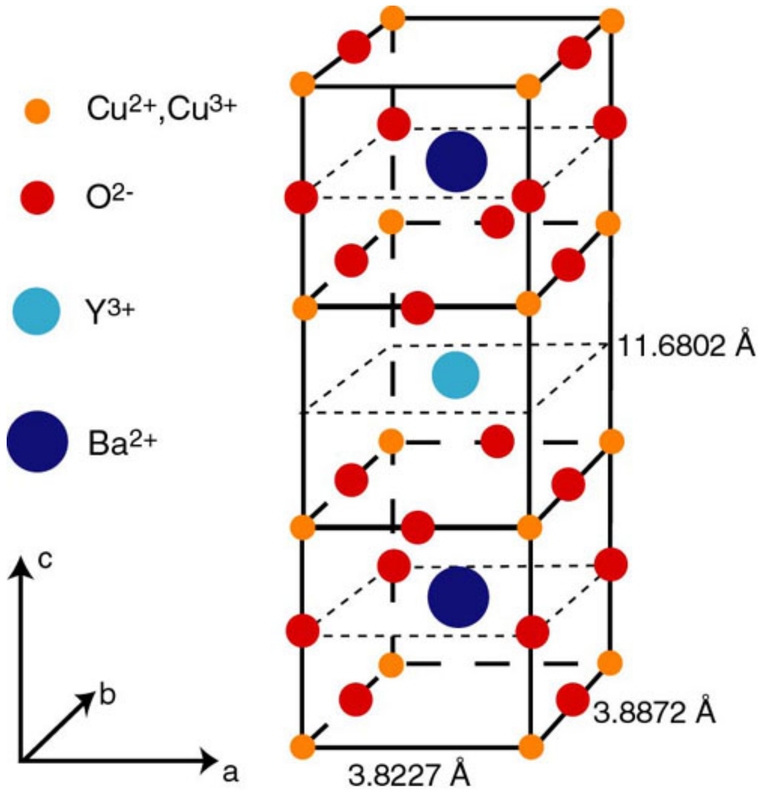
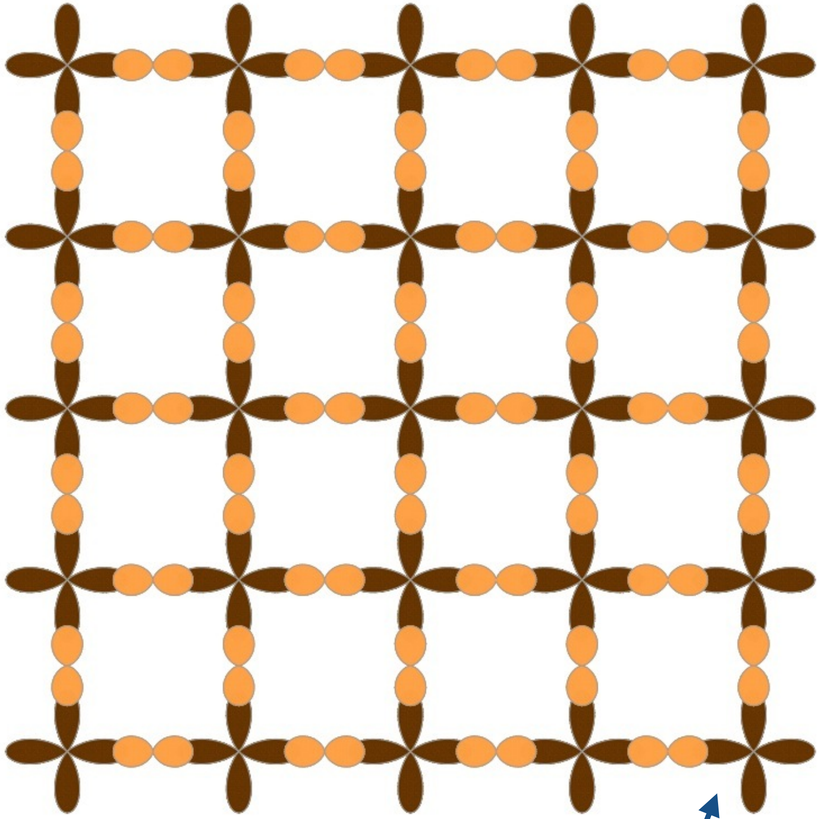
Detecting spin liquid quantum entanglement by measuring Fermi surfaces

Young Investigators Meet on Quantum Condensed Matter Theory 2025
Indian Institute of Science Education and Research, Tirupati
December 21, 2025

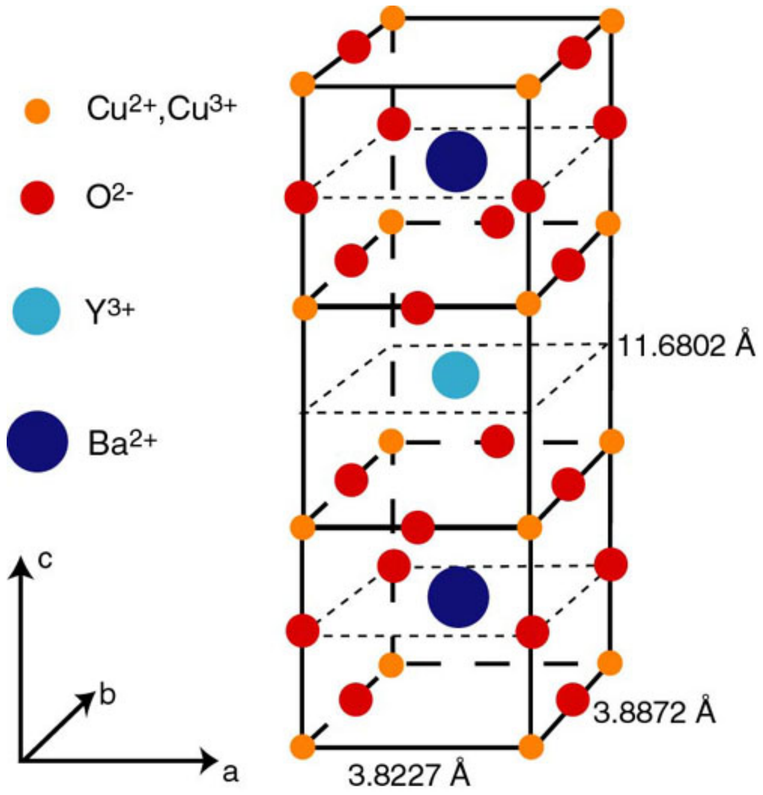
Subir Sachdev

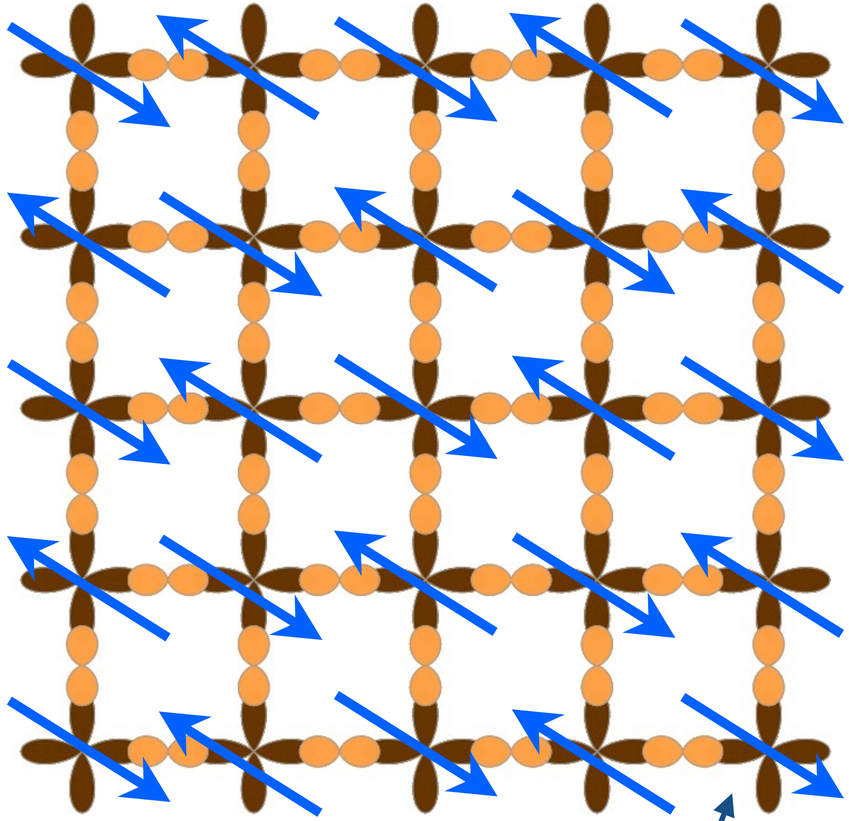




$YBa_2Cu_3O_{6+x}$

Cu

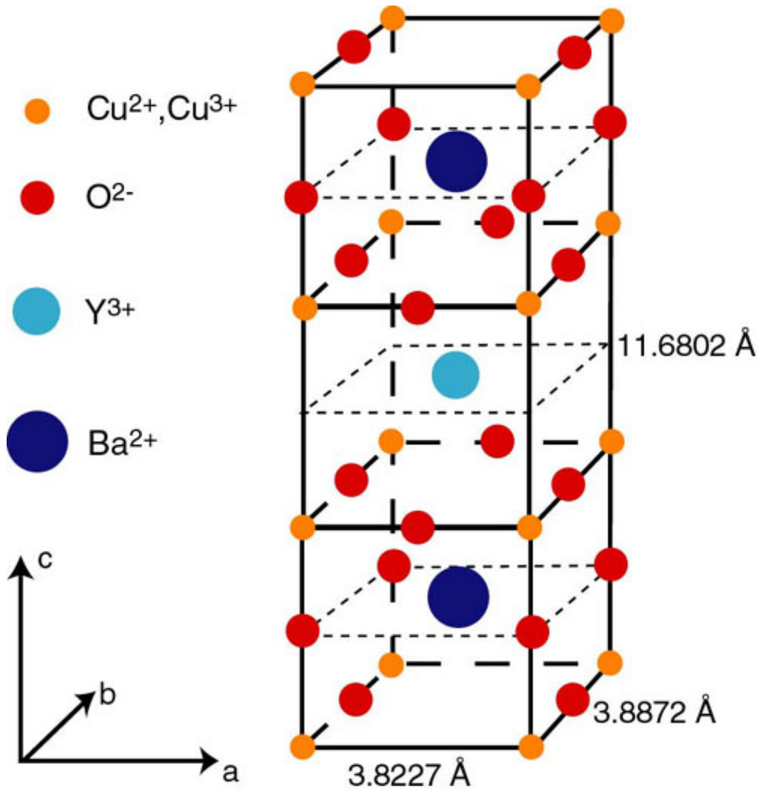




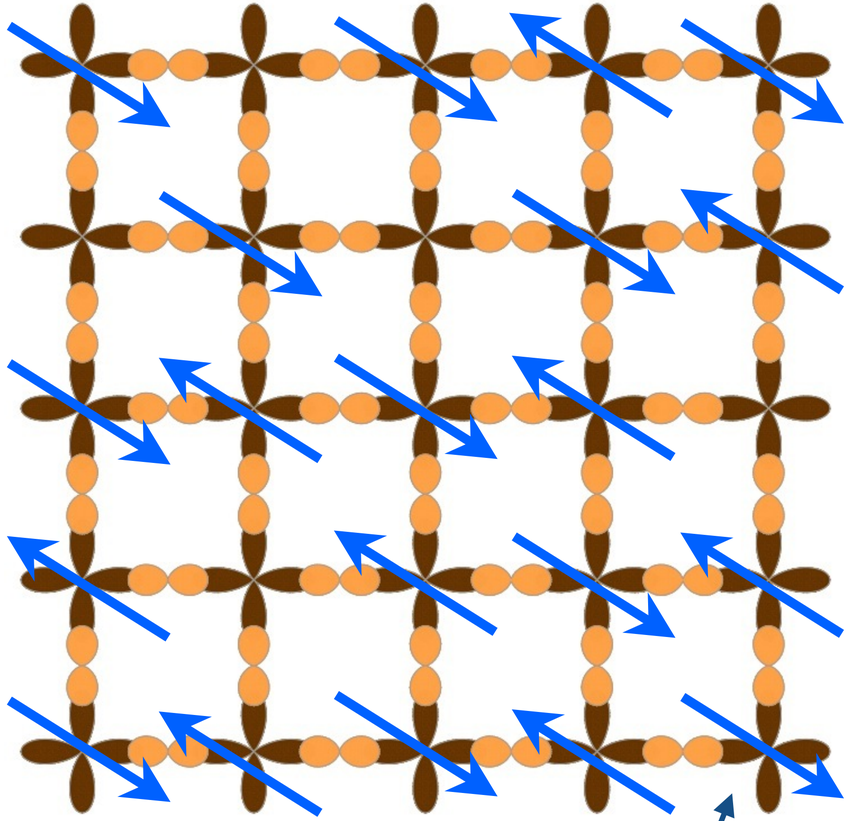
$YBa_2Cu_3O_{6+x}$

Cu

Insulating antiferromagnet with one electron per site

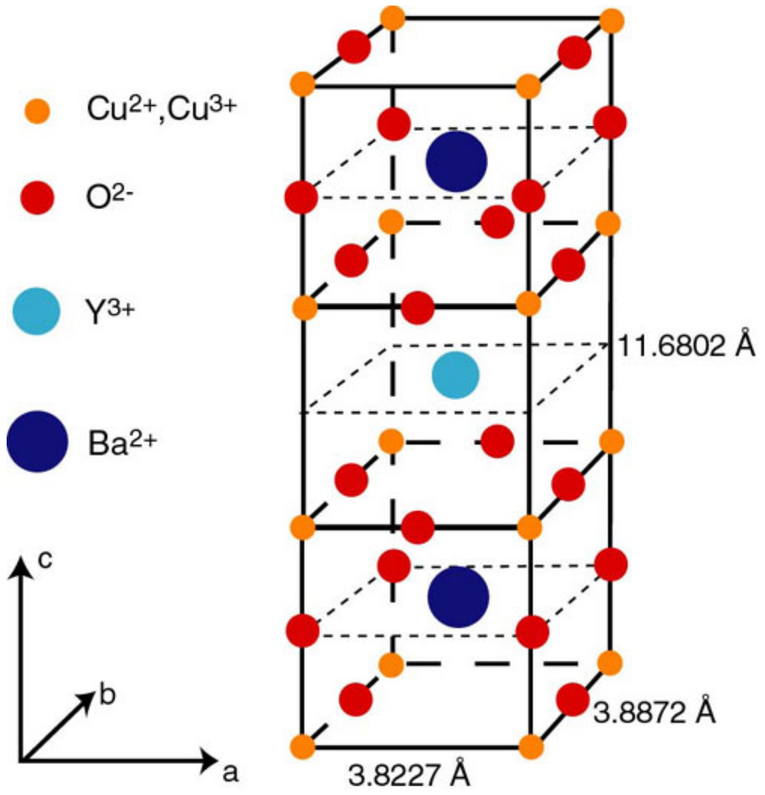


$YBa_2Cu_3O_{6+x}$

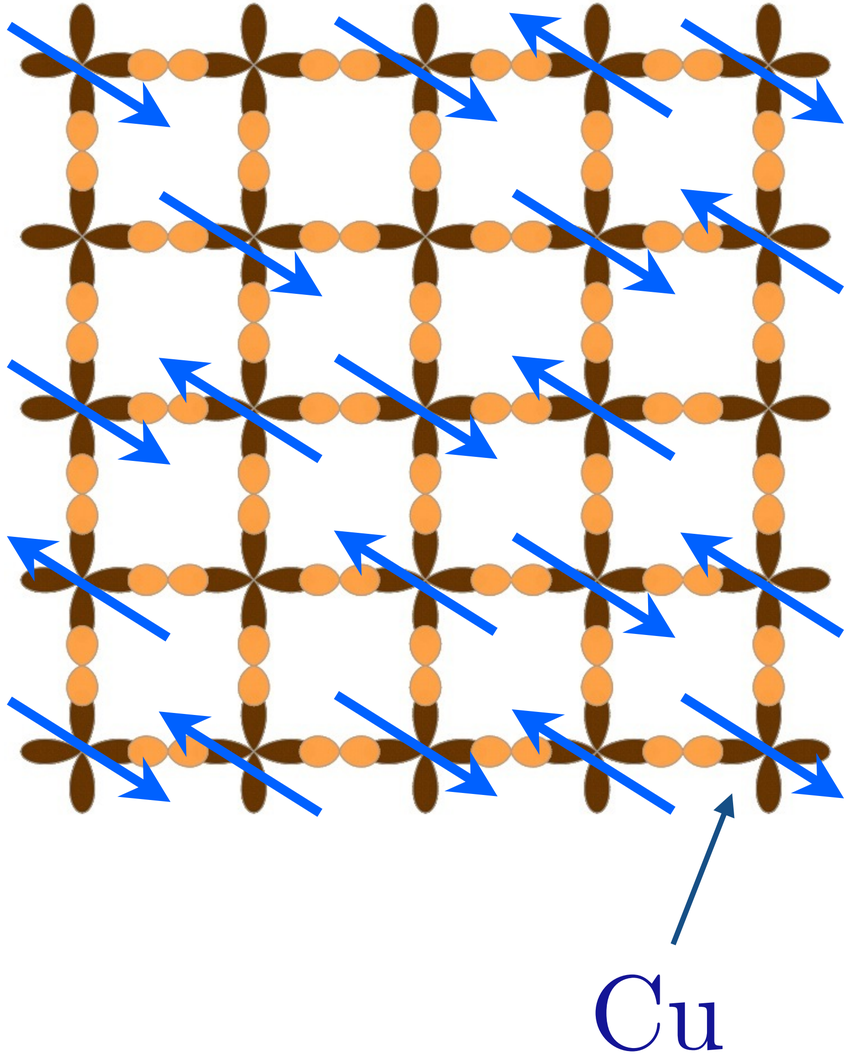


Cu

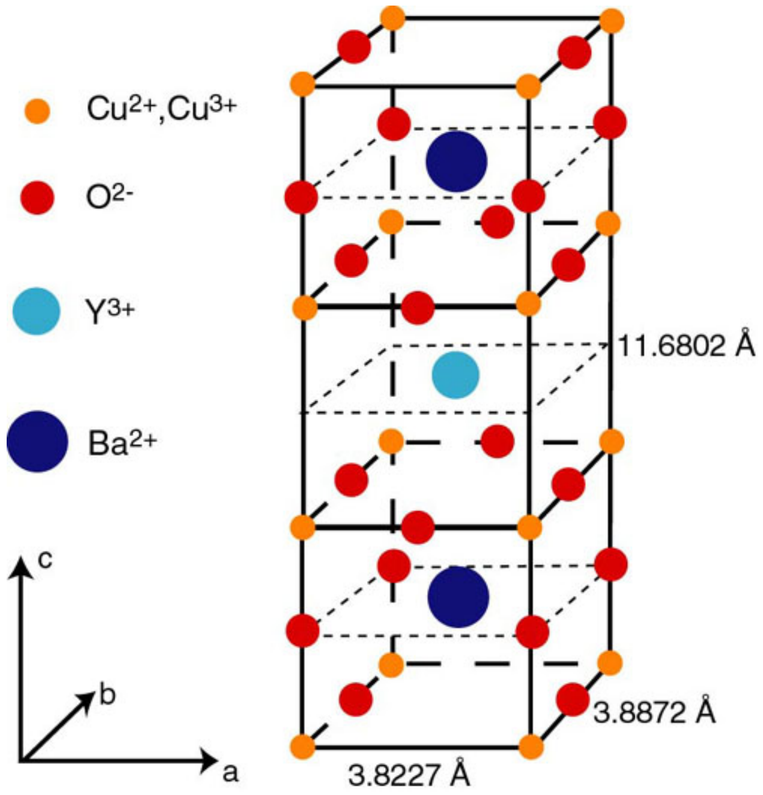
High temperature superconductor obtained upon doping the antiferromagnet with density p holes.

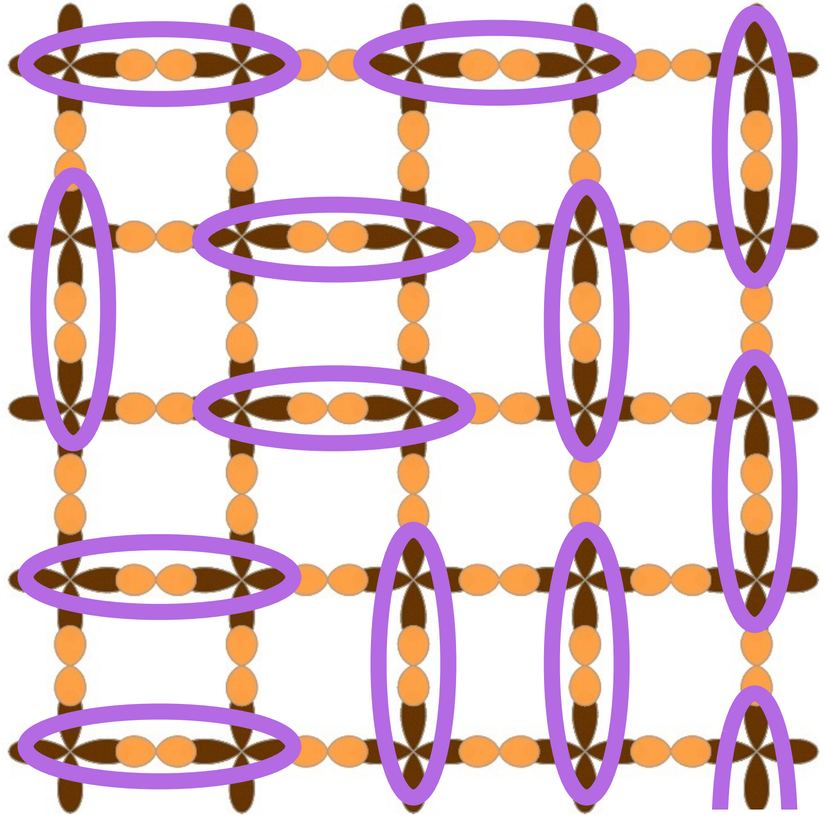


$YBa_2Cu_3O_{6+x}$



High temperature superconductor obtained upon doping the antiferromagnet with density p holes.
Hole density relative to the filled band $\rho = 1 + p$.





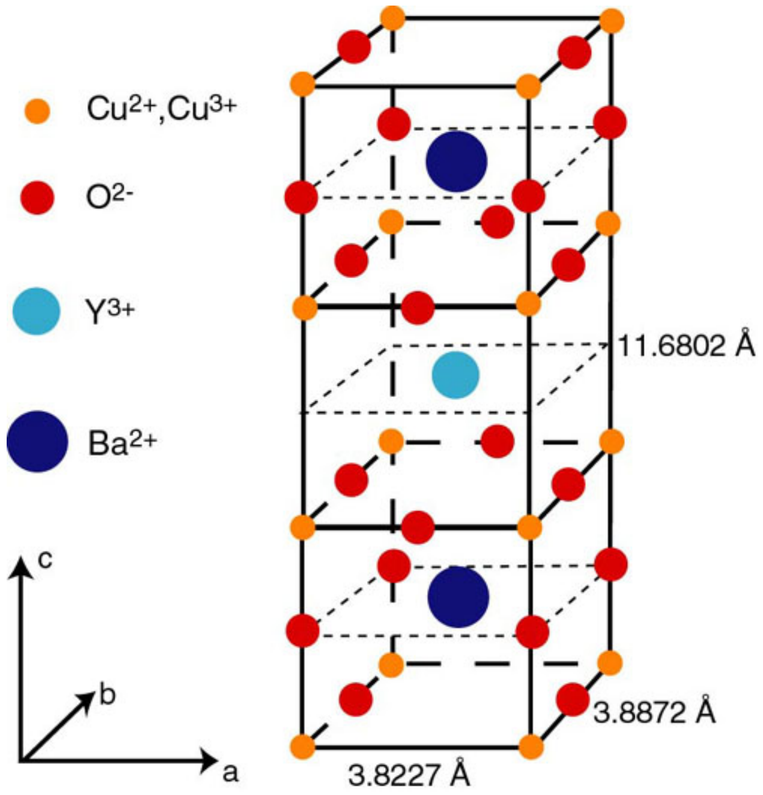
$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

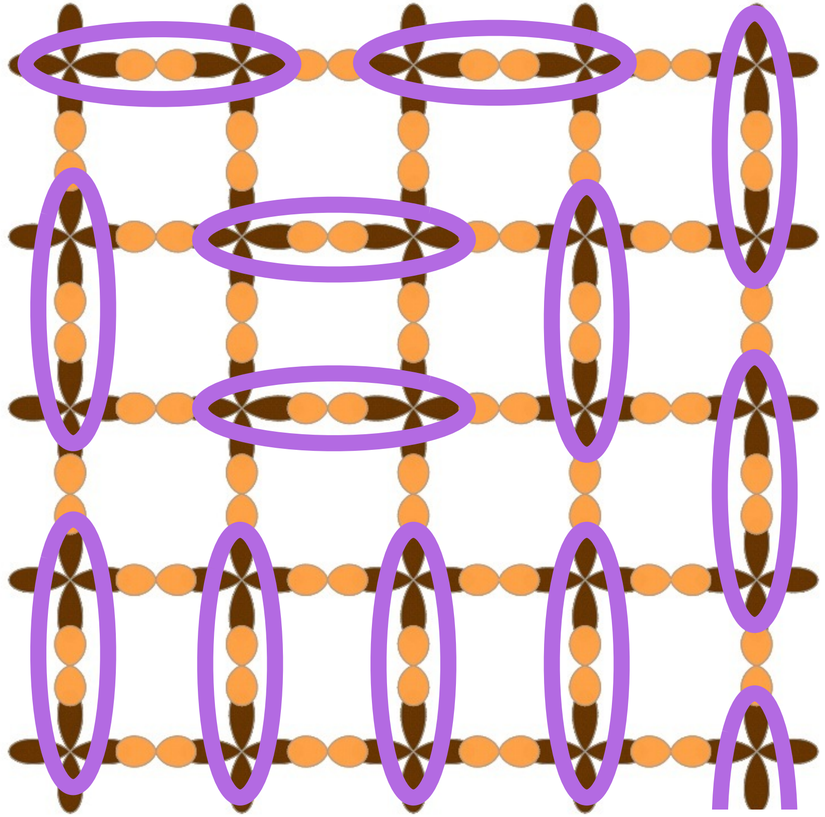
$\mathcal{D} \rightarrow$ dimer covering of lattice

$$YBa_2Cu_3O_{6+x}$$

$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

P.W. Anderson and G. Baskaran (1988): The key to high temperature superconductivity is the formation of a "resonating valence bond state" (a type of **quantum spin liquid**) which entangles the electrons on Cu





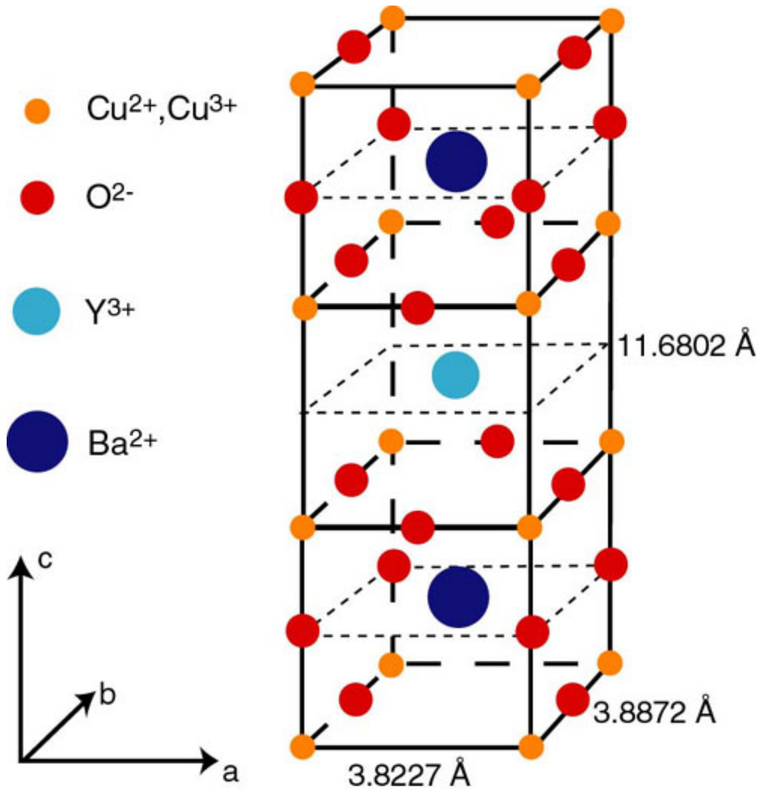
$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

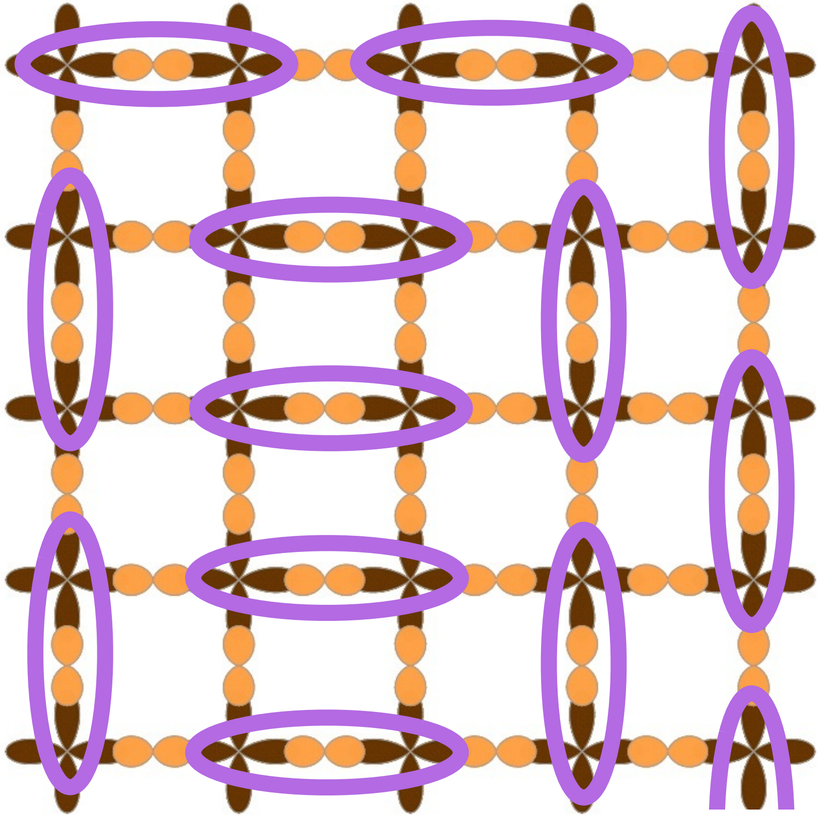
$\mathcal{D} \rightarrow$ dimer covering of lattice

$$YBa_2Cu_3O_{6+x}$$

$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

P.W. Anderson and G. Baskaran (1988): The key to high temperature superconductivity is the formation of a "resonating valence bond state" (a type of **quantum spin liquid**) which entangles the electrons on Cu





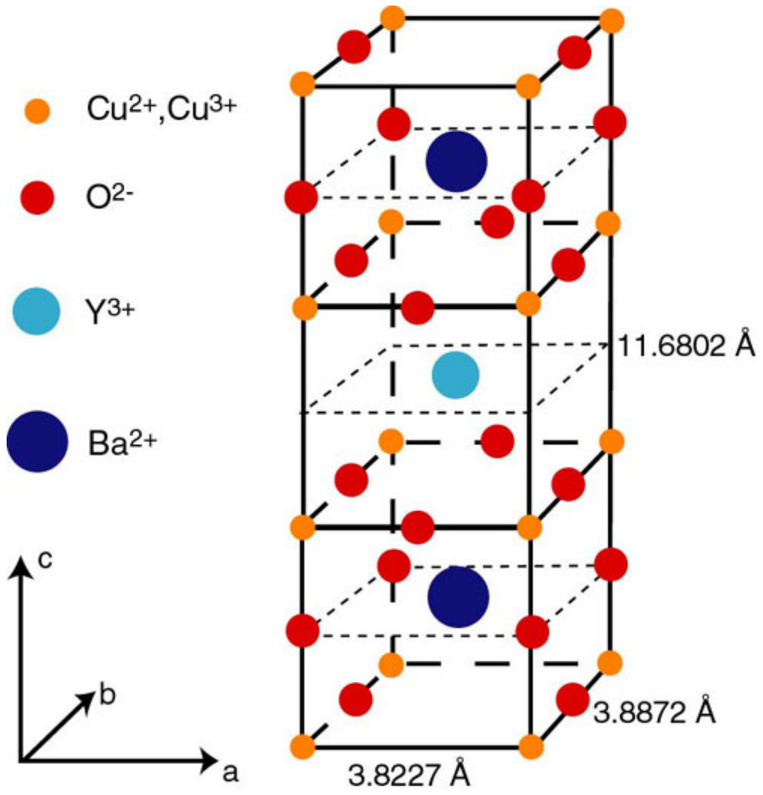
$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

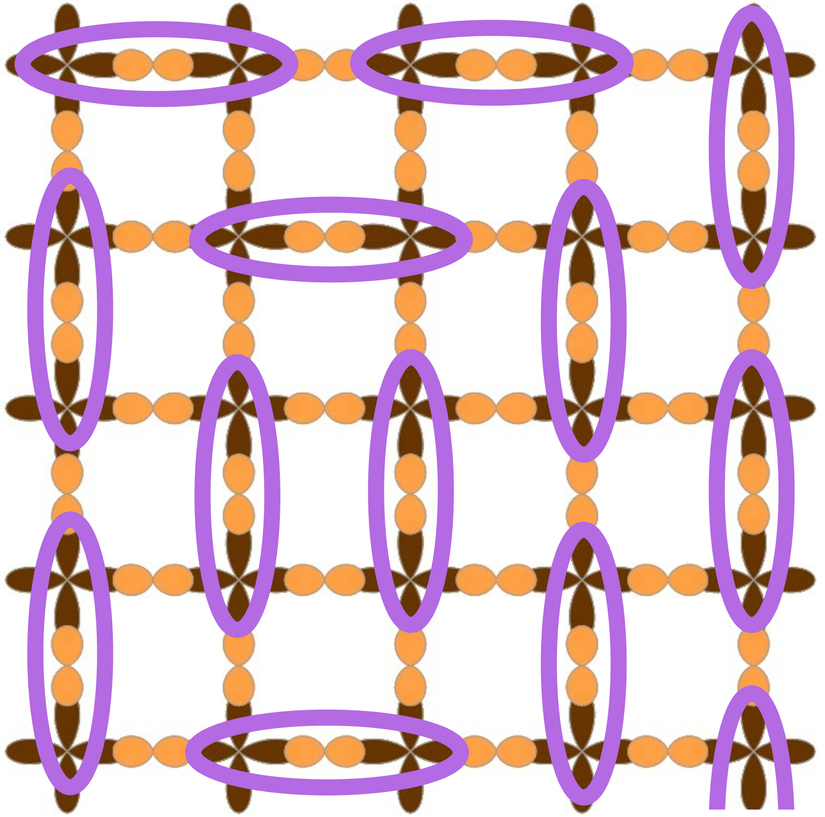
$\mathcal{D} \rightarrow$ dimer covering of lattice

$$YBa_2Cu_3O_{6+x}$$

$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

P.W. Anderson and G. Baskaran (1988): The key to high temperature superconductivity is the formation of a "resonating valence bond state" (a type of **quantum spin liquid**) which entangles the electrons on Cu





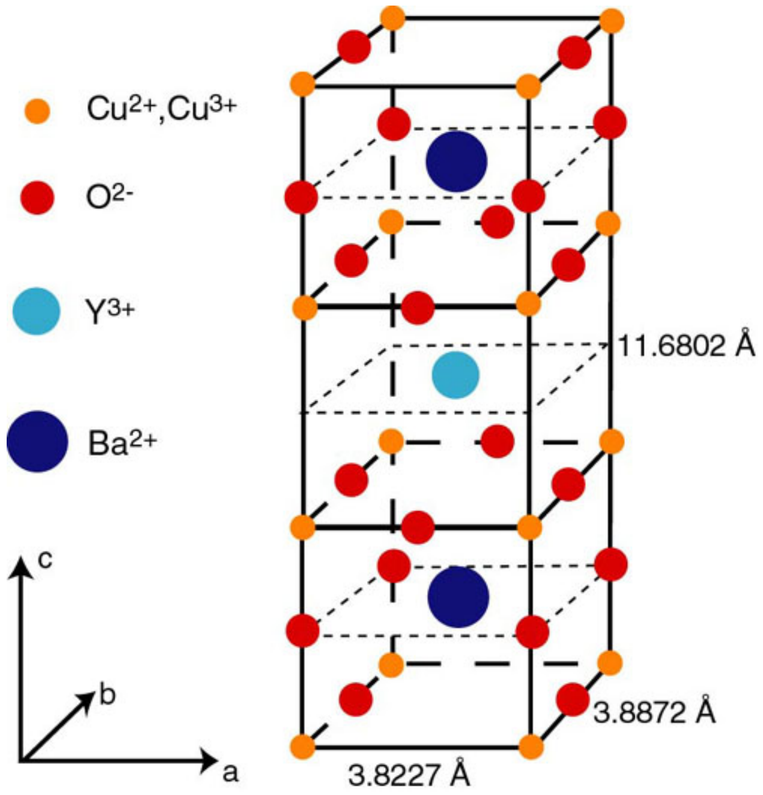
$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

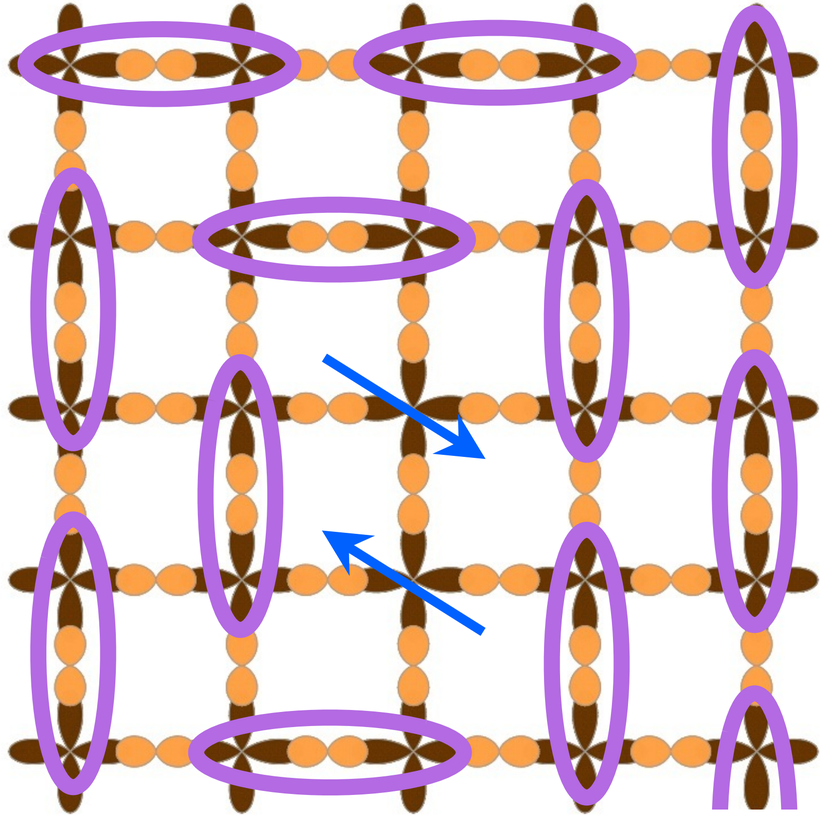
$\mathcal{D} \rightarrow$ dimer covering of lattice

$$YBa_2Cu_3O_{6+x}$$

$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

P.W. Anderson and G. Baskaran (1988): The key to high temperature superconductivity is the formation of a "resonating valence bond state" (a type of **quantum spin liquid**) which entangles the electrons on Cu

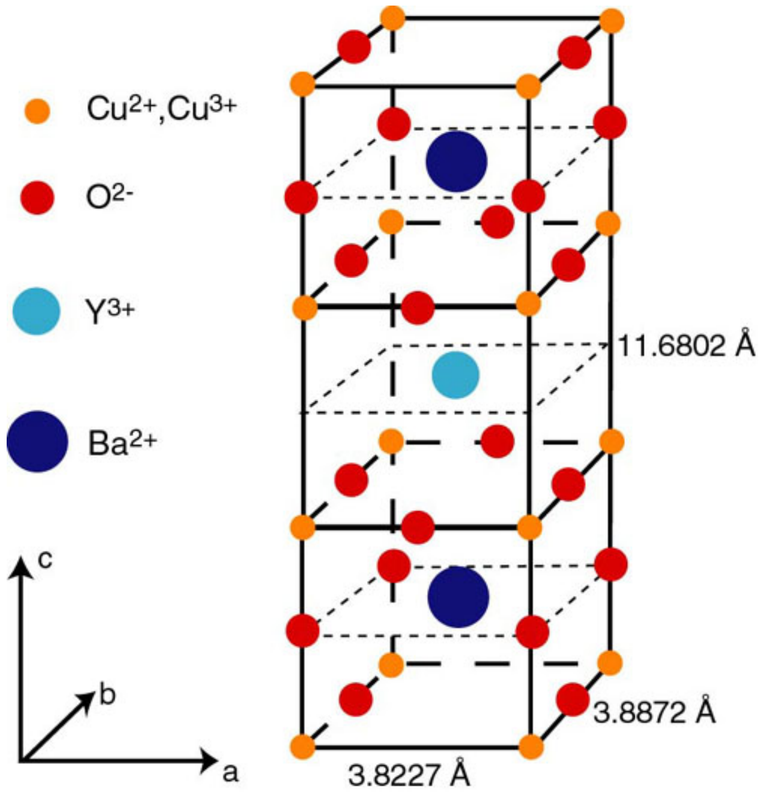


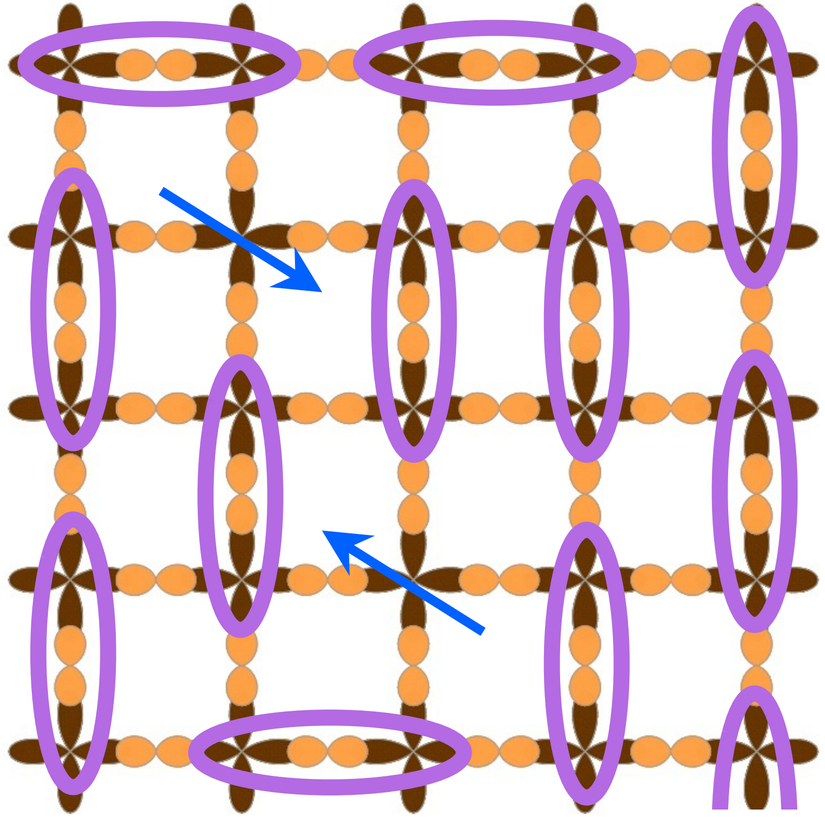


$YBa_2Cu_3O_{6+x}$

 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

Key feature: fractionalization. Excitations are particle-like, but cannot be created by local operators: they are classified under distinct superselection/anyon sectors.

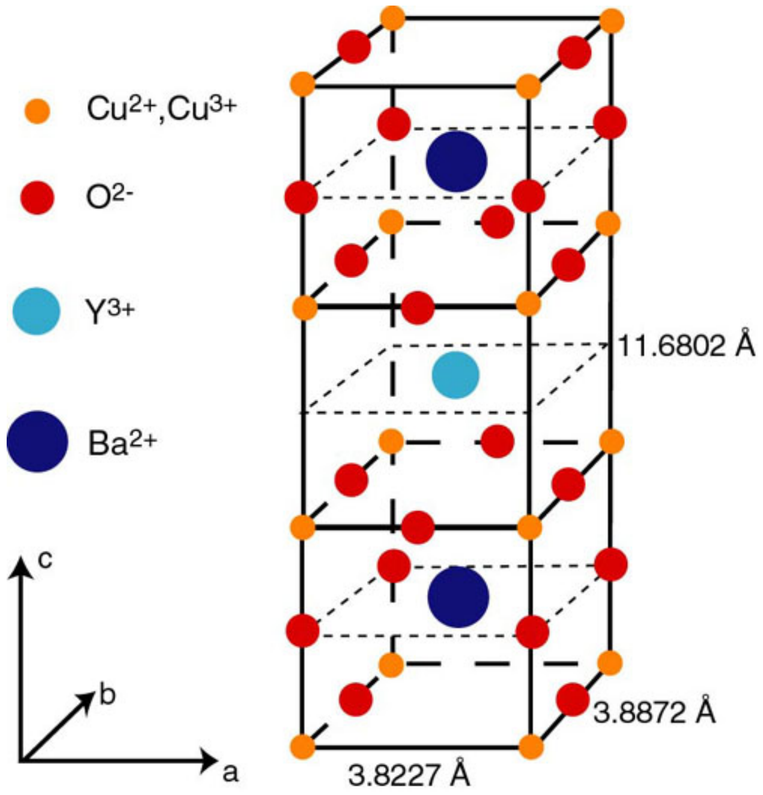


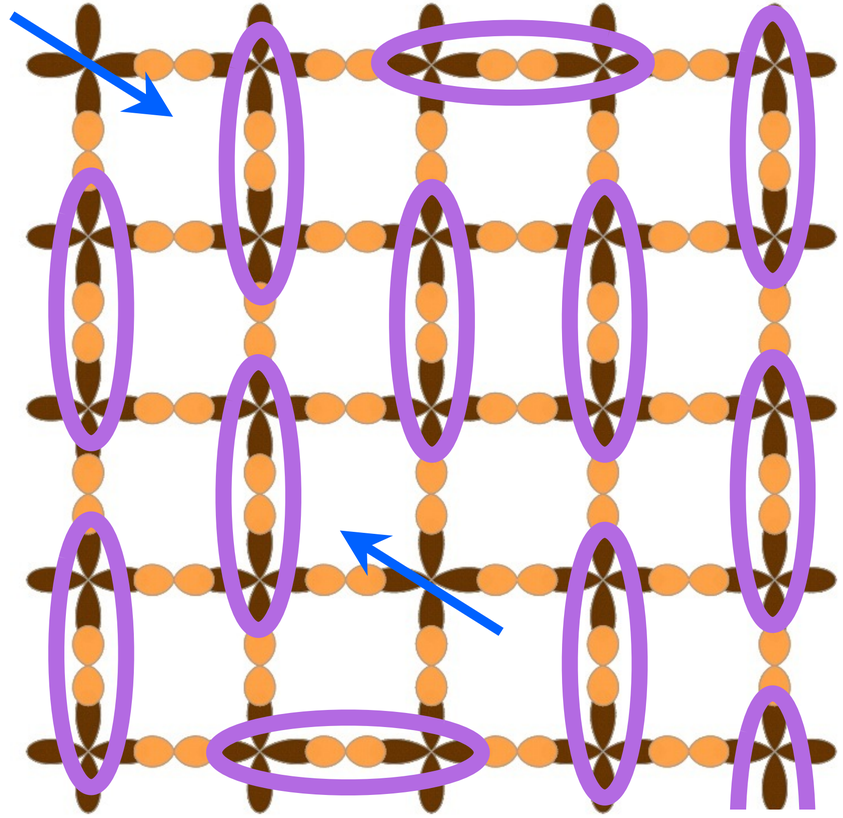


$YBa_2Cu_3O_{6+x}$

$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Key feature: fractionalization. Excitations are particle-like, but cannot be created by local operators: they are classified under distinct superselection/anyon sectors.

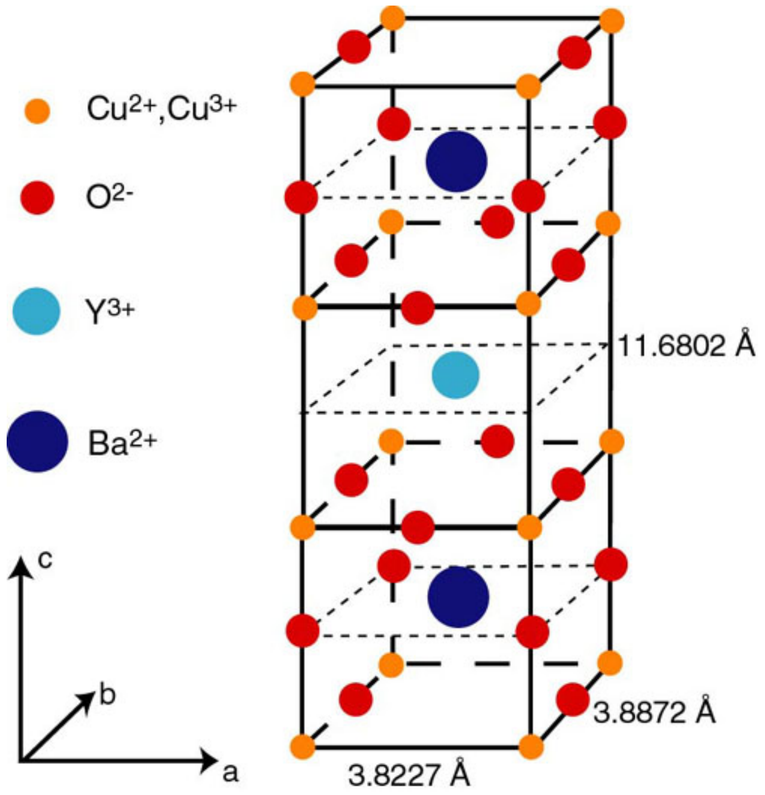


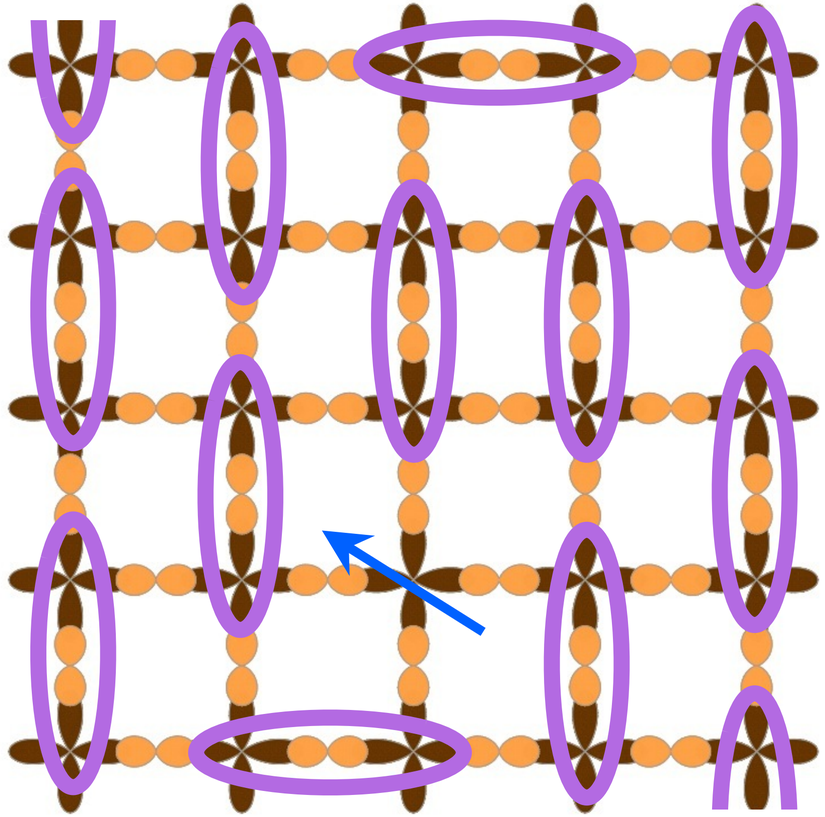


$YBa_2Cu_3O_{6+x}$

 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

Key feature: fractionalization. Excitations are particle-like, but cannot be created by local operators: they are classified under distinct superselection/anyon sectors.



