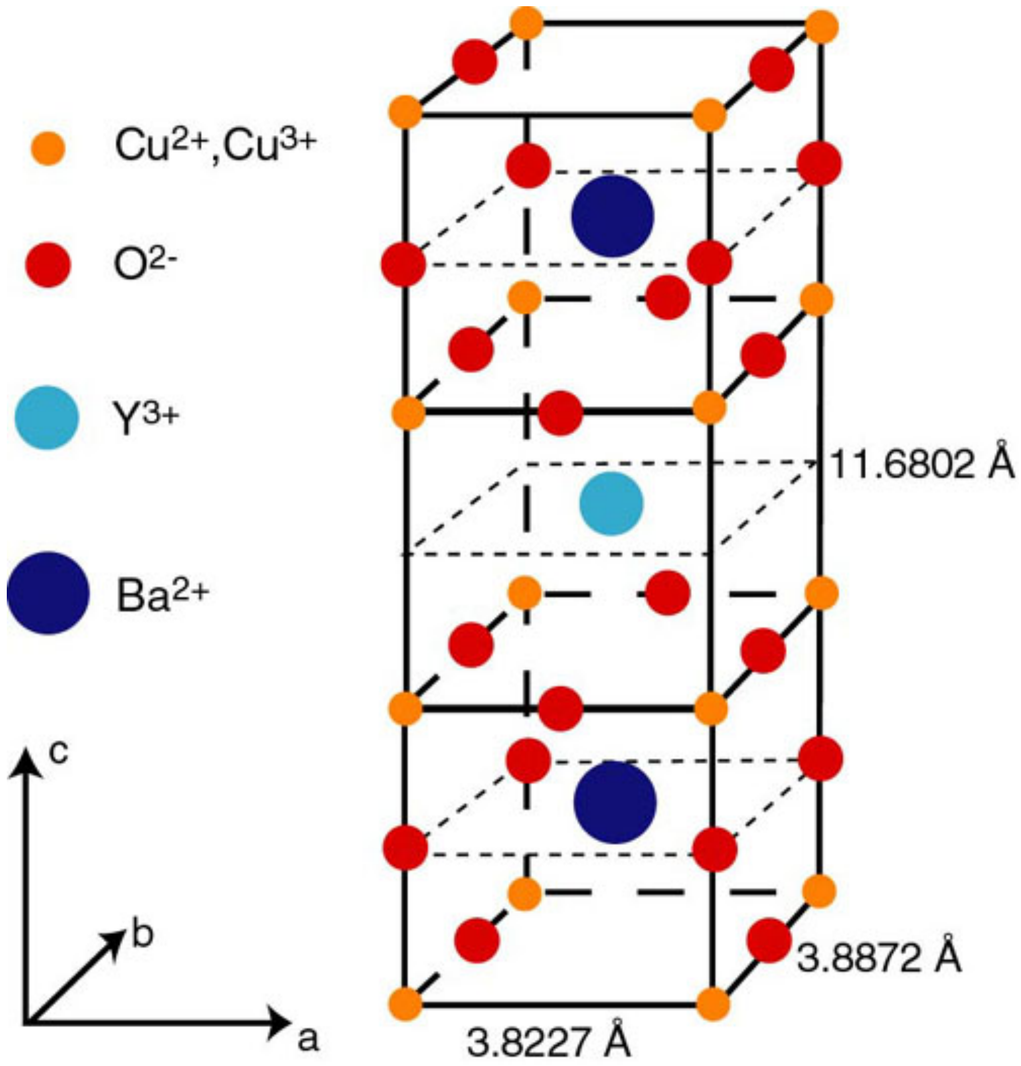
Many-fermion quantum entanglement in the cuprate superconductors

Imperial College, London

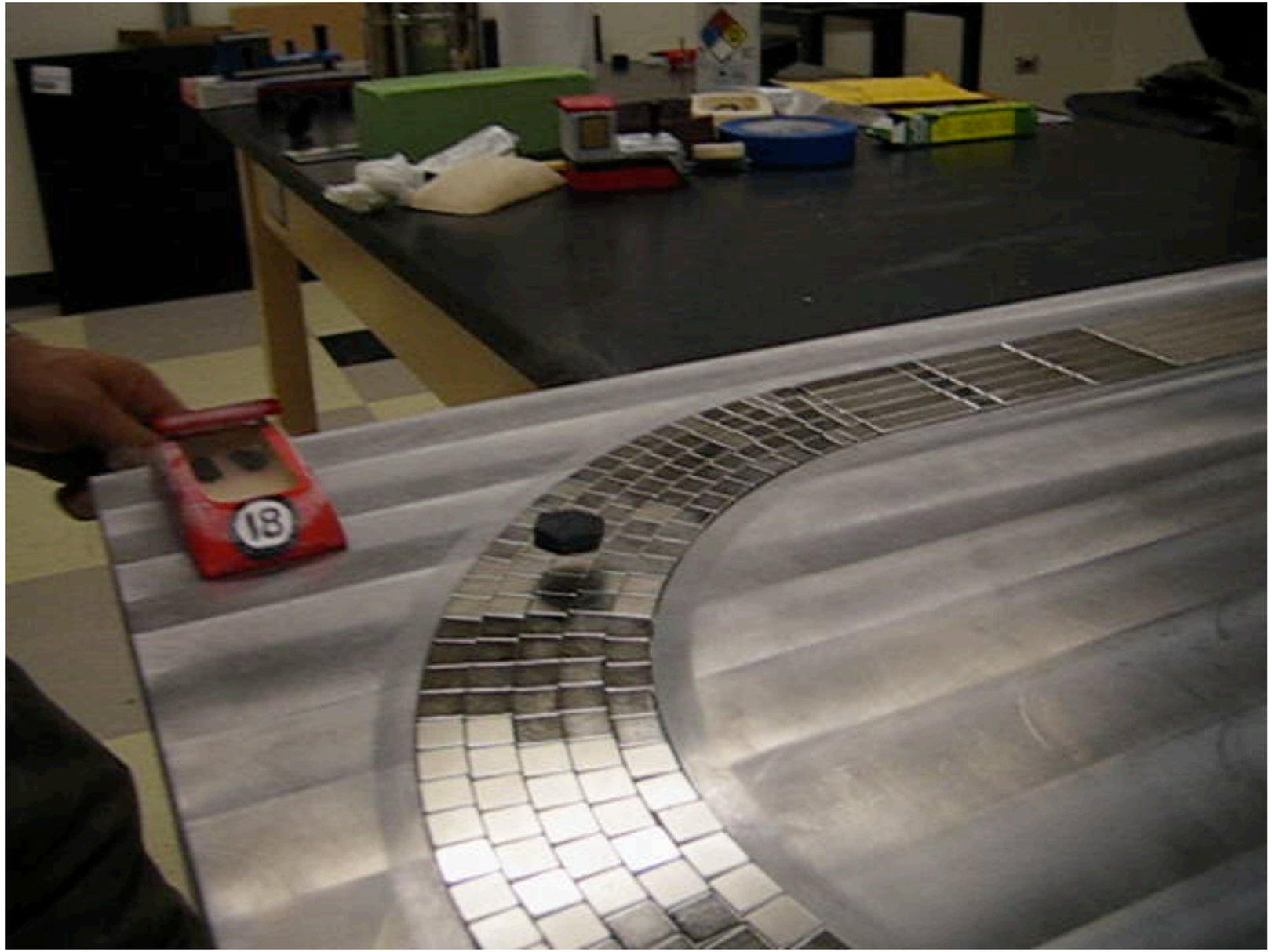
February 11, 2026

Subir Sachdev

Cuprate high temperature superconductors



$$YBa_2Cu_3O_{6+x}$$



Nd-Fe-B magnets, YBaCuO superconductor

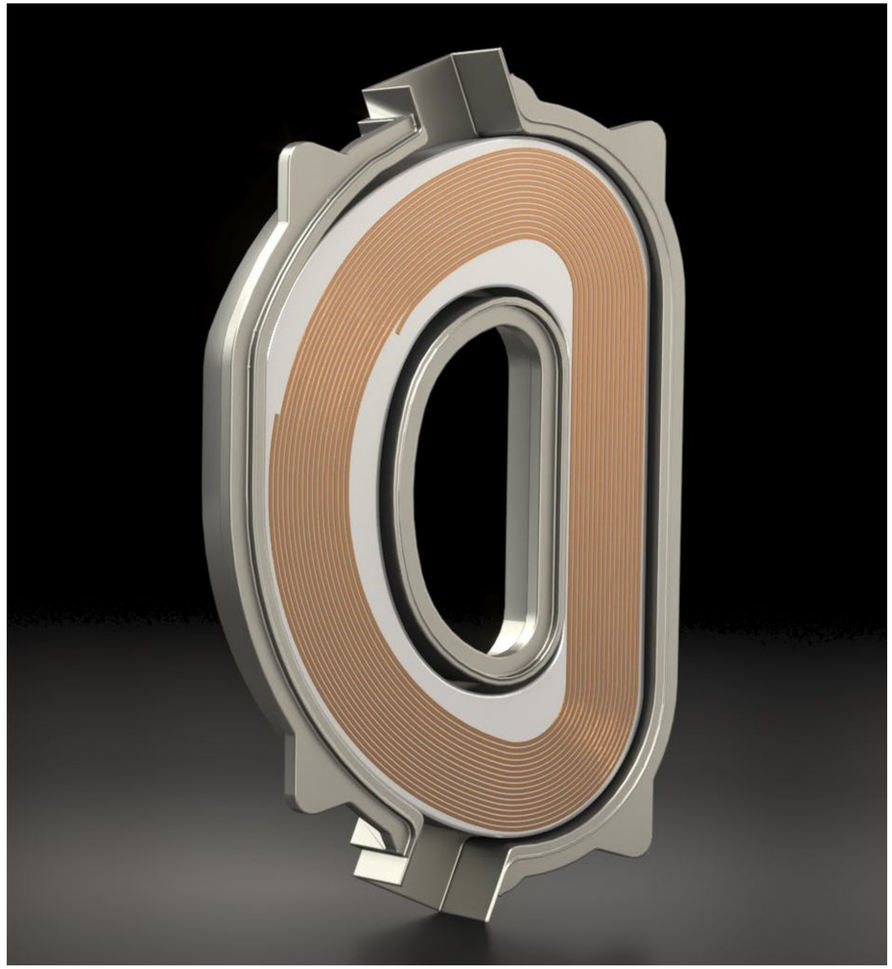
Julian Hetel and Nandini Trivedi, Ohio State University

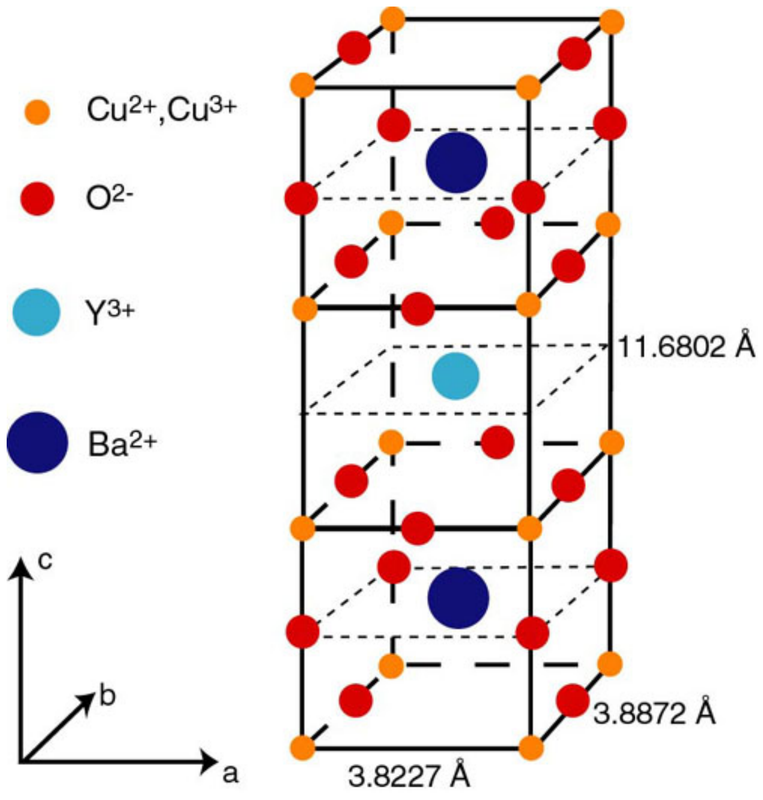
HTS Magnets: Enabling Technology

The surest path to limitless, clean, fusion energy

YBCO magnets allow for smaller, faster, and less expensive tokamaks for plasma fusion

Commonwealth Fusion Systems





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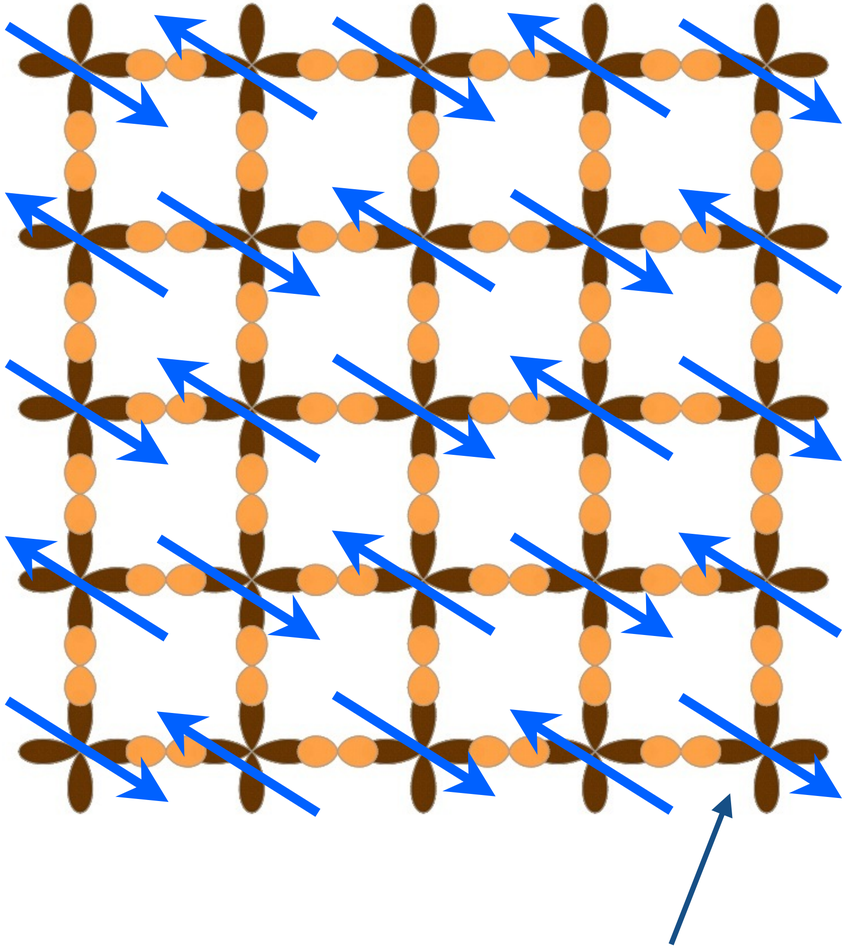


Cu

$$\mathcal{H} = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$S = 1/2$ on each site

$$|\uparrow\rangle, \; |\uparrow\rangle$$

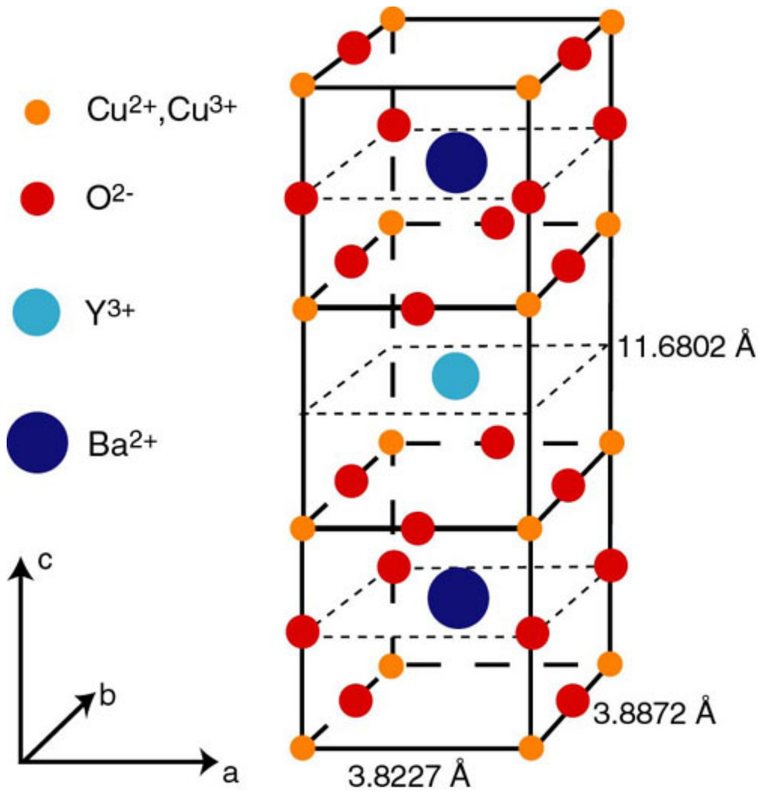
$$S_z |\uparrow\rangle = (1/2) |\uparrow\rangle$$

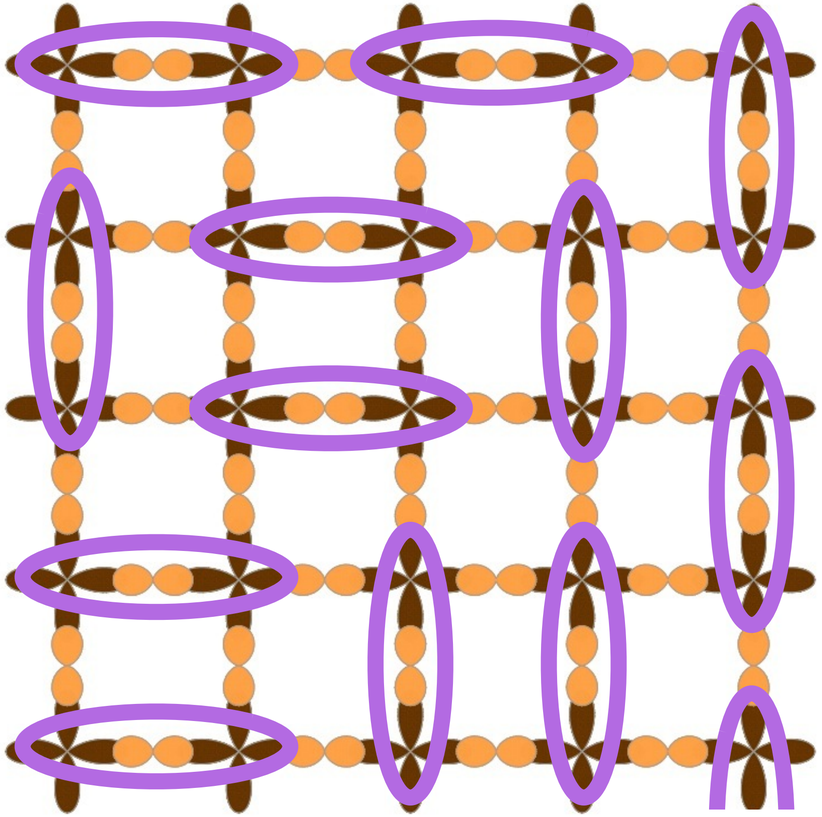
$$S_z |\downarrow\rangle = -(1/2) |\downarrow\rangle$$

$$(S_x + iS_y) |\downarrow\rangle = |\uparrow\rangle$$

$$(S_x - iS_y) |\uparrow\rangle = |\downarrow\rangle$$

Insulating antiferromagnet with one electron per site





$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

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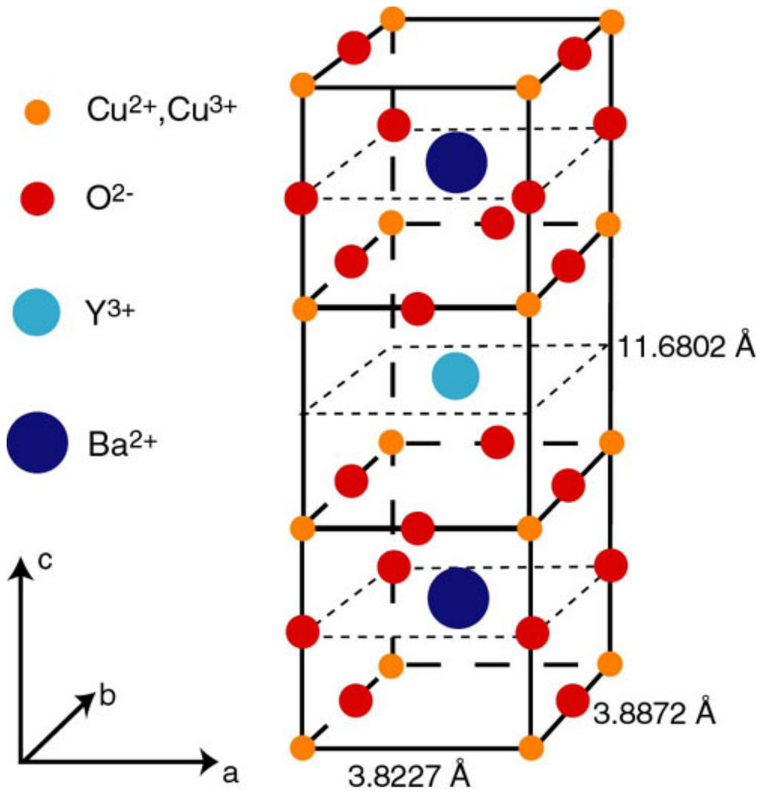
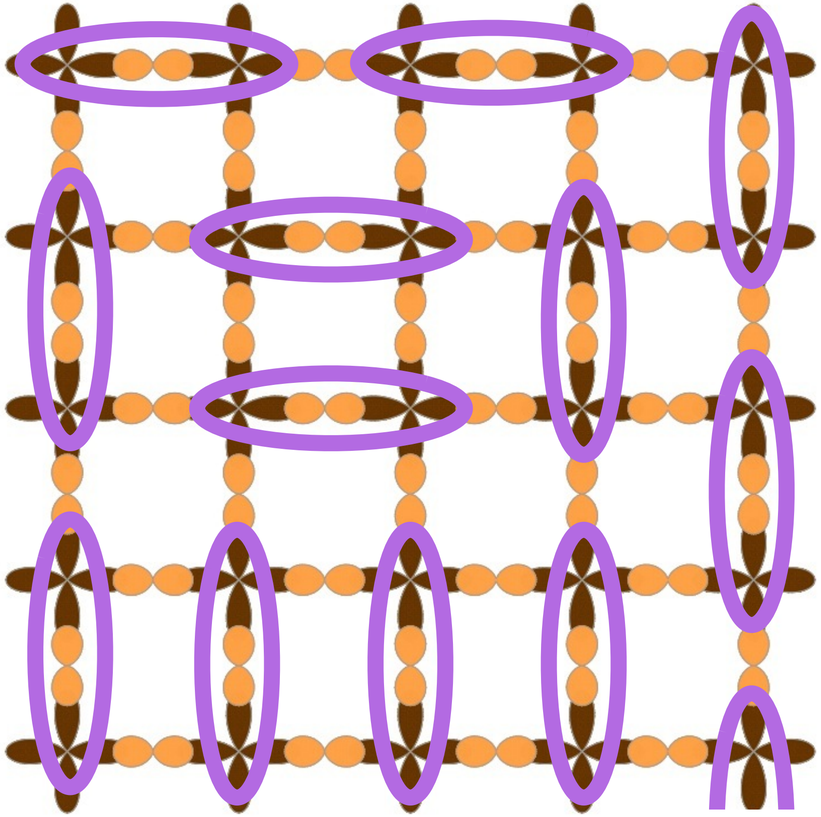
$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

P.W. Anderson (1987): The key to high temperature superconductivity is the formation of a "resonating valence bond state".

A quantum spin liquid with many-boson (spins on Cu) **entanglement**





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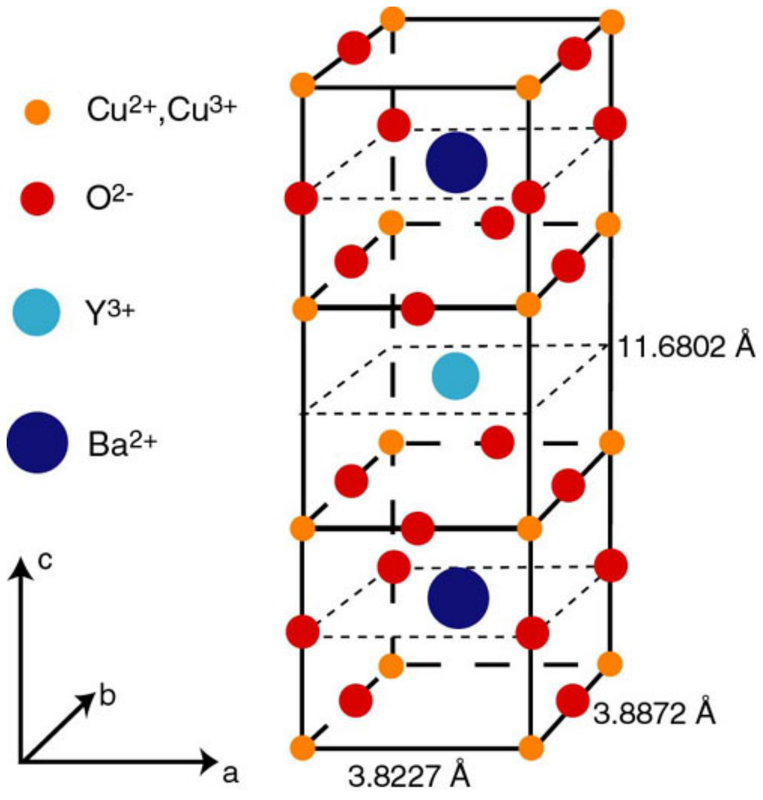

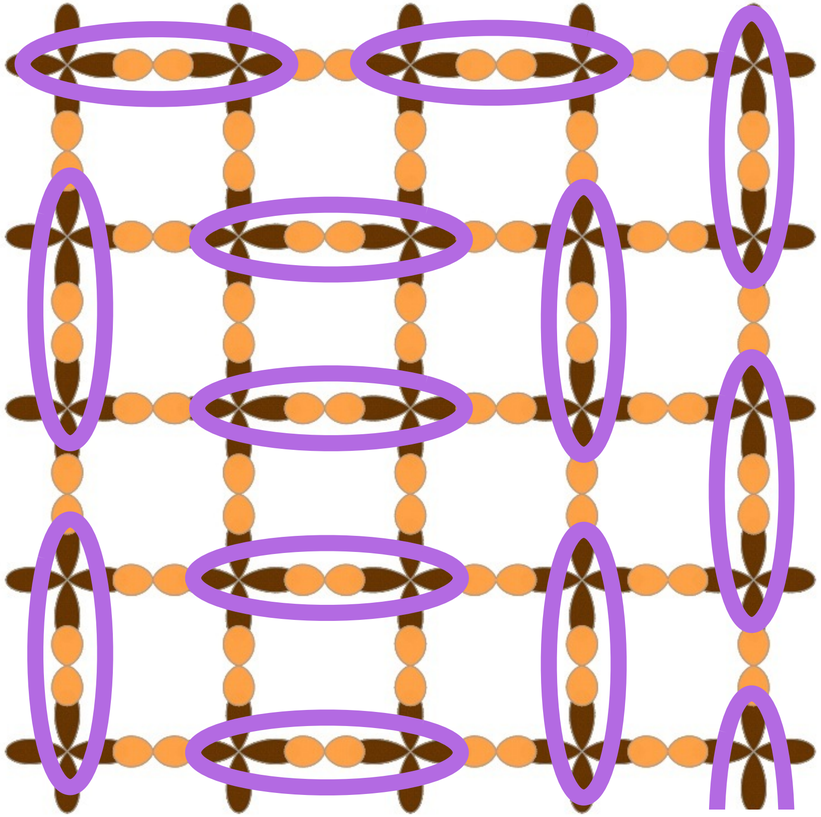

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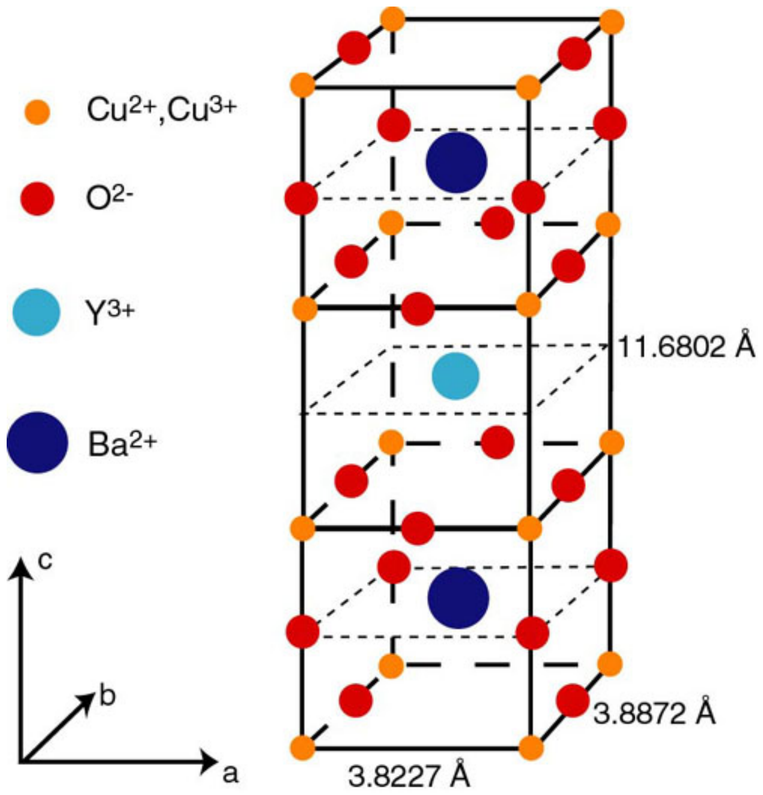

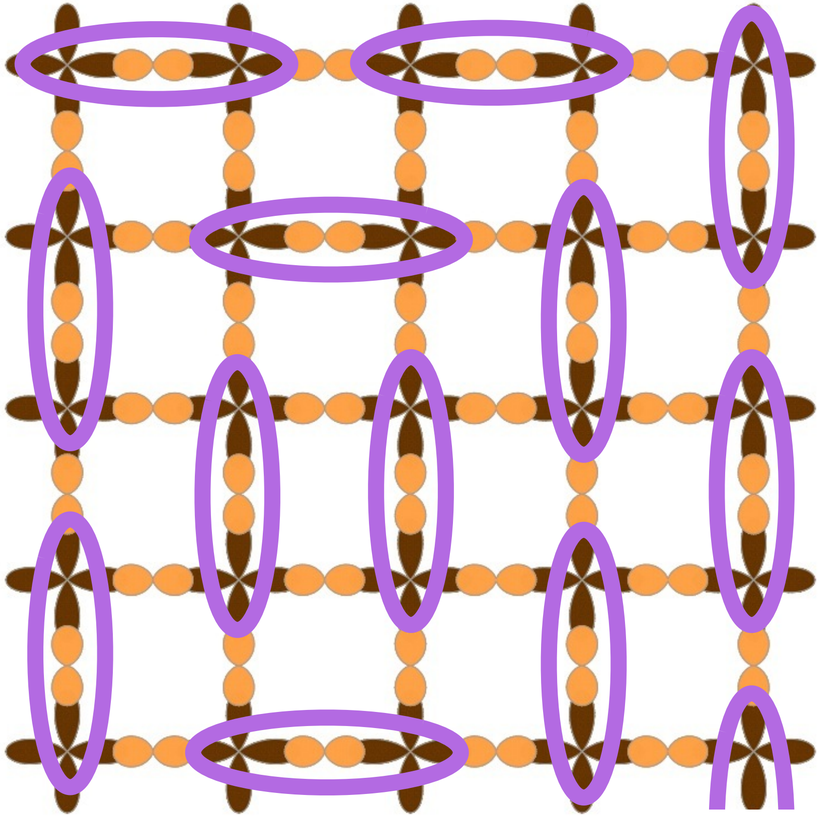

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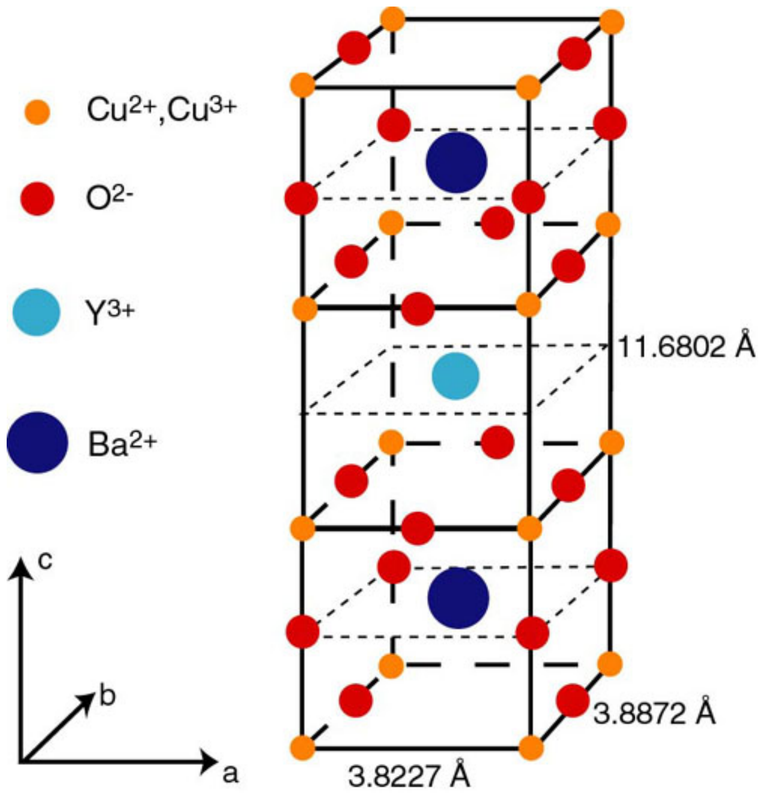

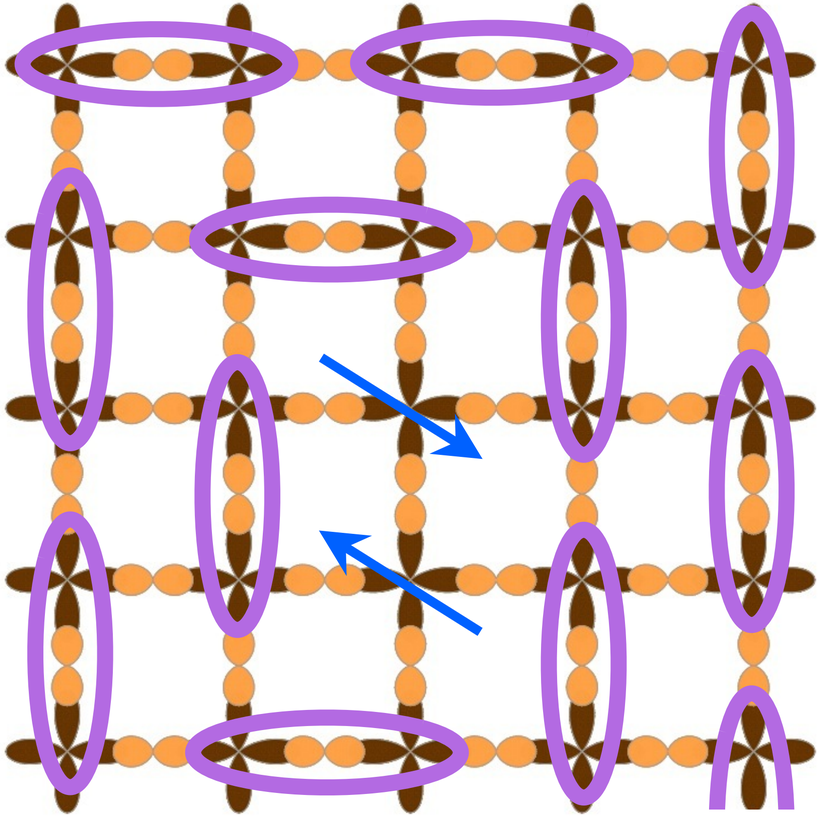

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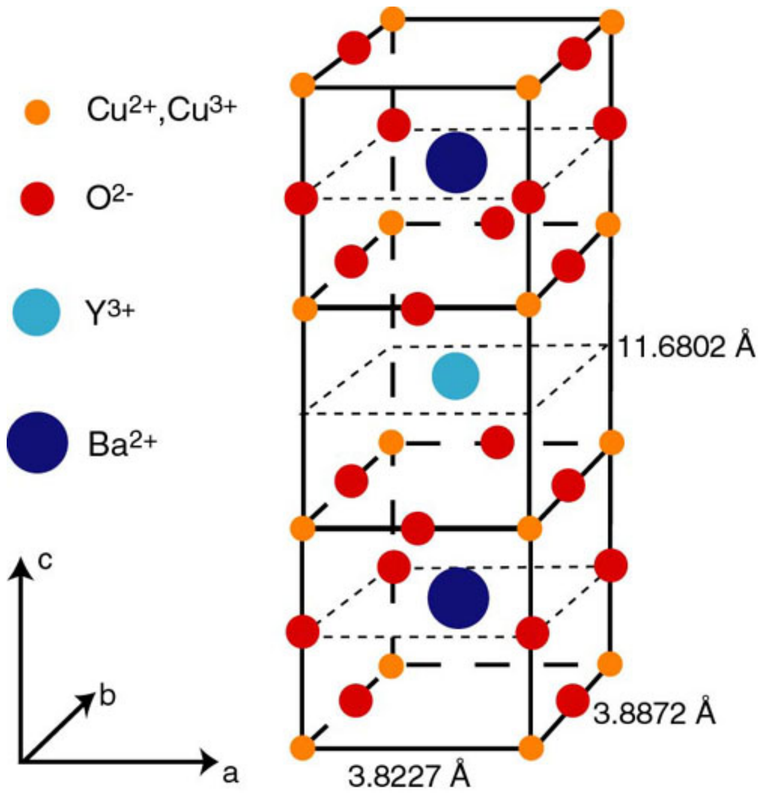
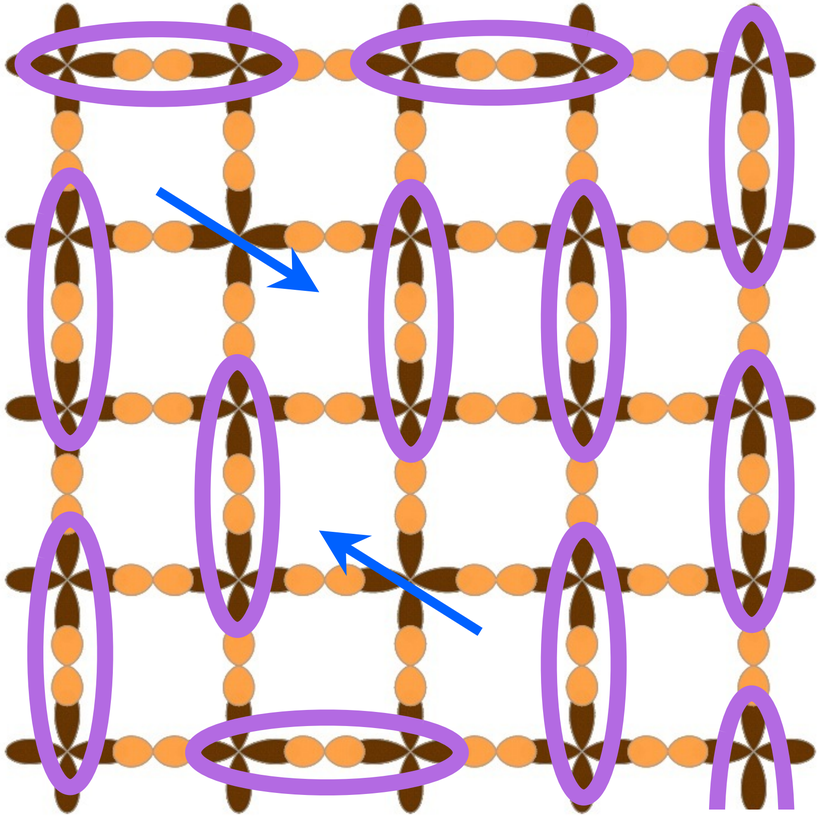
Key feature: fractionalization. Excitations are particle-like, but cannot be created by local operators: they are classified under distinct superselection/anyon sectors.





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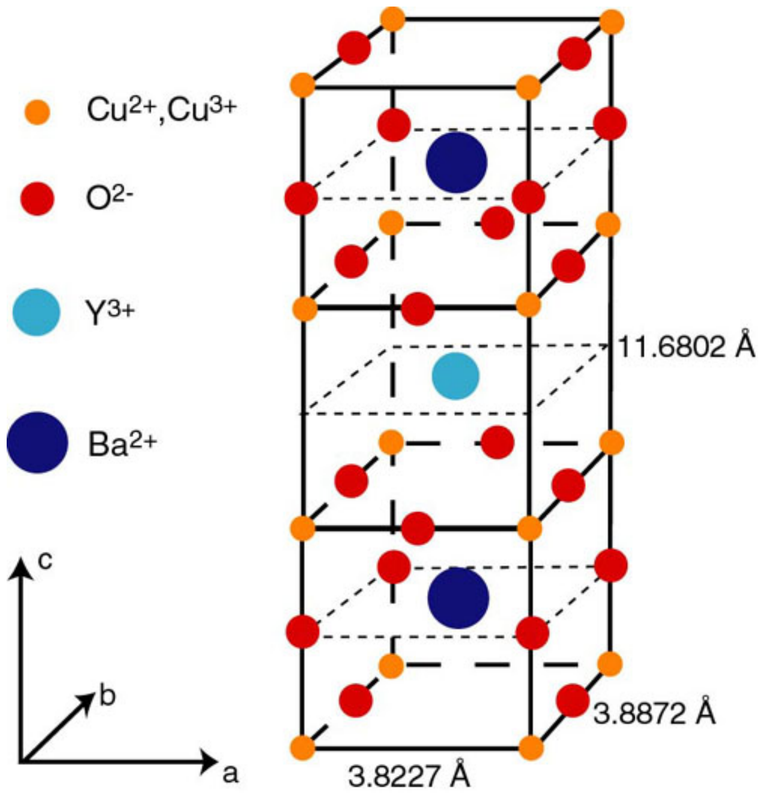

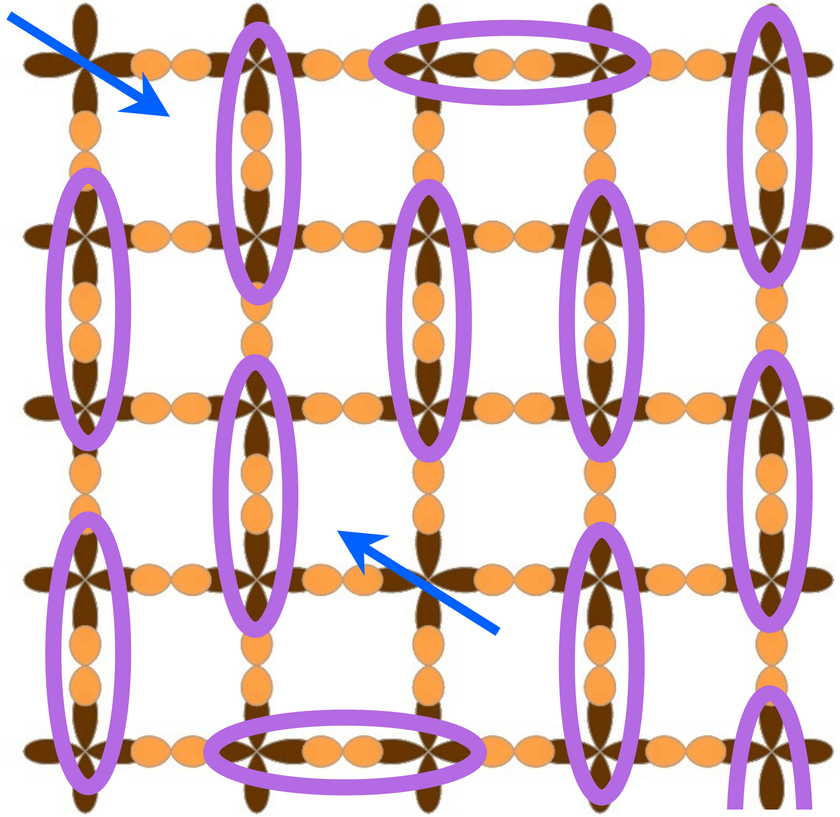

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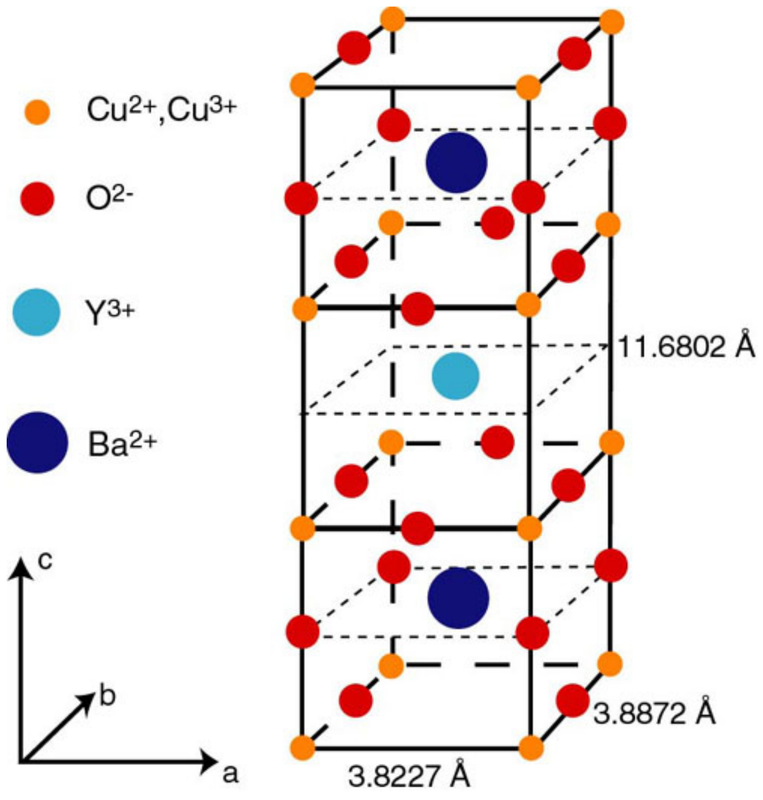

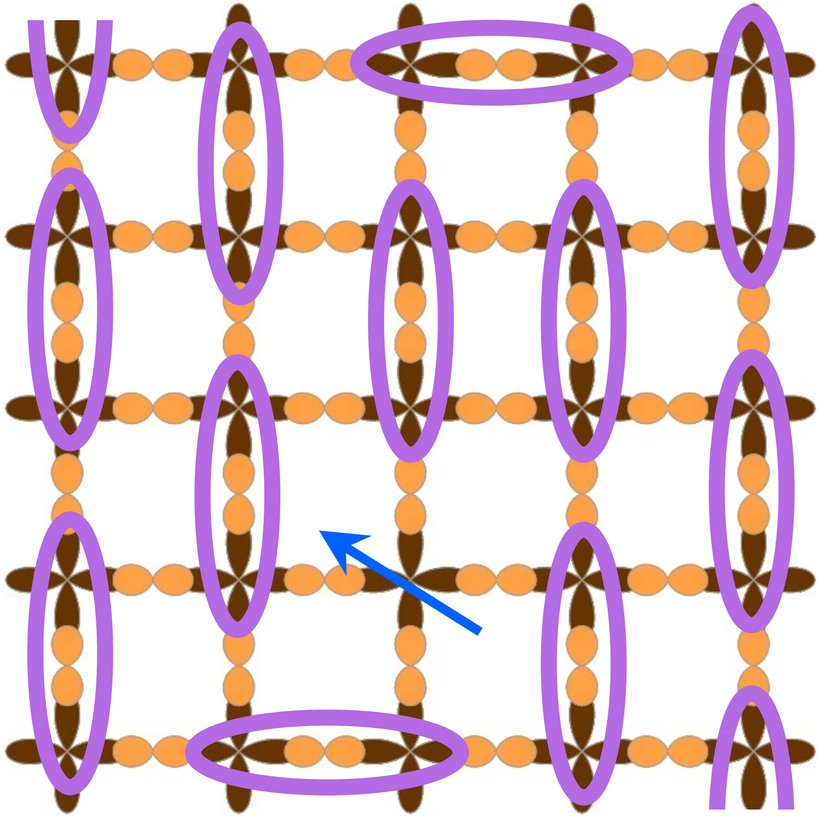

Spin S=1/2, charge neutral spinon

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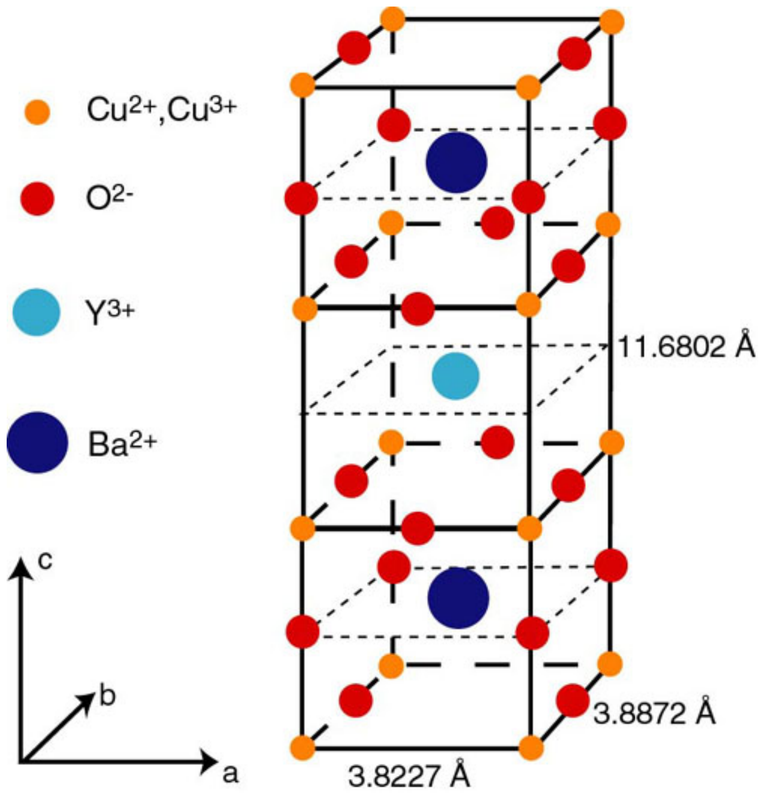

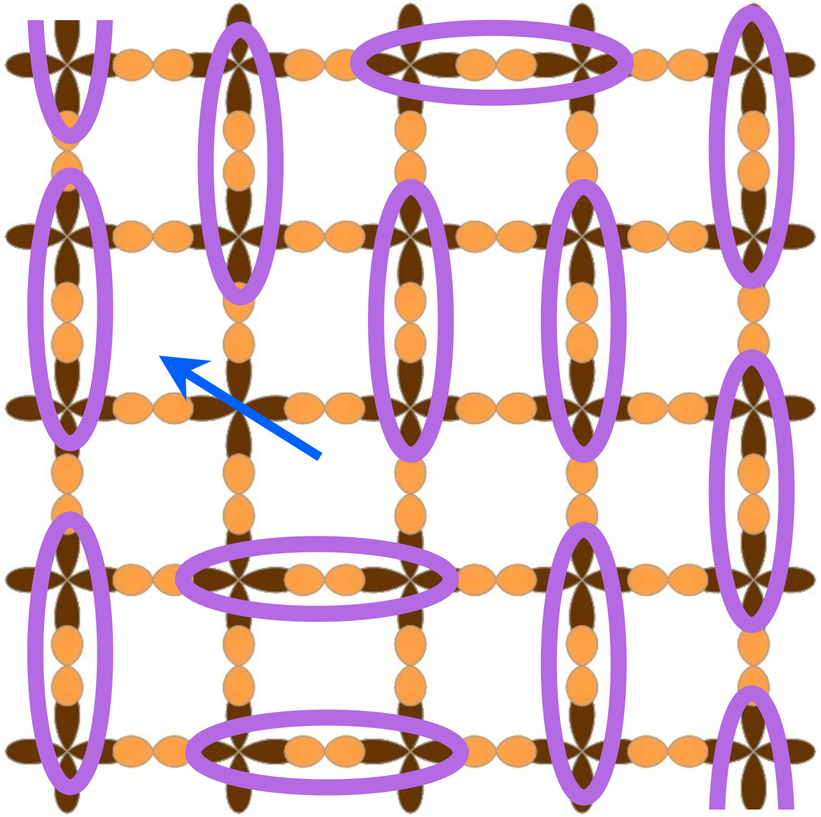

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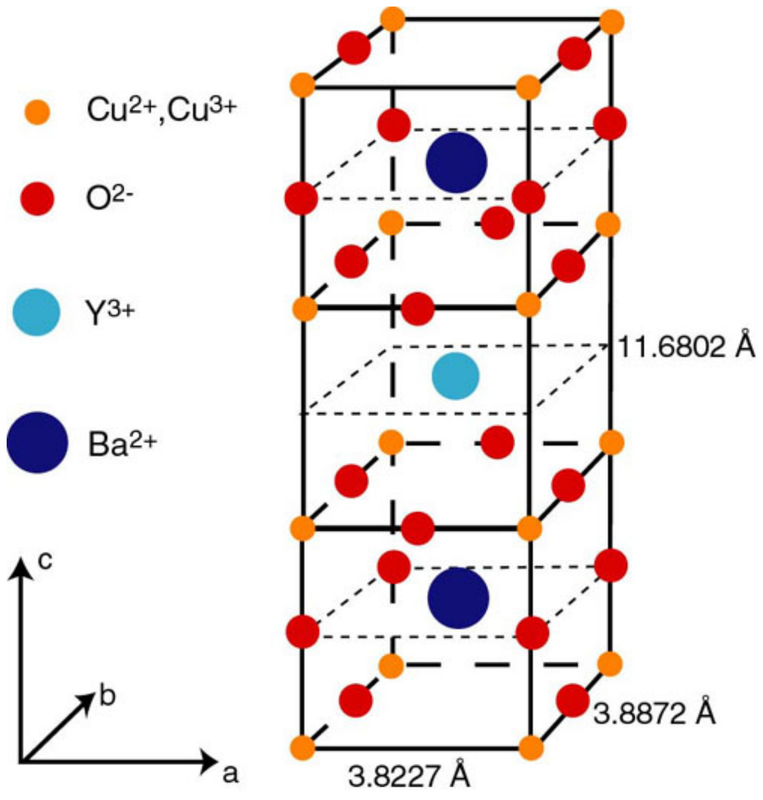

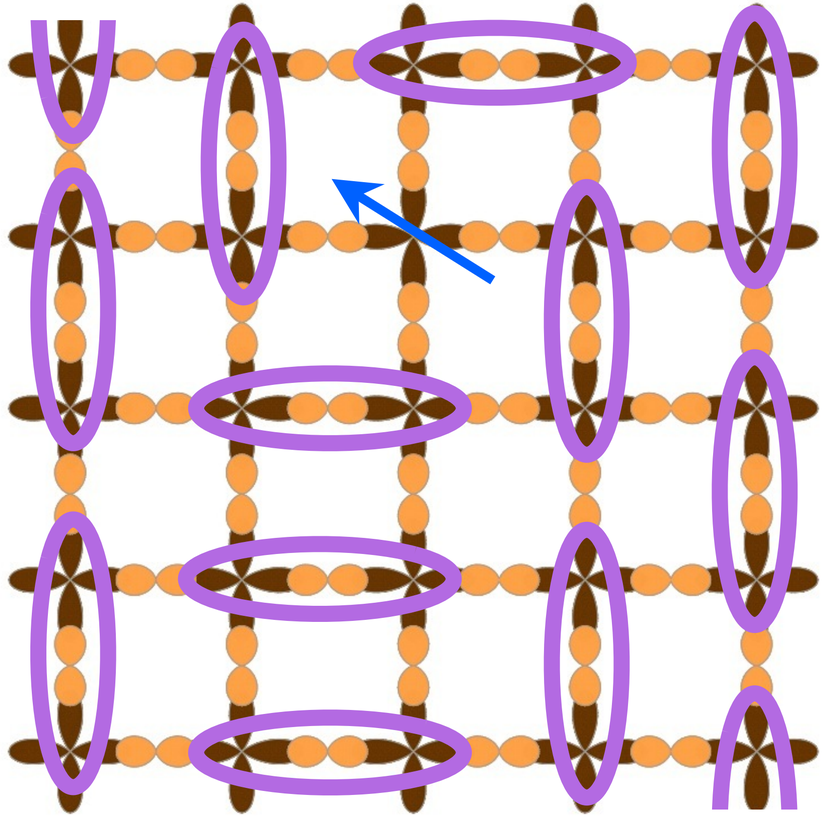

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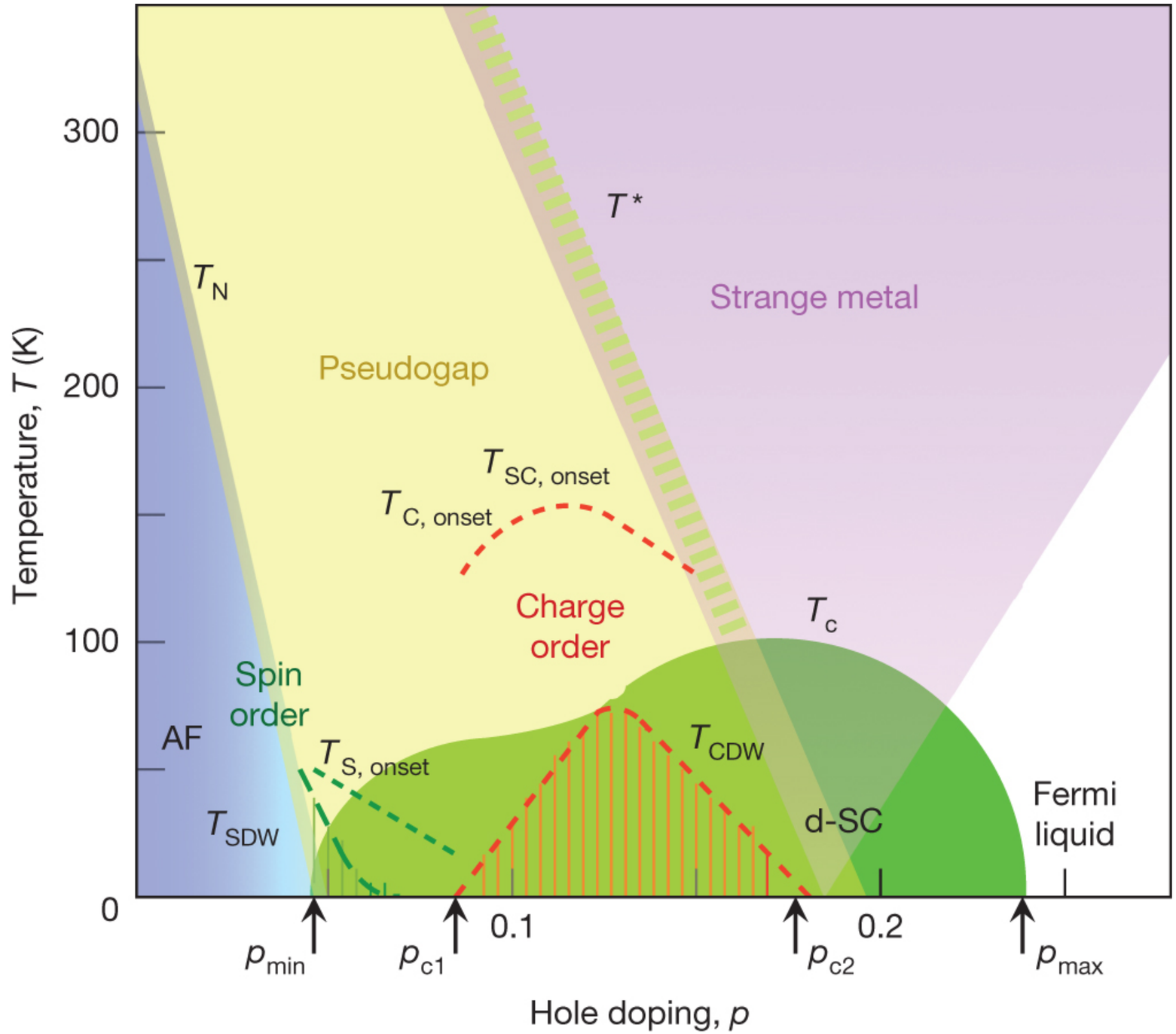
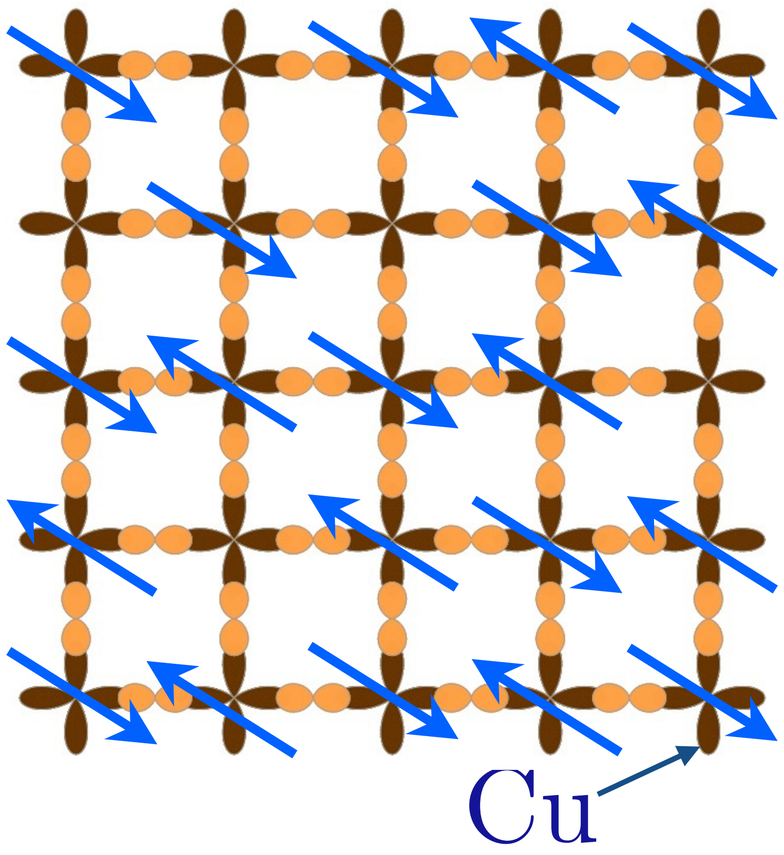
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





d-SC obtained upon doping AF with density p holes.

