A simple model of many-particle entanglement: how it describes black holes and superconductors

Mysteries of the Universe
Indian Institute of Technology, Roorkee
March 13, 2021

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

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in modern physics between
(A) the quantum theory of many interacting
particles (e.g. electrons in a crystal)
and
(B) the quantum theory of black holes.

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Among the many remarkable features:
gravitational forces are completely negligible in (A), while gravitational forces are extremely strong in (B).

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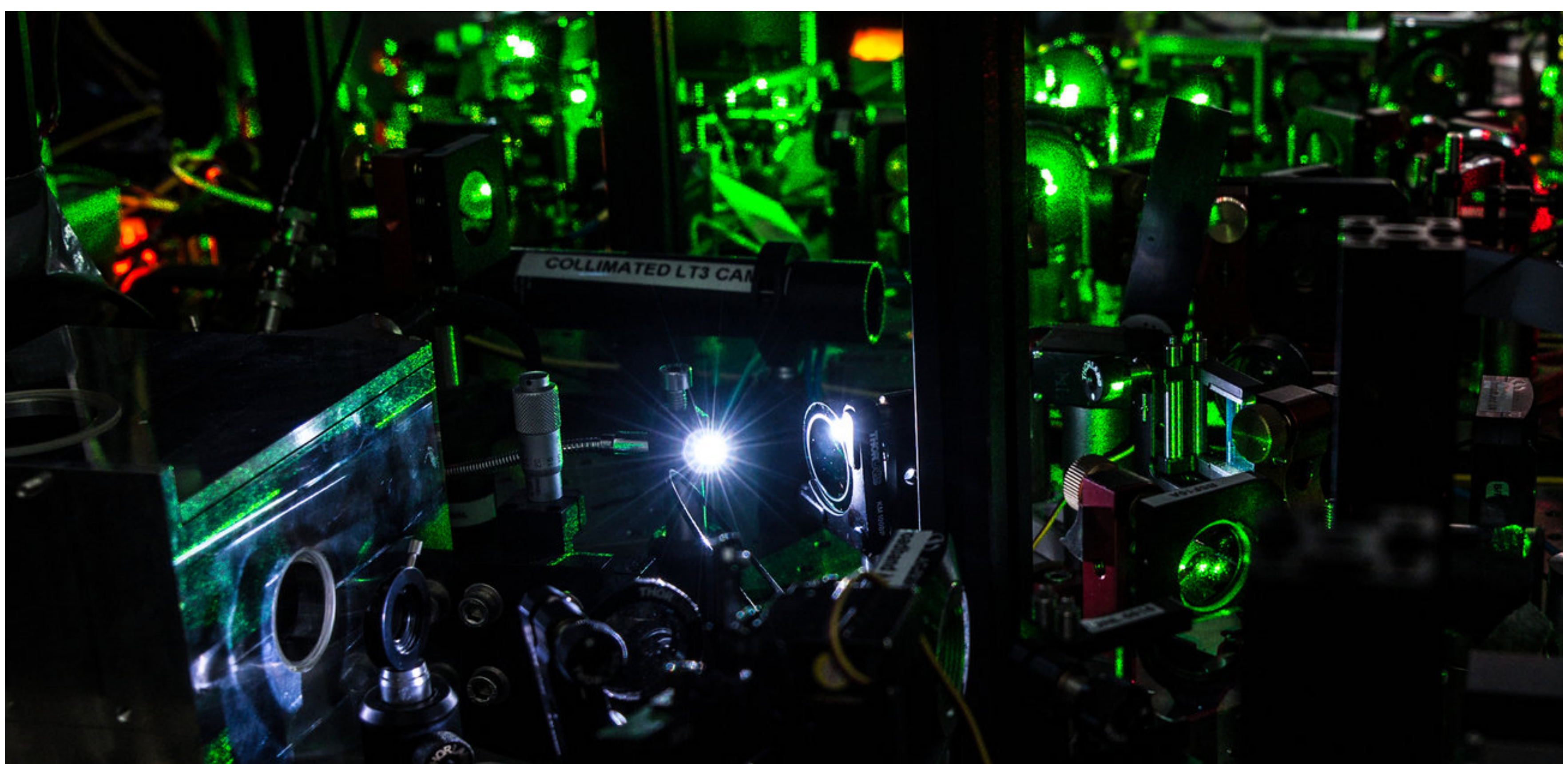
I will illustrate this connection using
a simple example: the Sachdev-Ye-Kitaev (SYK) model which describes
certain quantum many particle systems
and
certain black holes

Sorry, Einstein. Quantum Study Suggests ‘Spooky Action’ Is Real.

By **JOHN MARKOFF** OCT. 21, 2015

In a landmark study, scientists at Delft University of Technology in the Netherlands reported that they had conducted an experiment that they say proved one of the most fundamental claims of quantum theory — that objects separated by great distance can instantaneously affect each other’s behavior.

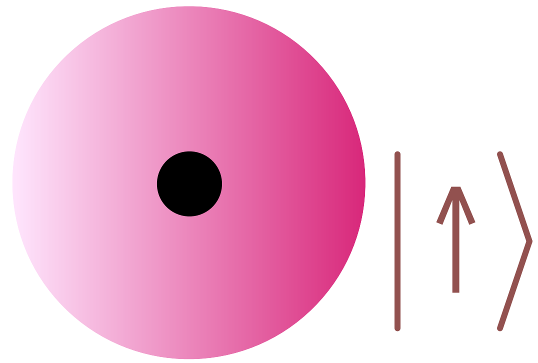


Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

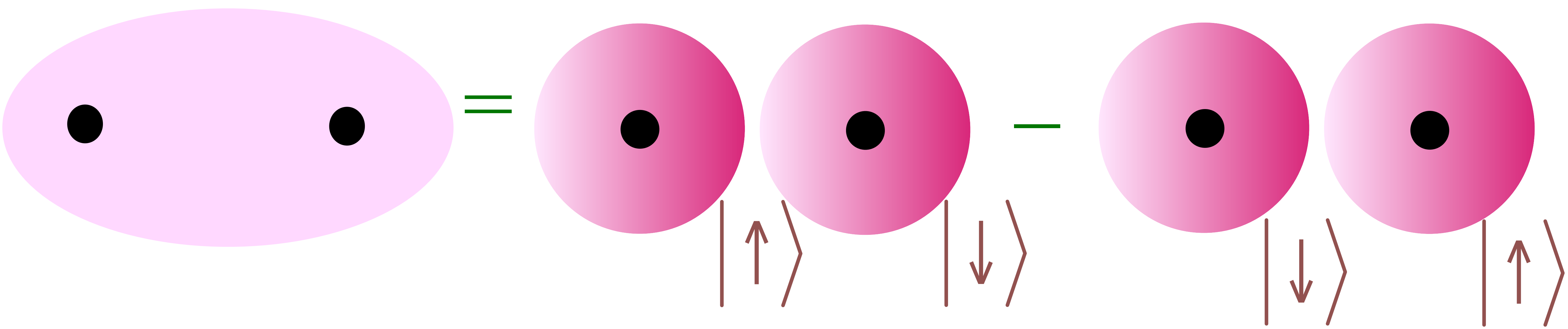
Quantum entanglement

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

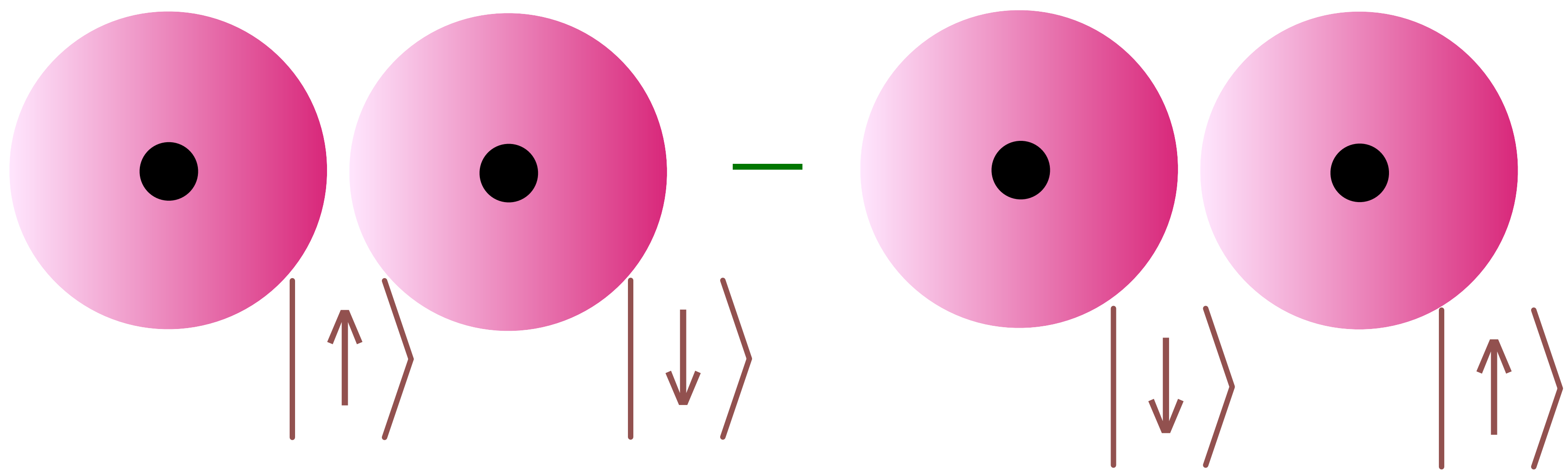


Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Quantum Entanglement: quantum superposition with more than one particle



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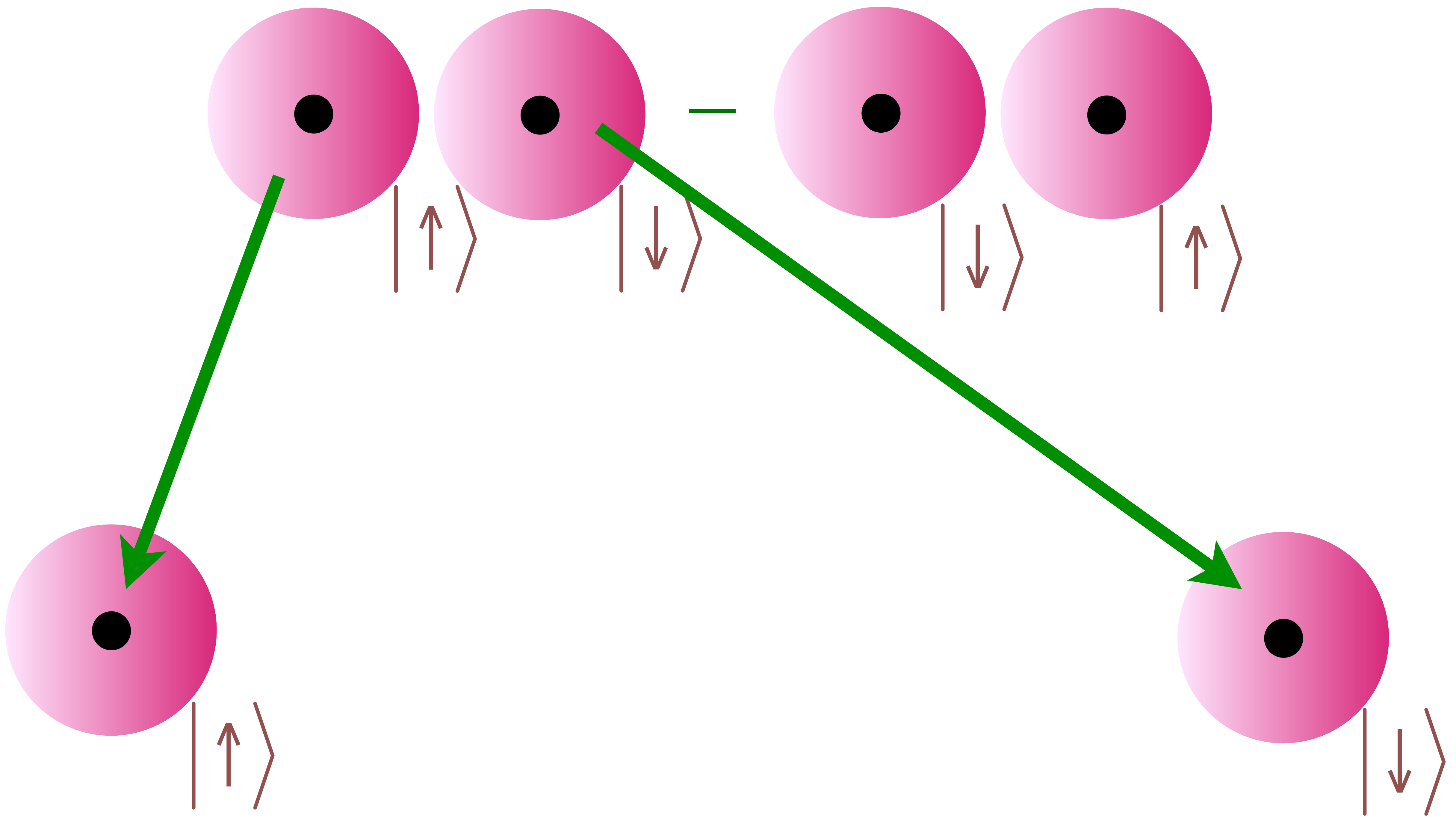

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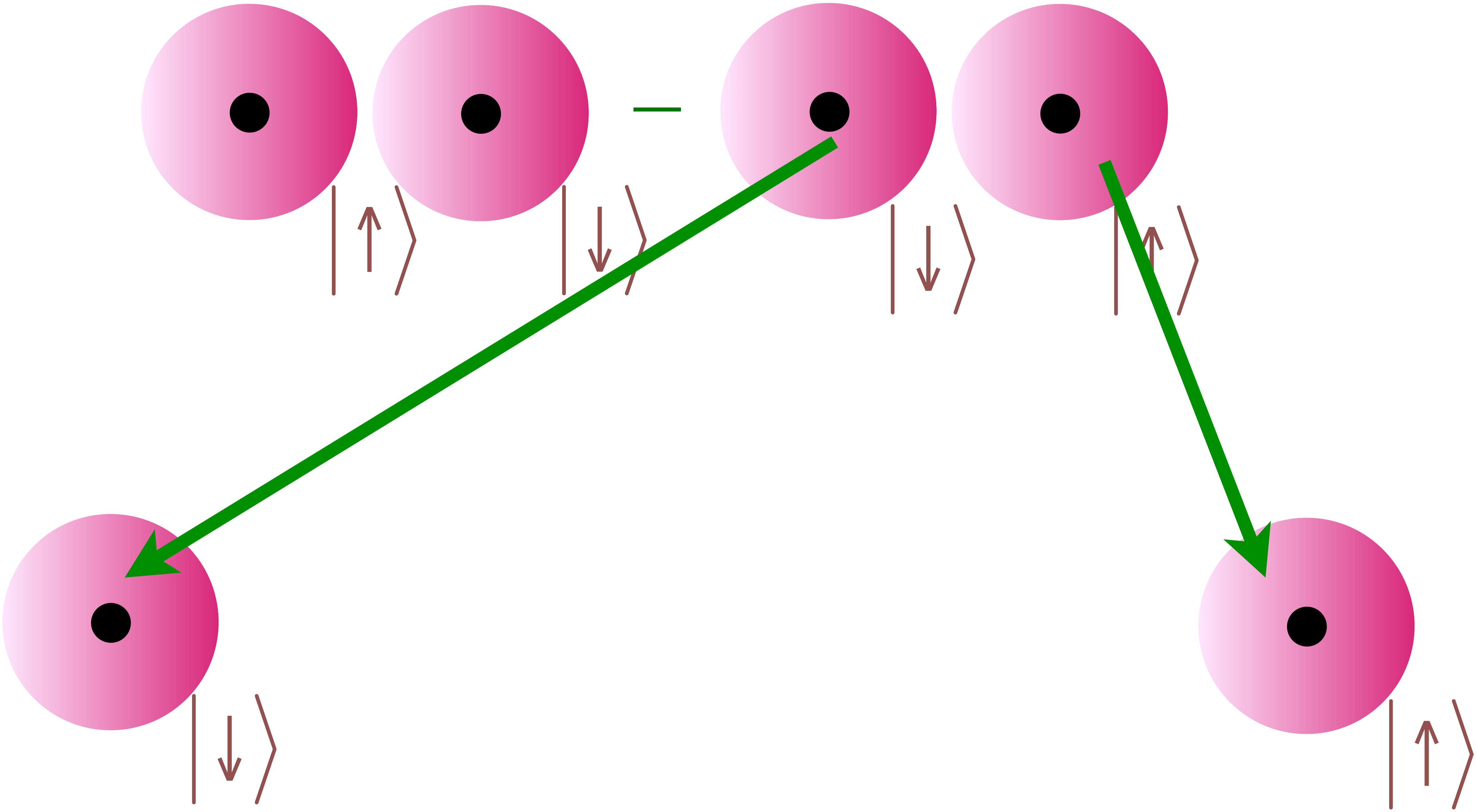

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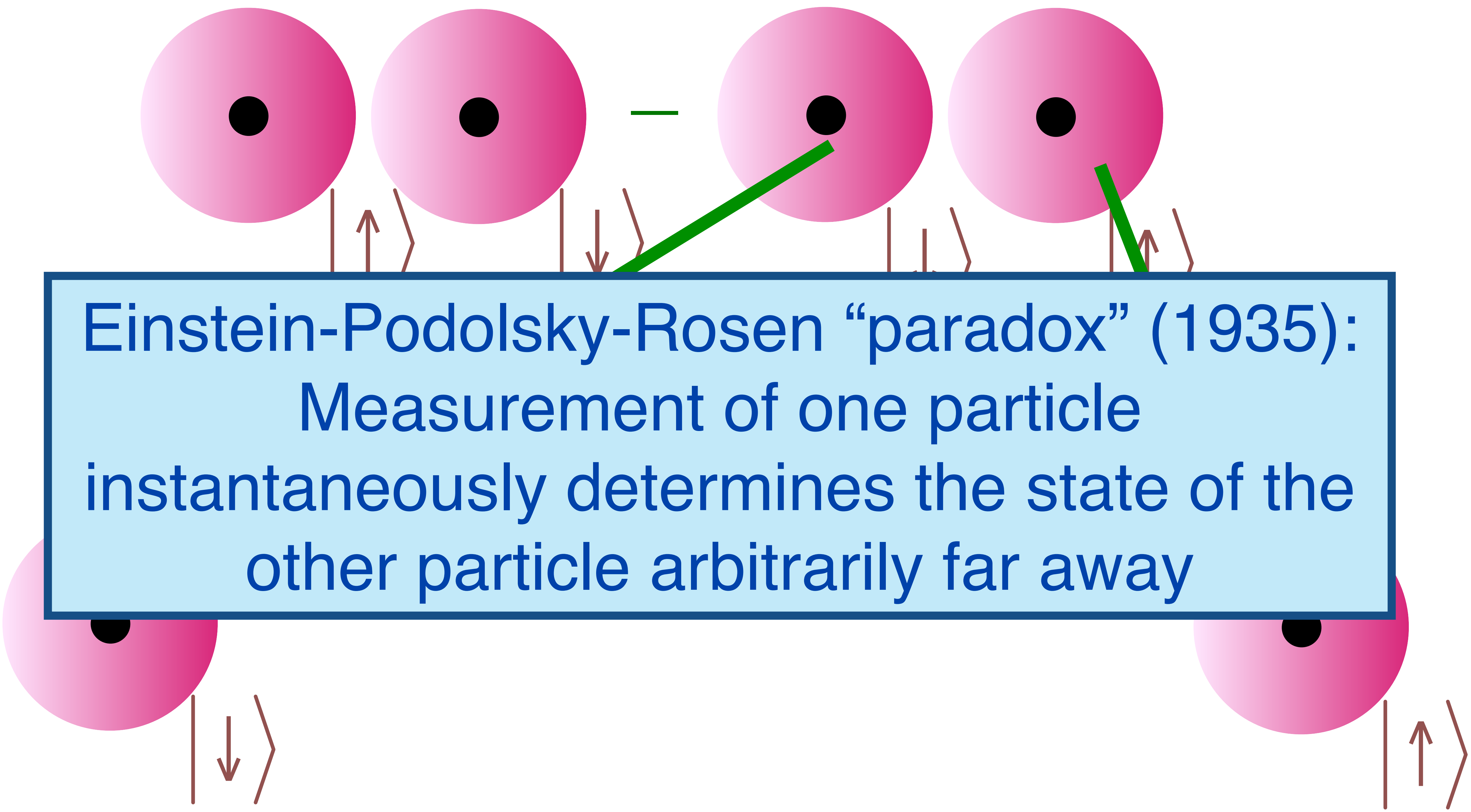
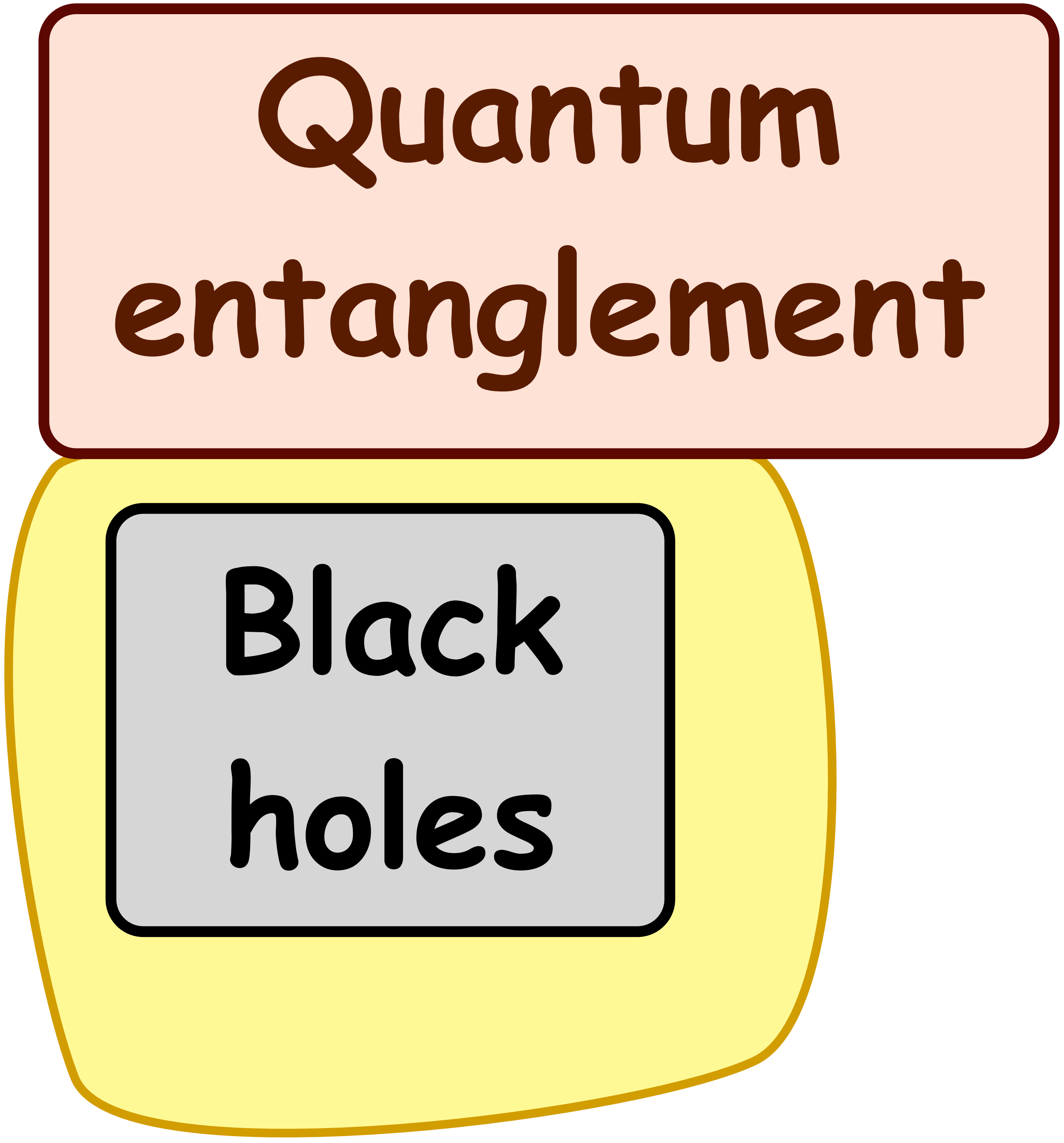
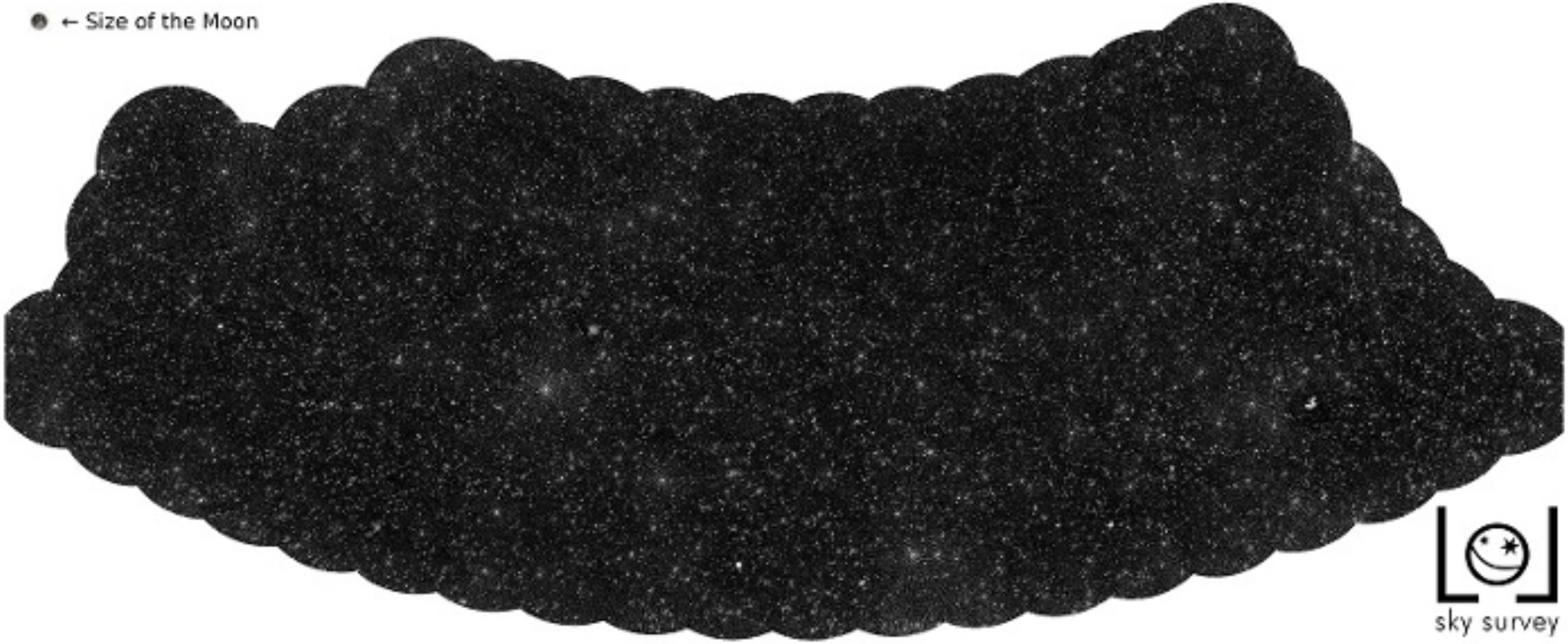


Einstein-Podolsky-Rosen "paradox" (1935): Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away

Quantum entanglement


Quantum entanglement
Black holes



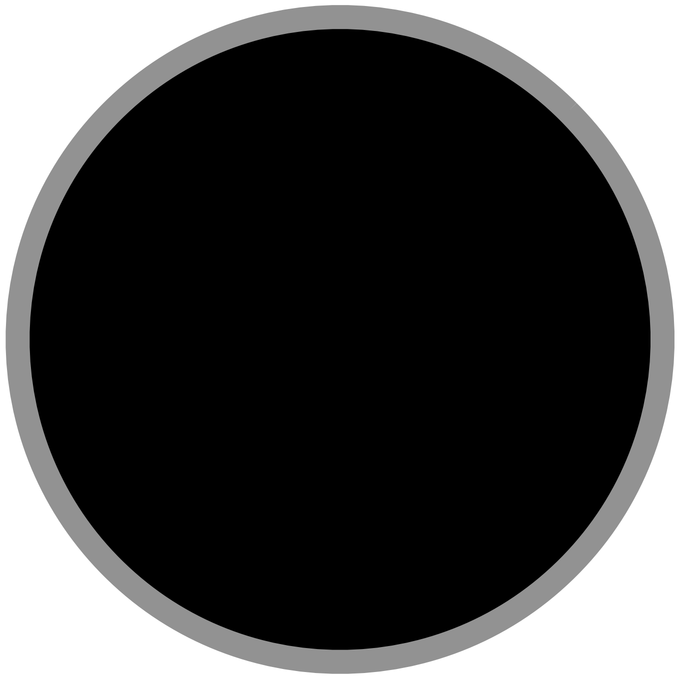
LOFAR LBA Sky Survey showing 25000 supermassive black holes on 4% of the northern sky. Obtained by 52 radio telescopes across Europe

de Gasperin et al. (2021)

Black Holes

Objects so dense that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.



$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

G Newton's constant, c velocity of light, M mass of black hole

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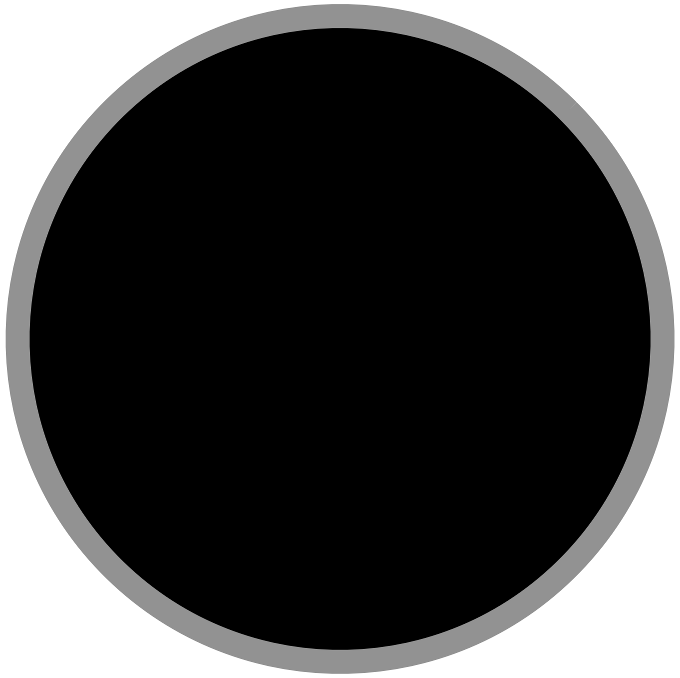

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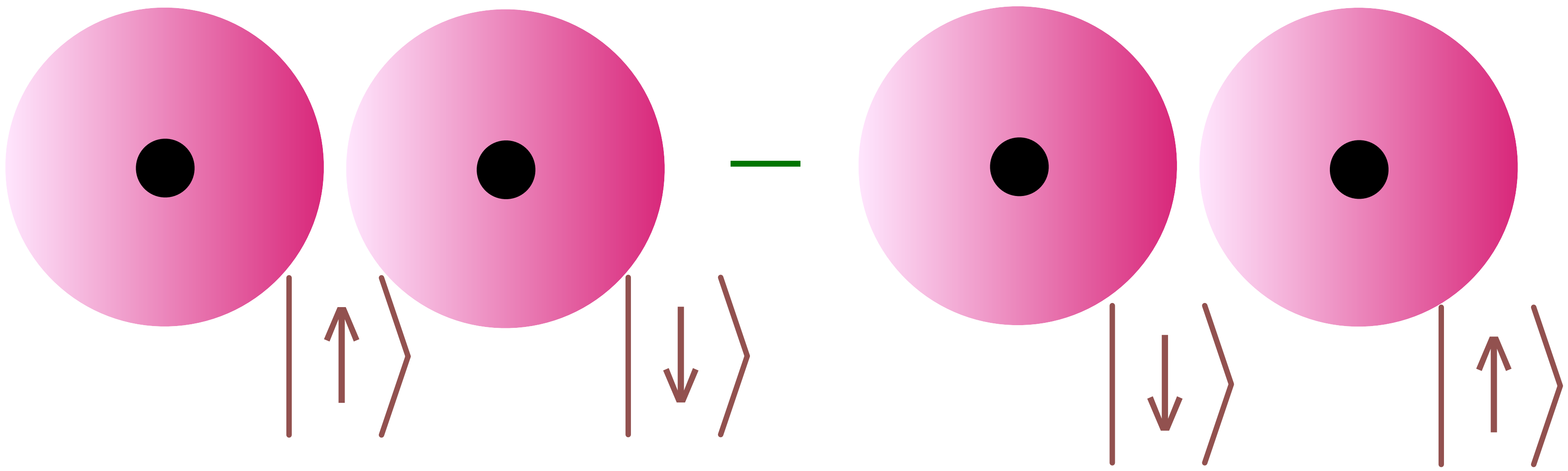
For $M =$ earth's mass, $R \approx 9\,mm$!

Quantum Entanglement across a black hole horizon



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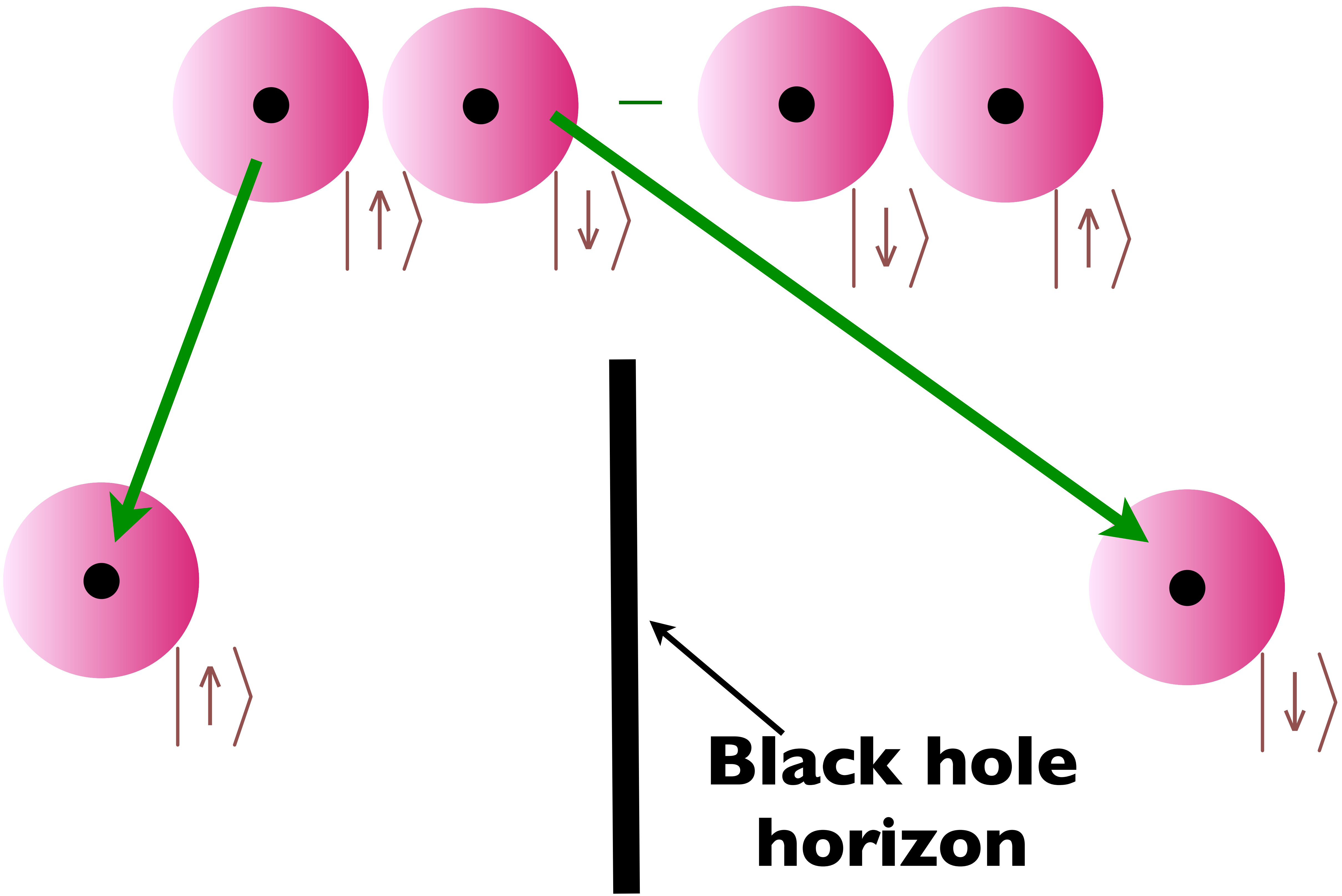

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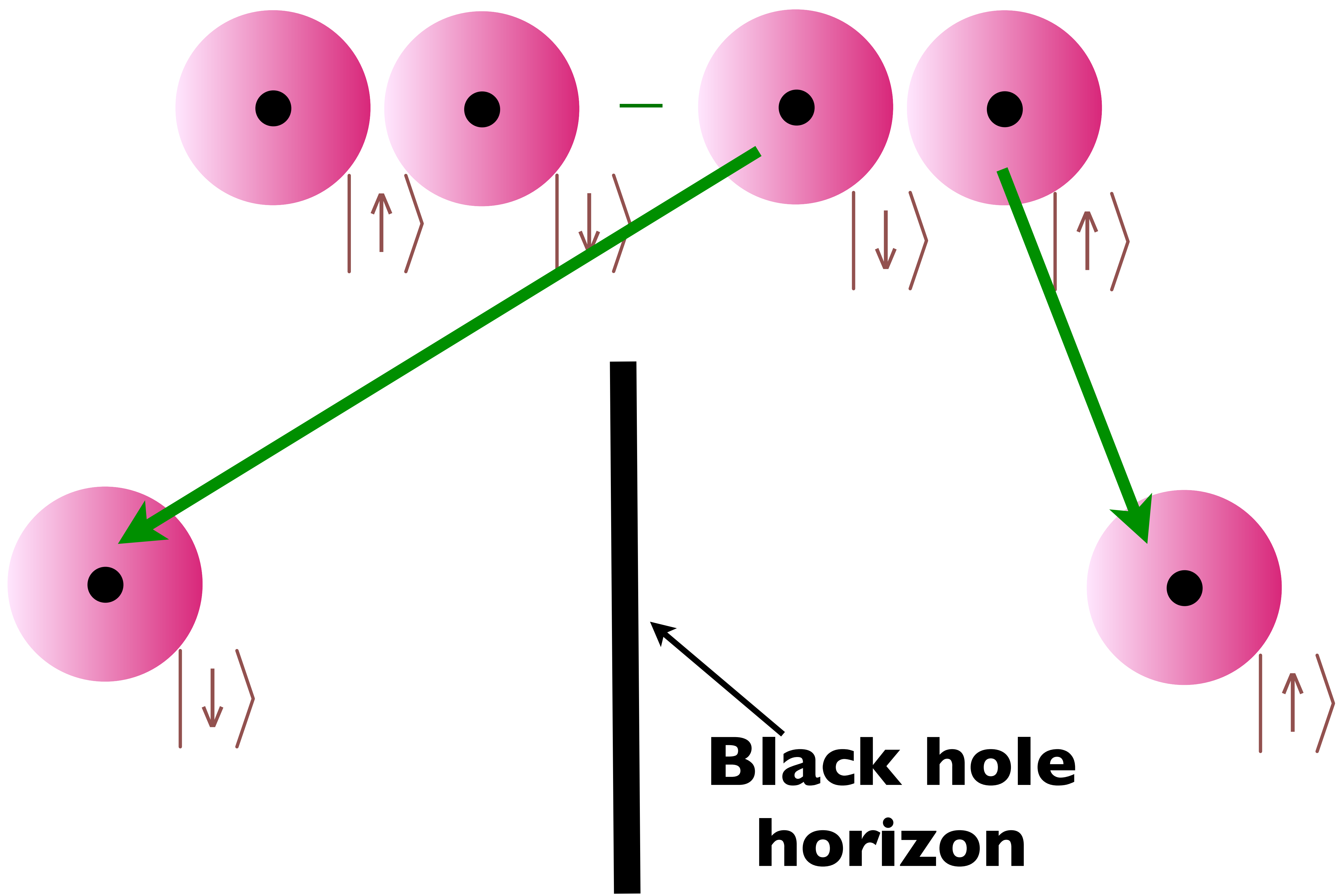
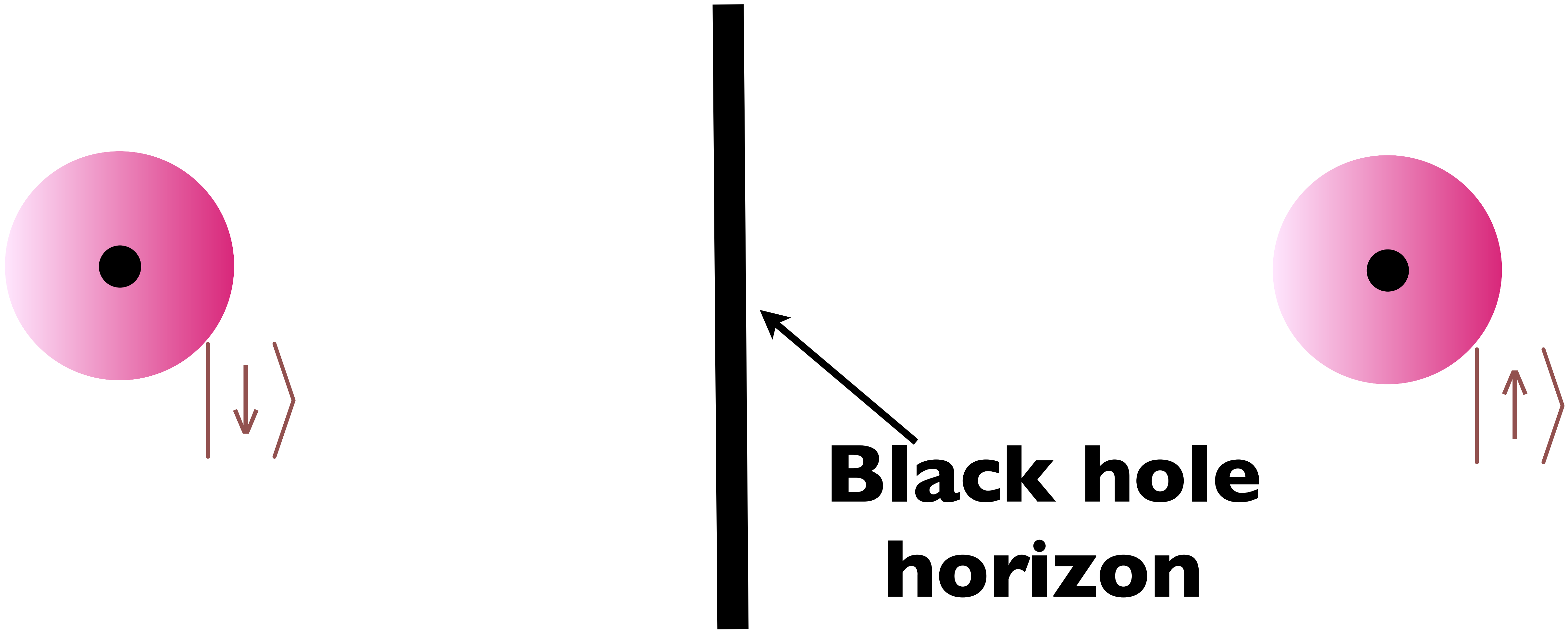


Quantum Entanglement across a black hole horizon

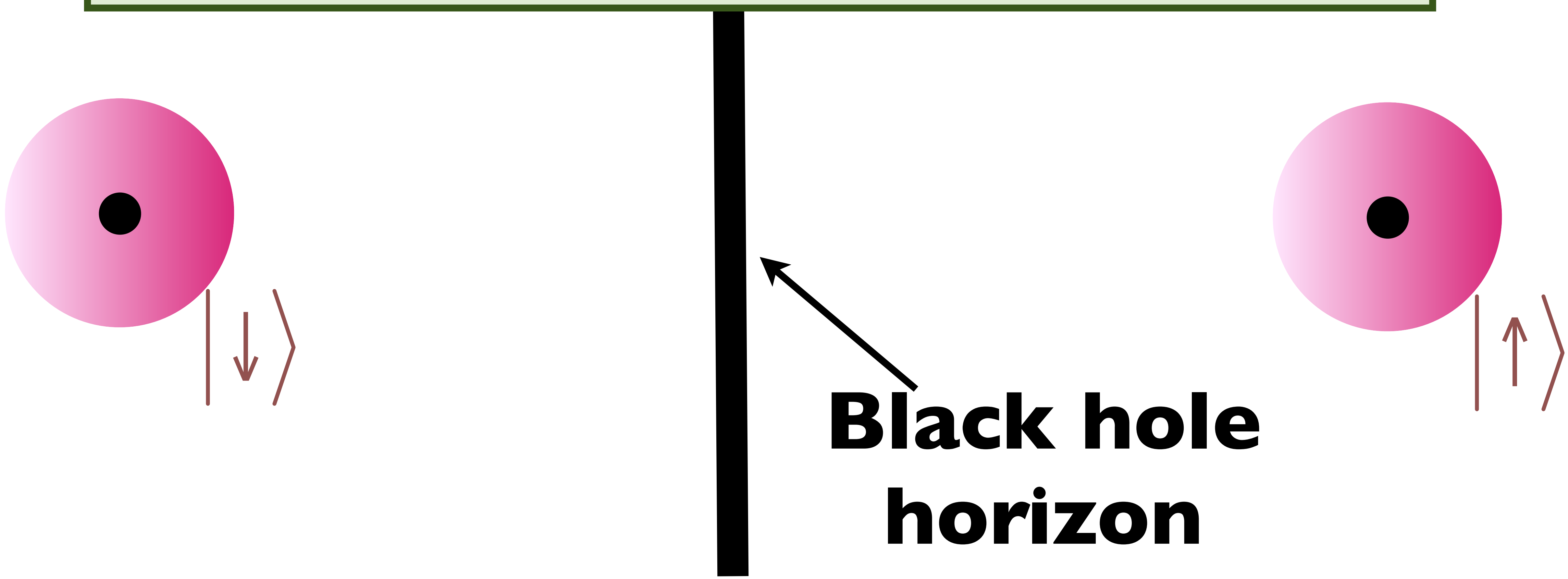
There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature

(because to an outside observer, the state of the electron inside the black hole is an unknown)



Quantum Black holes

- Black holes have an entropy and a temperature, T_H.
- The entropy, S_{BH} is proportional to their surface area.

