Fermi-volume-changing quantum phase transitions and the cuprate phase diagram

4th International Conference on
Evolution of Electronic Structure Theory and Experimental Realization
Indian Institute of Technology, Madras
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Subir Sachdev

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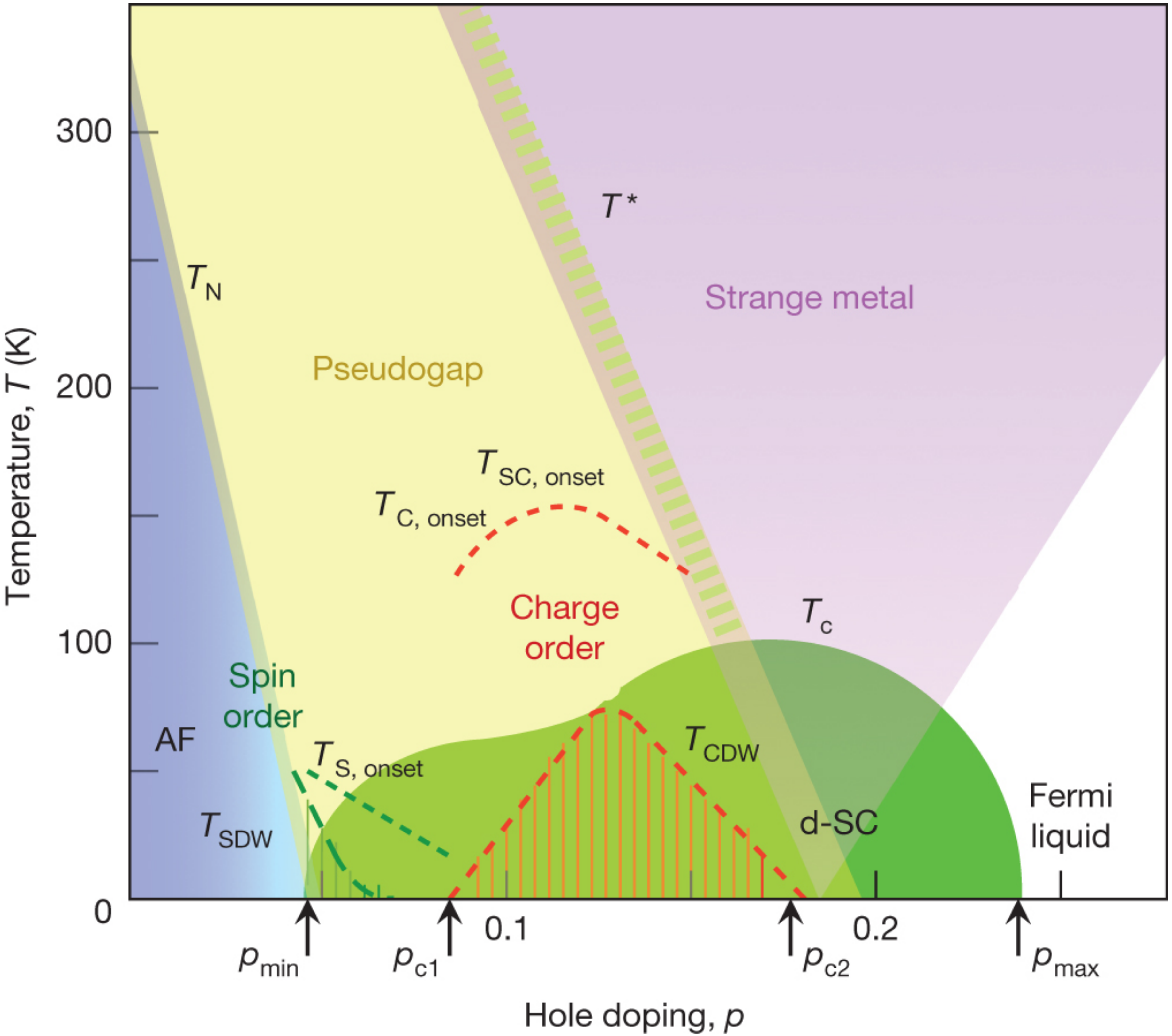
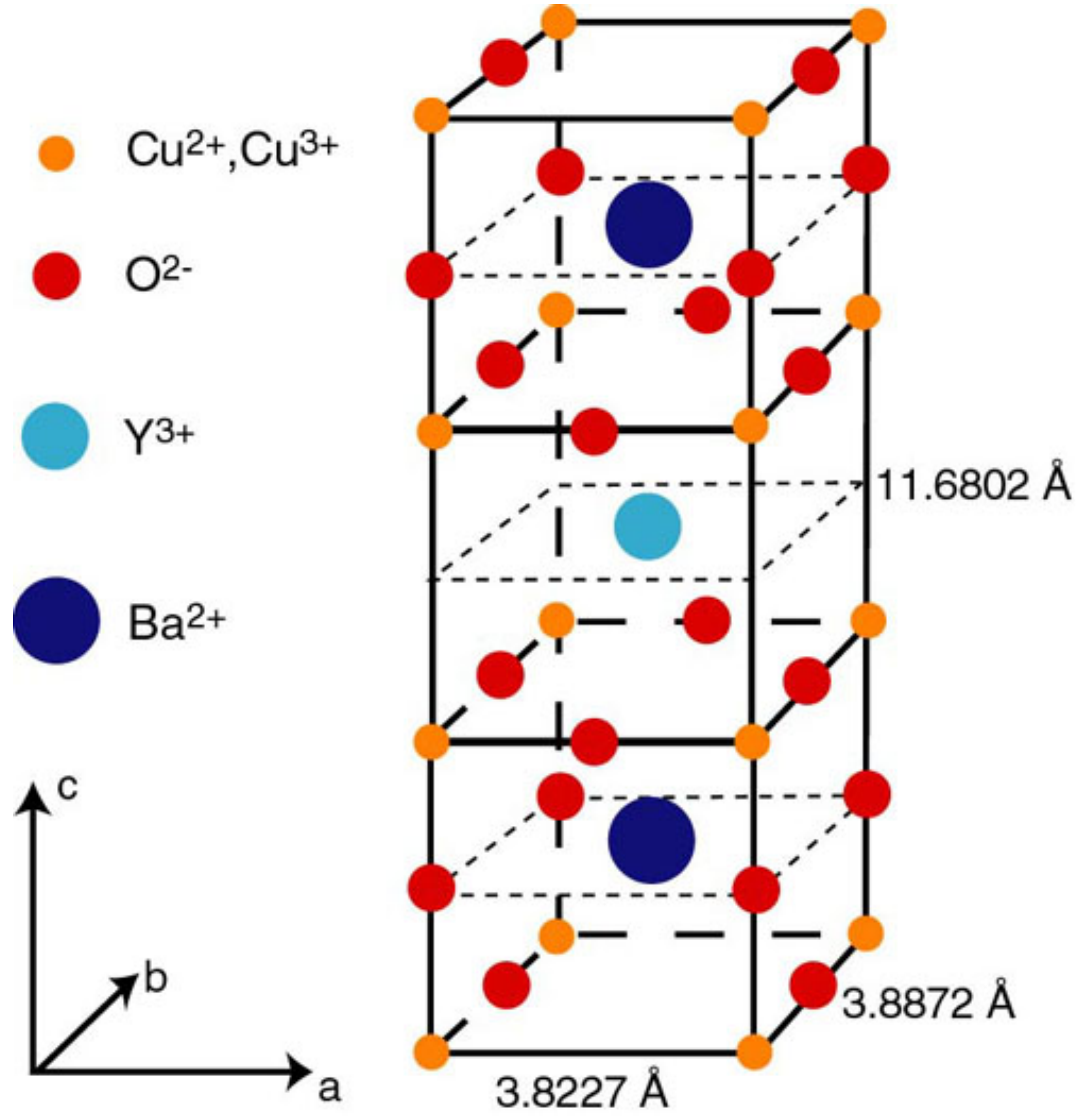
Talk online: sachdev.physics.harvard.edu

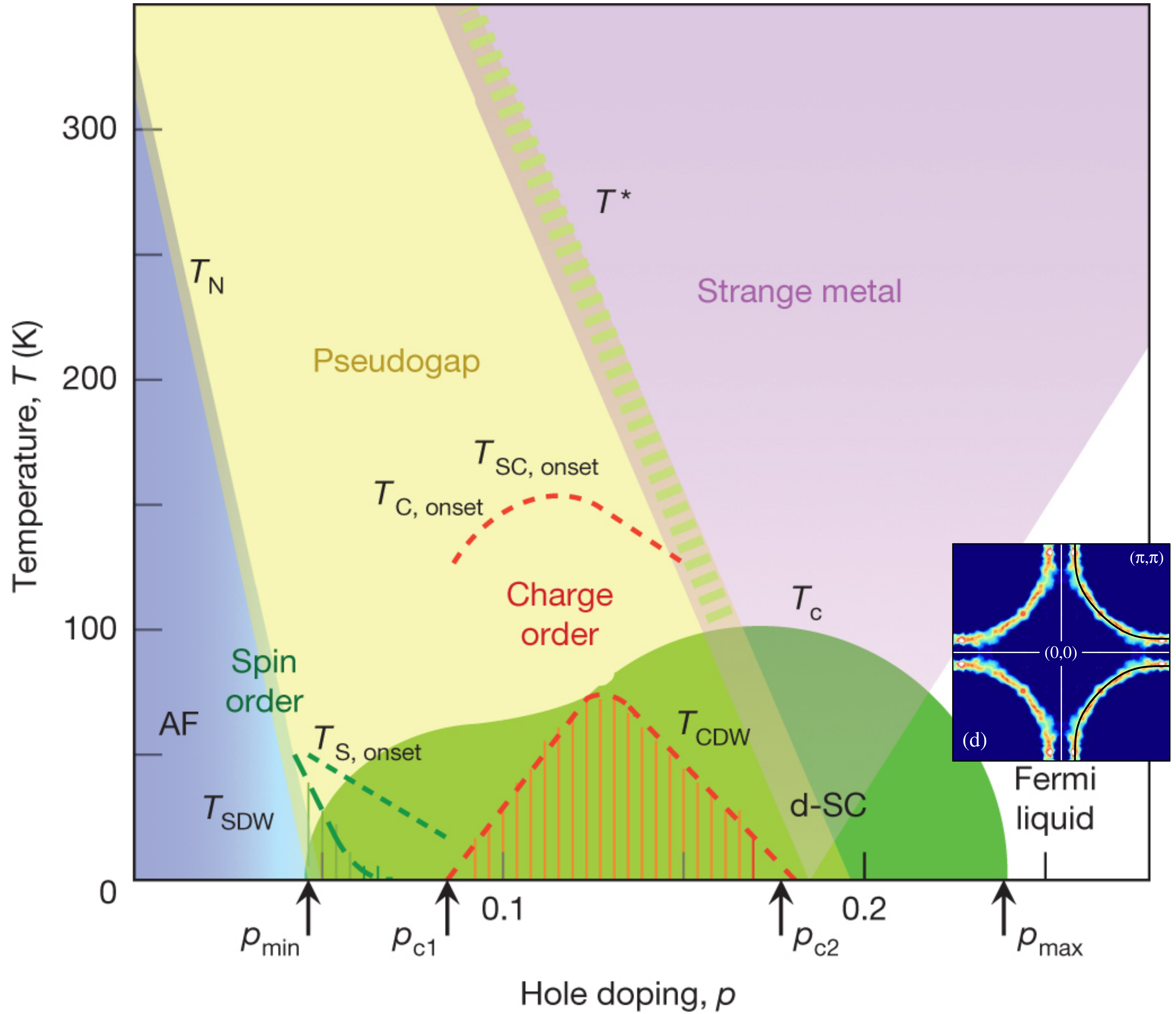
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





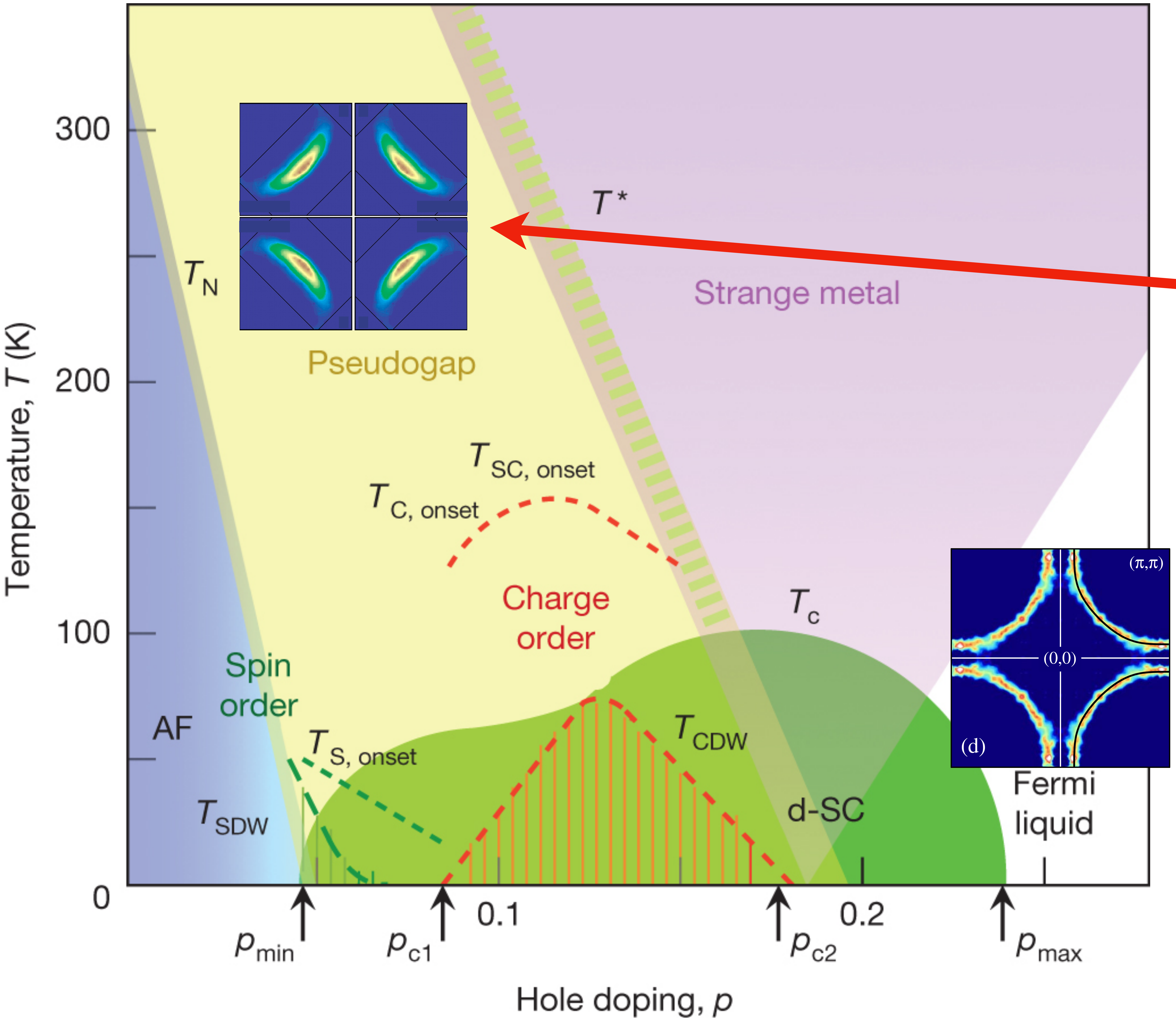
$YBa_2Cu_3O_{6+x}$

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



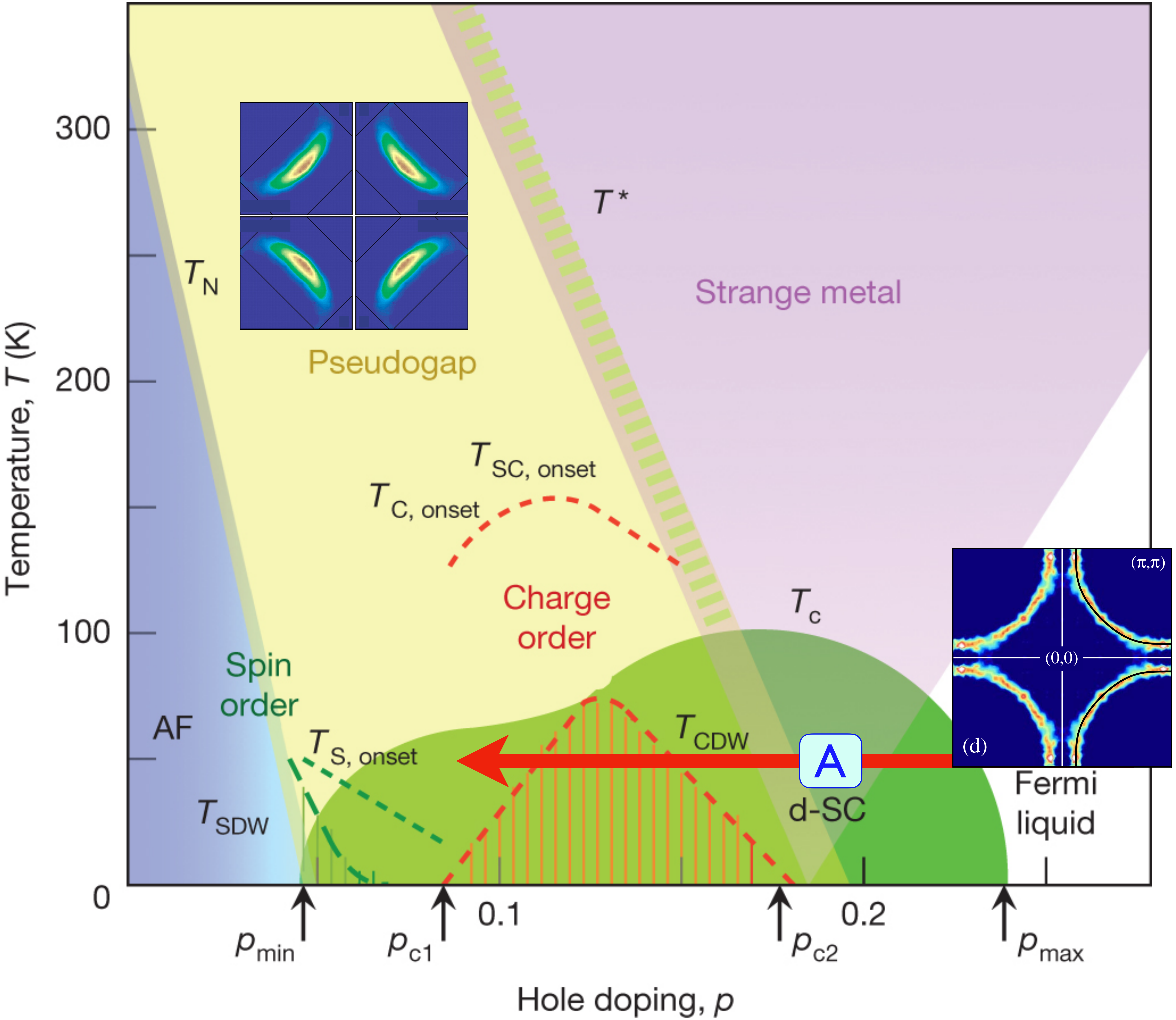
Fermi surface
as expected
in a model
of free electrons

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



"Pseudogap metal"
Fermi surface
modified by
electron-electron
interactions

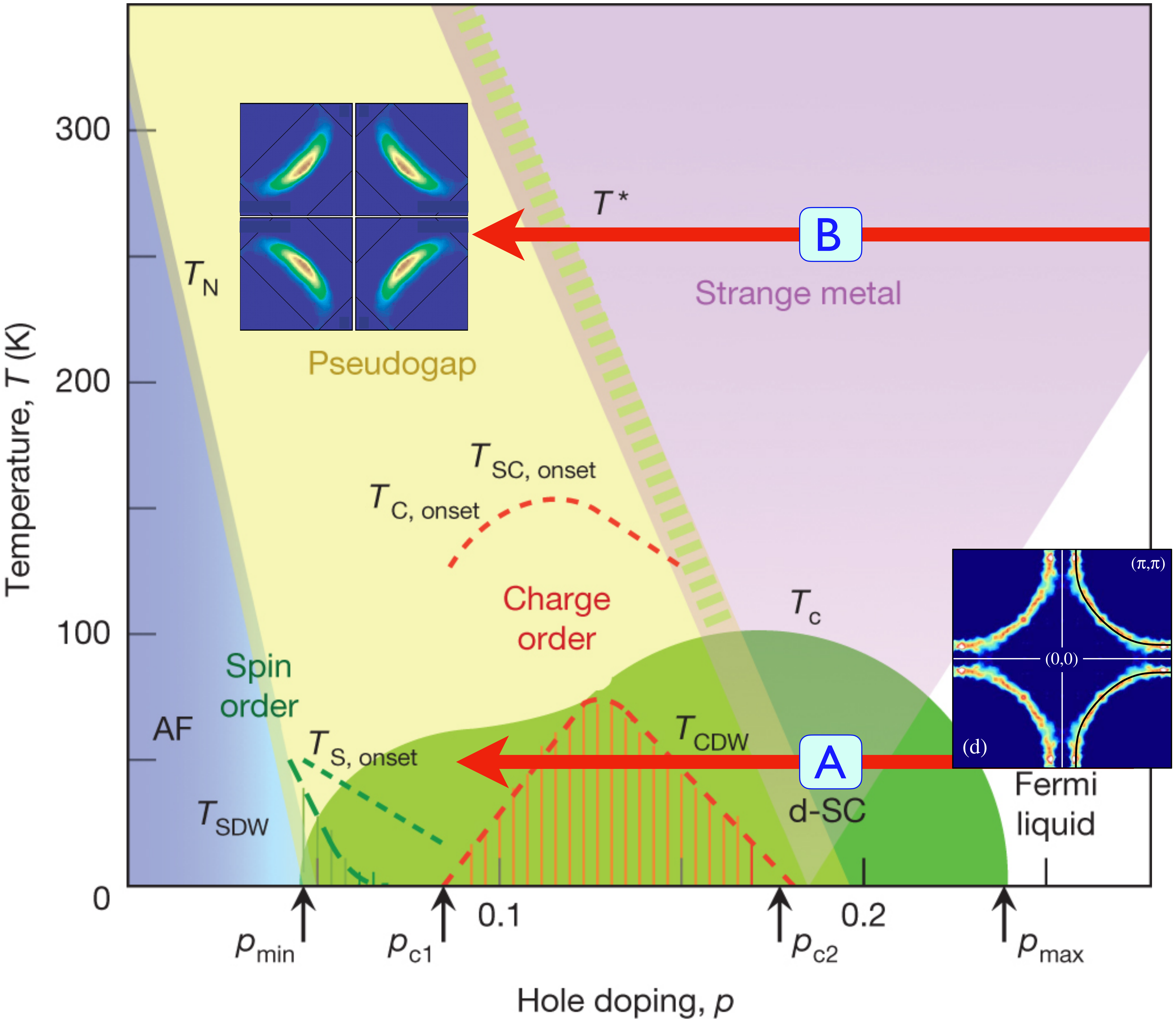
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Fermi-volume-changing QPT
with symmetry breaking
and with spatial disorder.

FL-SDW QPT

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Fermi-volume-changing QPT <u>without</u> symmetry breaking and with spatial disorder.

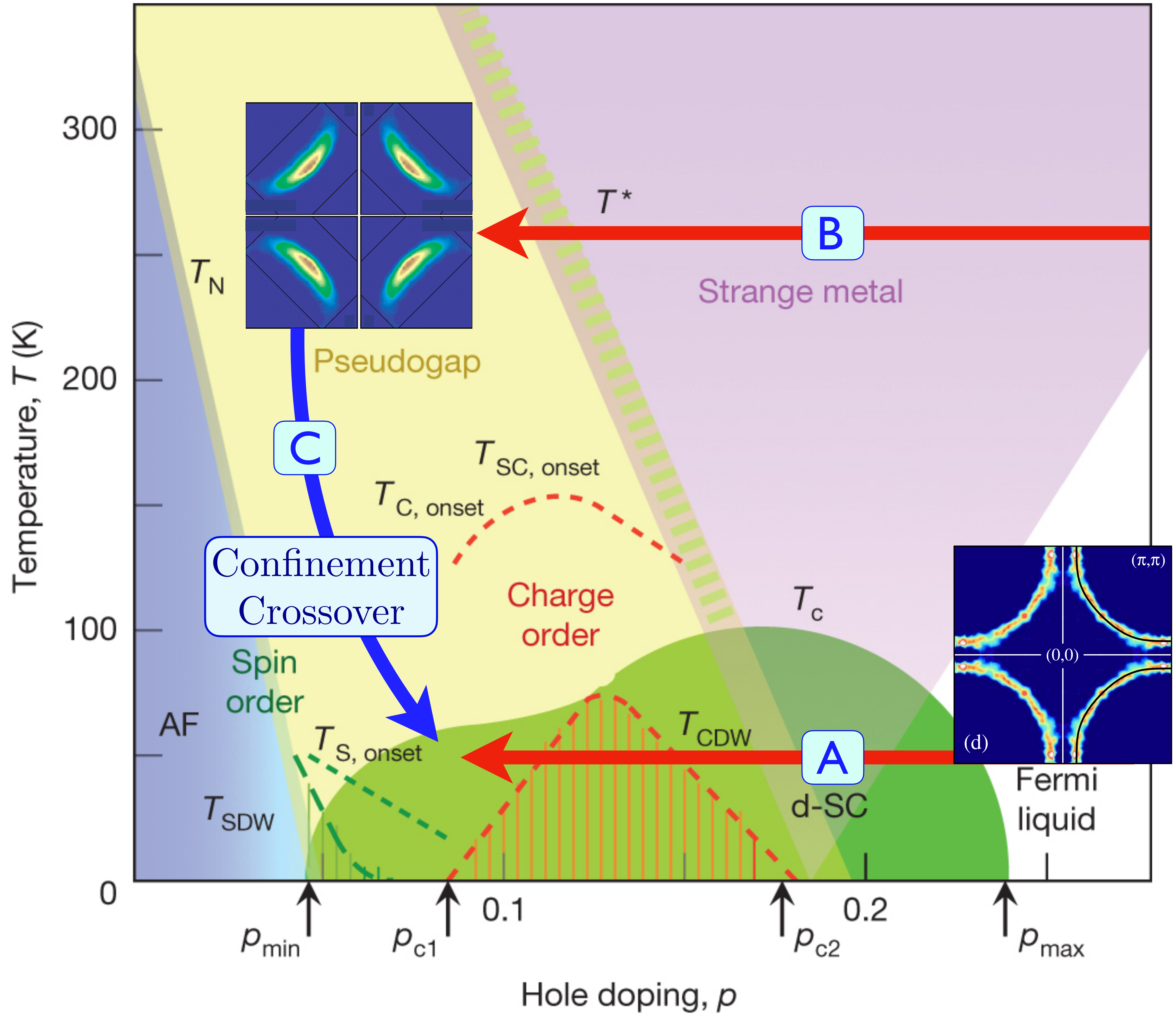
FL-FL* QPT

Requires fractionalization

Fermi-volume-changing QPT <u>with</u> symmetry breaking and with spatial disorder.

FL-SDW QPT

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Fermi-volume-changing QPT
without symmetry breaking
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FL-FL* QPT
Requires fractionalization

Fermi-volume-changing QPT
with symmetry breaking
and with spatial disorder.

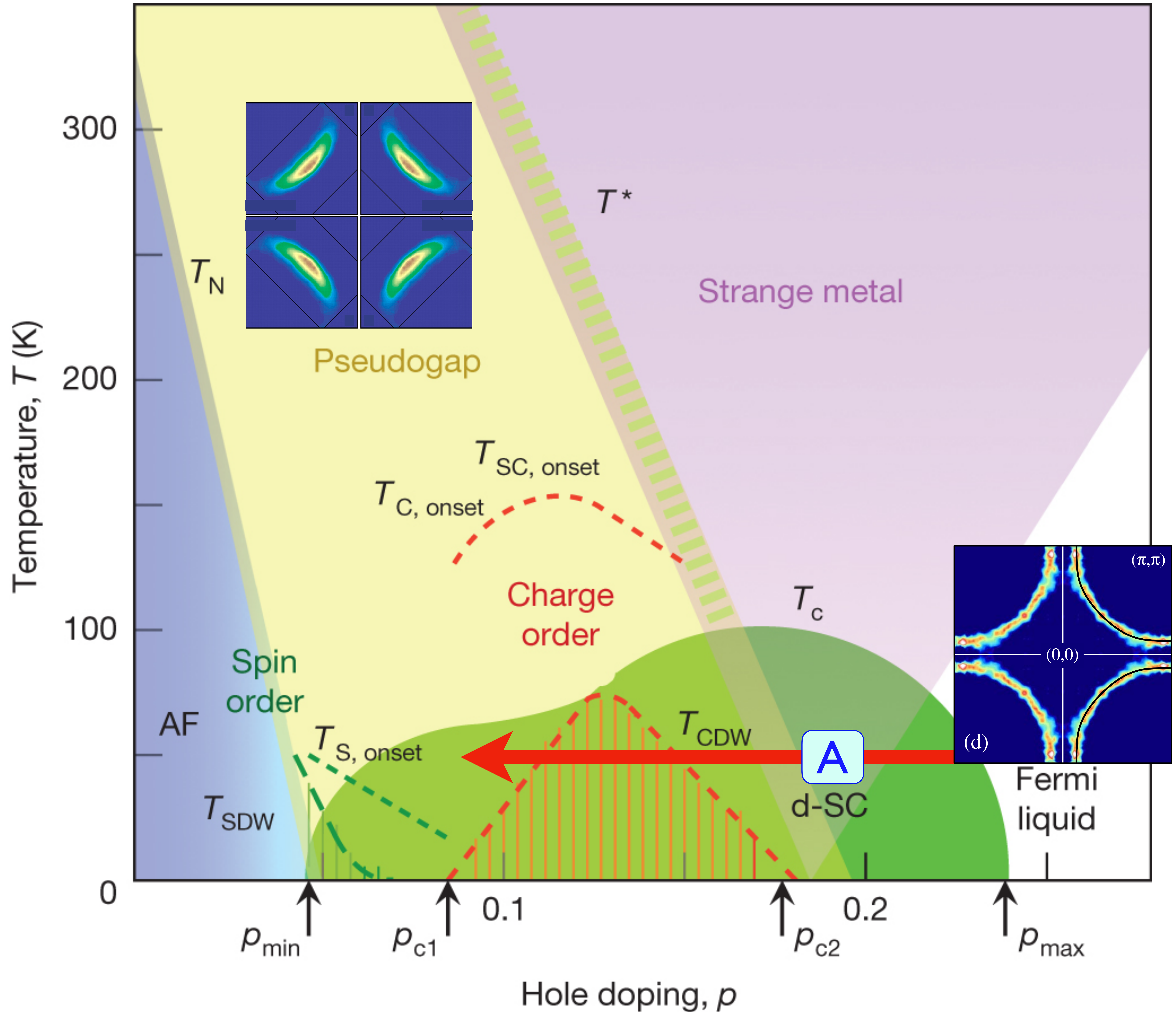
FL-SDW QPT

A. FL-SDW QPT

B. FL-FL* QPT

C. Confinement crossover

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



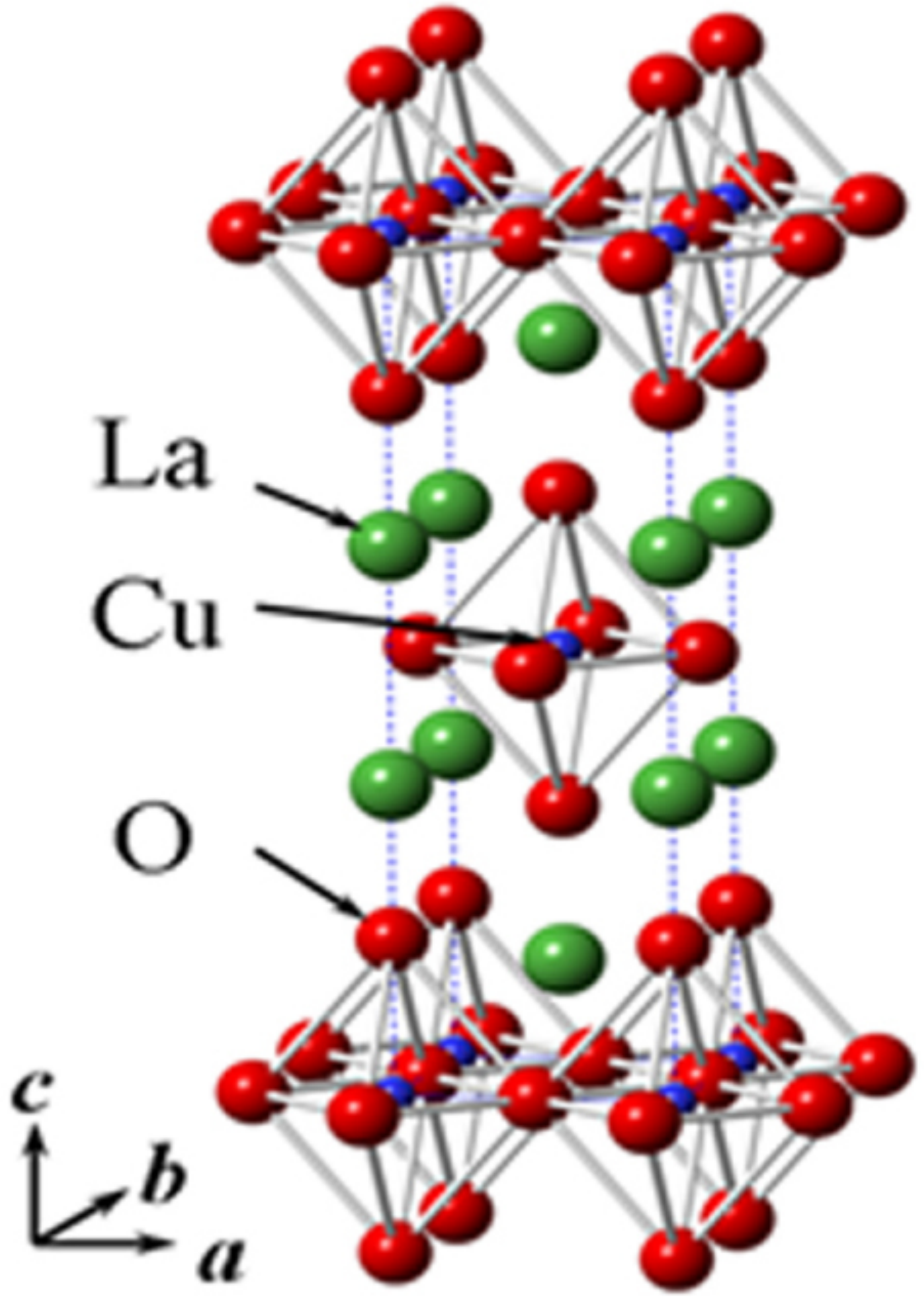
Fermi-volume-changing QPT
with symmetry breaking
and with spatial disorder.

