Planckian metals
and
deconfined quantum criticality
in the cuprates

Indian Institute of Science, Bengaluru
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Talk online: sachdev.physics.harvard.edu

PHYSICS

HARVARD

Remarkable recent observation of 'Planckian' strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity is associated with a universal scattering time $\approx \hbar/(k_B T)$.

Universal *T*-linear resistivity and Planckian dissipation in overdoped cuprates

NATURE PHYSICS | VOL 15 | FEBRUARY 2019 | 142–147

A. Legros[1,2], S. Benhabib[3], W. Tabis[3,4], F. Laliberté[1], M. Dion[1], M. Lizaire[1], B. Vignolle[3], D. Vignolles[3], H. Raffy[5], Z. Z. Li[5], P. Auban-Senzier[5], N. Doiron-Leyraud[1], P. Fournier[1,6], D. Colson[2], L. Taillefer[1,6*] and C. Proust[3,6*]

arXiv:1902.01034

Planckian dissipation and scale invariance in a quantum-critical disordered pnictide

Yasuyuki Nakajima,[1,2] Tristin Metz,[2] Christopher Eckberg,[2] Kevin Kirshenbaum,[2] Alex Hughes,[2] Renxiong Wang,[2] Limin Wang,[2] Shanta R. Saha,[2] I-Lin Liu,[2,3,4] Nicholas P. Butch,[2,4] Zhonghao Liu,[5,6] Sergey V. Borisenko,[5] Peter Y. Zavalij,[7] and Johnpierre Paglione[2,8]

Strange metal in magic-angle graphene with near Planckian dissipation

Yuan Cao,[1,*] Debanjan Chowdhury,[1,*] Daniel Rodan-Legrain,[1] Oriol Rubies-Bigordà,[1] Kenji Watanabe,[2] Takashi Taniguchi,[2] T. Senthil,[1,†] and Pablo Jarillo-Herrero[1,†]

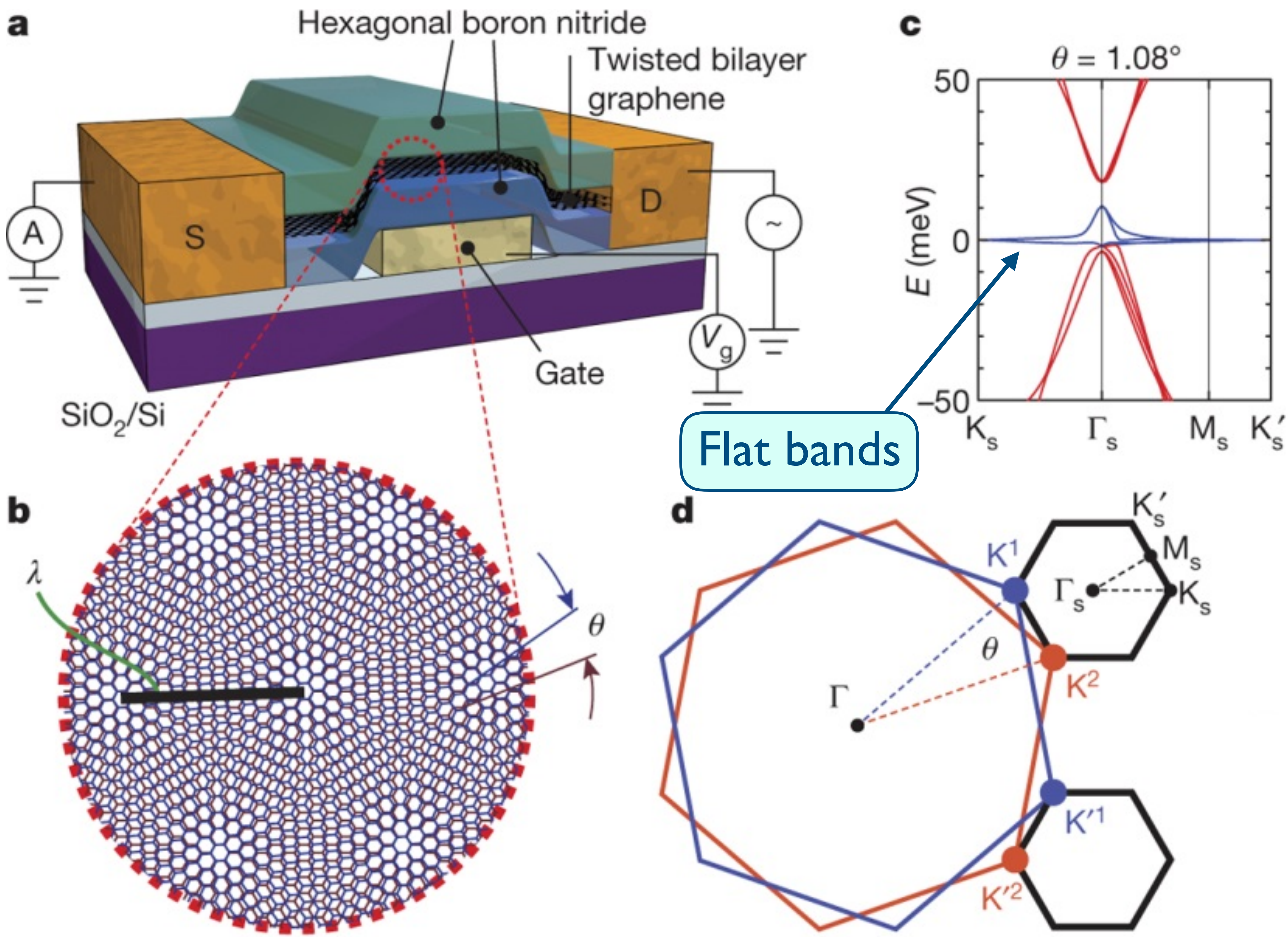
arXiv:1901.03710

Bad metallic transport in a cold atom Fermi-Hubbard system

Science **363**, 379–382 (2019)