J. D. Bekenstein, PRD **7**, 2333 (1973)
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All many-body quantum systems (without quantum gravity) have an entropy proportional to their volume !?!?

Thermodynamics of quantum black holes:

$$\int \mathcal{D} g_{\mu\nu} \exp\left(-\frac{1}{\hbar}\, \mathcal{S}^{(d+1)}_{\text{Einstein gravity}}[g_{\mu\nu}] \right)$$

Metric of spacetime

Quantum gravity: a summation over
all possible configurations of spacetime,
each weighted by a factor which is the exponential of
(the 'action' of Einstein gravity)/(Planck's constant)

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Metric of spacetime

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In general, this summation is not well defined, because to the uncontrollably large number of spacetime configurations. But in recent years, we have learnt how to evaluate the summation for Einstein gravity in some cases, and obtain the exact value of ....????.... thanks to connections to the *Sachdev-Ye-Kitaev* model.

Holography and duality

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$$= \exp\left(S_{BH}\right) \times \Big(\text{Many body quantum theory}$$

$$\text{in } d-1 \text{ spatial dimensions } \textit{without} \text{ gravity}\Big)$$

Metric of spacetime

Black holes are represented as a `*hologram*' by a quantum many-body system in one lower dimension.

Duality: a `*change of variables*' between the many-particle configurations and the metric of spacetime

Susskind, Maldacena…..

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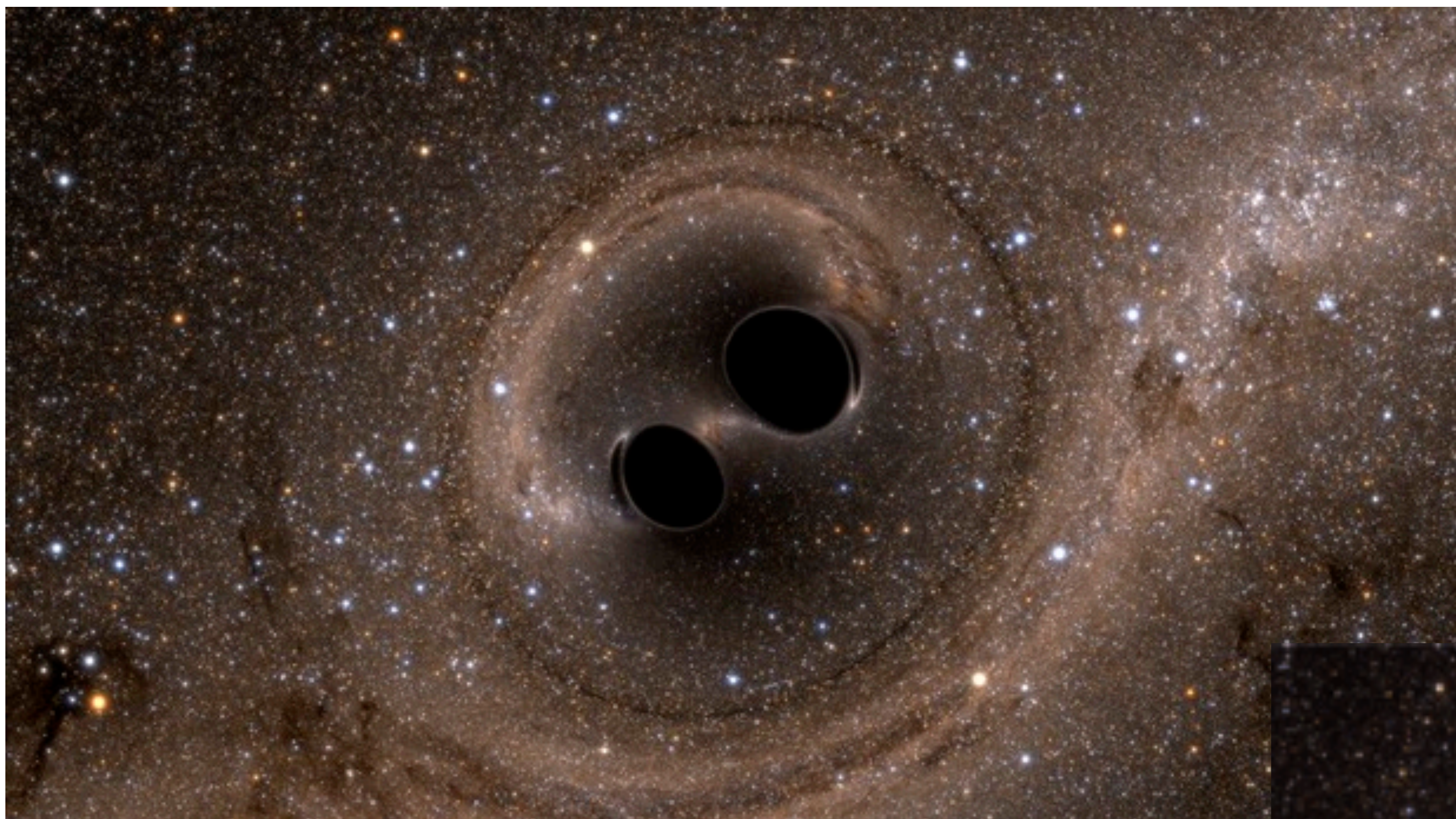
On September 14, 2015, LIGO detected the merger of two black holes, each weighing about 30 solar masses, with radii of about 100 km, 1.3 billion light years away



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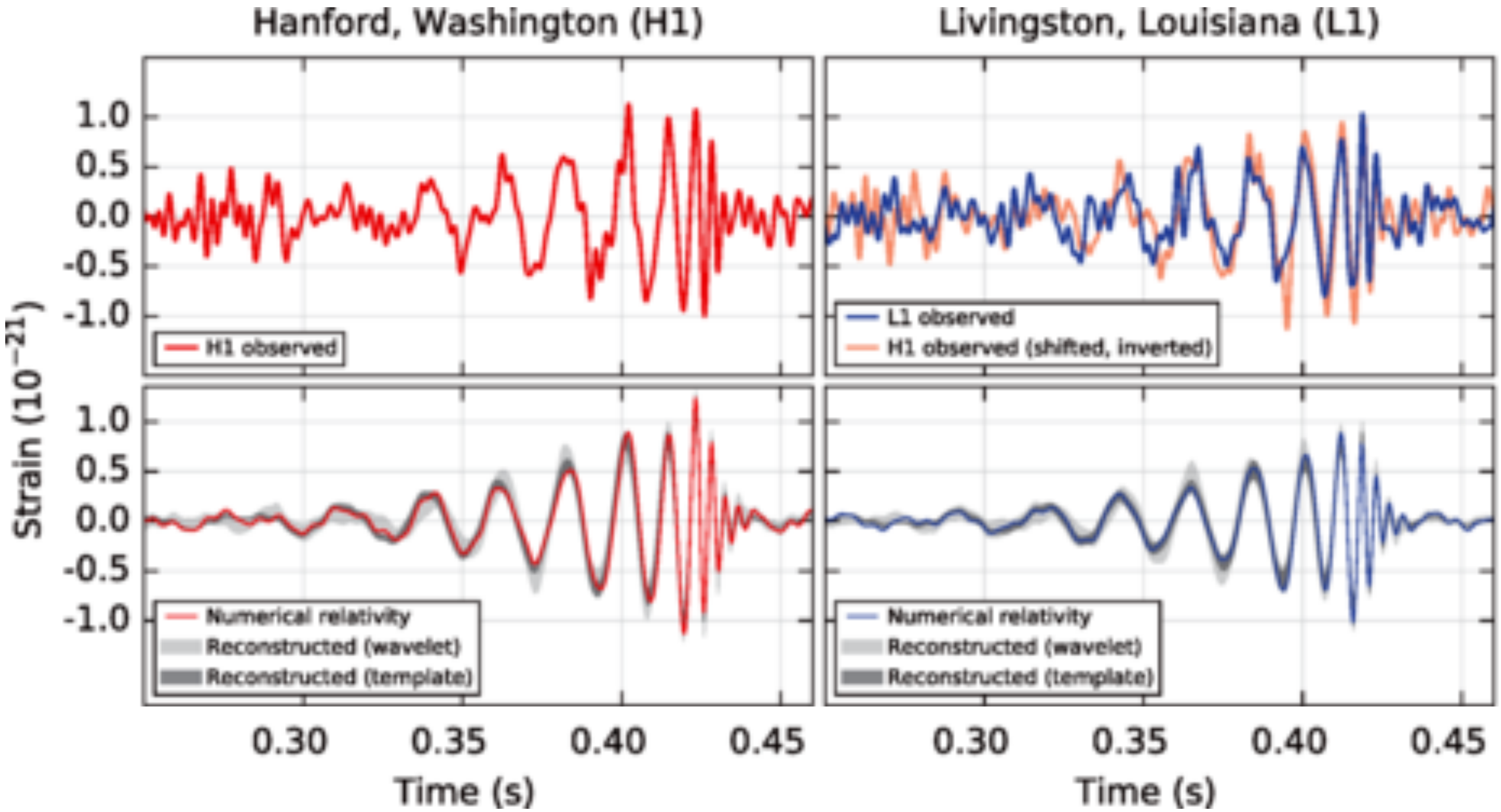

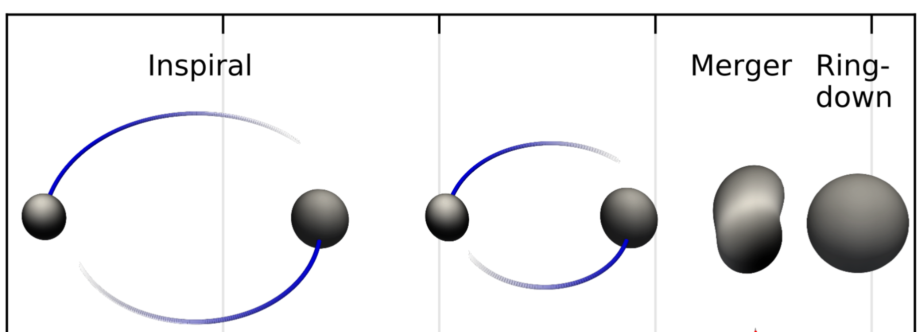

LIGO
September 14, 2015

- The ring-down time $\dfrac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously, for essentially all types of black holes, the ring-down time equals

$$\frac{\hbar}{k_B T_H}, \quad \hbar \text{ Planck's constant}, \quad k_B \text{ Boltzmann's constant}$$

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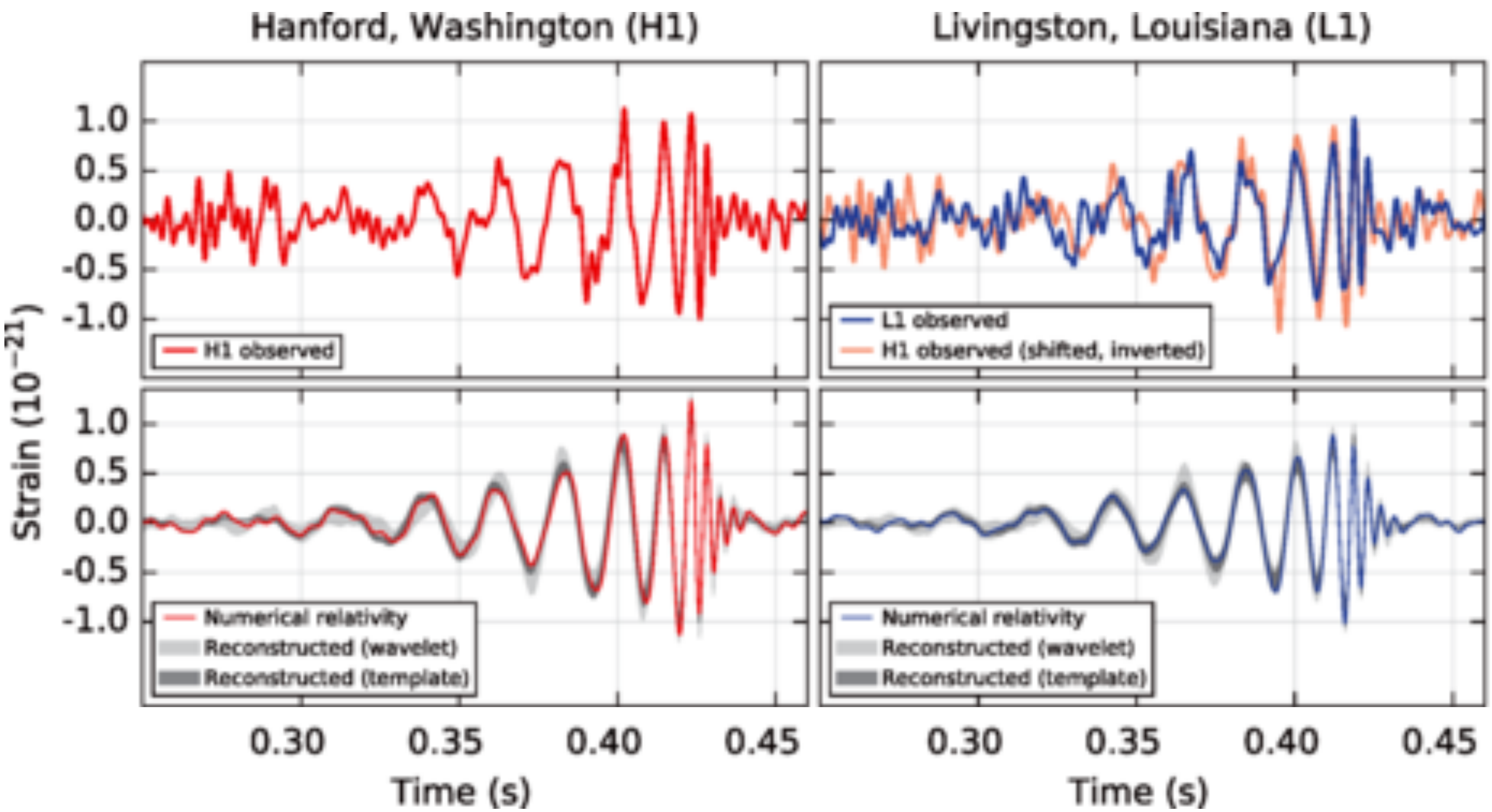

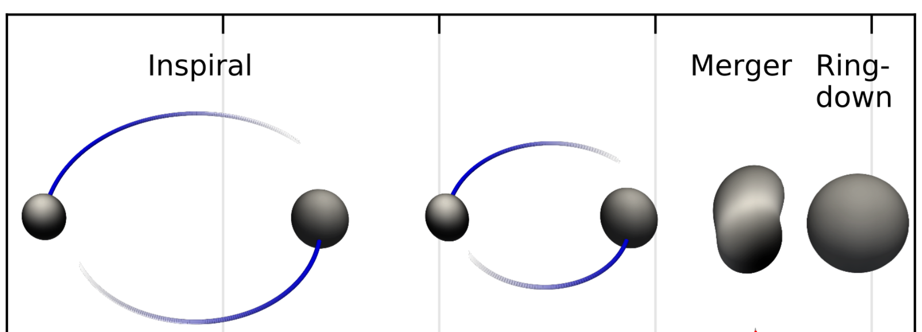

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The hologram of a black hole
in d dimensions
is a quantum many-particle system
in $(d-1)$ dimensions
which relaxes to thermal equilibrium
in a Planckian time $\sim \hbar/(k_B T)$

Quantum entanglement

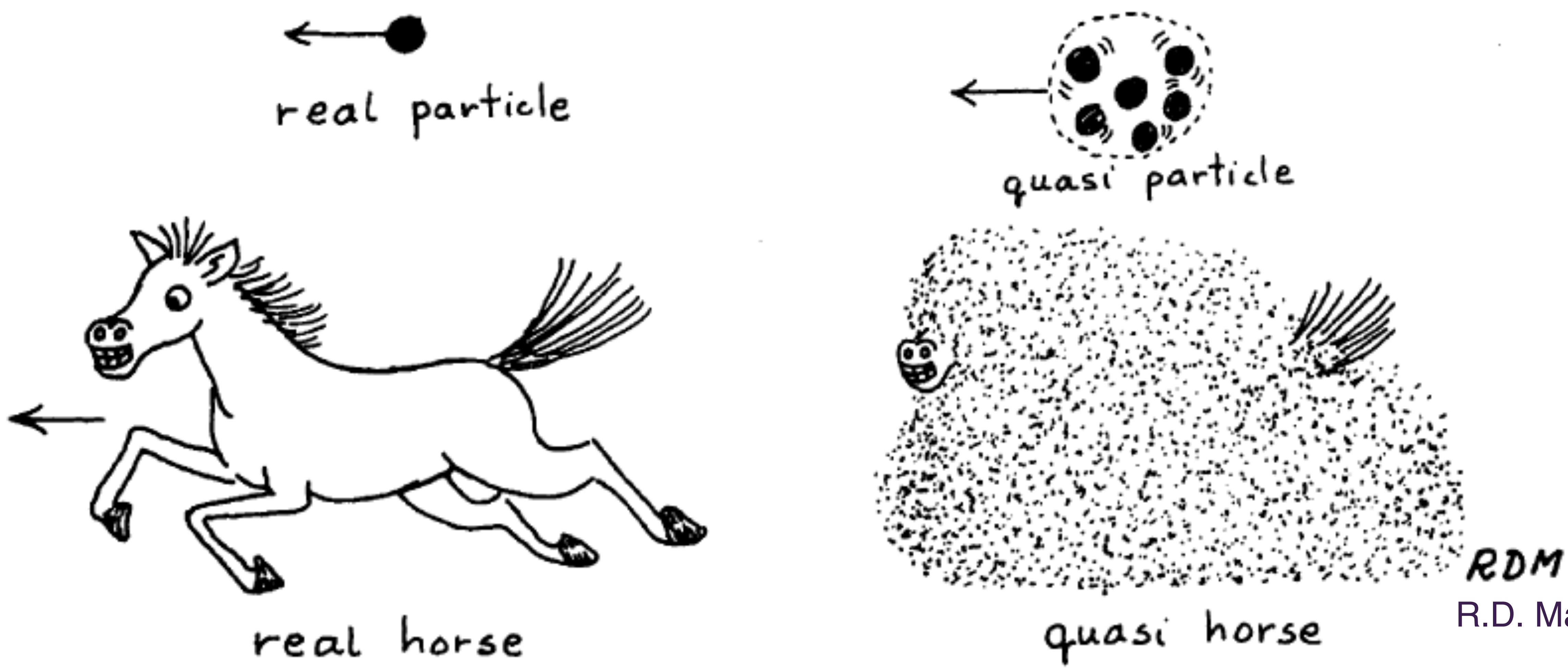
A simple many-particle (SYK) model

Black holes

Ordinary metals

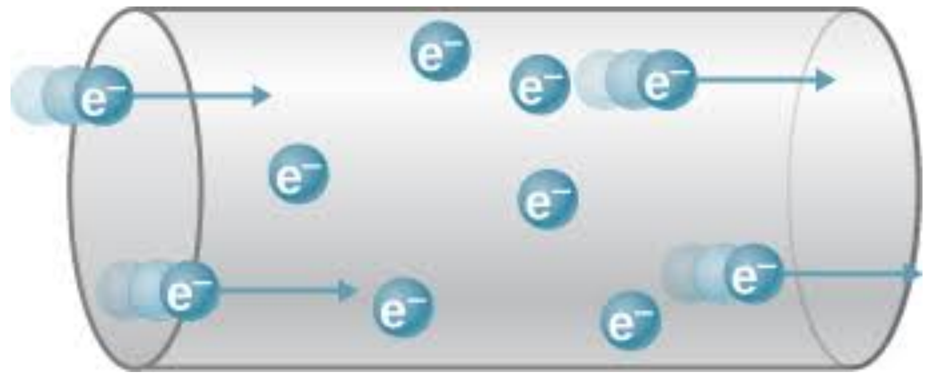
Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal

Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle. The existence of quasiparticles implies limited many-particle entanglement



R.D. Mattuck

Current flow with quasiparticles



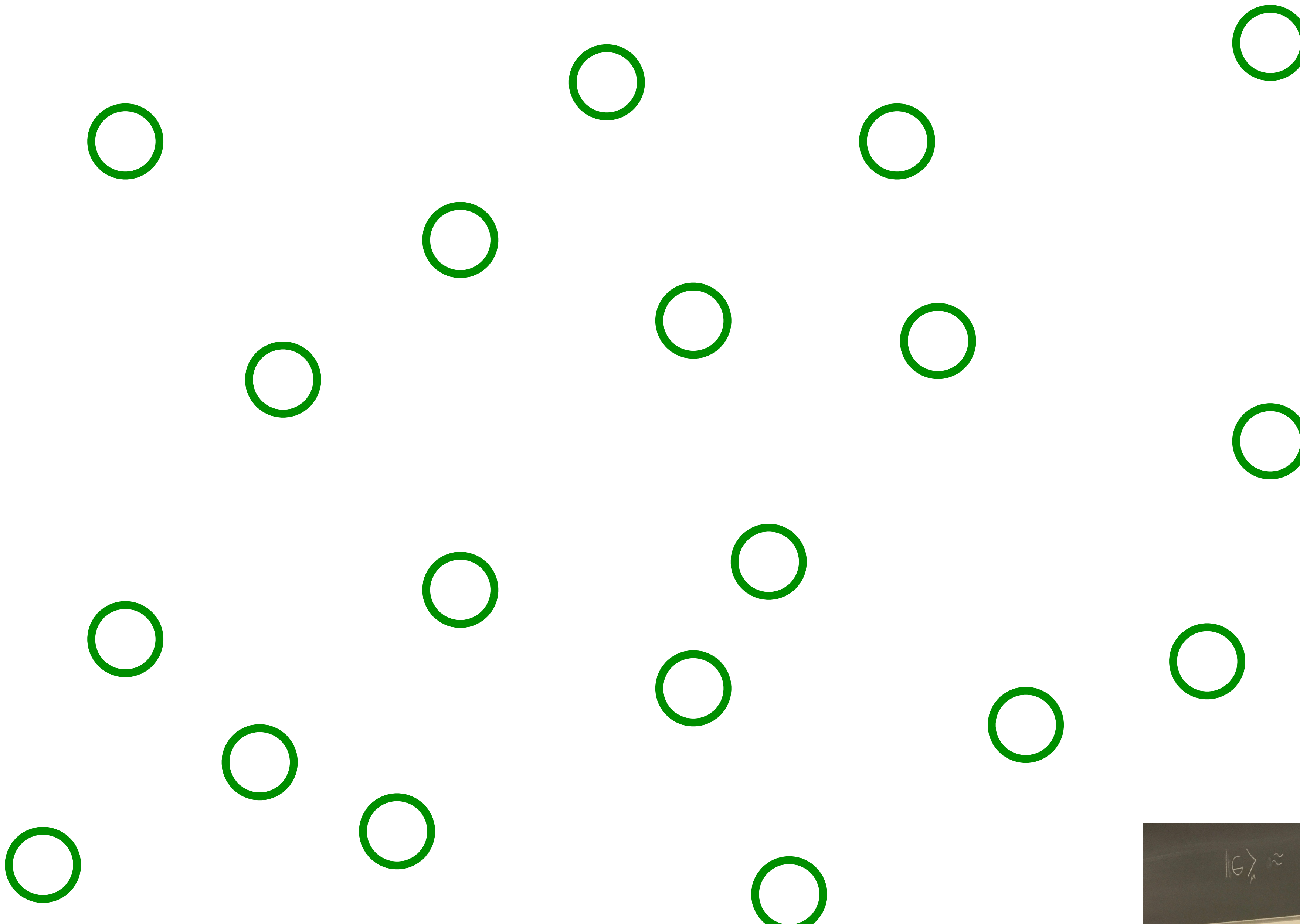
Flowing quasiparticles scatter off each other in a typical scattering time τ

This time is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

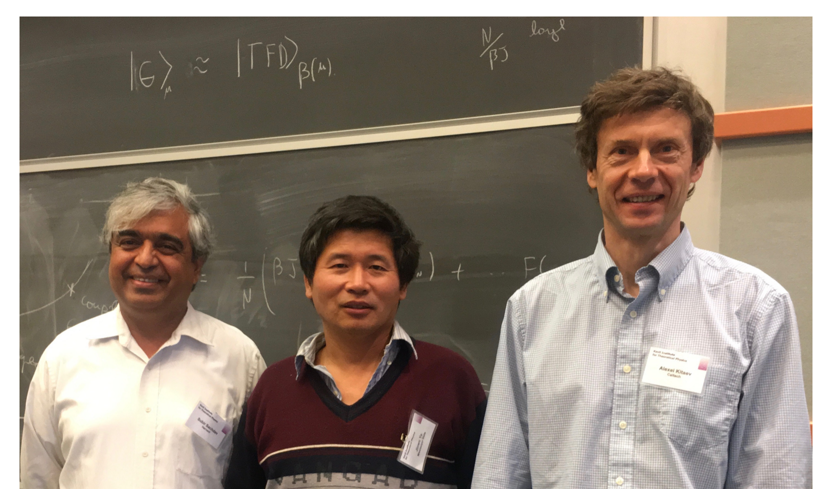
The long scattering time implies that quasiparticles are well-defined.

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)



Pick a set of random positions



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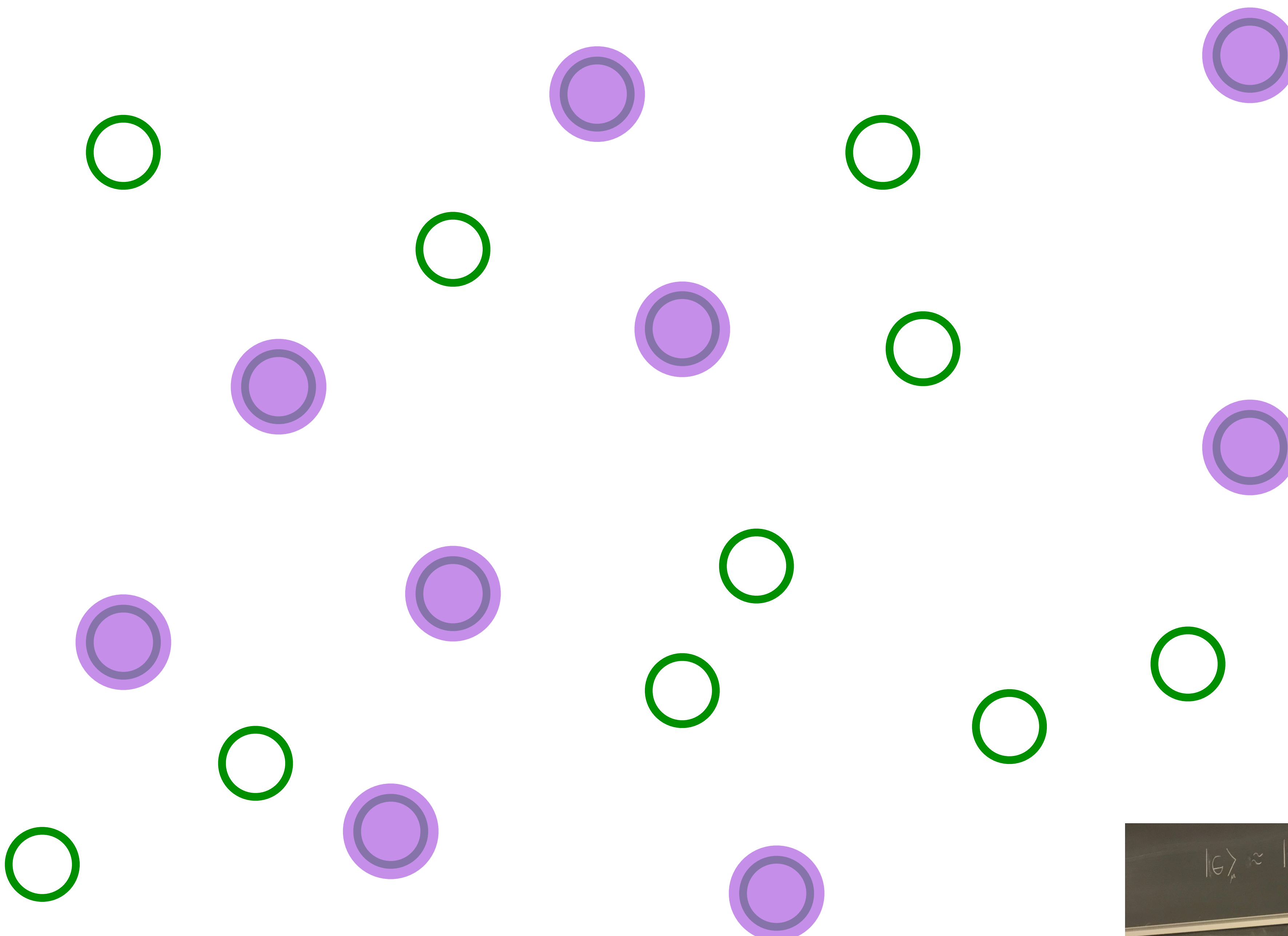
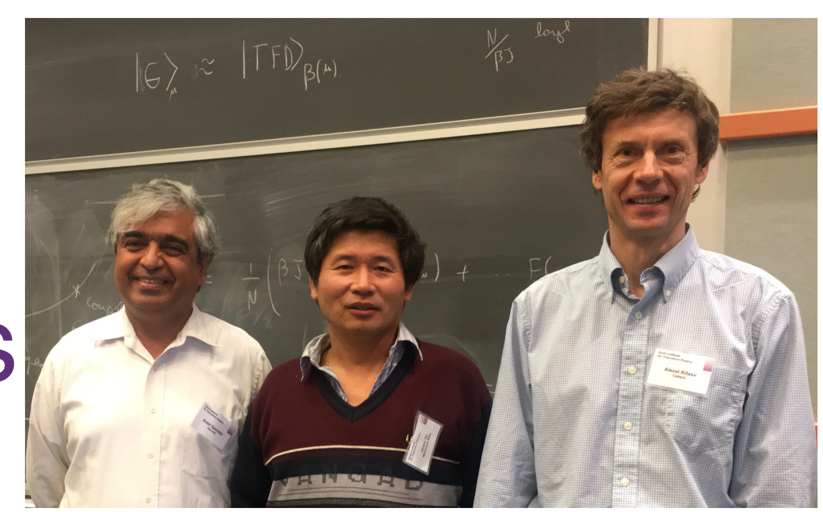


Place electrons randomly on some sites



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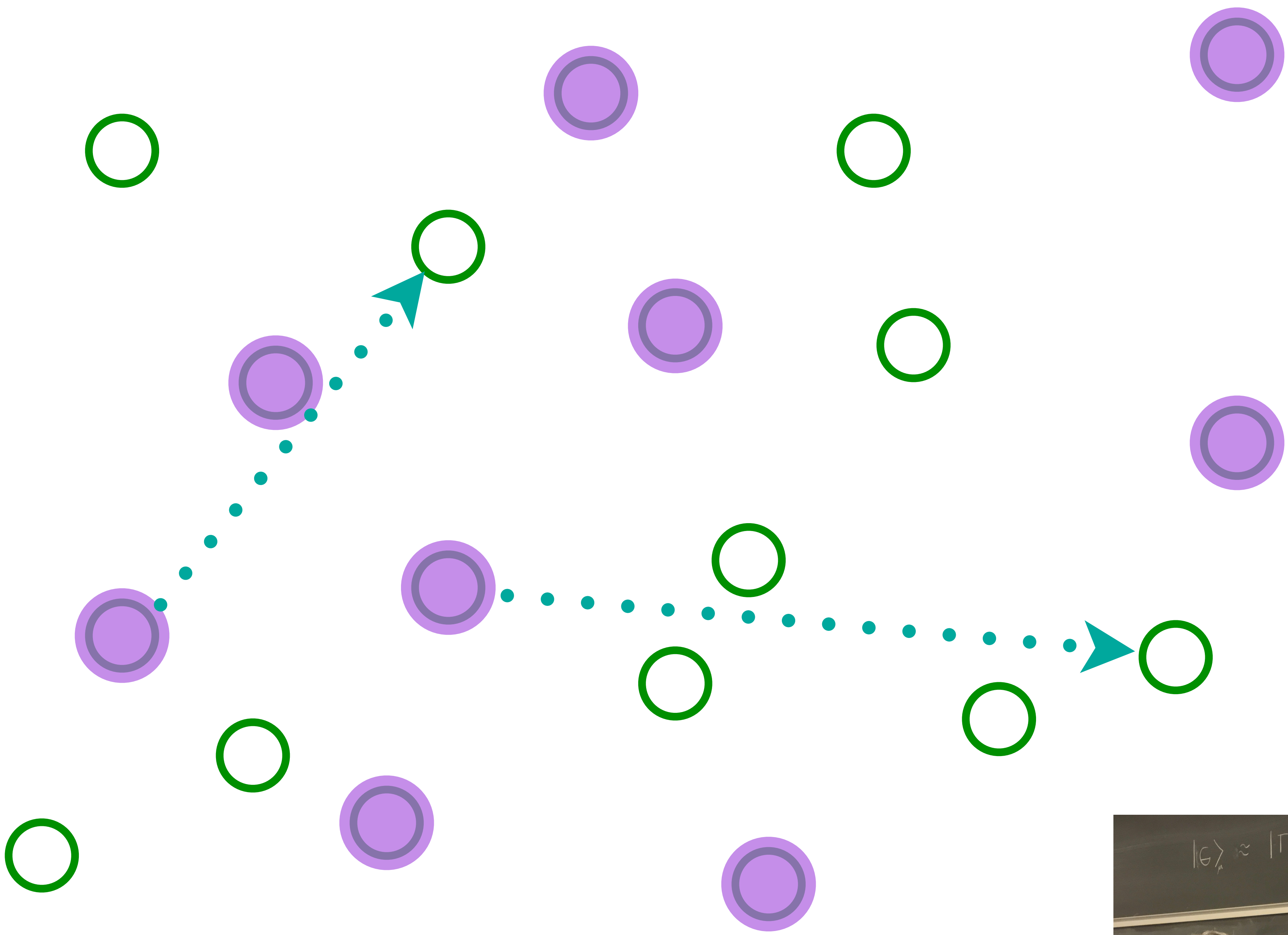

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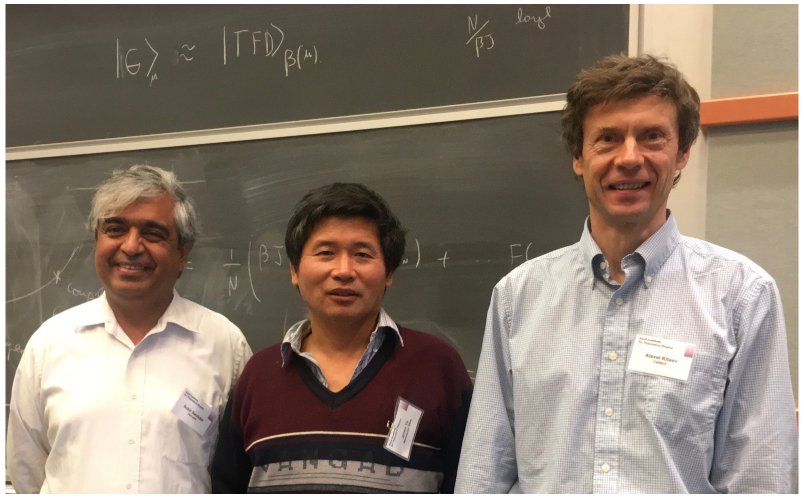

