Fermi surface reconstruction by emergent gauge fields, without translational symmetry breaking

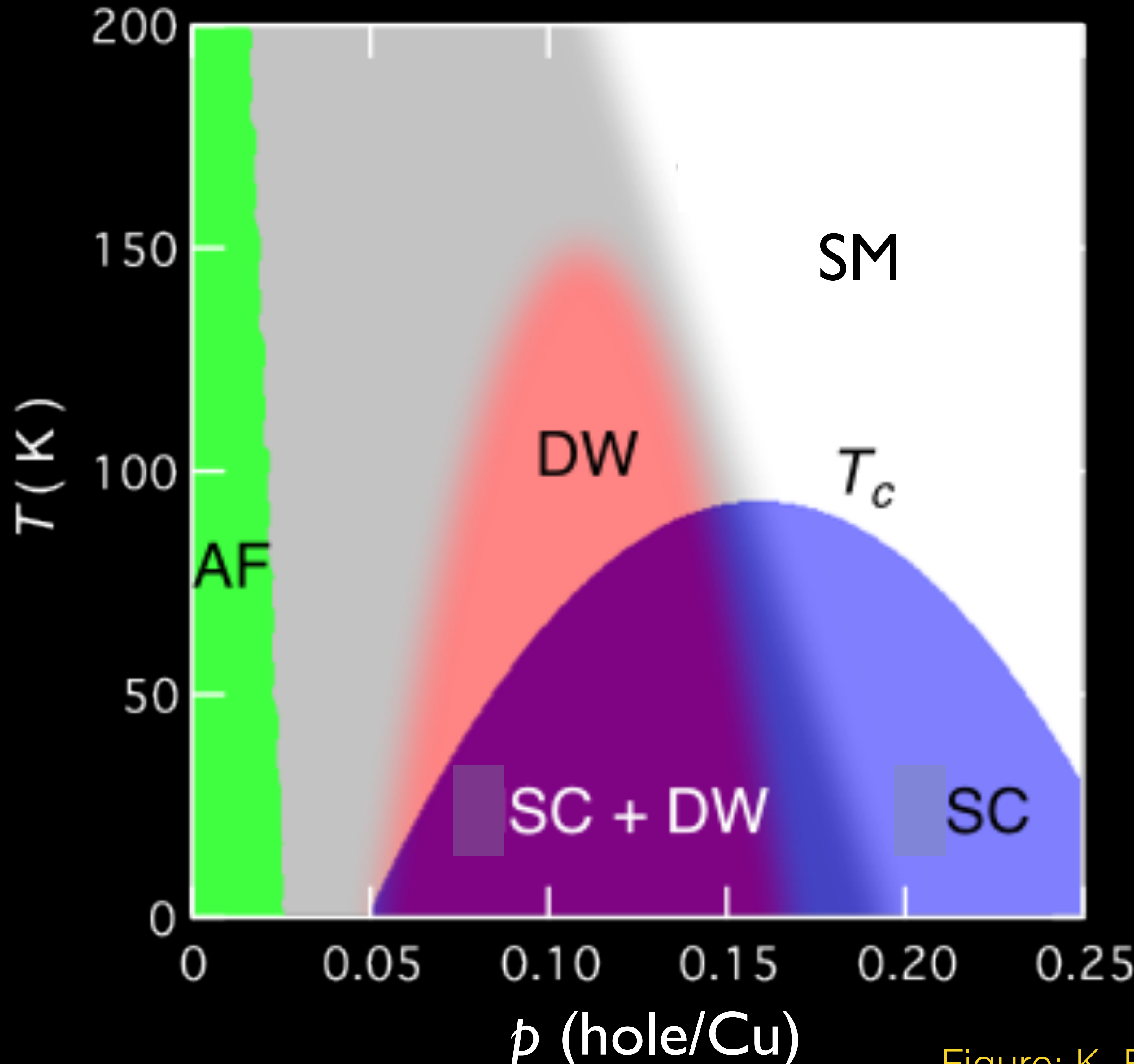
National Conference on Condensed Matter
Indian Institute of Science Education and Research, Mohali
July 27, 2018

Subir Sachdev

PHYSICS

HARVARD

Talk online: sachdev.physics.harvard.edu



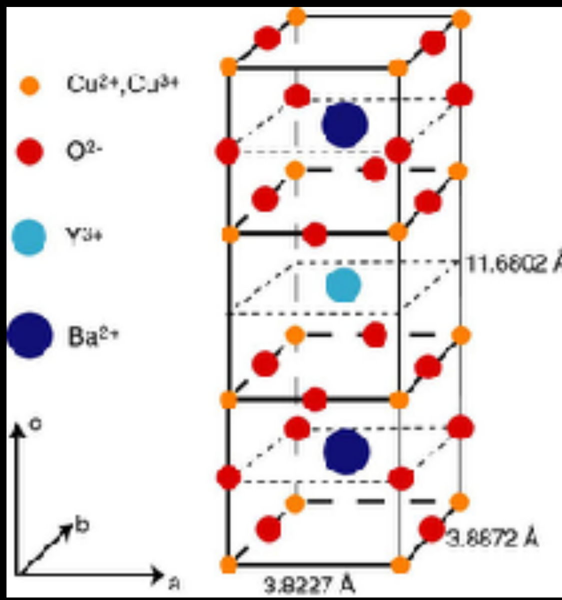
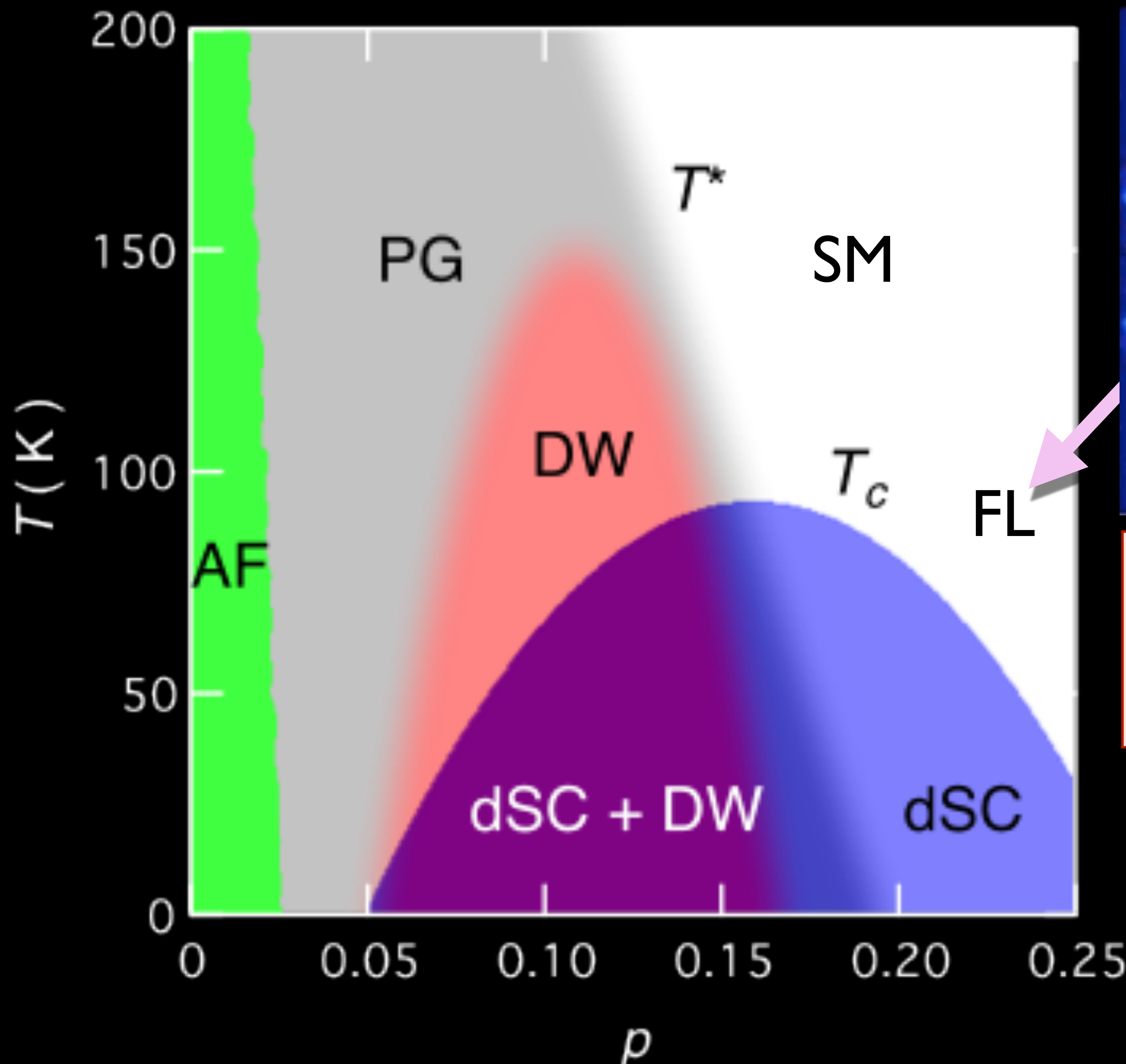
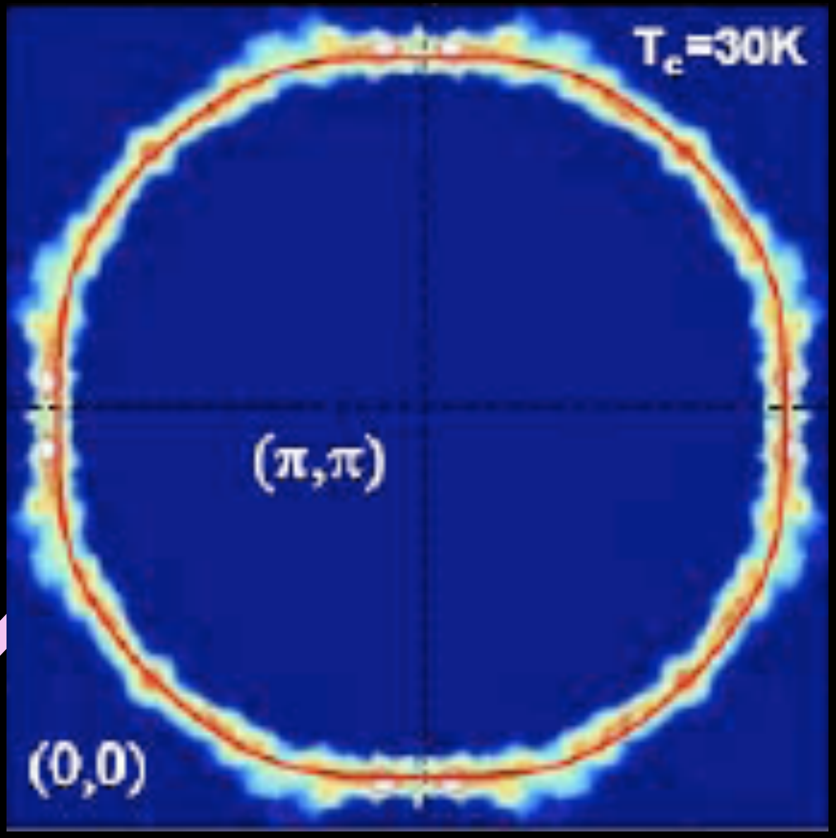


Figure: K. Fujita and J. C. Seamus Davis

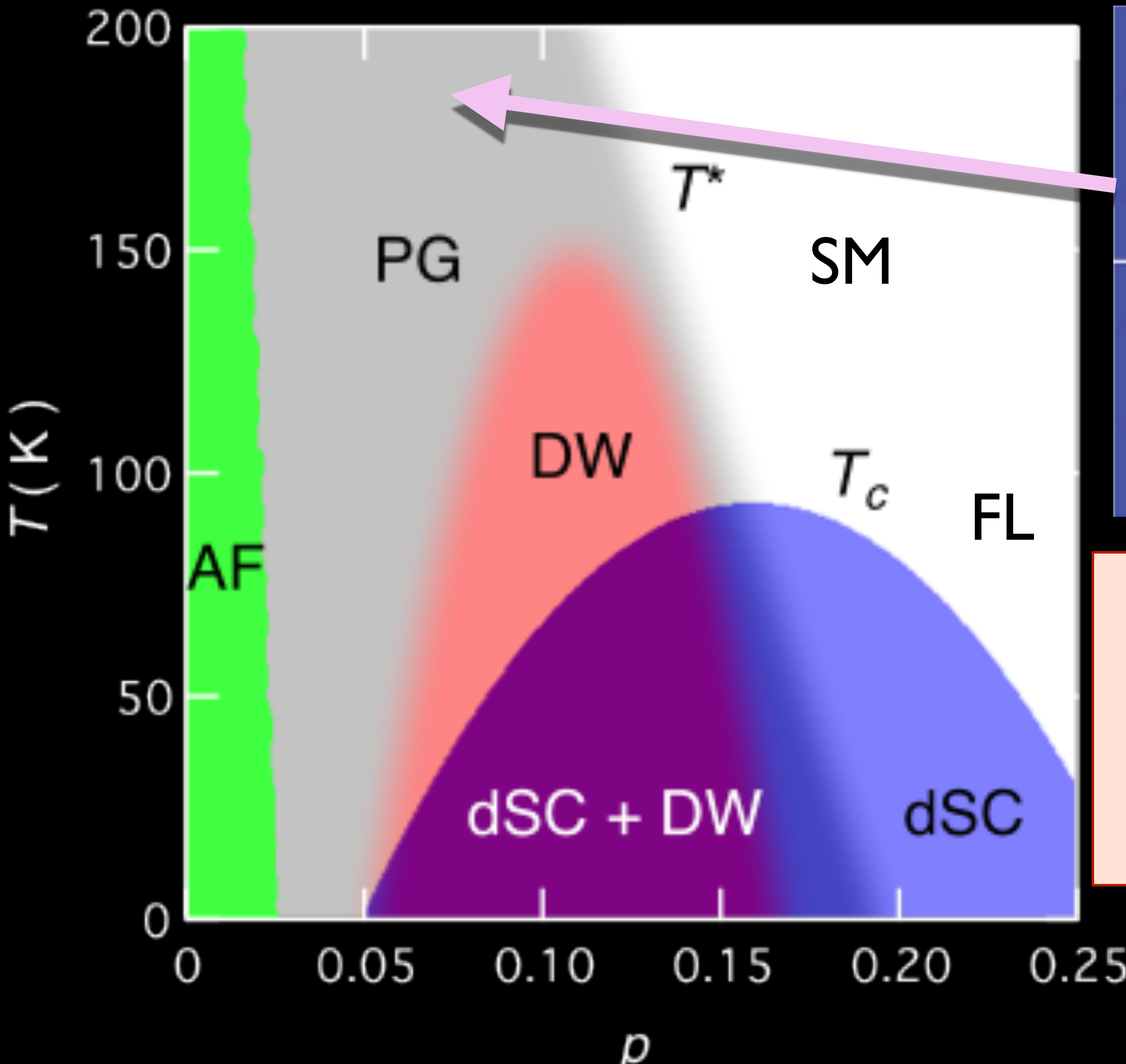
M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

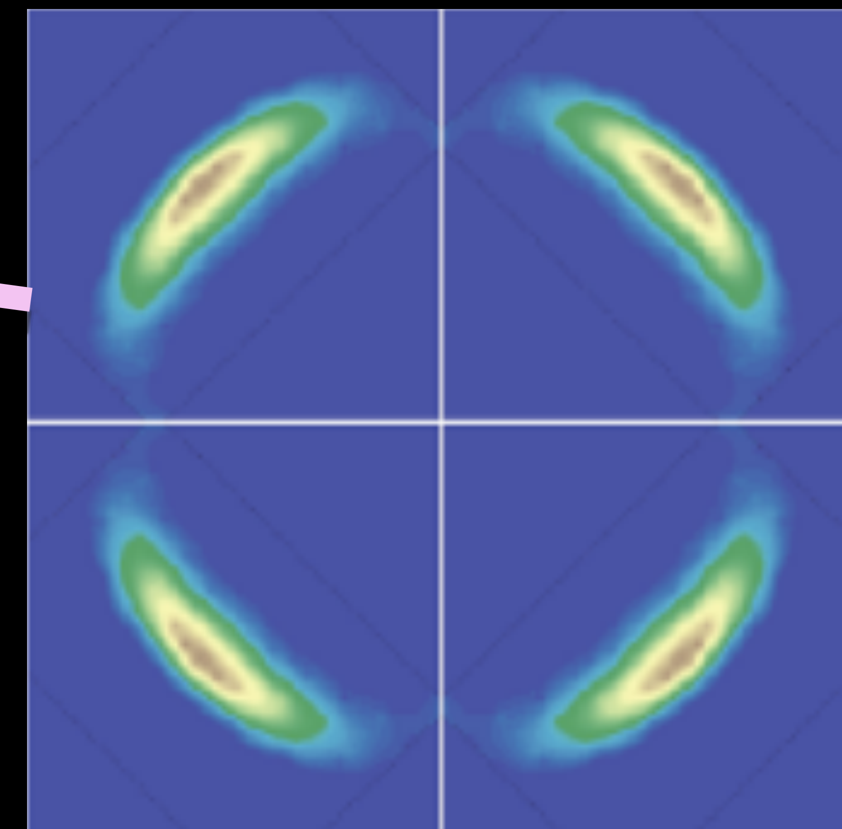




Fermi liquid
Area enclosed by
Fermi surface =1+p

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)





<u>Pseudogap metal</u>
at low p

The Hubbard Model

$$H = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha}$$

$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

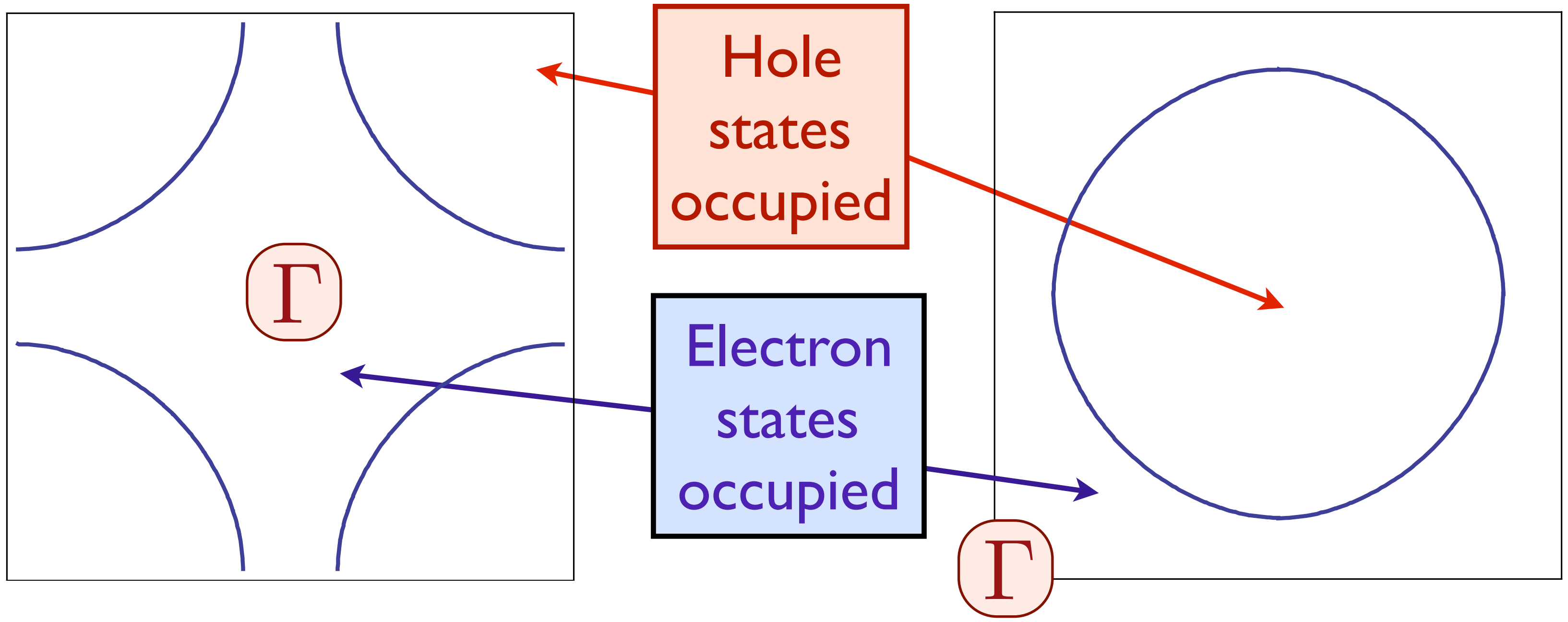
Spin index $\alpha = \uparrow, \downarrow$

$$n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha}$$

$$c_{i\alpha}^\dagger c_{j\beta} + c_{j\beta} c_{i\alpha}^\dagger = \delta_{ij} \delta_{\alpha\beta}$$
$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0$$

Will study on the square lattice

Fermi surfaces in electron- and hole-doped cuprates



Effective Hamiltonian for quasiparticles:

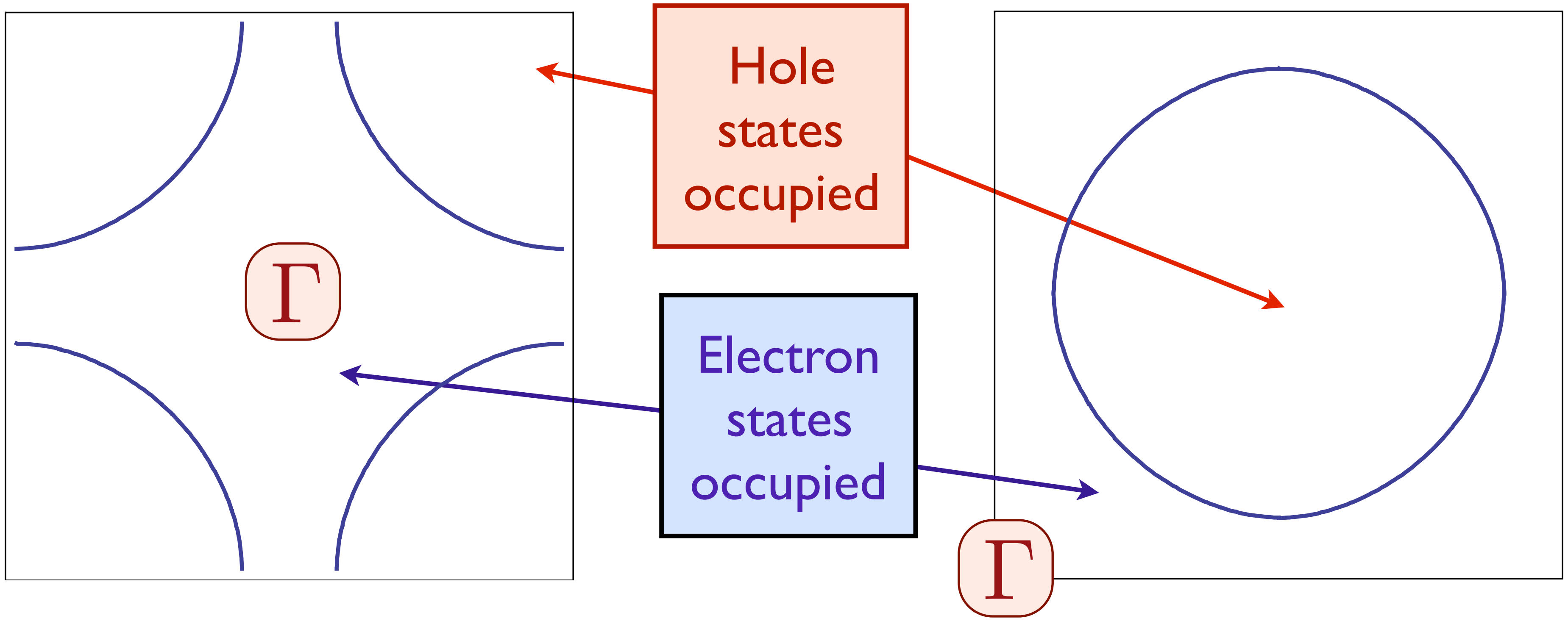
$$H_0 = -\sum_{i<j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha}$$

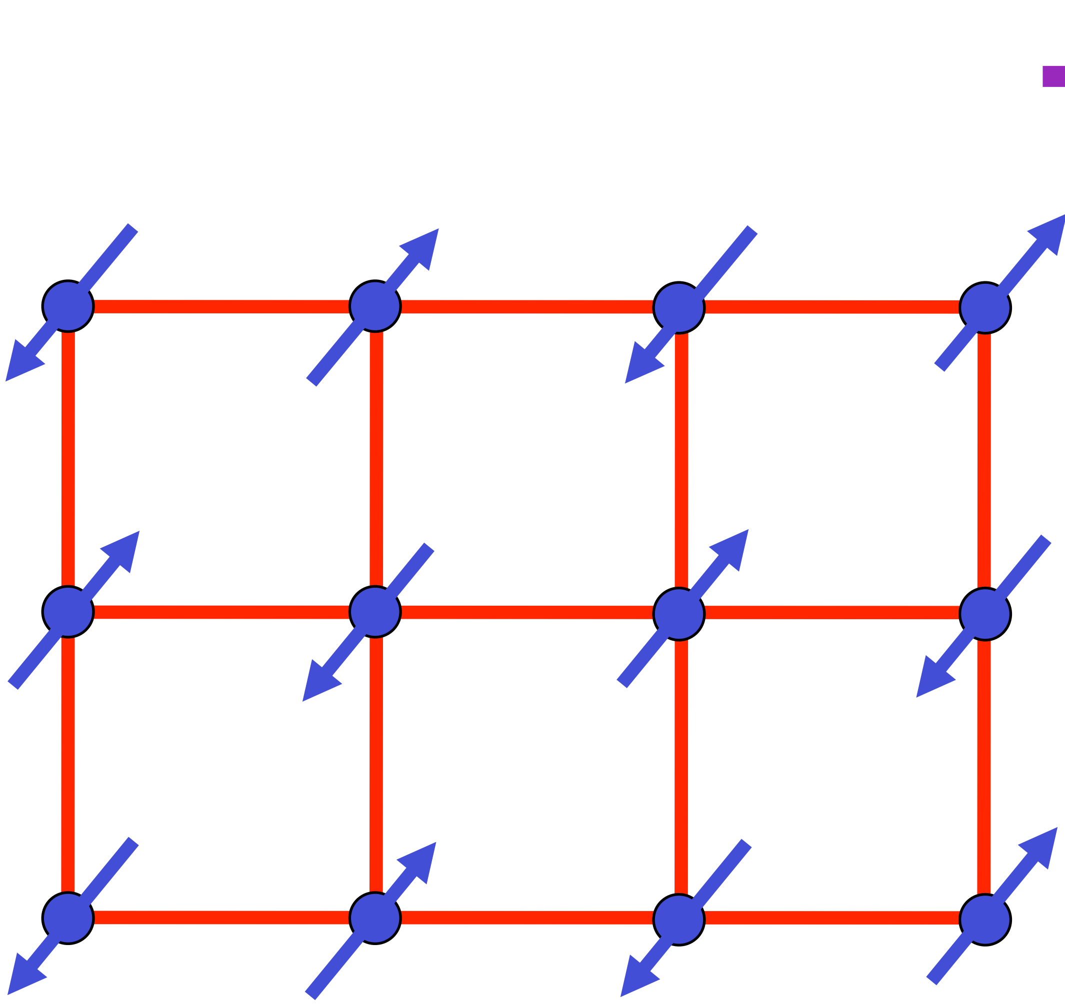
with t_{ij} non-zero for first, second and third neighbor, leads to satisfactory agreement with experiments. The area of the occupied electron states, $\mathcal{A}_e$, from Luttinger's theory is

$$\frac{\mathcal{A}_e}{2\pi^2} = \begin{cases} (1-p)(\mathrm{mod}2) & \text{for hole-doping } p \\ (1+x)(\mathrm{mod}2) & \text{for electron-doping } x \end{cases}$$

The area of the occupied hole states, $\mathcal{A}_h$, which form a closed Fermi surface and so appear in quantum oscillation experiments is $\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$.