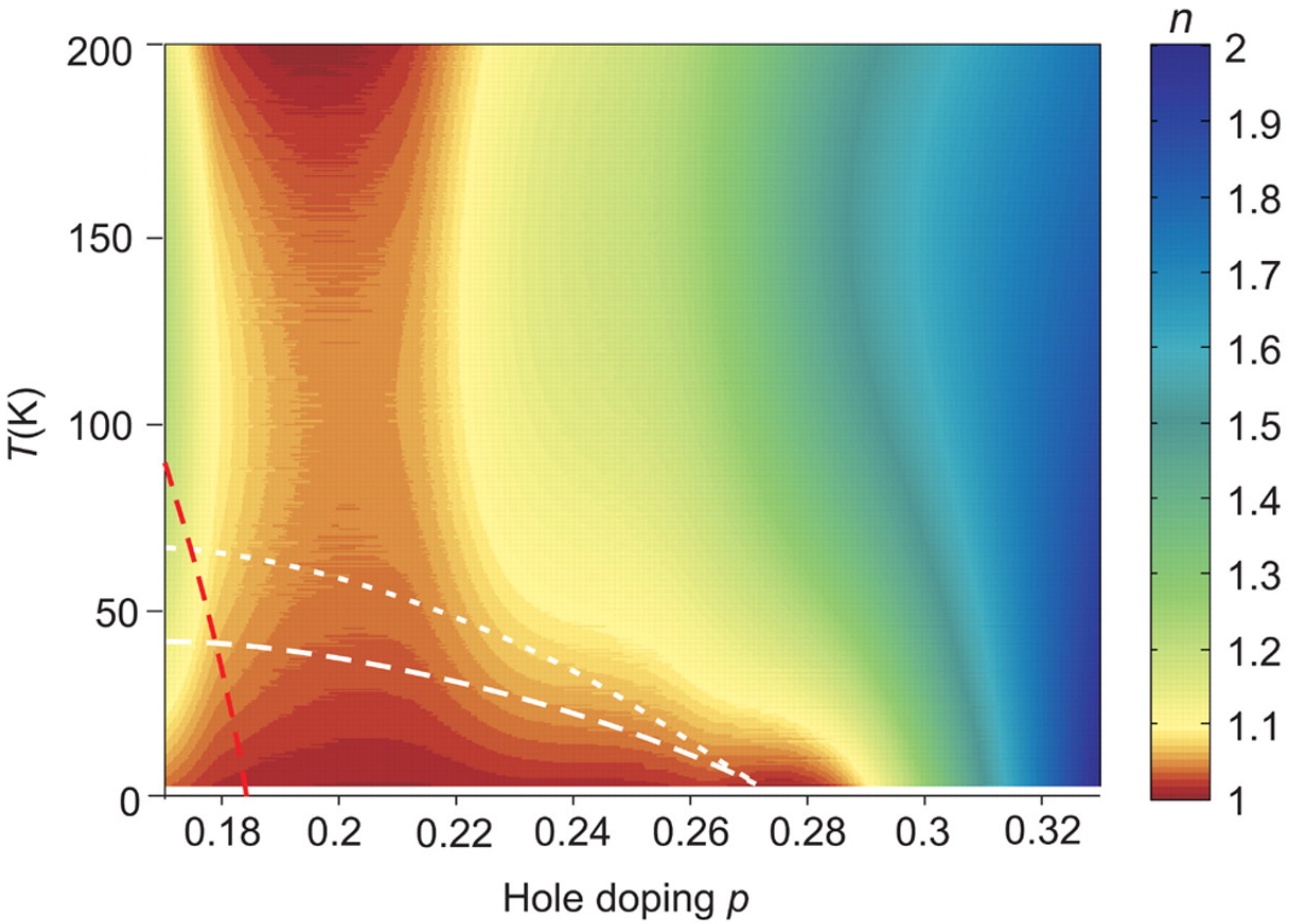
FL-SDW QPT

Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4]* H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

SCIENCE VOL 323 603 2009





FL-SDW QPT with Harris disorder provides a theory of the "foot"

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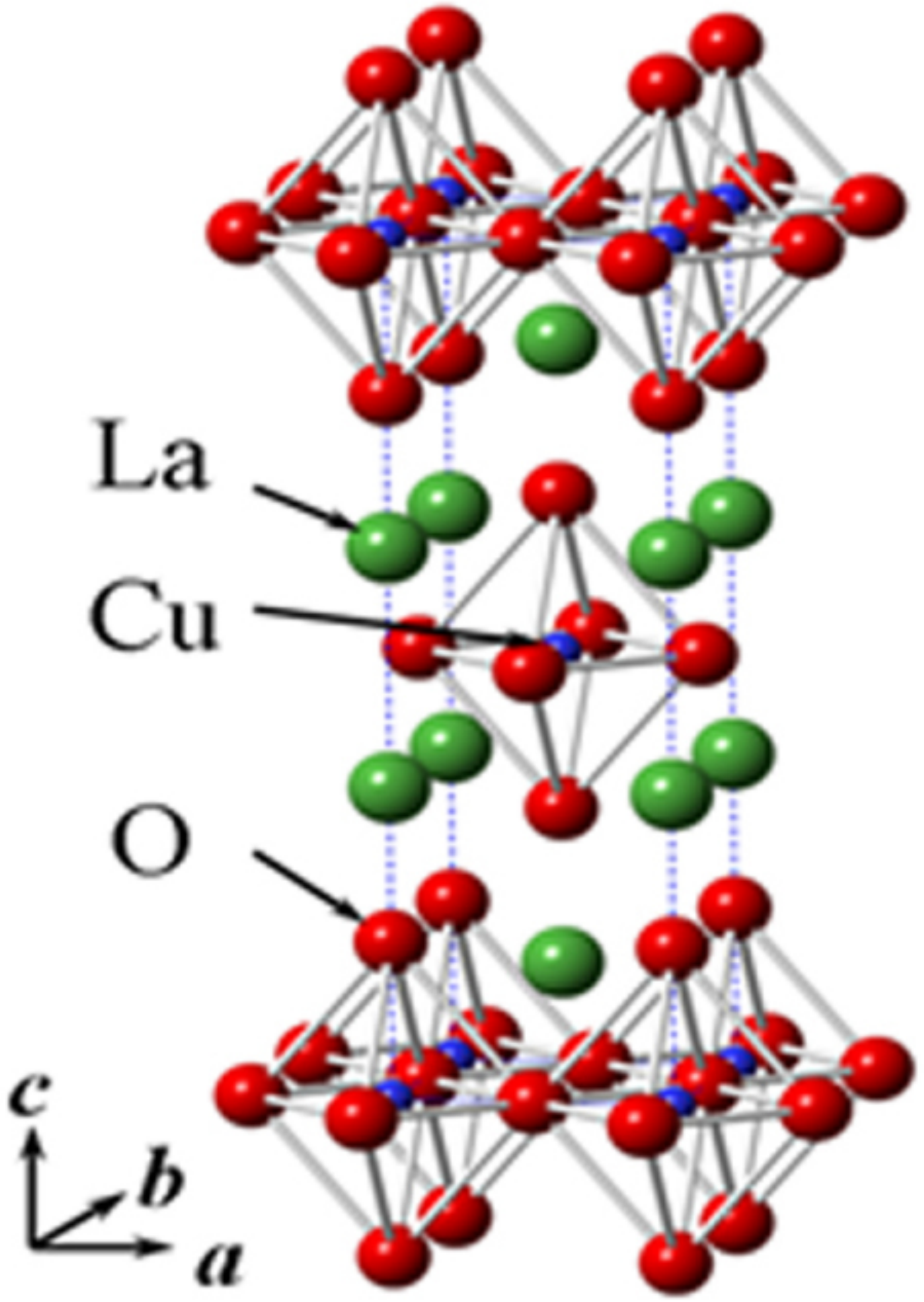
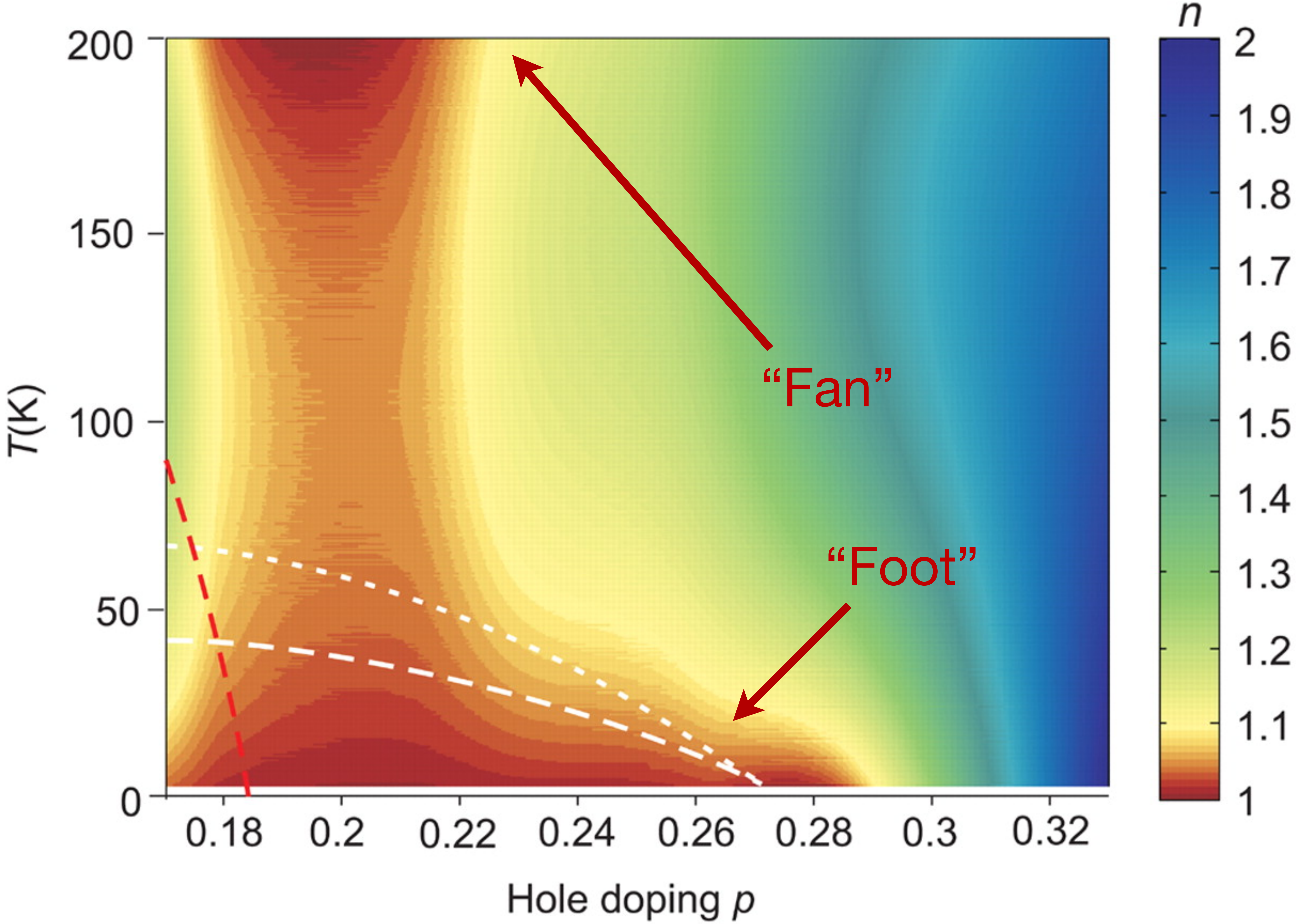
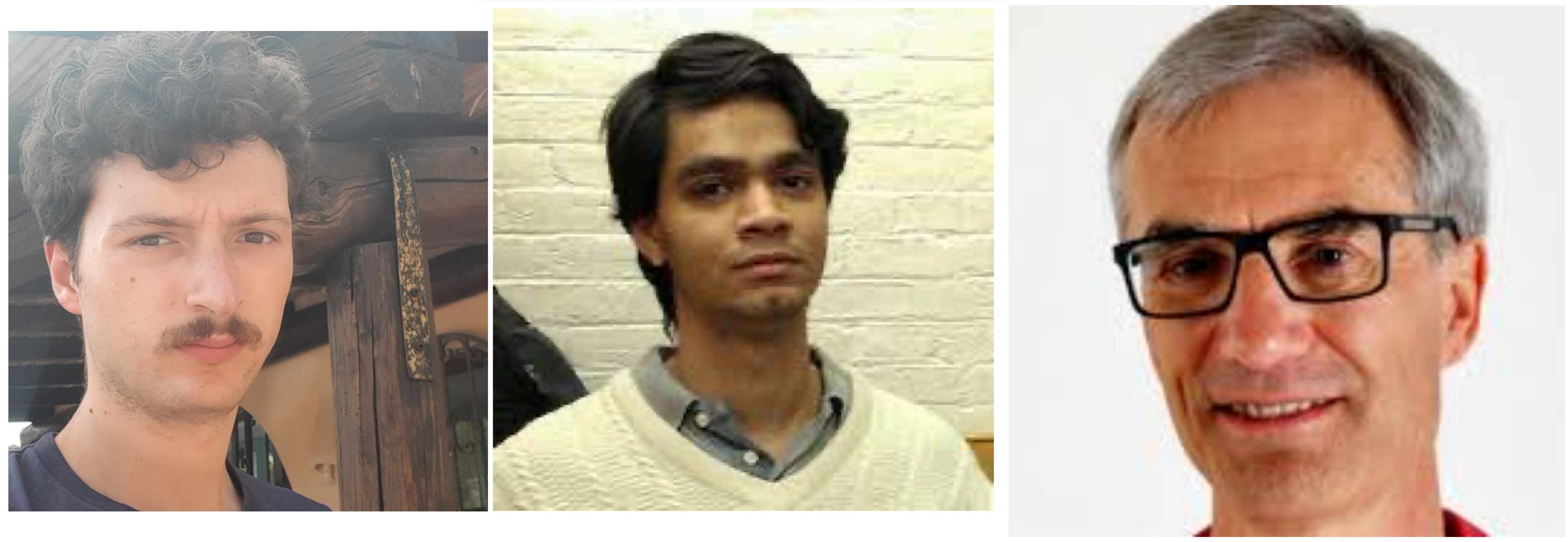
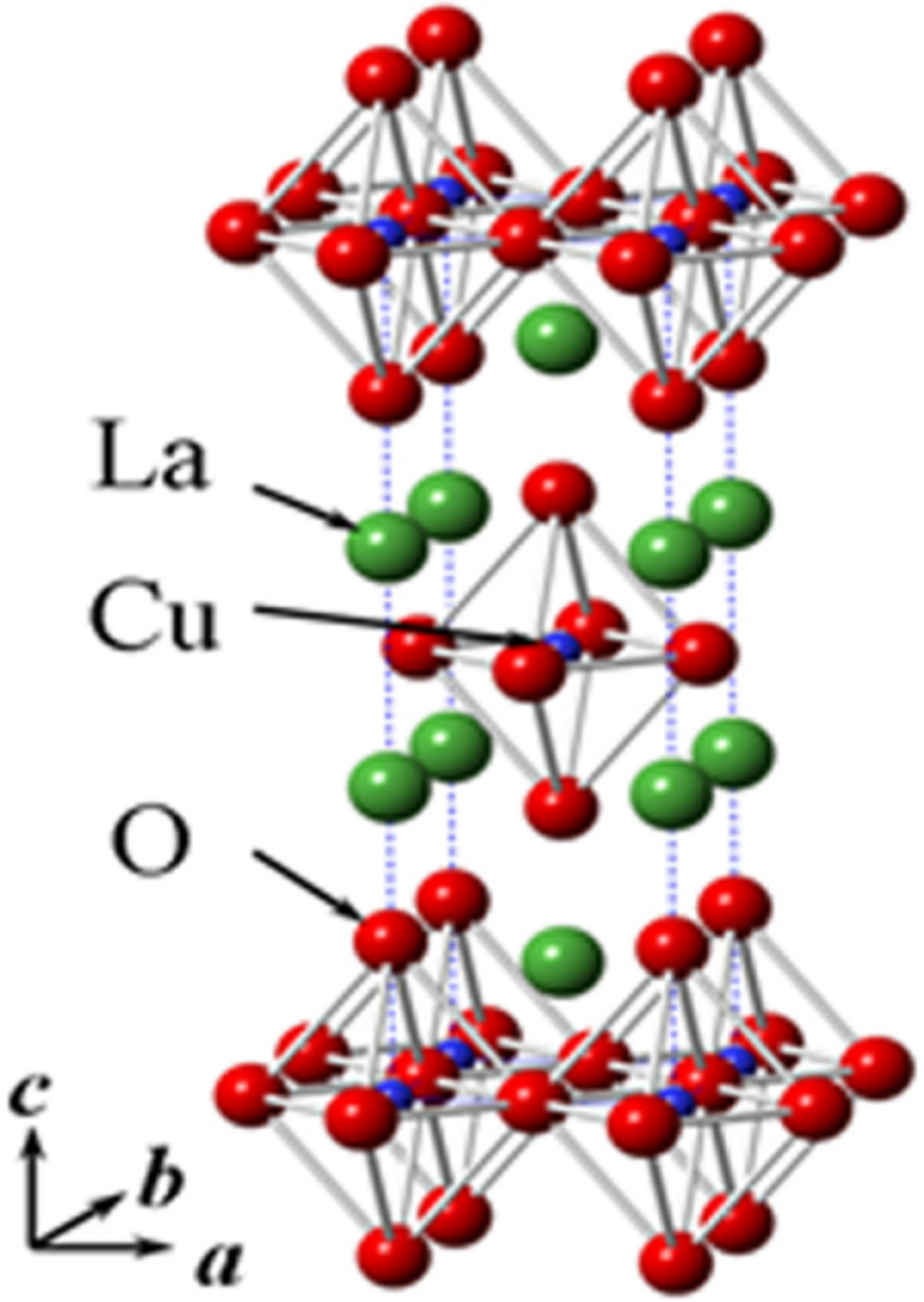


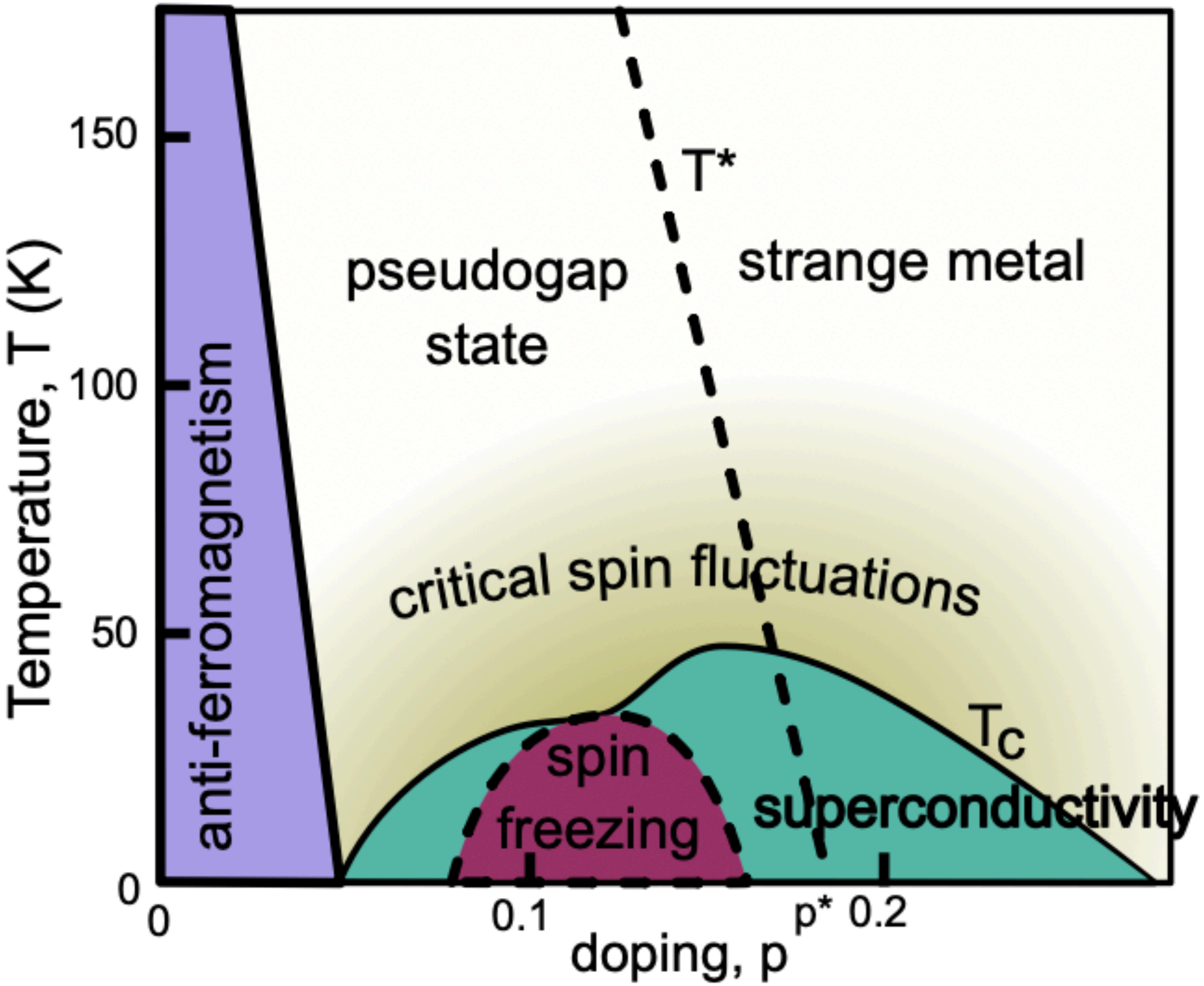


Neutron scattering in LSCO

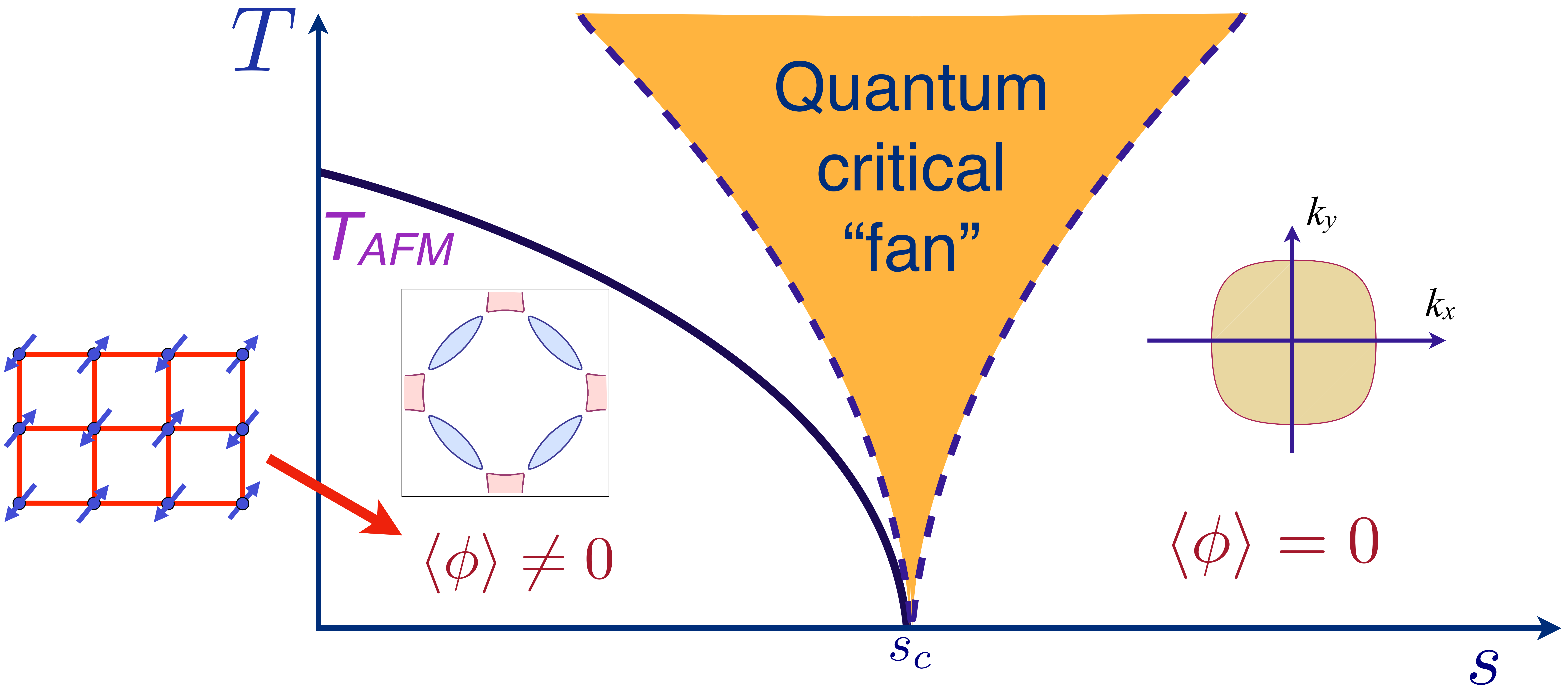
Jacopo Radaelli, Aavishkar A. Patel, ….S. S., Stephen Hayden, to appear



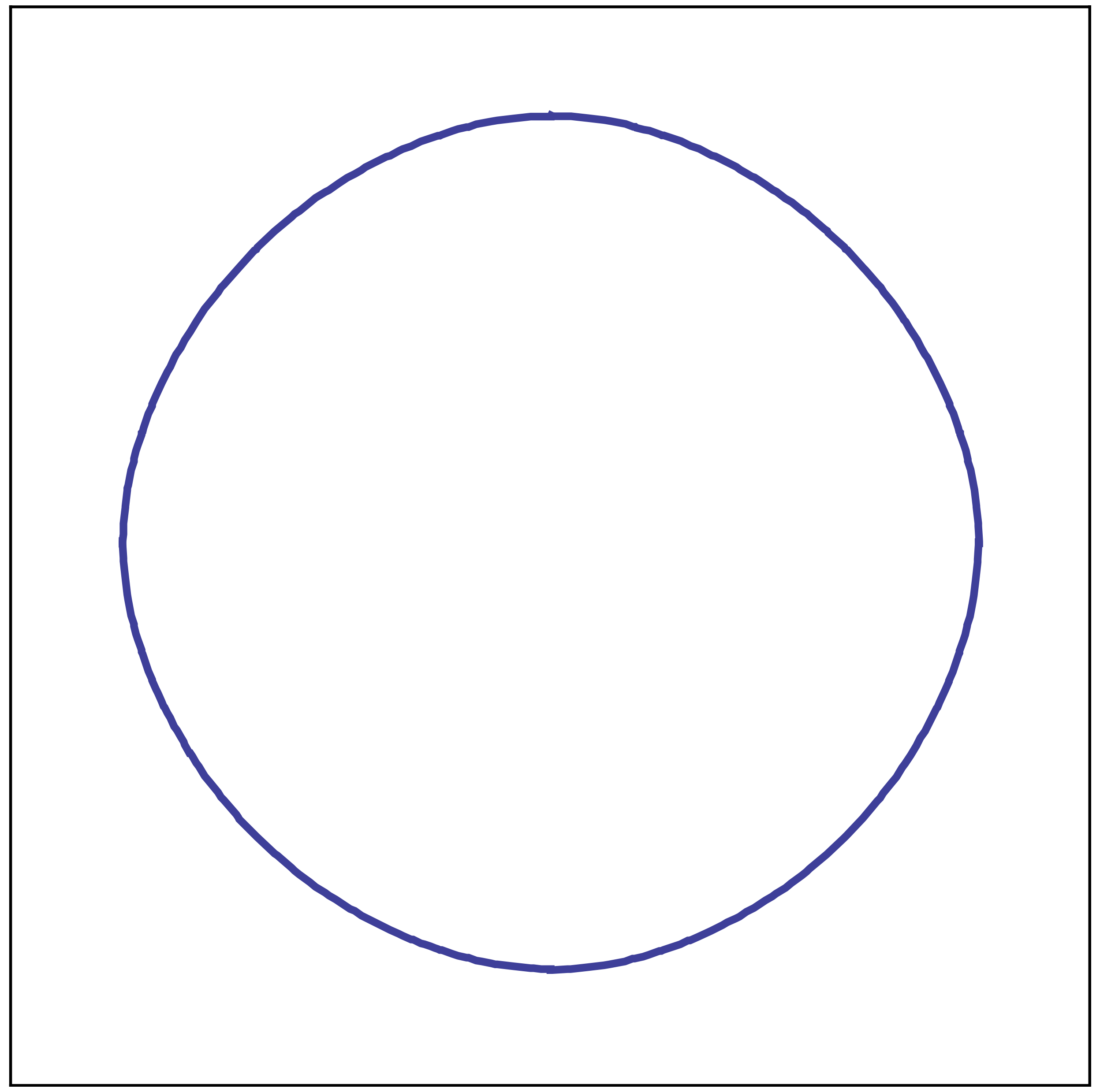




Fermi surface reconstruction from spin density wave (SDW) order

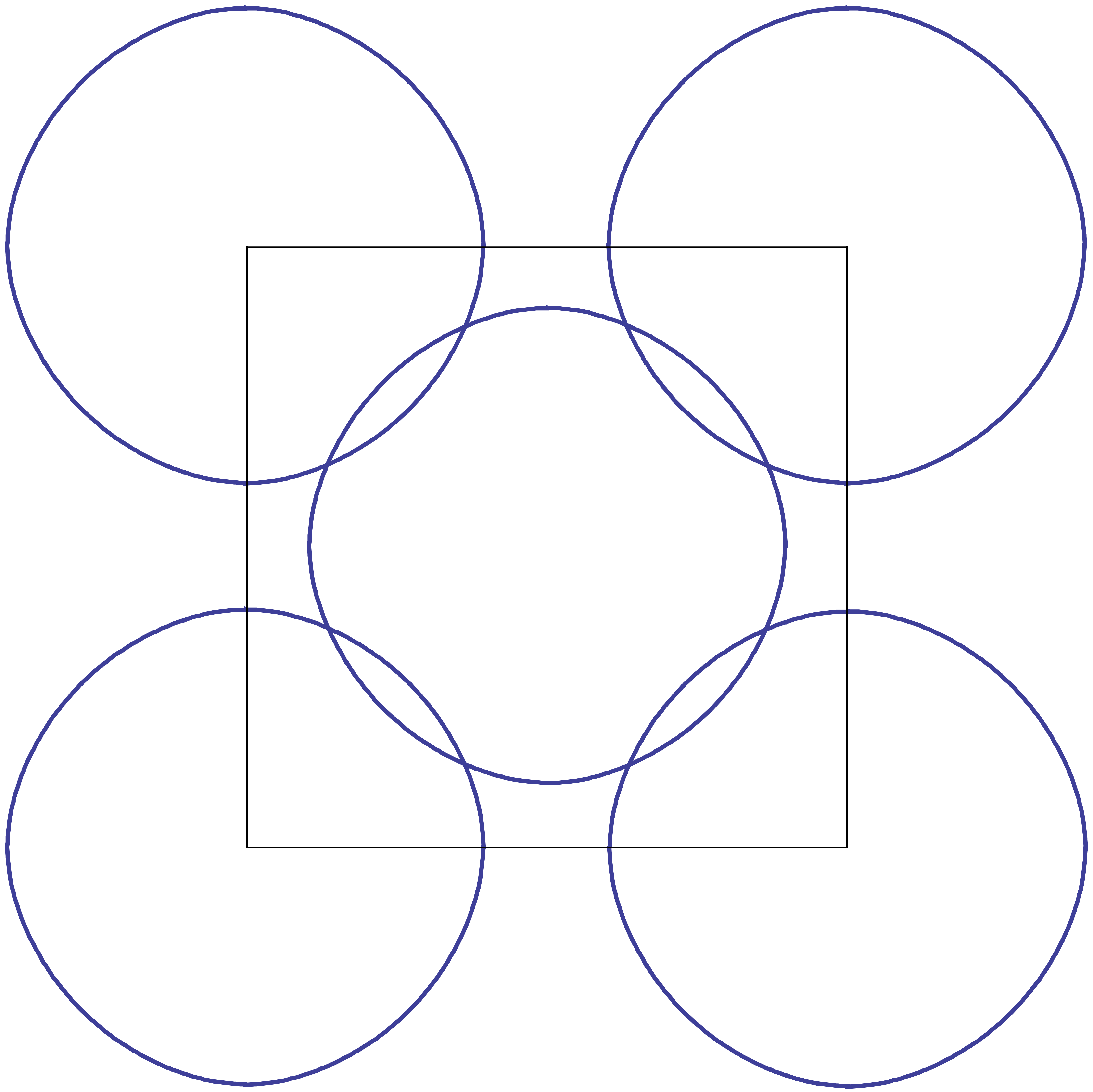


Fermi surface+antiferromagnetism



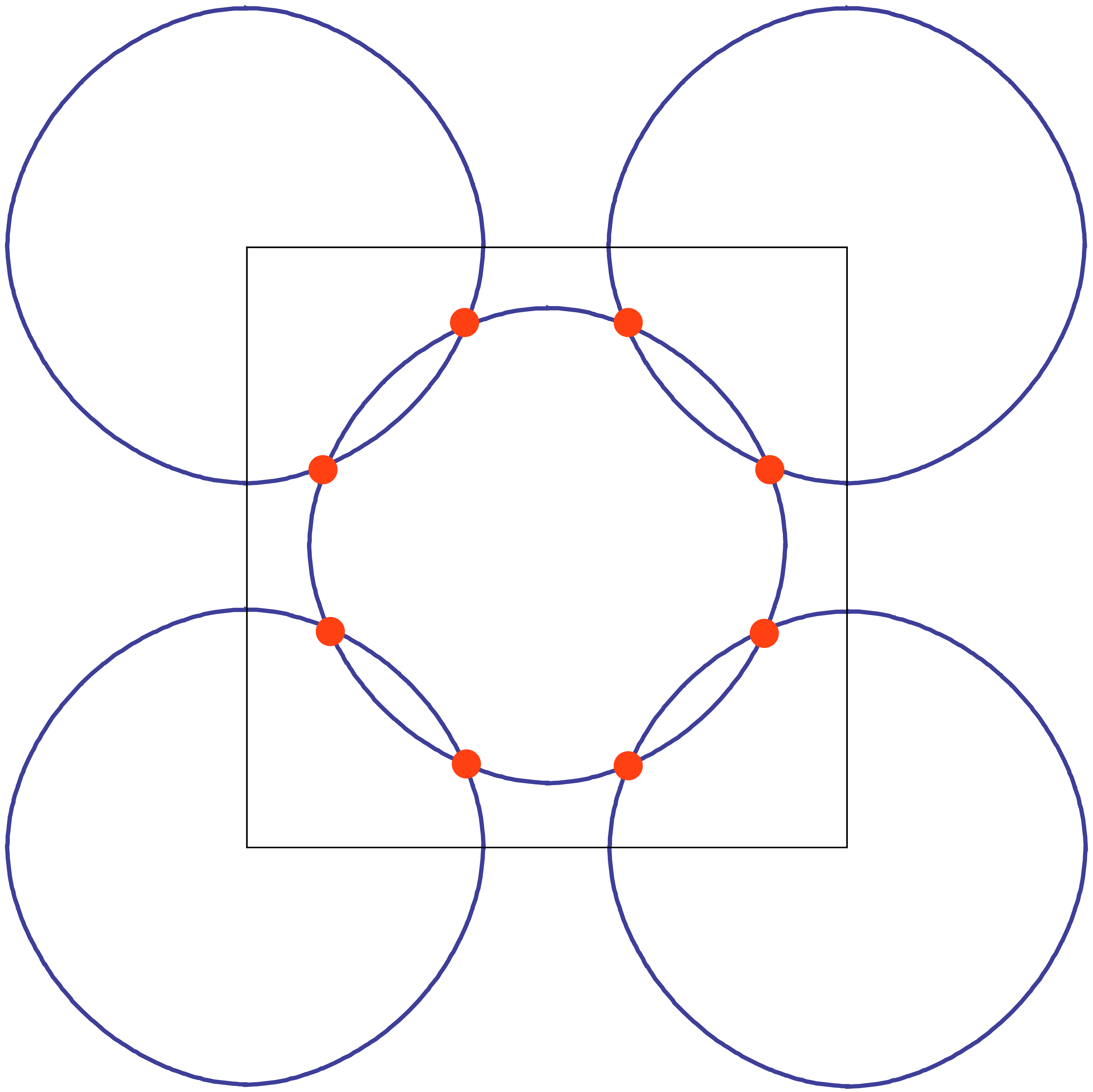
Metal with "large" Fermi surface

Fermi surface+antiferromagnetism



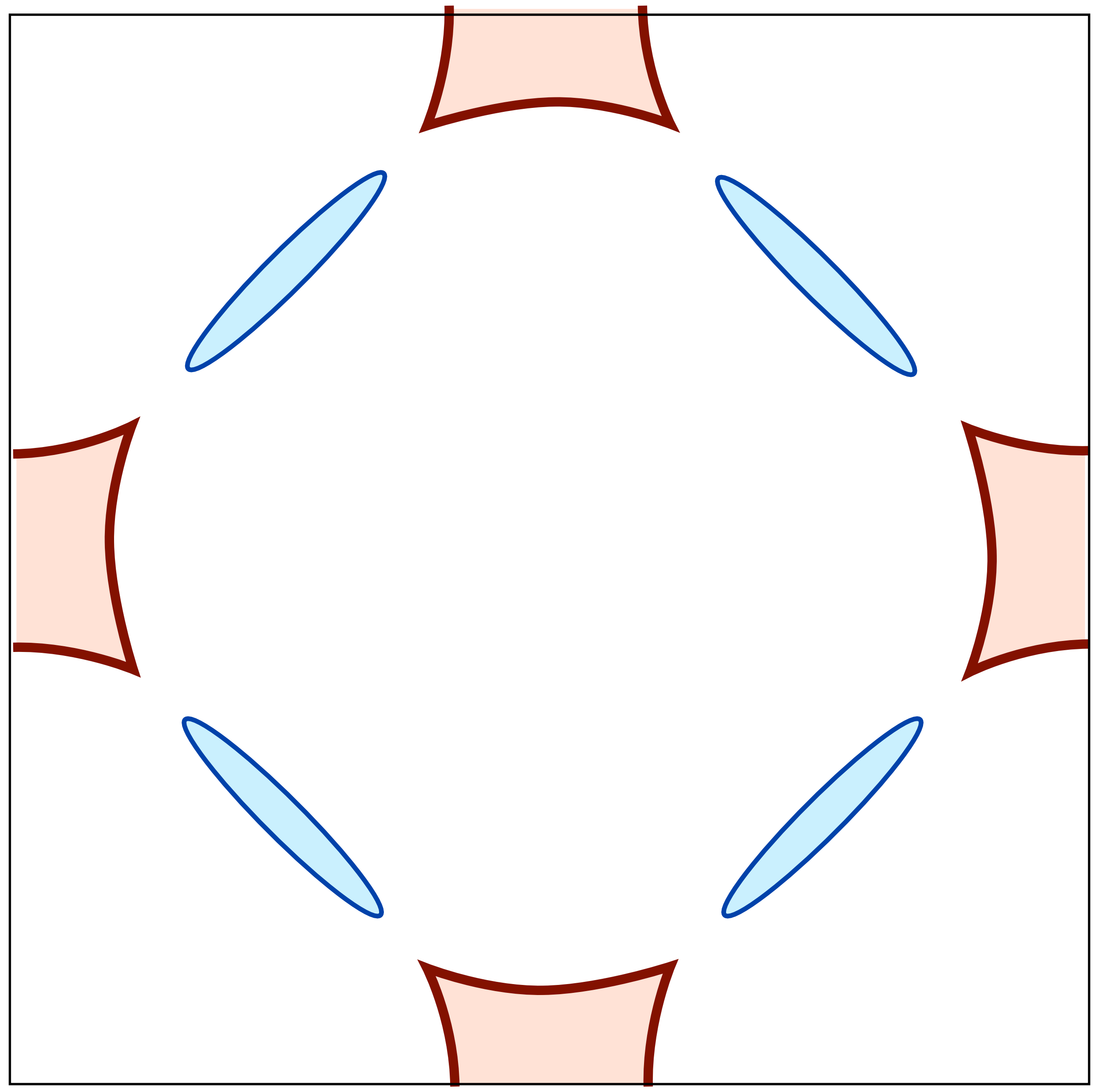
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



