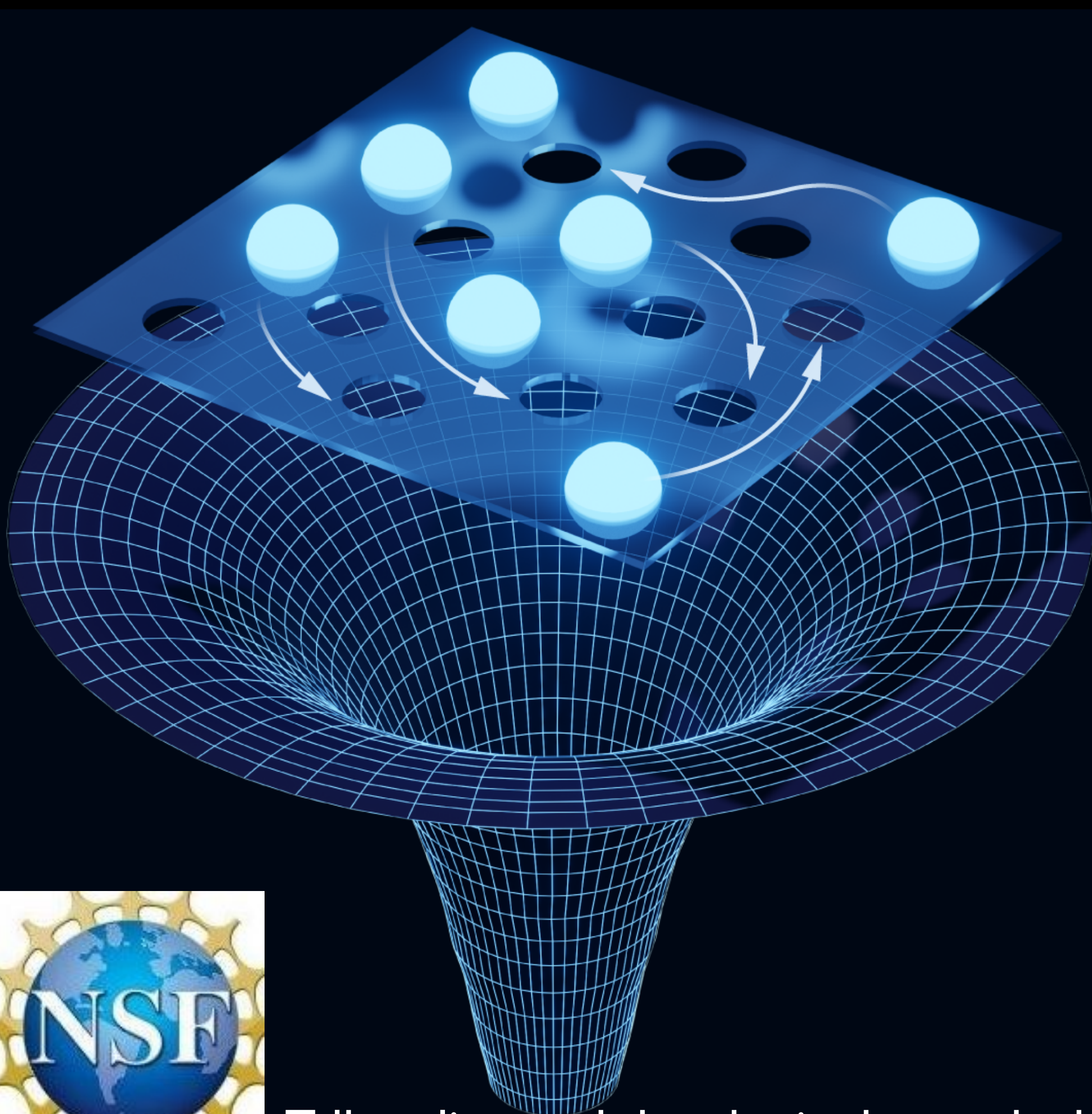


Quantum entanglement in nature:

superconductors and black holes



Institute Lecture
Indian Institute of Technology
Delhi
January 3, 2024

Subir Sachdev



Talk online: sachdev.physics.harvard.edu



Great discoveries in physics

Entropy (1870)

Superconductivity (1911)

Black holes (1916)

Quantum entanglement (1935)

**What is entropy
and temperature ?
(1870)**

Clausius (1865):

Second Law of Thermodynamics:

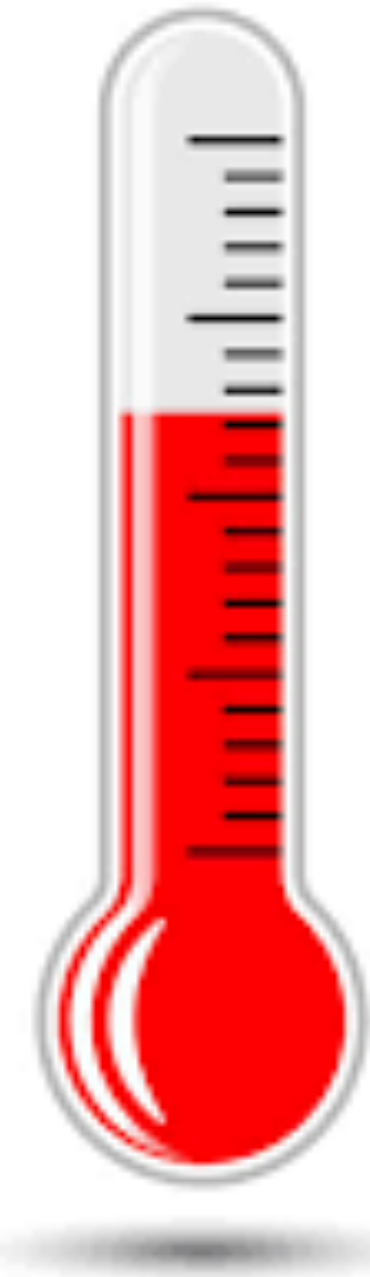
Every macroscopic system has an
“entropy” which cannot decrease.

➔ No perpetual motion machines!



Clausius (1865):
Second Law of Thermodynamics:
Every macroscopic system has an
“entropy” which cannot decrease.

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Temperature

Statistical interpretation of entropy (1870)

$$S = k_B \log W$$



Ludwig Boltzmann

20 February 1844 - September 5, 1906

Vienna, Austria

Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

$$\frac{1}{T} = \frac{dS}{dE}$$



Ludwig Boltzmann

20 February 1844 - September 5, 1906

Vienna, Austria

Ordinary conductors
(metals)
and
superconductors
(1911 - today)

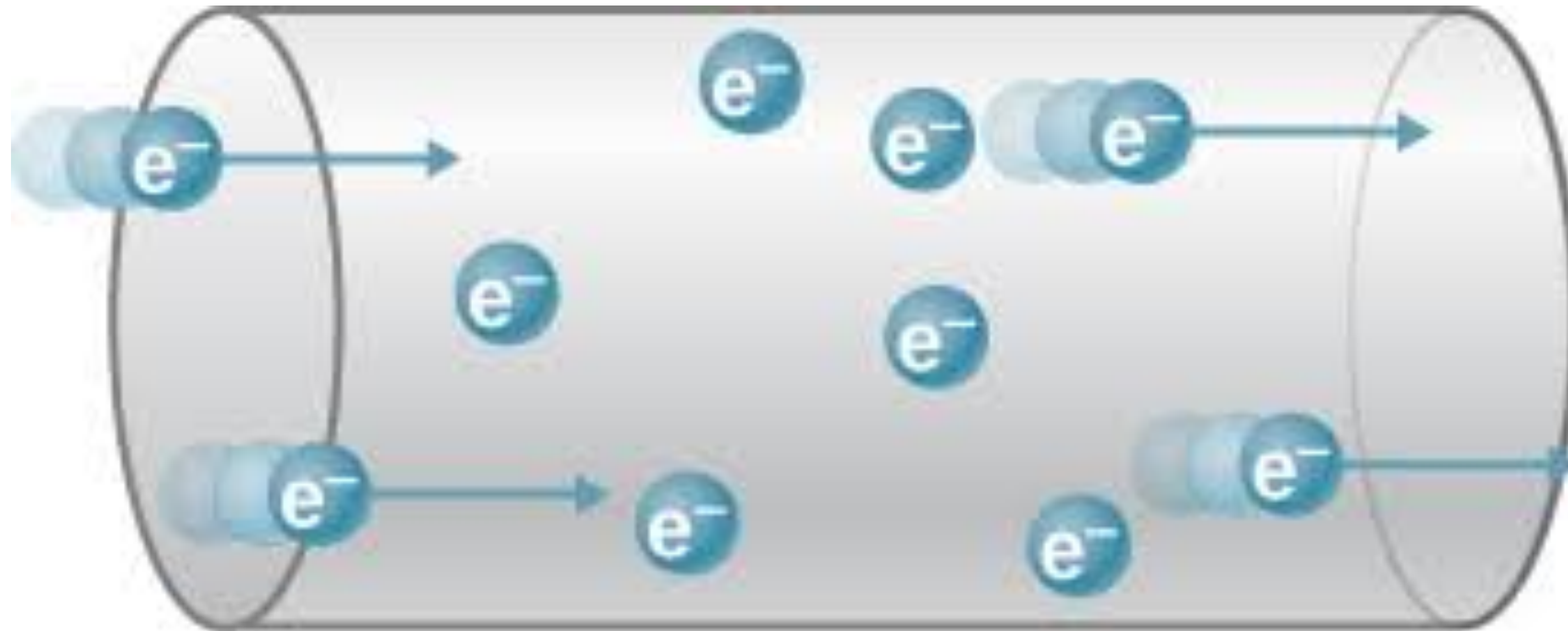


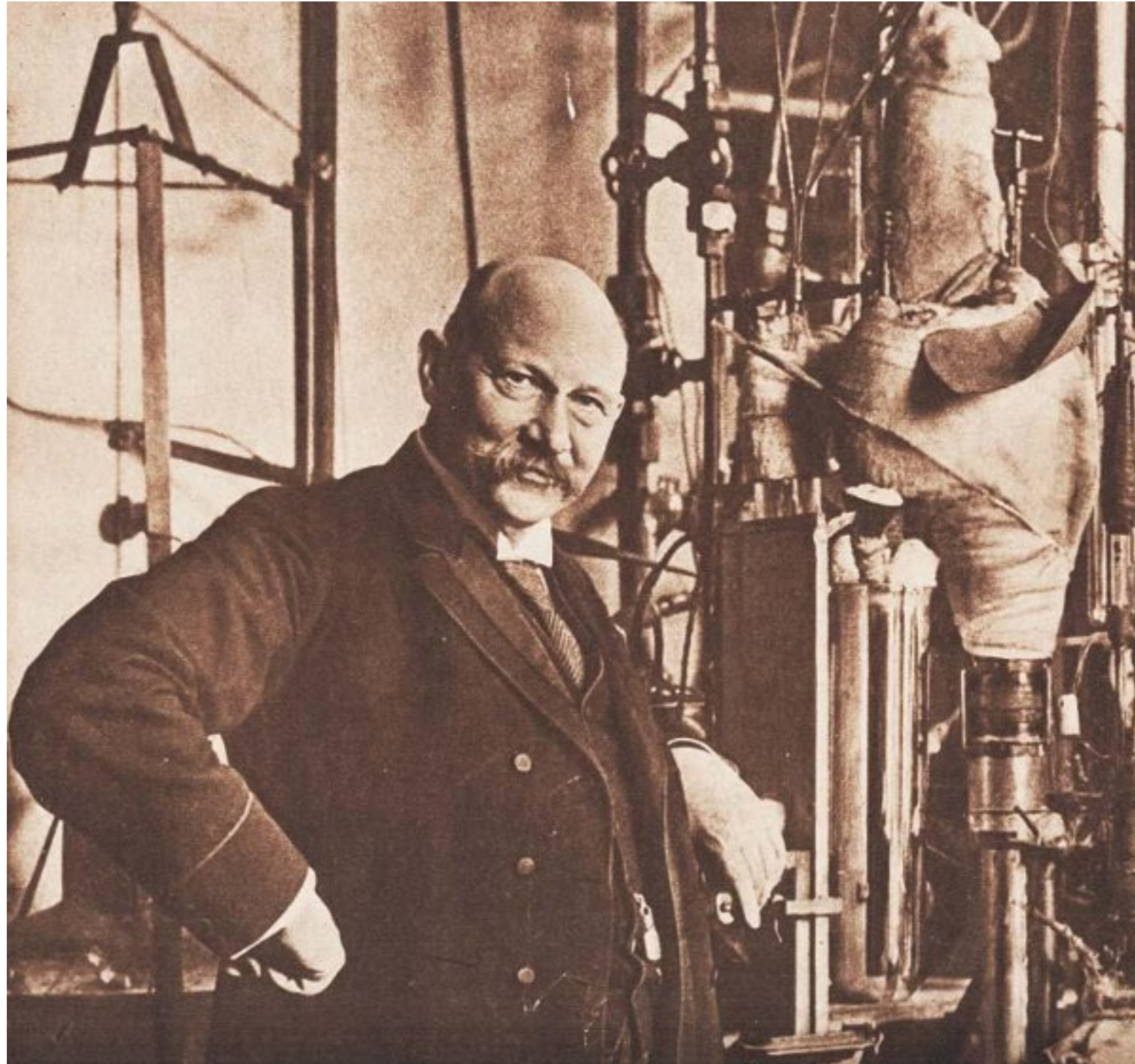
CONDUCTOR

SEMICONDUCTOR

SUPERCONDUCTOR

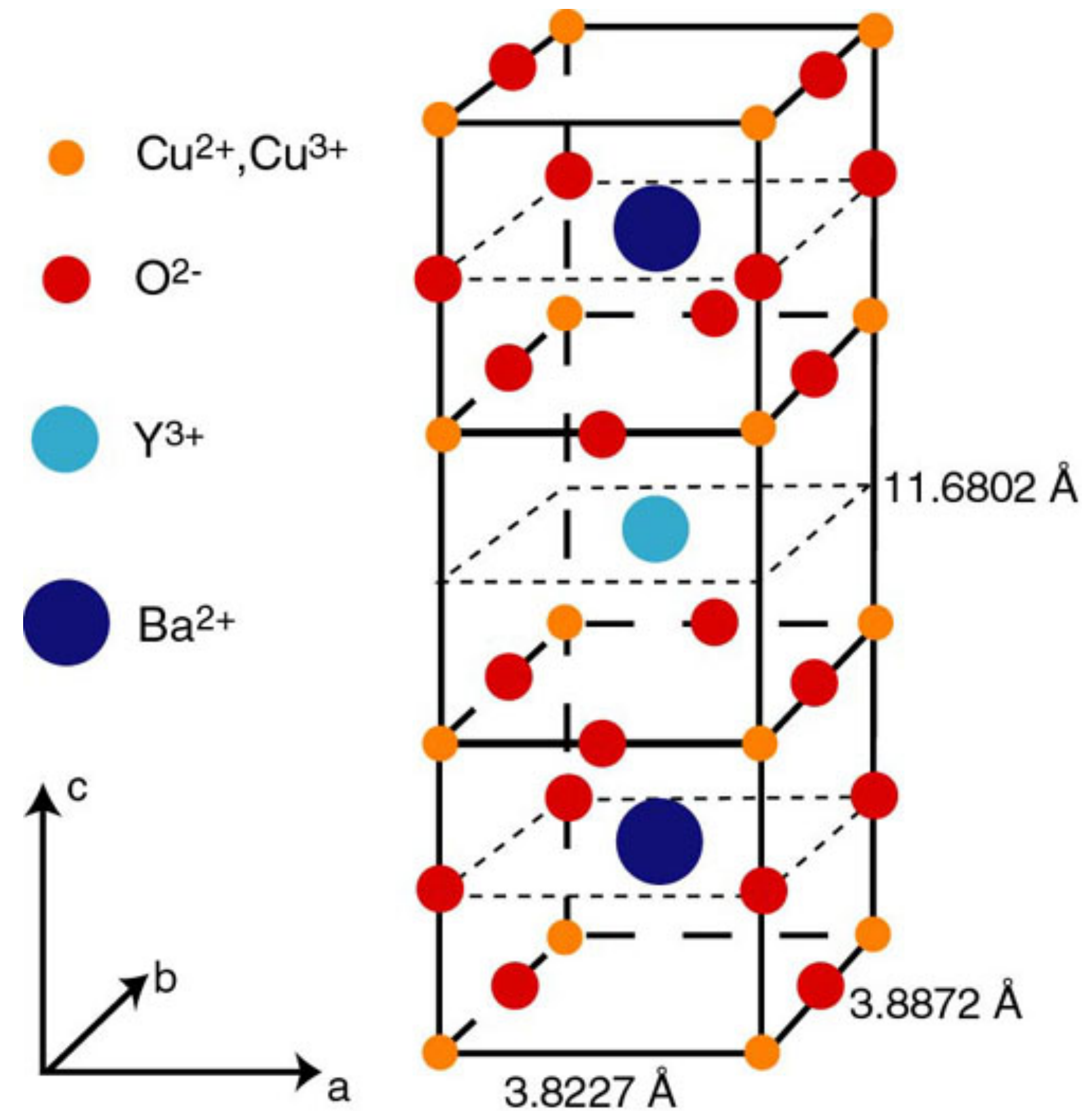
Copper: ordinary metal quantum matter

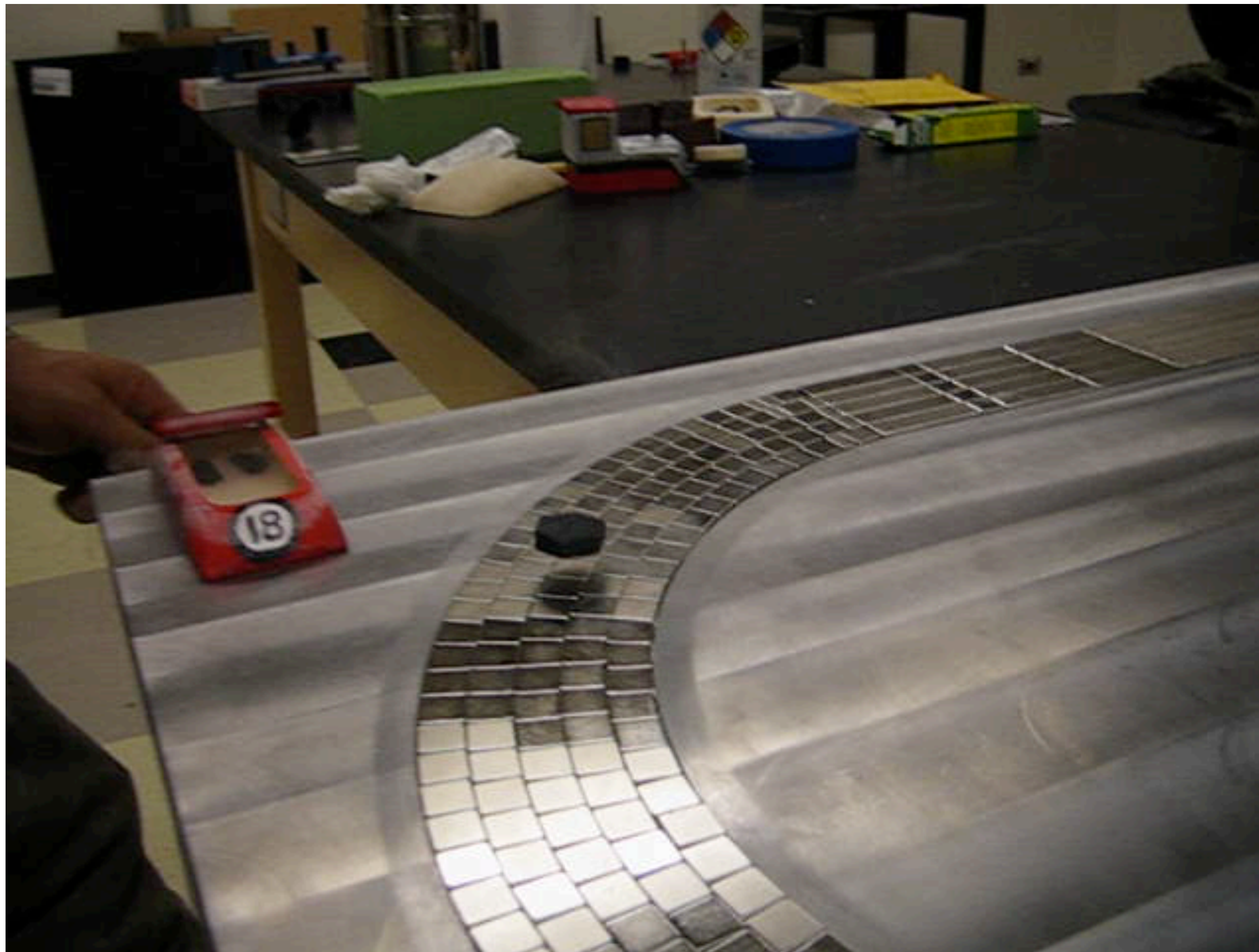




Kamerlingh Onnes 1911:
Mercury is a superconductor below $-269\text{ }^{\circ}\text{C}$

Cuprate high temperature superconductors





Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

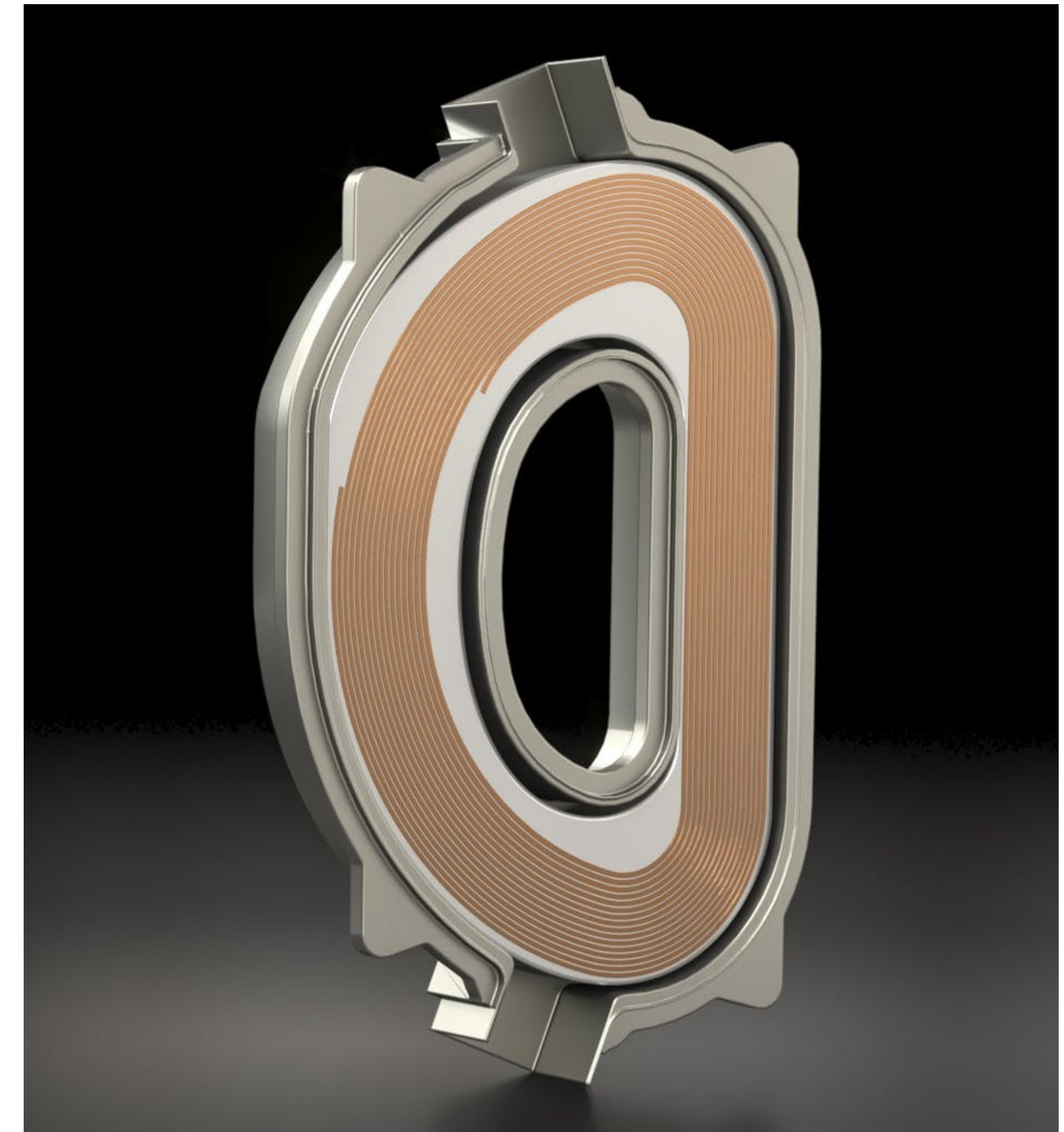
HTS Magnets: Enabling Technology

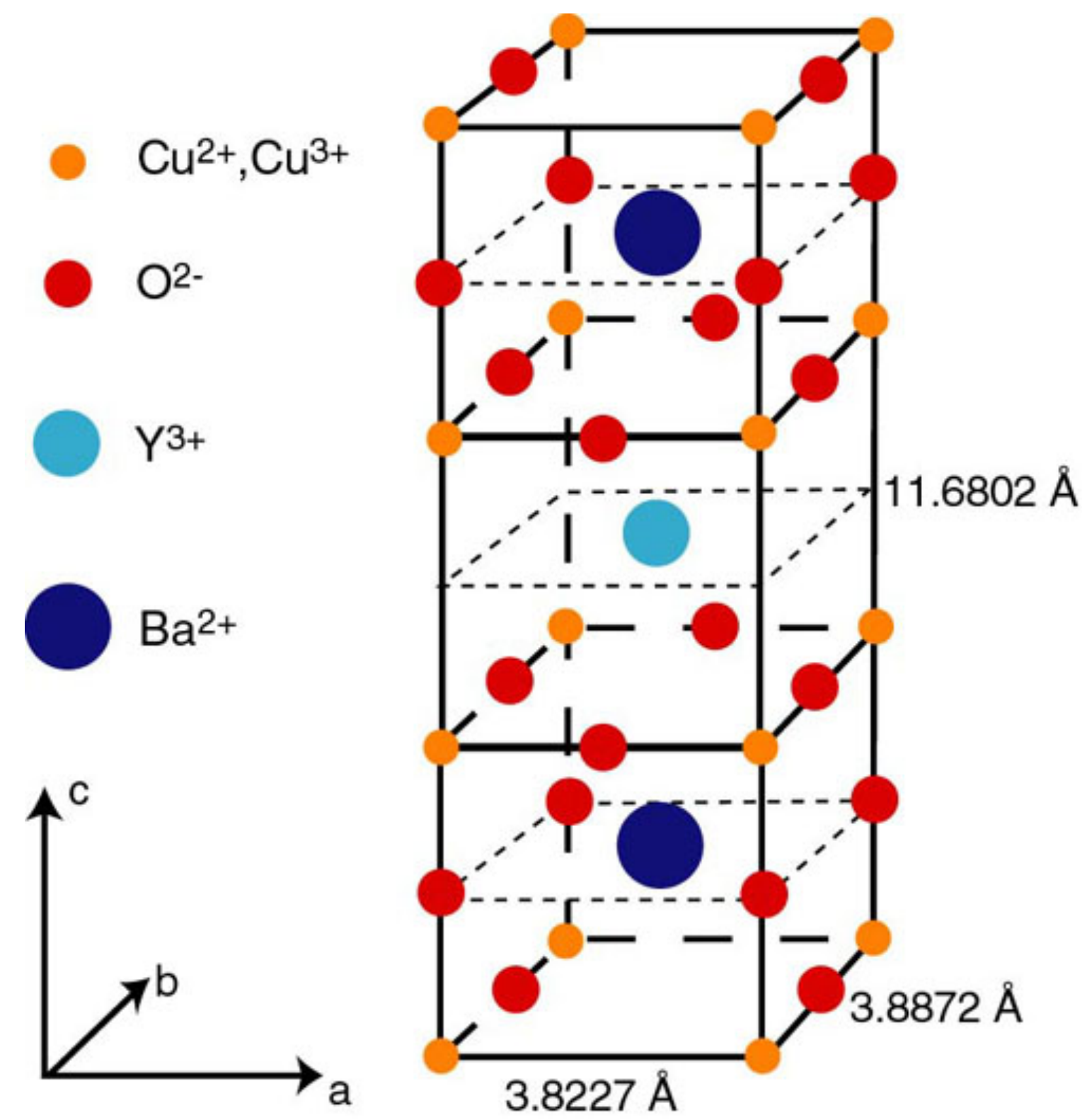
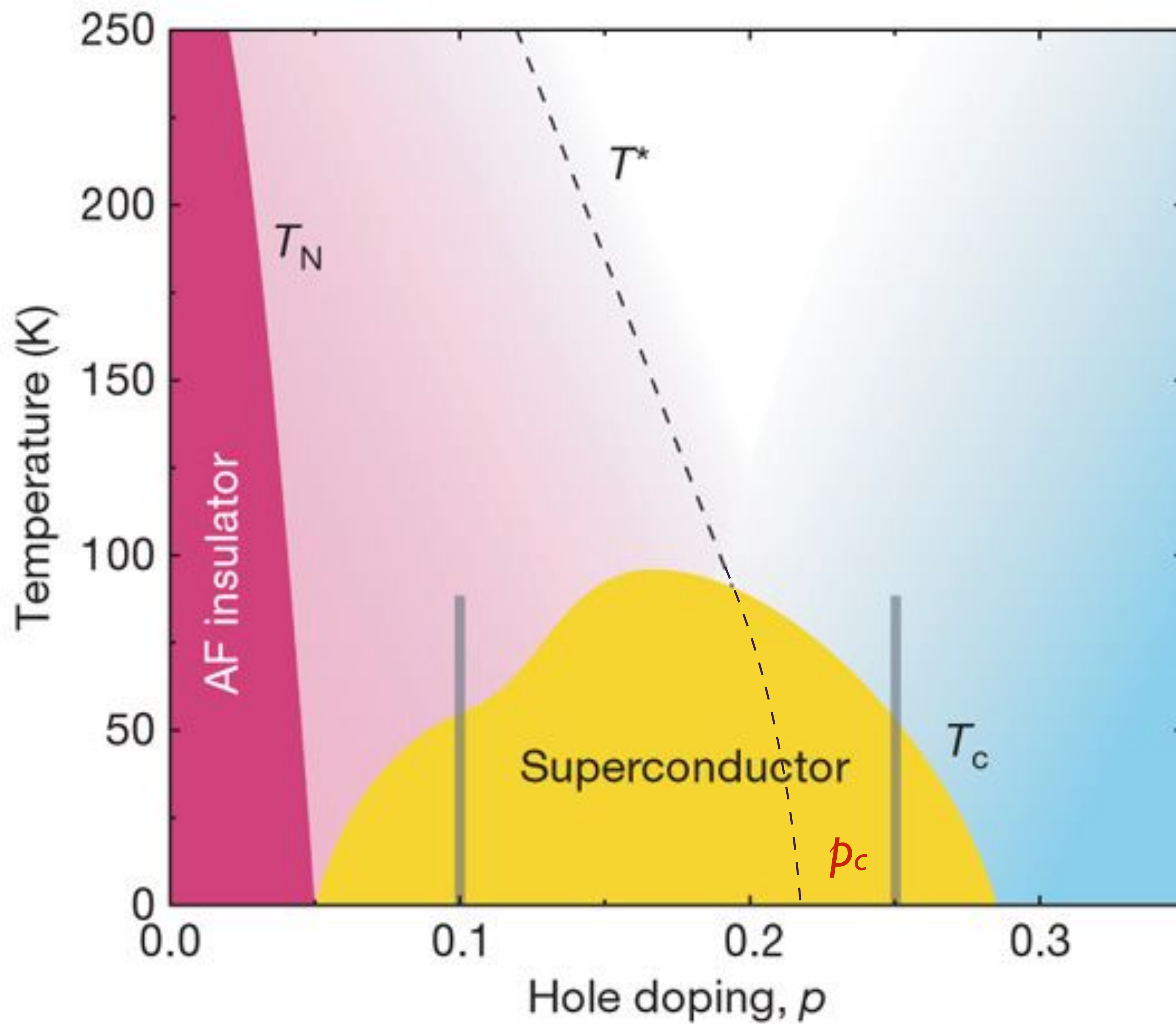
The surest path to limitless,
clean, fusion energy

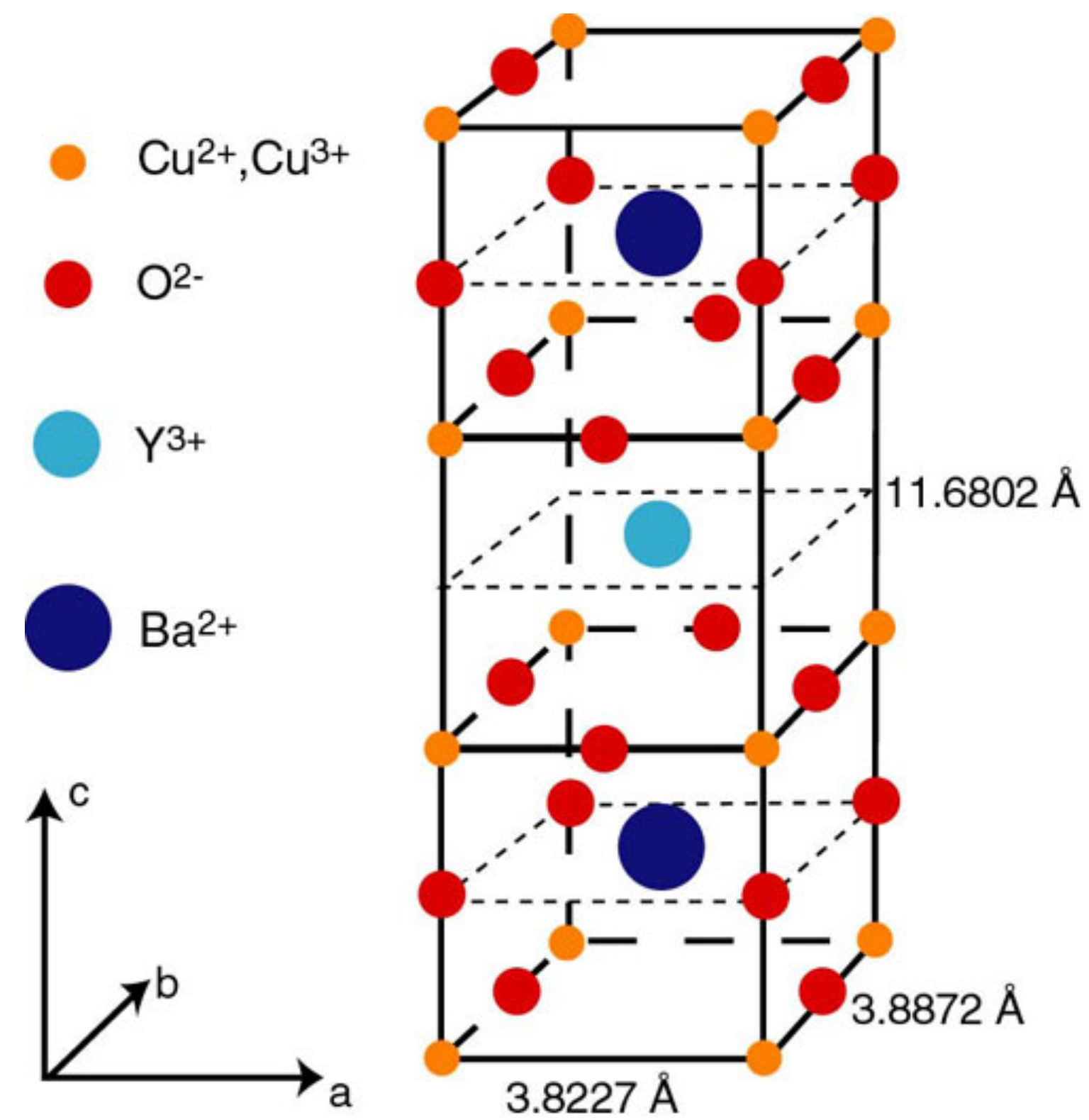
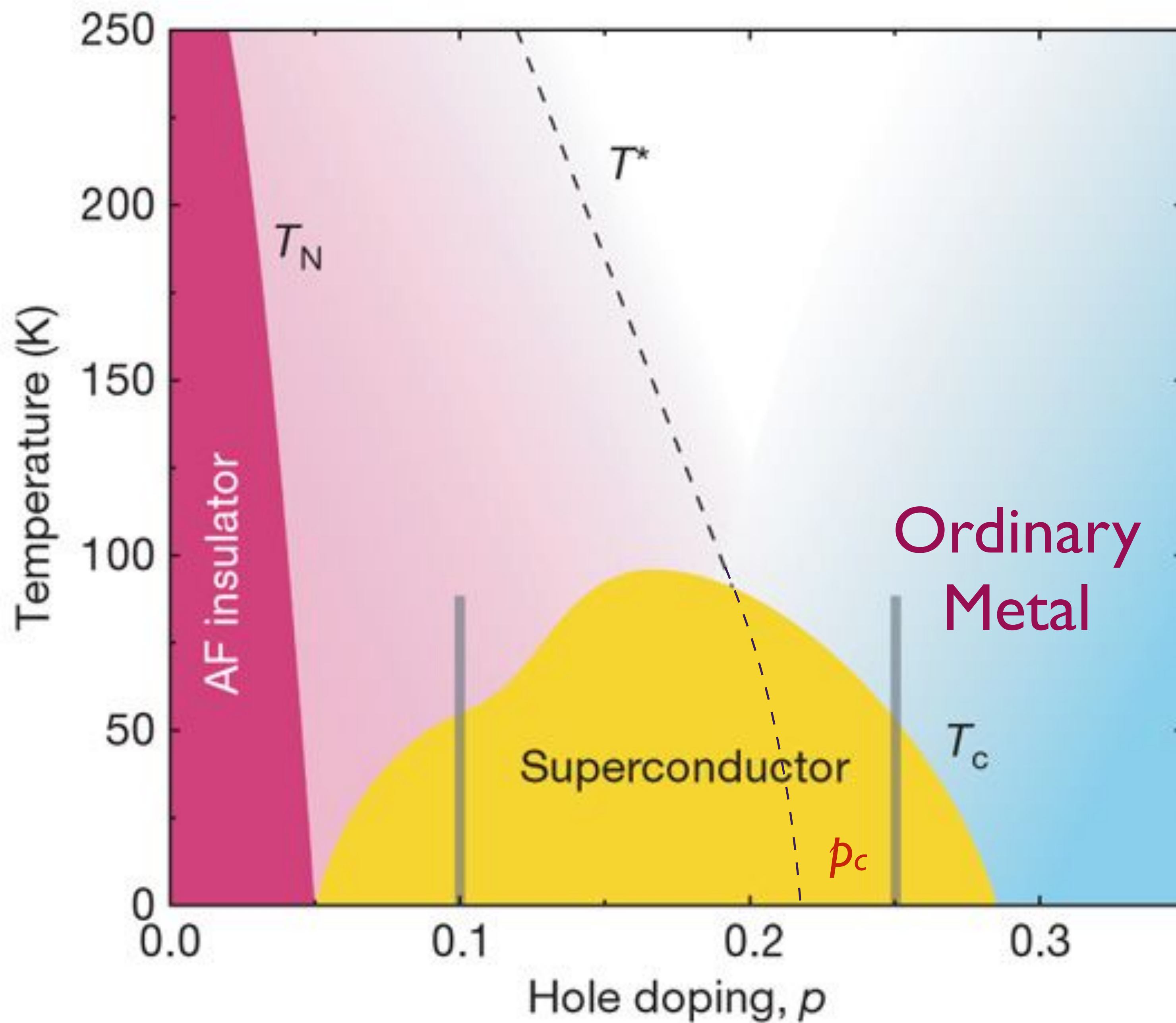
YBCO magnets allow for smaller,
faster, and less expensive
tokamaks for plasma fusion

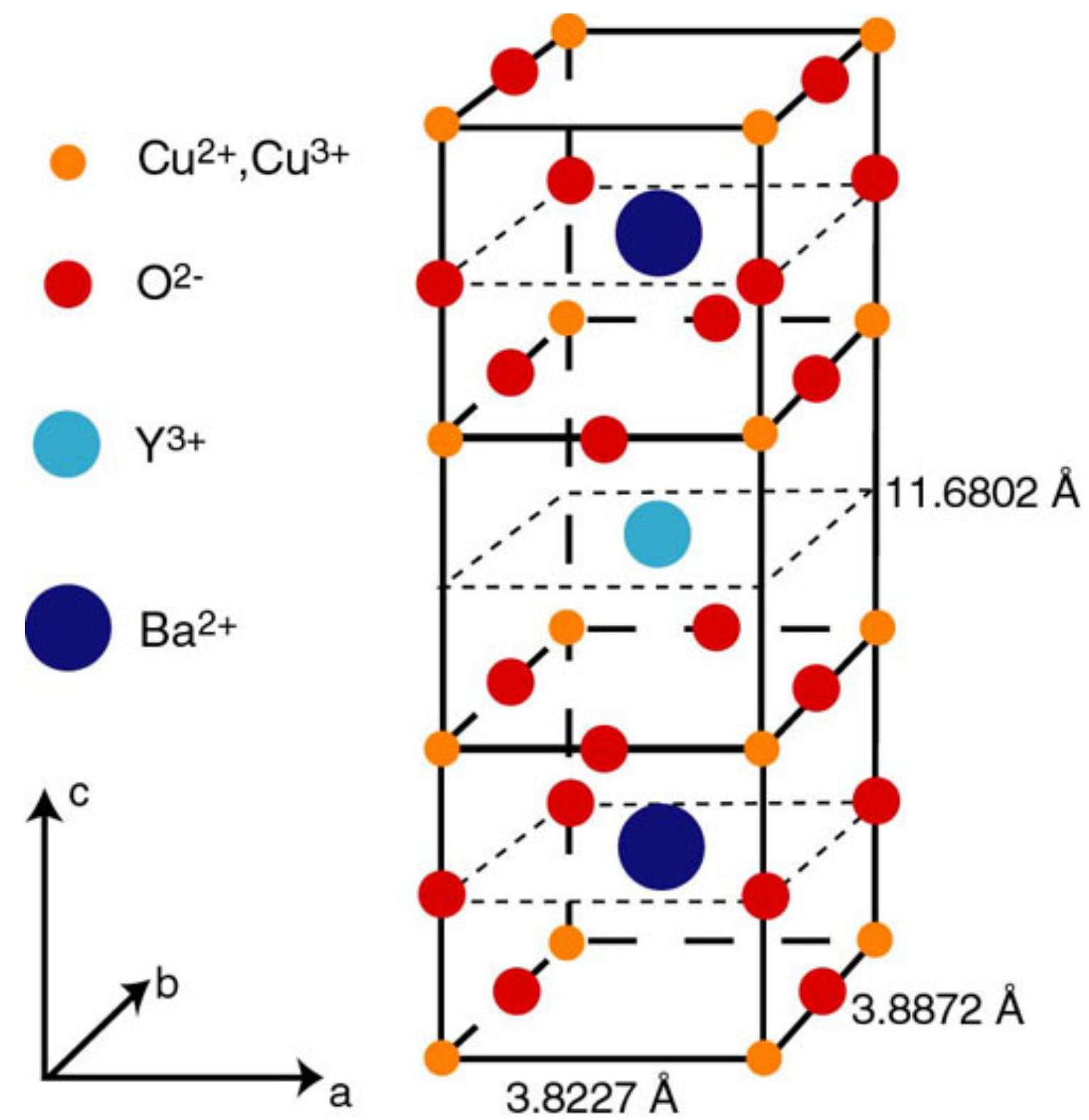
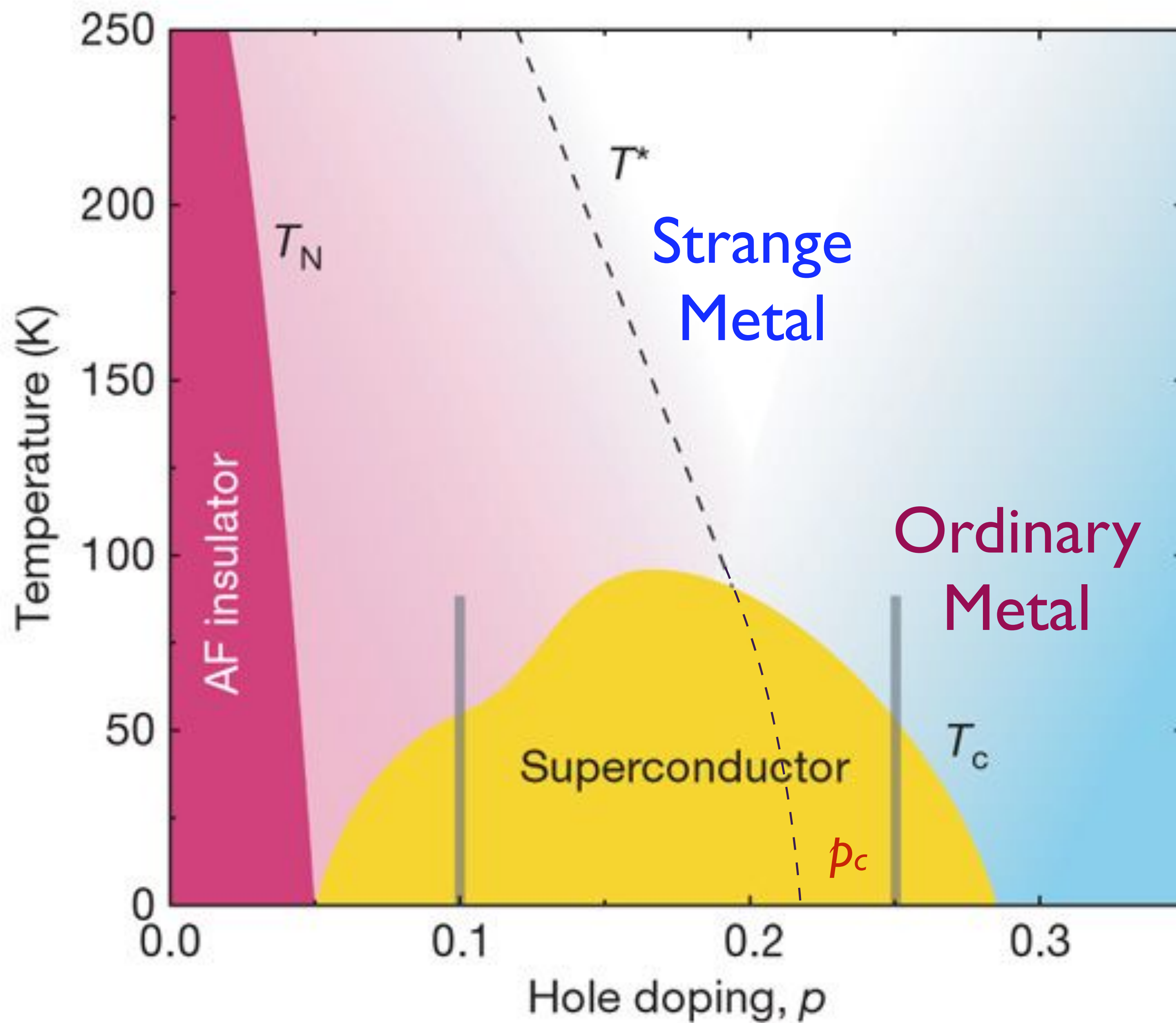