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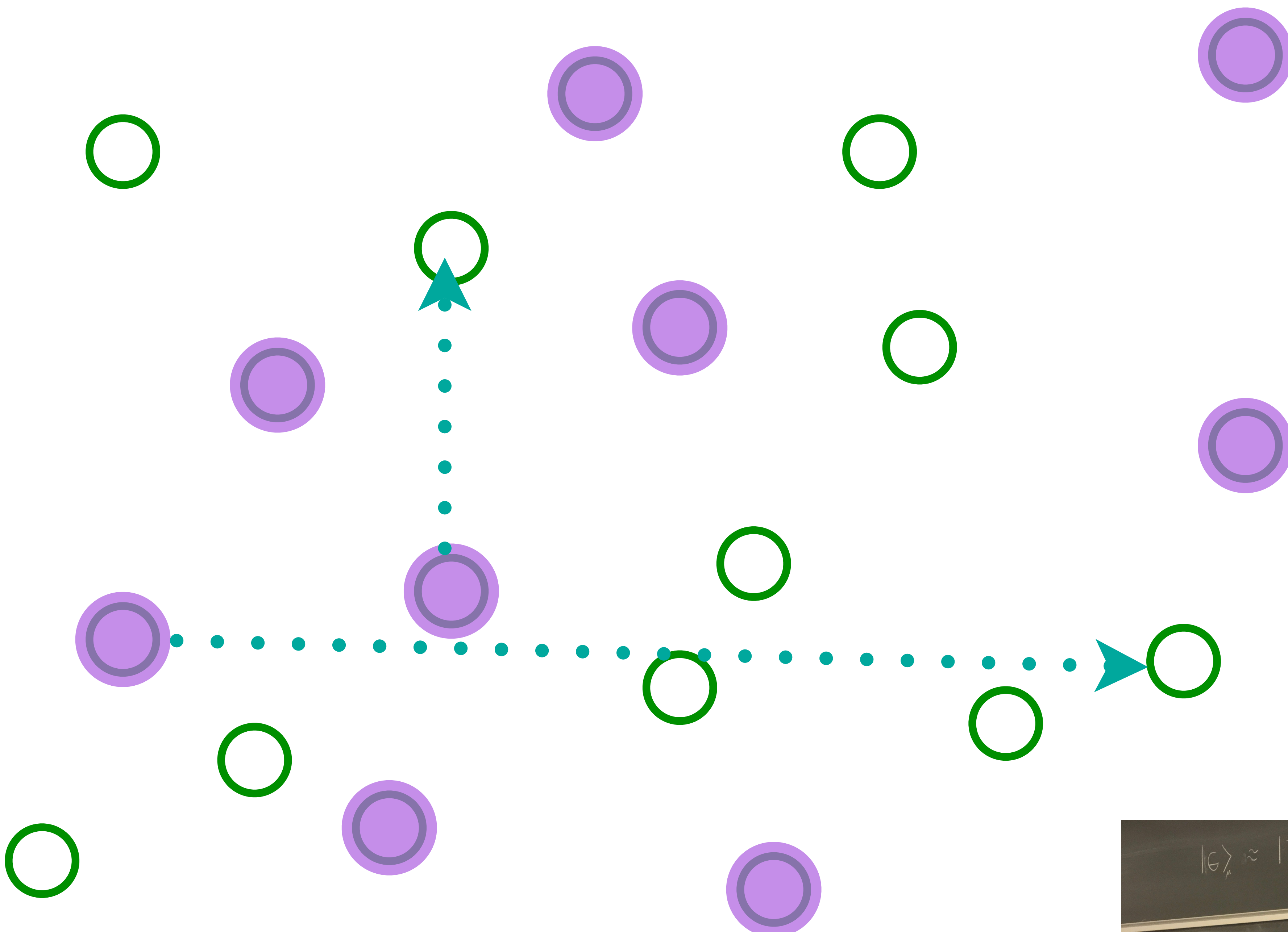

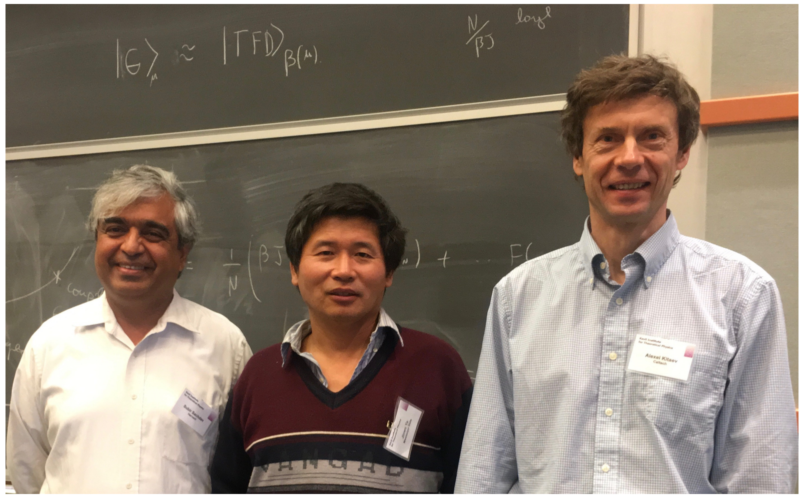

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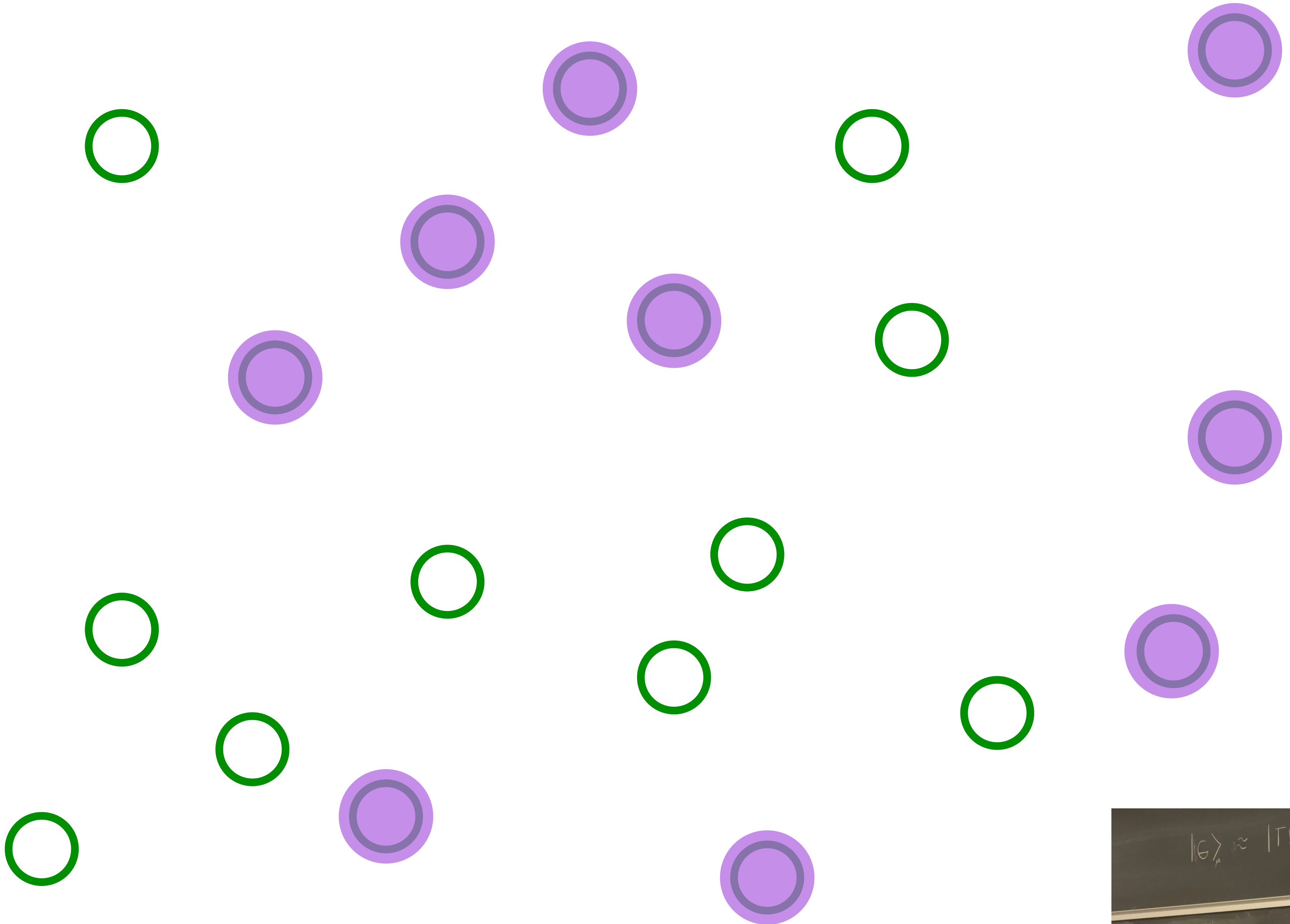
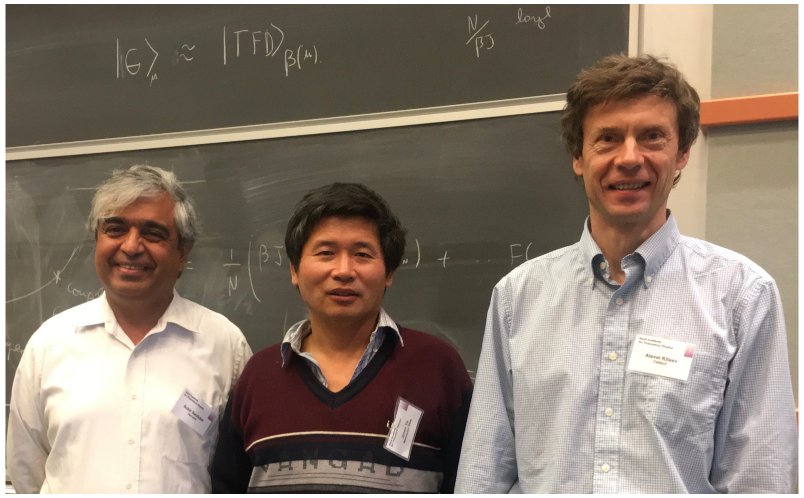


Entangle electrons pairwise randomly



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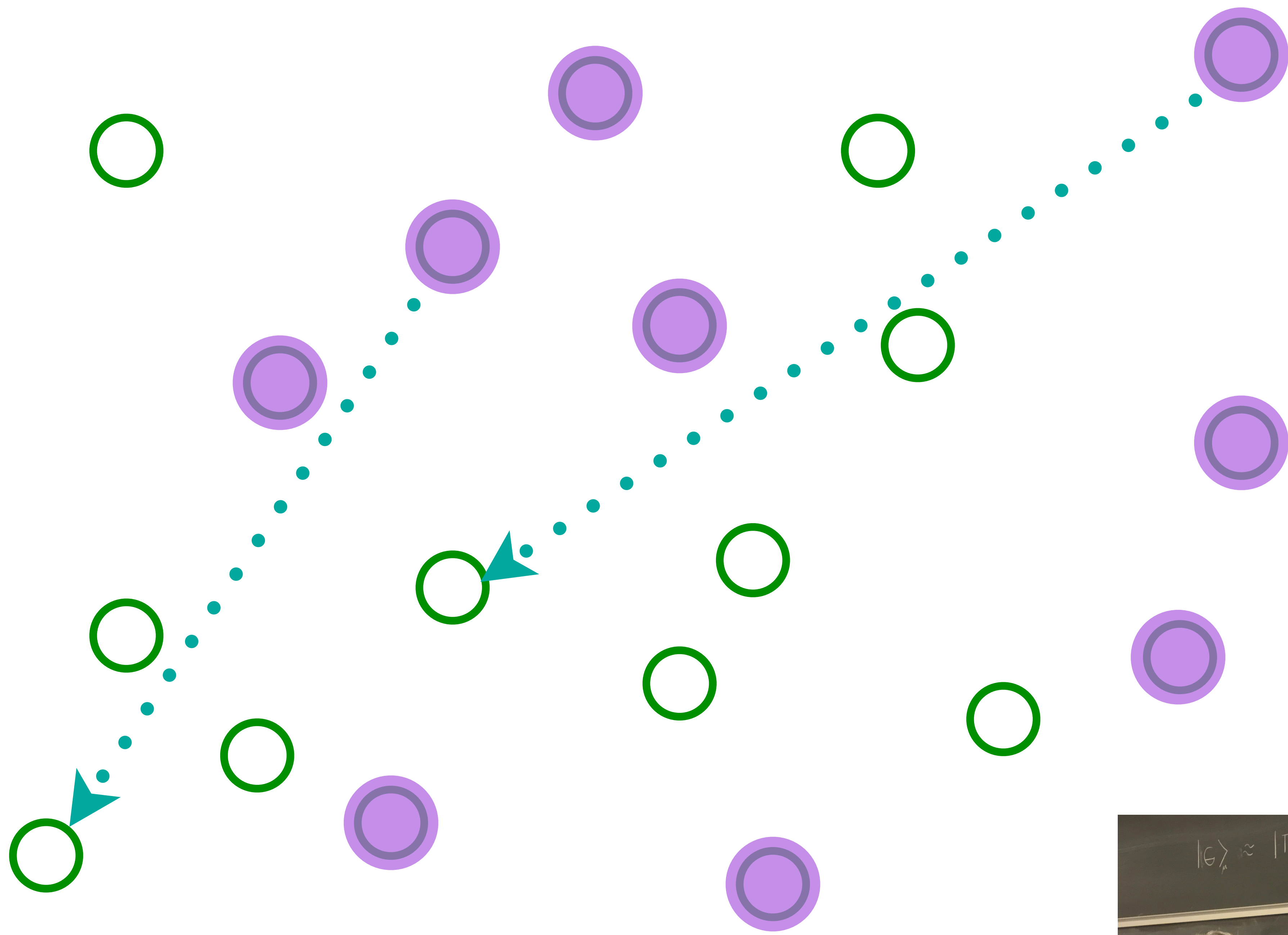

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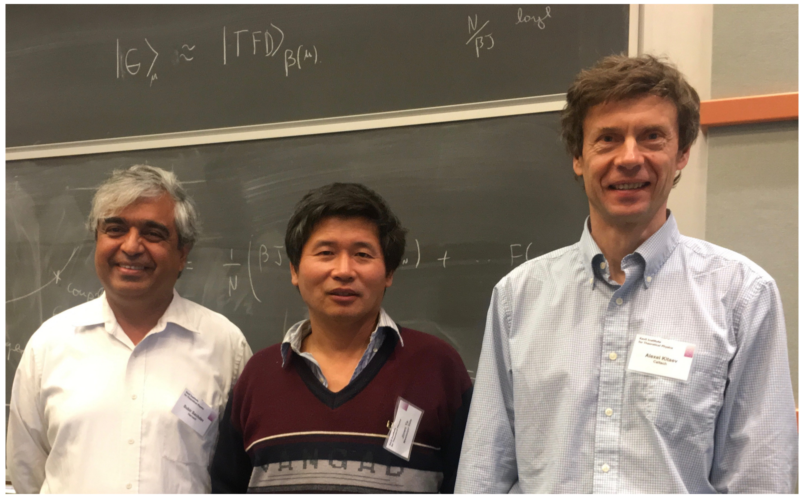

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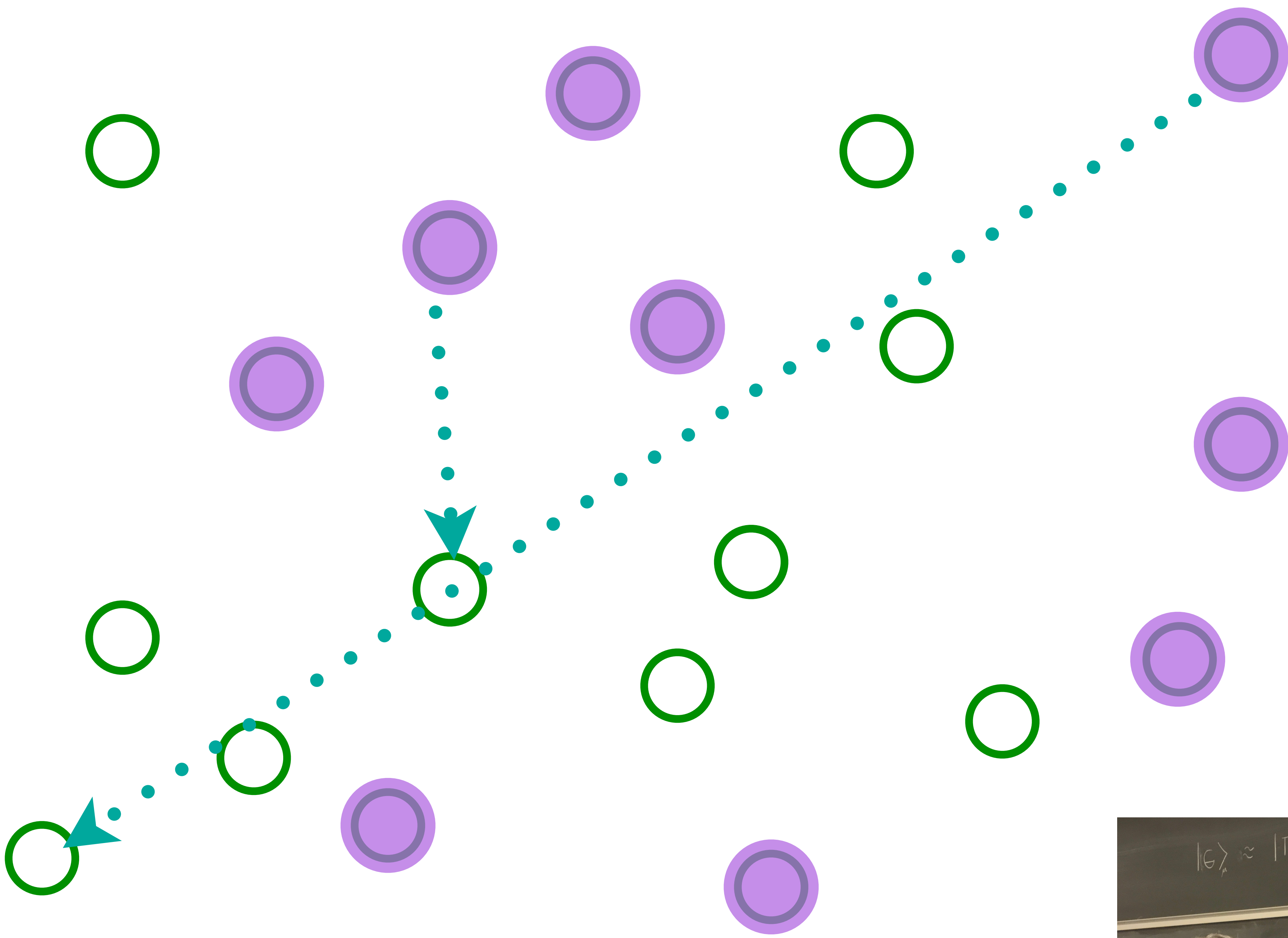

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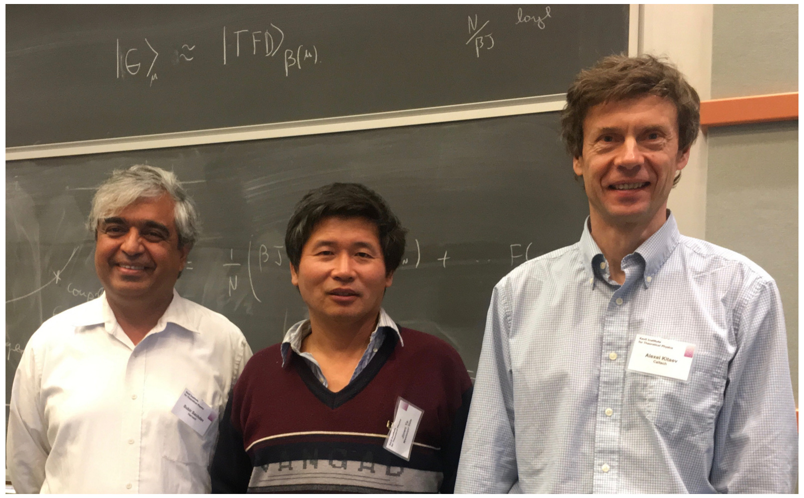

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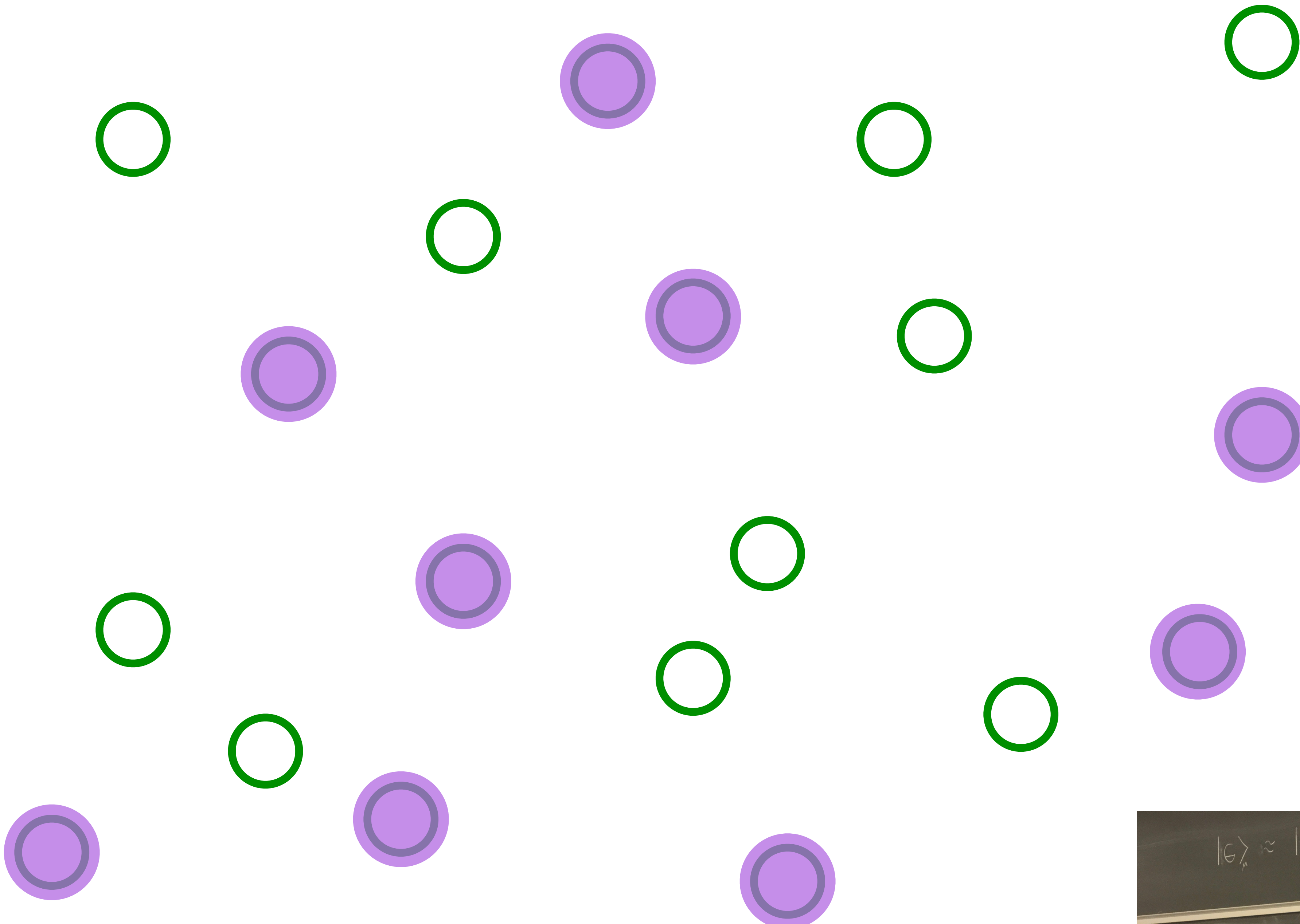

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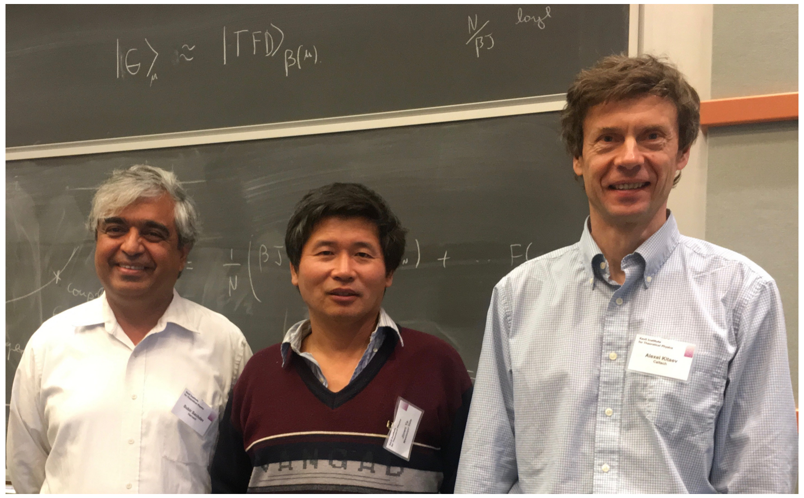

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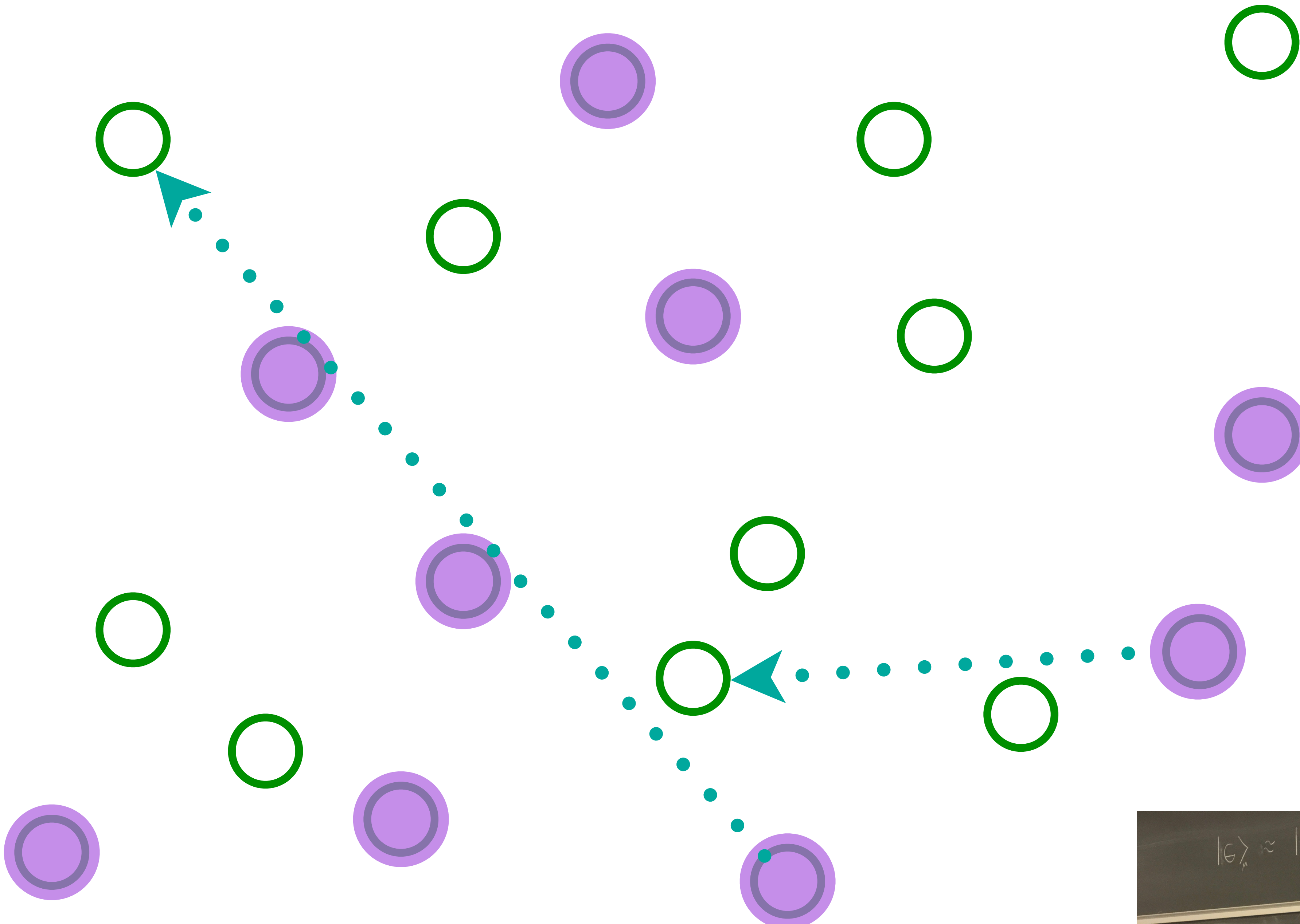

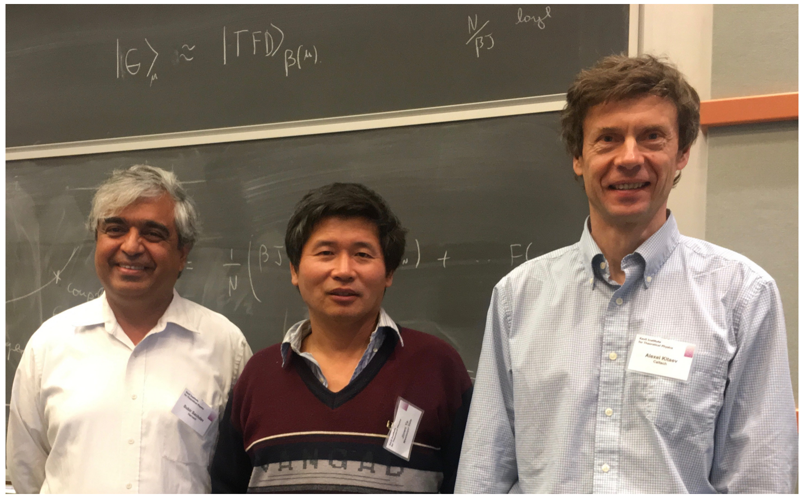

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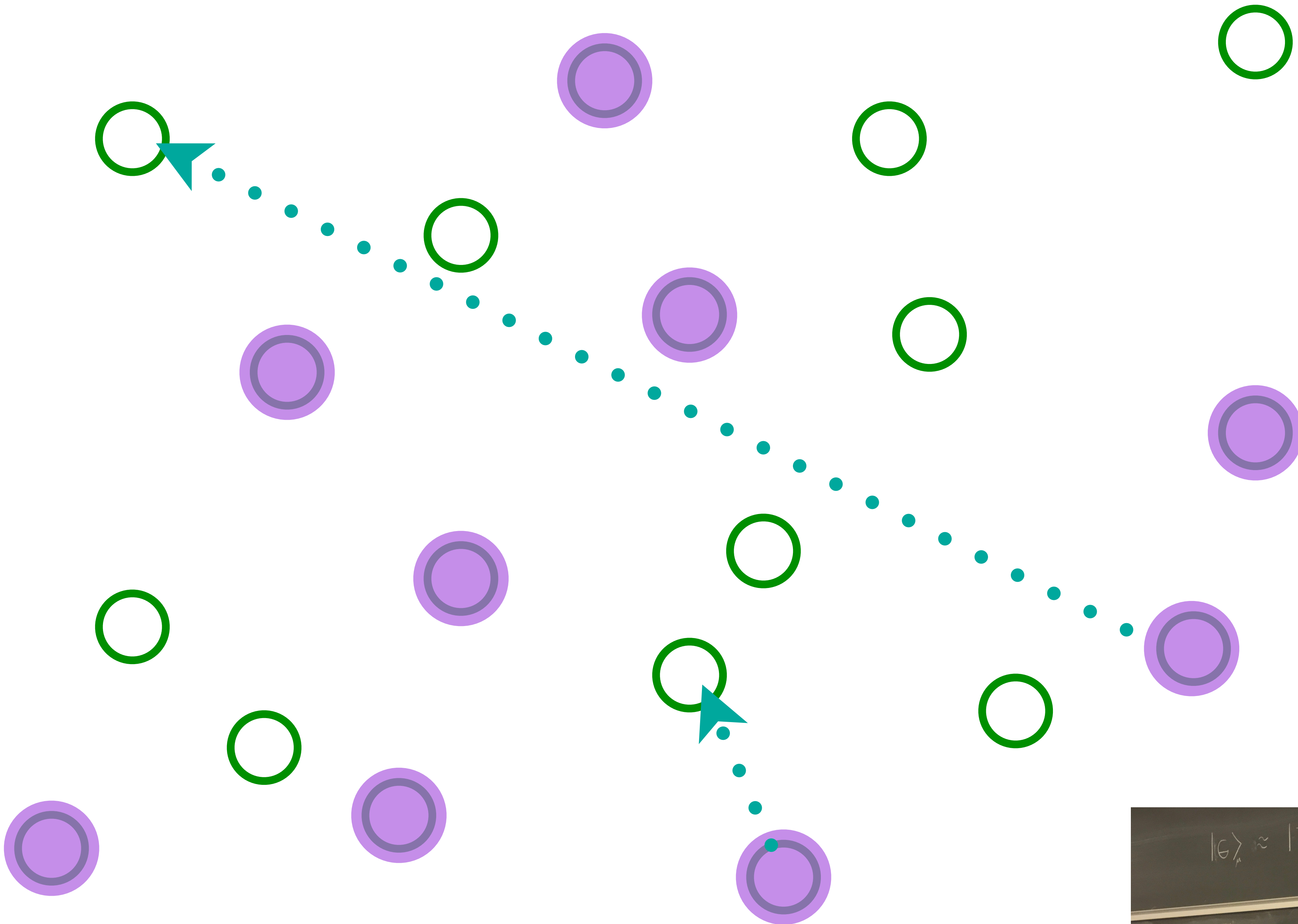

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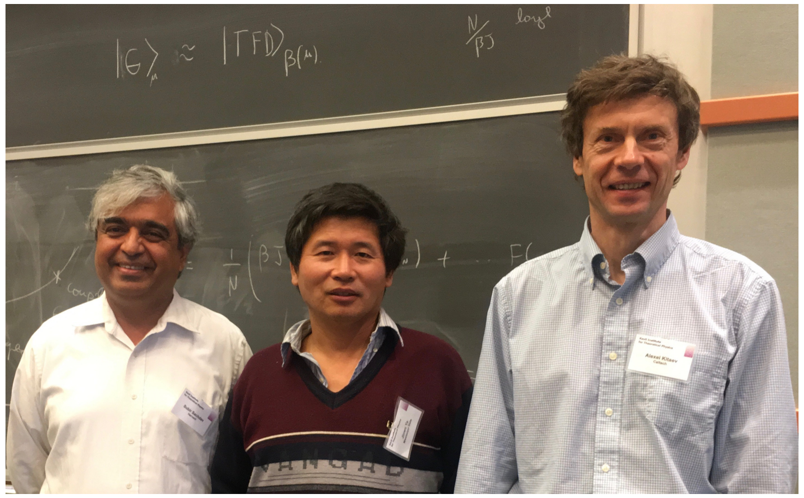

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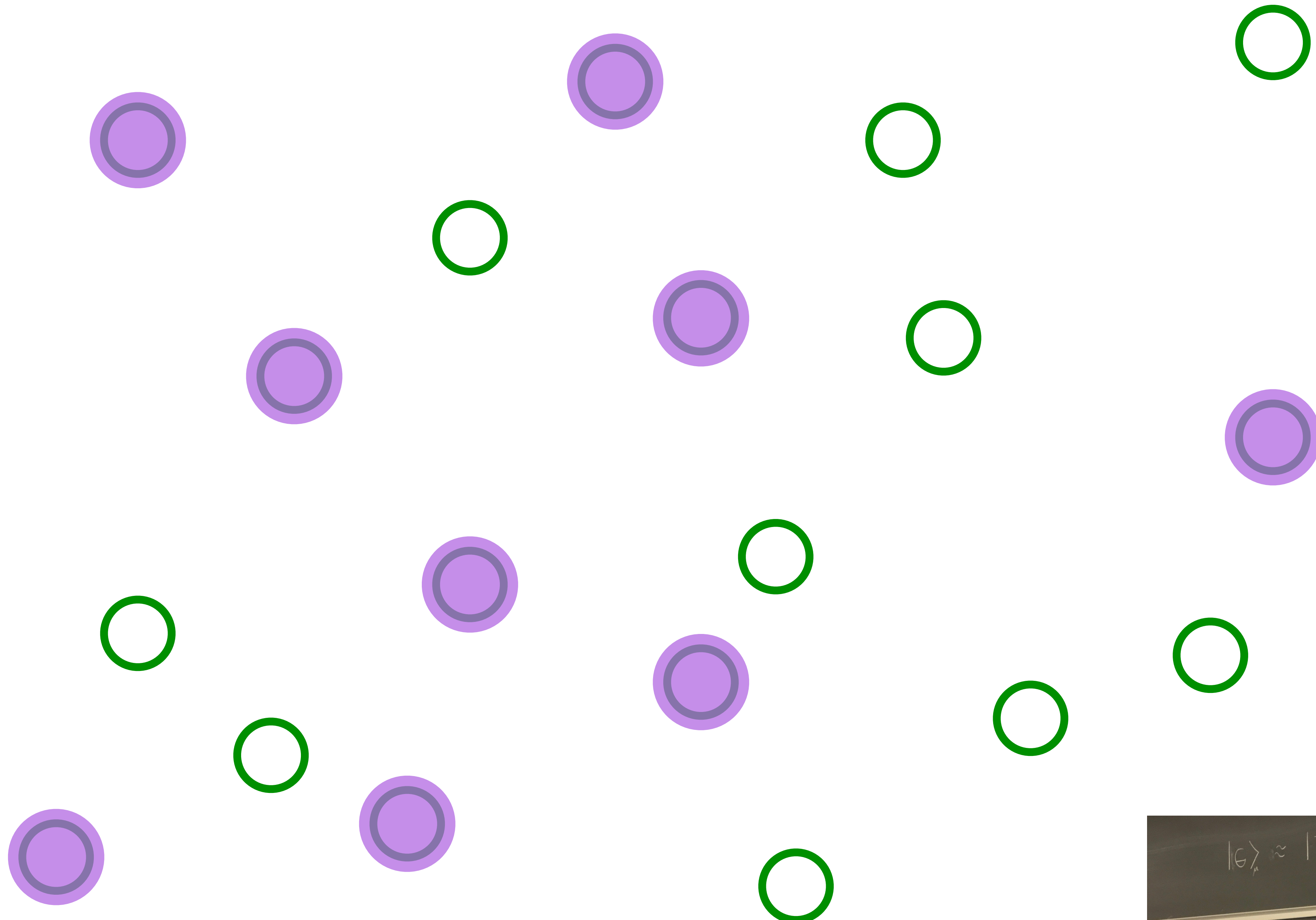

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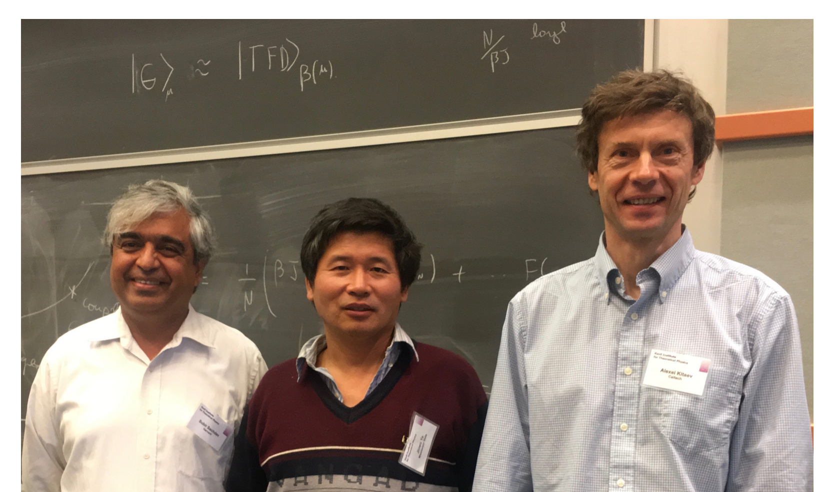

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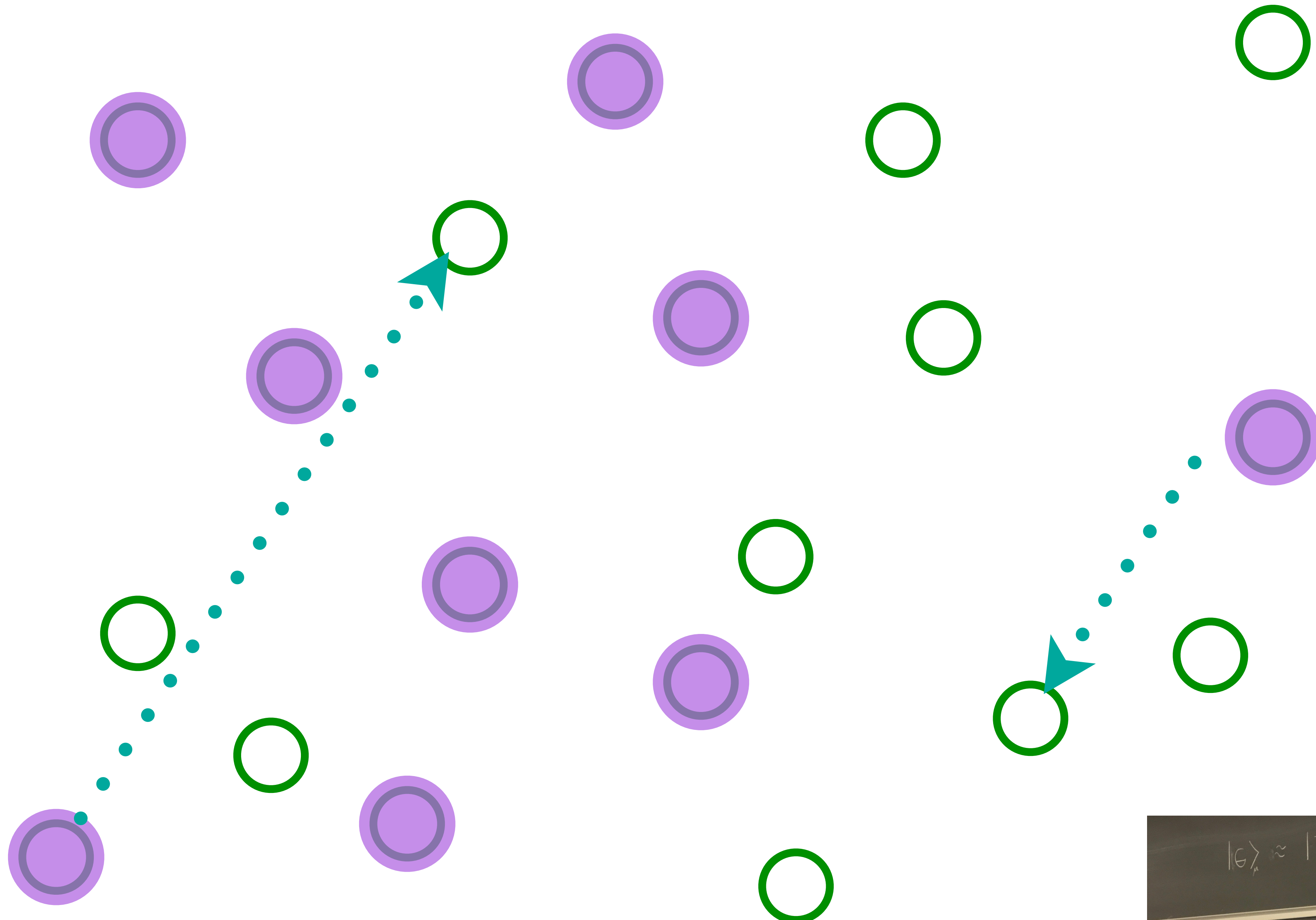

