Fermi-volume-changing quantum phase transitions and the cuprate phase diagram

Indian Institute of Science, Bengaluru
January 2, 2025
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PHYSICS
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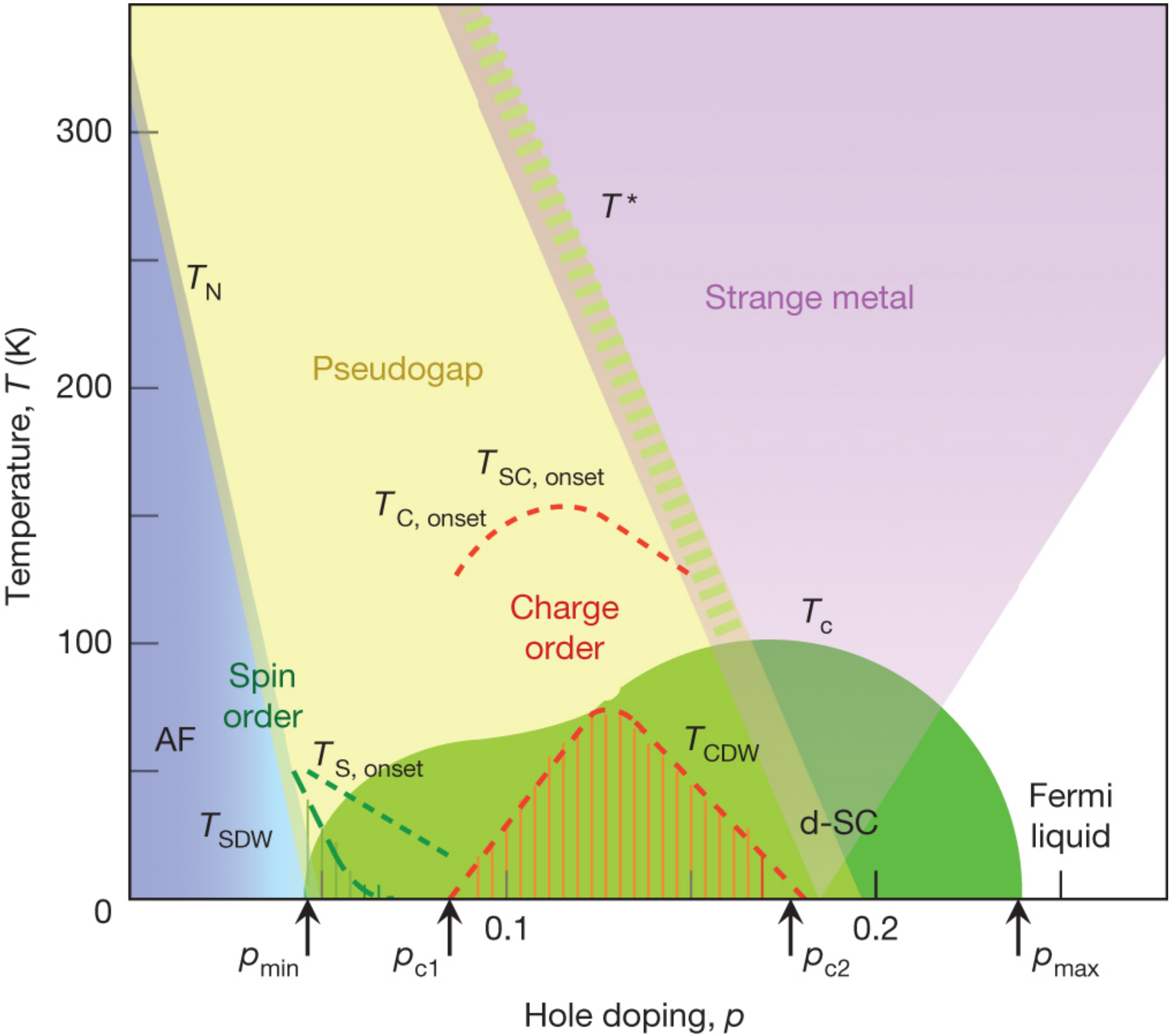
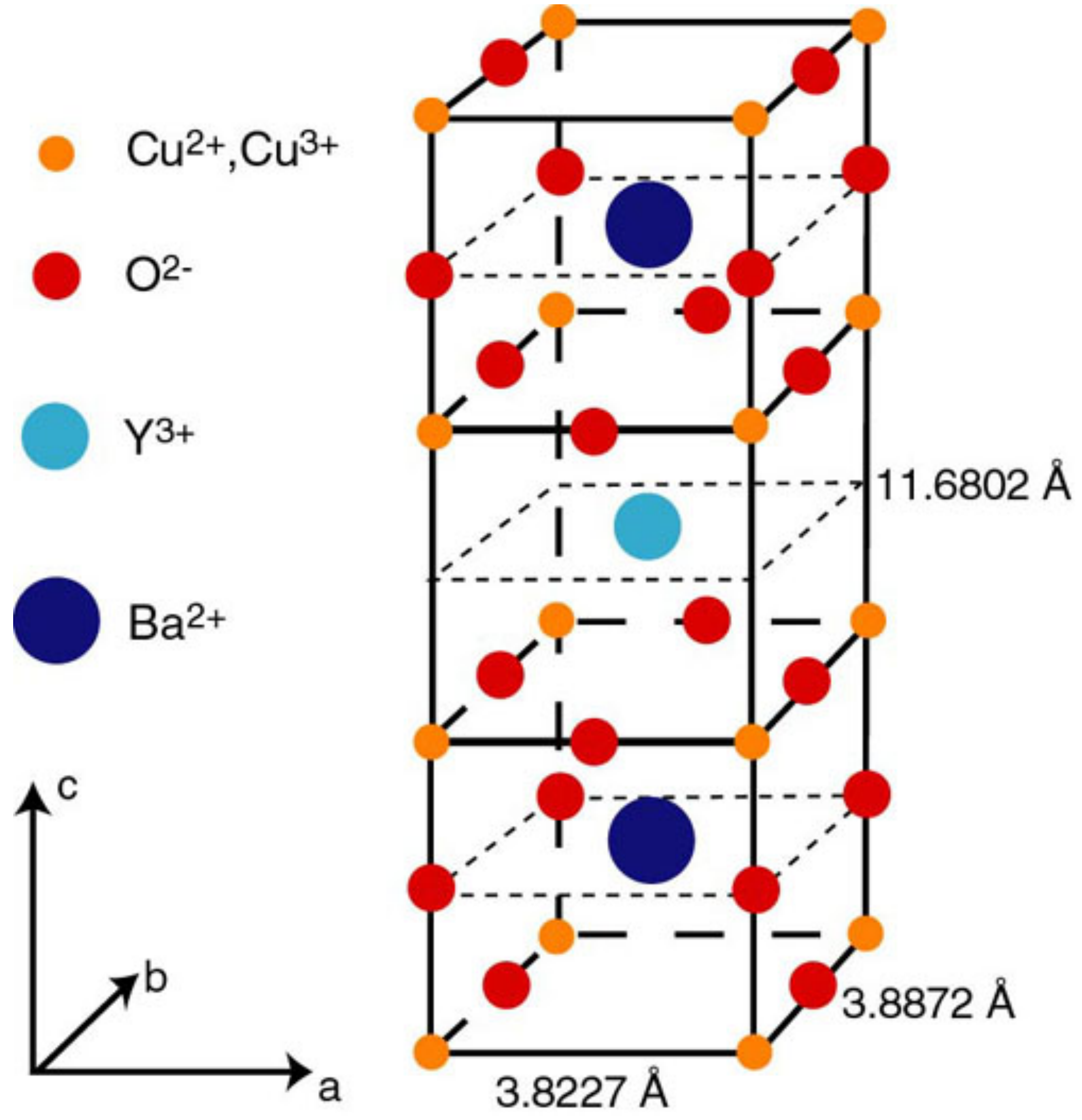
HARVARD

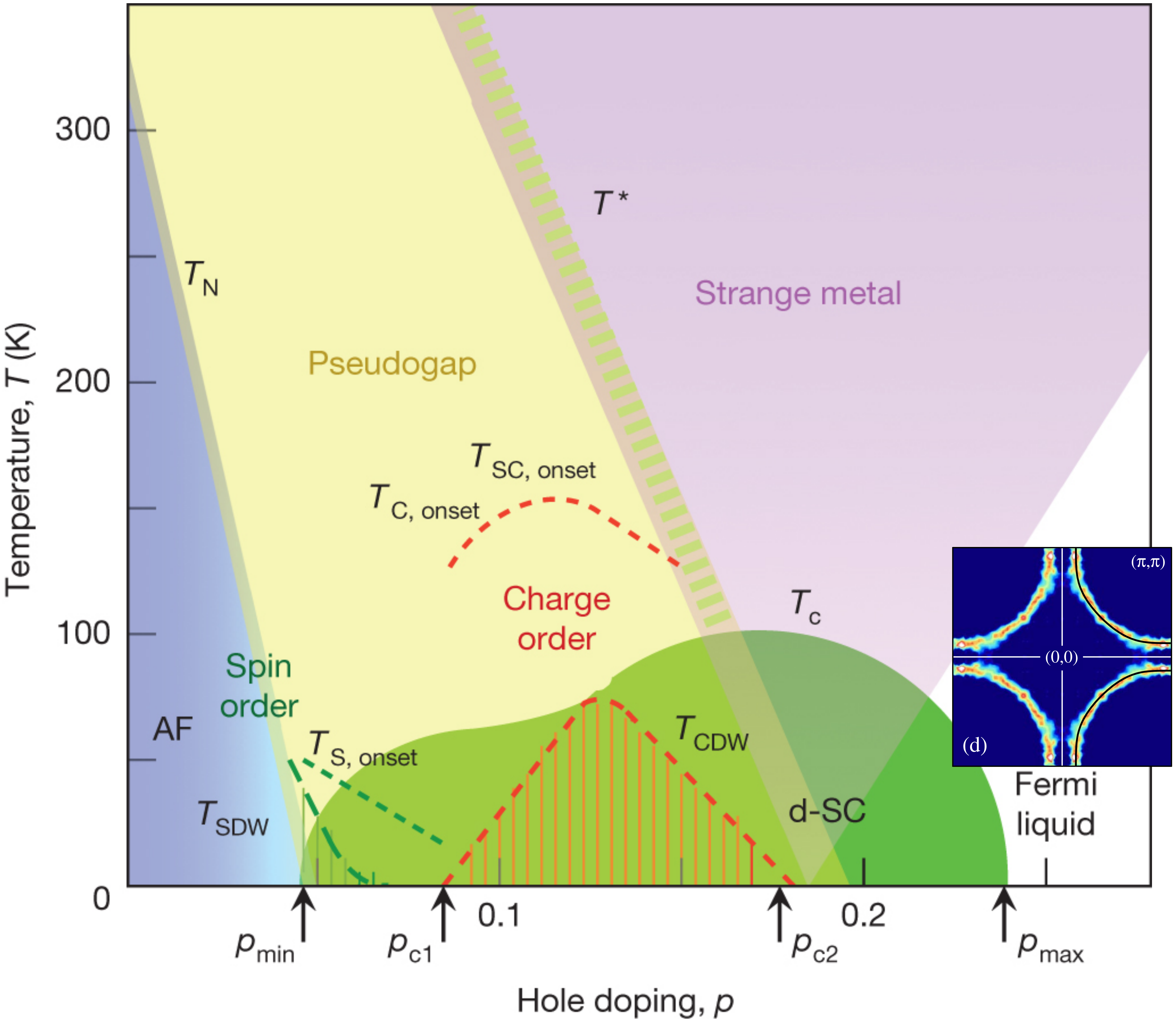
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





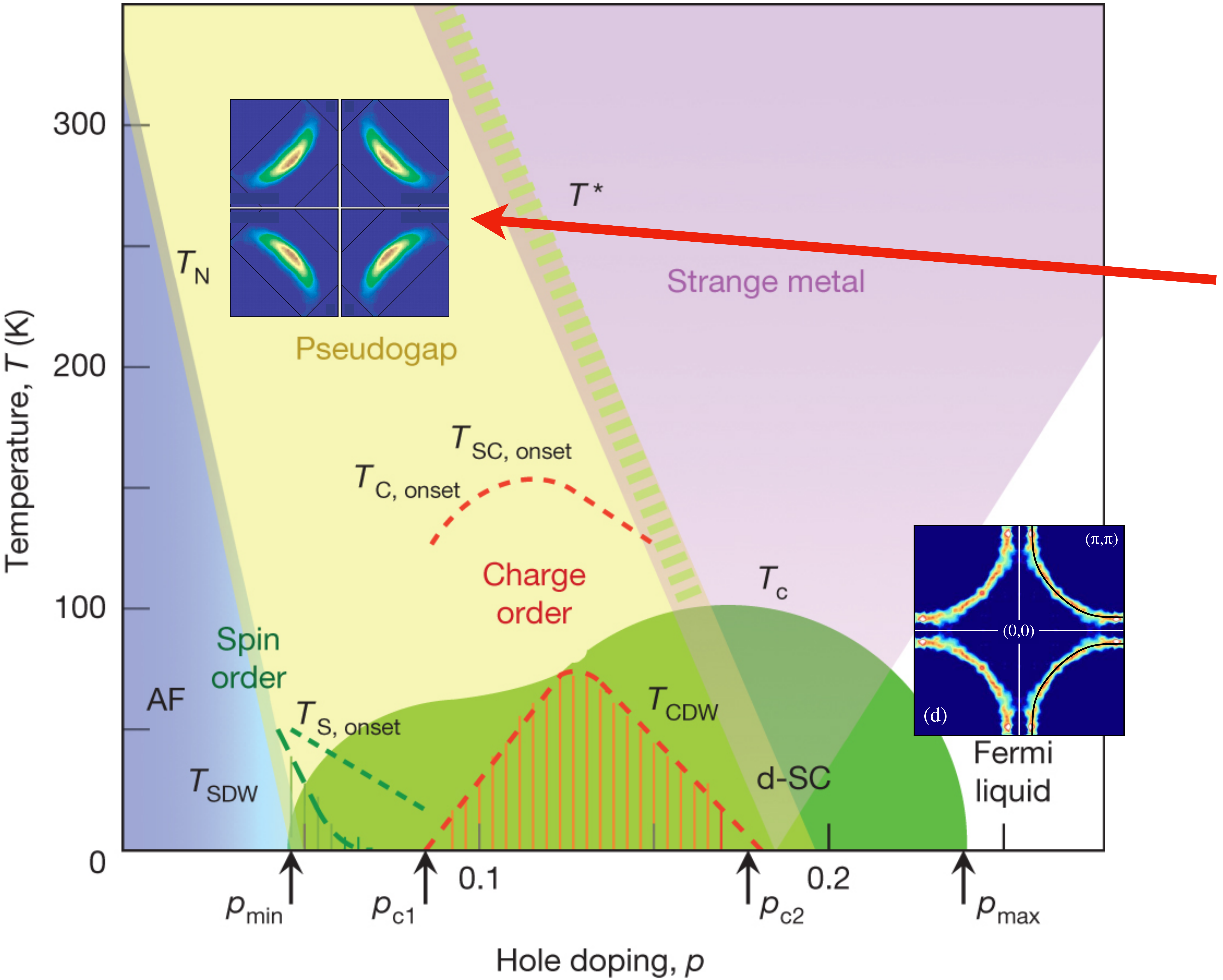
$YBa_2Cu_3O_{6+x}$

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



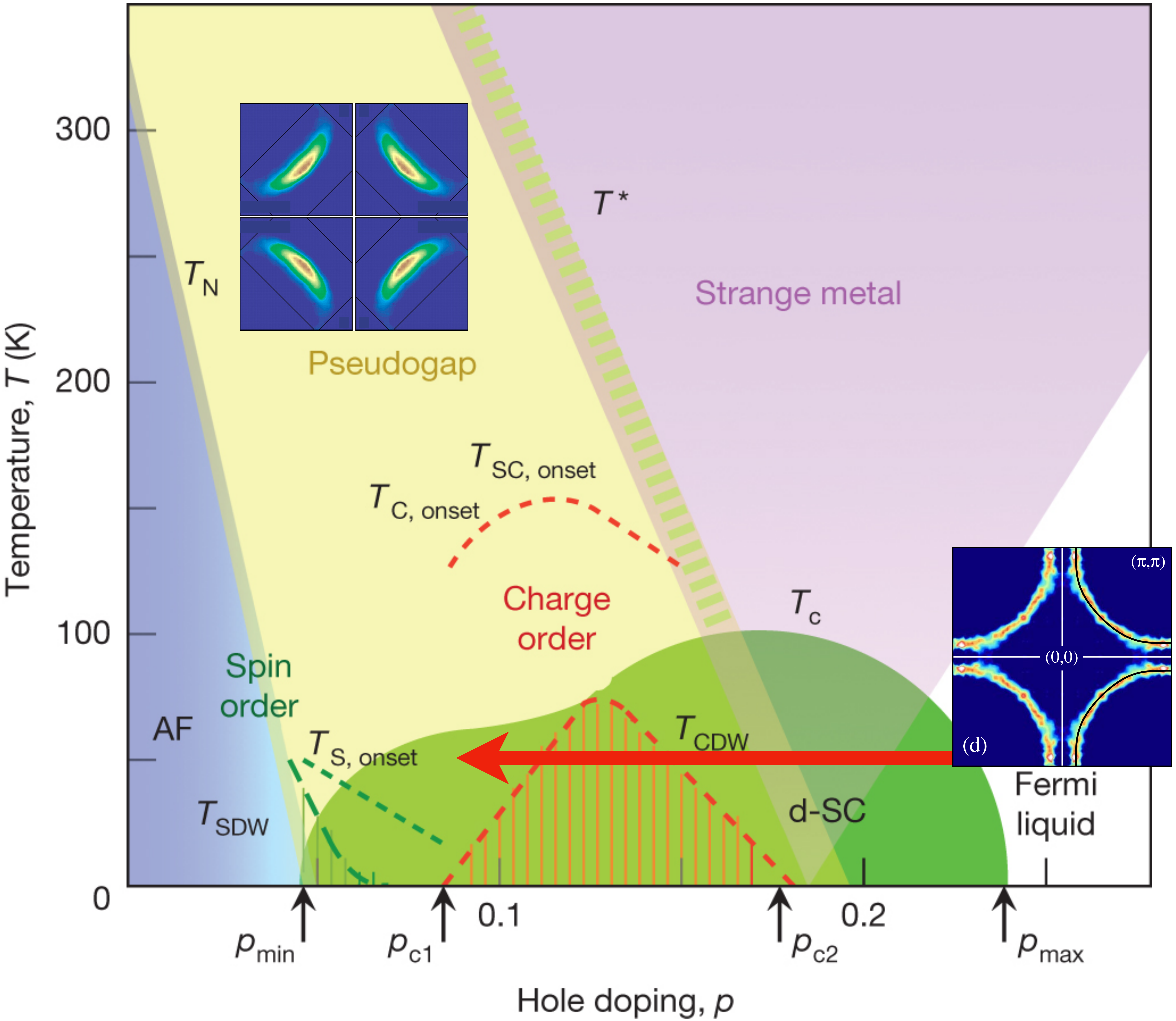
Fermi surface
as expected
in a model
of free electrons

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



"Pseudogap metal"
Fermi surface
modified by
electron-electron
interactions

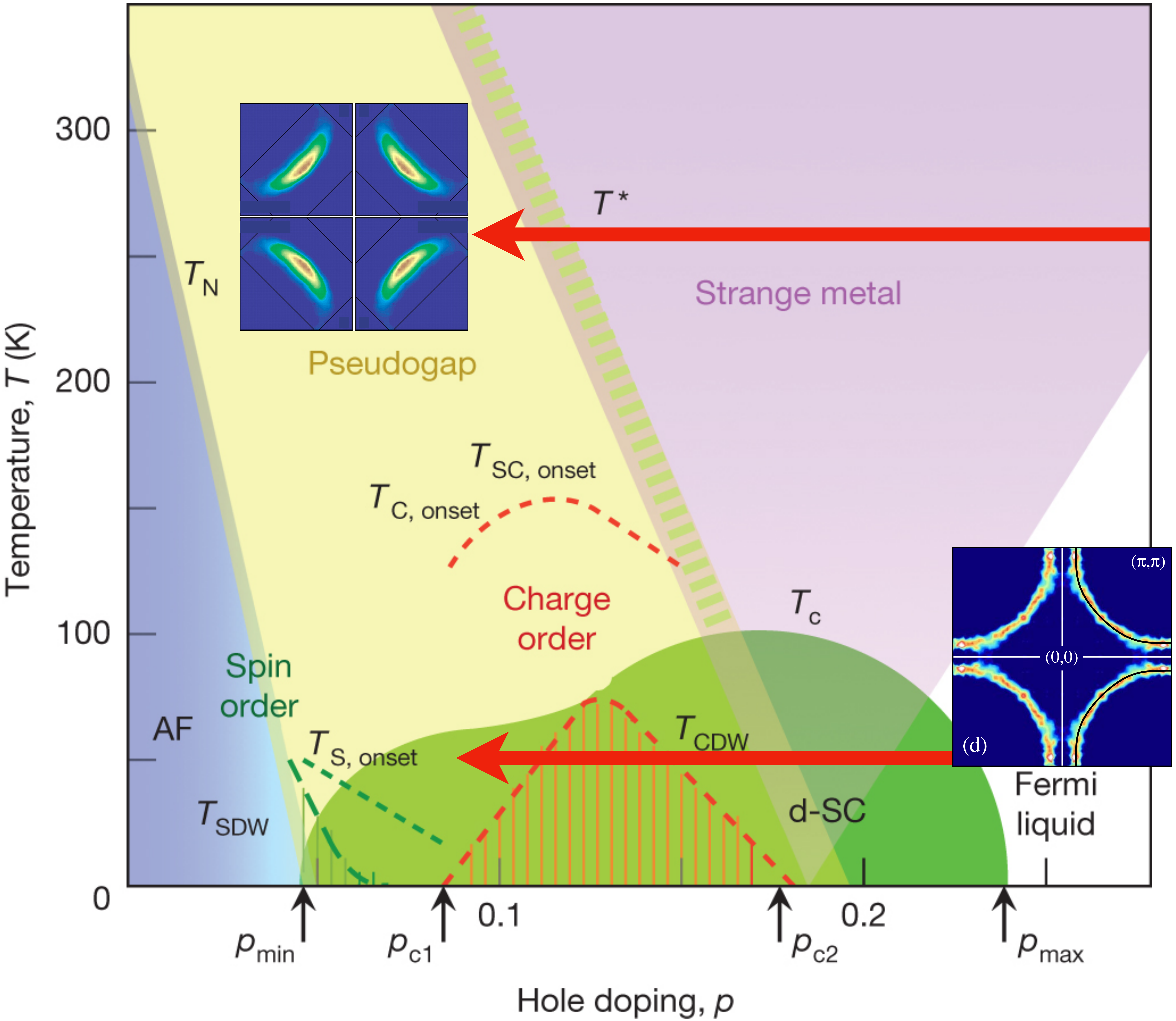
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Fermi-volume-changing QPT
with symmetry breaking
and with spatial disorder.

FL-SDW QPT

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Fermi-volume-changing QPT
<u>without</u> symmetry breaking
and with spatial disorder.

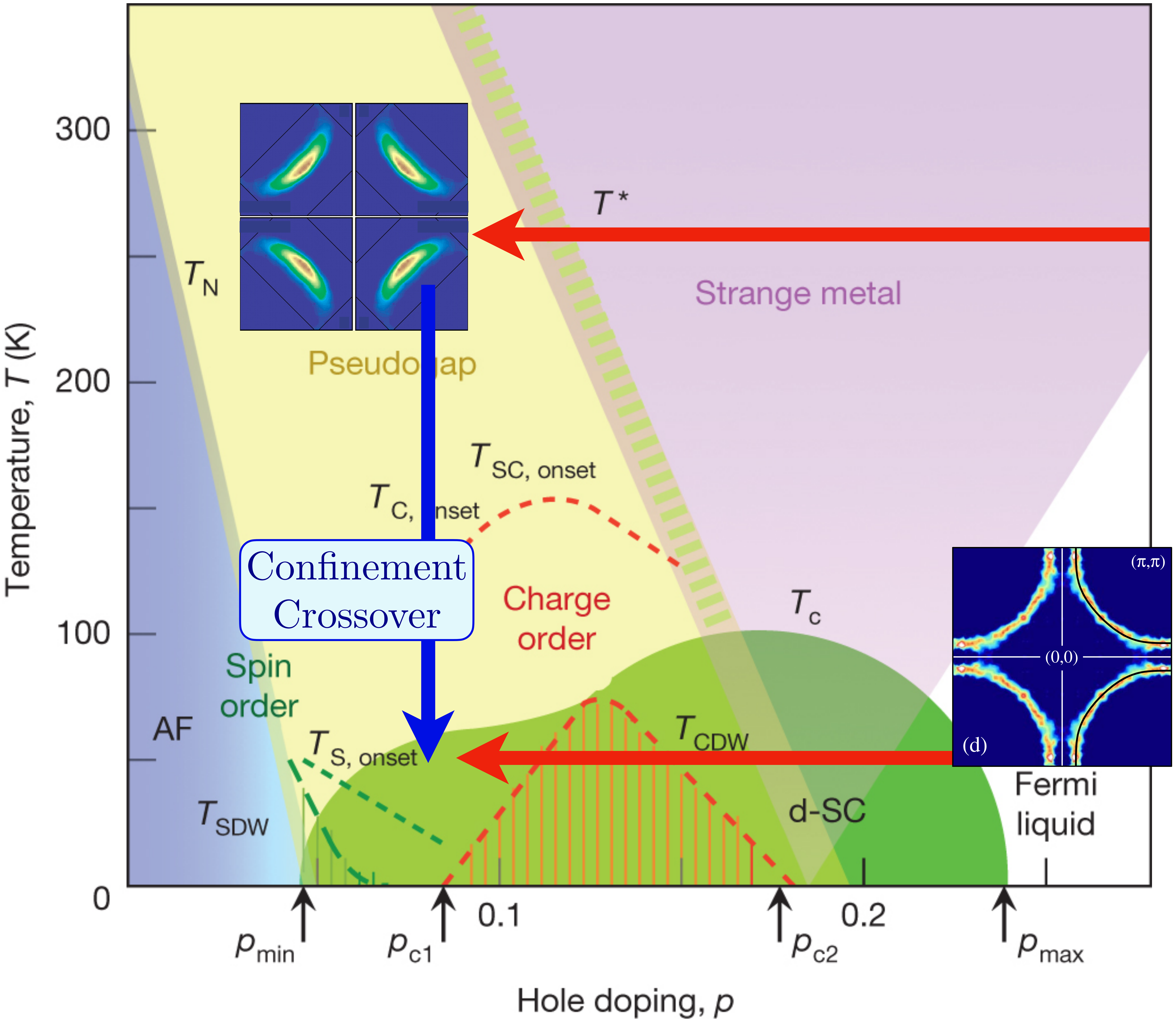
FL-FL* QPT

Requires fractionalization

Fermi-volume-changing QPT
<u>with</u> symmetry breaking
and with spatial disorder.

FL-SDW QPT

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Fermi-volume-changing QPT
without symmetry breaking
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FL-FL* QPT

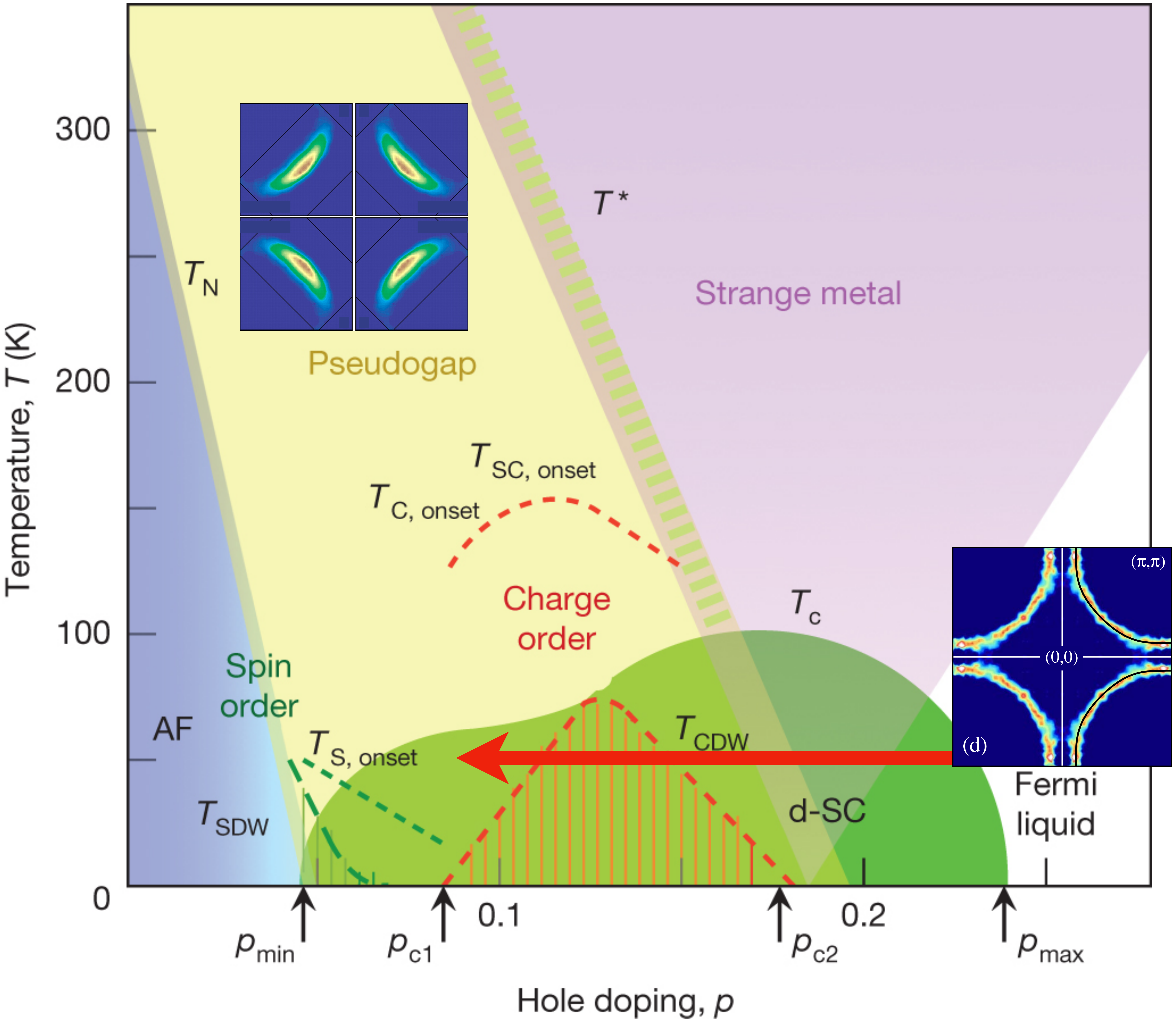
Requires fractionalization

Fermi-volume-changing QPT
with symmetry breaking
and with spatial disorder.

FL-SDW QPT

1. FL-SDW QPT
2. FL-FL* QPT
3. Confinement crossover

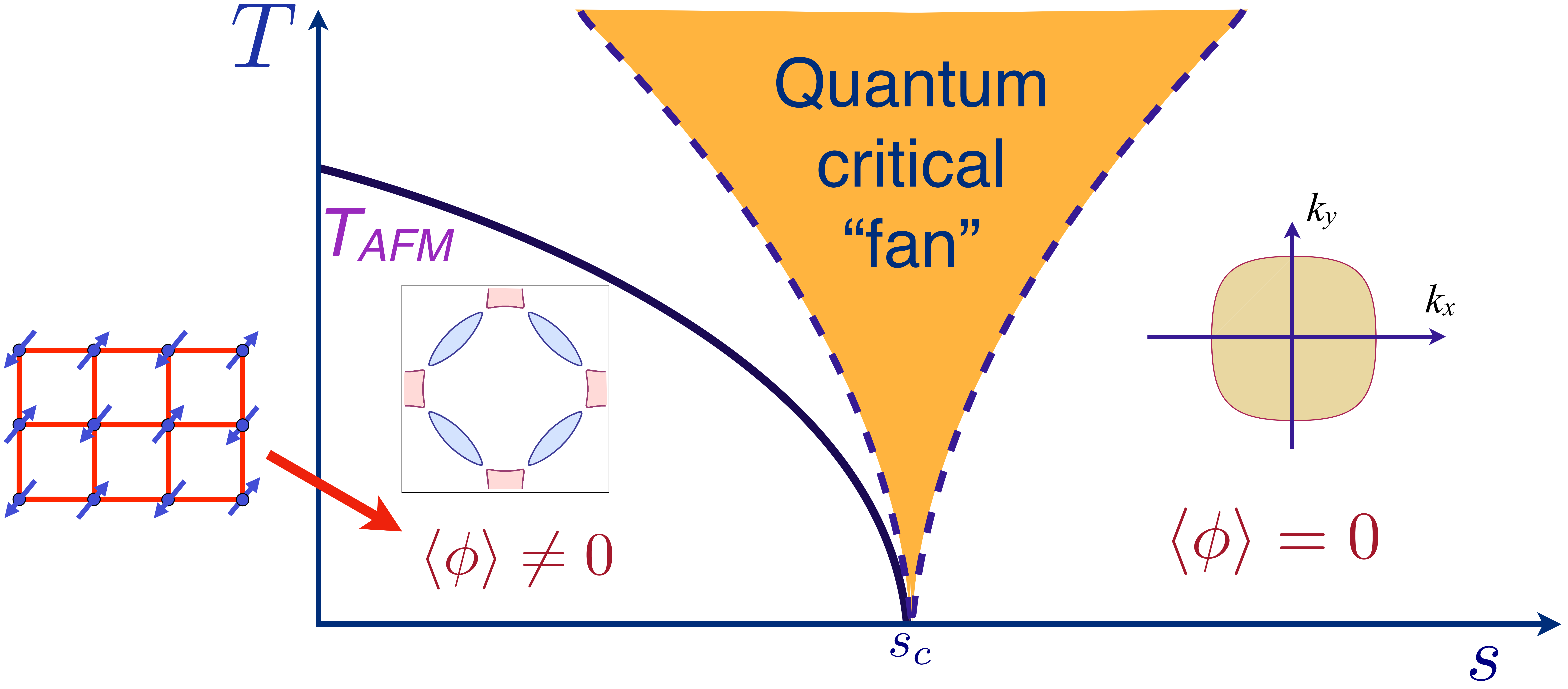
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



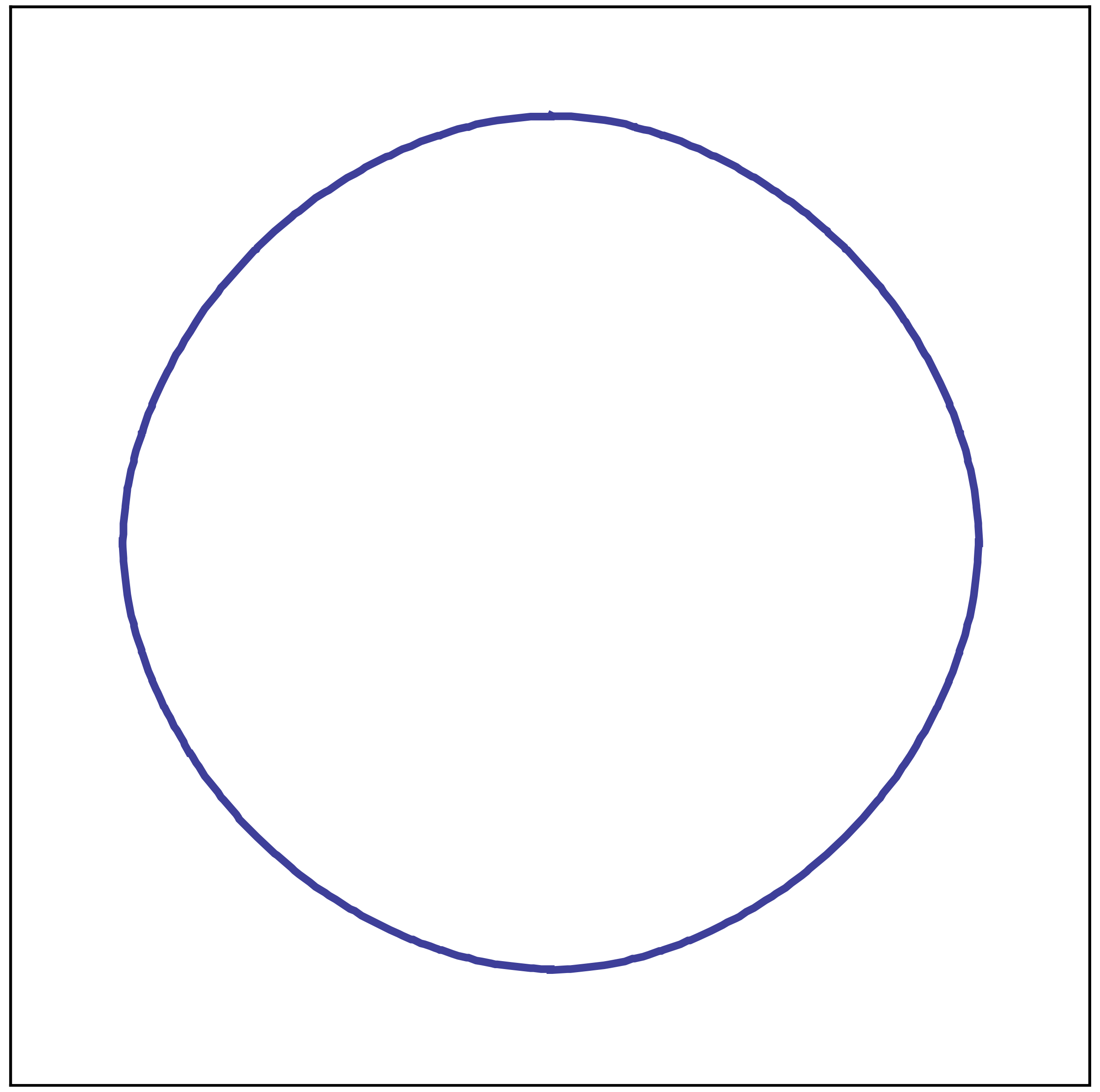
Fermi-volume-changing QPT
with symmetry breaking
and with spatial disorder.