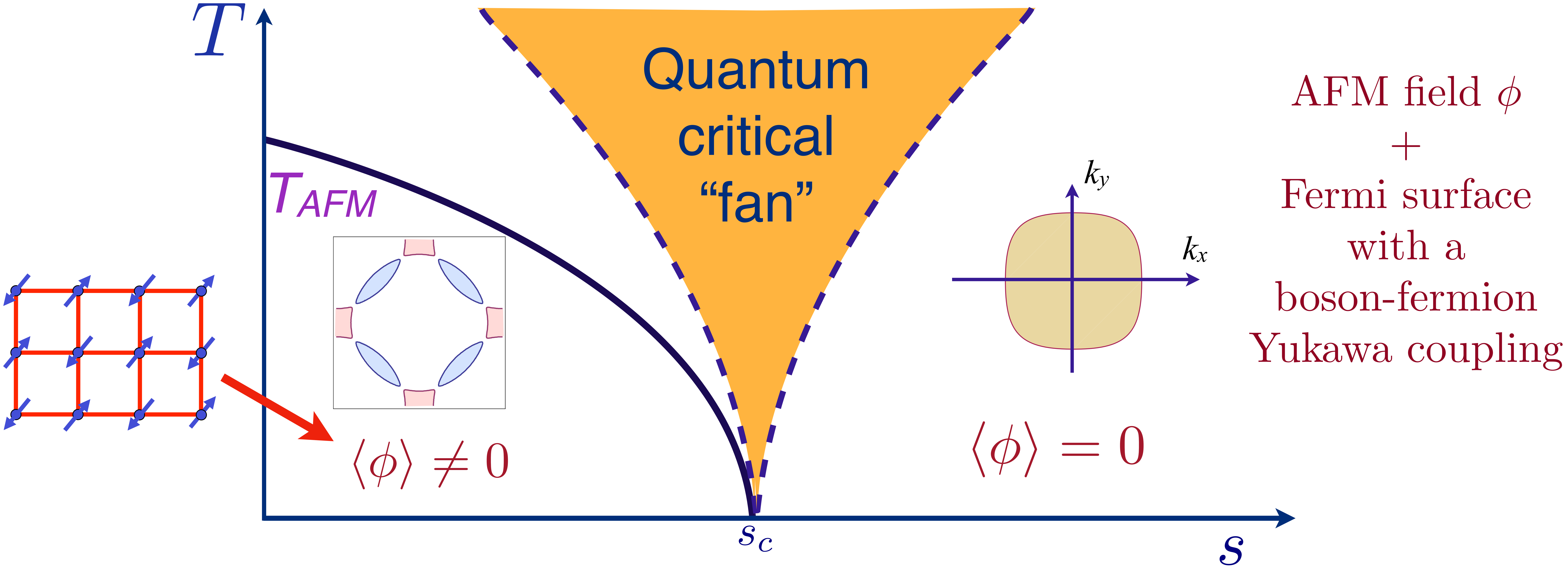
“Hot” spots

Fermi surface+antiferromagnetism



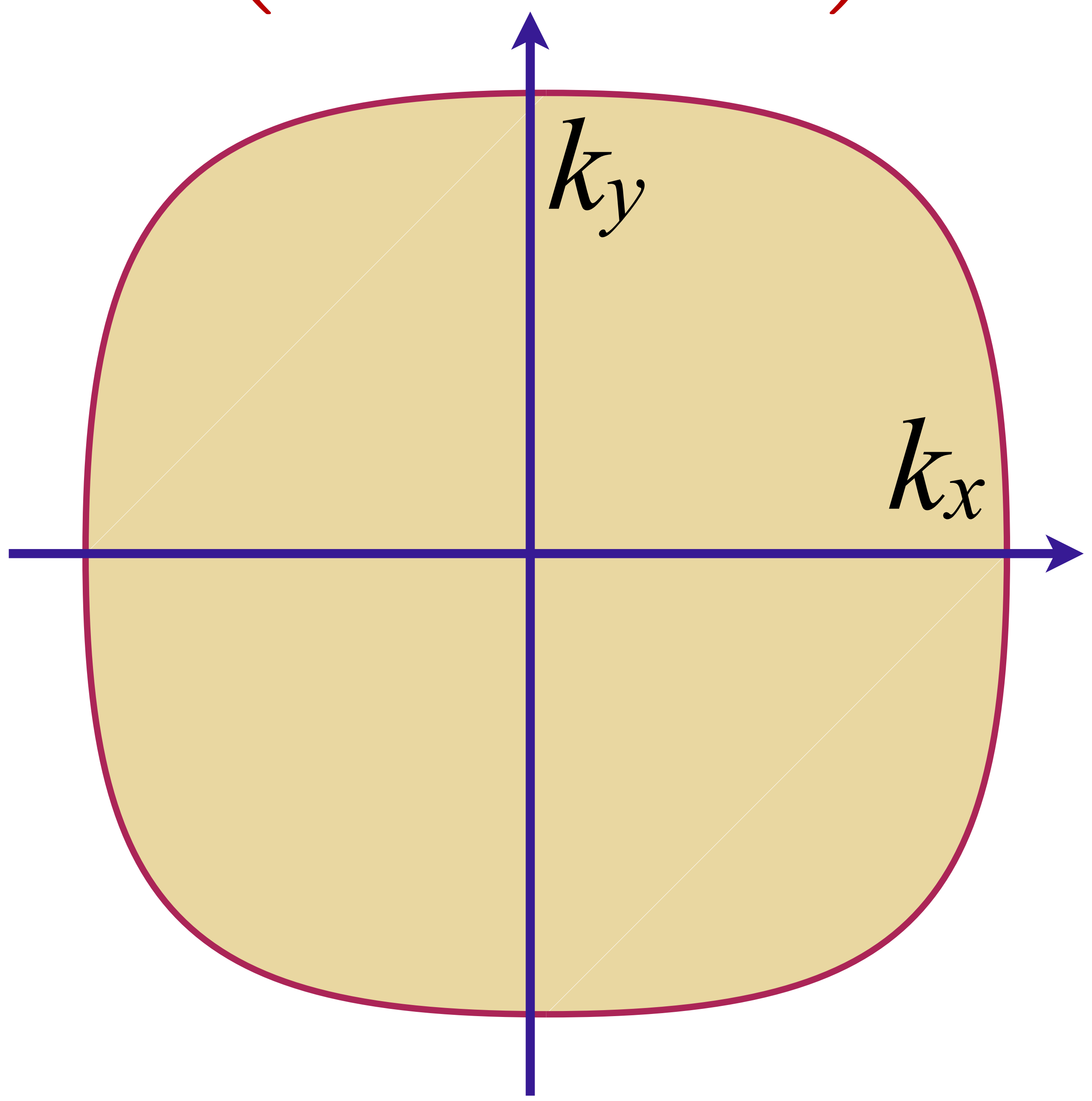
Electron and hole pockets in antiferromagnetic phase with $\langle \phi \rangle \neq 0$

Fermi surface reconstruction from spin density wave (SDW) order



Fermi surface + critical boson with no spatial disorder

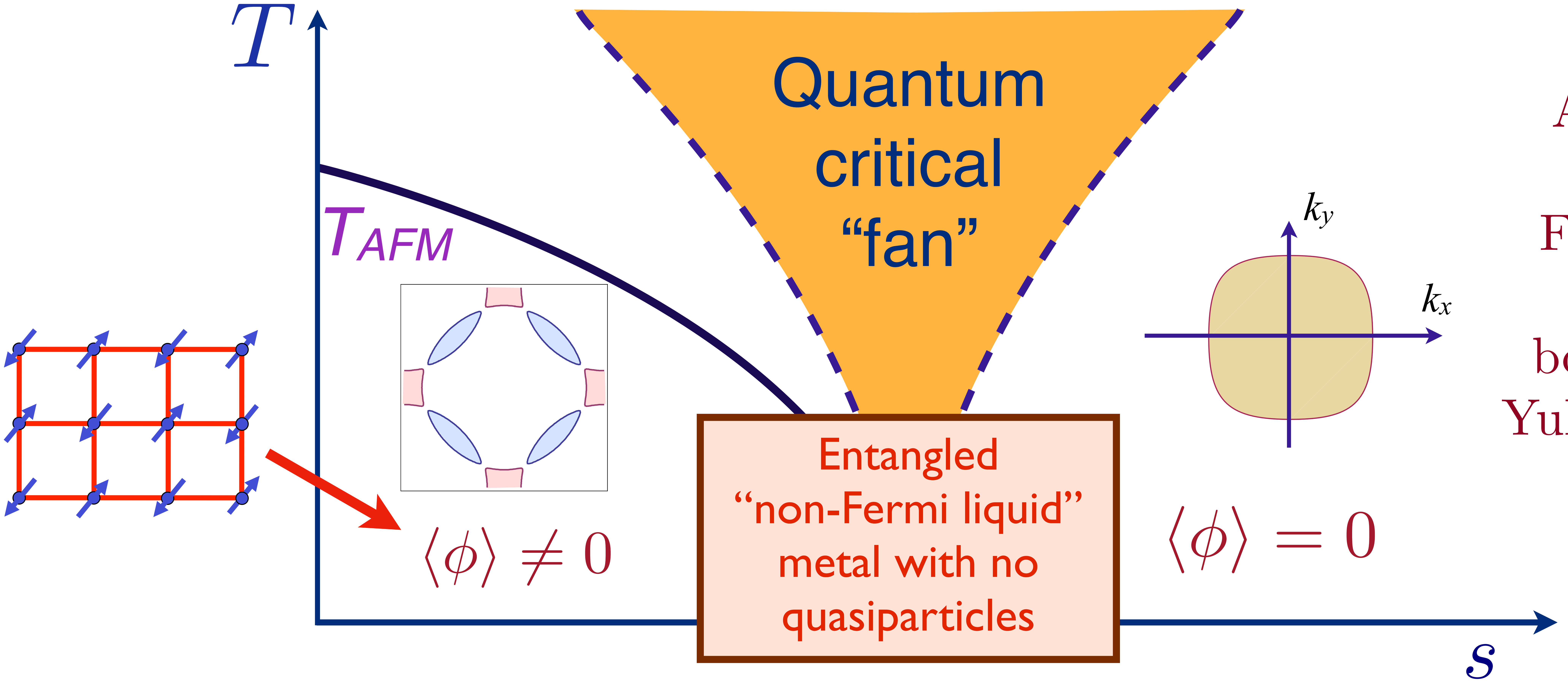
$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$



$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})\tau^a_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

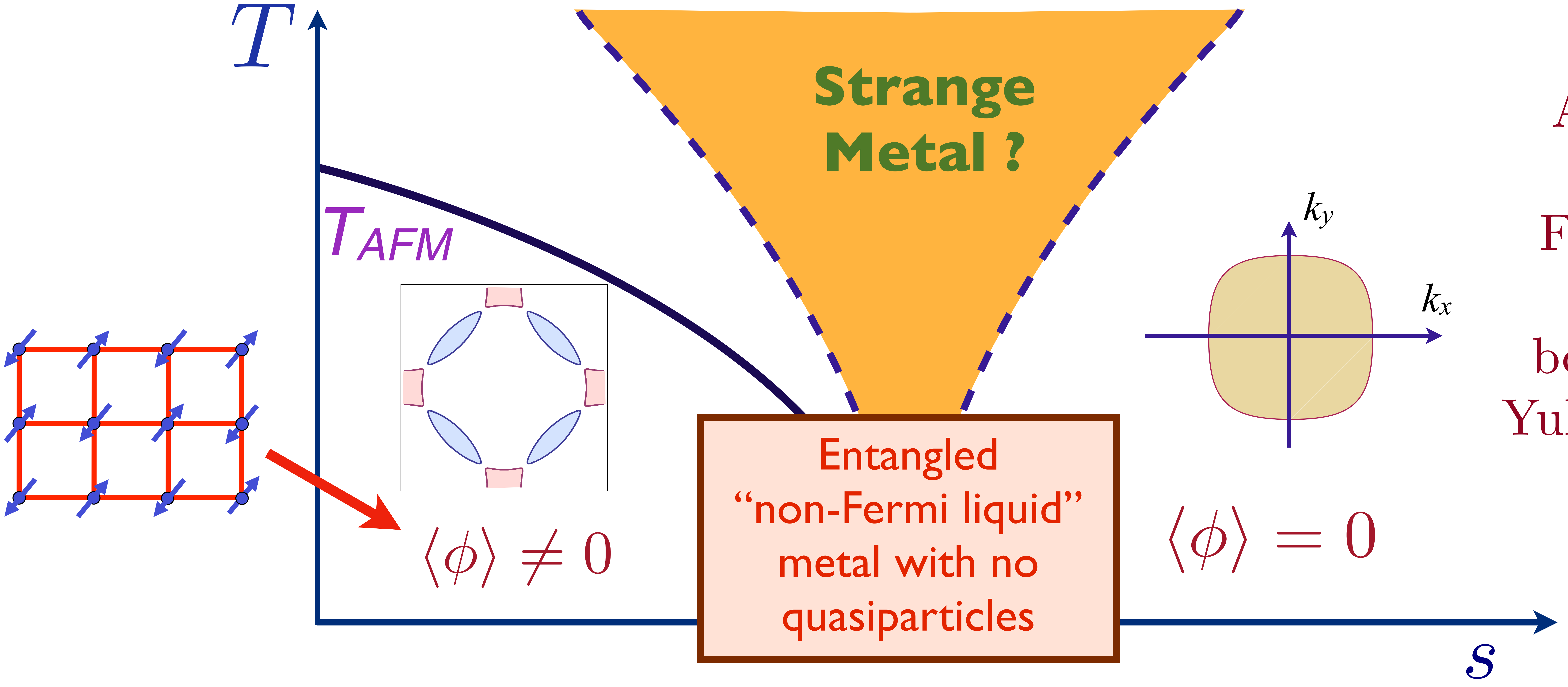
$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2+u\,[\phi(\boldsymbol{r})]^4$$

Fermi surface reconstruction from spin density wave (SDW) order



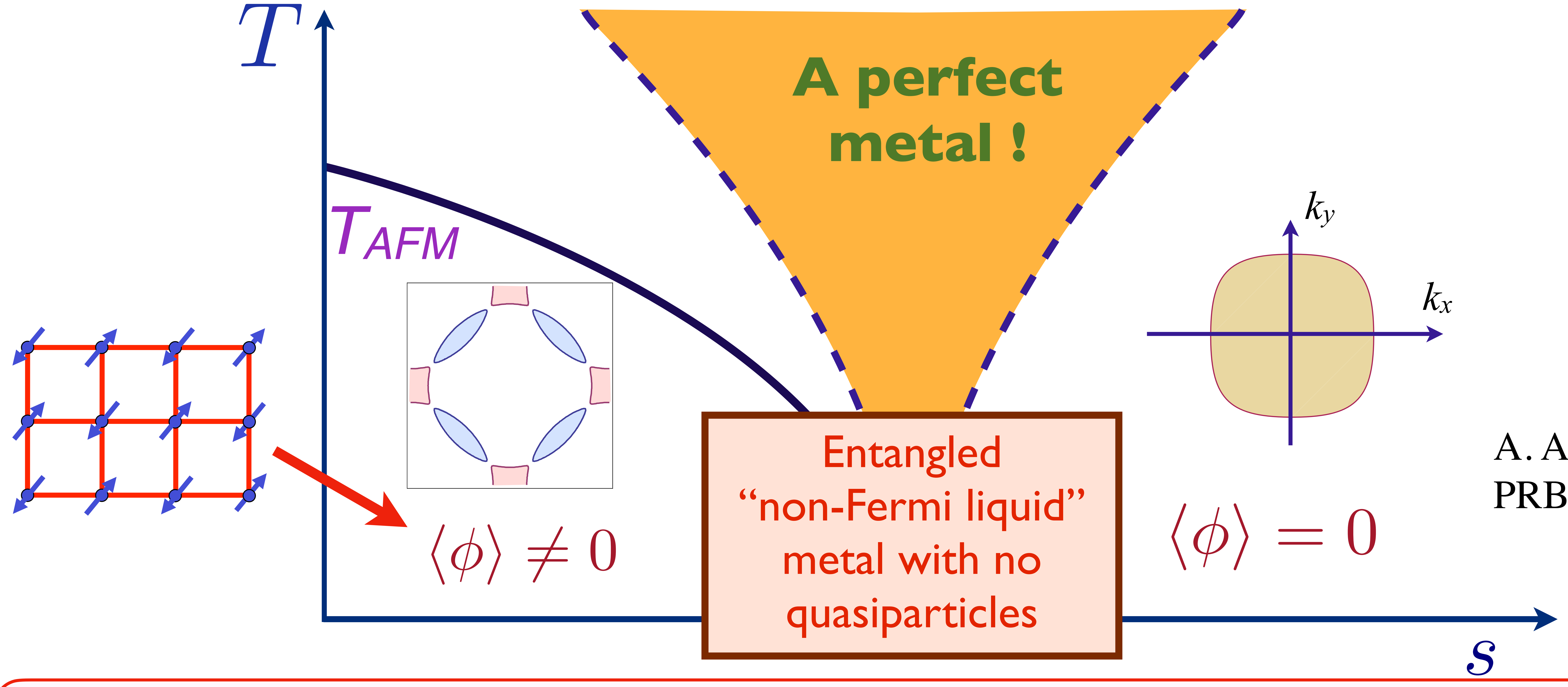
AFM field ϕ
+
Fermi surface
with a
boson-fermion
Yukawa coupling

Fermi surface reconstruction from spin density wave (SDW) order



AFM field ϕ
$+$
Fermi surface with a boson-fermion Yukawa coupling

Fermi surface reconstruction from spin density wave (SDW) order

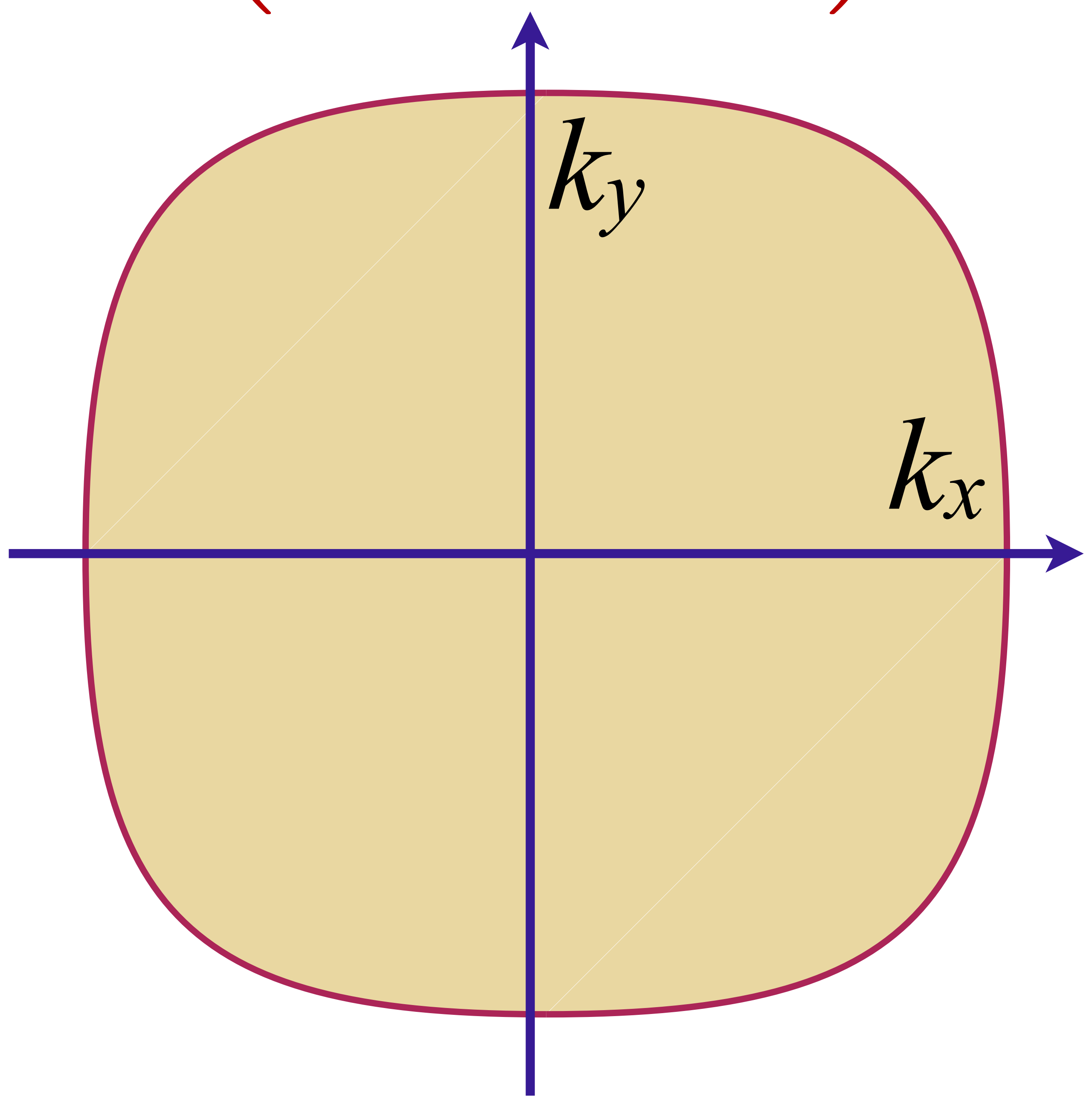


A. A. Patel and S. S., PRB **90**, 165146 (2014)

Extreme drag: the fermions c "drag" the bosons ϕ as they move, and so electrical current does not relax, even though strong c-ϕ scattering leads to absence of c quasiparticles.

Fermi surface + critical boson with no spatial disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$

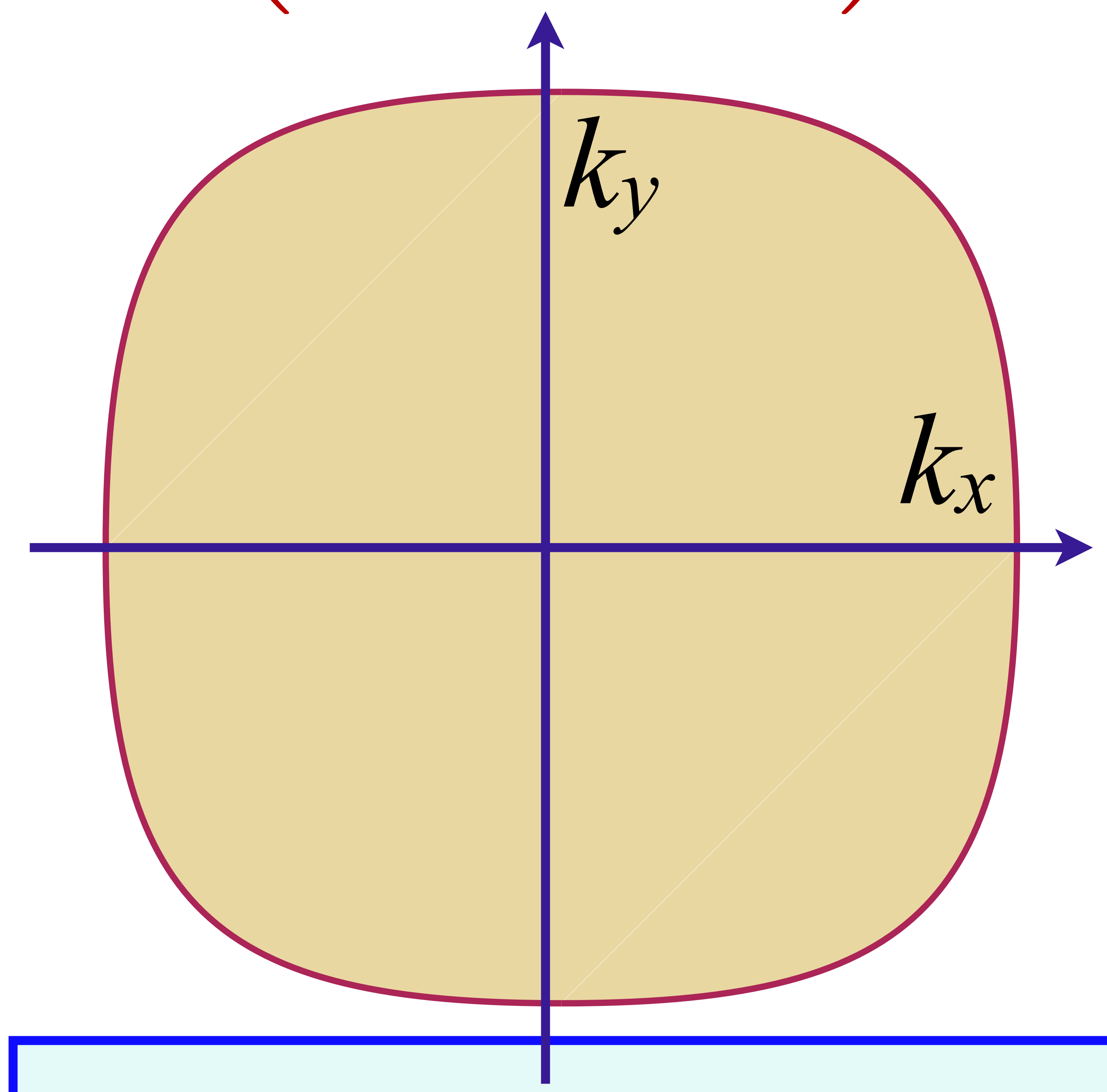


$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})\tau^a_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2+u\,[\phi(\boldsymbol{r})]^4$$

Fermi surface + critical boson with potential disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$



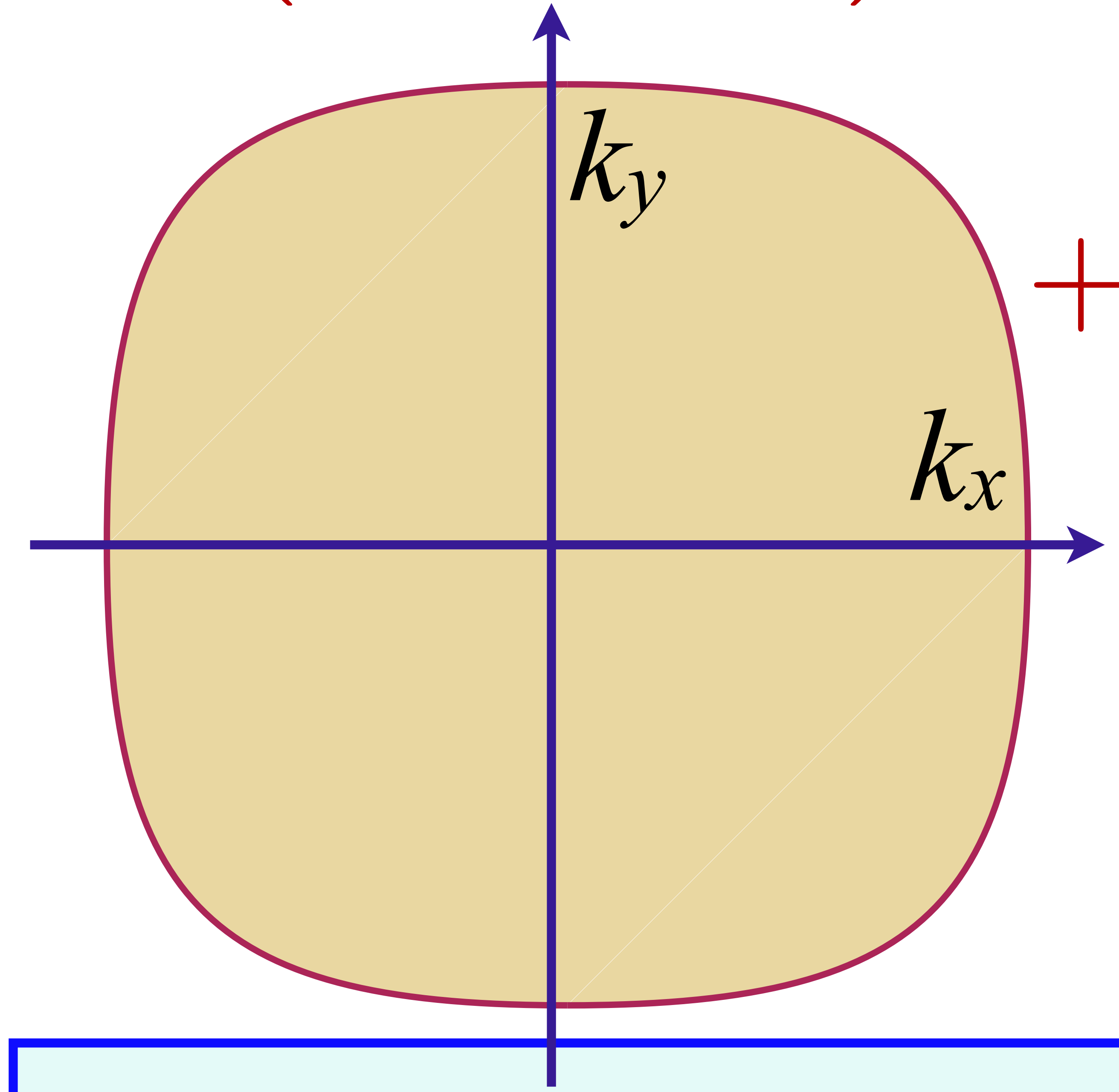
$$+s\left[\phi(\boldsymbol{r})\right]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})\tau^a_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

$$+K\left[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})\right]^2+u\left[\phi(\boldsymbol{r})\right]^4 \;+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0,\ \overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Fermi surface + critical boson with potential and interaction disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$



$$+[s+\delta s(\boldsymbol{r})][\phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})\tau^a_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

$$+K\left[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})\right]^2+u\left[\phi(\boldsymbol{r})\right]^4+v(\boldsymbol{r})c^\dagger_\sigma(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})}=0$, $\overline{\delta s(\boldsymbol{r})\delta s(\boldsymbol{r}')}=\delta s^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

RG analysis (Harris criterion) shows that $\delta s(\boldsymbol{r})$ is most relevant disorder.

Bosonic eigenmodes in random mass Hertz theory

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_\phi = \int d\tau \Big[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \Big(\frac{s + s'_j}{2} \phi_{ja}^2 + \frac{u}{4M} \left(\phi_{ja}^2 \right)^2 \Big) \Big]$$

$$\mathcal{S}_{\phi d} = \frac{T}{2} \sum_\Omega \sum_j \left(\gamma |\Omega| + \Omega^2 / c^2 \right) |\phi_{ja}(i\Omega)|^2 \, ,$$

where $a = 1 \ldots M$ is a flavor index for an order parameter with O(M) symmetry.

J. A. Hoyos, Chetan Kotabage, Thomas Vojta
Phys. Rev. Lett. **99**, 230601 (2007)
T. Vojta, J.A. Hoyos, Priyanka Mohan, Rajesh Narayanan,
J. Phys.: Condens. Matter **23**, 094206 (2011)

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where $a = 1 \ldots M$ is a flavor index for an order parameter with $\mathrm{O}(M)$ symmetry. Analyze in a self-consistent quadratic theory, treating disorder numerically exactly

$$\overline{\mathcal{S}}_\phi = \int d\tau \Big[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \frac{\overline{s}'_j}{2} \phi_{ja}^2 \Big]$$

Similar analysis in $d = 1$ works very well
A. Del Maestro, B. Rosenow, M. Müller and S. Sachdev, Phys. Rev. Lett. **101**, 035701 (2008).

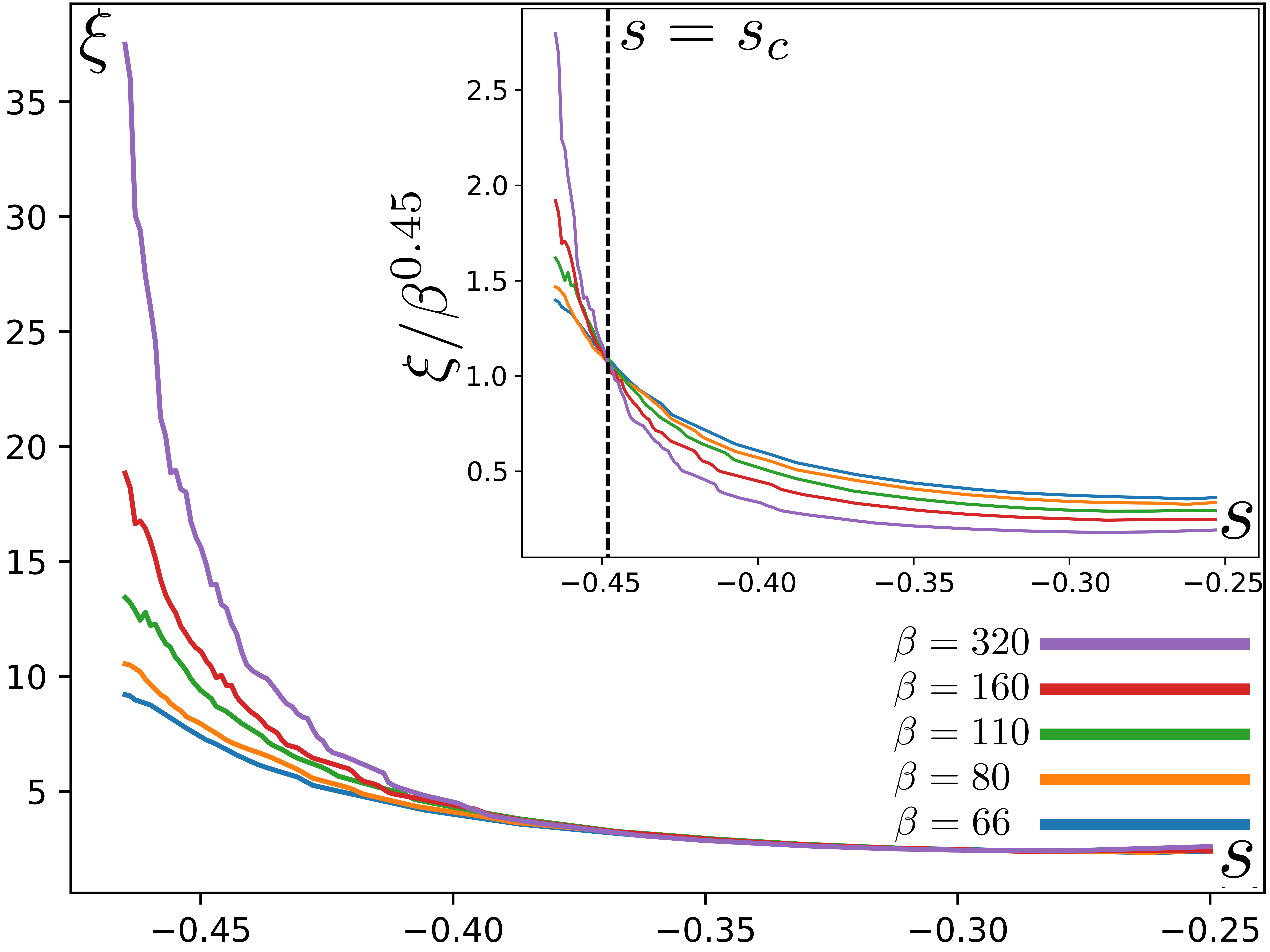
$$\overline{s}'_j = s + s'_j + \frac{u}{M} \sum_a \langle \phi_{ja}^2 \rangle_{\overline{\mathcal{S}}_\phi + \mathcal{S}_{\phi d}} = s + s'_j + uT \sum_\Omega \sum_\alpha \frac{\psi_{\alpha i} \psi_{\alpha j}}{\gamma |\Omega| + \Omega^2 / c^2 + e_\alpha}$$

where e_α and $\psi_{\alpha j}$ are eigenvalues and eigenfunctions of the ϕ quadratic form in $\overline{\mathcal{S}}_\phi$, labeled by the index $\alpha = 1 \ldots L^2$ for a $L \times L$ sample.

