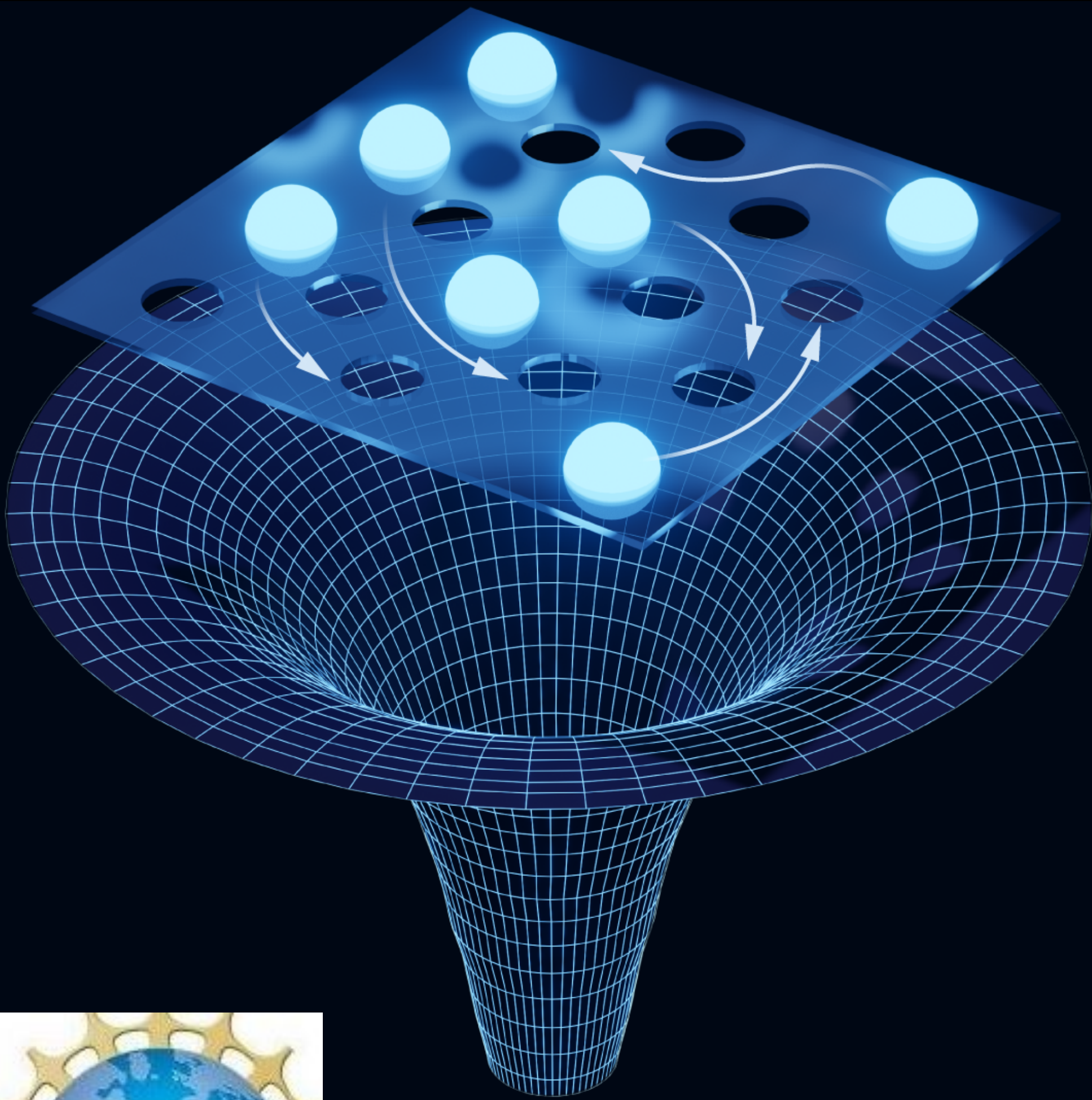


# From the Sachdev-Ye-Kitaev model to theories of strange metals and black holes



Spotlight Lecture, Shaastra 2025  
Indian Institute of Technology, Madras  
January 6, 2025

Subir Sachdev



Talk online: [sachdev.physics.harvard.edu](https://sachdev.physics.harvard.edu)



Boltzmann-Landau theory of  
ordinary metals: Cu, Ag

# Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states  $D(E) = \exp(S(E)/k_B)$

$$\frac{1}{T} = \frac{dS}{dE}$$



Ludwig Boltzmann

20 February 1844 - September 5, 1906  
Vienna, Austria



No perpetual  
motion machines!

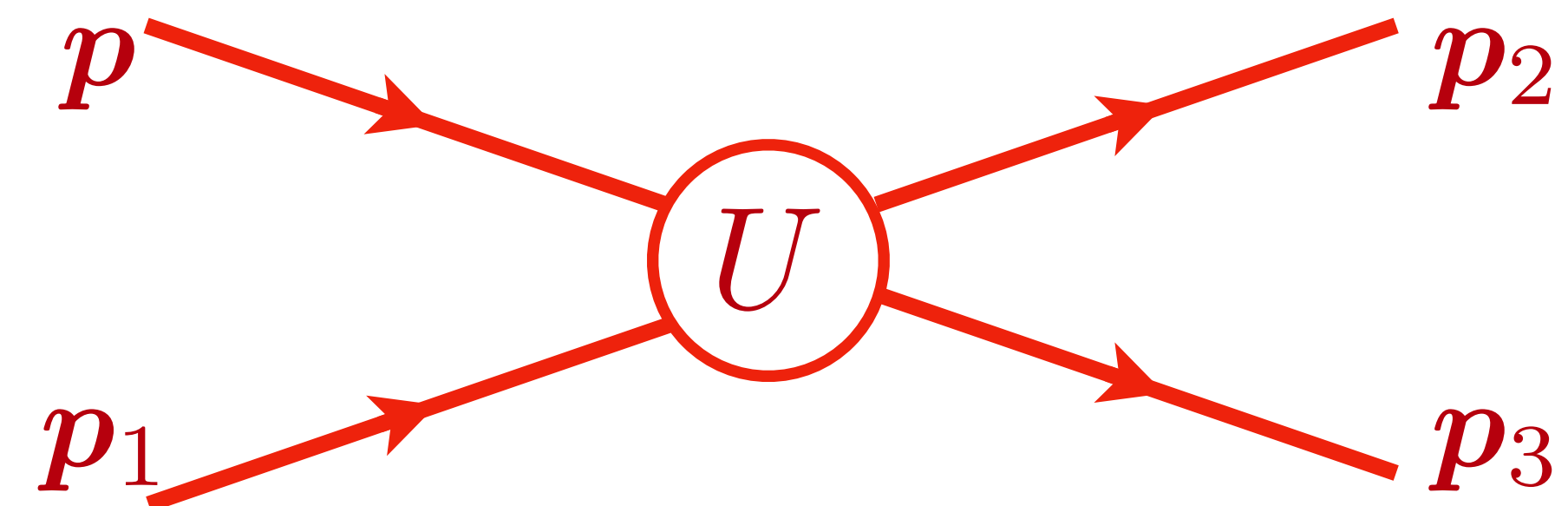
# Boltzmann equation (1872)

## Dilute classical gas

Molecular chaos:  
successive collisions are rare  
and statistically independent



Ludwig Boltzmann  
20 February 1844 - September 5, 1906  
Vienna, Austria

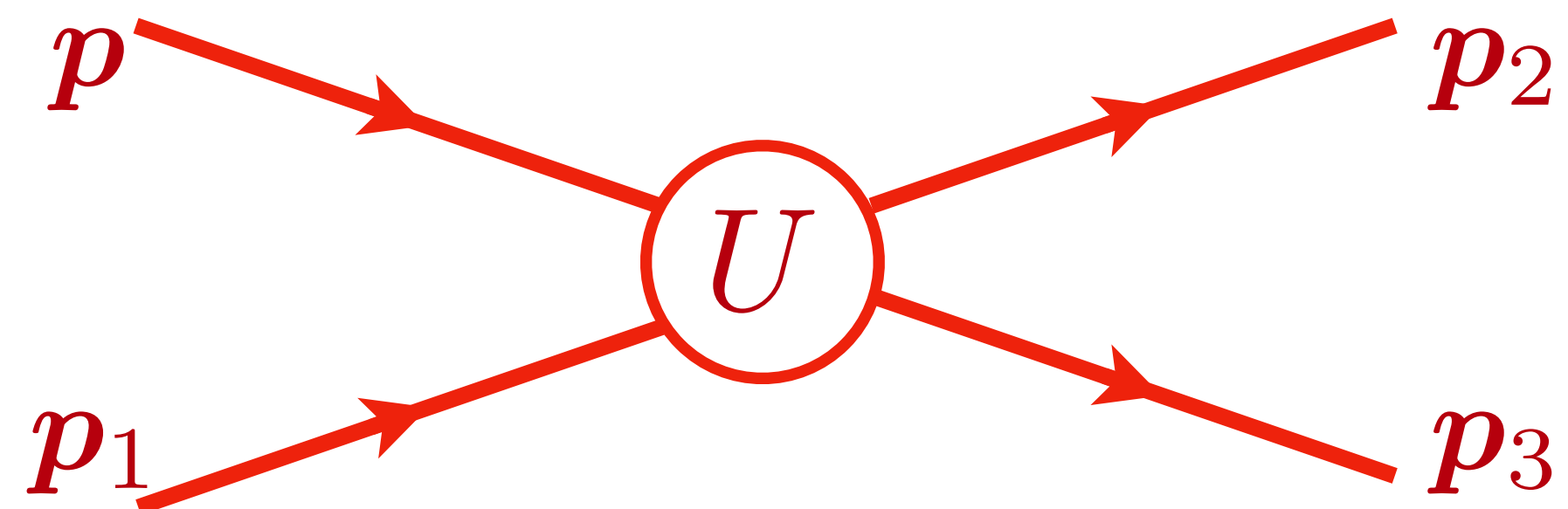


# Quantum Boltzmann equation (Landau)

## Dense gas of electrons

Collisions are also rare in a dense quantum gas at low temperatures because of the Pauli exclusion principle.

Neglect quantum interference (entanglement) between successive collisions

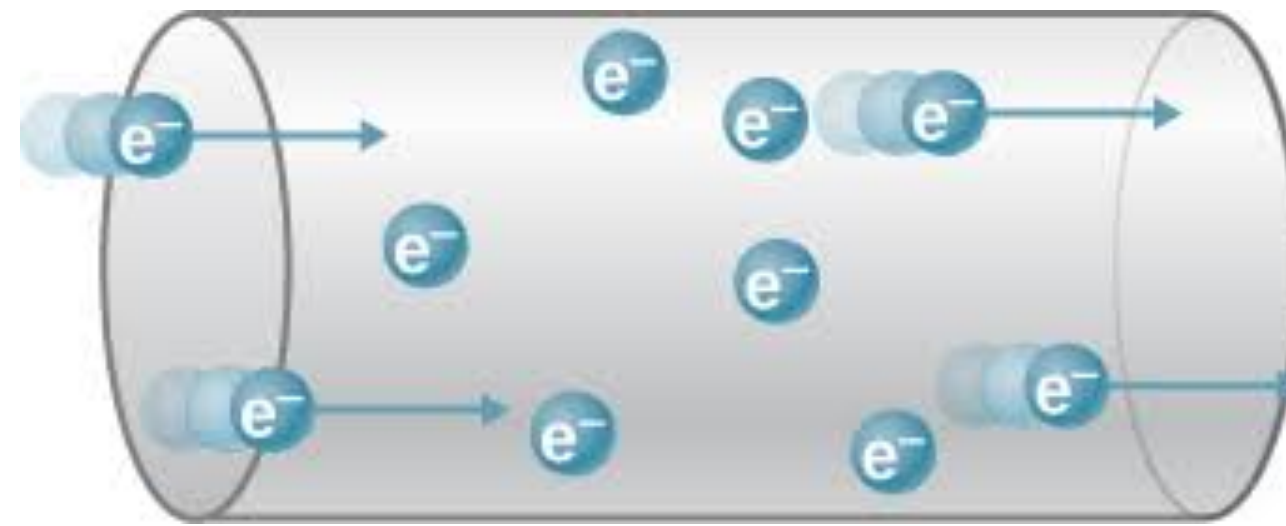


Ludwig Boltzmann

20 February 1844 - September 5, 1906

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## Current flow with electrons in ordinary metals



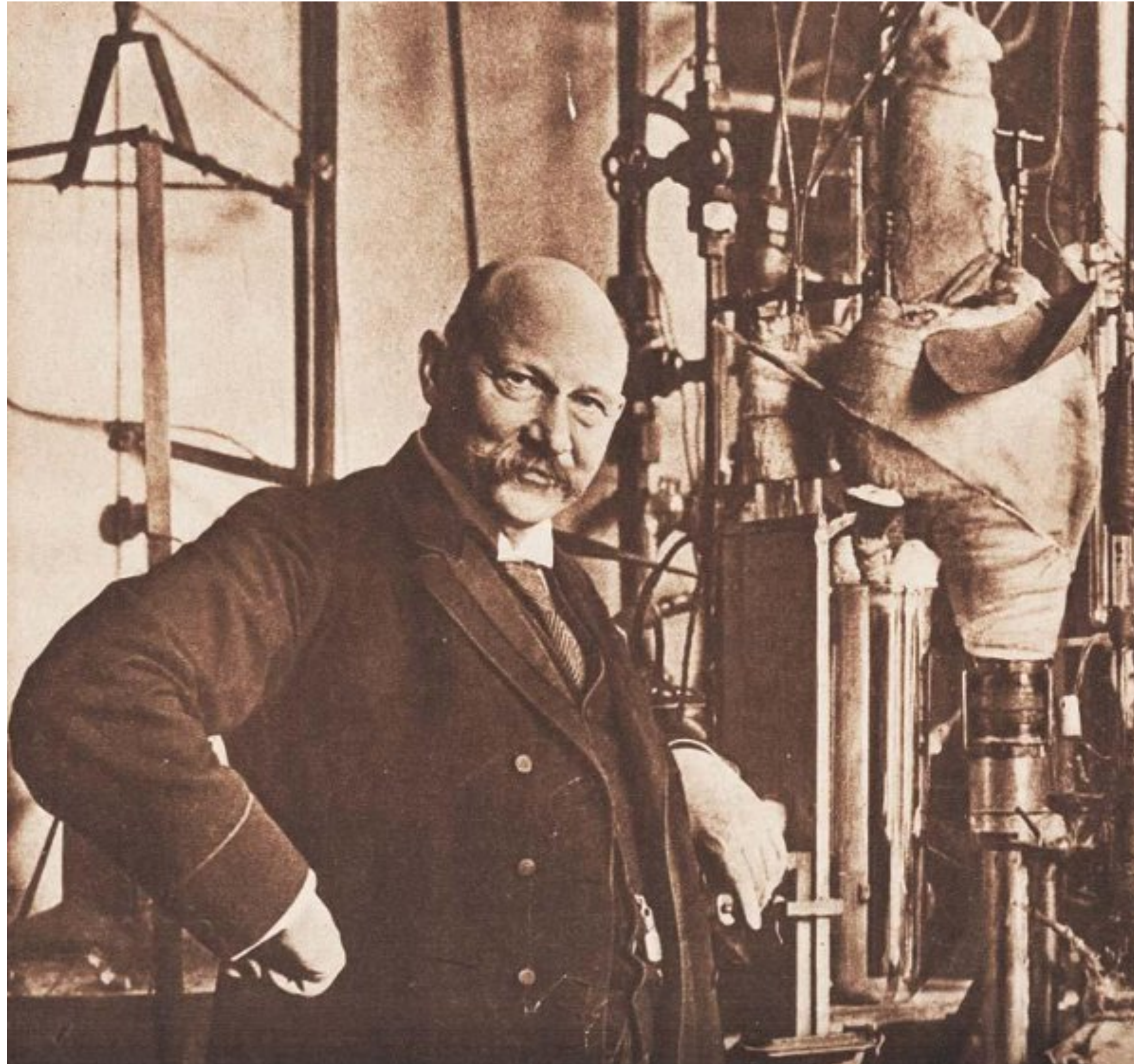
Flow of electrons described by Boltzmann equation  $\Rightarrow$   
typical scattering time  $\tau \sim 1/(UT)^2$  ( $U$  is the strength of interactions),  
resistivity  $\rho(T) = \rho(0) + AT^2$

The time  $\tau$  is much longer than a limiting ‘Planckian time’  $\frac{\hbar}{k_B T}$ .

The long scattering time implies that individual electrons are well-defined.

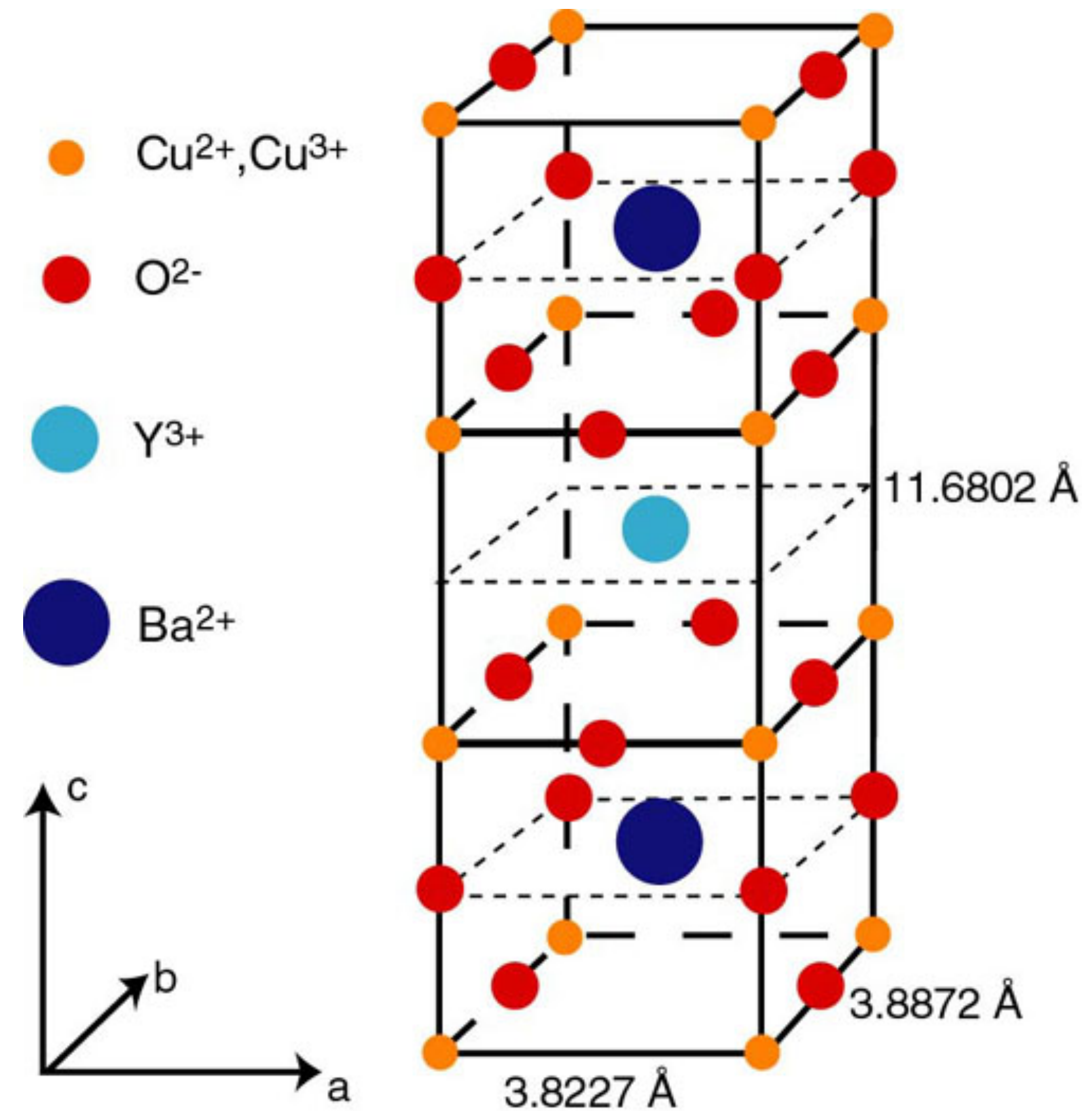
The motion of electrons is ‘ballistic’ or ‘integrable’  
up to the long time  $\tau$ , after which it is chaotic.

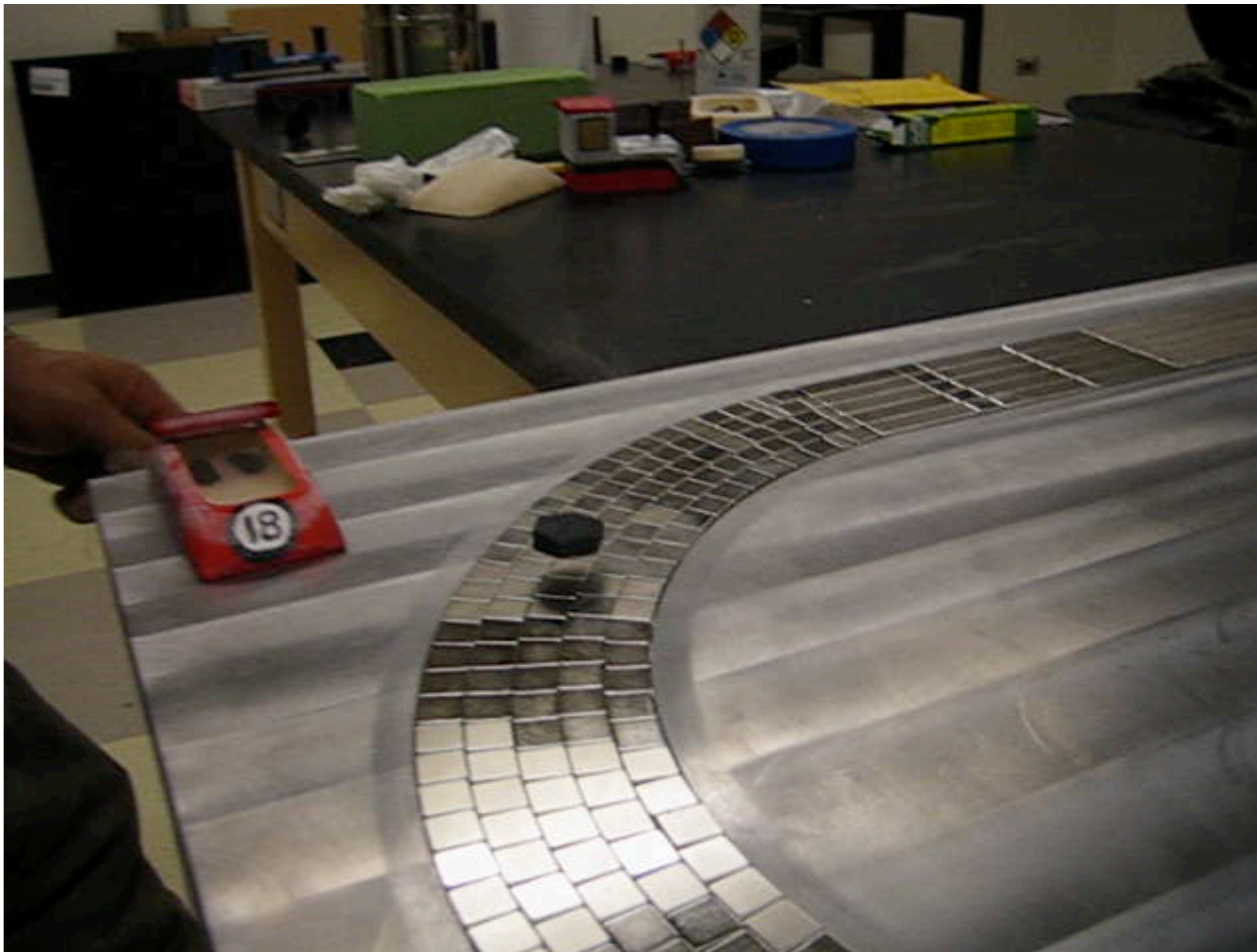
High temperature  
superconductivity



Kamerlingh Onnes 1911:  
Mercury is a superconductor below  $-269\text{ }^{\circ}\text{C}$

# Cuprate high temperature superconductors





Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

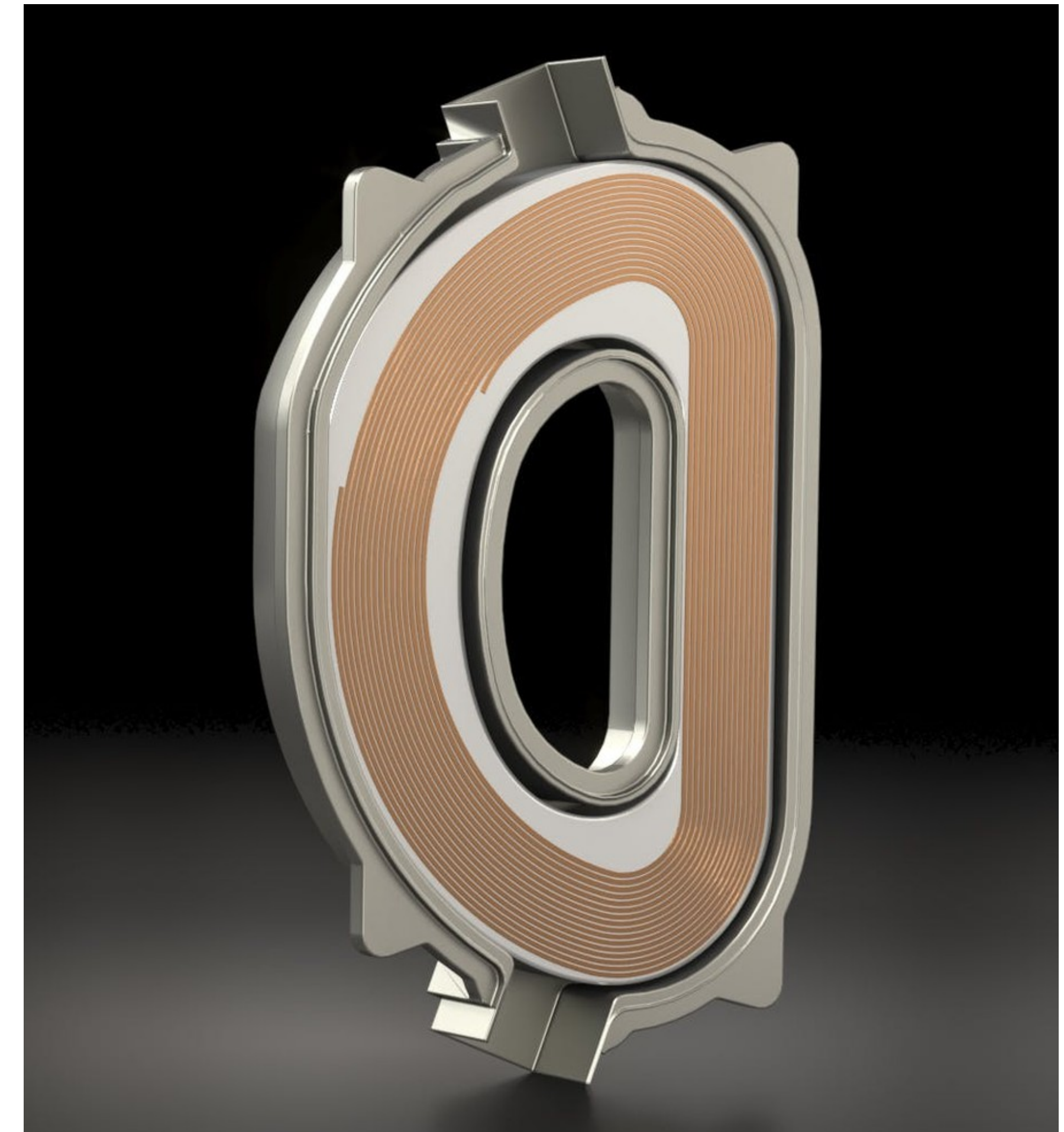
# HTS Magnets: Enabling Technology

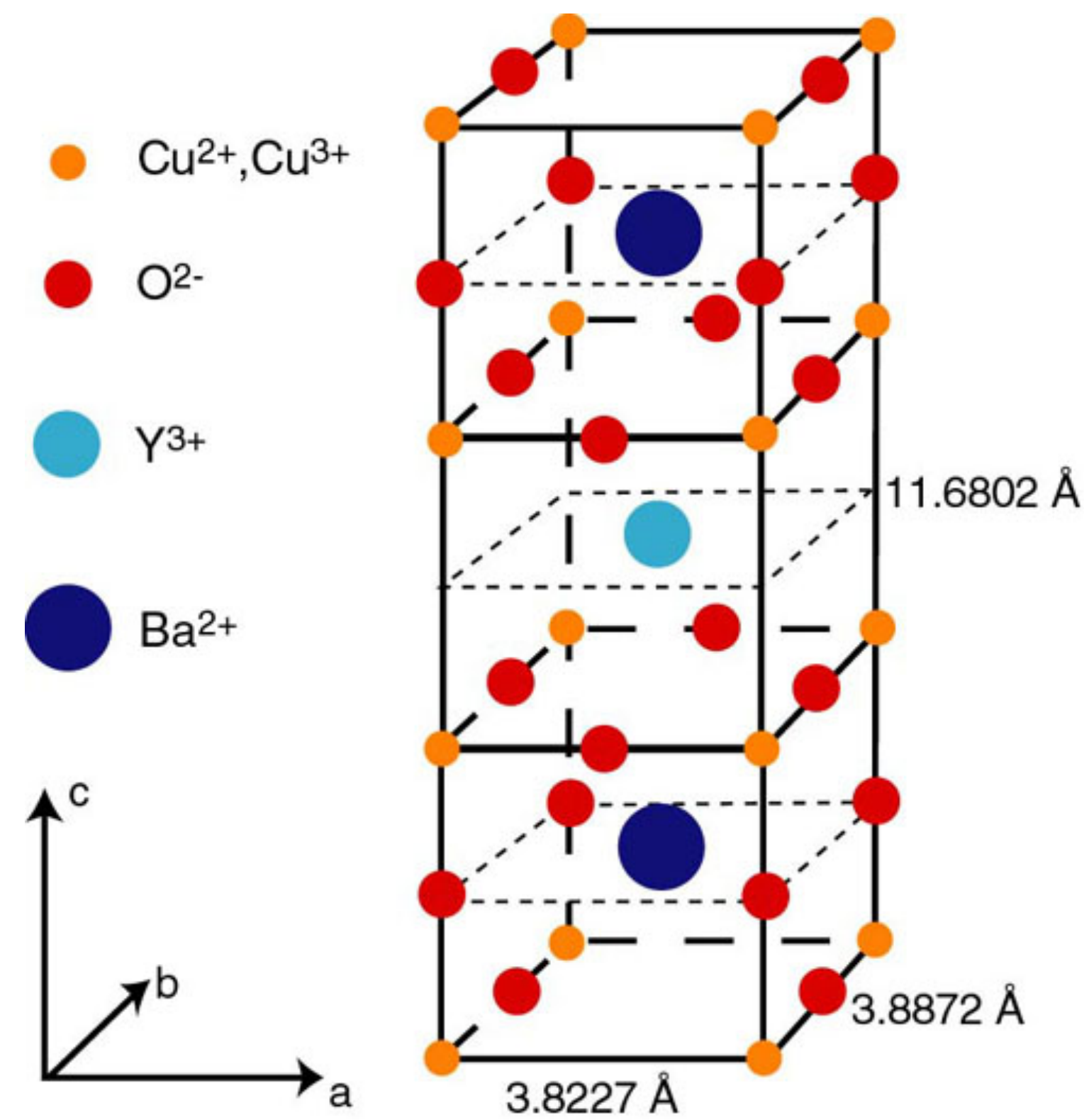
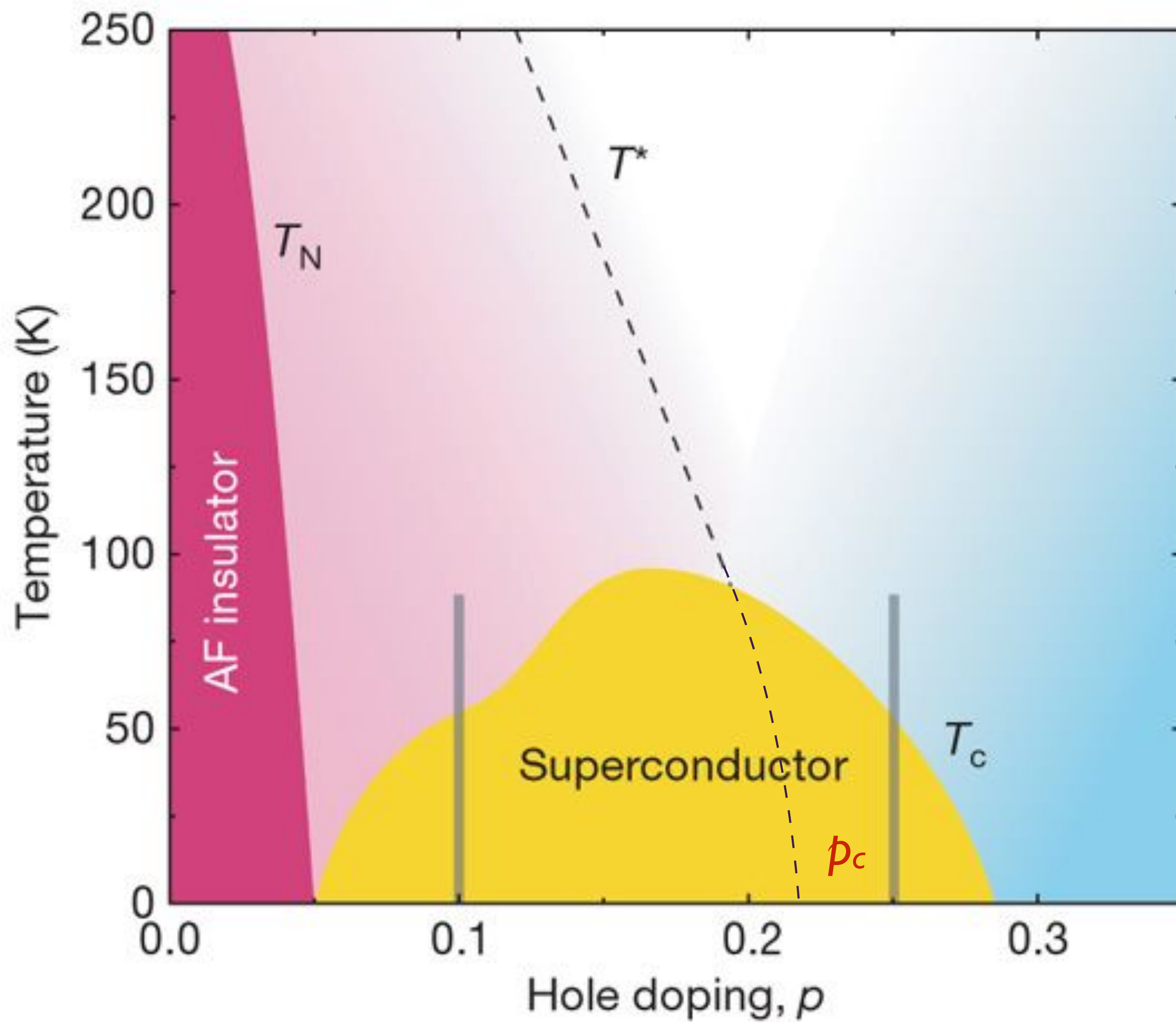
The surest path to limitless,  
clean, fusion energy