FL-SDW QPT

Fermi surface reconstruction from spin density wave (SDW) order

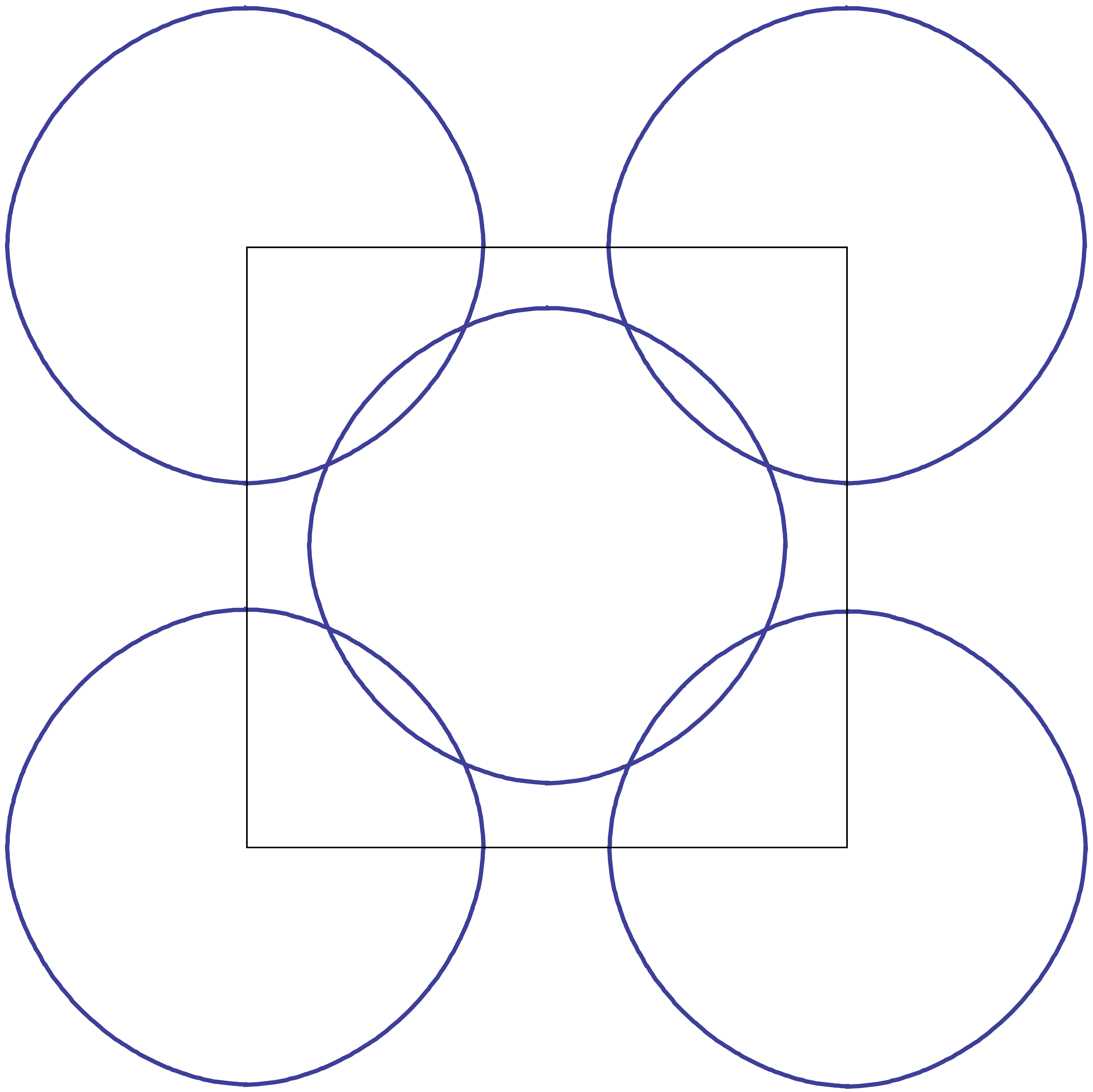


Fermi surface+antiferromagnetism



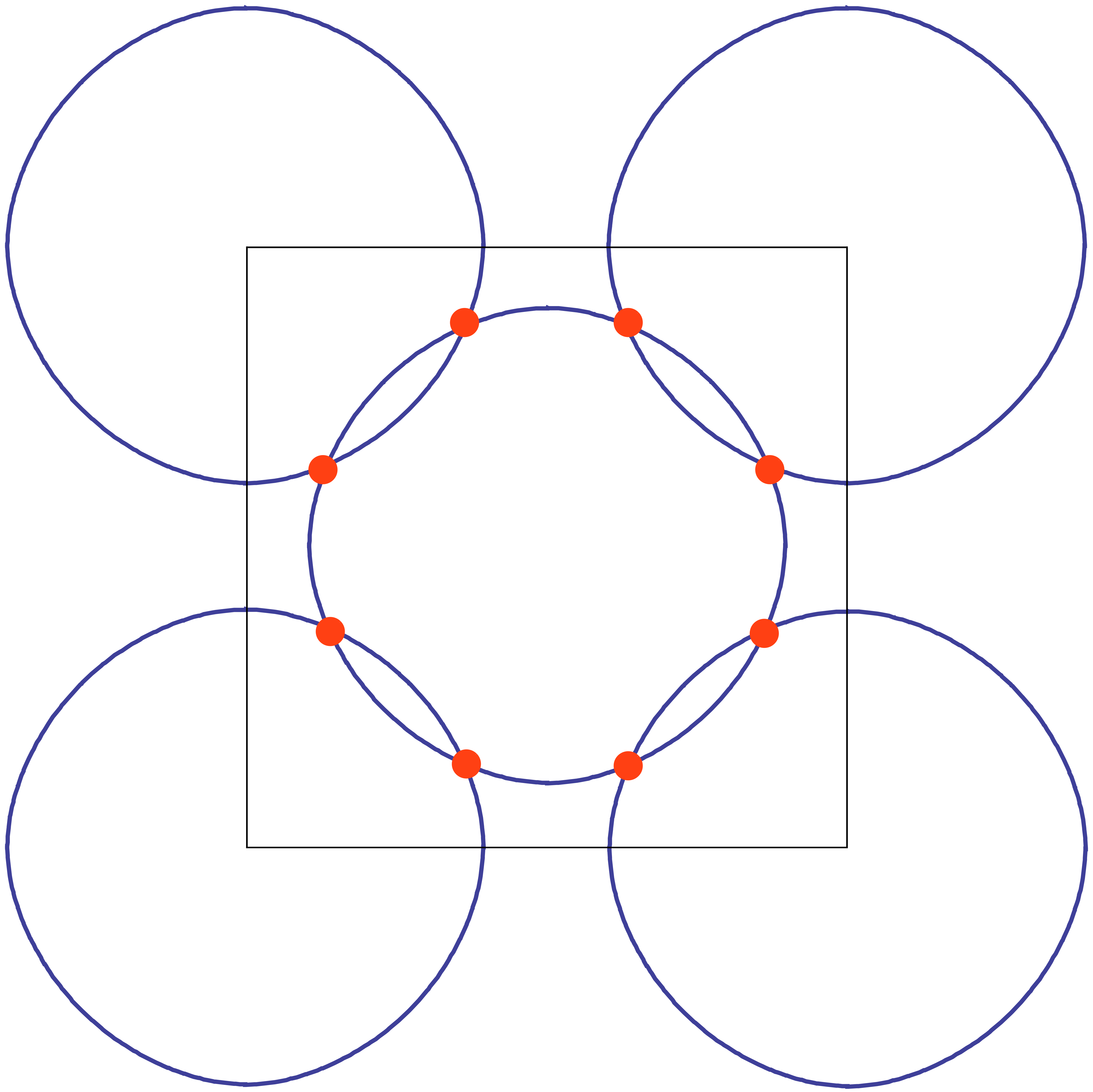
Metal with “large” Fermi surface

Fermi surface+antiferromagnetism



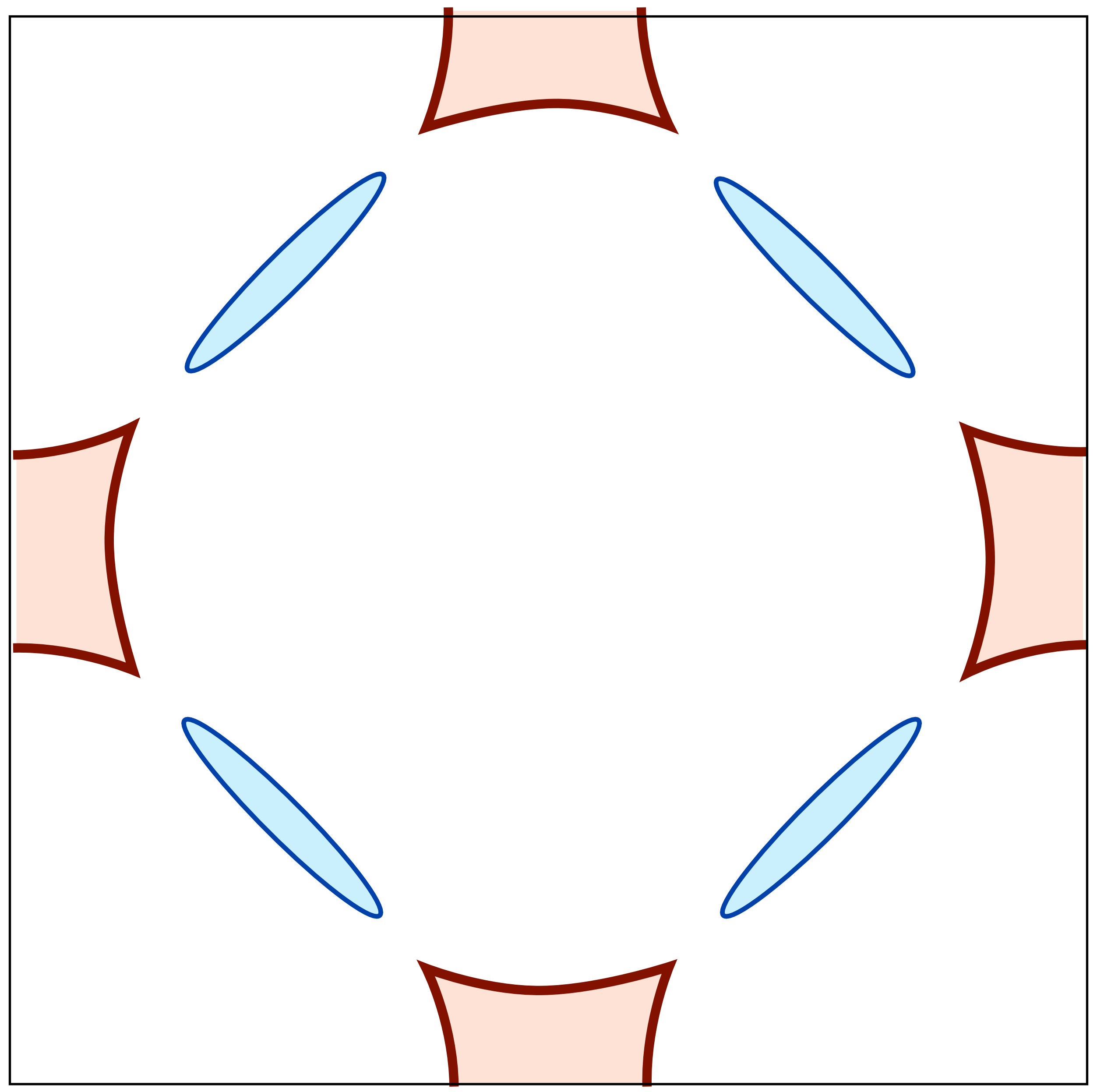
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



“Hot” spots

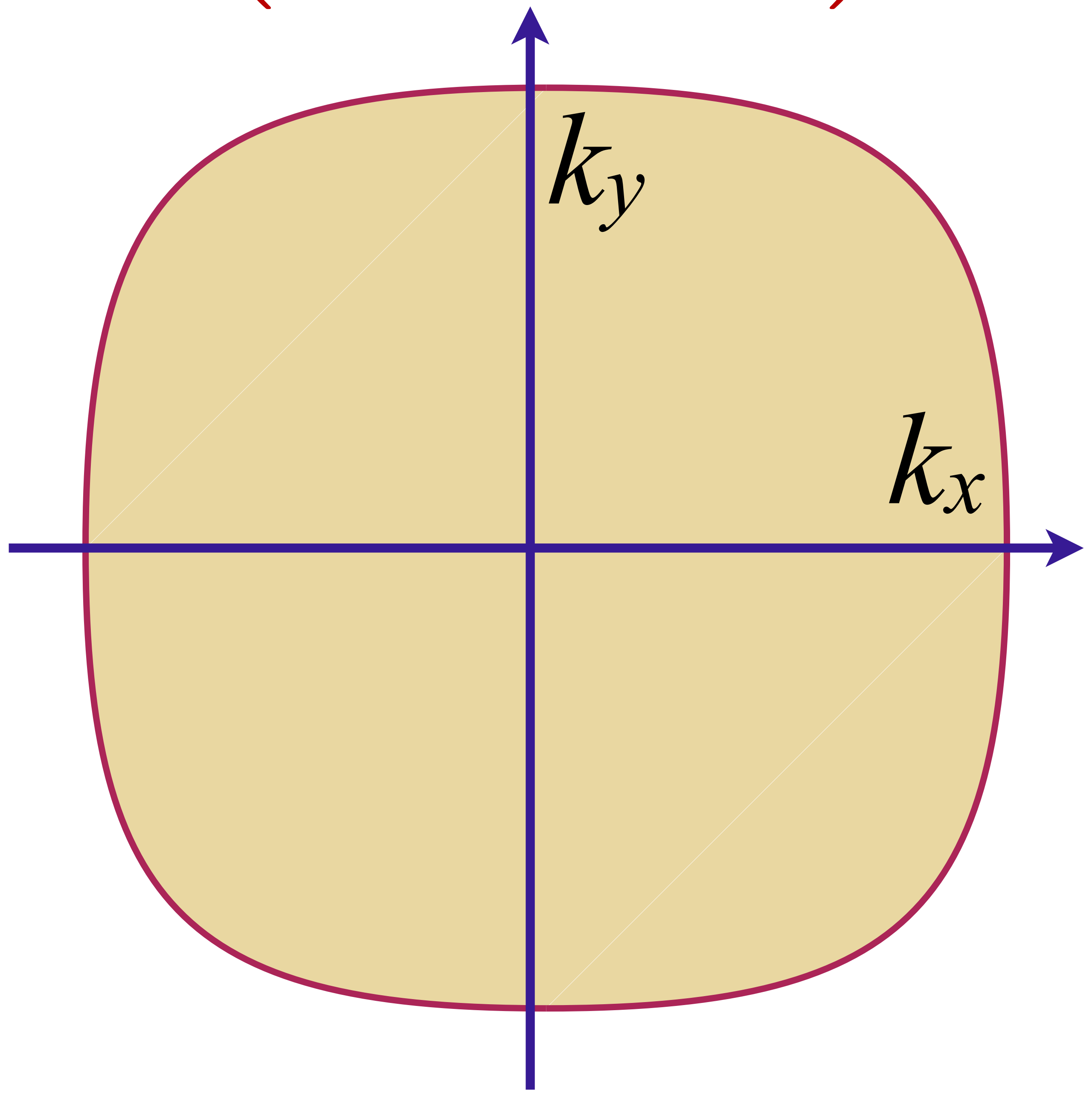
Fermi surface+antiferromagnetism



Electron and hole pockets in antiferromagnetic phase with $\langle \phi \rangle \neq 0$

Fermi surface + critical boson with no spatial disorder

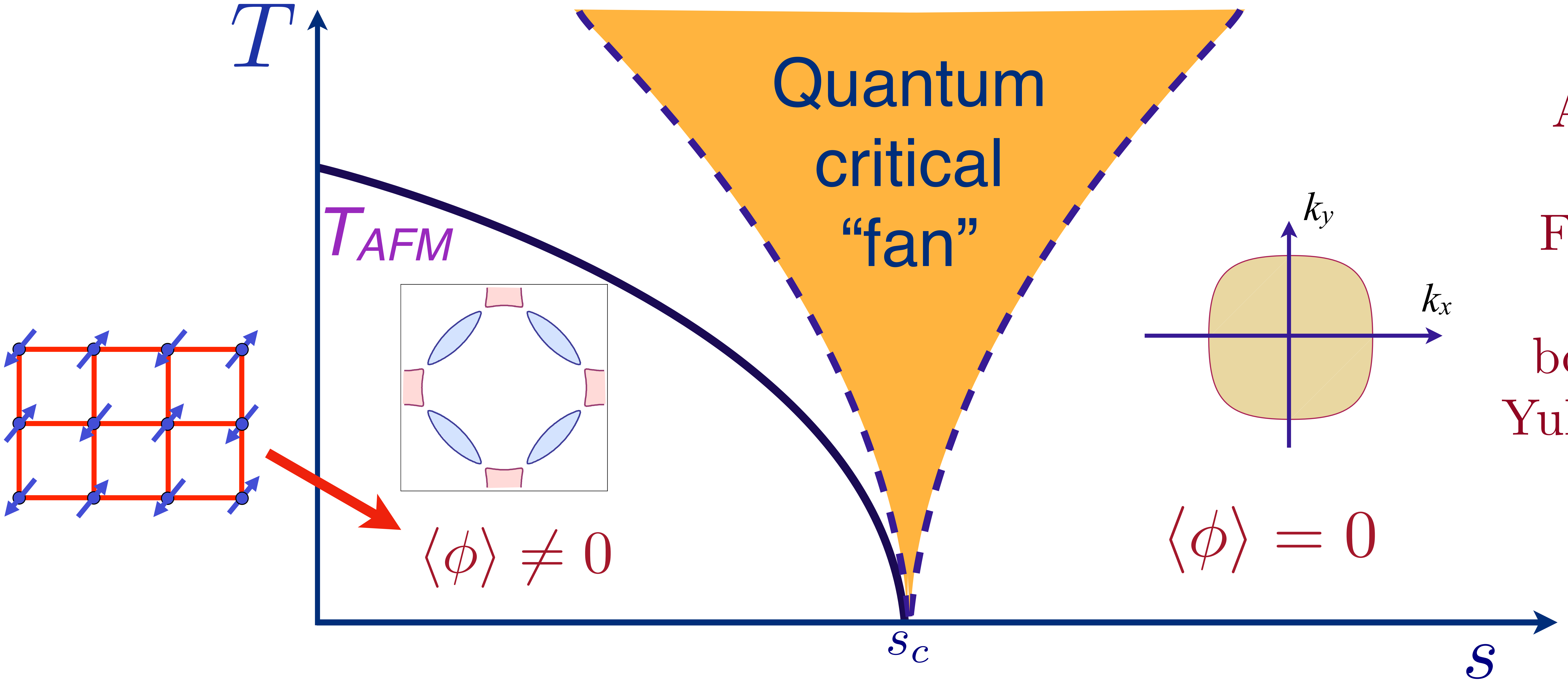
$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$



$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})\tau^a_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

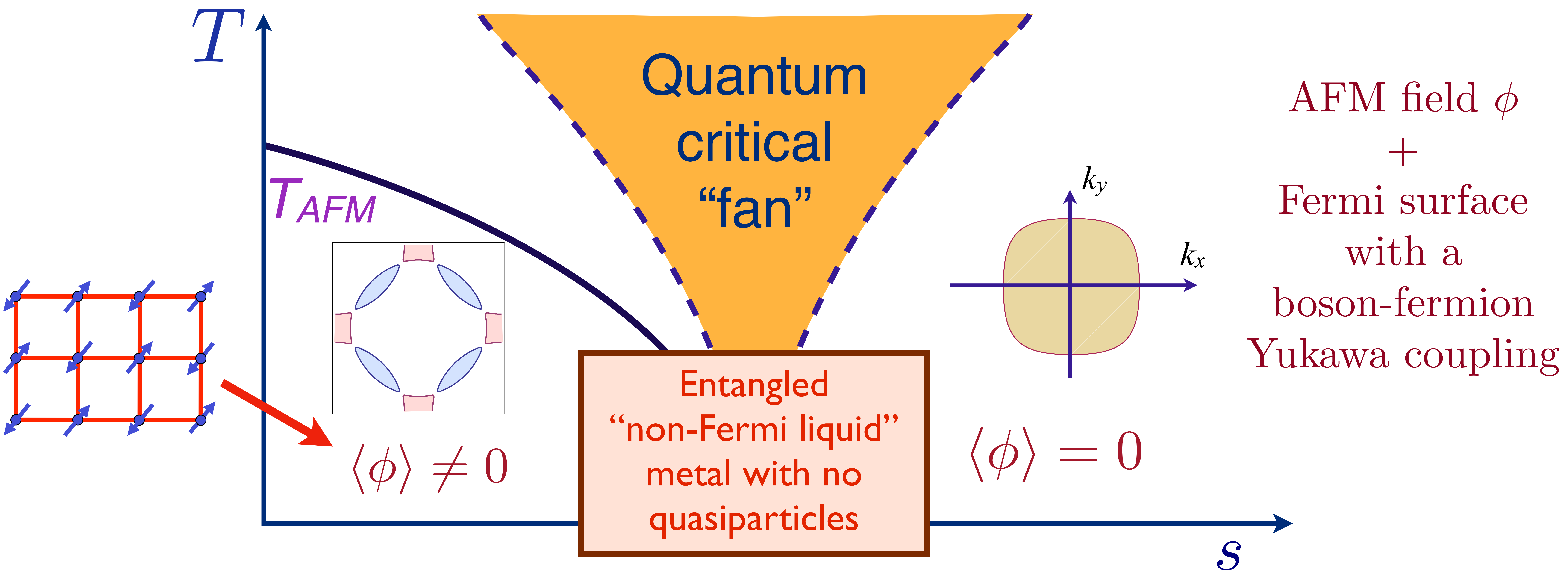
$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2+u\,[\phi(\boldsymbol{r})]^4$$

Fermi surface reconstruction from spin density wave (SDW) order

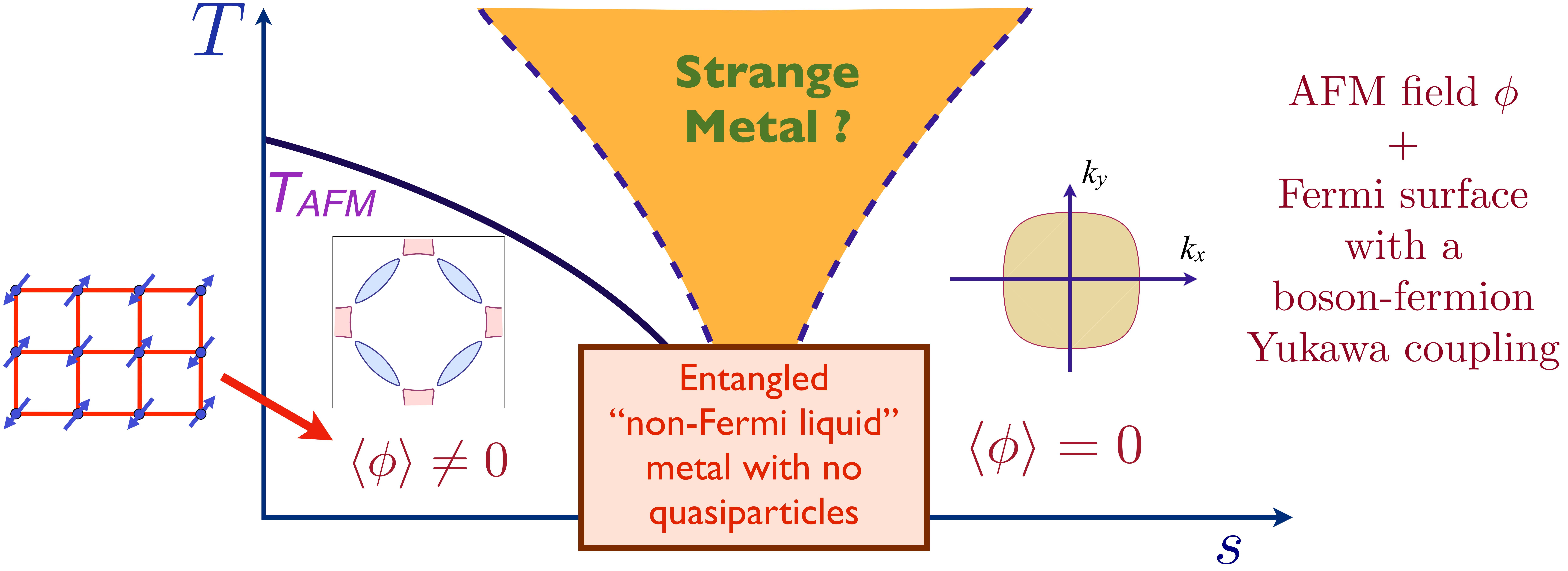


AFM field ϕ
$+$
Fermi surface
with a
boson-fermion
Yukawa coupling

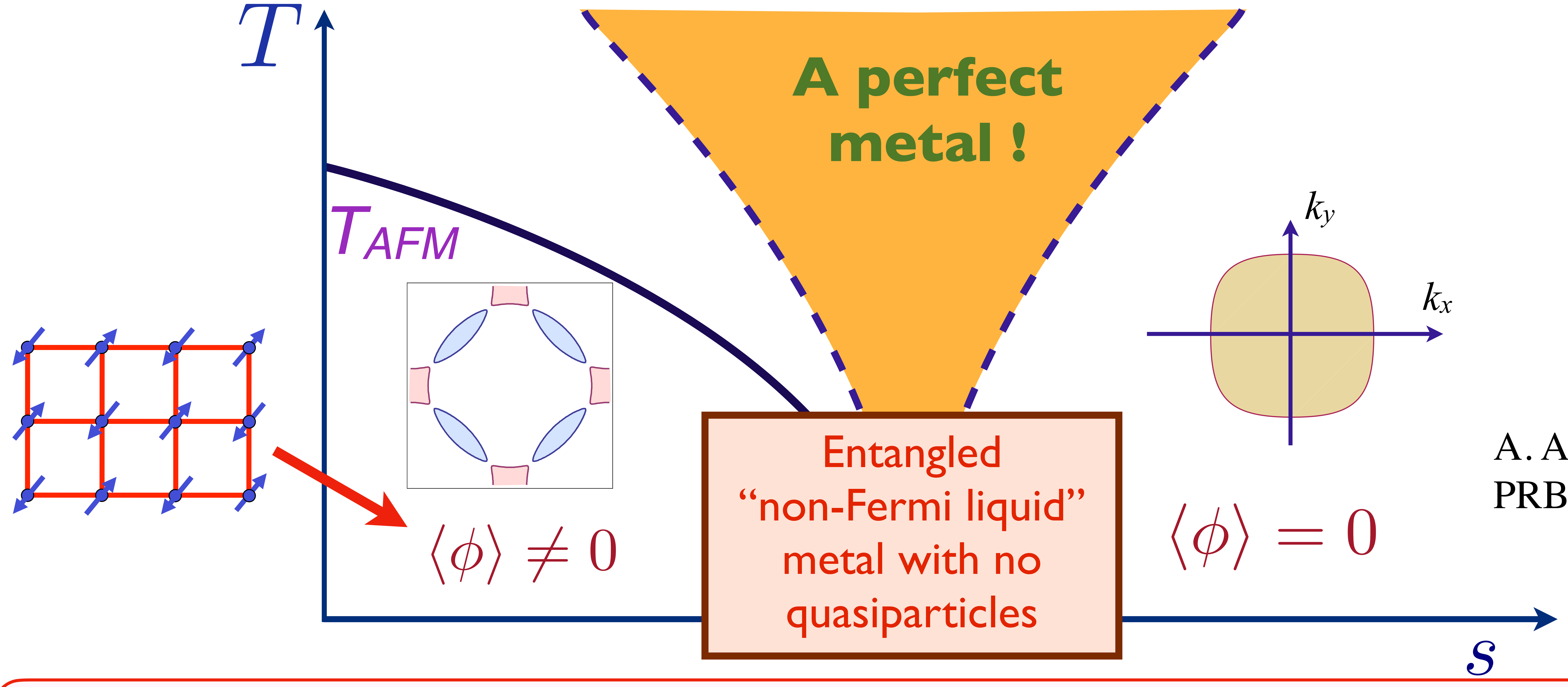
Fermi surface reconstruction from spin density wave (SDW) order



Fermi surface reconstruction from spin density wave (SDW) order



Fermi surface reconstruction from spin density wave (SDW) order

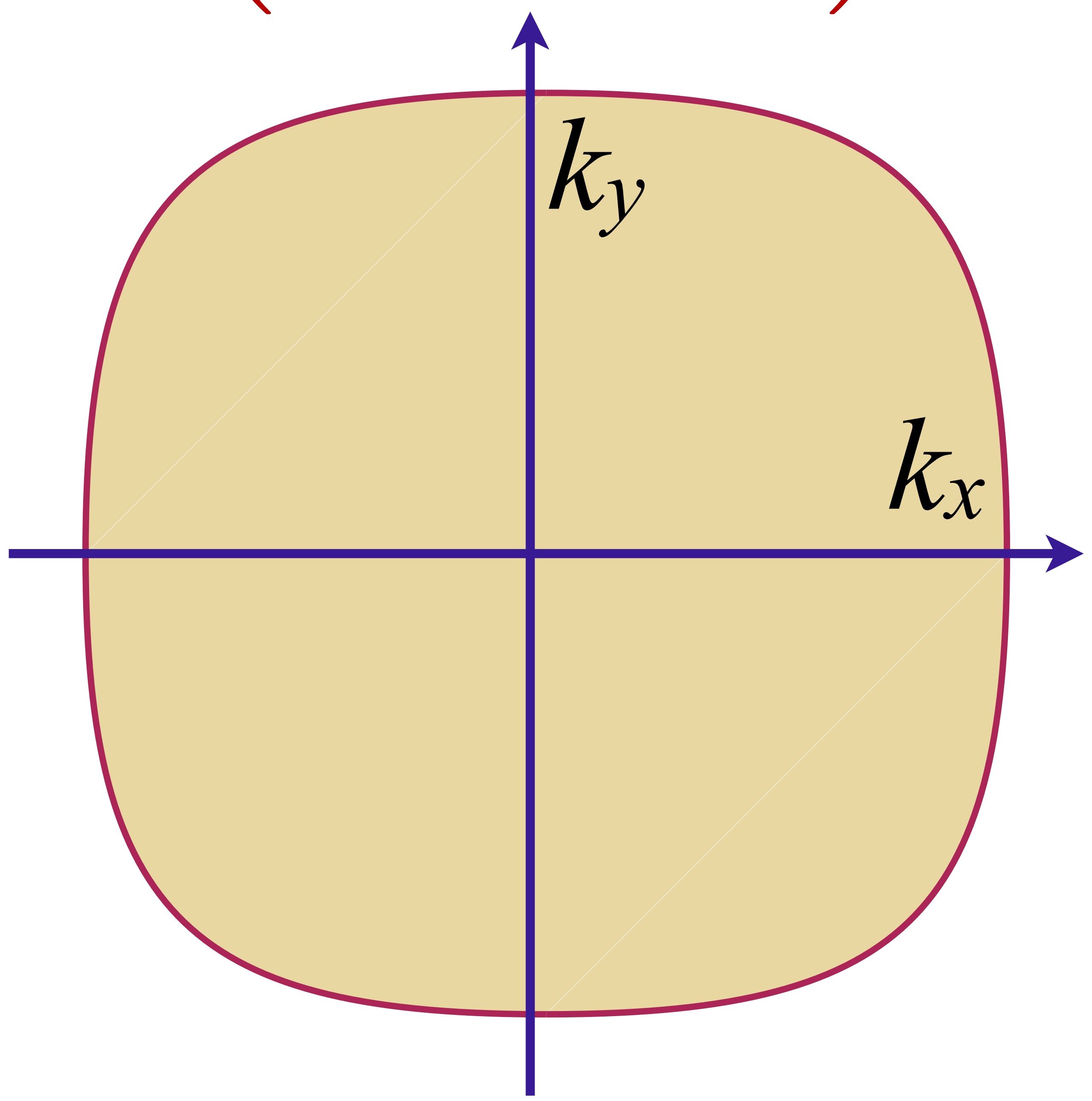


A. A. Patel and S. S., PRB **90**, 165146 (2014)

Extreme drag: the fermions c "drag" the bosons ϕ as they move, and so electrical current does not relax, even though strong c-ϕ scattering leads to absence of c quasiparticles.

Fermi surface + critical boson with no spatial disorder

$$c_{\boldsymbol{k}\sigma}^{\dagger}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$

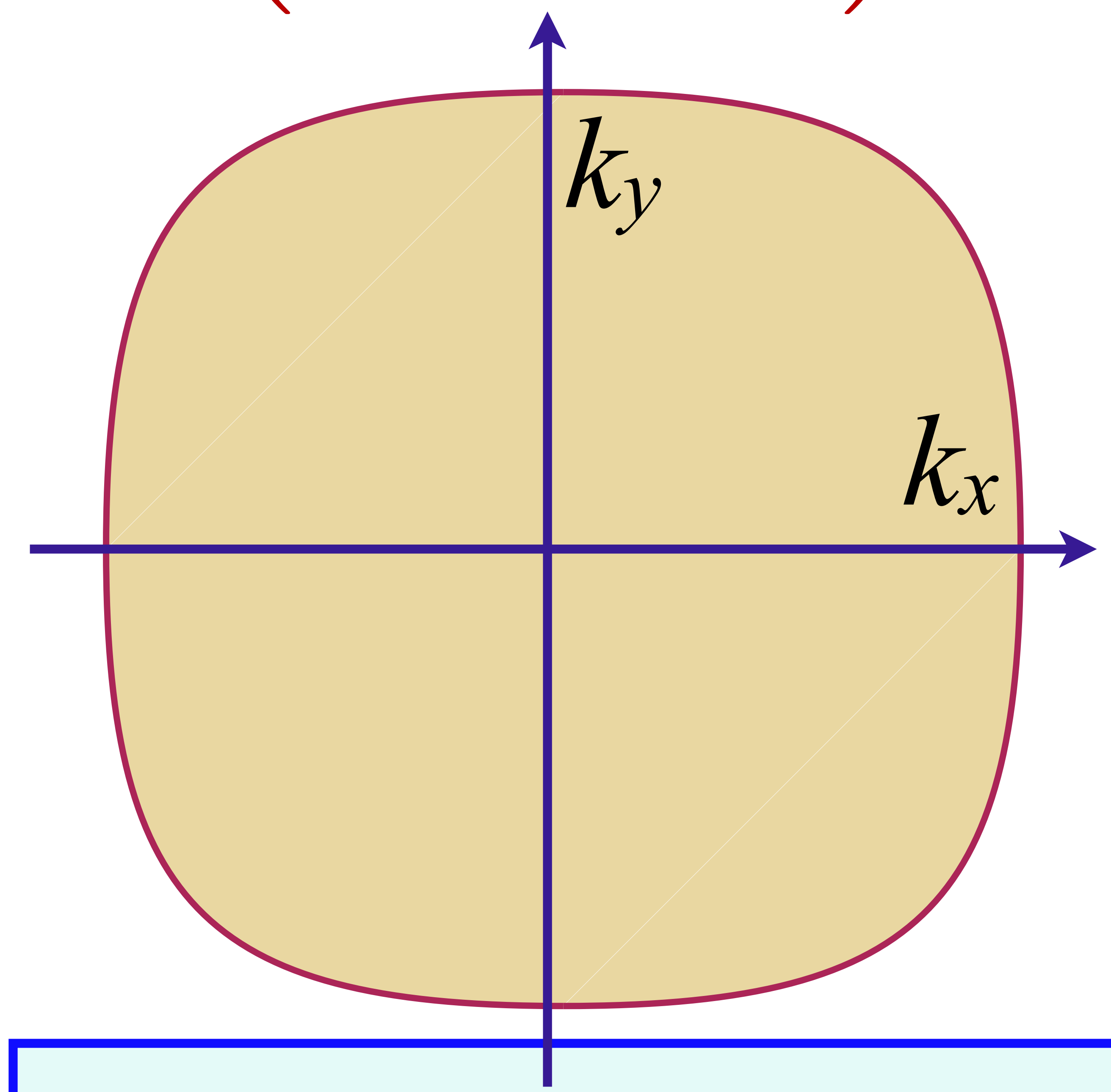


$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,c_{\sigma}^{\dagger}(\boldsymbol{r})\tau^{a}_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2+u\,[\phi(\boldsymbol{r})]^4$$

Fermi surface + critical boson with potential disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$



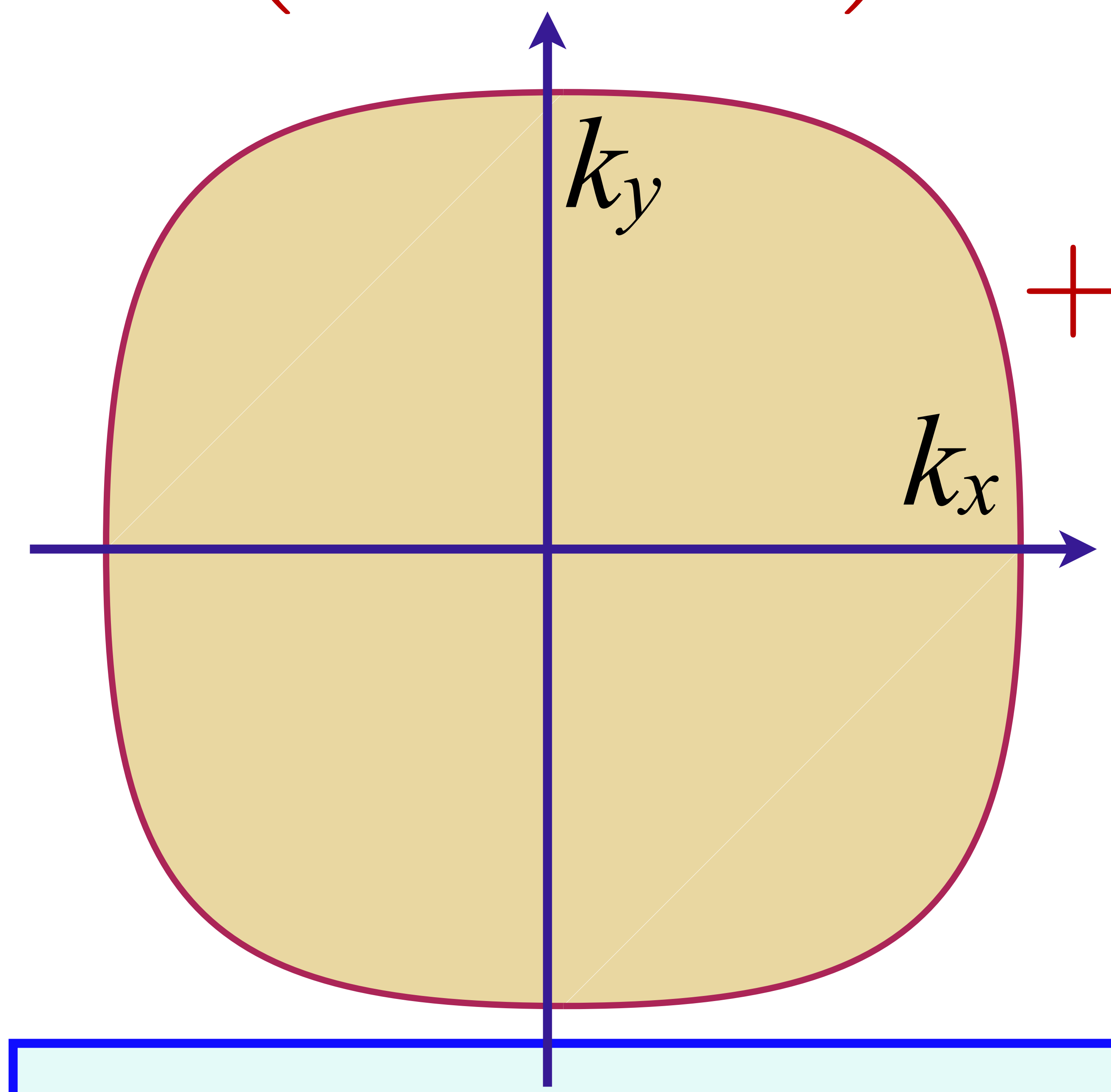
$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})\tau^a_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2+u\,[\phi(\boldsymbol{r})]^4 \; +v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Fermi surface + critical boson with potential and interaction disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$



$$+[s+\delta s(\boldsymbol{r})][\phi(\boldsymbol{r})]^2 \qquad +g\, c^\dagger_\sigma(\boldsymbol{r})\tau^a_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2+u\,[\phi(\boldsymbol{r})]^4\;+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0,\ \overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})}=0,\ \overline{\delta s(\boldsymbol{r})\delta s(\boldsymbol{r}')}=\delta s^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

RG analysis (Harris criterion) shows that $\delta s(\boldsymbol{r})$ is most relevant disorder.

Bosonic eigenmodes in random mass Hertz theory

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_\phi = \int d\tau \Big[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \big(\frac{s + s'_j}{2} \phi_{ja}^2 + \frac{u}{4M} \left(\phi_{ja}^2\right)^2 \big) \Big]$$

$$\mathcal{S}_{\phi d} = \frac{T}{2} \sum_{\Omega} \sum_j \left(\gamma |\Omega| + \Omega^2 / c^2 \right) |\phi_{ja}(i\Omega)|^2 \,,$$

where $a = 1 \dots M$ is a flavor index for an order parameter with O(M) symmetry.

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where $a = 1 \ldots M$ is a flavor index for an order parameter with $\mathrm{O}(M)$ symmetry. Analyze in a self-consistent quadratic theory, treating disorder numerically exactly

$$\overline{\mathcal{S}}_\phi = \int d\tau \Big[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \frac{\overline{s}'_j}{2} \phi_{ja}^2 \Big]$$

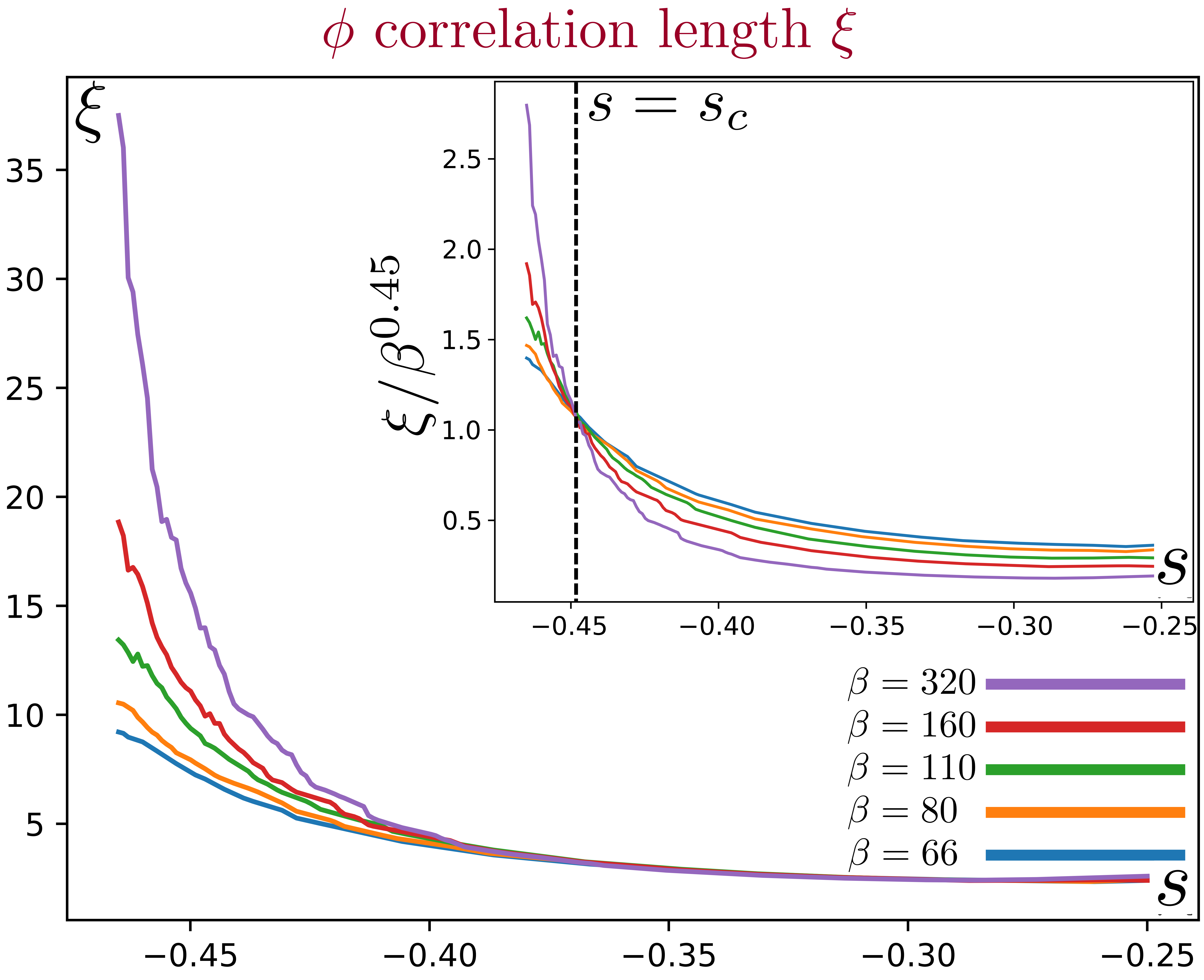
$$\overline{s}'_j = s + s'_j + \frac{u}{M} \sum_a \langle \phi_{ja}^2 \rangle_{\overline{\mathcal{S}}_\phi + \mathcal{S}_{\phi d}} = s + s'_j + uT \sum_\Omega \sum_\alpha \frac{\psi_{\alpha i} \psi_{\alpha j}}{\gamma |\Omega| + \Omega^2/c^2 + e_\alpha}$$

Similar analysis in $d = 1$ works very well
A. Del Maestro, B. Rosenow, M. Müller and S. Sachdev, Phys. Rev. Lett. **101**, 035701 (2008).

where e_α and $\psi_{\alpha j}$ are eigenvalues and eigenfunctions of the ϕ quadratic form in $\overline{\mathcal{S}}_\phi$, labeled by the index $\alpha = 1 \ldots L^2$ for a $L \times L$ sample.