Fermi surface+antiferromagnetism





The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r}, \tau) \right\rangle = \vec{\Phi}(\mathbf{r}, \tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K} = (\pi, \pi)$ is the ordering wavevector.

Fermi surface+antiferromagnetism

We use the operator equation (valid on each site i):

$$U\left(n_\uparrow - \frac{1}{2}\right)\left(n_\downarrow - \frac{1}{2}\right) = -\frac{2U}{3}\vec{S}^2 + \frac{U}{4} \quad (1)$$

Then we decouple the interaction via

$$\exp\left(\frac{2U}{3}\sum_i \int d\tau \vec{S}_i^2\right) = \int \mathcal{D}\vec{J}_i(\tau)\exp\left(-\sum_i \int d\tau \left[\frac{3}{8U}\vec{J}_i^2 - \vec{J}_i\vec{S}_i\right]\right) \quad (2)$$

We now integrate out the fermions, and look for the saddle point of the resulting effective action for $\vec{J}_i$. At the saddle-point we find that the lowest energy is achieved when the vector has opposite orientations on the A and B sublattices. Anticipating this, we look for a continuum limit in terms of a field $\vec{\Phi}_i$ where

$$\vec{J}_i = \vec{\Phi}_i \, e^{i\mathbf{K}\cdot\mathbf{r}_i} \quad (3)$$

Fermi surface+antiferromagnetism

In this manner, we obtain the "spin-fermion" model

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\Phi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$- \lambda \int d\tau \sum_i c^\dagger_{i\alpha} \vec{\Phi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i}$$

$$+ \int d\tau d^2 r \left[\frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\Phi} \right)^2 + \frac{1}{2} \left(\partial_\tau \vec{\Phi} \right)^2 + \frac{s}{2} \vec{\Phi}^2 + \frac{u}{4} \vec{\Phi}^4 \right]$$

Fermi surface+antiferromagnetism

In the Hamiltonian form (ignoring, for now, the time dependence of $\vec{\Phi}$), the coupling between $\vec{\Phi}$ and the electrons takes the form

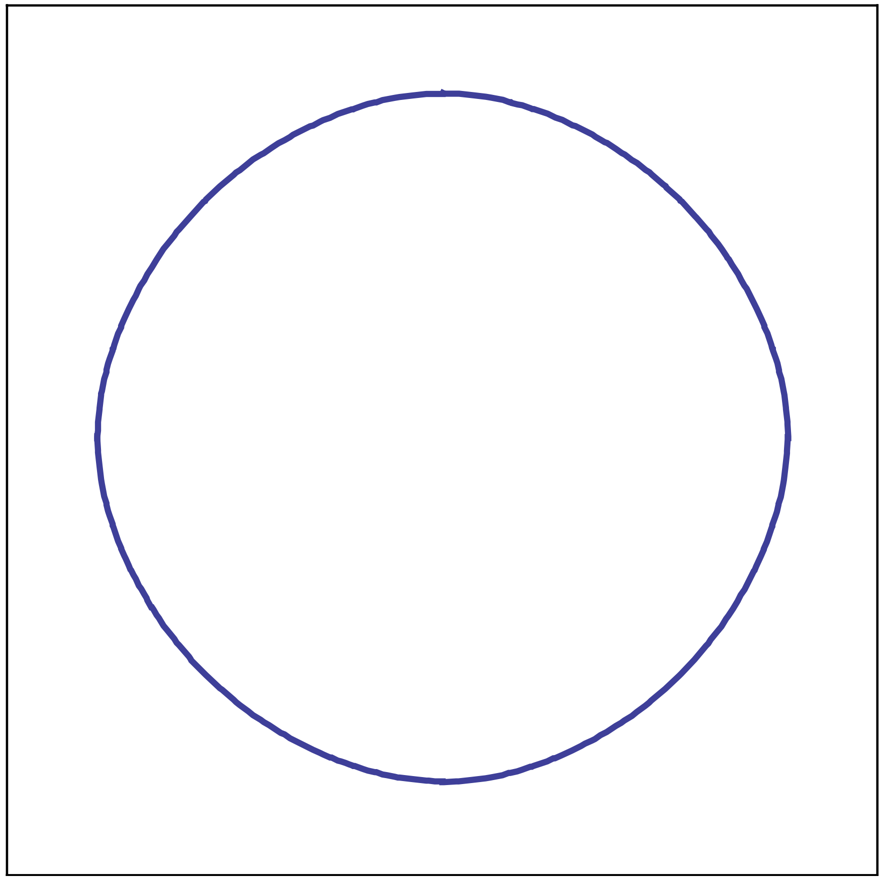
$$H_{\rm sdw} = \lambda \sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\Phi}_{\mathbf{q}} \cdot c^\dagger_{\mathbf{k}+\mathbf{q},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

where $\vec{\sigma}$ are the Pauli matrices, the boson momentum $\mathbf{q}$ is small, while the fermion momenum $\mathbf{k}$ extends over the entire Brillouin zone. In the antiferromagnetically ordered state, we may take $\vec{\Phi} \propto (0,0,1)$, and the electron dispersions obtained by diagonalizing $H_0 + H_{\rm sdw}$ are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right)^2 + \lambda^2 |\vec{\Phi}|^2}$$

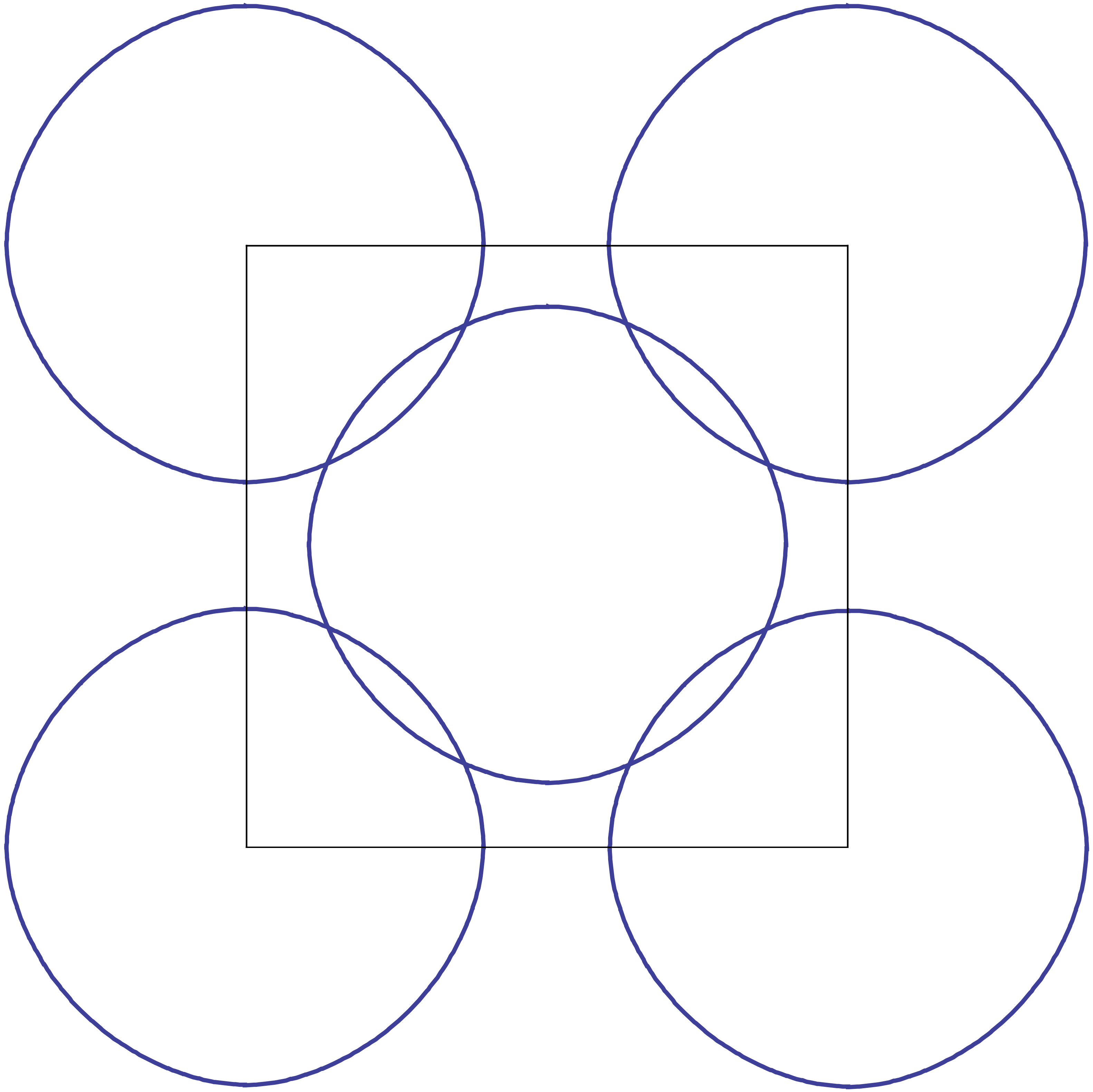
This leads to the Fermi surfaces shown in the following slides as a function of increasing $|\vec{\Phi}|$.

Fermi surface+antiferromagnetism



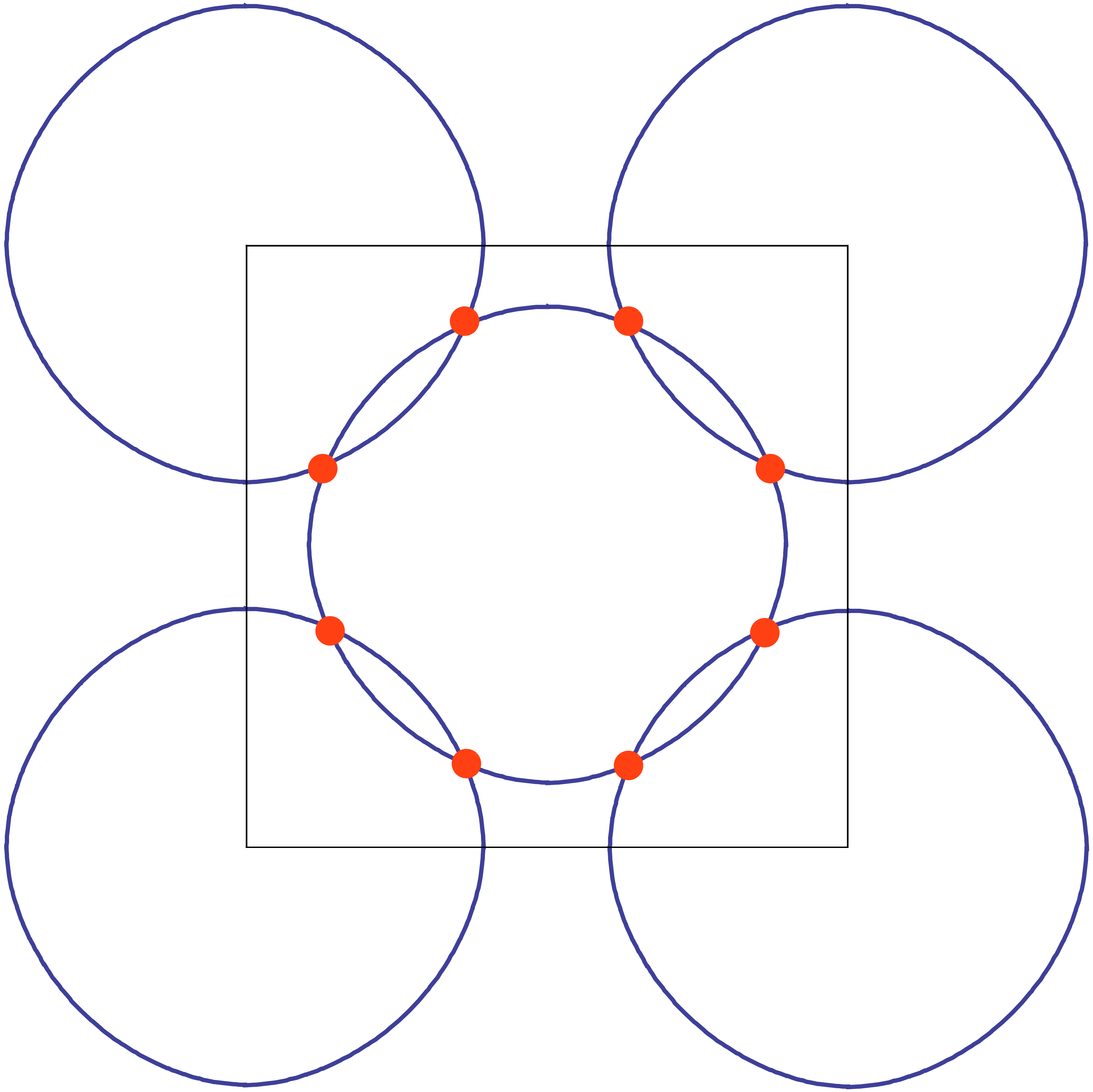
Metal with "large" Fermi surface

Fermi surface+antiferromagnetism



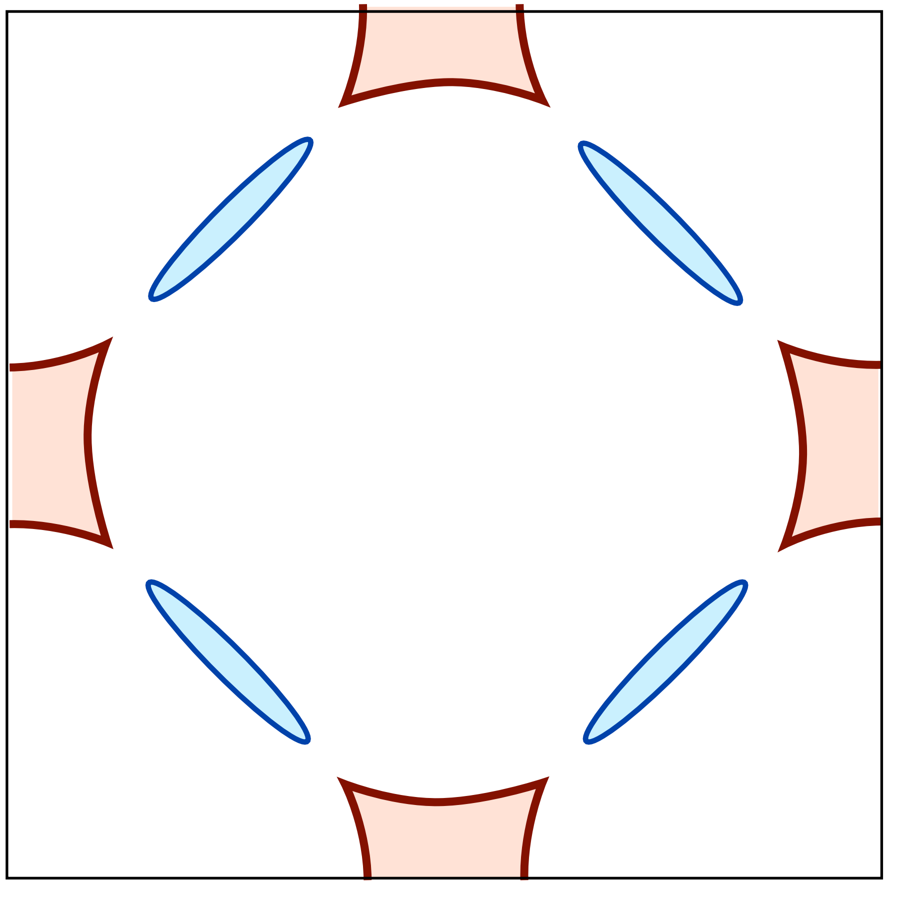
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



"Hot" spots

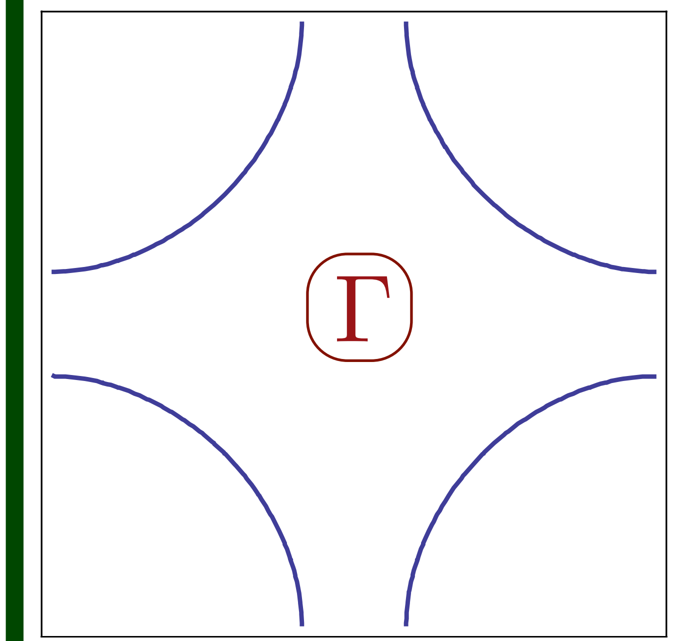
Fermi surface+antiferromagnetism



Electron and hole pockets in
antiferromagnetic phase with $\langle \vec{\Phi} \rangle \neq 0$

Square lattice Hubbard model with hole doping

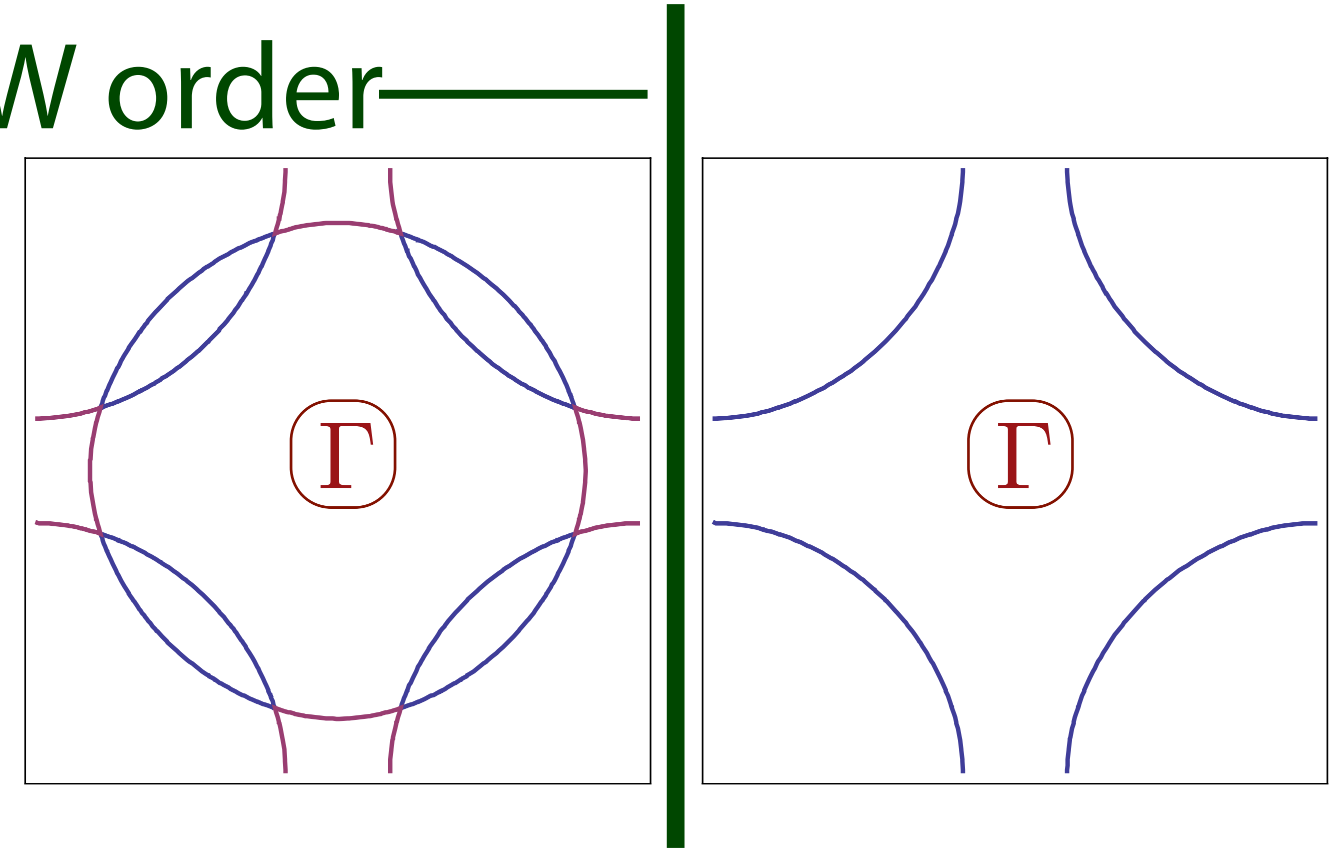
Increasing SDW order



S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Square lattice Hubbard model with hole doping

⟵ Increasing SDW order ⟶



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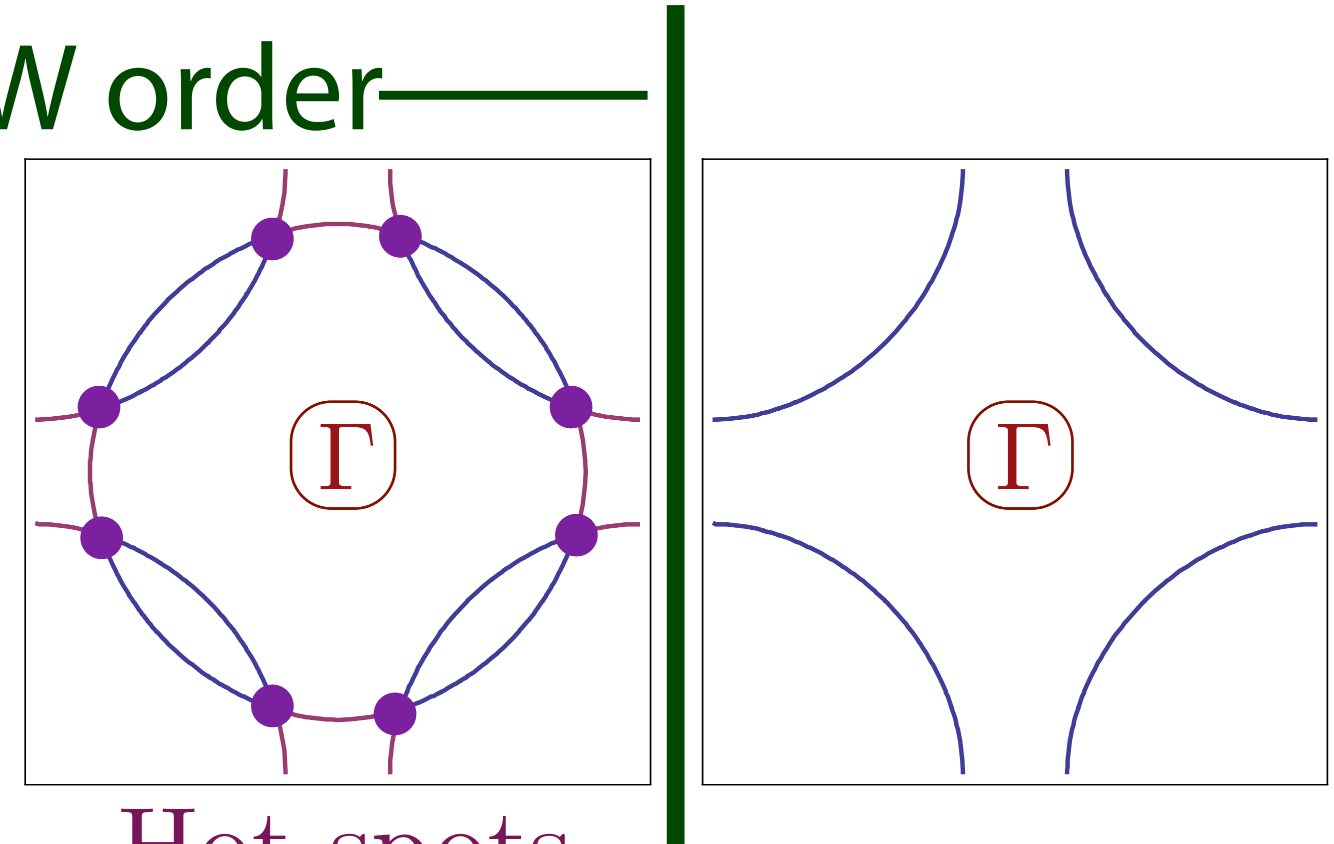
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Square lattice Hubbard model with hole doping