Aavishkar A. Patel, Peter Lunts, S.S., PNAS **121**, e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

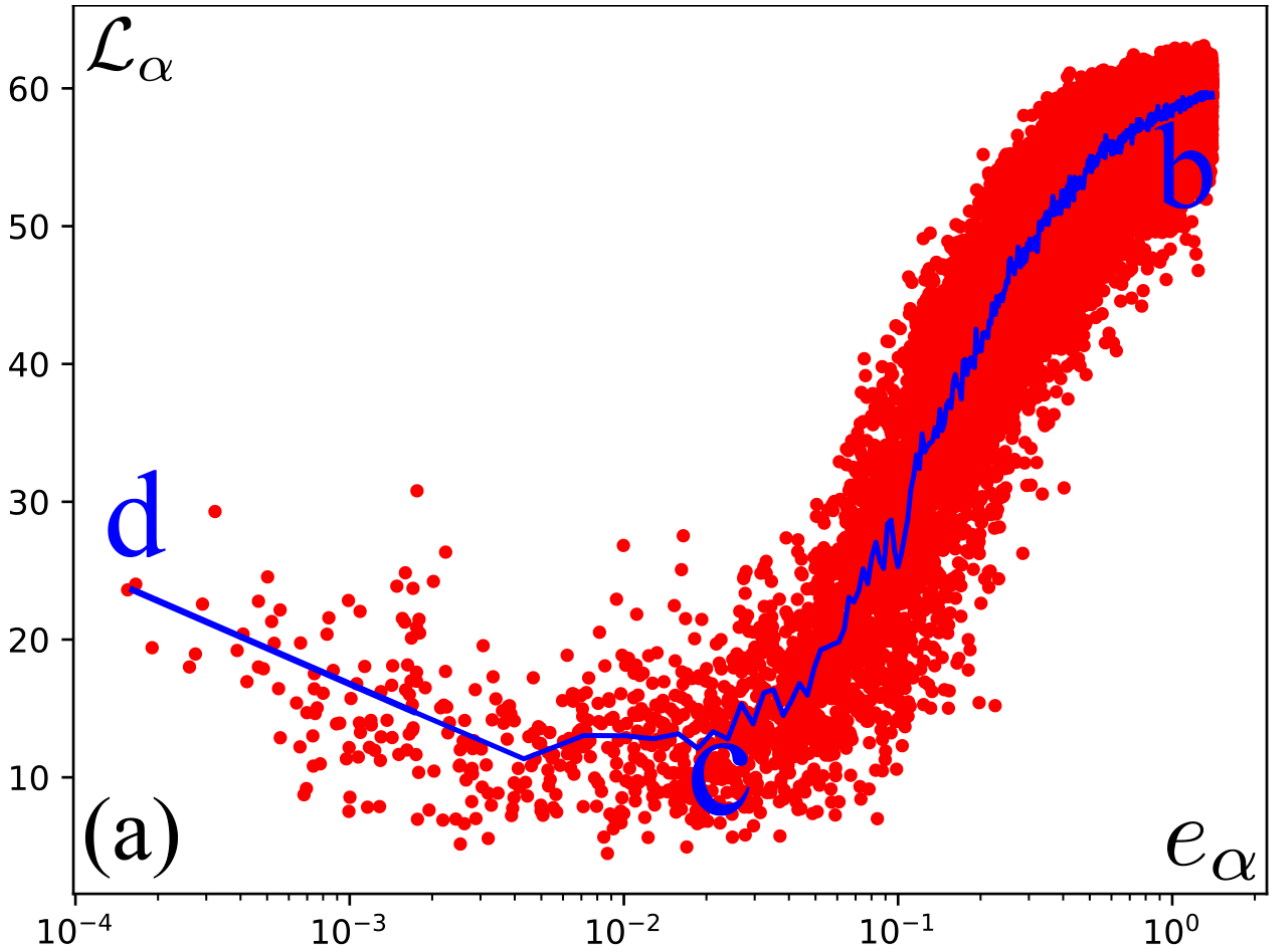
ϕ correlation length ξ



Aavishkar A. Patel, Peter Lunts, S.S., *PNAS* **121**, e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$



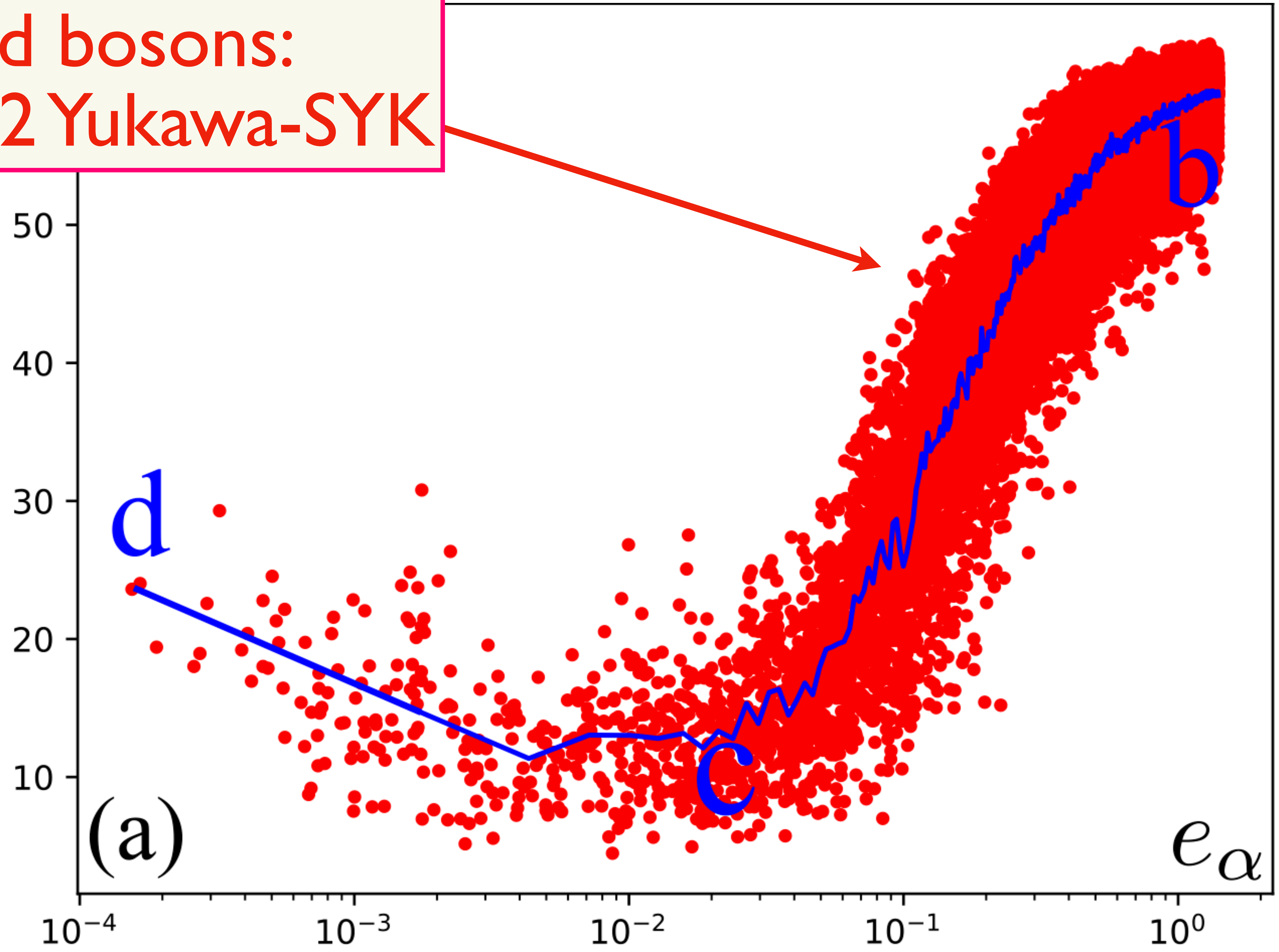
QMC:
Aavishkar A. Patel,
Peter Lunts, and
Michael Albergo,
arXiv:2410.05365

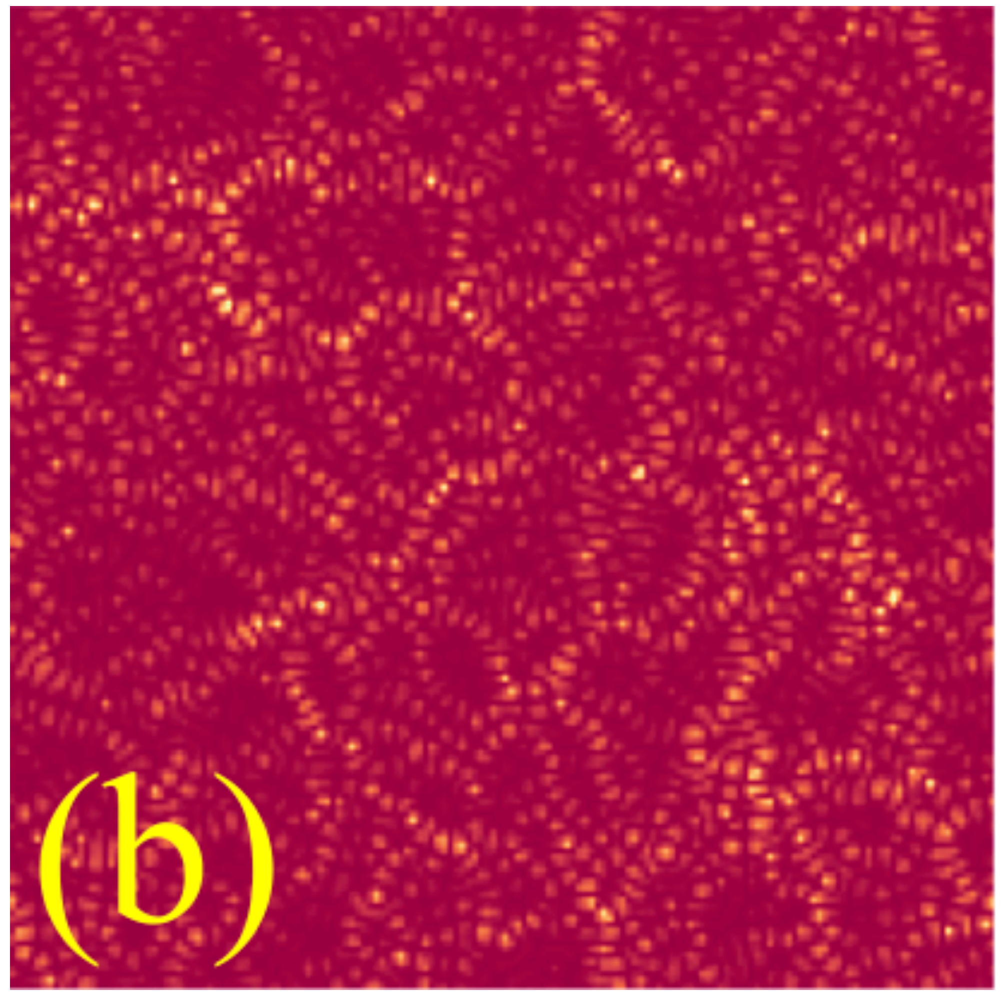
Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$

Extended bosons: physics of d=2 Yukawa-SYK





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Aavishkar A. Patel, Peter Lunts, and Michael Albergo, arXiv:2410.05365

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Bosonic eigenmodes in random mass Hertz theory

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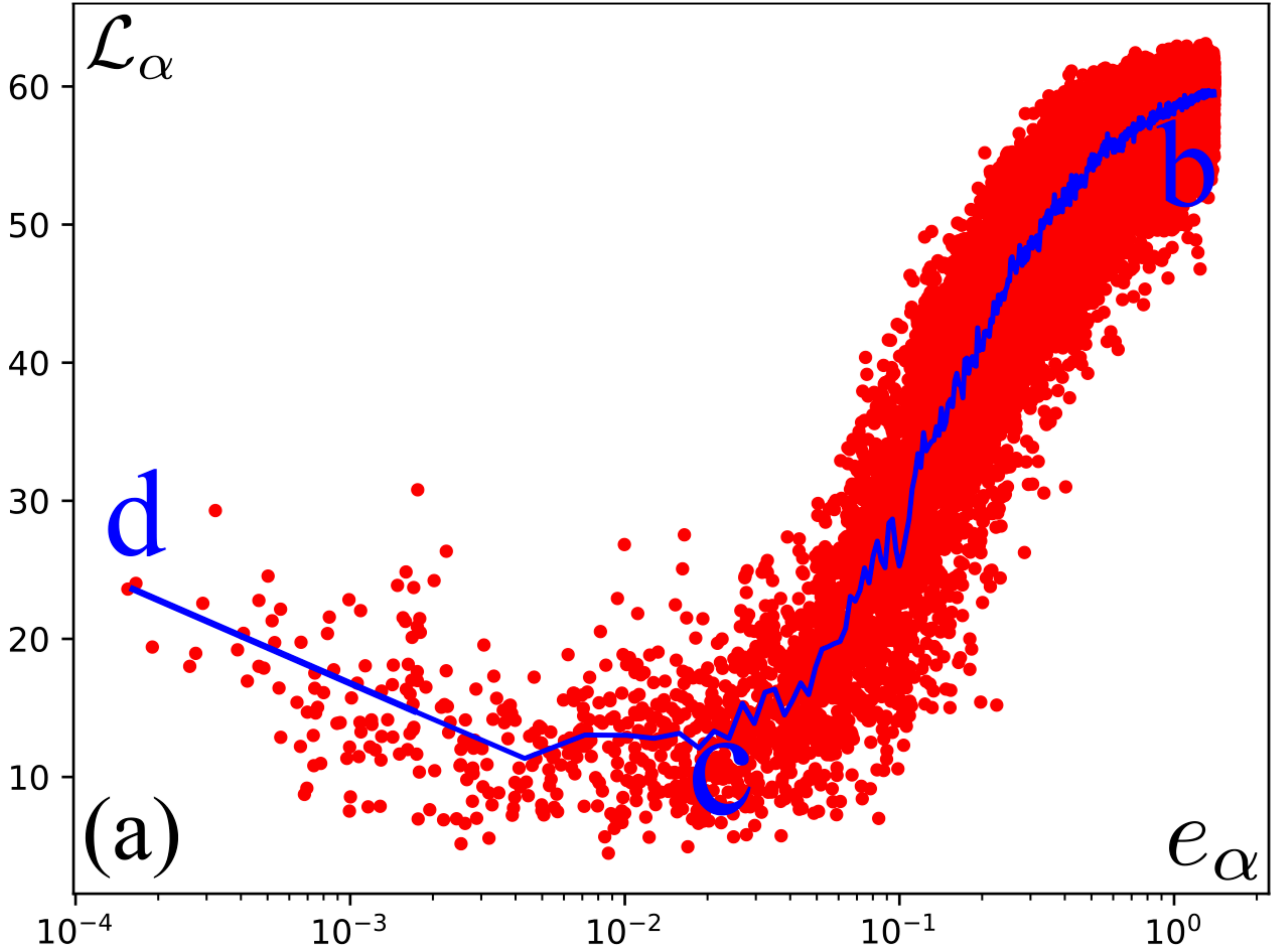

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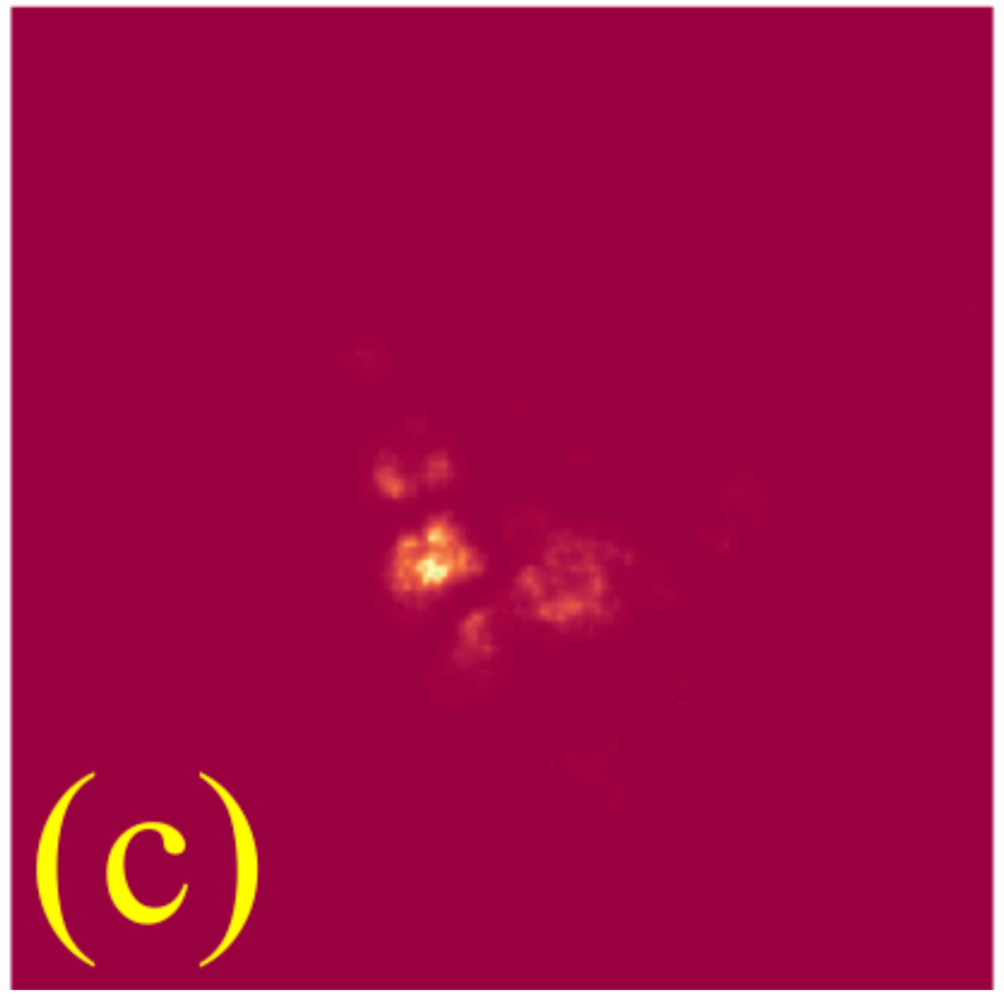

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e2402052121 (2024)

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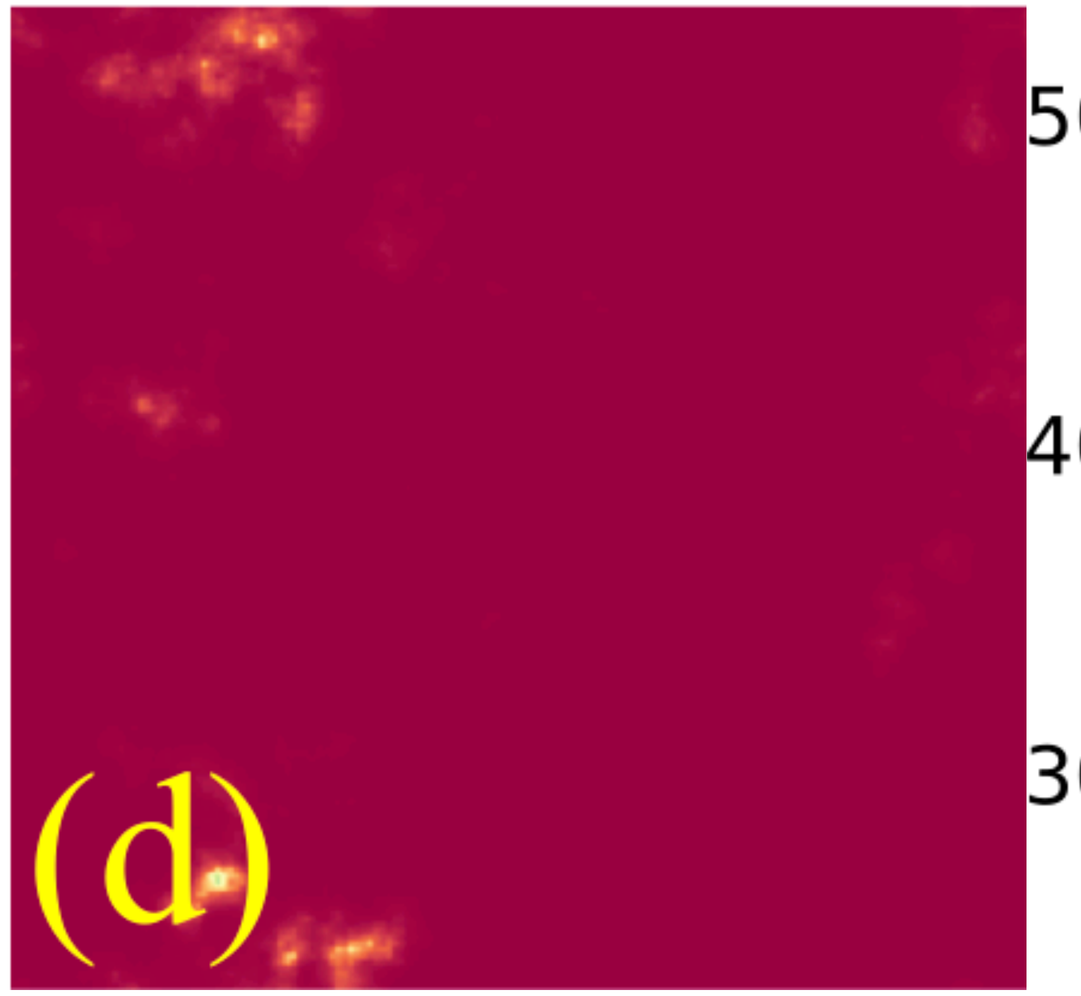

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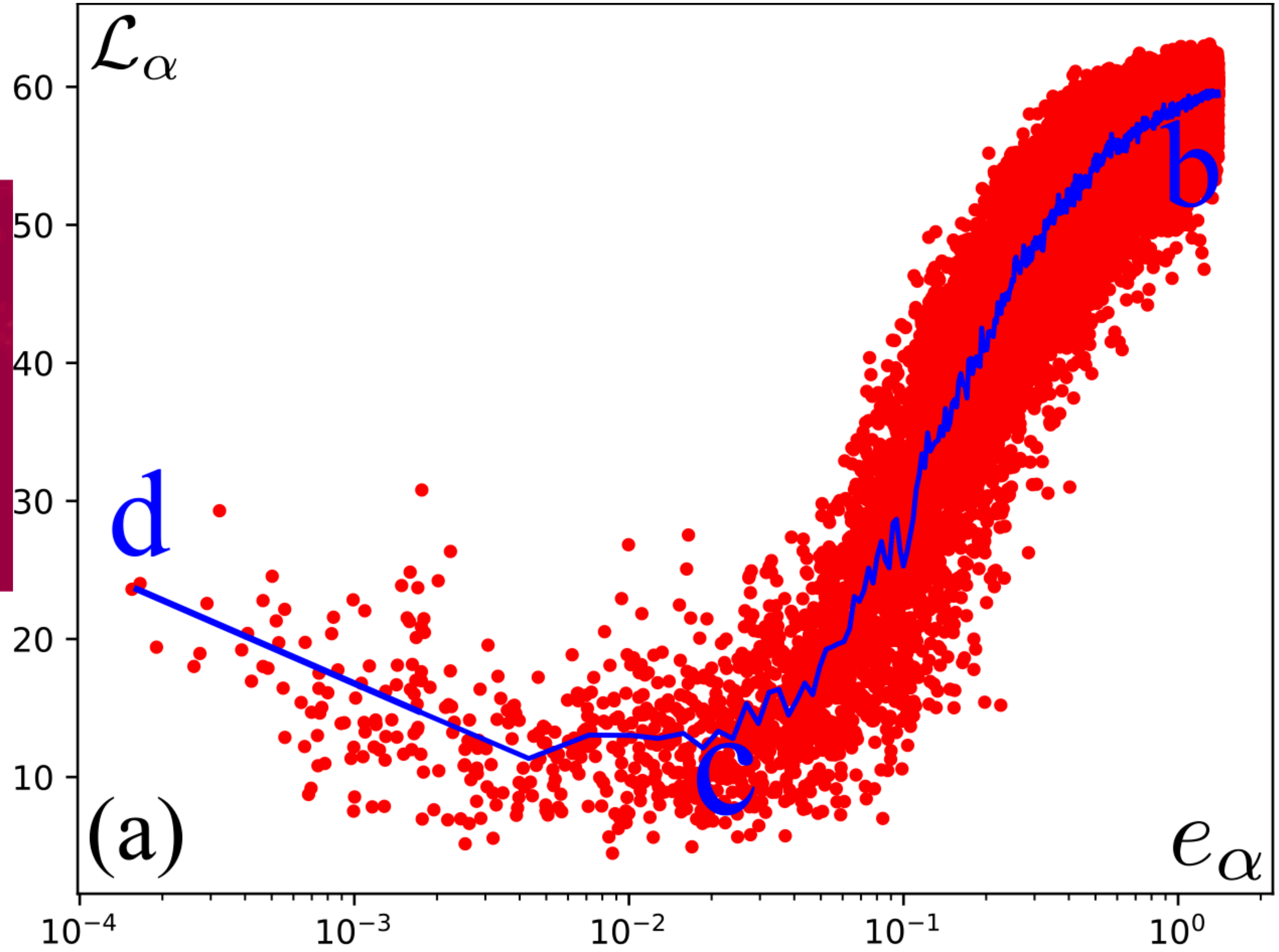
e2402052121 (2024)



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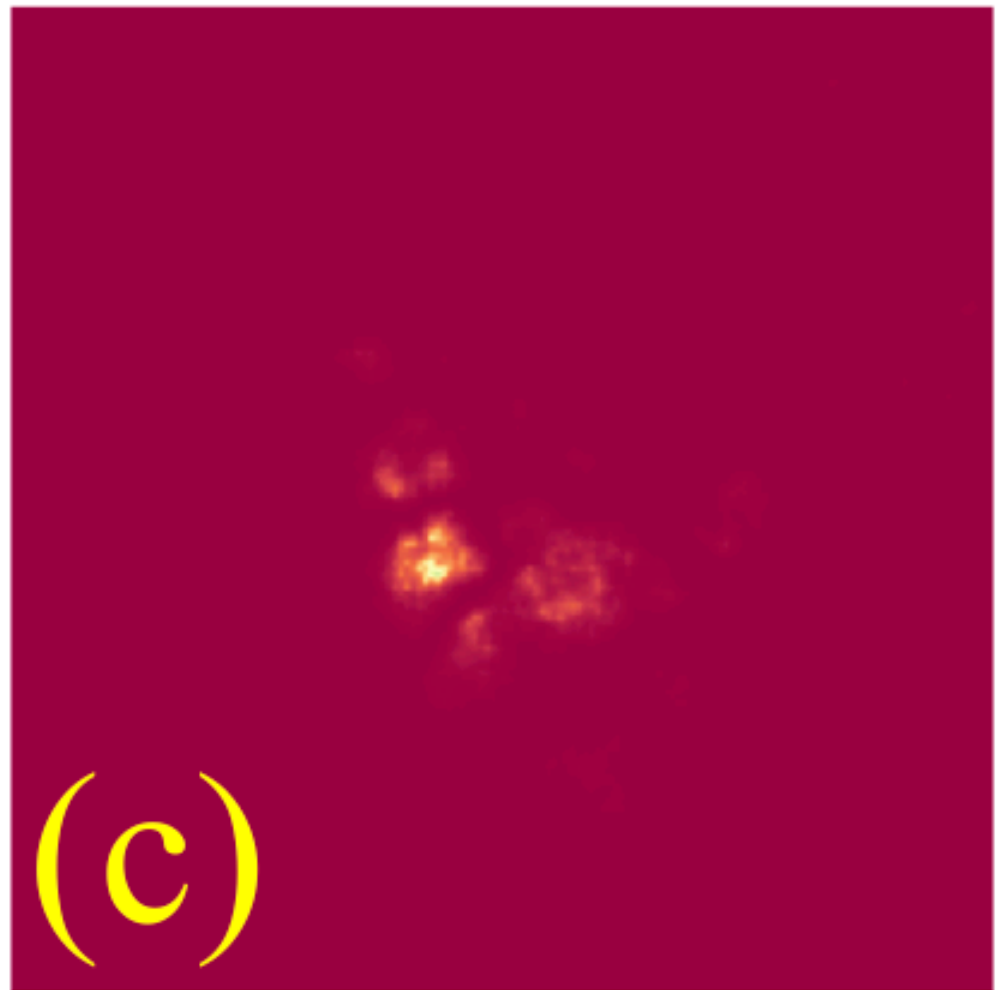
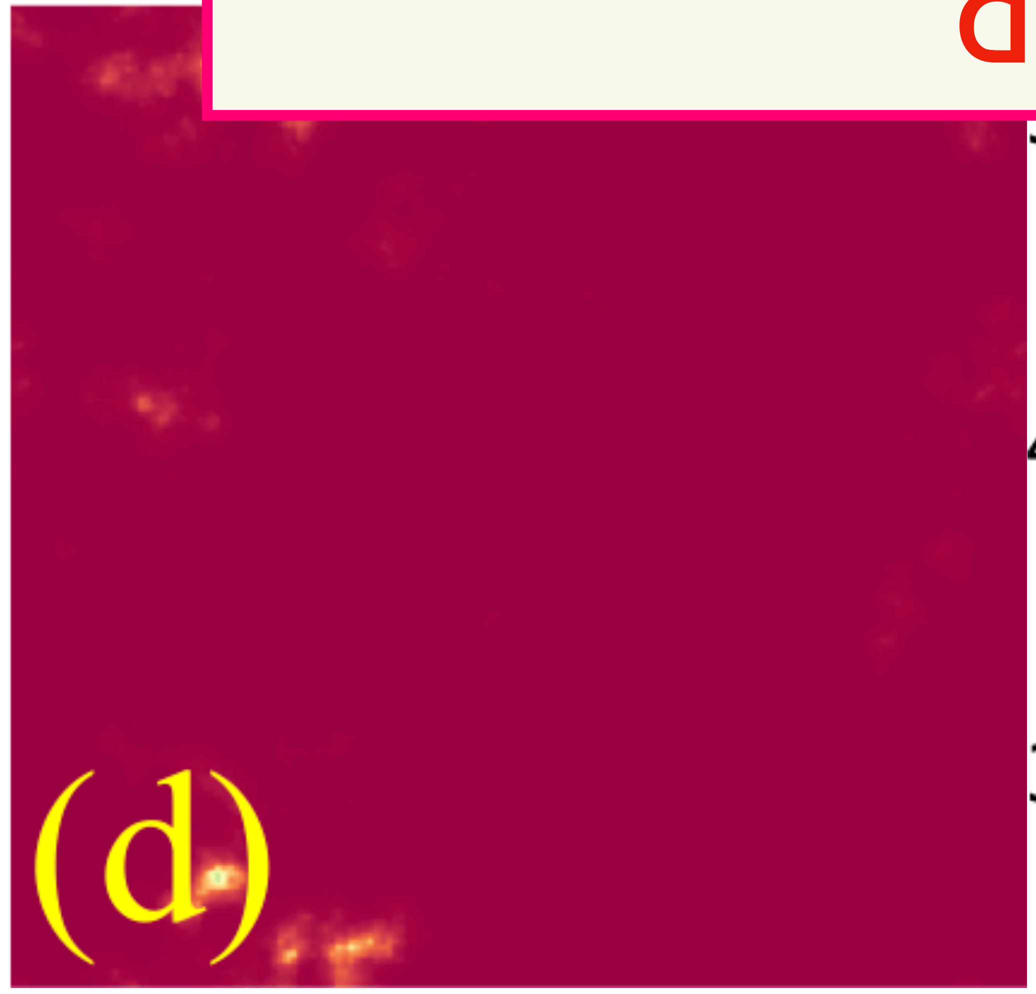
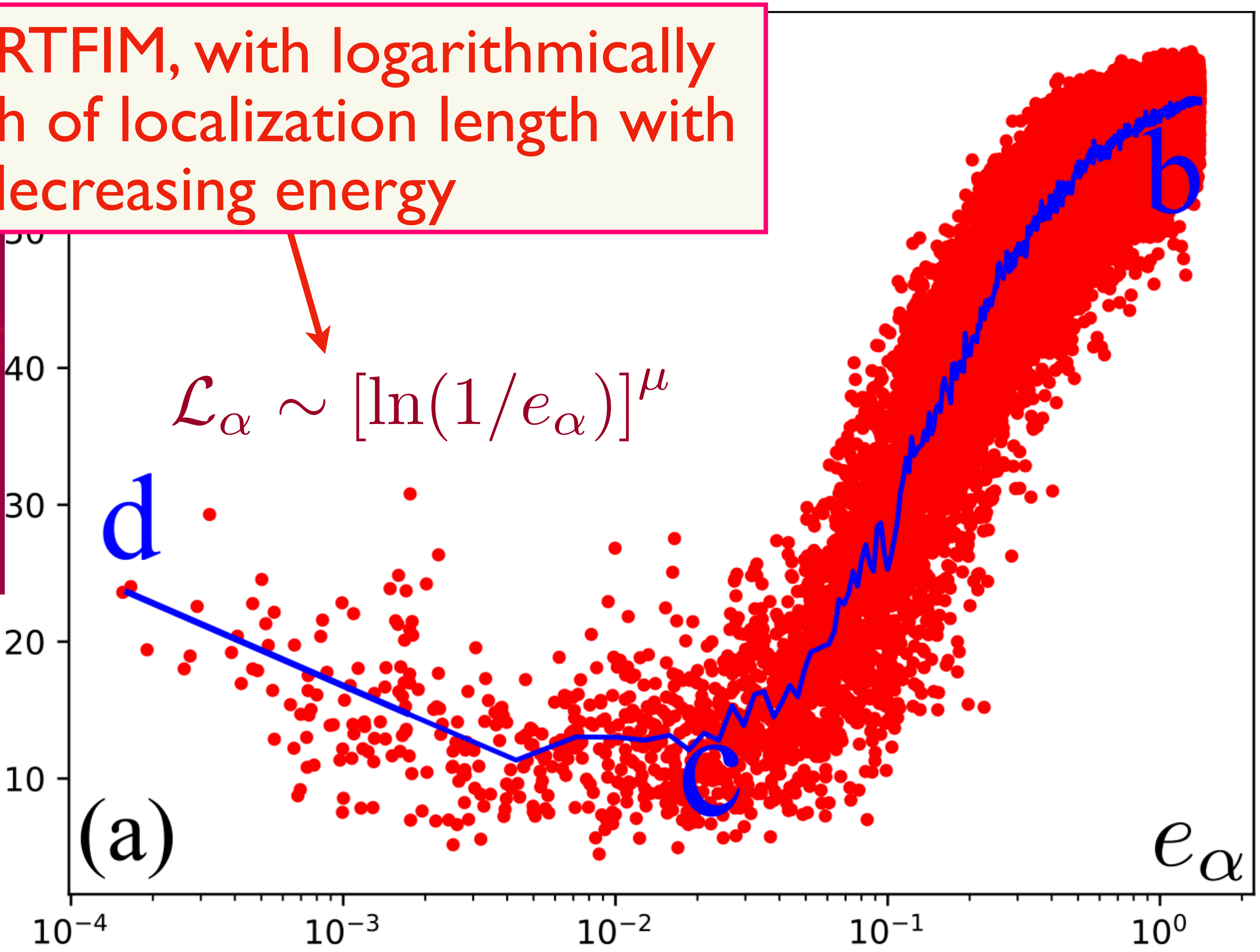


