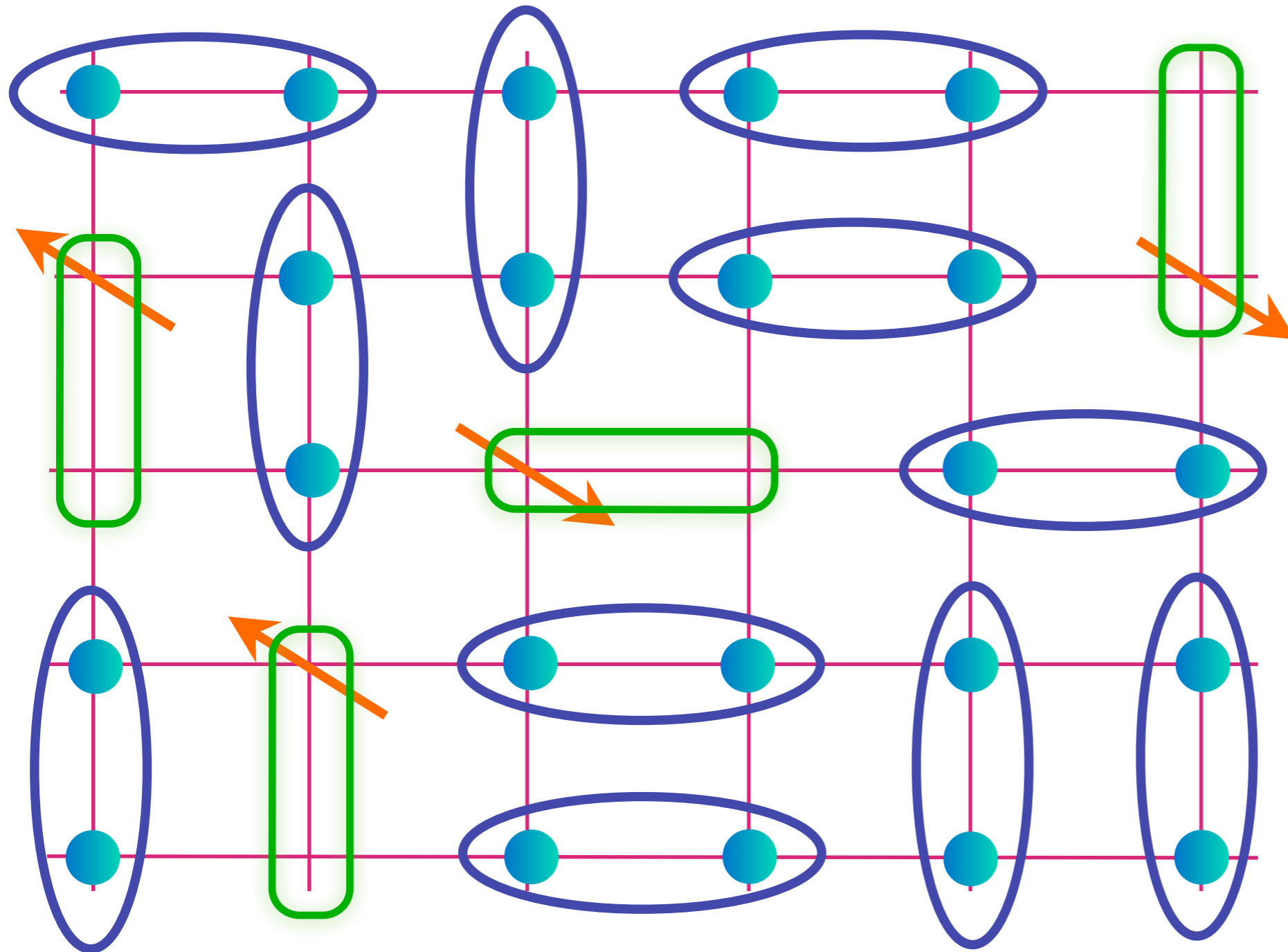


**FL\*** !

Spin  
singlets  
*and*  
charge  $+e$   
 $S=1/2$   
holes  
of density  $p$

Charge  $+e$ , spin  $S = 1/2$  holes form Fermi surfaces of total volume  $p$   
(and not  $1 + p$  as in a Fermi liquid).

# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)