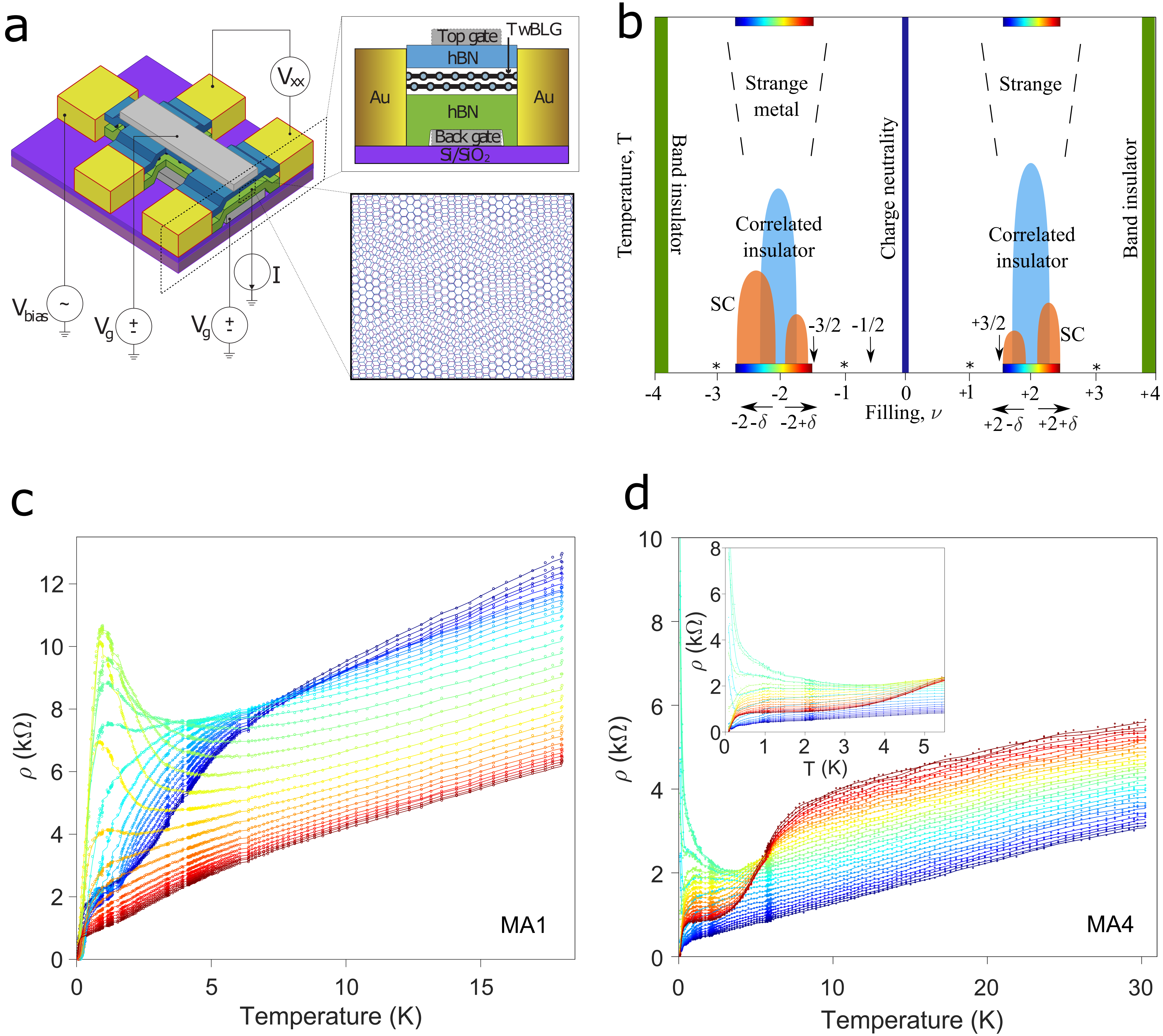
Peter T. Brown[1], Debayan Mitra[1], Elmer Guardado-Sanchez[1], Reza Nourafkan[2], Alexis Reymbaut[2], Charles-David Hébert[2], Simon Bergeron[2], A.-M. S. Tremblay[2,3], Jure Kokalj[4,5], David A. Huse[1], Peter Schauß[1*], Waseem S. Bakr[1]†

Twisted bilayer graphene



Cao et al., Nature **556**, 80 (2018)

