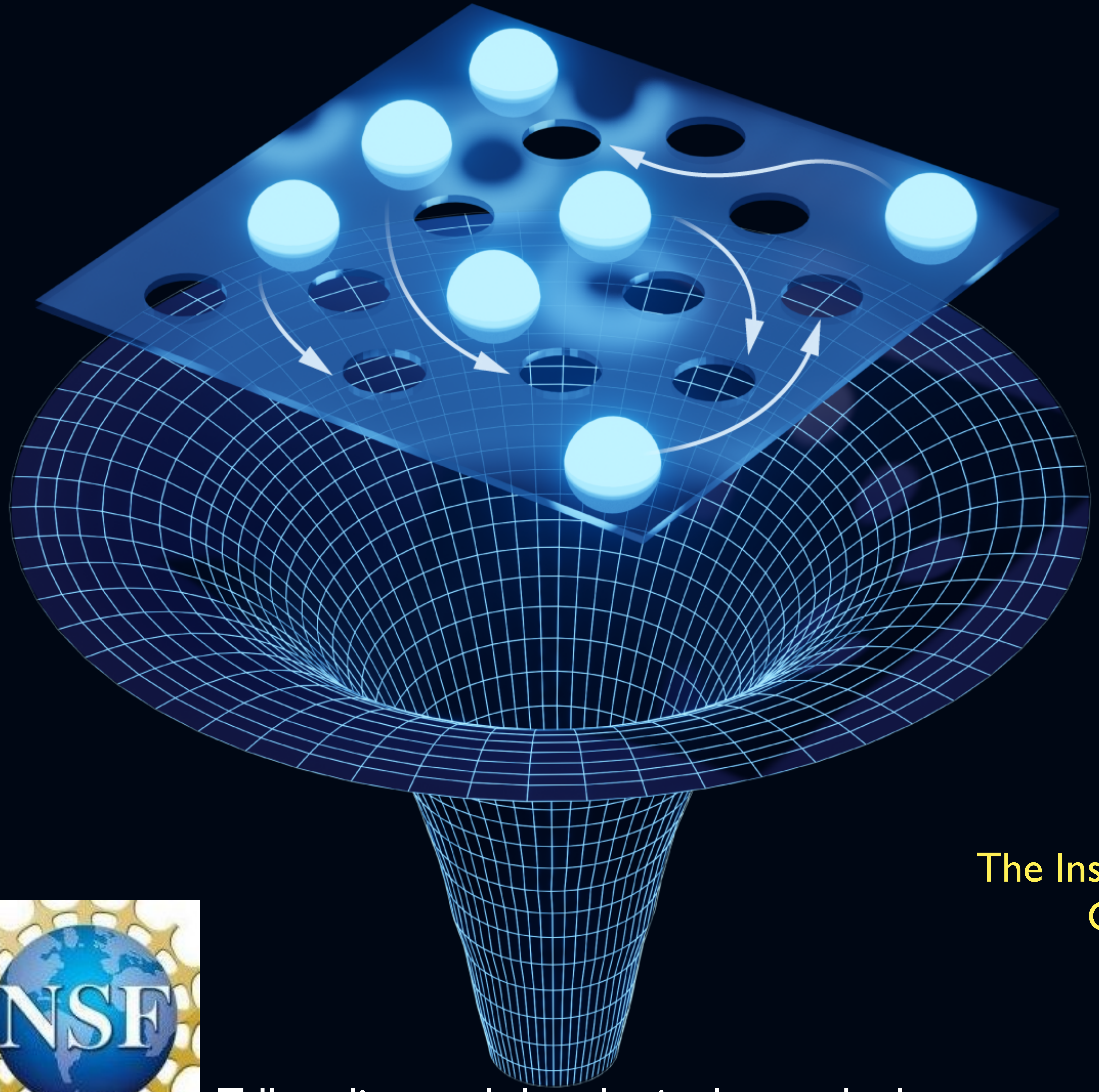

Graphene flakes, strange metals, and black holes: insights from the SYK model

Institute Colloquium
The Institute of Mathematical Sciences
Chennai, January 16, 2023

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

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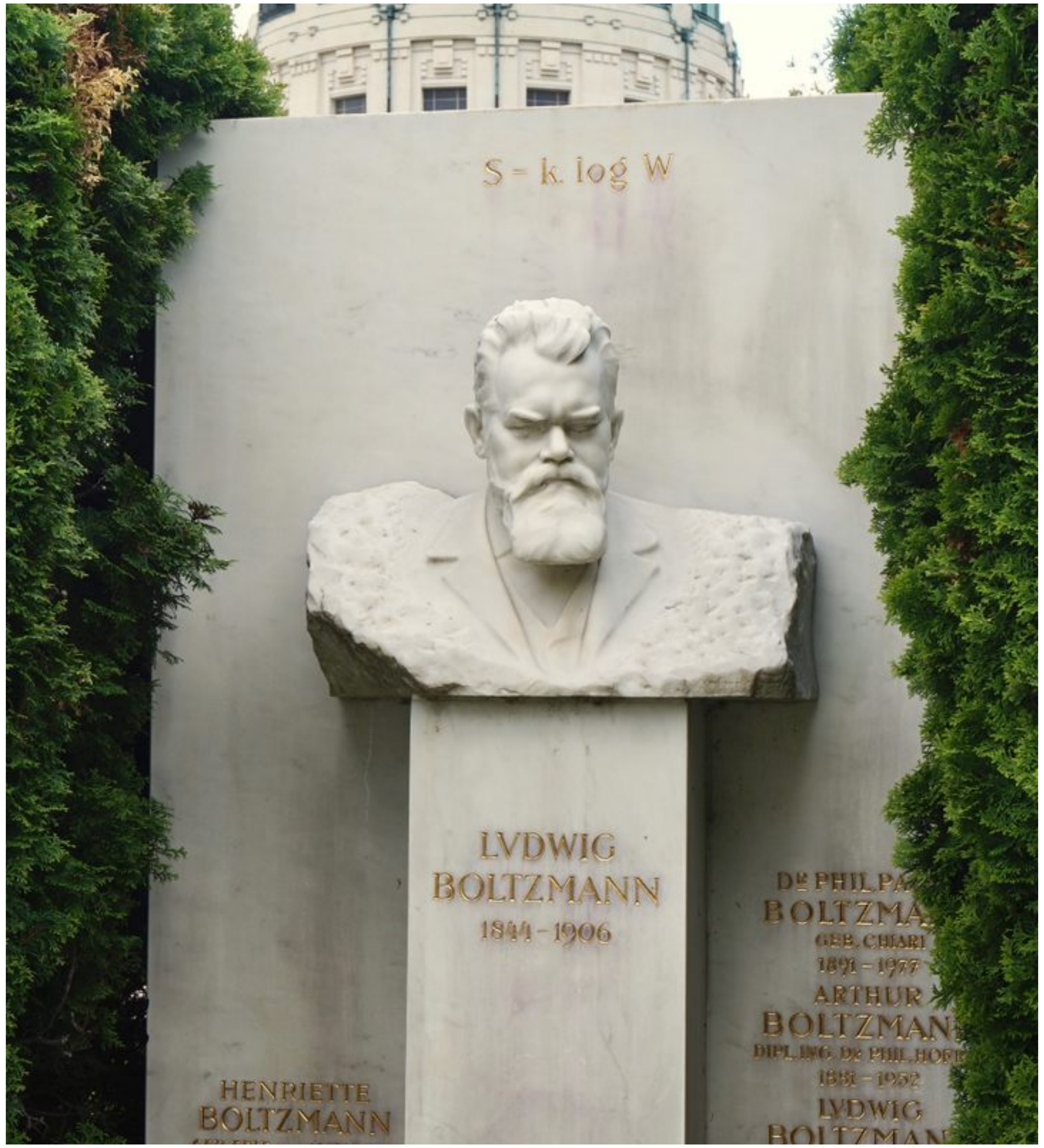
Foundations by Boltzmann



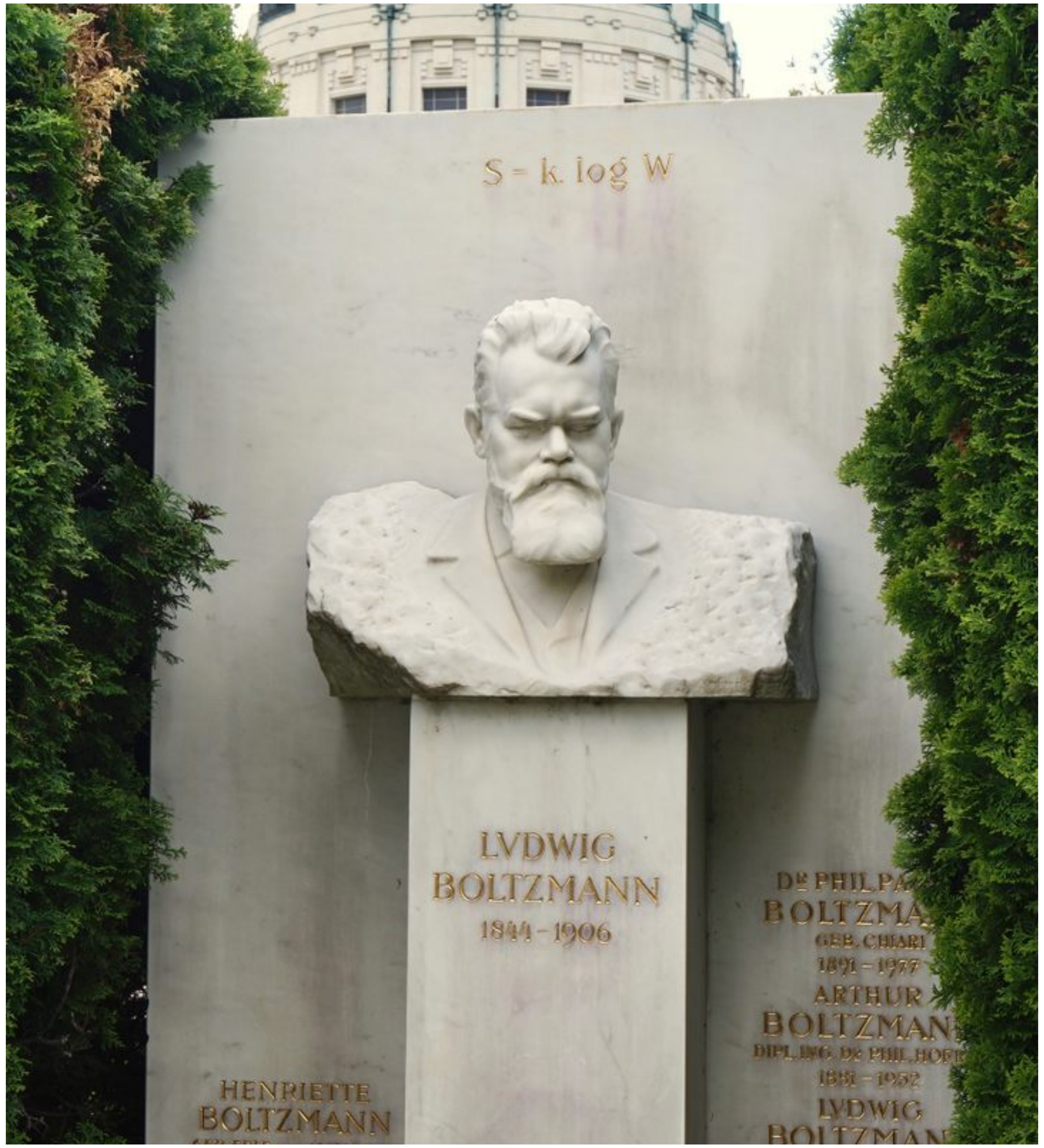
Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states $D(E) = \exp(S(E)/k_B)$

Boltzmann equation (1872)
Dilute classical gas



Ludwig Boltzmann
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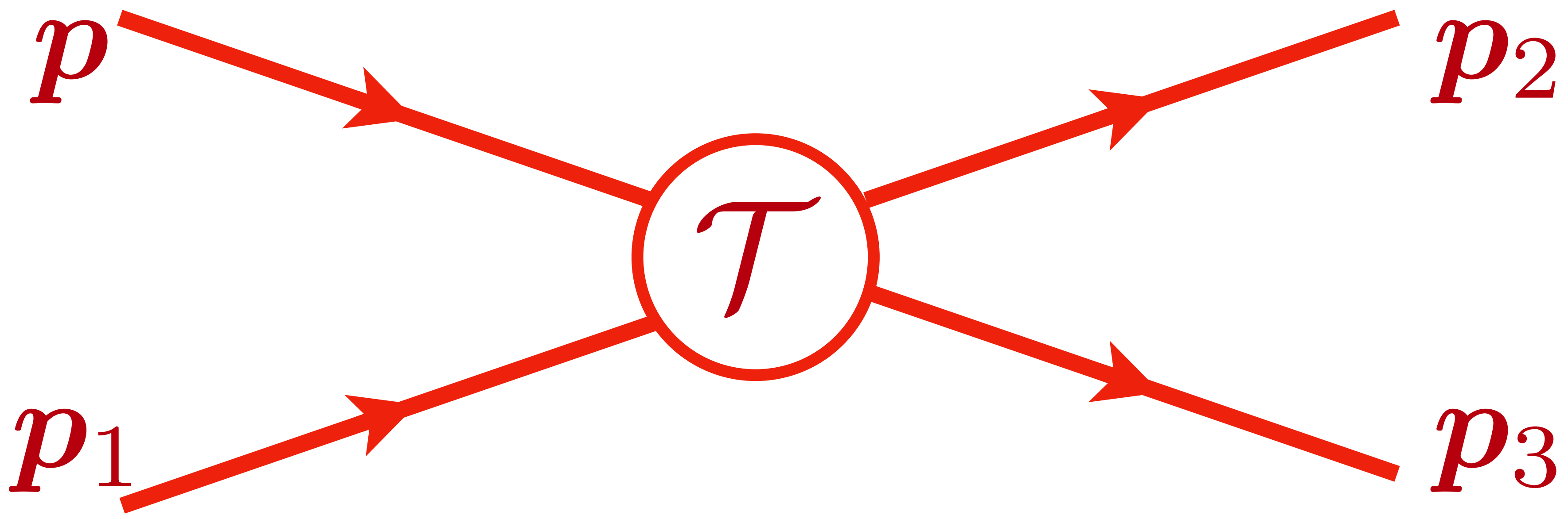
Molecular chaos: successive collisions are statistically independent

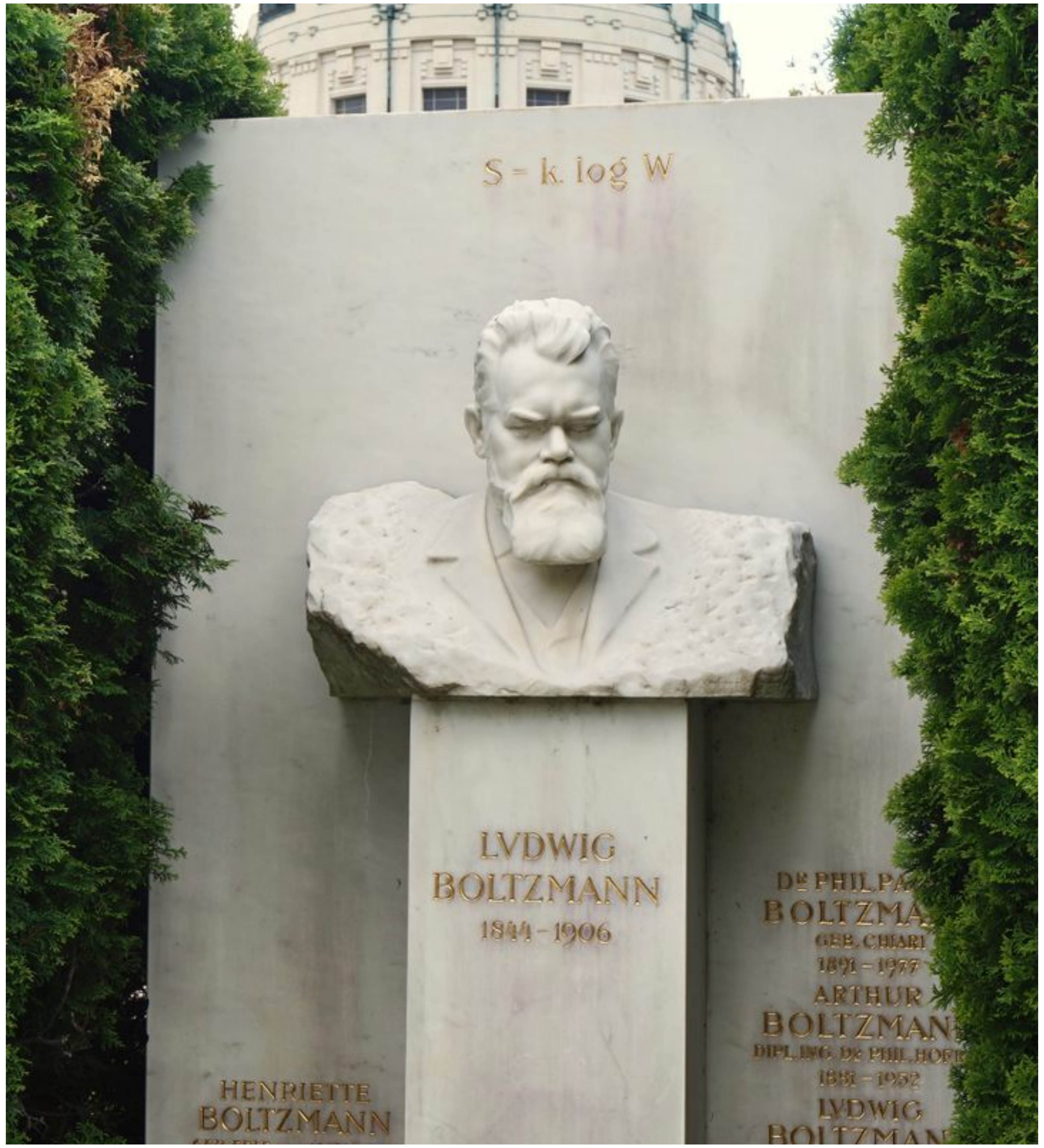
$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$

$$- 2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$

$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} \right]$$



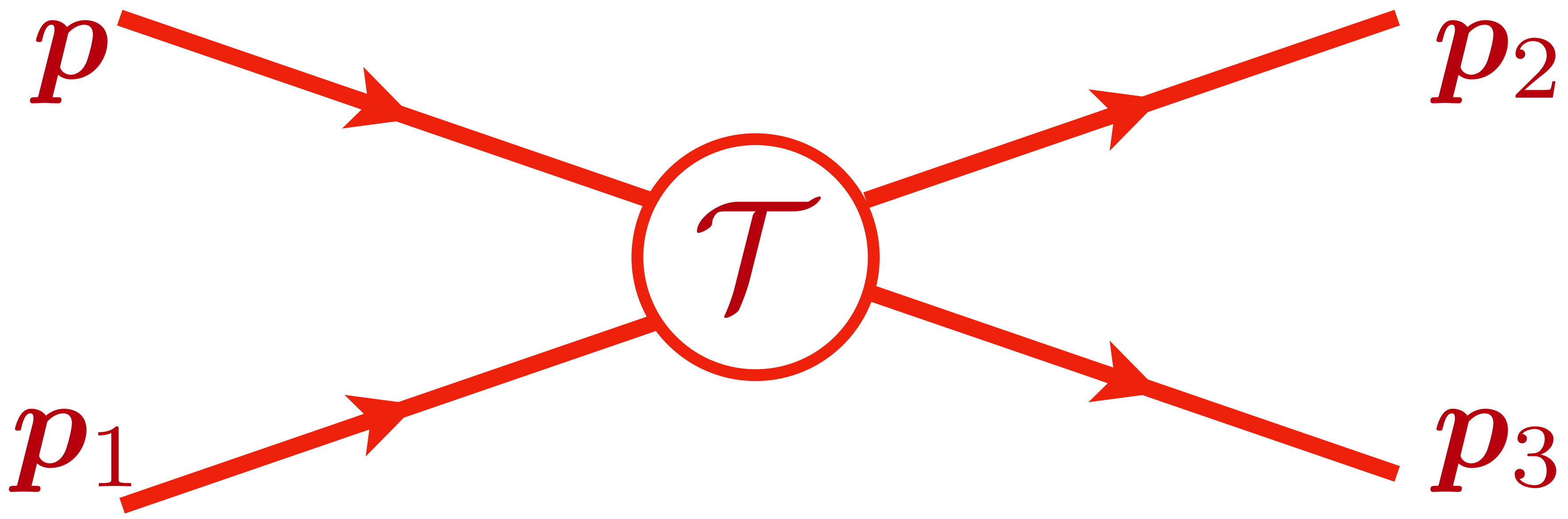
Quantum Boltzmann equation (Landau)
Dense gas of electrons



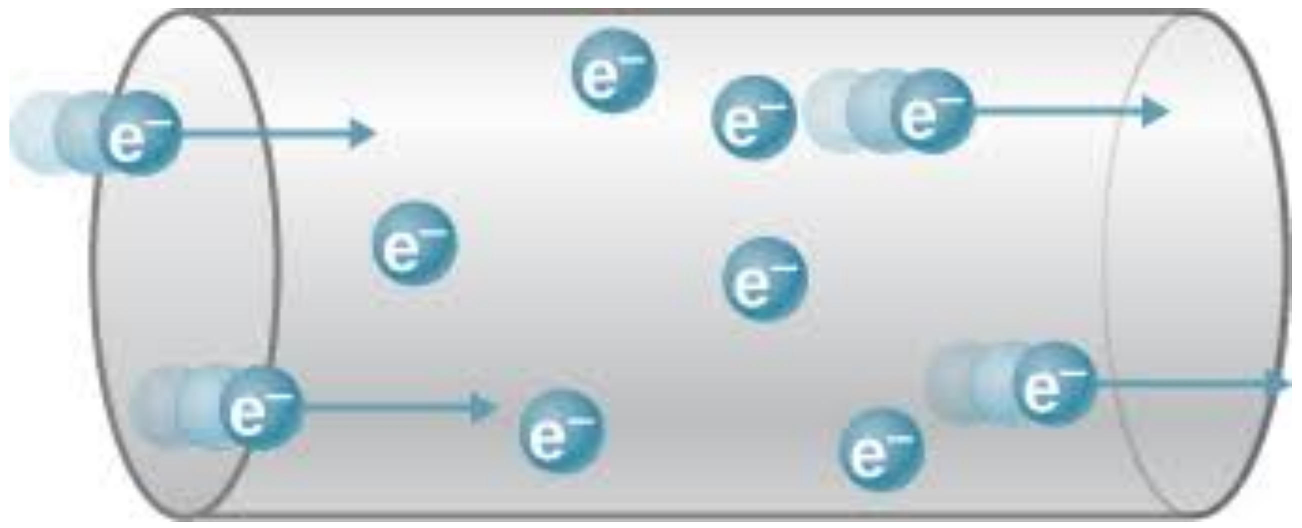
Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Neglects quantum interference (entanglement)
between successive collisions

$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$
$$- 2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$
$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} (1 - f_{\boldsymbol{p}_2})(1 - f_{\boldsymbol{p}_3}) - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} (1 - f_{\boldsymbol{p}})(1 - f_{\boldsymbol{p}_1}) \right]$$



Current flow with electrons in Copper

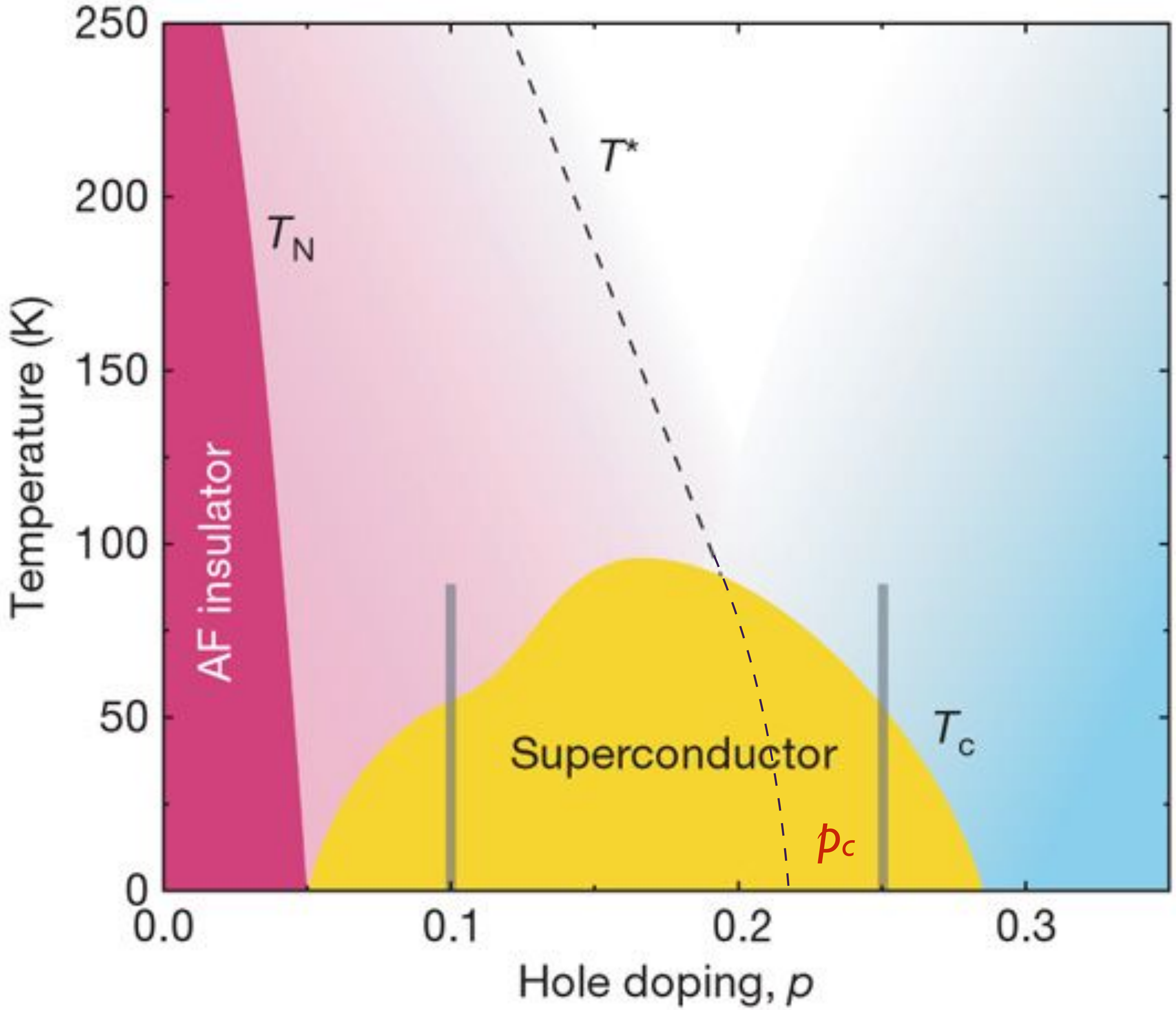


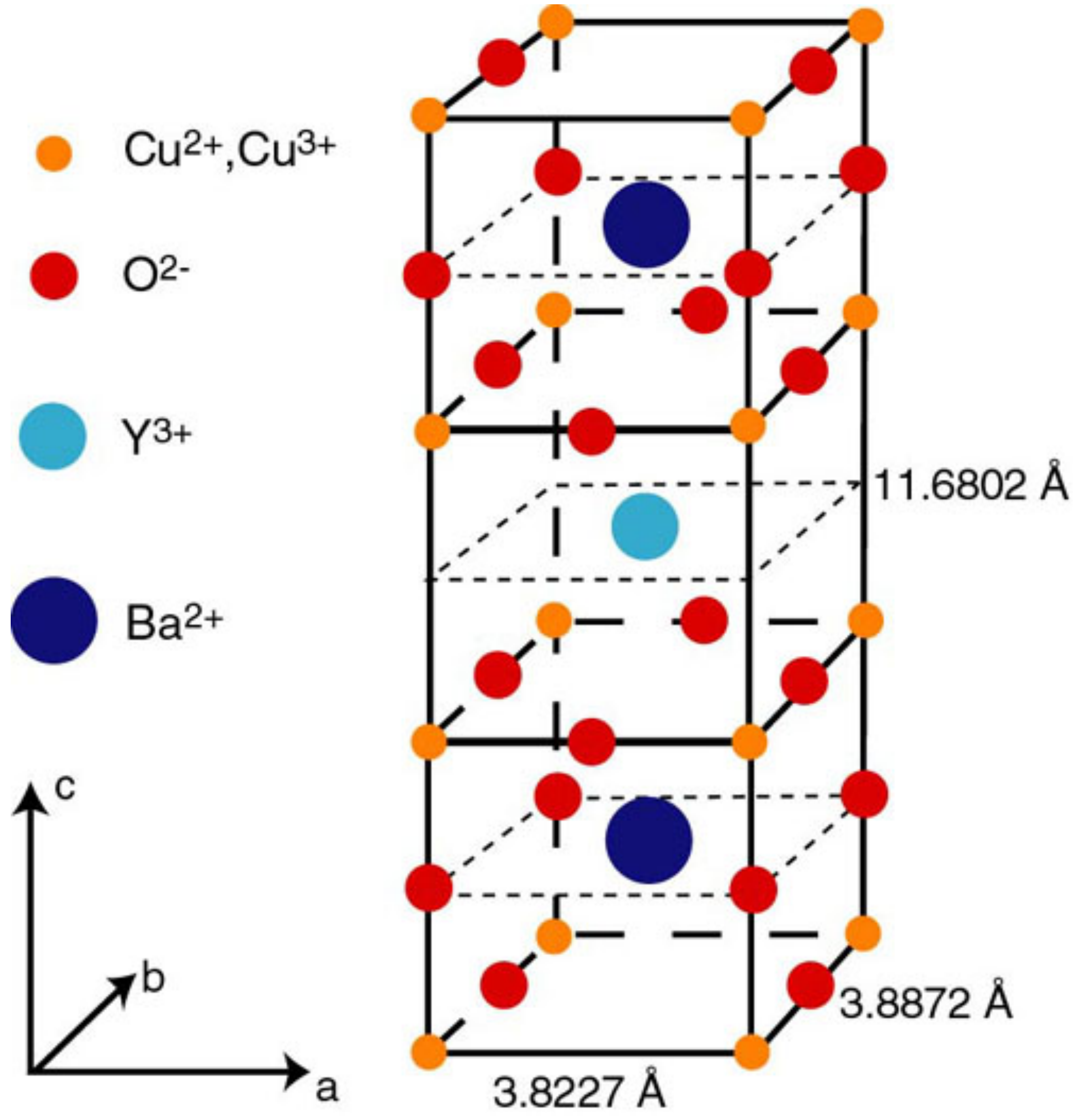
Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

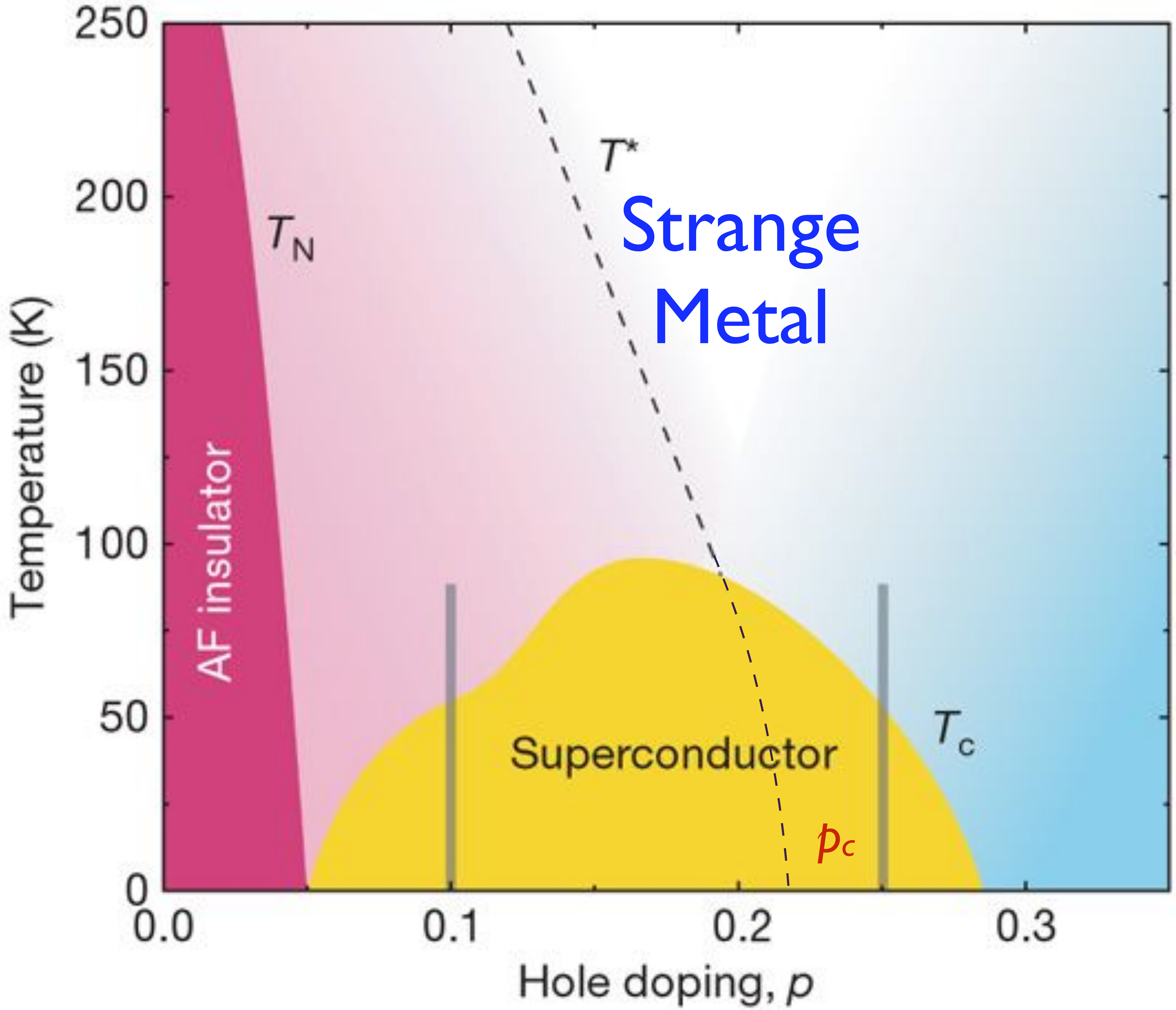
The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

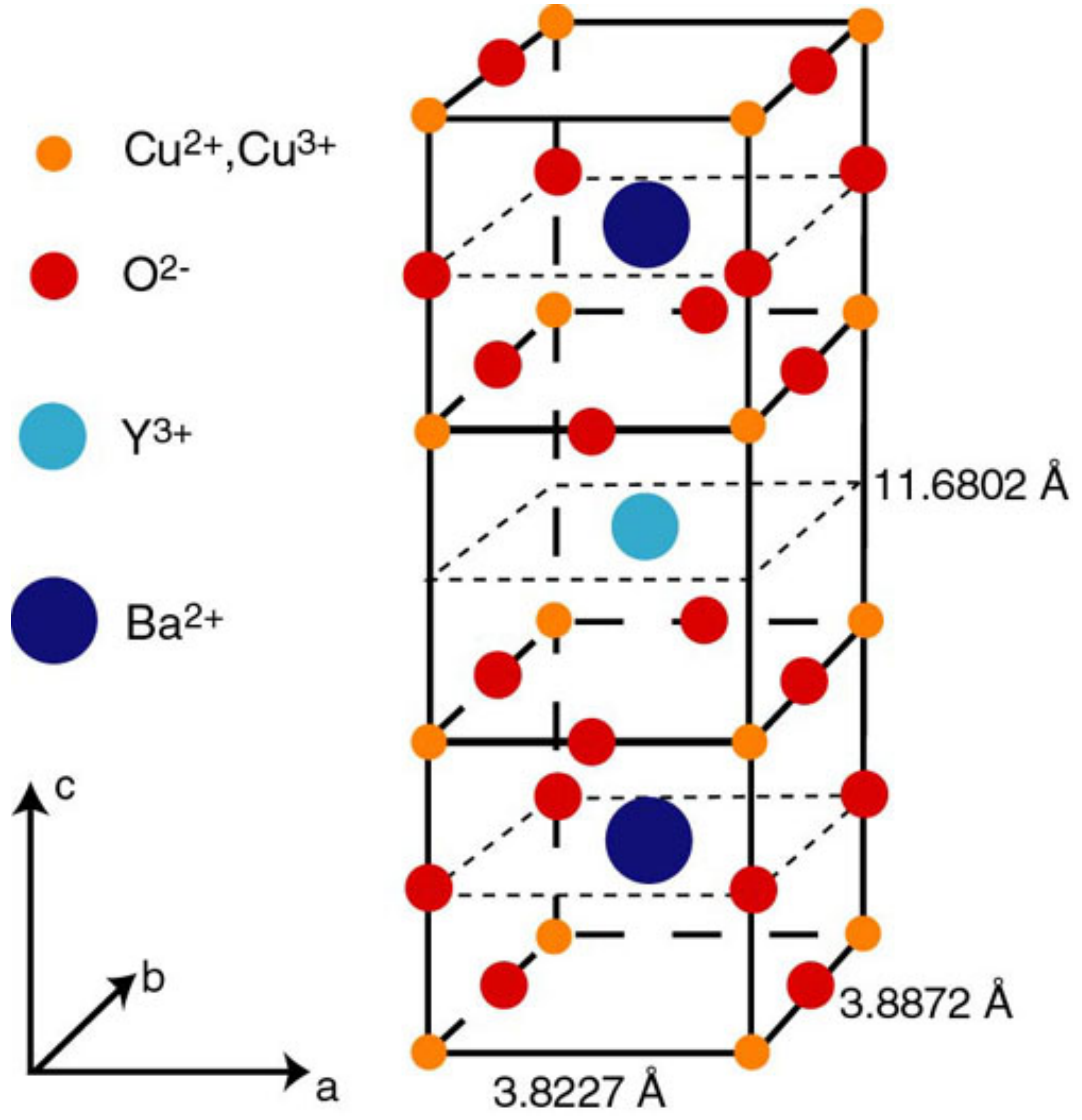
The long scattering time implies that individual electrons are well-defined.