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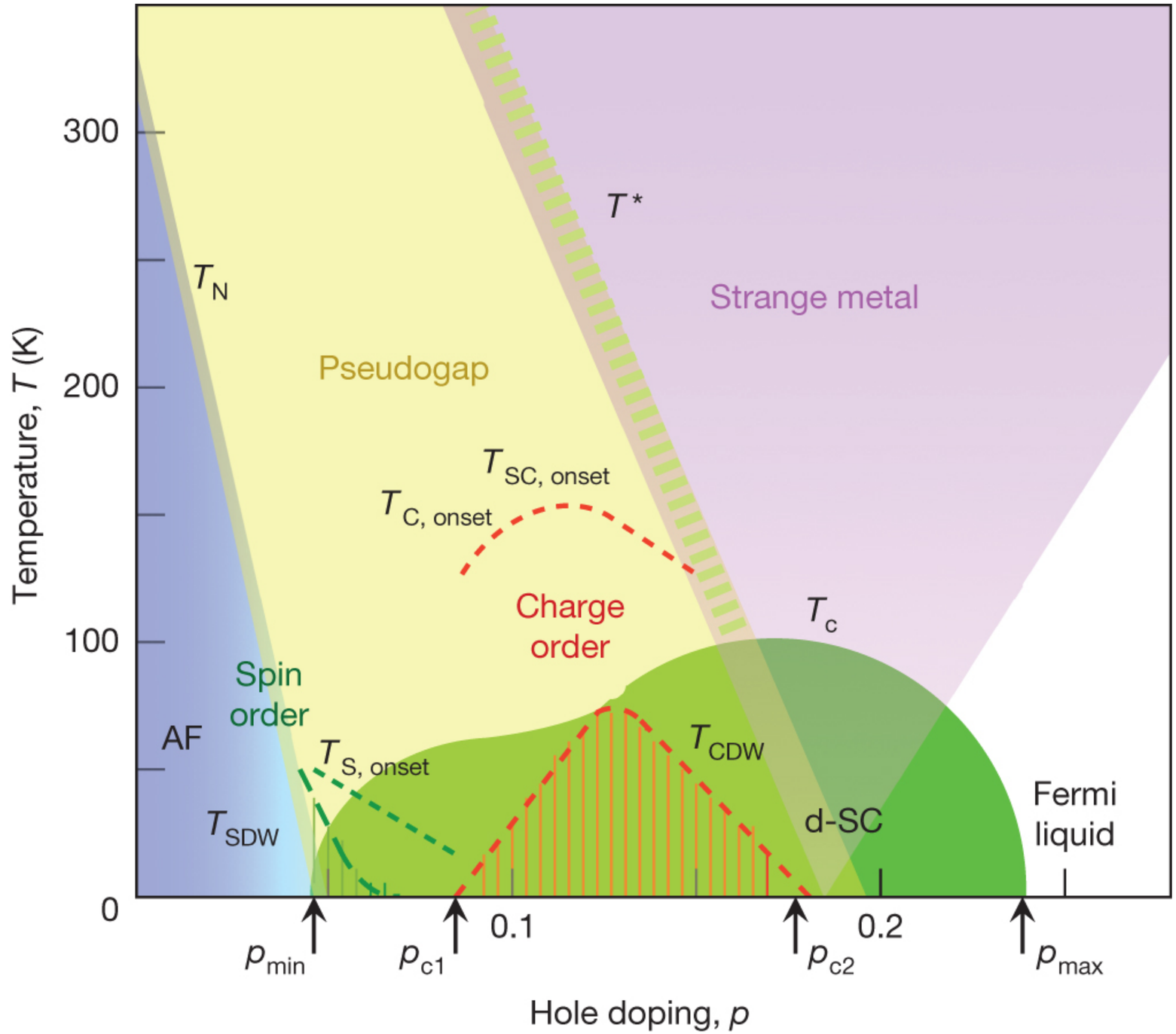

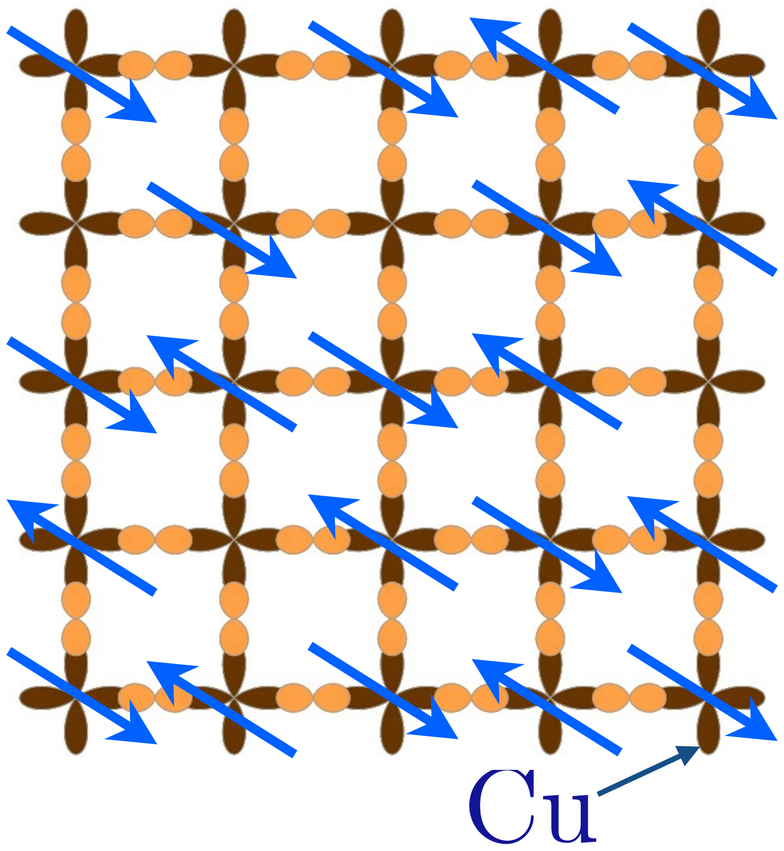

d-SC obtained upon doping AF with density p holes.
Hole density relative to the filled band $\rho = 1 + p$.
Electron density relative to the empty band $\rho_e = 1 - p$.

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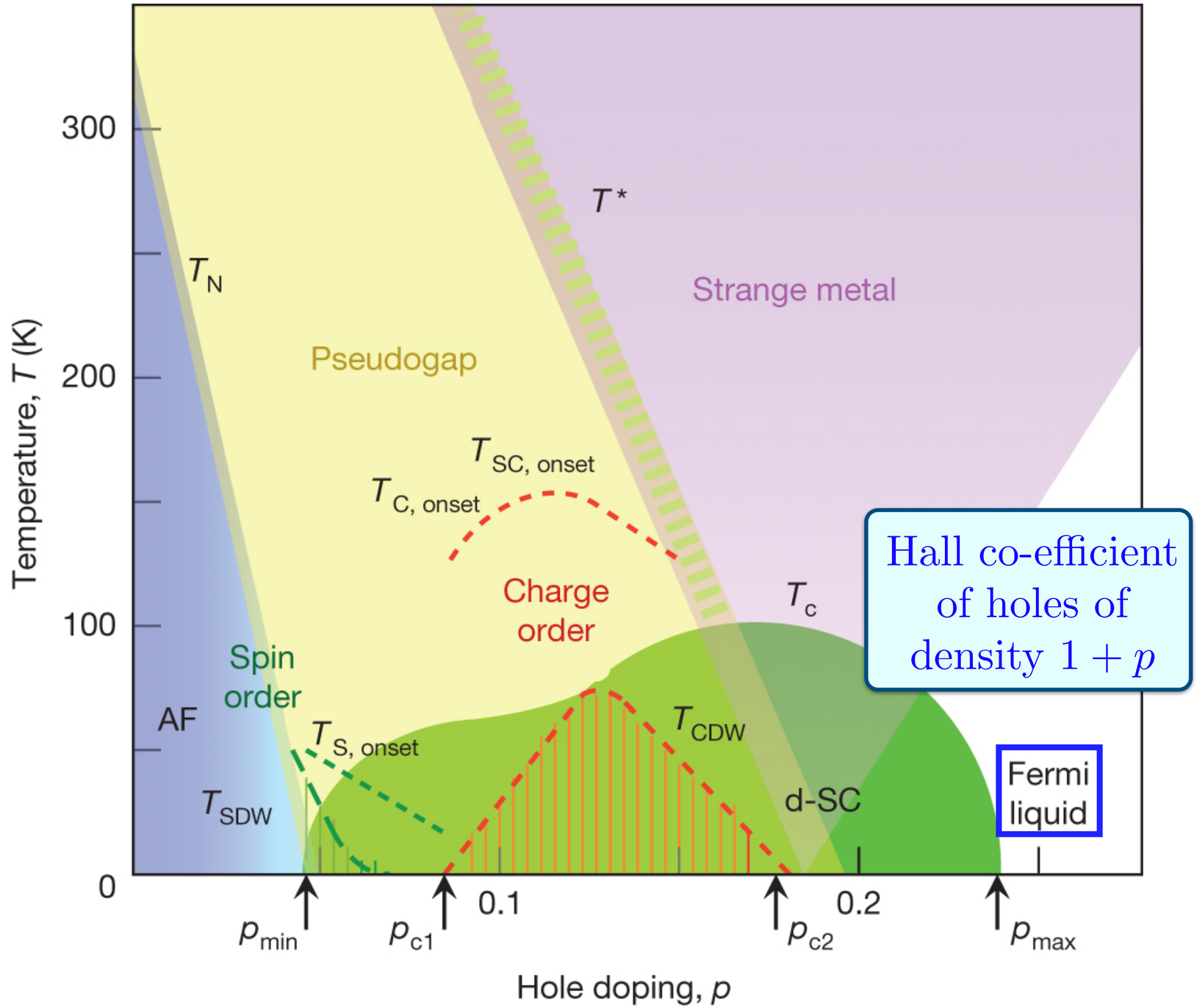
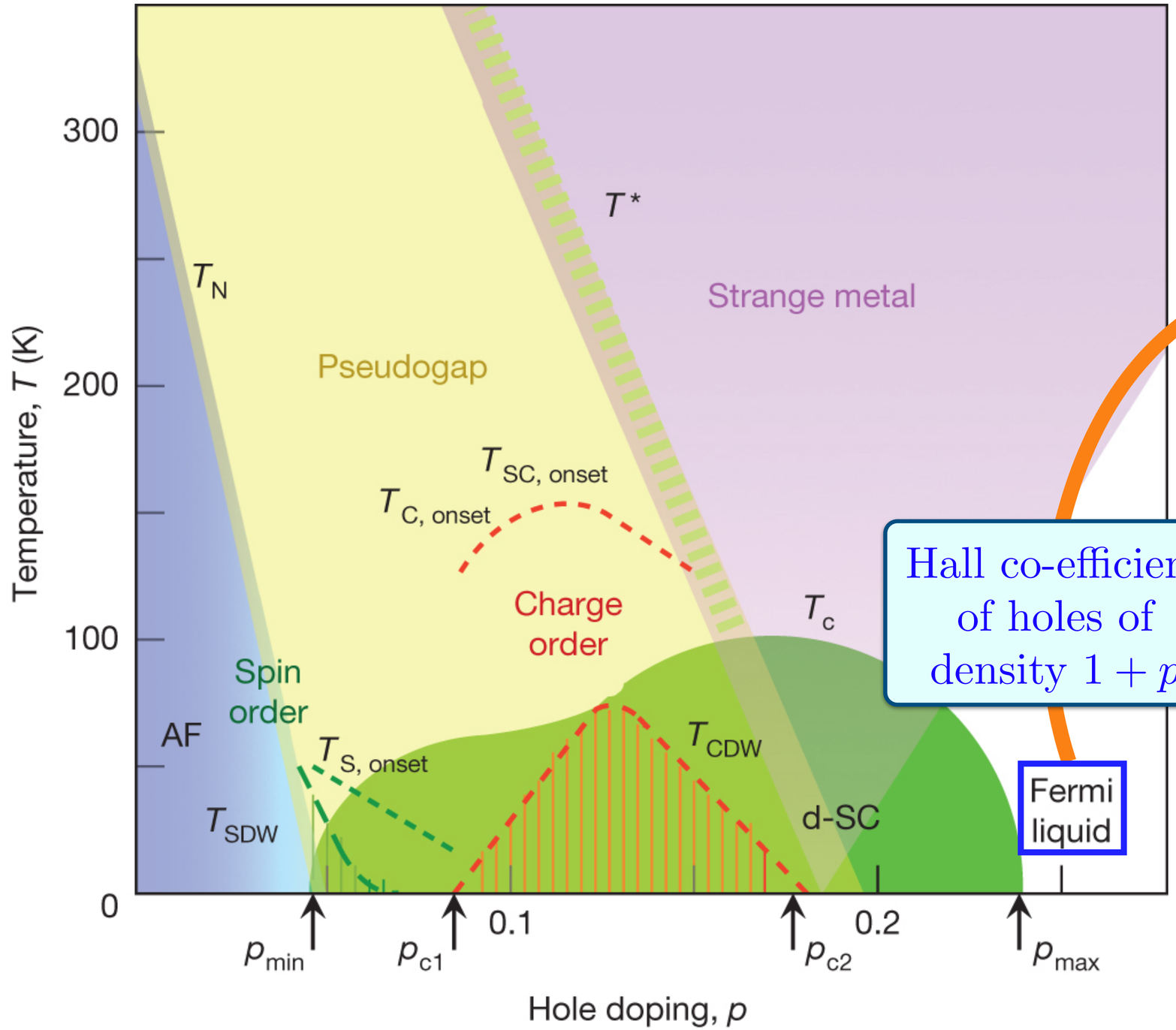


Hall co-efficient of holes of density $1+p$

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

M. Platé *et al.*, PRL **95**, 077001 (2005)



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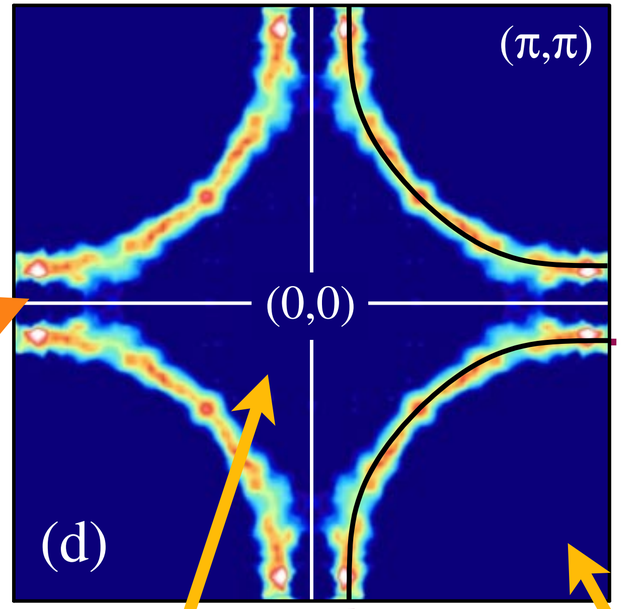

1-p electrons

1+p holes

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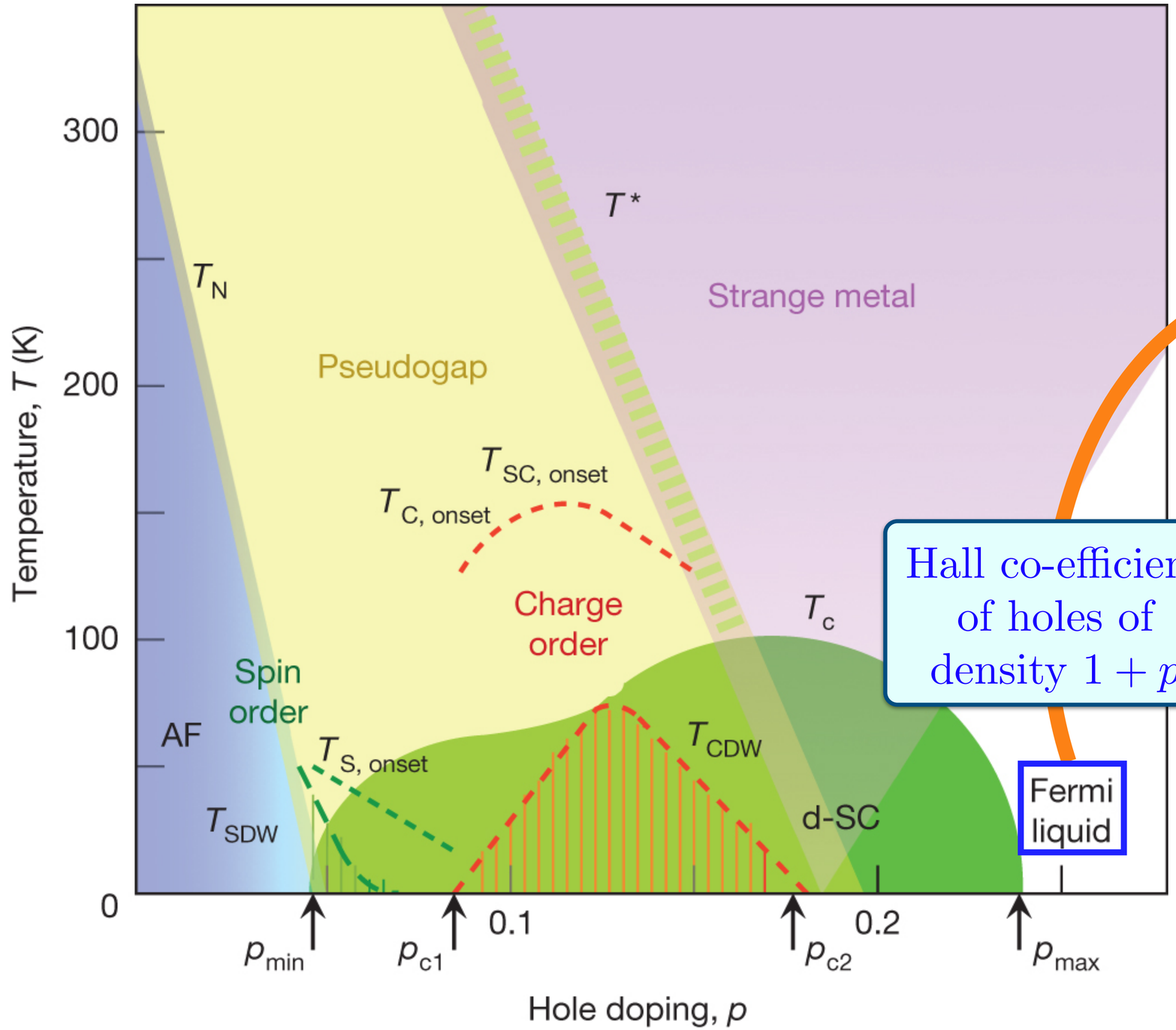

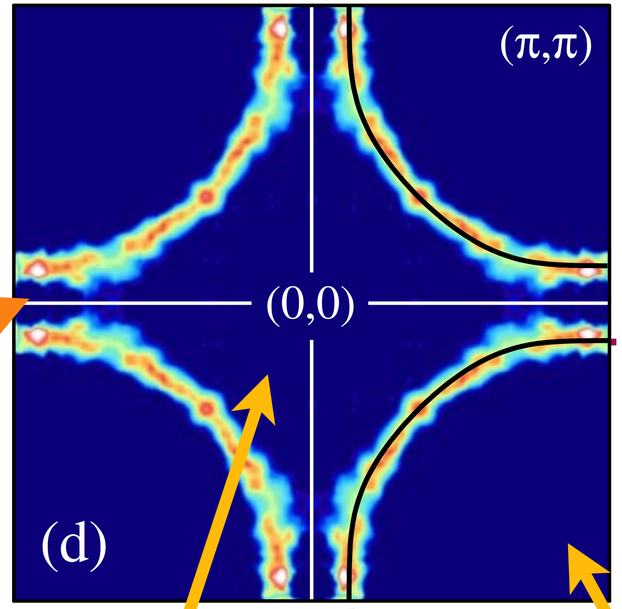

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Luttinger, 1960: Area enclosed by the Fermi surface is the same as that for free fermions *with the same symmetry.*

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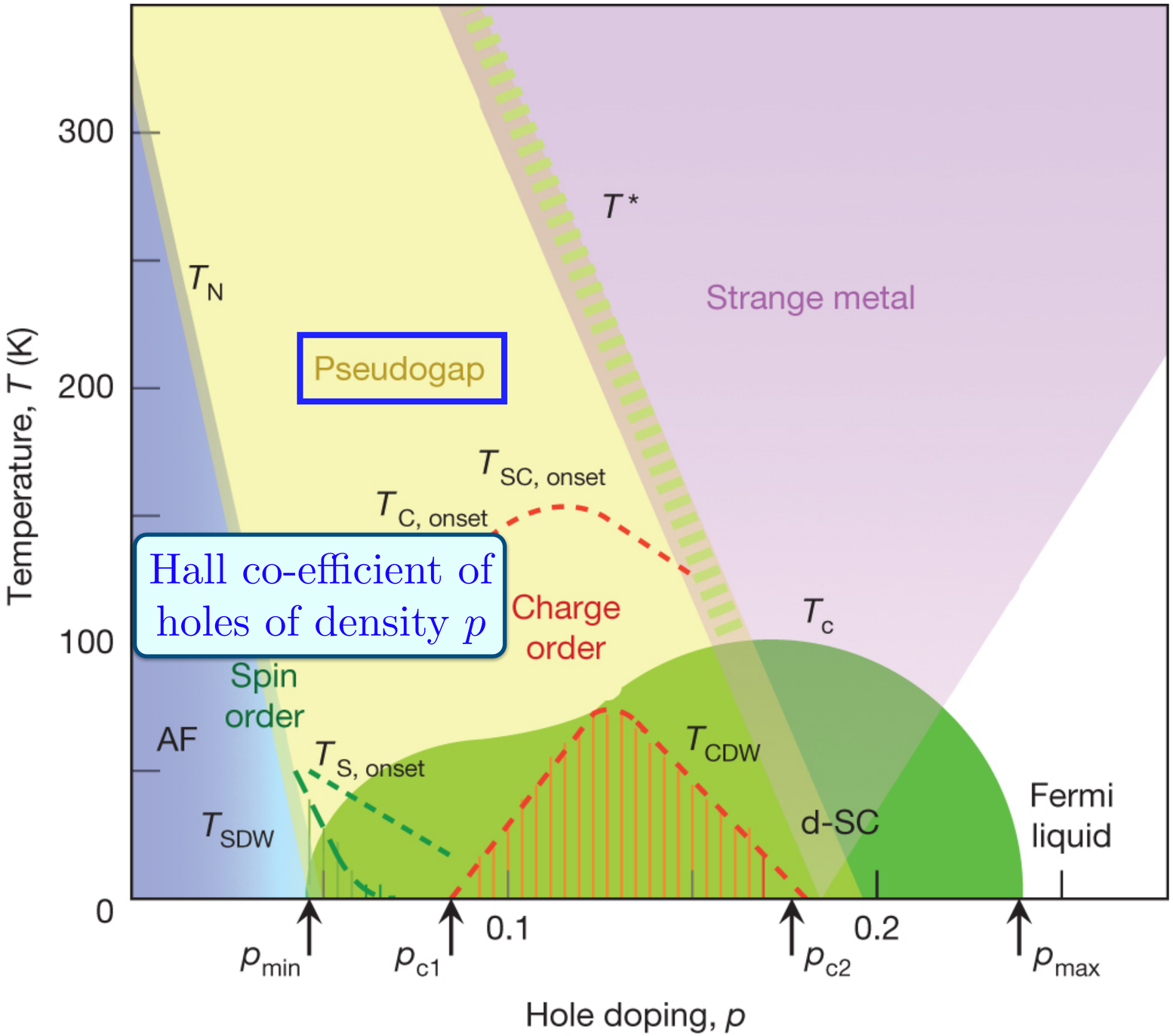
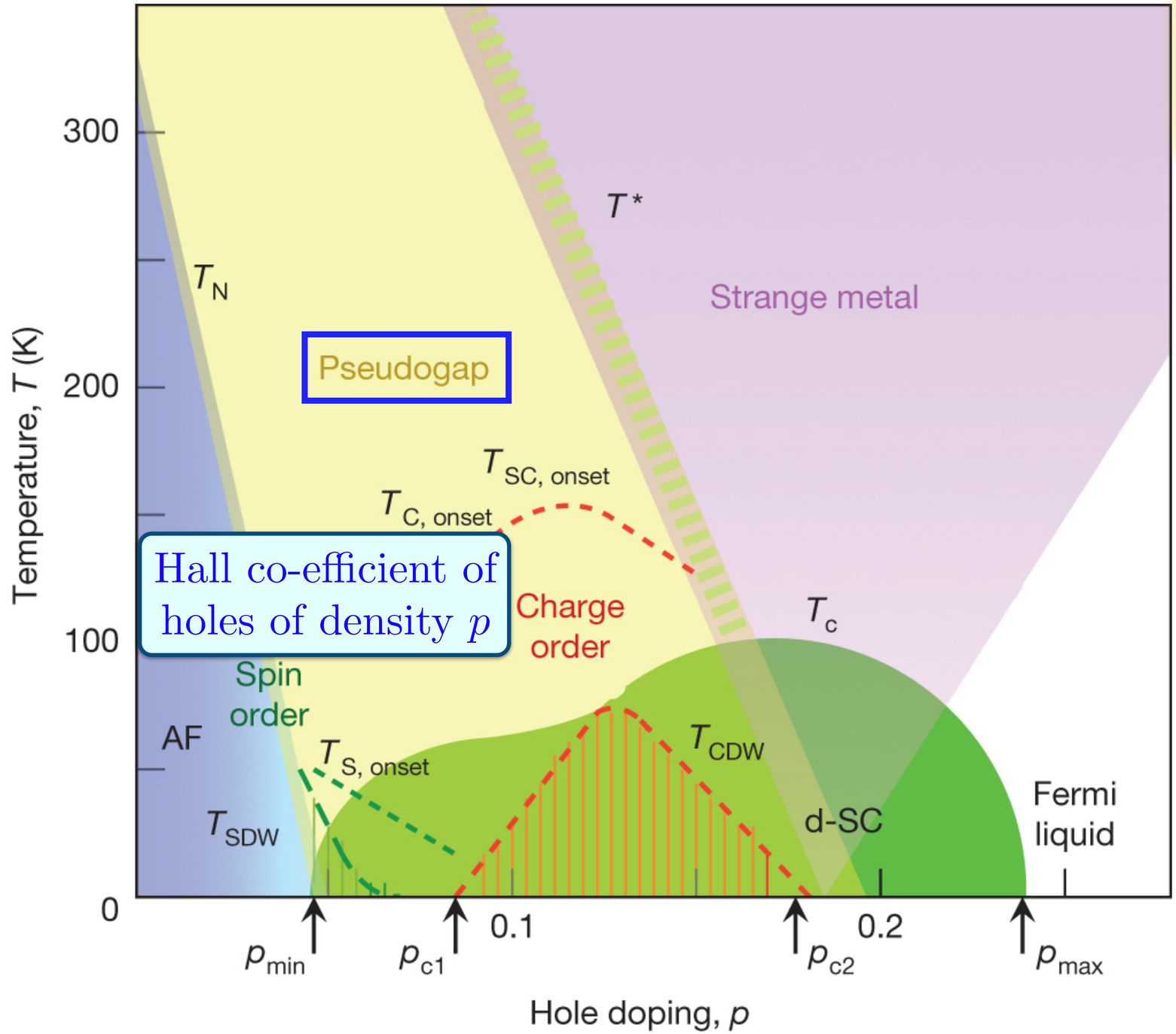


But there is no antiferromagnetic order to justify carrier density p

Many theories with fluctuating AFM, d-SC and charge orders.

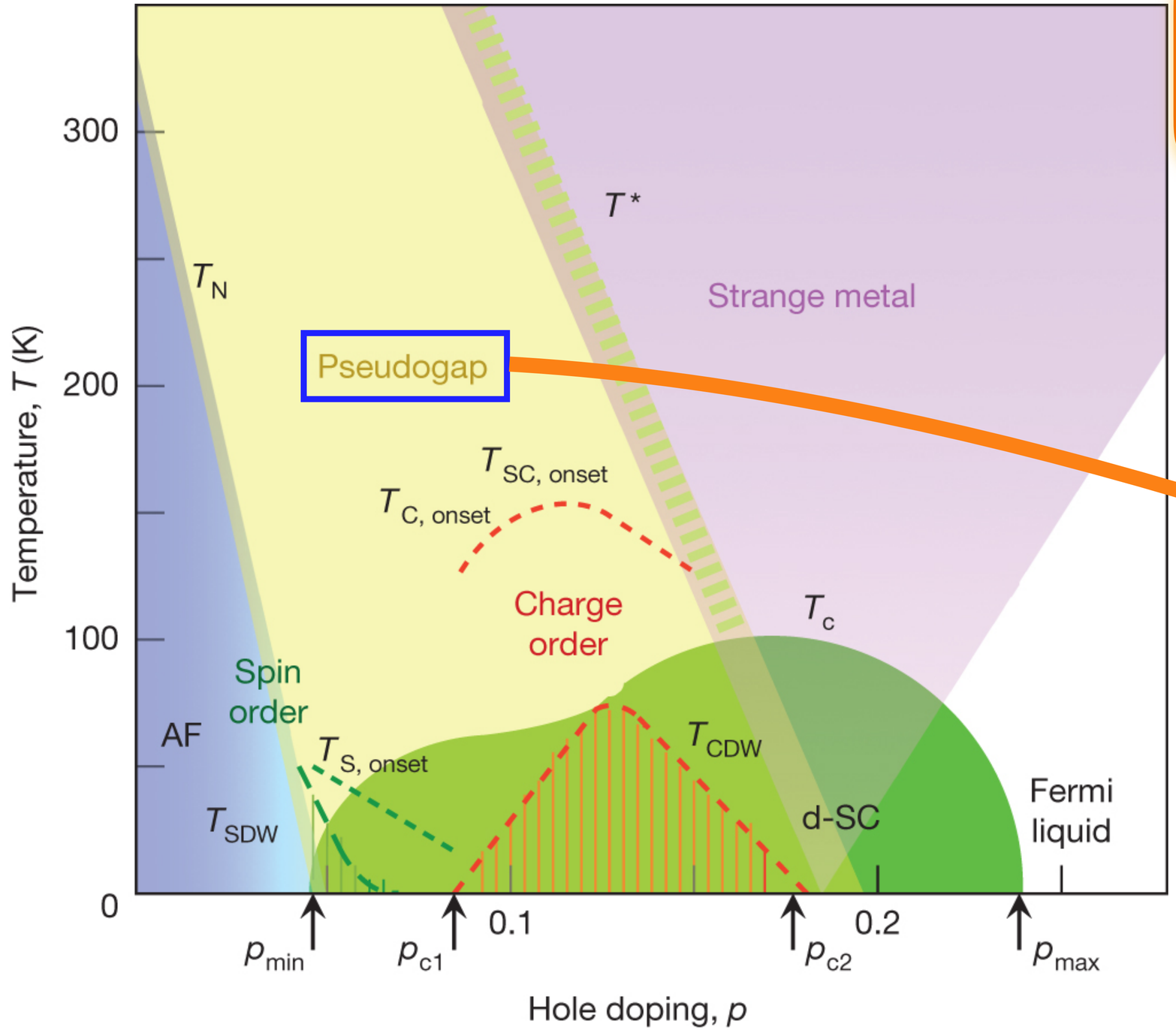
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Quantum entanglement of mobile fermions

I argue that a better starting point is a novel quantum ground state with no broken symmetry.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

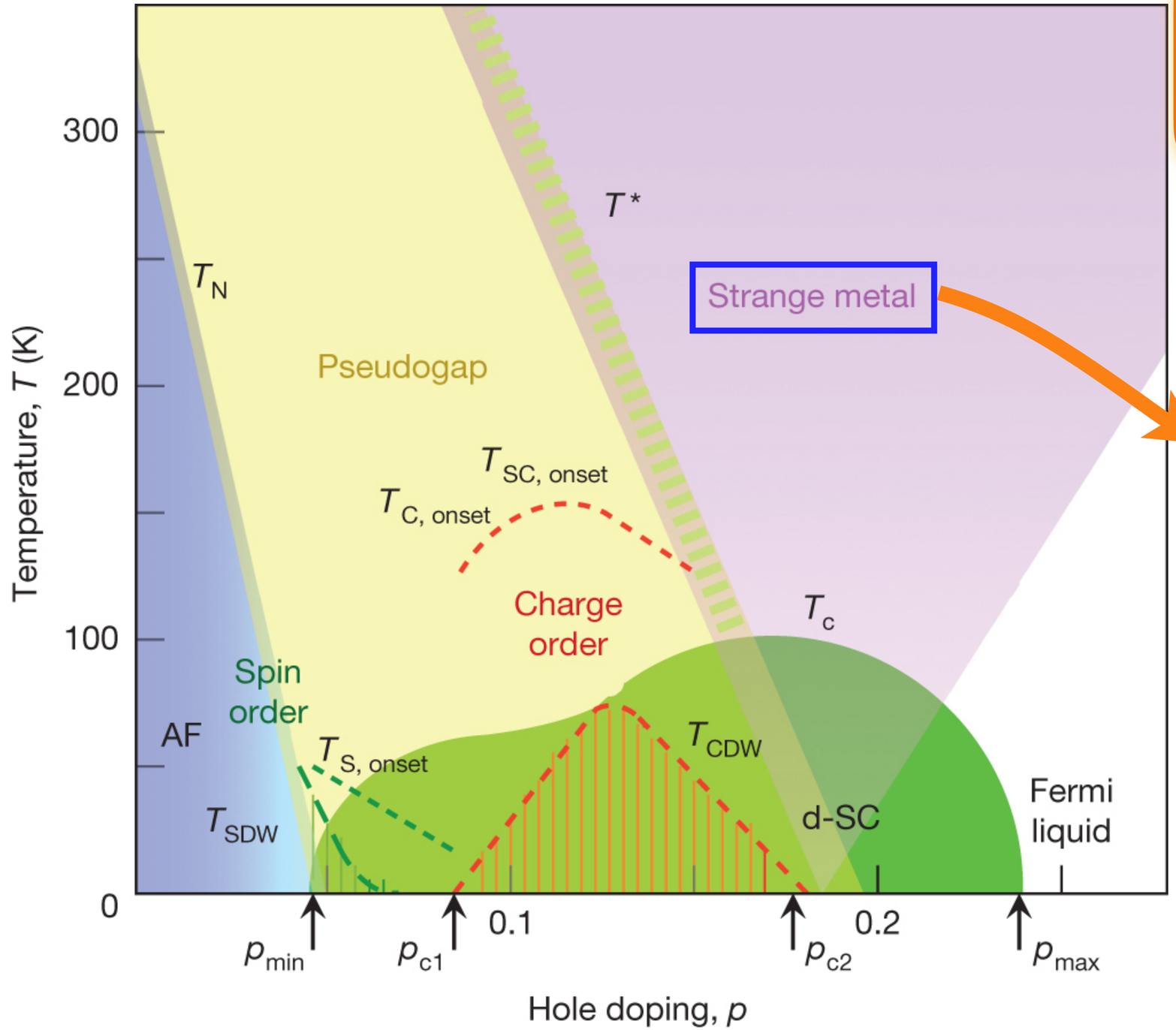


Quantum entanglement of mobile fermions

Fractionalized Fermi Liquid (FL*)

Entanglement of a (critical or gapped) quantum spin liquid coexisting with electronic Landau quasiparticles. Charge is carried by ordinary electrons, but there are also fractionalized (anyonic) spinon excitations.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

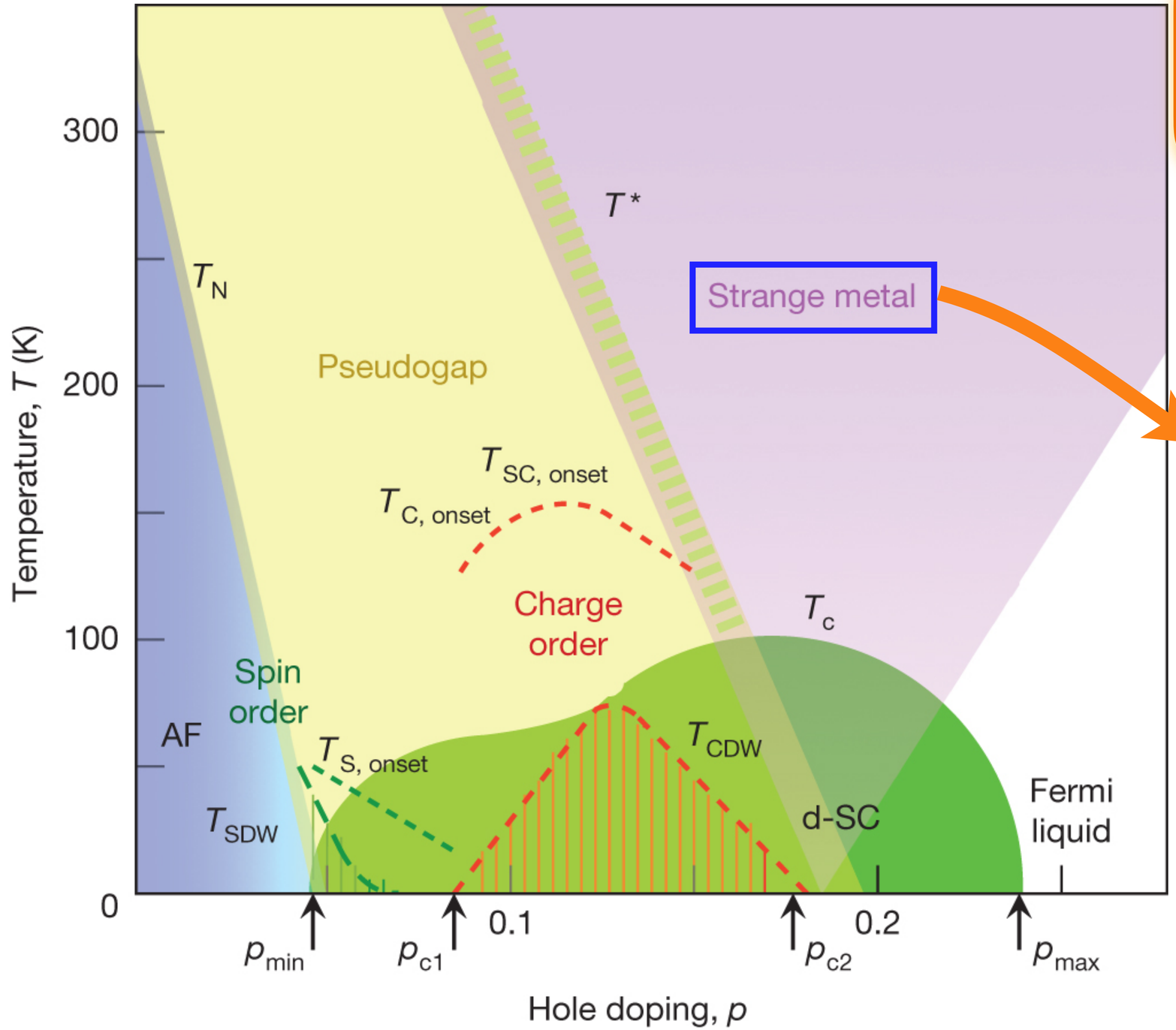


Quantum entanglement of mobile fermions

Sachdev-Ye-Kitaev (SYK) liquid

- Compressible state with no quasiparticles.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



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- 2D-YSYK: universal theory of strange metals

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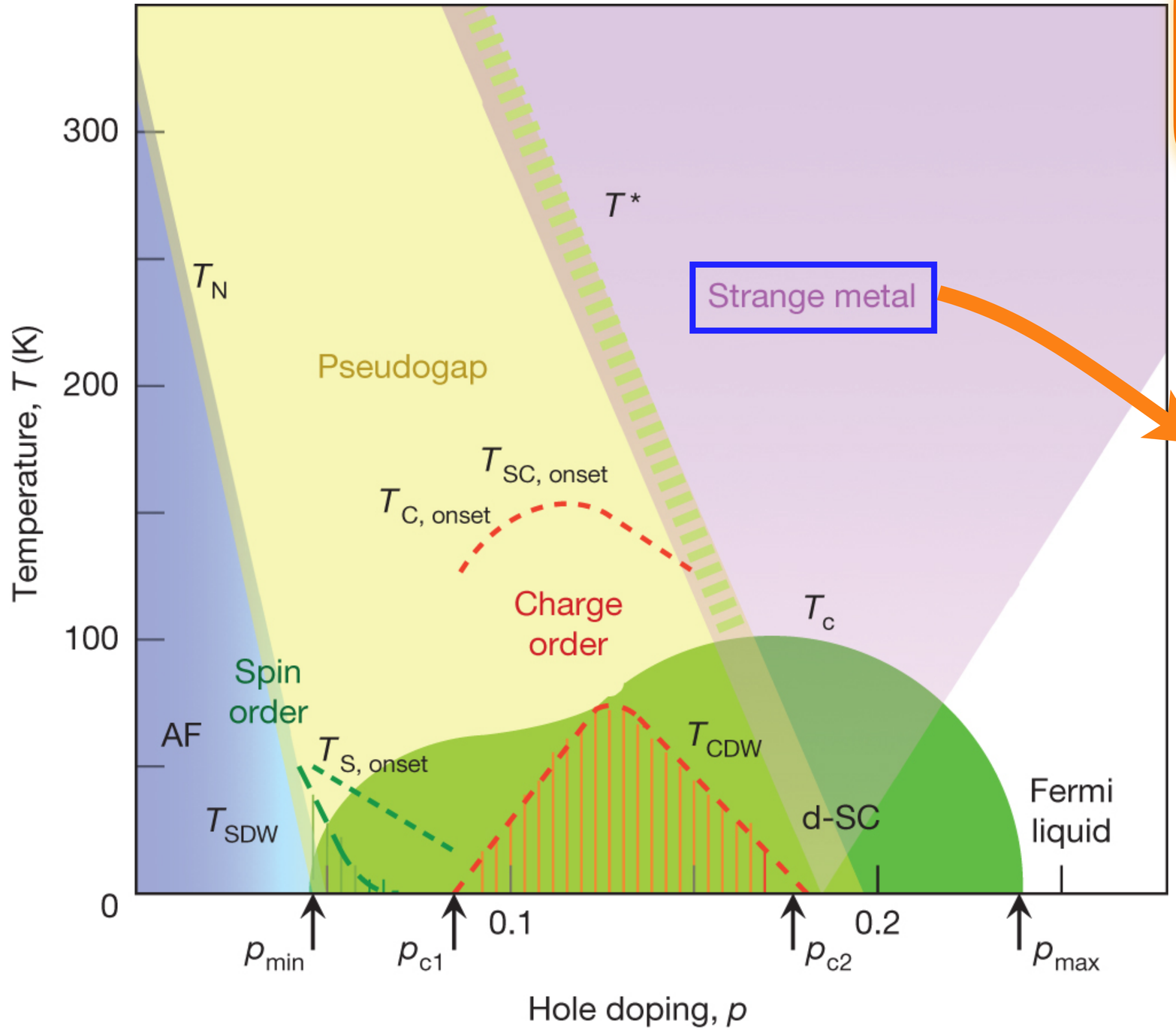


Quantum entanglement of mobile fermions

Sachdev-Ye-Kitaev (SYK) liquid

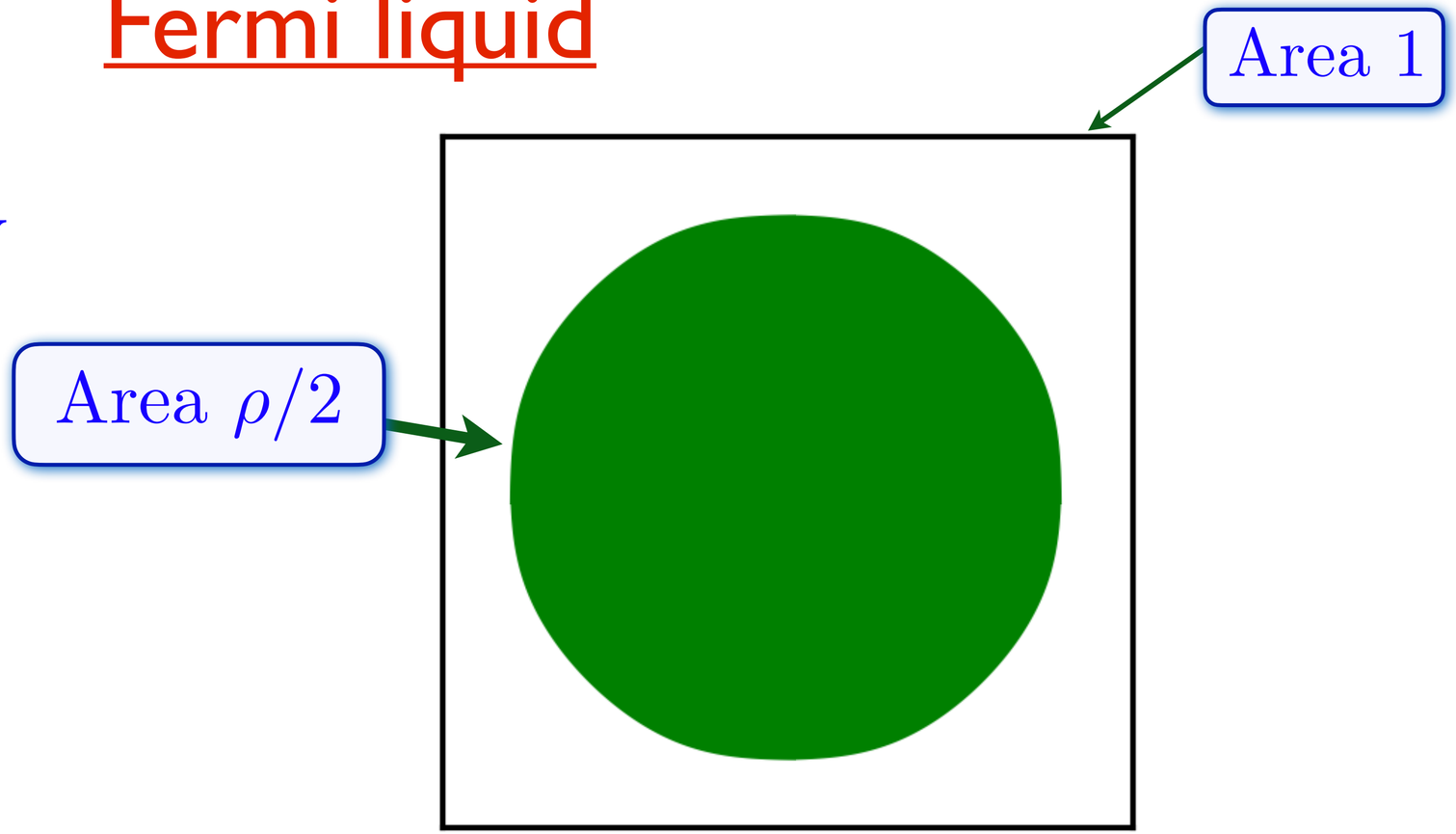
- Compressible state with no quasiparticles.
- 2D-YSYK: universal theory of strange metals
- SYK: low energy theory of generic charged black holes in asymptotically flat 3+1 dimensional space.

Fractionalized Fermi liquids (FL*)

Fermi liquid

Spin-1/2 holes of density
$\rho = 1 + p$

Positive Hall coefficient
of carrier density ρ



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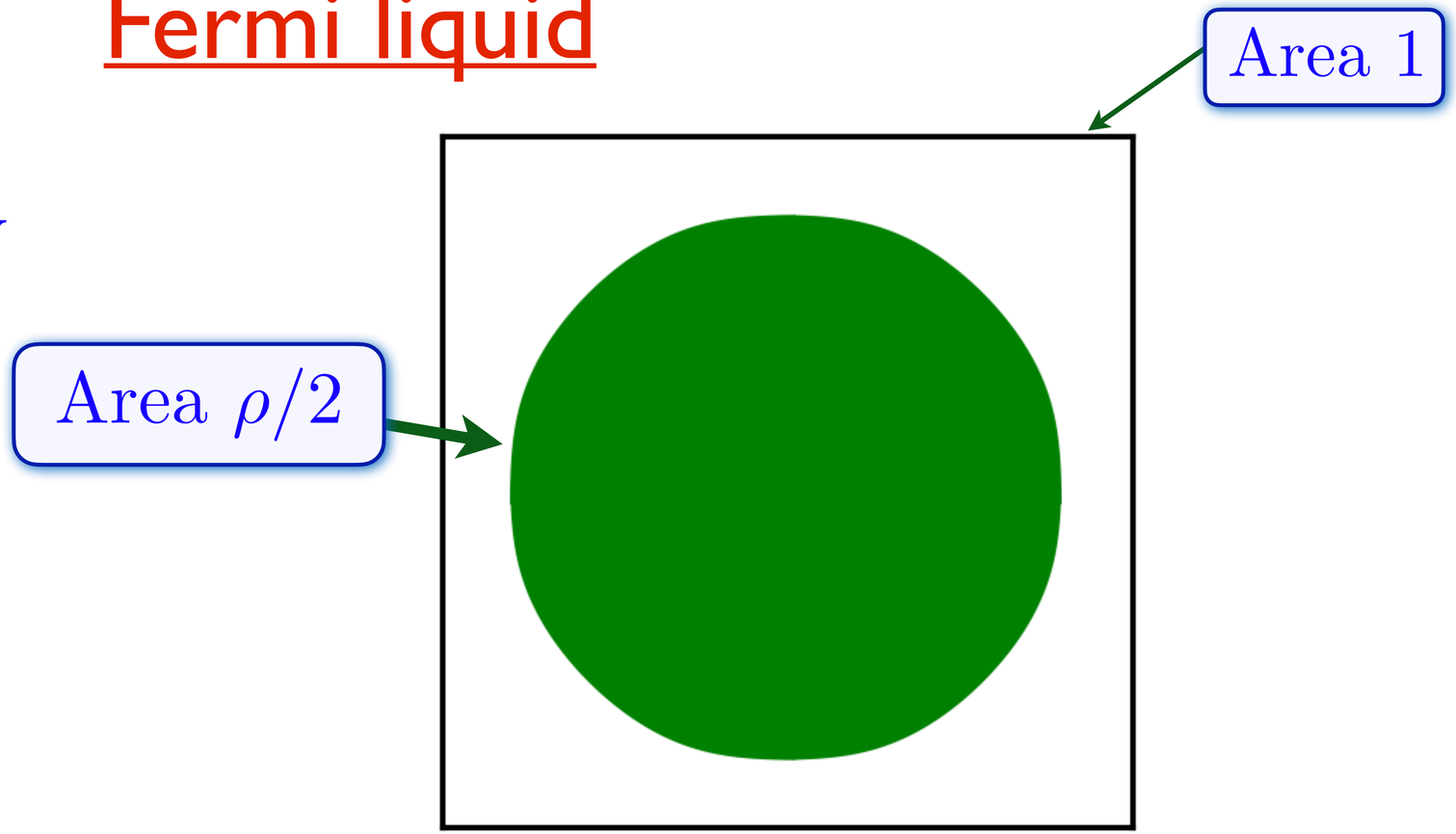


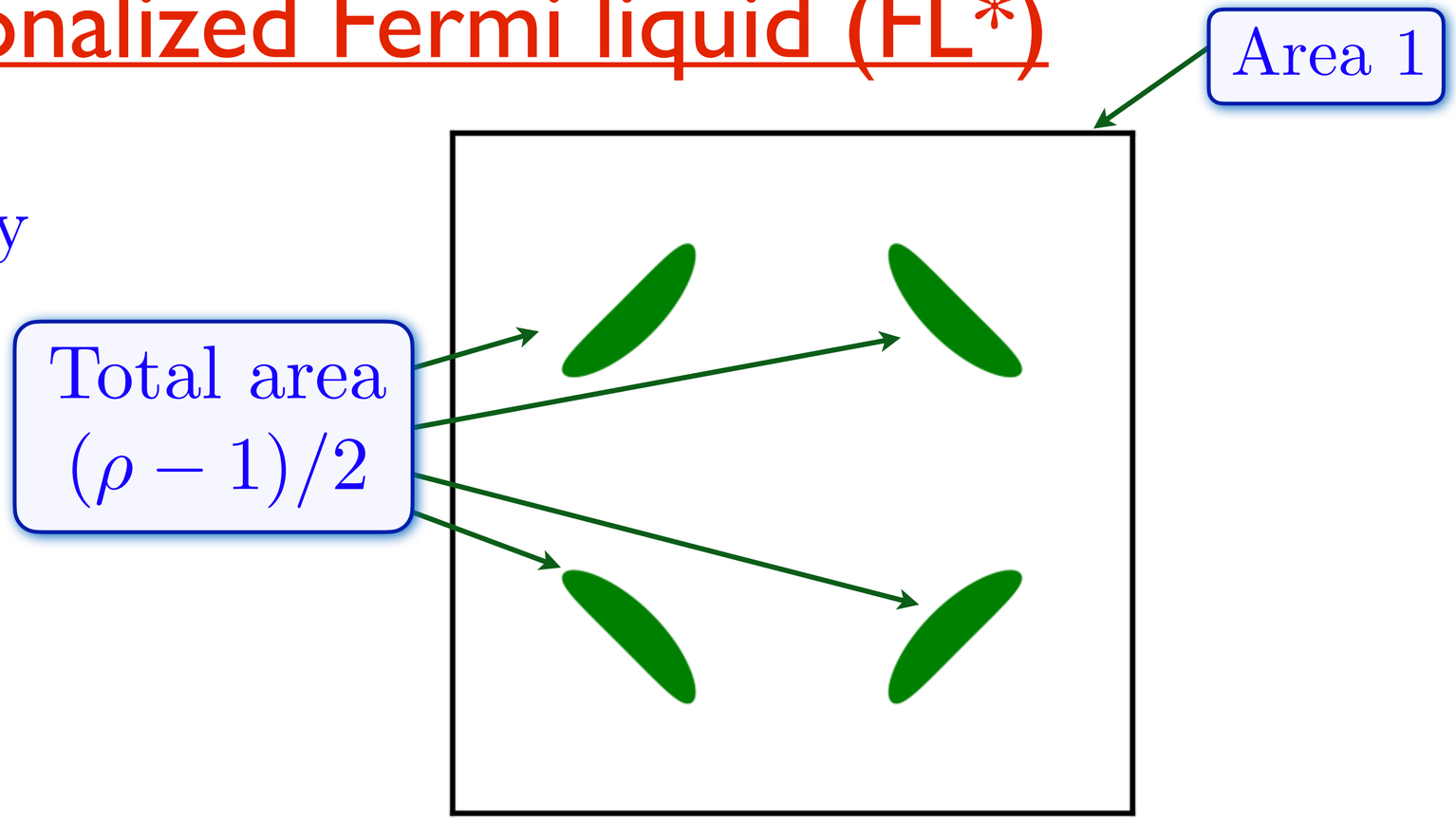
Luttinger, 1960: Area enclosed by the Fermi surface is the same as that for free fermions *with the same symmetry.*

Oshikawa, 2000: Area constrained by an anomaly-argument of global U(1) and translations

Fractionalized Fermi liquid (FL*)

Spin-1/2 holes of density
$$\rho = 1 + p$$

Positive Hall coefficient of carrier density $\rho - 1$



No broken symmetry

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

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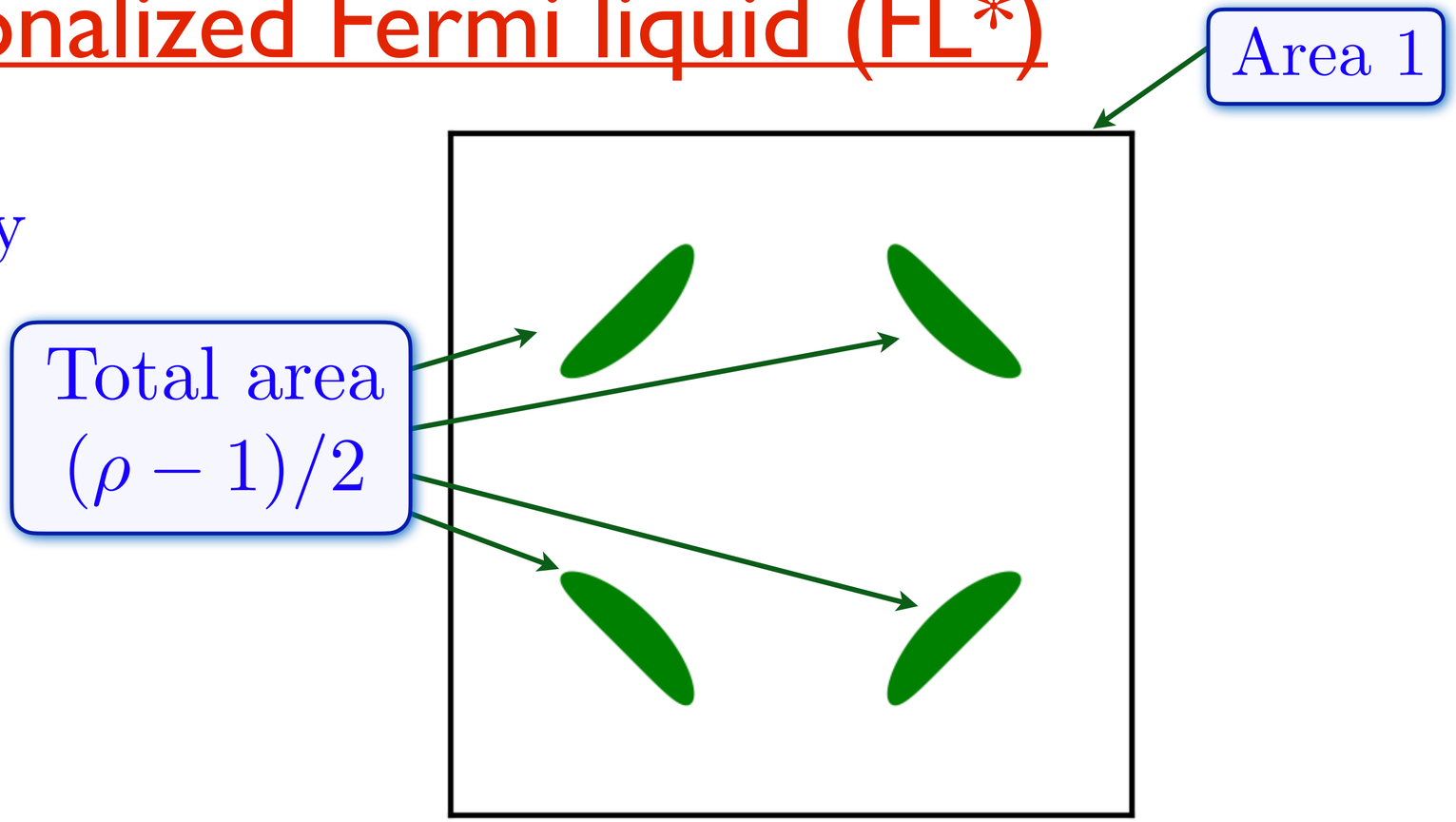

Oshikawa anomaly-argument is satisfied by
the sum of spin liquid (1) and
Fermi surface anomalies $(\rho - 1)$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

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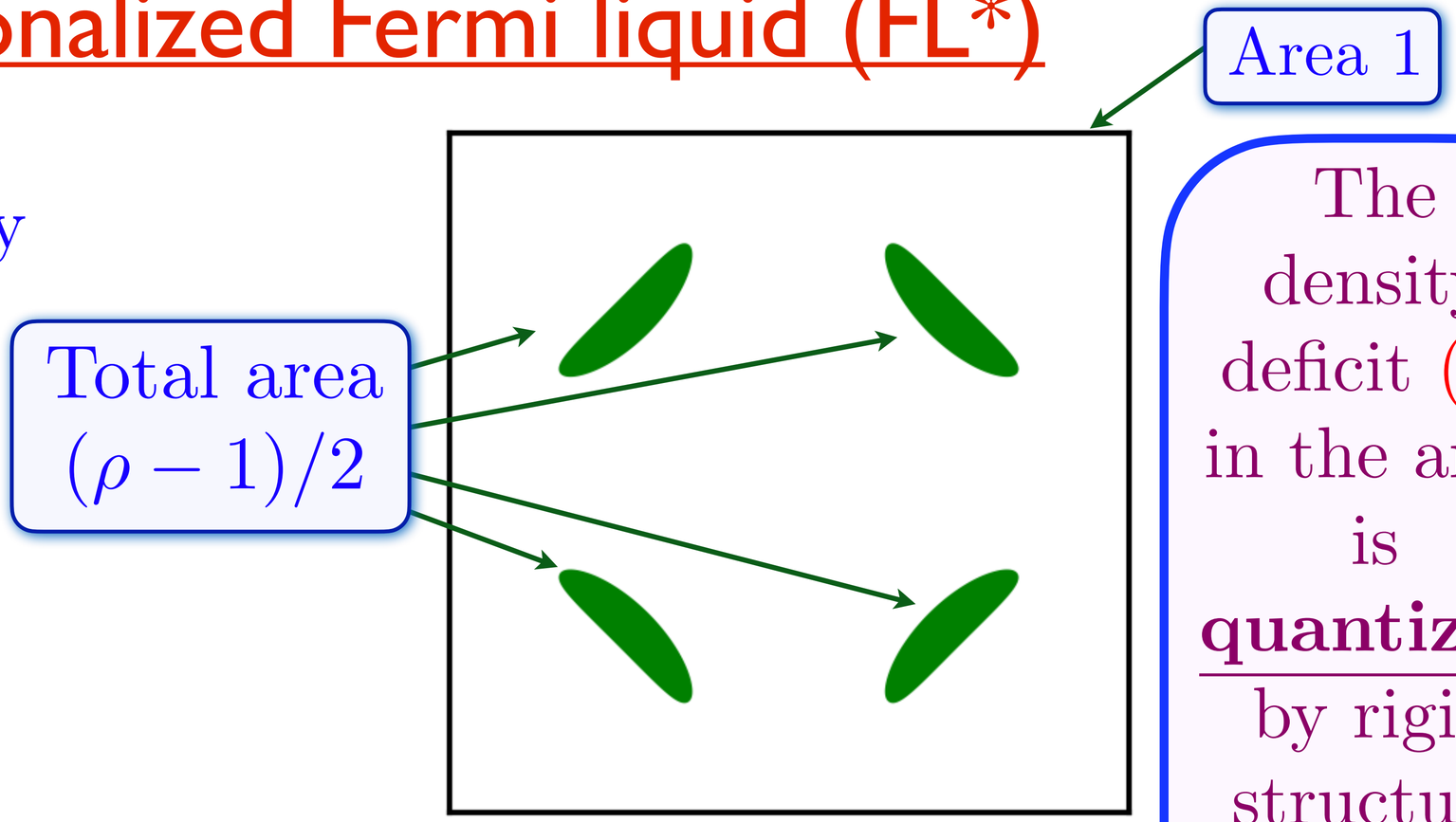

The density deficit (1) in the area is **quantized** by rigid structure of the spin liquid.