Aavishkar A. Patel, Peter Lunts, S.S., PNAS **121**, e2402052121 (2024)

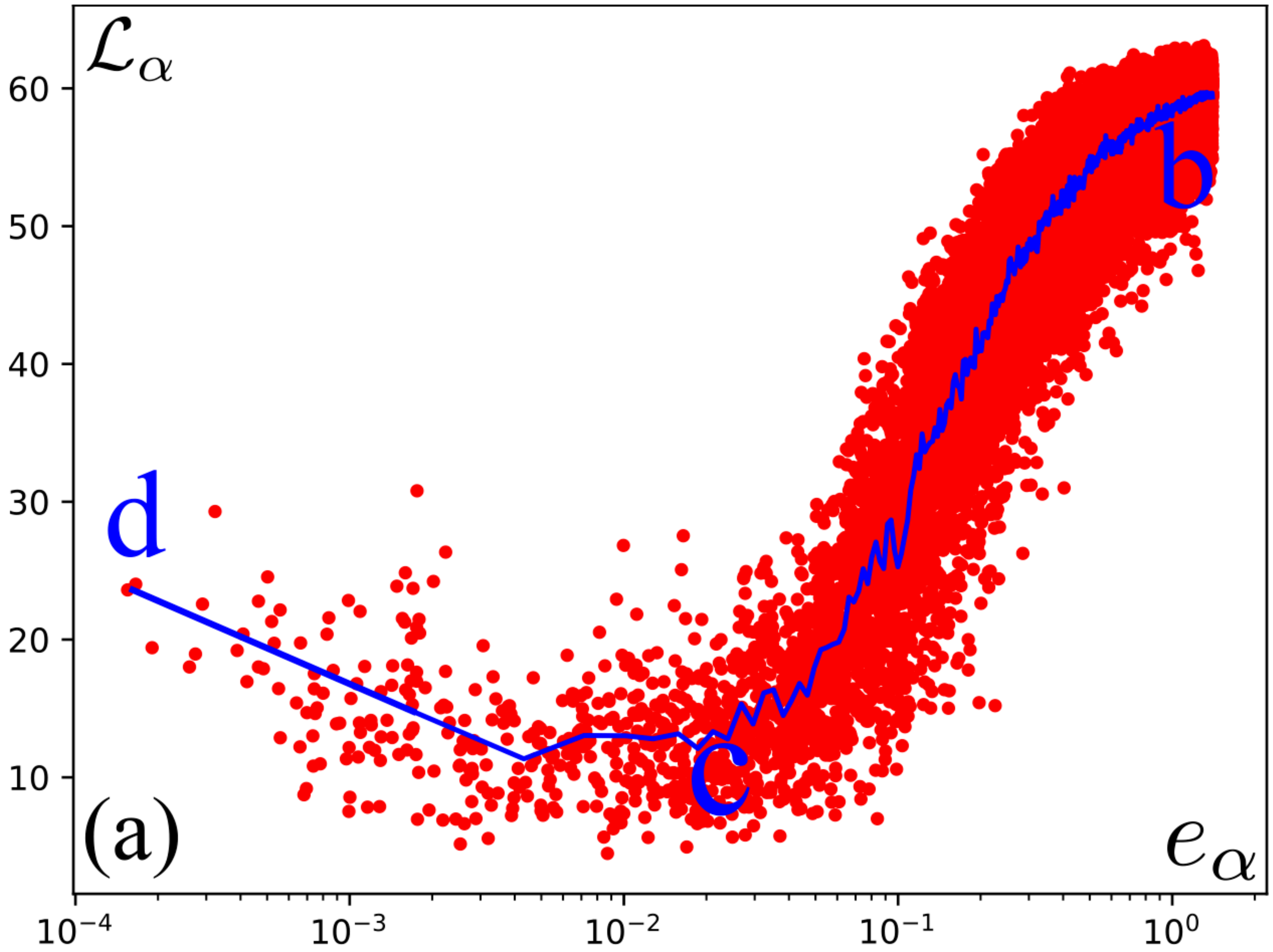
Bosonic eigenmodes in random mass Hertz theory



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$



See also: QMC results of Aavishkar A. Patel, Peter Lunts, Michael S. Albergo, arXiv:2410.05365

Aavishkar A. Patel, Peter Lunts, S.S., *PNAS* **121**, e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

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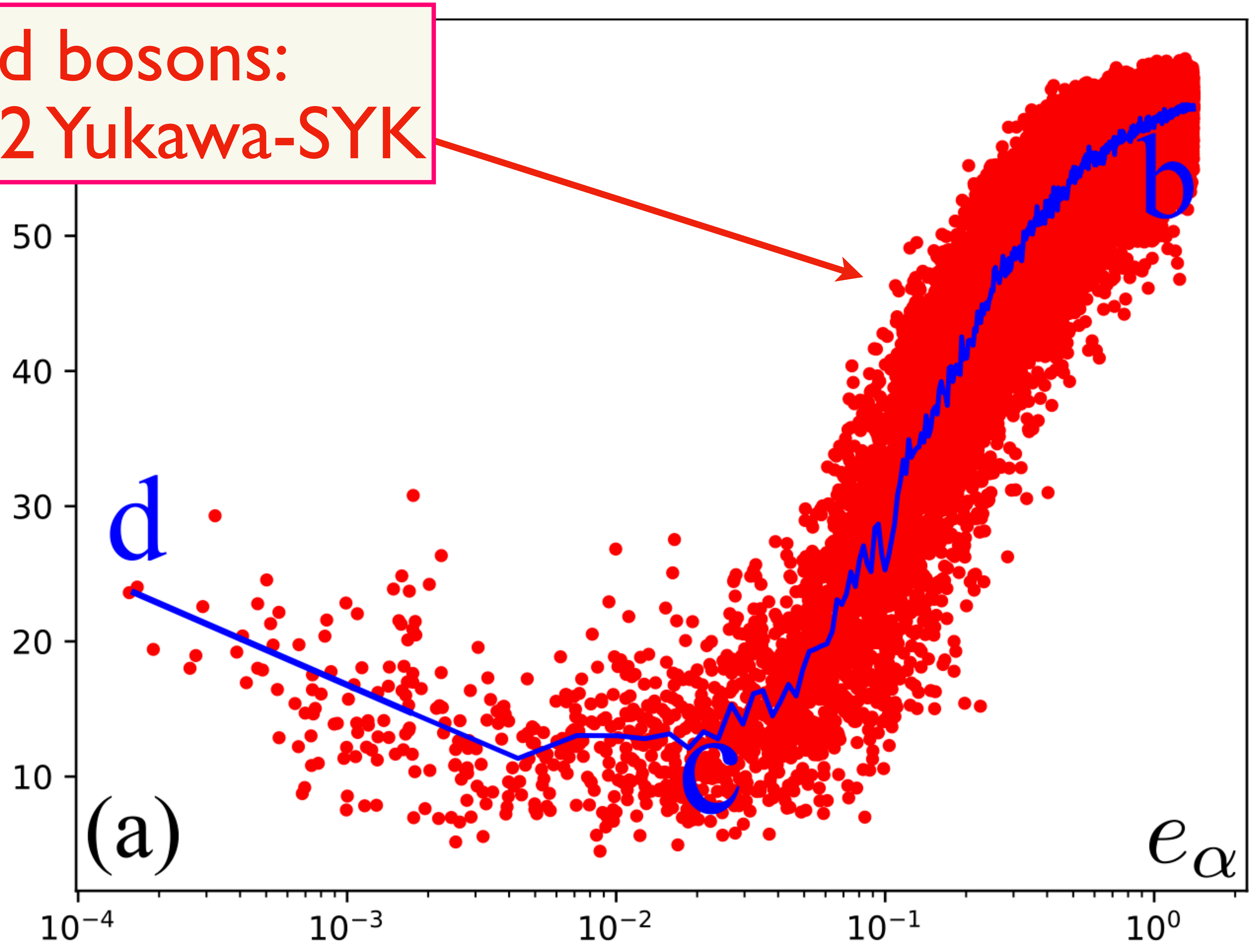
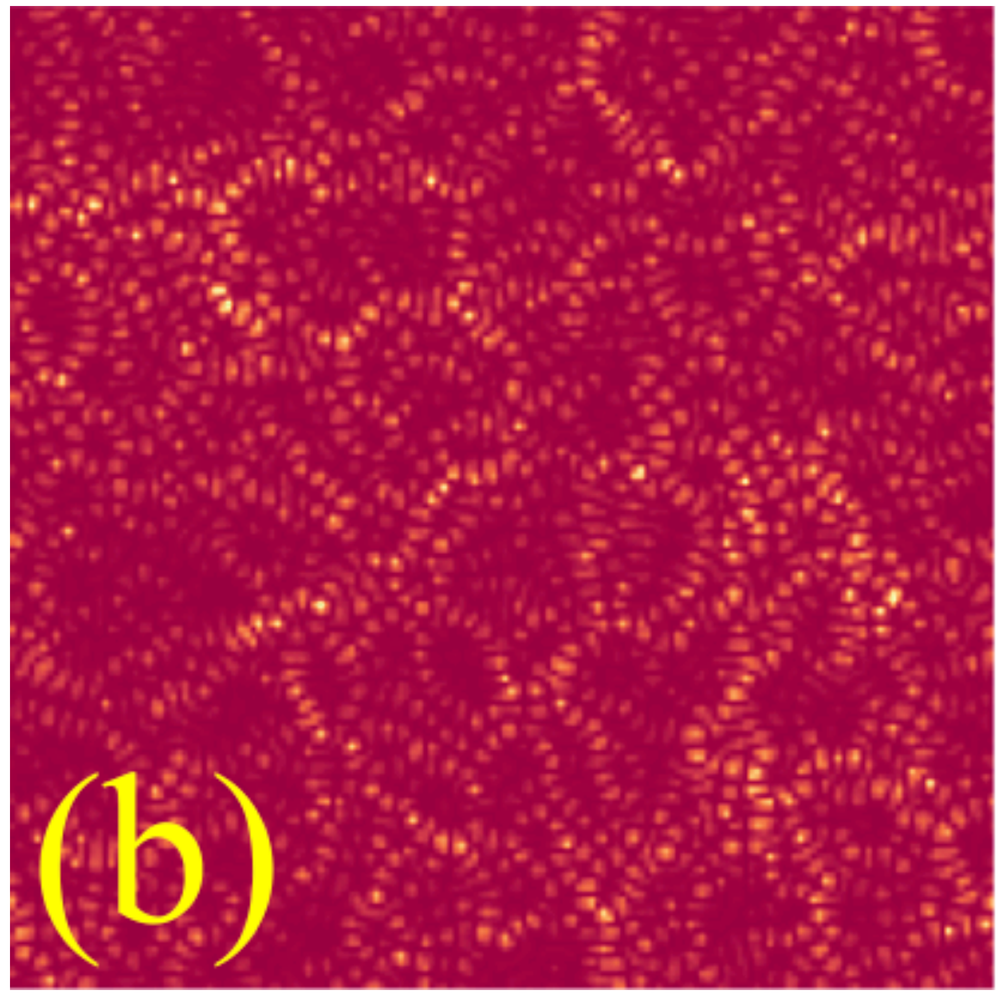
Extended bosons: physics of d=2 Yukawa-SYK





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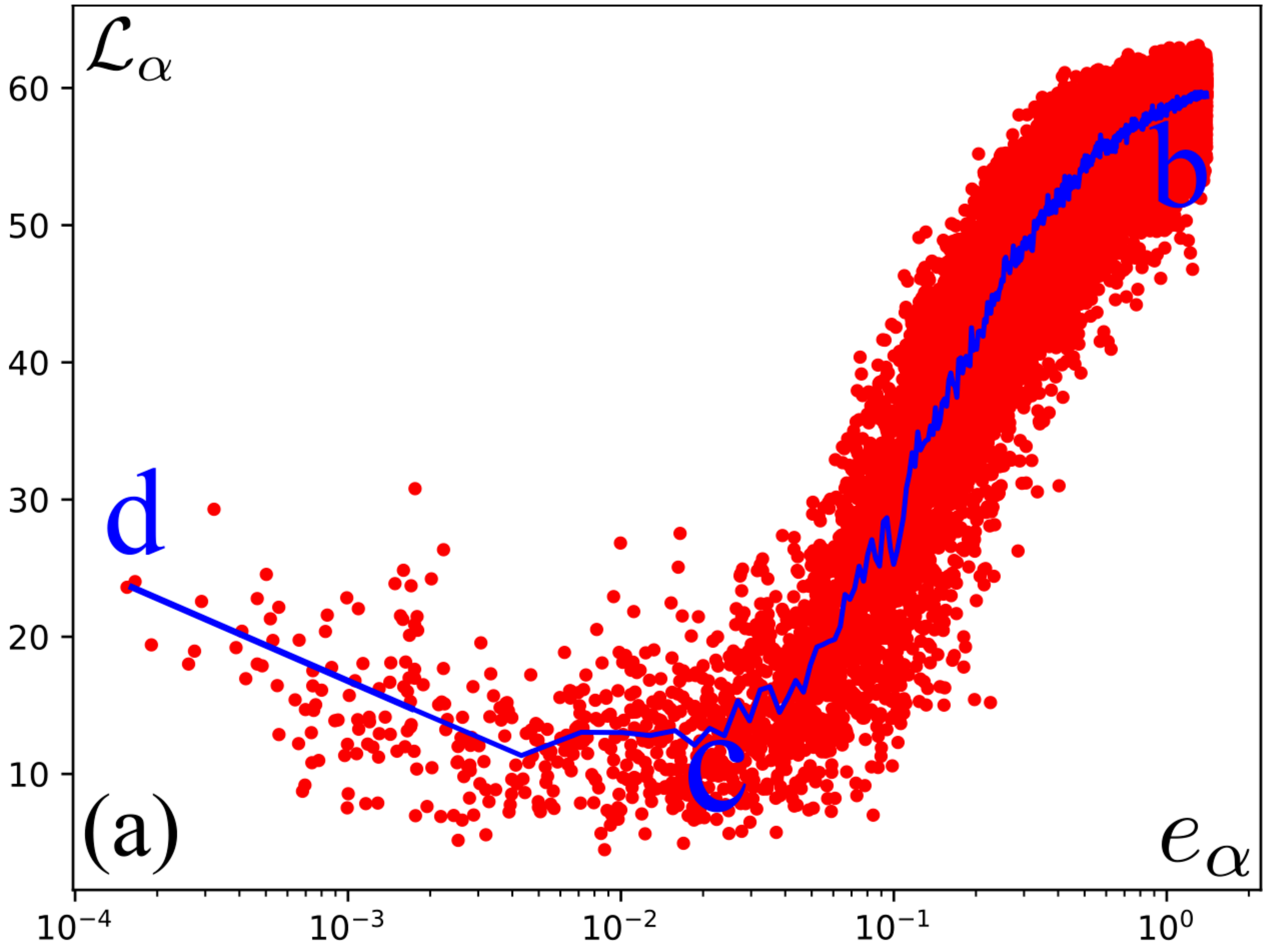

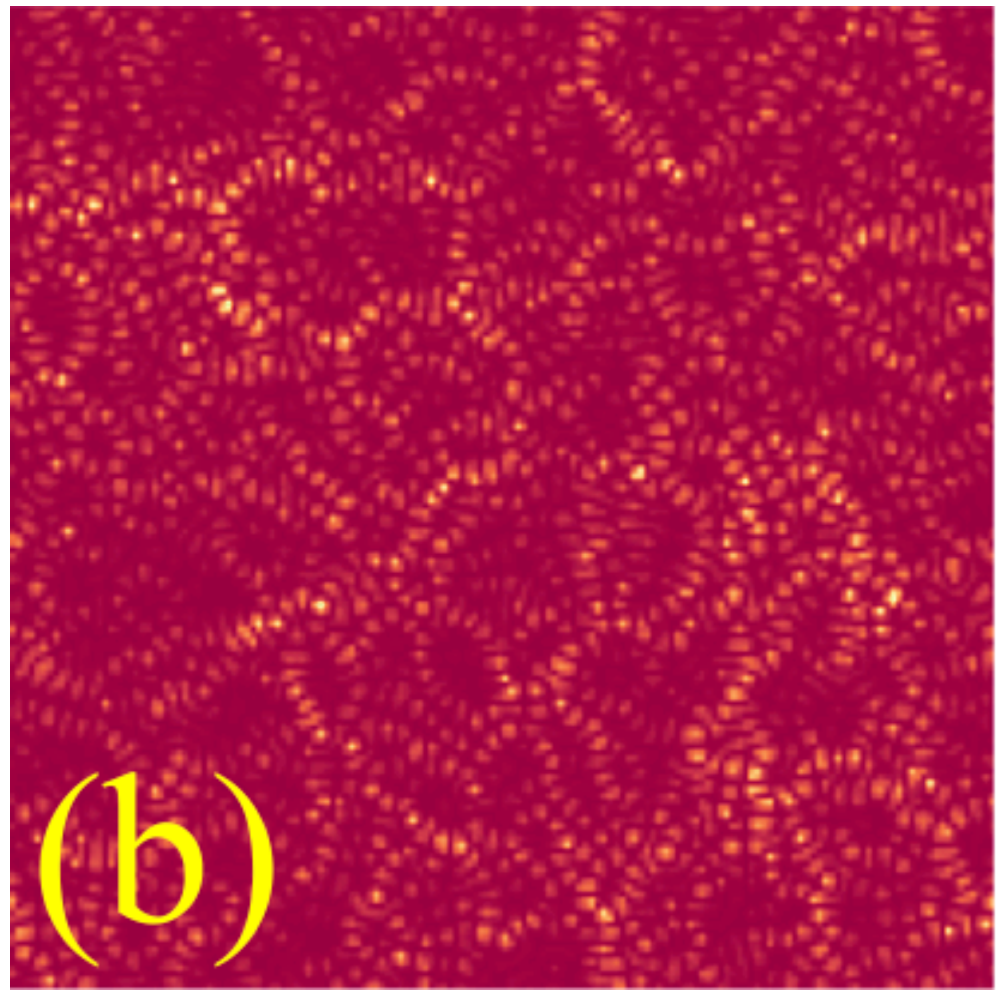

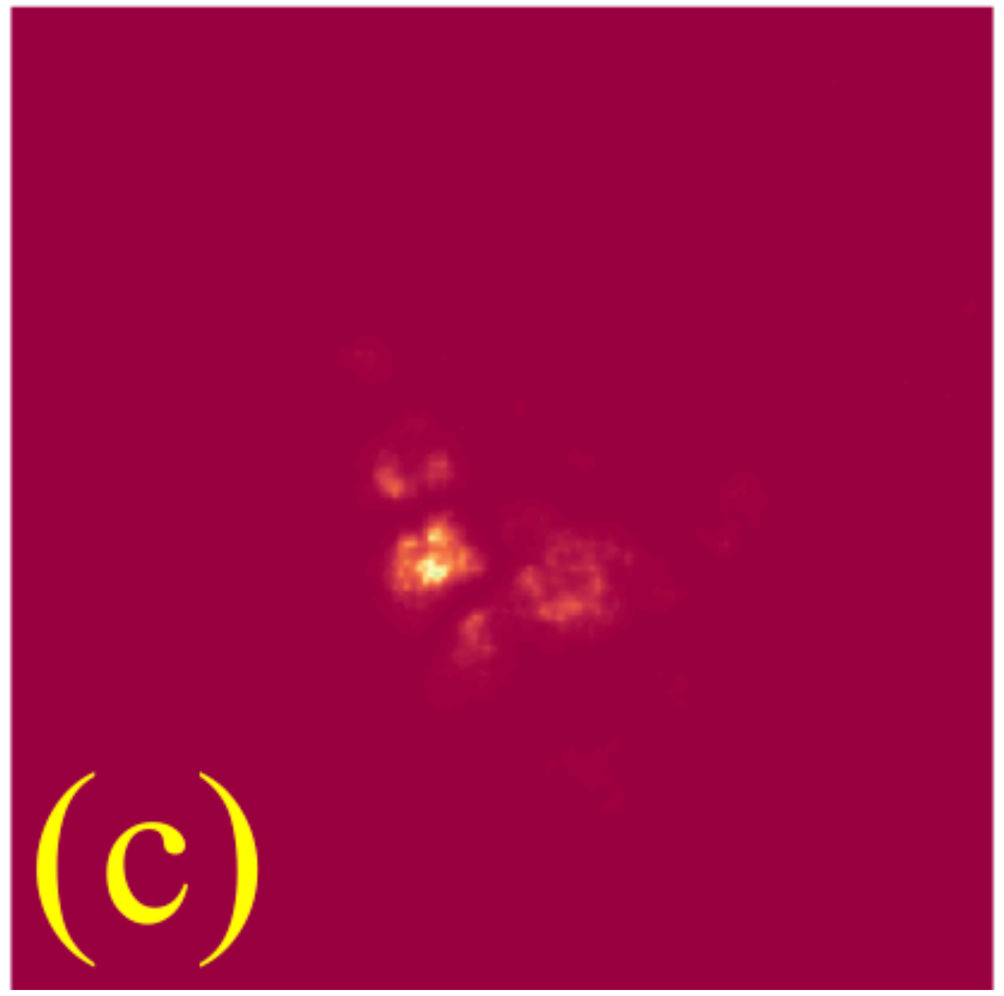

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Bosonic eigenmodes in random mass Hertz theory

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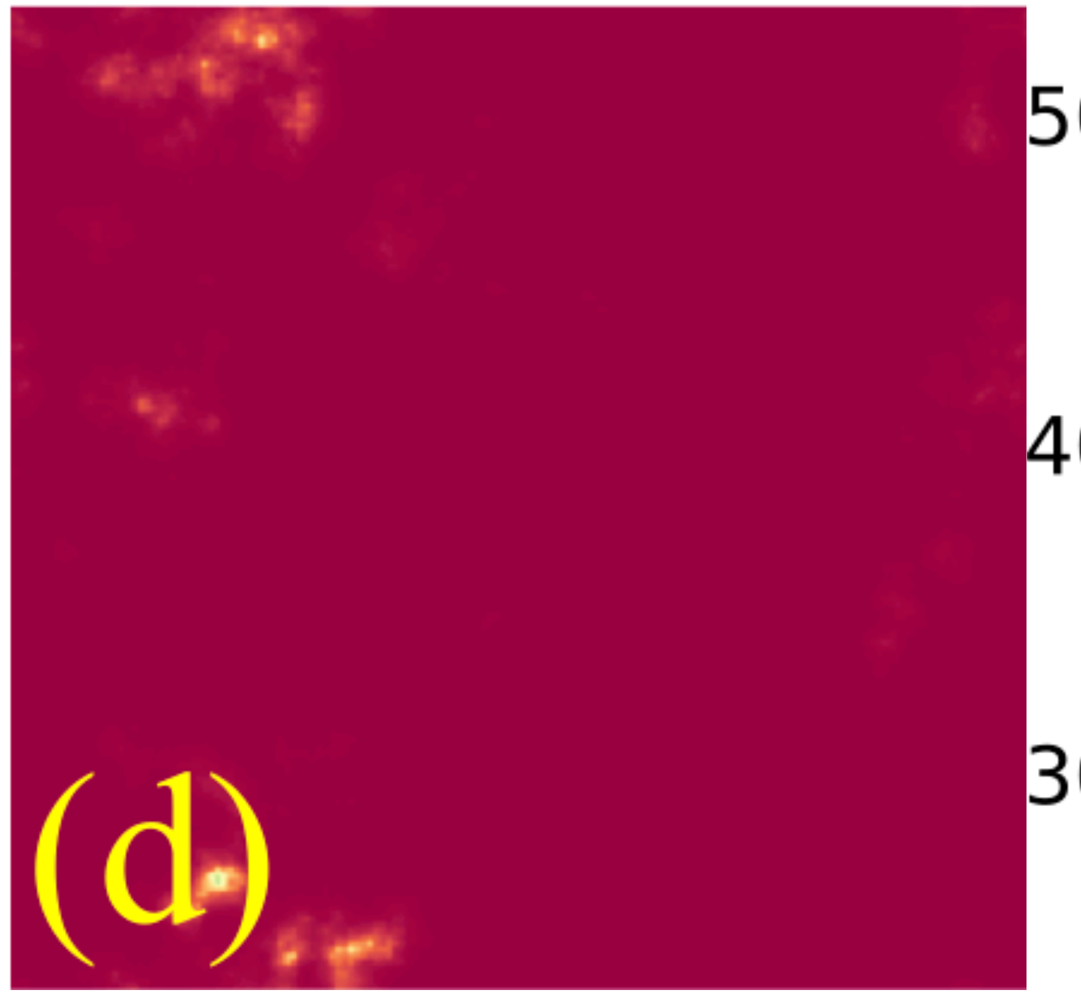

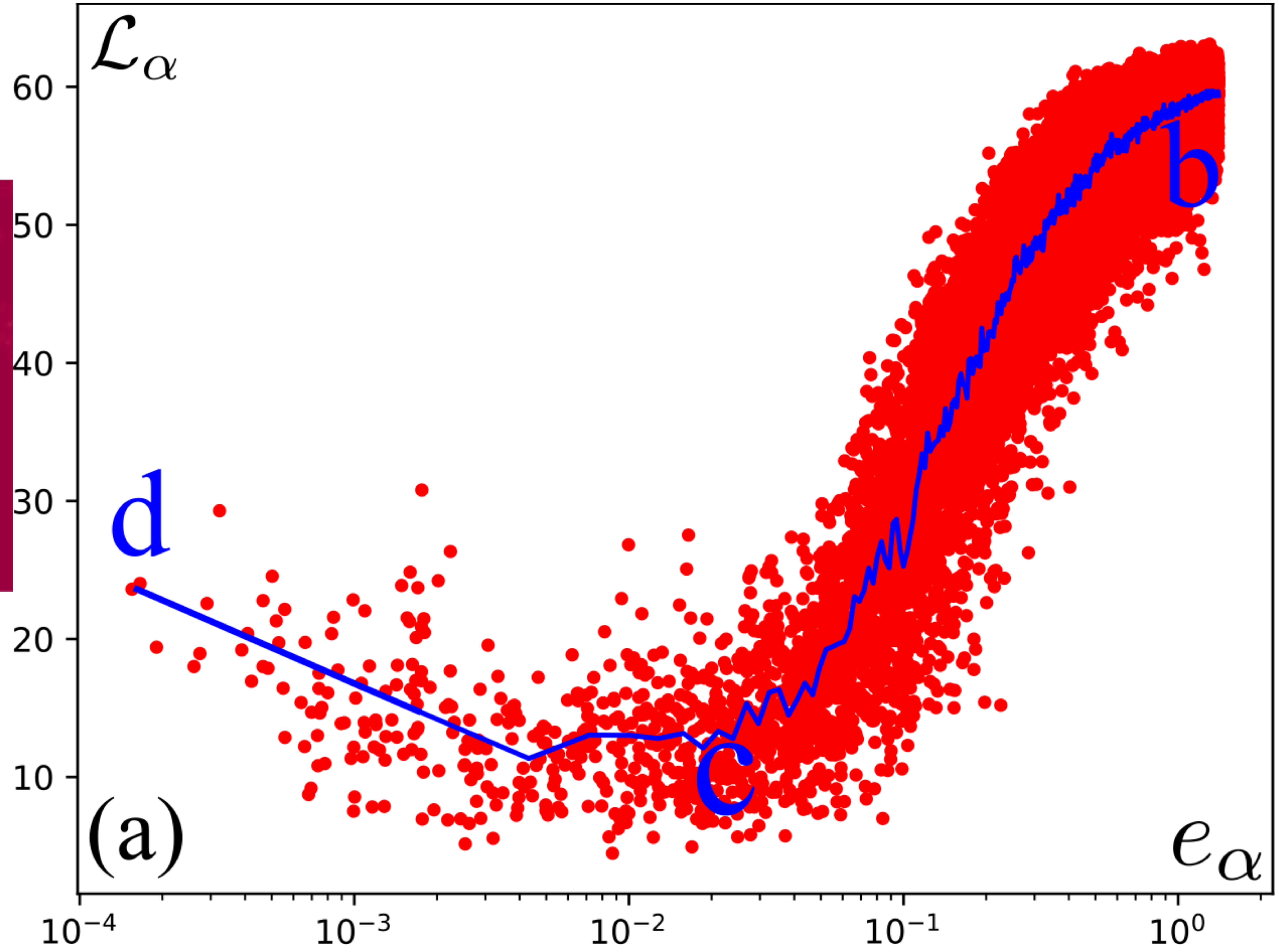

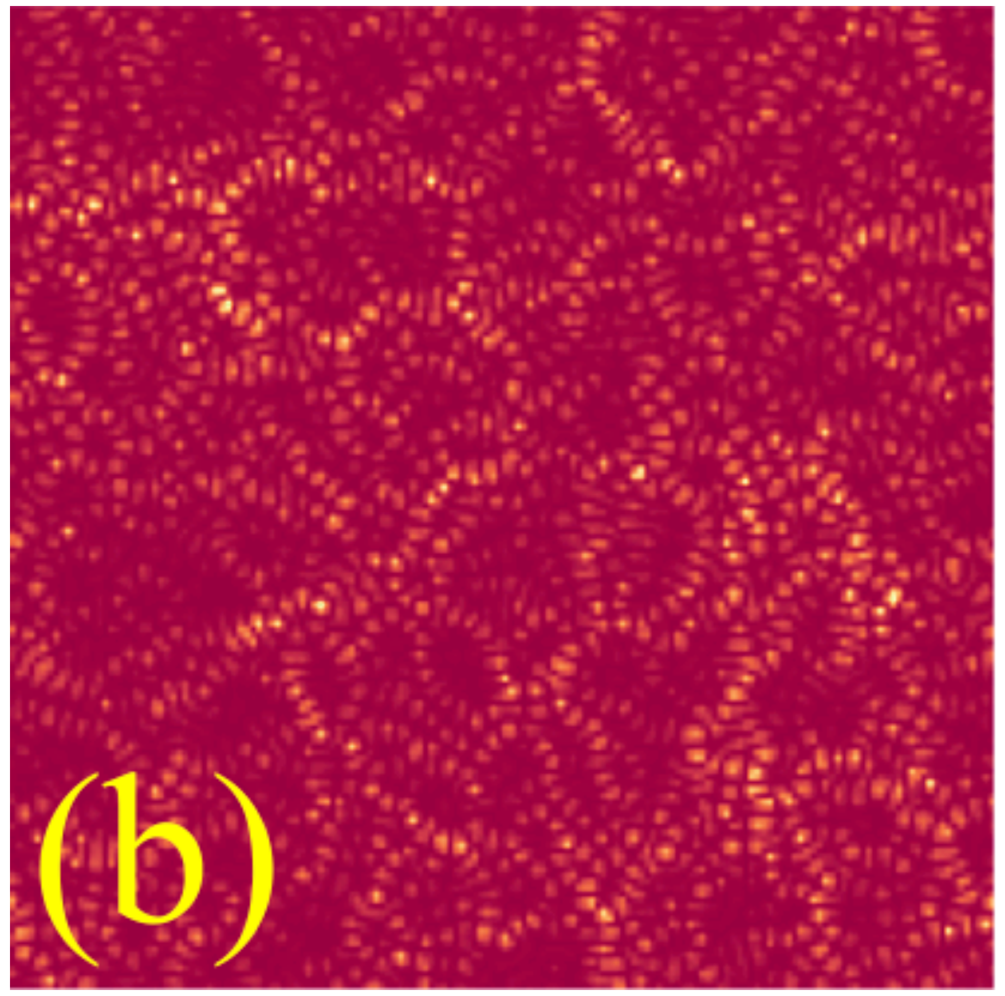

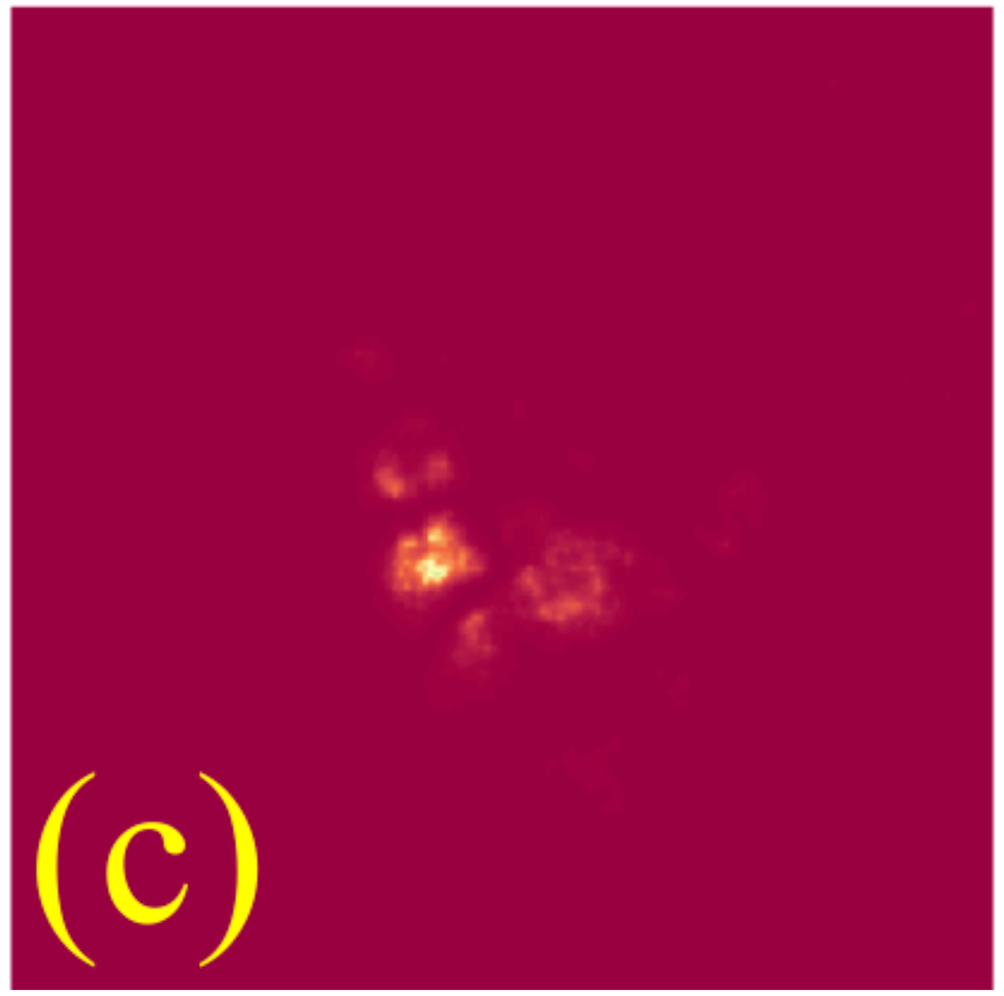

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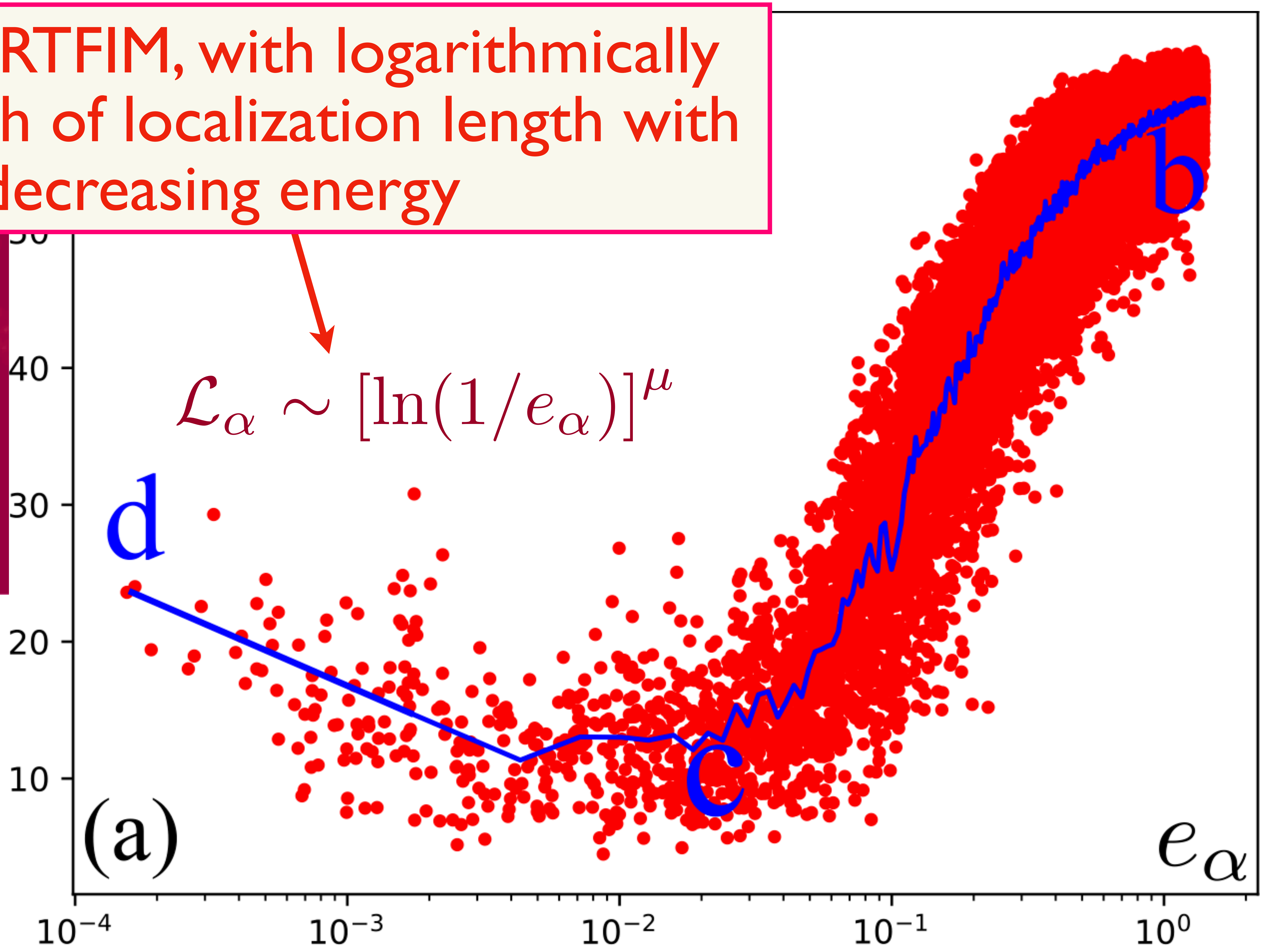
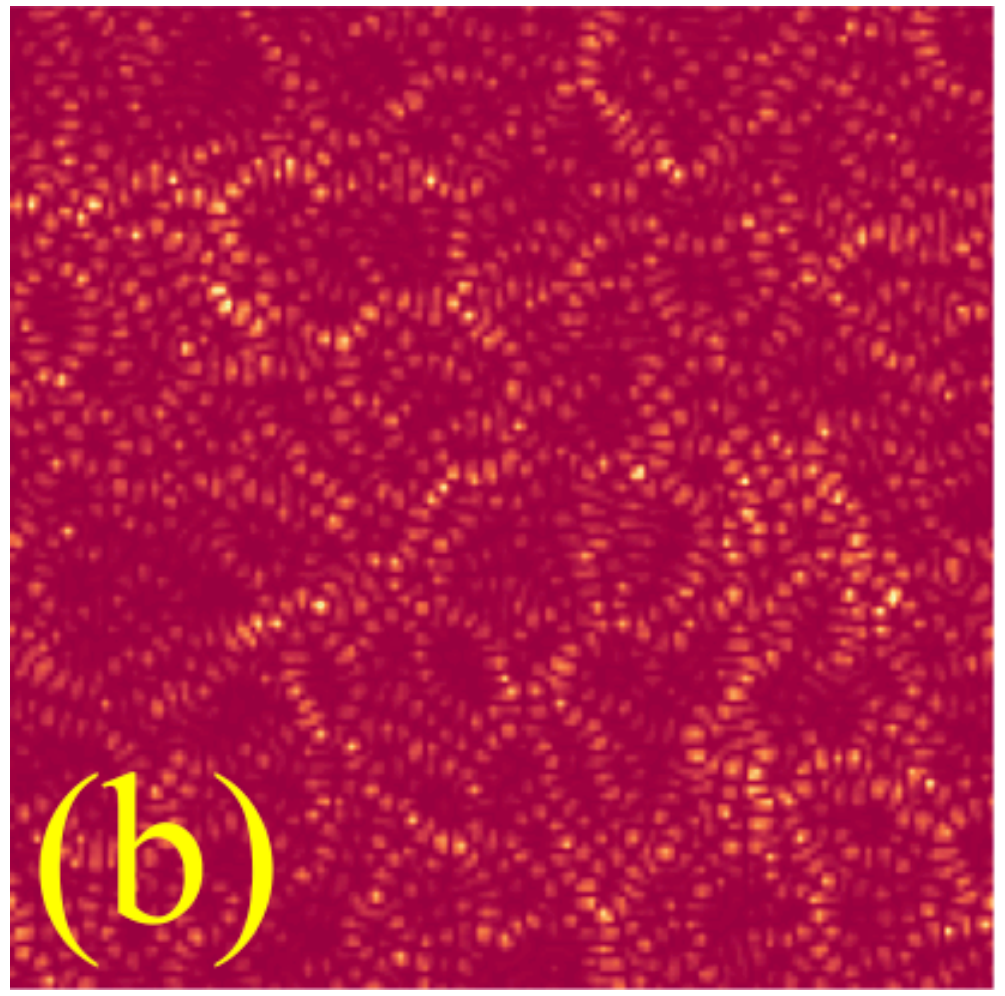
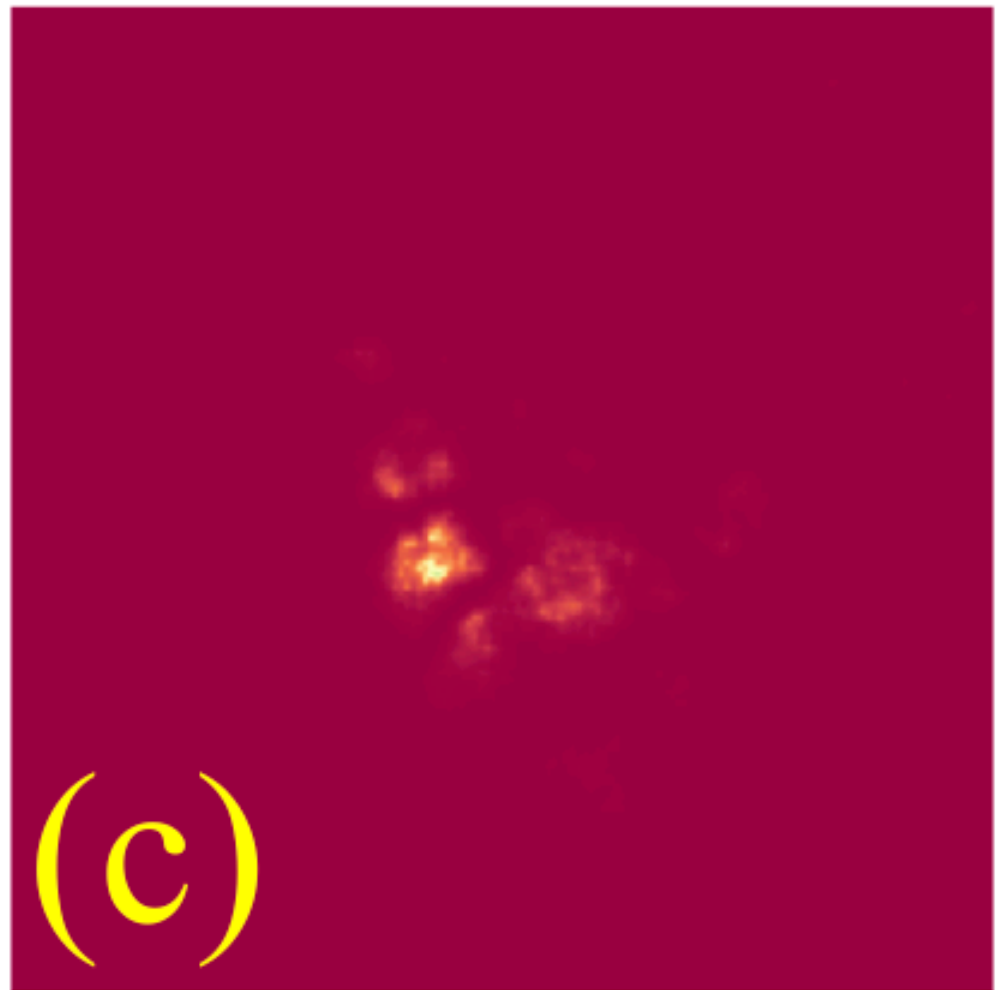
Bosonic eigenmodes in random mass Hertz theory