Bosonic eigenmodes in random mass Hertz theory

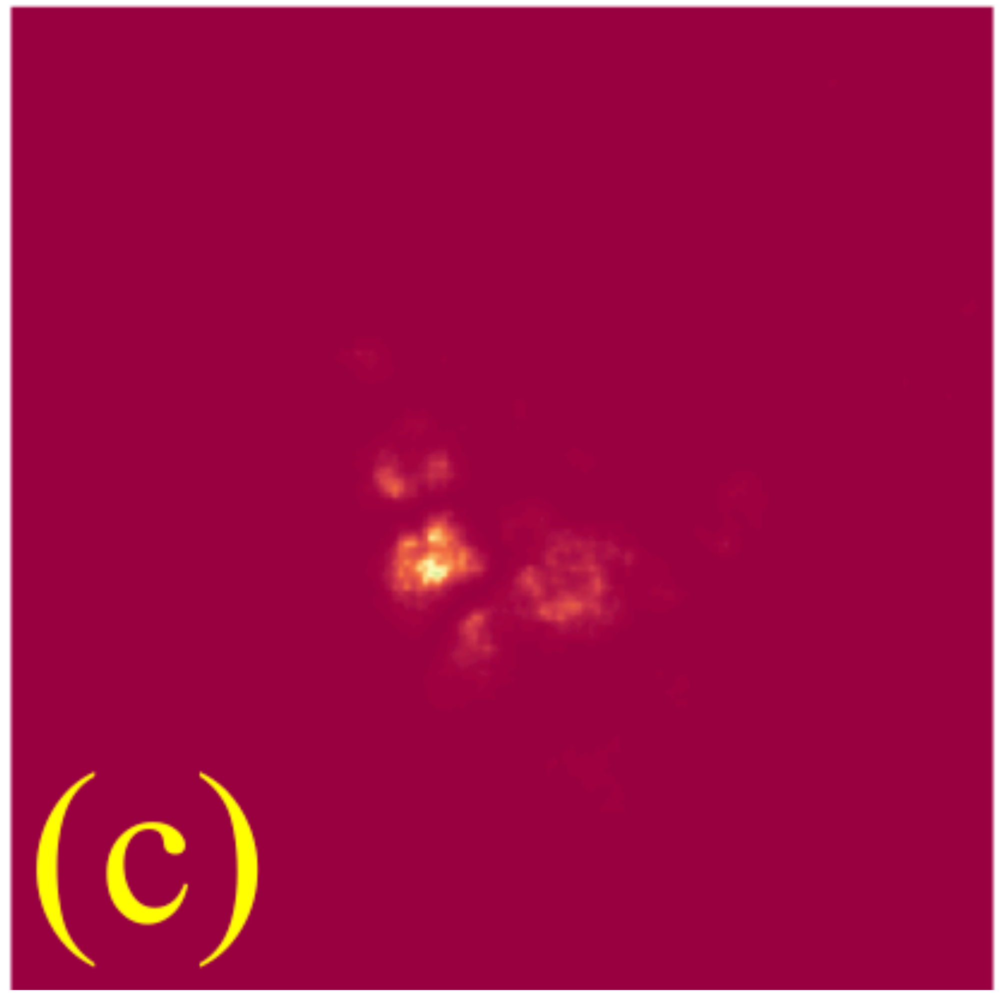
ϕ eigenmodes localization length $\mathcal{L}_\alpha$

Physics of RTFIM, with logarithmically slow growth of localization length with decreasing energy





$$\mathcal{L}_\alpha \sim [\ln(1/e_\alpha)]^\mu$$



Aavishkar A. Patel, Peter Lunts, S.S., *PNAS* **121**, e2402052121 (2024)

J. A. Hoyos, C. Kotabage, T. Vojta PRL **99**, 230601 (2007)

QMC: Aavishkar A. Patel, Peter Lunts, and Michael Albergo, arXiv:2410.05365

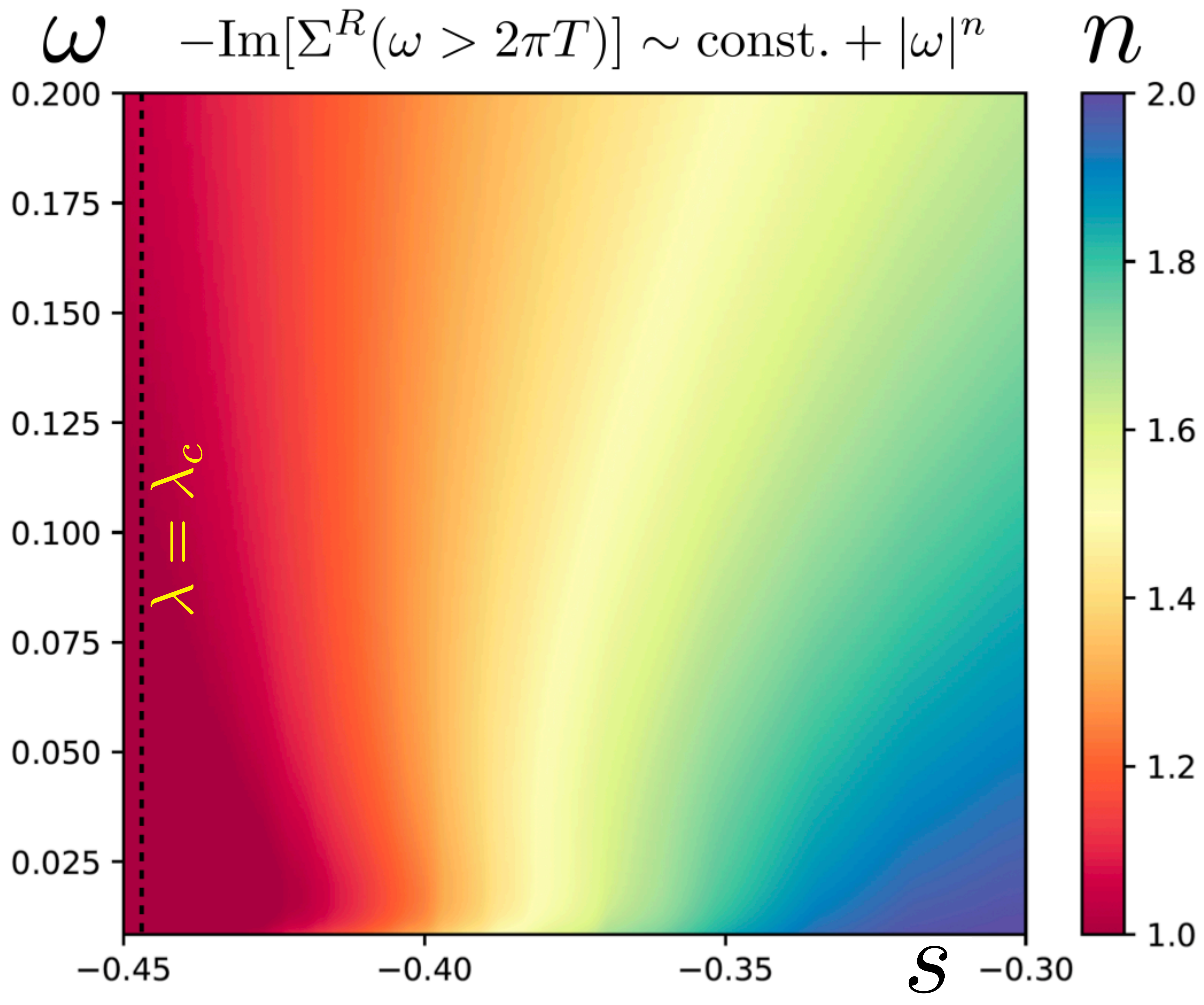
Bosonic eigenmodes in random mass Hertz theory

Transport scattering rate

$$\Sigma(i\omega) = -i\pi g'^2 \mathcal{N}_0 \frac{T}{L^2} \sum_{\alpha,\Omega} \frac{\mathrm{sgn}(\omega+\Omega)}{\gamma|\Omega| + \Omega^2/c^2 + e_\alpha} \,.$$

Aavishkar A. Patel, Peter Lunts, S.S., *PNAS* **121**, e2402052121 (2024)

See also: QMC results of Aavishkar A. Patel, Peter Lunts, Michael S. Albergo, arXiv:2410.05365



$L = 160, \beta = 800$, 10 disorder samples

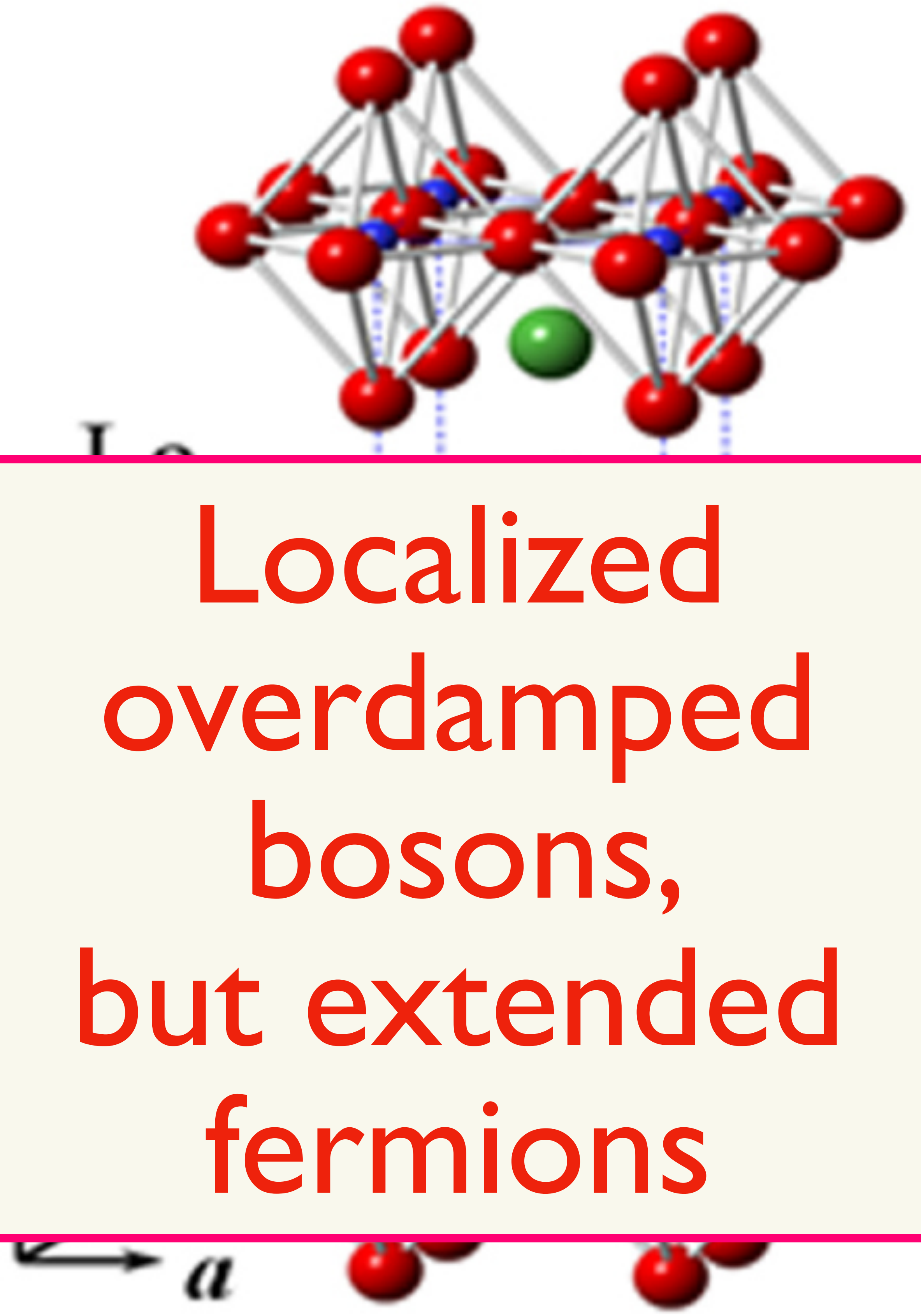
Extended region in λ with $n \approx 1$ - a strange metal *phase*

Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

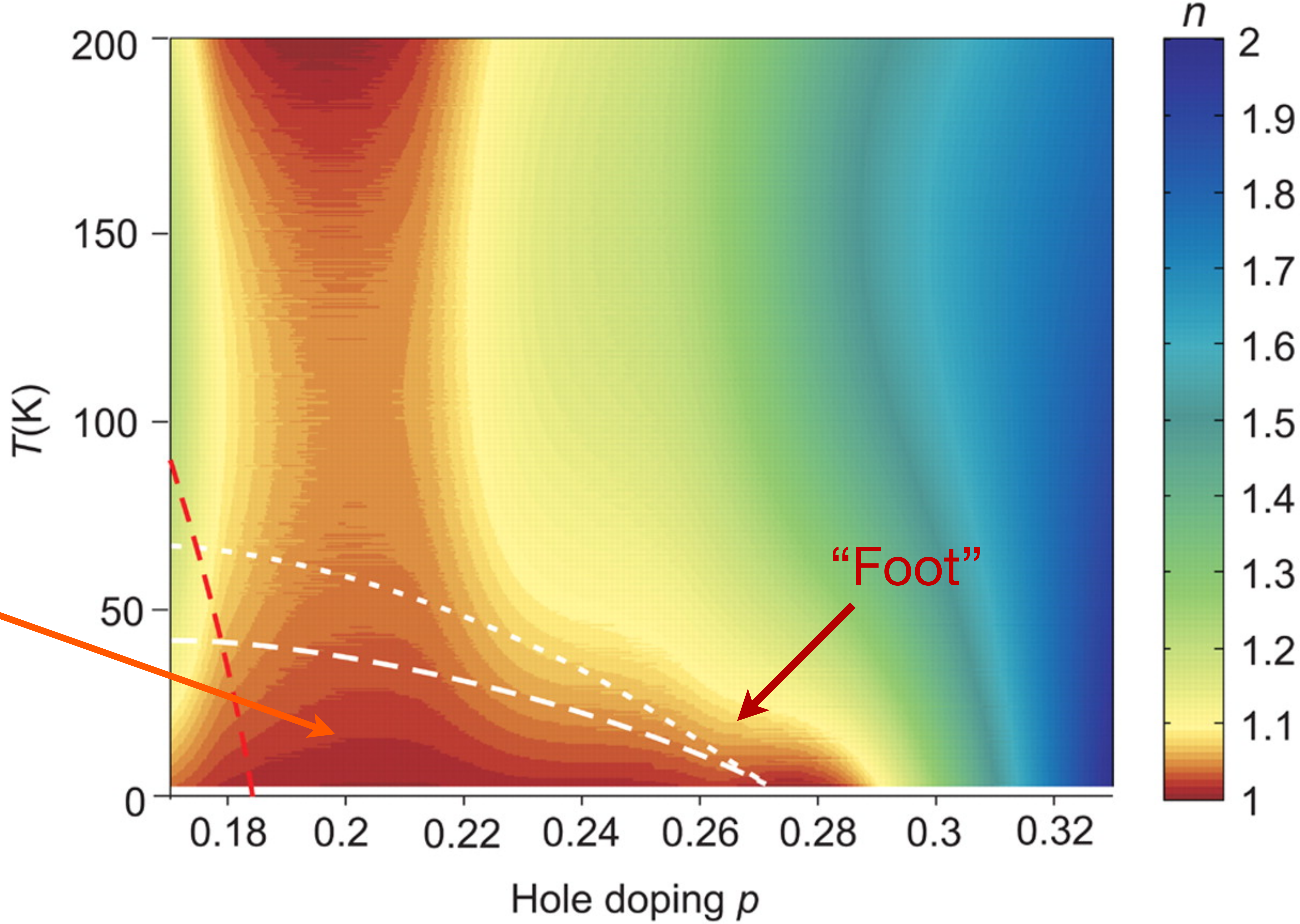
R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4]* H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

SCIENCE VOL 323 603 2009

FL-SDW QPT with Harris disorder provides a theory of the "foot"

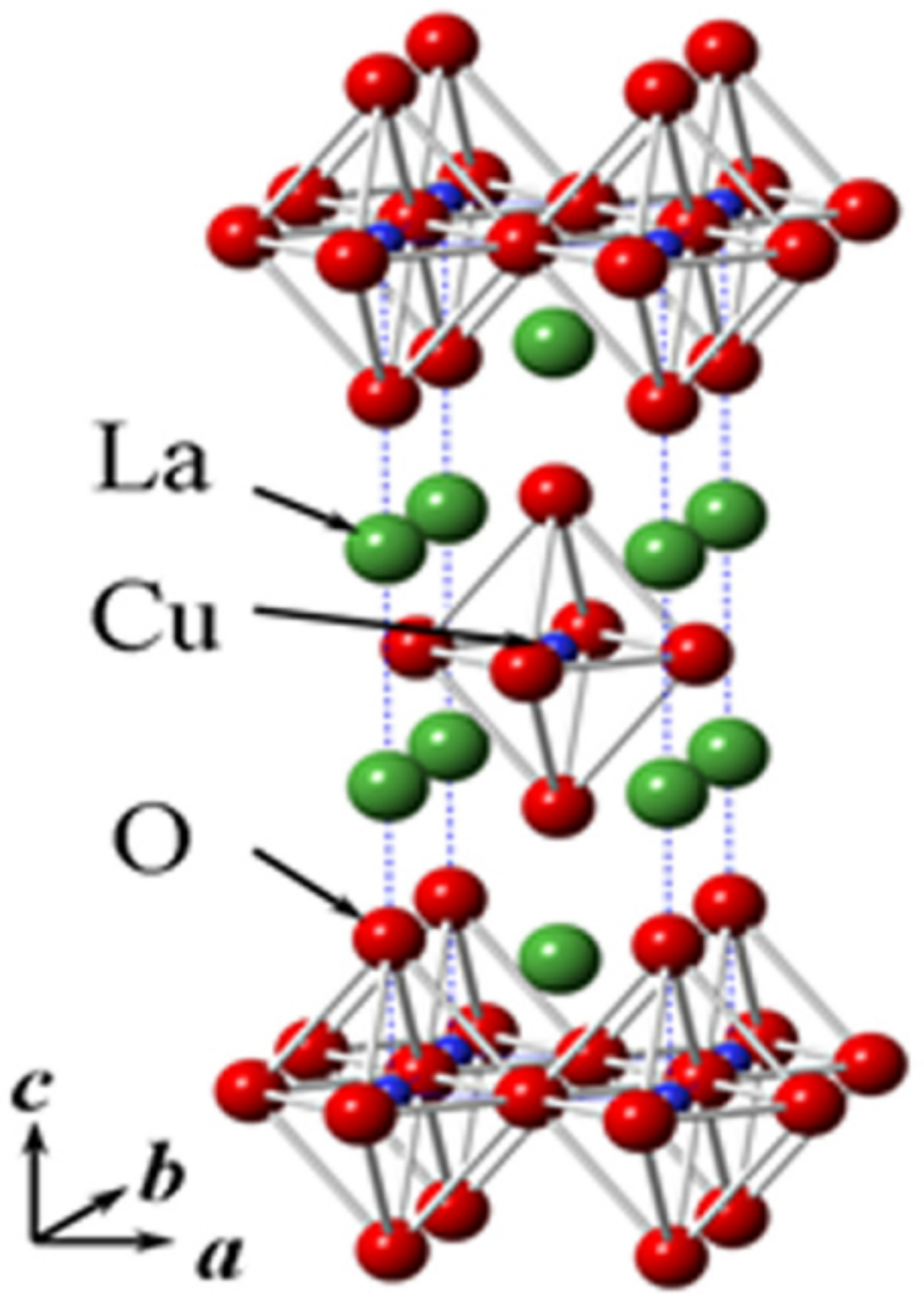


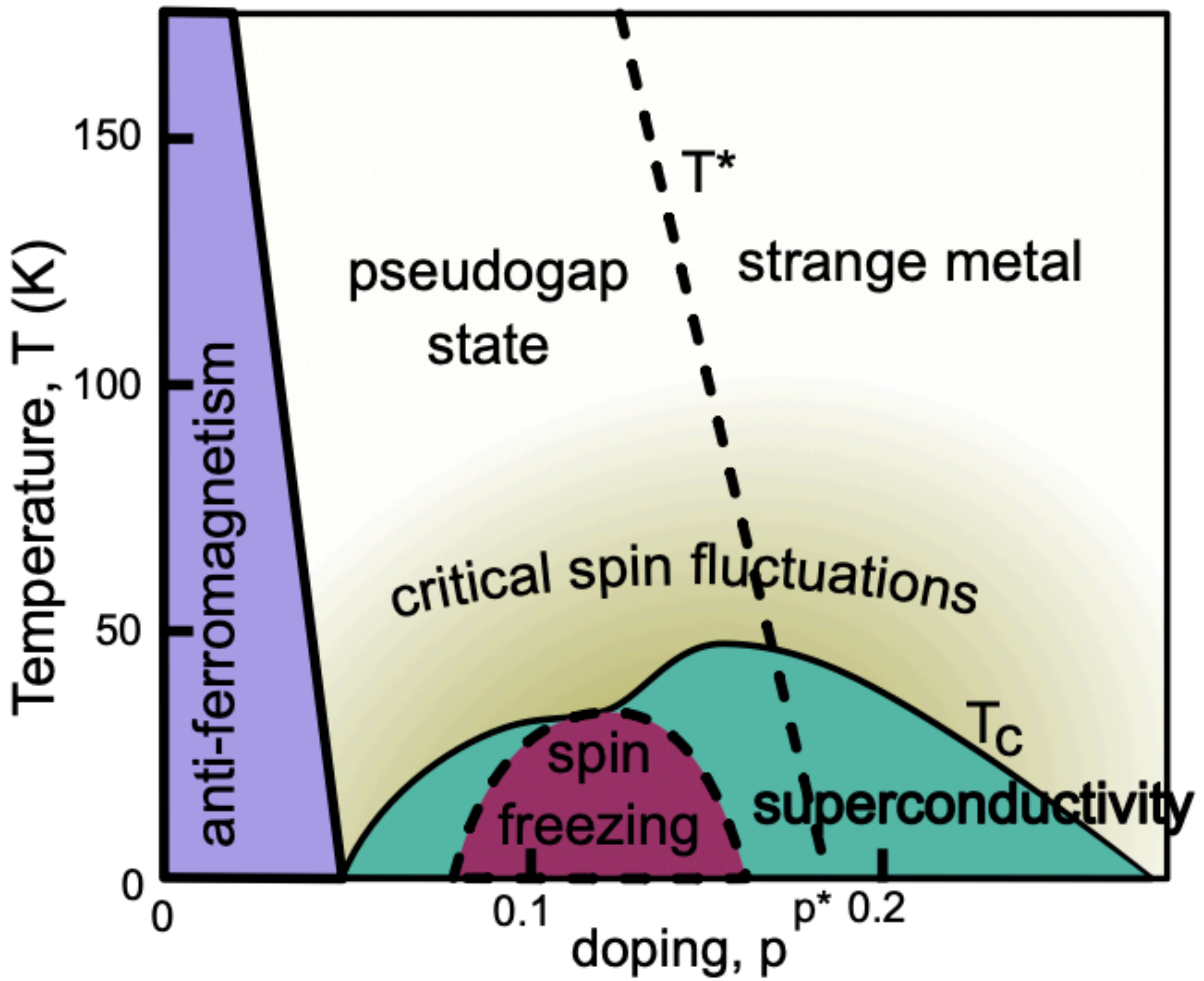
Localized overdamped bosons, but extended fermions



Neutron scattering in LSCO

Jacopo Radaelli, Aavishkar A. Patel, ….S. S., Stephen Hayden, to appear







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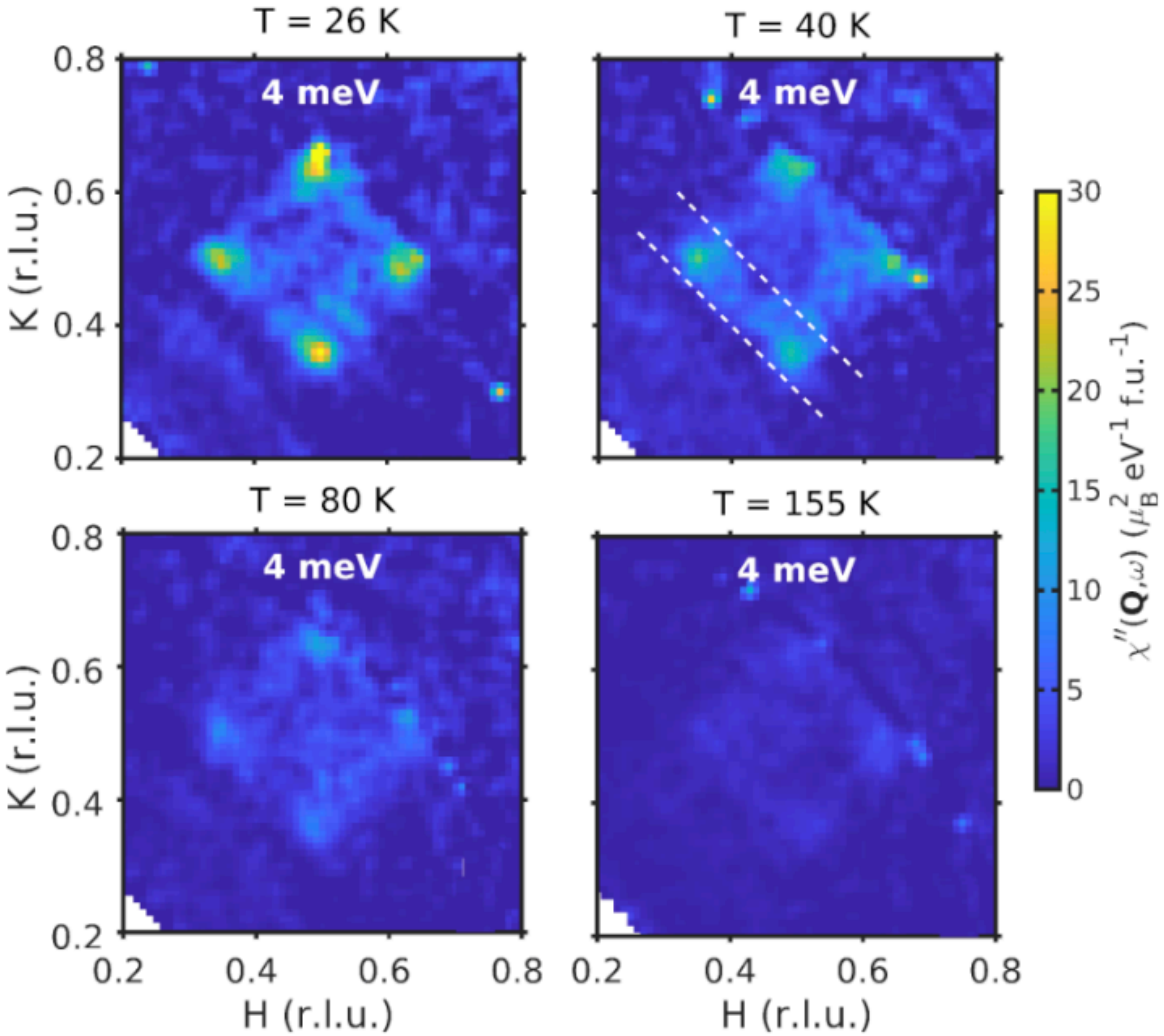
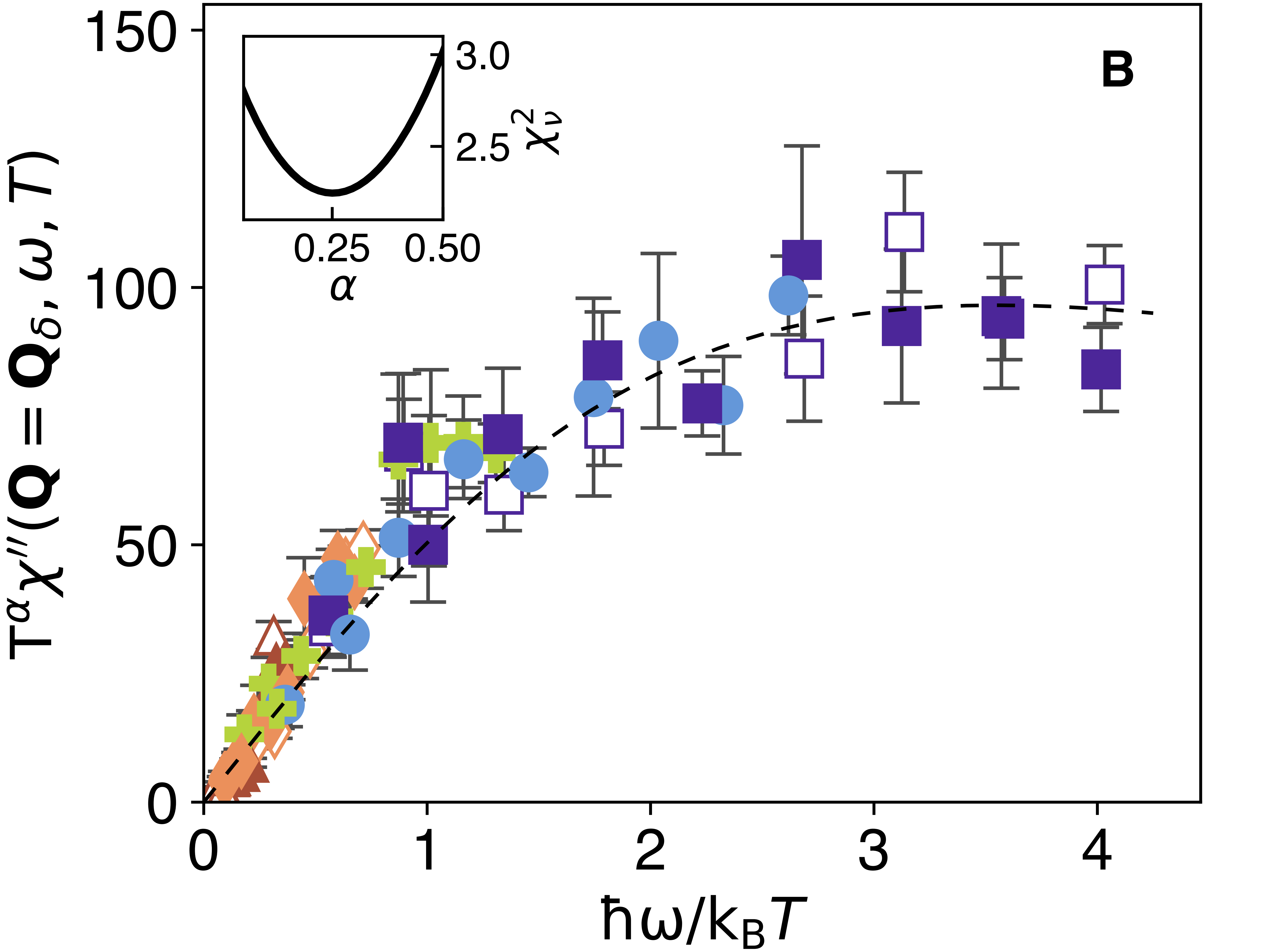
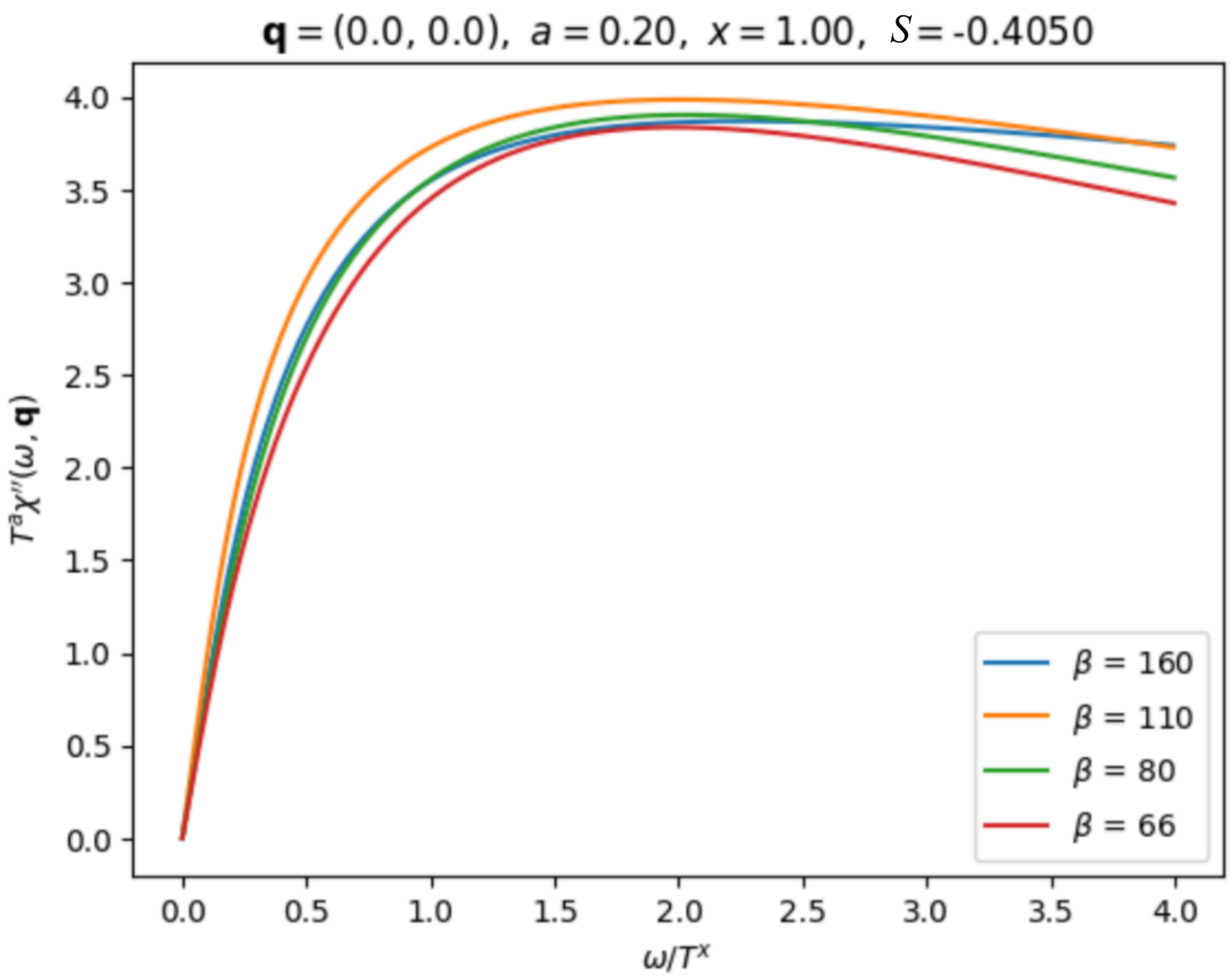