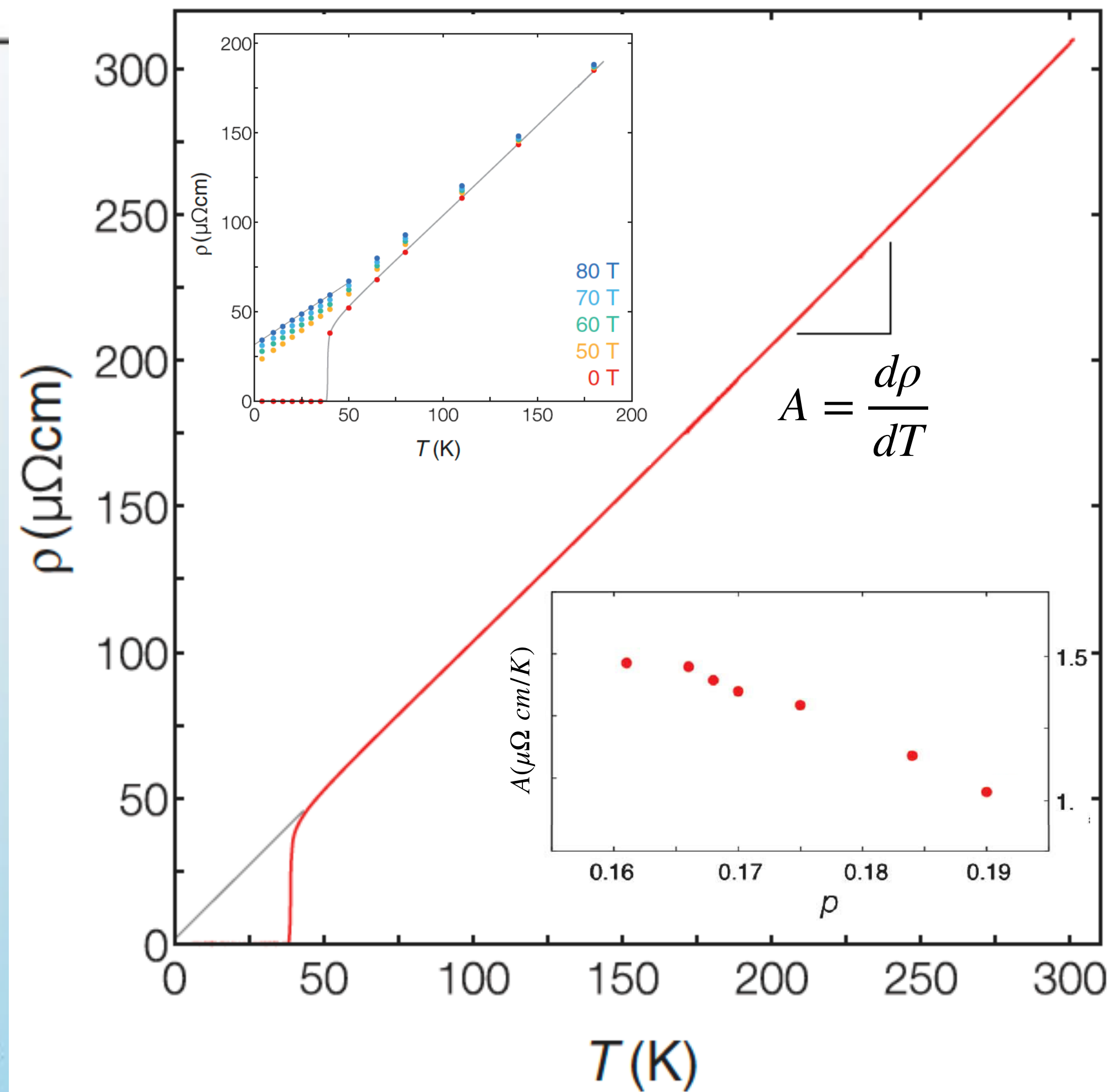
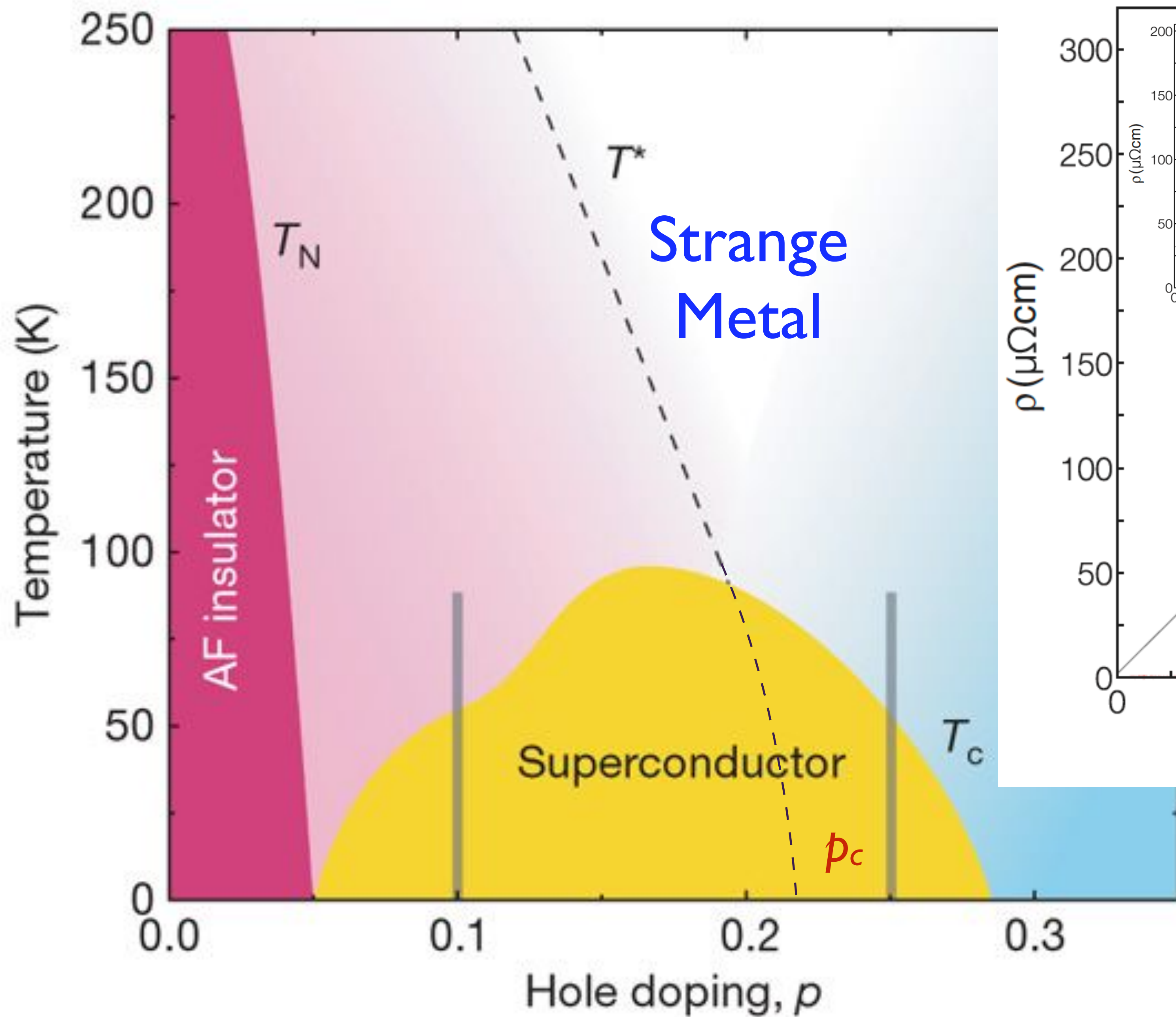
Commonwealth
Fusion Systems











LSCO: Giraldo-Gallo et al. 2018

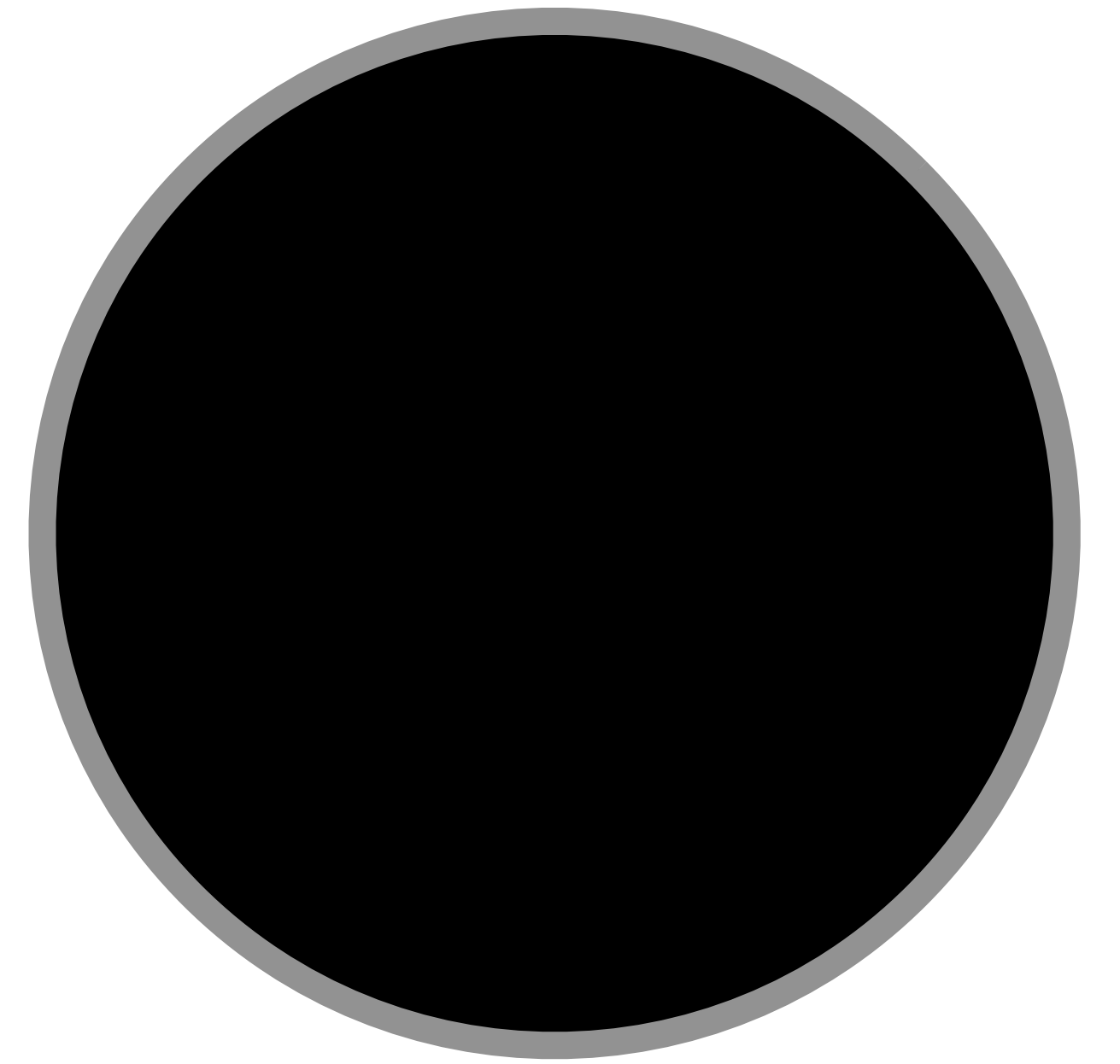
Black holes
(1916-today)

Black Holes

Objects so dense that light is gravitationally bound to them.



Horizon radius $R = \frac{2GM}{c^2}$

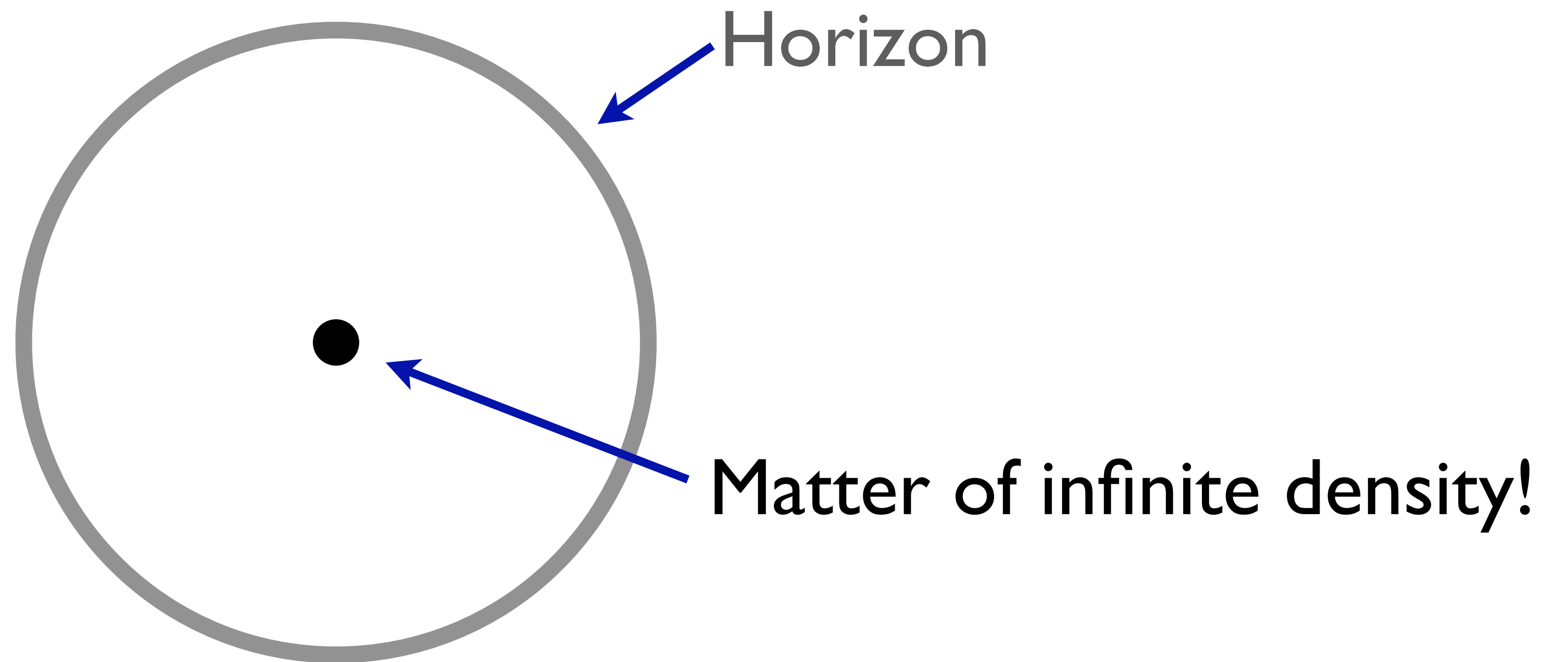


Karl Schwarzschild (1916)

G Newton's constant, c velocity of light, M mass of black hole
For $M = \text{earth's mass}$, $R \approx 9 \text{ mm!}$

What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.

This singularity convinced many early on that black holes were unphysical solutions of Einstein's equations, and did not exist in our universe.



The supermassive black hole lurking at the heart of the Milky Way – Sagittarius A* contains about 4.3 million solar masses

$$R = 1.3 \times 10^{11} \text{ m}$$

\approx earth's orbit

Event Horizon Telescope
May 12, 2022

What is inside a black hole ???

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This singularity convinced many early on that black holes were unphysical solutions of Einstein's equations, and did not exist in our universe.

In any case, it was clear that quantum theory should be applied to the collapsed matter, but no one knew how to.

What is inside a black hole ???

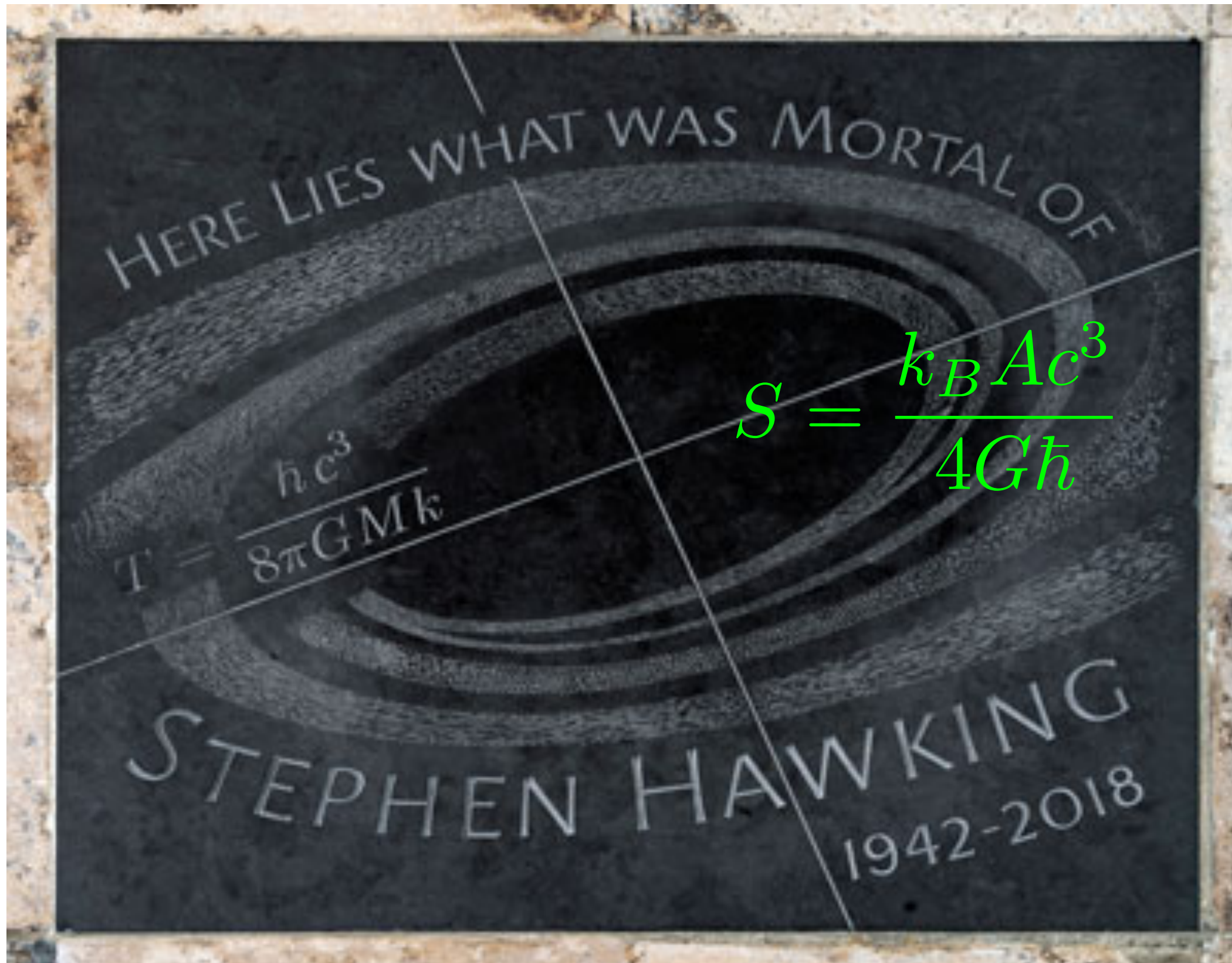
Hawking (1975): when viewed from the outside, black holes have an entropy and a temperature, and slowly evaporate like any thermal object



$$T = \frac{\hbar c^3}{8\pi G M k_B}$$

What is inside a black hole ???

Hawking (1975): when viewed from the outside, black holes have an entropy and a temperature, and slowly evaporate like any thermal object



$$T = \frac{\hbar c^3}{8\pi G M k_B}$$
$$S = \frac{k_B A c^3}{4G\hbar}$$

What is
quantum entanglement?
(1935-2015)

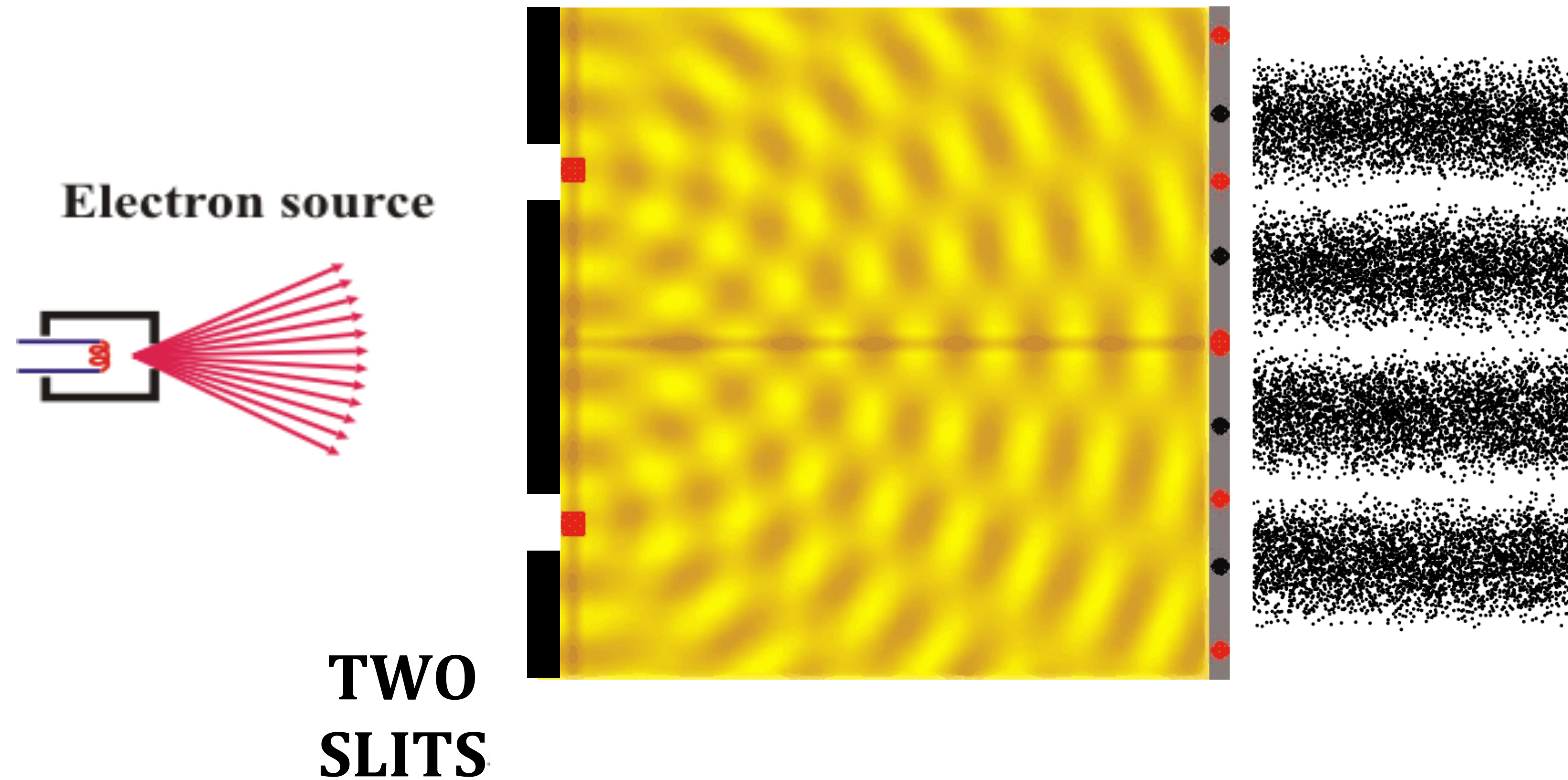


“About your cat, Mr. Schrödinger—I have good news and bad news.”

The most remarkable new idea in the quantum theory is the
principle of superposition:
a physical system can be in a
superposition of two (or more) distinct states.

Principles of Quantum Mechanics: I. Quantum Superposition

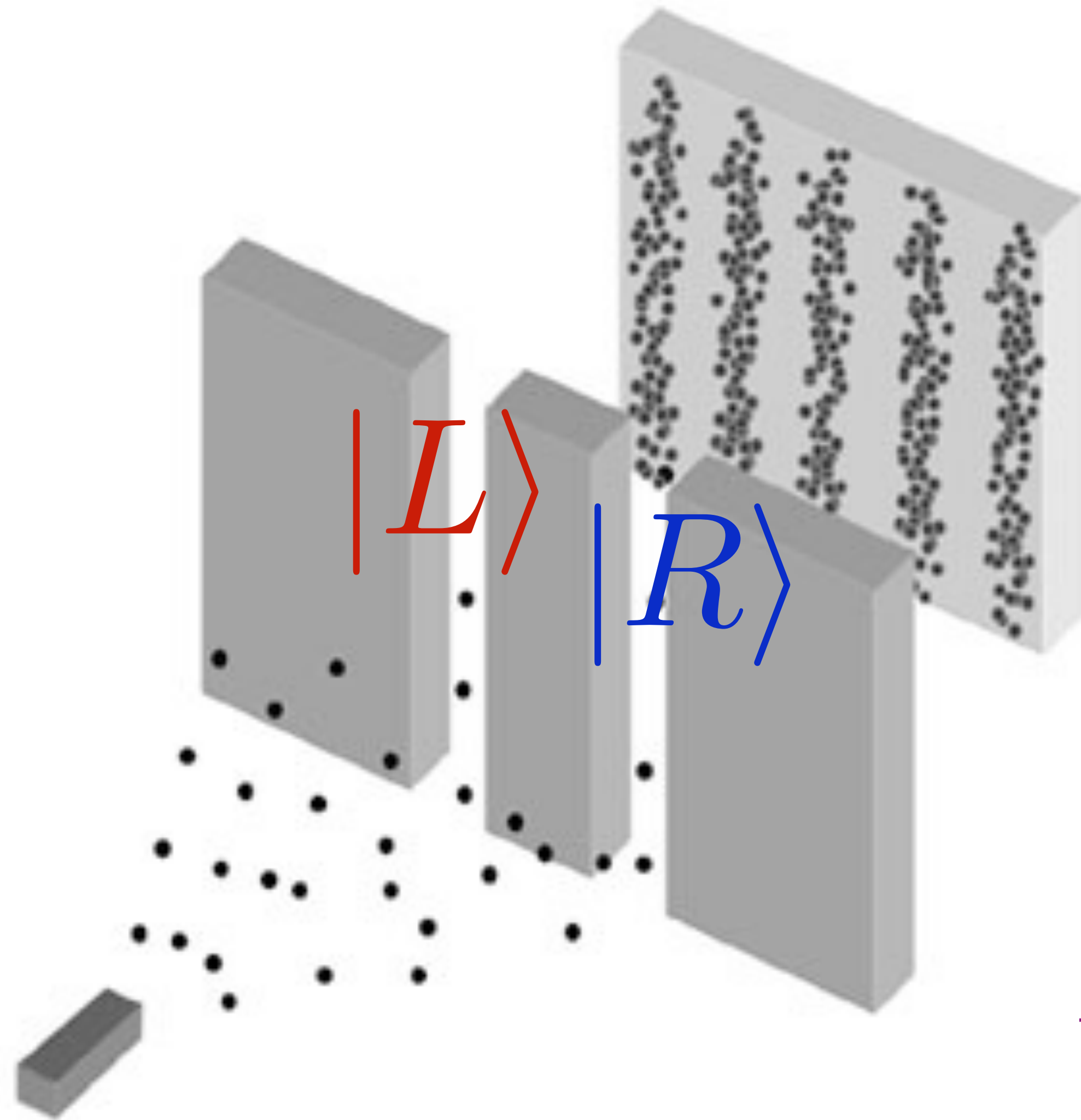
The double slit experiment



Unlike water waves, electrons arrive one-by-one (so is it like a particle ?)

Interference of electrons

The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

Actual state of *each* electron is

$$|L\rangle + |R\rangle$$

MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

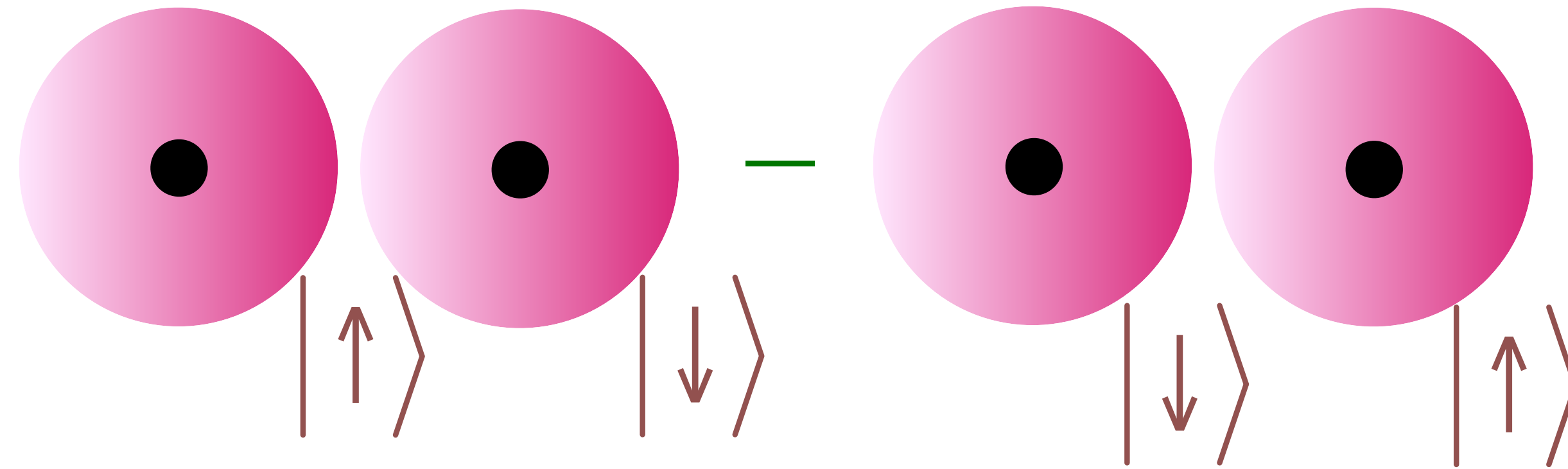
Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

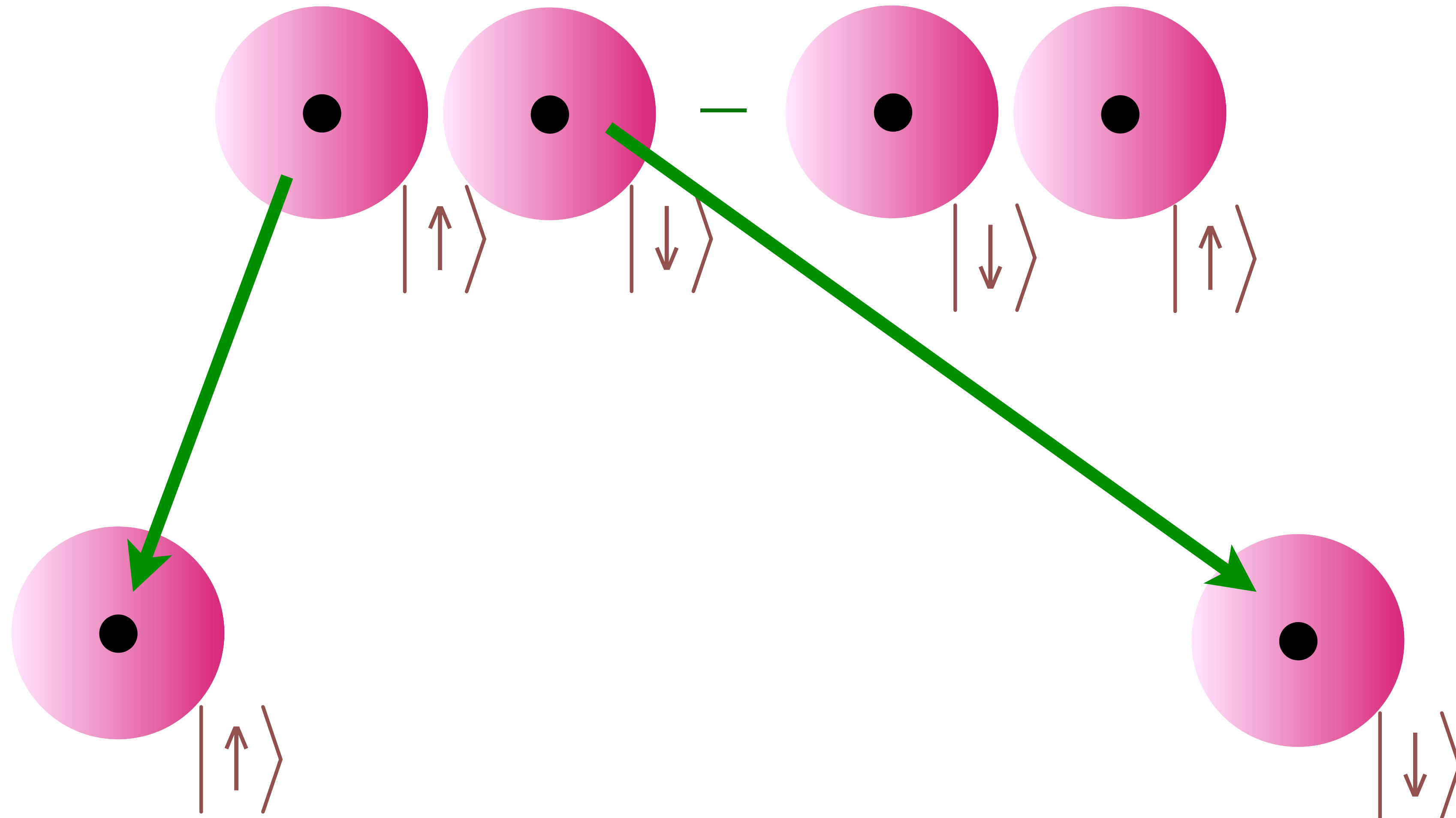
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



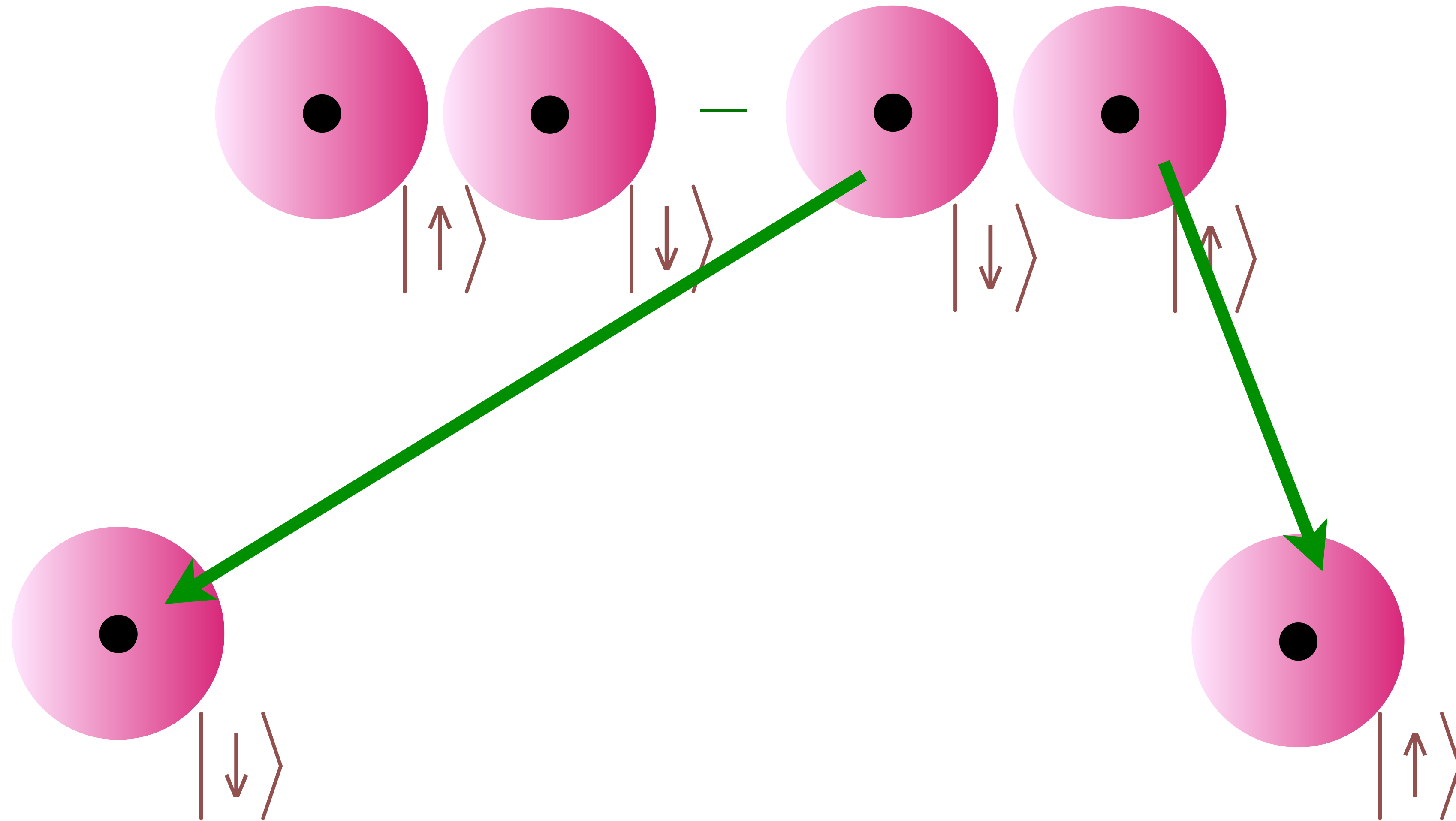
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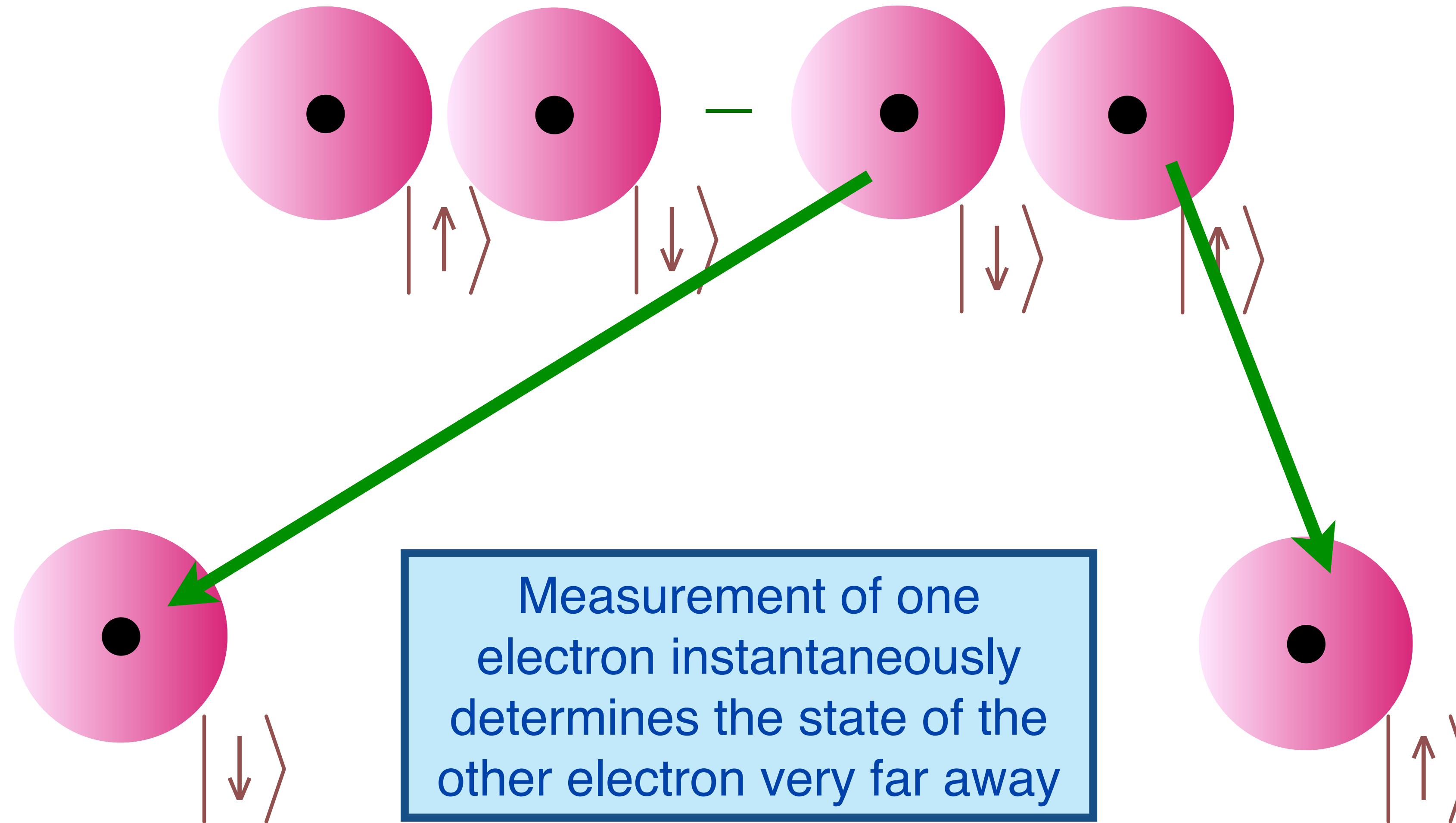
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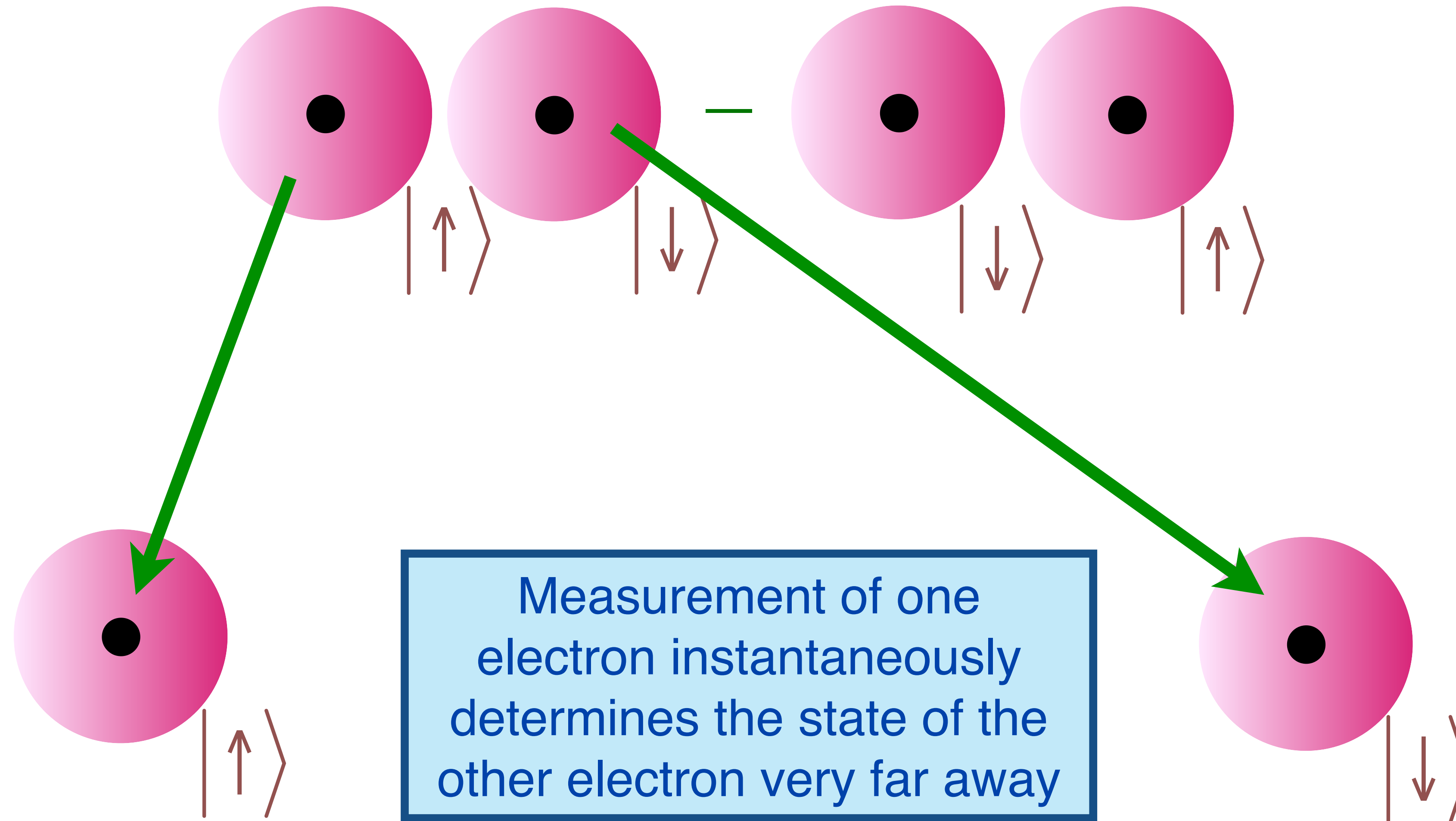
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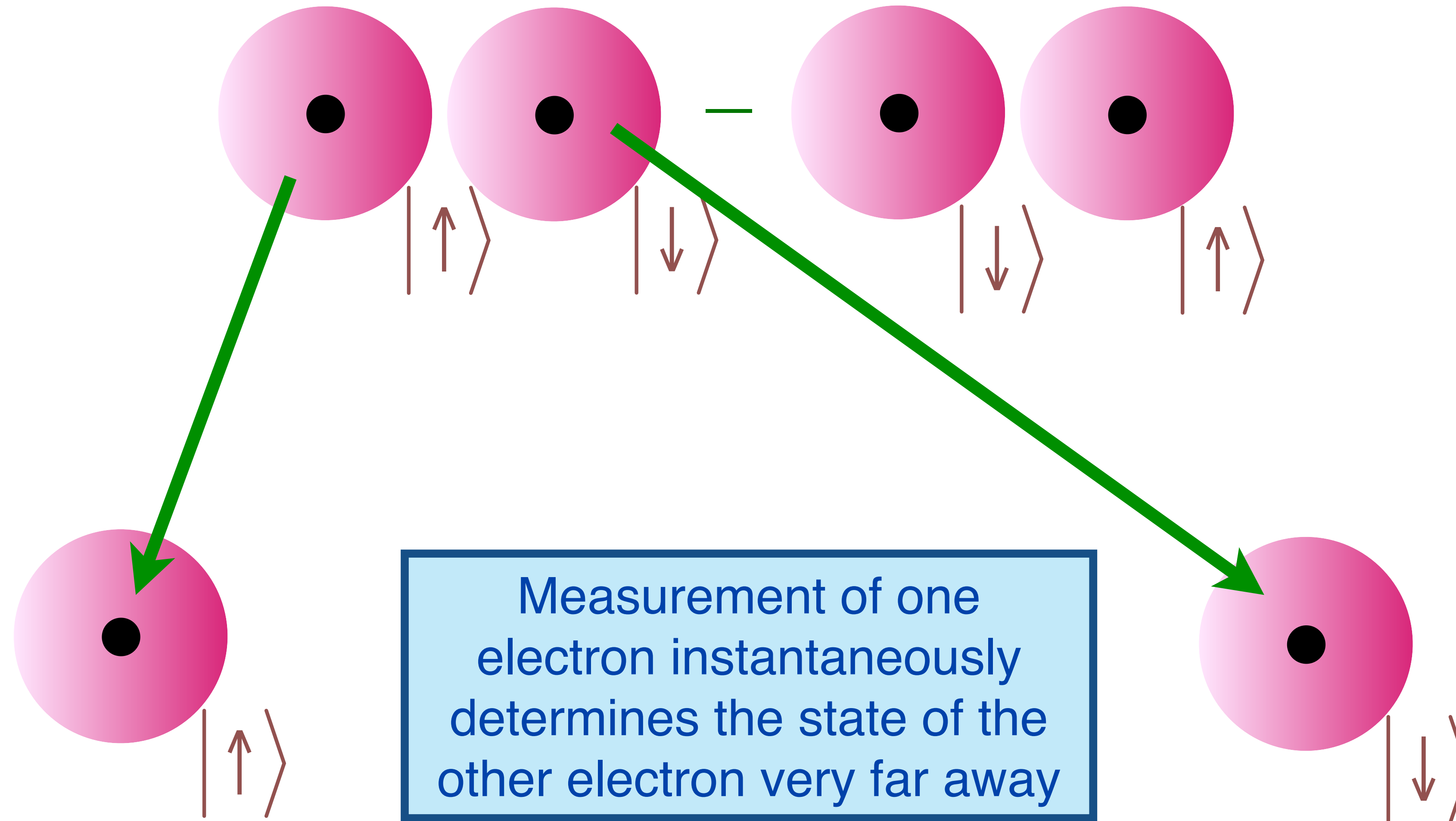
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



Spooky action at a distance !

natürlicher
deren Notwendigkeit im
mus ja zuerst von Dir klar erkannt wurde, einen Bedeutung
Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft dar-
an glauben, weil die Theorie mit dem Grundsatz unvereinbar
ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstel-
len soll, ohne spukhafte Fernwirkungen. Allerdings bin ich
überzeugt, daß es wirklich mit der Theorie

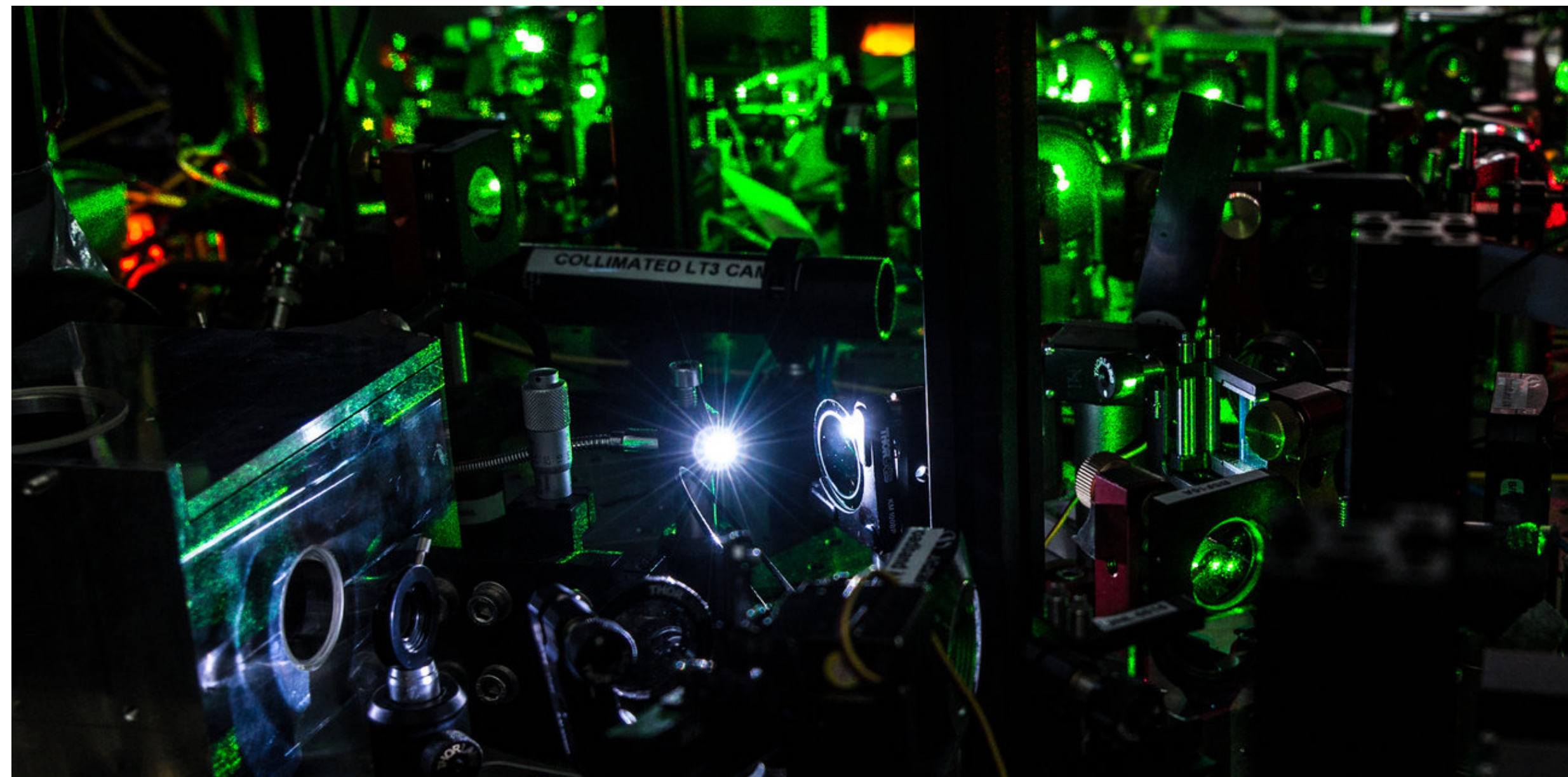
I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

Albert Einstein to Max Born, 3 March 1947

The New York Times

Sorry, Einstein. Quantum Study Suggests ‘Spooky Action’ Is Real.