Oshikawa anomaly-argument is satisfied by the sum of spin liquid (1) and Fermi surface anomalies $(\rho - 1)$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

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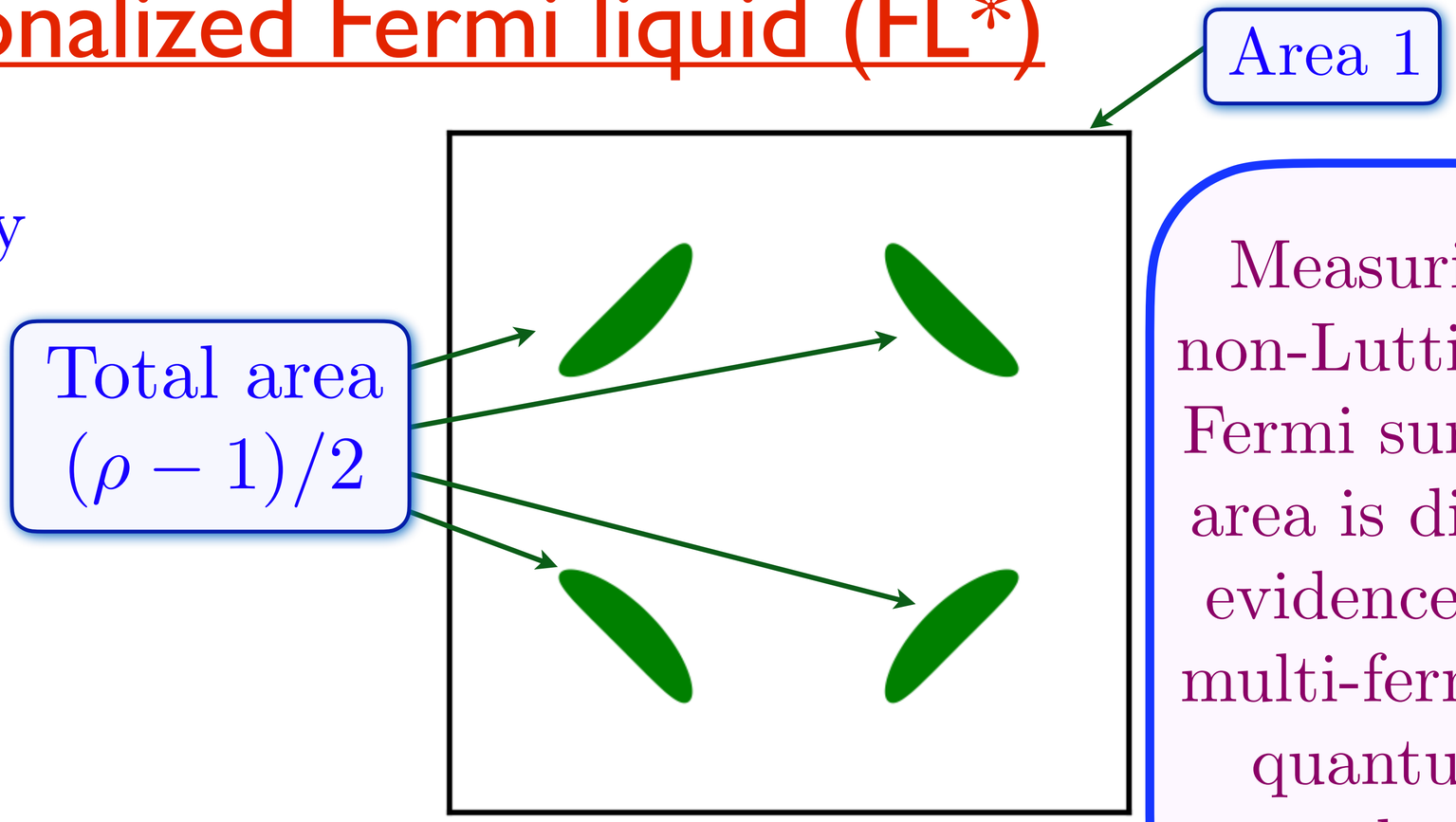

Measuring non-Luttinger Fermi surface area is direct evidence for multi-fermion quantum entanglement.

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T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

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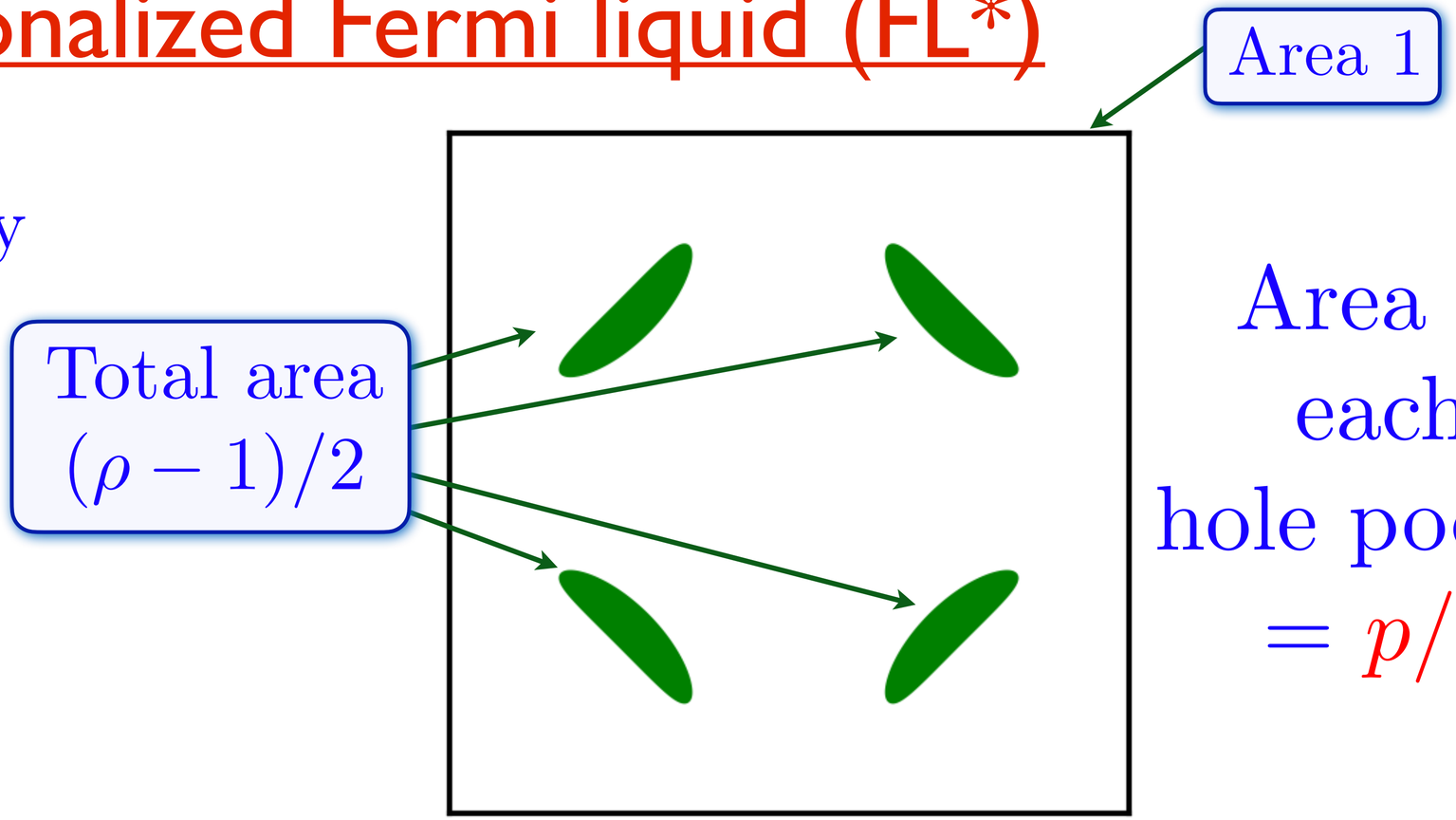

Area of
each
hole pocket
$= p/8$

Oshikawa anomaly-argument is satisfied by
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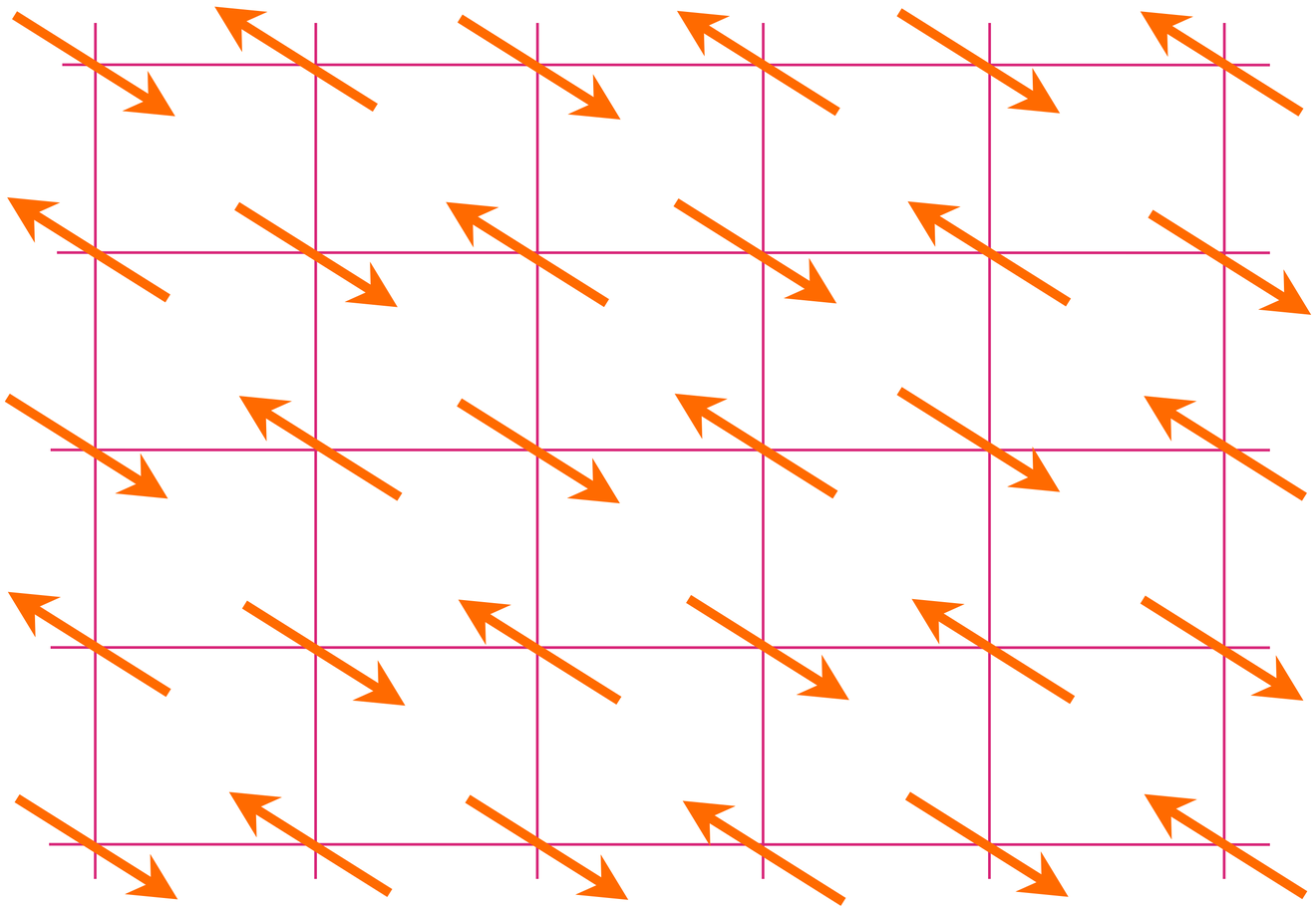
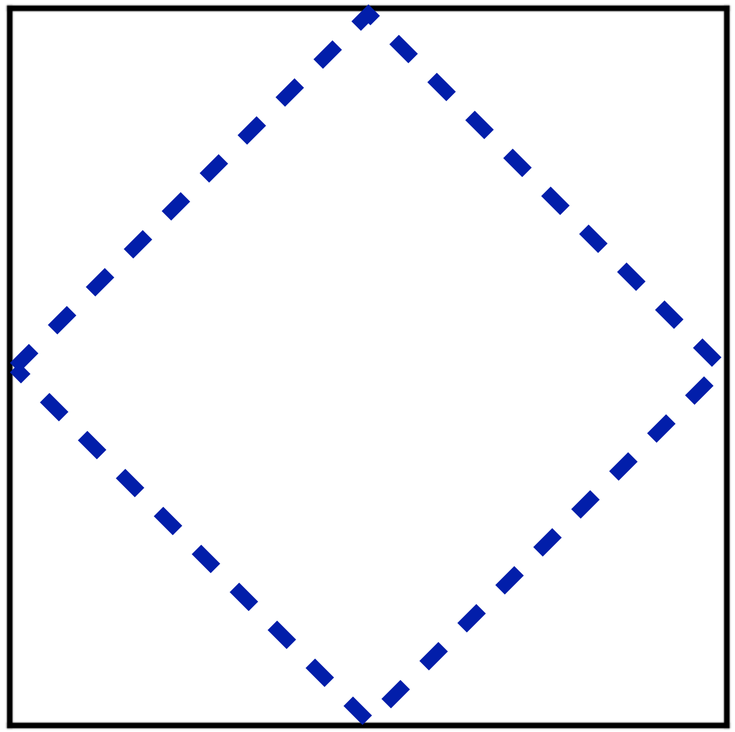
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Insulating antiferromagnet



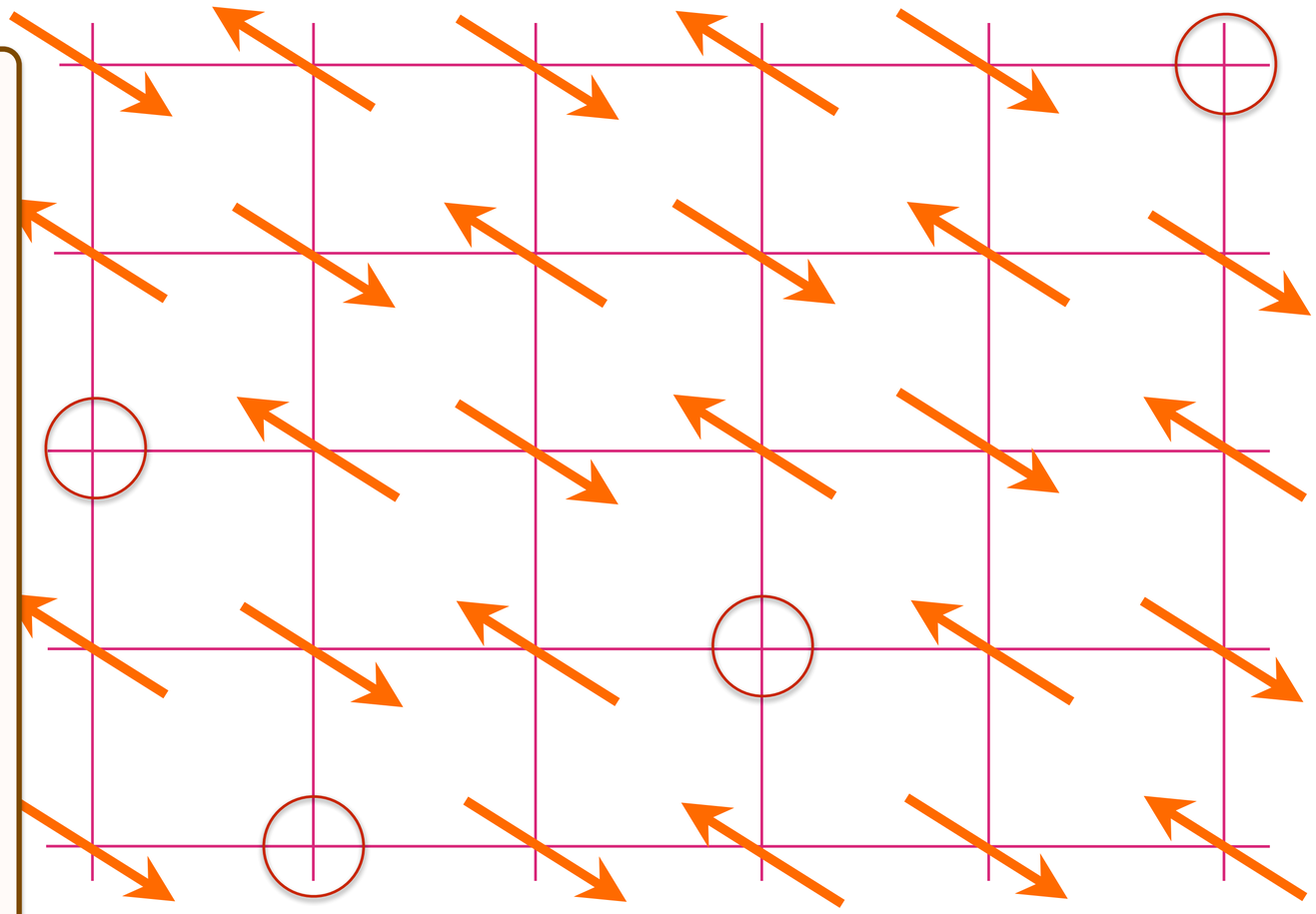
Reduced Brillouin Zone.
Broken symmetry



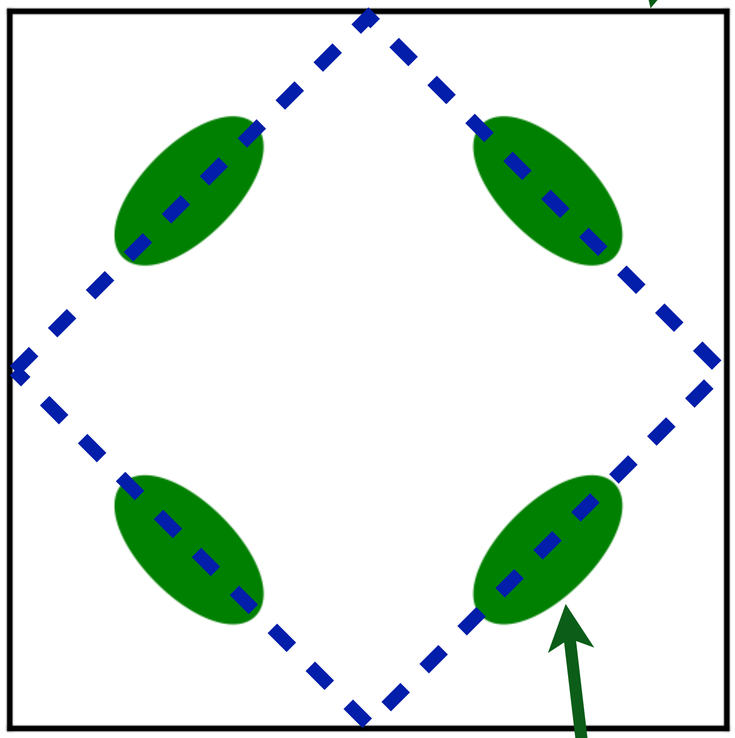
Doping an insulating antiferromagnet with holes of density p

AF metal

Fermi liquid with density p of spin 1/2, charge $+e$ holes. Coherent inter-layer transport requires inter-layer spin correlations.



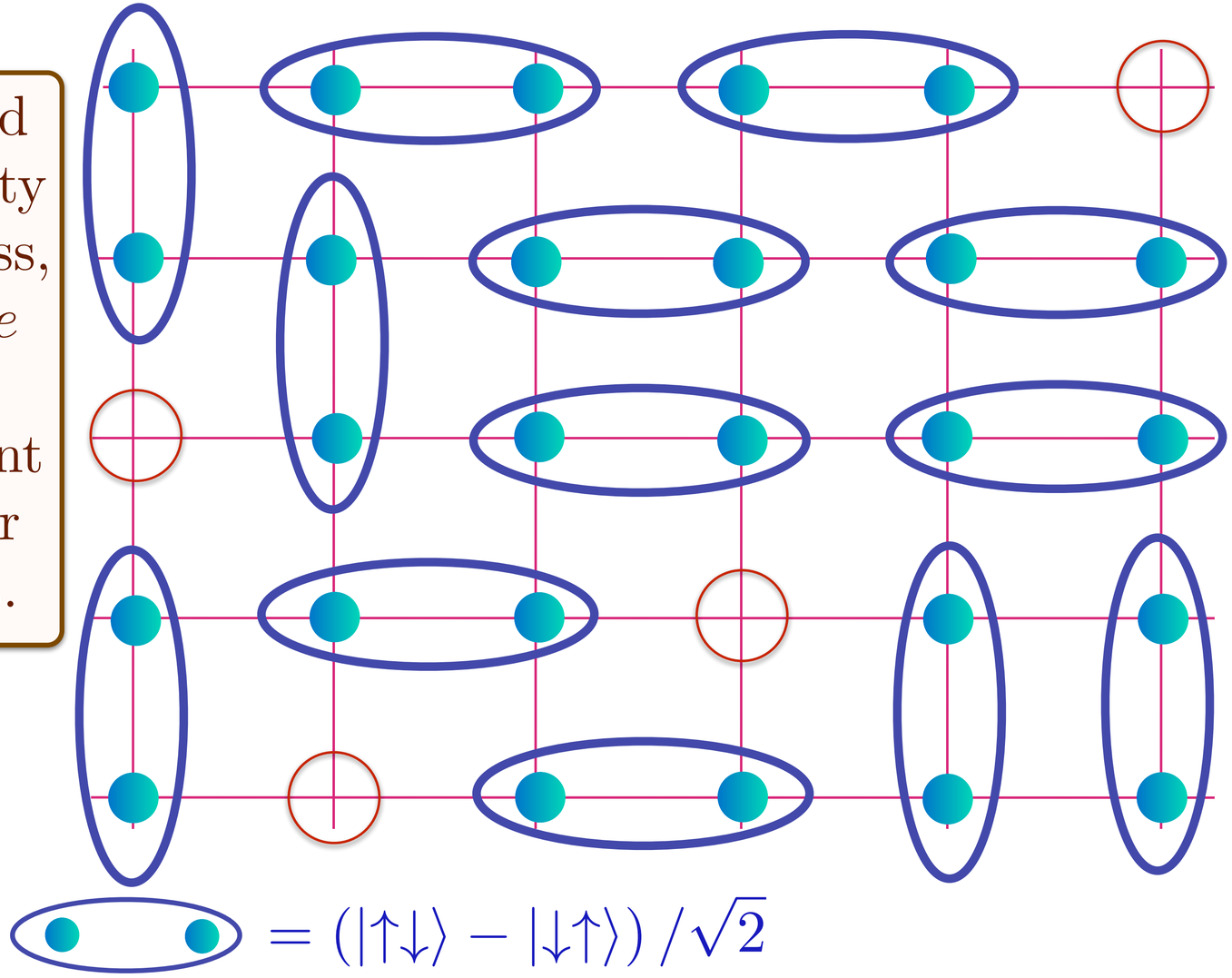
Luttinger area.
Broken symmetry



Doping an insulating antiferromagnet with holes of density p

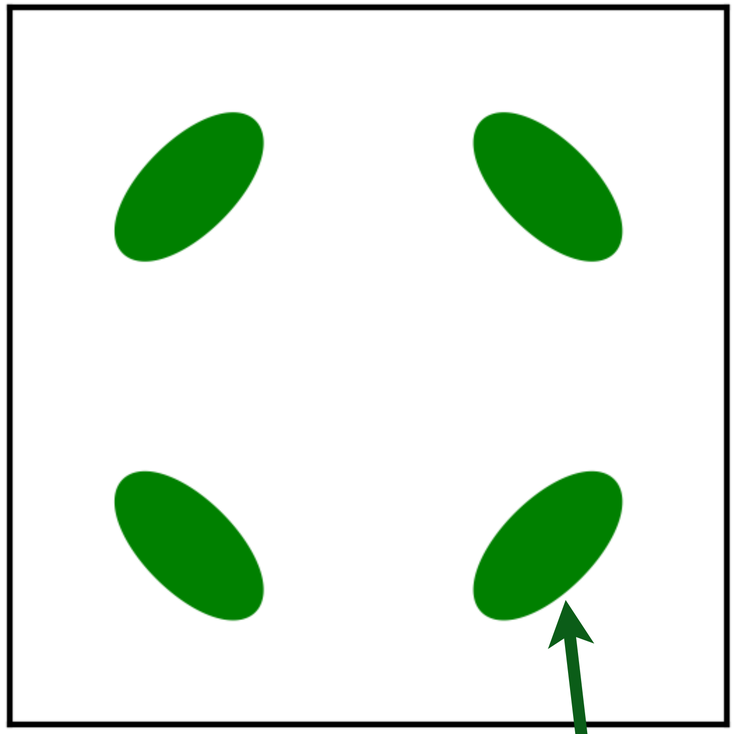
Holon metal

Spin liquid with density p of spinless, charge $+e$ holons. No coherent inter-layer transport.



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)



Area $p/4$

R. K. Kaul, Y. B. Kim, S. S., T. Senthil, Nature Physics **4**, 28 (2008); S.S. M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

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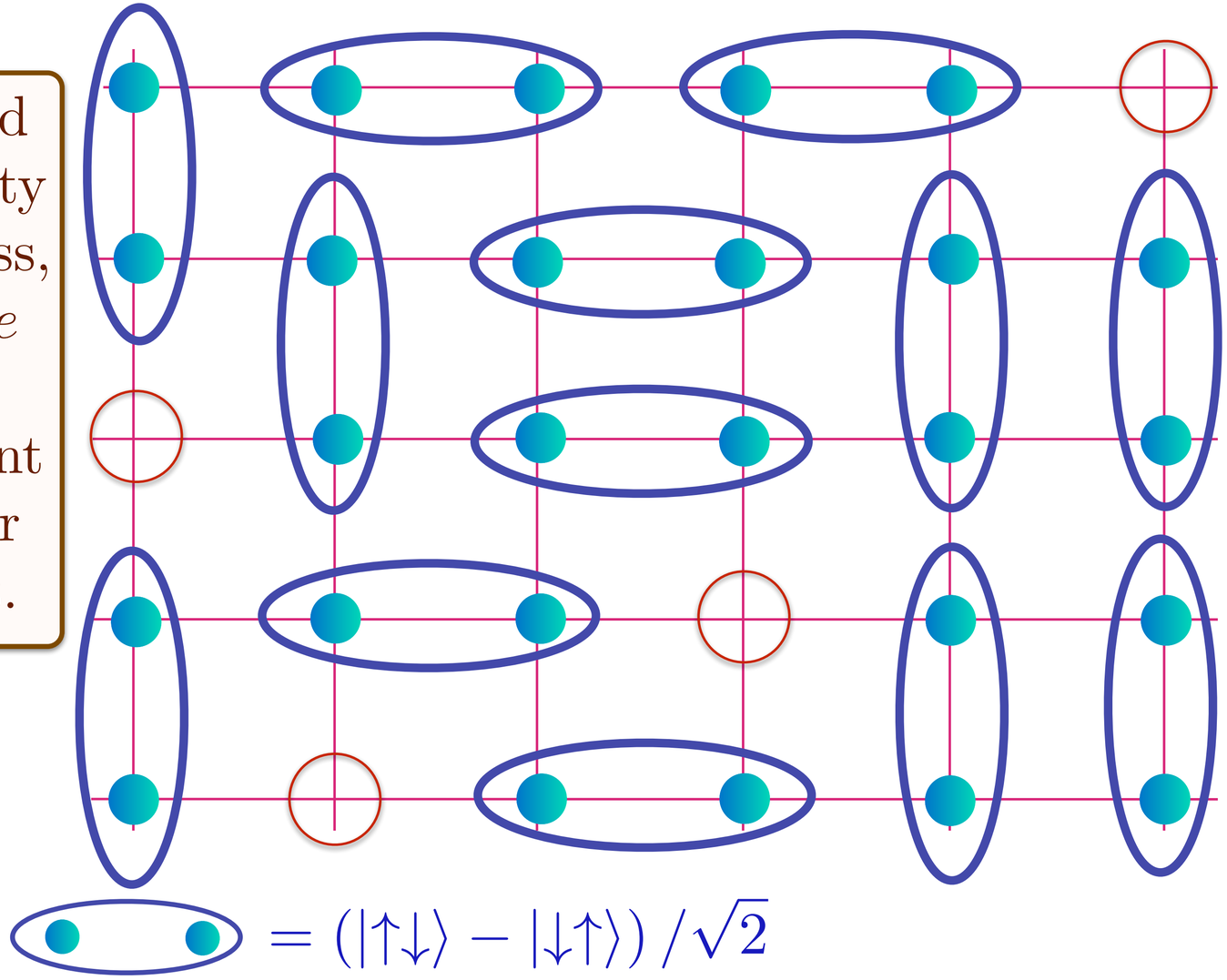

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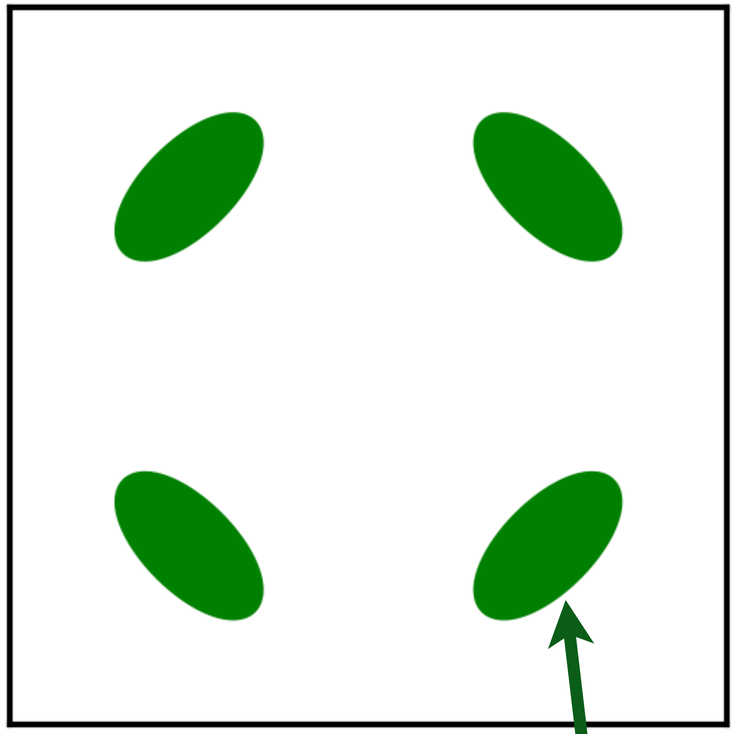

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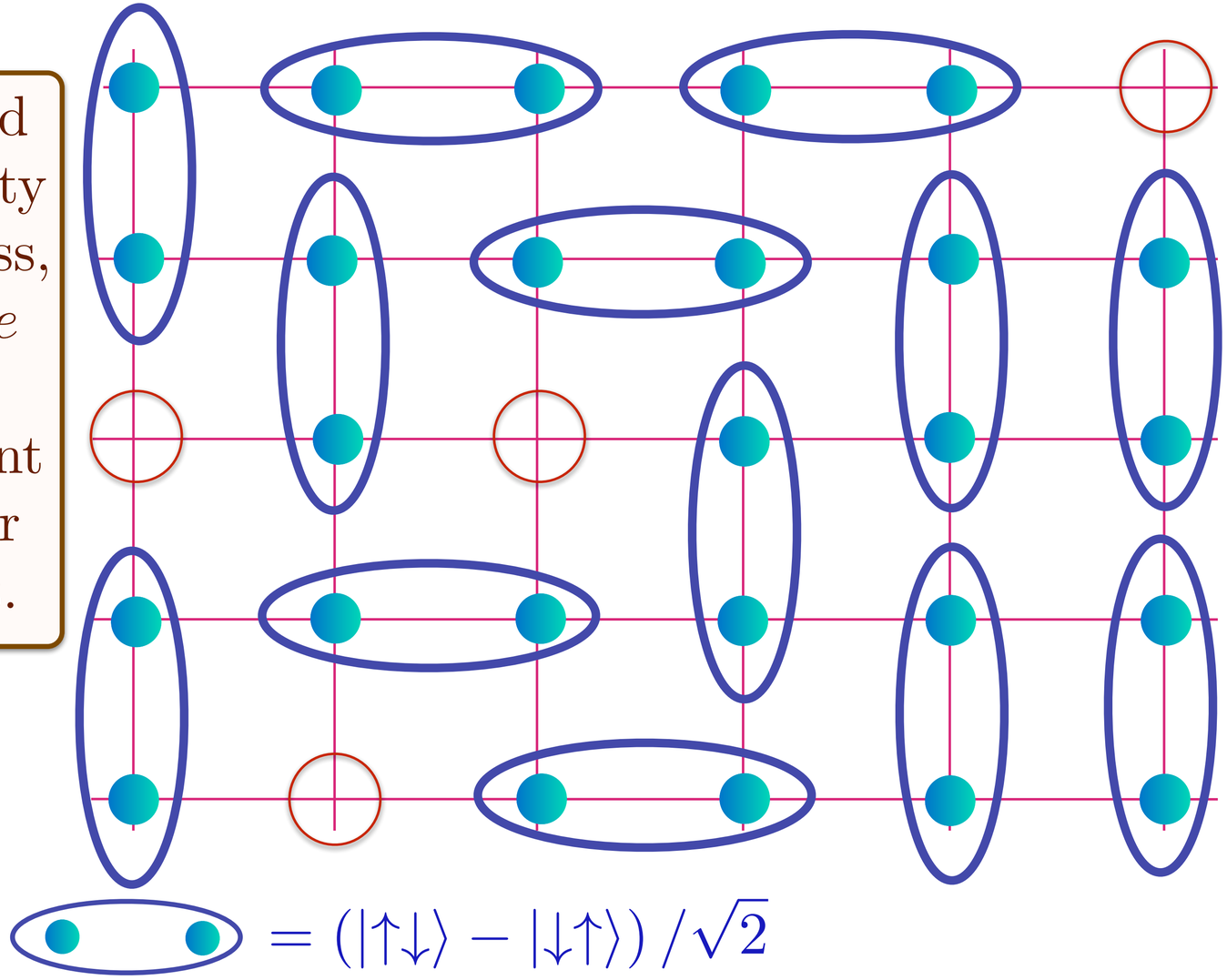

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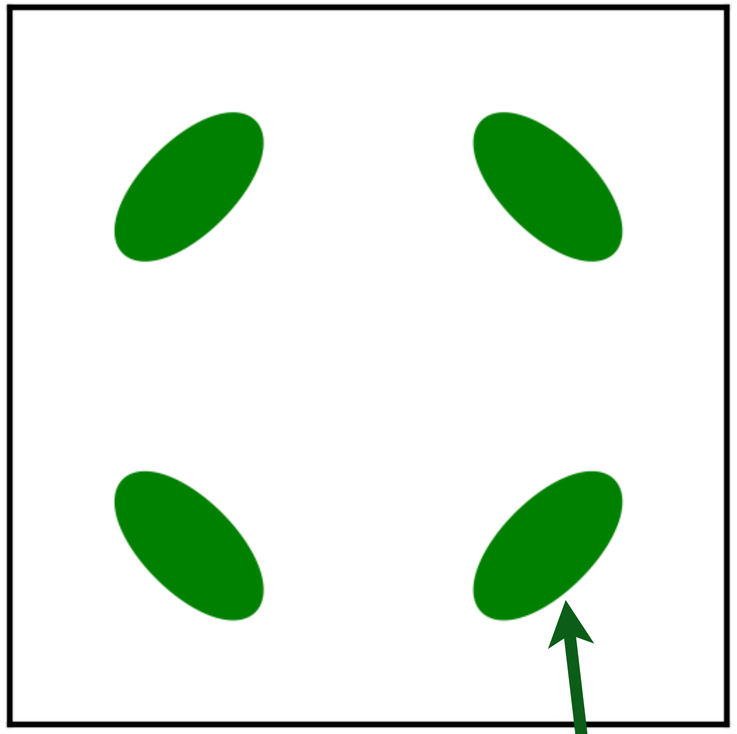

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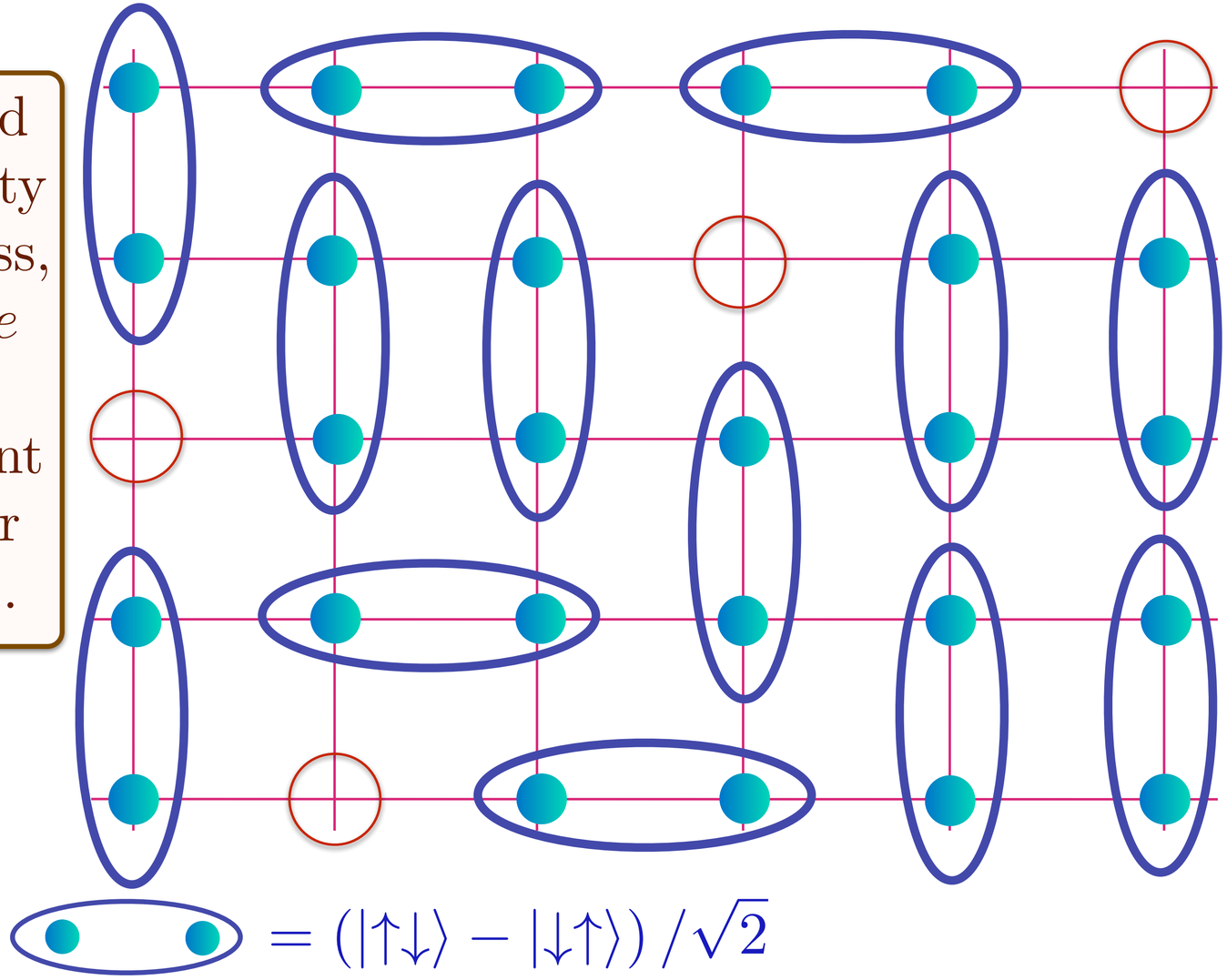

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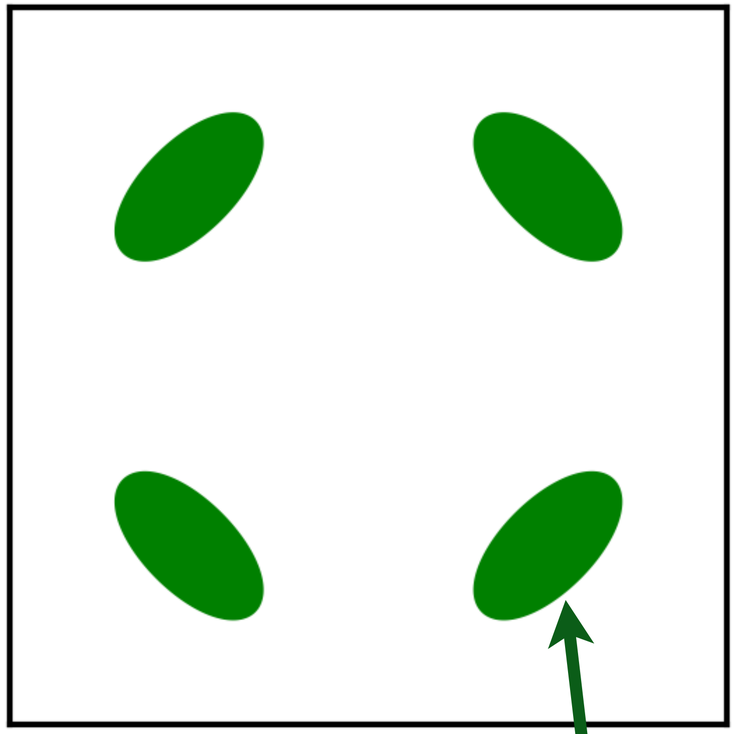

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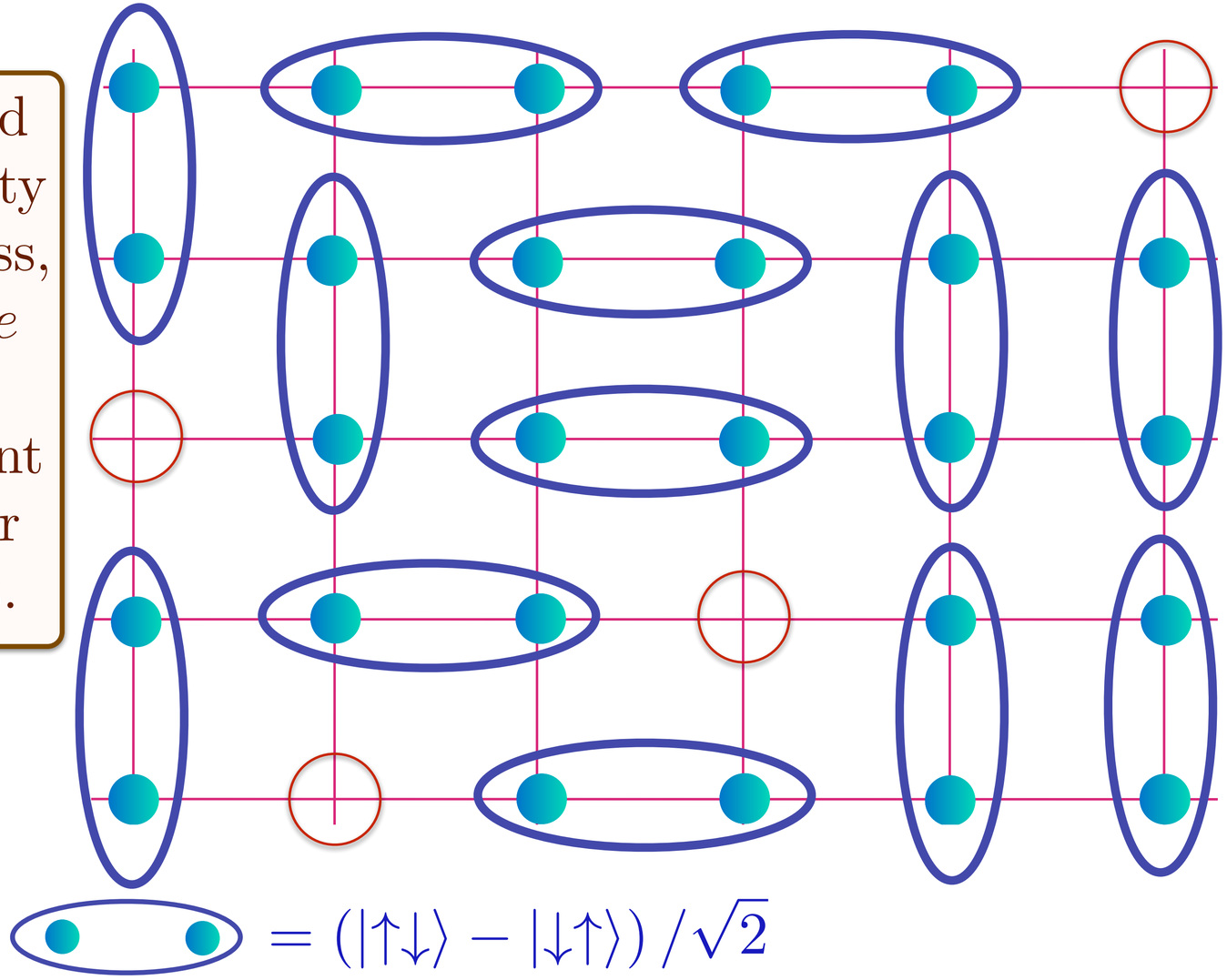

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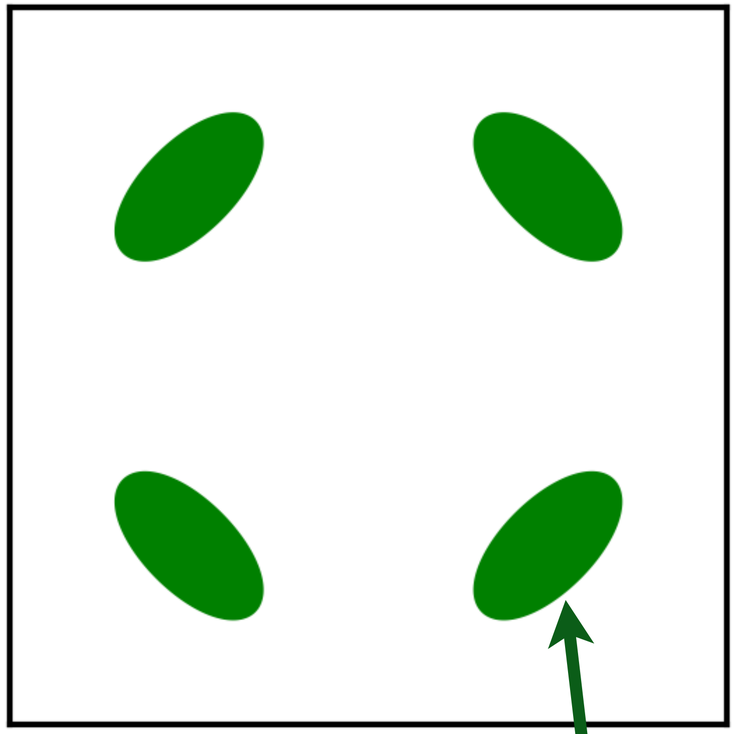

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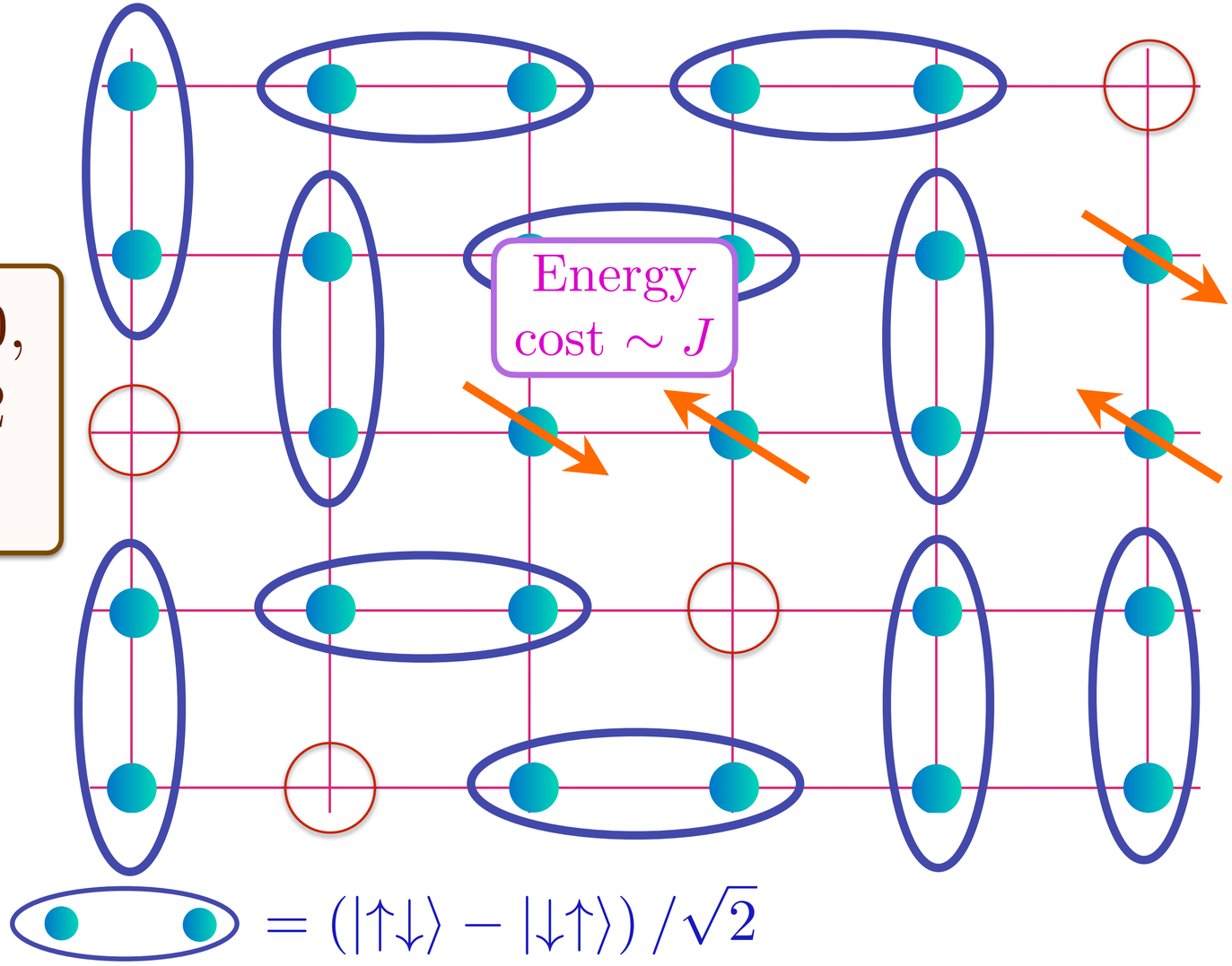
Doping an insulating antiferromagnet with holes of density p

Holon metal excited states

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Charge 0, spin-1/2 spinons



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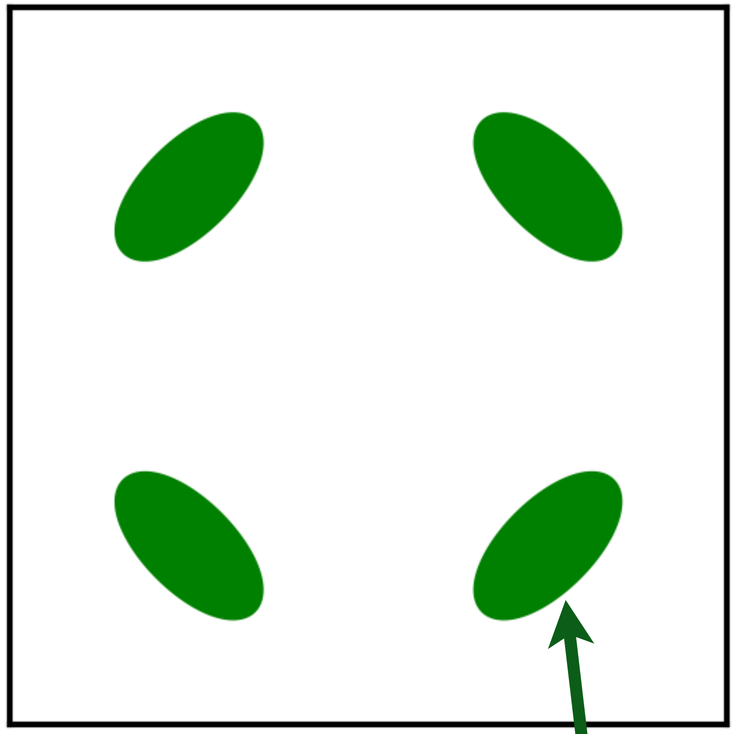

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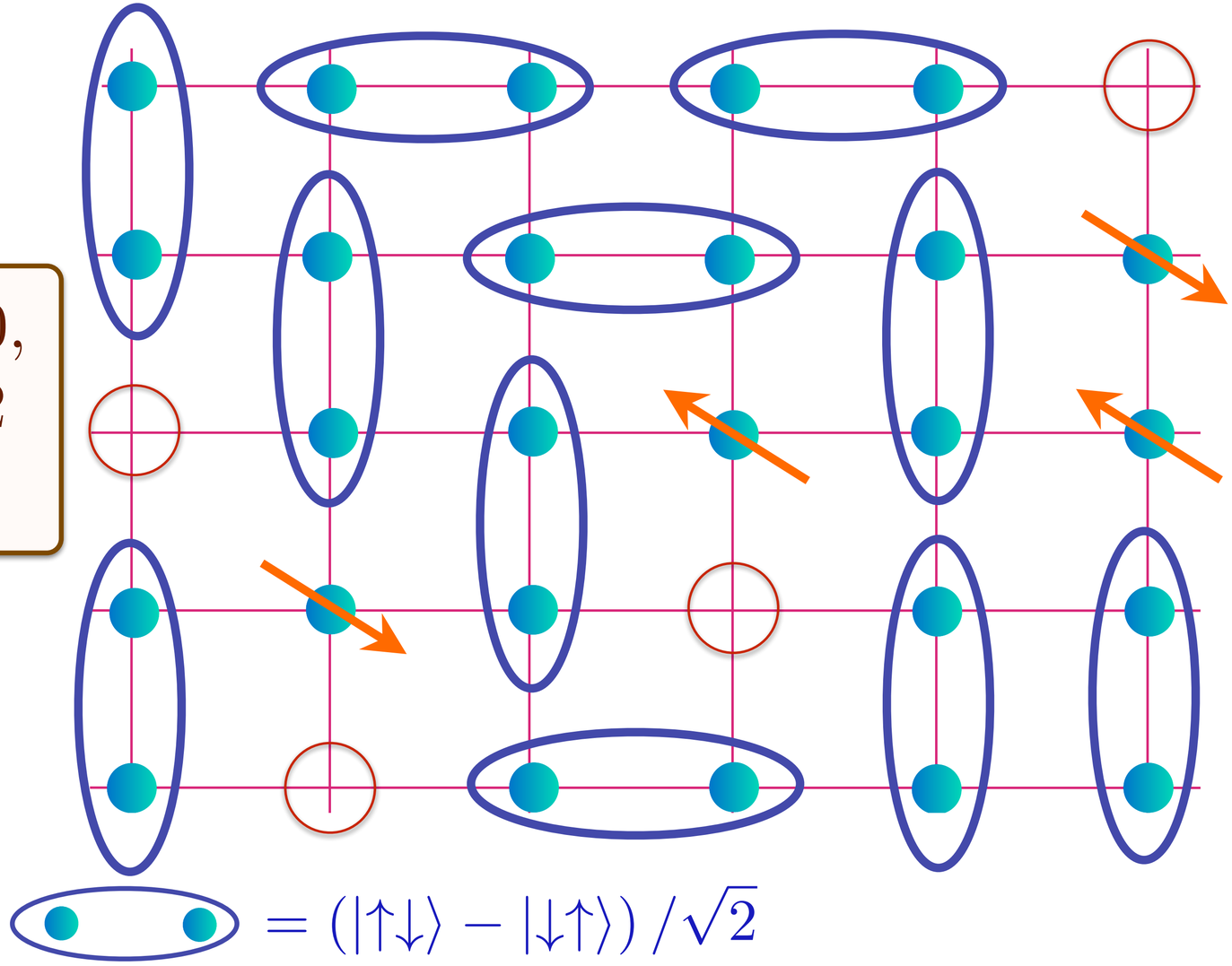