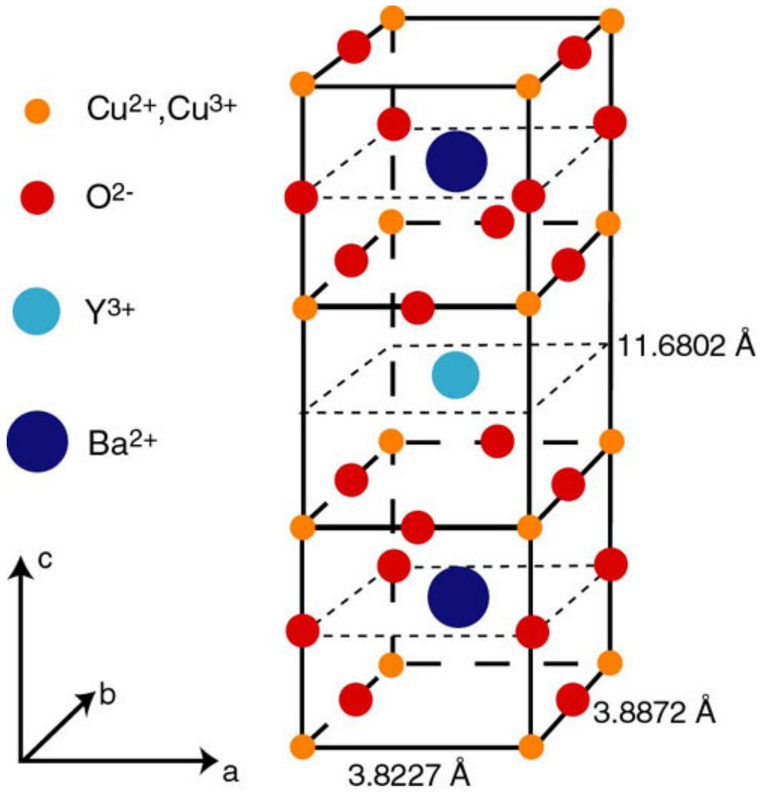


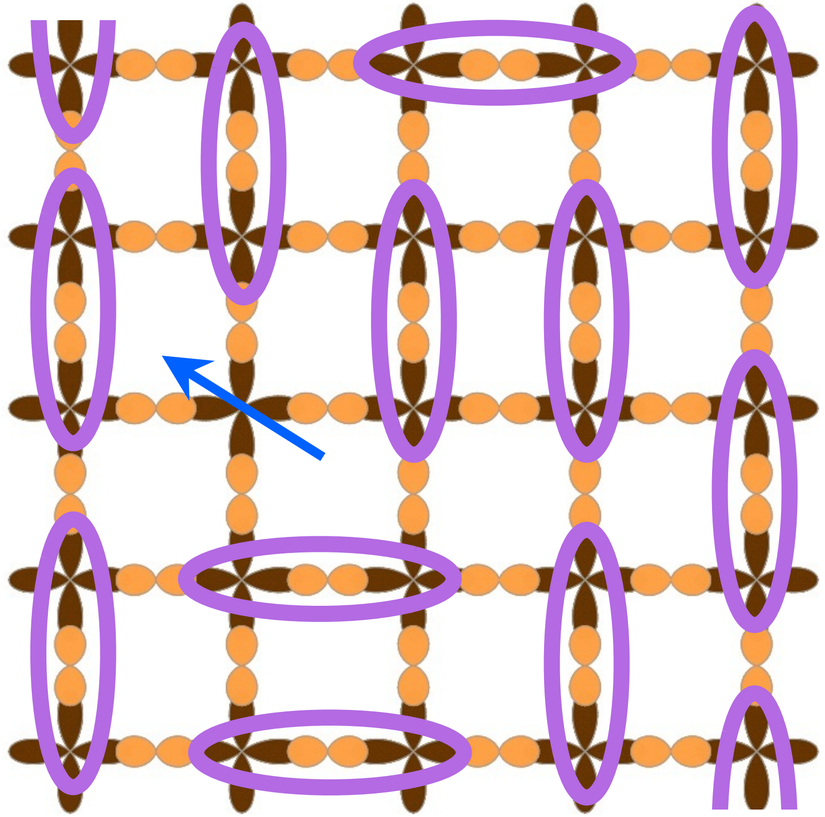
Spin S=1/2, charge neutral spinon

$YBa_2Cu_3O_{6+x}$

$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Key feature: fractionalization. Excitations are particle-like, but cannot be created by local operators: they are classified under distinct superselection/anyon sectors.



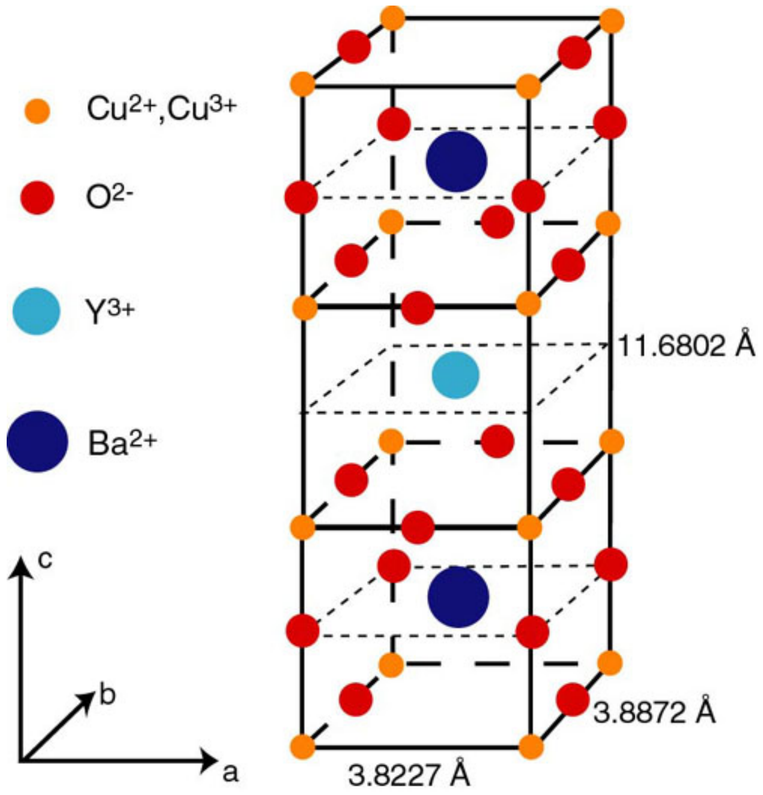


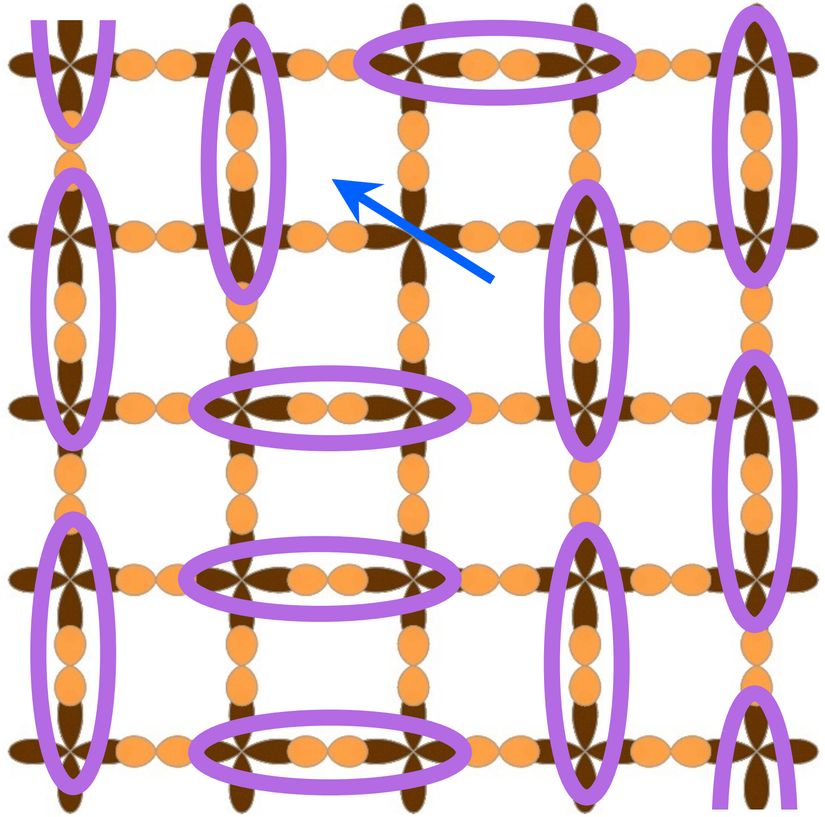
Spin S=1/2, charge neutral spinon

$YBa_2Cu_3O_{6+x}$

$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Key feature: fractionalization. Excitations are particle-like, but cannot be created by local operators: they are classified under distinct superselection/anyon sectors.





Spin S=1/2, charge neutral spinon

$YBa_2Cu_3O_{6+x}$

$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

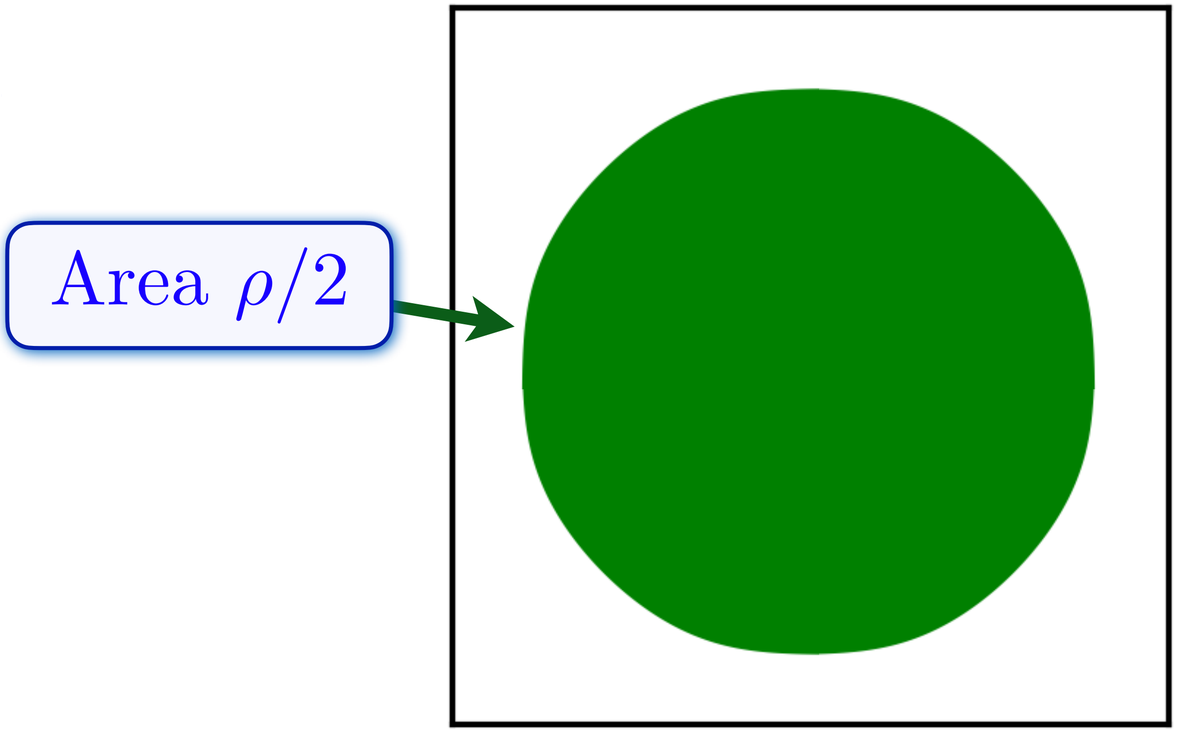
Key feature: fractionalization. Excitations are particle-like, but cannot be created by local operators: they are classified under distinct superselection/anyon sectors.

Quantum entanglement in metals: Fractionalized Fermi liquids (FL*)

Fermi liquid

Spin-1/2 holes of density
$\rho = 1 + p$

Positive Hall coefficient
of carrier density ρ

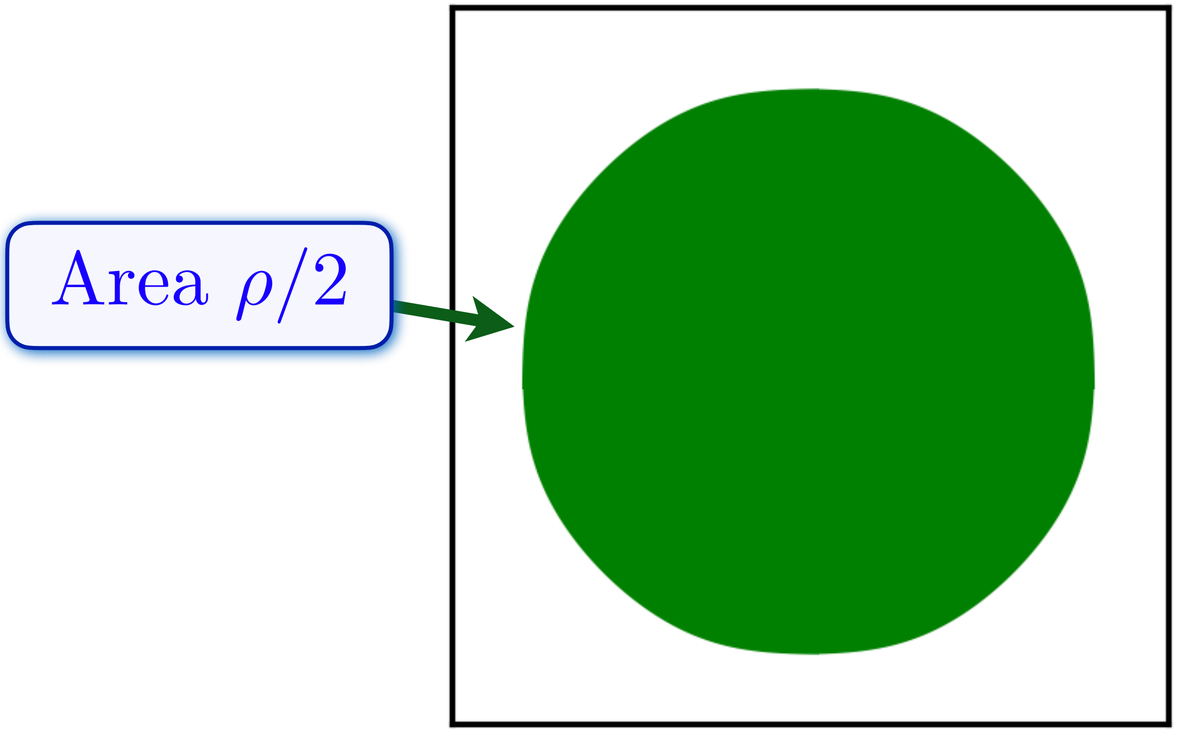


Luttinger, 1960: Area enclosed by the Fermi surface is the same as that for free fermions *with the same symmetry.*

Fermi liquid

Spin-1/2 holes of density
$\rho = 1 + p$

Positive Hall coefficient
of carrier density ρ



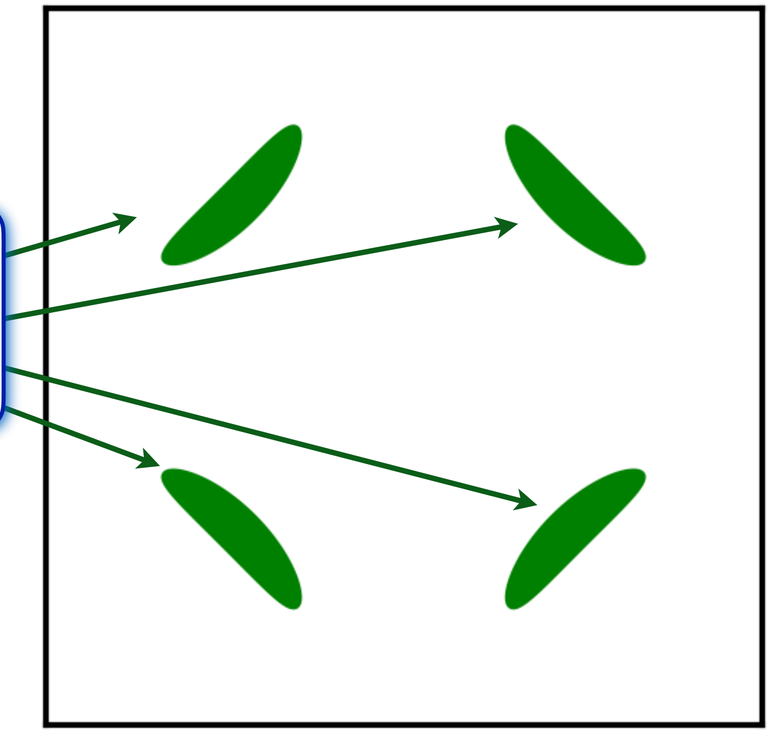
Luttinger, 1960: Area enclosed by the Fermi surface is the same as that for free fermions *with the same symmetry.*

Oshikawa, 2000: Area constrained by a 't Hooft anomaly of global U(1) and translations

Fractionalized Fermi liquid (FL*)

Spin-1/2 holes of density
$\rho = 1 + p$

Positive Hall coefficient
of carrier density $\rho - 1$



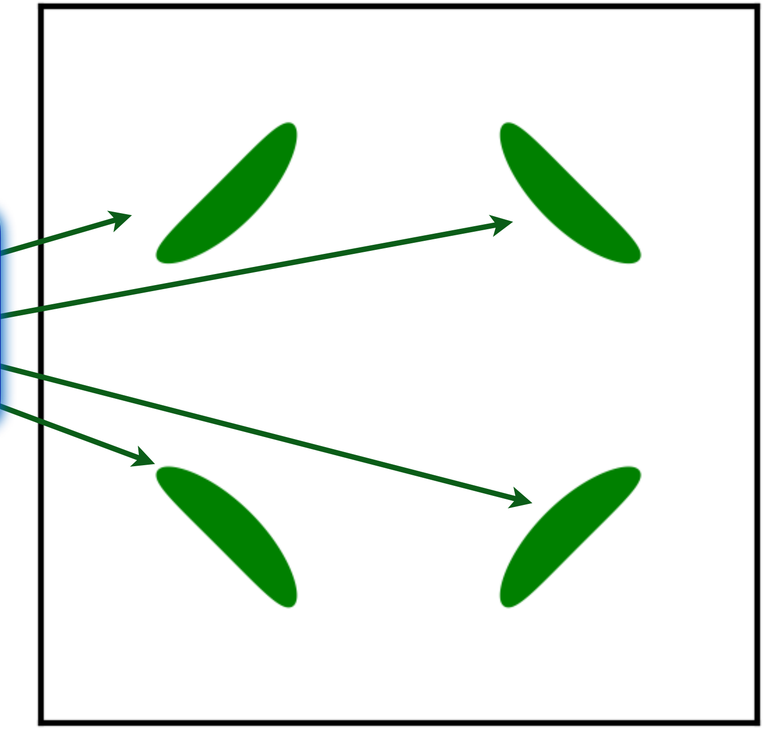
Oshikawa anomaly is satisfied by the sum of
spin liquid (1) and
Fermi surface anomalies $(\rho - 1)$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Fractionalized Fermi liquid (FL*)

Spin-1/2 holes of density
$\rho = 1 + p$

Positive Hall coefficient
of carrier density $\rho - 1$



Area of
each
hole pocket
$= p/8$

Oshikawa anomaly is satisfied by the sum of
spin liquid (1) and
Fermi surface anomalies $(\rho - 1)$

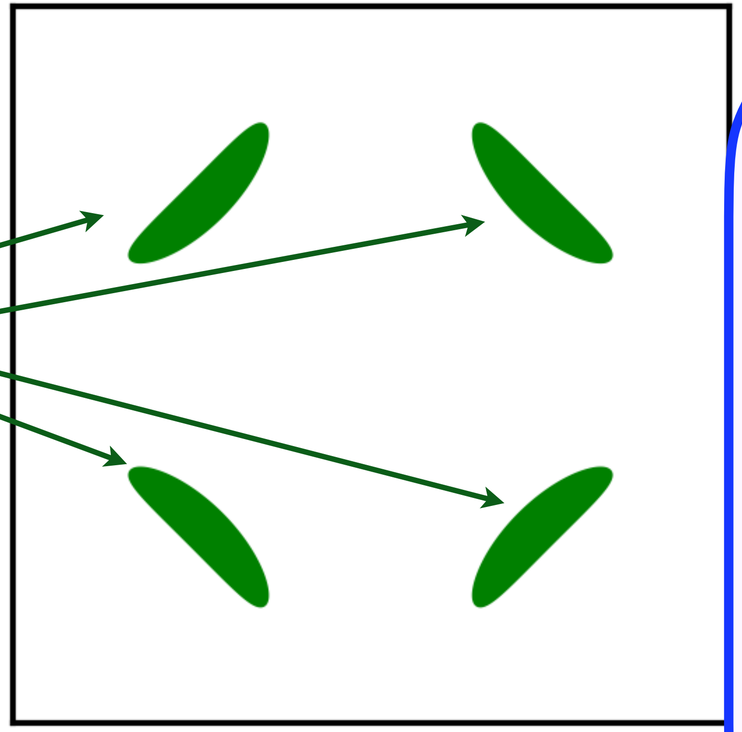
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Fractionalized Fermi liquid (FL*)

Spin-1/2 holes of density
$\rho = 1 + p$

Positive Hall coefficient
of carrier density $\rho - 1$

Total area
$(\rho - 1)/2$

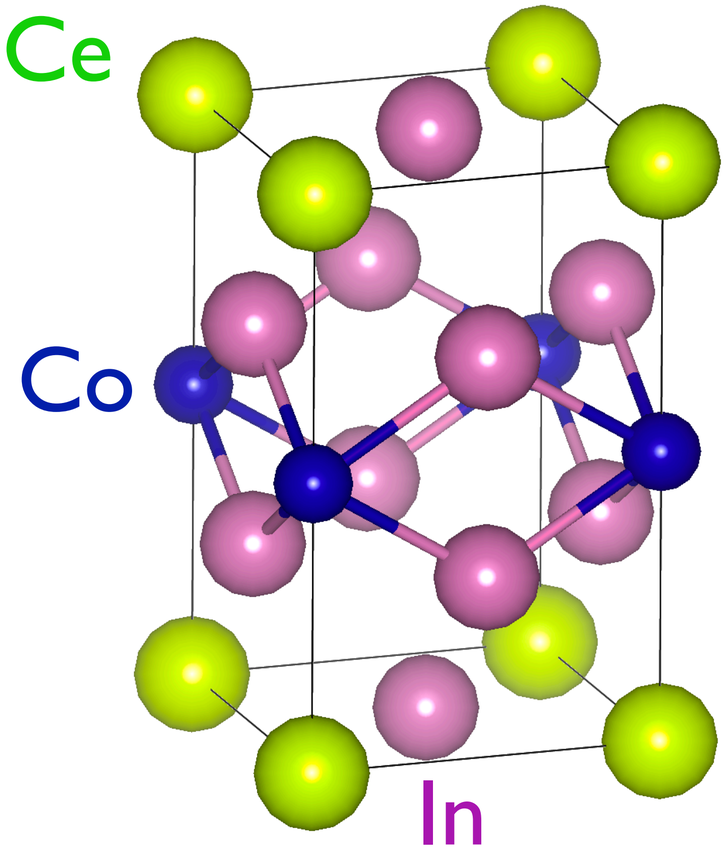


Measuring non-Luttinger Fermi surface area is direct evidence for spin liquid quantum entanglement.

Oshikawa anomaly is satisfied by the sum of
spin liquid (1) and
Fermi surface anomalies $(\rho - 1)$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

FL* in $CeCoIn_5$



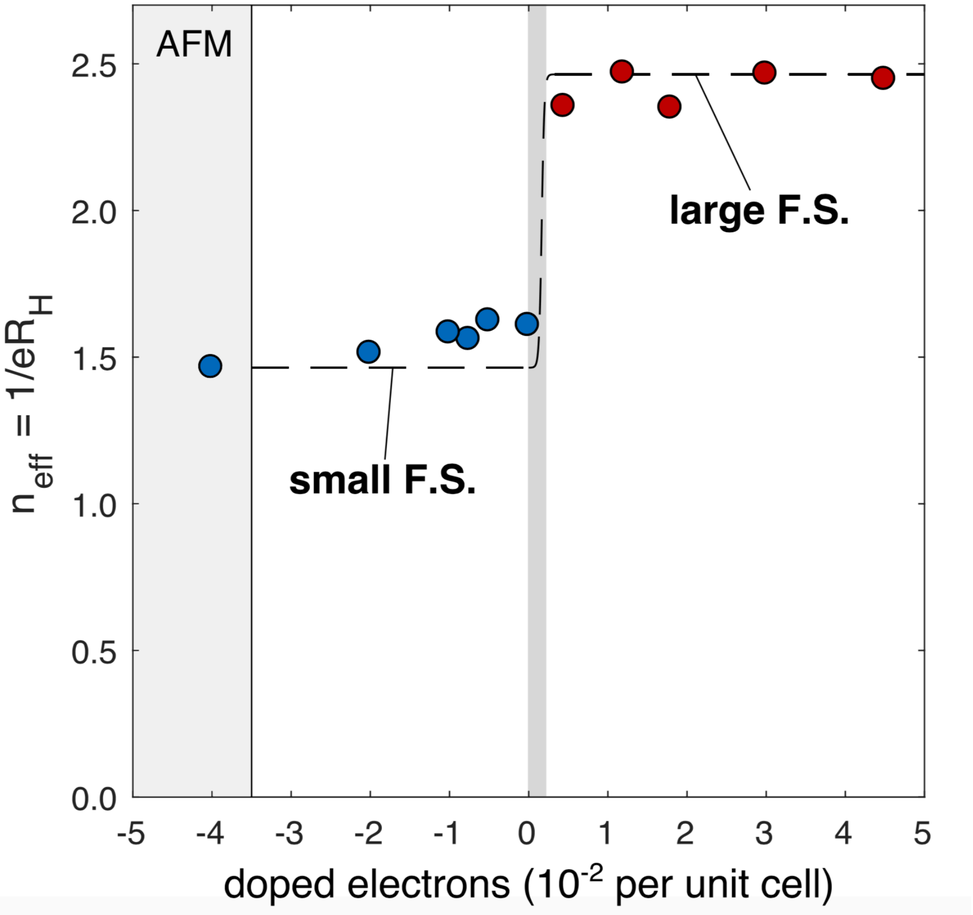
Strong on-site repulsion *U* only on Ce sites

$CeCoIn_5$

Evidence for a delocalization quantum phase transition without symmetry breaking in $CeCoIn_5$

N. Maksimovic, D.H. Eilbott, T. Cookmeyer, Fanghui Wan, J. Rusz. V. Nagarajan, S.C. Haley, E. Maniv. Amanda Gong. S. Faubel, I.M. Hayes, A. Bangura. J. Singleton. J. C. Palmstrom, L. Winter, R. McDonald, Sooyoung Jang. Ping Ai. Yi Lin, S. Ciocys, J. Gobbo, Y. Werman, P.M. Oppeneer, E. Altman, A. Lanzara, J. G. Analytis

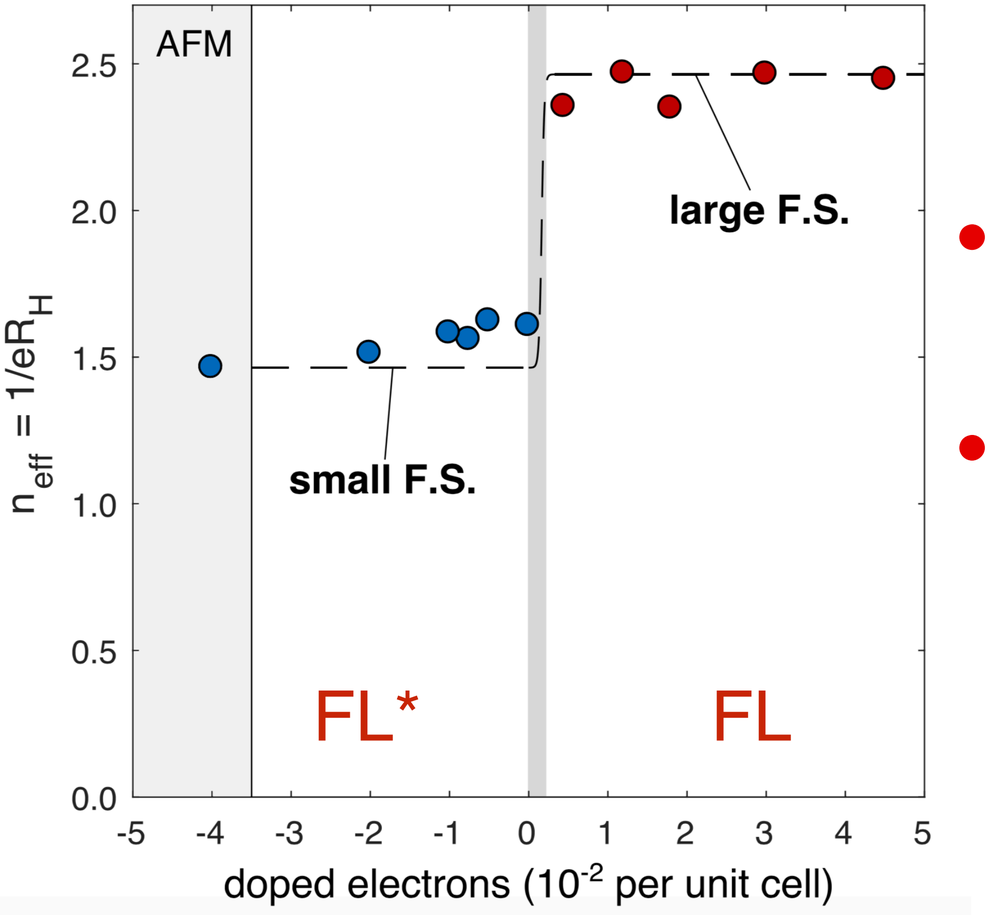
Science **375**, 76 (2021)



Evidence for a delocalization quantum phase transition without symmetry breaking in $CeCoIn_5$

N. Maksimovic, D.H. Eilbott, T. Cookmeyer, Fanghui Wan, J. Rusz. V. Nagarajan, S.C. Haley, E. Maniv. Amanda Gong. S. Faubel, I.M. Hayes, A. Bangura. J. Singleton. J. C. Palmstrom, L. Winter, R. McDonald, Sooyoung Jang. Ping Ai. Yi Lin, S. Ciocys, J. Gobbo, Y. Werman, P.M. Oppeneer, E. Altman, A. Lanzara, J. G. Analytis

Science **375**, 76 (2021)

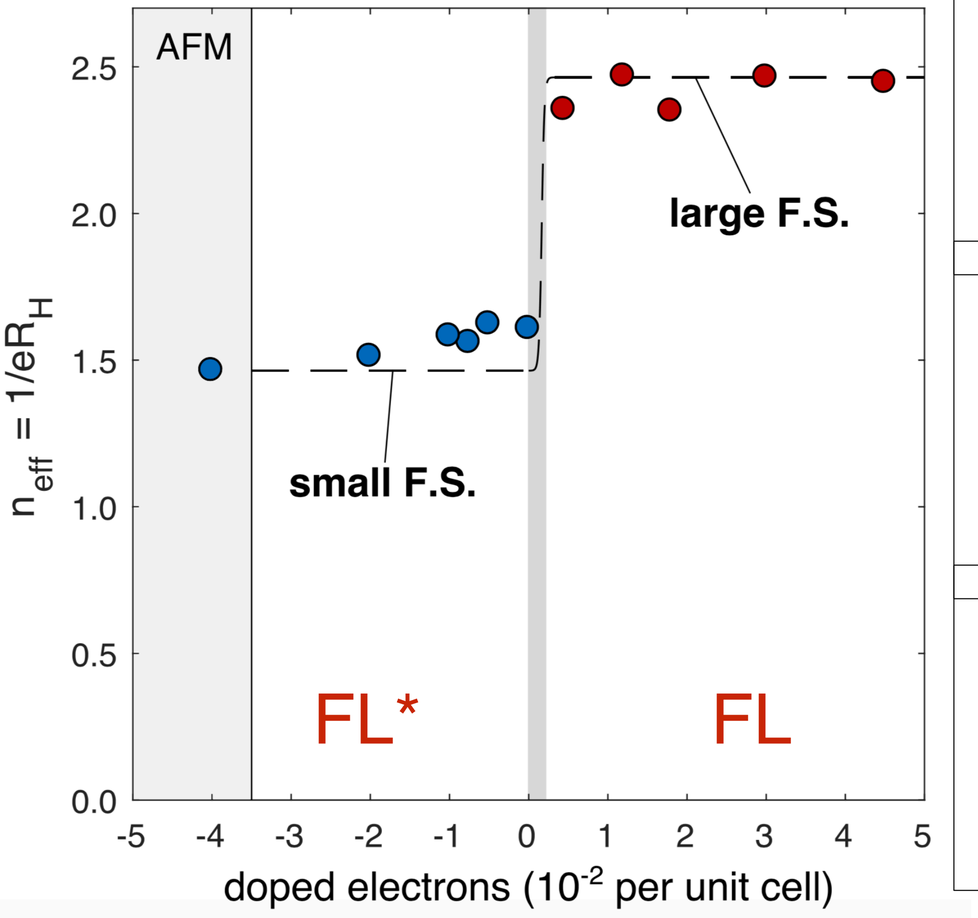


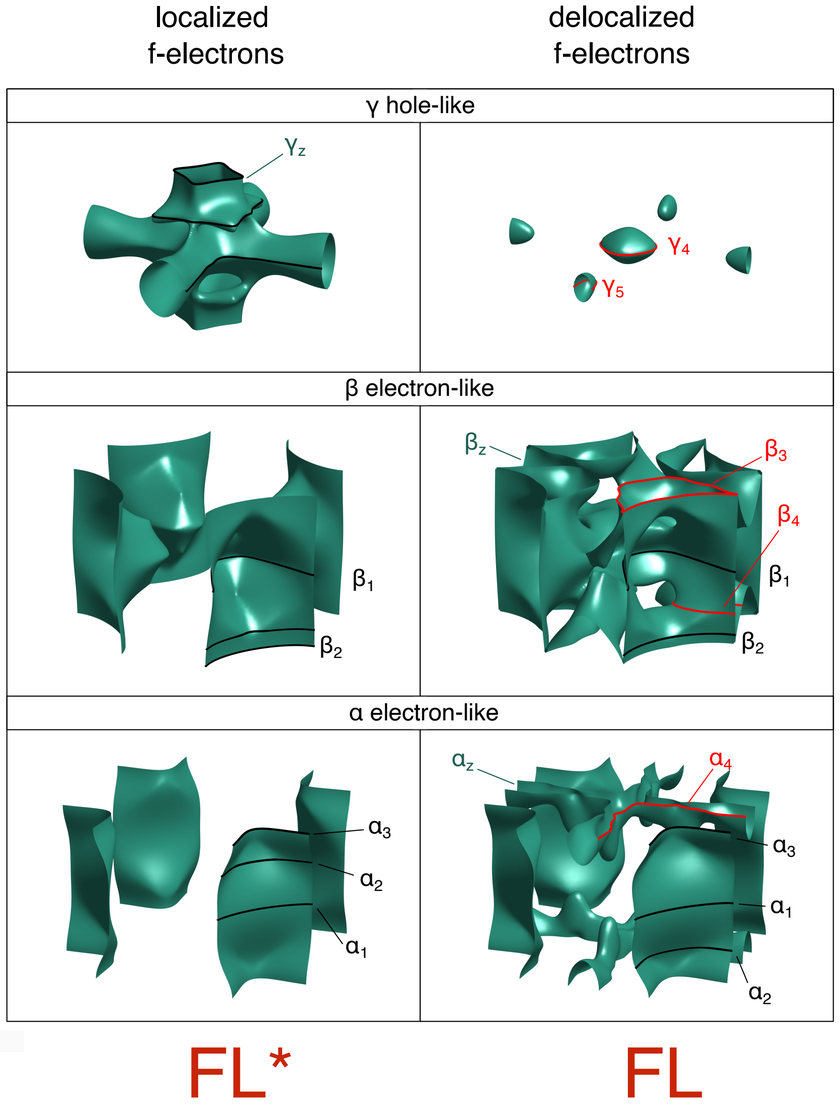
- FL $\Rightarrow$ One electron per Ce site in included in volume enclosed by Fermi surface.
- FL* $\Rightarrow$ One electron per Ce site forms a spin liquid, and is not part of the Fermi surface.

Evidence for a delocalization quantum phase transition without symmetry breaking in $CeCoIn_5$

N. Maksimovic, D.H. Eilbott, T. Cookmeyer, Fanghui Wan, J. Rusz. V. Nagarajan, S.C. Haley, E. Maniv. Amanda Gong. S. Faubel, I.M. Hayes, A. Bangura. J. Singleton. J. C. Palmstrom, L. Winter, R. McDonald, Sooyoung Jang. Ping Ai. Yi Lin, S. Ciocys, J. Gobbo, Y. Werman, P.M. Oppeneer, E. Altman, A. Lanzara, J. G. Analytis

Science **375**, 76 (2021)

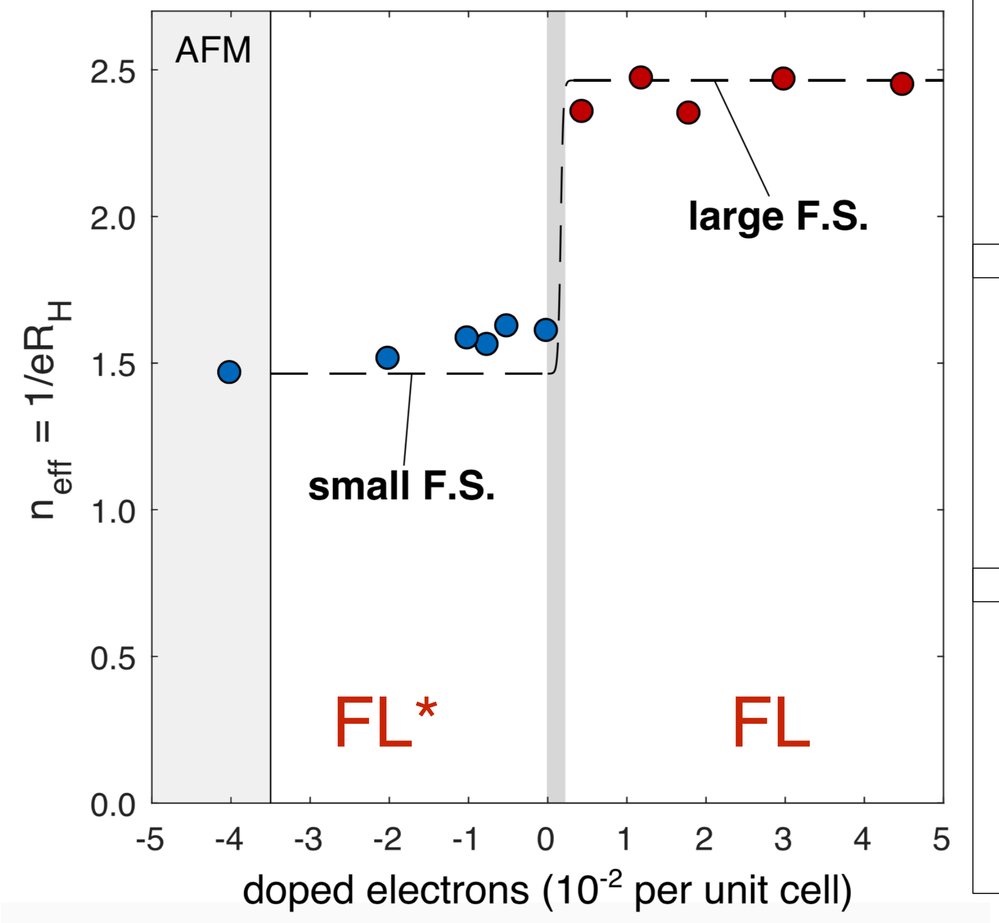


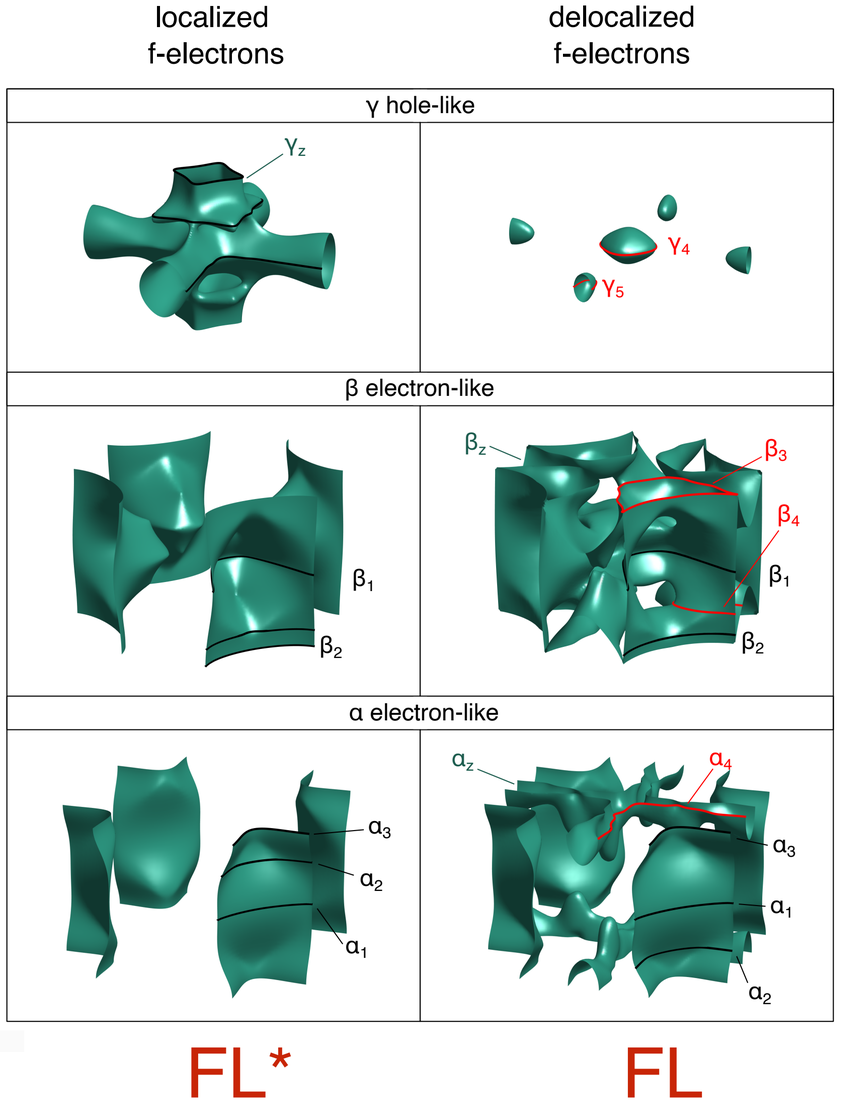


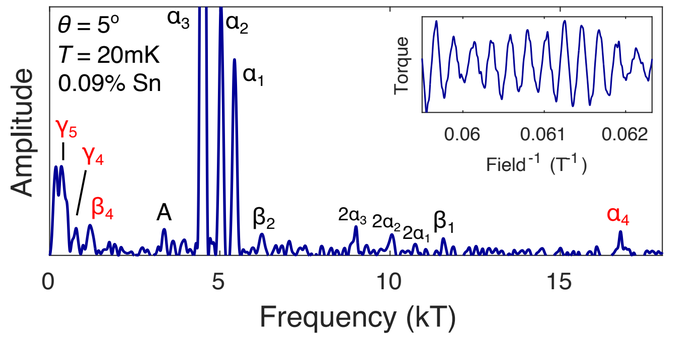
Evidence for a delocalization quantum phase transition without symmetry breaking in $CeCoIn_5$

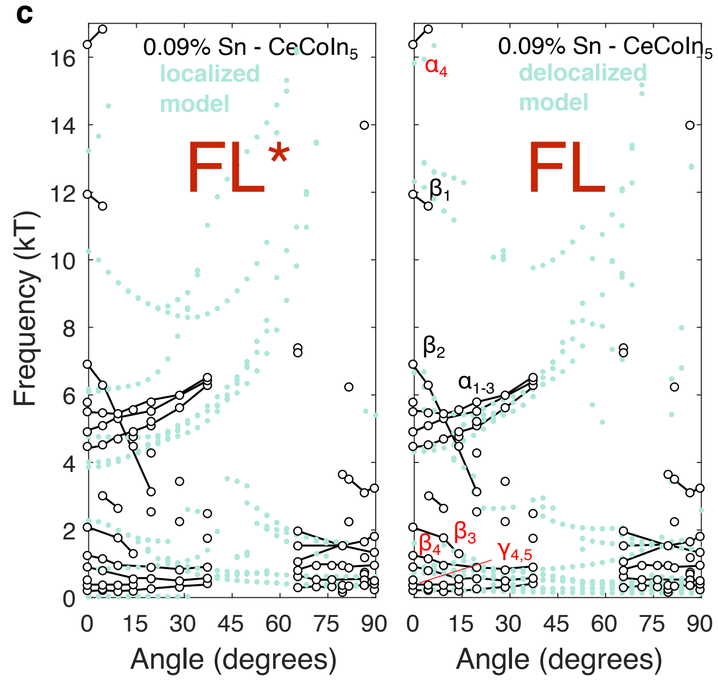
N. Maksimovic, D.H. Eilbott, T. Cookmeyer, Fanghui Wan, J. Rusz. V. Nagarajan, S.C. Haley, E. Maniv. Amanda Gong. S. Faubel, I.M. Hayes, A. Bangura. J. Singleton. J. C. Palmstrom, L. Winter, R. McDonald, Sooyoung Jang. Ping Ai. Yi Lin, S. Ciocys, J. Gobbo, Y. Werman, P.M. Oppeneer, E. Altman, A. Lanzara, J. G. Analytis

Science **375**, 76 (2021)







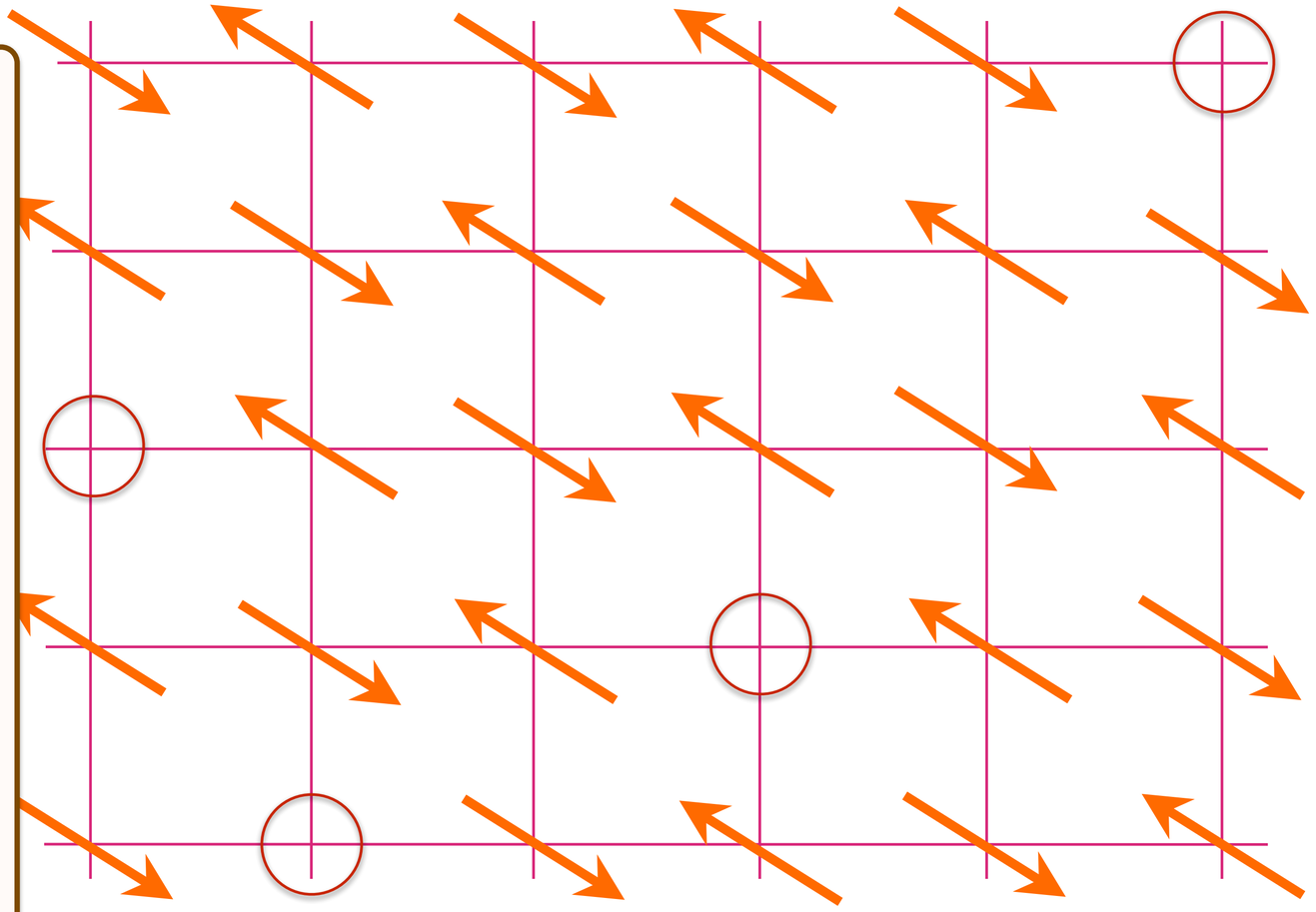


FL* on the square lattice

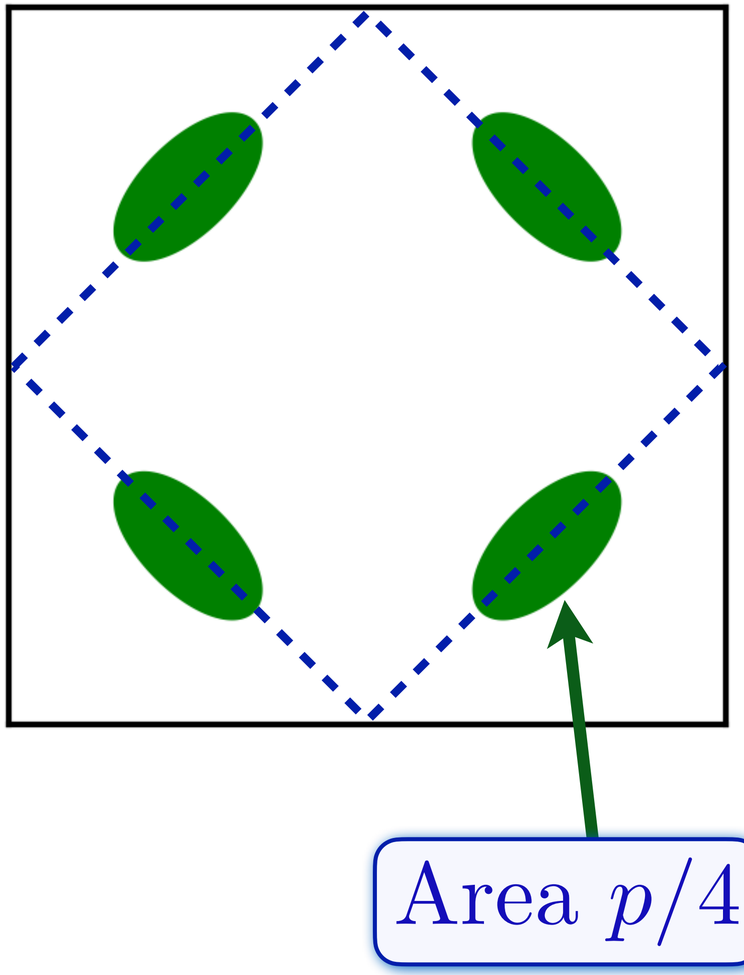
Doping an insulating antiferromagnet with holes of density p

AF metal

Fermi liquid with density p of spin 1/2, charge $+e$ holes. Yamaji effect requires inter-layer spin correlations (not present in Hg-1201).



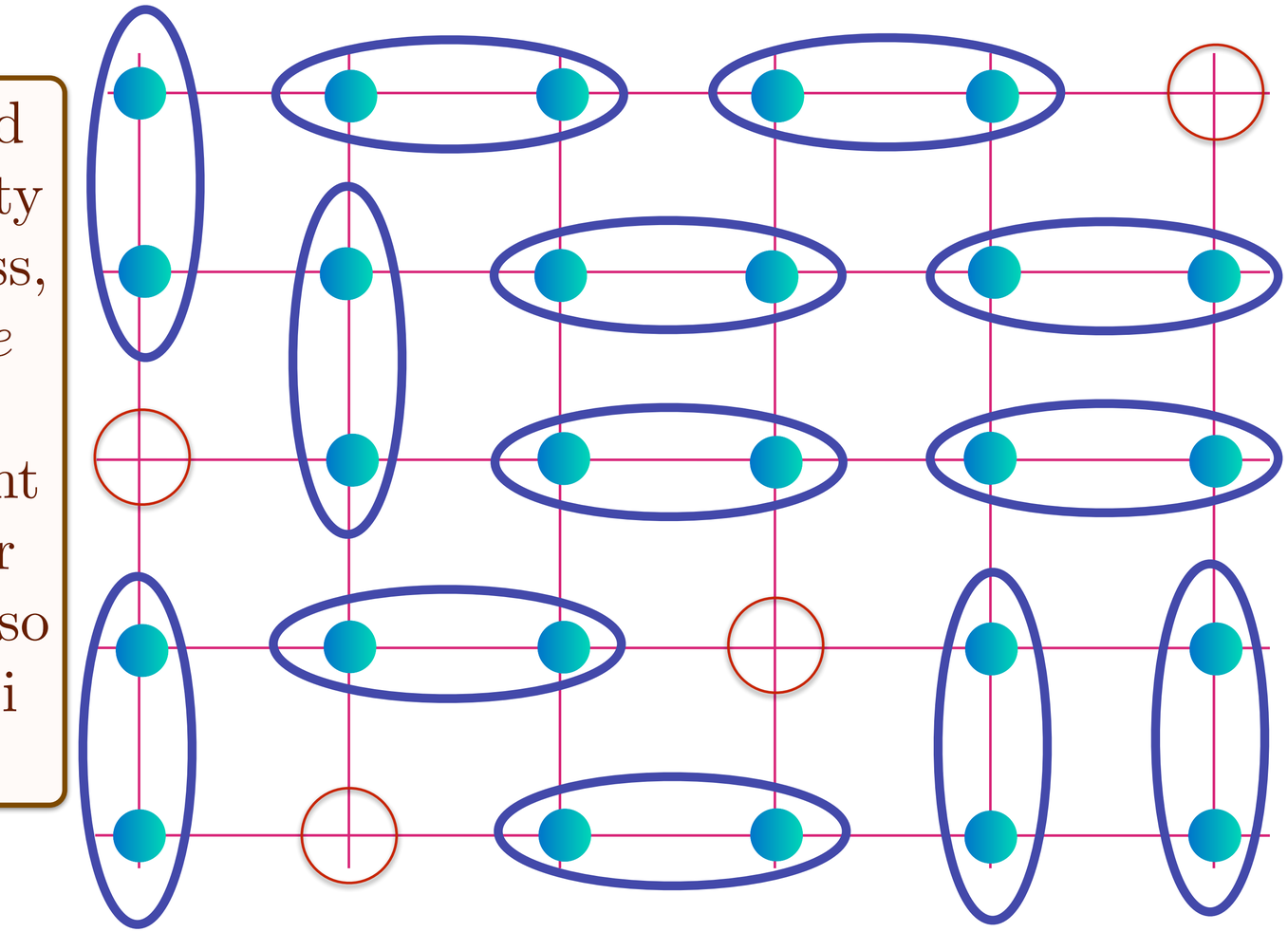
Luttinger area.
Broken symmetry



Doping an insulating antiferromagnet with holes of density p

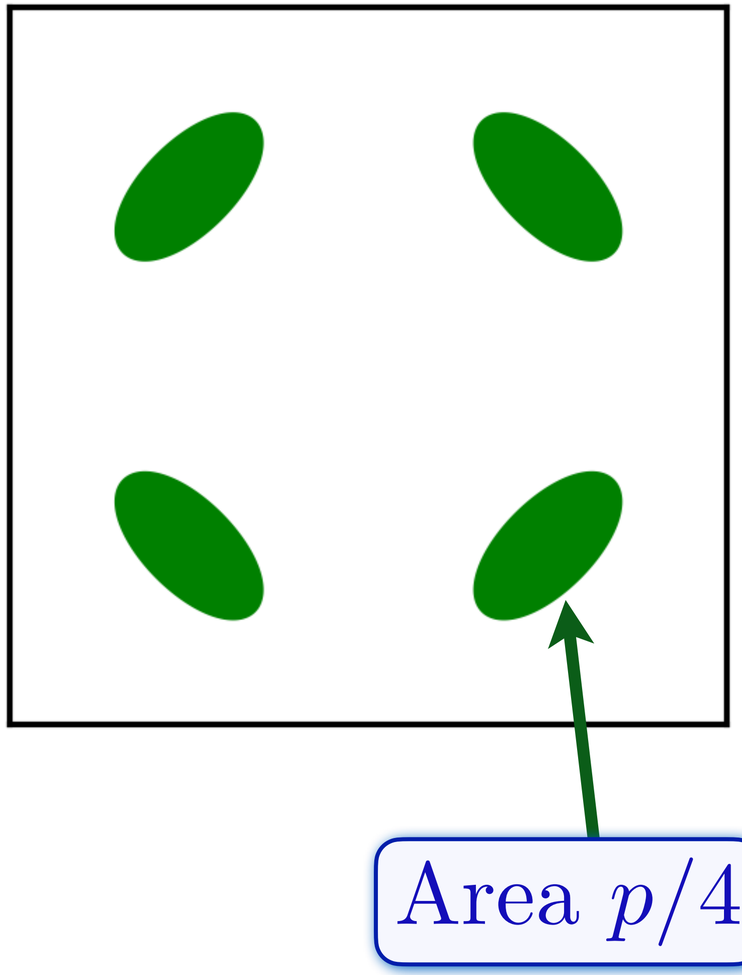
Holon metal

Spin liquid with density p of spinless, charge $+e$ holons. No coherent inter-layer transport, so no Yamaji effect.