← Increasing SDW order



Hot spots

where $\varepsilon_{\mathbf{k}} = \varepsilon_{\mathbf{k}+\mathbf{K}}$

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
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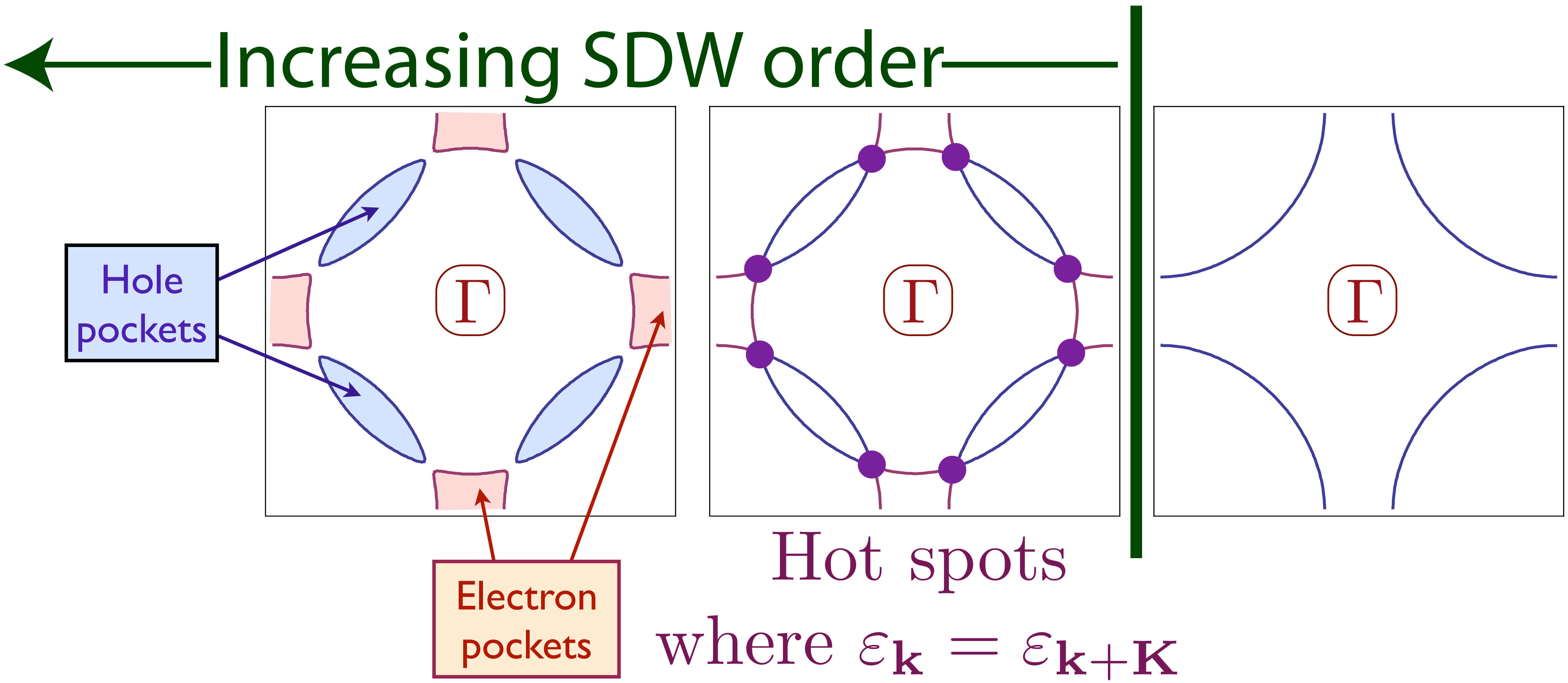
Square lattice Hubbard model with hole doping



Fermi surface breaks up at hot spots
into electron and hole "pockets"

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
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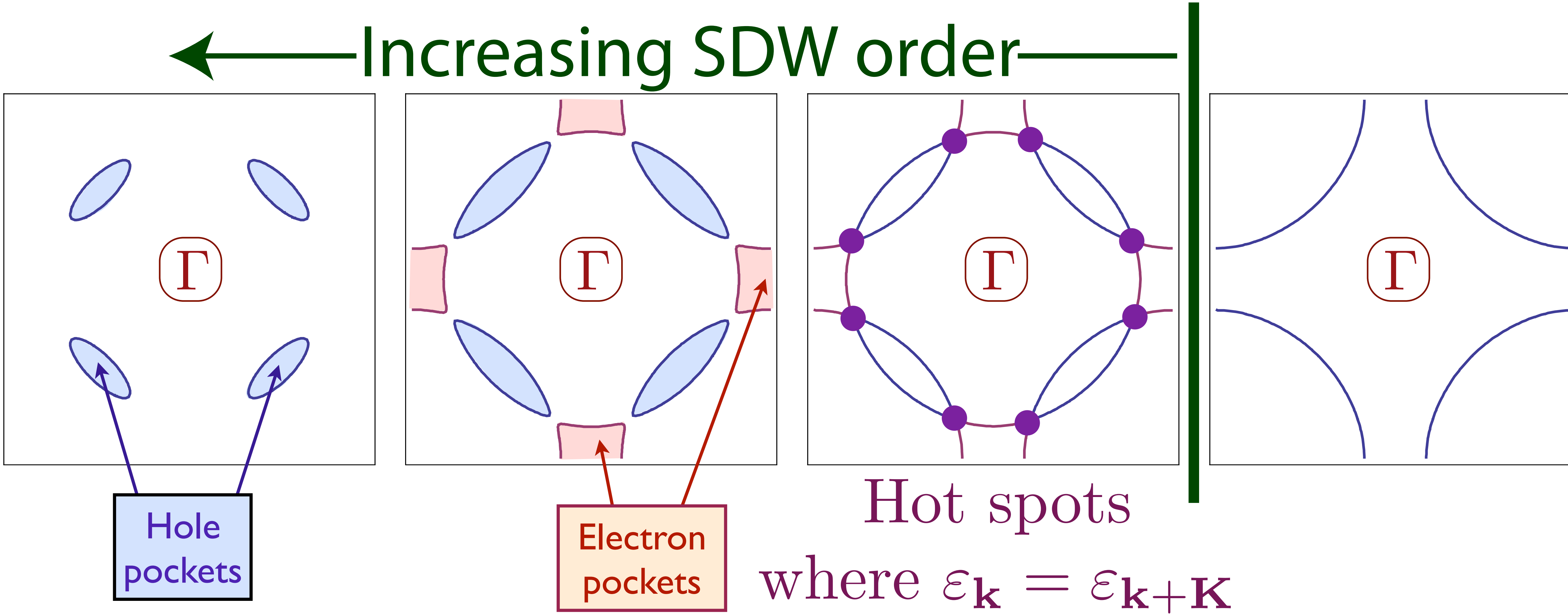
Square lattice Hubbard model with hole doping



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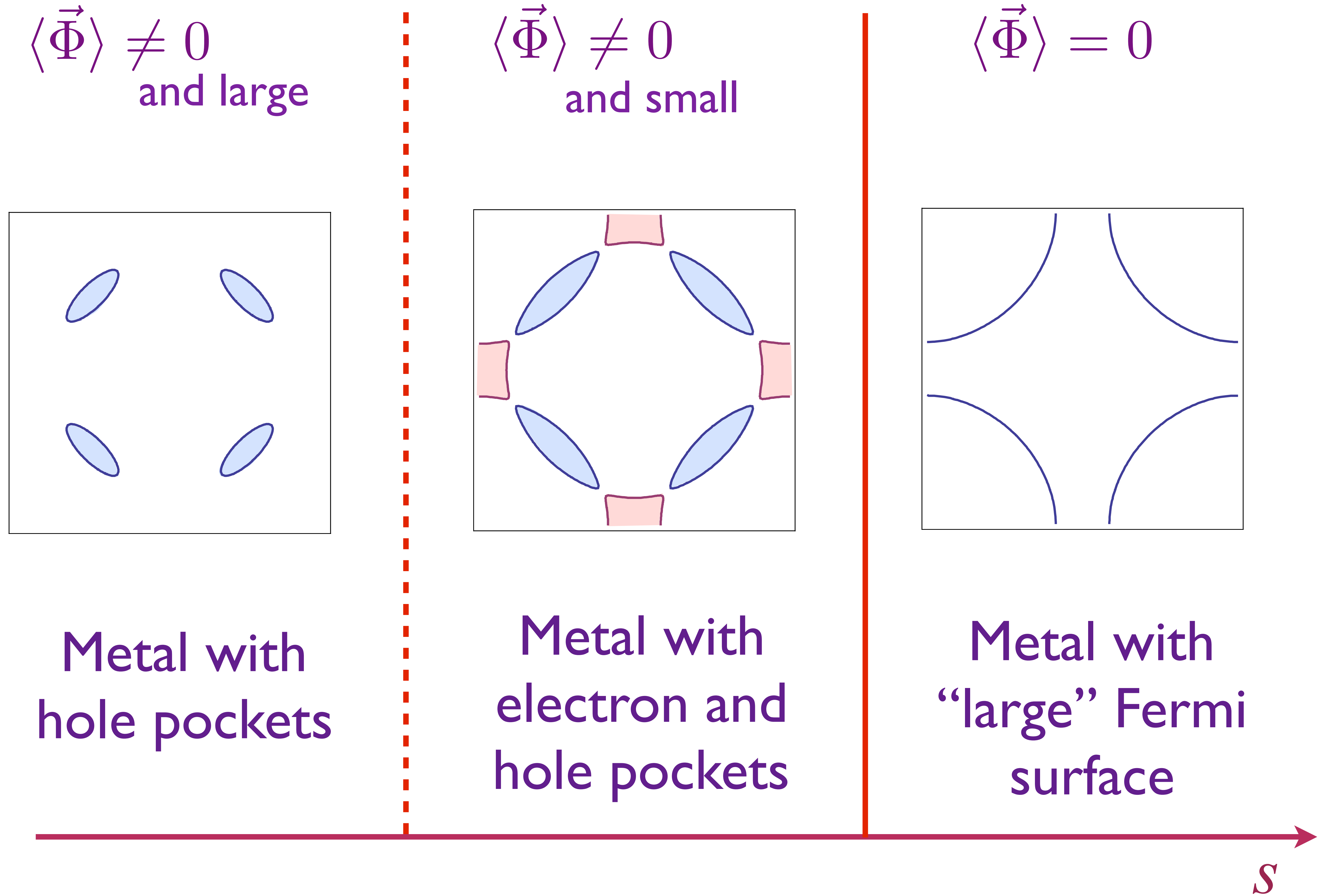
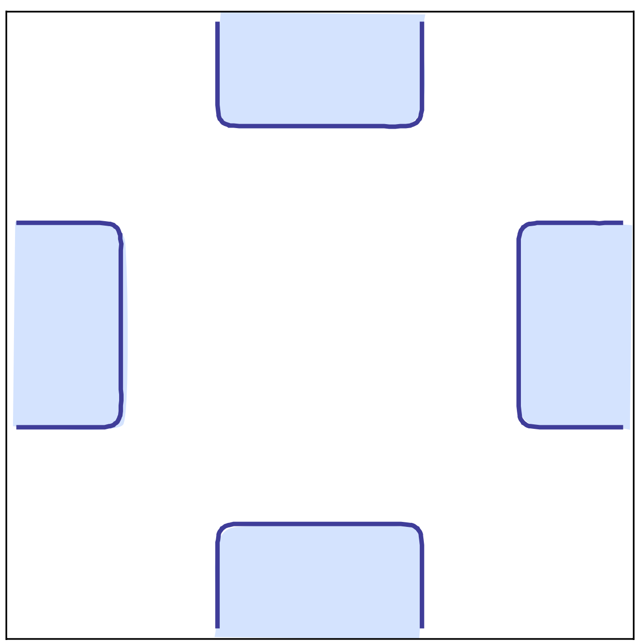
Square lattice Hubbard model with hole doping



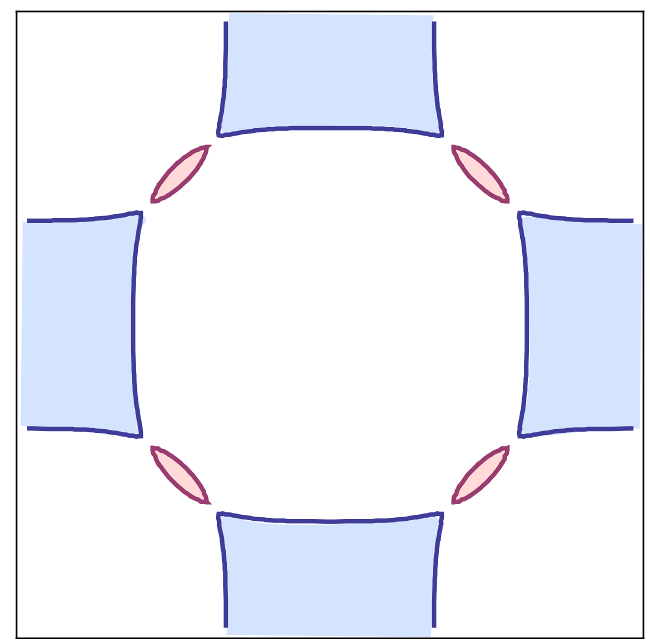
Square lattice Hubbard model with electron doping

$\langle \vec{\Phi} \rangle \neq 0$ and large



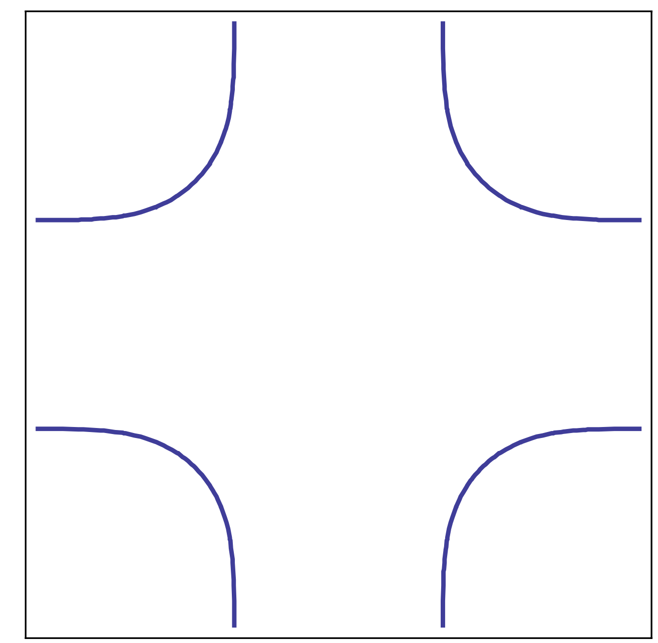
Metal with electron pockets

$\langle \vec{\Phi} \rangle \neq 0$ and small



Metal with electron and hole pockets

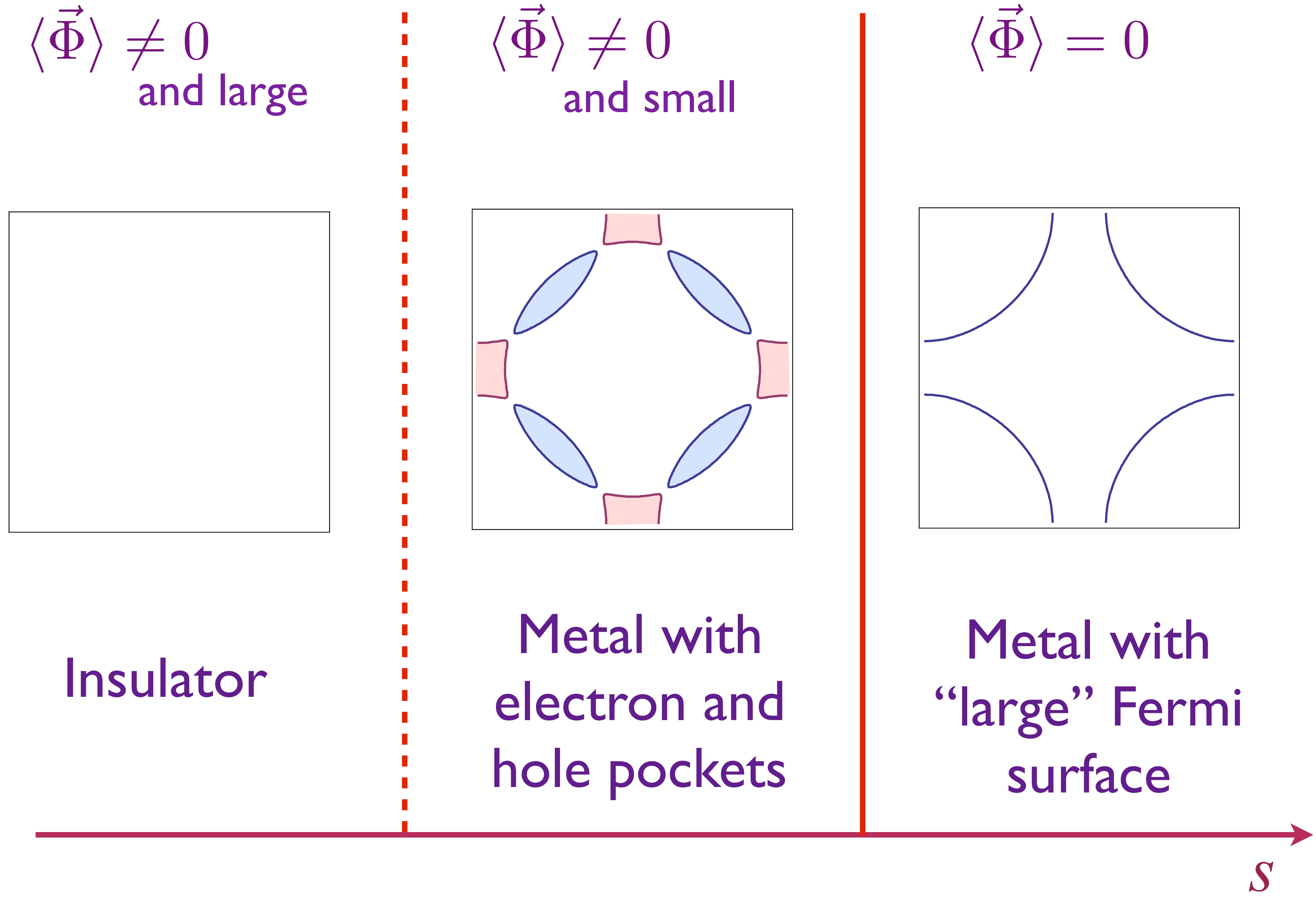
$\langle \vec{\Phi} \rangle = 0$



Metal with "large" Fermi surface

s

Square lattice Hubbard model with no doping



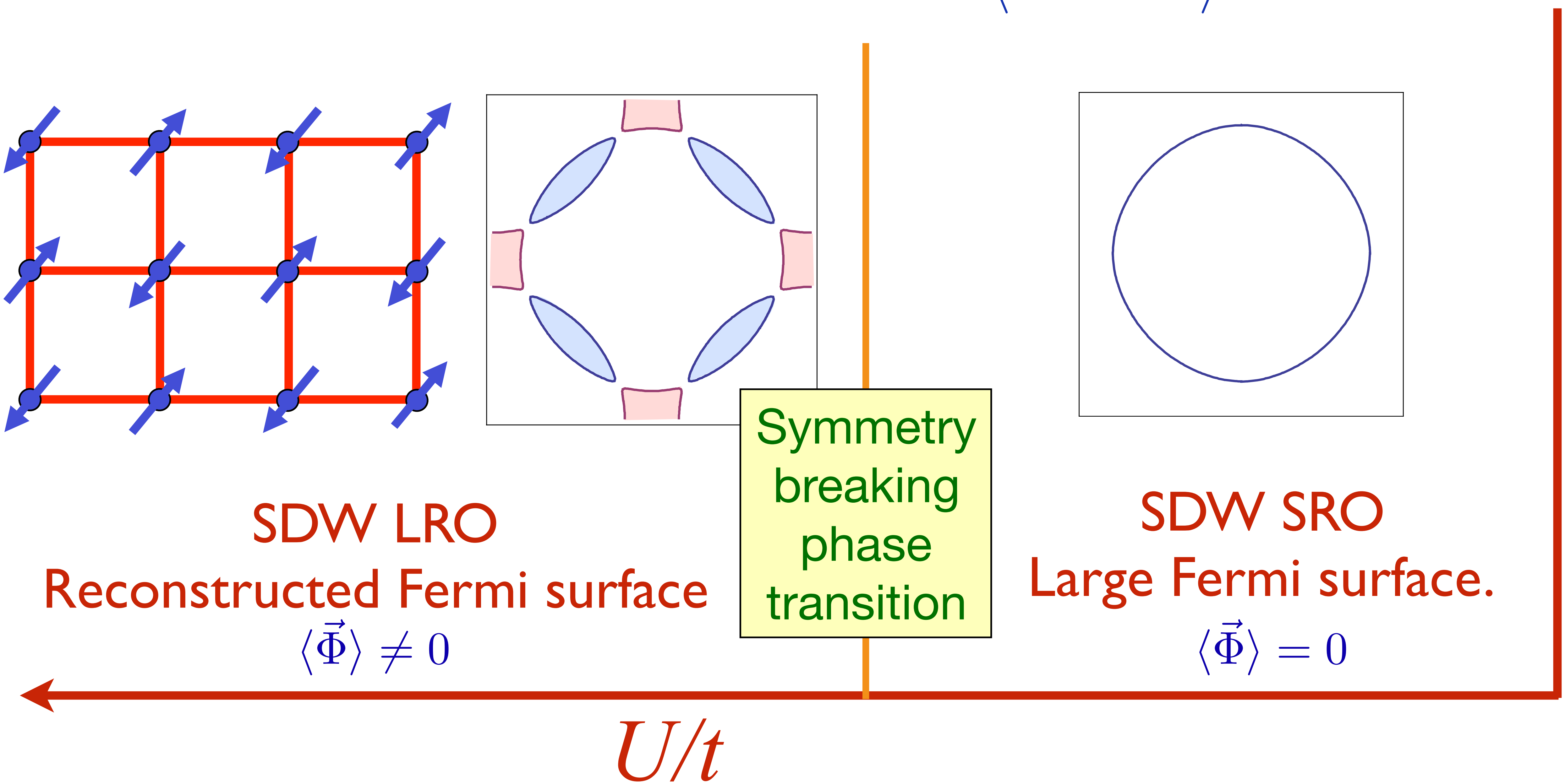
Antiferromagnetism in the Hubbard Model

$$H = -\sum_{i<j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c^\dagger_{i\alpha} c_{i\alpha}$$

$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

Mean-field theory with a spin density wave (SDW)

order parameter $\vec{\Phi}_i = (-1)^{i_x+i_y} \left\langle c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta} \right\rangle / 2$

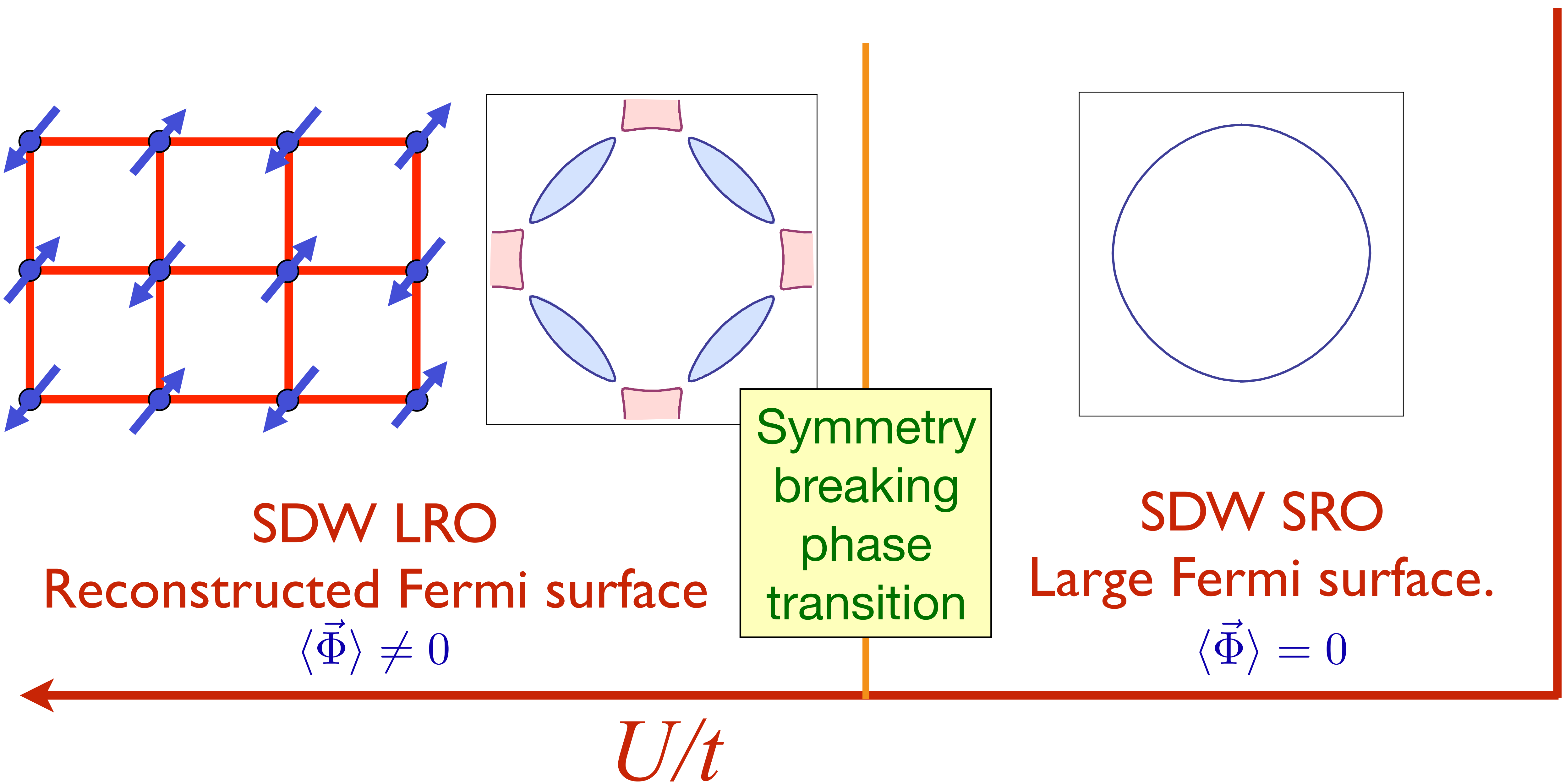


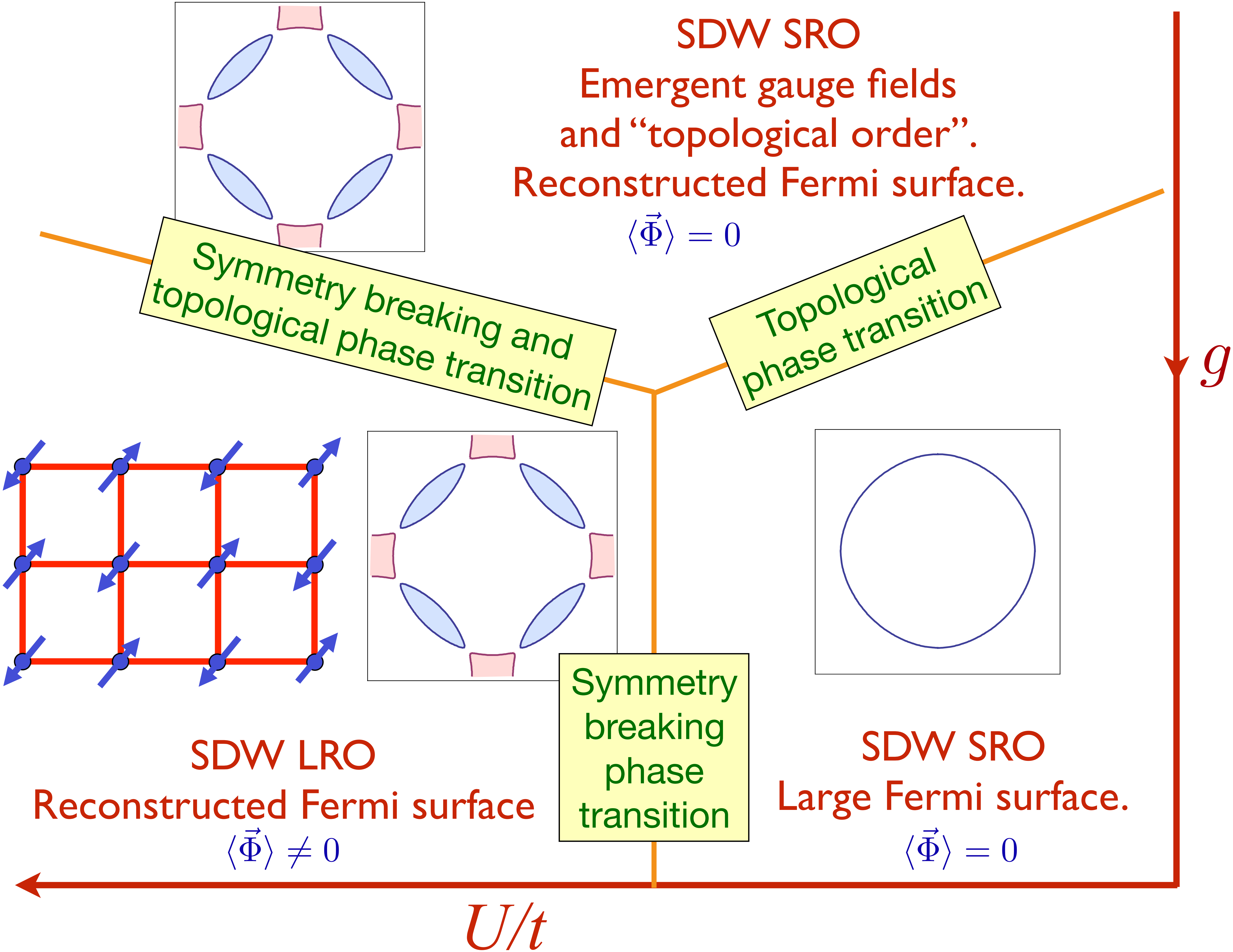
Antiferromagnetism in the Hubbard Model

$$H = -\sum_{i<j} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + U \sum_{i} \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_{i} c_{i\alpha}^{\dagger} c_{i\alpha}$$