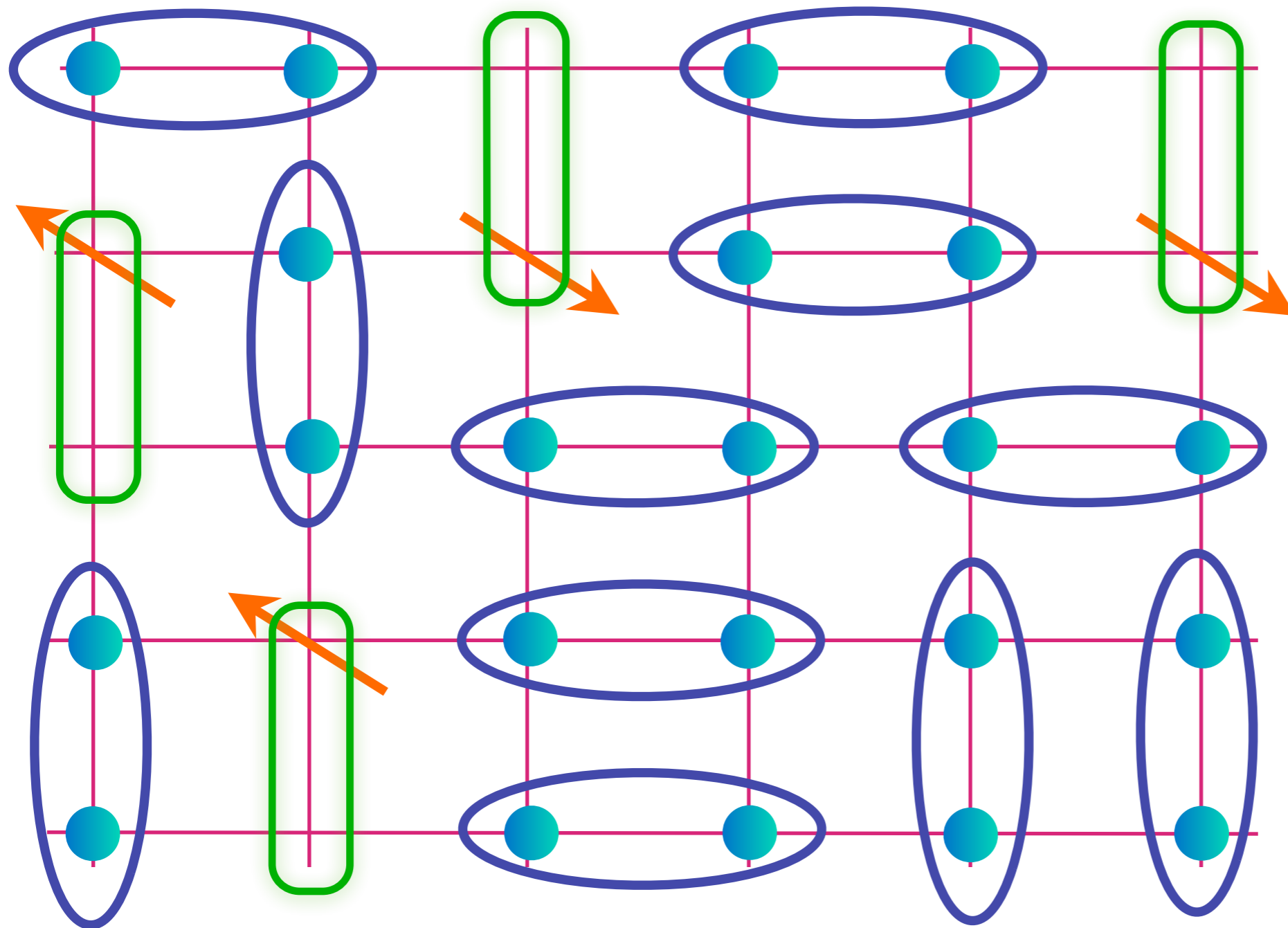


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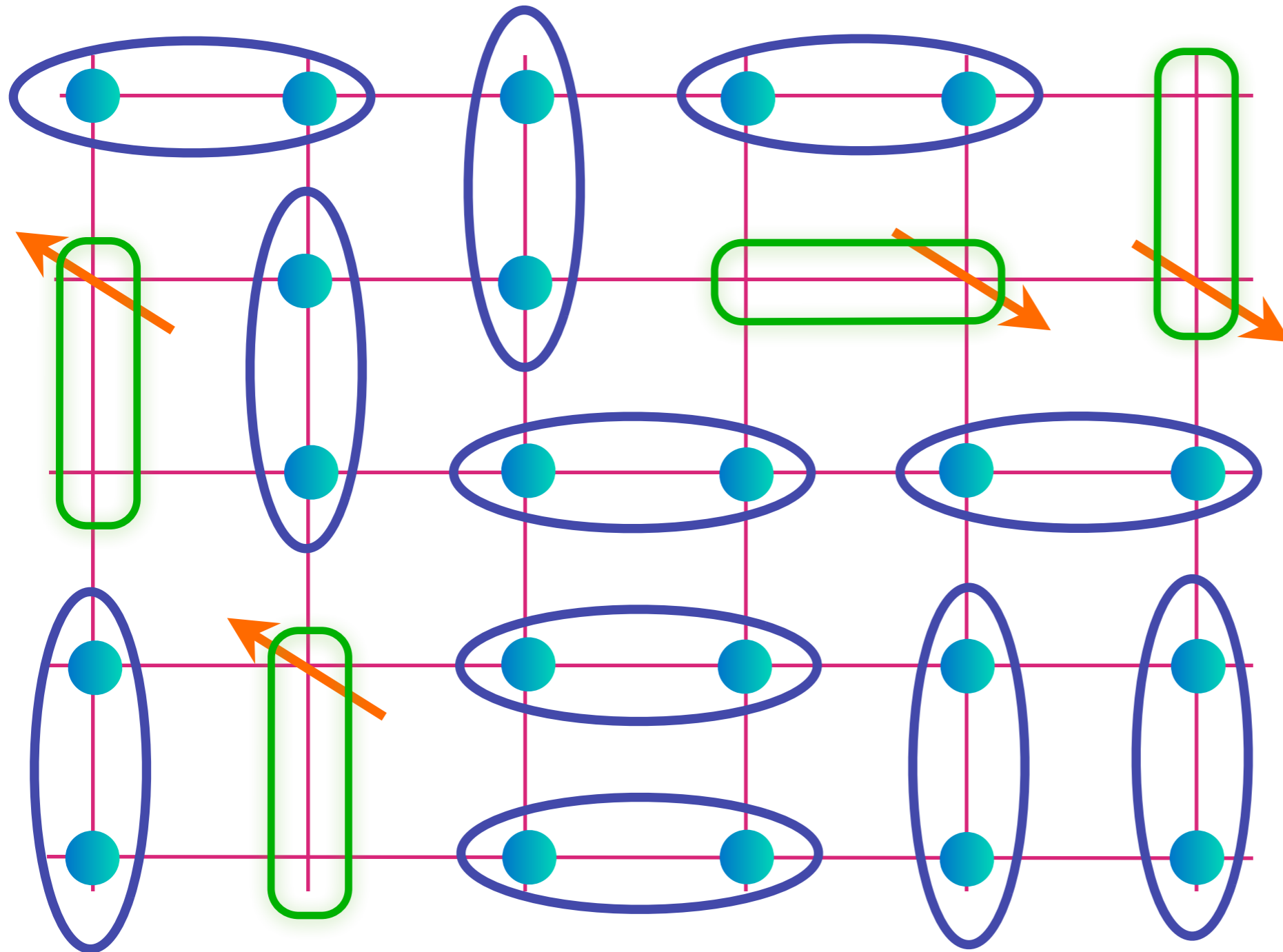
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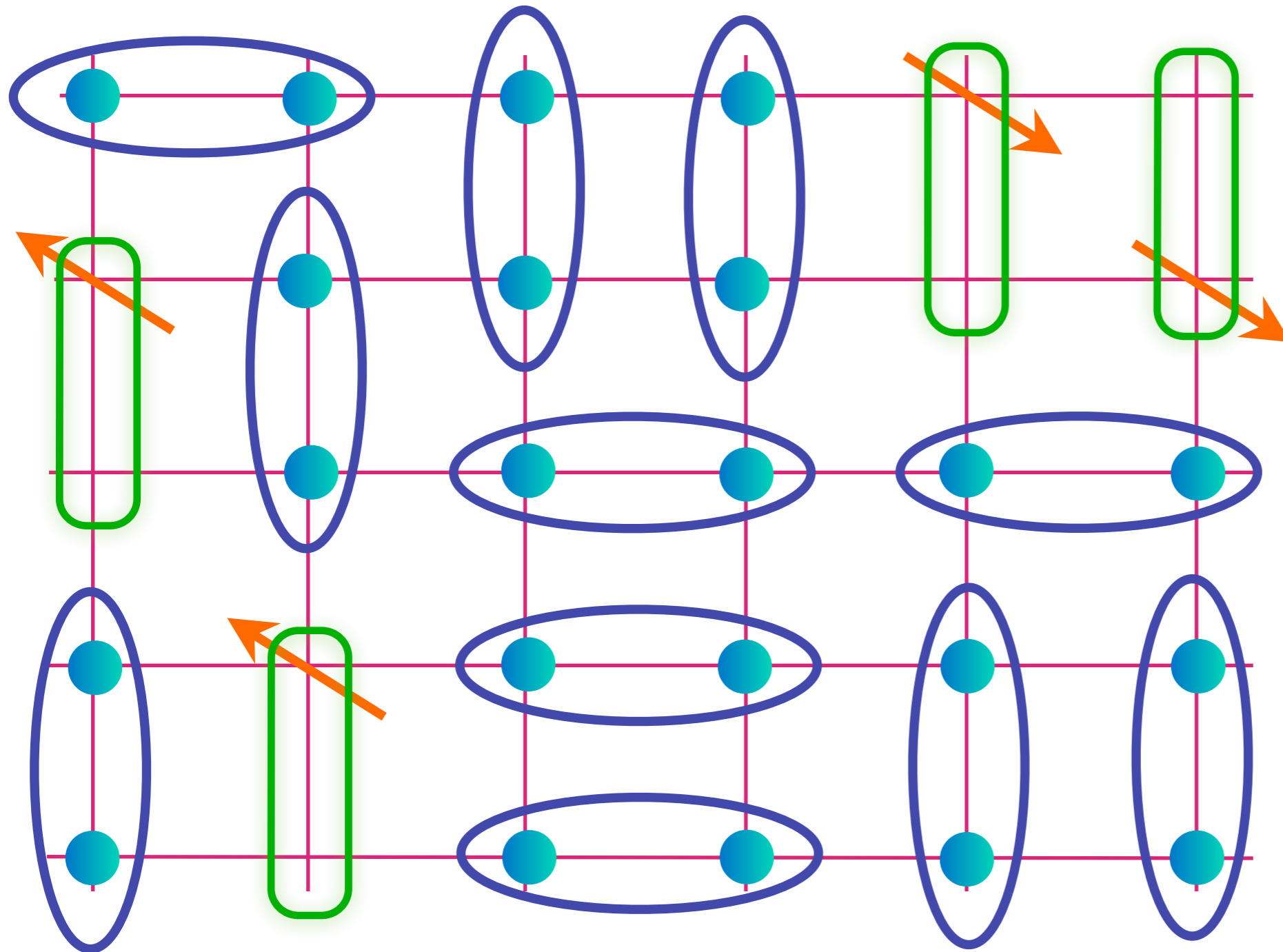