The motion of electrons is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å


Temperature (K)
250
200
150
100
50
0
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Strange
Metal
Superconductor
T_c
p_c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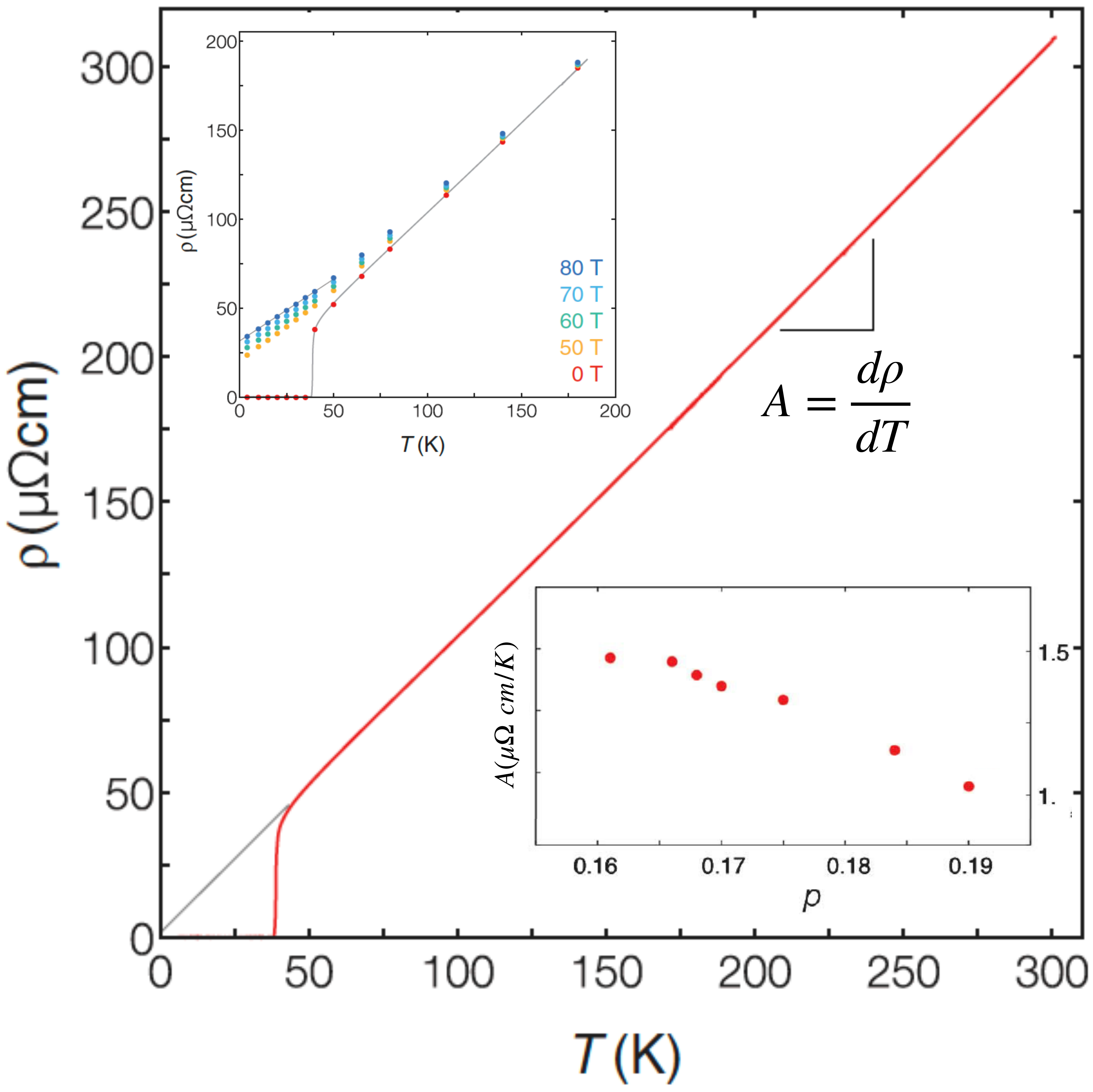
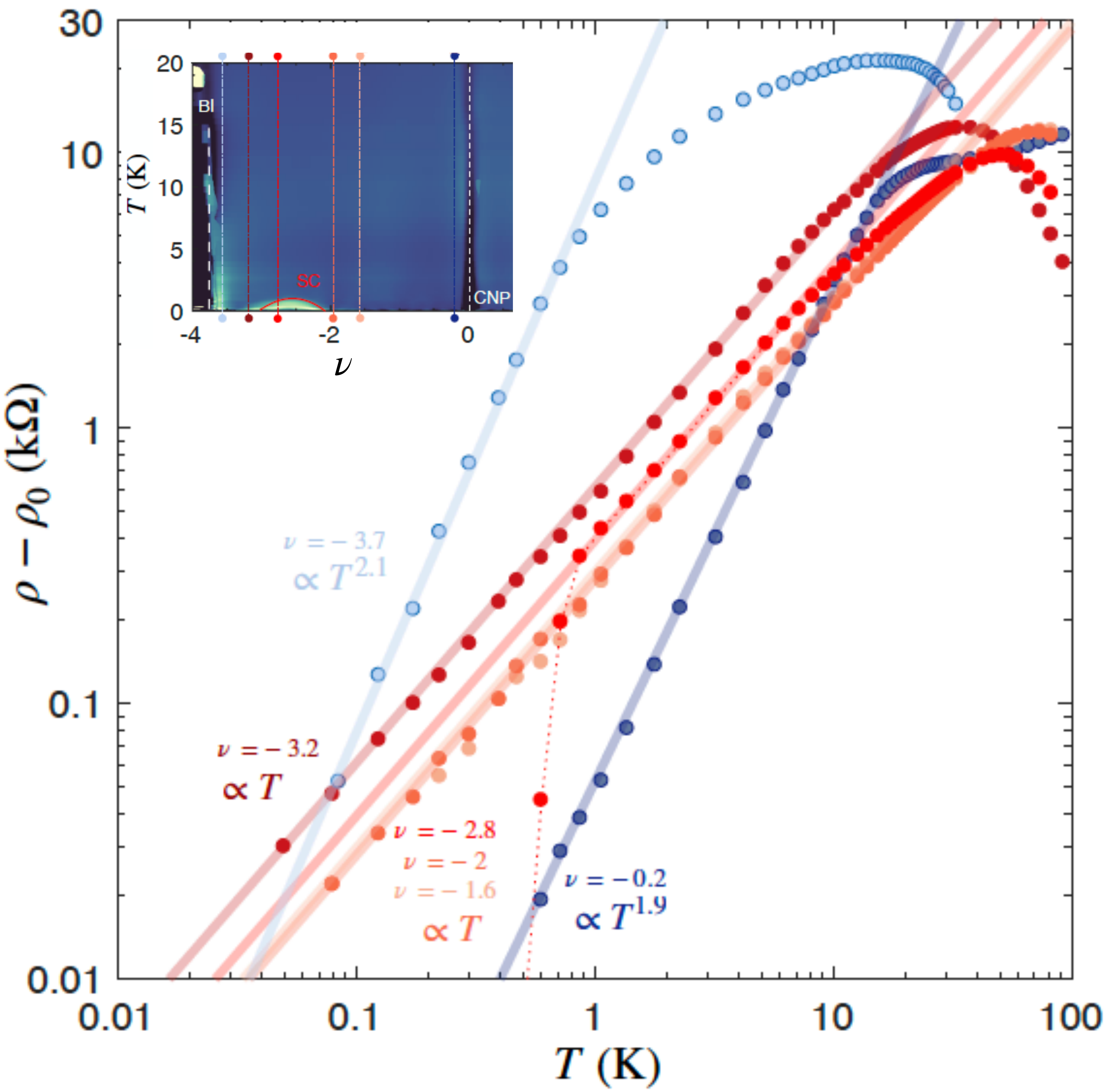


LSCO: Giraldo-Gallo et al. 2018

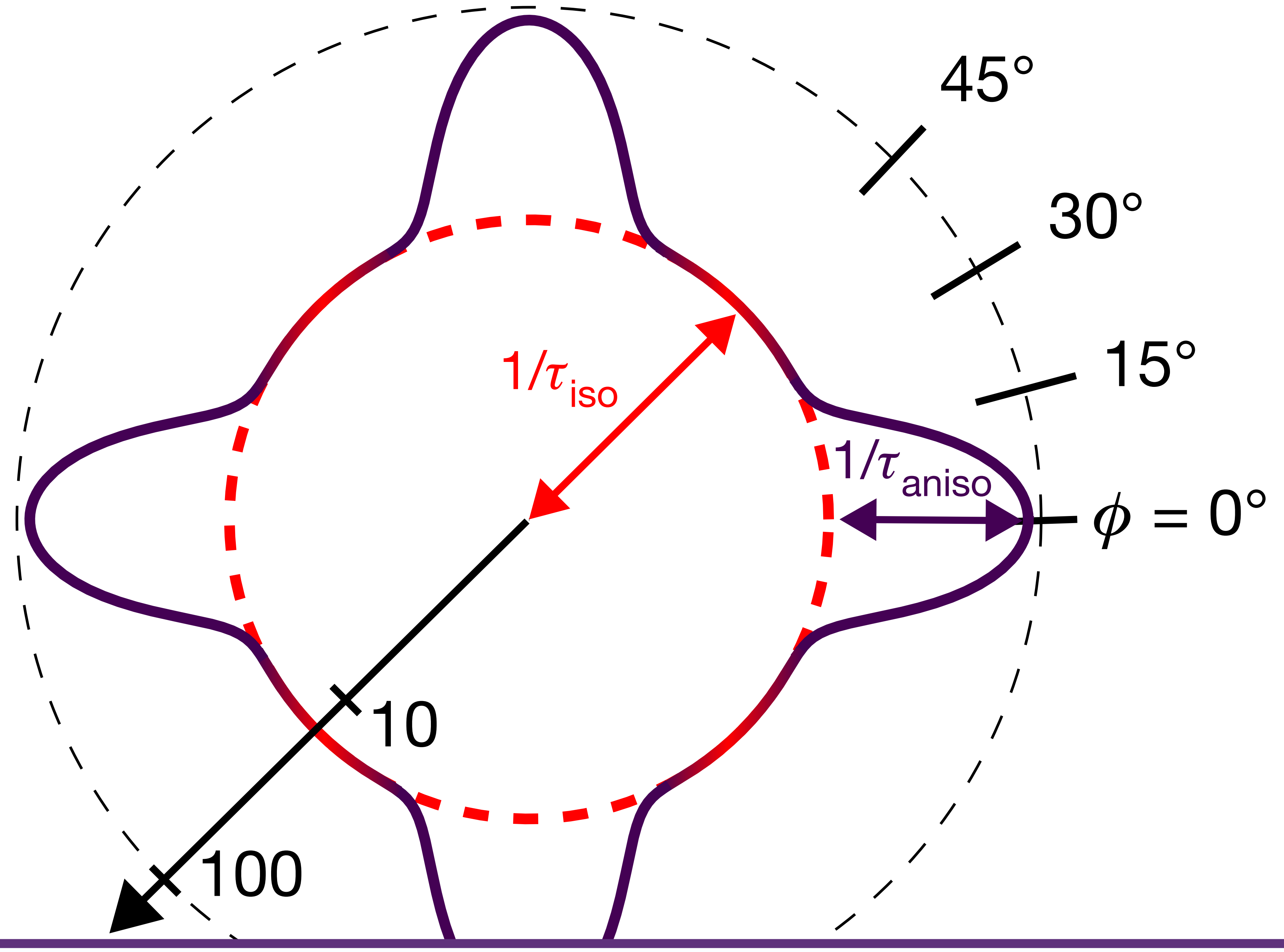


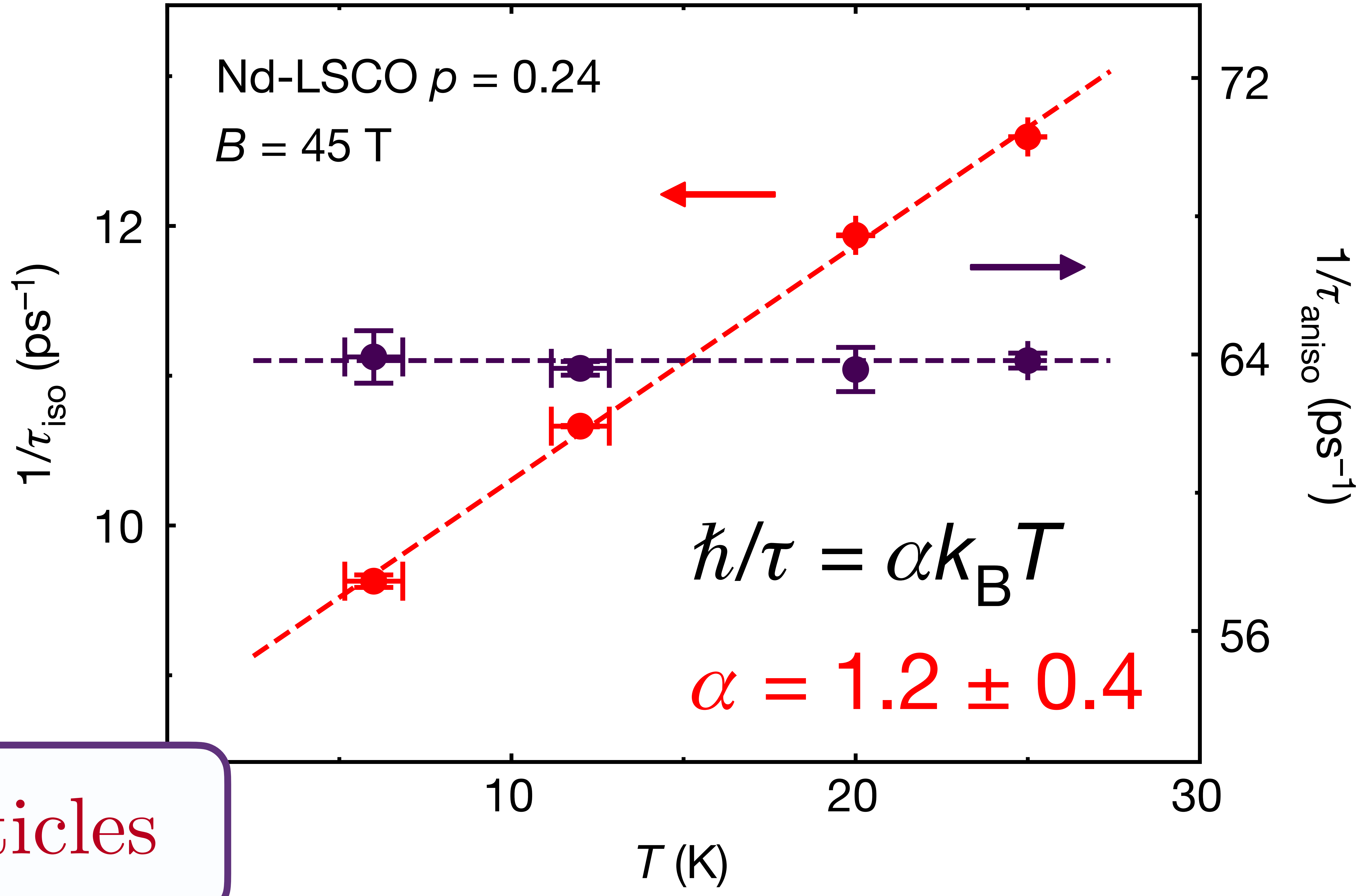
MATBG: Jaoui et al. 2021

Linear-in temperature resistivity from an isotropic Planckian scattering rate

Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw





Current flow without quasiparticles

No Boltzmann-Landau quasiparticle description $\Rightarrow$
Many particle quantum entanglement
from quantum interference between "collisions"

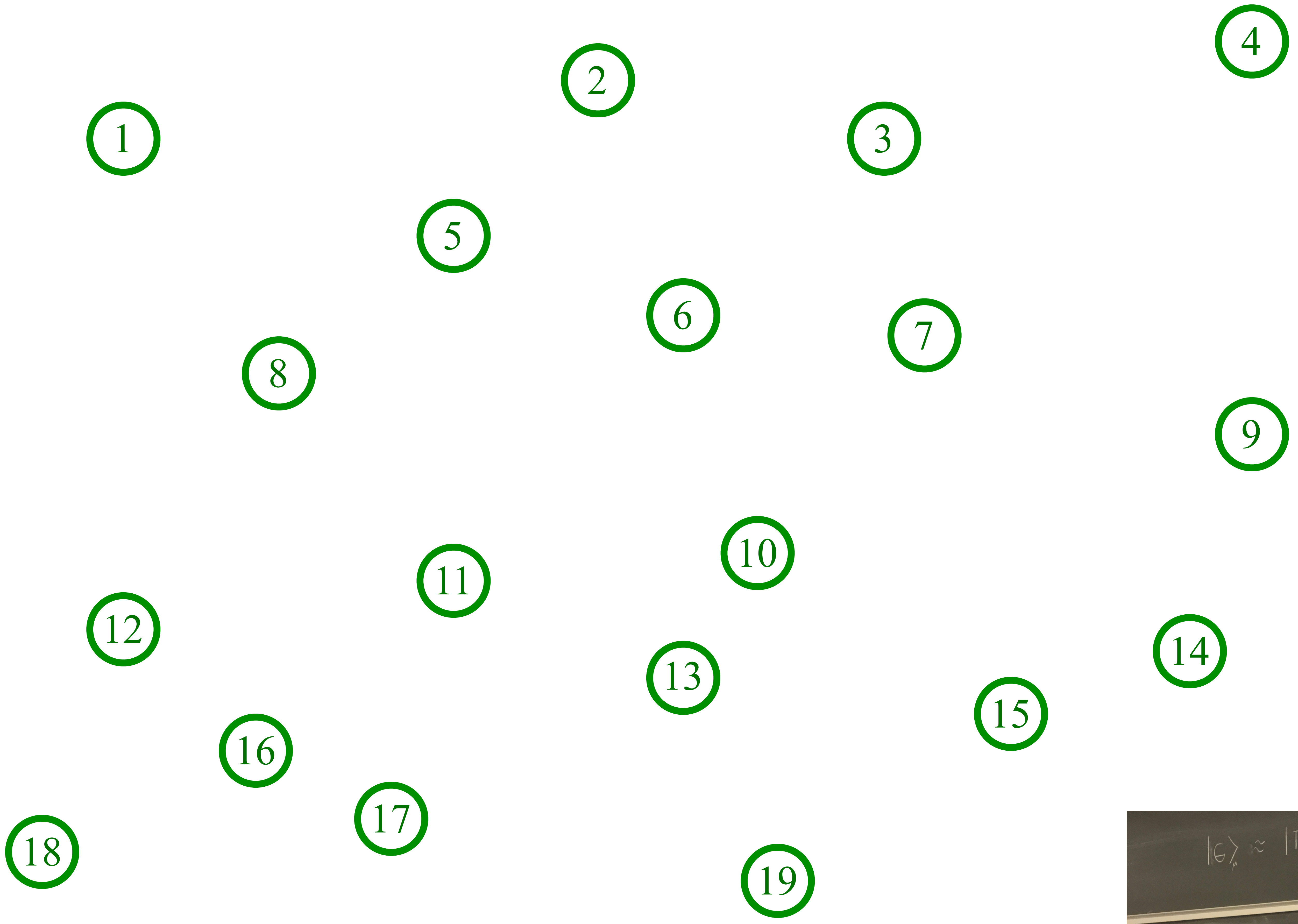
Sachdev-Ye-Kitaev Model

A solvable model of multi-particle entanglement which accounts for quantum interference between successive collisions:

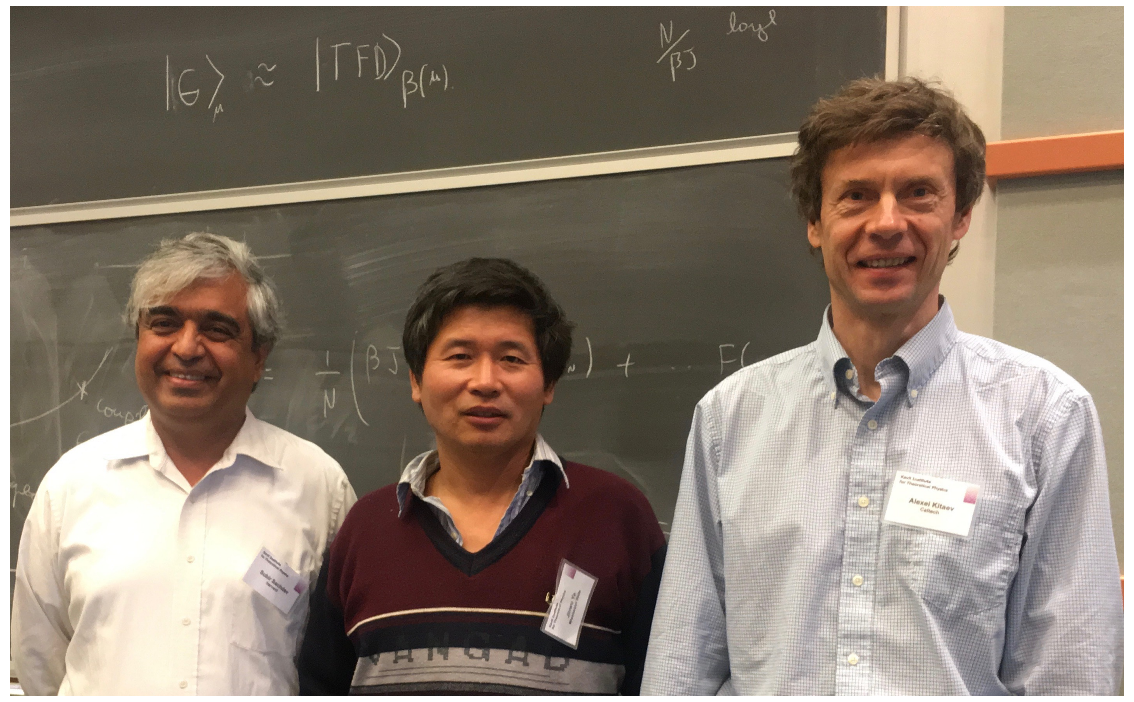
leading to a metal with no particle-like excitations

The SYK model

Sachdev, Ye (1993); Kitaev (2015)



Pick a set of random positions



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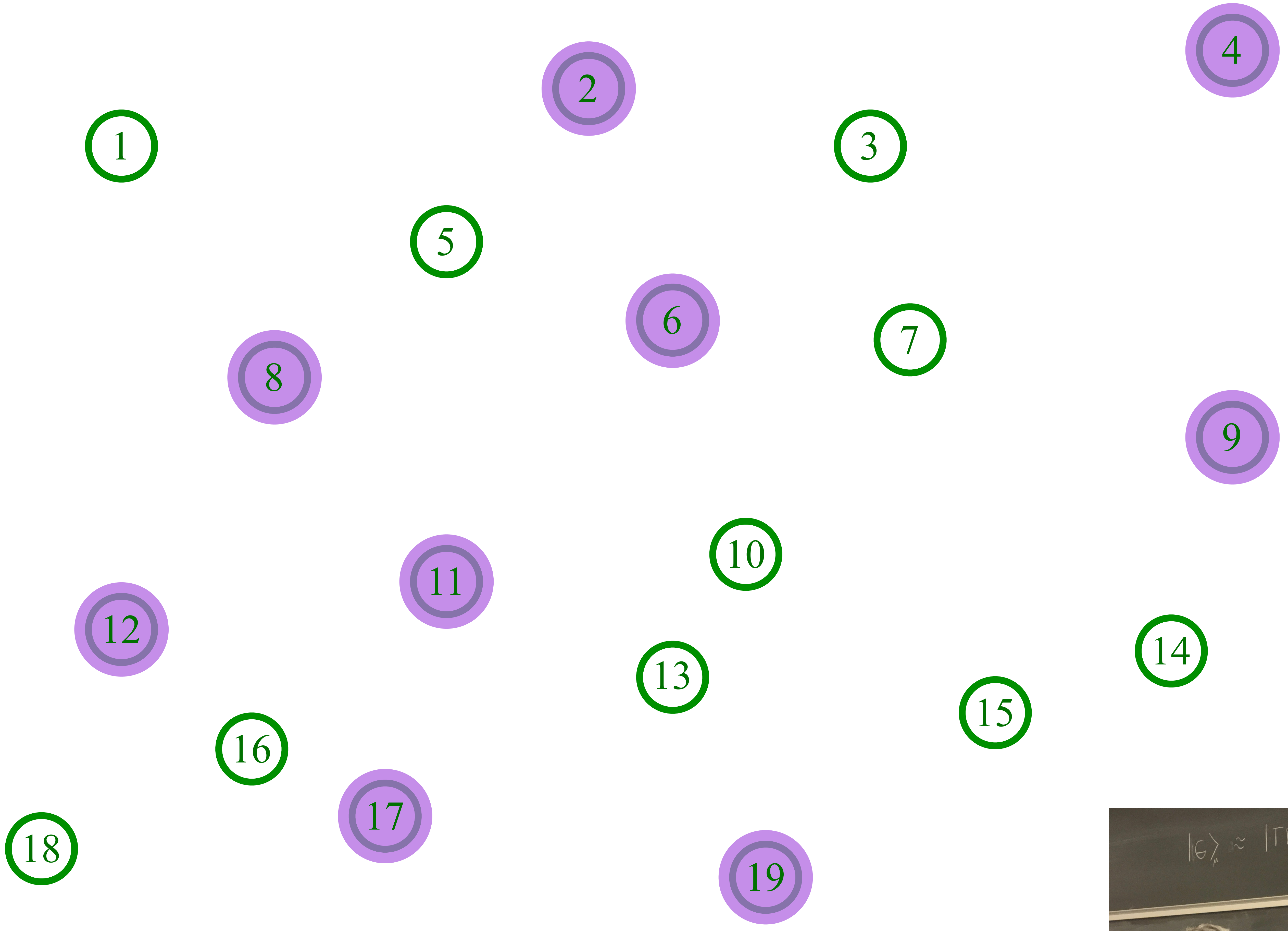
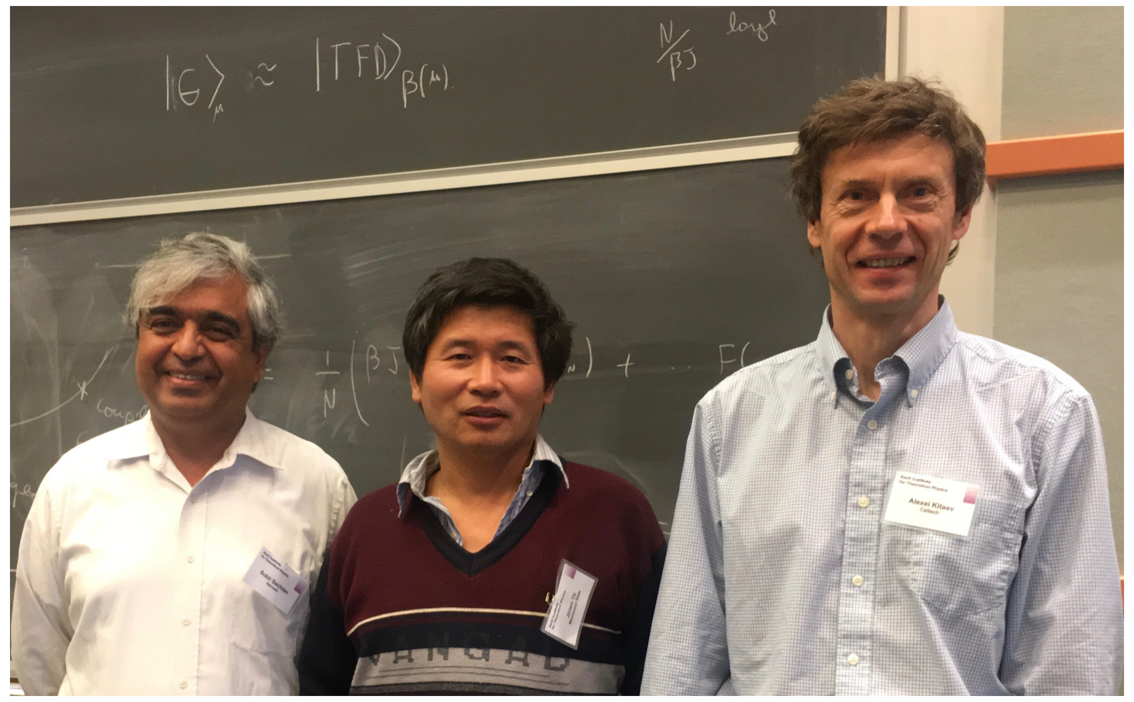


Place electrons randomly on some sites



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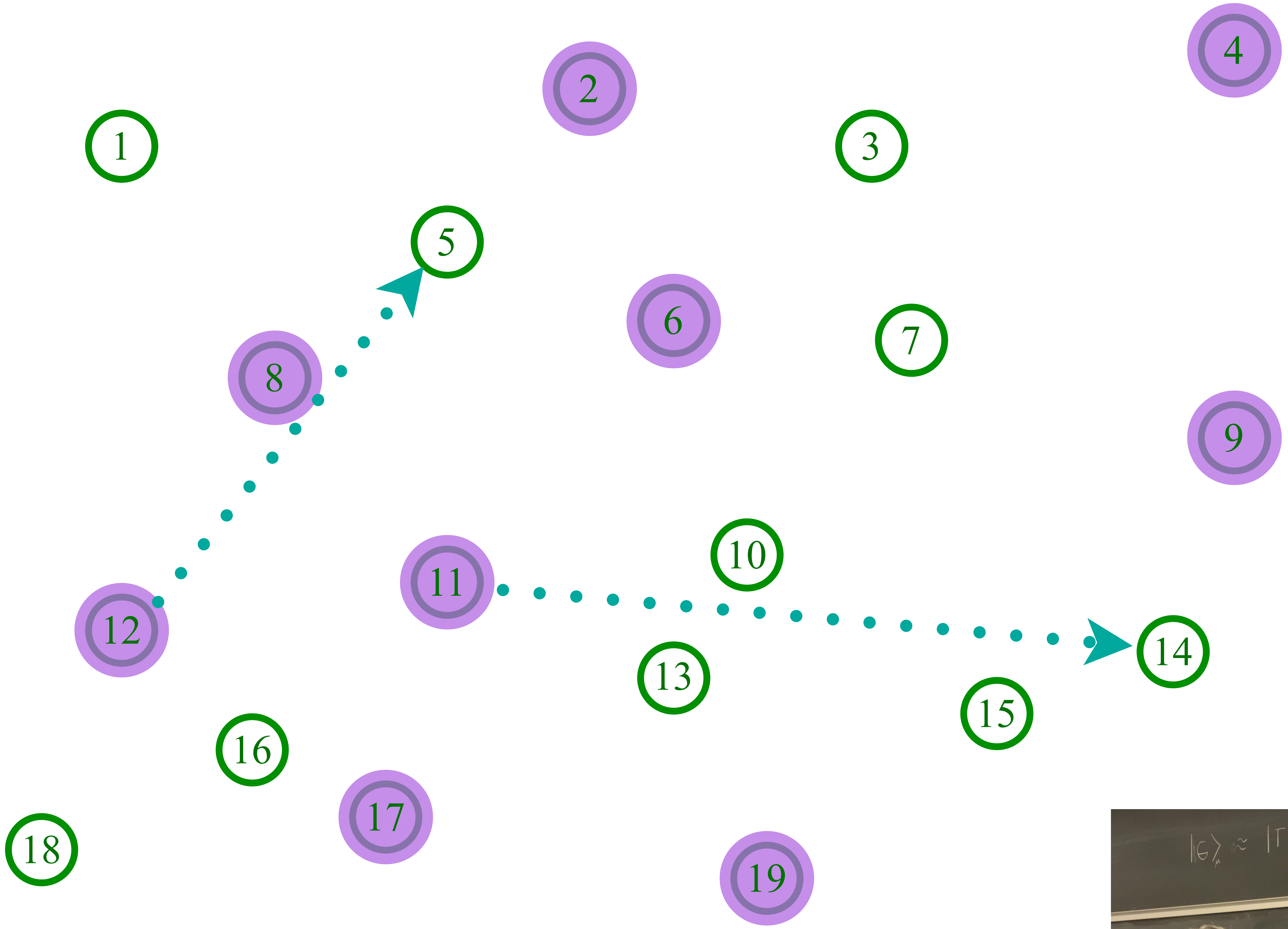
Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



