From the SYK model to a theory of the strange metal

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Talk online: sachdev.physics.harvard.edu

Quantum matter <u>with</u> quasiparticles:

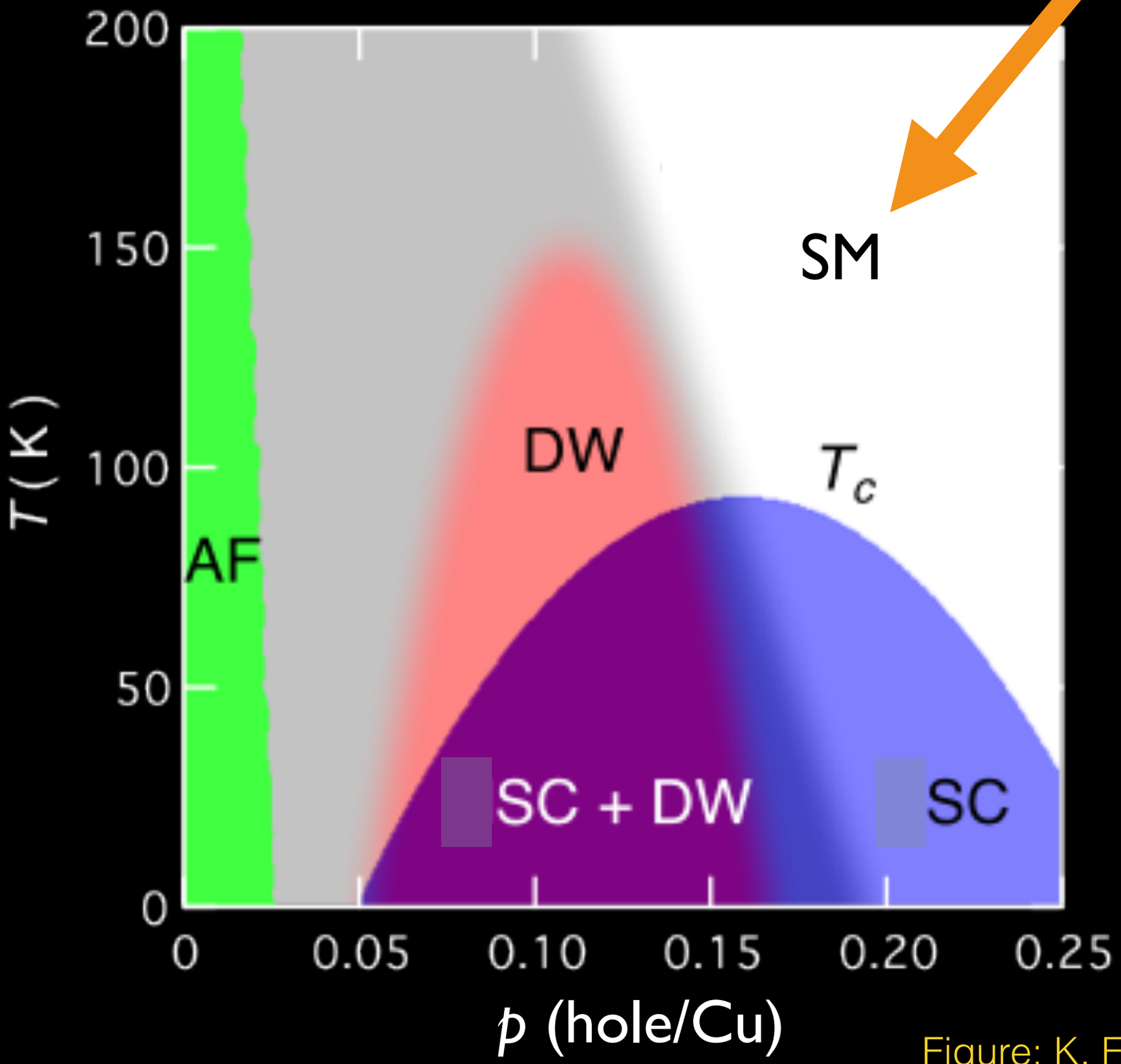
The quasiparticle idea is the key reason for the many successes of quantum condensed matter physics:

- Fermi liquid theory of metals, insulators, semiconductors
- Theory of superconductivity (pairing of quasiparticles)
- Theory of disordered metals and insulators (diffusion and localization of quasiparticles)
- Theory of metals in one dimension (collective modes as quasiparticles)
- Theory of the fractional quantum Hall effect (quasiparticles which are `fractions' of an electron)

Quantum matter <u>without</u> quasiparticles

Strange metal

Entangled electrons lead to "strange" temperature dependence of resistivity and other properties



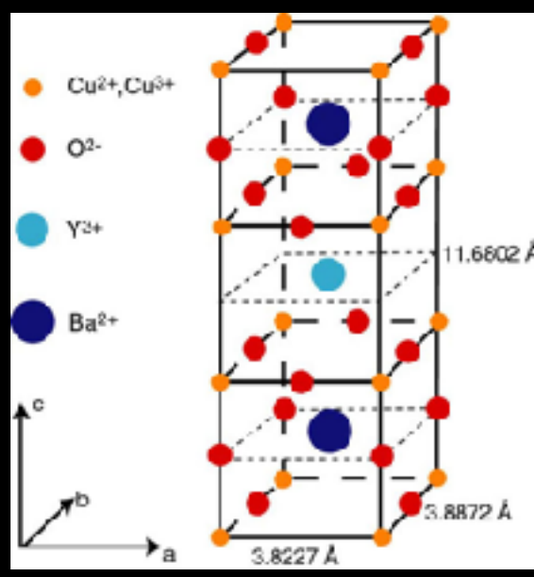


Figure: K. Fujita and J. C. Seamus Davis

"Strange",

"Bad",

or "Incoherent",

metal has a resistivity, ρ, which obeys

$$\rho \sim T,$$

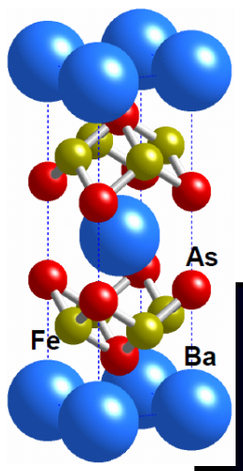
and

$$\rho \gg h/e^2$$

(in two dimensions),
where h/e^2 is the quantum unit of resistance.

Quantum matter <u>without</u> quasiparticles

Resistivity
$\sim \rho_0 + AT^\alpha$



Strange
(or "bad"
or "incoherent")
Metal
$\rho \sim T$
$\rho \gg h/e^2$

$BaFe_2(As_{1-x}P_x)_2$

AF
+nematic

Superconductivity

T (K)
200
100
0

0
0.2
0.4
0.6
x

α
2.0
1.0
0

Superconductivity in Bad Metals

V. J. Emery and S. A. Kivelson
Phys. Rev. Lett. **74**, 3253 – Published 17 April 1995

S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *PRB* **81**, 184519 (2010)

Quantum matter <u>with</u> quasiparticles:

- **Quasiparticles are additive excitations:**
  The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \dots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.

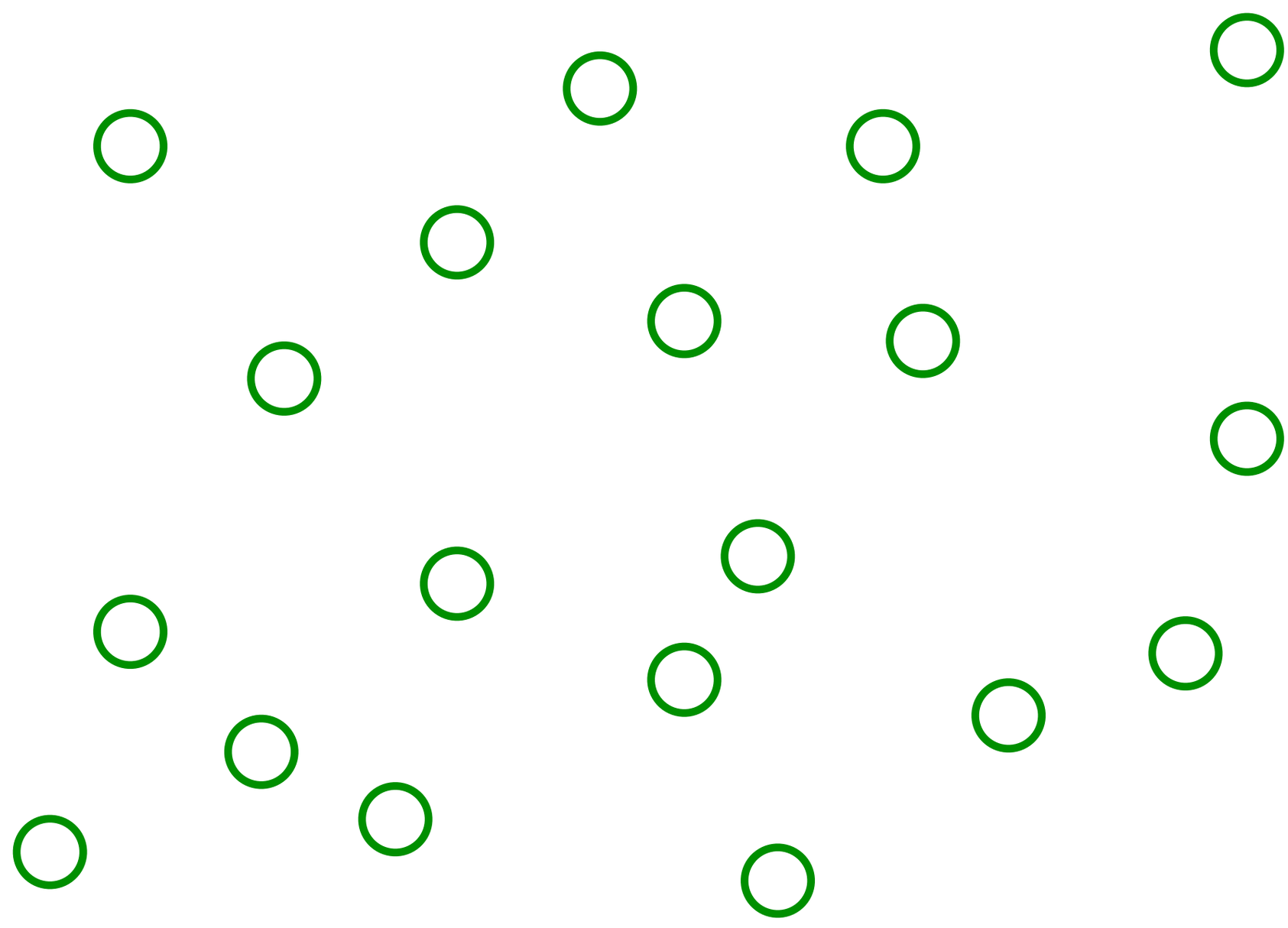
Quantum matter with quasiparticles:

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\rm eq} \sim \frac{\hbar E_F}{(k_B T)^2} \quad , \quad \text{as } T \to 0,$$

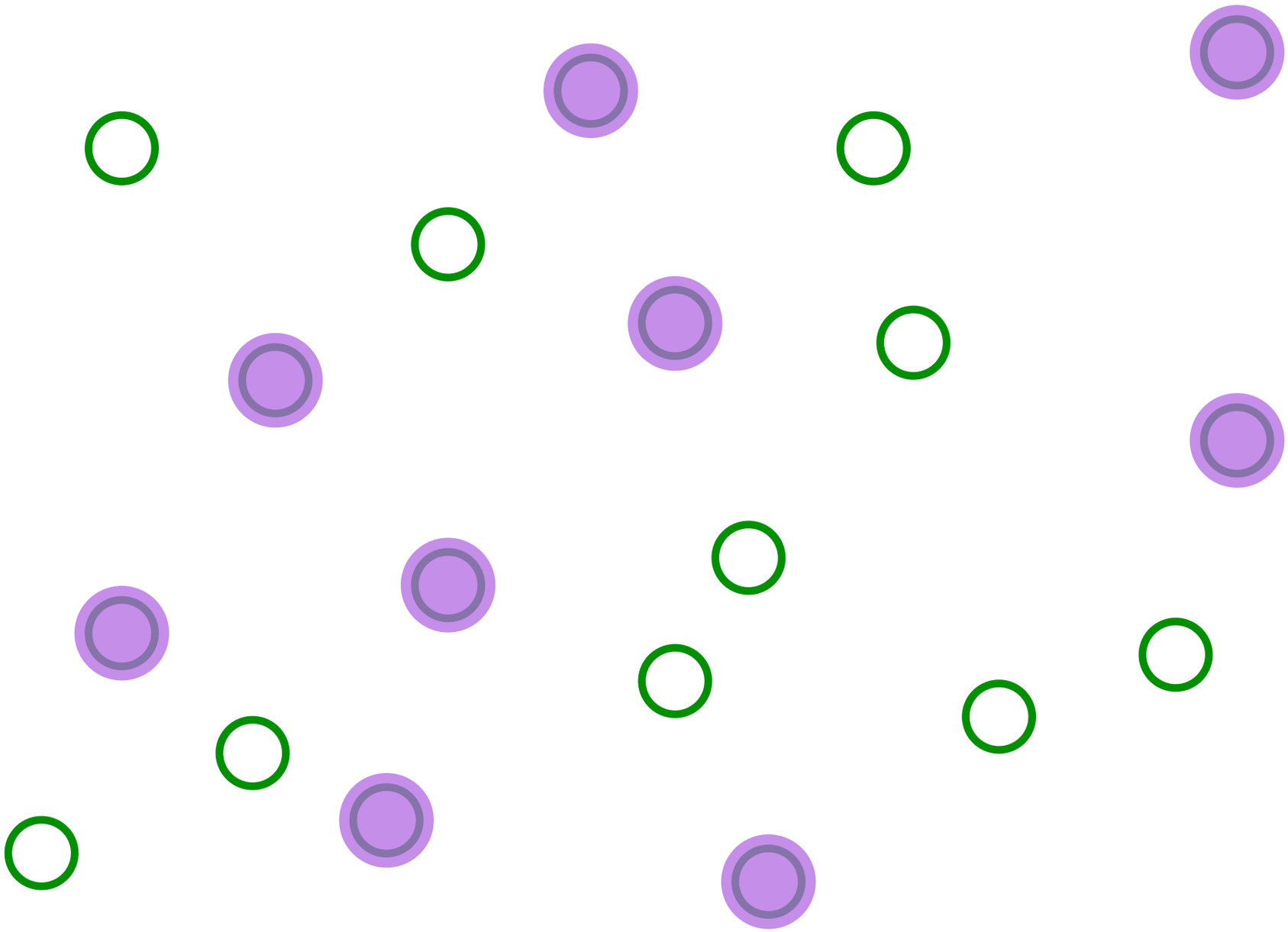
where E_F is the Fermi energy.