By JOHN MARKOFF OCT. 21, 2015



Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

Great discoveries in physics

Entropy (1870)

Superconductivity (1911)

Black holes (1916)

Quantum entanglement (1935)

Needed,
to solve open problems in the theory of
superconductivity and black holes:

A solvable model of quantum entanglement
of 3, 4, 5, ... ∞ particles

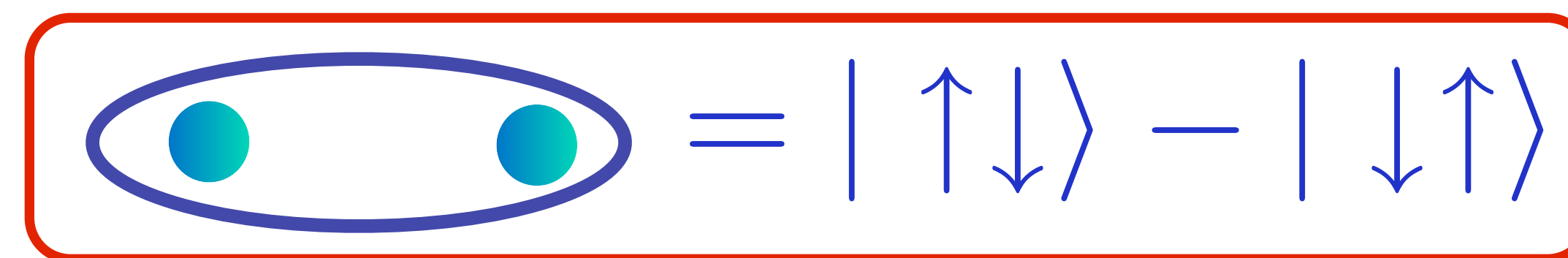
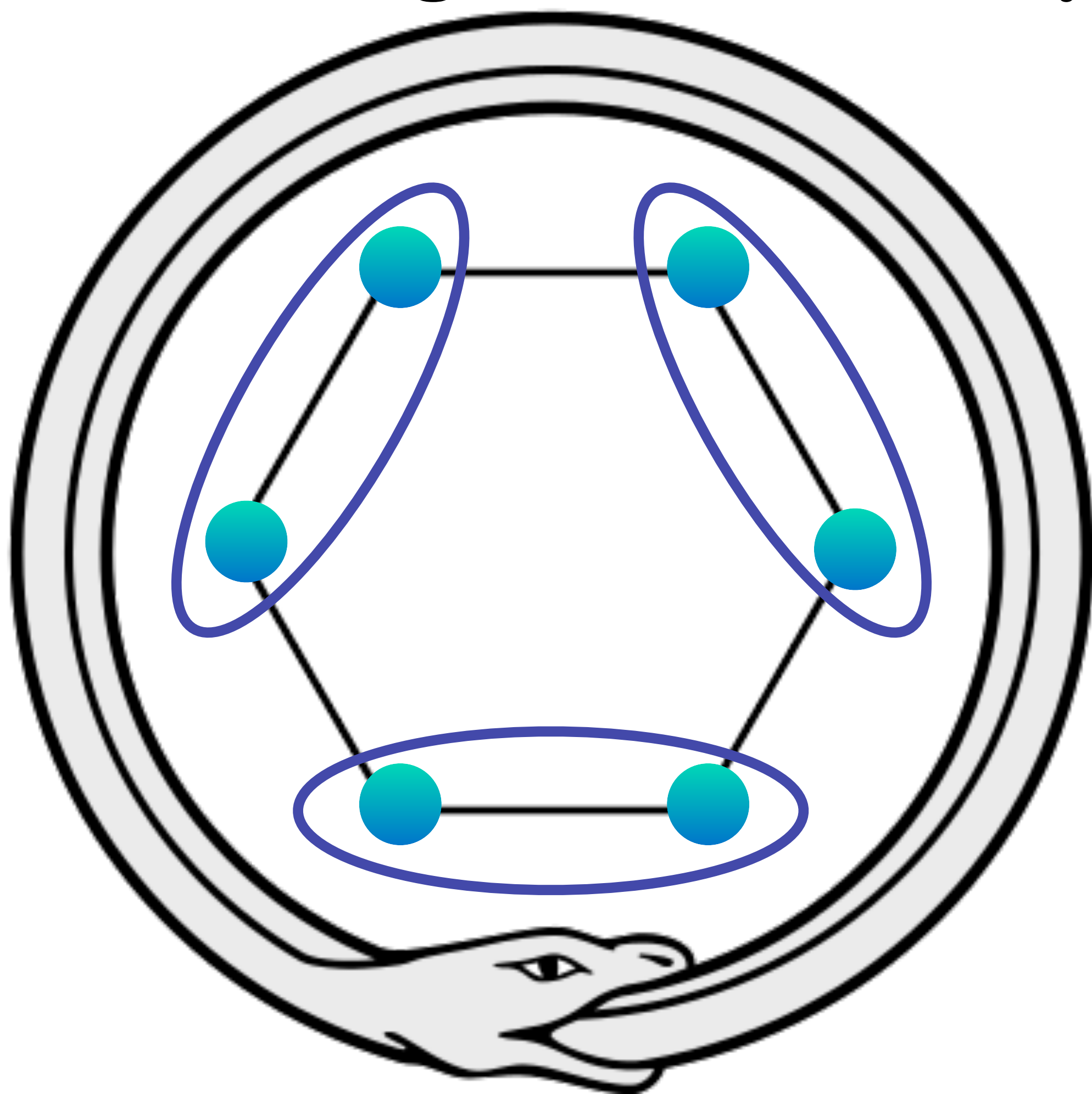
Needed,
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**The Sachdev-Ye-Kitaev model
of many-particle entanglement**

Kekulé's spooky dream (1865)

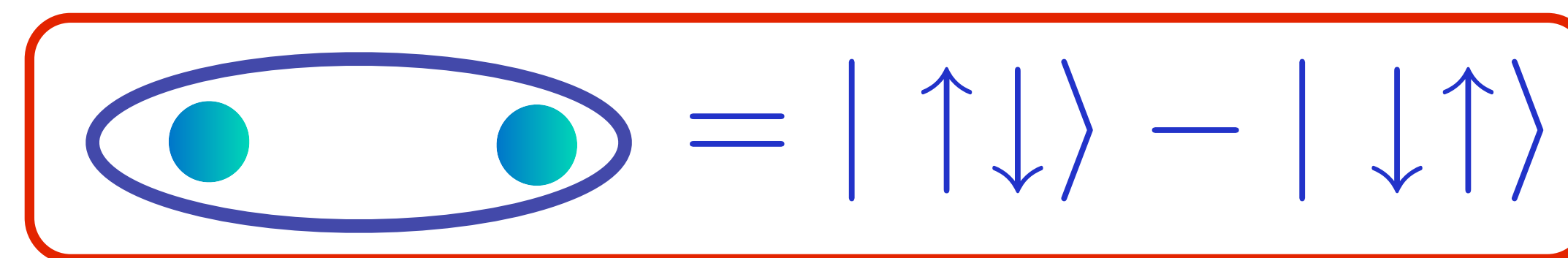
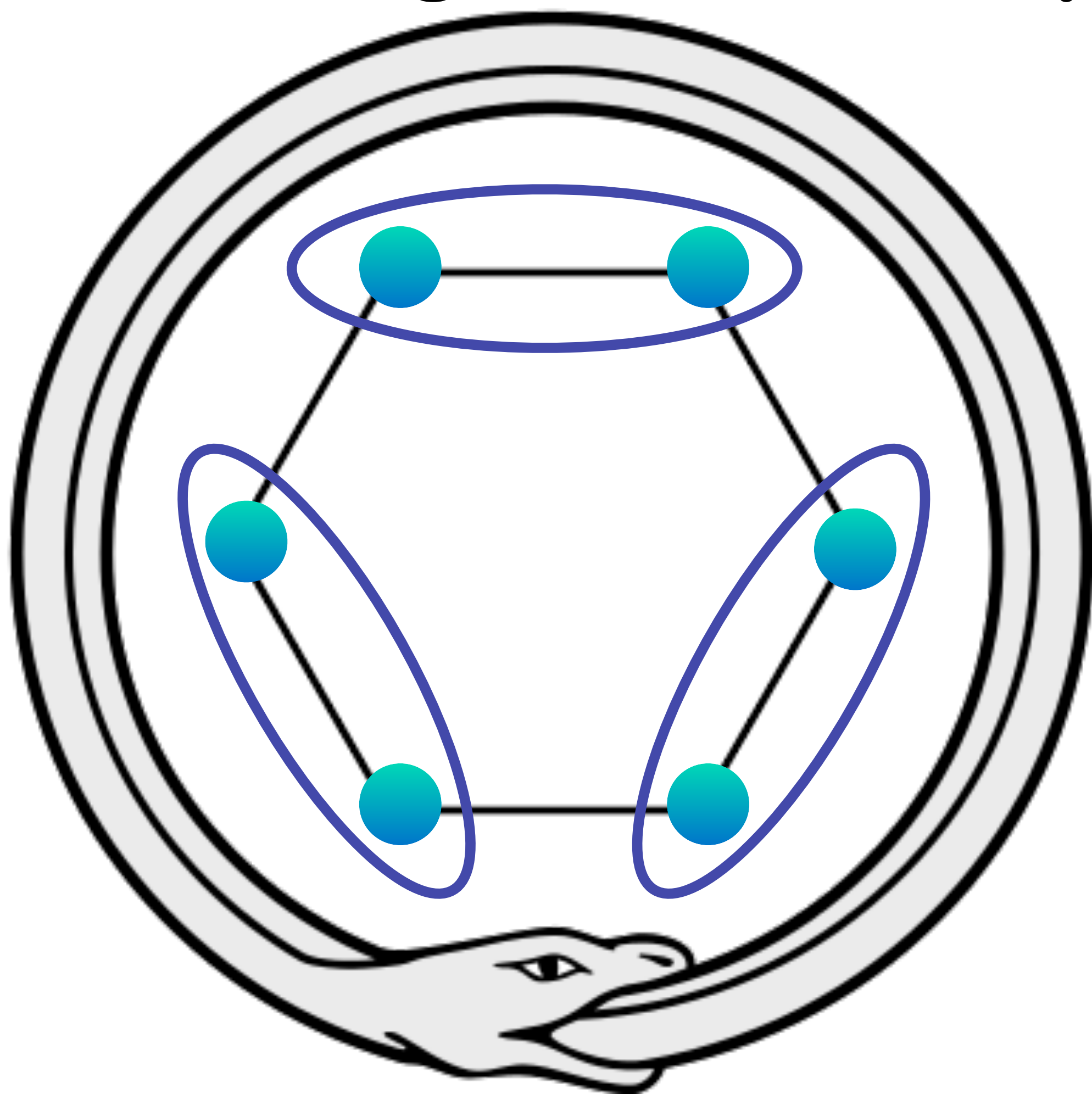
Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*



Benzene

Kekulé's spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*

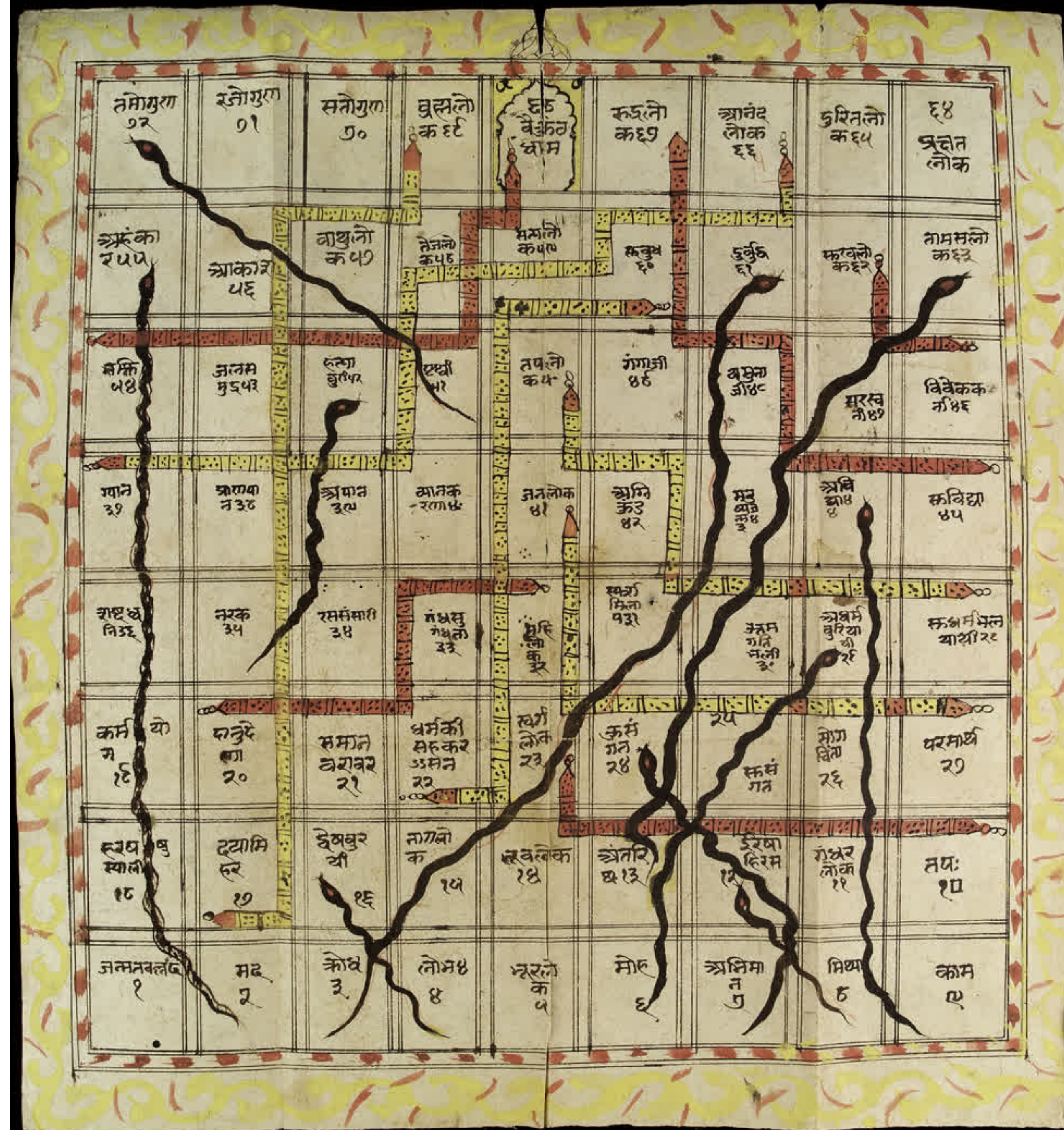


Benzene

My
spooky
dream*

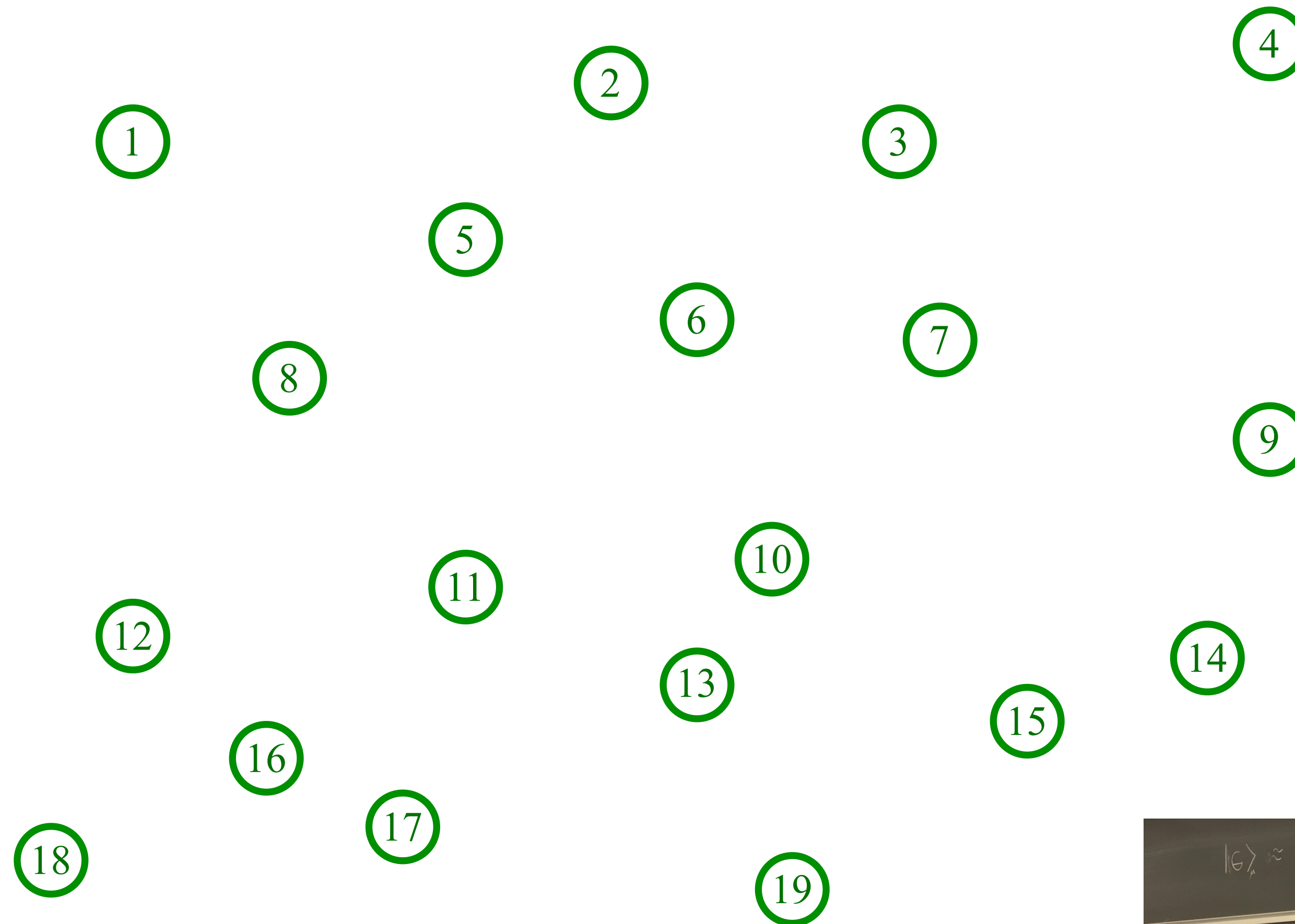
Ancient
Indian
game of
Snakes
and
Ladders

*Not true

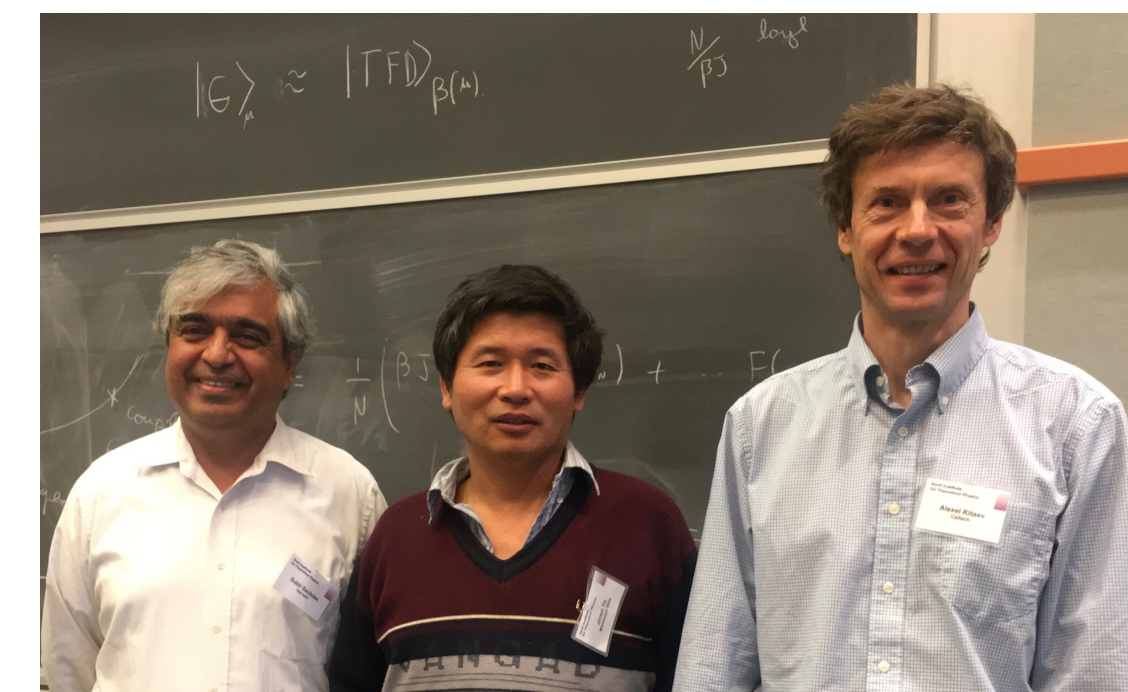


The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

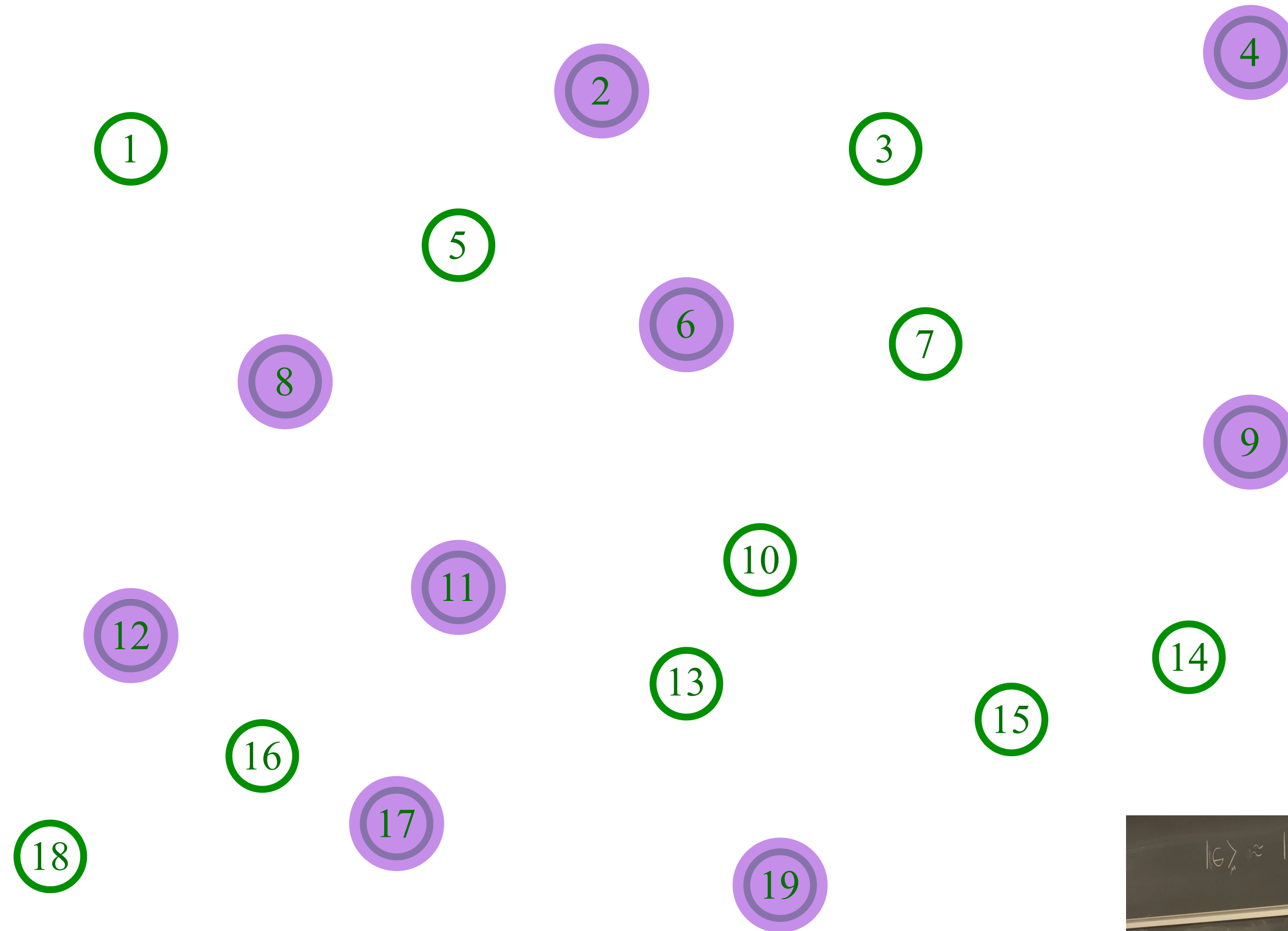


Pick a set of random positions

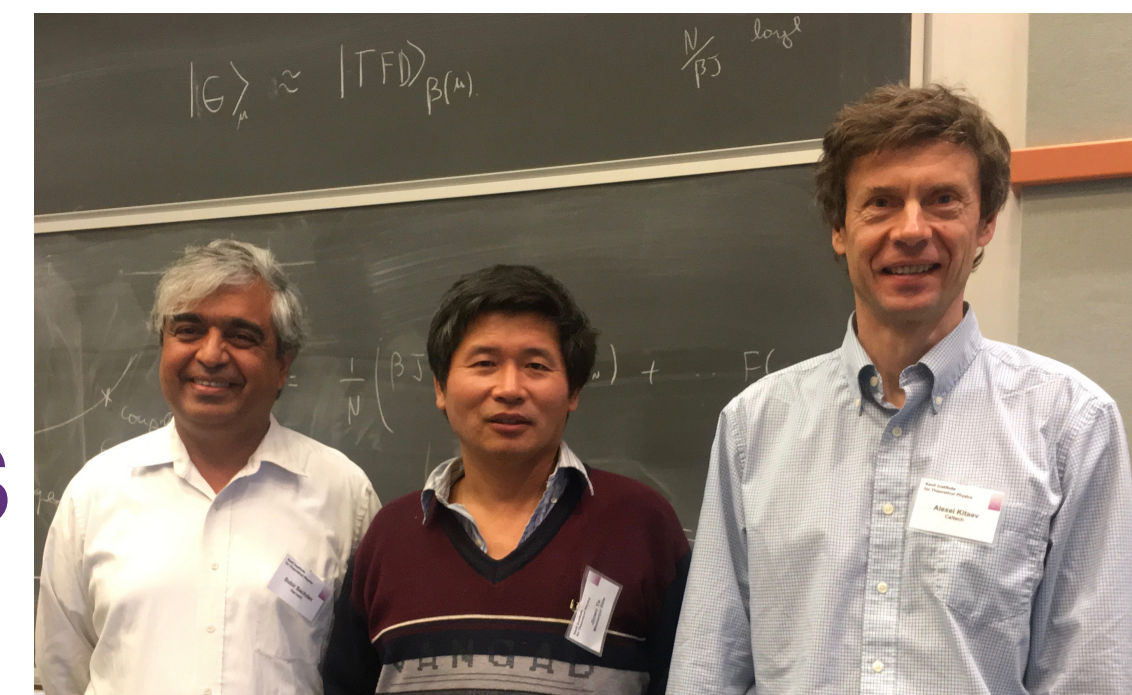


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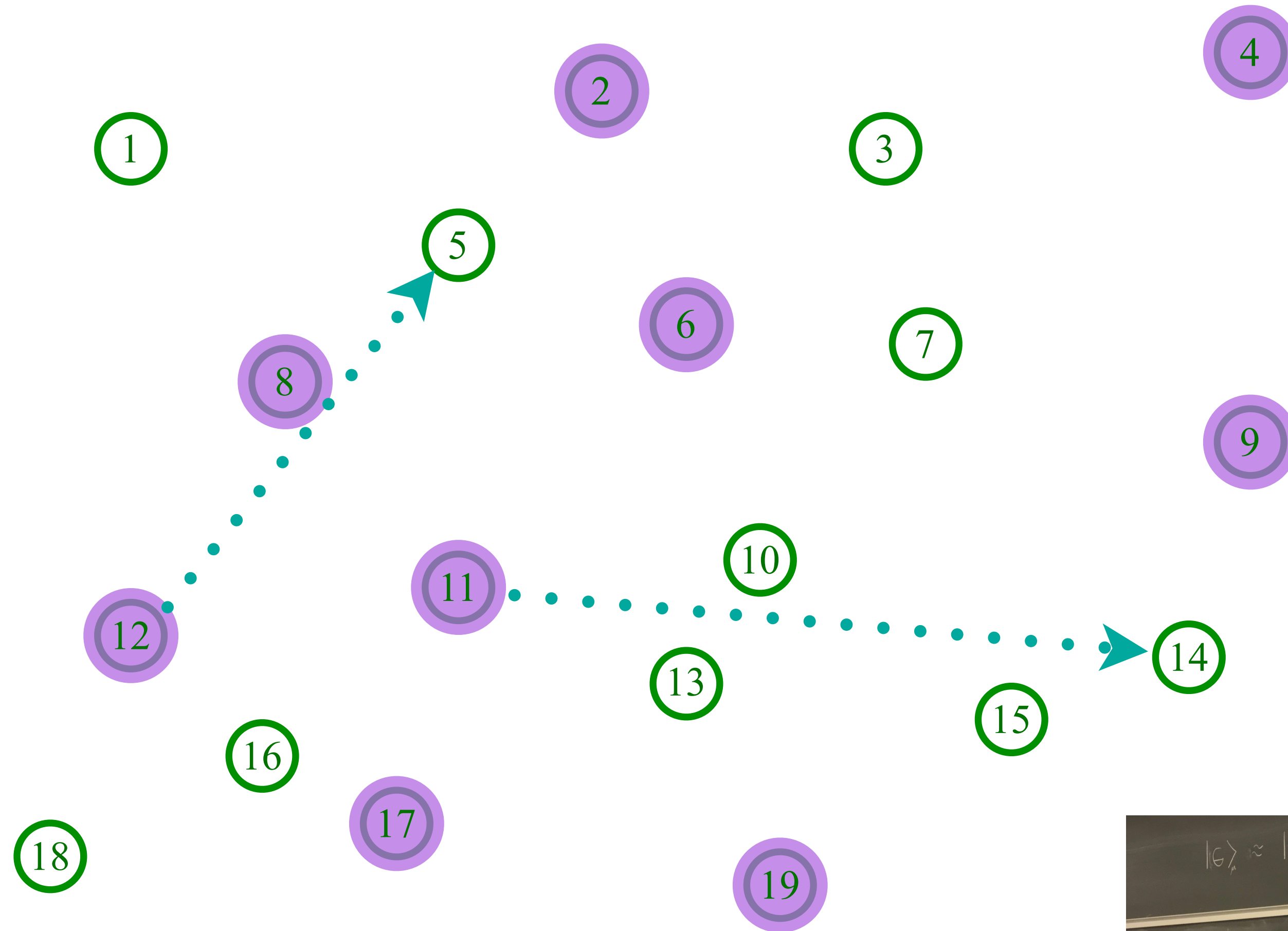
Place electrons randomly on some sites



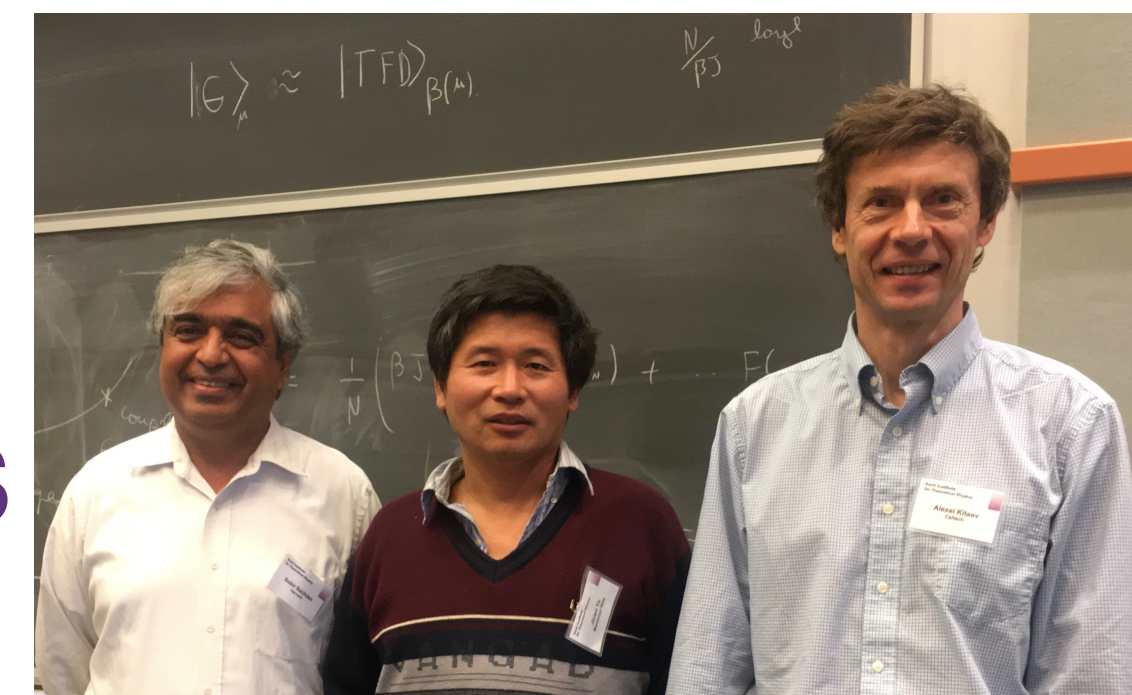
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



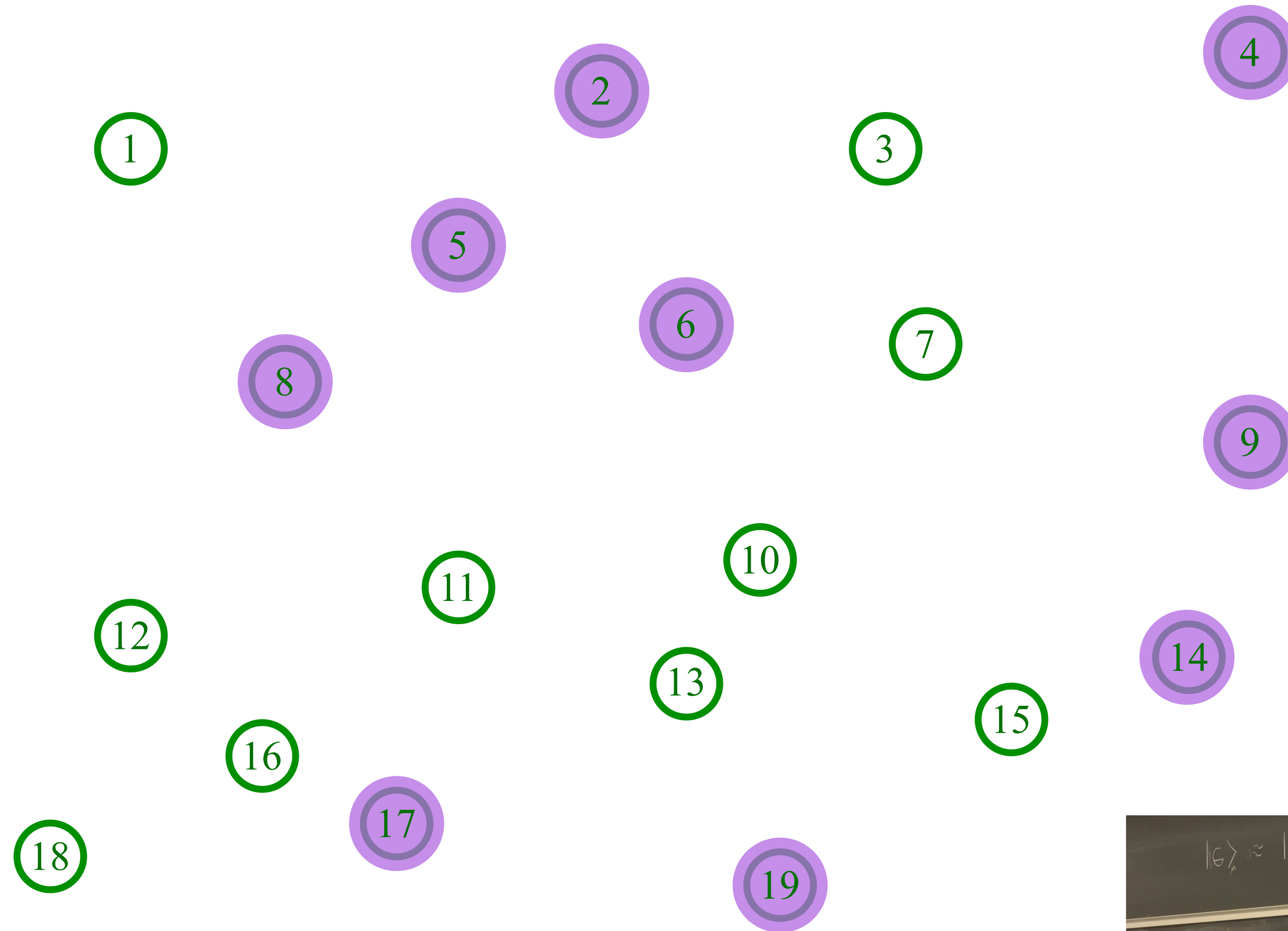
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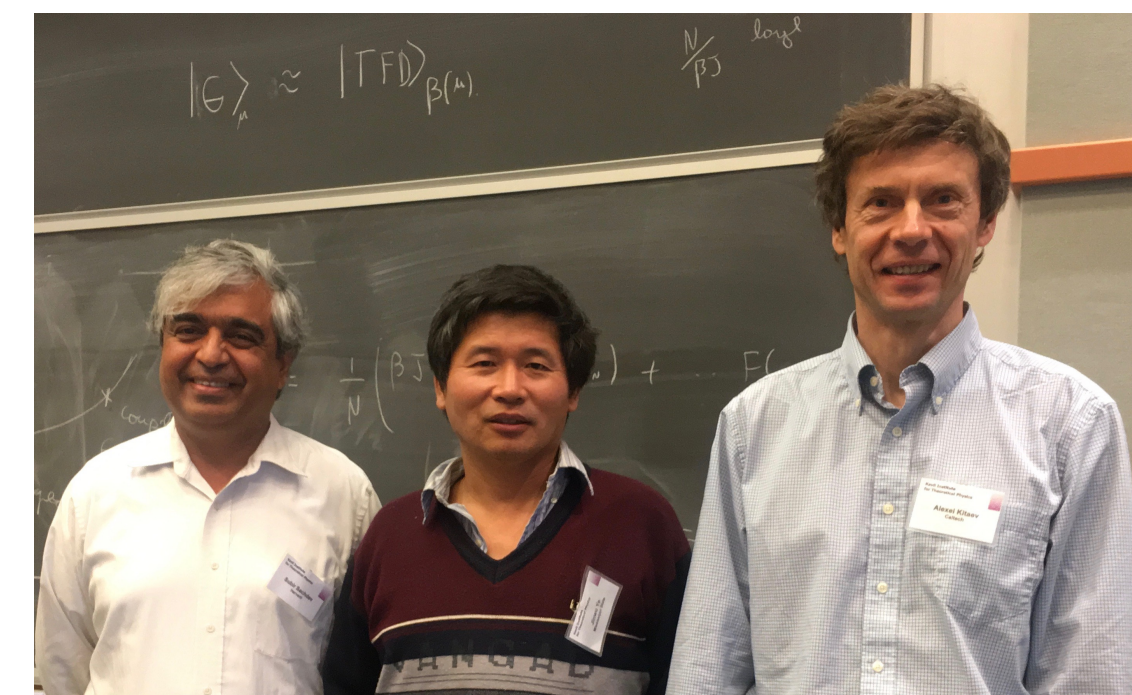
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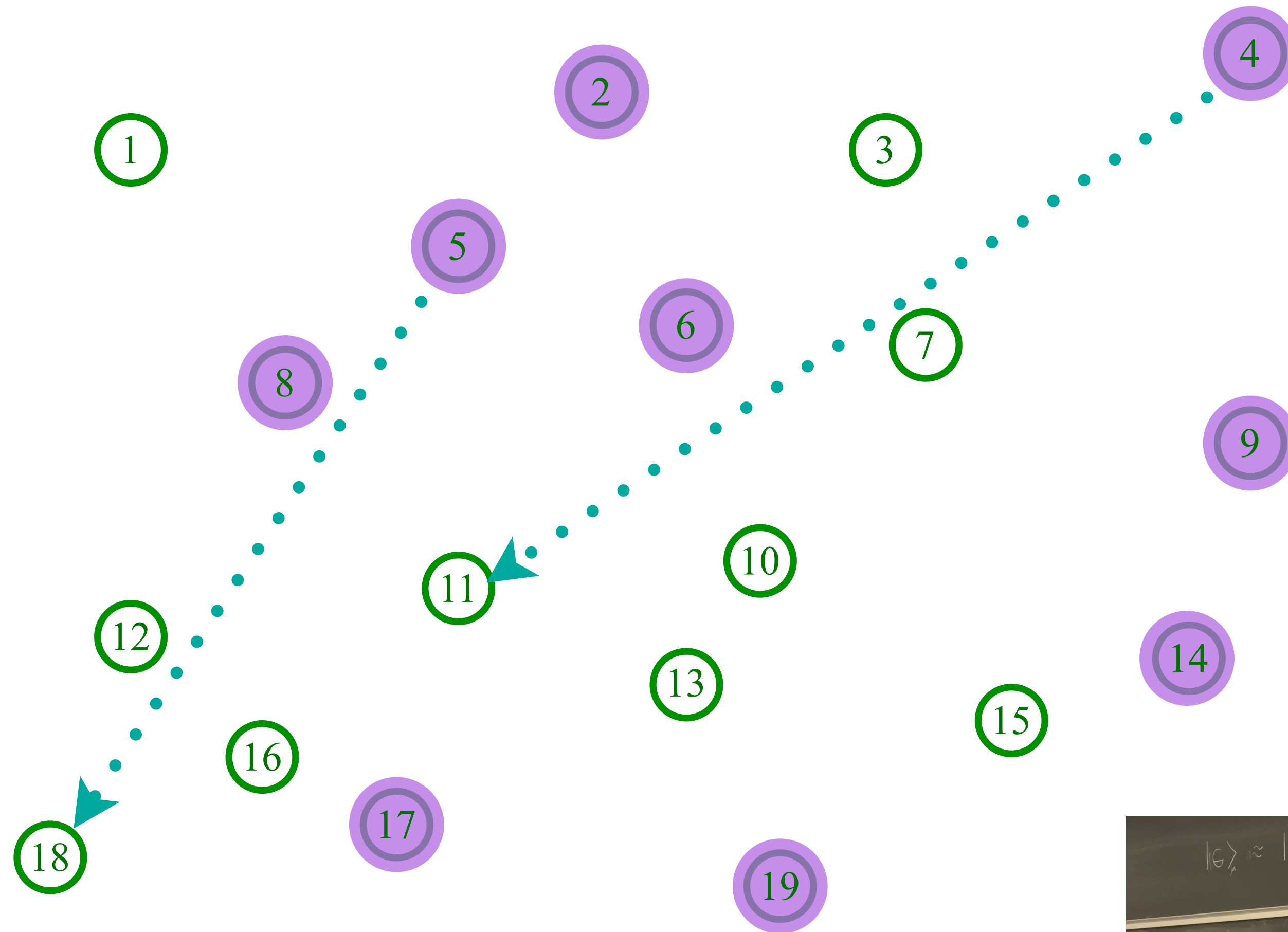
Entangle electrons pairwise randomly



