Quantum entanglement: strange metals and black holes

National Conference on Condensed Matter

Indian Institute of Science Education and Research, Mohali

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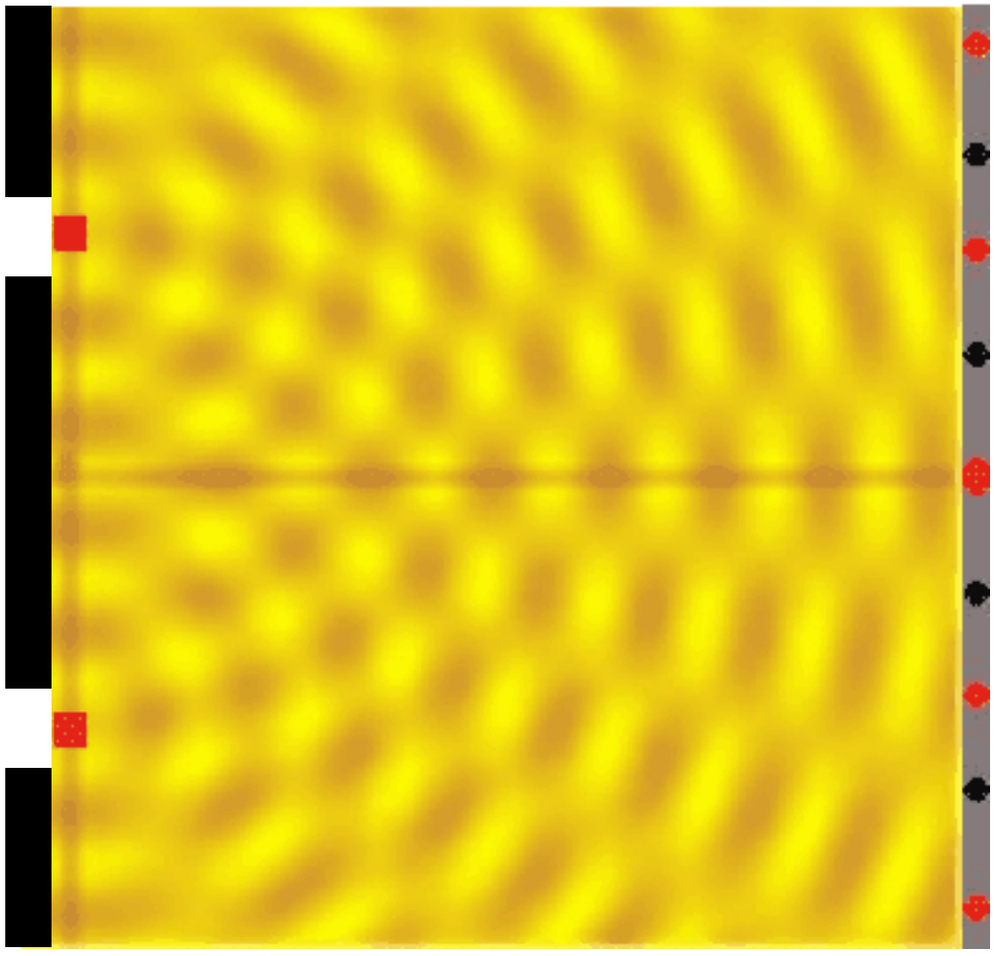
Talk online: sachdev.physics.harvard.edu

Quantum entanglement

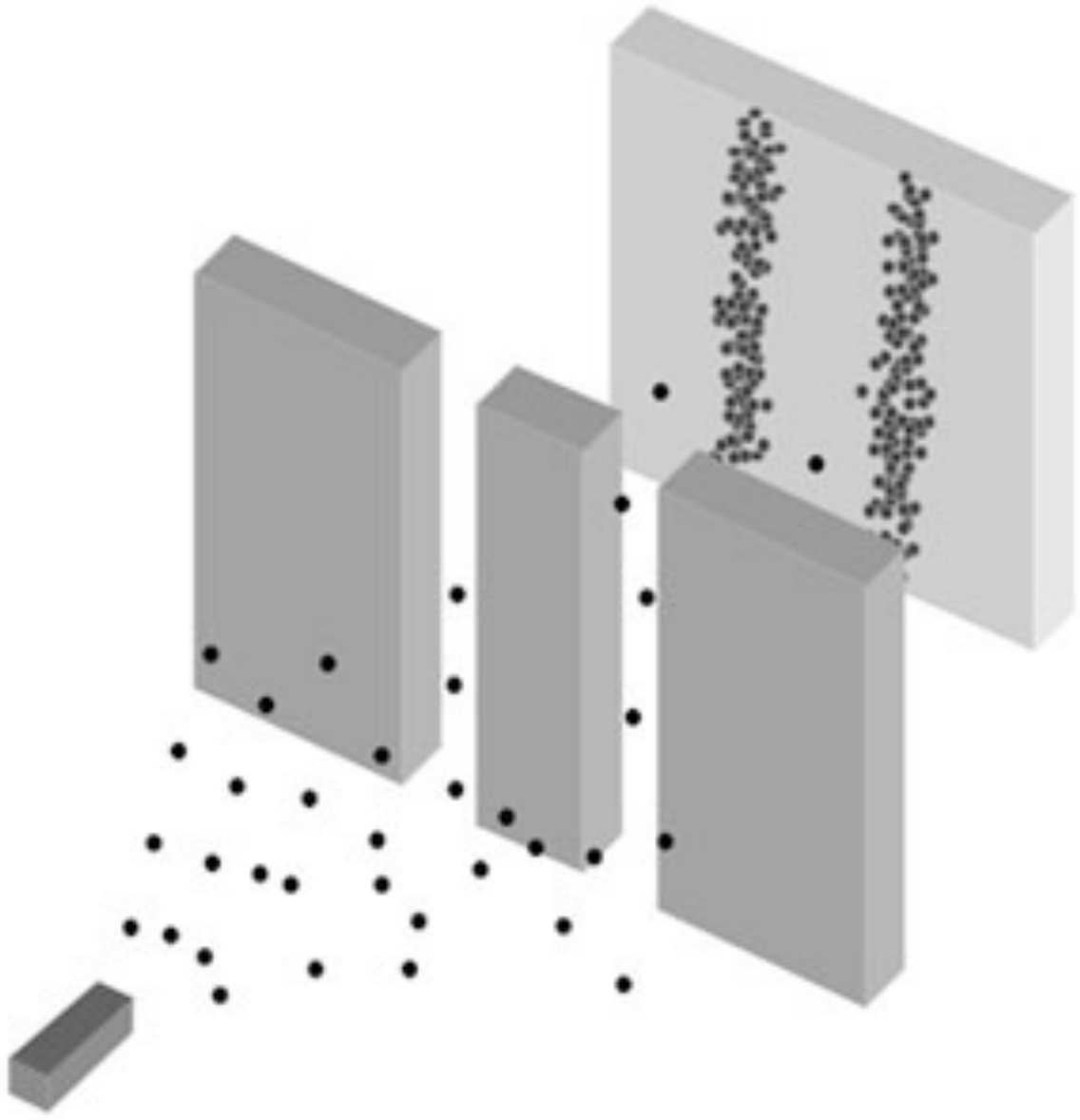
The double slit experiment



TWO SLITS

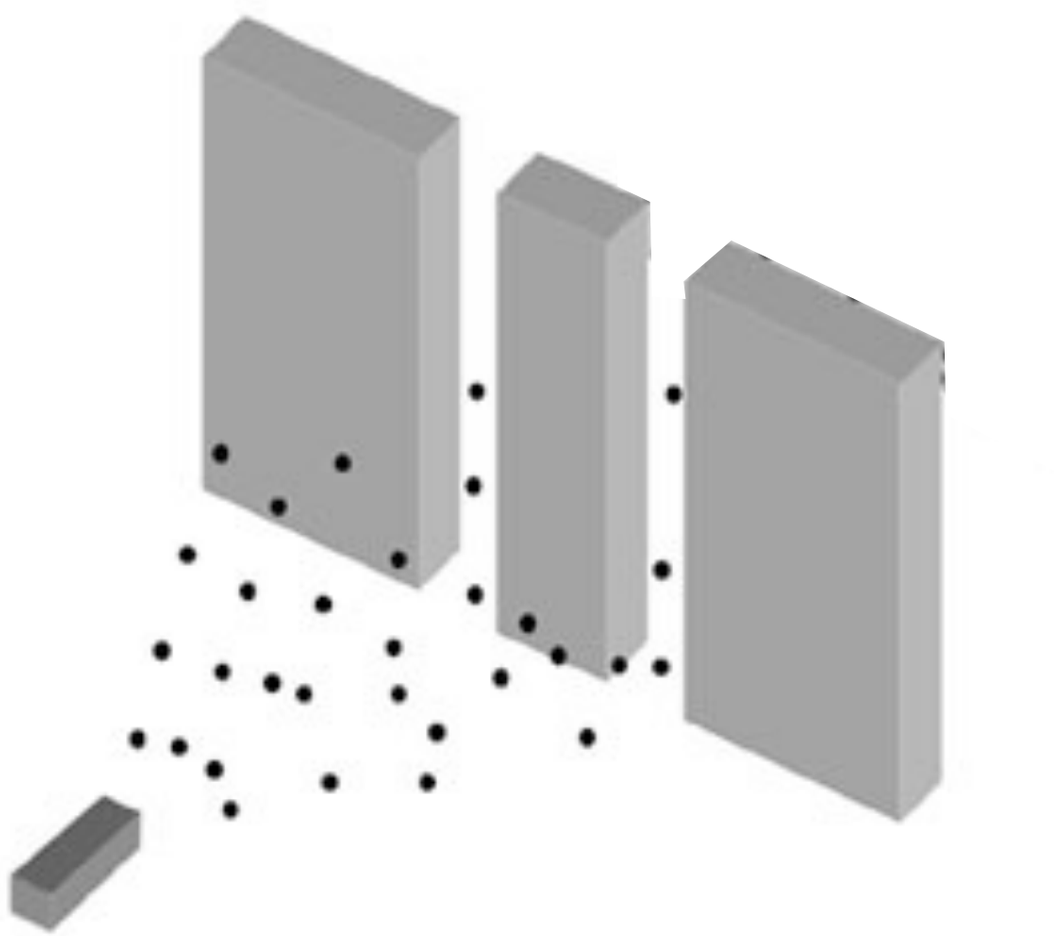
Interference of water waves

The double slit experiment



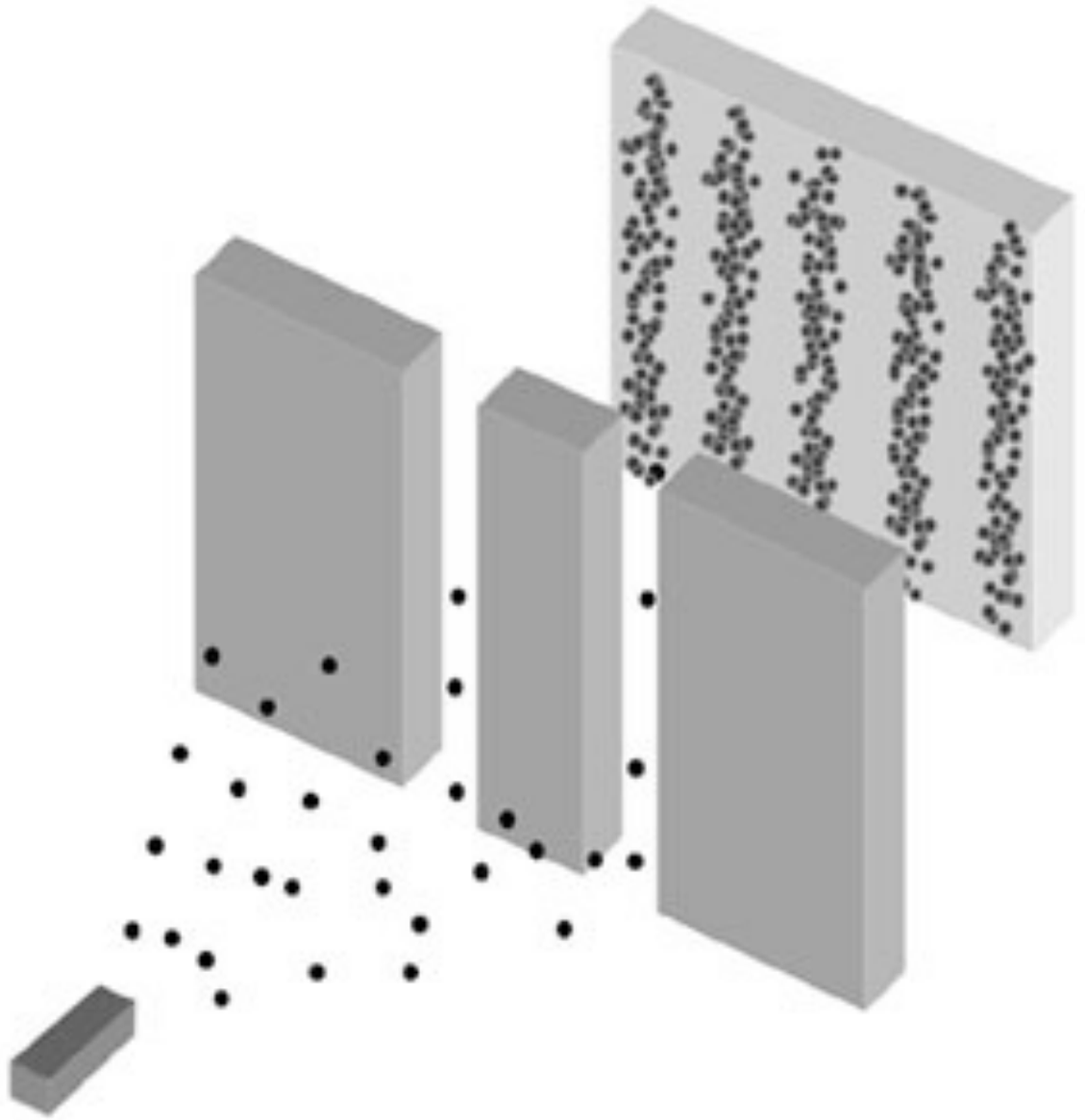
Bullets

The double slit experiment



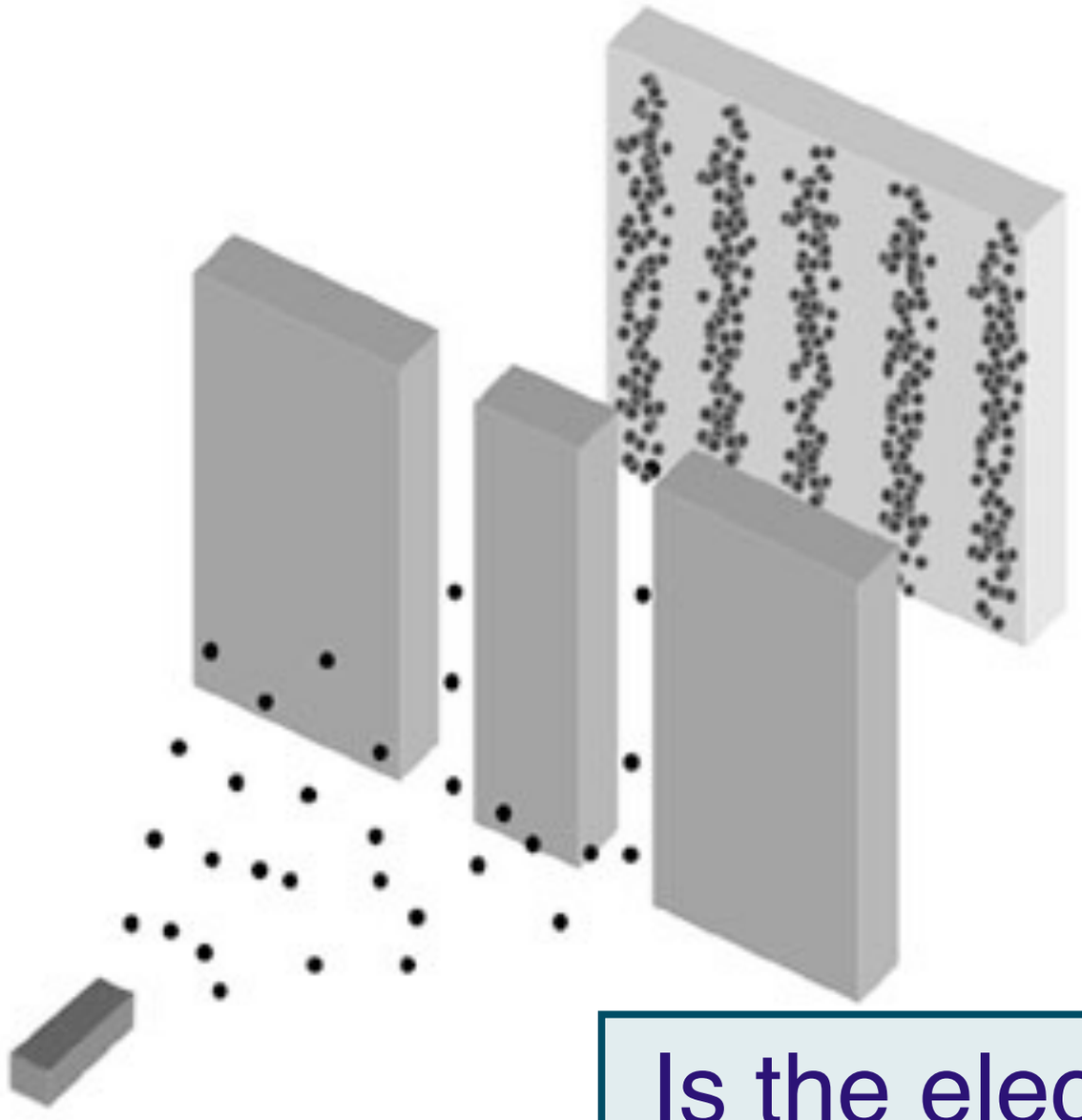
Send electrons through the slits

The double slit experiment



Interference of electrons

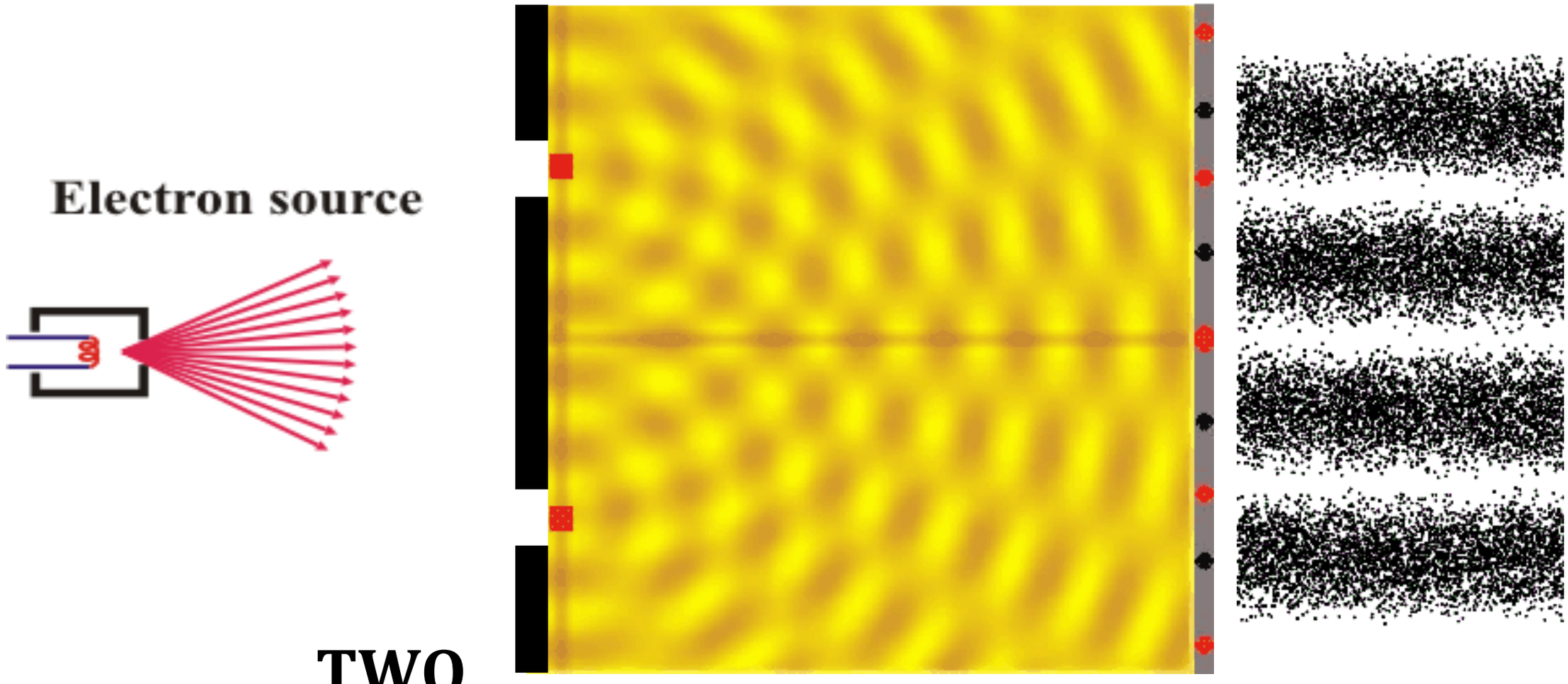
The double slit experiment



Is the electron a wave ?

Interference of electrons

The double slit experiment

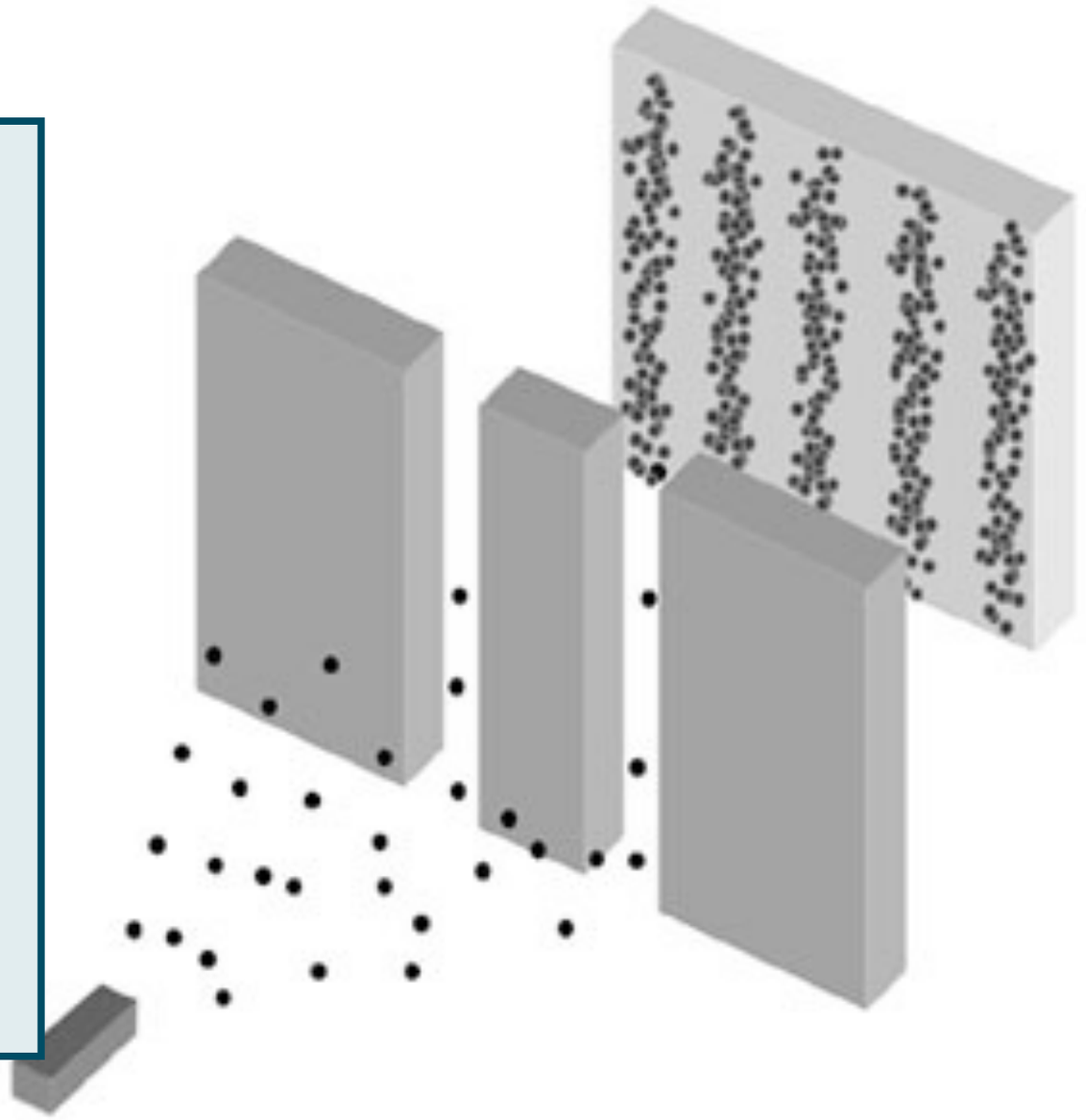


Unlike water waves, electrons arrive one-by-one (so is it like a particle ?)

Interference of electrons

The double slit experiment

But if it is like a particle, which slit does each electron pass through ?