**FL\*** !

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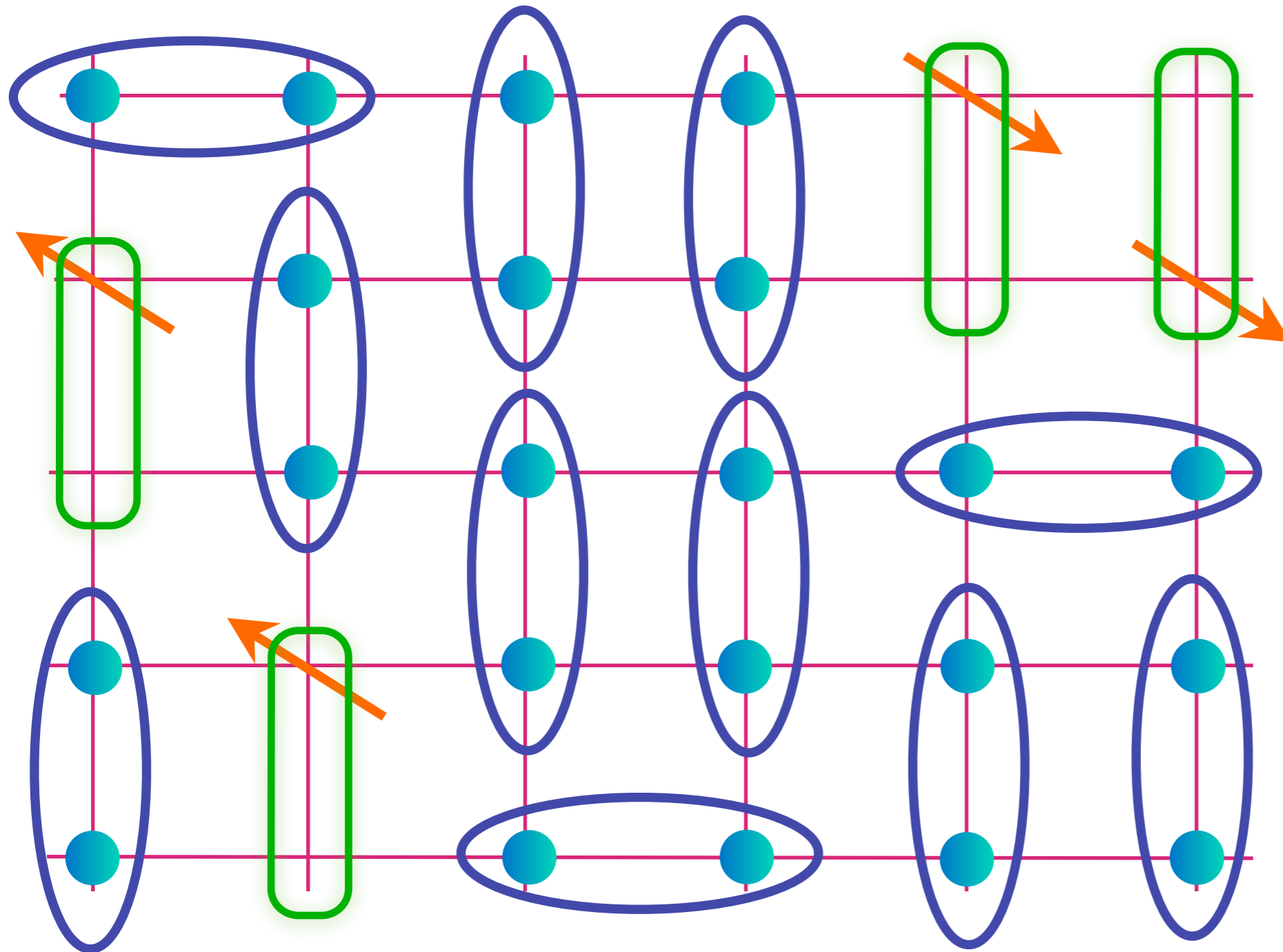


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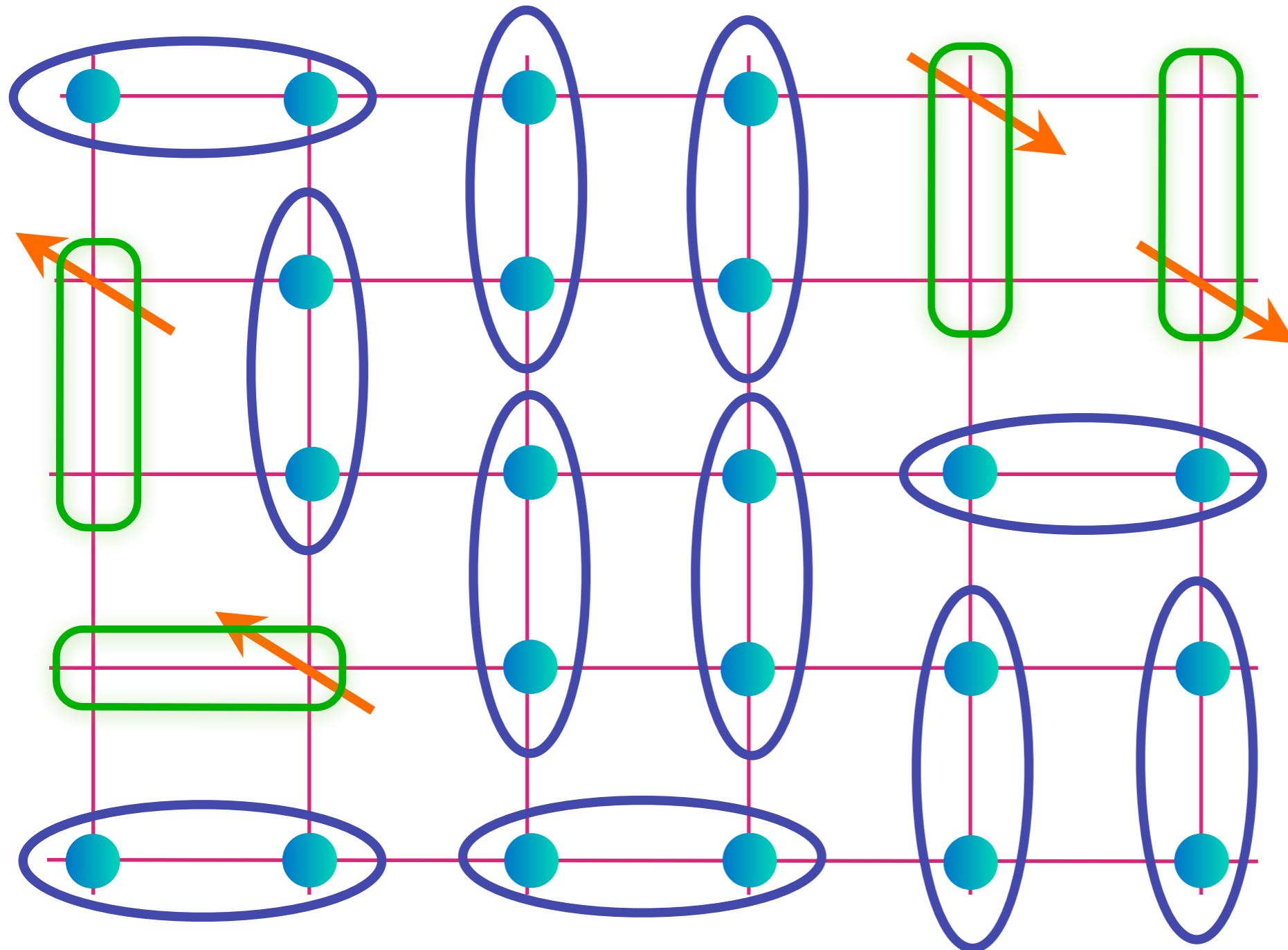