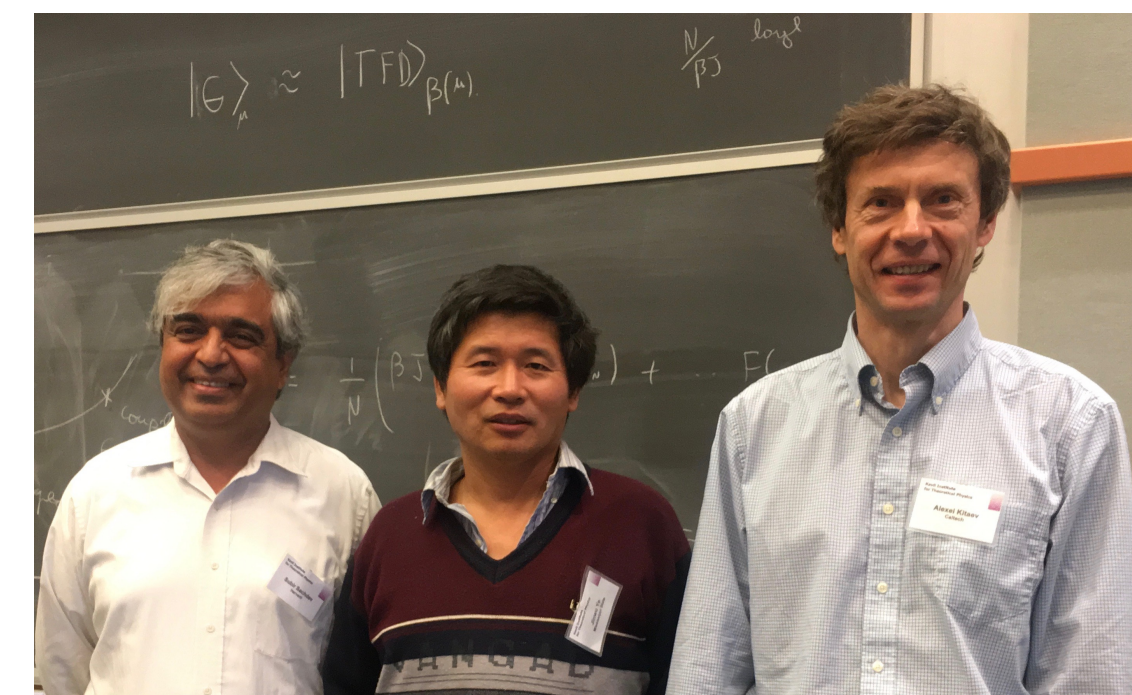
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



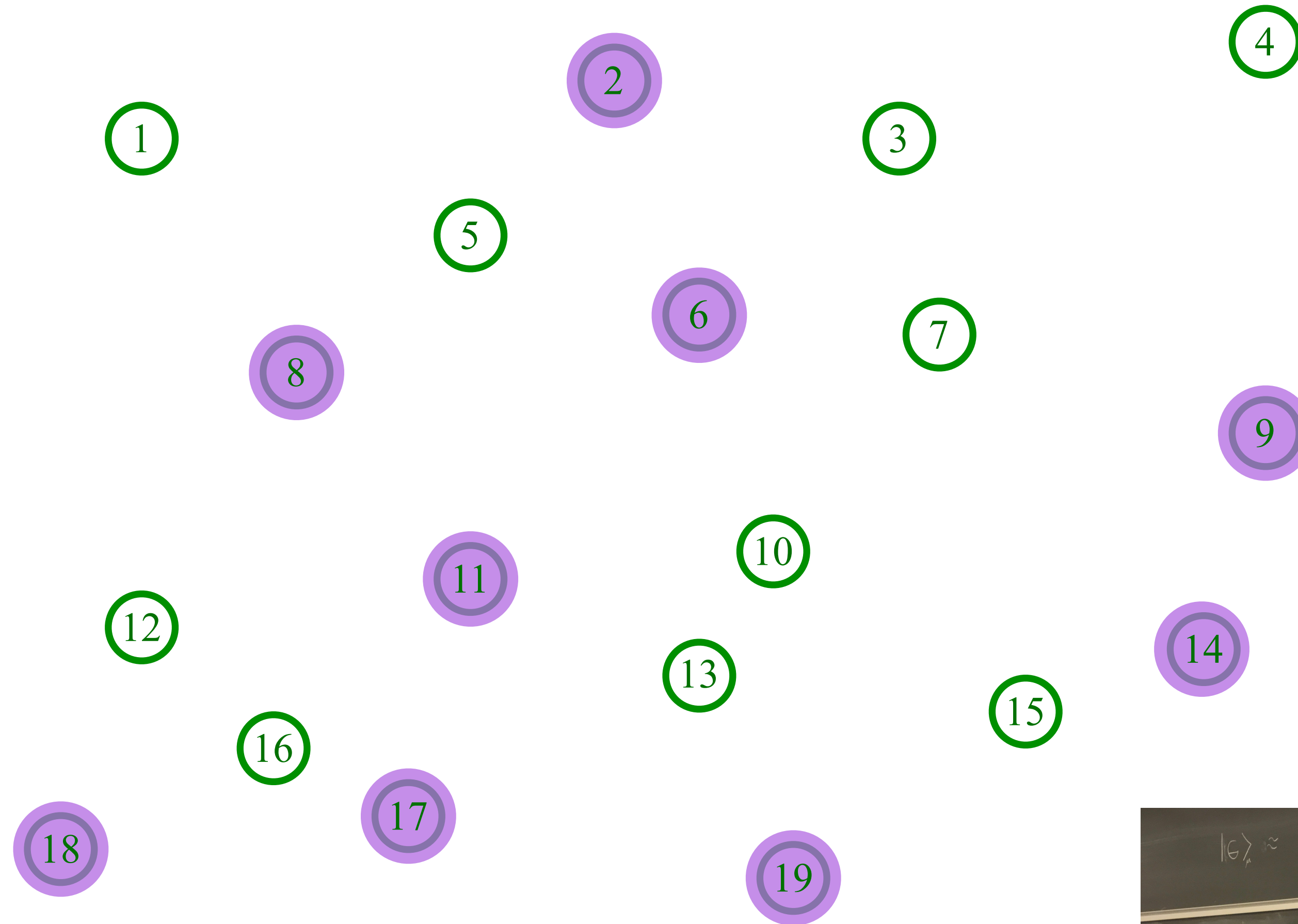
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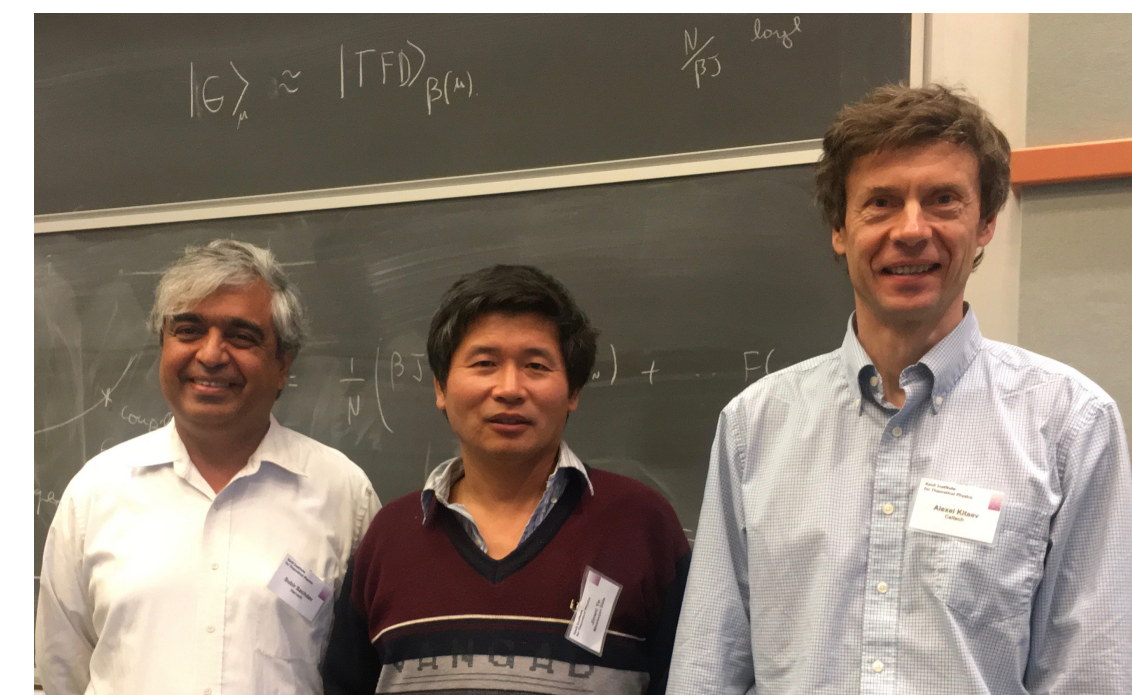
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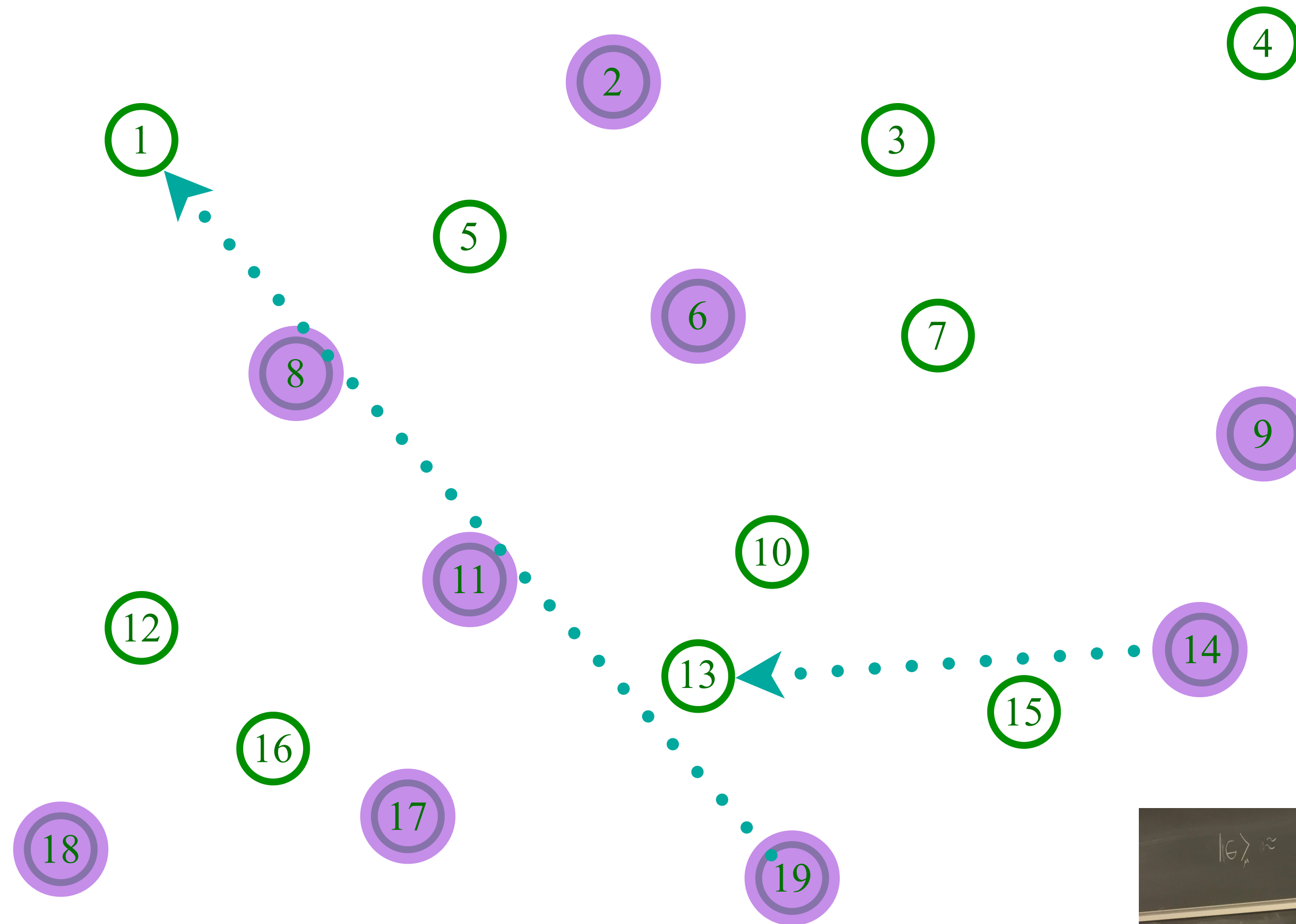
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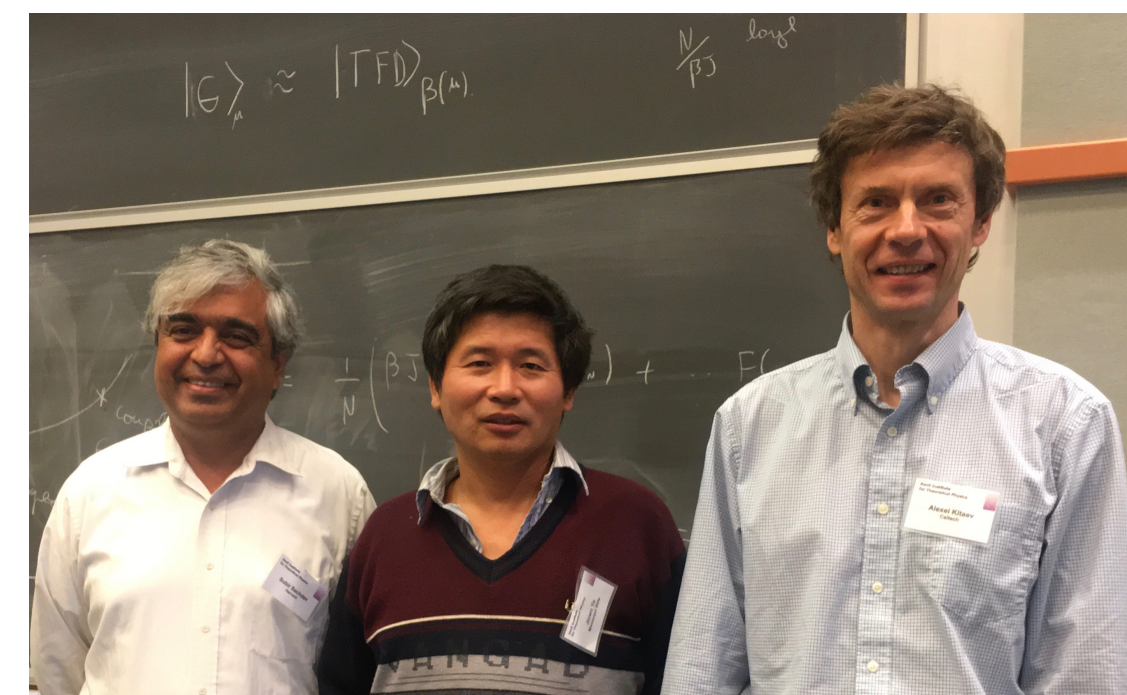
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Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



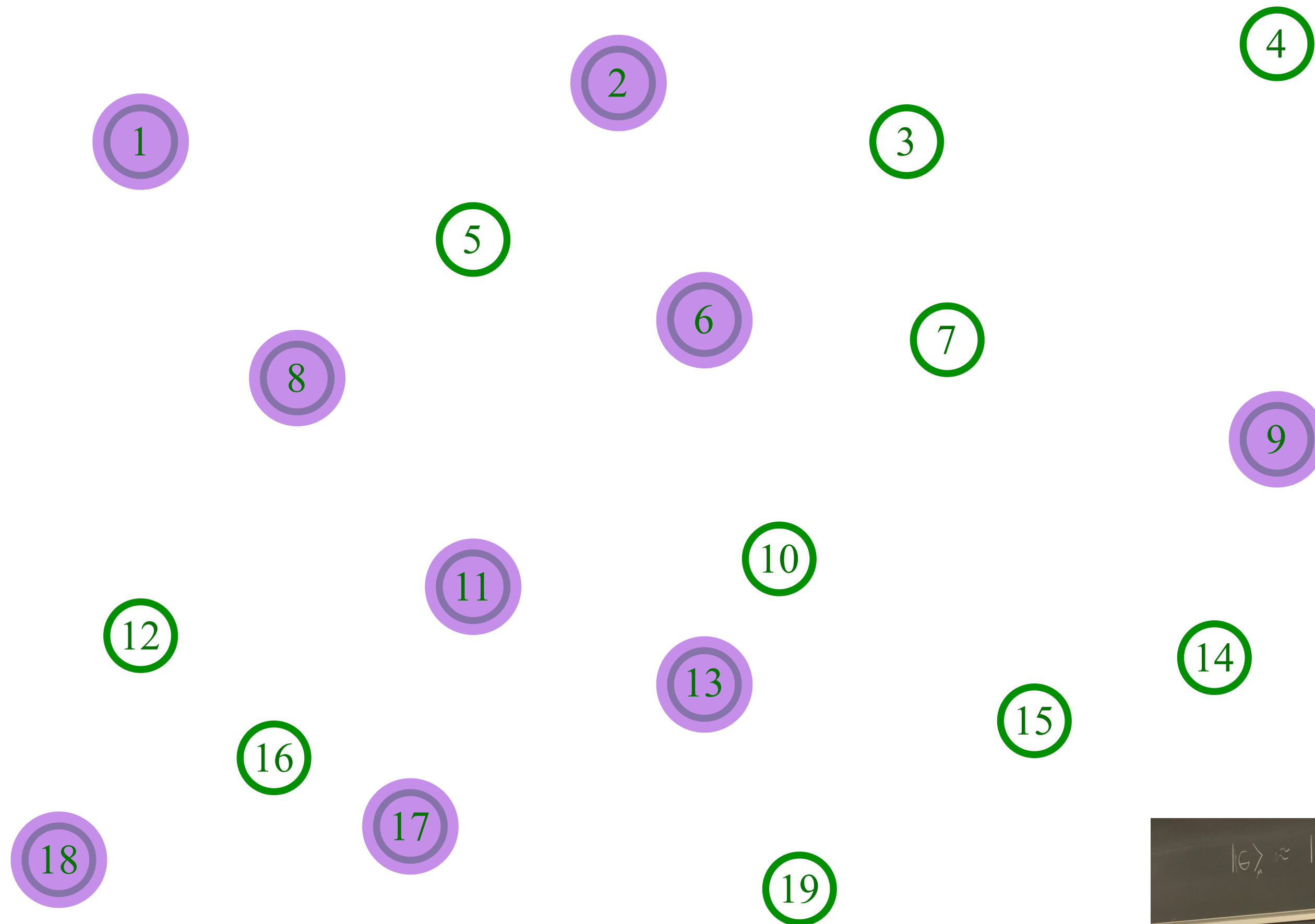
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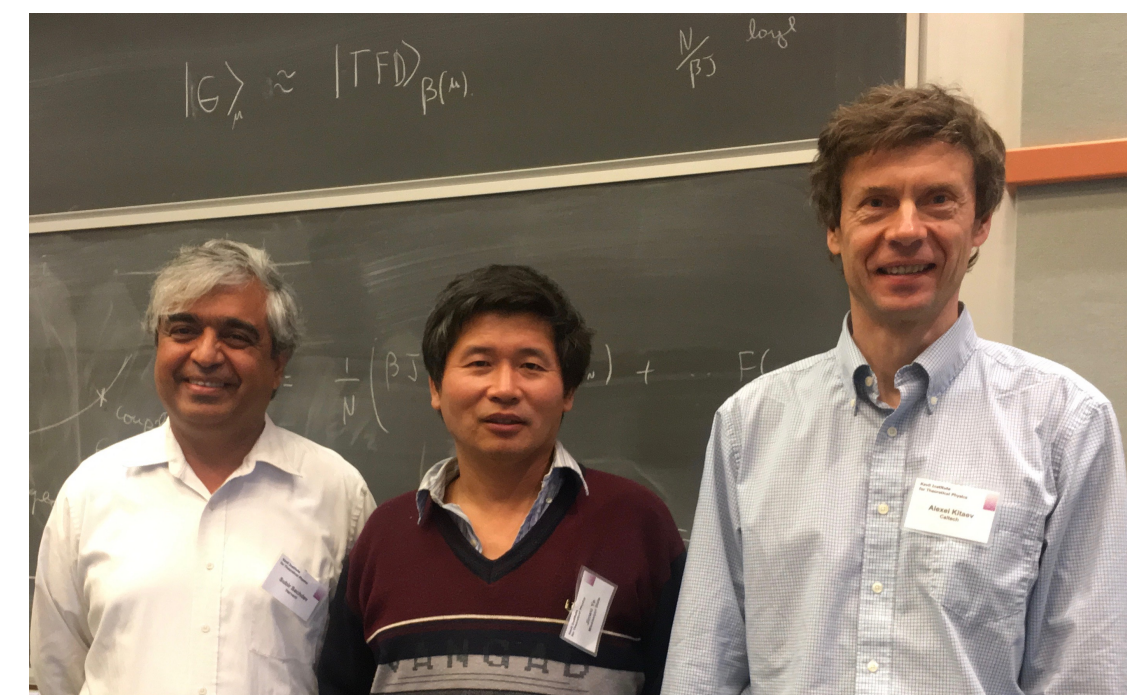
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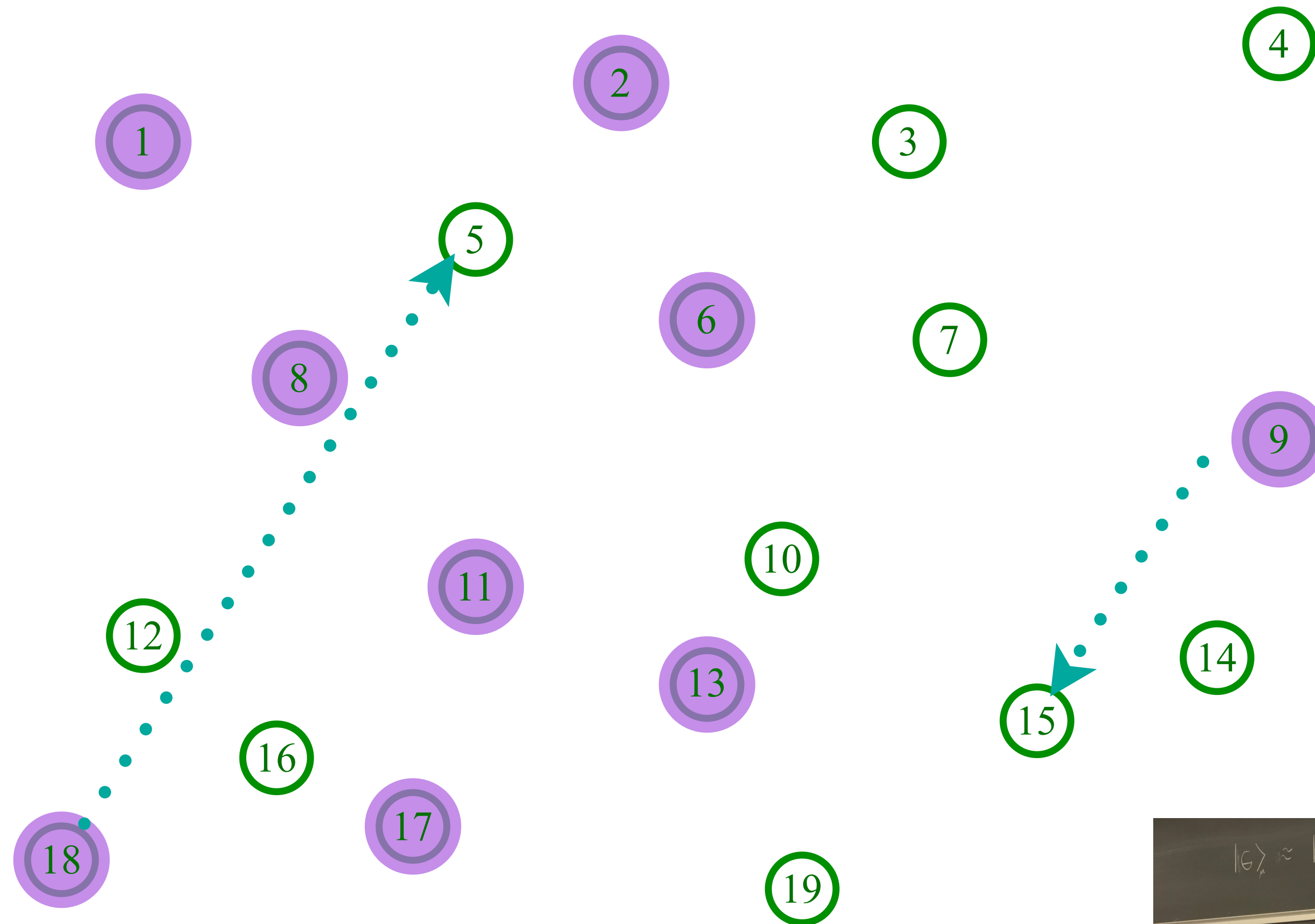
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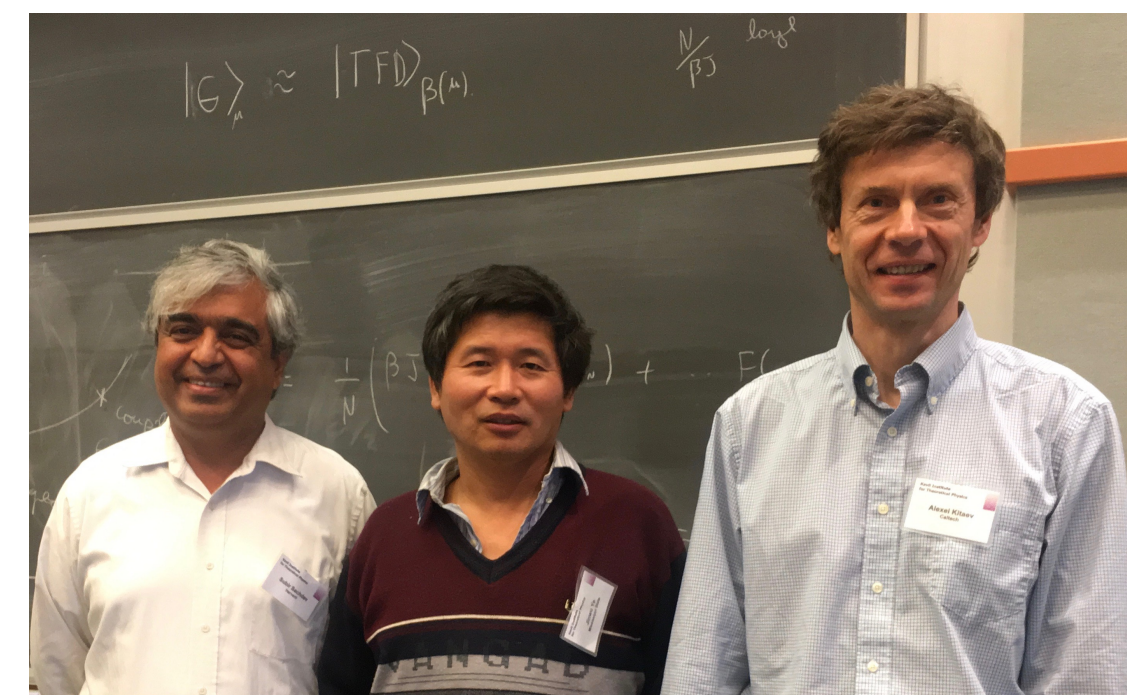
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



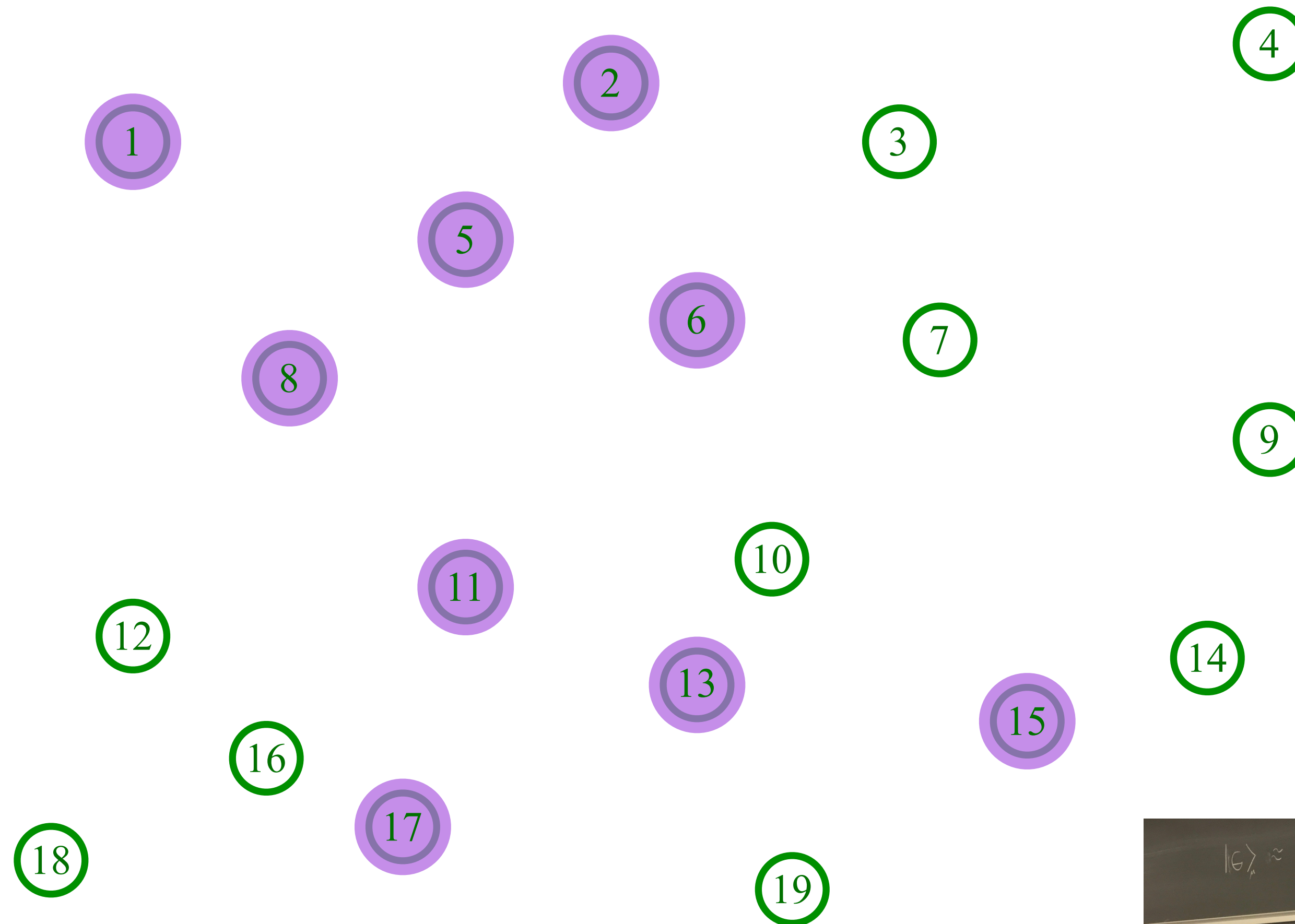
Entangle electrons pairwise randomly



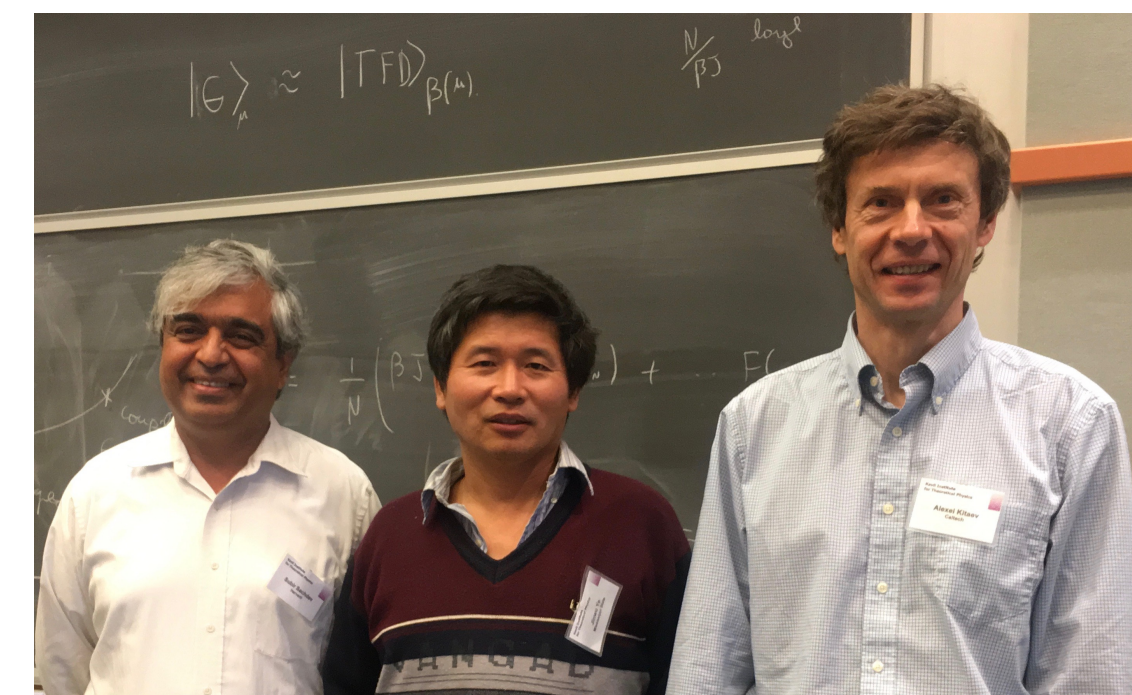
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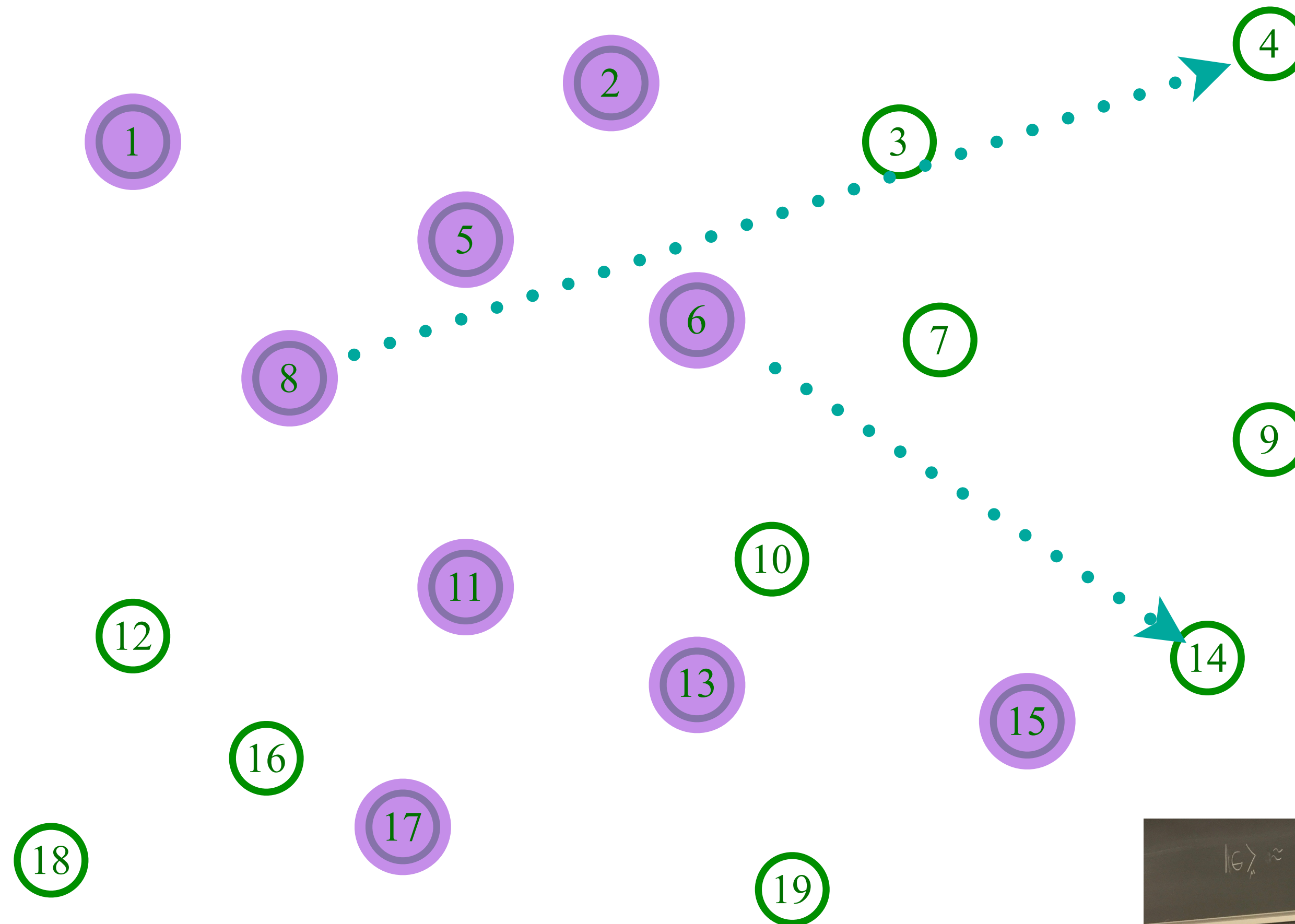
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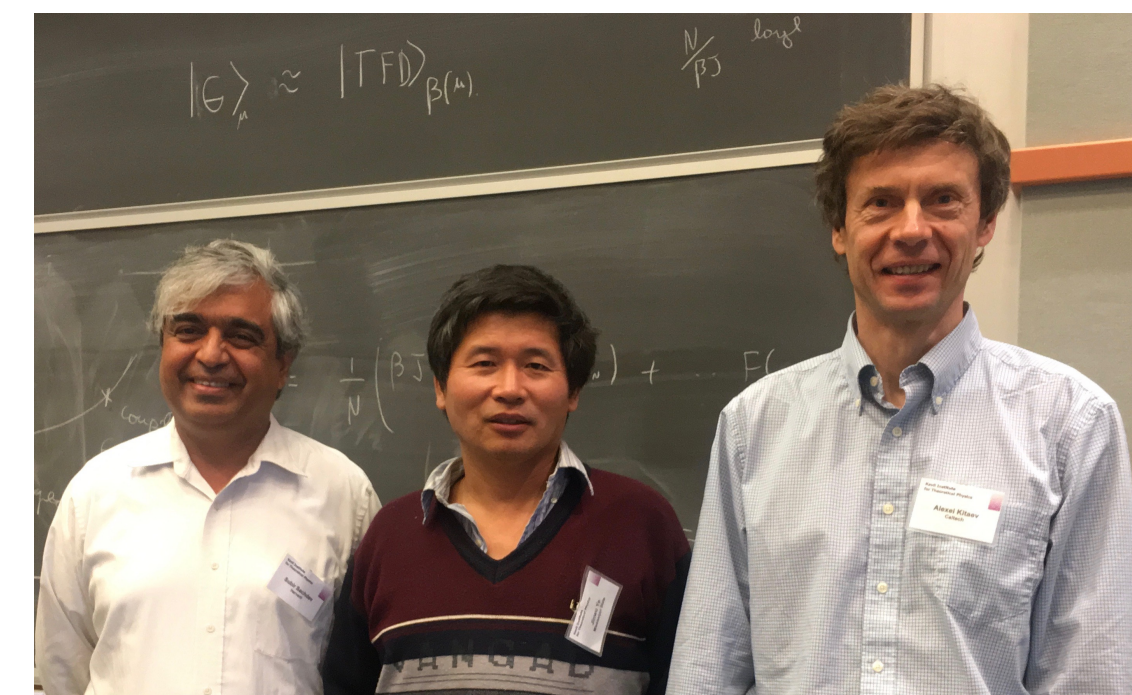
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