Interference of electrons

The double slit experiment

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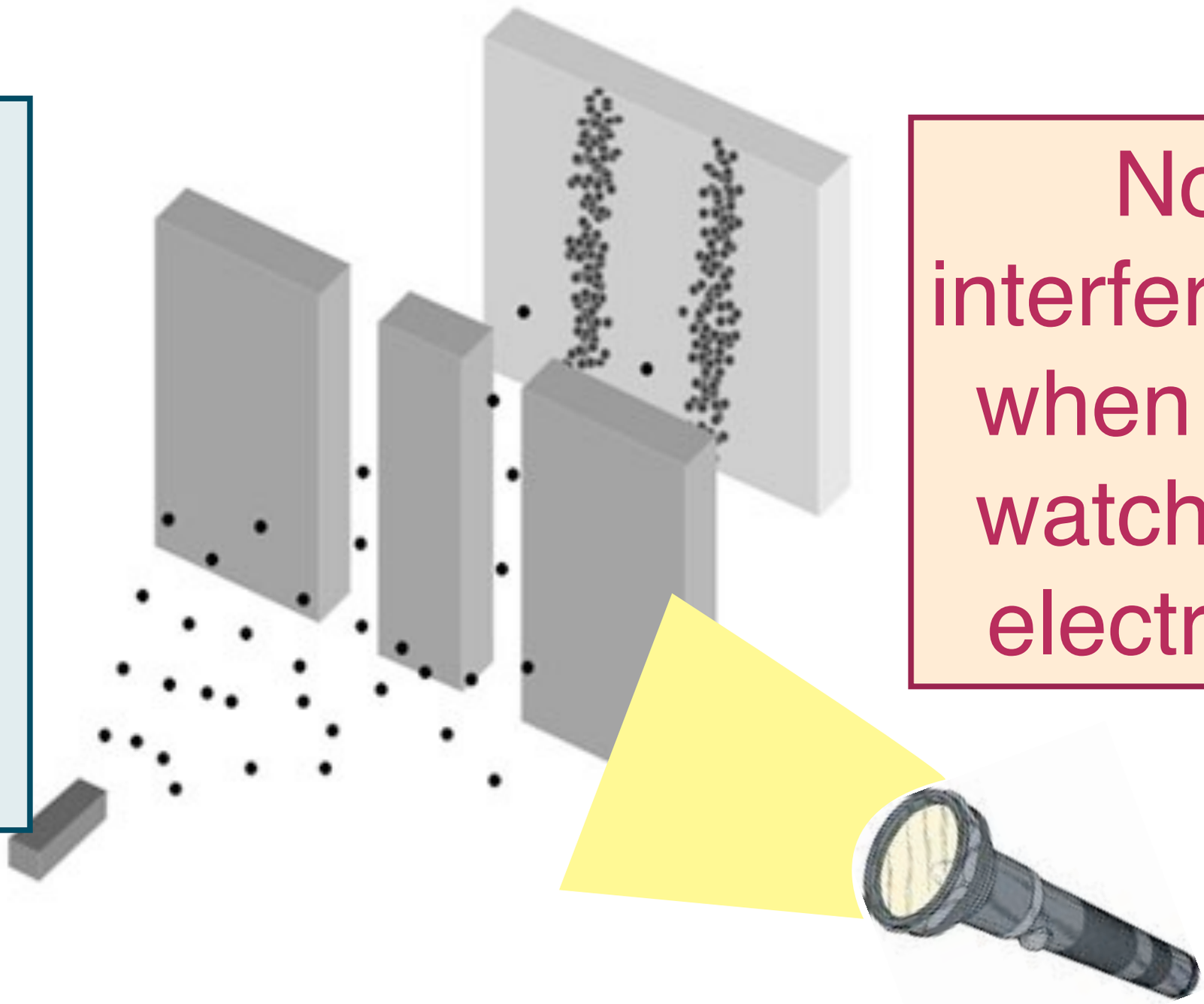
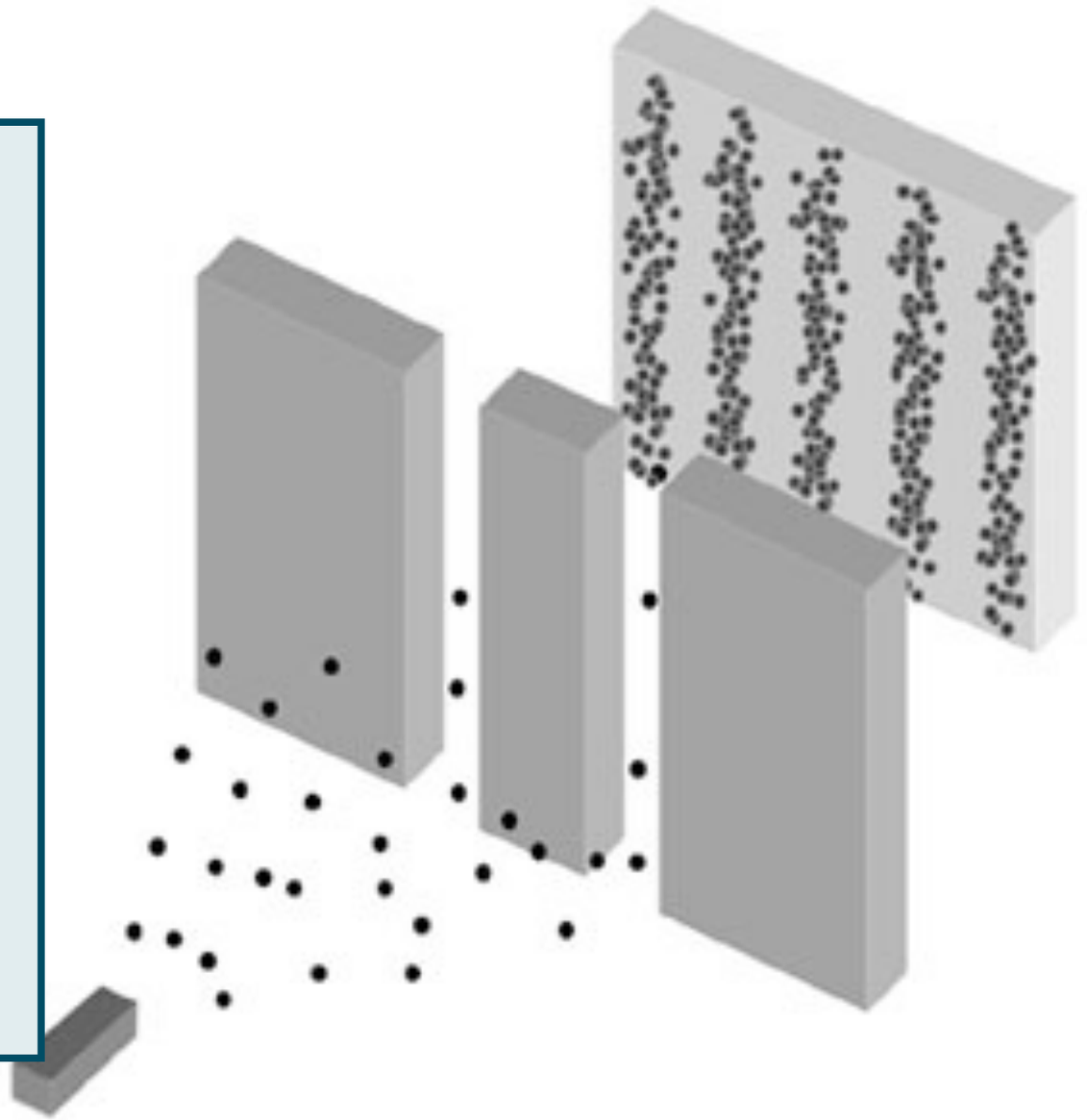


No interference when you watch the electrons

Interference of electron

The double slit experiment

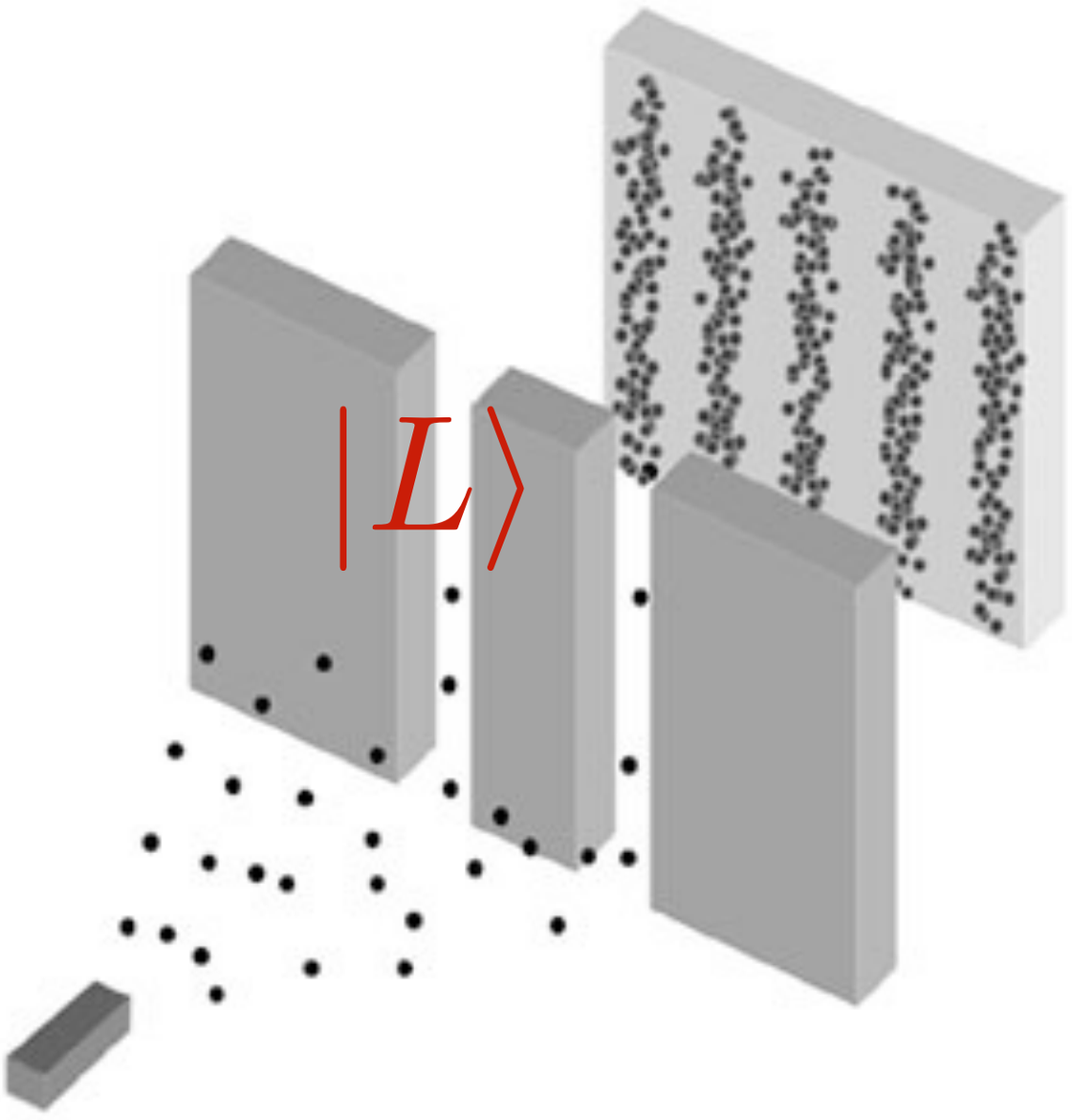
But if it is like a particle, which slit does each electron pass through ?



Each electron passes through both slits !

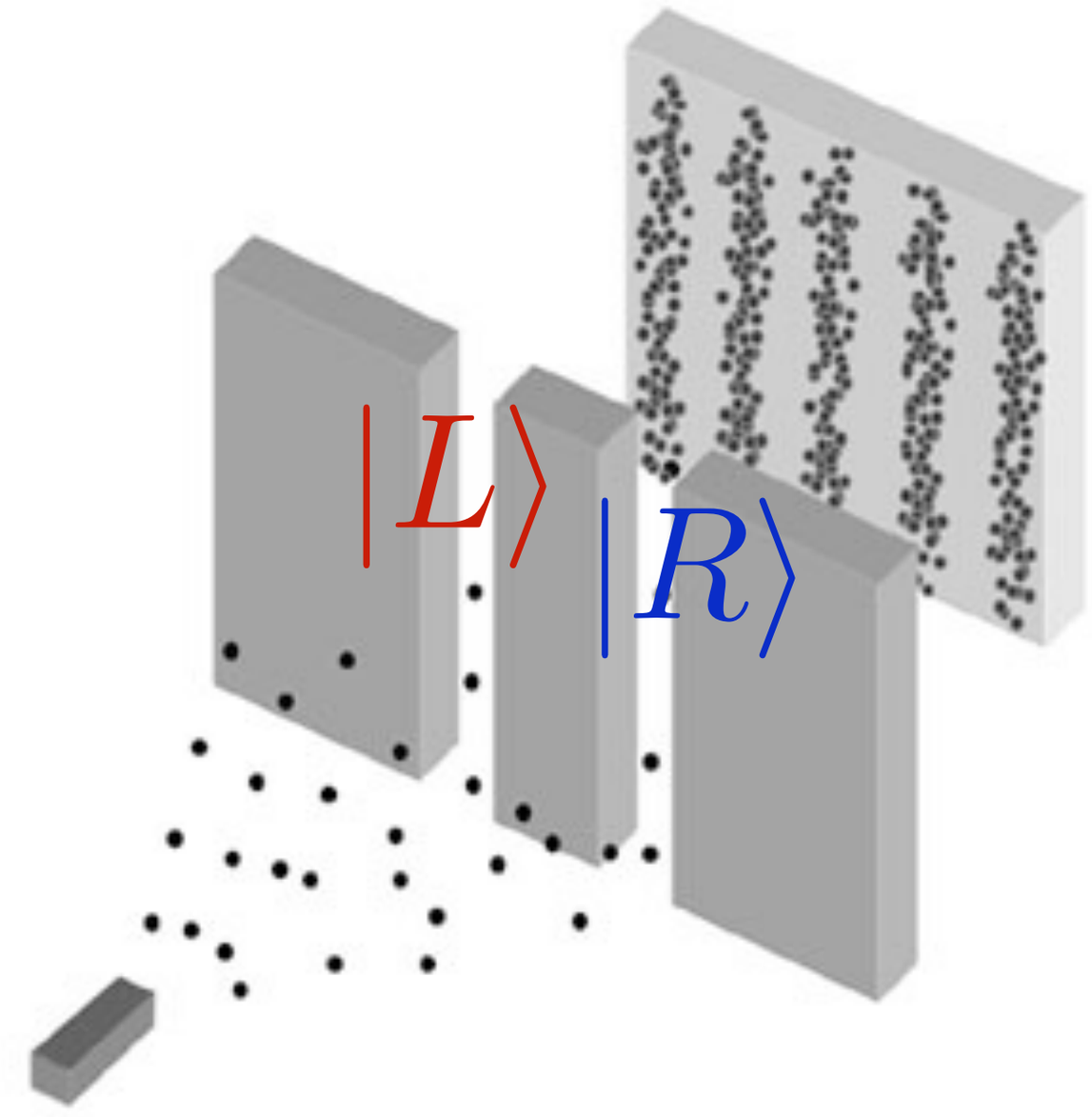
Interference of electrons

The double slit experiment



Let $|L\rangle$ represent the state
with the electron in the left slit

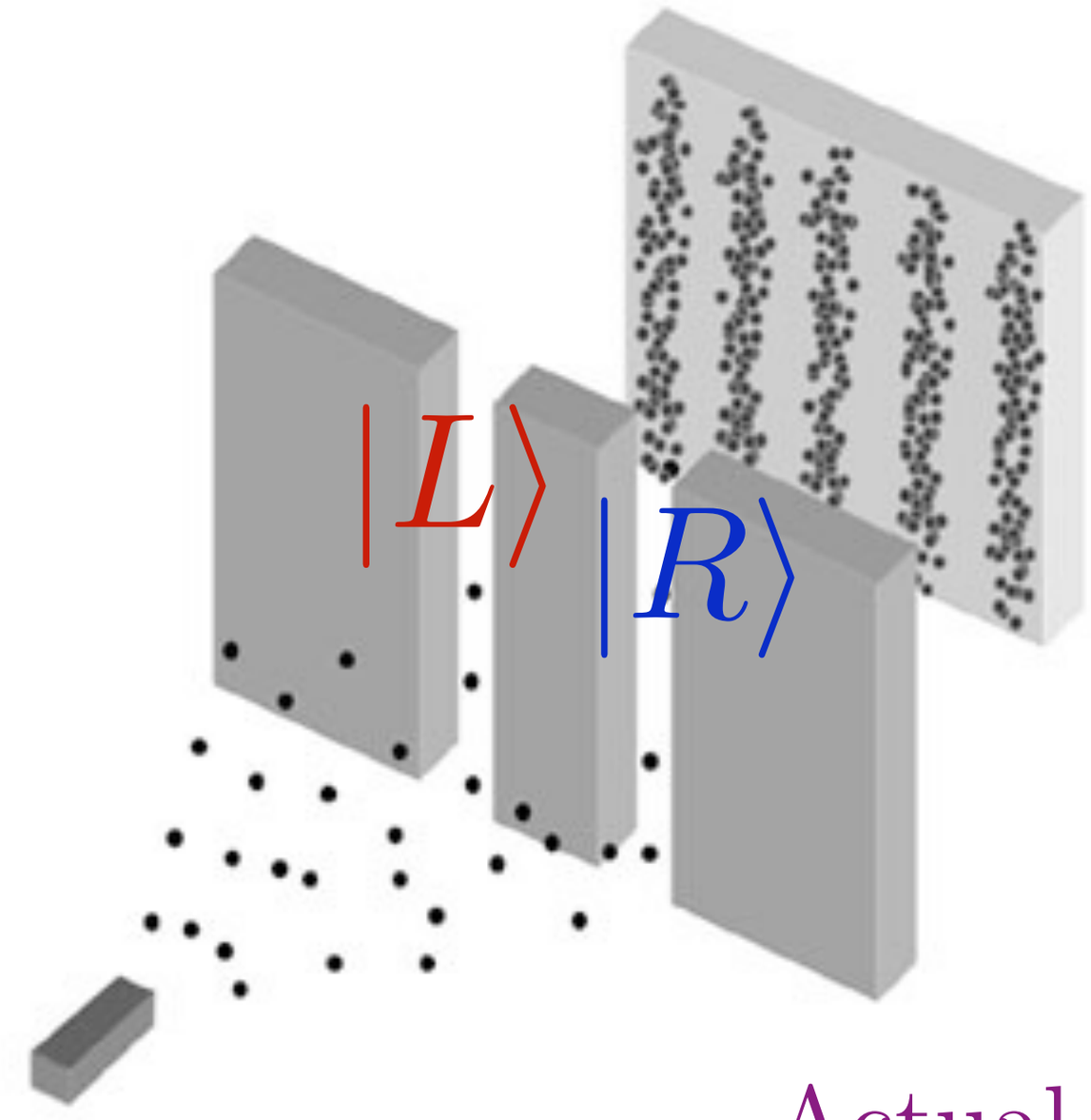
The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

The double slit experiment



Let $|L\rangle$ represent the state
with the electron in the left slit

And $|R\rangle$ represents the state
with the electron in the right slit

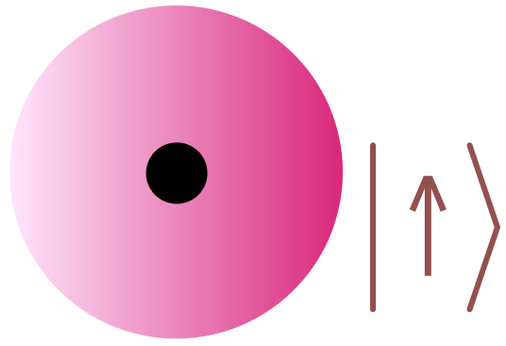
Actual state of *each* electron is

$$|L\rangle + |R\rangle$$

Quantum Entanglement: quantum superposition with more than one particle

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:



Quantum Entanglement: quantum superposition with more than one particle

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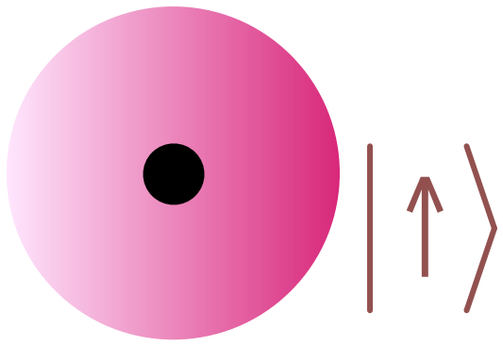
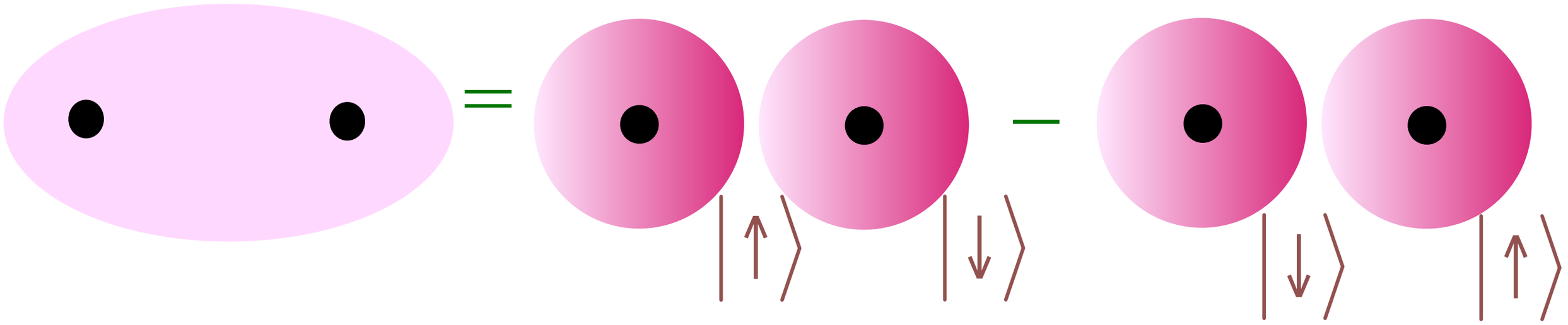
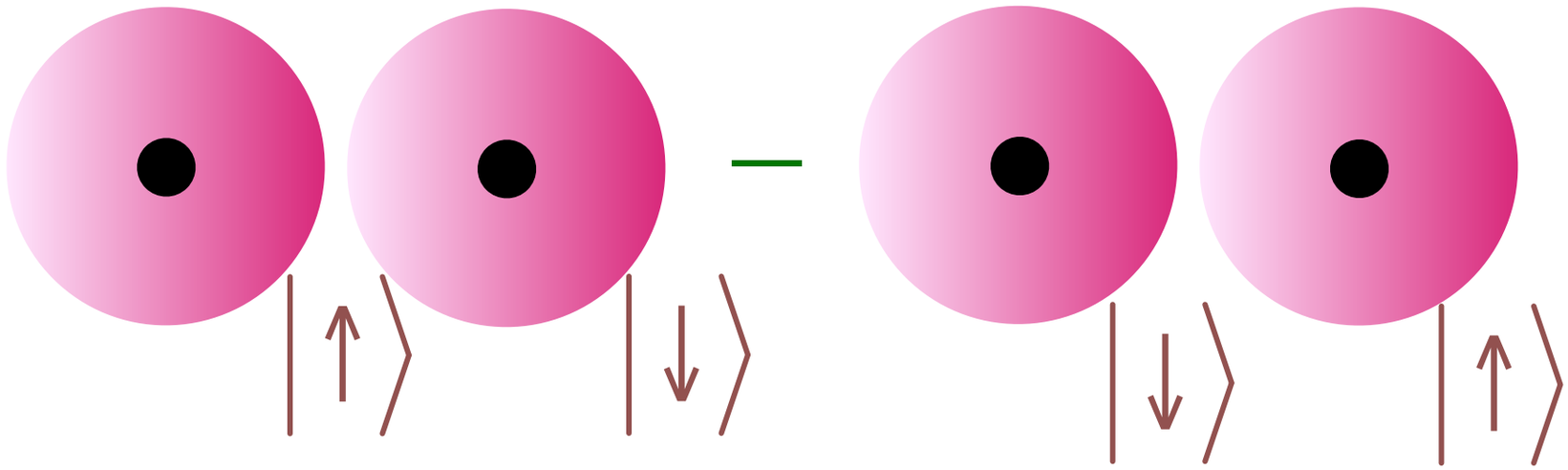


Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Quantum Entanglement: quantum superposition with more than one particle



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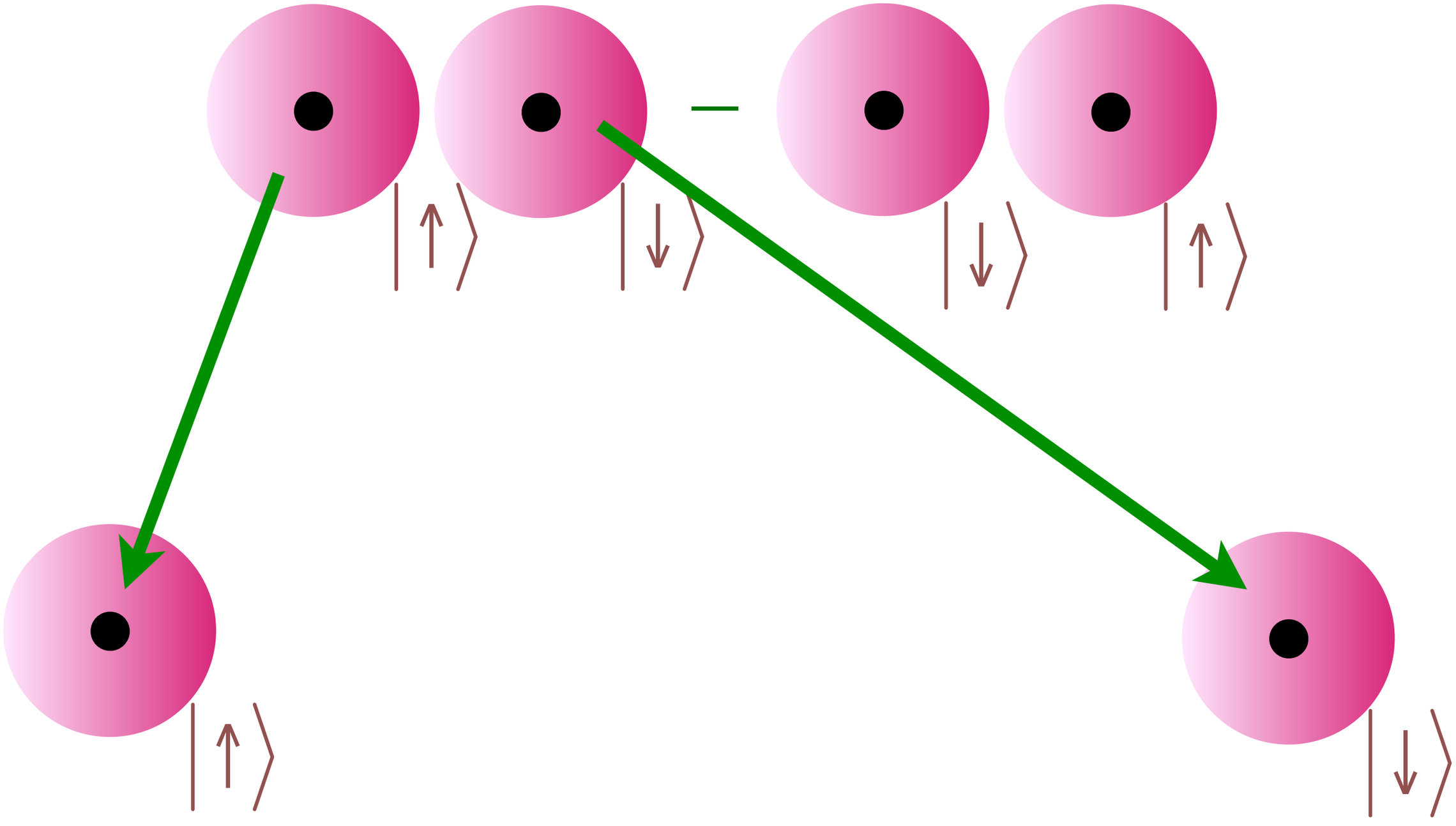