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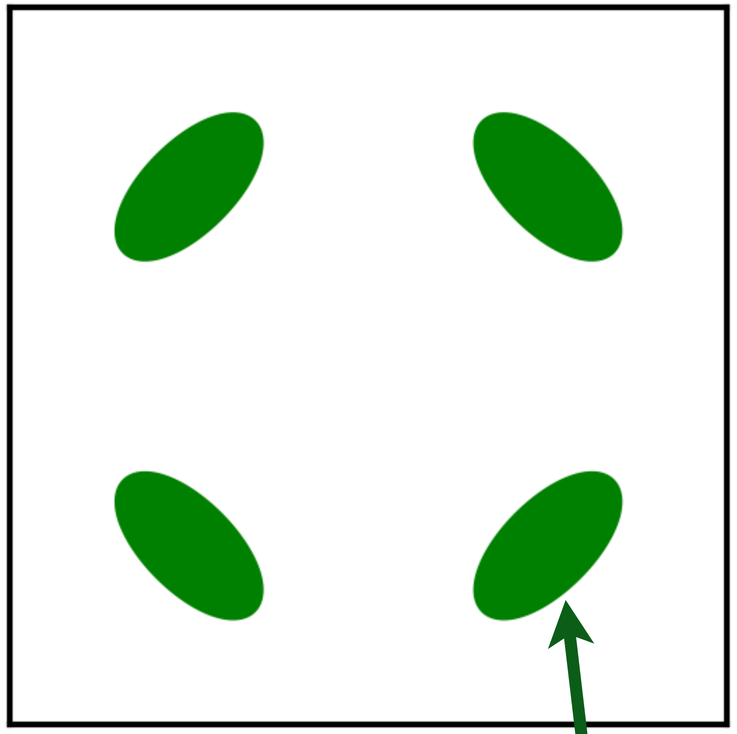

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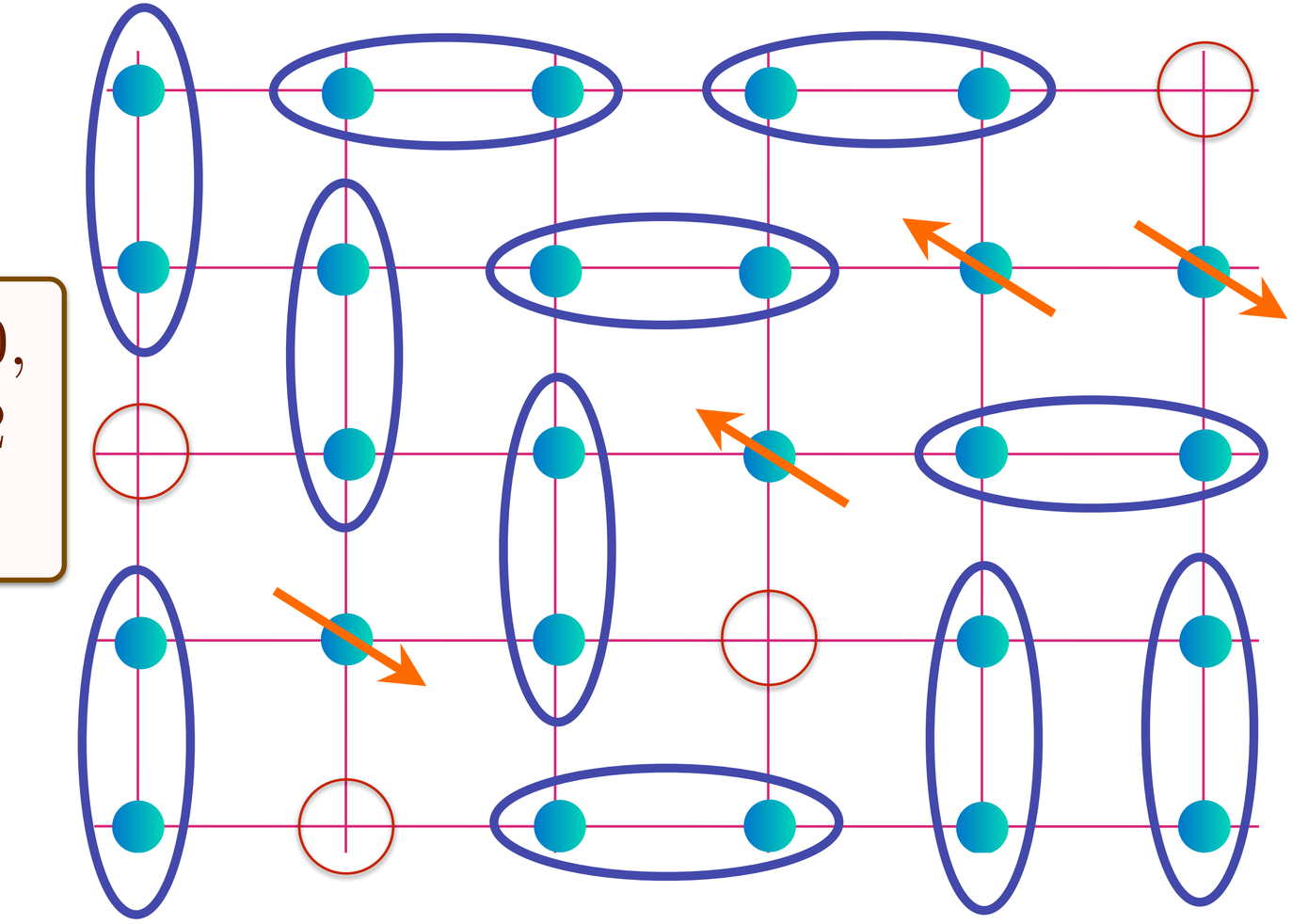

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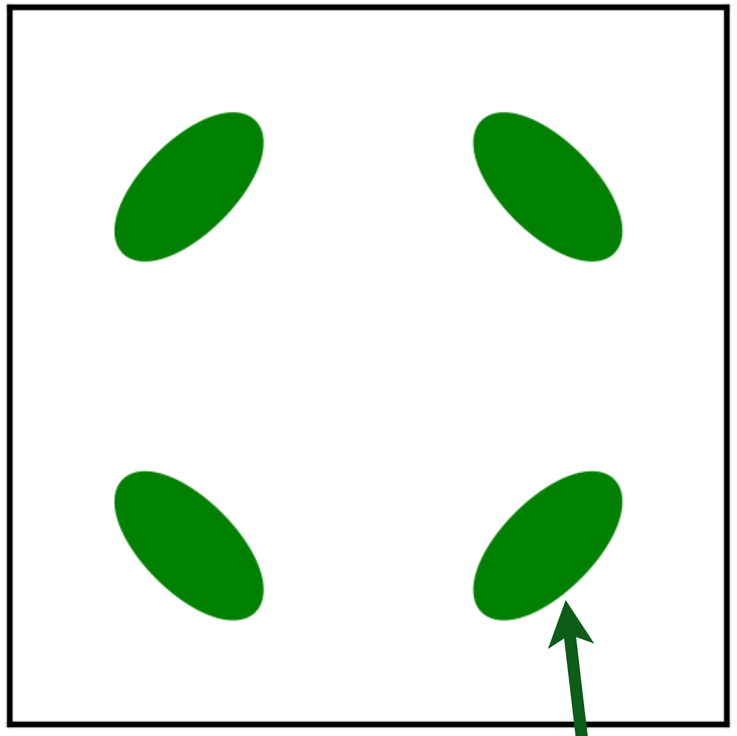

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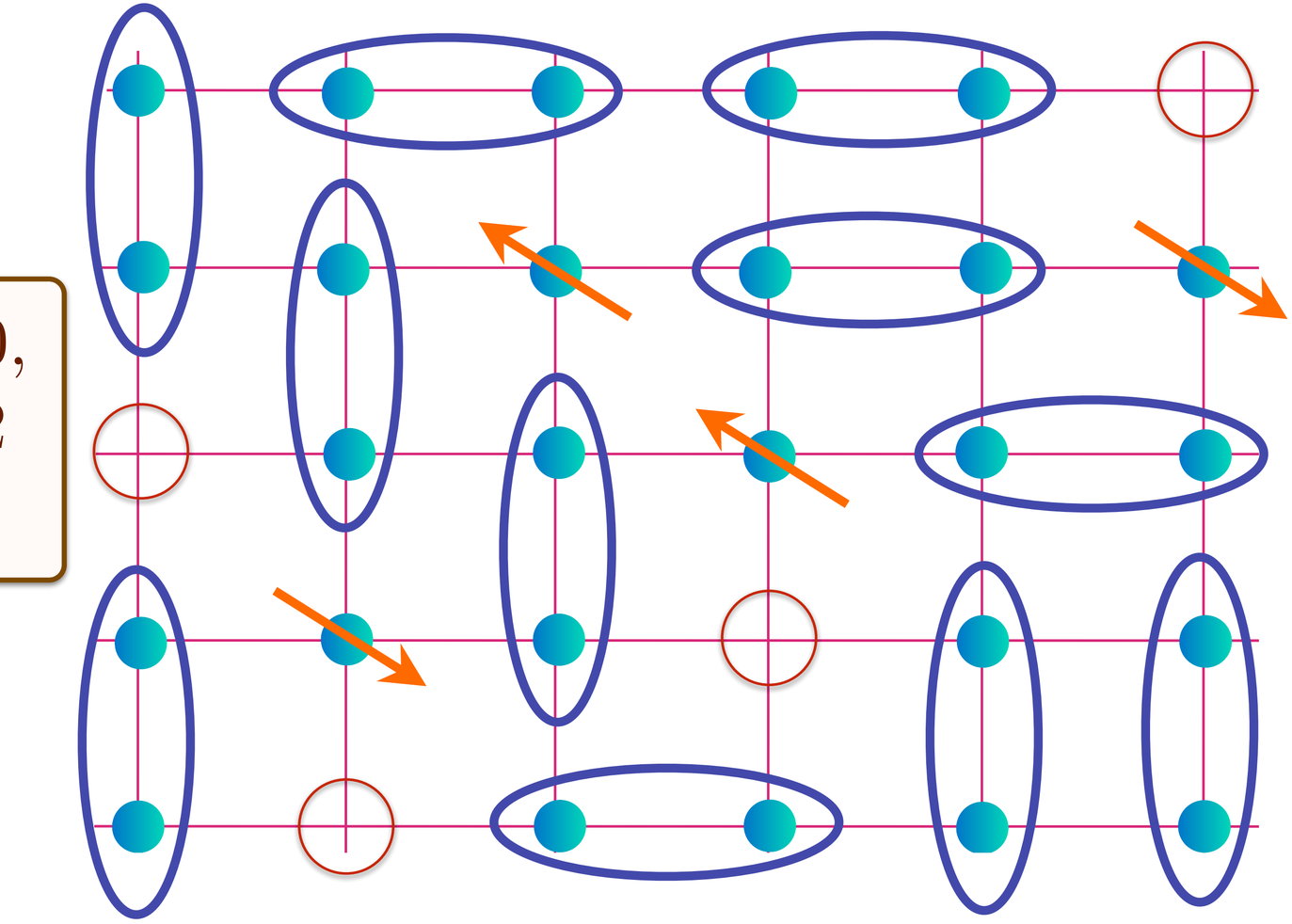

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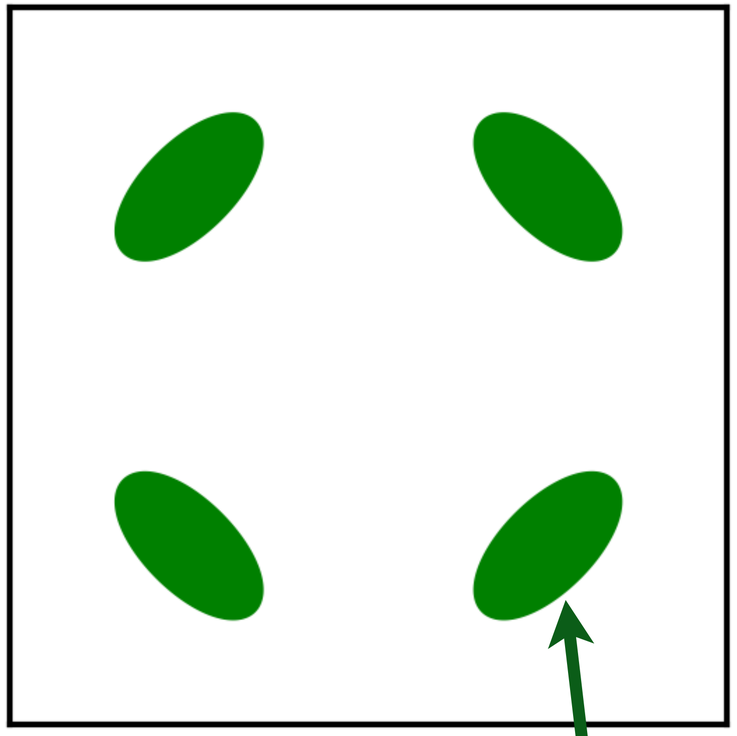

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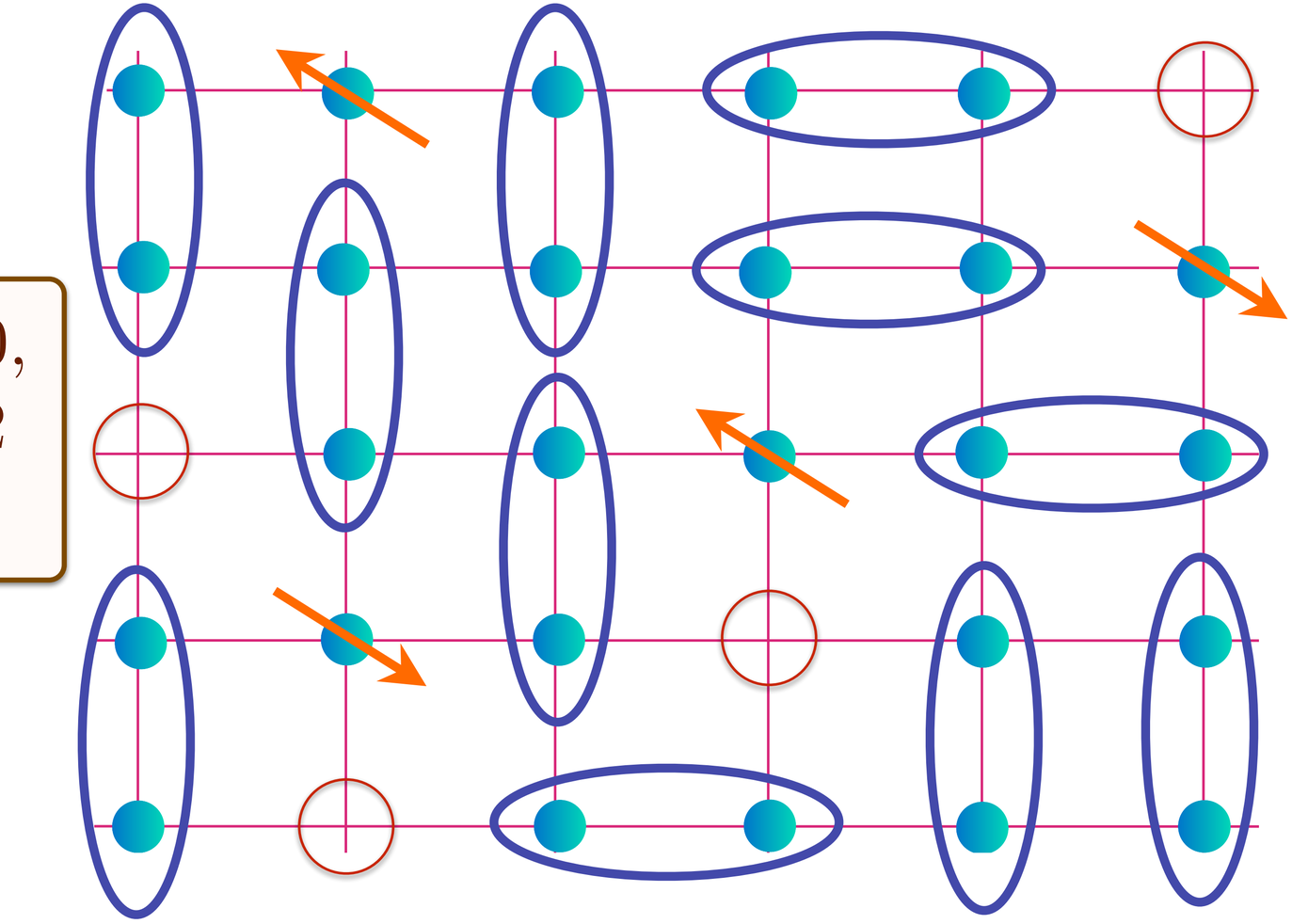

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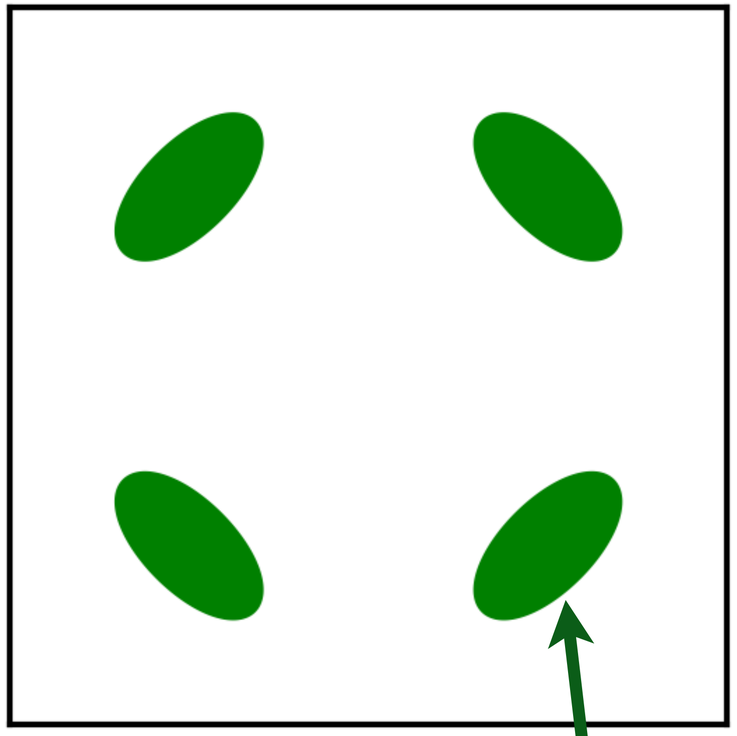

Area $p/4$

Doping an insulating antiferromagnet with holes of density p

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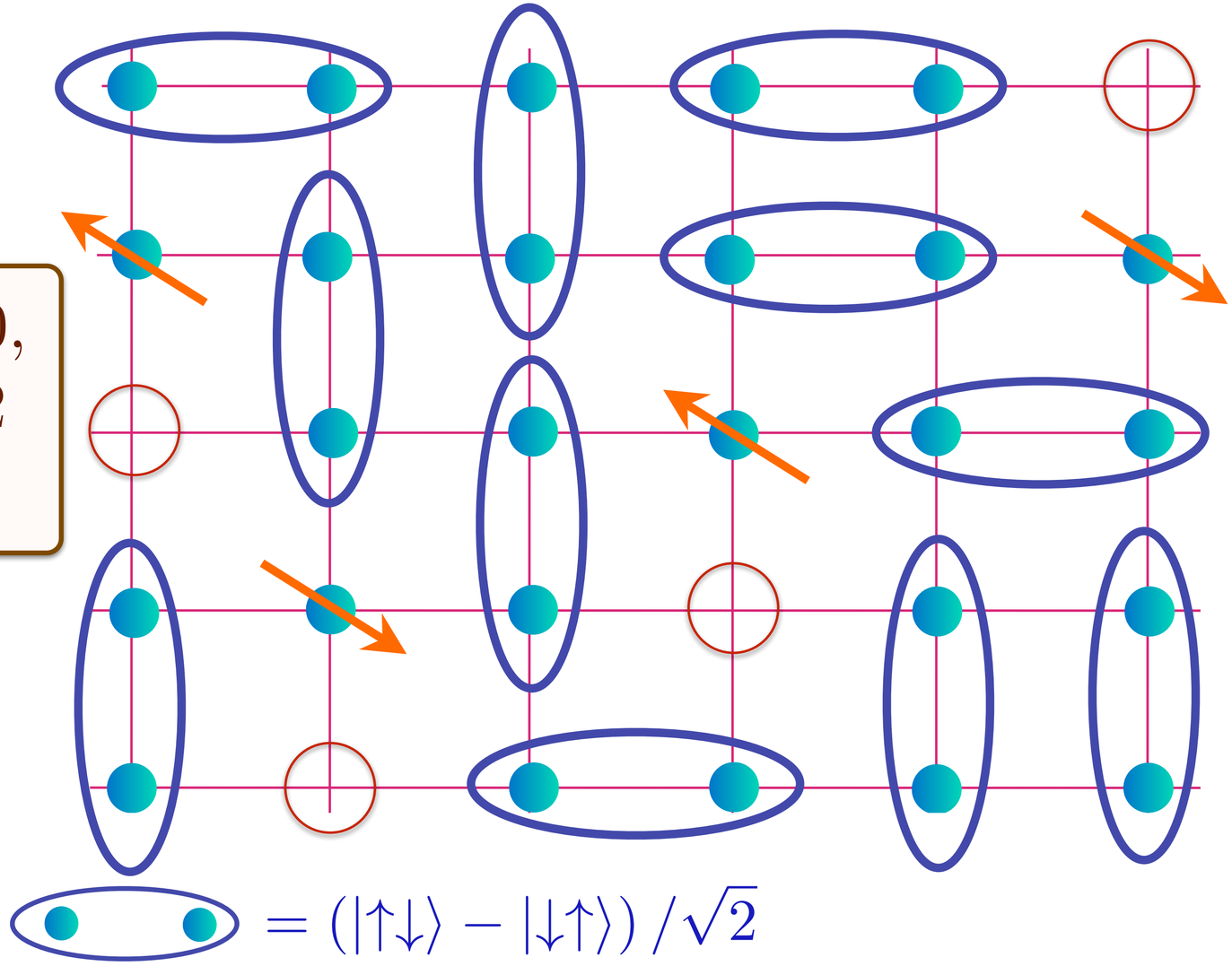
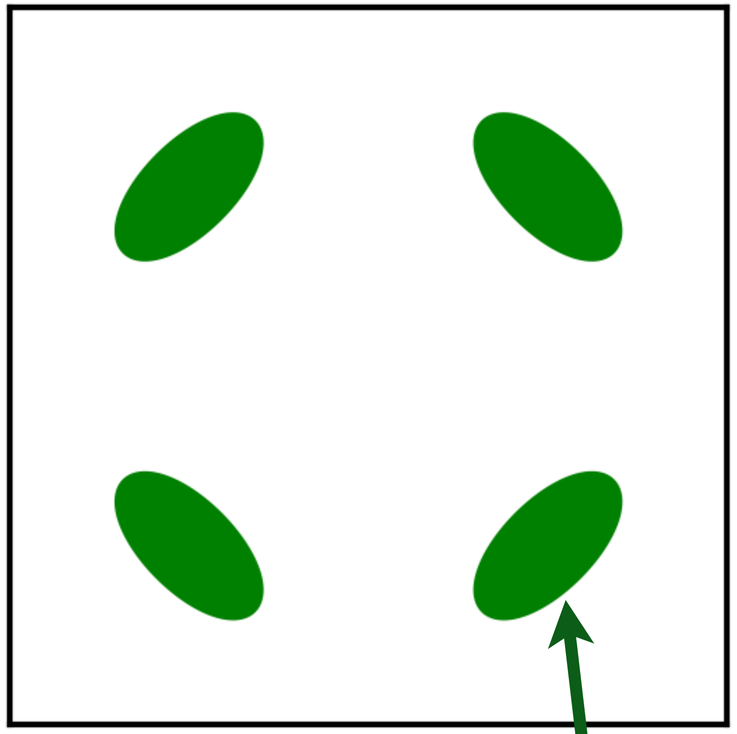


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



Area $p/4$

Doping an insulating antiferromagnet with holes of density p

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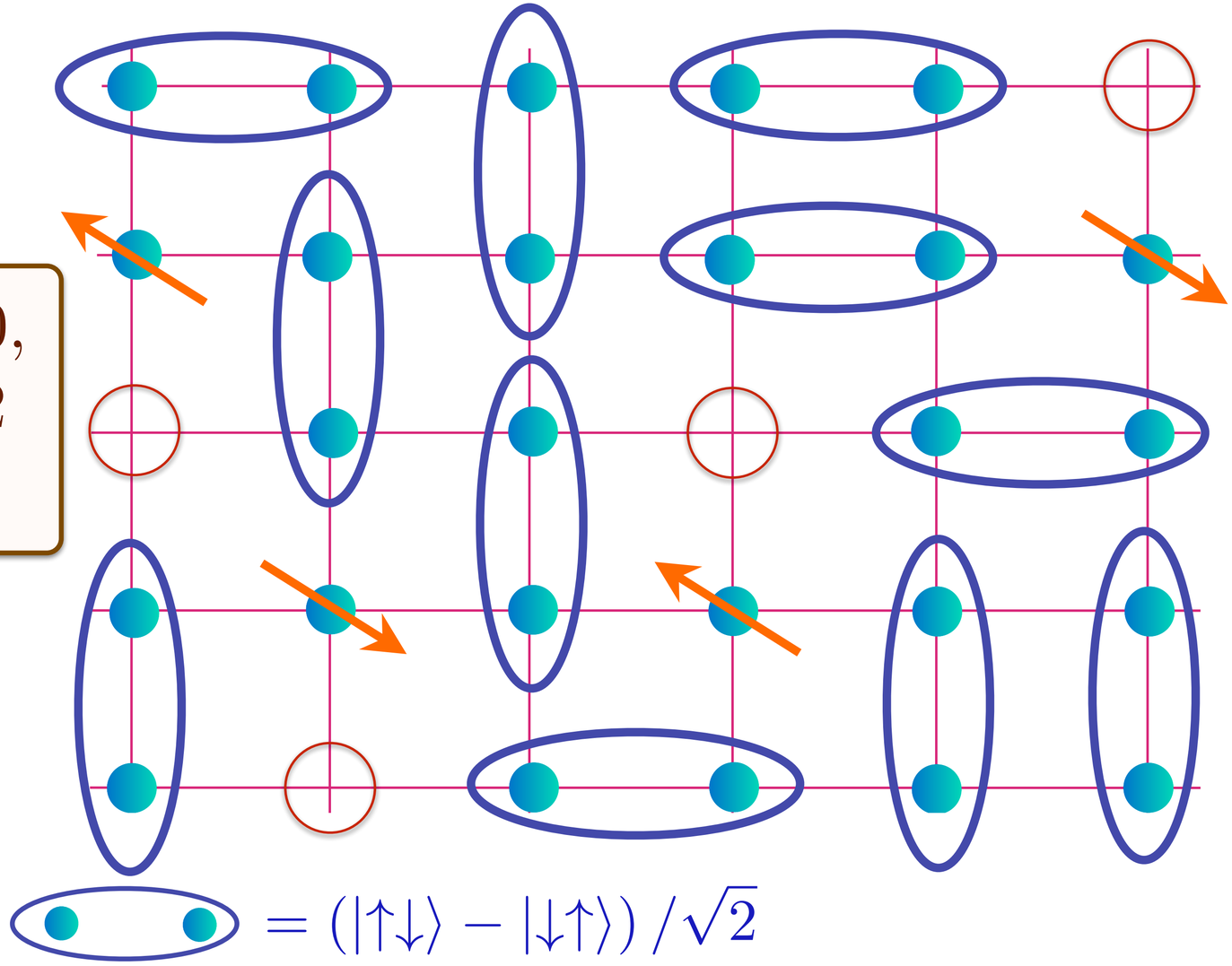
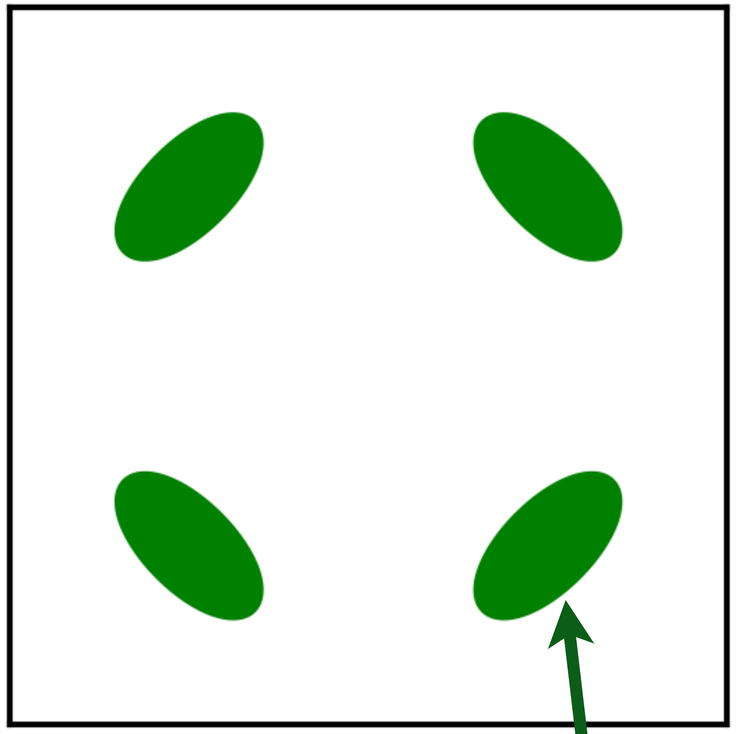


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



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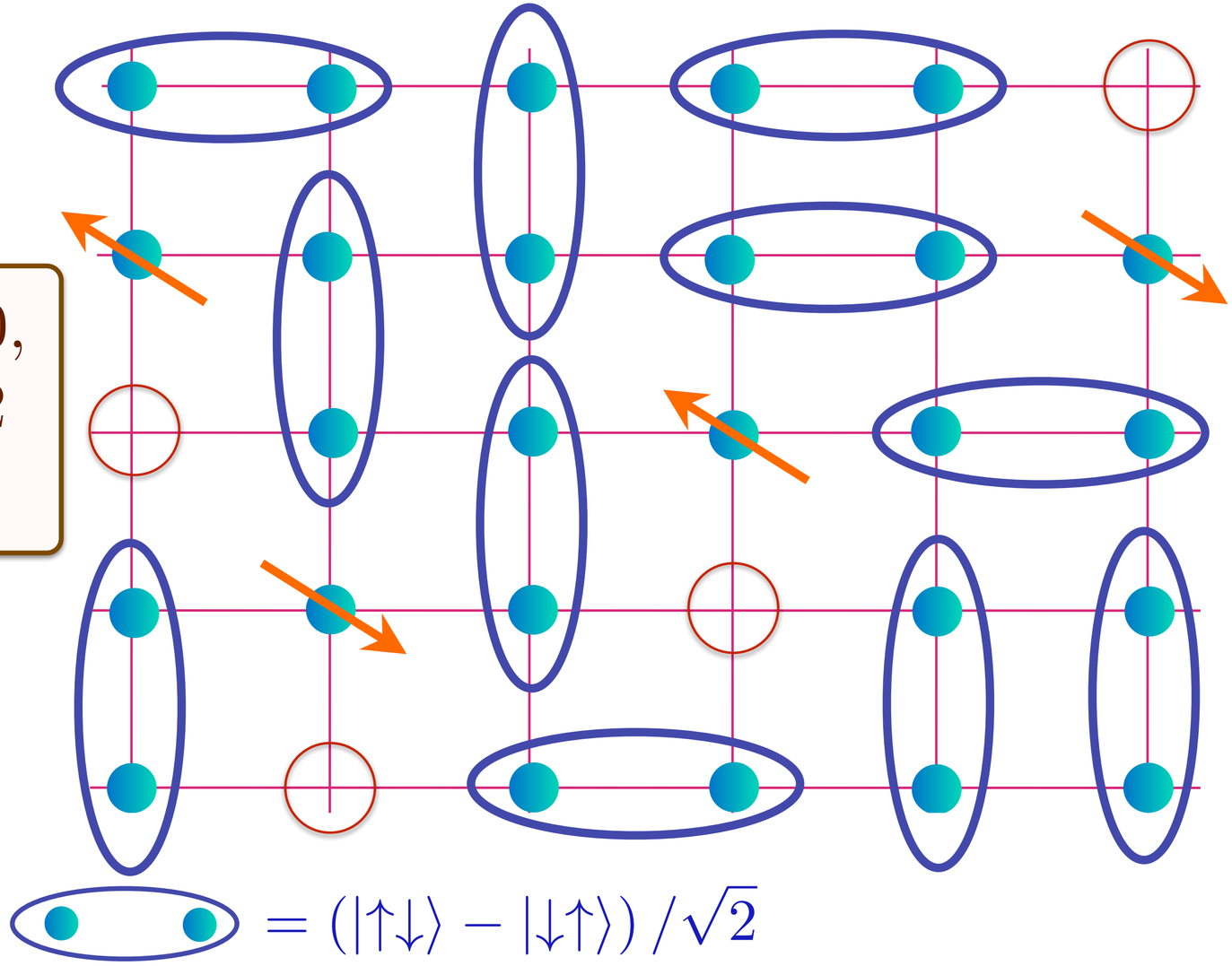

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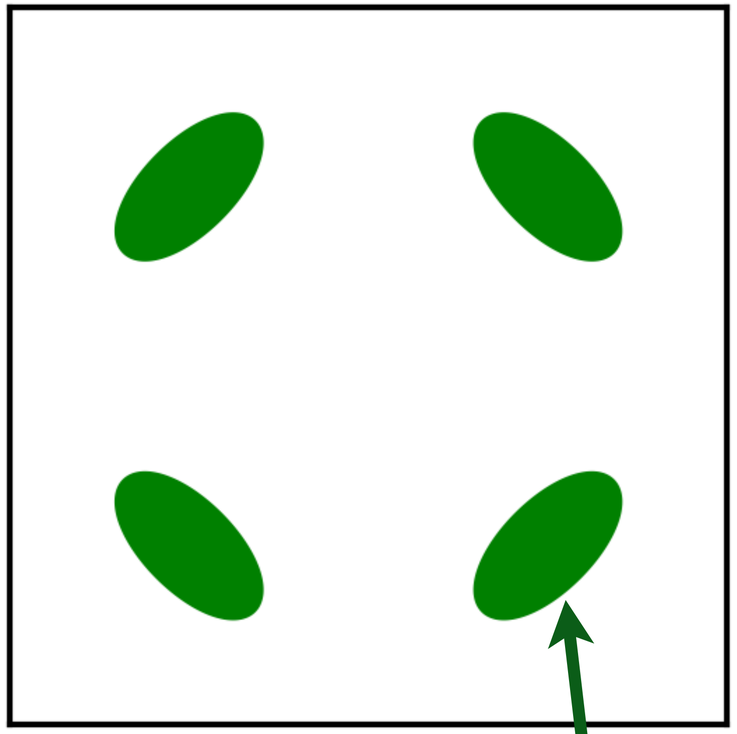


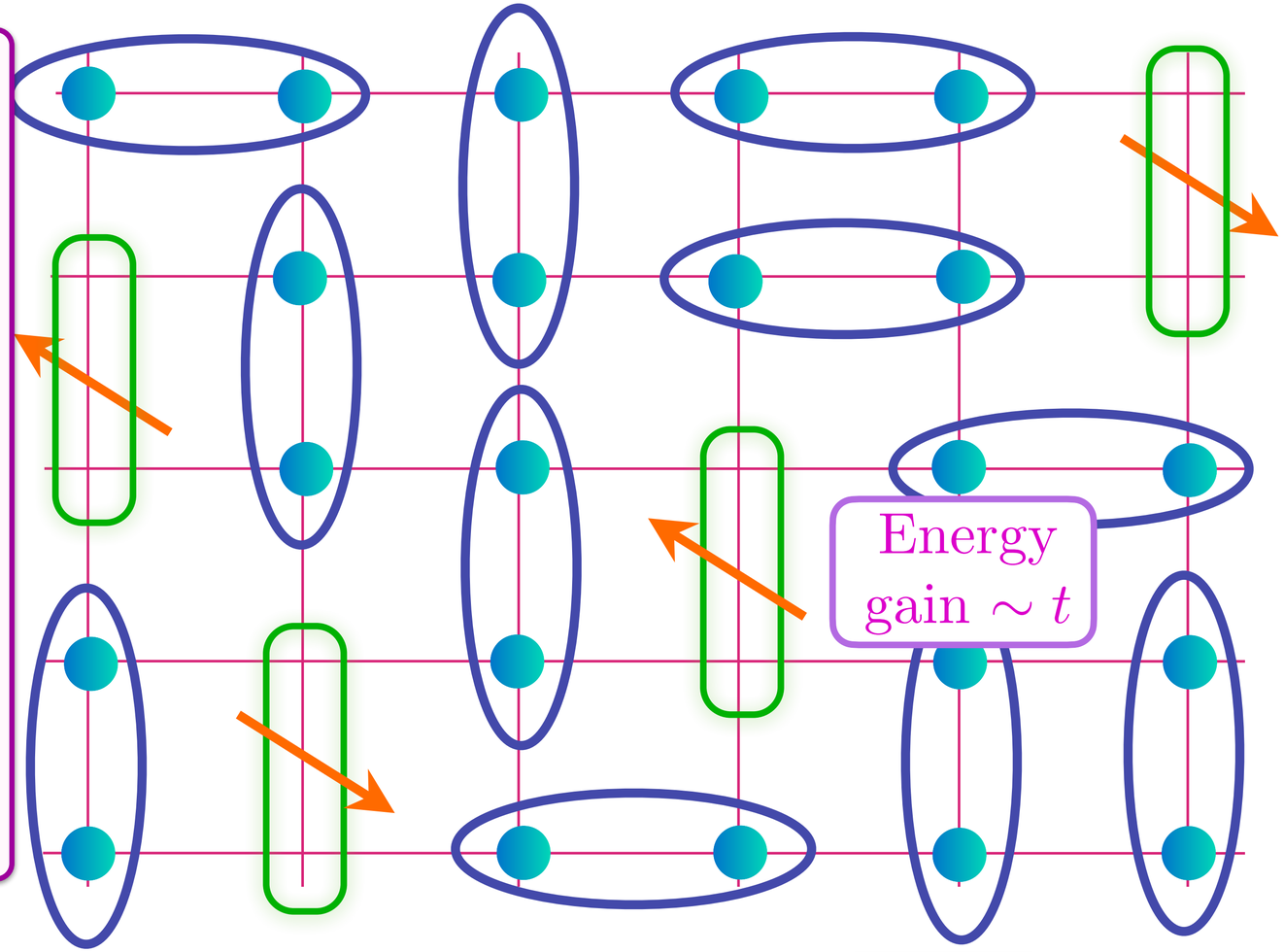
Area $p/4$

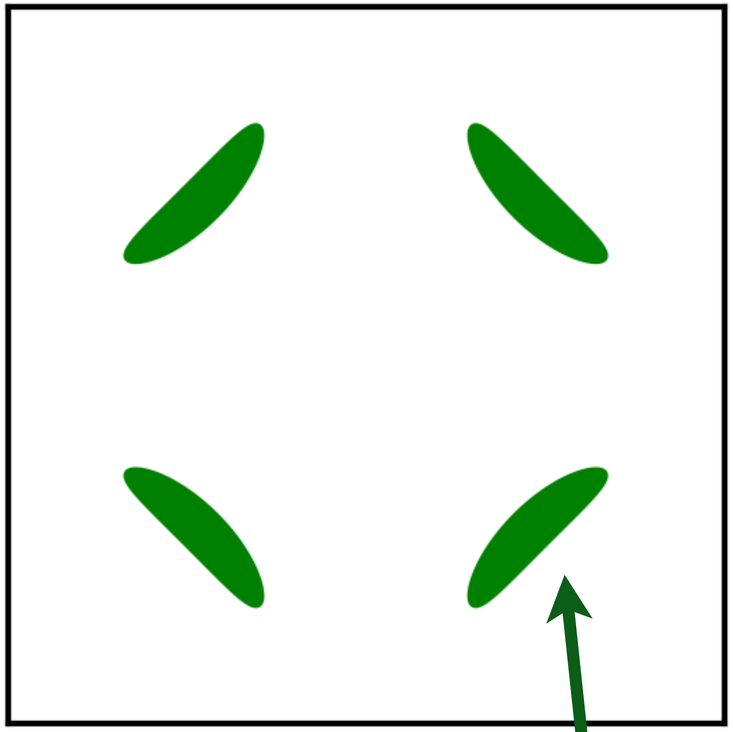
Doping an insulating antiferromagnet with holes of density p

FL*

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport.





Area $p/8$

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

$= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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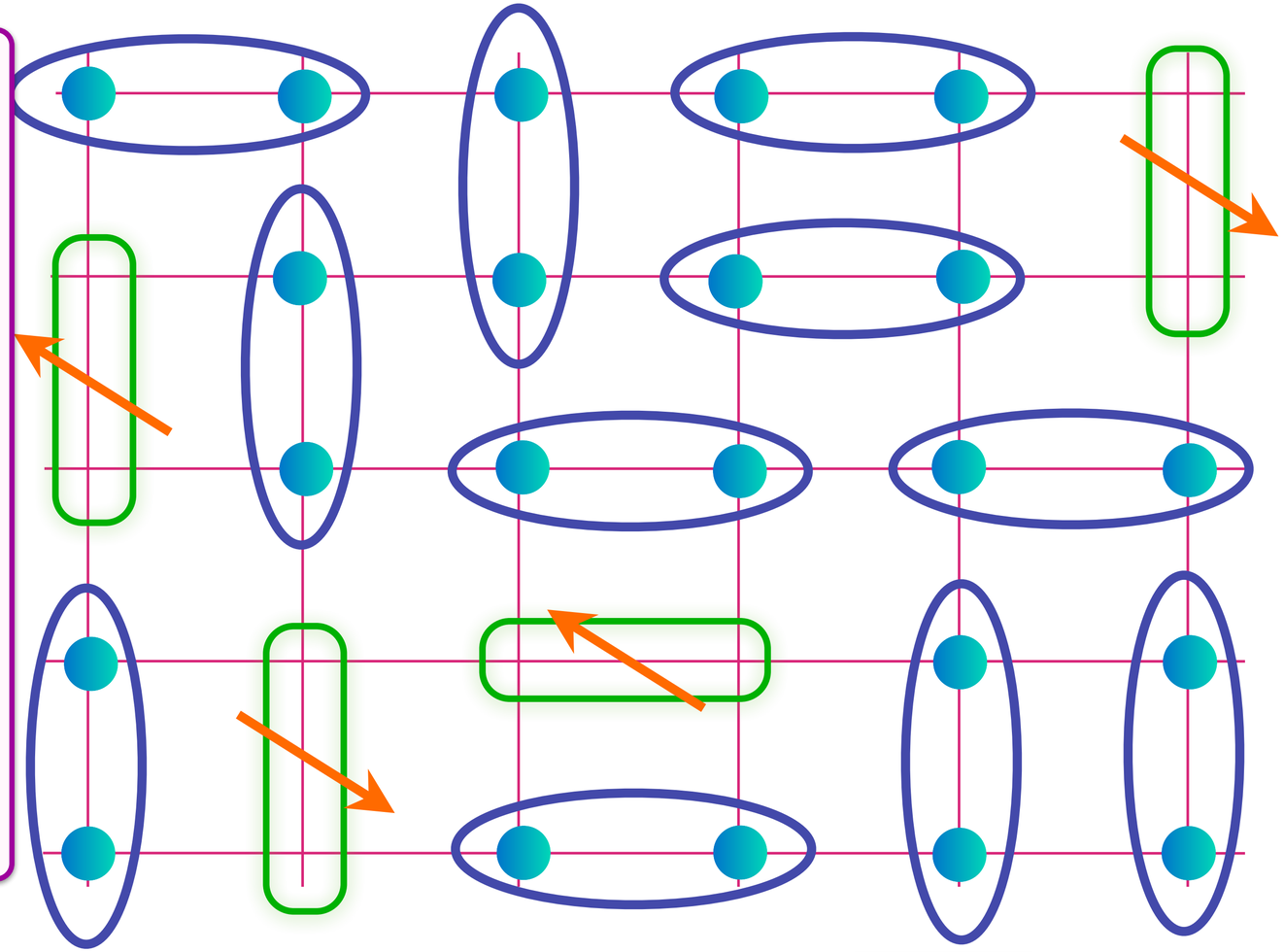

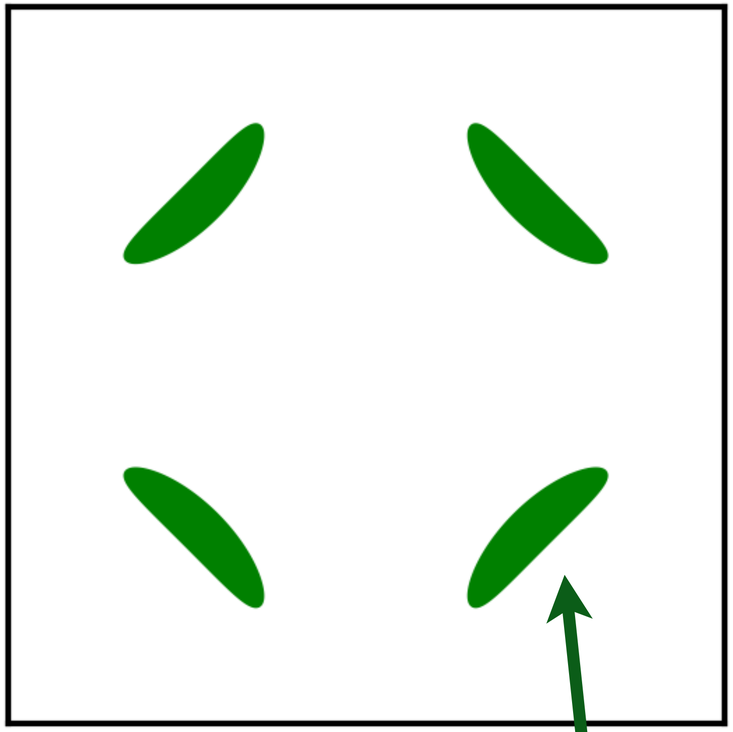

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T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
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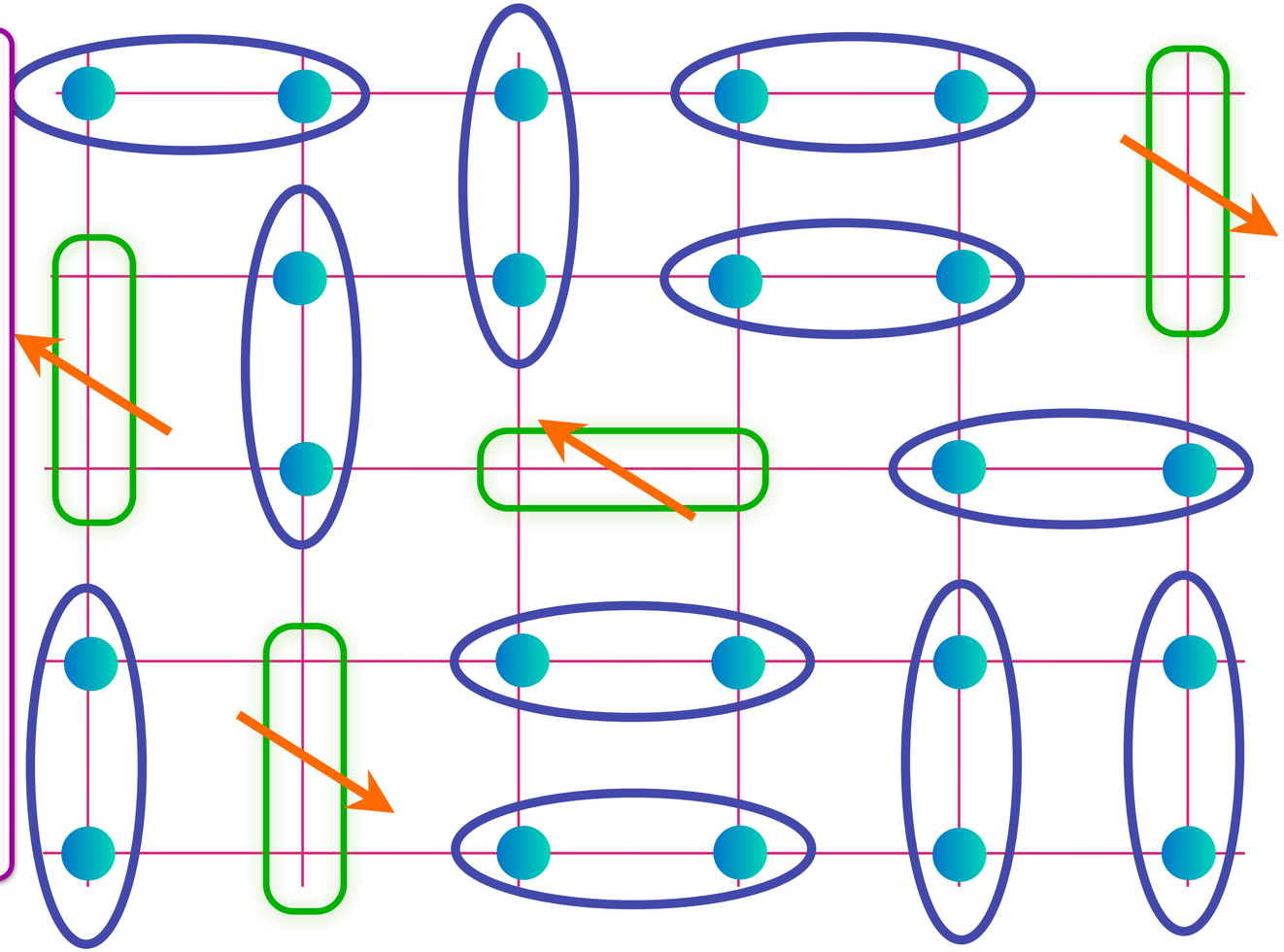

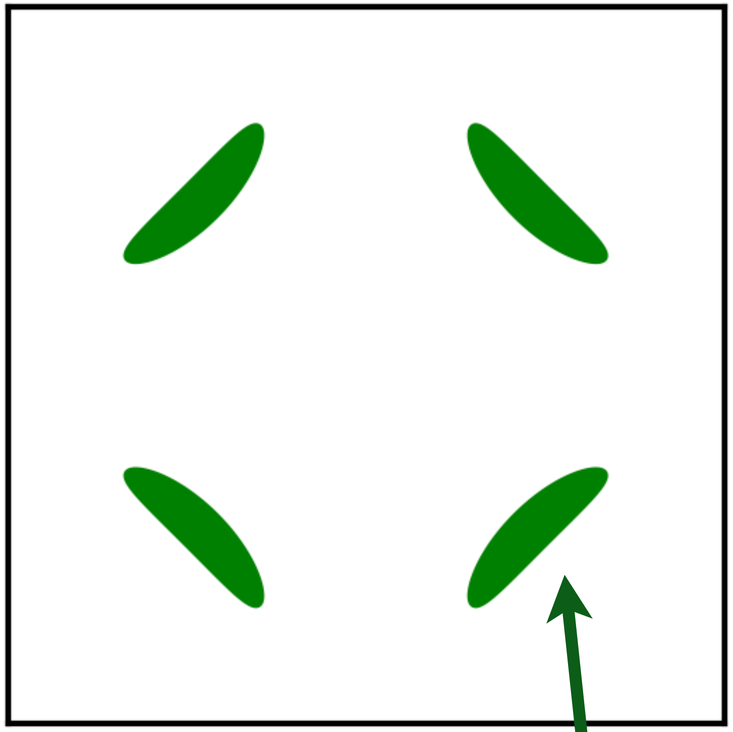

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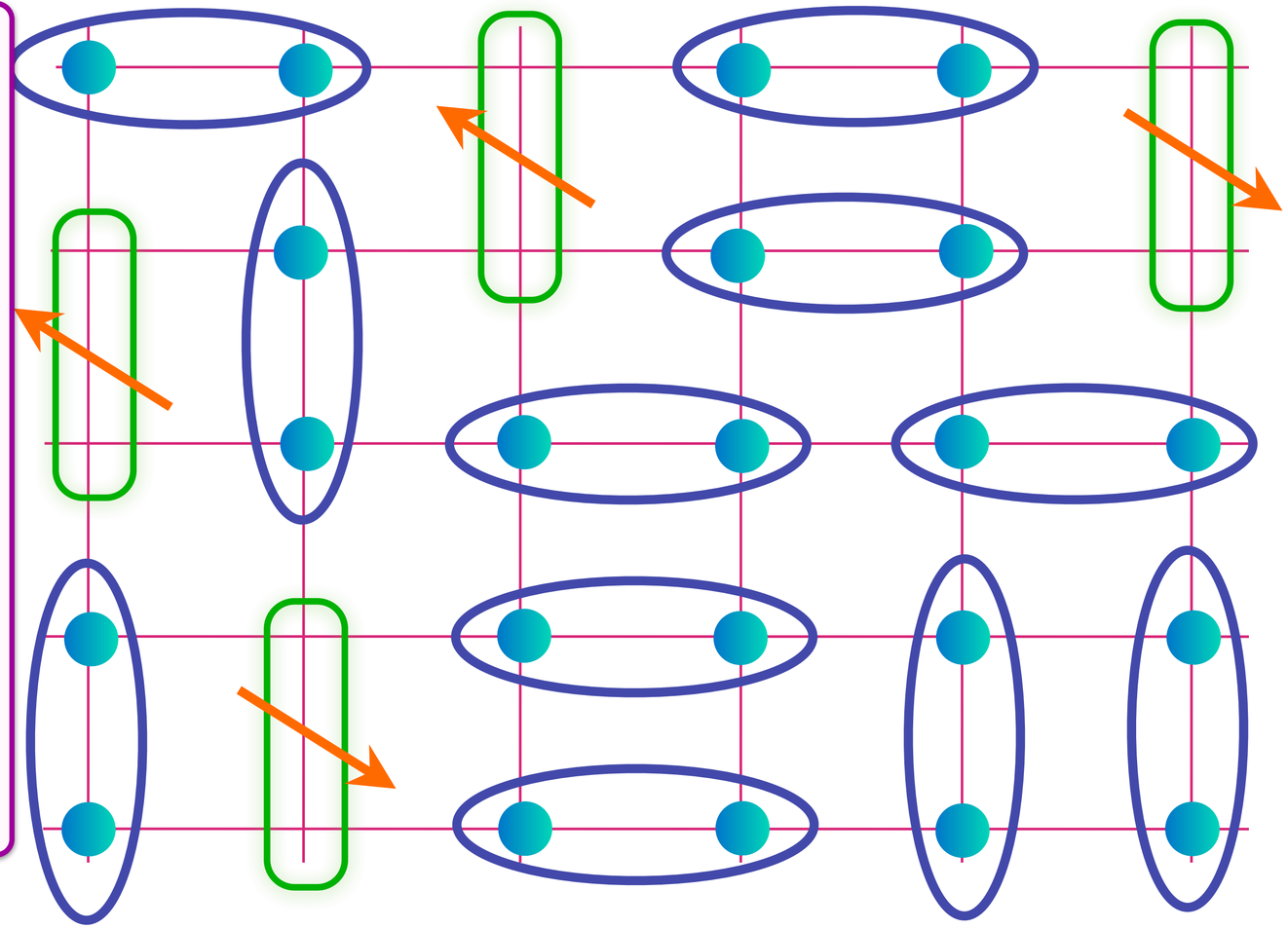

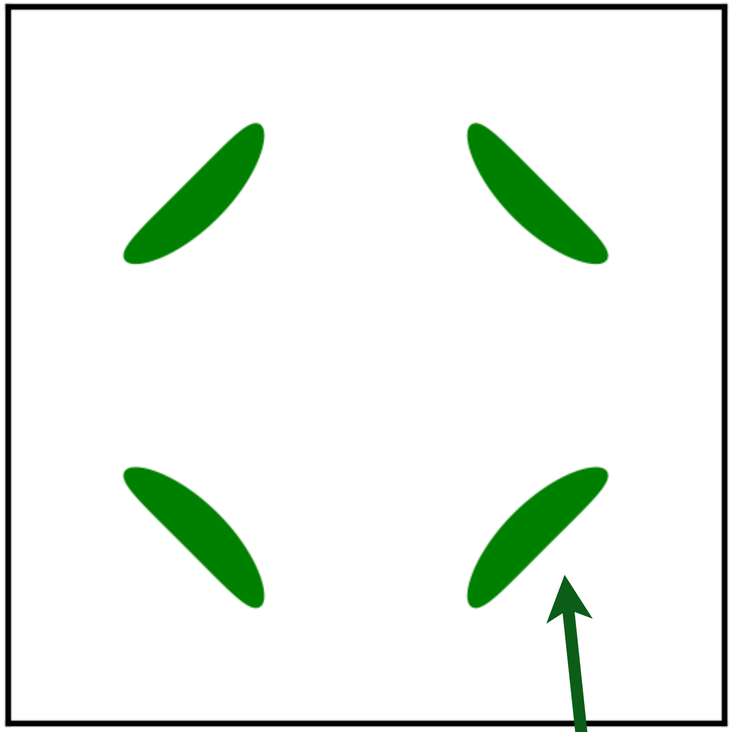

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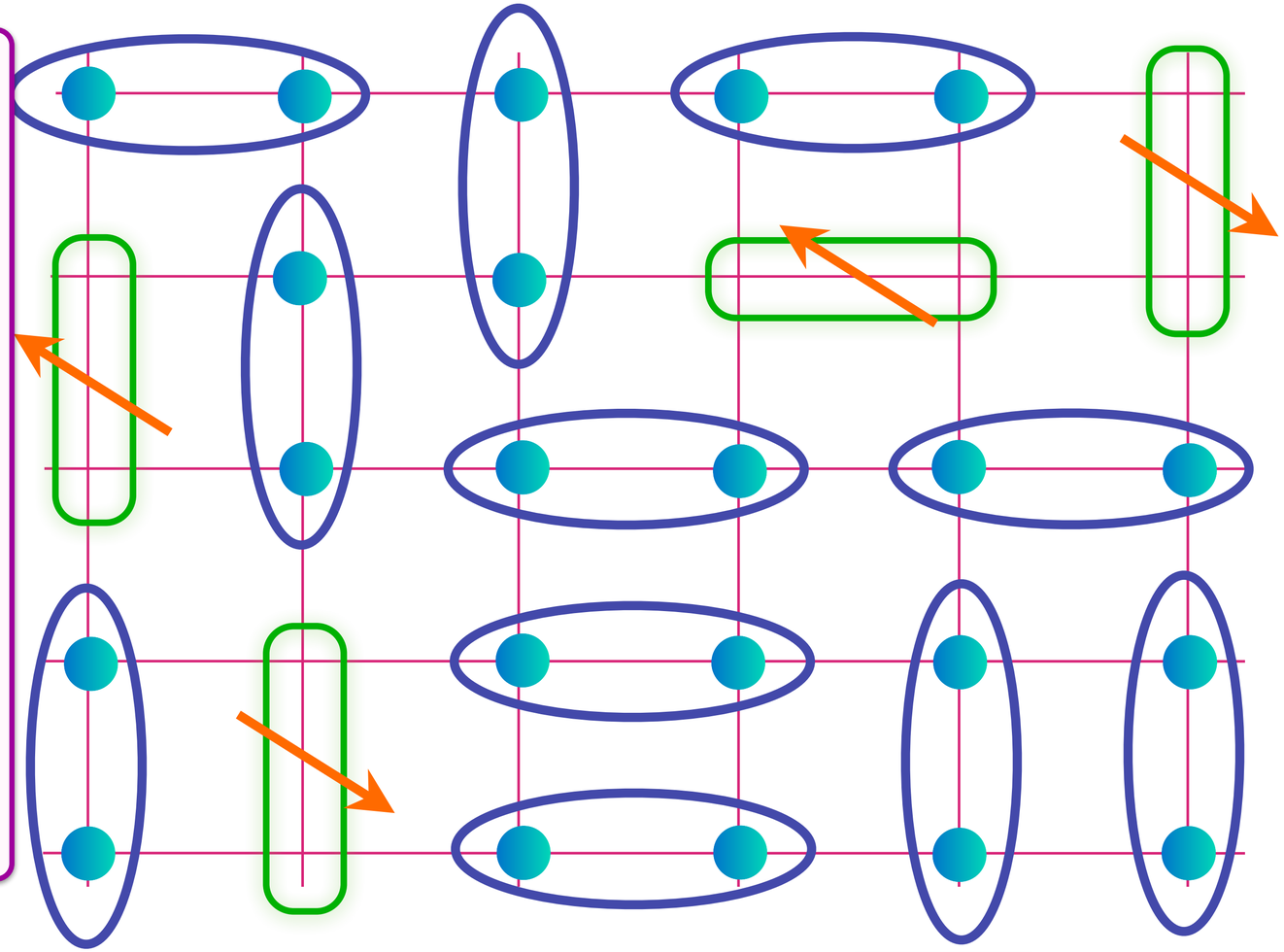