$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

Both states have Luttinger volume Fermi surfaces





SDW SRO
Emergent gauge fields
and "topological order".
Reconstructed Fermi surface.
$\langle \vec{\Phi} \rangle = 0$
Symmetry breaking and topological phase transition
Topological phase transition
g
SDW LRO
Reconstructed Fermi surface
$\langle \vec{\Phi} \rangle \neq 0$
Symmetry breaking phase transition
SDW SRO
Large Fermi surface.
$\langle \vec{\Phi} \rangle = 0$
U/t

1. Review of spin-density-wave theory

2. Phase transitions in classical XY models

3. SU(2) gauge theory of fluctuating spin density waves

4. Photoemission on the electron-doped cuprates

5. Comparison with cluster-DMFT studies

1. Review of spin-density-wave theory

2. Phase transitions in classical XY models

3. SU(2) gauge theory of fluctuating spin density waves

4. Photoemission on the electron-doped cuprates

5. Comparison with cluster-DMFT studies

Classical XY model

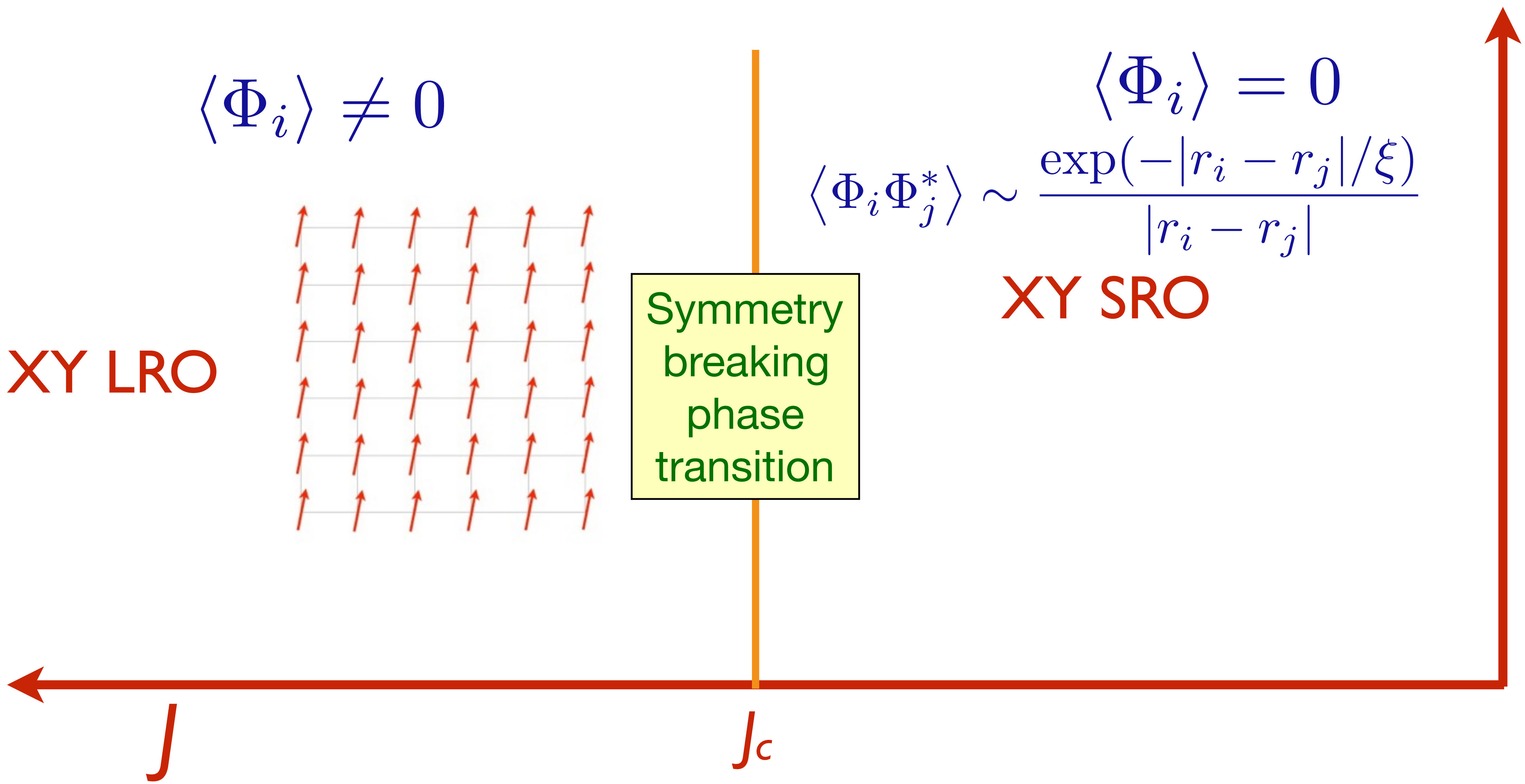
$$\mathcal{Z}_{XY} = \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-H/T\right)$$

$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

$$\Phi_i \equiv e^{i\theta_i}$$

Describes non-zero *T* phase transitions of superfluids, magnets with `easy-plane' spins, ...

Classical XY model in D=3



Classical XY model in D=2

Ordering, metastability and phase transitions in two-dimensional systems

J. Phys. C 1973

J M Kosterlitz and D J Thouless

A new definition of order called topological order is proposed for two-dimensional systems in which no long-range order of the conventional type exists.

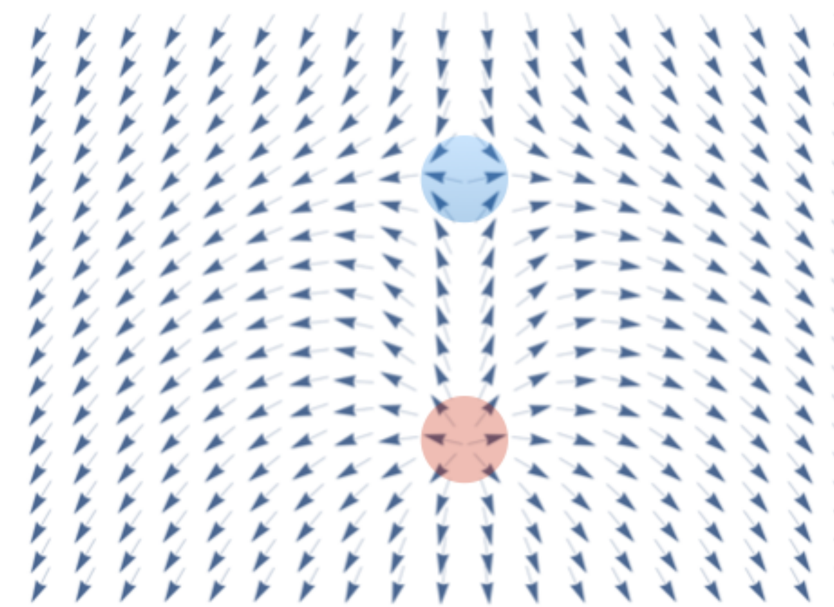
$$\langle \Phi_i \Phi_j^* \rangle \sim \frac{1}{|r_i - r_j|^\alpha}$$

$$\langle \Phi_i \Phi_j^* \rangle \sim \frac{\exp(-|r_i - r_j|/\xi)}{|r_i - r_j|^{1/2}}$$

XY QLRO
Topological order

Topological phase transition: Kosterlitz Thouless

XY SRO
No topological order



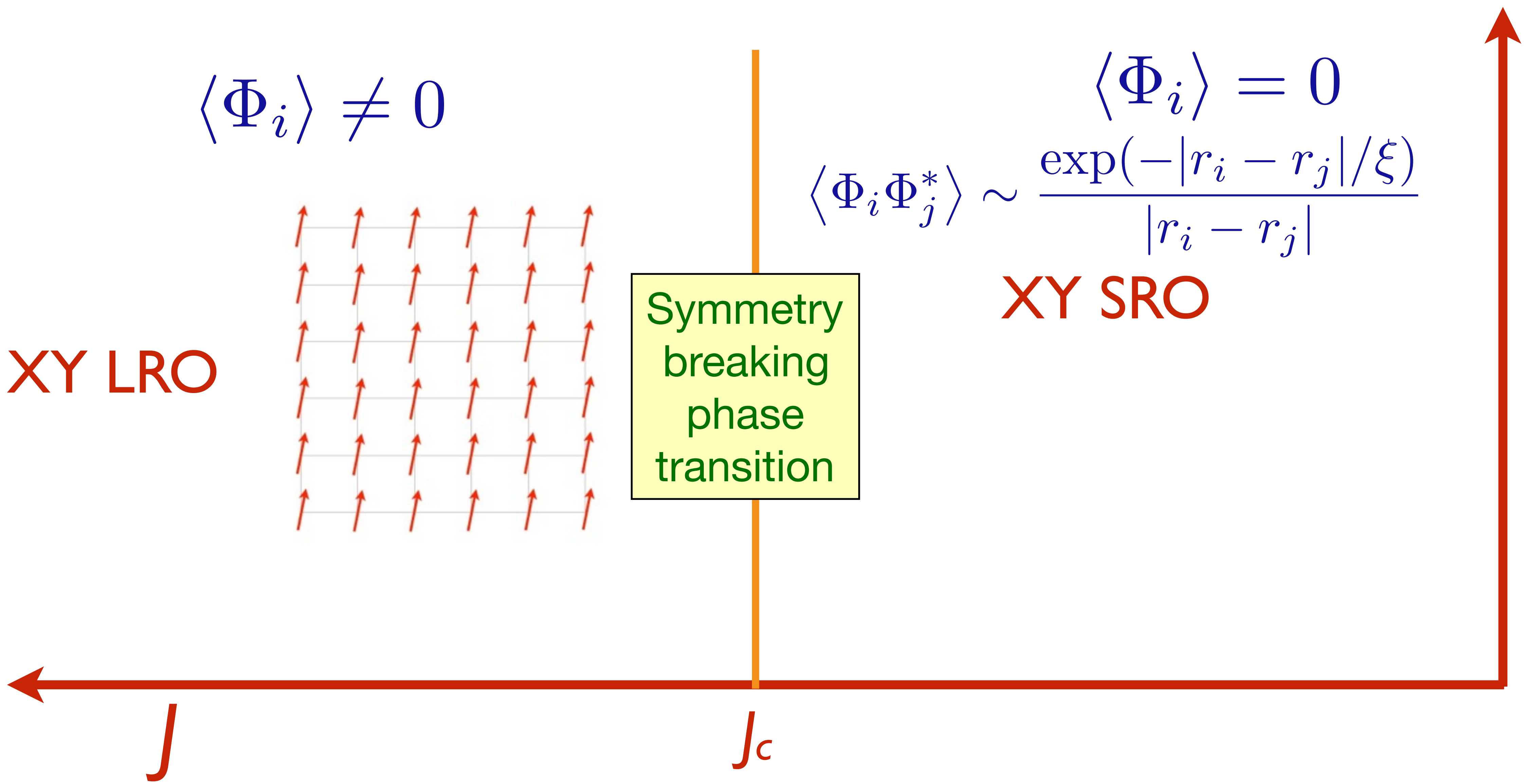
Vortices expelled

Vortices proliferate

T_{KT} $\quad$ T

Classical XY model in D=3

Can we have a topological phase transition in a XY model in D=3?



$$\widetilde{\mathcal{Z}}_{XY} = \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

$$+ \sum_{ijk\ell} K_{ijk\ell} \cos(\theta_i + \theta_j - \theta_k - \theta_\ell) + \ldots\ldots$$

Add terms which suppress $\pm 2\pi$ but not $\pm 4\pi$ vortices.

$$\widetilde{\mathcal{Z}}_{XY} = \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

$$+ \sum_{ijk\ell} K_{ijk\ell} \cos(\theta_i + \theta_j - \theta_k - \theta_\ell) + \ldots\ldots$$

Add terms which suppress $\pm 2\pi$ but not $\pm 4\pi$ vortices.
A convenient form is obtained using an auxiliary variable
$\sigma_{ij} = \pm 1$ on the links of the cubic lattice.

$$\widetilde{\mathcal{Z}}_{XY} = \sum_{\{\sigma_{ij}\}=\pm 1} \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \sigma_{ij} \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\square} \prod_{(ij)\in\square} \sigma_{ij}$$

$$\widetilde{\mathcal{Z}}_{XY} = \sum_{\{\sigma_{ij}\}=\pm 1} \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \sigma_{ij} \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\Box} \prod_{(ij)\in\Box} \sigma_{ij}$$

- At small K, we can explicitly sum over σ_{ij}, order-by-order in K, and the theory reduces to an ordinary XY model with multi-site interactions. The resulting effective action of the XY model is periodic in $\theta_i \to \theta_i + 2\pi$ (for any site i), and preserves the symmetry $\theta_i \to \theta_i + c$ (for all sites i).

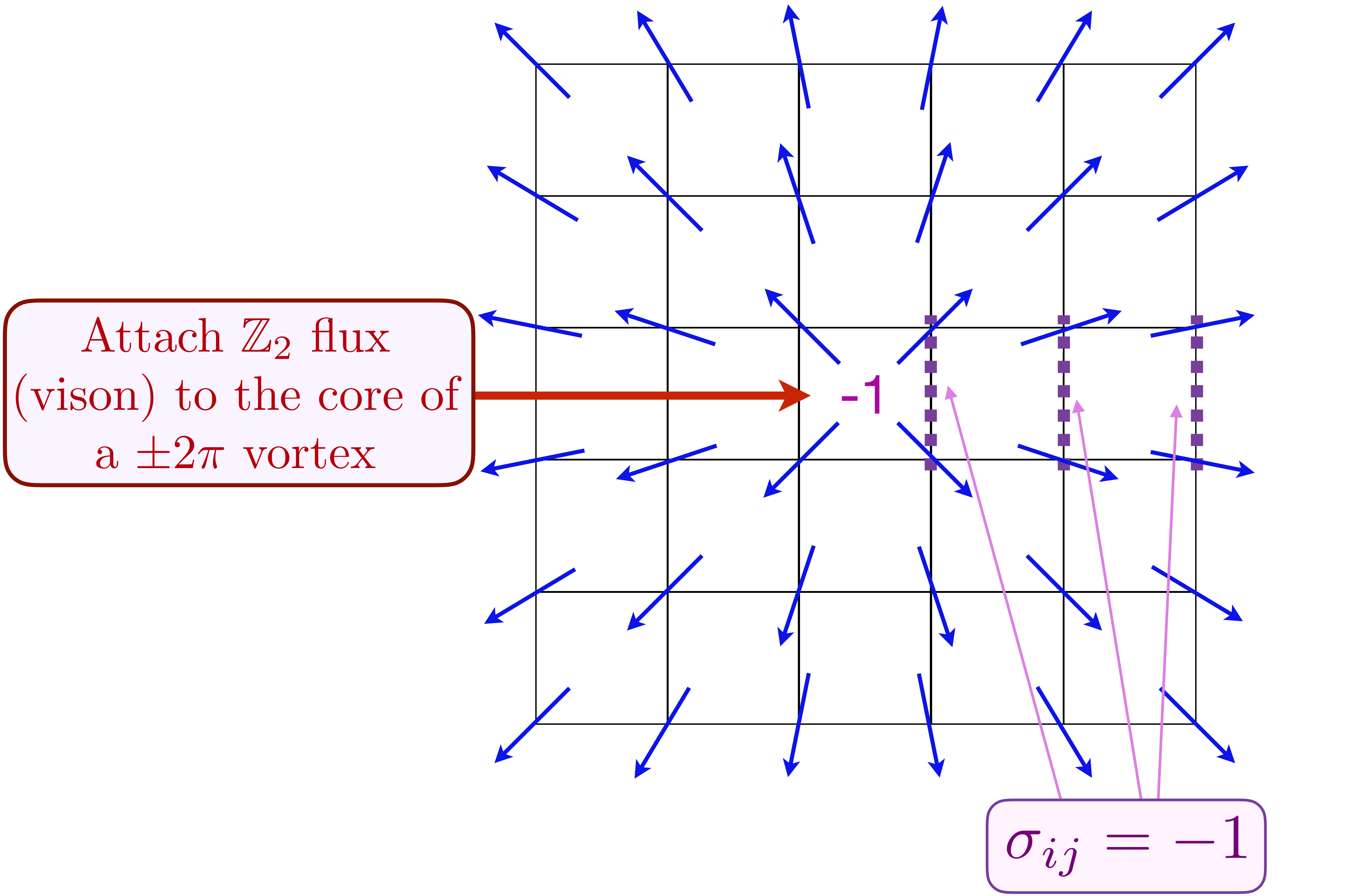
$$
\begin{aligned}
\widetilde{\mathcal{Z}}_{XY} &= \sum_{\{\sigma_{ij}\}=\pm 1} \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right) \\
\widetilde{H} &= -J \sum_{\langle ij \rangle} \sigma_{ij} \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\Box} \prod_{(ij)\in\Box} \sigma_{ij}
\end{aligned}
$$

- The theory has a $\mathbb{Z}_2$ gauge invariance: we can change

$$
\begin{aligned}
\theta_i &\to \theta_i + \pi(1 - \eta_i) \\
\sigma_{ij} &\to \eta_i \sigma_{ij} \eta_j \, ,
\end{aligned}
$$

  with $\eta_i = \pm 1$, and the energy remains unchanged.

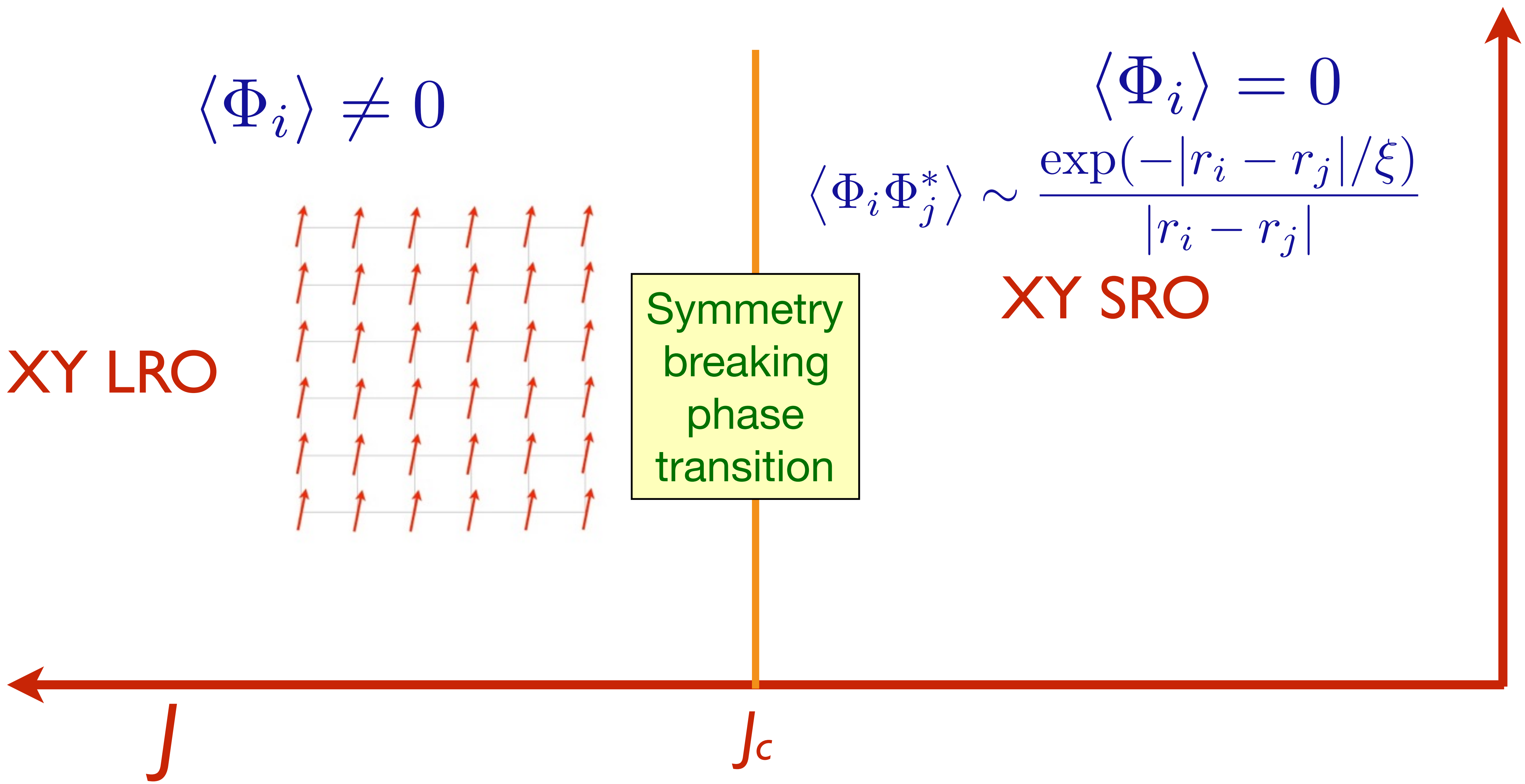
- The XY order parameter $\Psi_i = e^{i\theta_i}$ is gauge invariant, as are all physical observables. So this is an XY model with a modified Hamiltonian, and no additional degrees of freedom have been introduced.