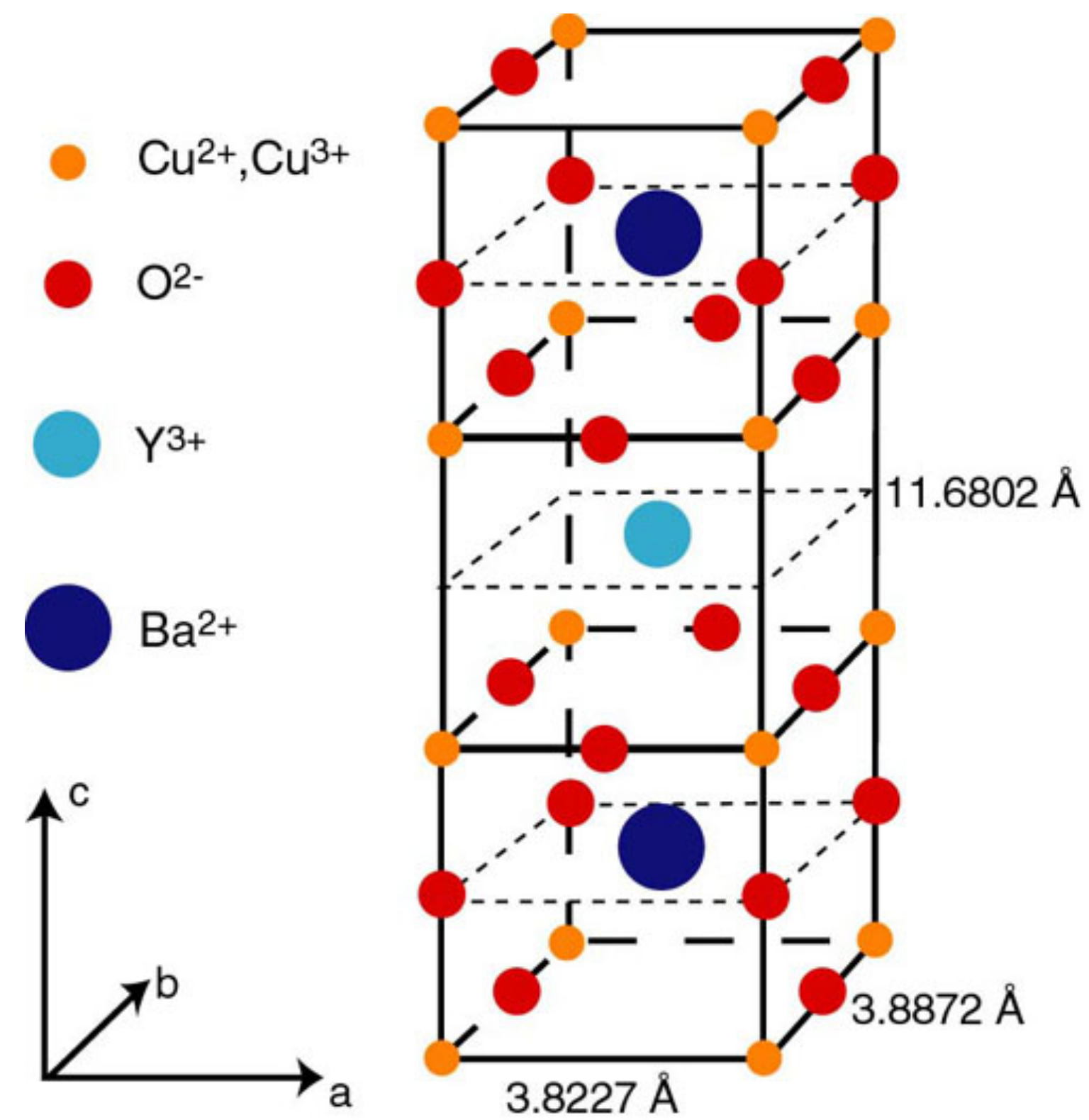
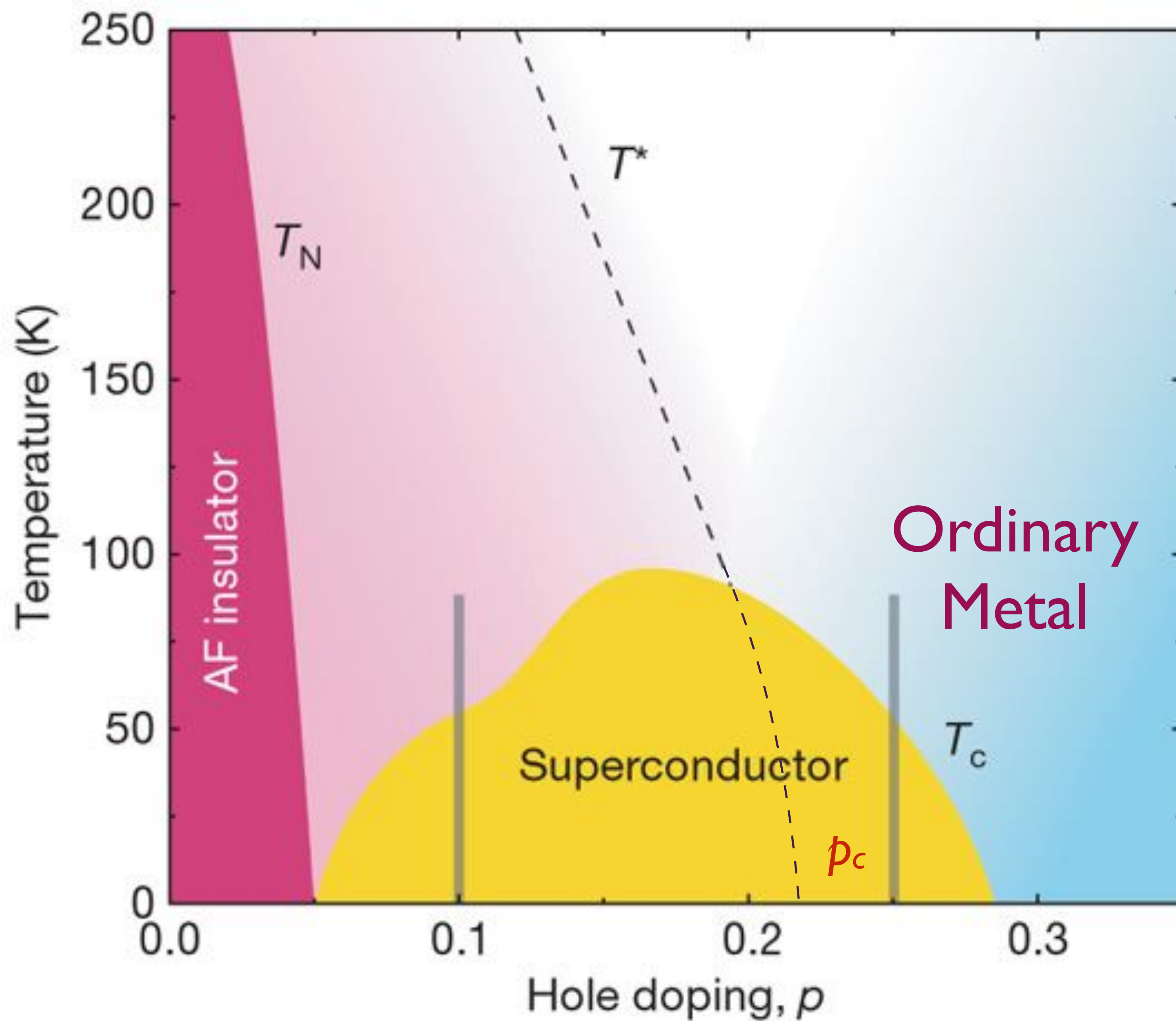
YBCO magnets allow for smaller,  
faster, and less expensive  
tokamaks for plasma fusion

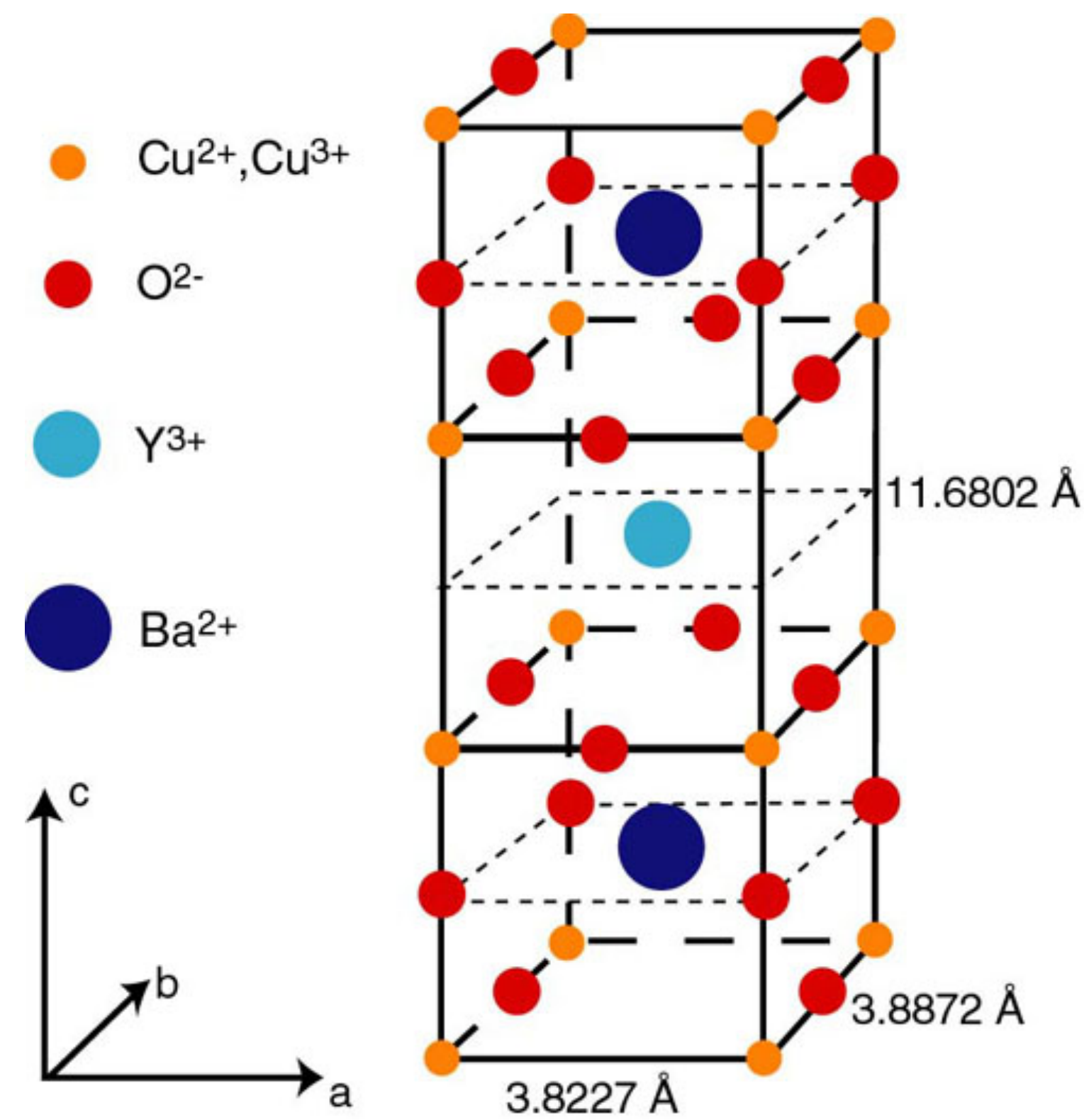
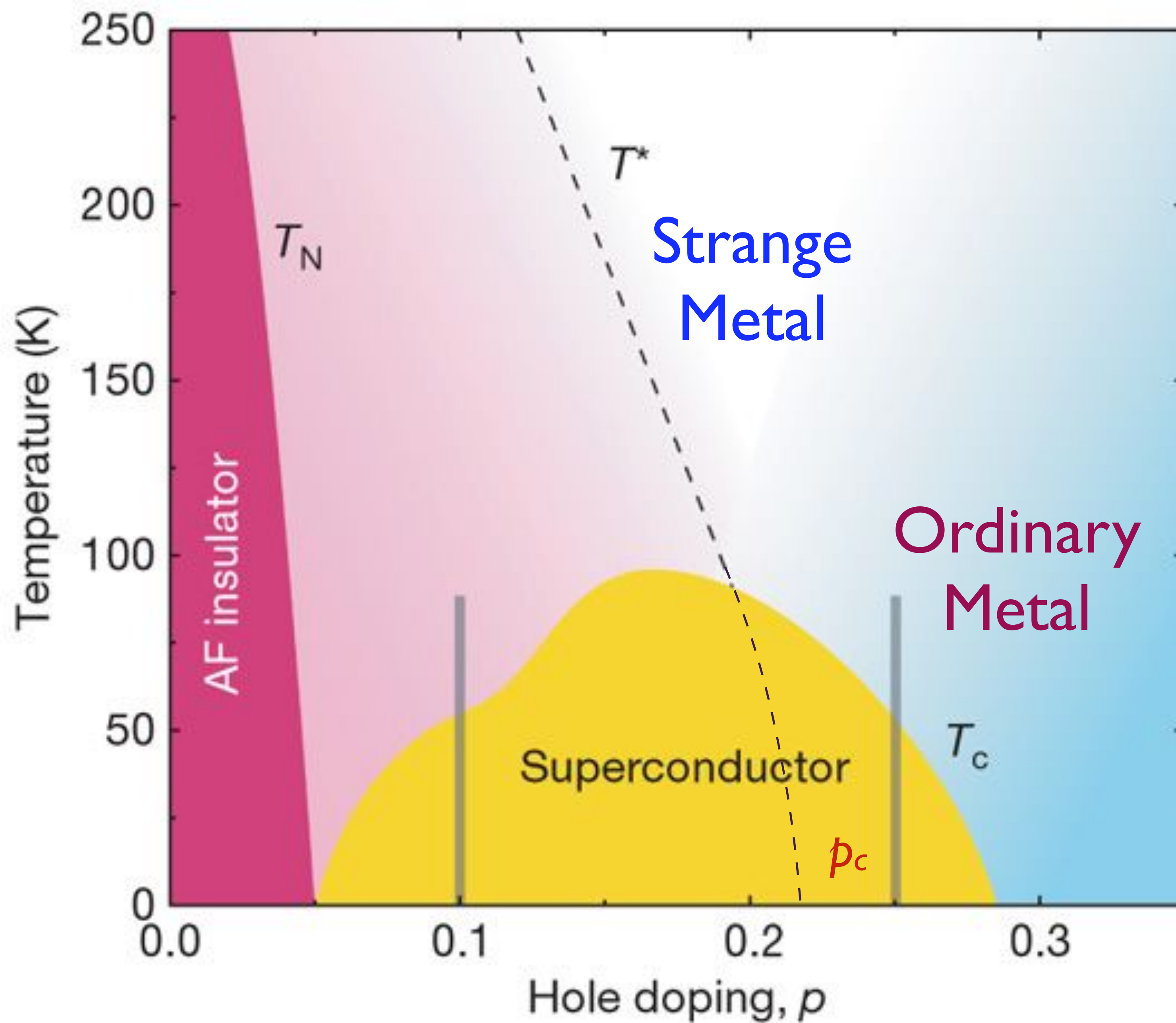


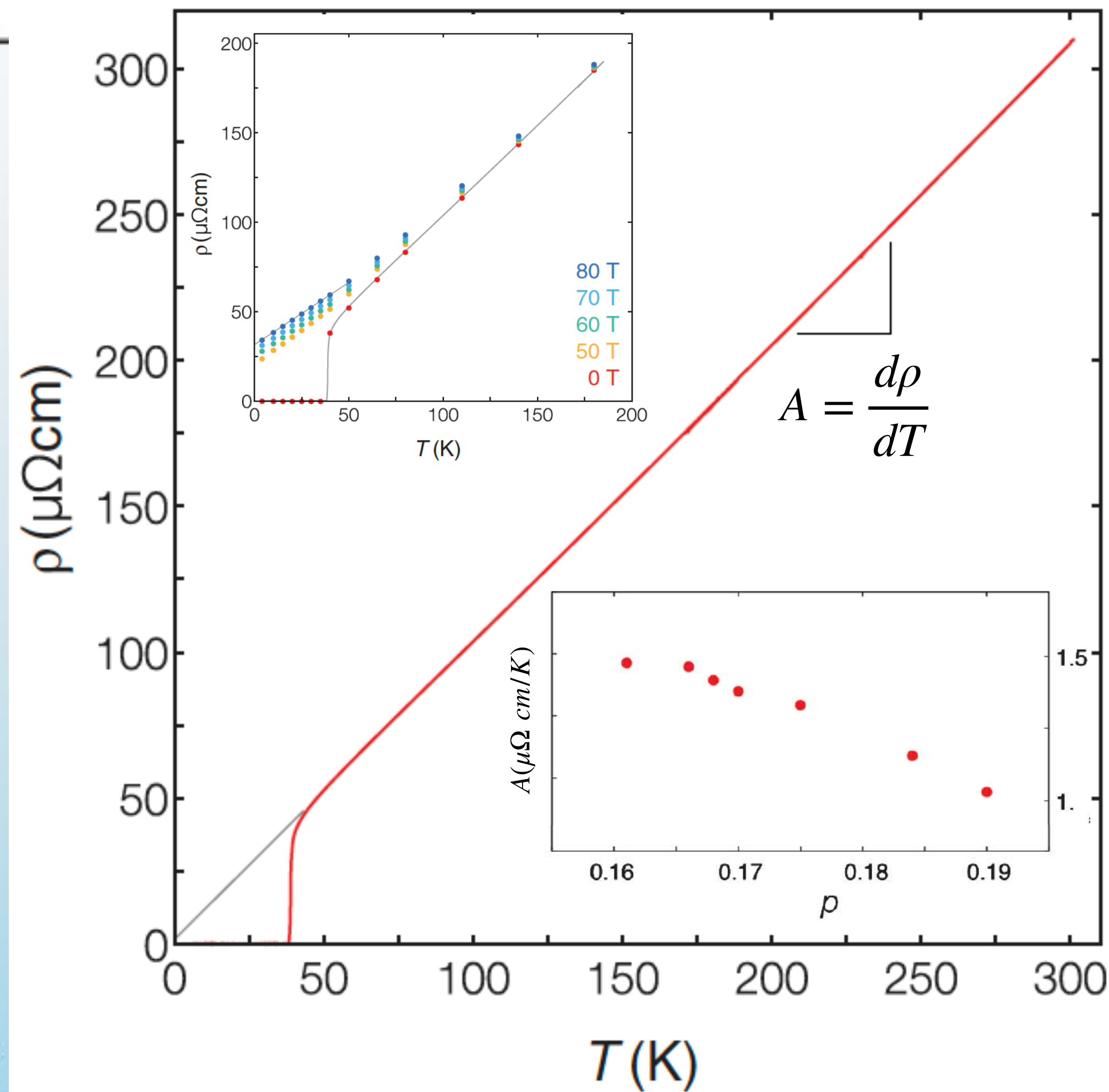
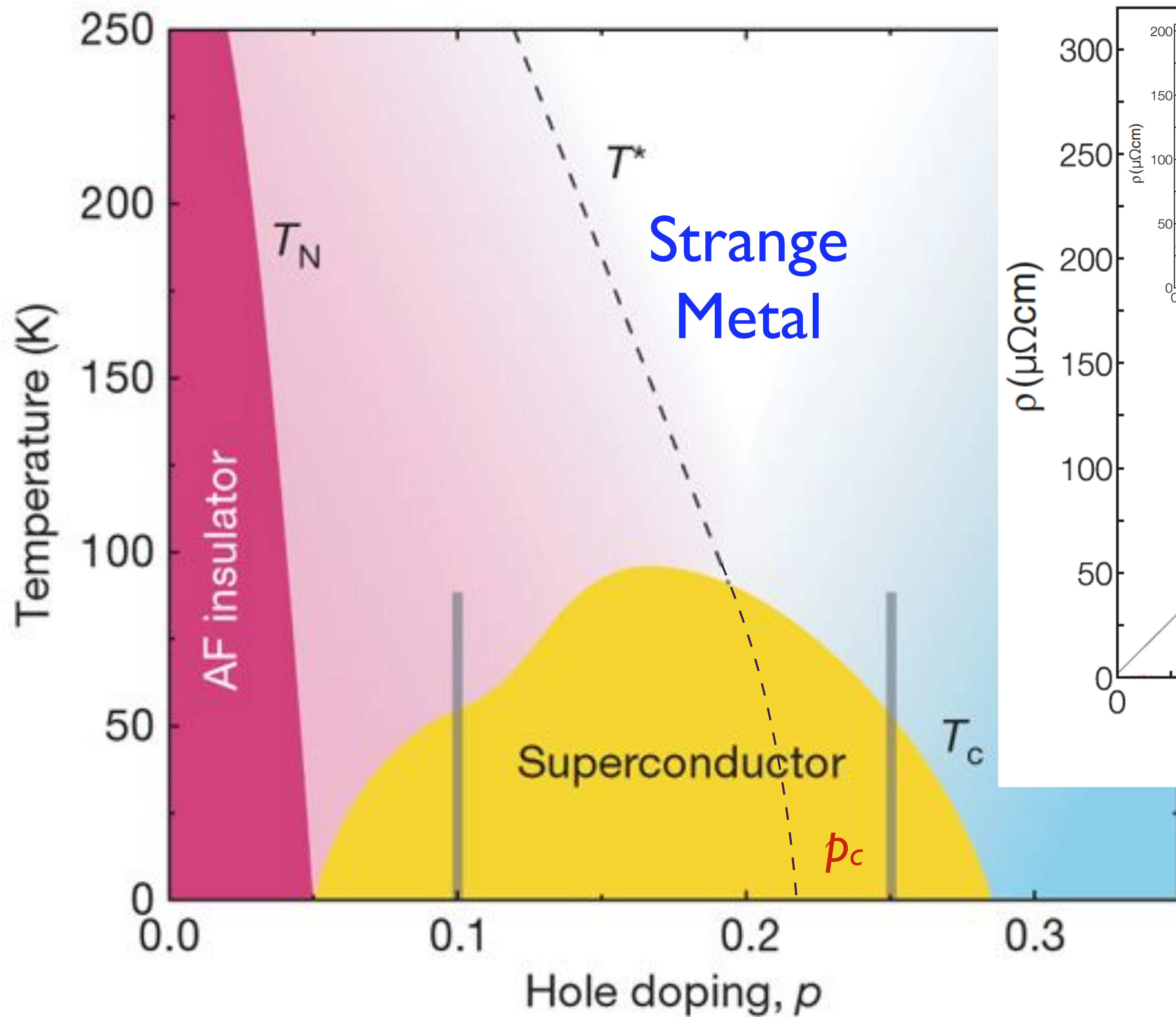
Commonwealth  
Fusion Systems











LSCO: Giraldo-Gallo et al. 2018

# Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

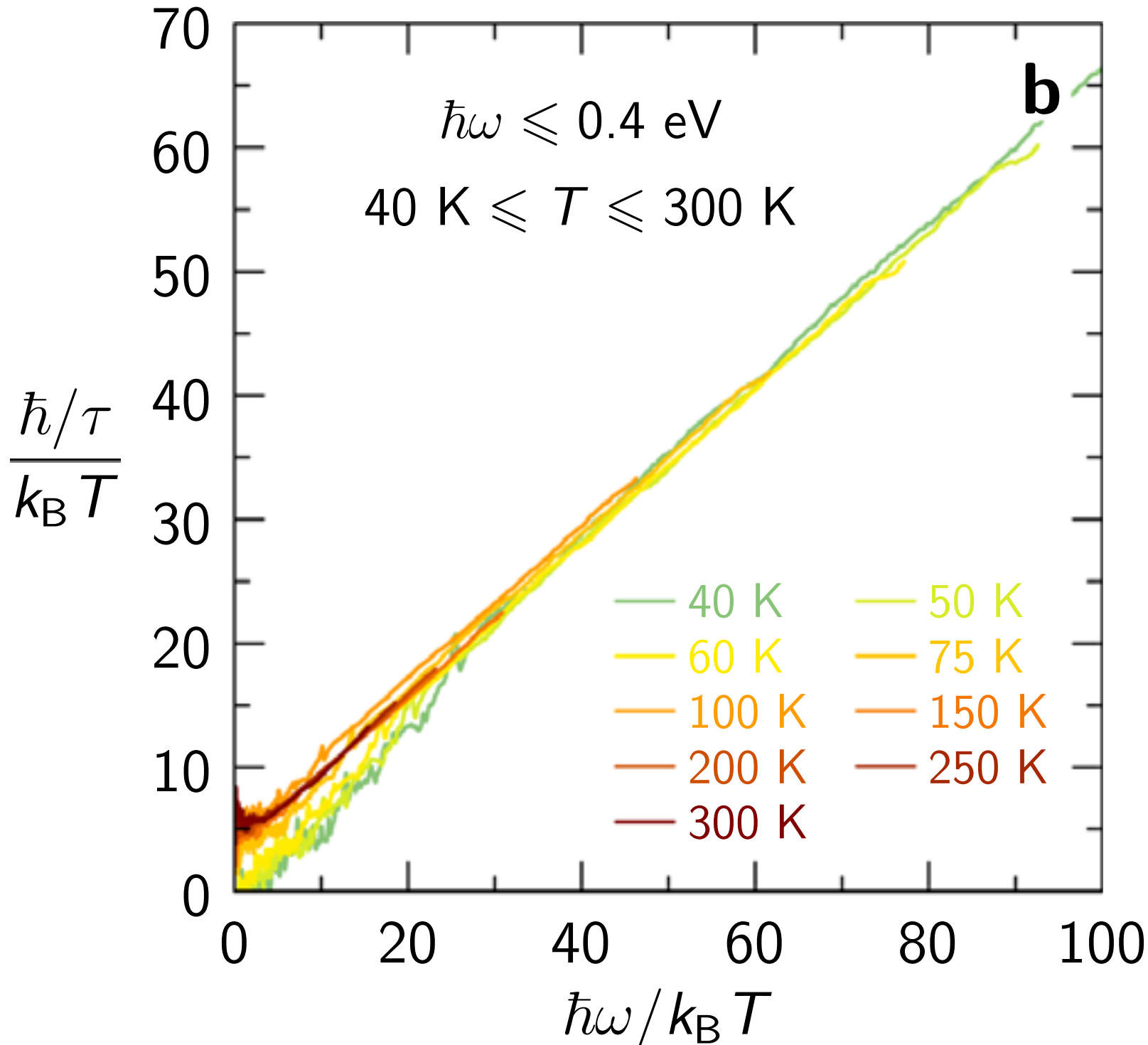
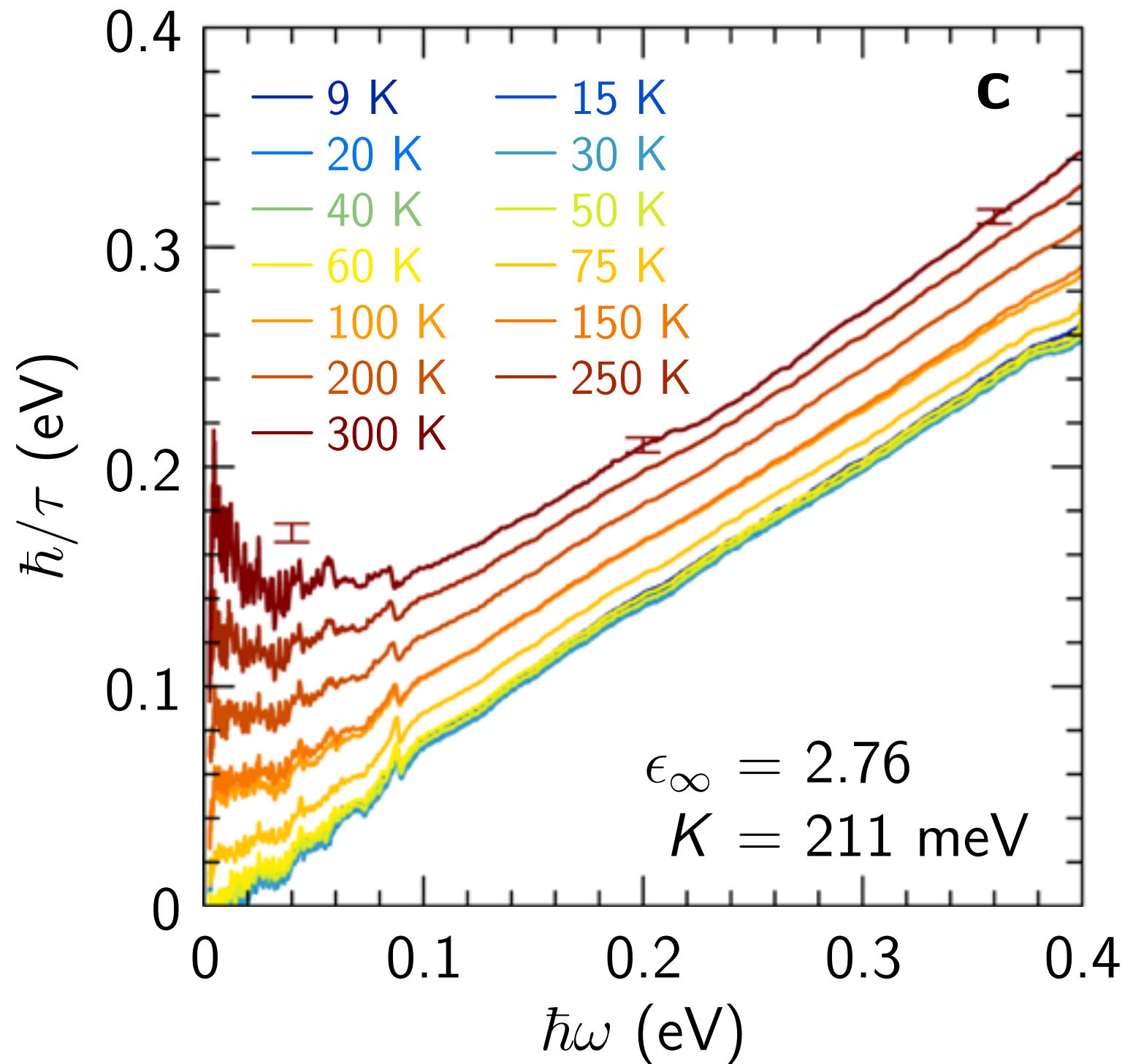
B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

*Nature Communications* **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i \frac{e^2 K / (\hbar d_c)}{\hbar \omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$

Planckian dynamics!

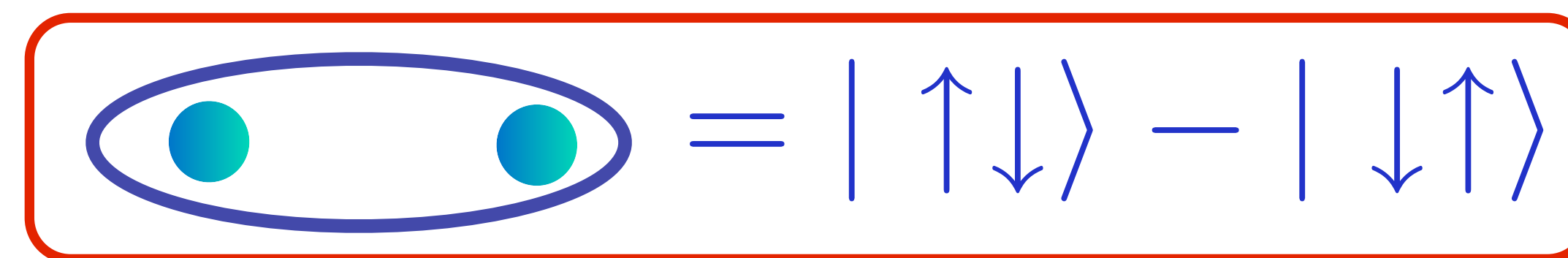
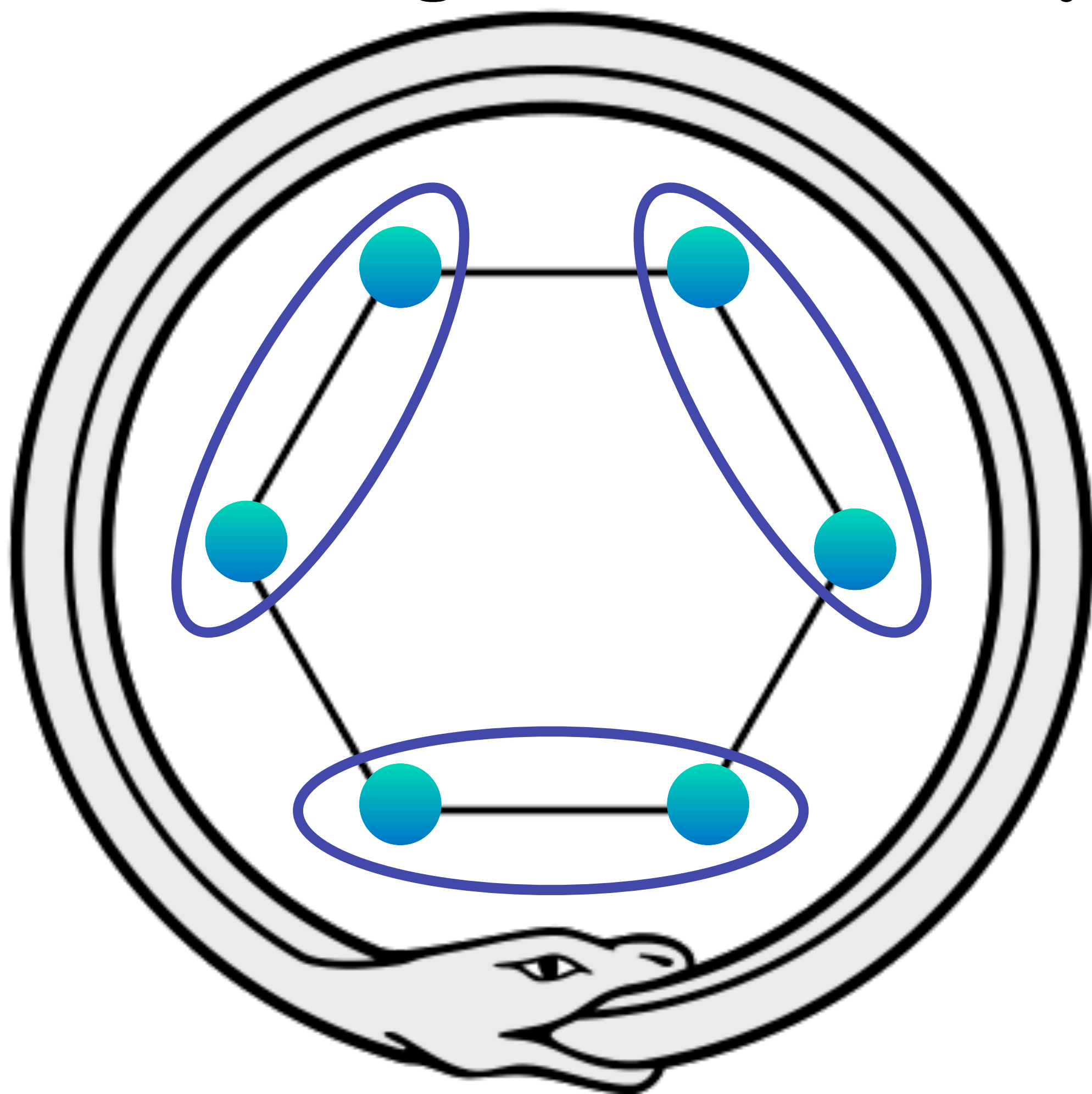
$$\tau(\omega) = \frac{\hbar}{k_B T} F \left( \frac{\hbar \omega}{k_B T} \right)$$



Quantum  
entanglement  
(1865)

# Kekulé's spooky dream (1865)

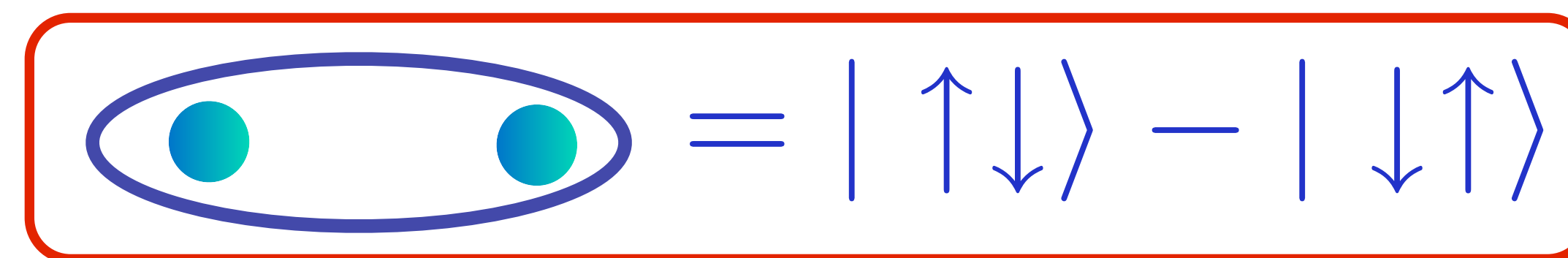
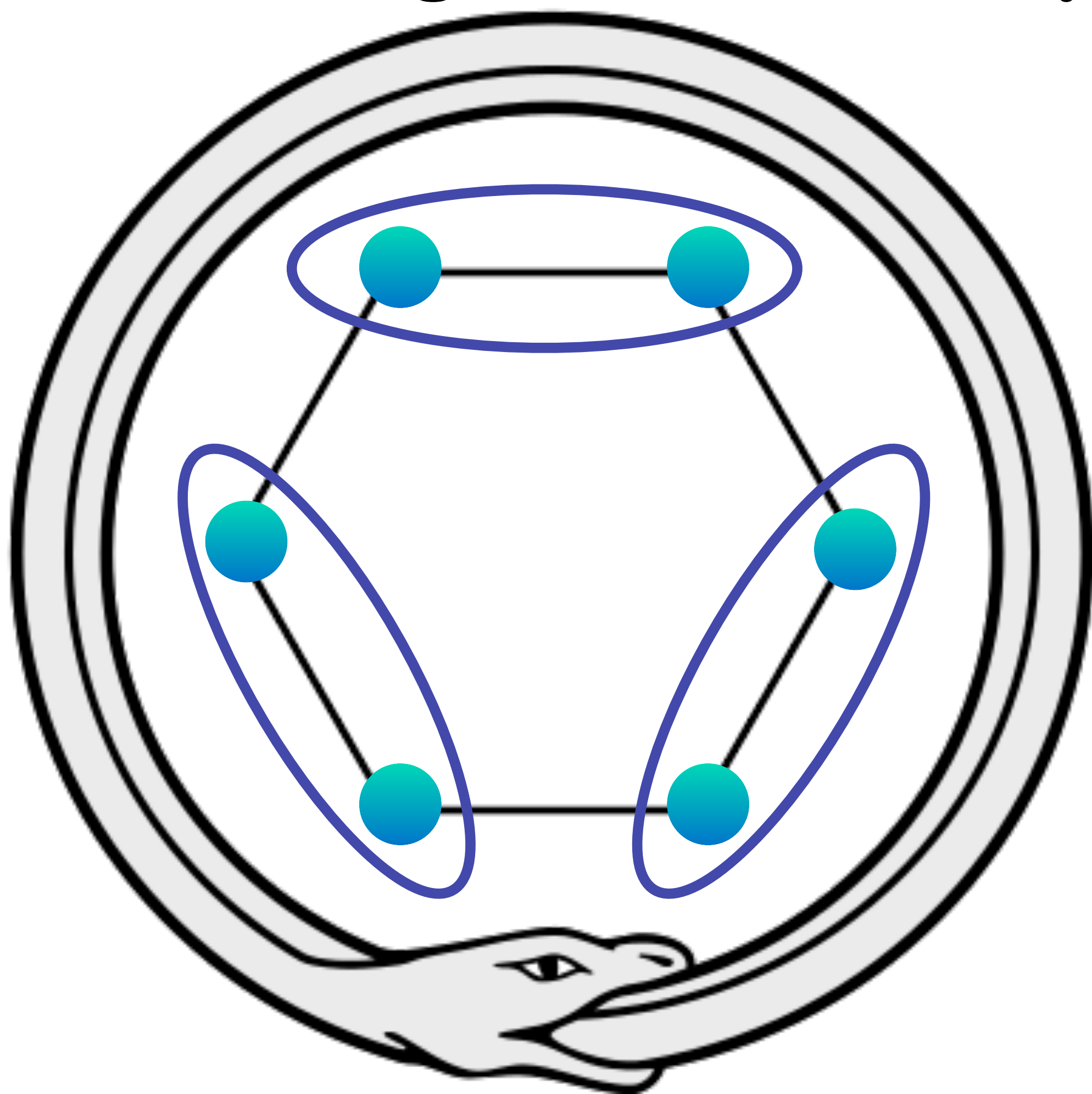
Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail\*