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

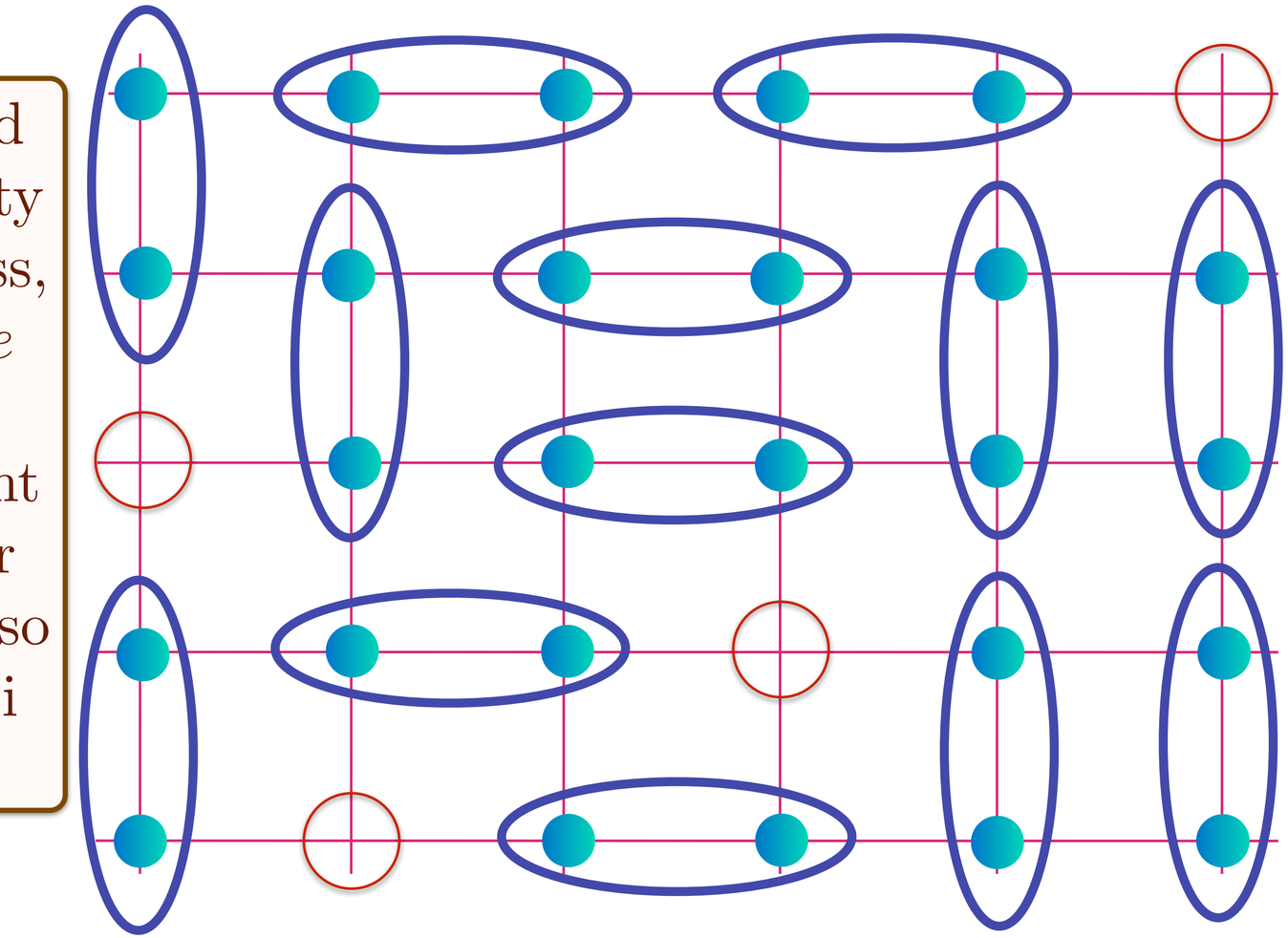


R. K. Kaul, Y. B. Kim, S. S., T. Senthil, Nature Physics **4**, 28 (2008); S.S. M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Doping an insulating antiferromagnet with holes of density p

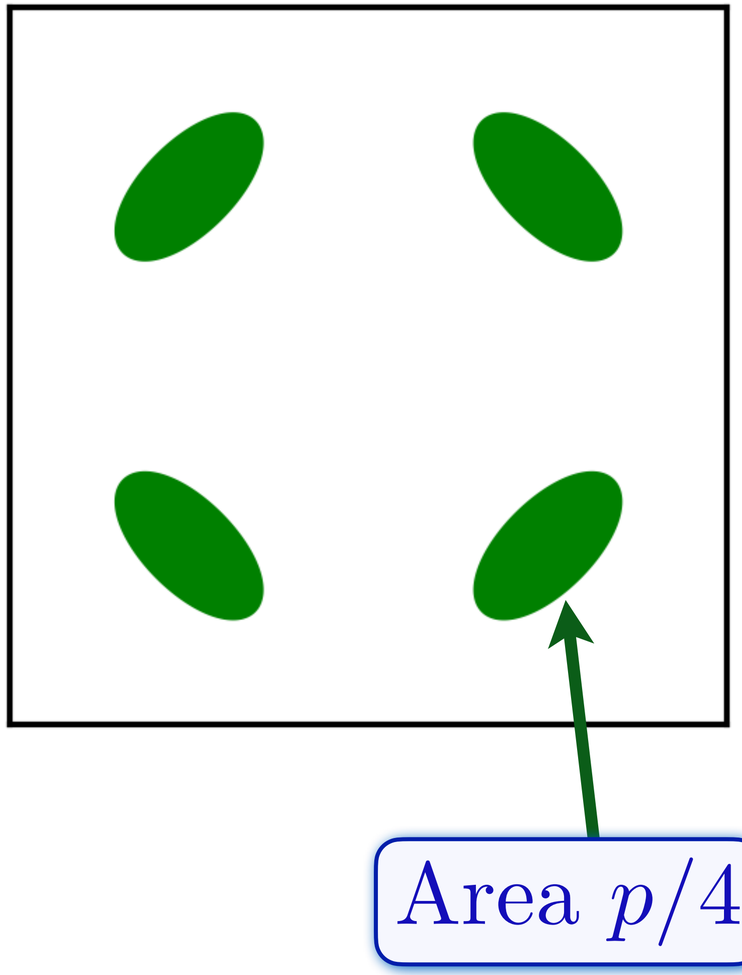
Holon metal

Spin liquid with density p of spinless, charge $+e$ holons. No coherent inter-layer transport, so no Yamaji effect.



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

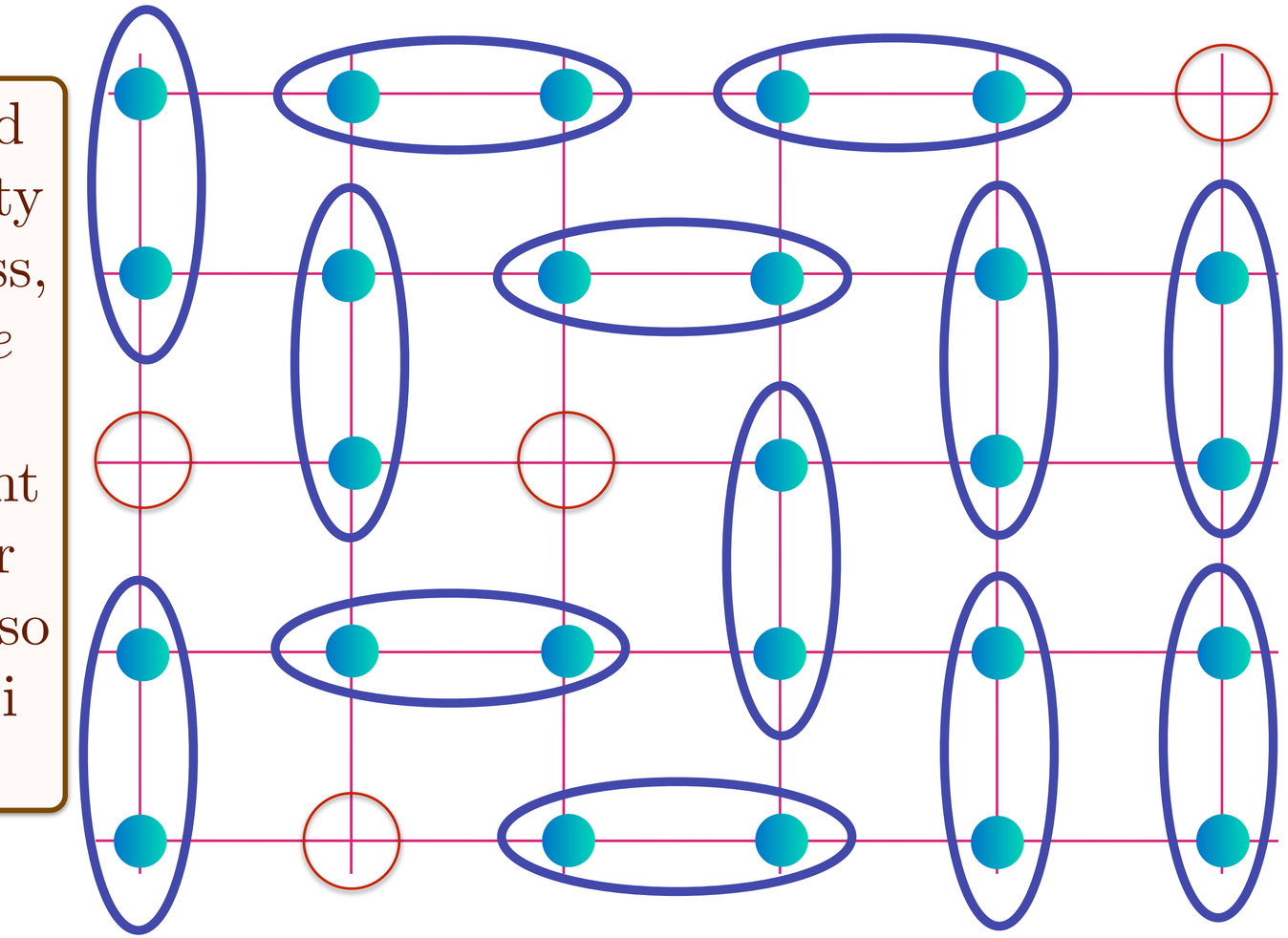


R. K. Kaul, Y. B. Kim, S. S., T. Senthil, Nature Physics **4**, 28 (2008); S.S. M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Doping an insulating antiferromagnet with holes of density p

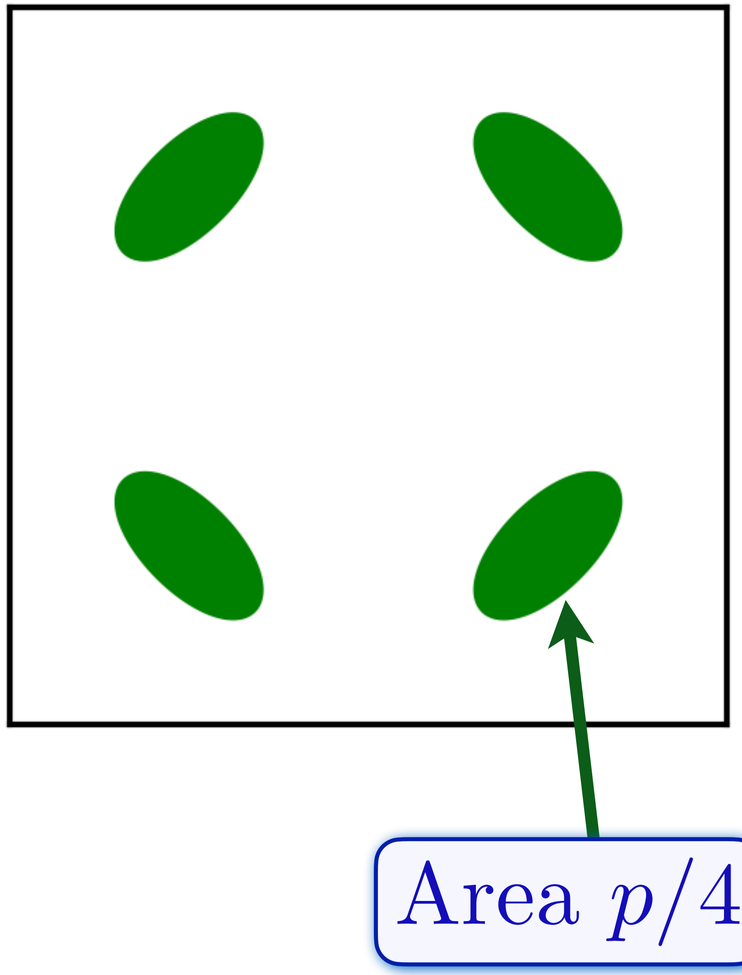
Holon metal

Spin liquid with density p of spinless, charge $+e$ holons. No coherent inter-layer transport, so no Yamaji effect.



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

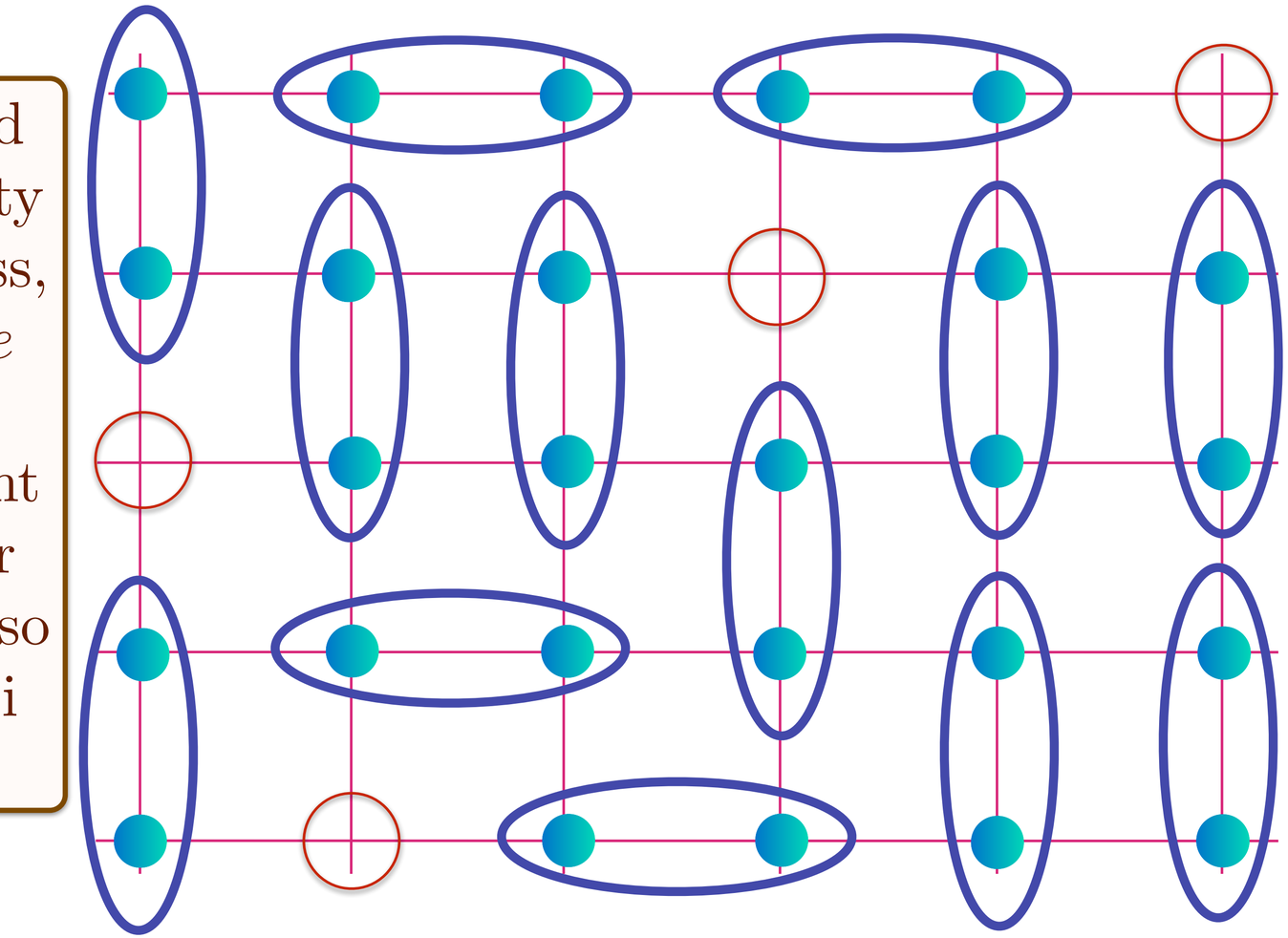


R. K. Kaul, Y. B. Kim, S. S., T. Senthil, Nature Physics **4**, 28 (2008); S.S. M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Doping an insulating antiferromagnet with holes of density p

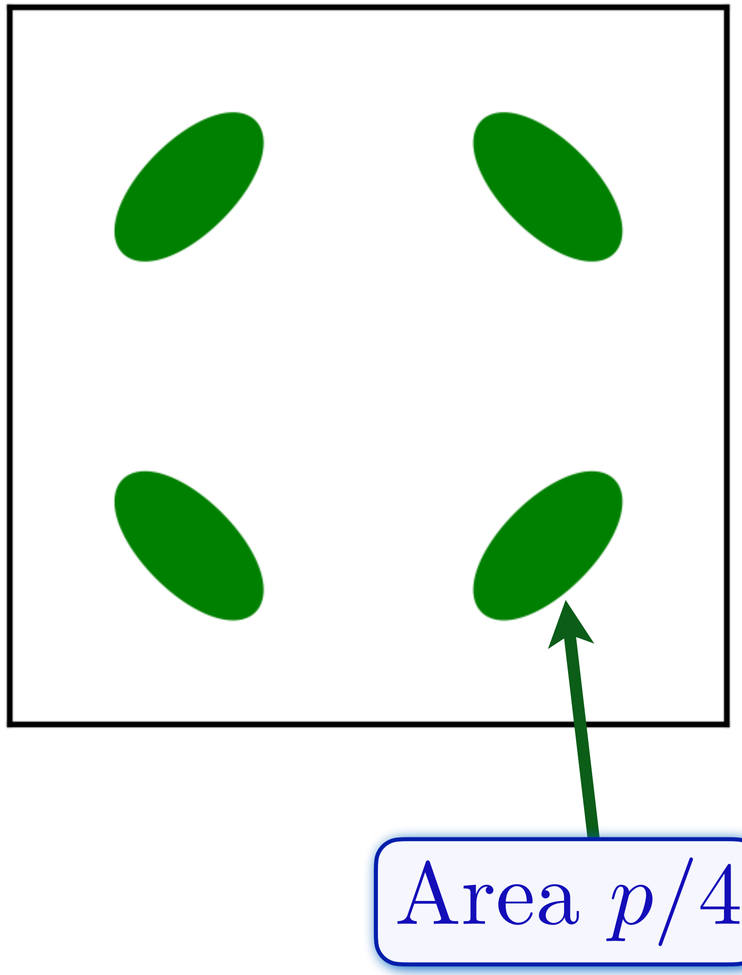
Holon metal

Spin liquid with density p of spinless, charge $+e$ holons. No coherent inter-layer transport, so no Yamaji effect.



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

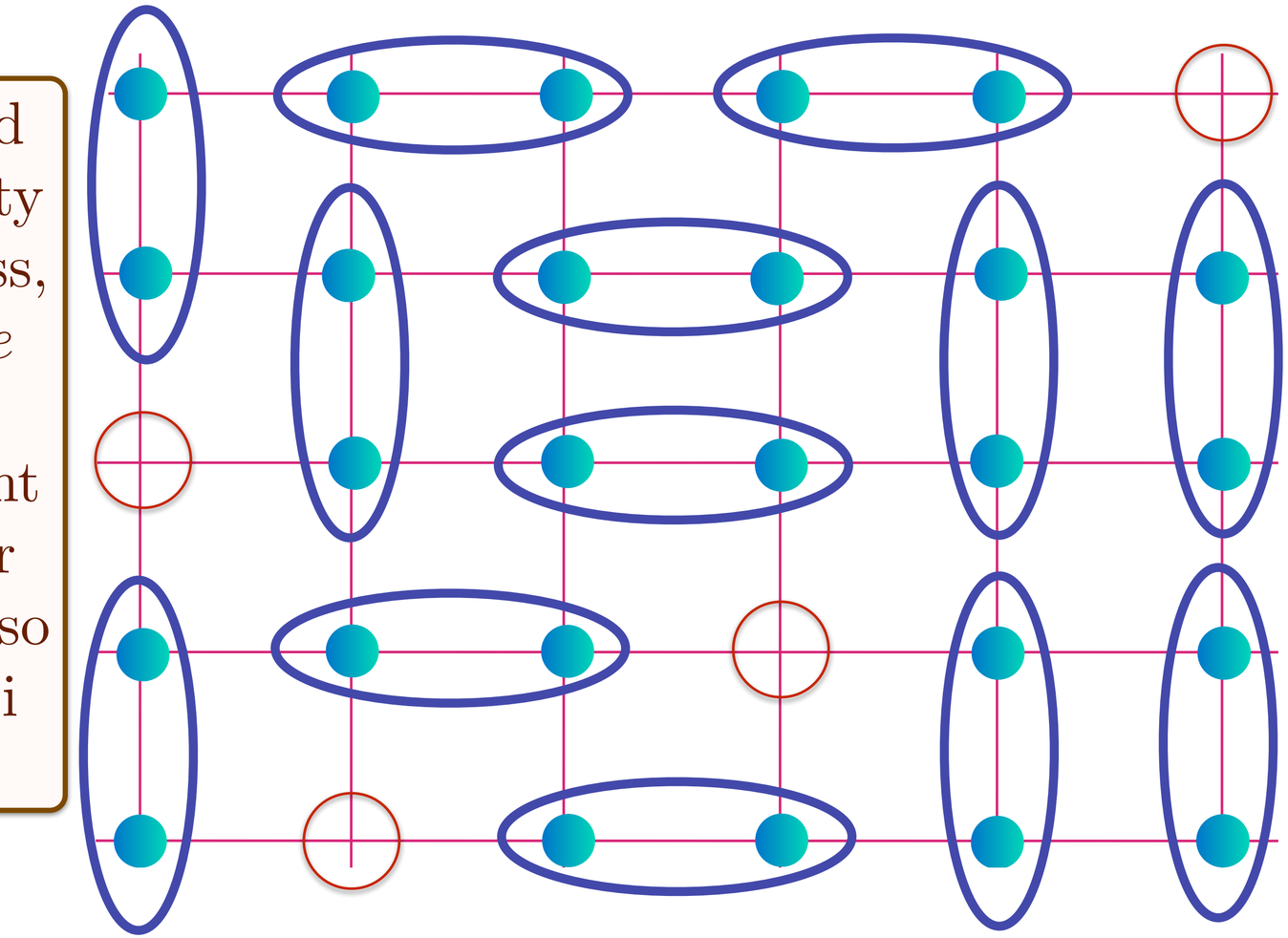


R. K. Kaul, Y. B. Kim, S. S., T. Senthil, Nature Physics **4**, 28 (2008); S.S. M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Doping an insulating antiferromagnet with holes of density p

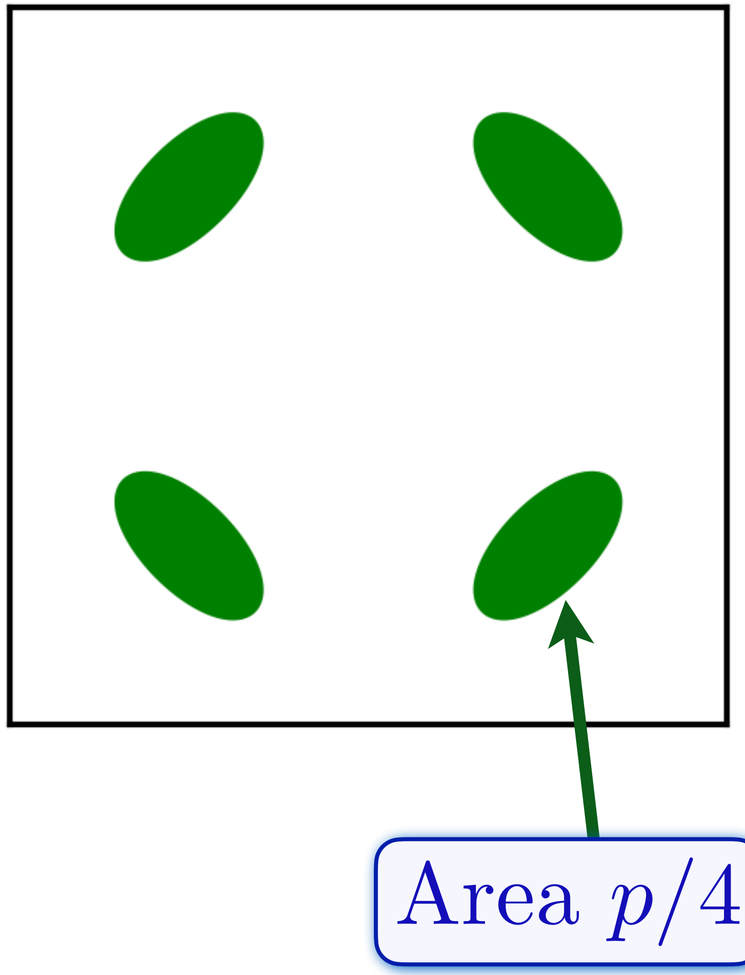
Holon metal

Spin liquid with density p of spinless, charge $+e$ holons. No coherent inter-layer transport, so no Yamaji effect.



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)



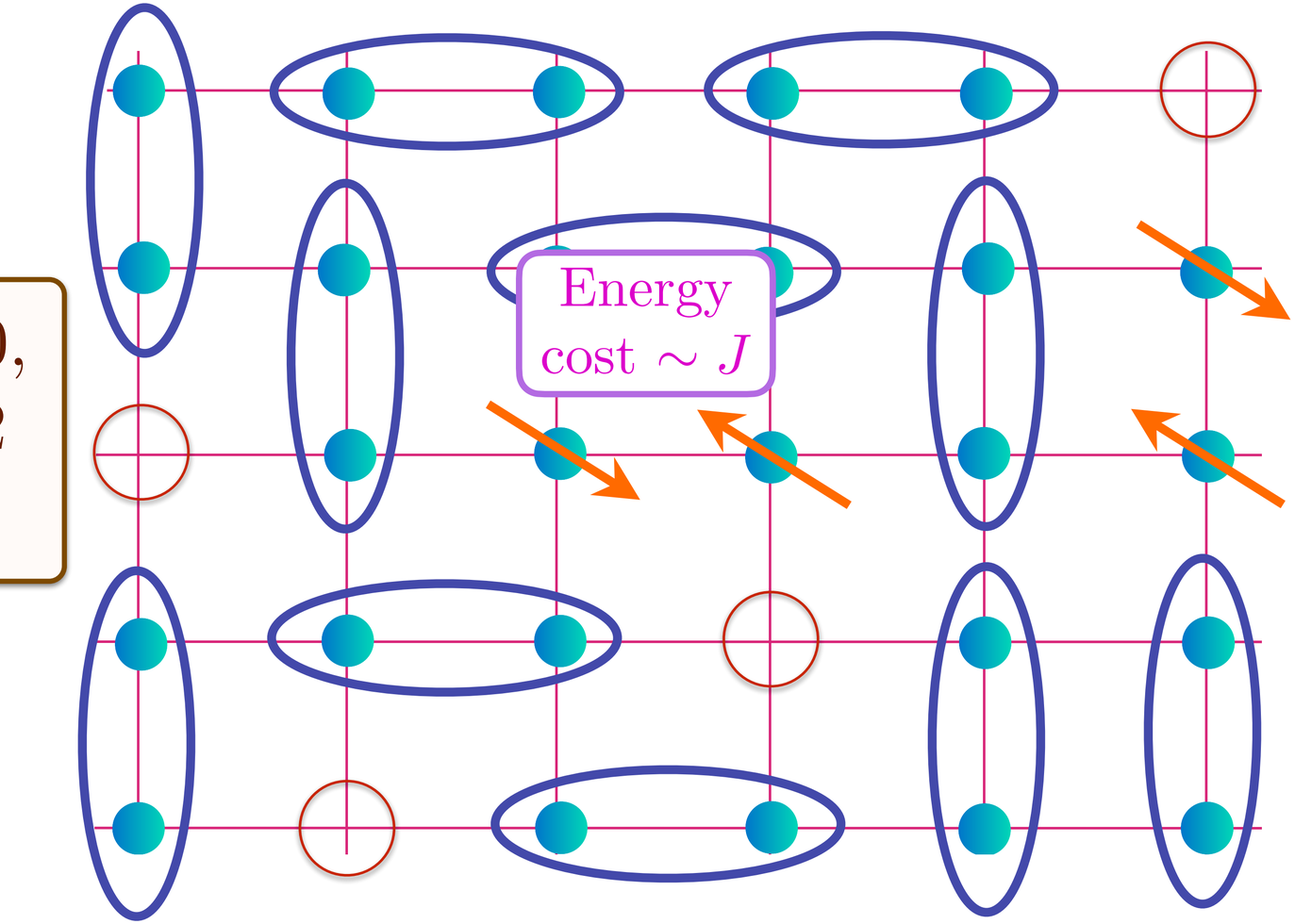
R. K. Kaul, Y. B. Kim, S. S., T. Senthil, Nature Physics **4**, 28 (2008); S.S. M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Doping an insulating antiferromagnet with holes of density p