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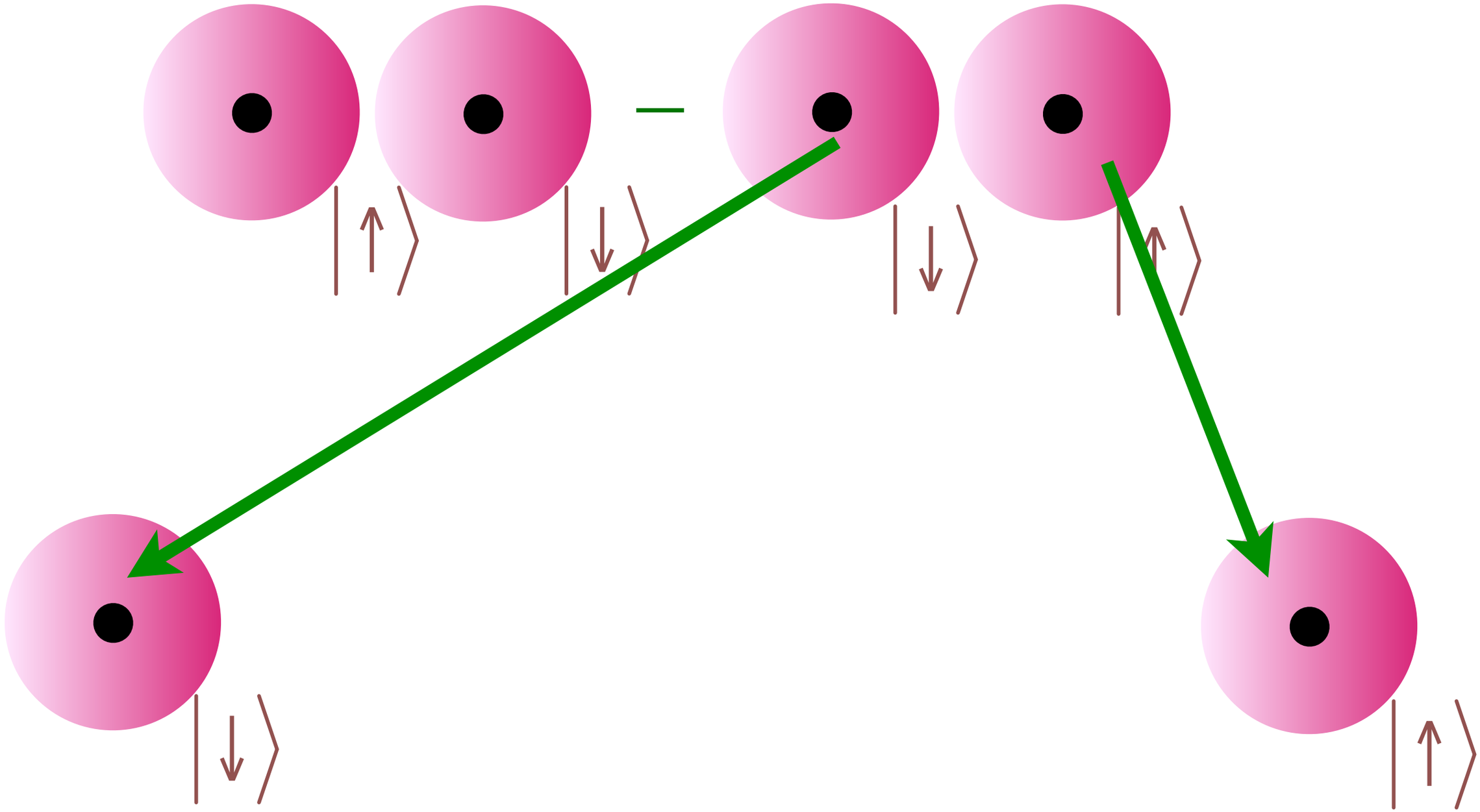
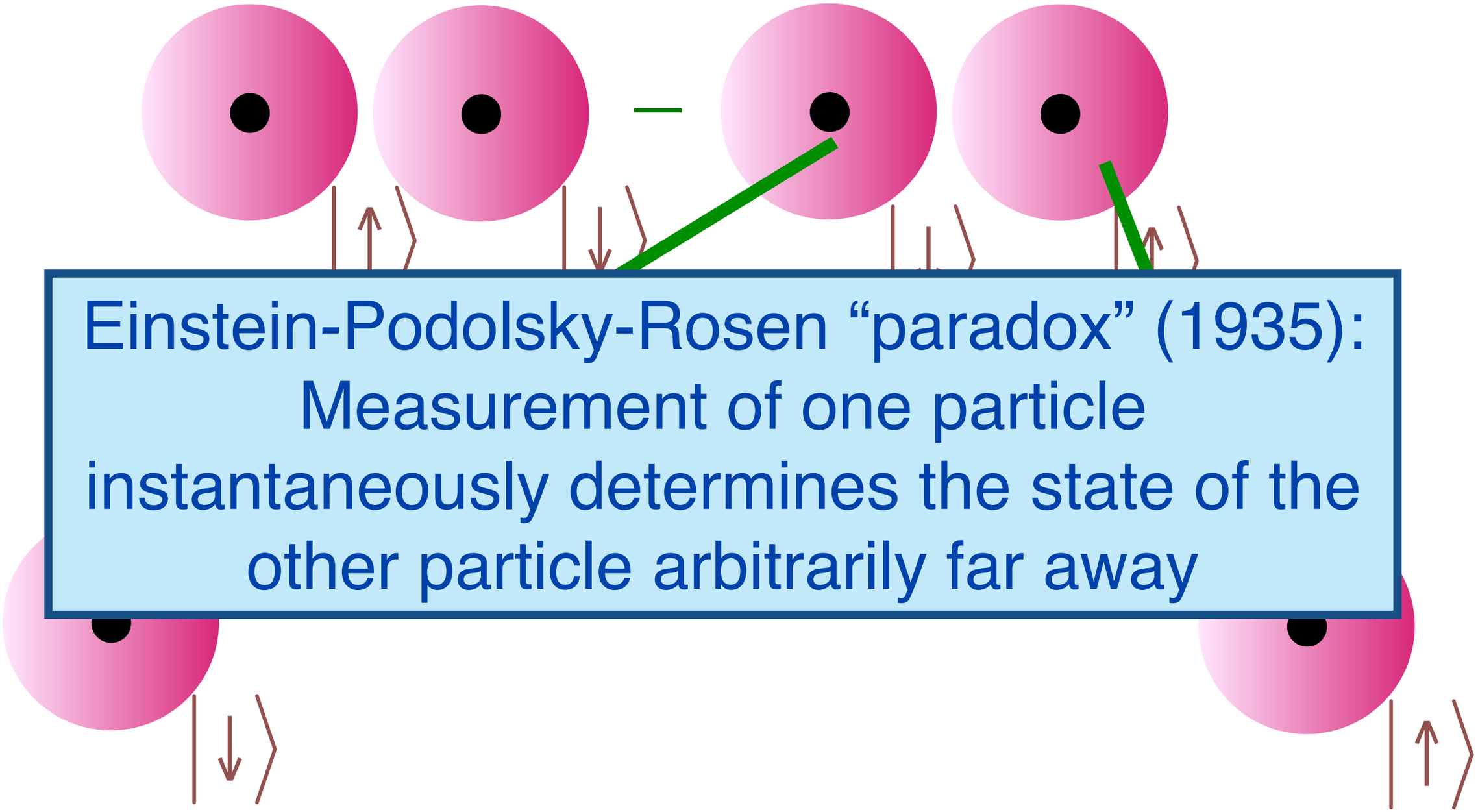


Quantum Entanglement: quantum superposition with more than one particle



Einstein-Podolsky-Rosen "paradox" (1935): Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away

Quantum entanglement

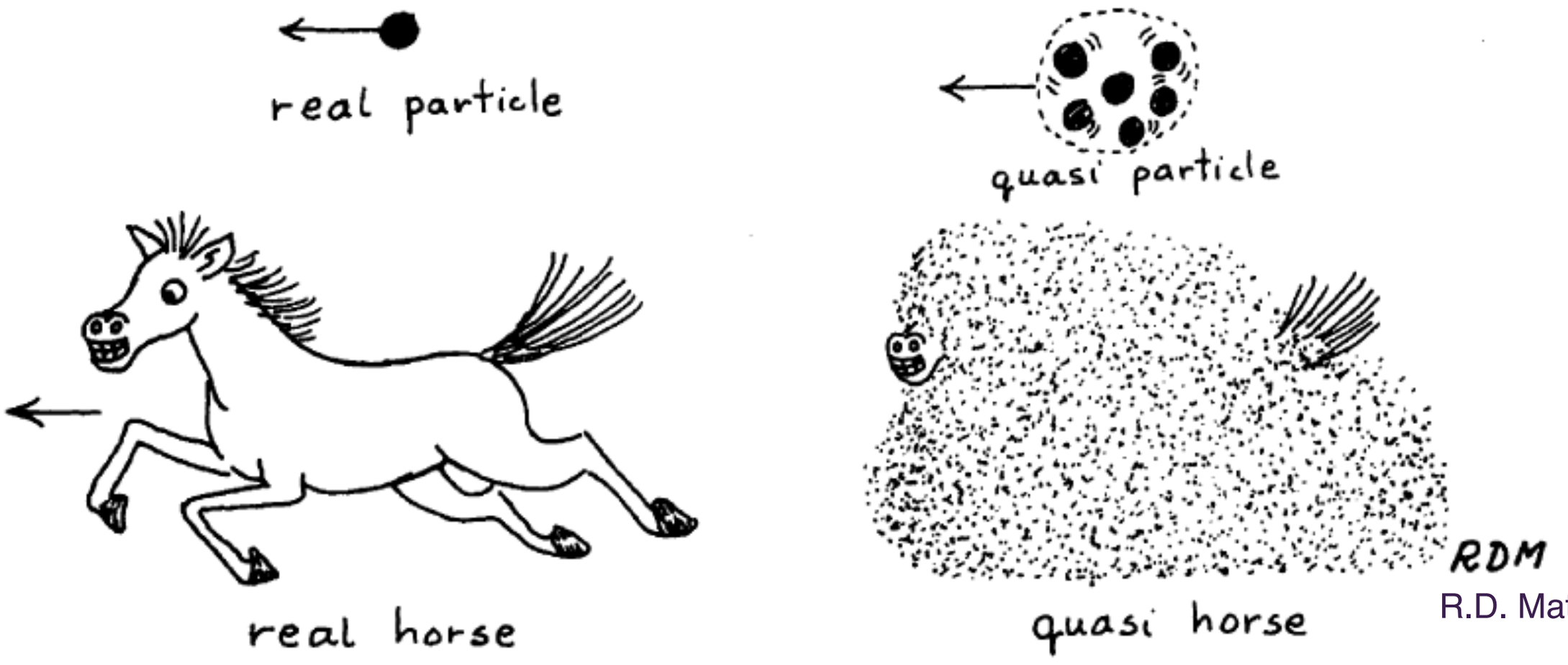
Quantum entanglement

Strange metals

Ordinary metals

Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal

Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.



R.D. Mattuck

Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\rm eq} \sim \frac{\hbar E_F}{(k_B T)^2} \quad , \quad \text{as } T \to 0,$$

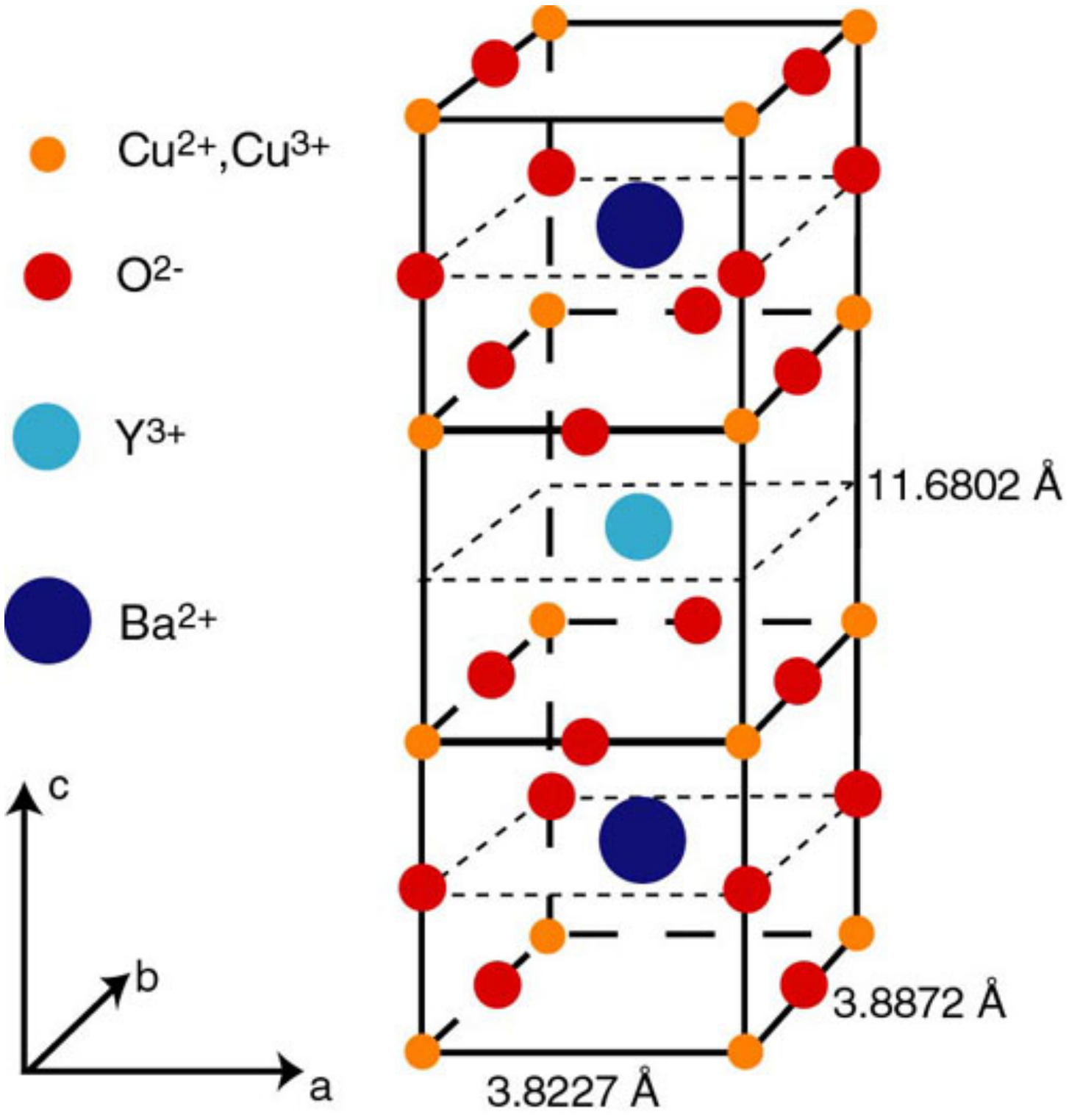
where E_F is the Fermi energy.

Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.

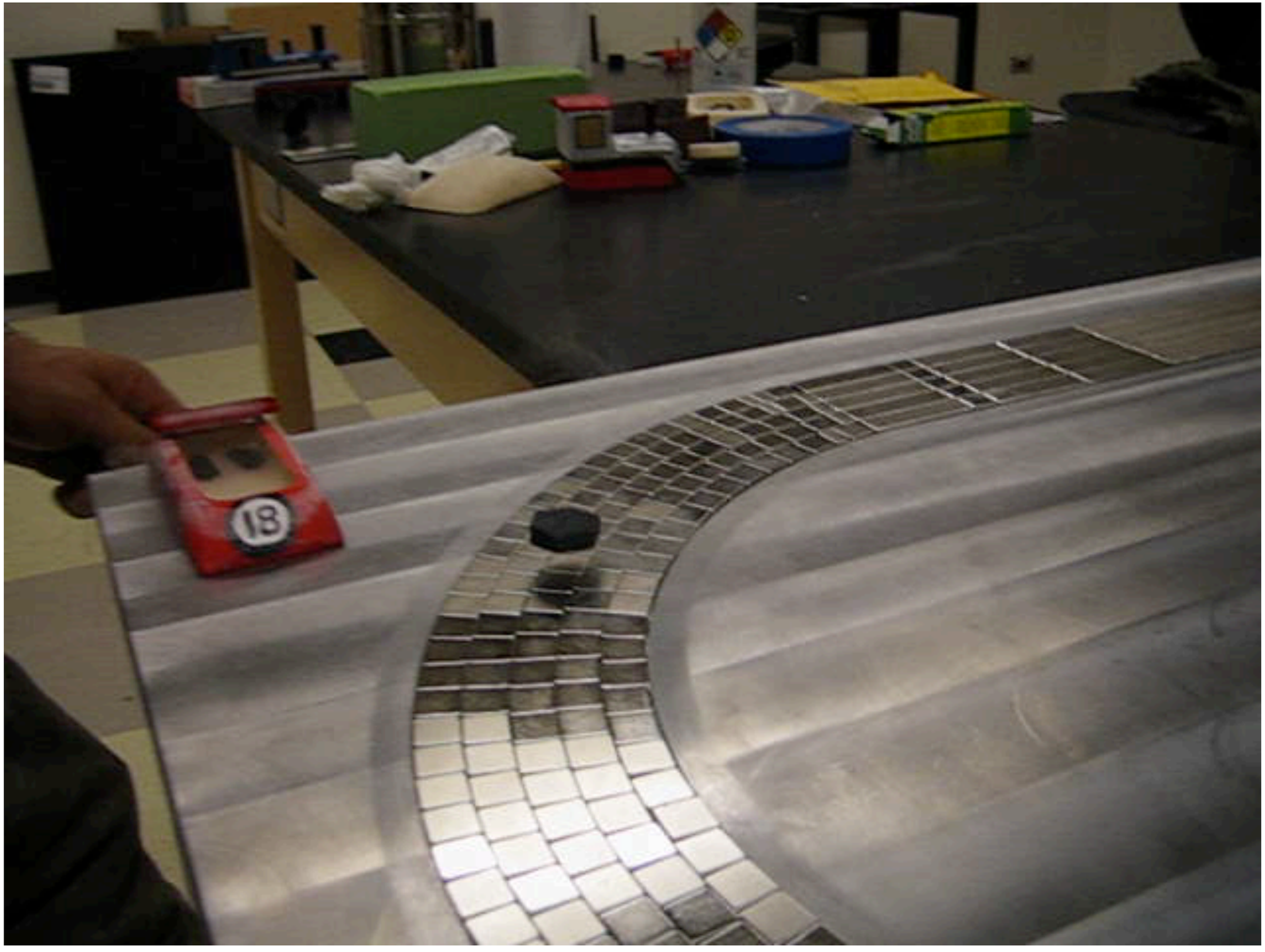
- The equilibration time for quasiparticles is much longer than the 'Planckian time' $\hbar/(k_B T)$

$$\tau_{\rm eq} \gg \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

High temperature superconductors



$$YBa_2Cu_3O_{6+x}$$



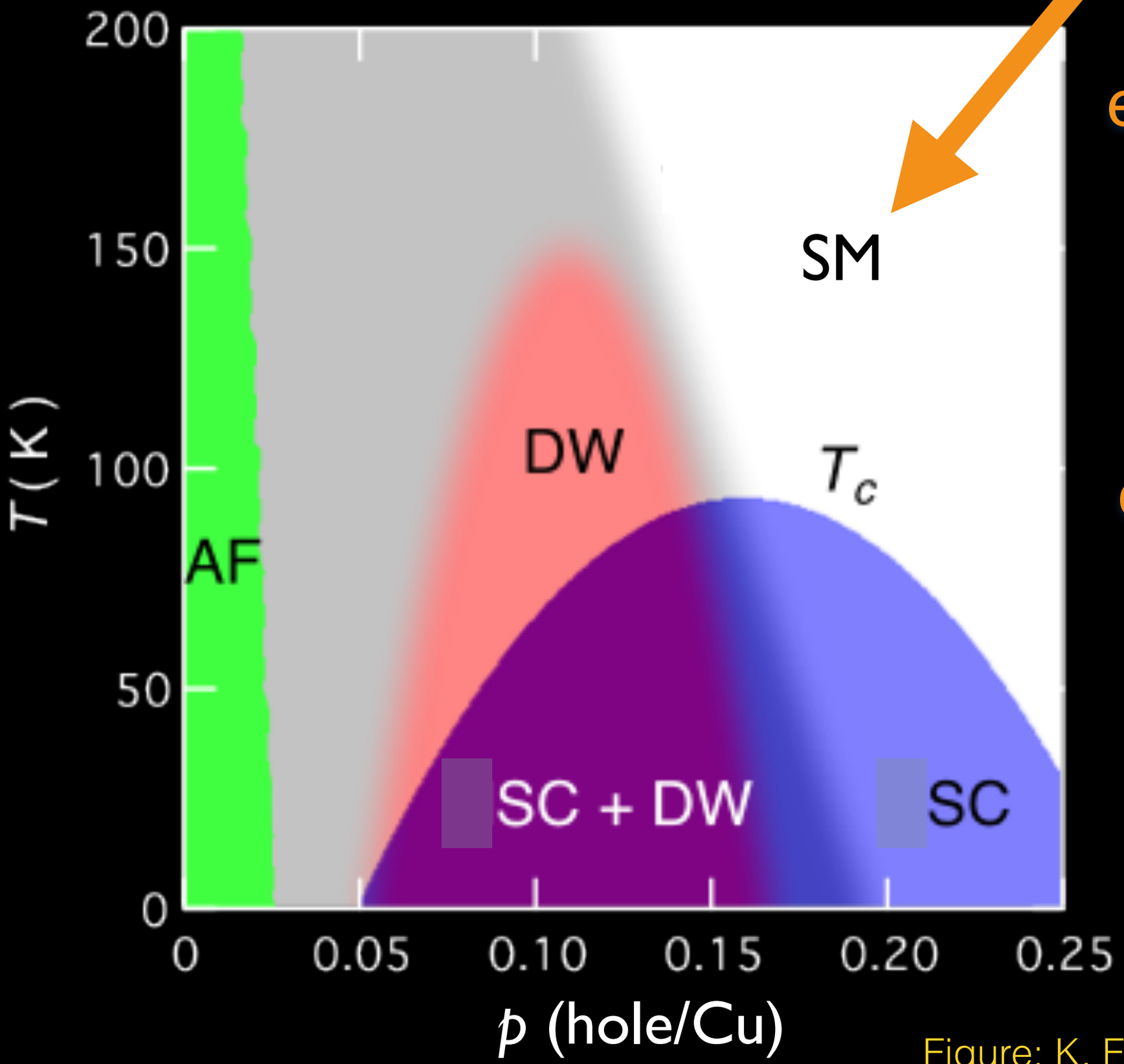
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

Quantum matter <u>without</u> quasiparticles

Strange metal

Entangled electrons lead to "strange" temperature dependence of resistivity and other properties



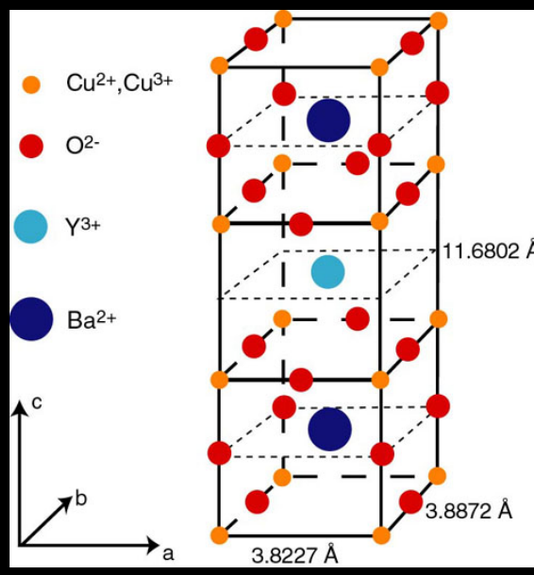


Figure: K. Fujita and J. C. Seamus Davis

"Strange",

"Bad",

or "Incoherent",

metal has a resistivity, ρ, which obeys

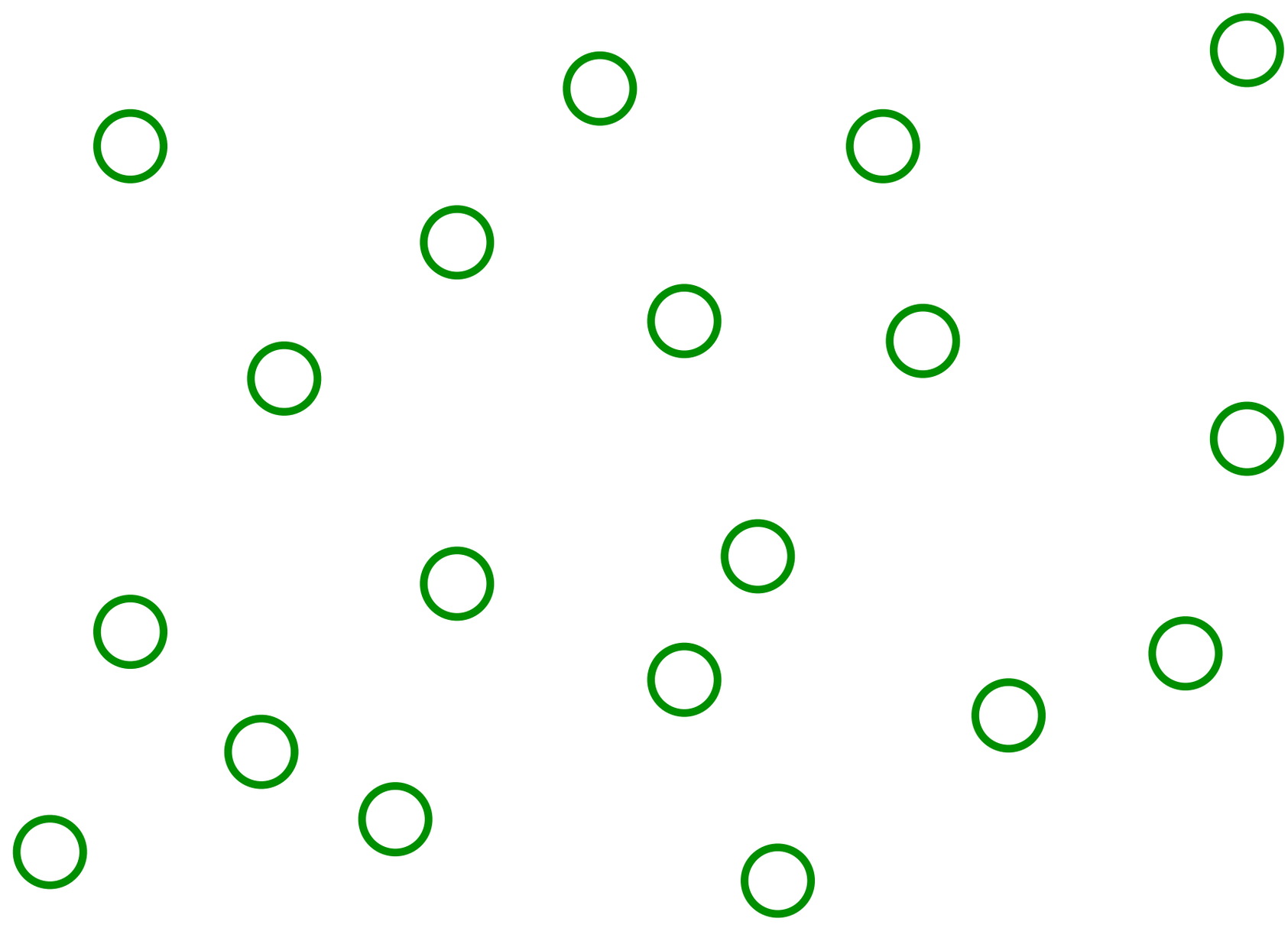
$$\rho \sim T,$$

and

in some cases $\rho \gg h/e^2$

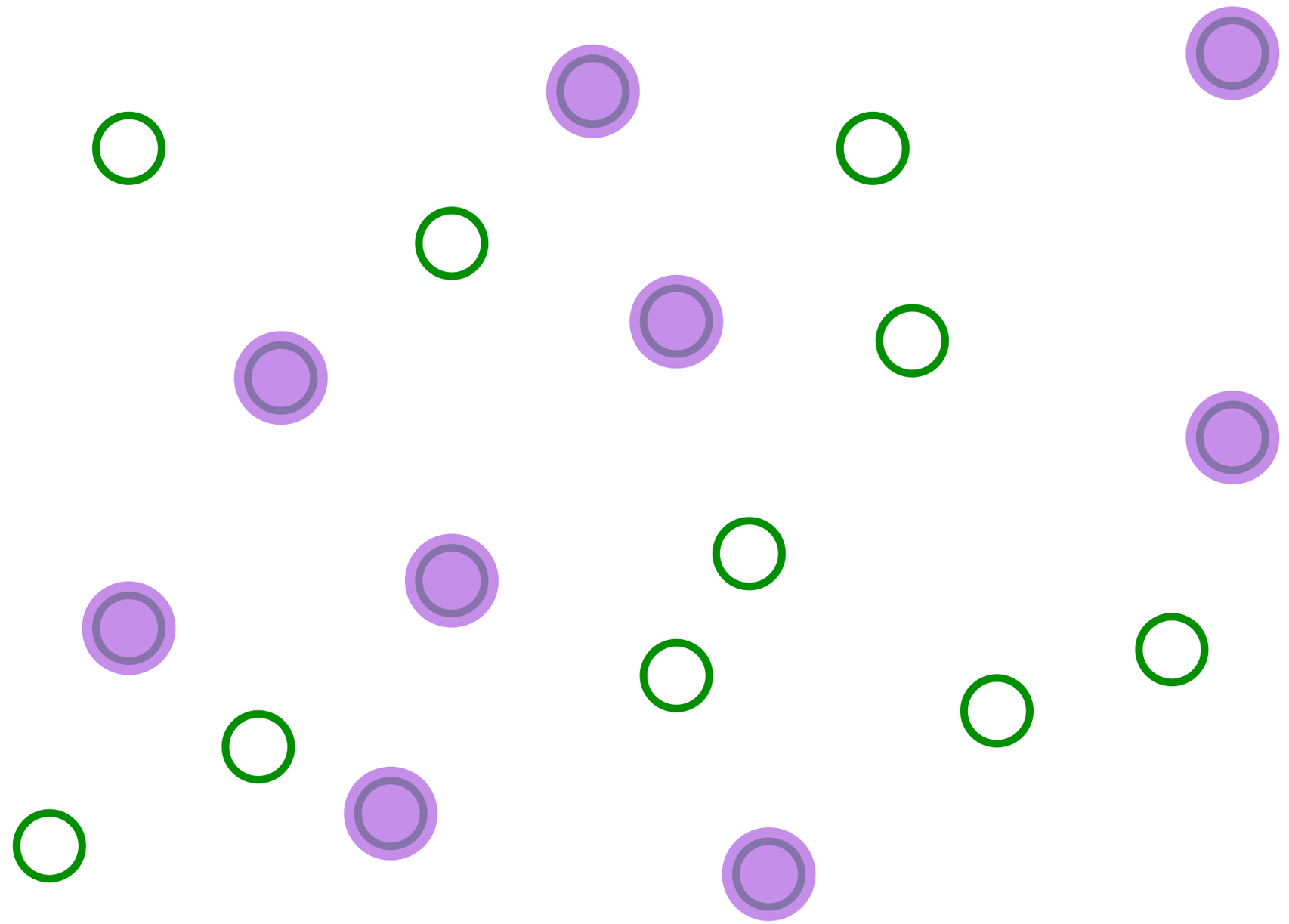
(in two dimensions),
where h/e^2 is the quantum unit of resistance.