Twisted bilayer graphene



Cao *et al.*, arXiv:1901.03710

Remarkable recent observation of 'Planckian' strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, ρ, is

$$\rho = \frac{m^*}{ne^2} \frac{1}{\tau}$$

with a universal scattering rate

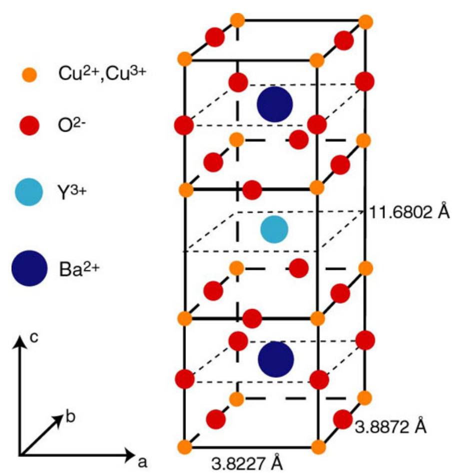
$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar}\,,$$

independent of the strength of interactions!

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Slope of *T*-linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$



A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

1. SYK models

2. Resonant SYK models
and Planckian metals

3. Deconfined quantum criticality of
random *t*-*J* models

1. SYK models

2. Resonant SYK models and Planckian metals

3. Deconfined quantum criticality of random *t*-*J* models

The complex SYK model

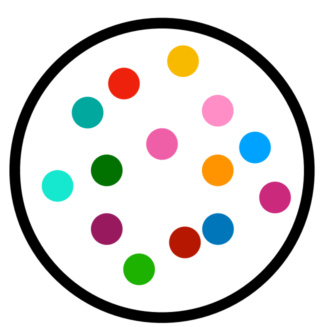
$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta} \, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta + e \sum_\alpha c_\alpha^\dagger c_\alpha$$

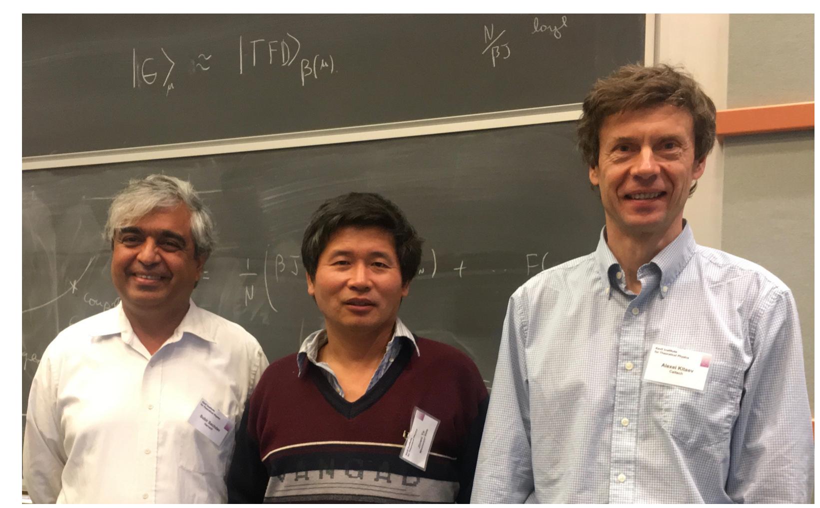
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$





S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

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Flat band

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

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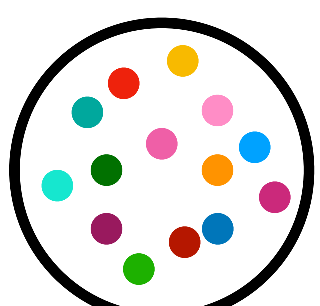

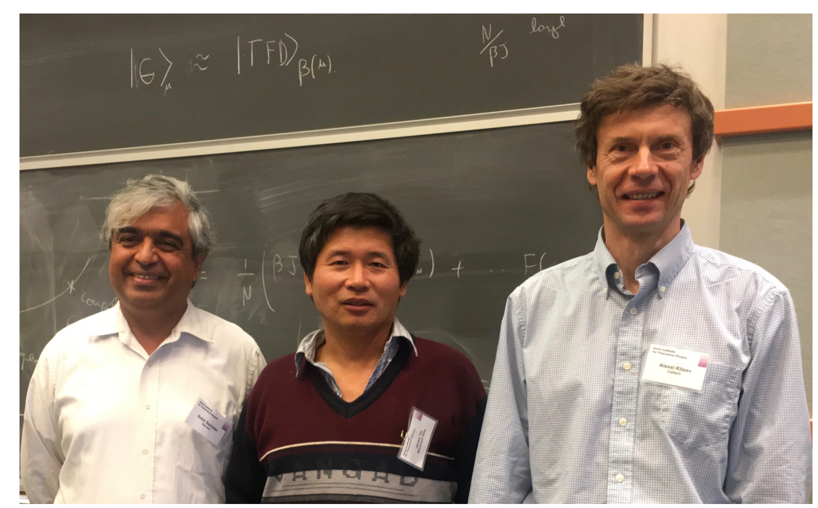

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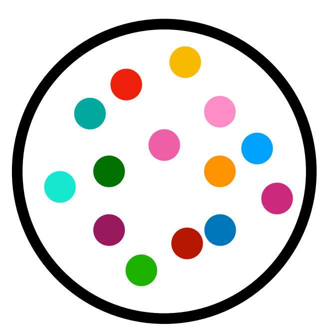
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