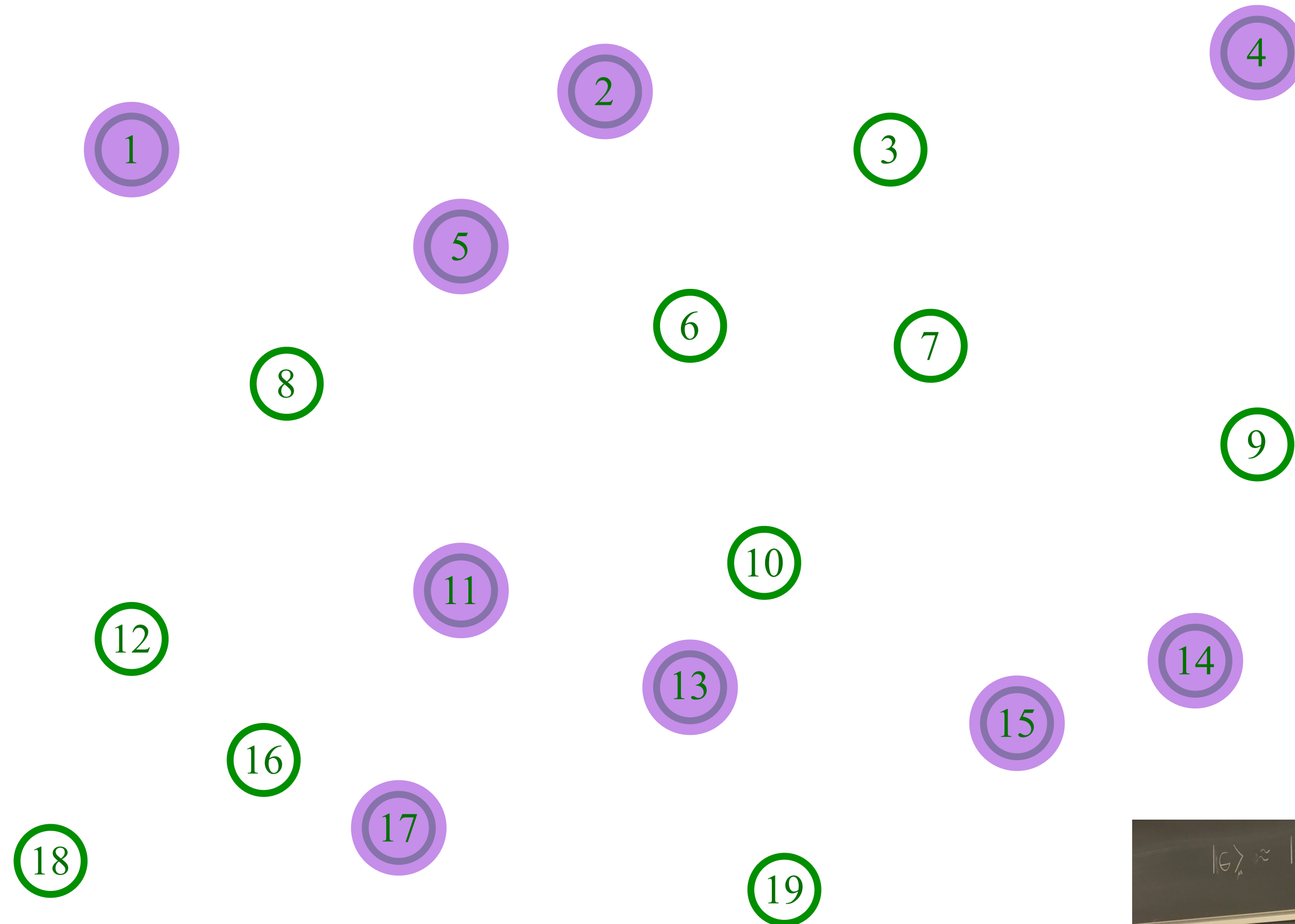
Entangle electrons pairwise randomly



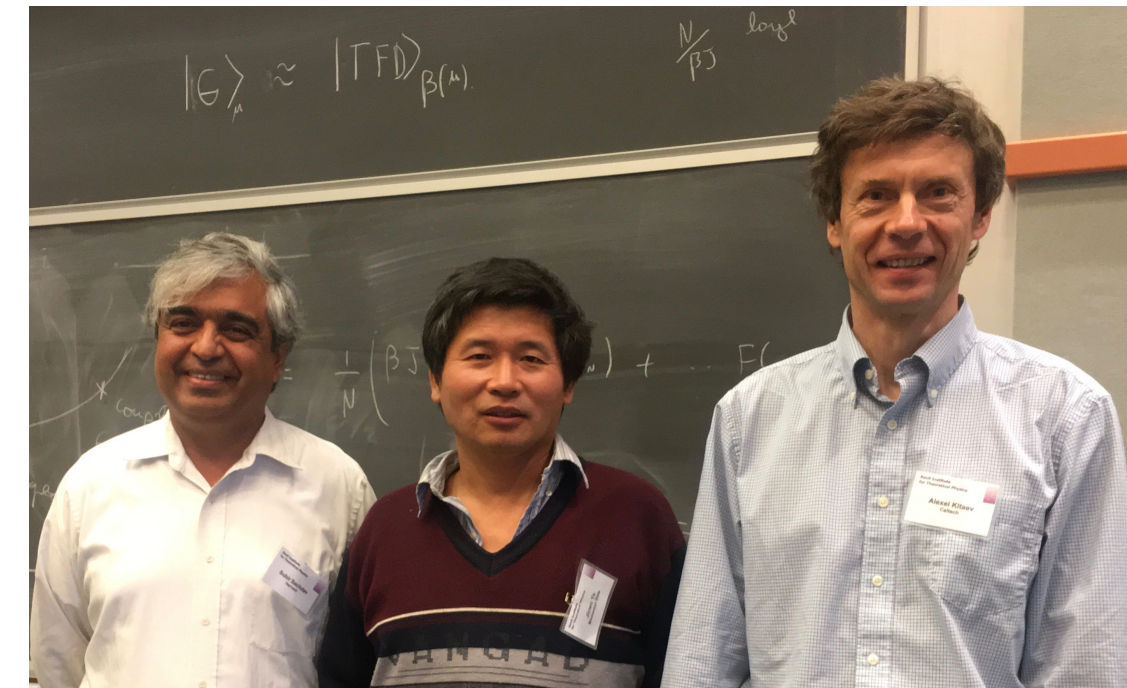
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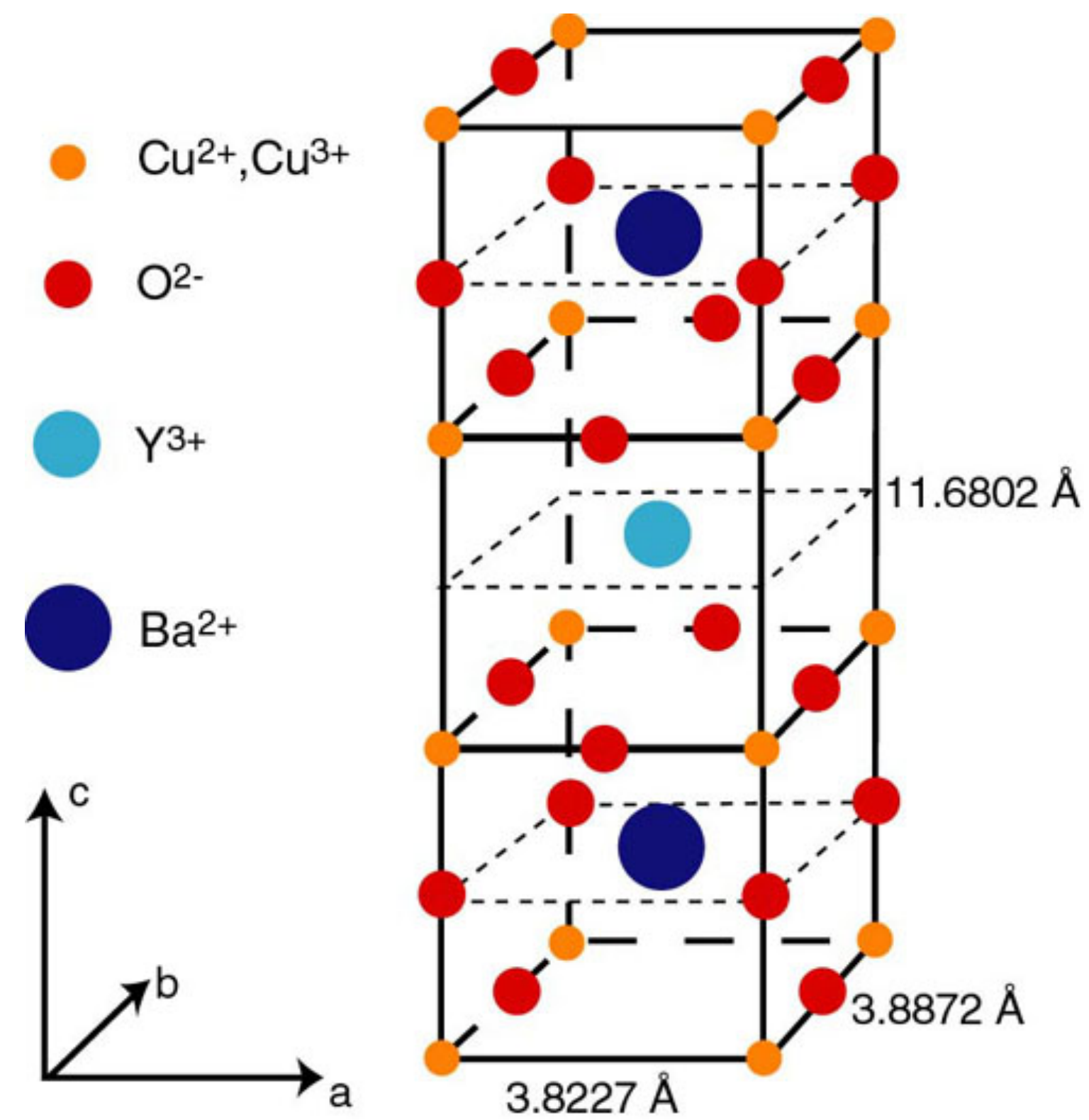
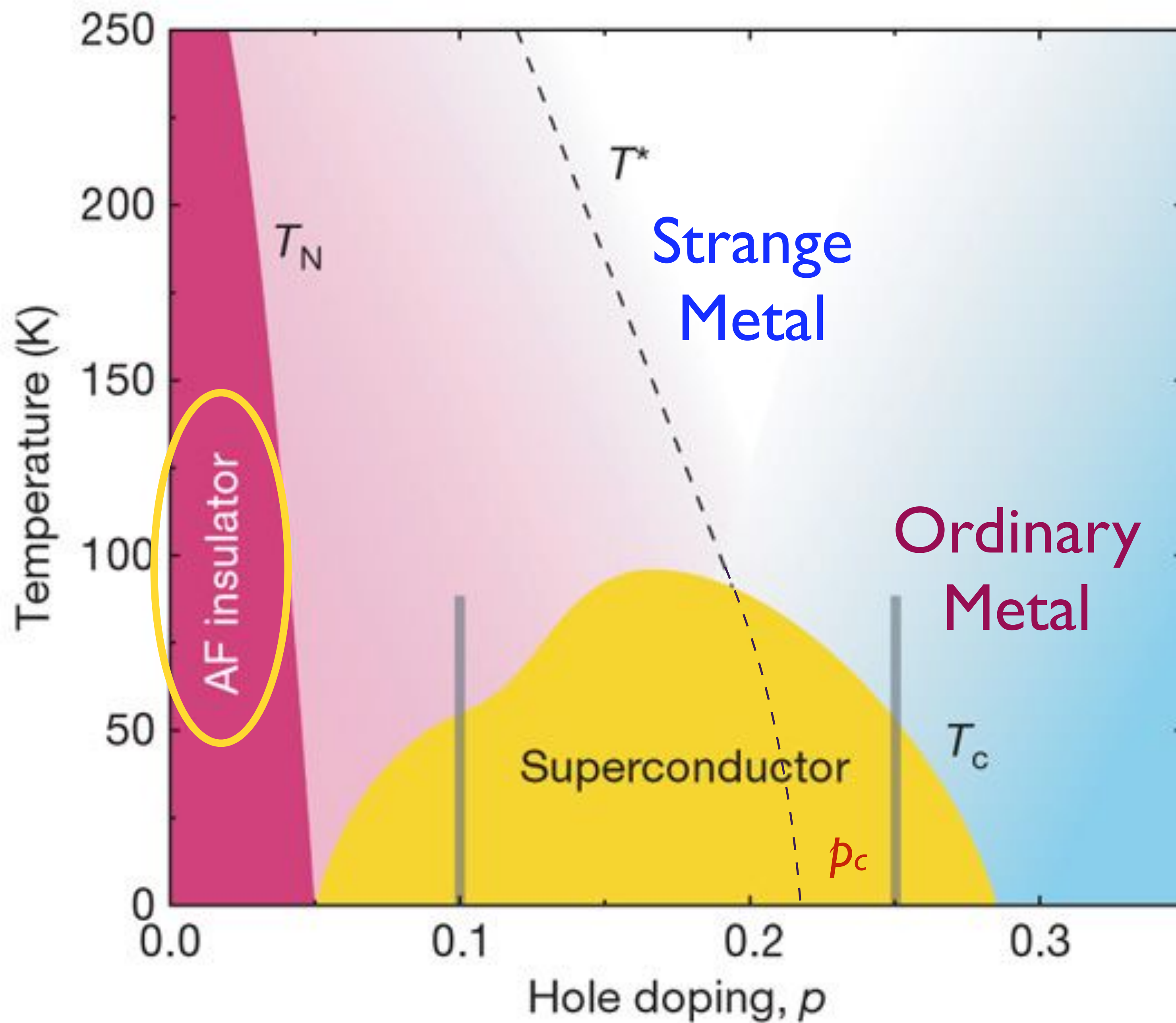
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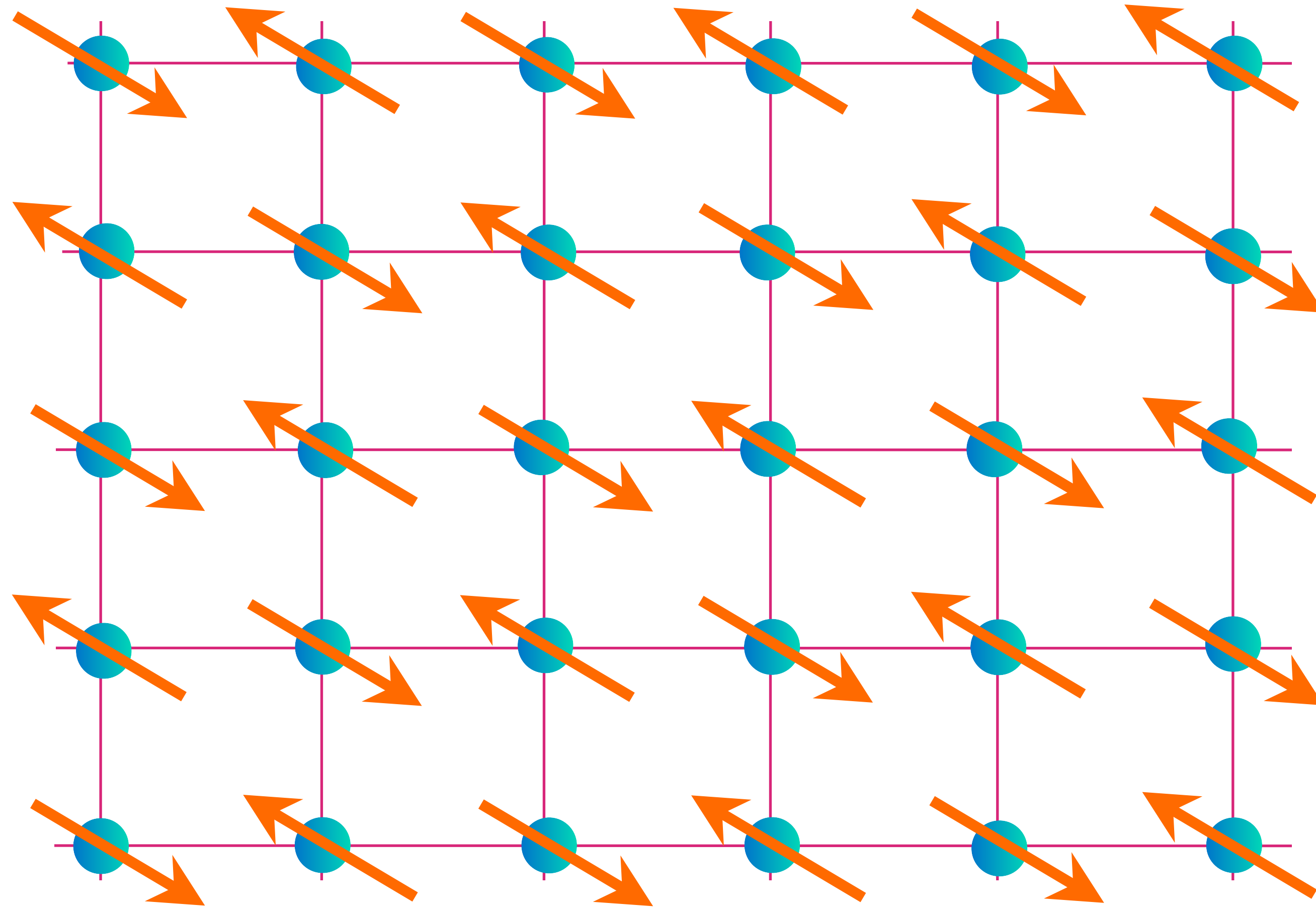
A solvable model of multi-particle
quantum entanglement.

Yields a metal in which current is carried
not by individual electrons,
but by an entangled “quantum soup”

Quantum entanglement and superconductivity

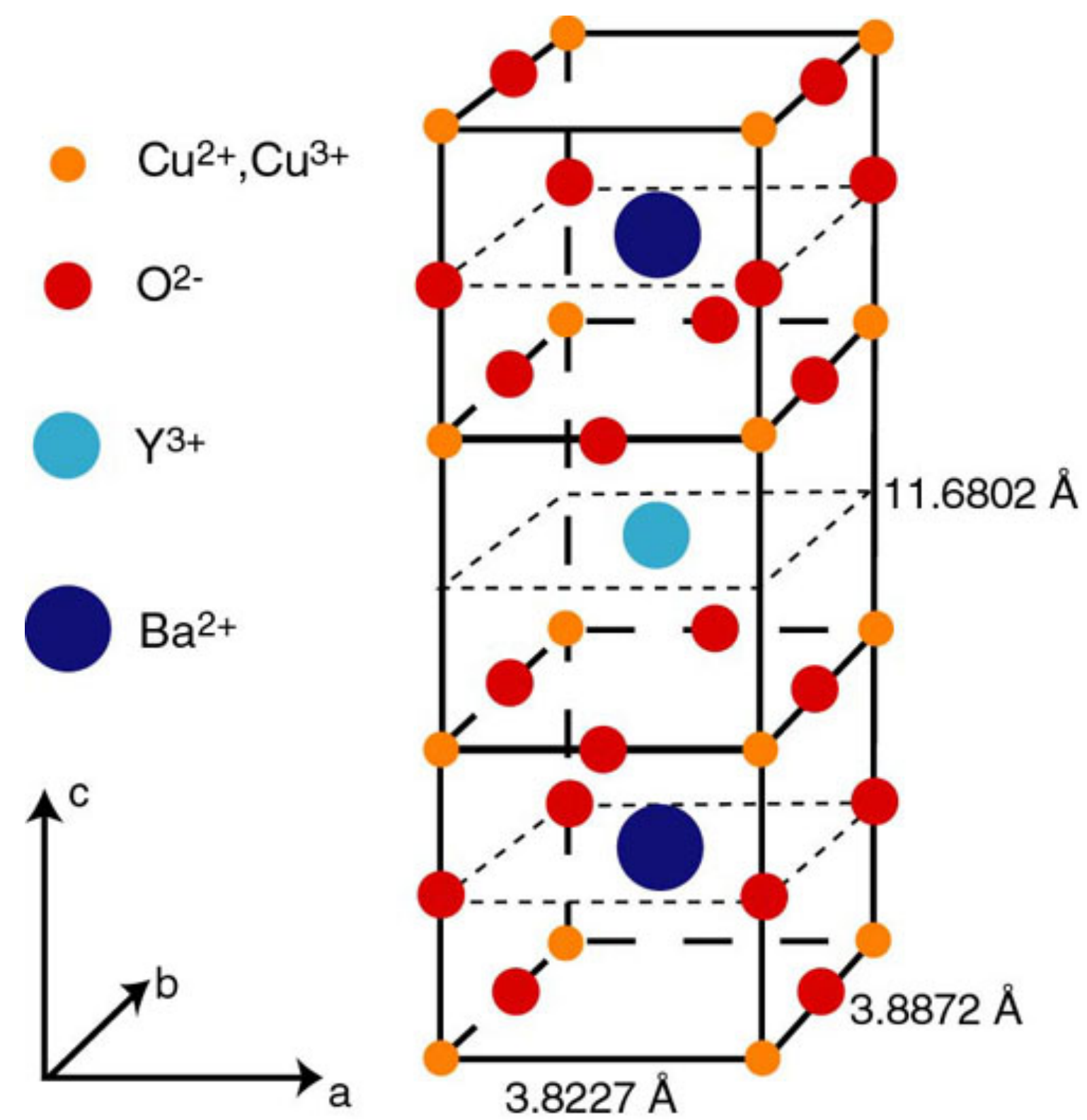
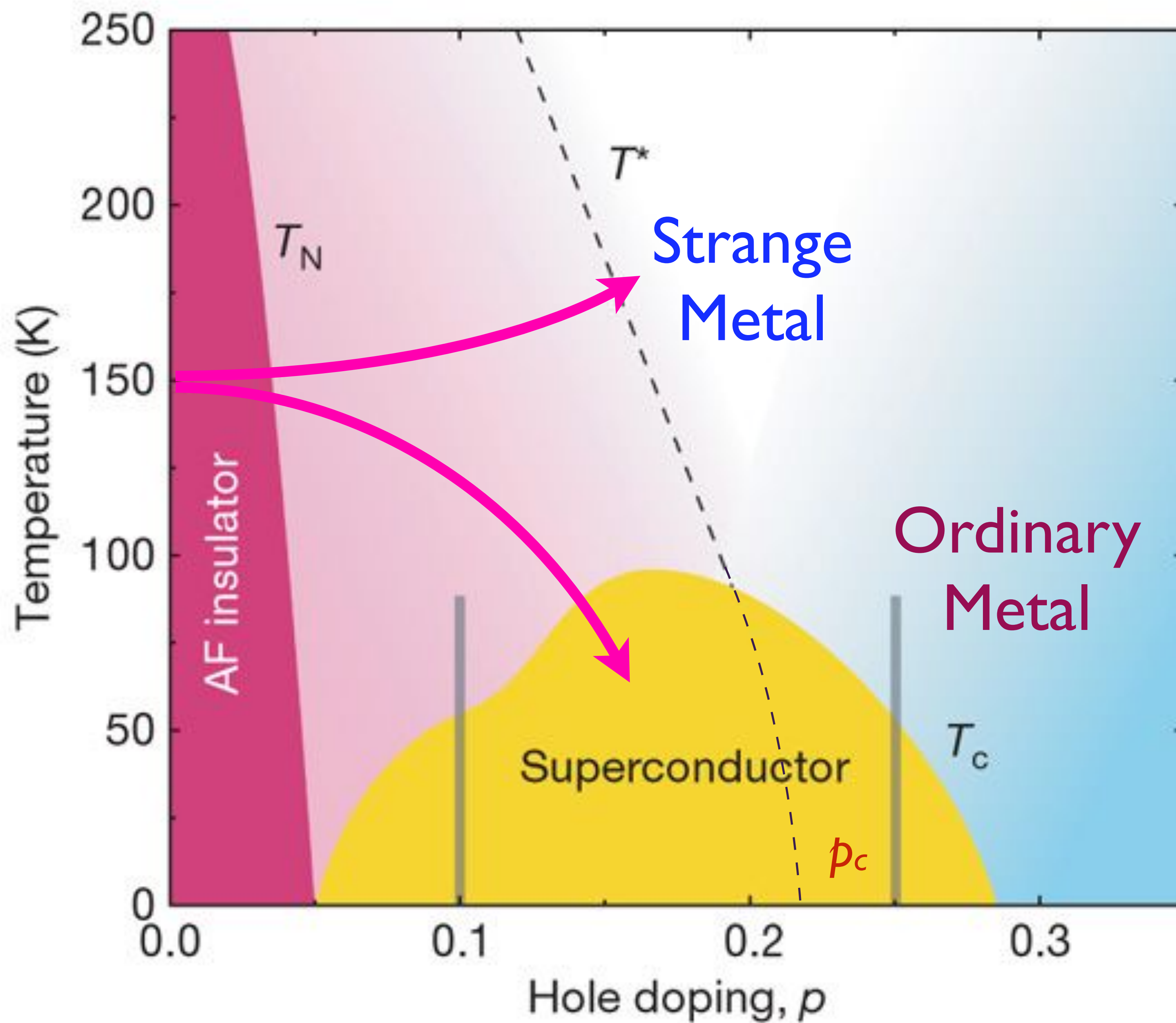


The dance of electrons on Cu atoms in YBCO

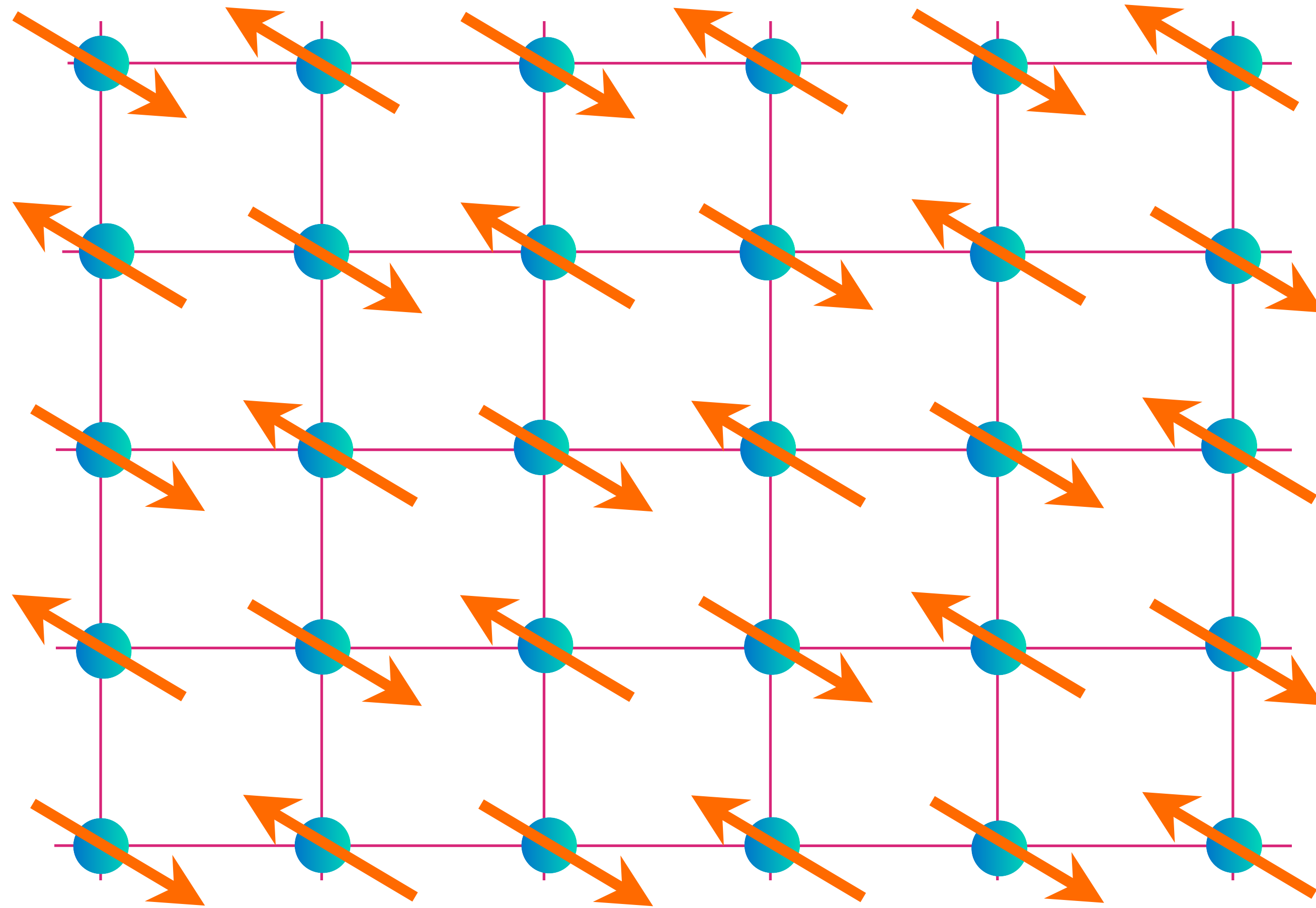


Antiferromagnetism

All nearest-neighbor pairs of electrons have opposite spins



The dance of electrons on Cu atoms in YBCO



Antiferromagnetism

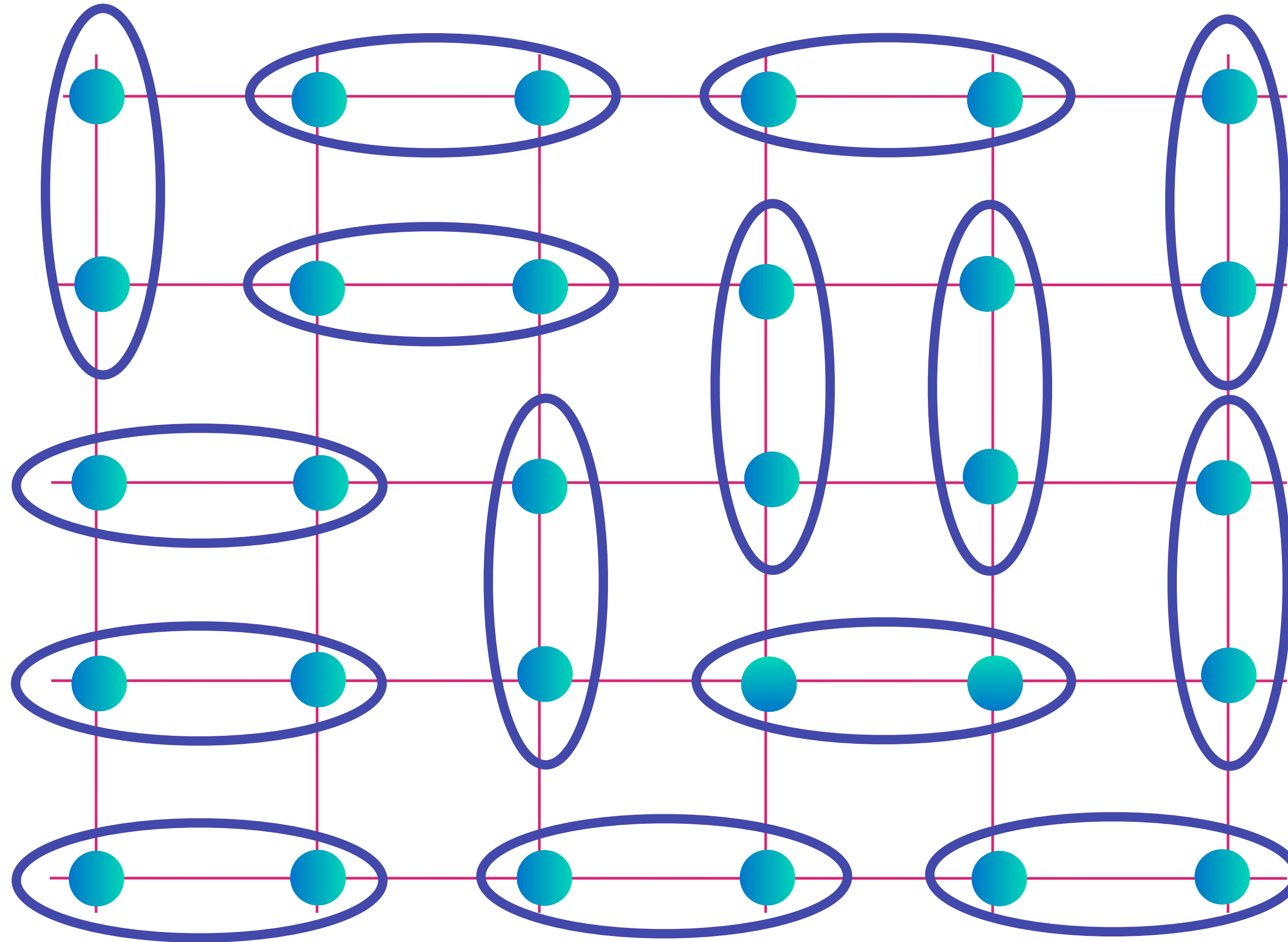
All nearest-neighbor pairs of electrons have opposite spins

The dance of electrons on Cu atoms in YBCO

Baskaran+Anderson (1987)

Spin liquid

Electrons form entangled pairs, and the pairs entangle across the entire sample



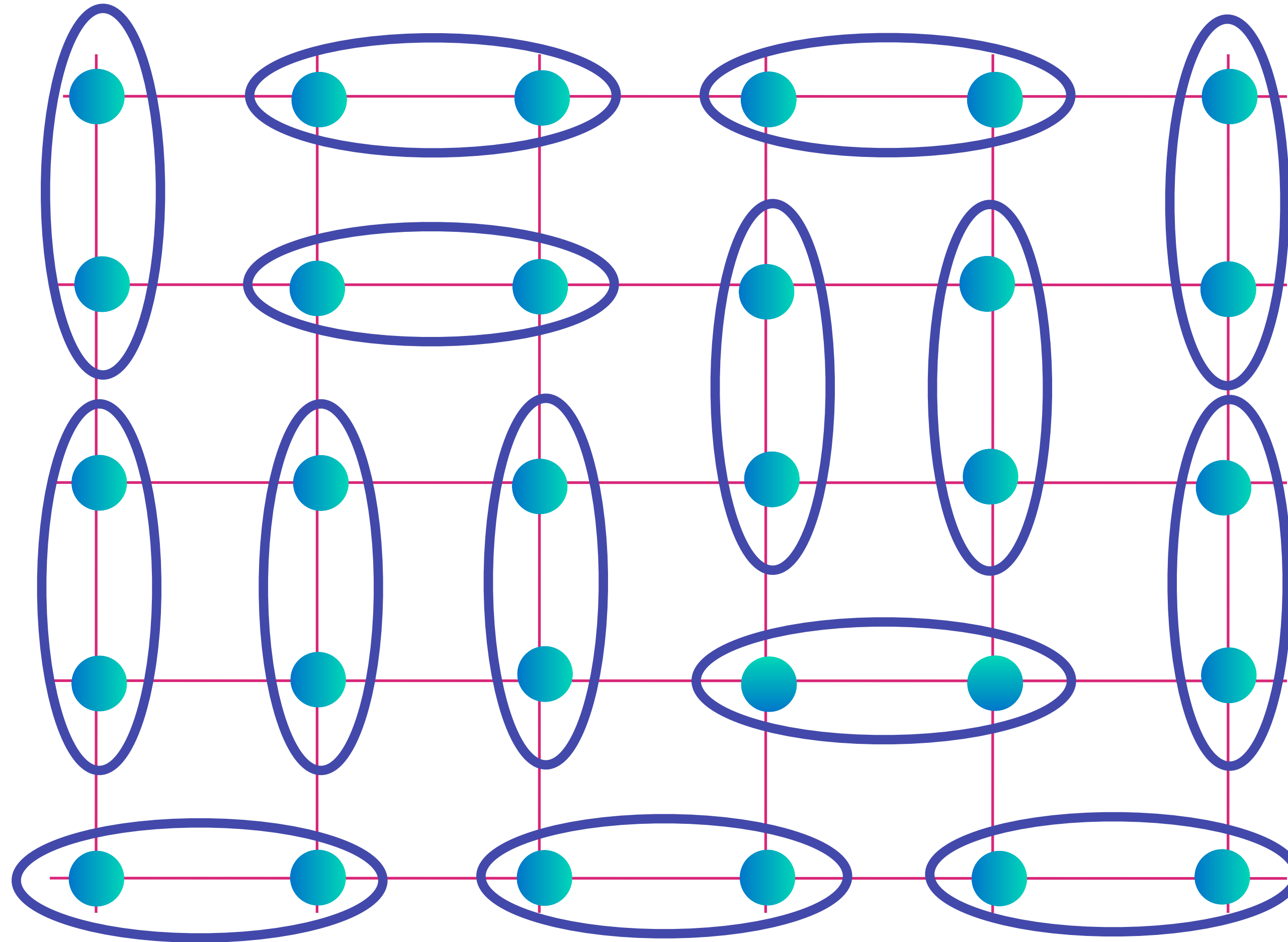
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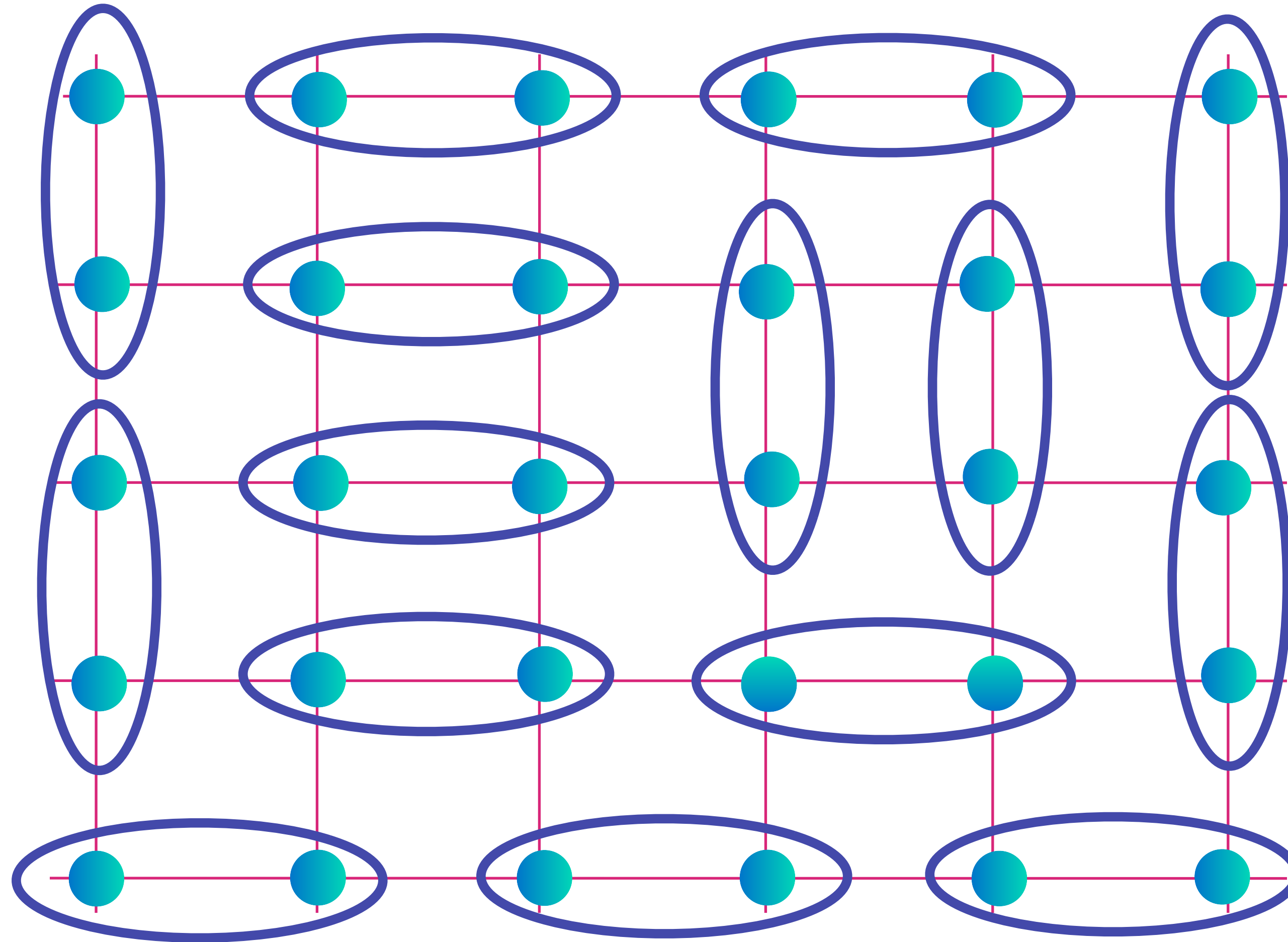
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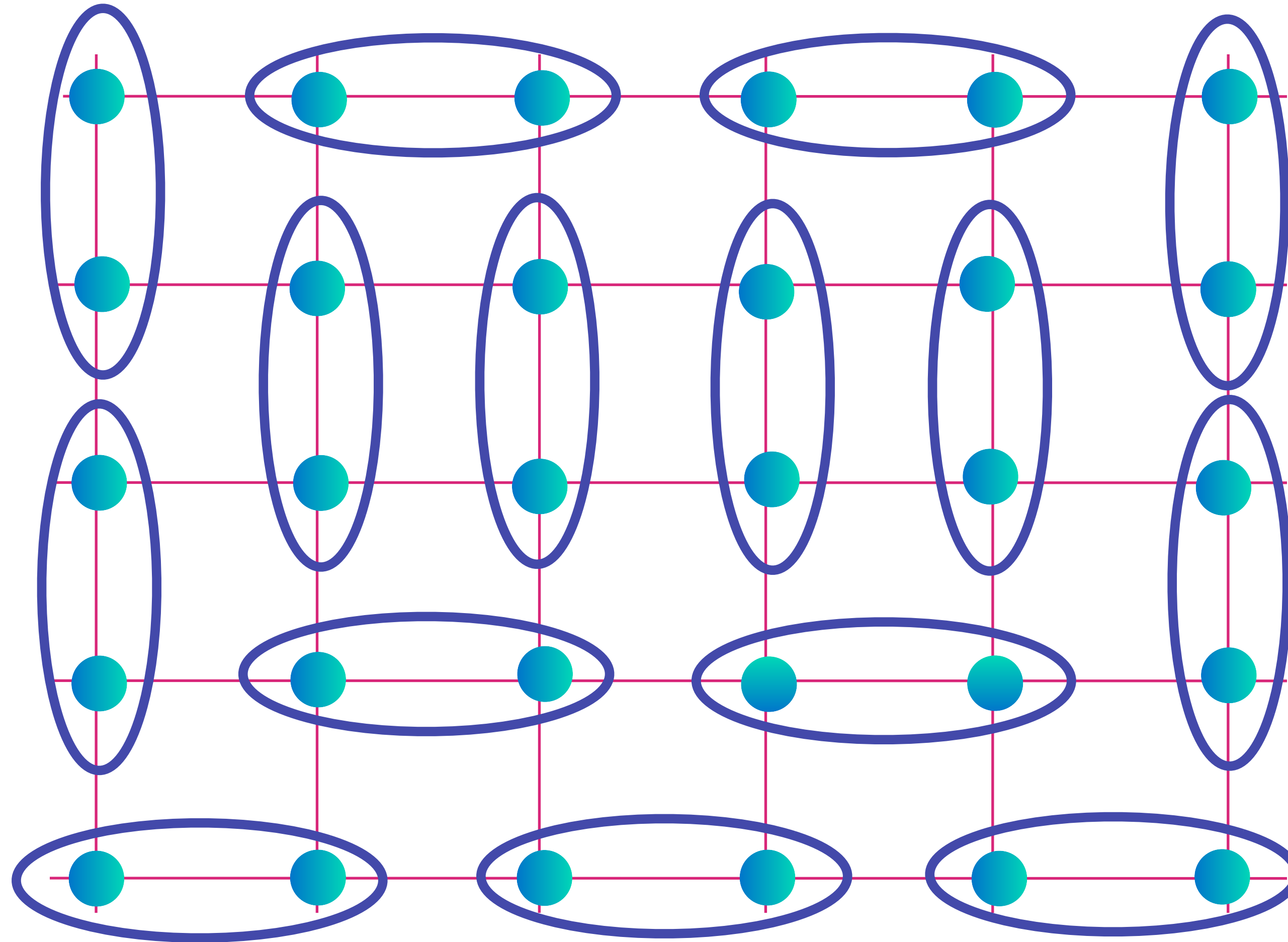
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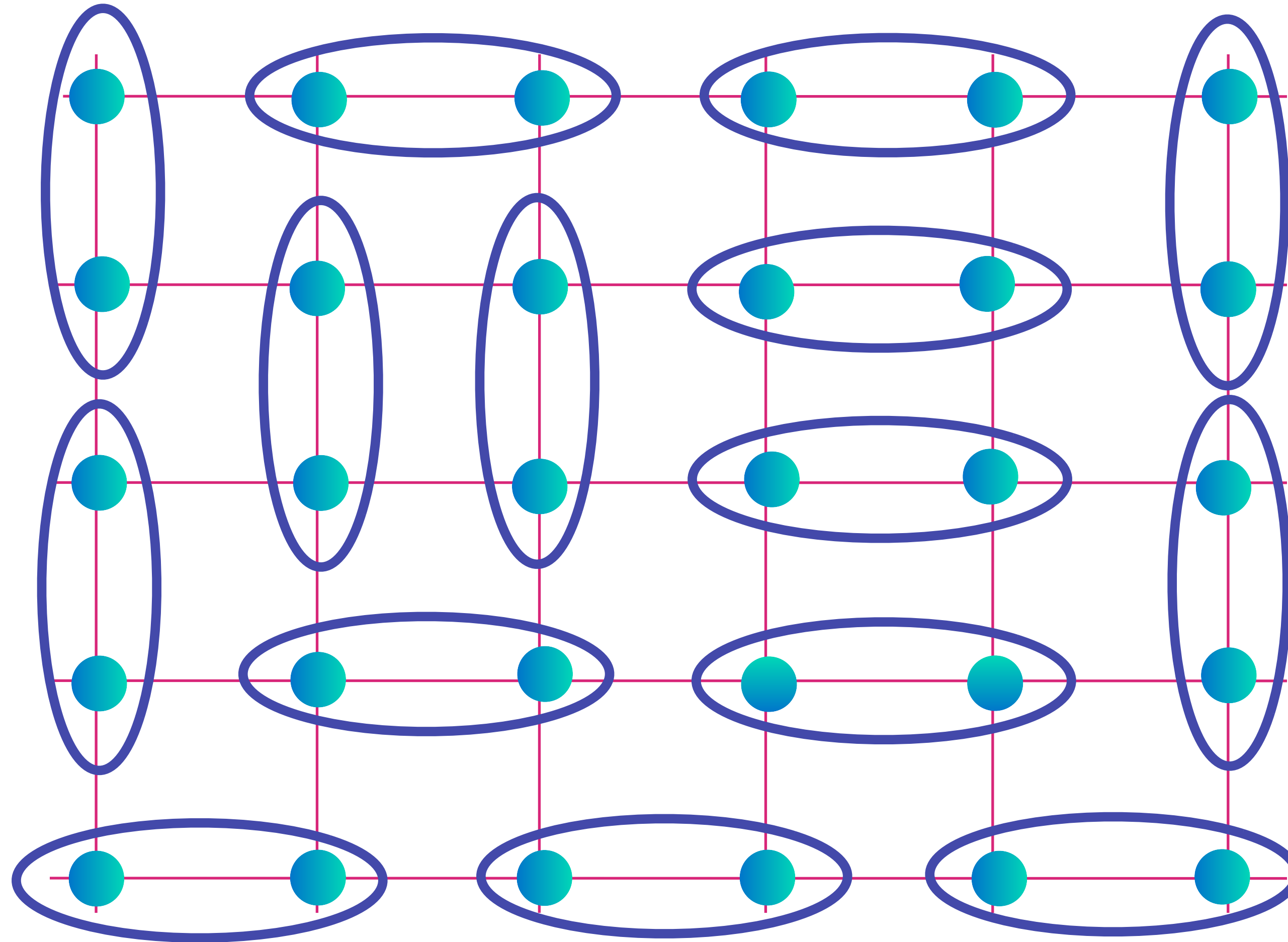
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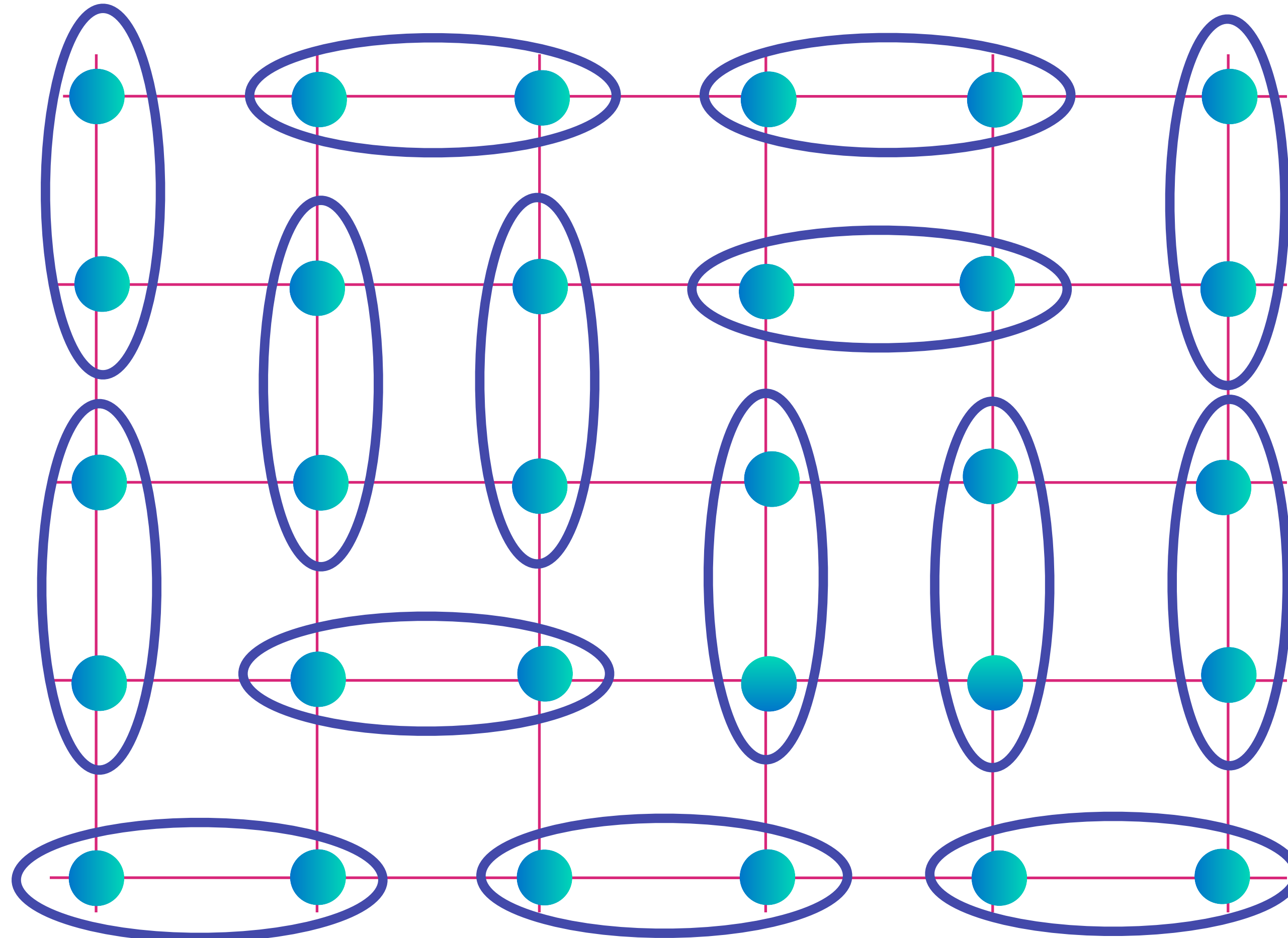
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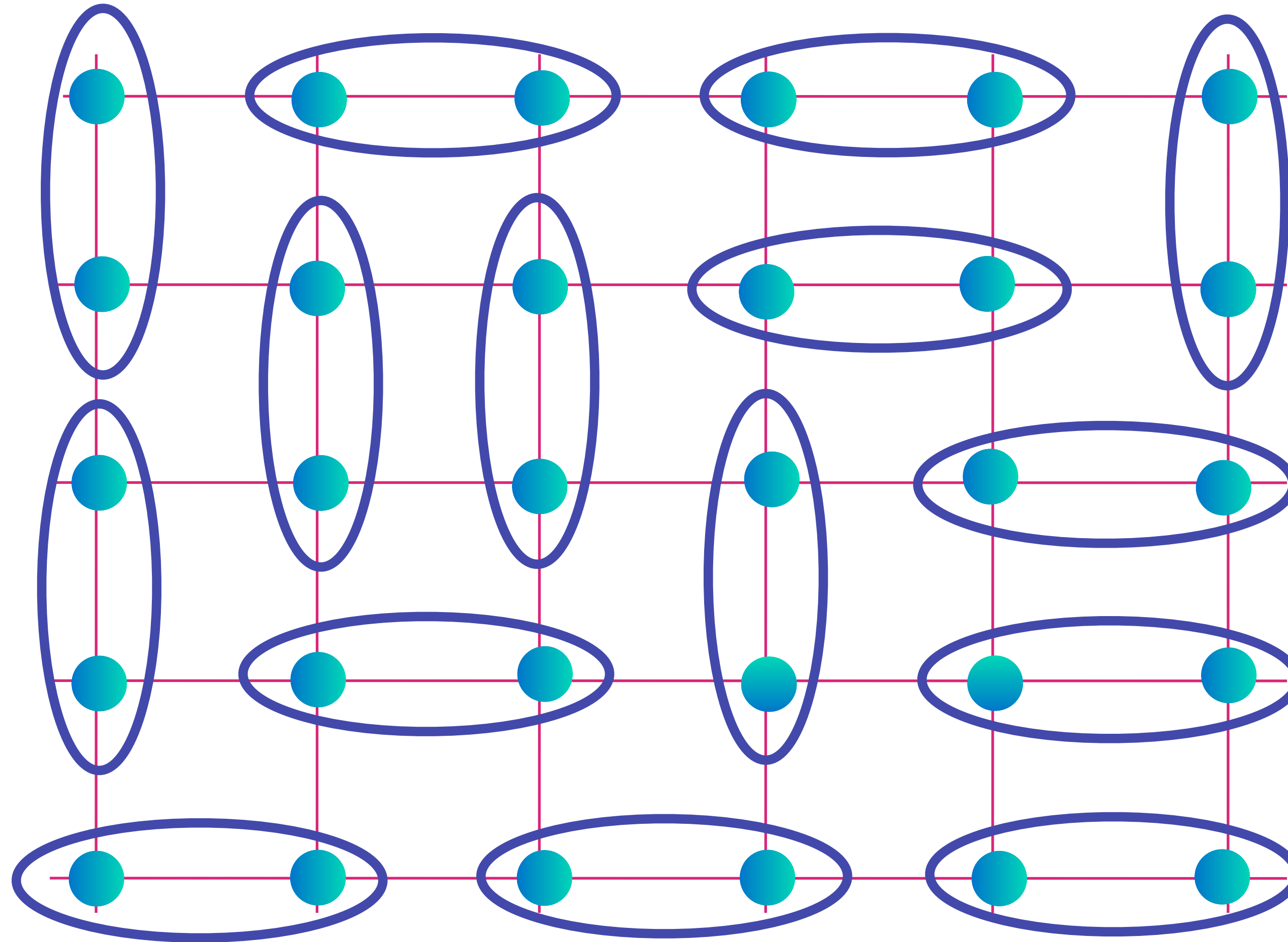
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The dance of electrons on Cu atoms in YBCO