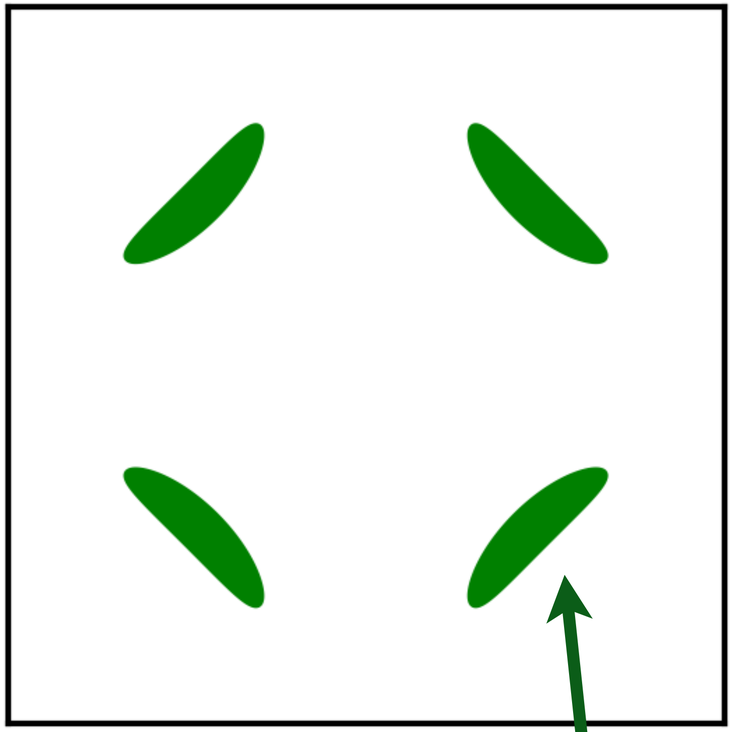

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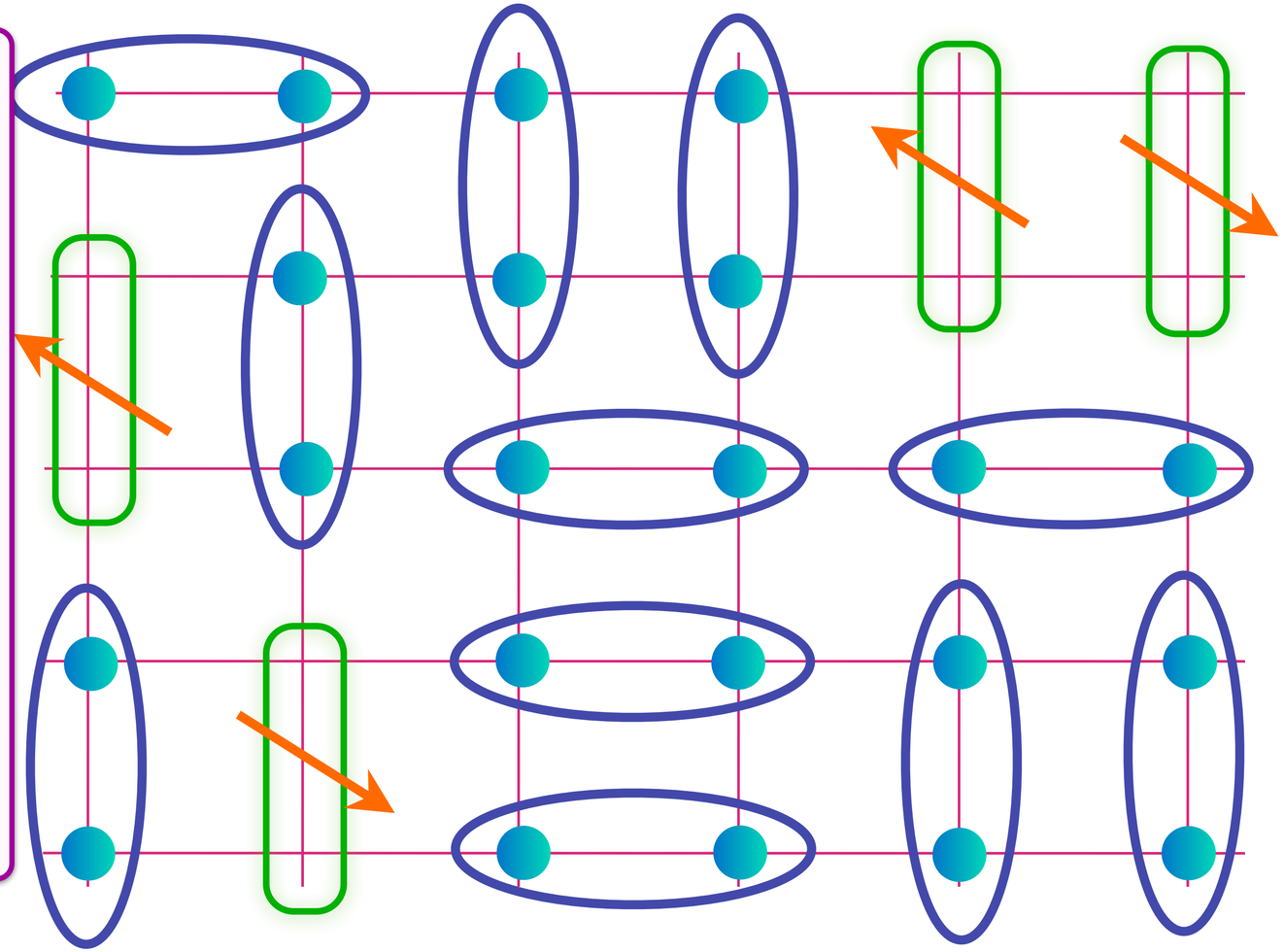

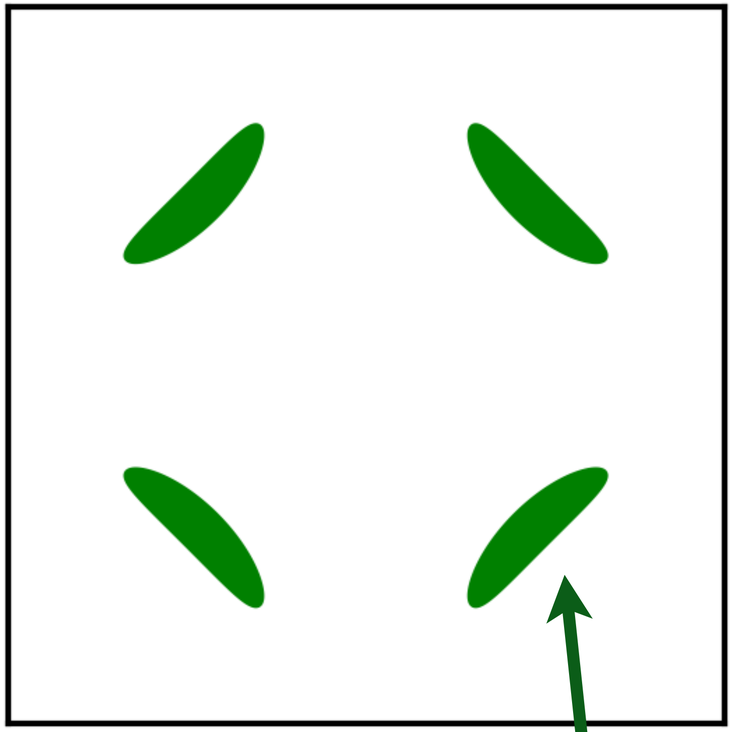

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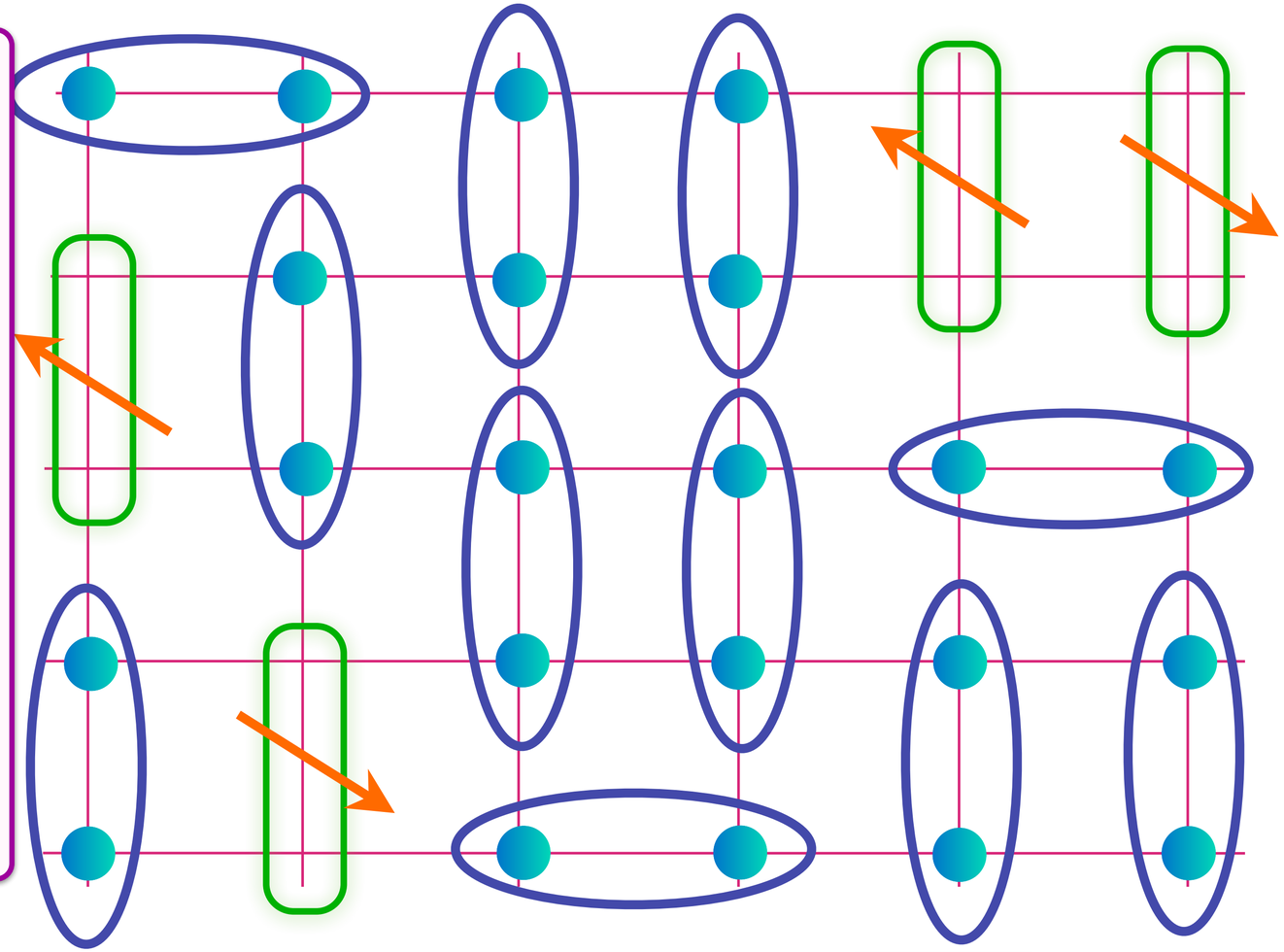

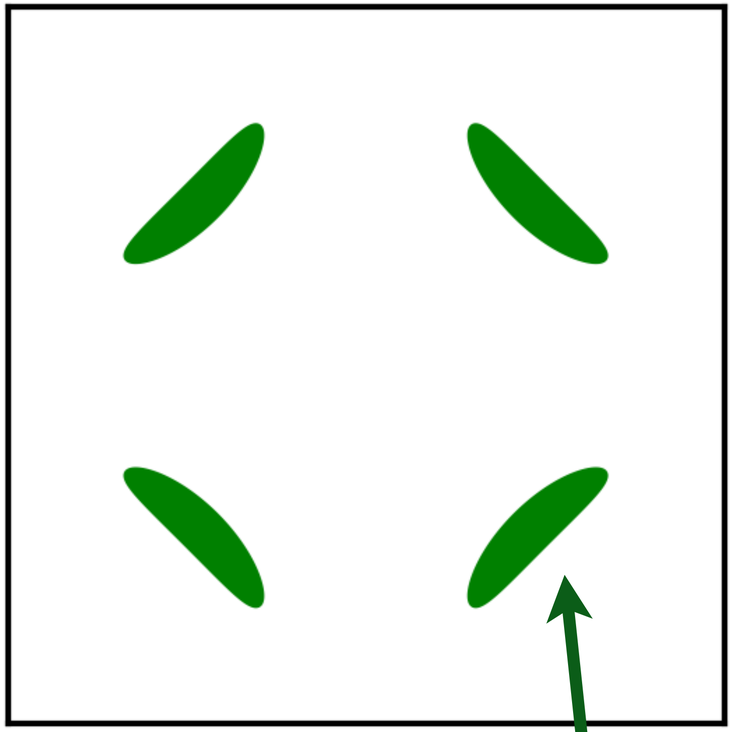

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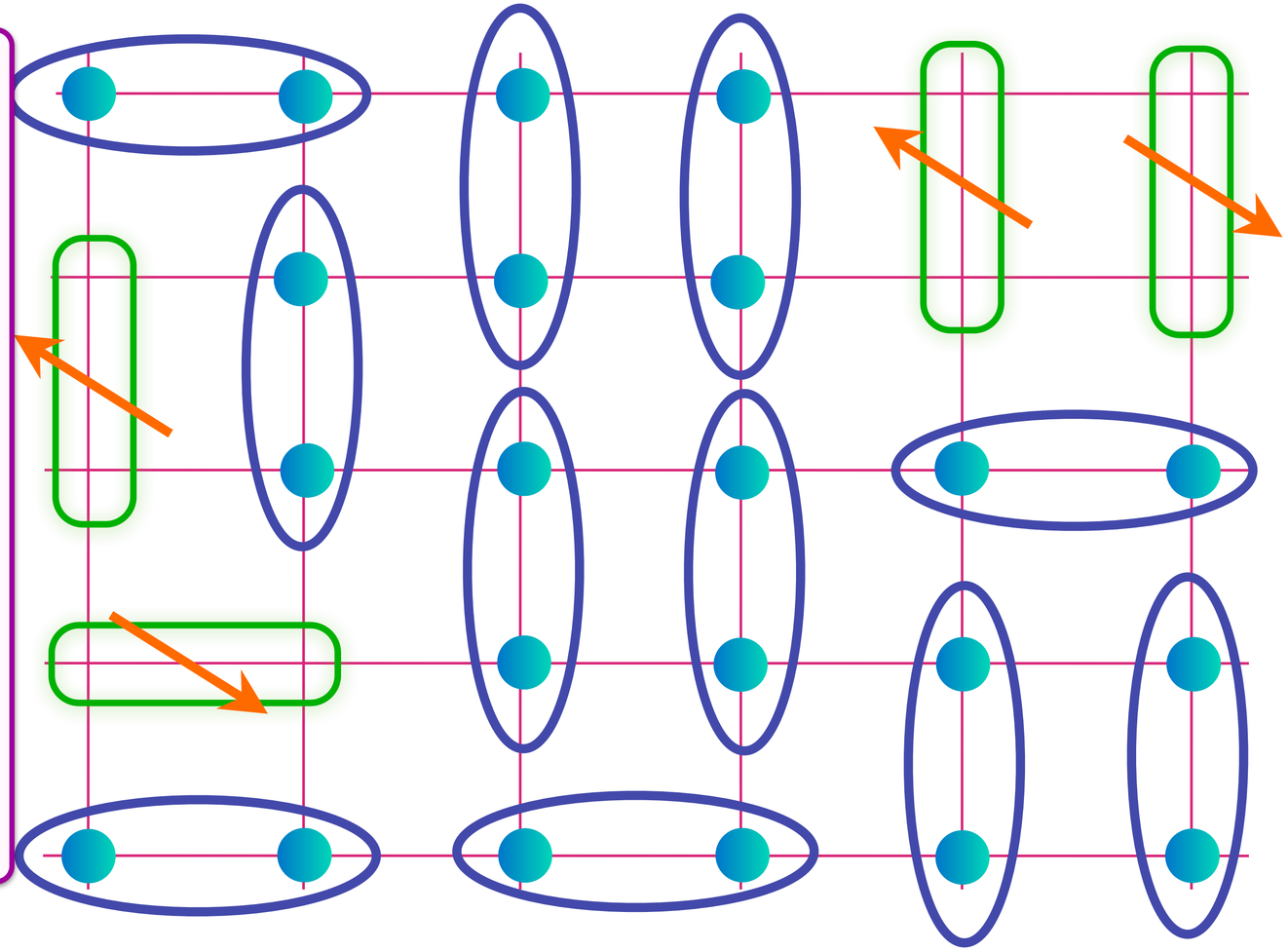

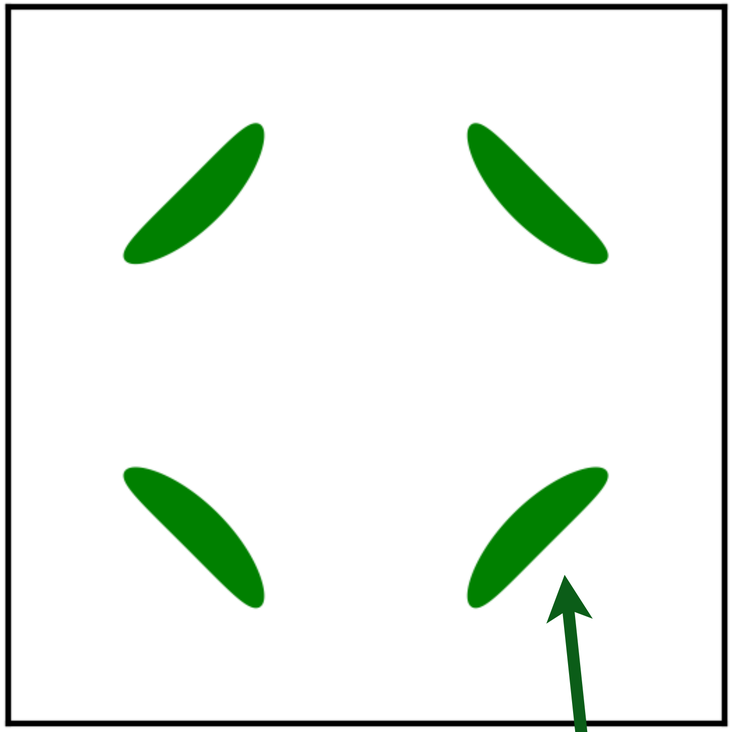

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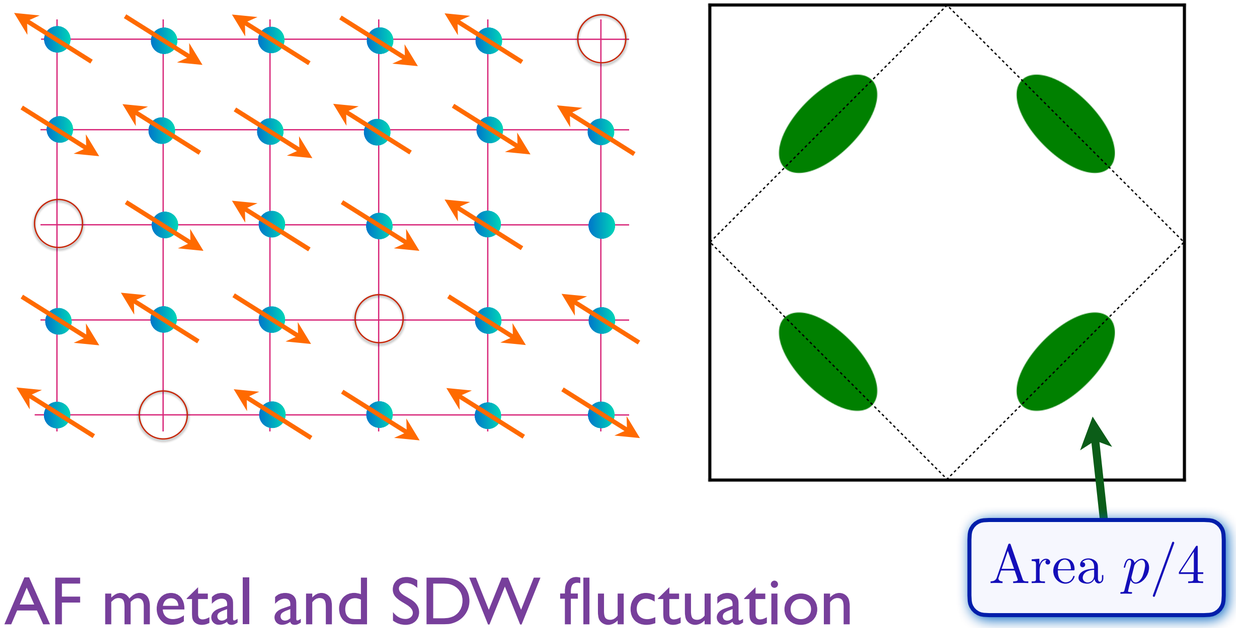
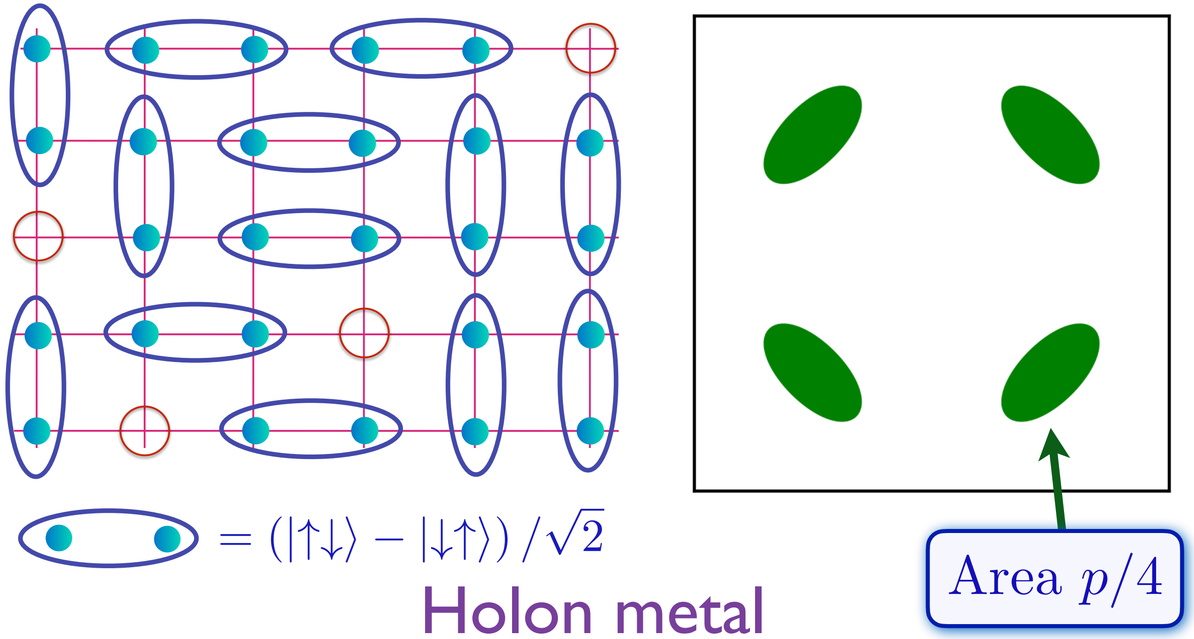


AF metal and SDW fluctuation



Holon metal

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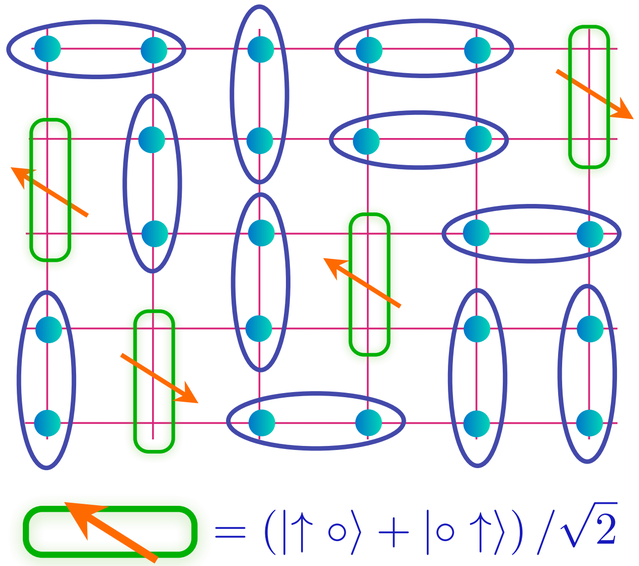
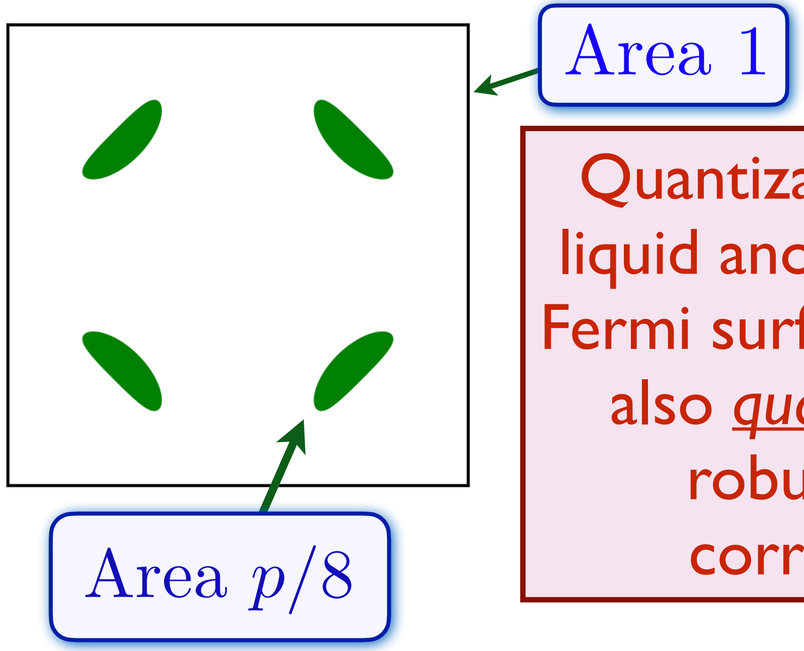


FL*

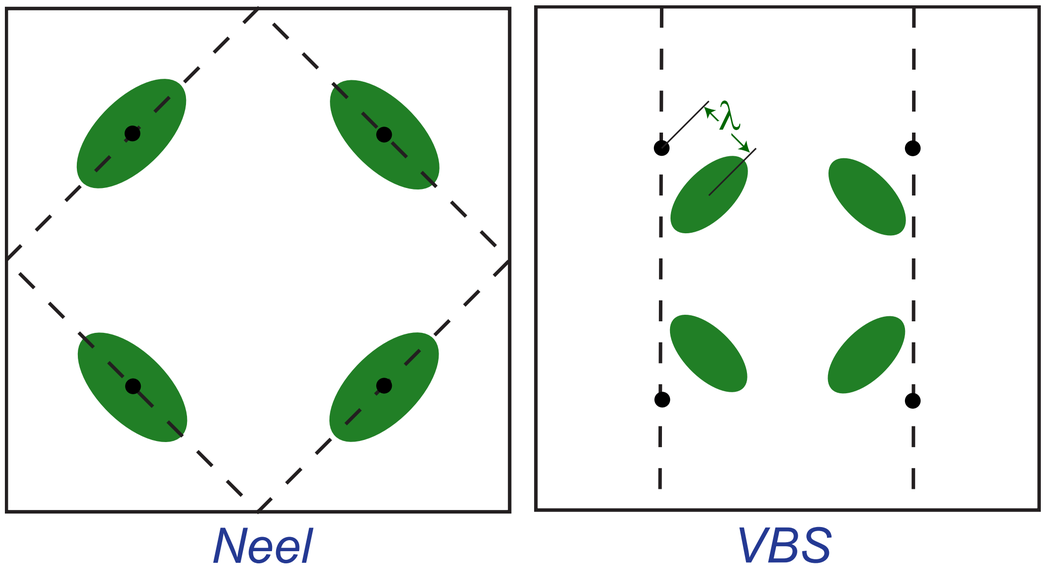


Quantization of spin liquid anomaly implies Fermi surface areas are also *quantized* and robust to all corrections.

PHYSICAL REVIEW B **75**, 235122 (2007)

Hole dynamics in an antiferromagnet across a deconfined quantum critical point

Ribhu K. Kaul,[1] Alexei Kolezhuk,[1,2] Michael Levin,[1] Subir Sachdev,[1] and T. Senthil[3,4]



The dashed line in the Néel phase indicates the boundary of the magnetic Brillouin zone. Only the Fermi surfaces within this zone contribute to the Luttinger counting, and so the area of each ellipse is $\mathcal{A}_F = (2\pi)^2 \delta/4$. In the VBS phase, all four pockets are inequivalent, and so the area of each ellipse is $\mathcal{A}_F = (2\pi)^2 \delta/8$.

Factor of 2 between SDW fluctuation and FL*

Observation of the Yamaji effect in the cuprate pseudogap

See also:

Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, *Nature Physics* **18**, 558 (2022)

Angle-dependent magnetoresistance (ADMR) of $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$

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nature physics
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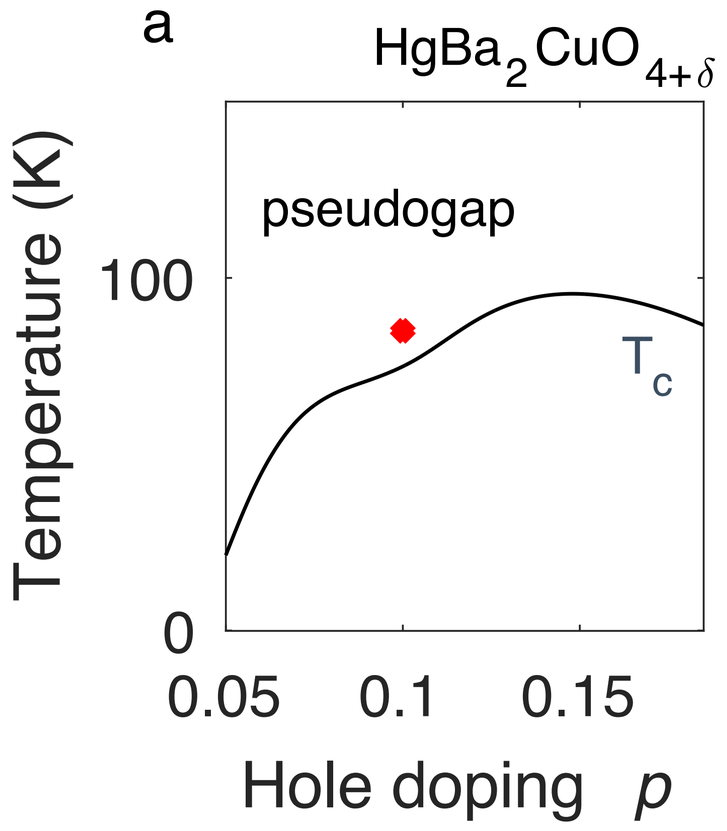

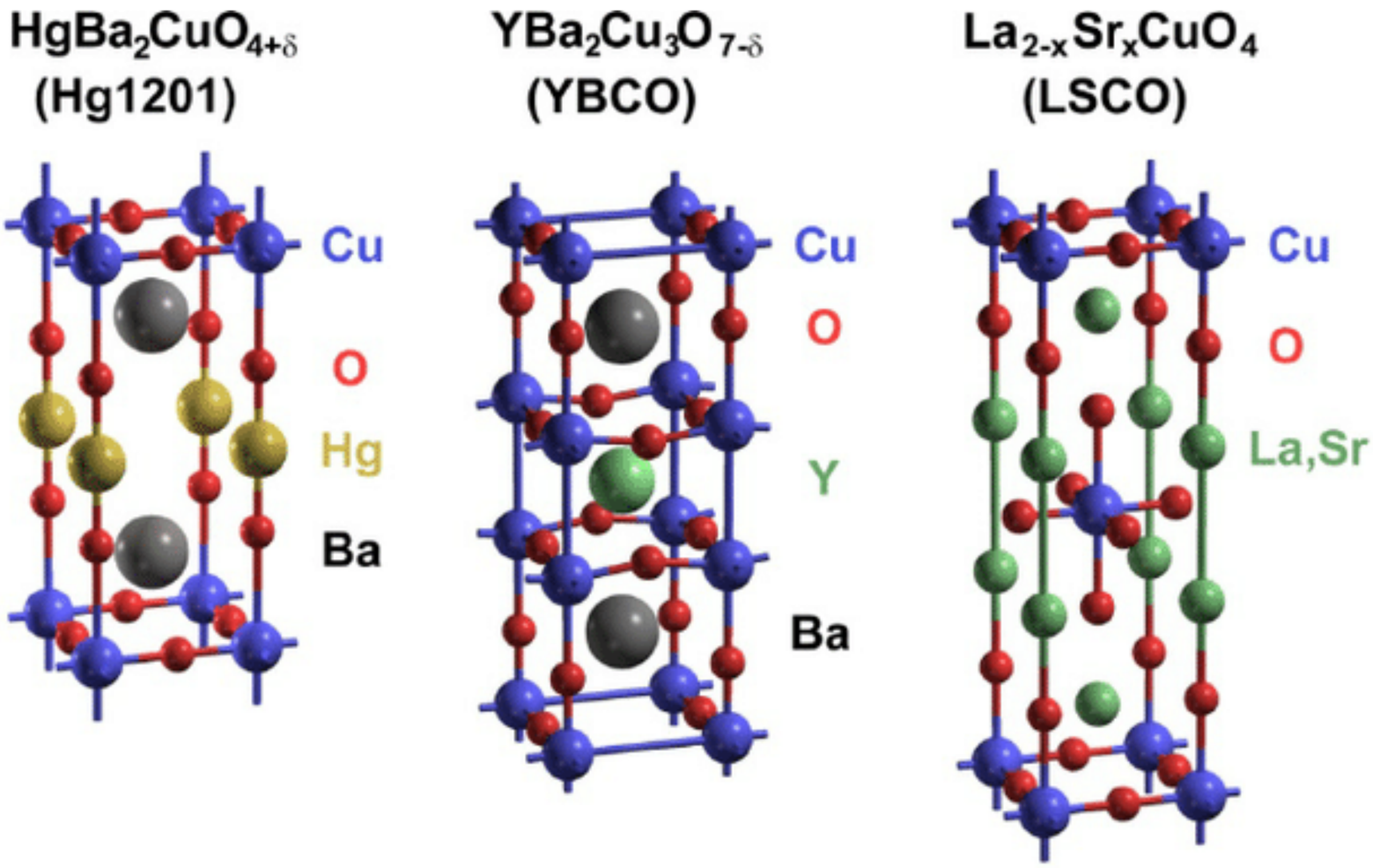

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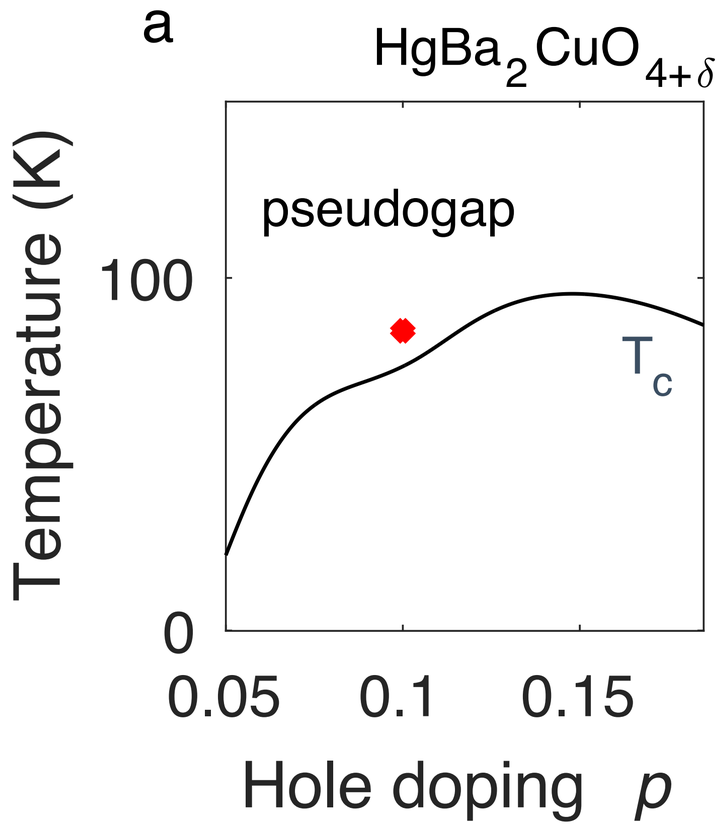

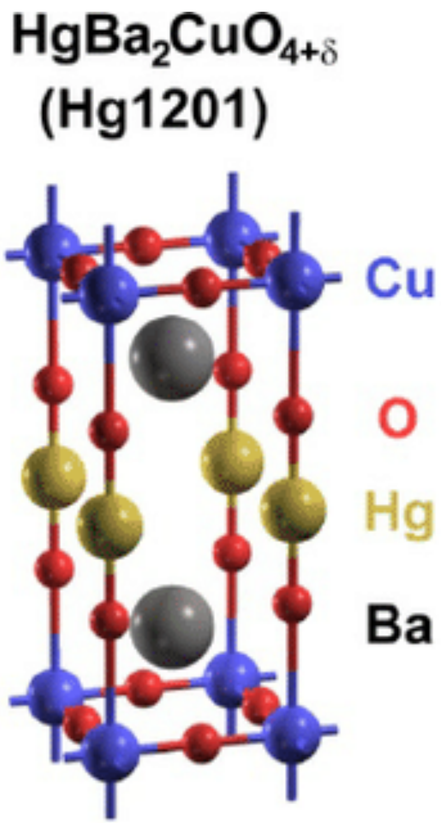

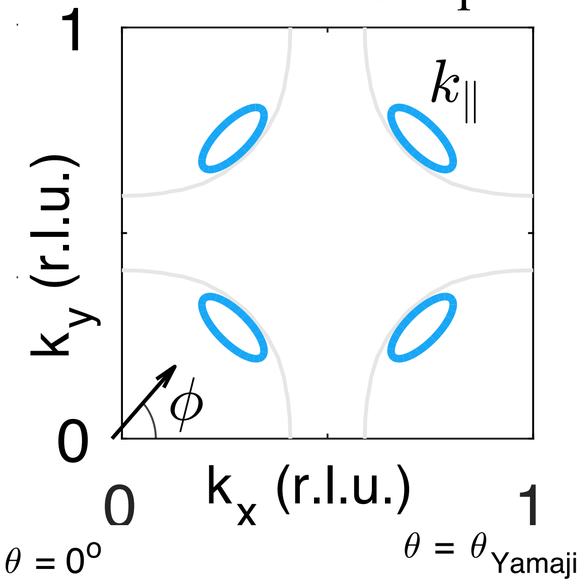

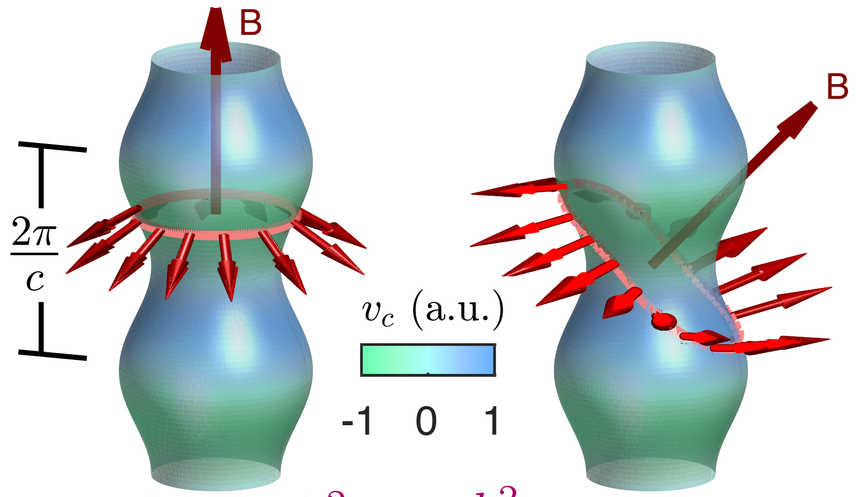

At the Yamaji angle, the orbits in the plane orthogonal to $\boldsymbol{B}$ have an area which is independent of momentum in the c direction, to first order in the hopping along the c direction.

K.Yamaji JPSJ **58**, 1520 (1989)

$$\epsilon(\mathbf{k}) = \frac{k_x^2}{2m_1} + \frac{k_y^2}{2m_2} - 2t_\perp \cos(k_z c)$$

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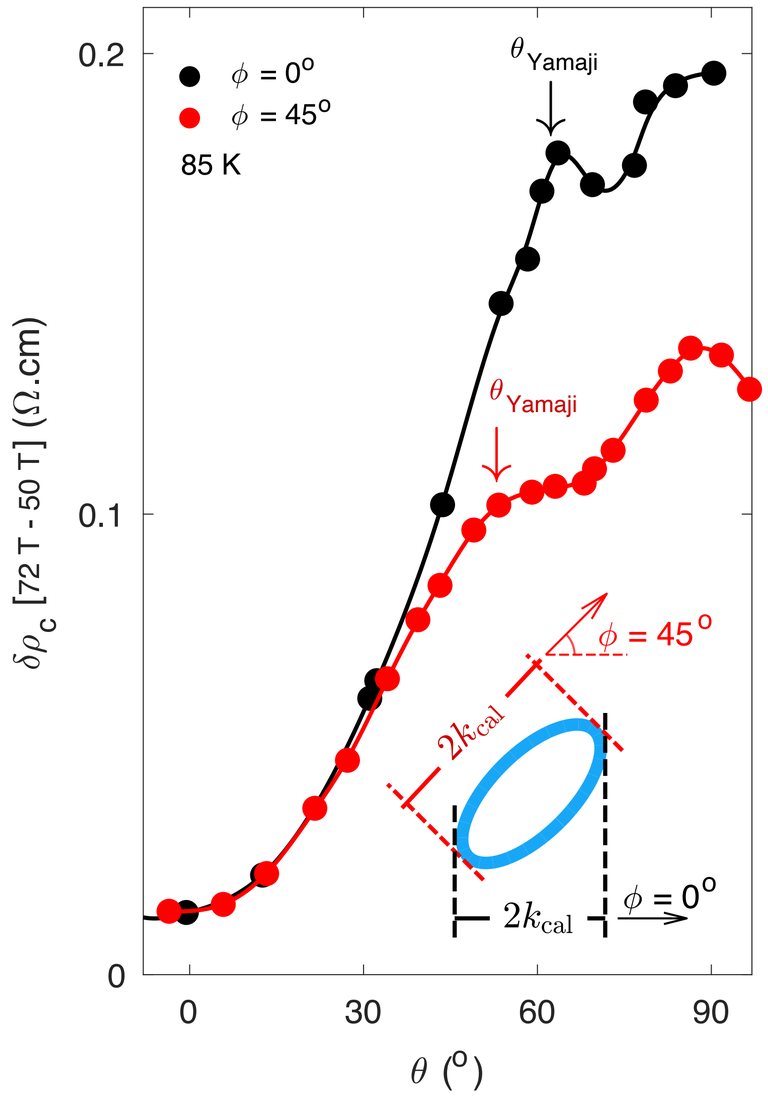

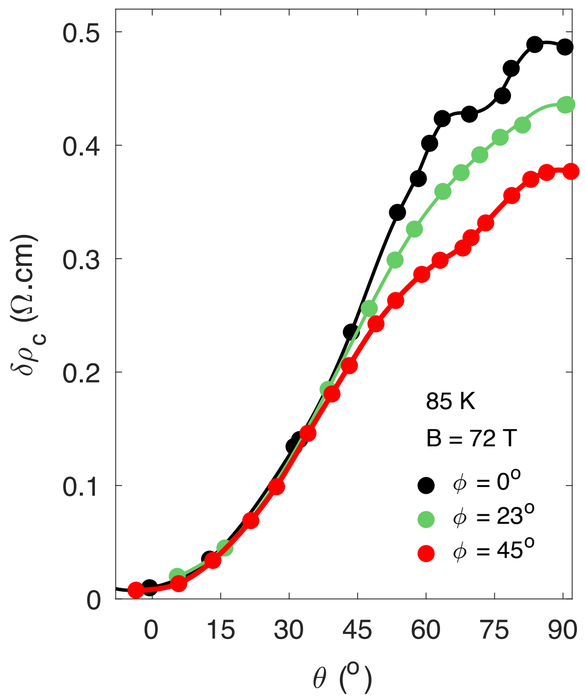

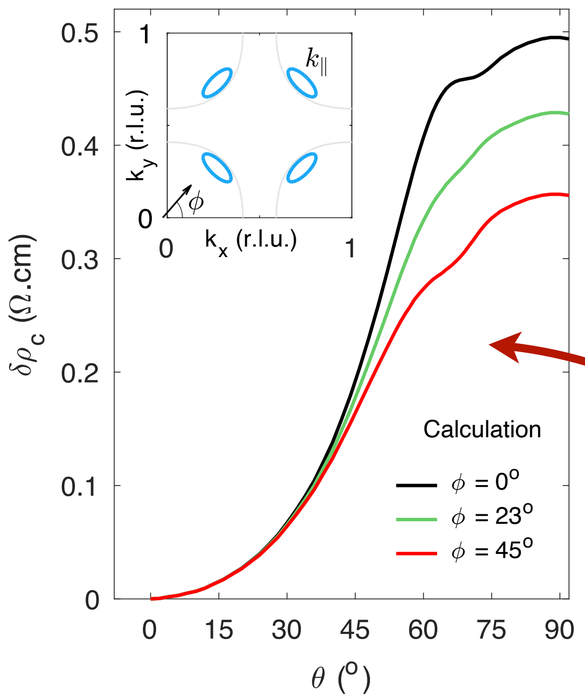

Doping
$p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase.

$$\frac{\partial f}{\partial t} + e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \mathbf{v} \left(-\frac{\partial f}{\partial \epsilon} \right) = -\frac{f - f_0}{\tau}$$

$$\mathbf{v} = \nabla_{\mathbf{k}} \epsilon(\mathbf{k}) \; ; \; f_0(\epsilon) = \frac{1}{e^{(\epsilon - \mu)/T} + 1}$$

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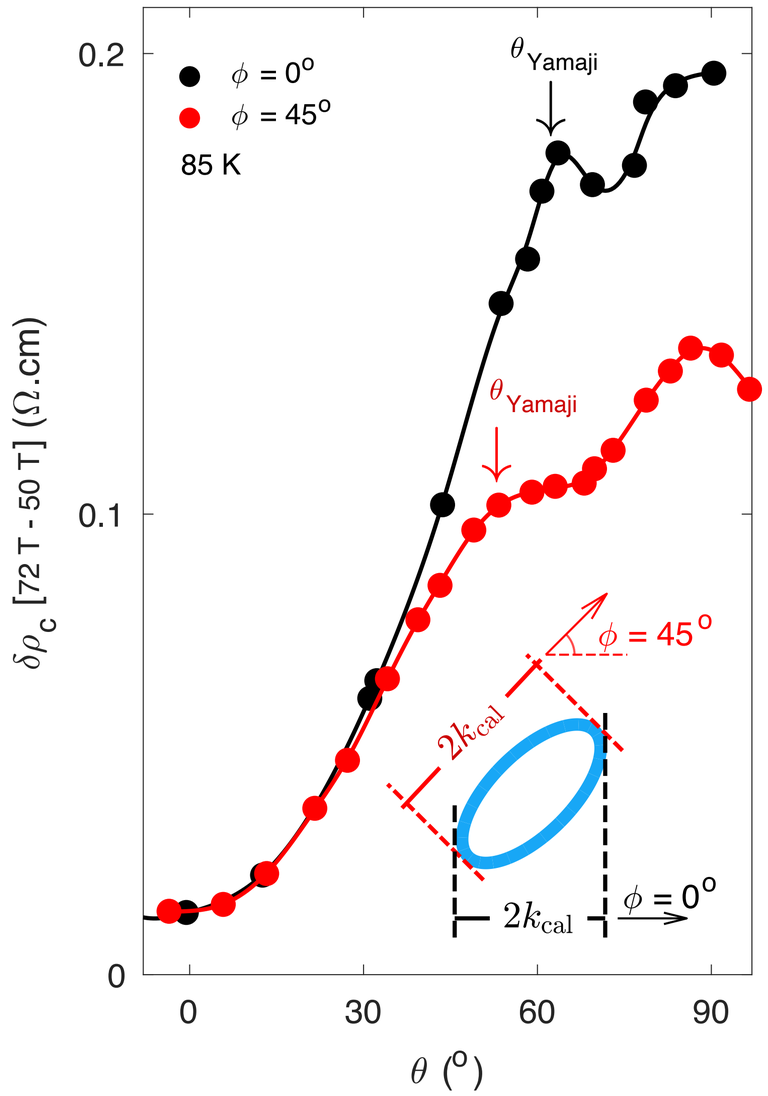

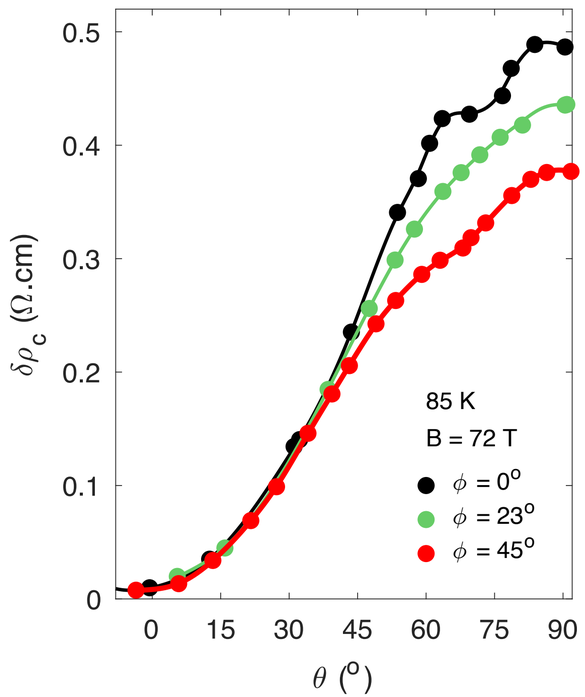

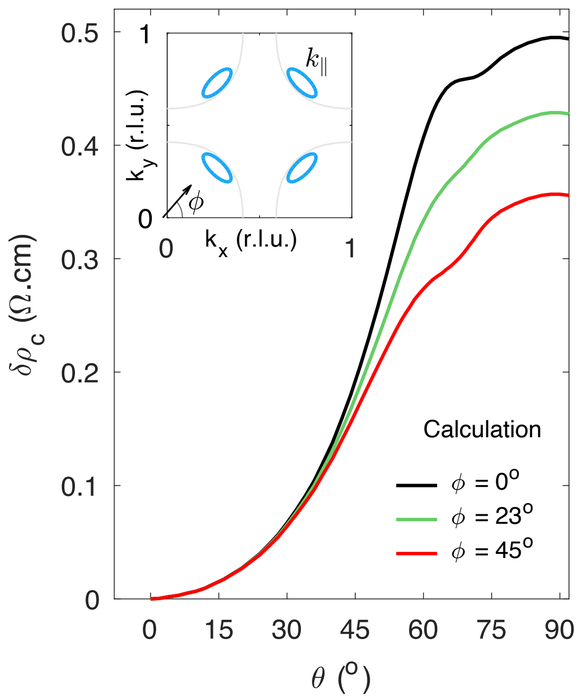

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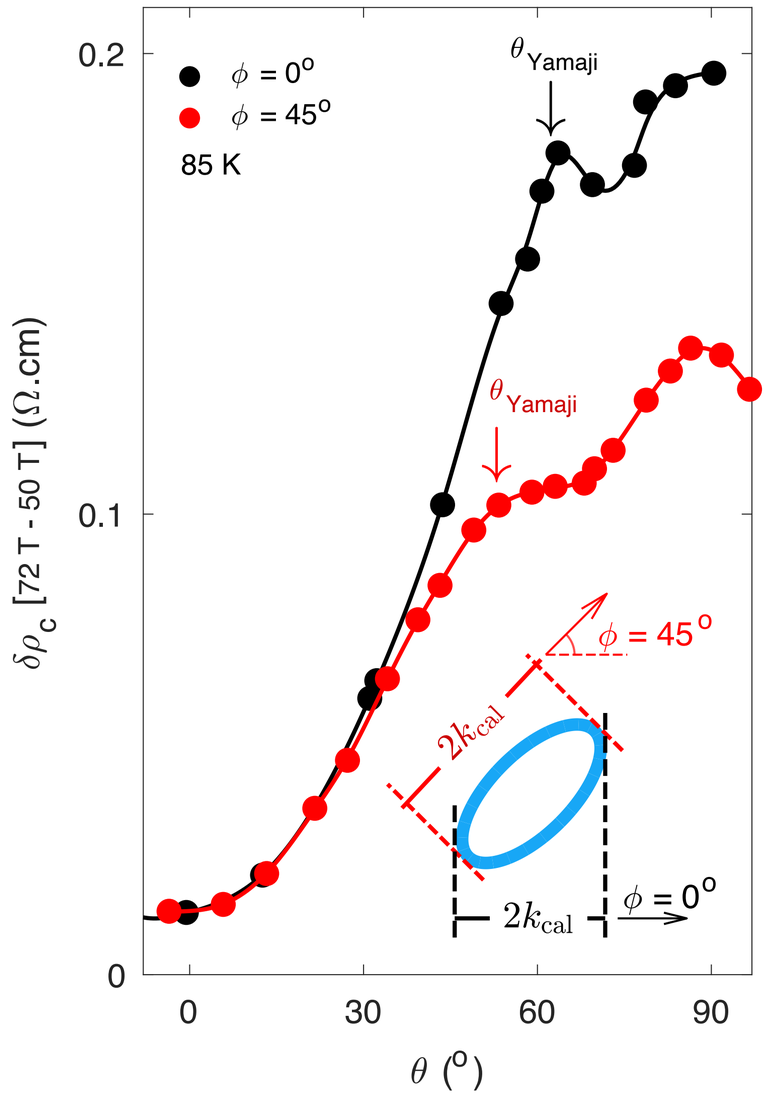

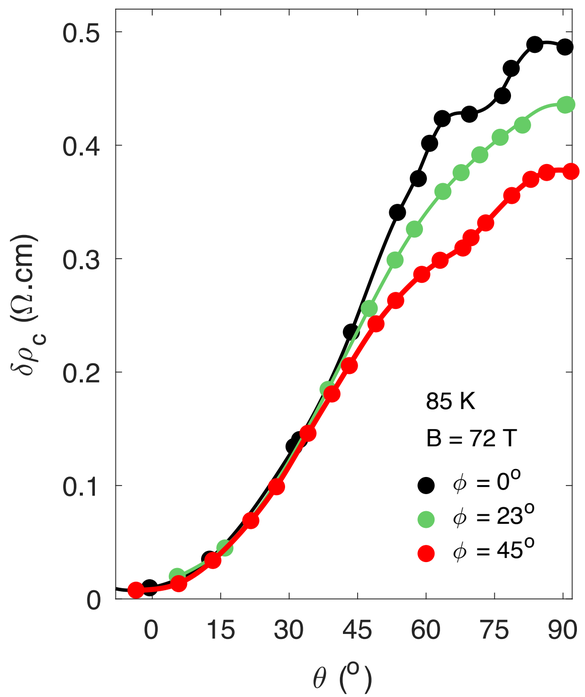

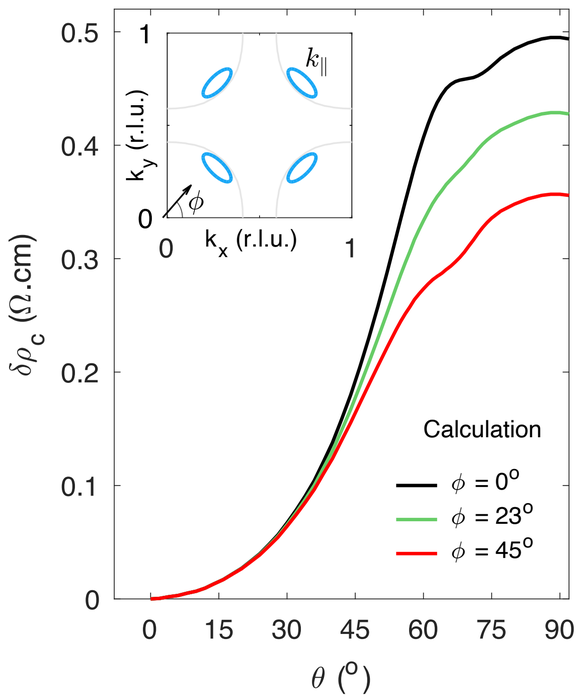

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Rules out holon metal

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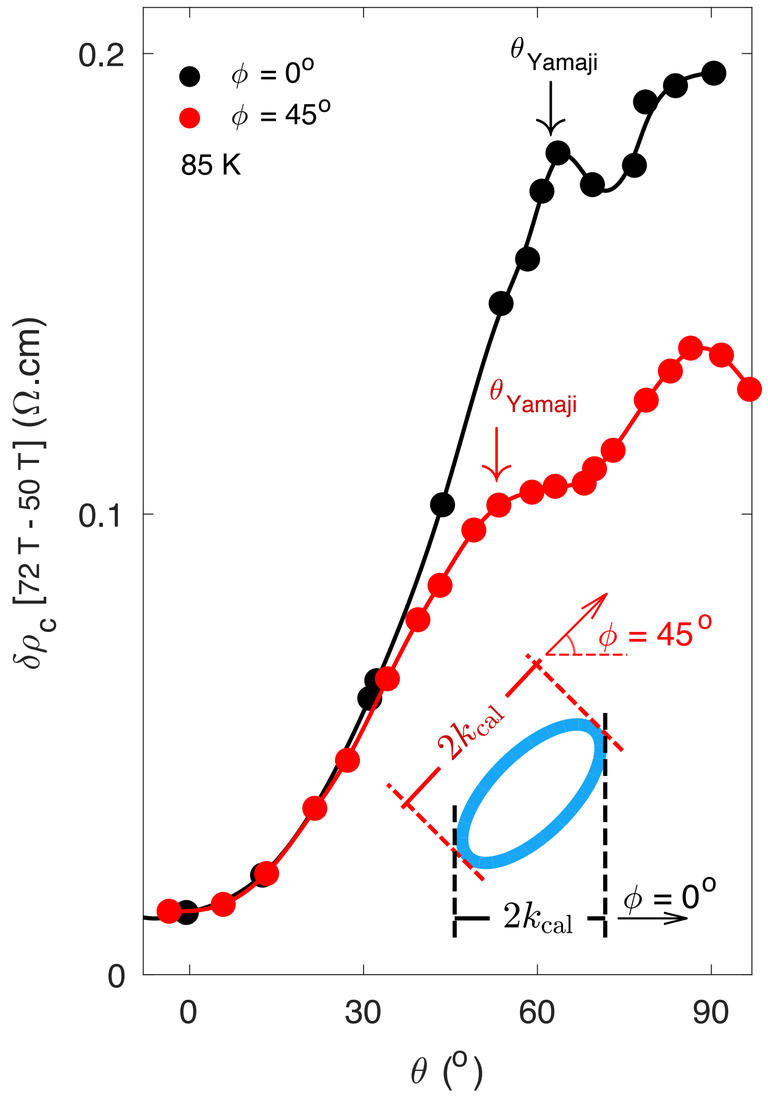

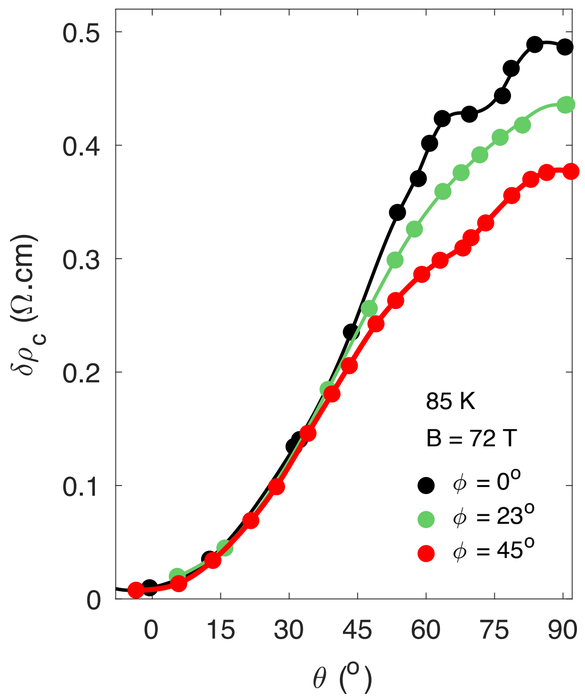

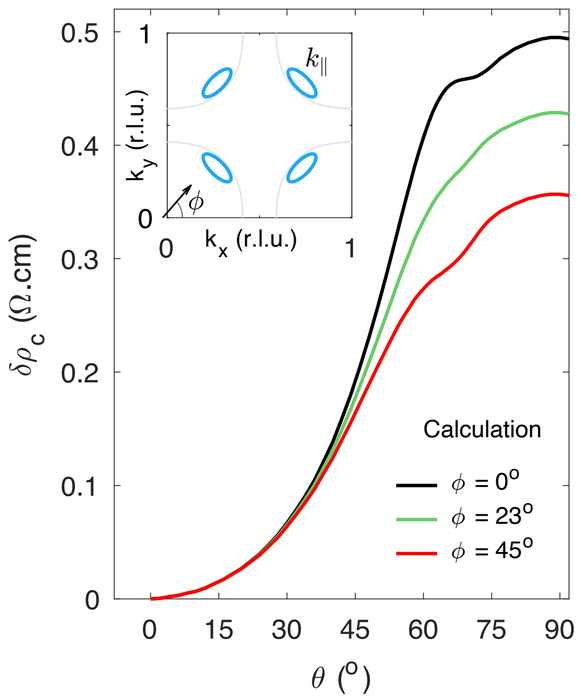

Doping
$p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase.