The Sachdev-Ye-Kitaev (SYK) model



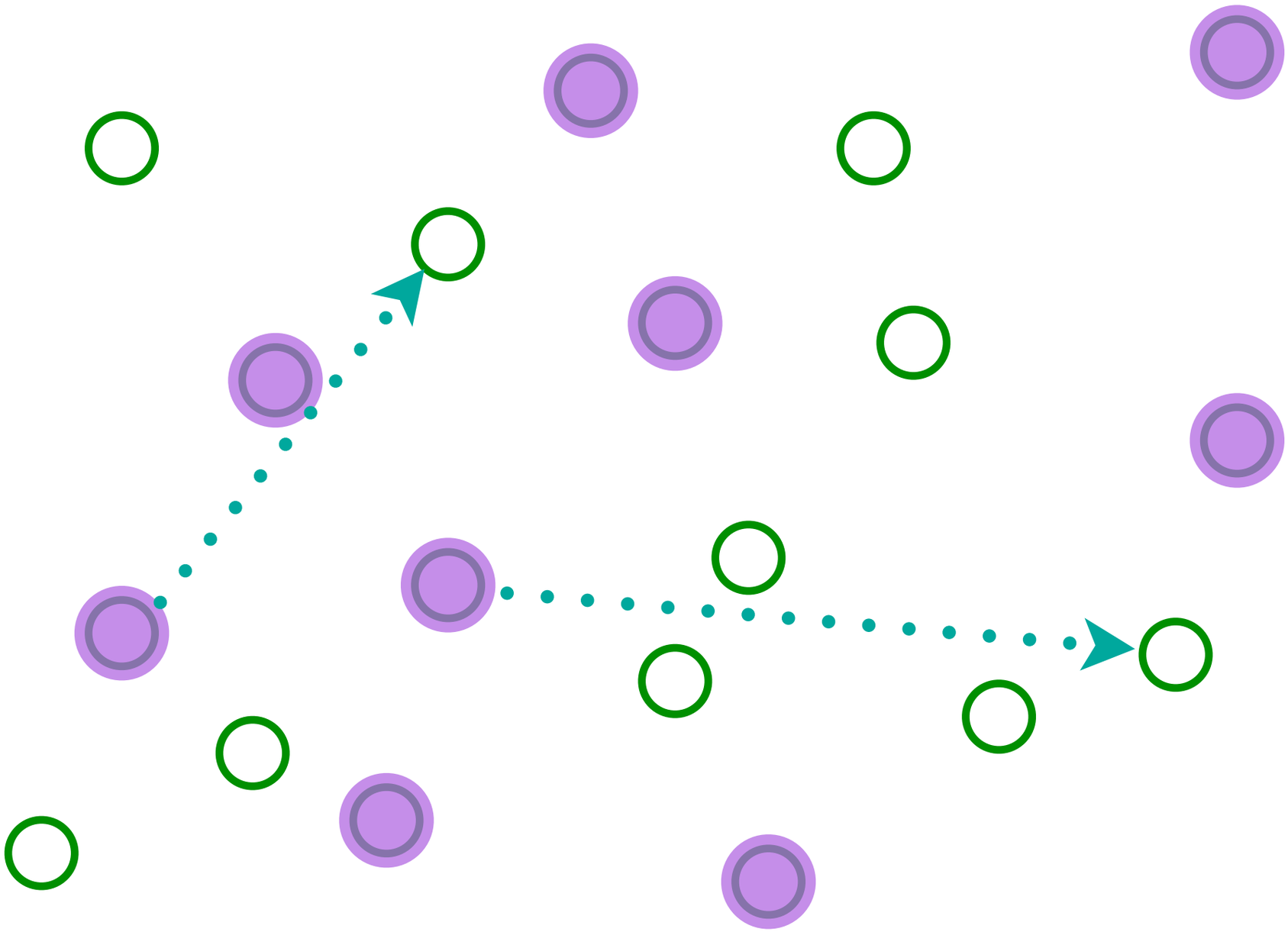
Pick a set of random positions

The SYK model



Place electrons randomly on some sites

The SYK model



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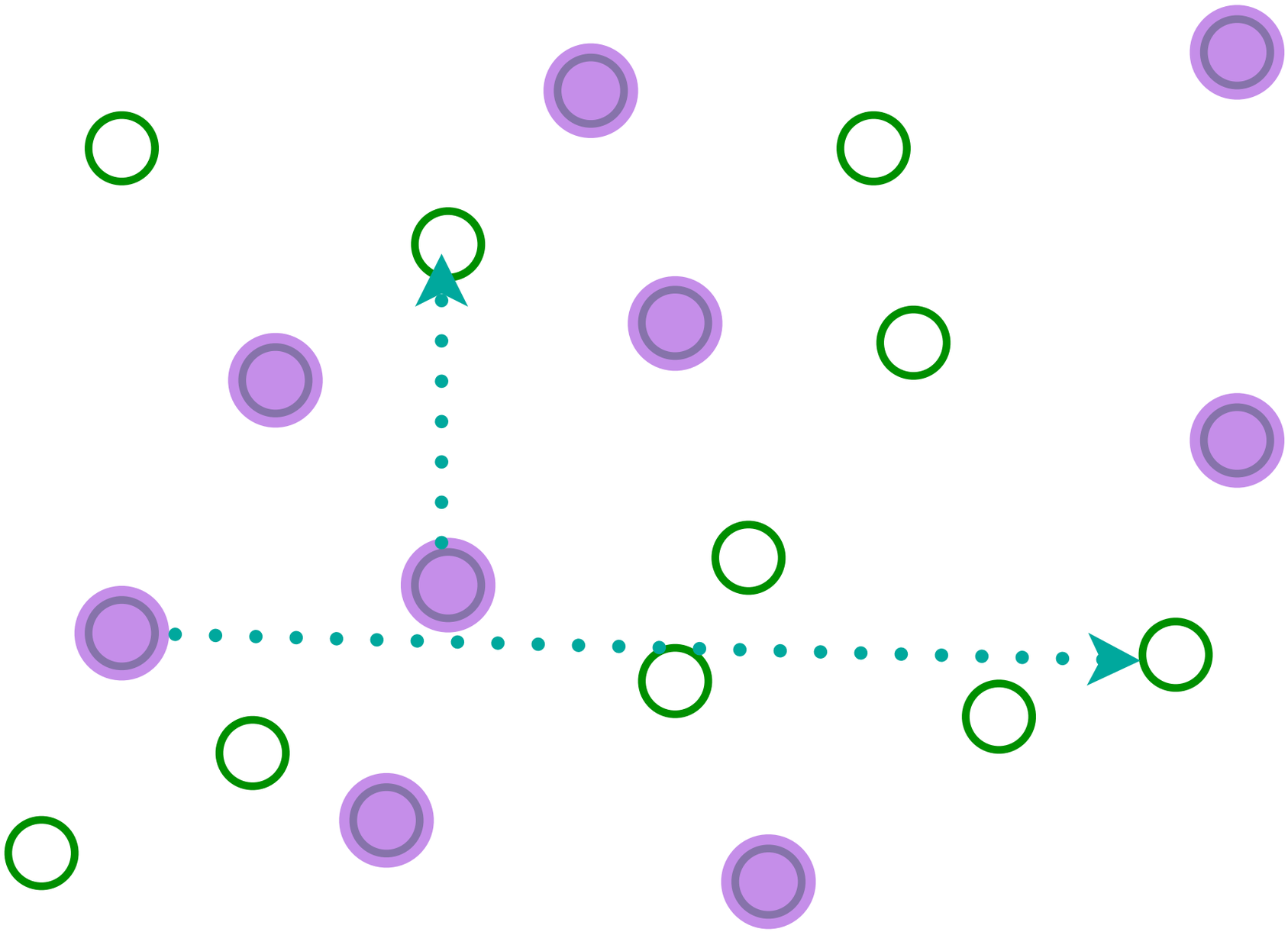

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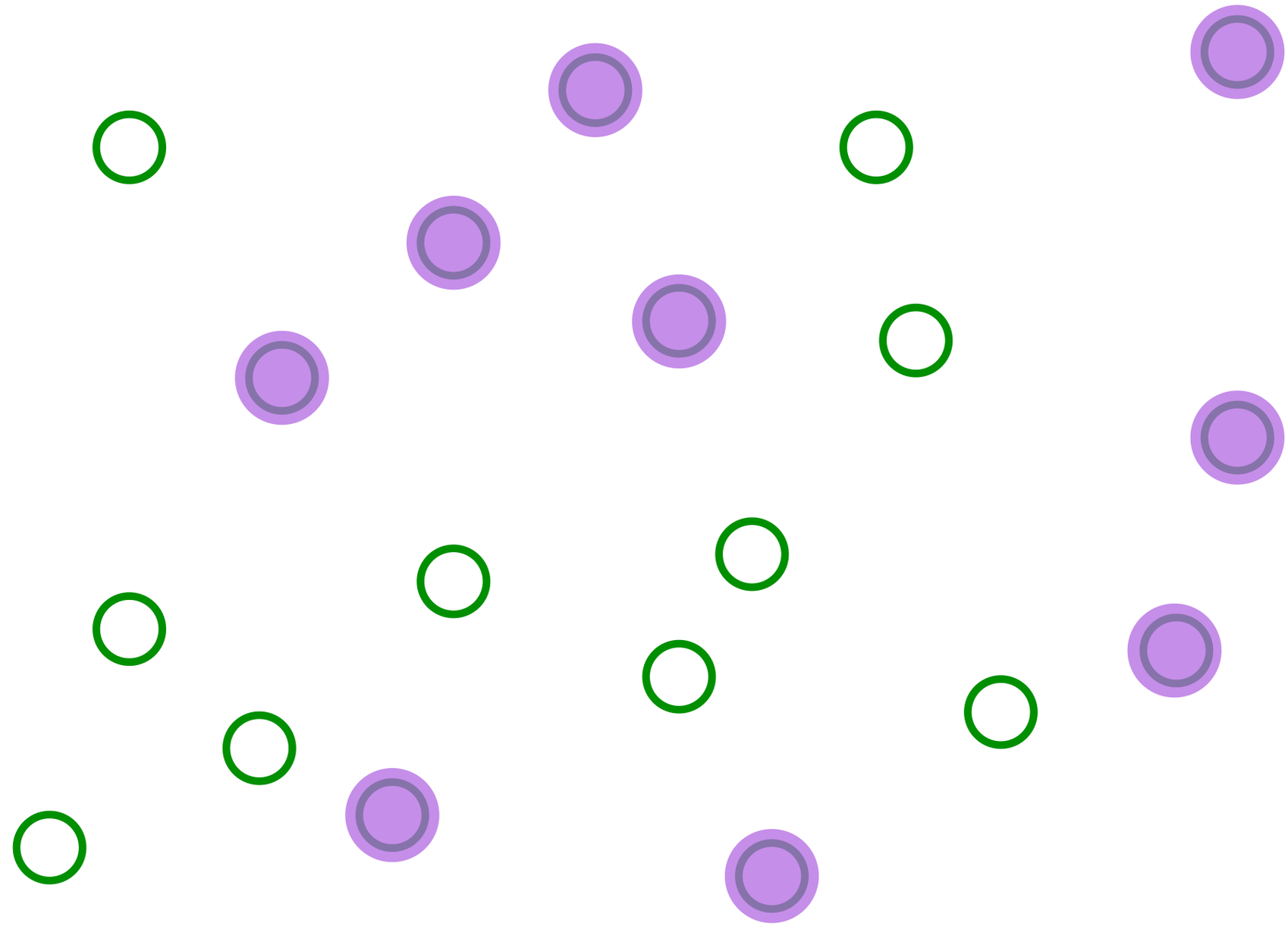

Entangle electrons pairwise randomly

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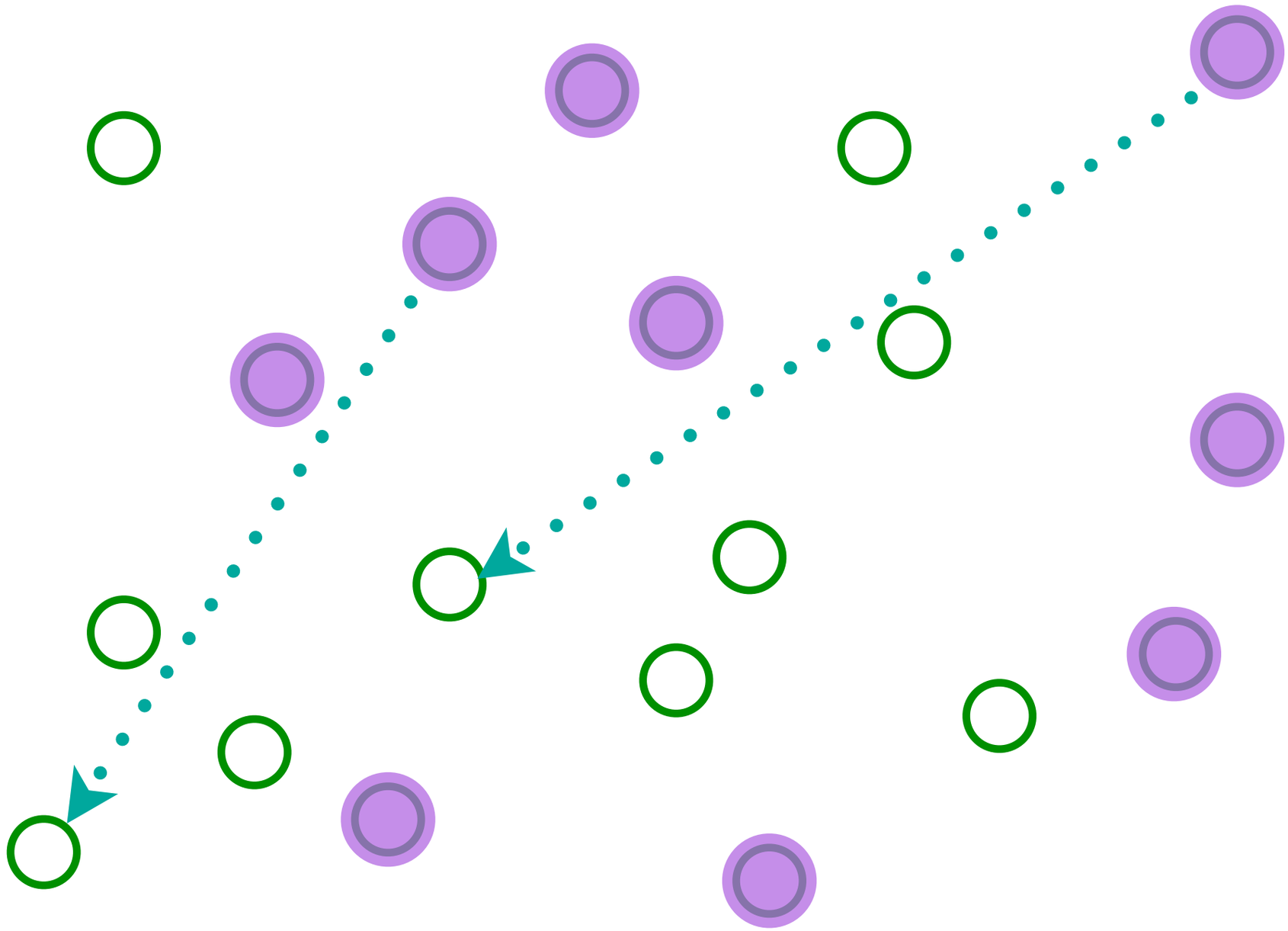

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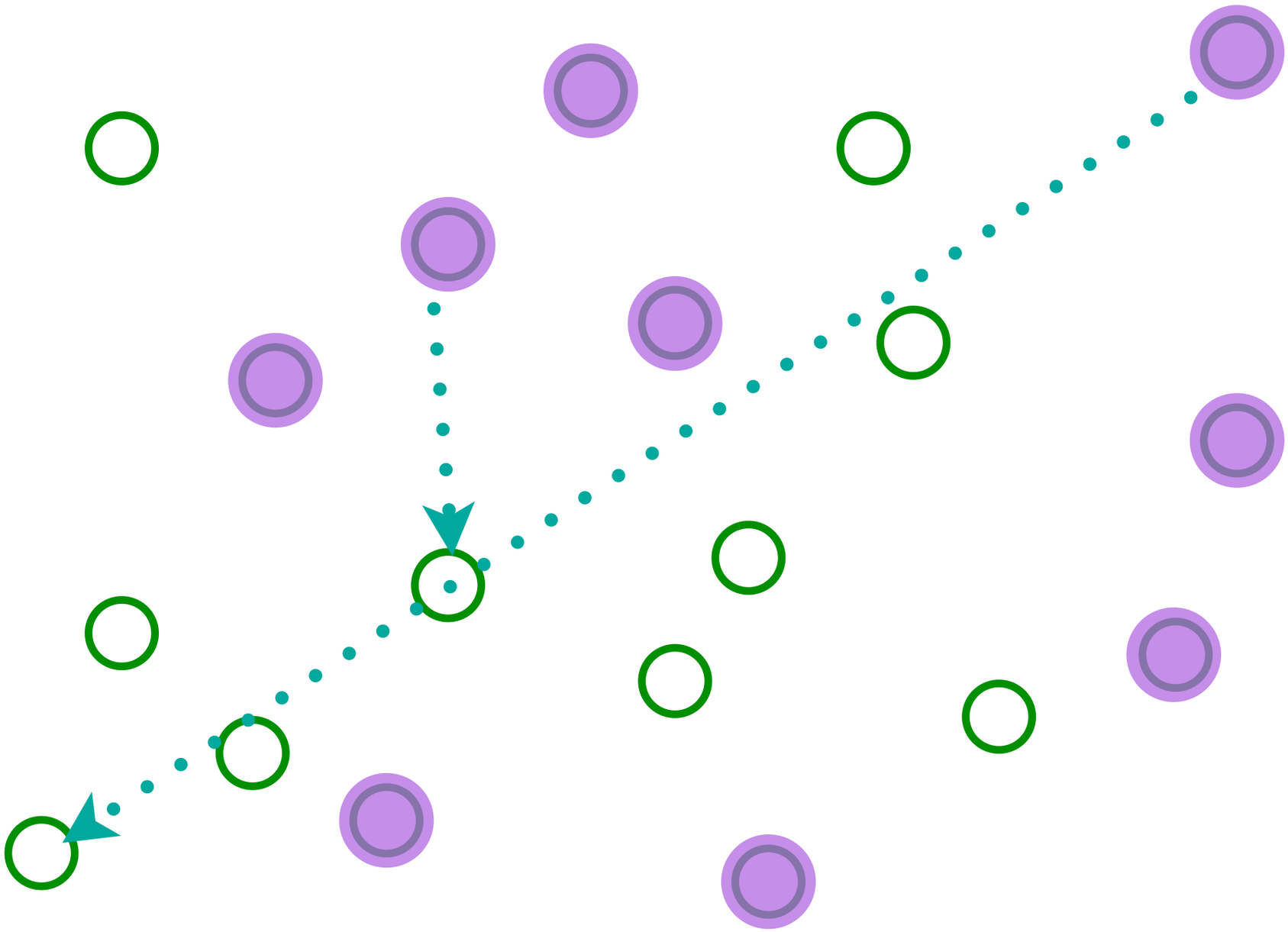

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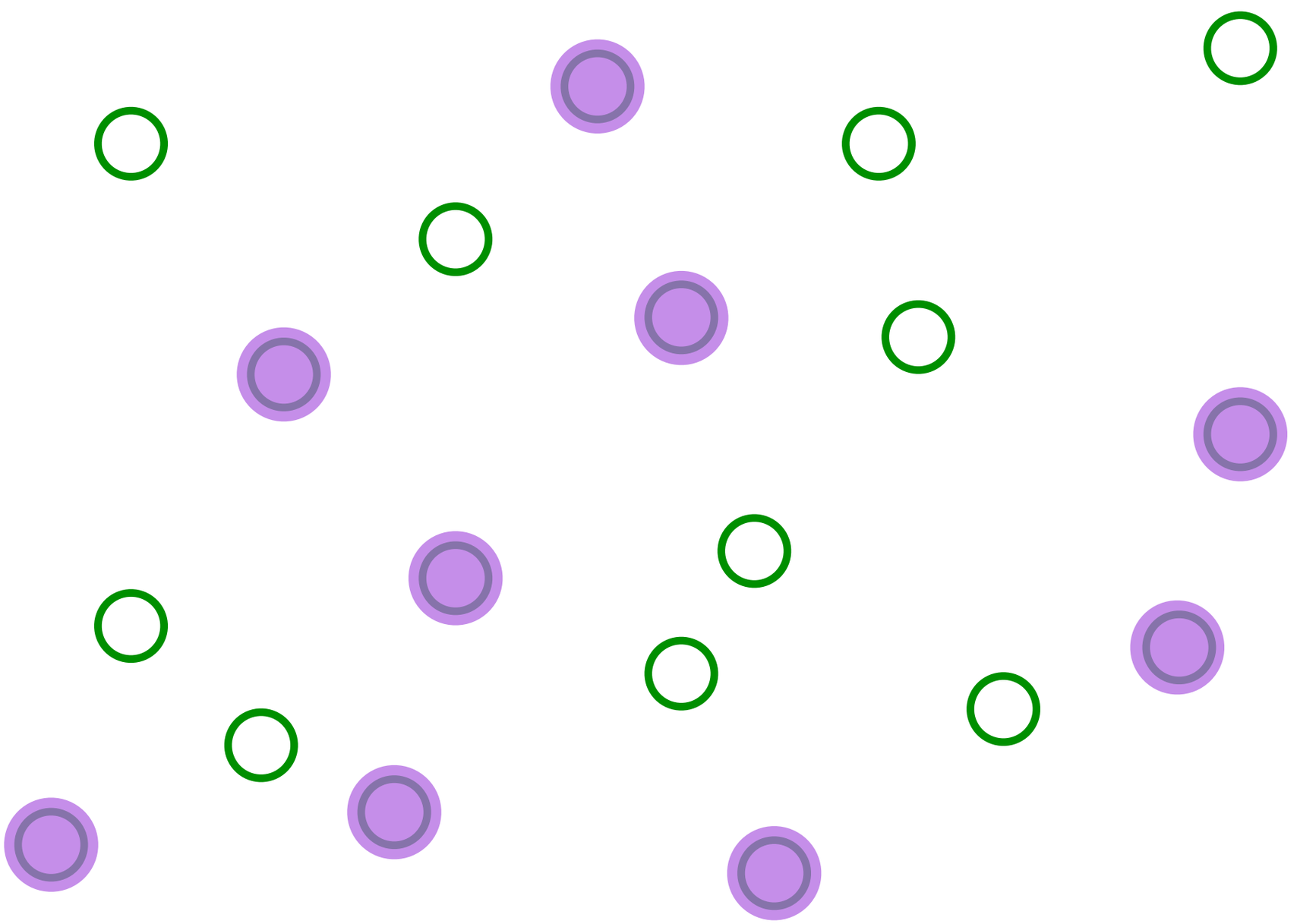

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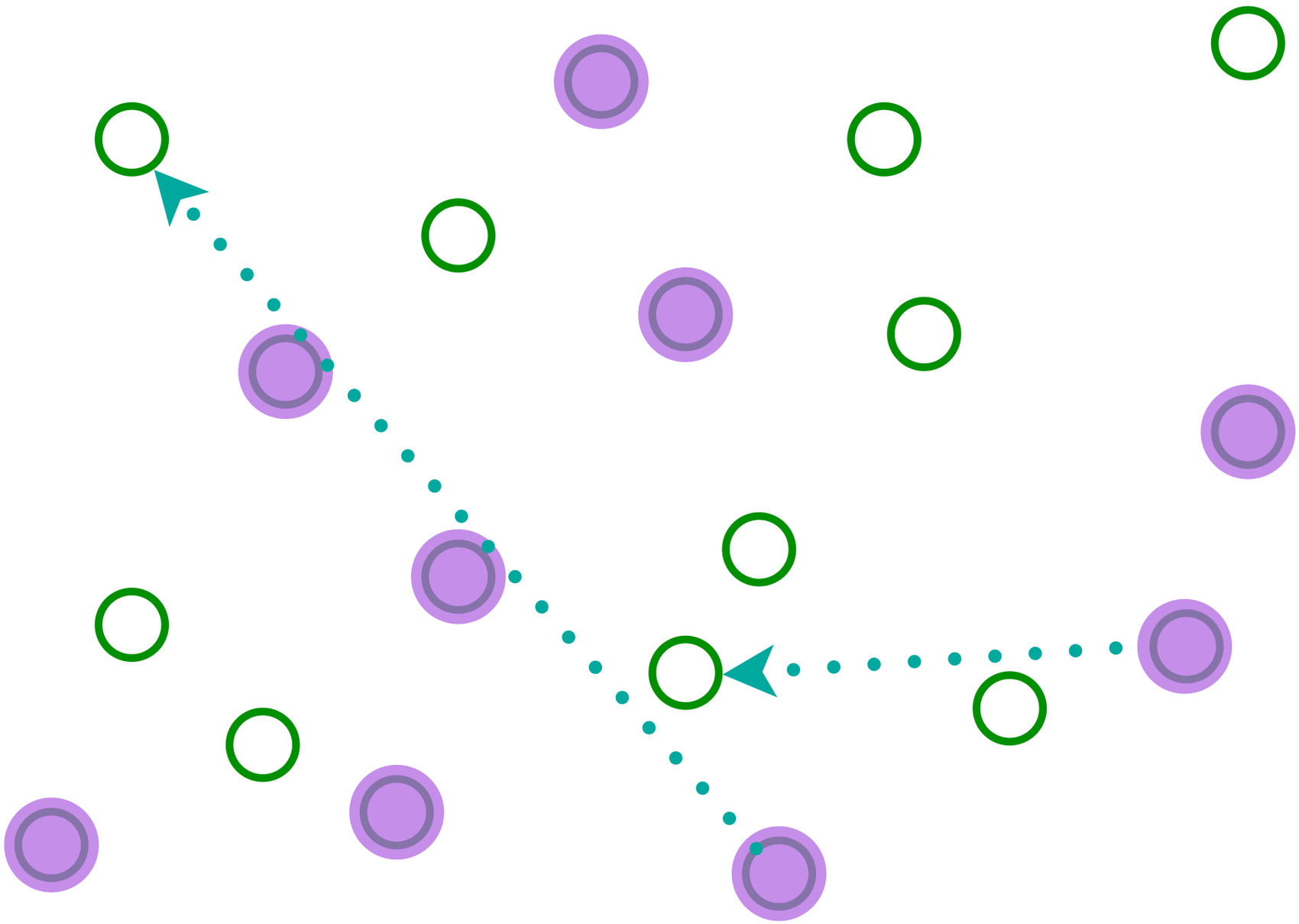