A simple model of a metal with quasiparticles



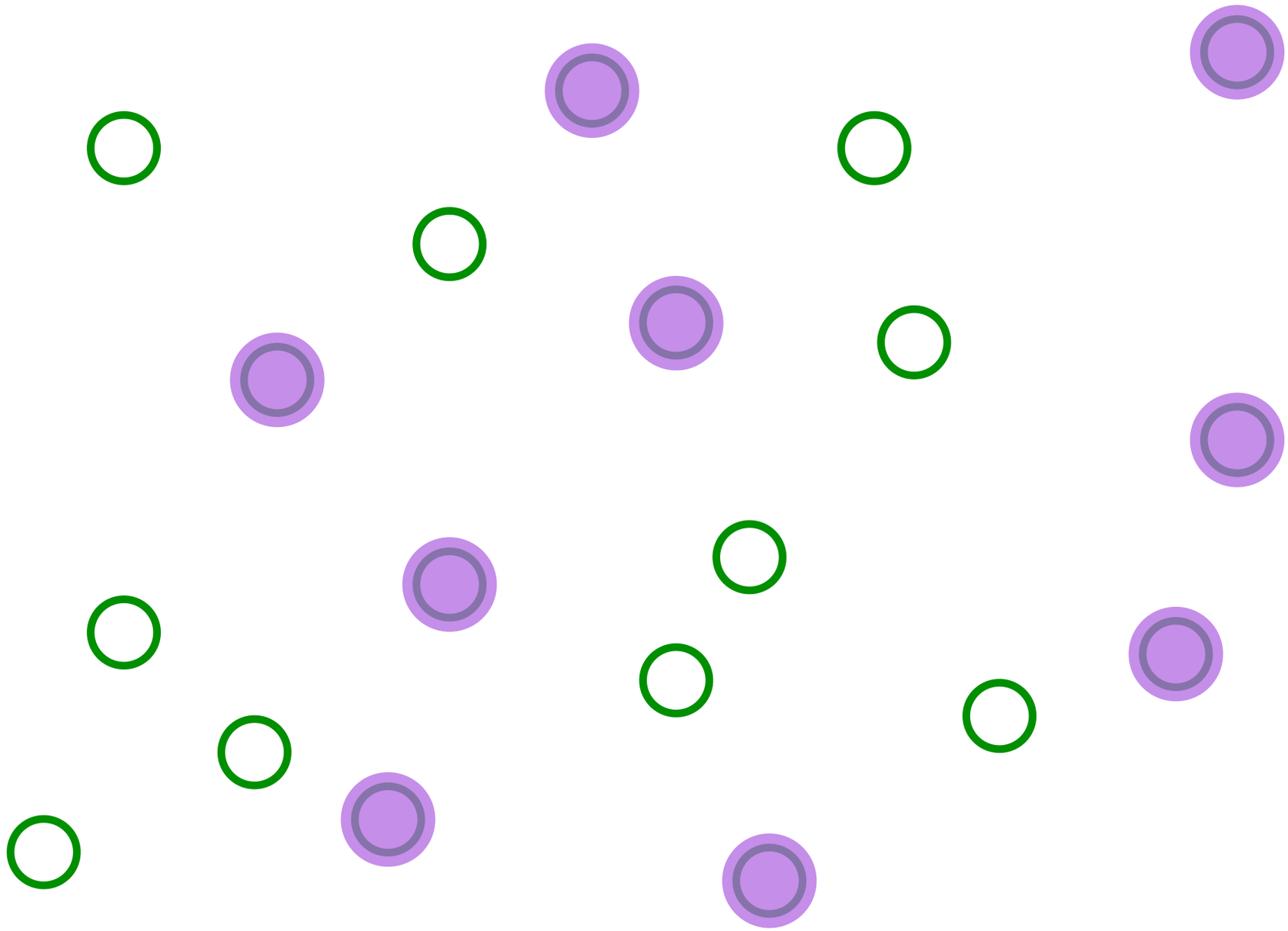
Pick a set of random positions

A simple model of a metal with quasiparticles



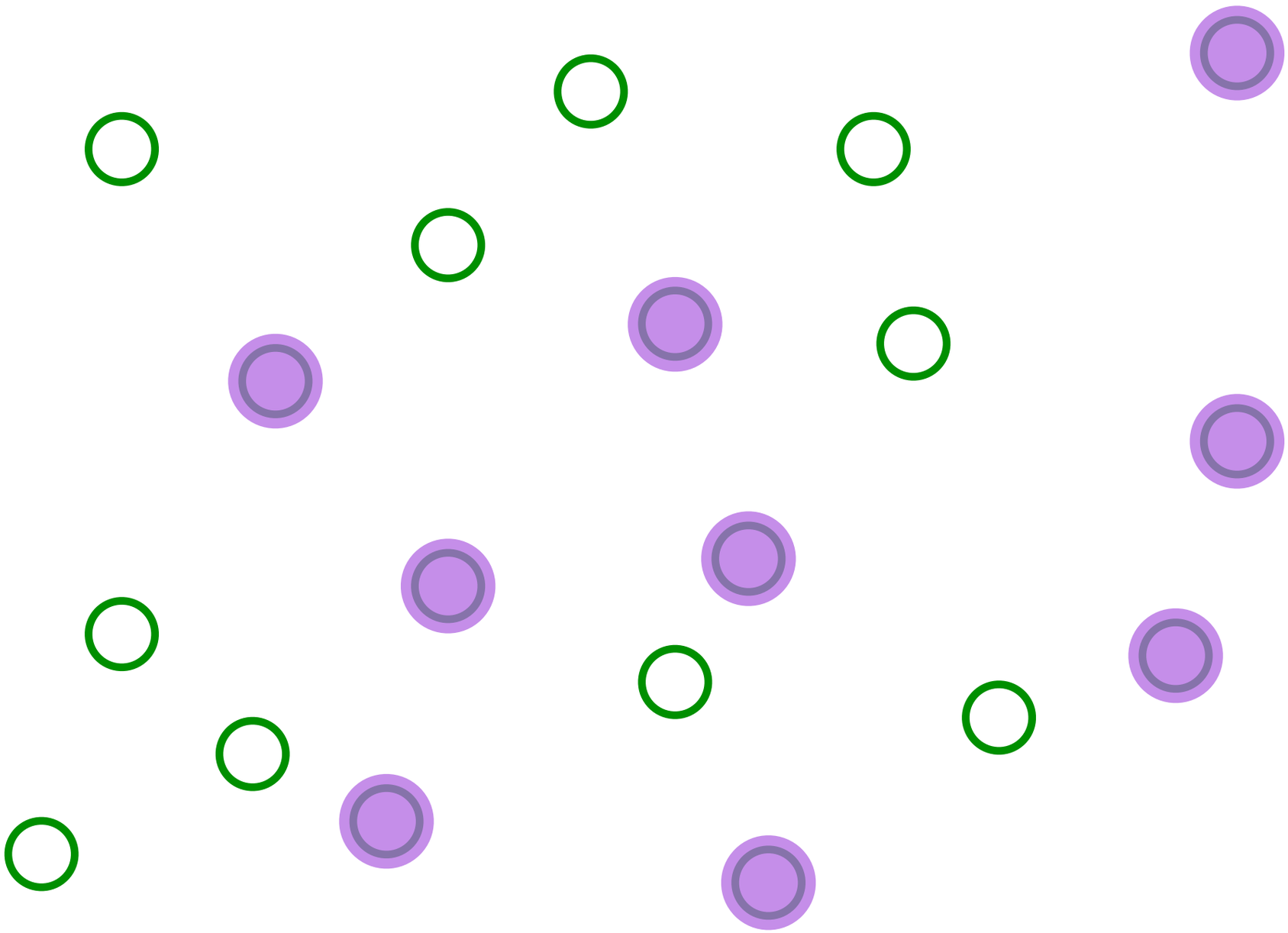
Place electrons randomly on some sites

A simple model of a metal with quasiparticles



Electrons move one-by-one randomly

A simple model of a metal with quasiparticles



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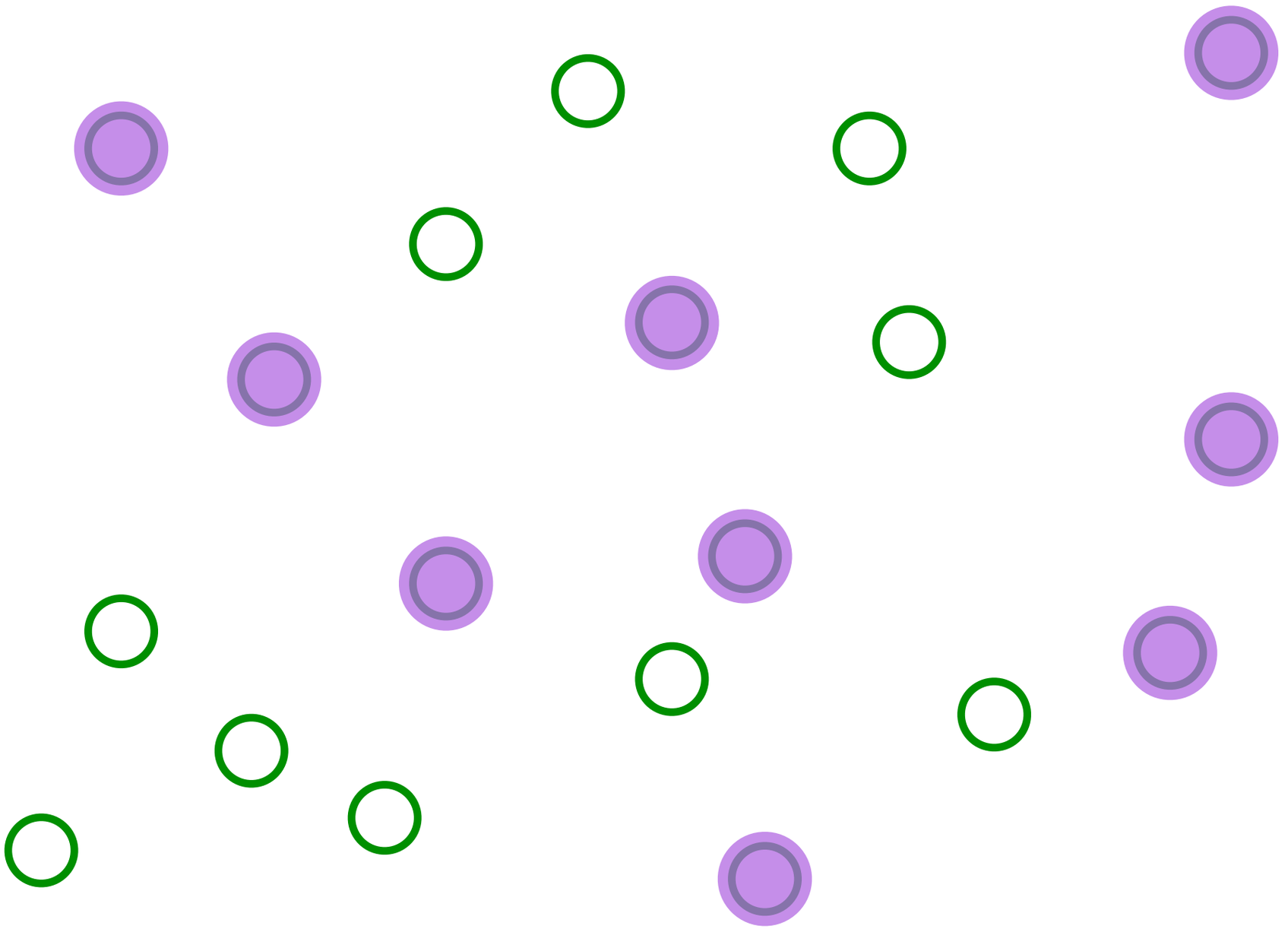

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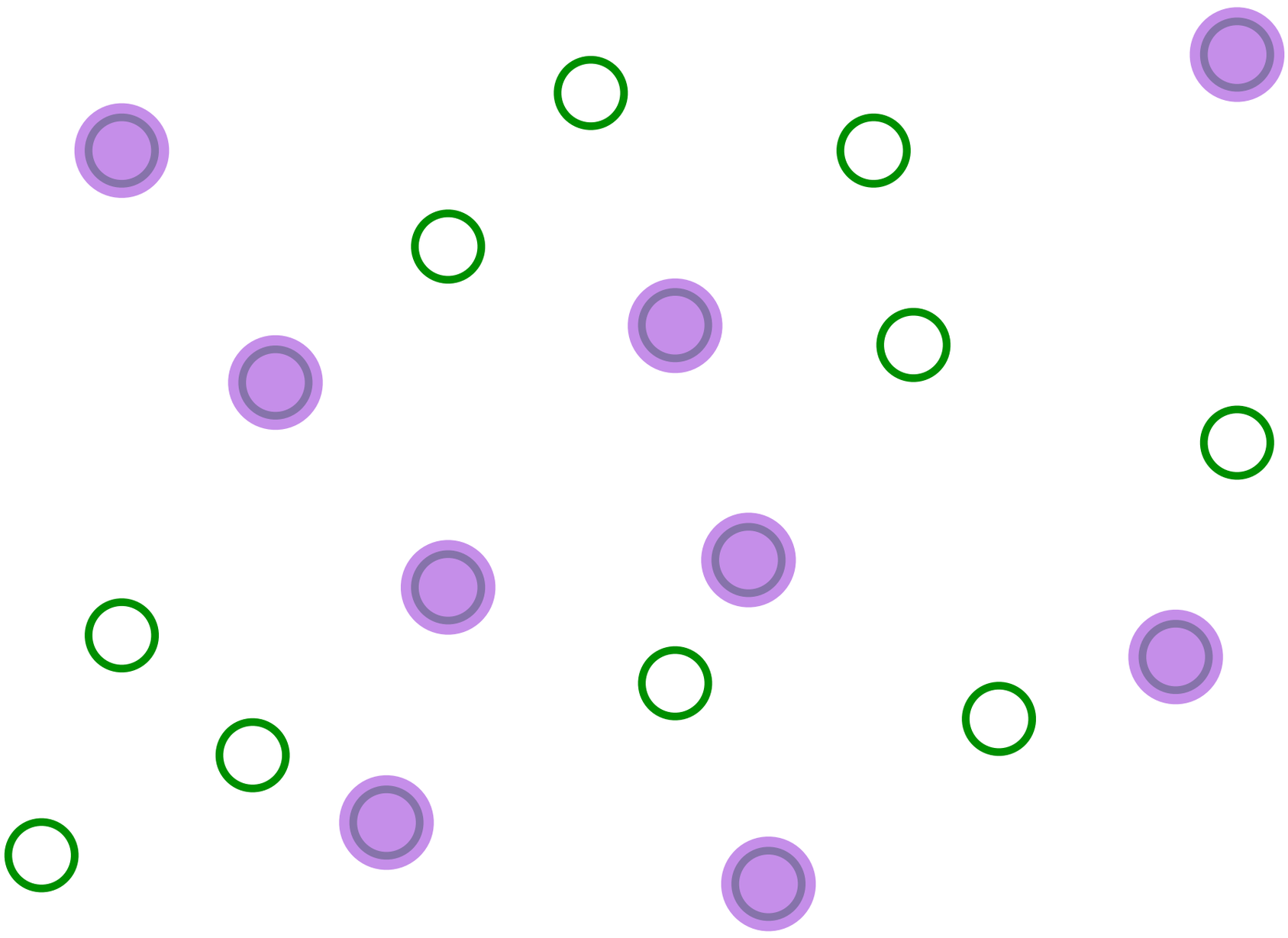


Electrons move one-by-one randomly

A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \dots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

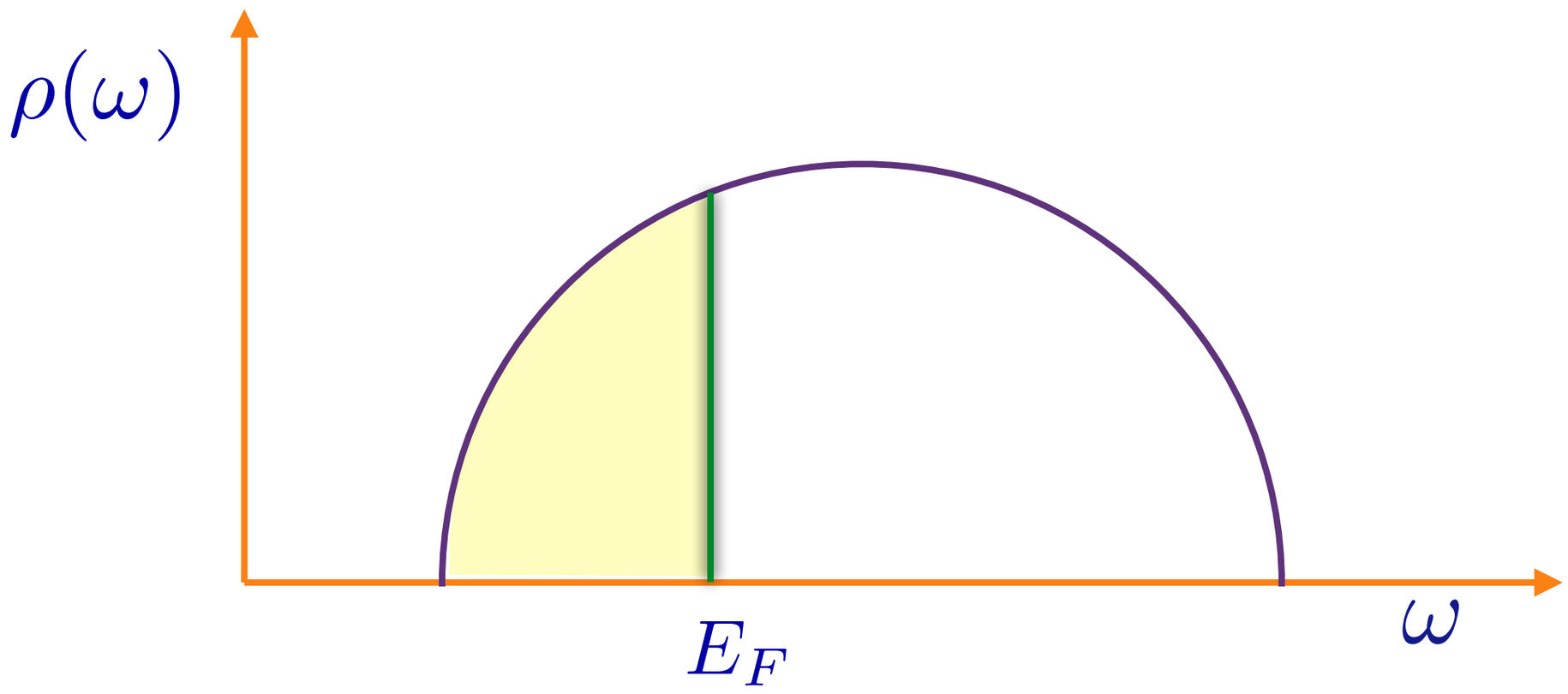
$$\frac{1}{N} \sum_i c_i^\dagger c_i = Q$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

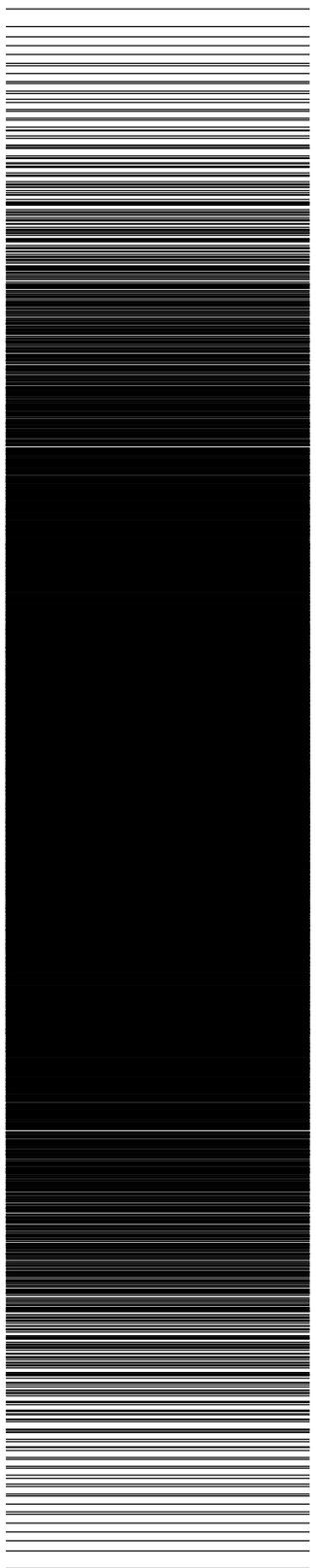
Fermions occupying the eigenstates of a
N x *N* random matrix

A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The density of states is $\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$.



A simple model of a metal with quasiparticles



Many-body level spacing $\sim 2^{-N}$

Quasiparticle excitations with spacing $\sim 1/N$

There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha ,$$

where $n_\alpha = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_α have a level spacing $\sim 1/N$.