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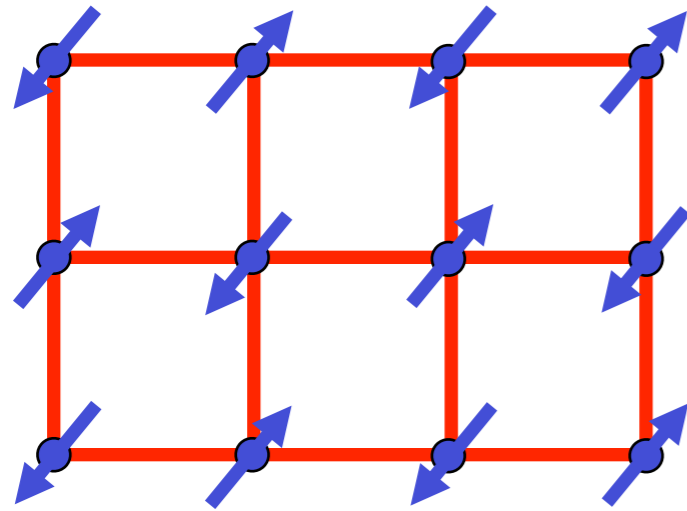


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# Quantum “disordering” magnetic order



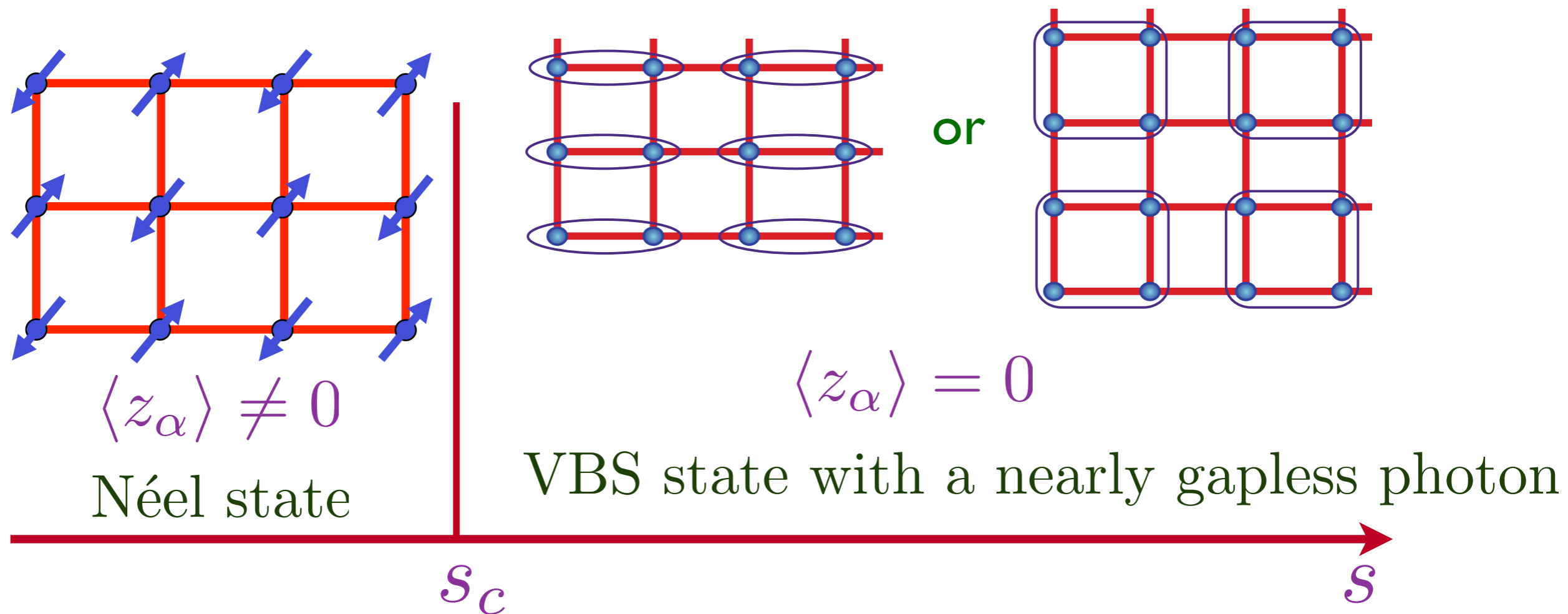
collinear Néel state

Spin liquid with a “**photon**”, which is unstable to the appearance of valence bond solid (VBS) order