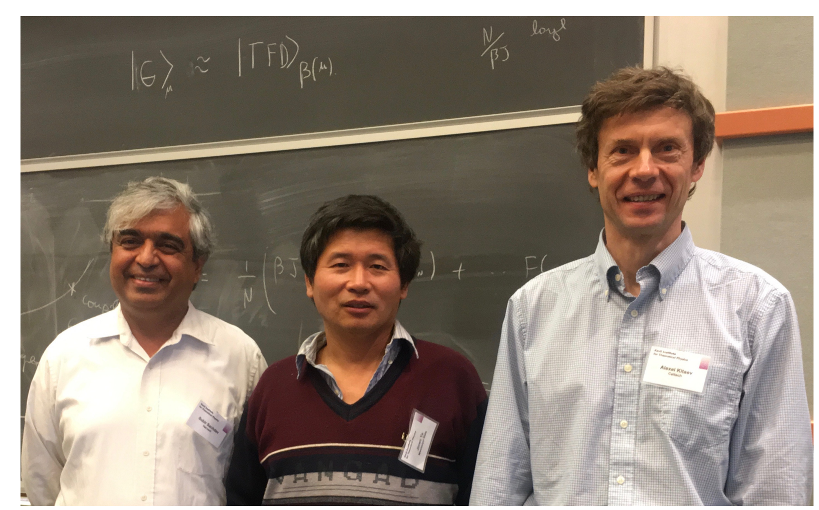
Random interactions

Flat band

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$





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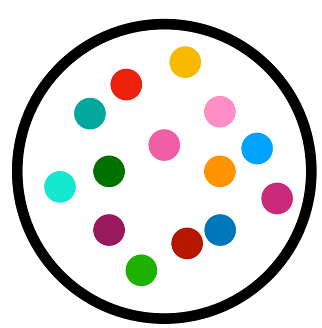
Random interactions