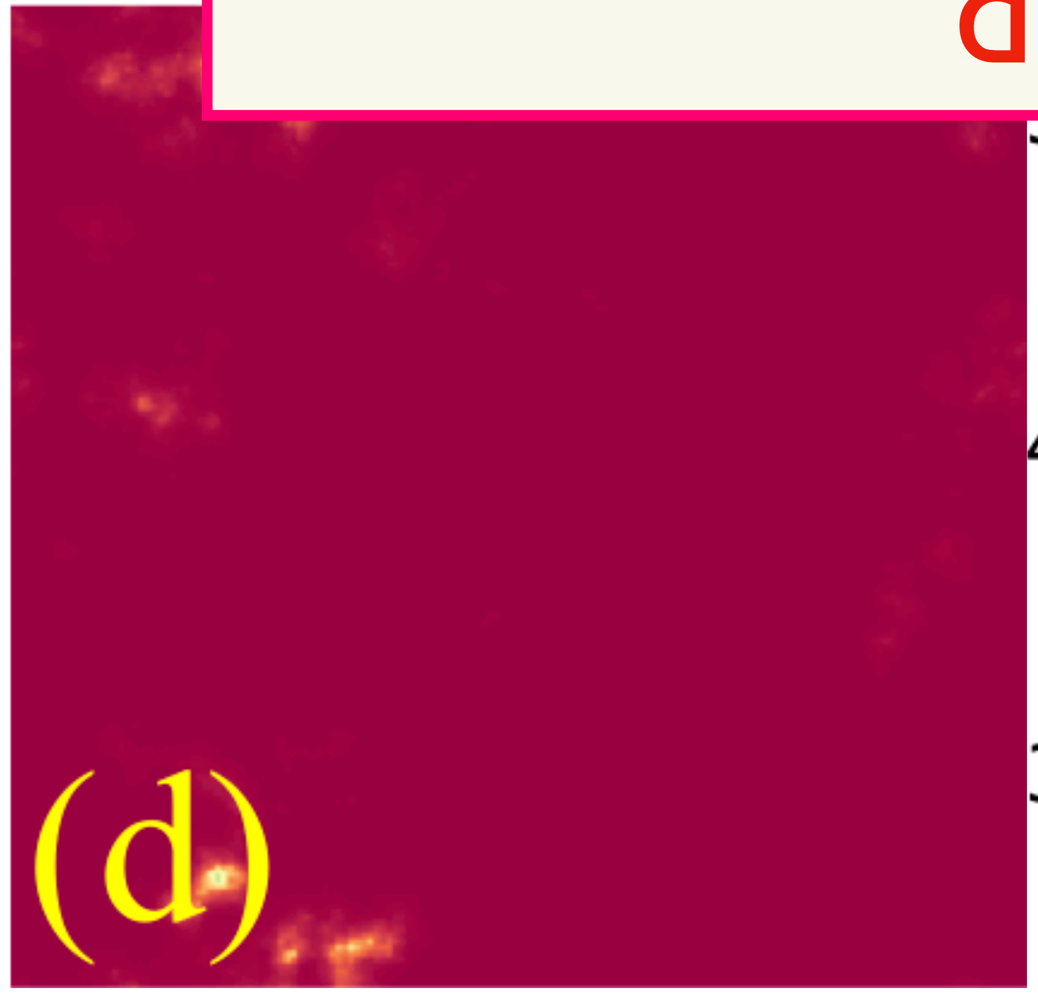
ϕ eigenmodes localization length $\mathcal{L}_\alpha$

Physics of RTFIM, with logarithmically slow growth of localization length with decreasing energy









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Bosonic eigenmodes in random mass Hertz theory

Transport scattering rate

$$\Sigma(i\omega) = -i\pi g'^2 \mathcal{N}_0 \frac{T}{L^2} \sum_{\alpha,\Omega} \frac{\mathrm{sgn}(\omega+\Omega)}{\gamma|\Omega| + \Omega^2/c^2 + e_\alpha}\,.$$

Aavishkar A. Patel, Peter Lunts, S.S., *PNAS* **121**, e2402052121 (2024)

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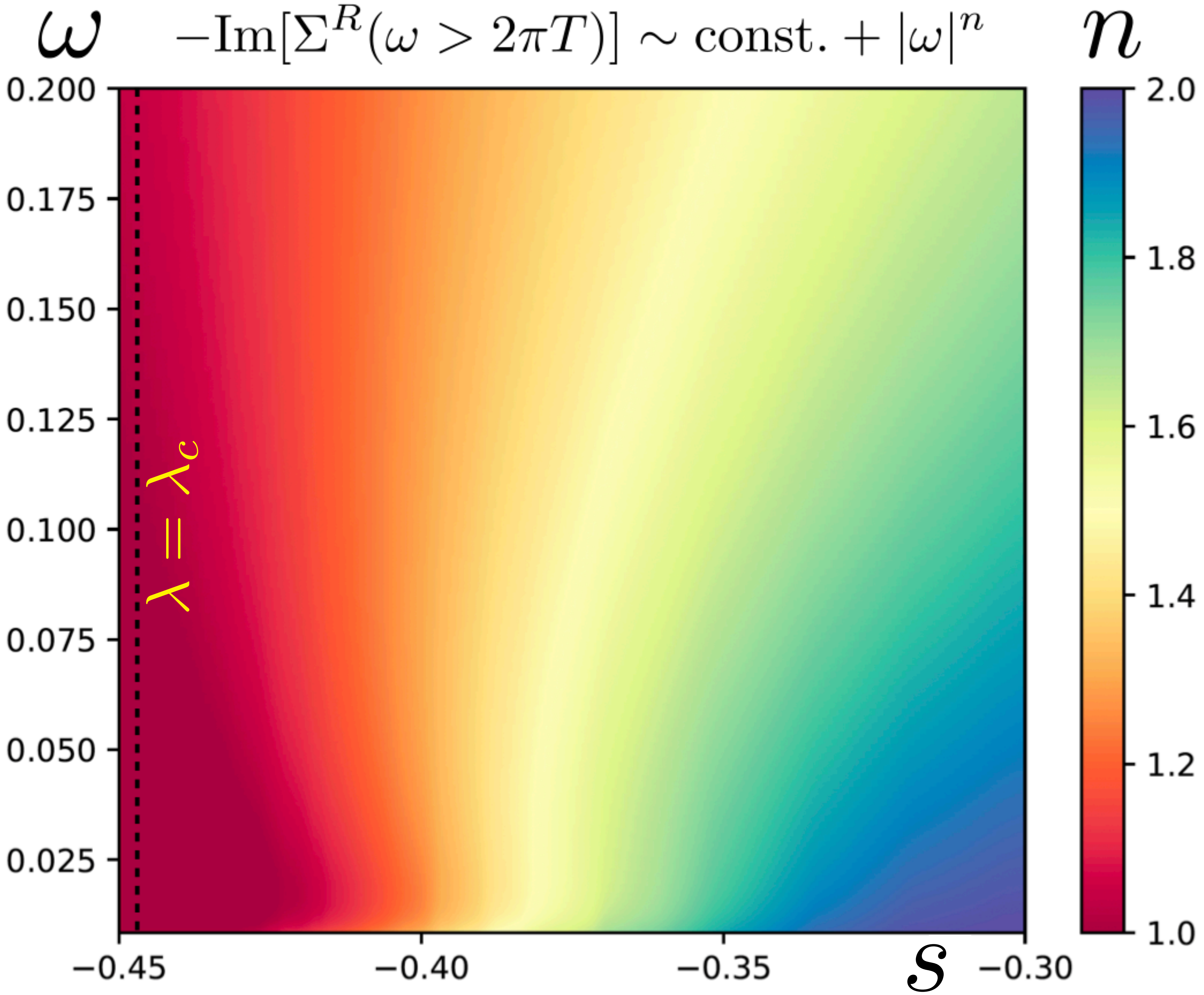
$-\mathrm{Im}[\Sigma^R(\omega > 2\pi T)] \sim \mathrm{const.} + |\omega|^n$



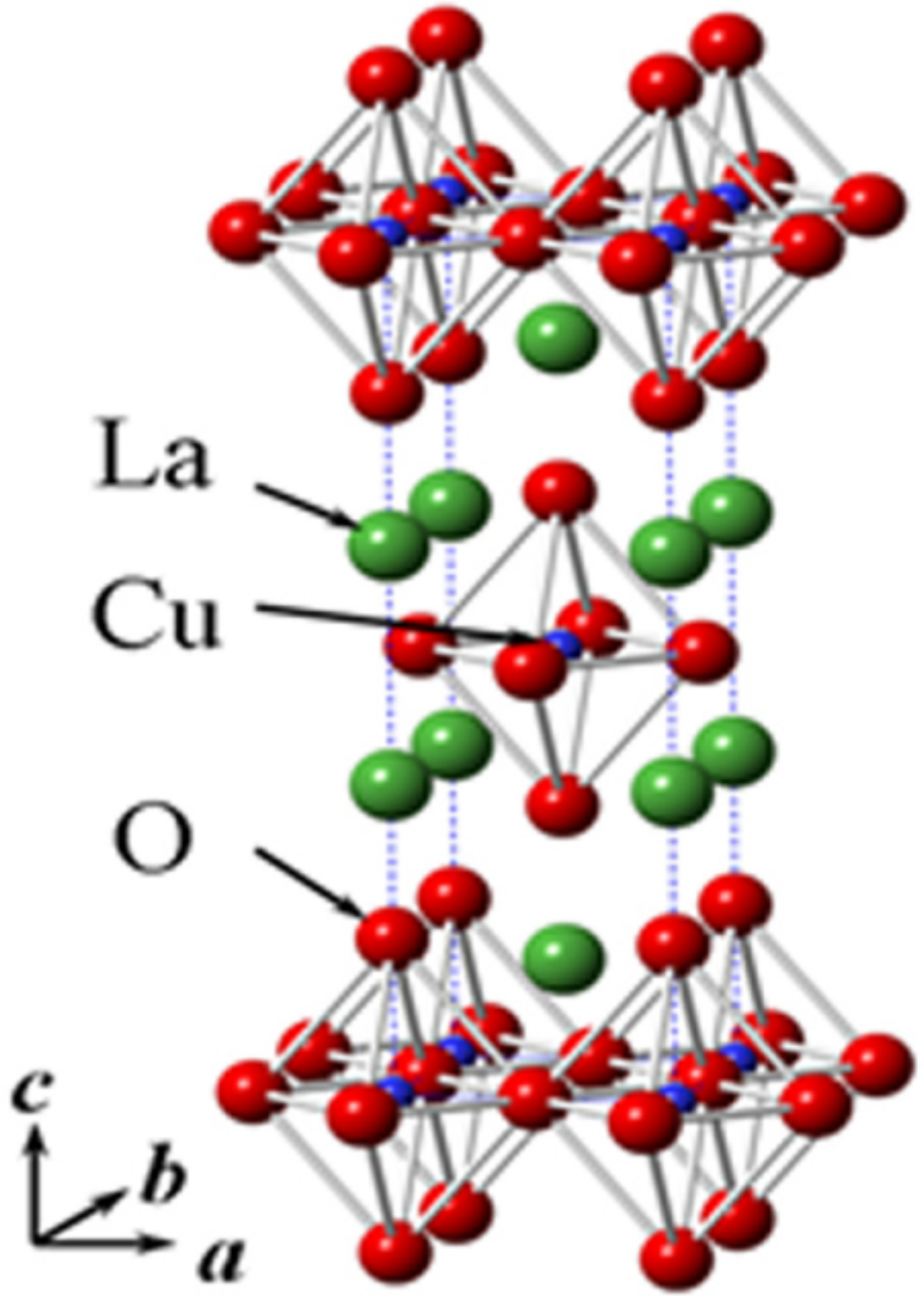
$L = 160, \beta = 800$, 10 disorder samples

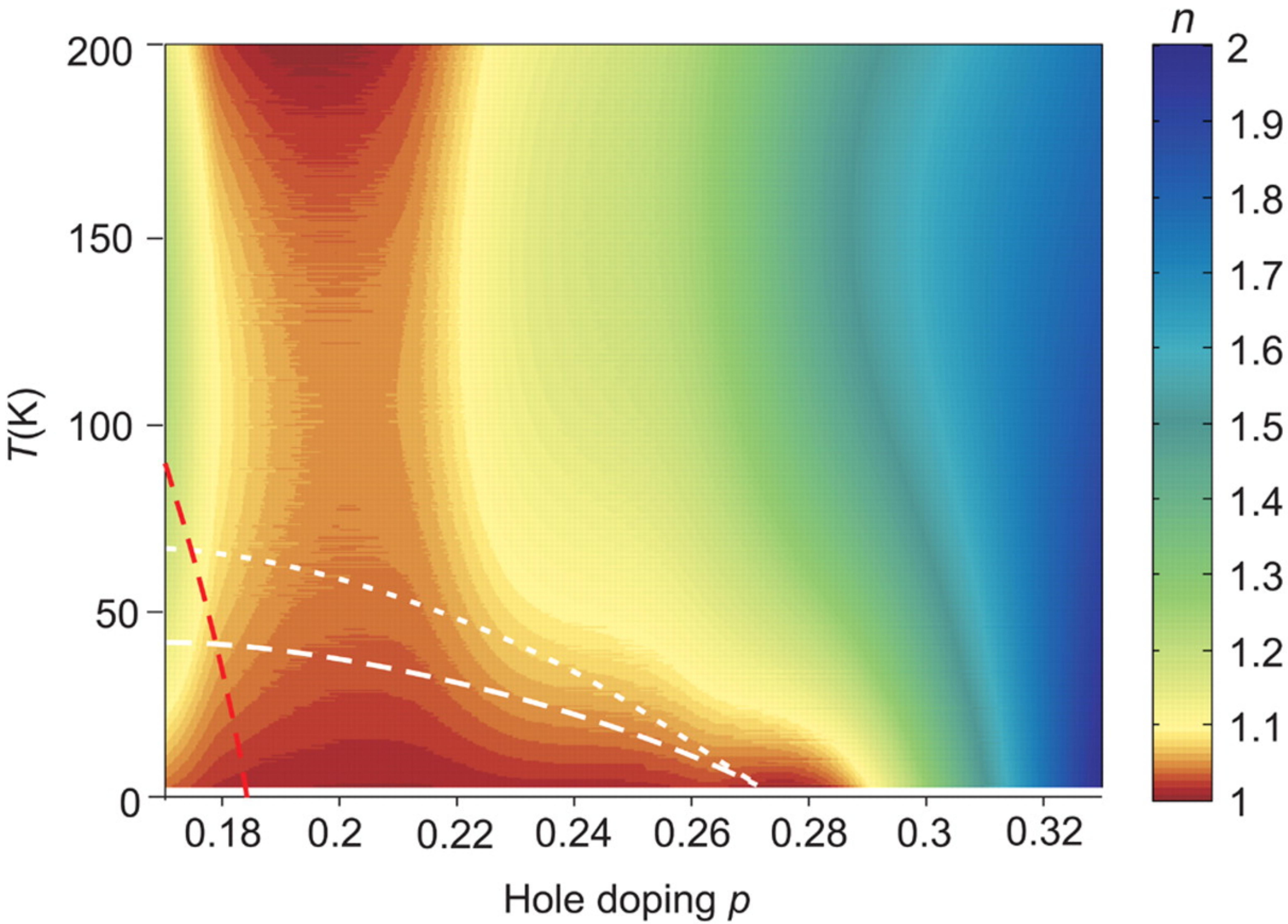
Extended region in λ with $n \approx 1$ - a strange metal *phase*

Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4*] H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1†]

SCIENCE VOL 323 603 2009





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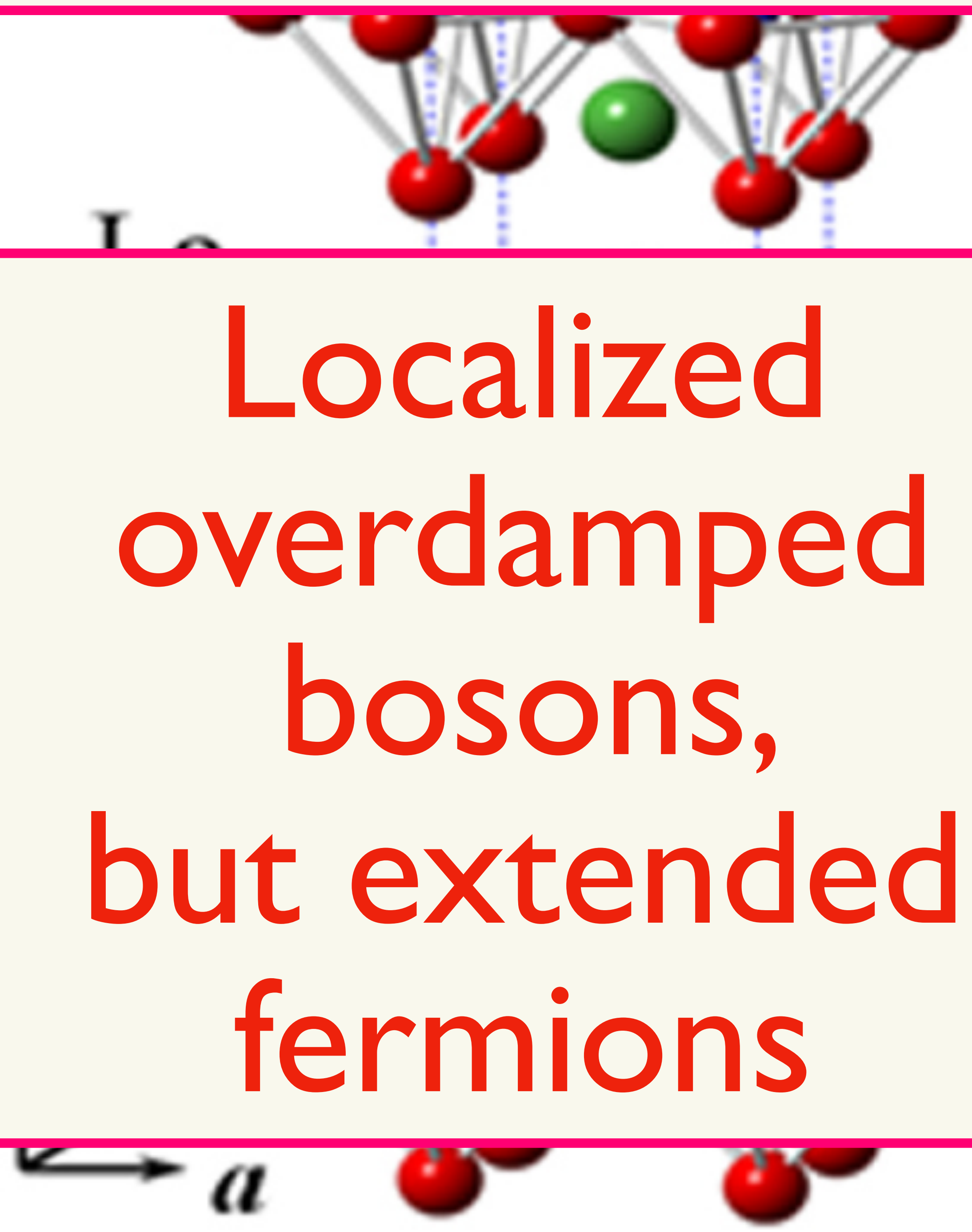
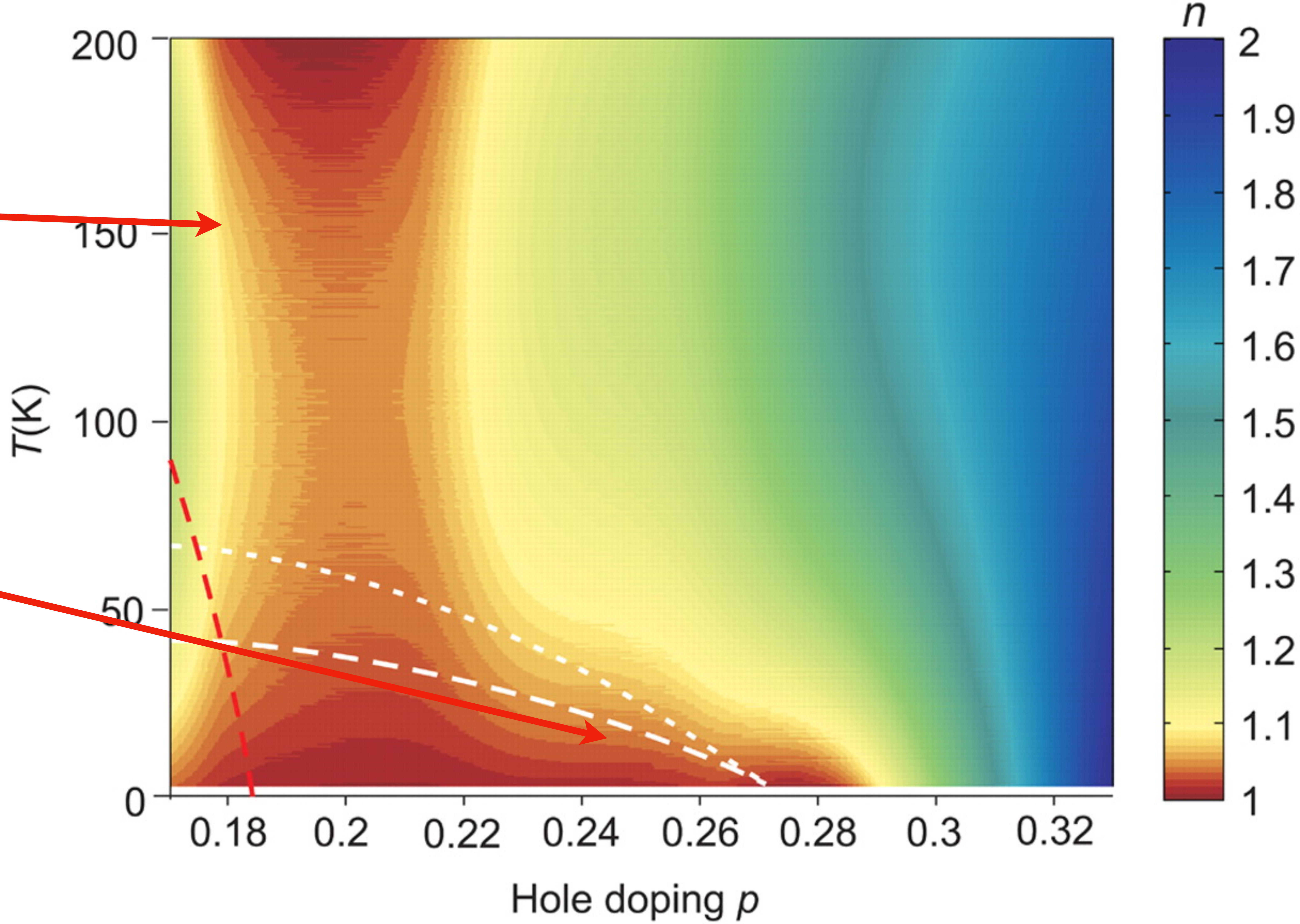
Two-dimensional metals with Harris disorder

Extended fermions and bosons

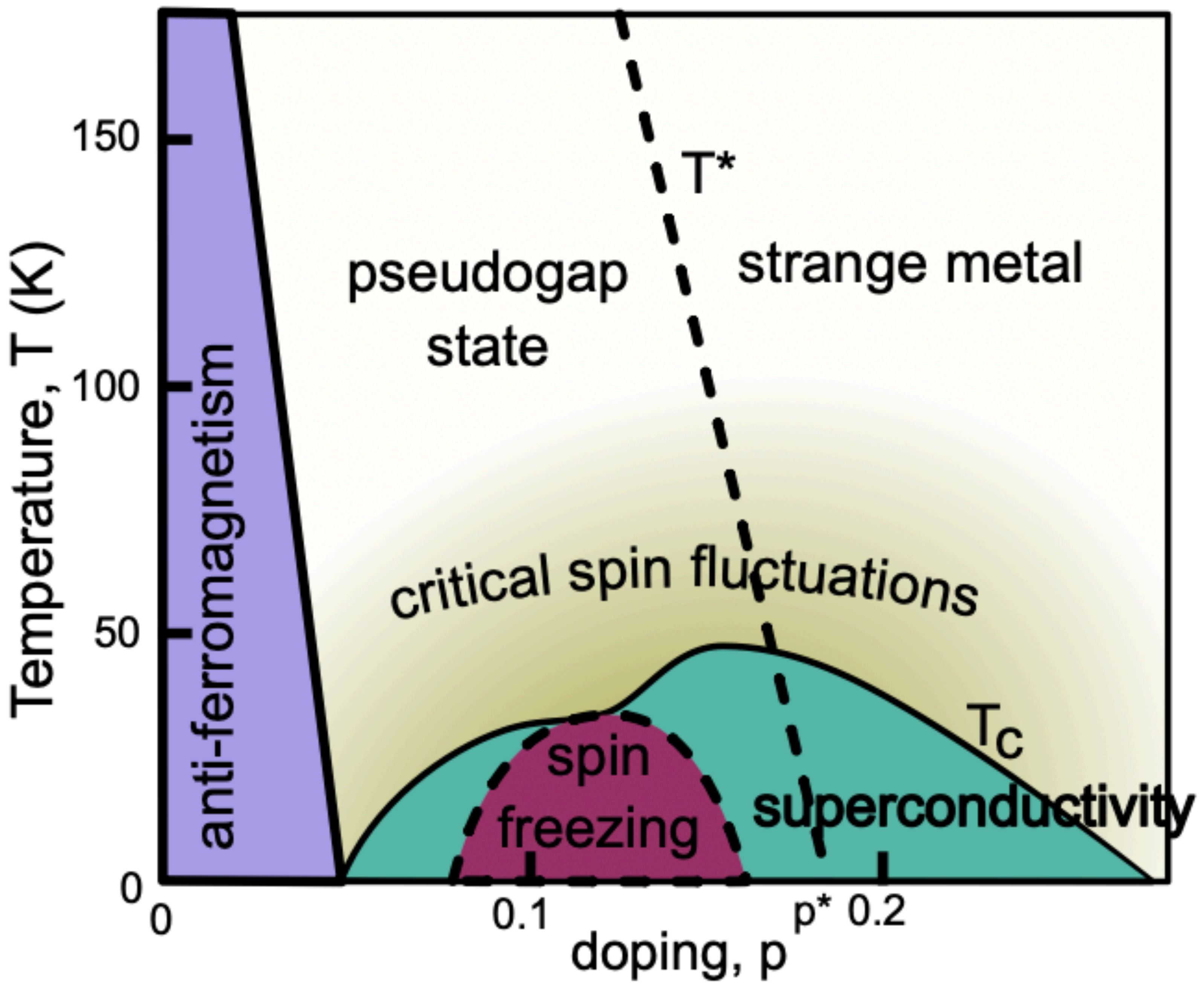


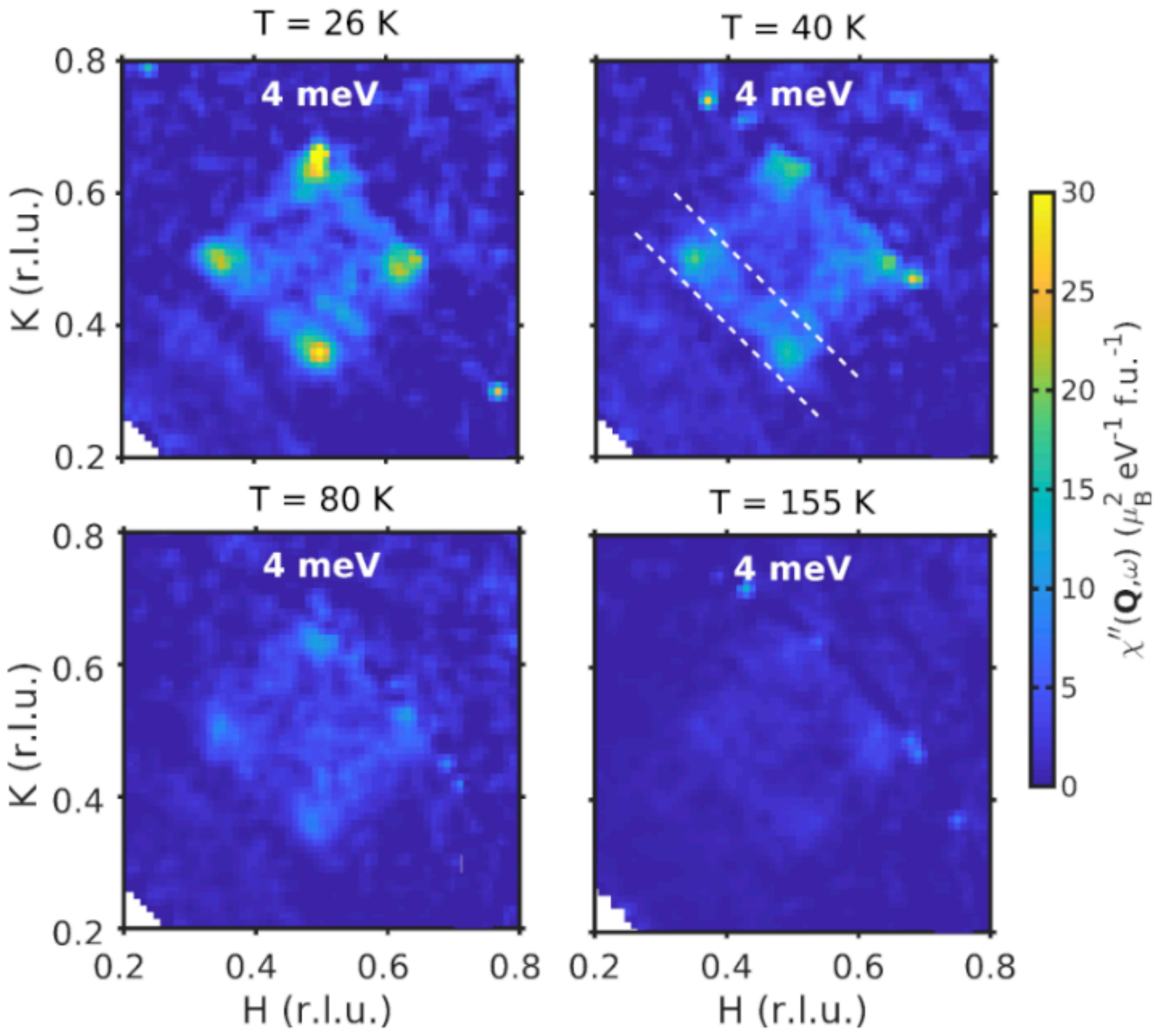
Localized overdamped bosons, but extended fermions