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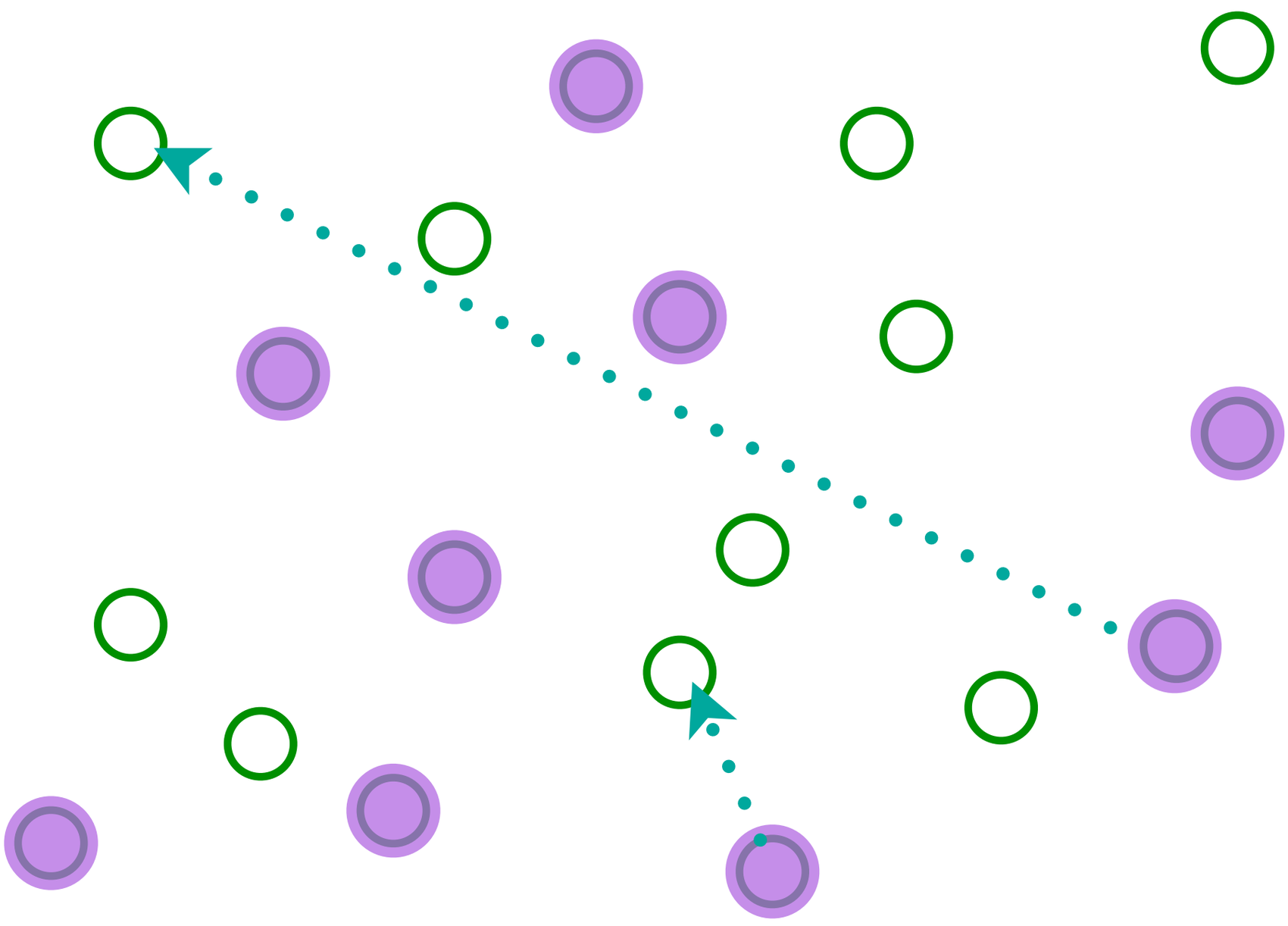

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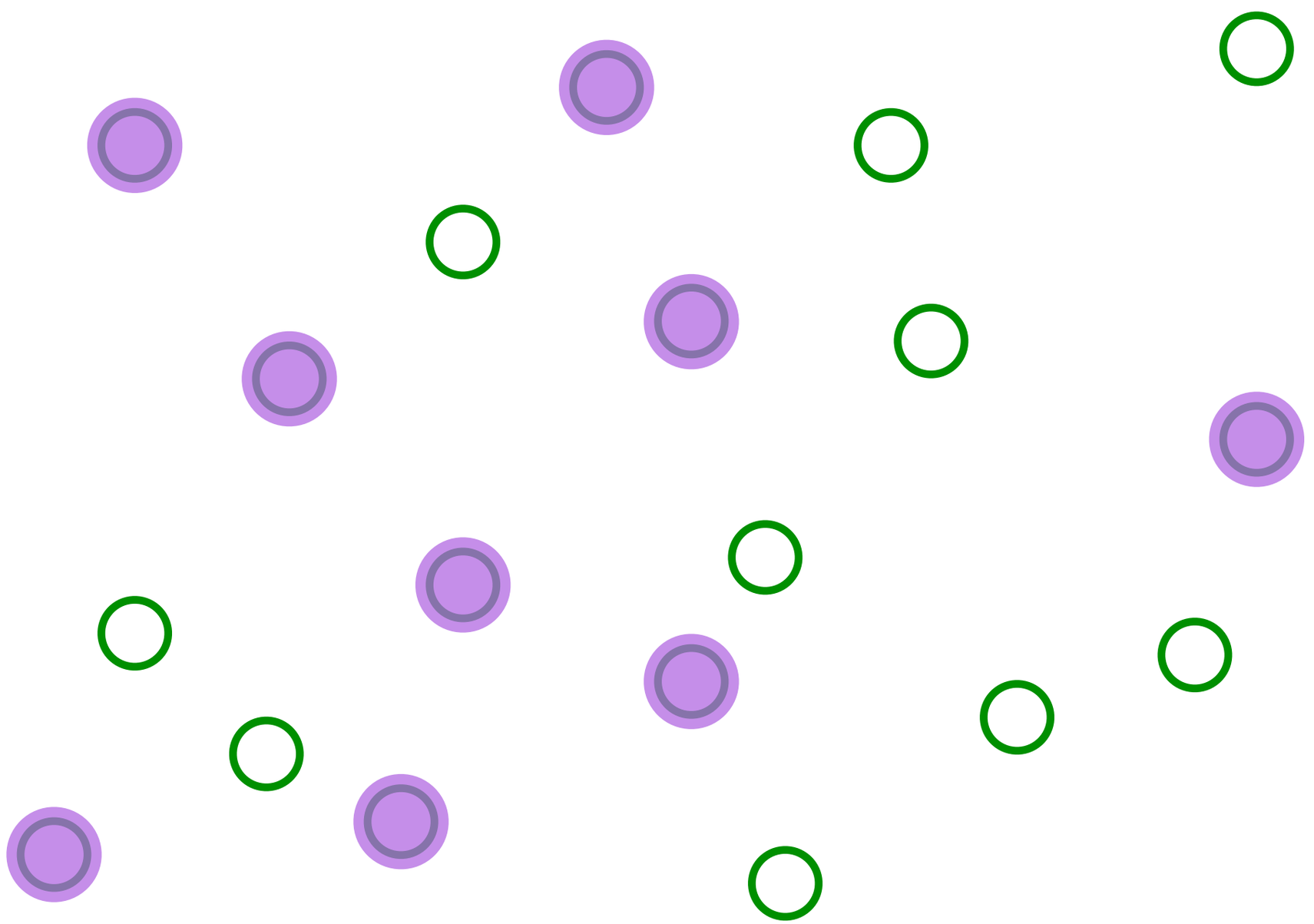

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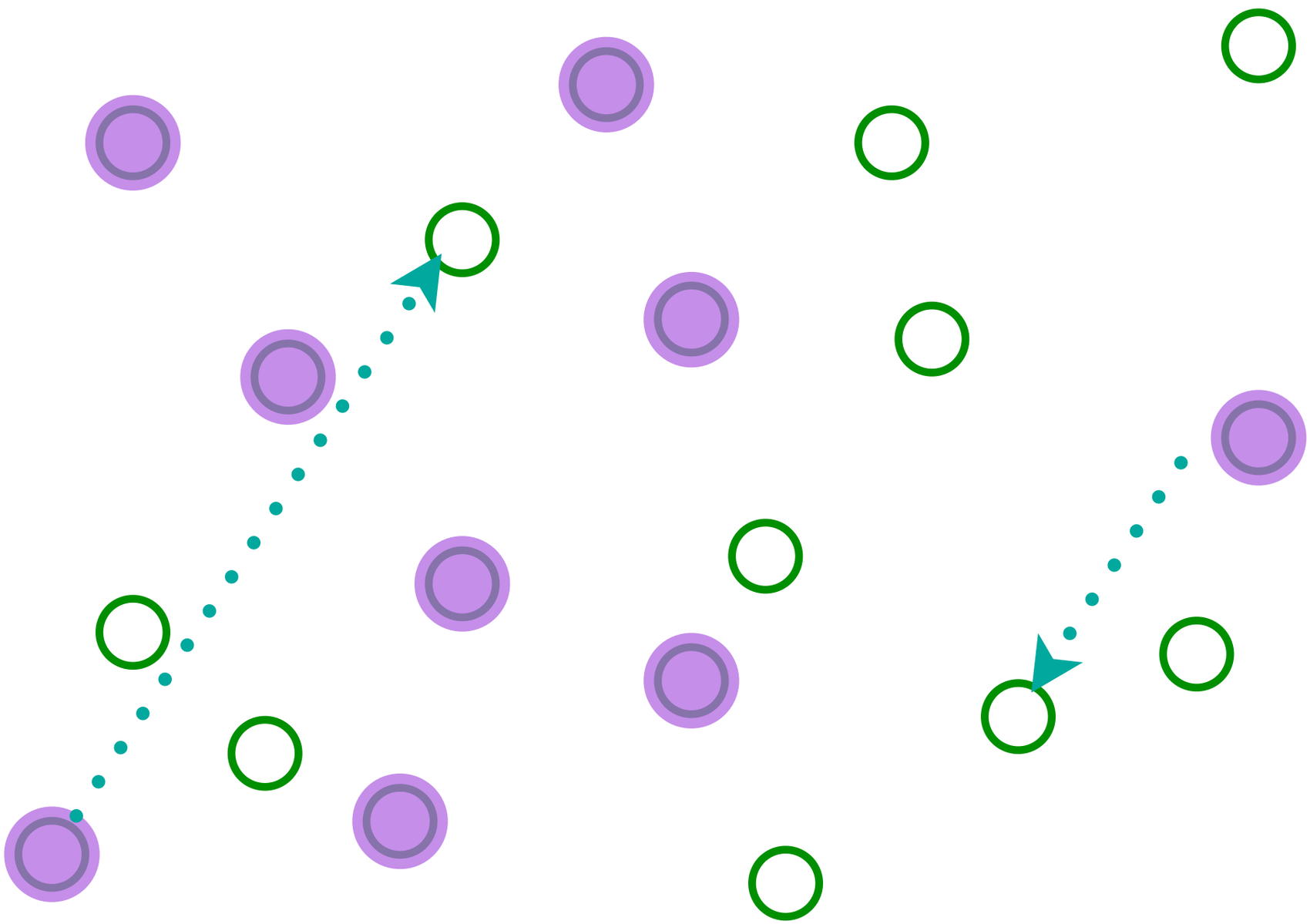

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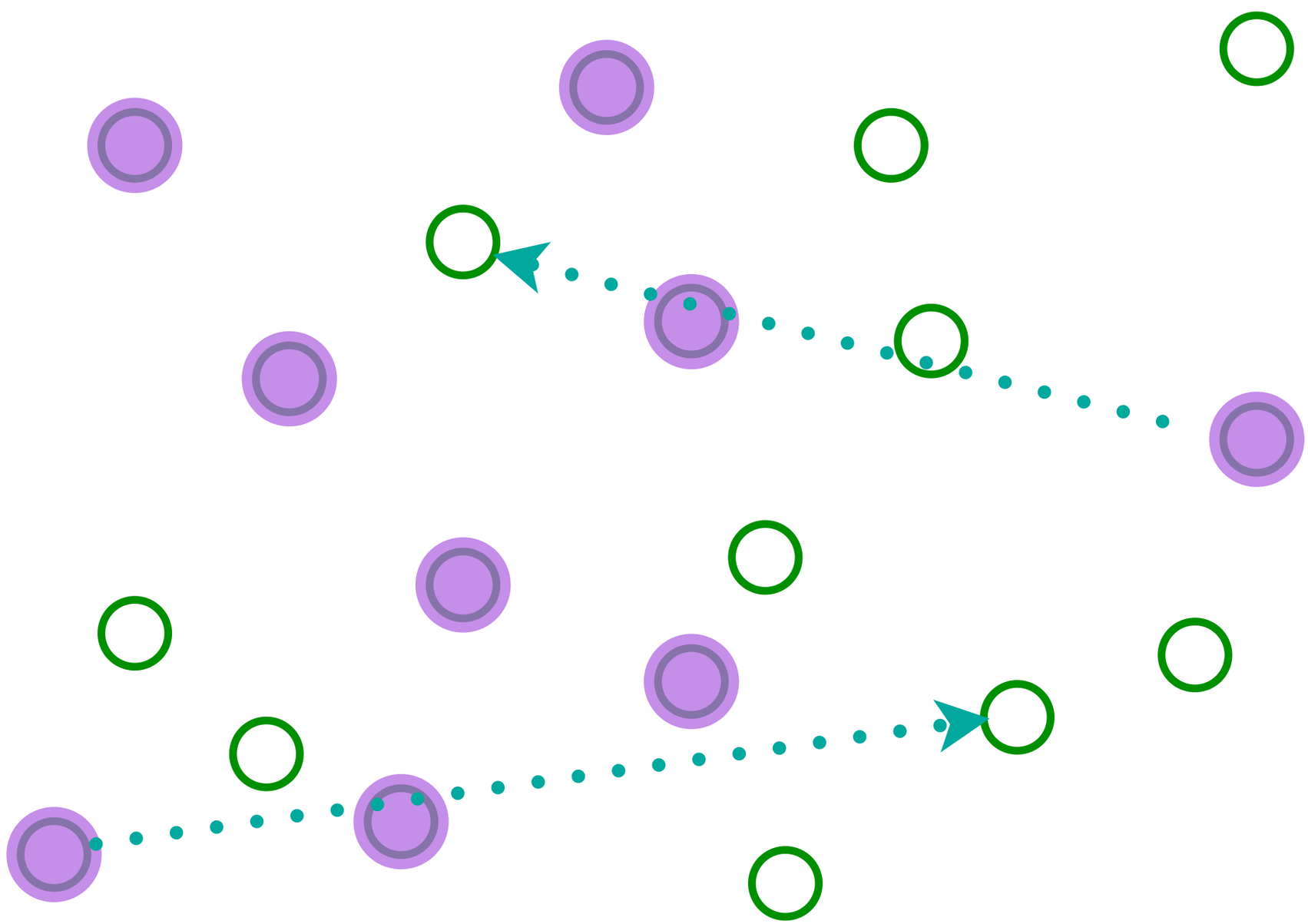

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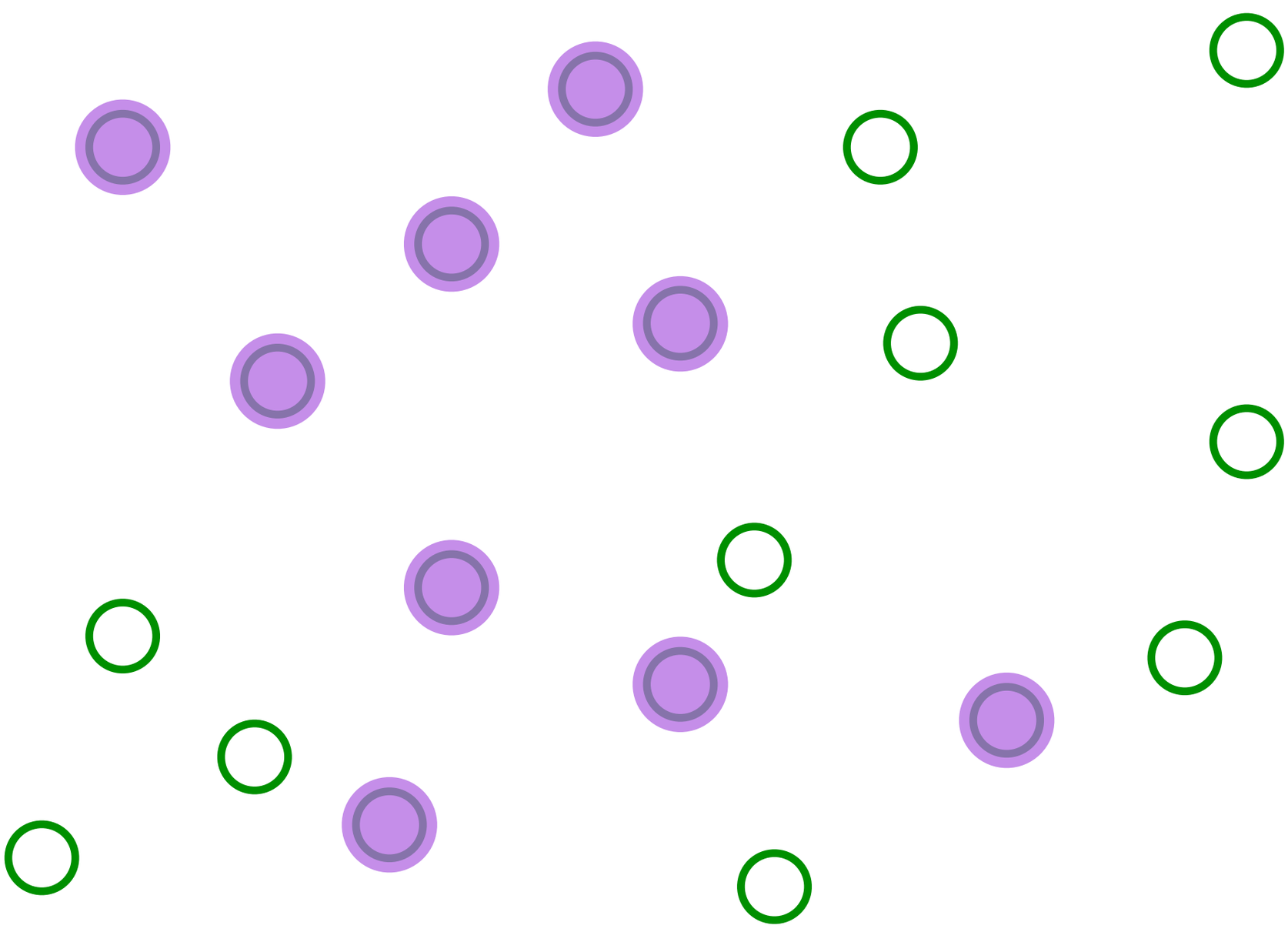

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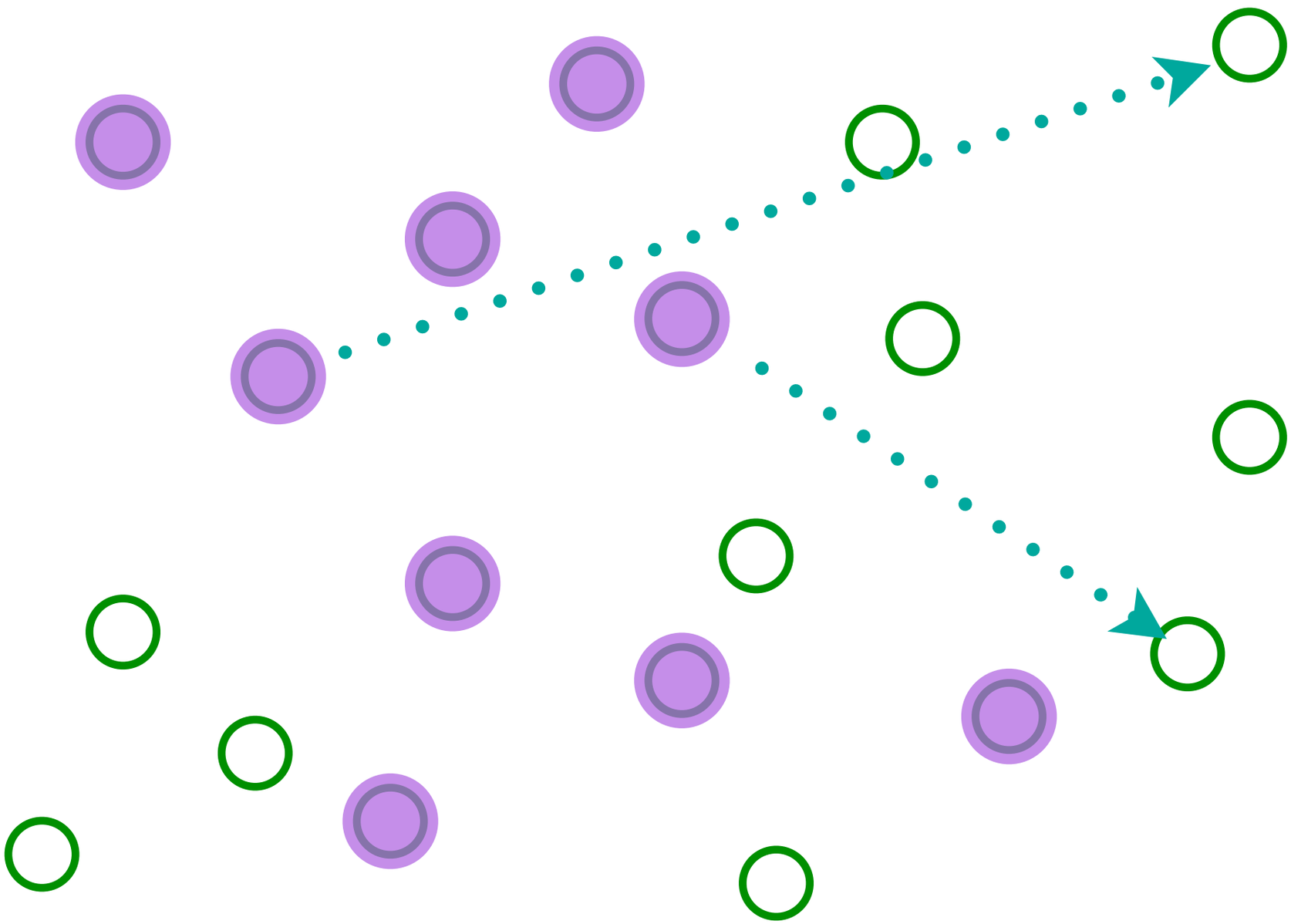

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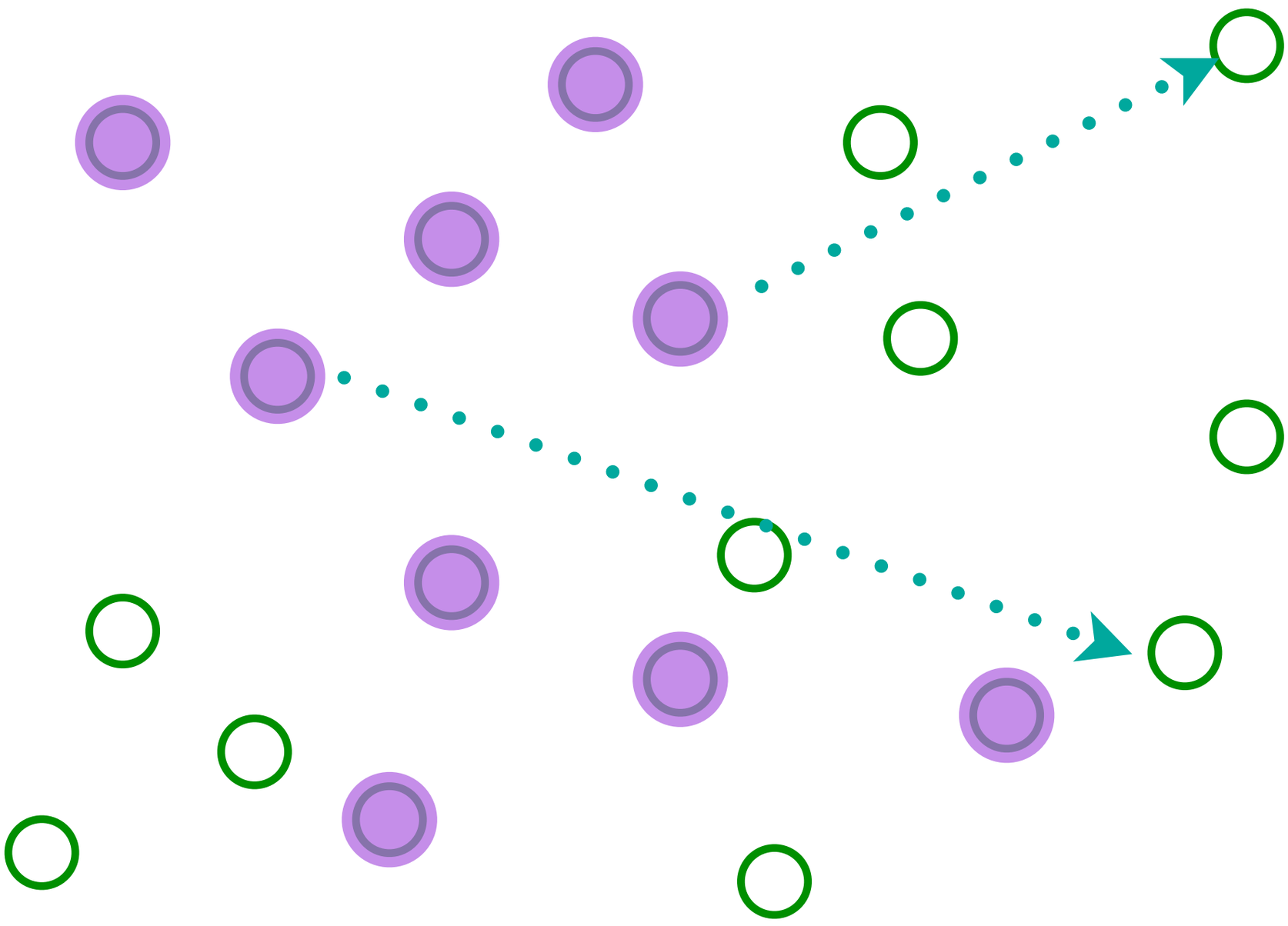