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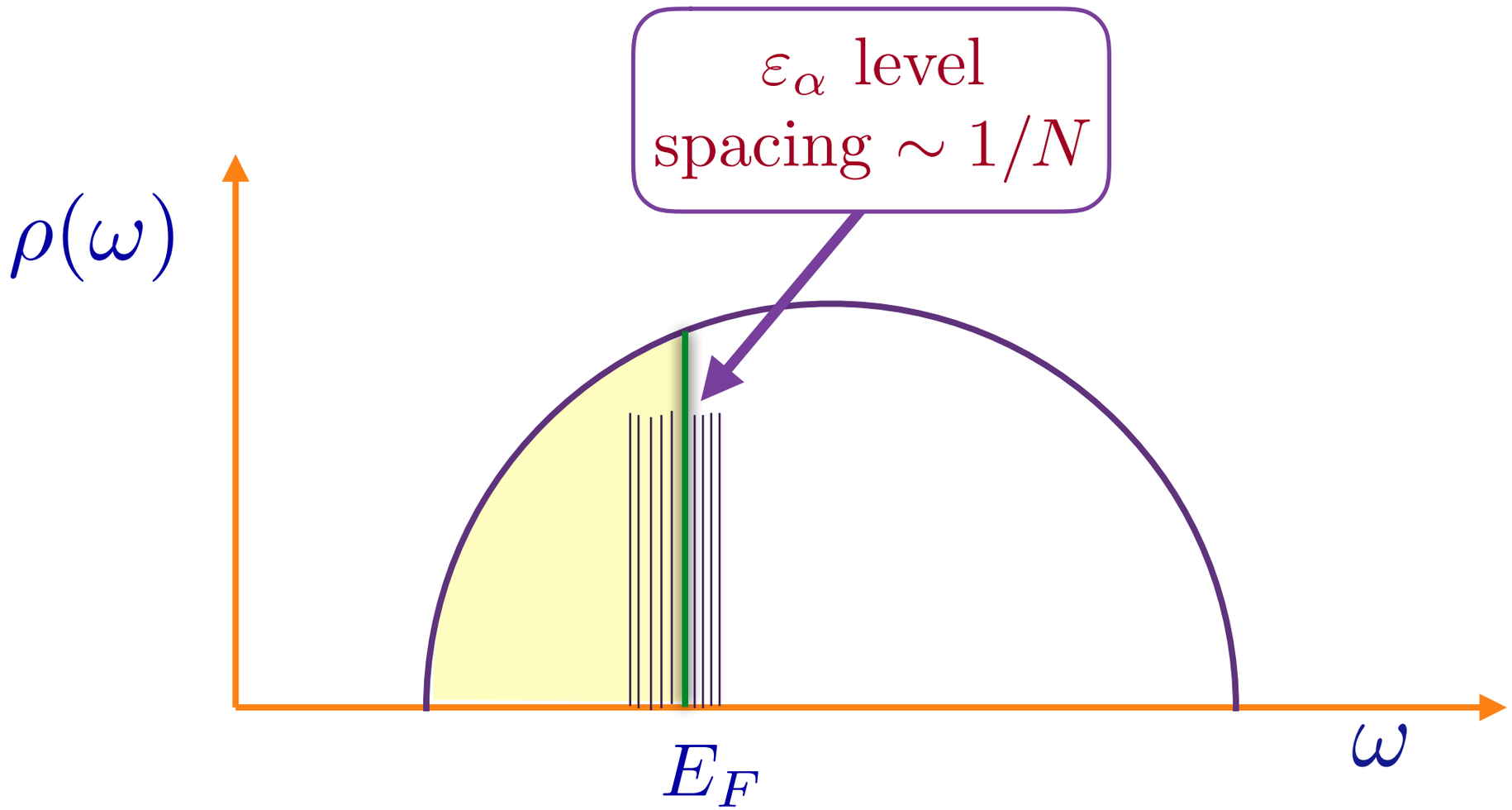
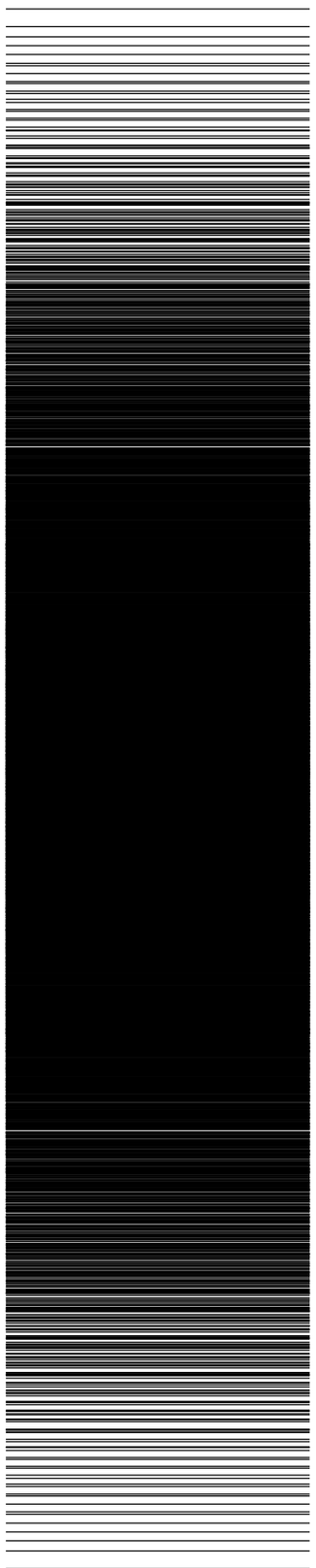


A simple model of a metal with quasiparticles



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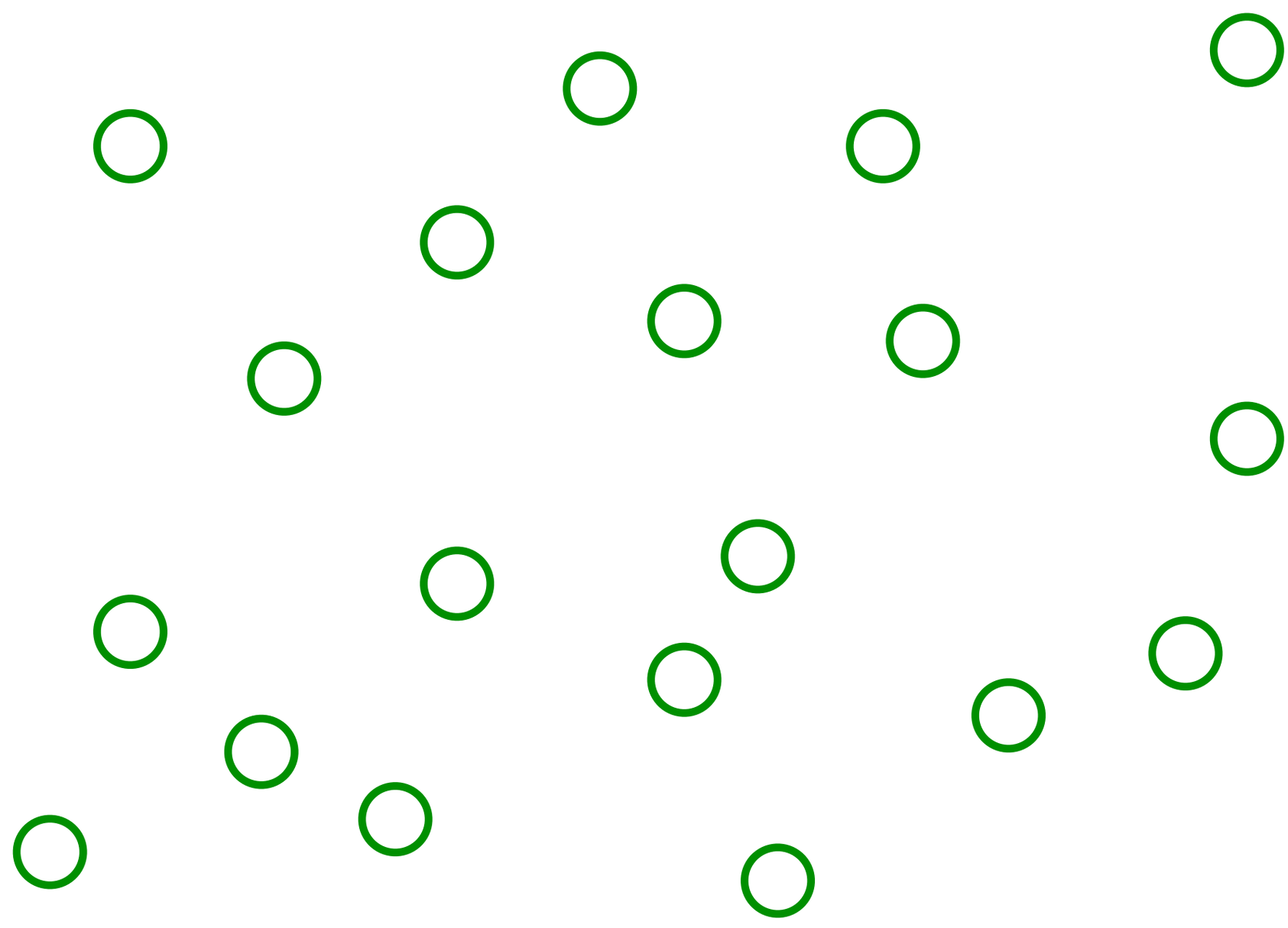
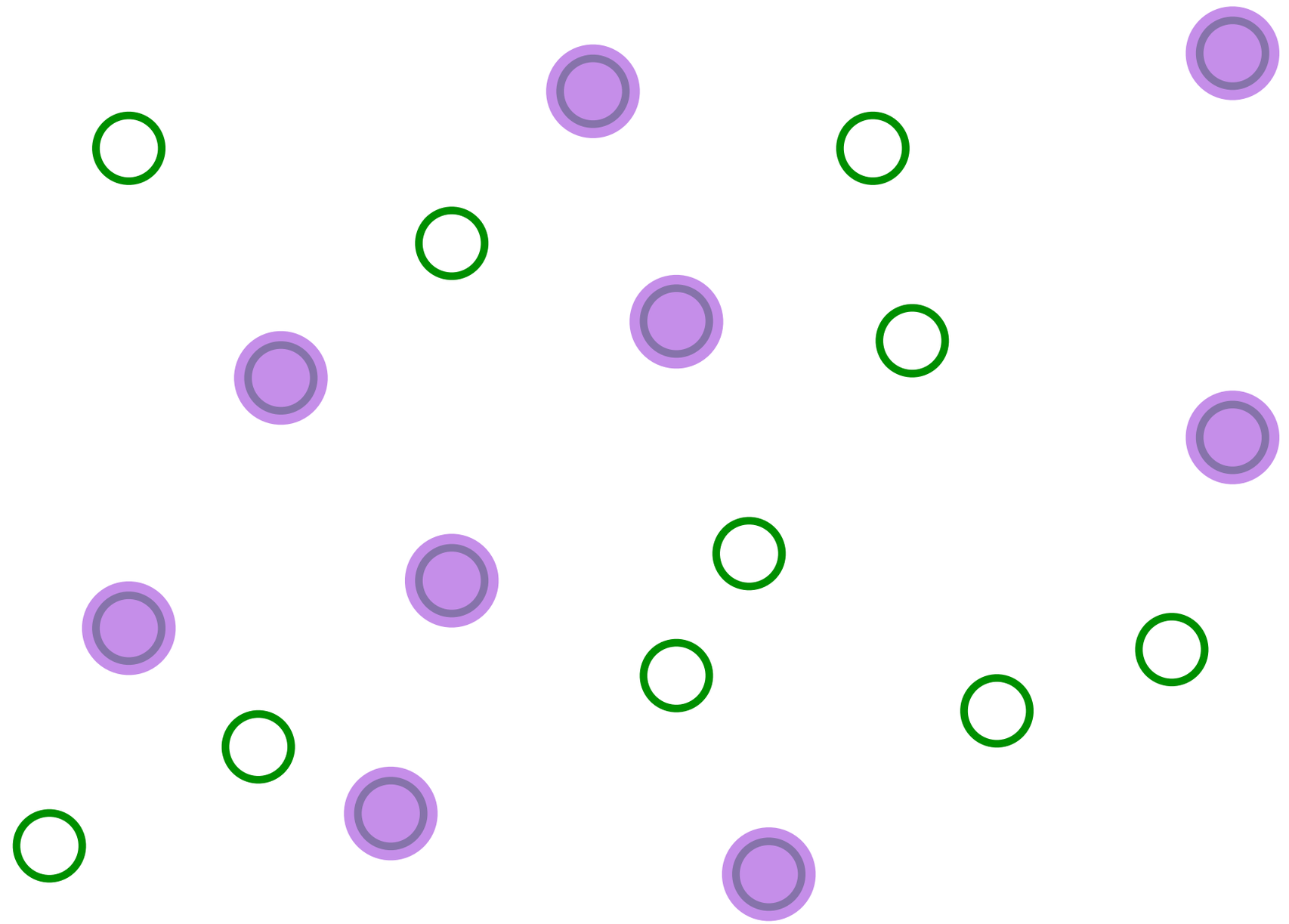
The Sachdev-Ye-Kitaev (SYK) model



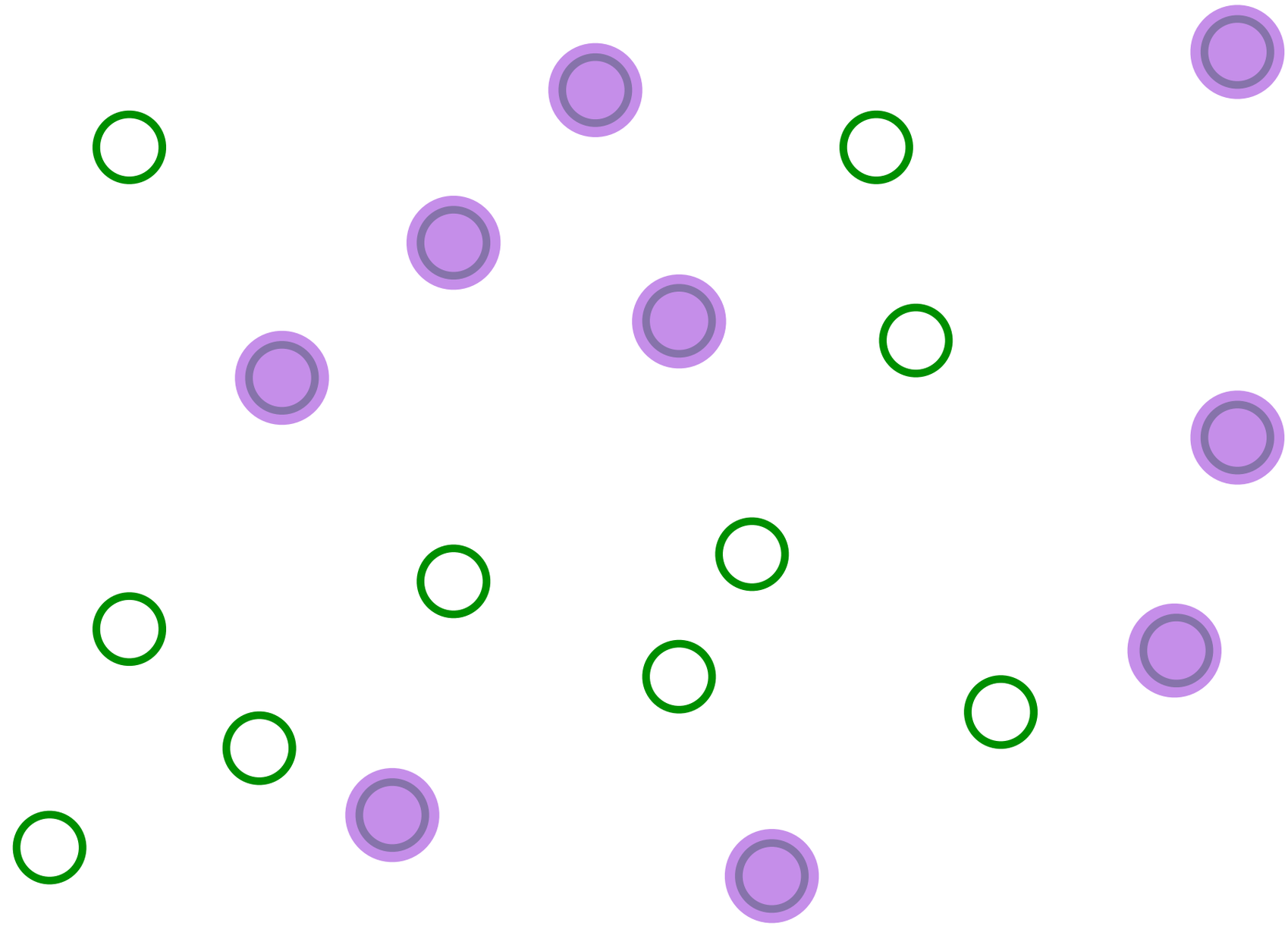
Pick a set of random positions

The SYK model



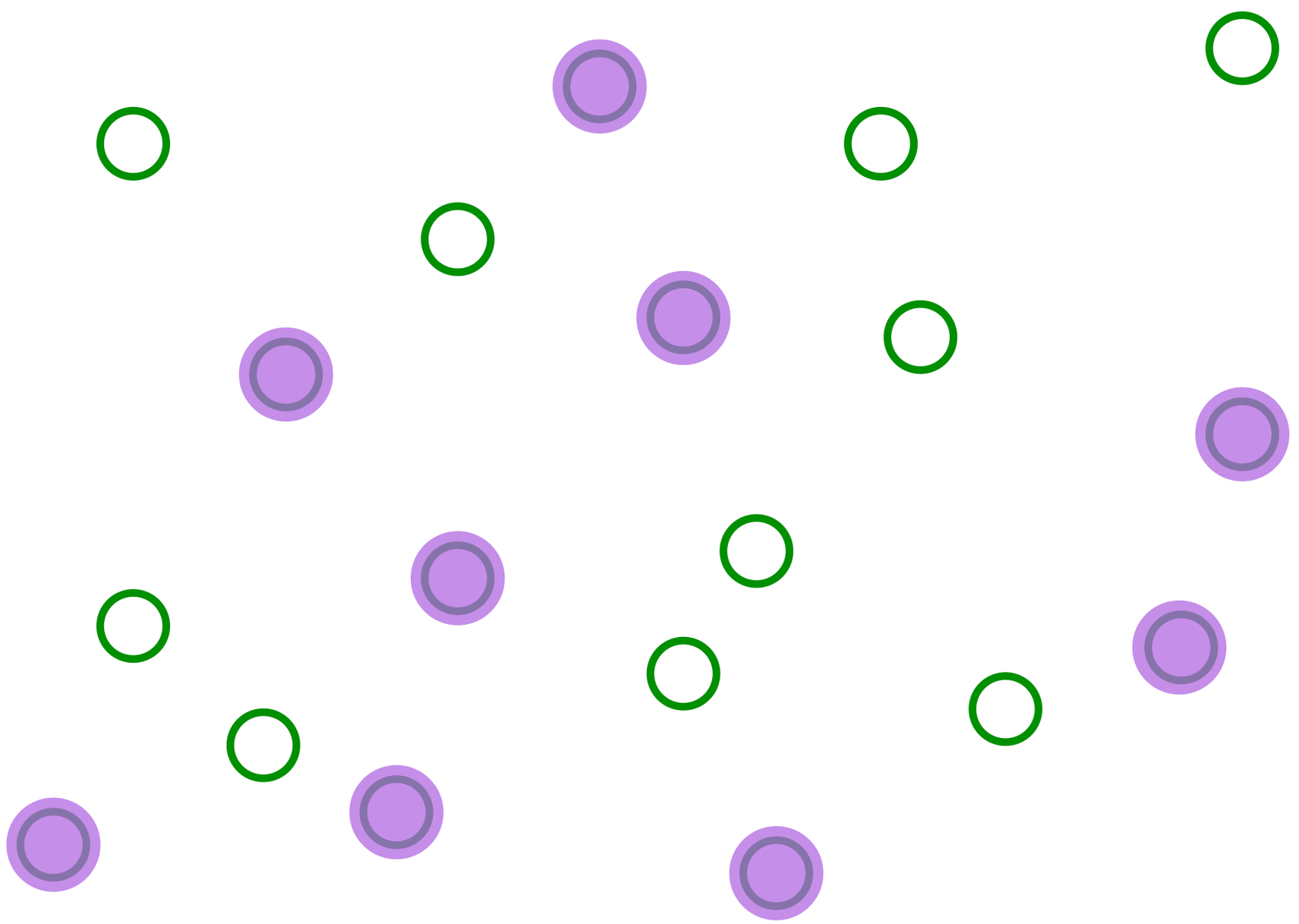
Place electrons randomly on some sites

The SYK model



Entangle electrons pairwise randomly

The SYK model



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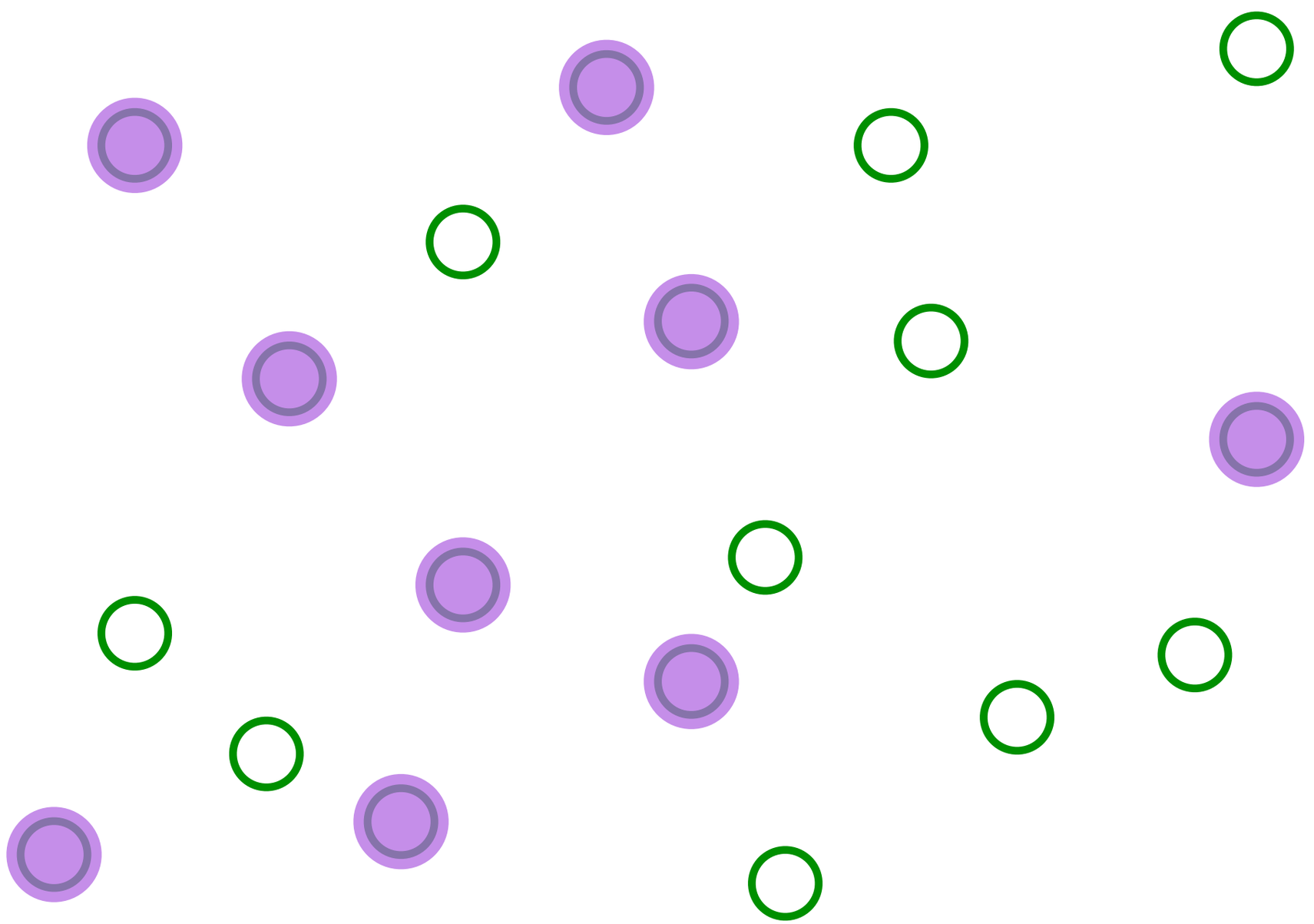