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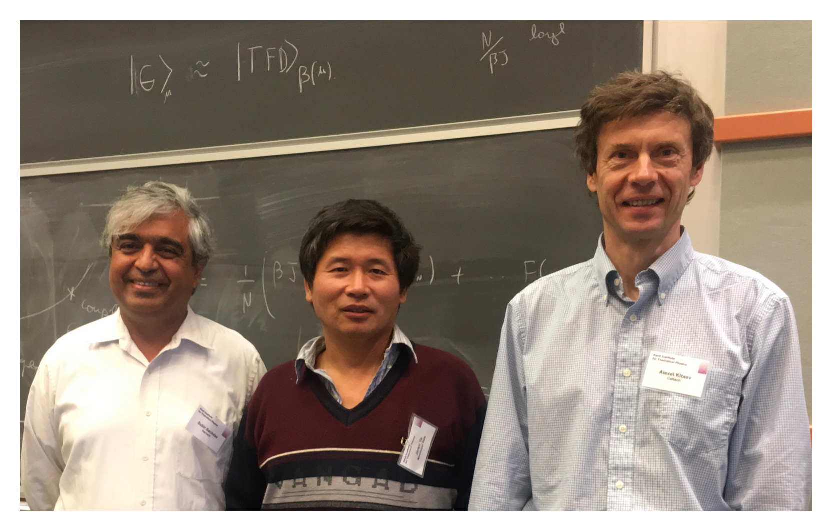

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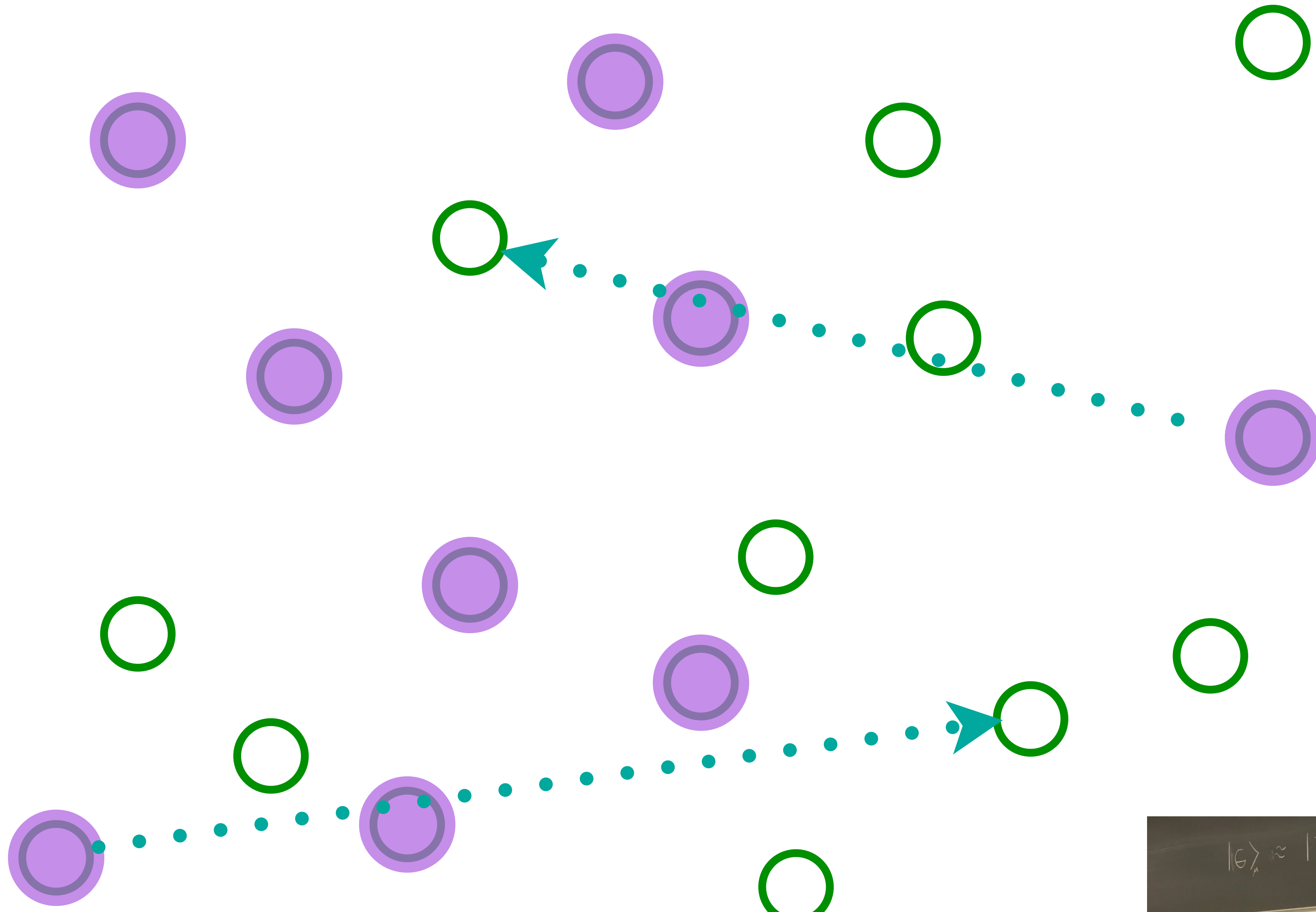

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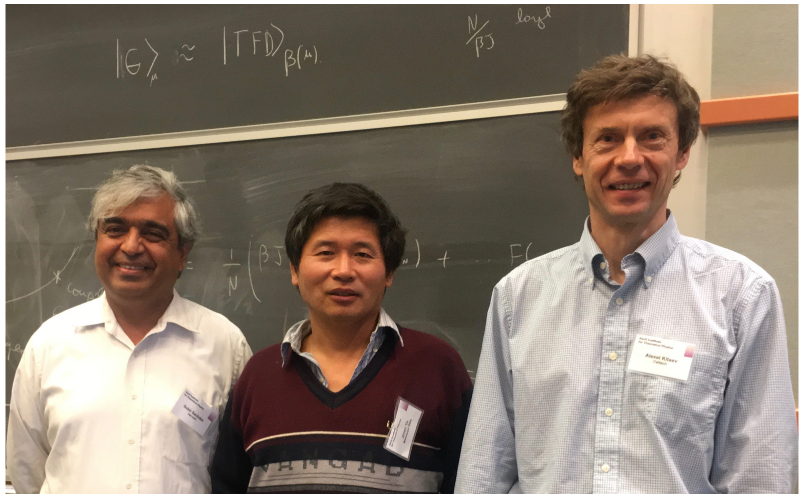

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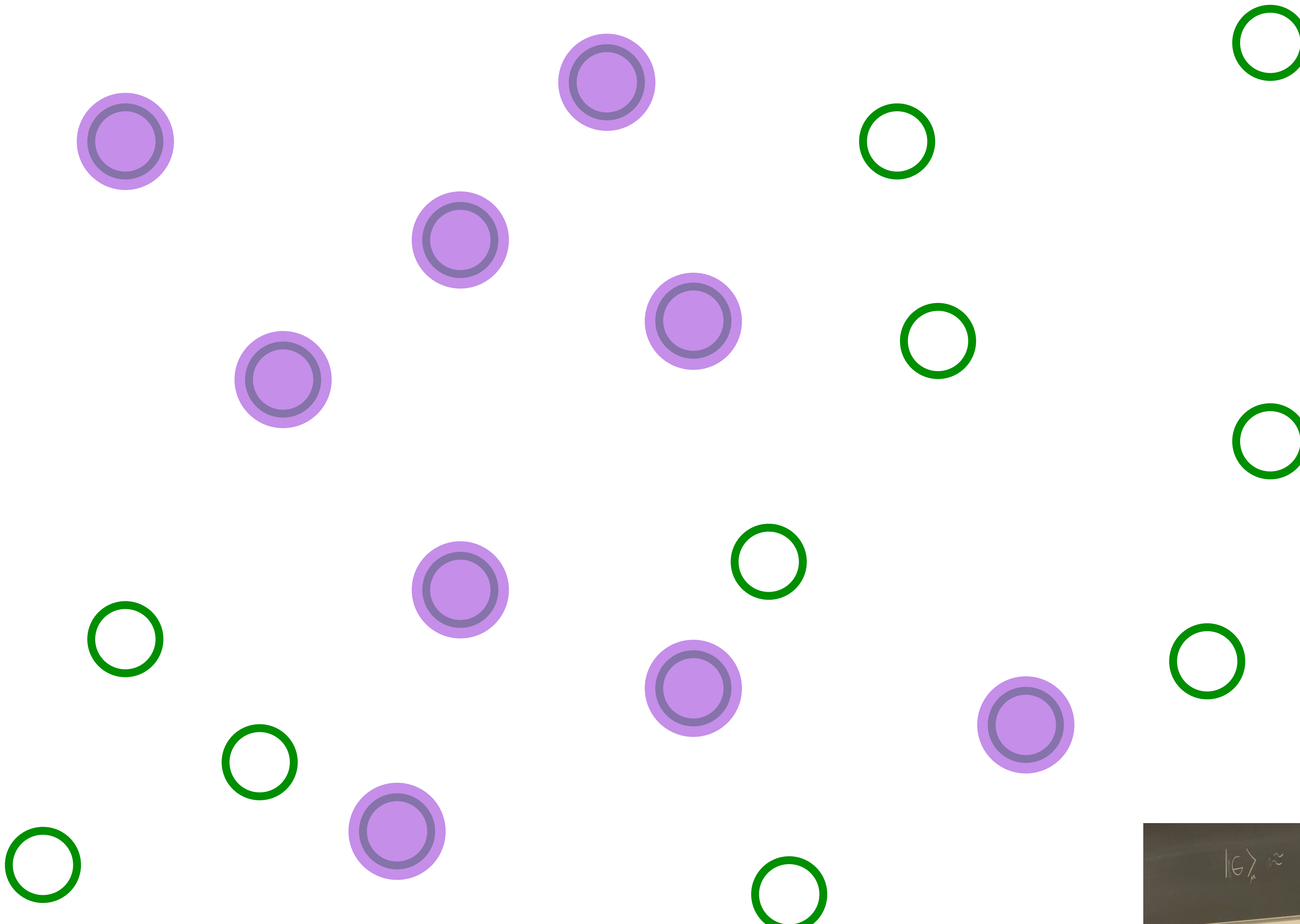
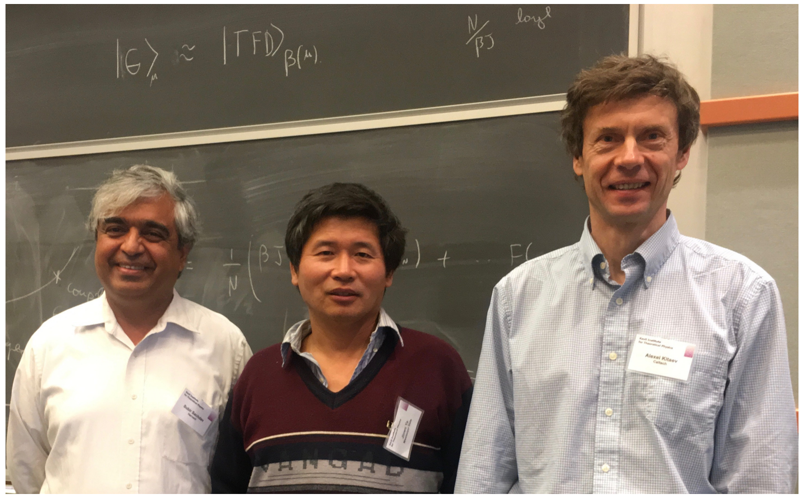


Entangle electrons pairwise randomly



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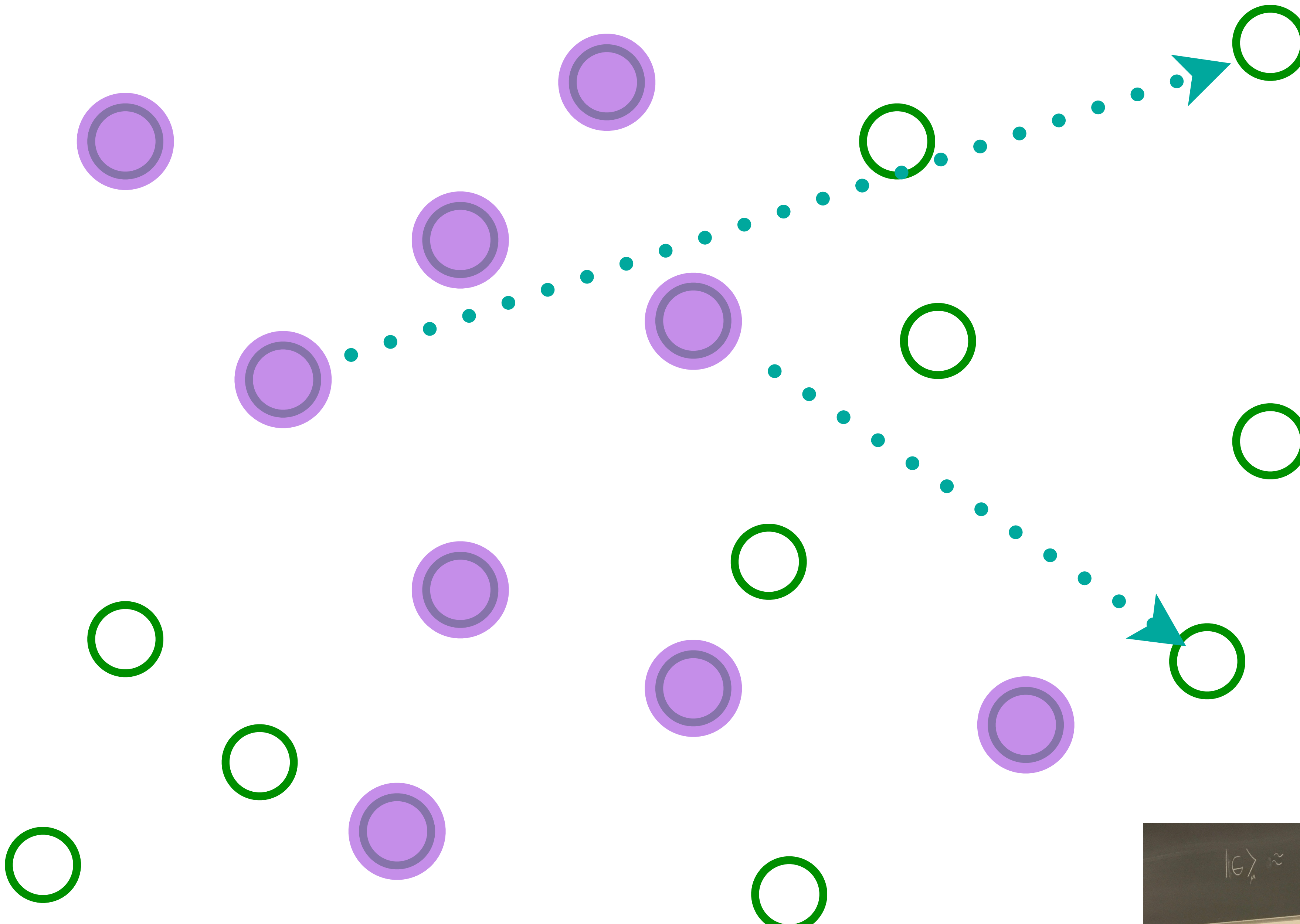

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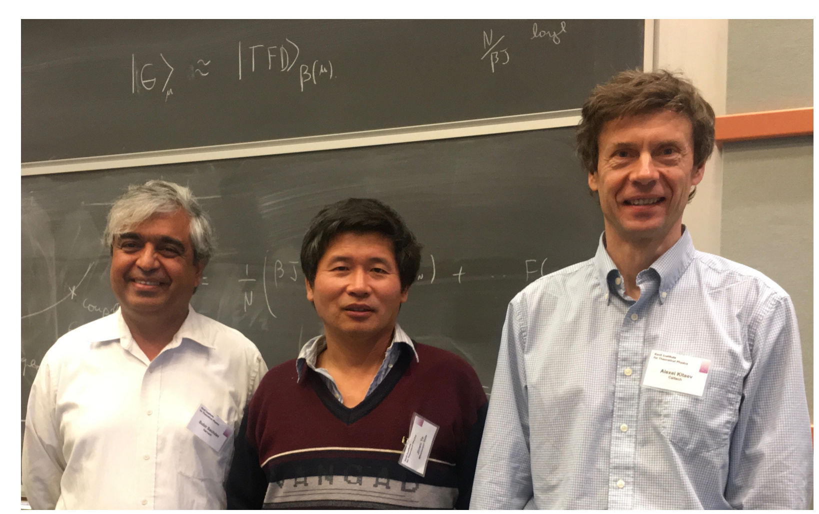

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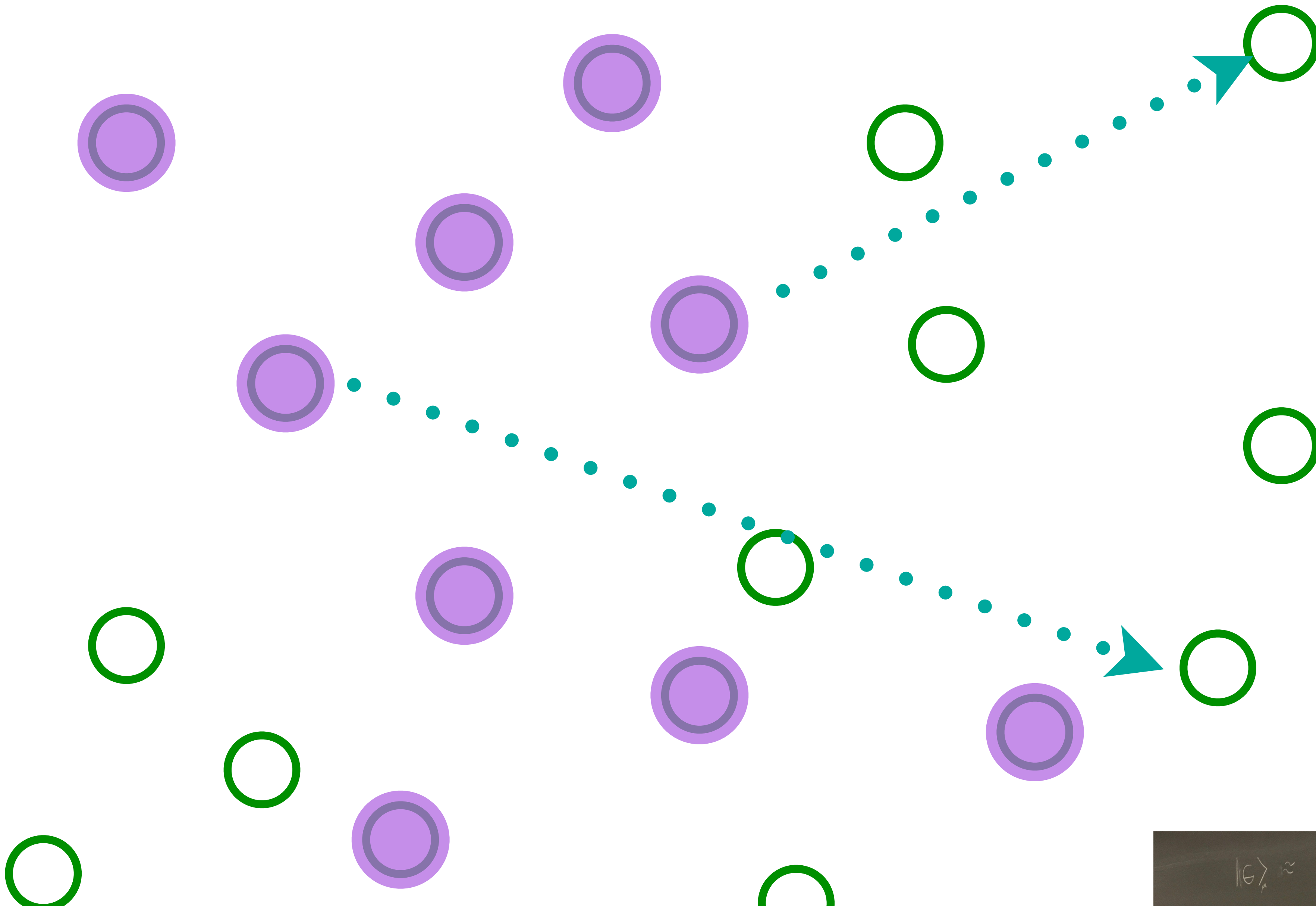

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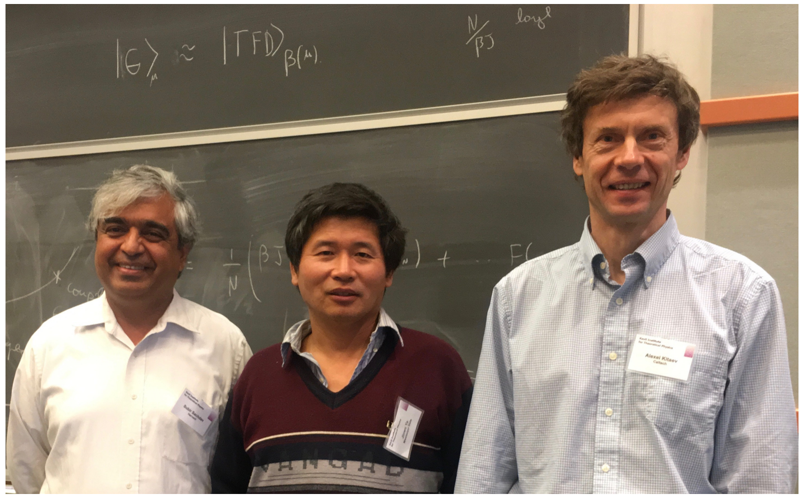

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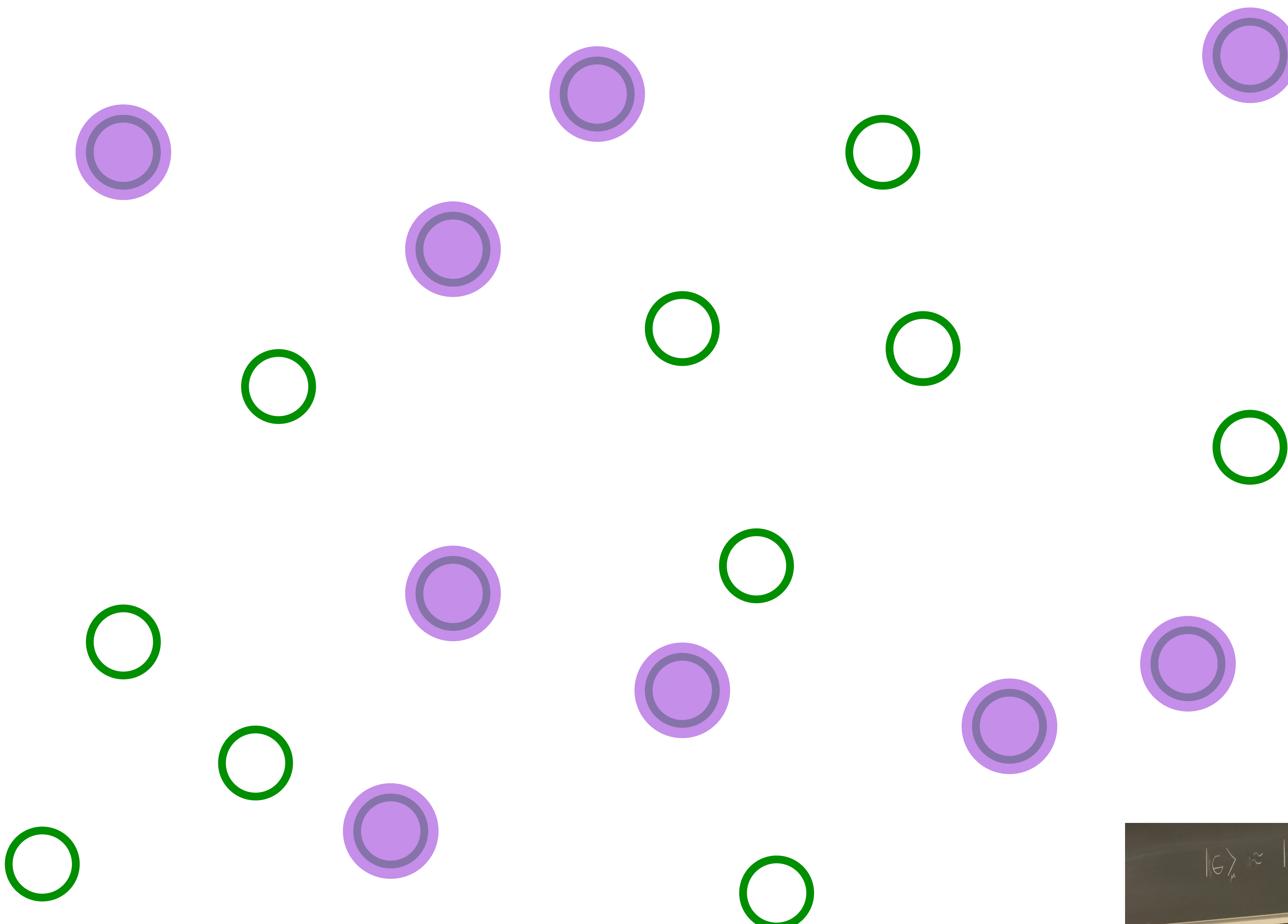

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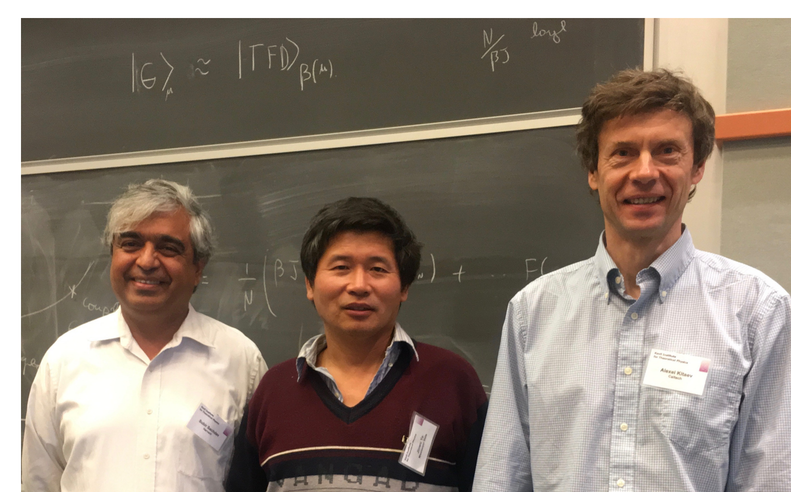

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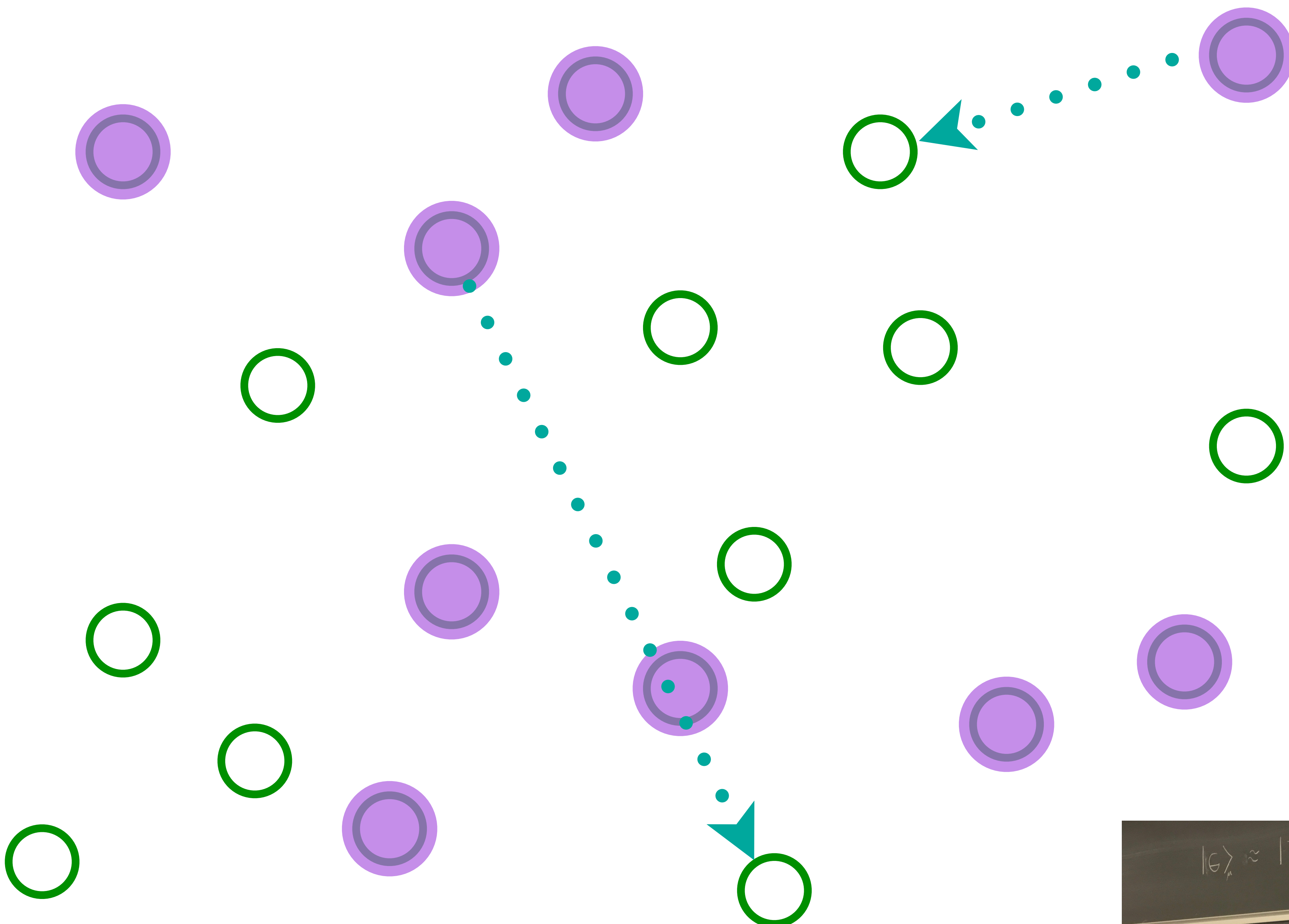

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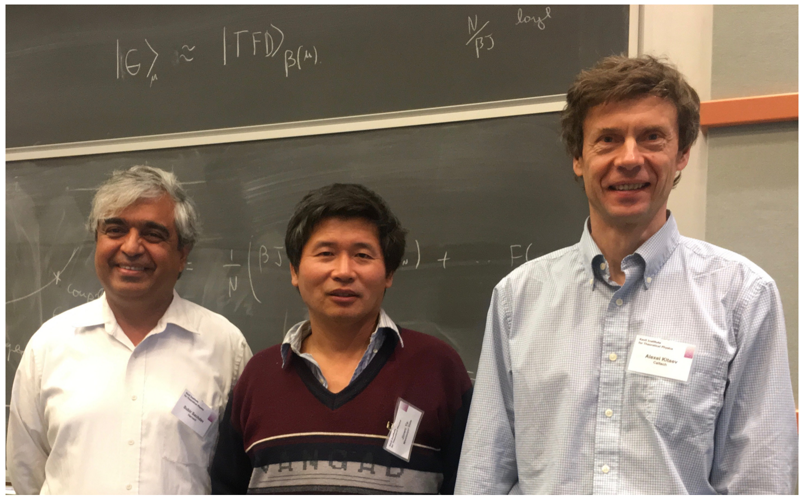

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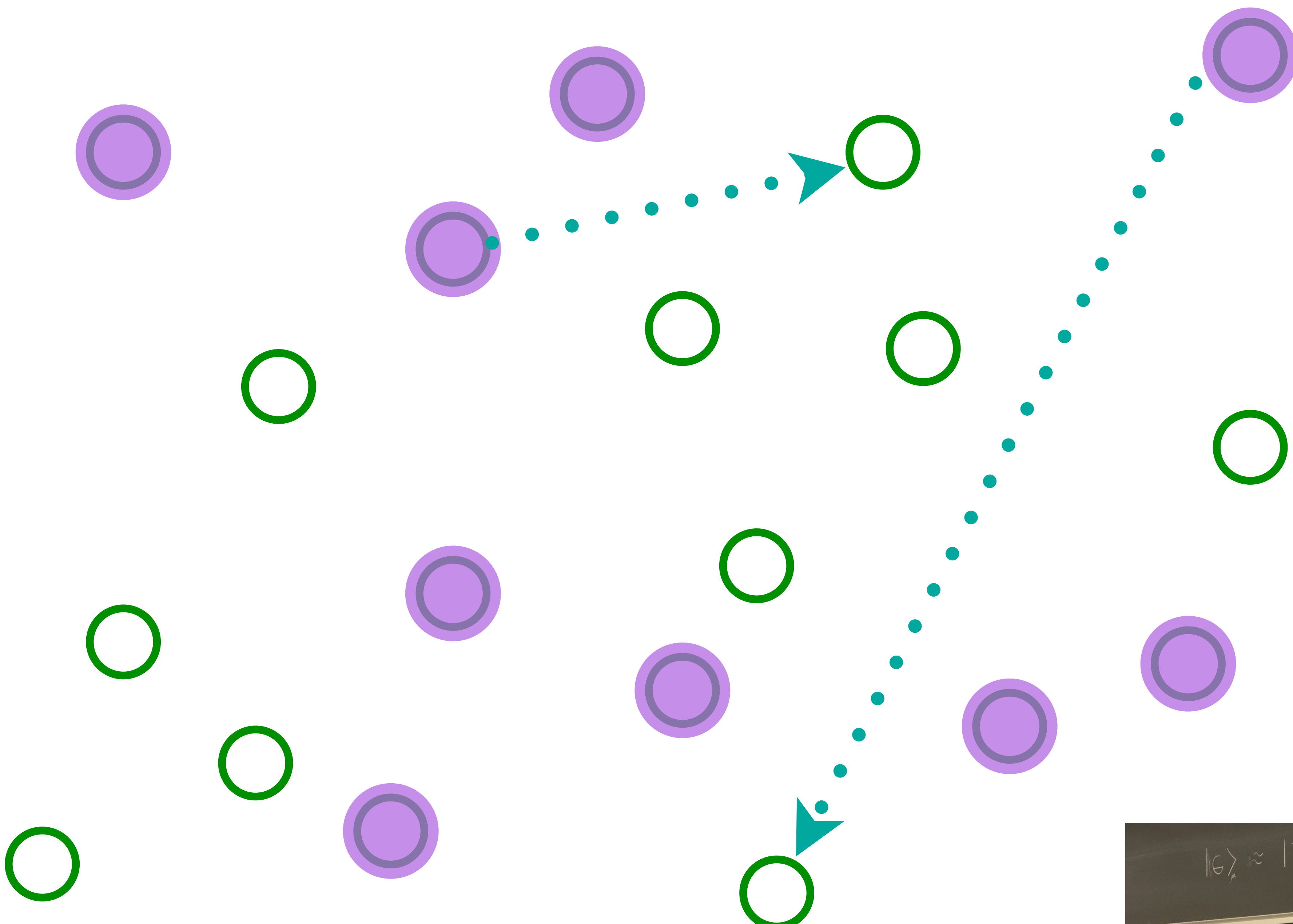

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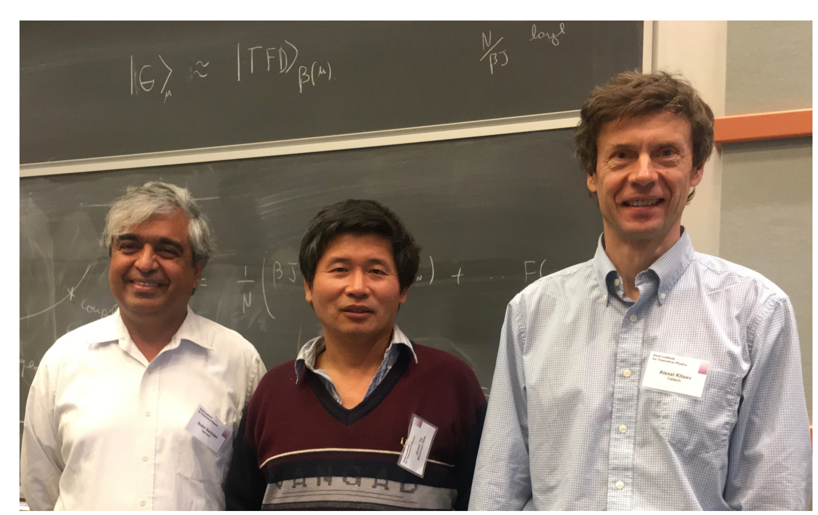

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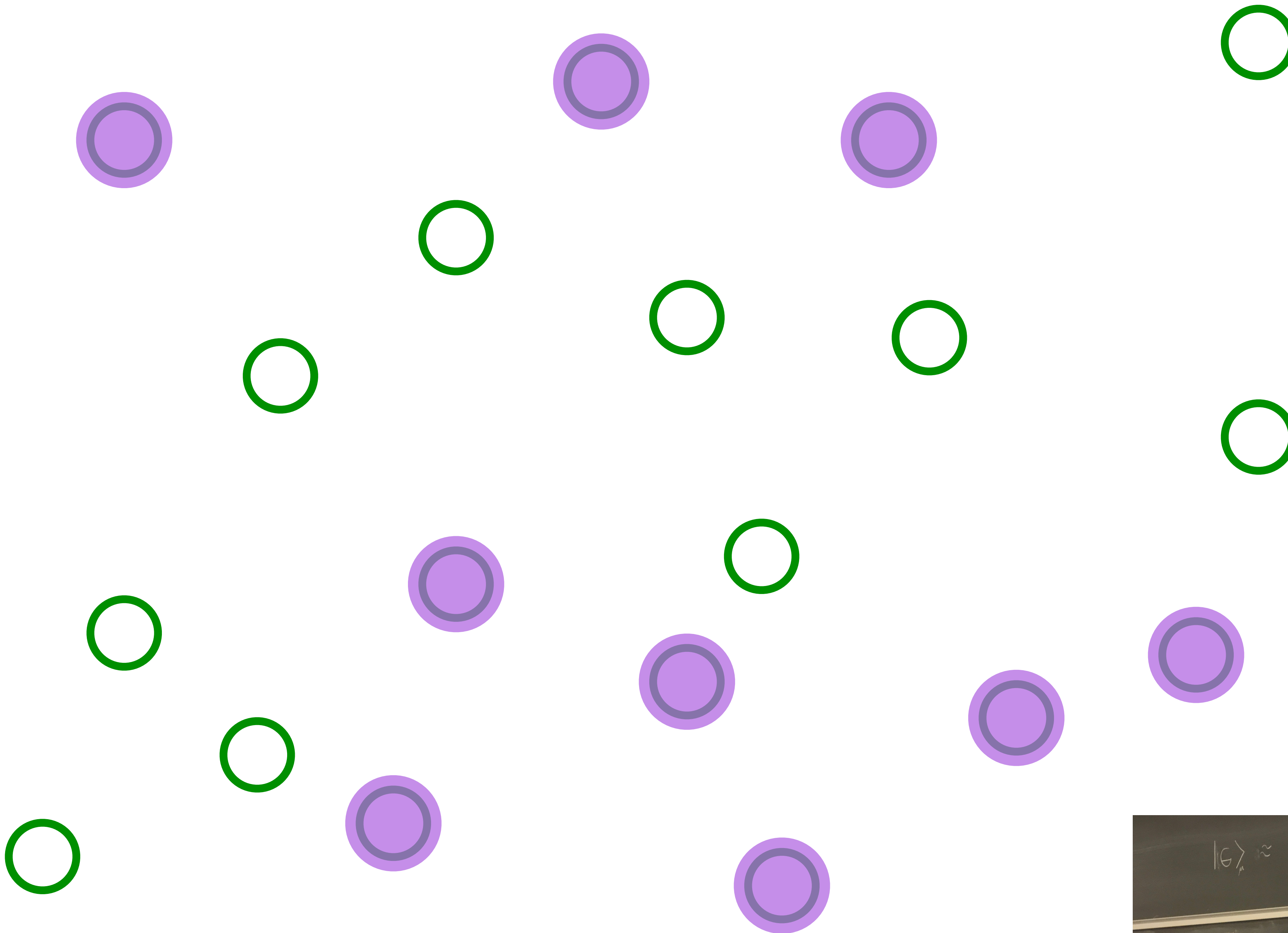