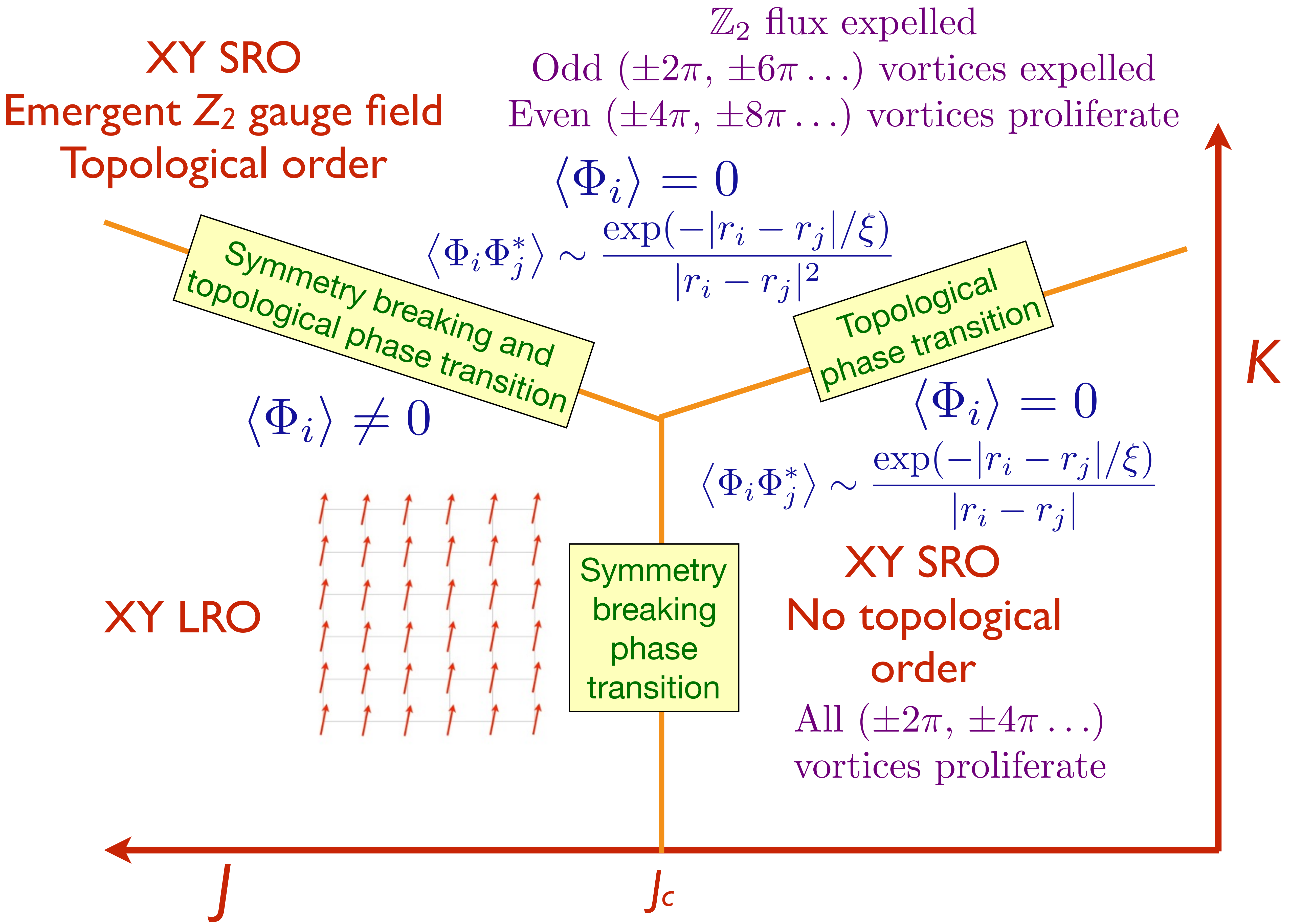
Attach $\mathbb{Z}_2$ flux (vison) to the core of a $\pm 2\pi$ vortex
-1
$\sigma_{ij} = -1$

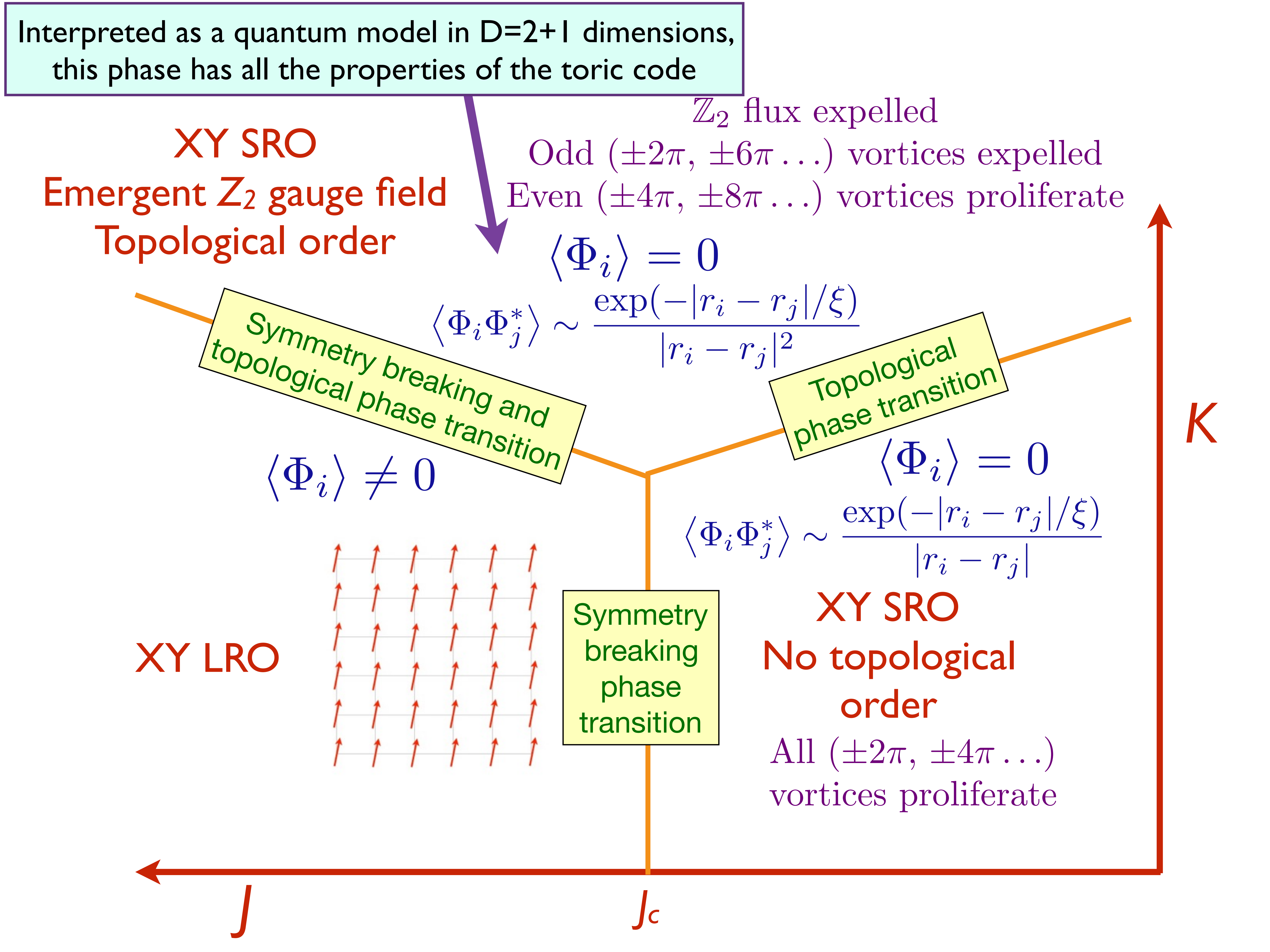
Classical XY model in D=3

Can we have a topological phase transition in D=3?

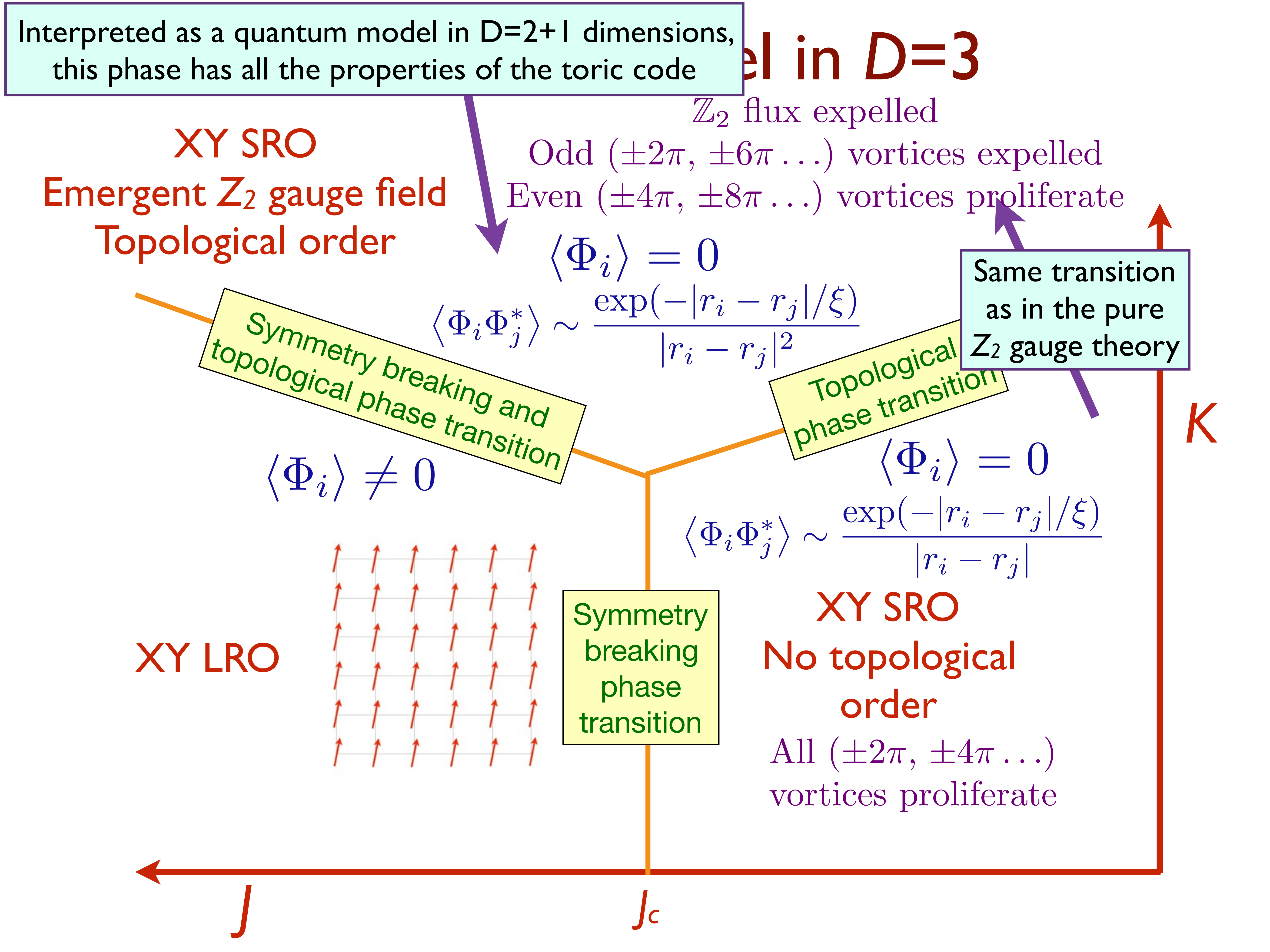


Classical XY model in D=3





Interpreted as a quantum model in D=2+1 dimensions, this phase has all the properties of the toric code
XY SRO
Emergent Z_2 gauge field
Topological order
$\mathbb{Z}_2$ flux expelled
Odd $(\pm 2\pi, \pm 6\pi \ldots)$ vortices expelled
Even $(\pm 4\pi, \pm 8\pi \ldots)$ vortices proliferate
$\langle \Phi_i \rangle = 0$
$\langle \Phi_i \Phi_j^* \rangle \sim \frac{\exp(-|r_i - r_j|/\xi)}{|r_i - r_j|^2}$
Symmetry breaking and topological phase transition
Topological phase transition
$\langle \Phi_i \rangle \neq 0$
$\langle \Phi_i \rangle = 0$
$\langle \Phi_i \Phi_j^* \rangle \sim \frac{\exp(-|r_i - r_j|/\xi)}{|r_i - r_j|}$
XY LRO
Symmetry breaking phase transition
XY SRO
No topological order
All $(\pm 2\pi, \pm 4\pi \ldots)$ vortices proliferate
K
J
J_c



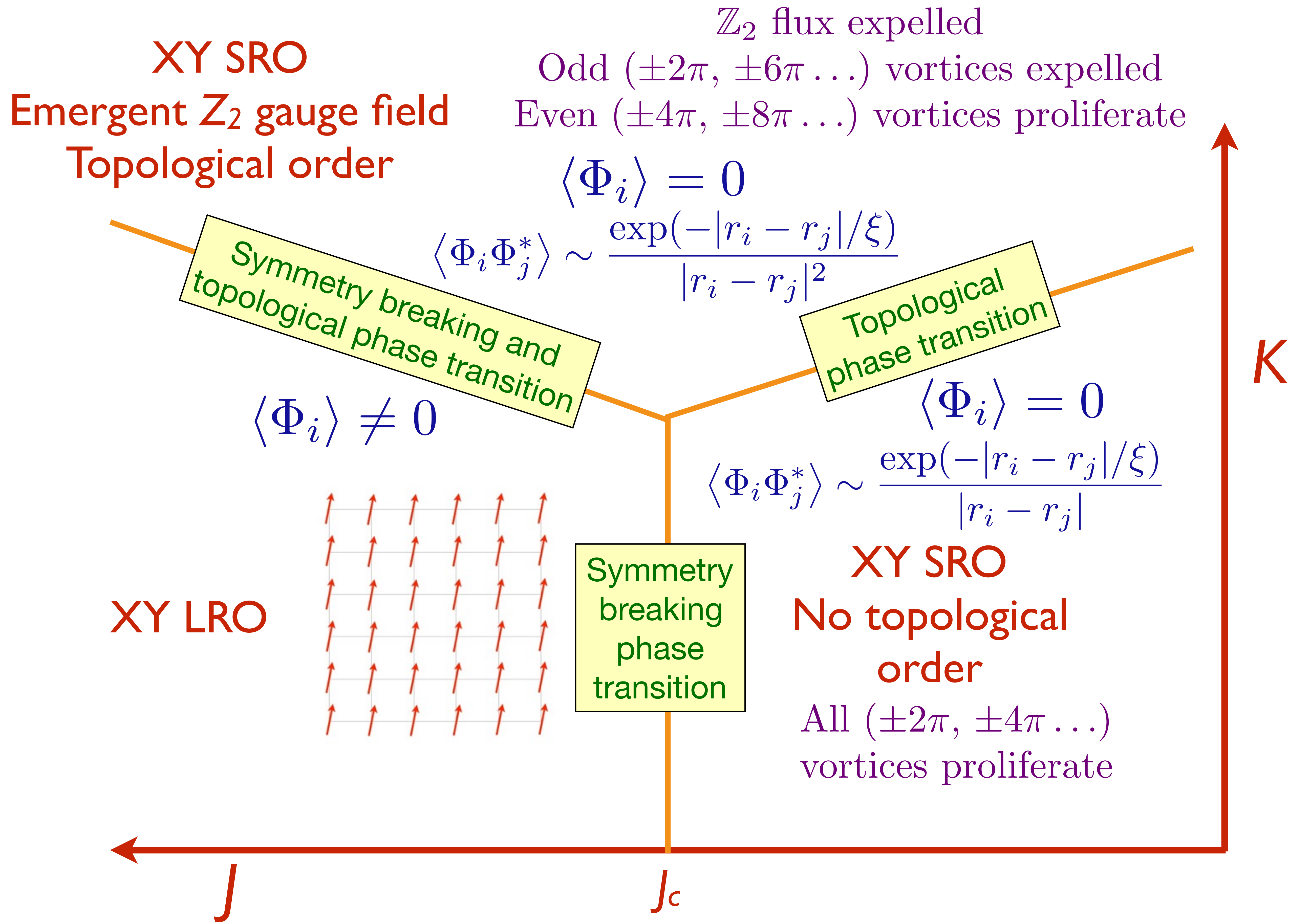
el in D=3
Interpreted as a quantum model in D=2+1 dimensions, this phase has all the properties of the toric code
$\mathbb{Z}_2$ flux expelled
Odd $(\pm 2\pi, \pm 6\pi \ldots)$ vortices expelled
Even $(\pm 4\pi, \pm 8\pi \ldots)$ vortices proliferate
XY SRO
Emergent Z_2 gauge field
Topological order
$\langle \Phi_i \rangle = 0$
$\langle \Phi_i \Phi_j^* \rangle \sim \frac{\exp(-|r_i - r_j|/\xi)}{|r_i - r_j|^2}$
Same transition as in the pure Z_2 gauge theory
Symmetry breaking and topological phase transition
Topological phase transition
K
$\langle \Phi_i \rangle \neq 0$
$\langle \Phi_i \rangle = 0$
$\langle \Phi_i \Phi_j^* \rangle \sim \frac{\exp(-|r_i - r_j|/\xi)}{|r_i - r_j|}$
XY LRO
Symmetry breaking phase transition
XY SRO
No topological order
All $(\pm 2\pi, \pm 4\pi \ldots)$ vortices proliferate
J
J_c

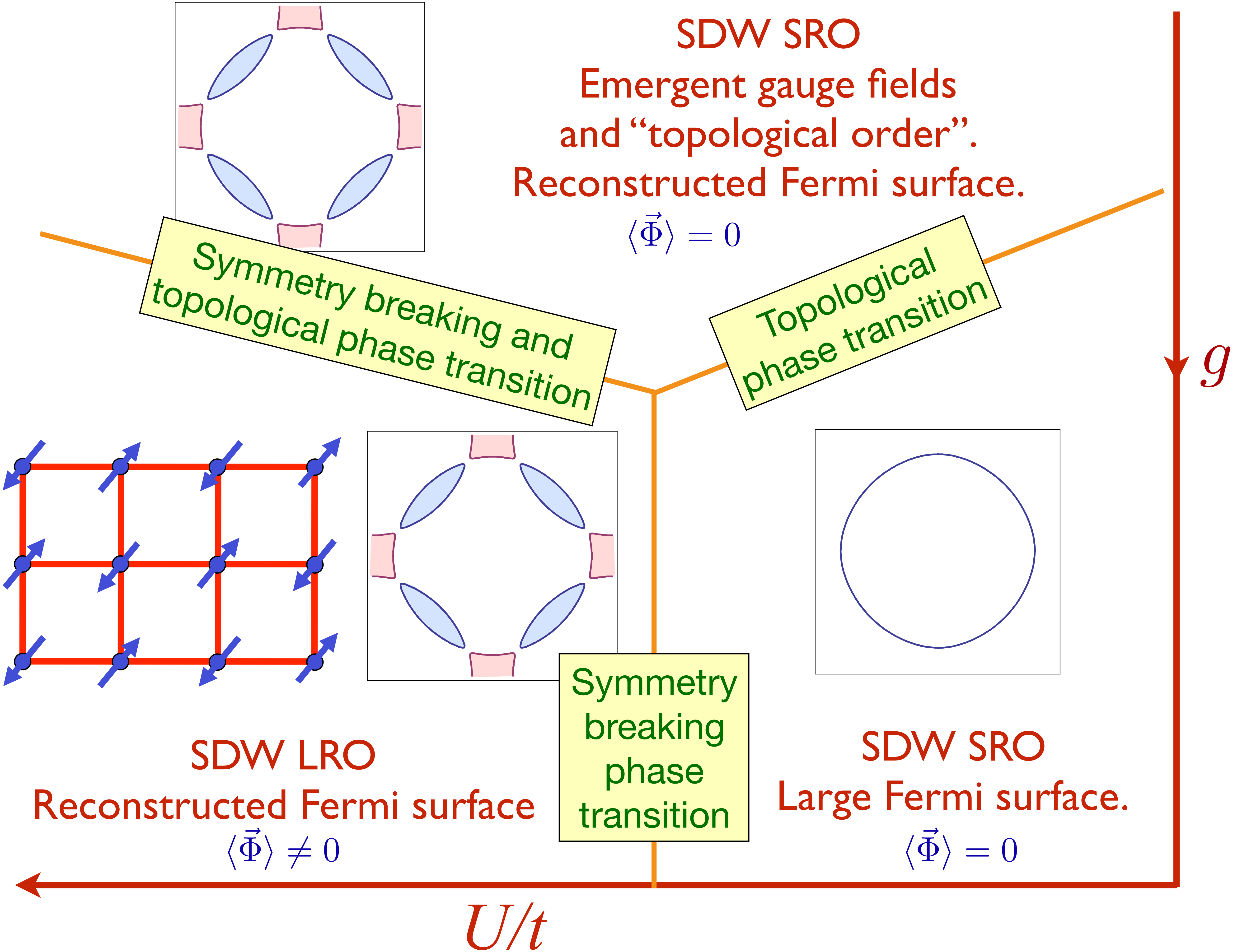
$$\widetilde{\mathcal{Z}}_{XY} = \sum_{\{\sigma_{ij}\}=\pm 1} \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \sigma_{ij} \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\square} \prod_{(ij)\in\square} \sigma_{ij}$$

- At $J = 0$, we have a "pure" emergent $\mathbb{Z}_2$ gauge theory. This theory has a confinement-deconfinement phase transition (Wegner, 1971) which also describes the topological phase transition between the two SRO phases of the XY model.

Classical XY model in D=3





SDW SRO
Emergent gauge fields
and “topological order”.
Reconstructed Fermi surface.
$\langle \vec{\Phi} \rangle = 0$
Symmetry breaking and
topological phase transition
Topological
phase transition
g
SDW LRO
Reconstructed Fermi surface
$\langle \vec{\Phi} \rangle \neq 0$
Symmetry
breaking
phase
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SDW SRO
Large Fermi surface.
$\langle \vec{\Phi} \rangle = 0$
U/t

1. Review of spin-density-wave theory

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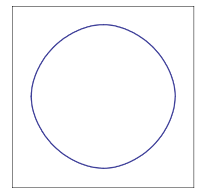
5. Comparison with cluster-DMFT studies

We can (exactly) transform the Hubbard model to the “spin-fermion” model: **electrons** $c_{i\alpha}$ on the square lattice with dispersion

$$\mathcal{H}_c = -\sum_{i,\rho} t_\rho \left(c_{i,\alpha}^\dagger c_{i+\boldsymbol{v}_\rho,\alpha} + c_{i+\boldsymbol{v}_\rho,\alpha}^\dagger c_{i,\alpha} \right) - \mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} + \mathcal{H}_{\text{int}}$$



are coupled to an **antiferromagnetic SDW order parameter** $\Phi^\ell(i)$**,** $\ell = x, y, z$

$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \eta_i \Phi^\ell(i) c_{i,\alpha}^\dagger \sigma_{\alpha\beta}^\ell c_{i,\beta} + V_\Phi$$

where $\eta_i = \pm 1$ on the two sublattices. (For suitable V_Φ, integrating out the Φ^ℓ yields back the Hubbard model).

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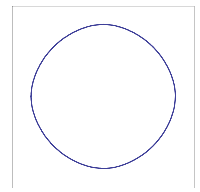

are coupled to an **antiferromagnetic SDW order parameter** $\Phi^\ell(i)$**,** $\ell = x, y, z$

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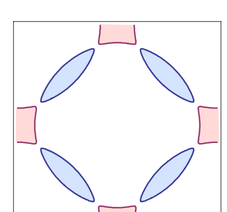
where $\eta_i = \pm 1$ on the two sublattices. (For suitable V_Φ, integrating out the Φ^ℓ yields back the Hubbard model).

When $\Phi^\ell(i) =$ (non-zero constant) independent of i, we have long-range SDW order, which transforms the Fermi surfaces from large to small.



For (fluctuating) SDW SRO, we transform to a rotating reference frame using the SU(2) rotation R_i

$$\left(\begin{array}{c} c_{i\uparrow} \\ c_{i\downarrow} \end{array} \right) = R_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right),$$

in terms of fermionic "chargons" ψ_s and a **Higgs field** $H^a(i)$

$$\sigma^\ell \Phi^\ell(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

The Higgs field is the SDW order in the rotating reference frame.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

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The Higgs field is the SDW order in the rotating reference frame. Note that this representation is ambiguous up to a SU(2) gauge transformation, V_i

$$\begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix} \to V_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix}$$

$$R_i \to R_i V_i^\dagger$$

$$\sigma^a H^a(i) \to V_i \, \sigma^b H^b(i) \, V_i^\dagger.$$

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Fluctuating SDW

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the SDW order replaced by the Higgs field.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi^\dagger_{i,s} \psi_{i+\boldsymbol{v}_\rho,s} + \psi^\dagger_{i+\boldsymbol{v}_\rho,s} \psi_{i,s} \right) - \mu \sum_i \psi^\dagger_{i,s} \psi_{i,s} + \mathcal{H}_{\rm int}$$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i H^a(i) \psi^\dagger_{i,s} \sigma^a_{ss'} \psi_{i,s'} + V_H$$

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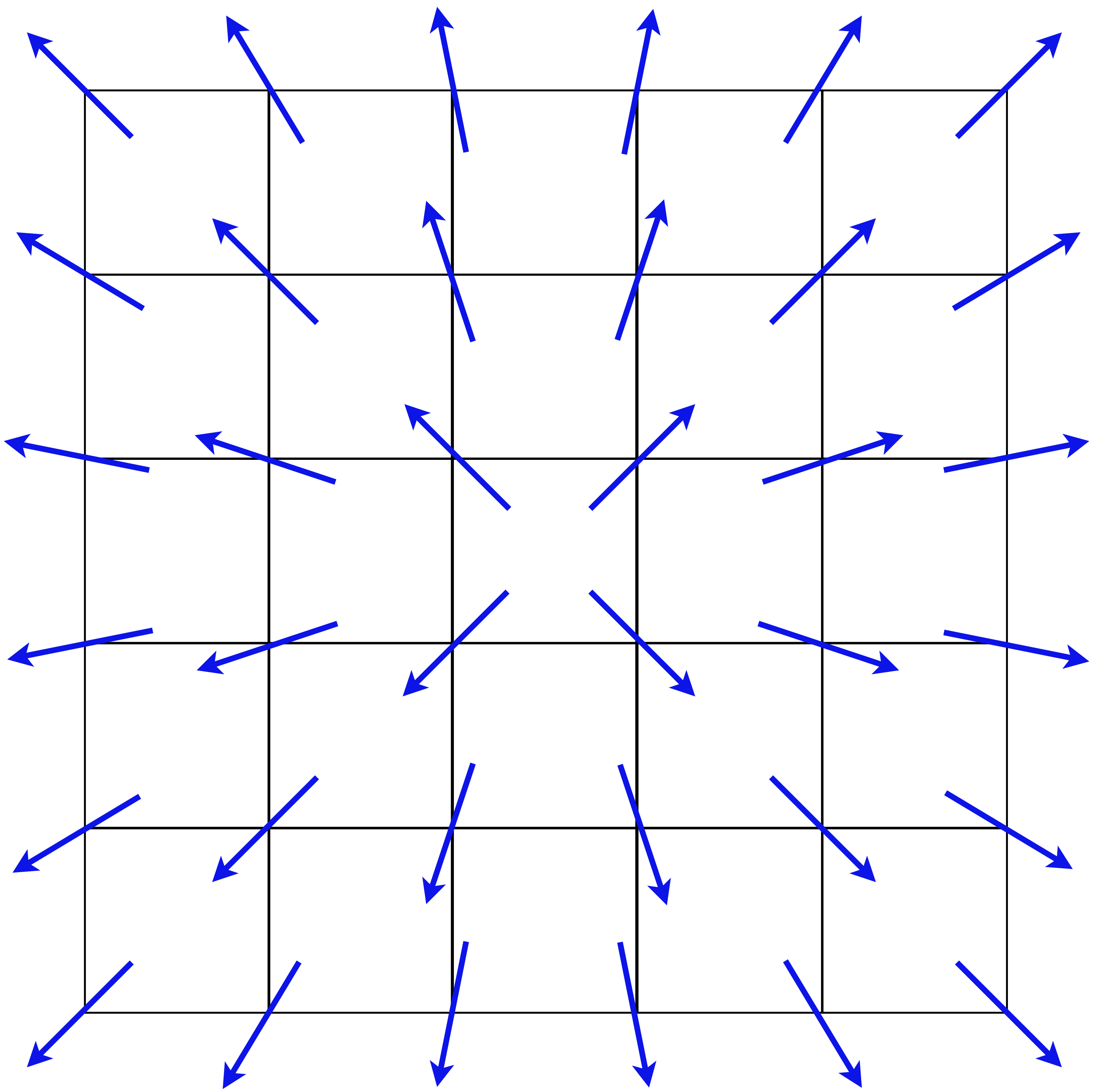
$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i H^a(i) \psi_{i,s}^\dagger \sigma_{ss'}^a \psi_{i,s'} + V_H$$

IF we can transform to a rotating reference frame in which $H^a(i) =$ a constant independent of i and time, **THEN** the ψ fermions in the presence of (fluctuating) SDW SRO will inherit the small Fermi surfaces of the electrons in the presence of SDW LRO.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Fluctuating SDW

We cannot always find a single-valued SU(2) rotation R_i to make the Higgs field $H^a(i)$ a constant !



vortex in AFM order

(assume easy-plane AFM for simplicity)