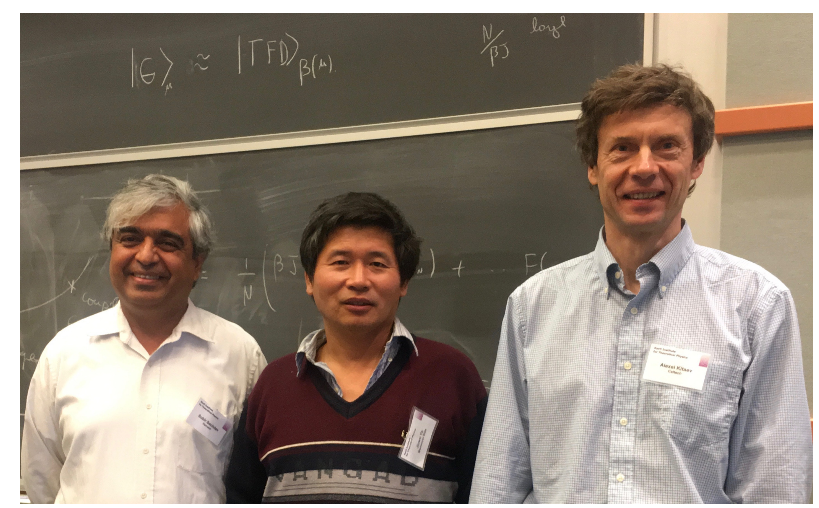
Flat band

Density

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$





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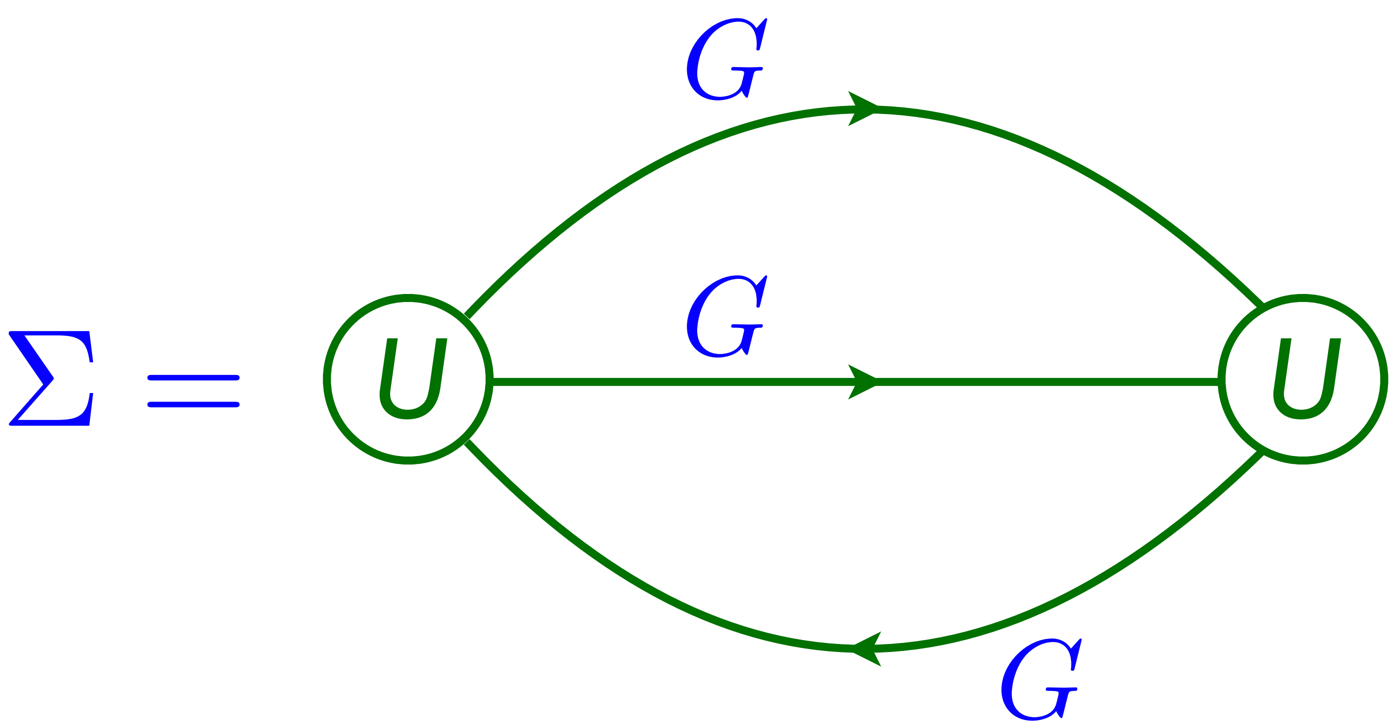
The complex SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega - e - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

The complex SYK model

There is a one-parameter family of critical solutions with varying e/U, yielding different $0 < \mathcal{Q} < 1$.

For long (imaginary) times $\tau > 0$

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle = A e^{-2\pi\mathcal{E}T\tau} \times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

In a Fermi liquid,

$$\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \frac{T}{\sin(\pi T \tau)}$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

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Determines the particle-hole asymmetry, and $\mathcal{E} = \mathbb{C} e/U$, with $\mathbb{C} = 0.41$ from a numerical solution.

In a Fermi liquid,

$$\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \frac{T}{\sin(\pi T \tau)}$$

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For long (imaginary) times $\tau > 0$

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle = A e^{-2\pi \mathcal{E} T \tau} \times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

Known dimensionless constant

In a Fermi liquid,

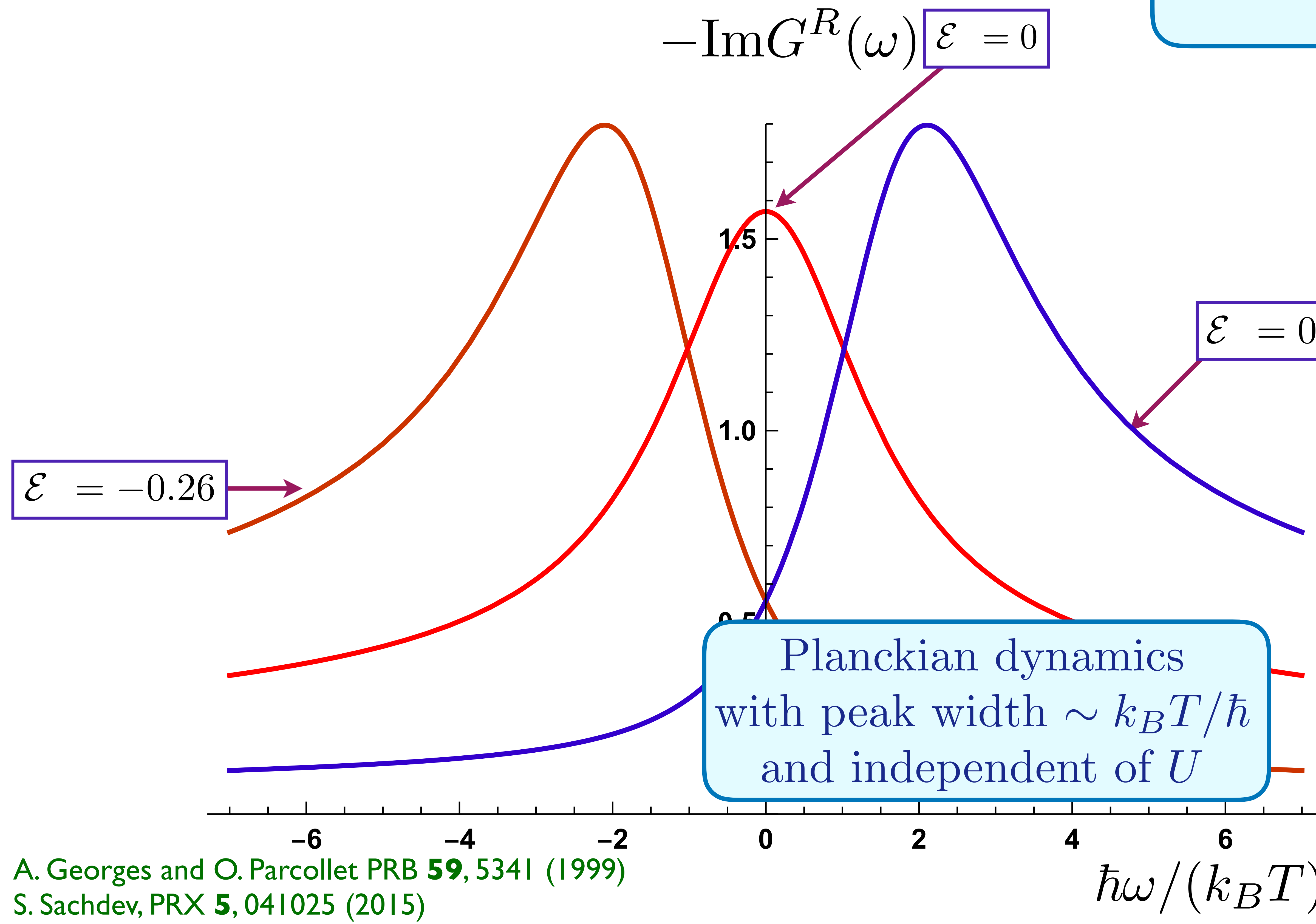
$$\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \frac{T}{\sin(\pi T \tau)}$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