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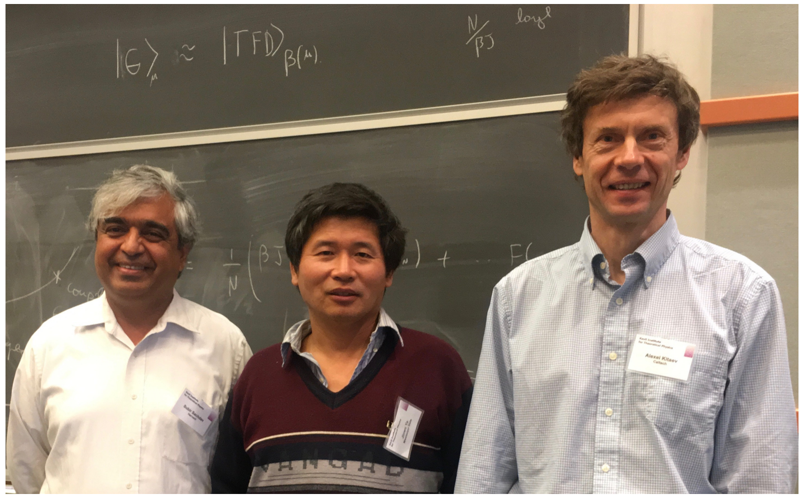

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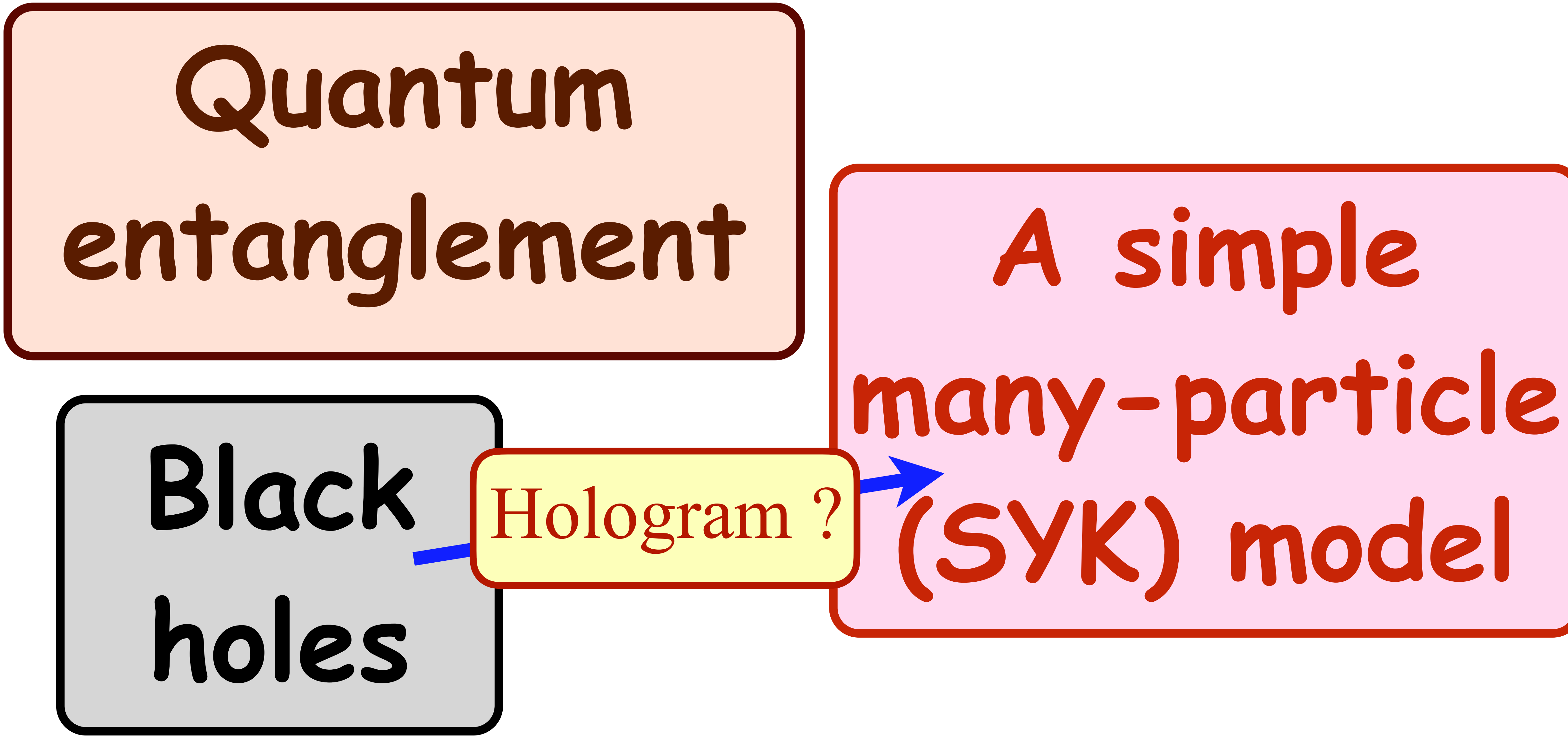
Many-body chaos and thermal equilibration in the shortest possible

$$\text{Planckian time} \sim \frac{\hbar}{k_B T}.$$


Quantum entanglement
Black holes
Hologram ?
A simple many-particle (SYK) model

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge

(Similar considerations also apply
to rapidly rotating black holes,
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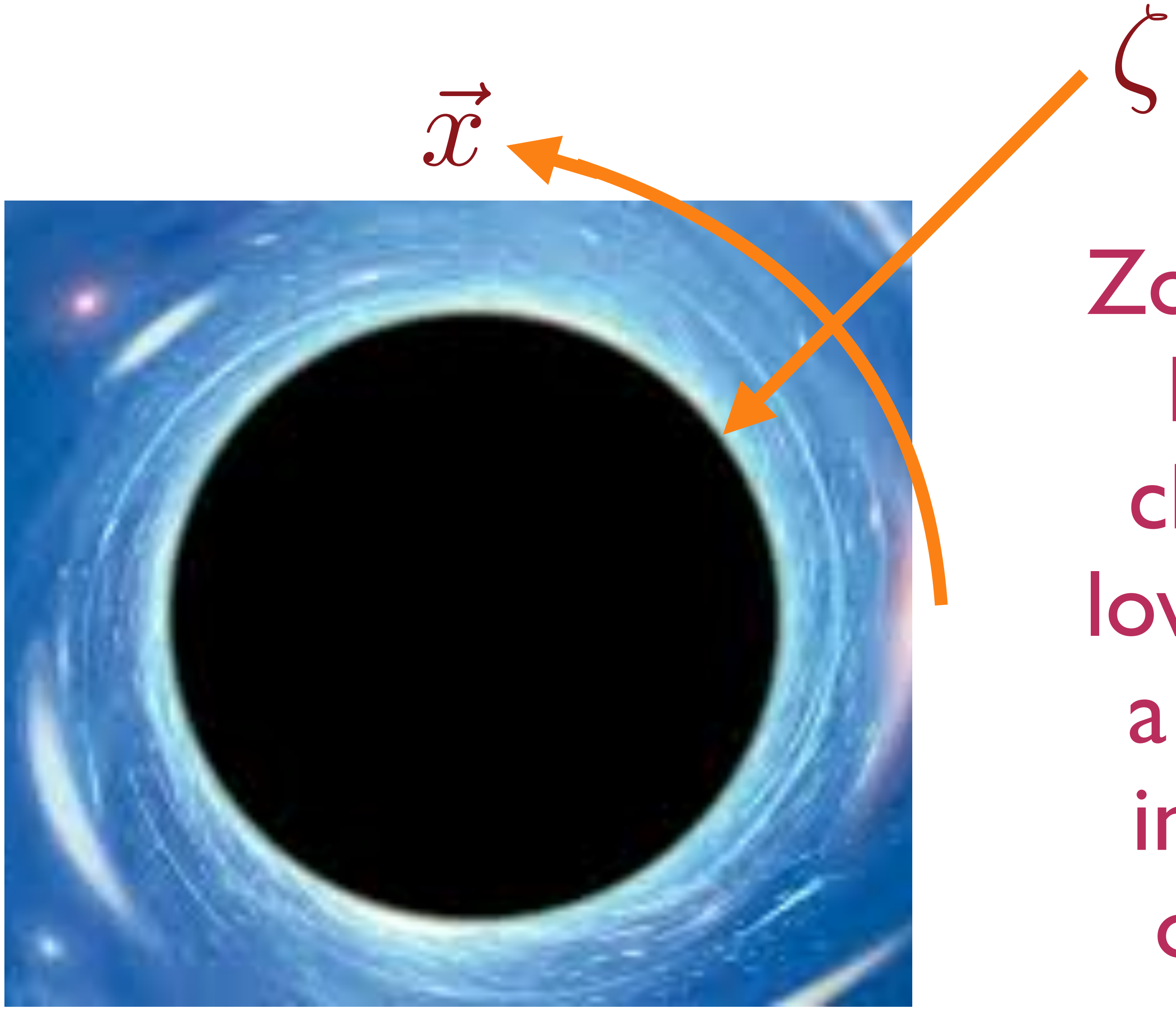
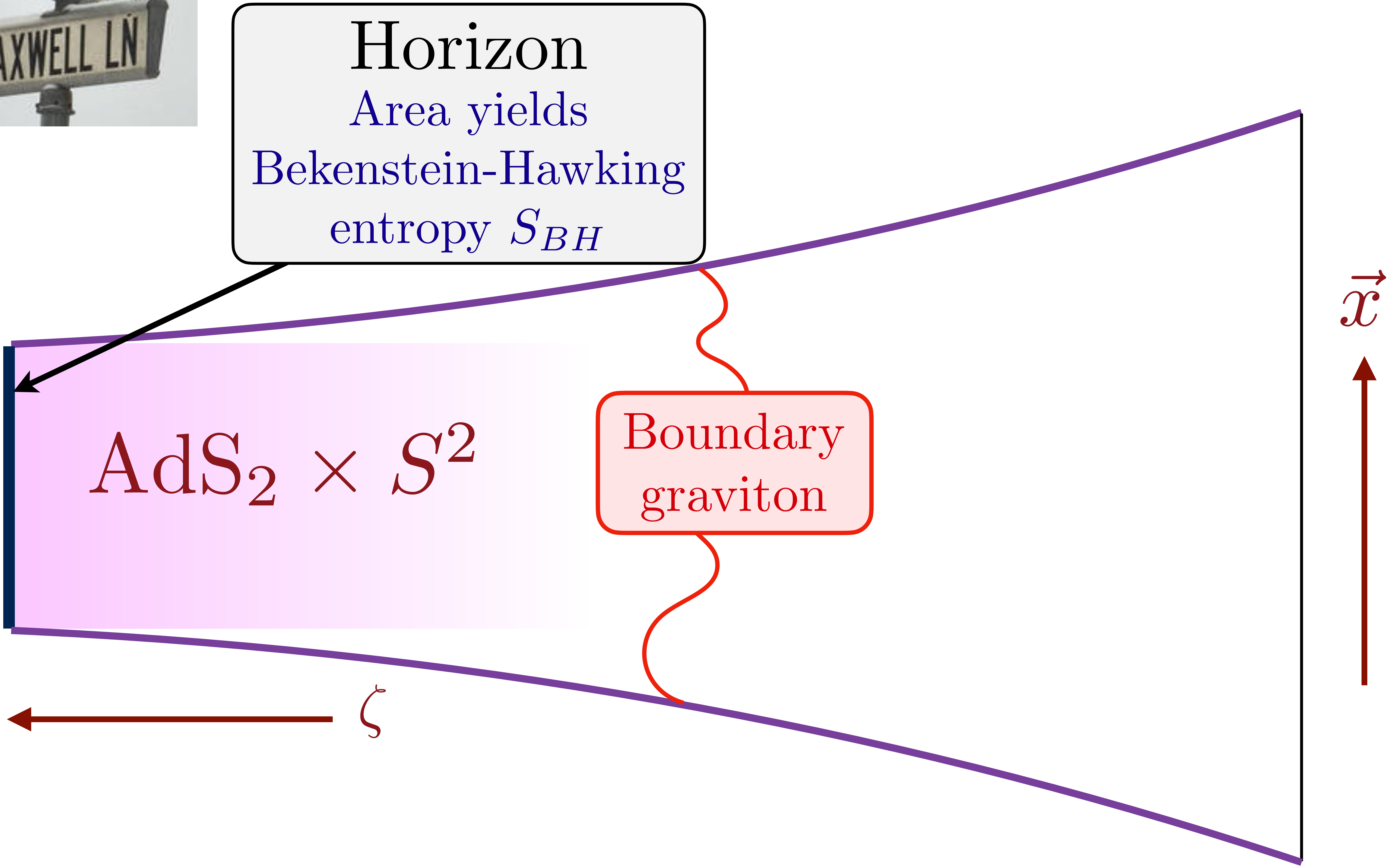


Zooming into the near-horizon region of a charged black hole at low temperature, yields a gravitational theory in one space (ζ) and one time dimension

SYK model and charged black holes



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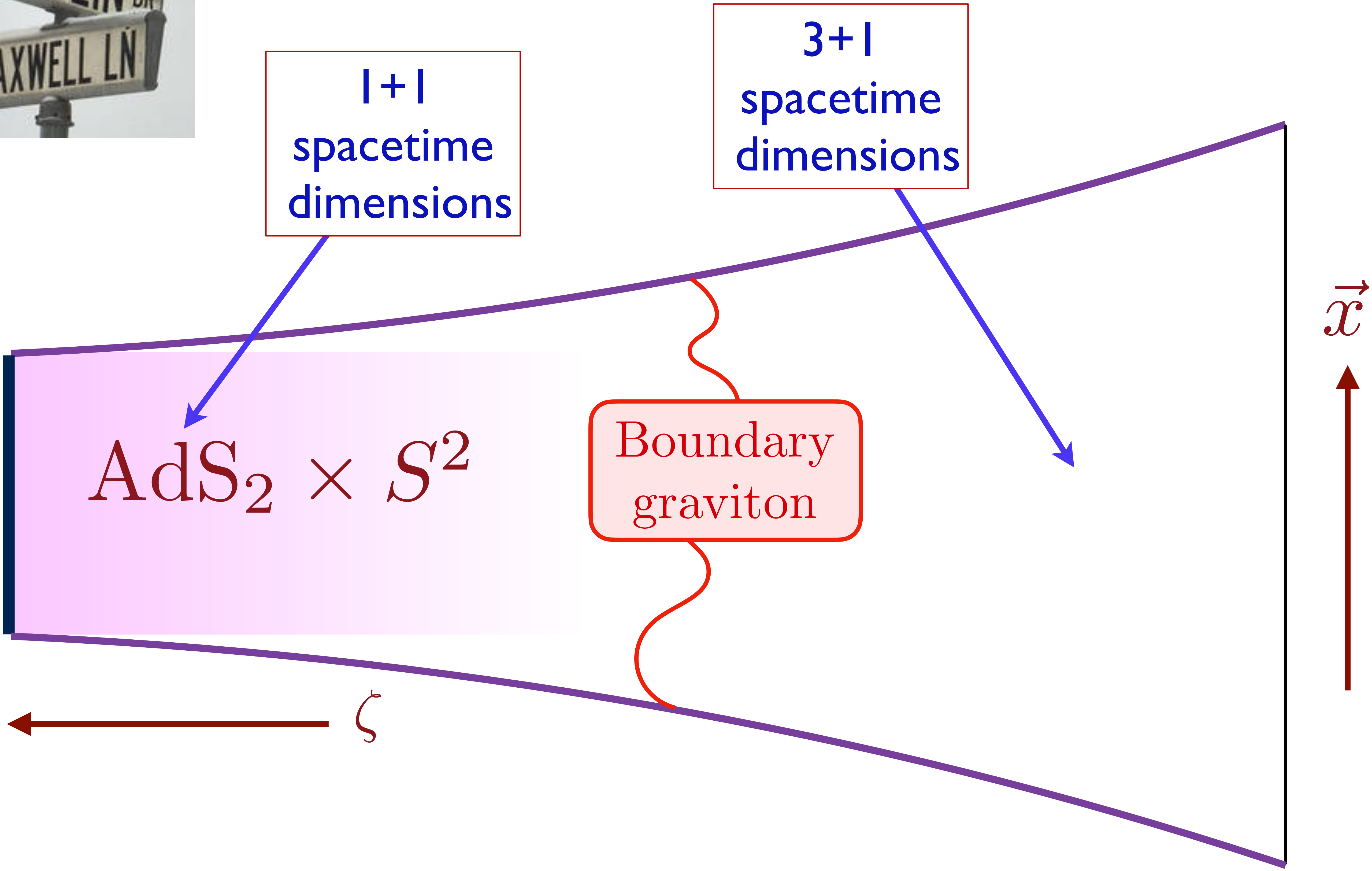

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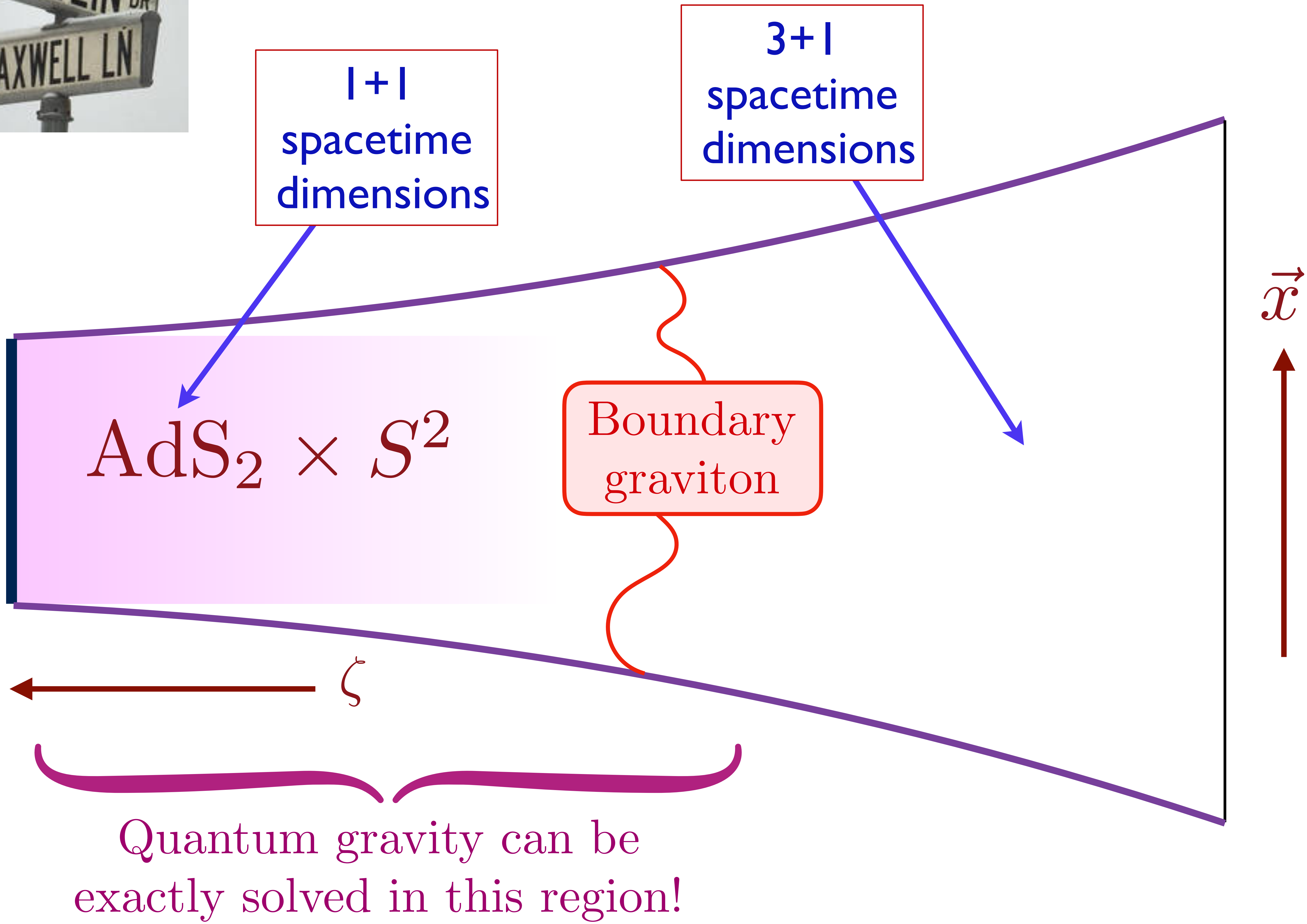

SYK model and charged black holes

Thermodynamics of charged quantum black holes

$$\int \mathcal{D}g_{\mu\nu} \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(3+1)}_{\text{Einstein-Maxwell theory}}[g_{\mu\nu}]\right) \quad T \to 0,$$

$$\approx \int \mathcal{D}g_{\mu\nu} \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(1+1)}_{\text{Gravity of AdS}_2\text{ and boundary}}[g_{\mu\nu}]\right)$$

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$$= \exp\Big(S_{BH}\Big) \times \exp\left(-\frac{1}{T} \times \text{Free energy of SYK model}\right)$$

The hologram of the 1+1 dimensional gravity near the horizon of a charged black hole is the 0+1 dimensional SYK model

Sachdev (2010); Kitaev (2015); Sachdev (2015); Maldacena, Stanford, Yang (2016) ; Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Iliesiu, Turaci (2020)

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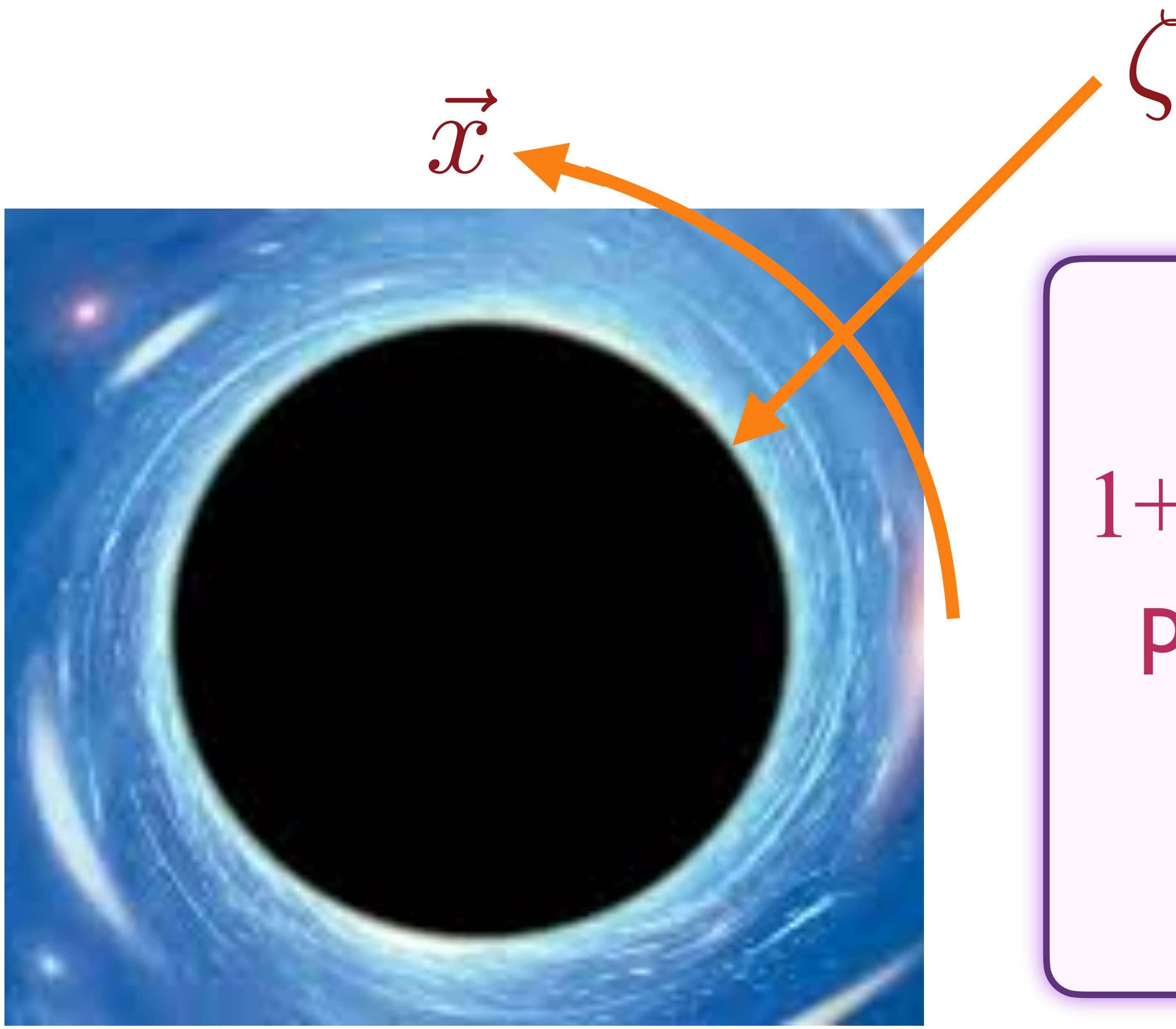
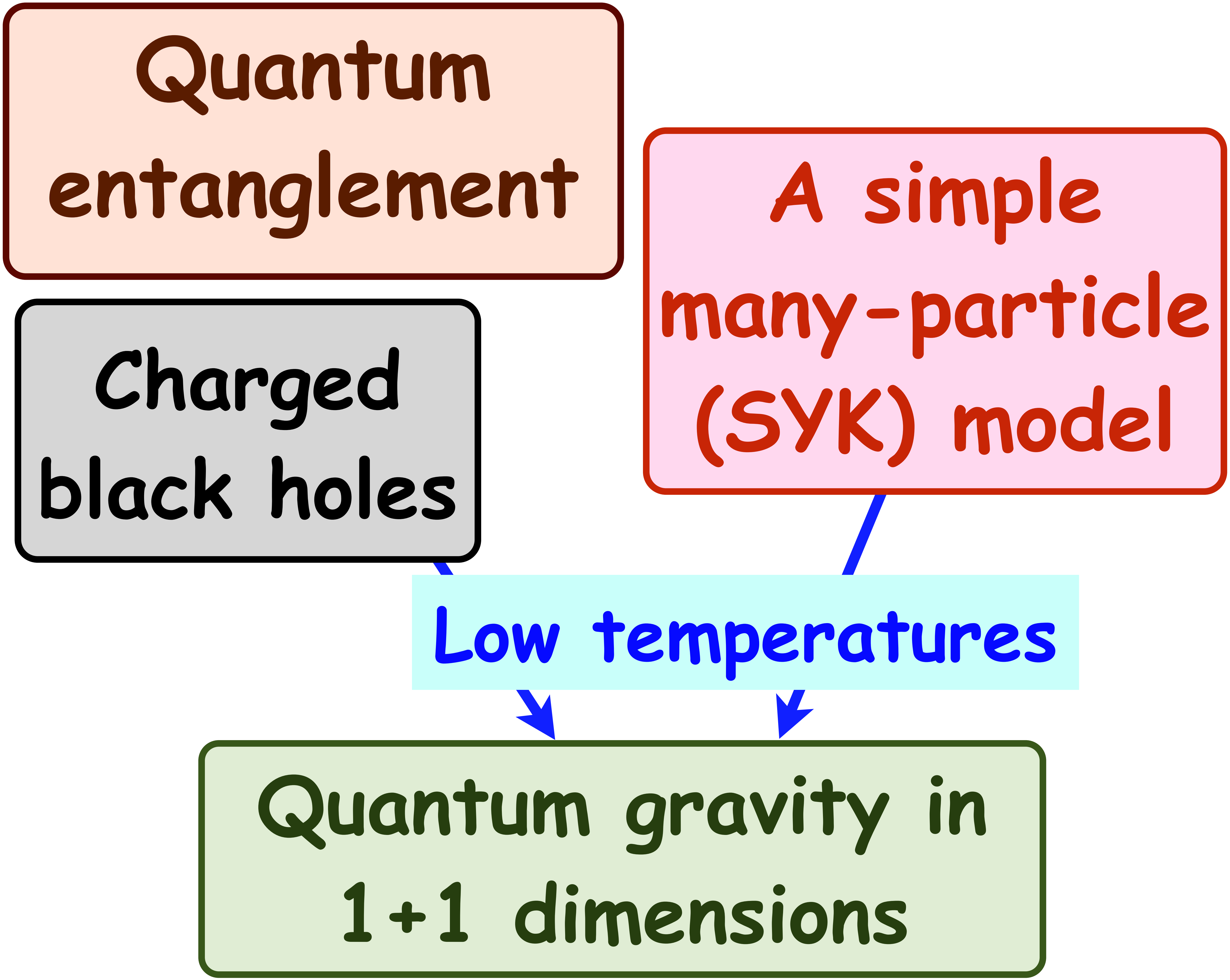


The near-horizon 1+1D-gravity theory is precisely that of the low T limit of the SYK models


Quantum entanglement
Charged black holes
A simple many-particle (SYK) model
Low temperatures
Quantum gravity in 1+1 dimensions

Complex multi-particle entanglement leads to quantum systems without quasiparticle excitations.

Many-body chaos and thermal equilibration in the shortest possible Planckian time $\sim \dfrac{\hbar}{k_B T}$.

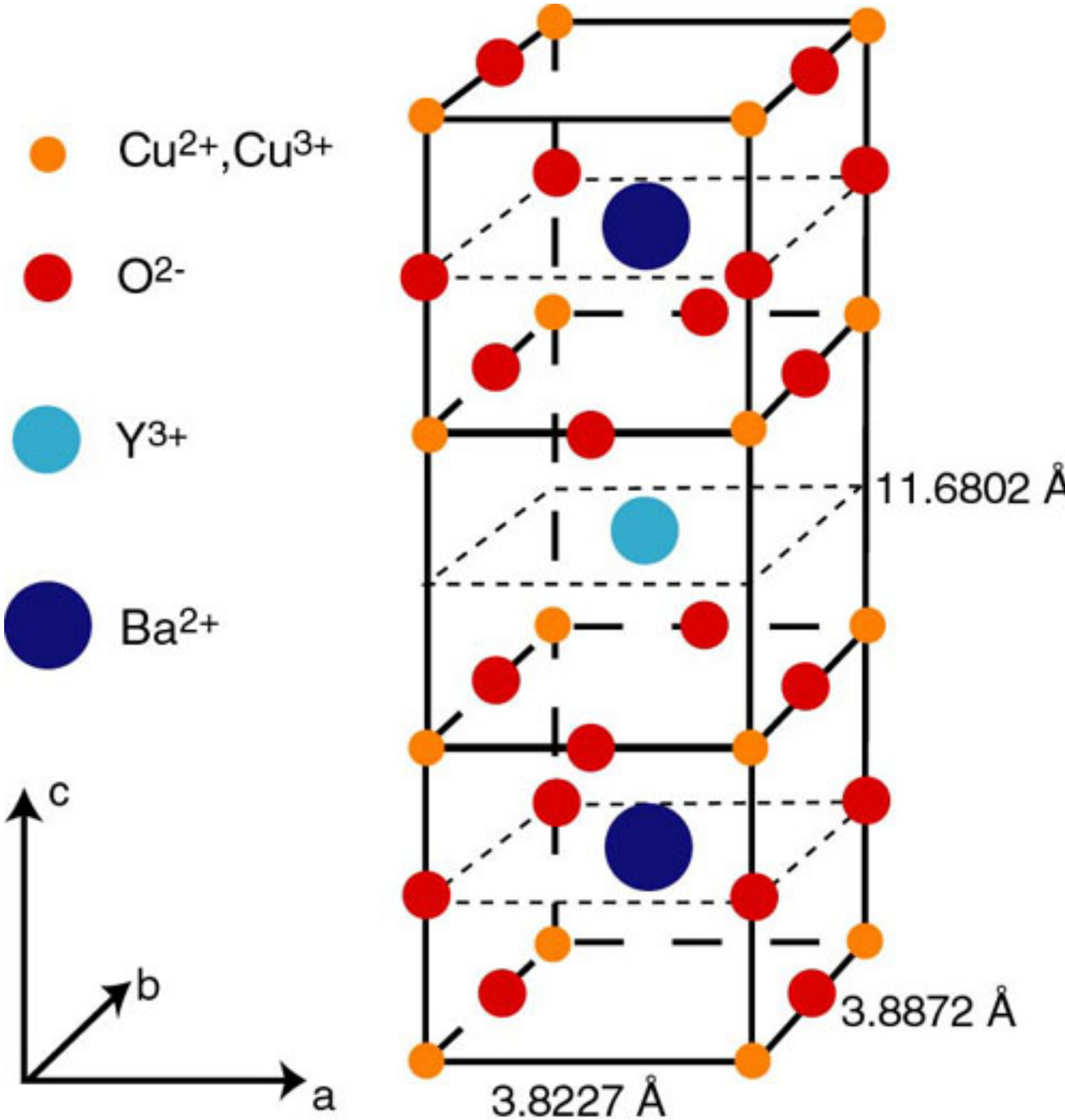
Quantum entanglement

A simple many-particle (SYK) model

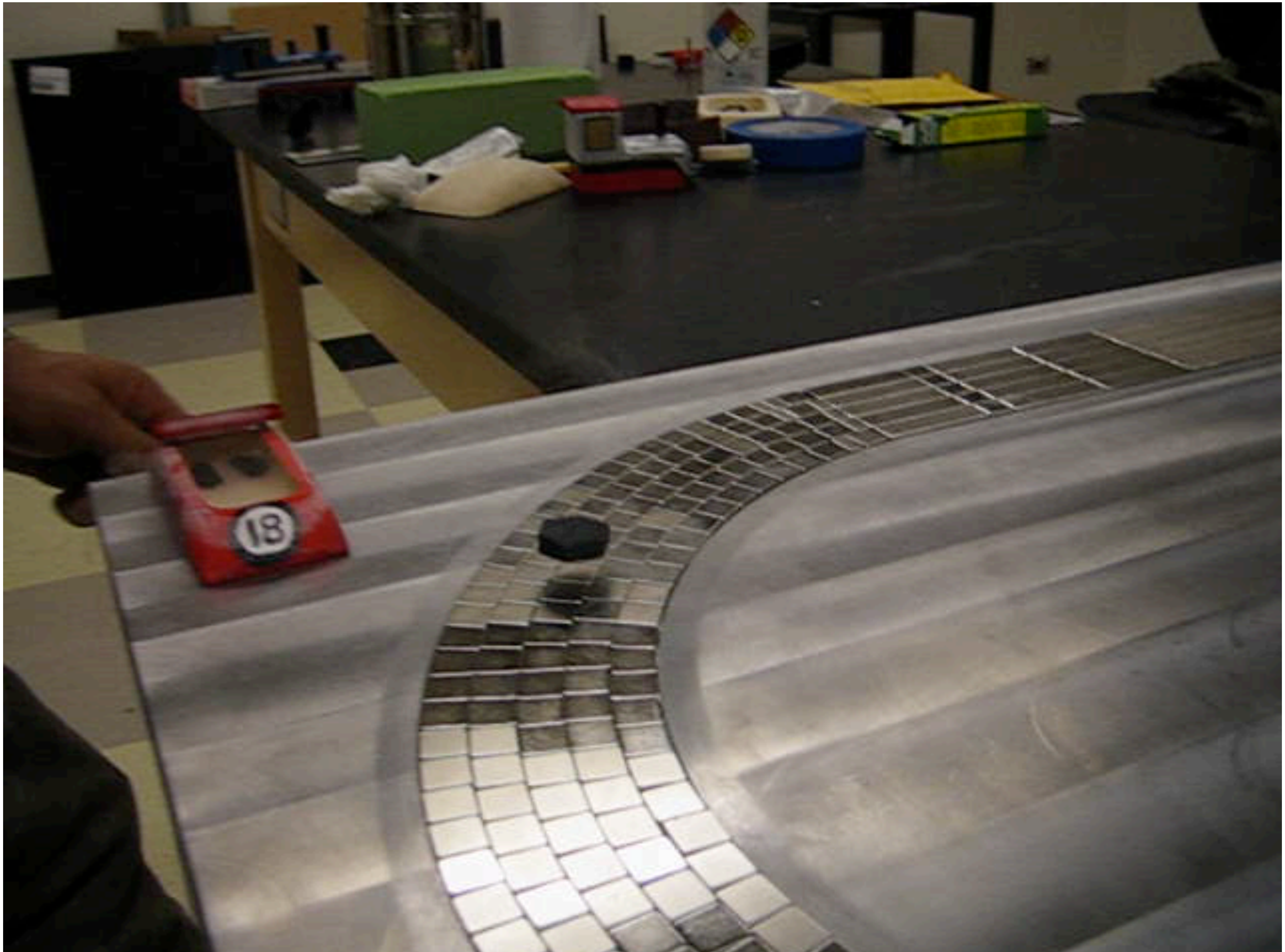
Charged black holes

Copper-based superconductors

High temperature superconductors



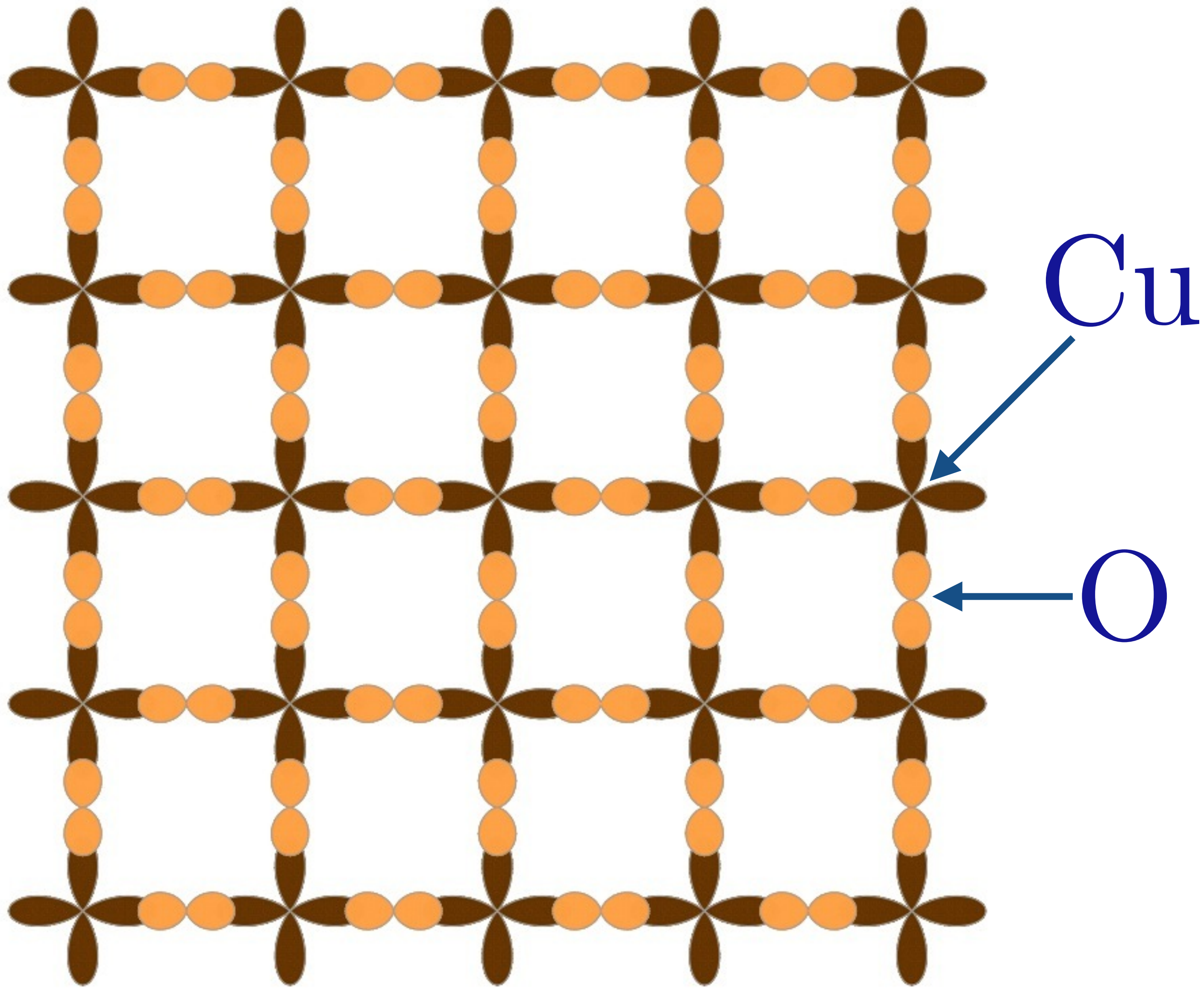
$$YBa_2Cu_3O_{6+x}$$



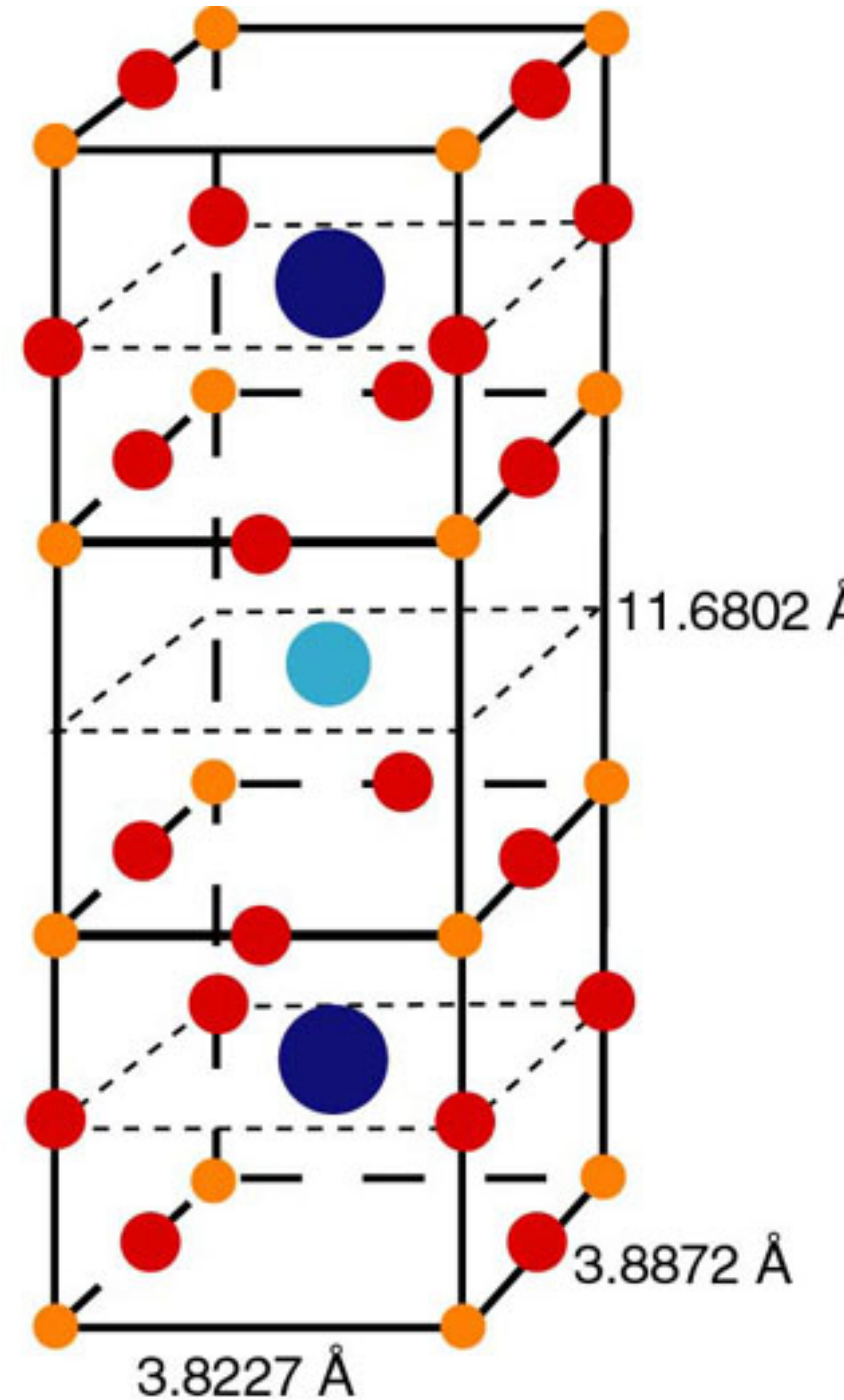
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