Baskaran+Anderson (1987)

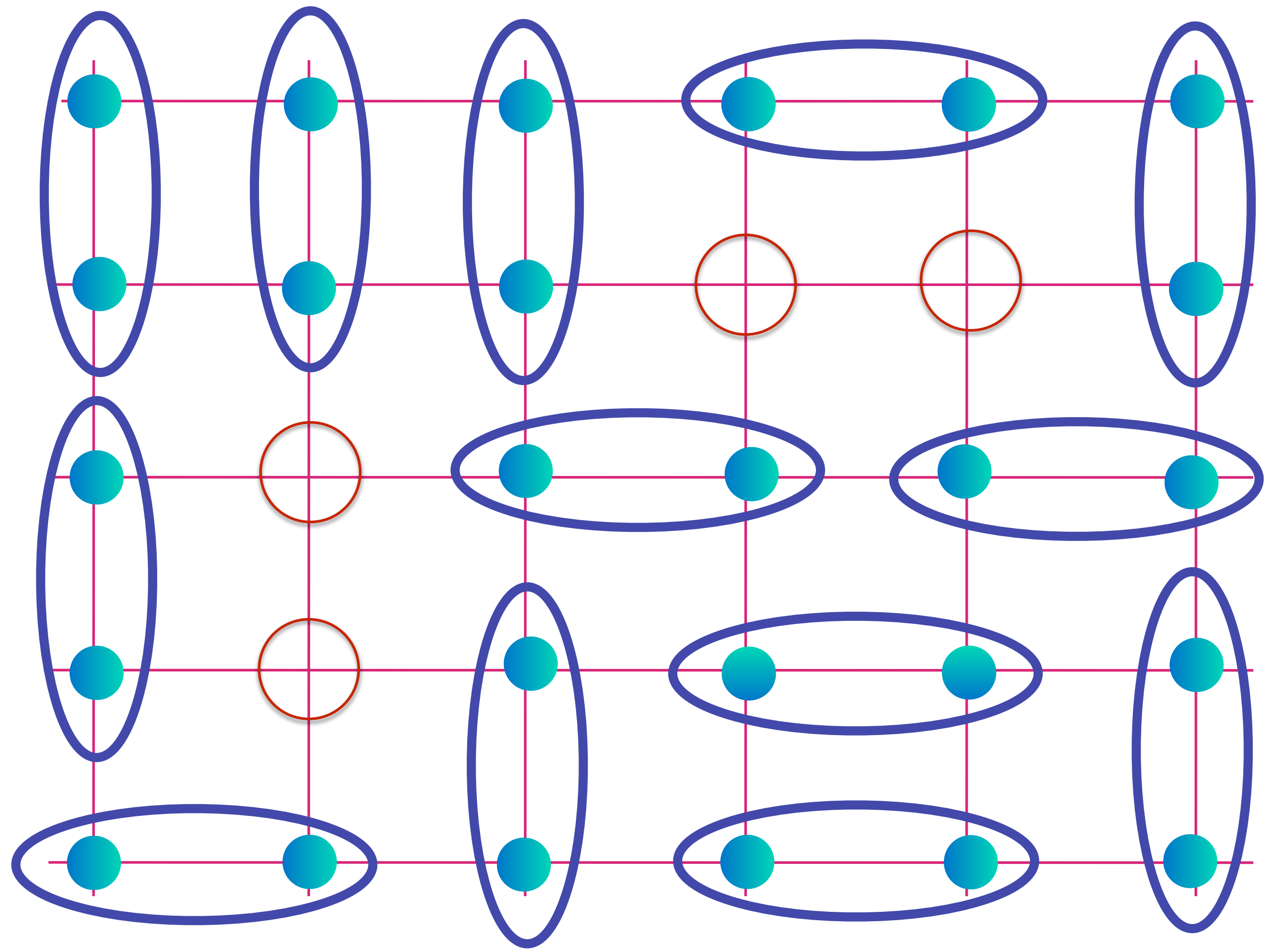
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Superconductivity

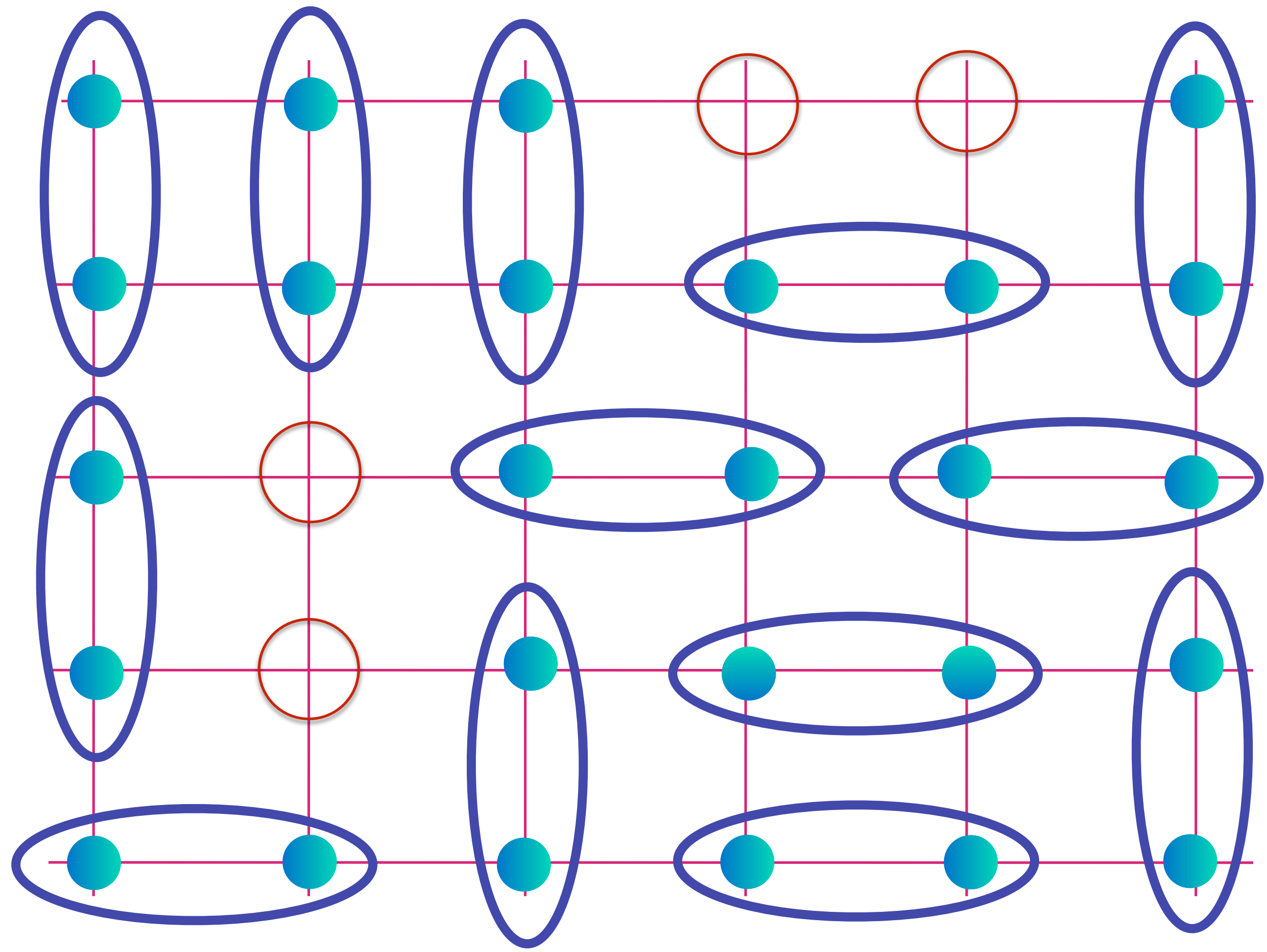
Bose condensation of electron pairs



S.N. Bose (1924)

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The dance of electrons on Cu atoms in YBCO



Superconductivity

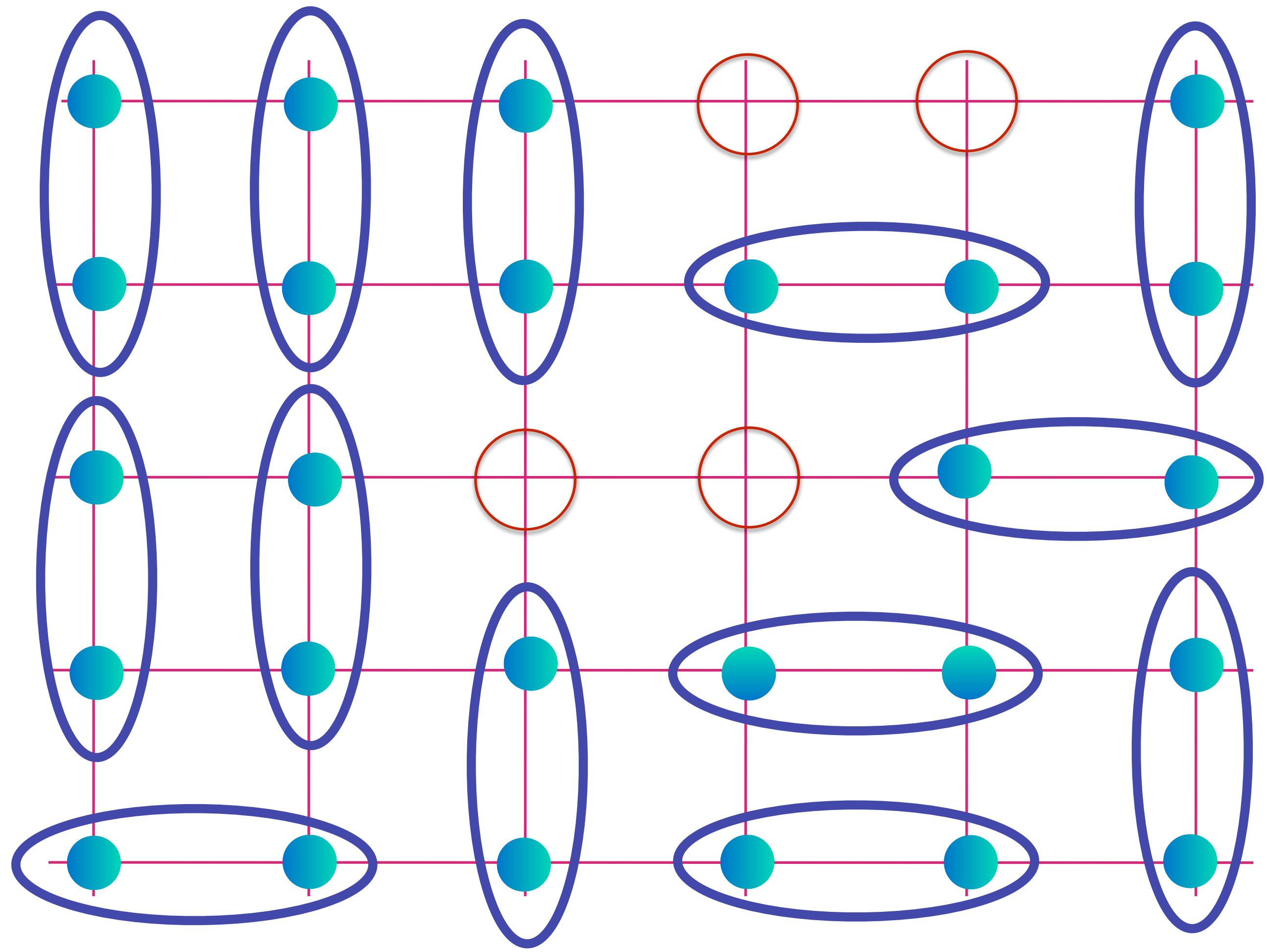
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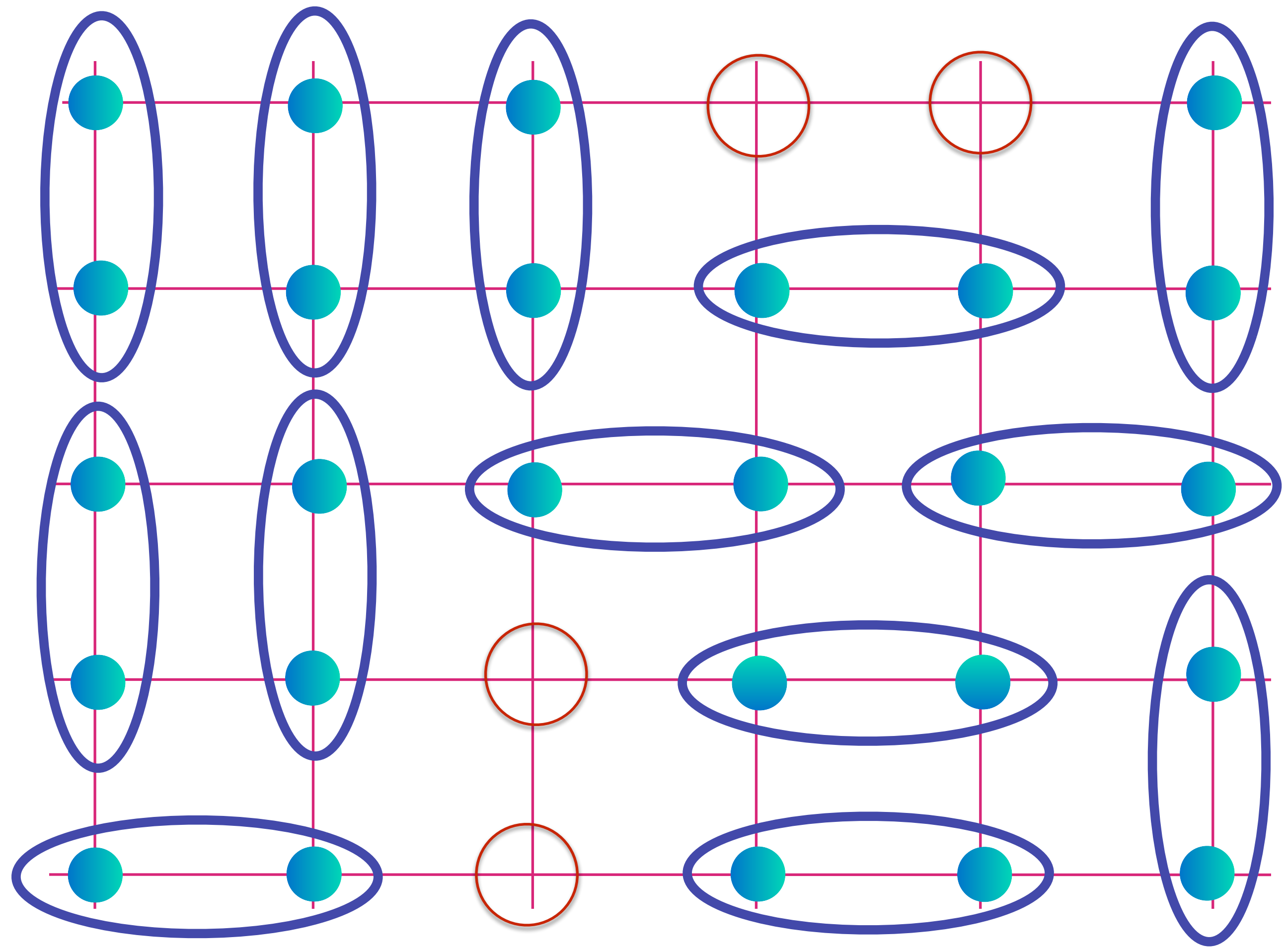
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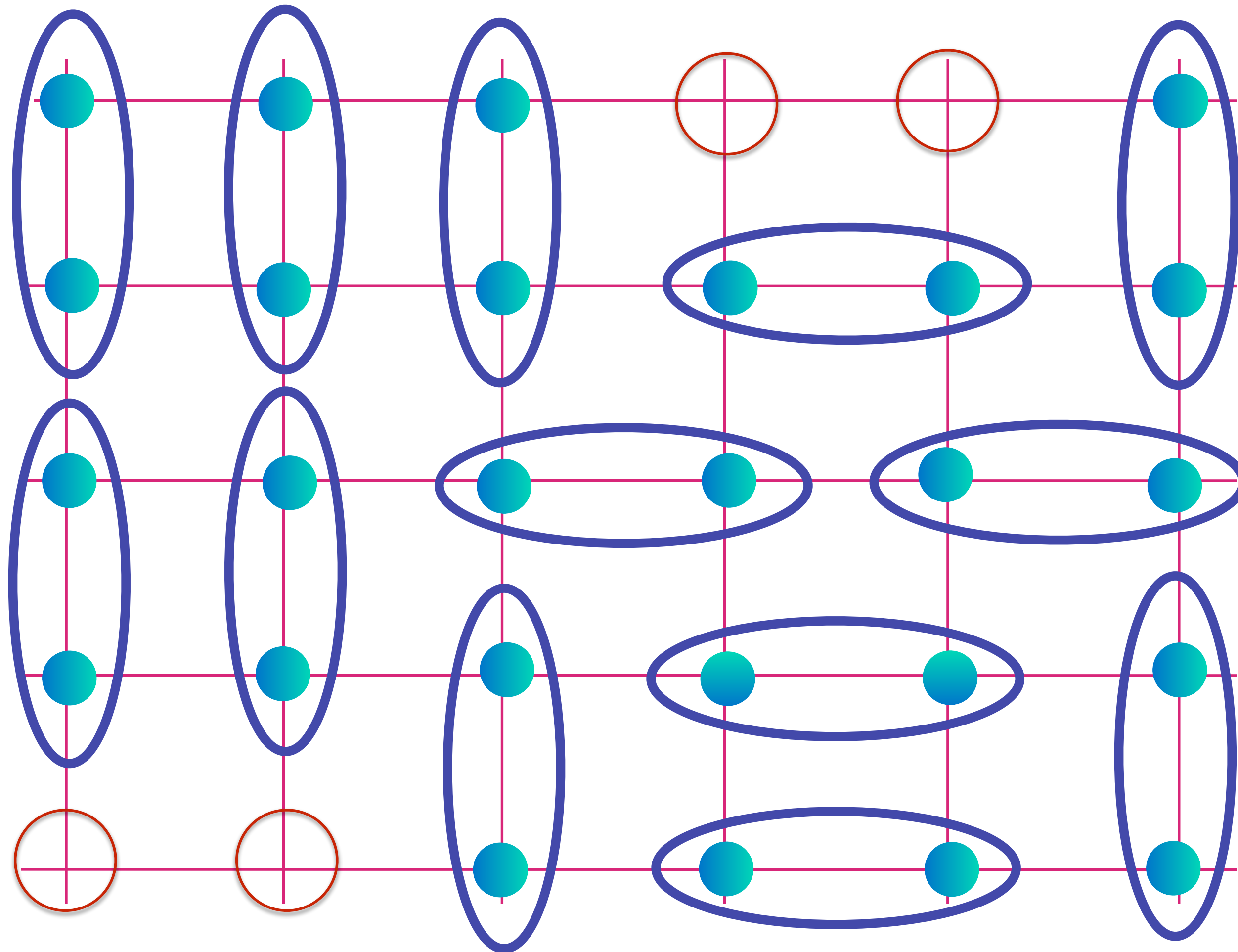
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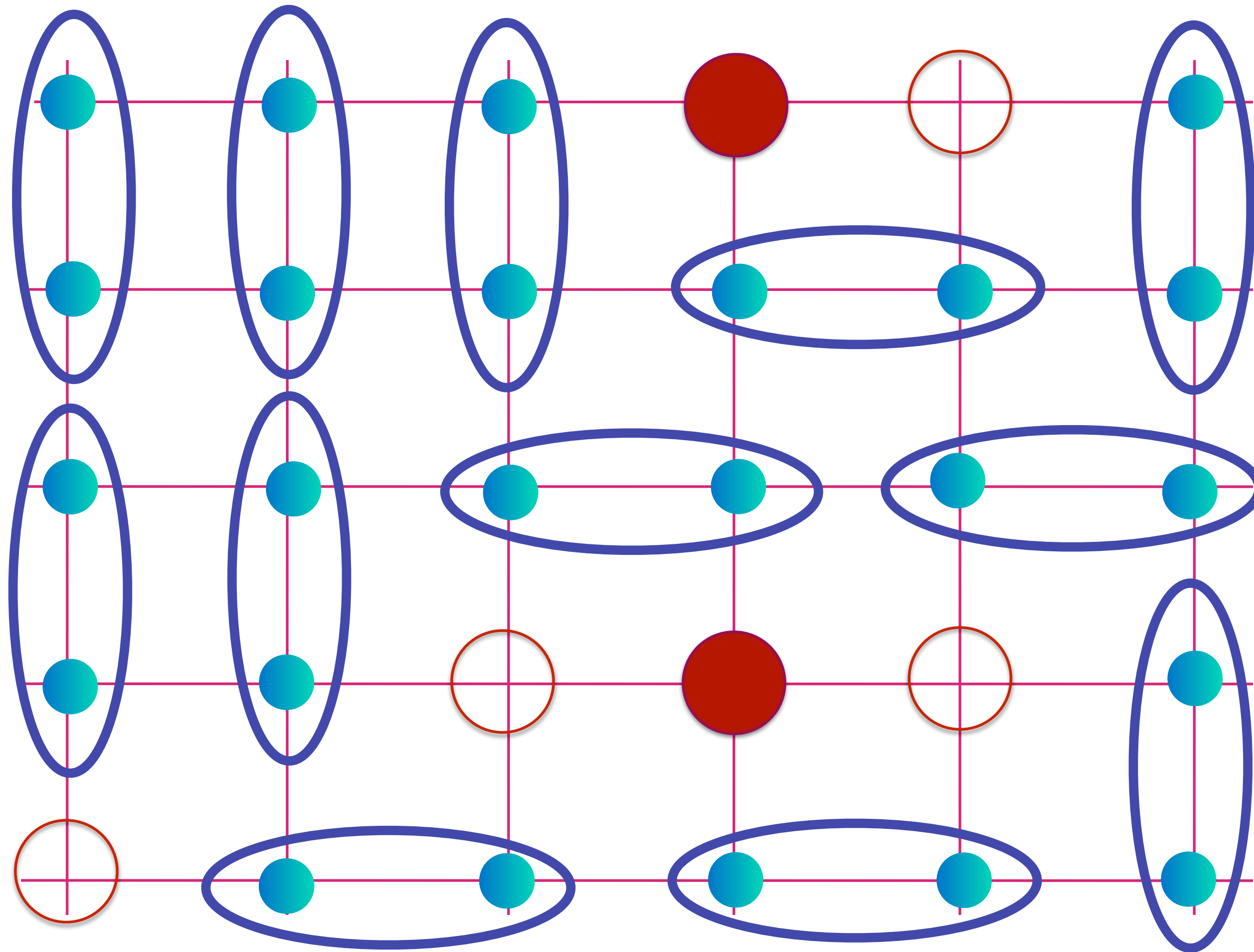
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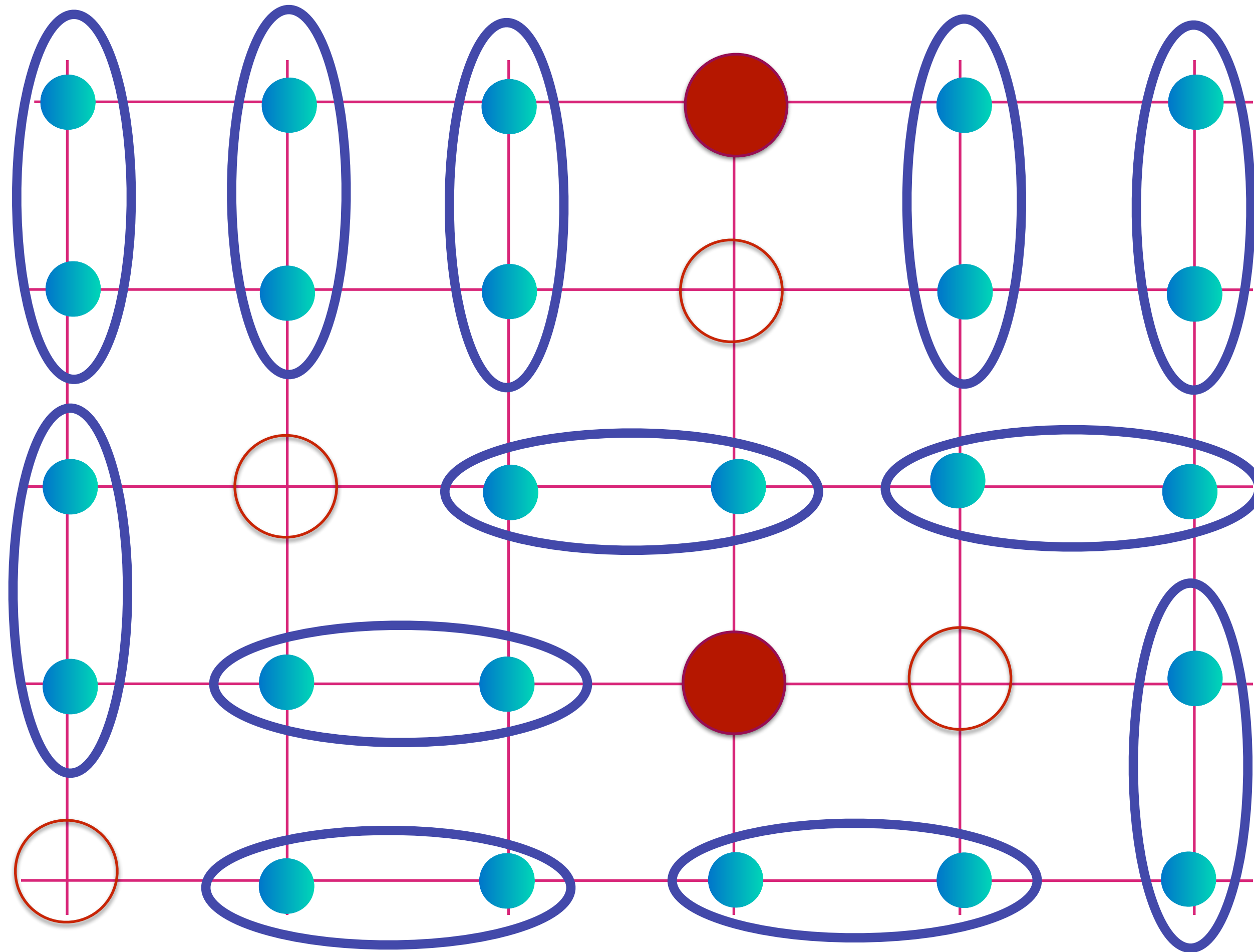


Strange metal

Complex entanglement in the presence of impurities, similar to that in the SYK model

$$\text{[Diagram of two electrons in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

The dance of electrons on Cu atoms in YBCO

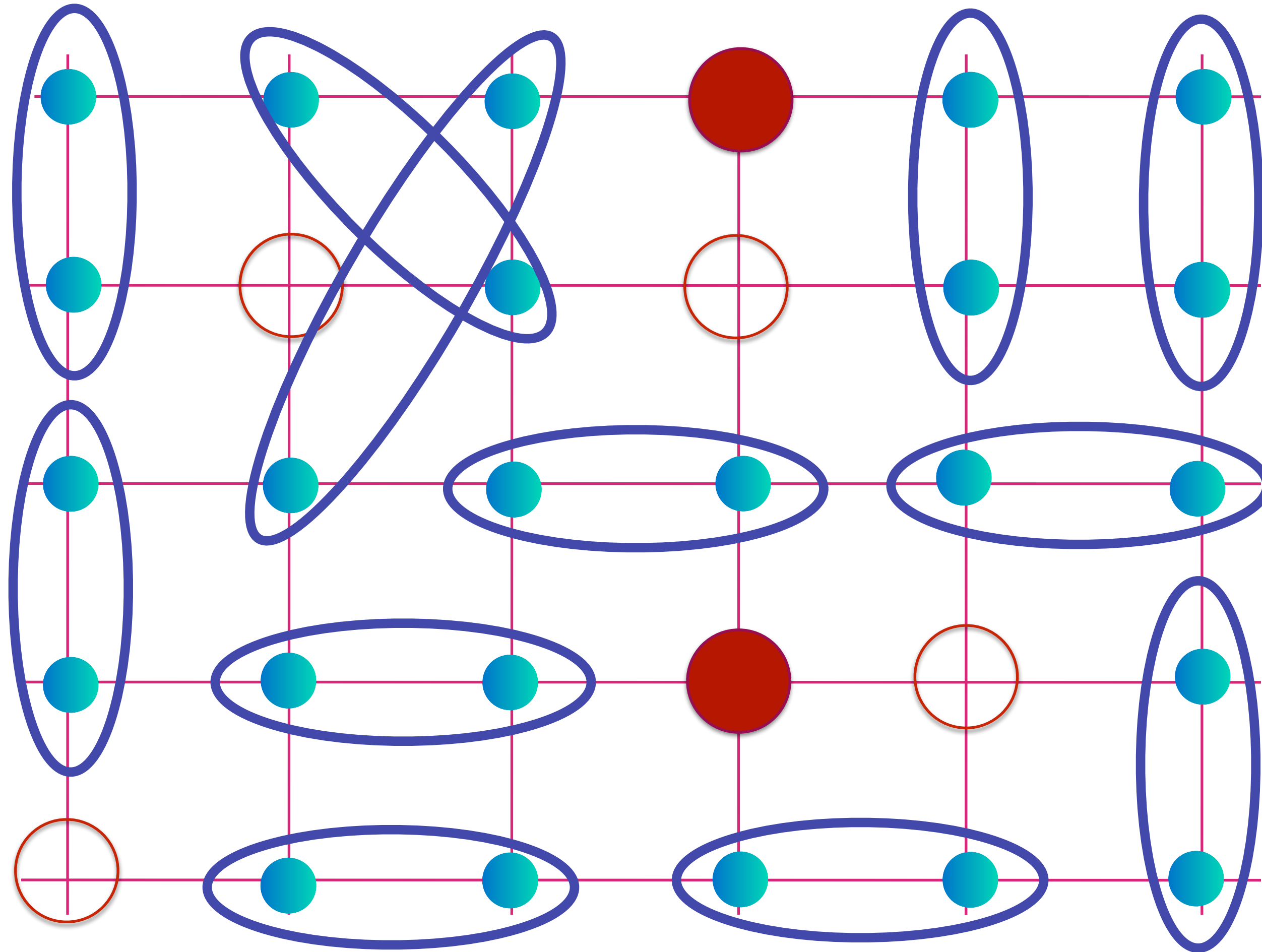


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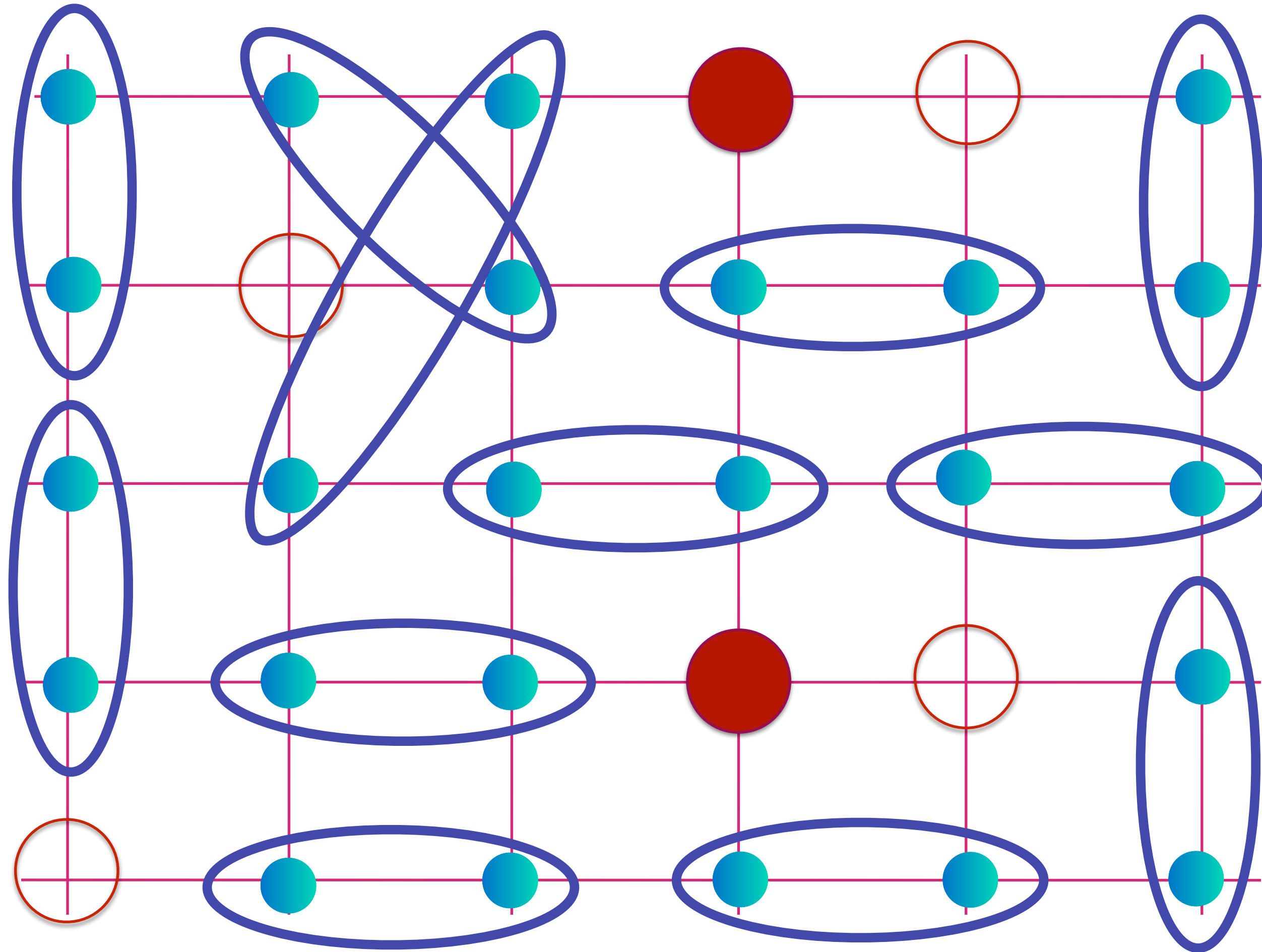


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The dance of electrons on Cu atoms in YBCO



Strange metal

Complex entanglement in the presence of impurities, similar to that in the SYK model

$$\text{Diagram of two electrons in an orbital} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



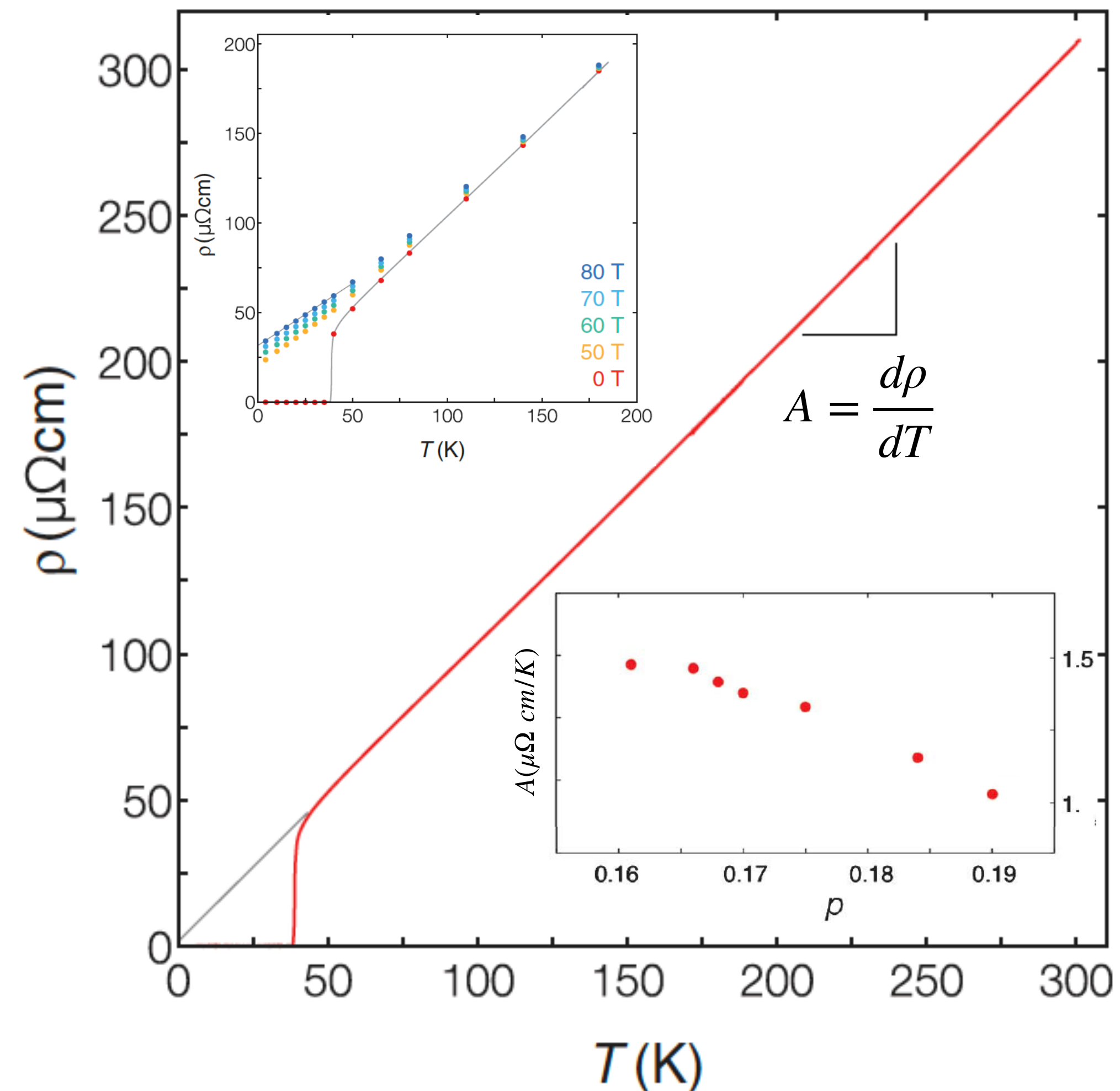
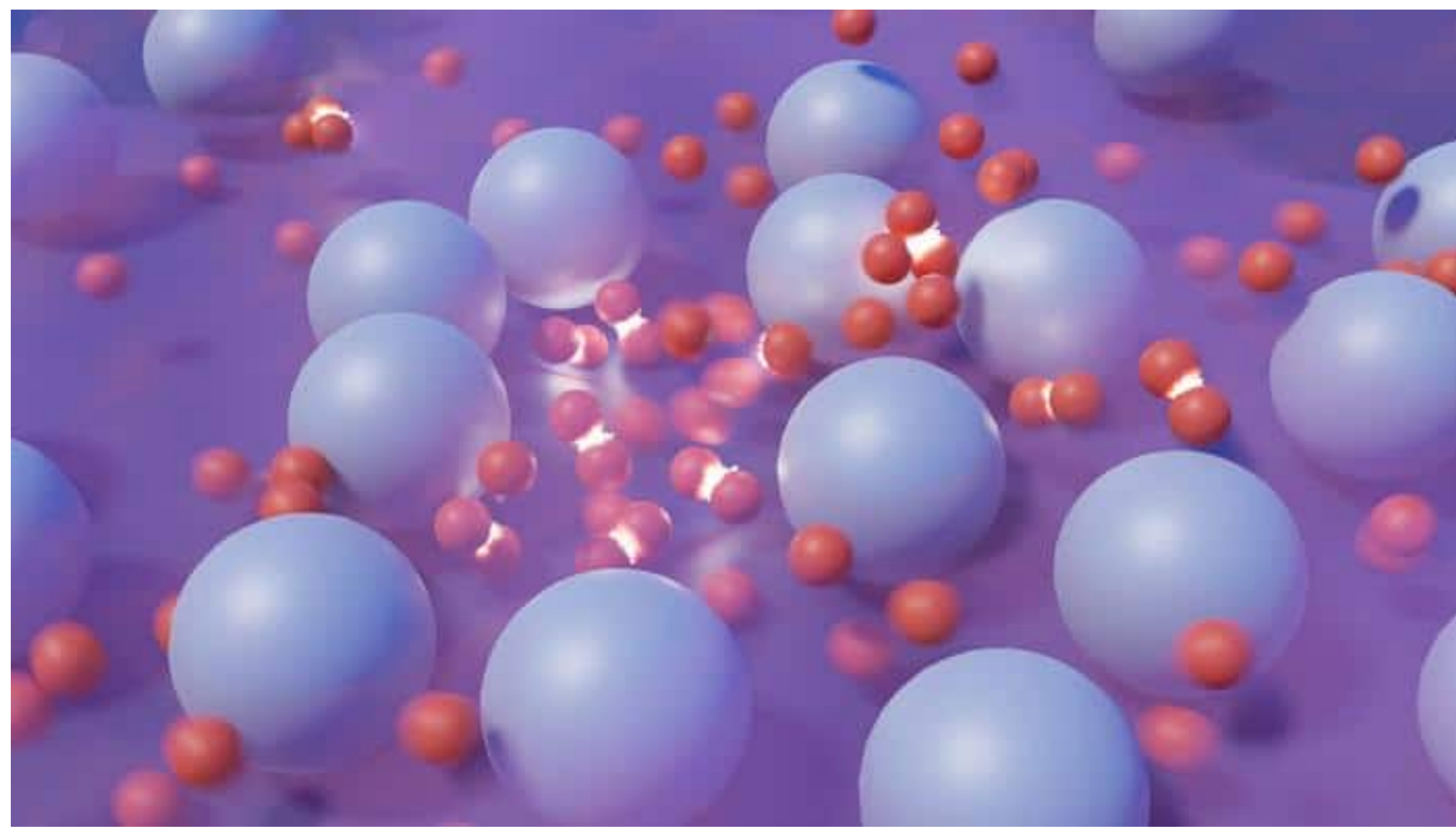
Aavishkar Patel
Flatiron Institute