The SYK model

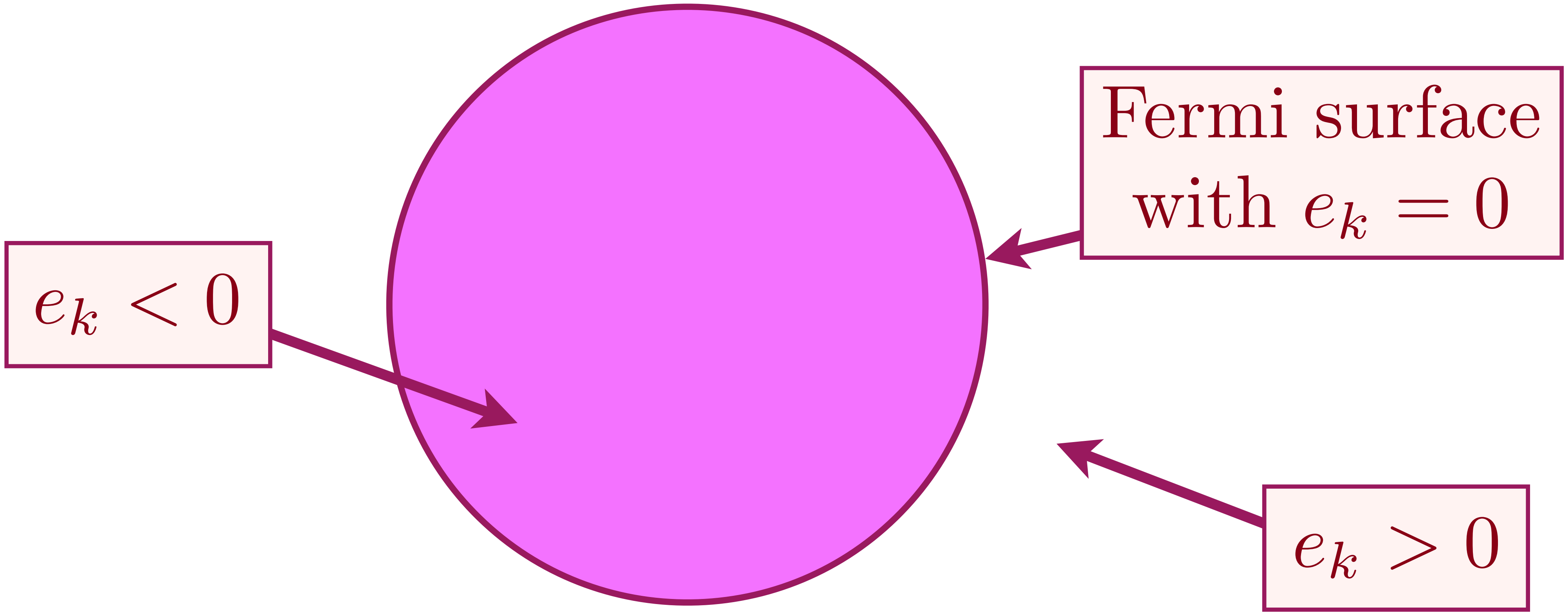
$$\mathcal{E} = \mathbb{C}\frac{e}{U}$$



Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of U

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

Adding dispersion



- All electrons in the (flat band) SYK model have the same e
- In a more realistic metal, the electrons have a dispersion e_k (k is momentum), and $e_k = 0$ is the Fermi surface.