**Benzene**

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**Benzene**

MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

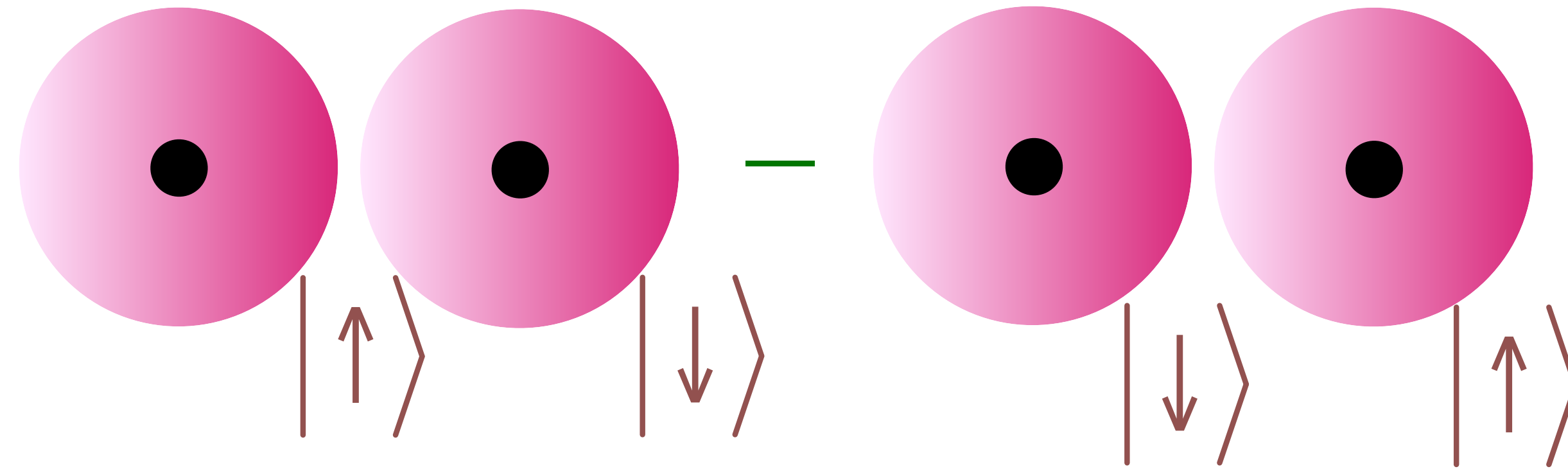
# Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

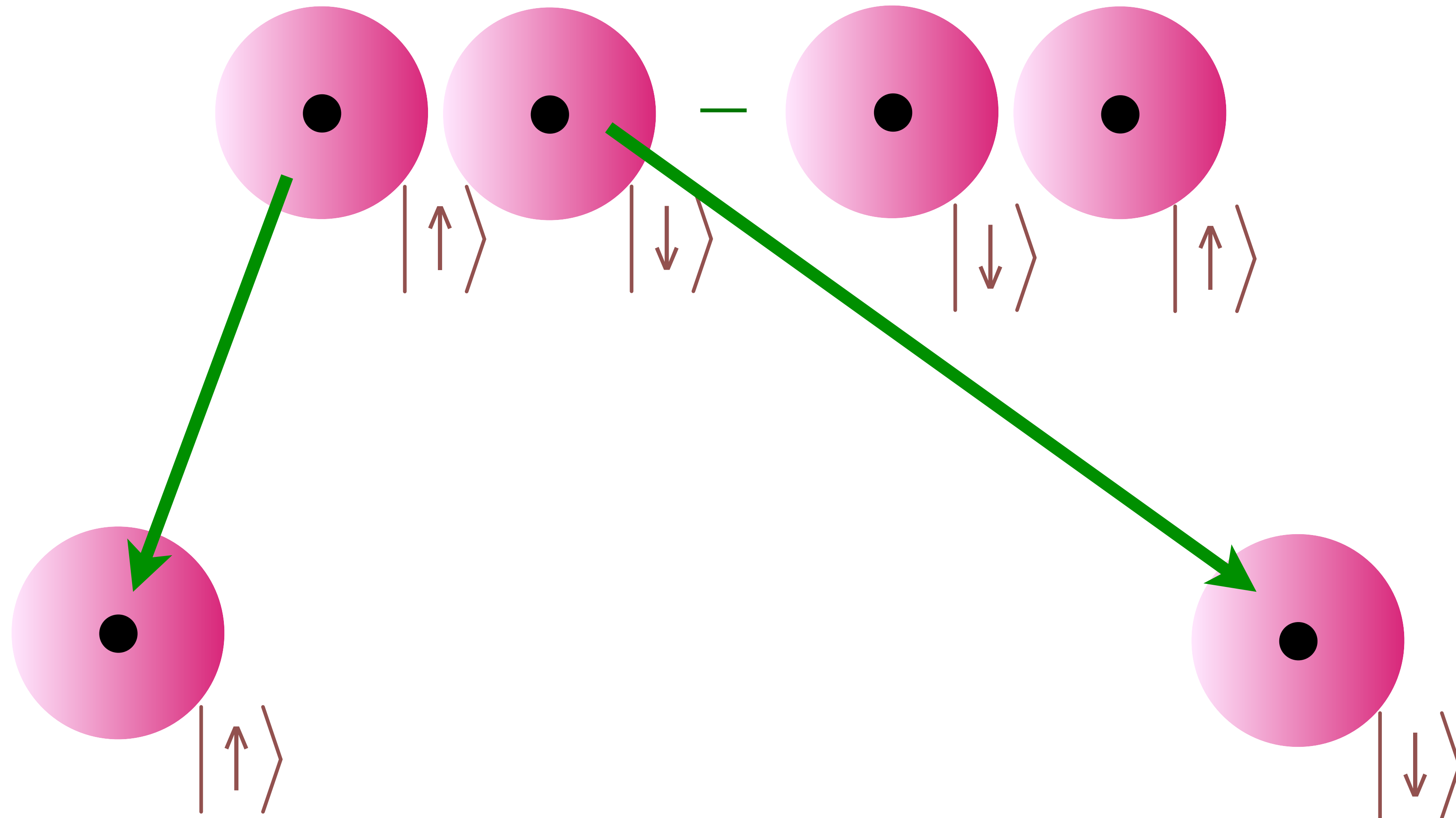
# Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



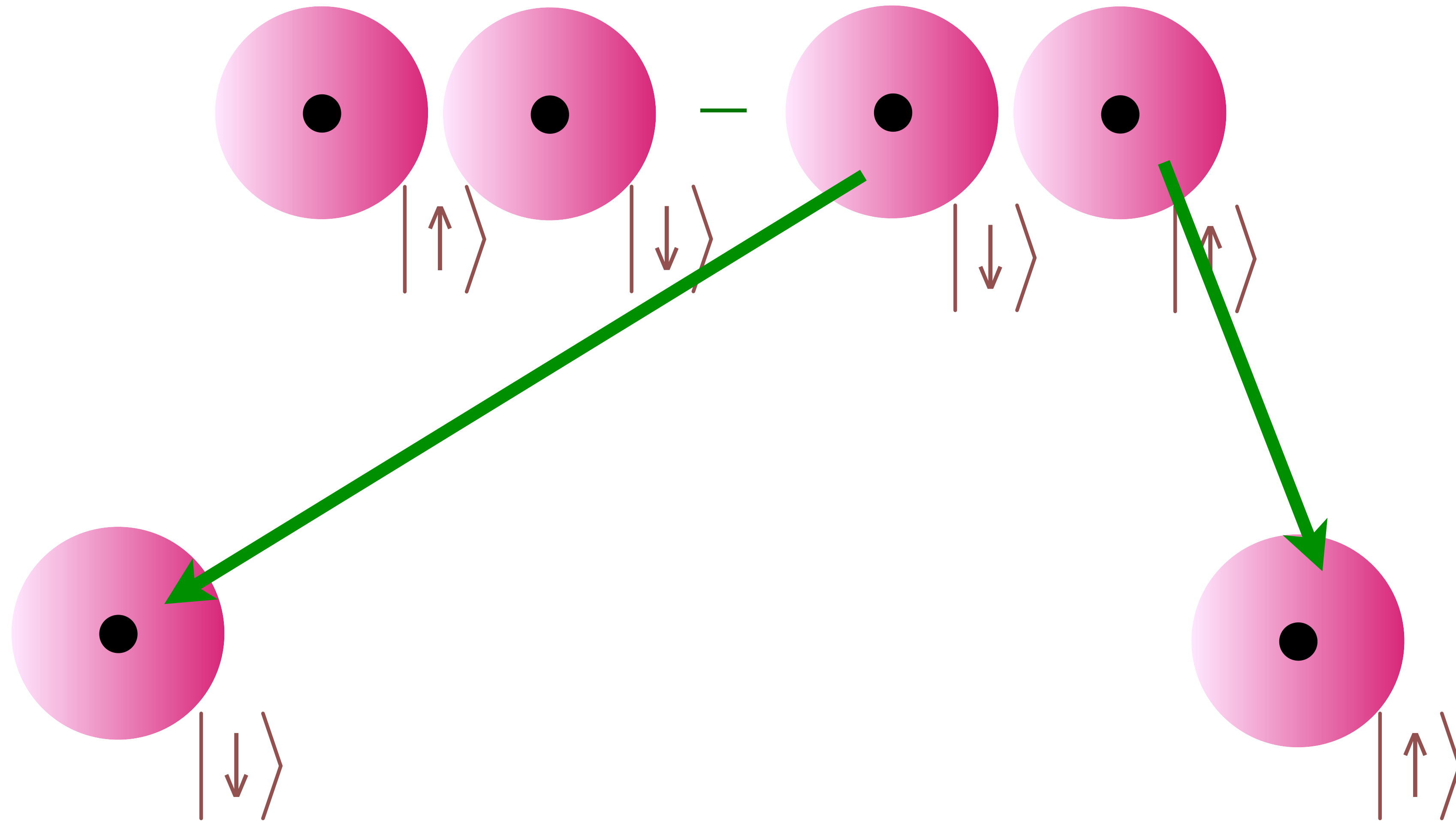
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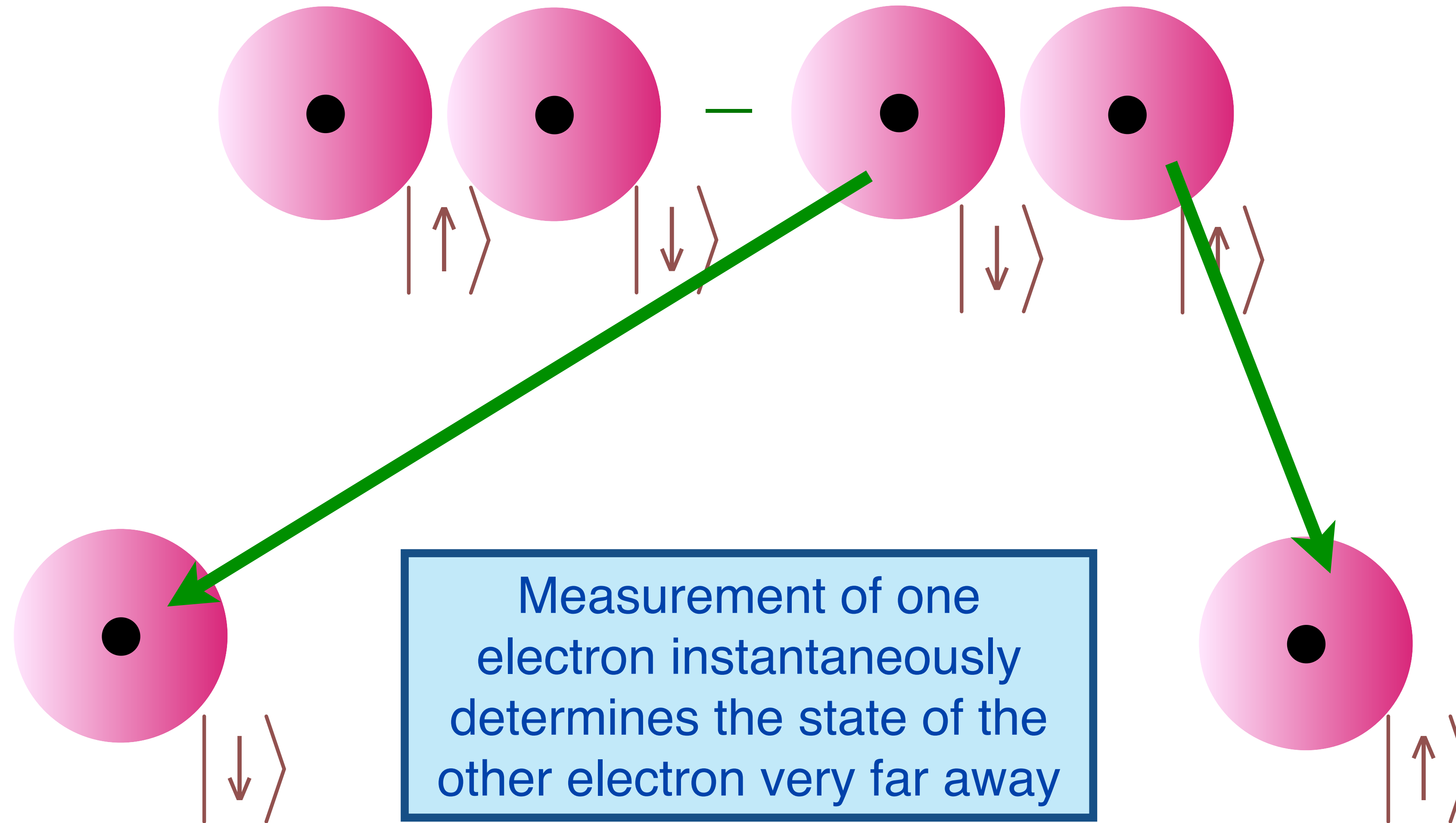
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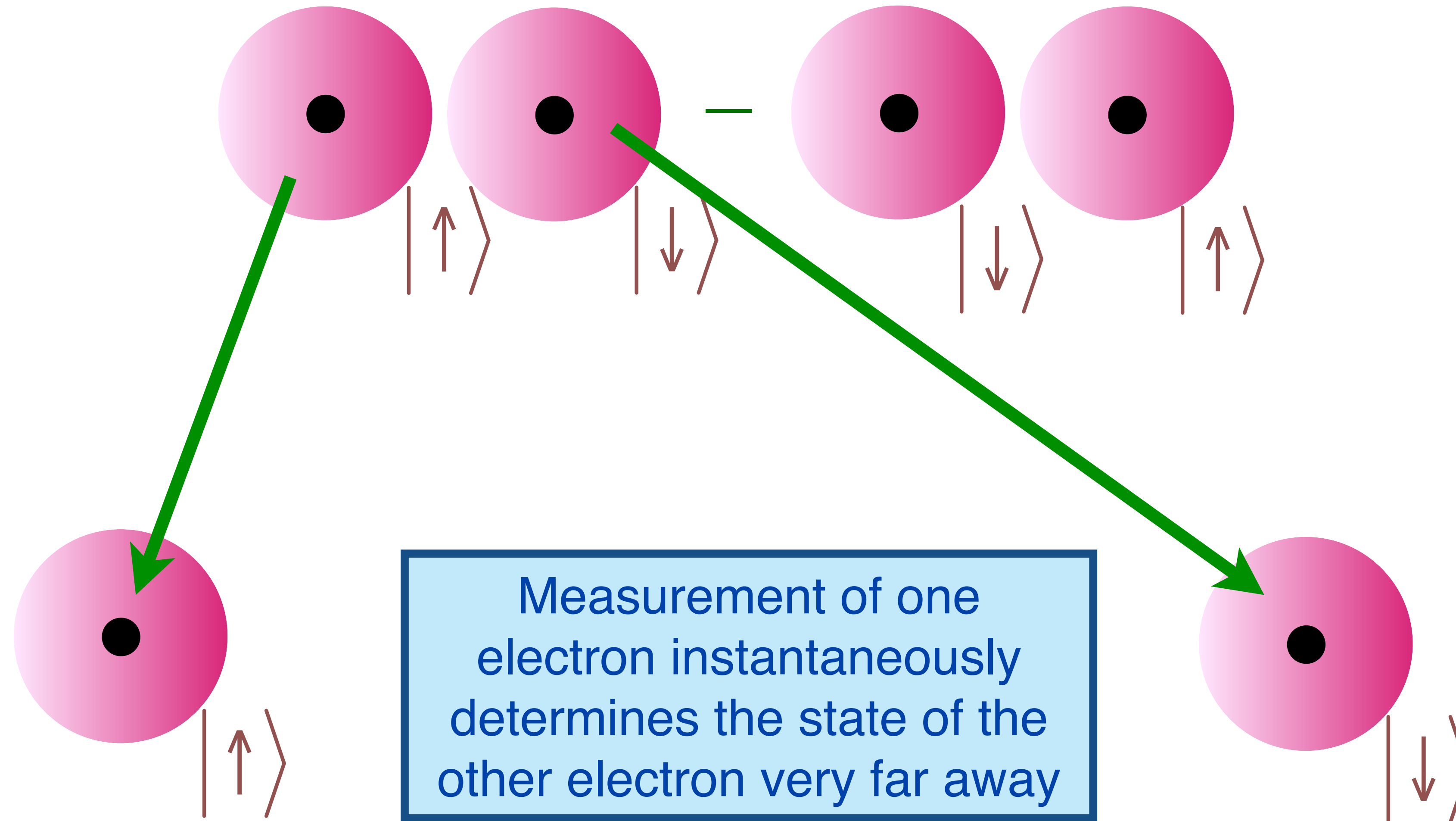
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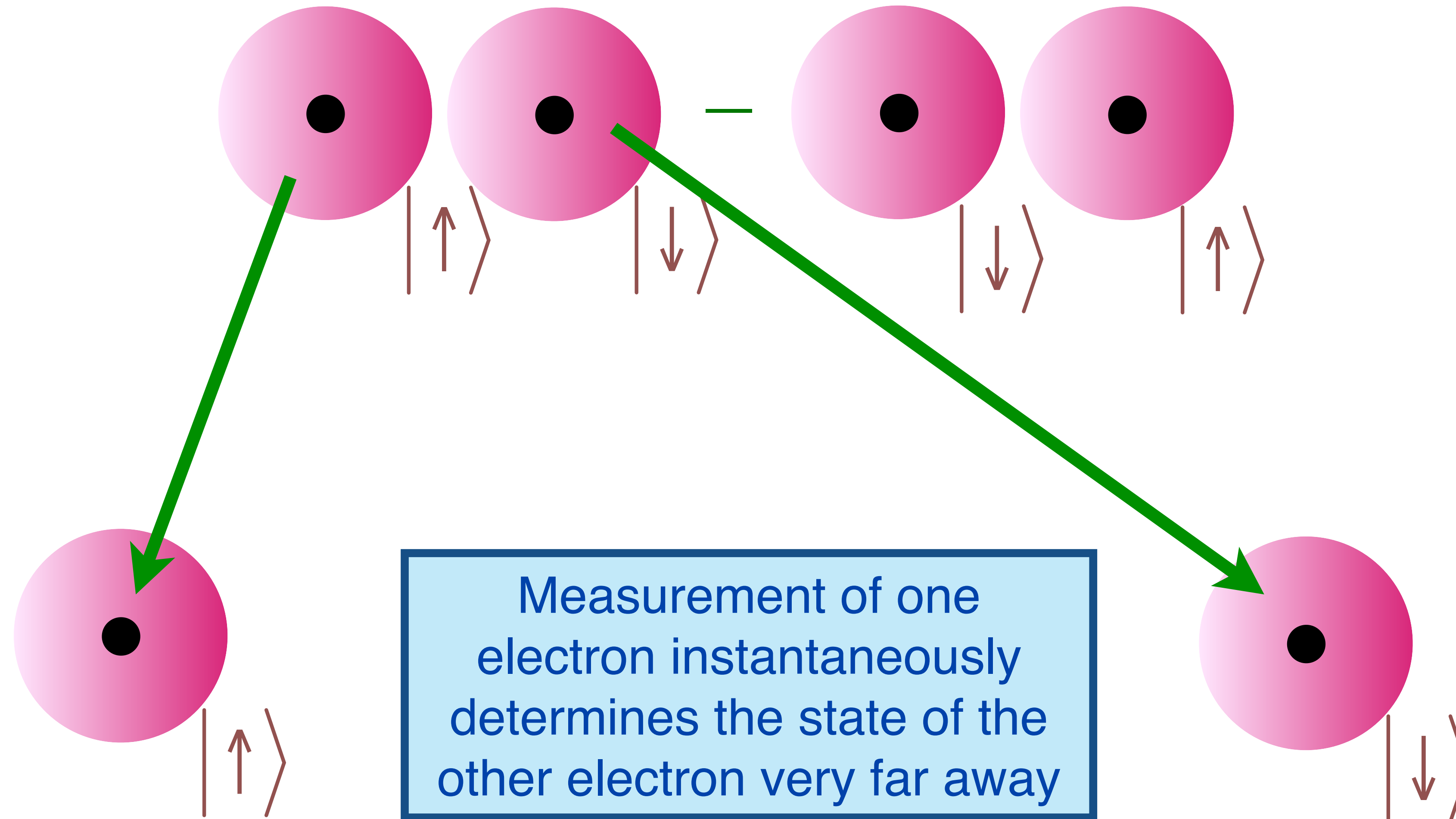
# Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



# Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



**Spooky action at a distance !**

natürlicher  
deren Notwendigkeit im  
mus ja zuerst von Dir klar erkannt wurde, einen Bedeutung  
Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft dar-  
an glauben, weil die Theorie mit dem Grundsatz unvereinbar  
ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstel-  
len soll, ohne spukhafte Fernwirkungen. Allerdings bin ich  
überzeugt daß es wirklich mit der Theorie

I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

Albert Einstein to Max Born, 3 March 1947

Needed,  
to solve open problems in the theory of  
superconductivity and black holes:

A solvable model of quantum entanglement  
of 3, 4, 5, ...  $\infty$  particles

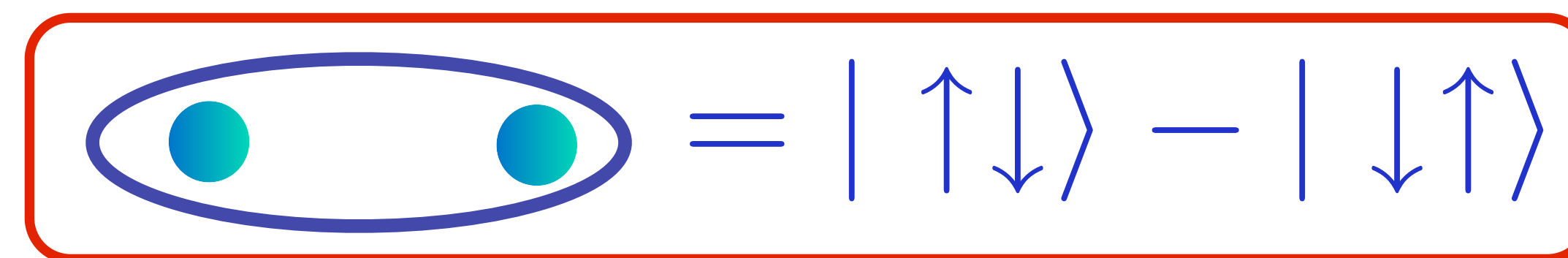
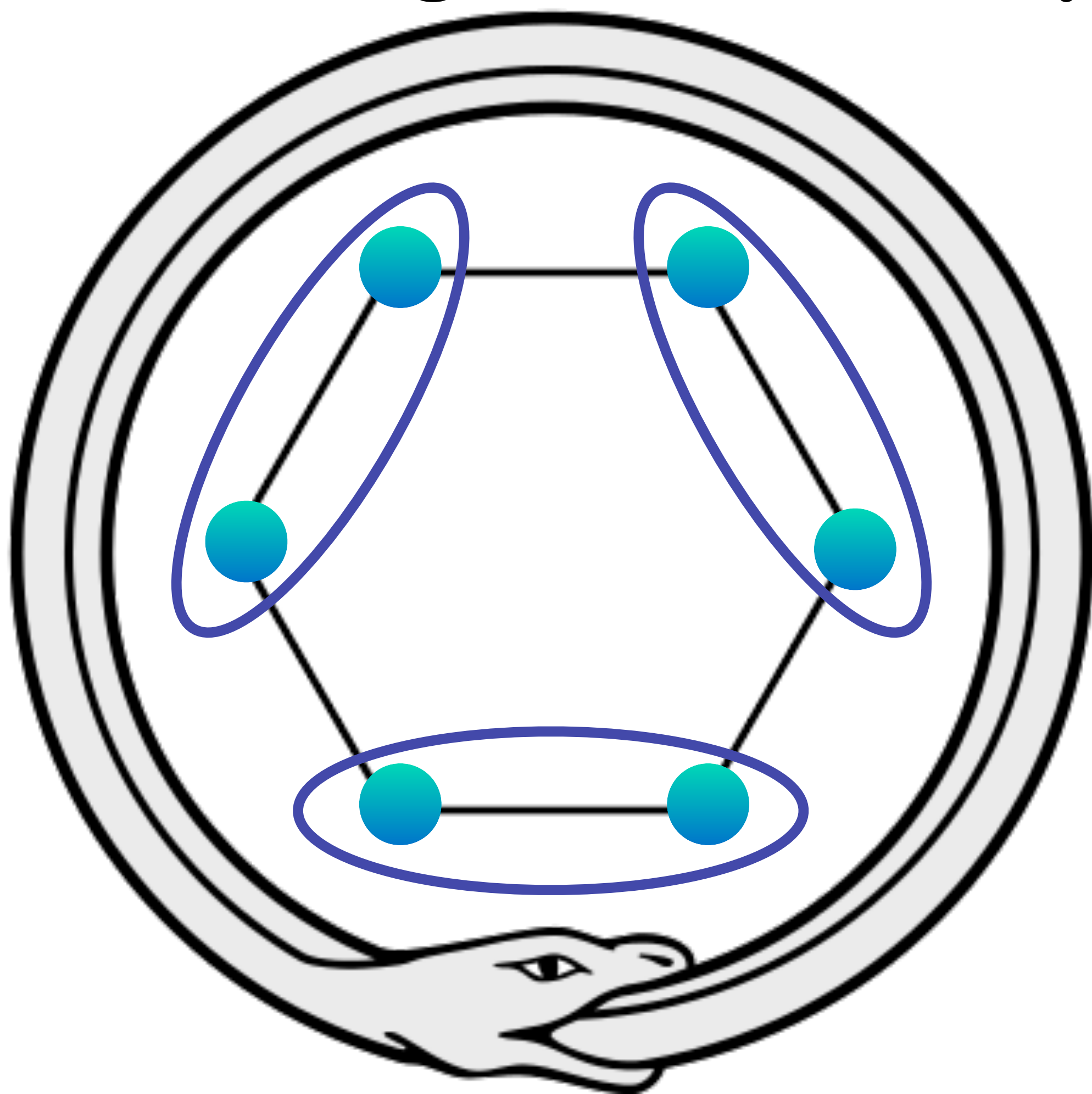
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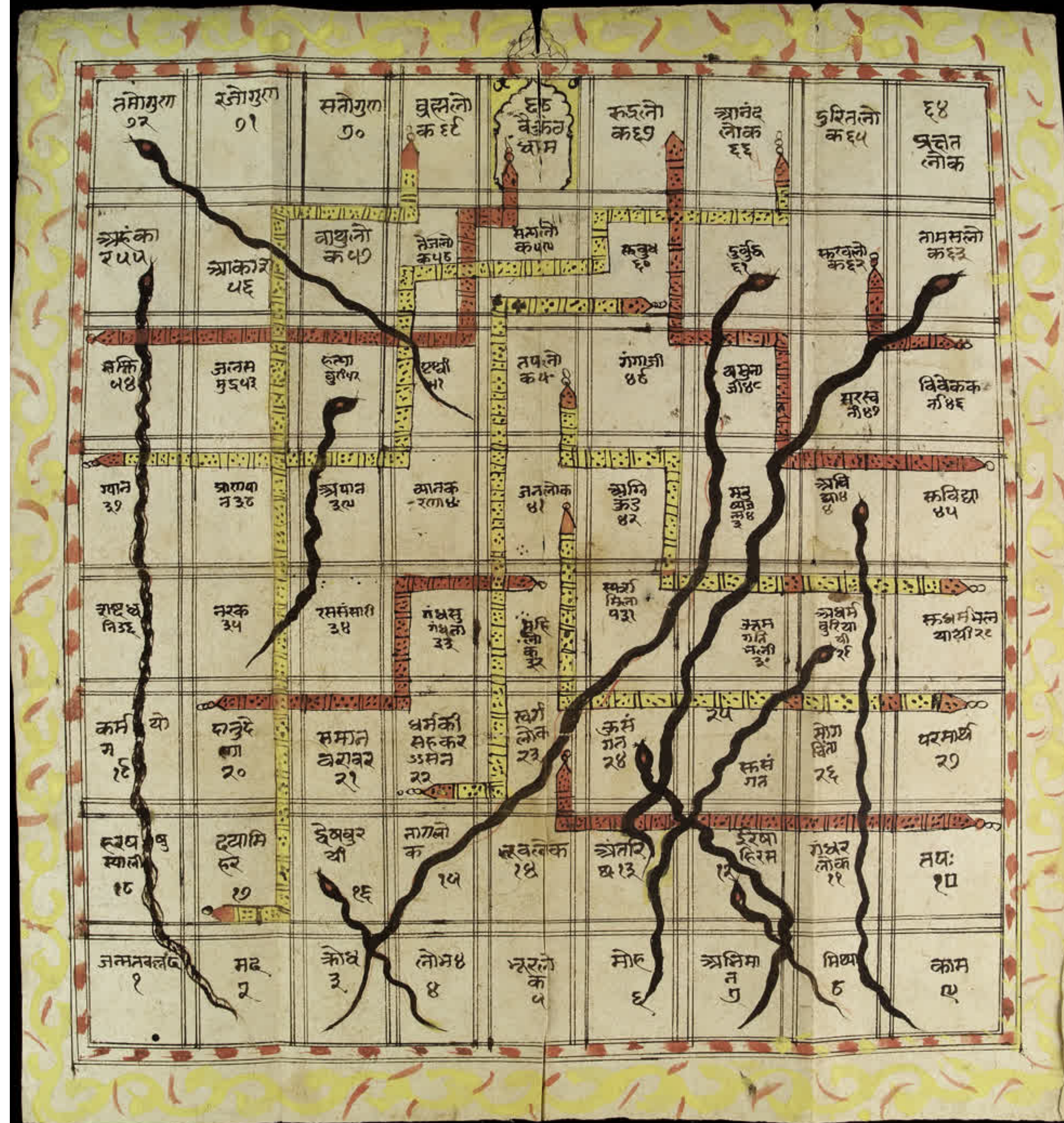
**The Sachdev-Ye-Kitaev model  
of many-particle entanglement**

# Kekulé's spooky dream (1865)

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**Benzene**

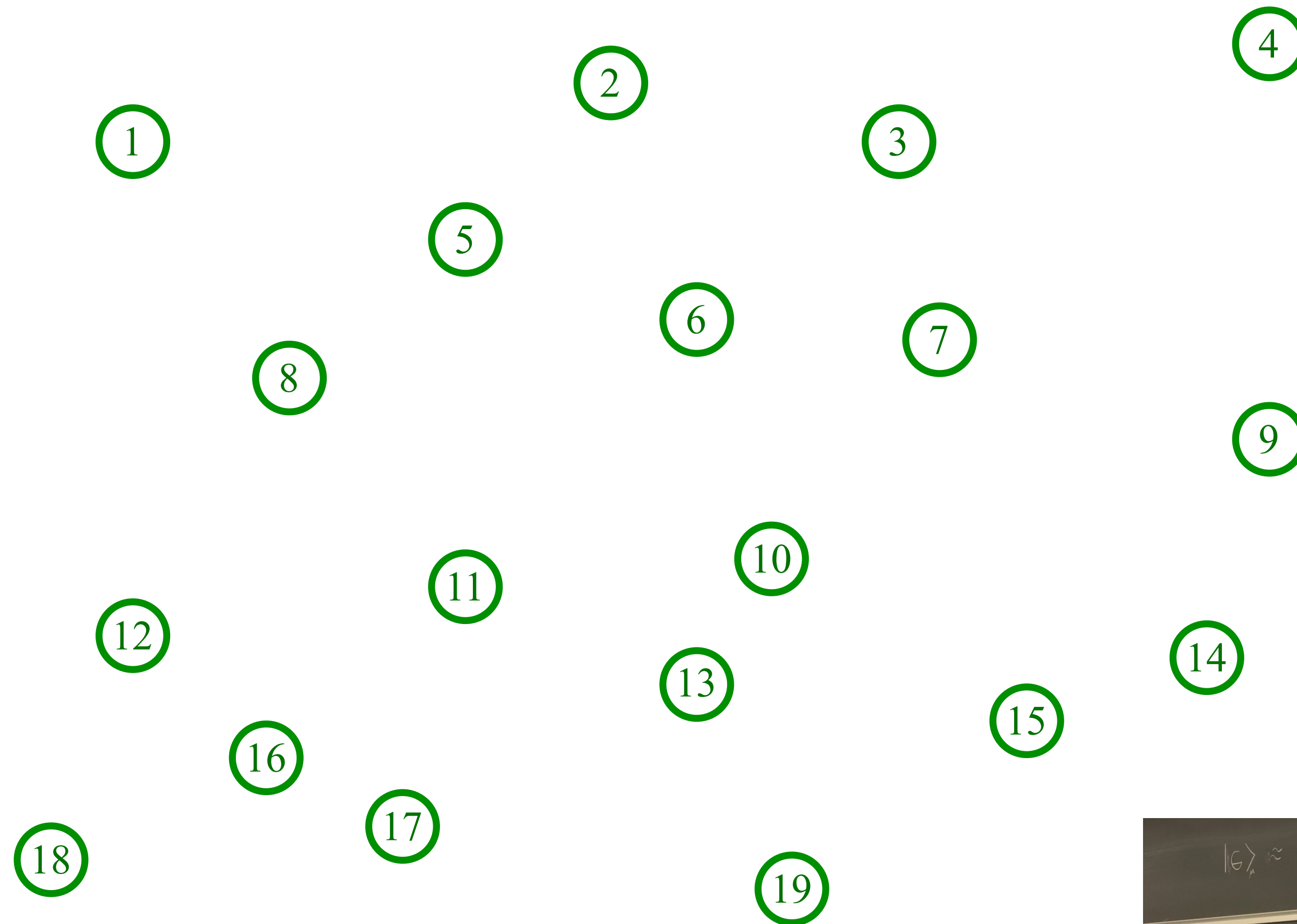


My  
spooky  
dream  
(1992)\*  
Ancient  
Indian  
game of  
Snakes  
and  
Ladders