High temperature superconductors

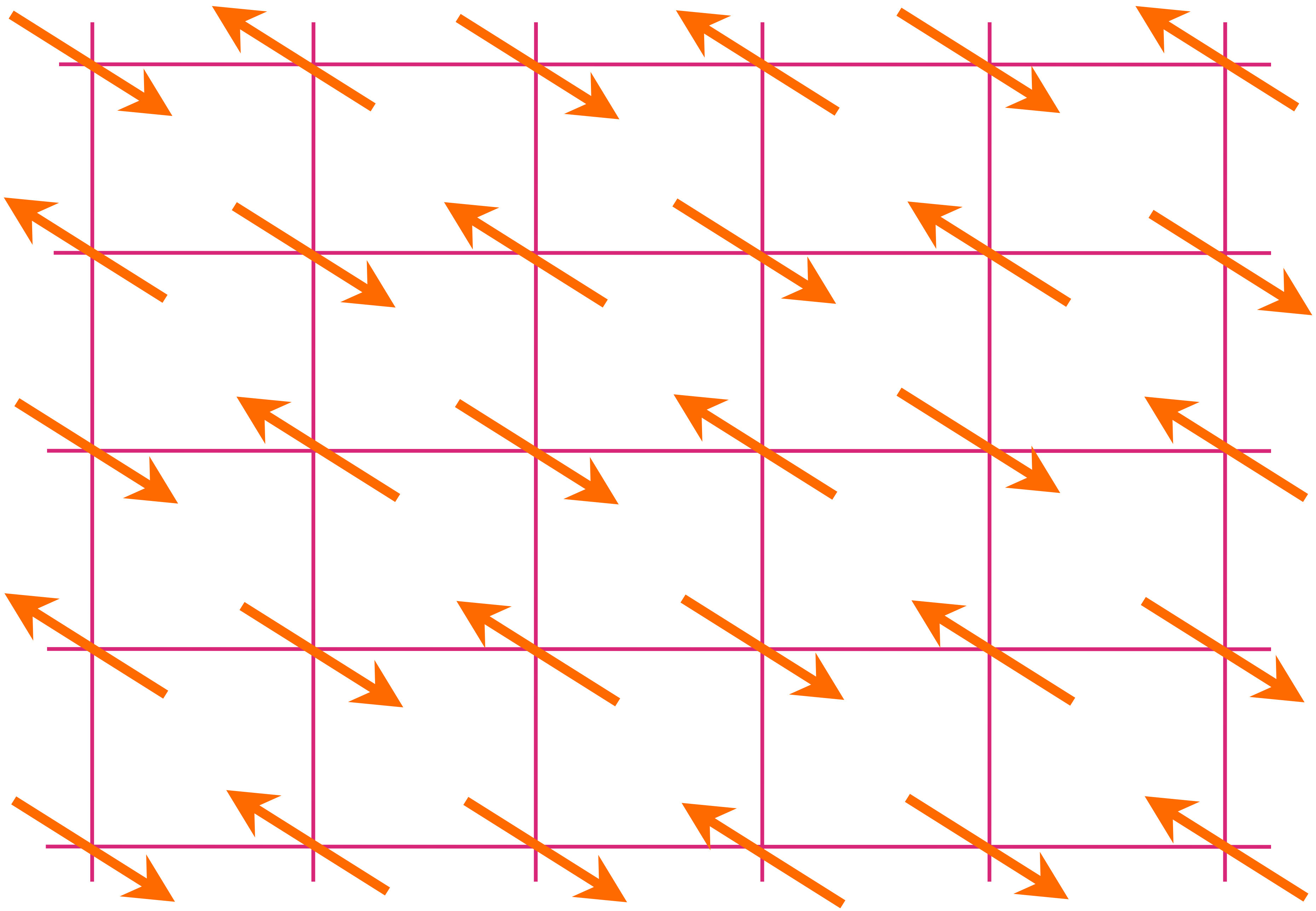


CuO_2 plane

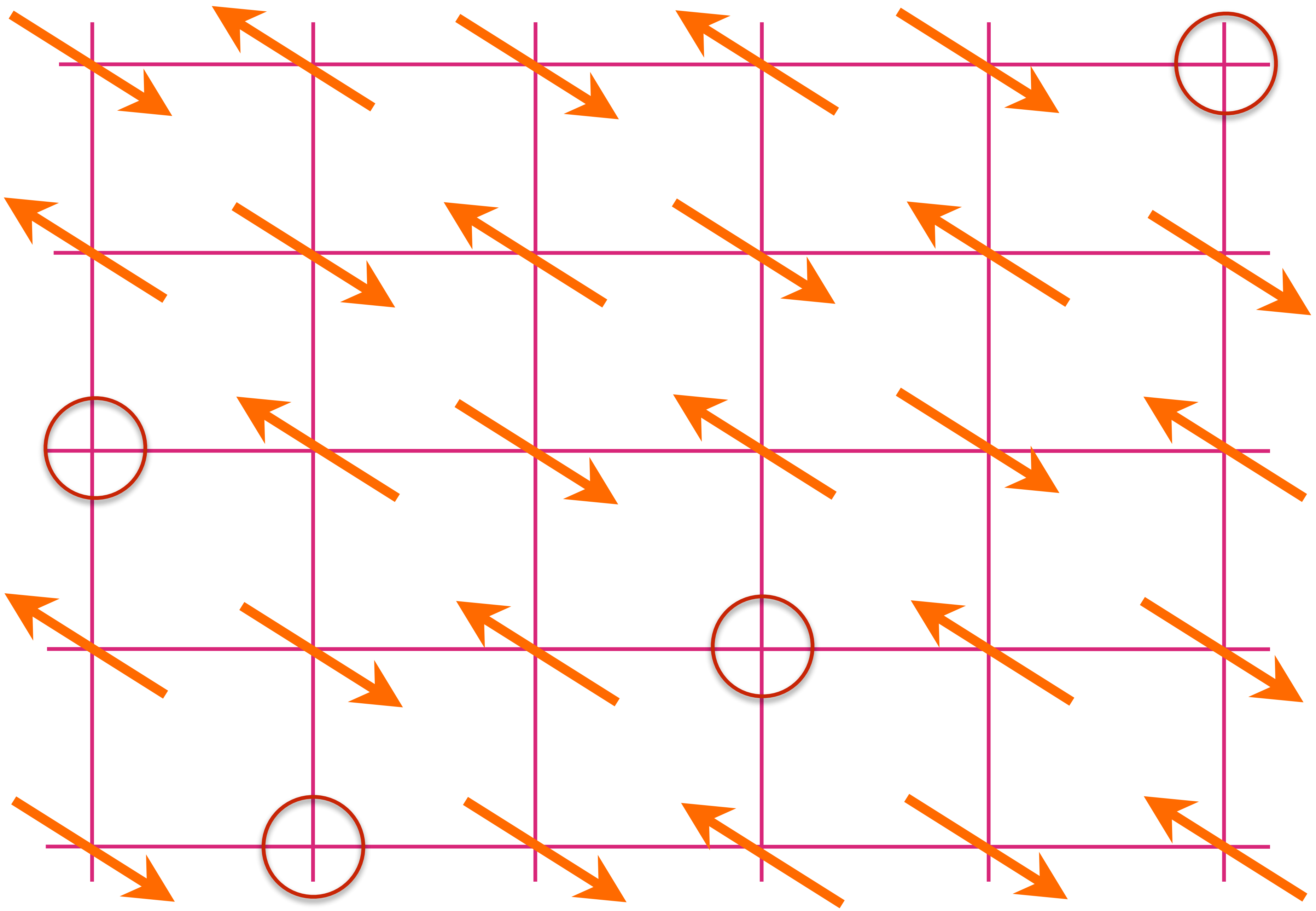


$YBa_2Cu_3O_{6+x}$

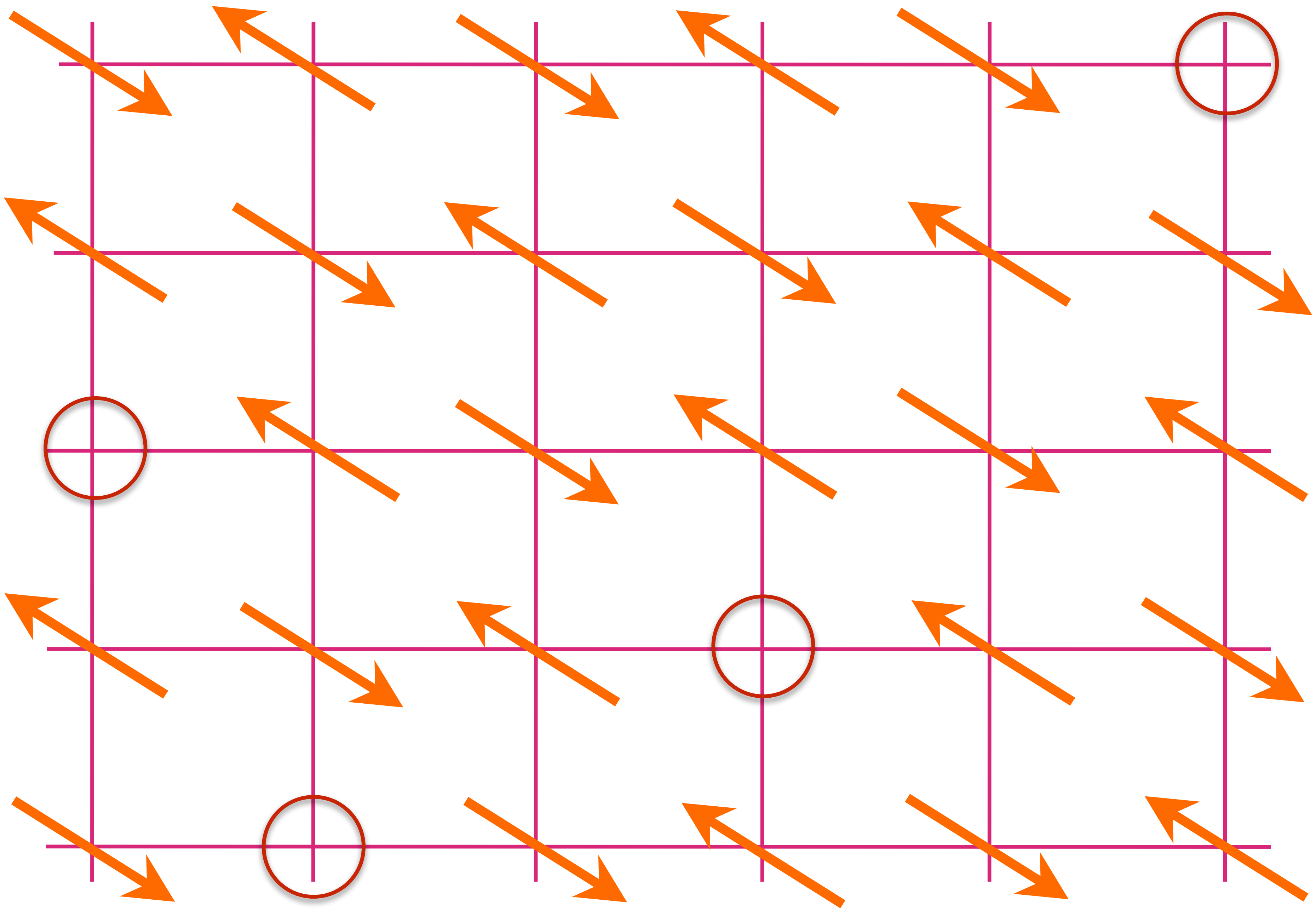
Insulating antiferromagnet



Antiferromagnet doped with hole density p



Real-space view



p mobile holes in a background of fluctuating spins

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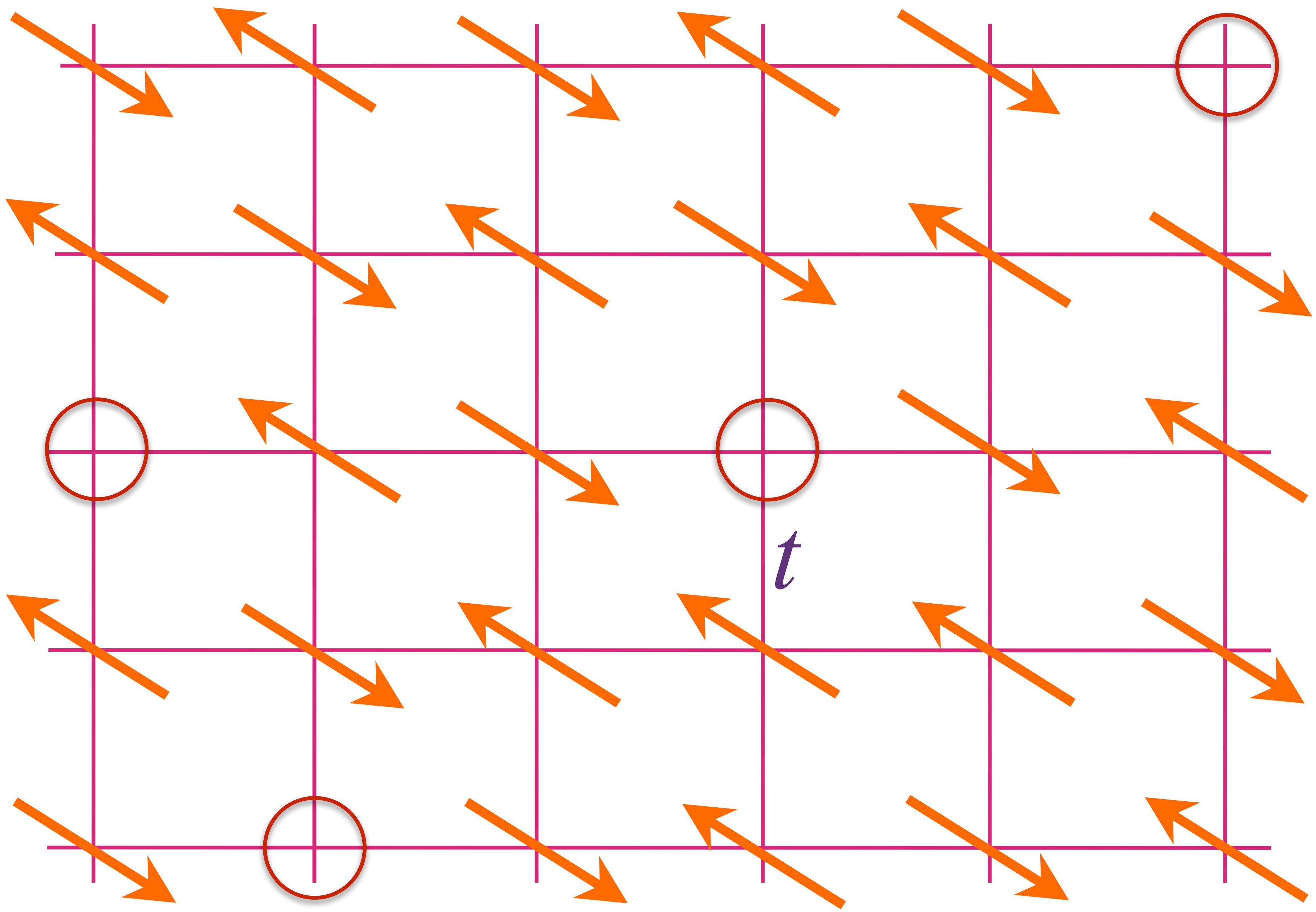

Baskaran, Anderson (1988)

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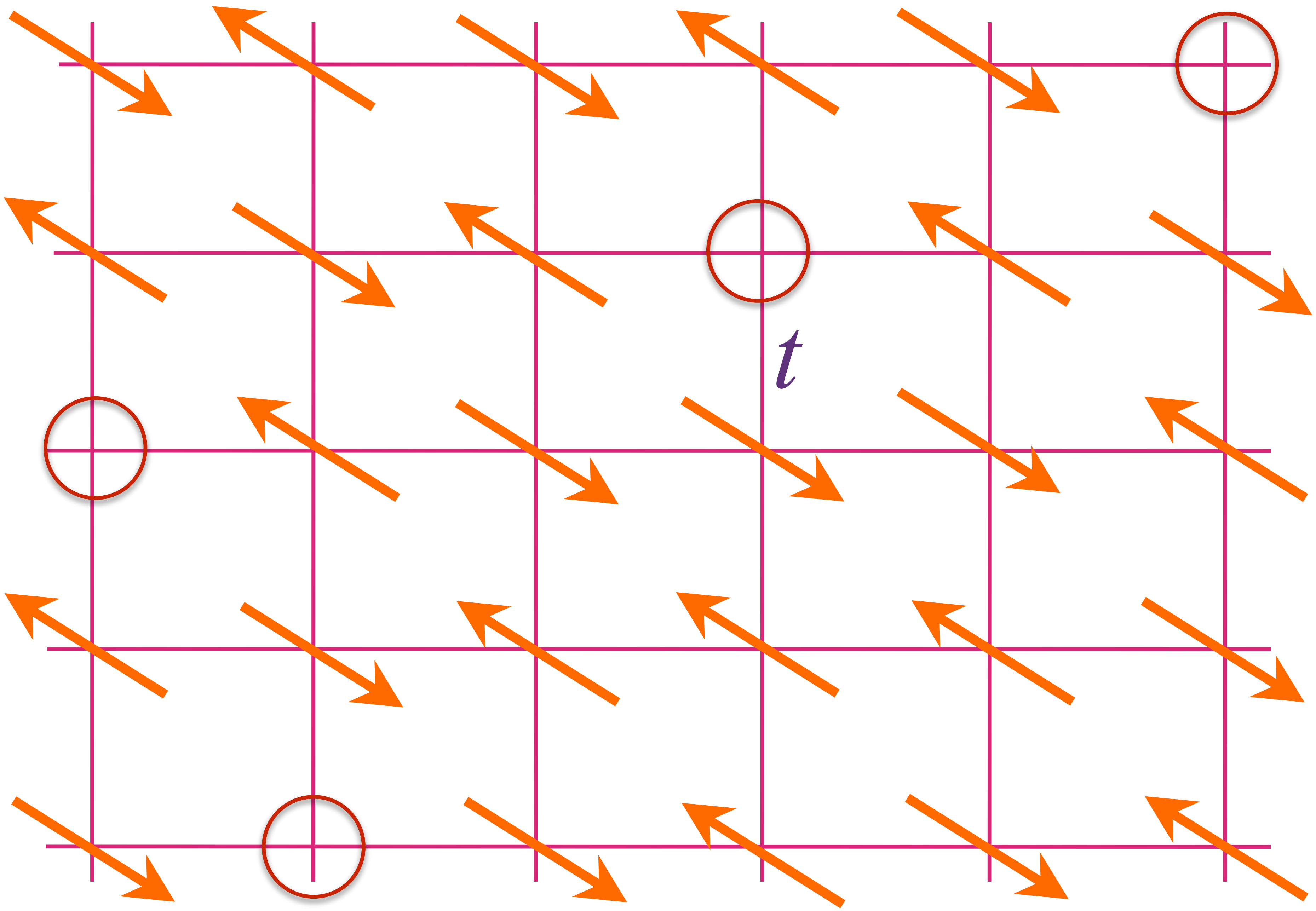

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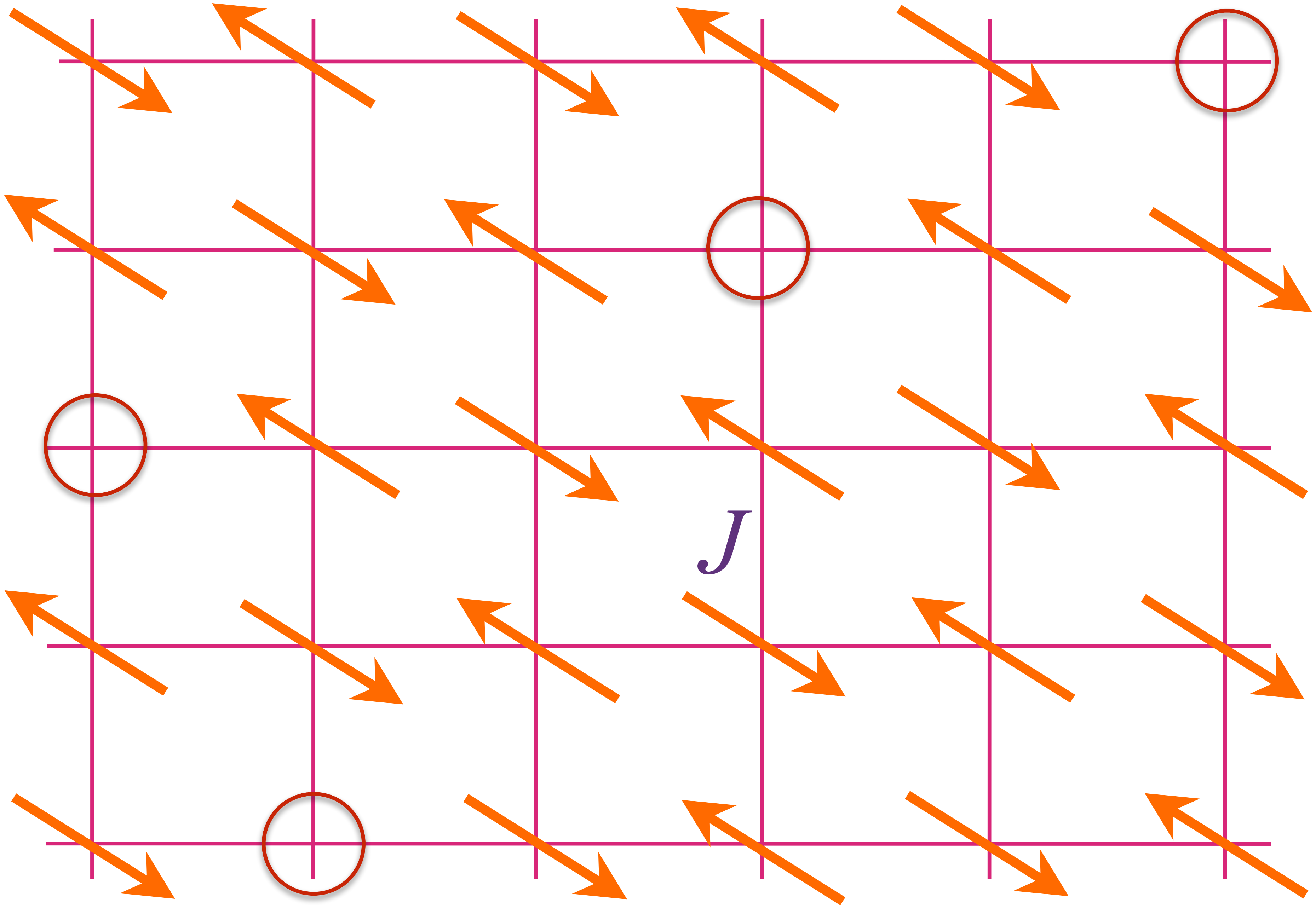

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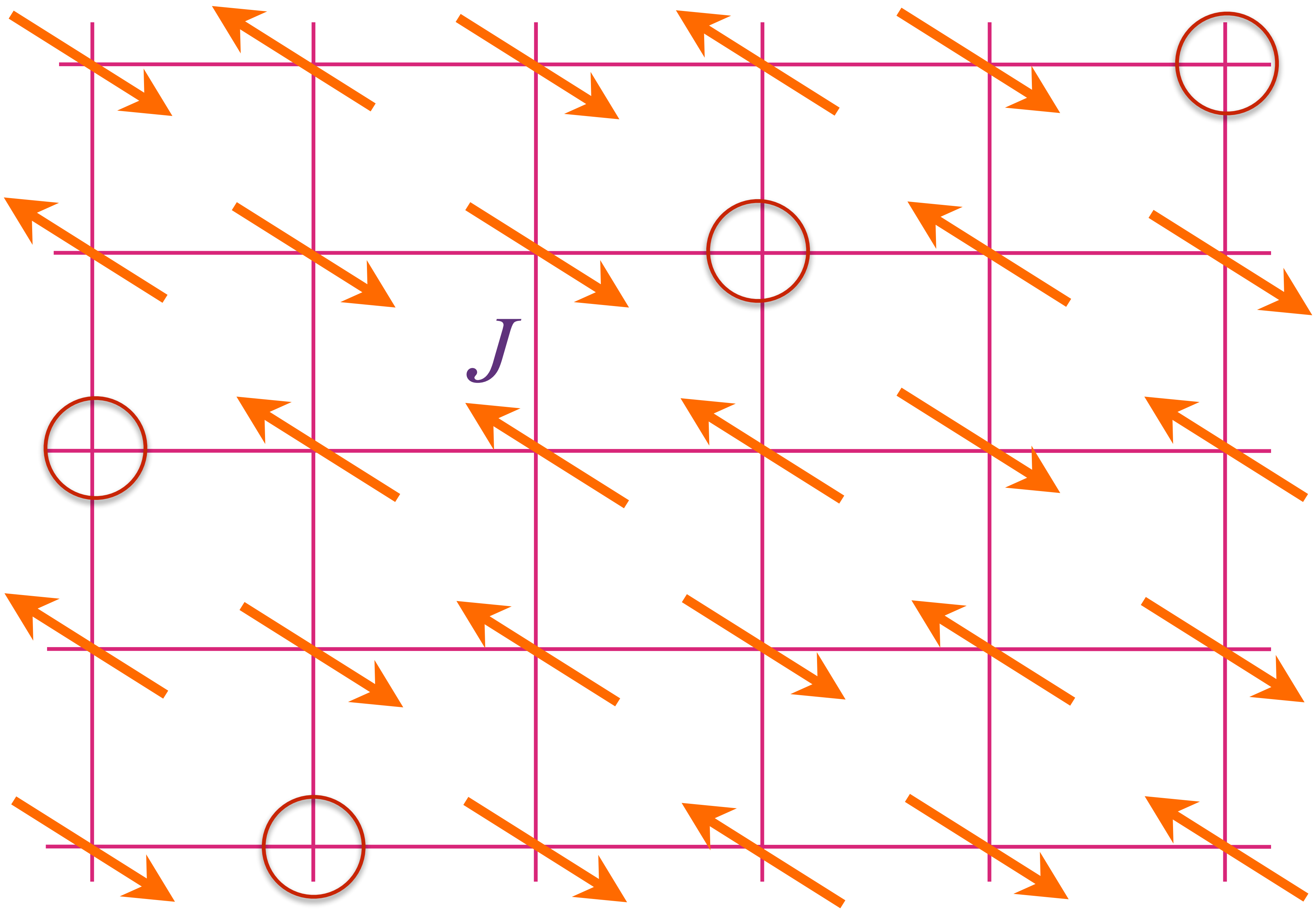

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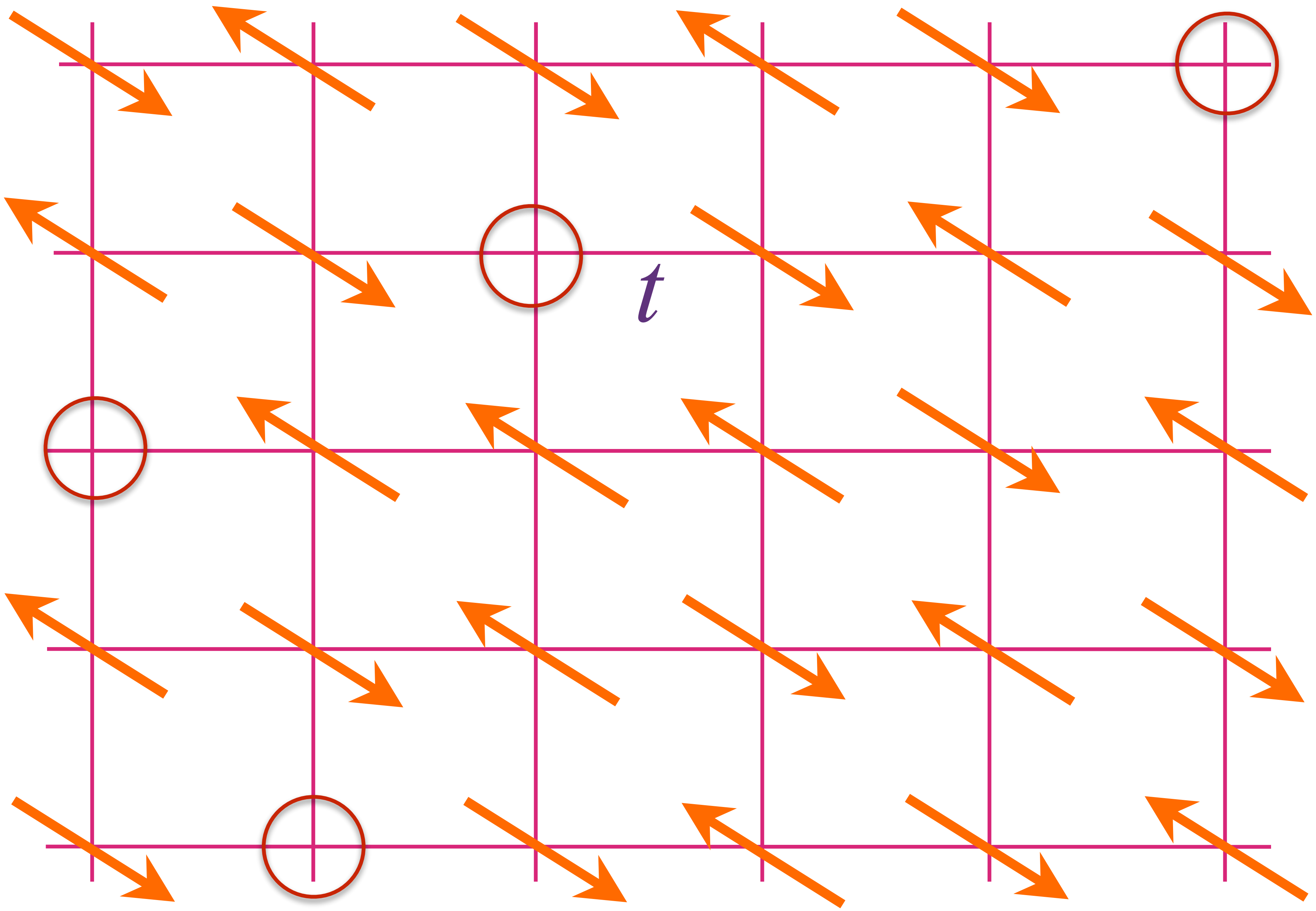

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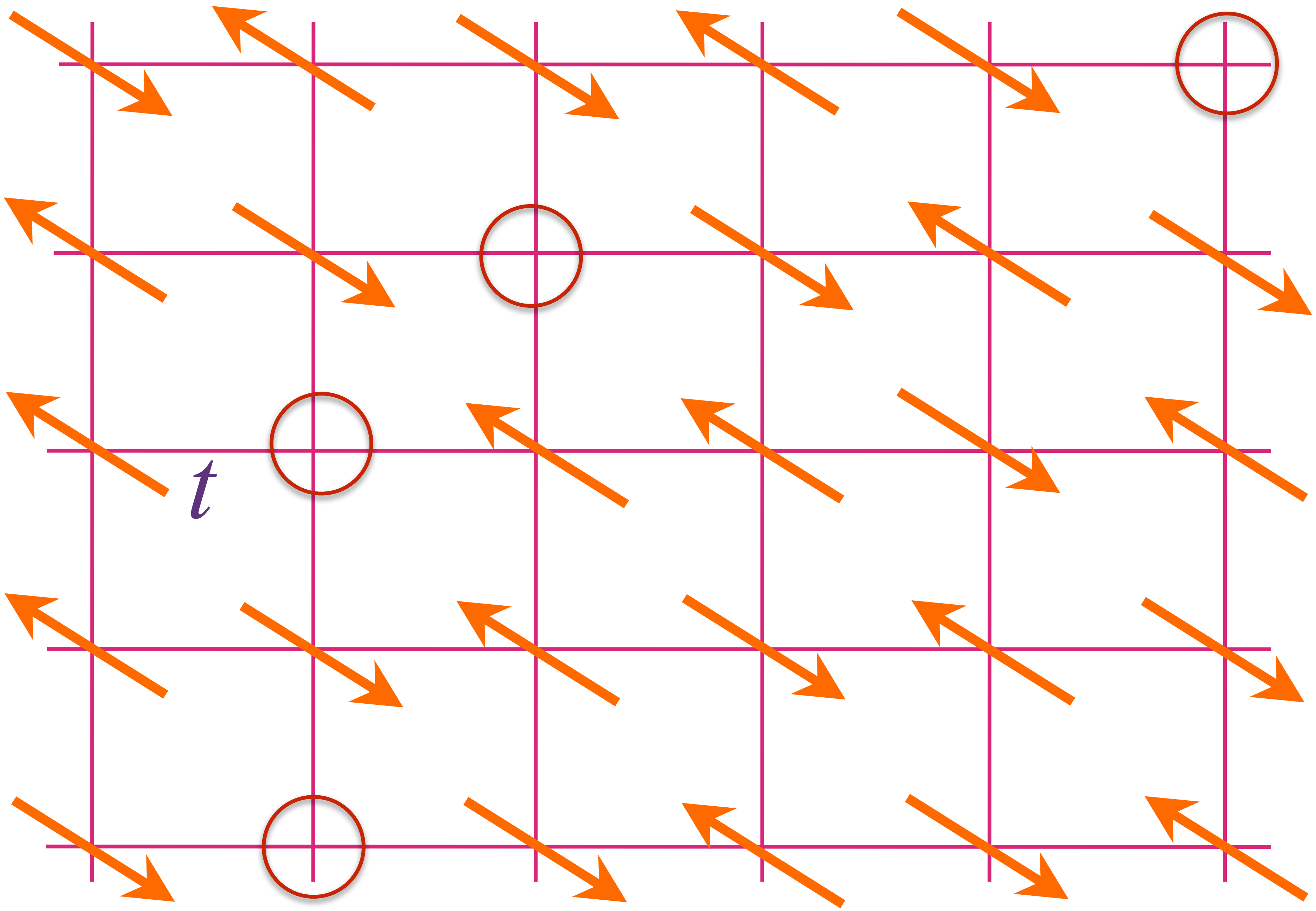

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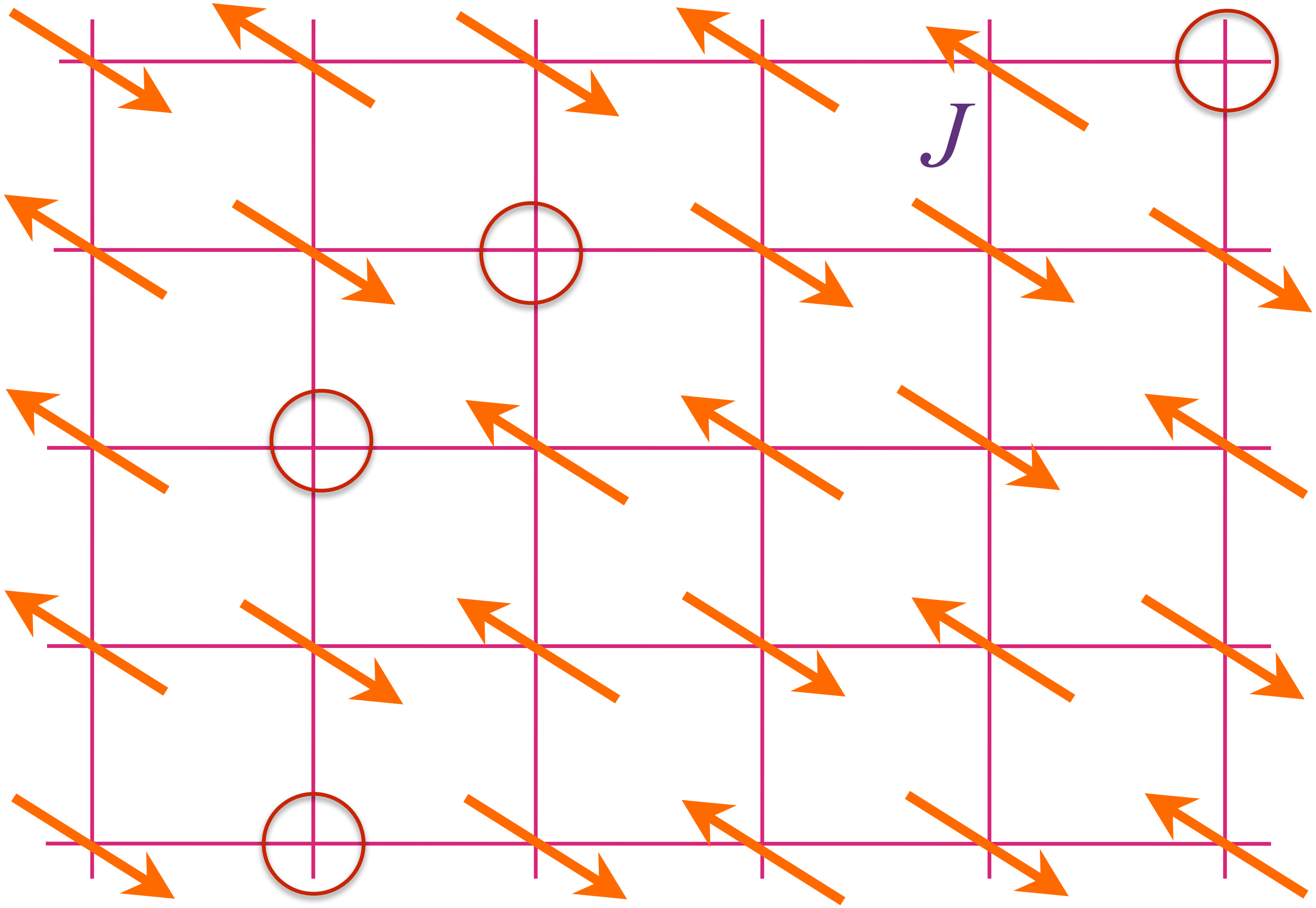