$S_c$

$S$

# Neel-VBS quantum transition

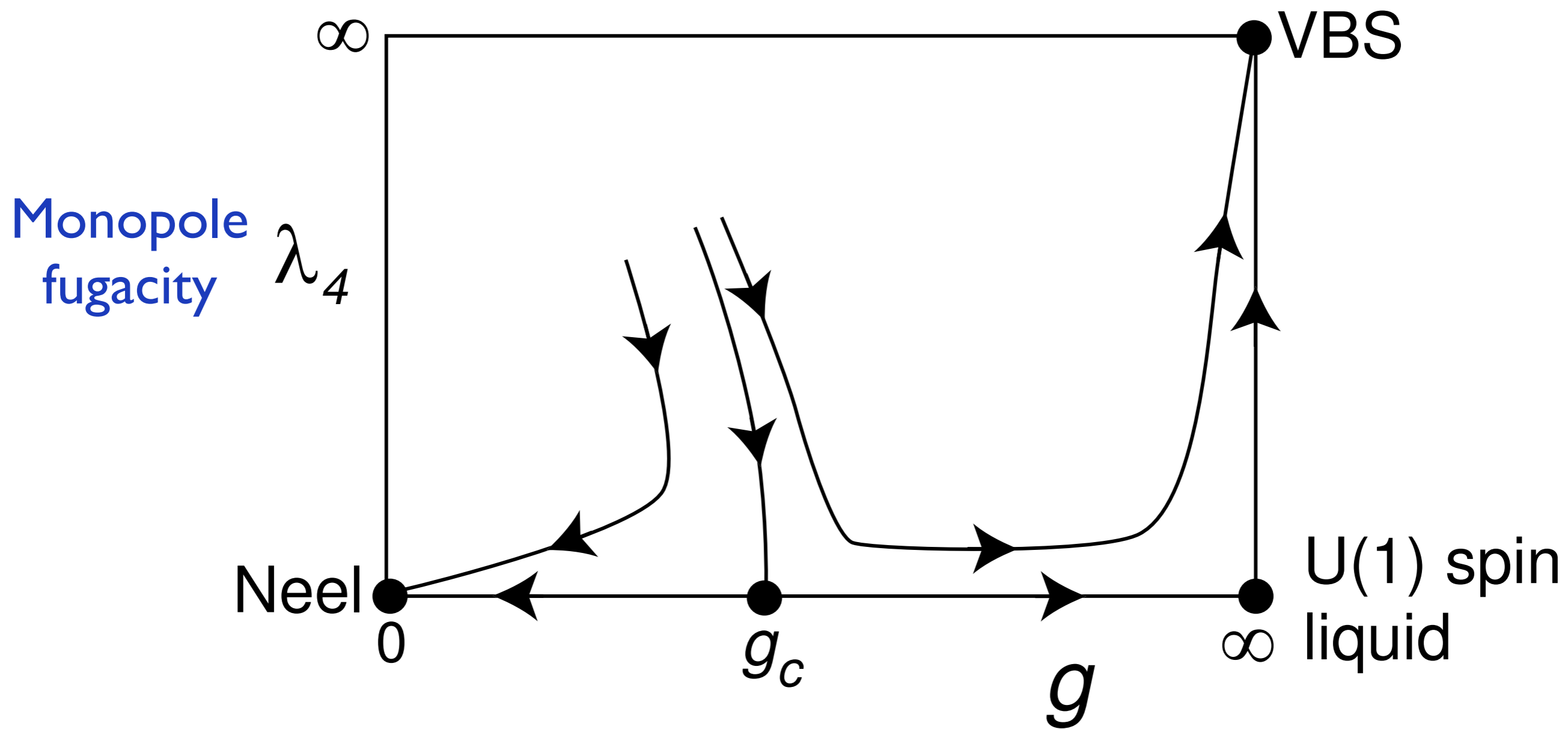


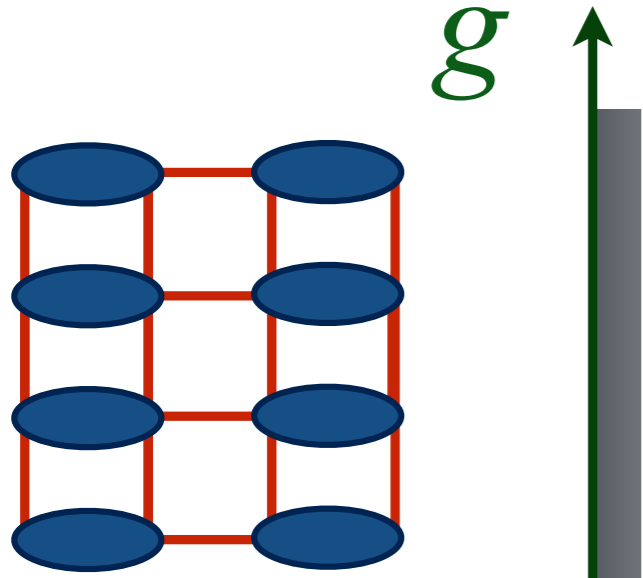
Critical theory for photons and deconfined spinons:

$$\mathcal{S}_z = \int d^2r d\tau \left[ |(\partial_\mu - iA_\mu)z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right]$$

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).

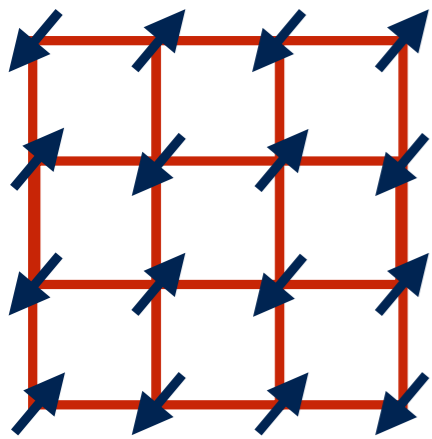
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).