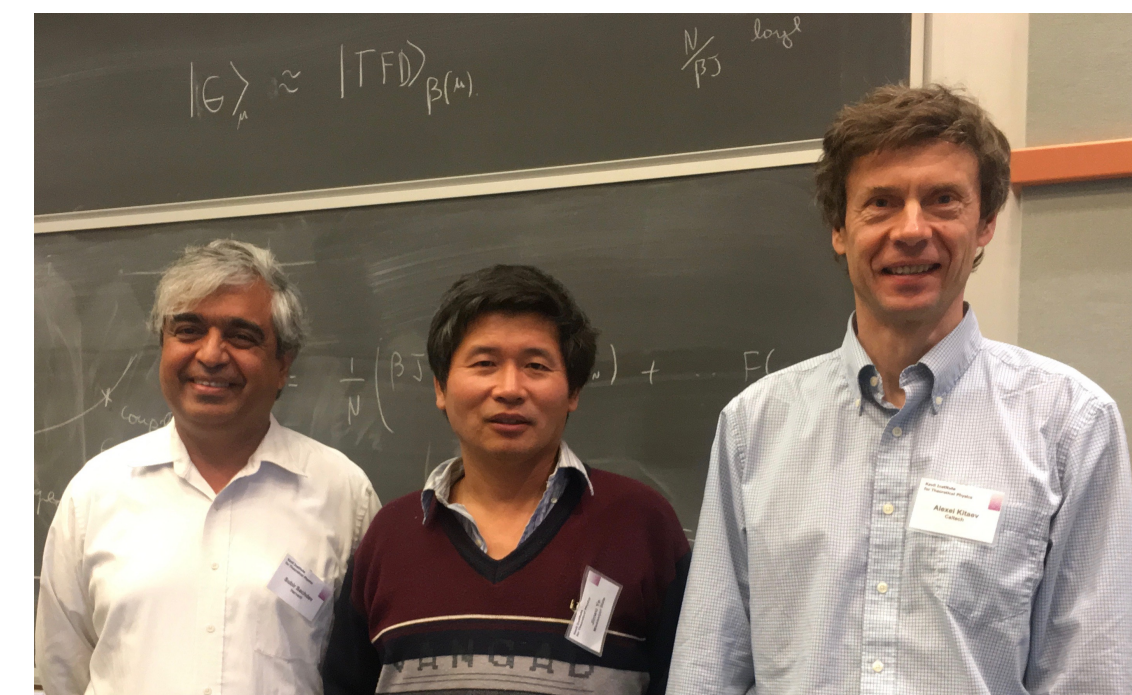
\*Not true

# The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

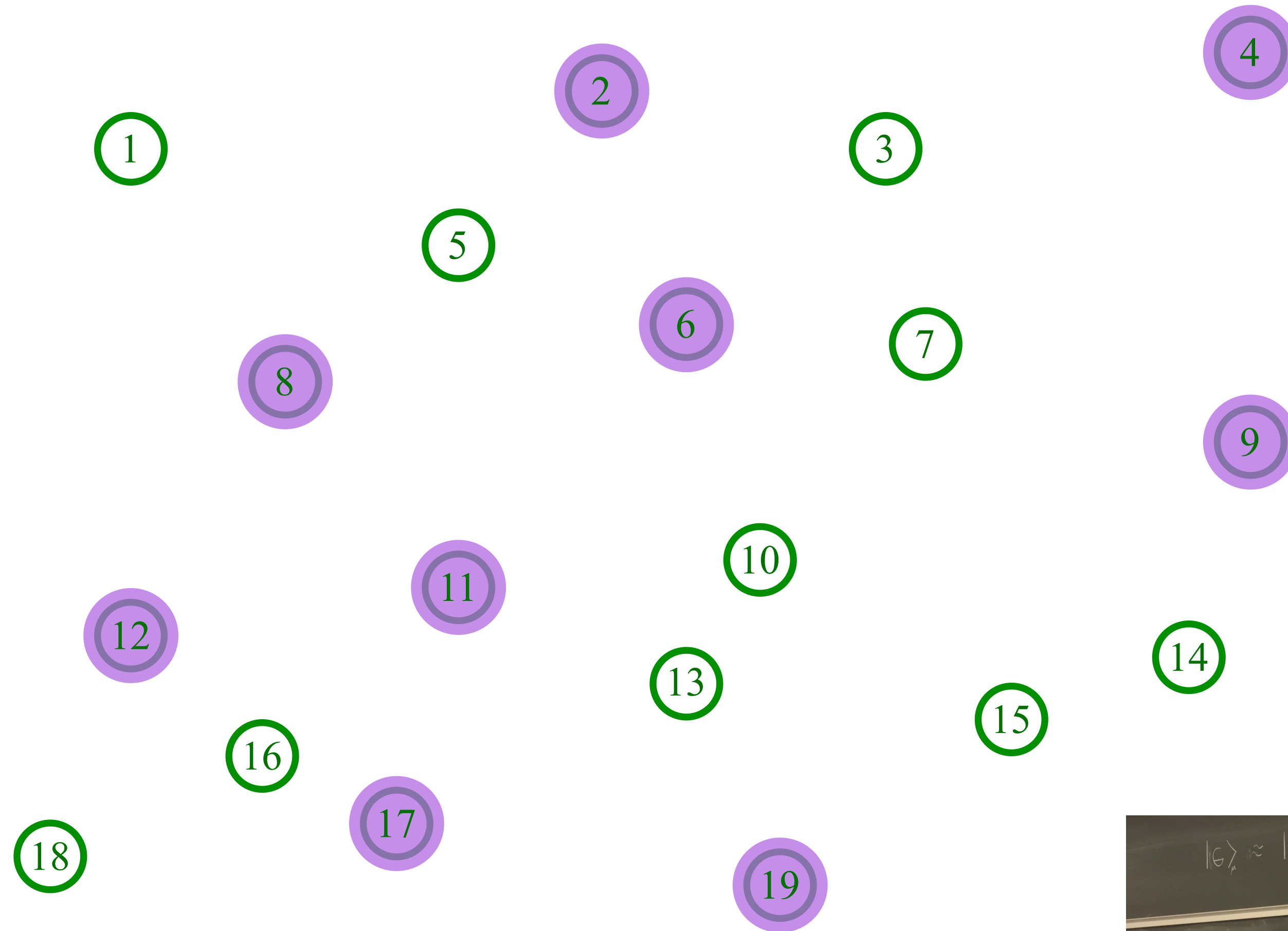


Pick a set of random positions

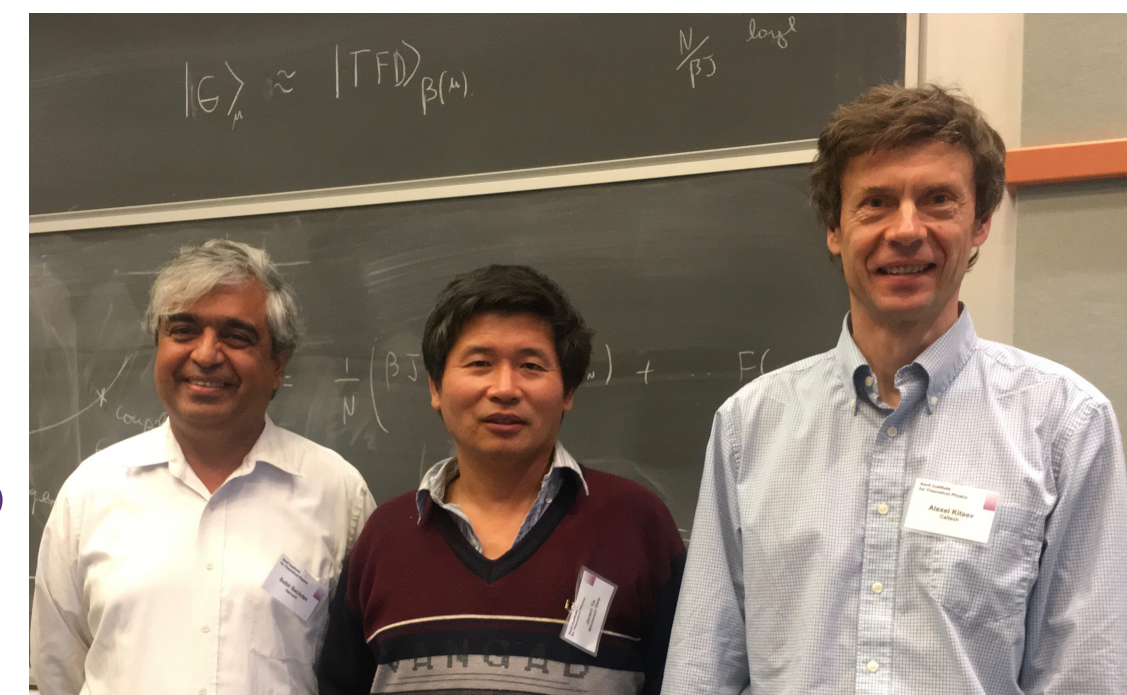


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Sachdev, Ye (1993); Kitaev (2015)



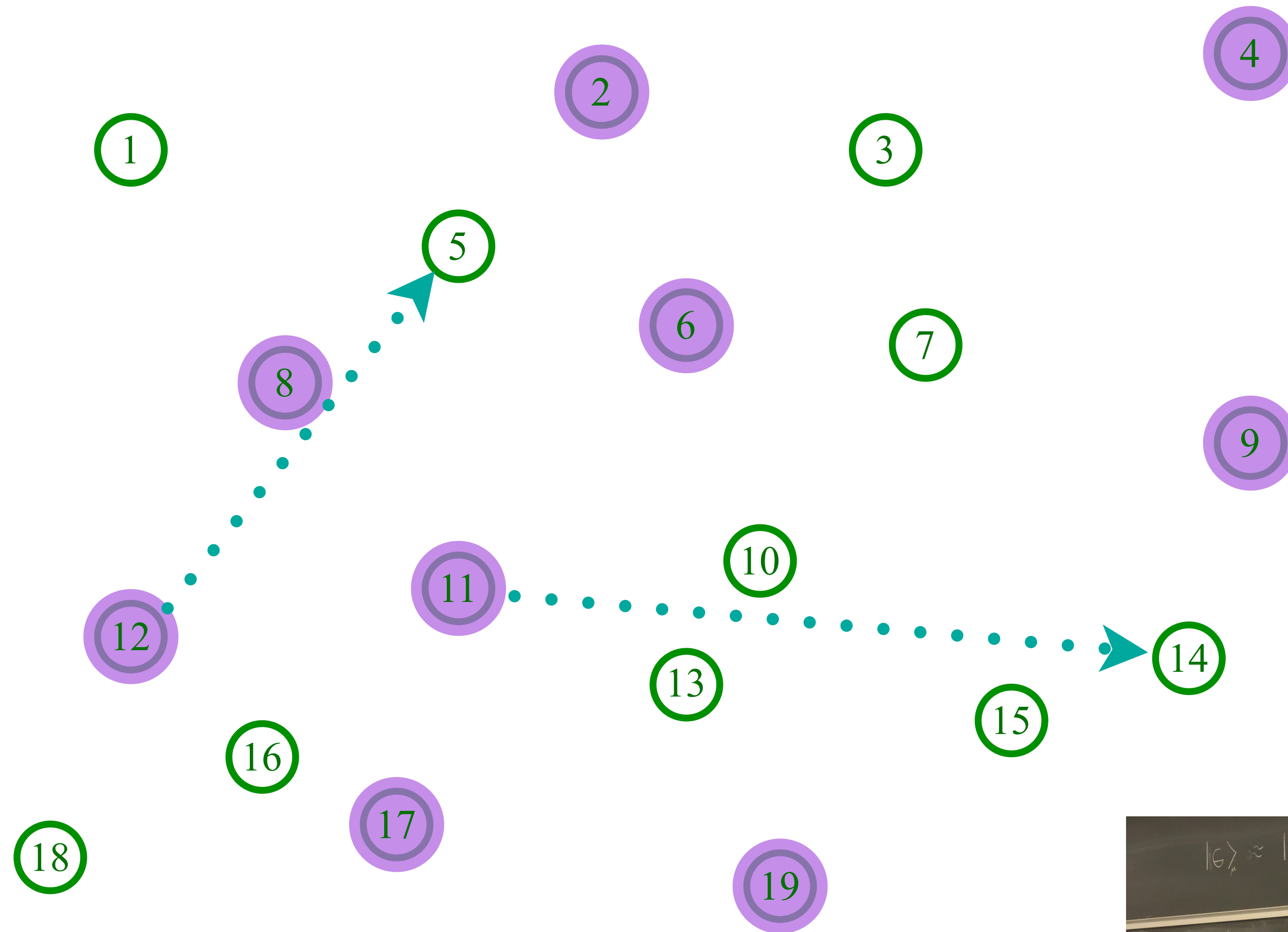
Place electrons randomly on some sites



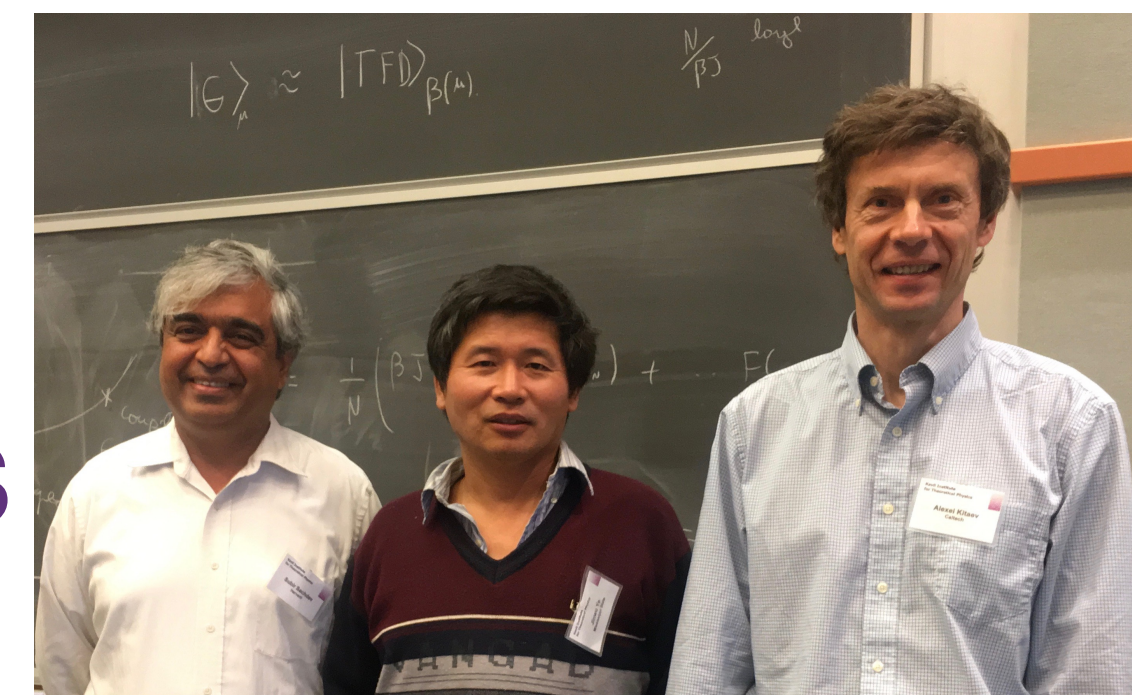
# The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



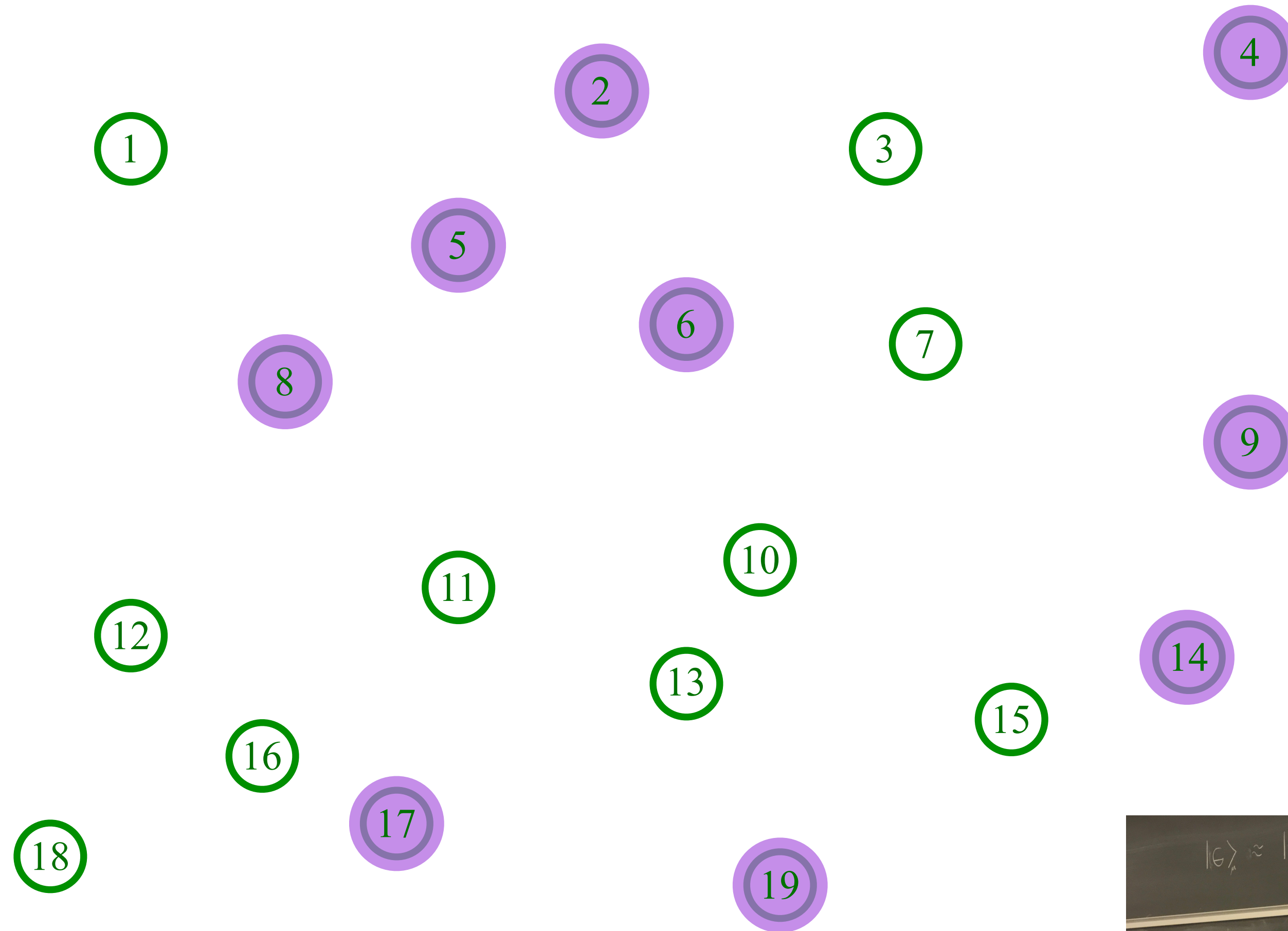
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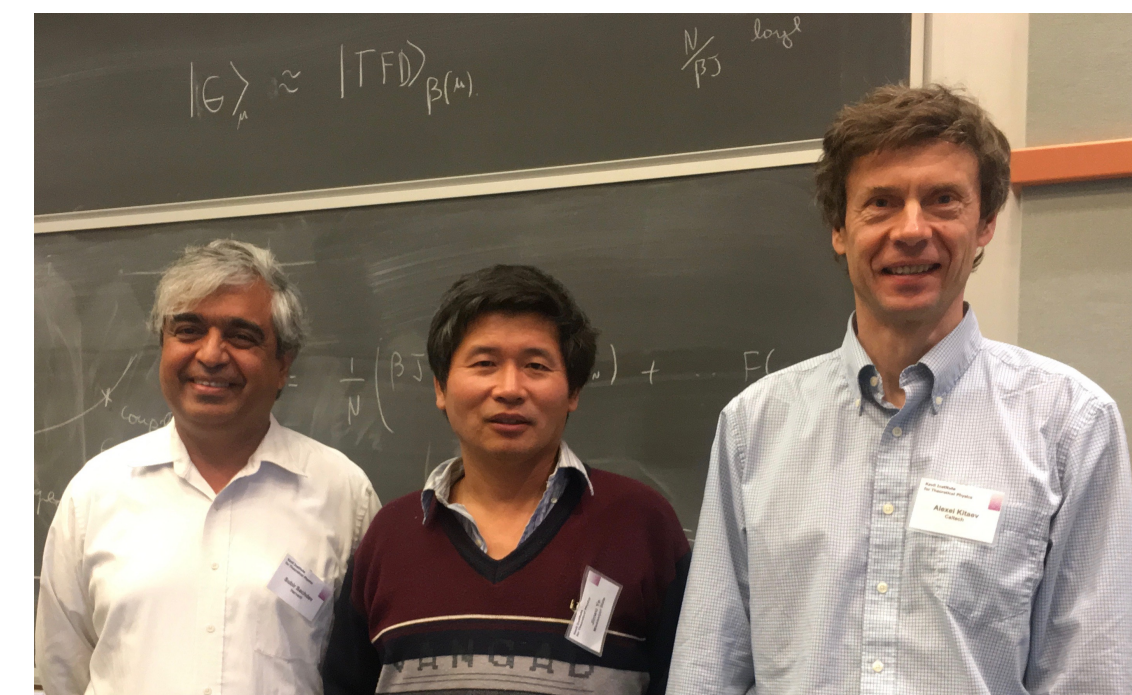
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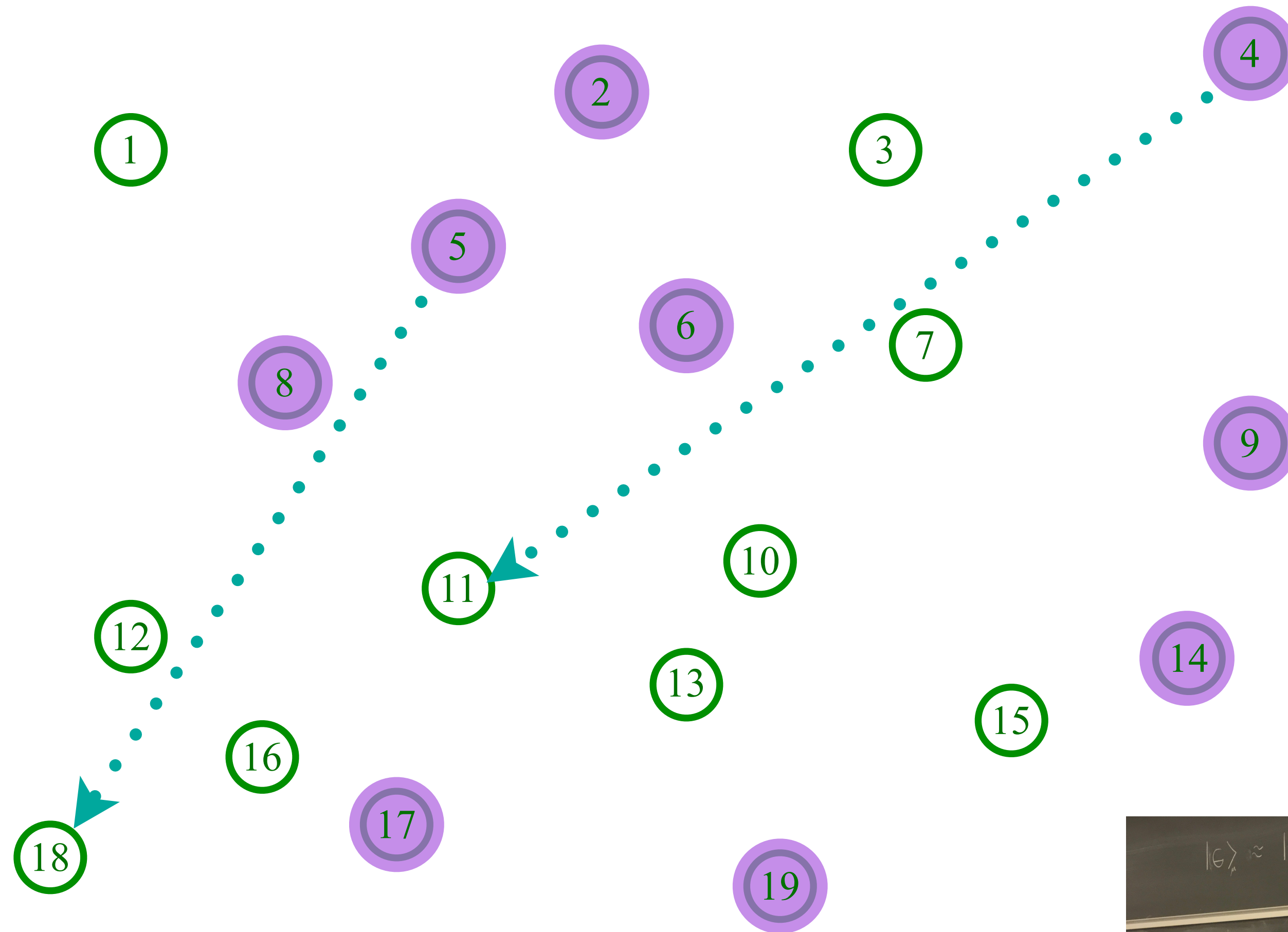
Entangle electrons pairwise randomly



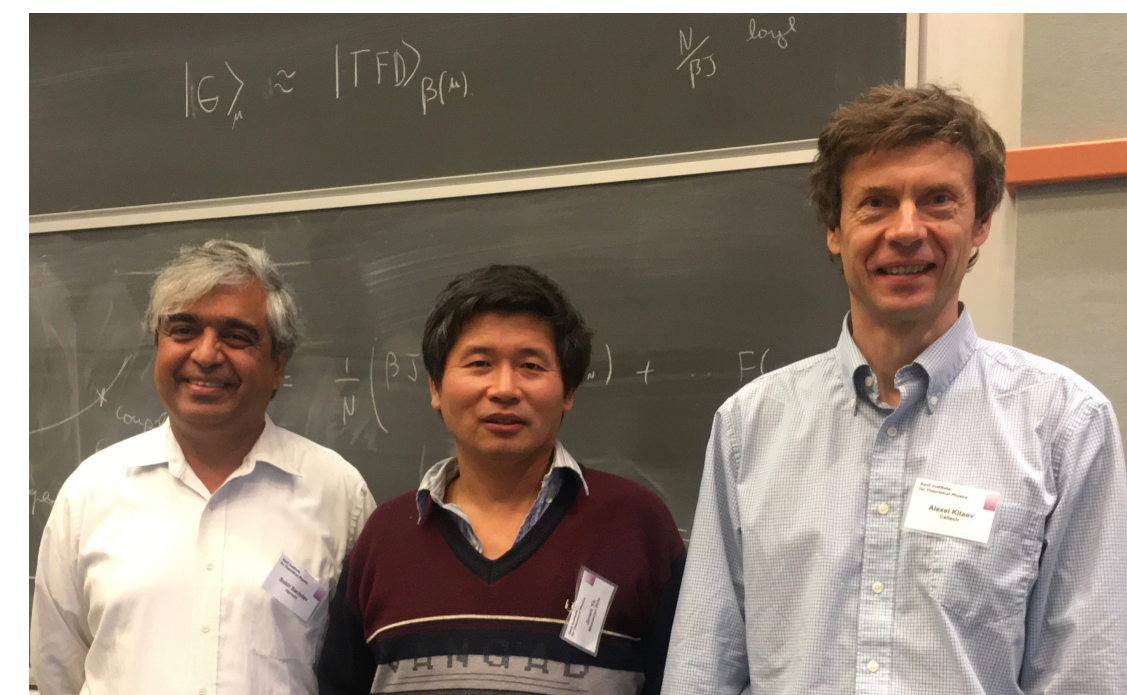
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Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



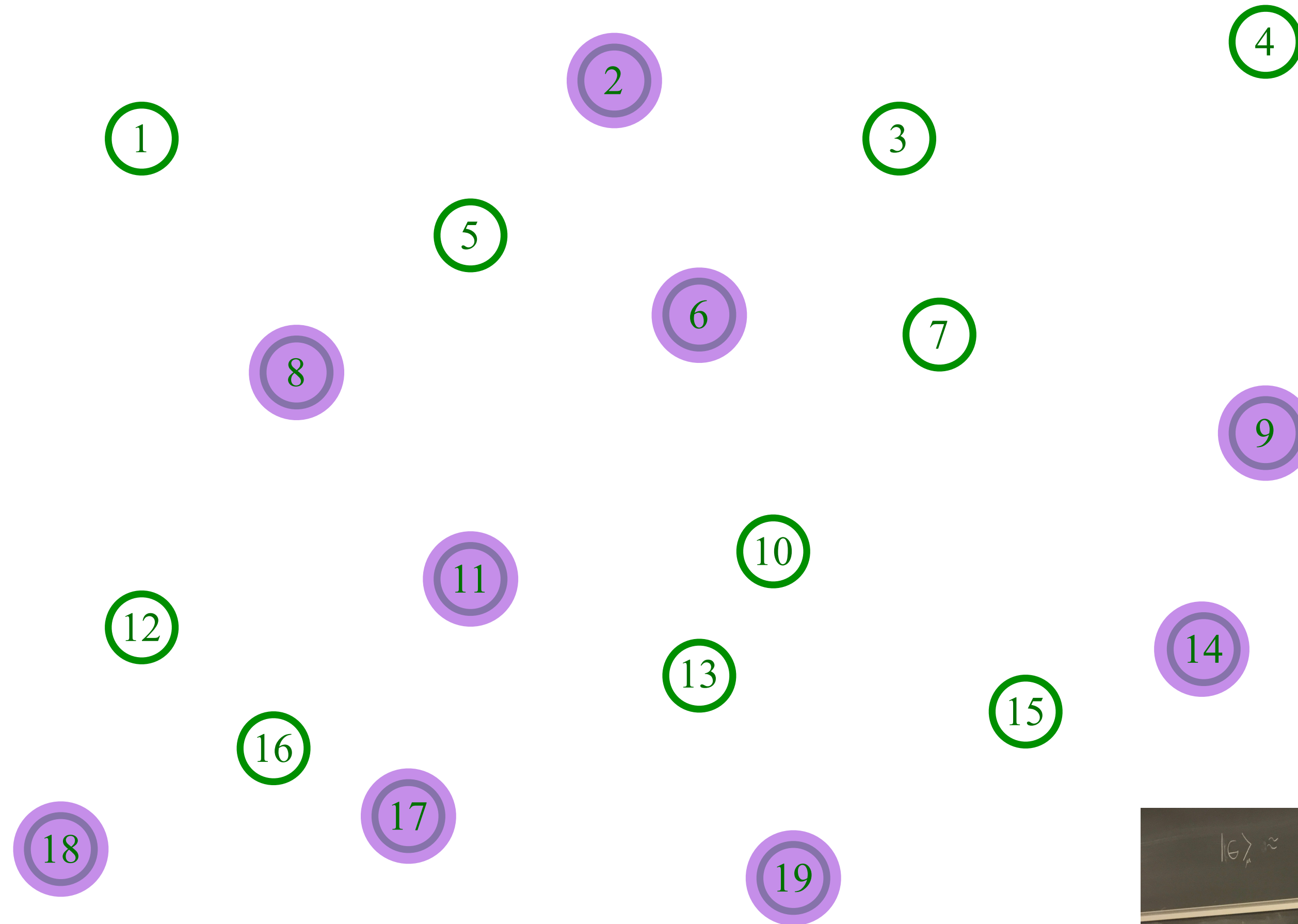
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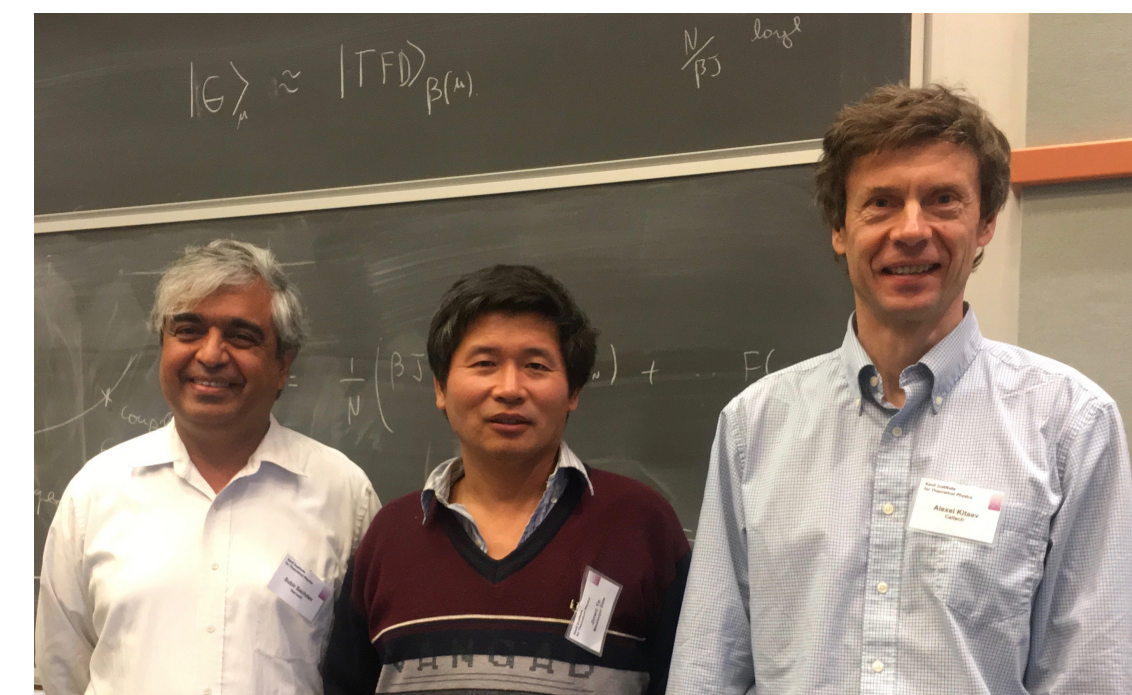
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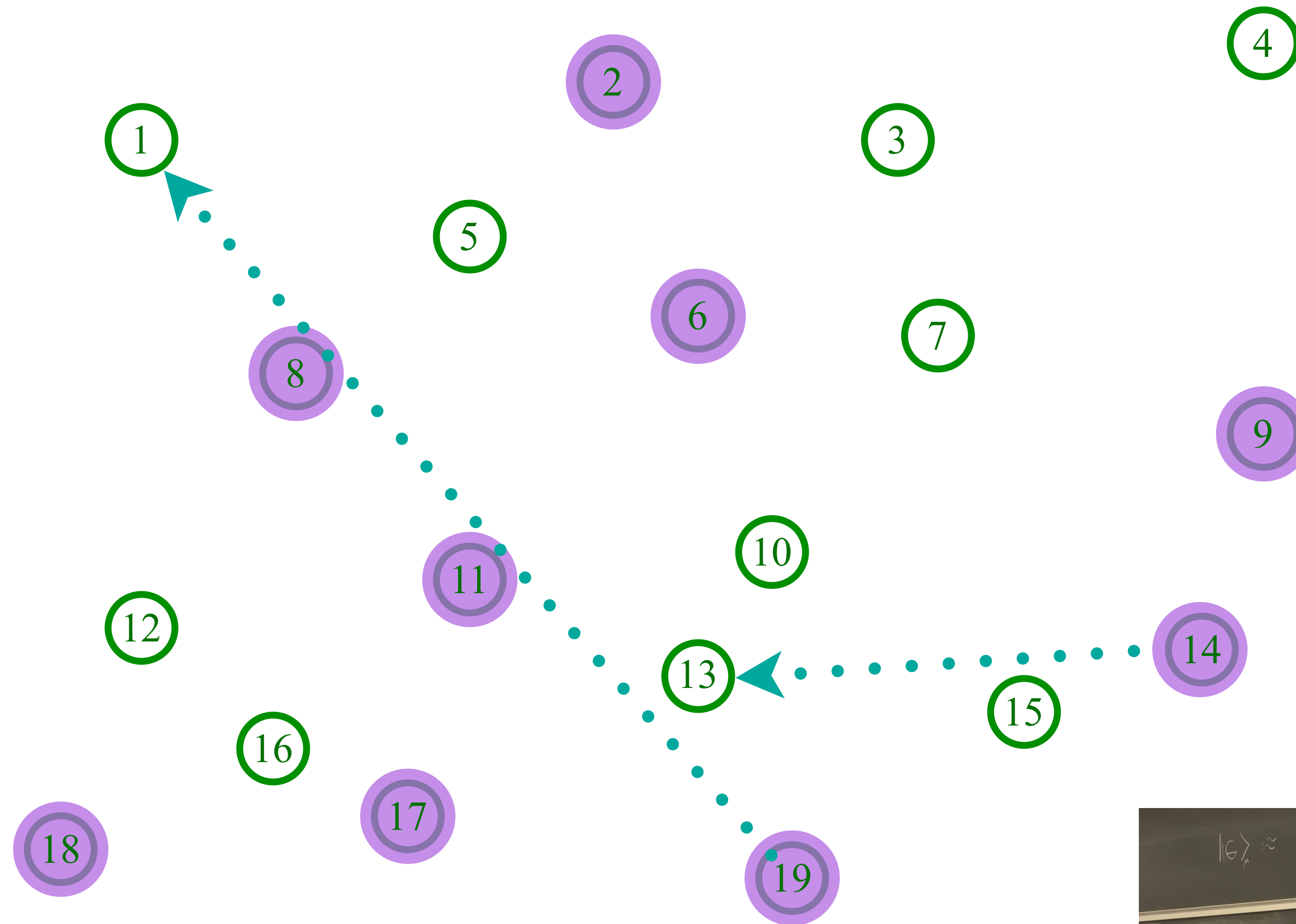
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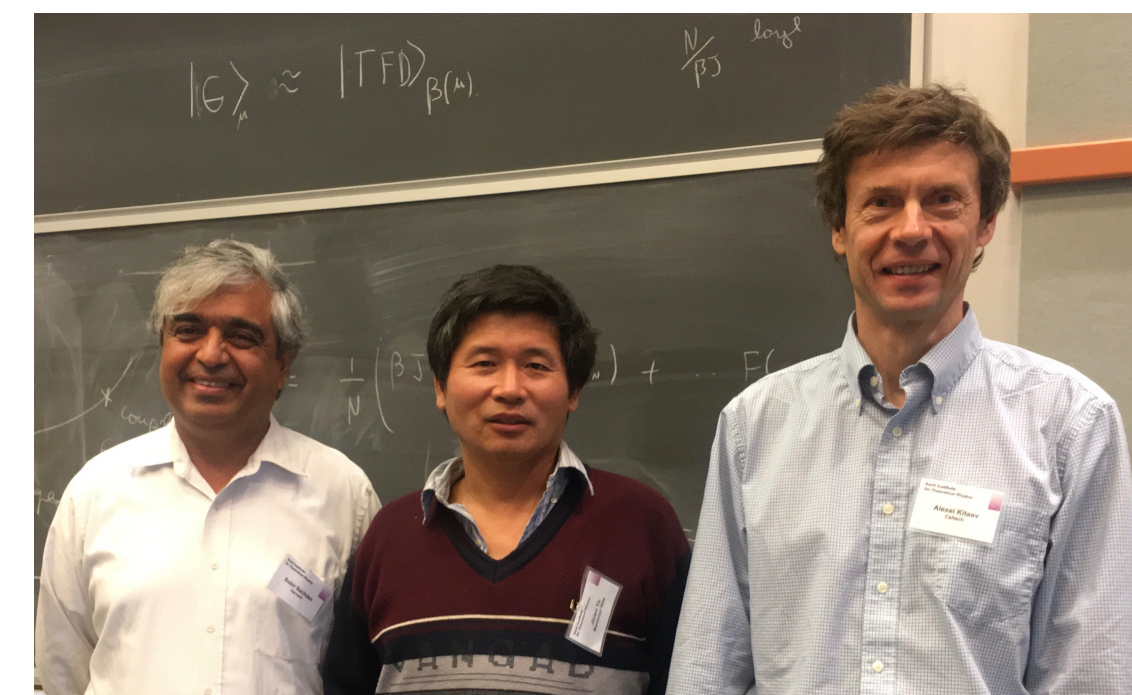
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Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