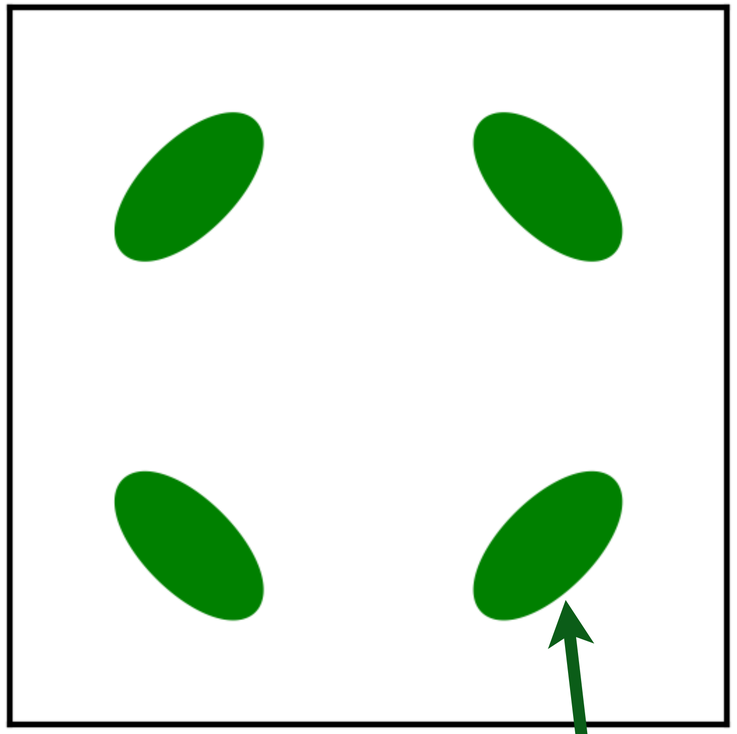
Holon metal excited states

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Charge 0, spin-1/2 spinons



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



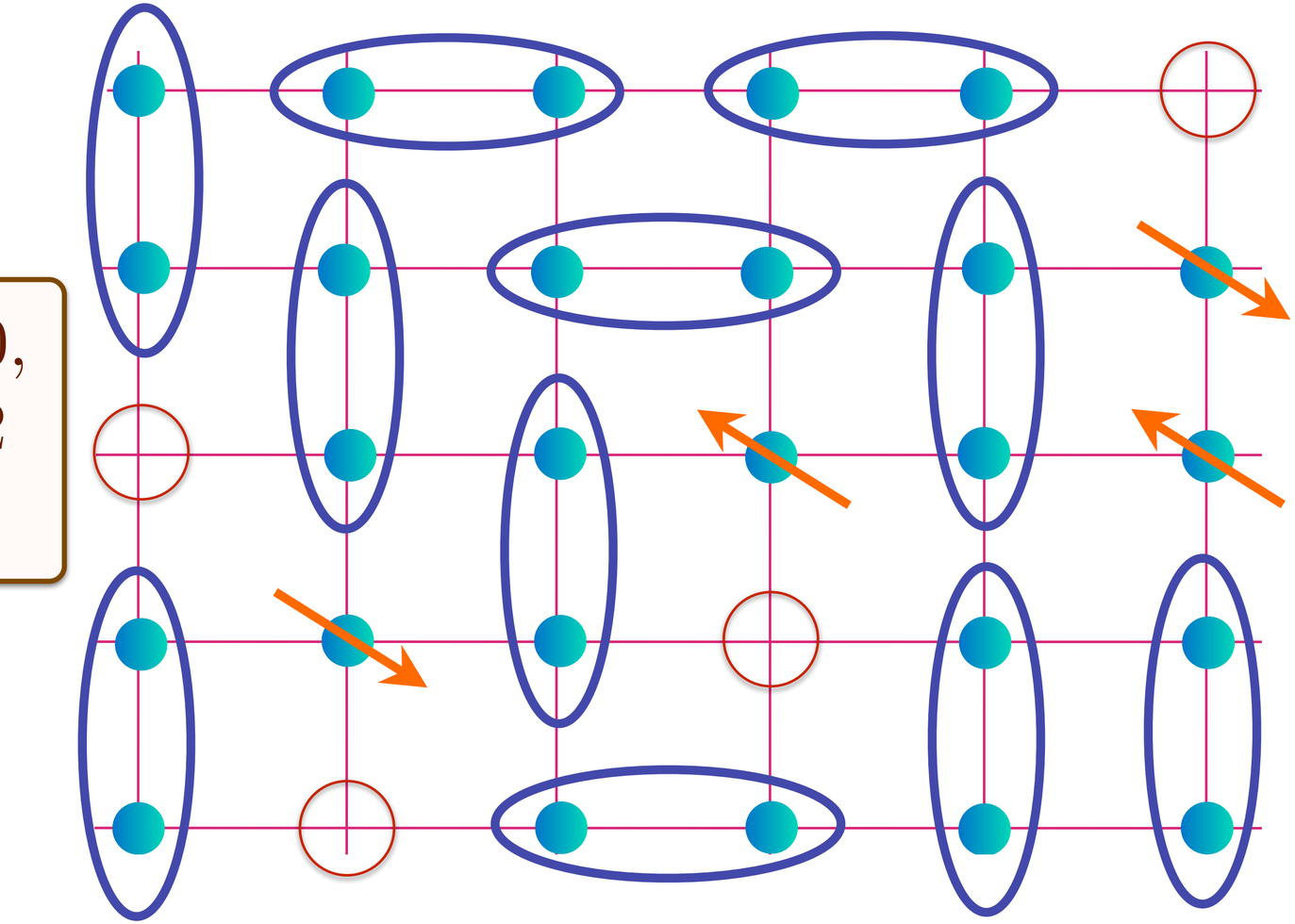
Area $p/4$

Doping an insulating antiferromagnet with holes of density p

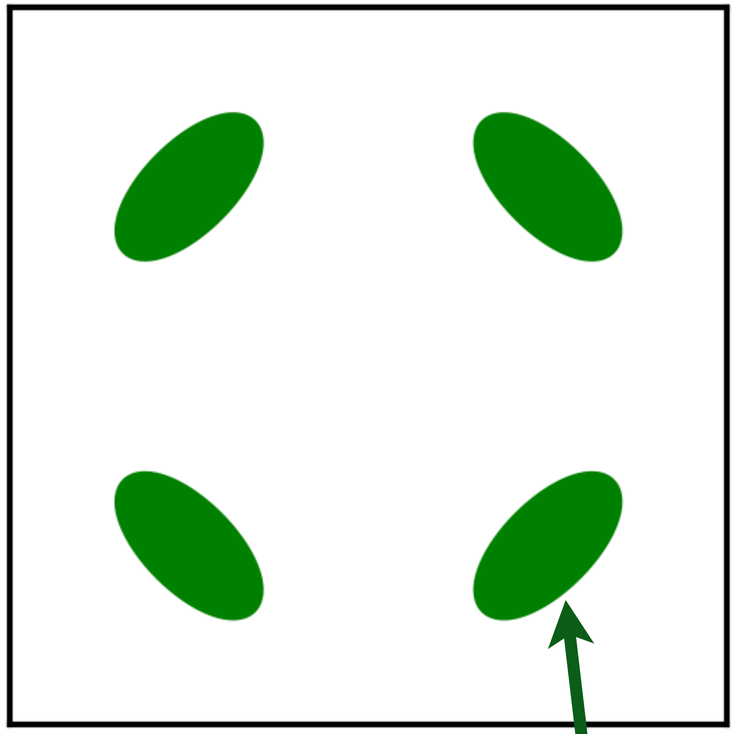
Holon metal excited states

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Charge 0, spin-1/2 spinons



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



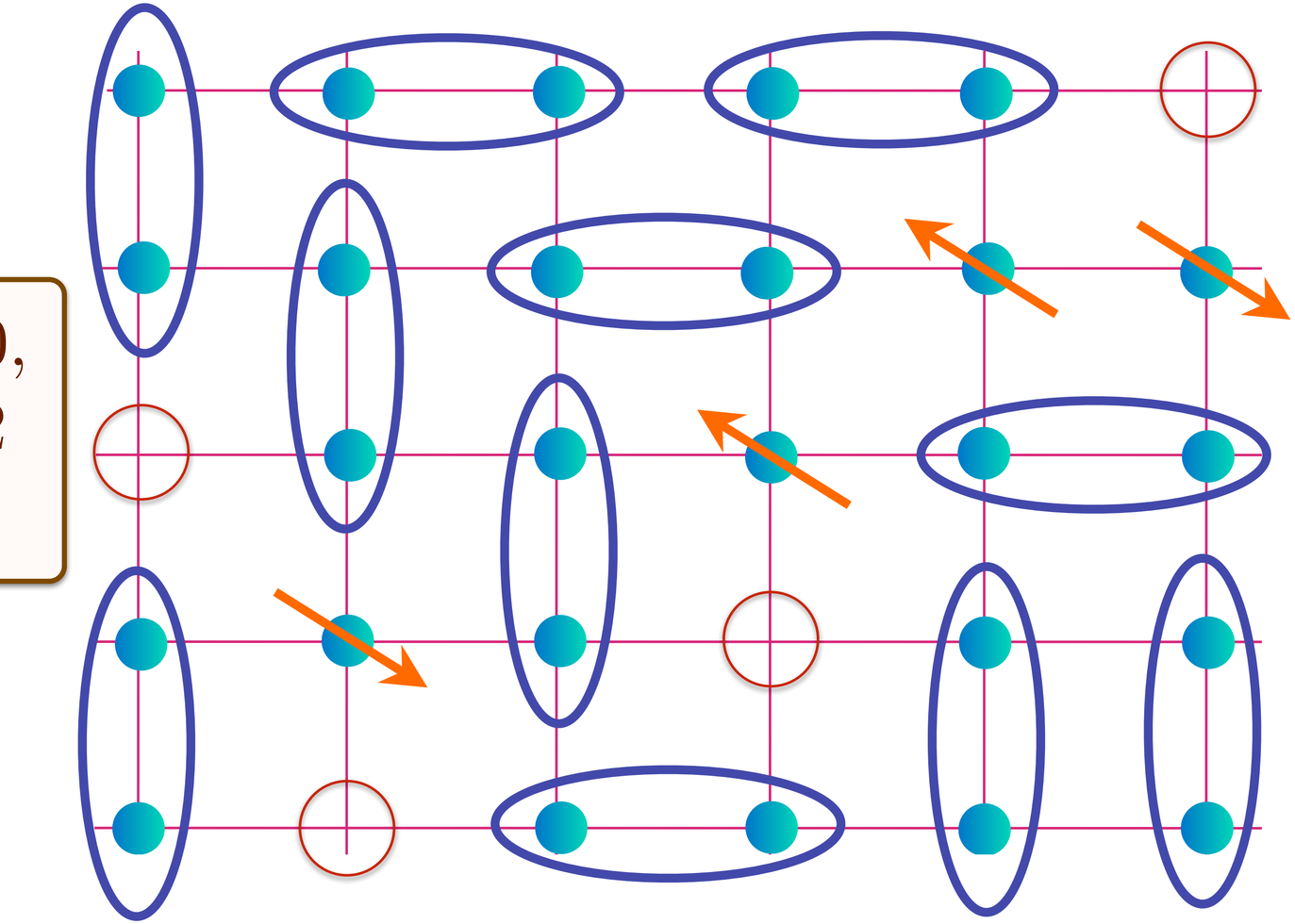
Area $p/4$

Doping an insulating antiferromagnet with holes of density p

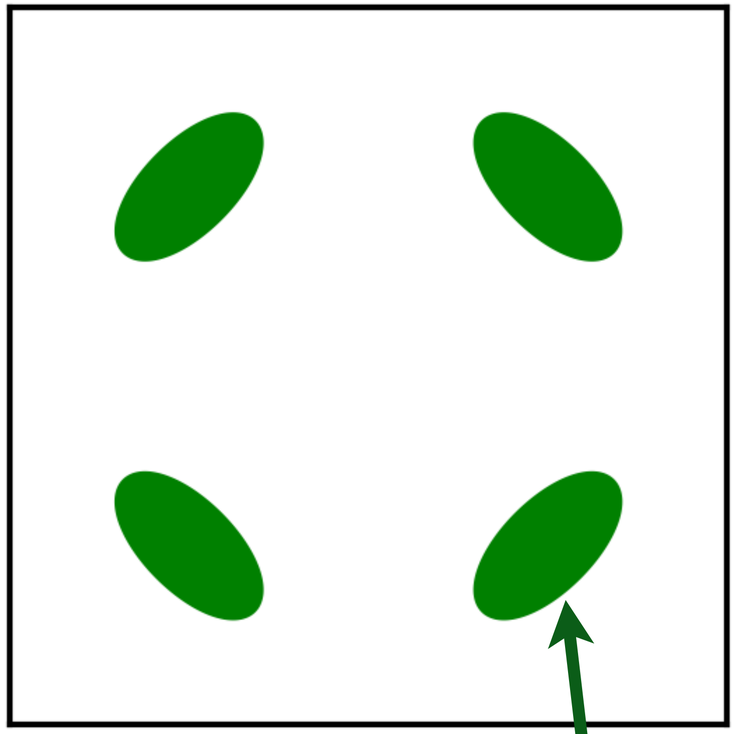
Holon metal excited states

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Charge 0, spin-1/2 spinons



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



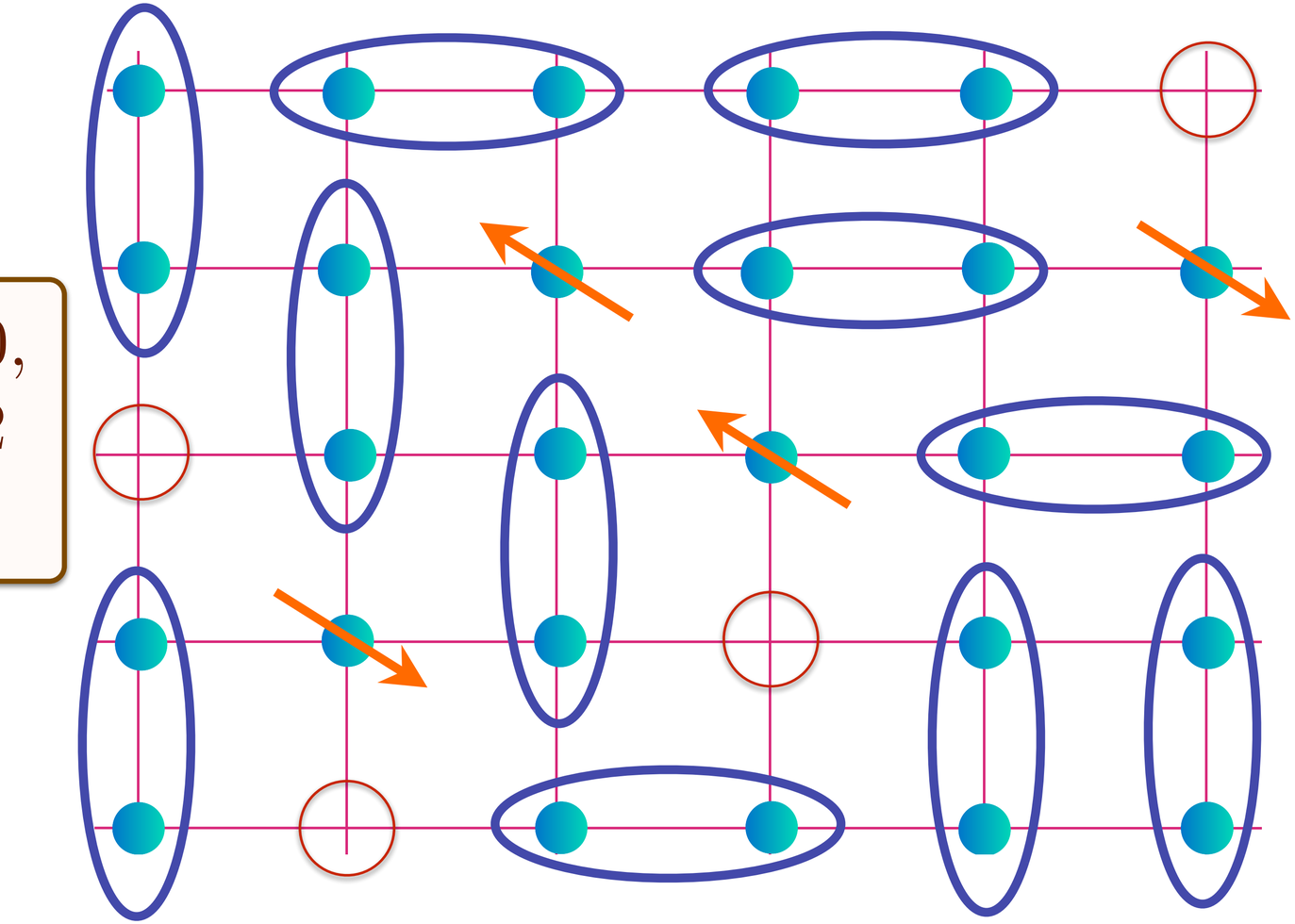
Area $p/4$

Doping an insulating antiferromagnet with holes of density p

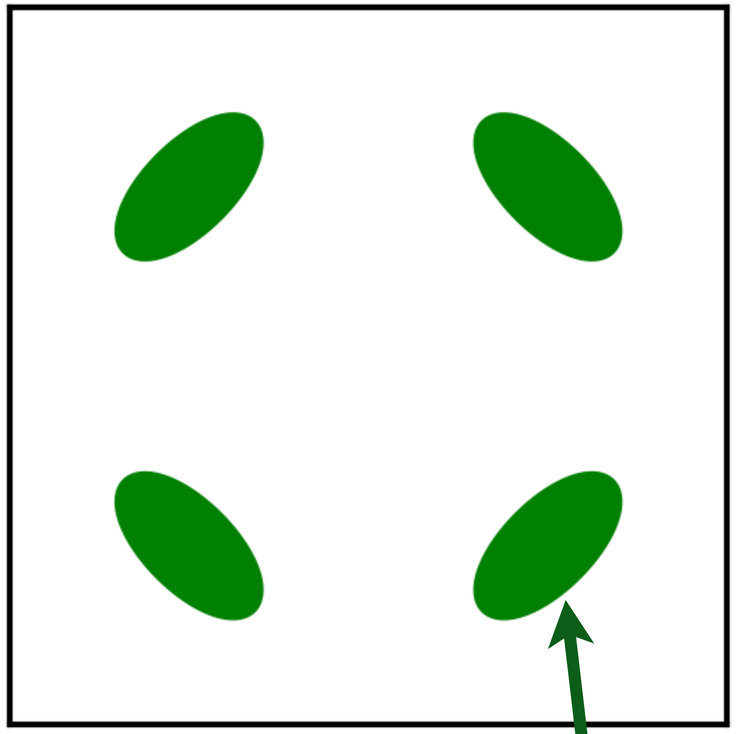
Holon metal excited states

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Charge 0, spin-1/2 spinons



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



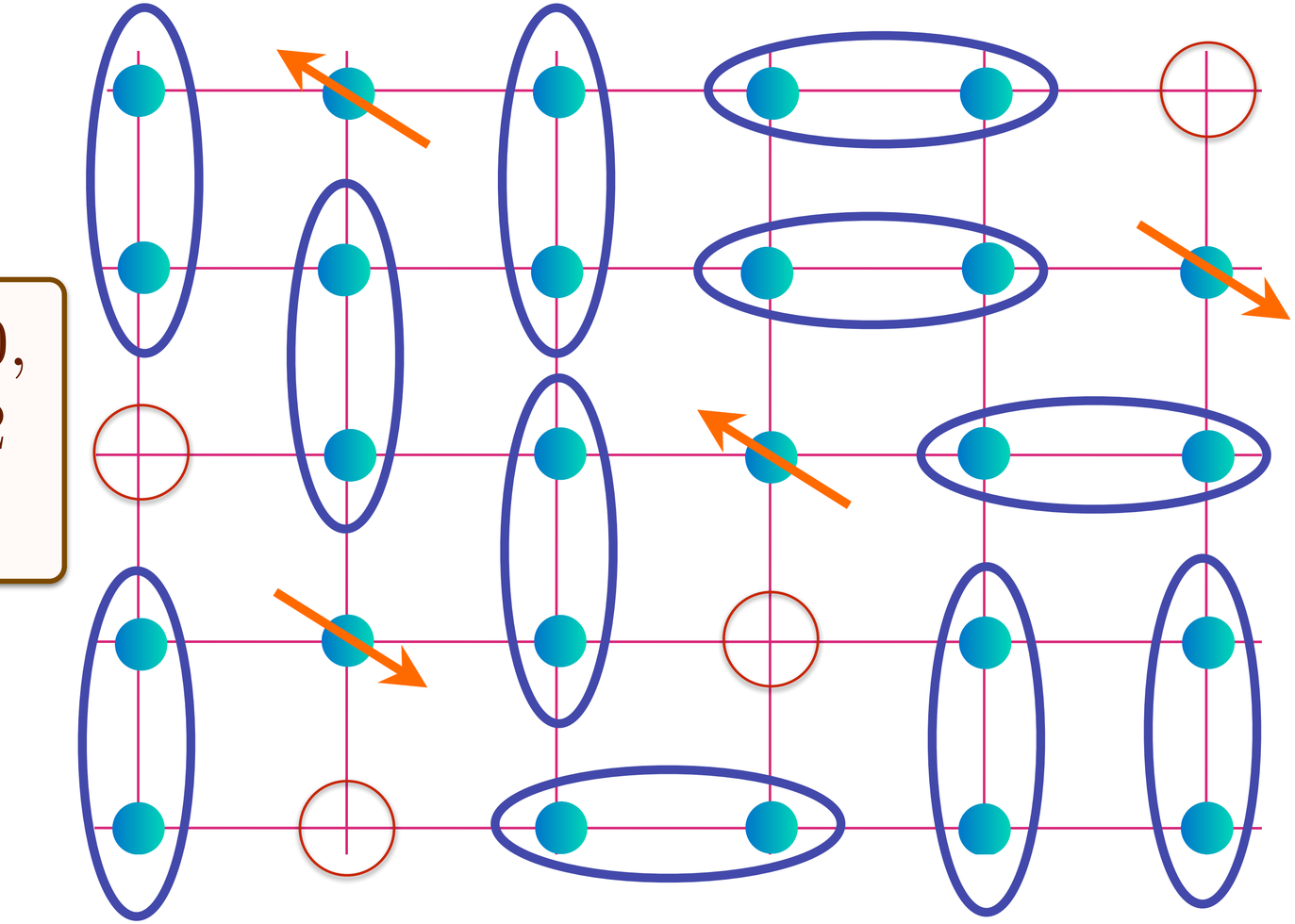
Area $p/4$

Doping an insulating antiferromagnet with holes of density p

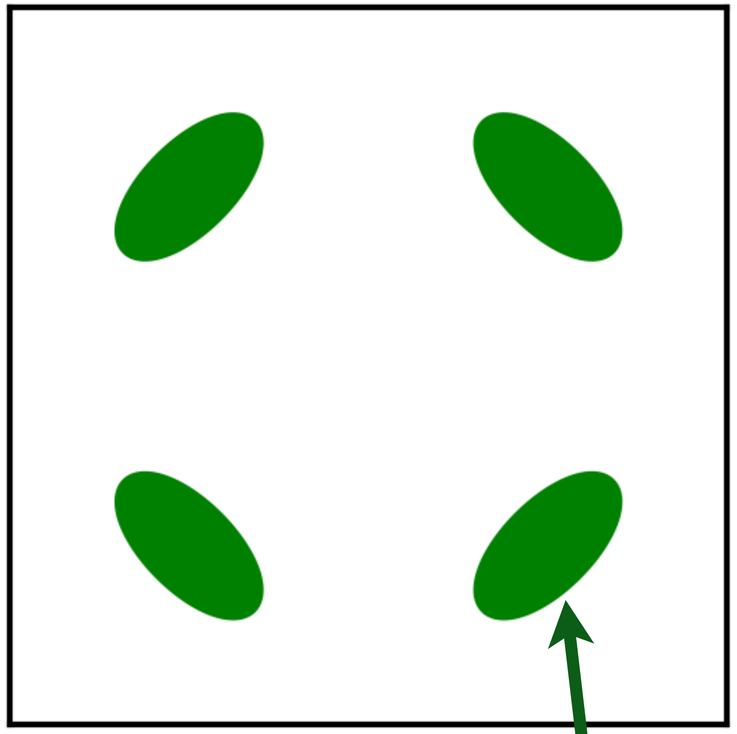
Holon metal excited states

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Charge 0, spin-1/2 spinons



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



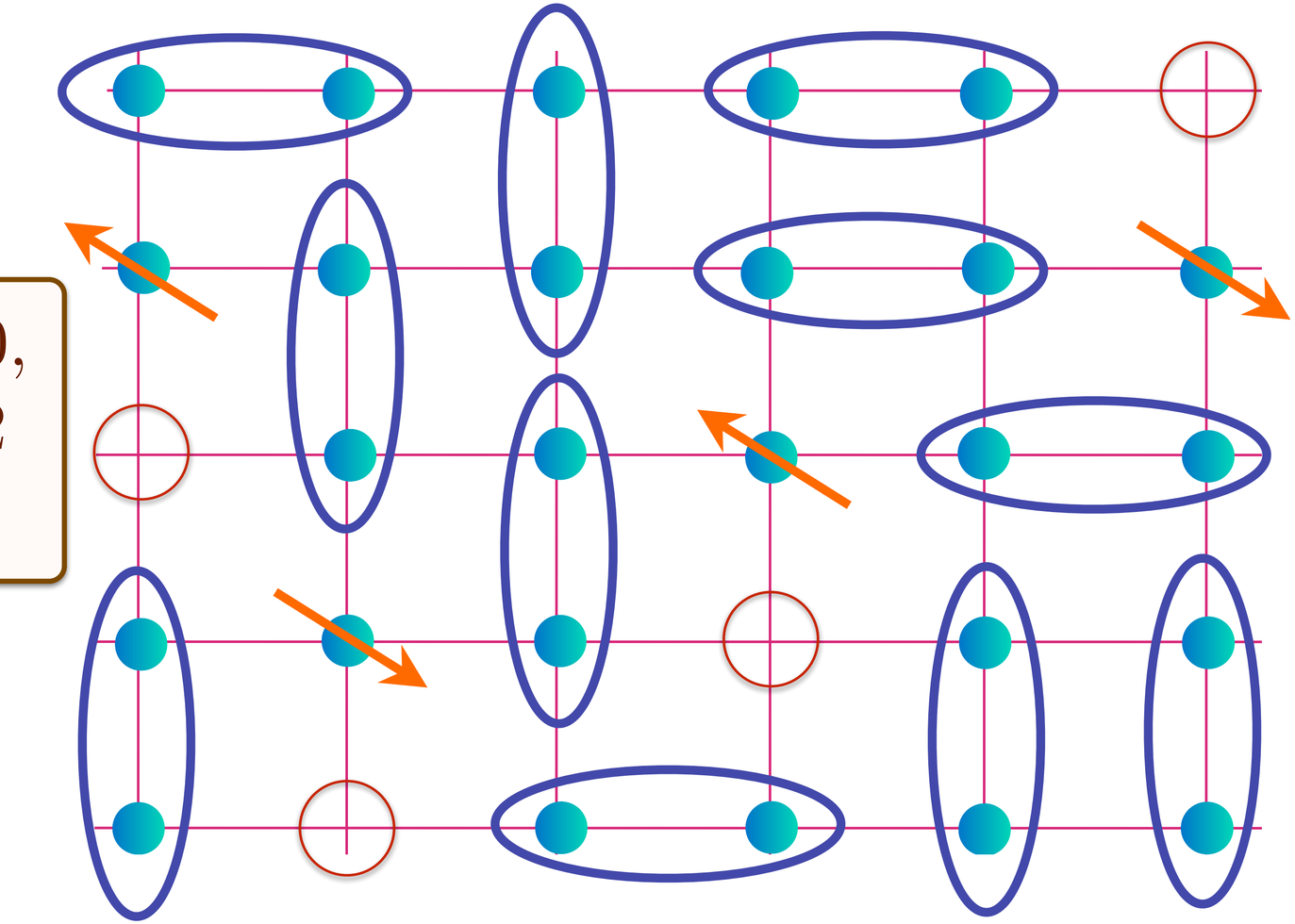
Area $p/4$

Doping an insulating antiferromagnet with holes of density p

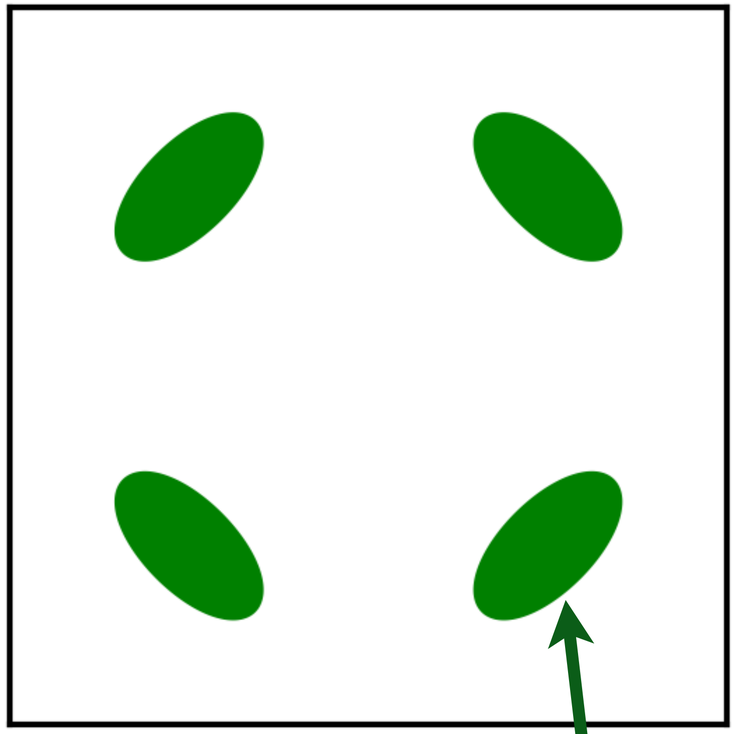
Holon metal excited states

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Charge 0, spin-1/2 spinons



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



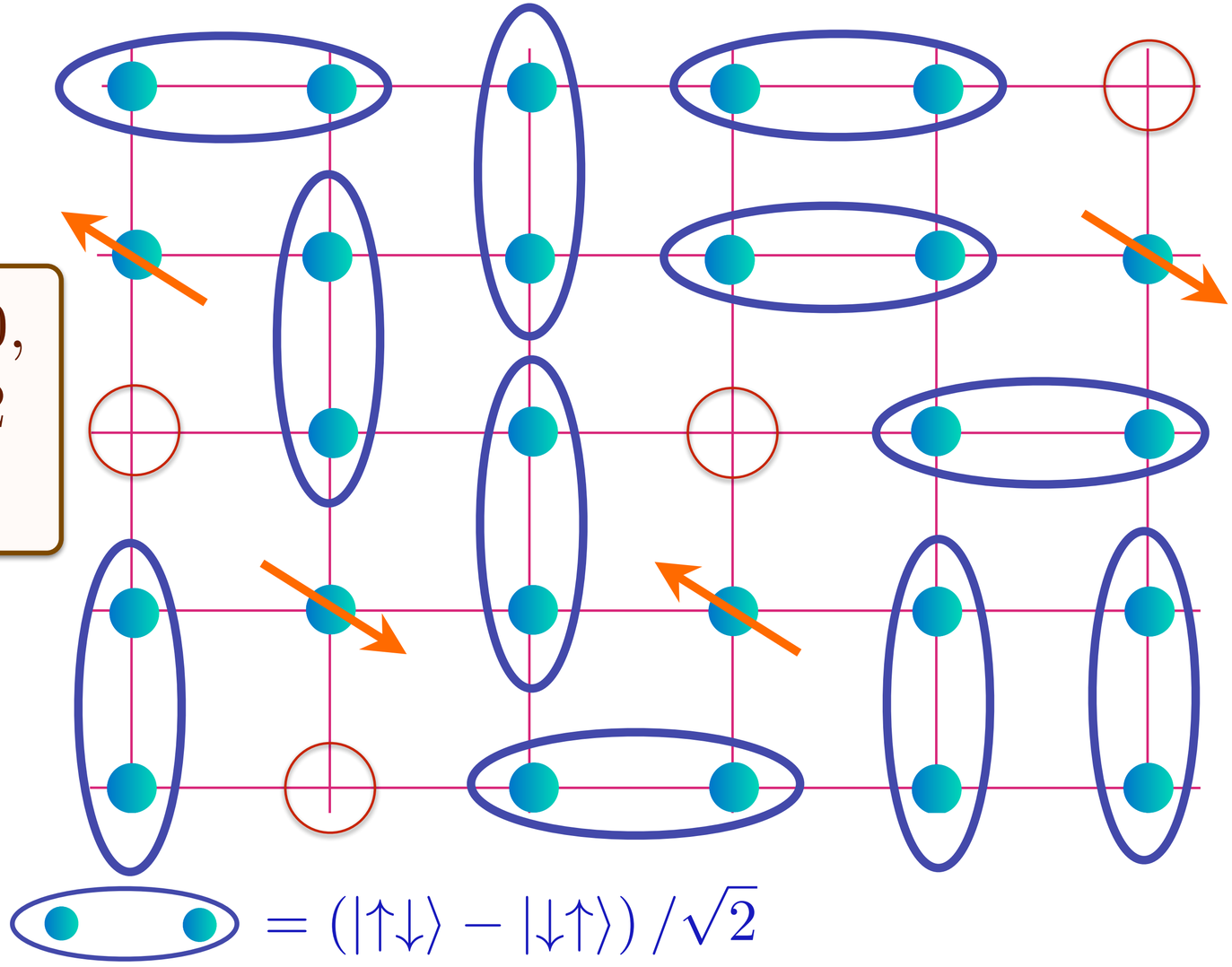
Area $p/4$

Doping an insulating antiferromagnet with holes of density p

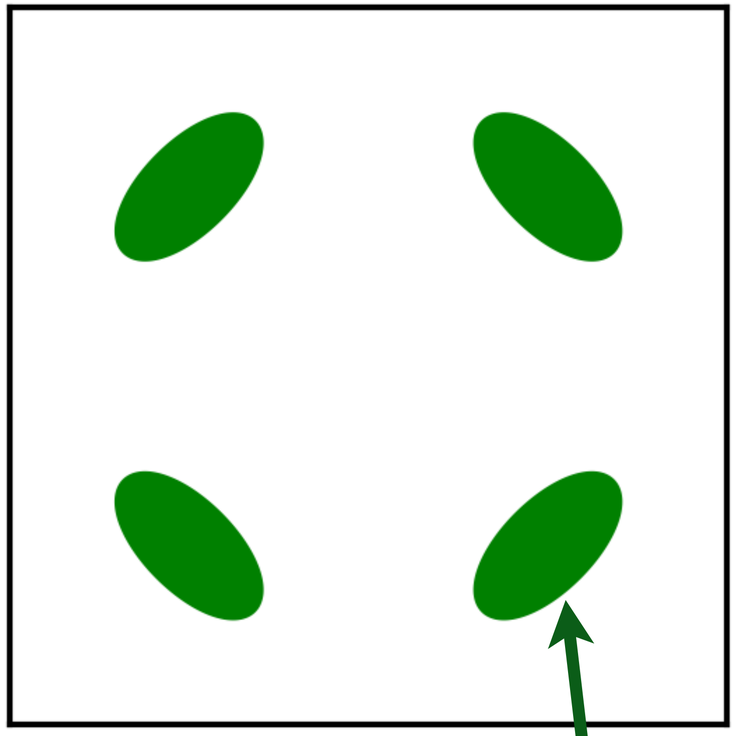
Holon metal excited states

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Charge 0, spin-1/2 spinons



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



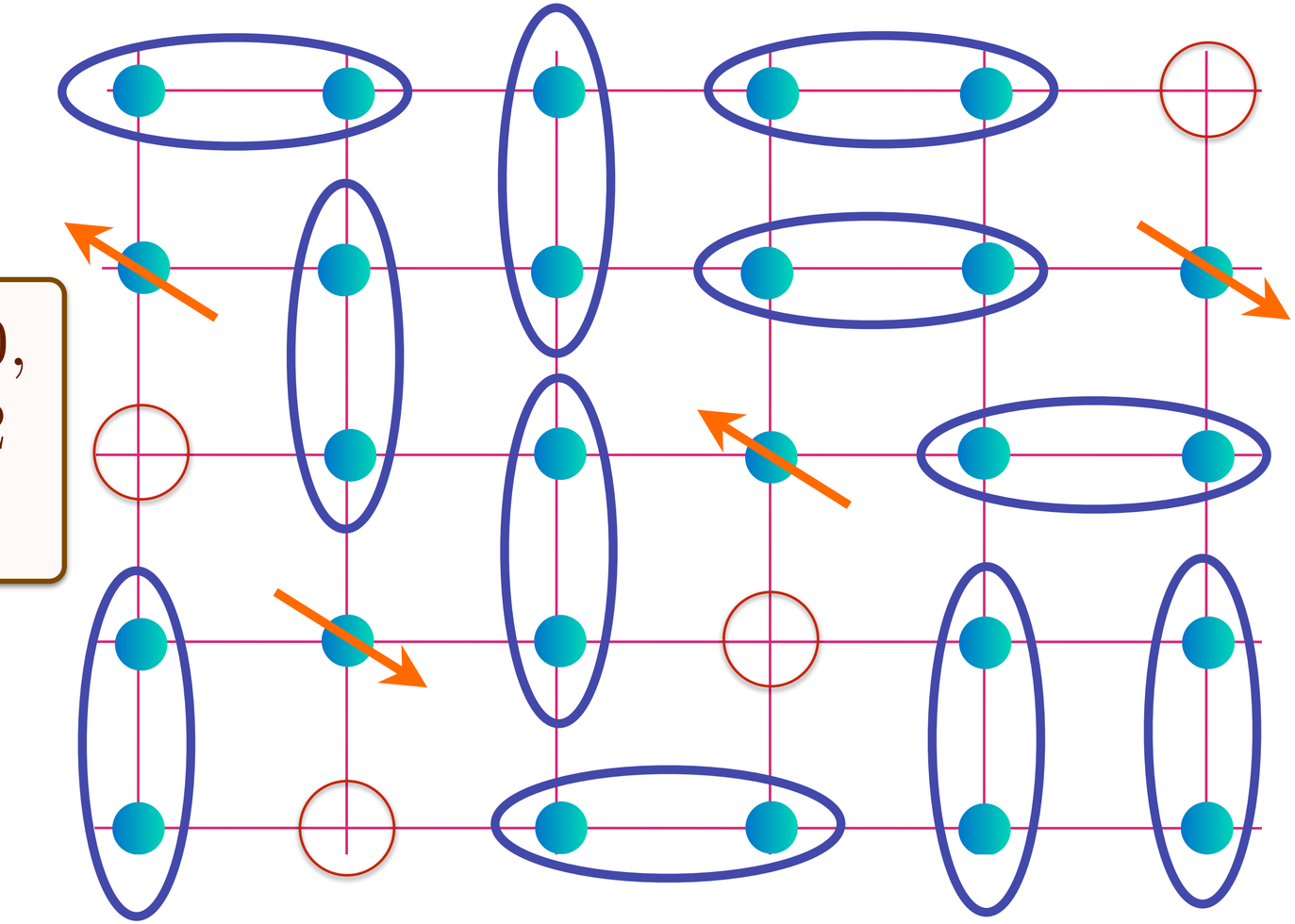
Area $p/4$

Doping an insulating antiferromagnet with holes of density p

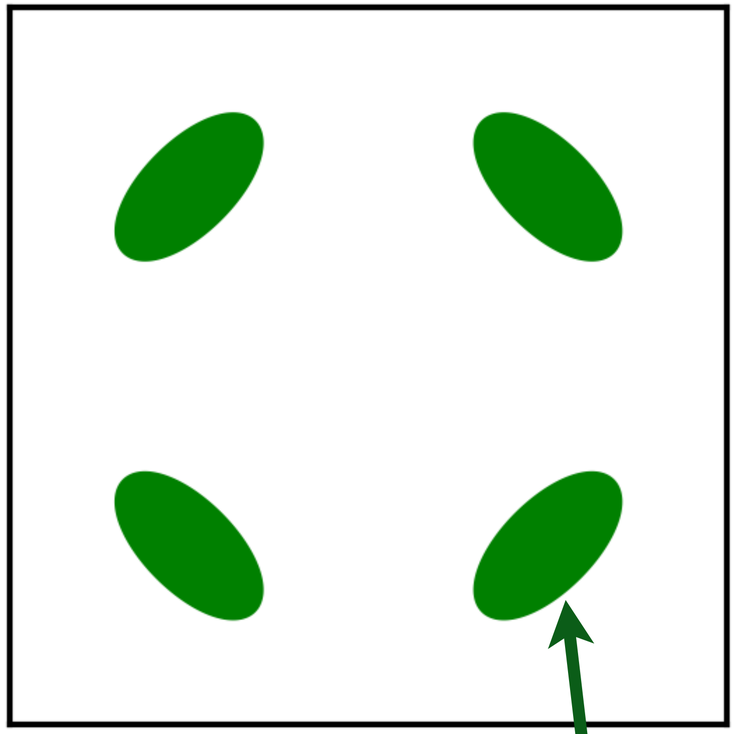
Holon metal excited states

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Charge 0, spin-1/2 spinons



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



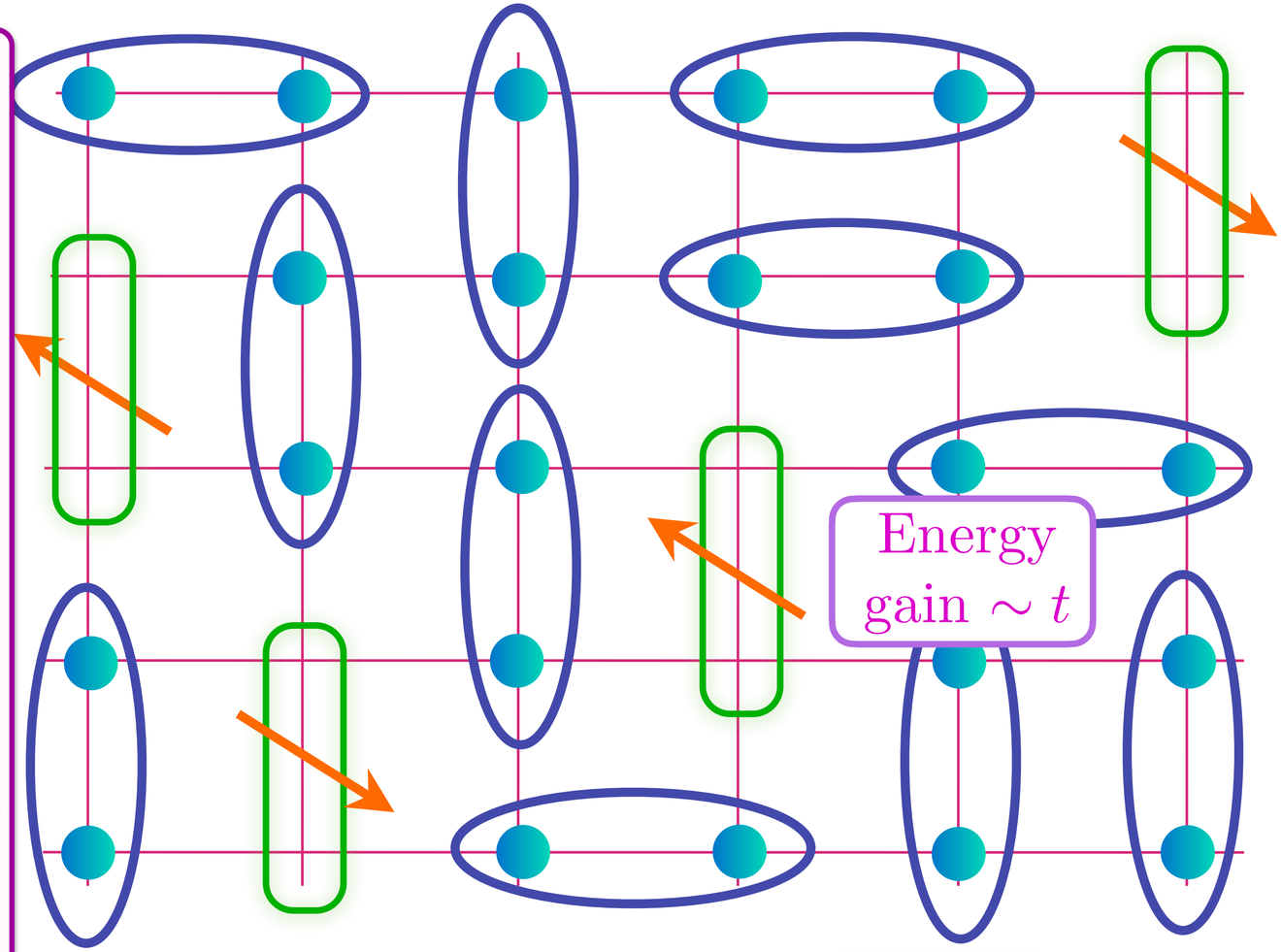
Area $p/4$

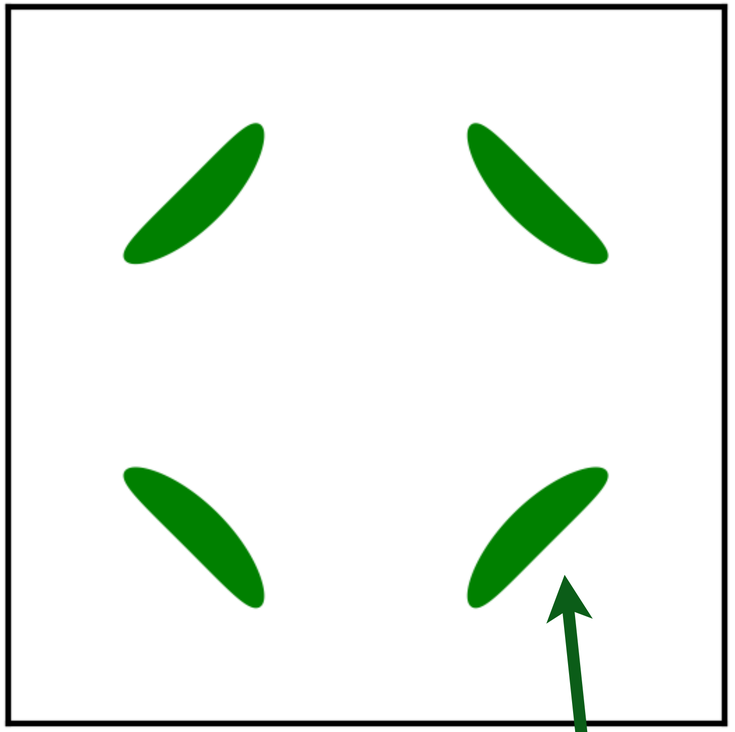
Doping an insulating antiferromagnet with holes of density p

FL*

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport for Yamaji effect.





Area $p/8$

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ $= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

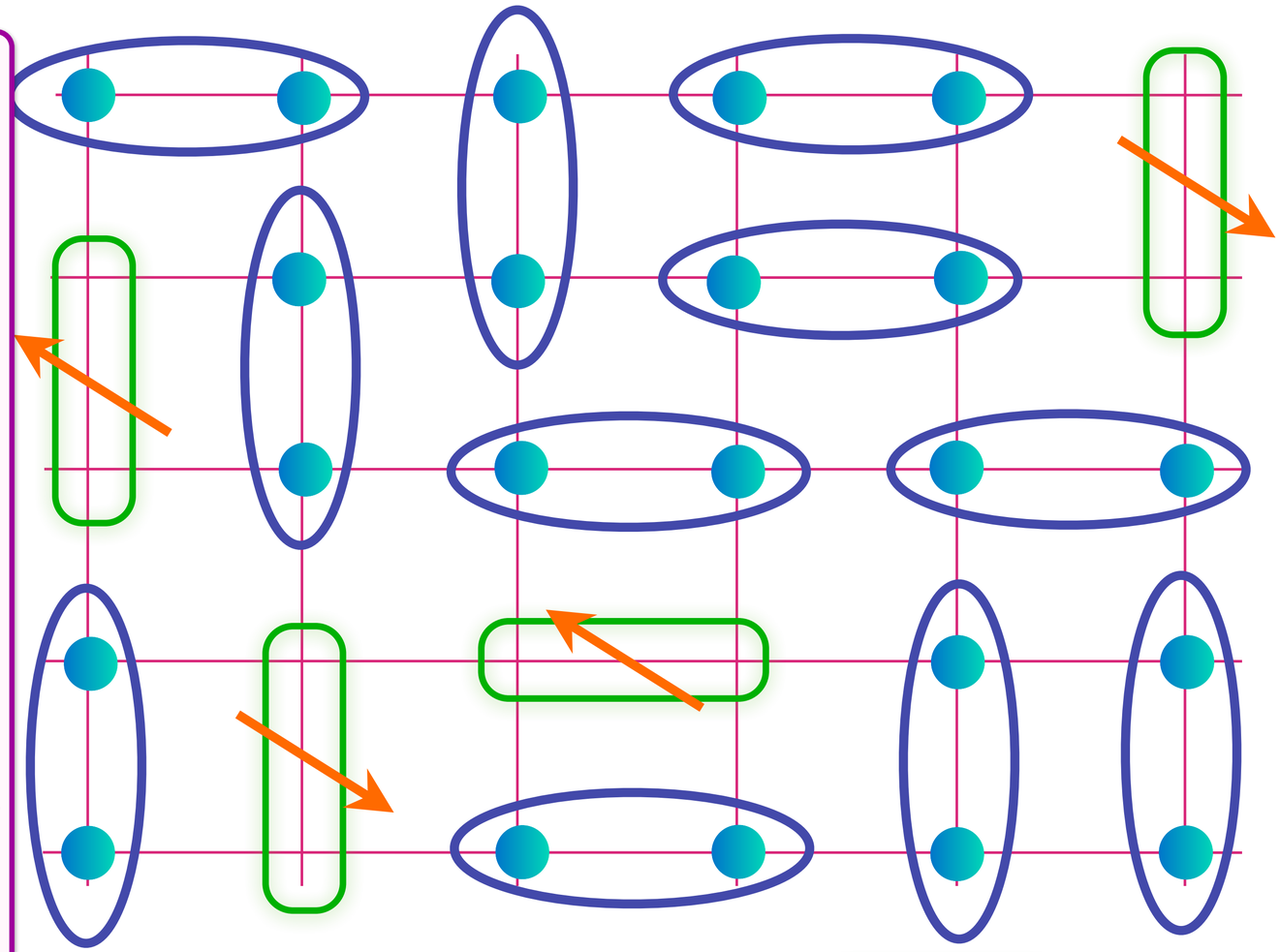
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

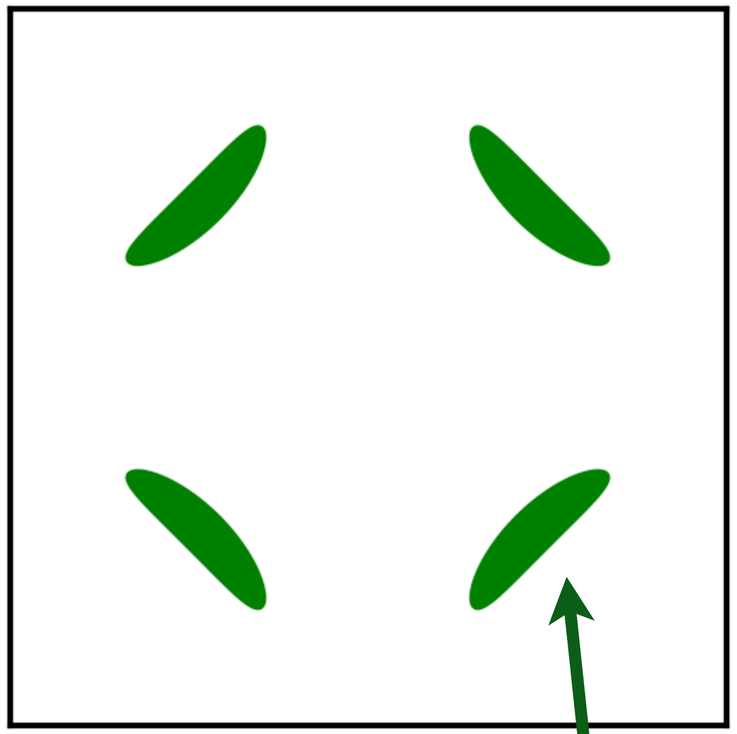
Doping an insulating antiferromagnet with holes of density p

FL*

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport for Yamaji effect.





Area $p/8$

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ $= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

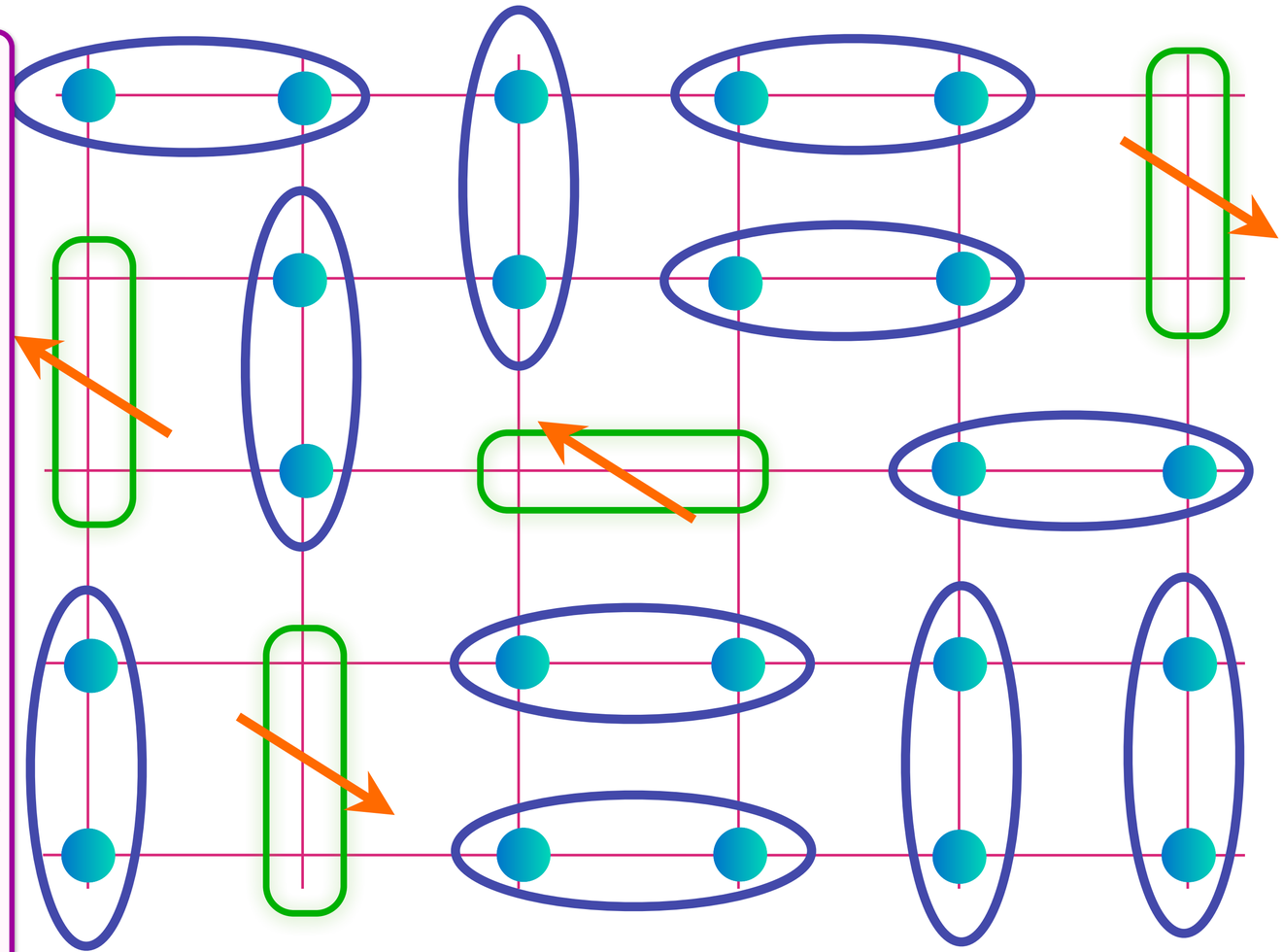
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

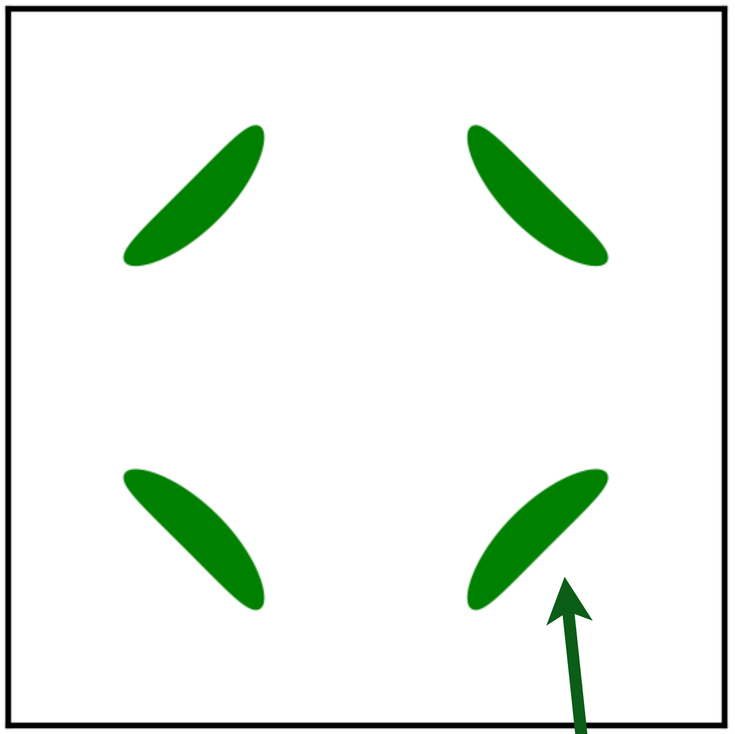
Doping an insulating antiferromagnet with holes of density p

FL*

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport for Yamaji effect.





Area $p/8$

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ $= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

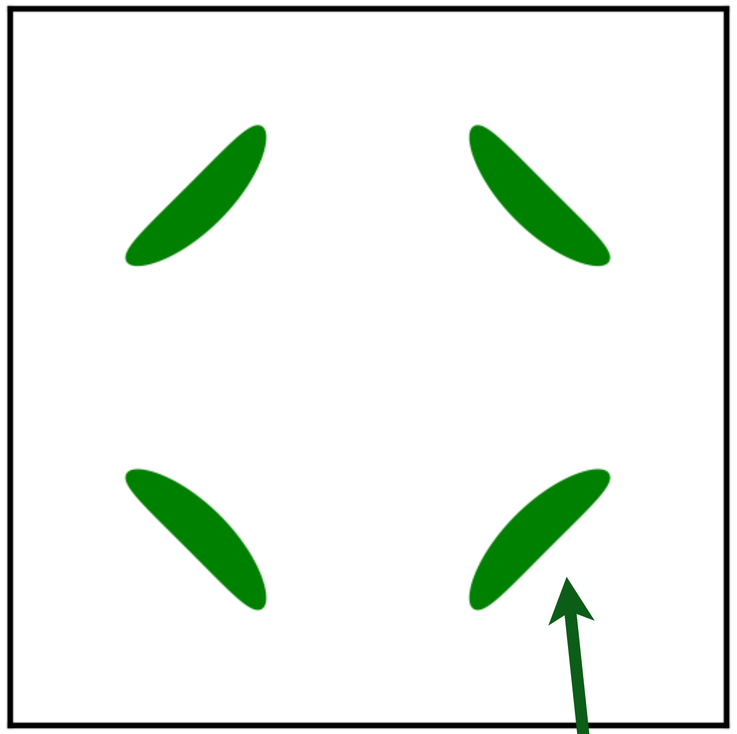
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Doping an insulating antiferromagnet with holes of density p

FL*

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport for Yamaji effect.

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)



Area $p/8$

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ $= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

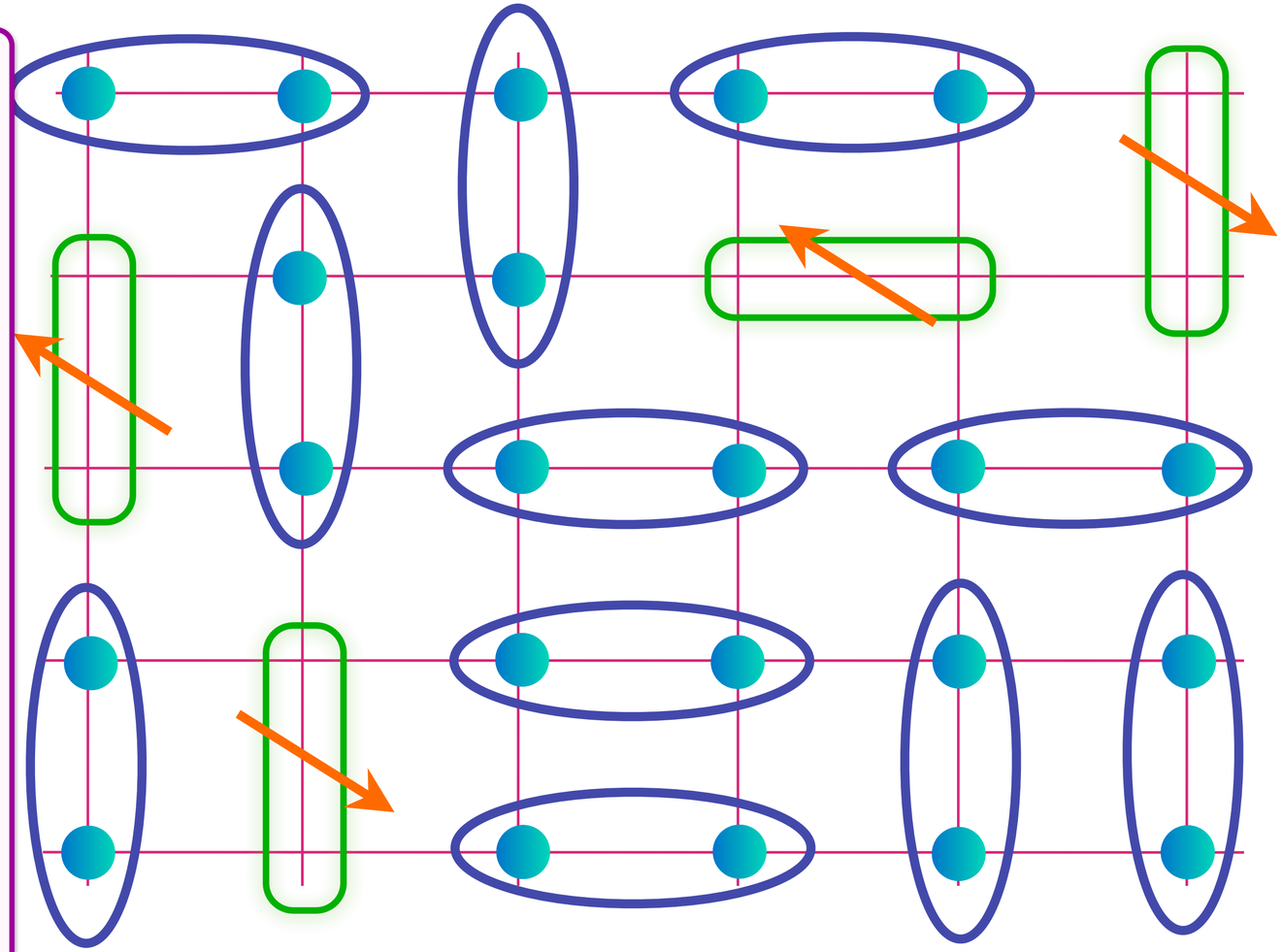
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

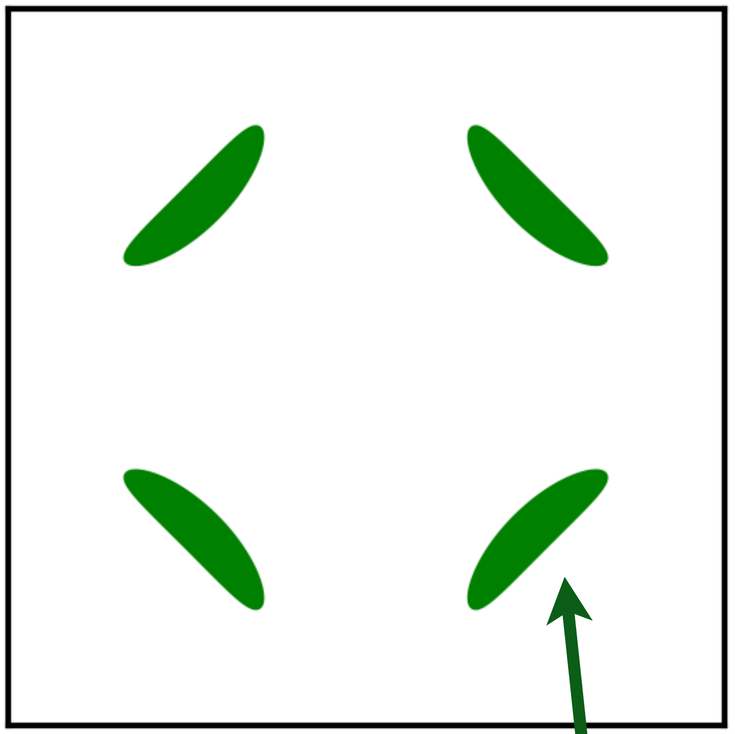
Doping an insulating antiferromagnet with holes of density p

FL*

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport for Yamaji effect.

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)





Area $p/8$

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ $= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

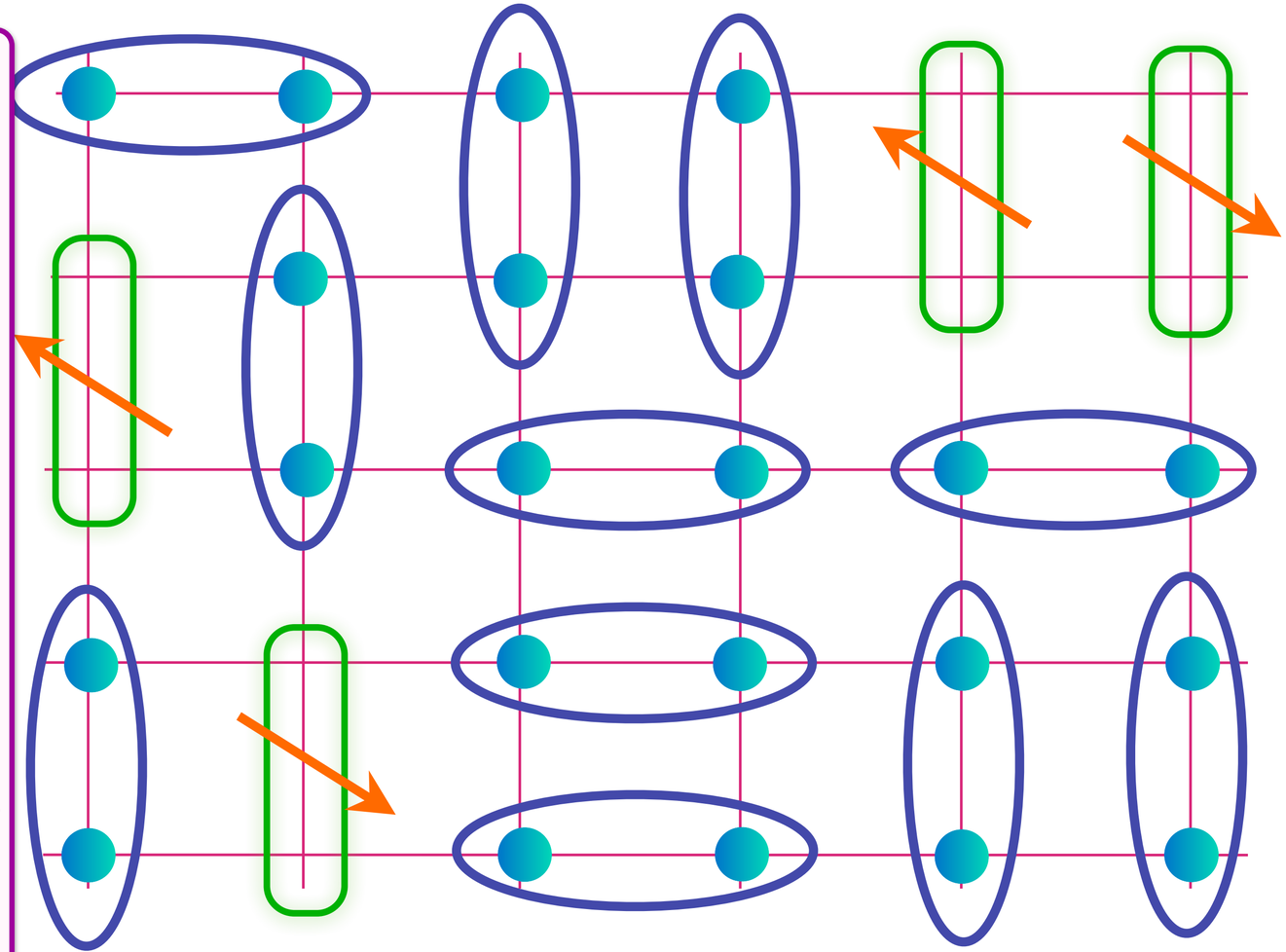
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

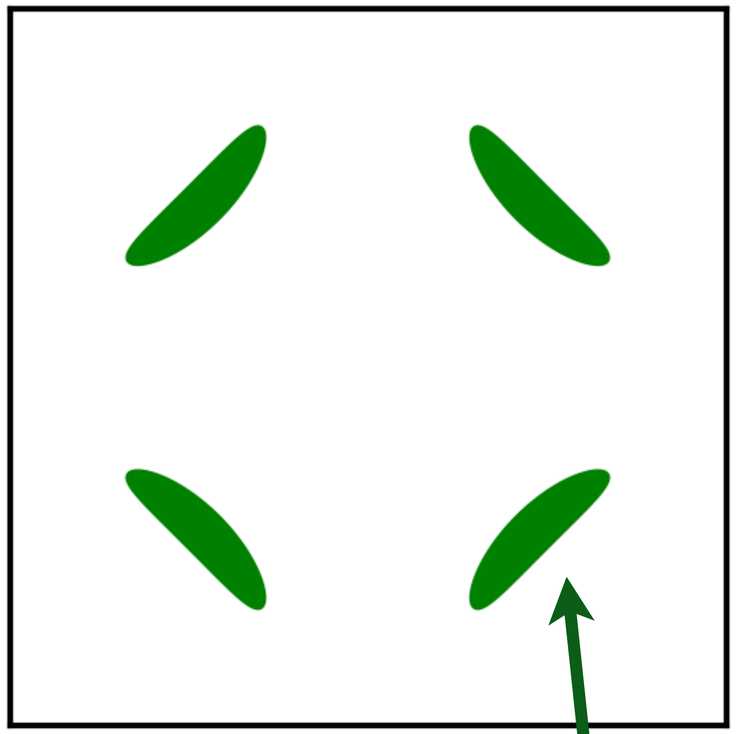
Doping an insulating antiferromagnet with holes of density p

FL*

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport for Yamaji effect.

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)





Area $p/8$

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

$= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

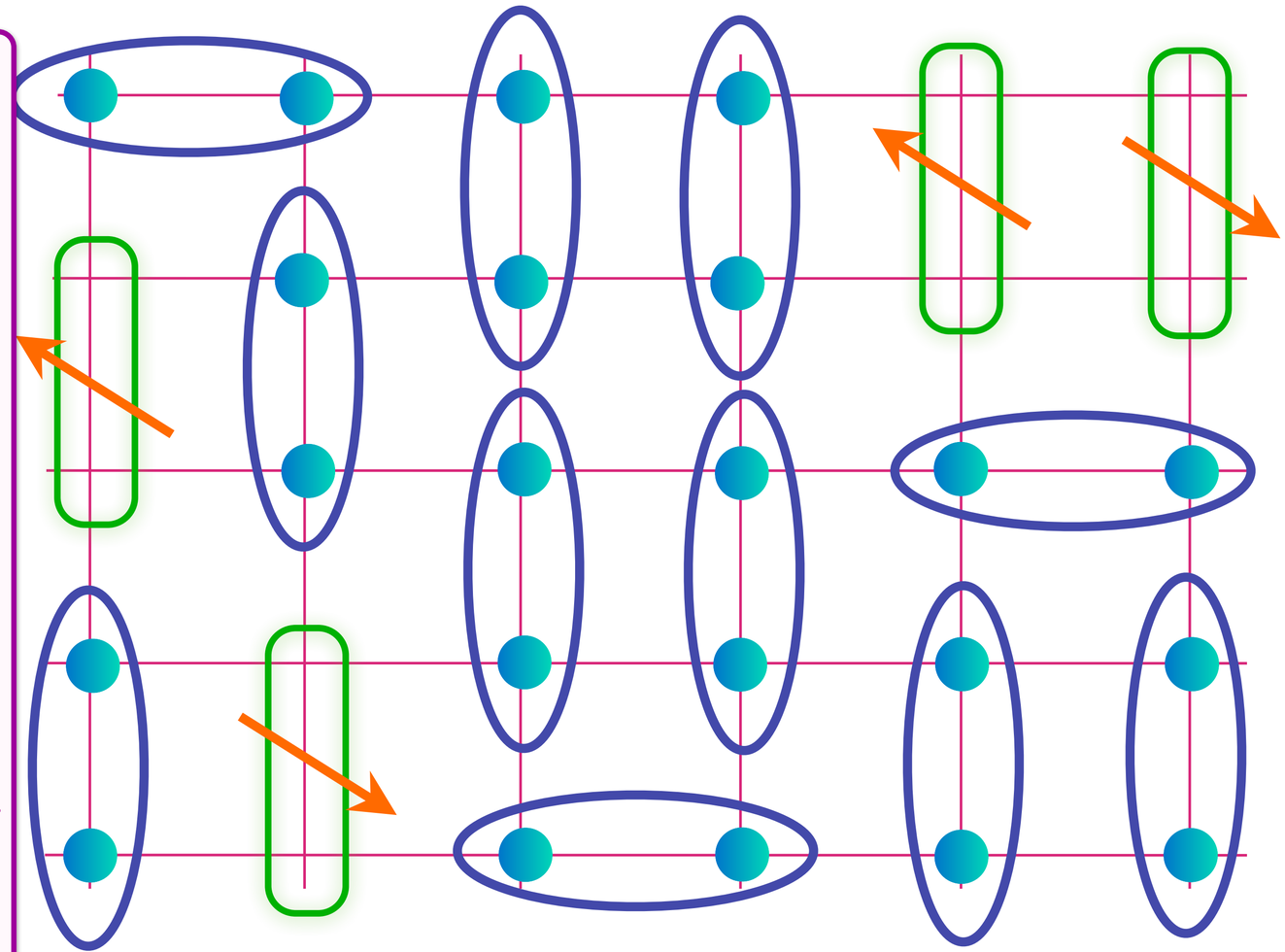
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

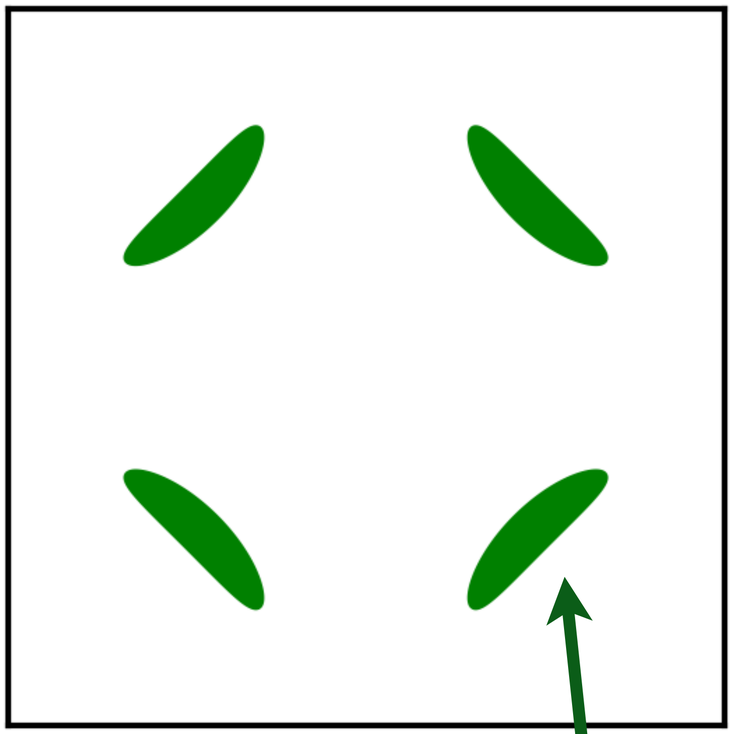
Doping an insulating antiferromagnet with holes of density p

FL*

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport for Yamaji effect.