S. Sachdev, E. Berg, S. Chatterjee, and Y. Schattner, PRB **94**, 115147 (2016)

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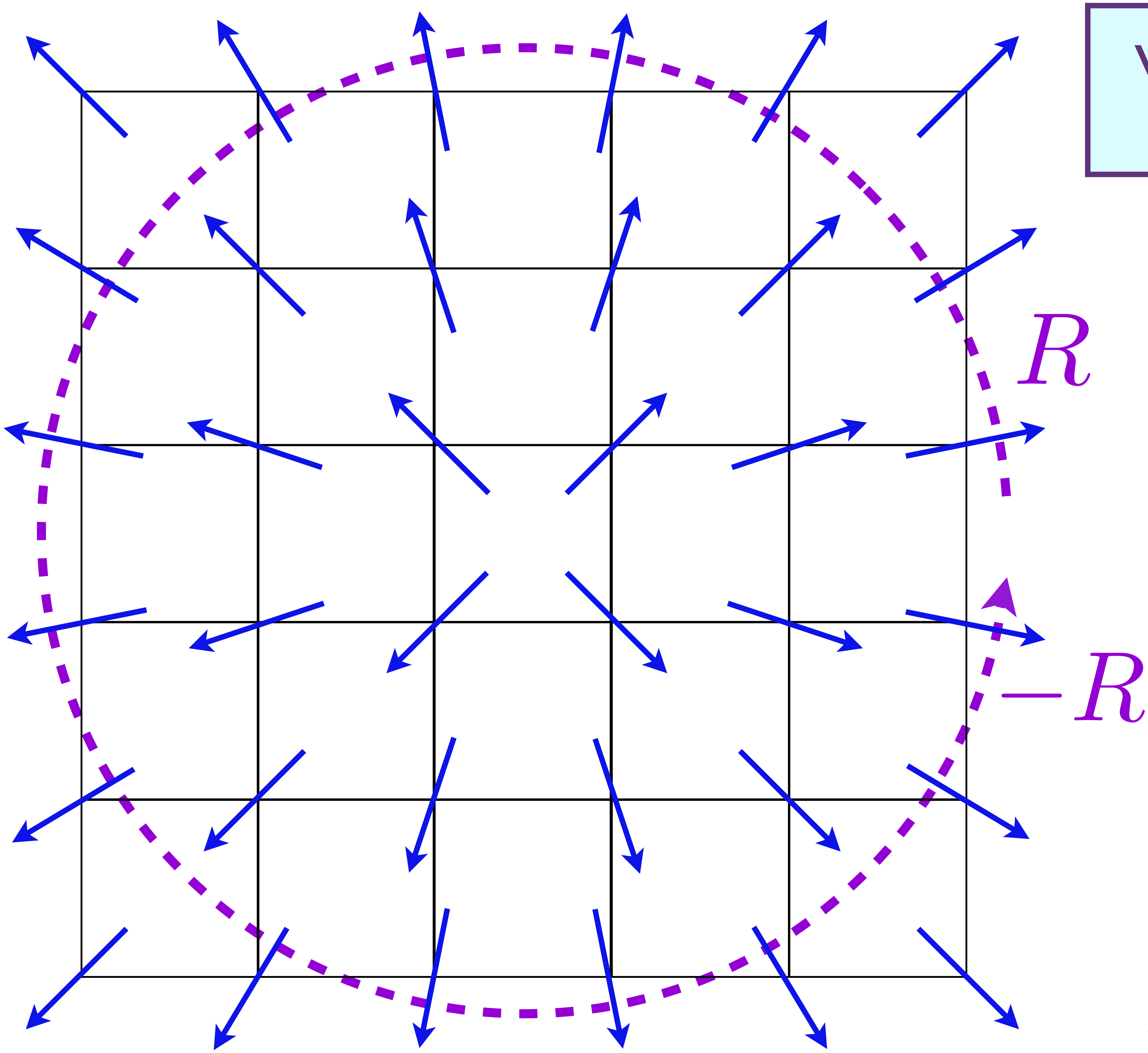

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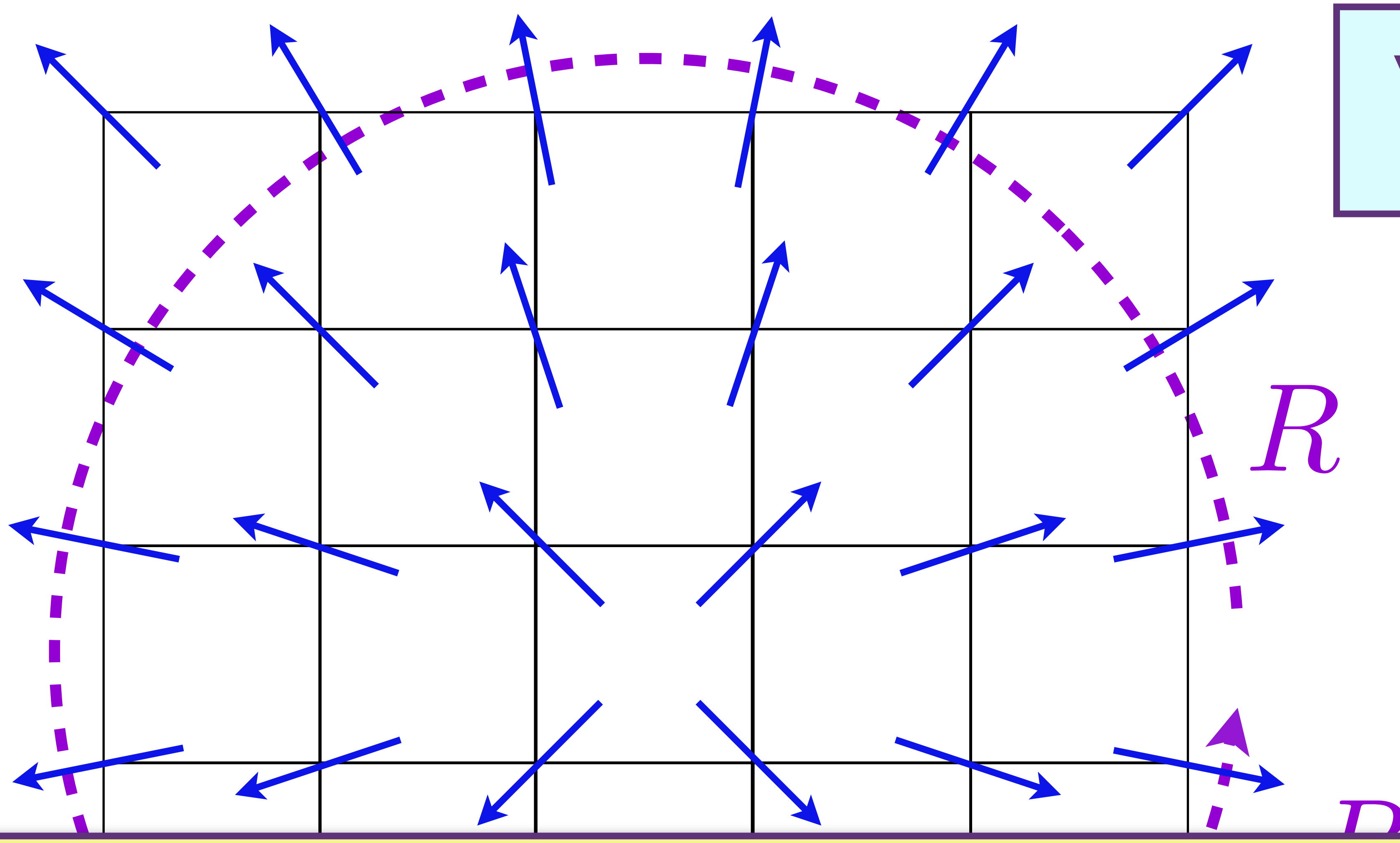
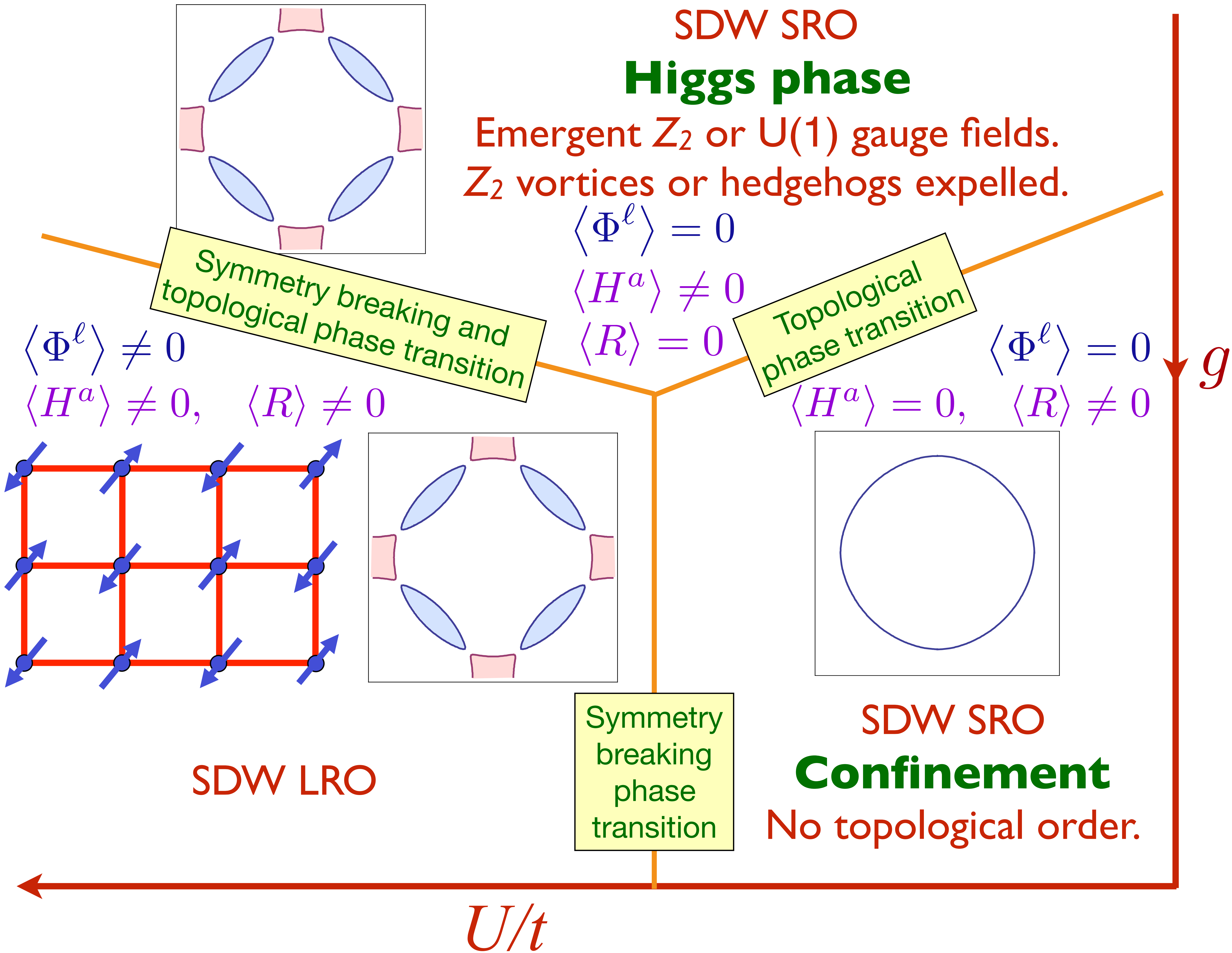


vortex in AFM order

(assume easy-plane AFM for simplicity)

The HIGGS PHASE, with H^a condensed, has fluctuating R and SDW SRO with odd vortices expelled (for easy-plane SDW). Such a metal has topological order and the fermions which inherit the small Fermi surfaces of the metal with SDW LRO.

...jee, and T. Schattner, PRB **94**, 115147 (2016)


SDW SRO
Higgs phase
Emergent Z_2 or U(1) gauge fields.
Z_2 vortices or hedgehogs expelled.
$\langle \Phi^\ell \rangle = 0$
$\langle H^a \rangle \neq 0$
$\langle R \rangle = 0$
Symmetry breaking and topological phase transition
Topological phase transition
$\langle \Phi^\ell \rangle \neq 0$
$\langle H^a \rangle \neq 0, \quad \langle R \rangle \neq 0$
$\langle \Phi^\ell \rangle = 0$
$\langle H^a \rangle = 0, \quad \langle R \rangle \neq 0$
g
SDW LRO
Symmetry breaking phase transition
SDW SRO
Confinement
No topological order.
U/t

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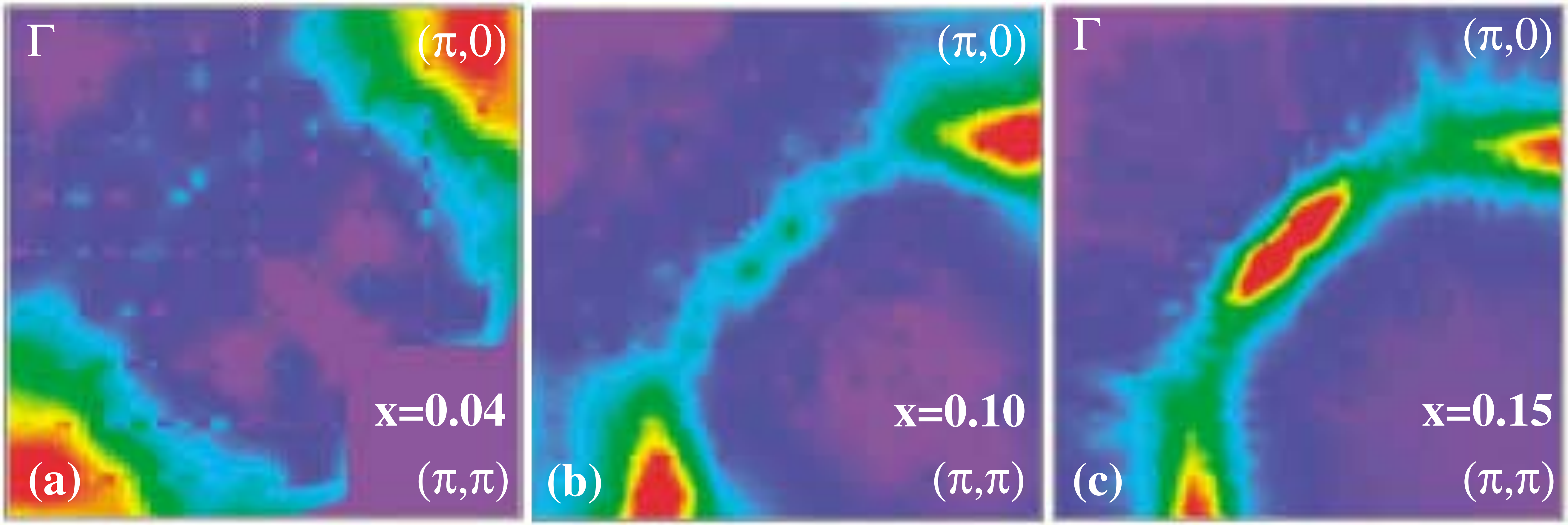
$Nd_{2-x}Ce_xCuO_{4\pm\delta}$

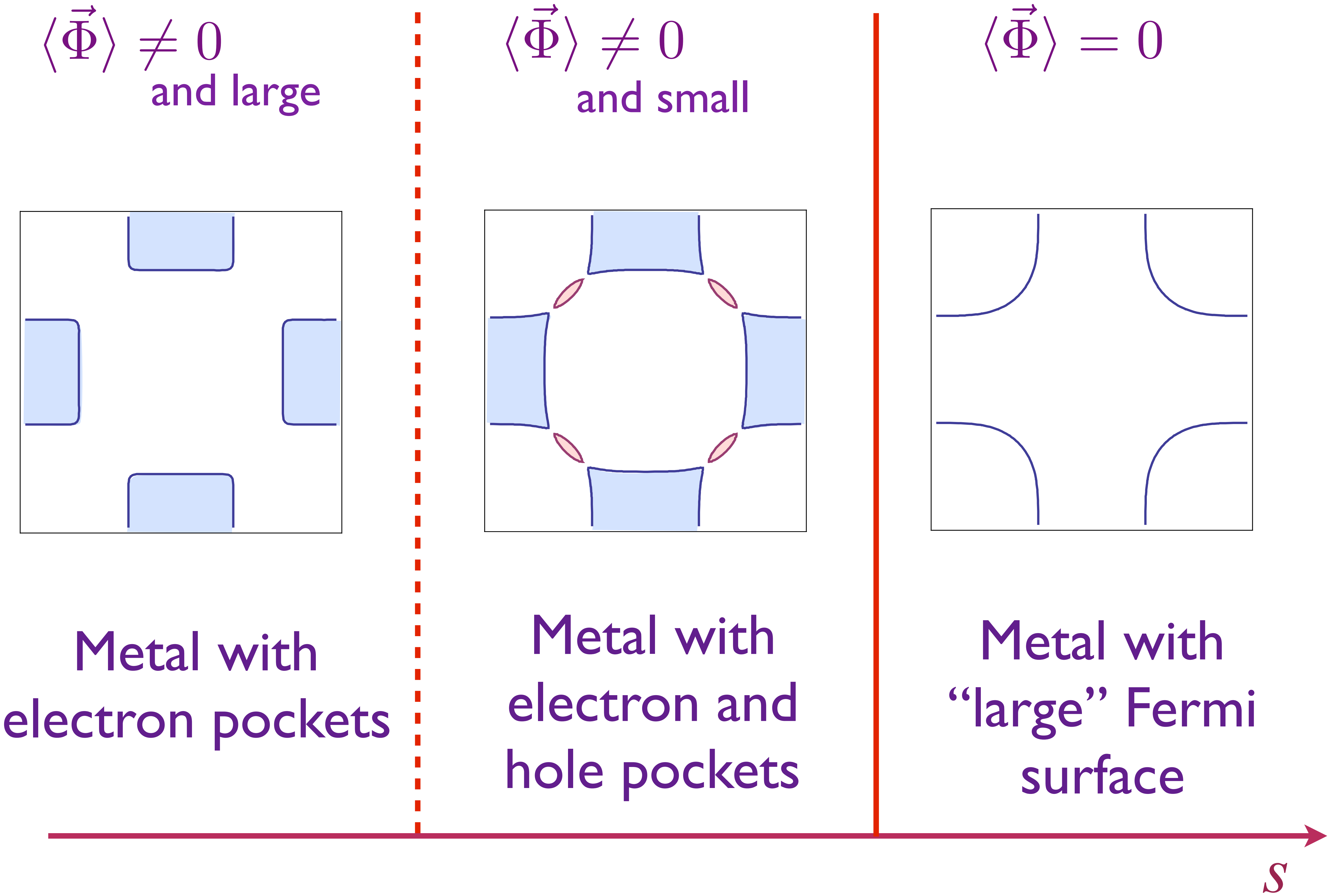


Doping Dependence of an n-Type Cuprate Superconductor Investigated by Angle-Resolved Photoemission Spectroscopy

N. P. Armitage, F. Ronning, D. H. Lu, C. Kim, A. Damascelli, K. M. Shen, D. L. Feng, H. Eisaki, Z.-X. Shen, P. K. Mang, N. Kaneko, M. Greven, Y. Onose, Y. Taguchi, and Y. Tokura

Phys. Rev. Lett. **88**, 257001 (2002)

Square lattice Hubbard model with electron doping



Fermi surface reconstruction in electron-doped cuprates without long-range order

J.-F. He[1,2], C. R. Rotundu[1,2], M. S. Scheurer[3], Y. He[1,2], M. Hashimoto[4], K. Xu[2], Y. Wang[1,3], E. W. Huang[1,2], T. Jia[2], S.-D. Chen[1,2], B. Moritz[1,2], D.-H. Lu[4], Y. S. Lee[1,2], T. P. Devereaux[1,2] & Z.-X. Shen[1,2,*]

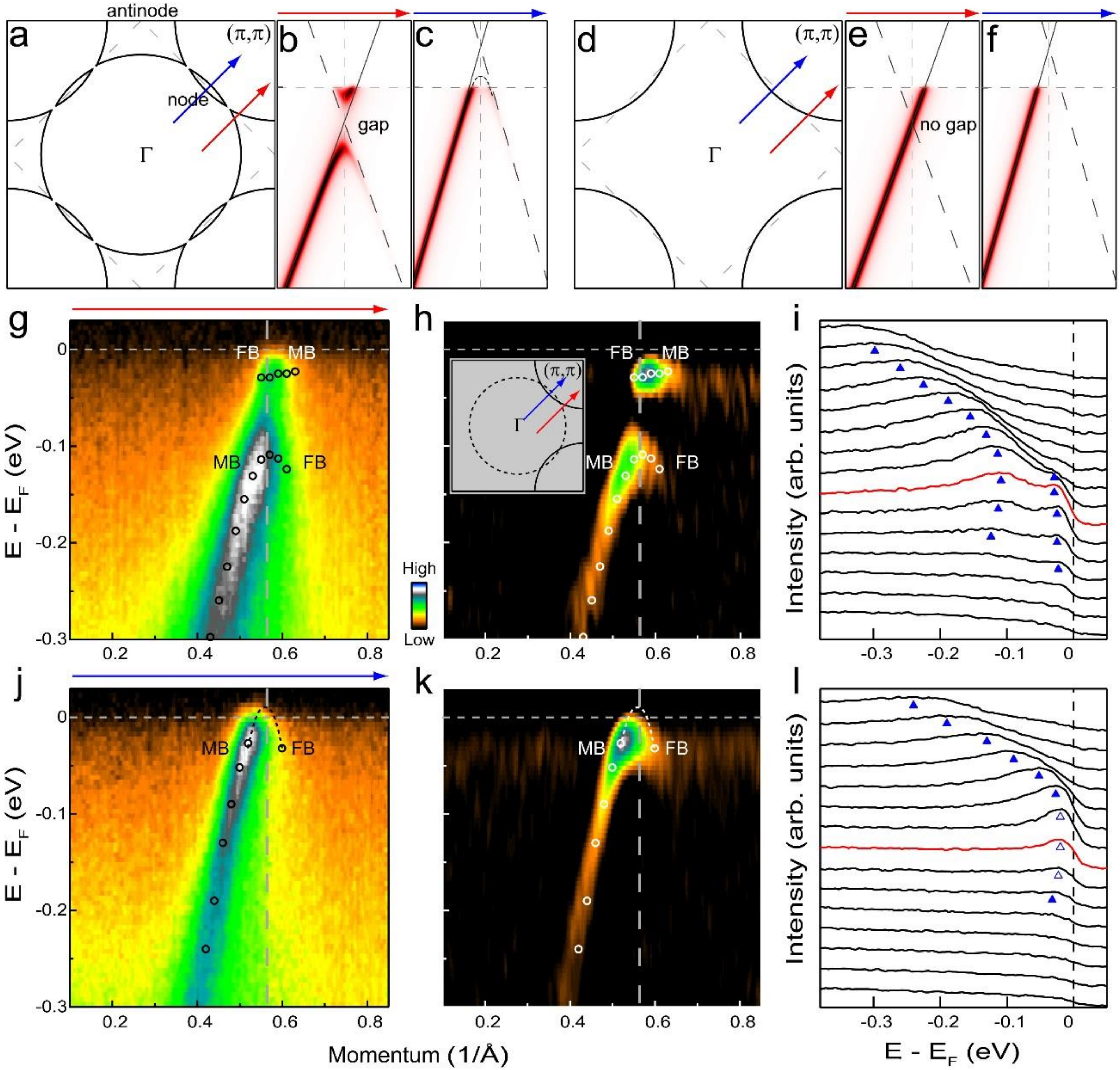
[1]Stanford Institute for Materials and Energy Sciences, SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, California 94025, USA

[2]Geballe Laboratory for Advanced Materials, Departments of Physics and Applied Physics, Stanford University, Stanford, California 94305, USA

[3]Department of Physics, Harvard University, Cambridge MA 02138, USA

[4]Stanford Synchrotron Radiation Lightsource, SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, California 94025, USA

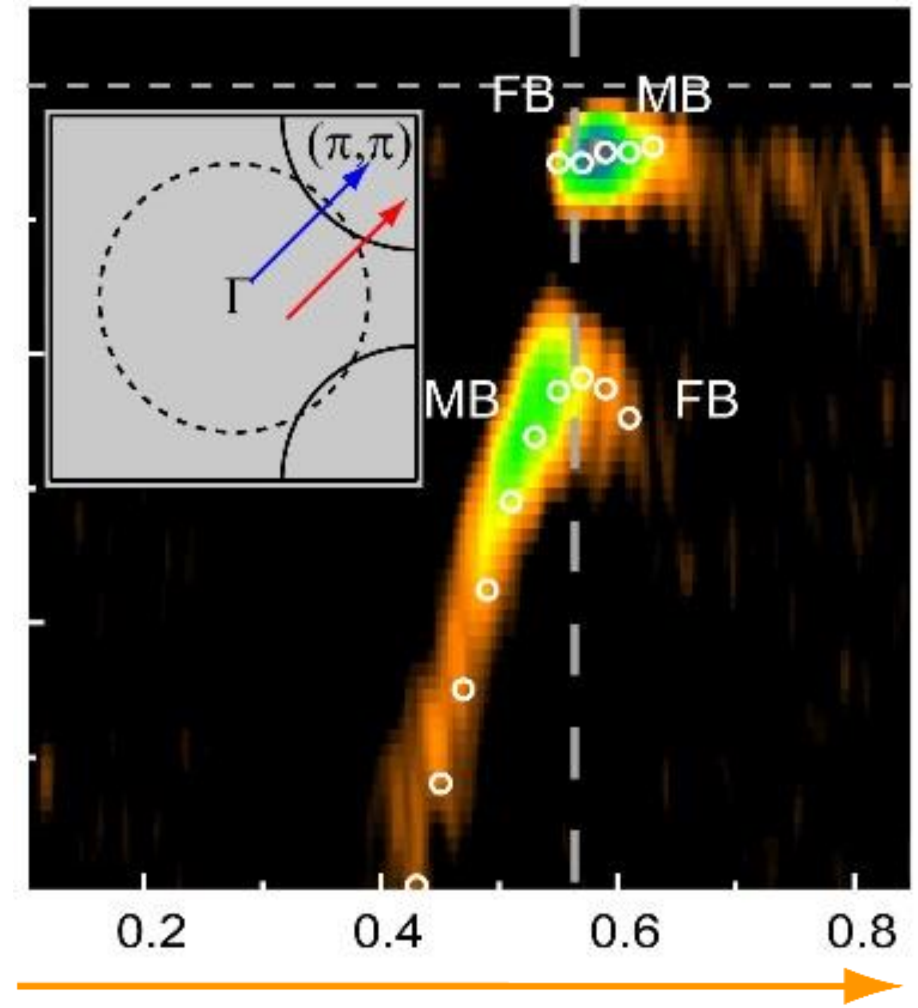
Fermi surface (FS) topology is a fundamental property of materials that determines the nature of carriers. In copper oxide superconductors, the doping evolution of FS topology remains a mystery[1,2]. This is partly due to the coexistence of multiple electronic orders, which may reconstruct the FS. In electron-doped cuprates, it is generally agreed that the antiferromagnetic (AFM) long-range order in the underdoped regime[3-6] splits the large FS into small pockets[2,7]. However, its doping dependence remains highly debated. Inelastic neutron-scattering measurements on $Nd_{2-x}Ce_xCuO_4$ (NCCO) suggest that the long-range order vanishes before superconductivity appears near x=0.14 doping[4], but magnetic quantum oscillations indicate that the FS reconstruction extends to the over-doped regime until superconductivity is strongly suppressed near x=0.17 doping[8-12]. To reconcile the discrepancy, an external magnetic field-induced AFM long-range order has been widely discussed to explain the quantum oscillation results[2,8-12]. However, this makes the doping evolution of intrinsic FS topology inconclusive. Here, we report angle-resolved photoemission (ARPES) evidence of an energy gap, band folding and resulting FS reconstruction in optimal- and over-doped NCCO at x=0.15 and 0.16. Strikingly, the observed hole pockets are in quantitative agreement with those measured by quantum oscillations. Further, the gap collapses near x=0.16-0.17, providing a microscopic reinforcement for reports of a quantum critical point (QCP)[13-16]. Our observation of intrinsic FS reconstruction at zero field and QCP suggests the presence of long-range order below x=0.17. AFM order is the simplest explanation, but it is disfavored by neutron experiment on the superconducting samples beyond x=0.14 (ref. 4). As such, the totality of the data suggests an intriguing possibility of a correlation driven topological order[17-19].