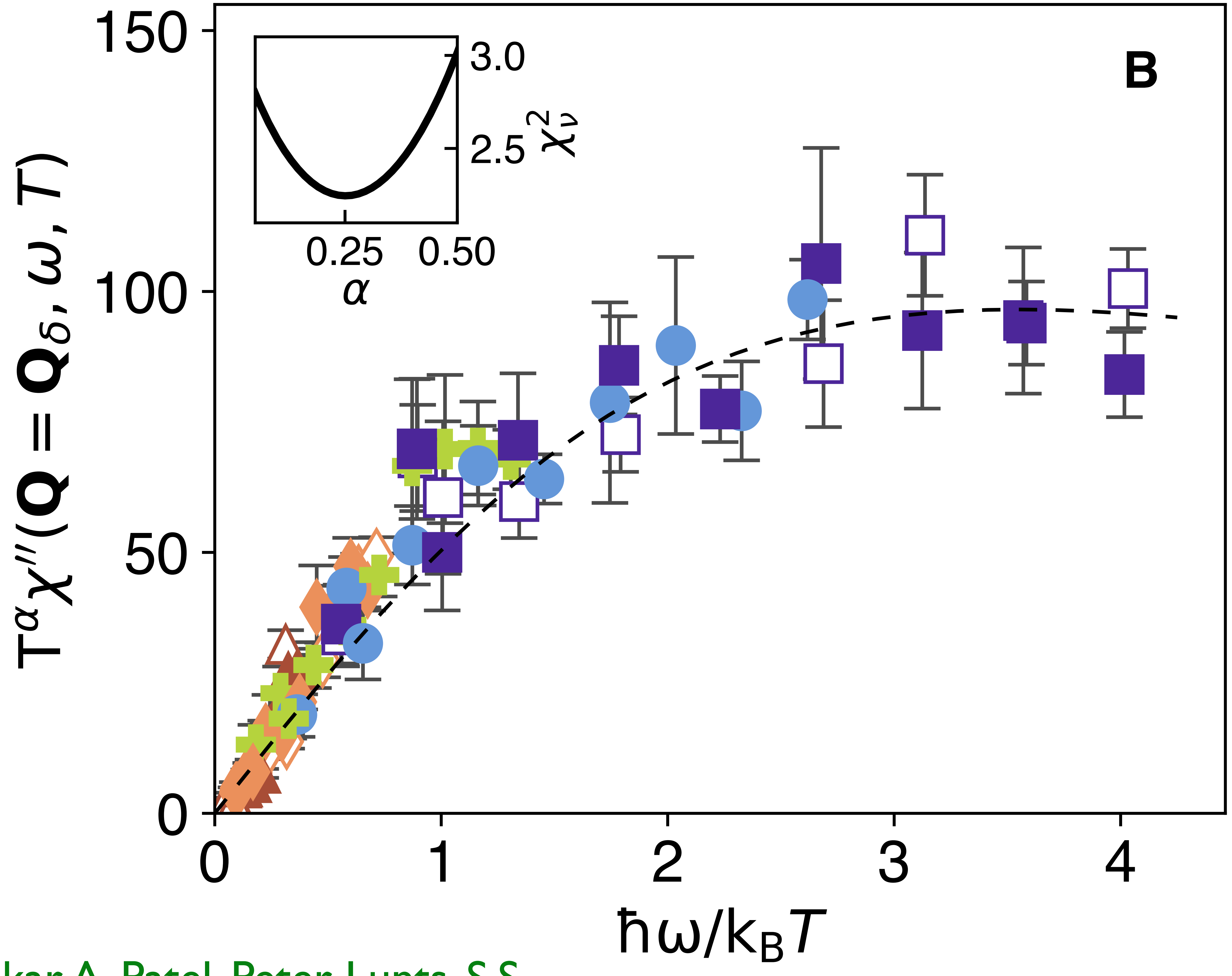


Neutron scattering in LSCO



Jacopo Radaelli, Aavishkar A. Patel, ….S. S., Stephen Hayden, to appear





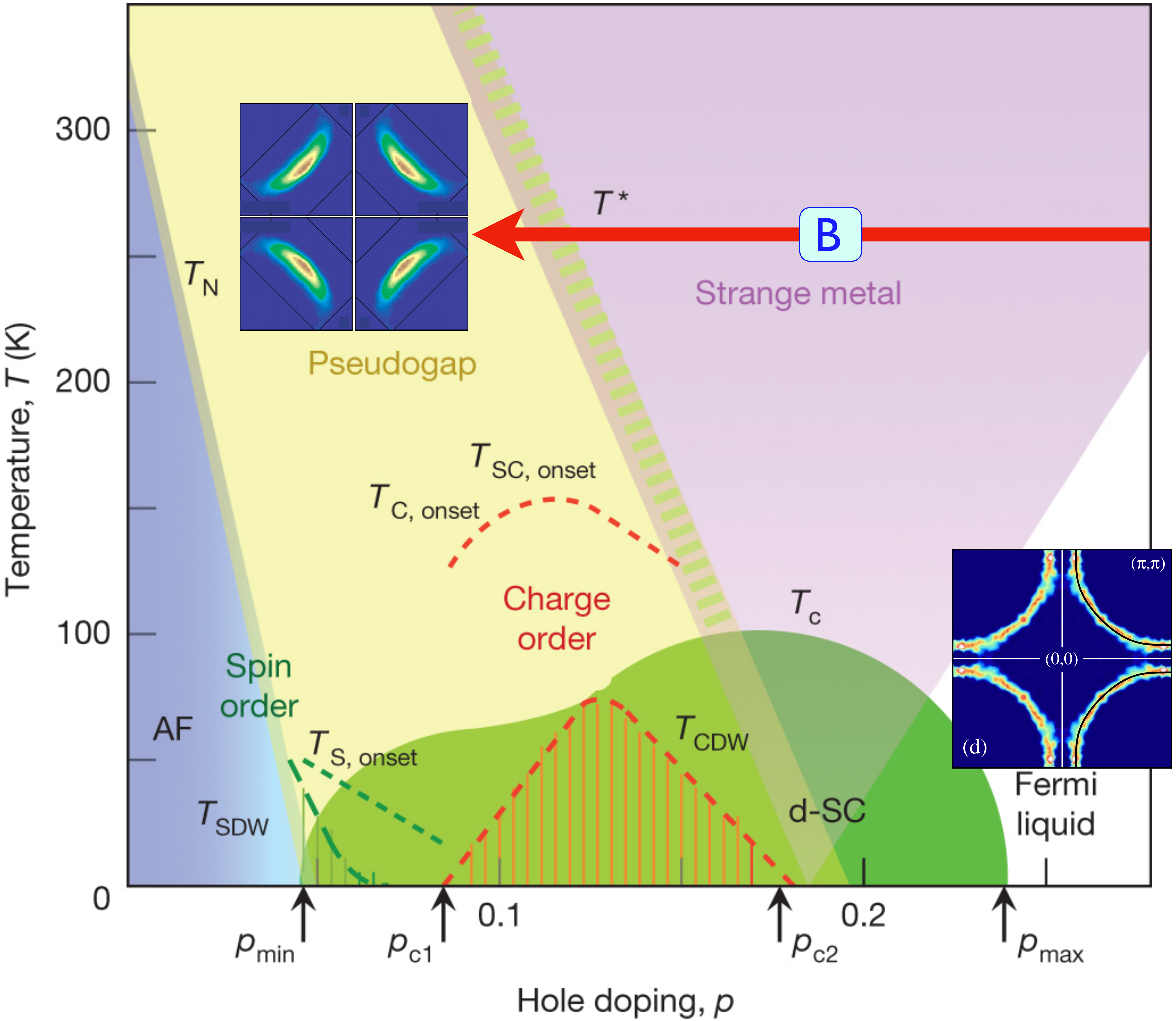
Aavishkar A. Patel, Peter Lunts, S.S.,
PNAS **121**, e2402052121 (2024)

A. FL-SDW QPT

B. FL-FL* QPT

C. Confinement crossover

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



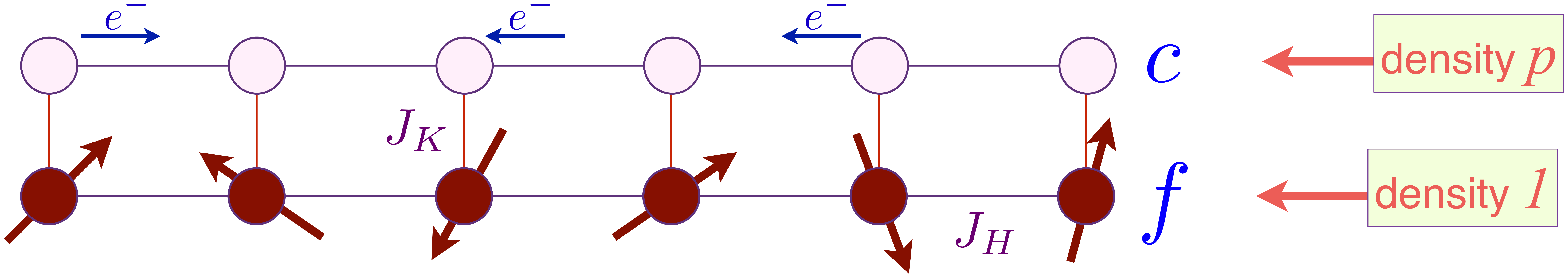
Fermi-volume-changing QPT
without symmetry breaking
and with spatial disorder.

FL-FL* QPT

Requires fractionalization

Fermi-volume-changing QPT in the Kondo lattice

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^{\dagger}_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_i J_K \, c^{\dagger}_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_i + \sum_{\langle ij \rangle} J_H \, \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



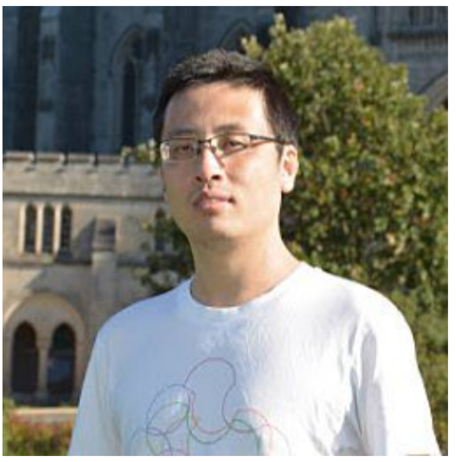
Assume J_H is chosen so that at $J_K = 0$
the $\boldsymbol{S}_i$ spins have a fractionalized spin liquid ground state.

Represent $\boldsymbol{S}_i$ by fermionic spinons: $\boldsymbol{S}_i = \frac{1}{2} f^{\dagger}_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} f_{i\sigma'}$

$$\sum_{\sigma} f^{\dagger}_{i\sigma} f_{i\sigma} = 1 \text{ for all } i.$$

Fermi-volume-changing QPT in the Kondo lattice

T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)



Kondo lattice

FL* $\langle \Phi \rangle = 0$ | $\langle \Phi \rangle \neq 0$ FL

0 — Small Fermi surface of size p | Large Fermi surface of size $1 + p$ — J_K

One-band model

FL* $\langle \Phi \rangle \neq 0$ | $\langle \Phi \rangle = 0$ FL

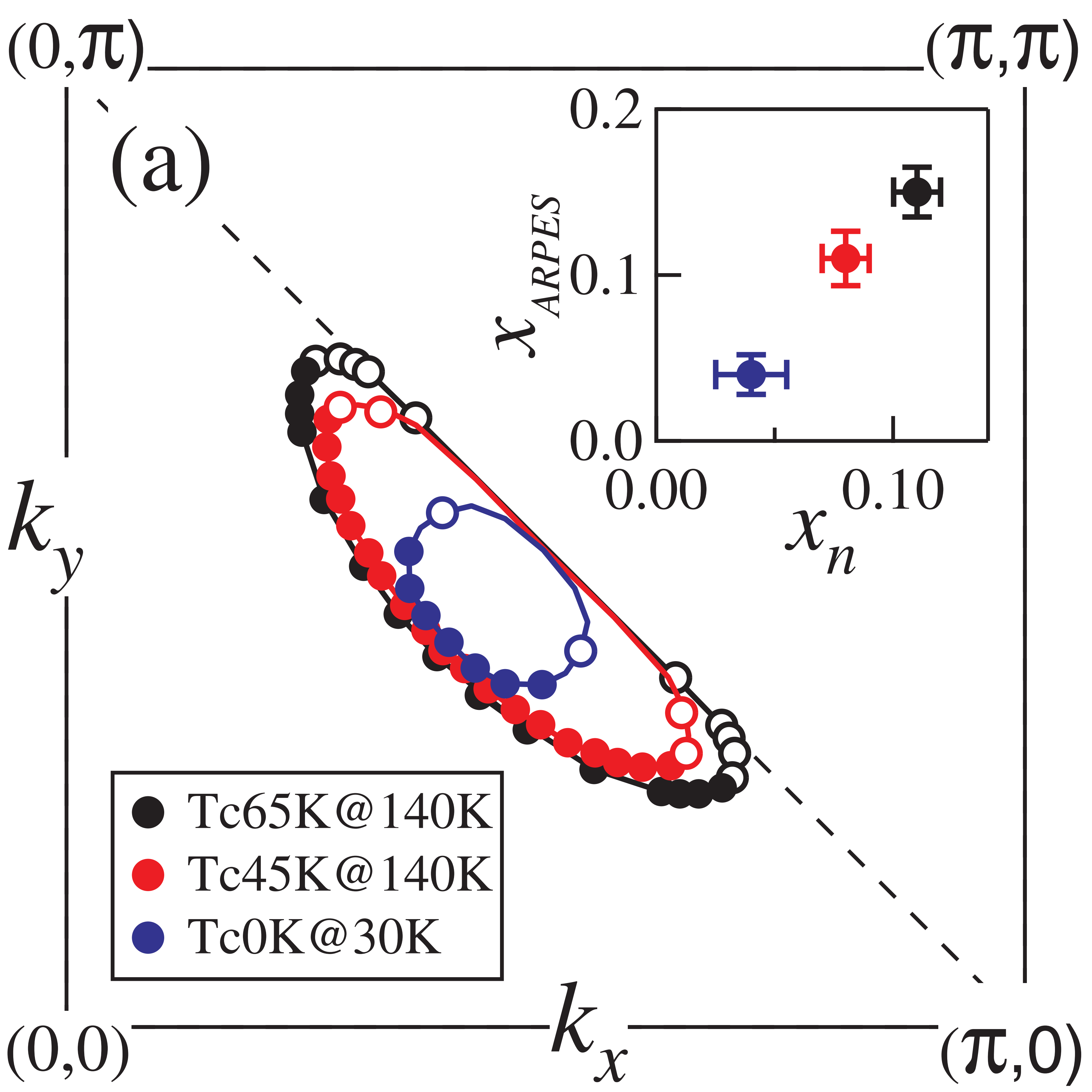
J_K — Small Fermi surface of size p | Large Fermi surface of size $1 + p$ — 0

One-band model has an **'inverted' Kondo lattice transition** in a theory using a bilayer of ancilla qubits

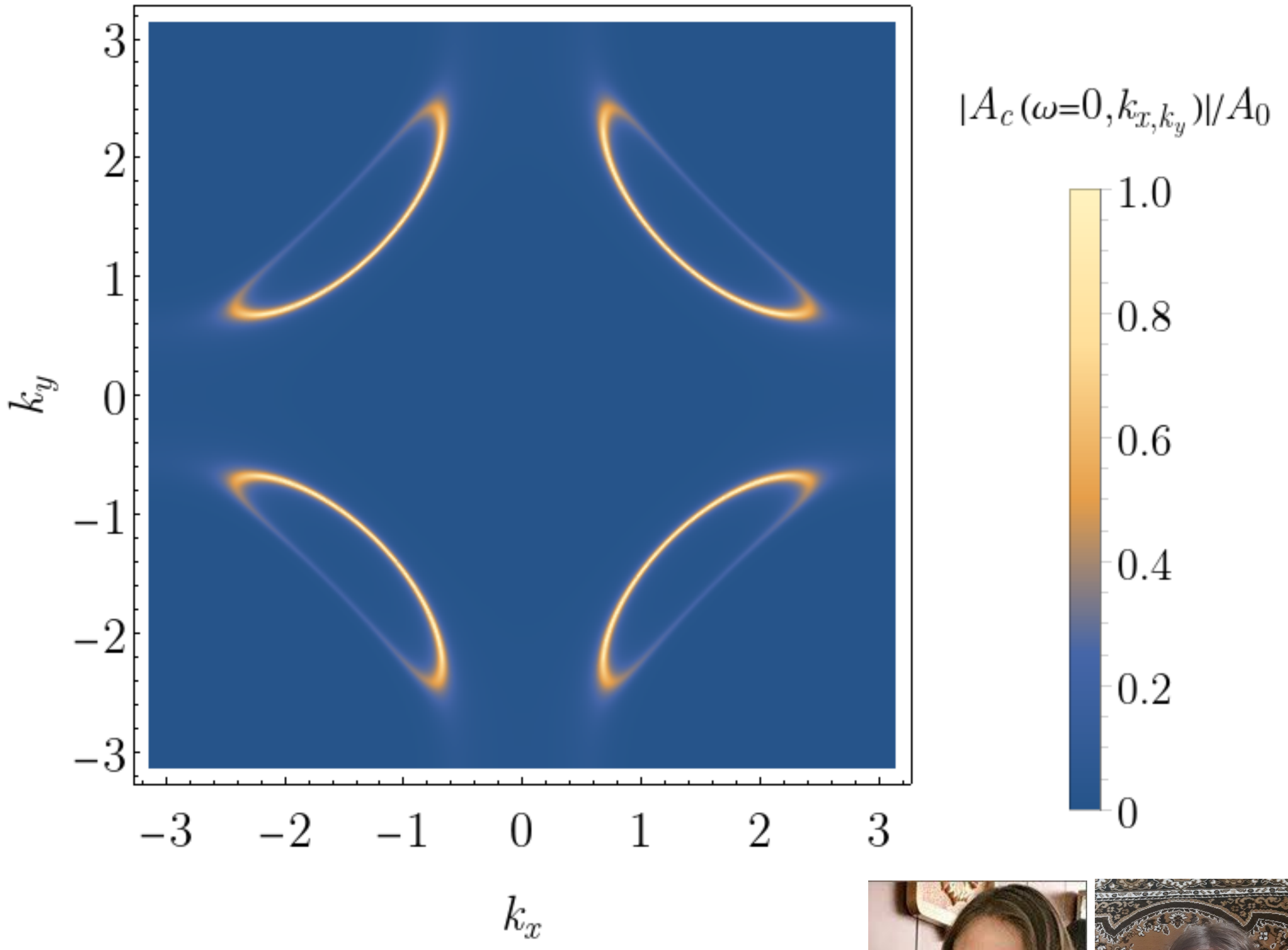
Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)

Ancilla theory of photoemission

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)



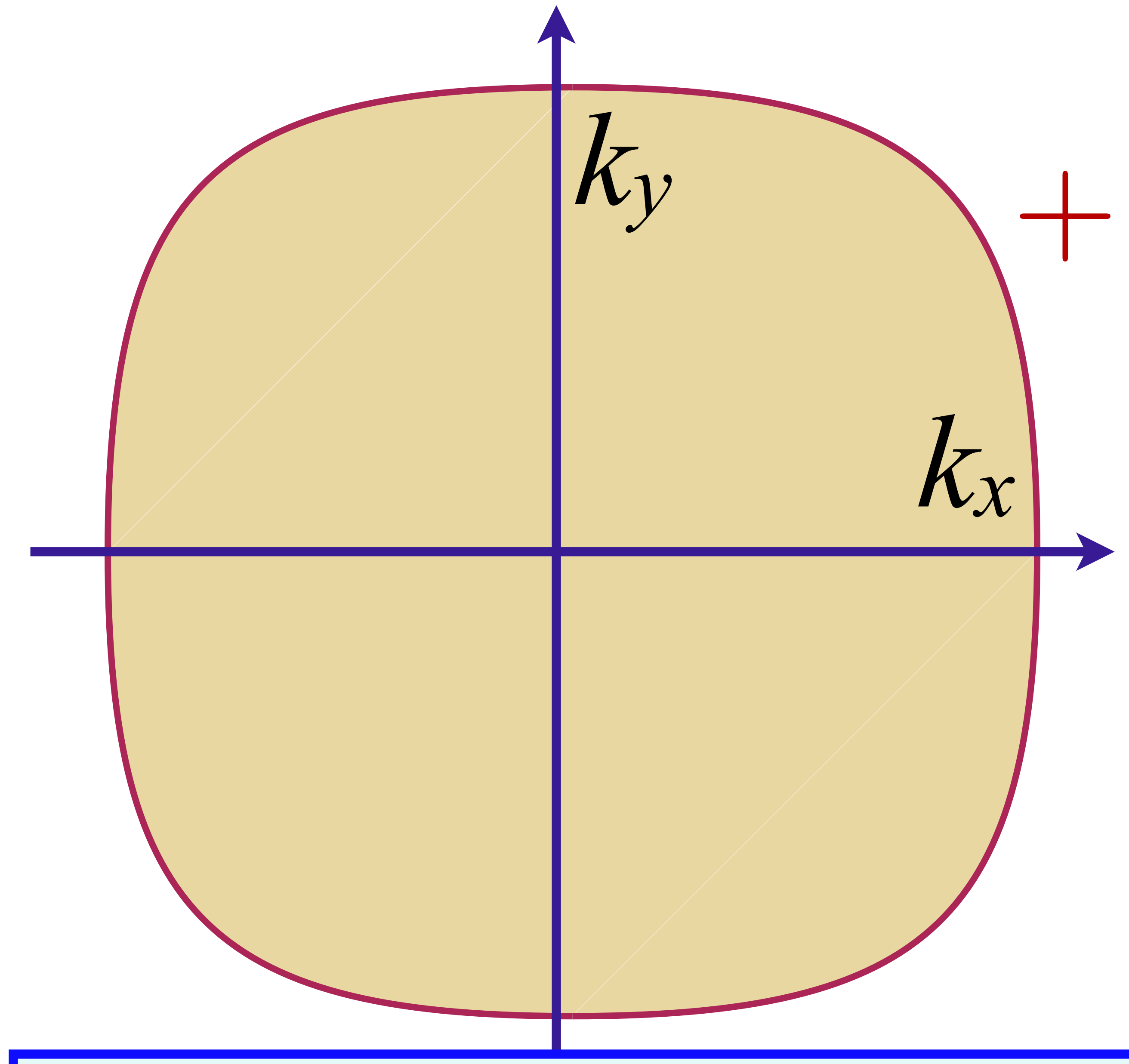
Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).



$$H_{\rm mf} = -\sum_{i,j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} - \sum_{i,j} t_{1,ij} f^\dagger_{1i\sigma} f_{1j\sigma} - \sum_i \Phi \left(c^\dagger_{i\sigma} f_{1i\sigma} + f^\dagger_{1i\sigma} c_{i\sigma} \right)$$


Kondo lattice + critical boson with potential and interaction disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}+f^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon_1(\boldsymbol{k})\right)f_{\boldsymbol{k}\sigma}$$



$$+[s+\delta s(\boldsymbol{r})]\,[\Phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})f_\sigma(\boldsymbol{r})\,\Phi(\boldsymbol{r})+\text{H.c.}$$

$$+K\,[\nabla_{\boldsymbol{r}}\Phi(\boldsymbol{r})]^2+u\,[\Phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c^\dagger_\sigma(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

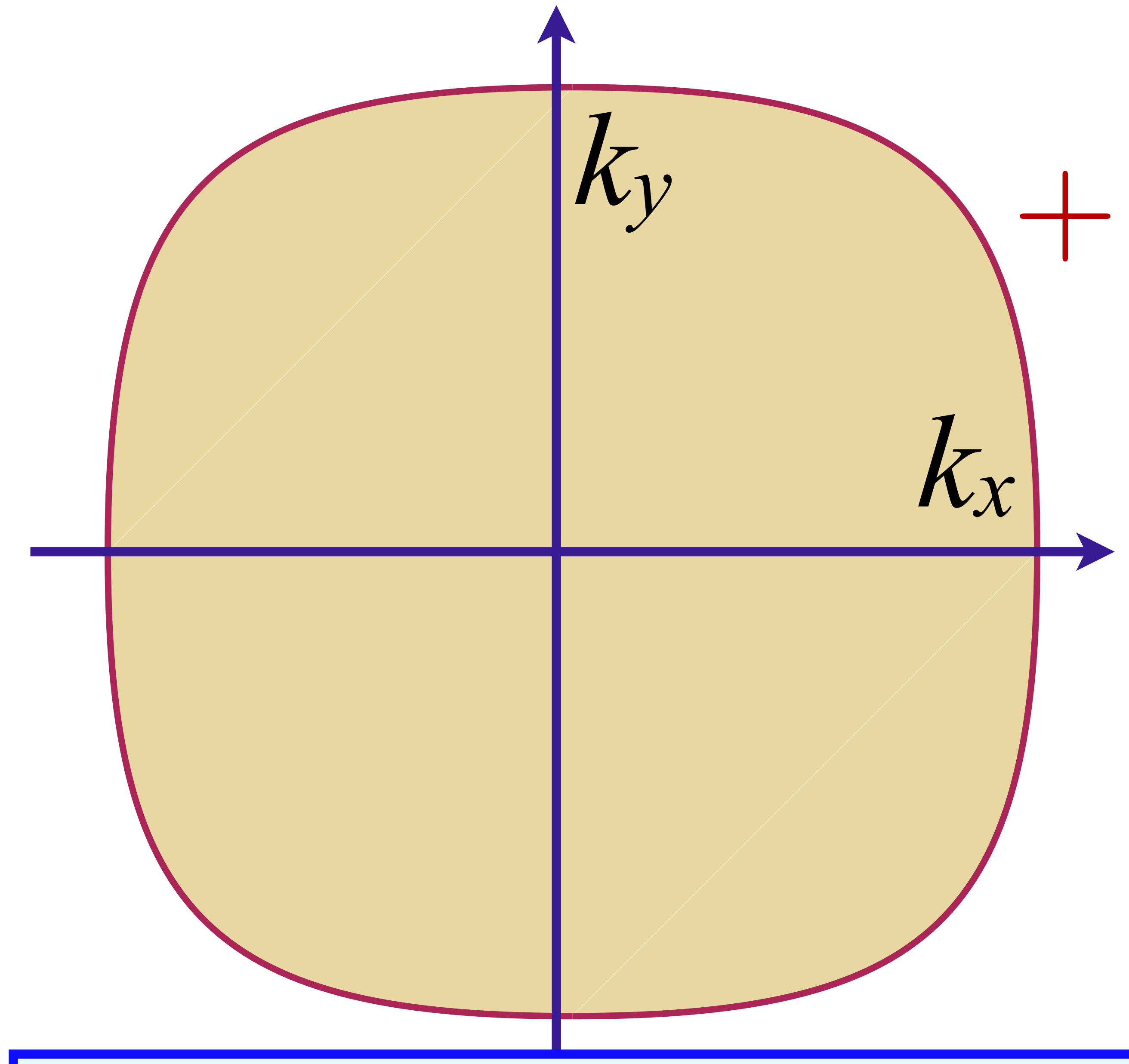
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0,\ \overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})}=0,\ \overline{\delta s(\boldsymbol{r})\delta s(\boldsymbol{r}')}=\delta s^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

$v(\boldsymbol{r})$ leads to elastic scattering of c_σ and ‘Altshuler-Aronov’ corrections; localization of c_σ only at long length scales, not relevant for experiments

Kondo lattice + critical boson with potential and interaction disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}+f^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon_1(\boldsymbol{k})\right)f_{\boldsymbol{k}\sigma}$$



$$+[s+\delta s(\boldsymbol{r})]\,[\Phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})f_\sigma(\boldsymbol{r})\,\Phi(\boldsymbol{r})+\text{H.c.}$$

Rescale Φ

$$+K\,[\nabla_{\boldsymbol{r}}\Phi(\boldsymbol{r})]^2+u\,[\Phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c^\dagger_\sigma(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

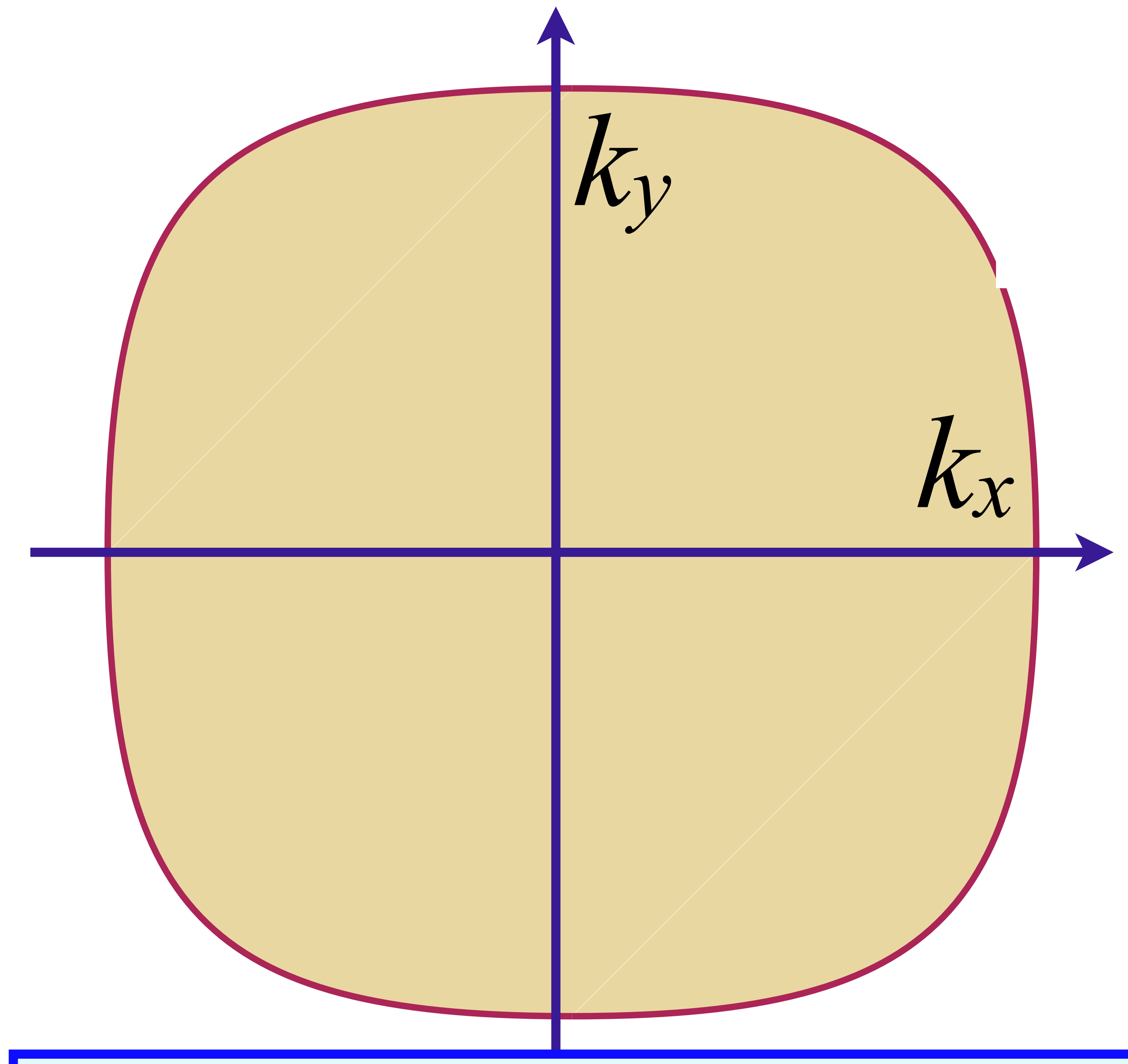
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})}=0$, $\overline{\delta s(\boldsymbol{r})\delta s(\boldsymbol{r}')}=\delta s^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

$v(\boldsymbol{r})$ leads to elastic scattering of c_σ and ‘Altshuler-Aronov’ corrections; localization of c_σ only at long length scales, not relevant for experiments

Kondo lattice + critical boson with potential and interaction disorder

$$c_{\boldsymbol{k}\sigma}^{\dagger}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}+f_{\boldsymbol{k}\sigma}^{\dagger}\left(\frac{\partial}{\partial\tau}+\varepsilon_1(\boldsymbol{k})\right)f_{\boldsymbol{k}\sigma}$$



$$+s\,[\Phi(\boldsymbol{r})]^2+[g+g'(\boldsymbol{r})]\,c_\sigma^\dagger(\boldsymbol{r})f_\sigma(\boldsymbol{r})\,\Phi(\boldsymbol{r})+\text{H.c.}$$

$$+K\,[\nabla_{\boldsymbol{r}}\Phi(\boldsymbol{r})]^2+u\,[\Phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c_\sigma^\dagger(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

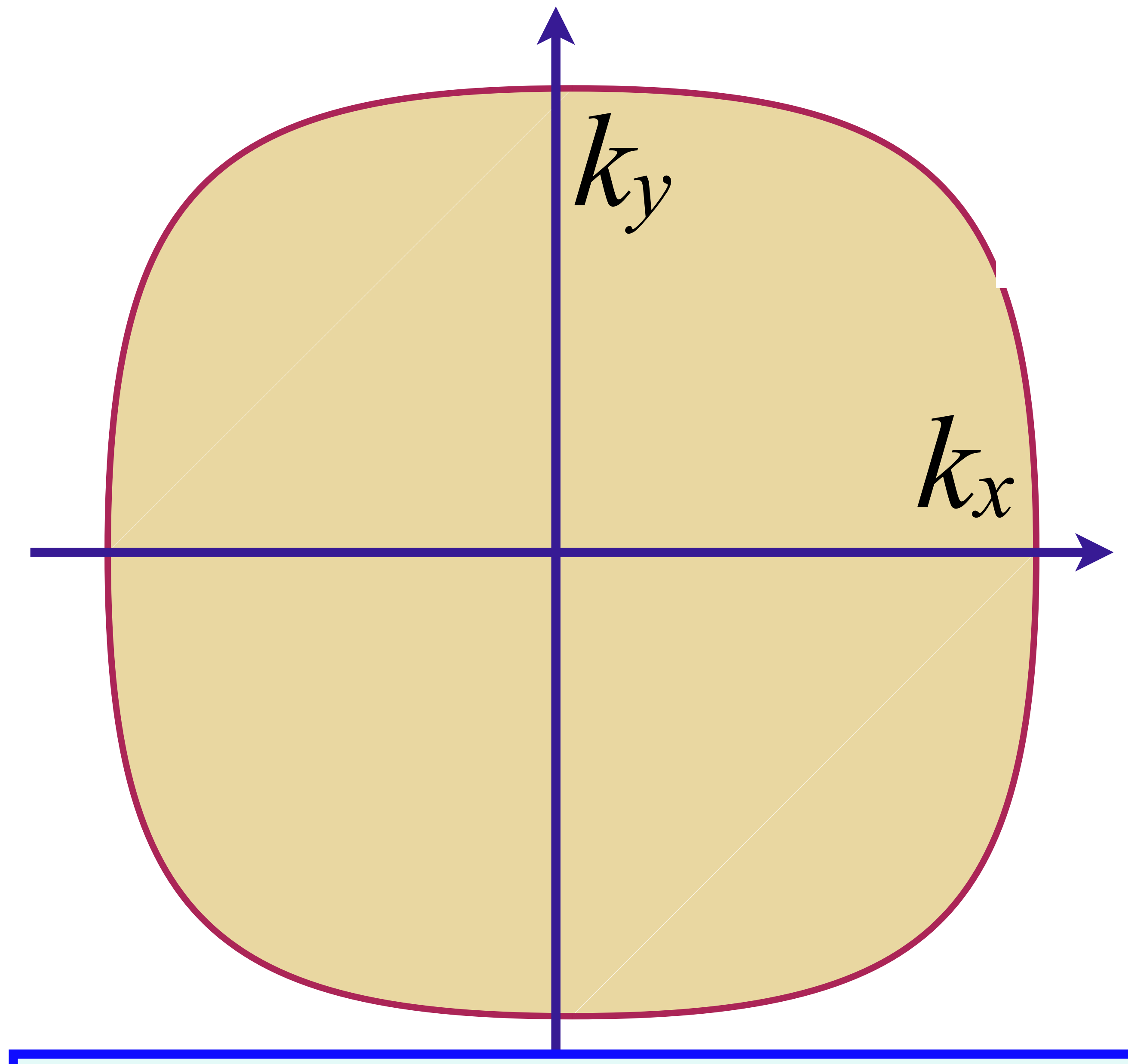
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})}=0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')}=g'^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

$g'(\boldsymbol{r})$ creates inhomogeneity in the position of QCP (Harris disorder): the two-dimensional Yukawa-Sachdev-Ye-Kitaev model.

Kondo lattice + critical boson with potential and interaction disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}+f^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon_1(\boldsymbol{k})\right)f_{\boldsymbol{k}\sigma}$$



$$+s\,[\Phi(\boldsymbol{r})]^2+[g+g'(\boldsymbol{r})]\,c^\dagger_\sigma(\boldsymbol{r})f_\sigma(\boldsymbol{r})\,\Phi(\boldsymbol{r})+\text{H.c.}$$

$$+K\,[\nabla_{\boldsymbol{r}}\Phi(\boldsymbol{r})]^2+u\,[\Phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c^\dagger_\sigma(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

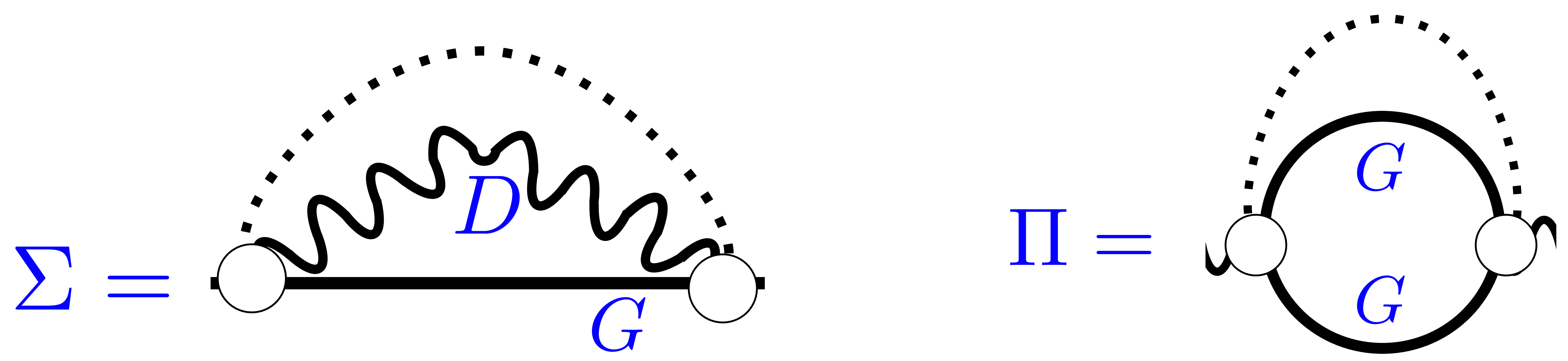
Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

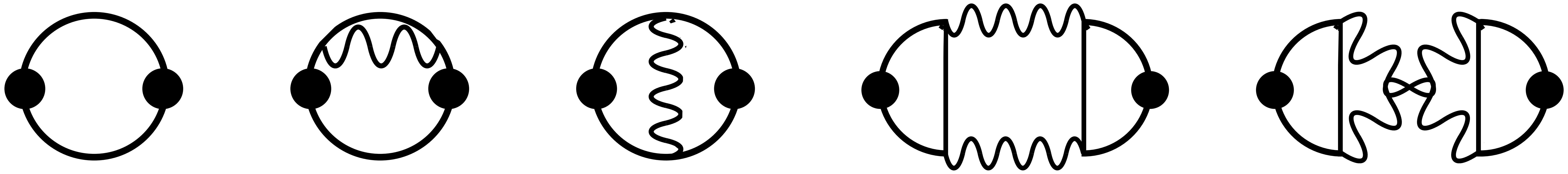
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})}=0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')}=g'^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Analyze 2d-YSYK model in a self-averaging manner as in the SYK model.
Should be applicable as long as eigenmodes of $\Phi(\boldsymbol{r})$ are extended.