Area $p/8$

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ $= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

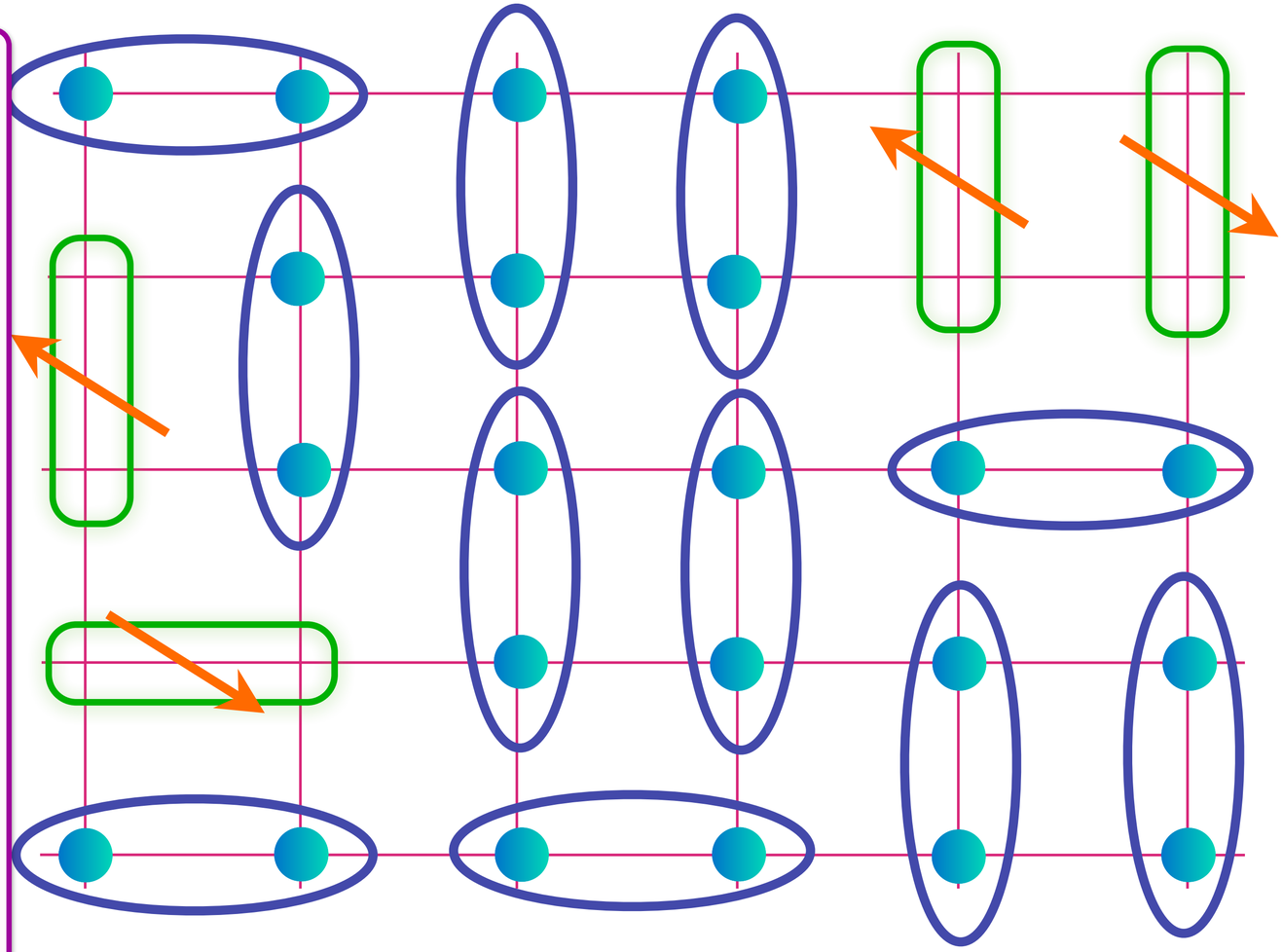
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

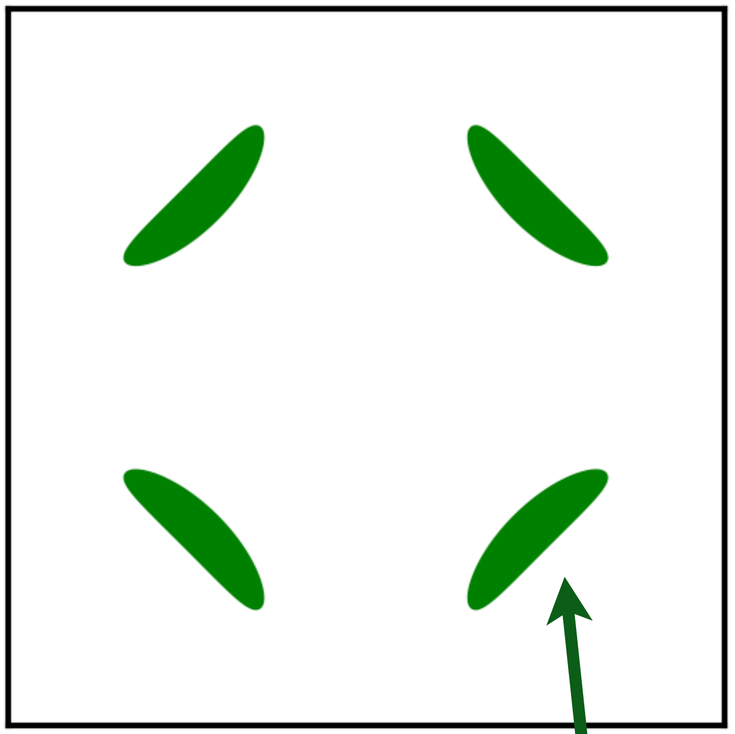
Doping an insulating antiferromagnet with holes of density p

FL*

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport for Yamaji effect.

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)





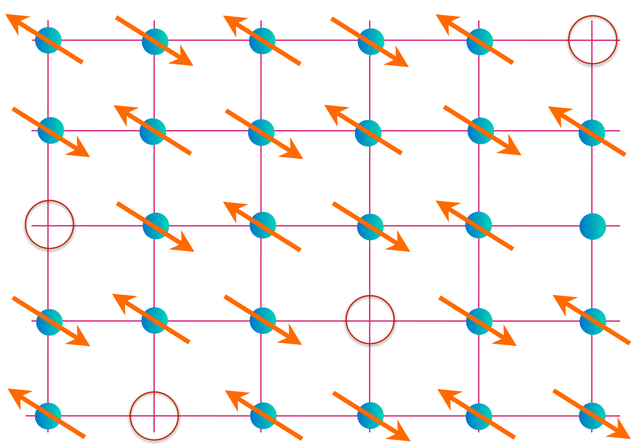
Area $p/8$

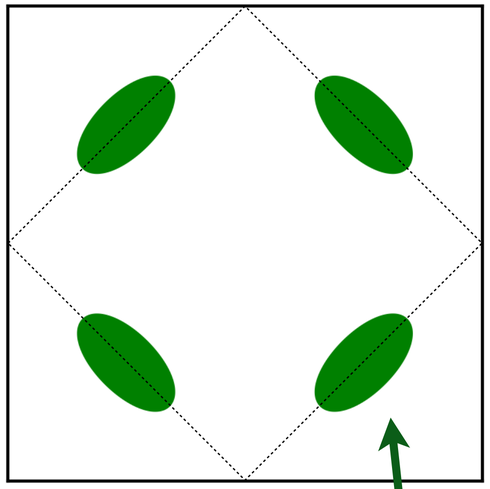
$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

$= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

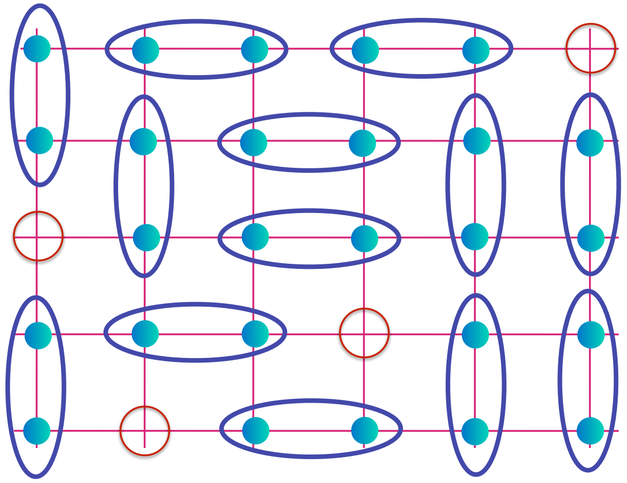
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Doping an insulating antiferromagnet with holes of density p

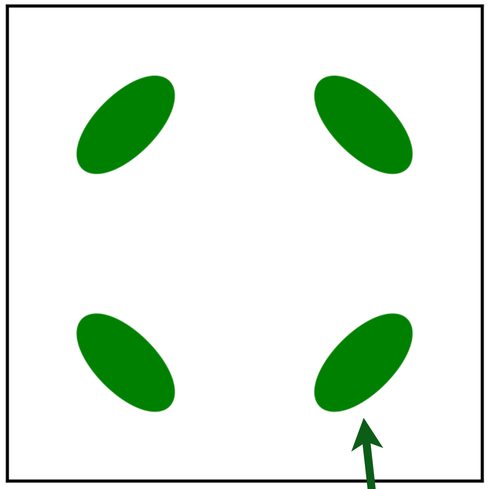




AF metal and SDW fluctuation

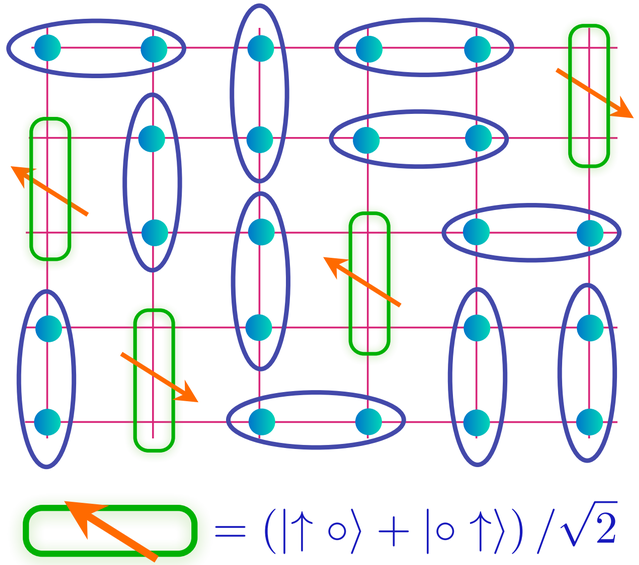


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



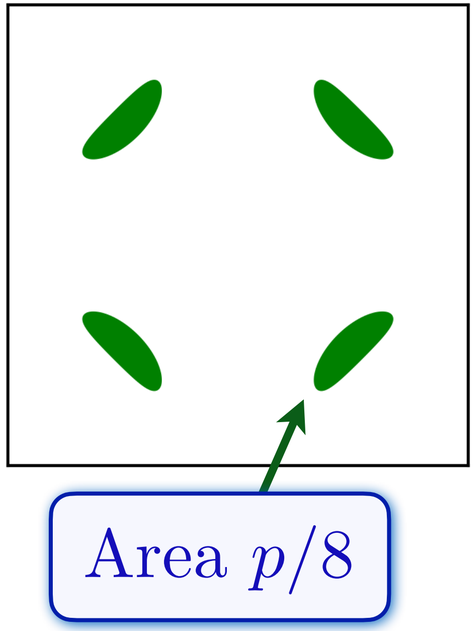
Holon metal

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003);
R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, S. S., PRB **105**, 075146 (2022)



$= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

FL*

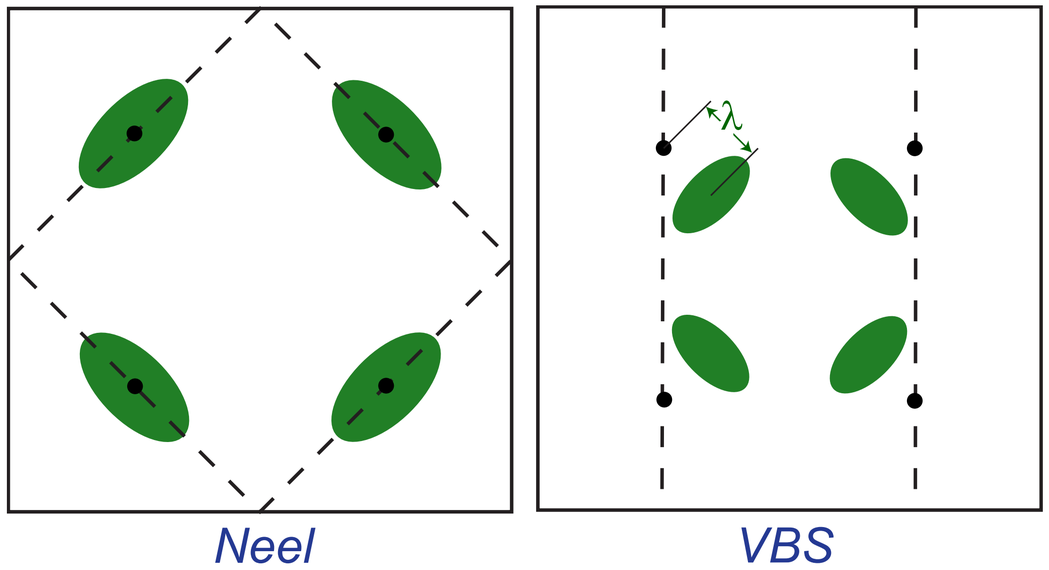


Quantization of spin liquid anomaly implies Fermi surface areas are also *quantized* and robust to all corrections.

PHYSICAL REVIEW B **75**, 235122 (2007)

Hole dynamics in an antiferromagnet across a deconfined quantum critical point

Ribhu K. Kaul,[1] Alexei Kolezhuk,[1,2] Michael Levin,[1] Subir Sachdev,[1] and T. Senthil[3,4]

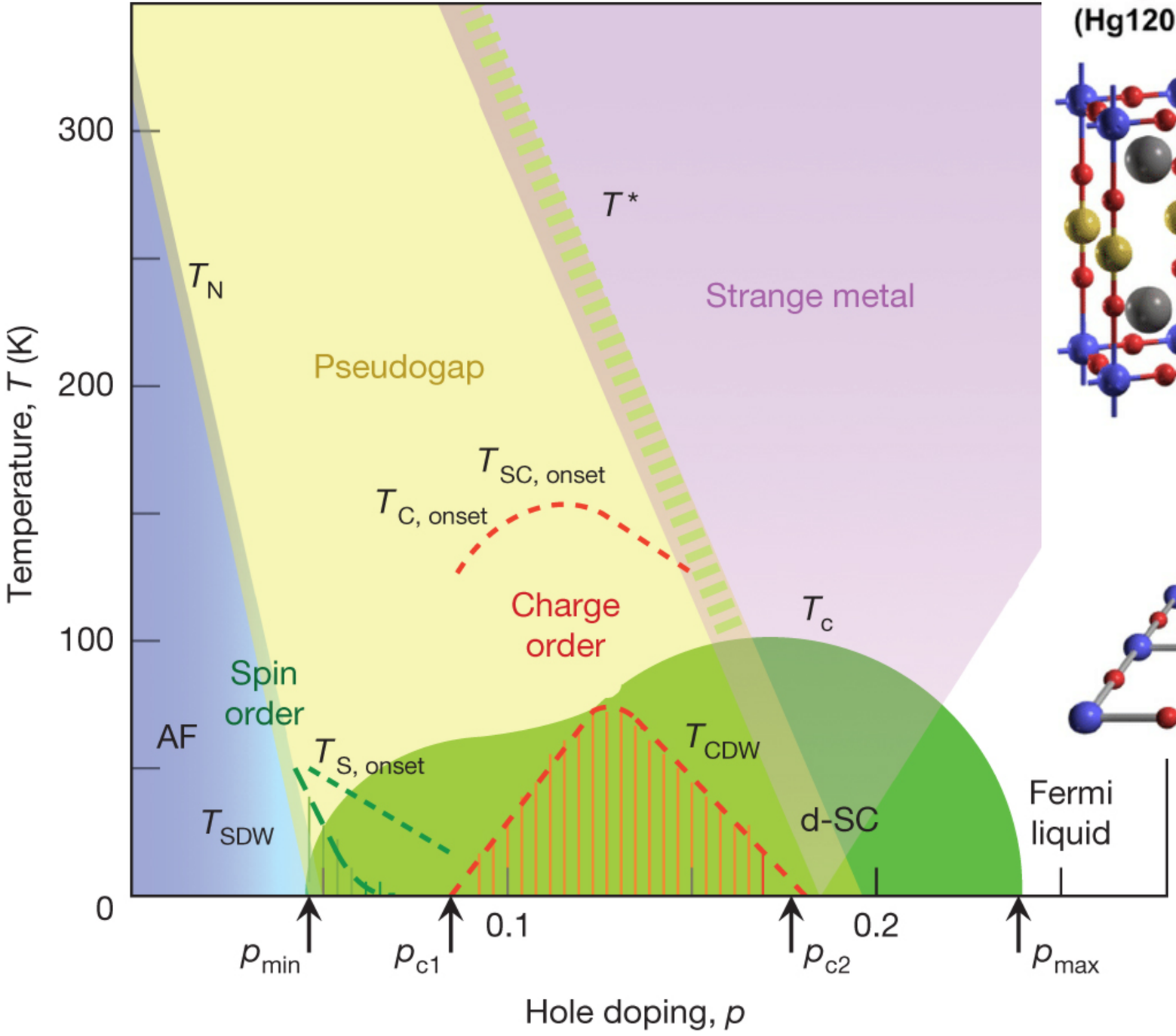


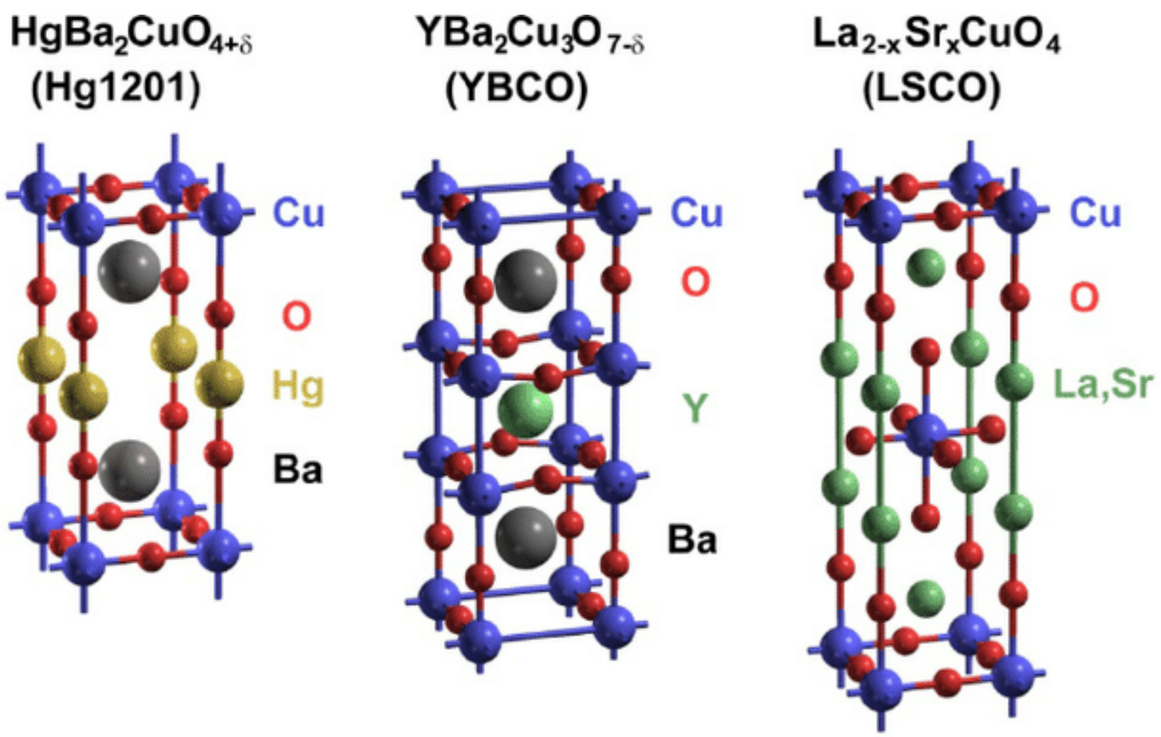
The dashed line in the Néel phase indicates the boundary of the magnetic Brillouin zone. Only the Fermi surfaces within this zone contribute to the Luttinger counting, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/4$. In the VBS phase, all four pockets are inequivalent, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/8$.

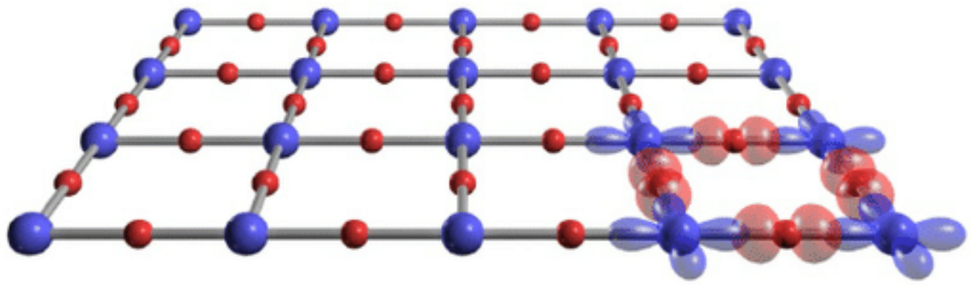
Factor of 2 between SDW fluctuation and FL*

Observation of the Yamaji effect in a cuprate

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

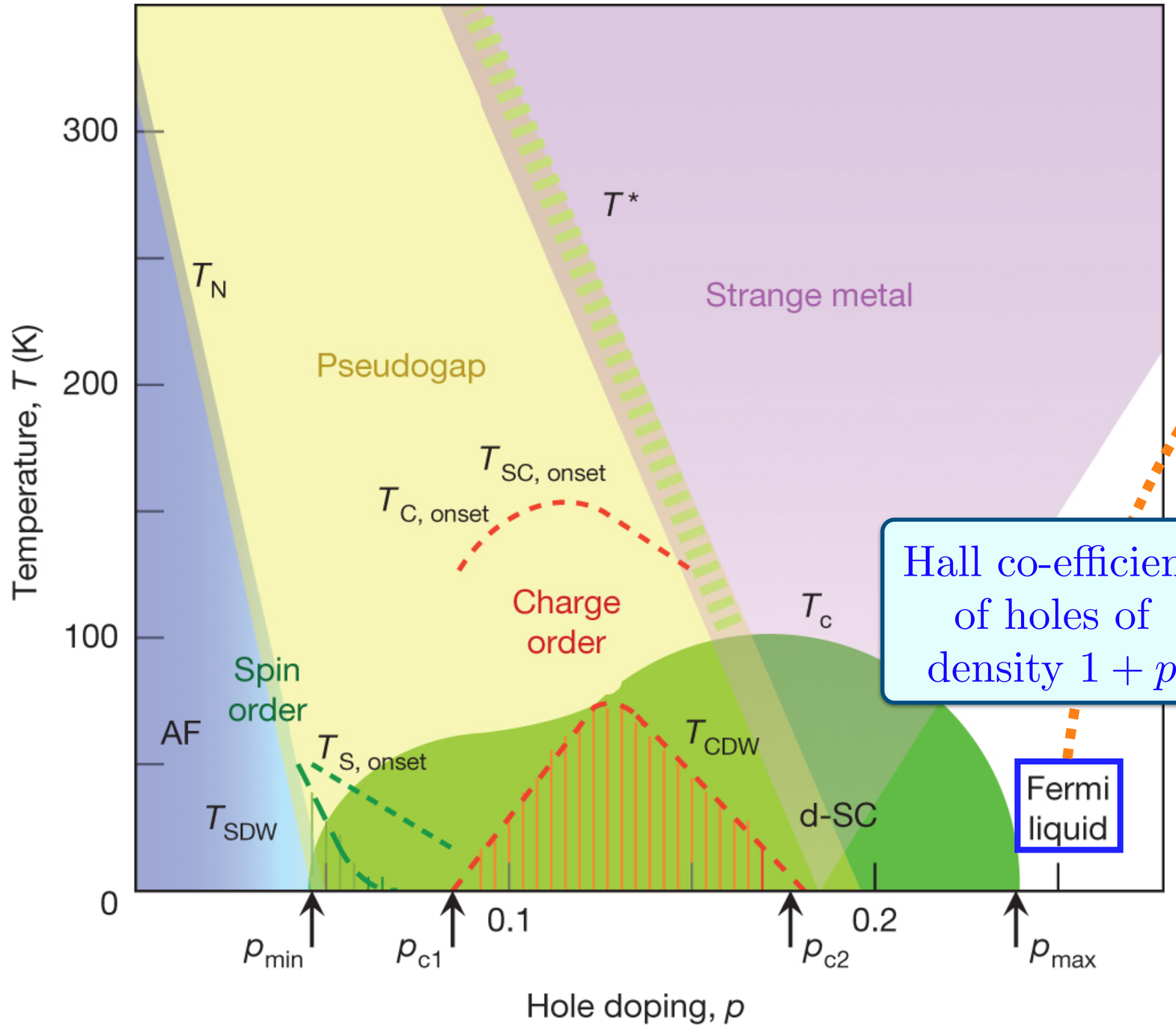


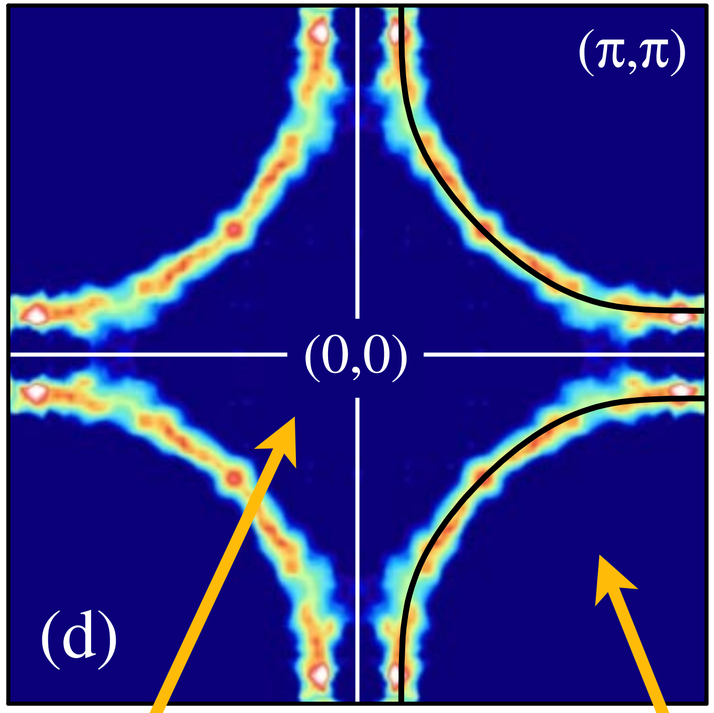




Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

M. Platé *et al.*, PRL **95**, 077001 (2005)



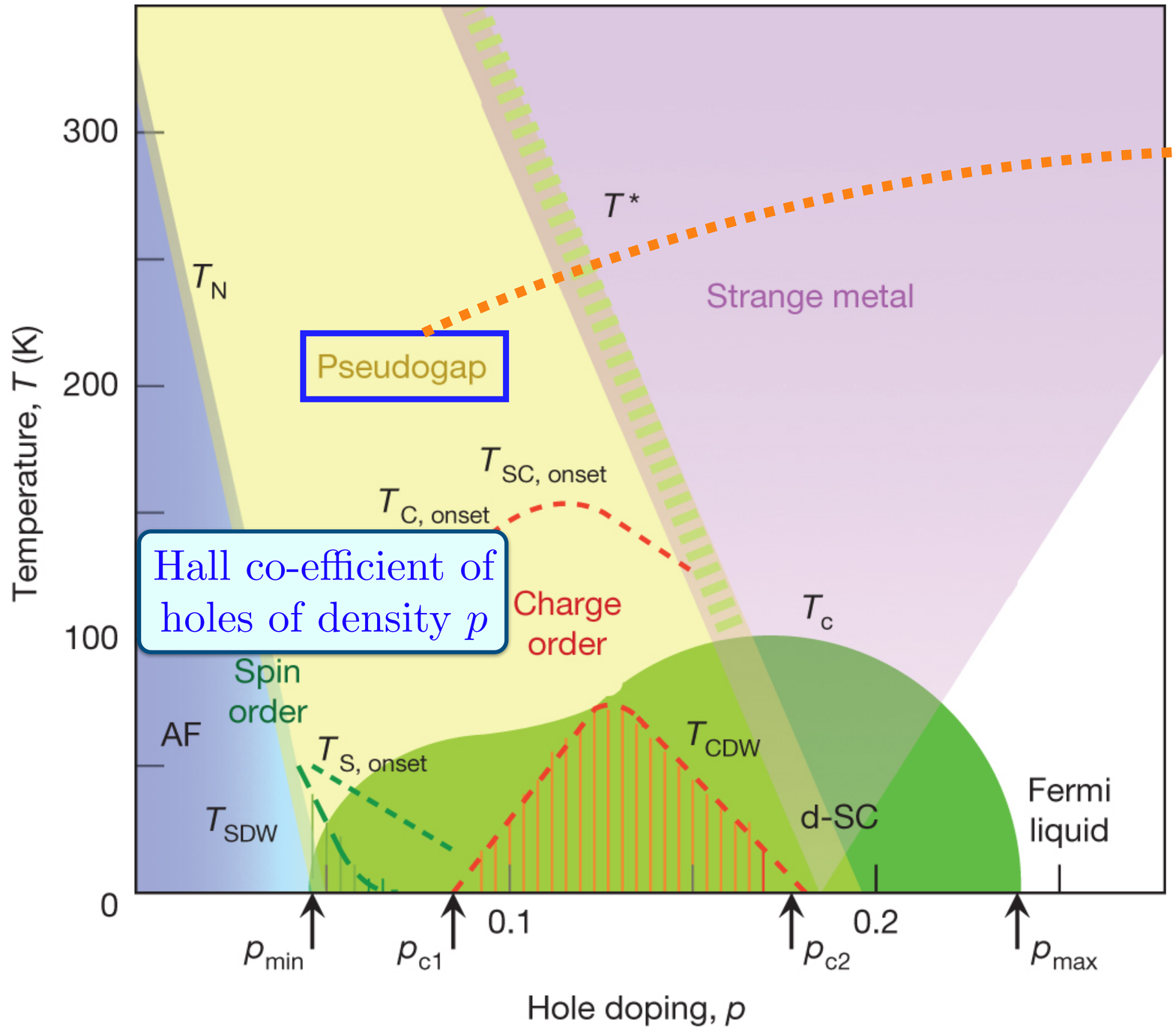


Hall co-efficient of holes of density $1 + p$

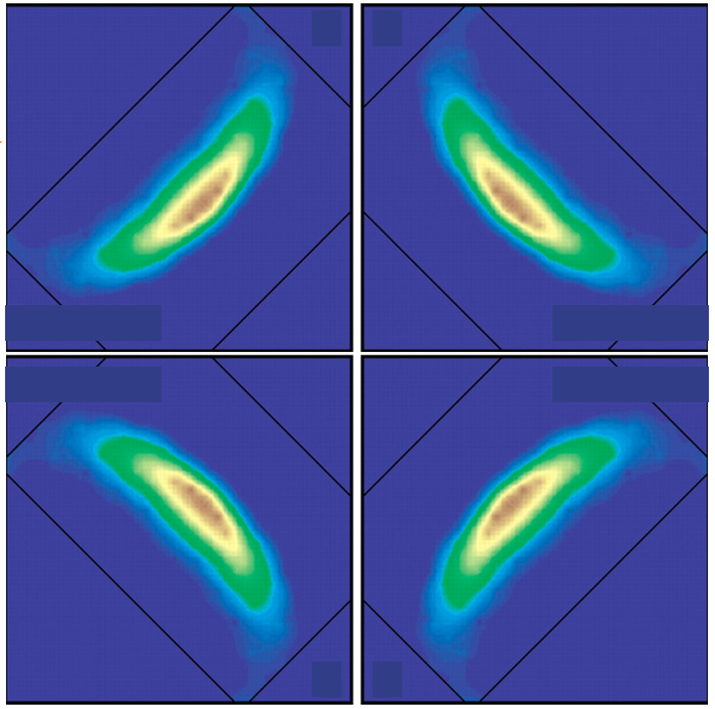
1-p electrons

1+p holes

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Kyle M. Shen *et al.*, Science **307**, 901 (2005)

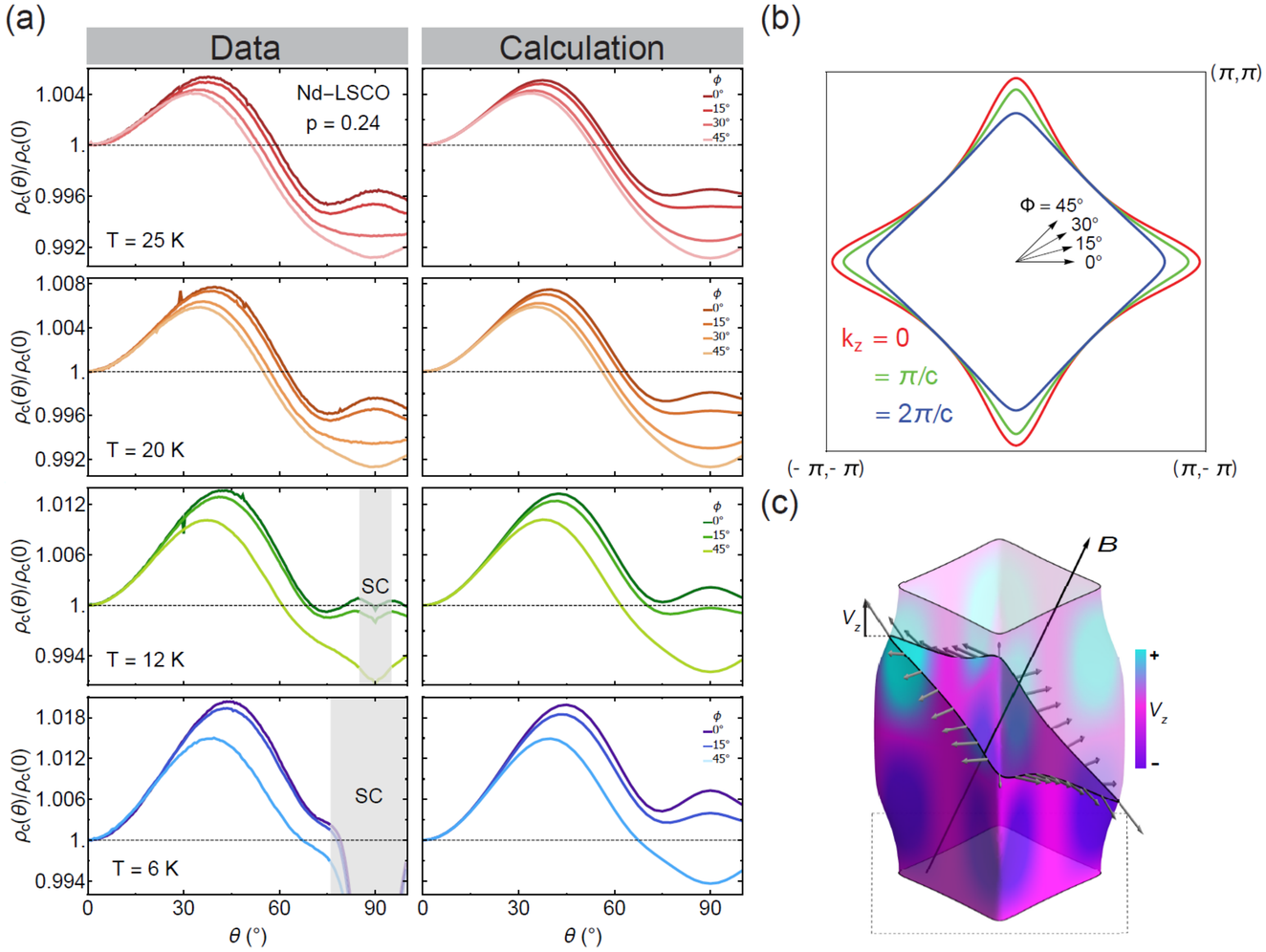


`Fermi arcs' ?

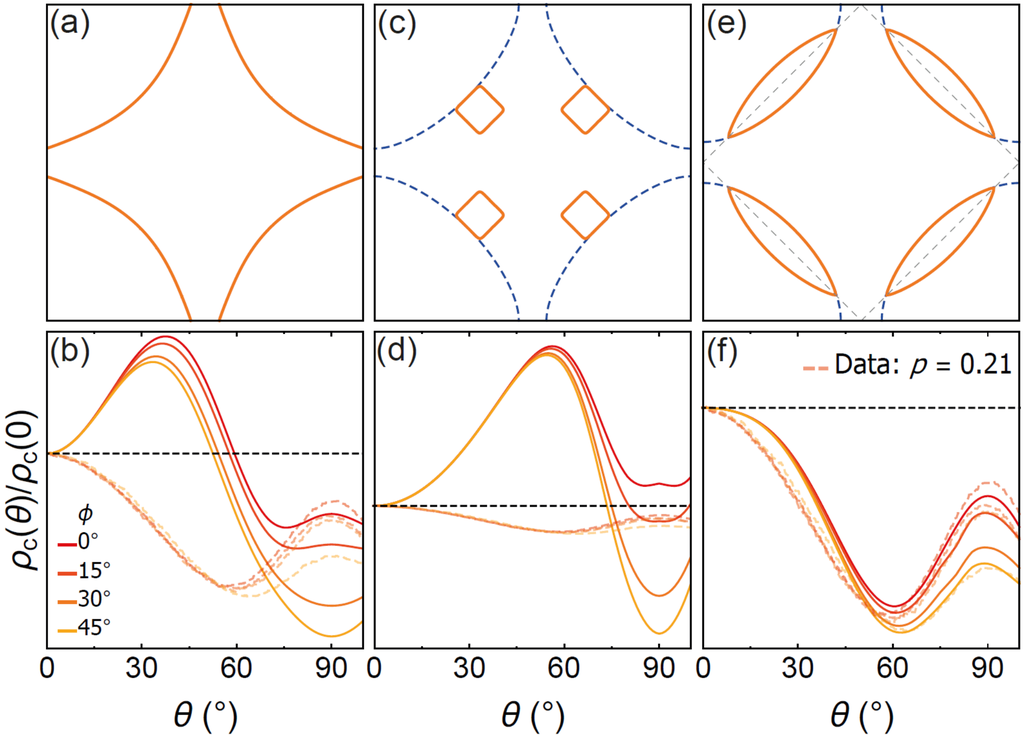
Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, *Nature Physics* **18**, 558 (2022)

Angle-dependent magnetoresistance (ADMR) of $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$



$p > p_c$ Large Fermi surface

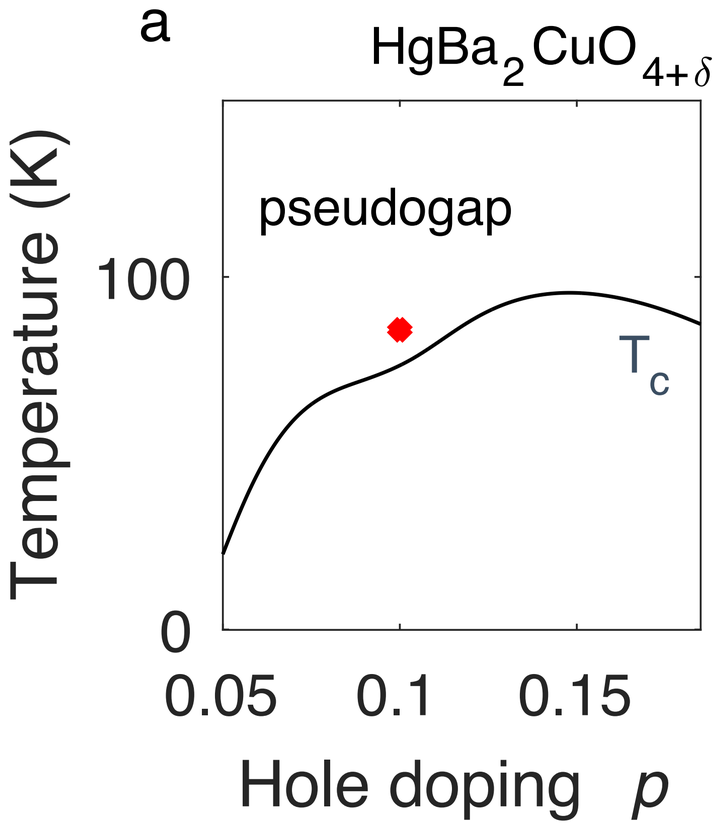


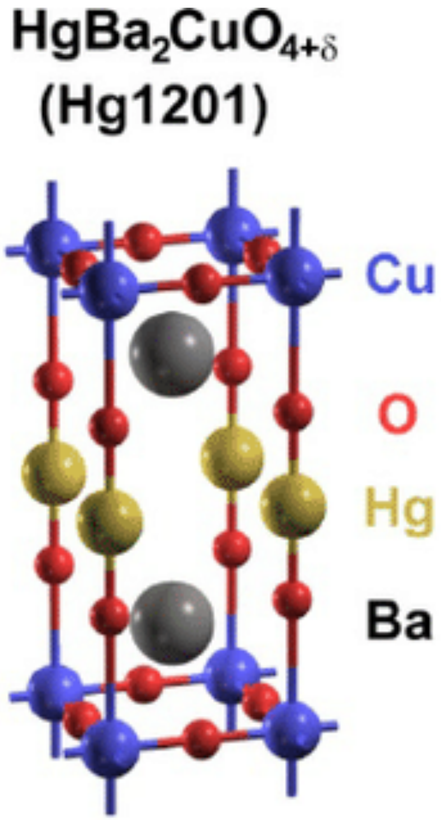
$p < p_c$ Reconstructed Fermi surface

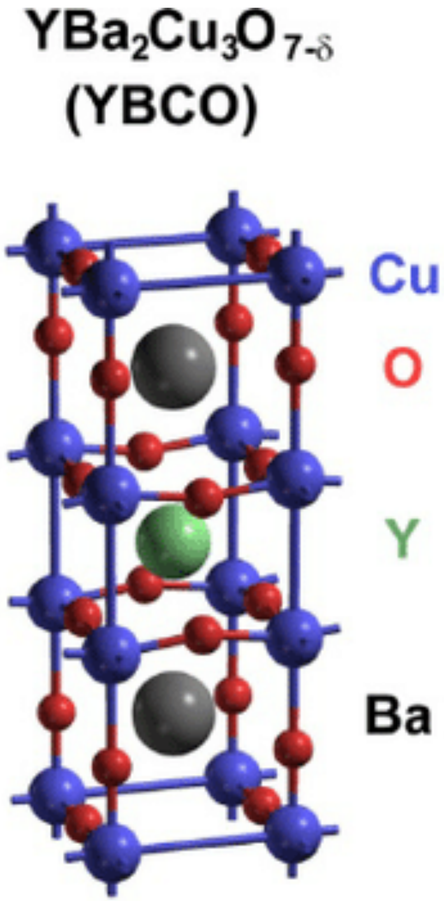
Observation of the Yamaji effect in a cuprate superconductor

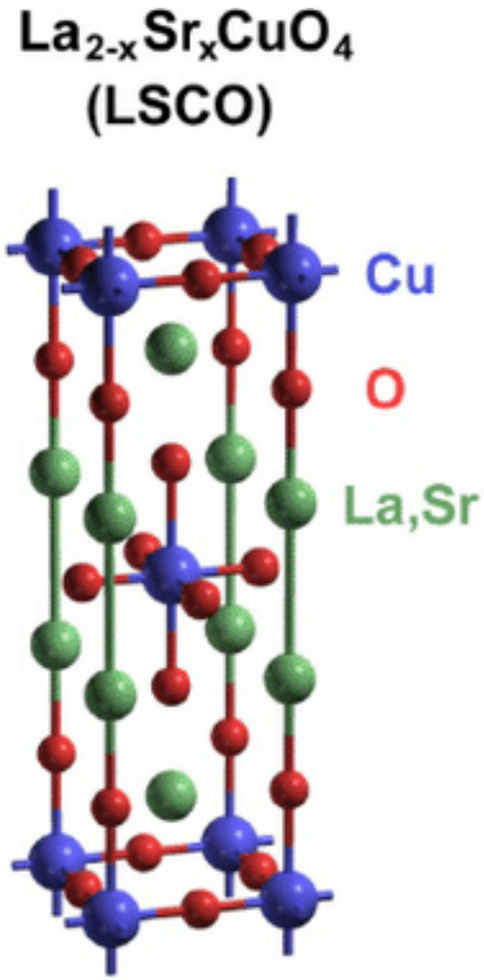
Mun K. Chan[1] ✉, Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
Published online: 16 September 2025





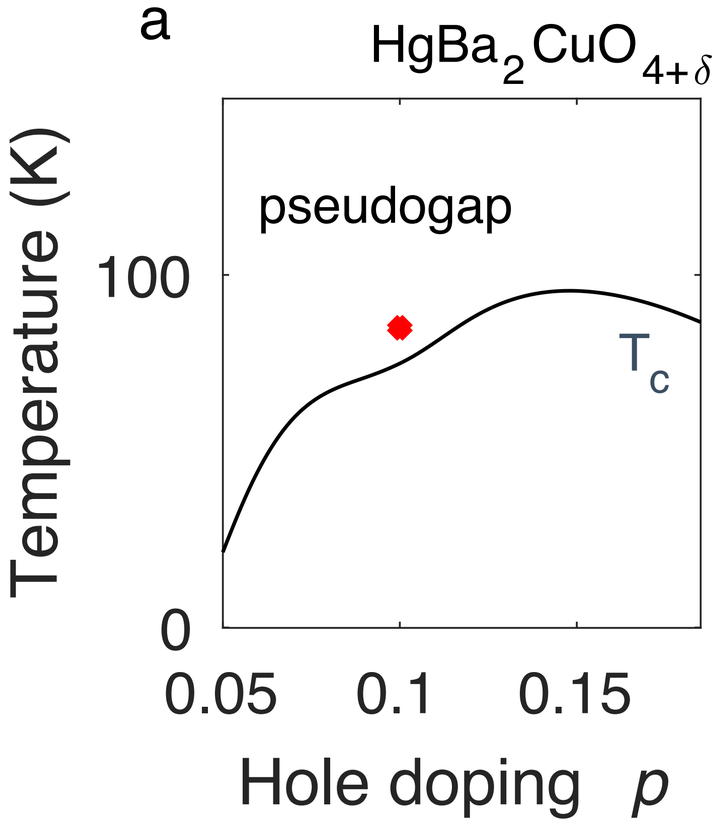


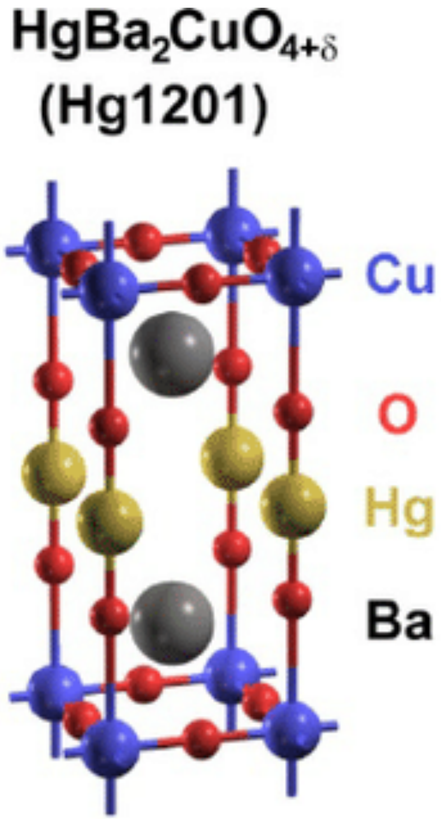


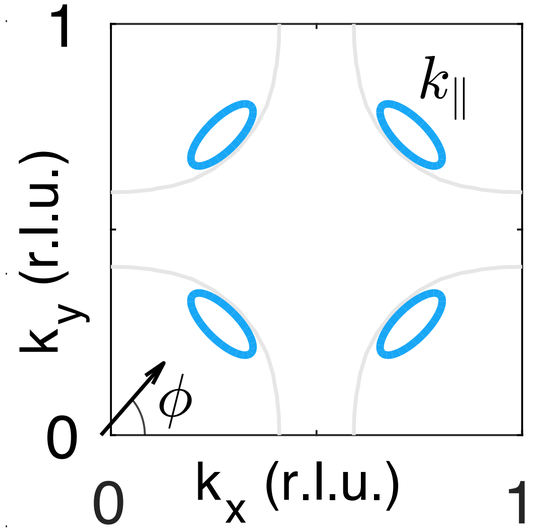
Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1] ✉, Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
Published online: 16 September 2025

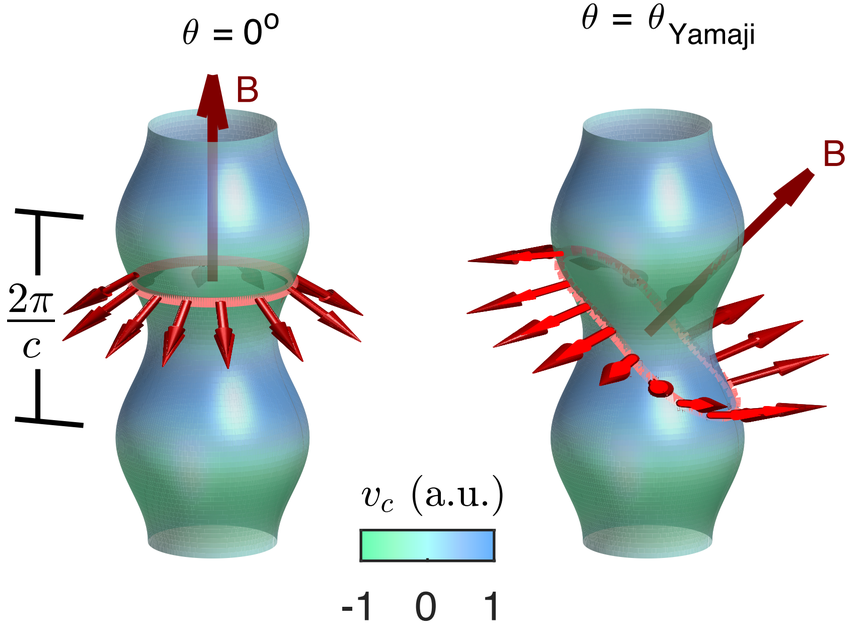






At the Yamaji angle, the orbits in the plane orthogonal to $\boldsymbol{B}$ have an area which is independent of momentum in the c direction, to first order in the hopping along the c direction.

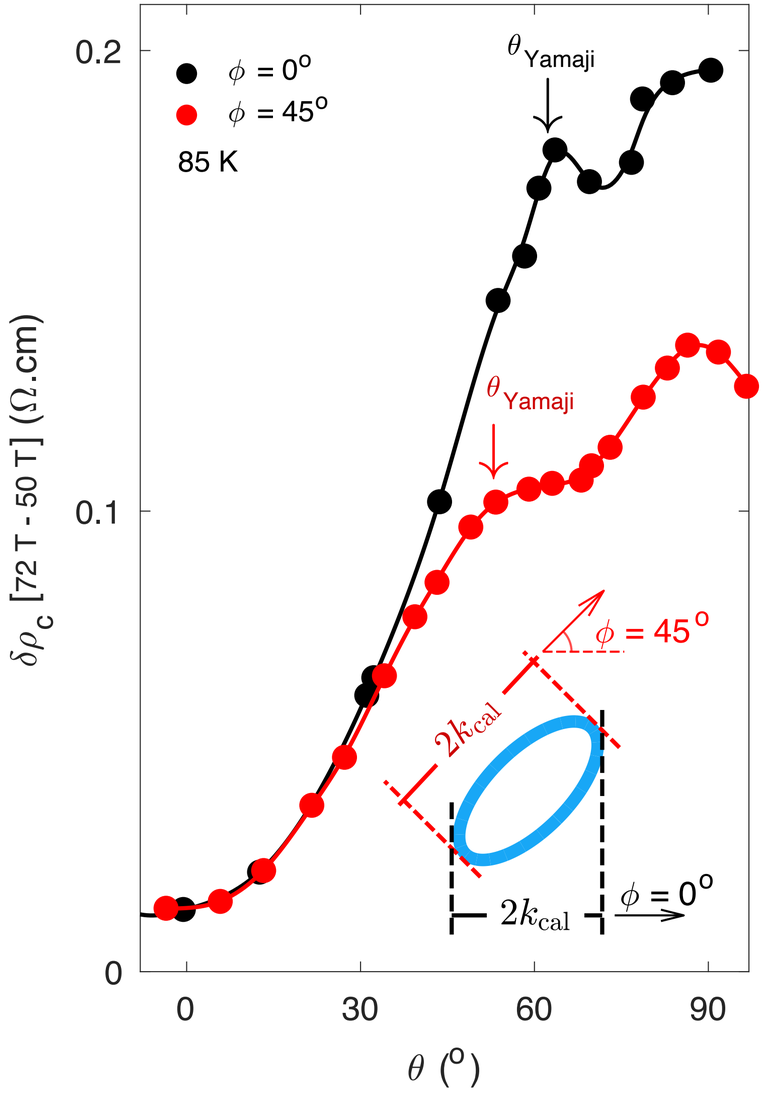
K. Yamaji JPSJ **58**, 1520 (1989)

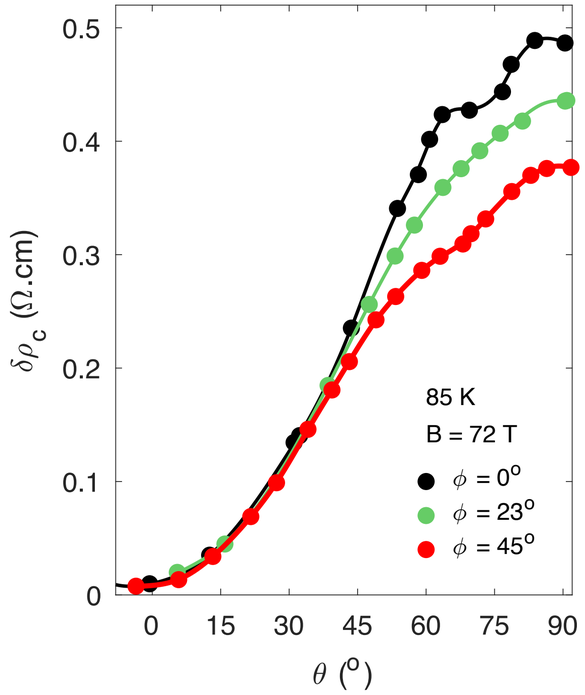


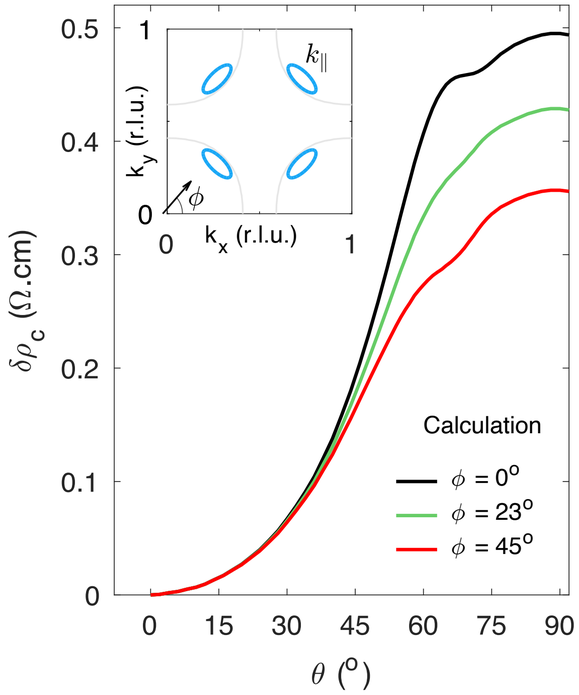
Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1] ✉, Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
Published online: 16 September 2025







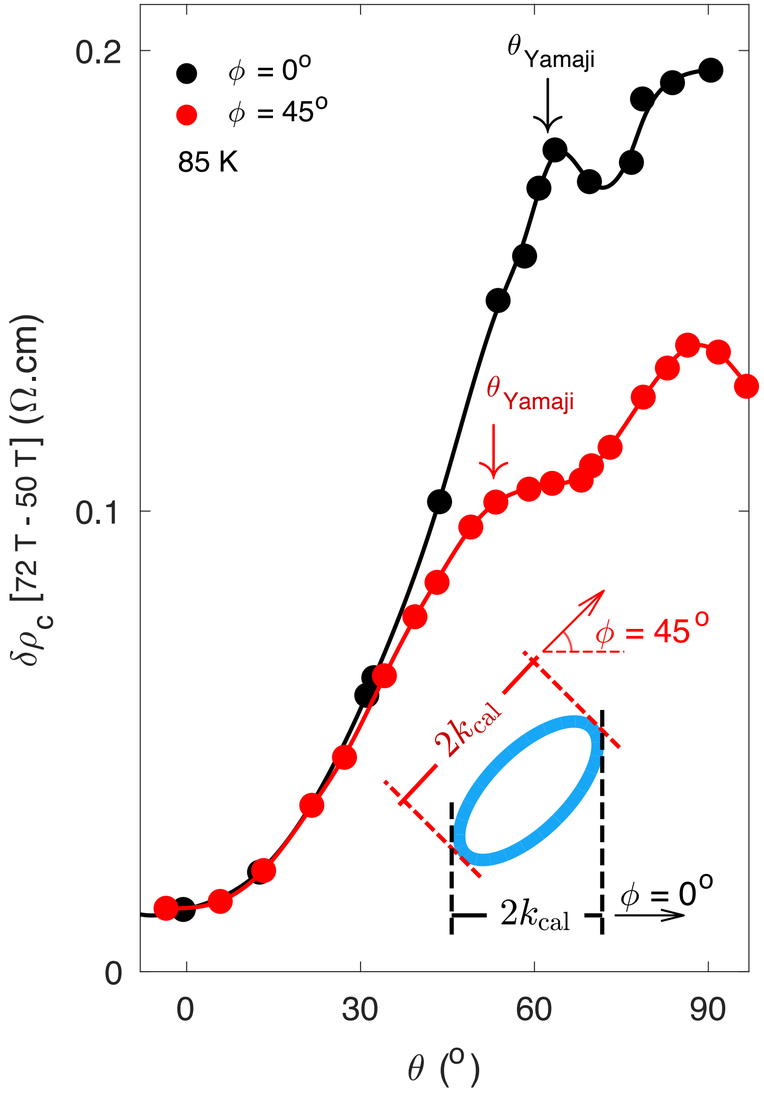
Doping
$p = 0.1$

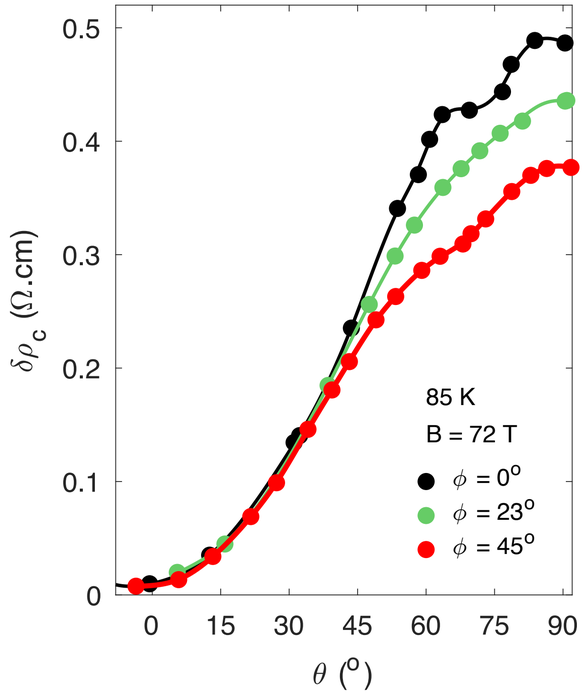
The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase.