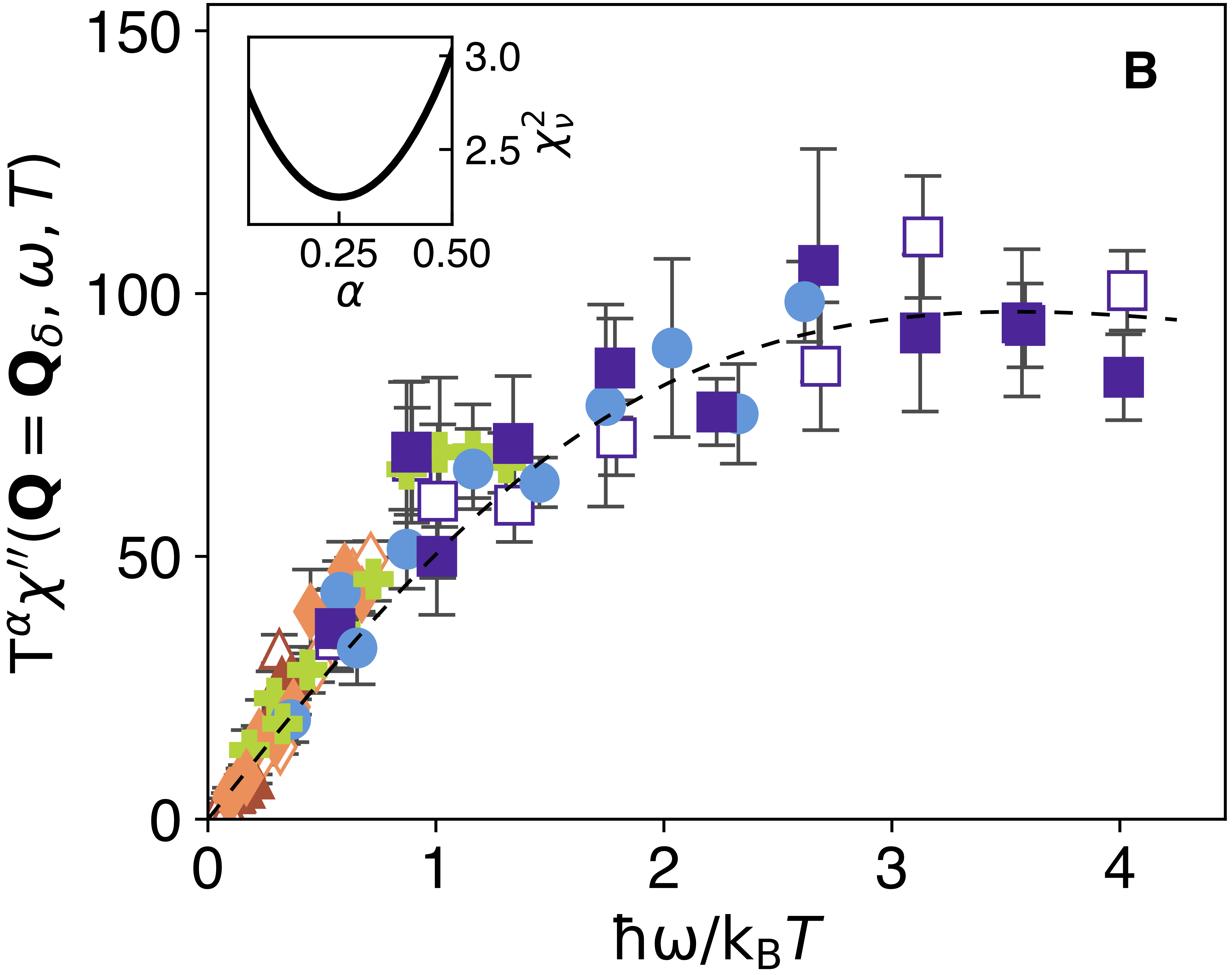


Neutron scattering in LSCO

Jacopo Radaelli, Aavishkar A. Patel, S. S., Stephen Hayden, to appear

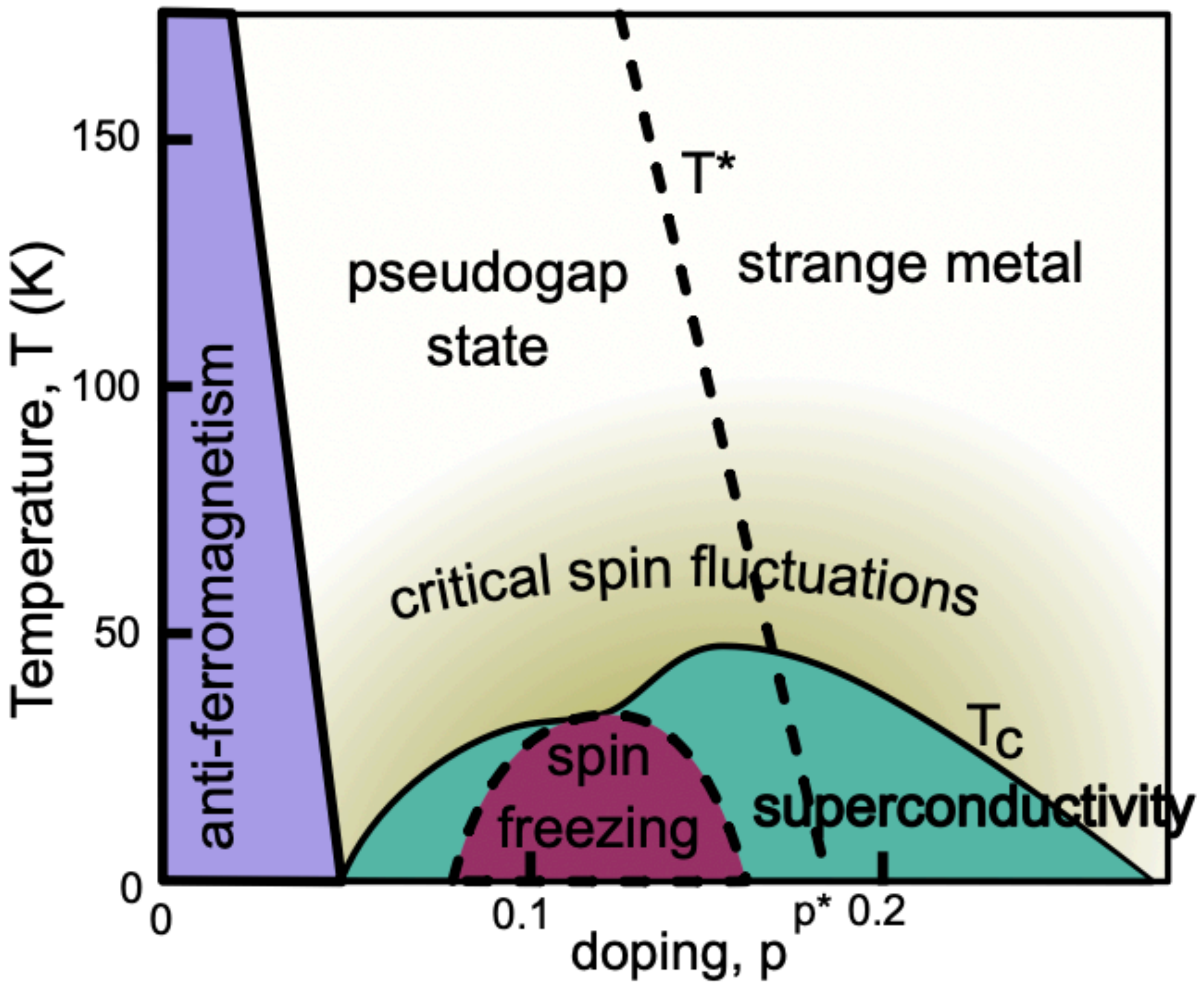


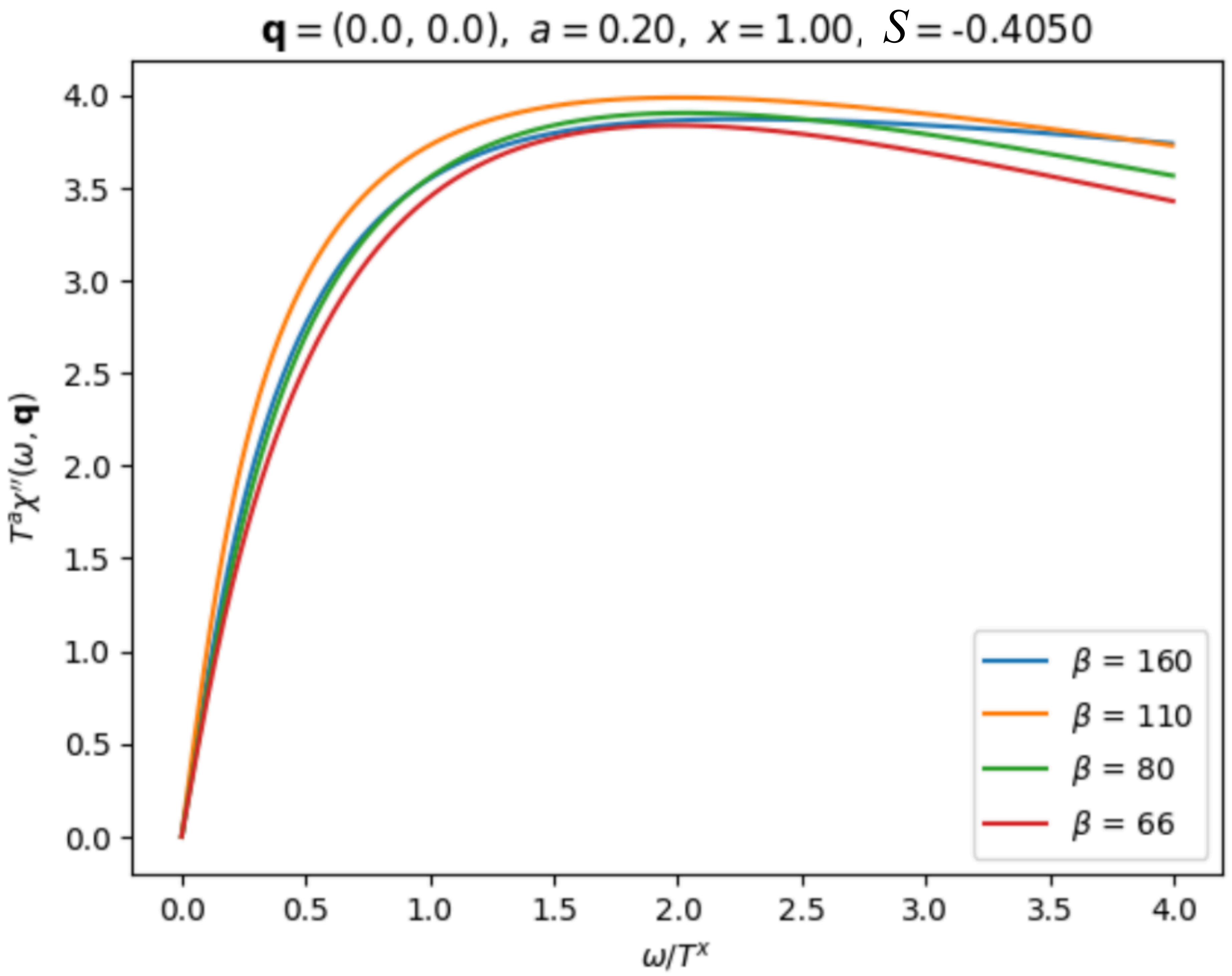


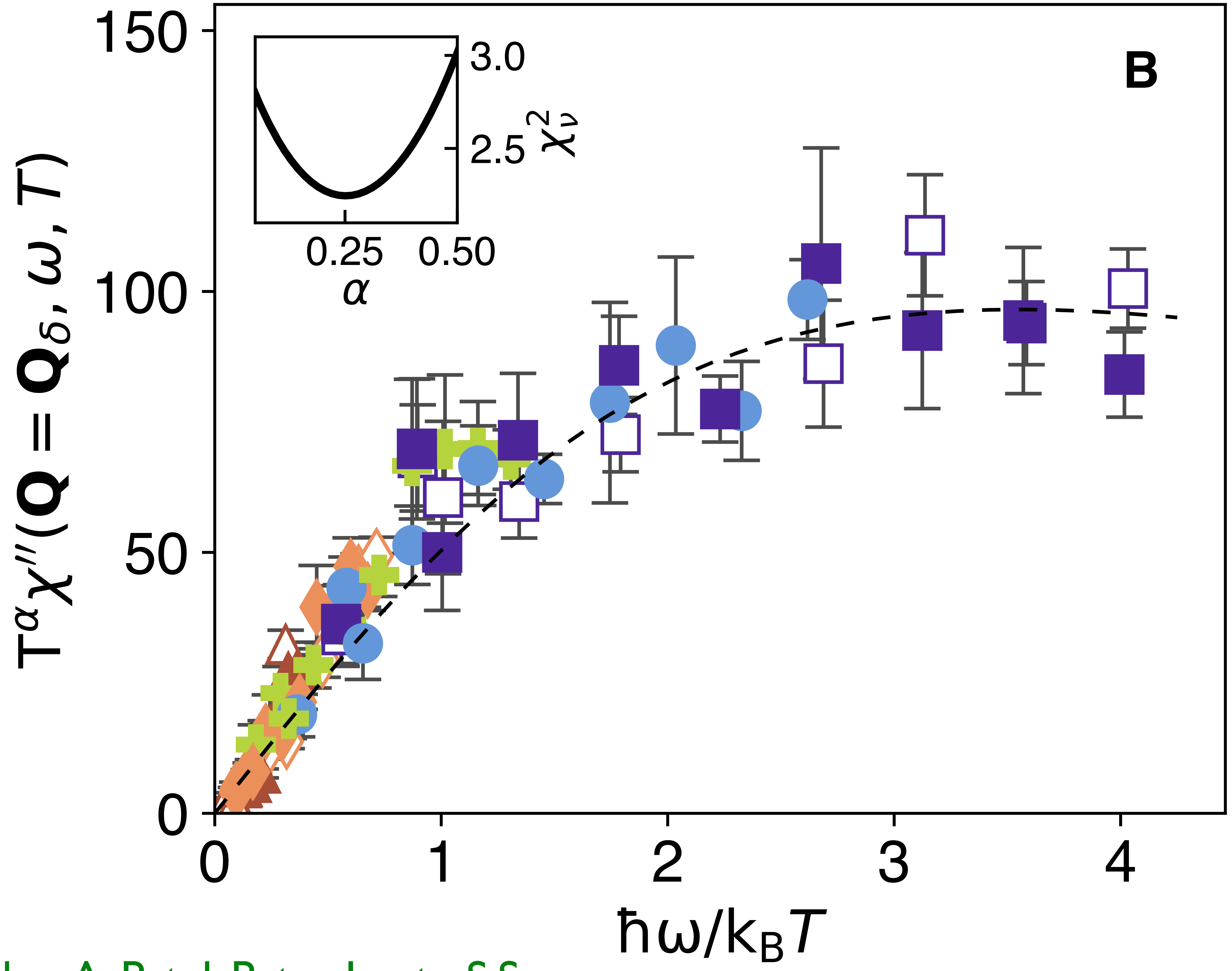


Neutron scattering in LSCO

Jacopo Radaelli, Aavishkar A. Patel, S. S., Stephen Hayden, to appear



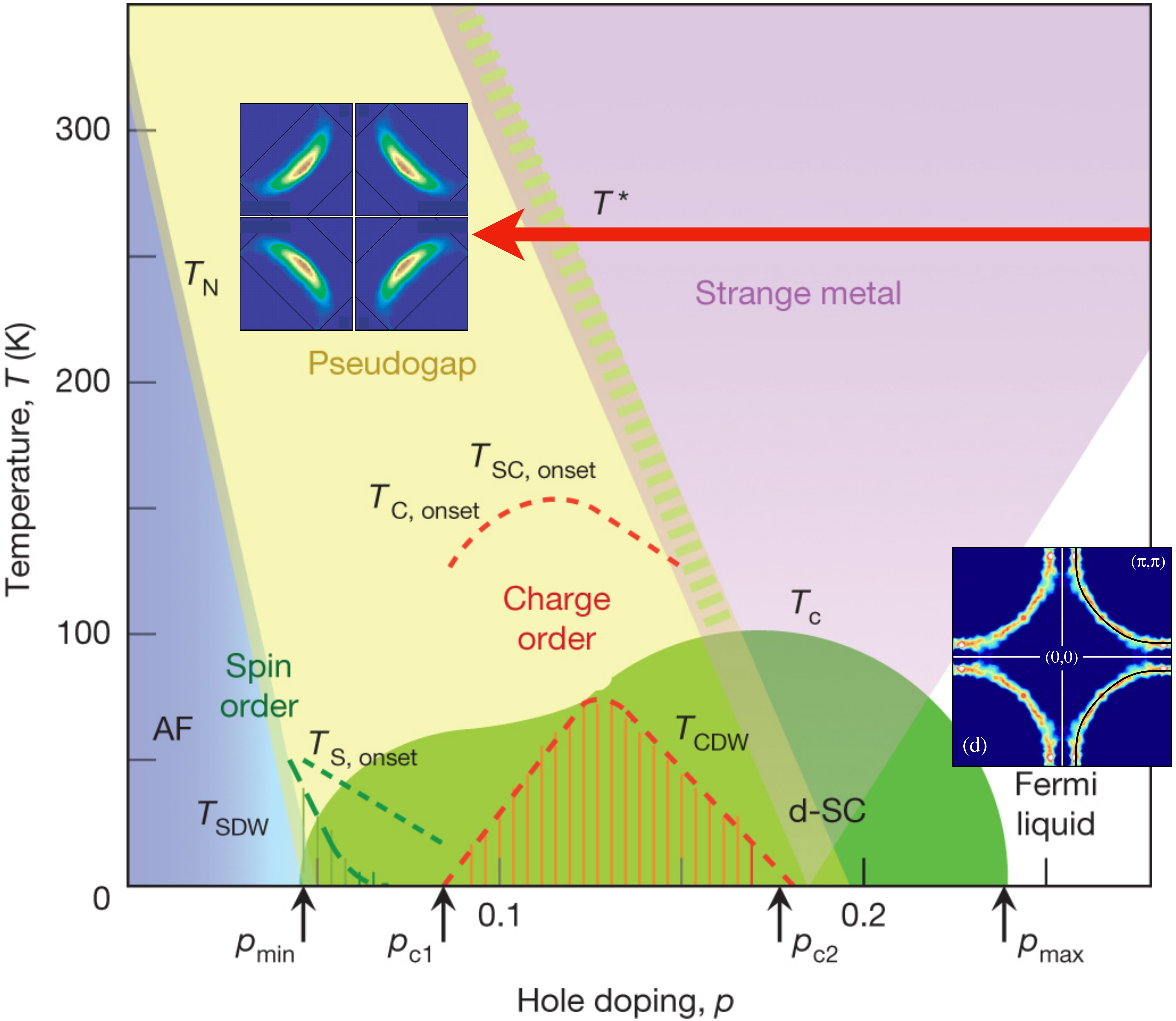




Aavishkar A. Patel, Peter Lunts, S.S., *PNAS* **121**, e2402052121 (2024)

1. FL-SDW QPT
2. FL-FL* QPT
3. Confinement crossover

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

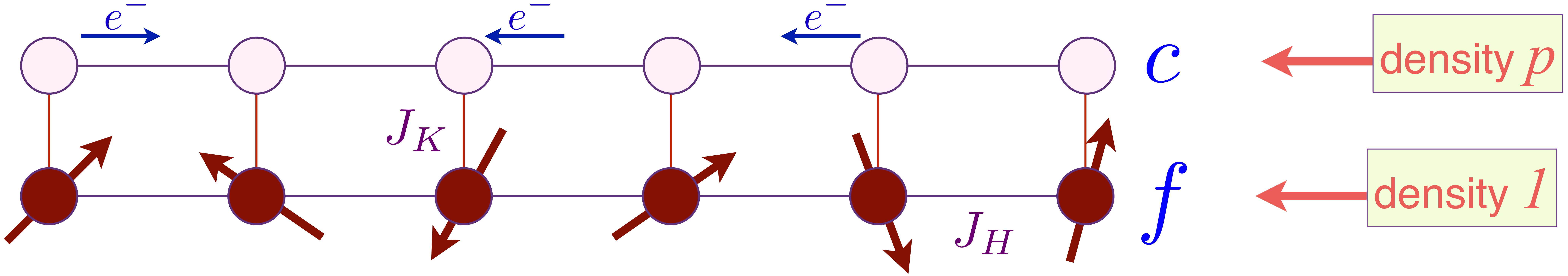


Fermi-volume-changing QPT
without symmetry breaking
and with spatial disorder.

FL-FL* QPT
Requires fractionalization

Fermi-volume-changing QPT in the Kondo lattice

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_i J_K \, c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_i + \sum_{\langle ij \rangle} J_H \, \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



Assume J_H is chosen so that at $J_K = 0$
the $\boldsymbol{S}_i$ spins have a fractionalized spin liquid ground state.

Represent $\boldsymbol{S}_i$ by fermionic spinons: $\boldsymbol{S}_i = \frac{1}{2} f^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} f_{i\sigma'}$

$$\sum_\sigma f^\dagger_{i\sigma} f_{i\sigma} = 1 \text{ for all } i.$$

Fermi-volume-changing QPT in the Kondo lattice

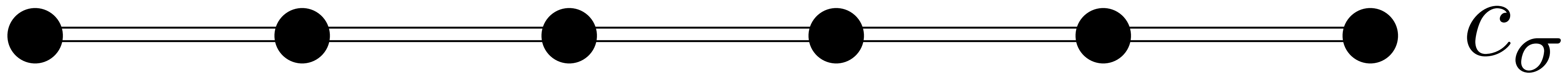
T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Fermi-volume-changing QPT in the Kondo lattice

T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Fermi-volume-changing QPT in a one-band model

(Luttinger-Oshikawa for dummies)

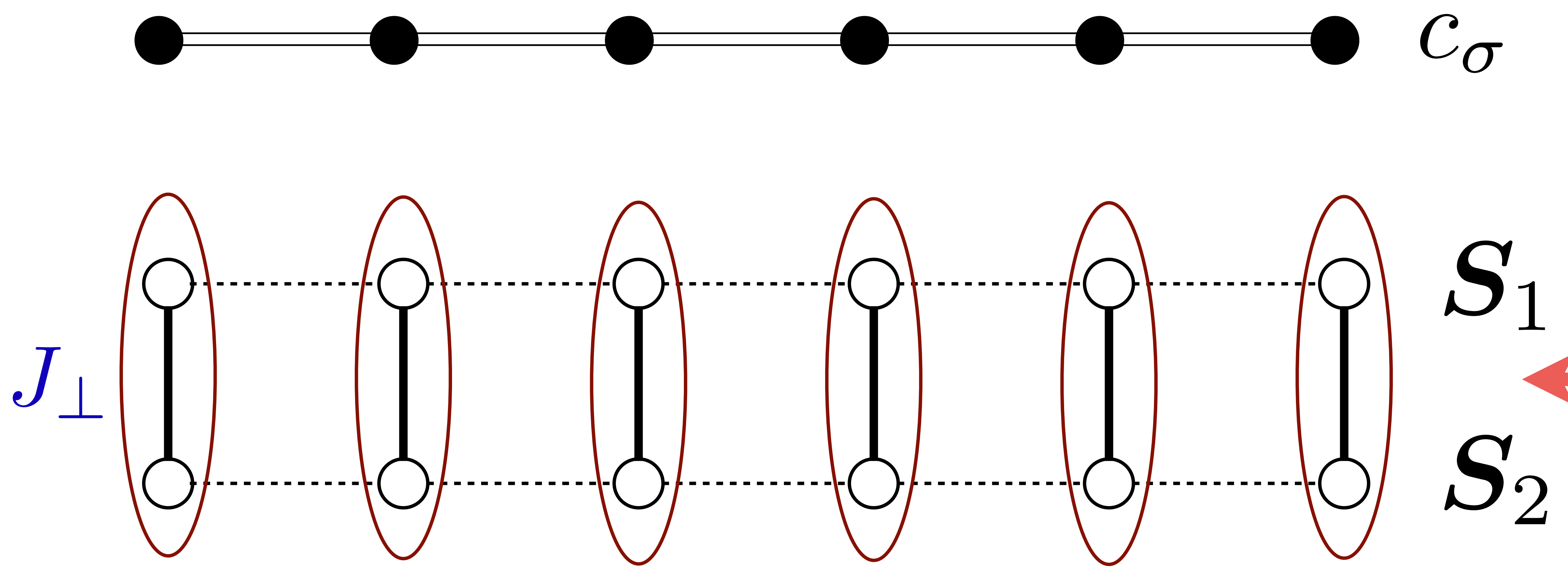


Hubbard model of hole density 1+p

Fermi-volume-changing QPT in a one-band model

(Luttinger-Oshikawa for dummies)

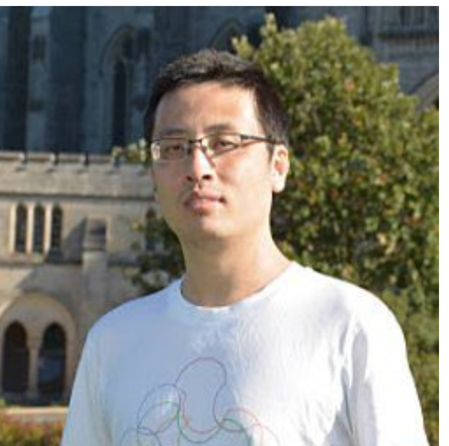
Hubbard model of hole density 1+p



Ancilla qubits in a "trivial" gapped insulator

$$\mathcal{H}_{\text{Hubbard}} + \mathcal{H}_{\text{trivial insulator}}$$

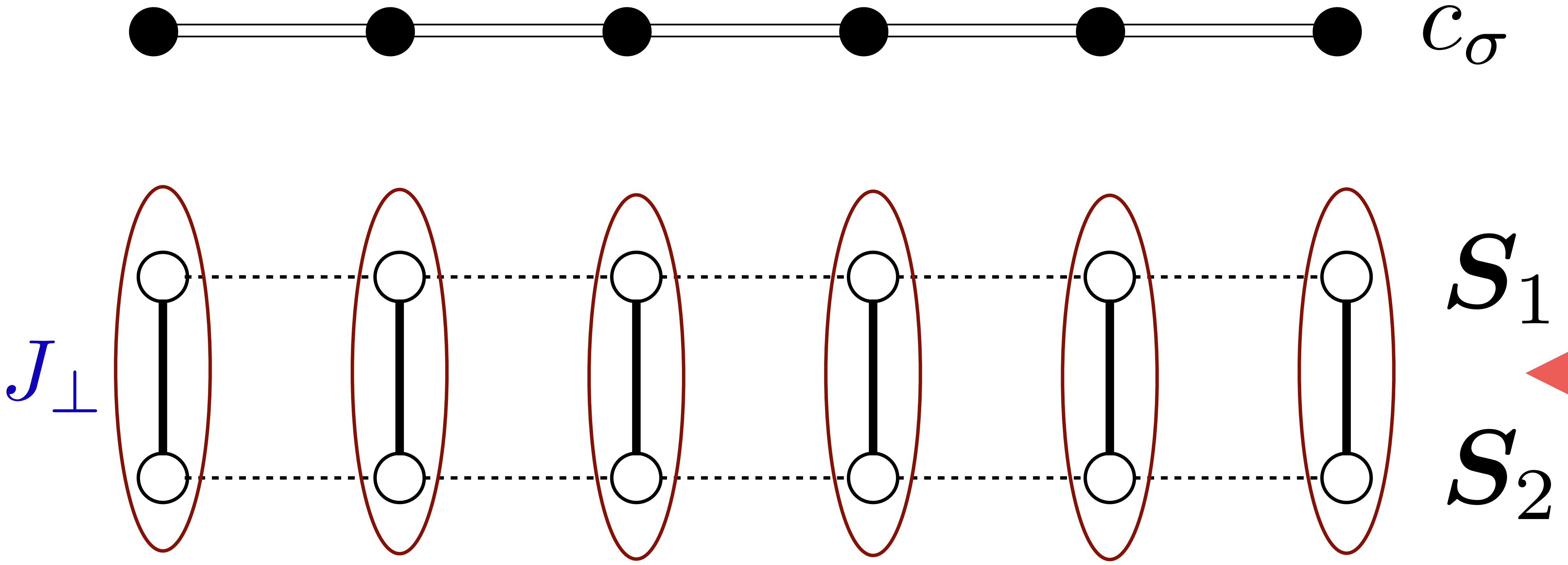
Ya-Hui Zhang



Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. Sachdev, and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Fermi-volume-changing QPT in a one-band model

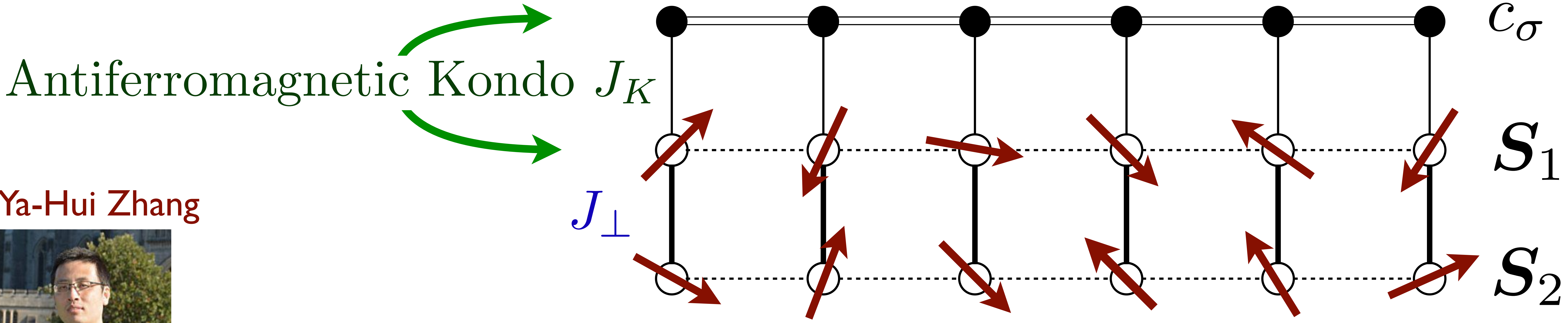
(Luttinger-Oshikawa for dummies )



Hubbard model of hole density 1+p

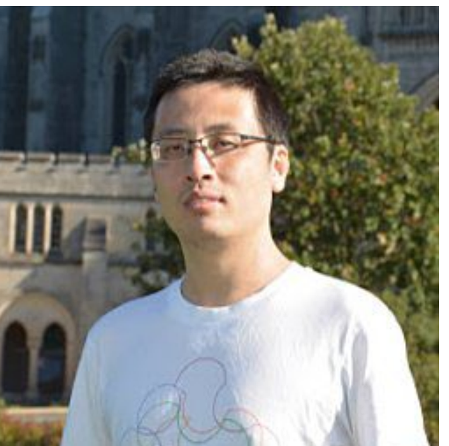
Ancilla qubits in a “trivial” gapped insulator

$$\mathcal{U}\left(\mathcal{H}_{\rm Hubbard} + \mathcal{H}_{\rm trivial\ insulator}\right)\mathcal{U}^{-1} = \mathcal{H}_{\rm ancilla}$$



Free holes of density 1+p

Ya-Hui Zhang

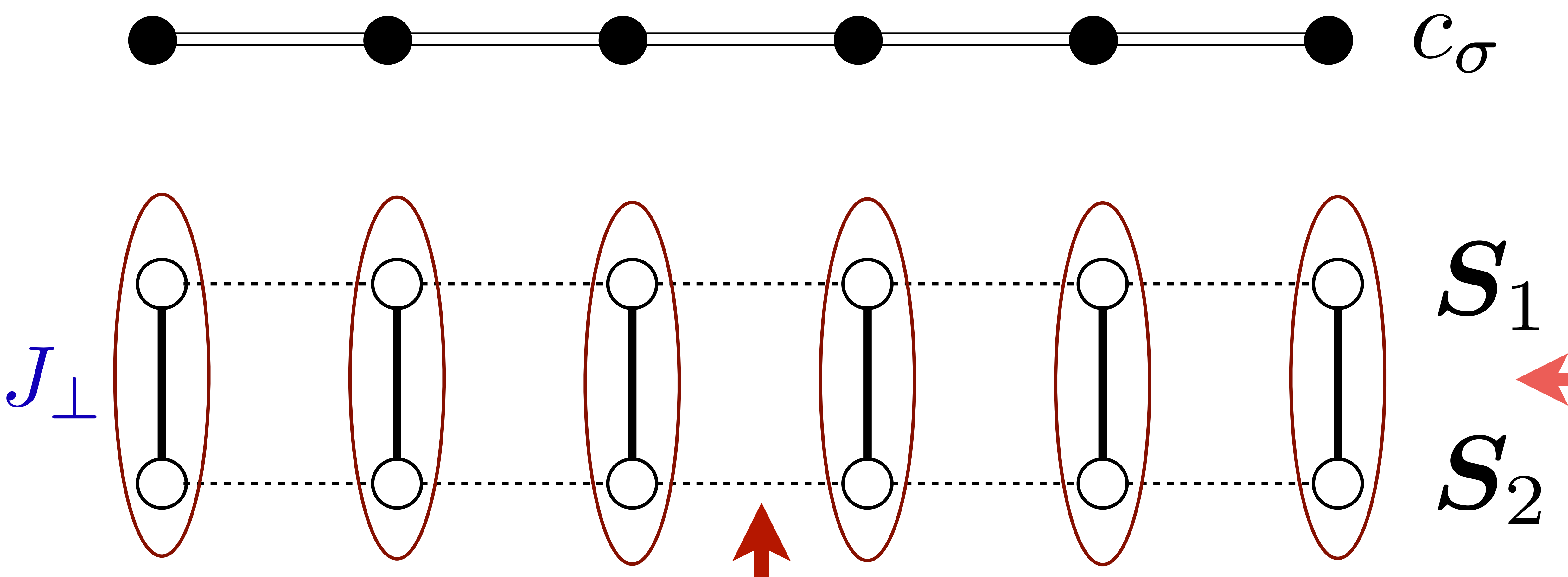


Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)

A. Nikolaenko, M. Tikhanovskaya, S. Sachdev, and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Fermi-volume-changing QPT in a one-band model

(Luttinger-Oshikawa for dummies )

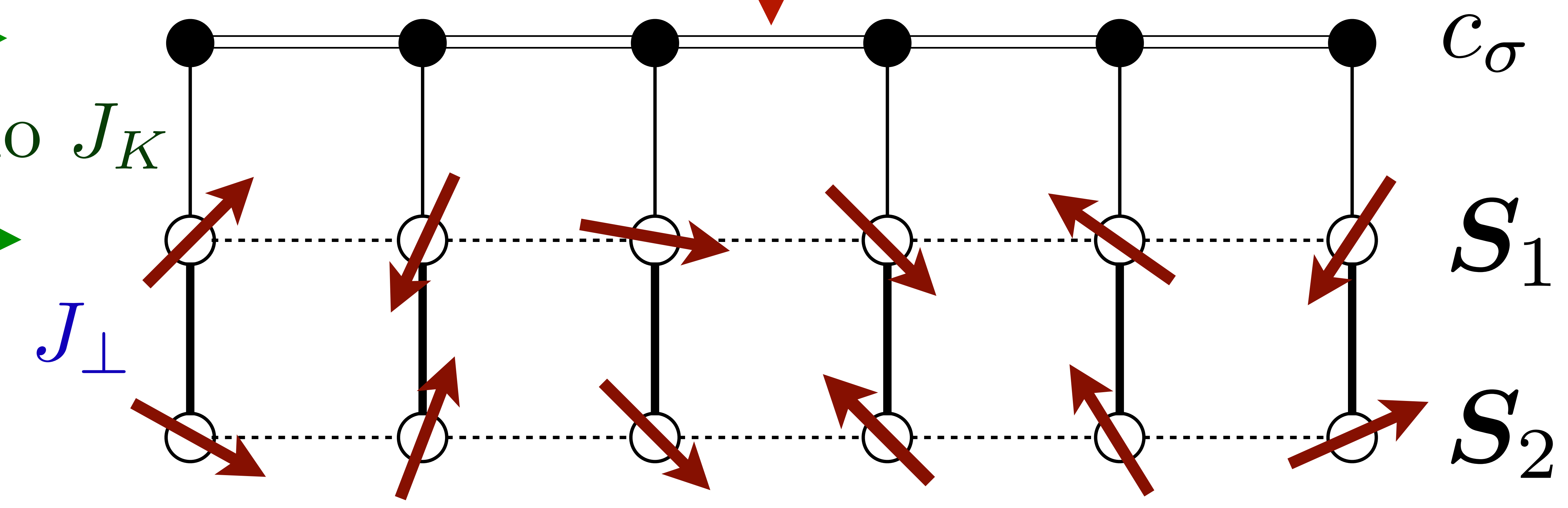


Hubbard model of hole density 1+p

Ancilla qubits in a “trivial” gapped insulator

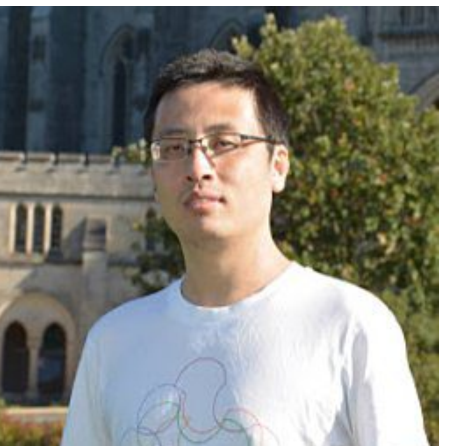
Schrieffer-Wolff transformation at large $J_\perp$ yields $U = \dfrac{3J_K^2}{8J_\perp} + \dfrac{3J_K^3}{16J_\perp^2} + \ldots$

Antiferromagnetic Kondo J_K



Free holes of density 1+p

Ya-Hui Zhang

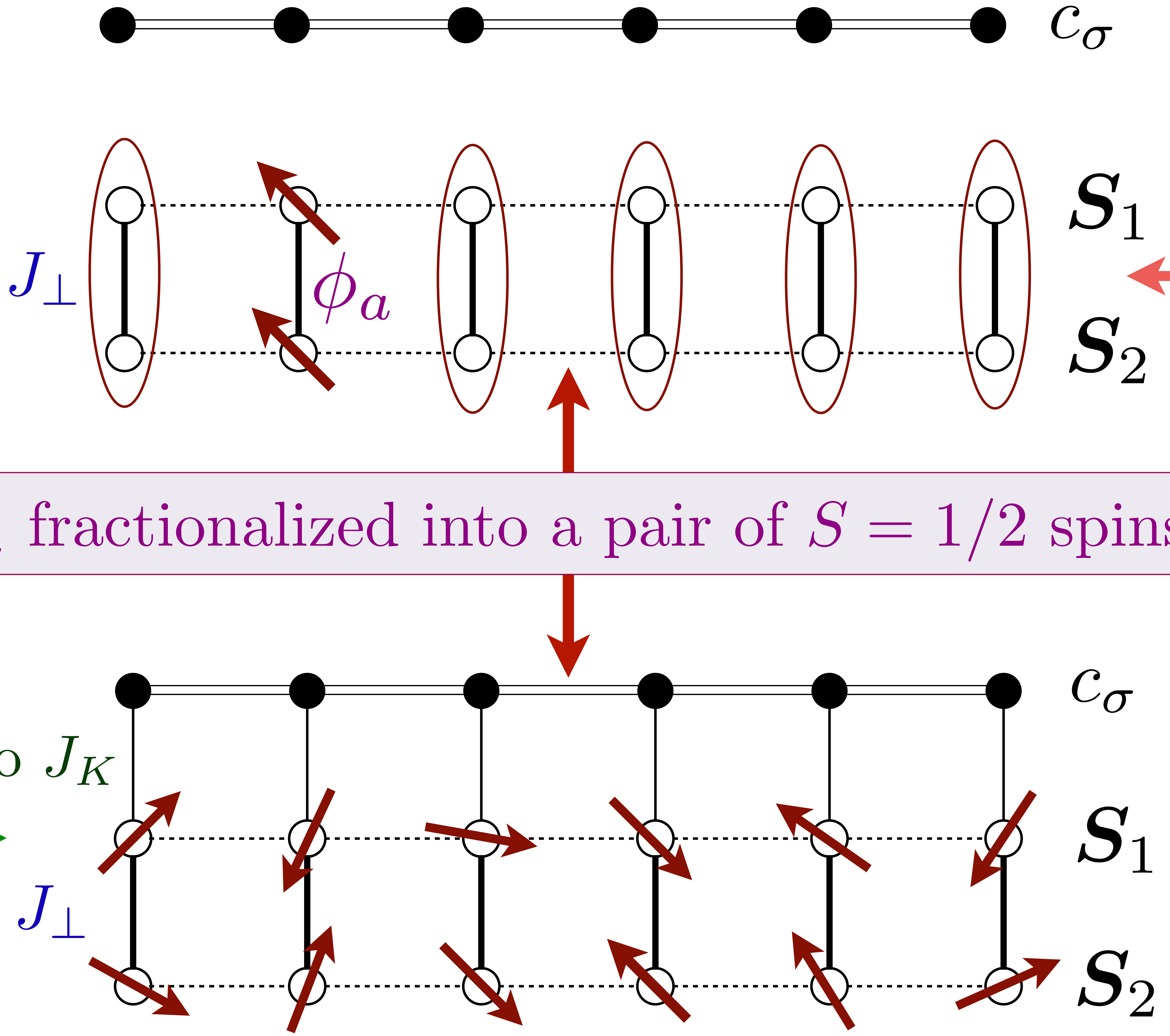


Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)

A. Nikolaenko, M. Tikhanovskaya, S. Sachdev, and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Fermi-volume-changing QPT in a one-band model

(Luttinger-Oshikawa for dummies )



Hubbard model of hole density 1+p

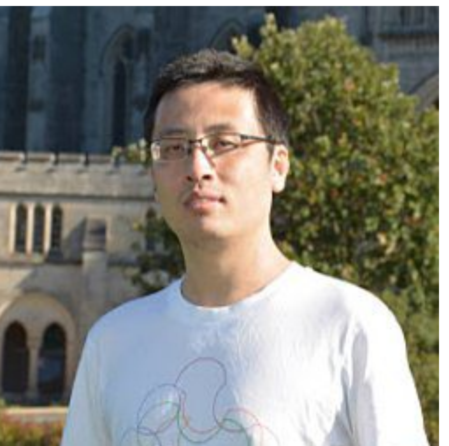
Ancilla qubits in a "trivial" gapped insulator

SDW order ϕ_a fractionalized into a pair of $S = 1/2$ spins, $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$.

Antiferromagnetic Kondo J_K

Free holes of density 1+p

Ya-Hui Zhang



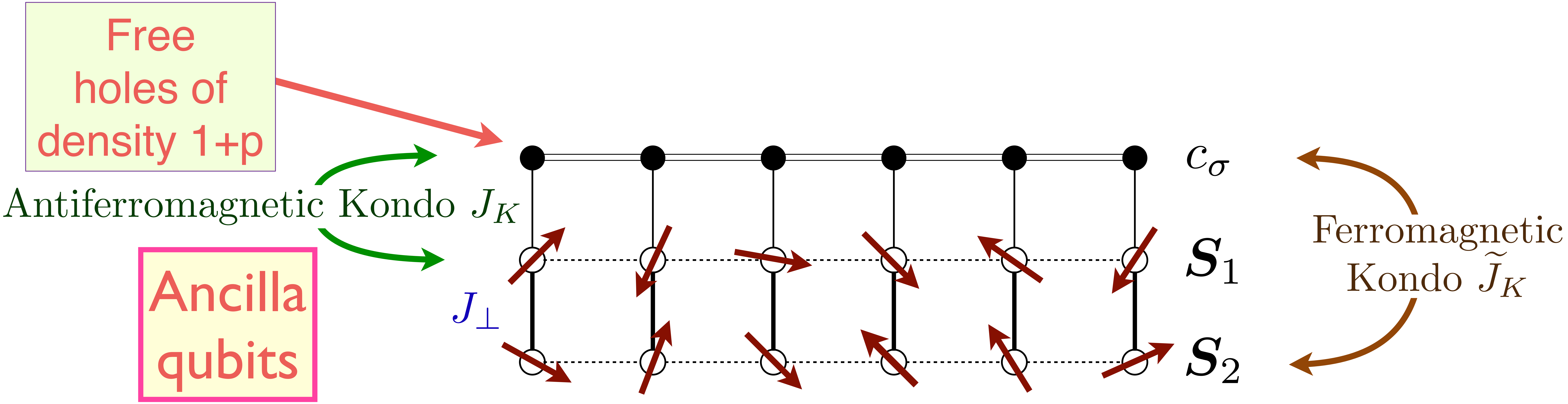
Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)

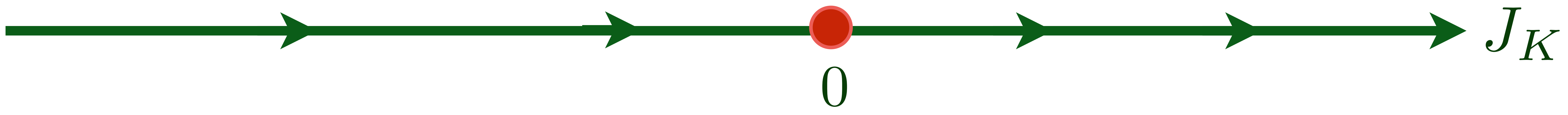
A. Nikolaenko, M. Tikhanovskaya, S. Sachdev, and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Fermi-volume-changing QPT in a one-band model

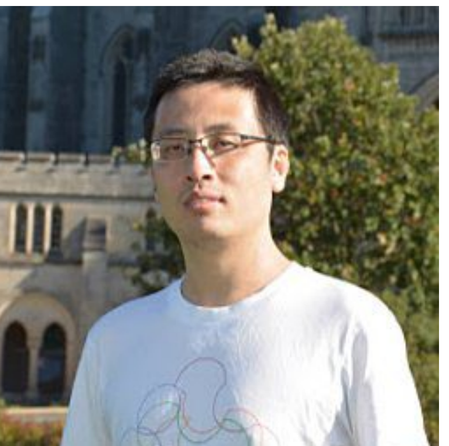
(Luttinger-Oshikawa for dummies )

Ya-Hui Zhang and S. S.,
PRR **2**, 023172 (2020)





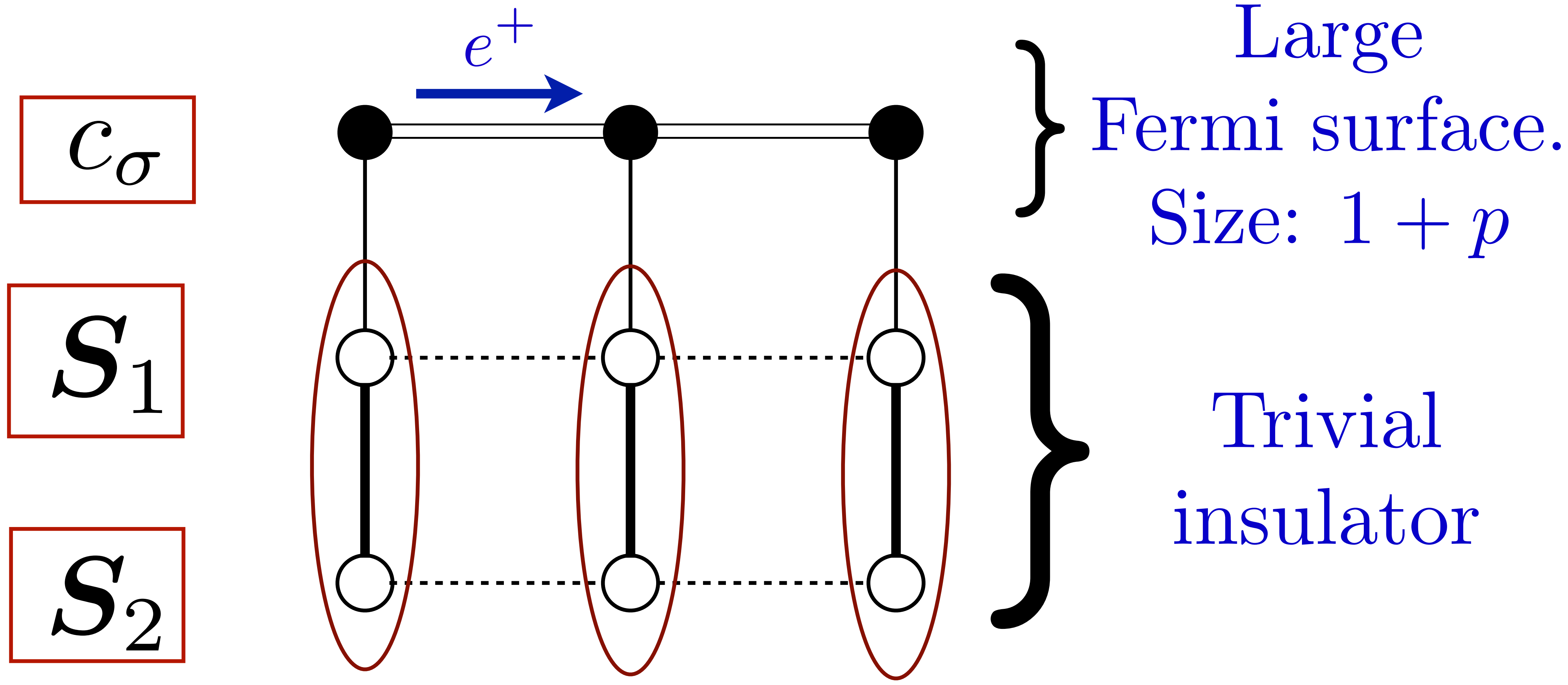
Ya-Hui Zhang



$$\mathcal{H}_{\rm ancilla} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + J_K \sum_i c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Fermi-volume-changing QPT in a one-band model

Ya-Hui Zhang and S. S., *PRR* **2**, 023172 (2020)

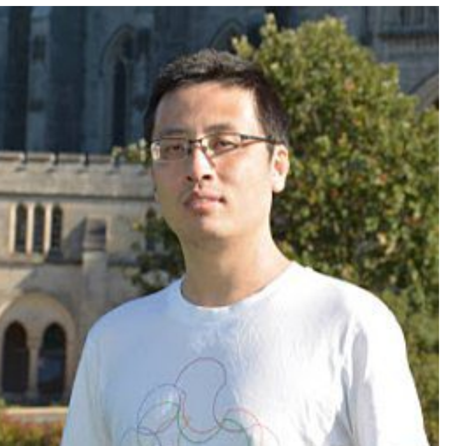


FL

J_K

doping p

Ya-Hui Zhang



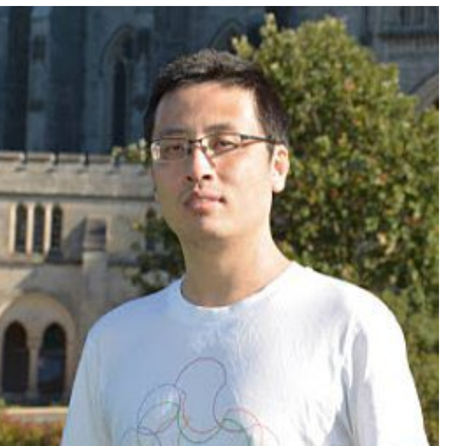
Fermi-volume-changing QPT in a one-band model

Ya-Hui Zhang and S. S., *PRR* **2**, 023172 (2020)

Pseudogap metal = Kondo Lattice Heavy Fermi Liquid $\oplus$ Spin Liquid

Fractionalized excitations of layer $\boldsymbol{S}_1$ confined by condensation of Higgs boson $\Phi \sim f_{1\sigma}^\dagger c_\sigma$.