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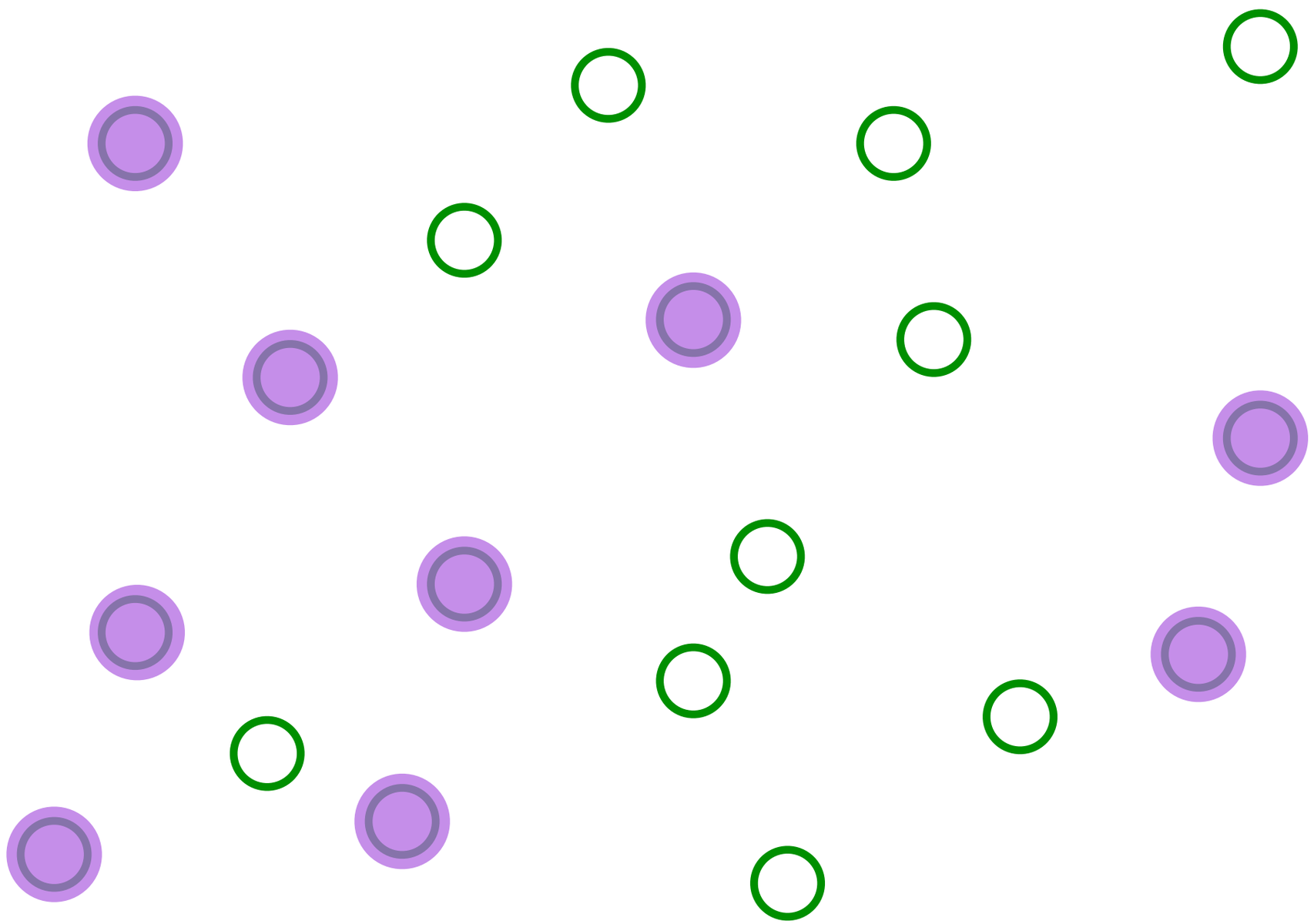

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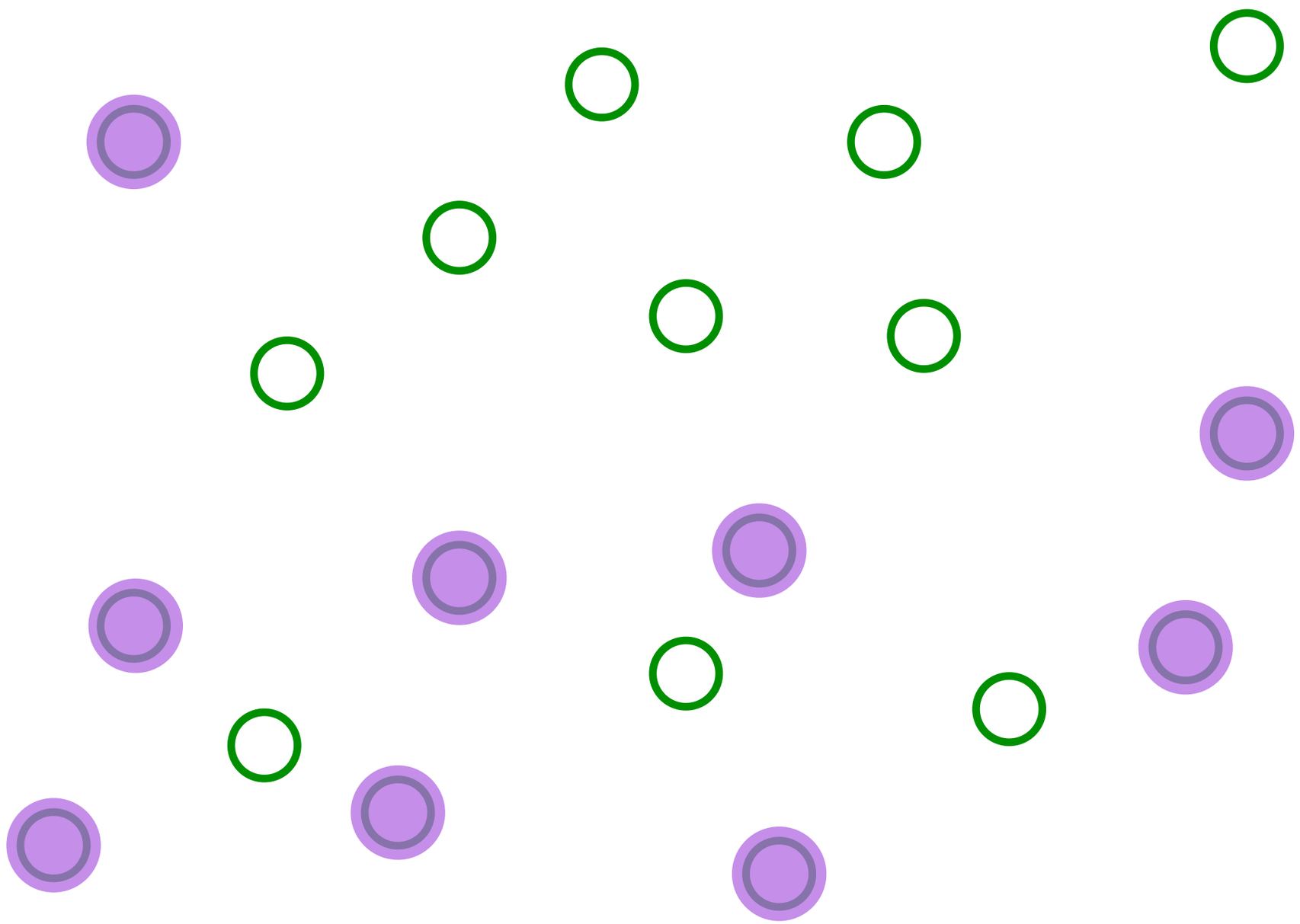

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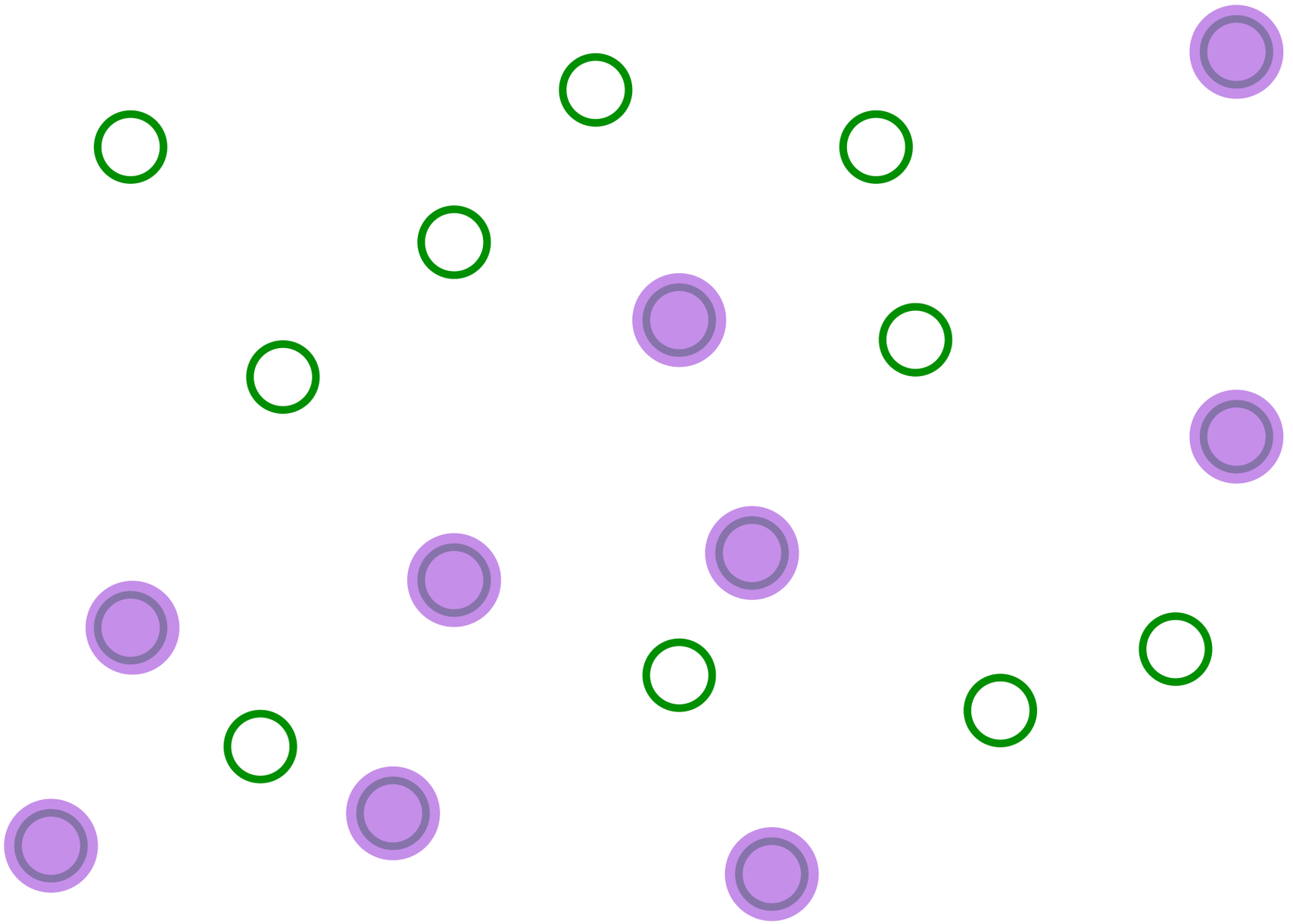

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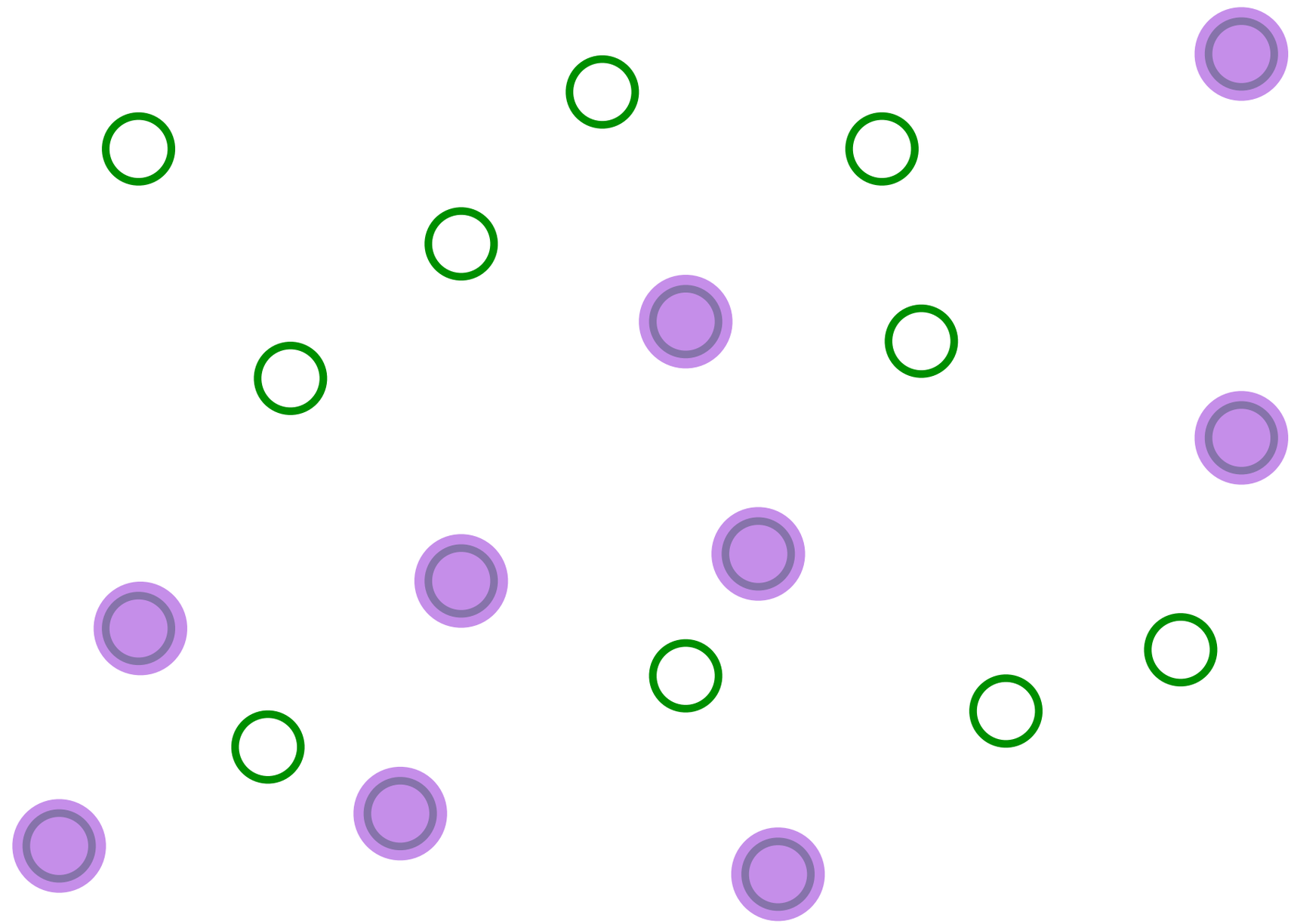


This describes both a strange metal and a black hole!