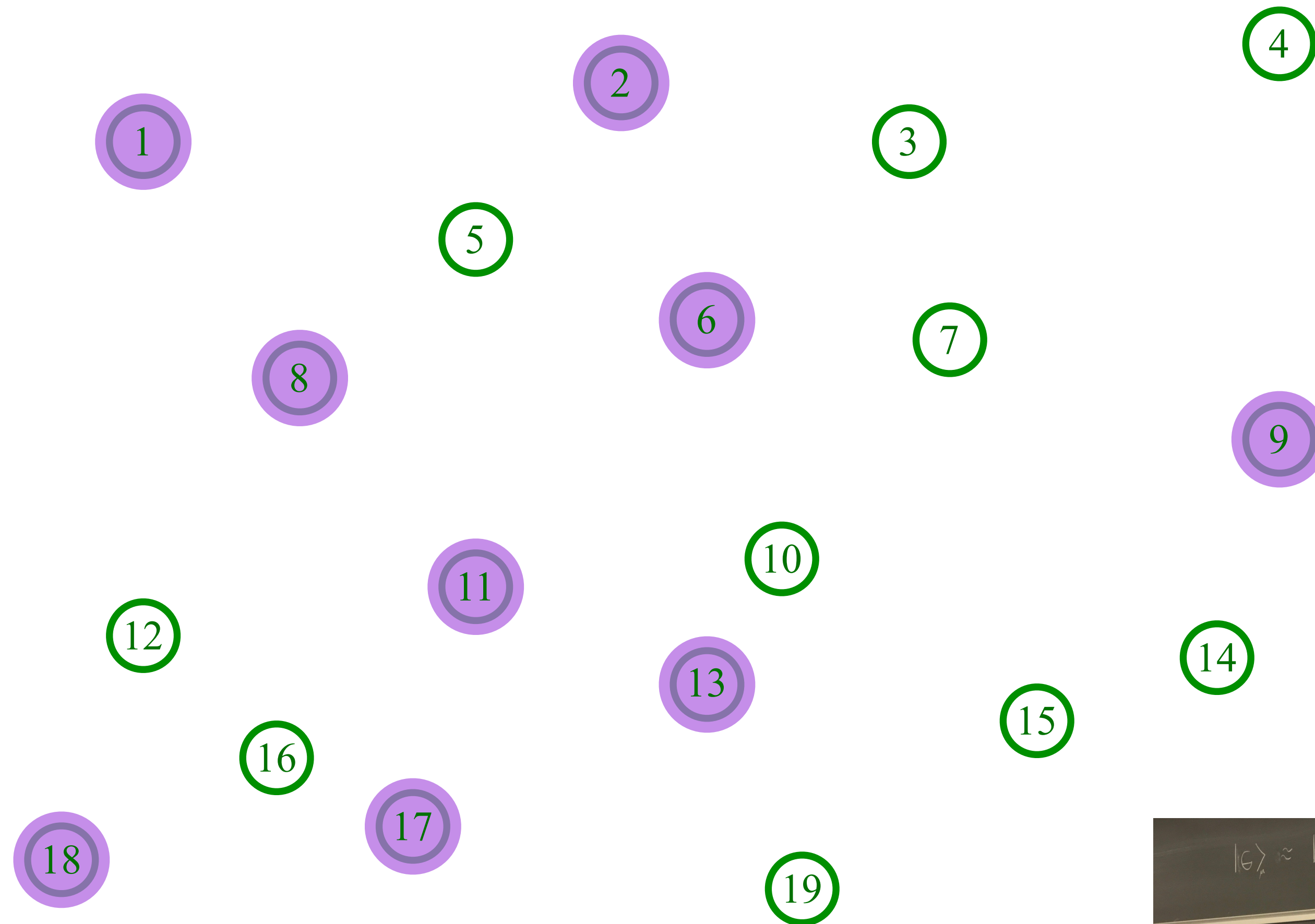
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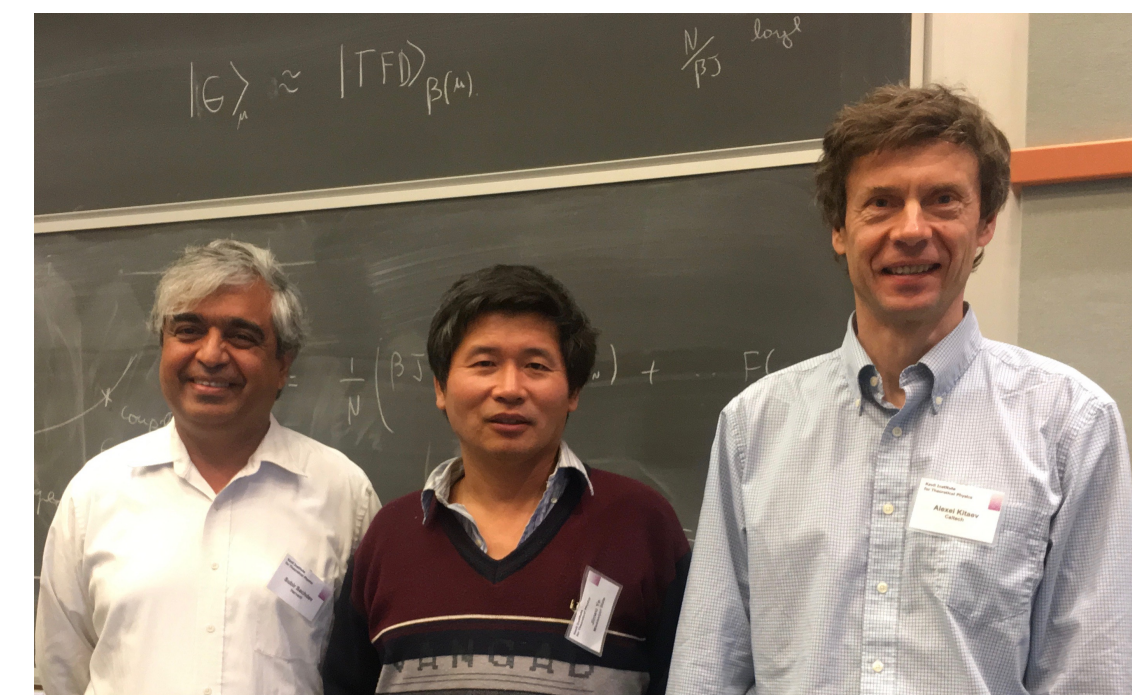
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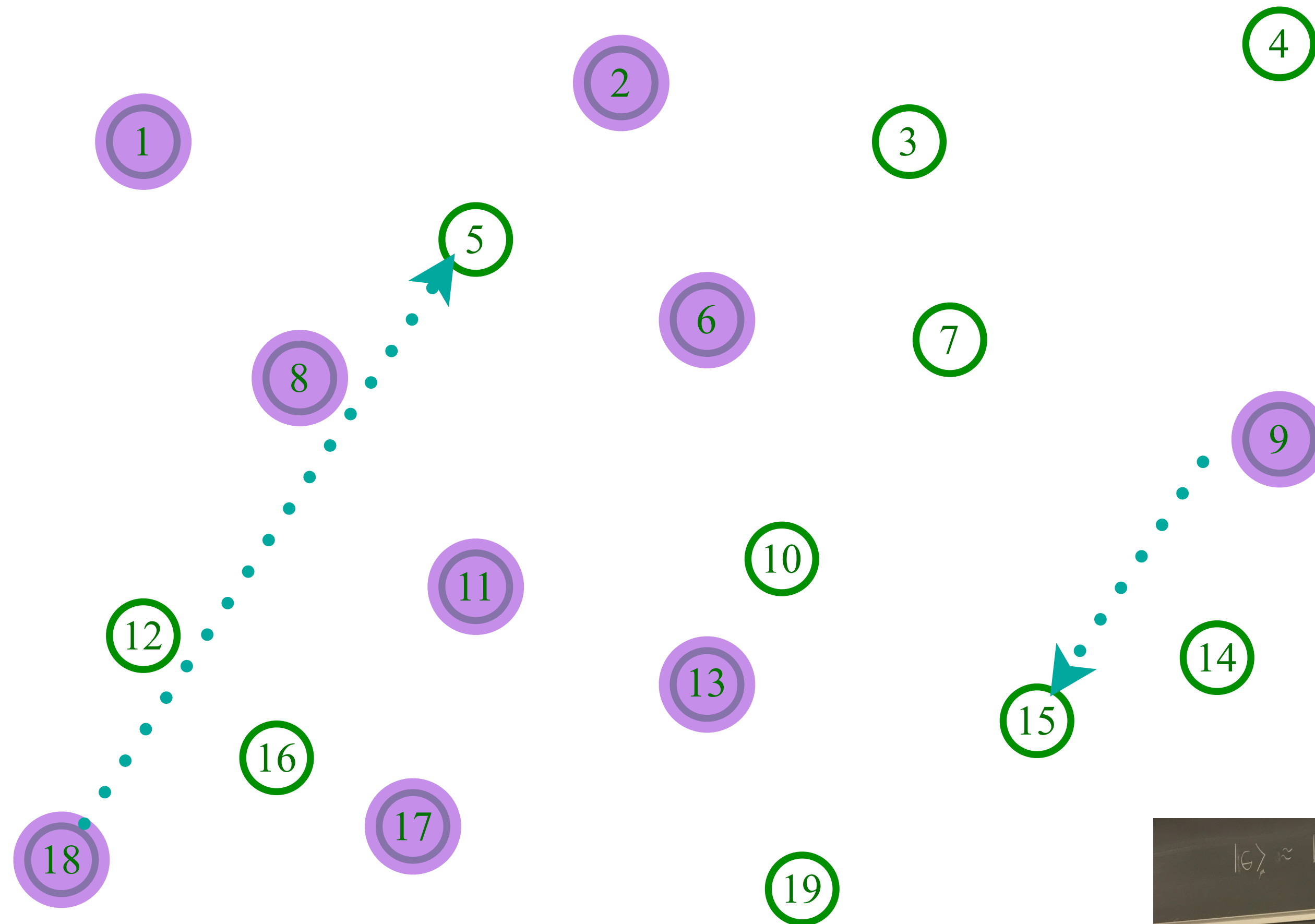
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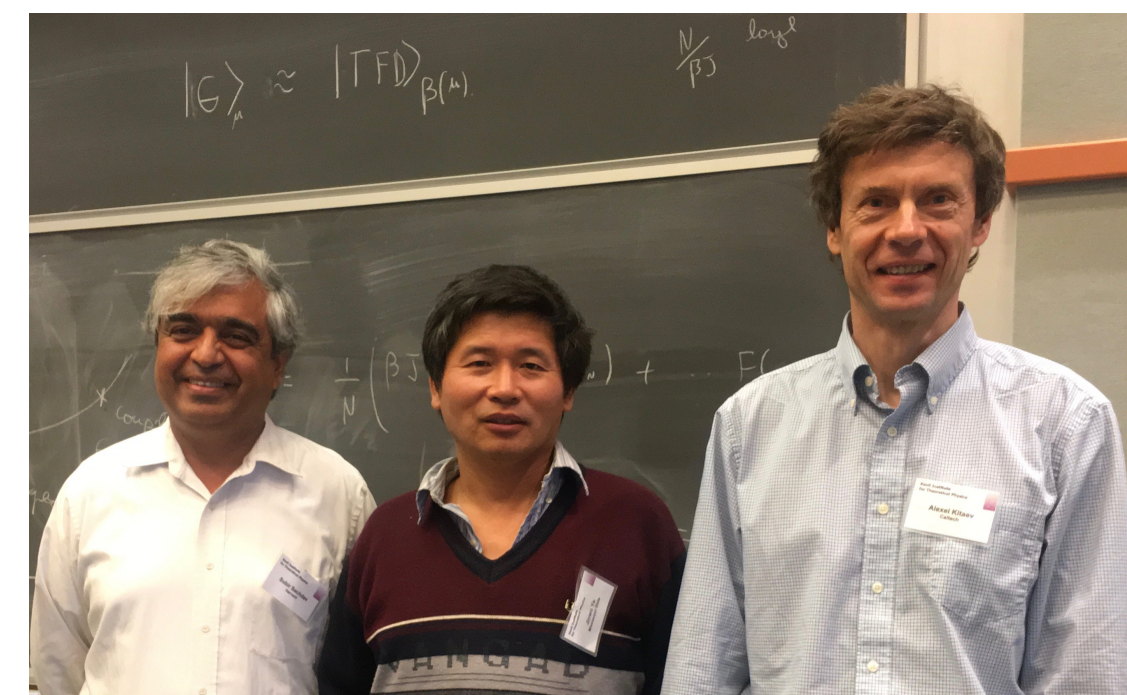
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Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



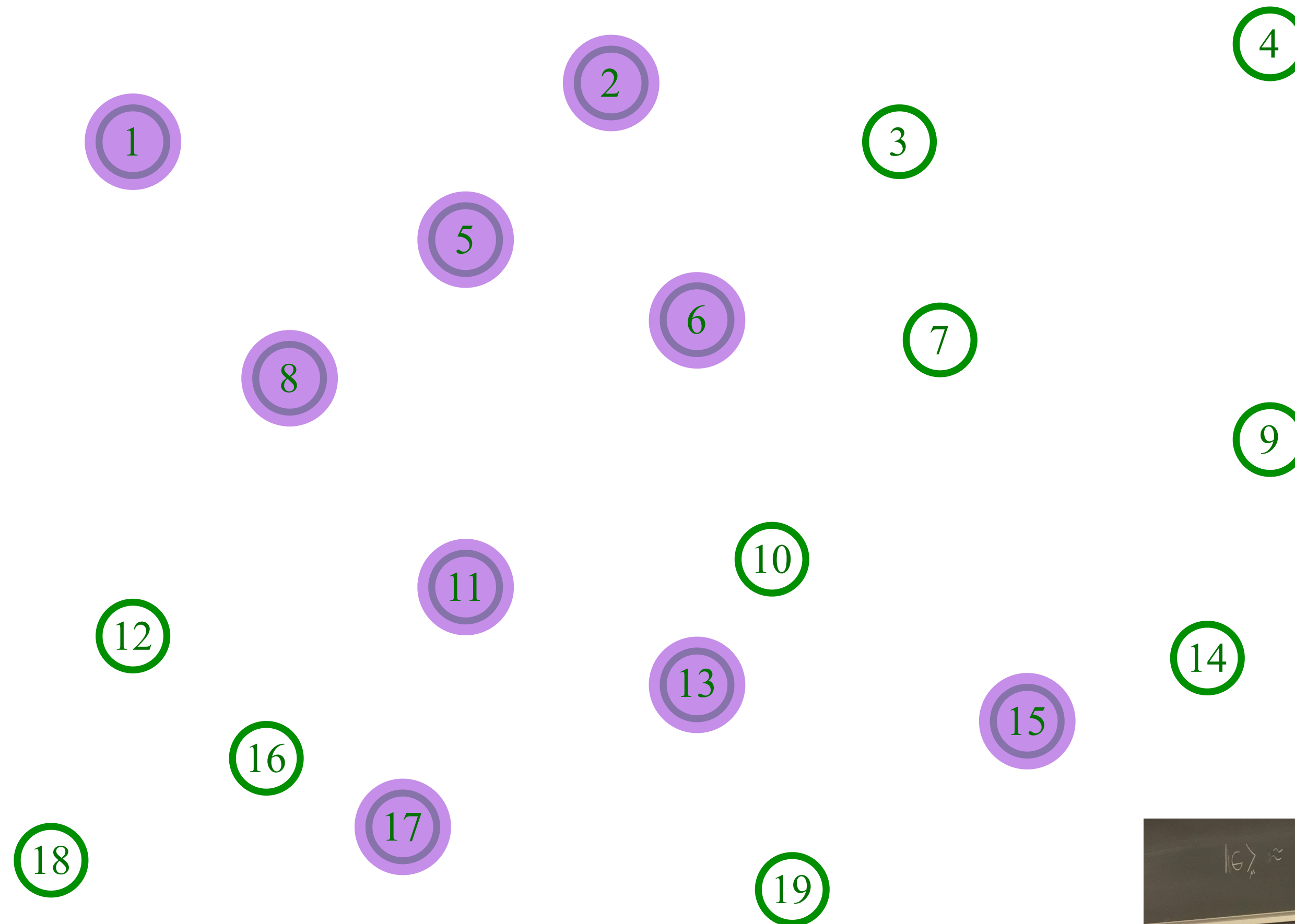
Entangle electrons pairwise randomly



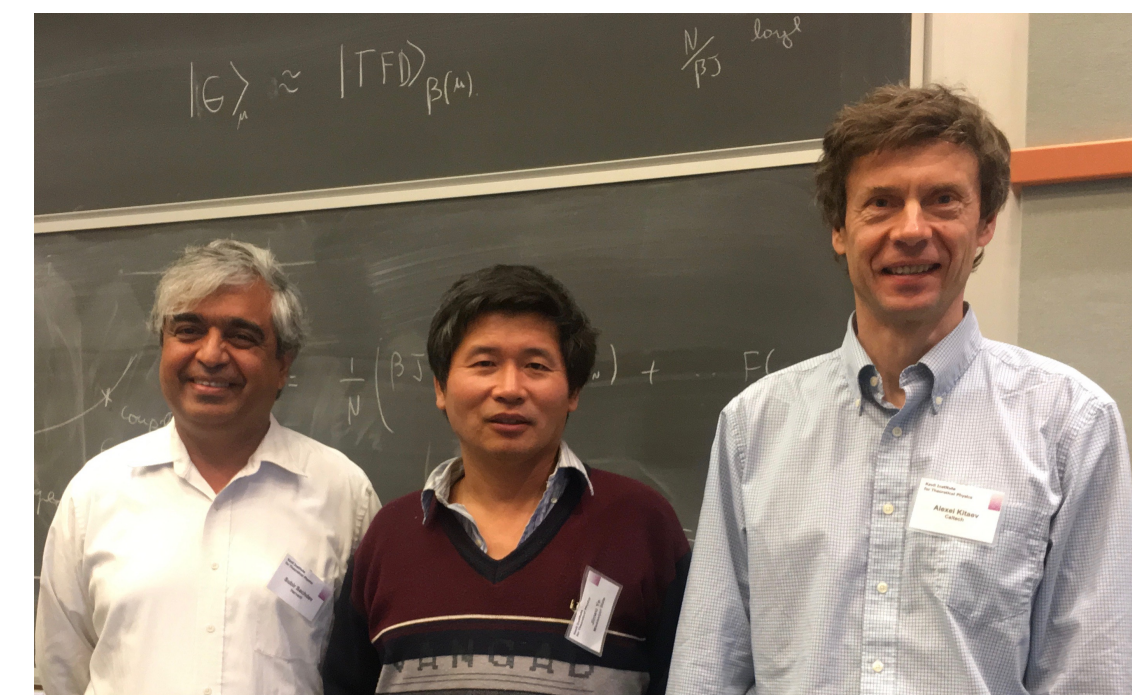
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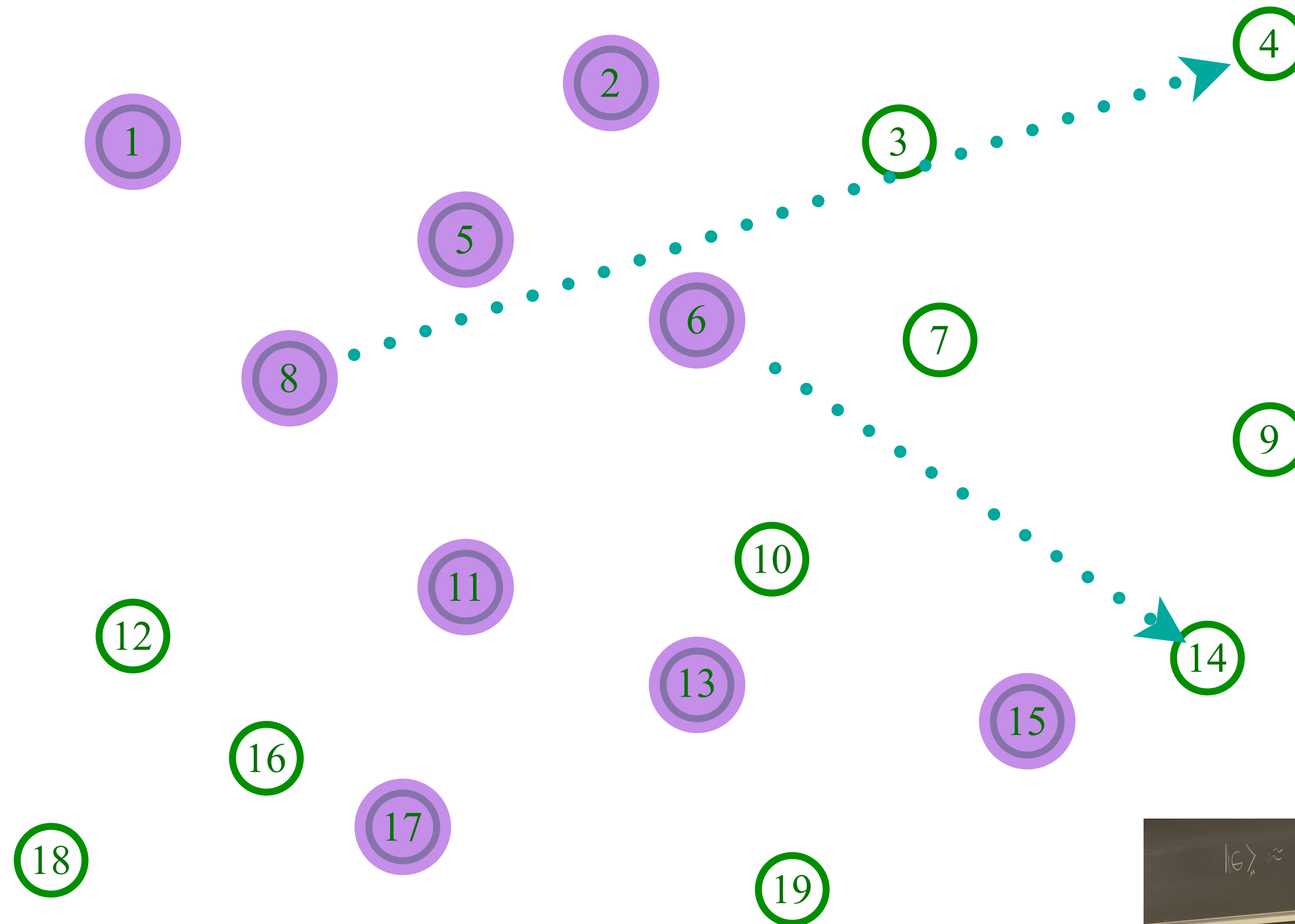
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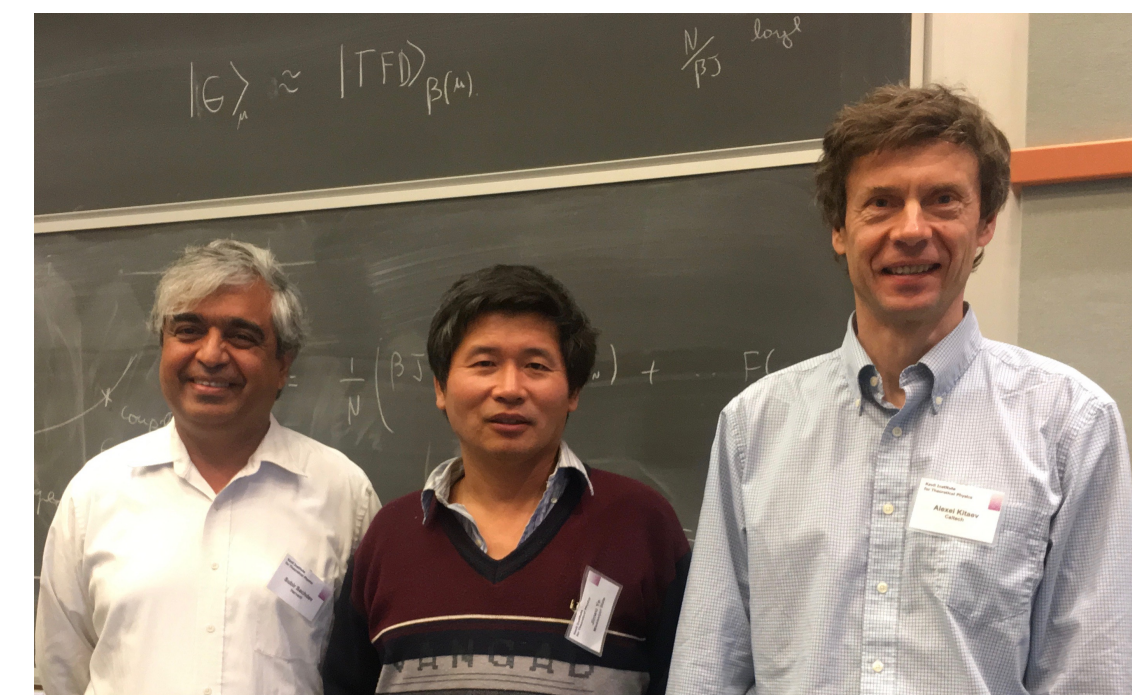
# The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



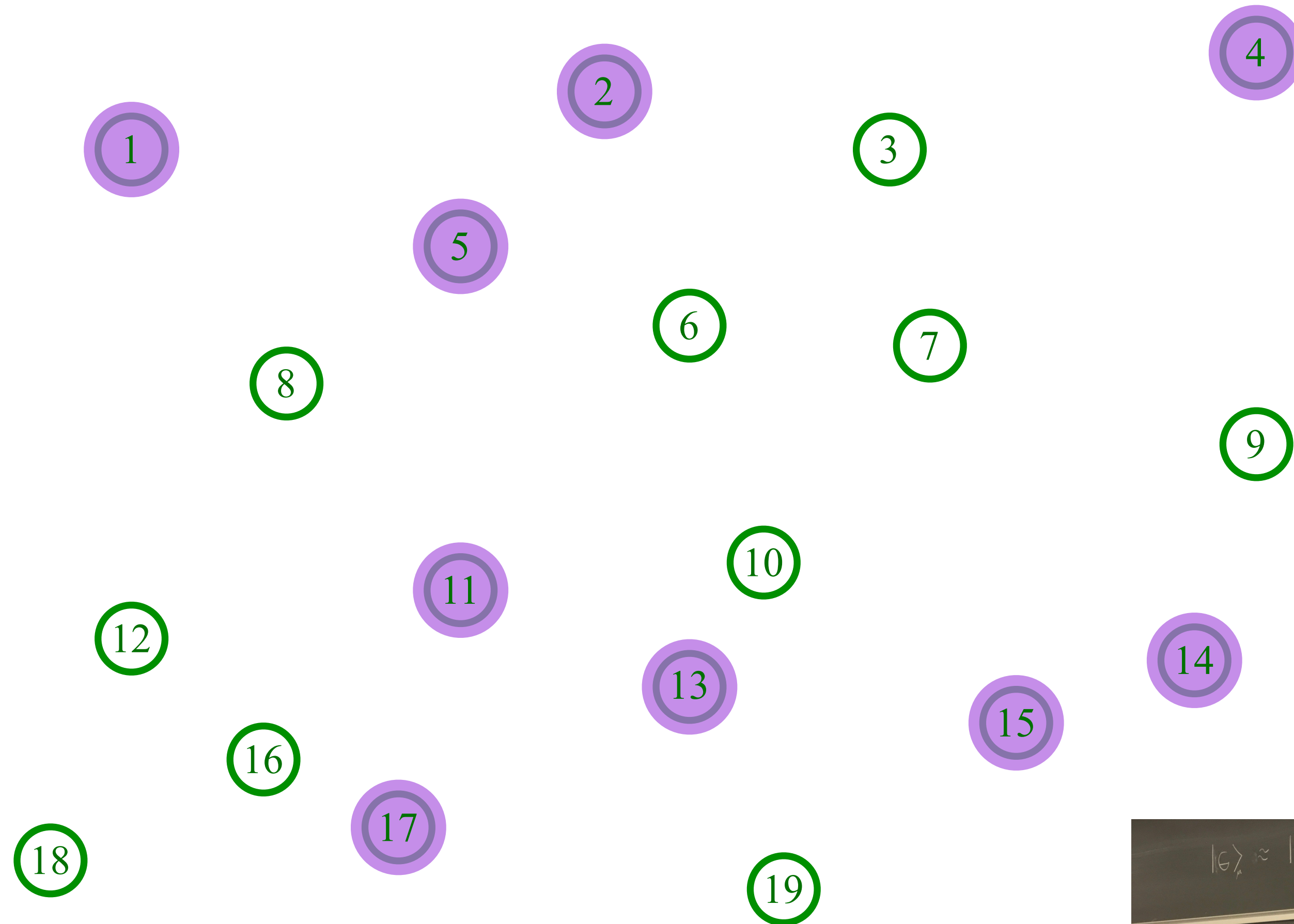
Entangle electrons pairwise randomly



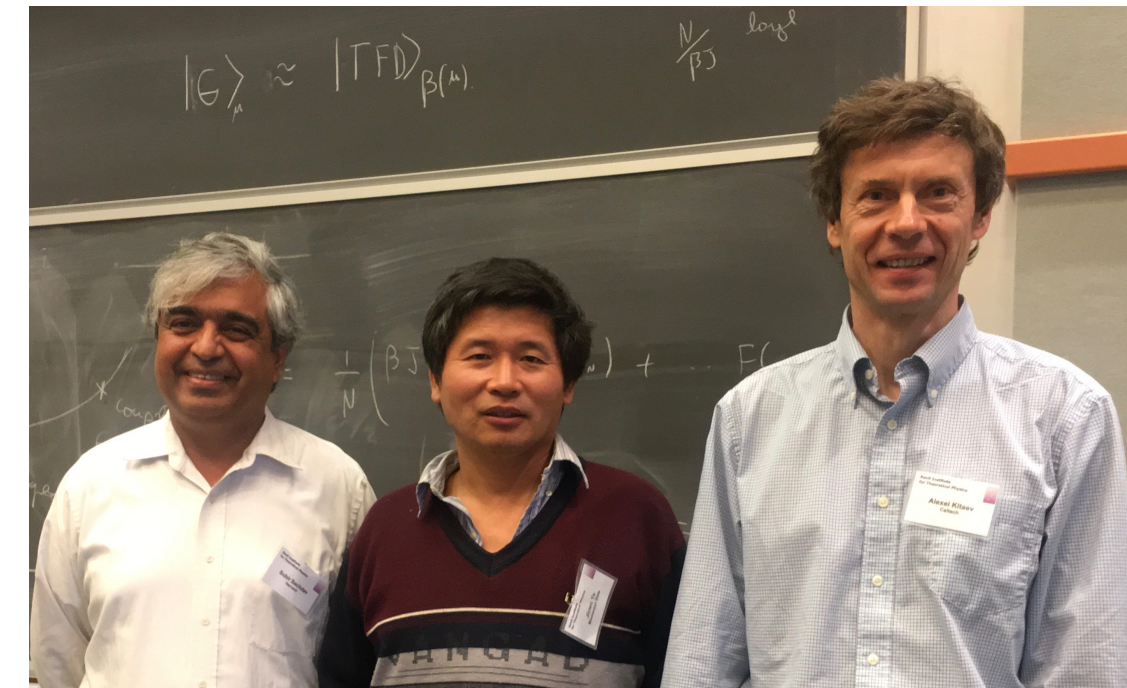
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Sachdev, Ye (1993); Kitaev (2015)

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Entangle electrons pairwise randomly



# The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle  
quantum entanglement.

No quasiparticles: yields a metal in which  
current is carried  
not by individual electrons,  
but by an entangled “quantum soup”

# The SYK model

Consequences of emergent time-reparameterization and conformal symmetries  
in low-energy theory in 0+1 spacetime dimensions:

## 1. Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right) \text{ independent of } U.$$

No bosons, fermions, anyons ...

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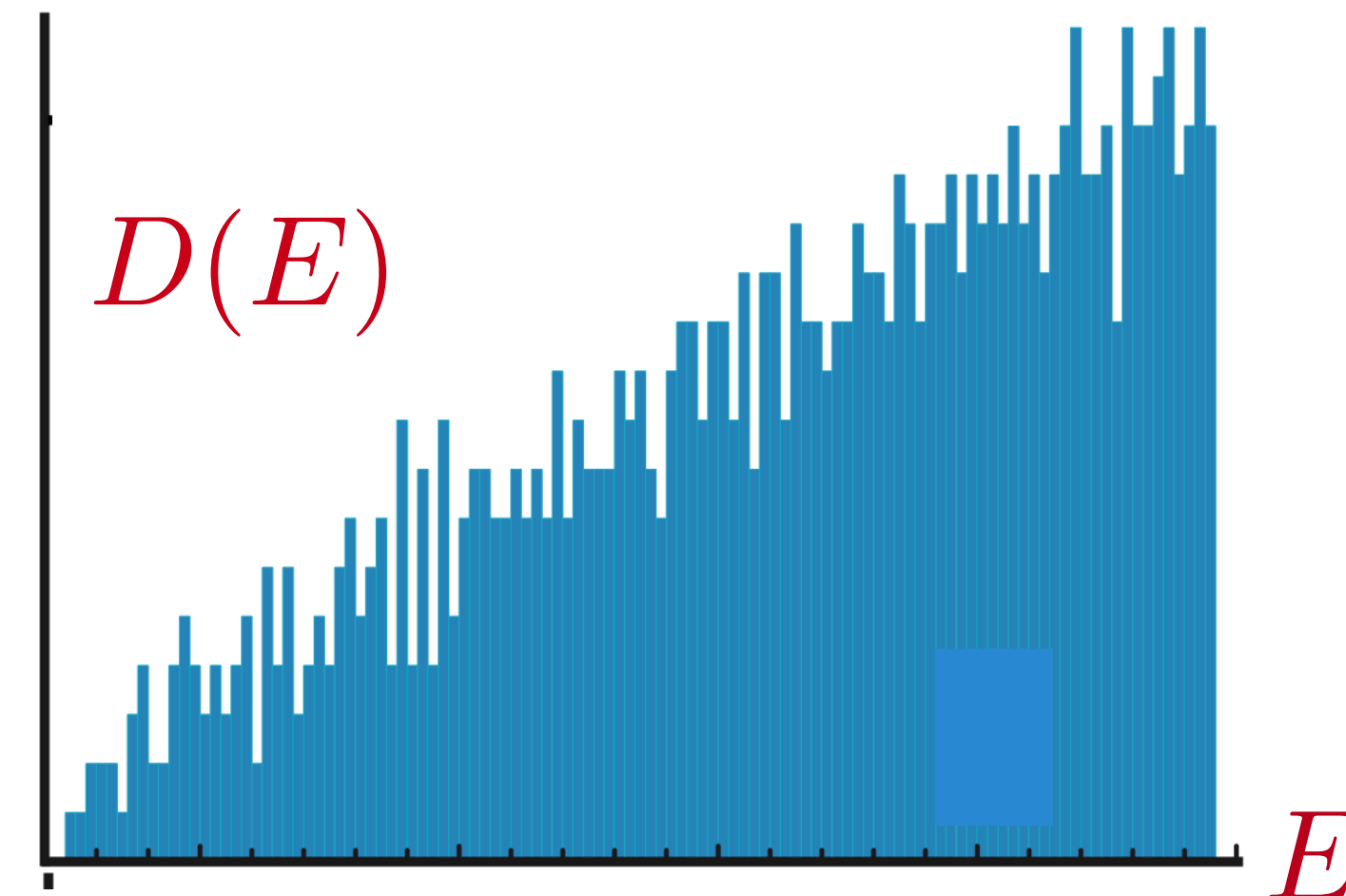
No bosons, fermions, anyons ...



## 2. Zero temperature entropy without exponential ground state degeneracy!

$$\lim_{T \rightarrow 0} \lim_{N \rightarrow \infty} \frac{1}{N} S(T) = s_0 \quad , \quad D(E \rightarrow 0) = e^{N s_0} f_{\text{smooth}}(E)$$

$$s_0 = 0.46484769917080510749\dots \text{ for } Q = 1/2.$$



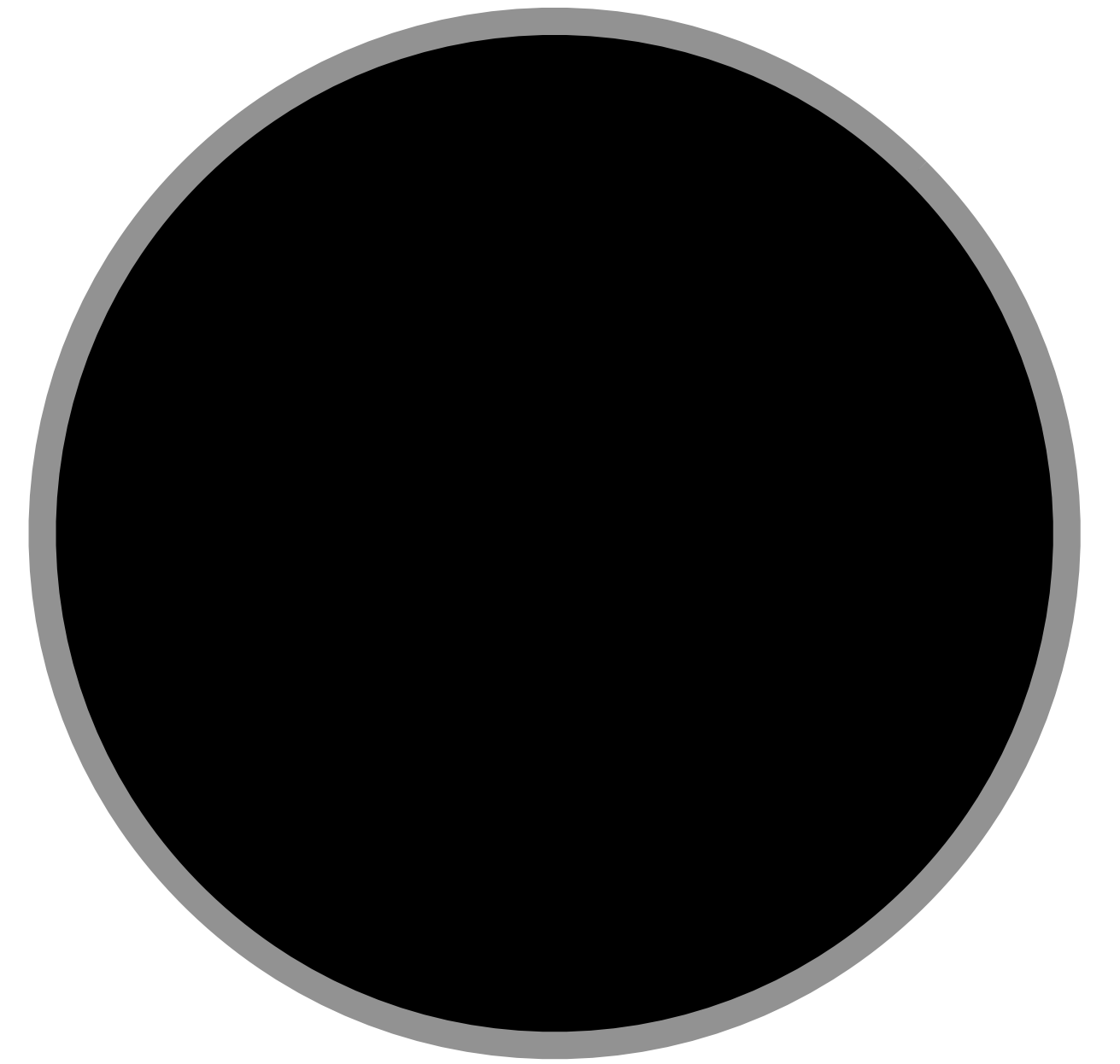
From  
the SYK model  
to  
black holes

# Black Holes

Objects so dense that light is gravitationally bound to them.