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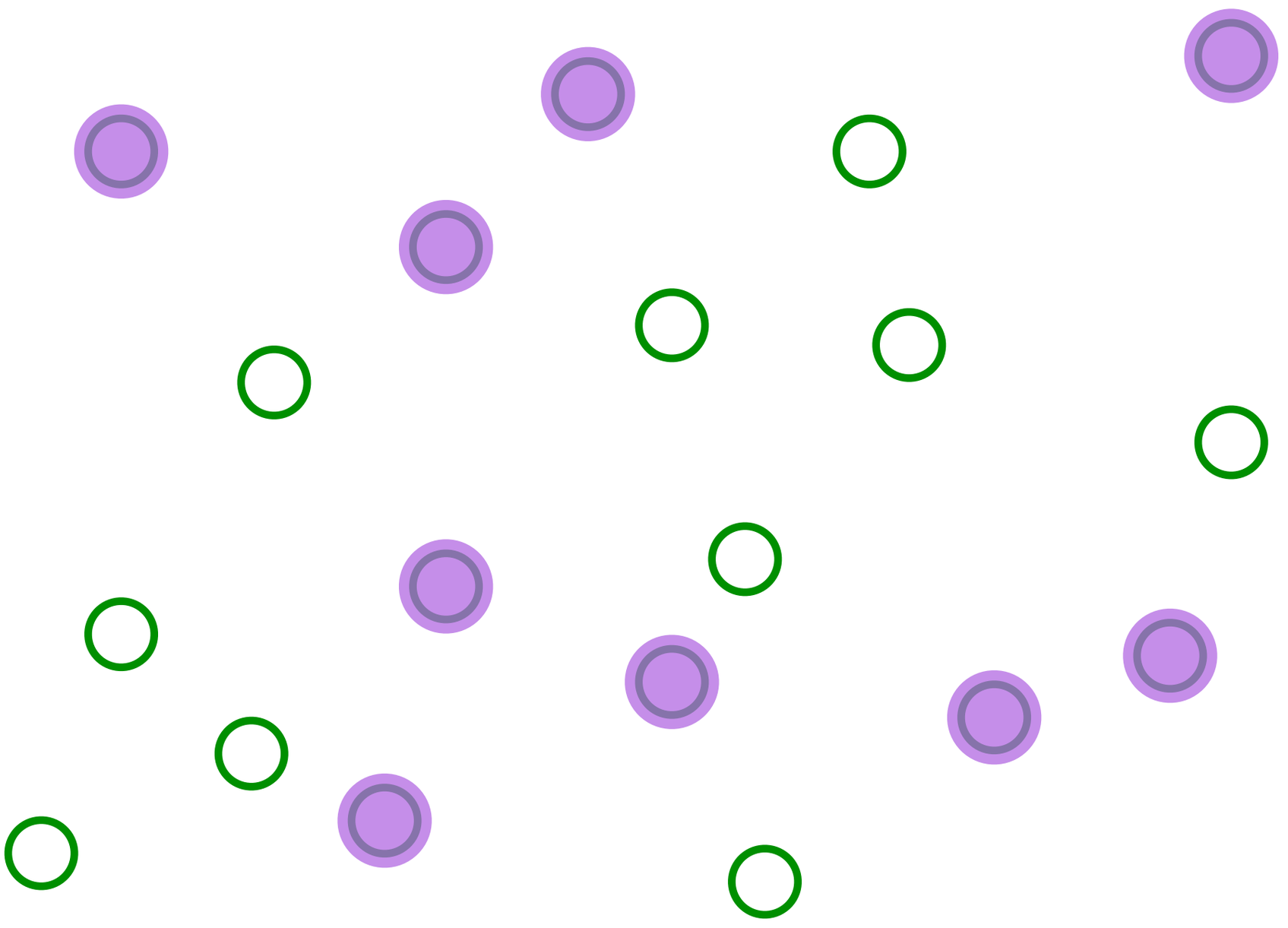

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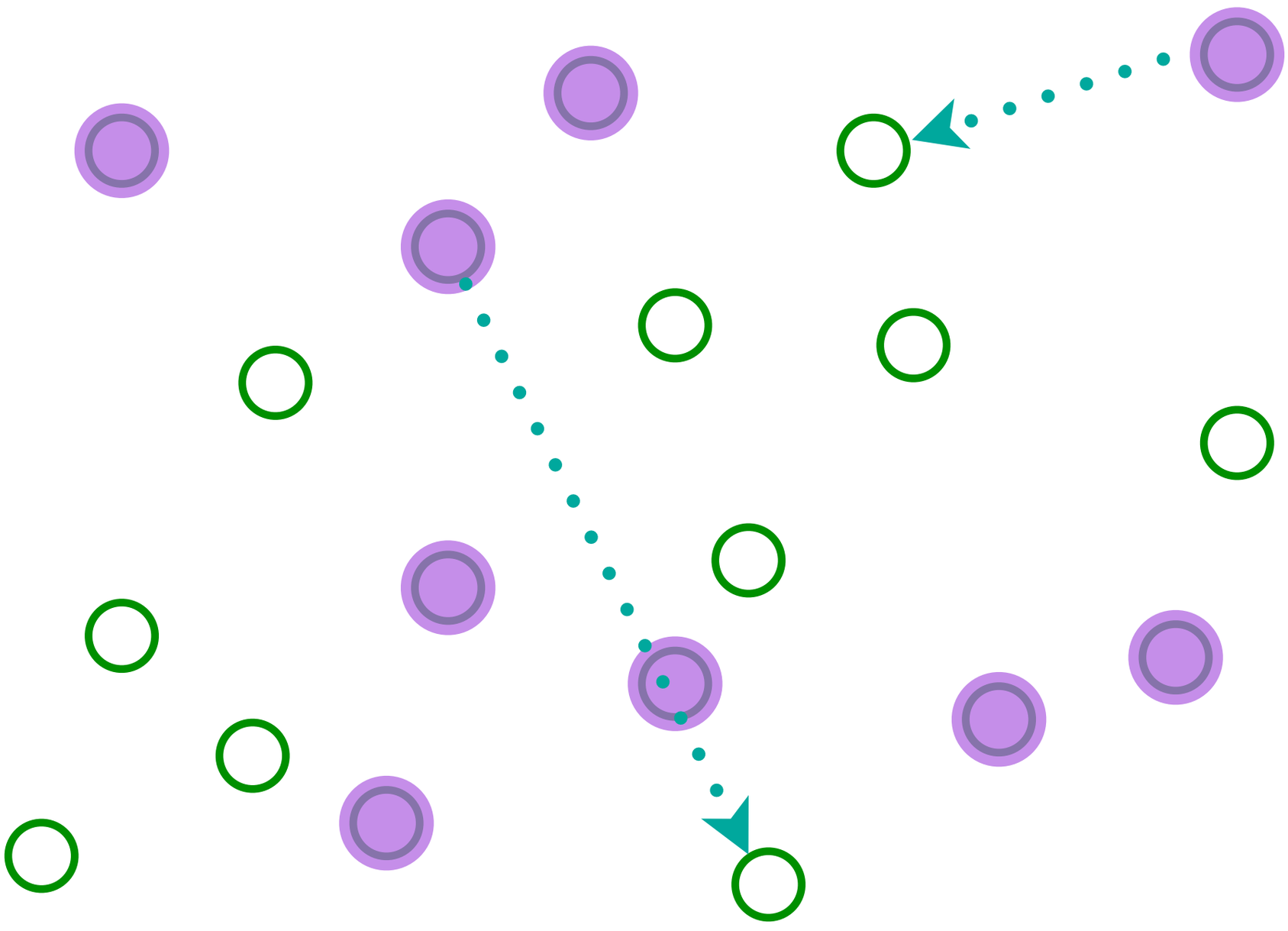

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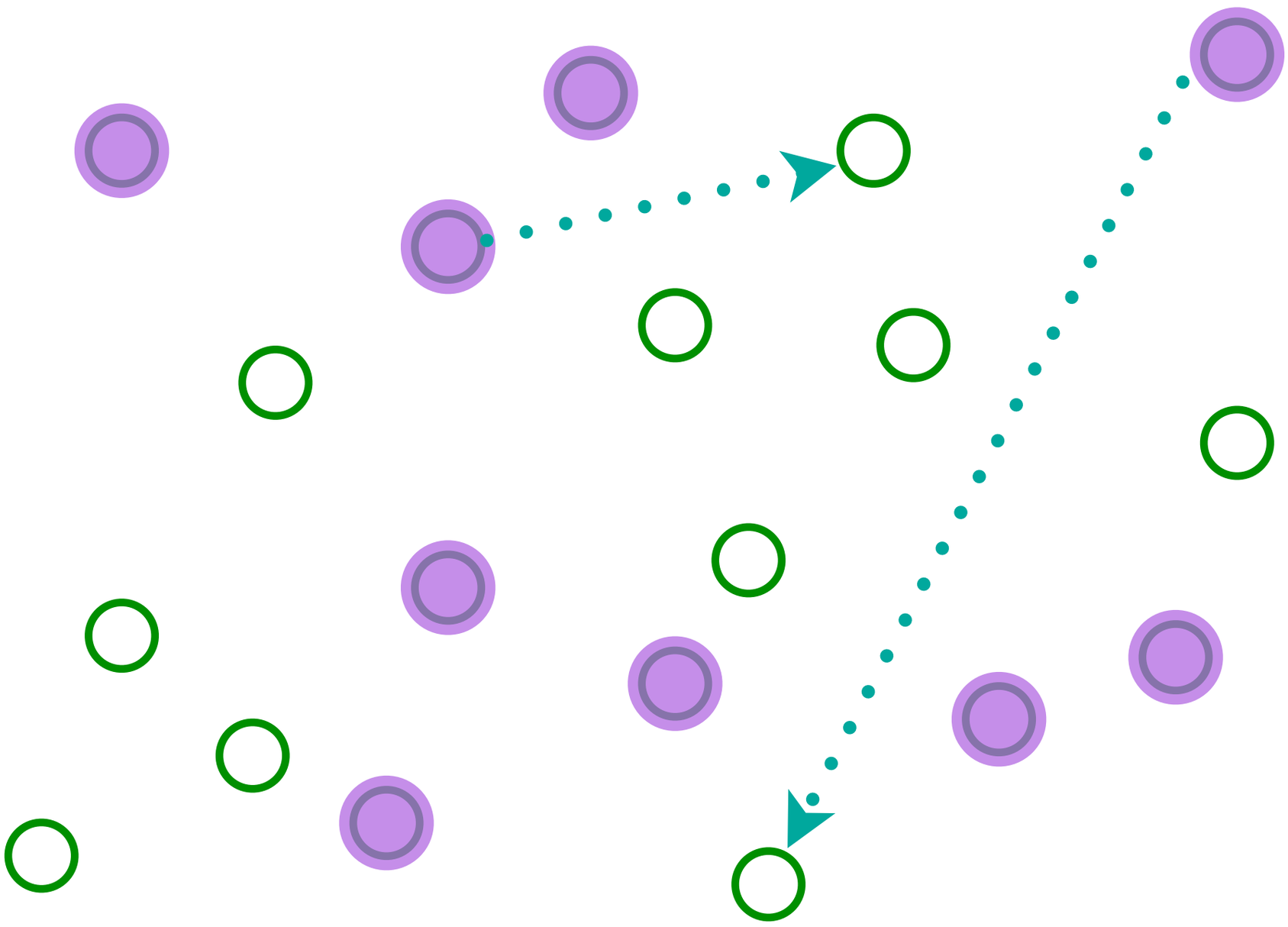

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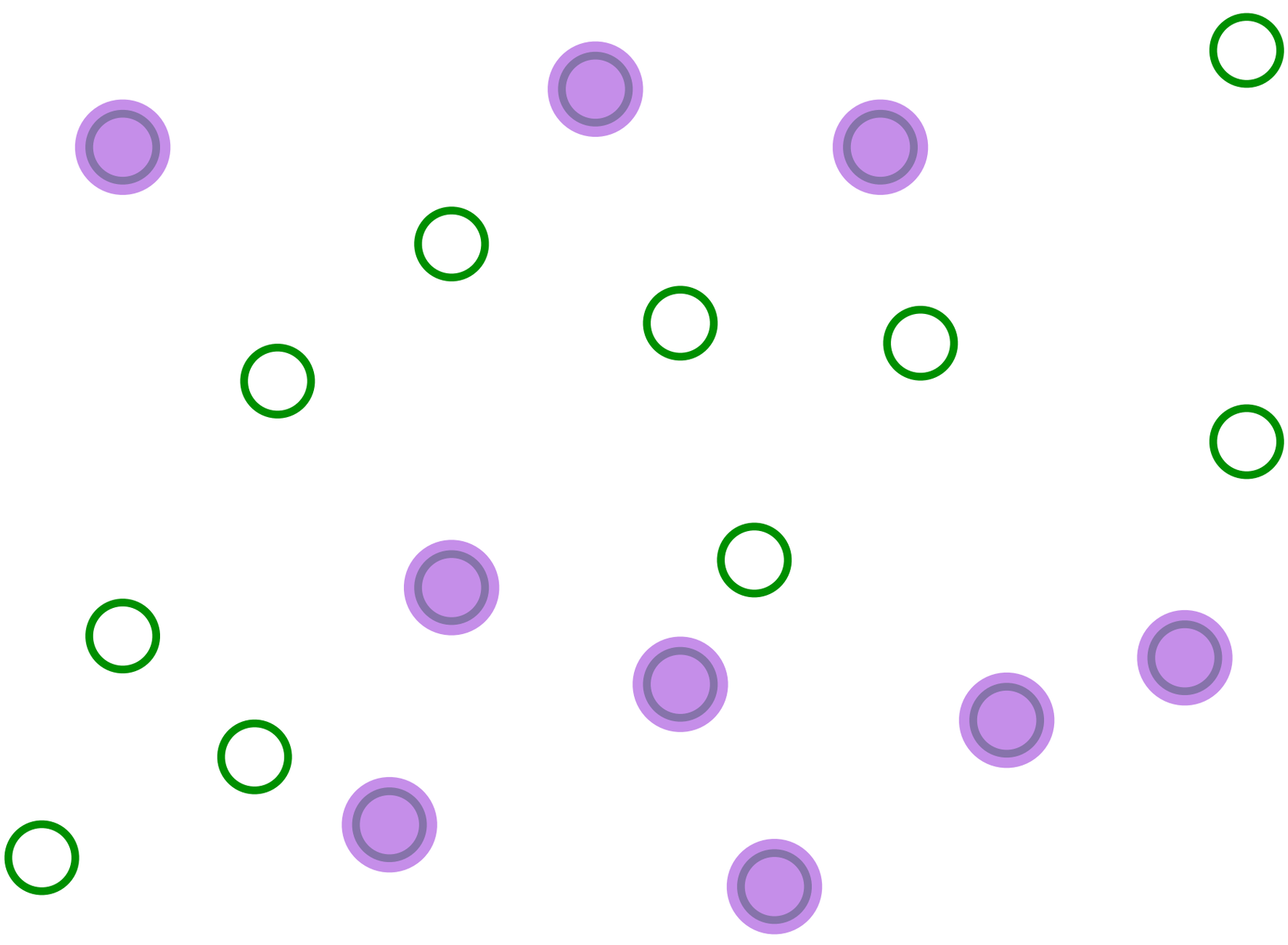

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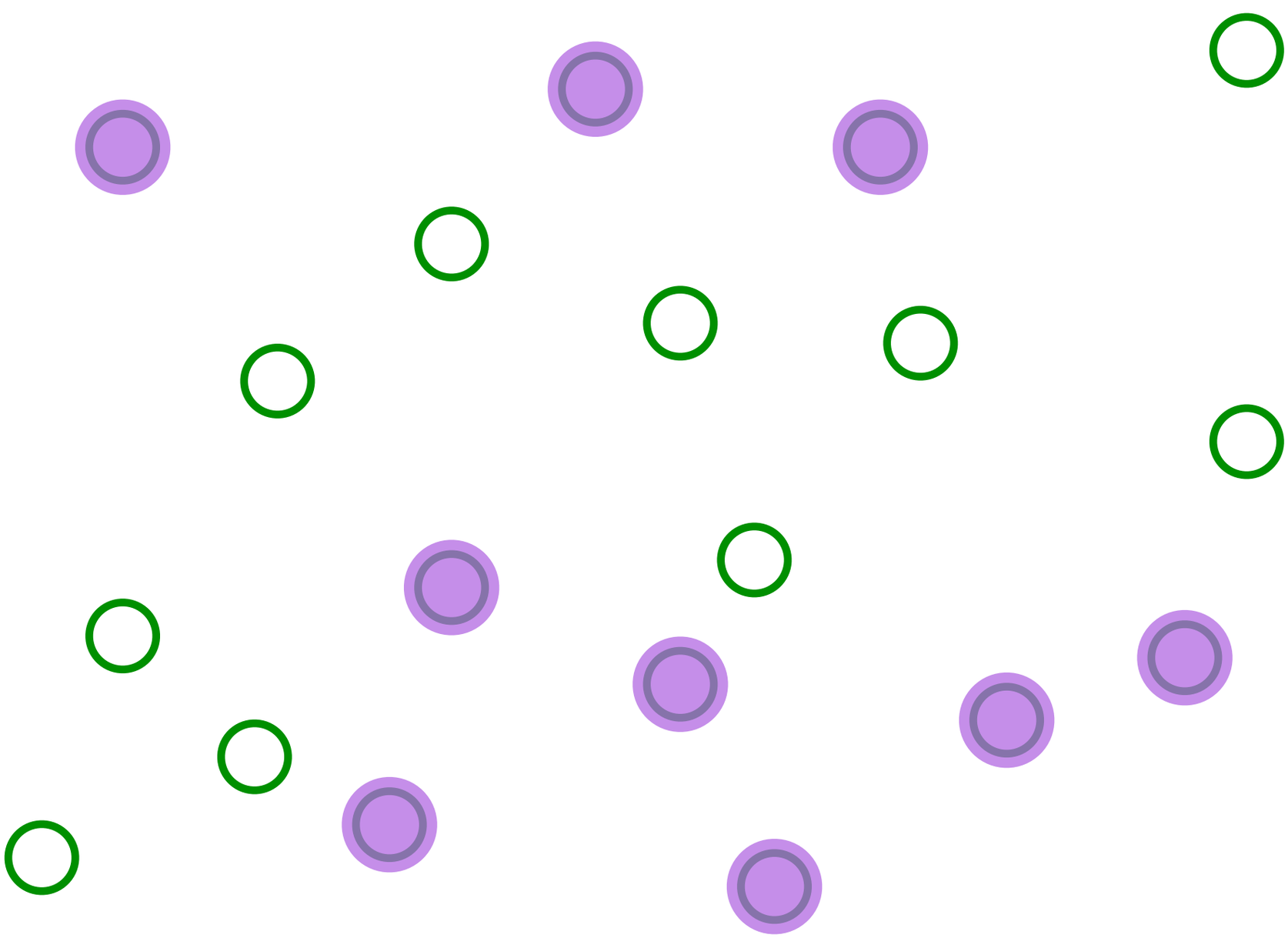

This describes both a strange metal and a black hole!

The SYK model leads to an `extreme' entanglement of the electrons. Consequently, there are no quasiparticle excitations

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- The equilibration time for the SYK model is of order the 'Planckian time' $\hbar/(k_B T)$

$$\tau_{\rm eq} \sim \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

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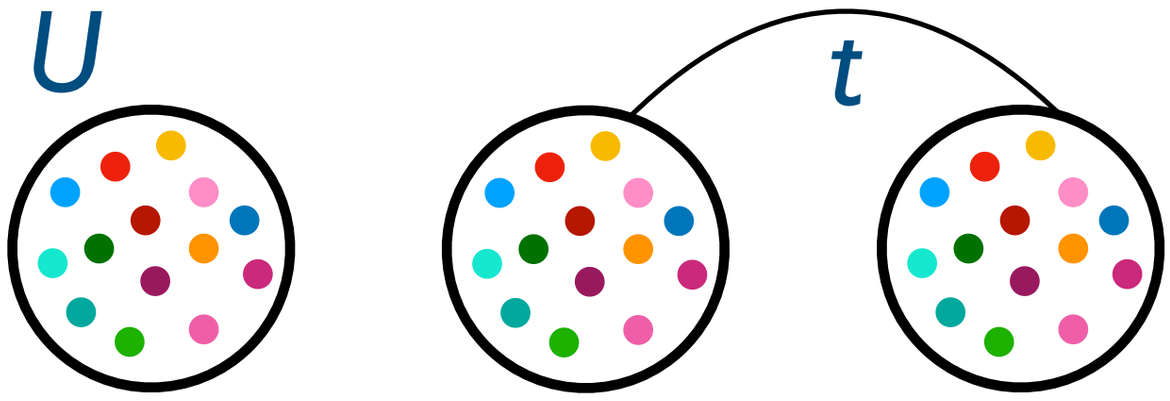
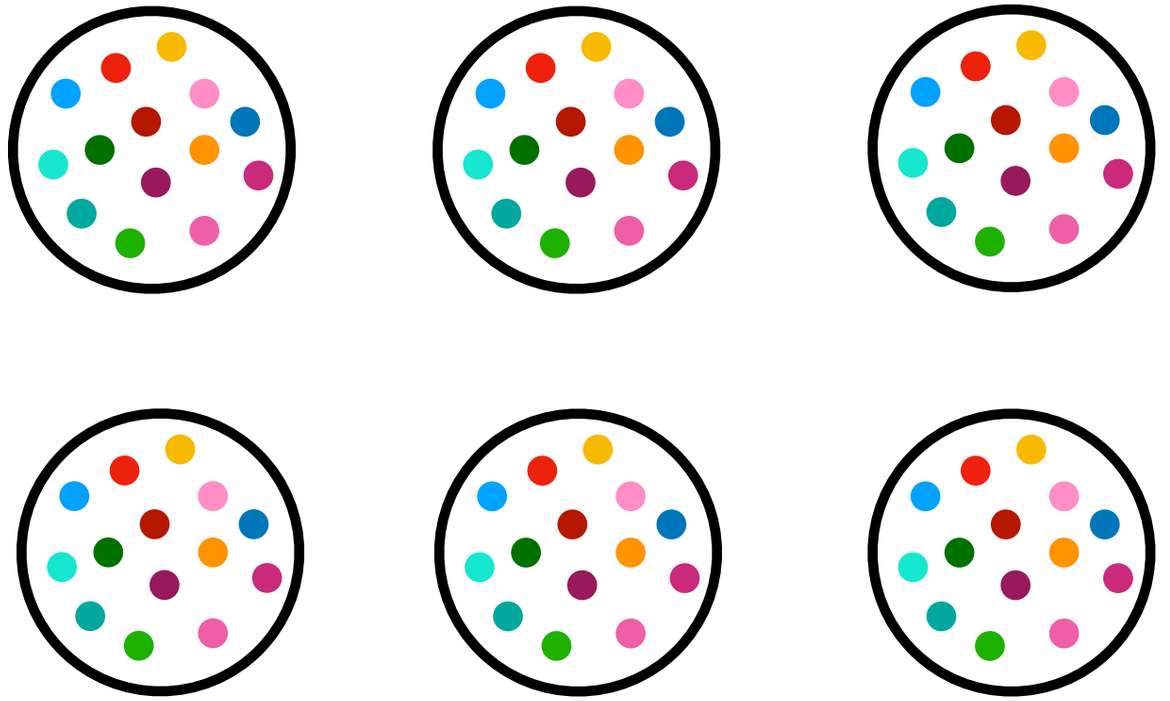
- Because of the all-to-all entanglement, the low energy quantum theory has a high degree of symmetry. It is expressed as an integral over all possible reparameterizations of time (*i.e.* rates of the running of a clock), modulo the symmetry group SL(2, R):

$$\tau \to \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1$$

Coupled SYK Islands

SYK quantum islands of electrons with single particle hopping between them.