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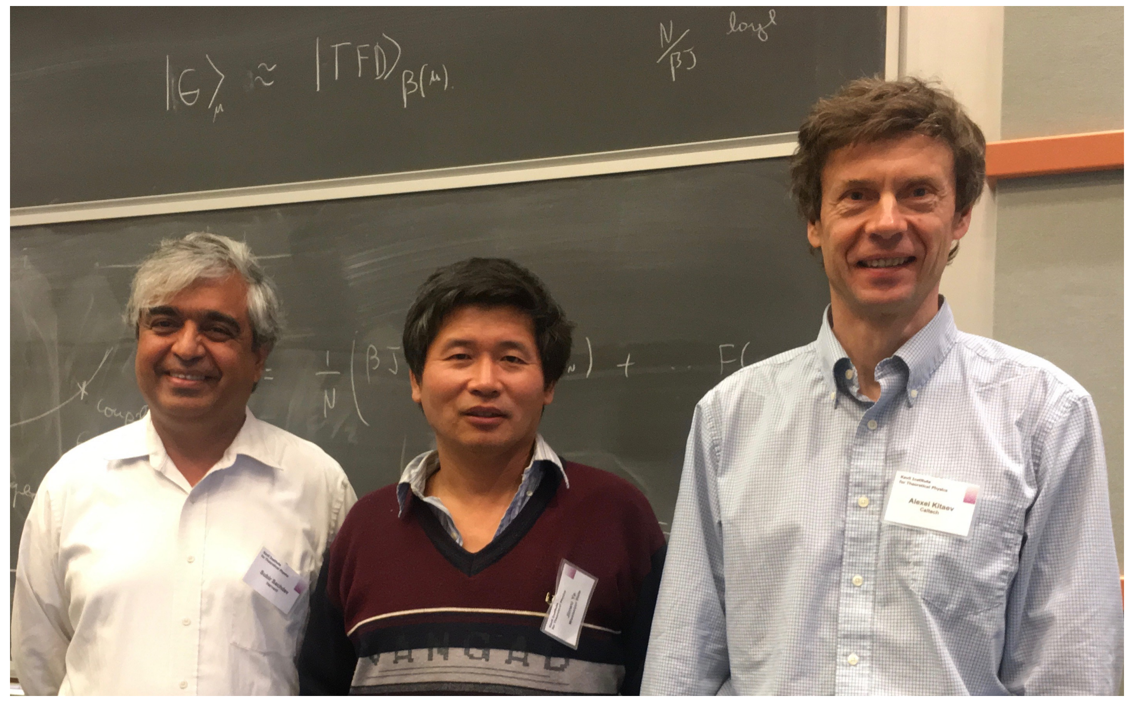

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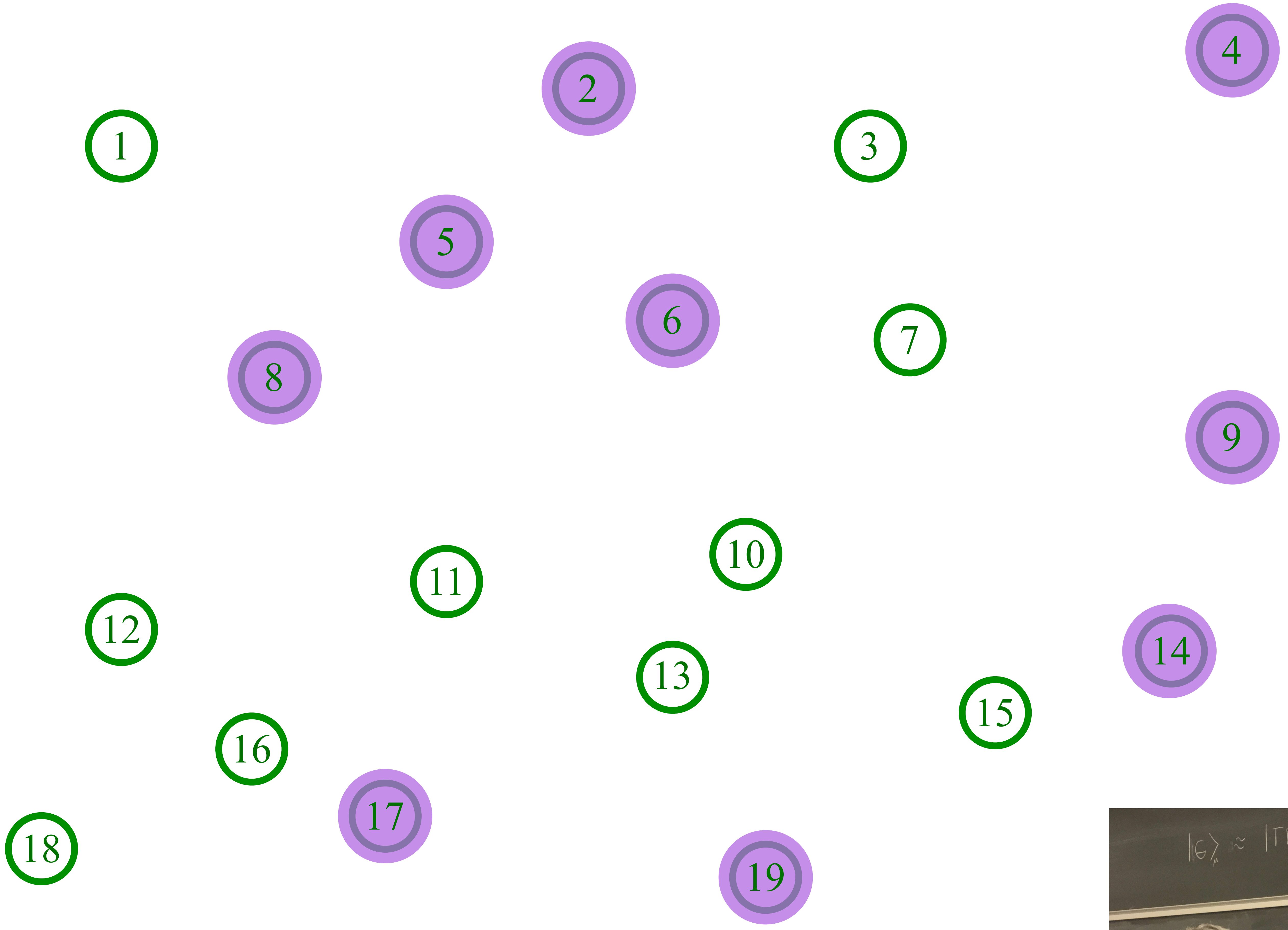
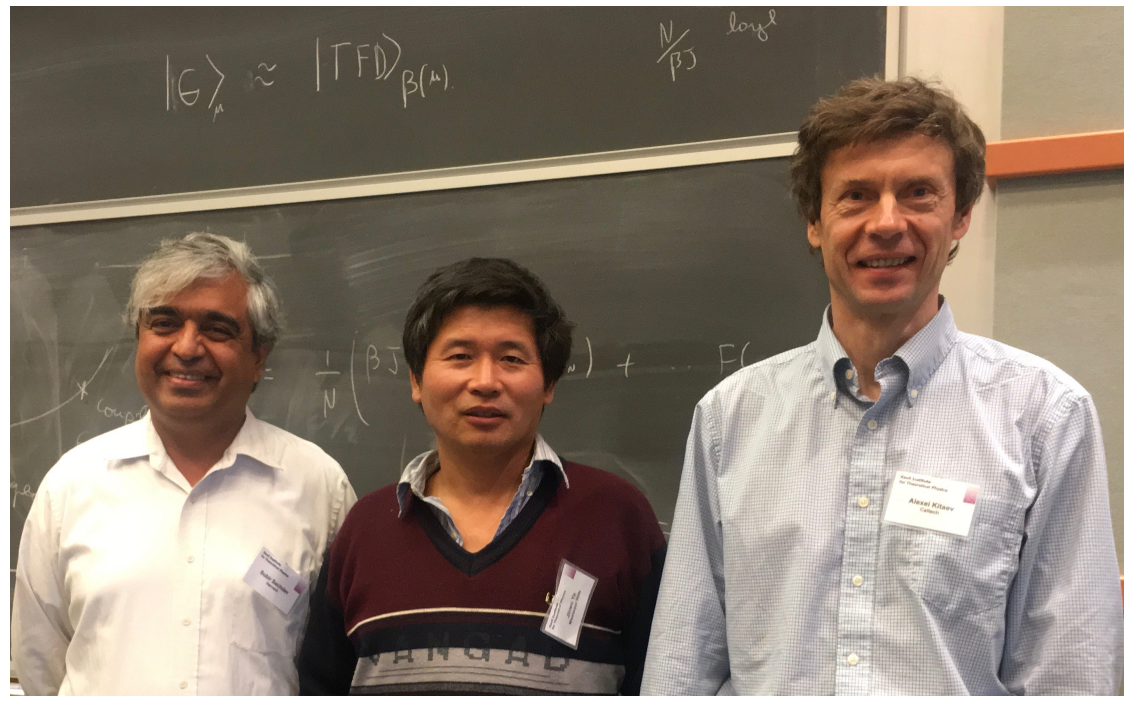


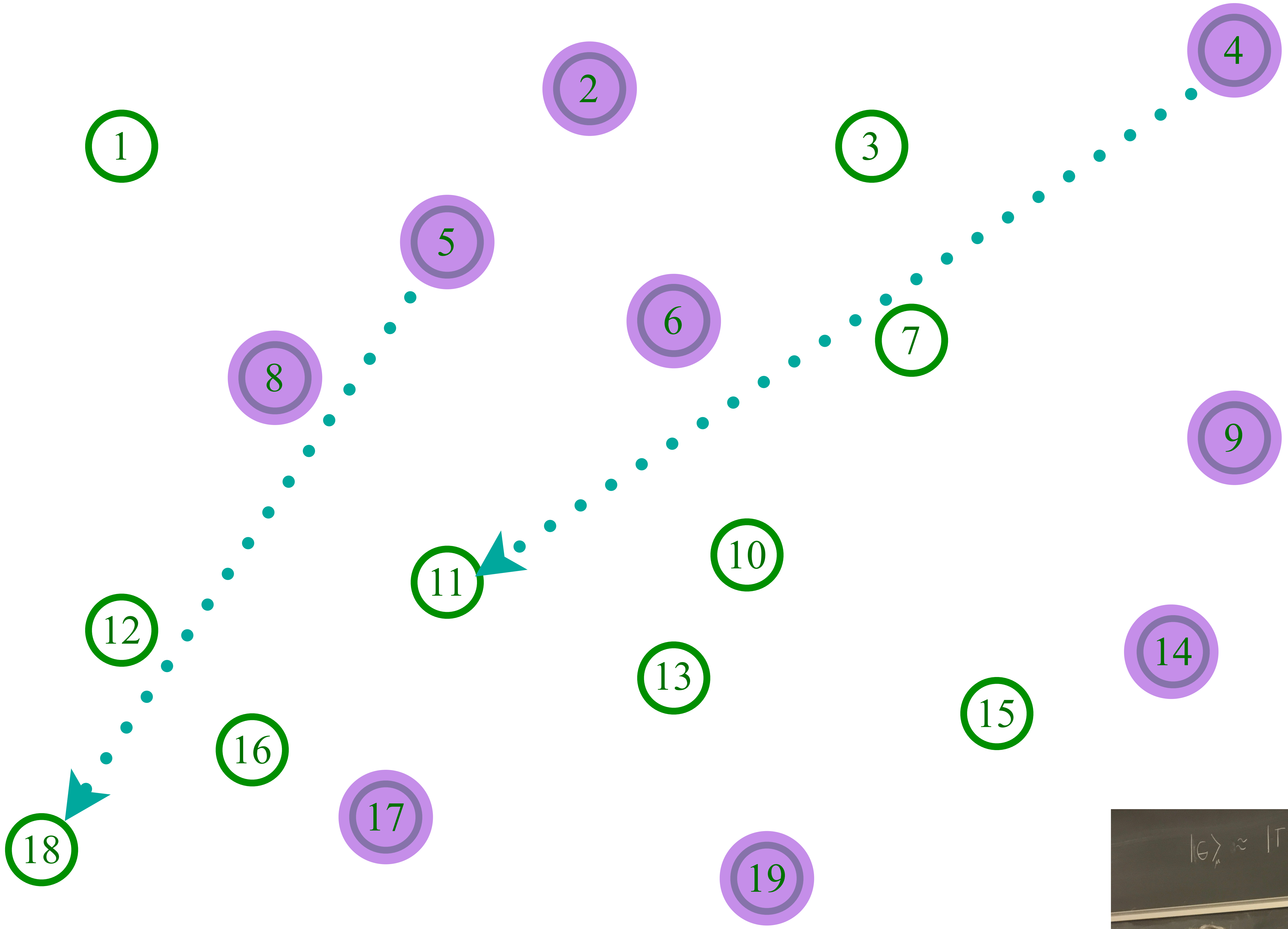
Entangle electrons pairwise randomly



The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{4,5;11,18}$



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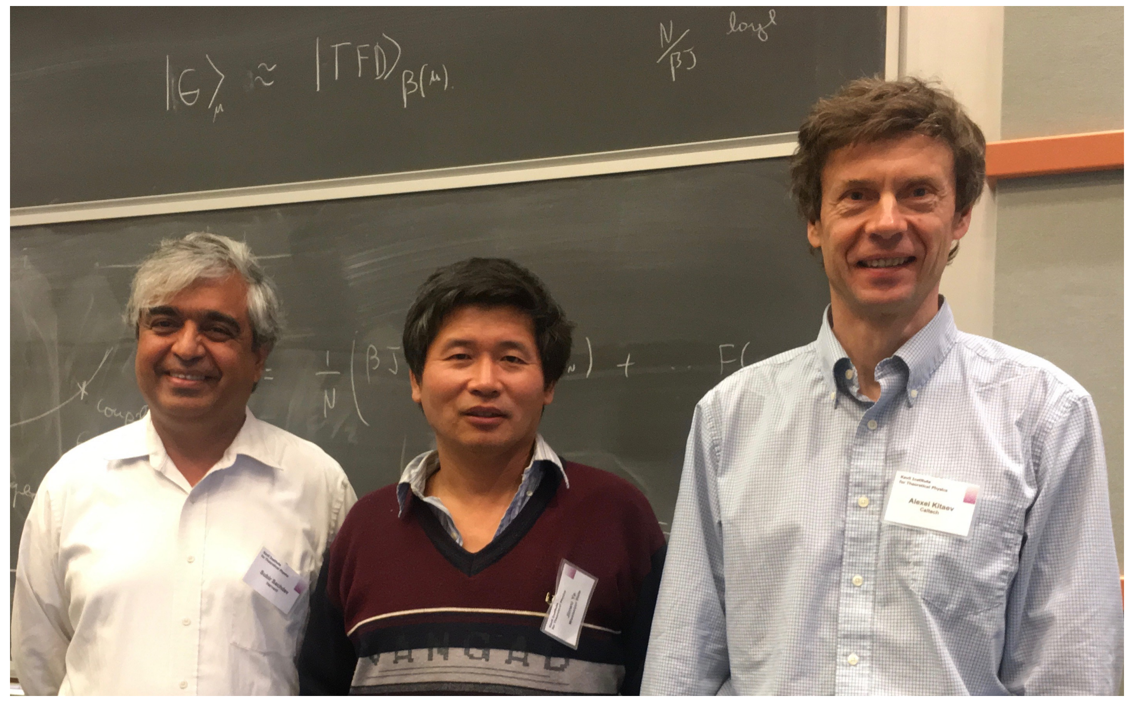

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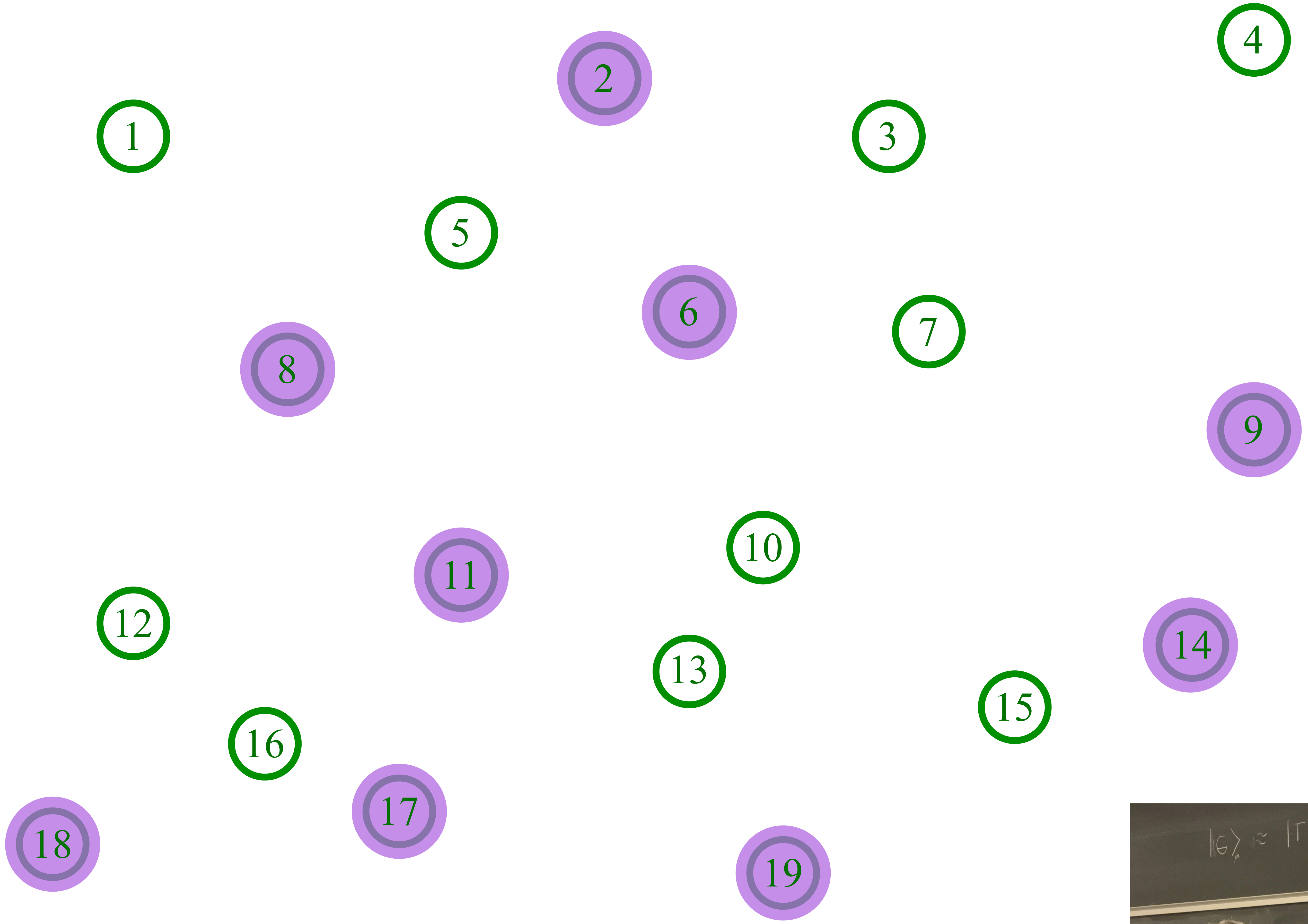

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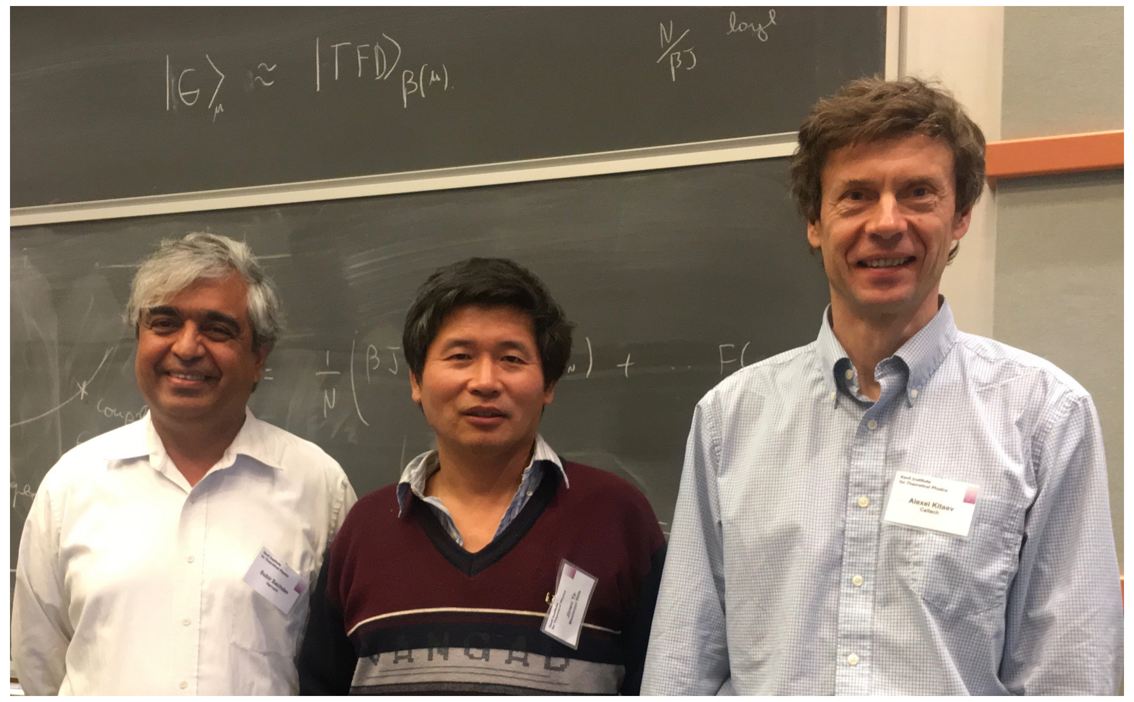
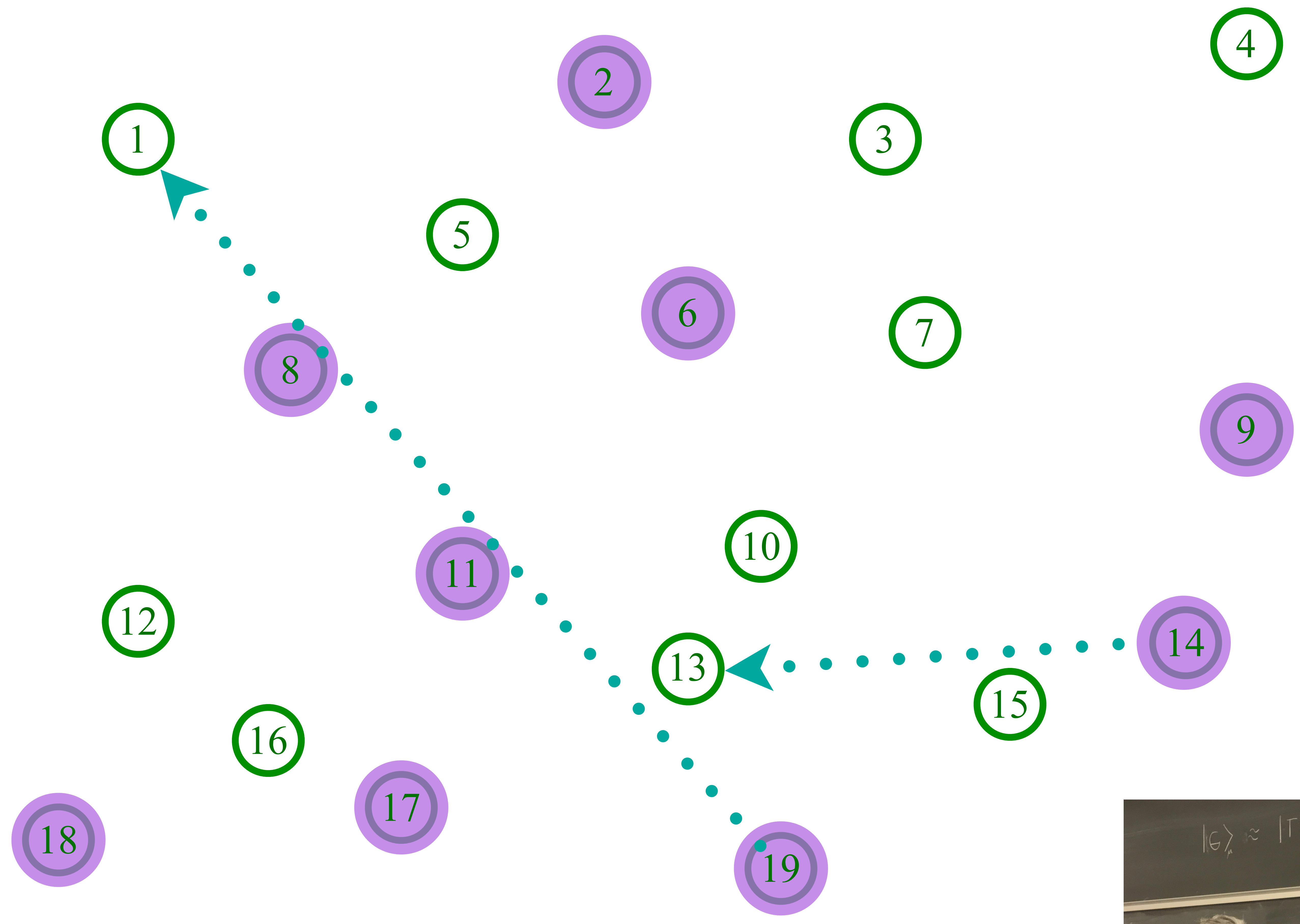


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{14,19;1,13}$



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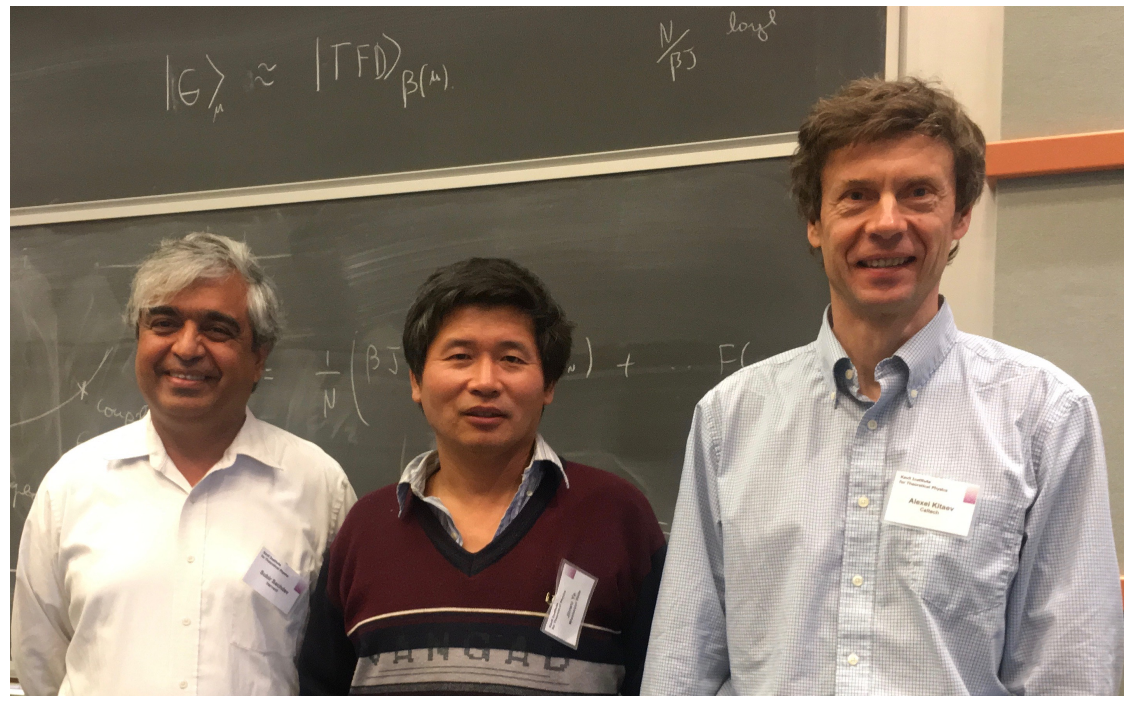

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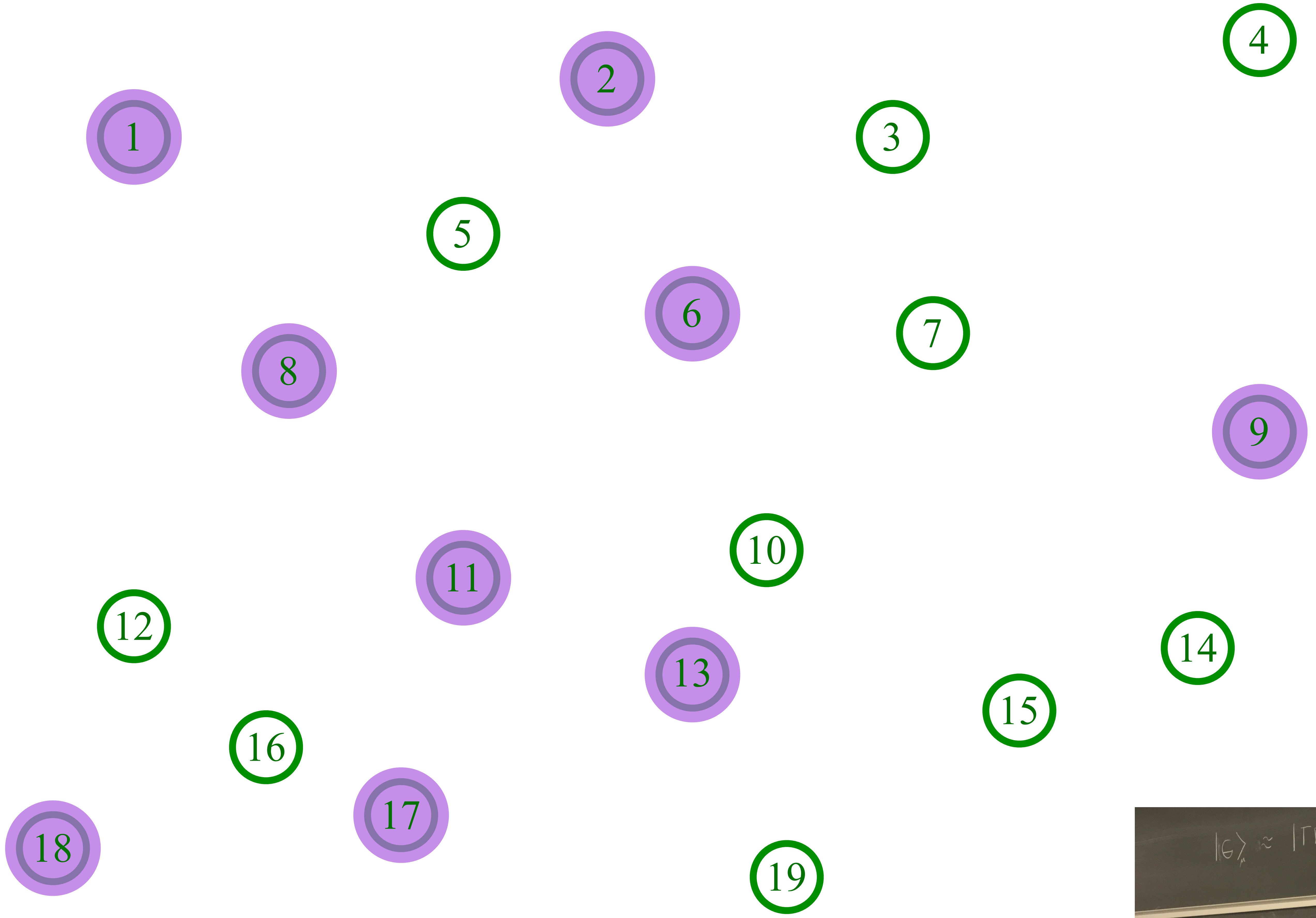

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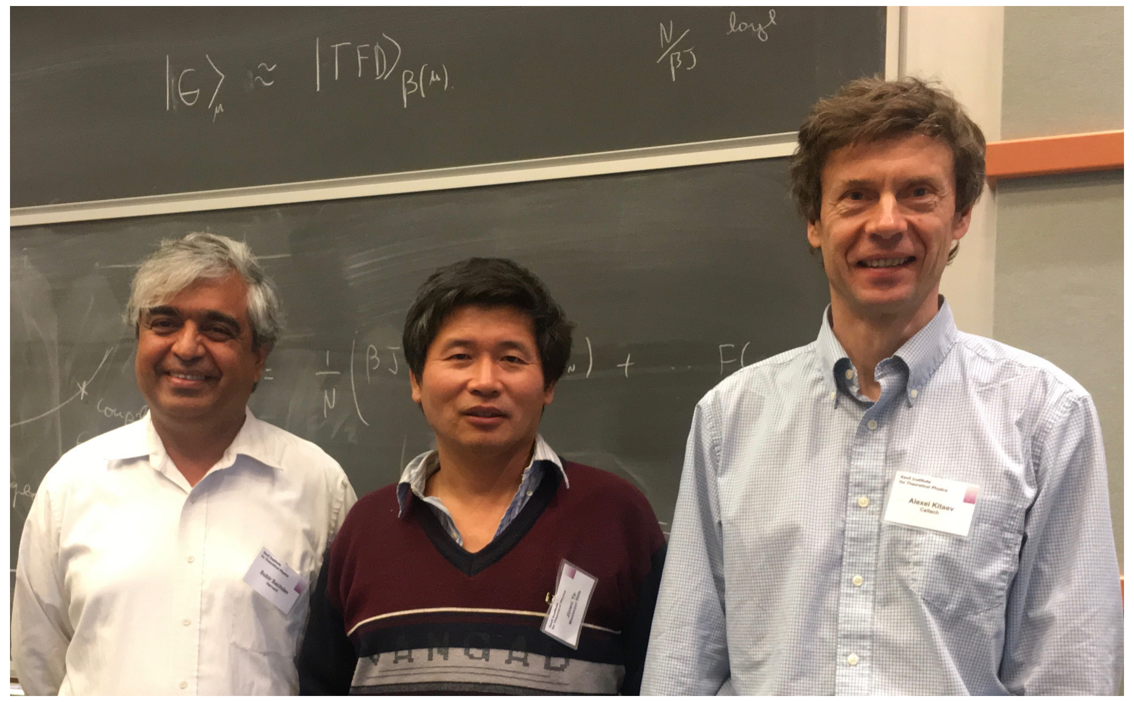
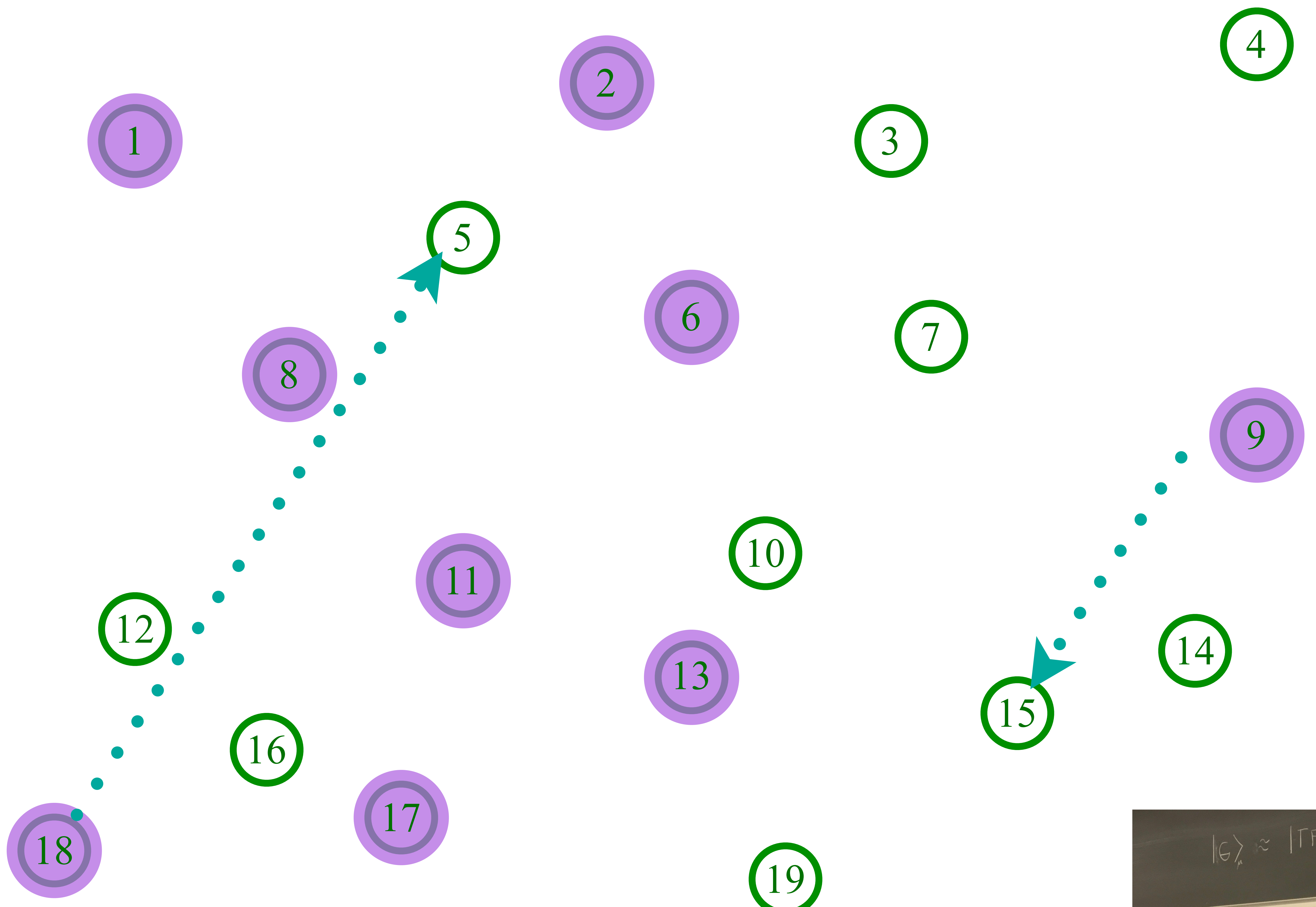