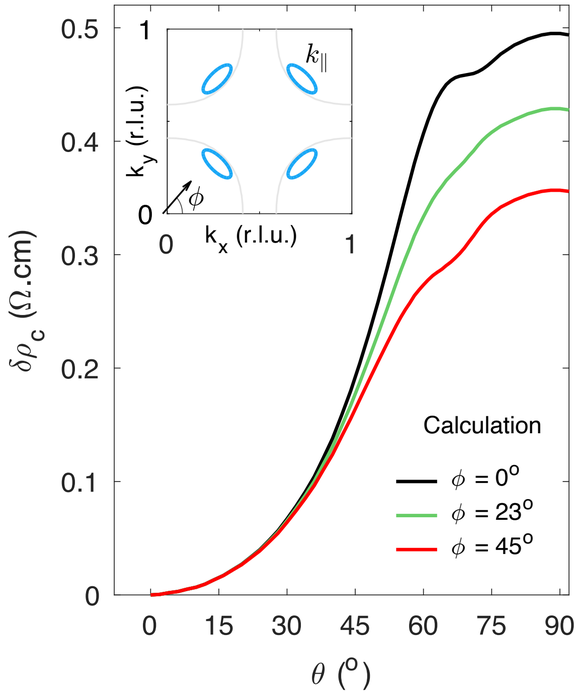
Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1] ✉, Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
Published online: 16 September 2025







Doping
$p = 0.1$

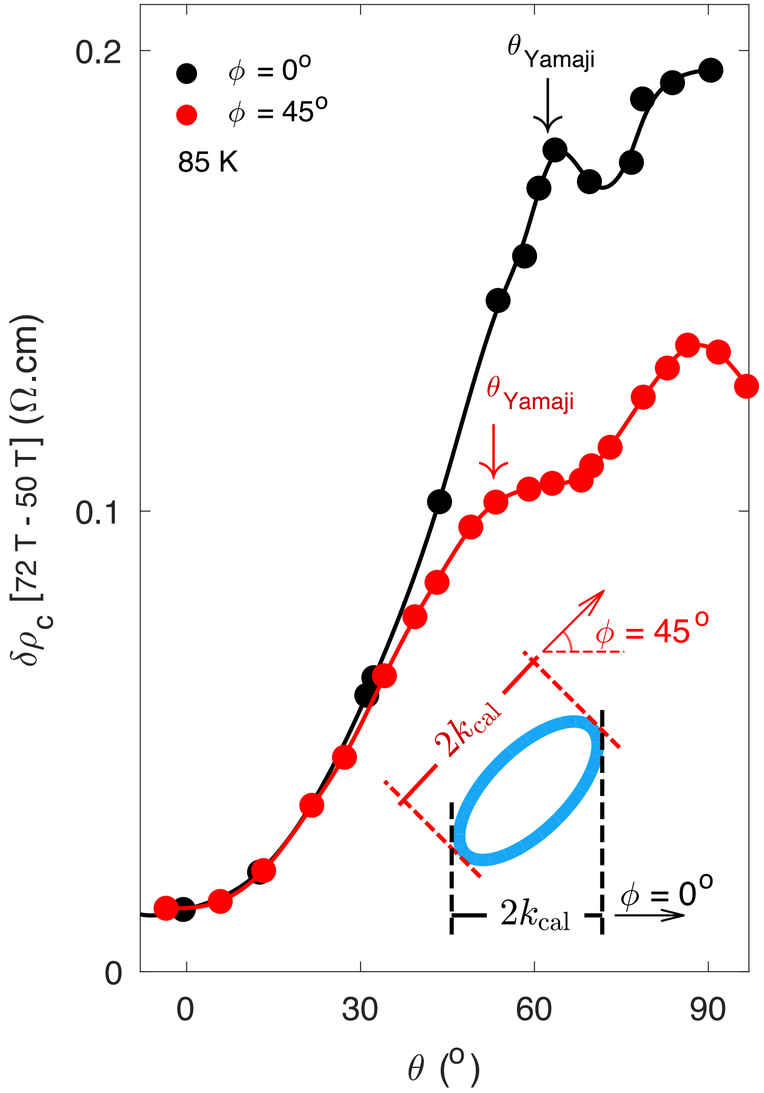
The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase.

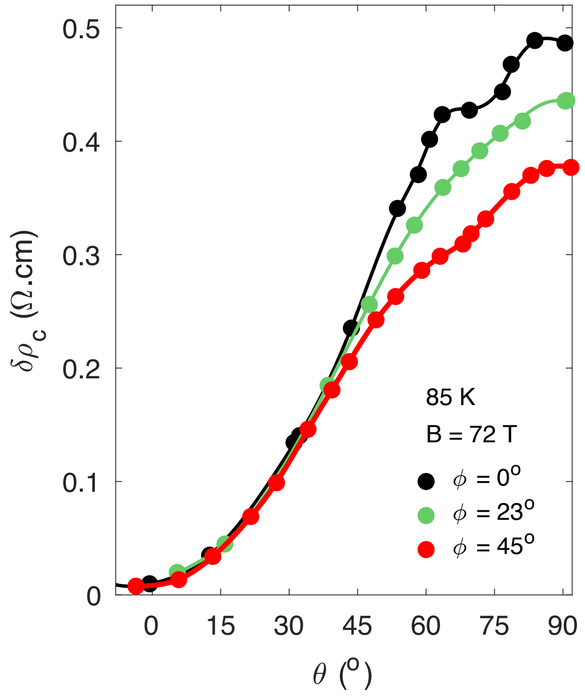
Excellent evidence for hole pockets with coherent interlayer-transport.

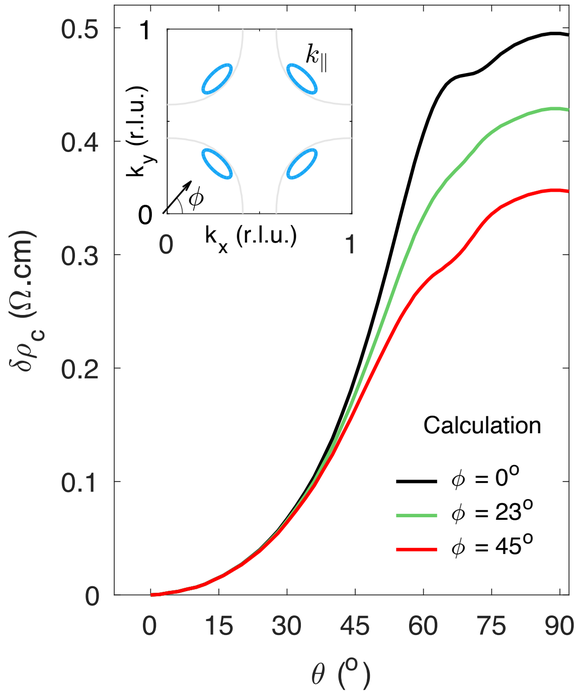
Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1] ✉, Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
Published online: 16 September 2025







Doping
$p = 0.1$

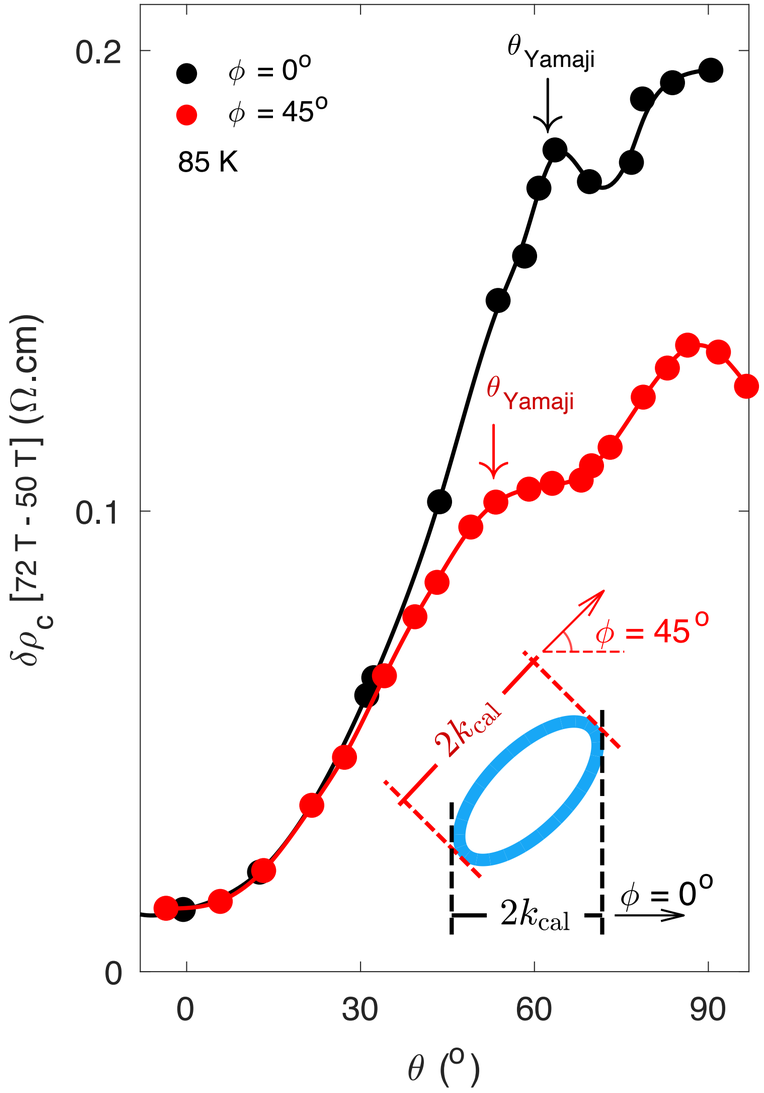
The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase.

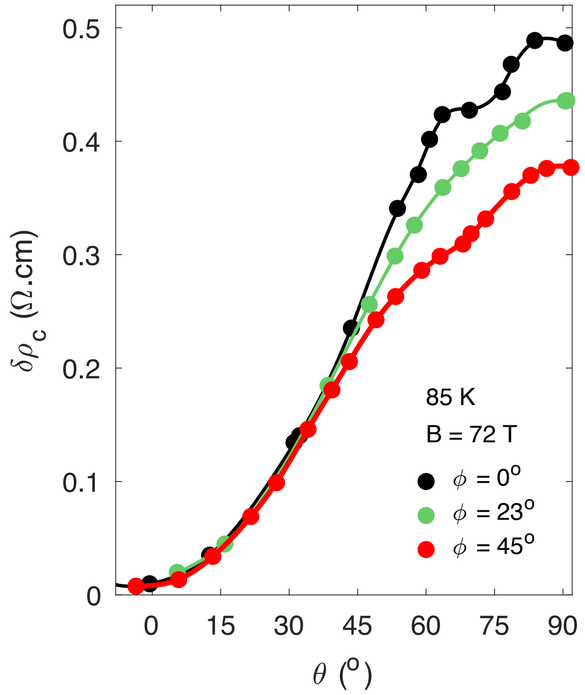
Excellent evidence for hole pockets with coherent interlayer-transport.
Rules out holon metal

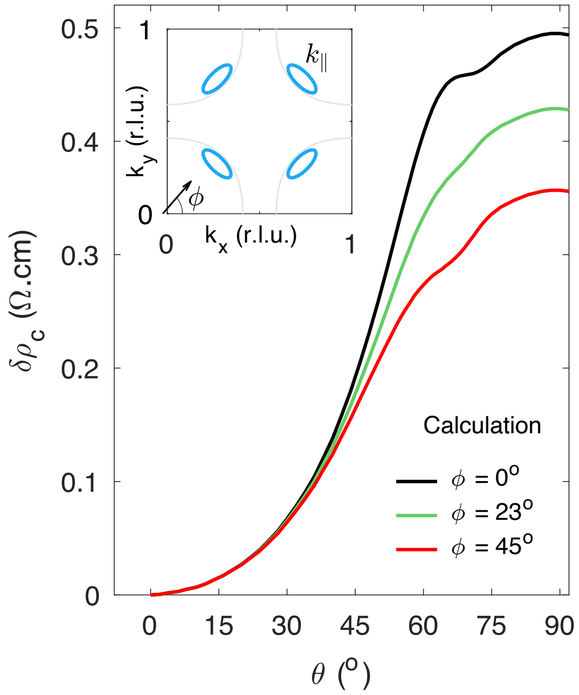
Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1] ✉, Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
Published online: 16 September 2025







Doping
$p = 0.1$

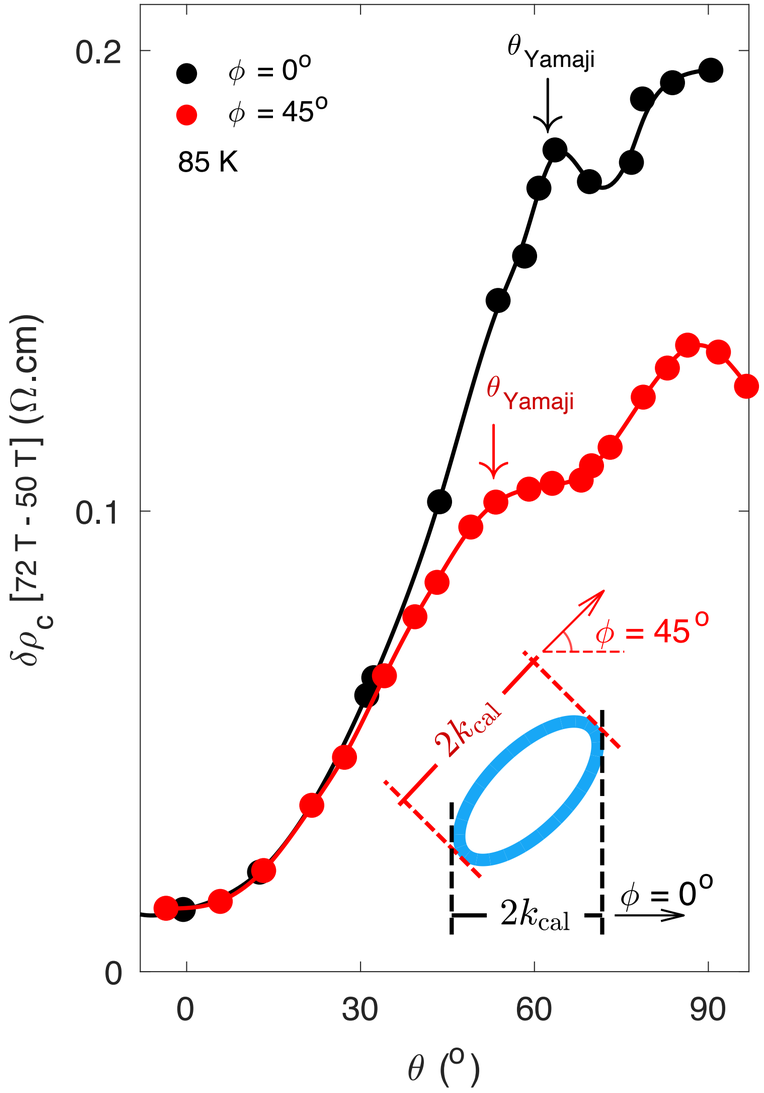
The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase.

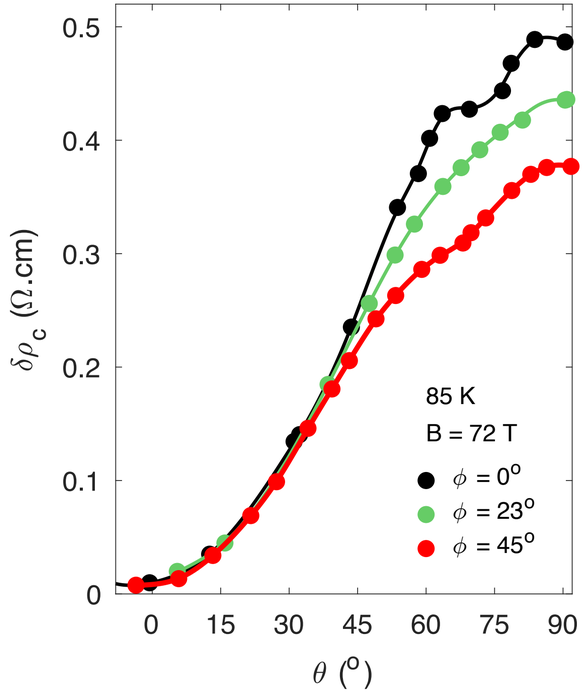
Excellent evidence for hole pockets with coherent interlayer-transport.
Rules out holon metal
and possibly SDW metal

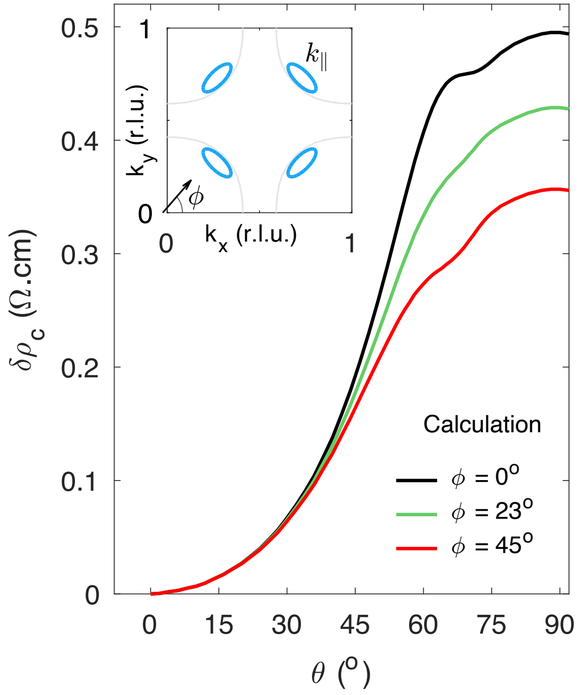
Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1] ✉, Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
Published online: 16 September 2025







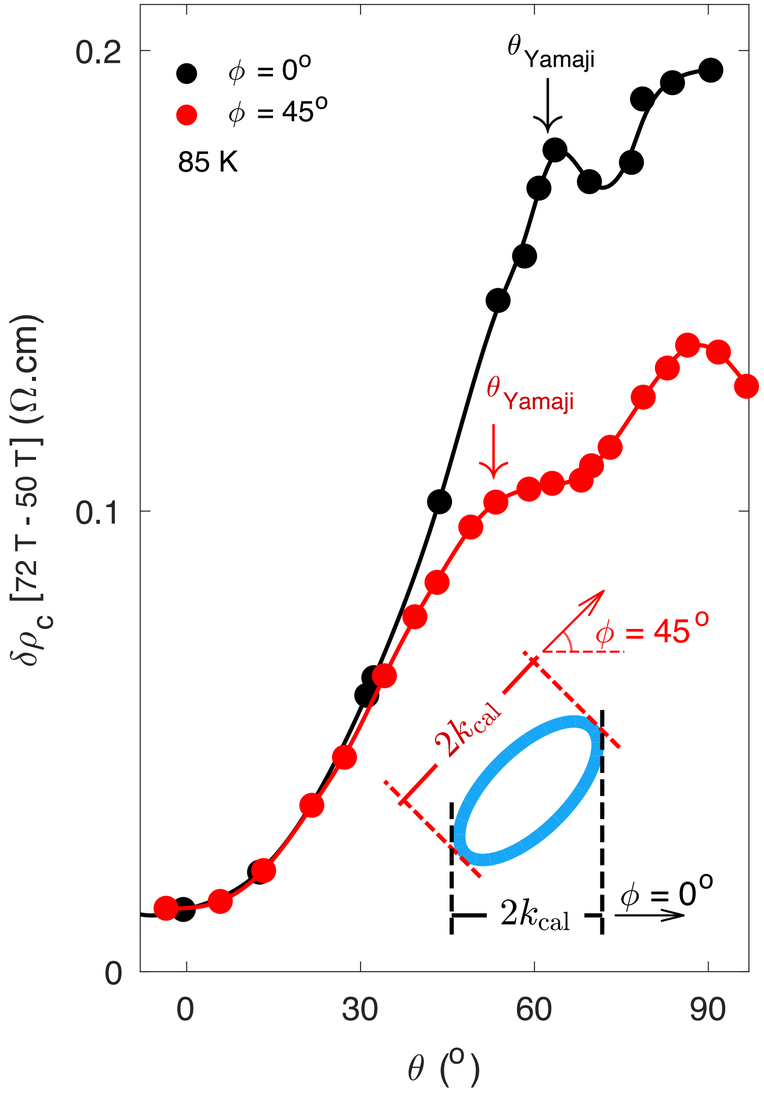
Doping
$p = 0.1$

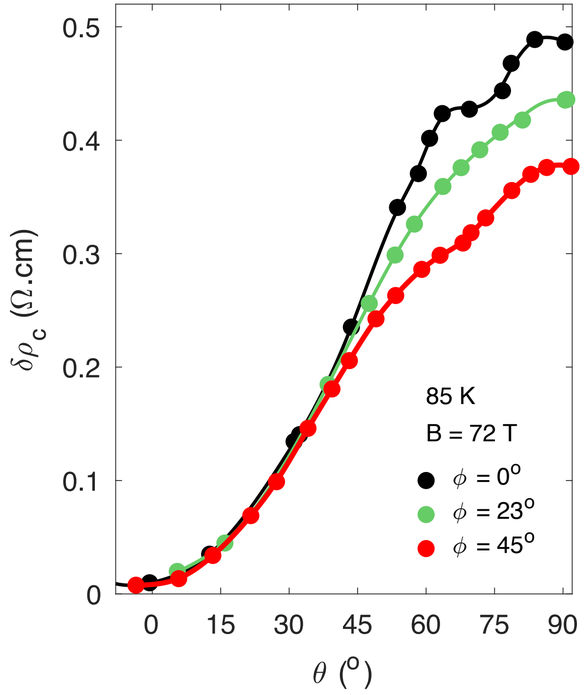
The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase. The small size of the pockets, each estimated to occupy only 1.3% of the Brillouin zone area, is not expected given the absence of long-range broken translational symmetry.

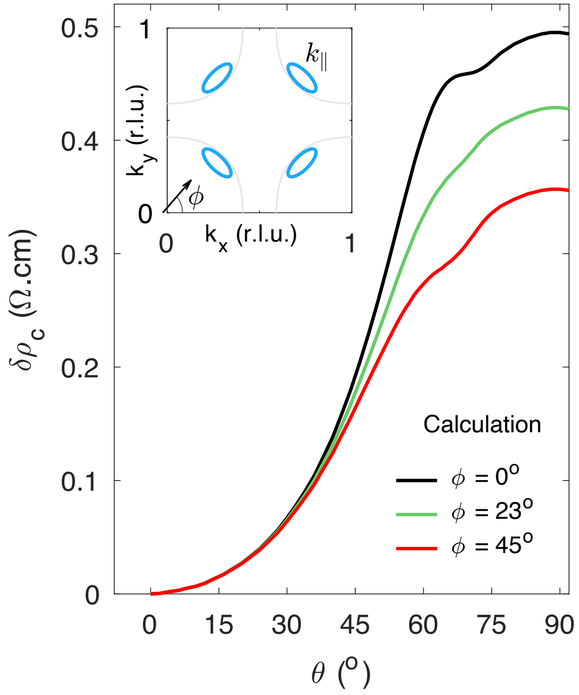
Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1] ✉, Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
Published online: 16 September 2025







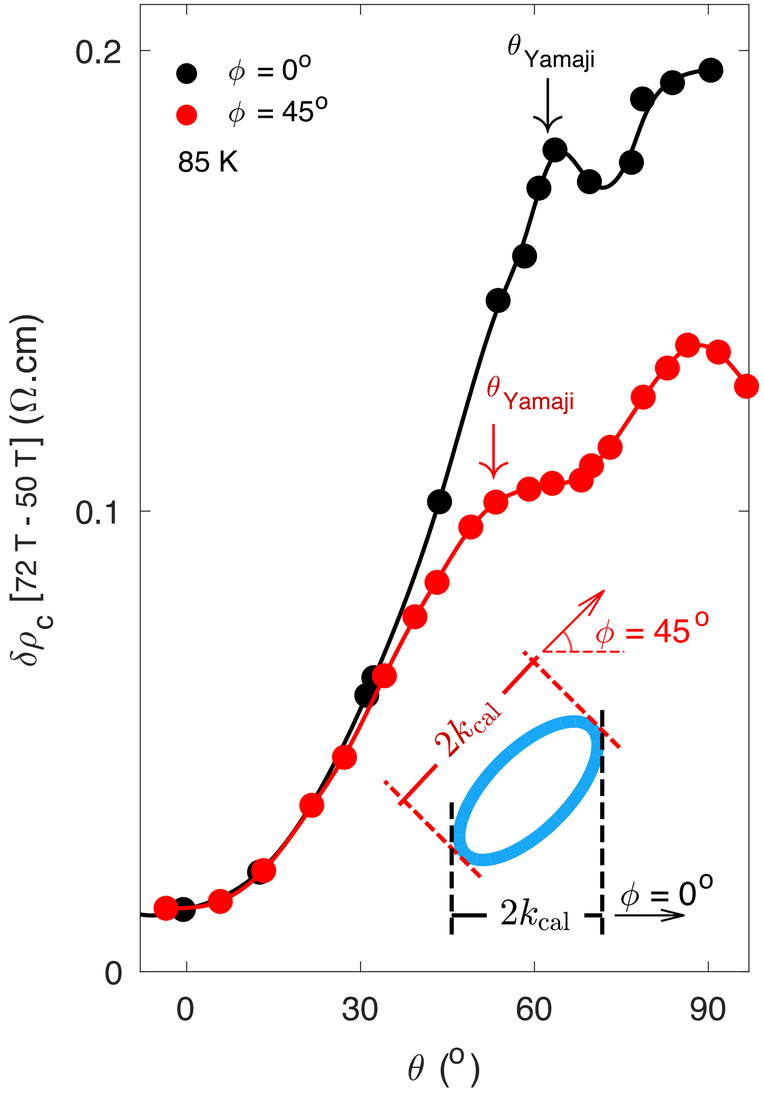
Doping
$p = 0.1$

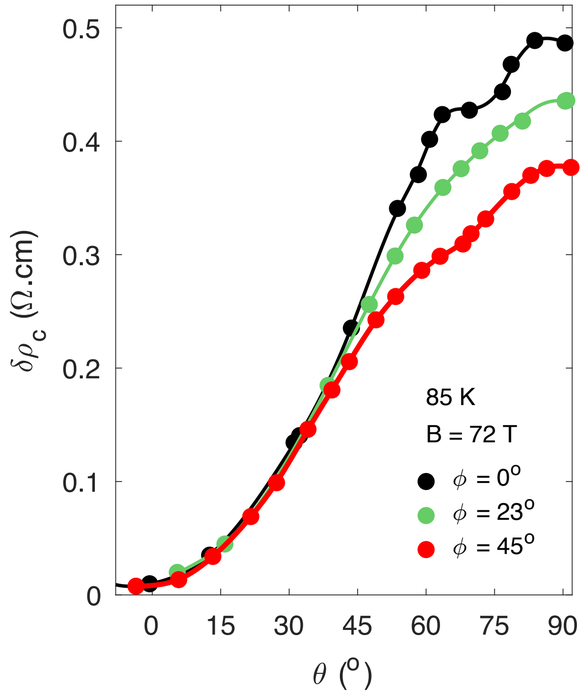
The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase. The small size of the pockets, each estimated to occupy only 1.3% of the Brillouin zone area, is not expected given the absence of long-range broken translational symmetry.

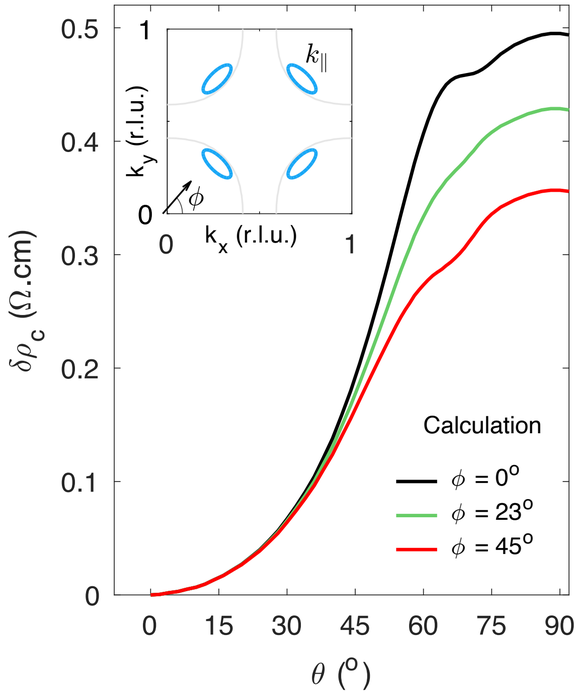
Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1] ✉, Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
Published online: 16 September 2025







Doping
$p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase. The small size of the pockets, each estimated to occupy only 1.3% of the Brillouin zone area, is not expected given the absence of long-range broken translational symmetry.

Predicted FL* pocket fraction $= p/8 = 1.25\%$!

Fluctuating AF metal fraction $= p/4 = 2.5\%$.

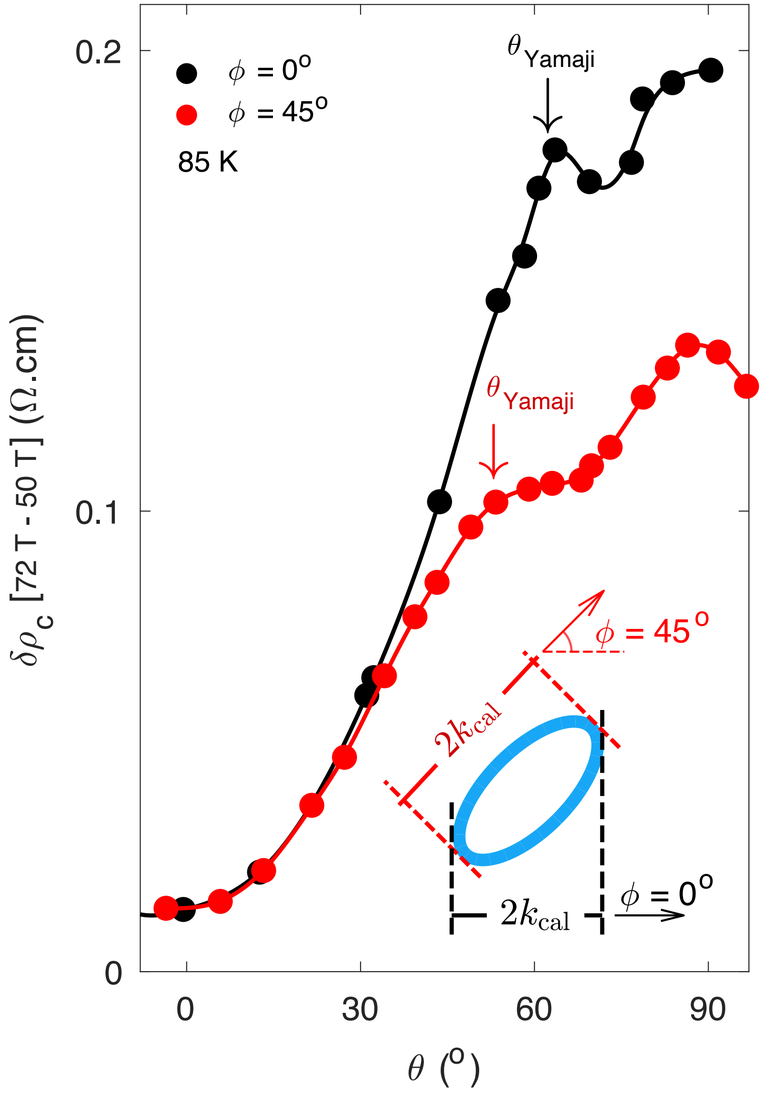
($p/8$ also in YRZ ansatz, Peter Johnson photoemission, and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar)

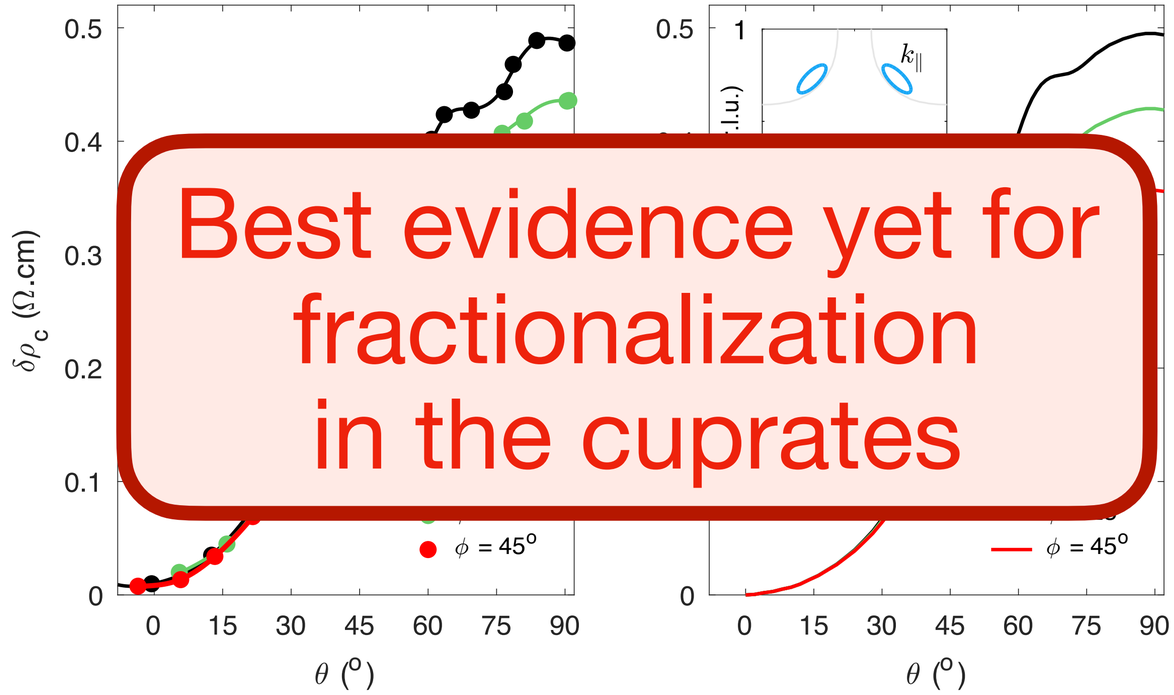
Jing-Yu Zhao, S. Chatterjee, S. S., Ya-Hui Zhang, arXiv:2510.13943

Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1] ✉, Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
Published online: 16 September 2025





Best evidence yet for fractionalization in the cuprates

Doping $p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase. The small size of the pockets, each estimated to occupy only 1.3% of the Brillouin zone area, is not expected given the absence of long-range broken translational symmetry.

Predicted FL* pocket fraction $= p/8 = 1.25\%$!

Fluctuating AF metal fraction $= p/4 = 2.5\%$.

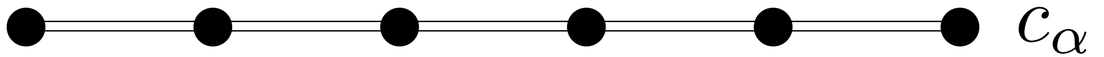
($p/8$ also in YRZ ansatz, Peter Johnson photoemission, and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar)

Jing-Yu Zhao, S. Chatterjee, S. S., Ya-Hui Zhang, arXiv:2510.13943

Ancilla Layer Model (ALM) of FL*

Ancilla Layer Model of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)



Hubbard model of hole density 1+p

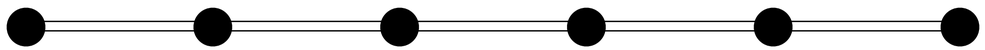
$$\mathcal{H}_{\text{Hubbard}} = -\sum_{i,j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + U \sum_i (c^\dagger_{i\uparrow} c_{i\uparrow})(c^\dagger_{i\downarrow} c_{i\downarrow})$$

Ya-Hui Zhang

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla Layer Model of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)



Hubbard model of hole density 1+p

$$\mathcal{H}_{\rm Hubbard} = -\sum_{i,j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + \sum_i \left[\frac{3U}{8} \boldsymbol{\mathcal{P}}_i^2 + U \boldsymbol{\mathcal{P}}_i \cdot c^\dagger_{i\alpha} \frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} c_{i\beta} \right] .$$

$$\boldsymbol{\mathcal{P}}_i \quad \Rightarrow \quad \text{Paramagnon}$$

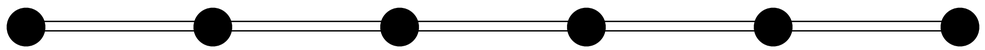
Ya-Hui Zhang

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla Layer Model of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)



Hubbard model of hole density 1+p

$$\mathcal{H}_{\rm Hubbard} = -\sum_{i,j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + \sum_i \left[\frac{3U}{8} \boldsymbol{\mathcal{P}}_i^2 + U \boldsymbol{\mathcal{P}}_i \cdot c^\dagger_{i\alpha} \frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} c_{i\beta} + \frac{m_{\boldsymbol{\mathcal{P}}}}{2} \left(\partial_\tau \boldsymbol{\mathcal{P}}_i\right)^2 \right] .$$

$$\boldsymbol{\mathcal{P}}_i \quad \Rightarrow \quad \text{Paramagnon}$$

Ya-Hui Zhang

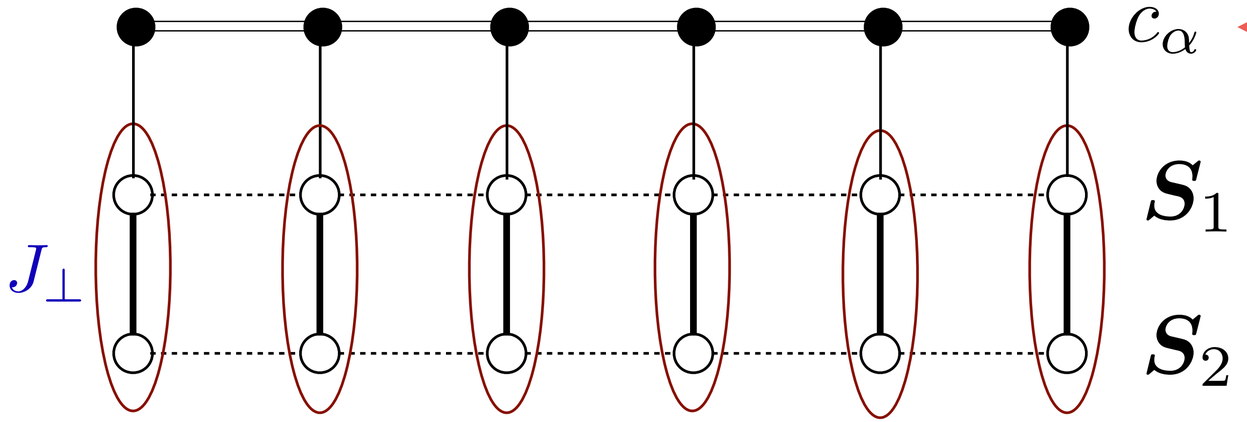
Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla Layer Model of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)

Free holes of density 1+p



$$\mathcal{H}_{\rm Hubbard} = -\sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{ij}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} + \sum_{\boldsymbol{i}} \left[\frac{3U}{8} \boldsymbol{\mathcal{P}}^2_{\boldsymbol{i}} + U \boldsymbol{\mathcal{P}}_{\boldsymbol{i}} \cdot c^\dagger_{\boldsymbol{i}\alpha} \frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} c_{\boldsymbol{i}\beta} + \frac{m_{\boldsymbol{\mathcal{P}}}}{2} \left(\partial_\tau \boldsymbol{\mathcal{P}}_{\boldsymbol{i}}\right)^2 \right] .$$

$3\ \boldsymbol{\mathcal{P}}$ oscillators' states: $|0,0,0\rangle,\ |1,0,0\rangle,\ |0,1,0\rangle,\ |0,0,1\rangle$

$\boldsymbol{S}_{1,2}$ ancilla qubits states : $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2},\ |\uparrow\uparrow\rangle,\ (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2},\ |\downarrow\downarrow\rangle$

$$\boldsymbol{\mathcal{P}} \sim \boldsymbol{S}_1 - \boldsymbol{S}_2$$

Ya-Hui Zhang

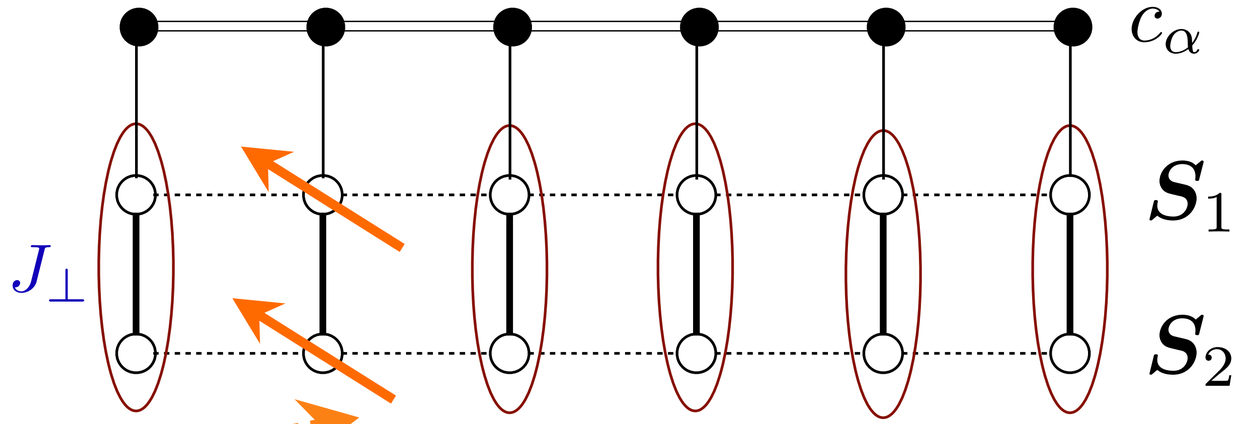
Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla Layer Model of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)

Free holes of density 1+p



Paramagnon

$$\mathcal{H}_{\rm Hubbard} = -\sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{ij}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} + \sum_{\boldsymbol{i}} \left[\frac{3U}{8} \boldsymbol{\mathcal{P}}^2_{\boldsymbol{i}} + U \boldsymbol{\mathcal{P}}_{\boldsymbol{i}} \cdot c^\dagger_{\boldsymbol{i}\alpha} \frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} c_{\boldsymbol{i}\beta} + \frac{m_{\boldsymbol{\mathcal{P}}}}{2} \left(\partial_\tau \boldsymbol{\mathcal{P}}_{\boldsymbol{i}}\right)^2 \right] .$$

$3\ \boldsymbol{\mathcal{P}}$ oscillators' states: $|0,0,0\rangle,\ |1,0,0\rangle,\ |0,1,0\rangle,\ |0,0,1\rangle$

$\boldsymbol{S}_{1,2}$ ancilla qubits states : $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2},\ |\uparrow\uparrow\rangle,\ (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2},\ |\downarrow\downarrow\rangle$

$$\boldsymbol{\mathcal{P}} \sim \boldsymbol{S}_1 - \boldsymbol{S}_2$$

Ya-Hui Zhang

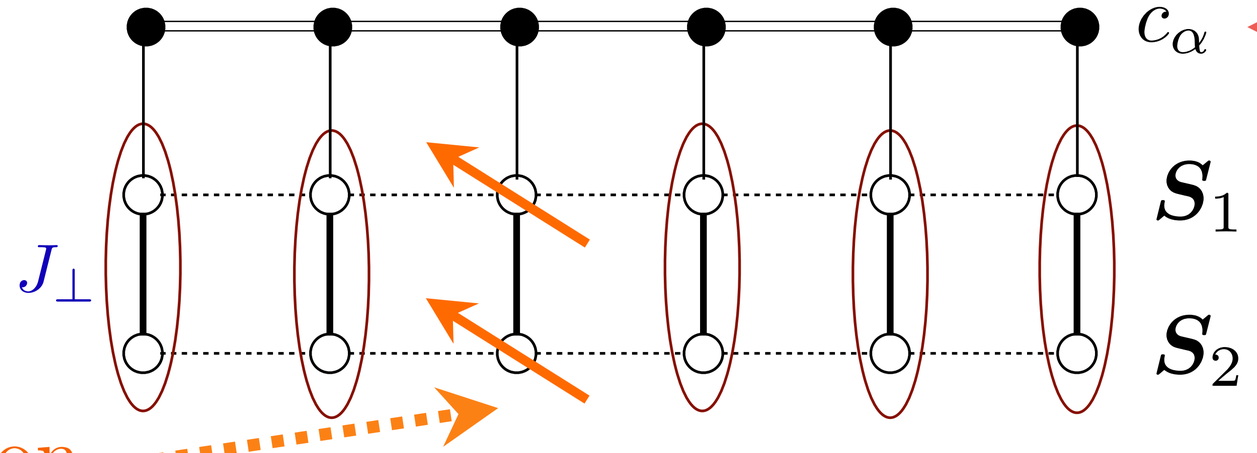
Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla Layer Model of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)

Free holes of density 1+p



Paramagnon

$$\mathcal{H}_{\rm Hubbard} = -\sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{ij}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} + \sum_{\boldsymbol{i}} \left[\frac{3U}{8} \boldsymbol{\mathcal{P}}^2_{\boldsymbol{i}} + U \boldsymbol{\mathcal{P}}_{\boldsymbol{i}} \cdot c^\dagger_{\boldsymbol{i}\alpha} \frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} c_{\boldsymbol{i}\beta} + \frac{m_{\boldsymbol{\mathcal{P}}}}{2} \left(\partial_\tau \boldsymbol{\mathcal{P}}_{\boldsymbol{i}}\right)^2 \right] .$$

$3\ \boldsymbol{\mathcal{P}}$ oscillators' states: $|0,0,0\rangle,\ |1,0,0\rangle,\ |0,1,0\rangle,\ |0,0,1\rangle$

$\boldsymbol{S}_{1,2}$ ancilla qubits states : $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2},\ |\uparrow\uparrow\rangle,\ (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2},\ |\downarrow\downarrow\rangle$

$$\boldsymbol{\mathcal{P}} \sim \boldsymbol{S}_1 - \boldsymbol{S}_2$$

Ya-Hui Zhang

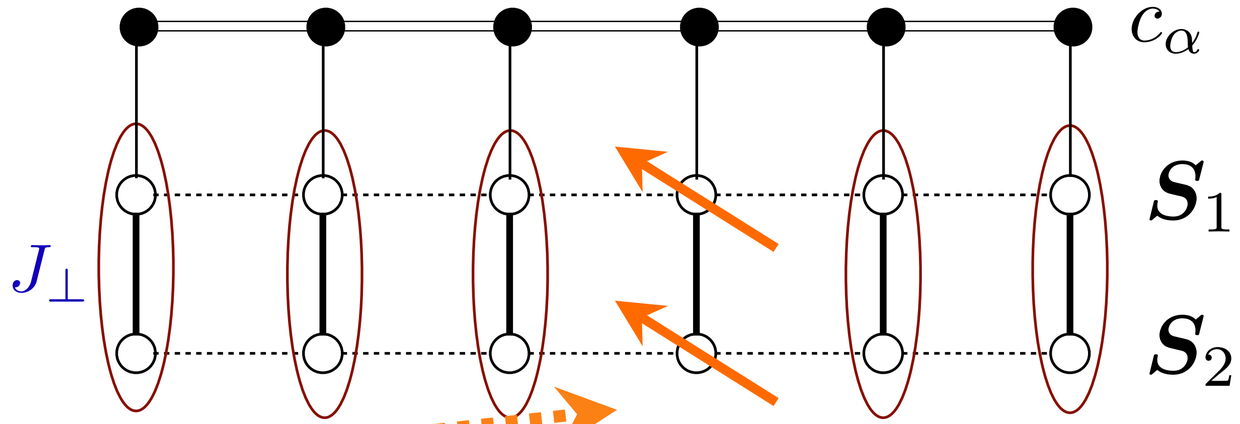
Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla Layer Model of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)

Free holes of density 1+p



Paramagnon

$$\mathcal{H}_{\rm Hubbard} = -\sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{ij}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} + \sum_{\boldsymbol{i}} \left[\frac{3U}{8} \boldsymbol{\mathcal{P}}^2_{\boldsymbol{i}} + U \boldsymbol{\mathcal{P}}_{\boldsymbol{i}} \cdot c^\dagger_{\boldsymbol{i}\alpha} \frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} c_{\boldsymbol{i}\beta} + \frac{m_{\boldsymbol{\mathcal{P}}}}{2} \left(\partial_\tau \boldsymbol{\mathcal{P}}_{\boldsymbol{i}}\right)^2 \right] .$$

$3\ \boldsymbol{\mathcal{P}}$ oscillators' states: $|0,0,0\rangle,\ |1,0,0\rangle,\ |0,1,0\rangle,\ |0,0,1\rangle$

$\boldsymbol{S}_{1,2}$ ancilla qubits states : $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2},\ |\uparrow\uparrow\rangle,\ (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2},\ |\downarrow\downarrow\rangle$

$$\boldsymbol{\mathcal{P}} \sim \boldsymbol{S}_1 - \boldsymbol{S}_2$$

Ya-Hui Zhang

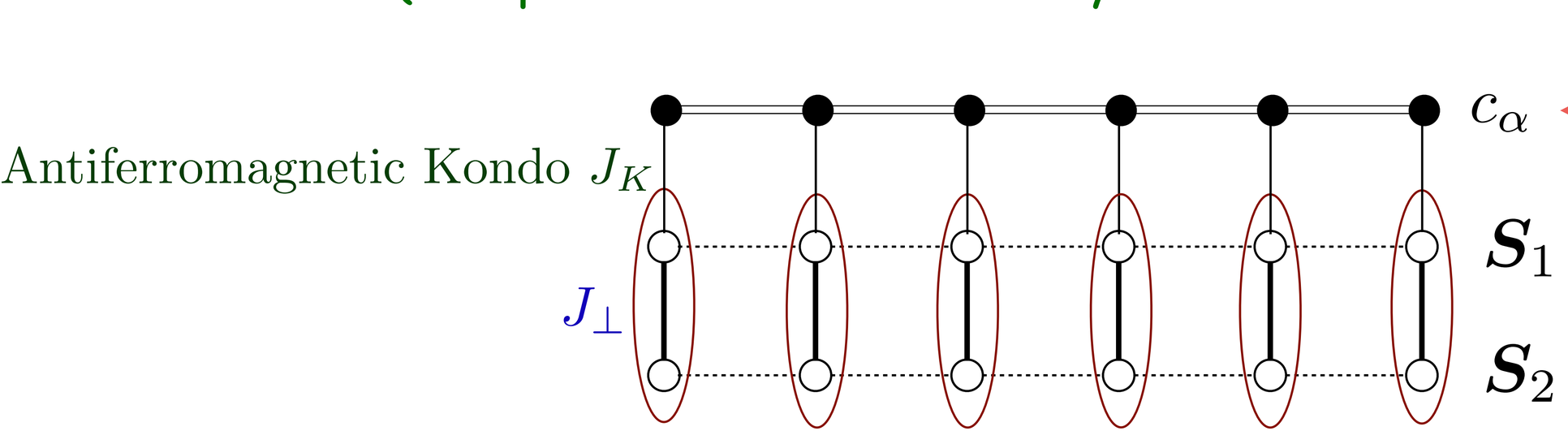
Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla Layer Model of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)

Free holes of density 1+p



$$\mathcal{H}_{\rm ALM} = -\sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{ij}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} + \sum_{\boldsymbol{i}} \frac{J_K}{2} \boldsymbol{S}_{1i} \cdot c^\dagger_{\boldsymbol{i}\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{\boldsymbol{i}\beta} + J_\perp \sum_{\boldsymbol{i}} \boldsymbol{S}_{1\boldsymbol{i}} \cdot \boldsymbol{S}_{2\boldsymbol{i}} \,.$$

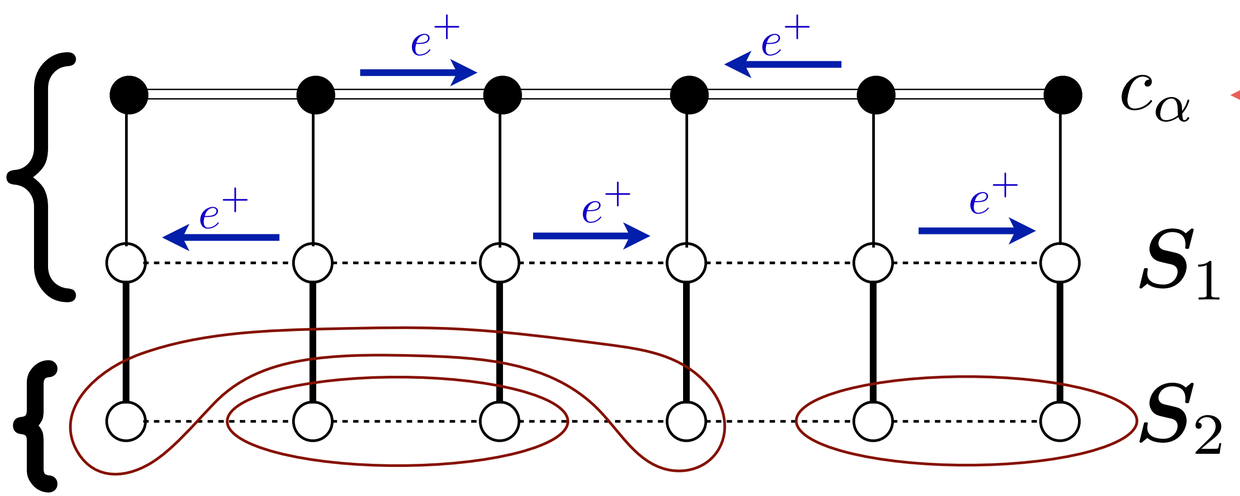
Ya-Hui Zhang

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

ALM of FL* of Hubbard model

Kondo lattice heavy Fermi liquid.
Area $(1+p+1)/2$ $= p/2 \pmod 1$.
Small Fermi surface!