Frationalized excitations of layer $\boldsymbol{S}_2$ remain deconfined



Ya-Hui Zhang

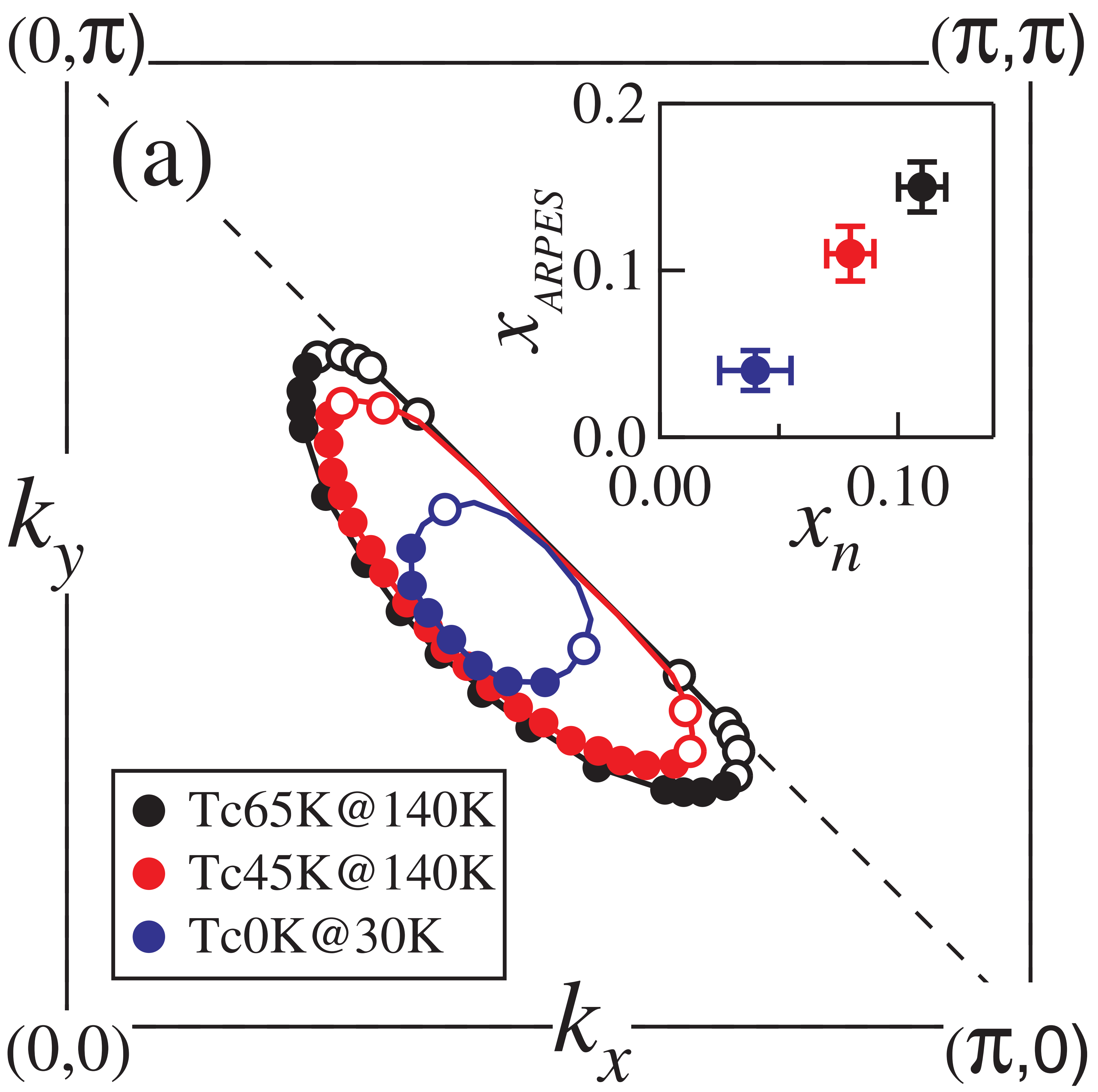
Kondo lattice

One-band model

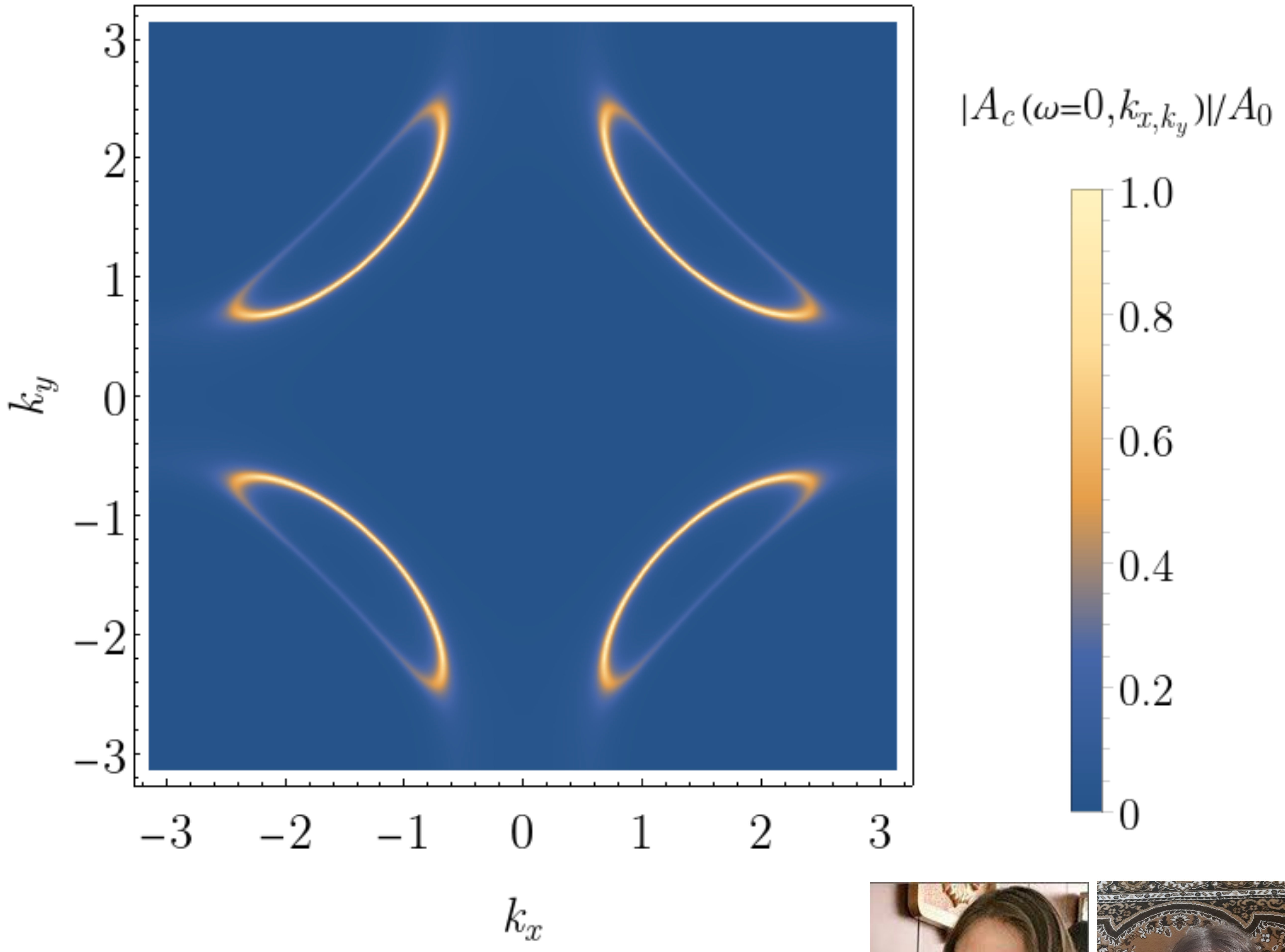
One-band model has an 'inverted' Kondo lattice transition

Ancilla theory of photoemission

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)



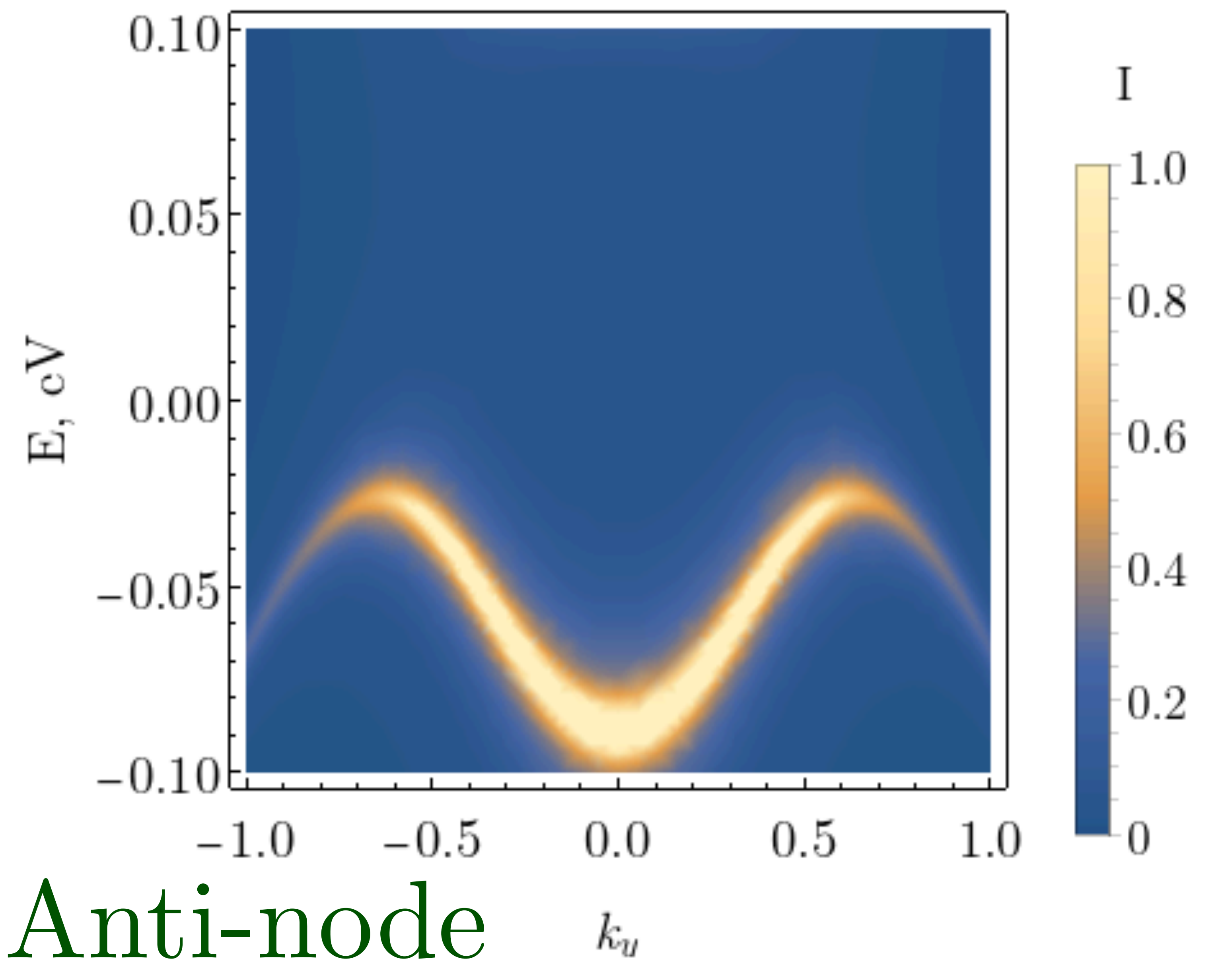
Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors,
H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).



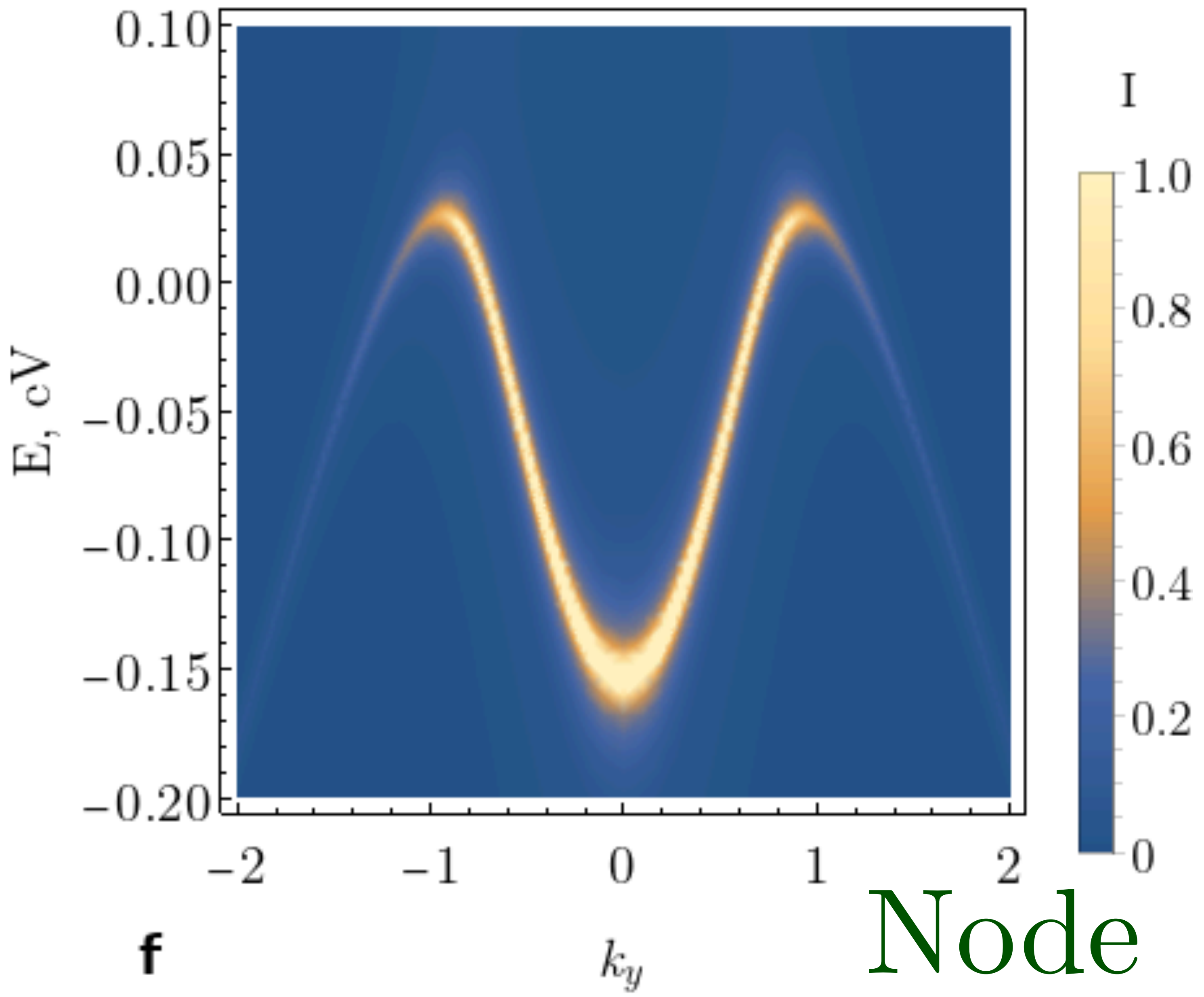
$$H_{\rm mf} = -\sum_{i,j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} - \sum_{i,j} t_{1,ij} f^\dagger_{1i\sigma} f_{1j\sigma} - \sum_i \Phi \left(c^\dagger_{i\sigma} f_{1i\sigma} + f^\dagger_{1i\sigma} c_{i\sigma} \right)$$

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Ancilla theory of photoemission

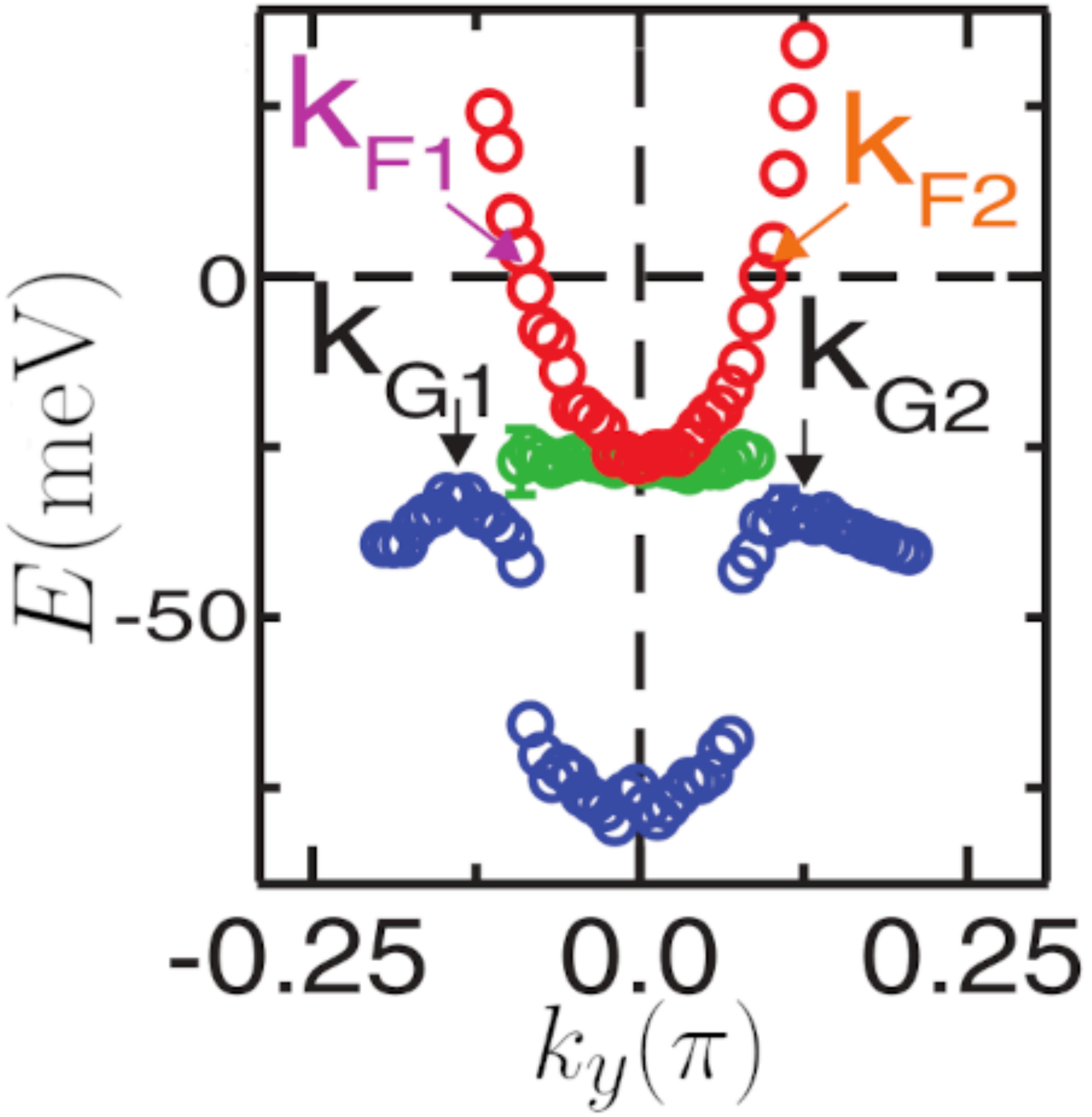


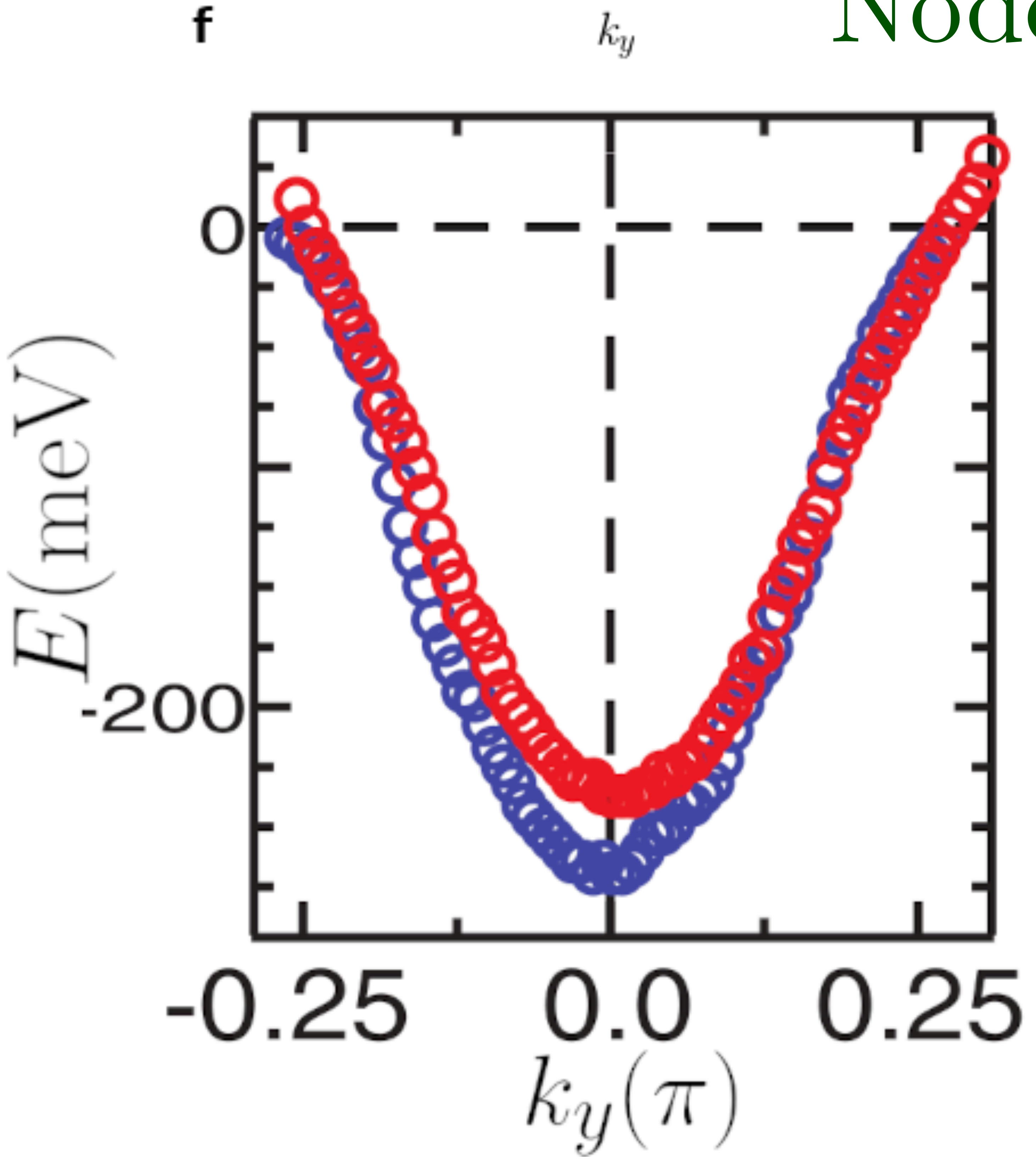
Anti-node



Node

$$H_{\rm mf} = -\sum_{i,j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} - \sum_{i,j} t_{1,ij} f_{1i\sigma}^\dagger f_{1j\sigma} - \sum_i \Phi \left(c_{i\sigma}^\dagger f_{1i\sigma} + f_{1i\sigma}^\dagger c_{i\sigma} \right)$$



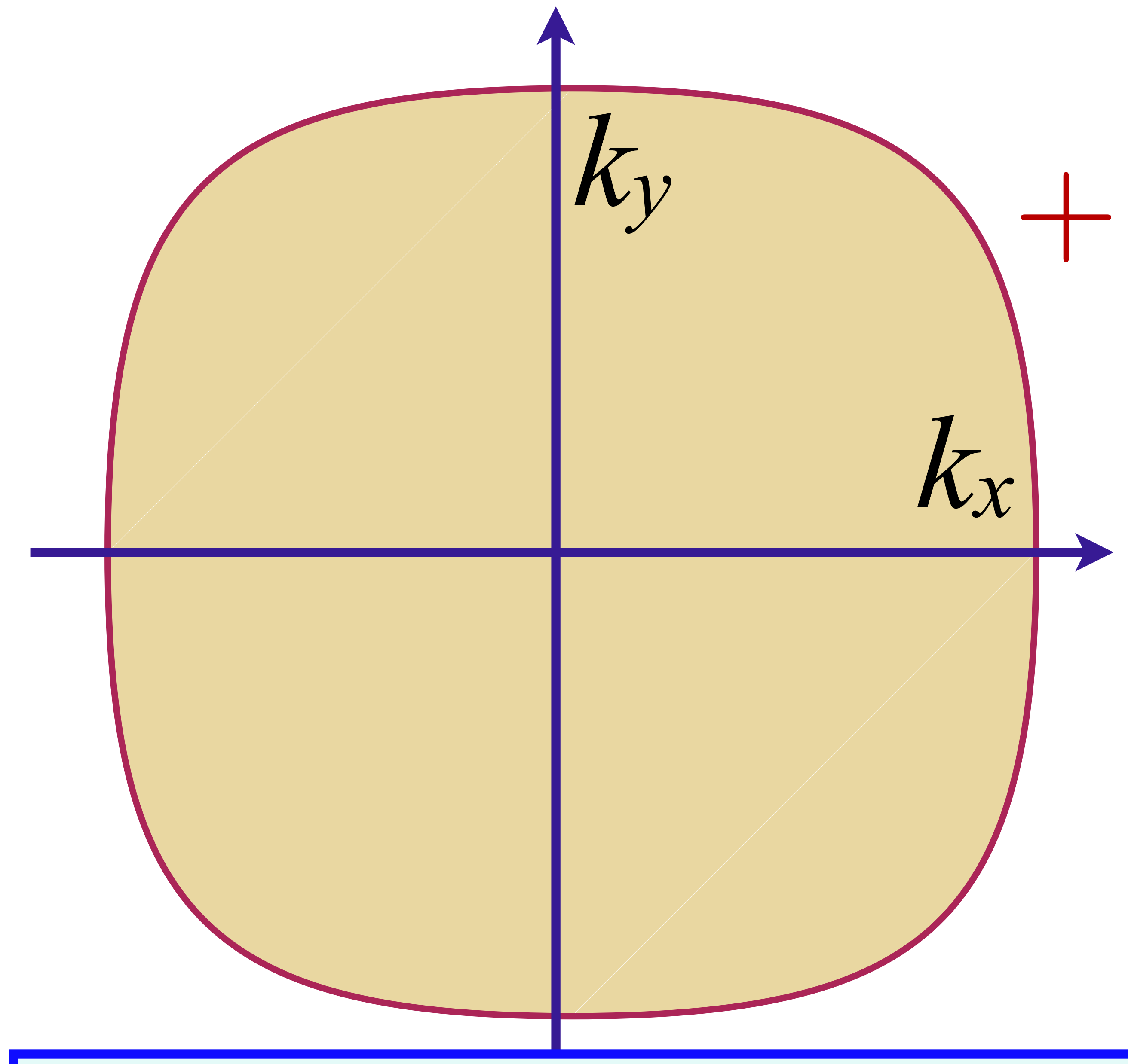


R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

ARPES on Bi2201

Kondo lattice + critical boson with potential and interaction disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}+f^\dagger_{1\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon_1(\boldsymbol{k})\right)f_{1\boldsymbol{k}\sigma}$$



$$+[s+\delta s(\boldsymbol{r})]\,[\Phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})f_{1\sigma}(\boldsymbol{r})\,\Phi(\boldsymbol{r})+\text{H.c.}$$

$$+K\,[\nabla_{\boldsymbol{r}}\Phi(\boldsymbol{r})]^2+u\,[\Phi(\boldsymbol{r})]^4 + v(\boldsymbol{r})c^\dagger_\sigma(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

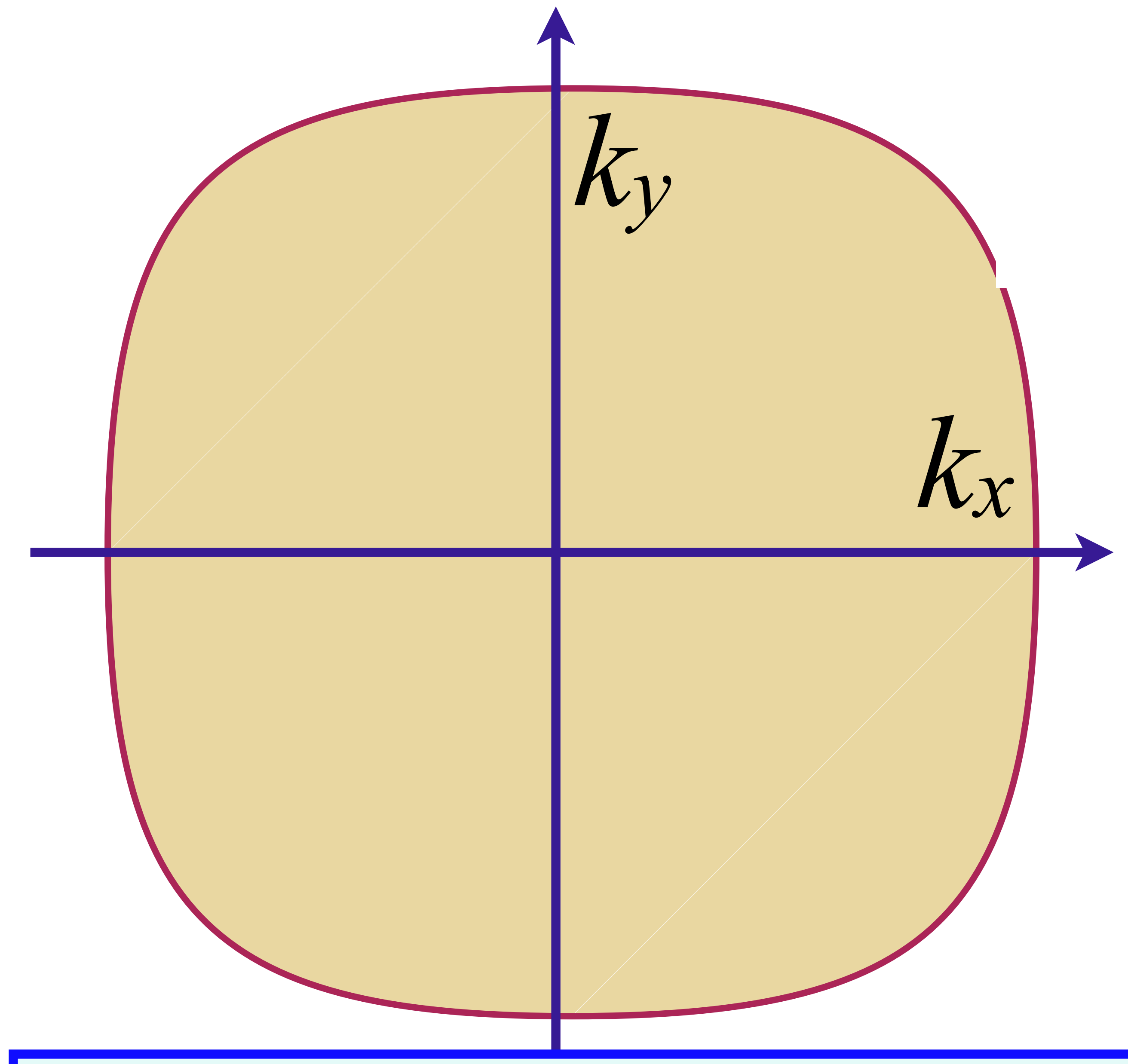
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0,\ \overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})}=0,\ \overline{\delta s(\boldsymbol{r})\delta s(\boldsymbol{r}')}=\delta s^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

$v(\boldsymbol{r})$ leads to elastic scattering of c_σ and 'Altshuler-Aronov' corrections; localization of c_σ only at long length scales, not relevant for experiments

Kondo lattice + critical boson with potential and interaction disorder

$$c_{\boldsymbol{k}\sigma}^{\dagger}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}+f_{1\boldsymbol{k}\sigma}^{\dagger}\left(\frac{\partial}{\partial\tau}+\varepsilon_1(\boldsymbol{k})\right)f_{1\boldsymbol{k}\sigma}$$



$$+s\,[\Phi(\boldsymbol{r})]^2+[g+g'(\boldsymbol{r})]\,c_{\sigma}^{\dagger}(\boldsymbol{r})f_{1\sigma}(\boldsymbol{r})\,\Phi(\boldsymbol{r})+\text{H.c.}$$

$$+K\,[\nabla_{\boldsymbol{r}}\Phi(\boldsymbol{r})]^2+u\,[\Phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c_{\sigma}^{\dagger}(\boldsymbol{r})c_{\sigma}(\boldsymbol{r})$$

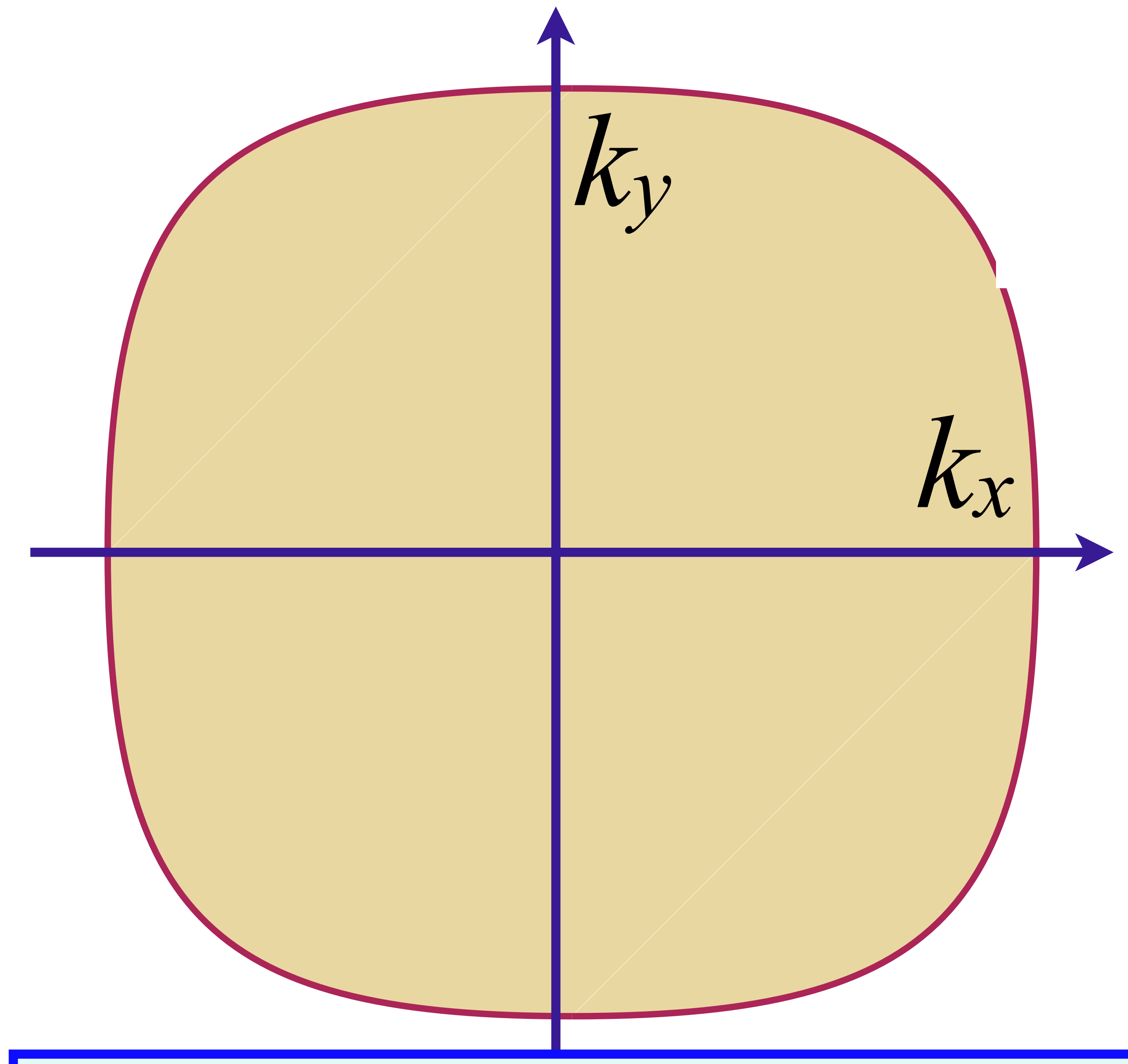
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})}=0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')}=g'^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

$g'(\boldsymbol{r})$ creates inhomogeneity in the position of QCP (Harris disorder): the two-dimensional Yukawa-Sachdev-Ye-Kitaev model.

Kondo lattice + critical boson with potential and interaction disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}+f^\dagger_{1\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon_1(\boldsymbol{k})\right)f_{1\boldsymbol{k}\sigma}$$



$$+s\,[\Phi(\boldsymbol{r})]^2+[g+g'(\boldsymbol{r})]\,c^\dagger_\sigma(\boldsymbol{r})f_{1\sigma}(\boldsymbol{r})\,\Phi(\boldsymbol{r})+\text{H.c.}$$

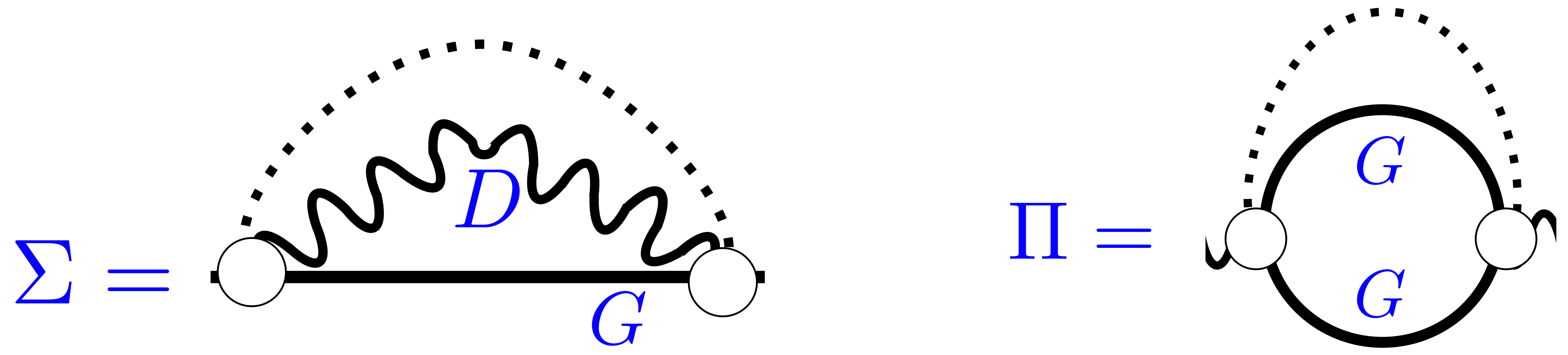
$$+K\,[\nabla_{\boldsymbol{r}}\Phi(\boldsymbol{r})]^2+u\,[\Phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c^\dagger_\sigma(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

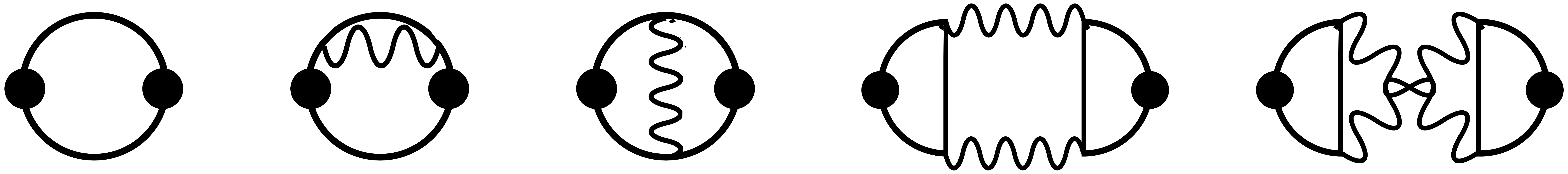
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})}=0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')}=g'^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Analyze 2d-YSYK model in a self-averaging manner as in the SYK model.
Should be applicable as long as eigenmodes of $\Phi(\boldsymbol{r})$ are extended.