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{9,18;5,15}$



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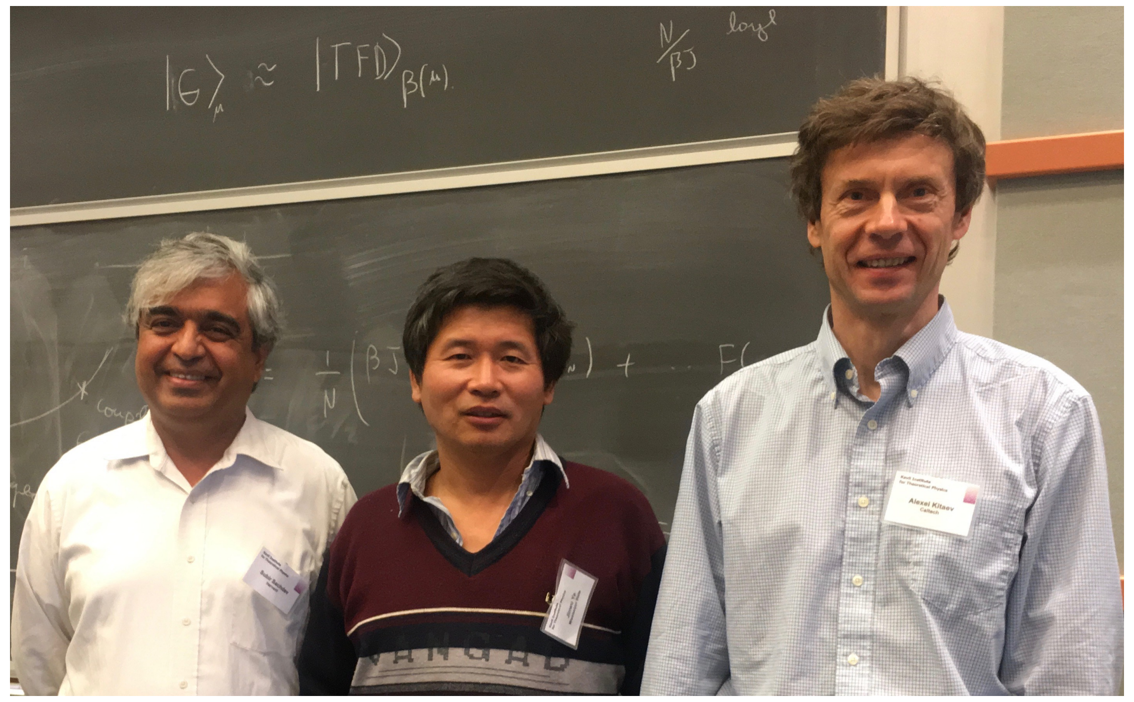

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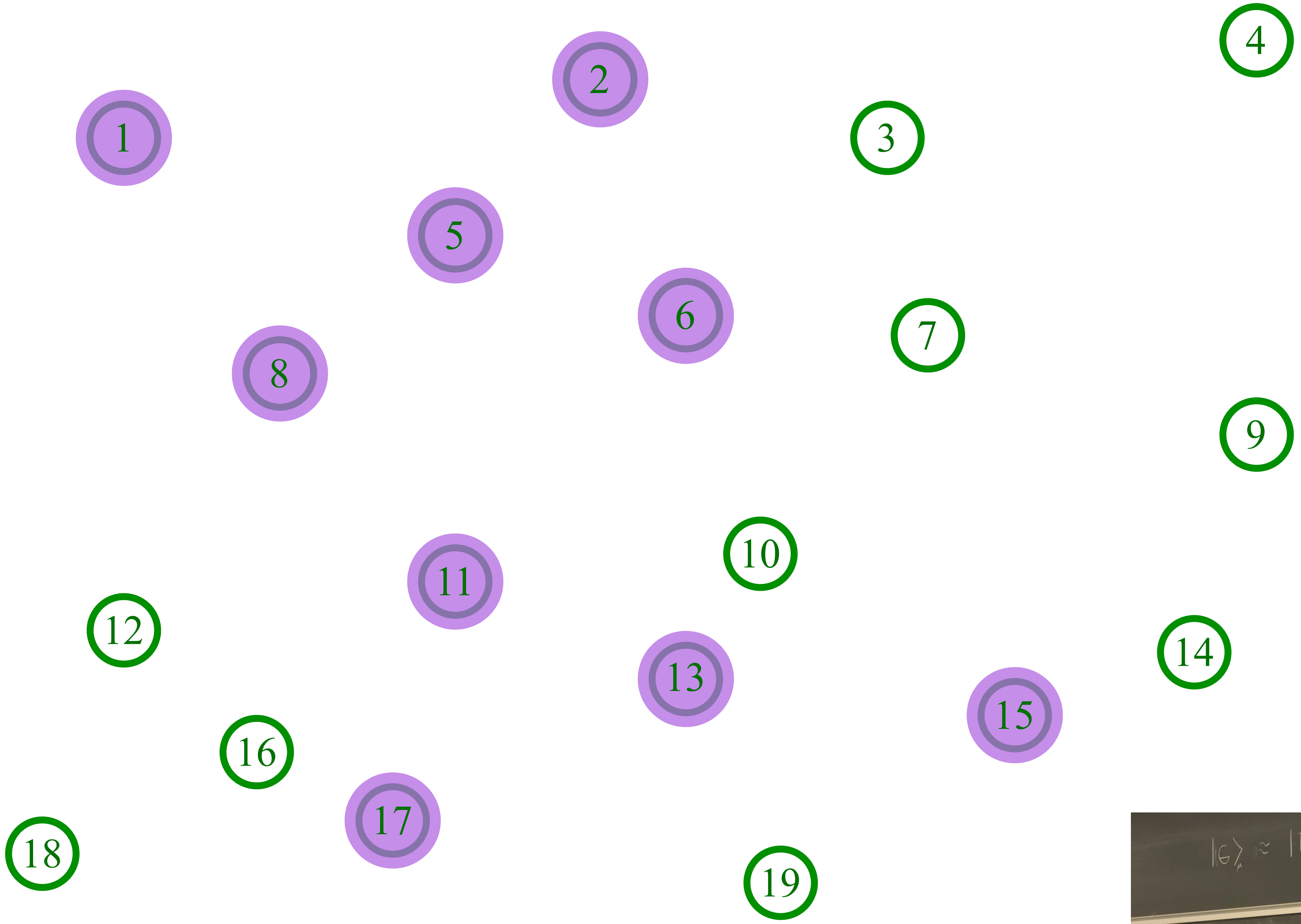

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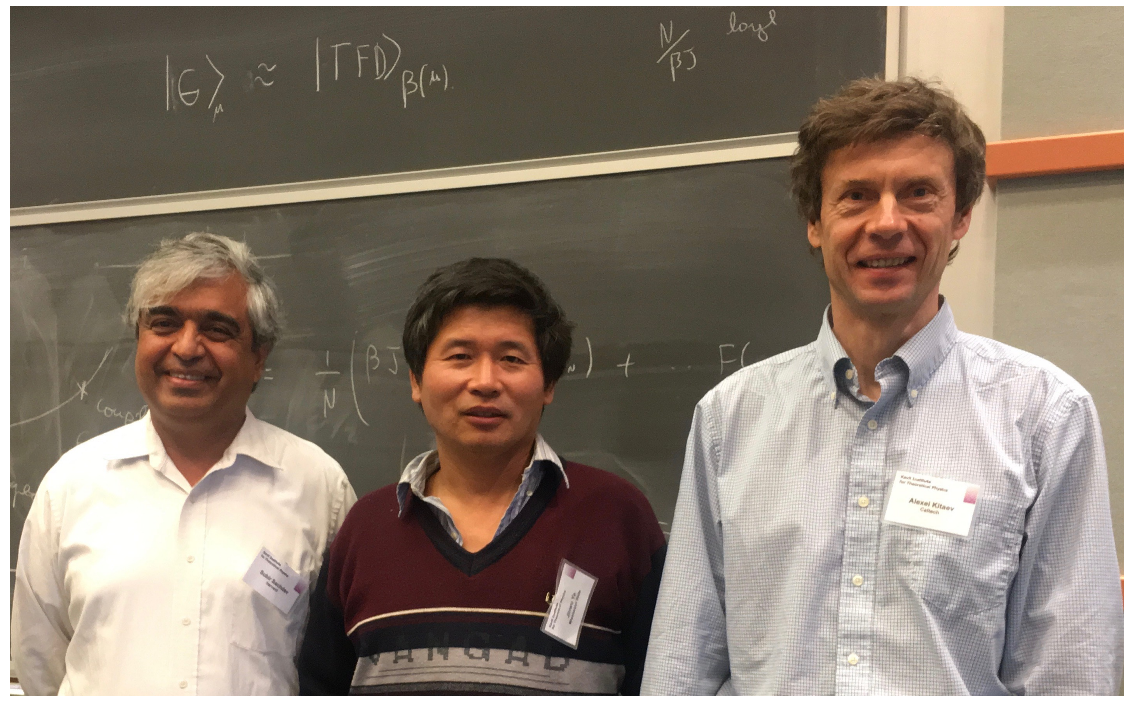
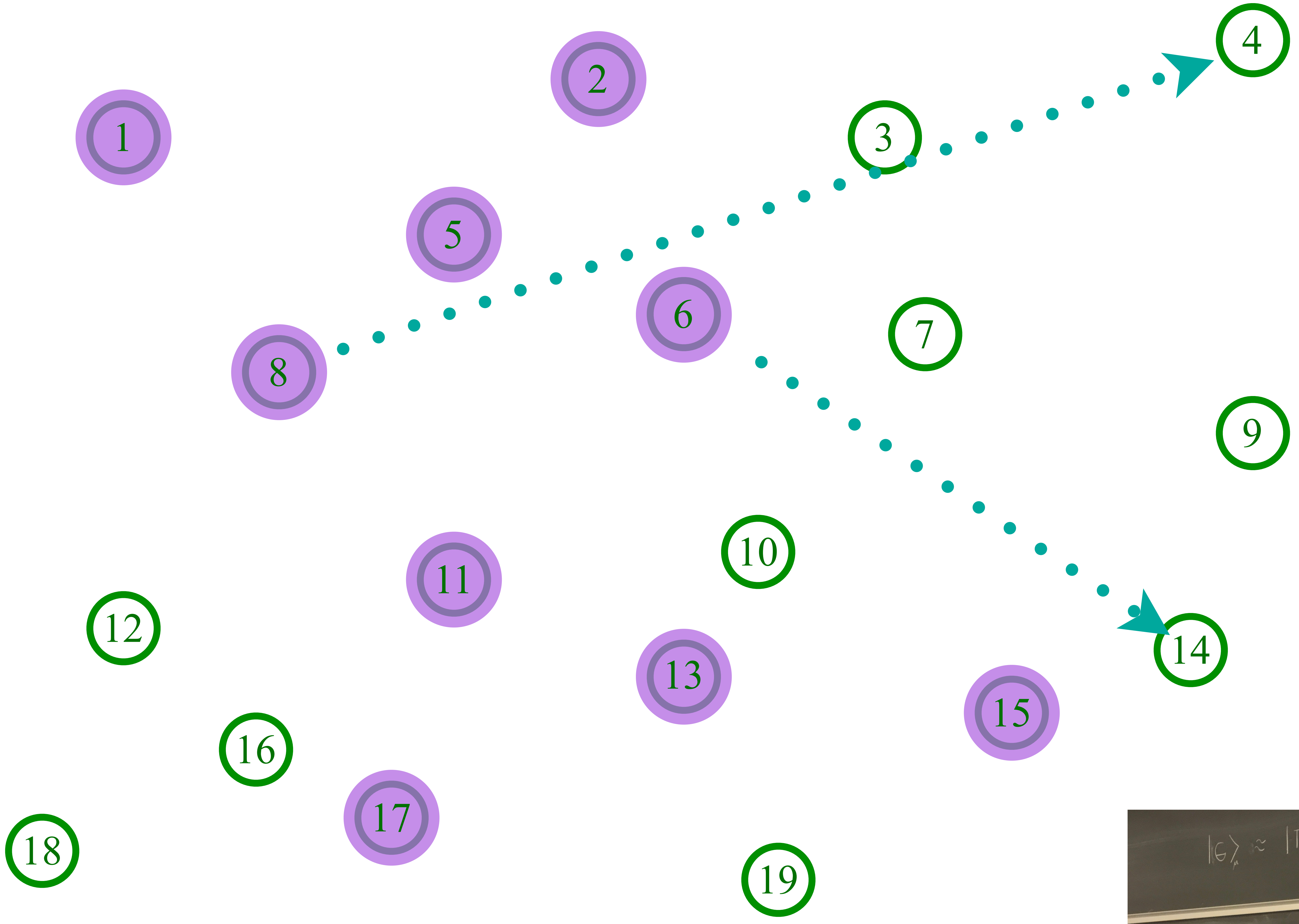


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



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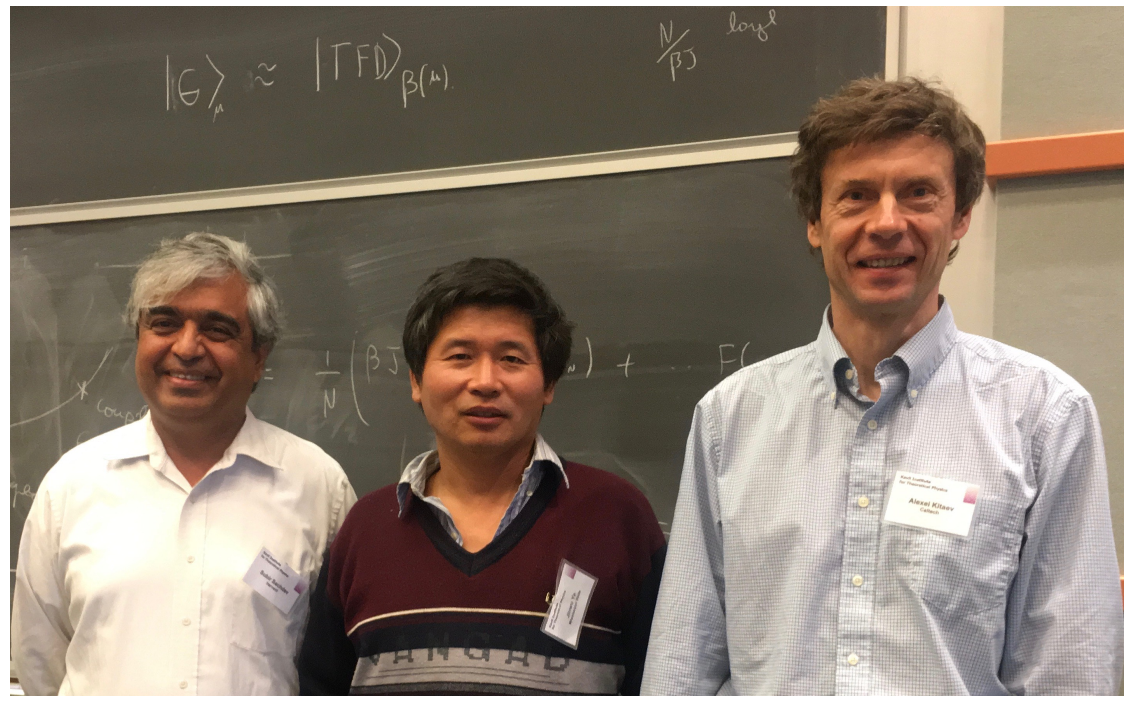

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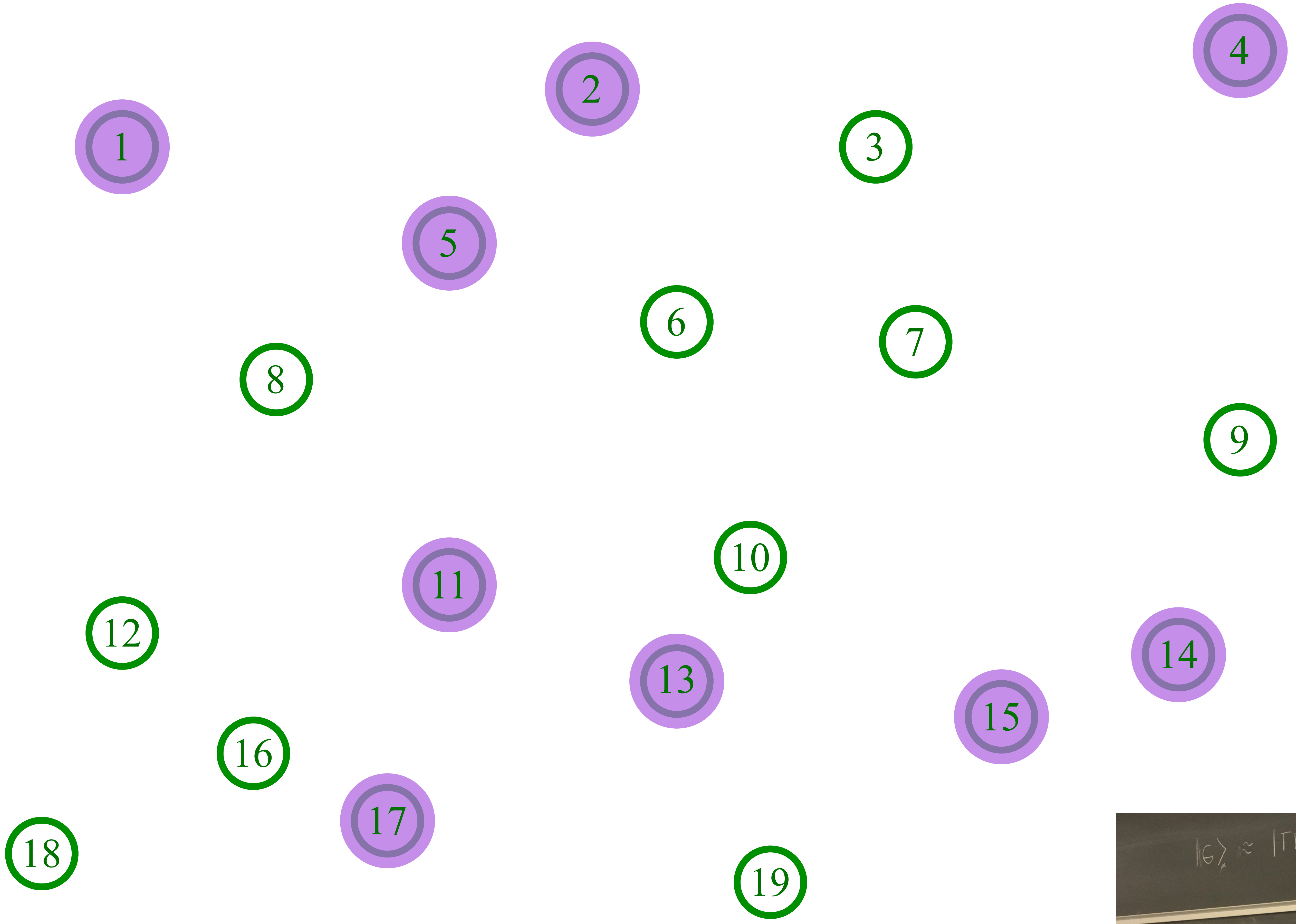

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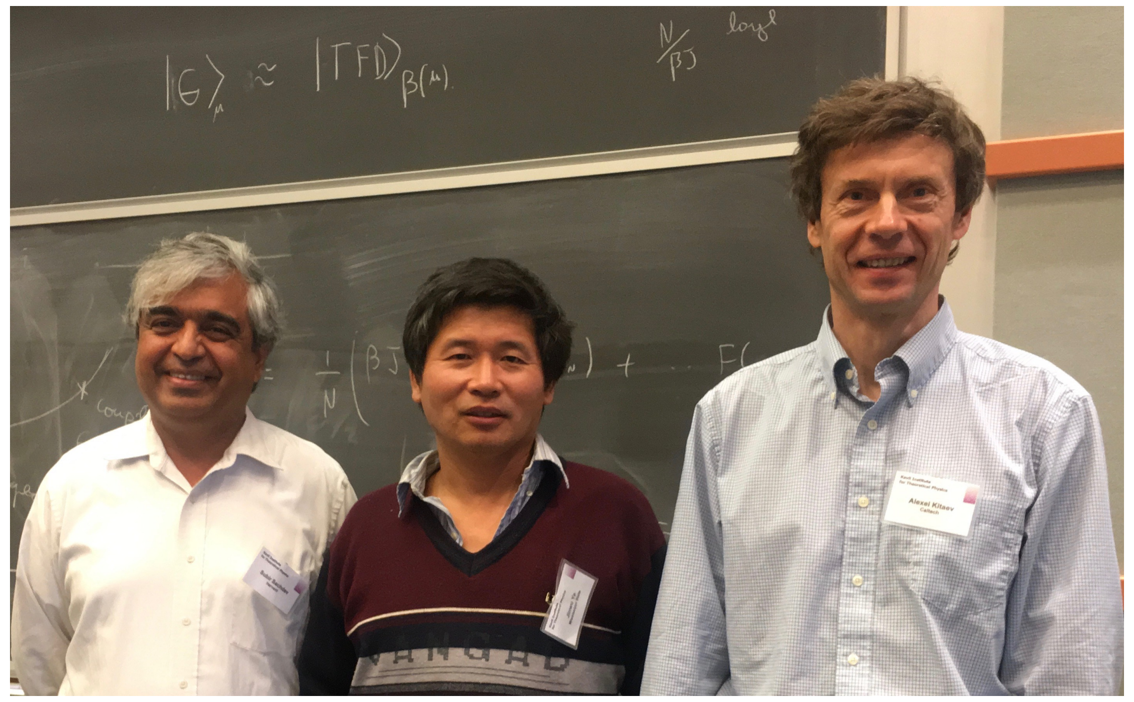


The SYK model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

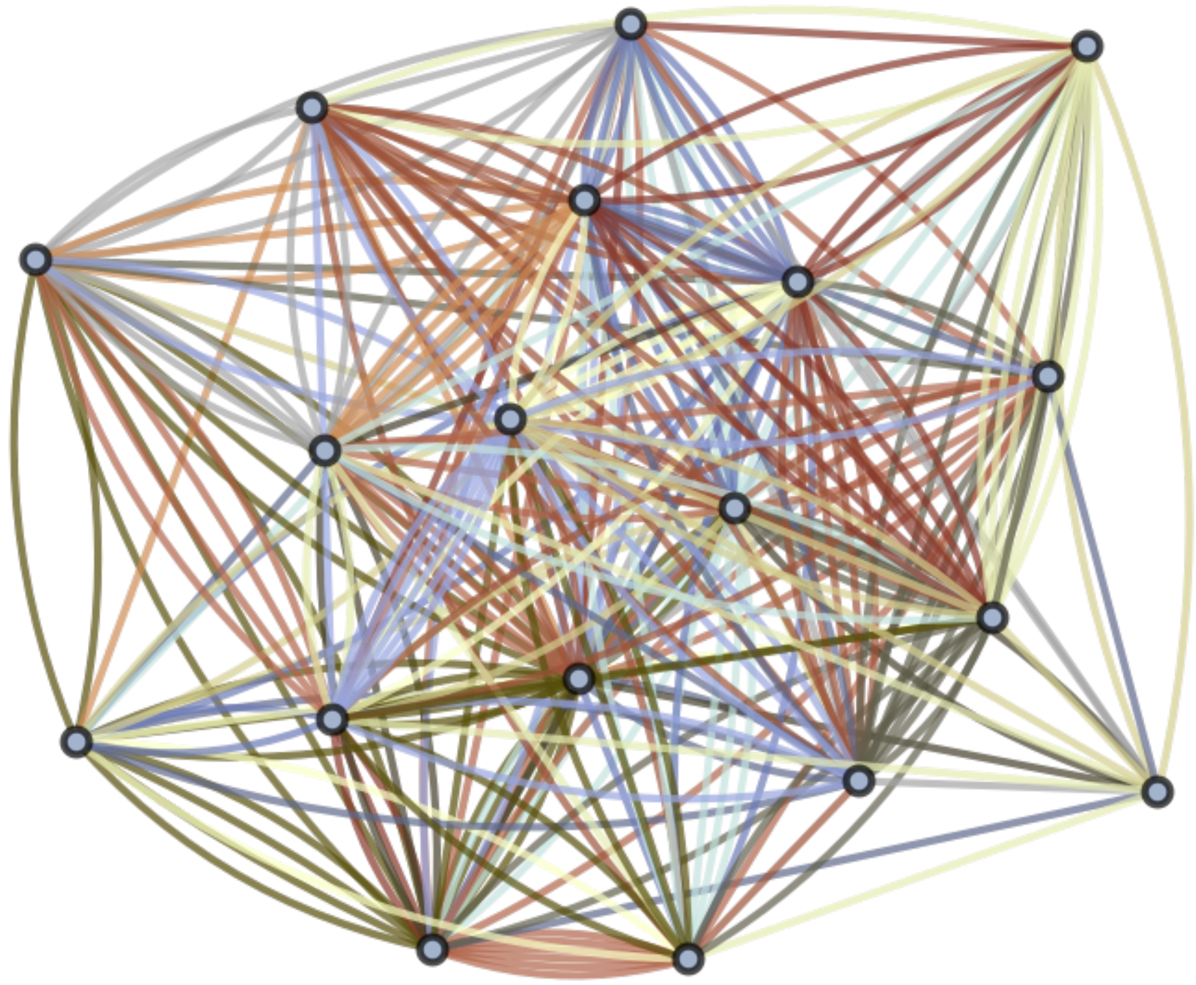
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

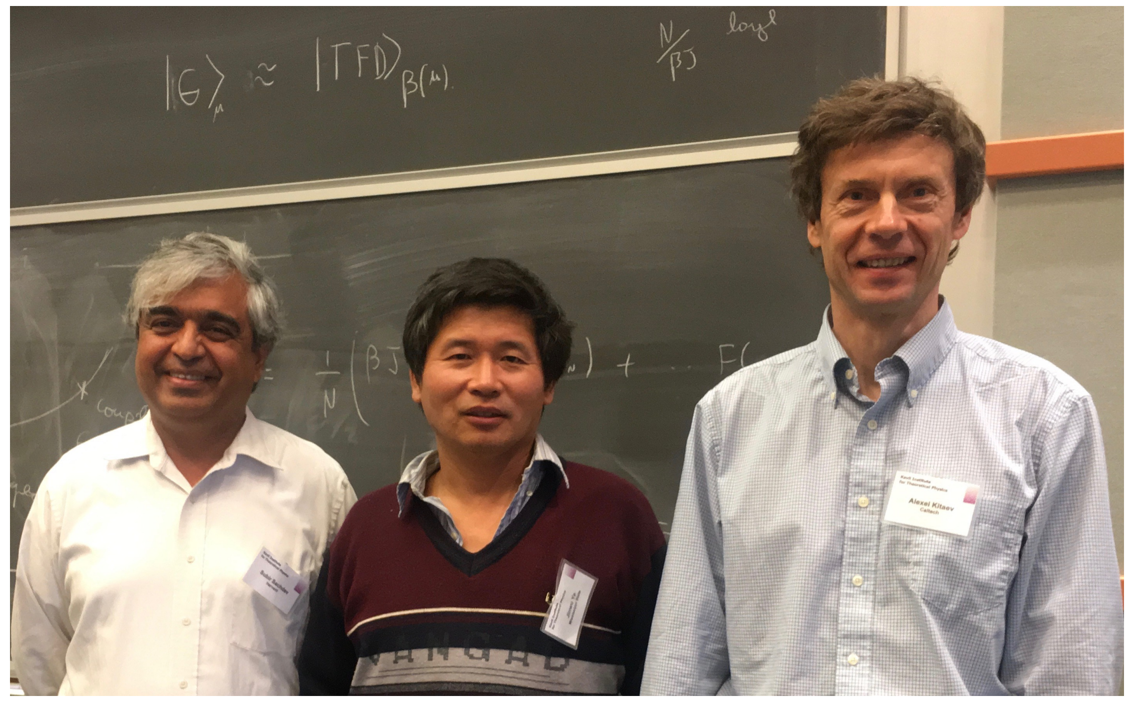
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha \, ; \quad [\mathcal{H}, \mathcal{Q}] = 0 \, ; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





The SYK model

- Planckian time dynamics without quasiparticles with a relaxation time $\sim \hbar/(k_B T)$ when $k_B T \ll U$.
  Fermion Green's function $G(\omega) \sim T^{-1/2} F(\hbar\omega/k_B T)$

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- There is an extensive entropy as $T \to 0$

$$
\begin{aligned}
s_0/k_B &= \lim_{T\to 0} \lim_{N\to\infty} S/(k_B N) \neq 0 \\
&= \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4} = 0.464847699170805107492692486833\ldots .
\end{aligned}
$$

However, the ground state is not extensively degenerate.

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\end{aligned}
$$

  However, the ground state is not extensively degenerate.