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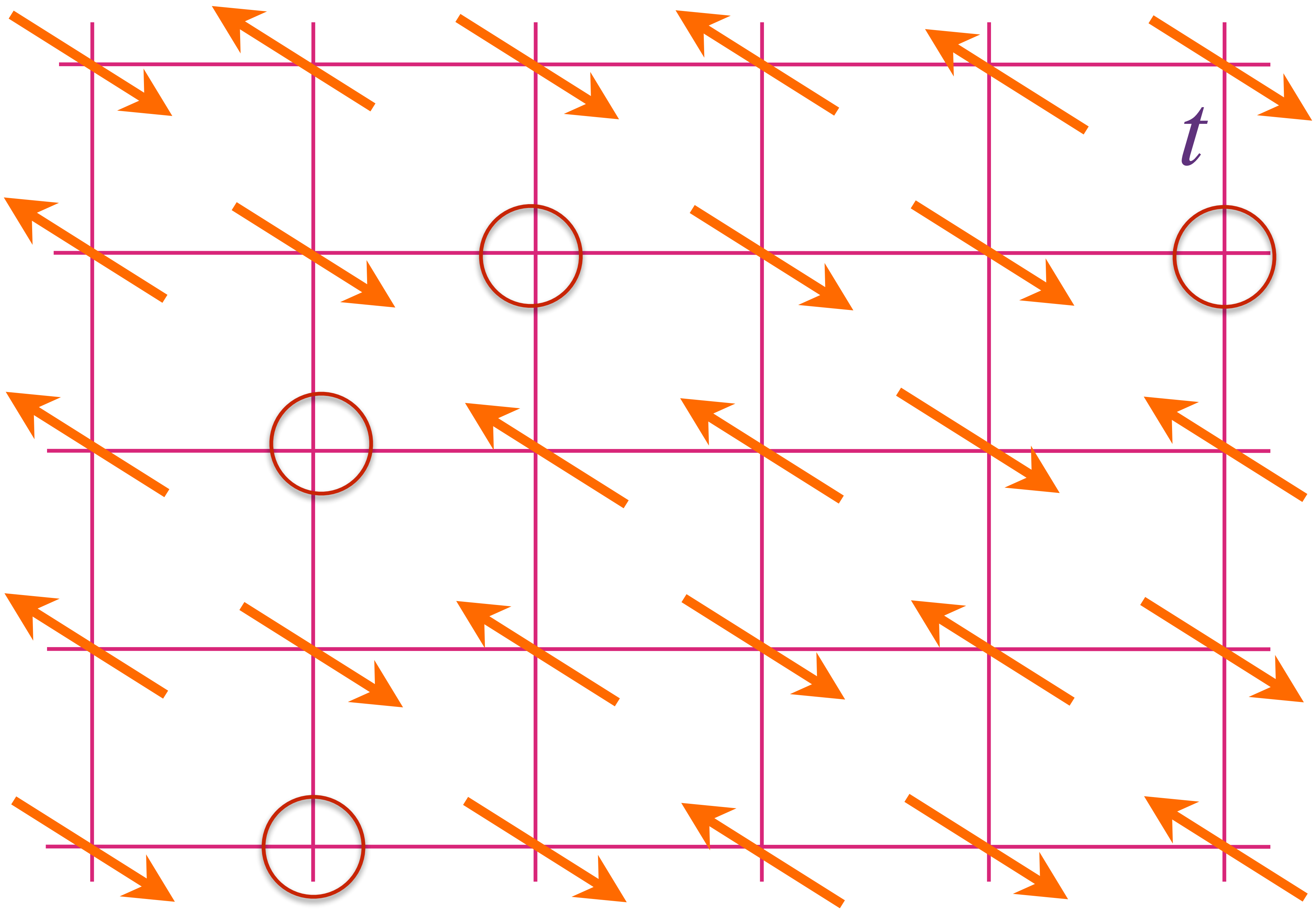

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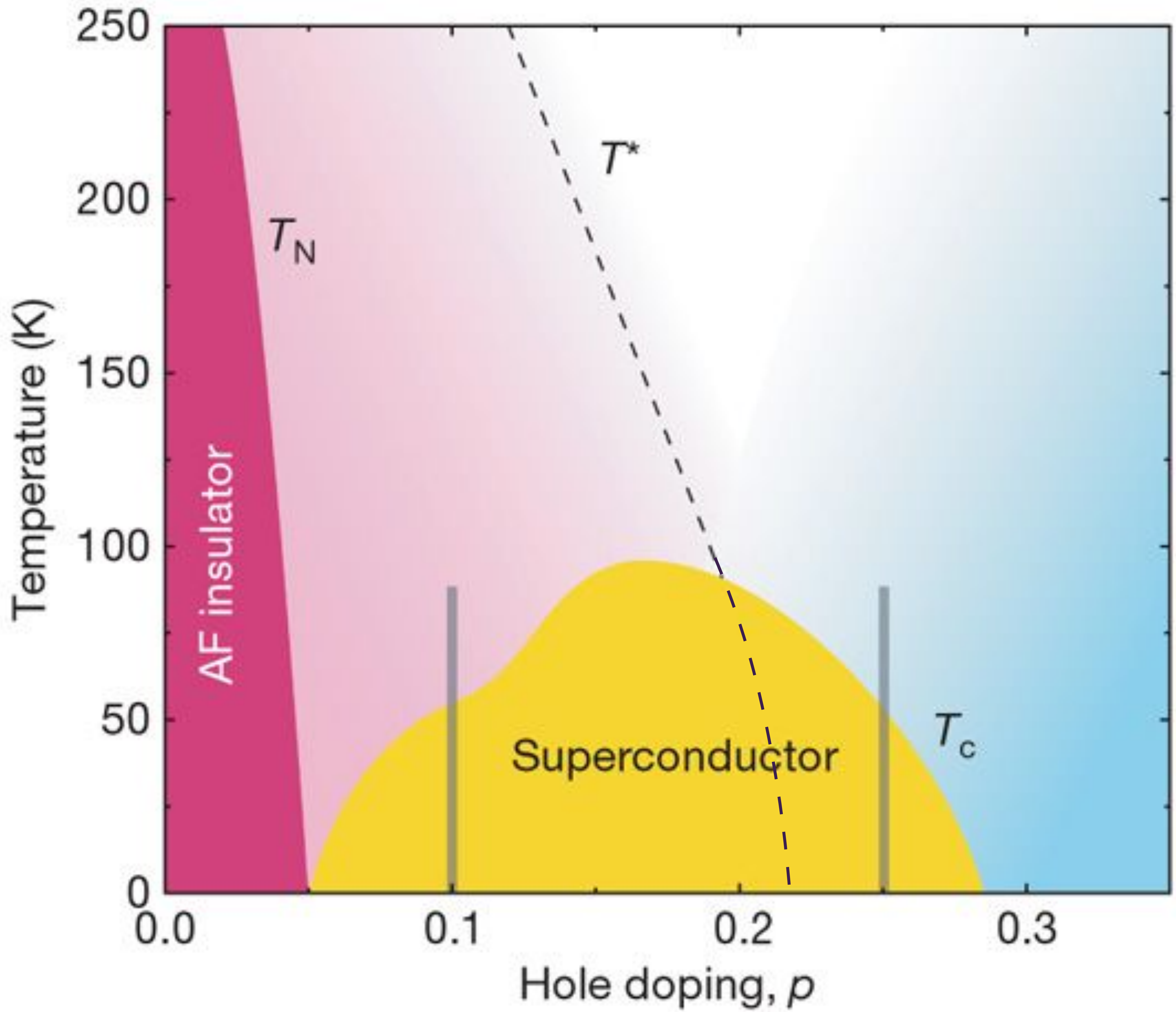
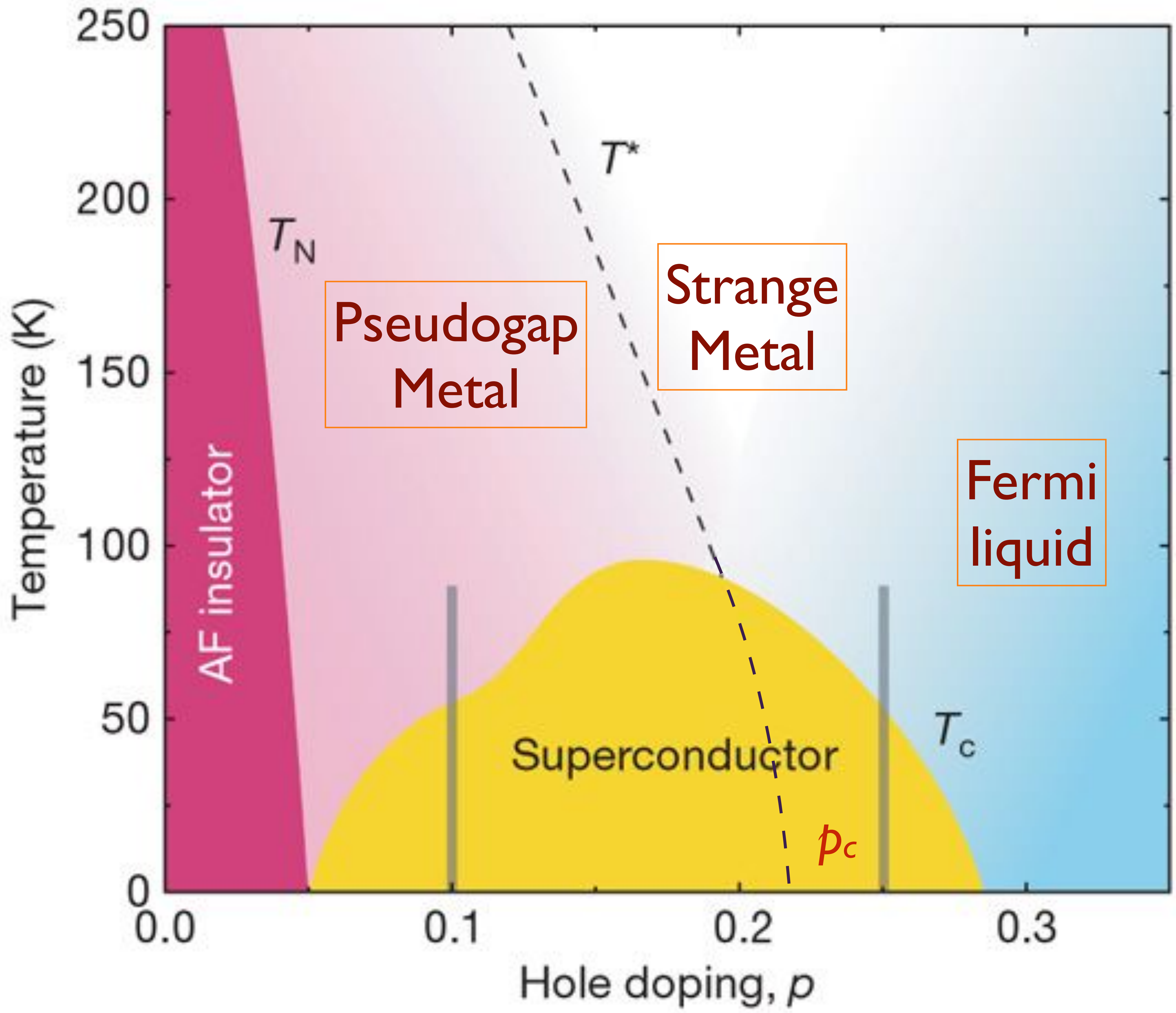


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Superconductor
T_c



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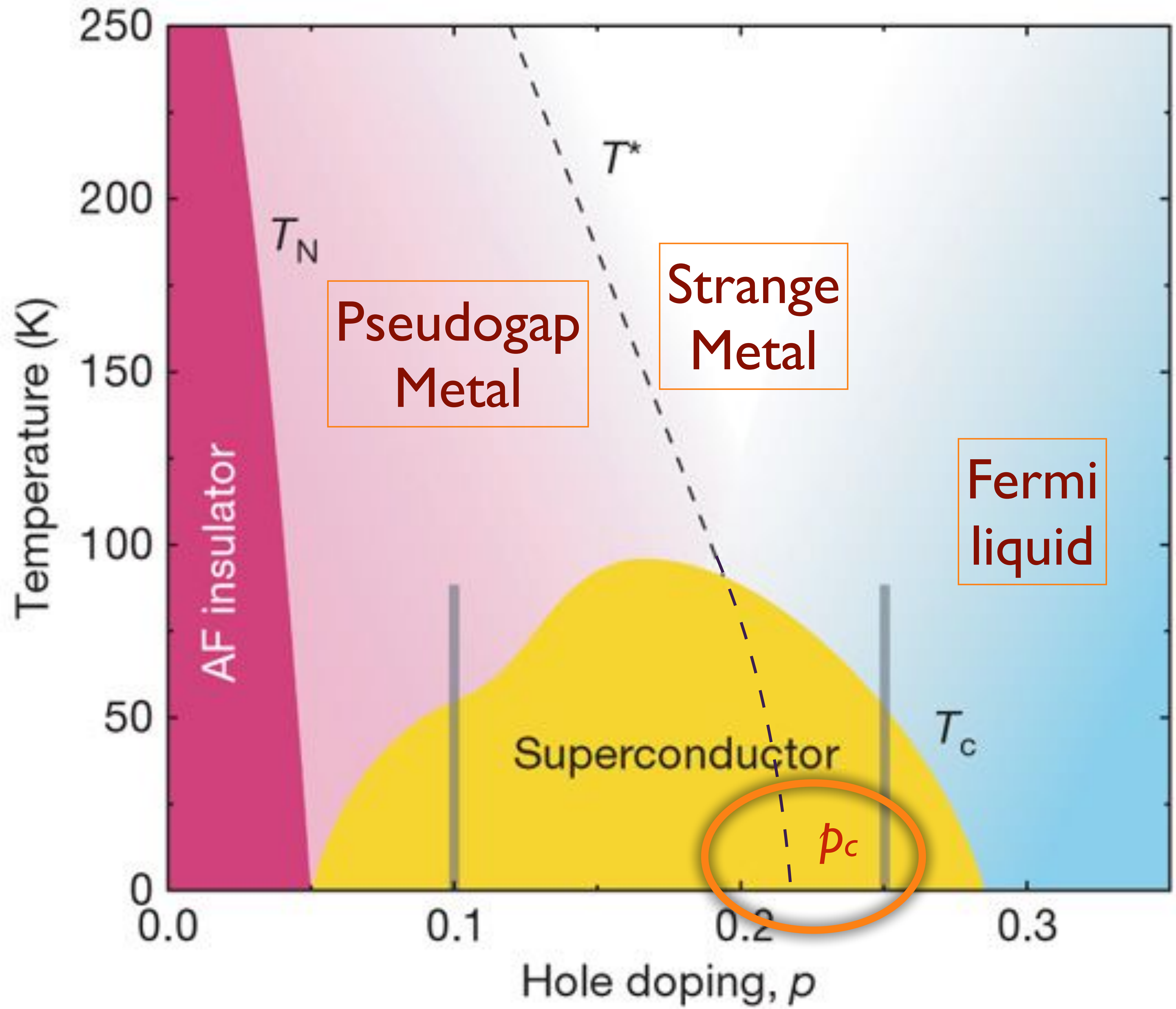

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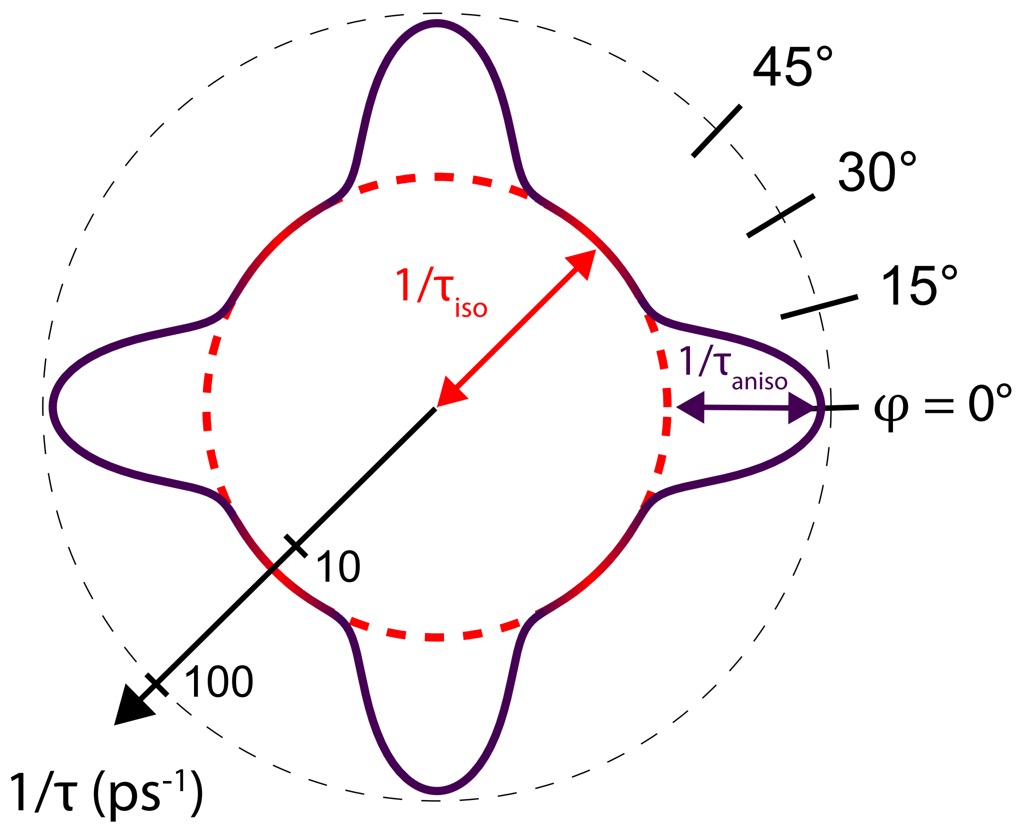
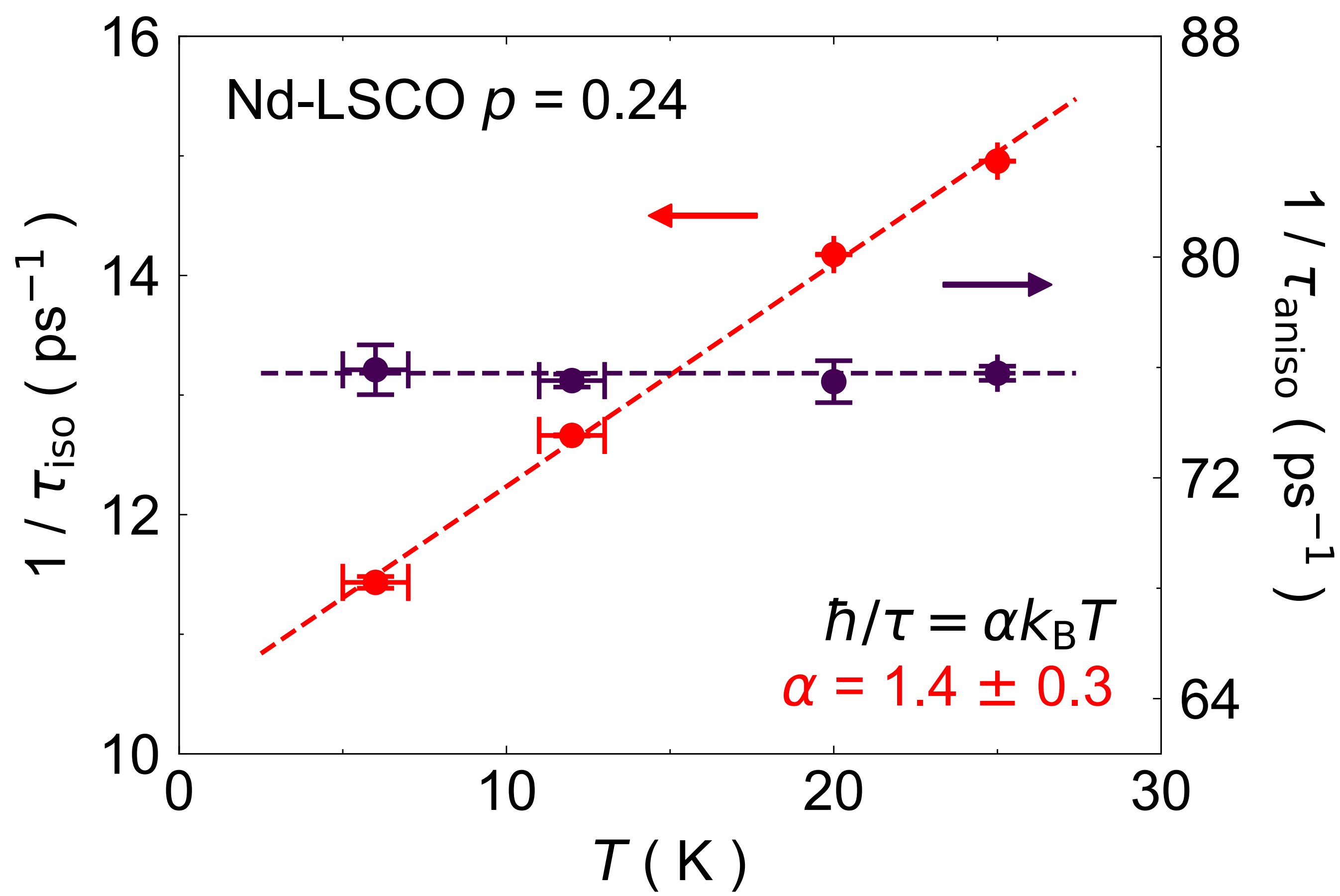
Measurement of the Planckian Scattering Rate

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw, arXiv:2011.13054

Angle-dependent magnetoresistance in Nd-LSCO near $p = p_c \approx 0.23$.





$$\frac{1}{\tau} = \frac{1}{\tau_{\text{aniso}}(\vec{k})} + \frac{\alpha}{\hbar} k_B T$$

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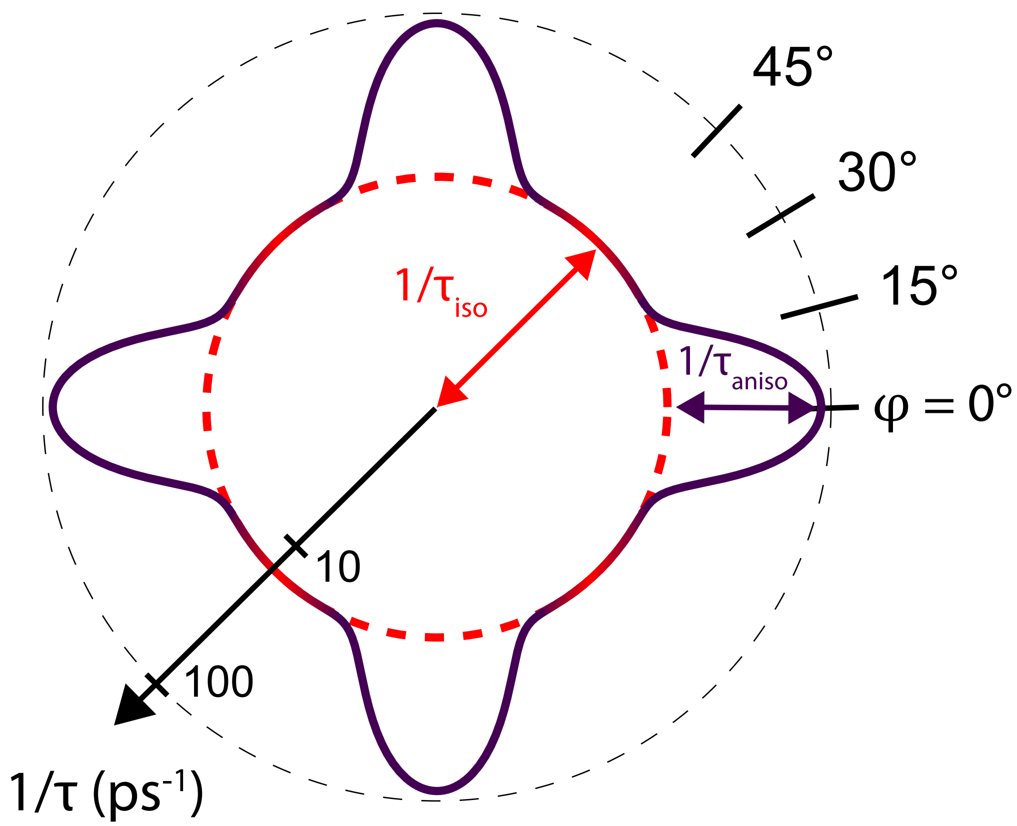

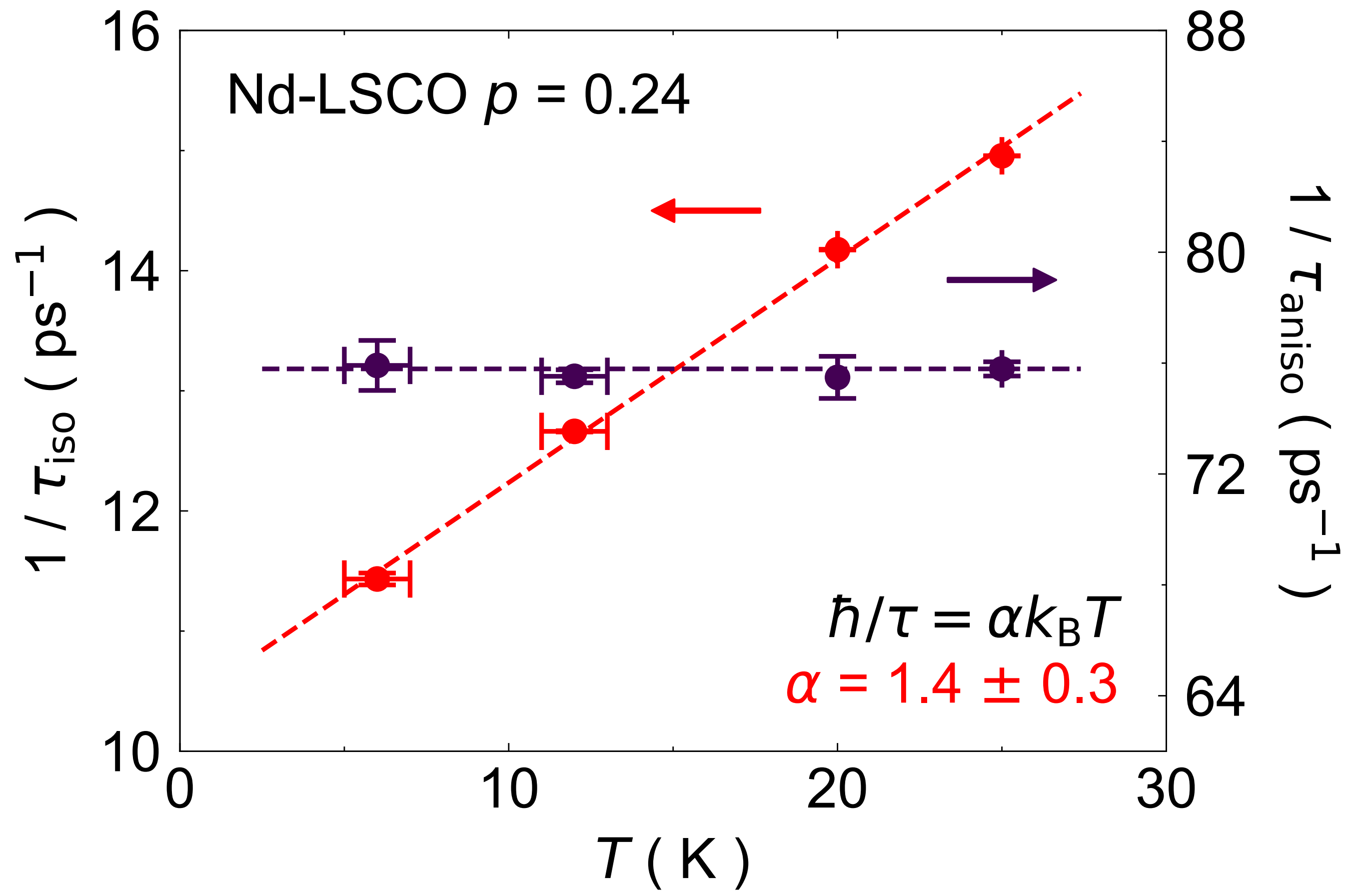

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Henry Shackleton

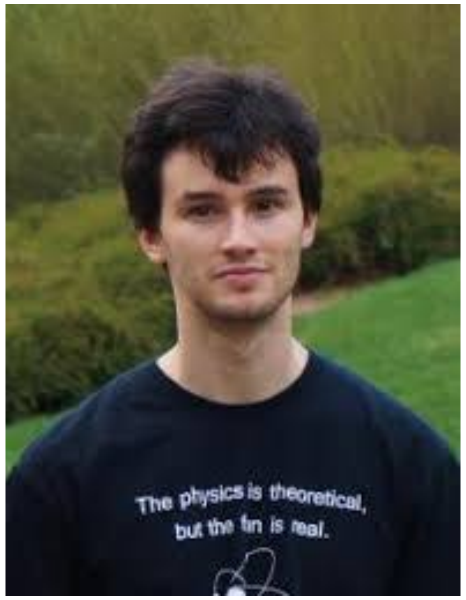
Alexander Wietek

Antoine Georges

arXiv:2012.06589

Maria Tikhanovskaya

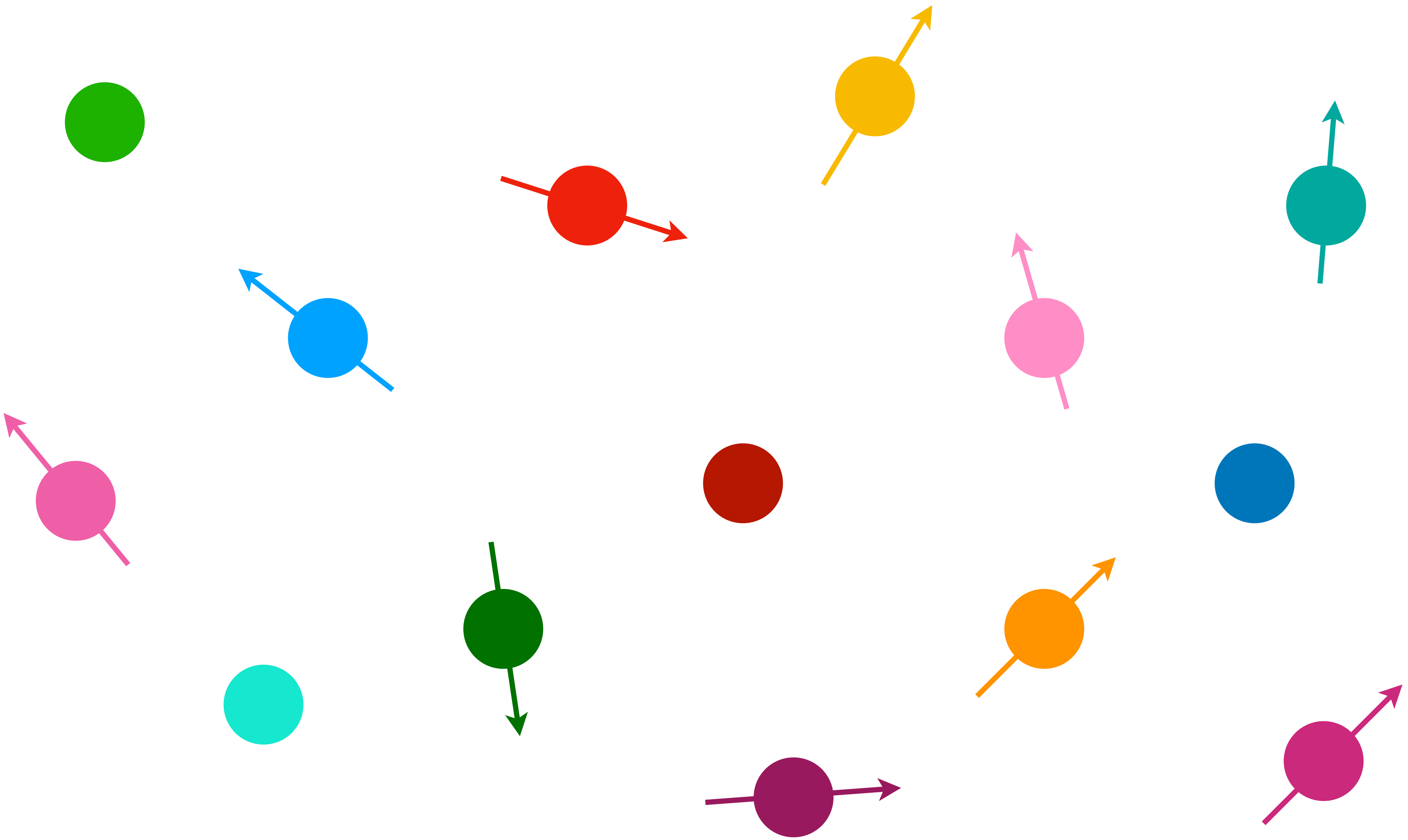
Haoyu Guo



Grigory Tarnopolsky

arXiv:2010.09742
arXiv:2012.14449

Random *t*-*J* model



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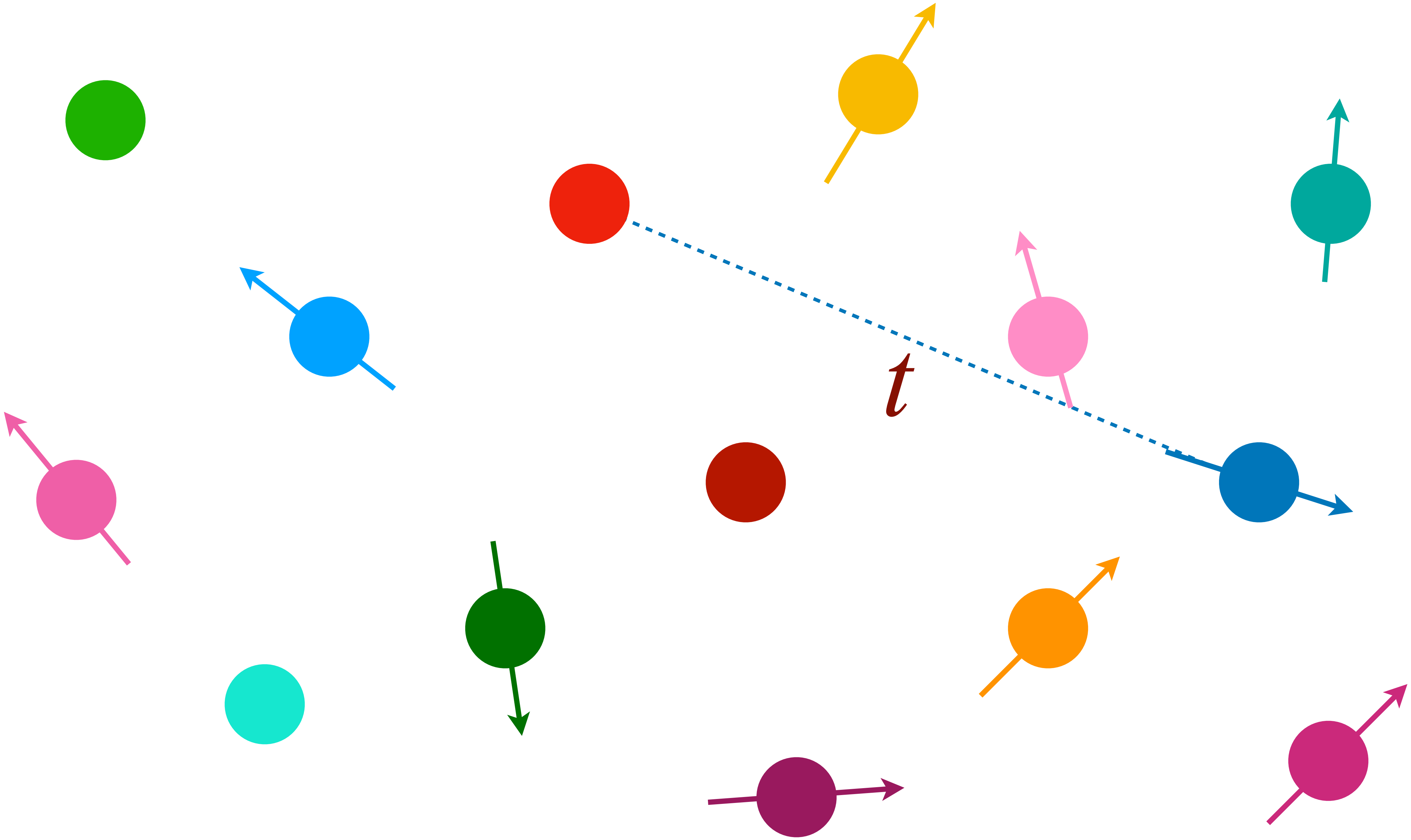

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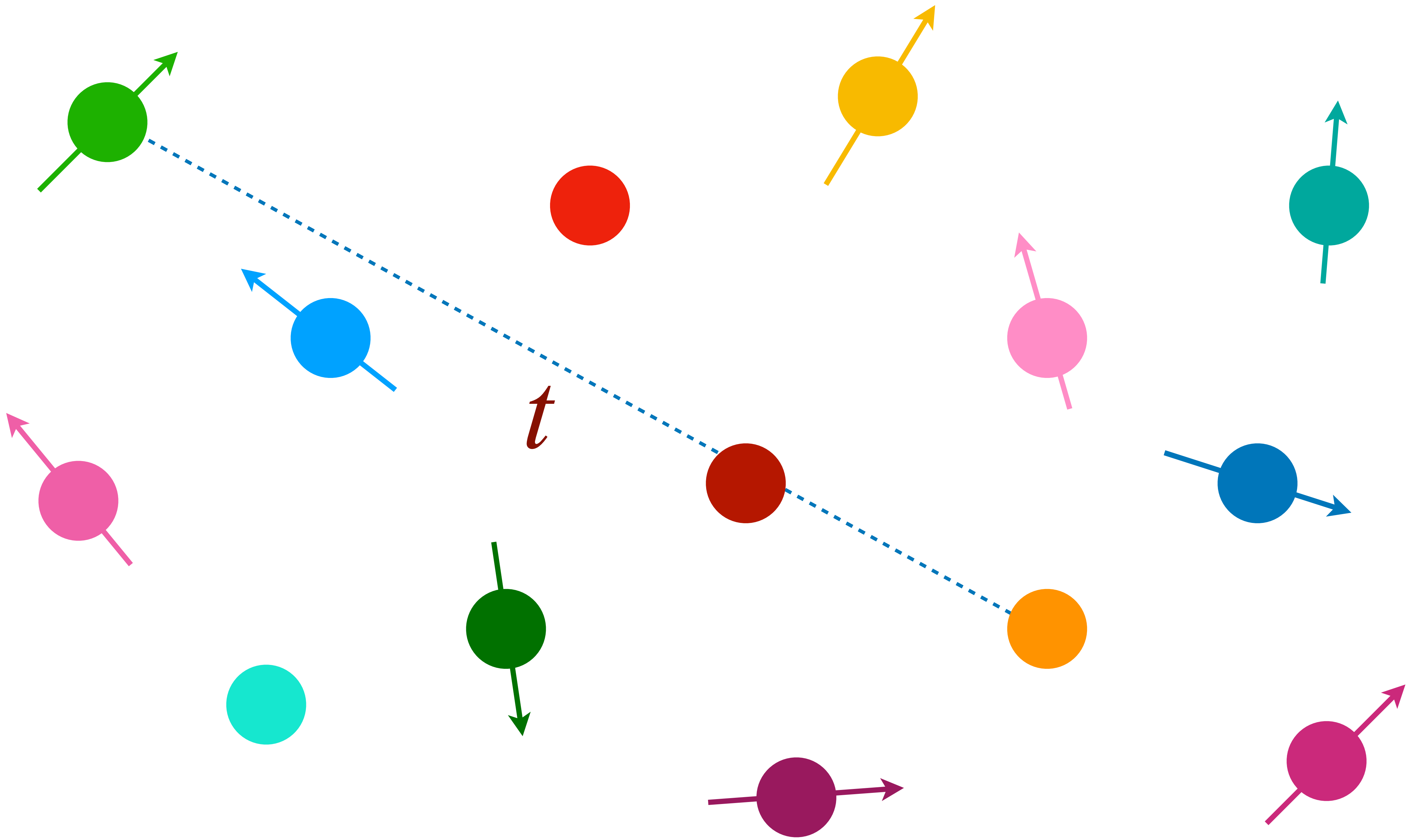