Pengfei Zhang, PRB **96**, 205138 (2017)

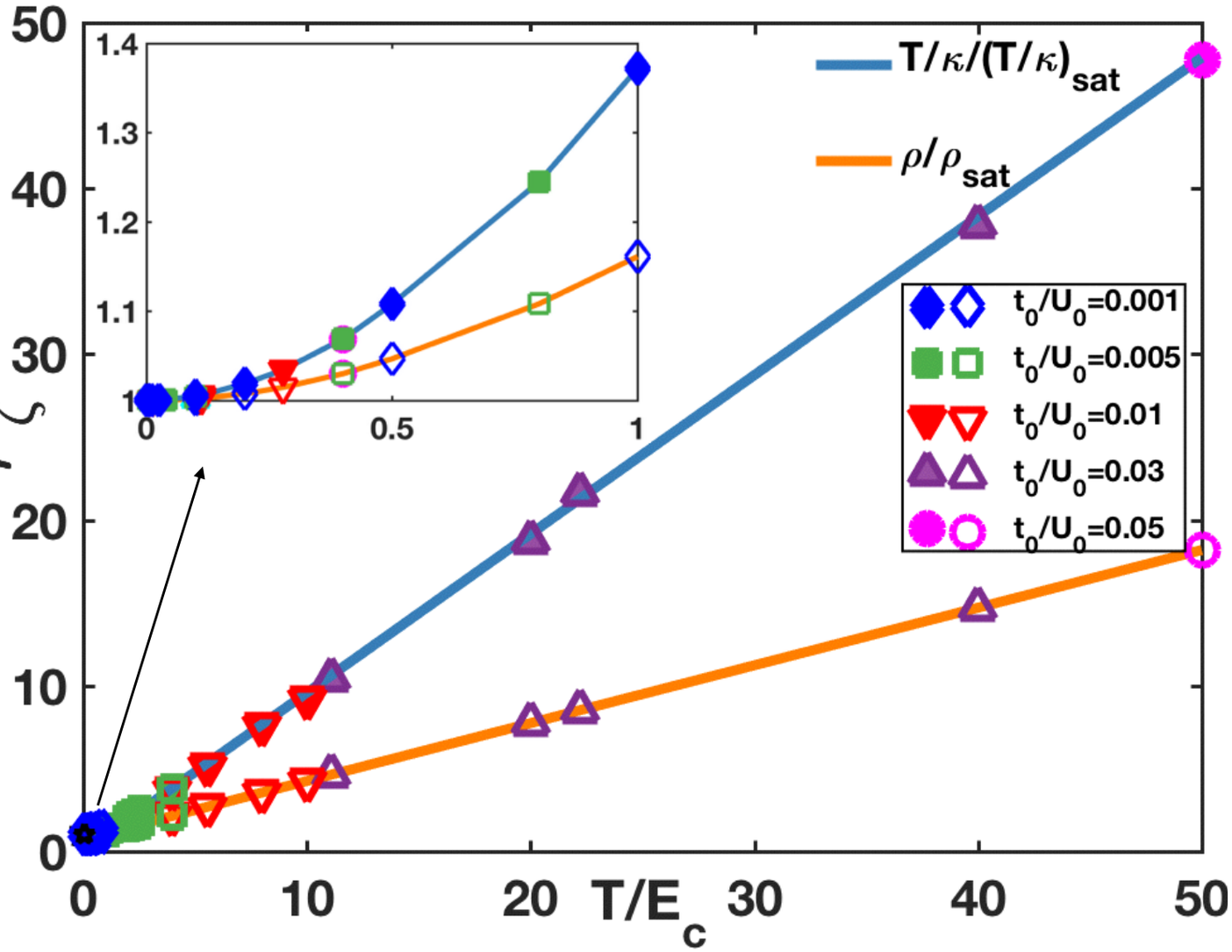
Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018)

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Coupled SYK Islands

Low 'coherence' scale

$$E_c \sim \frac{t_0^2}{U}$$



Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

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Low 'coherence' scale

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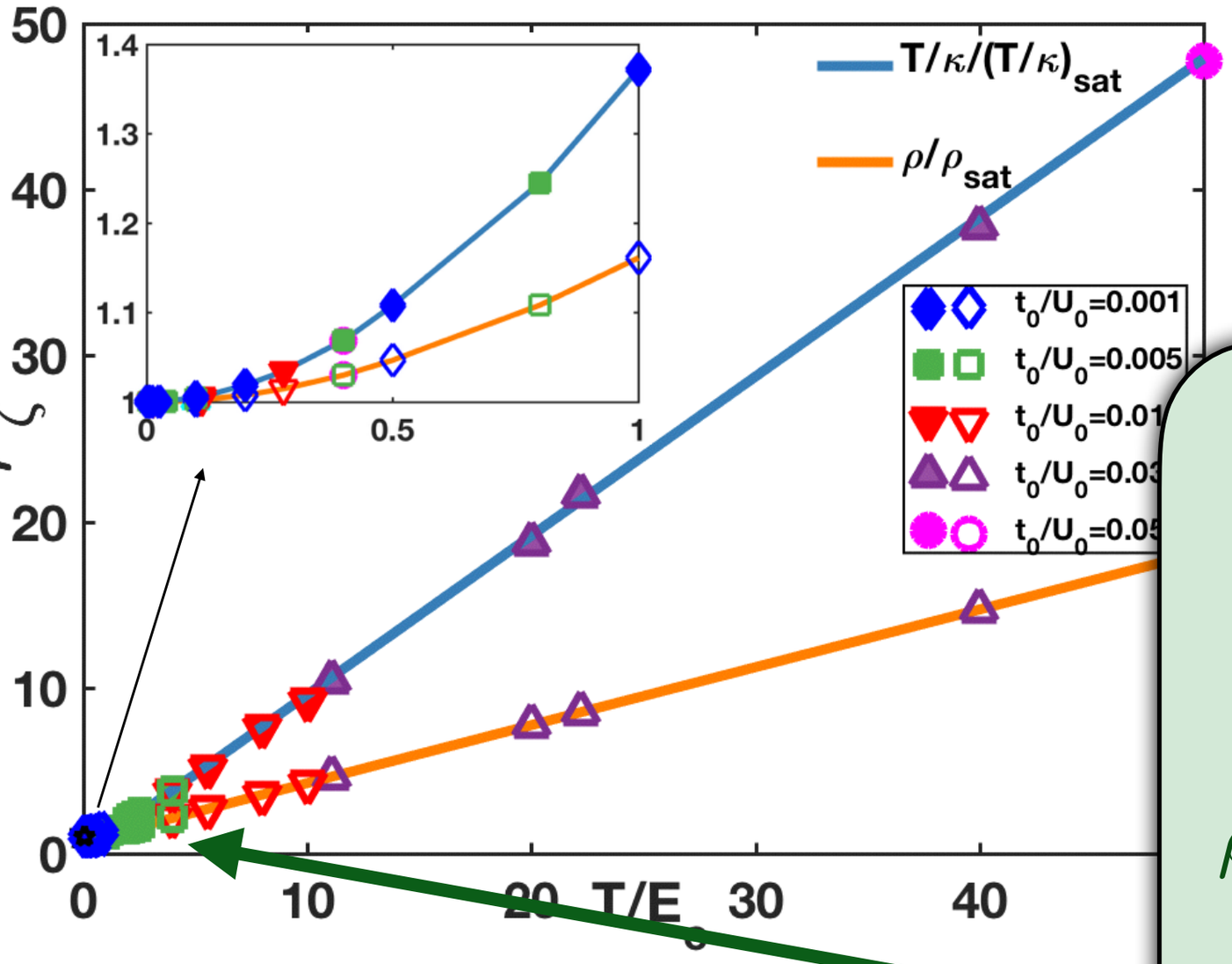

For $T < E_c$, the resistivity, ρ, and entropy density, s, are

$$\rho = \frac{h}{e^2}\left[c_1 + c_2\left(\frac{T}{E_c}\right)^2\right]$$

$$s \sim s_0\left(\frac{T}{E_c}\right)$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Coupled SYK Islands

Low 'coherence' scale

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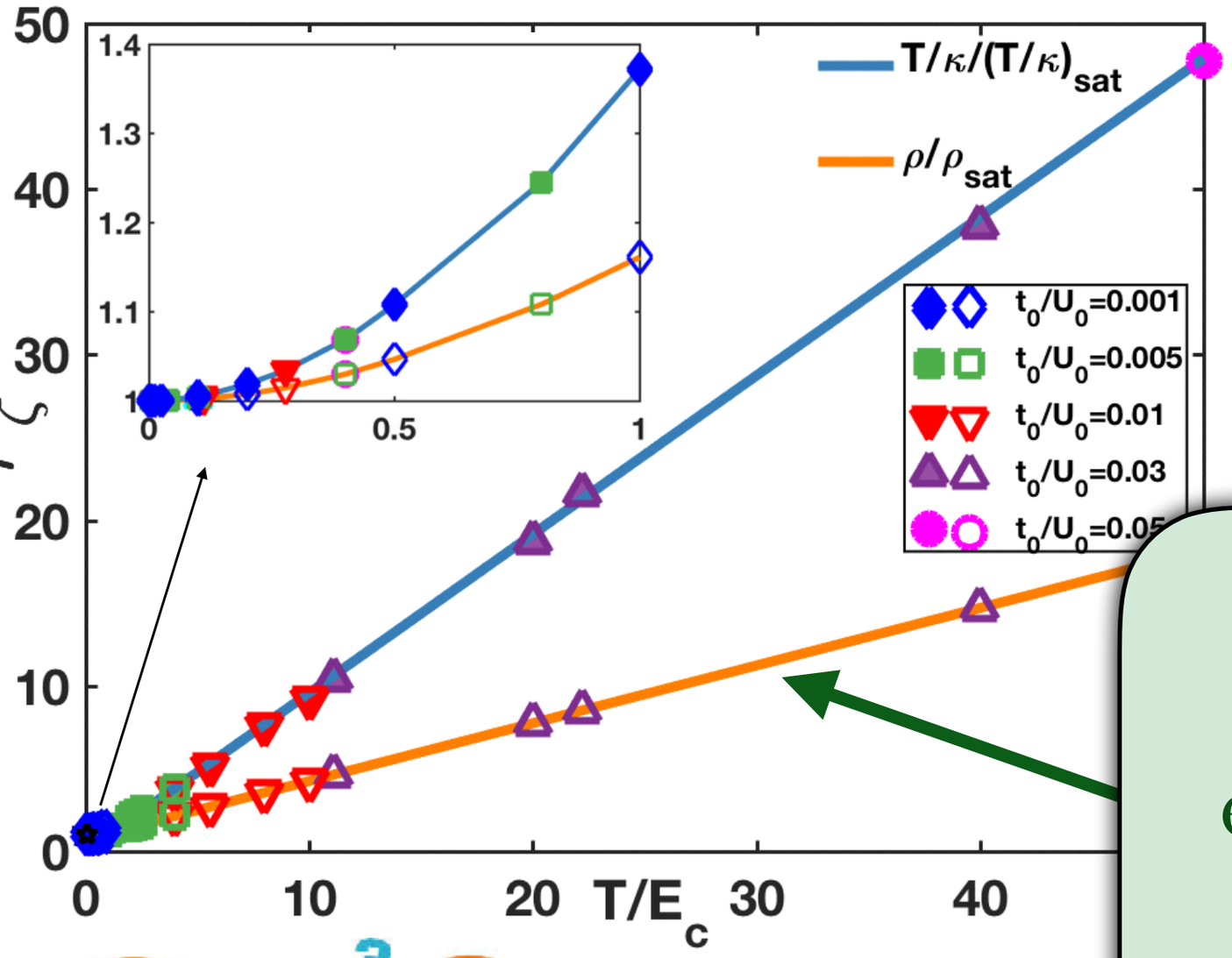

For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right), \quad s = s_0$$

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Quantum entanglement

Strange metals

Quantum entanglement

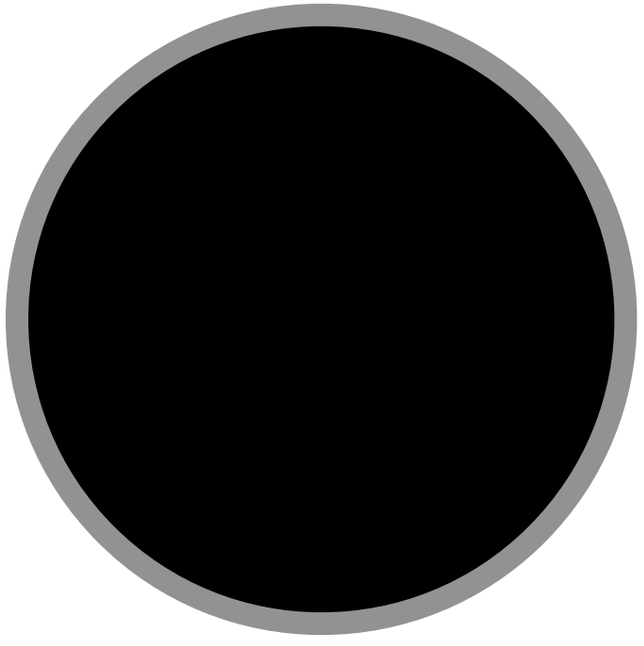
Black holes

Strange metals

Black Holes

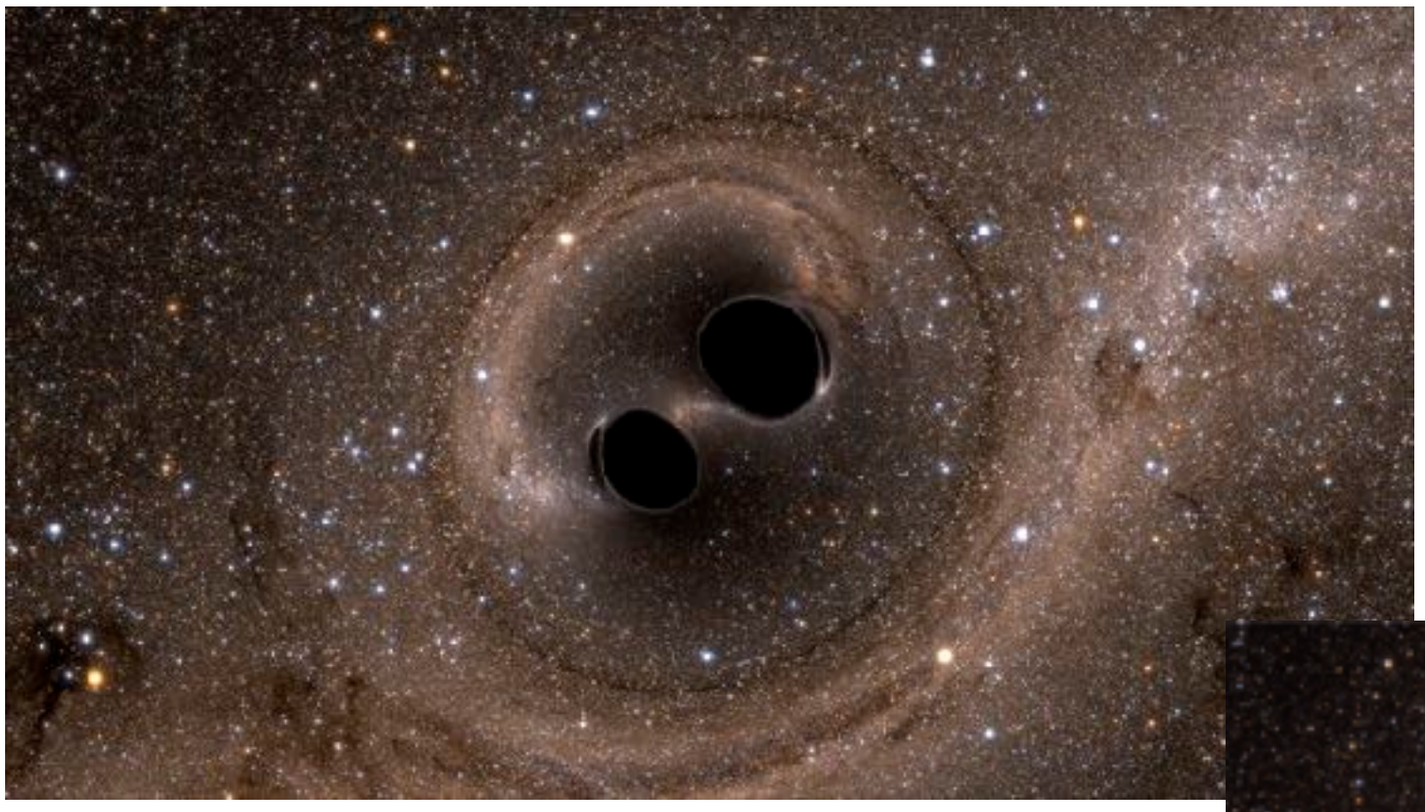
Objects so dense that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole horizon is disconnected from the rest of the universe.

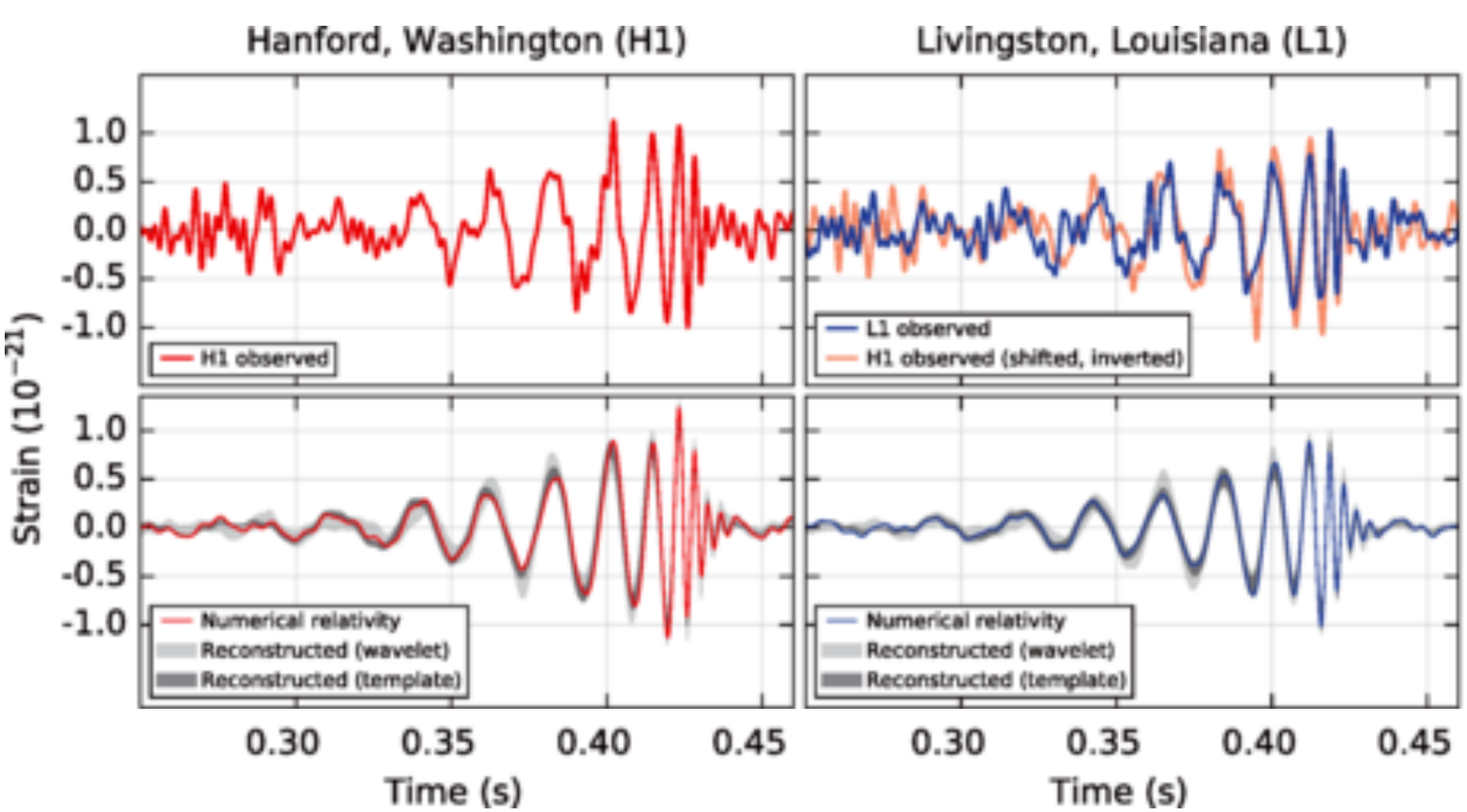


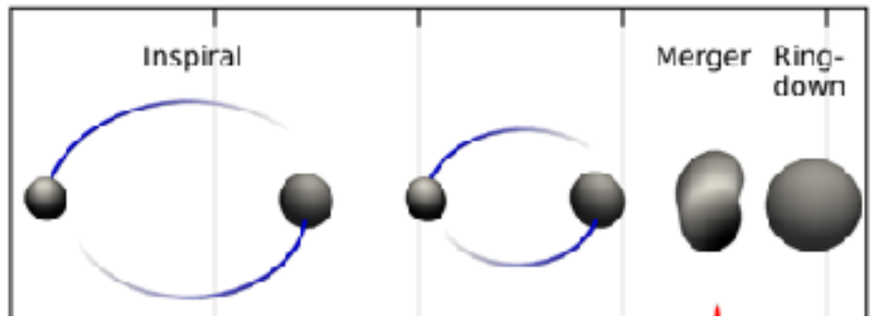
$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

On September 14, 2015, LIGO detected the merger of two black holes, each weighing about 30 solar masses, with radii of about 100 km, 1.3 billion light years away



0.1 seconds later !





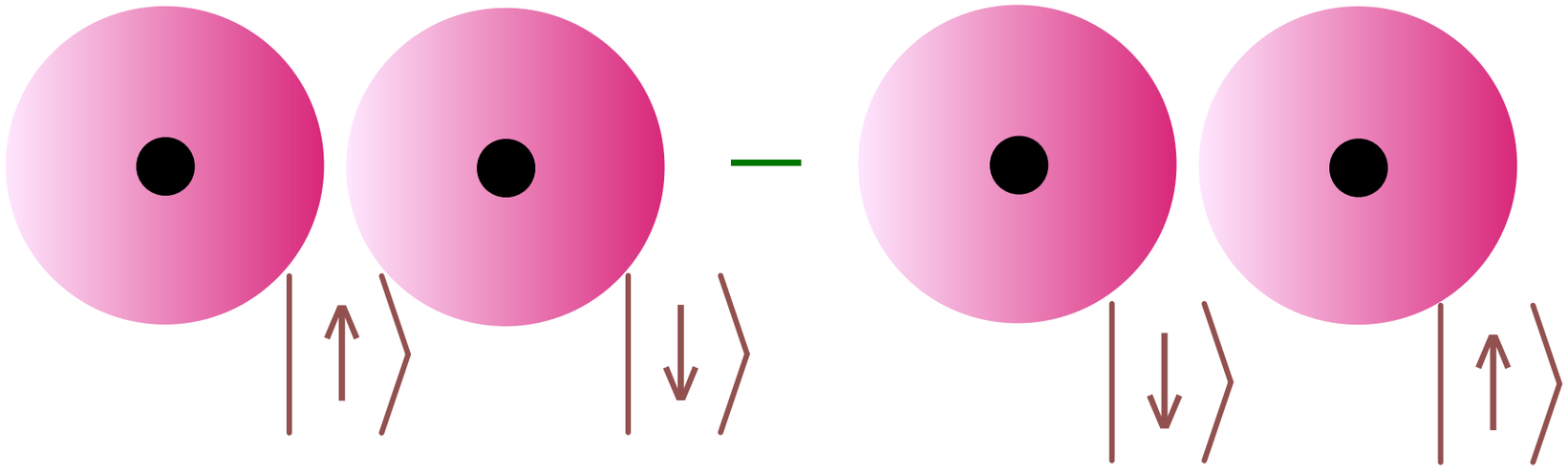
LIGO
September 14, 2015

- The ring-down is predicted by General Relativity to happen in a time $\dfrac{8\pi GM}{c^3} \sim 8$ milliseconds.

Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

Quantum Entanglement across a black hole horizon



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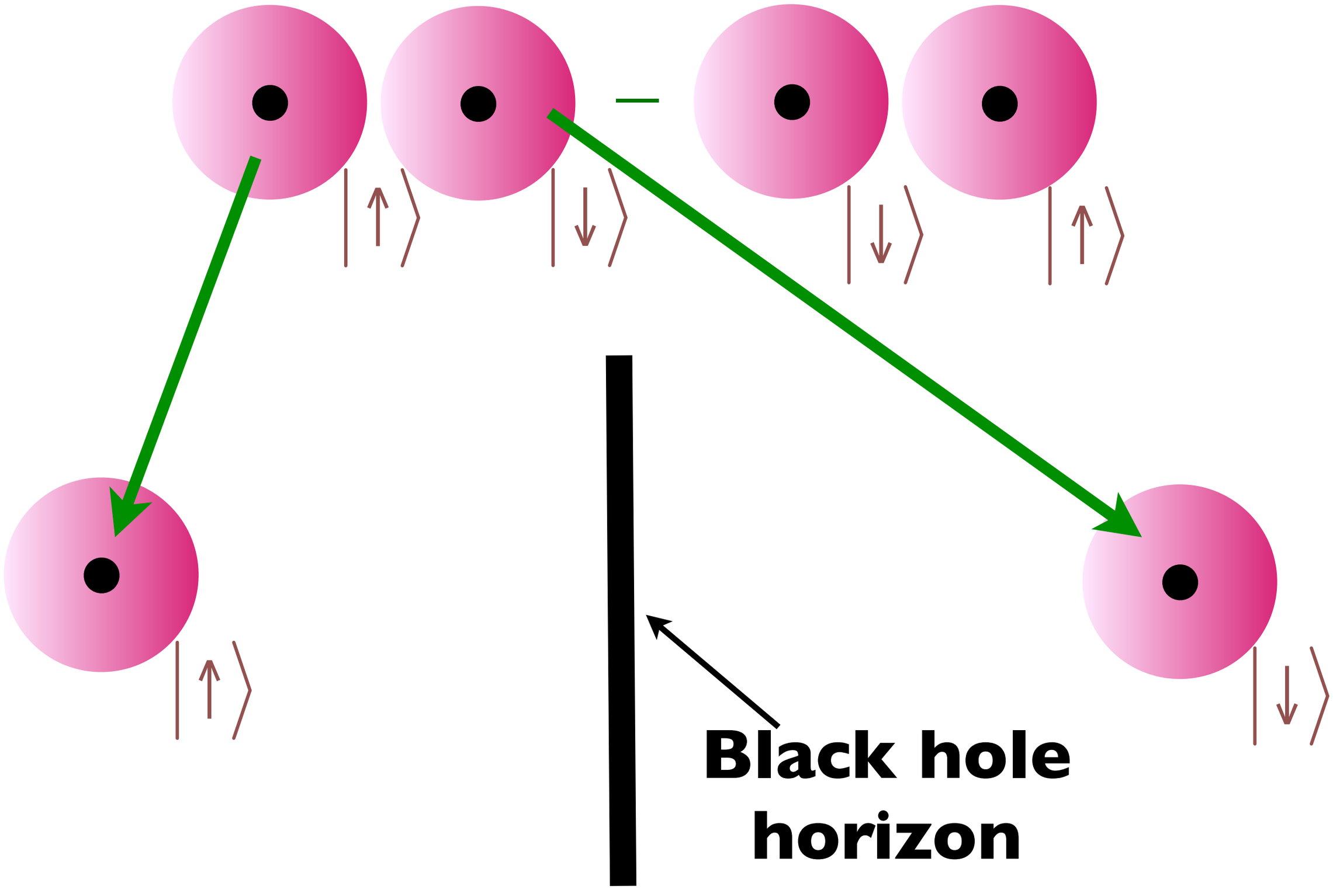
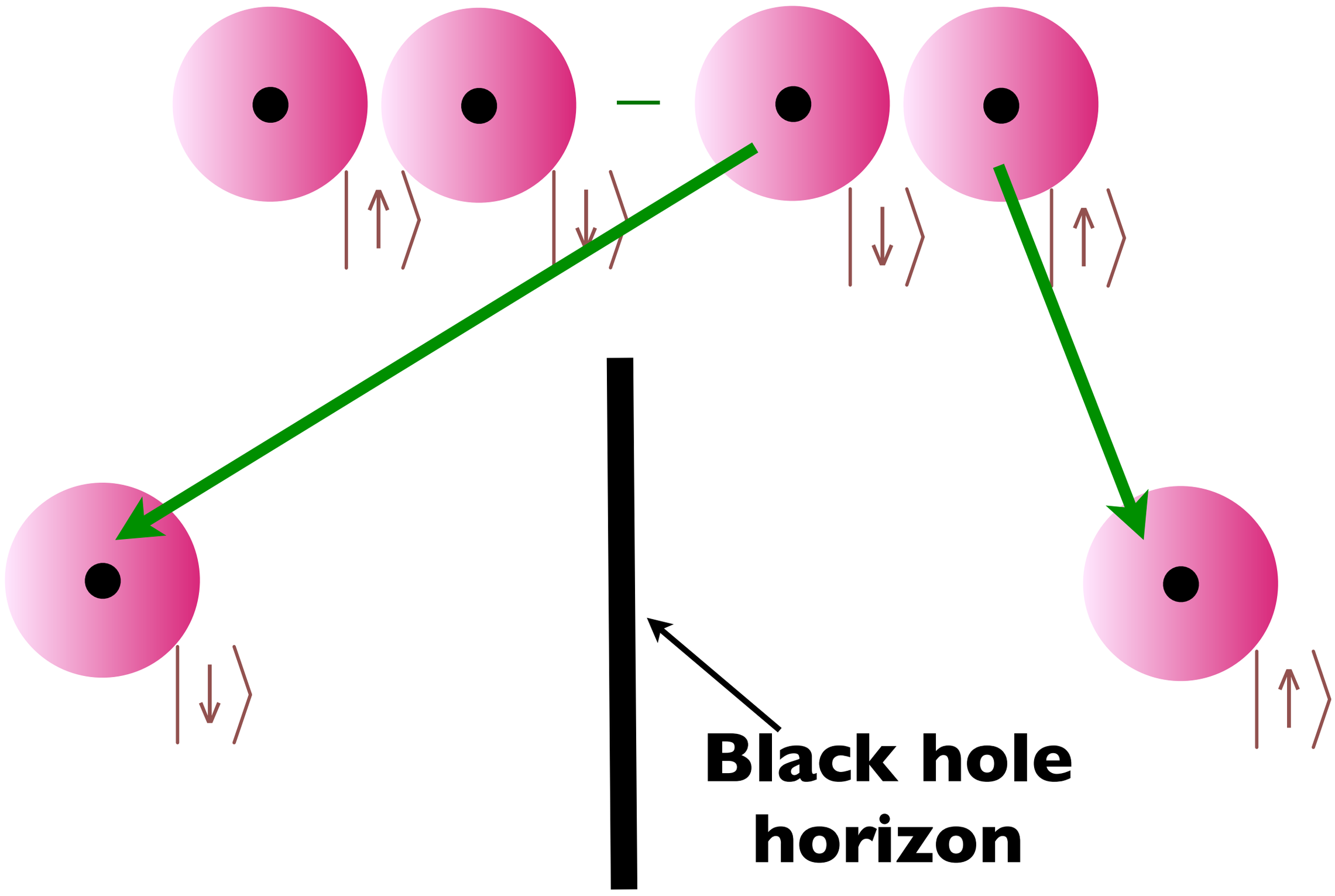
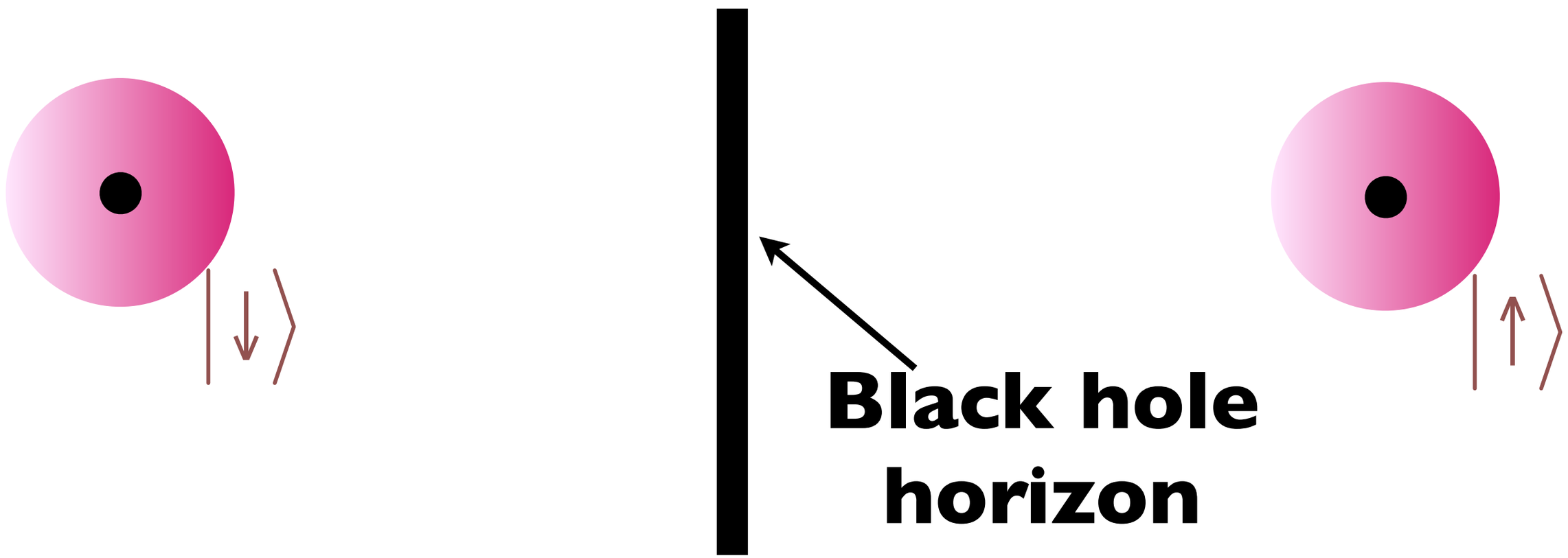


Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

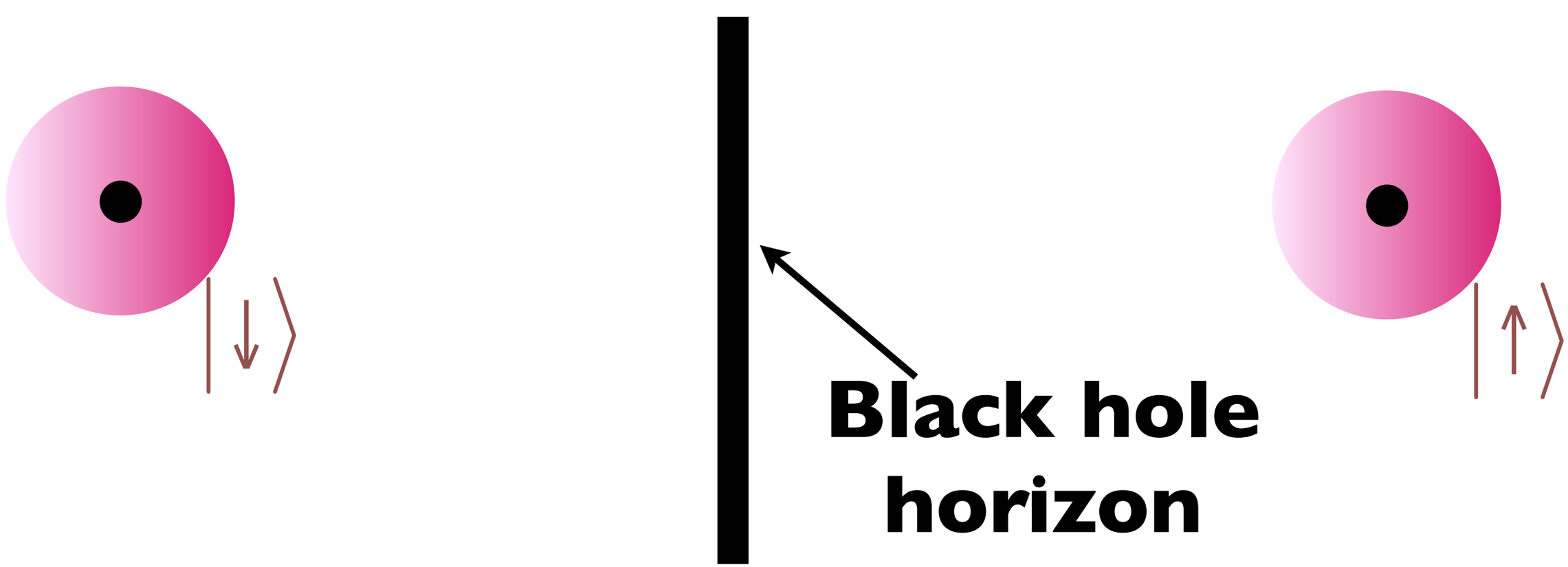
There is long-range quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

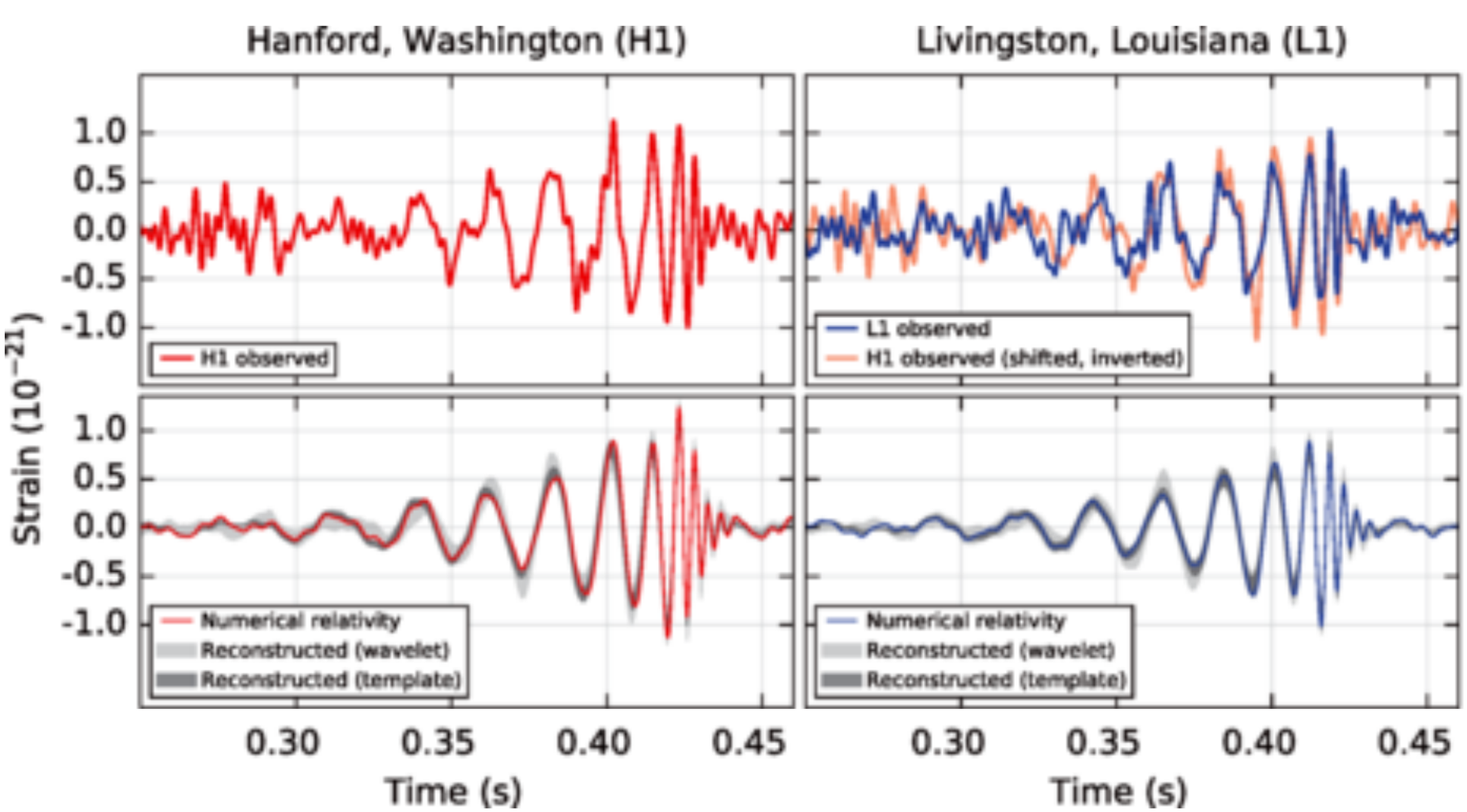
Hawking used this to show that black hole horizons have an entropy and a temperature

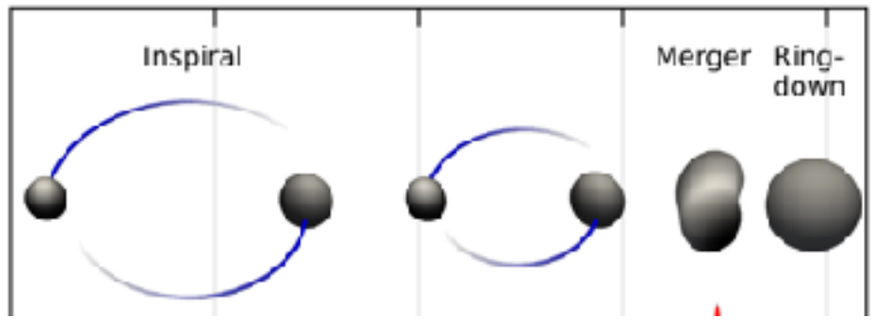
(because to an outside observer, the state of the electron inside the black hole is an unknown)



Black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- The entropy is proportional to their surface area.





LIGO
September 14, 2015

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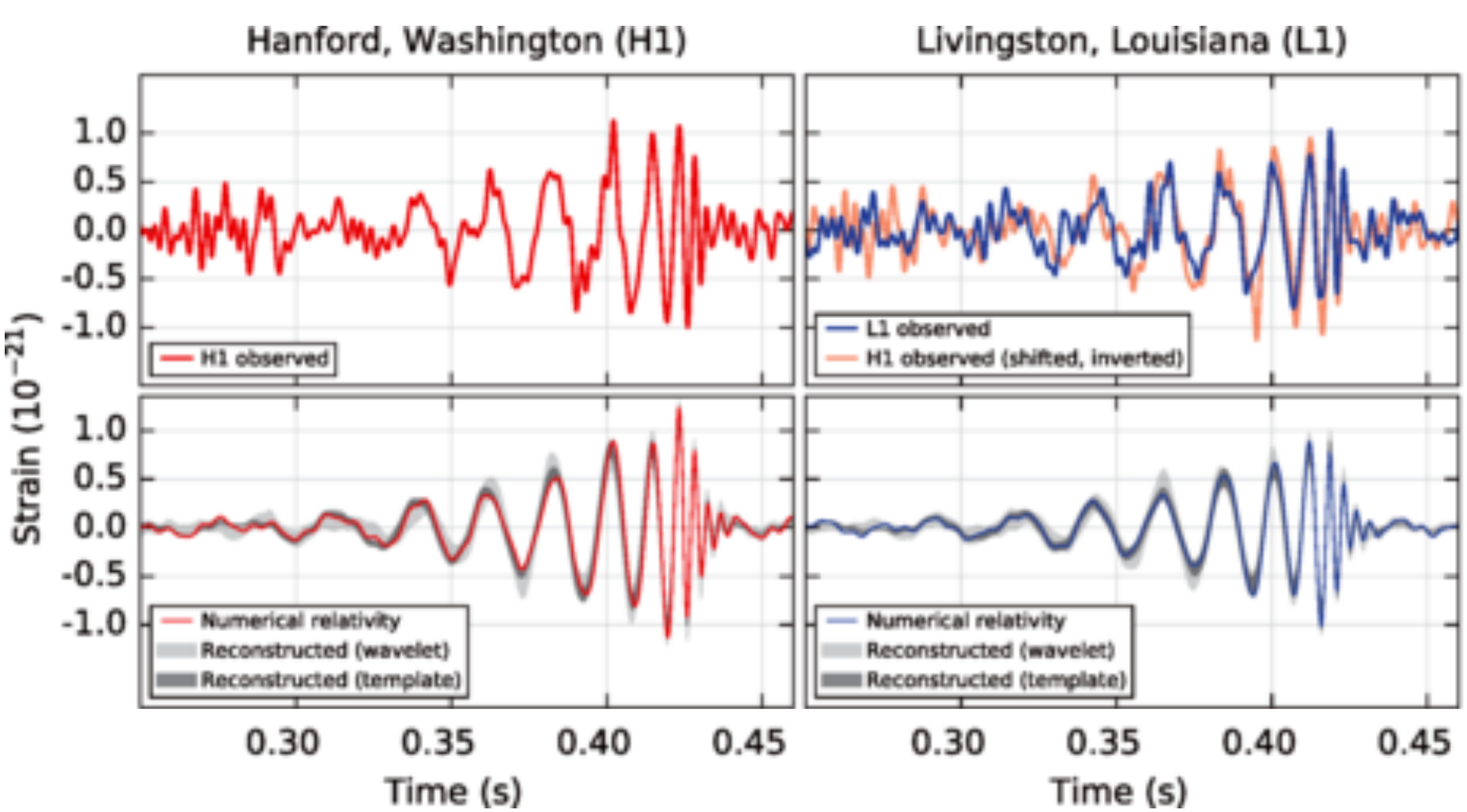

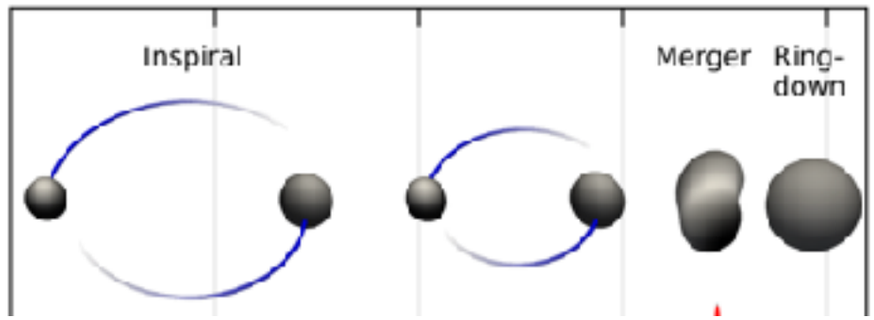

LIGO
September 14, 2015

- The ring-down is predicted by General Relativity to happen in a time $\dfrac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously this happens to equal $\dfrac{\hbar}{k_B T_H}$: so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate!

Black holes

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Holography:

Quantum black holes "look like" quantum many-particle systems without quasiparticle excitations, residing "on" the surface of the black hole

Quantum entanglement

Black holes

Strange metals

Quantum entanglement

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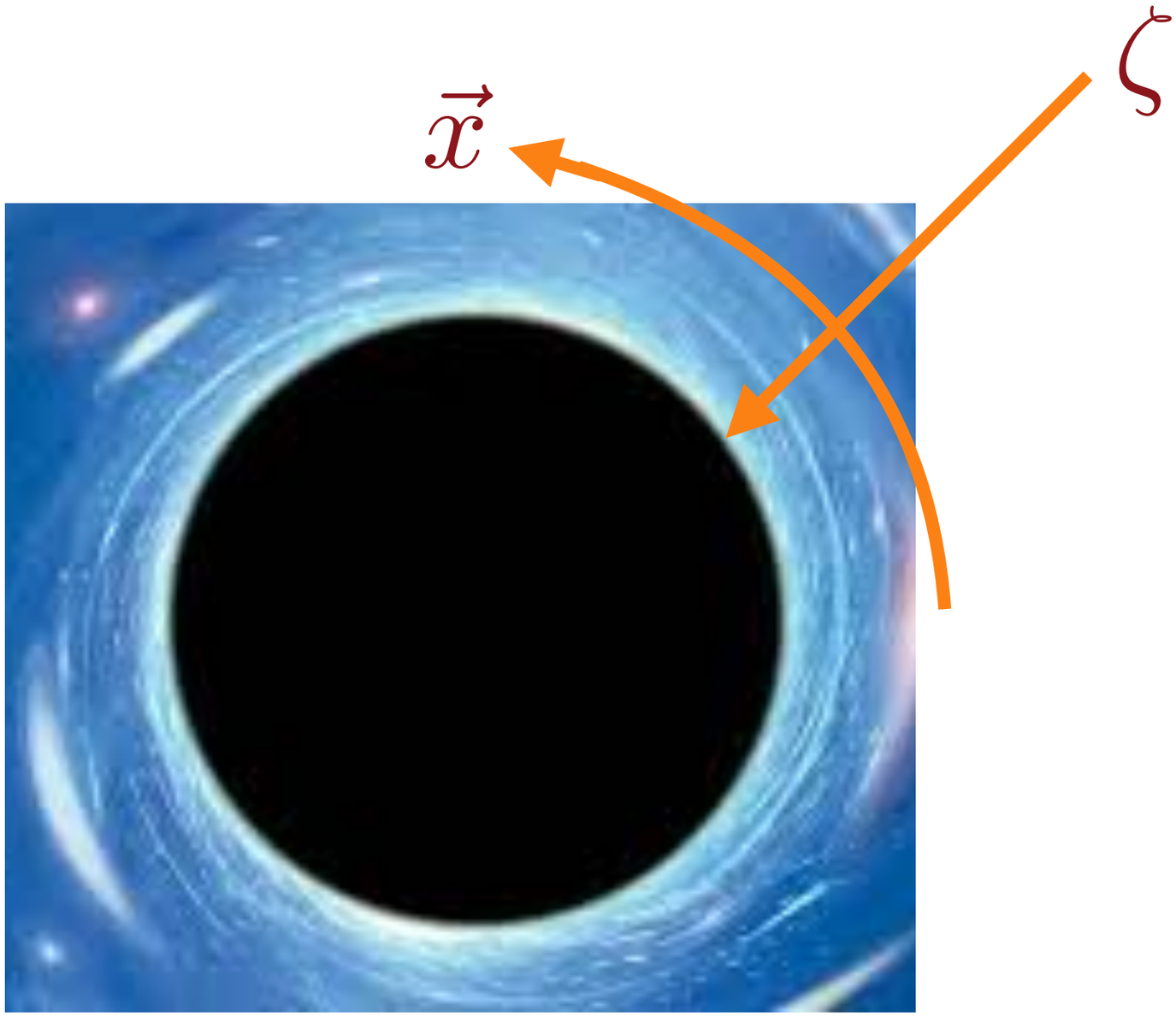
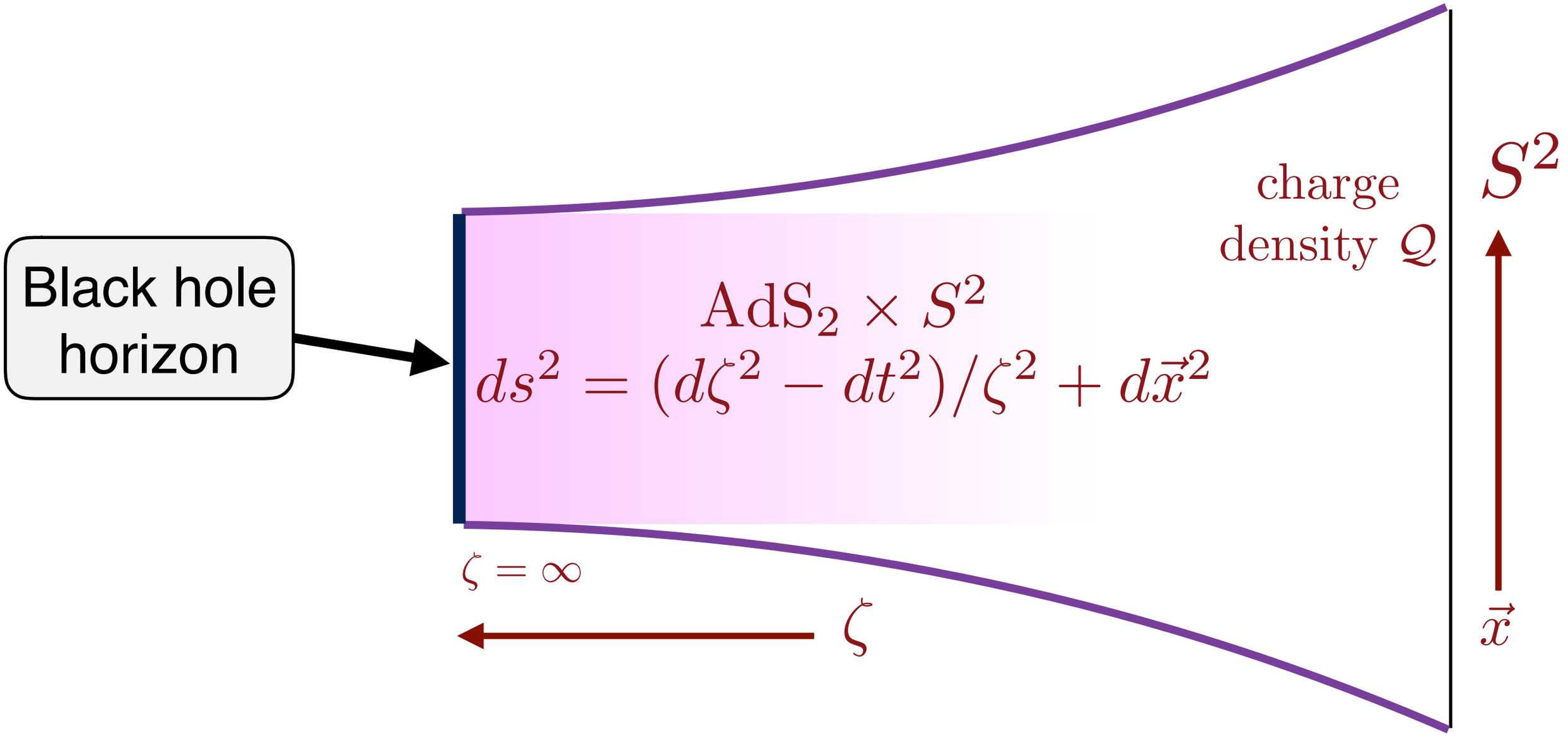
Strange metals

The SYK model also describes extremal charged black holes !



Consider a charged black hole with the smallest possible mass: the extremal limit. Zoom in to the near-horizon region at low energies. In this limit, the quantum theory lives in one space (ζ) and one time dimension



The near-horizon region of an extremal charged black hole has the geometry of (1+1)-dimensional anti-de Sitter spacetime. By holography, this should map to a zero-dimensional quantum system: this turns out to be the SYK model

SYK models and black holes

- Reparameterization invariance is a defining property of Einstein's theory of gravity

SYK models and black holes

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- In imaginary time, AdS_2 is the homogeneous hyperbolic space: two-dimensional surface of constant negative curvature. Its metric is invariant under SL(2,R)

$ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$ is invariant under

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d} \text{ with } ad - bc = 1.$$

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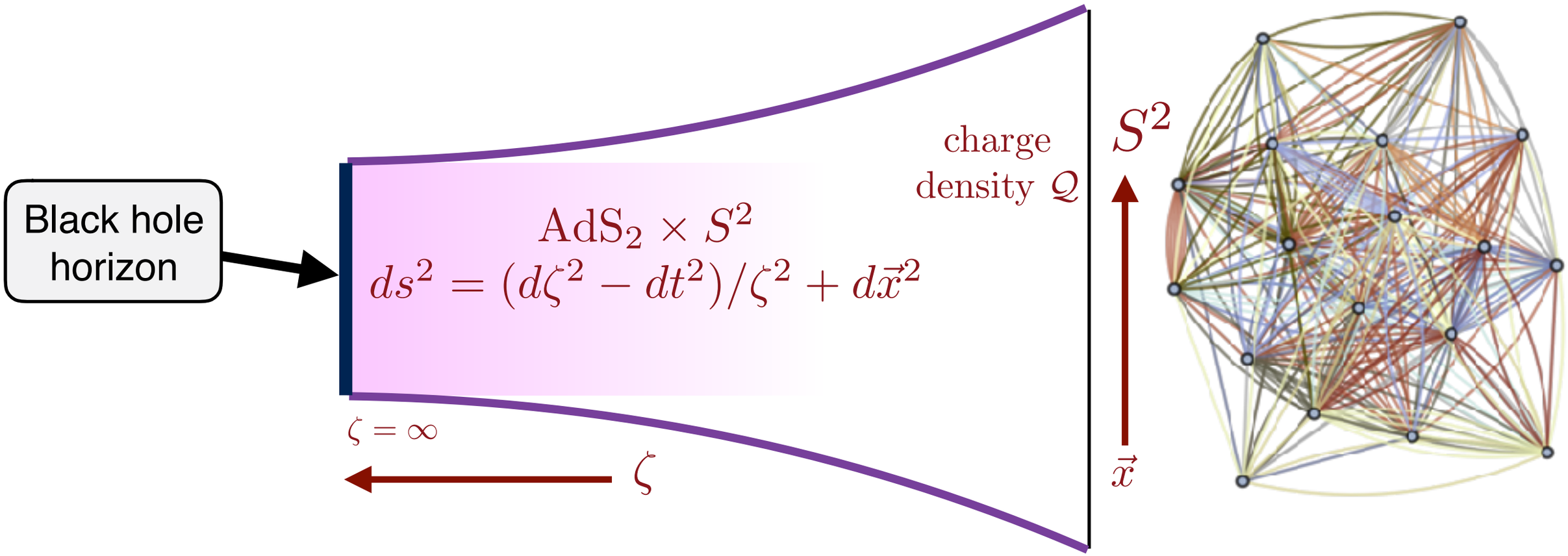
Their identical symmetries lead to the same low energy quantum theory for the SYK model and extremal charged black holes !



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Quantum entanglement

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Strange metals

The SYK model describes a strange metal and extremal charged black holes

Quantum matter <u>without</u> quasiparticles:

- No quasiparticle decomposition of low-lying states.

- Thermalization and many-body chaos in the shortest possible time of order $\hbar/(k_B T)$.

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- These are also characteristics of black holes in quantum gravity.