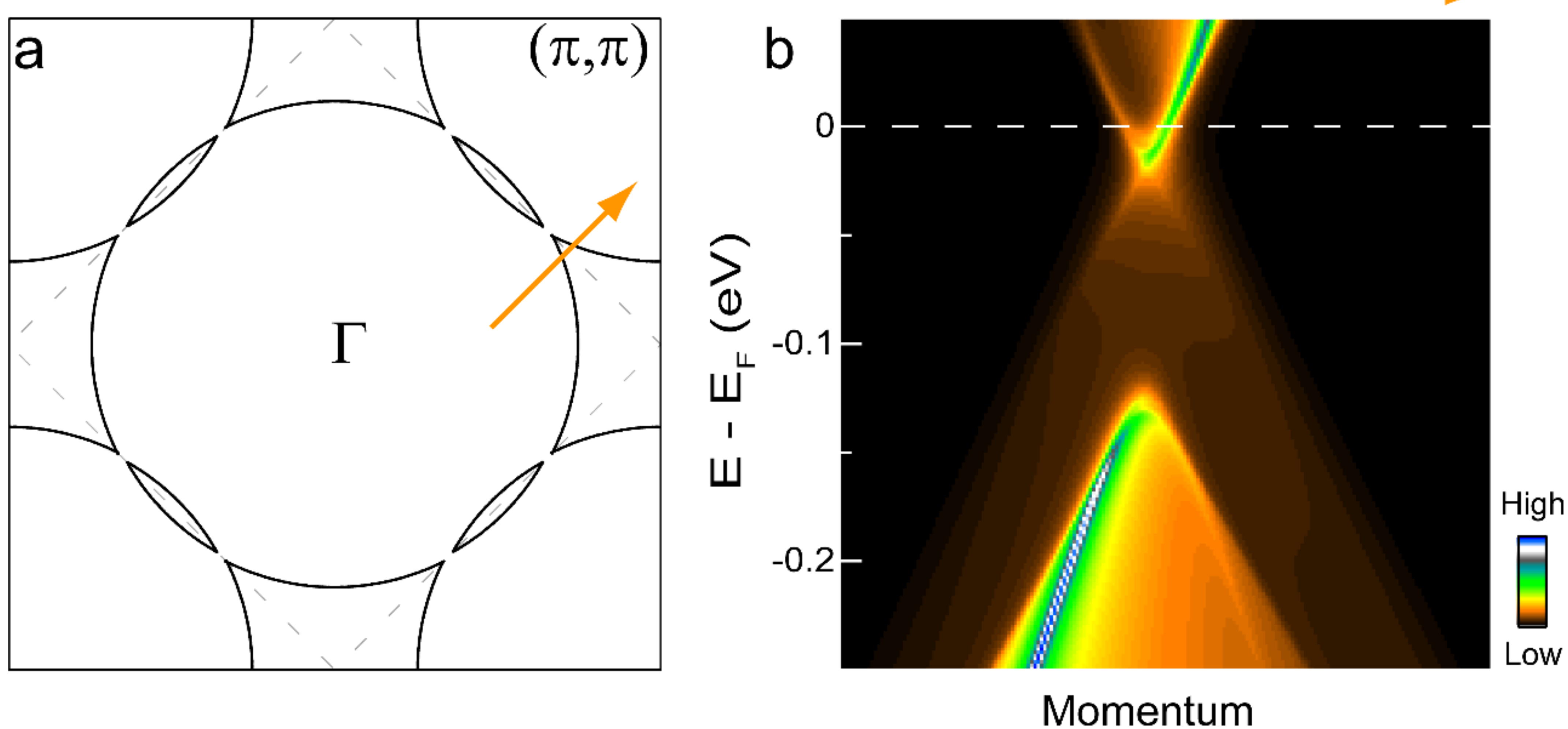


antinode
a
(π,π)
node
Γ
b
gap
c
d
(π,π)
Γ
e
no gap
f
g
FB
MB
MB
FB
E - E_F (eV)
h
FB
MB
(π,π)
Γ
MB
FB
High
Low
i
Intensity (arb. units)
j
MB
FB
E - E_F (eV)
k
MB
FB
l
Intensity (arb. units)
Momentum (1/Å)
E - E_F (eV)

S. Sachdev, Topological order and Fermi surface reconstruction, arXiv:1801.01125

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, Proceedings of the National Academy of Sciences **115**, E3665 (2018)





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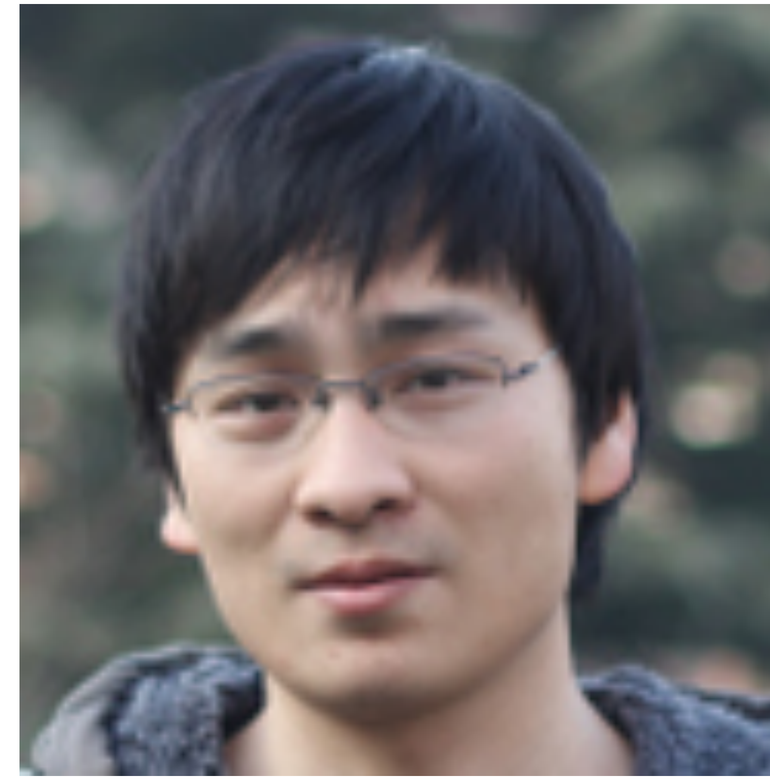


Wei Wu

Michel Ferrero

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, Proceedings of the National Academy of Sciences **115**, E3665 (2018)

Wei Wu, M. S. Scheurer, S. Chatterjee, S. Sachdev, A. Georges, and M. Ferrero, Physical Review X **8**, 021048 (2018)

Antoine Georges

Cluster DMFT studies of hole-doped cuprates (Hubbard model)

- Momentum-space differentiation: electron self-energy is enhanced at low frequencies in the anti-nodal region (apparent pole in self-energy), and vanishes in the nodal region.
- Gapped spectrum in the anti-nodal region
- Fermi arcs in the nodal region
- Apparent zero of Green's function on a "Luttinger surface".

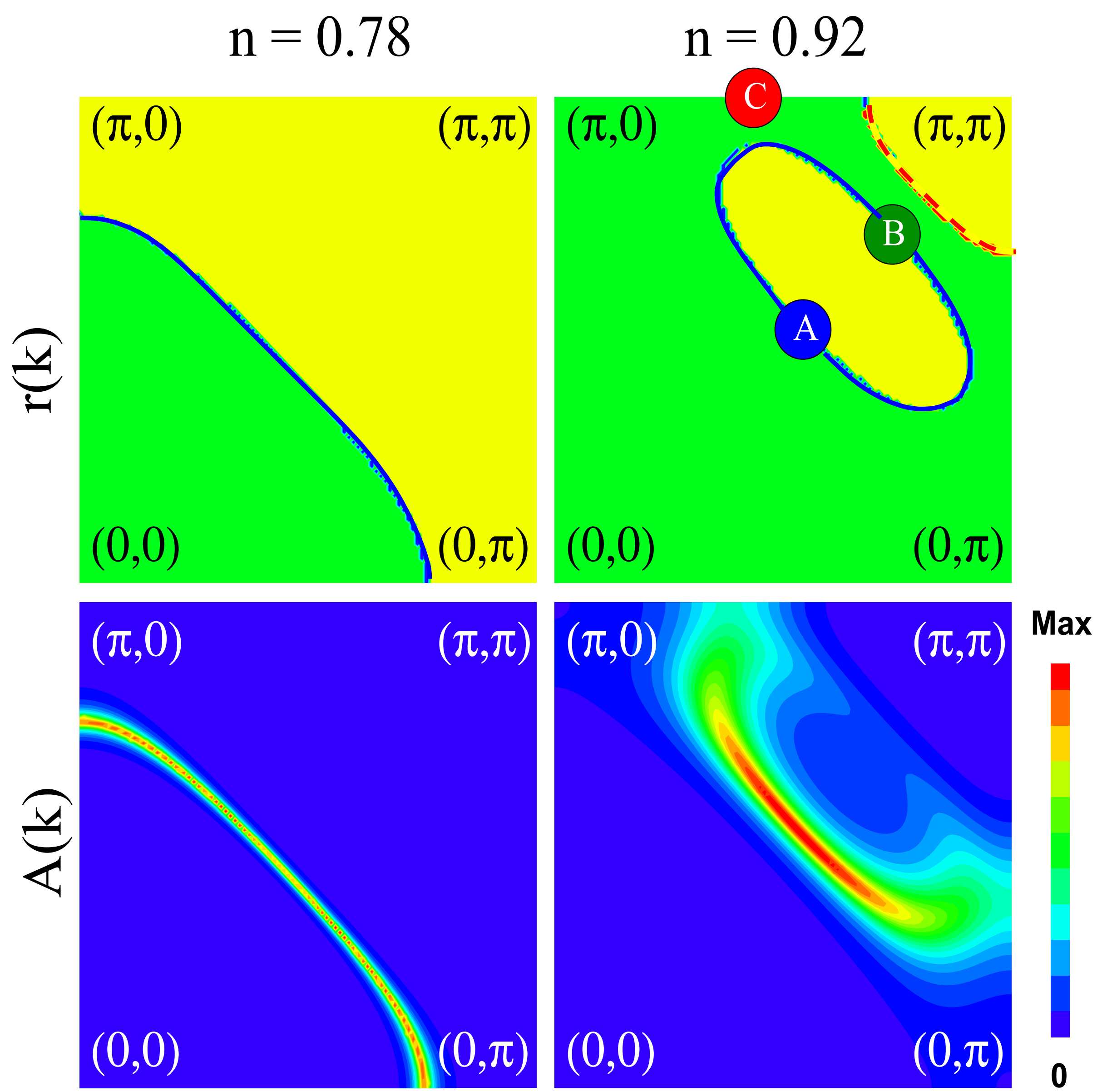


FIG. 4. (Color online) Renormalized energy $r(\mathbf{k})$ (upper panels) and spectral function $A(\mathbf{k})$ (lower panels) for the 2D Hubbard model with $U=8t$ and $T=0$. The color code for the upper panels is green/gray ($r<0$), blue/dark gray line ($r=0$), yellow/light gray ($r>0$), red dashed line ($r\rightarrow\infty$).

T.D. Stanescu and G. Kotliar, PRB **74**, 125110 (2006)

cf. previous work by Parcollet, Kotliar, Stanescu et al. Gull, Millis et al., Sakai, Imada et al., Tsvelik et al., Civelli, YRZ phenomenology; Berthod, Biermann, Giamarchi

Electron Green's function in Higgs phase of SU(2) gauge theory

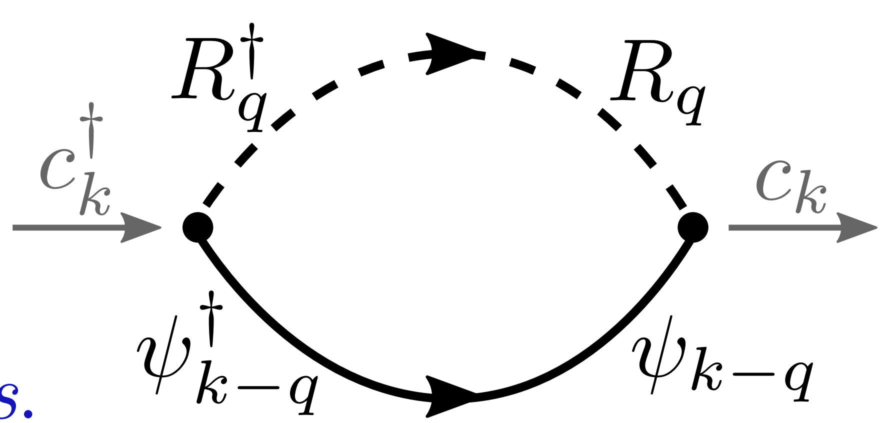
The effective Hamiltonian of the chargons in a constant Higgs potential $\langle H^a \rangle = H_0^a$ is (the hoppings have been renormalized by $\langle R_i^\dagger R_j \rangle$):

$$\mathcal{H}_\psi = -\sum_{i,\rho} \tilde{t}_\rho \left(\psi^\dagger_{i,s} \psi_{i+\boldsymbol{v}_\rho,s} + \psi^\dagger_{i+\boldsymbol{v}_\rho,s} \psi_{i,s} \right) - \mu \sum_i \psi^\dagger_{i,s} \psi_{i,s} - \lambda \sum_i (-1)^{i_x+i_y} H_0^a \psi^\dagger_{i,s} \sigma^a_{ss'} \psi_{i,s'}$$

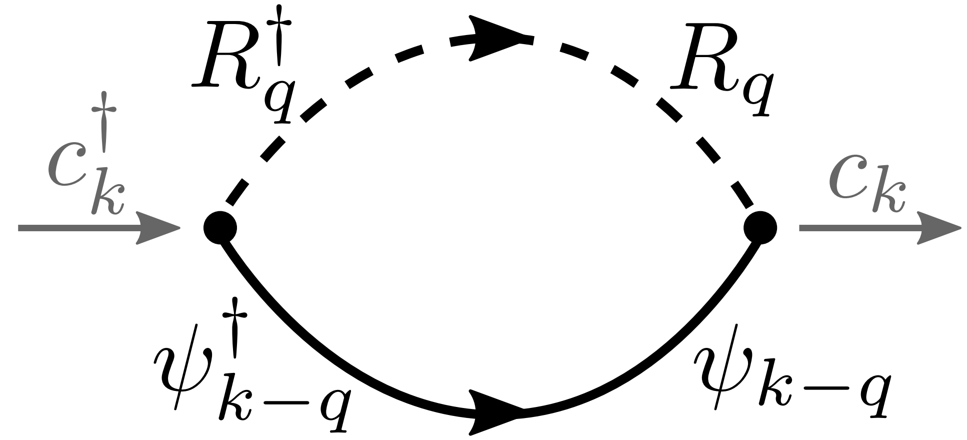
The chargon Fermi surface reconstructs into "small pockets", even though translational and spin rotation symmetries remain *unbroken*. The diagonal chargon Green's function is

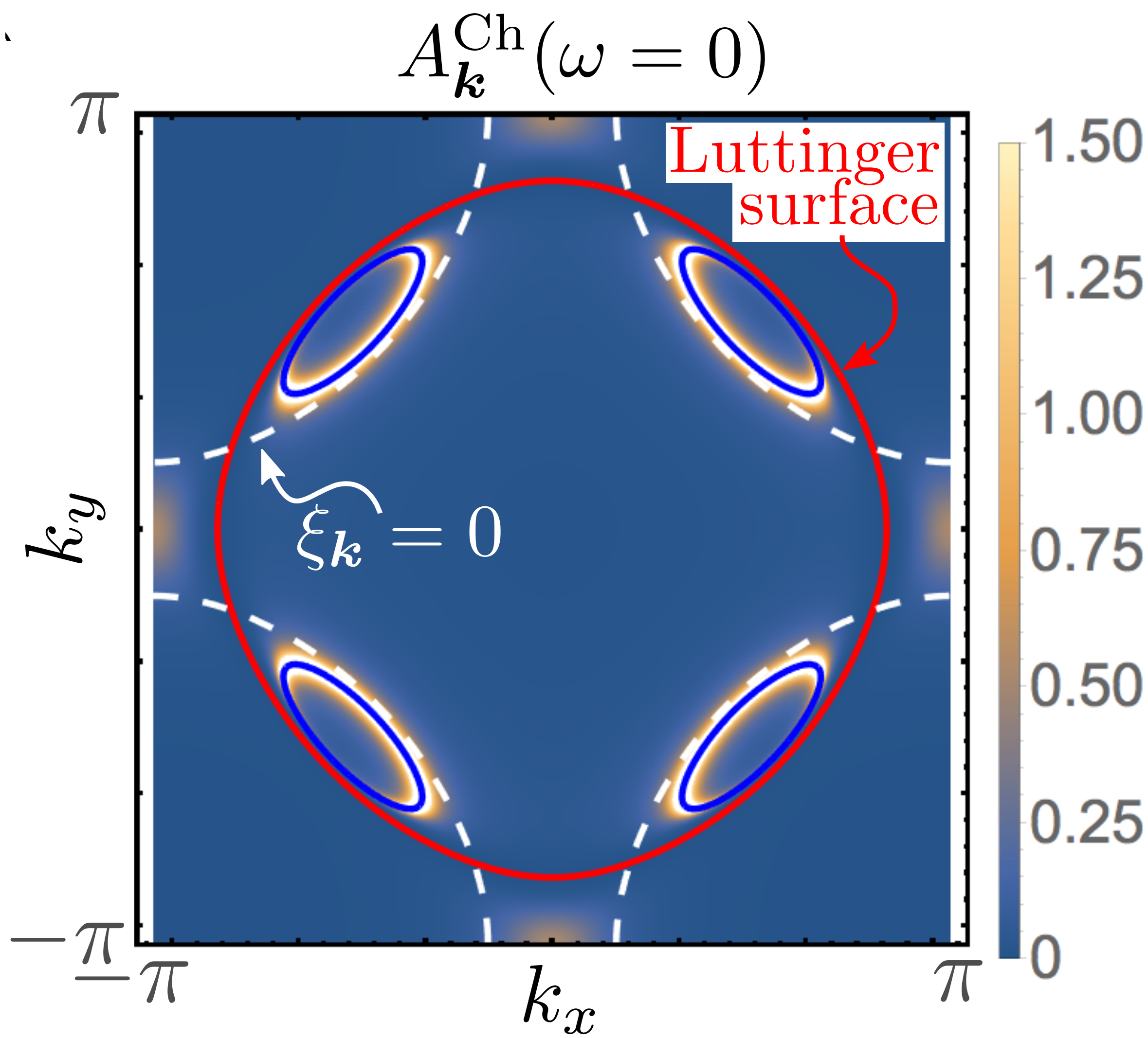
$$G_\psi(\omega, \vec{k}) = \frac{1}{\omega - \varepsilon_{\vec{k}} - \Sigma_\psi(\omega, \vec{k})}, \quad \Sigma_\psi(\omega, \vec{k}) = \frac{H_0^2}{\omega - \varepsilon_{\vec{k}+\vec{Q}}}, \quad \vec{Q} = (\pi, \pi).$$

This has poles at the pocket Fermi surfaces, and zeros at $\varepsilon_{\vec{k}+\vec{Q}}$. The electron Green's function is computed via a convolution with the spinons (R), and then the zeros are smeared to *approximate zeros*.

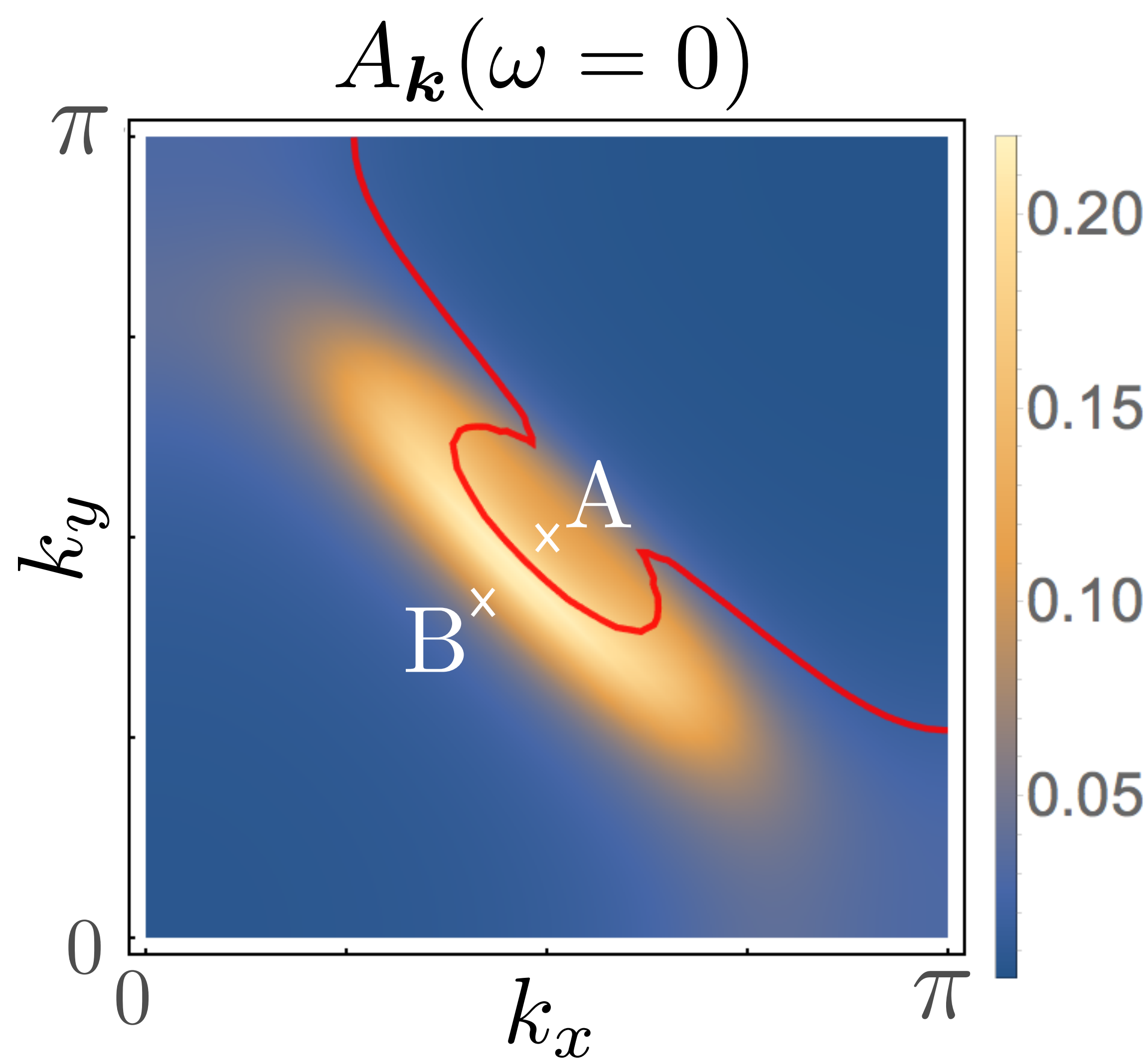


Electron Green's function in Higgs phase of SU(2) gauge theory





Red line indicates the locus of $G(\mathbf{k}, \omega = 0) = 0$



Red line indicates the locus of $\mathrm{Re}\, G(\mathbf{k}, \omega = 0) = 0$

Full Brillouin zone spectra of chargons (ψ) and electrons (c)

Luttinger relation in Higgs phase of SU(2) gauge theory

- The electron density is equal to the chargon density: $c_i^\dagger c_i = \psi_i^\dagger \psi_i$

- In the Higgs phase, the chargons experience an effective Hamiltonian that is the same as that of electrons in the presence of long-range AFM order.

- Apply the Luttinger argument to the chargons: volume enclosed by chargon Fermi surfaces equals the chargon density (= electron density) modulo filled bands in the AFM Brillouin zone.

Luttinger relation in Higgs phase of SU(2) gauge theory

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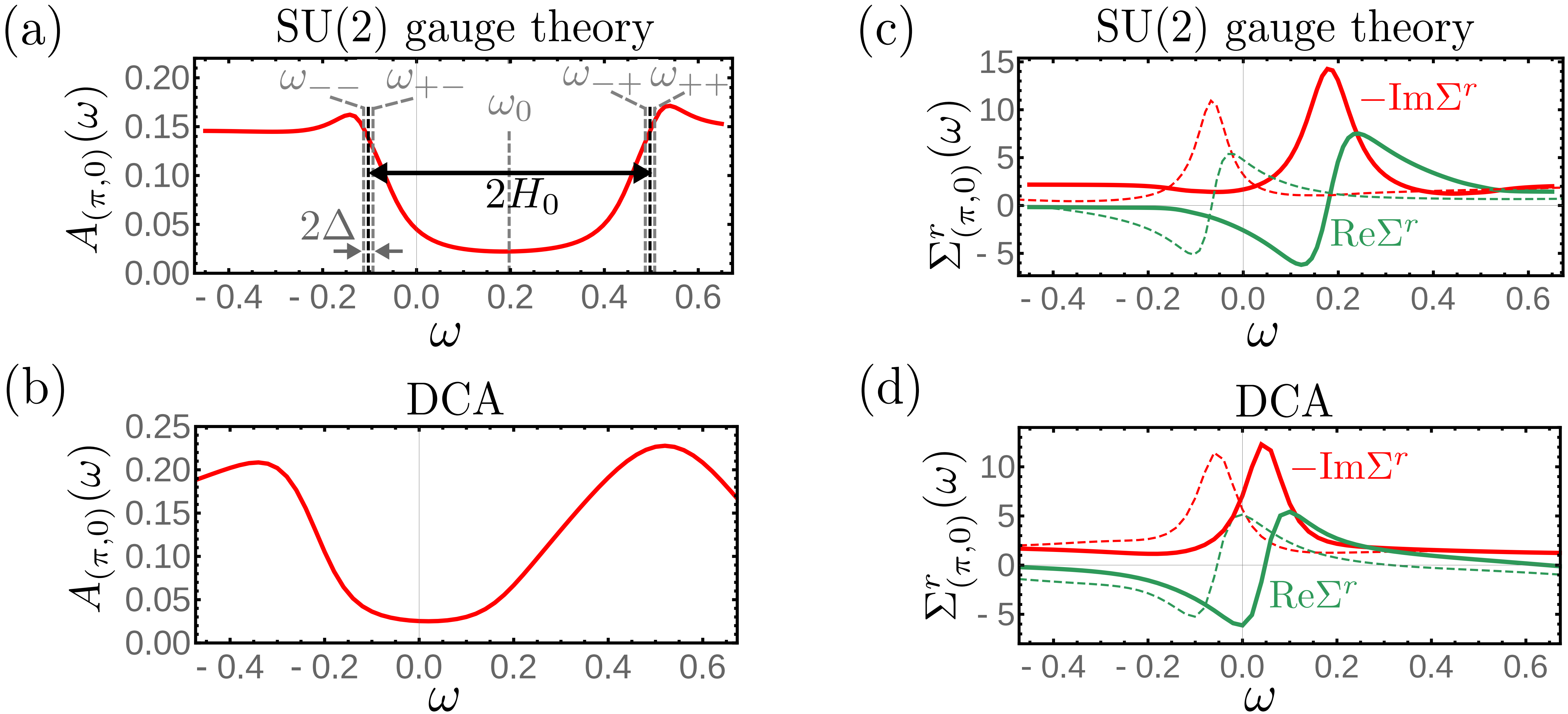
- The topological order allows one to 'break' translational symmetry in gauge-dependent quantities, but preserve translation invariance in all gauge-invariant observables. This is sufficient to obtain a non-Luttinger volume enclosed by all Fermi services of fermions carrying the global U(1) charge.