Haoyu Guo
Cornell



Ilya Esterlis
Wisconsin

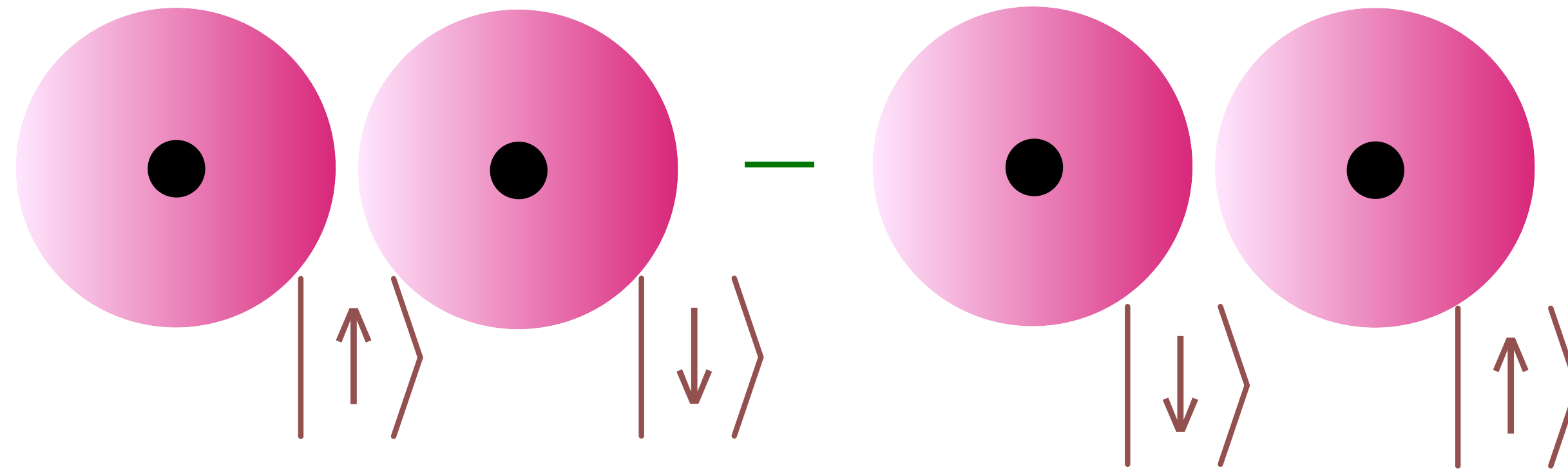


LSCO: Giraldo-Gallo et al. 2018

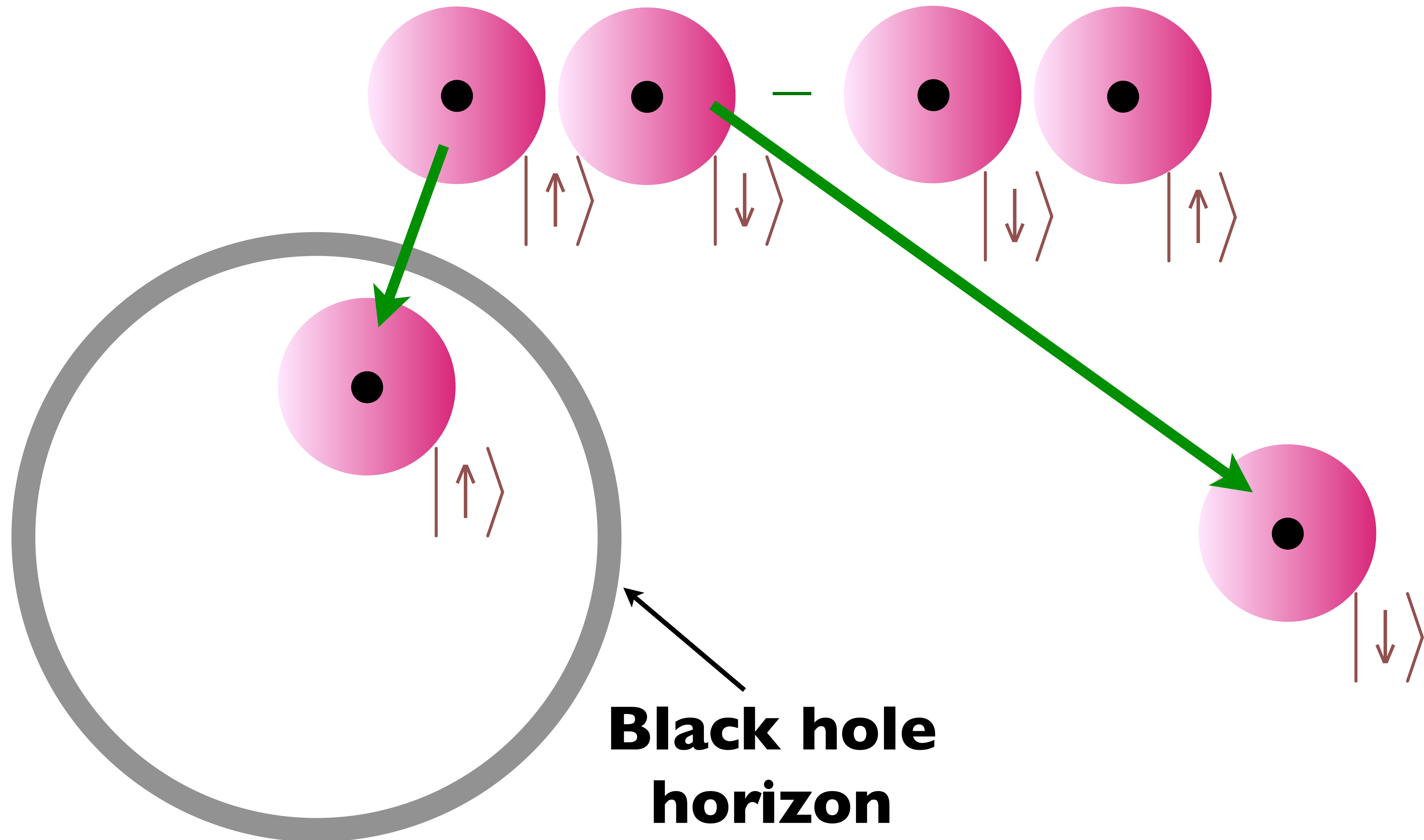
Universal theory of strange metals from
spatially random interactions,
Aavishkar A. Patel, Haoyu Guo,
Ilya Esterlis, and S. Sachdev,
Science **381**, 790 (2023)

Quantum entanglement,
the SYK model,
and black holes
holes

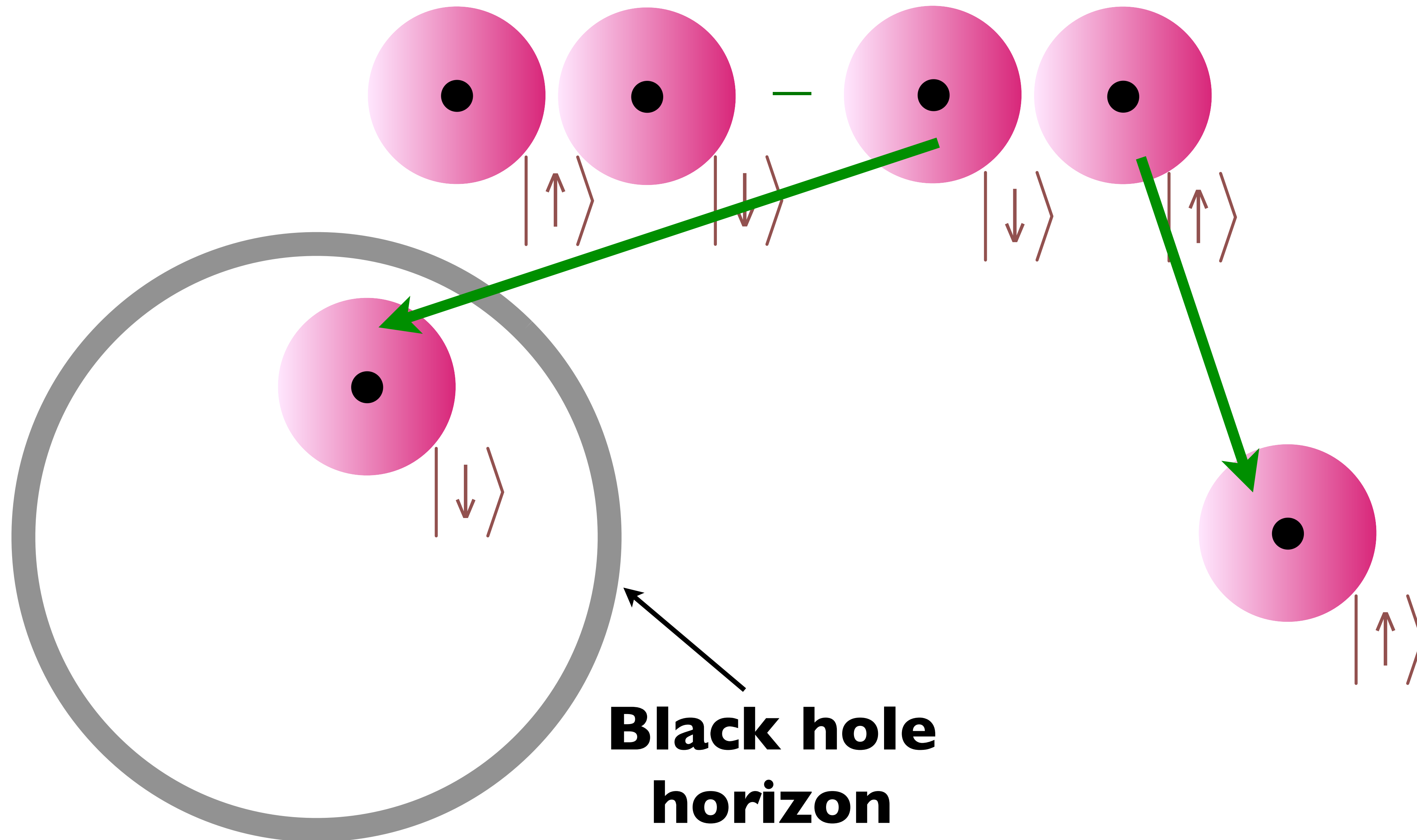
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

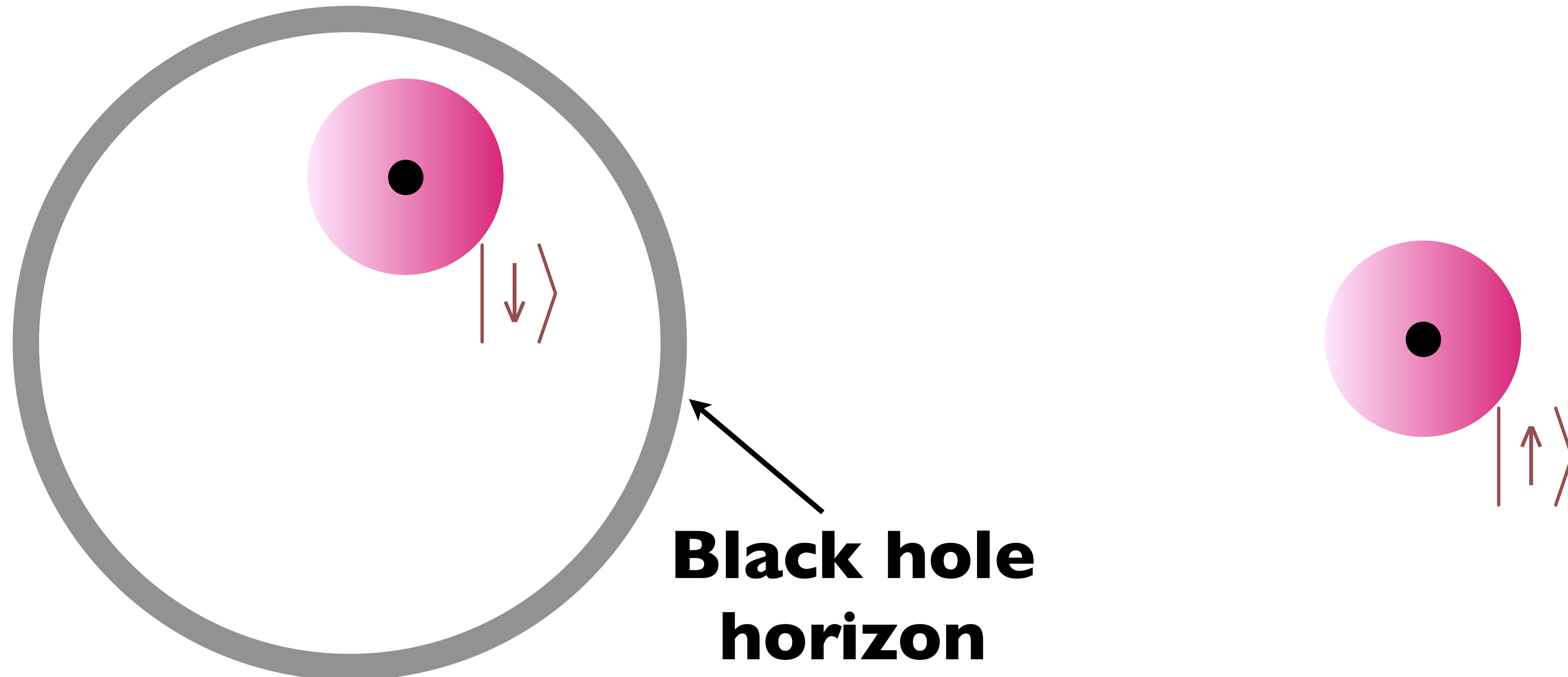


Quantum Entanglement across a black hole horizon



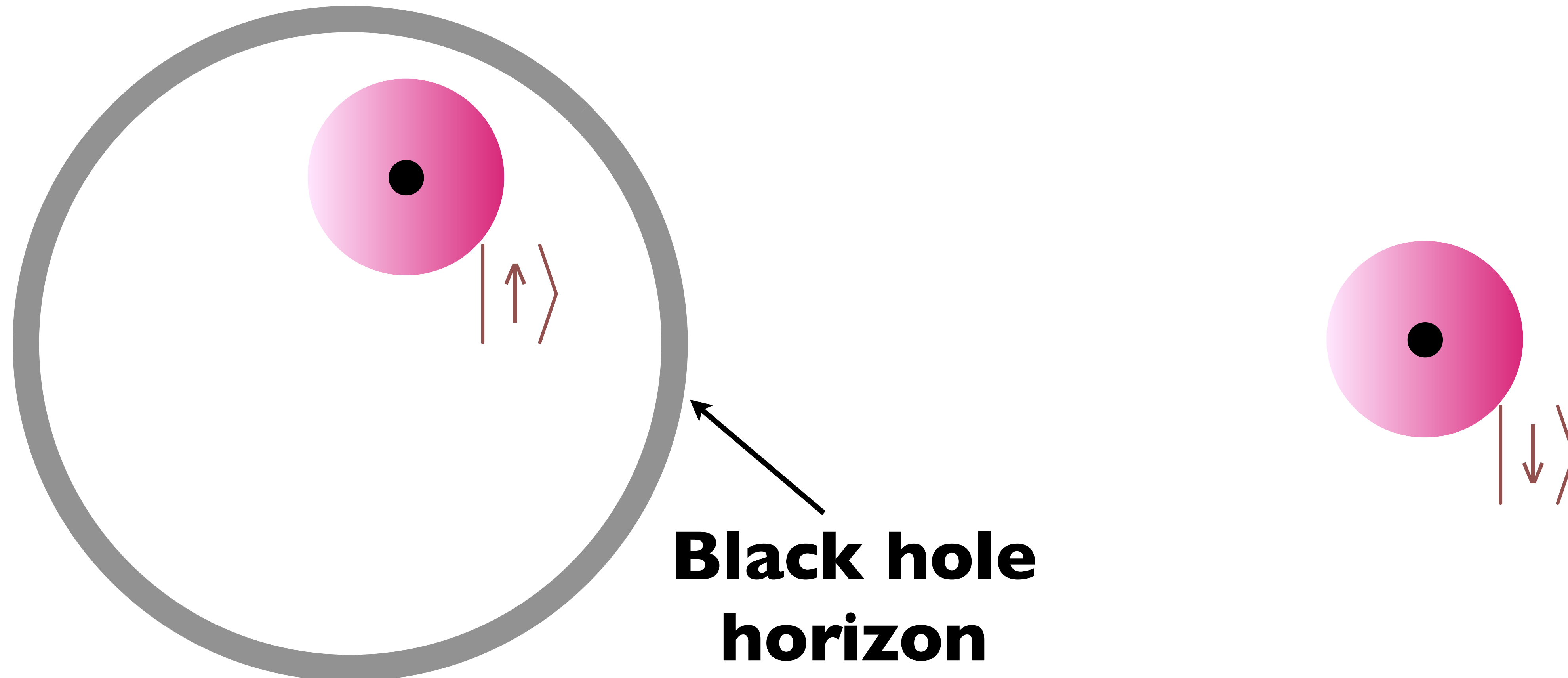
Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

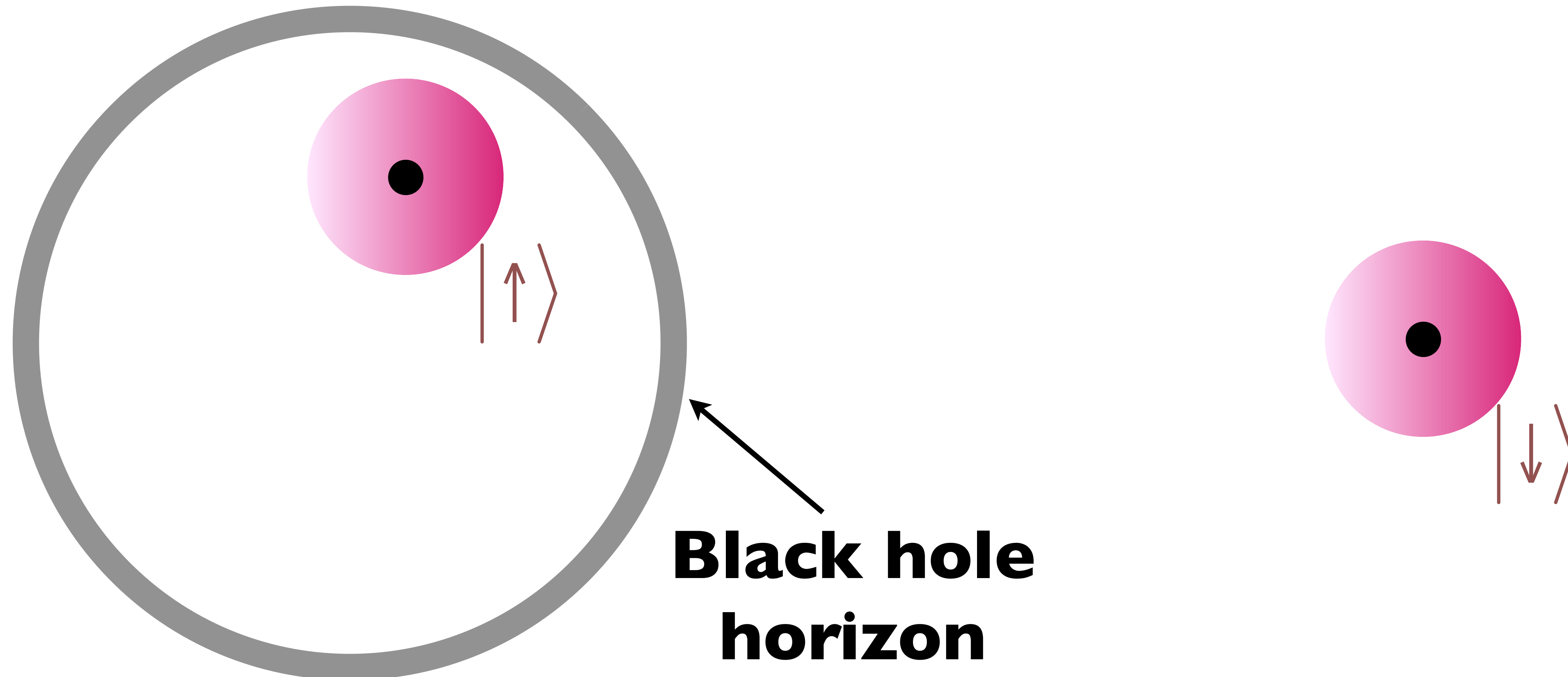
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Quantum Entanglement across a black hole horizon

Hawking (1975): Black holes have a temperature and an entropy!

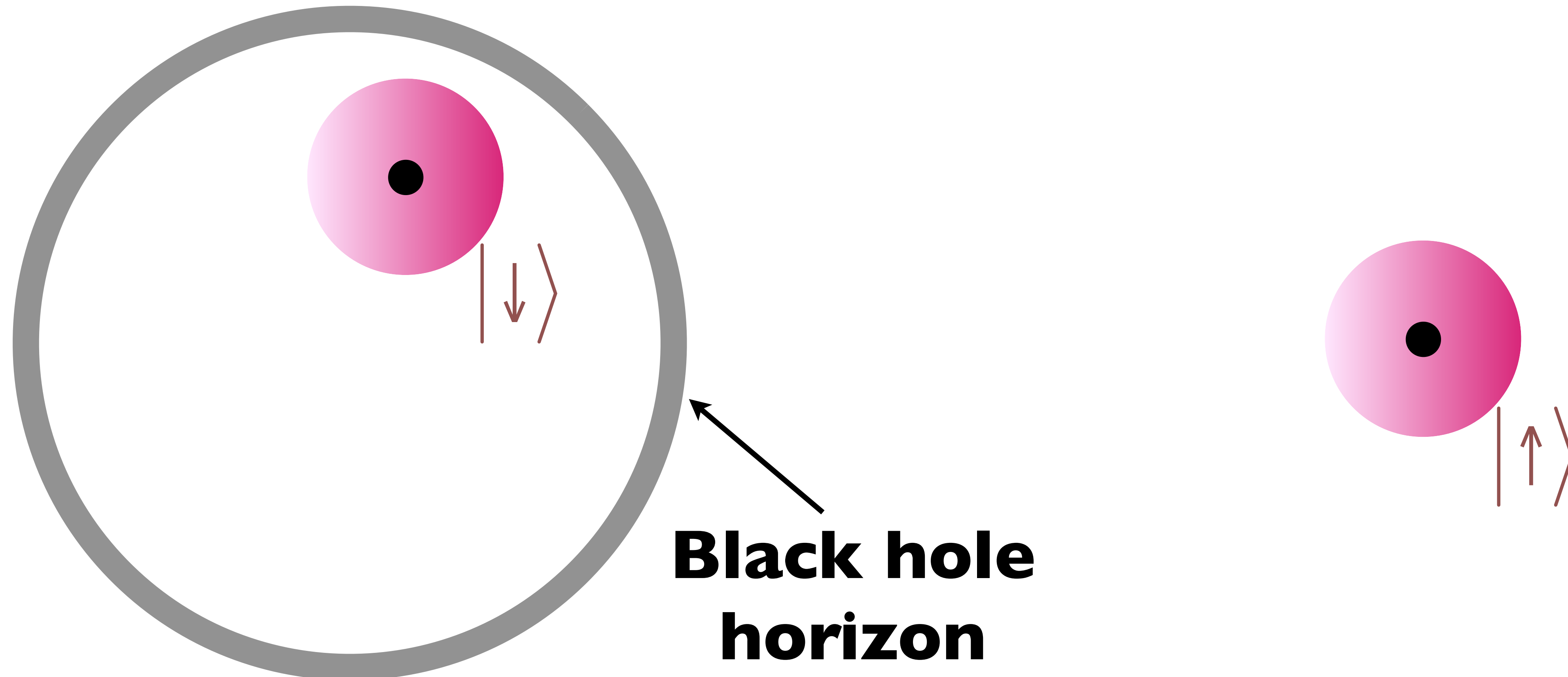
To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.



Quantum Entanglement across a black hole horizon

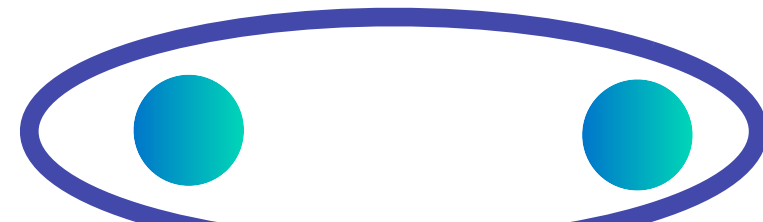
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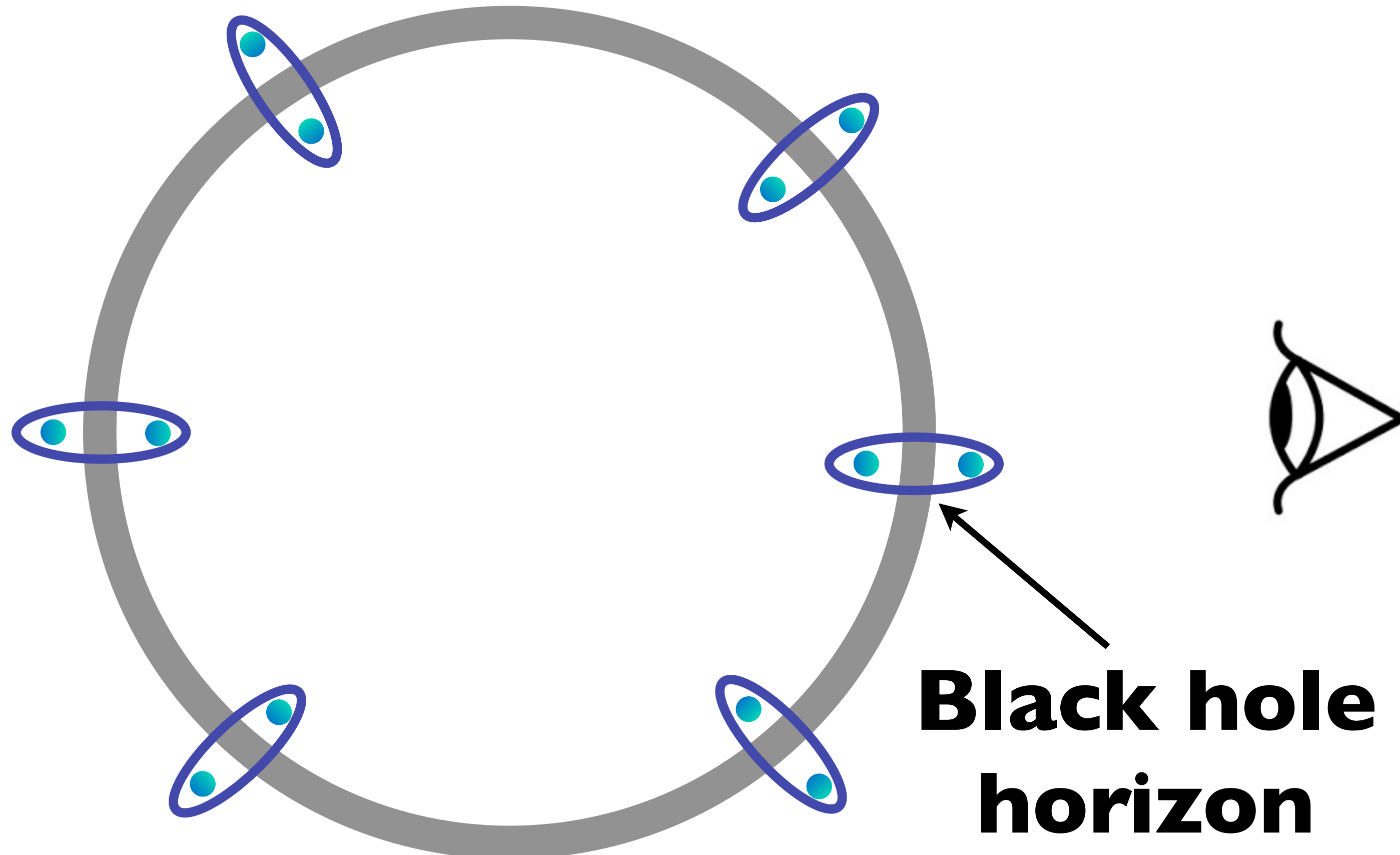
To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.



Quantum Entanglement across a black hole horizon

Quantum entanglement
on the surface


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



By computations *outside*
the black hole,
Hawking obtained

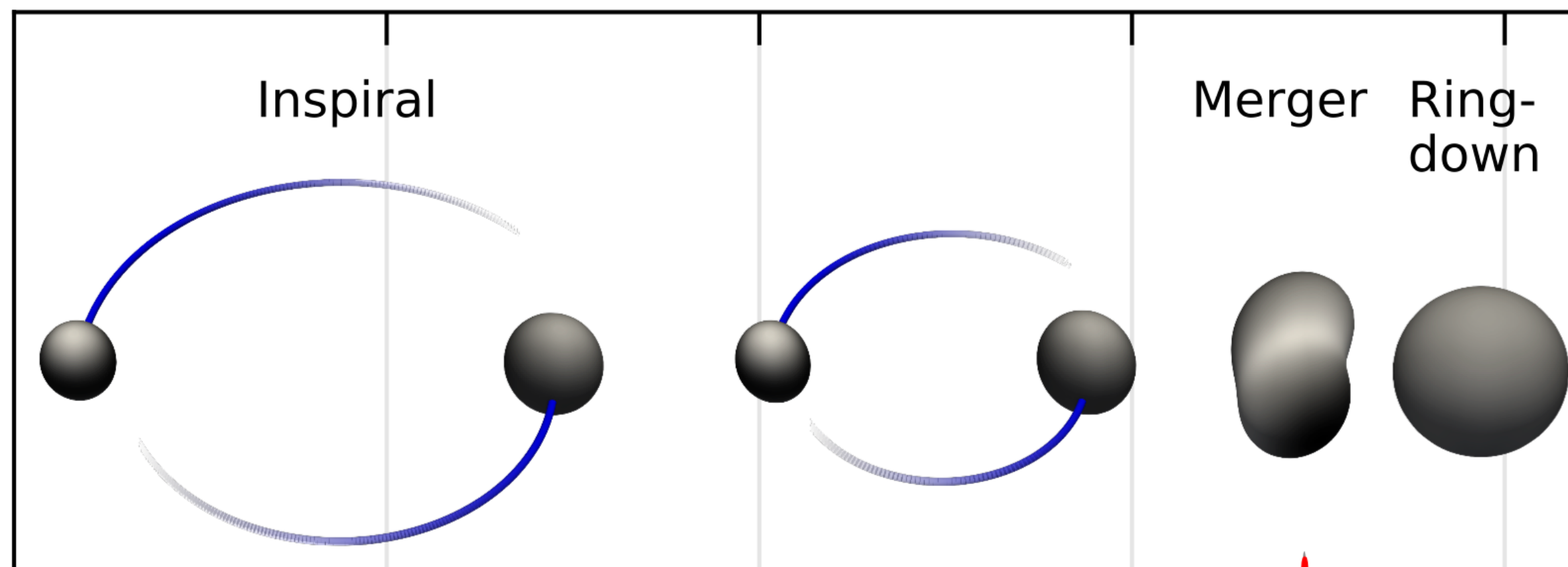
$$S = \frac{k_B A c^3}{4G\hbar}$$

where A is area of the
black hole horizon.

All other systems have
entropy proportional to
their volume.

Quantum black holes

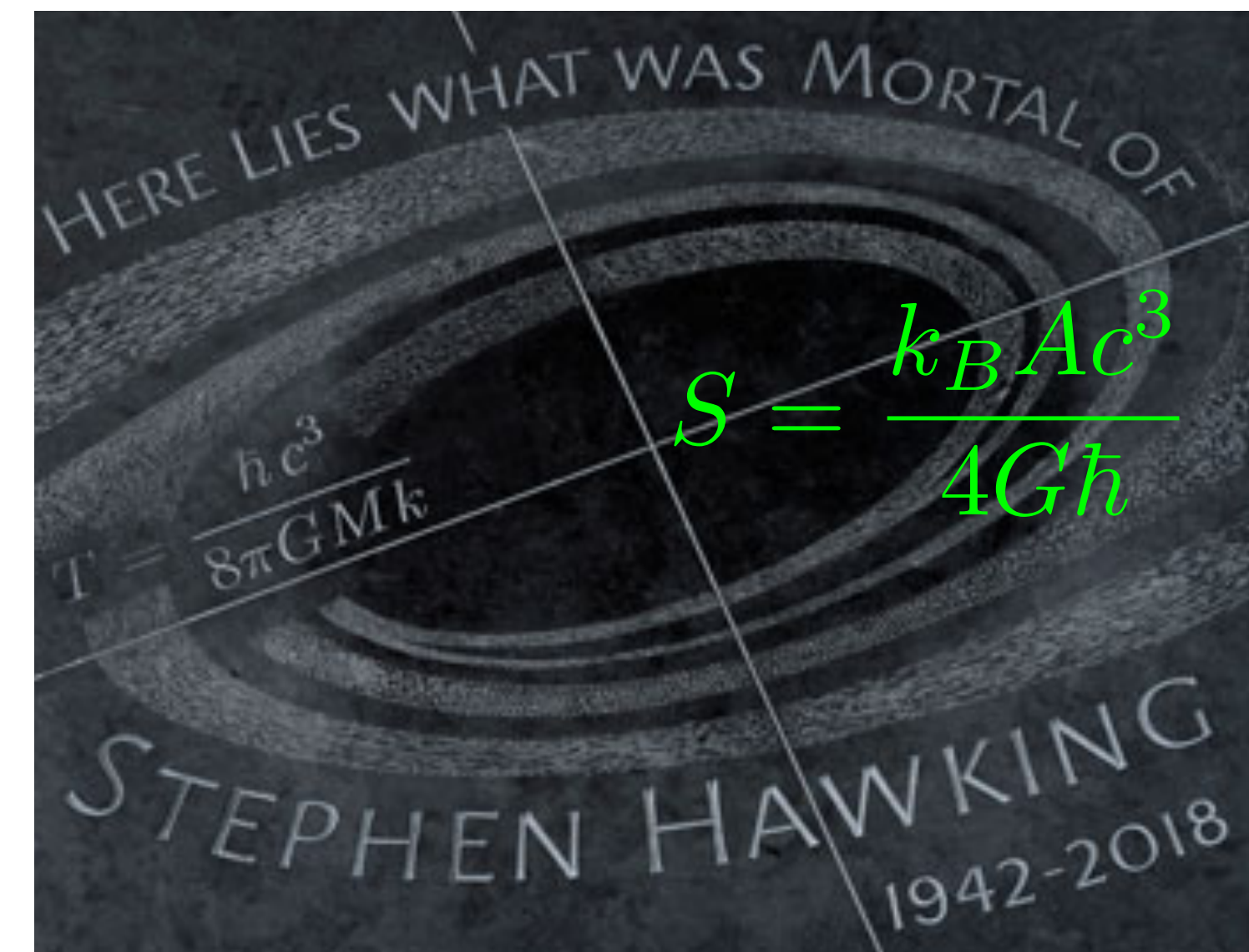
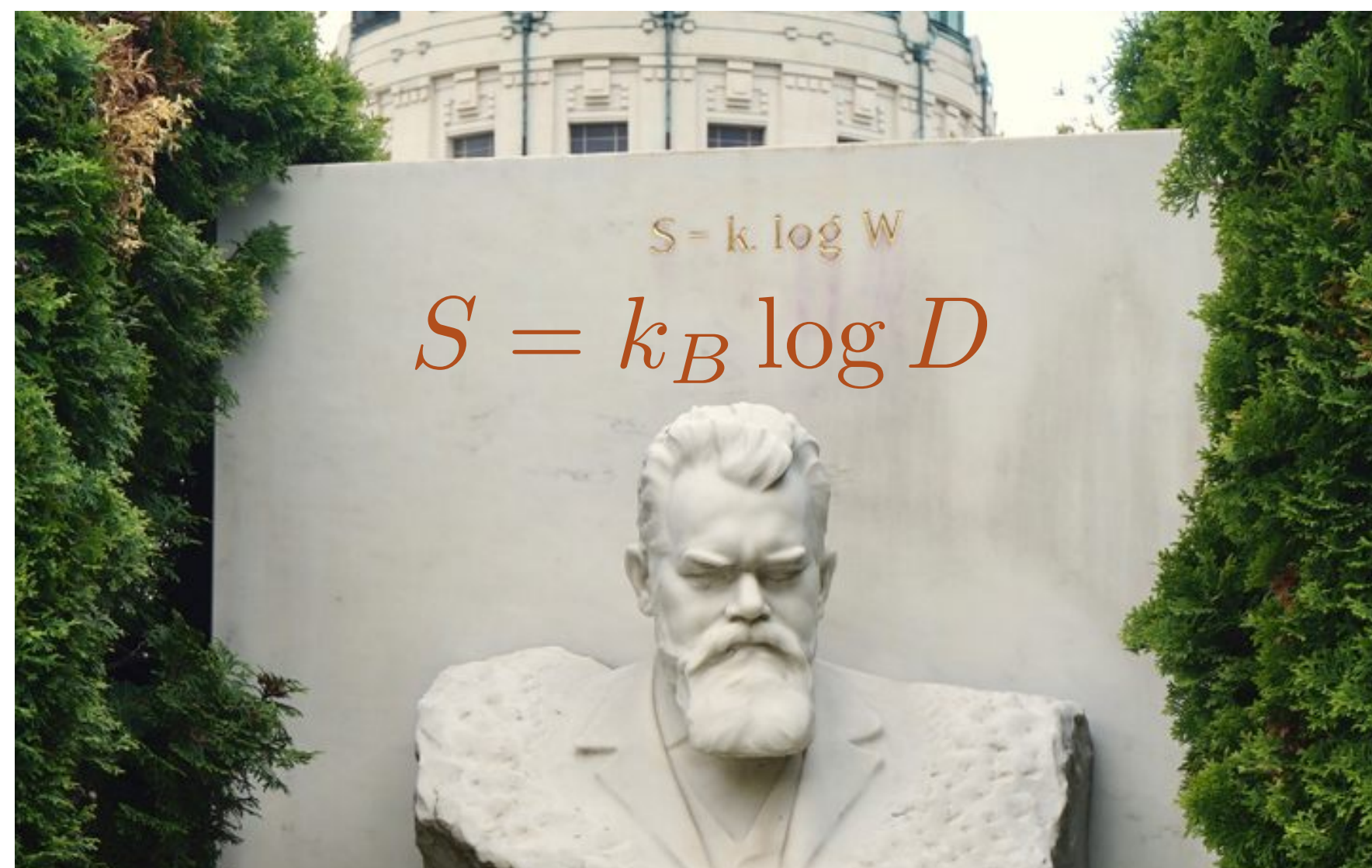
- Black holes have an entropy and a temperature, $T_H = \hbar c^3 / (8\pi G M k_B)$.
- The entropy is proportional to their surface area. $S = A k_B c^3 / (4G\hbar)$.
- They relax to thermal equilibrium in a time $\sim 8\pi G M / c^3 = \hbar / (k_B T_H)$, the Planckian time also found in strange metals, which suggests a connection to the SYK model!



J. D. Bekenstein, PRD **7**, 2333 (1973)
S.W. Hawking, Nature **248**, 30 (1974)
C.V. Vishveshwara, Nature **227**, 936 (1970)

Quantum Black Holes

- Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* $D(E)$ [the quantum analog of Boltzmann's W] matches Hawking's entropy, in accordance with Boltzmann's principles of statistical mechanics, $S(E) = k_B \log D(E)$?



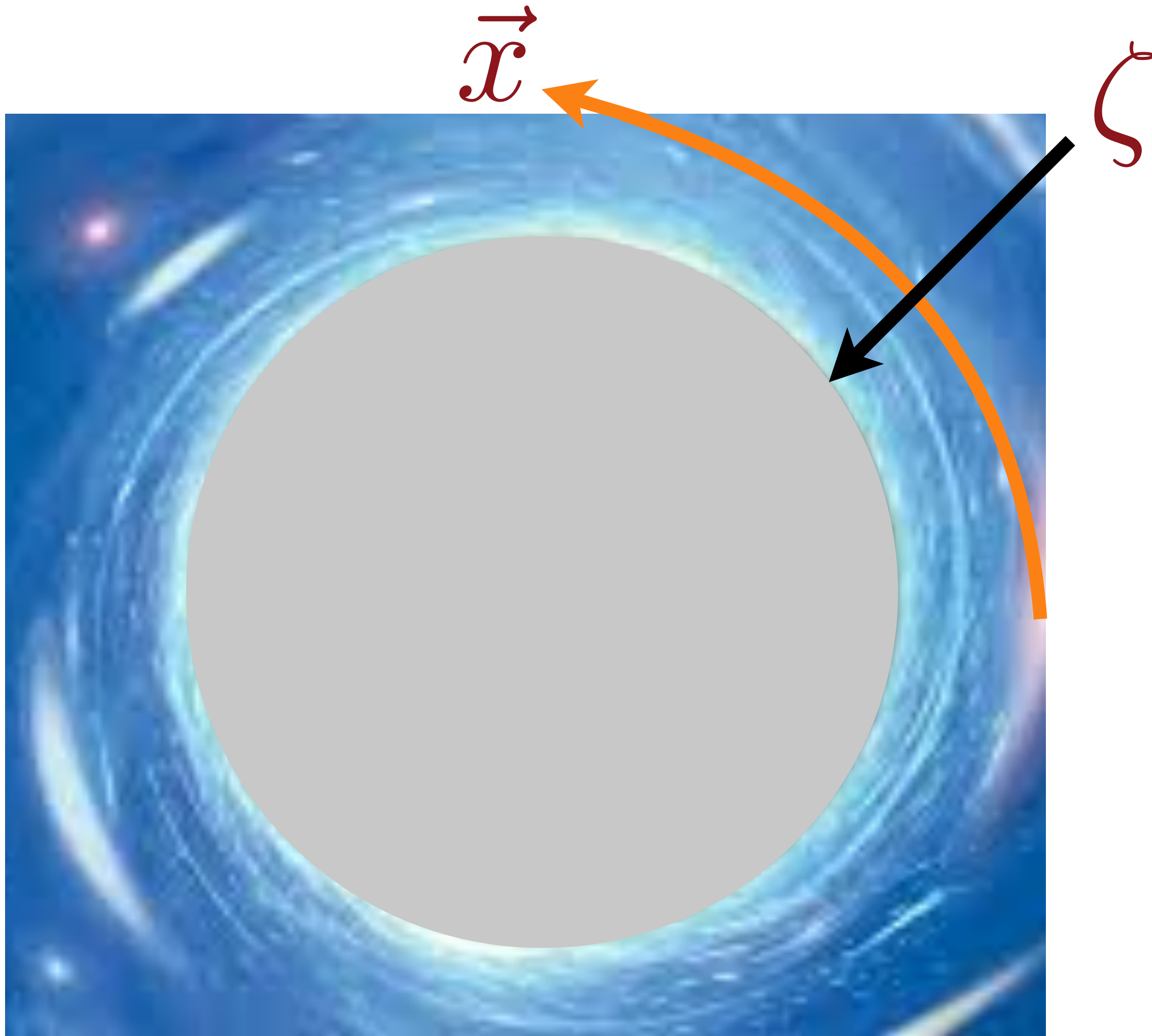
Quantum Black Holes

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- Answer from string theory for 'supersymmetric' charged black holes: $D(E) = e^S \delta(E)$ *i.e.* all the states required by Hawking's entropy have exactly the same energy.