2d-YSYK model: Fermi surface + critical boson with interaction disorder





Residual resistivity is determined by v^2

Linear-in-T resistivity determined by g'^2

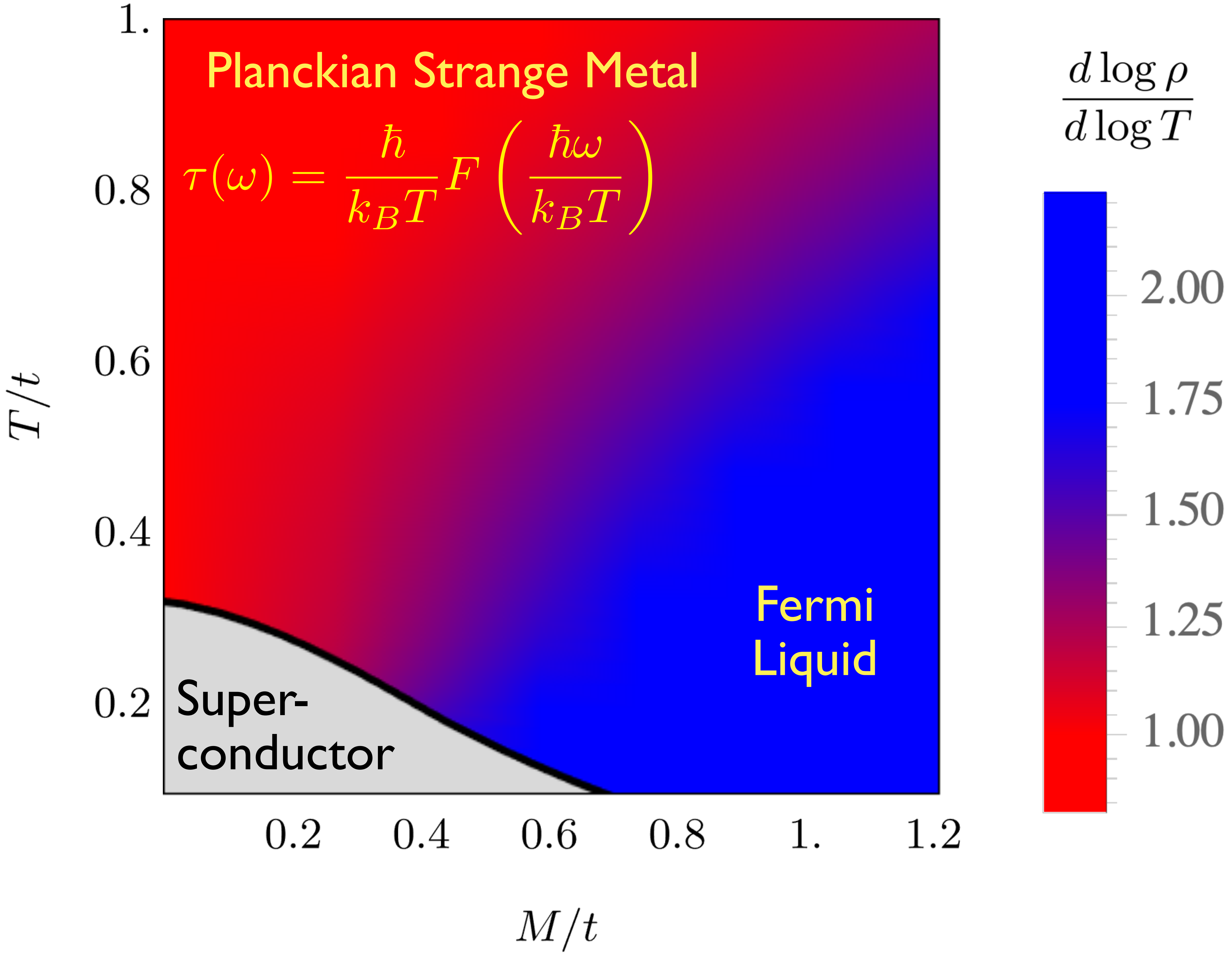
Transport insensitive to g

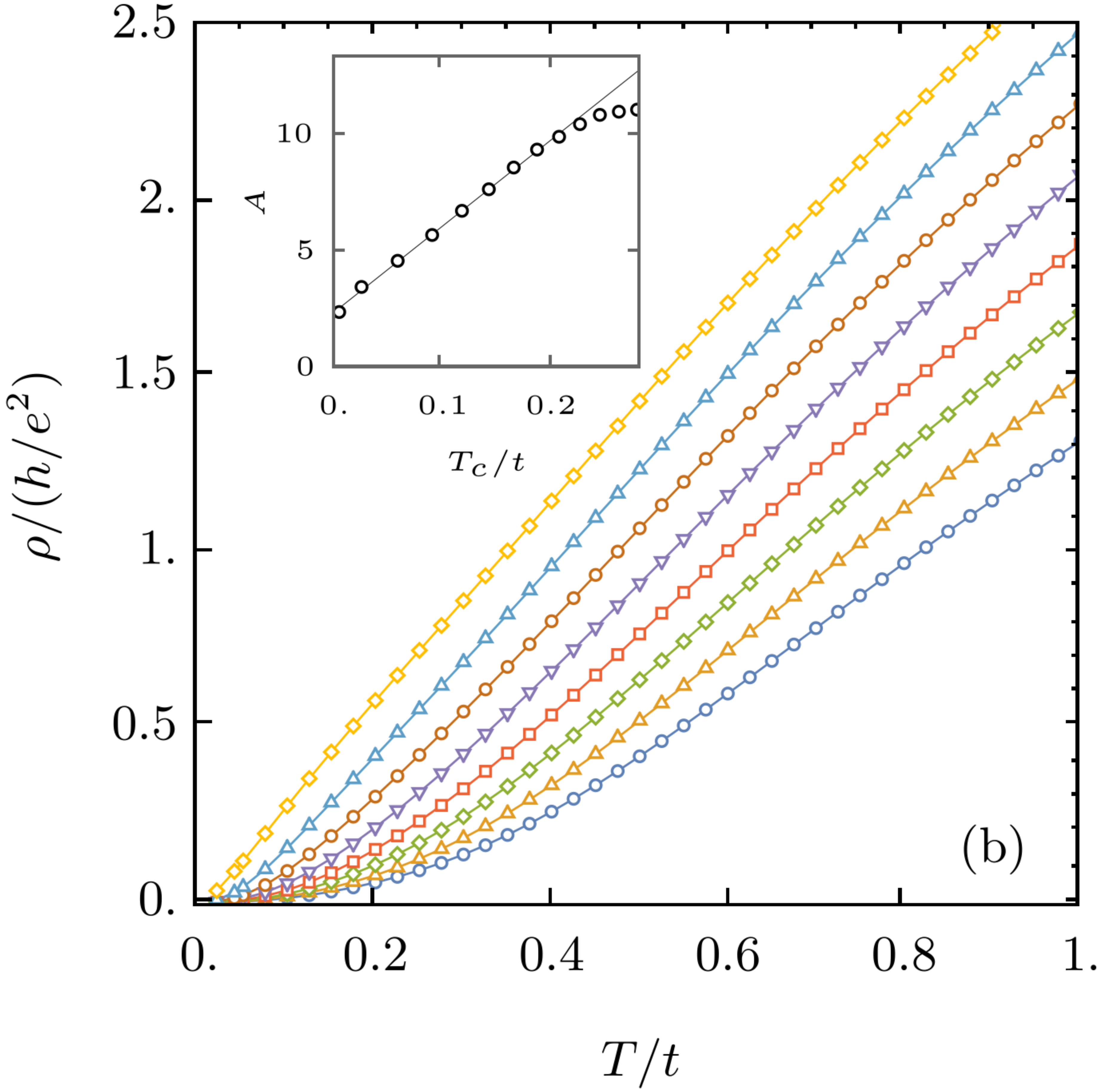
Marginal Fermi liquid self energy $\Sigma \sim \omega \ln \omega$

$T \ln(1/T)$ specific heat

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL in press; arXiv:2406.07608





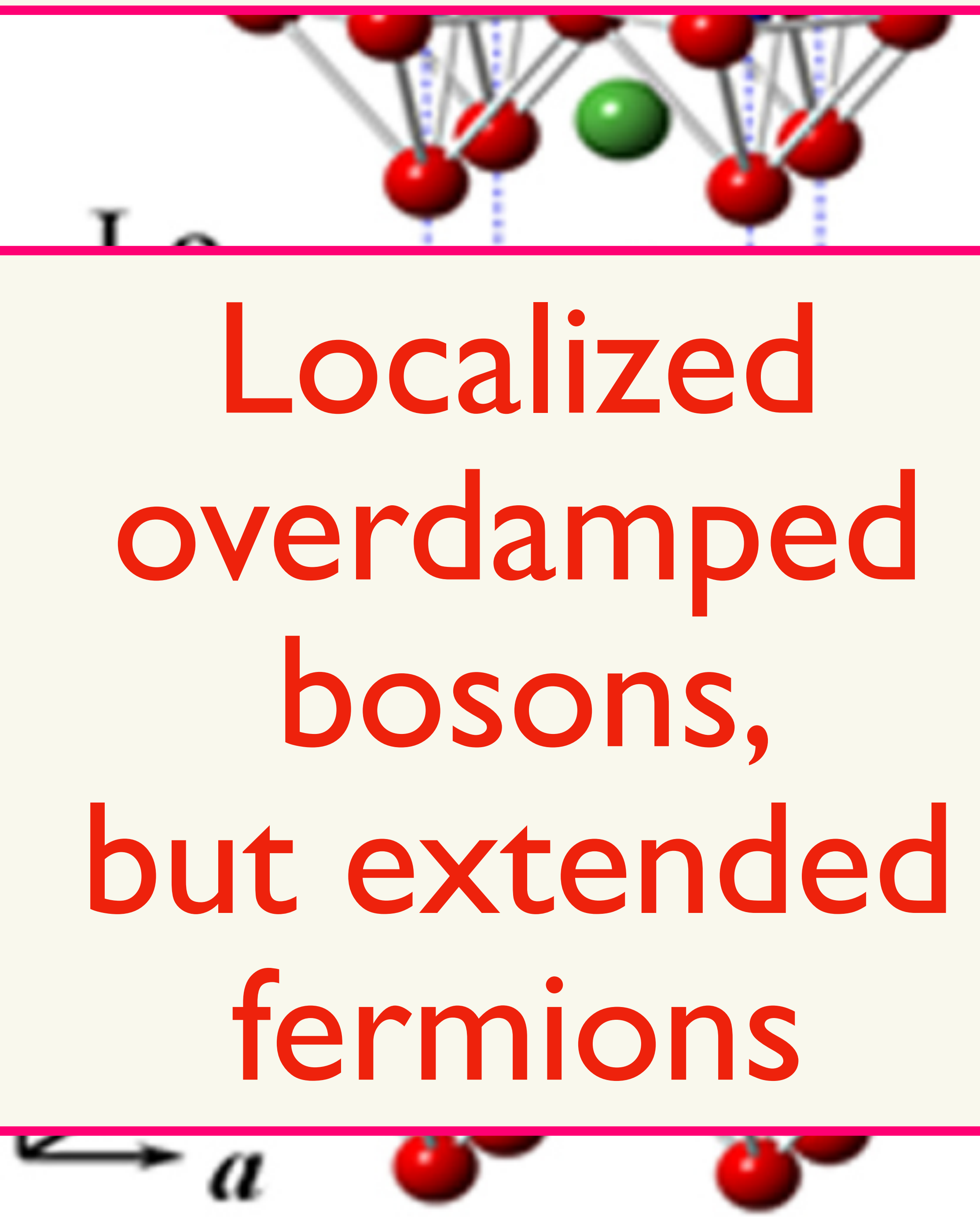
Two-dimensional metals with Harris disorder

Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

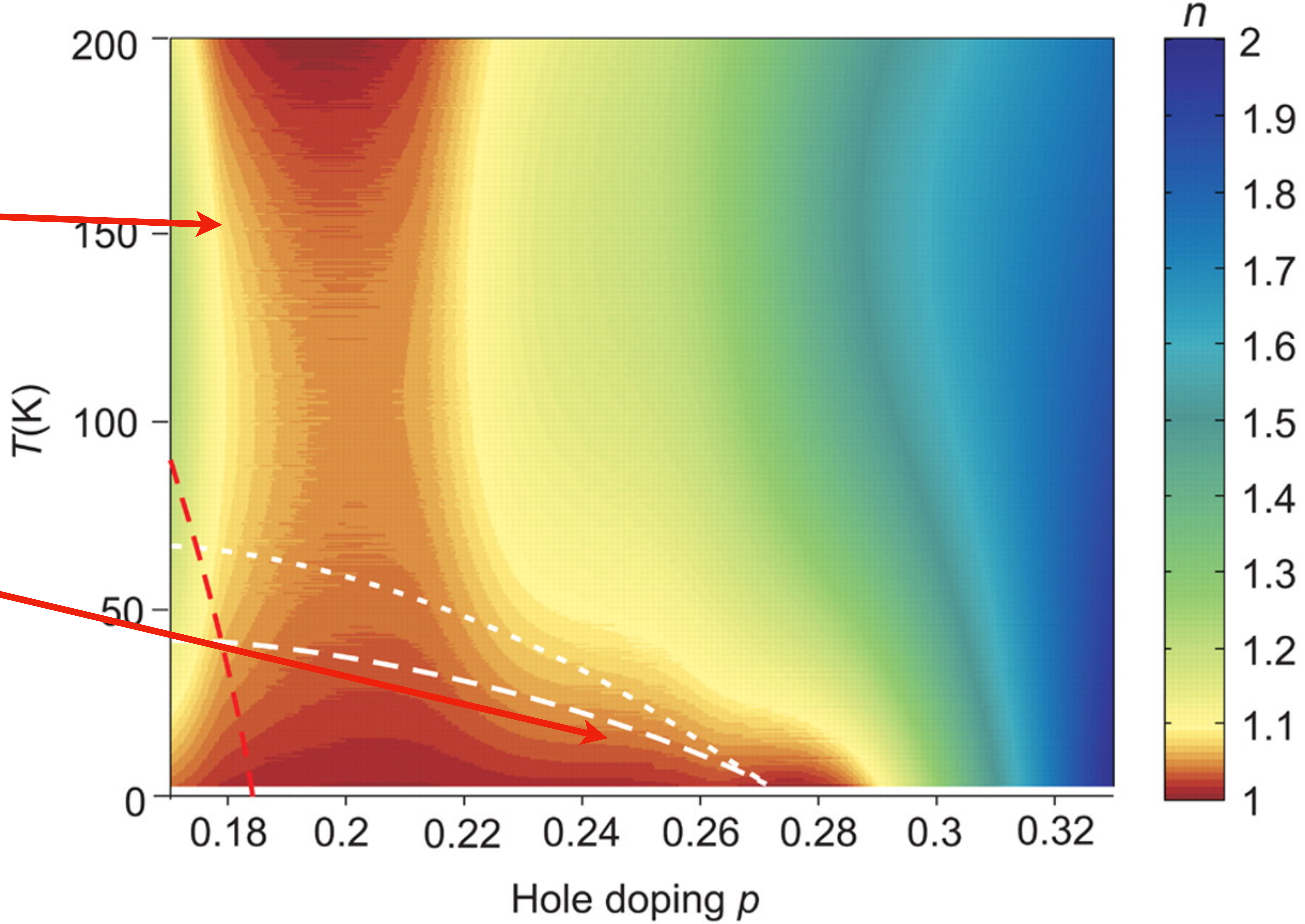
R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4*] H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1†]

SCIENCE VOL 323 603 2009

Extended fermions and bosons



Localized overdamped bosons, but extended fermions

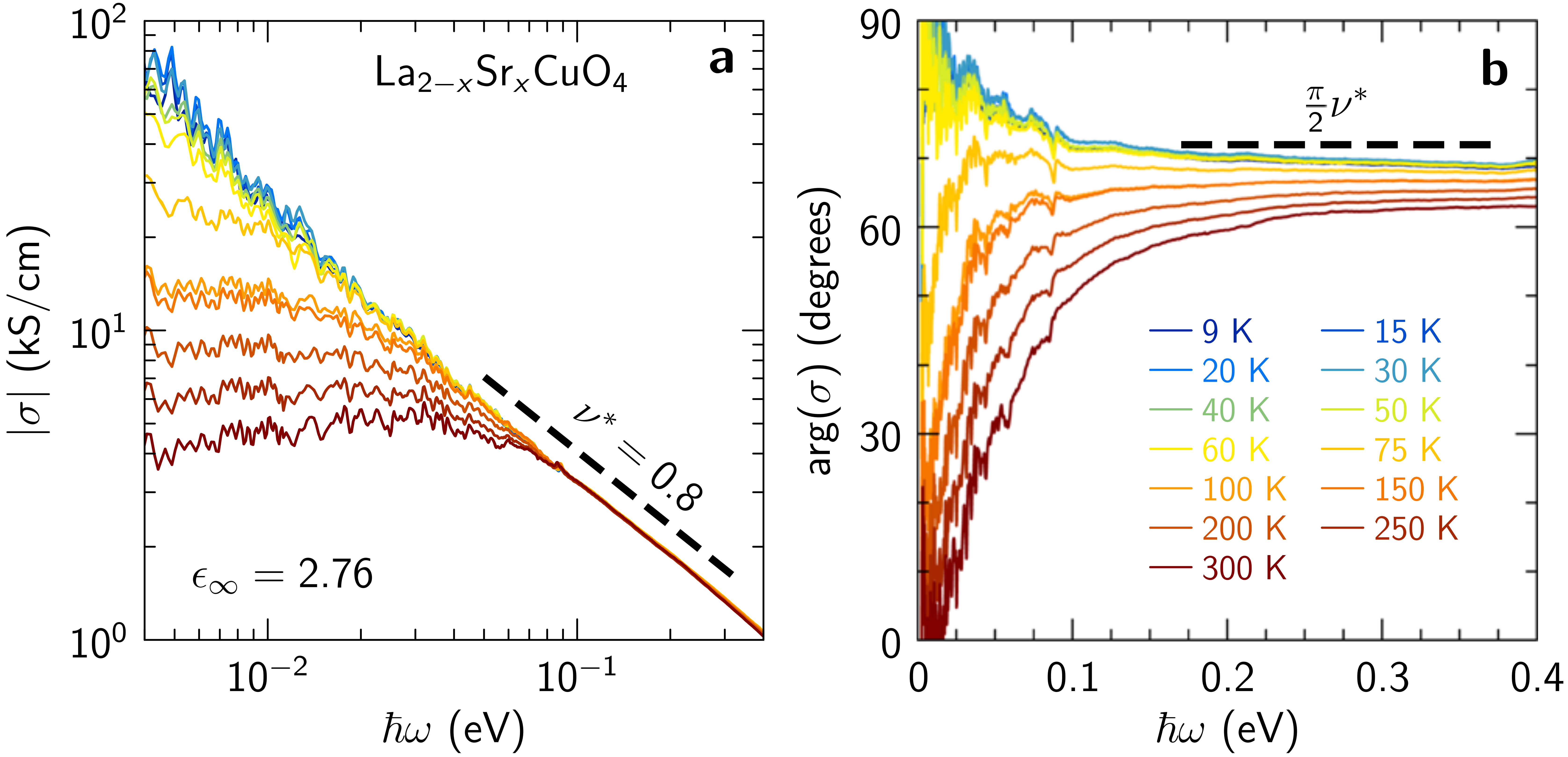


Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\dfrac{m^*(\omega)}{m} + i\dfrac{\hbar}{\tau(\omega)}}$$



$La_{2-x}Sr_xCuO_4$

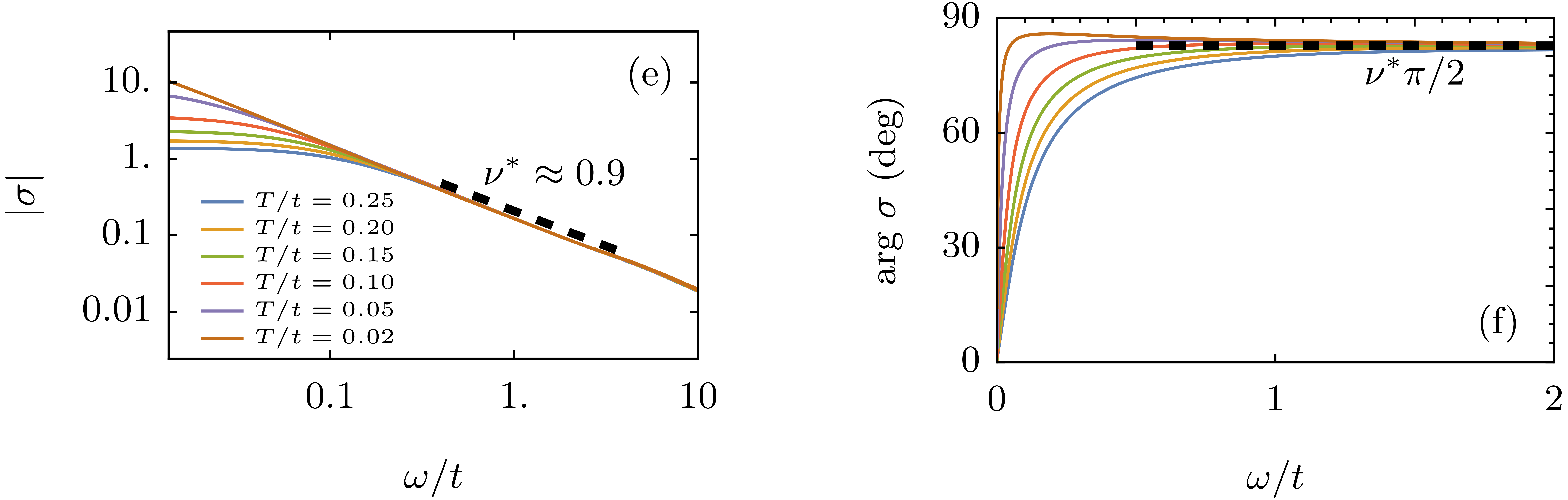
$p = 0.24$

$T_c = 19$ K

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL in press; arXiv:2406.07608

$$\sigma(\omega) = i \frac{e^2 K/(\hbar d_c)}{\hbar\omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$



Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

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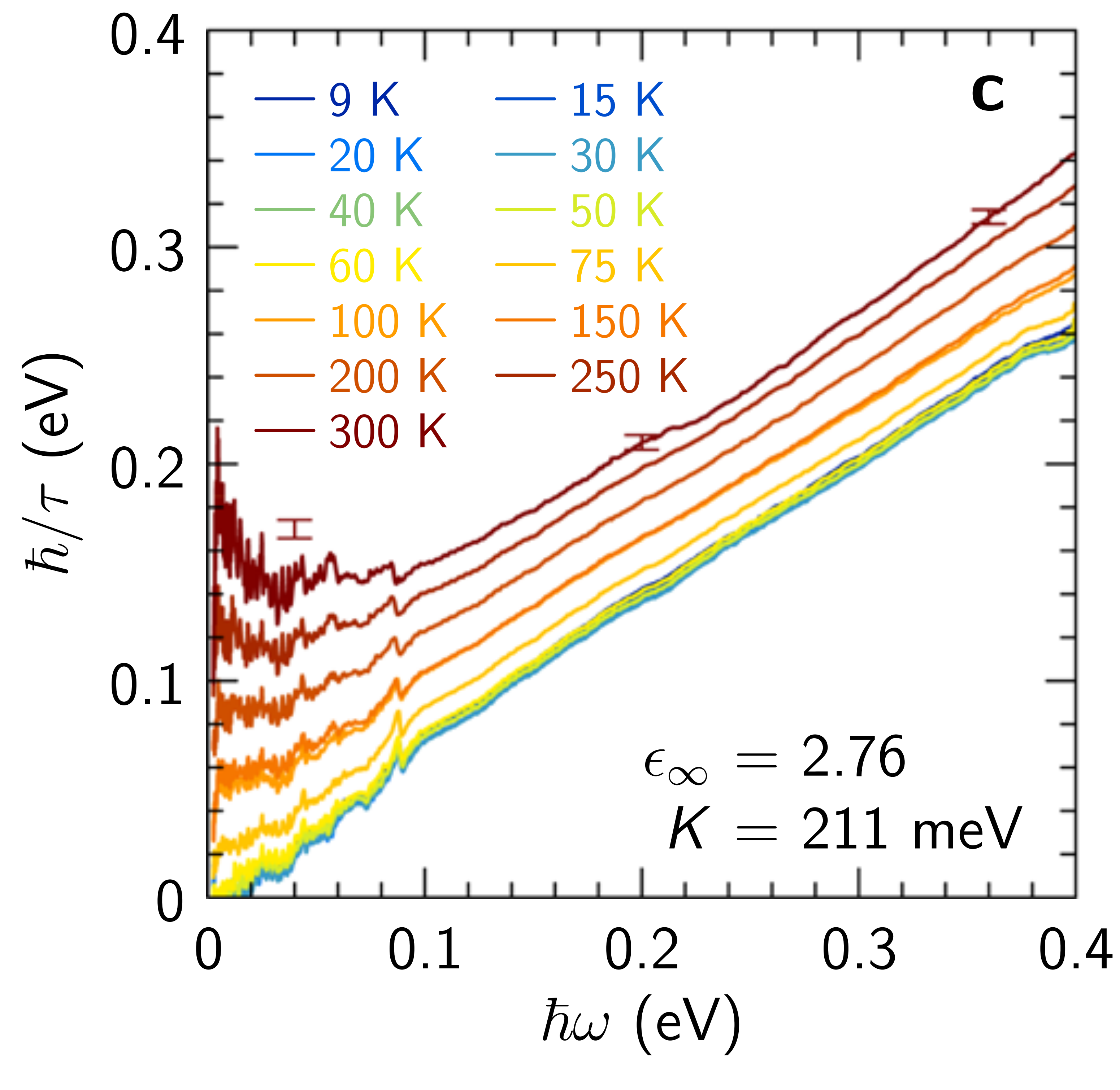

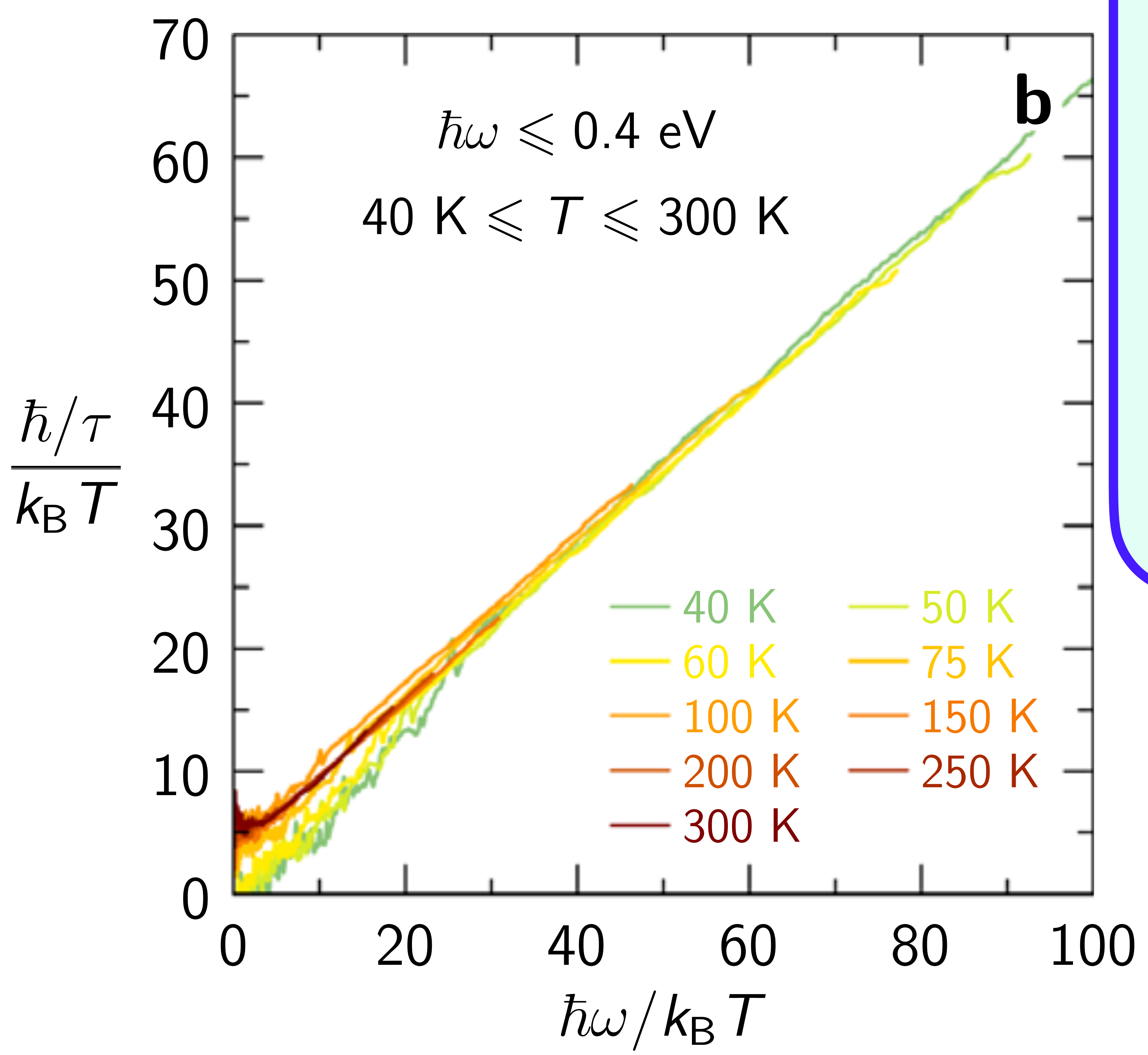

Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy
$S(T \to 0) \sim T \ln(1/T)$.

$La_{2-x}Sr_xCuO_4$
$p = 0.24$
$T_c = 19$ K

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

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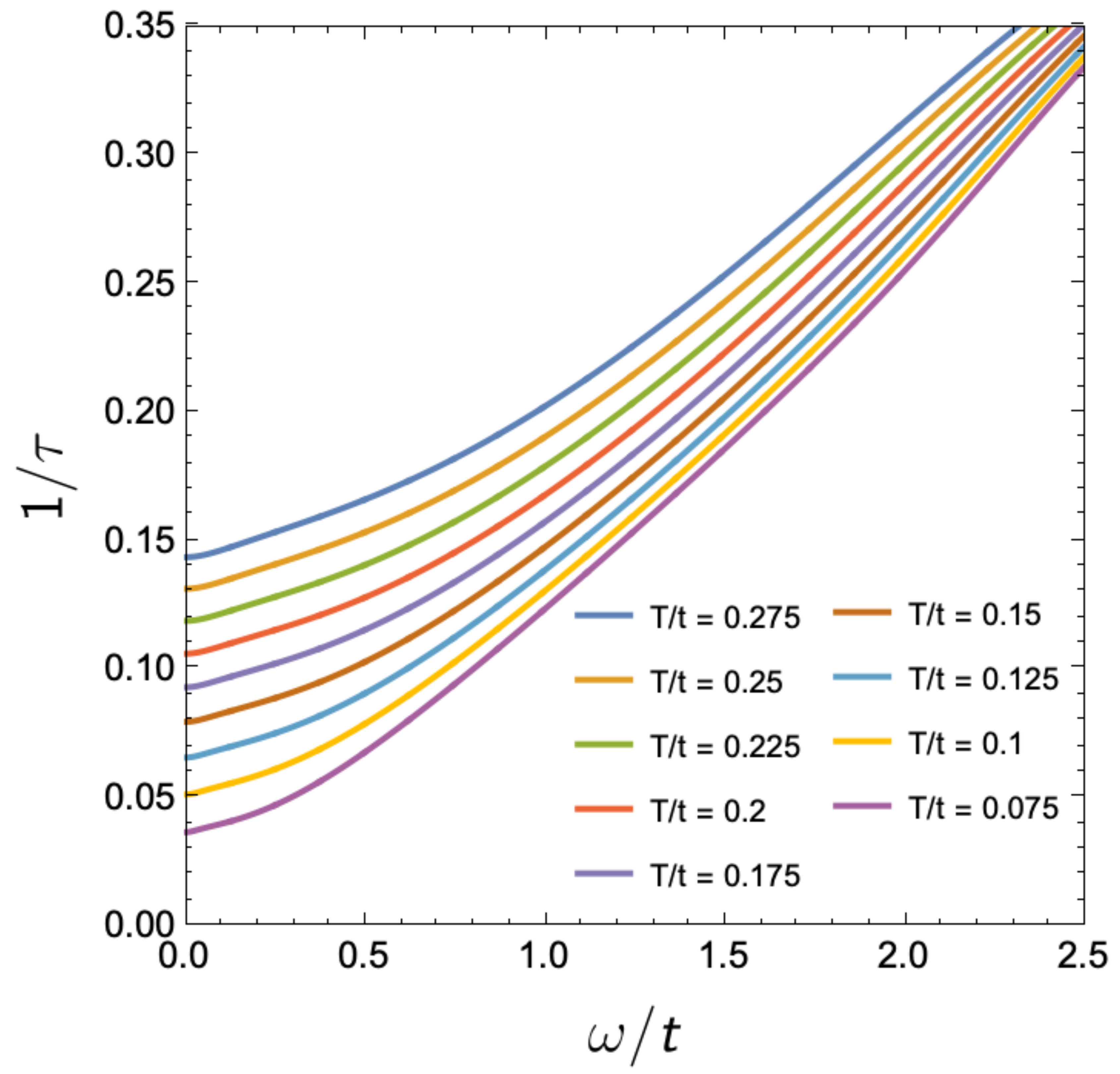
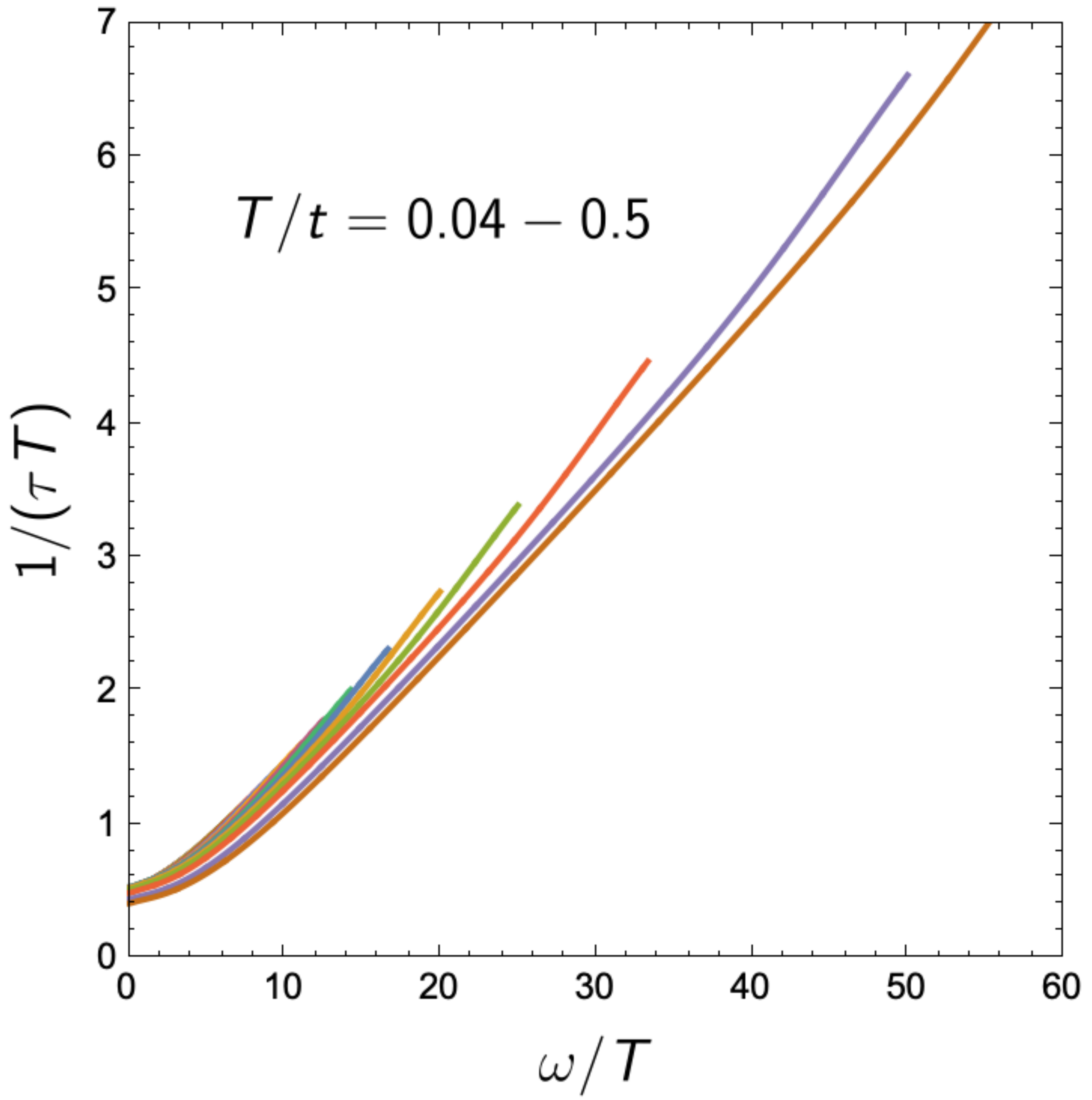




Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy $S(T \to 0) \sim T\ln(1/T)$ in 2d-YSYK model (unlike zero temperature entropy in SYK model).

Seebeck Coefficient in a Cuprate Superconductor: Particle-Hole Asymmetry in the Strange Metal Phase and Fermi Surface Transformation in the Pseudogap Phase

A. Gourgout,[1,*] G. Grissonnanche,[1,2,3,*,†] F. Laliberté,[1] A. Ataei,[1] L. Chen,[1] S. Verret,[1] J.-S. Zhou,[4] J. Mravlje,[5] A. Georges,[6,7,8,9] N. Doiron-Leyraud,[1] and Louis Taillefer[1,10,‡]

PHYSICAL REVIEW X **12**, 011037 (2022)

Skewed non-Fermi liquids and the Seebeck effect

Antoine Georges[1,2,3,4] and Jernej Mravlje[5]

PRR **3**, 043132 (2021)

We consider non-Fermi liquids in which the inelastic scattering rate has an intrinsic particle-hole asymmetry and obeys ω/T scaling. We show that, in contrast to Fermi liquids, this asymmetry influences the low-temperature behavior of the thermopower even when the impurity scattering dominates. Implications for the unconventional sign and temperature dependence of the thermopower in cuprates in the strange metal (Planckian) regime are emphasized.

The particle-hole asymmetry in the Φ propagator (for the FL*-FL transition)

$$\sim 1/(-i\omega + q^2 + \gamma|\omega| + \alpha)$$

leads to a **skewed marginal Fermi liquid**.

The ϕ propagator (for the SDW-FL transition) does not have the $-i\omega$ term, and so is *not* skewed.