Generalized SYK models

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}(k_a)\, c^\dagger_{k_1\alpha} c^\dagger_{k_2\beta} c_{k_3\gamma} c_{k_4\delta}$$

$$+ \sum_{k\alpha} e_k c^\dagger_{k\alpha} c_{k\alpha}$$

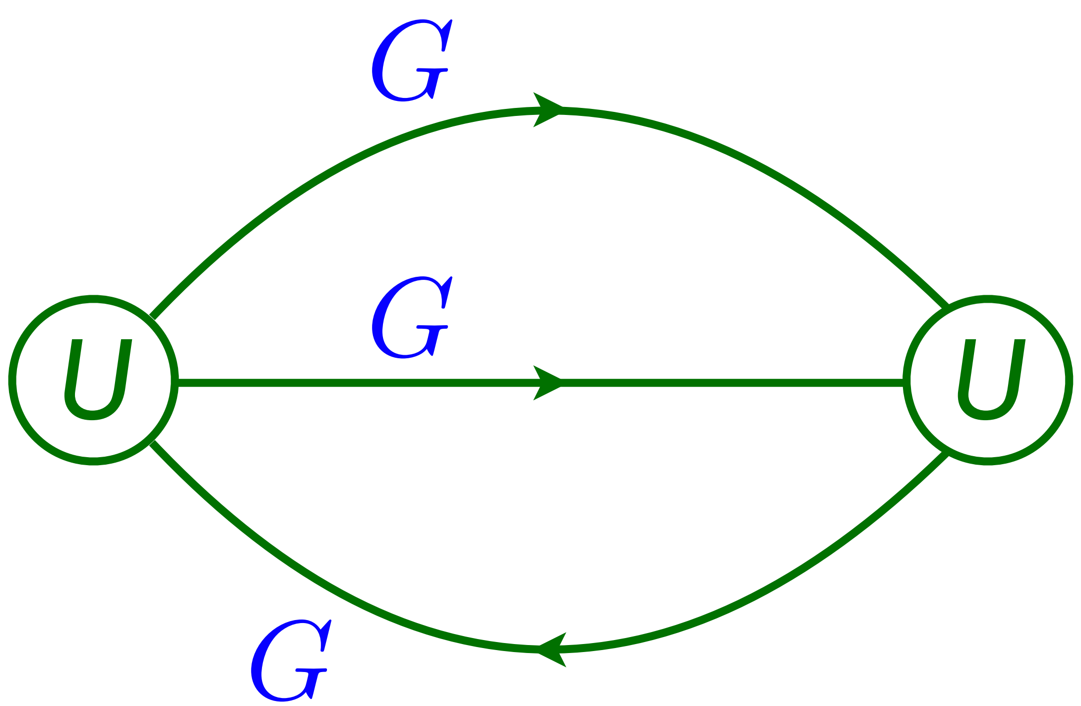
$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$

e_k has a bandwidth W.

The large N limit is still given by the sum of "melon" diagrams.

$$G(k, i\omega) = \frac{1}{i\omega - e_k - \Sigma(k, i\omega)}$$

$$\Sigma =$$



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$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$

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The large N limit is still given by the sum of "melon" diagrams.

For many generic models in this class with $U \gg W$, $\hbar\omega/(k_B T)$ scaling of SYK holds for $W^2/U \ll k_B T \ll U$, and Fermi liquid theory is recovered for $k_B T \ll W^2/U$.

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017); Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)
See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999); Yingfei Gu, Xiao-Liang Qi, D. Stanford, JHEP (2017) 125

Generalized SYK models

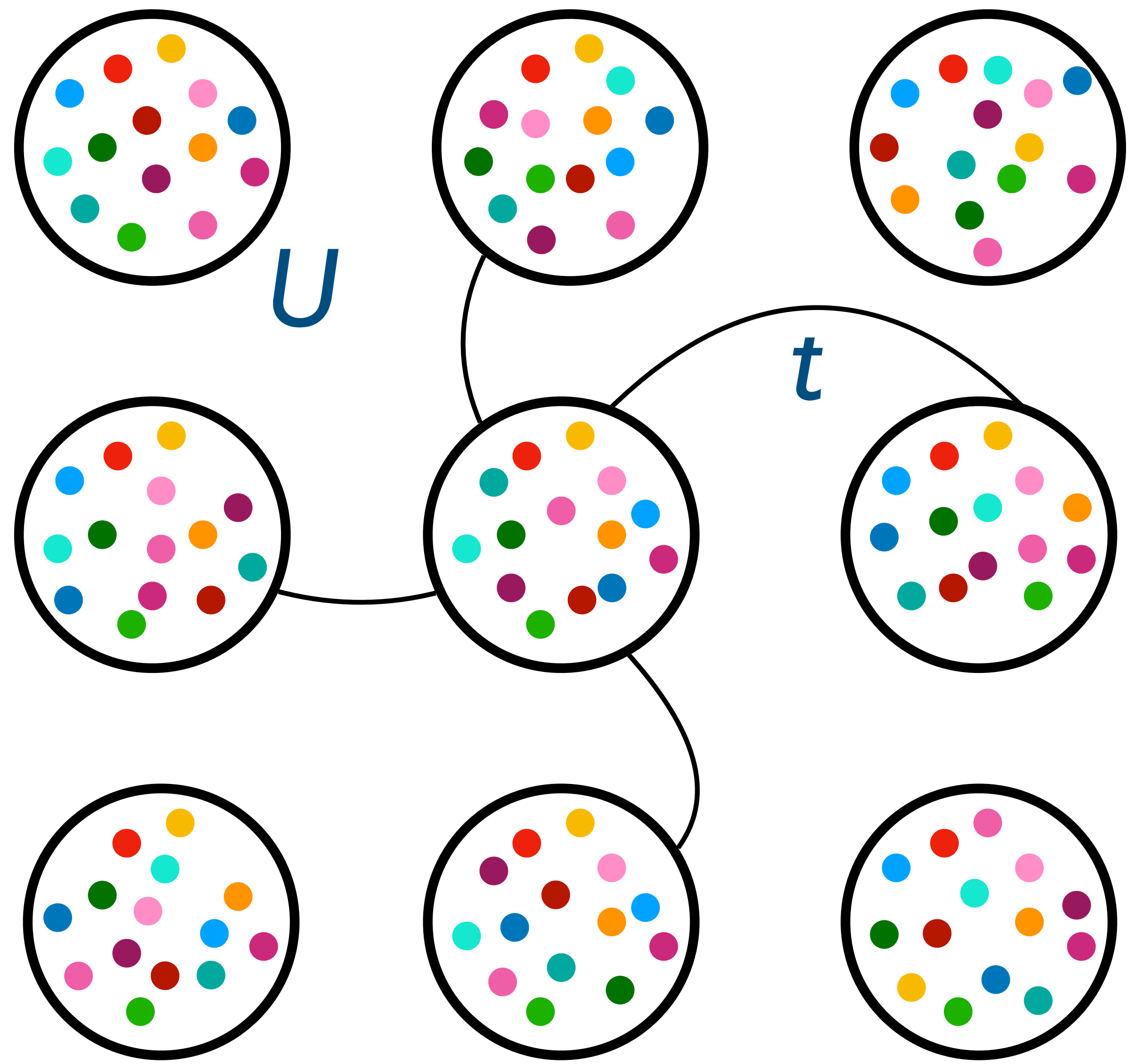
$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{i,\alpha\beta;\gamma\delta}\, c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_{\alpha} c_{i\alpha}^\dagger c_{j\alpha}$$