Strominger, Vafa (1996)

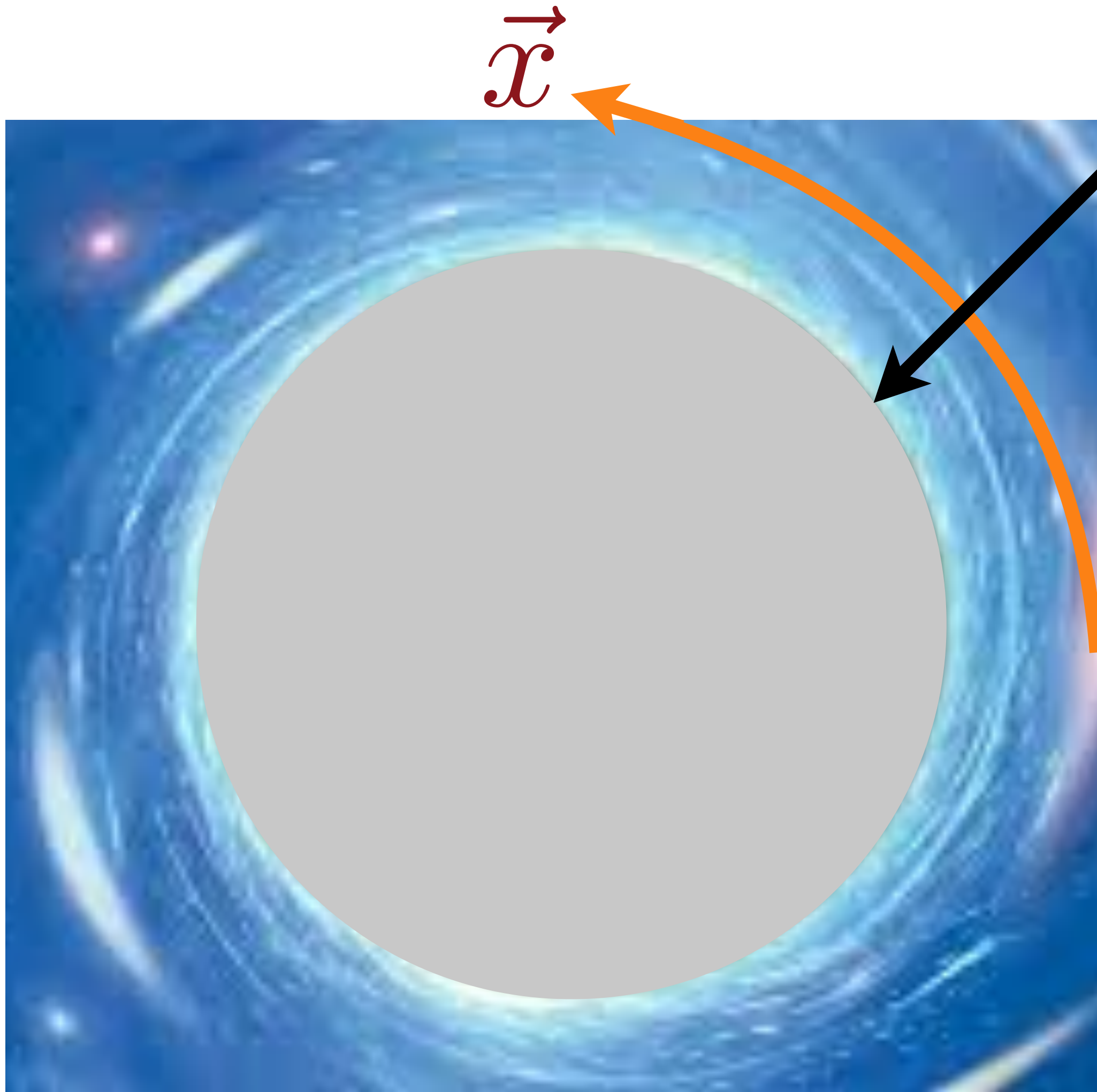


Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge





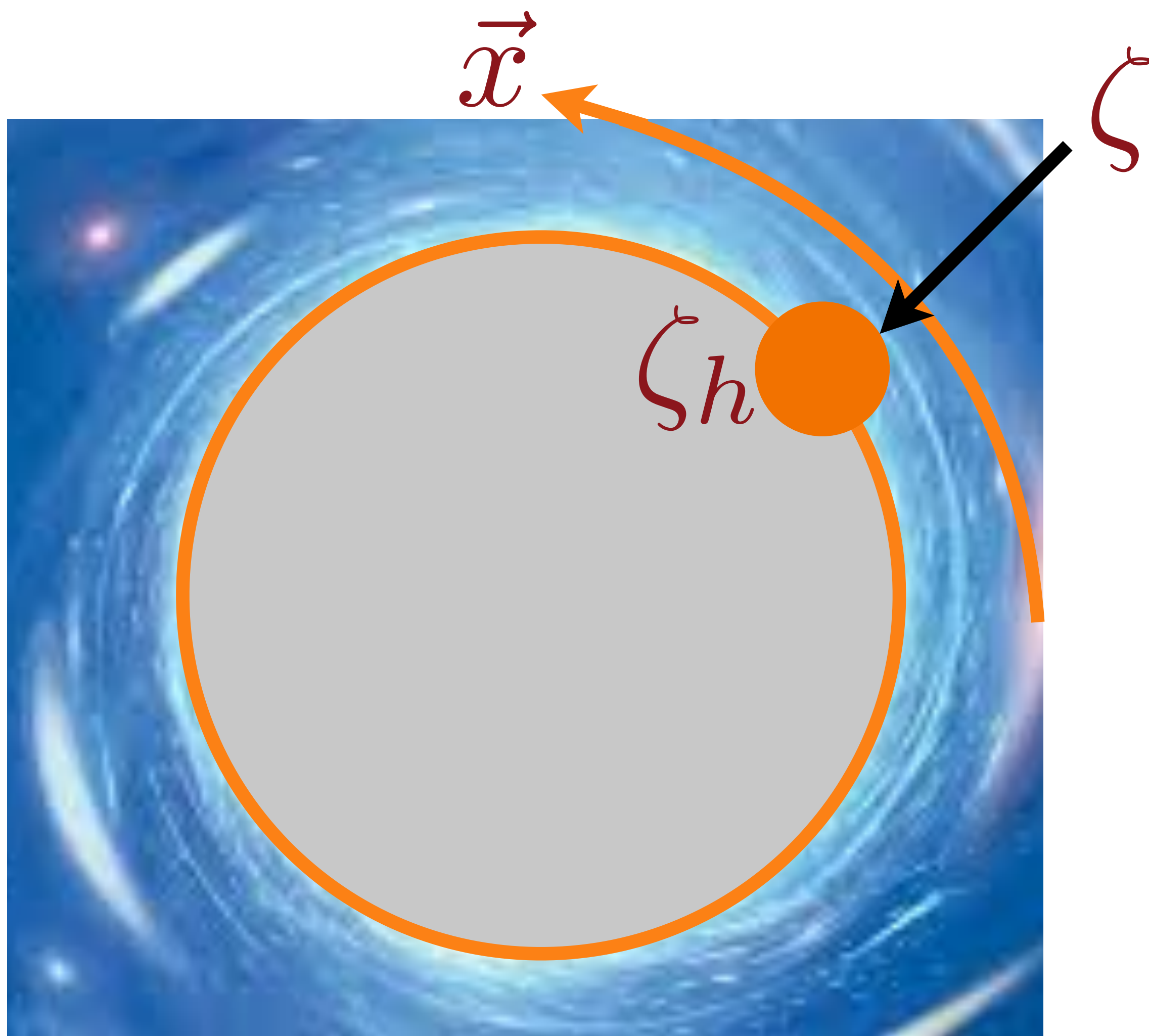
Maxwell's electromagnetism
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Zooming into the
near-horizon region
of a charged black hole
at low temperature,
yields a theory
in one space (ζ) and
one time dimension



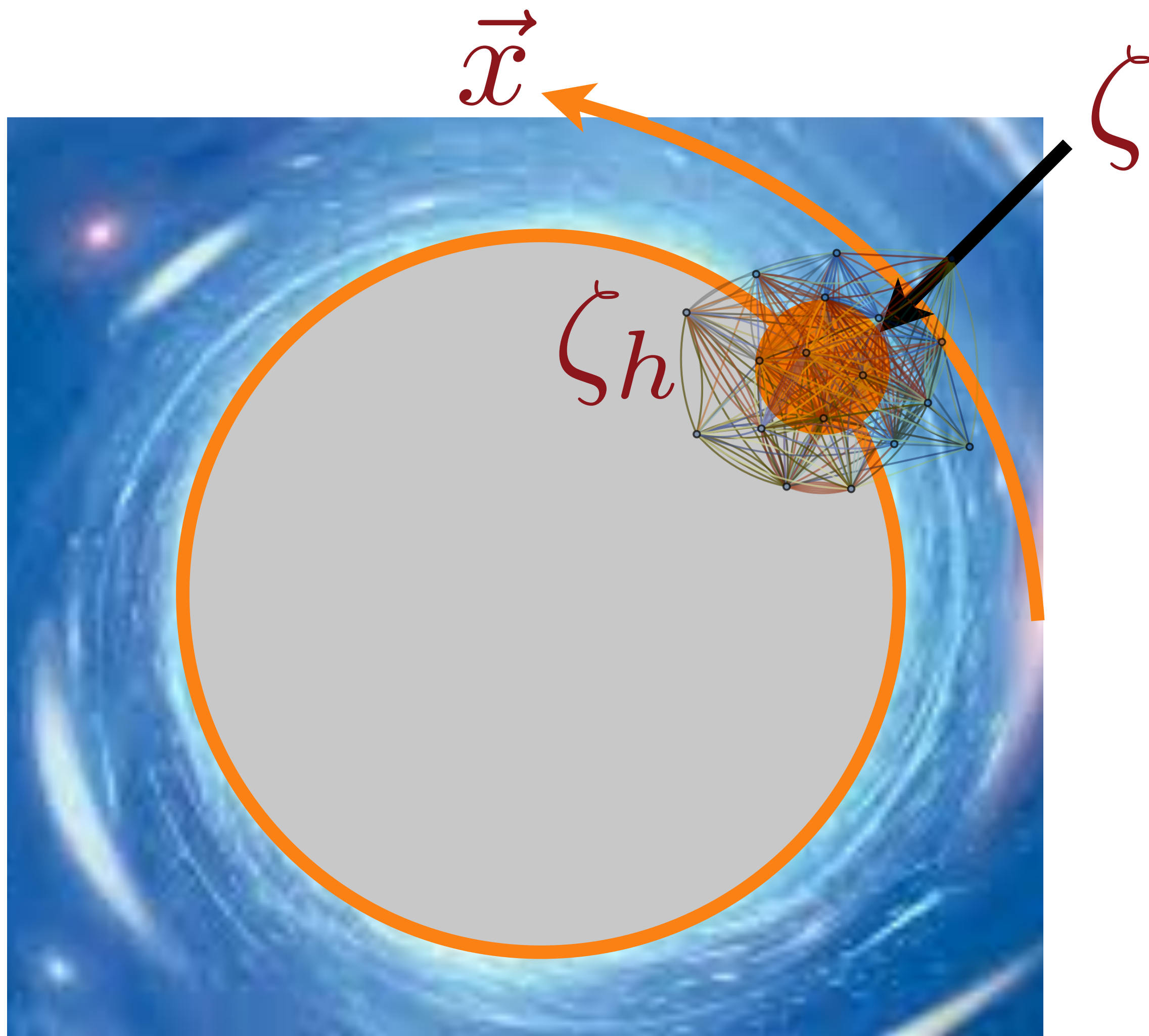
Maxwell's electromagnetism
and Einstein's general relativity
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So we need only consider
complex entanglement at
one spatial “point”
on the horizon ($\zeta = \zeta_h$),
just as is described
by the SYK model



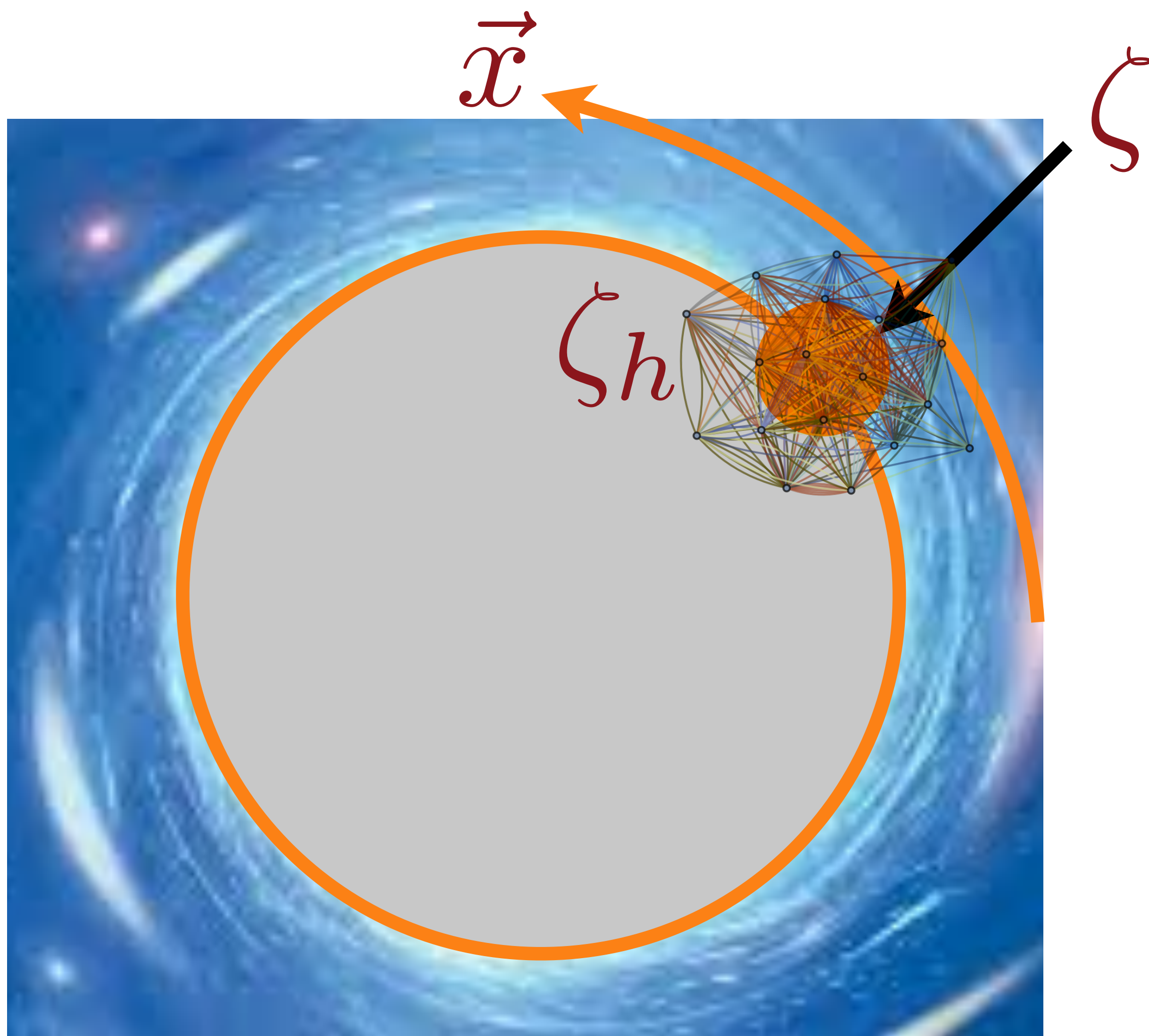
Maxwell's electromagnetism
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The quantum versions of
Maxwell's and Einstein's
equations in this
two-dimensional spacetime are
also the equations describing
electron entanglement in the
SYK model!



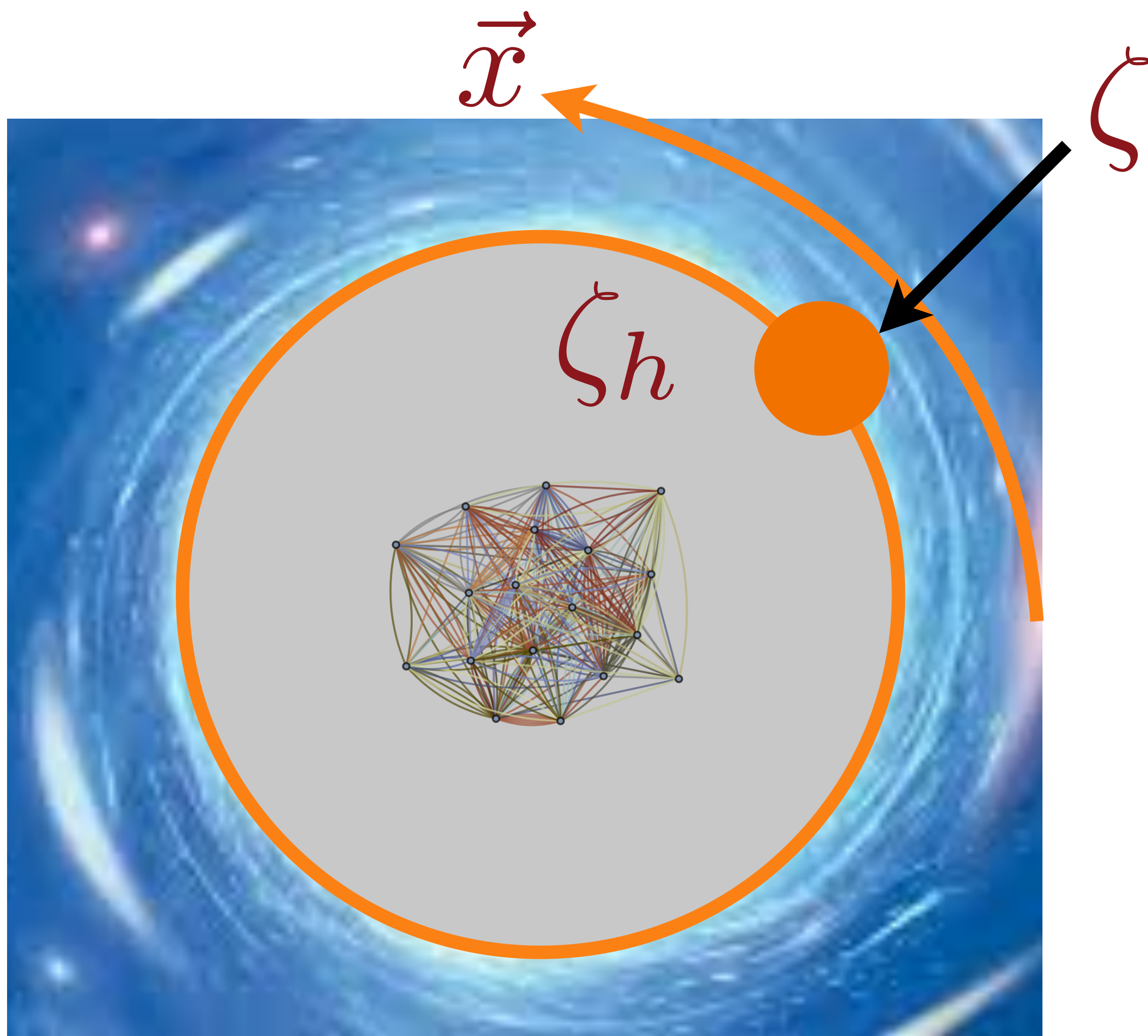
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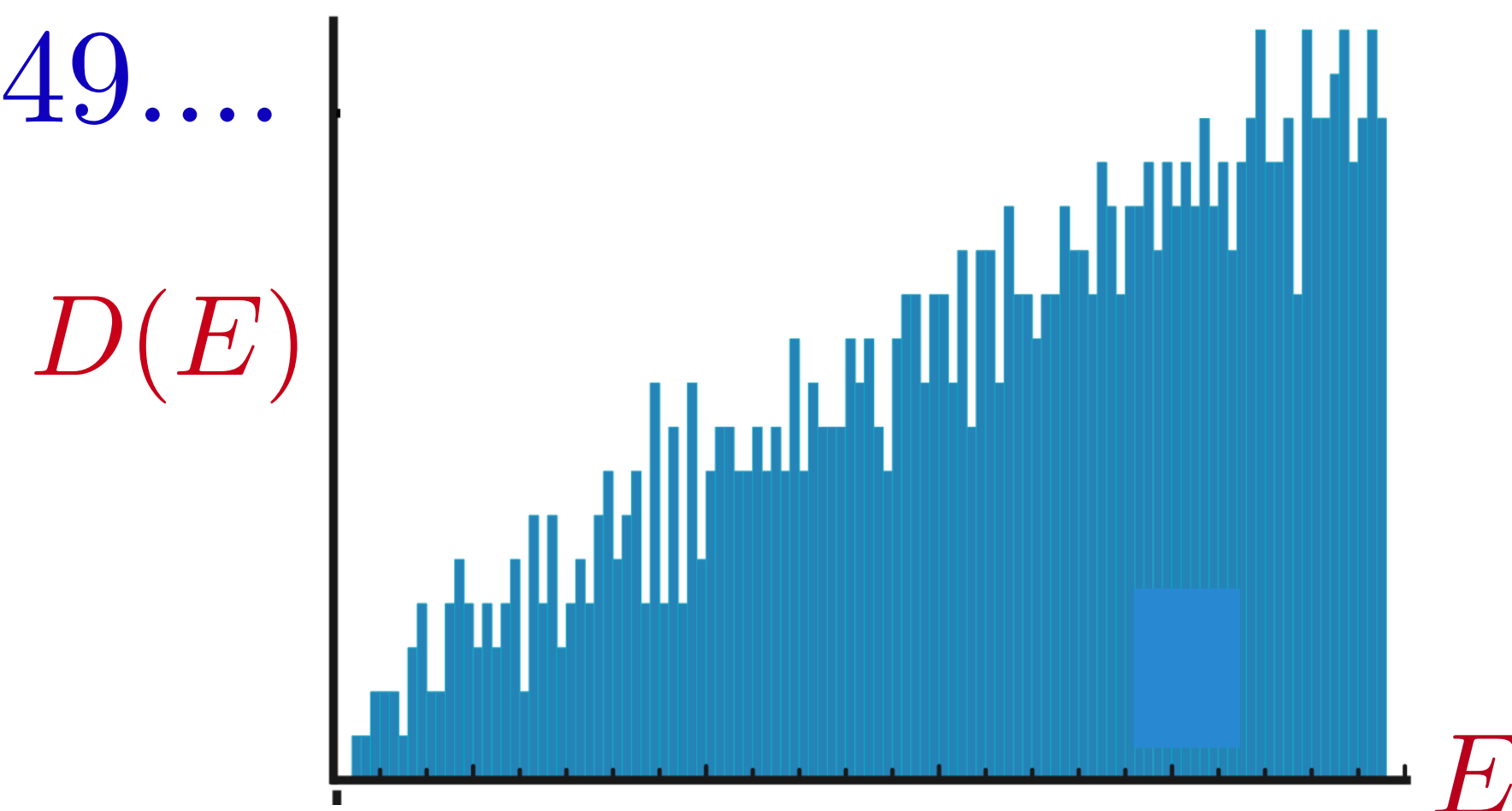
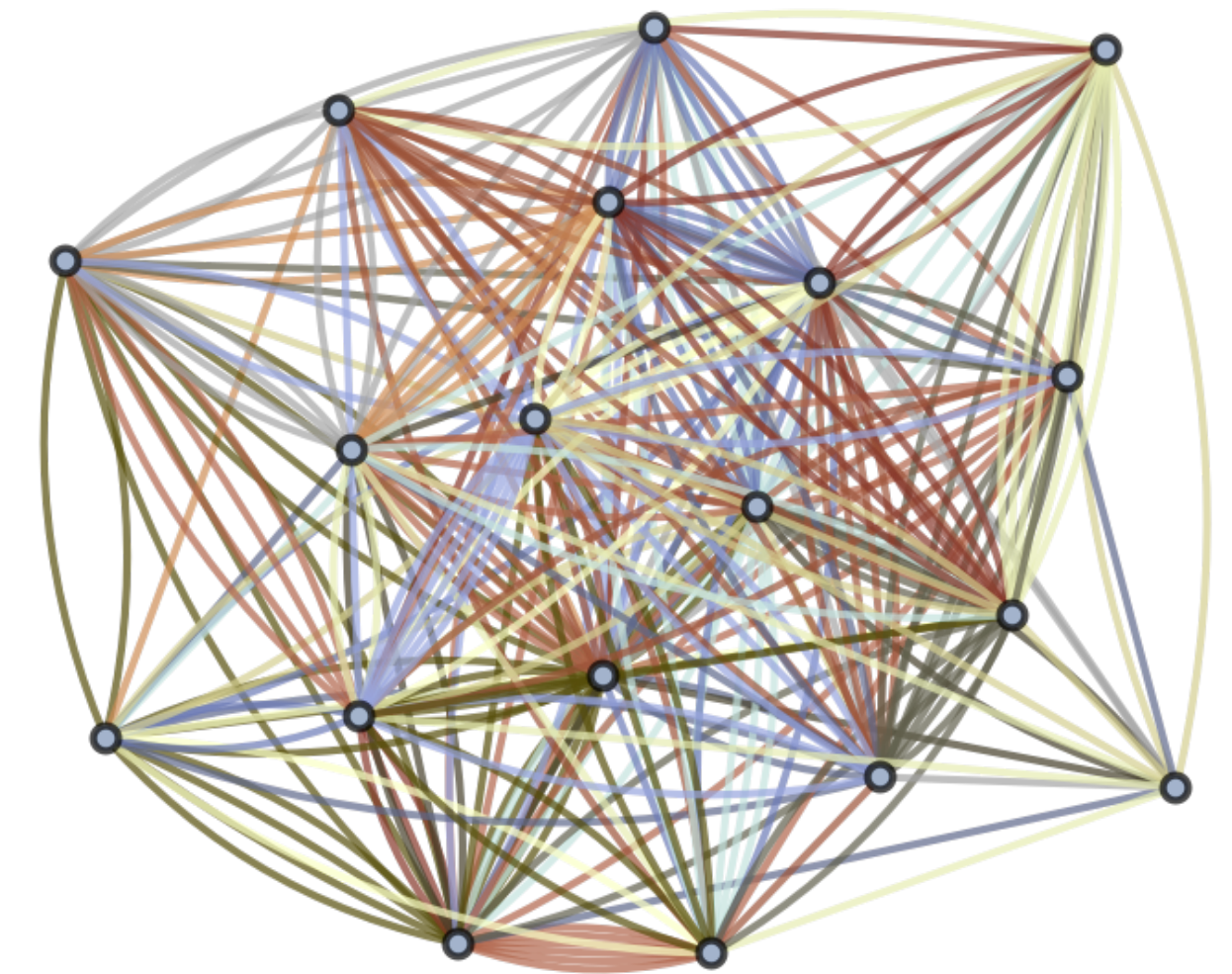
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The Sachdev-Ye-Kitaev (SYK) model

- Density of quantum states of the SYK model with N sites

$$D(E) \sim \frac{1}{N} \exp(N s_0) \sinh\left([2N\gamma E]^{1/2}\right)$$

where $s_0 = 0.46484769917080510749\dots$



D(E) of charged black holes from the SYK model

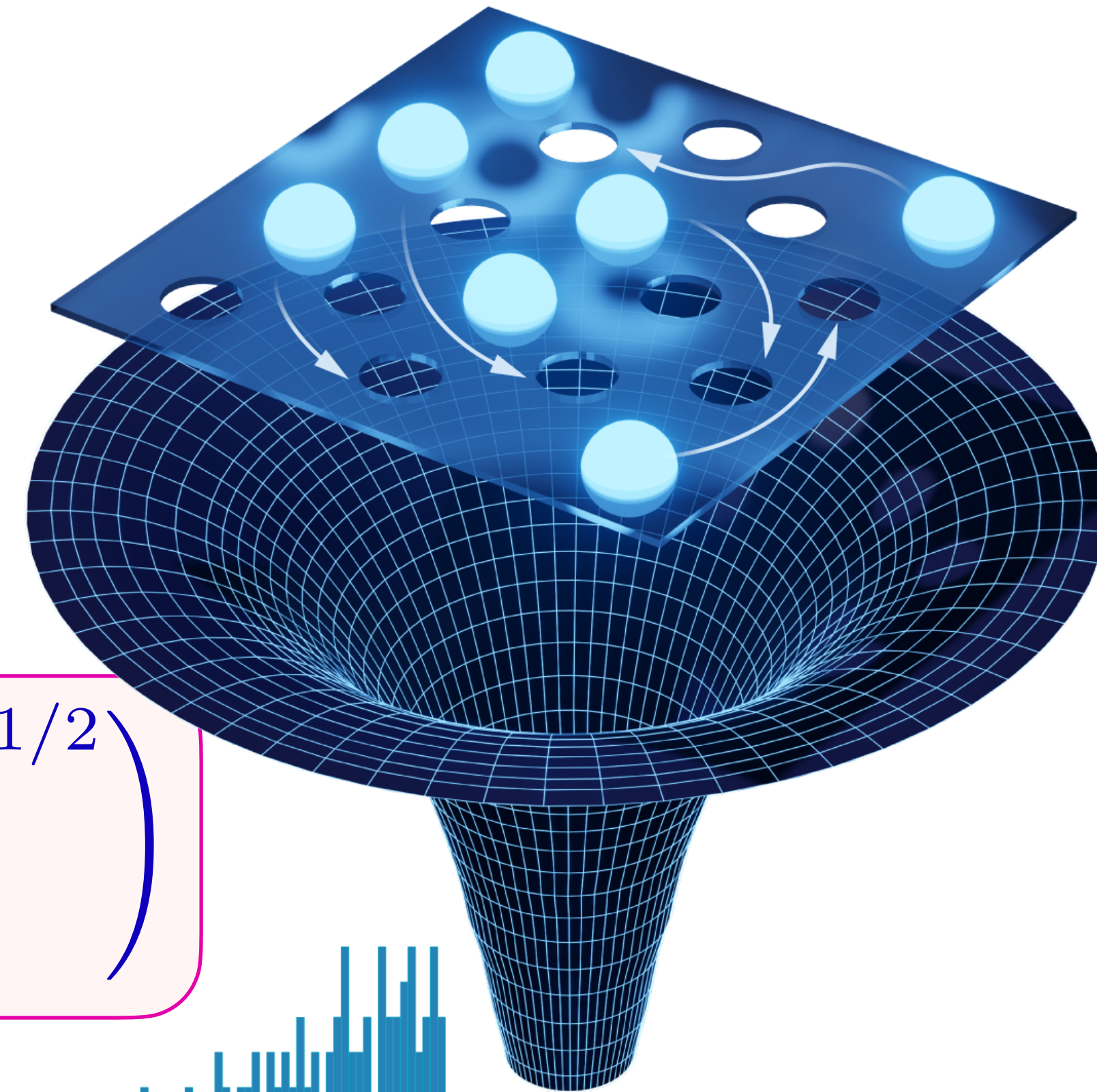
- For generic charged black holes in 3+1 dimensions with horizon area A at $T = 0$ and fixed charge Q ($A = 2GQ^2/c^4$), the density of quantum states at small energy E is

$$D(E) \sim \left(\frac{Ac^3}{\hbar G} \right)^{-347/90} \exp \left(\frac{Ac^3}{4\hbar G} \right) \sinh \left(\left[\frac{\sqrt{\pi} A^{3/2} c^2}{\hbar^2 G} E \right]^{1/2} \right)$$

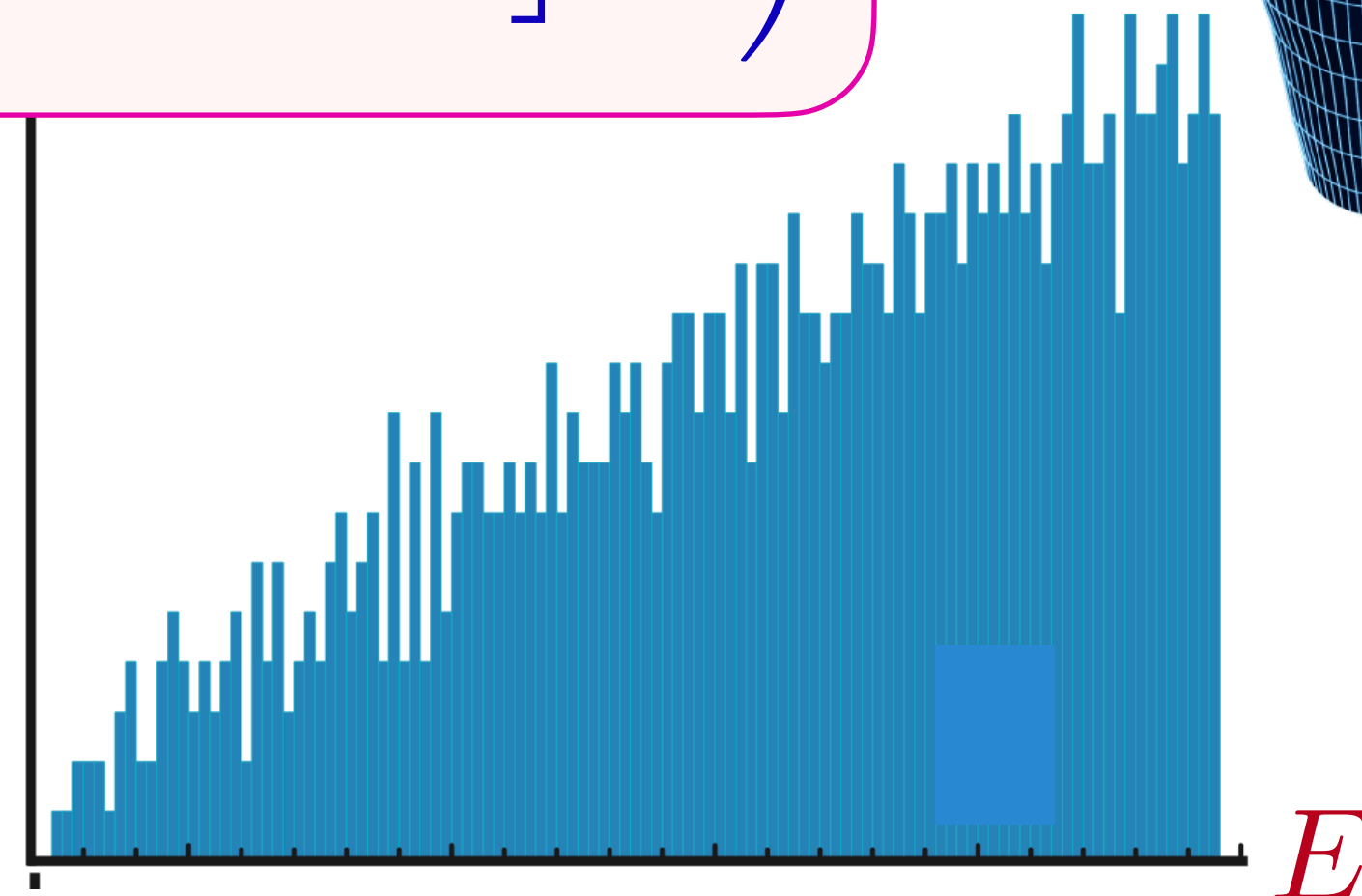
Bekenstein-Hawking

Iliesiu, Murthy, Turiaci (2022)

Developments from the SYK model



$D(E)$



Recap

Great discoveries in physics

Entropy (1870)

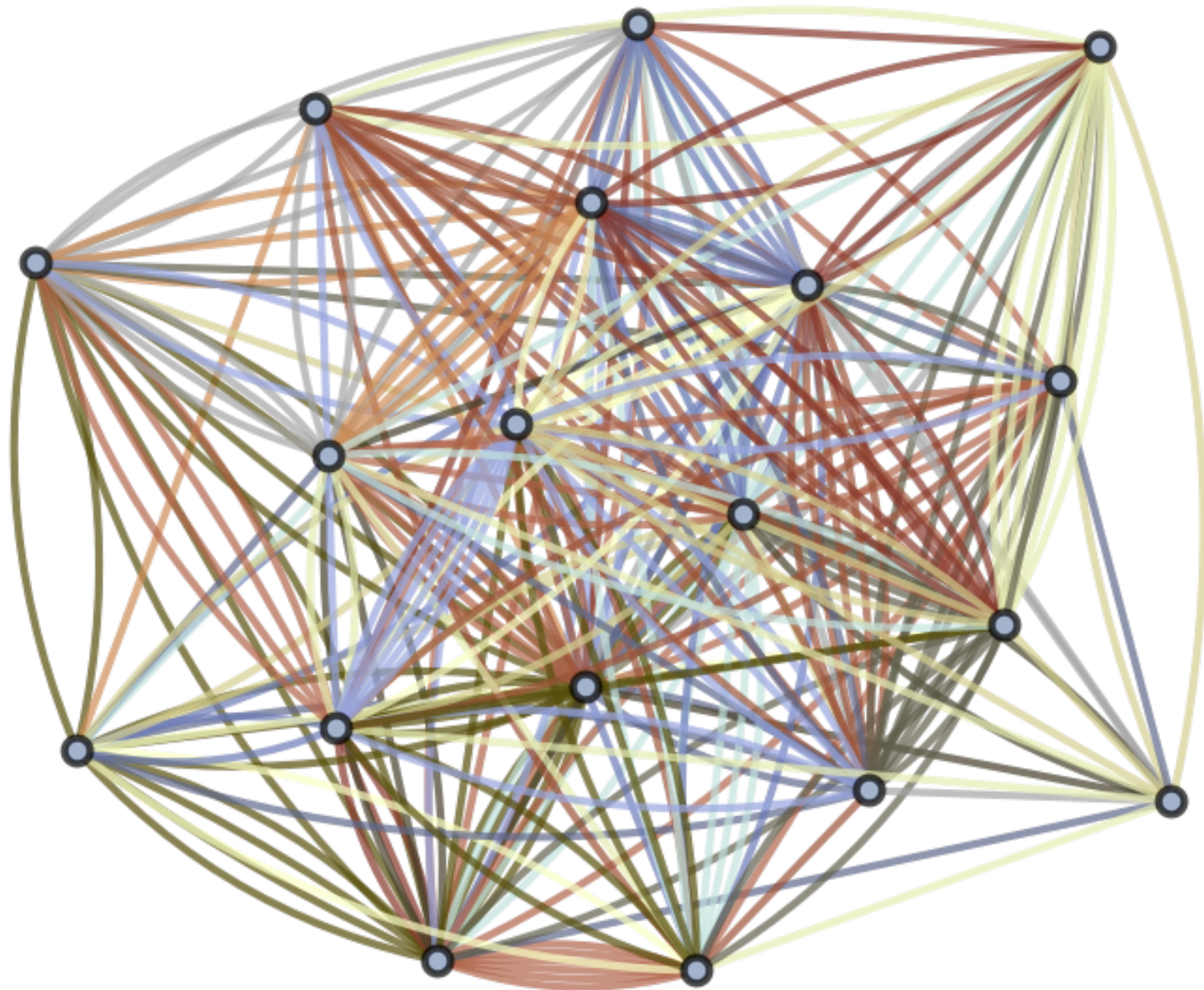
Superconductivity (1911)

Black holes (1916)

Quantum entanglement (1935)

The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

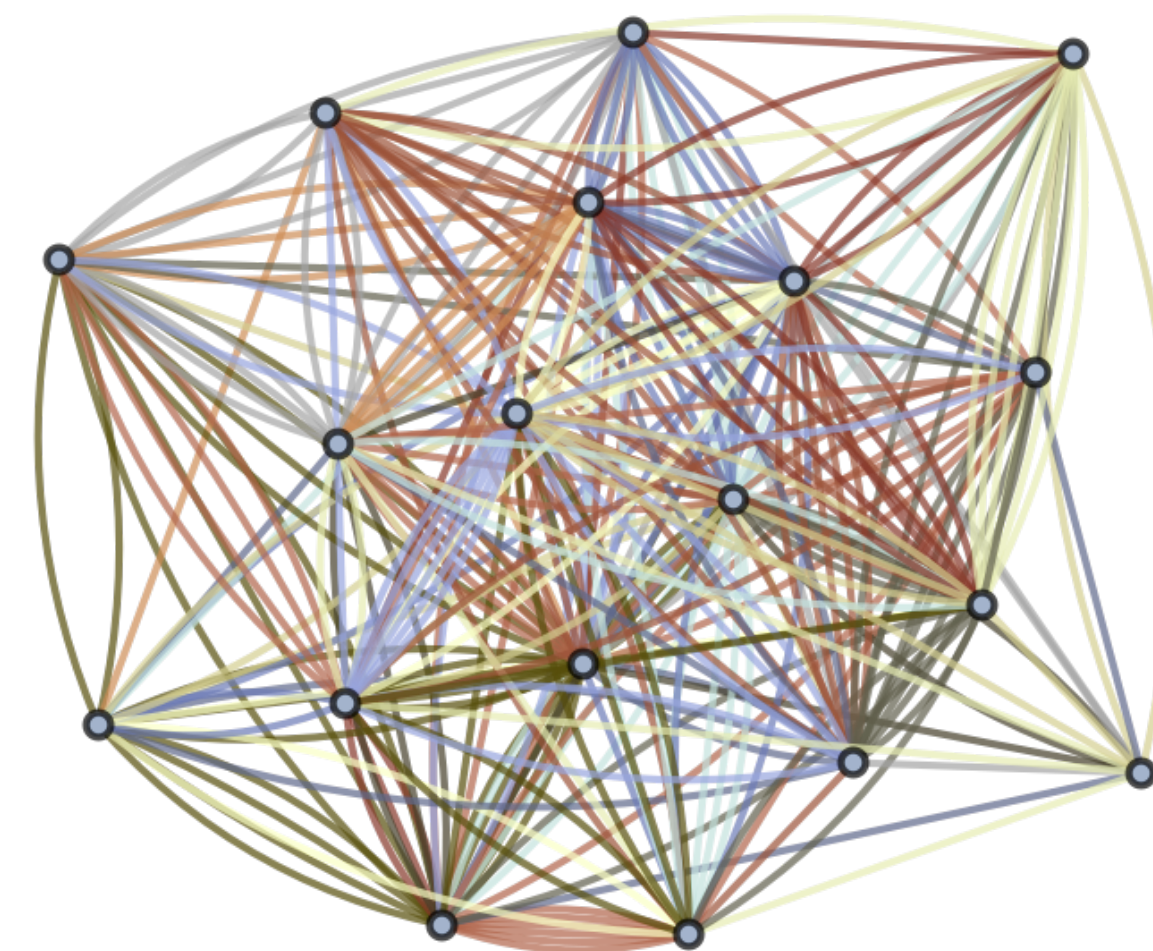
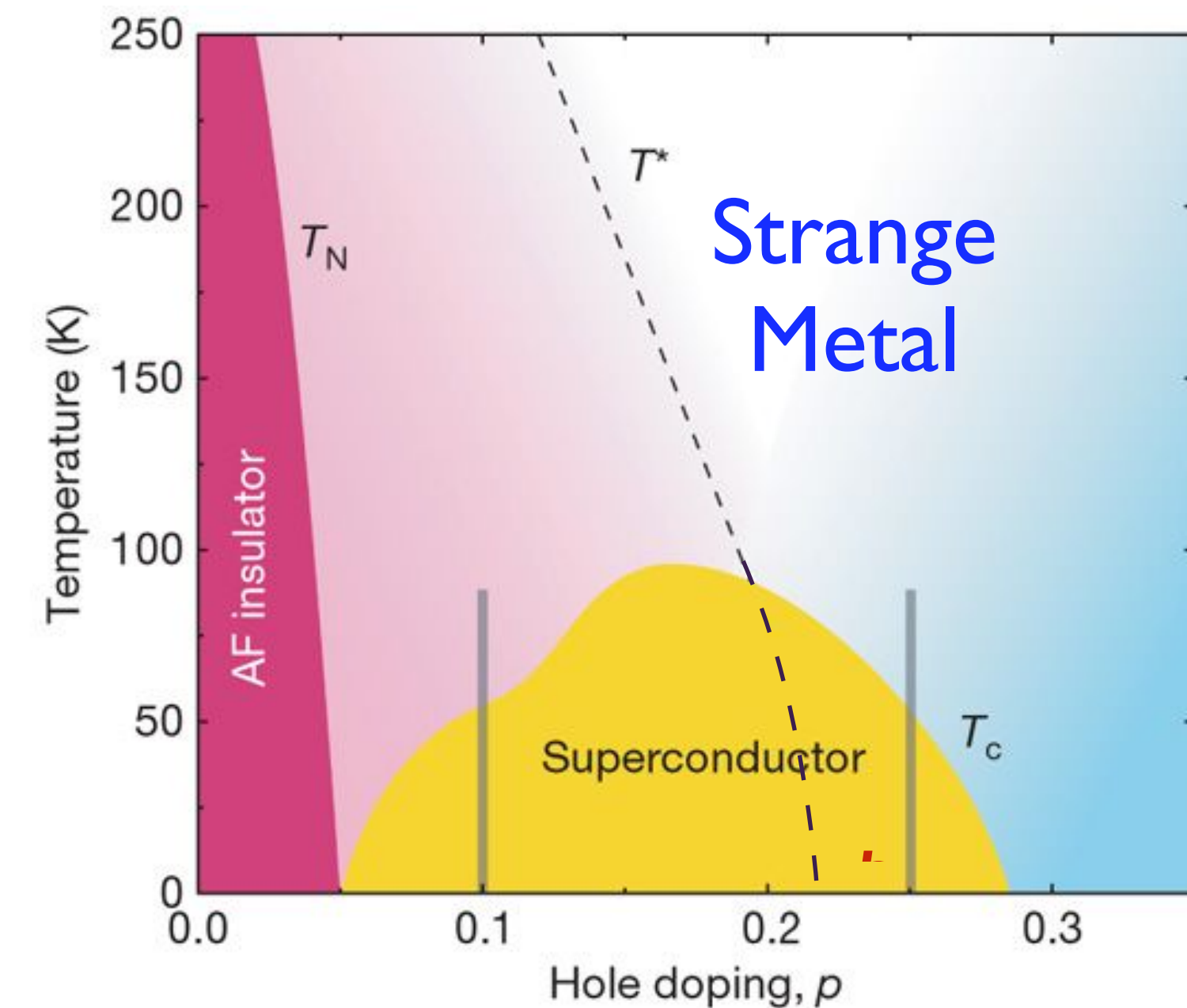


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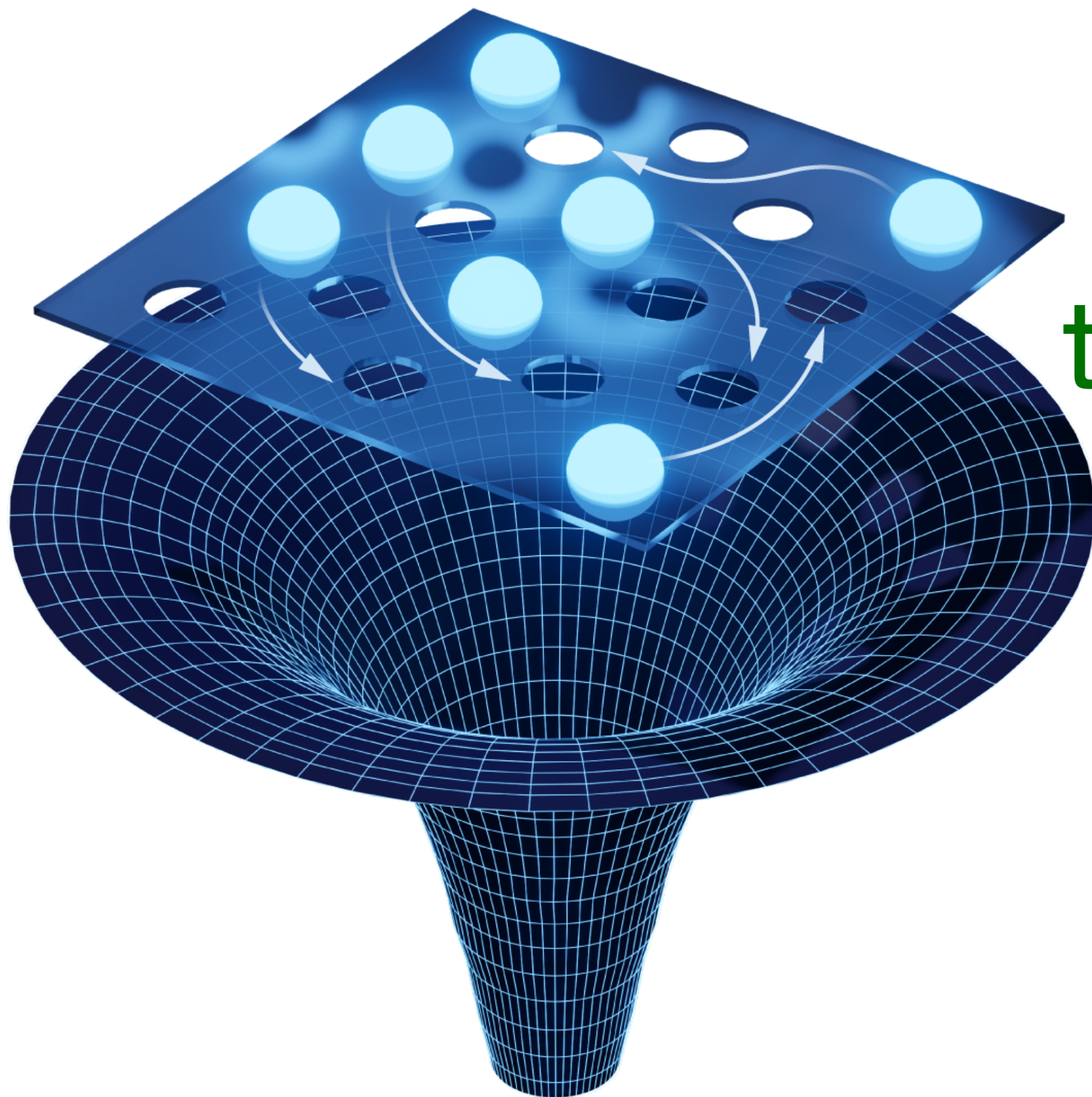
In one set of variables, it helps describe the *strange* electrical properties of YBCO

Sachdev, Ye (1993)



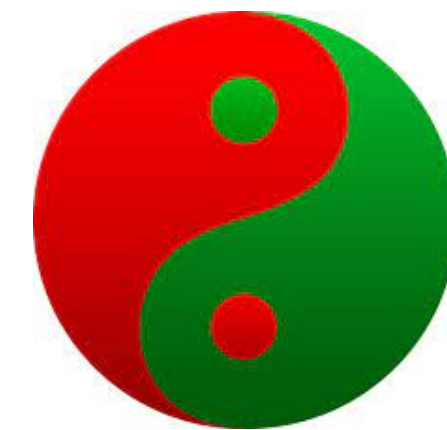
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In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)



In a ***dual*** set of variables it describes the interior of ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)