Horizon radius  $R = \frac{2GM}{c^2}$



Karl Schwarzschild (1916)

$G$  Newton's constant,  $c$  velocity of light,  $M$  mass of black hole  
For  $M = \text{earth's mass}$ ,  $R \approx 9 \text{ mm!}$



The supermassive black hole lurking at the heart of the Milky Way – Sagittarius A\* contains about 4.3 million solar masses

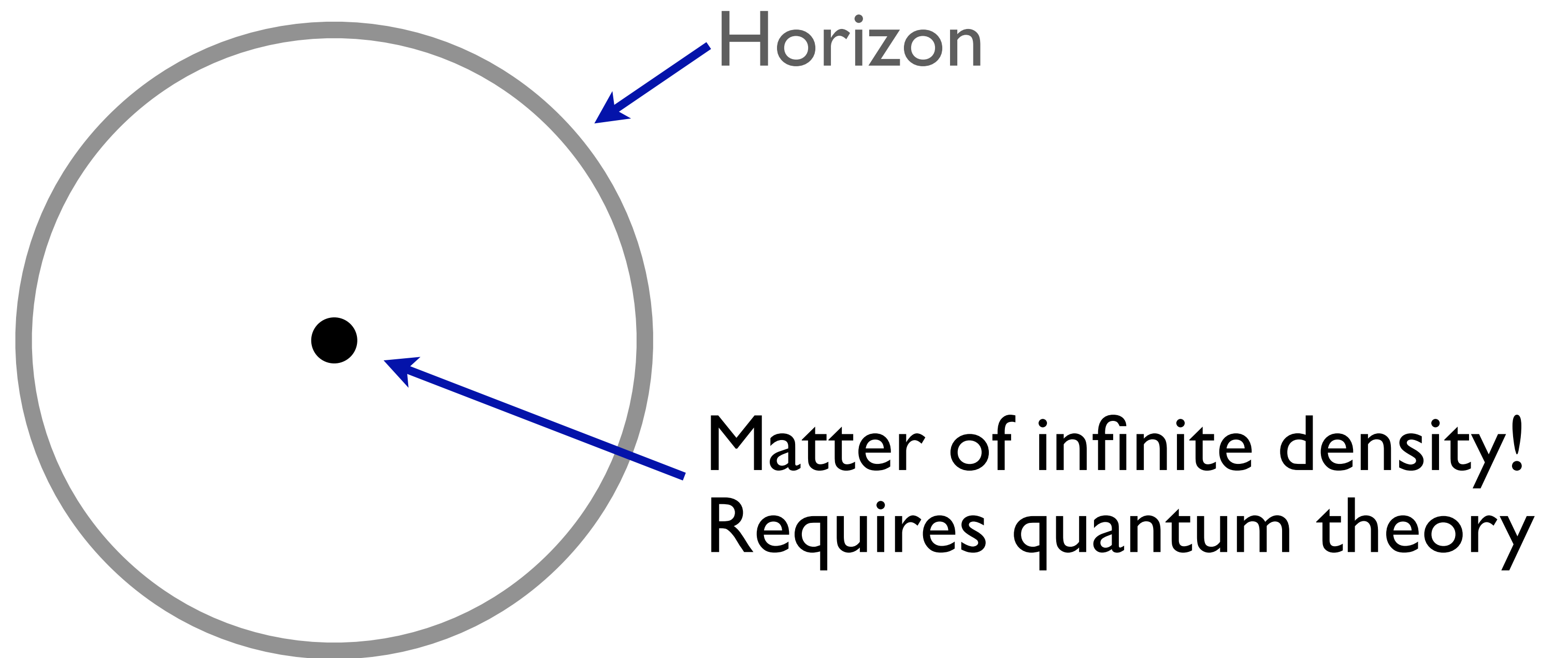
$$R = 1.3 \times 10^{11} \text{ m}$$

$\approx$  earth's orbit

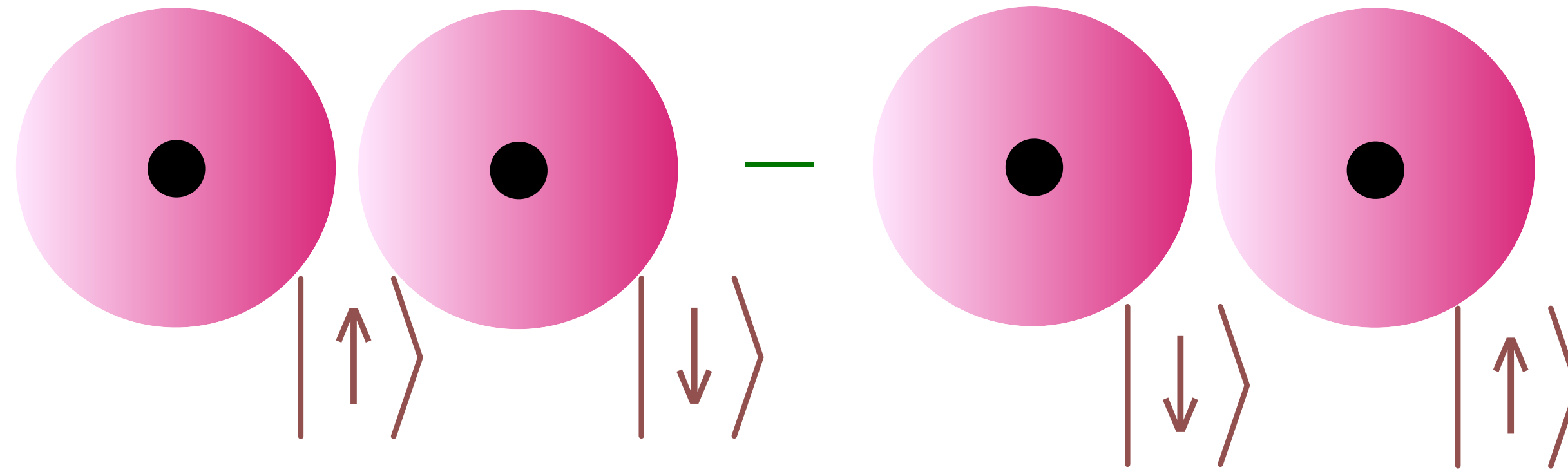
Event Horizon Telescope  
May 12, 2022

# What is inside a black hole ???

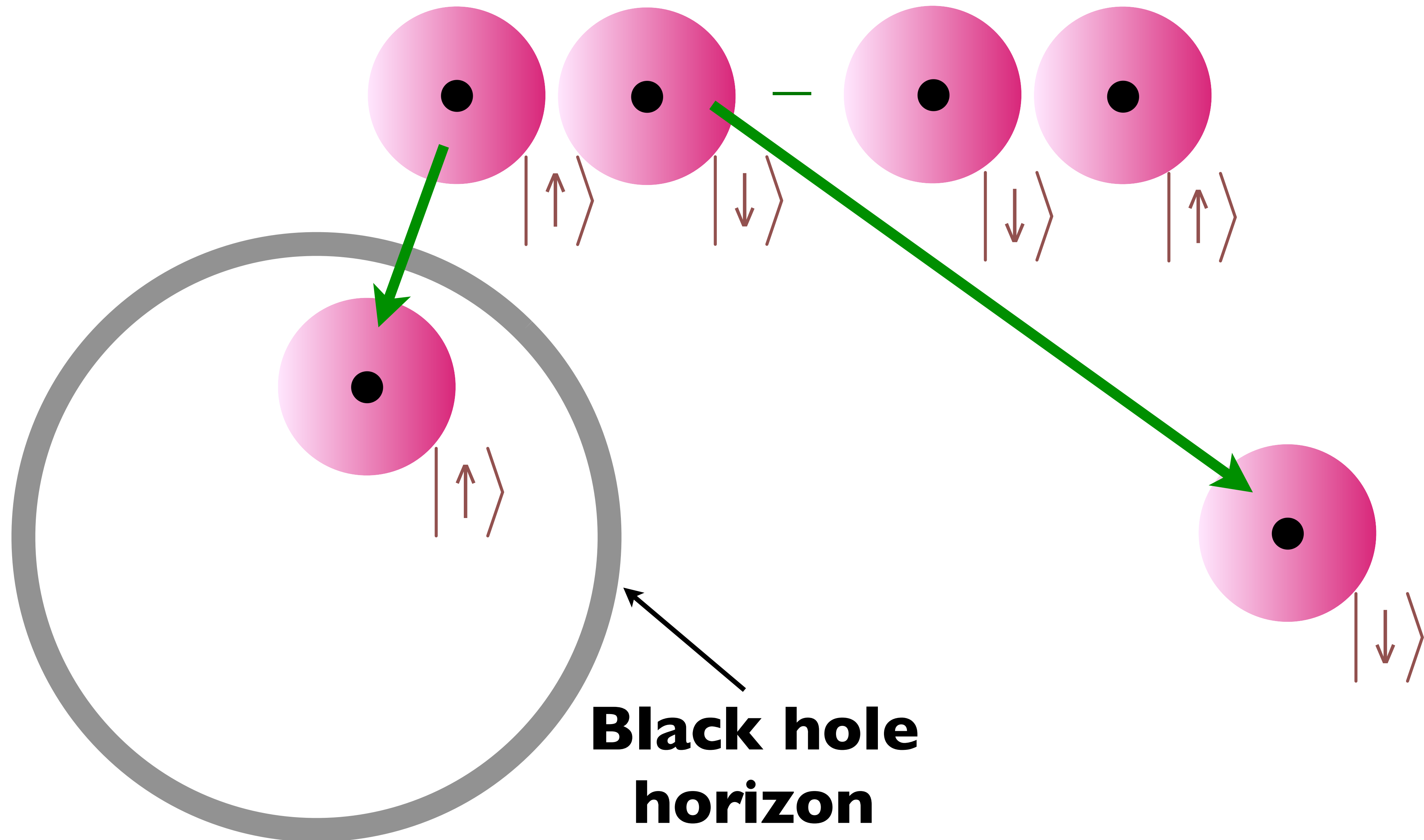
In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



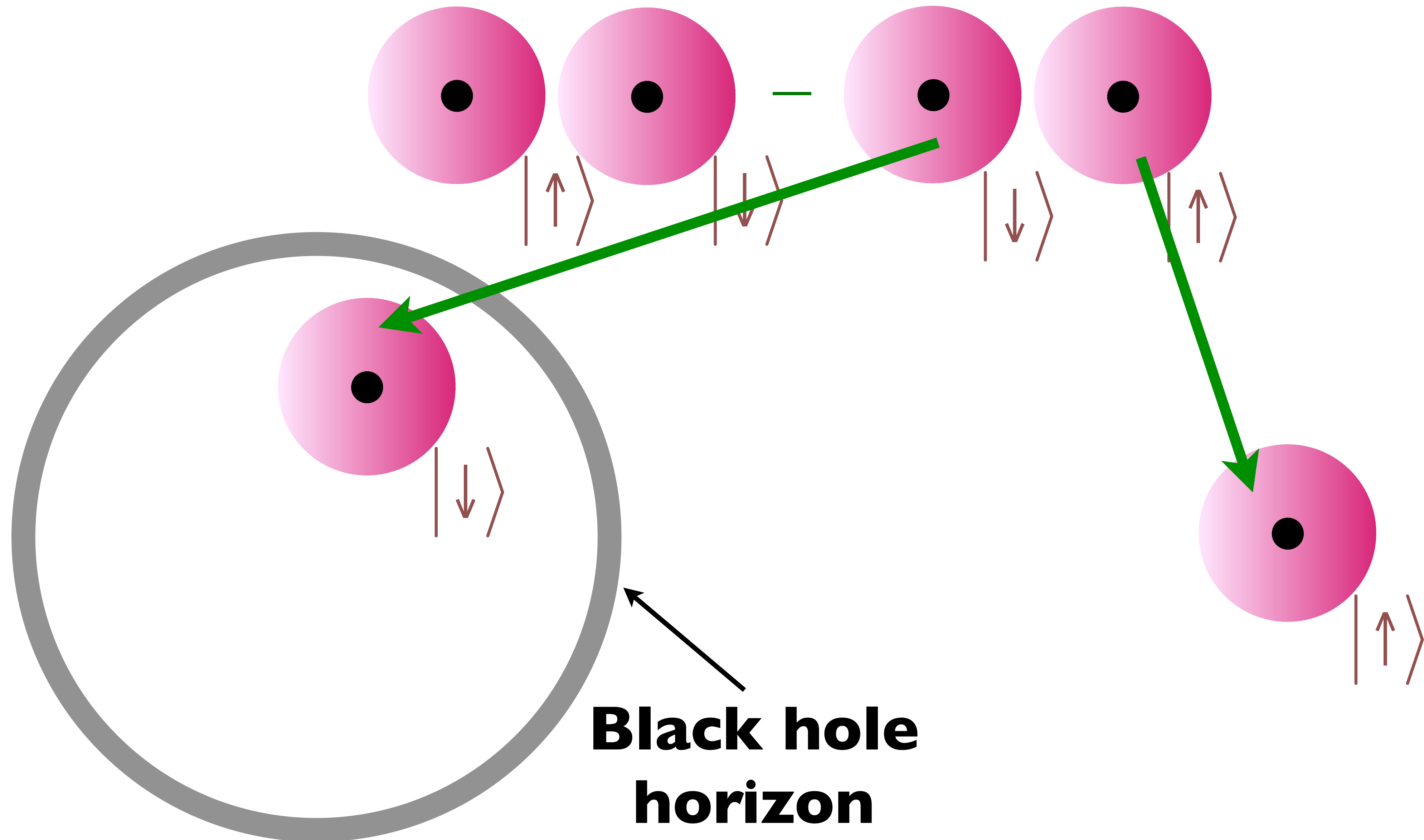
# Quantum Entanglement across a black hole horizon



# Quantum Entanglement across a black hole horizon



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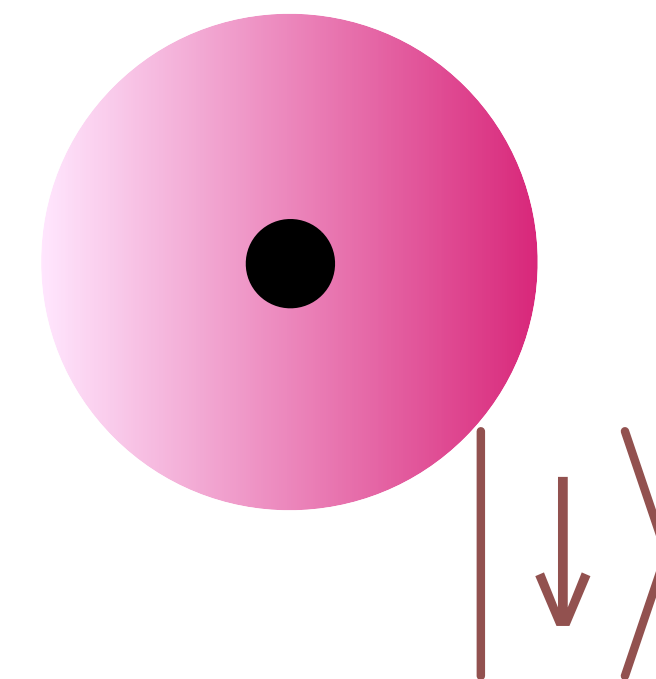
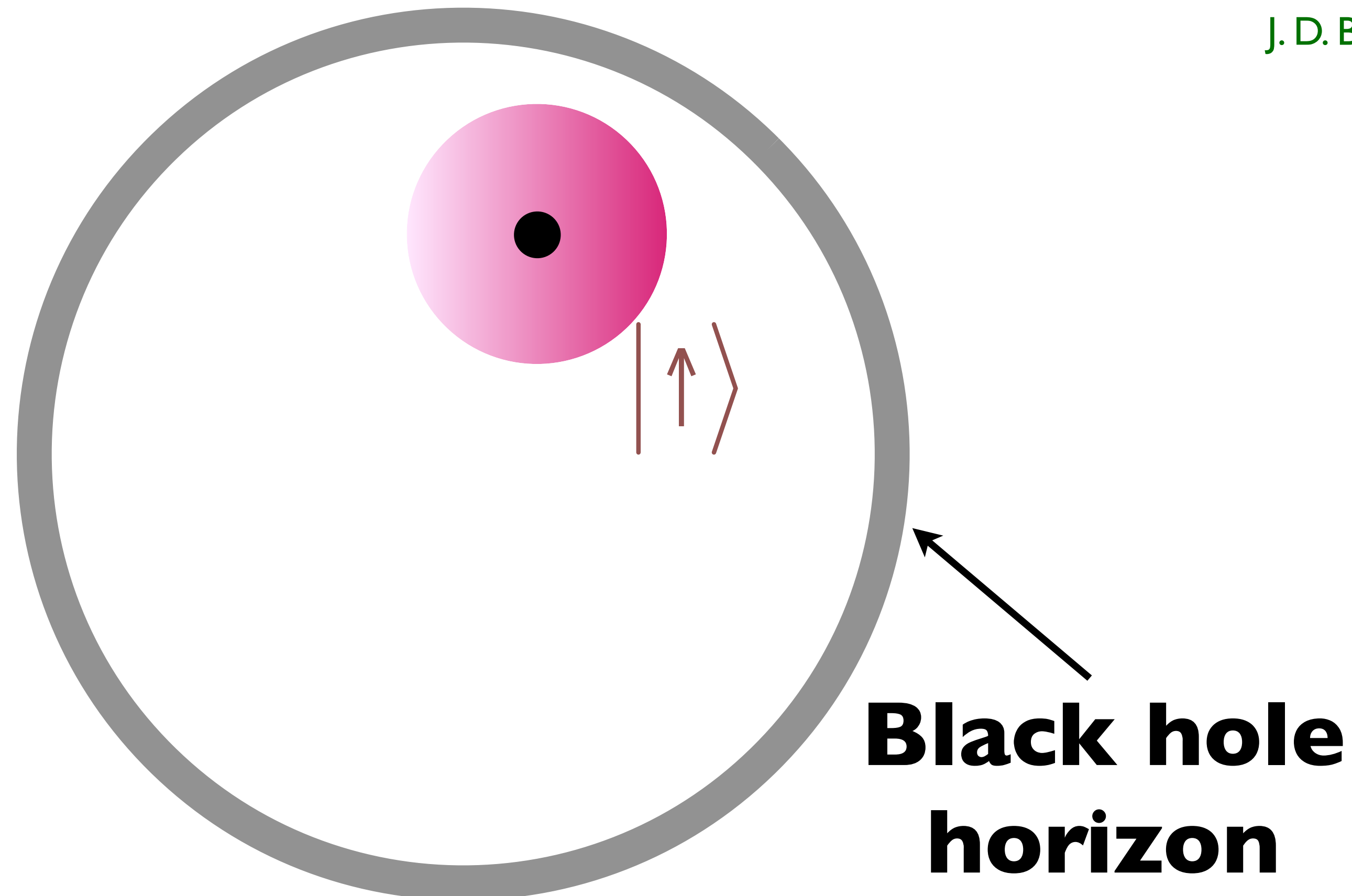


# Quantum Entanglement across a black hole horizon

*Bekenstein, Hawking: Black holes have a temperature and an entropy!*

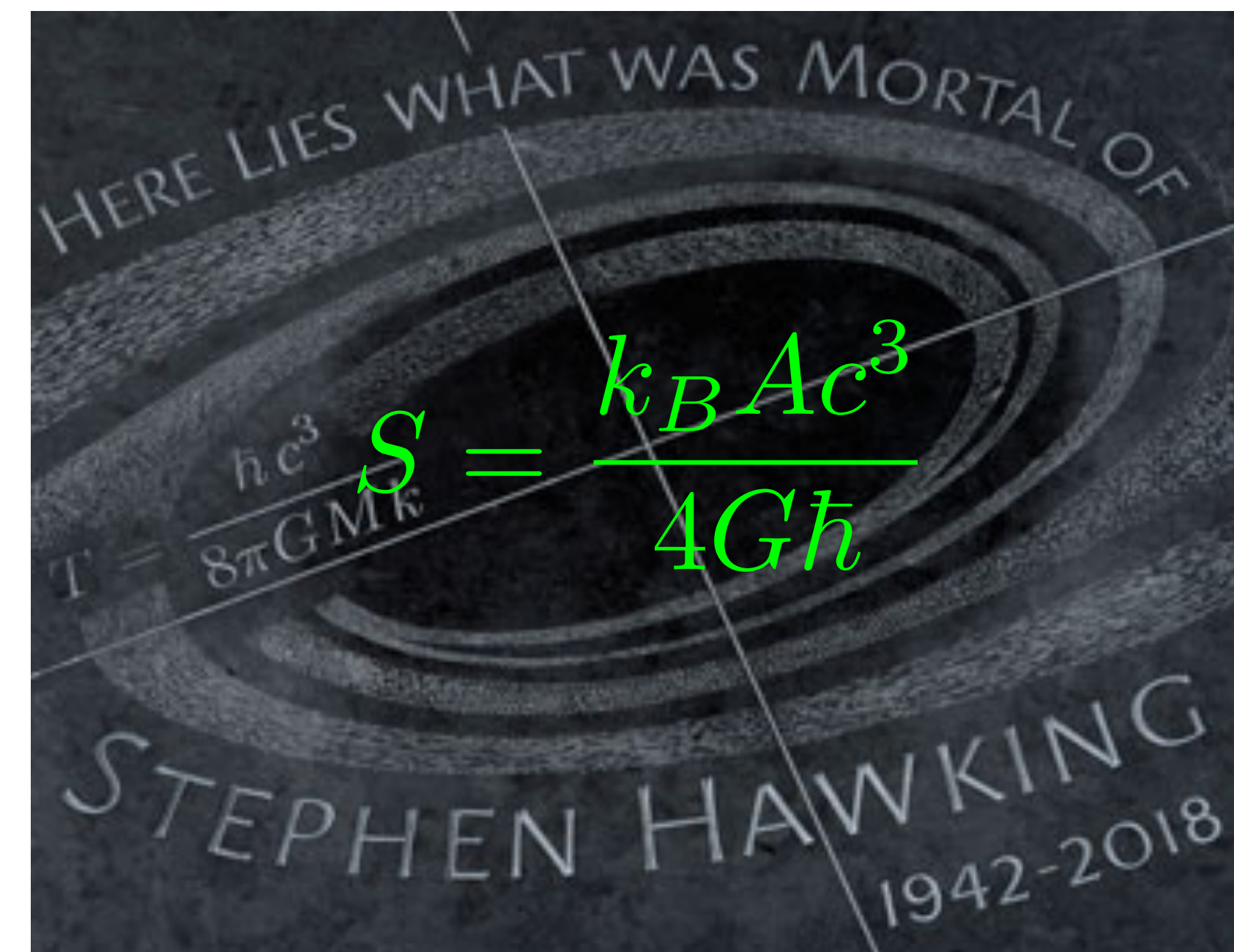
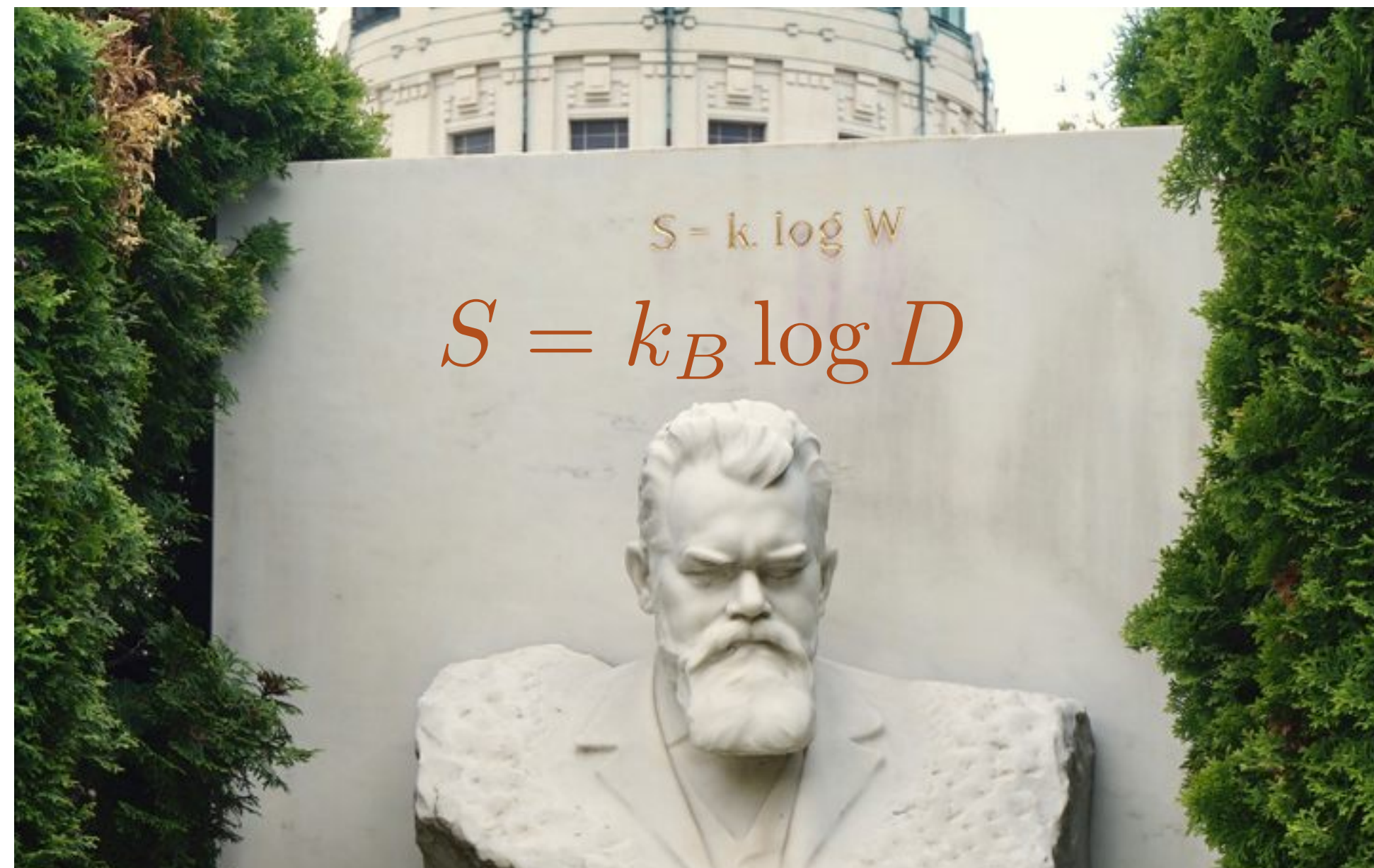
To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.

J. D. Bekenstein, PRD **7**, 2333 (1973); S.W. Hawking, Nature **248**, 30 (1974)



# Quantum Black Holes

- Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states*  $D(E)$  [the quantum analog of Boltzmann's  $W$ ] matches Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics,  $S(E) = k_B \log D(E)$  ?



# The SYK model

Consequences of emergent time-reparameterization and conformal symmetries  
in low-energy theory in 0+1 spacetime dimensions:

## 1. Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right) \text{ independent of } U.$$

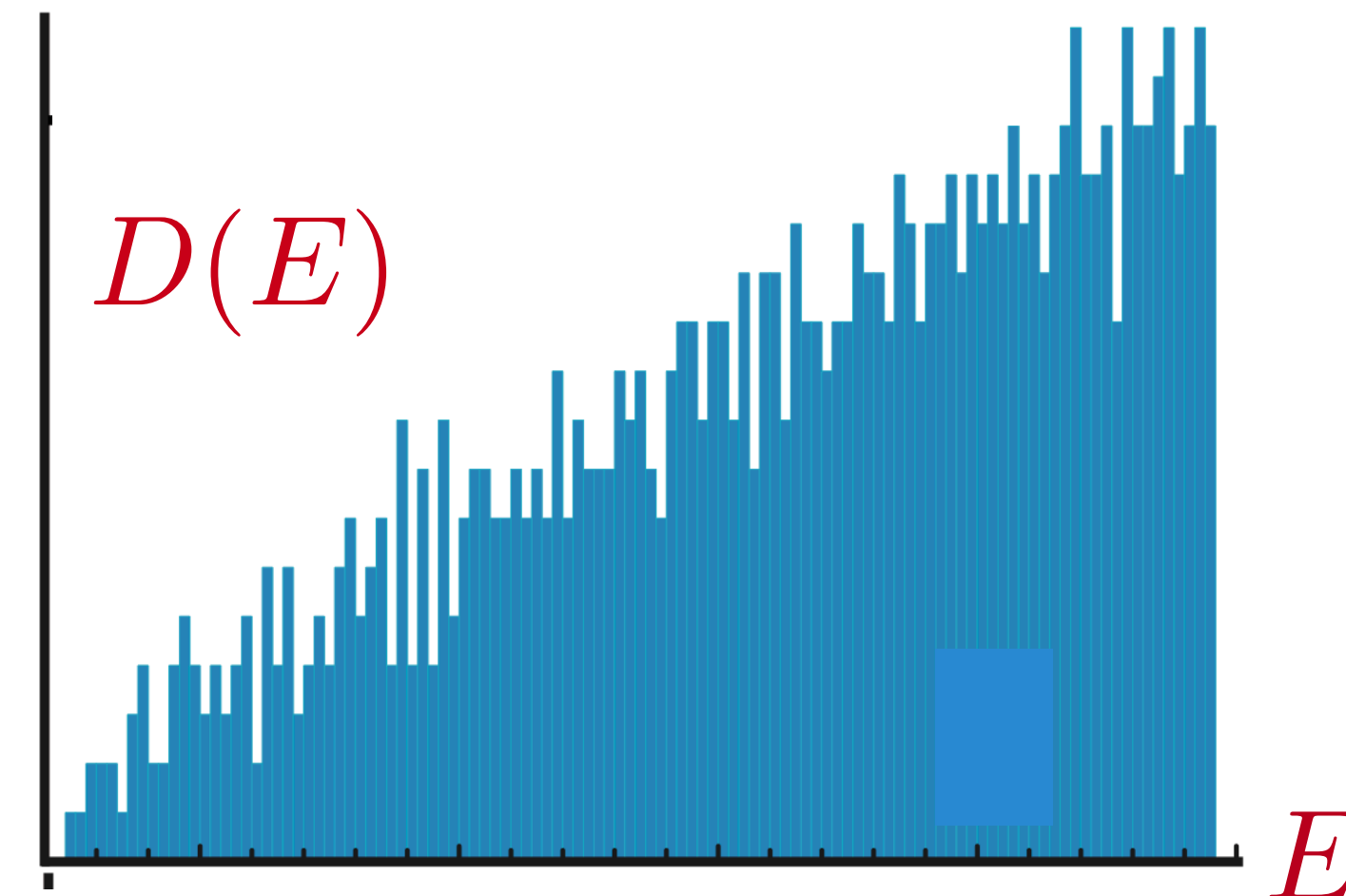
No bosons, fermions, anyons ...



## 2. Zero temperature entropy without exponential ground state degeneracy!

$$\lim_{T \rightarrow 0} \lim_{N \rightarrow \infty} \frac{1}{N} S(T) = s_0 \quad , \quad D(E \rightarrow 0) = e^{N s_0} f_{\text{smooth}}(E)$$

$$s_0 = 0.46484769917080510749\dots \text{ for } Q = 1/2.$$



# Connections between the SYK model and black holes

- Black hole ‘ring-down’ or ‘quasinormal mode damping’ or ‘chaos’ times are Planckian  $\sim \hbar/(k_B T)$

C.V. Vishveshwara, Nature **227**, 936 (1970)

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- Charged black holes have a non-zero Bekenstein-Hawking entropy in the limit  $T \rightarrow 0$ :

$S_{BH} = A_0 c^3 / (4\hbar G)$  where  $A_0 = 2GQ^2/c^4$  is the area of the charged black hole horizon at  $T = 0$ .

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Also applies to rotating neutral black holes.

U. Moitra, S.K. Sake, S.P.Trivedi and V.Vishal, JHEP 11 (2019) 047.

D. Kapec, A. Sheta, A. Strominger and C. Toldo, PRL 133 (2024) 021601

M. Kolanowski, D. Marolf, I. Rakic, M. Rangamani and G.J.Turiaci, arXiv:2409.16248

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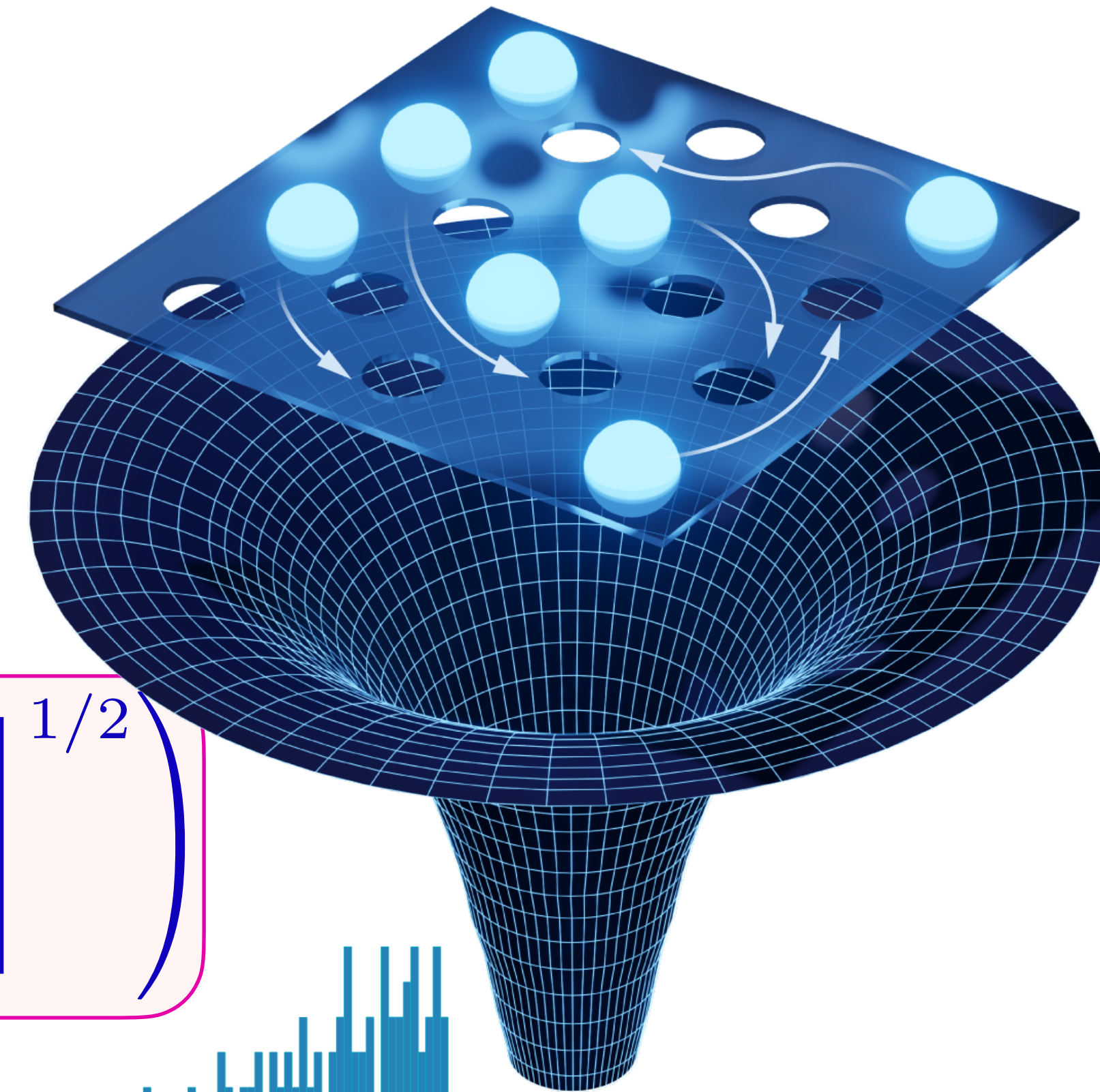
Also applies to rotating neutral black holes.

- The example of the SYK model implies that  $S_{BH}$  is *not* realized by an exponentially large ground state degeneracy (as is the case in all earlier string-theoretic computations).