Your favorite spin liquid



Free holes of density 1+p

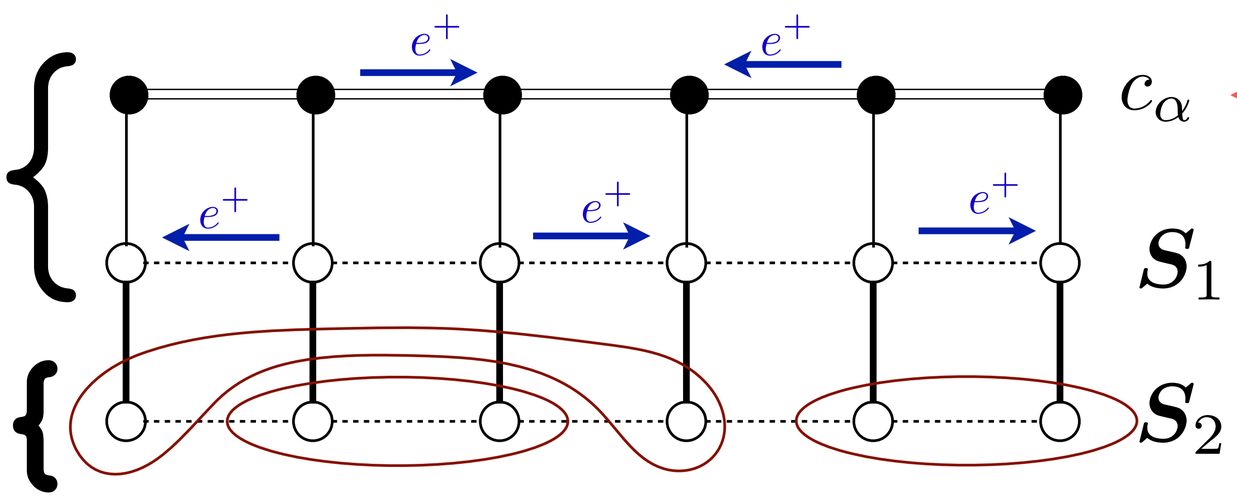
Ya-Hui Zhang

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

ALM of FL* of Hubbard model

Kondo lattice heavy Fermi liquid.
Area $(1+p+1)/2$ $= p/2 \pmod 1$.
Small Fermi surface!

Your favorite spin liquid



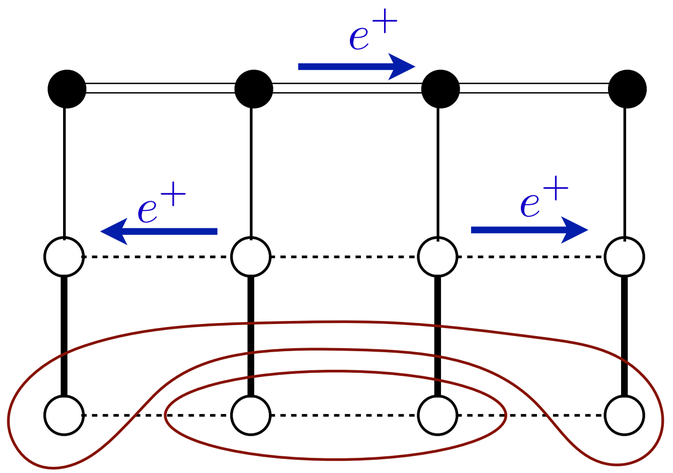
Free holes of density 1+p

Pseudogap metal = Kondo Lattice FL of c_α and $\boldsymbol{S}_1$ $\oplus$ Spin Liquid of $\boldsymbol{S}_2$

Ya-Hui Zhang

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla Layer Model of the Hubbard model



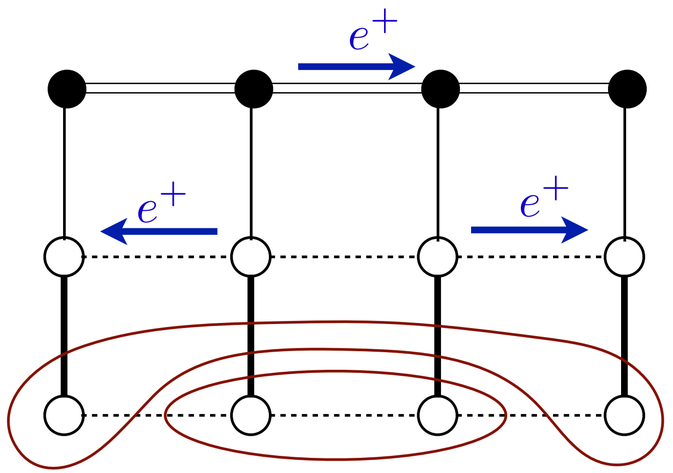
Φ

Higgs field Φ determines the pseudogap. In FL* $\langle\Phi\rangle \neq 0$, antinodal pseudogap is determined by $\langle\Phi\rangle$, and electrons c_α are in 4 area $p/8$ hole pockets.

$$|\mathrm{FL}^*\rangle = |\text{Slater determinant of top two layers with hybridization } \Phi\rangle$$

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla Layer Model of the Hubbard model



Φ

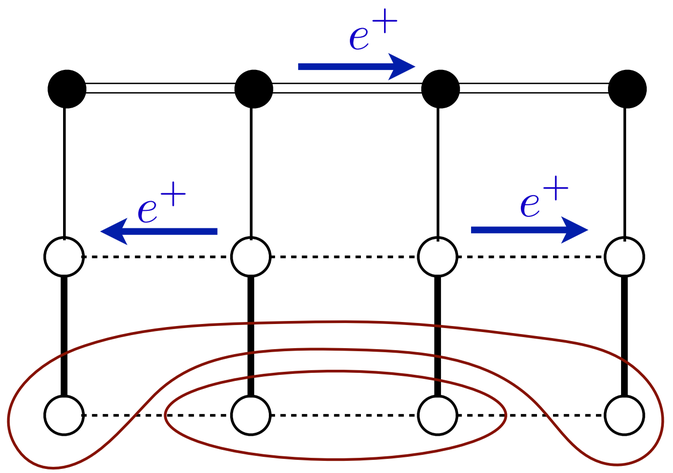
Higgs field Φ determines the pseudogap. In FL* $\langle\Phi\rangle \neq 0$, antinodal pseudogap is determined by $\langle\Phi\rangle$, and electrons c_α are in 4 area $p/8$ hole pockets.

- Spinons f_α in bottom layer are in a π-flux spin liquid

$$|\mathrm{FL}^*\rangle = \bowtie |\text{Slater determinant of top two layers with hybridization } \Phi\rangle \otimes |\text{Slater determinant to spinons } f_\alpha\rangle$$

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla Layer Model of the Hubbard model



Φ

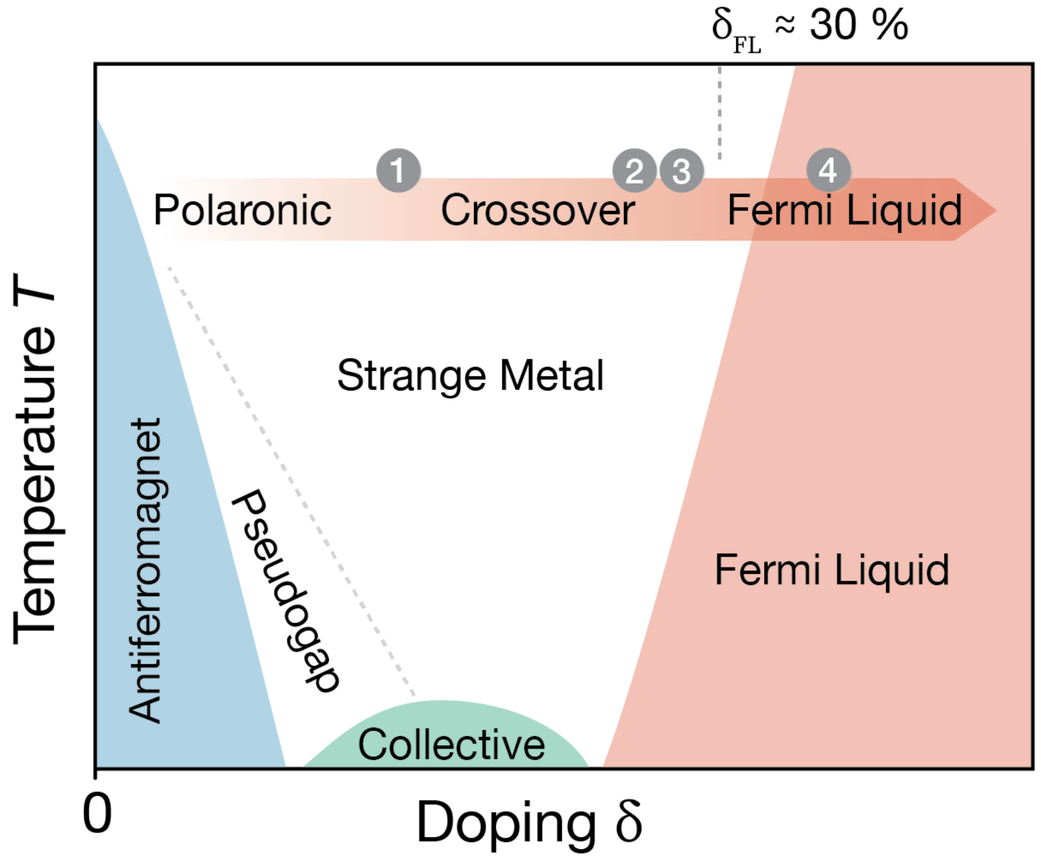
Higgs field Φ determines the pseudogap. In FL* $\langle\Phi\rangle \neq 0$, antinodal pseudogap is determined by $\langle\Phi\rangle$, and electrons c_α are in 4 area $p/8$ hole pockets.

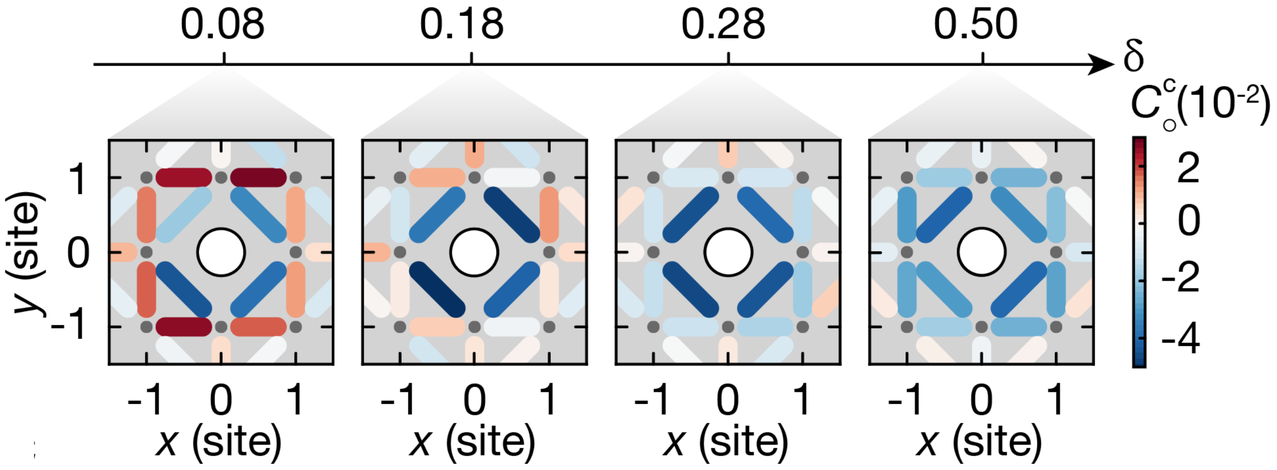
- Spinons f_α in bottom layer are in a π-flux spin liquid

$$|\mathrm{FL}^*\rangle = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2] \bowtie |\text{Slater determinant of top two layers with hybridization } \Phi\rangle \otimes |\text{Slater determinant to spinons } f_\alpha\rangle$$

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)





Ultracold fermionic atoms in optical lattices

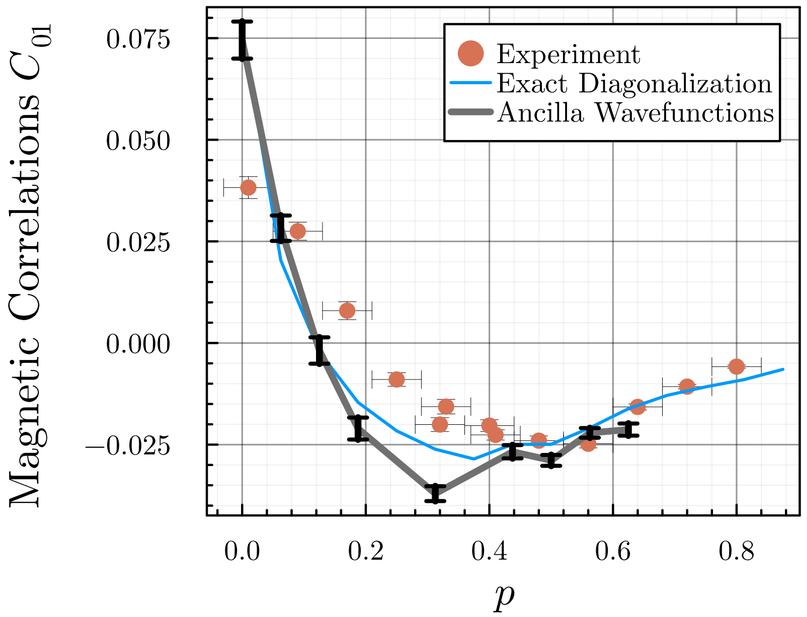
Microscopic evolution of doped Mott insulators from polaronic metal to Fermi liquid

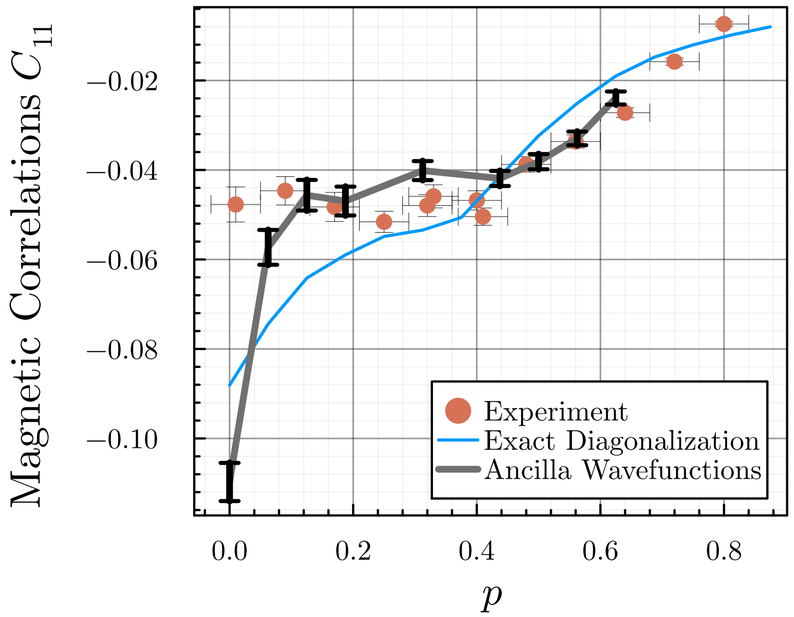
Joannis Koepsell, Dominik Bourgund, Pimonpan Sompet, Sarah Hirthe, Annabelle Bohrdt, Yao Wang, Fabian Grusdt, Eugene Demler, Guillaume Salomon, Christian Gross, Immanuel Bloch

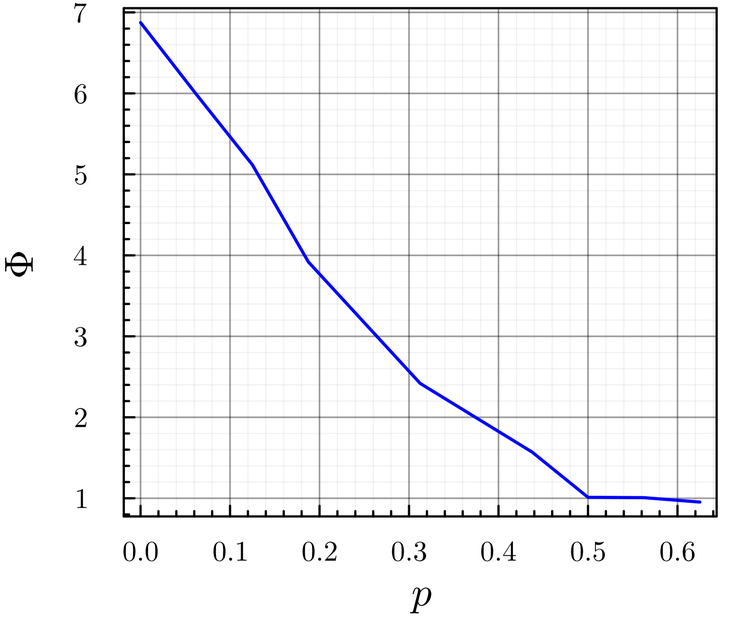
Science **374** (2021) 82

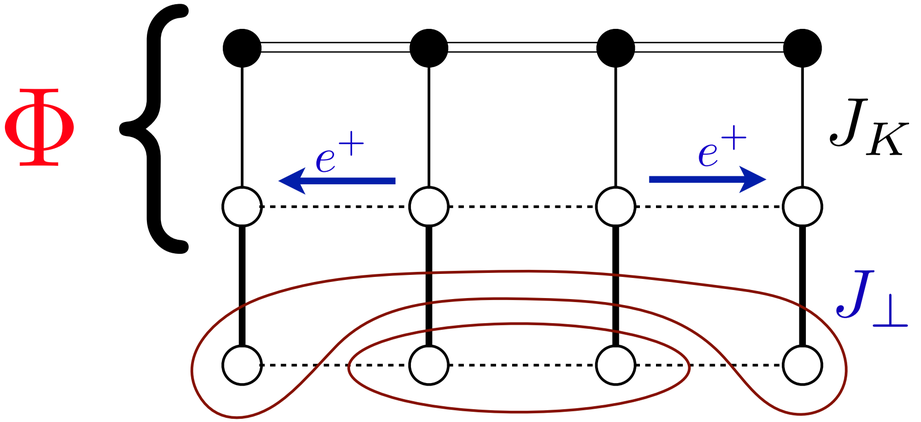
Max Planck Institute of Quantum Optics, Garching

Trial wavefunction for FL* of Hubbard model







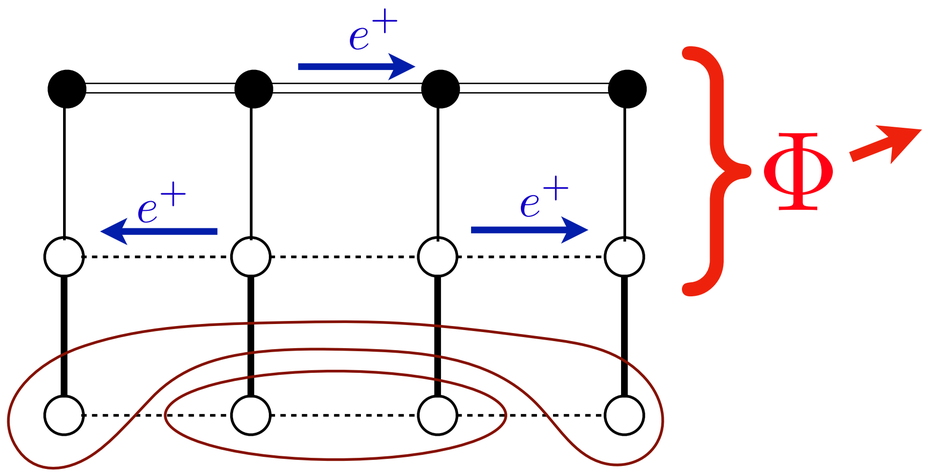


L. Shackleton and Shiwei Zhang, arXiv:2408.02190

Tobias Müller, Yasir Iqbal, S.S., Ronny Thomale, PNAS **122**, e2504261122 (2025)

From FL* to d-wave superconductivity and charge order in the ALM

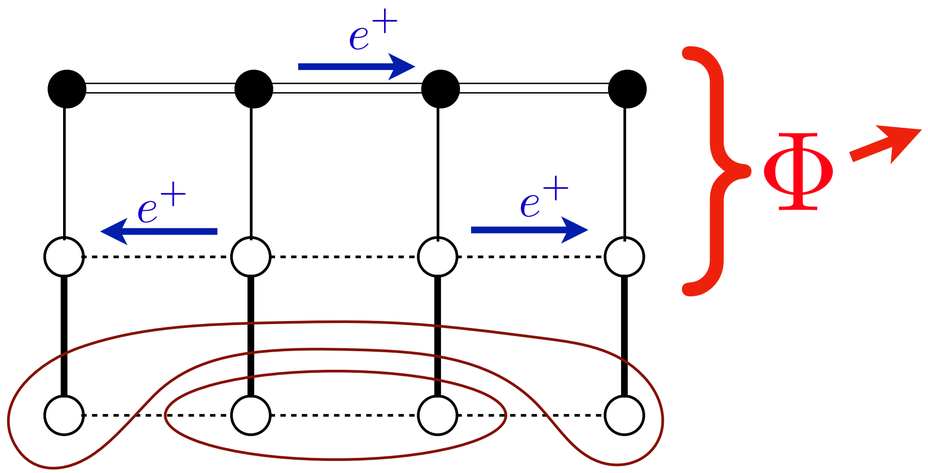
Ancilla Layer Model of the Hubbard model



Higgs field Φ determines the pseudogap. In FL* $\langle \Phi \rangle \neq 0$, antinodal pseudogap is determined by $\langle \Phi \rangle$, and electrons c_α are in 4 area $p/8$ hole pockets.

- Spinons f_α in bottom layer are in a π-flux spin liquid

Ancilla Layer Model of the Hubbard model

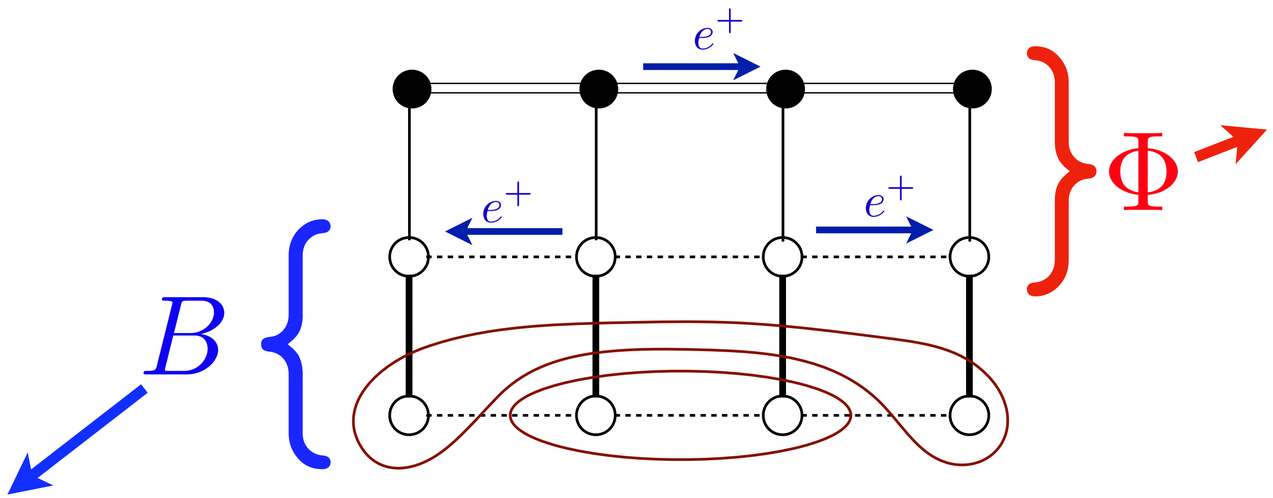


Higgs field Φ determines the pseudogap. In FL* $\langle \Phi \rangle \neq 0$, antinodal pseudogap is determined by $\langle \Phi \rangle$, and electrons c_α are in 4 area $p/8$ hole pockets.

- Spinons f_α in bottom layer are in a π-flux spin liquid with a SU(2) gauge field U.

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

Ancilla Layer Model of the Hubbard model

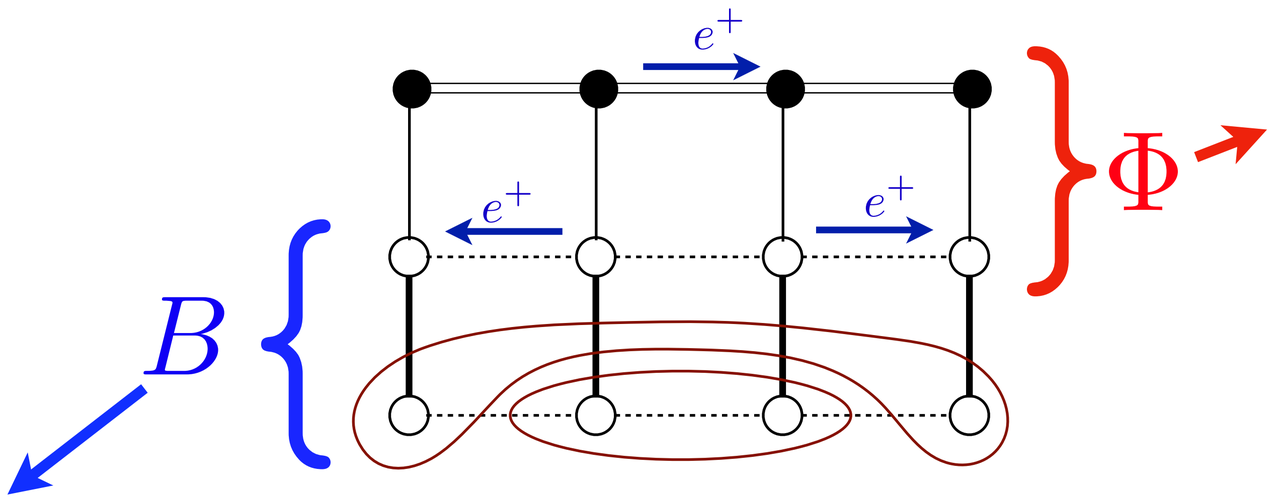


Higgs field Φ determines the pseudogap. In FL* $\langle \Phi \rangle \neq 0$, antinodal pseudogap is determined by $\langle \Phi \rangle$, and electrons c_α are in 4 area $p/8$ hole pockets.

- Spinons f_α in bottom layer are in a π-flux spin liquid with a SU(2) gauge field U.
- Higgs boson B has charge e, and is a SU(2) fundamental.

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

Ancilla Layer Model of the Hubbard model

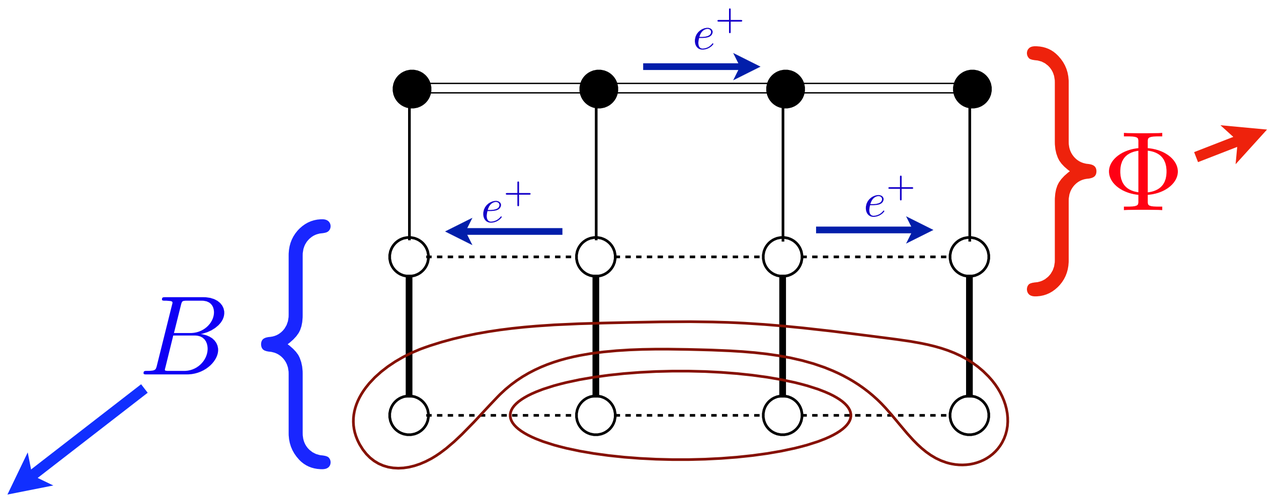


Higgs field Φ determines the pseudogap. In FL* $\langle \Phi \rangle \neq 0$, antinodal pseudogap is determined by $\langle \Phi \rangle$, and electrons c_α are in 4 area $p/8$ hole pockets.

- Spinons f_α in bottom layer are in a π-flux spin liquid with a SU(2) gauge field U.
- Higgs boson B has charge e, and is a SU(2) fundamental.
- Yukawa coupling between c_α, f_α and B.

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

Ancilla Layer Model of the Hubbard model



Higgs field Φ determines the pseudogap. In FL* $\langle \Phi \rangle \neq 0$, antinodal pseudogap is determined by $\langle \Phi \rangle$, and electrons c_α are in 4 area $p/8$ hole pockets.

- Spinons f_α in bottom layer are in a π-flux spin liquid with a SU(2) gauge field U.
- Higgs boson B has charge e, and is a SU(2) fundamental.
- Yukawa coupling between c_α, f_α and B.
- B is a **fractionalized order parameter**, whose composites describe numerous superconducting and charge order parameters!

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

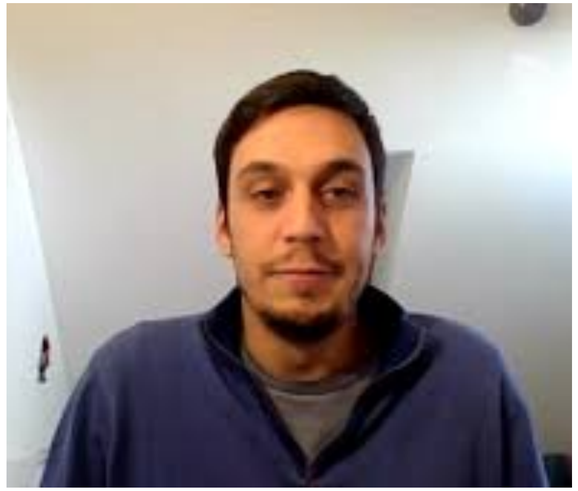
Monte-Carlo of Born-Oppenheimer approx.

Classical B, U, quantum c_α, f_α.

Thermal SU(2) lattice gauge theory of the cuprate pseudogap: reconciling Fermi arcs and hole pockets

H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

Maine Christos
Caltech



Pietro Bonetti

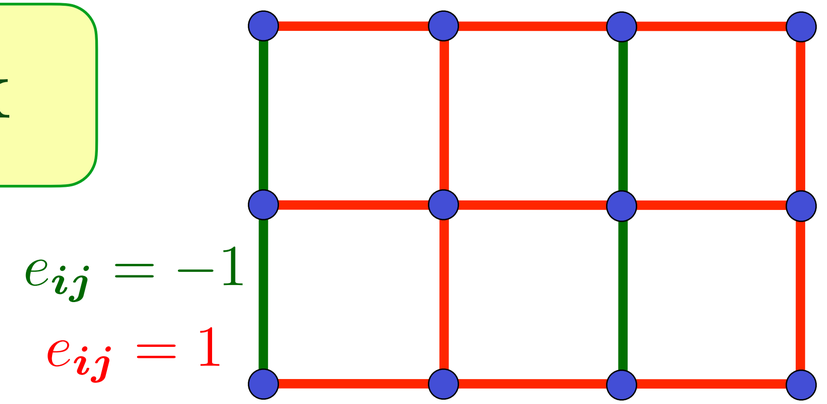
Harshit Pandey

Ravi Shanker

Sayantan Sharma

The Institute of Mathematical Sciences, Chennai

f_α and B both move in π-flux



Symmetry	f_α	B_a
T_x	$(-1)^y f_\alpha$	$(-1)^y B_a$
T_y	f_α	B_a
P_x	$(-1)^x f_\alpha$	$(-1)^x B_a$
P_y	$(-1)^y f_\alpha$	$(-1)^y B_a$
P_{xy}	$(-1)^{xy} f_\alpha$	$(-1)^{xy} B_a$
$\mathcal{T}$	$(-1)^{x+y} \varepsilon_{\alpha\beta} f_\beta$	$(-1)^{x+y} B_a$

Projective transformations of the f spinons and B chargons on lattice sites $\boldsymbol{i} = (x, y)$ under the symmetries

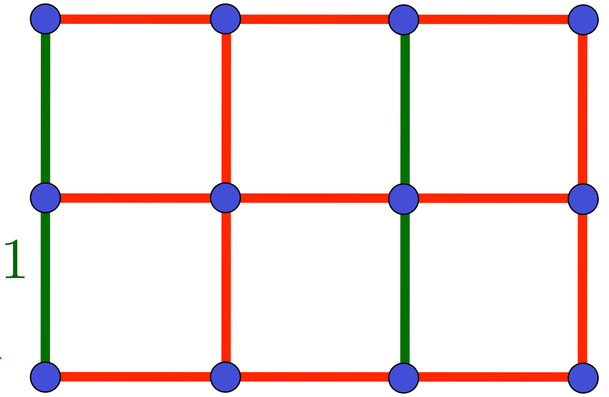
$T_x : (x, y) \to (x+1, y)$; $T_y : (x, y) \to (x, y+1)$;
$P_x : (x, y) \to (-x, y)$; $P_y : (x, y) \to (x, -y)$;
$P_{xy} : (x, y) \to (y, x)$; and time-reversal $\mathcal{T}$.
The indices α, β refer to global SU(2) spin, while the index $a = 1, 2$ refers to gauge SU(2).

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

f_α and B both move in π-flux

Symmetry	f_α	B_a
T_x	$(-1)^y f_\alpha$	$(-1)^y B_a$
T_y	f_α	B_a
P_x	$(-1)^x f_\alpha$	$(-1)^x B_a$
P_y	$(-1)^y f_\alpha$	$(-1)^y B_a$
P_{xy}	$(-1)^{xy} f_\alpha$	$(-1)^{xy} B_a$
$\mathcal{T}$	$(-1)^{x+y} \varepsilon_{\alpha\beta} f_\beta$	$(-1)^{x+y} B_a$

Projective transformations of the f spinons and B chargons on lattice sites $\boldsymbol{i} = (x, y)$ under the symmetries
$T_x : (x, y) \to (x+1, y)$; $T_y : (x, y) \to (x, y+1)$;
$P_x : (x, y) \to (-x, y)$; $P_y : (x, y) \to (x, -y)$;
$P_{xy} : (x, y) \to (y, x)$; and time-reversal $\mathcal{T}$.
The indices α, β refer to global SU(2) spin, while the index $a = 1, 2$ refers to gauge SU(2).



Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim$

$$\Delta_{ij} = \Delta_{ji} = \varepsilon_{ab} B_{ai} e_{ij} U_{ij} B_{bj}$$

site charge density: $\left\langle c^\dagger_{i\alpha} c_{i\alpha} \right\rangle \sim \rho_i = B^\dagger_i B_i$

bond density: $\left\langle c^\dagger_{i\alpha} c_{j\alpha} + c^\dagger_{j\alpha} c_{i\alpha} \right\rangle$

$$\sim Q_{ij} = Q_{ji} = \mathrm{Im}\left(B^\dagger_i e_{ij} U_{ij} B_j \right)$$

bond current: $i \left\langle c^\dagger_{i\alpha} c_{j\alpha} - c^\dagger_{j\alpha} c_{i\alpha} \right\rangle$

$$\sim J_{ij} = -J_{ji} = \mathrm{Re}\left(B^\dagger_i e_{ij} U_{ij} B_j \right)$$

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

Energy functional for B and U: $\mathcal{E}[B,U] = \mathcal{E}_2[B,U] + \mathcal{E}_4[B,U] + \mathcal{E}_{YM}[U]$

$$\mathcal{E}_2[B,U] = (r + 2\sqrt{2}w) \sum_{\boldsymbol{i}} B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}} + iw \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(B_{\boldsymbol{i}}^\dagger U_{\boldsymbol{ij}} B_{\boldsymbol{j}} - B_{\boldsymbol{j}}^\dagger U_{\boldsymbol{ji}} B_{\boldsymbol{i}} \right)$$

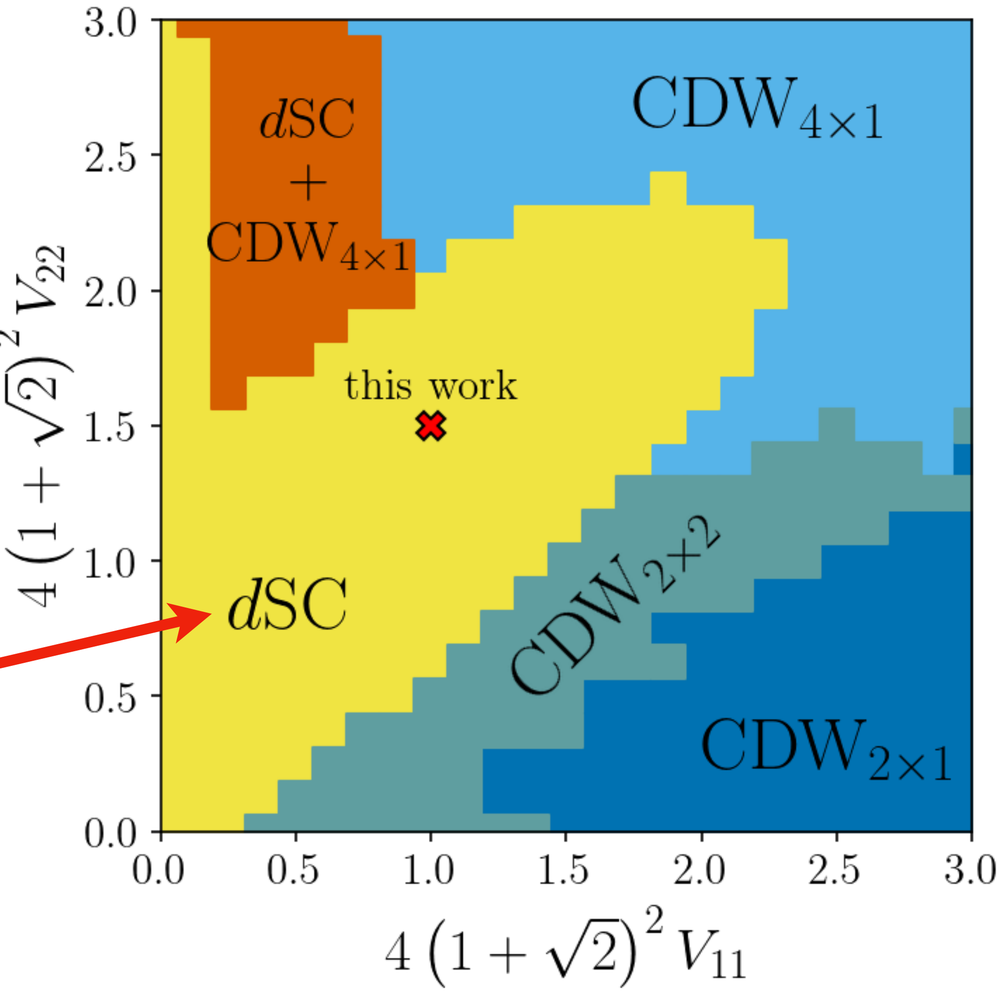
$$\mathcal{E}_4[B,U] = \frac{u}{2} \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}}^2 + V_1 \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{y}}} \right) + g \sum_{\langle \boldsymbol{ij} \rangle} |\Delta_{\boldsymbol{ij}}|^2 + J_1 \sum_{\langle \boldsymbol{ij} \rangle} Q_{\boldsymbol{ij}}^2 + K_1 \sum_{\langle \boldsymbol{ij} \rangle} J_{\boldsymbol{ij}}^2$$
$$+ V_{11} \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}+\hat{\boldsymbol{y}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}-\hat{\boldsymbol{y}}} \right) + V_{22} \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+2\hat{\boldsymbol{x}}+2\hat{\boldsymbol{y}}} + \rho_{\boldsymbol{i}+2\hat{\boldsymbol{x}}-2\hat{\boldsymbol{y}}} \right)$$

$$\mathcal{E}_{YM}[U] = \kappa \sum_{\square} \left[1 - \frac{1}{2} \mathrm{ReTr} \prod_{\boldsymbol{ij} \in \square} U_{\boldsymbol{ij}} \right]$$

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

At $T = 0$, minimize $\mathcal{E}[B, U]$.

d-SC with
4 nodal quasiparticles
and $v_F \gg v_\Delta$.



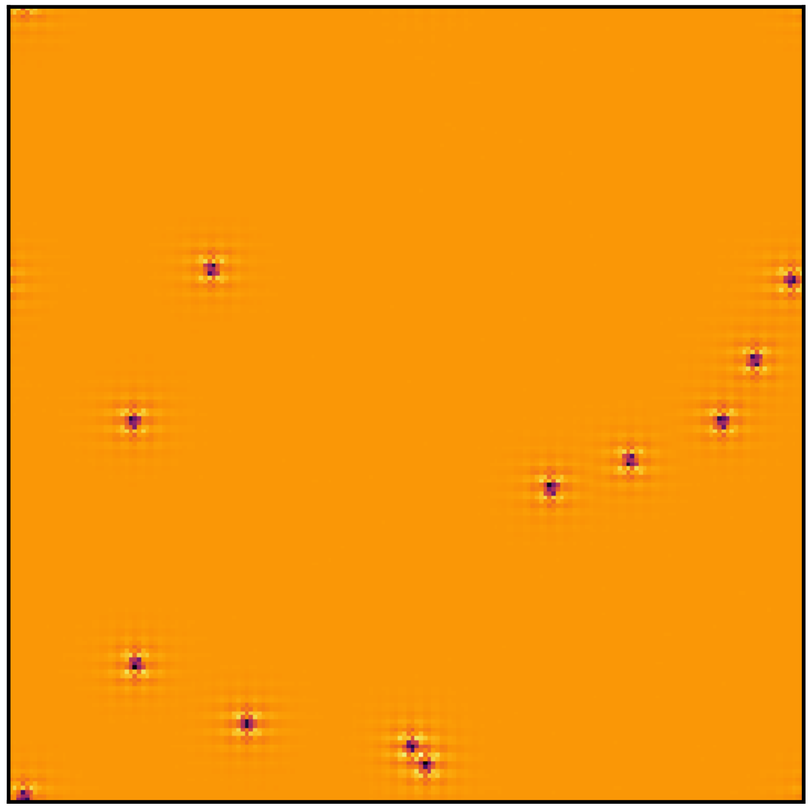
Parameters chosen so that the ground state is a d-wave superconductor,
and second best state is a period-4 stripe.

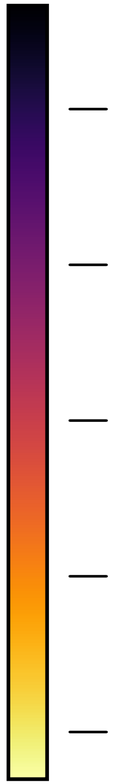
Monte Carlo at a temperature T

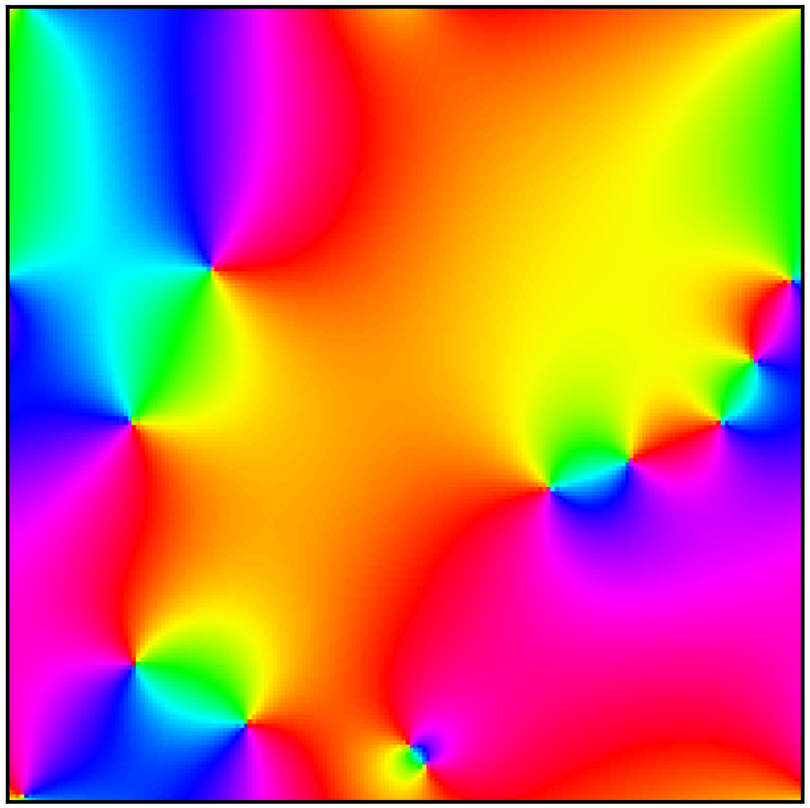
$$\mathcal{Z}_{2+0} = \int \prod_{i} \mathcal{D}B_{i} \int \prod_{\langle ij \rangle} \mathcal{D}U_{ij} \exp\left[-\mathcal{E}[B,U]/T\right]$$

- Simulation of classical, thermal theory for bosons B, U defined by $\mathcal{Z}_{2+0}$
- Diagonalize 3-layer fermion Hamiltonian for c, f_1, f for each snapshot of B, U, and average.