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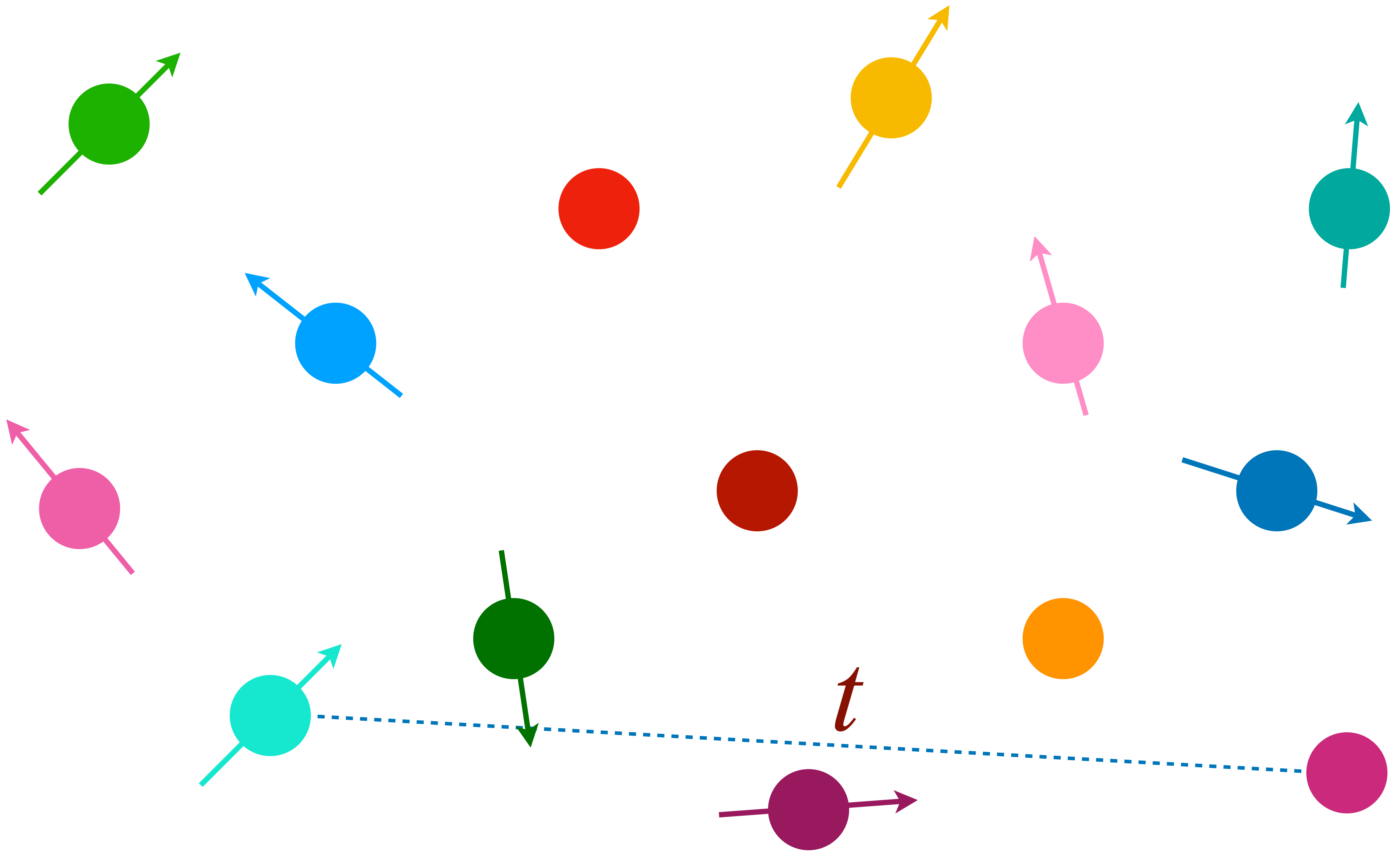

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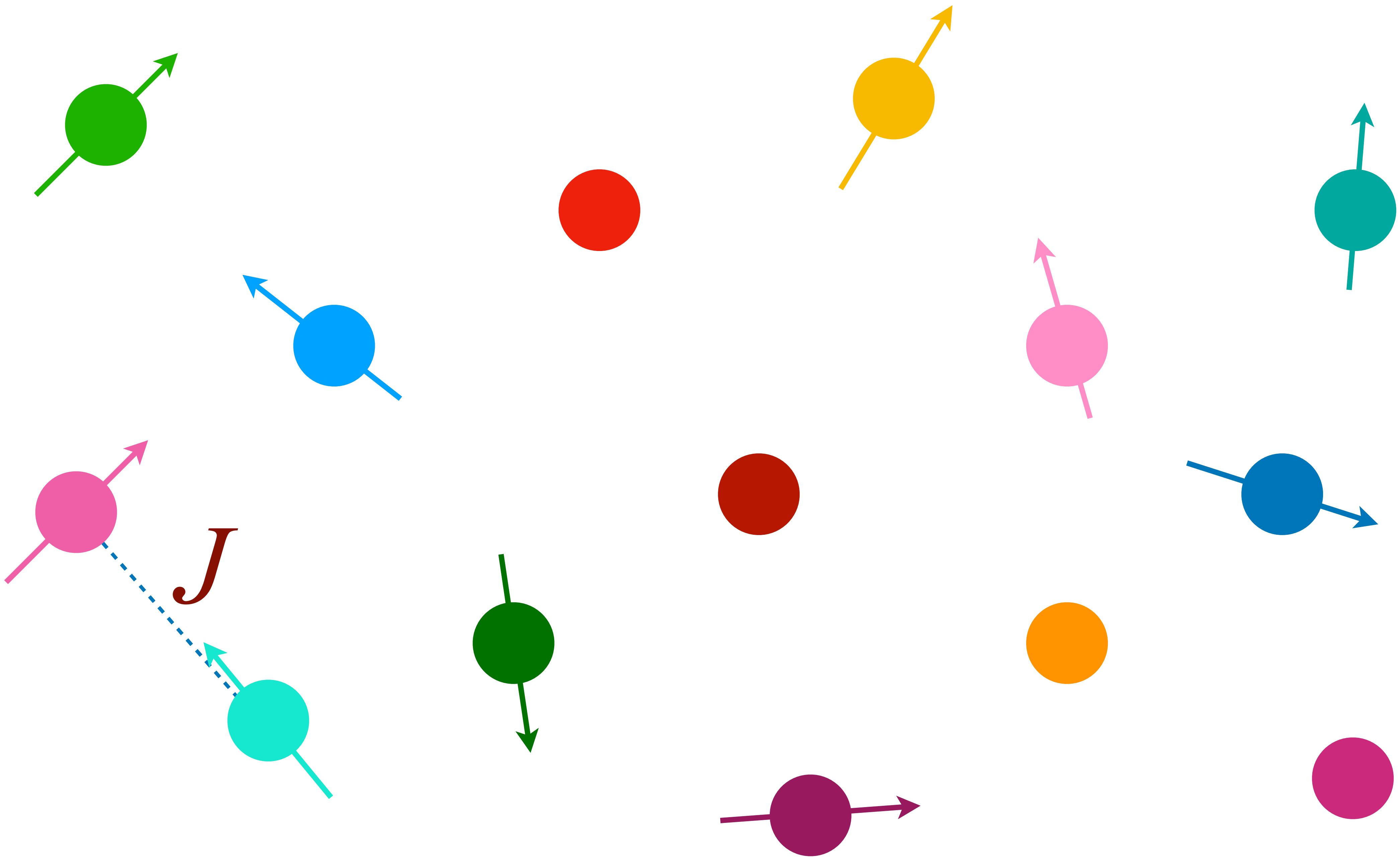

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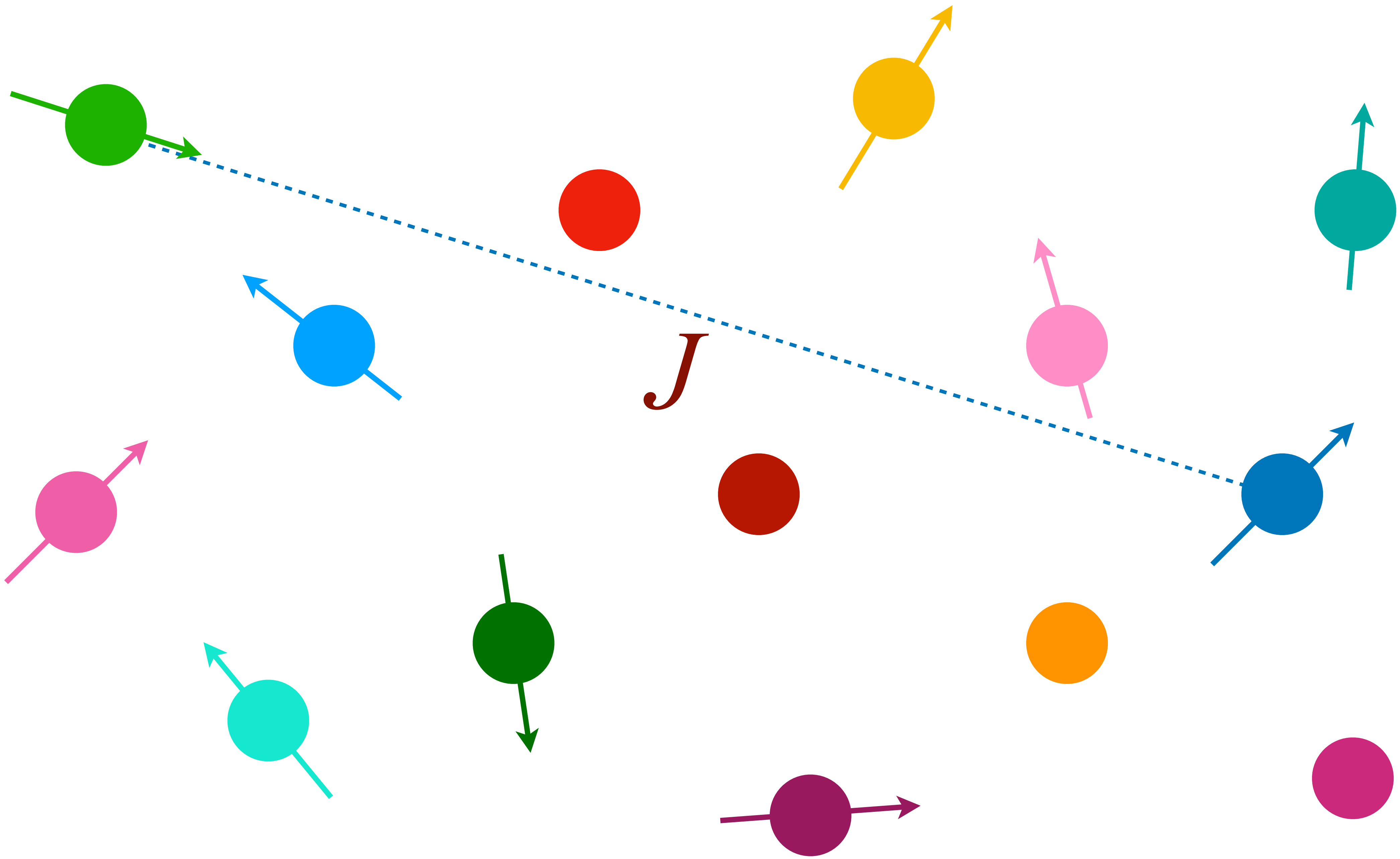

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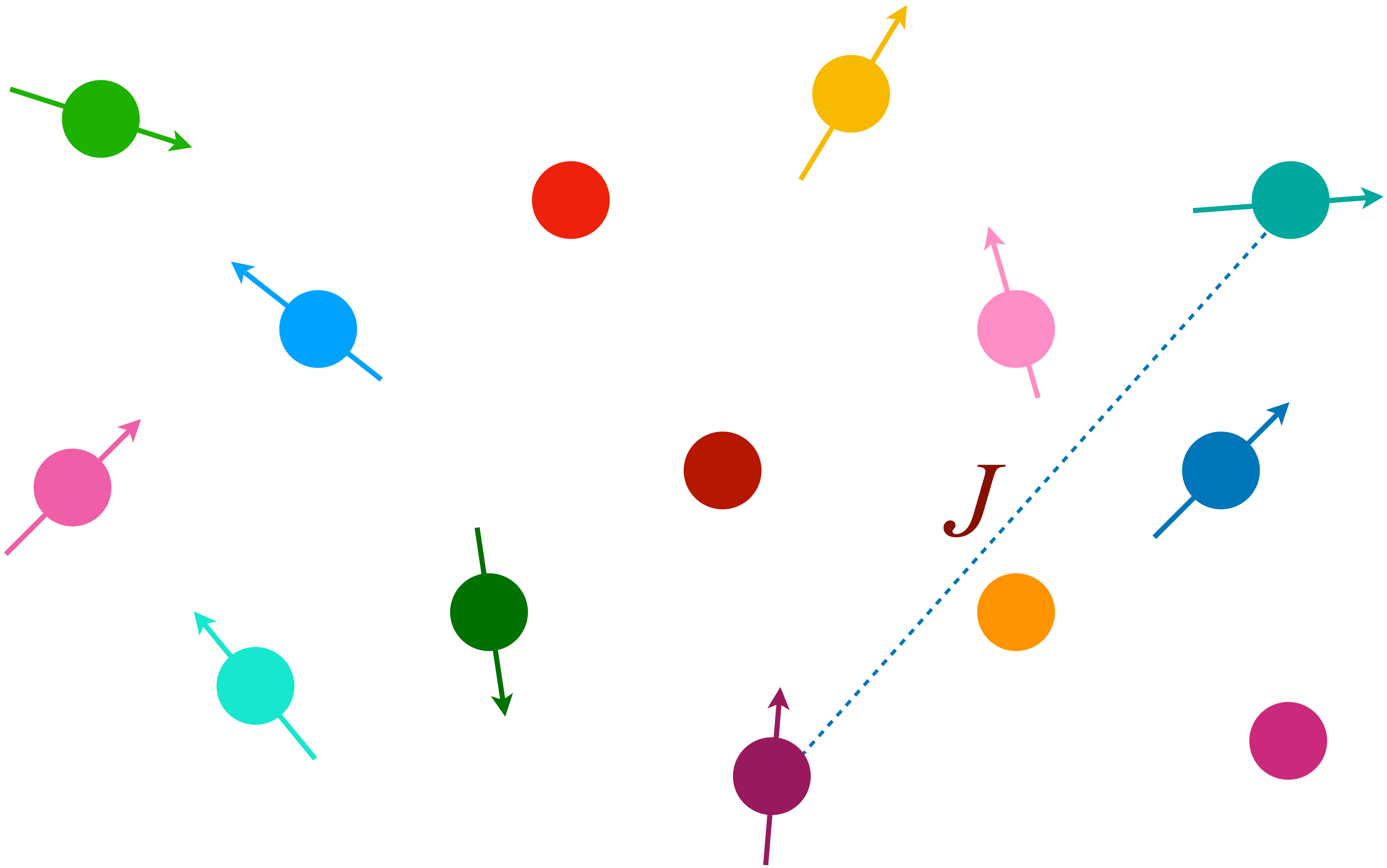
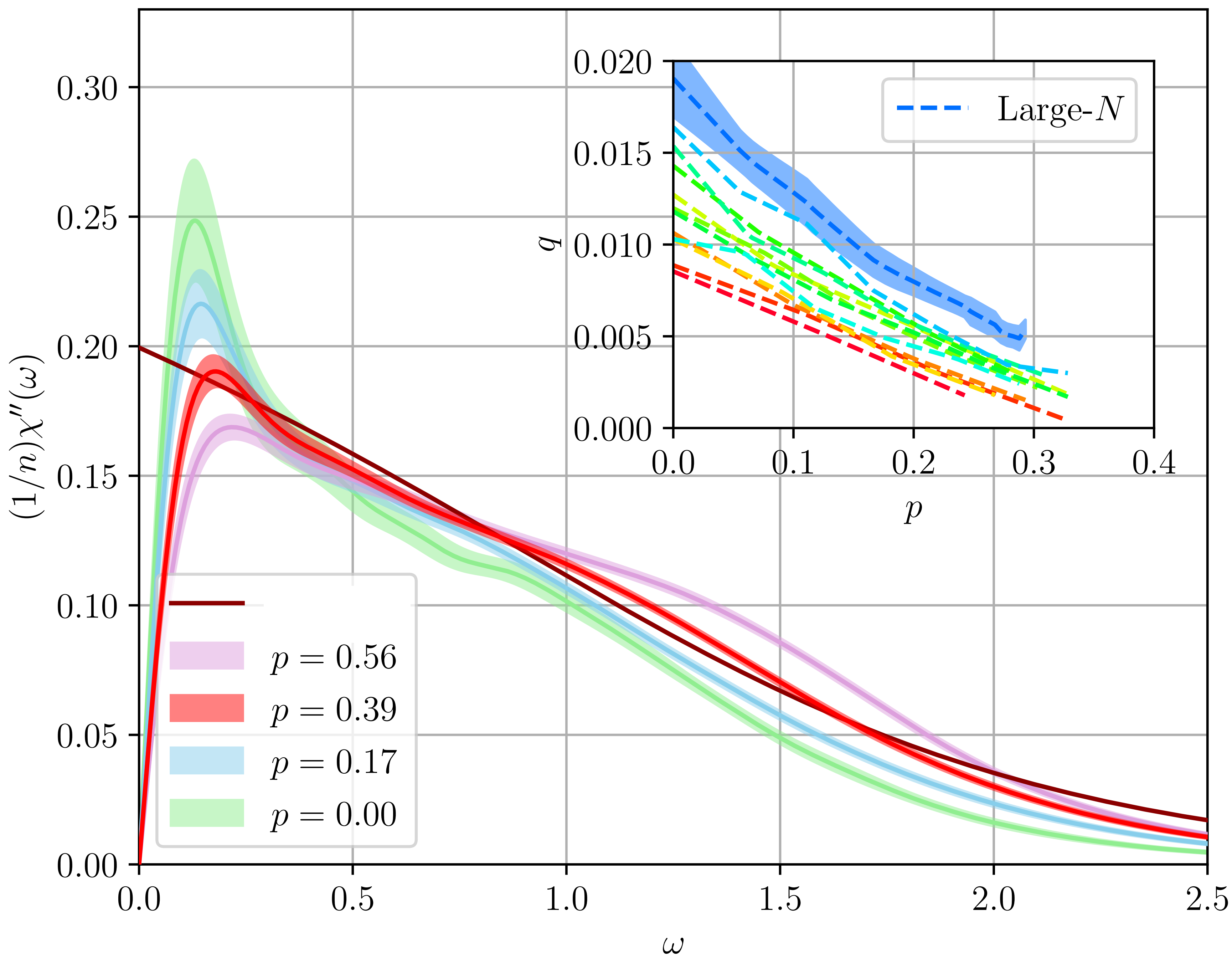


Dynamic spin susceptibility

Probability to flip an electron spin while absorbing energy $\hbar\omega$



Dynamic spin susceptibility

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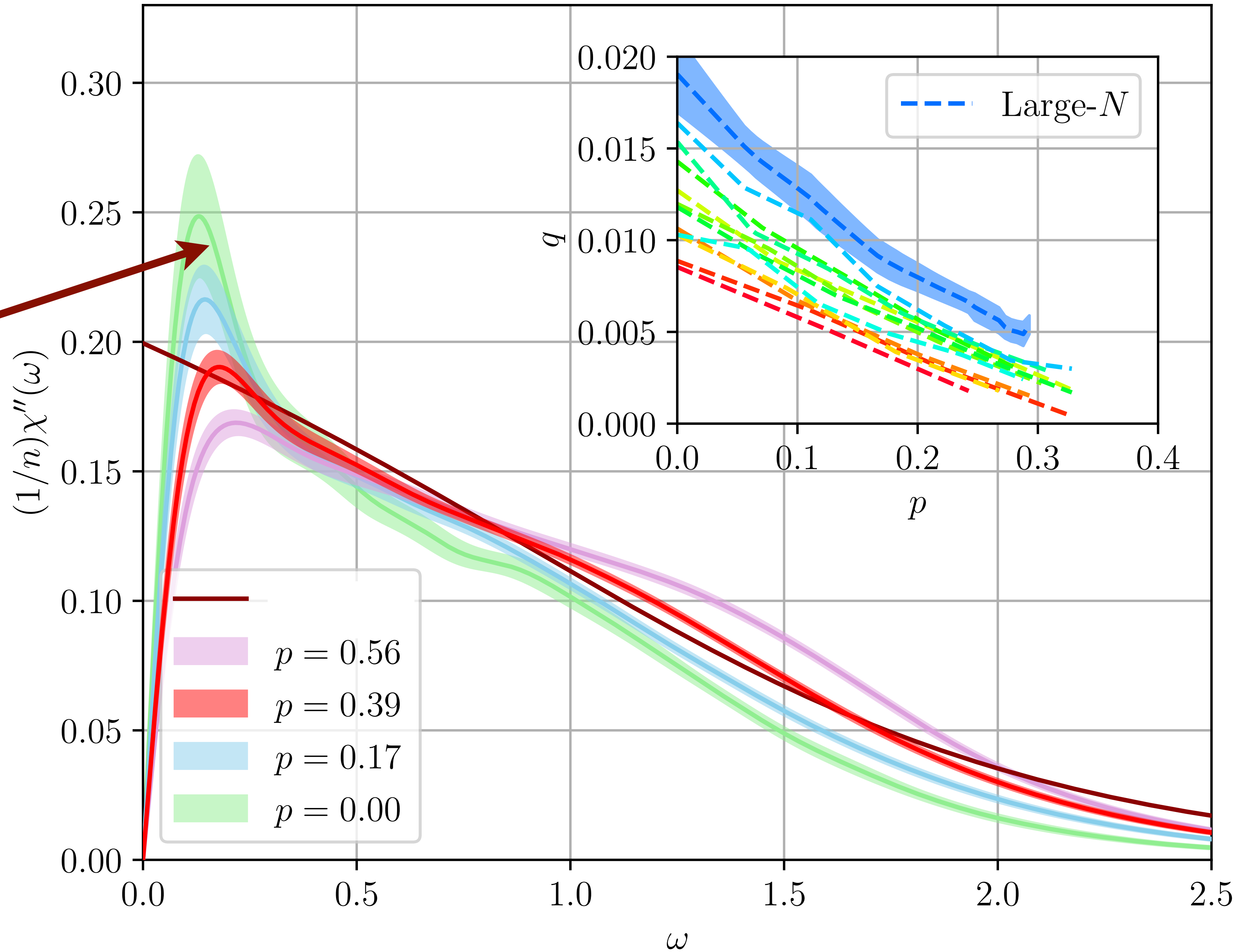
Spin glass order

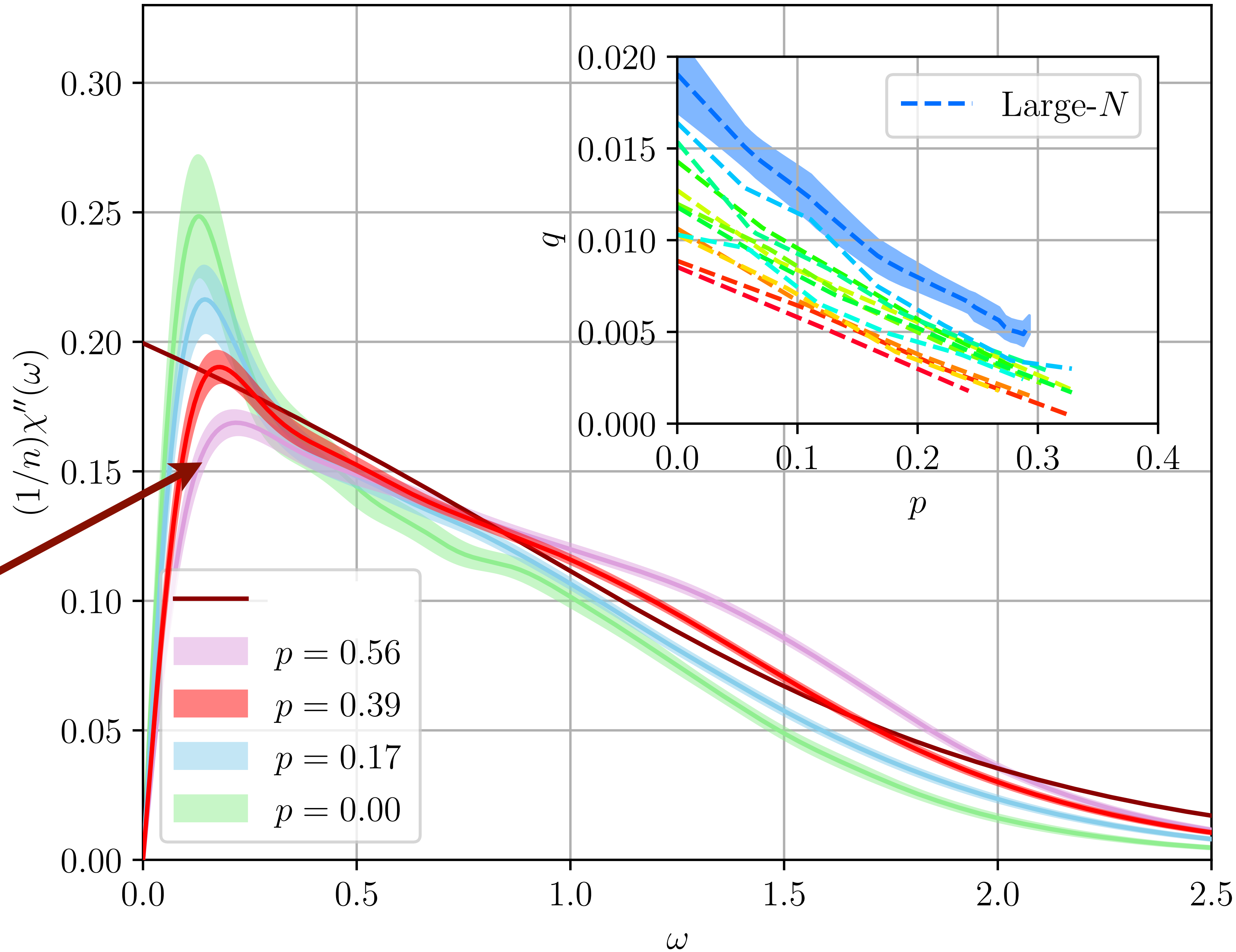


Spin glass order q non-zero for $p < p_c \approx 0.4$

Dynamic spin susceptibility

Probability to flip an electron spin while absorbing energy $\hbar\omega$

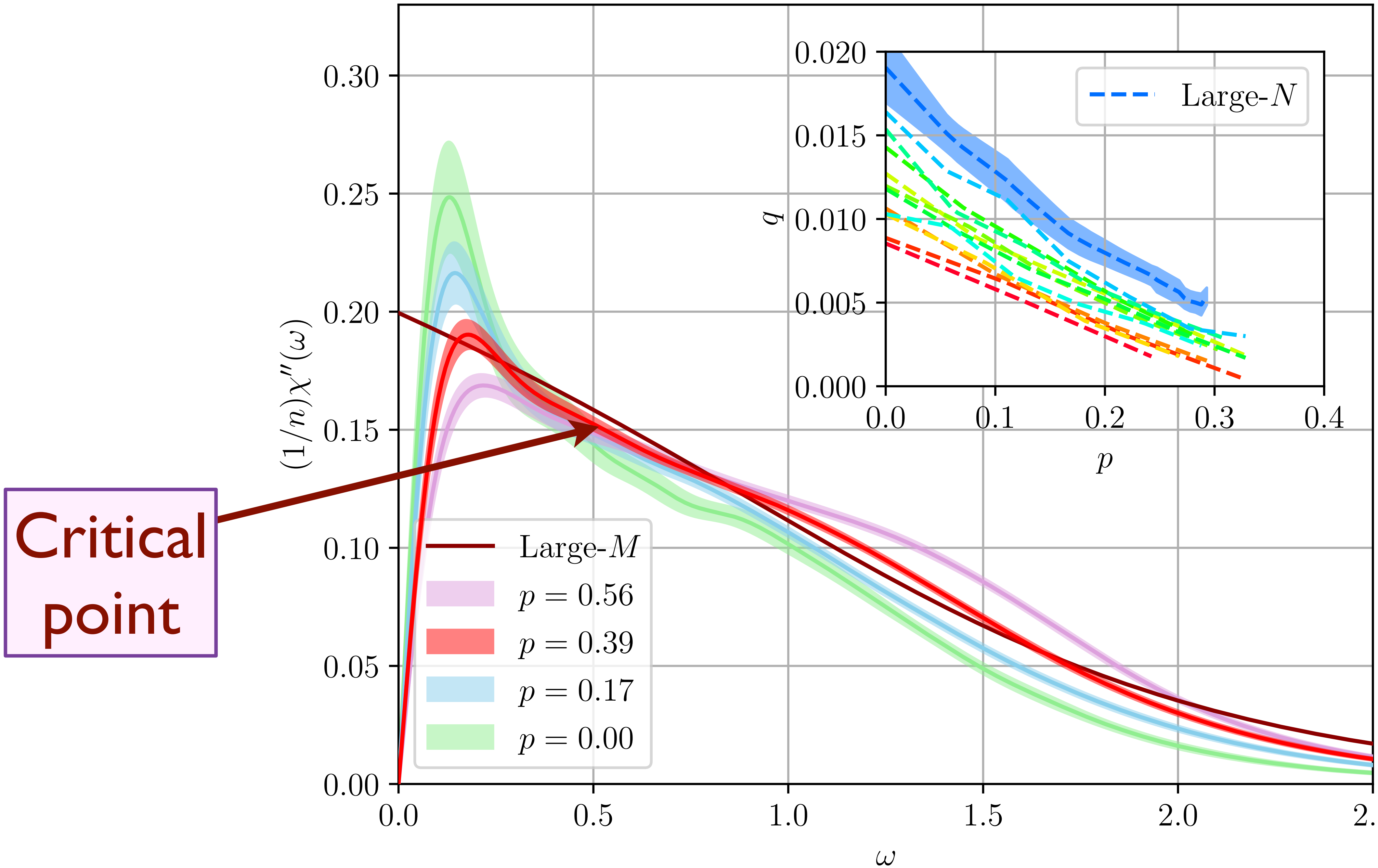
Ordinary metal



Spin susceptibility and other properties match those of an ordinary metal $p > p_c$

Dynamic spin susceptibility

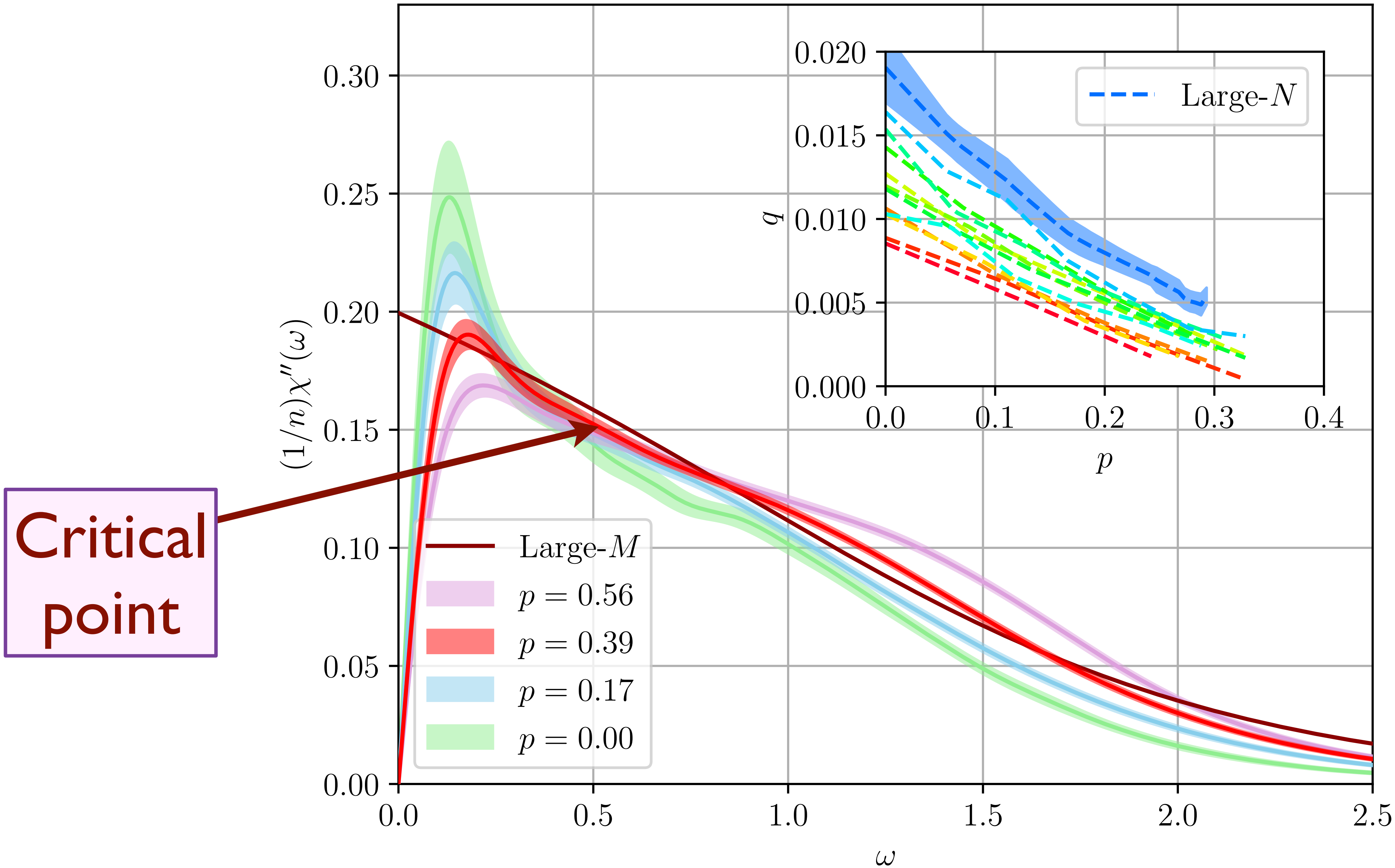
Probability to flip an electron spin while absorbing energy $\hbar\omega$



Critical spin susceptibility matches the SYK model!
Planckian dissipation in time $\sim \hbar/(k_B T)$,
and frequency dependence $\sim \text{sgn}(\omega)\,[1 - \mathcal{C}\gamma|\omega| + \ldots]$
matches contribution of boundary graviton.

Dynamic spin susceptibility

Probability to flip an electron spin while absorbing energy $\hbar\omega$



D. G. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, and S. Sachdev, PRX **10**, 021033 (2020)

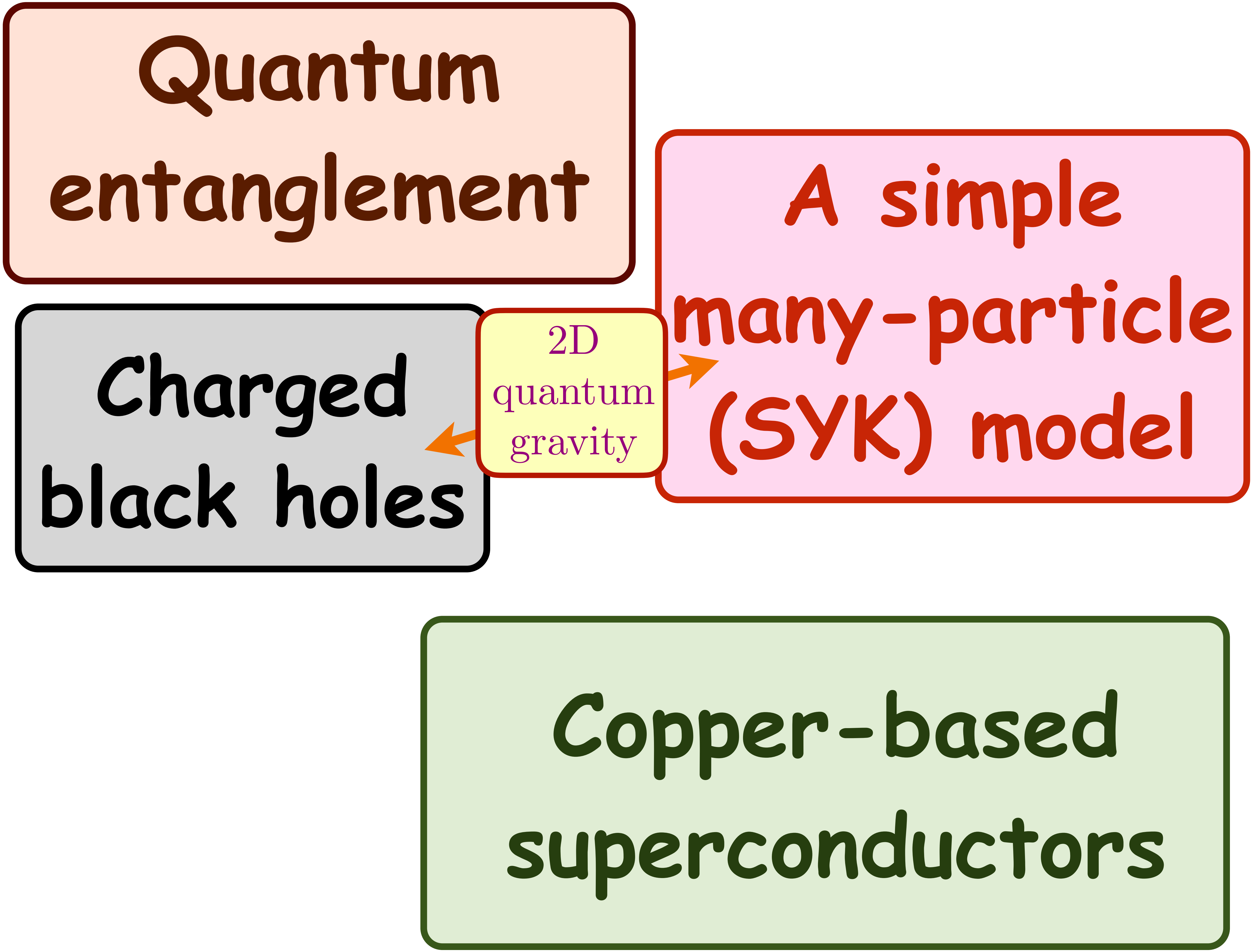
SYK criticality can be understood by the *fractionalization* of the electron into 'partons' carrying its spin and charge.
These partons obey an SYK-like model

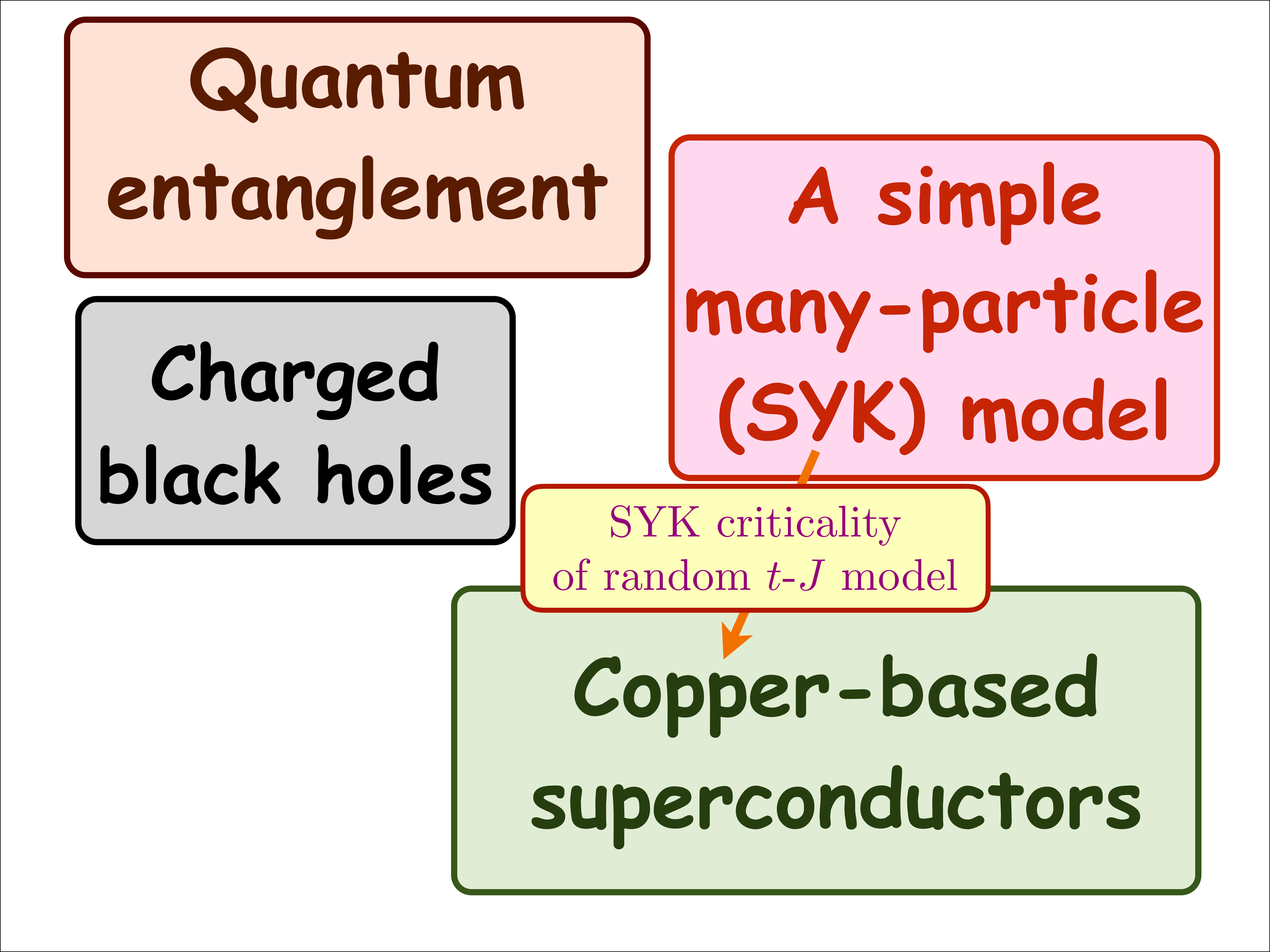
Quantum entanglement

A simple many-particle (SYK) model

Charged black holes

Copper-based superconductors


Quantum entanglement
A simple many-particle (SYK) model
Charged black holes
2D quantum gravity
Copper-based superconductors


Quantum entanglement
Charged black holes
A simple many-particle (SYK) model
SYK criticality of random t-J model
Copper-based superconductors

Complex multi-particle entanglement leads to quantum systems without quasiparticle excitations.

Many-body chaos and thermal equilibration in the shortest possible

$$\text{Planckian time} \sim \frac{\hbar}{k_B T}.$$