Electron Green's function in Higgs phase of SU(2) gauge theory

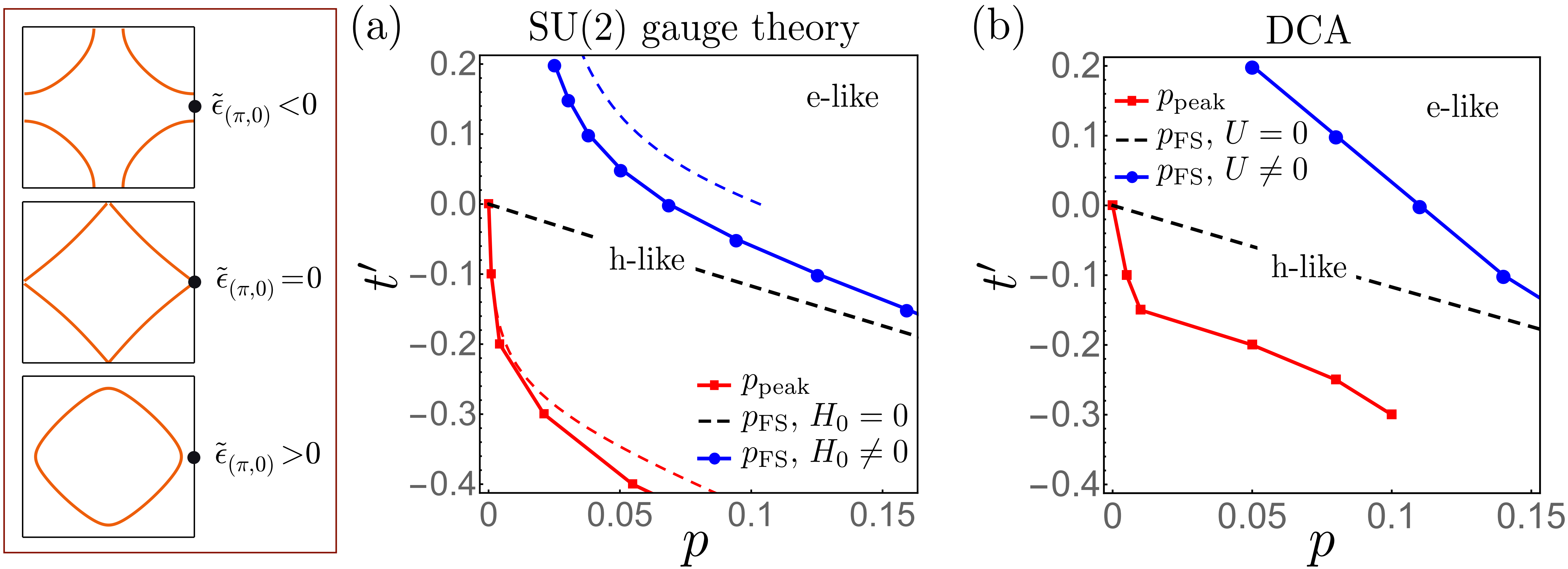


$$T = t/30 \quad , \quad U = 7t \quad , \quad p = 0.05$$

t' takes different negative values

Anti-nodal spectra compared to cluster DMFT

Lifshitz transition compared to cluster DMFT



$$\tilde{\epsilon}_{\vec{k}} = \epsilon_{\vec{k}} + \mathrm{Re}\,\Sigma_{\vec{k}}(\omega = 0) = -\mathrm{Re}\left(G_c(\omega = 0, \vec{k})\right)^{-1}$$

The p-t' dependence of the "interacting Lifshitz transition", defined by the sign change of the renormalized quasiparticle energy $\tilde{\epsilon}_{(\pi,0)}$ at $\omega_{\rm peak} > 0$, is shown as solid blue lines calculated from the SU(2) gauge theory, part (a), and DCA, part (b). The black dashed lines show the location of the same transition for non-interacting electrons. The red lines indicate where the particle-hole asymmetry of the self-energy changes, *i.e.*, where the peak position $\omega_{\rm peak}$ of the anti-nodal Im(self-energy) changes sign.

- Fermi surface reconstruction can be induced by emergent gauge fields and “topological order”

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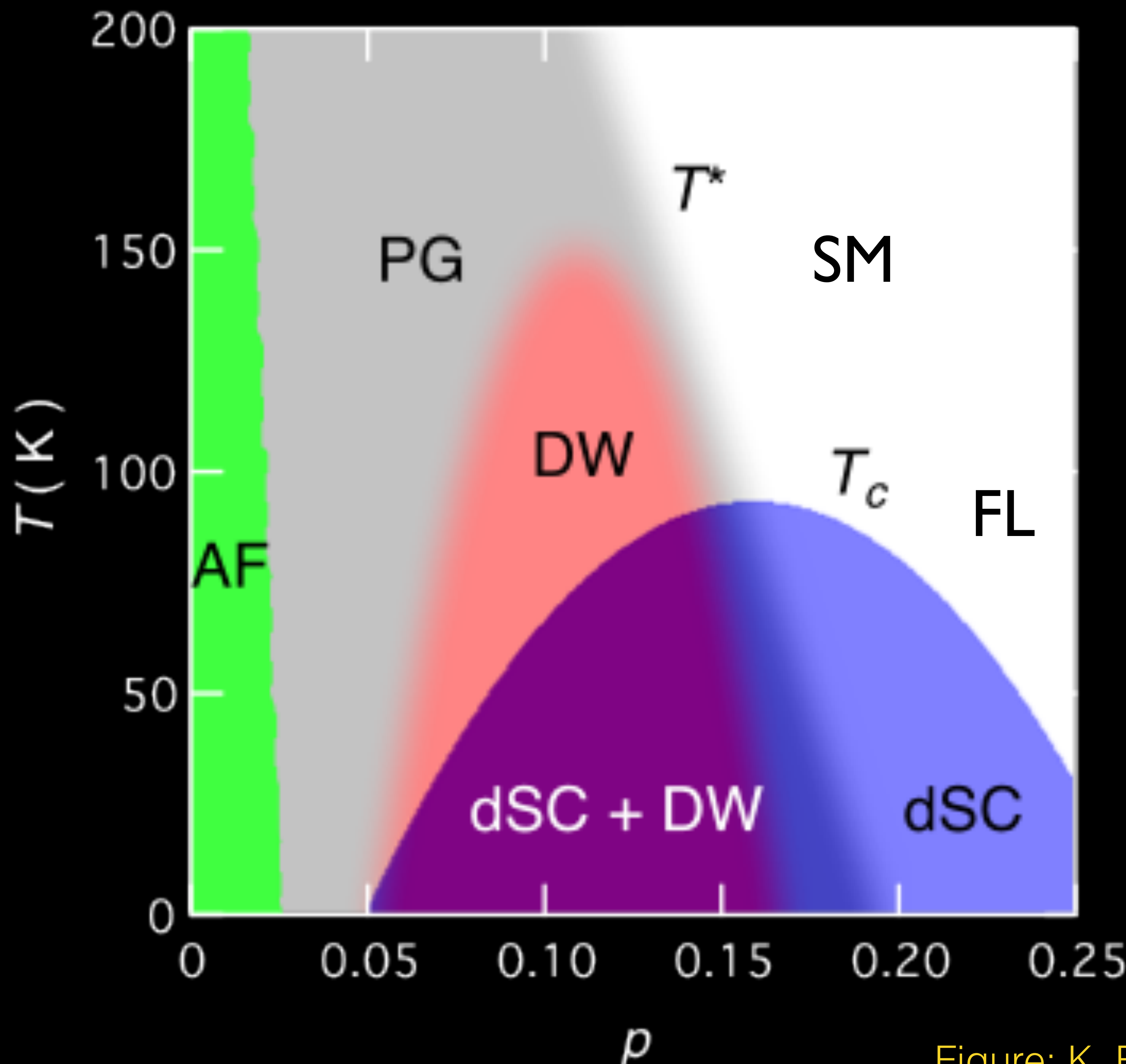
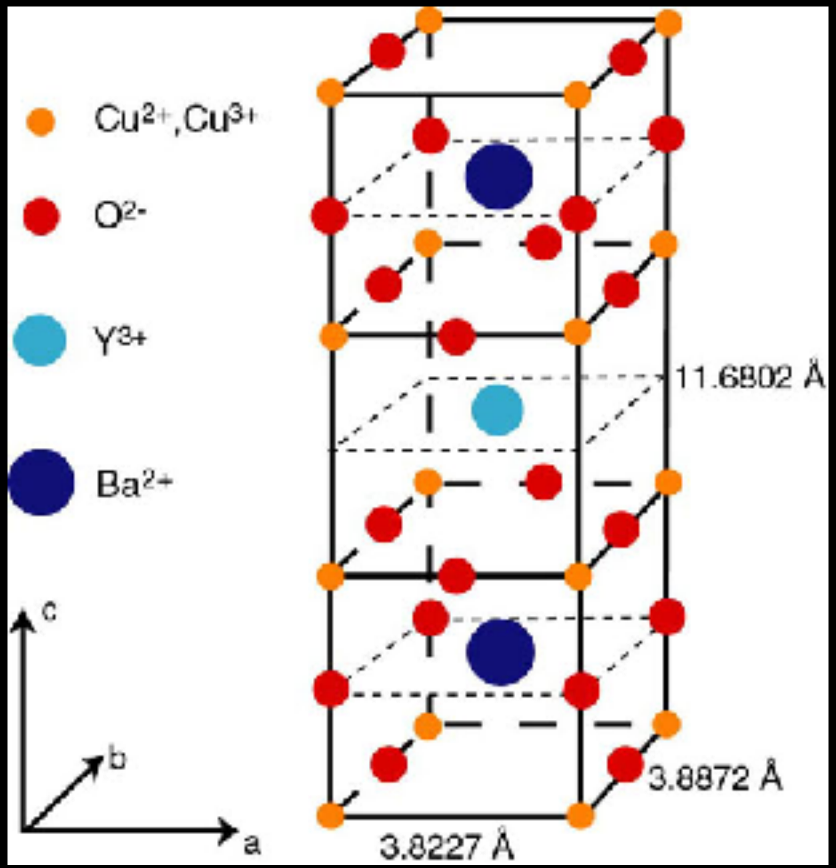
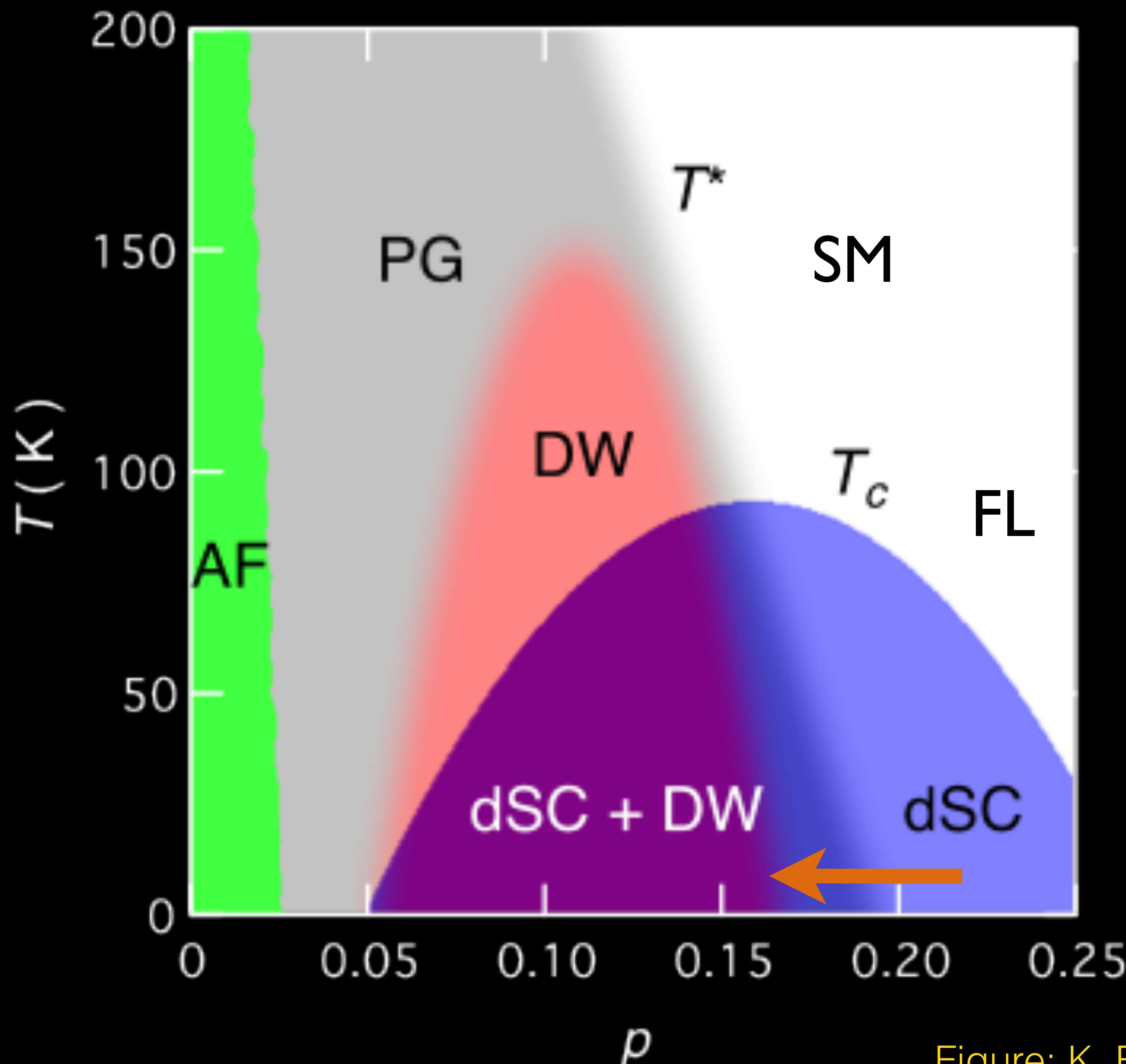
- A metal with emergent gauge fields is consistent with cluster-DMFT studies of the hole-doped Hubbard model, and with photoemission experiments on the electron-doped cuprates

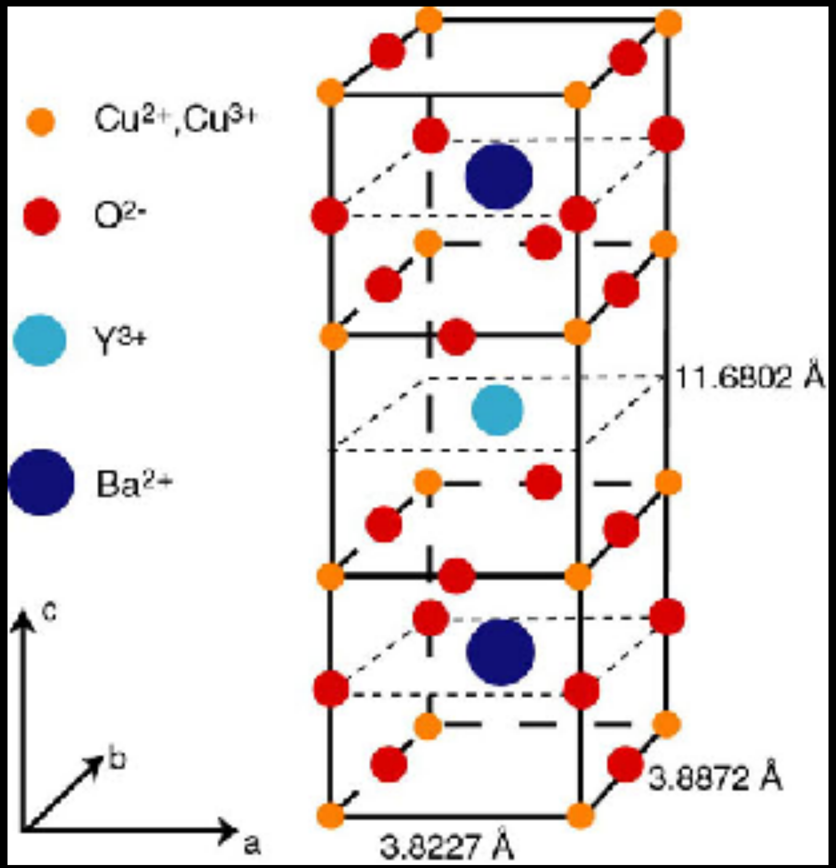




$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis





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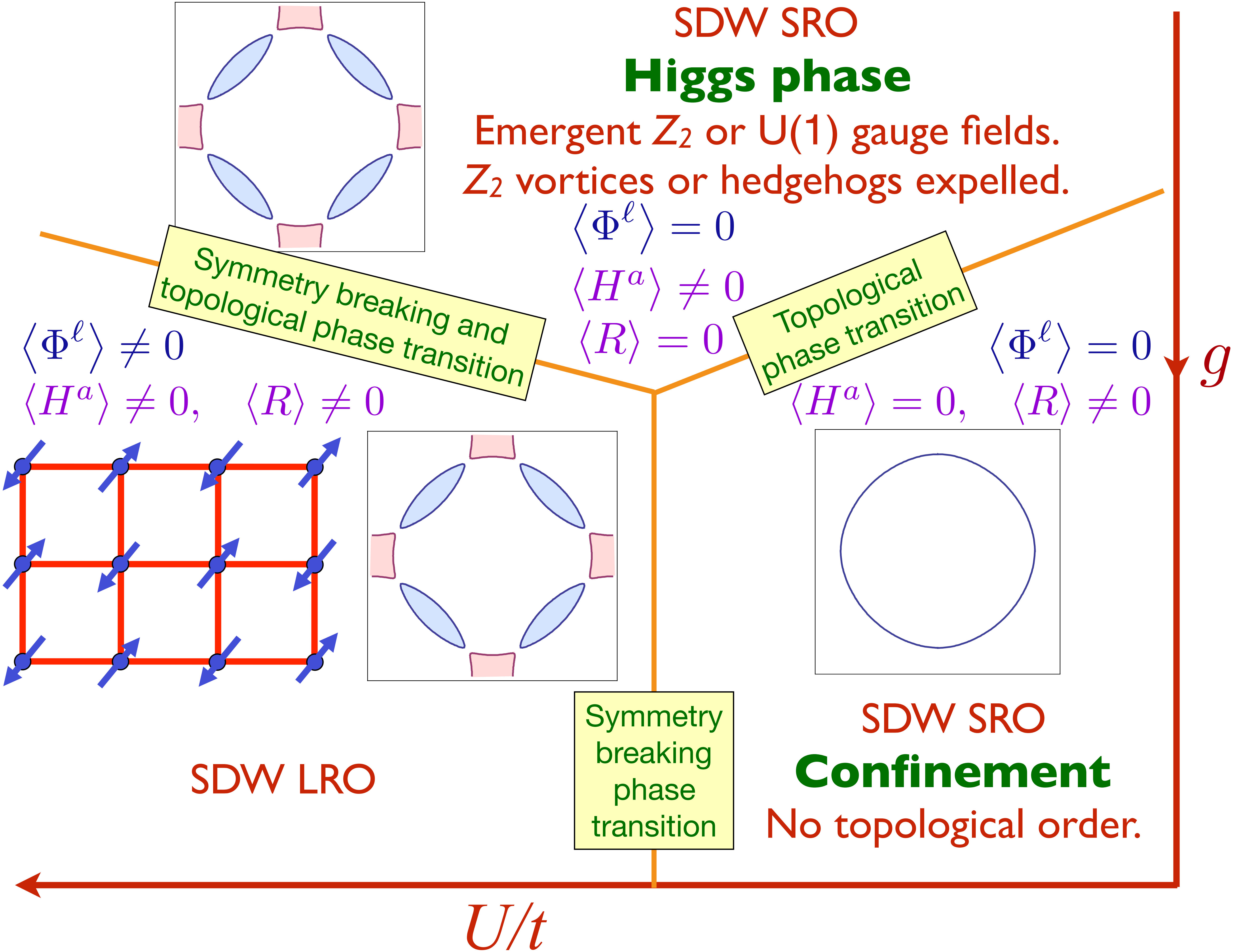

SDW SRO
Higgs phase
Emergent Z_2 or U(1) gauge fields.
Z_2 vortices or hedgehogs expelled.
$\langle \Phi^\ell \rangle = 0$
$\langle H^a \rangle \neq 0$
$\langle R \rangle = 0$
Symmetry breaking and topological phase transition
Topological phase transition
$\langle \Phi^\ell \rangle \neq 0$
$\langle H^a \rangle \neq 0, \quad \langle R \rangle \neq 0$
$\langle \Phi^\ell \rangle = 0$
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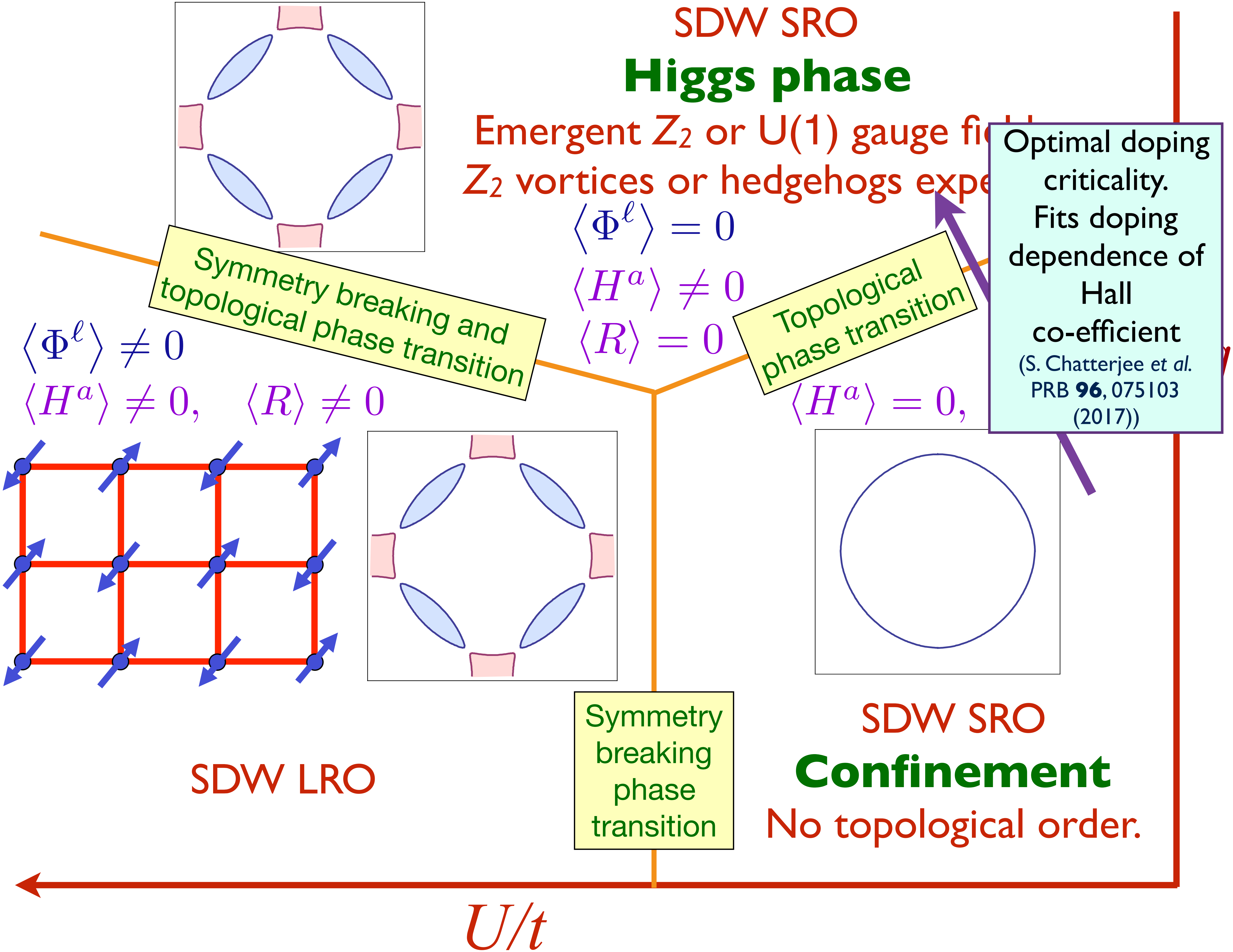
SDW LRO
Symmetry breaking phase transition
SDW SRO
Confinement
No topological order.
U/t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$\langle \Phi^\ell \rangle = 0$
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Symmetry breaking and topological phase transition
Topological phase transition
Optimal doping criticality.
Fits doping dependence of Hall co-efficient
(S. Chatterjee et al. PRB 96, 075103 (2017))
$\langle \Phi^\ell \rangle \neq 0$
$\langle H^a \rangle \neq 0, \quad \langle R \rangle \neq 0$
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Symmetry breaking phase transition
SDW LRO
SDW SRO
Confinement
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U/t

Change of carrier density at the pseudogap critical point of a cuprate superconductor

S. Badoux[1], W. Tabis[2,3], F. Laliberté[2], G. Grissonnanche[1], B. Vignolle[2], D. Vignolles[2], J. Béard[2], D. A. Bonn[4,5], W. N. Hardy[4,5], R. Liang[4,5], N. Doiron-Leyraud[1], Louis Taillefer[1,5] & Cyril Proust[2,5]

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$YBa_2Cu_3O_{6+x}$