# D(E) of charged black holes from the SYK model

- For generic charged black holes in 3+1 dimensions with horizon area  $A_0$  at  $T = 0$  and fixed charge  $Q$  ( $A_0 = 2GQ^2/c^4$ ), the density of quantum states at small energy  $E$  is

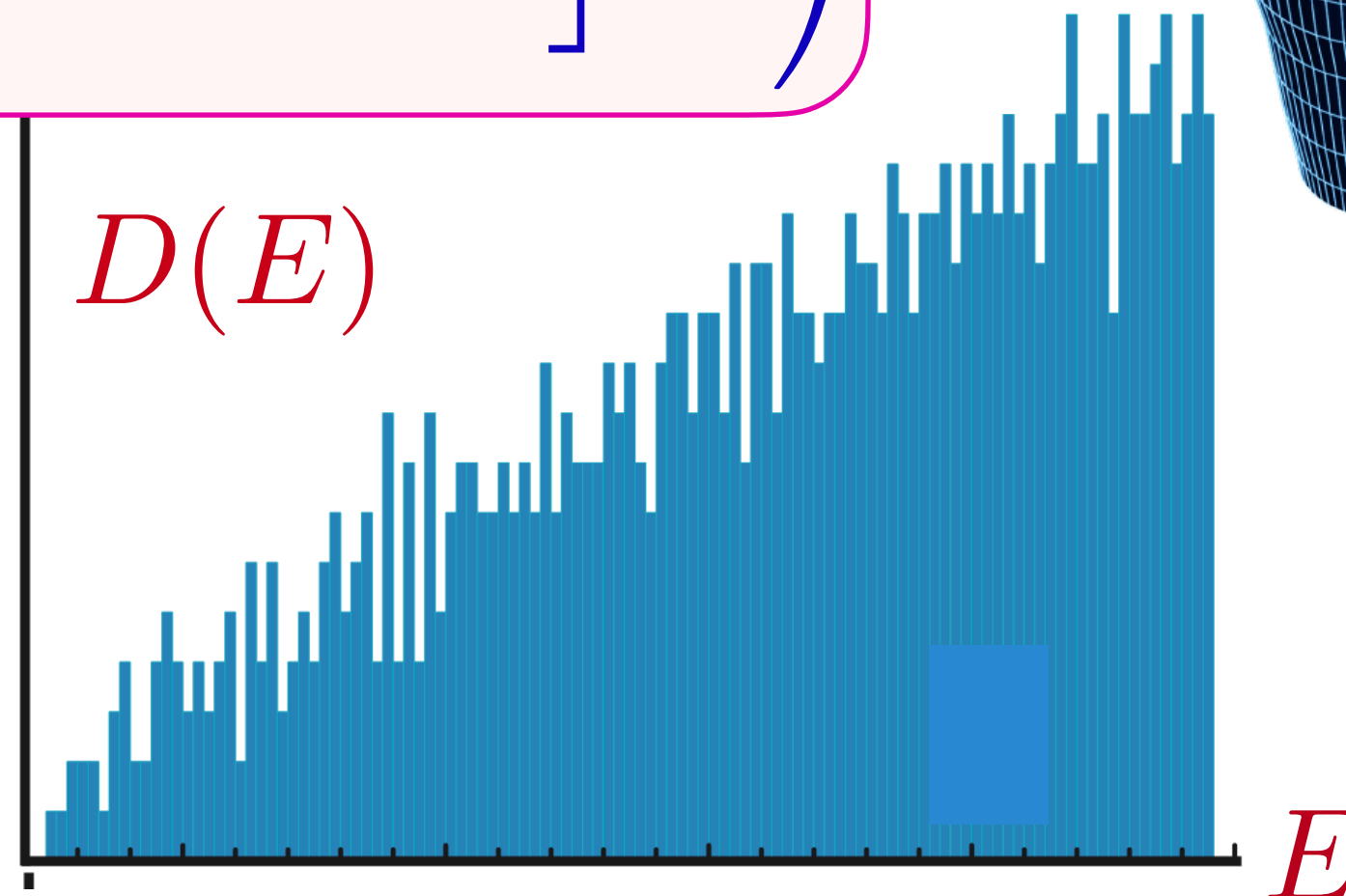
$$D(E) \sim \left( \frac{A_0 c^3}{\hbar G} \right)^{-347/90} \exp \left( \frac{A_0 c^3}{4\hbar G} \right) \sinh \left( \left[ \frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E \right]^{1/2} \right)$$



Bekenstein-Hawking

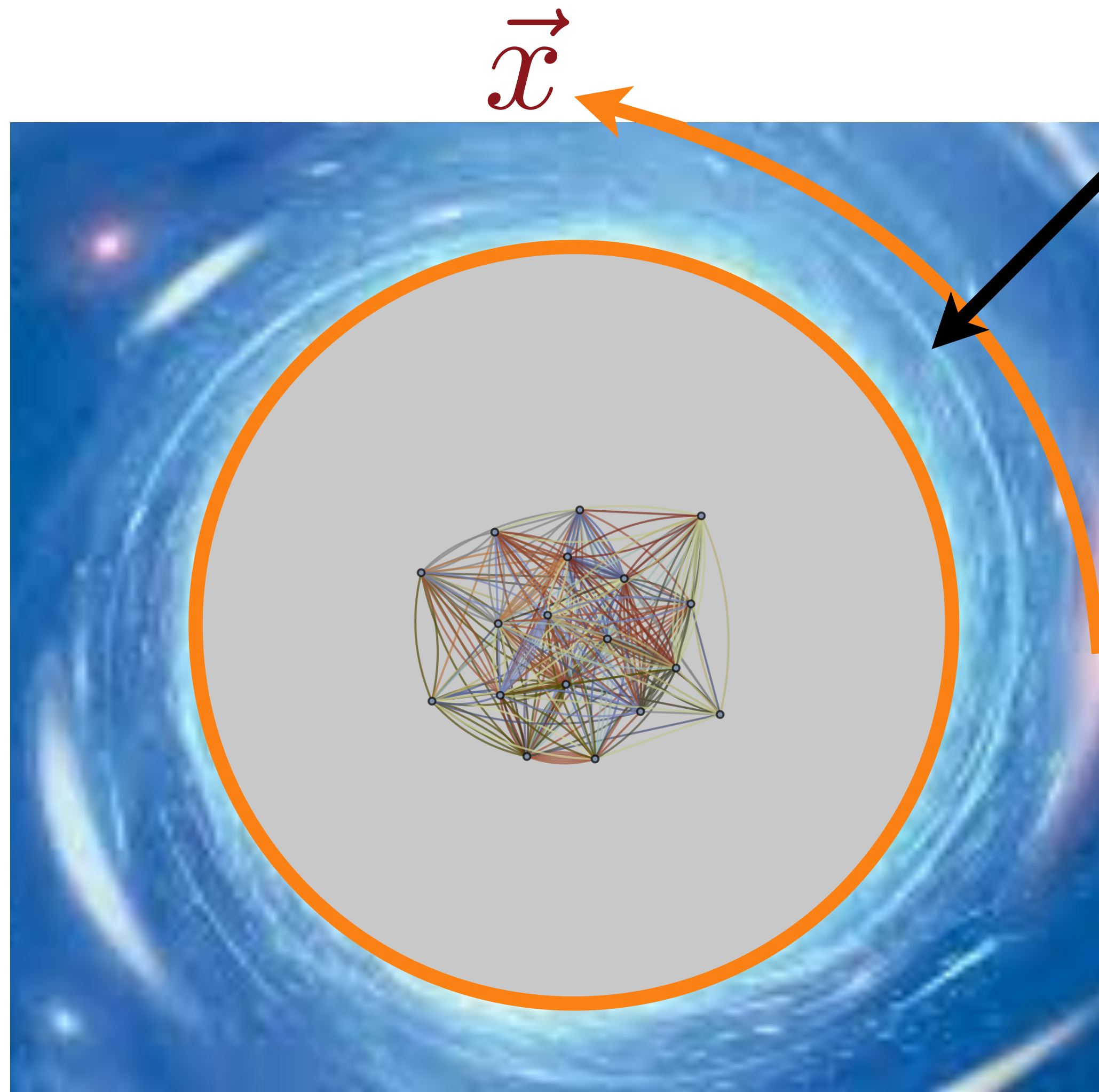
Iliesiu, Murthy, Turiaci (2022)

$f_{\text{smooth}}(E)$ : developments from the SYK model



Similar remarks apply to rotating neutral black holes.

# Quantum simulation of charged black holes by the SYK model



The SYK model simulates the low energy properties of the interior of the black hole for an outside observer in  $\zeta$ - $\tau$  co-ordinates.

# From the SYK model to the universal 2d-YSYK theory of strange metals

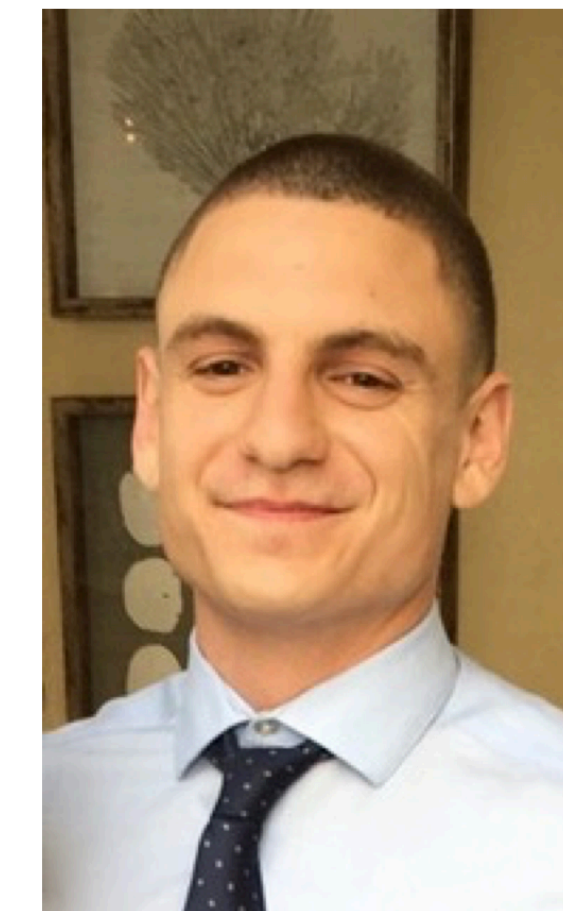
Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)



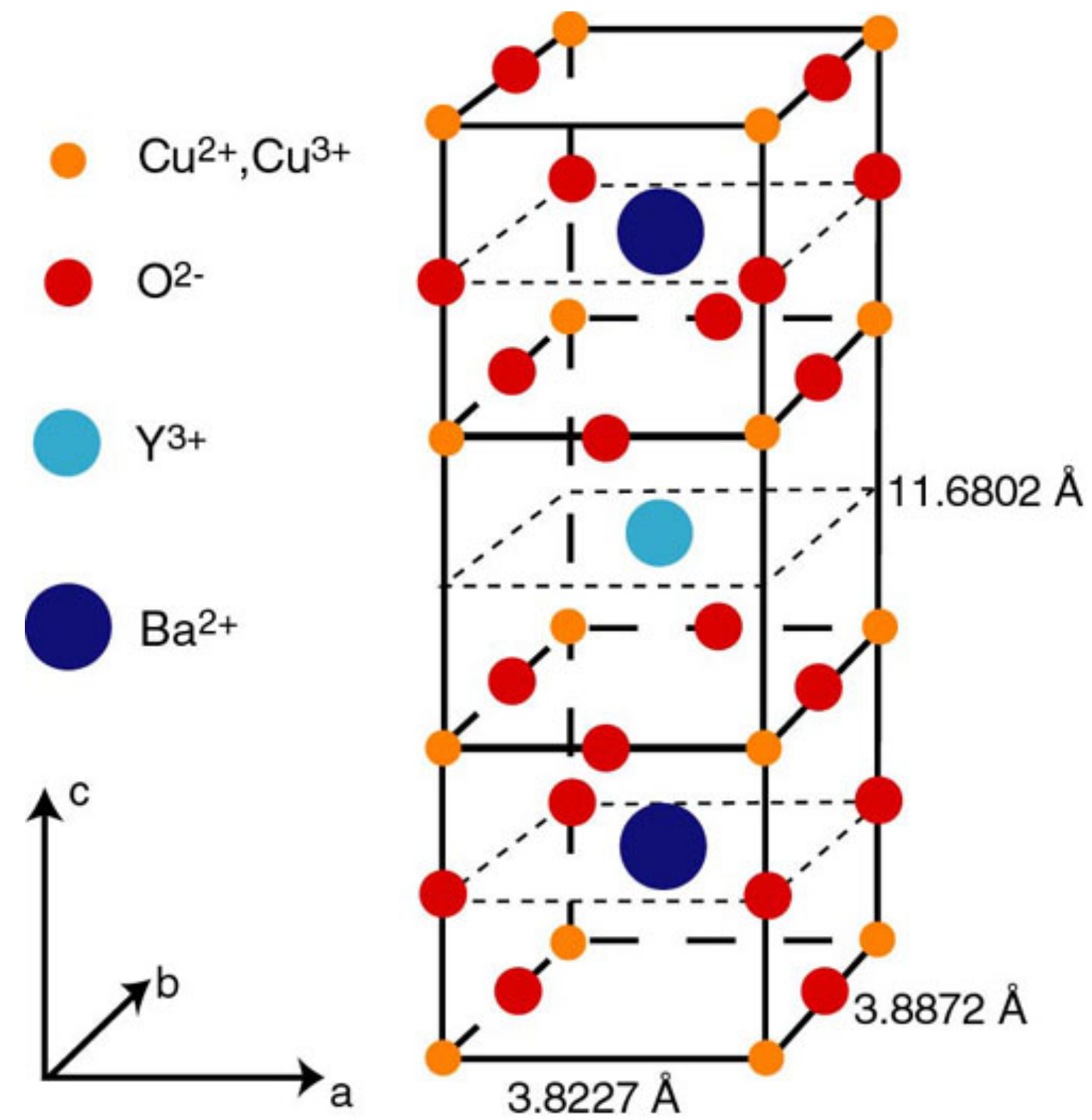
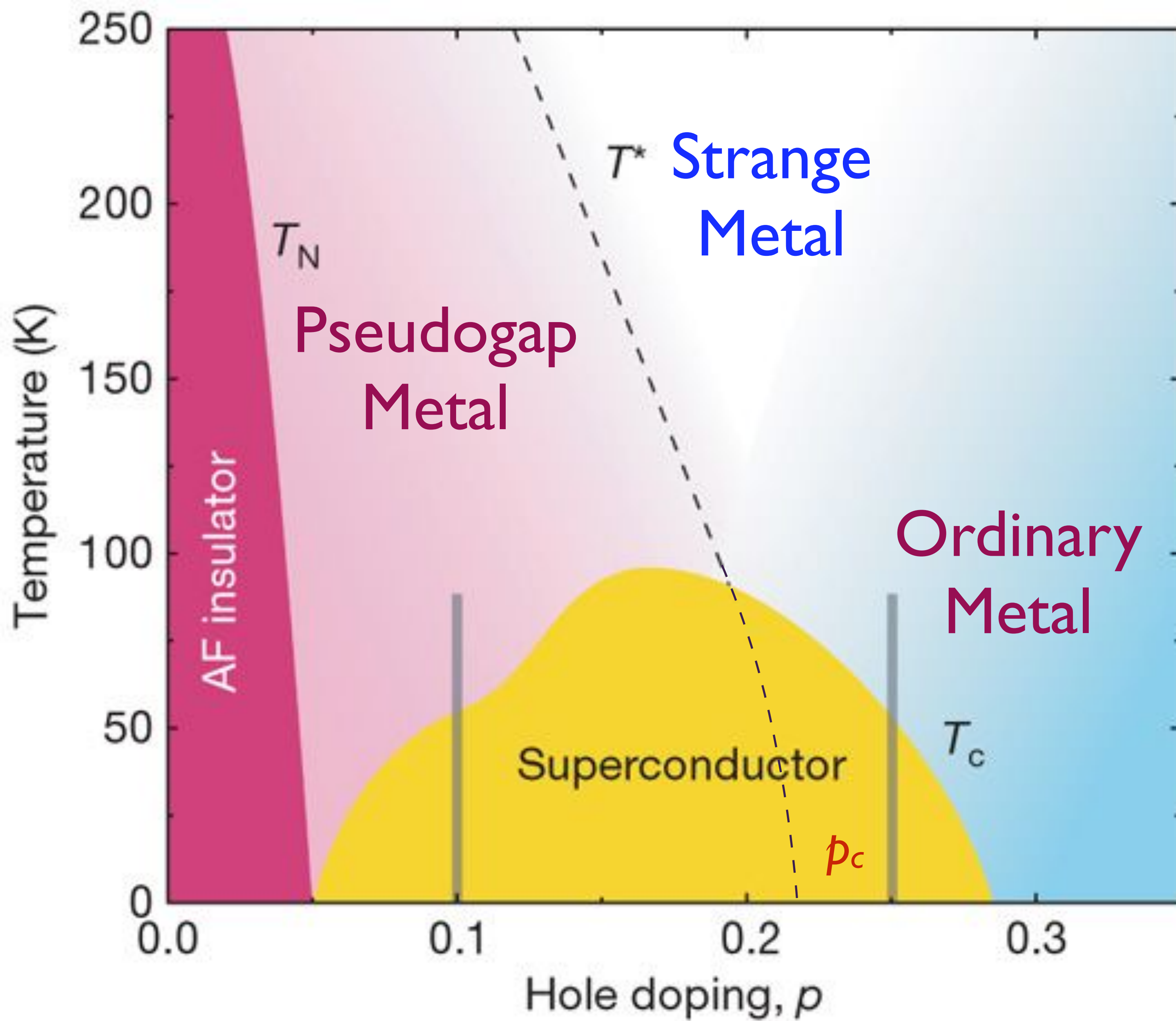
Aavishkar Patel  
Flatiron

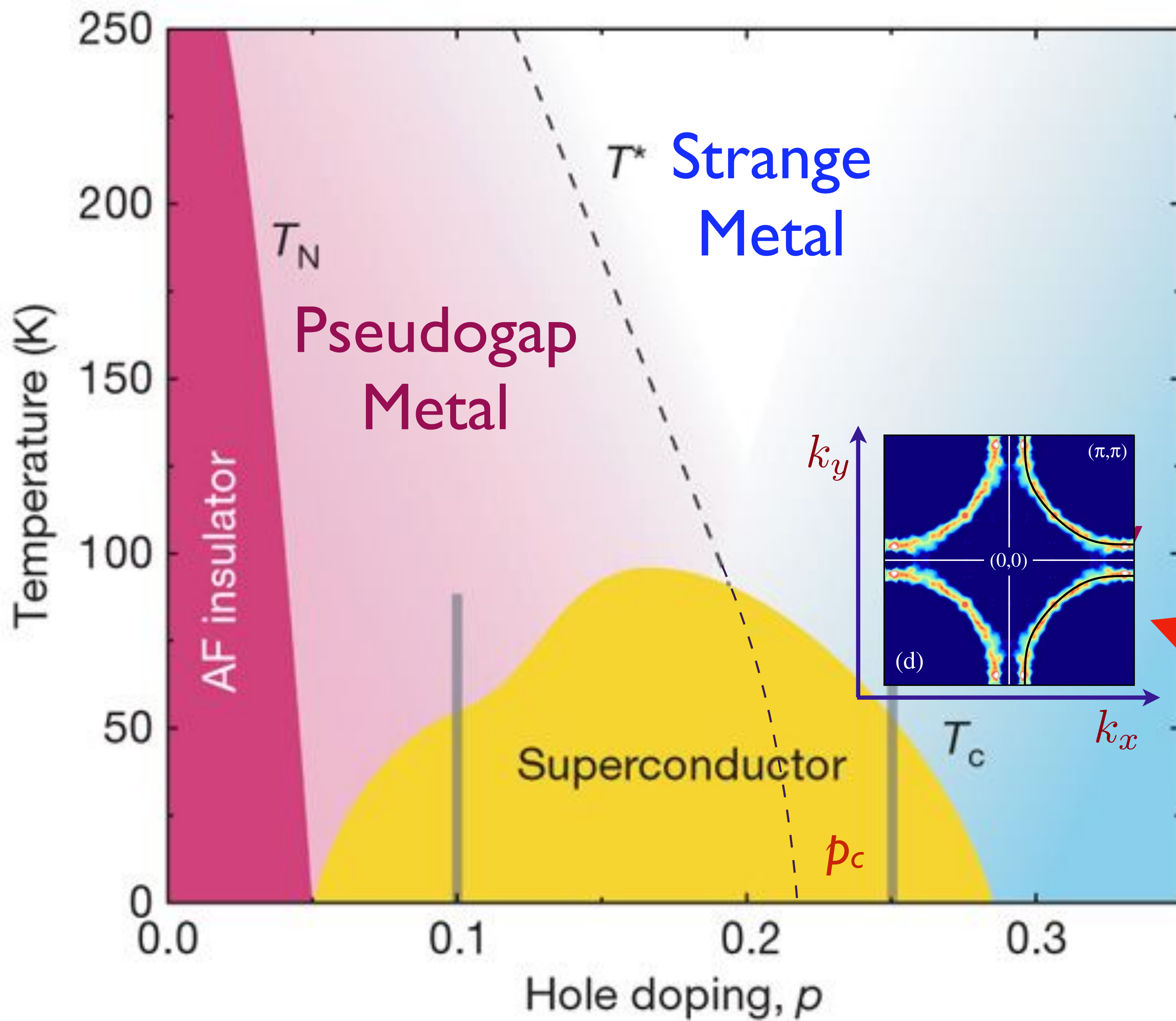


Haoyu Guo  
Cornell

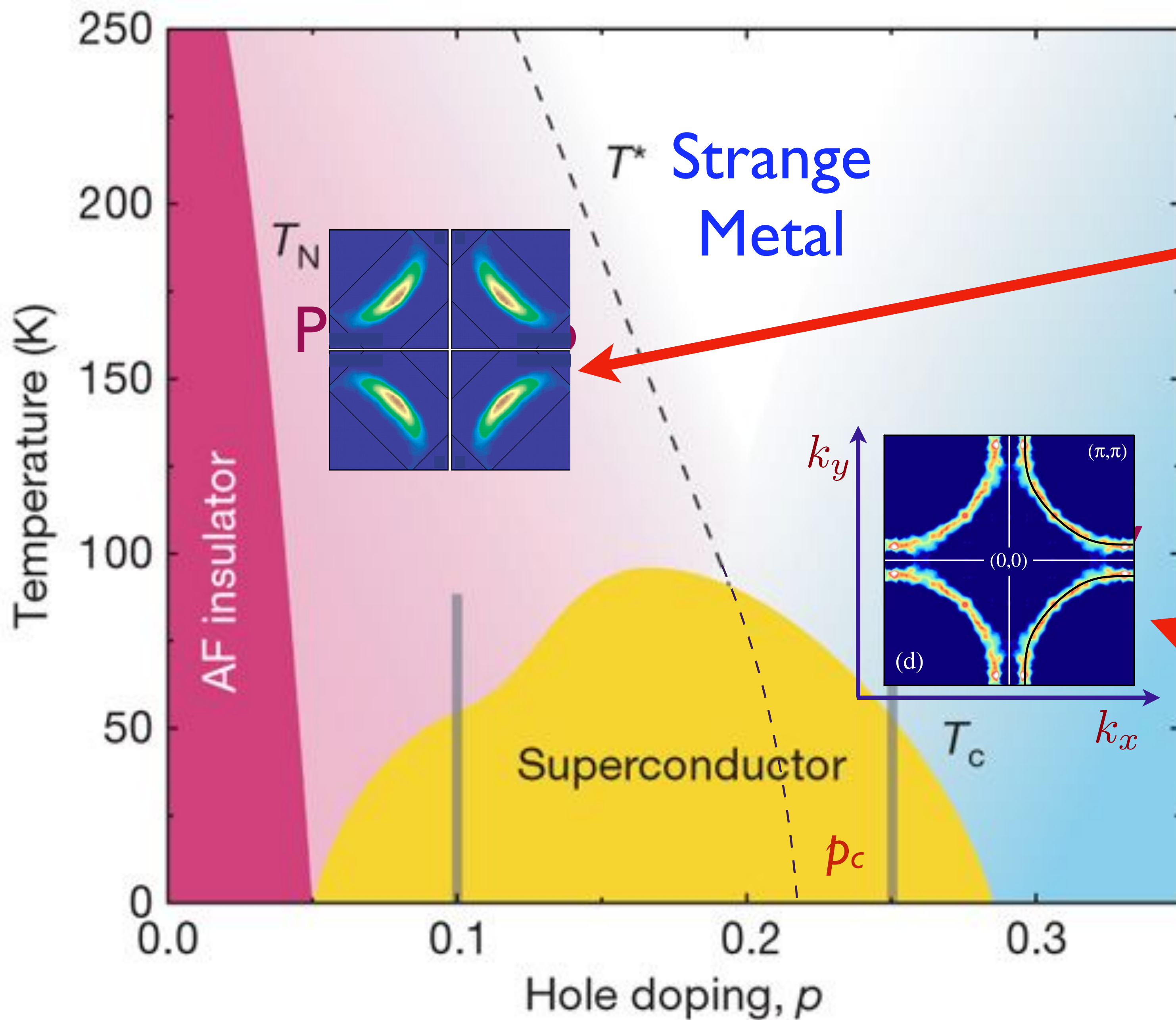


Ilya Esterlis  
Wisconsin





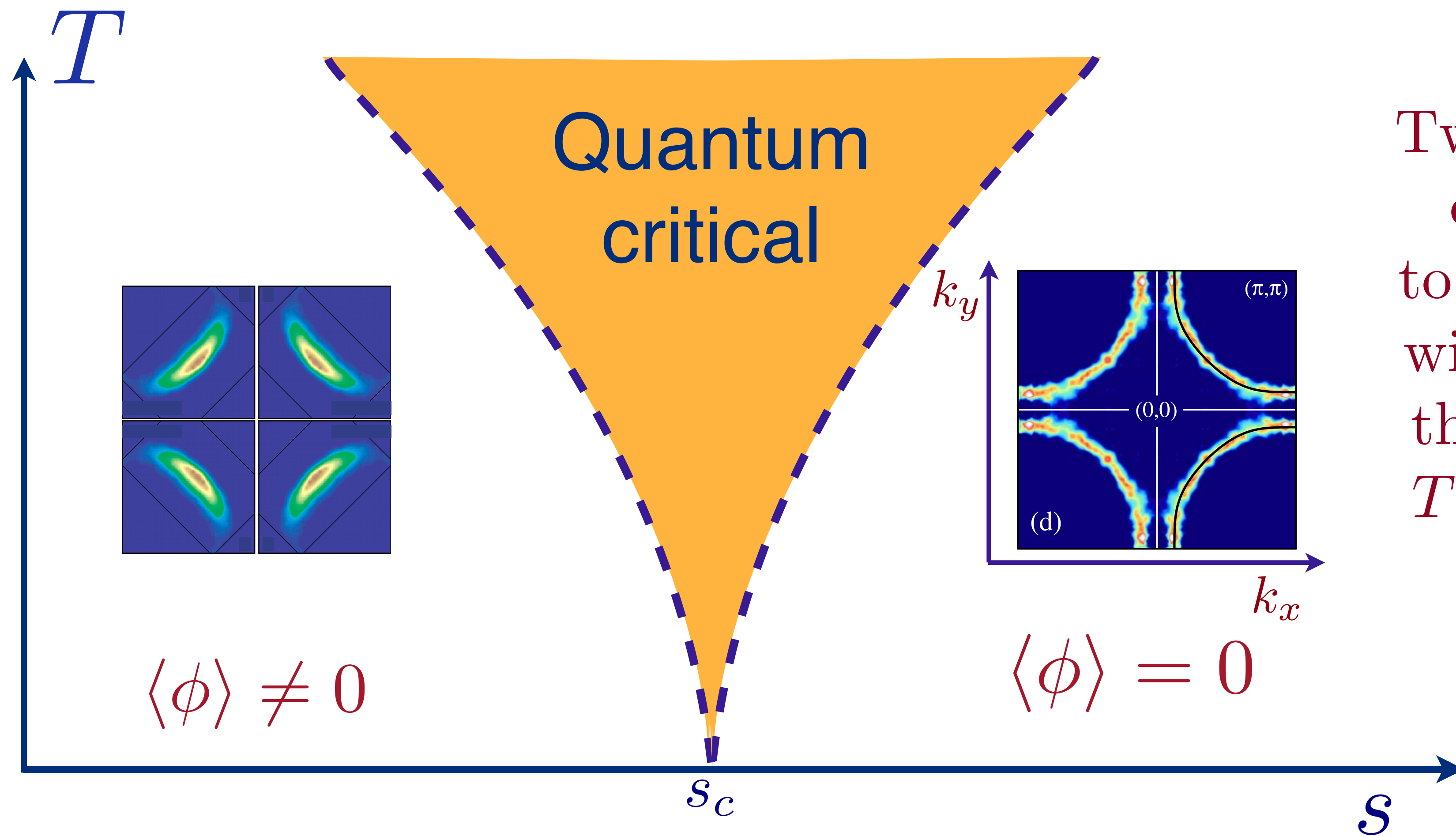
Fermi surface separating occupied and empty electron momenta. As expected, in an ordinary metal



“Pseudogap metal”  
 Fermi surface  
 modified by  
 electron-electron  
 interactions

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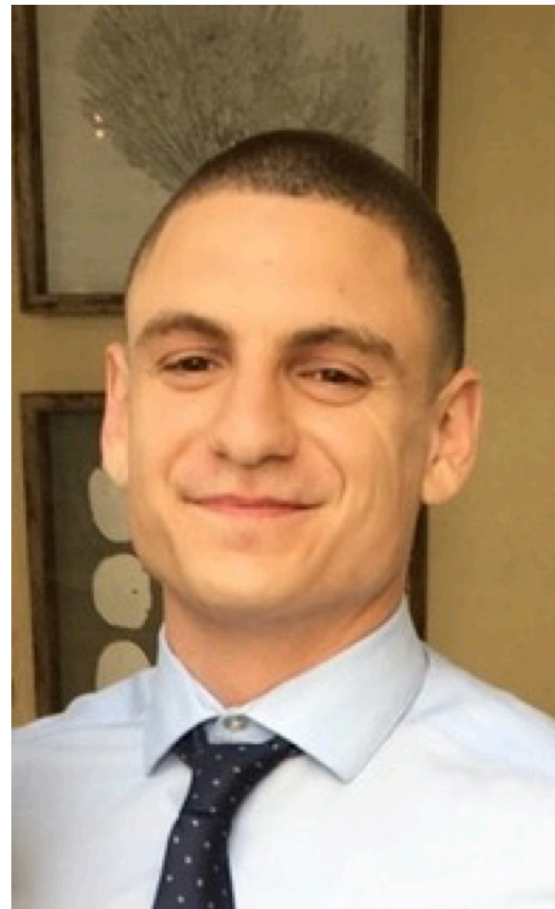
# Quantum phase transition of Fermi surface change



Two-dimensional YSYK model describes electrons coupled to a boson  $\phi$  driving the QPT, with spatial randomness in  $s_c$ , the position of the underlying  $T = 0$  quantum critical point.

# Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)



Ilya Esterlis  
Wisconsin



Haoyu Guo  
Cornell



Aavishkar Patel  
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Chenyuan Li  
Harvard → Rice



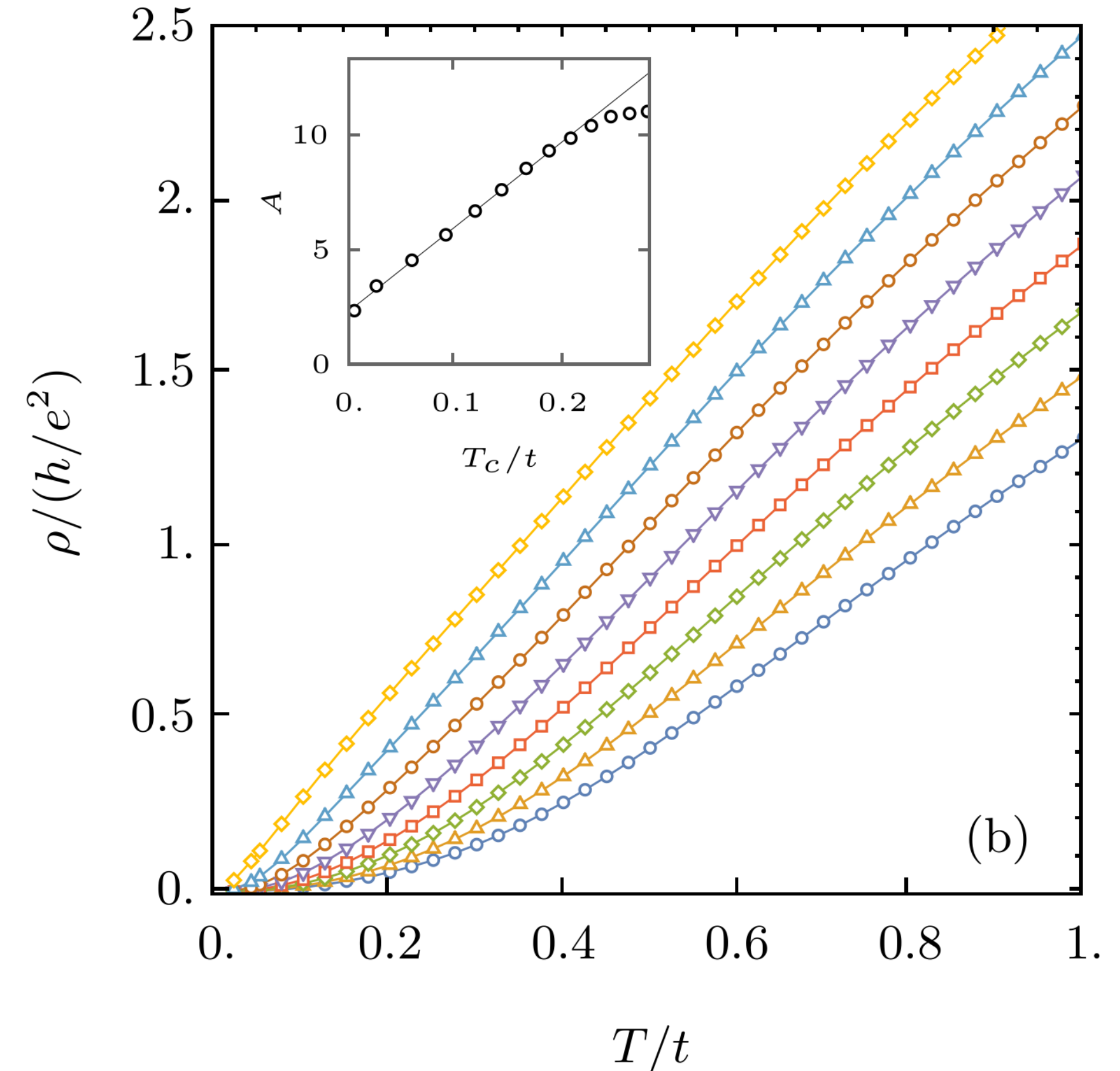
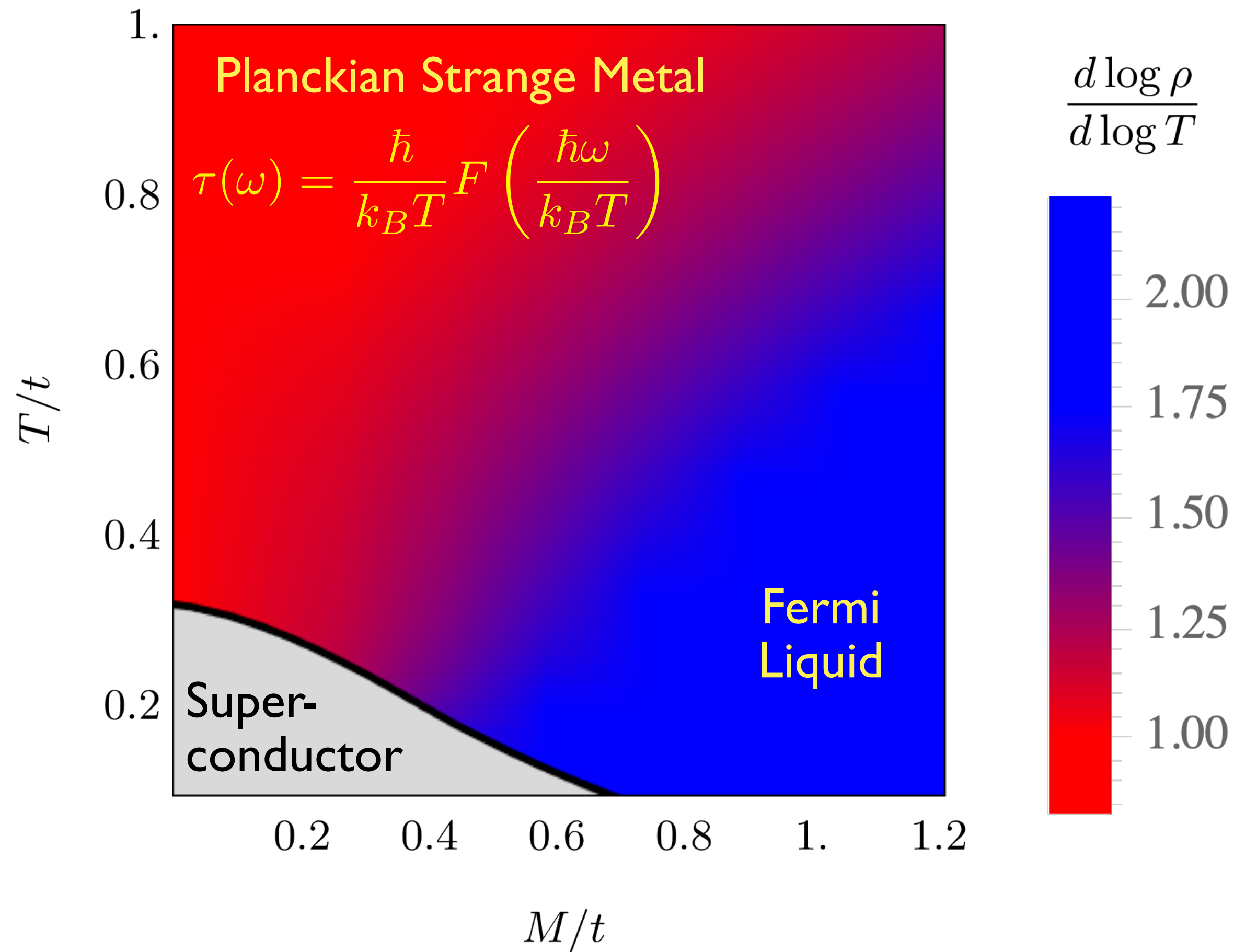
Davide Valentinis  
KIT