Excellent evidence for hole pockets with coherent interlayer-transport.
Rules out holon metal
and possibly SDW metal

Observation of the Yamaji effect in a cuprate superconductor

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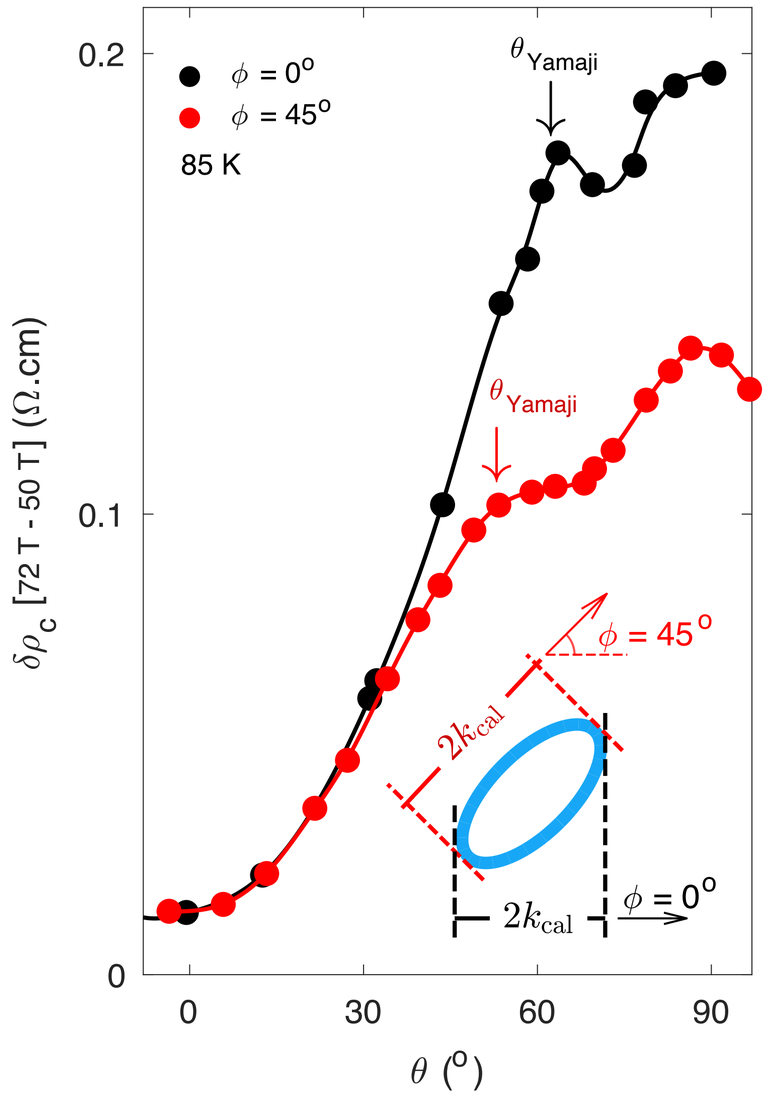

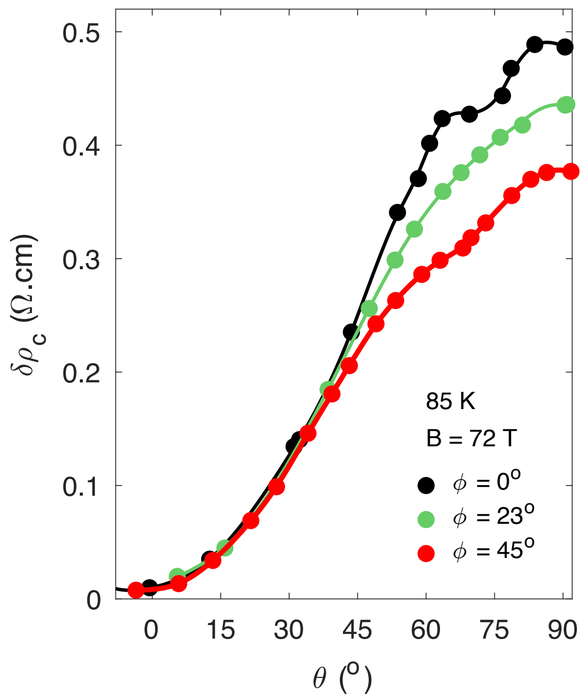

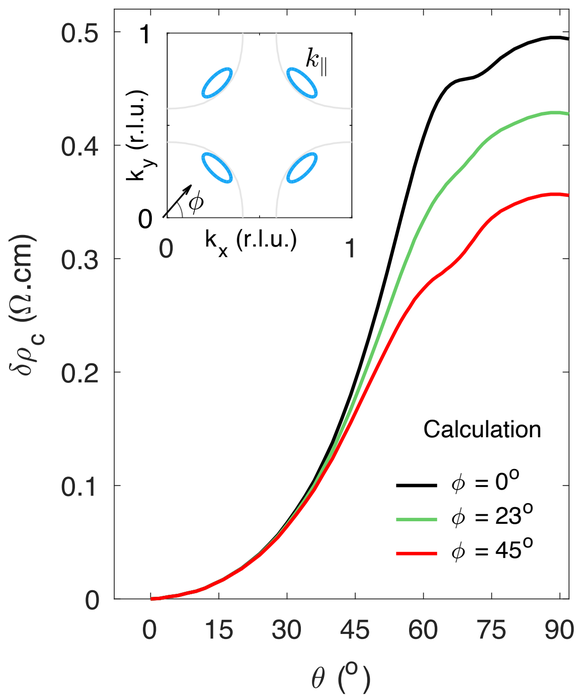

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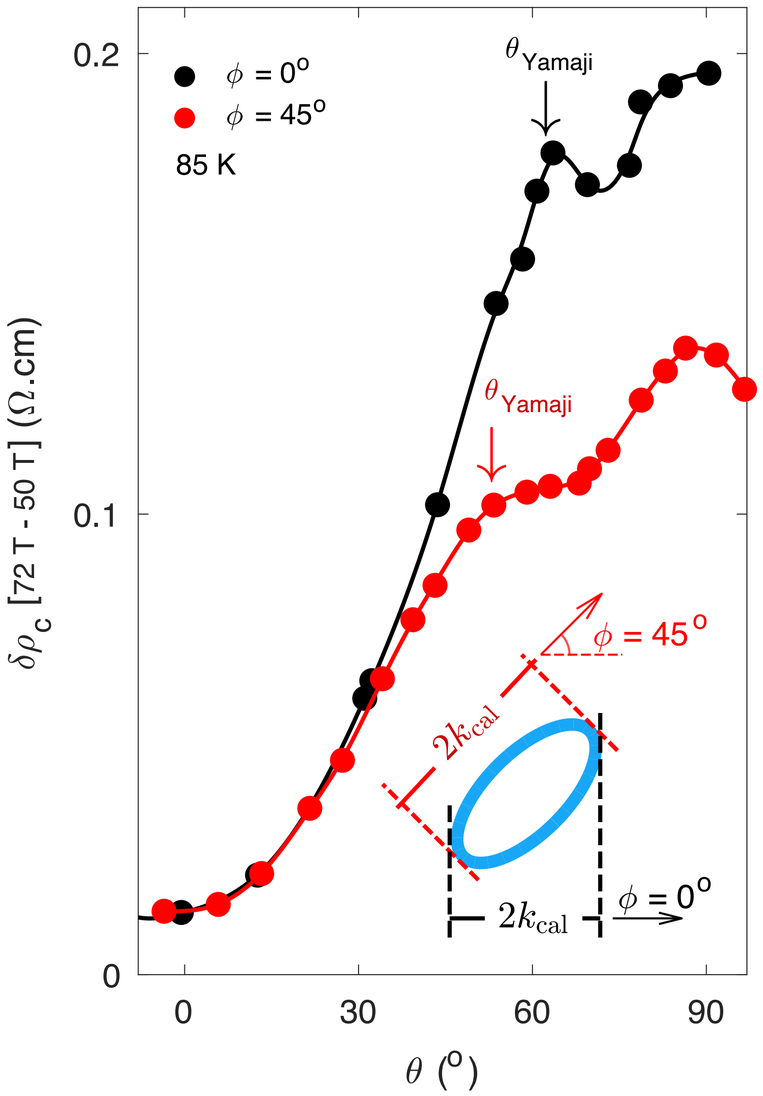

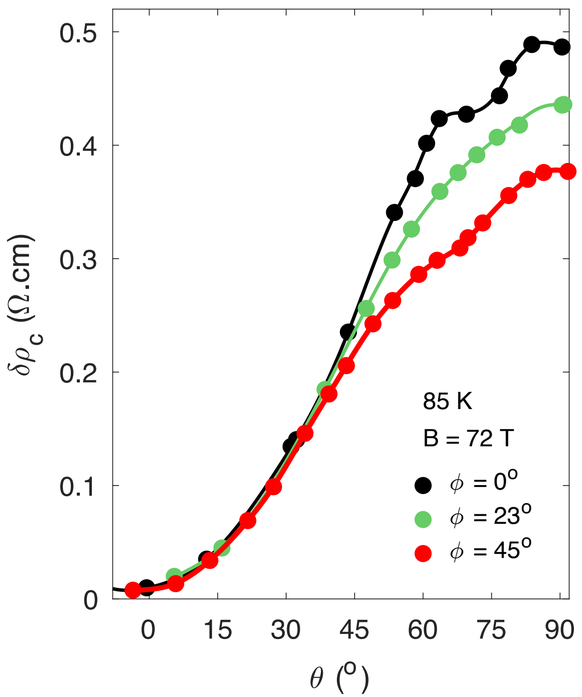

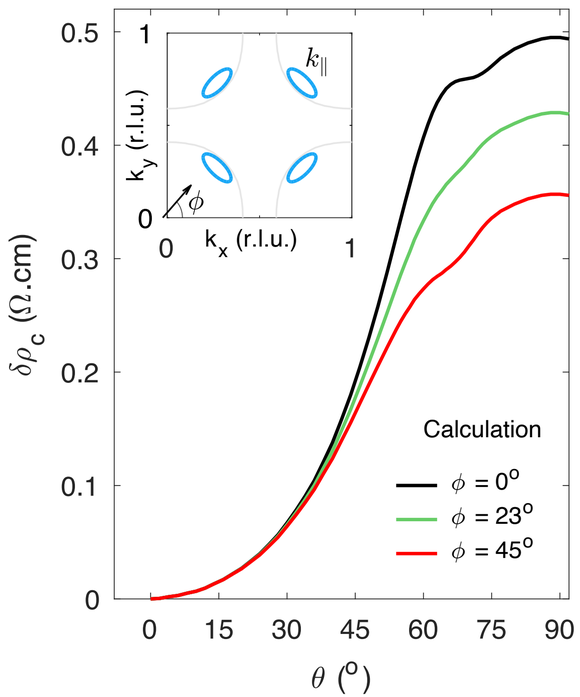

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(was expected by us!)

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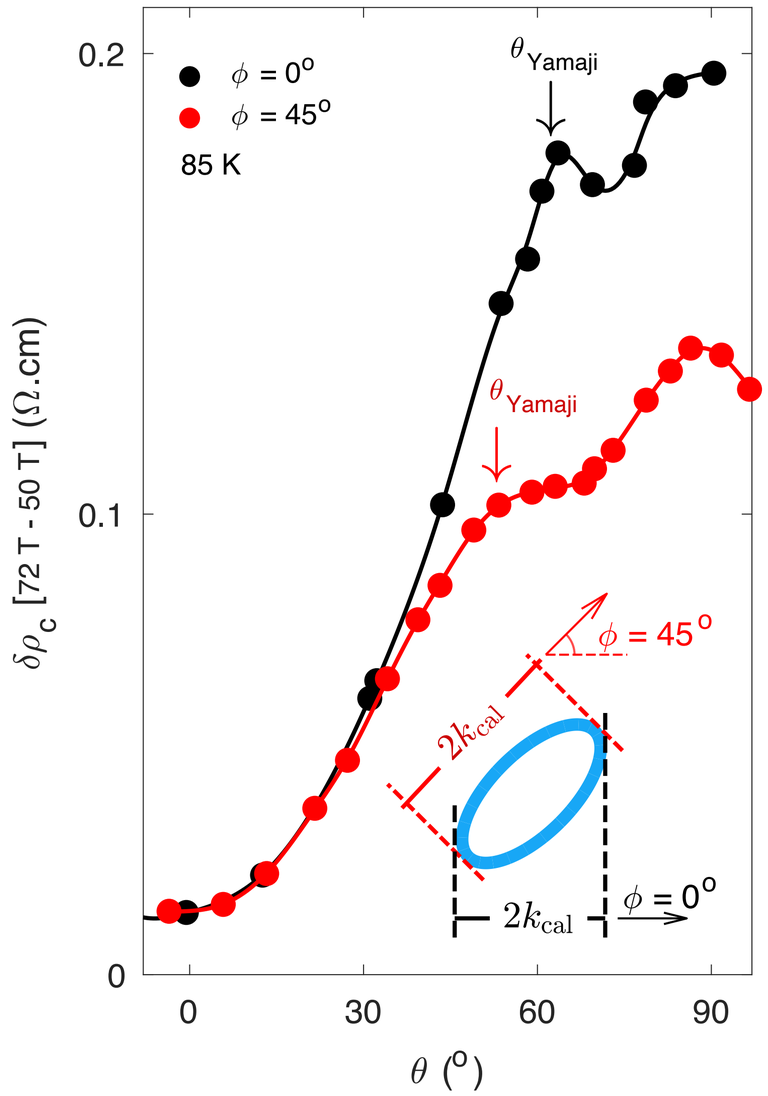

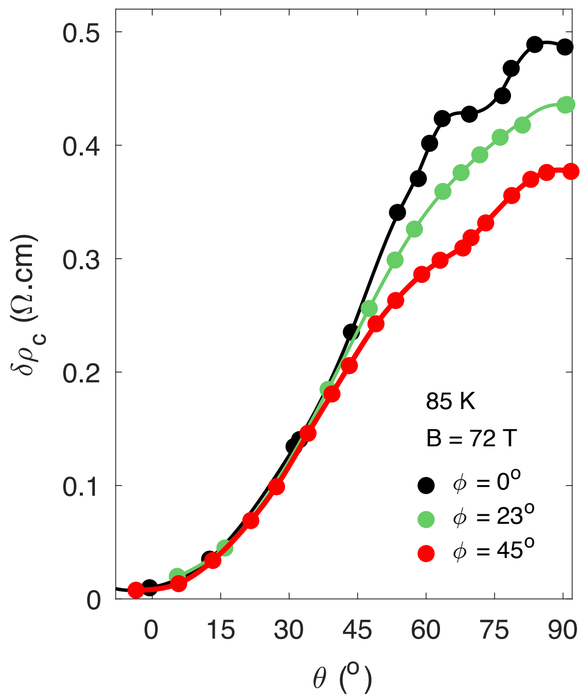

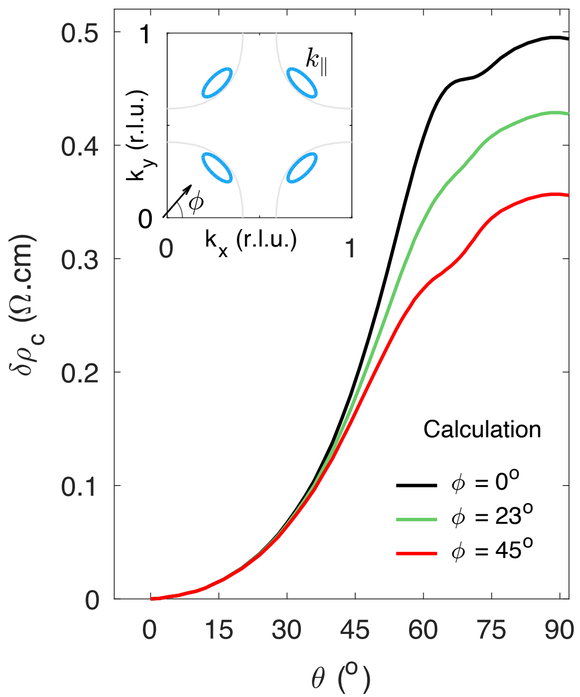

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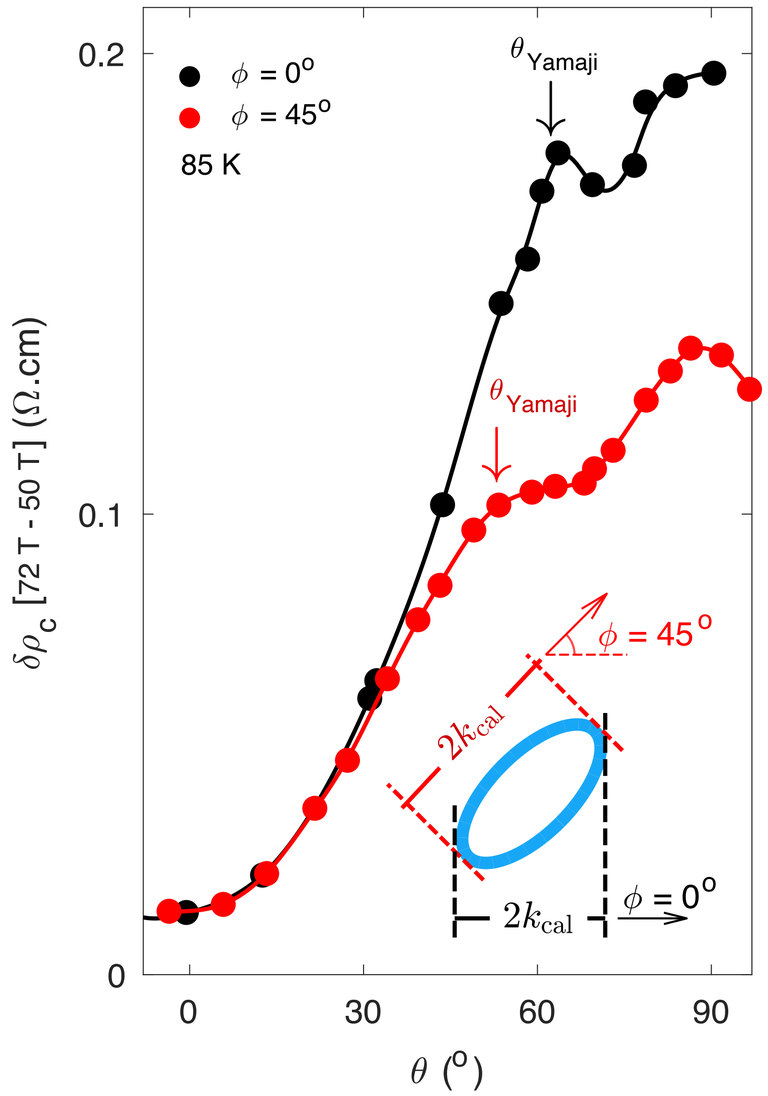

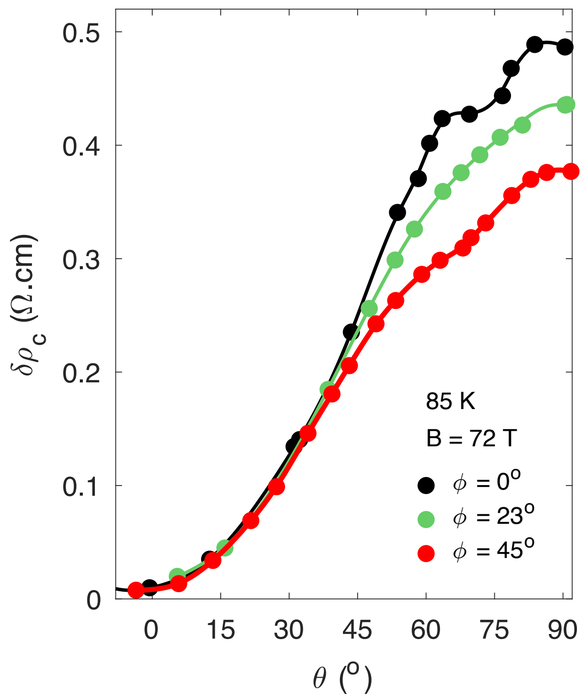

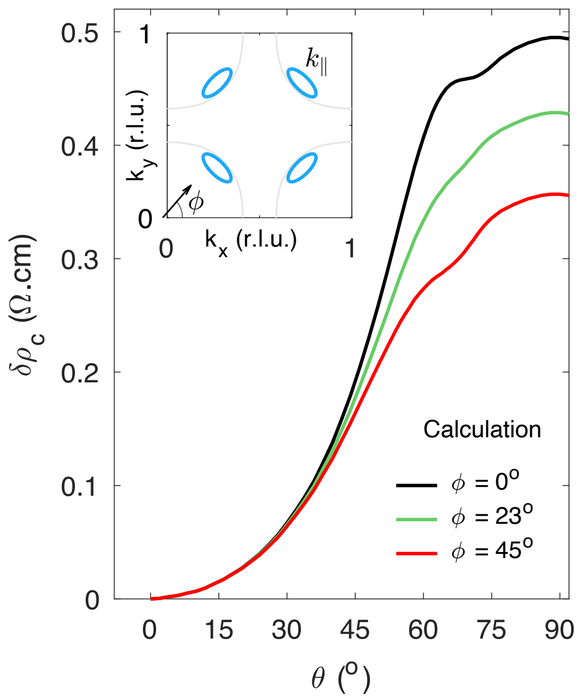

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Predicted FL* pocket fraction $= p/8 = 1.25\%$!

Fluctuating AF metal fraction $= p/4 = 2.5\%$.

($p/8$ also in Yang-Rice-Zjang ansatz, Peter Johnson photoemission, and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar)

Jing-Yu Zhao, S. Chatterjee, S. S., Ya-Hui Zhang, arXiv:2510.13943

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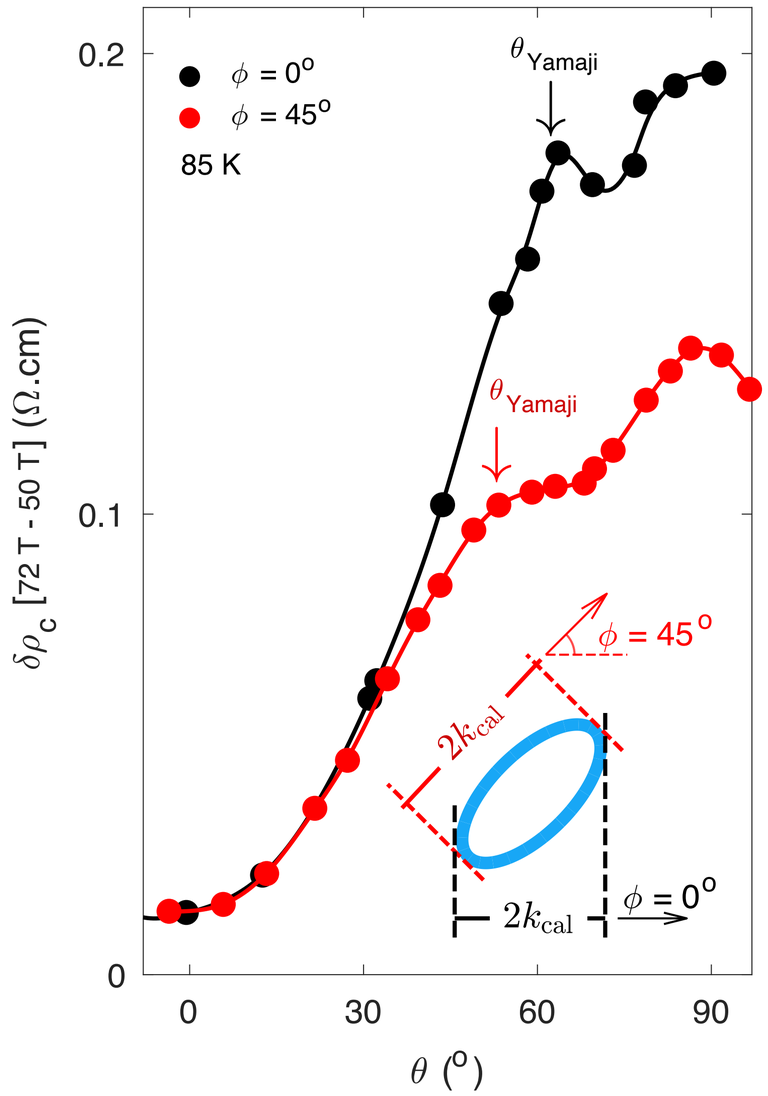

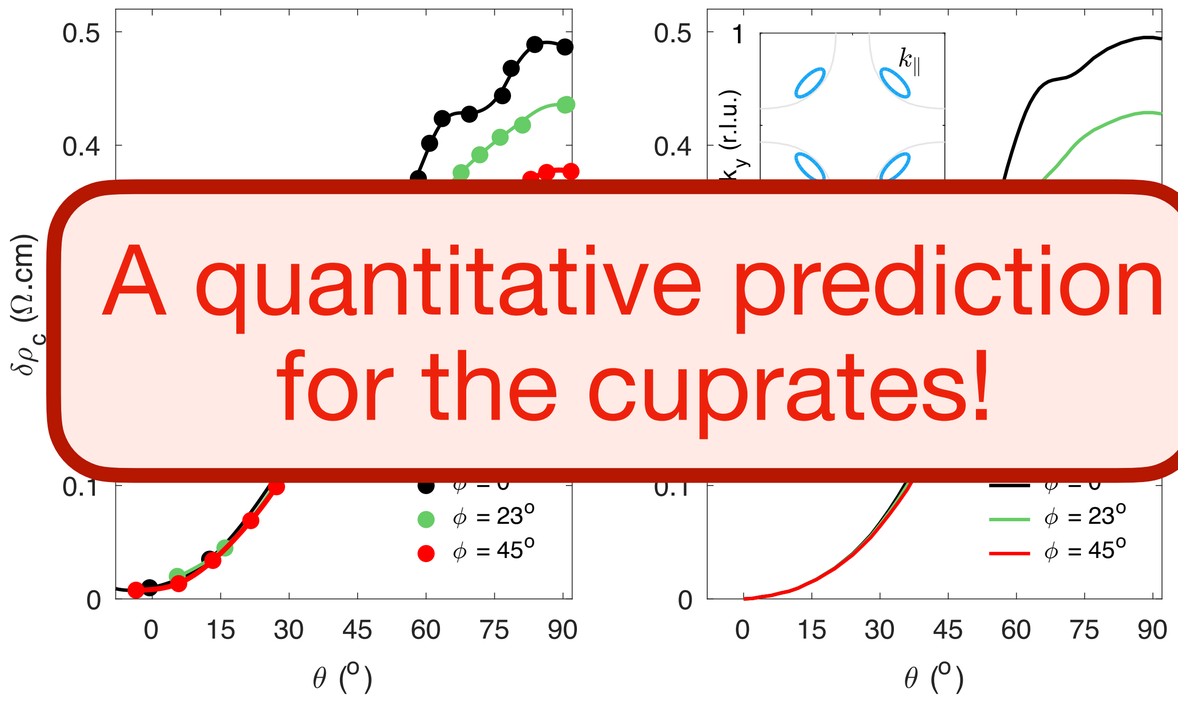

A quantitative prediction for the cuprates!

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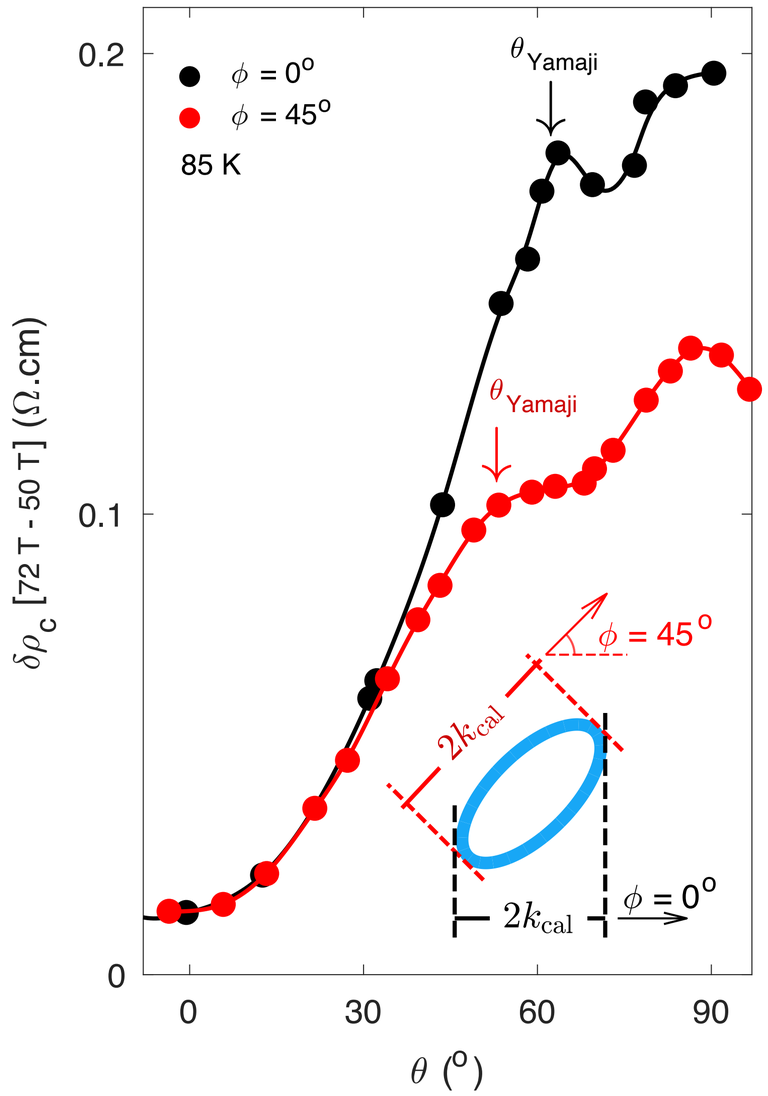

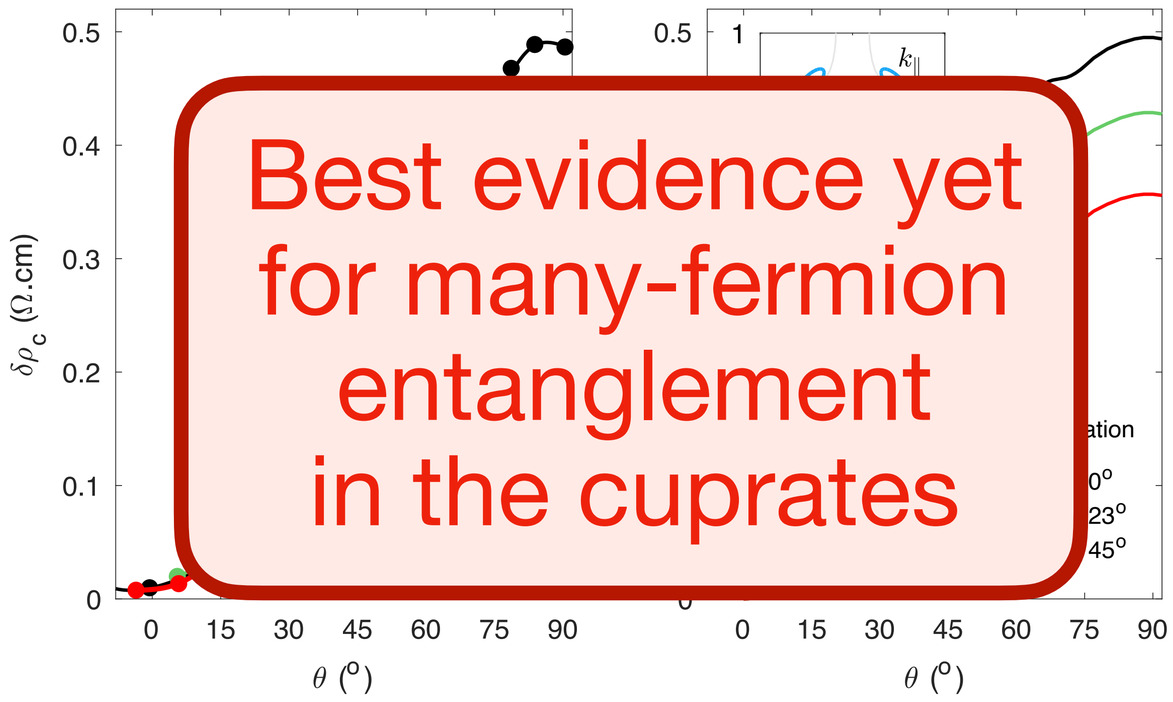

Best evidence yet for many-fermion entanglement in the cuprates

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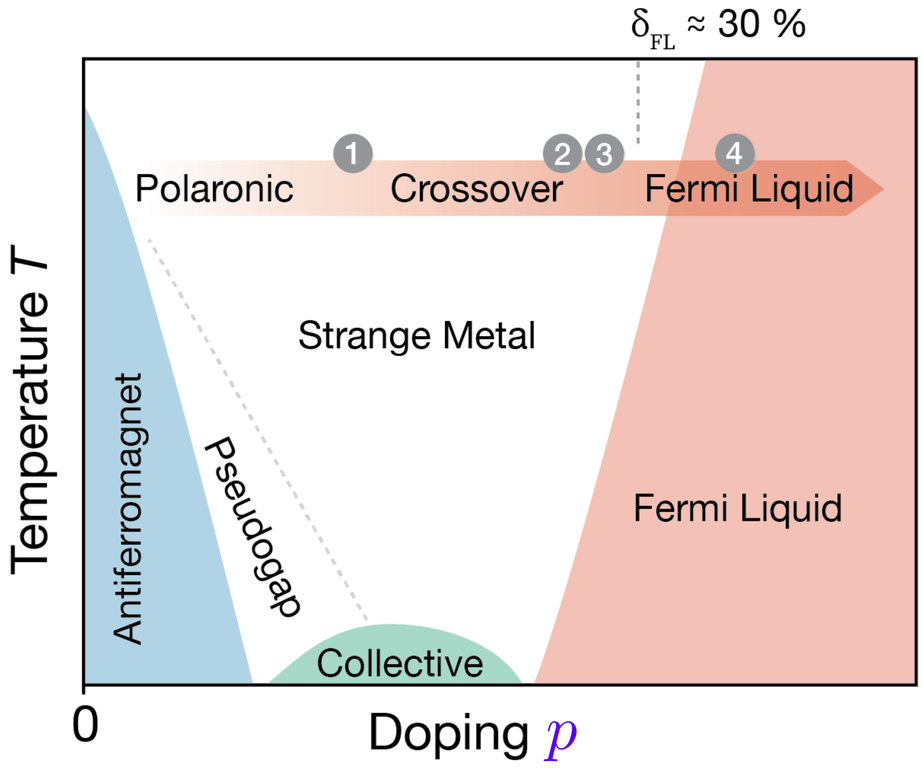
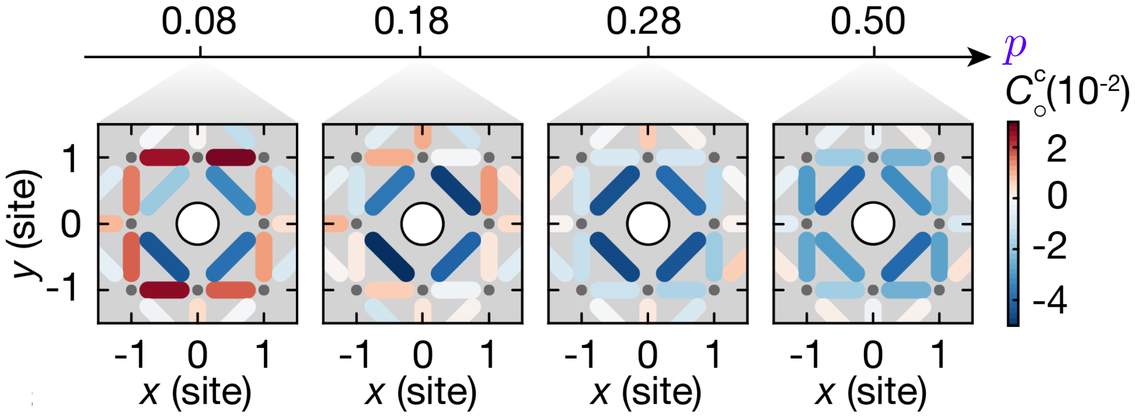
Wavefunction for FL*
and
observations on ultracold atoms





Microscopic evolution of doped Mott insulators from polaronic metal to Fermi liquid

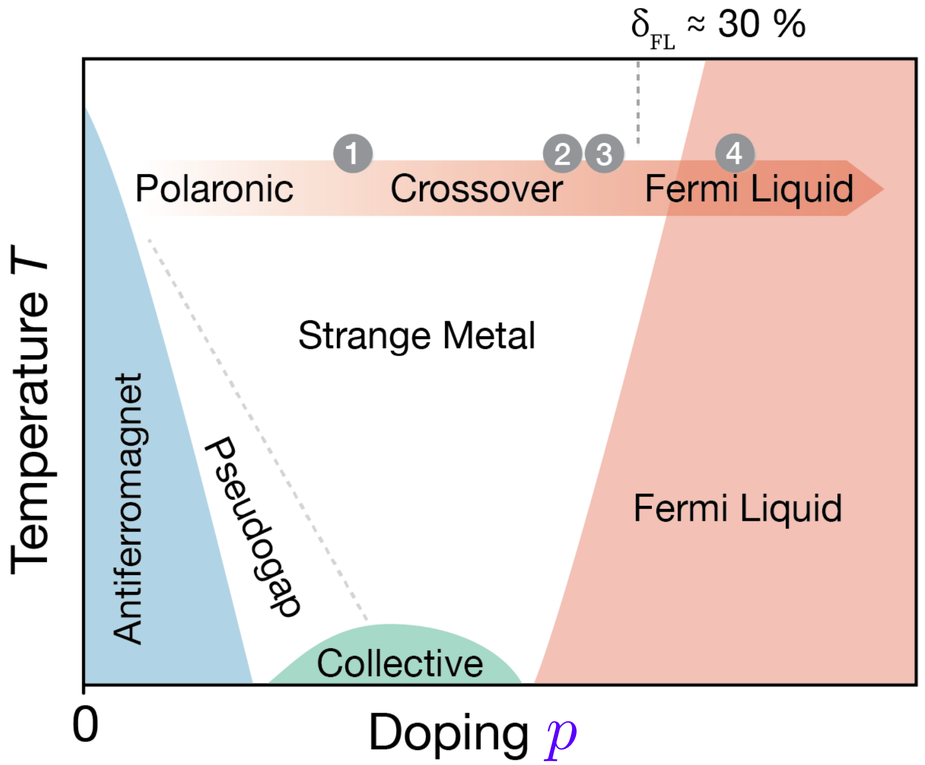
Joannis Koepsell, Dominik Bourgund, Pimonpan Sompet, Sarah Hirthe, Annabelle Bohrdt, Yao Wang, Fabian Grusdt, Eugene Demler, Guillaume Salomon, Christian Gross, Immanuel Bloch

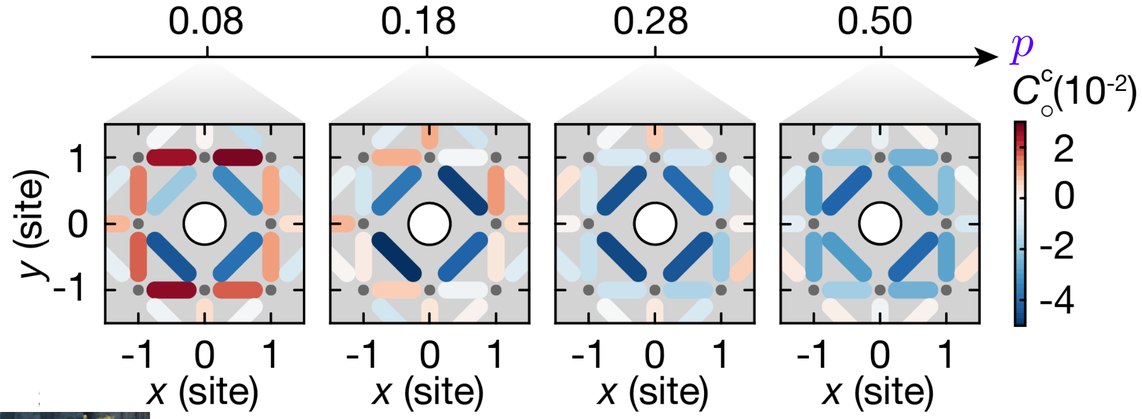
Science **374** (2021) 82

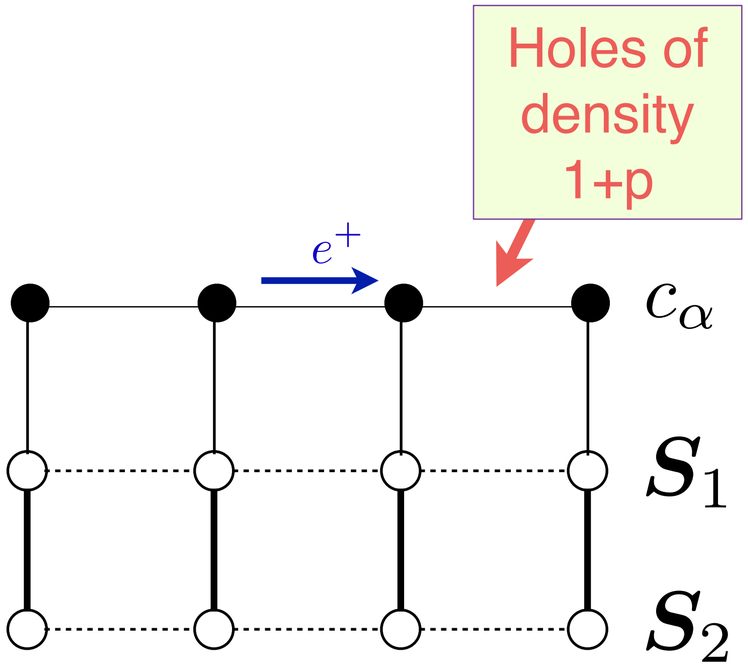
Chalopin….Bloch, PNAS **123**, e2525539123 (2026)

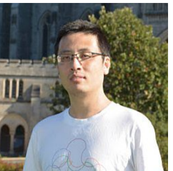
Max Planck Institute of Quantum Optics, Garching

Ancilla Layer Model









Ya-Hui Zhang

Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, S. Sachdev, PRB **105**, 075146 (2022)

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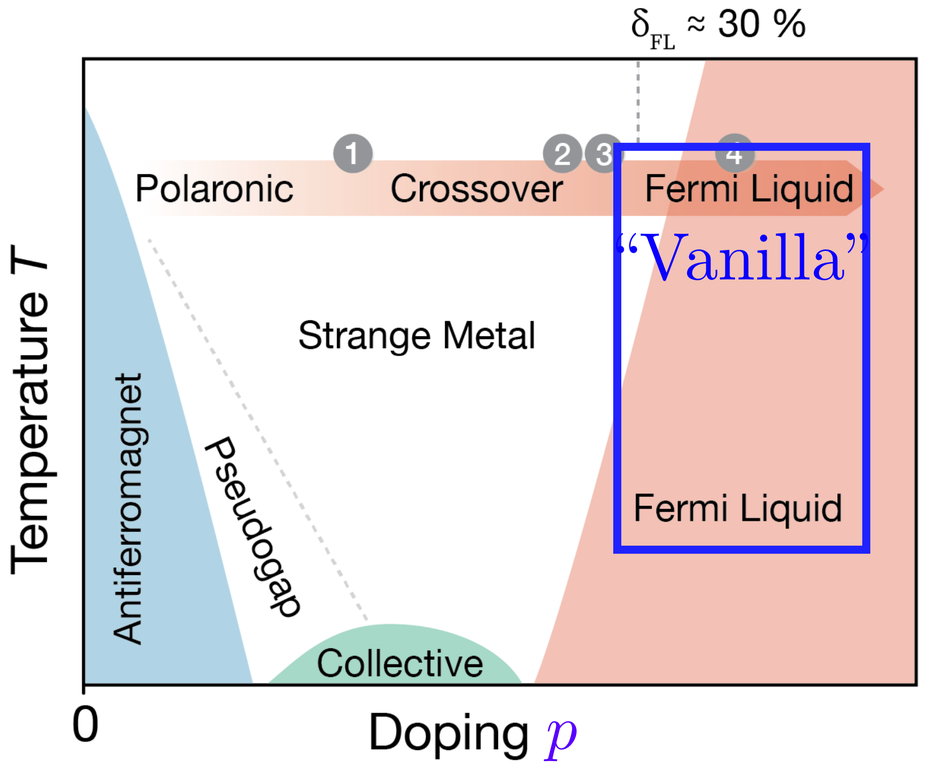
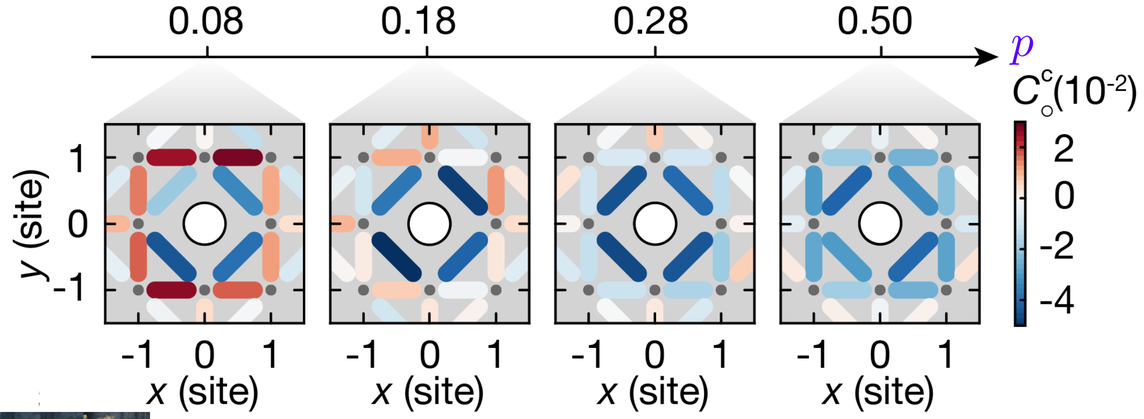
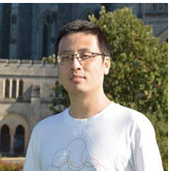


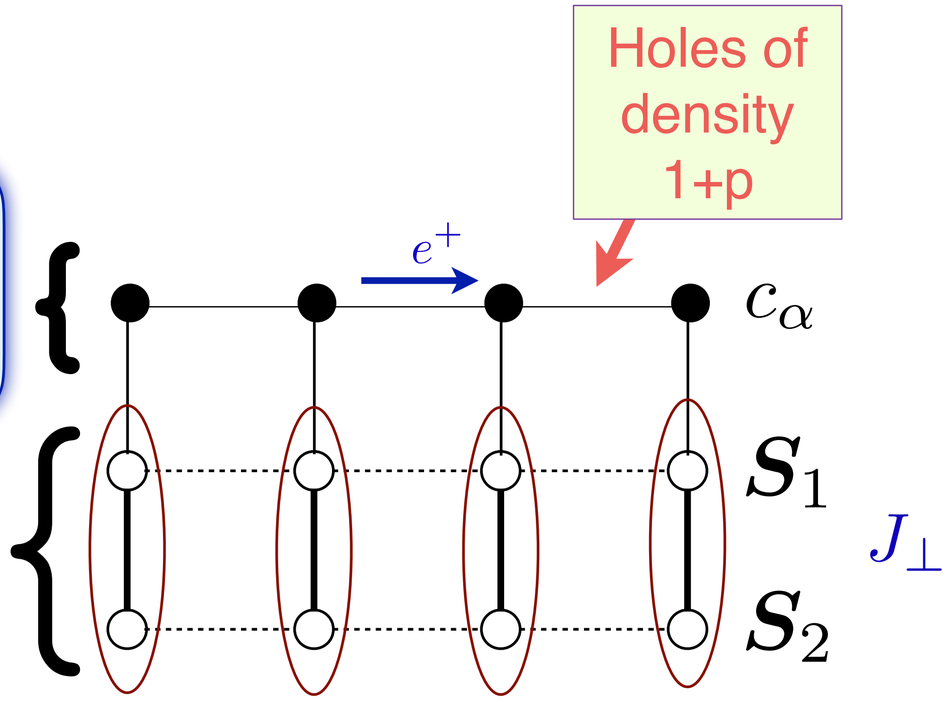




Ya-Hui Zhang

Fermi liquid. Fermi surface encloses area $(1+p)/2$.

Rung singlets of ancilla spins $\boldsymbol{S}_1$, $\boldsymbol{S}_2$.



Fermi liquid = FL of c_α $\oplus$ *Trivial, gapped* state of $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$

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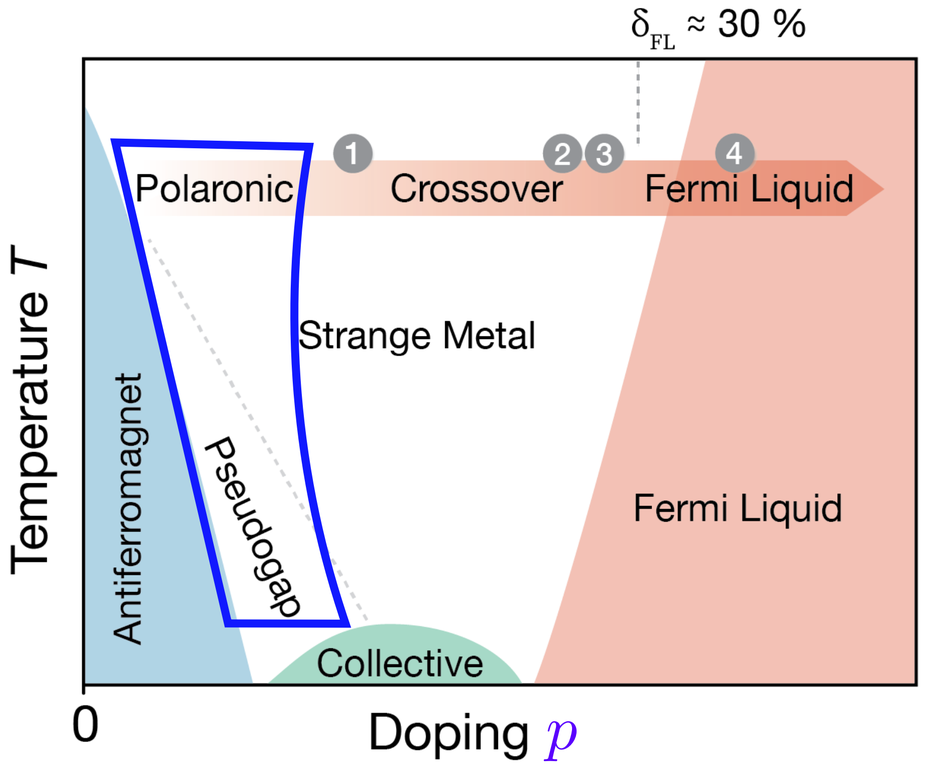
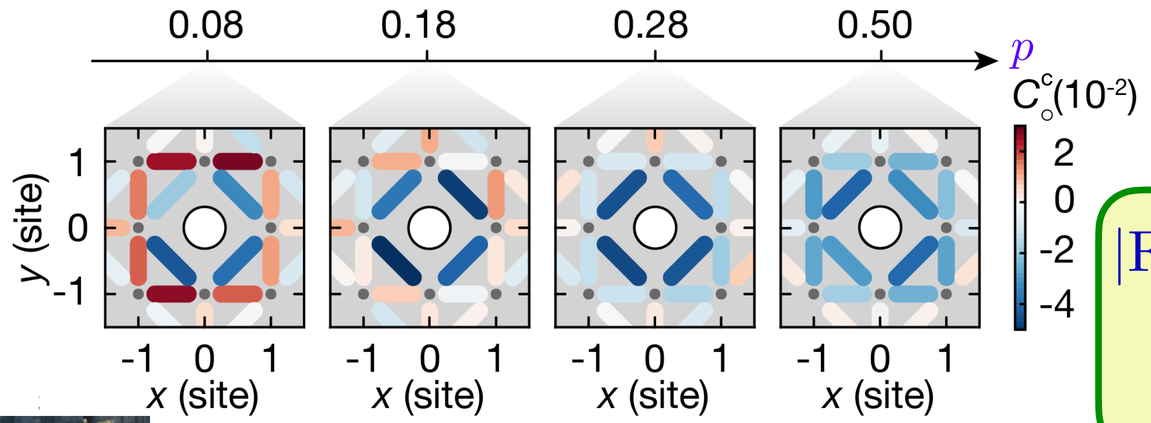
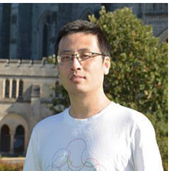
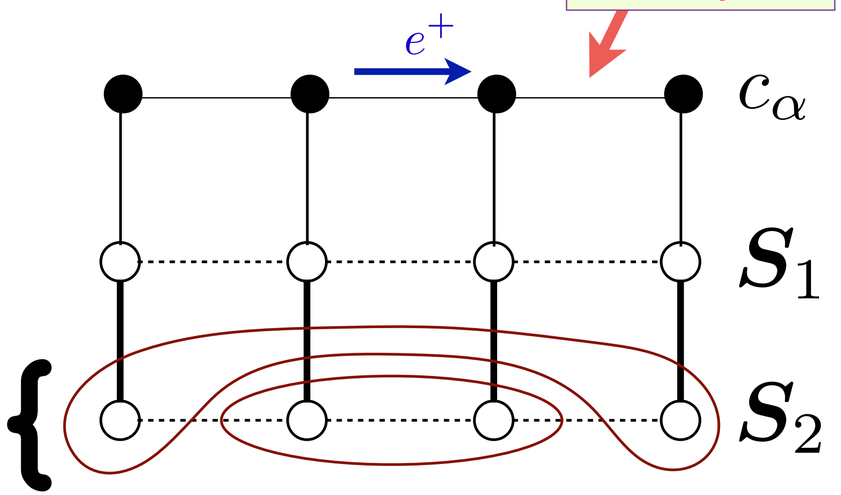






Ya-Hui Zhang

Holes of density 1+p



Your favorite spin liquid

$$S_{2i} = \frac{1}{2} f^\dagger_{i\alpha} \sigma_{\alpha\beta} f_{i\beta}$$

$$|\mathrm{FL}*\rangle = |\text{Slater determinant of } f\rangle$$

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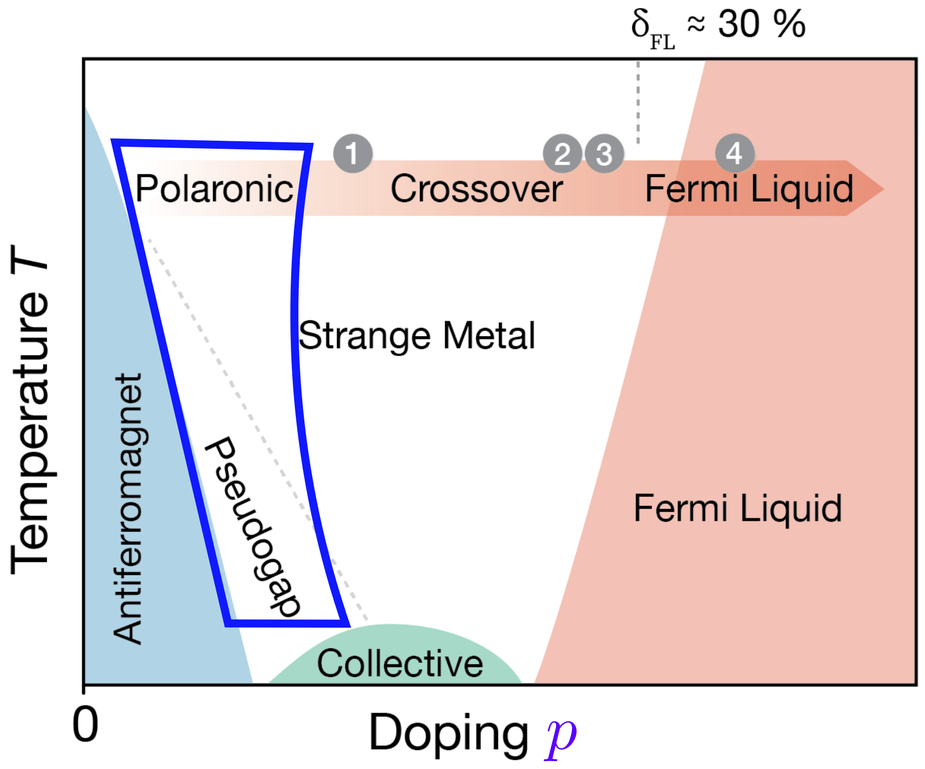
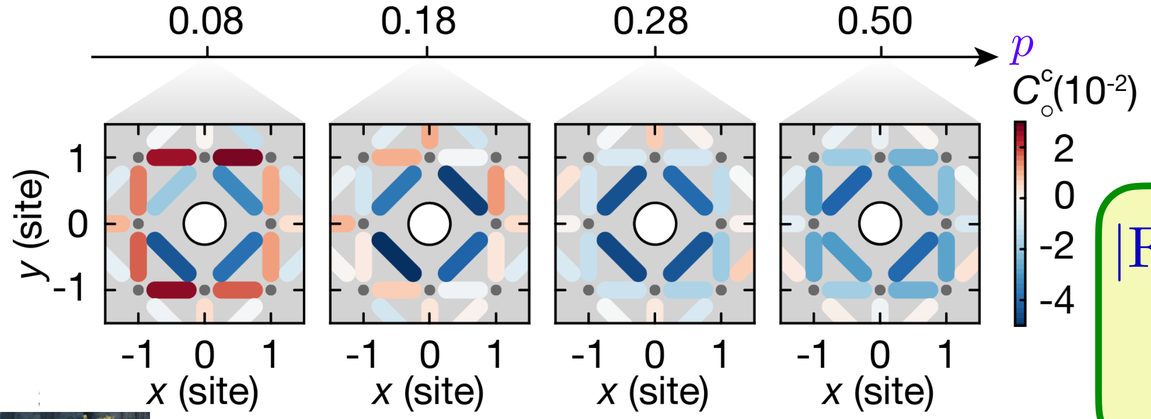
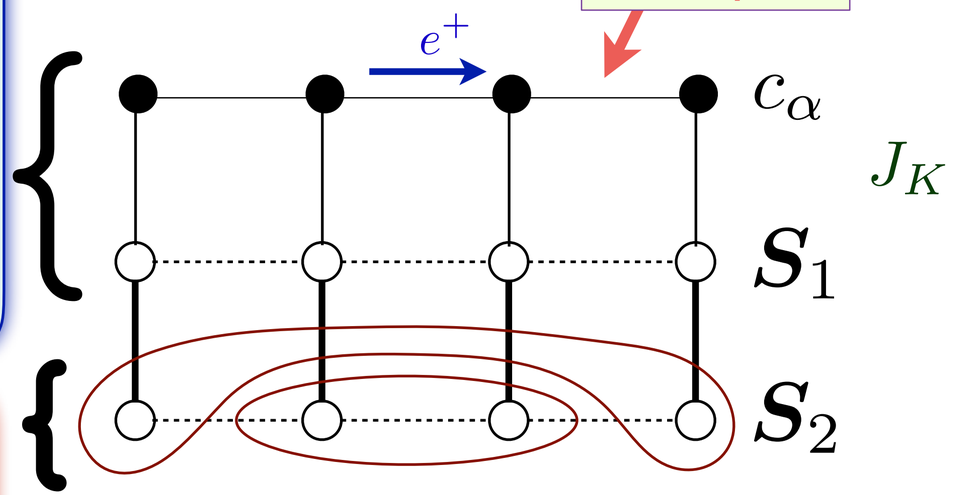




Kondo lattice heavy Fermi liquid. Hybridization $\Phi \sim f_{1\alpha}^\dagger c_\alpha$. Fermi surfaces enclose area $(1+p+1)/2 = p/2 \pmod 1$.