$U(1)$  SL  $\rightarrow$  VBS  
+ confinement

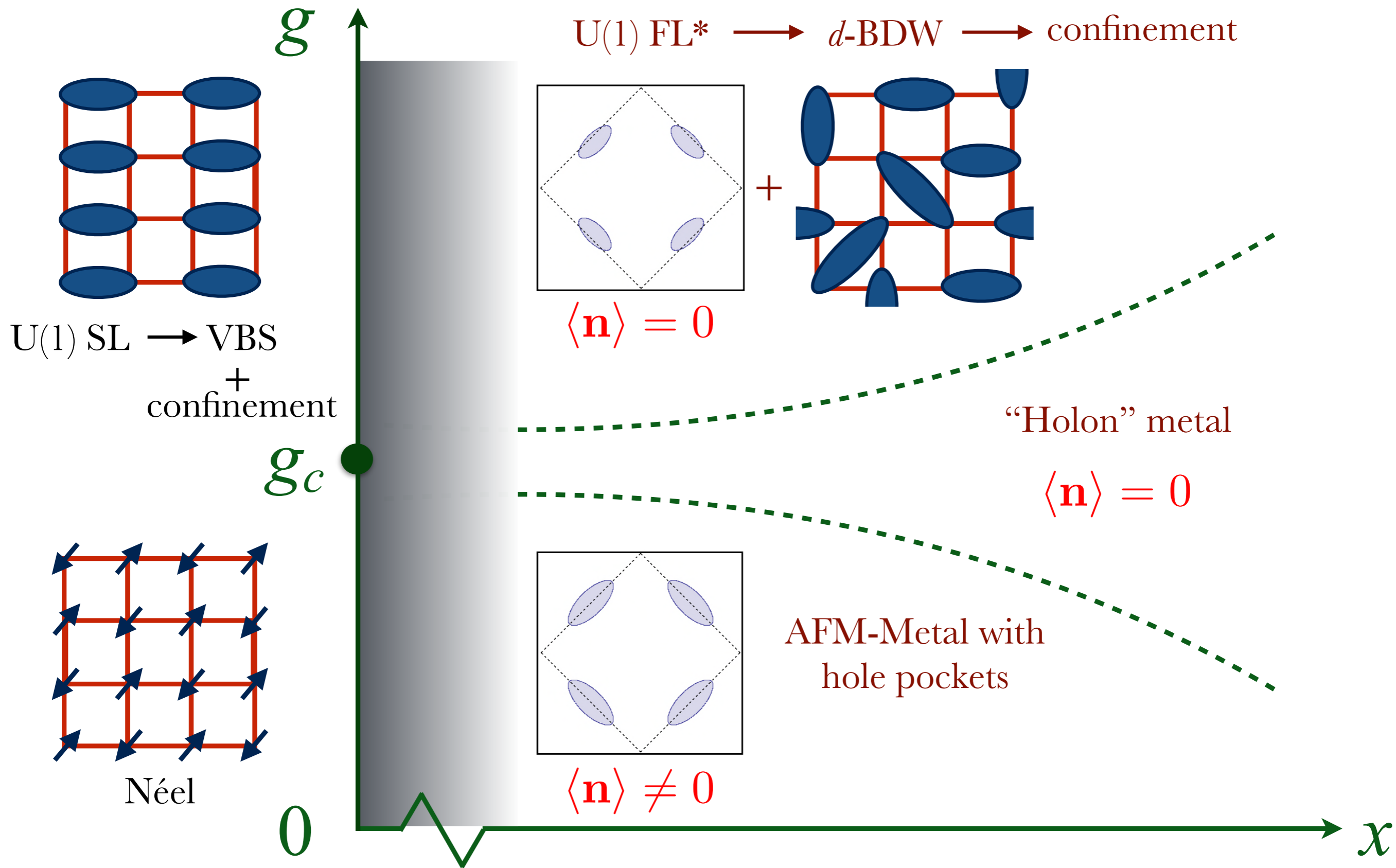
$g_c$



Néel

0



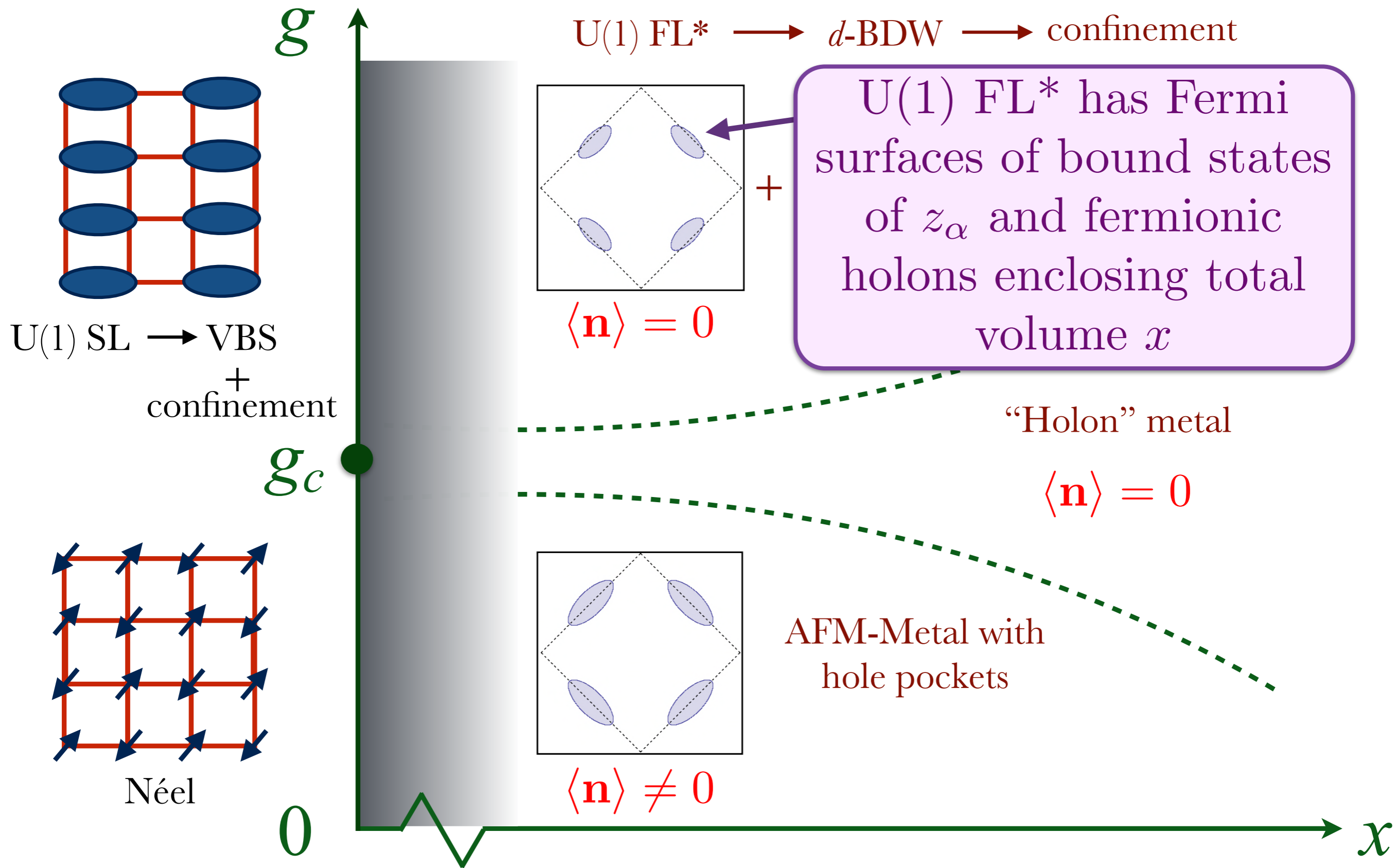


R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, Phys. Rev. B **75**, 235122 (2007)

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)