- The $D(E) \sim \exp(S/k_B)$ is determined by a time-reparameterization $\tau \to f(\tau)$ mode (similar to the graviton fluctuations of the spacetime metric), and a phase mode $\phi(\tau)$:

$$
\mathcal{Z}_{SYK} = e^{N s_0/k_B} \int \mathcal{D}f \mathcal{D}\phi \exp\left(-\frac{1}{\hbar} \int_0^{\hbar/(k_B T)} d\tau \, \mathcal{L}_{SYK}[f, \phi] \right)
$$

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-

$$D(E) \sim N^{-1} \exp\left(N s_0/k_B\right) \sinh(\sqrt{2N\gamma E})$$

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- A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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J. S. Cotler et al., JHEP 05 (2017) 118

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Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, JHEP 02 (2020) 157

Graphene flakes and the SYK model

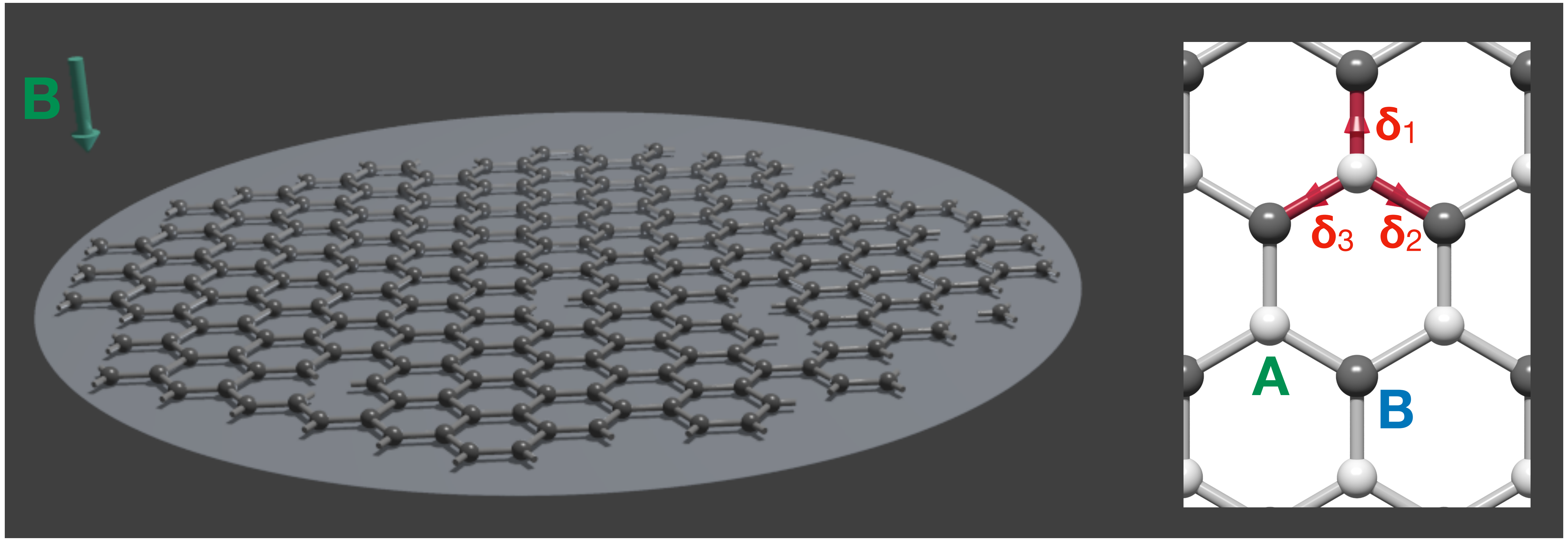
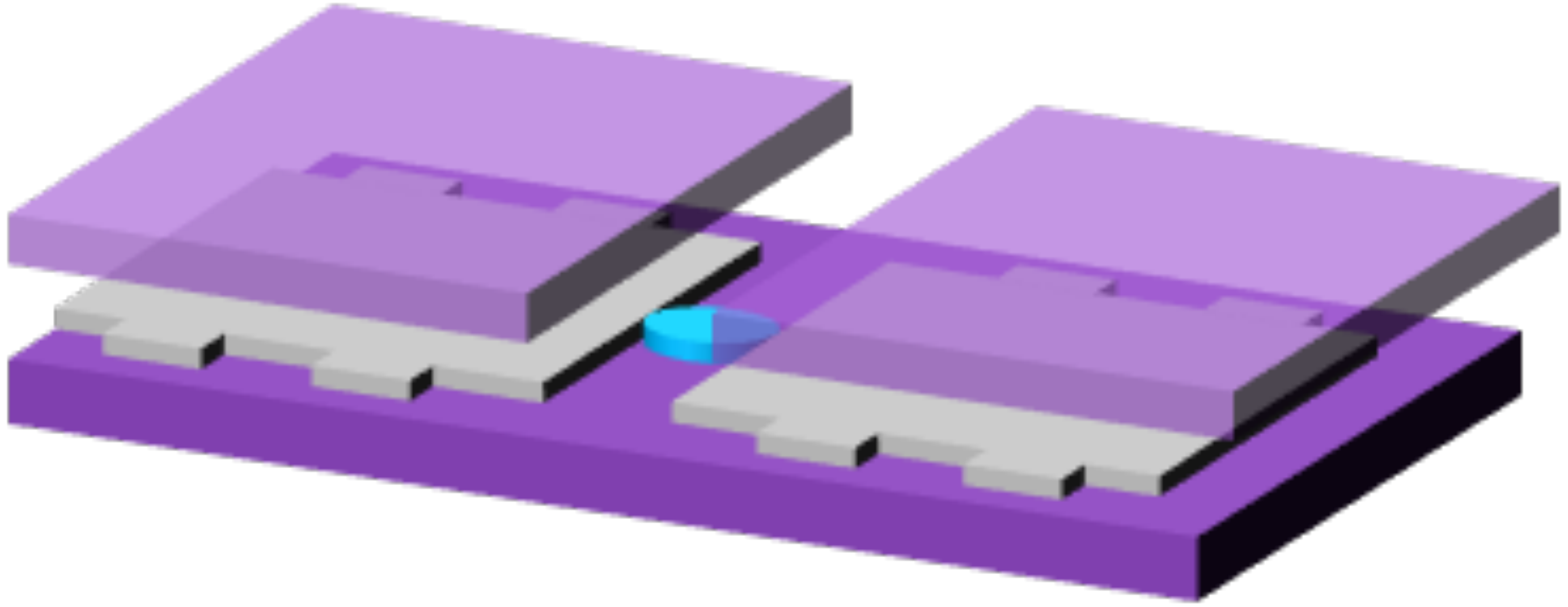


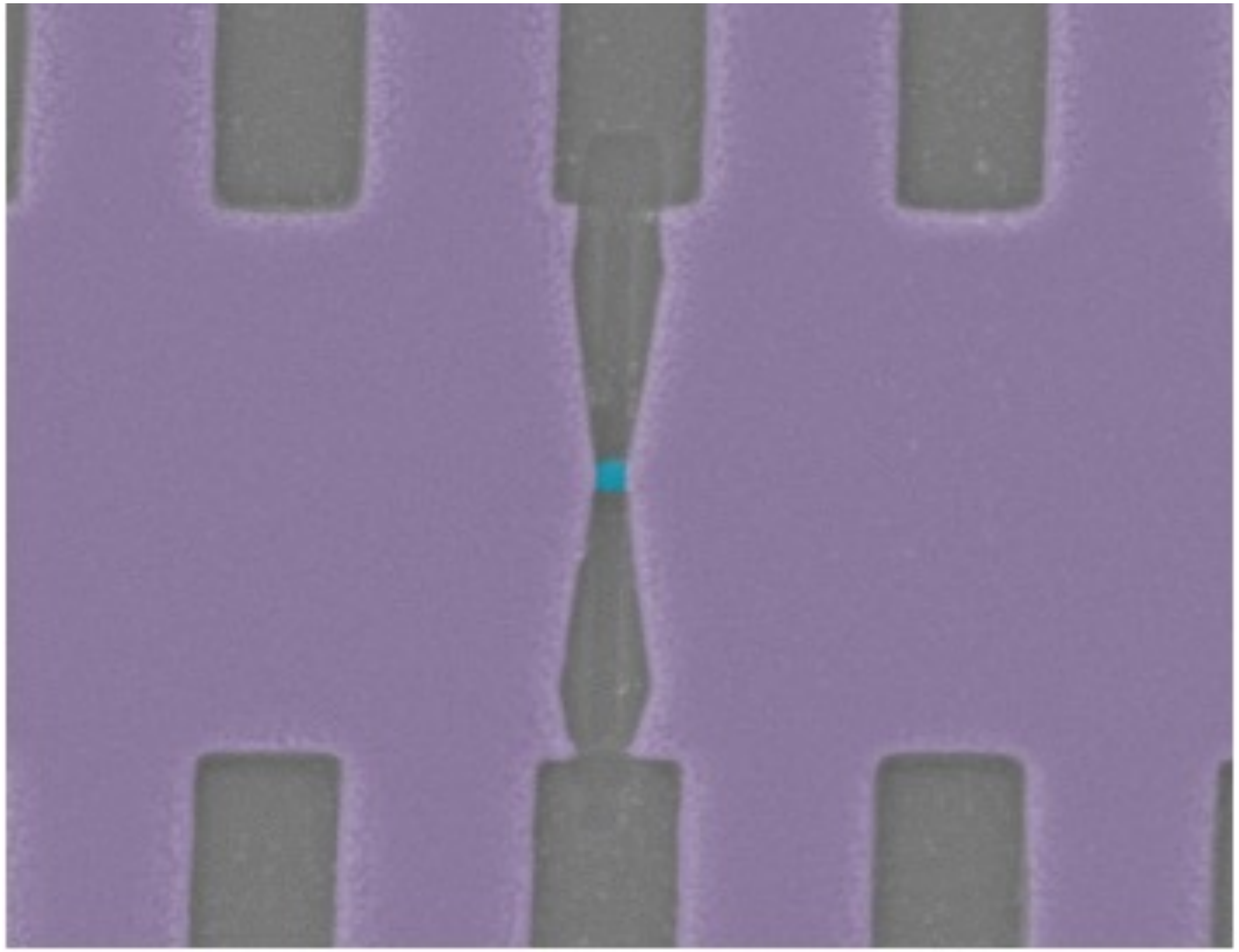
FIG. 1. Schematic depiction of the proposed device. Irregular shaped graphene flake in applied magnetic field B forms the (0+1) dimensional many-body system equivalent to a black hole in (1+1) anti-de Sitter space. Inset: lattice structure of graphene with A and B sublattices marked and nearest neighbor vectors denoted by $\boldsymbol{\delta}_j$.

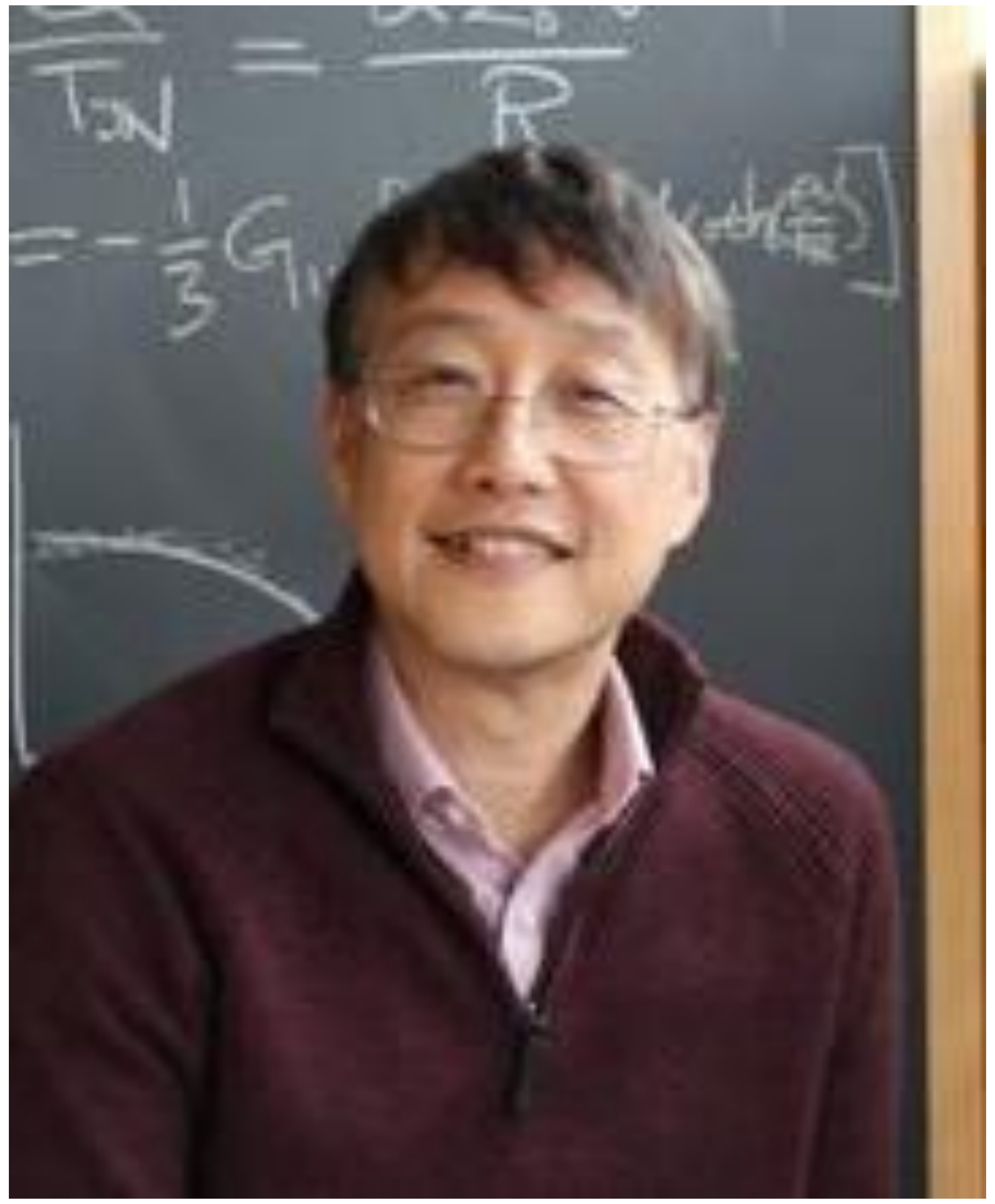
A. Chen, R. Ilan, F. de Juan, D. I. Pikulin, and M. Franz, *Phys. Rev. Lett.* **121**, 036403 (2018)

Laurel Anderson ……..
Philip Kim, to appear.

L. Anderson, Ph.D.
thesis, Harvard, 2022







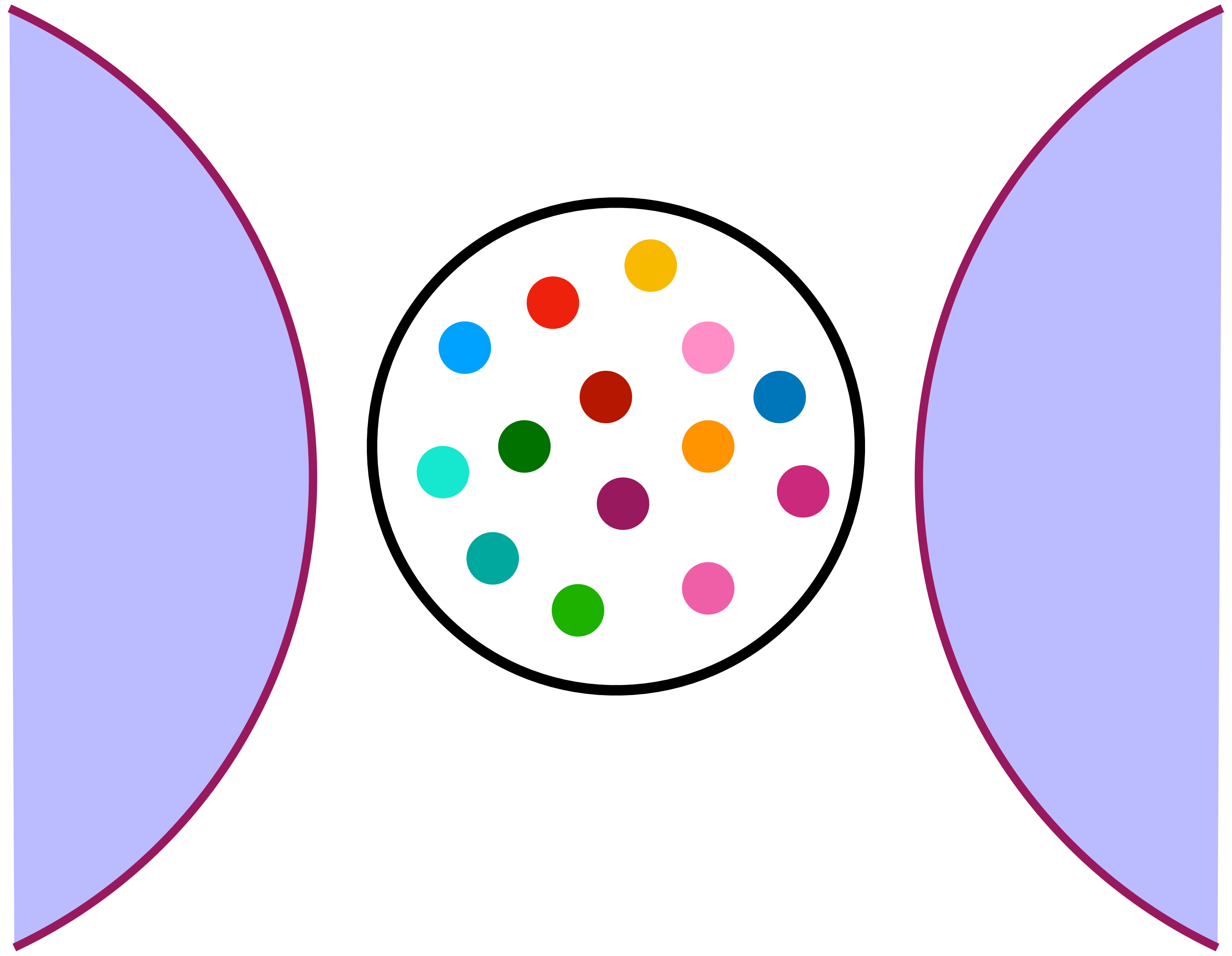
Thermopower

Apply a temperature difference ΔT, and measure the voltage difference ΔV, while no current is flowing. The thermopower is

$$\Theta = \frac{\Delta V}{\Delta T} = \frac{k_B}{e} \times (\text{a number})$$

Thermopower

A. Kruchkov, A. A. Patel, Philip Kim, and S. Sachdev, PRB **101**, 205148 (2020)



SYK metal island weakly coupled to normal metal leads

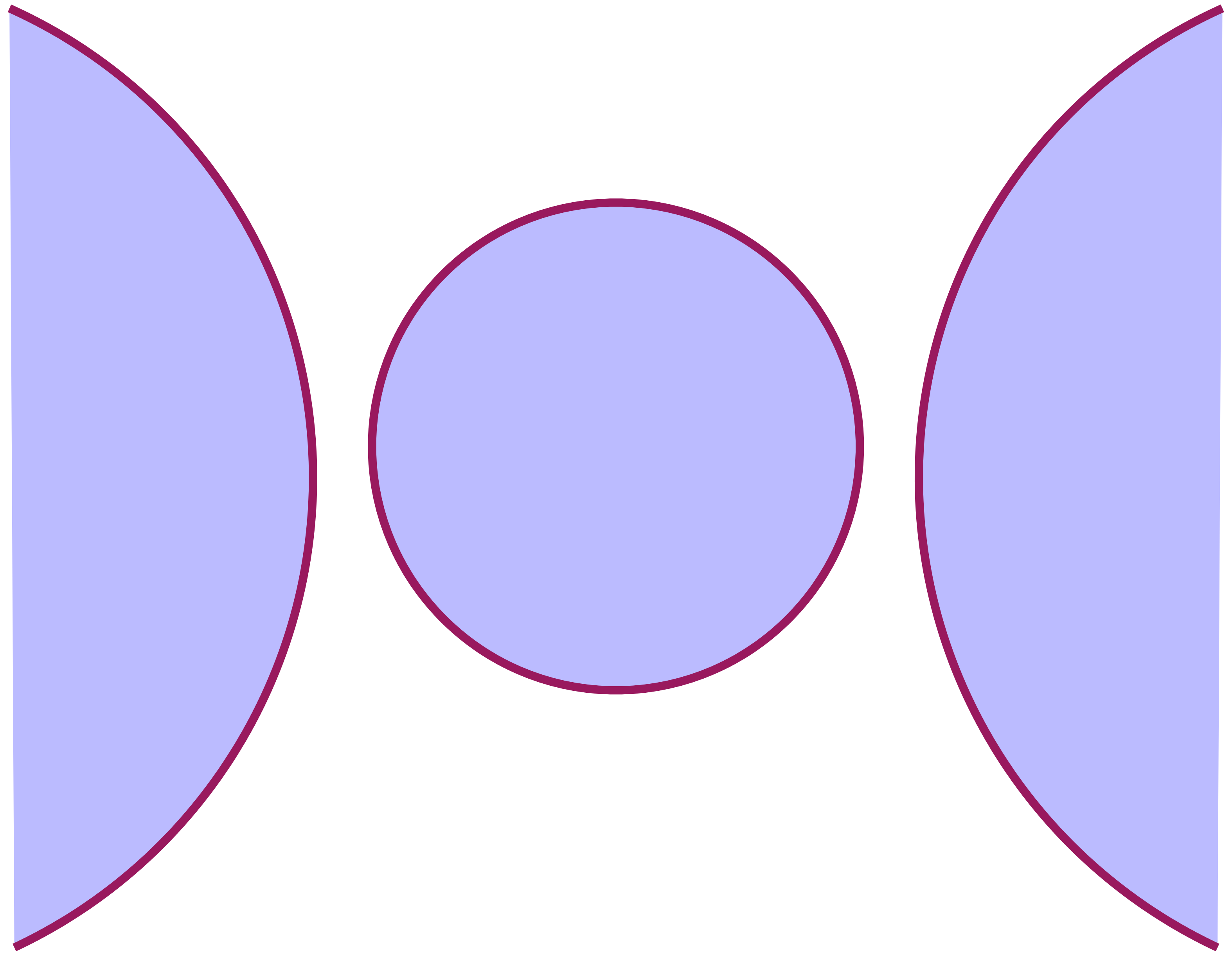
$\mathcal{E}$ is a dimensionless measure of the particle-hole asymmetry ($\mathcal{E} = 0$ for $\mathcal{Q} = 1/2$)

$$f(\omega) = \frac{1}{e^{\hbar\omega/k_B T} + 1}$$

$$\Theta = \frac{\hbar}{eT} \frac{\int d\omega\, \omega \, (-\partial f/\partial\omega) \left[\mathrm{Im}\overline{G(\omega)}\right]}{\int d\omega \, (-\partial f/\partial\omega) \left[\mathrm{Im}\overline{G(\omega)}\right]} = \frac{k_B}{e}\frac{4\pi\mathcal{E}}{3} = \frac{2}{3e}\frac{ds_0}{d\mathcal{Q}}$$

Thermopower is non-zero as $T \to 0$

Thermopower



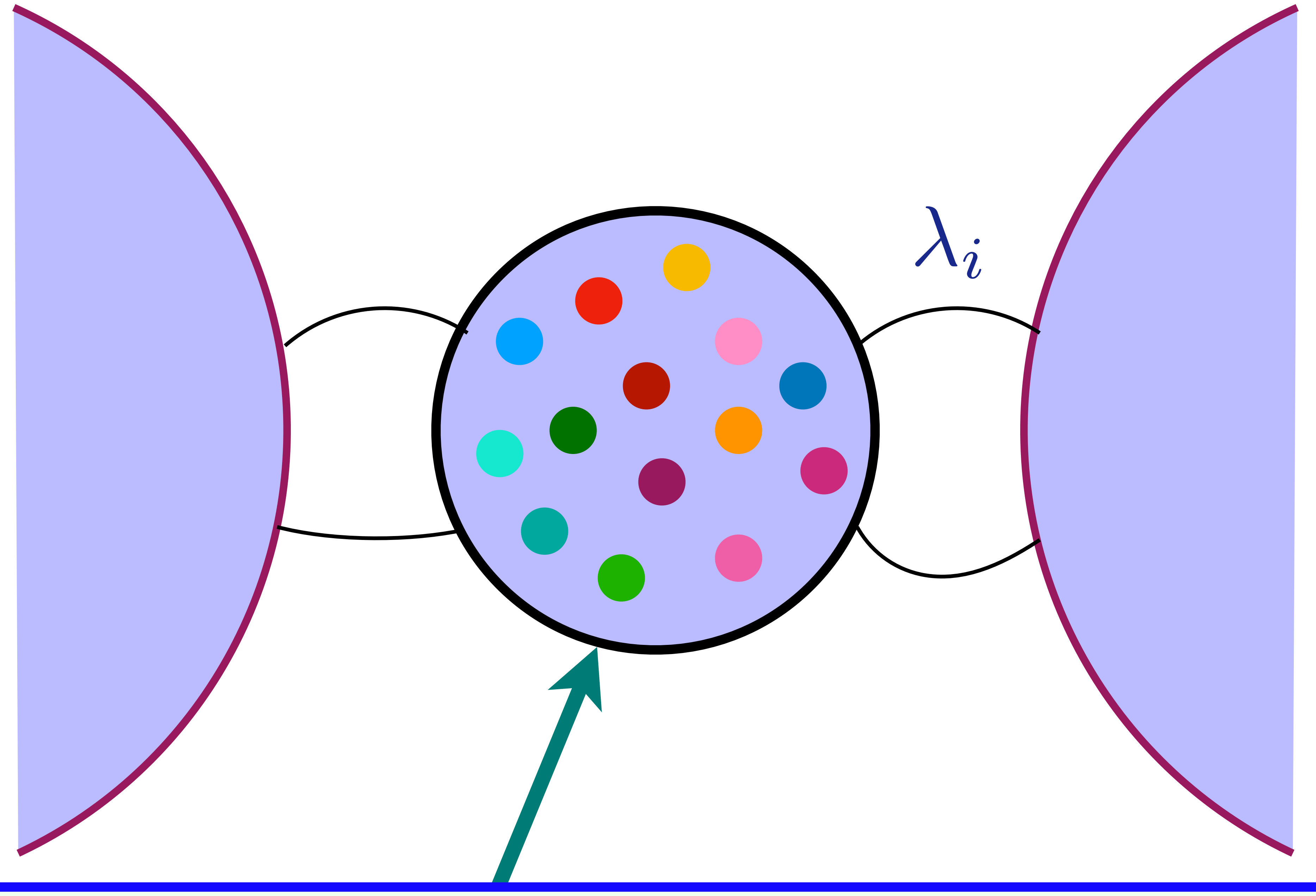
Normal metal island weakly coupled to normal metal leads

$$f(\omega) = \frac{1}{e^{\hbar\omega/k_B T} + 1}$$

$$\Theta = \frac{\hbar}{eT} \frac{\int d\omega\, \omega\, (-\partial f/\partial \omega) \left[\mathrm{Im}\overline{G(\omega)} \right]}{\int d\omega\, (-\partial f/\partial \omega) \left[\mathrm{Im}\overline{G(\omega)} \right]} = \frac{k_B}{e} \frac{\pi^2 \rho'(E_F) k_B T}{3\rho(E_F)} = \frac{1}{Ne} \frac{dS}{dQ}$$

Thermopower vanishes linearly as $T \to 0$

A more realistic model



$$\Gamma = \pi \rho_{\text{lead}} \sum_i |\lambda_i|^2$$

$$\overline{|h_{\alpha\beta}|^2} = h^2$$

$$H = \frac{1}{\sqrt{N}} \sum_{\alpha\beta=1}^{N} h_{\alpha\beta} c_\alpha^\dagger c_\beta + \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

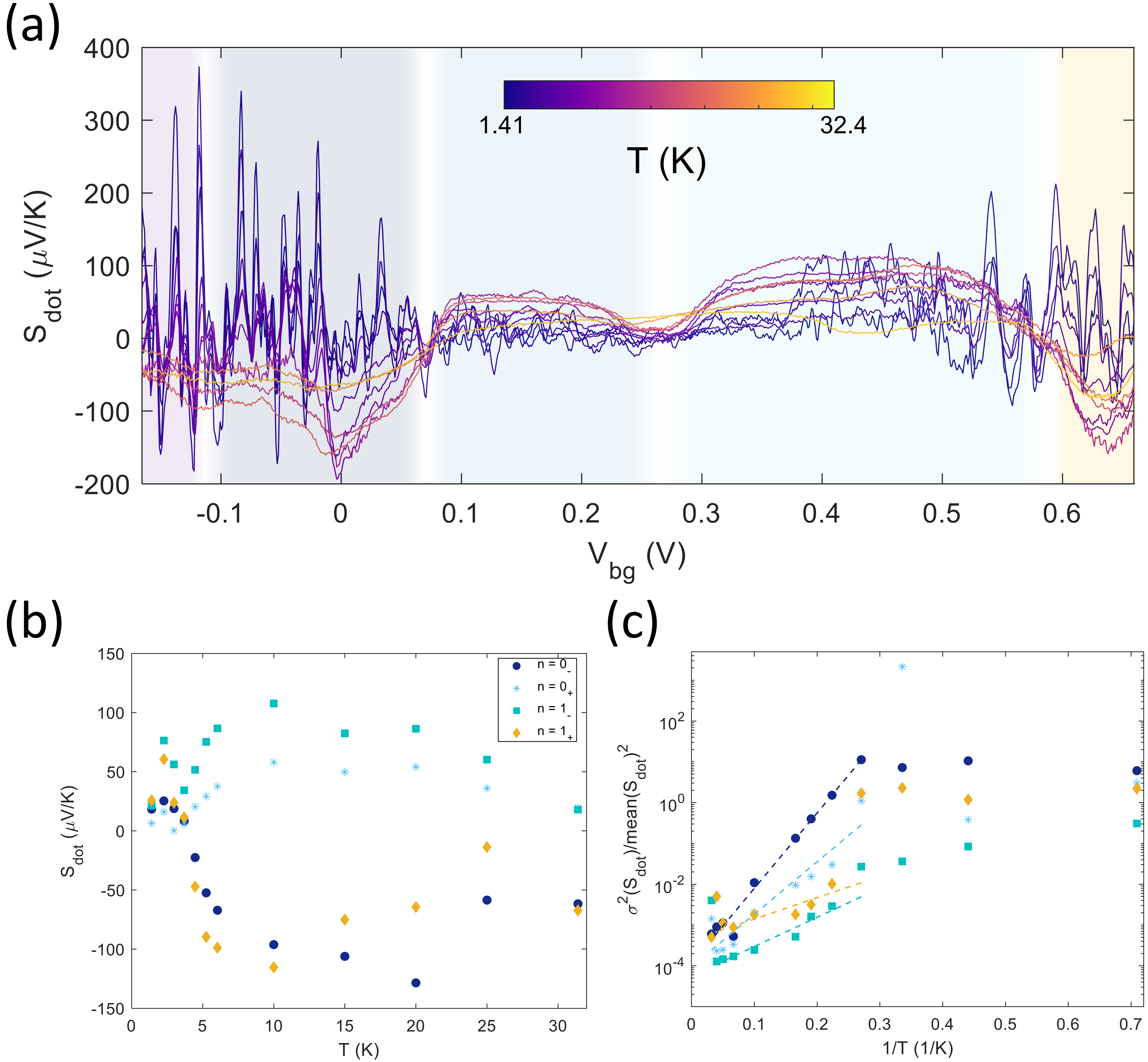
Normal metal behavior for $T < T_{\text{coh}} \sim h^2/U$

SYK behavior for $T > T_{\text{coh}} \sim h^2/U$

Notation: $S \to \Theta$

Laurel Anderson ……..
Philip Kim, to appear.

L. Anderson, Ph.D.
thesis, Harvard, 2022



Striking feature of observations at finite N:
fluctuations from random couplings are much larger
for $T < T_{\mathrm{coh}}$ (normal metal regime)
than for $T > T_{\mathrm{coh}}$ (possible SYK regime)

Striking feature of observations at finite N: fluctuations from random couplings are much larger for $T < T_{\text{coh}}$ (normal metal regime) than for $T > T_{\text{coh}}$ (possible SYK regime)

Henry Shackleton

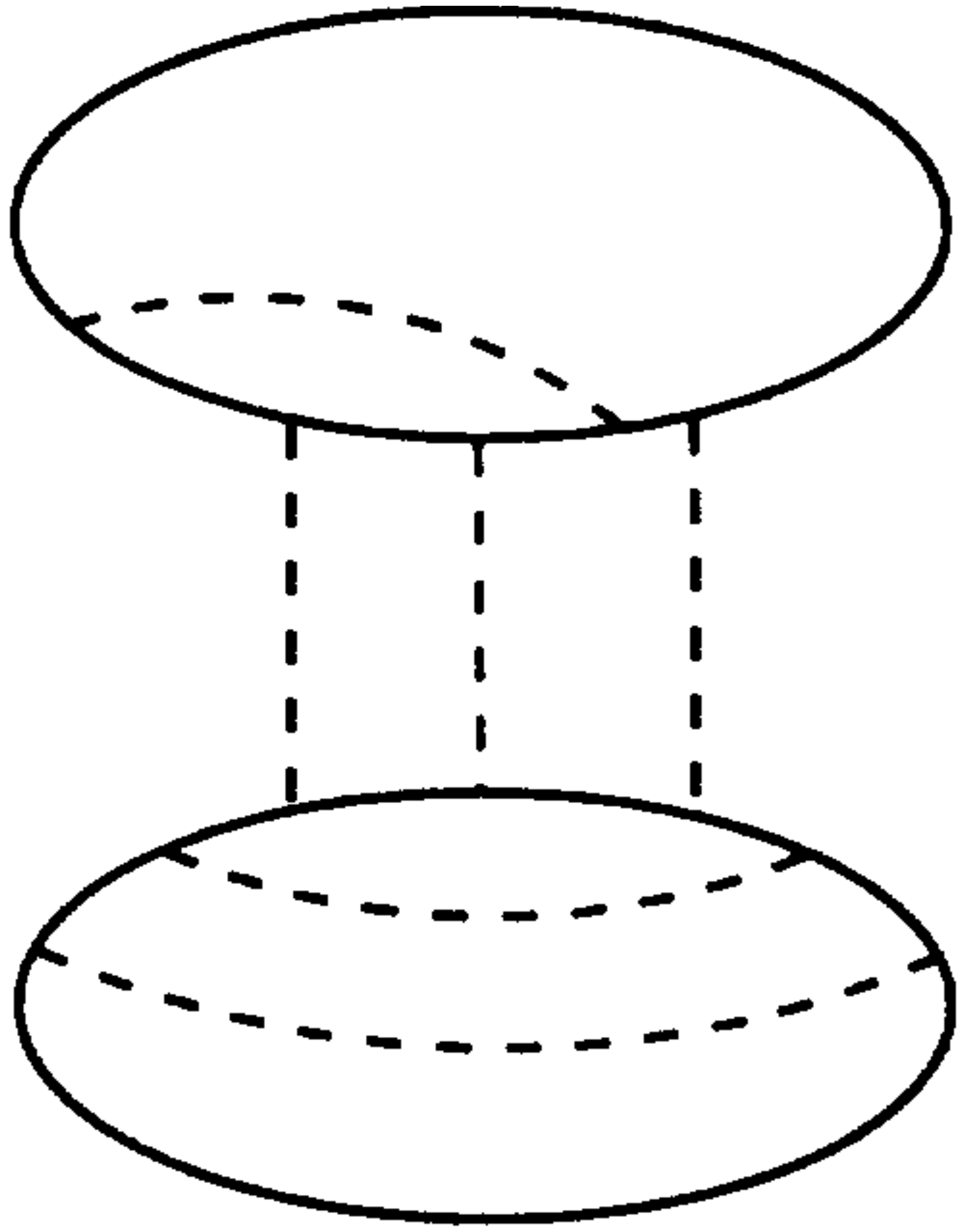
Conductivity σ

Normal metal regime:

$$\frac{\text{var}(\sigma) = \overline{(\sigma - \overline{\sigma})^2}}{\overline{\sigma}^2} = \frac{3\xi(3)}{16\pi^3}\left(\frac{h}{TN}\right)^2$$

Follows from Dyson-Mehta formula for single-particle RMT eigenvalue repulsion

$$\overline{\text{Im}G^r(\omega)\text{Im}G^r(\epsilon)} - \overline{\text{Im}G^r(\omega)}\,\overline{\text{Im}G^r(\epsilon)} = -\frac{1}{2(\omega-\epsilon)^2}$$



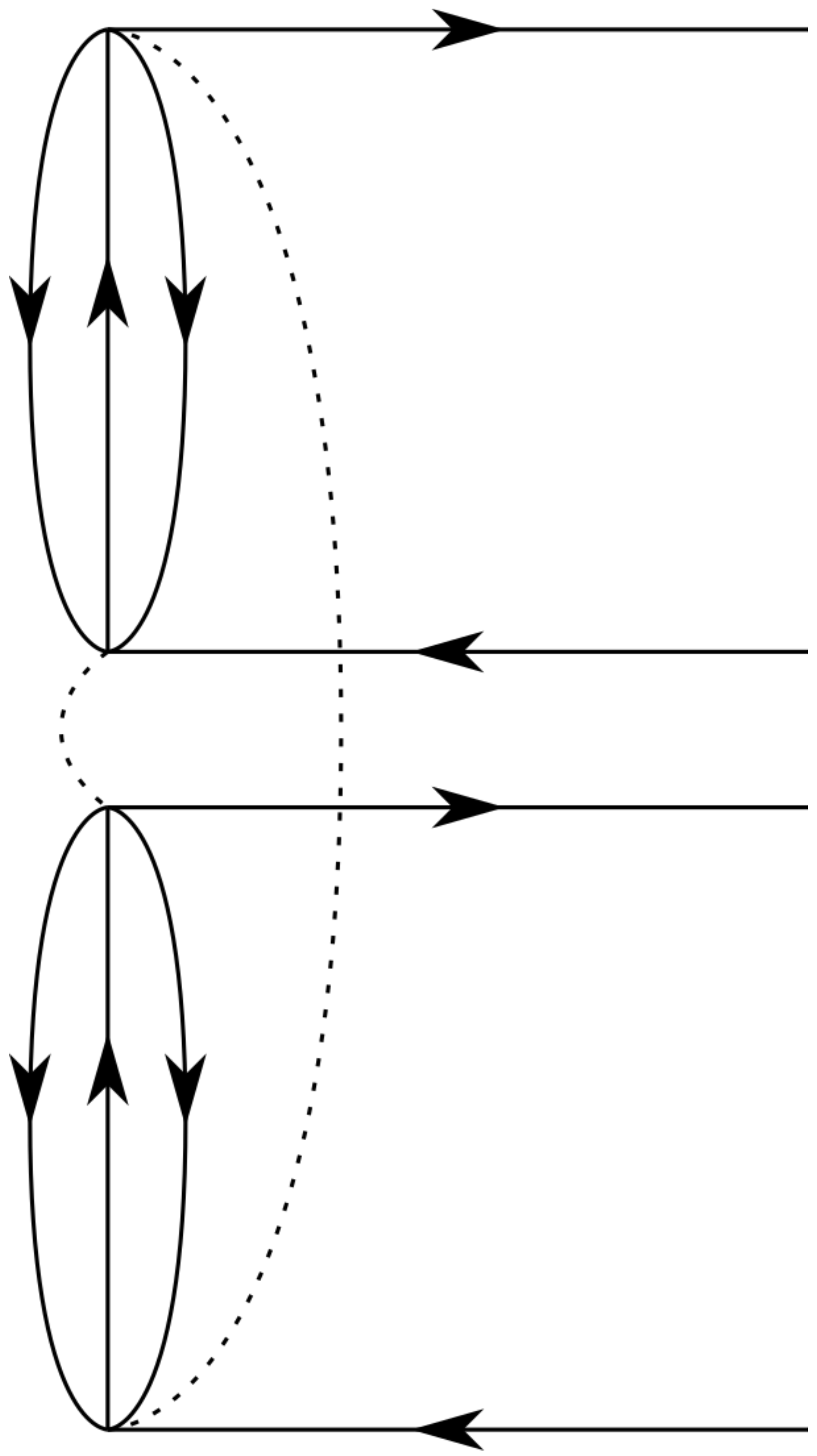
Striking feature of observations at finite N: fluctuations from random couplings are much larger for $T < T_{\rm coh}$ (normal metal regime) than for $T > T_{\rm coh}$ (possible SYK regime)