The SYK model

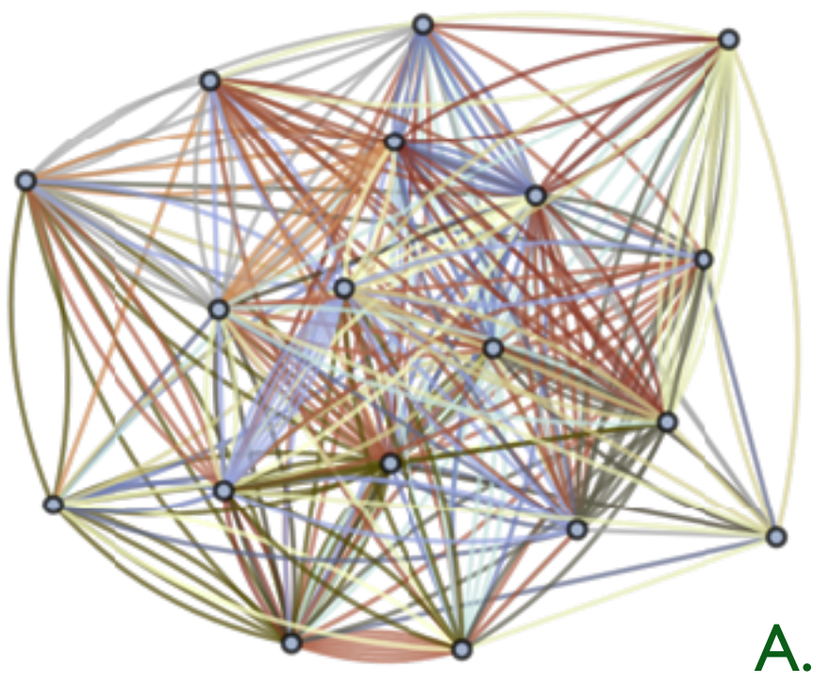
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_i c_i^\dagger c_i$$

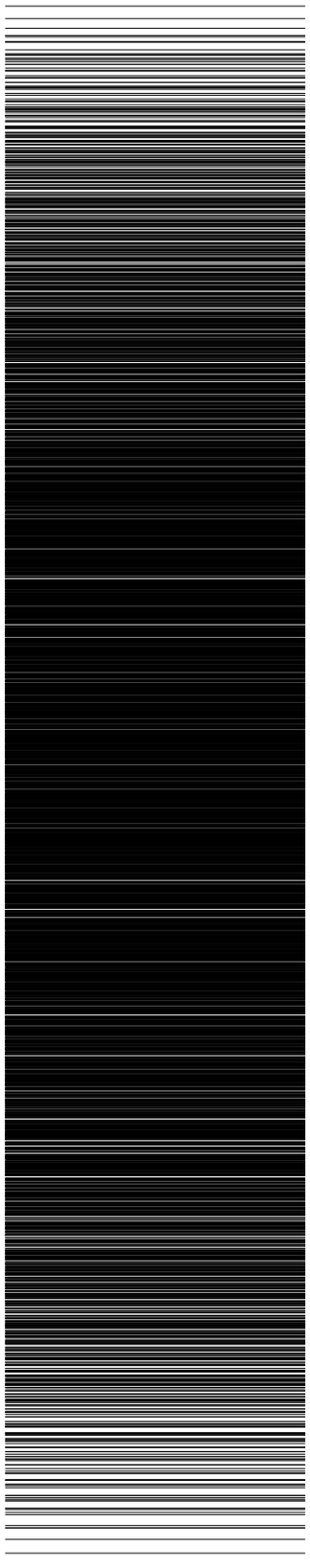
$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The SYK model



Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$ with

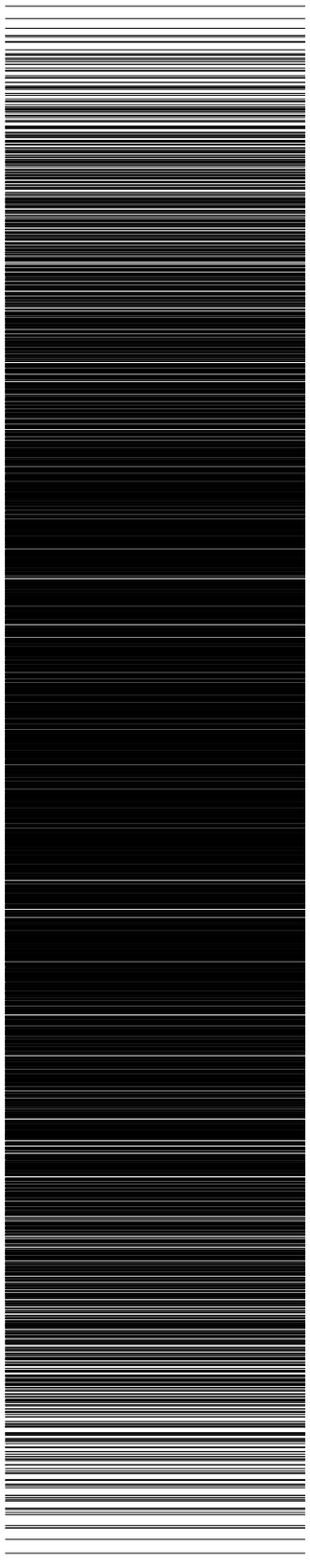
$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots < \ln 2$$

where G is Catalan's constant, for the half-filled case $Q = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The SYK model



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$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

No quasiparticles !

$$E \neq \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

ev,
PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

The SYK model

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$$G(i\omega) = \frac{1}{\cancel{i\omega} + \cancel{\mu} - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \cancel{\mu} - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

At frequencies $\ll J$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \, \frac{g(\sigma_1)}{g(\sigma_2)} \, \widetilde{G}(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \, \frac{g(\sigma_1)}{g(\sigma_2)} \, \widetilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Kitaev, unpublished

S. Sachdev, PRX **5,** 041025 (2015)

The SYK model

Let us write the large N saddle point solutions of S as

$$\begin{aligned} G_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-1/2} \\ \Sigma_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-3/2}. \end{aligned}$$

The saddle point will be invariant under a reperamaterization $f(\tau)$ when choosing $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ leads to a transformed $G(\sigma_1, \sigma_2) = G_s(\sigma_1 - \sigma_2)$ (and similarly for Σ). It turns out this is true only for the SL(2, R) transformations under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken down to SL(2, R) by the saddle point.

A. Kitaev, unpublished

The SYK model

Reparametrization and phase zero modes

We can write the path integral for the SYK model as

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_1)\mathcal{D}\Sigma(\tau_1, \tau_2)e^{-NS[G,\Sigma]}$$

for a known action $S[G, \Sigma]$. We find the saddle point, G_s, Σ_s, and only focus on the "Nambu-Goldstone" modes associated with breaking reparameterization and U(1) gauge symmetries by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2))e^{i\phi(\tau_1)-i\phi(\tau_2)}$$

(and similarly for Σ). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)e^{-NS_{\text{eff}}[f,\phi]}.$$

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5,** 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

The SYK model

$$\mathcal{Z} = \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau) e^{-NS_{\rm eff}[f,\phi]}.$$

Symmetry arguments, and explicit computations, show that the effective action is

$$S_{\rm eff}[f,\phi] = \frac{K}{2}\int_0^{1/T} d\tau(\partial_\tau\phi + i(2\pi\mathcal{E}T)\partial_\tau\epsilon)^2 - \frac{\gamma}{4\pi^2}\int_0^{1/T} d\tau\,\{\tan(\pi T(\tau+\epsilon(\tau)),\tau\},$$

where $f(\tau) \equiv \tau + \epsilon(\tau)$, the couplings K, γ, and $\mathcal{E}$ can be related to thermodynamic derivatives and we have used the Schwarzian:

$$\{g,\tau\} \equiv \frac{g'''}{g'} - \frac{3}{2}\left(\frac{g''}{g'}\right)^2.$$

Specifically, an argument constraining the effective at $T = 0$ is

$$S_{\rm eff}\left[f(\tau) = \frac{a\tau+b}{c\tau+d}, \phi(\tau) = 0\right] = 0,$$

and this is origin of the Schwarzian.

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5,** 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

The SYK model

- Low energy, many-body density of states
$\rho(E) \sim e^{N s_0} \sinh(\sqrt{2(E-E_0)N\gamma})$ (for Majorana model)

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)
D. Stanford and E. Witten, 1703.04612
A. M. Garica-Garcia, J.J.M. Verbaarschot, 1701.06593
D. Bagrets, A. Altland, and A. Kamenev, 1607.00694

The SYK model

- Low energy, many-body density of states
  $\rho(E) \sim e^{N s_0} \sinh(\sqrt{2(E-E_0)N\gamma})$

- Low temperature entropy $S = N s_0 + N\gamma T + \ldots$.

A. Kitaev, unpublished
J. Maldacena and D. Stanford, 1604.07818

The SYK model

- Low energy, many-body density of states
  $\rho(E) \sim e^{N s_0} \sinh(\sqrt{2(E - E_0)N\gamma})$

- Low temperature entropy $S = N s_0 + N\gamma T + \ldots$.

- $T = 0$ fermion Green's function is incoherent: $G(\tau) \sim \tau^{-1/2}$ at large τ. (Fermi liquids with quasiparticles have the coherent: $G(\tau) \sim 1/\tau$)

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

The SYK model

- Low energy, many-body density of states
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A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, arXiv:1706.07803

Quantum matter without quasiparticles:

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Quantum Phase Transitions,
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- Systems without quasiparticles are the fastest possible in reaching local equilibrium, and all many-body quantum systems obey, as $T \to 0$

$$\tau_{\rm eq} > C \frac{\hbar}{k_B T} \, .$$

S. Sachdev,
Quantum Phase Transitions,
Cambridge (1999)

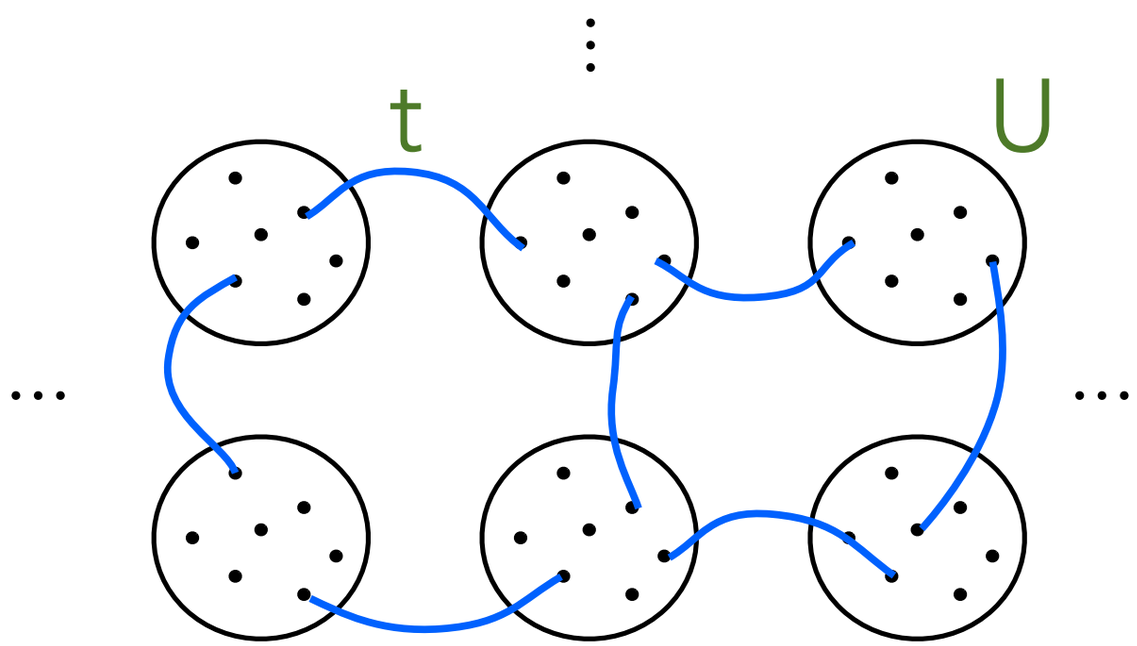
  - In Fermi liquids $\tau_{\rm eq} \sim 1/T^2$, and so the bound is obeyed as $T \to 0$.
  - This bound rules out quantum systems with *e.g.* $\tau_{\rm eq} \sim \hbar/(J k_B T)^{1/2}$.
  - There is no bound in classical mechanics ($\hbar \to 0$). By cranking up frequencies, we can attain equilibrium as quickly as we desire.

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

arXiv:1705.00117

Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models

Authors: Xue-Yang Song, Chao-Ming Jian, Leon Balents



$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

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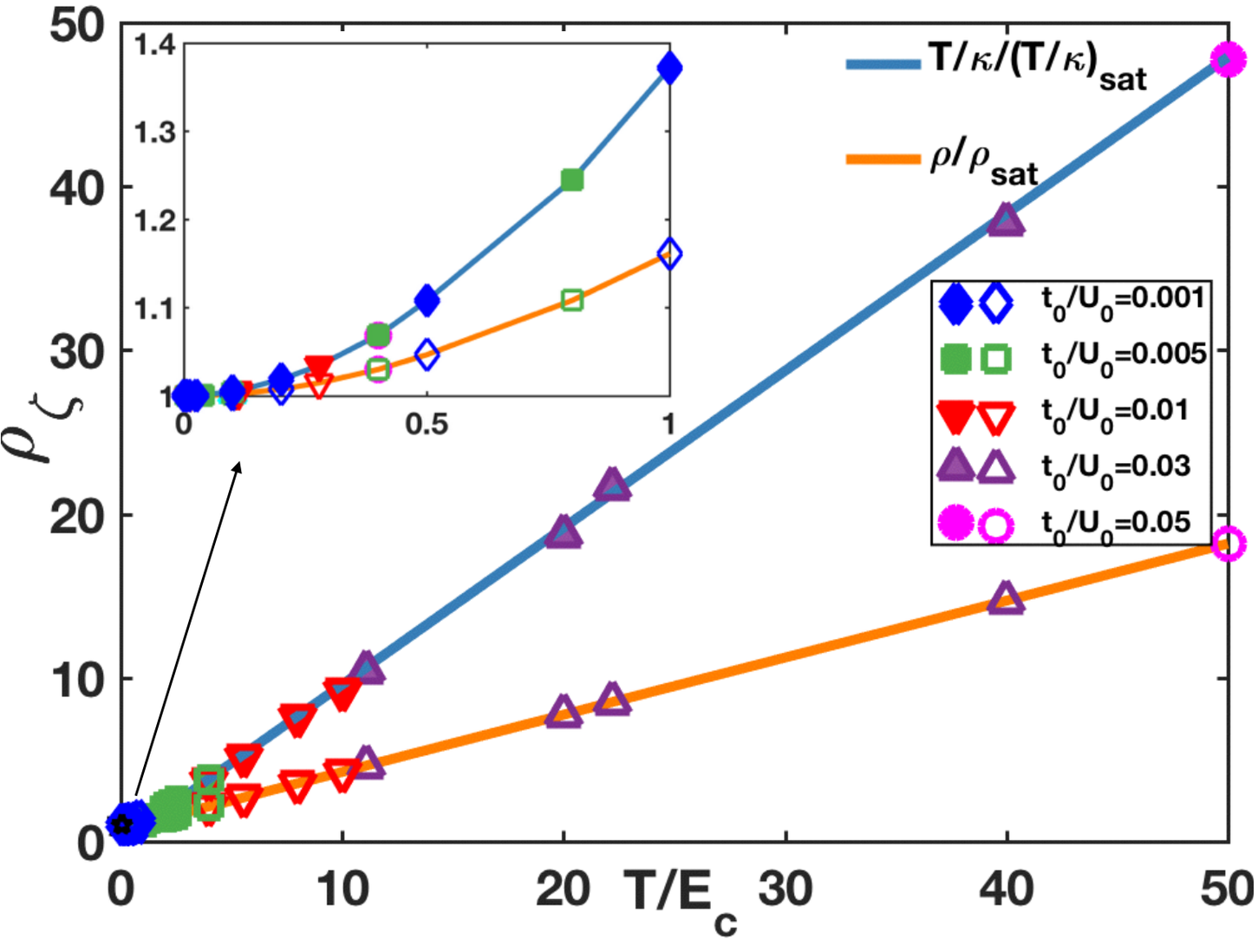
Low ‘coherence’ scale

$$E_c \sim \frac{t_0^2}{U}$$



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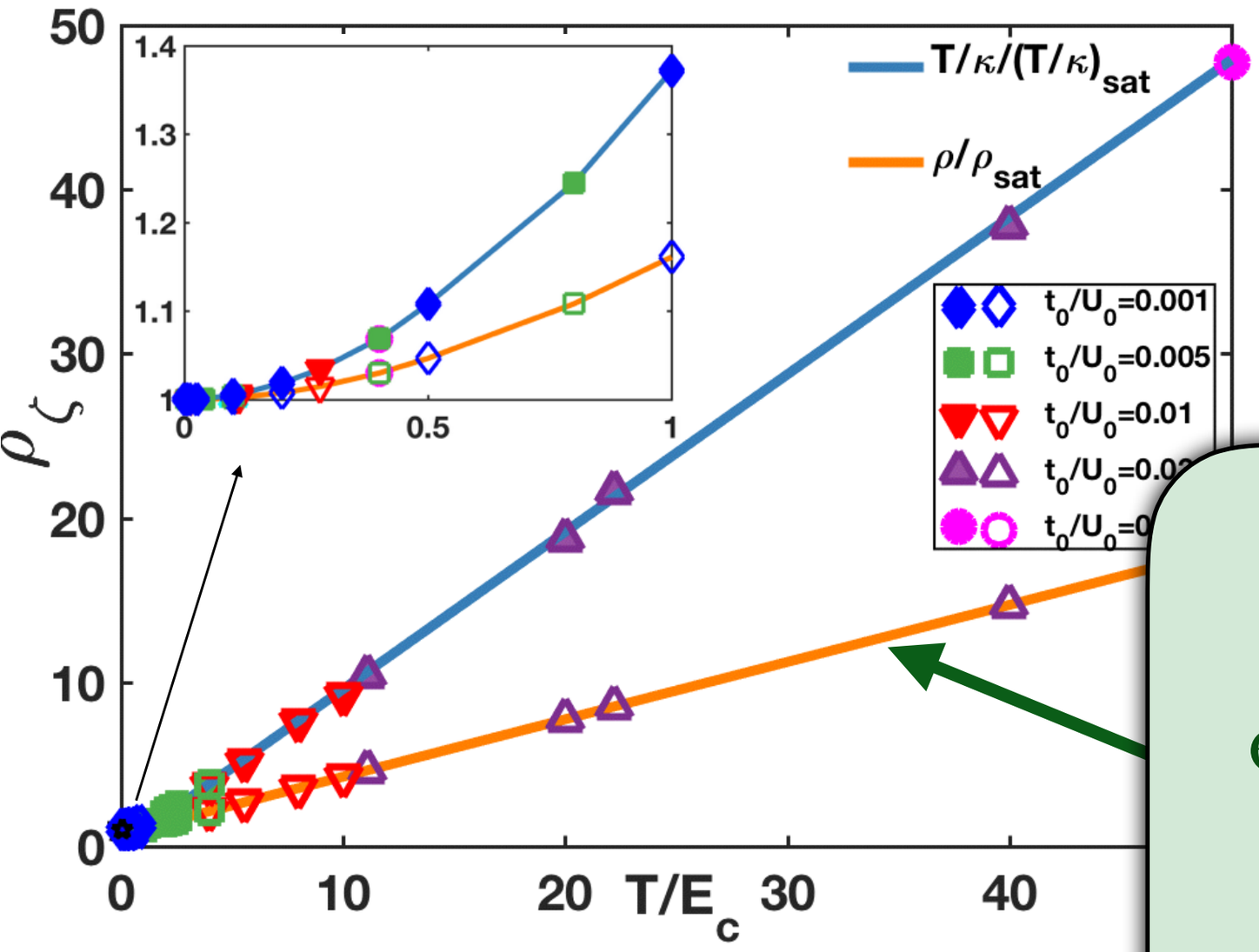