Holes of density 1+p



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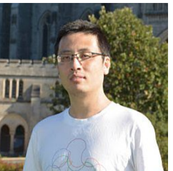

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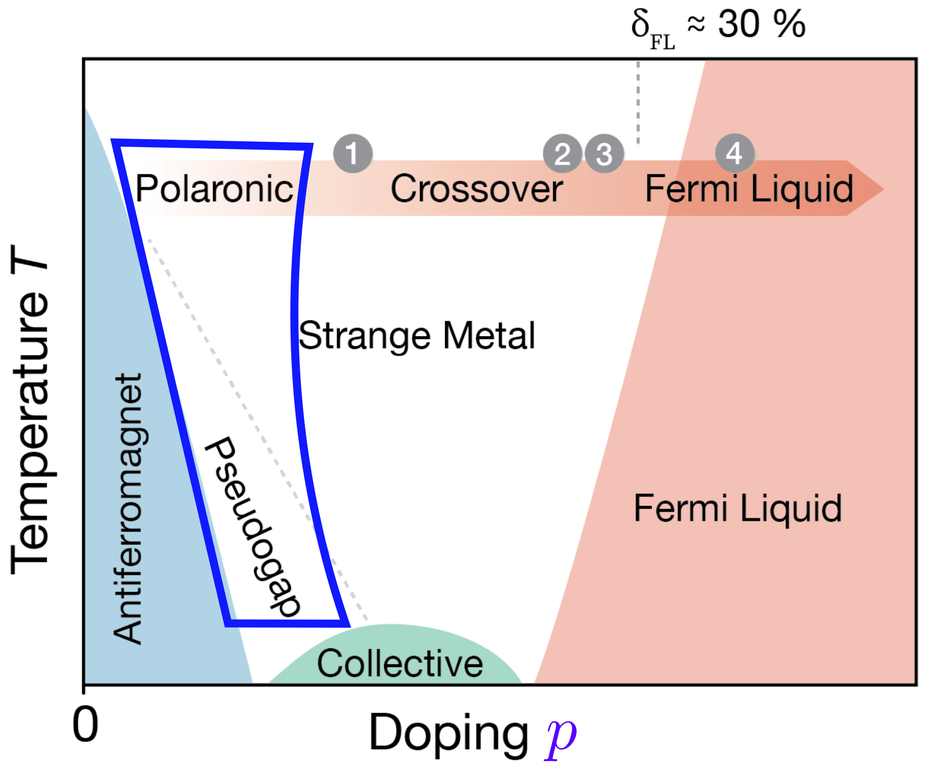

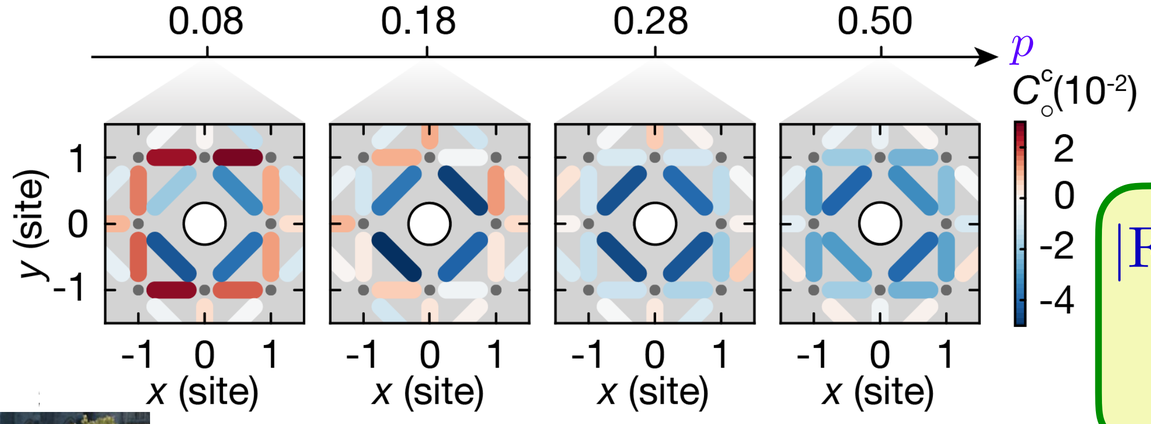

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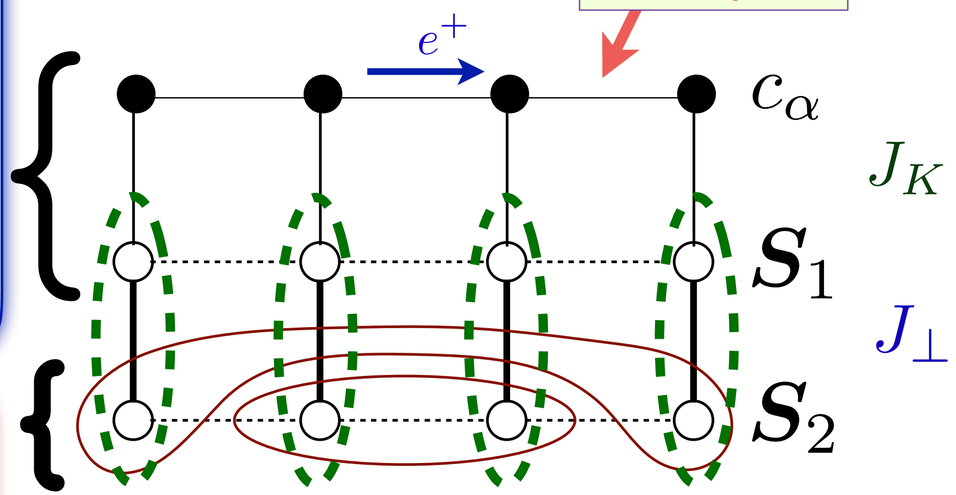

Your favorite spin liquid

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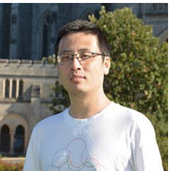
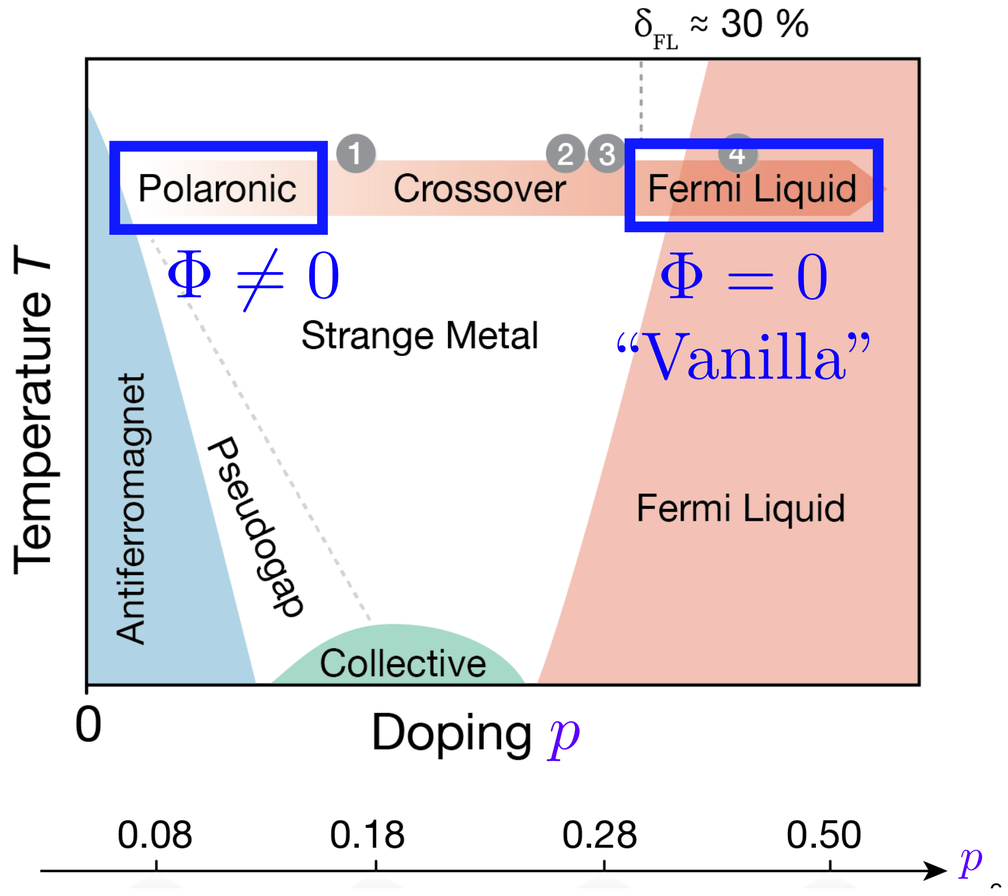
$$|\mathrm{FL}*\rangle_{\mathrm{Hubbard}} = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2] \bowtie |\text{Slater determinant of } (c, f_1)\rangle \otimes |\text{Slater determinant of } f\rangle$$



Ya-Hui Zhang

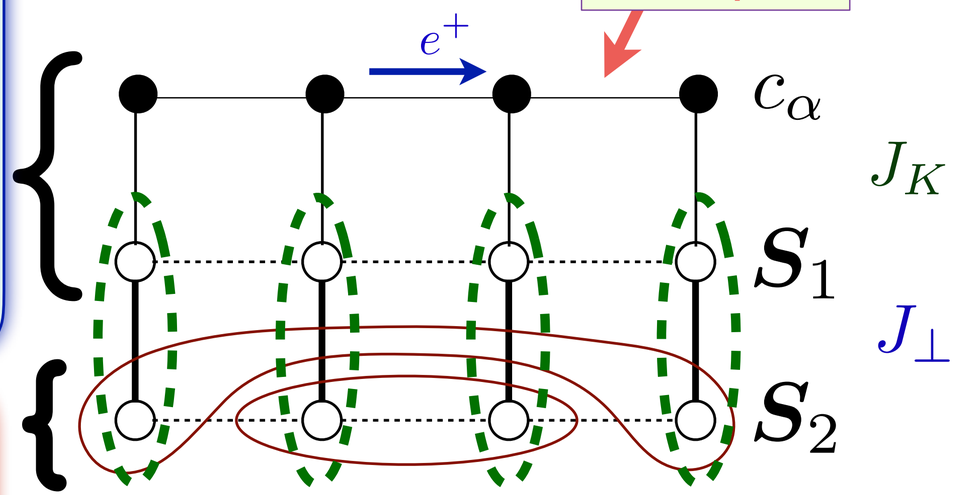
Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, S. Sachdev, PRB **105**, 075146 (2022)

Ancilla Layer Model



Kondo lattice heavy Fermi liquid. Hybridization $\Phi \sim f_{1\alpha}^\dagger c_\alpha$. Fermi surfaces enclose area $(1+p+1)/2 = p/2 \pmod 1$.

Holes of density 1+p

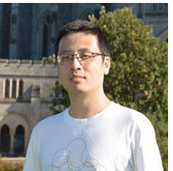


Your favorite spin liquid

$$\boldsymbol{S}_{1i} = \frac{1}{2} f_{1i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{1i\beta}, \quad \boldsymbol{S}_{2i} = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta}$$

$\Phi \Rightarrow$ Higgs order parameter for FL*-FL transition!

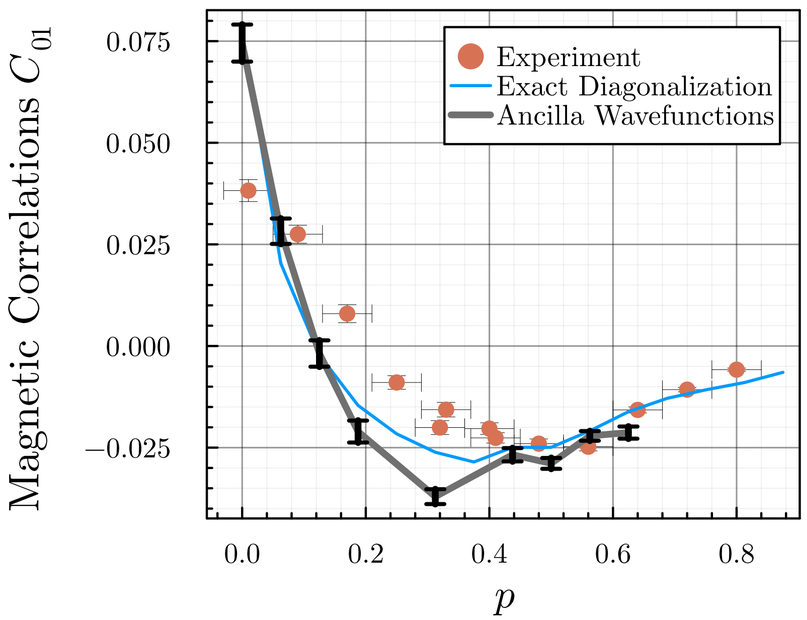
$$|\mathrm{FL}*\rangle_{\mathrm{Hubbard}} = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2] \bowtie |\text{Slater determinant of } (c, f_1)\rangle \otimes |\text{Slater determinant of } f\rangle$$

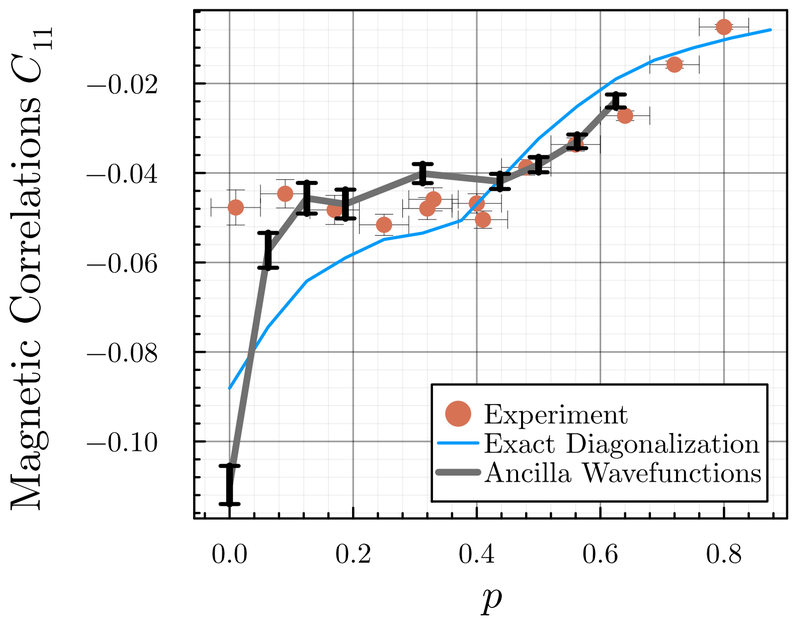


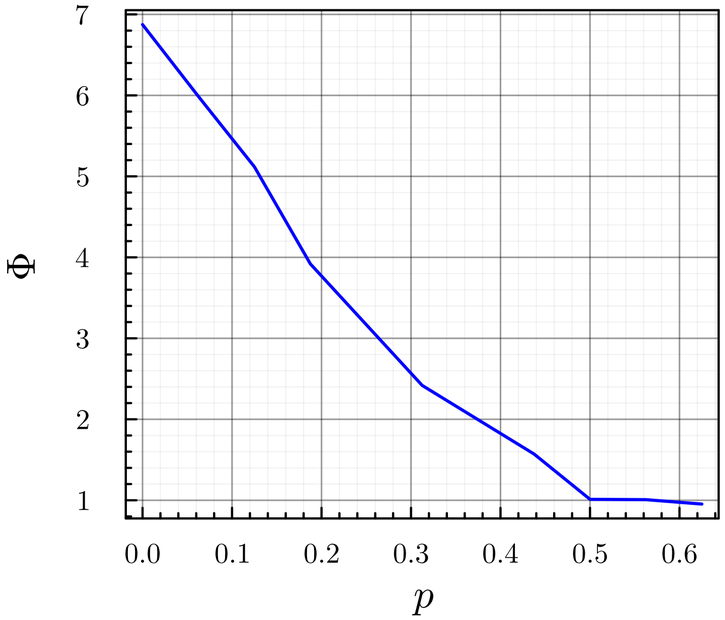
Ya-Hui Zhang

Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, S. Sachdev, PRB **105**, 075146 (2022)

Ancilla wavefunction for FL* of Hubbard model







Hybridization (Higgs boson) $\Phi \sim f^\dagger_{1\alpha} c_\alpha$.

FL*: $\Phi \neq 0$; FL: $\Phi = 0$

Φ: variational parameter with Hubbard Hamiltonian

$$|\mathrm{FL}*\rangle_{\mathrm{Hubbard}} = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2] \bowtie |\text{Slater determinant of } (c, f_1)\rangle \otimes |\text{Slater determinant of } f\rangle$$

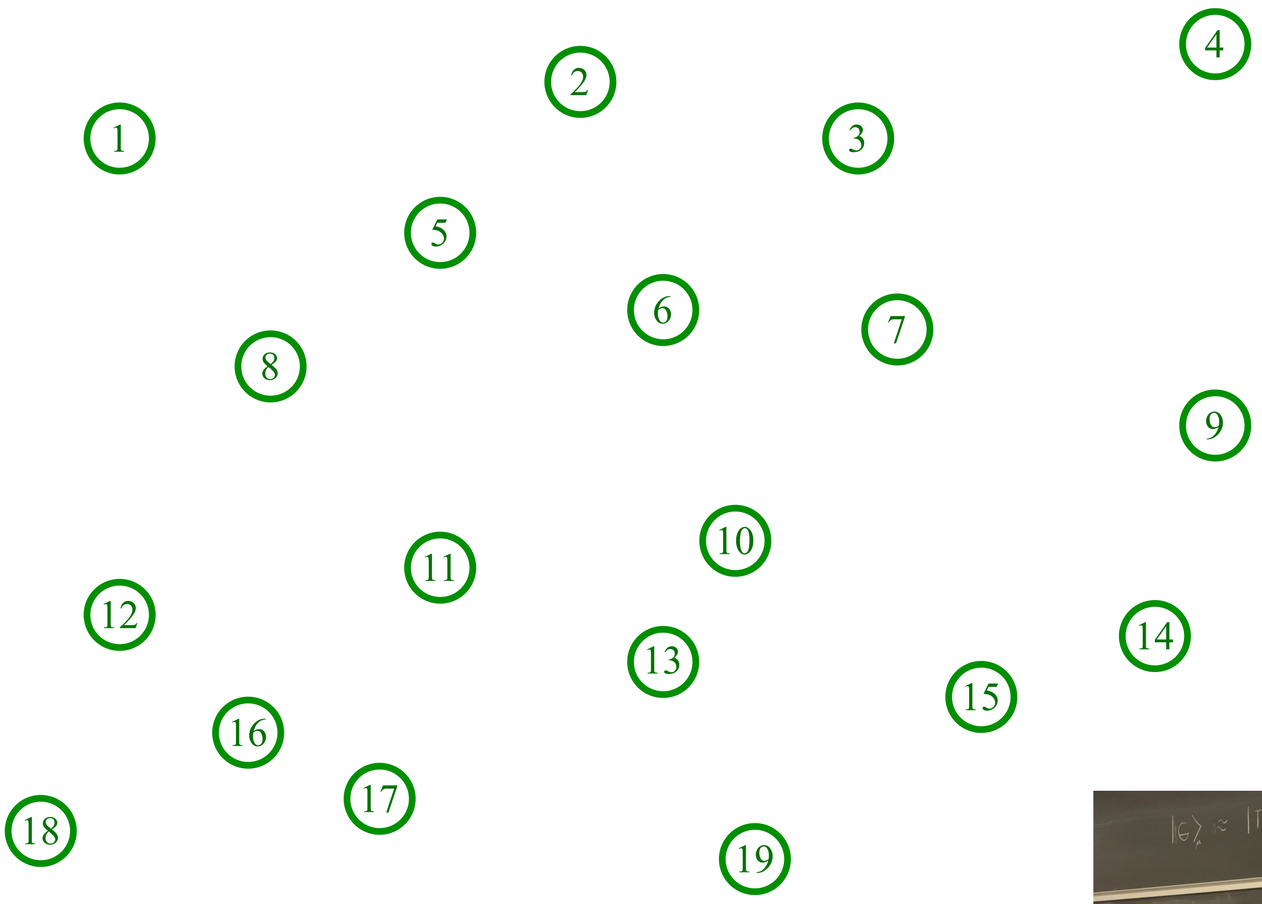
L. Shackleton and Shiwei Zhang, arXiv:2408.02190

Tobias Müller, Yasir Iqbal, S.S., Ronny Thomale, PNAS **122**, e2504261122 (2025)

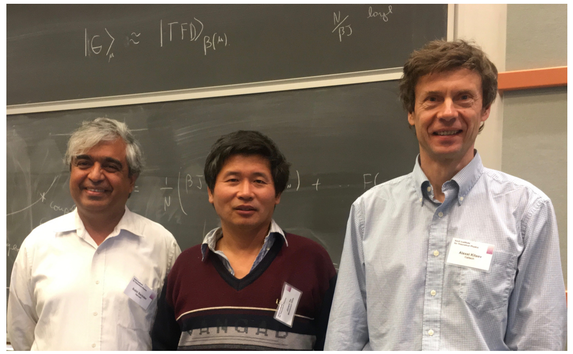
Sachdev-Ye-Kitaev liquids

The SYK model

Sachdev, Ye (1993); Kitaev (2015)

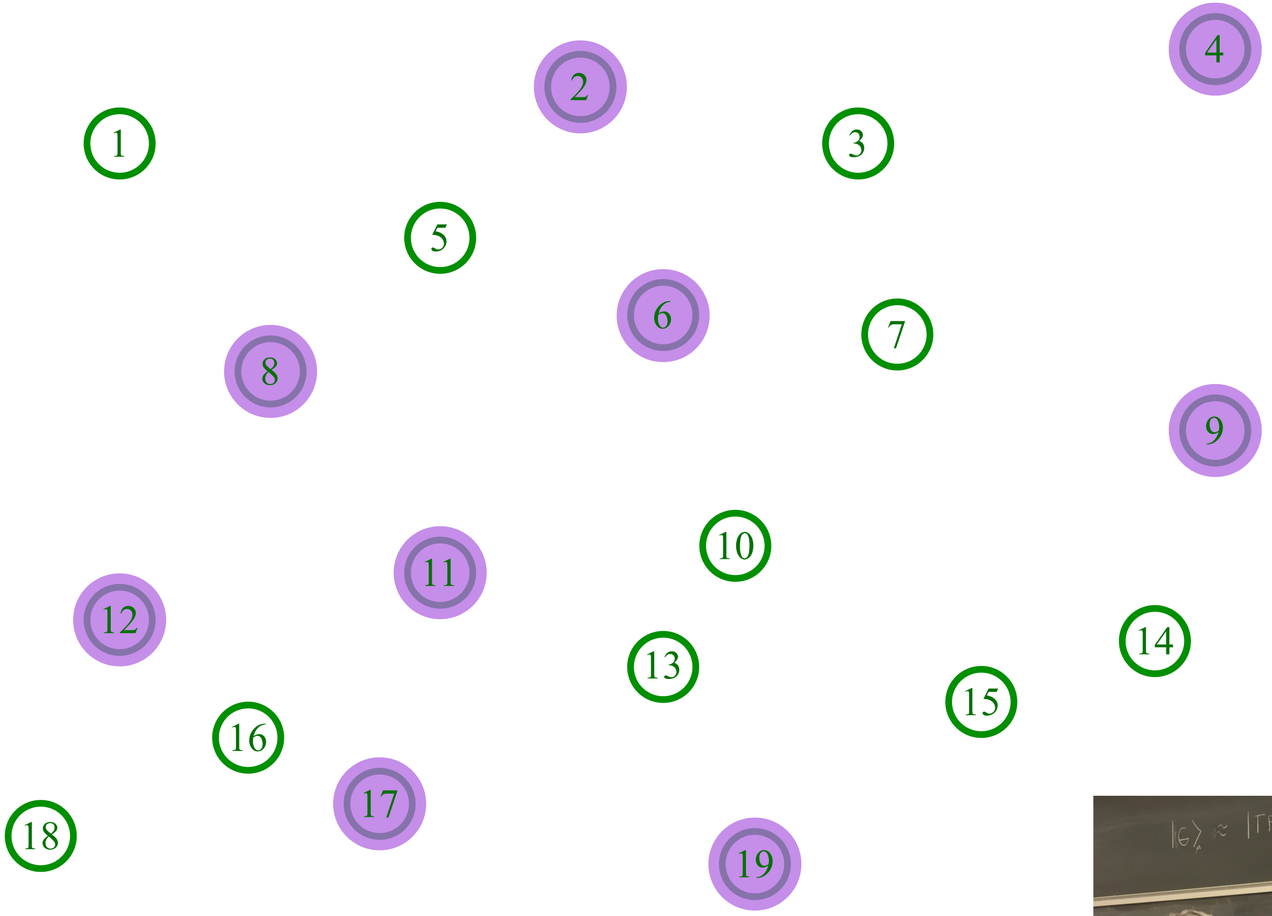


Pick a set of random positions

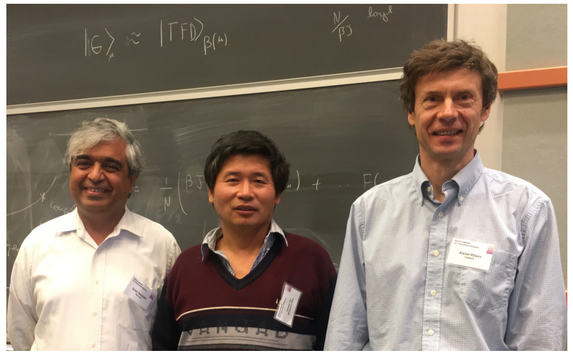


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



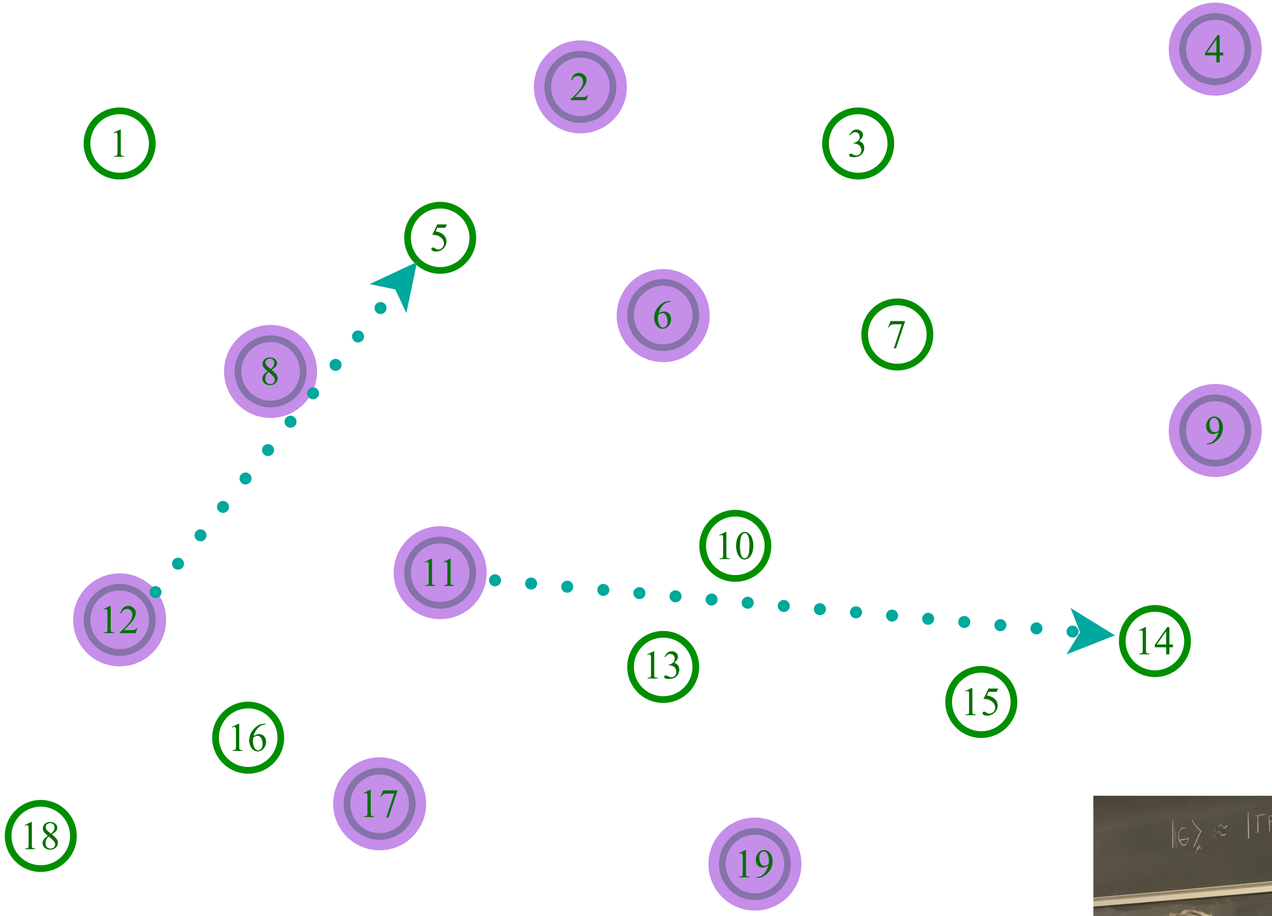
Place electrons randomly on some sites



The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



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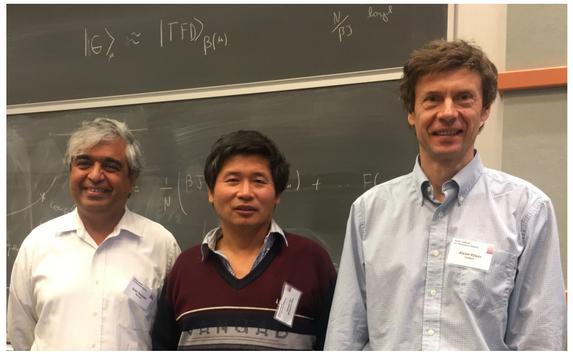

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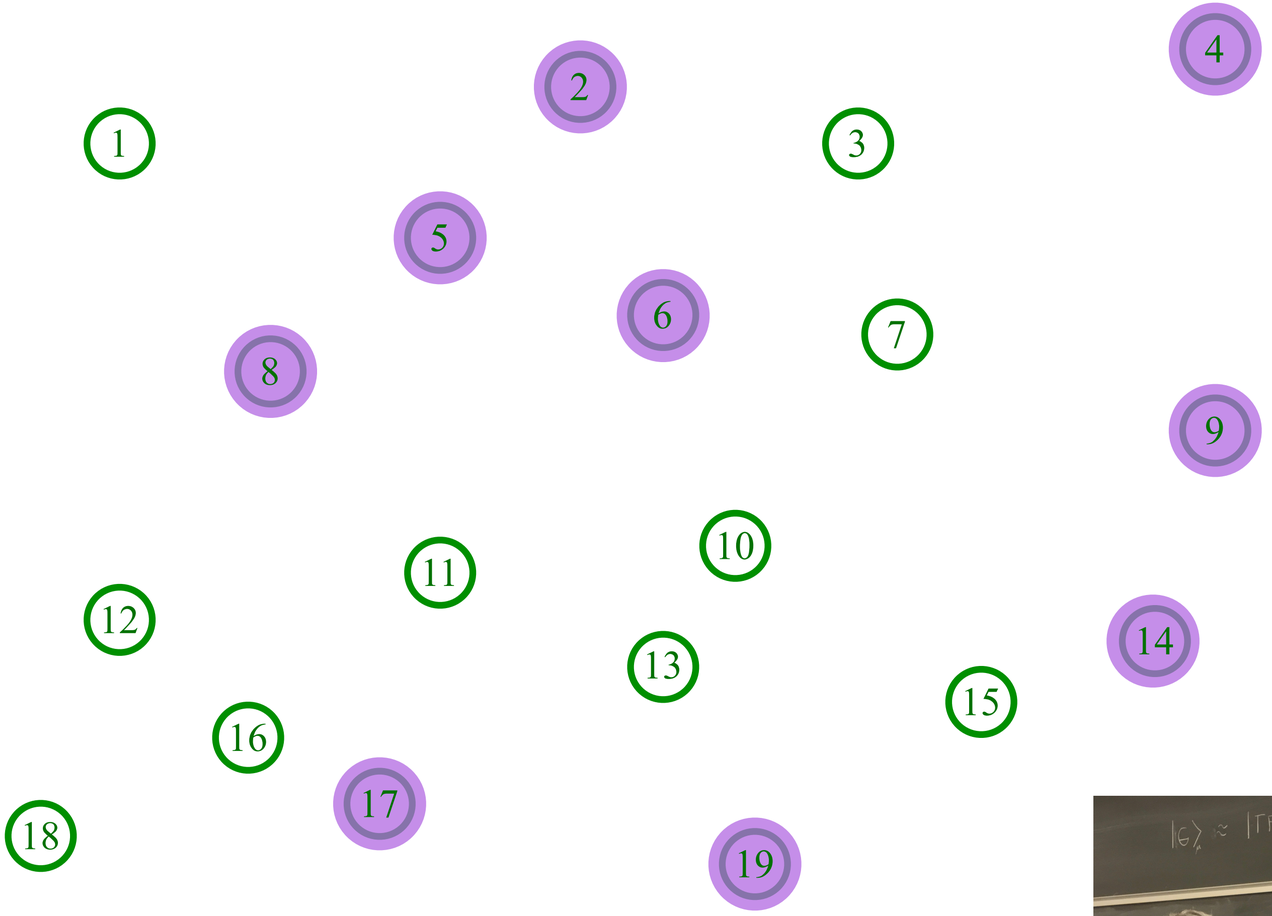
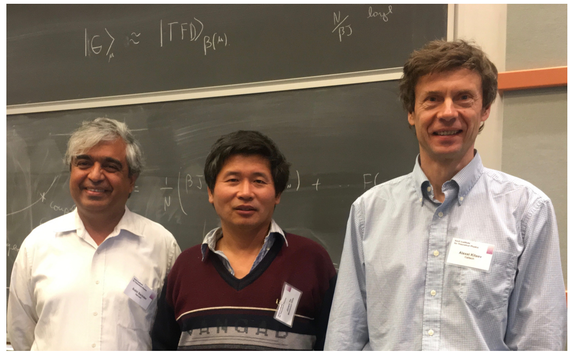


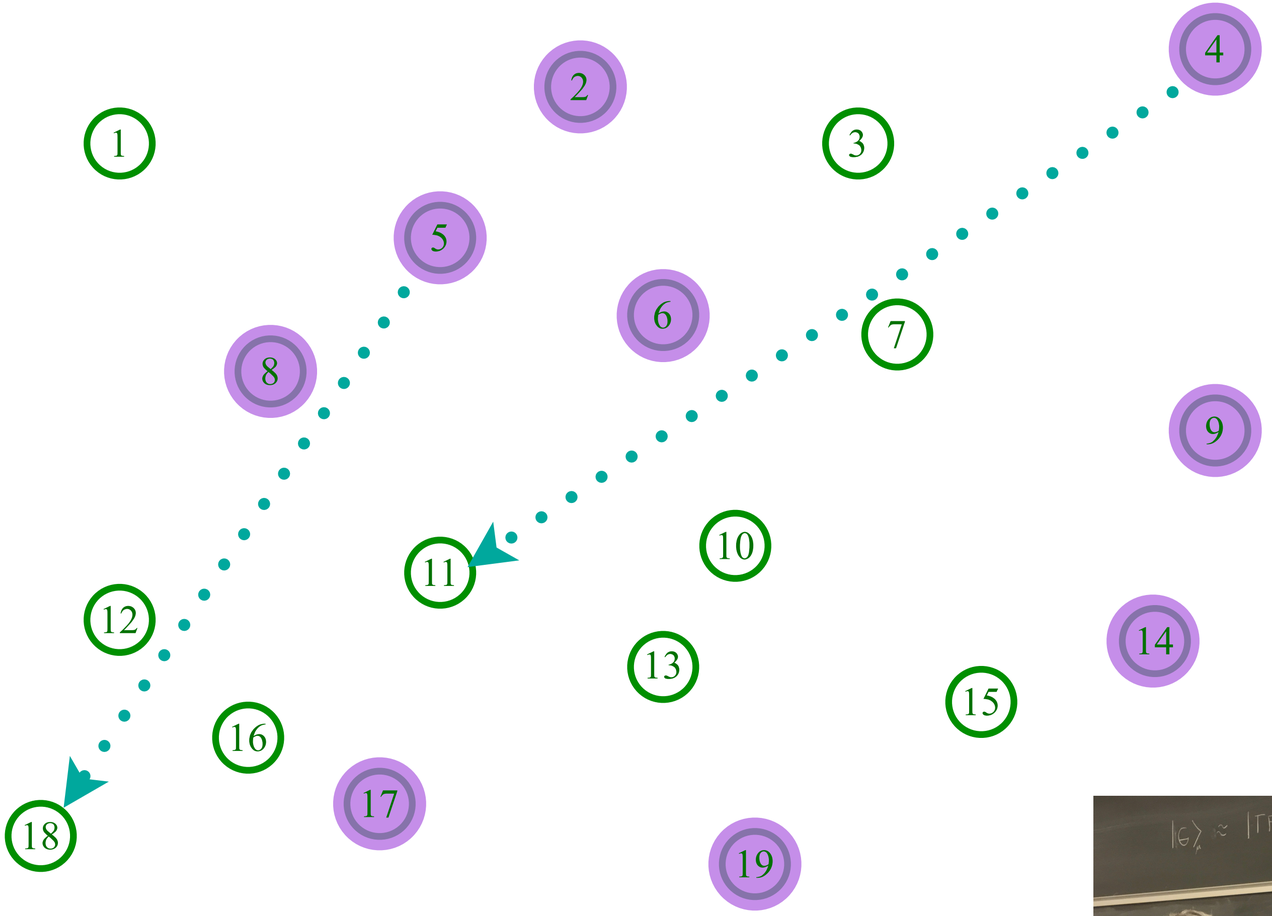
Entangle electrons pairwise randomly



The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{4,5;11,18}$



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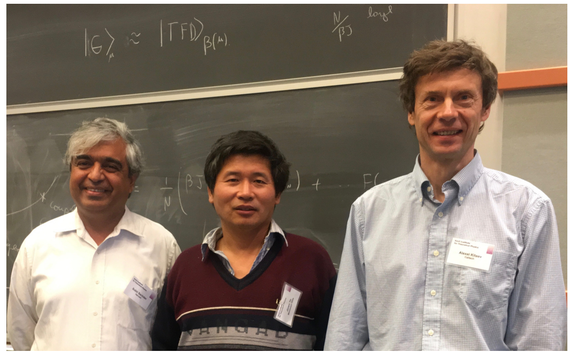

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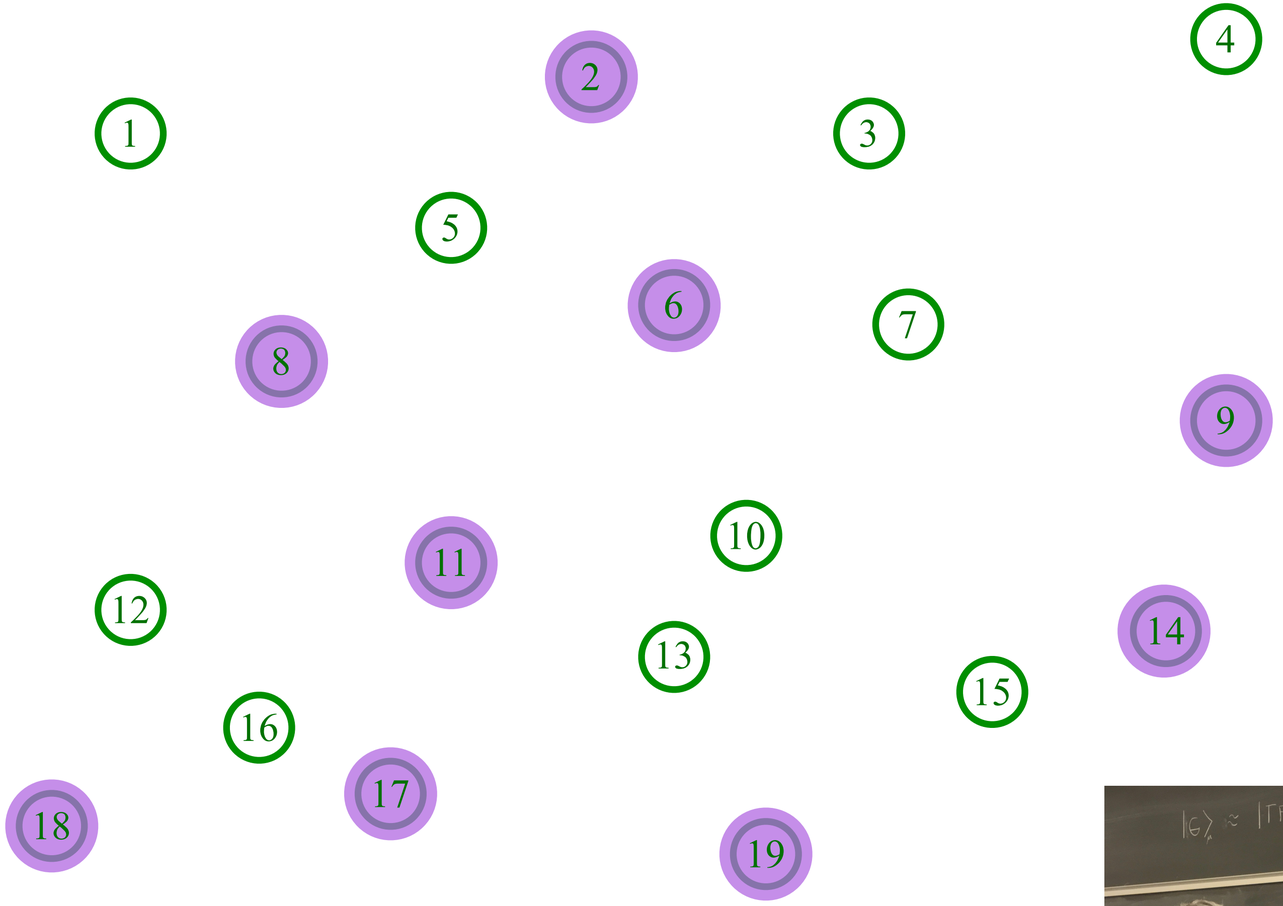

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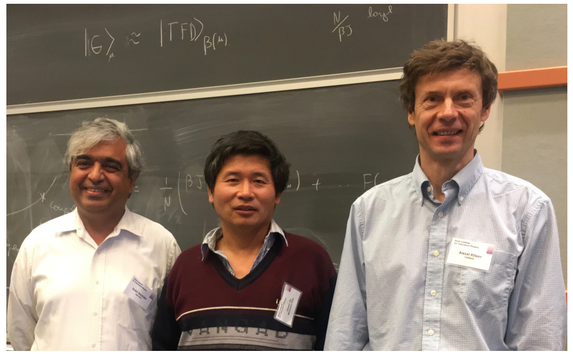

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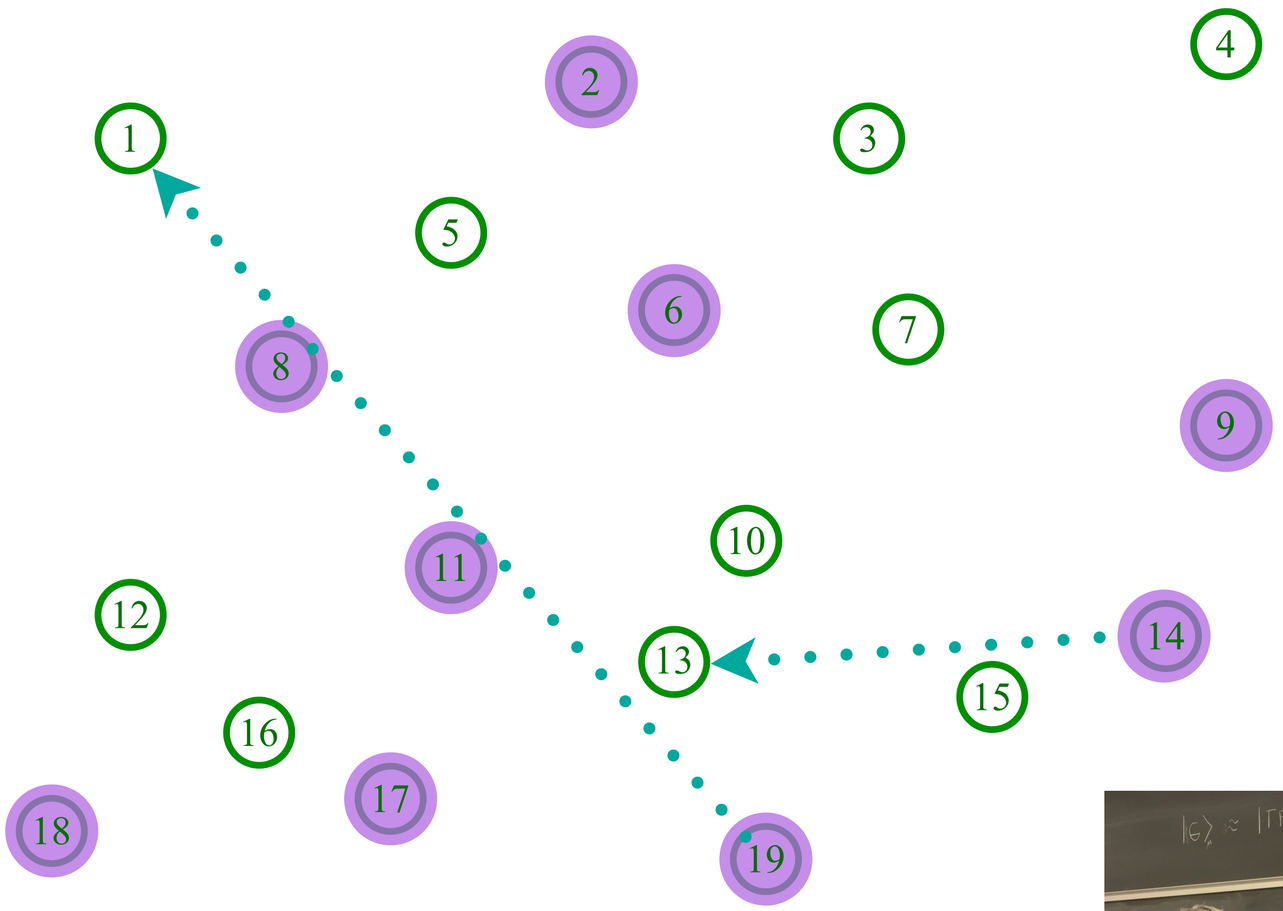
$$U_{14,19;1,13}$$



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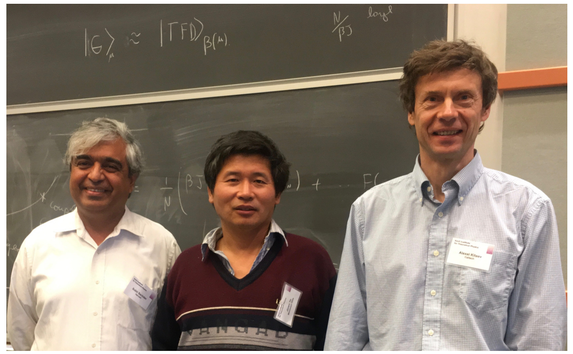

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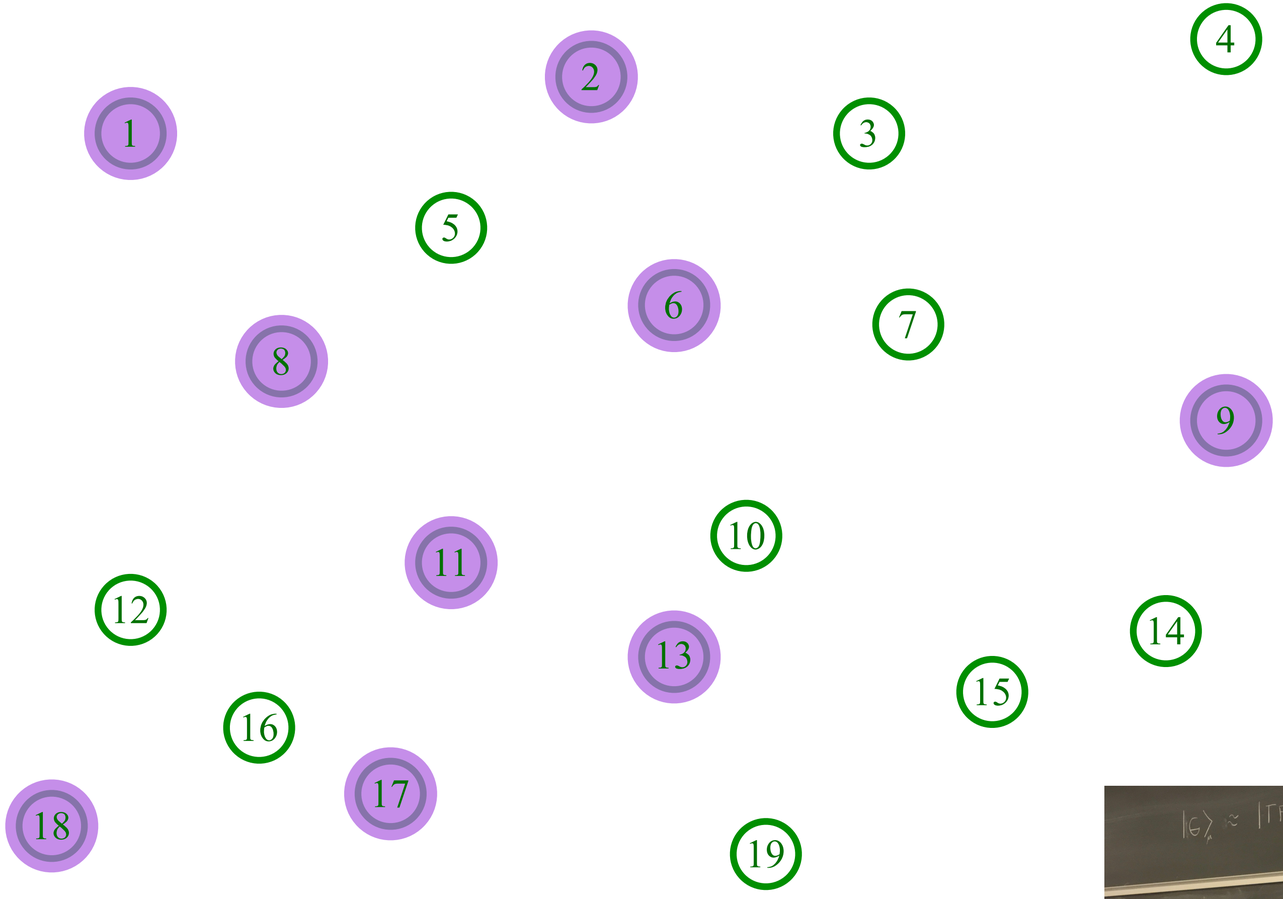

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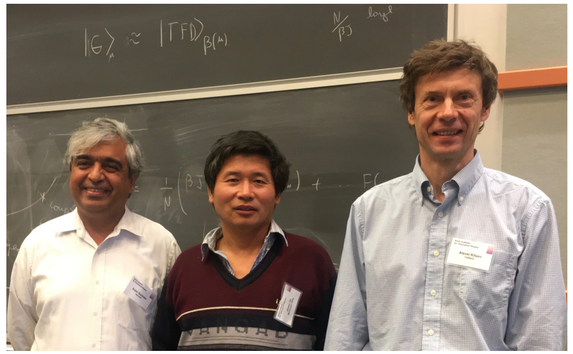
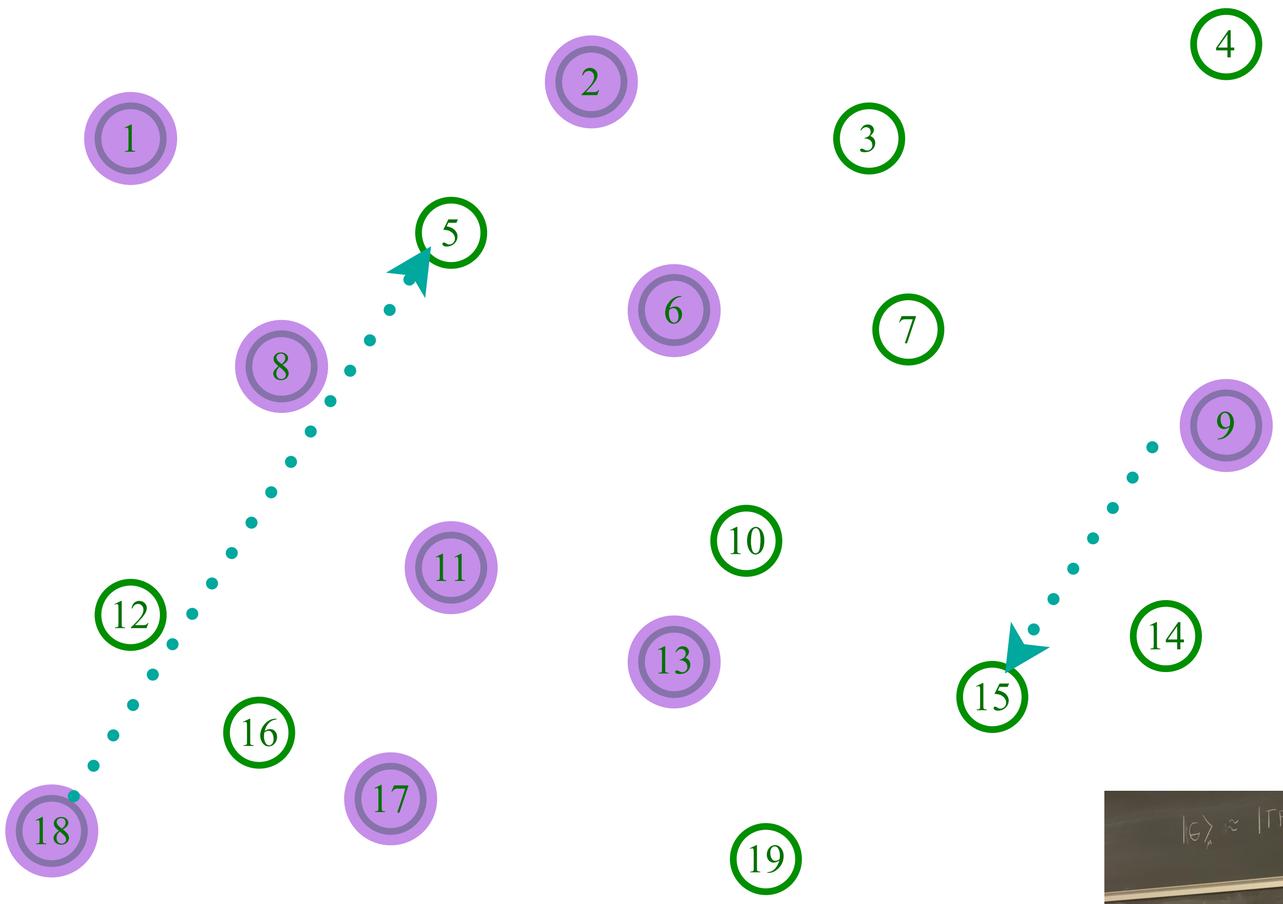


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{9,18;5,15}$



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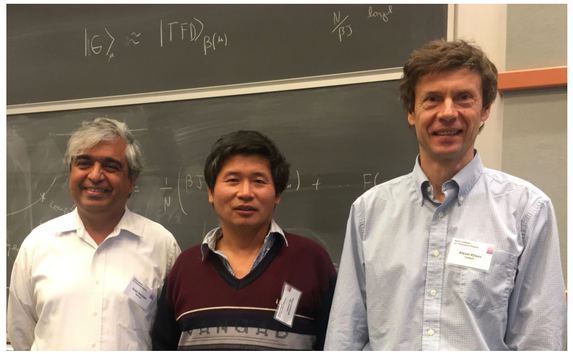

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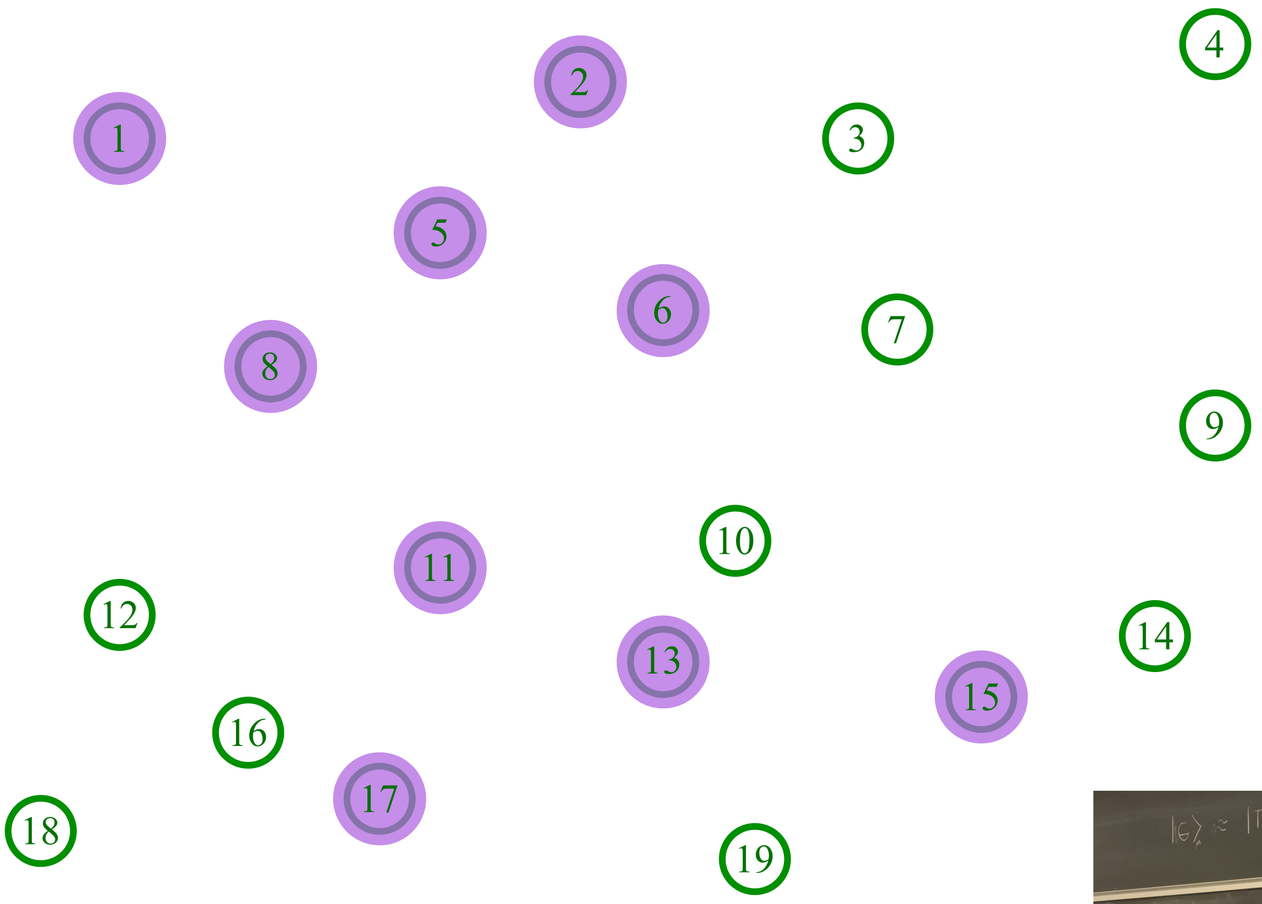

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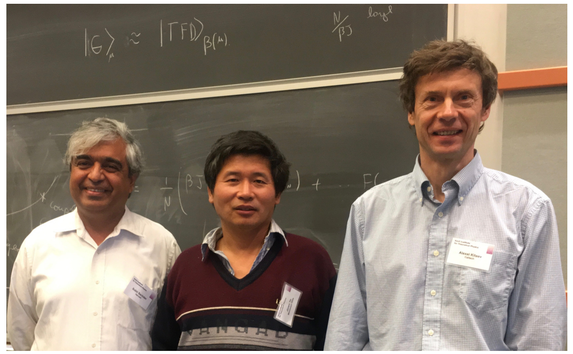

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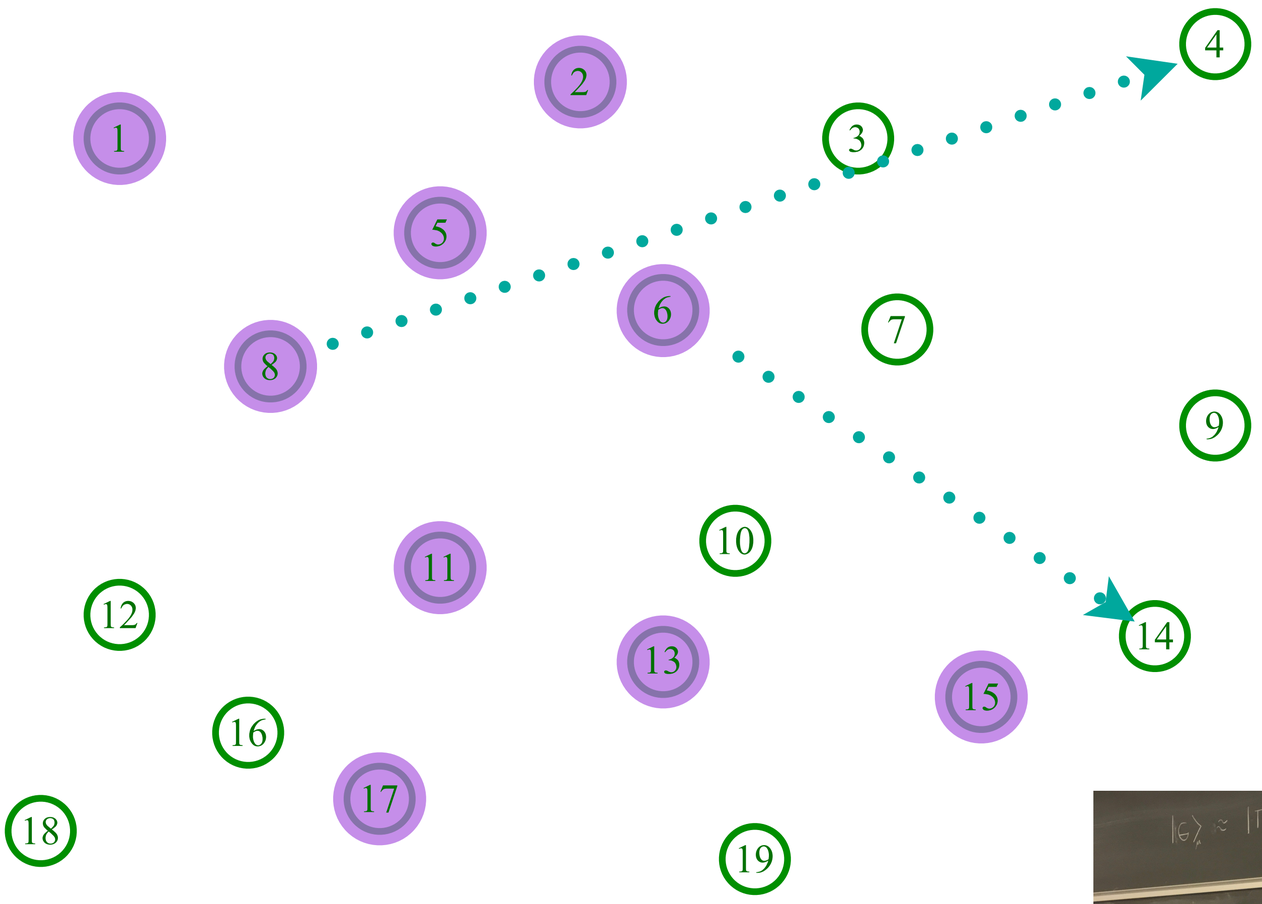
Sachdev, Ye (1993); Kitaev (2015)