Monte Carlo at a temperature T



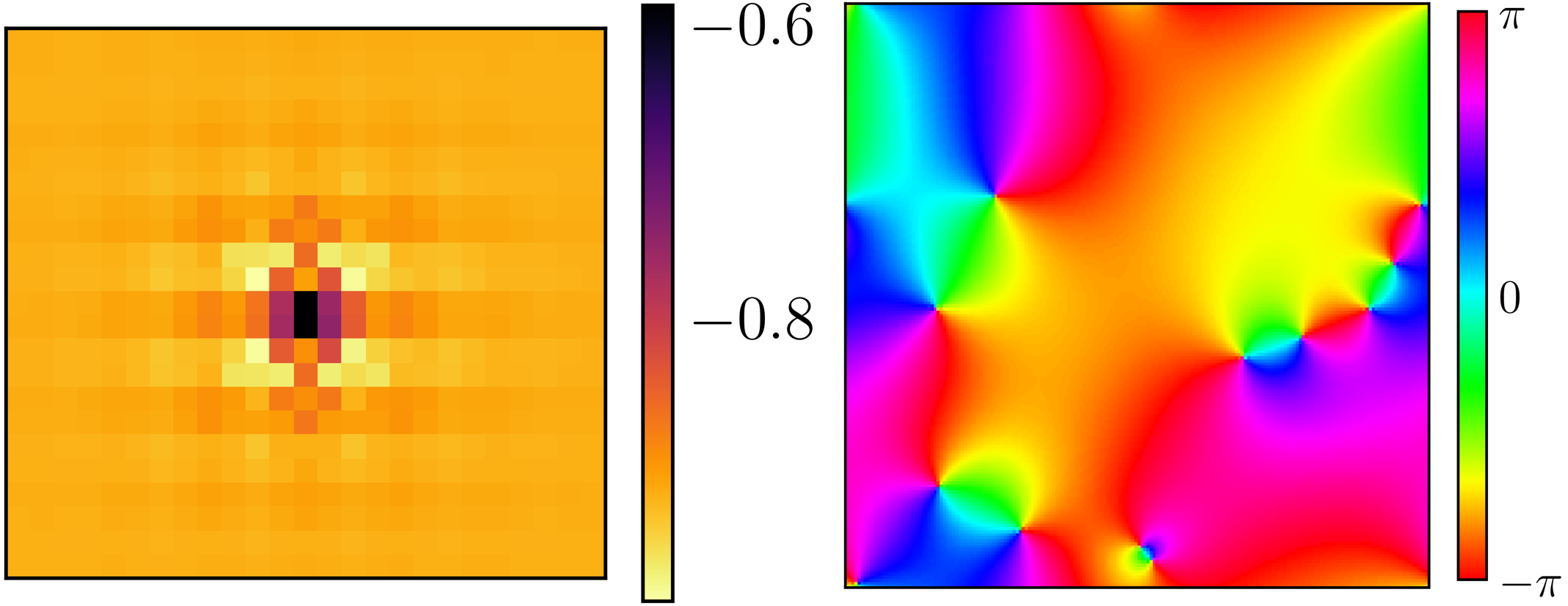




Bond density

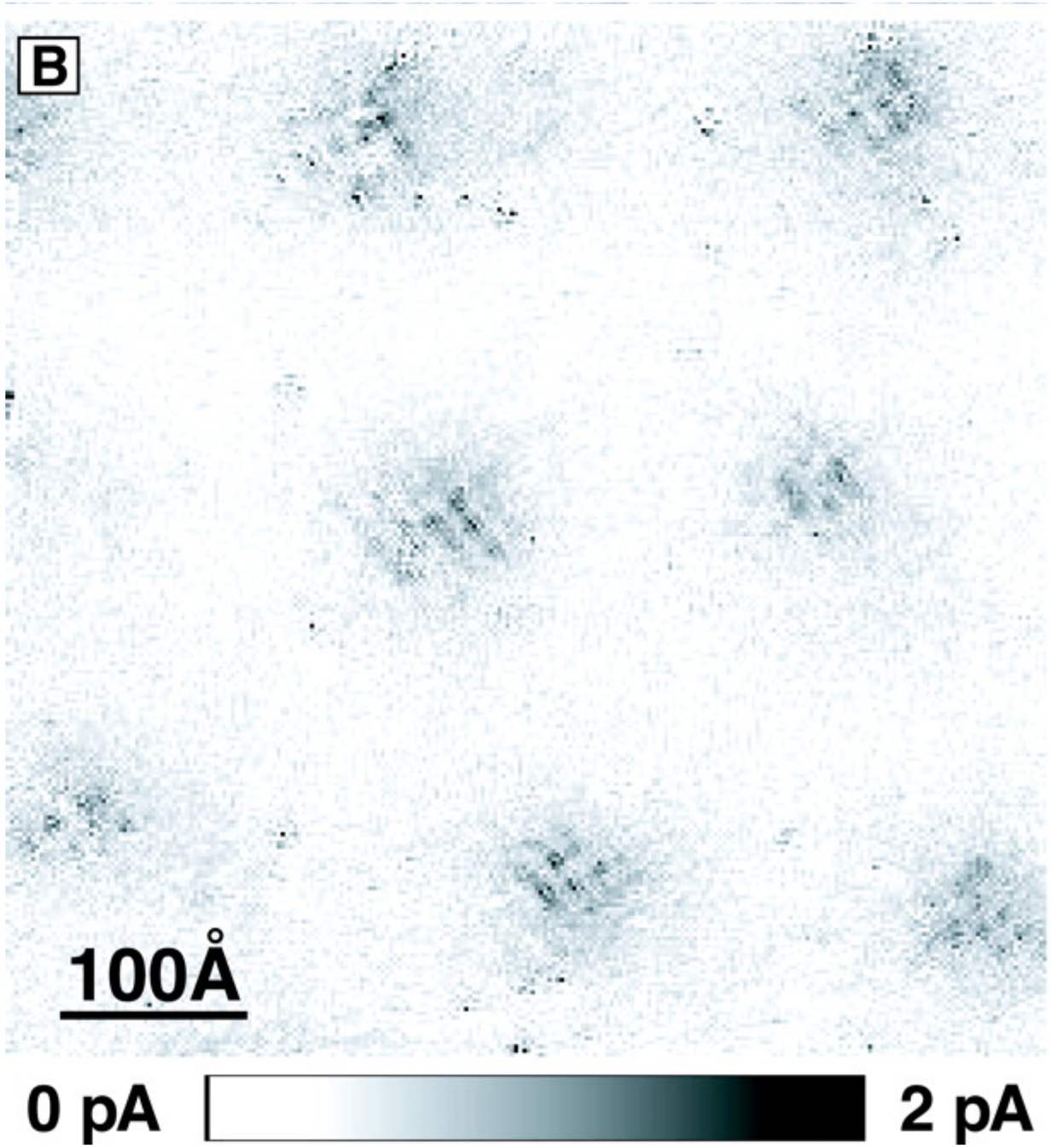
Phase of pairing amplitude

Monte Carlo at a temperature T



Bond density

Phase of pairing amplitude



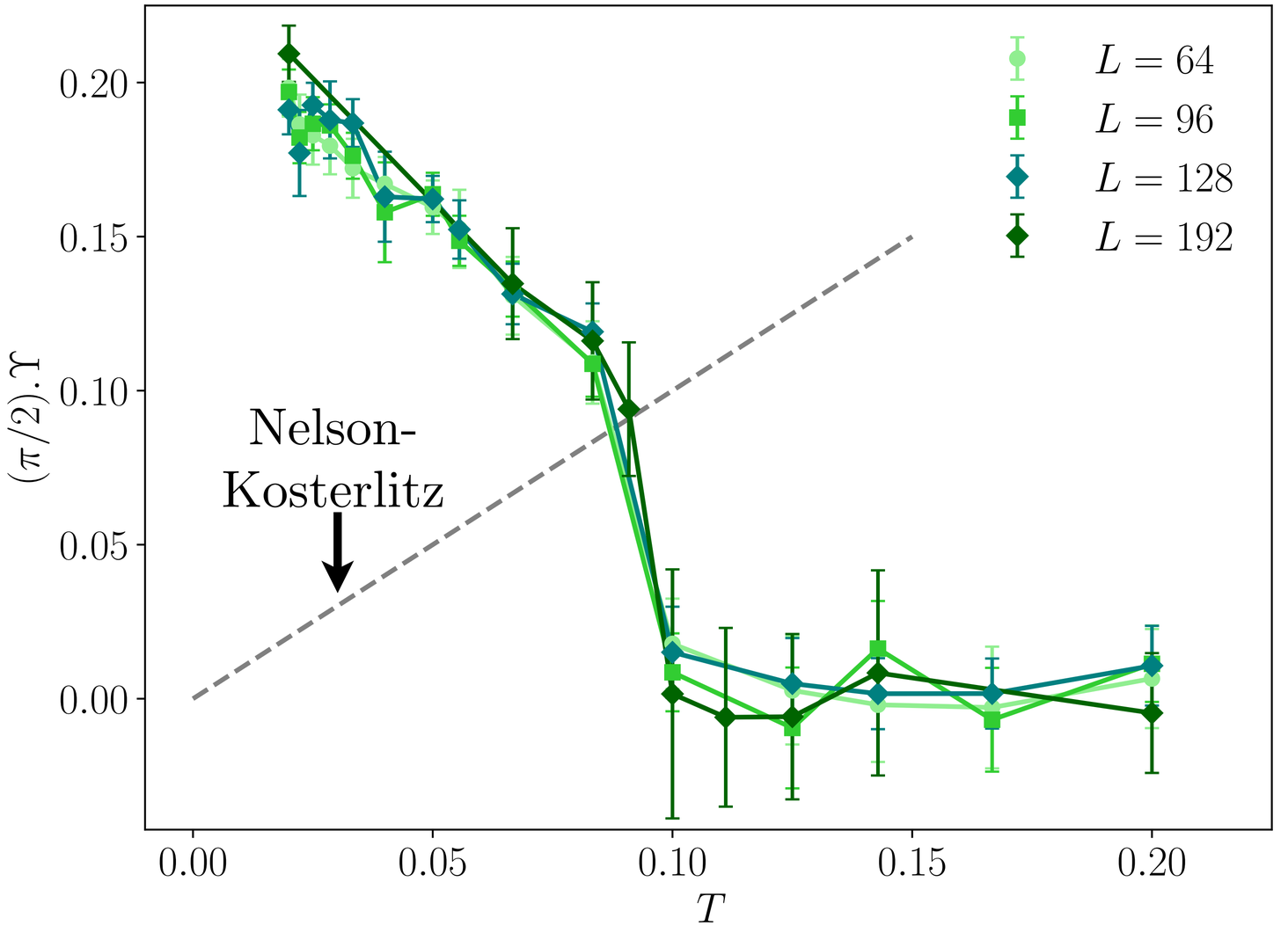
A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $Bi_2Sr_2CaCu_2O_{8+\delta}$

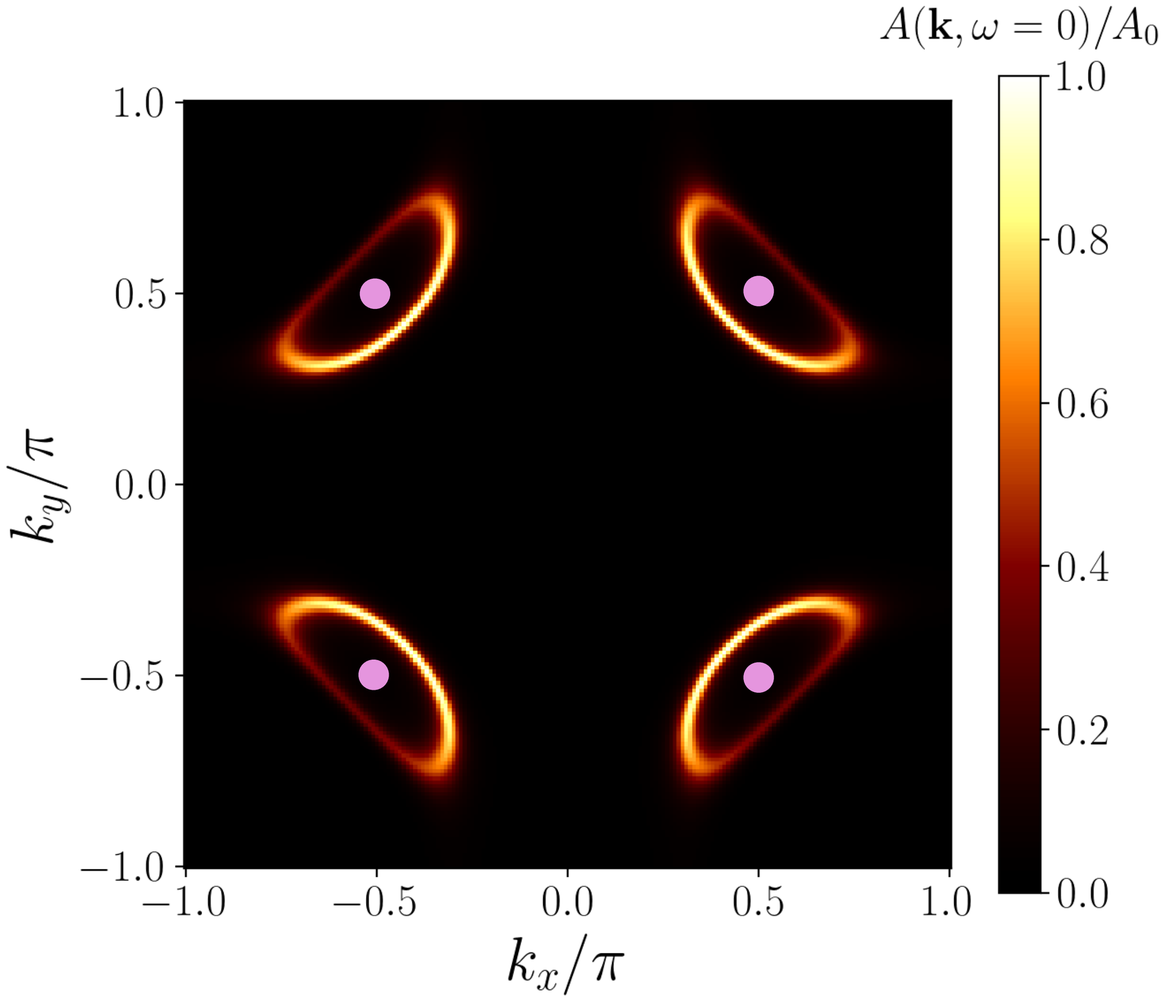
J. E. Hoffman, E. W. Hudson,
K. M. Lang, V. Madhavan,
H. Eisaki, S. Uchida, J.C. Davis

Science **295**, 466 (2002)

Monte Carlo at a temperature T

$\Upsilon =$ Helicity Modulus

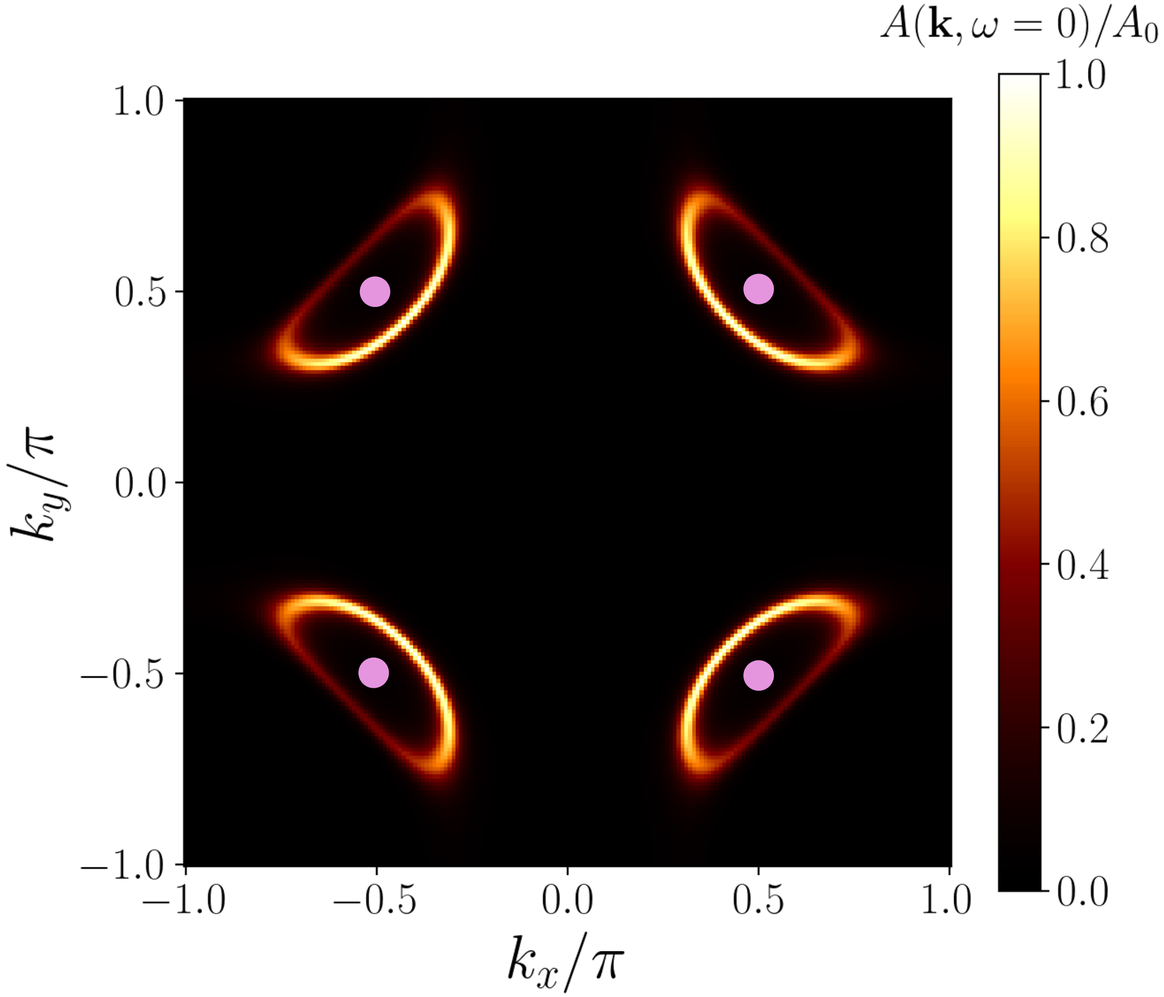


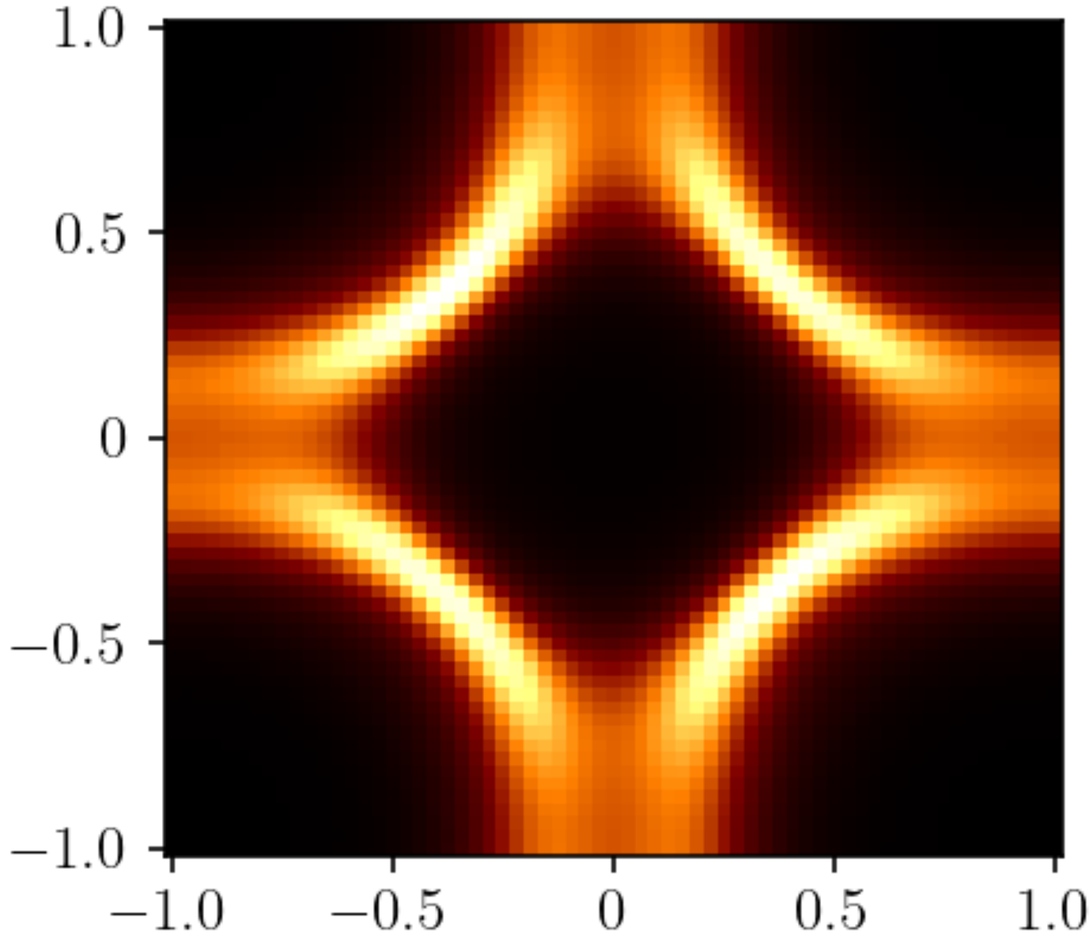


FL* fermionic spectrum with $B = 0$, $U = 1$

4 holes pockets of size $p/8$;

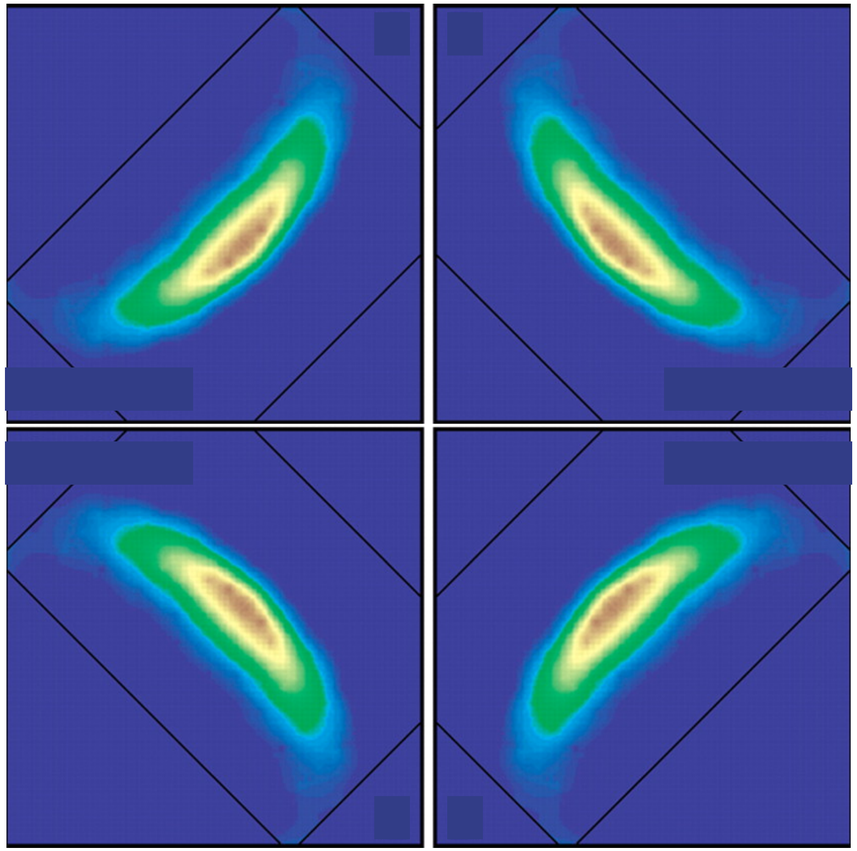
4 nodal spinons





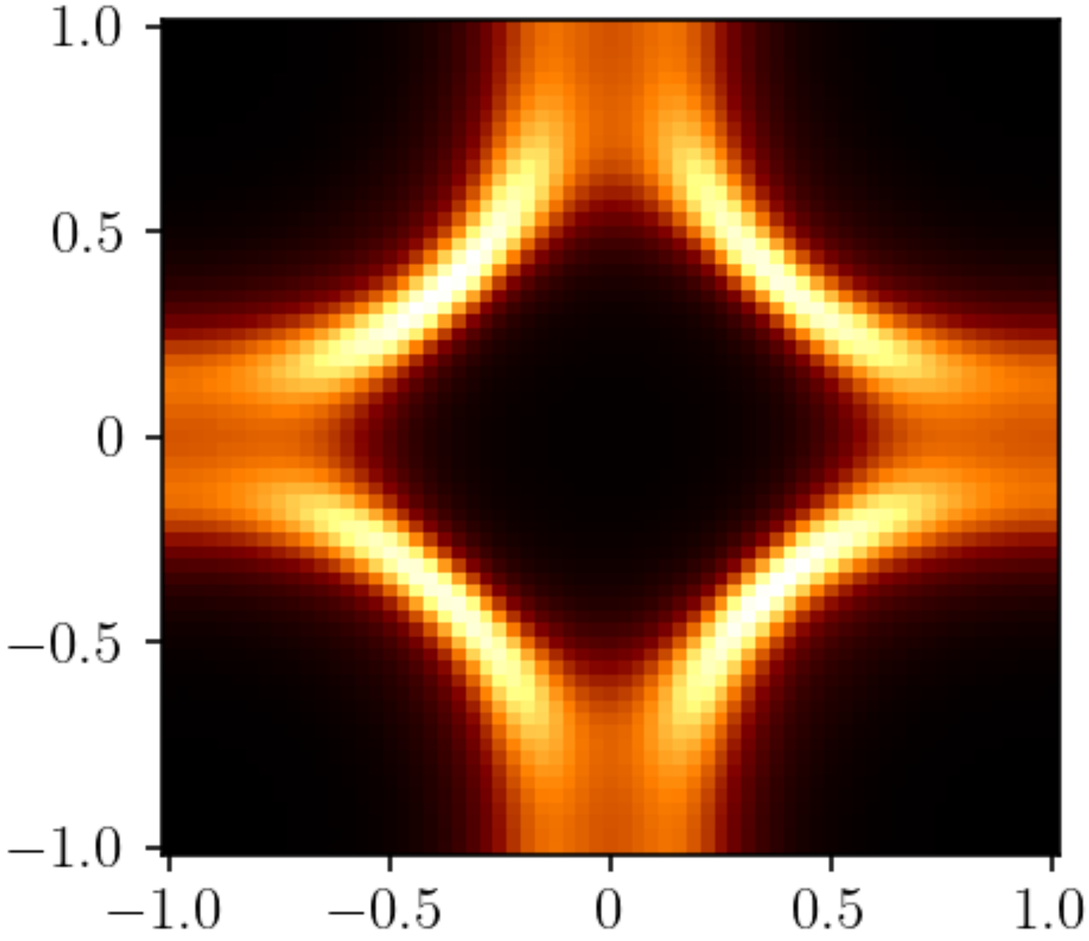
FL* fermionic spectrum with $B = 0$, $U = 1$
4 holes pockets of size $p/8$;
4 nodal spinons

Monte Carlo at a temperature $T > T_{KT}$

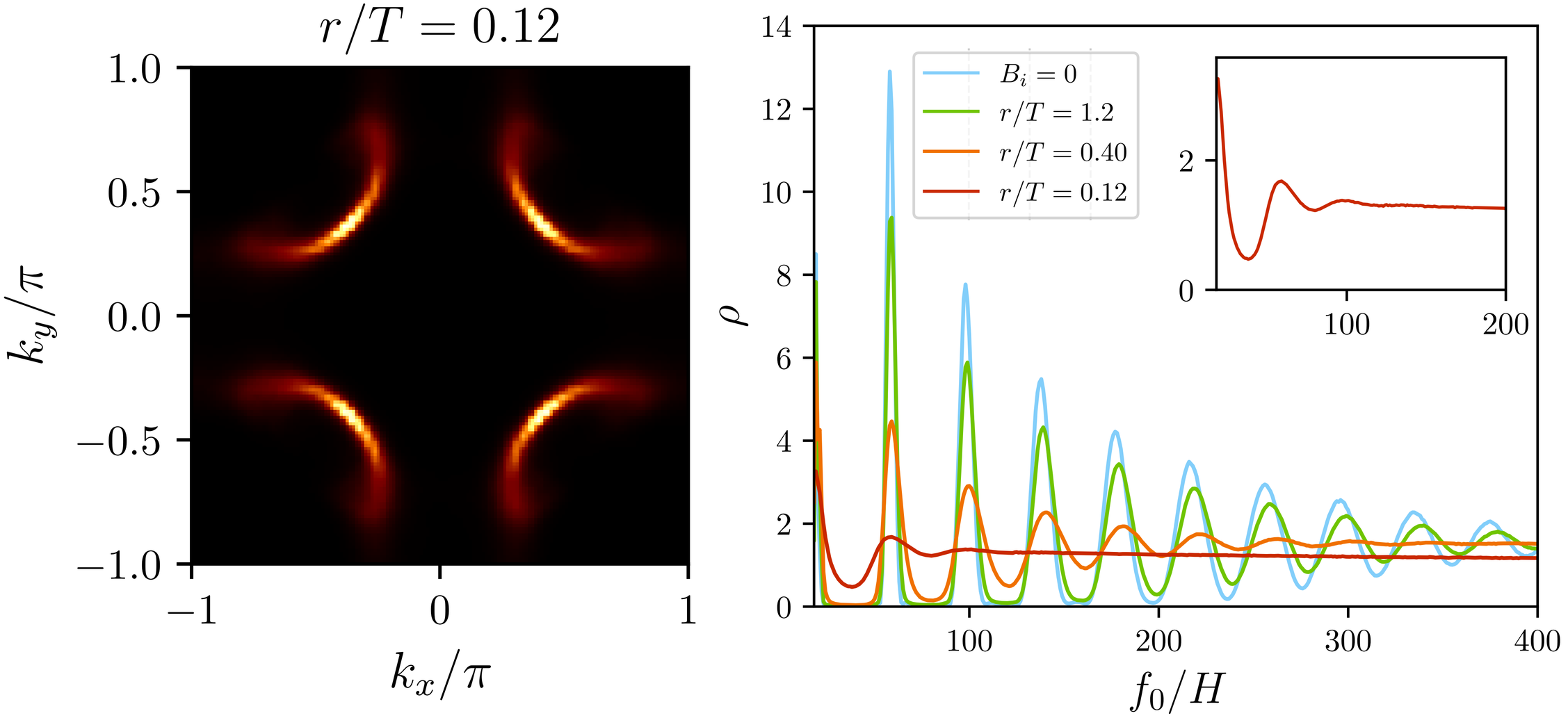


Kyle M. Shen, . . . Z.-X. Shen, Science **307**, 901 (2005)

Photoemission observations



Monte Carlo at a temperature $T > T_{KT}$

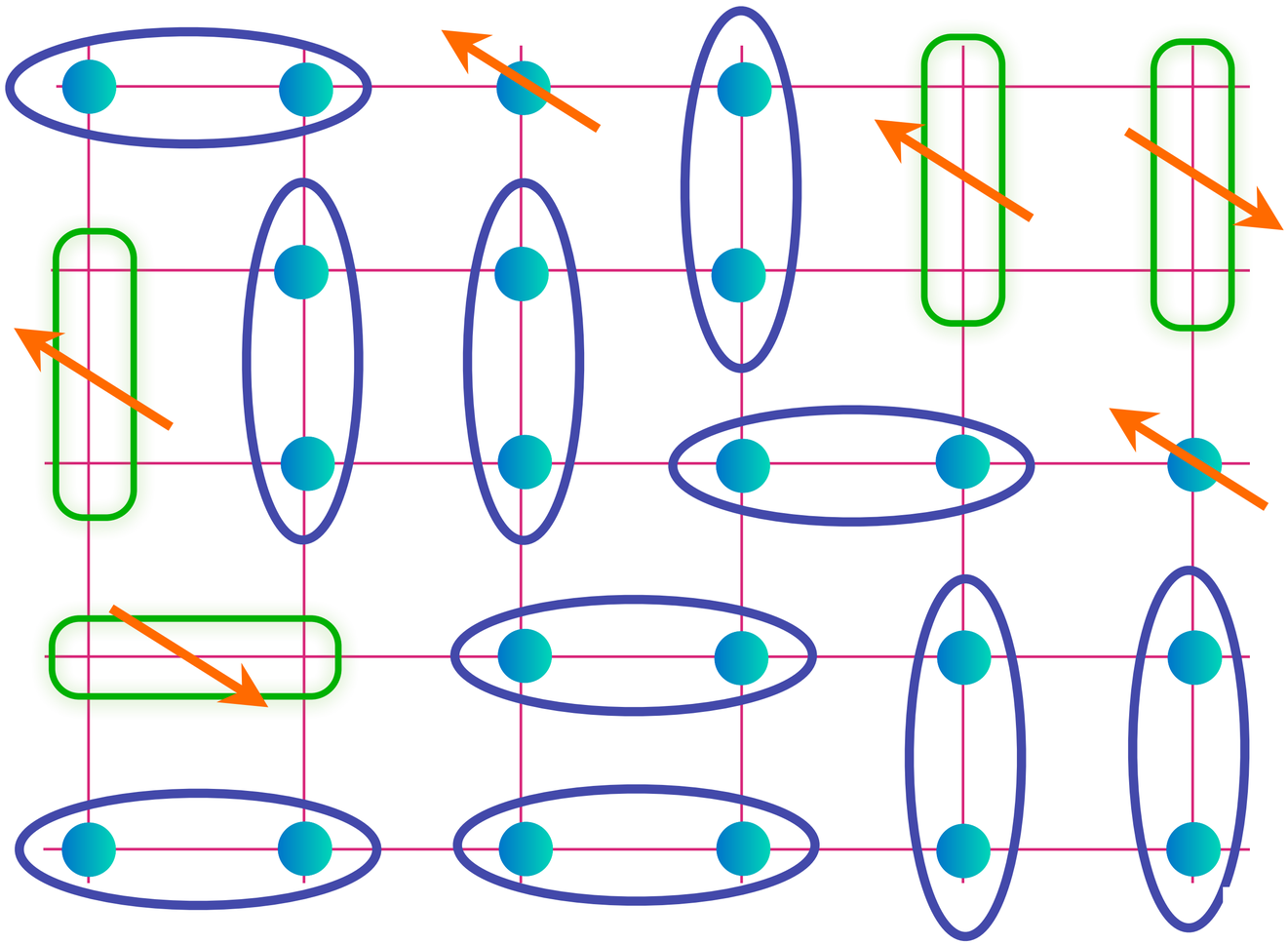


Thermal SU(2) gauge theory of mixing between holes and spinons mediated by Yukawa couplings to a SU(2) fundamental, charge $+e$ Higgs boson B.

Quantum oscillations survive even when pockets have turned to arcs in photoemission.

The cuprate phase diagram

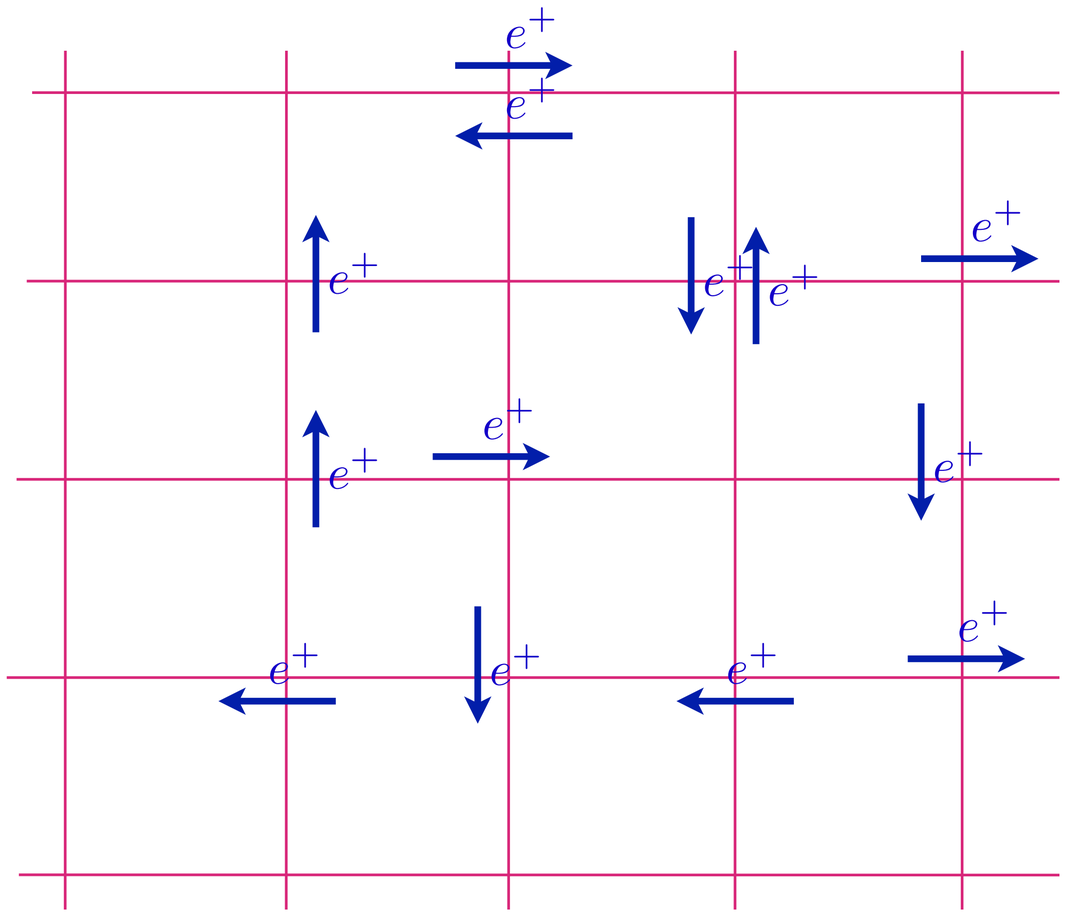
FL*



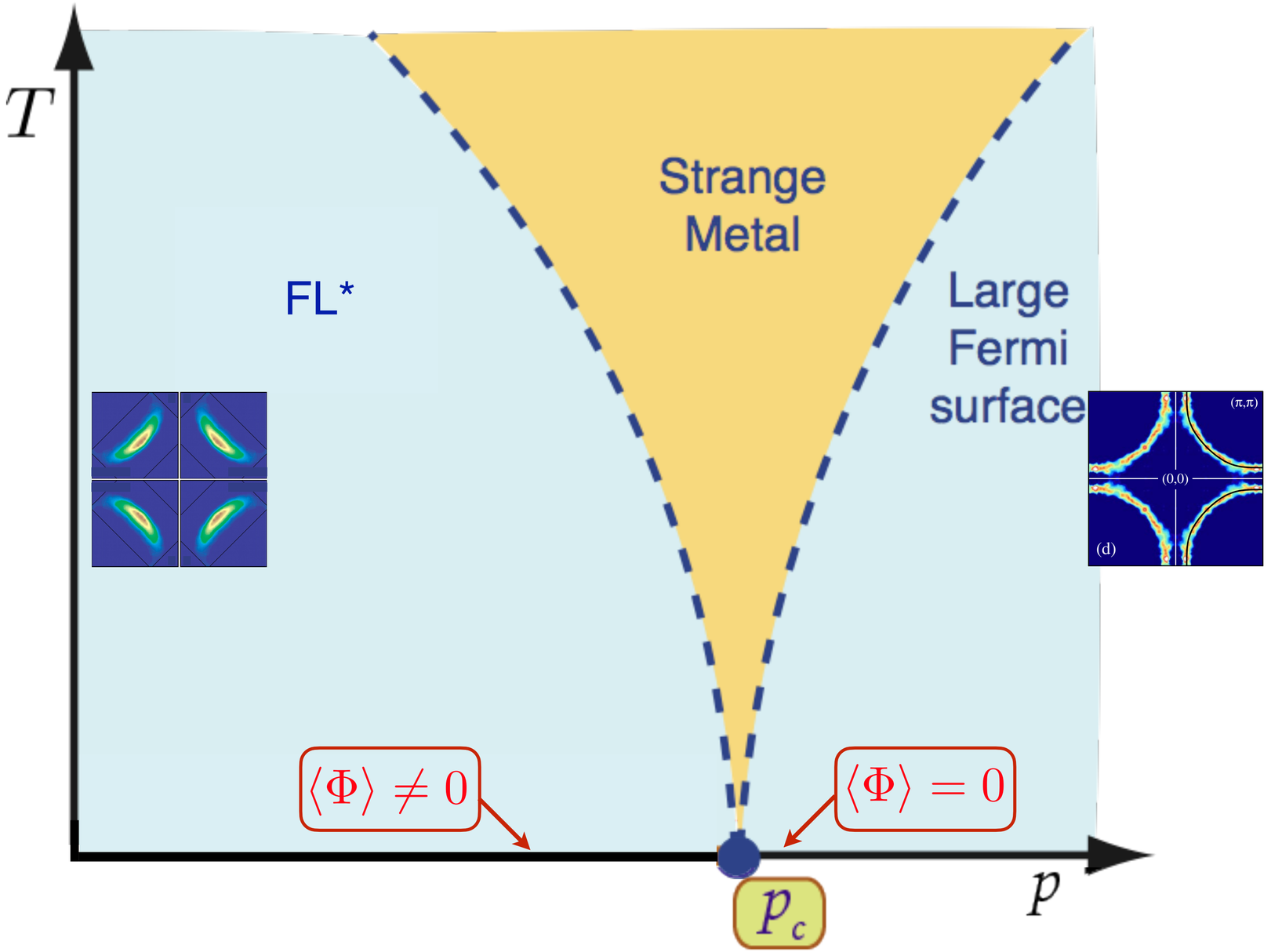
$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

$= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

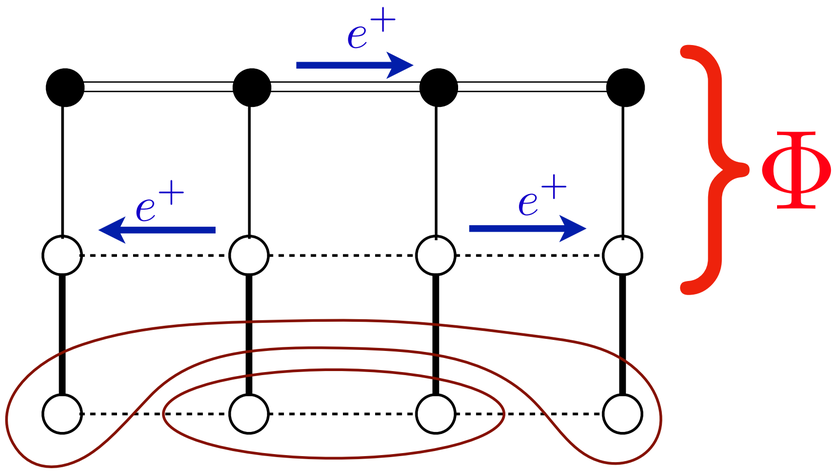
Ordinary metal



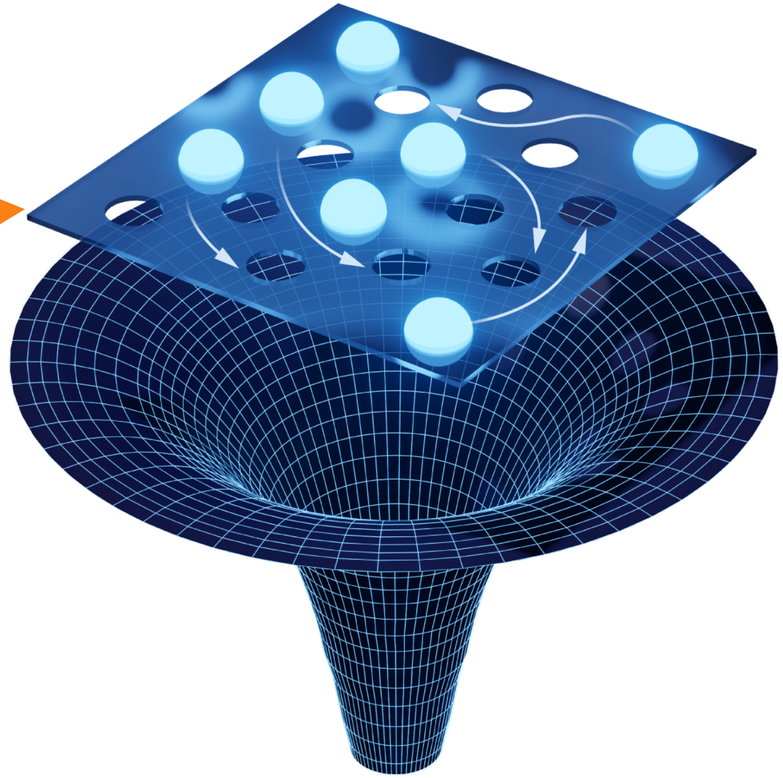
At large p, we obtain a gas of nearly free fermionic holes of density $1+p$ (relative to the filled band with 2 electrons per site)



Quantum-criticality
of a quantum phase transition
between two metals
(FL* and FL) at $p = p_c$,
with no symmetry breaking.



Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

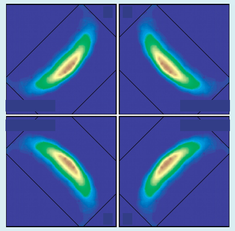


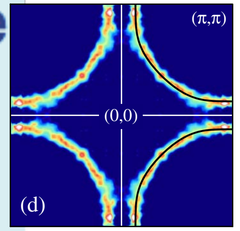
T

Strange Metal

FL*

Large Fermi surface





$\langle \Phi \rangle \neq 0$

$\langle \Phi \rangle = 0$

p_c

p

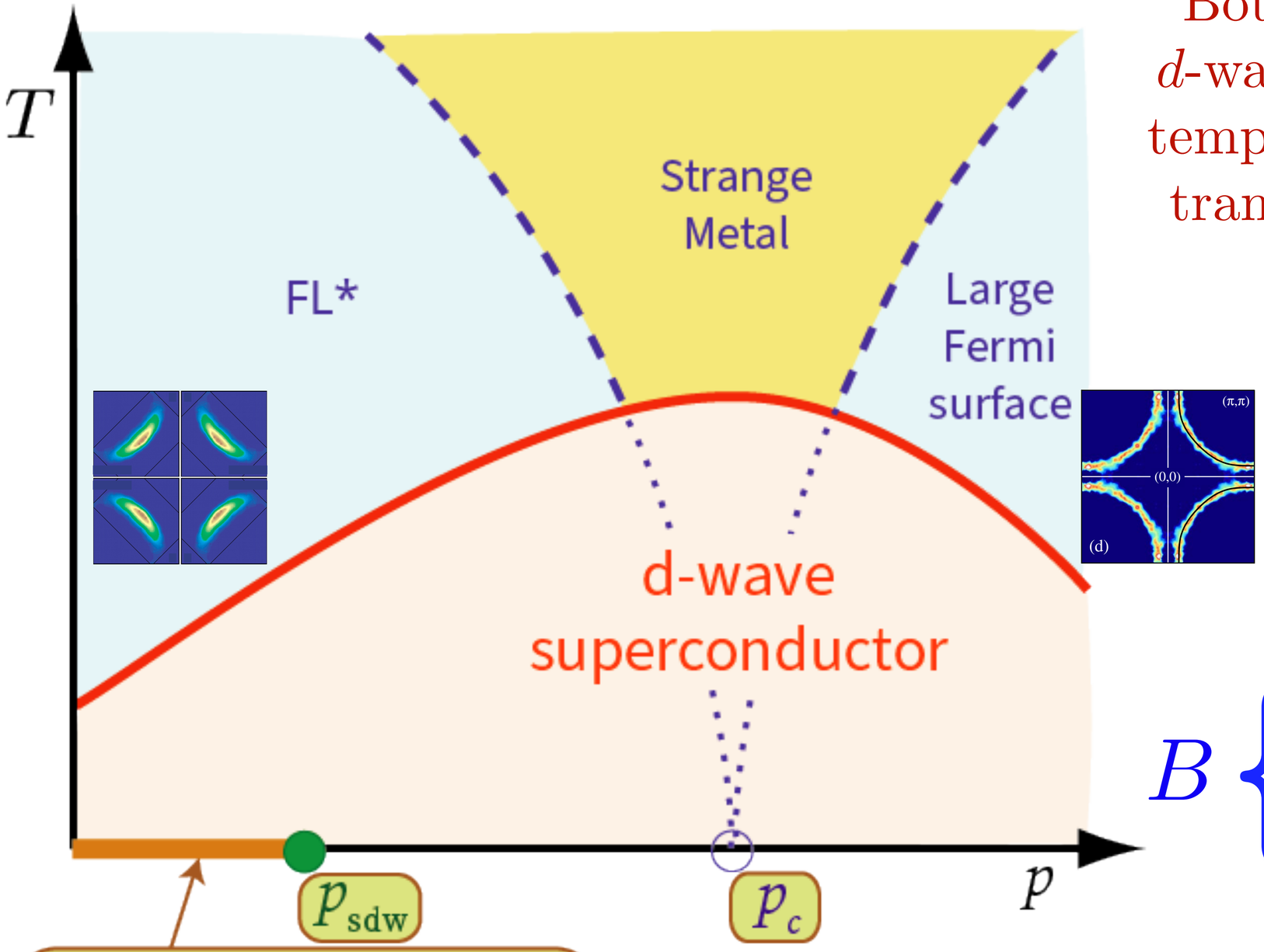
Explain with a local, two-dimensional extension of the Sachdev-Ye-Kitaev (SYK) model of mobile electrons, the 2D-Yukawa-SYK model with a spatially random Yukawa coupling between the large Fermi surface and Φ: a critical charge liquid

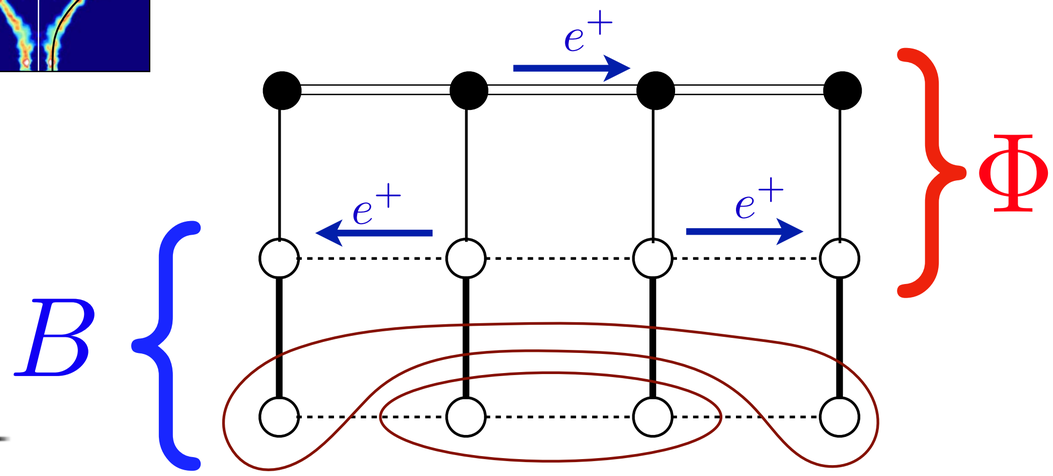
Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)

Both metals lead to the same d-wave superconductor at lower temperatures, and so there is no transition at $p = p_c$ within the superconducting state.

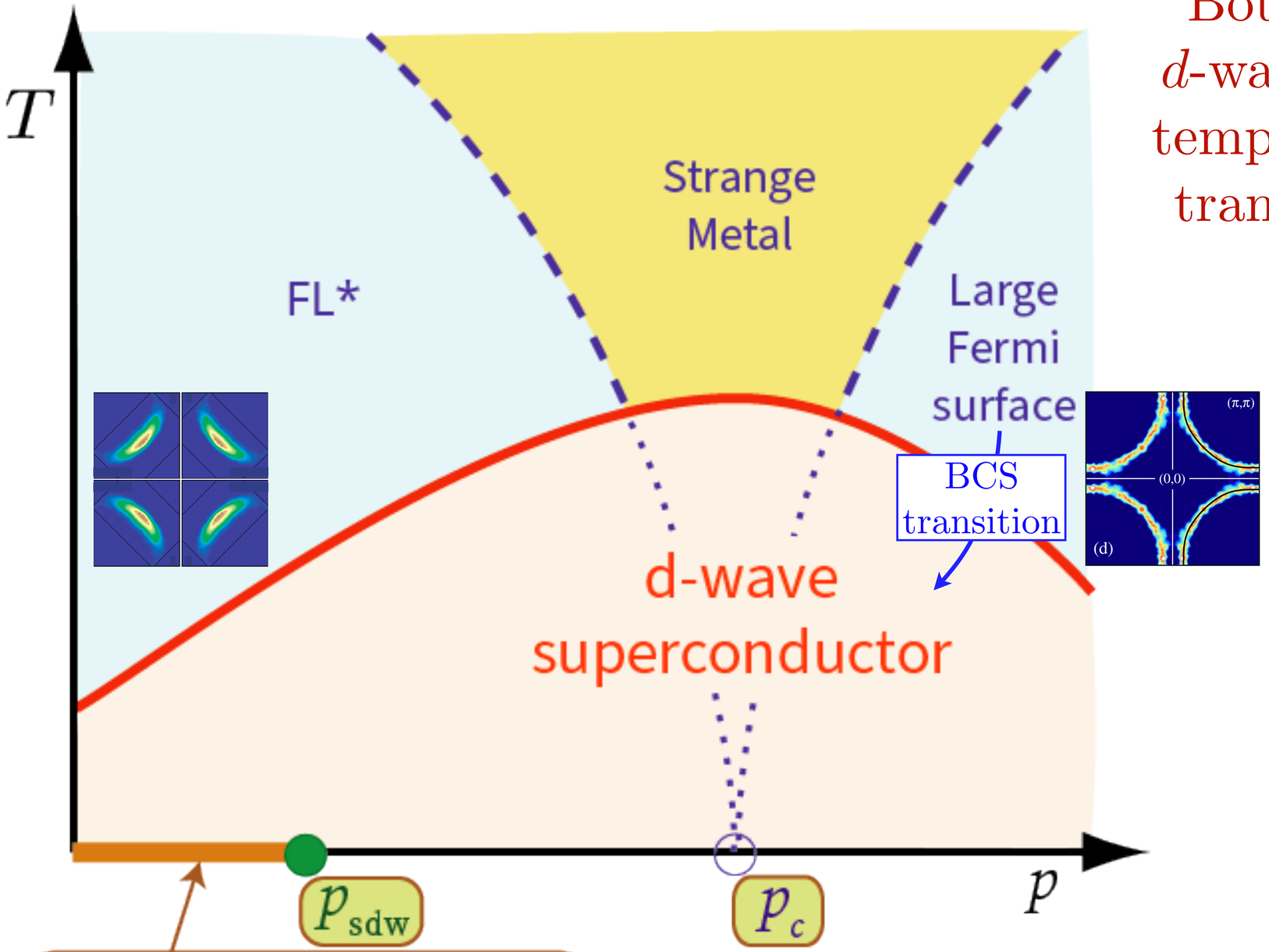


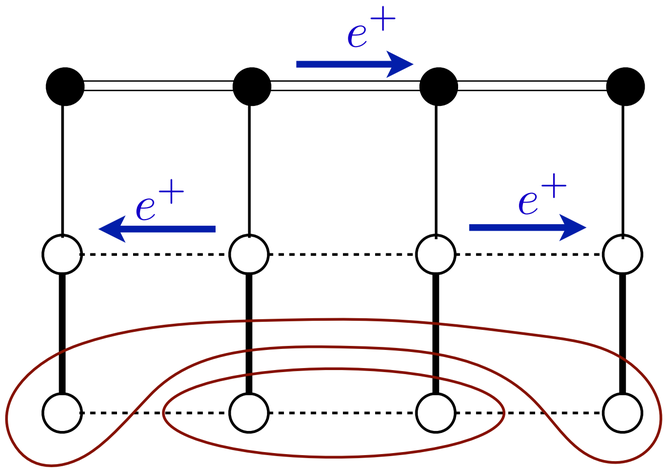


M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)

Both metals lead to the same d-wave superconductor at lower temperatures, and so there is no transition at $p = p_c$ within the superconducting state.

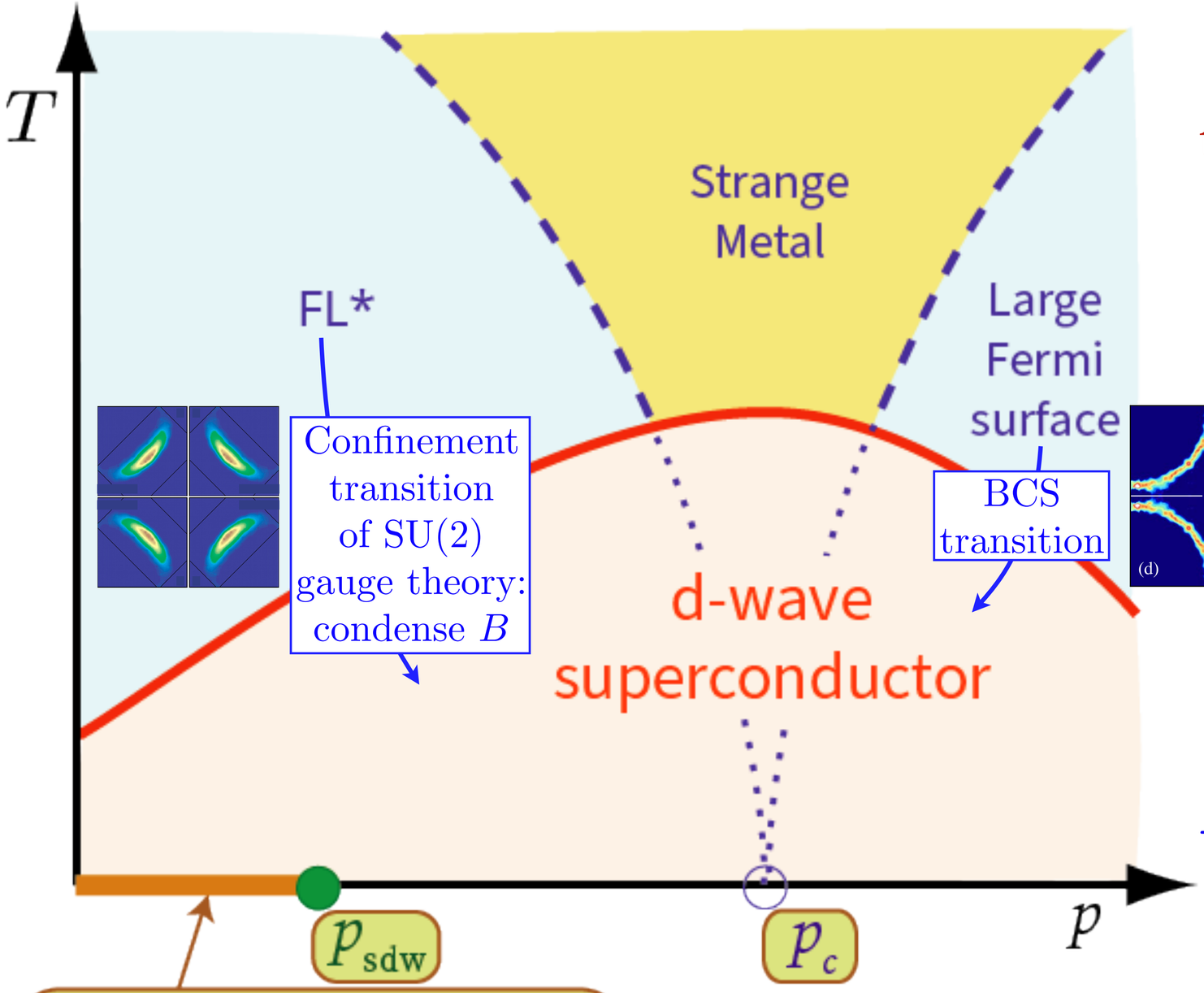


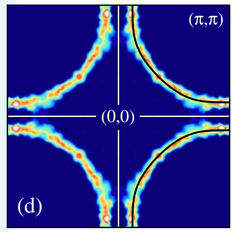


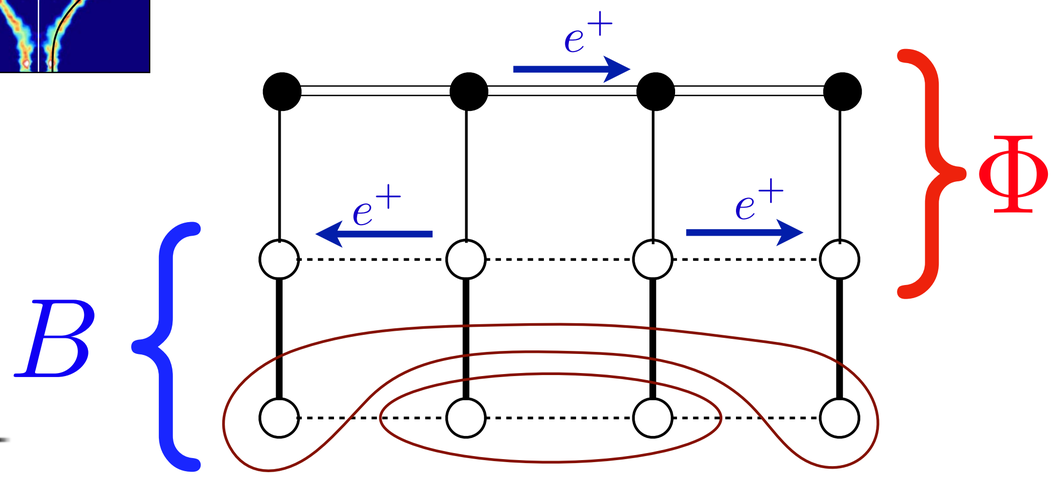
M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)

Both metals lead to the same d-wave superconductor at lower temperatures, and so there is no transition at $p = p_c$ within the superconducting state.







M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)