$$E_c \sim \frac{t_0^2}{U}$$

For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right), \quad s = s_0$$

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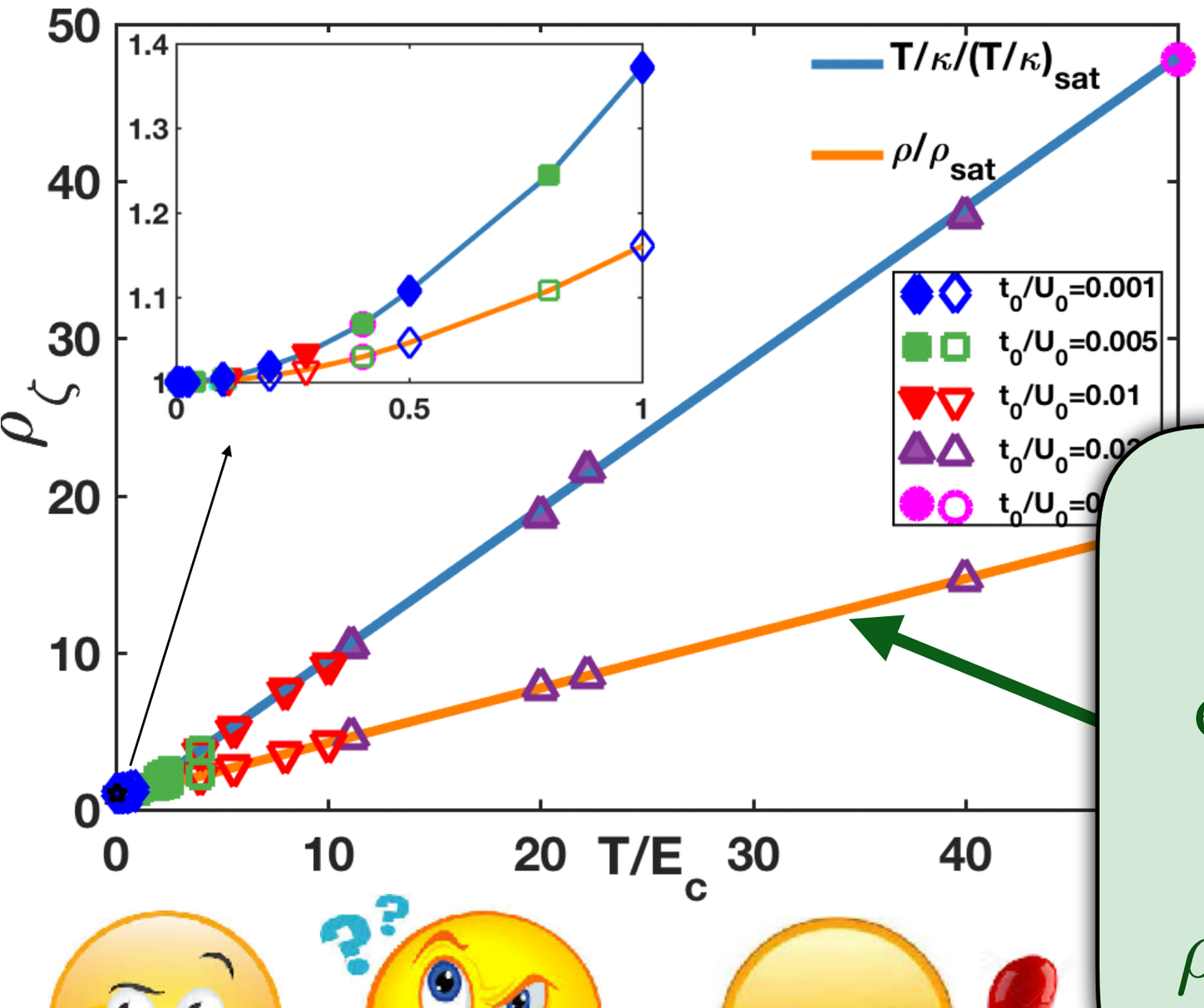

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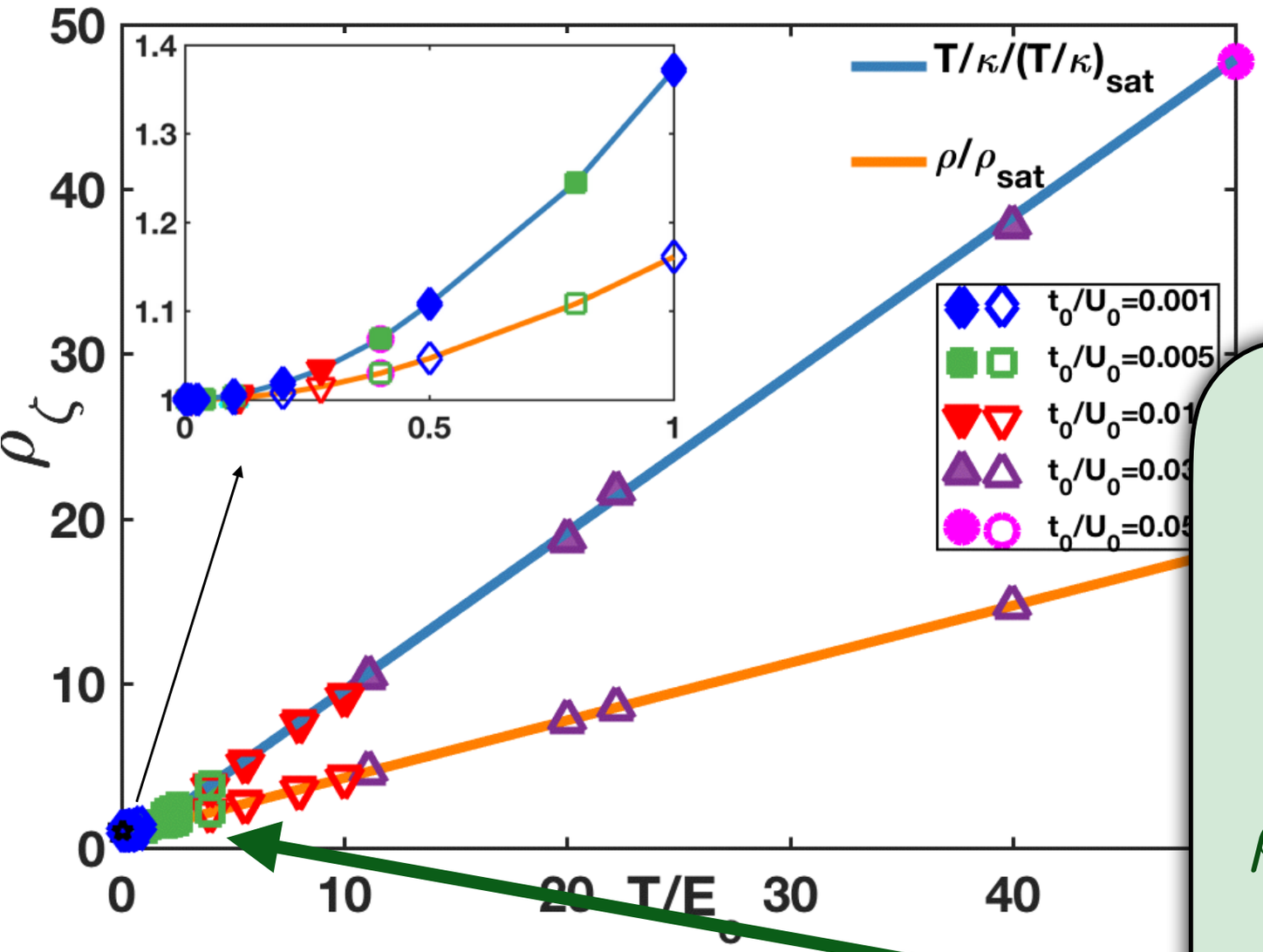


$$E_c \sim \frac{t_0^2}{U}$$

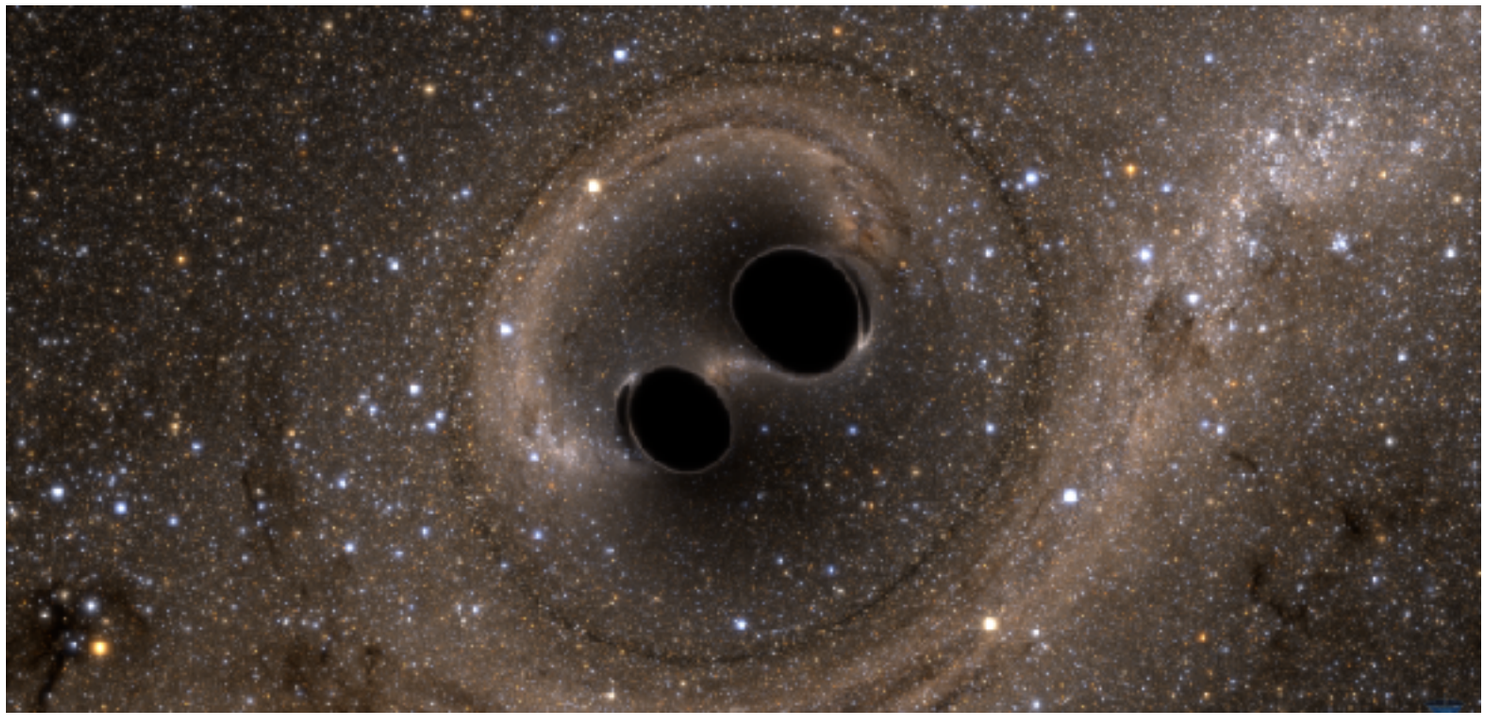
For $T < E_c$, the resistivity, ρ, and entropy density, s, are

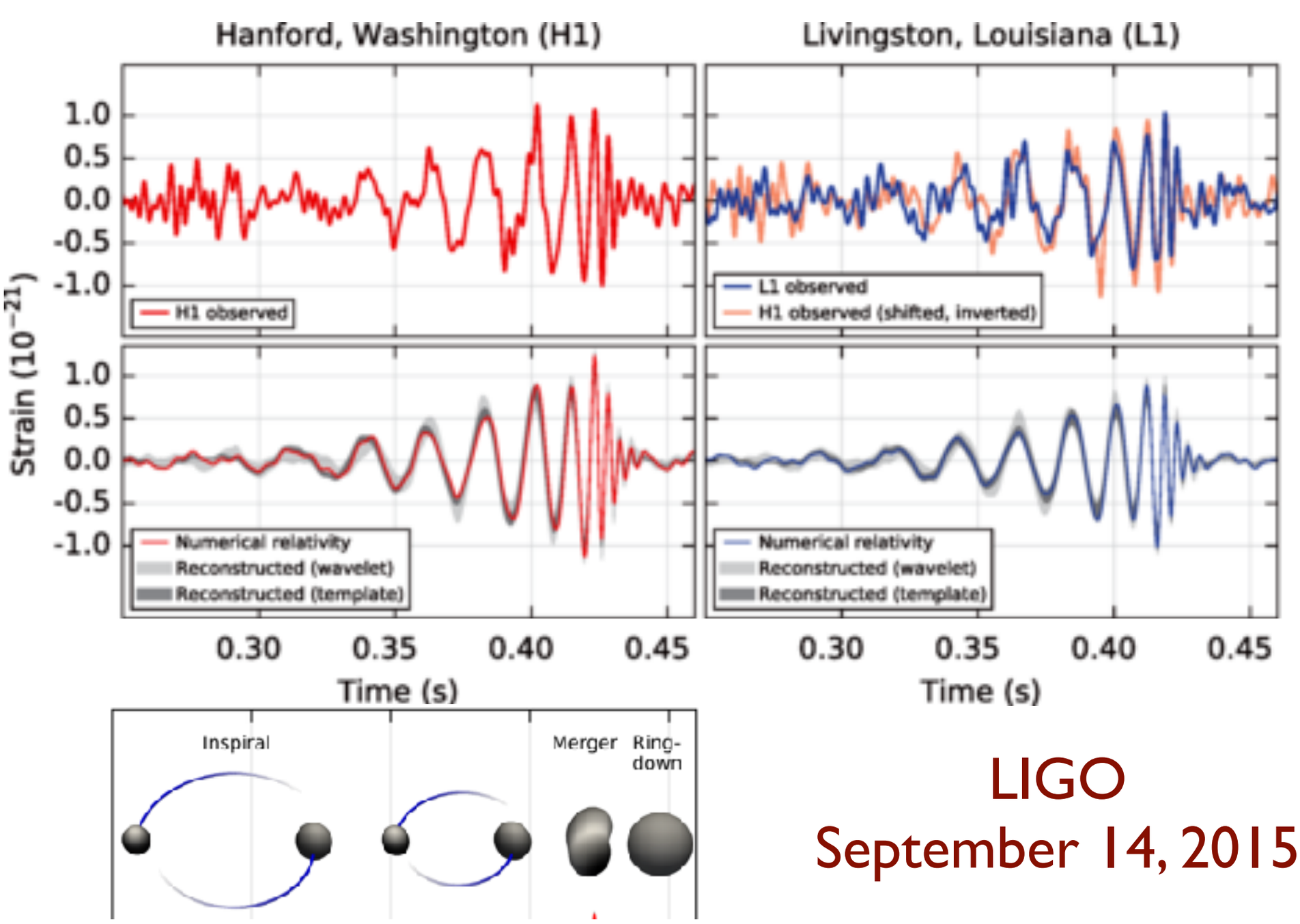
$$\rho = \frac{h}{e^2}\left[c_1 + c_2\left(\frac{T}{E_c}\right)^2\right]$$

$$s \sim s_0\left(\frac{T}{E_c}\right)$$

Black holes

- Black holes have an entropy and a temperature, T_H.
- The entropy is proportional to their surface area.





LIGO
September 14, 2015

- The Hawking temperature, T_H influences the radiation from the black hole at the very last stages of the ring-down (not observed so far). The ring-down (approach to thermal equilibrium) happens very rapidly in a time $\sim \dfrac{\hbar}{k_B T_H} = \dfrac{8\pi G M}{c^3} \sim 8$ milliseconds.

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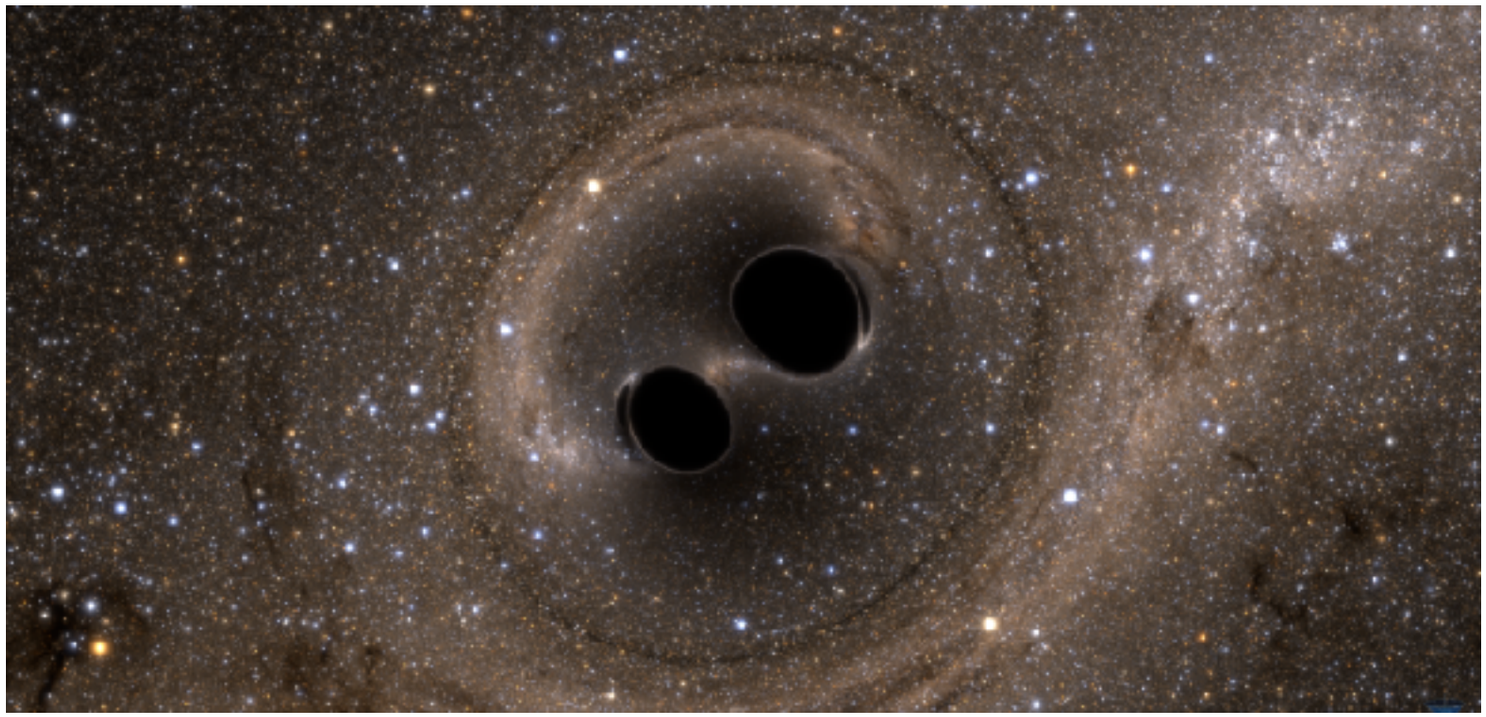