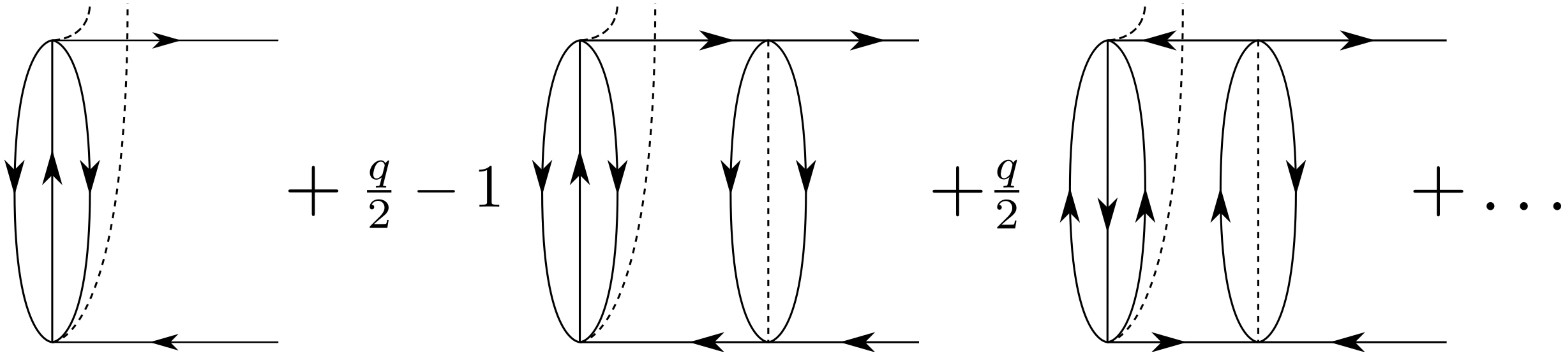
Henry Shackleton

Conductivity σ

SYK regime:

$$\frac{\text{var}(\sigma) = \overline{(\sigma - \overline{\sigma})^2}}{\overline{\sigma}^2} = \frac{1}{8N^4}$$



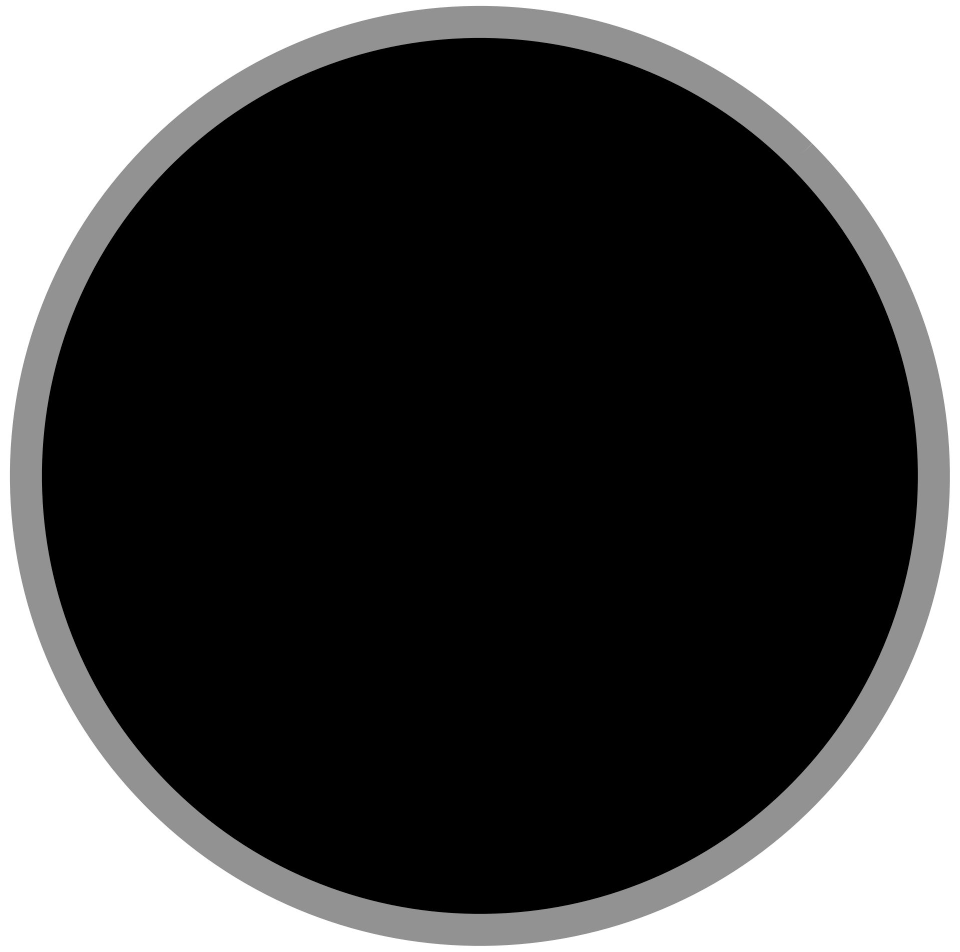


From the SYK model to a quantum theory of charged black holes

Black Holes

Objects so dense that light is gravitationally bound to them.

Horizon radius $R = \frac{2GM}{c^2}$

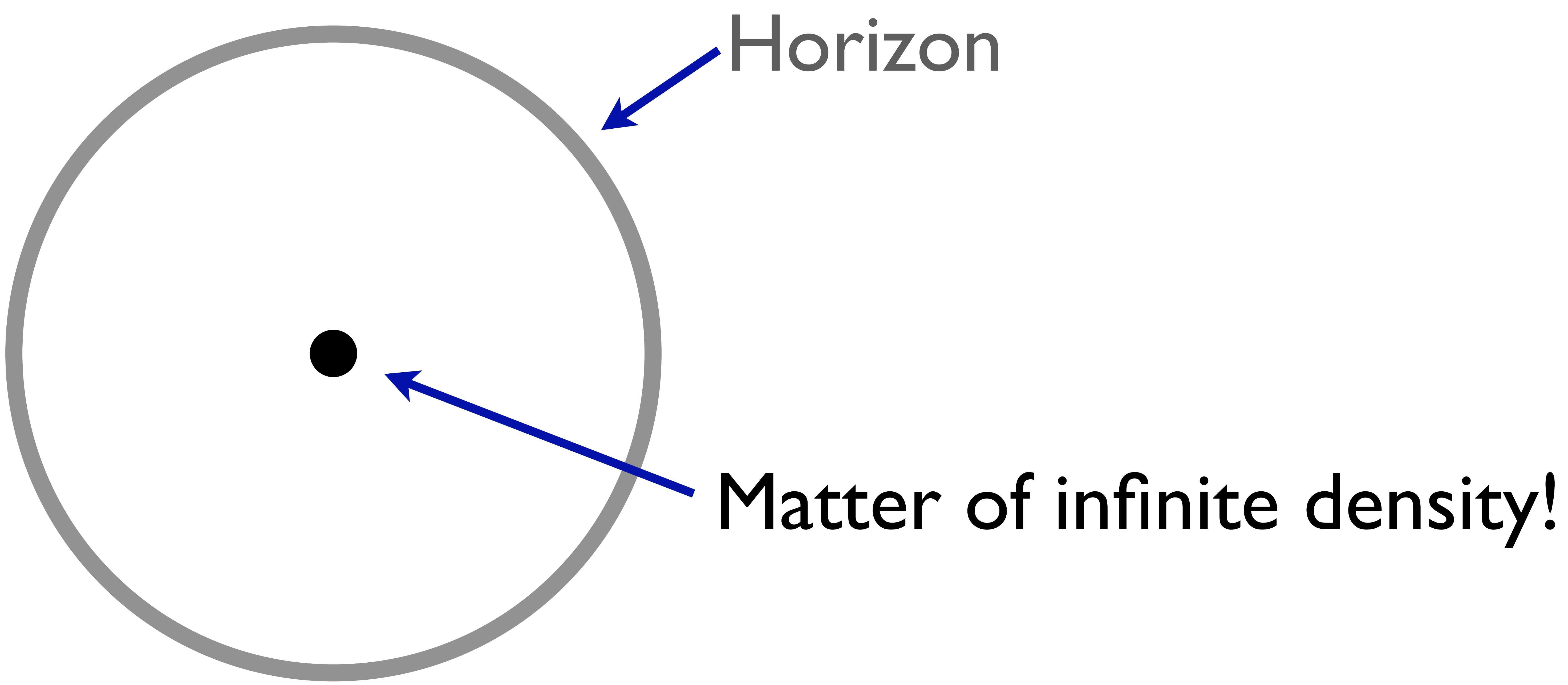


G Newton's constant, c velocity of light, M mass of black hole

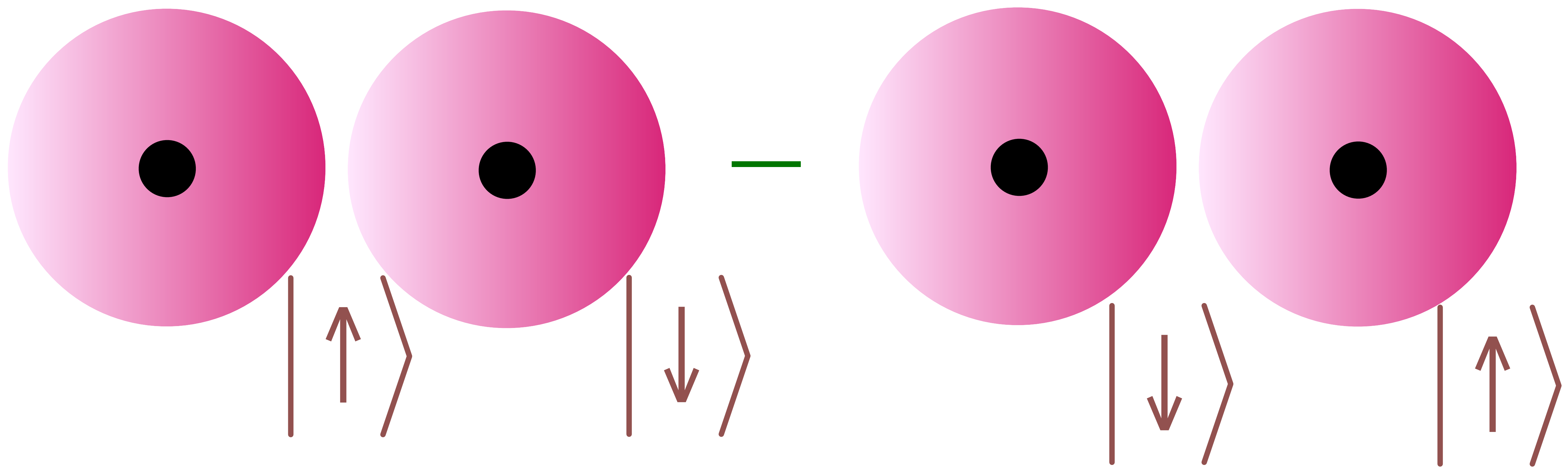
For $M =$ earth's mass, $R \approx 9\,mm$!

What is inside a black hole ???

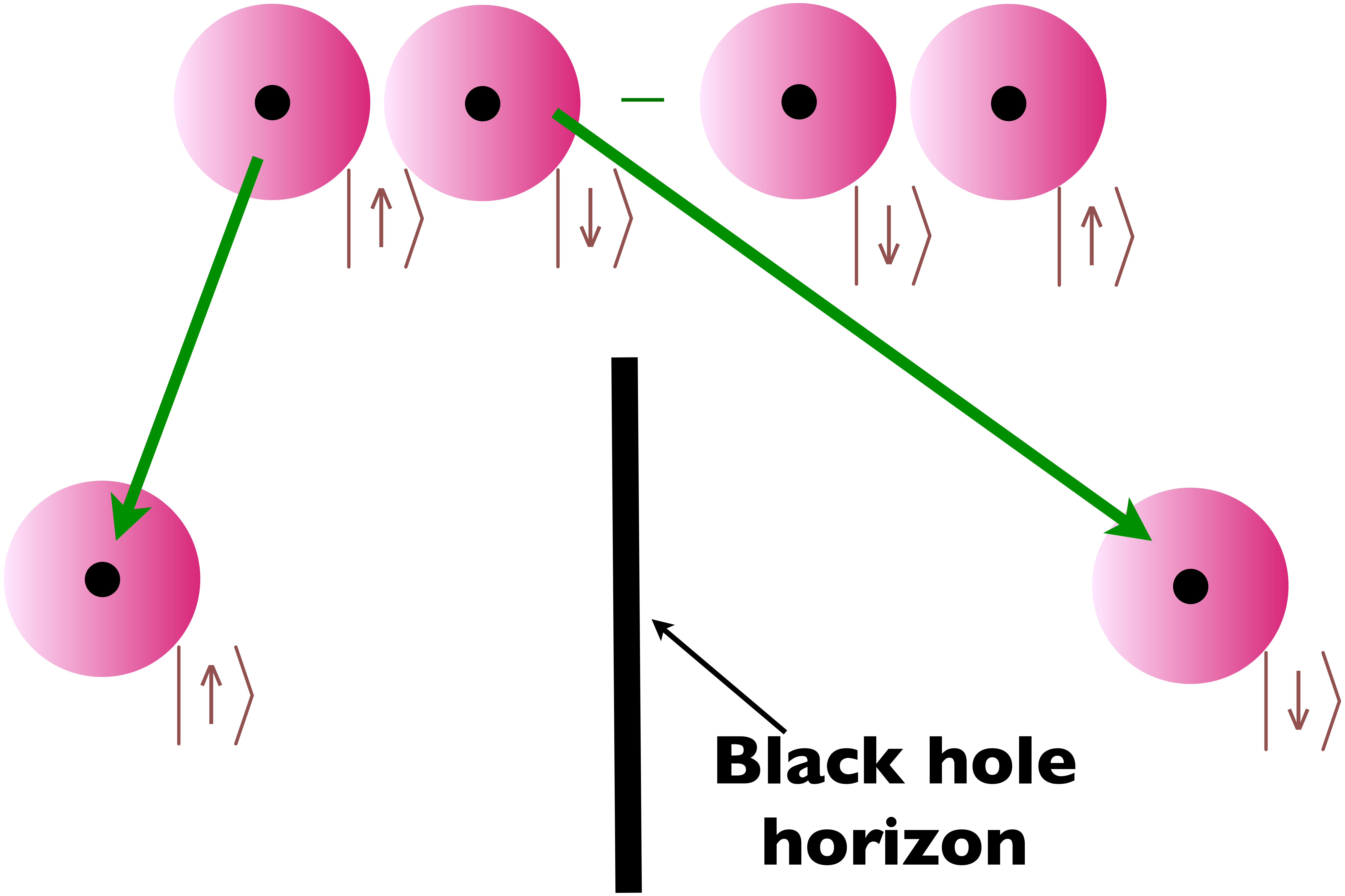
In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.
This singularity must be understood quantum mechanically



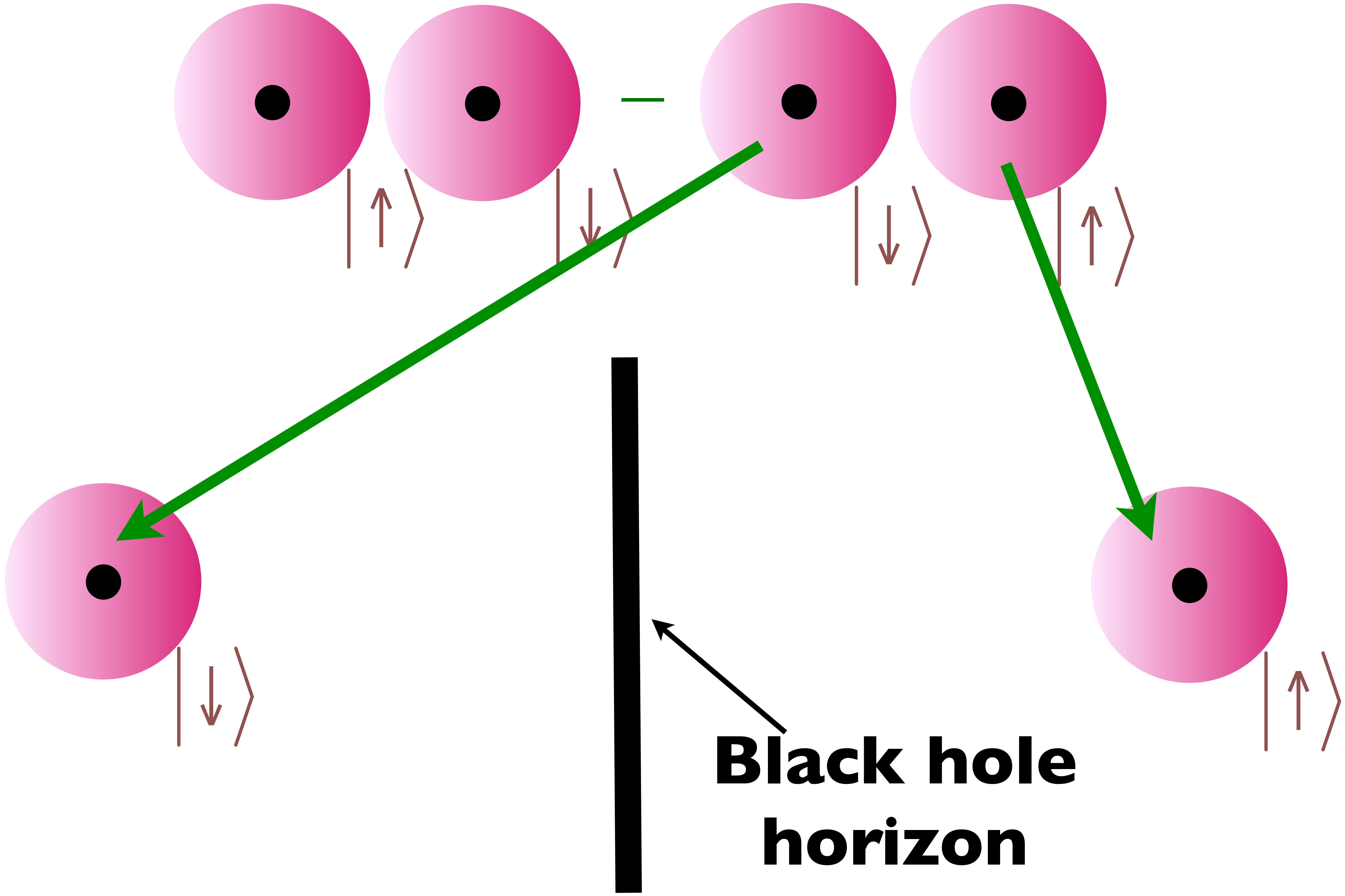
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

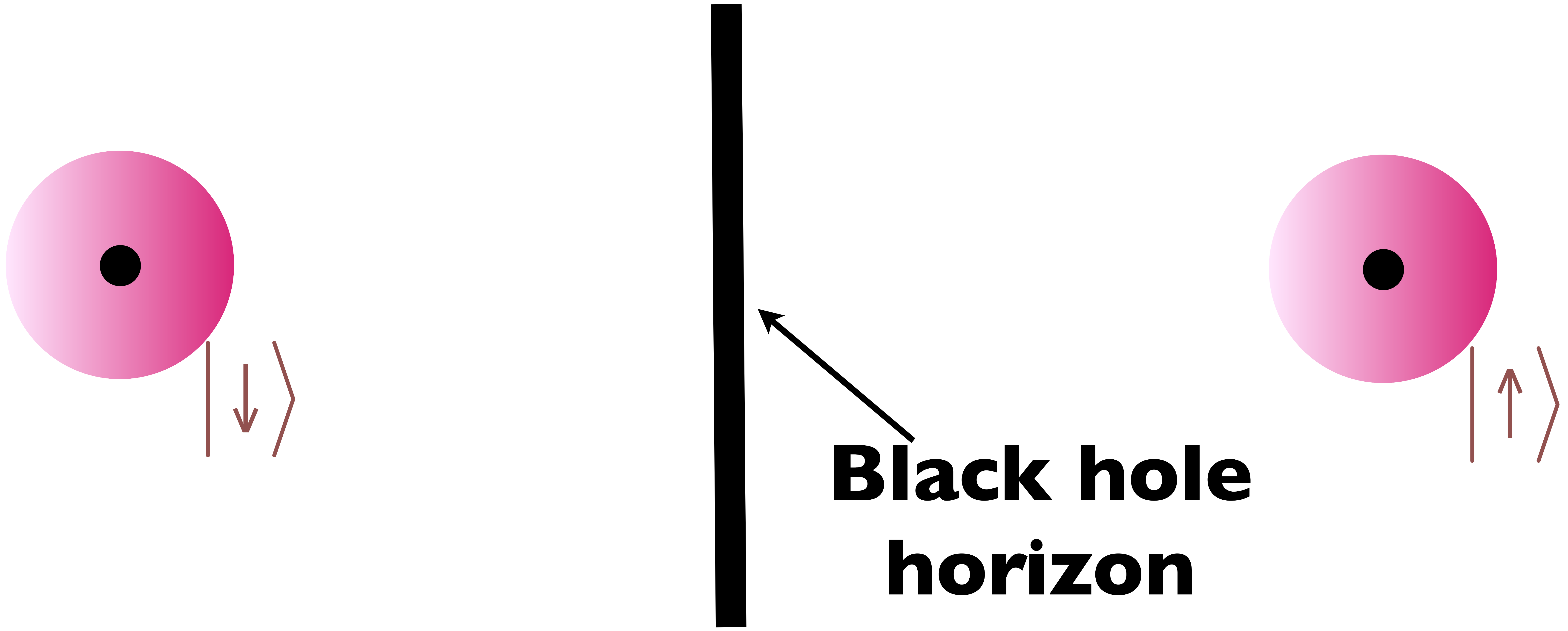


Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

Black hole horizons have
a temperature and an entropy (Hawking, 1974):
to an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.



Quantum Black Holes

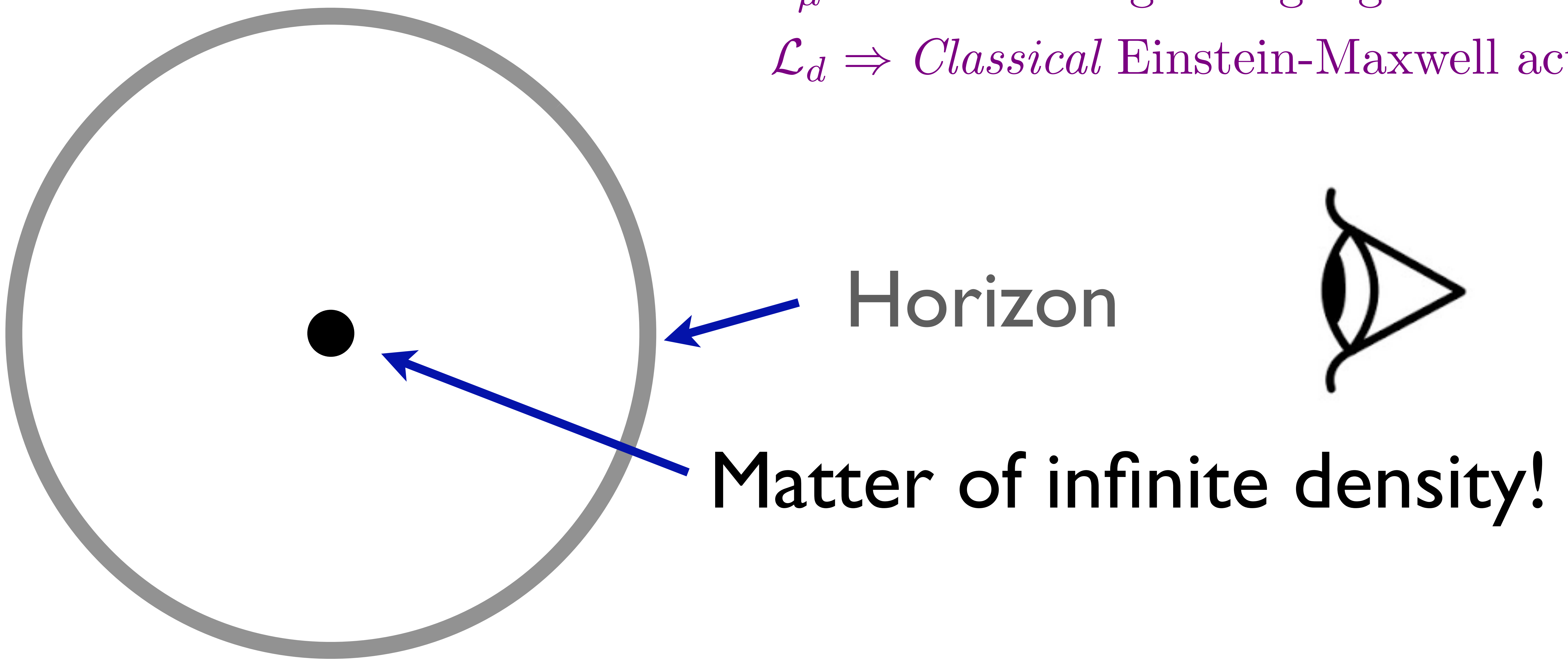
Hawking obtained the black hole entropy by the saddle-point of the gravitational path integral *outside* the black hole horizon.

$$\mathcal{Z} = \int \mathcal{D}g_{\mu\nu}\mathcal{D}a_\mu \exp\left(-\frac{1}{\hbar}\int d^dx \int_0^{\hbar/(k_BT)} d\tau \sqrt{g}\,\mathcal{L}_d[g_{\mu\nu}, a_\mu]\right)$$

$g_{\mu\nu} \Rightarrow$ spacetime metric, $\quad g = \det(g_{\mu\nu})$

$a_\mu \Rightarrow$ Electromagnetic gauge field

$\mathcal{L}_d \Rightarrow$ *Classical* Einstein-Maxwell action



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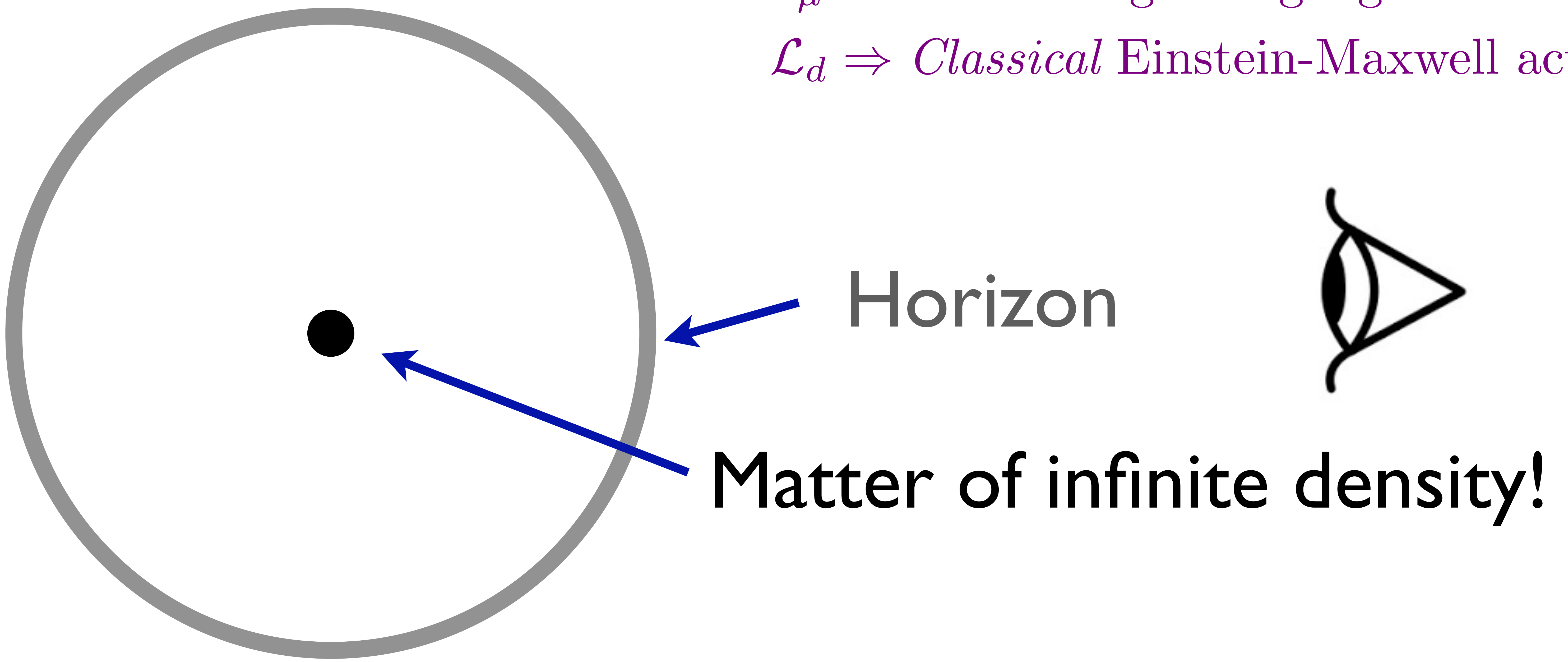
$g_{\mu\nu} \Rightarrow$ spacetime metric, $\quad g = \det(g_{\mu\nu})$

$a_\mu \Rightarrow$ Electromagnetic gauge field

$\mathcal{L}_d \Rightarrow$ *Classical* Einstein-Maxwell action

This allowed Hawking to avoid the contradictions associated with the singularity at the center of the black hole. The answer was independent of the nature of the matter inside.



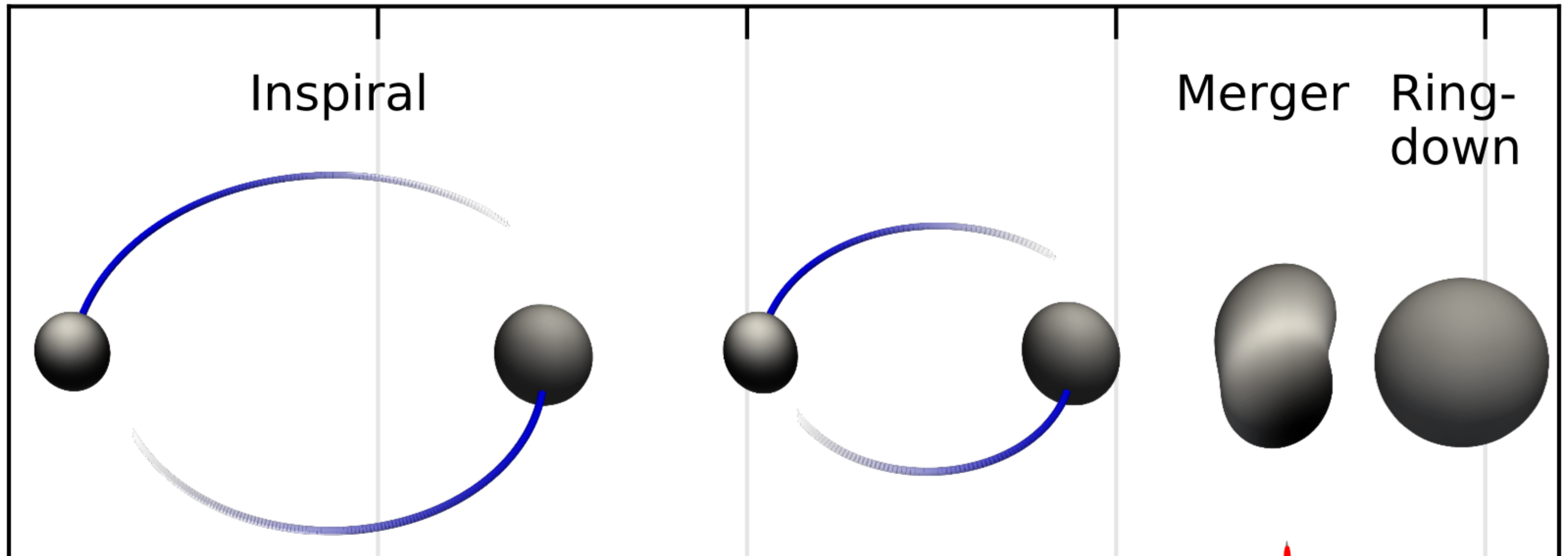
Quantum Black Holes

Hawking obtained the black hole entropy by the saddle-point of the gravitational path integral *outside* the black hole horizon.

Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* matches the Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics ?