D. Chowdhury and S. Sachdev, arXiv:1409.5430

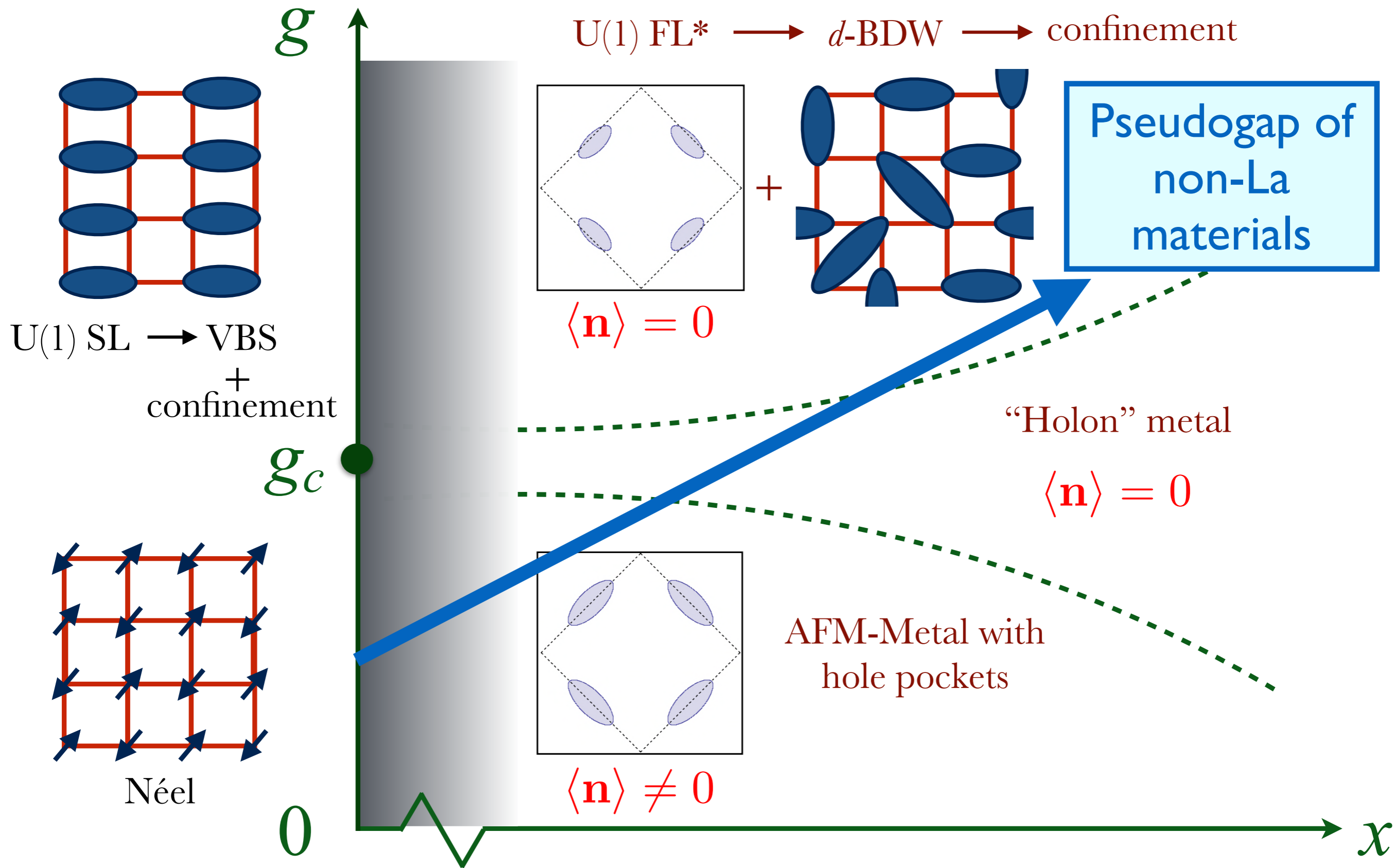


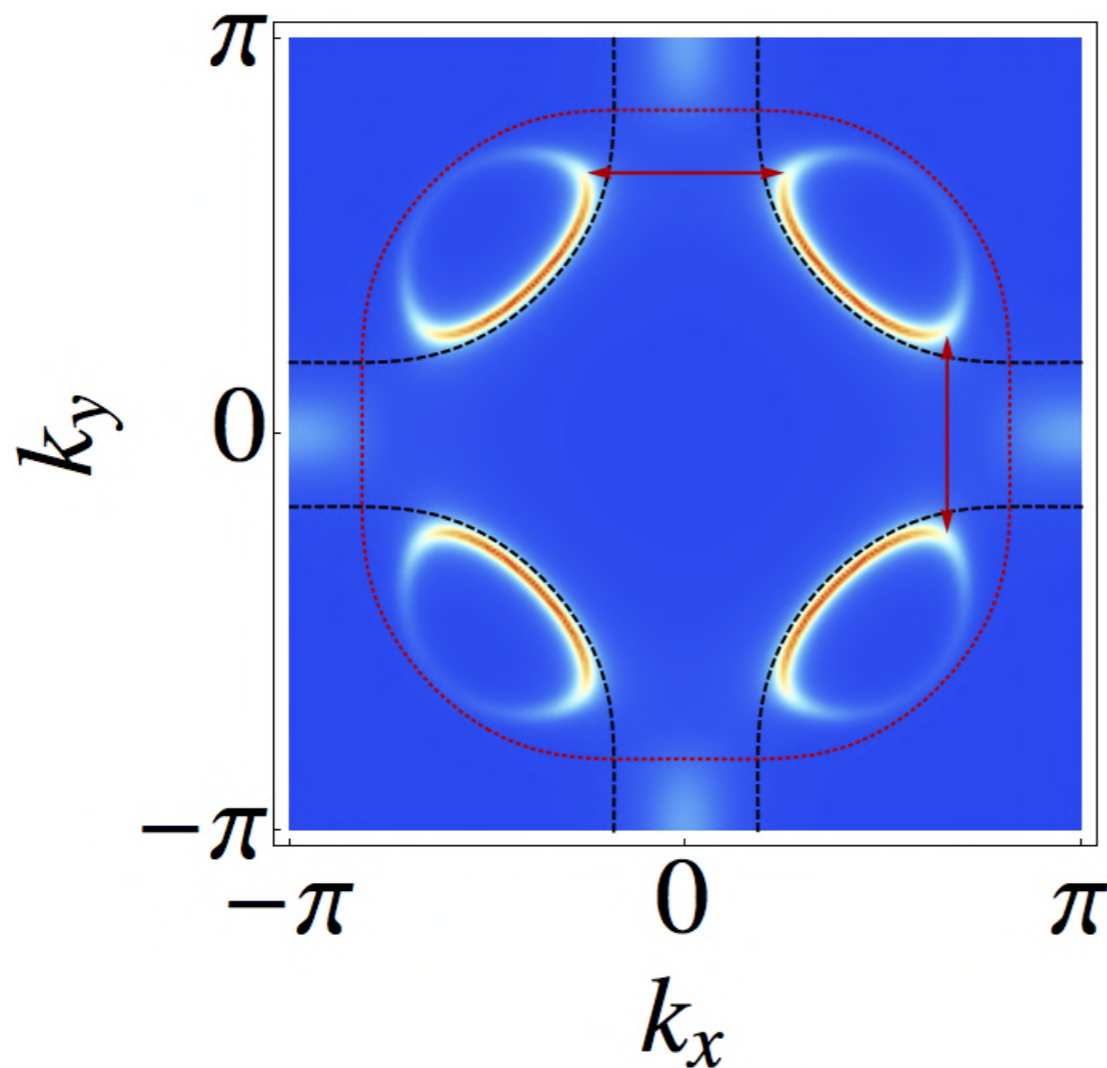


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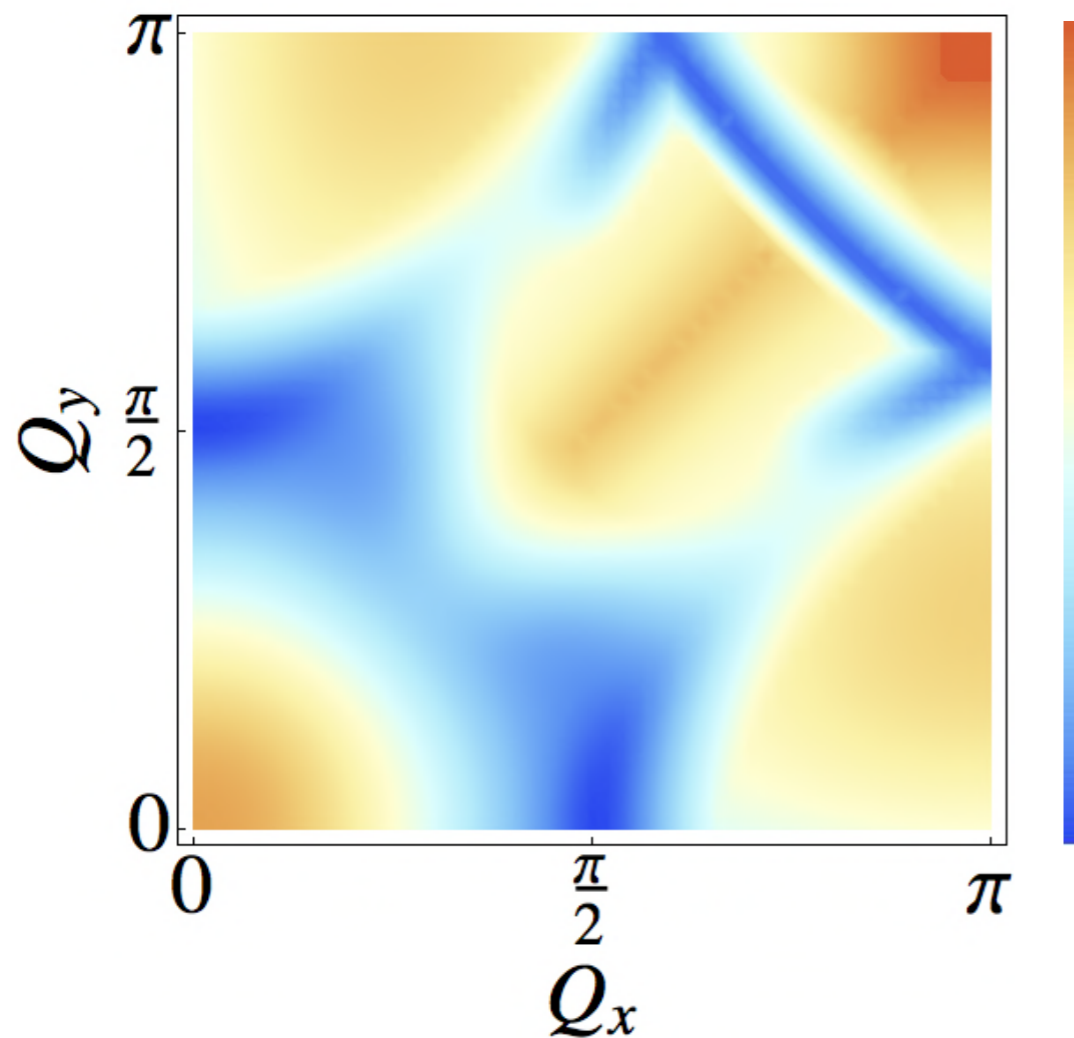




Electron spectral  
function of FL\*

The pseudogap is described by the U(1)-FL\*: a state with hole pockets on a background of a spin-liquid described by a U(1) gauge theory. Its dominant density wave instability is a predominantly  $d$ -form factor density wave with a wavevector  $\mathbf{Q}$  along the  $(1, 0)$  and  $(0, 1)$  square lattice directions, in agreement with observations on the non-La-based cuprates.

- Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)  
M. Punk and S. Sachdev, Phys. Rev. B **85**, 195123 (2012)  
D. Chowdhury and S. Sachdev, arXiv:1409.5430



Eigenvalues of spin-singlet,  
time-reversal-preserving  
particle-hole propagator

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# Conclusions

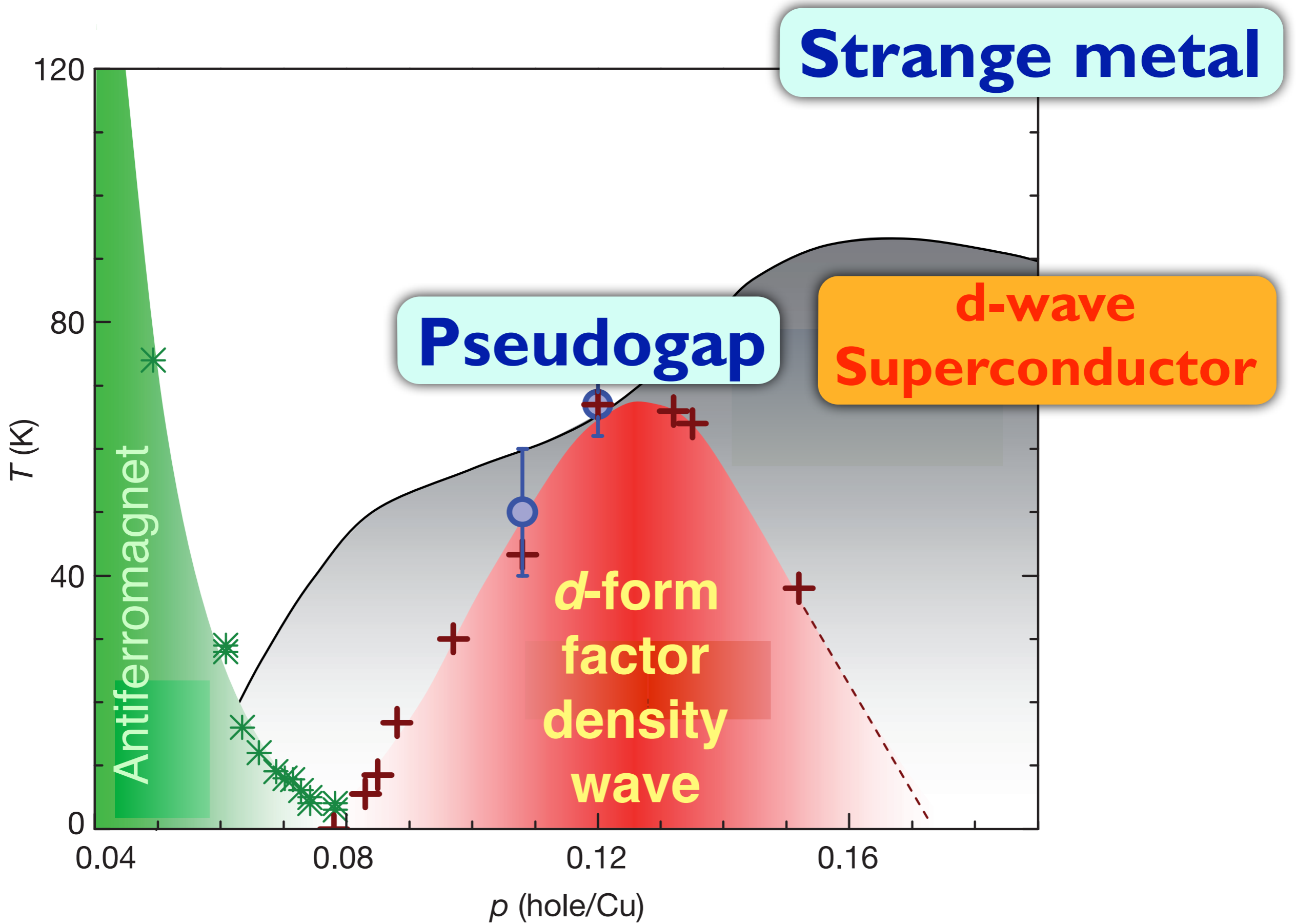
1.  $d$ -form factor density wave order observed in the non-La hole-doped cuprate superconductors.

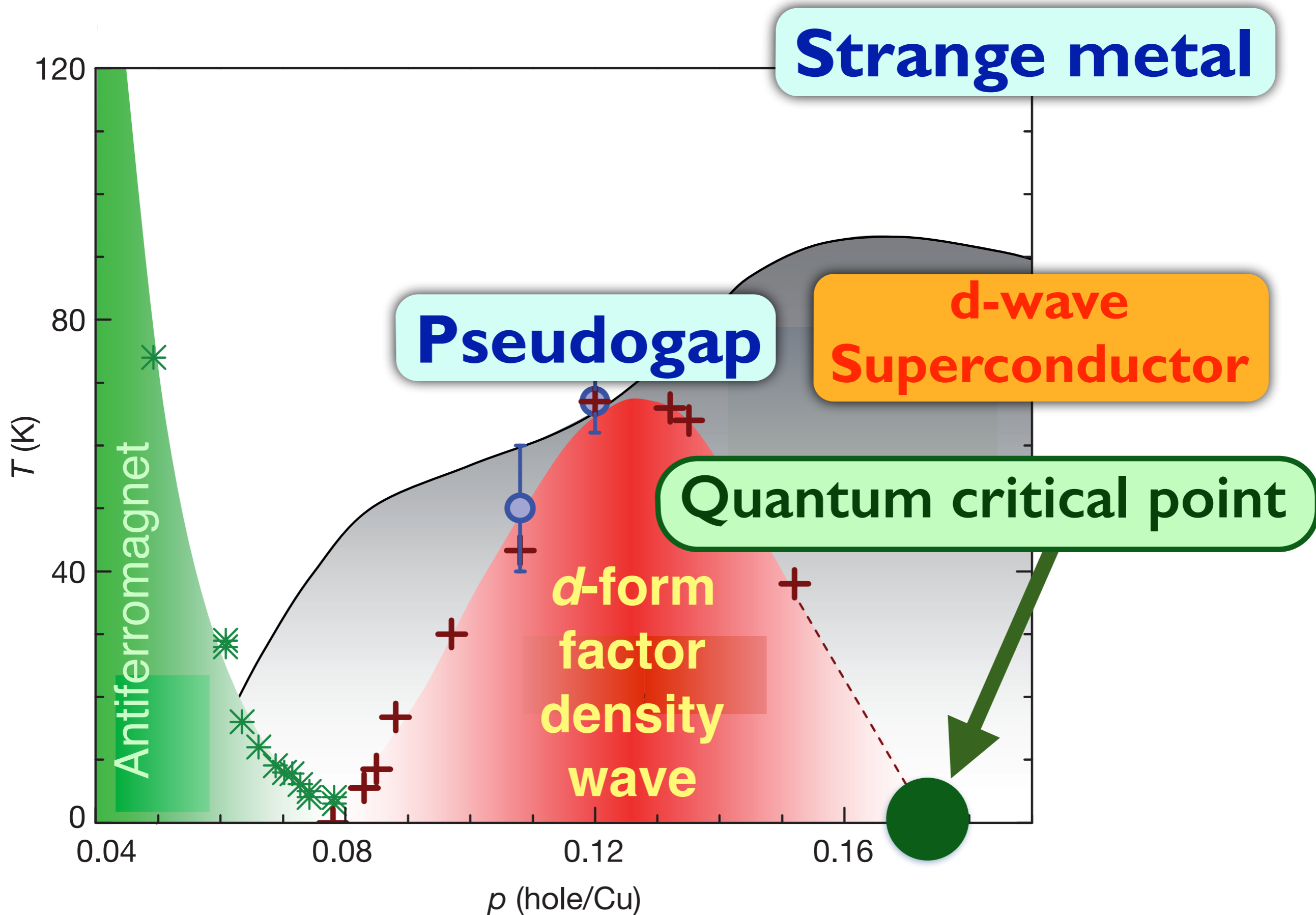
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3. The  $d$ -form factor appears to be an unexpected window into the spin-liquid physics of the pseudogap.





Y. He *et al.*, Science **344**, 608 (2014)  
 K. Fujita *et al.*, Science **344**, 612 (2014)