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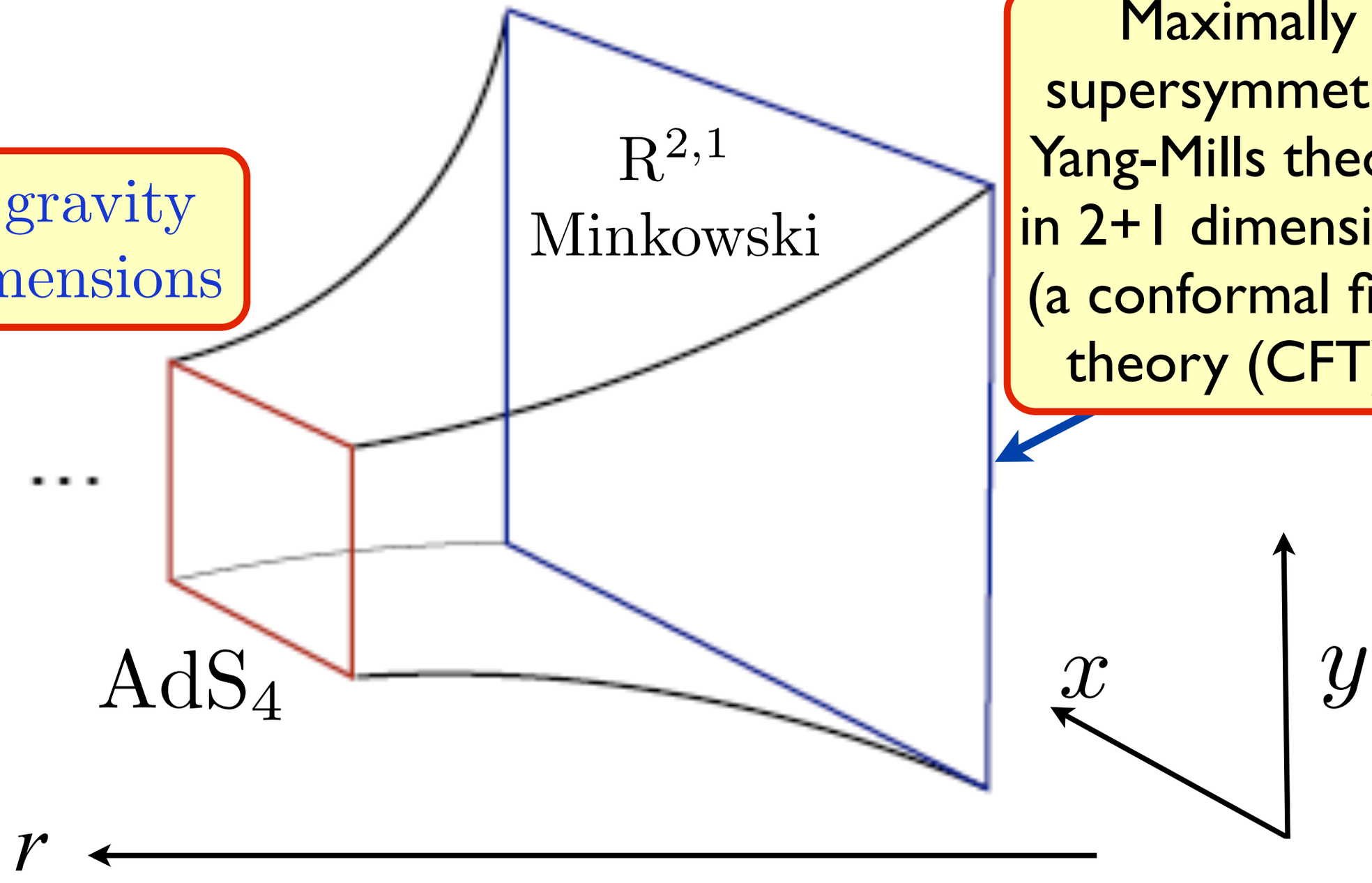
- They relax to thermal equilibrium in a time $\sim \hbar/(k_B T_H)$.

AdS/CFT correspondence at zero temperature

Quantum gravity in 3+1 dimensions

Maximally supersymmetric Yang-Mills theory in 2+1 dimensions (a conformal field theory (CFT))



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[dr^2 - dt^2 + dx^2 + dy^2\right]$$

Maldacena, Gubser, Klebanov, Polyakov, Witten

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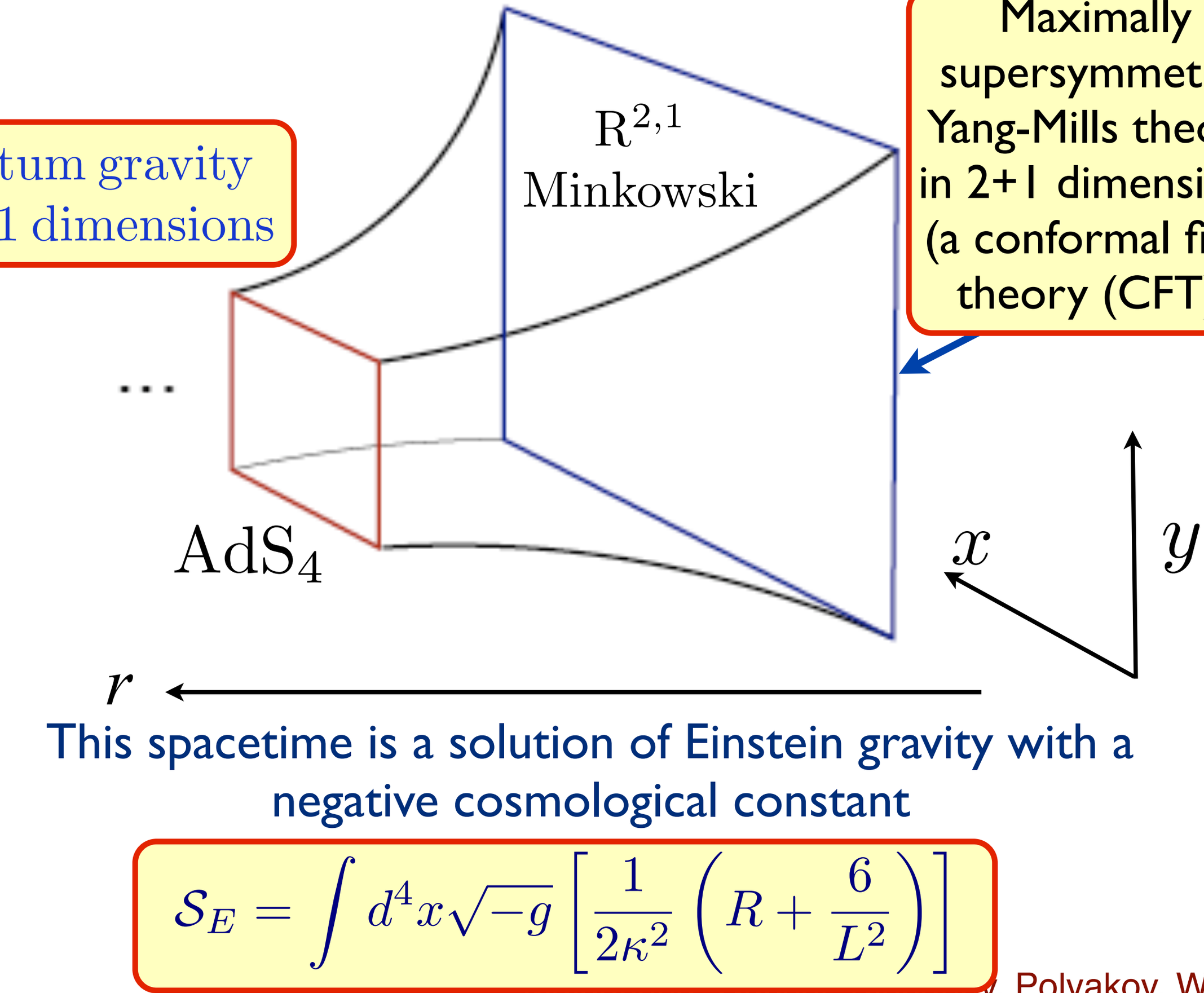


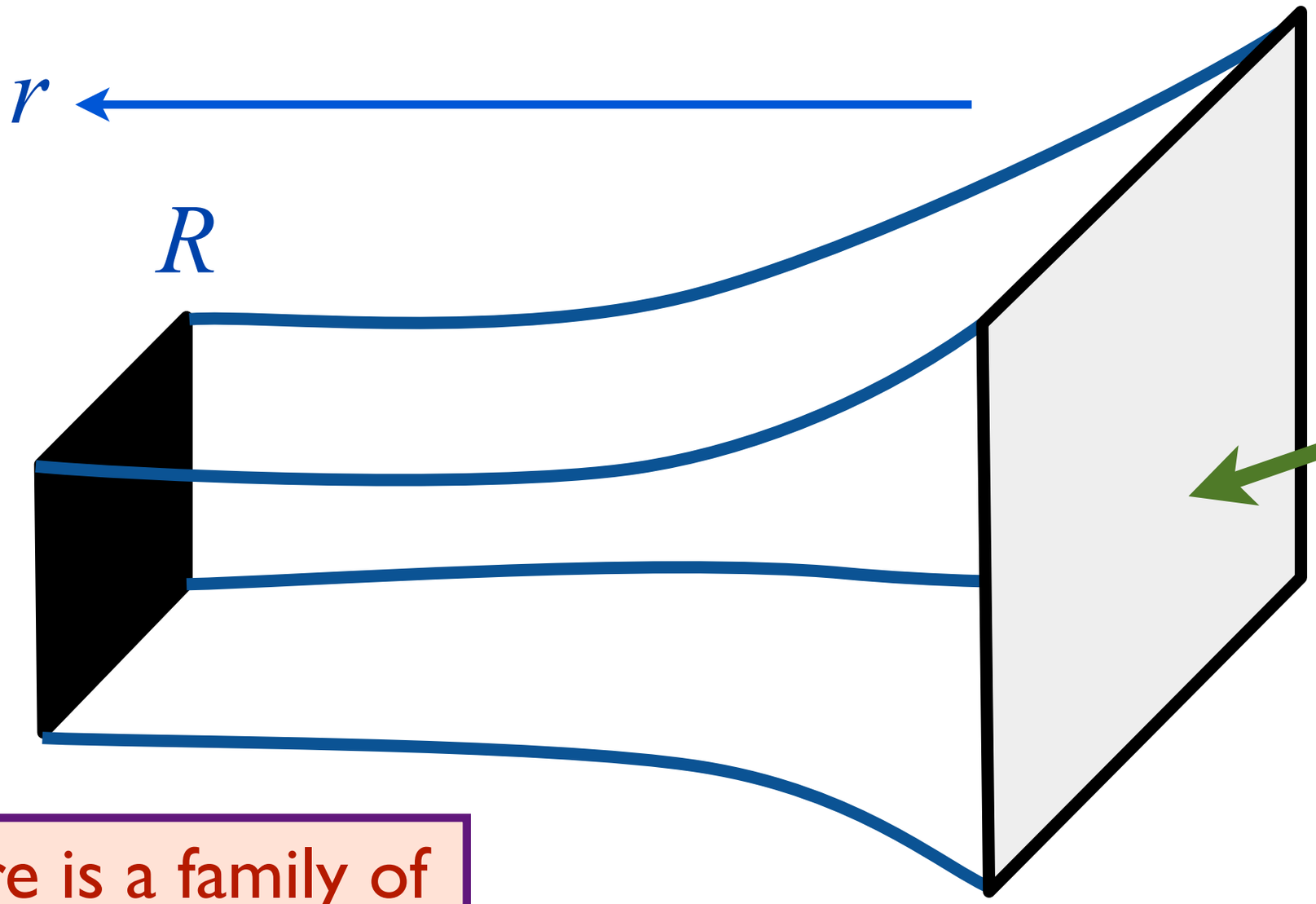
This spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right]$$

Maldacena; Gubser; Klebanov; Polyakov, Witten

AdS/CFT correspondence at non-zero temperatures

AdS_4-Schwarzschild black-brane



Maximally supersymmetric Yang-Mills at a temperature

$$k_B T = \frac{3\hbar}{4\pi R}.$$

There is a family of solutions of Einstein gravity which describe non-zero temperatures

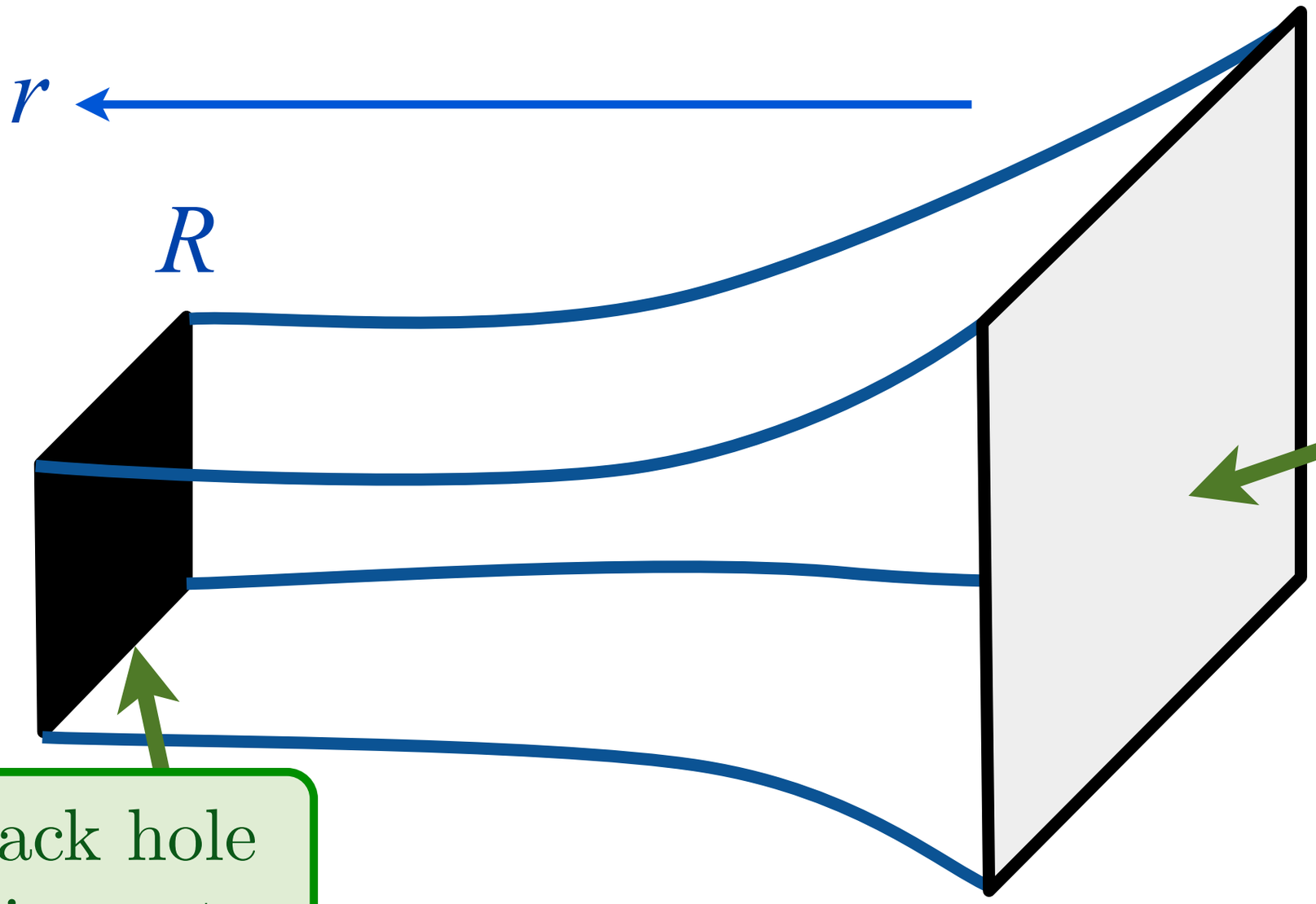
$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = 1 - (r/R)^3$

Maldacena, Gubser, Klebanov, Polyakov, Witten

AdS/CFT correspondence at non-zero temperatures

AdS_4-Schwarzschild black-brane



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Black hole horizon at a Hawking temperature

$$T_H = T$$

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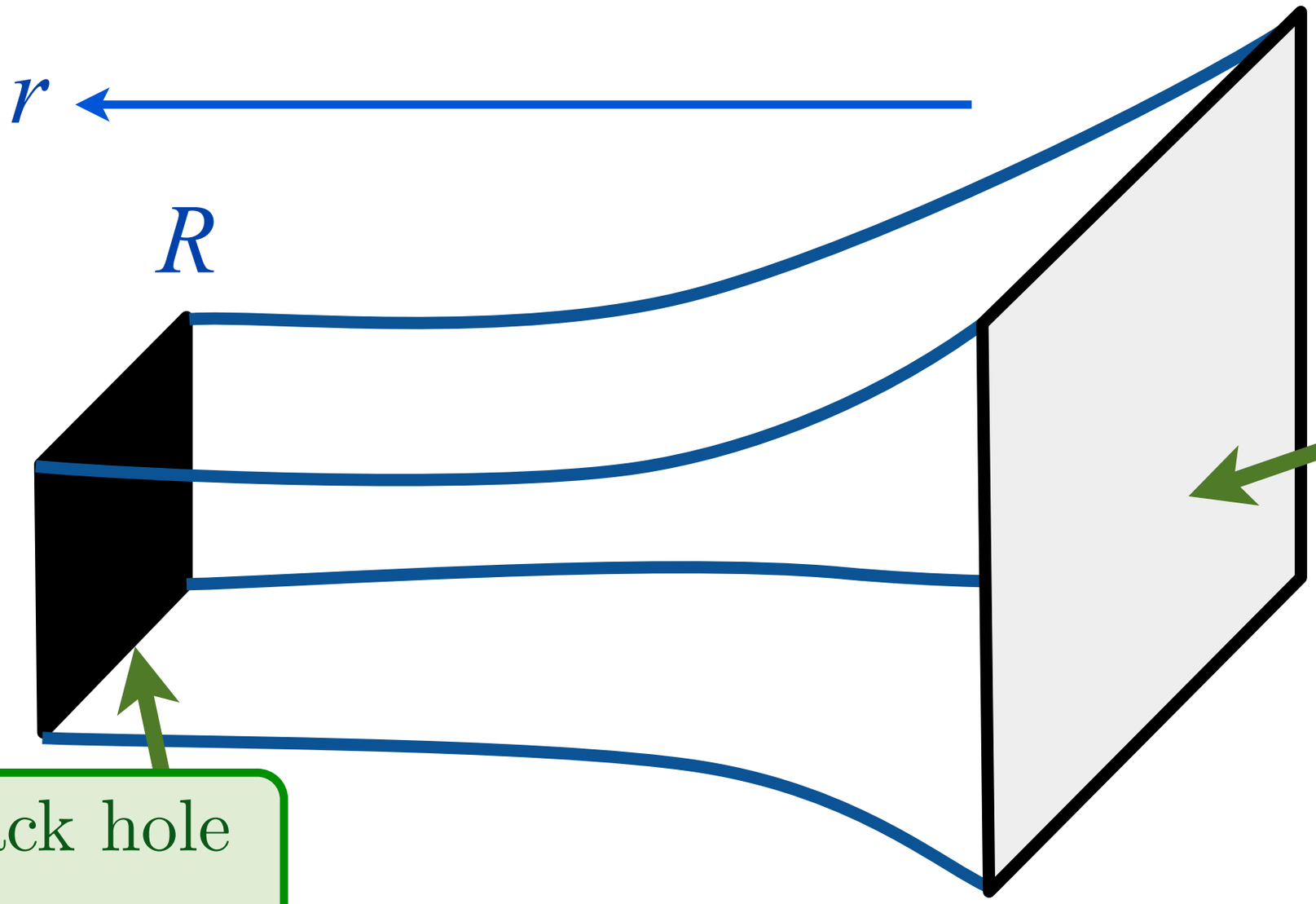