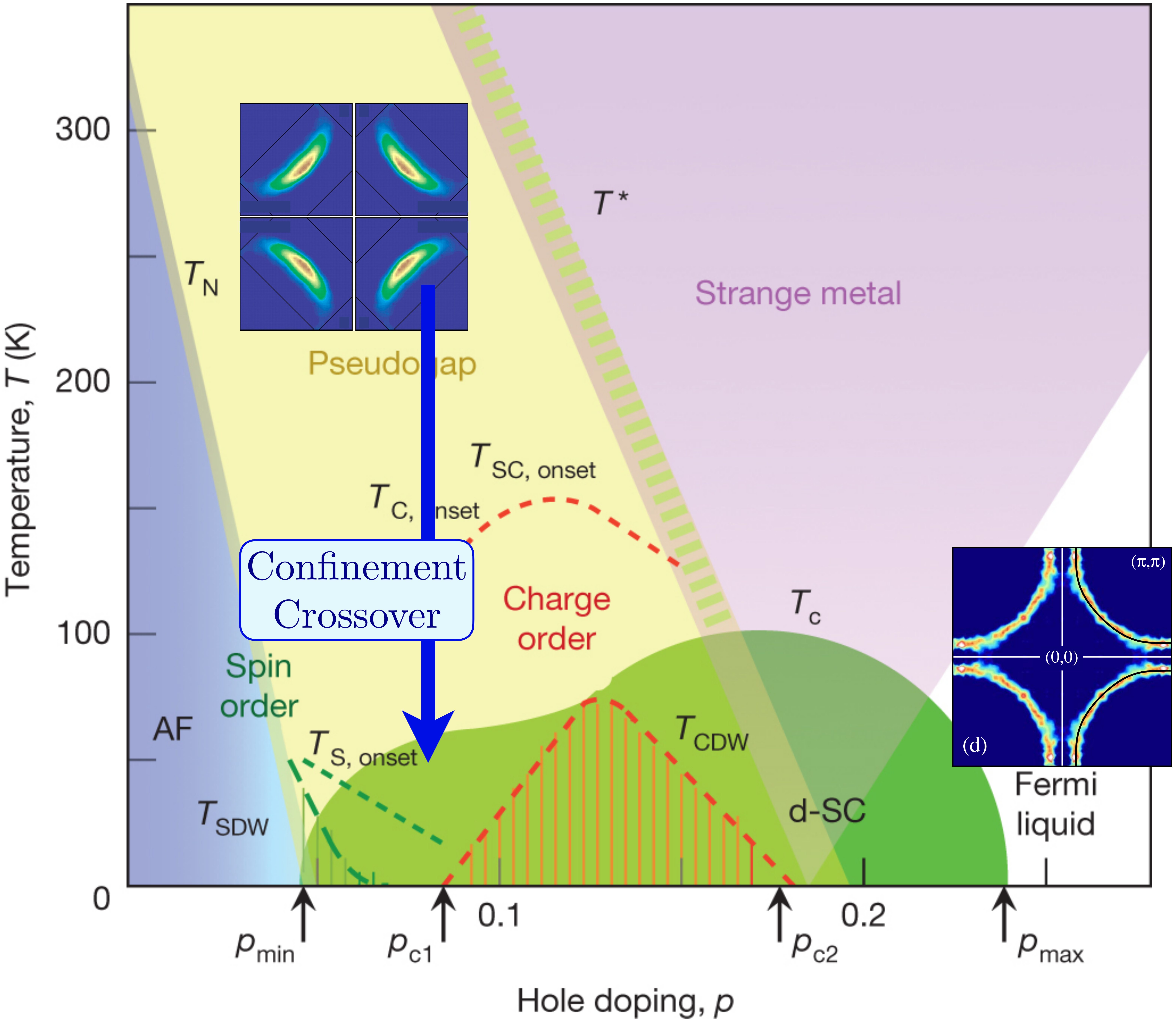
P. Lunts, A.A. Patel, and S.S., arXiv:2412.15330

1. FL-SDW QPT

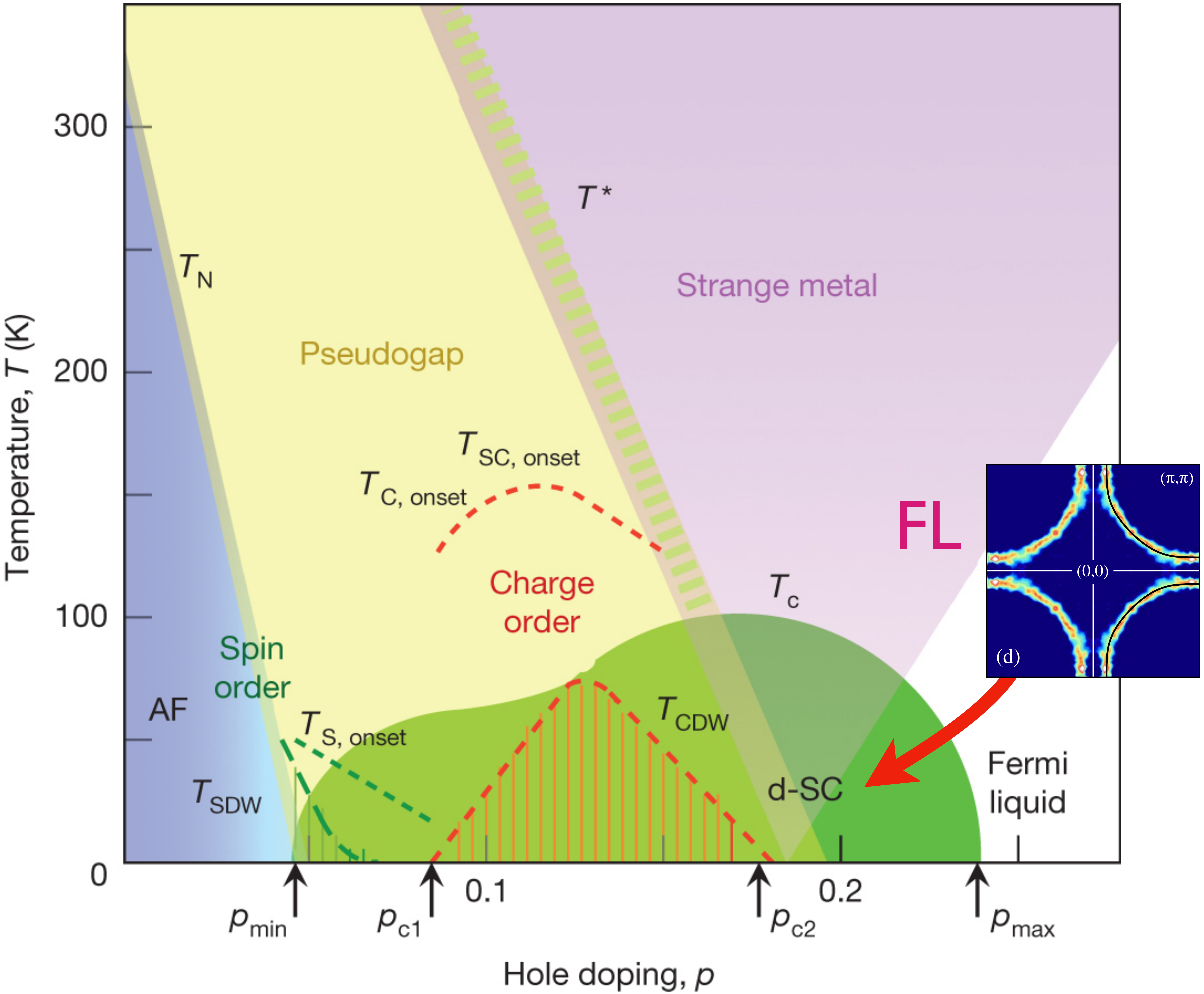
2. FL-FL* QPT

3. Confinement crossover

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

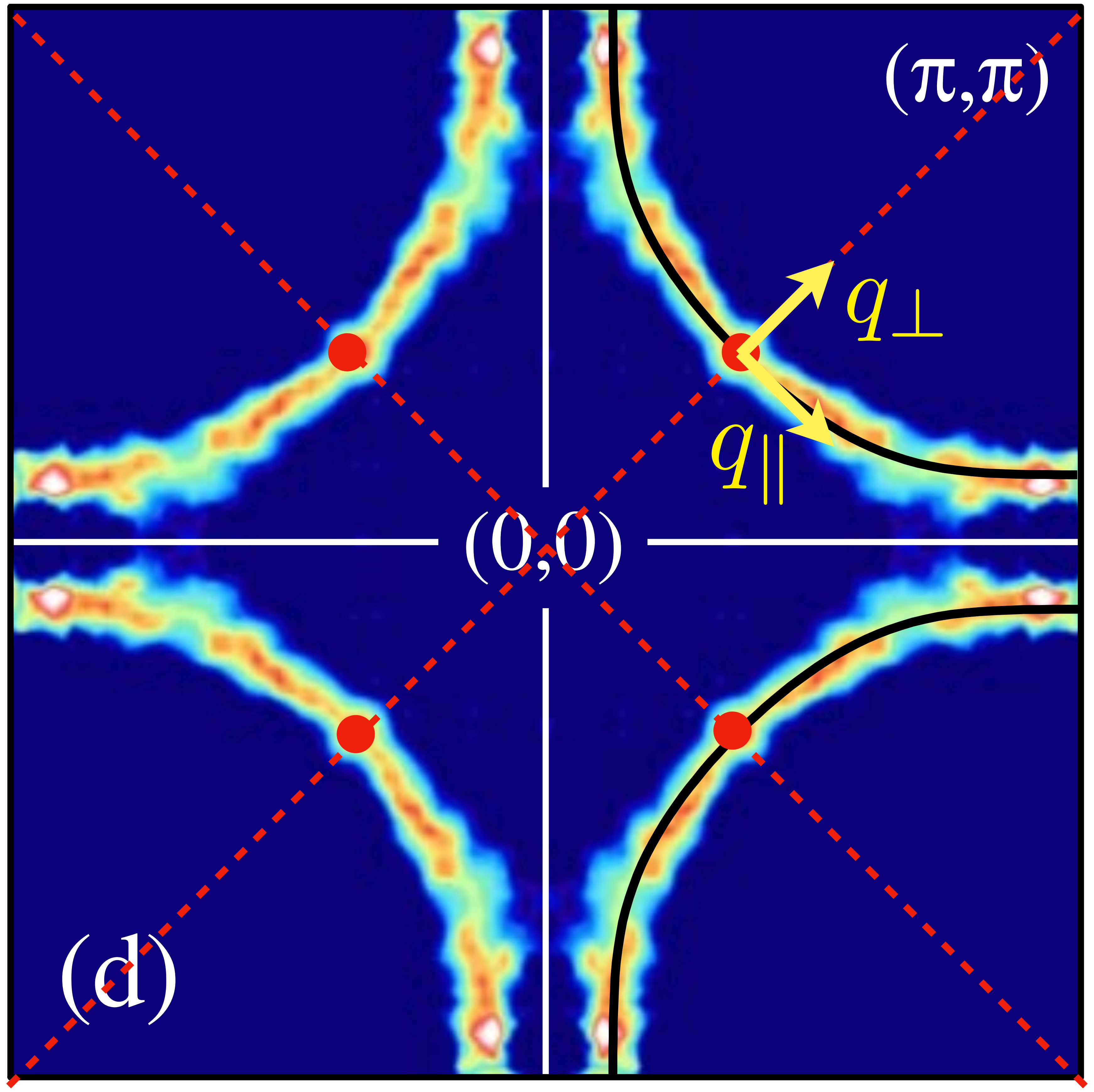


Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



BCS-type theory of *d*-wave superconductivity (and charge order) induced by antiferromagnetic spin fluctuations.

FL→dSC



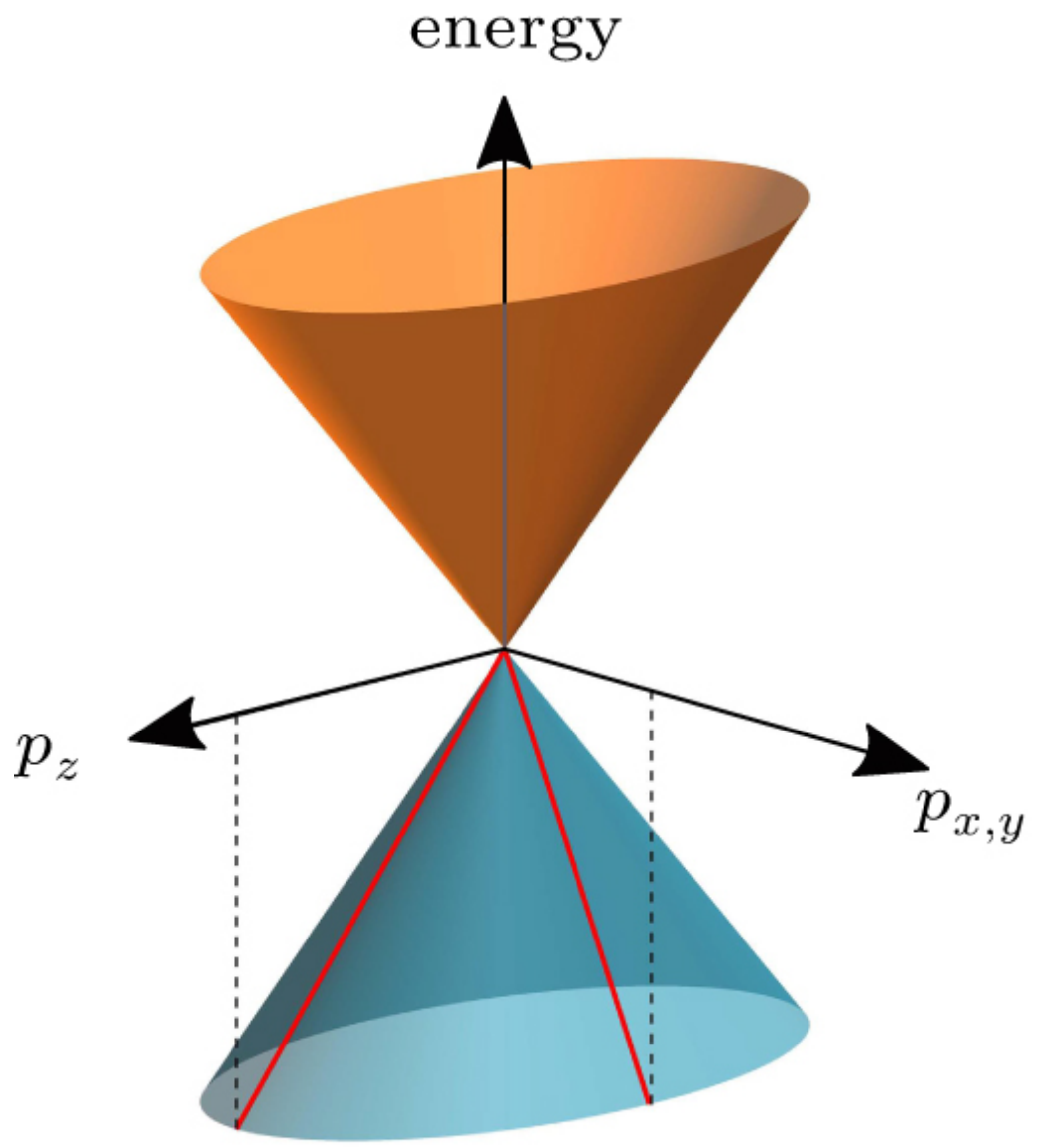
BCS/Bogoliubov quasiparticles in a d-wave superconductor

$$E_{\boldsymbol{k}} = \left(\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2\right)^{1/2}$$
$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$$

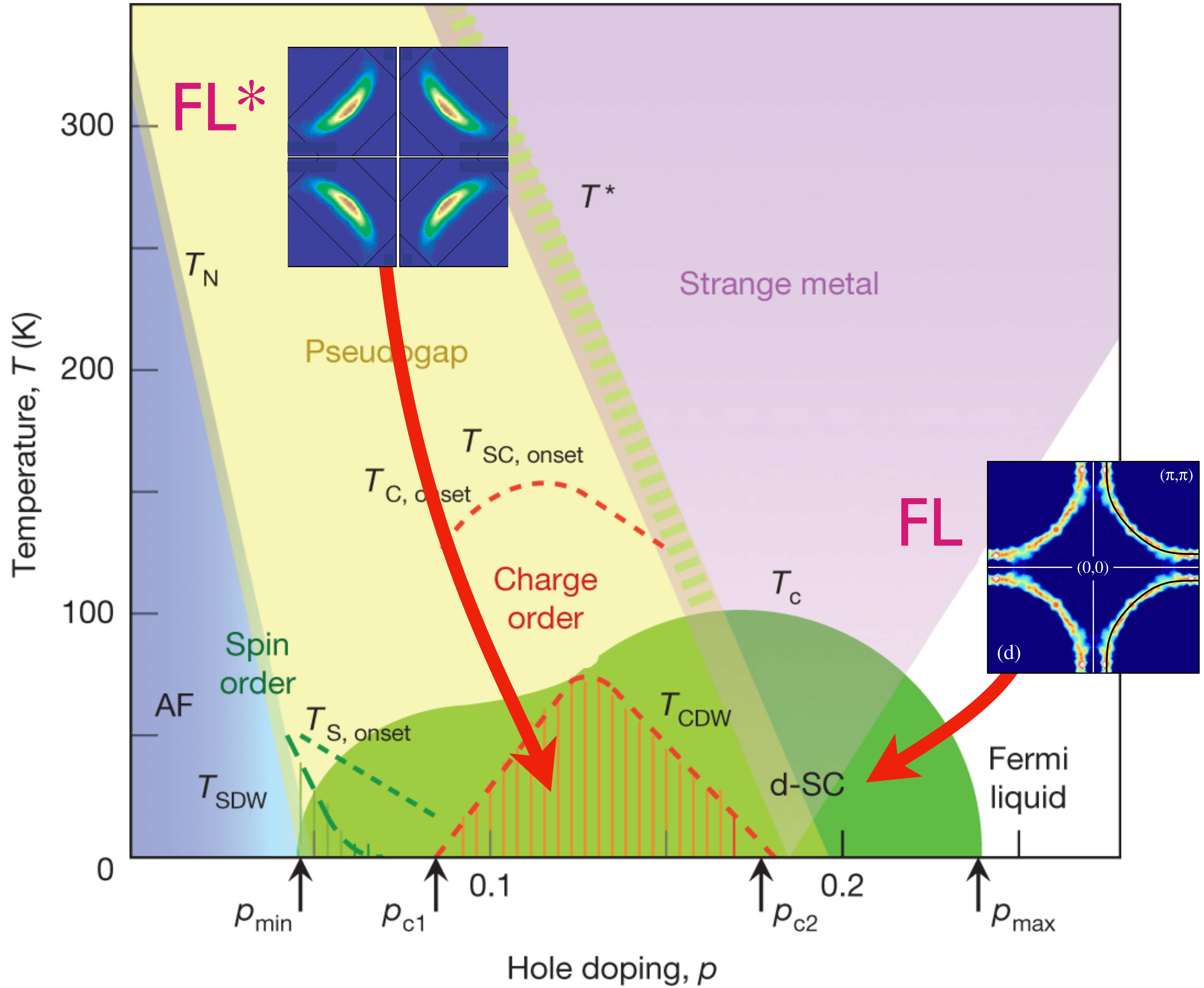
4 nodal points where

$$E_{\boldsymbol{k}_0+\boldsymbol{q}} = \left(v_F^2 q_\perp^2 + v_\Delta^2 q_\parallel^2\right)^{1/2}$$

with $v_F \gg v_\Delta$.



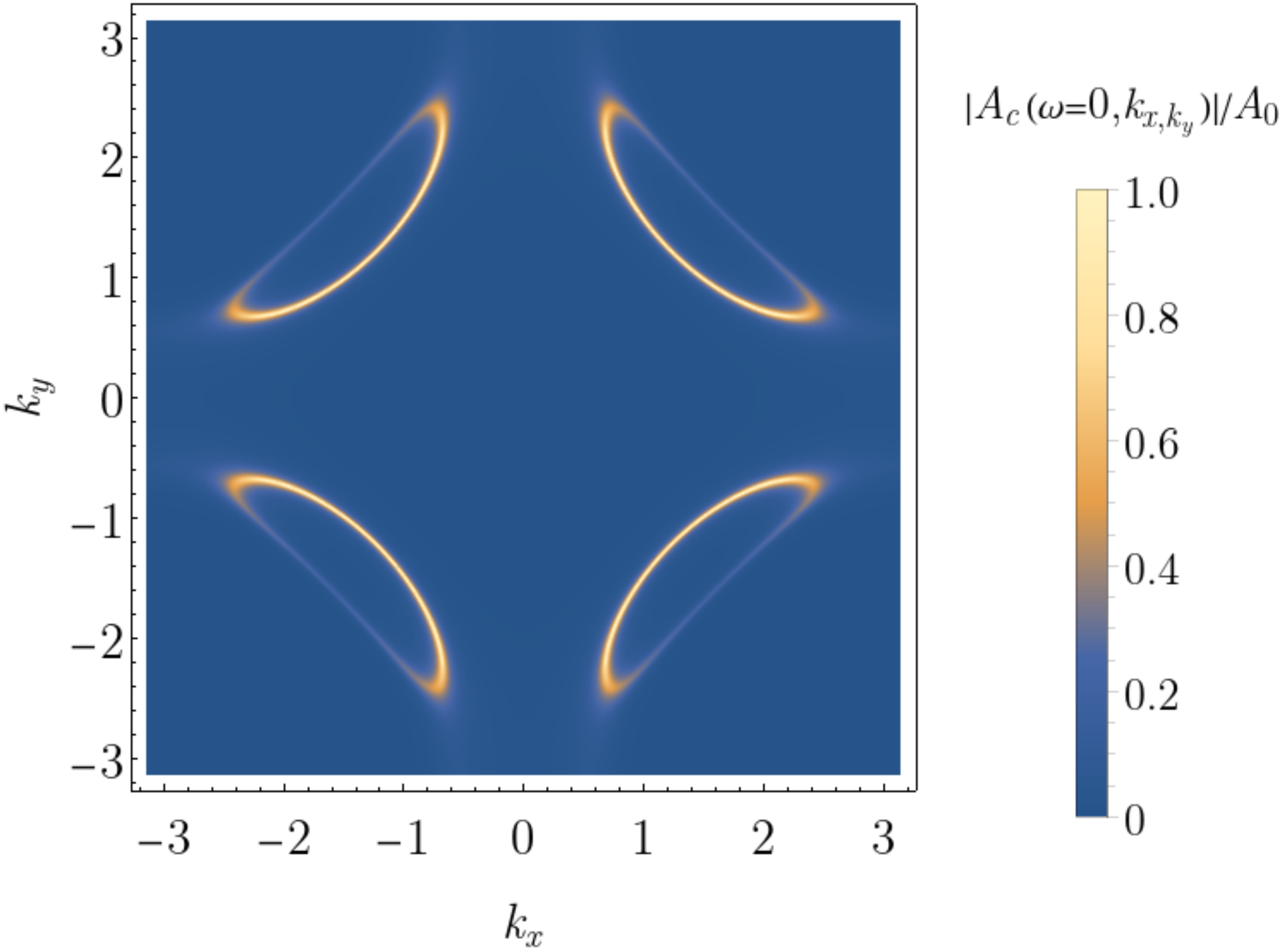
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Obtain d-wave superconductor and charge order from a theory of *confinement* instabilities of FL*.

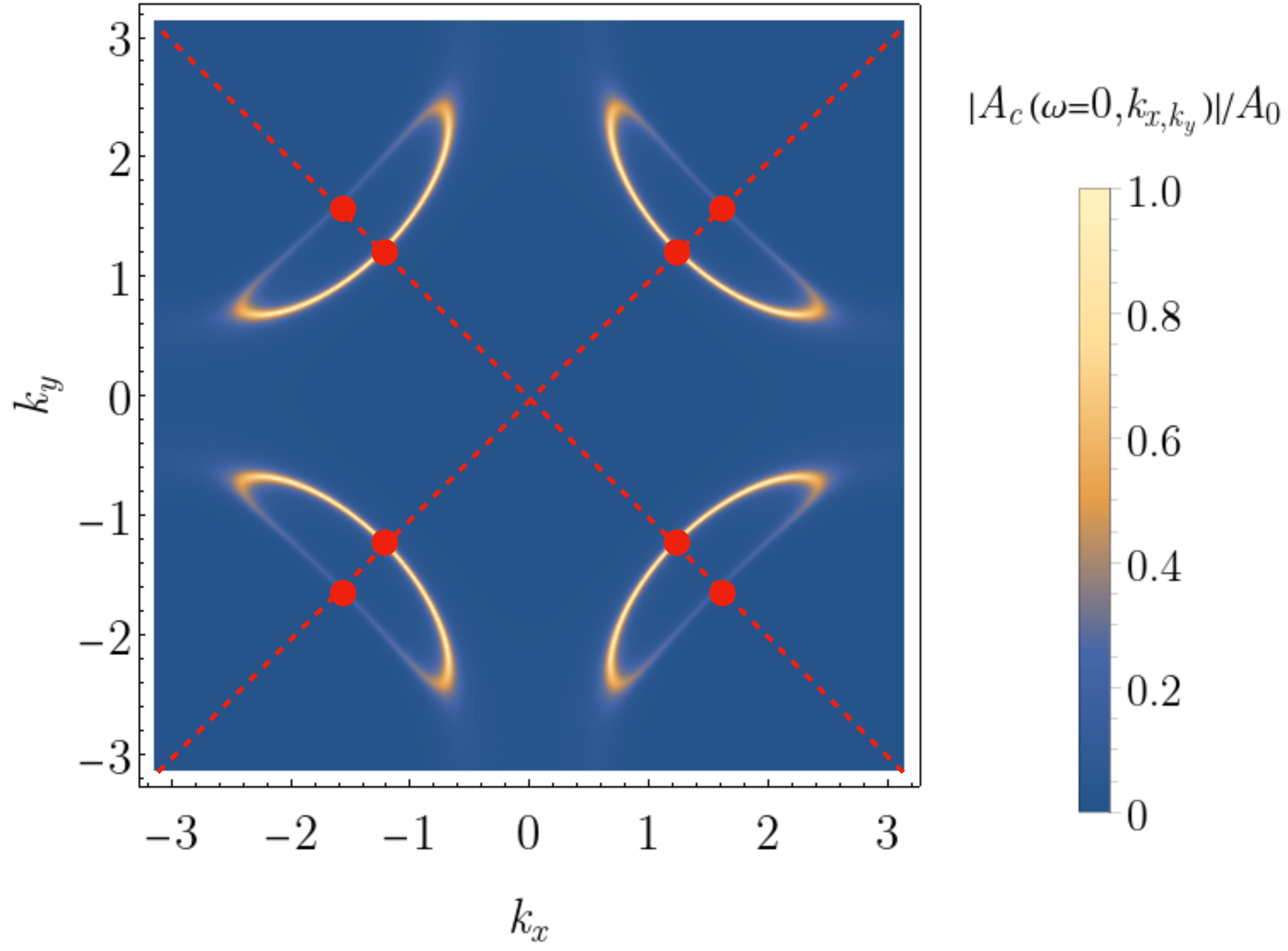
The resulting low T ordered states should be adiabatically connected to the corresponding states obtained from instabilities of FL.

FL*



E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

FL*→dSC*

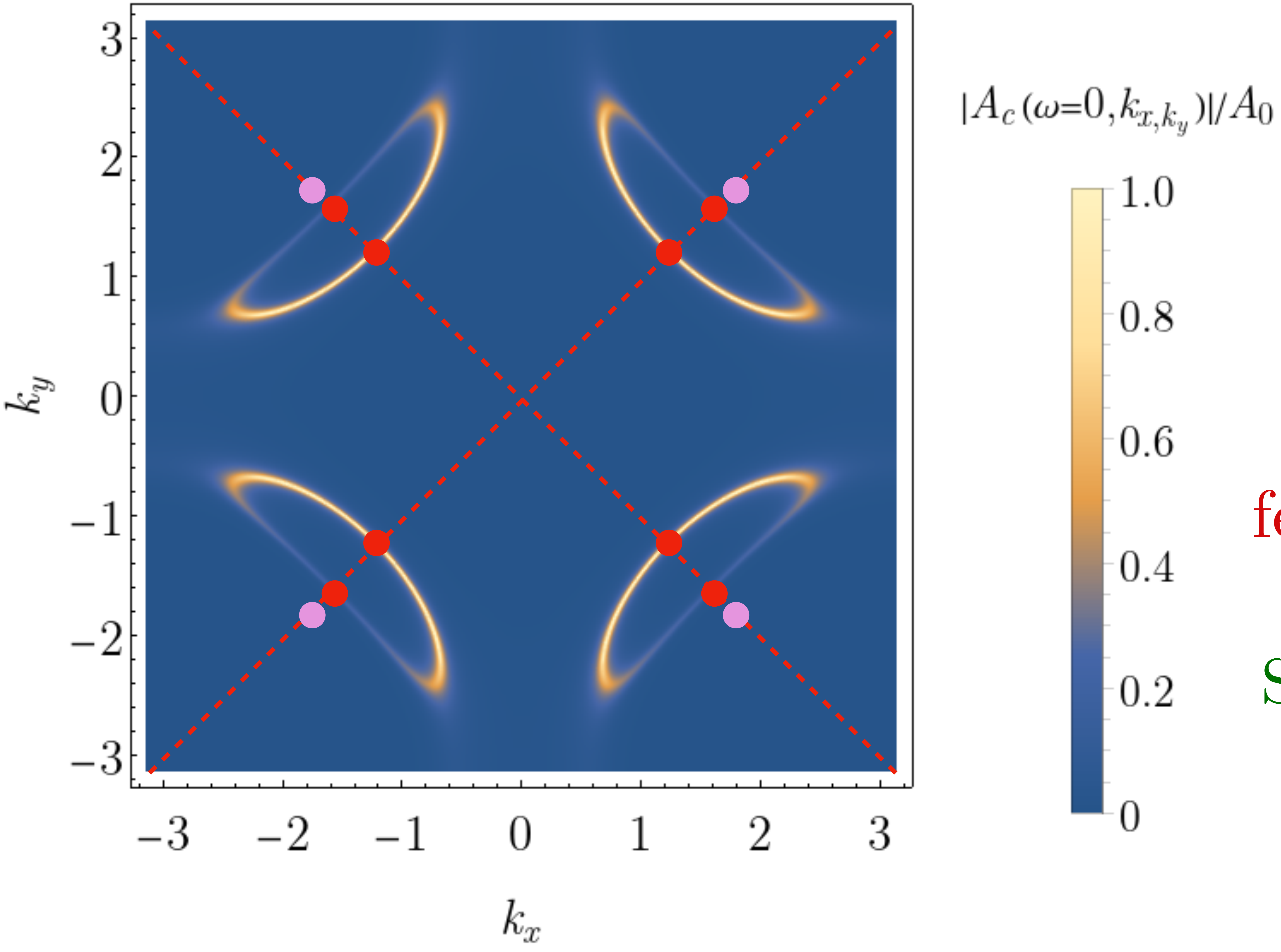


$$E_{\boldsymbol{k}} = \left(\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2\right)^{1/2}$$

$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$$

Adding *d*-wave pairing
to the hole pockets
leads to 8 nodal points???

FL*→dSC*

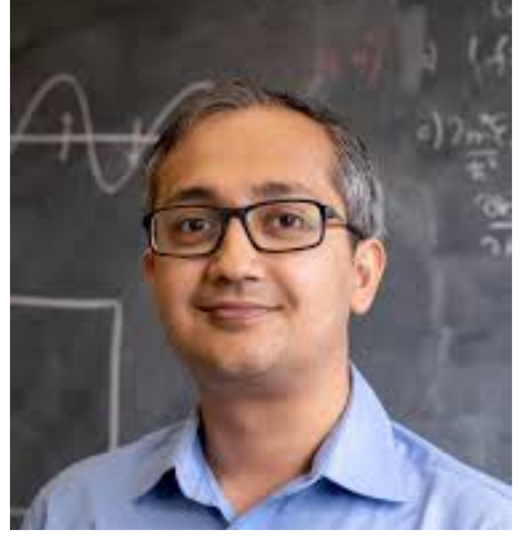


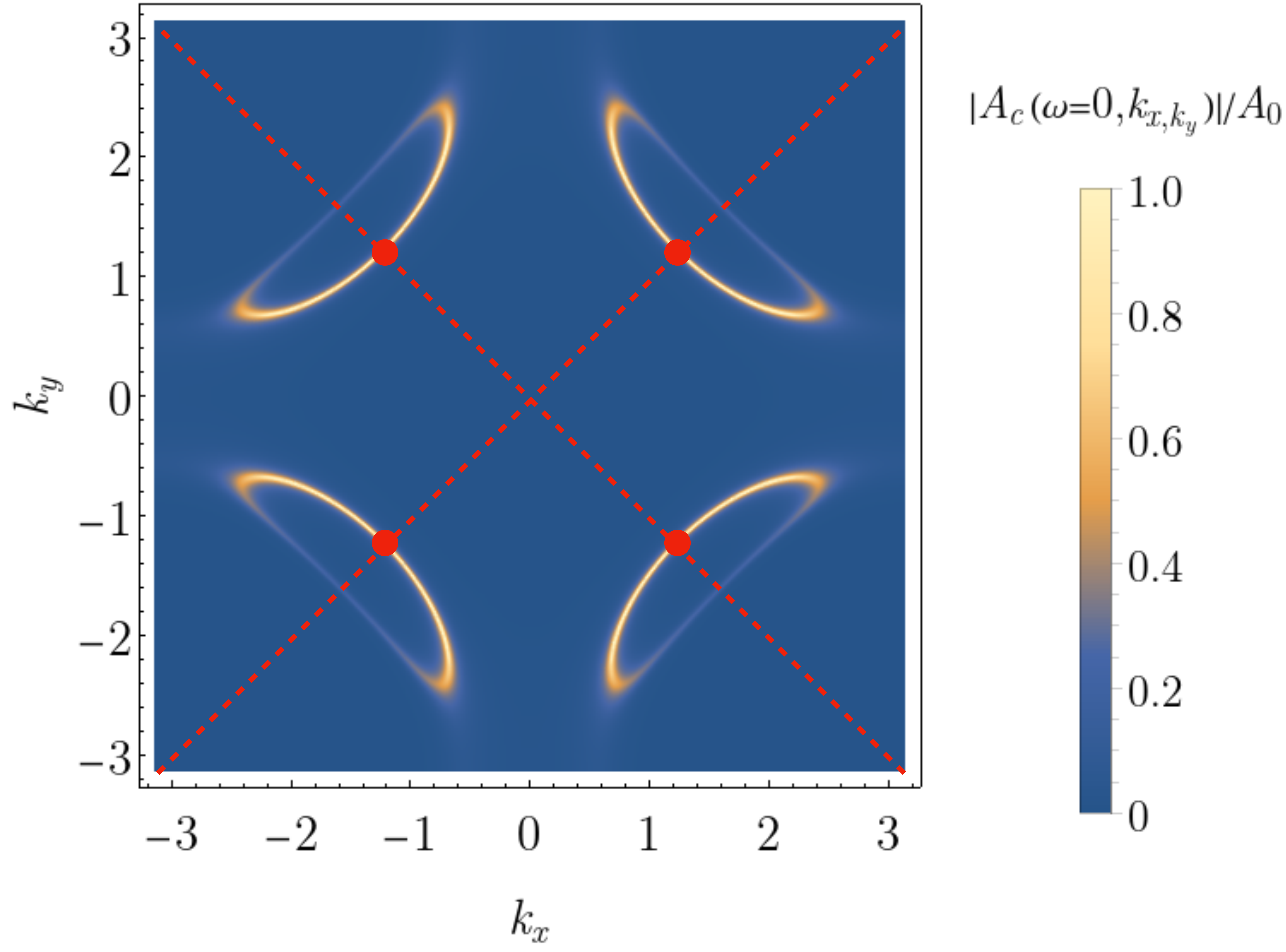
8 nodal points of Bogoliubov quasiparticles from the Fermi pockets
and
4 nodal points of fermionic Dirac spinons from the π-flux spin liquid

Such a spin liquid is required to be present in the background for FL*: realized by $\boldsymbol{S}_2$ spins.

FL*→dSC

Shubhayu Chatterjee and S. Sachdev,
PRB **94**, 205117 (2016)
Maine Christos and S.Sachdev,
npj Quantum Materials **9**, 4 (2024)





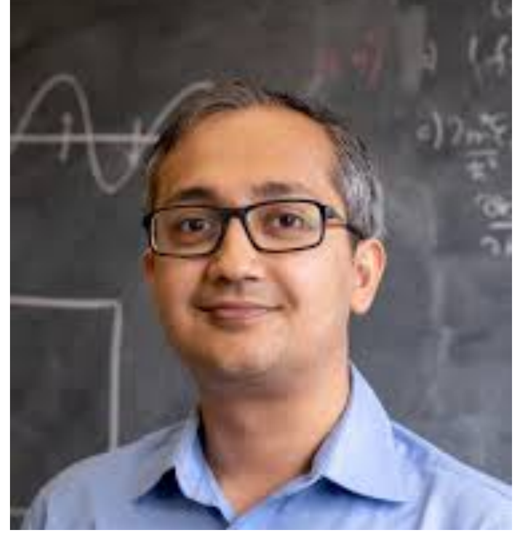
d-wave superconductor obtained by condensing charge-e, SU(2) fundamental boson B.

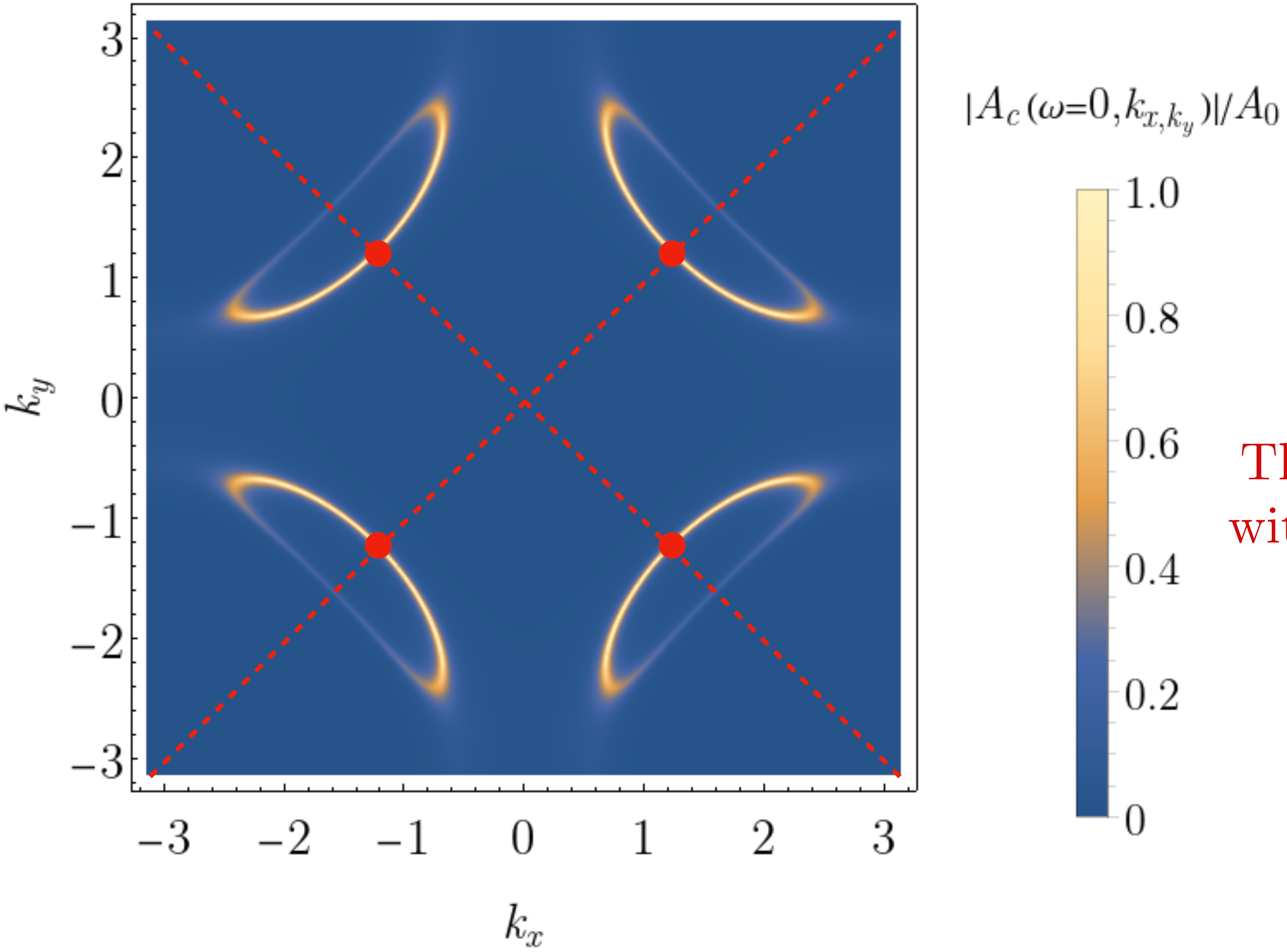
The B Higgs condensate allows spinons and Bogoliubov quasiparticles
to hybridize.

8 nodal points annihilate each other, leaving 4 nodal points with anisotropic velocities, just as in a BCS d-wave state.

FL*→dSC

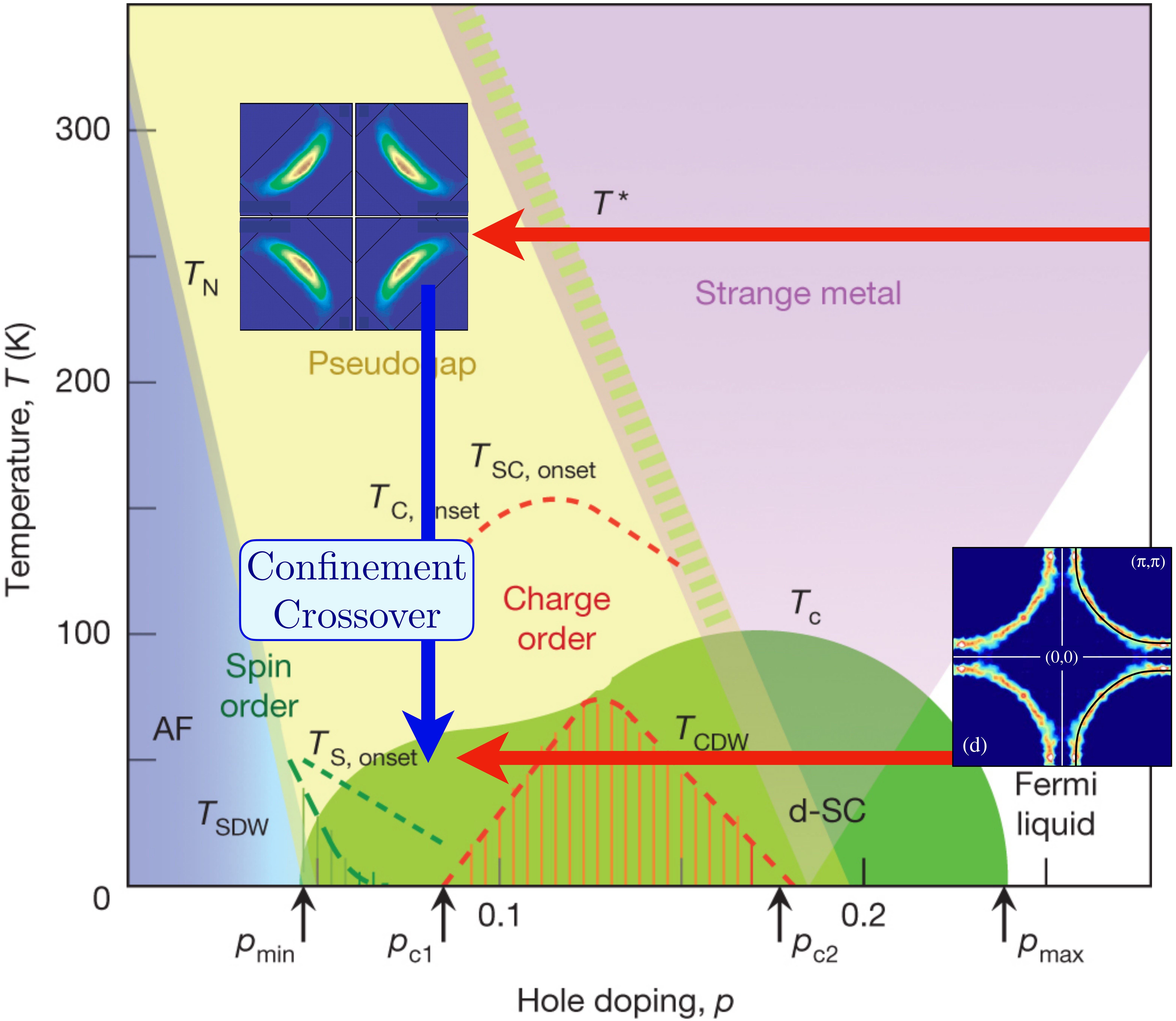
Shubhayu Chatterjee and S. Sachdev,
PRB **94**, 205117 (2016)
Maine Christos and S.Sachdev,
npj Quantum Materials **9**, 4 (2024)





The spinons do *not* become the Bogoliubov quasiparticles, they *annihilate* the unwanted Bogoliubov quasiparticles. This leads to a d-wave superconductor with 4 nodal Bogoliubov quasiparticles, with $v_F \gg v_\Delta$, consistent with observations.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Fermi-volume-changing QPT
without symmetry breaking
and with spatial disorder.

FL-FL* QPT
Requires fractionalization

Fermi-volume-changing QPT
with symmetry breaking
and with spatial disorder.

FL-SDW QPT