$U_{6,8;4,14}$



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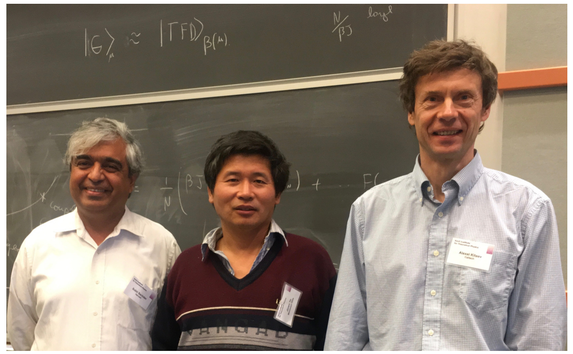

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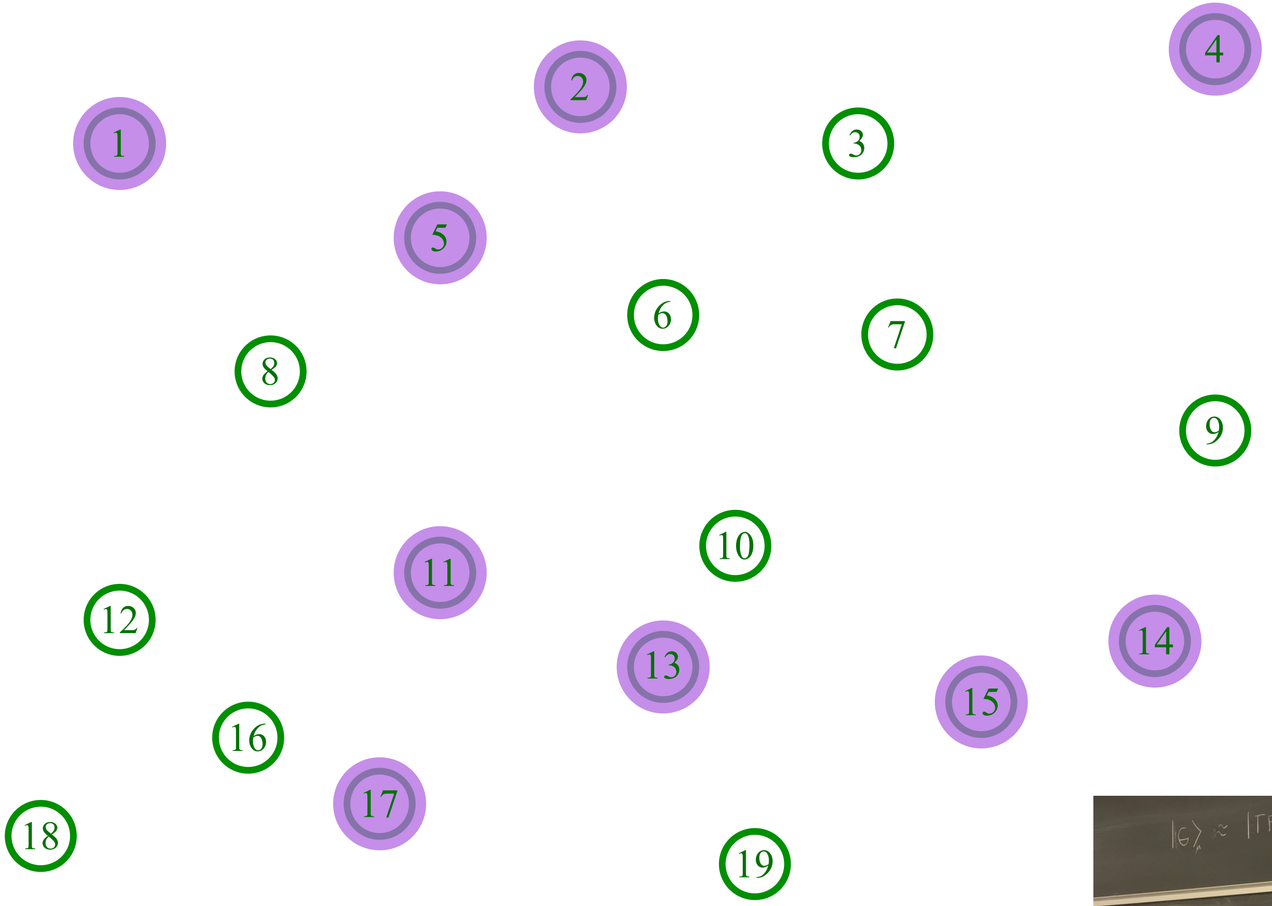

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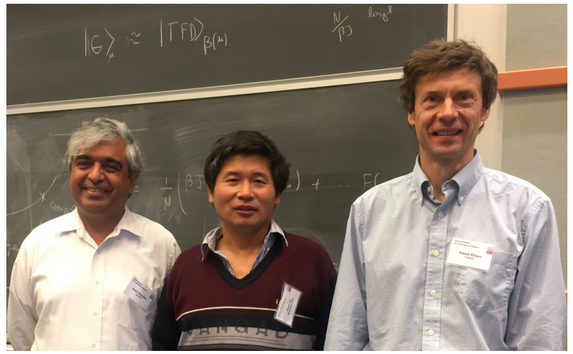
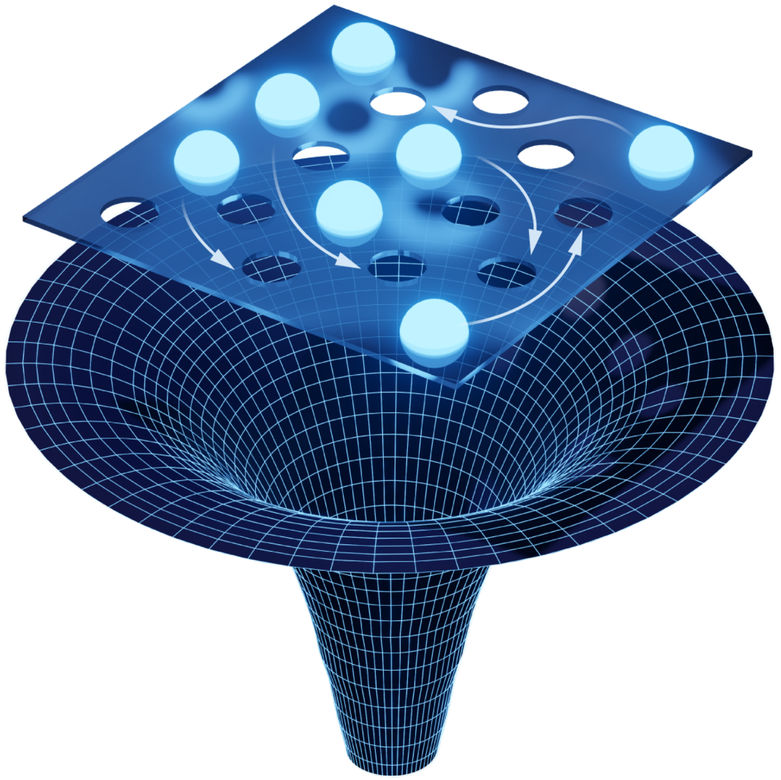


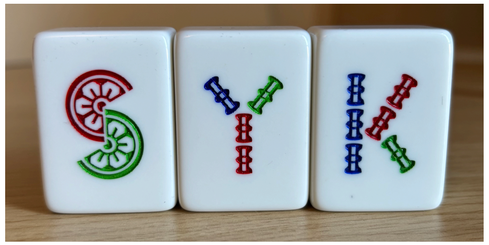
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

Solvable models of multi-particle quantum entanglement with mobile fermions.



Yields a metal whose excitations are not particle-like
i.e. no bosons, fermions, anyons….



Current is carried by an "entangled quantum soup"

The SYK model

At $T > 0$, solutions are fully characterized by a universal, frequency-dependent, 'Planckian', relaxation time,

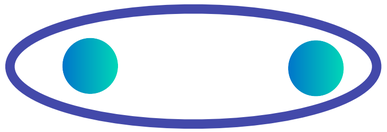
$$\frac{\hbar}{\tau(\omega)} = k_B T \, \Phi_\tau \left(\frac{\hbar\omega}{k_B T} \right)$$

where Φ_τ is a known universal function.

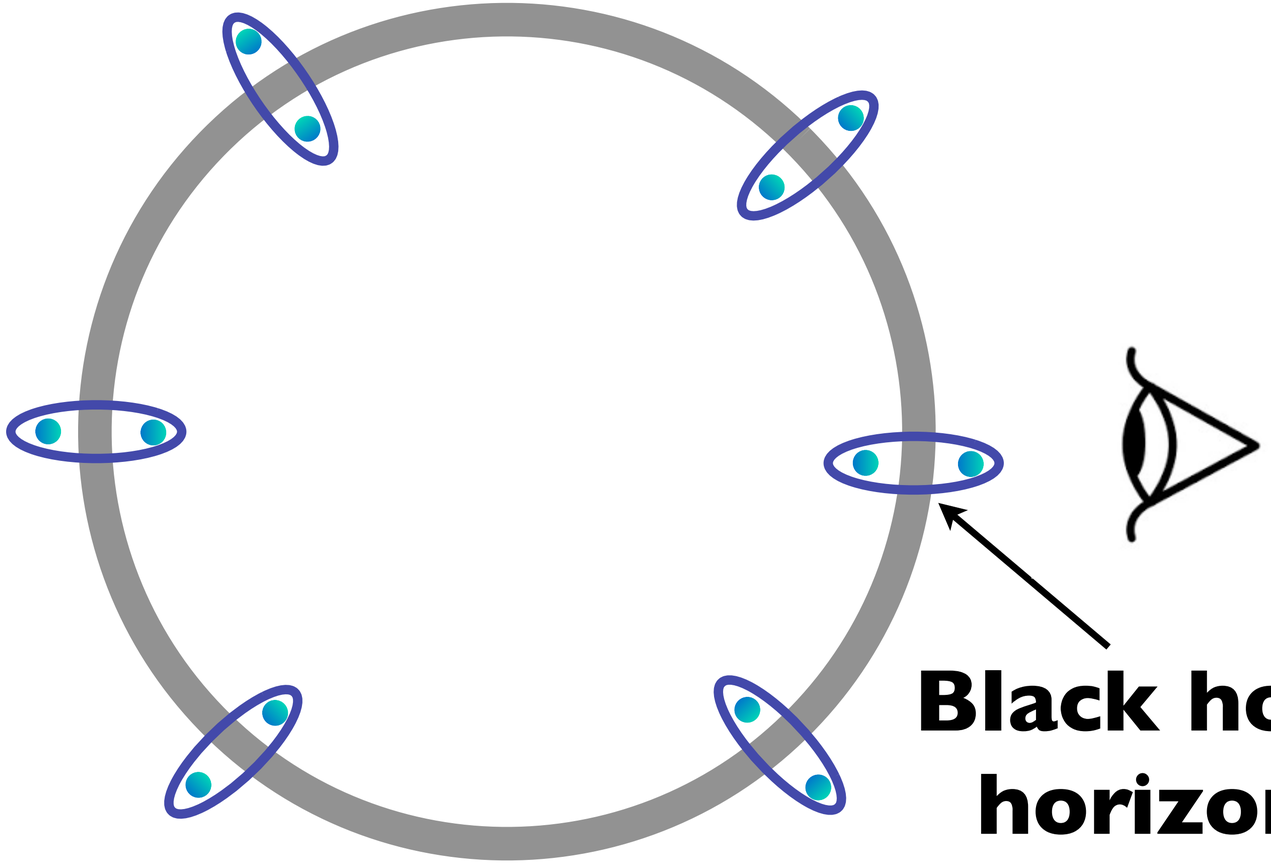
The SYK model and black holes

Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface



$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



By computations *outside* the black hole, Hawking obtained the black hole entropy

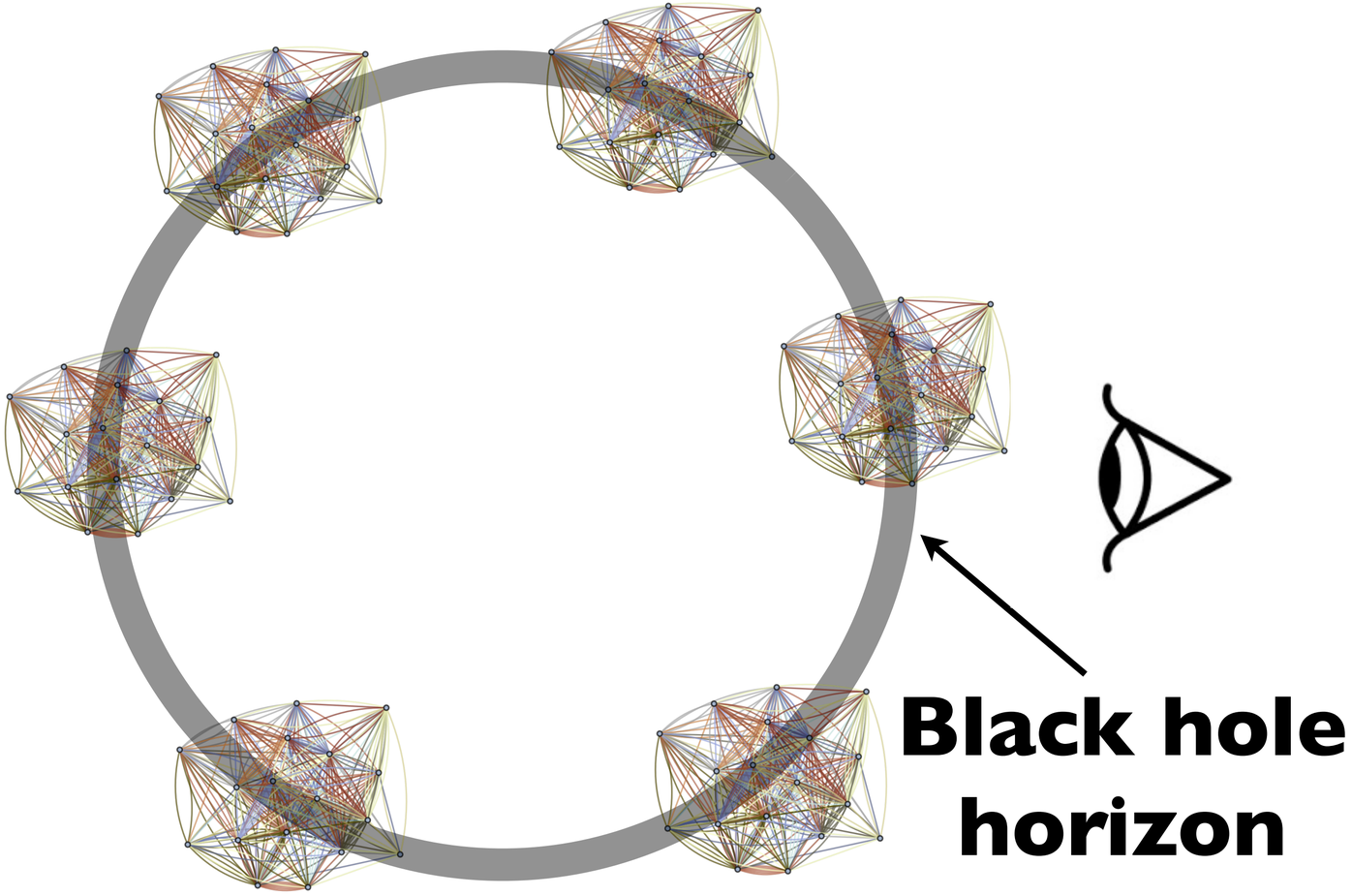
$$S = \frac{Ac^3}{4G\hbar}$$

where A is area of the black hole horizon.

All other systems have entropy proportional to their volume.

Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface



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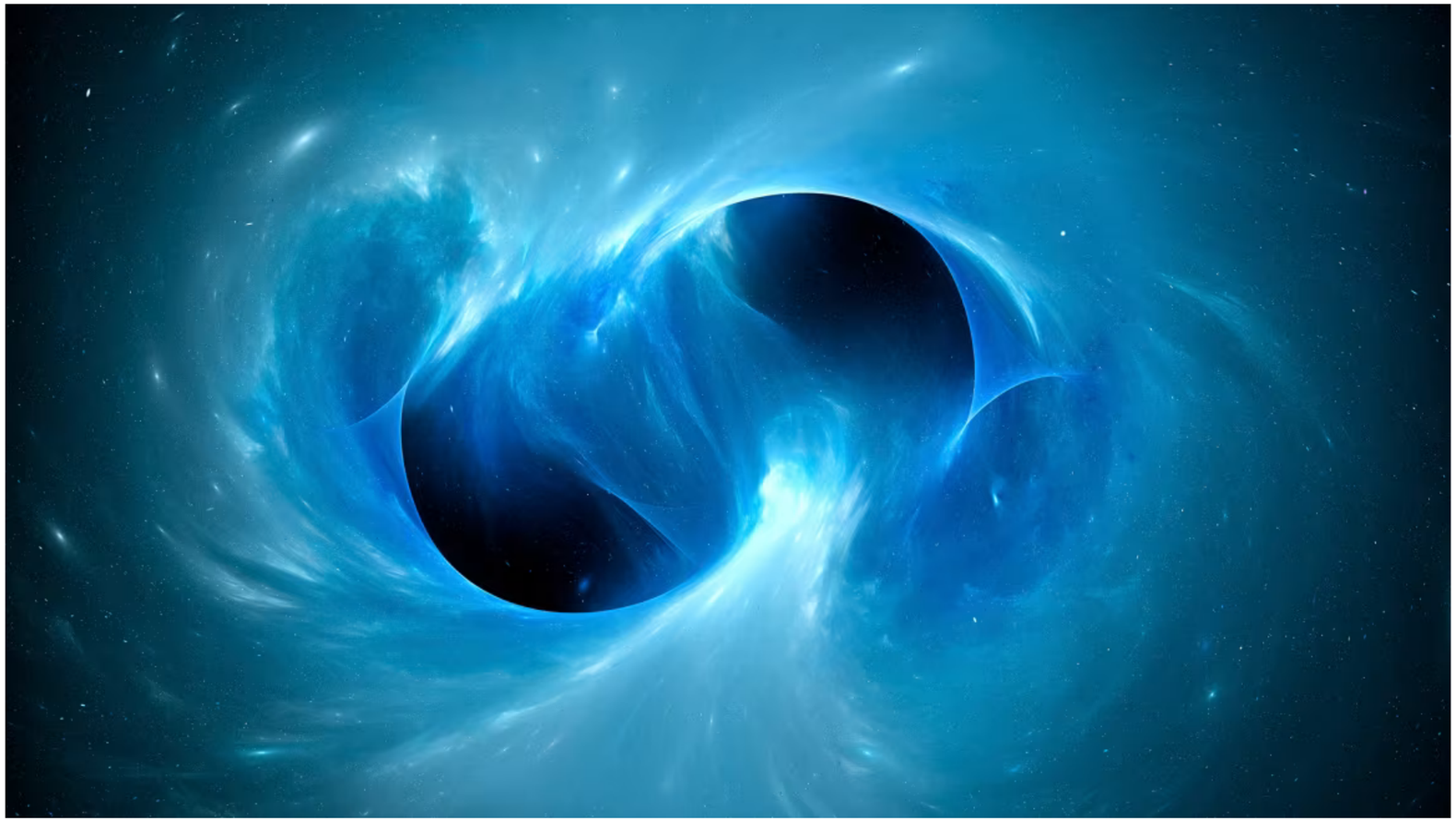


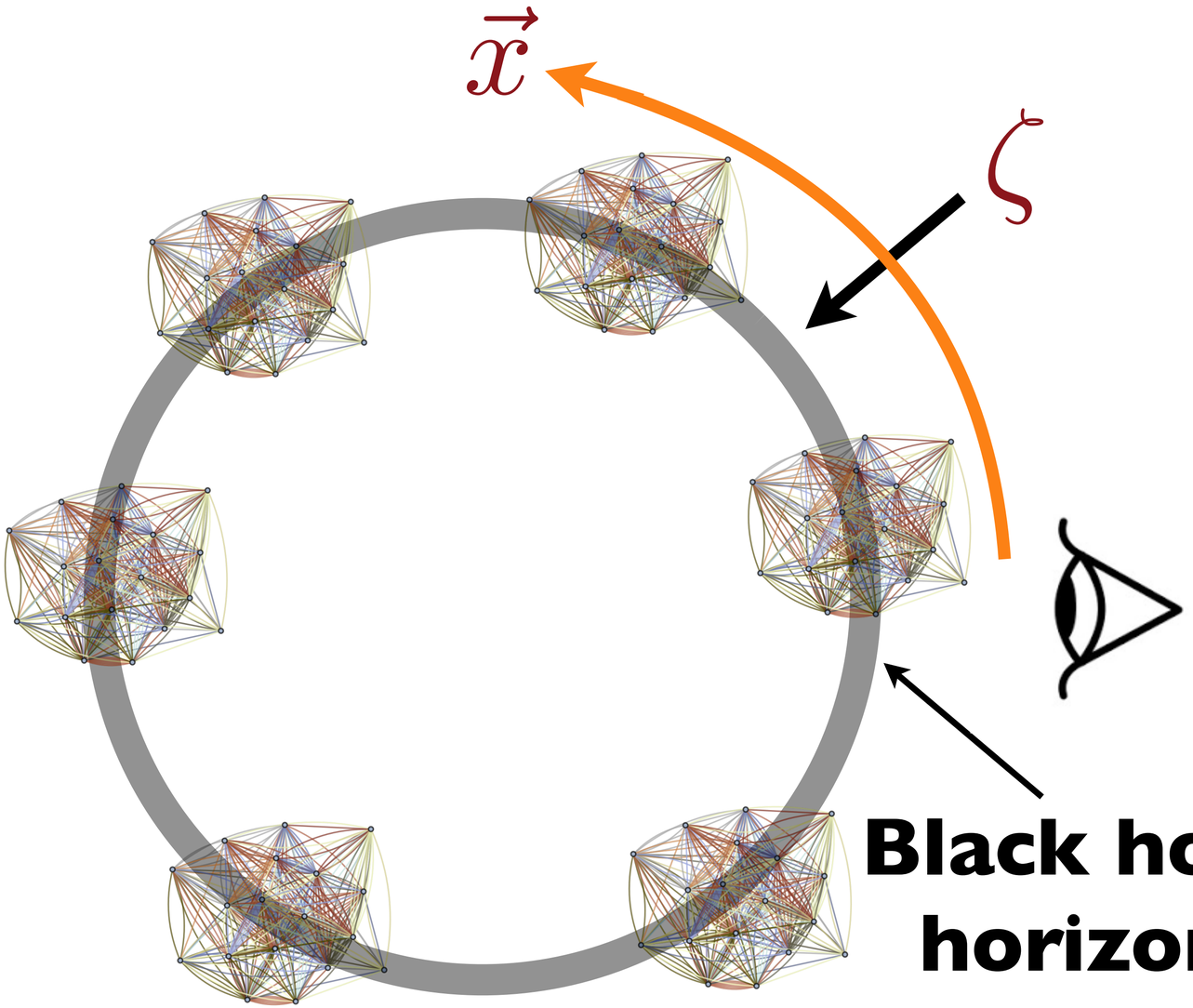
Sakkmesterke/Science Photo Library RF/Getty Images

$$\tau_{\mathrm{ring-down}} \sim \frac{\hbar}{k_B T}$$

Planckian dynamics!

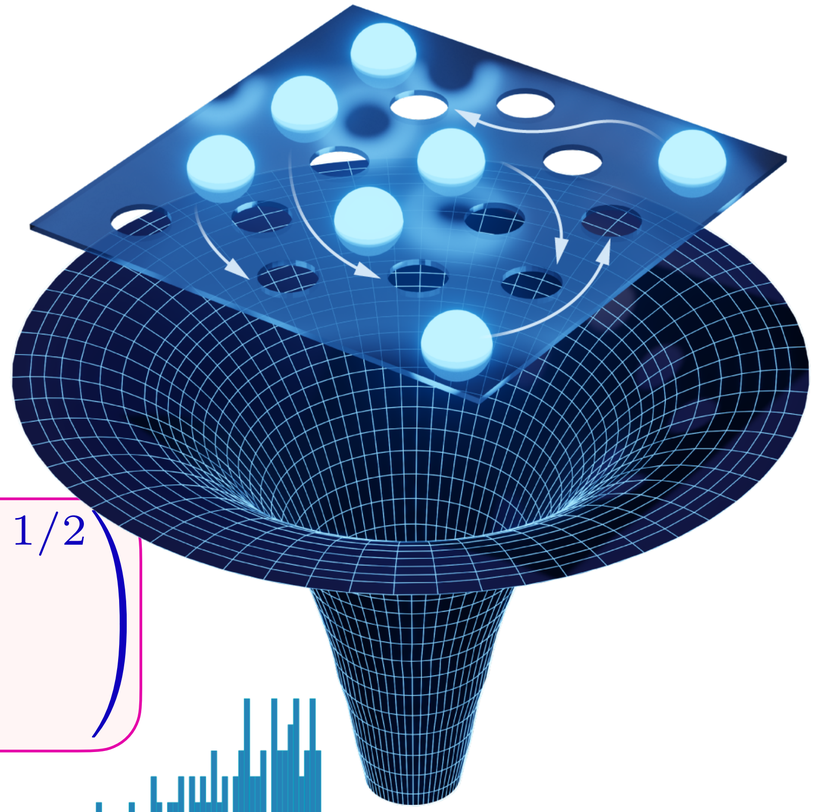
T is the Hawking temperature of the black hole

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



The quantum versions of Maxwell's and Einstein's equations in ζ space and time are also the equations describing electron entanglement in the SYK model!

D(E) of charged black holes from the SYK model



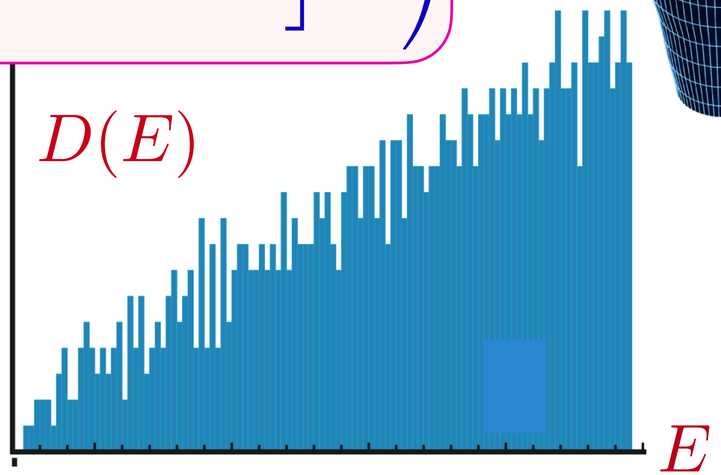
- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T = 0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy E is

Bekenstein-Hawking

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} {A_0}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

Iliesiu, Murthy, Turiaci (2022)

Developments from the SYK model

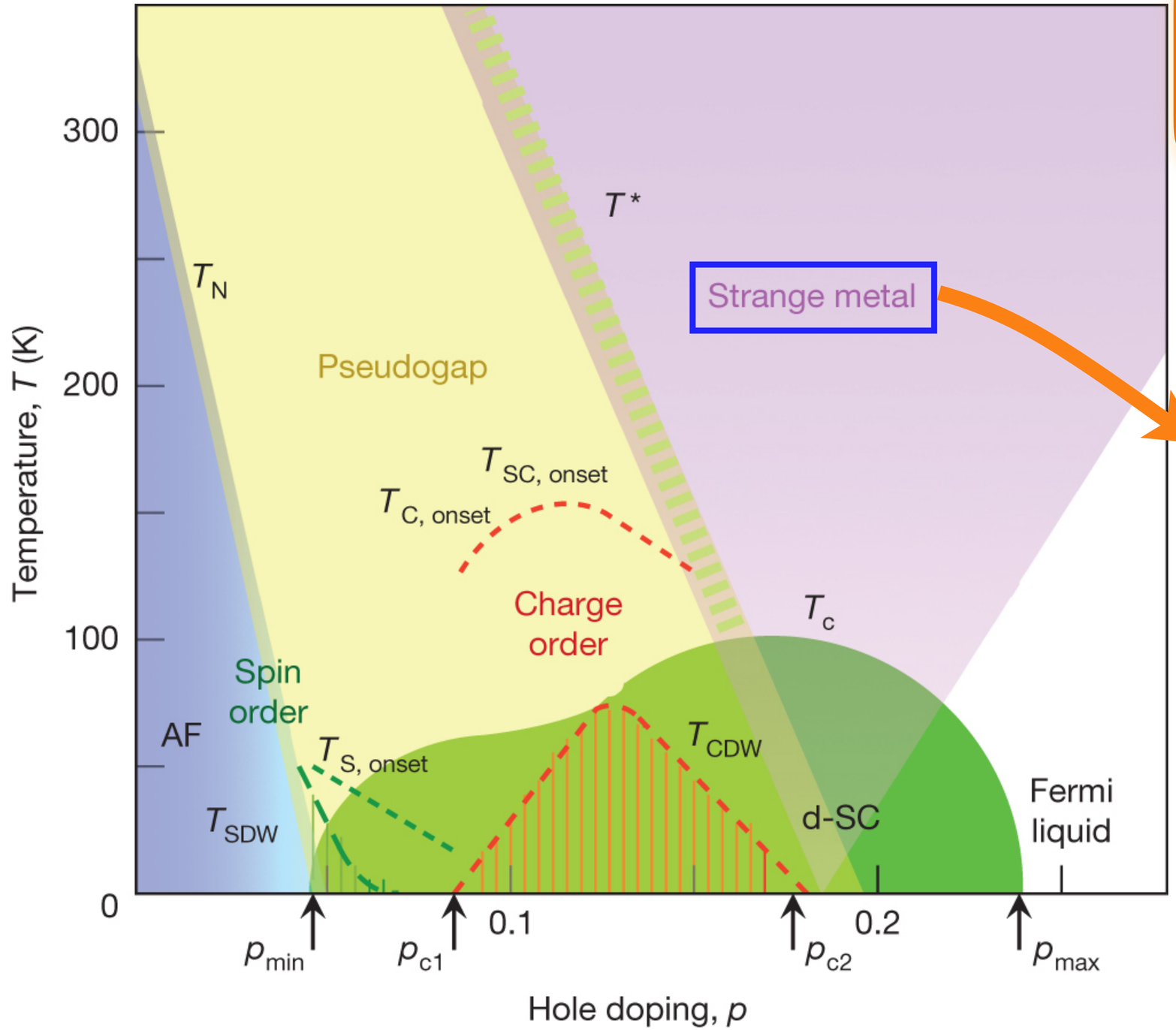


Similar remarks apply to rotating neutral black holes.

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

From FL* to FL
via
the strange metal
using the 2D-YSYK model

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Quantum entanglement of mobile fermions

Sachdev-Ye-Kitaev (SYK) liquid

- Compressible state with no quasiparticles.
- 2D-YSYK: universal theory of strange metals applied to FL*-FL transition

Yukawa-Sachdev-Ye-Kitaev model

$$\mathcal{H} = -\mu \sum_i c_i^\dagger c_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \Phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} c_i^\dagger c_j \Phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
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Yukawa-Sachdev-Ye-Kitaev model

$$\mathcal{H} = -\mu \sum_i c_i^\dagger c_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \Phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell} c_i^\dagger c_j \Phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N equations for the Green's functions and self energies of the fermions (G, Σ) and bosons (D, Π) are

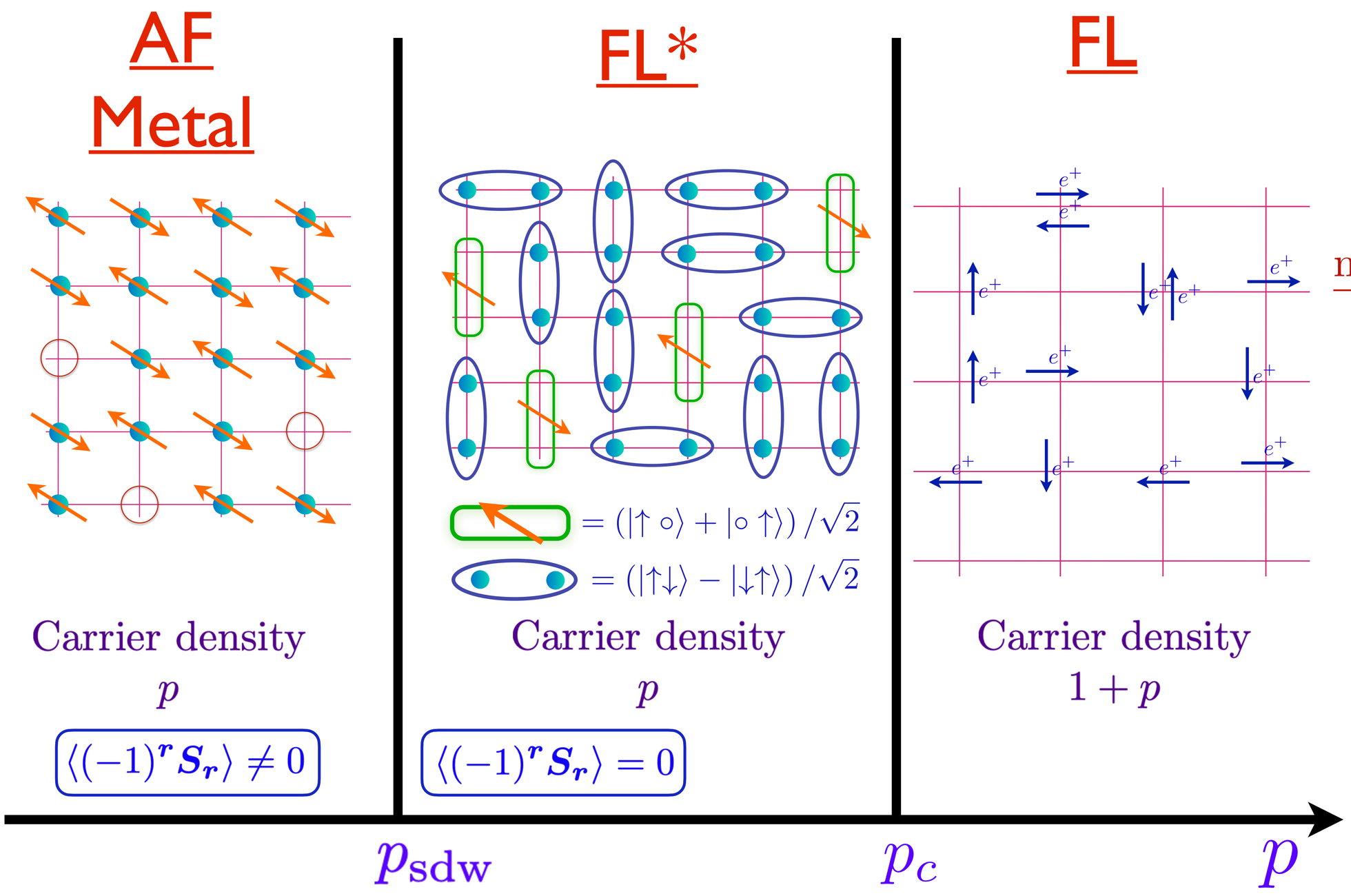
$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$

$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

At $T > 0$, solutions are fully characterized by a universal frequency-dependent relaxation time,

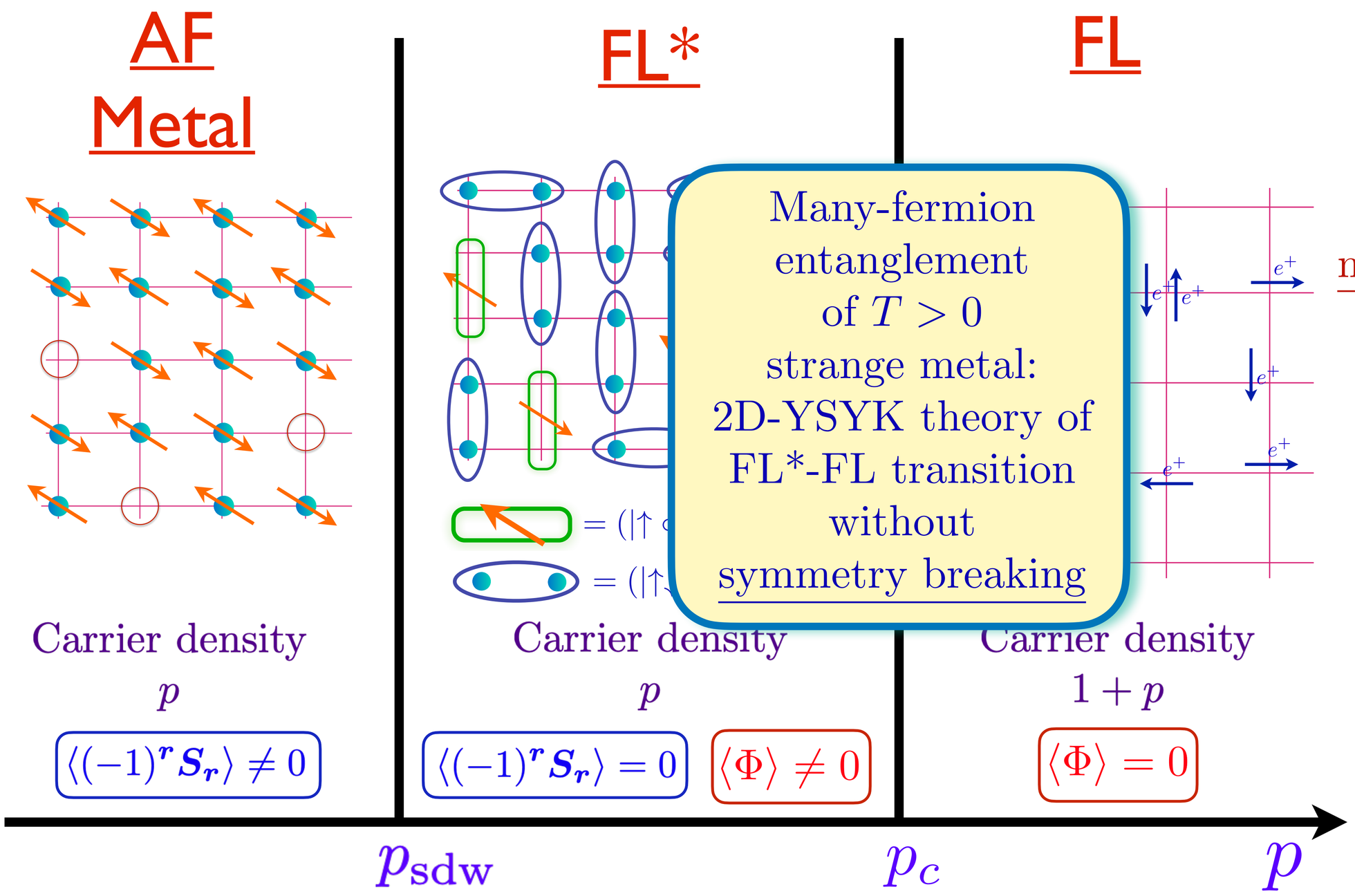
$$\frac{\hbar}{\tau(\omega)} = k_B T \, \Phi_\tau \left(\frac{\hbar\omega}{k_B T}\right)$$

where Φ_τ is a known universal function.



Quantum phase transition between two metals (FL* and FL) at $p = p_c$, with no symmetry breaking.

S. Sachdev, M. A. Metlitski and M. Punk, Journal of Physics Condensed Matter **24**, 294205 (2012)



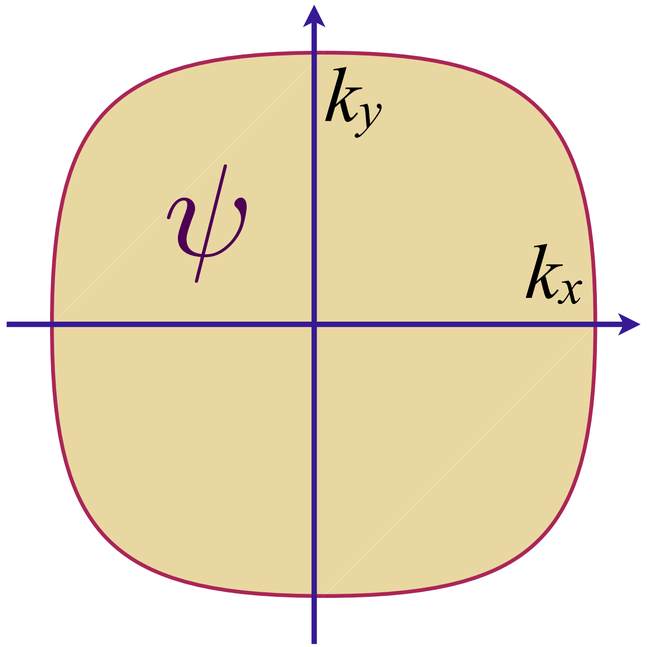
Quantum phase transition between two metals (FL* and FL) at $p = p_c$, with no symmetry breaking.

Described by the condensation of a *Higgs* field Φ.

S. Sachdev, M. A. Metlitski and M. Punk, Journal of Physics Condensed Matter **24**, 294205 (2012)
Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

2D-YSYK model: Fermi surface + Higgs boson with interaction disorder

$$\mathcal{L} = c^\dagger_{\boldsymbol{k}\alpha}\left(\frac{\partial}{\partial\tau} + \varepsilon(\boldsymbol{k})\right) c_{\boldsymbol{k}\alpha} + f^\dagger_{1\boldsymbol{k}\alpha}\left(\frac{\partial}{\partial\tau} + \widetilde{\varepsilon}(\boldsymbol{k})\right) f_{1\boldsymbol{k}\alpha}$$



$$+ [\nabla\Phi(\boldsymbol{r})]^2 + s\,[\Phi(\boldsymbol{r})]^2 + u\,[\Phi(\boldsymbol{r})]^4$$

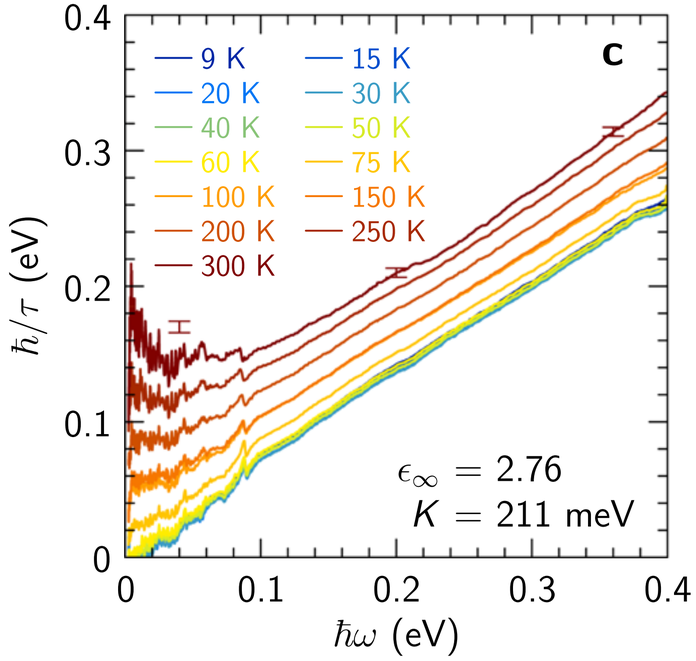
$$+ [g + g'(\boldsymbol{r})]\, c^\dagger_\alpha(\boldsymbol{r}) f_{1\alpha}(\boldsymbol{r})\, \Phi(\boldsymbol{r}) + \text{H.c.}$$

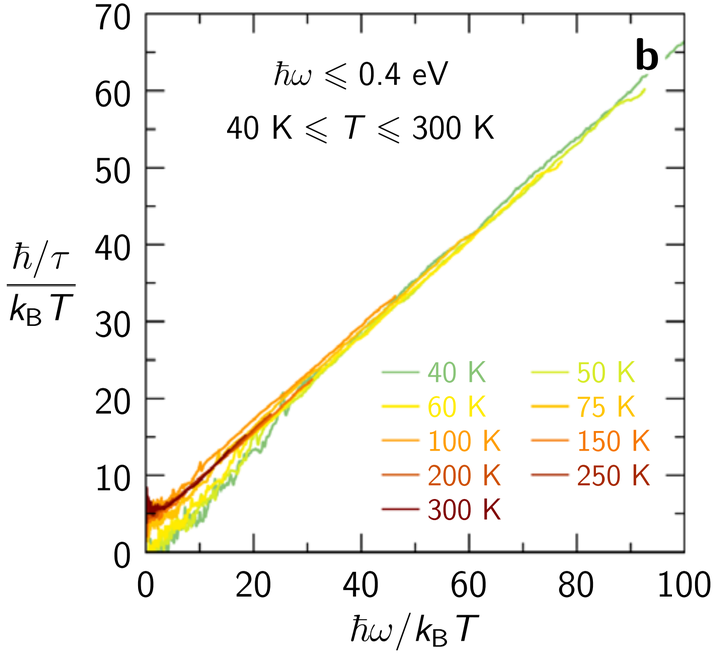
$$+ v(\boldsymbol{r}) c^\dagger_\alpha(\boldsymbol{r}) c_\alpha(\boldsymbol{r})$$

Φ^2 "mass" disorder $s \to s + \delta s(\boldsymbol{r})$ is strongly relevant; rescale Φ to move disorder to the Yukawa coupling.

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

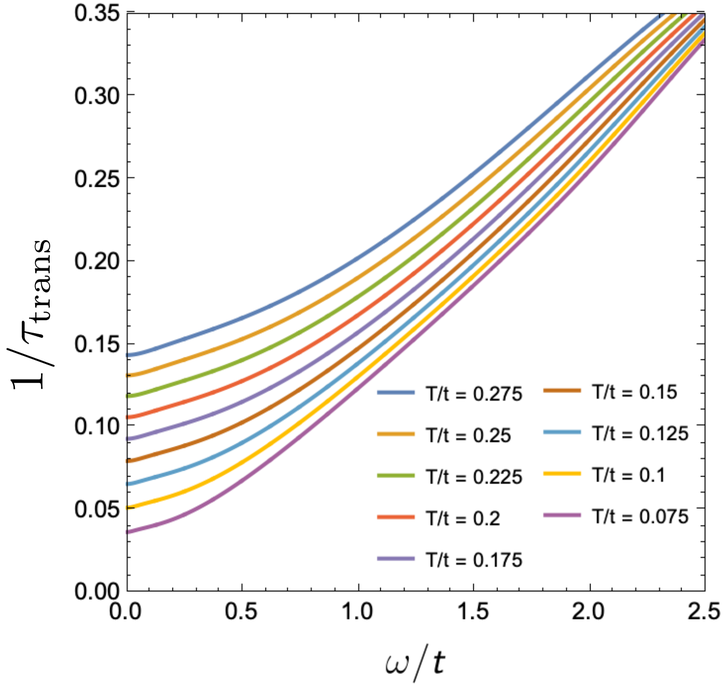


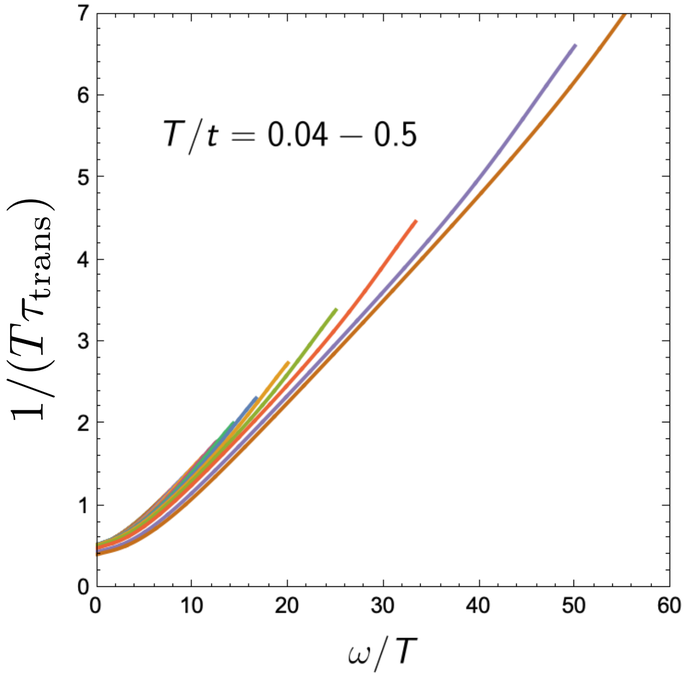


$$\sigma(\omega) = i\frac{e^2 K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$

From optical conductivity data of Michon et al. (2023)

$$\frac{\hbar}{\tau(\omega)} = k_B T\,\Phi_\tau\left(\frac{\hbar\omega}{k_B T}\right)$$





2d-YSYK theory

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

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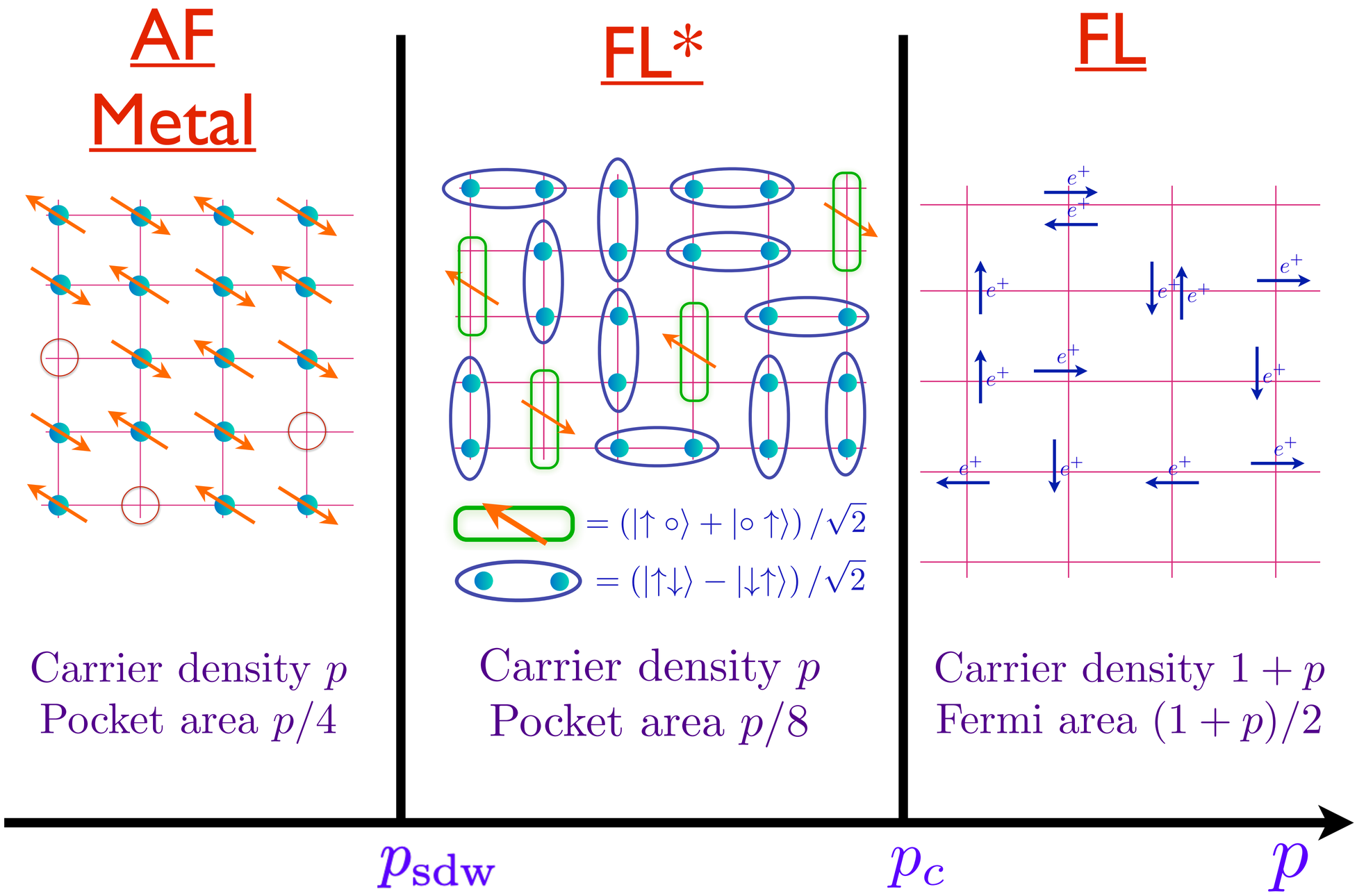

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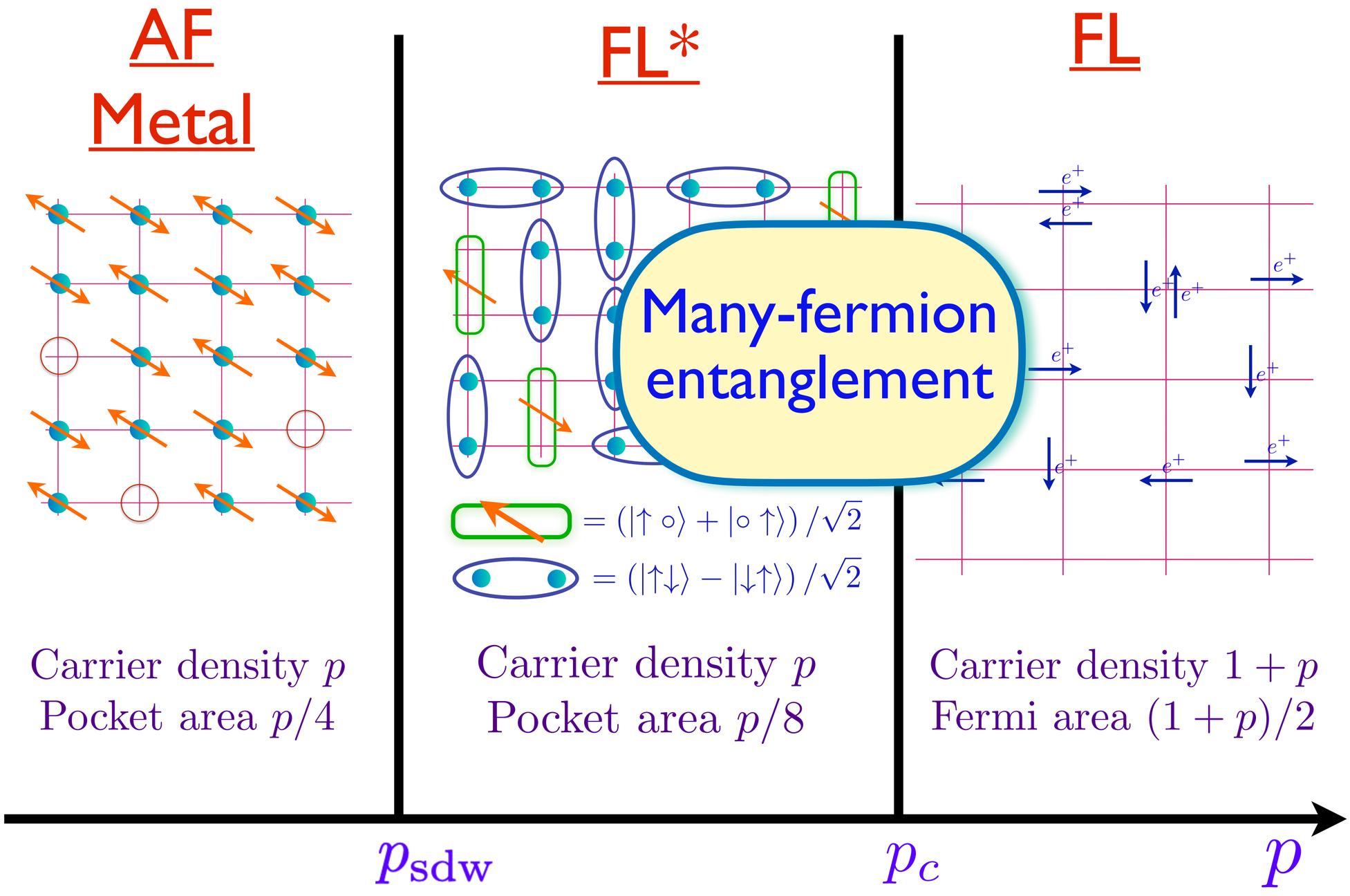
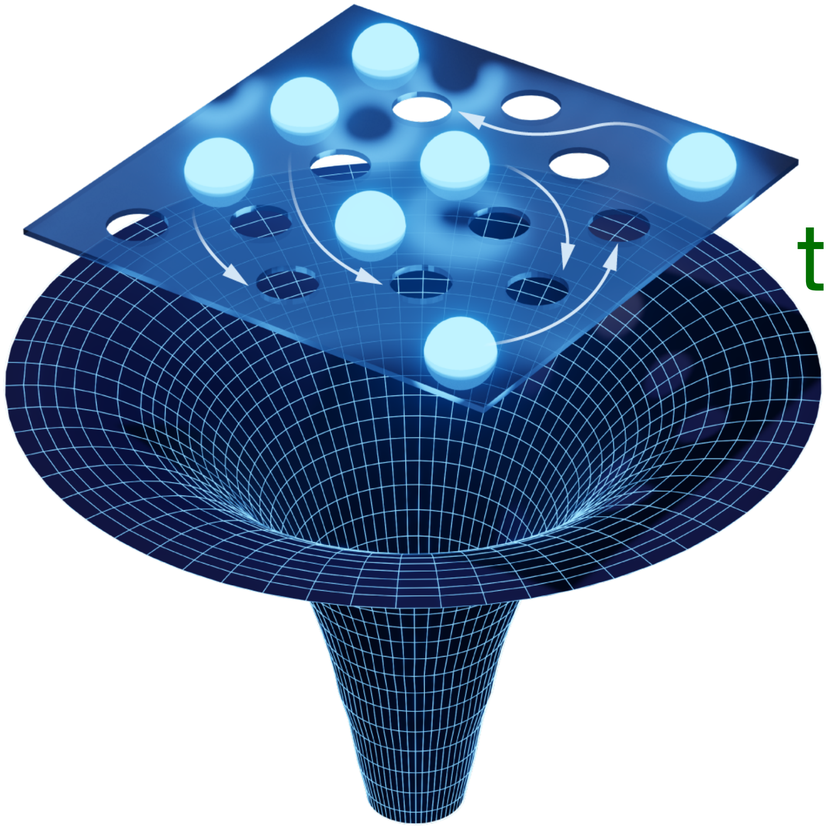


S. Sachdev, M. A. Metlitski and M. Punk, Journal of Physics Condensed Matter **24**, 294205 (2012)

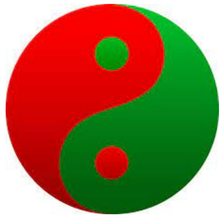
The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

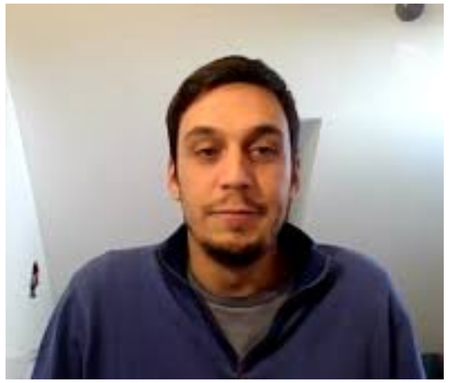
Sachdev, Ye (1993)



In a *dual* set of variables it describes the interior of ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)

Maine Christos
Caltech



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