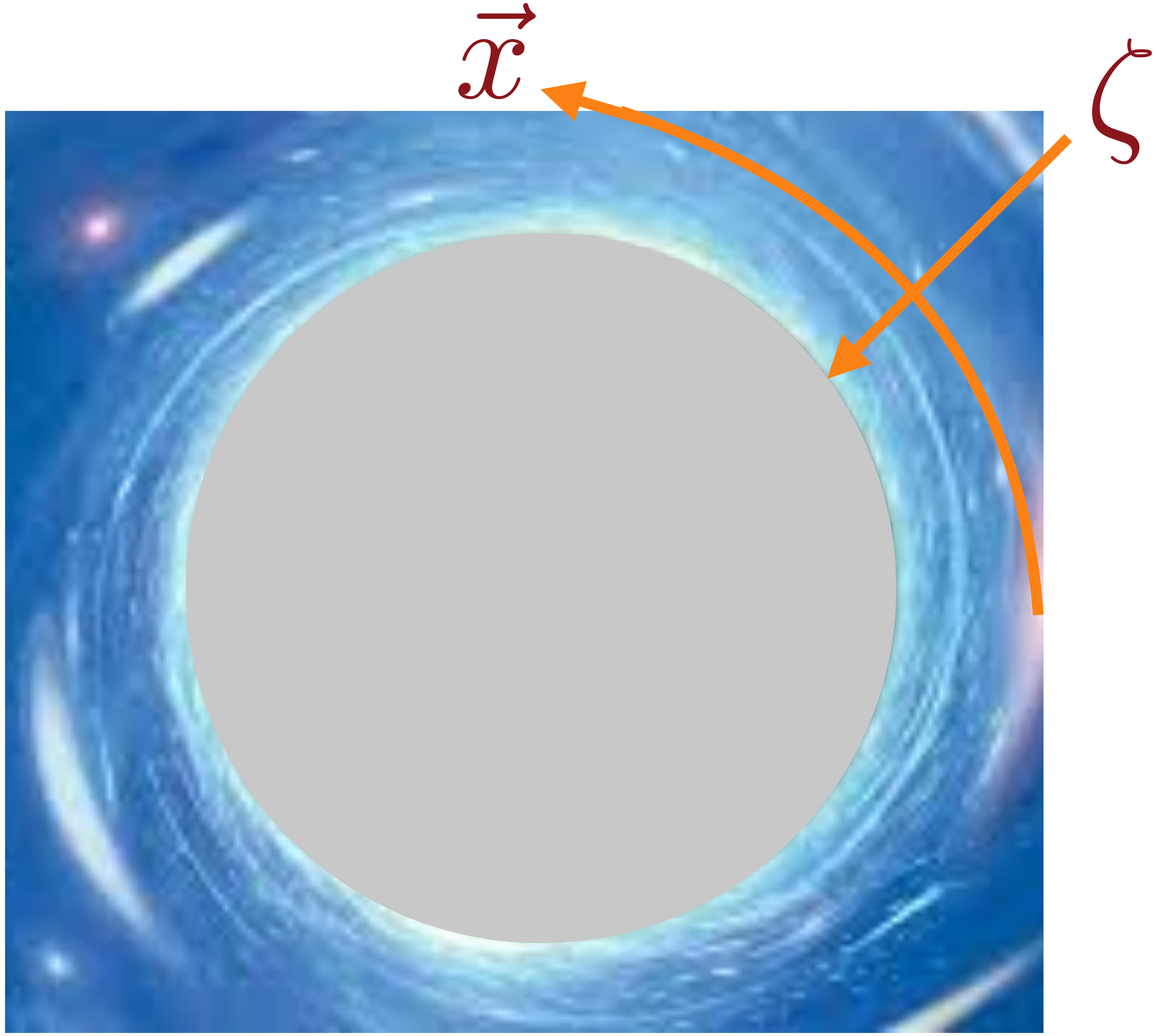
Quantum black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- The entropy is proportional to their surface area. $S = A k_B c^3/(4G\hbar)$.
- They relax to thermal equilibrium in a time $\sim 8\pi G M/c^3 = \hbar/(k_B T_H)$ which is Planckian!

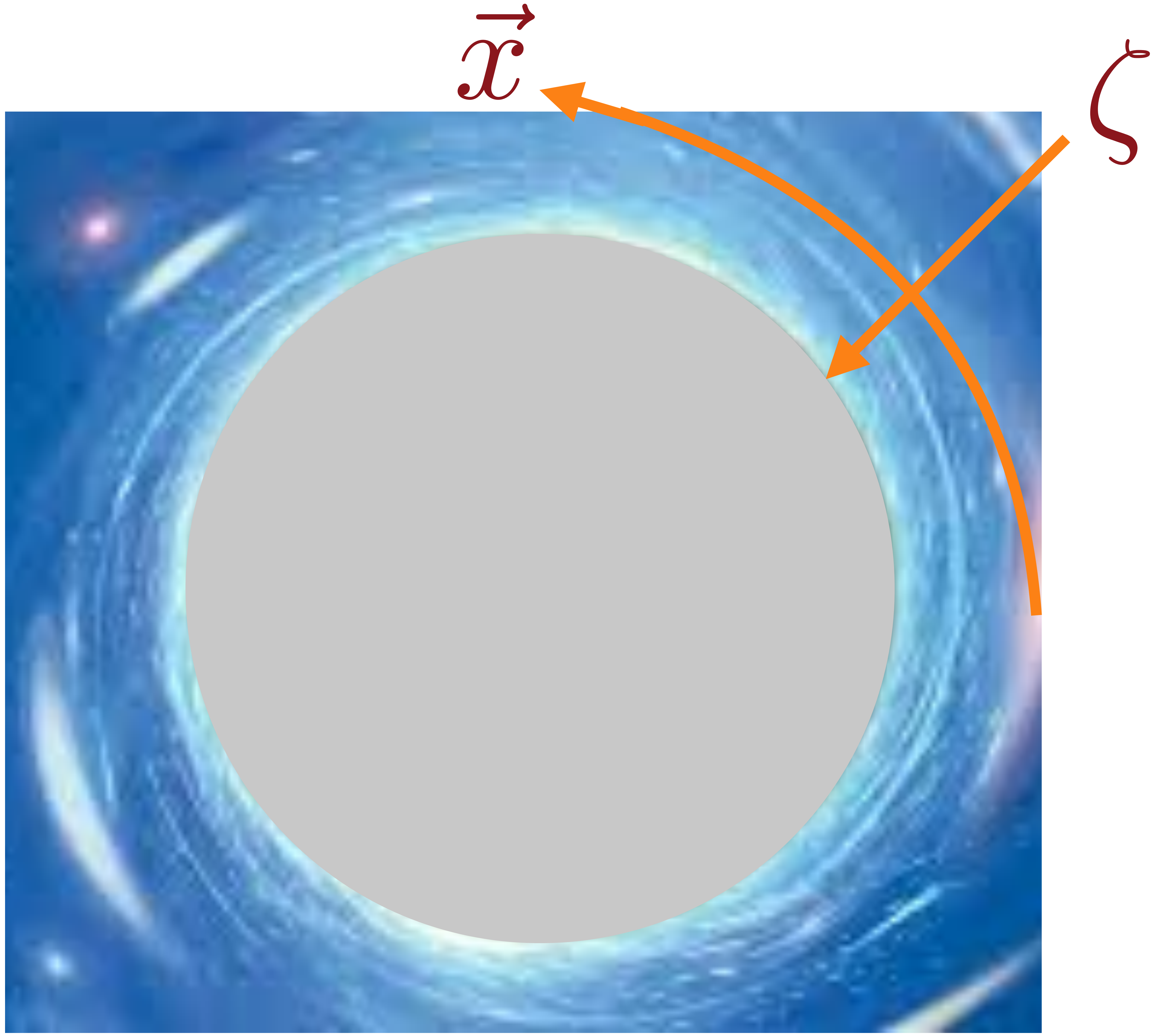


J. D. Bekenstein, PRD **7**, 2333 (1973)
S. W. Hawking, Nature **248**, 30 (1974)
C.V. Vishveshwara, Nature **227**, 936 (1970)

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge

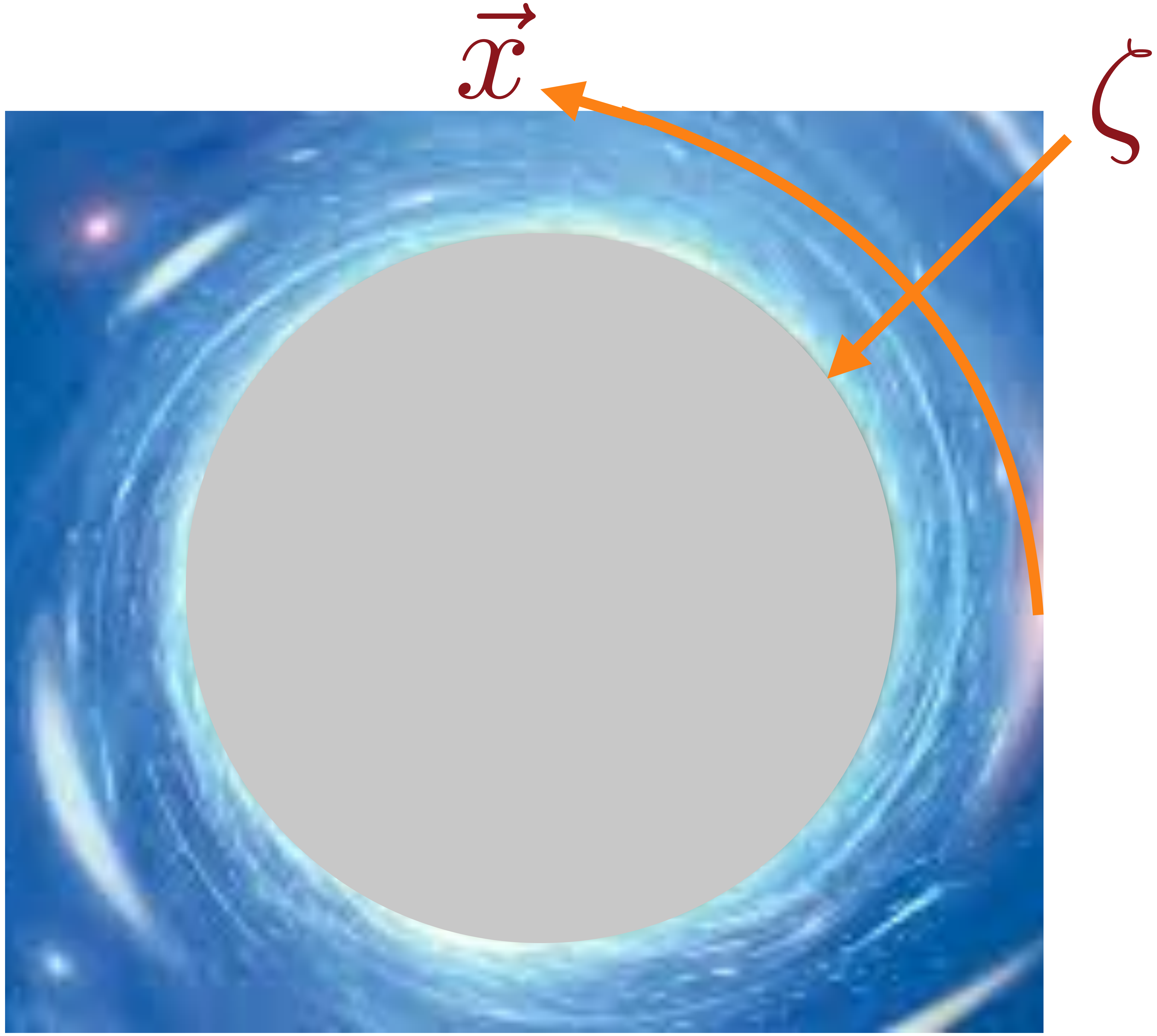


Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge



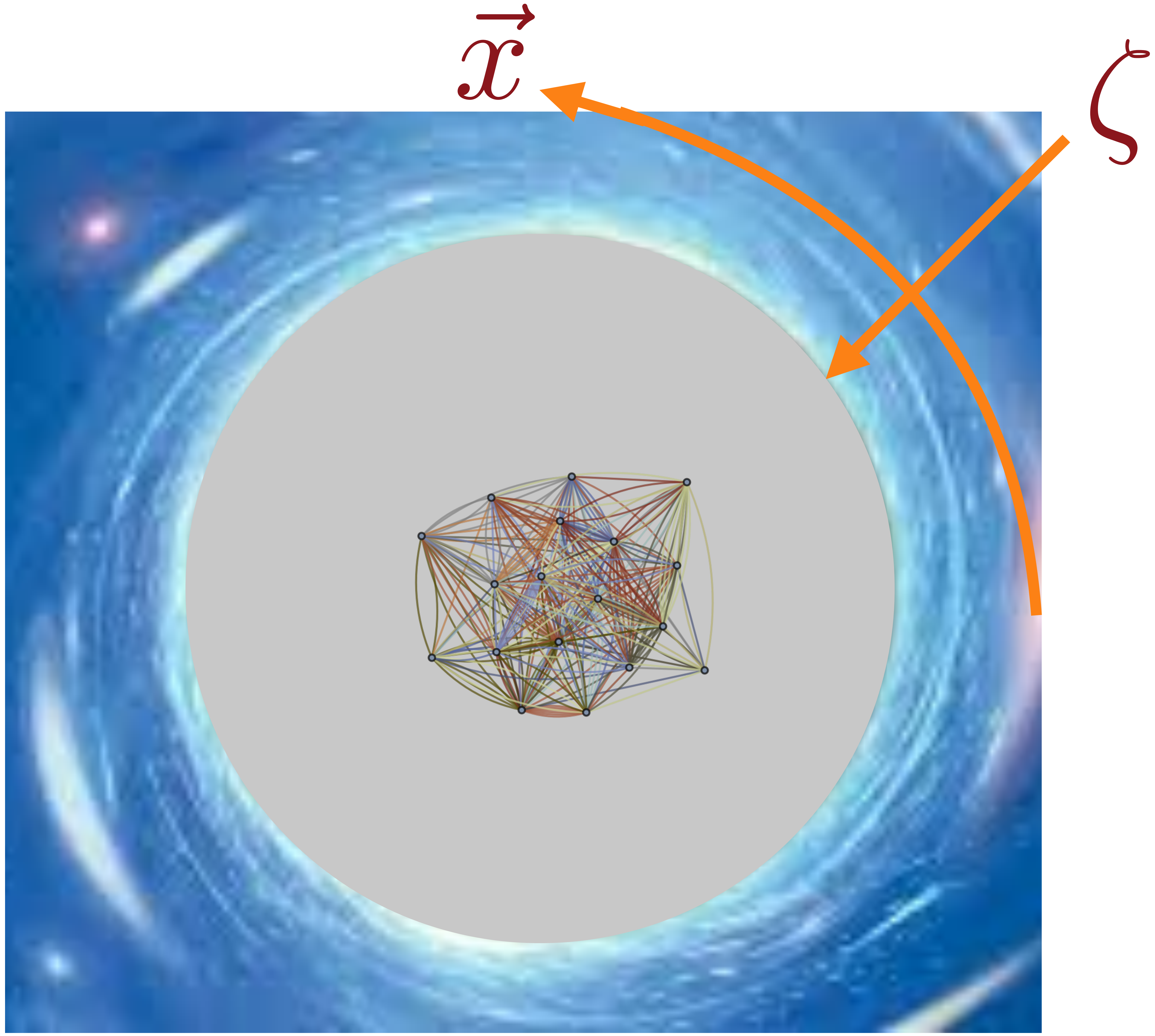
Zooming into the near-horizon region of a charged black hole at low temperature, yields a quantum theory in one space (ζ) and one time dimension

Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge



The quantum versions of Maxwell's and Einstein's equations in this two-dimensional spacetime are also the equations describing electron entanglement in the SYK model

Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge



The SYK provides a realization of the black hole interior, and its density of quantum states matches gravitational entropy computations for charged black holes !

The SYK model

- Planckian time dynamics without quasiparticles with a relaxation time $\sim \hbar/(k_B T)$ when $k_B T \ll U$.
  Fermion Green's function $G(\omega) \sim T^{-1/2} F(\hbar\omega/k_B T)$
- There is an extensive entropy as $T \to 0$

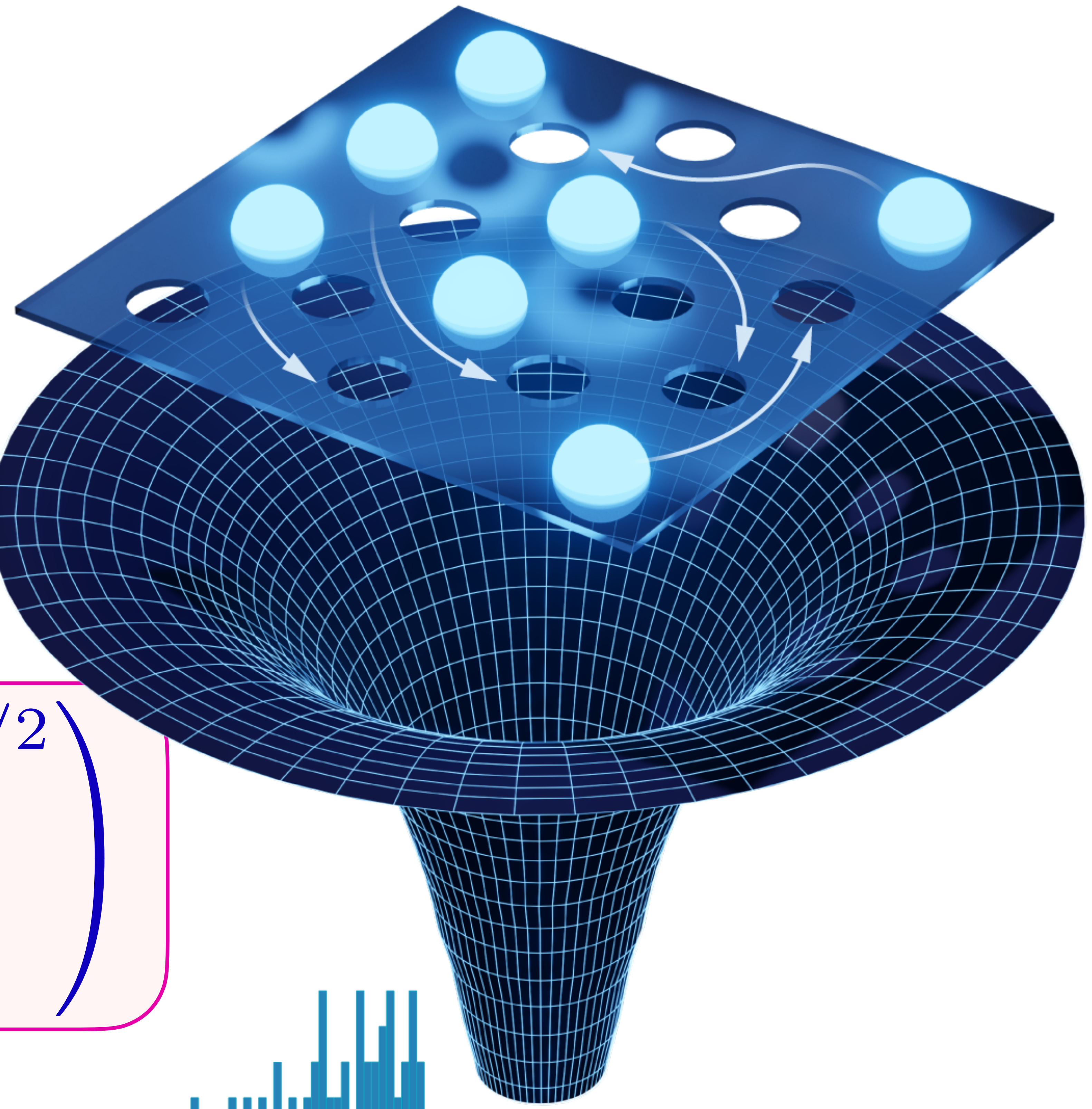
$$
\begin{aligned}
s_0/k_B &= \lim_{T\to 0} \lim_{N\to\infty} S/(k_B N) \neq 0 \\
&= \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4} = 0.464847699170805107492692486833\ldots .
\end{aligned}
$$

  However, the ground state is not extensively degenerate.

-

$$
D(E) \sim N^{-1} \exp\left(N s_0/k_B\right) \sinh(\sqrt{2N\gamma E})
$$

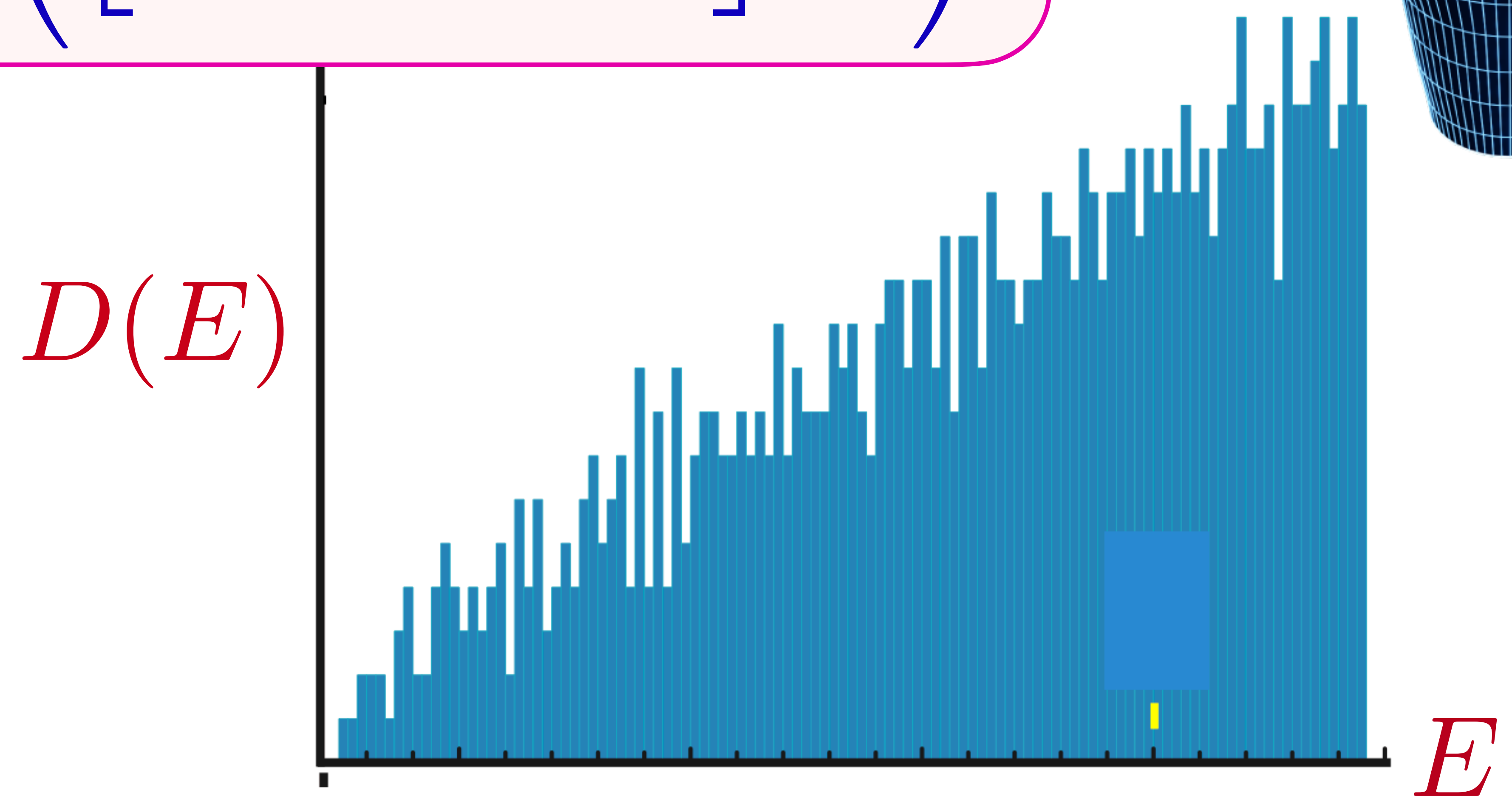
Quantum simulation of charged black holes by the SYK model



- For generic black holes with charge $\mathcal{Q}$ in asymptotically Minkowski 3+1 dimensions, the SYK model yields, in terms of $A_0 = 2G\mathcal{Q}^2/c^4$, the horizon area at $T = 0$:

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

There is no degeneracy, but an exponentially small level spacing down to the ground state.

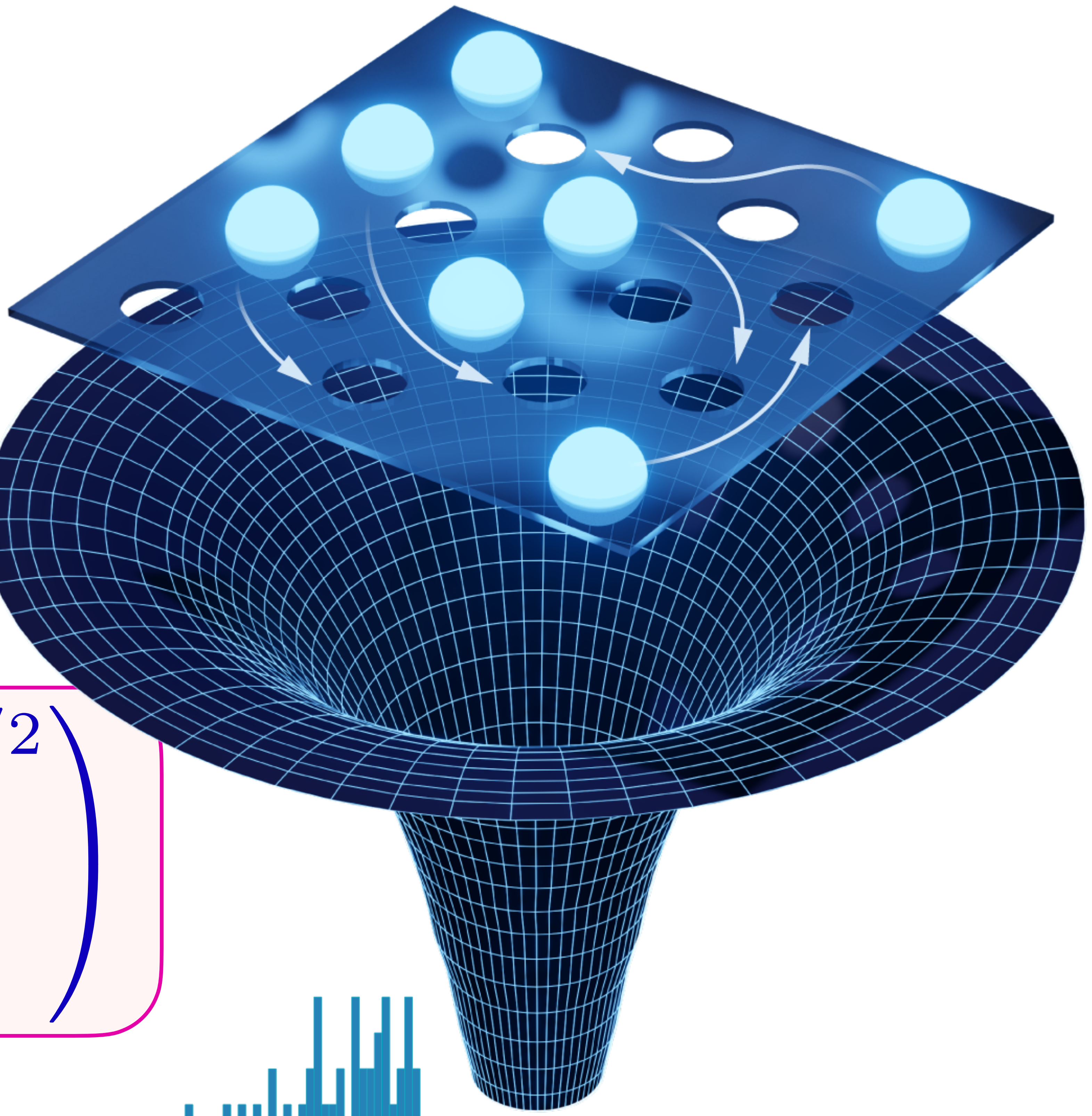


Quantum simulation of charged black holes by the SYK model

- For generic black holes with charge Q in asymptotically Minkowski 3+1 dimensions, the SYK model yields, in terms of $A_0 = 2GQ^2/c^4$, the horizon area at $T=0$:

Bekenstein-Hawking

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$



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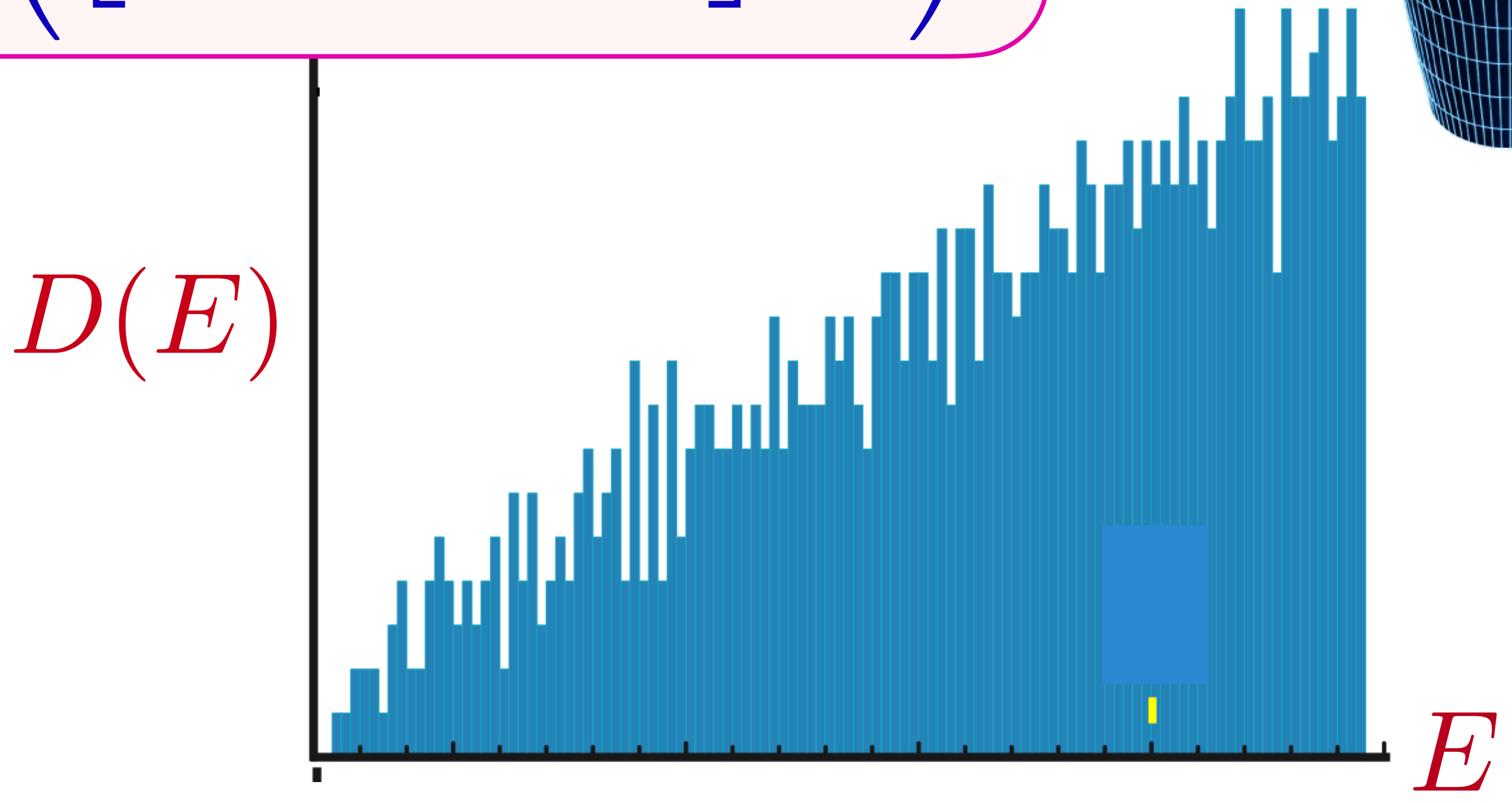

Quantum simulation of charged black holes by the SYK model

- For generic black holes with charge $\mathcal{Q}$ in asymptotically Minkowski 3+1 dimensions the SYK model yields, in terms of A_0 $2G\mathcal{Q}^2/c^4$, the horizon area at $T = 0$:

Developments from the SYK model
Moitra, Trivedi, Vishal (2018); Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019)

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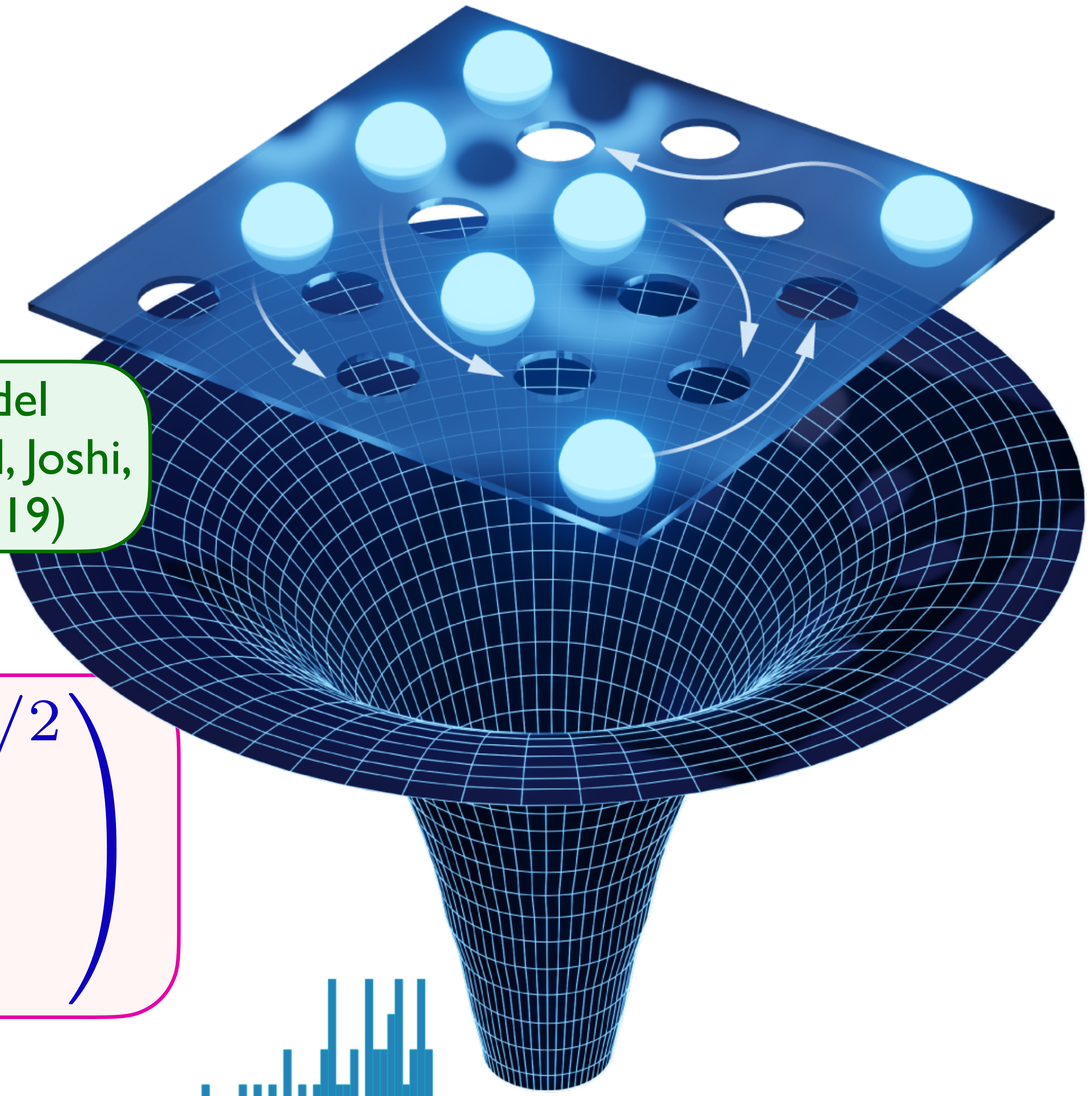