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Black hole entropy = Entropy of Yang-Mills theory

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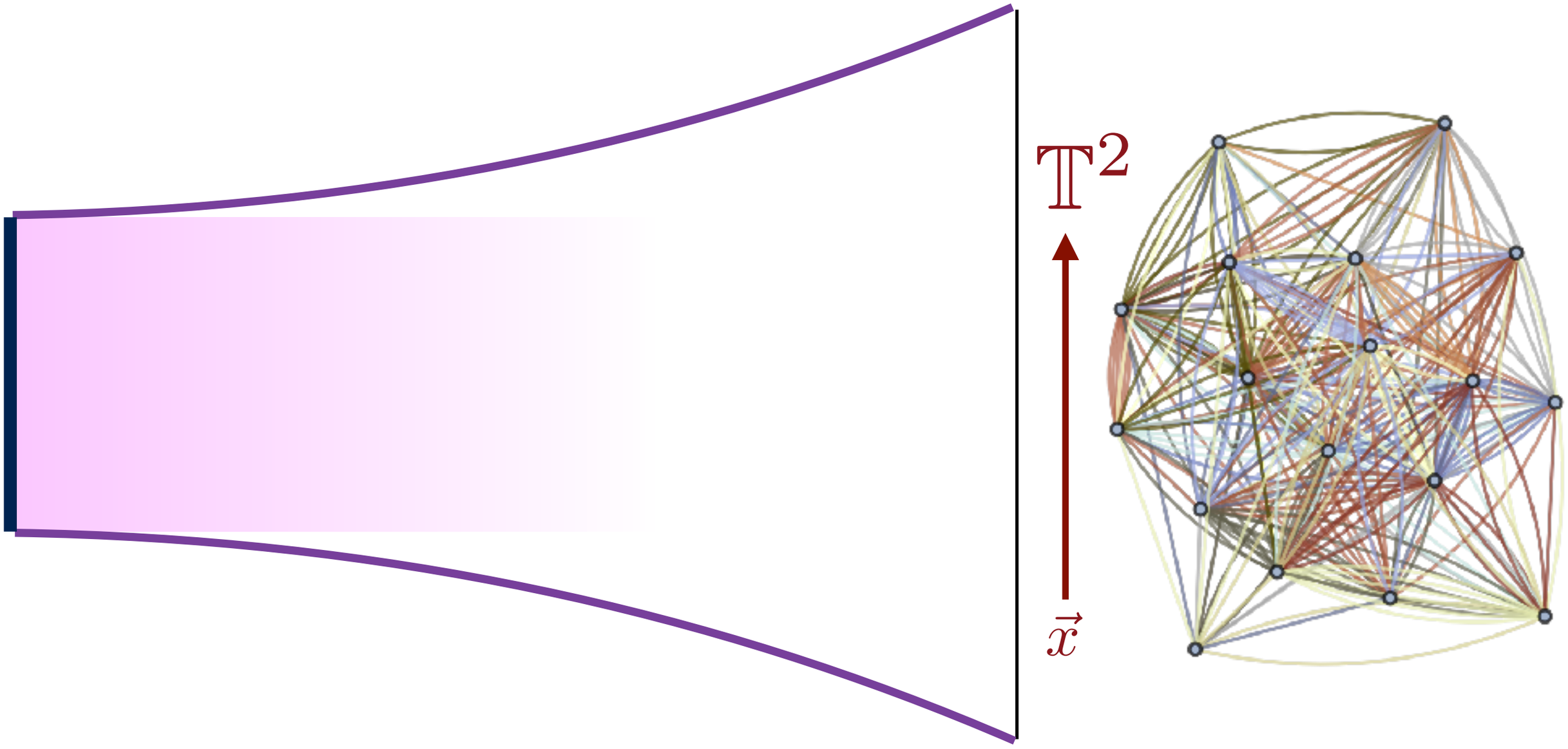
SYK and black holes



Is there a holographic quantum gravity dual of the SYK model ?

The SYK model

- Low energy, many-body density of states
  $\rho(E) \sim e^{N s_0} \sinh(\sqrt{2(E-E_0)N\gamma})$

- Low temperature entropy $S = N s_0 + N\gamma T + \ldots$.

- $T = 0$ fermion Green's function $G(\tau) \sim \tau^{-1/2}$ at large τ. (Fermi liquids with quasiparticles have $G(\tau) \sim 1/\tau$)

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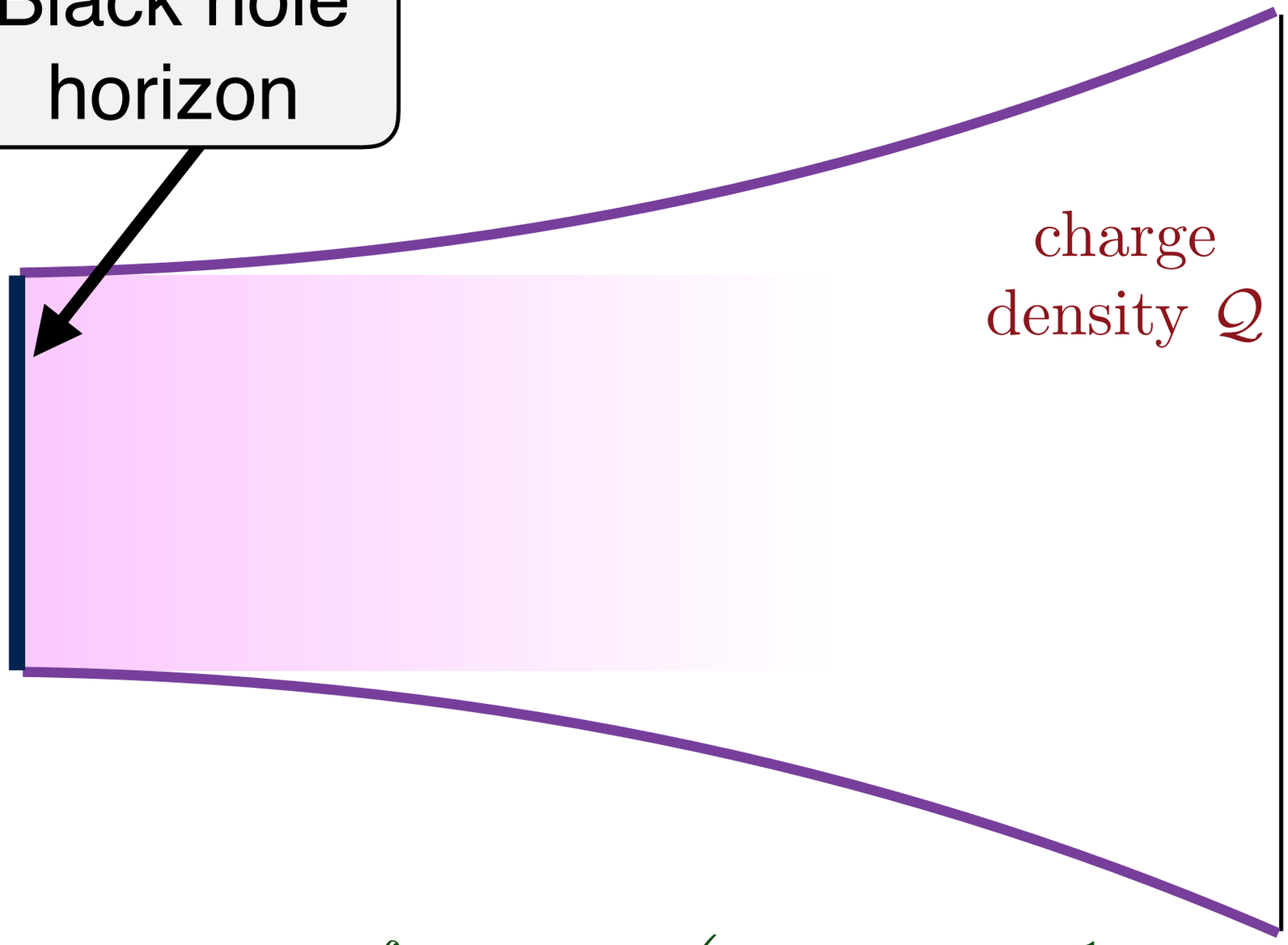
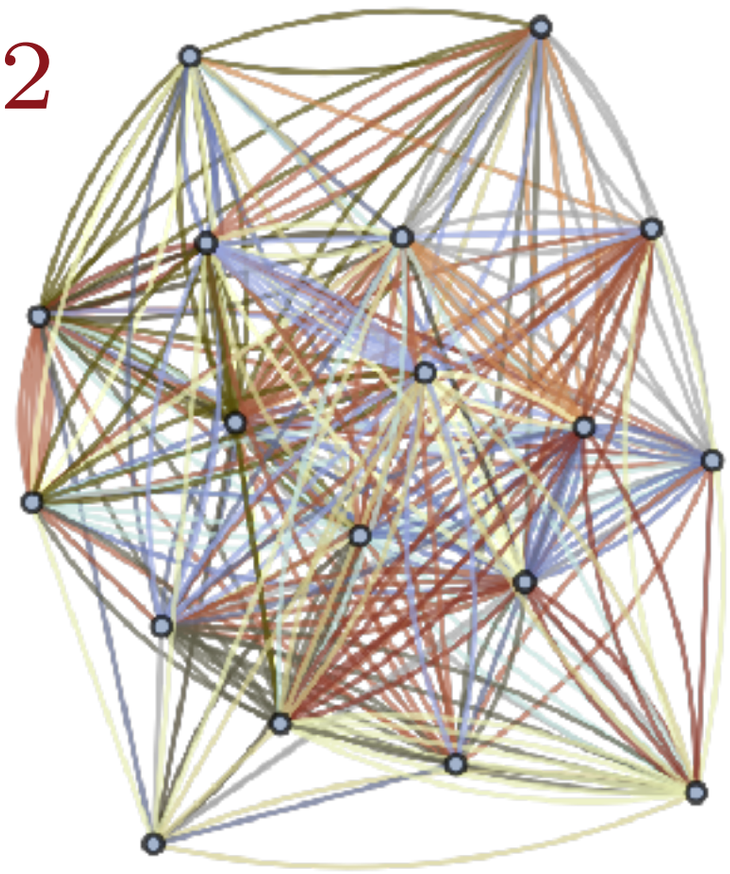
- Th
ha

Is there any black hole which holographically matches these properties ?

SYK and black holes

Black hole horizon



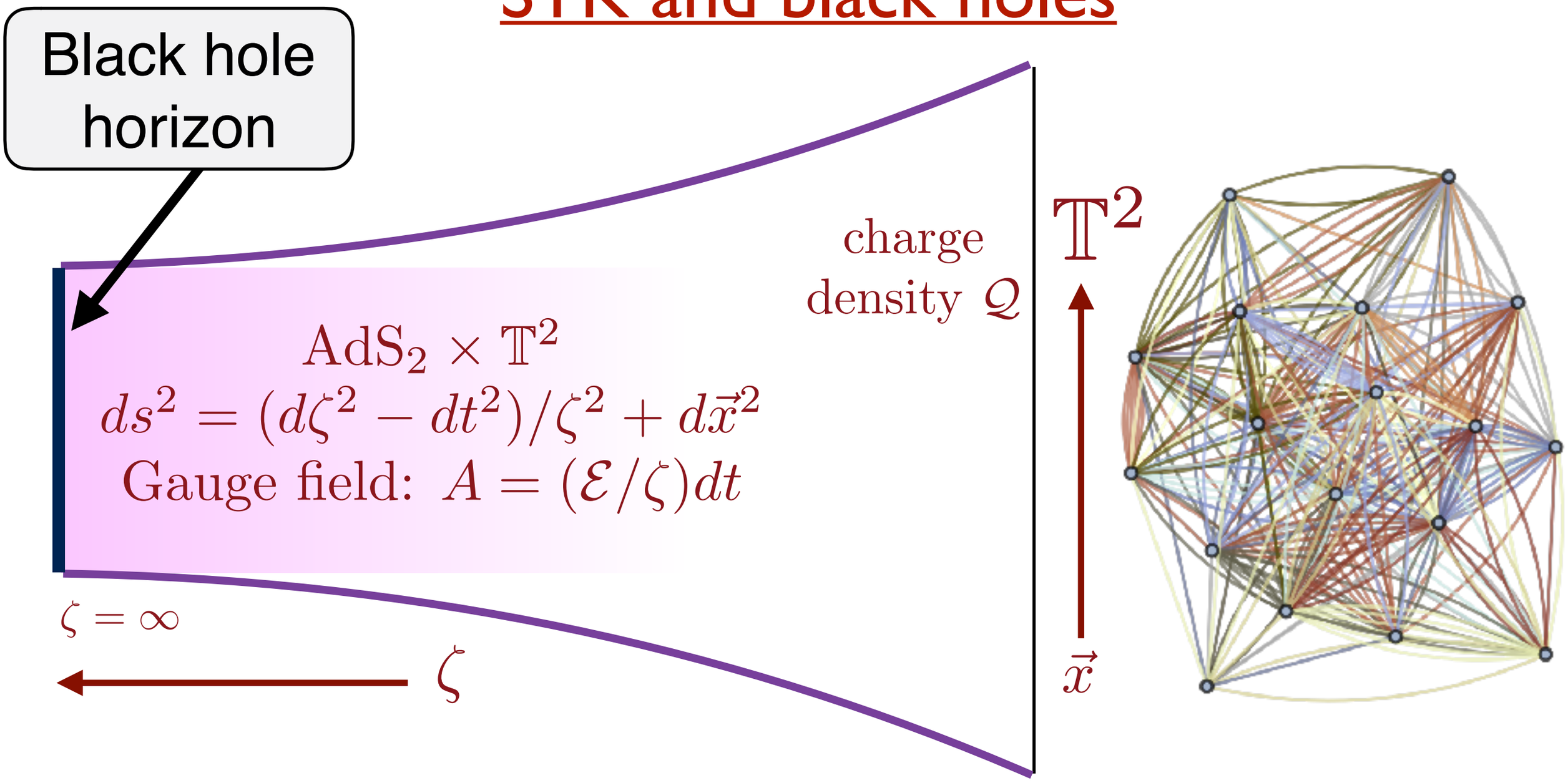


$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right)$$

Yes, a charged black hole in Einstein-Maxwell theory

S. Sachdev, PRL **105**, 151602 (2010)

SYK and black holes



The leading low temperature properties of an AdS-Reissner-Nordstrom black hole (as computed by T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694) match those of the SYK model.
The mapping applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$.

S. Sachdev, PRL **105**, 151602 (2010)

SYK and black holes

Black hole horizon

$\mathrm{AdS}_2 \times \mathbb{T}^2$

$$\boxed{ds^2 = (d\zeta^2 - dt^2)/\zeta^2} + d\vec{x}^2$$

Gauge field: $A = (\mathcal{E}/\zeta)dt$

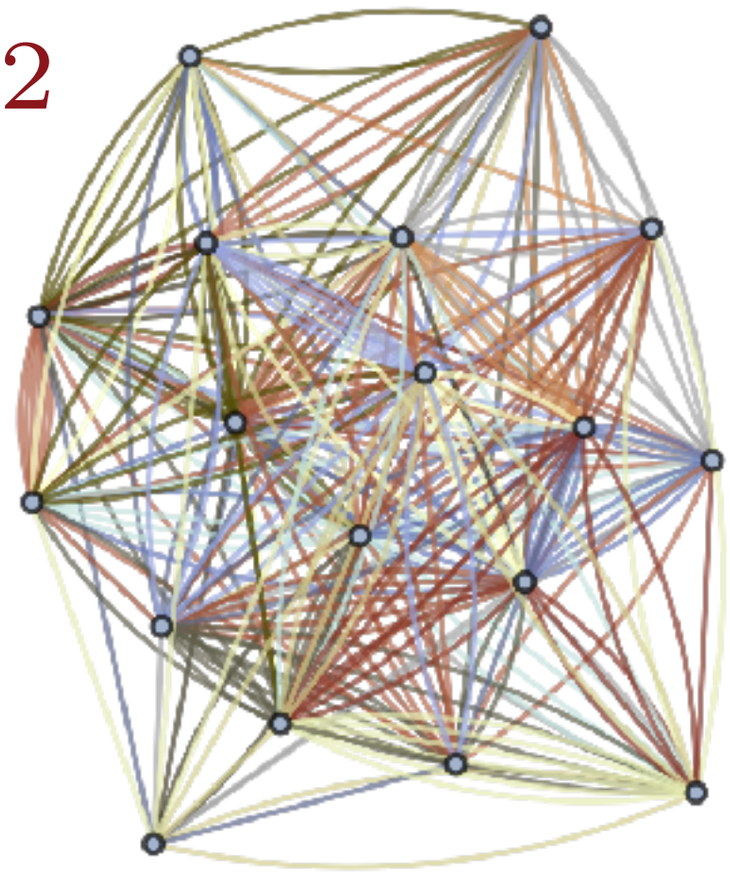
charge density $\mathcal{Q}$

$\zeta = \infty$

ζ

$\mathbb{T}^2$

$\vec{x}$



Quantum gravity on the 1+1 dimensional spacetime AdS_2 (when embedded in AdS_4) is holographically matched to the 0+1 dimensional SYK model

S. Sachdev, PRL **105**, 151602 (2010); A. Kitaev (unpublished); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857

SYK and AdS_2

Connections of SYK to gravity and AdS_2 horizons

- Reparameterization and gauge invariance are the 'symmetries' of the Einstein-Maxwell theory of gravity and electromagnetism

- SL(2,R) is the isometry group of AdS_2.

$ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$ is invariant under

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d}$$

with $ad - bc = 1$.

SYK and black holes

- Low energy, many-body density of states $\rho(E) \sim e^{N s_0}$

- Low temperature entropy $S = N s_0$

- $T = 0$ fermion Green's function $G(\tau) \sim \tau^{-1/2}$ at large τ. (Fermi liquids with quasiparticles have $G(\tau) \sim 1/\tau$)

- $T > 0$ Green's function has conformal invariance

- [illegible]

- [illegible]

All these properties of the SYK model match those of the AdS_2 horizon in Einstein-Maxwell theory

S. Sachdev, PRL **105**, 151602 (2010)

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Schwarzian theory of quantum gravity fluctuations also matches these corrections

A Kitaev, unpublished, J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
D. Stanford and E. Witten, arXiv:1703.04612

Many-body quantum chaos

- Using holographic analogies, Shenker and Stanford introduced the "Lyapunov time", τ_L, the time over which a generic many-body quantum system loses memory of its initial state.

S. Shenker and D. Stanford, arXiv:1306.0622

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$$\tau_L \geq \frac{\hbar}{2\pi k_B T}$$

J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

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J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

- The SYK model, and black holes in Einstein gravity, saturate the bound on the Lyapunov time

$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

A. Kitaev, unpublished
J. Maldacena and D. Stanford, arXiv:1604.07818

Quantum matter <u>without</u> quasiparticles:

- No quasiparticle decomposition of low-lying states:
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- Thermalization and many-body chaos in the shortest possible time of order $\hbar/(k_B T)$.

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- Thermalization and many-body chaos in the shortest possible time of order $\hbar/(k_B T)$.

- These are also characteristics of black holes in quantum gravity.