Joerg Schmalian  
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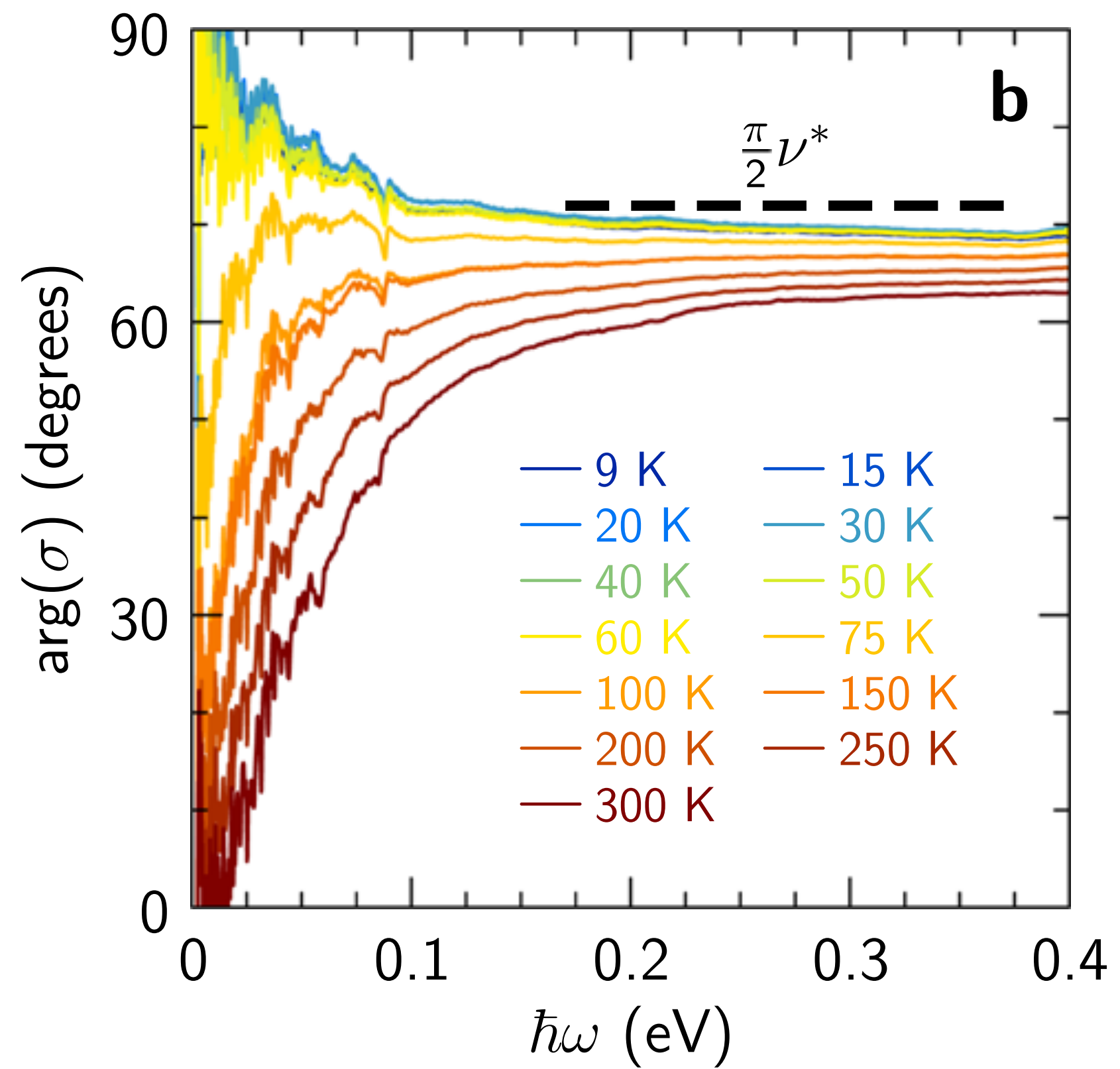
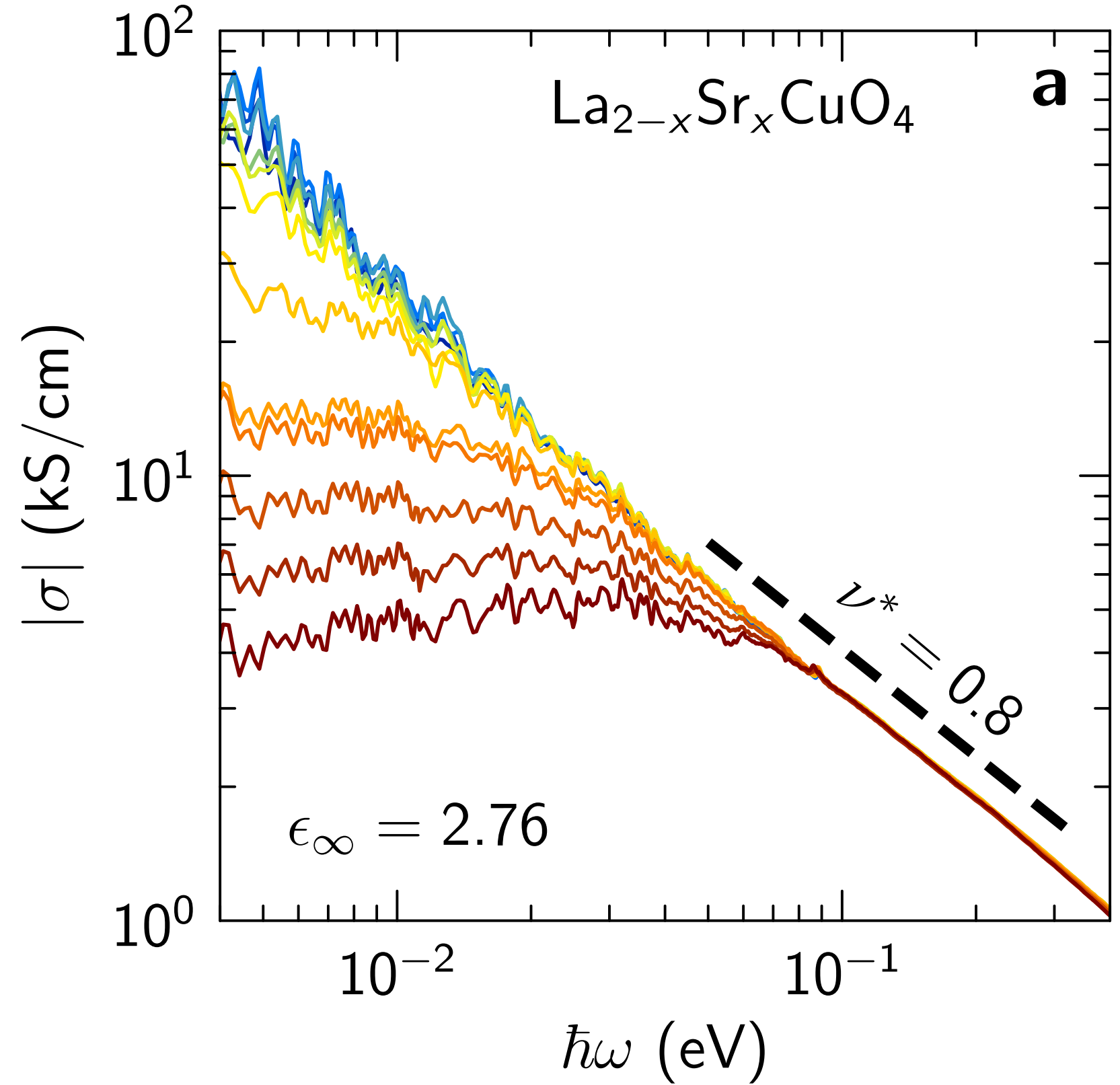


# Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

*Nature Communications* **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i \frac{e^2 K / (\hbar d_c)}{\hbar\omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$

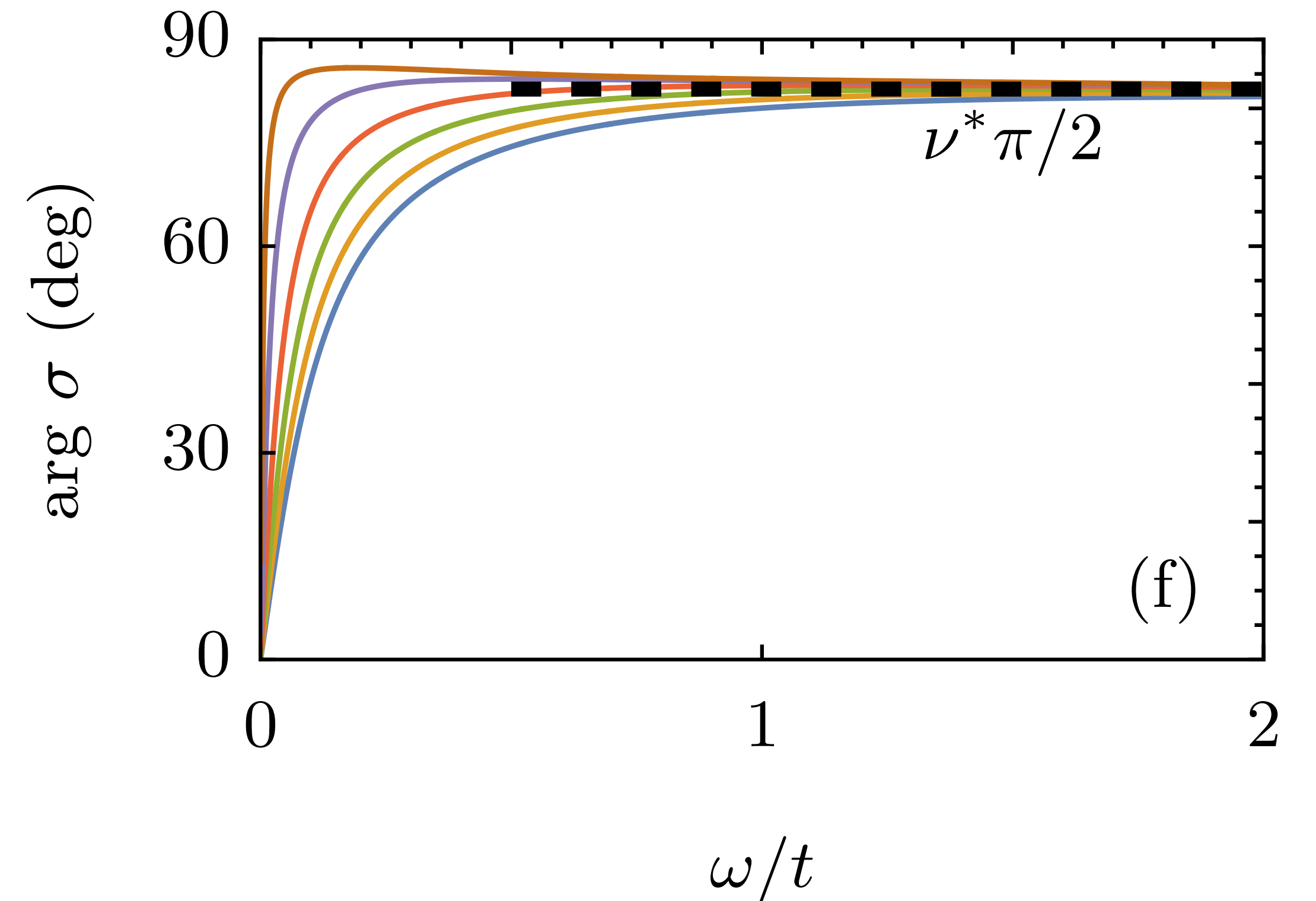
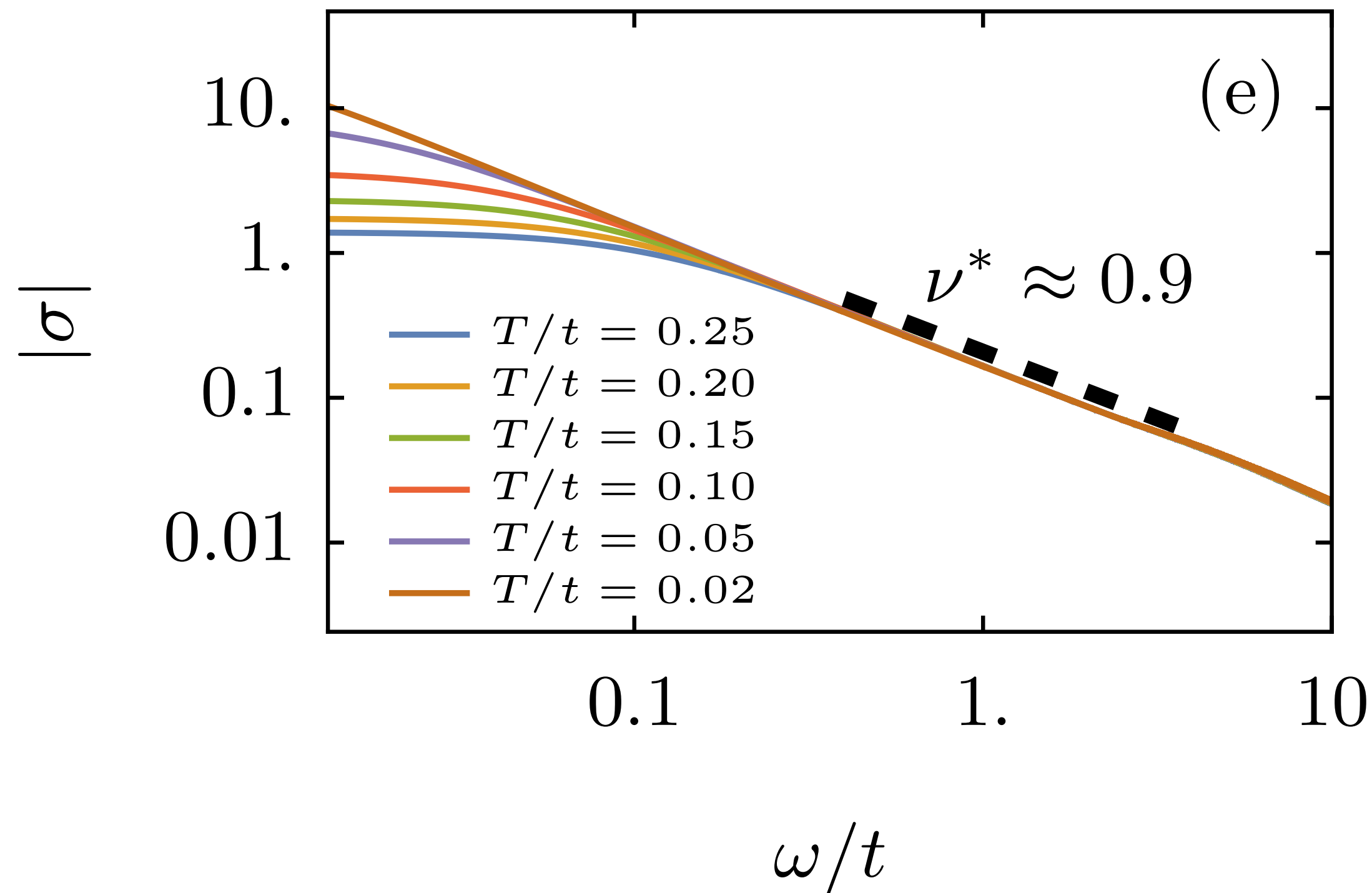


La<sub>2-x</sub>Sr<sub>x</sub>CuO<sub>4</sub>  
 $p = 0.24$   
 $T_c = 19$  K

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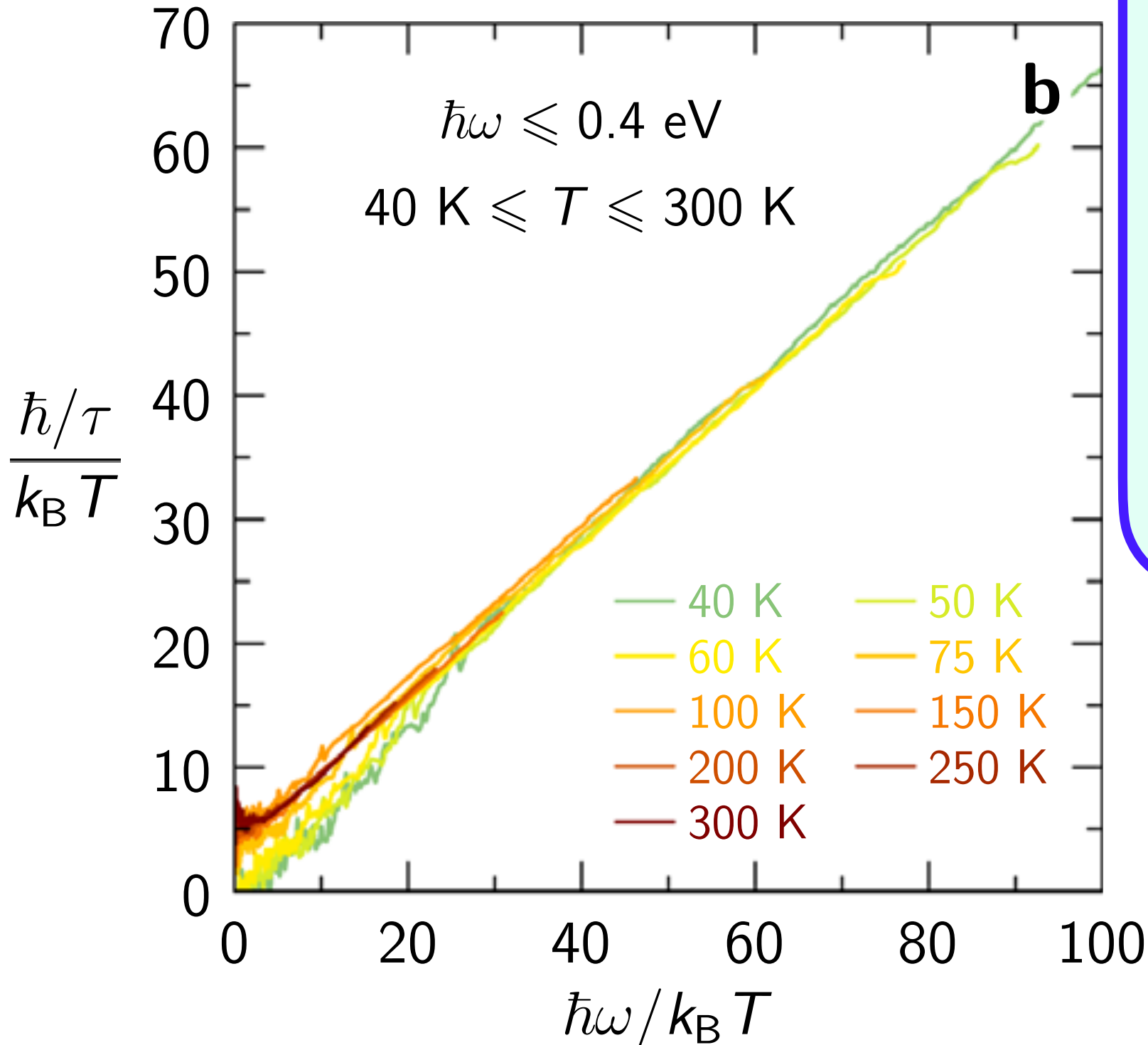
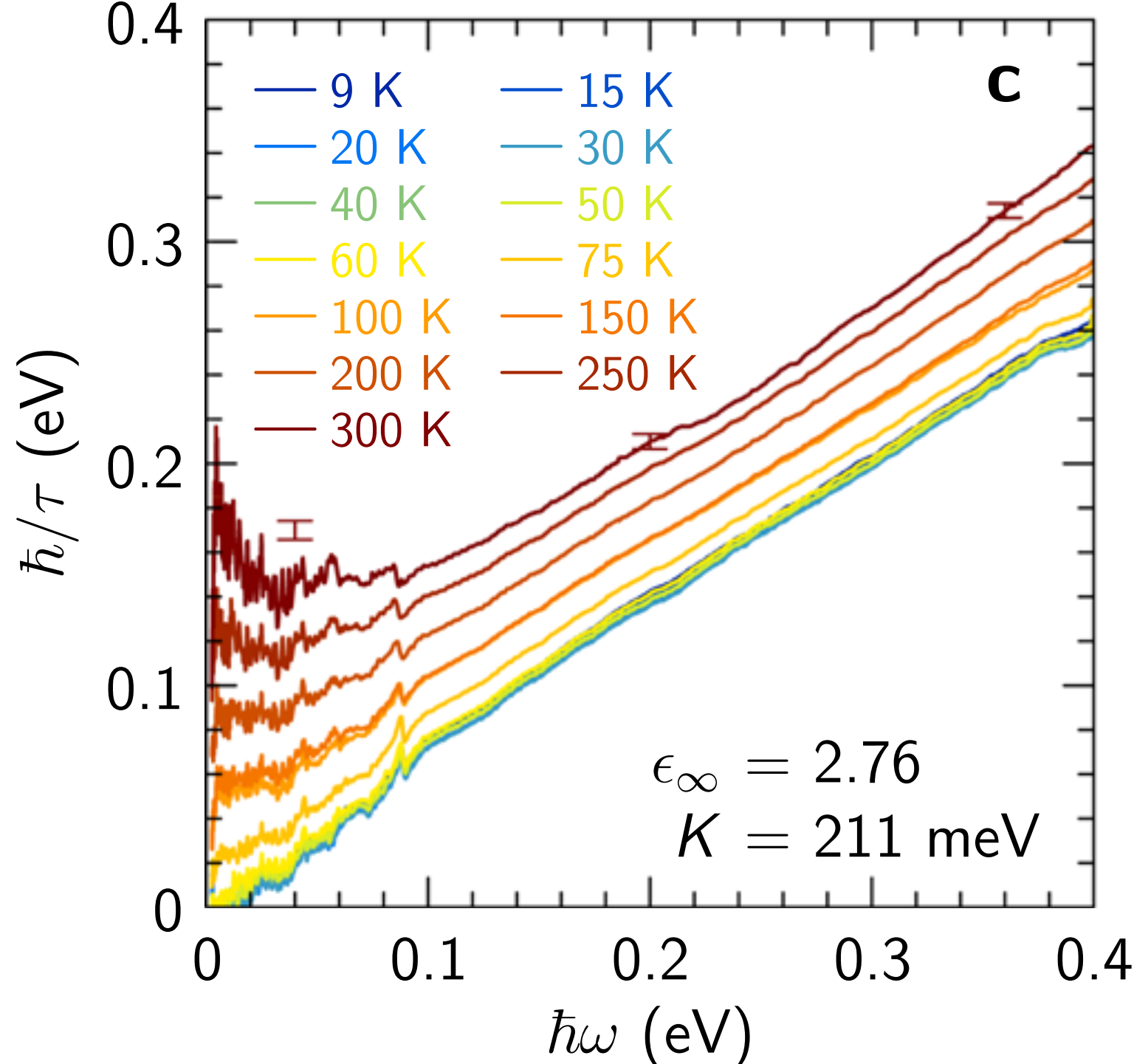


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Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy

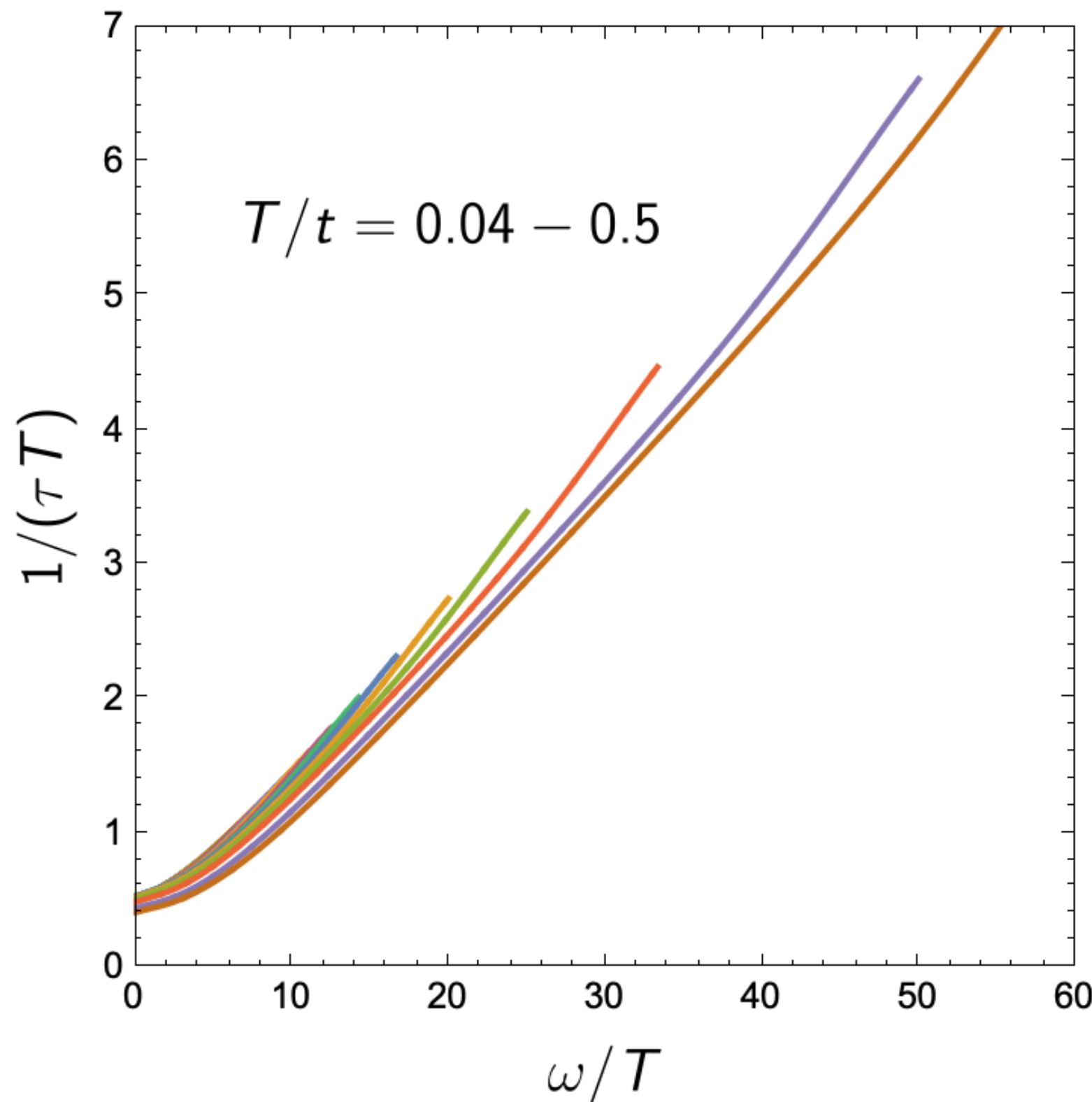
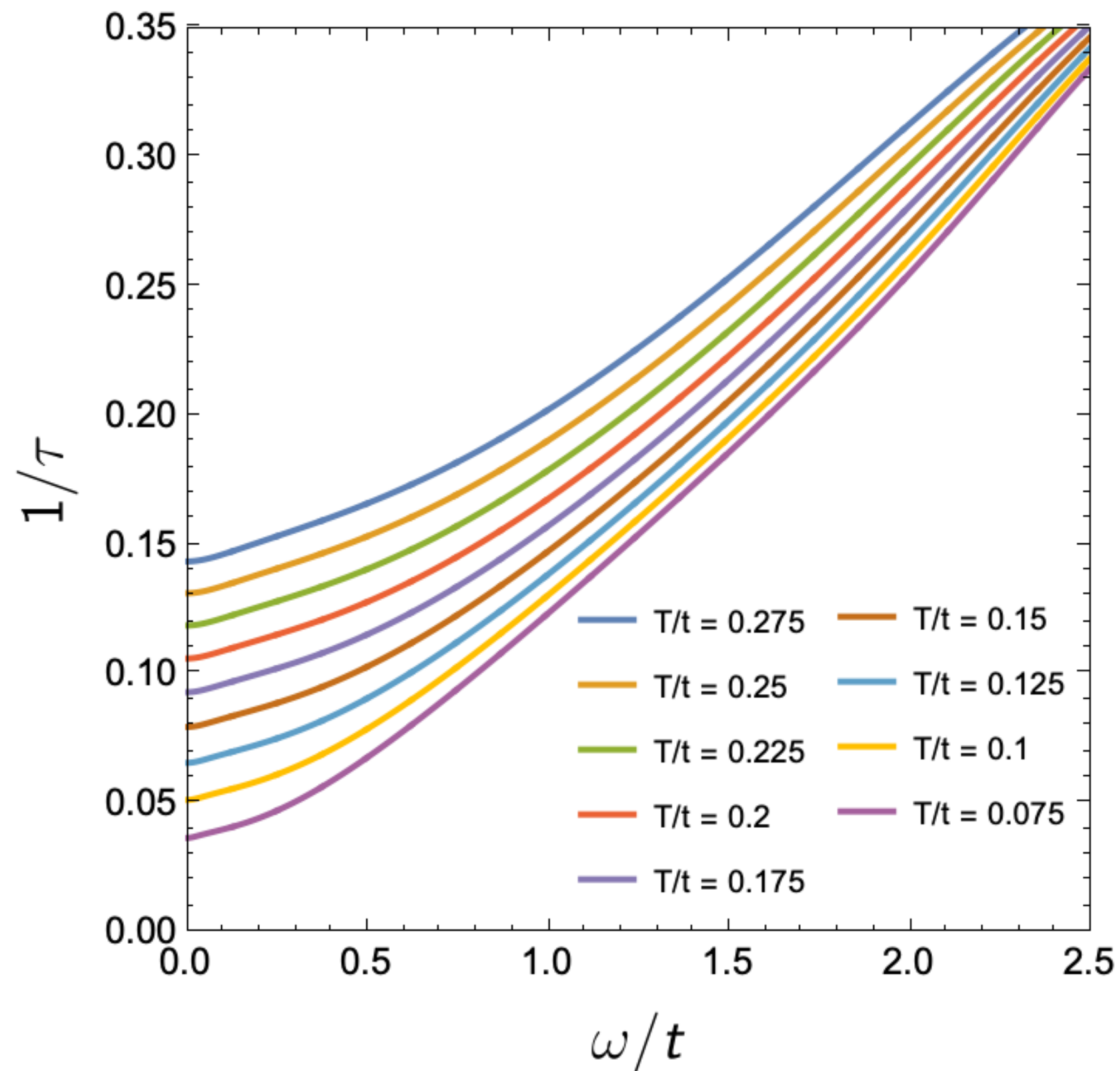
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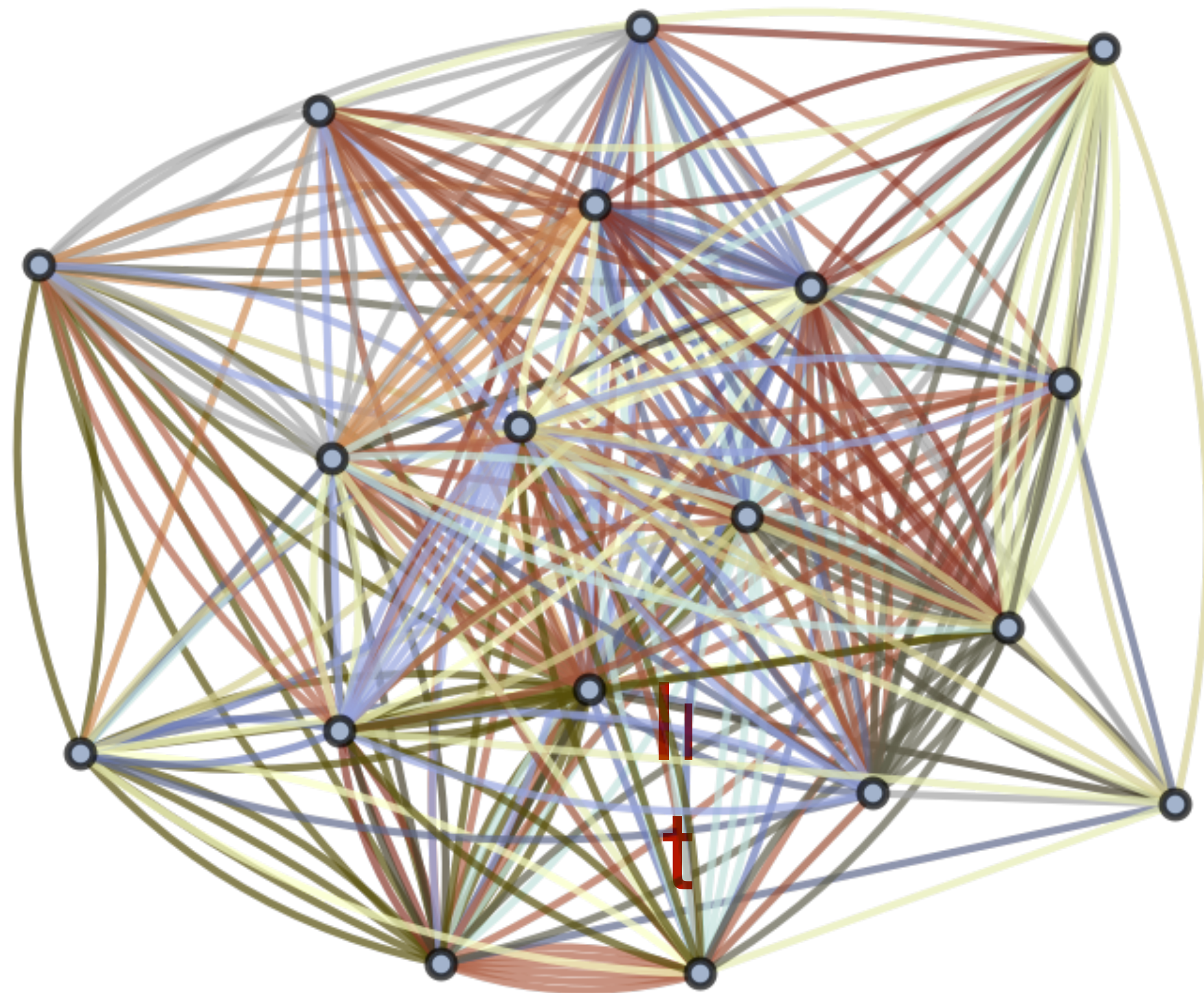
and entropy

$S(T \rightarrow 0) \sim T \ln(1/T)$   
in 2d-YSYK model  
(unlike zero temperature entropy in SYK model).

Recap

# The Sachdev-Ye-Kitaev (SYK) model

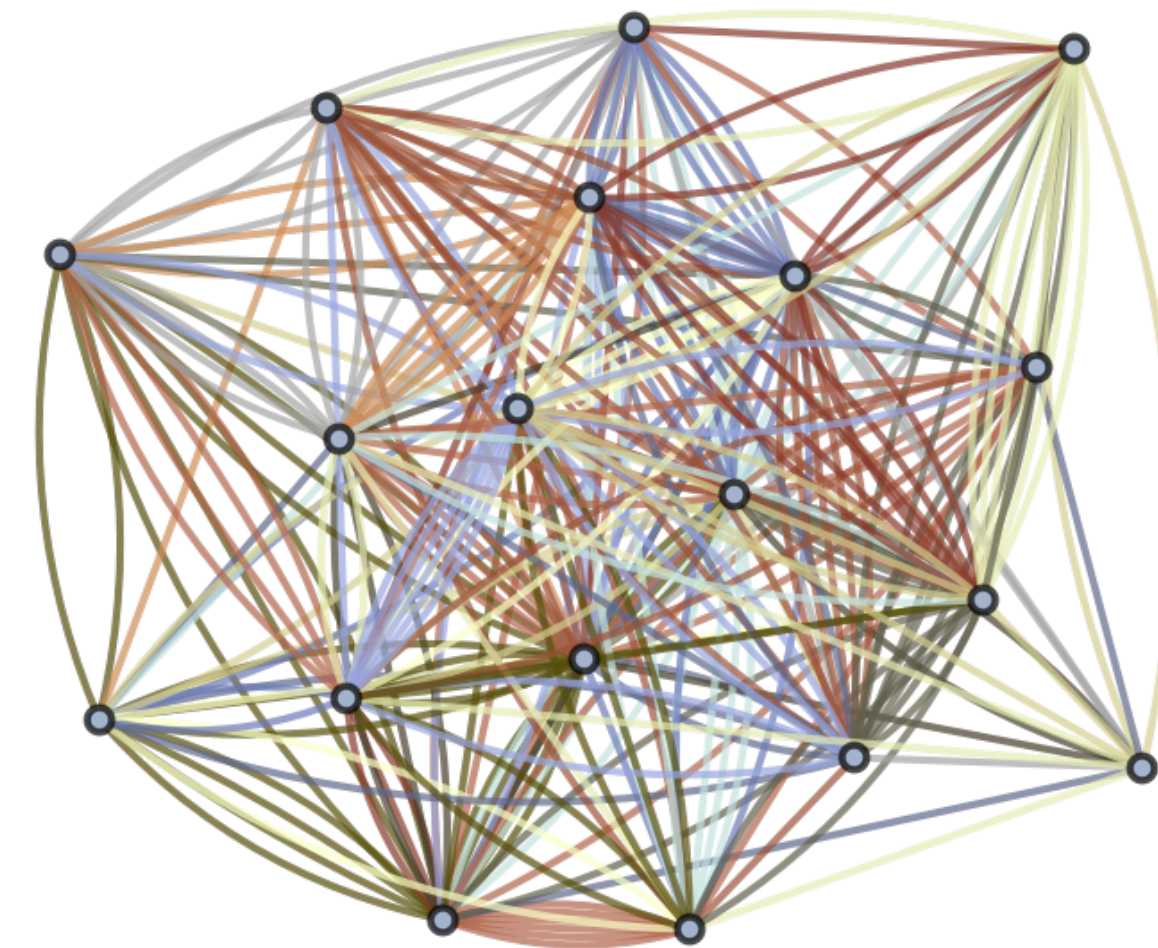
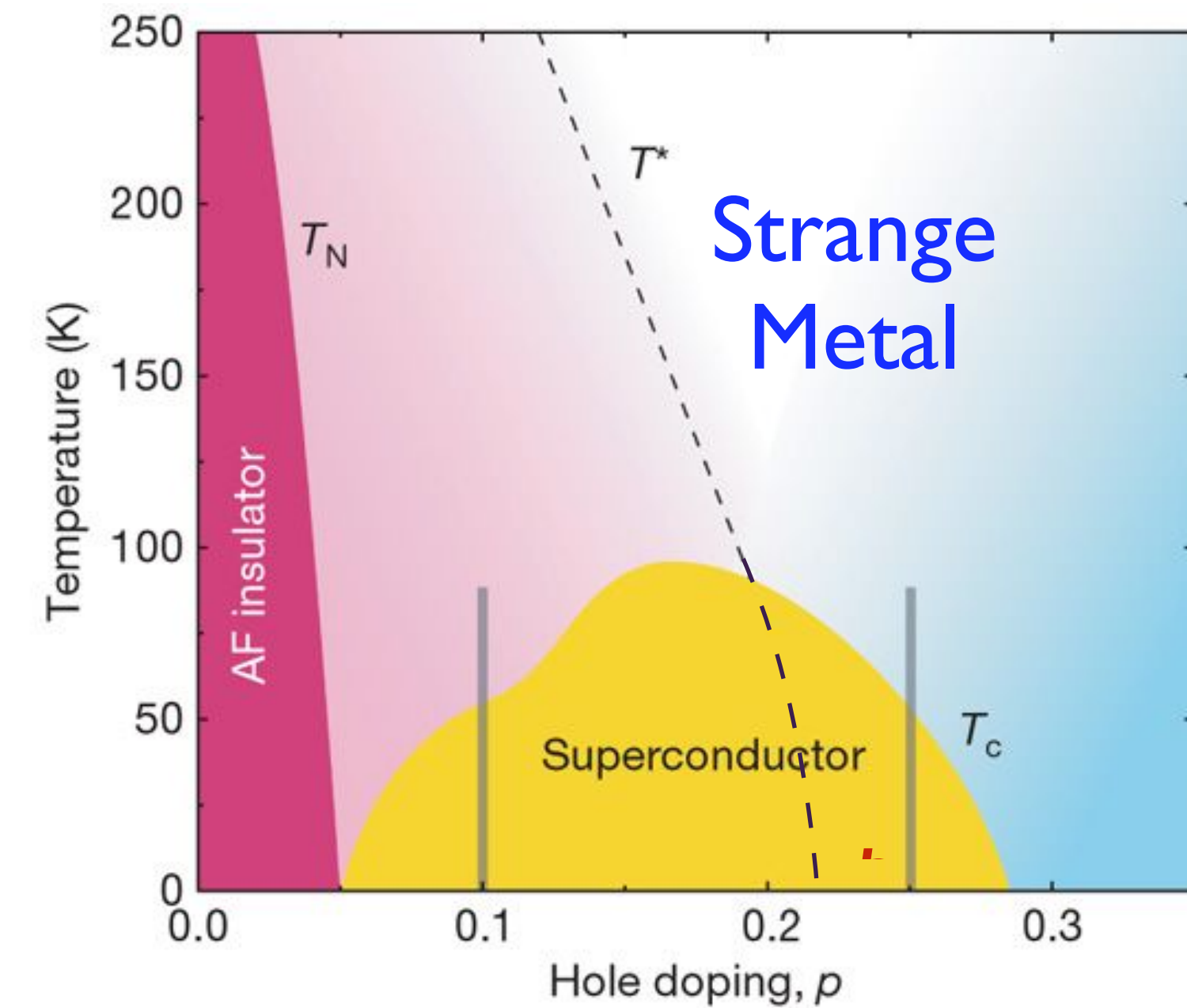
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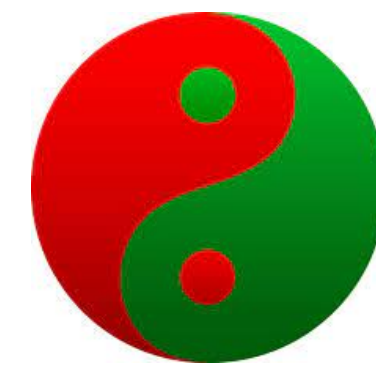
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A 2d-YSYK theory describes the **strange metal** behavior of numerous quantum materials



In a *dual* set of variables the SYK model has led to the computation of the low energy density of states of ***charged black holes***