2d-YSYK model: Fermi surface + critical boson with interaction disorder





Residual resistivity is determined by v^2

Linear-in-T resistivity determined by g'^2

Transport insensitive to g

Marginal Fermi liquid self energy $\Sigma \sim \omega \ln \omega$

$T \ln(1/T)$ specific heat

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

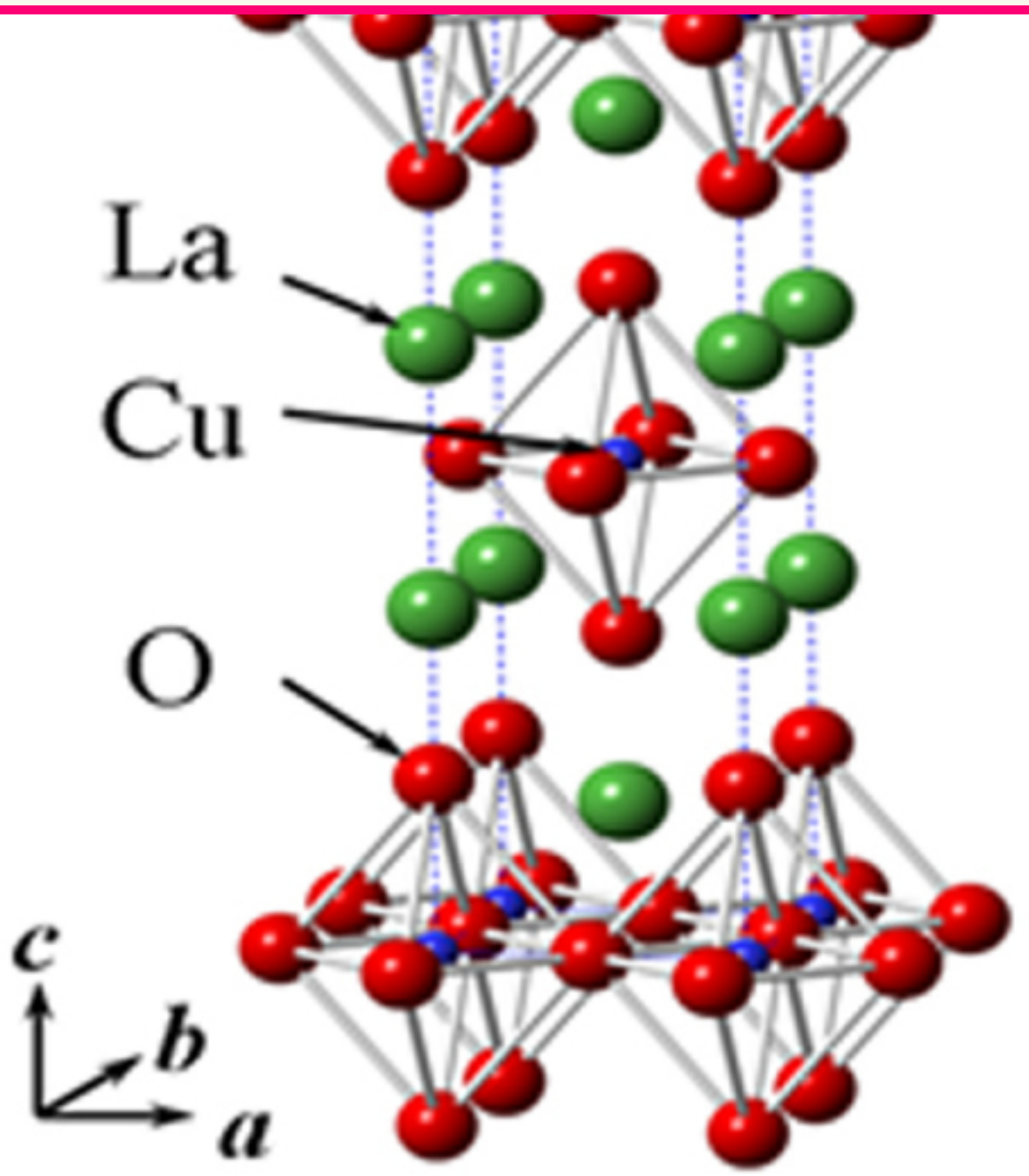
Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

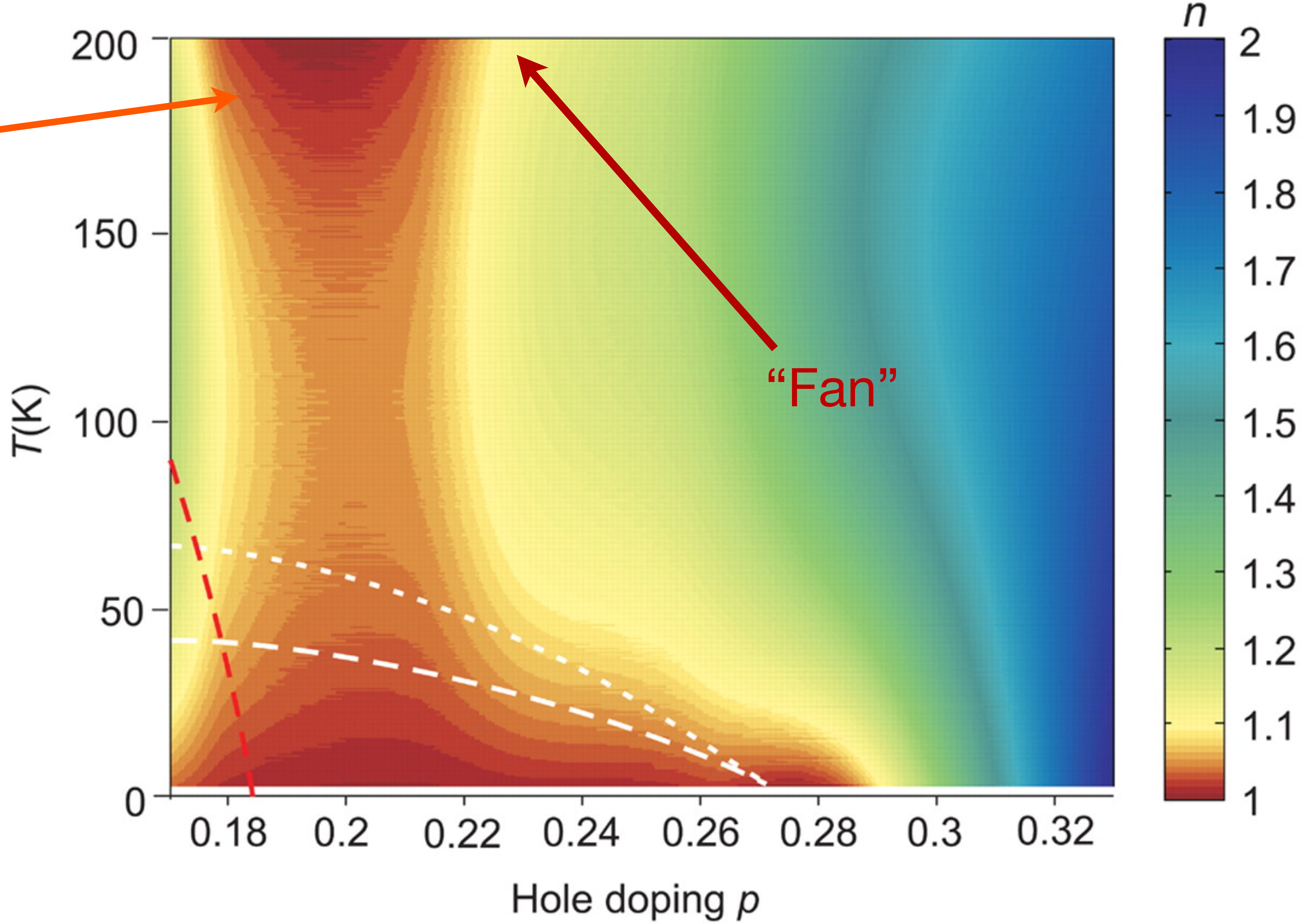
R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4]* H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

SCIENCE VOL 323 **603** 2009

2dYSYK theory of FL-FL* QPT provides a theory of the "fan"

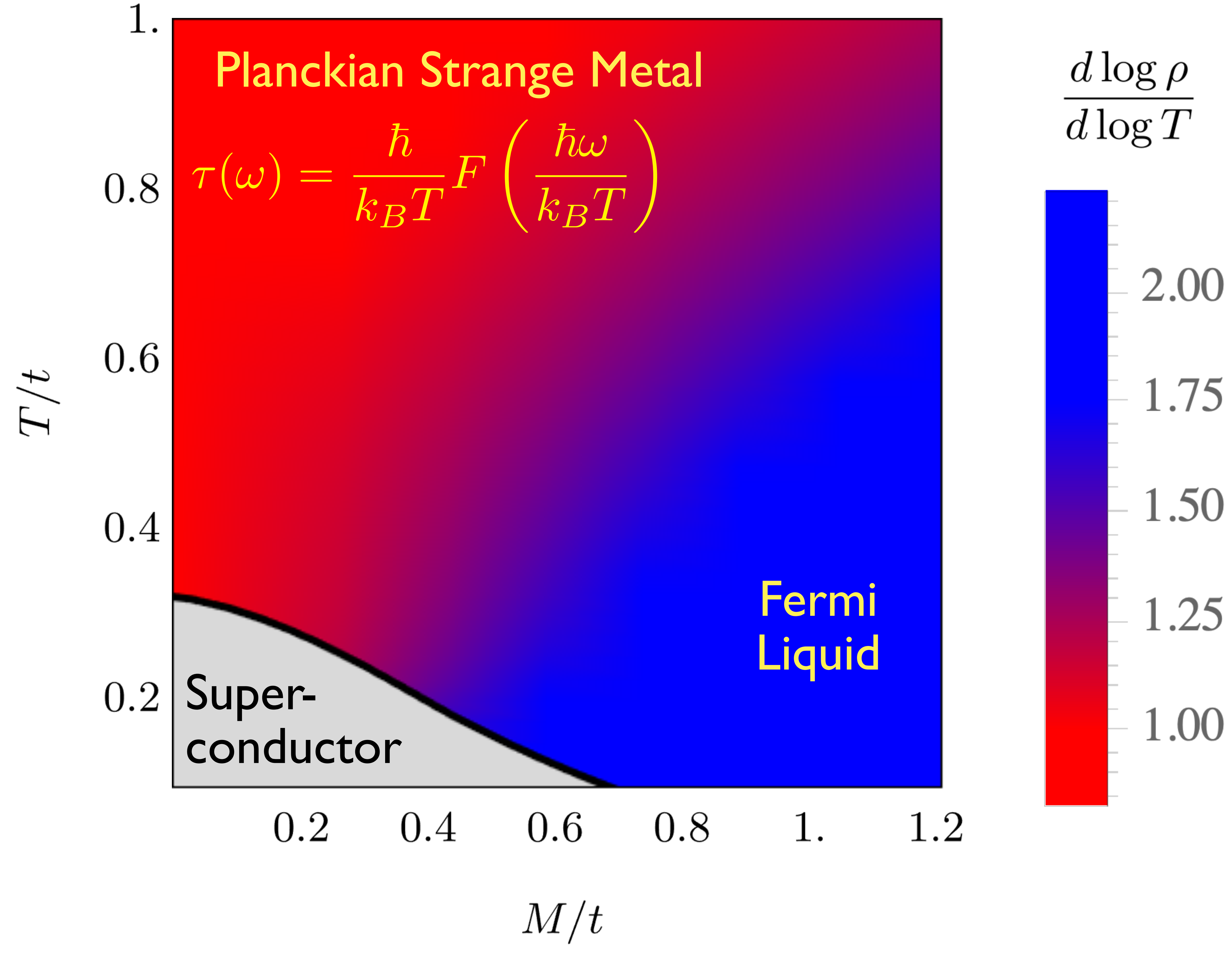
Extended fermions and bosons

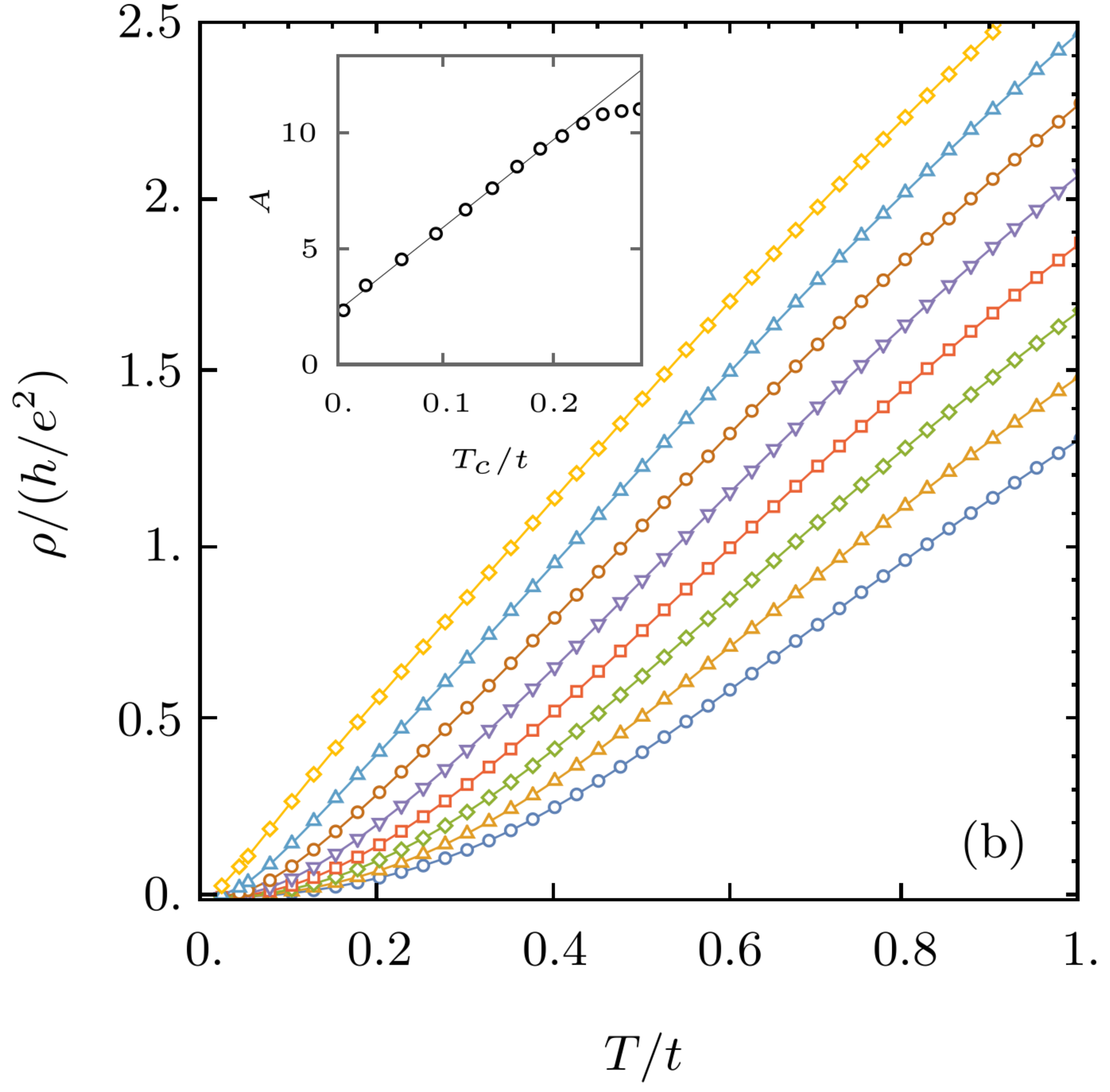




Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)



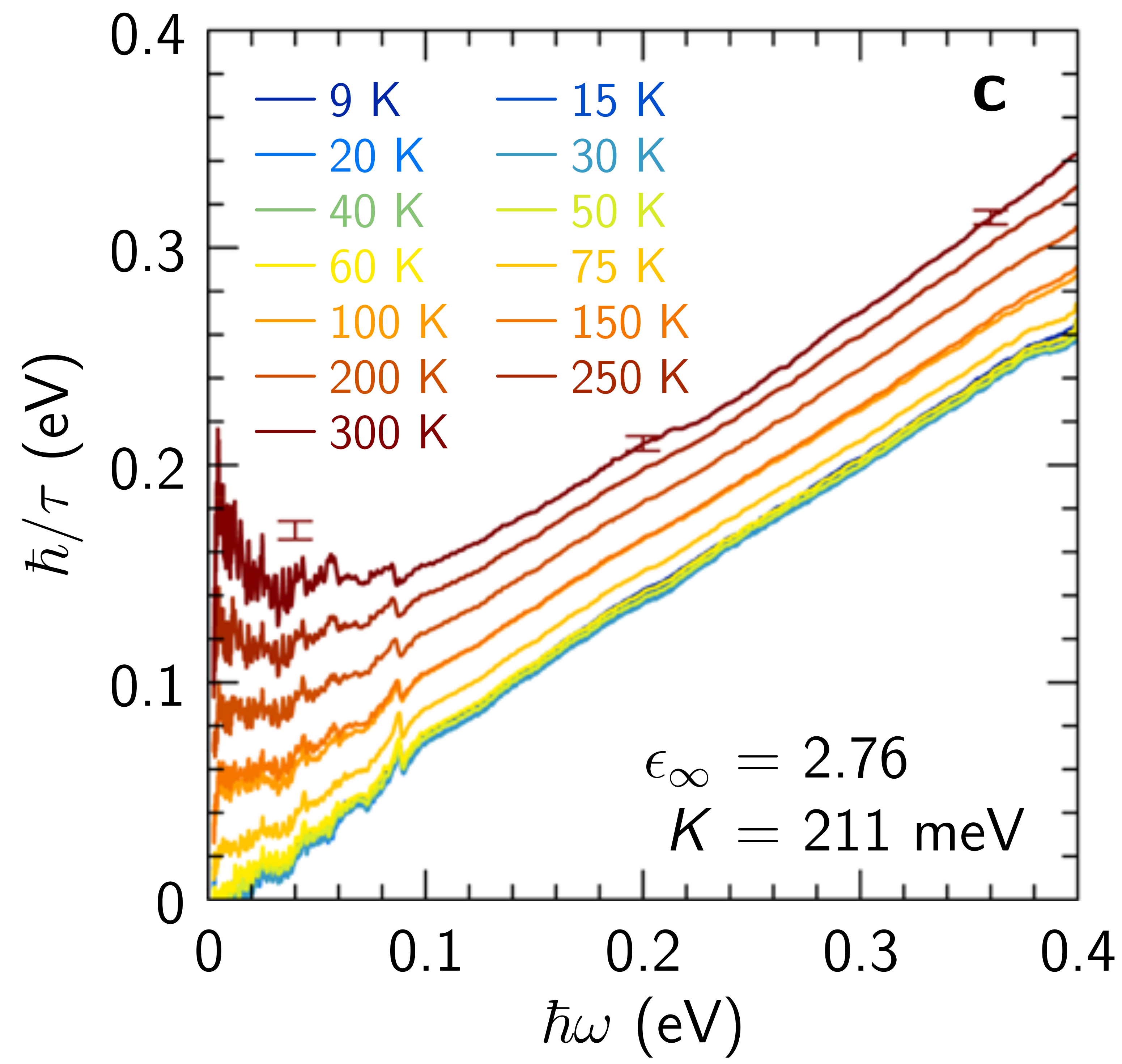


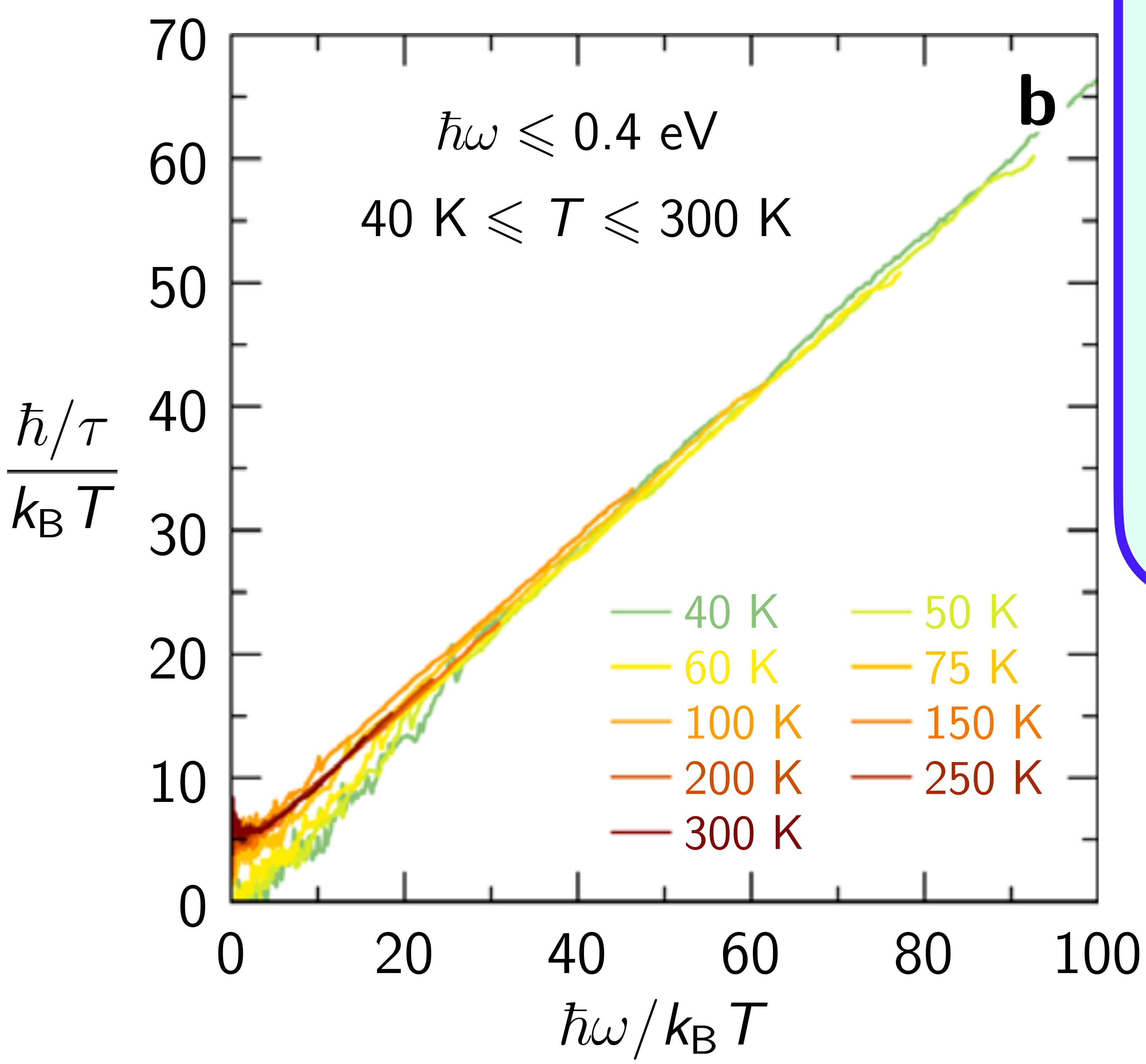
Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$





Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy

$S(T \to 0) \sim T\ln(1/T)$.

$La_{2-x}Sr_xCuO_4$

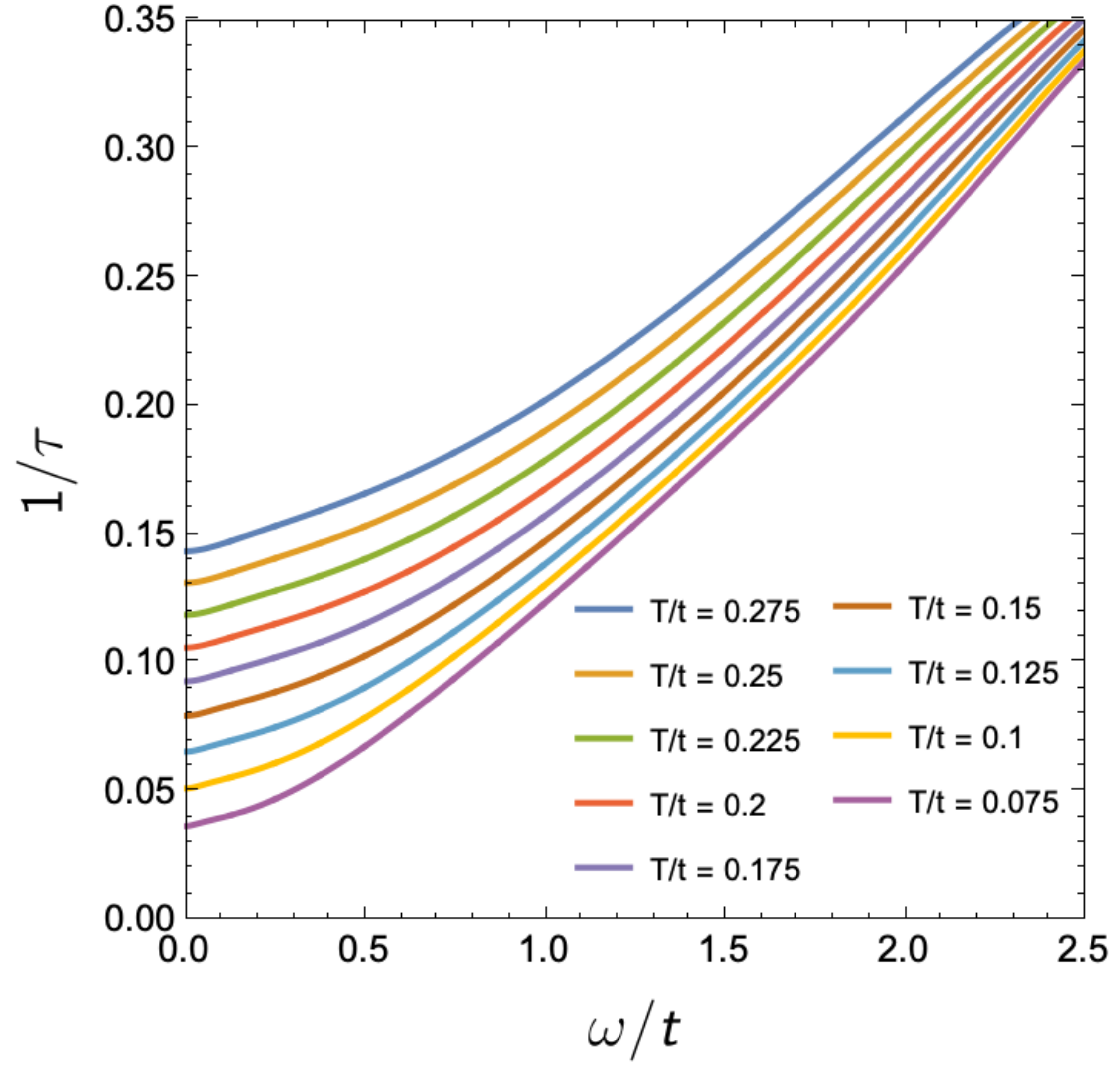
$p = 0.24$

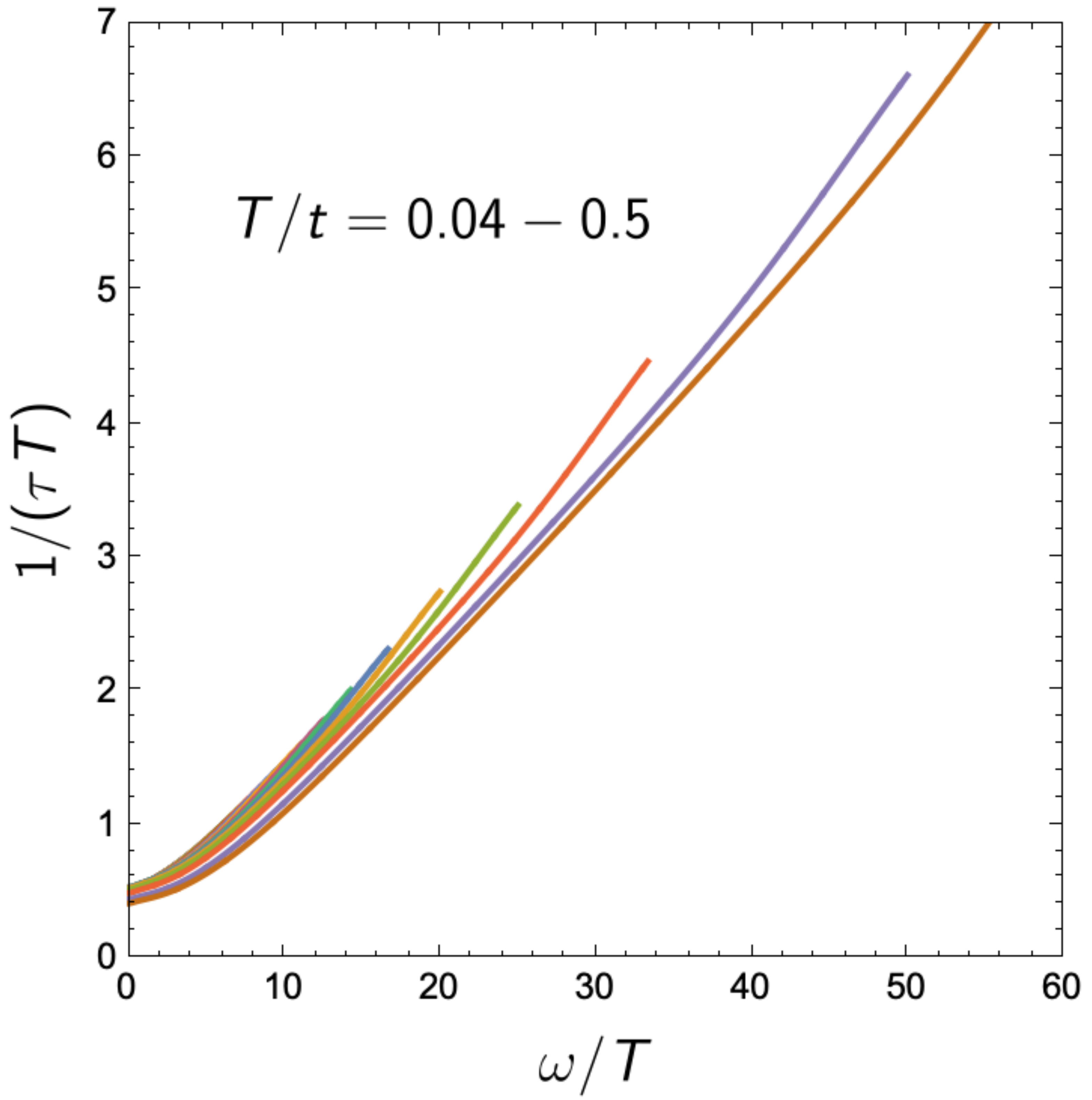
$T_c = 19$ K

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$





Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy $S(T \to 0) \sim T\ln(1/T)$ in 2d-YSYK model (unlike zero temperature entropy in SYK model).

Seebeck Coefficient in a Cuprate Superconductor: Particle-Hole Asymmetry in the Strange Metal Phase and Fermi Surface Transformation in the Pseudogap Phase

A. Gourgout,[1,*] G. Grissonnanche,[1,2,3,*,†] F. Laliberté,[1] A. Ataei,[1] L. Chen,[1] S. Verret,[1] J.-S. Zhou,[4] J. Mravlje,[5] A. Georges,[6,7,8,9] N. Doiron-Leyraud,[1] and Louis Taillefer[1,10,‡]

PHYSICAL REVIEW X **12,** 011037 (2022)

Skewed non-Fermi liquids and the Seebeck effect

Antoine Georges[1,2,3,4] and Jernej Mravlje[5]

PRR **3**, 043132 (2021)

The particle-hole asymmetry in the Φ propagator
(for the FL*-FL transition)
$\sim 1/(-i\omega + q^2 + \gamma|\omega| + \alpha)$
leads to a **skewed marginal Fermi liquid**.

P. Lunts, A.A. Patel, and S.S., arXiv:2412.15330

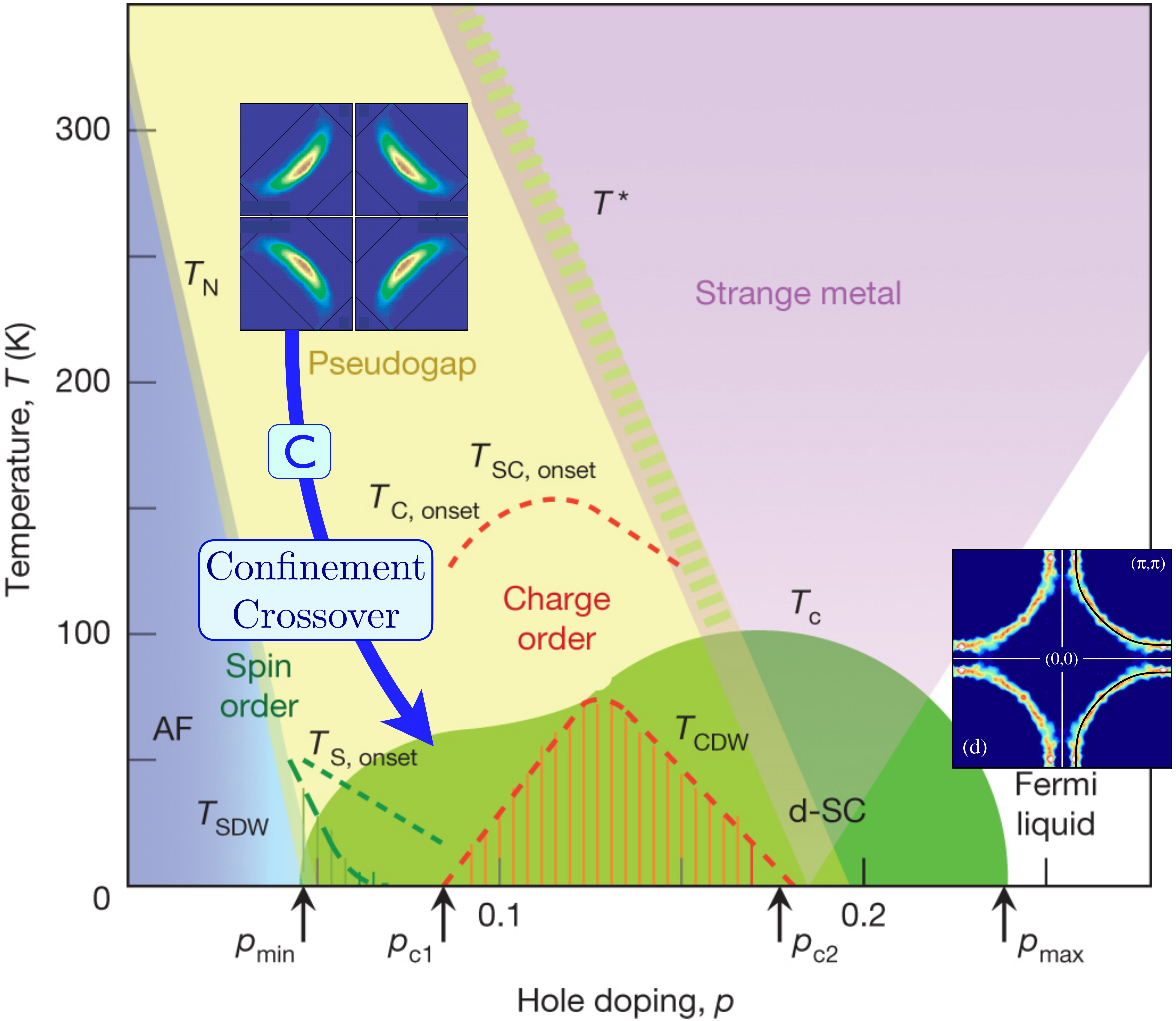
The ϕ propagator (for the SDW-FL transition)
does not have the $-i\omega$ term, and so is *not* skewed.