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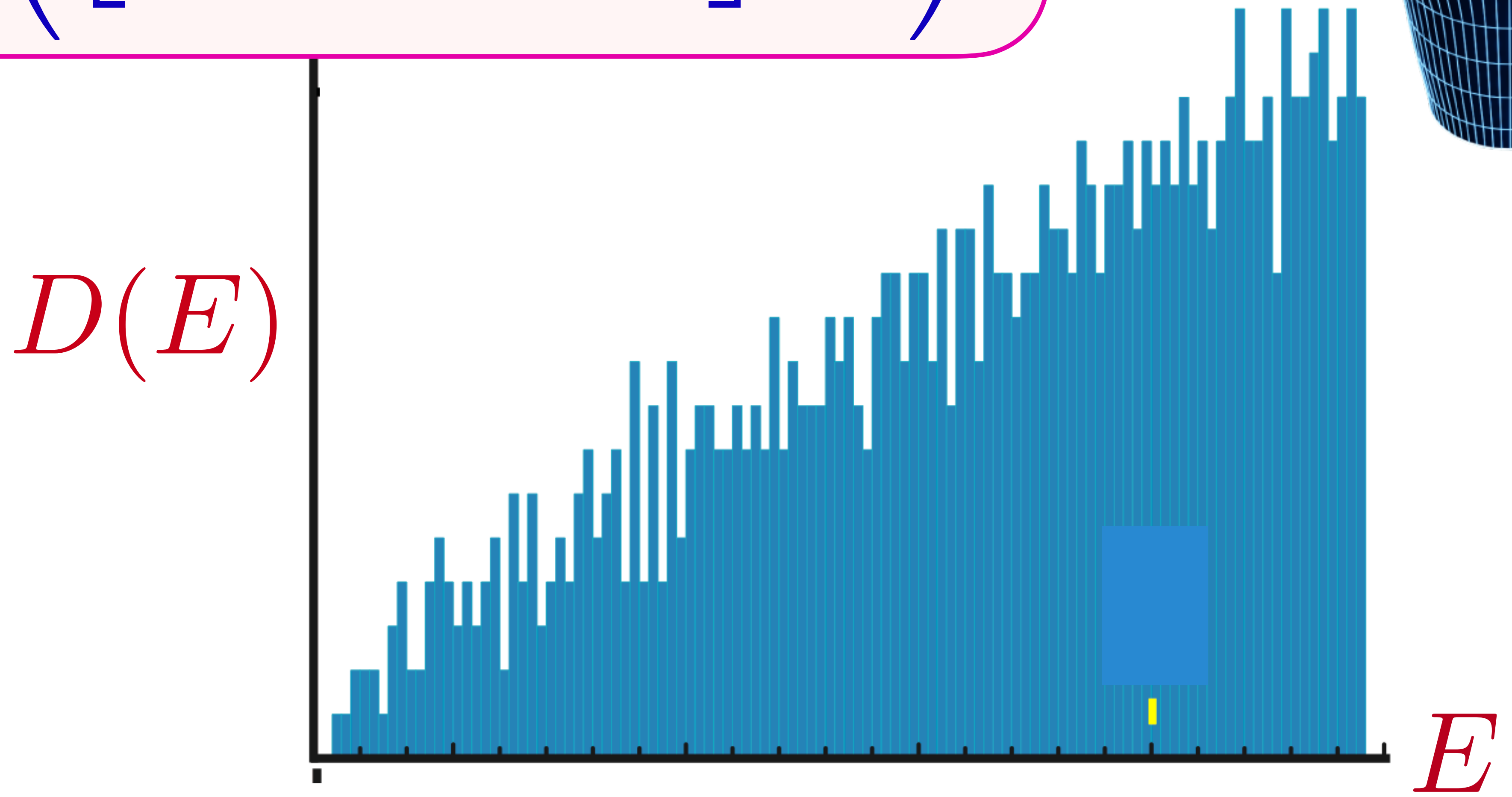

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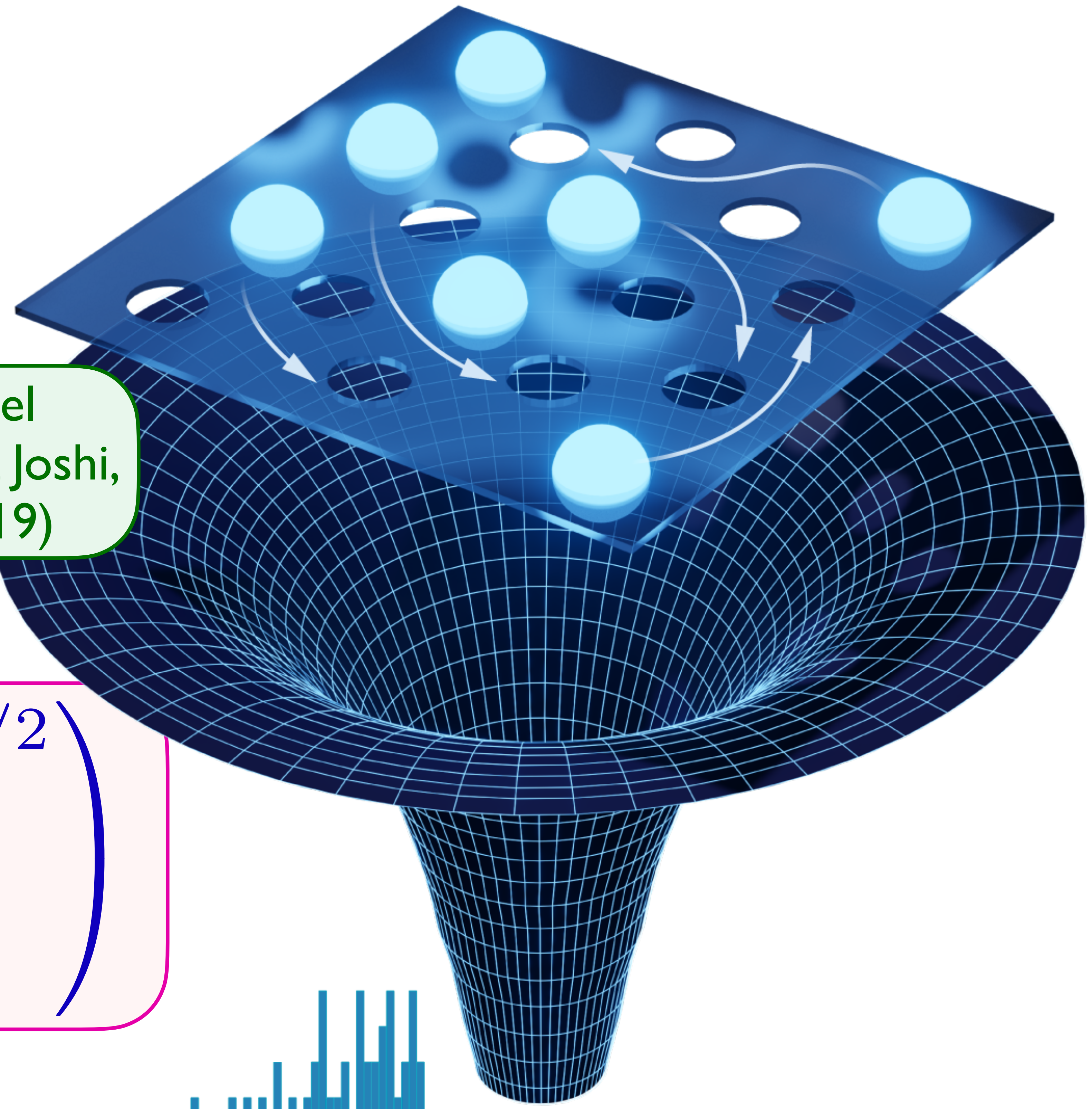

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Iliesiu, Murthy, Turiaci (2022)

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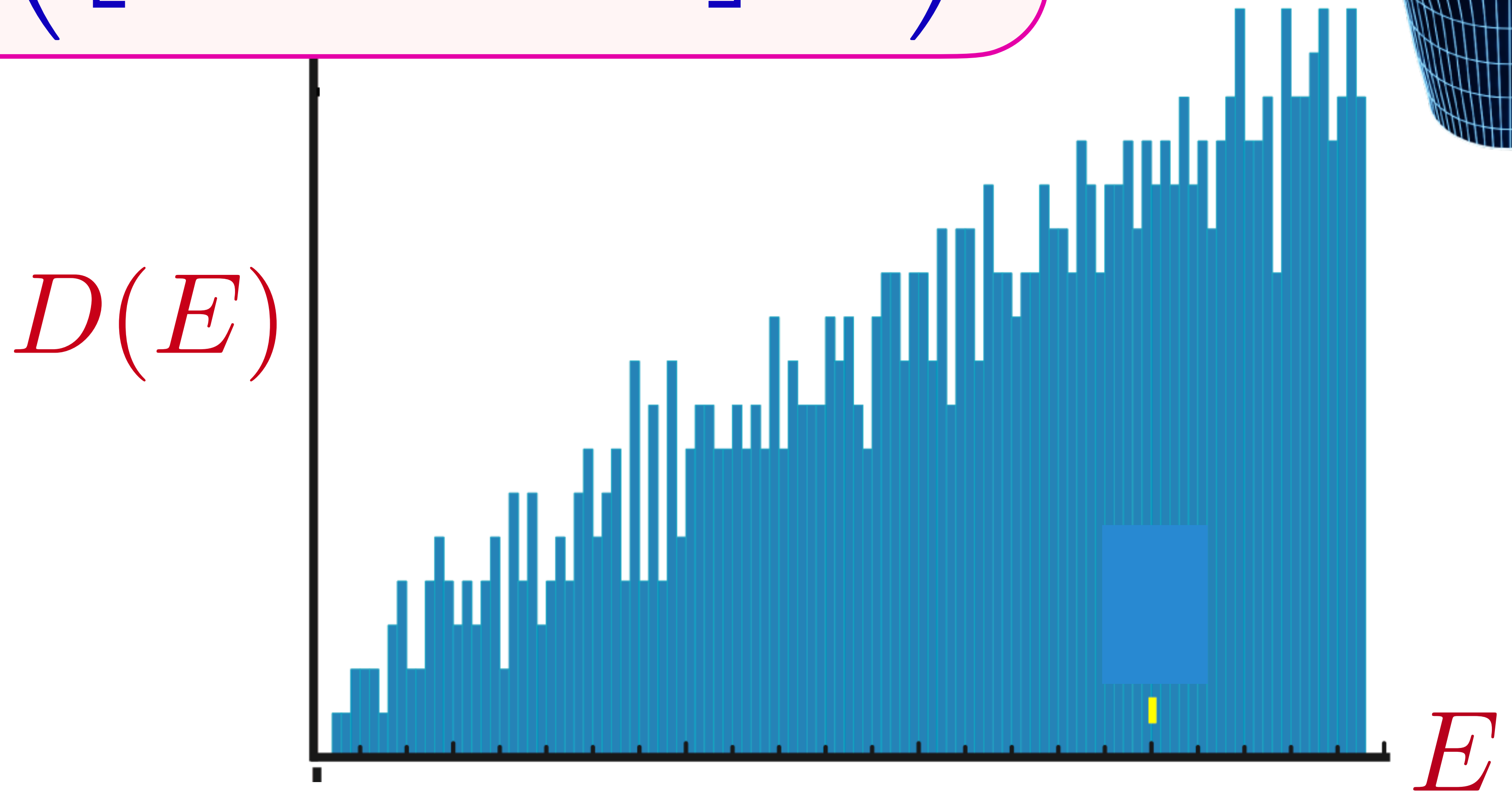
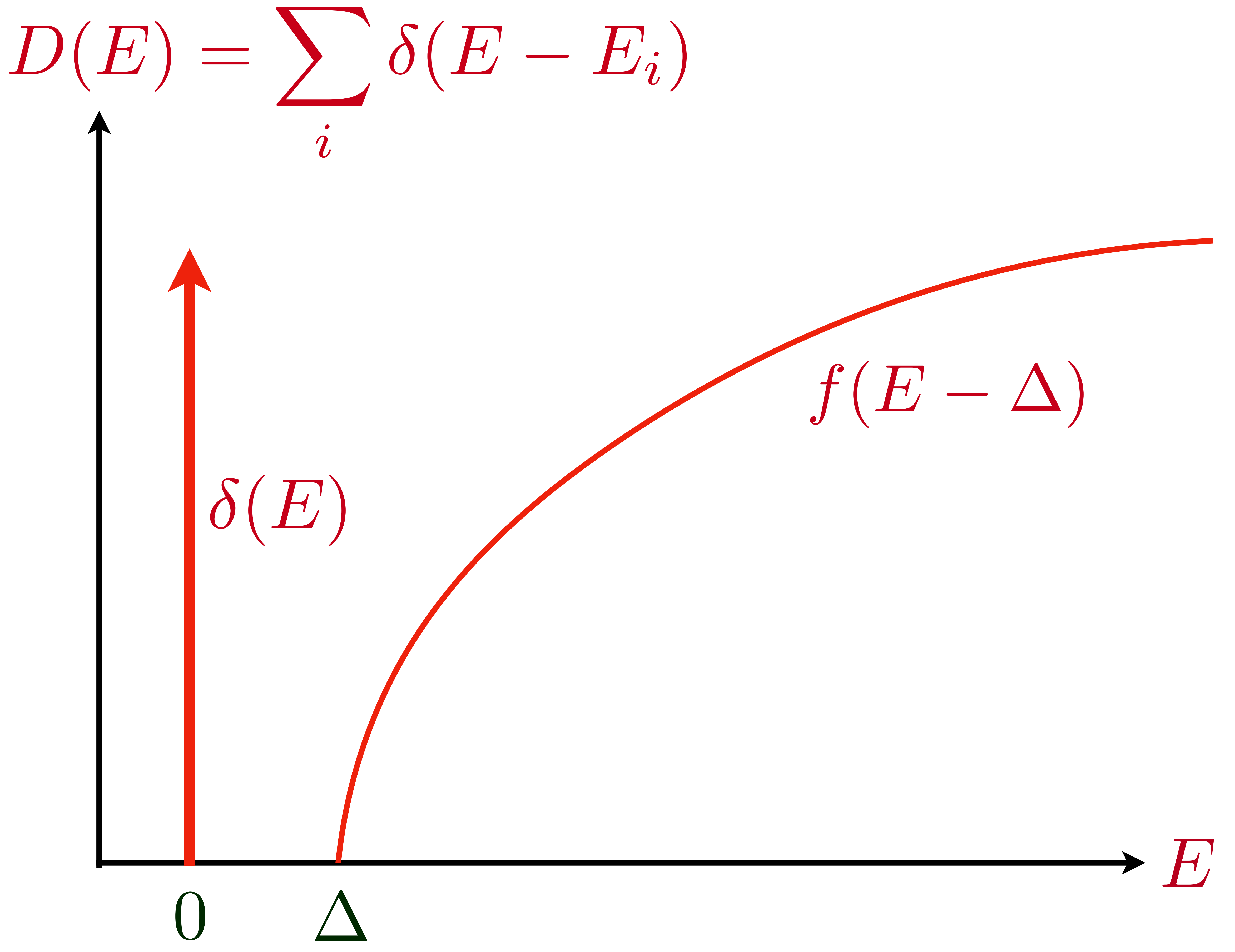


String theory of charged black holes

- With sufficient low energy supersymmetry:

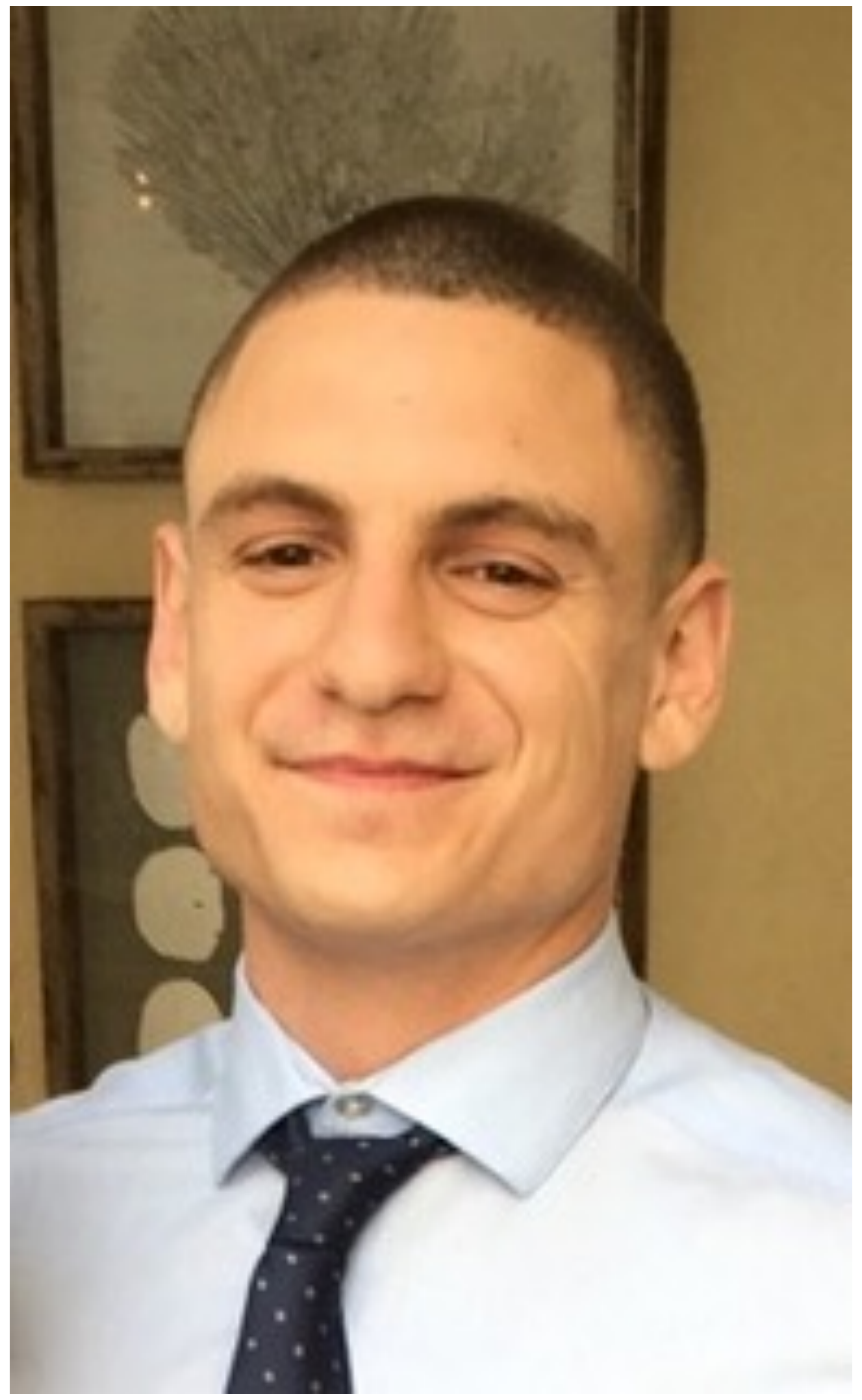
$$D(E) = \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\Big[\delta(E) + \theta(E-\Delta) f(E-\Delta) + \ldots\Big]$$

There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state. Similar results for supersymmetric SYK models



M. Heydeman, L.V. Iliesiu, G. J. Turiaci, and W. Zhao, 2020
L.V. Iliesiu, S. Murthy, G. J. Turiaci, 2022

From the SYK model to a universal theory of strange metals



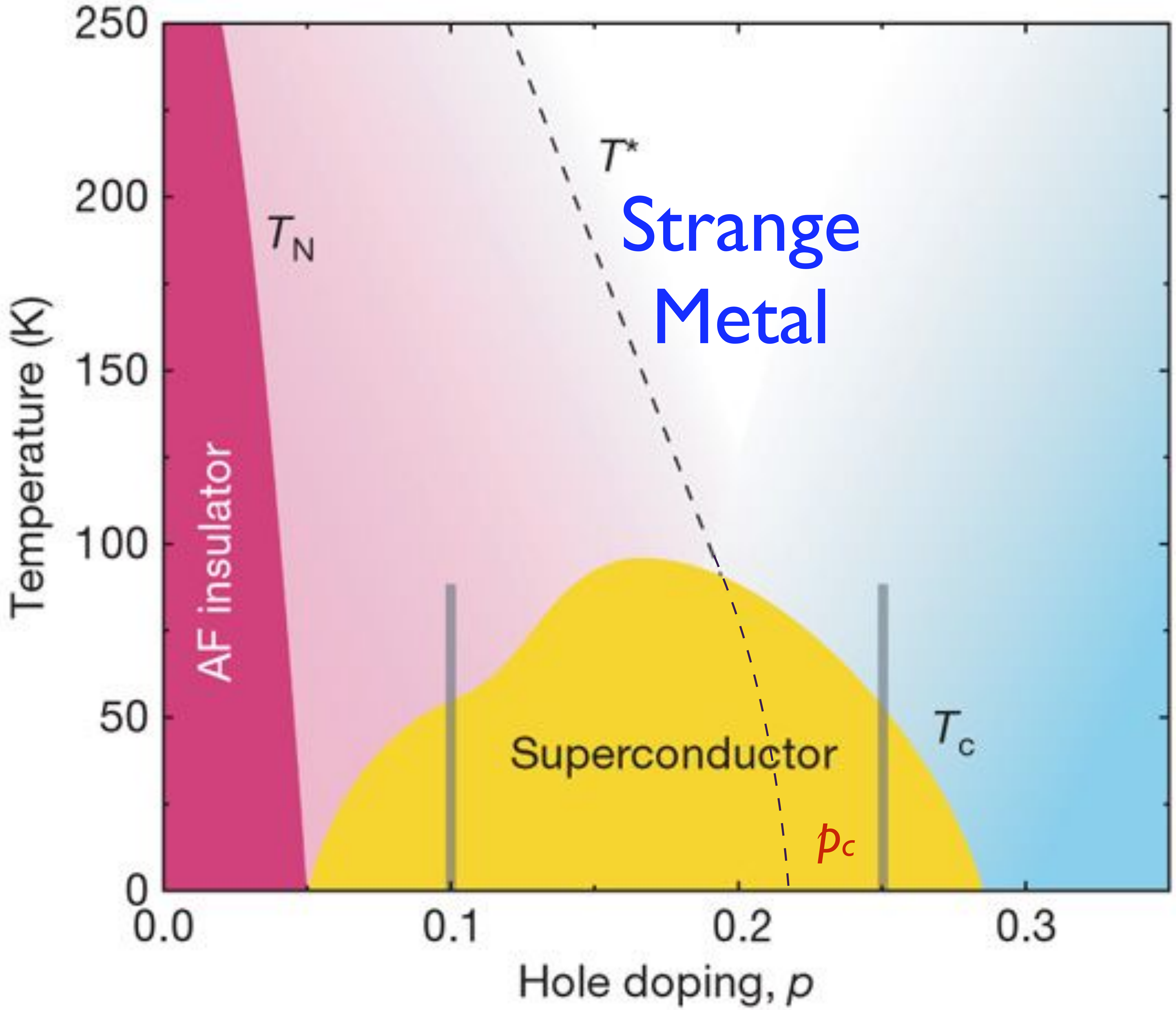
Aavishkar Patel
Flatiron Institute, NYC

Haoyu Guo
Harvard

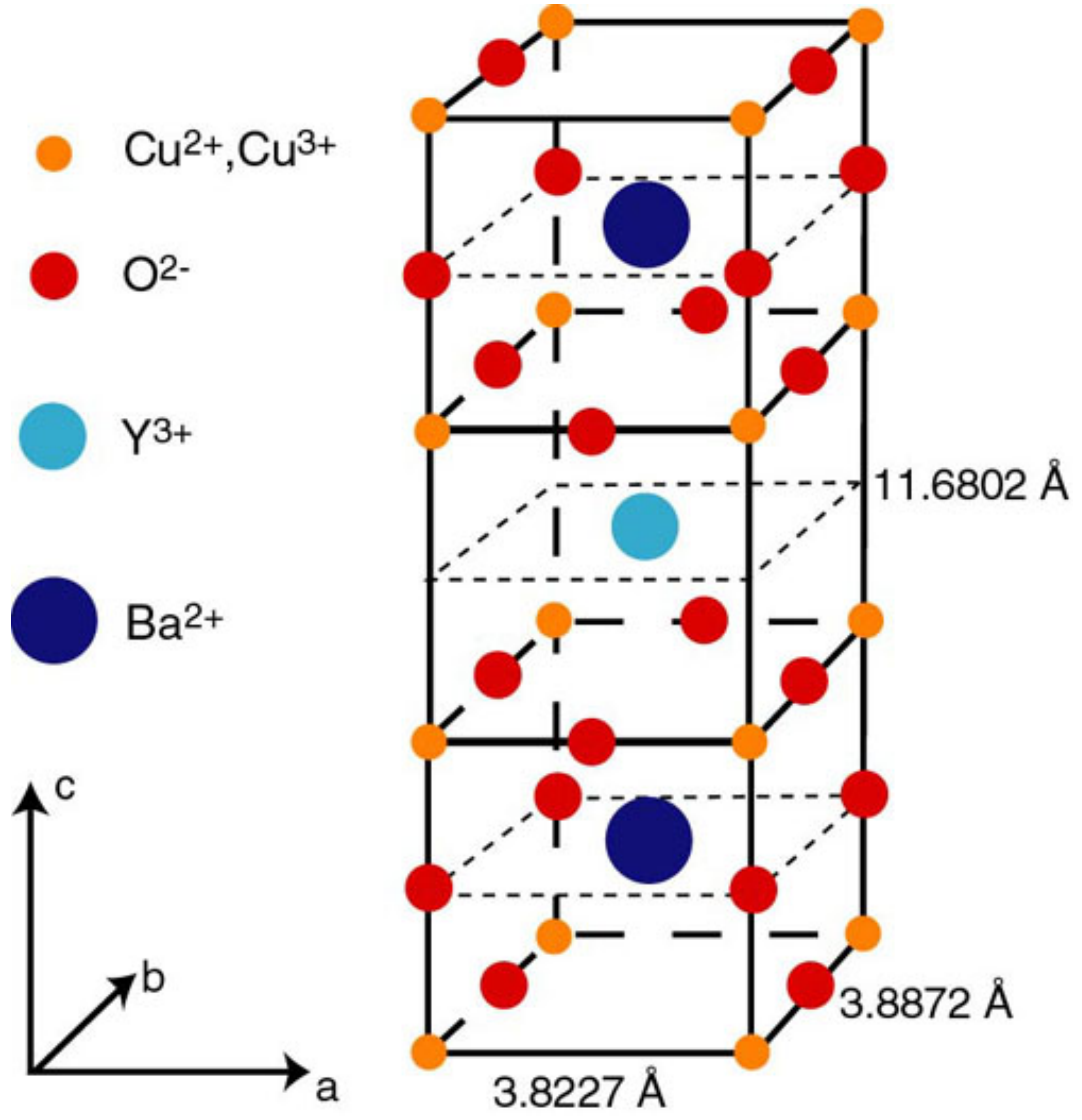
Ilya Esterlis
Harvard ➔ Wisconsin

arXiv: 2103.08615, 2203.04990, 2207.08841

E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, arXiv:2012.00763



250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Strange
Metal
Superconductor
T_c
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

3. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\text{trans}}(\omega)} - i\omega \dfrac{m^*_{\text{trans}}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon……A. Georges, arXiv:2205.04030

4. Photoemission: nearly "marginal Fermi liquid" electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau(\omega)} \sim |\omega| \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

T.J. Reber….D. Dessau, Nature Communications **10**, 5737 (2019)

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell}\psi_i^\dagger \psi_j \phi_\ell \,,$$

$g_{ij\ell}$ independent random numbers with zero mean. Large N limit leads to Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$.

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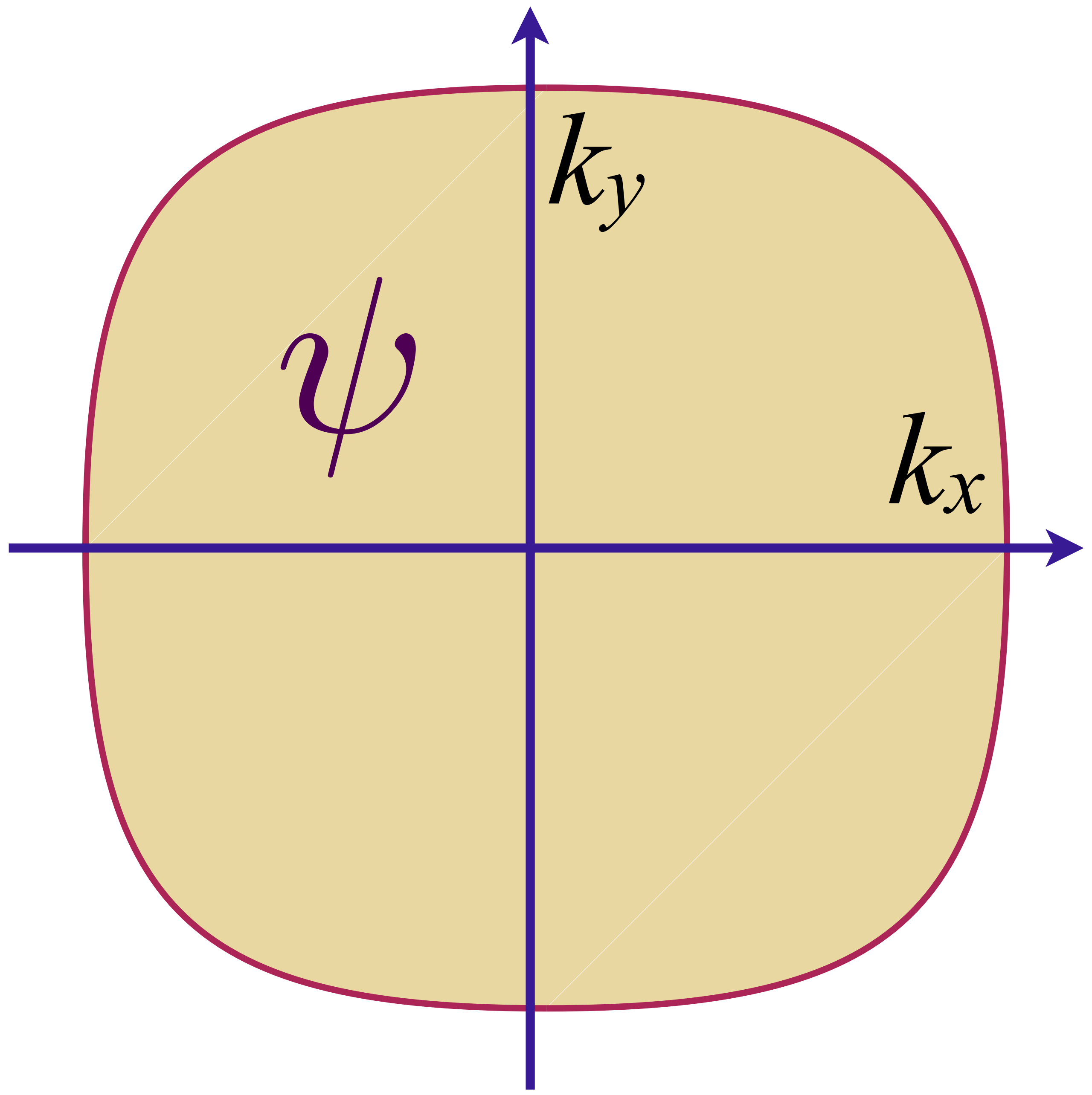
Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Fermi surface coupled to a critical boson with disorder

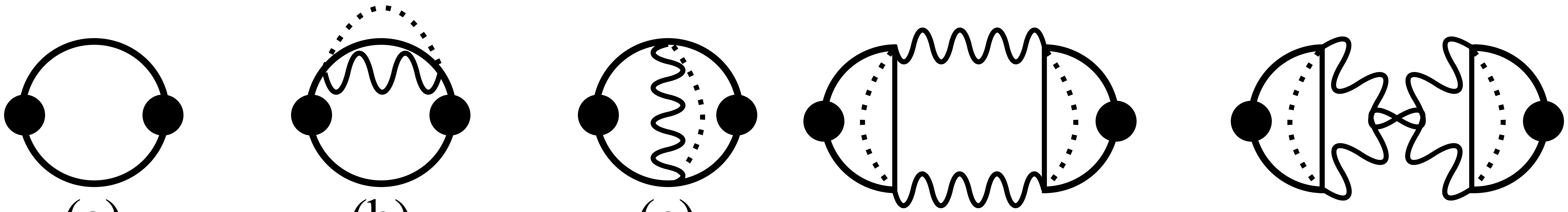
Electron Green's function: $G(\omega) \sim \dfrac{1}{\omega \dfrac{m^*(\omega)}{m} - \varepsilon(\boldsymbol{k}) + i\left(\dfrac{1}{\tau_e} + \dfrac{1}{\tau_{\rm in}(\omega)}\right) \text{sgn}(\omega)}$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2\right)|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi}\left(\frac{g^2}{v^2} + g'^2\right)\ln(\Lambda/\omega)$$

Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat.

Fermi surface coupled to a critical boson with disorder

Conductivity:



(a) σ_v (b) $\frac{\sigma_{\Sigma,g}}{2}, \frac{\sigma_{\Sigma,g'}}{2}$ (c) $\sigma_{V,g}$ (d) (e)

+ all ladders and bubbles.....

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Fermi surface coupled to a critical boson with disorder

$$\text{Conductivity: } \sigma(\omega) \sim \frac{1}{\dfrac{1}{\tau_{\text{trans}}(\omega)} - i\omega\,\dfrac{m^*_{\text{trans}}(\omega)}{m}}$$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*_{\text{trans}}(\omega)}{m} \sim \frac{2g'^2}{\pi}\ln(\Lambda/\omega)$$

$$\text{Electron Green's function: } G(\omega) \sim \frac{1}{\omega\,\dfrac{m^*(\omega)}{m} - \varepsilon(\boldsymbol{k}) + i\left(\dfrac{1}{\tau_e} + \dfrac{1}{\tau_{\text{in}}(\omega)}\right)\text{sgn}(\omega)}$$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2\right)|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi}\left(\frac{g^2}{v^2} + g'^2\right)\ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2;
Transport insensitive to g; Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat.

Summary

- Graphene flakes display a crossover from quasiparticle behavior with large disorder-induced fluctuations at low T, to a higher T regime with weak fluctuations and a T-independent thermopower. The latter behavior matches that of the SYK model

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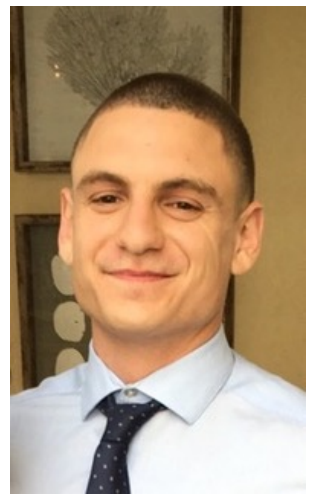
- Linear-T resistivity, $T \ln(1/T)$ specific heat, $\sim 1/\omega$ optical conductivity, and marginal Fermi liquid electron spectrum *all* arise from a SYK-like model with spatially random interactions in a two-dimensional quantum-critical metal.





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