A. FL-SDW QPT

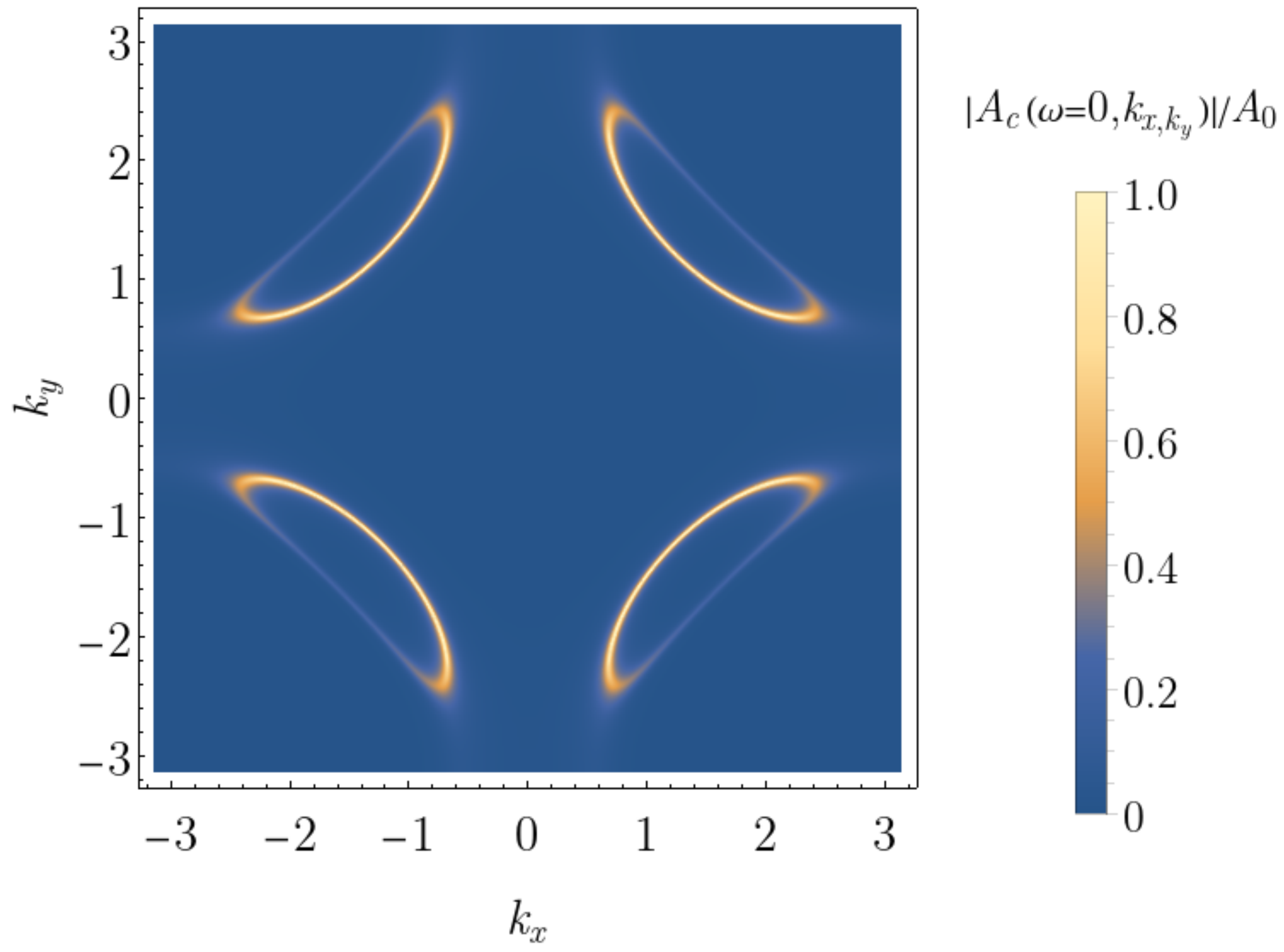
B. FL-FL* QPT

C. Confinement crossover

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

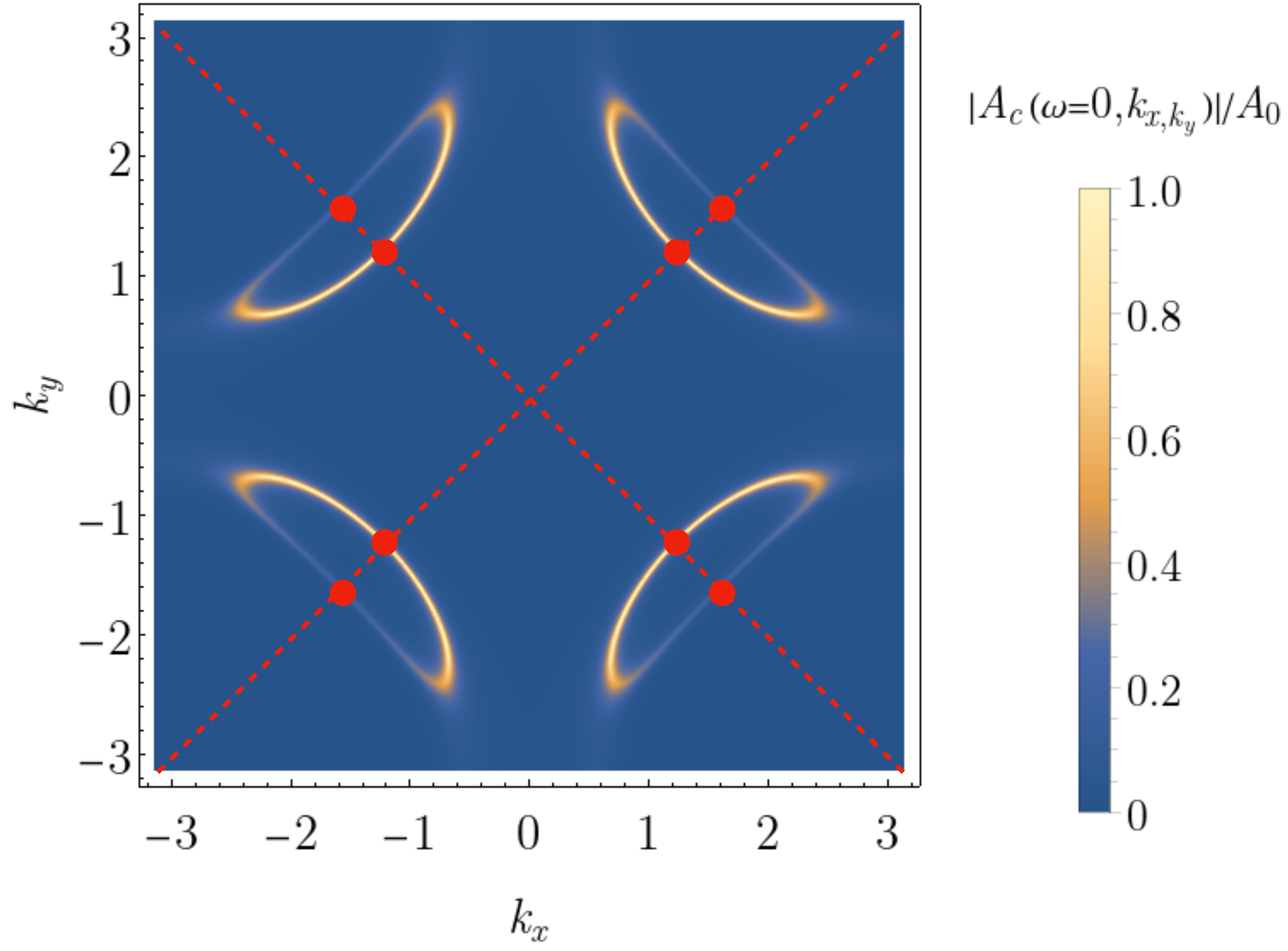


FL*



E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

FL*→dSC*

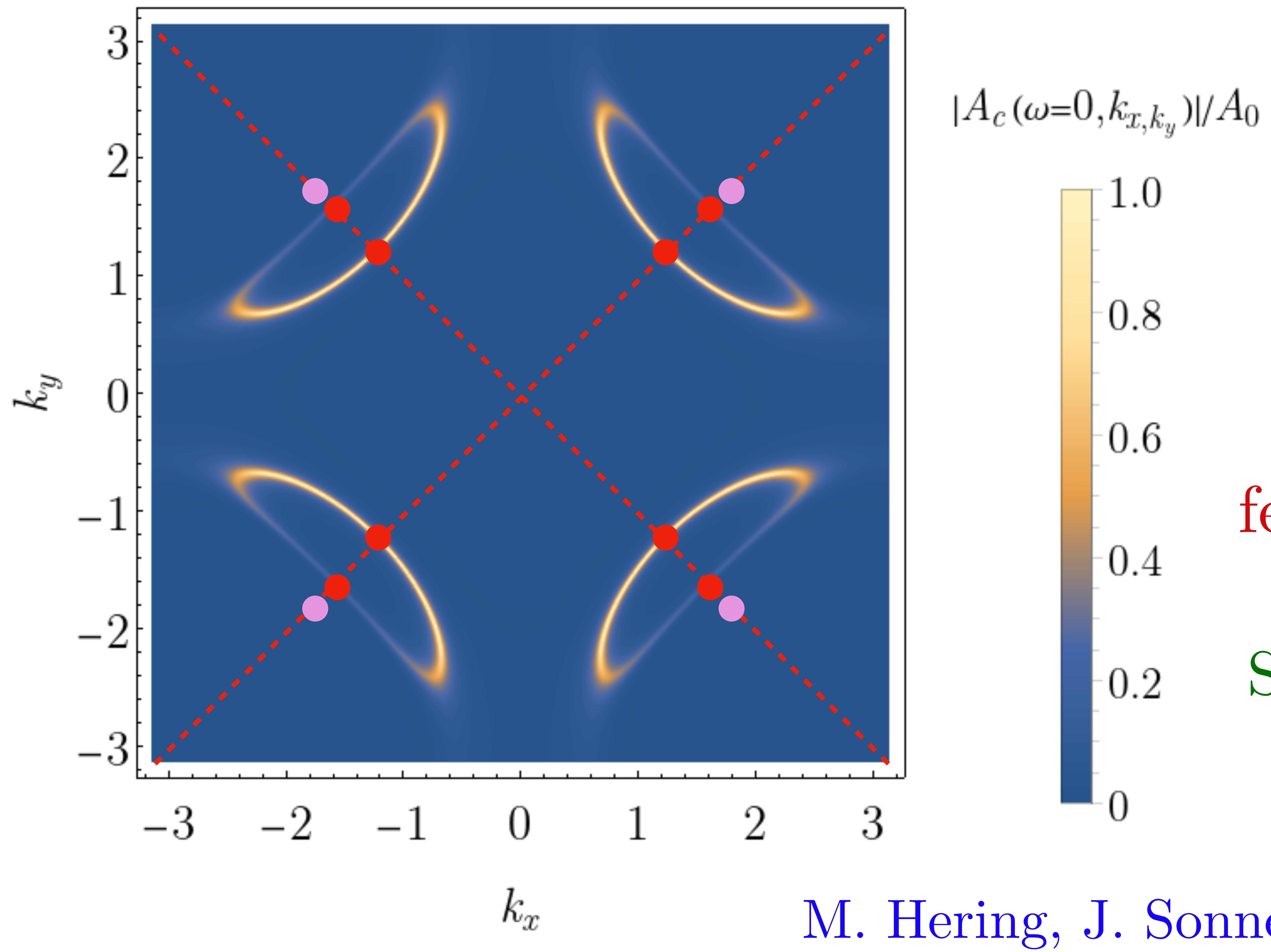


$$E_{\boldsymbol{k}} = \left(\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2\right)^{1/2}$$

$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$$

Adding d-wave pairing
to the hole pockets
leads to 8 nodal points???

FL*→dSC*



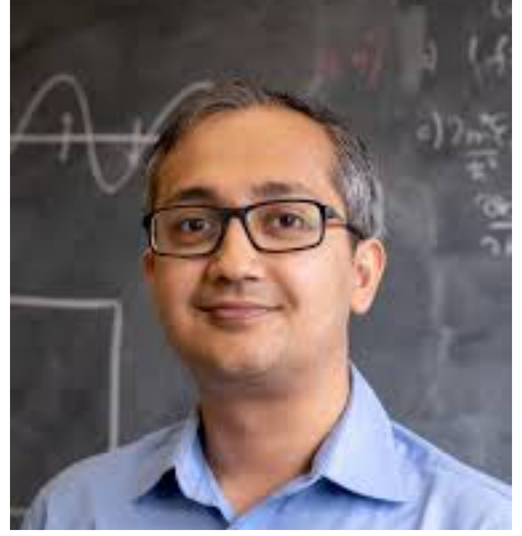
8 nodal points of Bogoliubov quasiparticles from the Fermi pockets
and
4 nodal points of fermionic Dirac spinons from the π-flux spin liquid

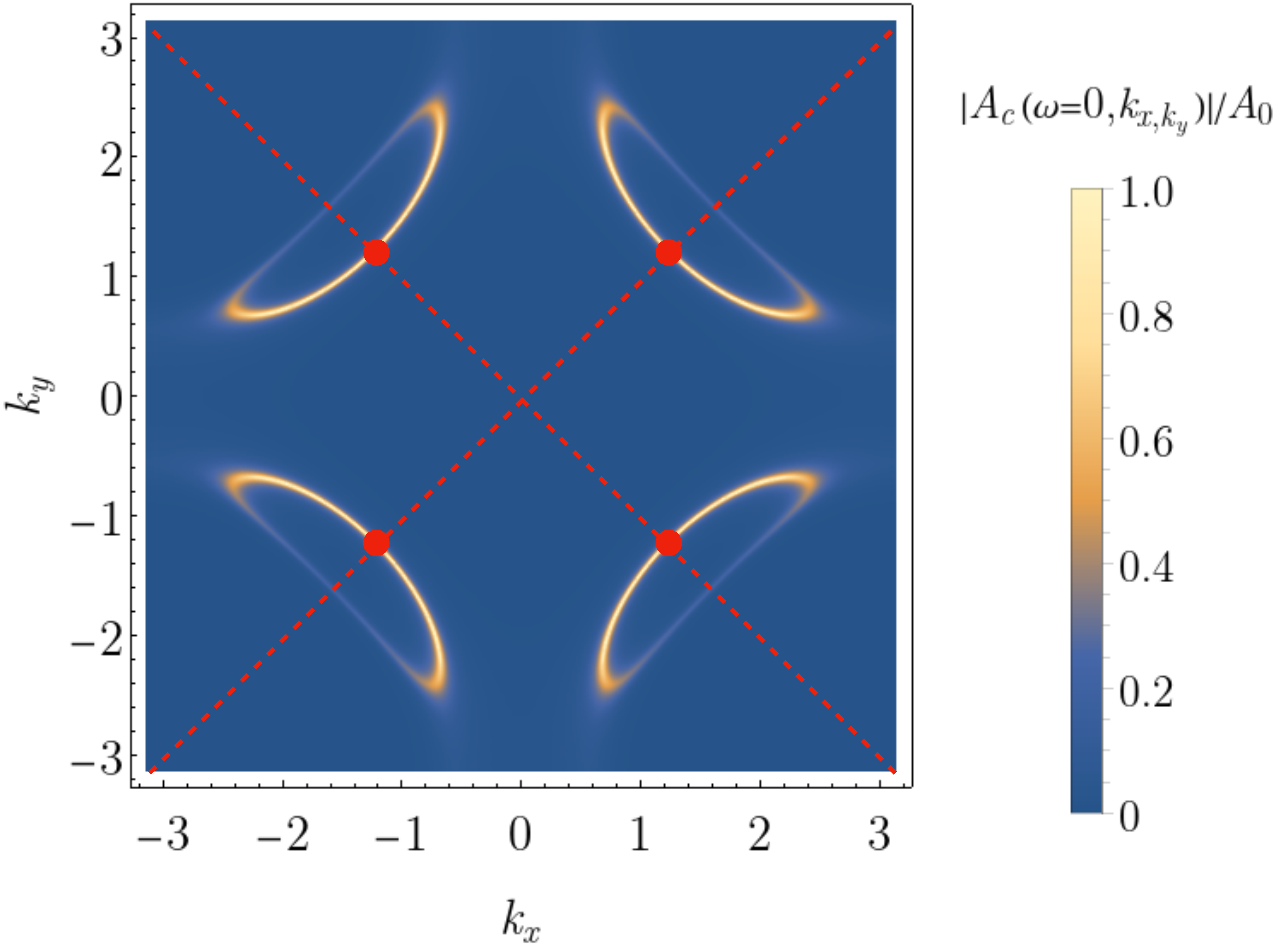
Such a spin liquid is required to be present in the background for FL*:

M. Hering, J. Sonnenschein, Y. Iqbal and J. Reuther, PRB **99**, 100405 (2019)

FL*→dSC

Shubhayu Chatterjee and S. Sachdev,
PRB **94**, 205117 (2016)
Maine Christos and S.Sachdev,
npj Quantum Materials **9**, 4 (2024)



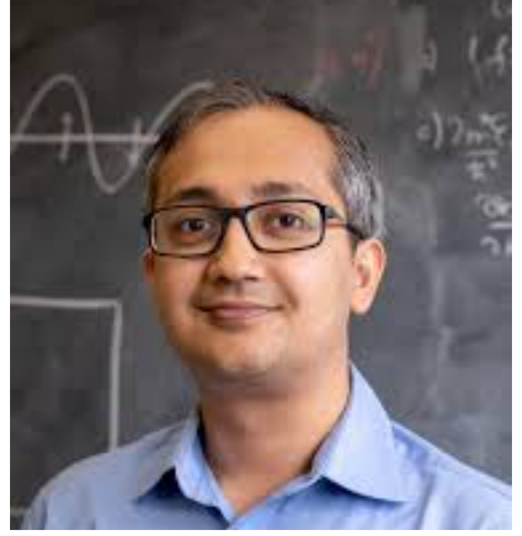
d-wave superconductor obtained by condensing charge-e, SU(2) fundamental boson B.

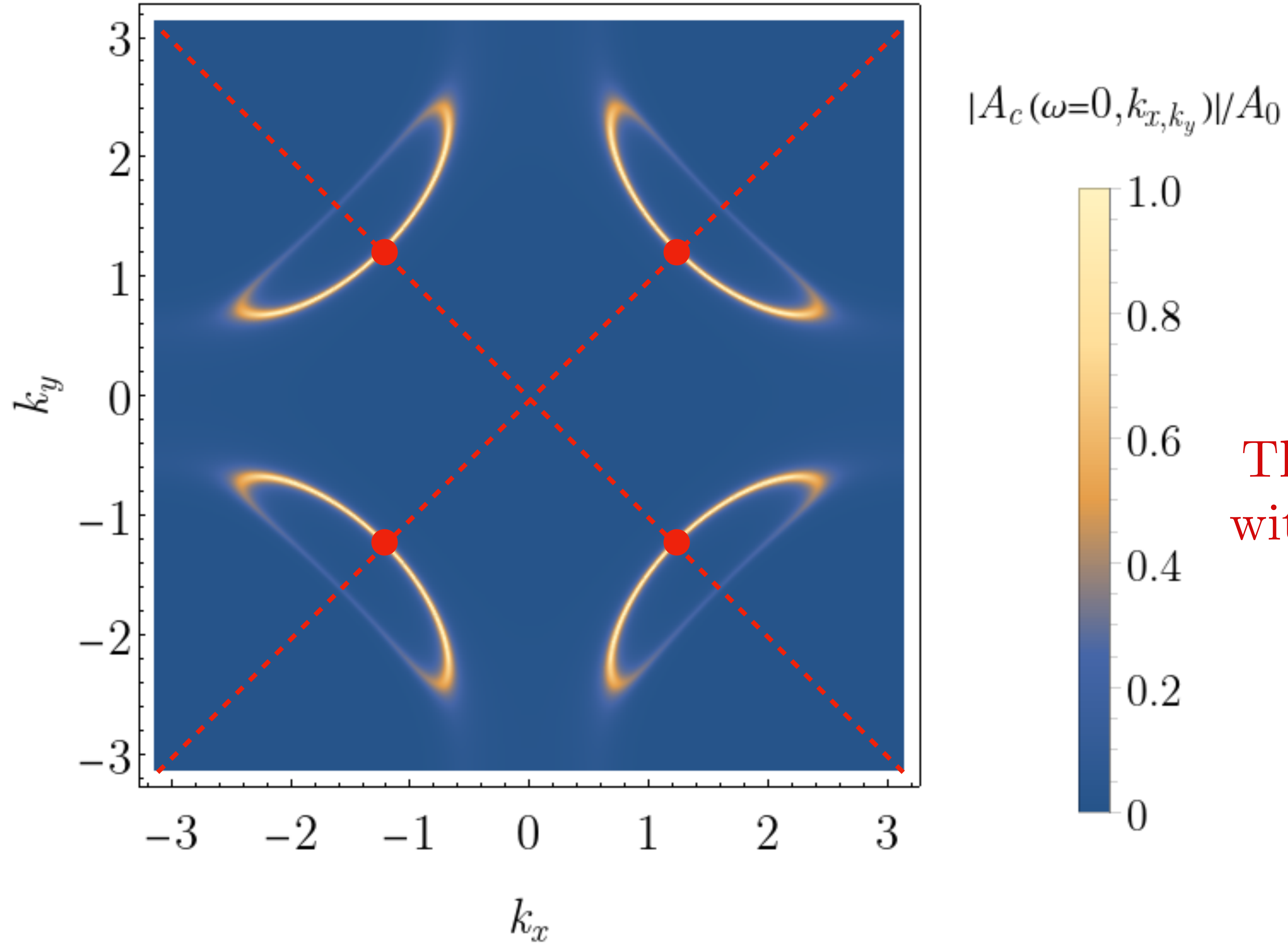
The B Higgs condensate allows spinons and Bogoliubov quasiparticles to hybridize.

8 nodal points annihilate each other, leaving 4 nodal points with anisotropic velocities, just as in a BCS d-wave state.

FL*→dSC

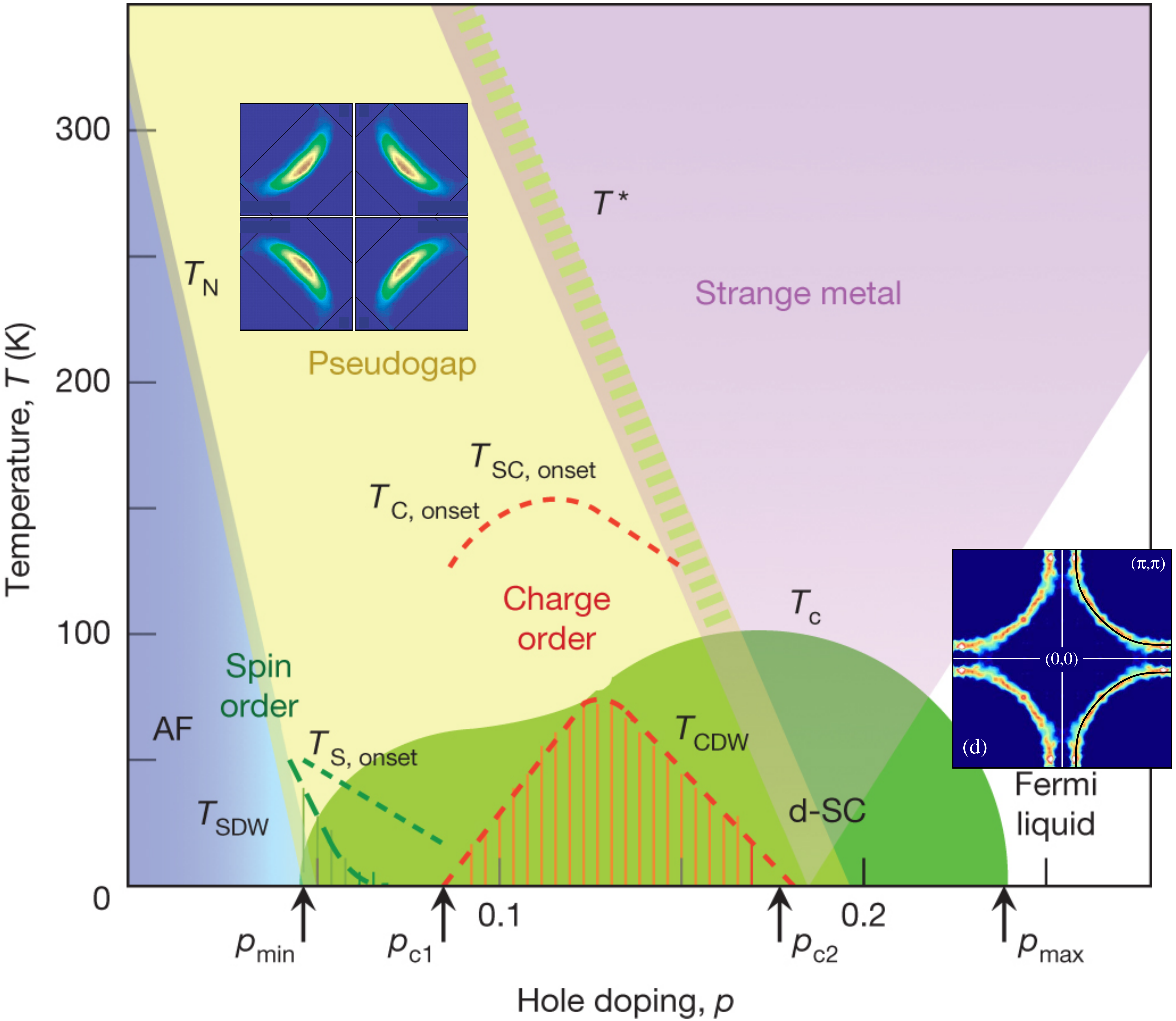
Shubhayu Chatterjee and S. Sachdev,
PRB **94**, 205117 (2016)
Maine Christos and S.Sachdev,
npj Quantum Materials **9**, 4 (2024)



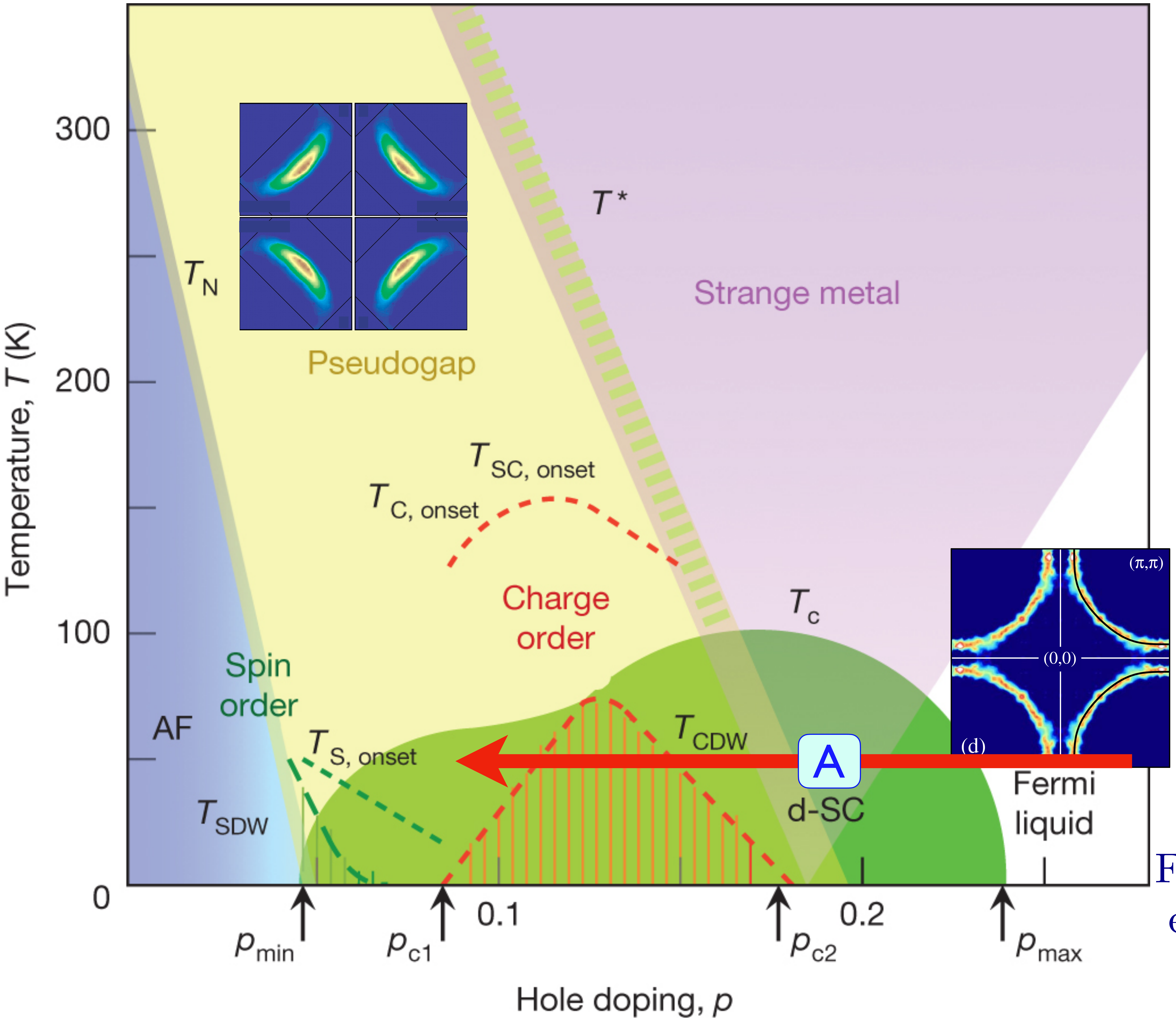


The spinons do *not* become the Bogoliubov quasiparticles, they *annihilate* the unwanted Bogoliubov quasiparticles. This leads to a d-wave superconductor with 4 nodal Bogoliubov quasiparticles, with $v_F \gg v_\Delta$, consistent with observations.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Fermi-volume-changing QPT
with symmetry breaking
and with spatial disorder.

FL-SDW QPT with Harris disorder with extended fermions and localized bosons provides a theory of the "foot"

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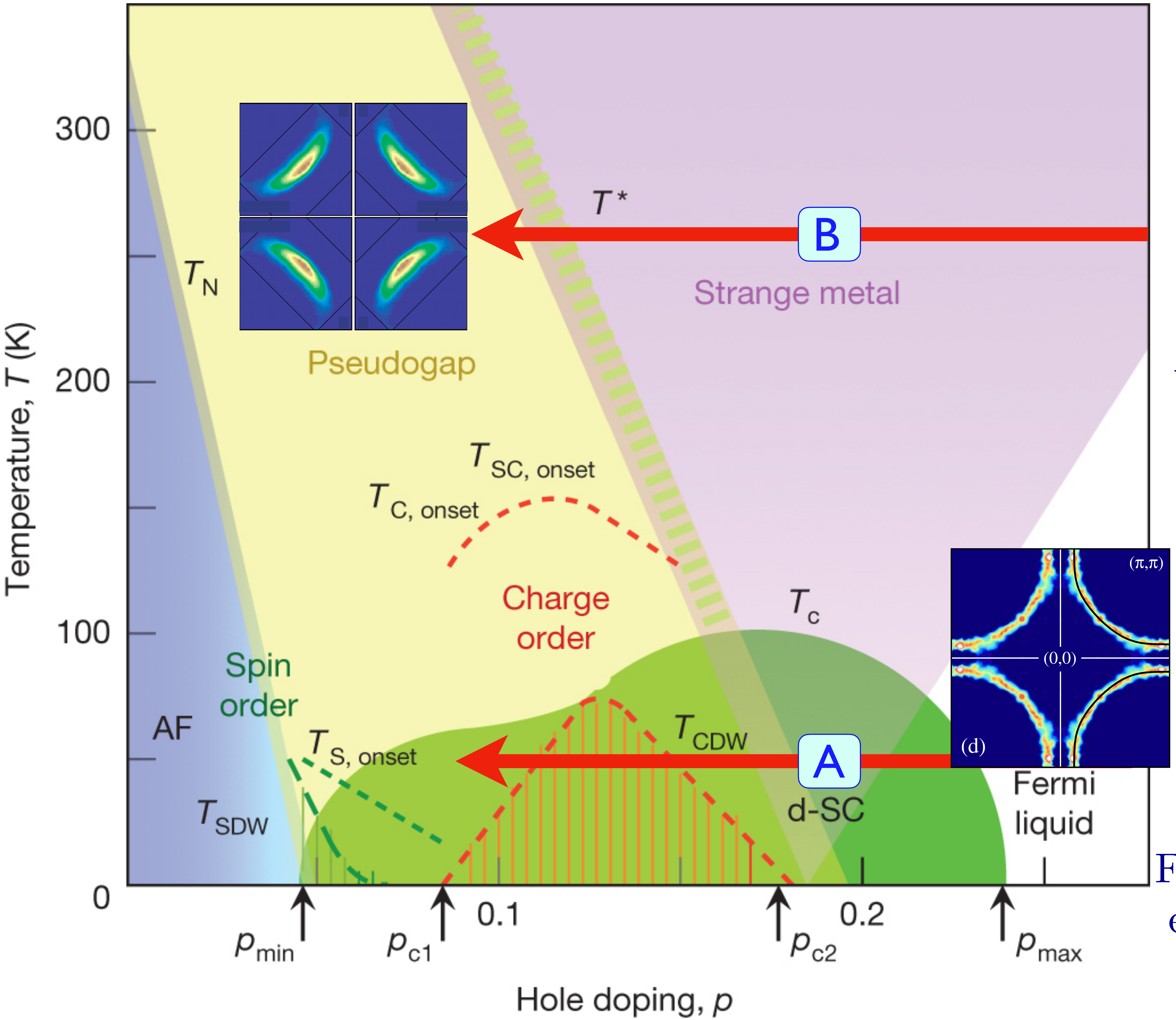

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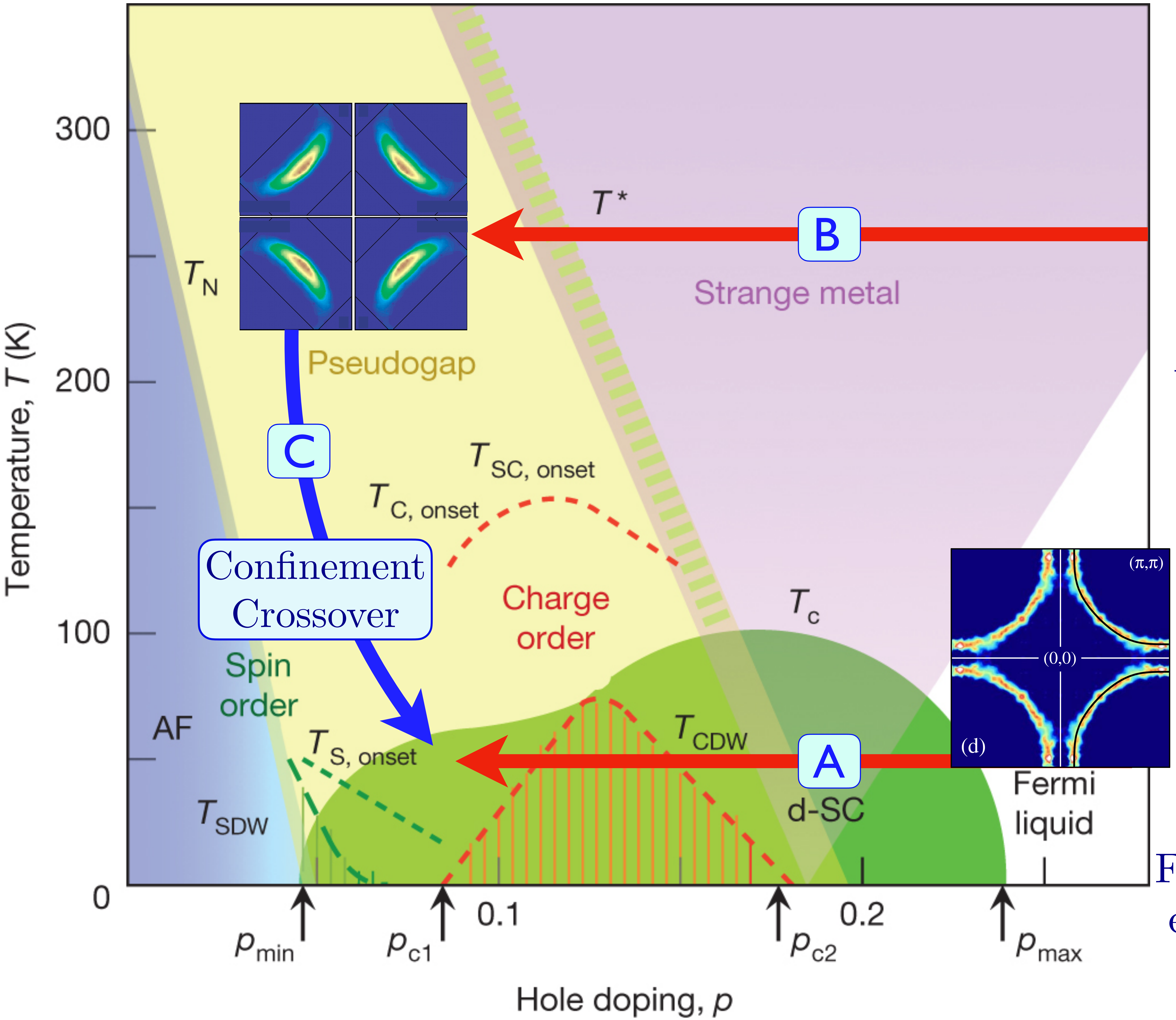

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