Choose $U \gg t$ on-site, and independent between sites; yields 'incoherent metal' with no Fermi surface for $t^2/U \ll k_B T \ll U$ with

$$G(\mathbf{k}, \omega) = G_{\rm SYK}(e, \hbar\omega/(k_B T))$$

independent of $\mathbf{k}$.
There is linear-in-T resistivity but only with bad metal behavior with $\rho > h/e^2$, and co-efficient dependent upon U:

$$\rho \sim \frac{h}{e^2} \frac{k_B T}{t^2/U}$$



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1. SYK models

2. Resonant SYK models
and Planckian metals

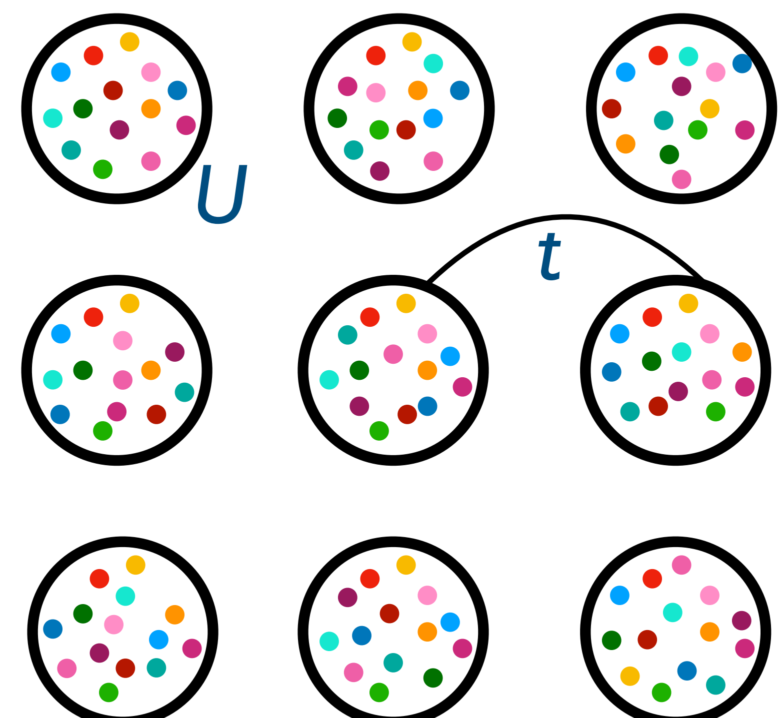
3. Deconfined quantum criticality of
random *t*-*J* models

Generalized SYK models

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}(k_a)\, c^\dagger_{k_1\alpha} c^\dagger_{k_2\beta} c_{k_3\gamma} c_{k_4\delta} + \sum_{k\alpha} e_k c^\dagger_{k\alpha} c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$

e_k has a bandwidth W.



Rewriting of lattice model of incoherent and bad metal in momentum space

$$\overline{U(k_1,k_2,k_3,k_4)U^*(k_5,k_6,k_7,k_8)} = U^2 \left[\delta(k_1+k_2-k_3-k_4-k_5-k_6+k_7+k_8)\right]$$

Generalized SYK models

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}(k_a)\, c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta}$$

$$+ \sum_{k\alpha} e_k c_{k\alpha}^\dagger c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$

e_k has a bandwidth W.

We examine a model with weaker $W \lesssim U$, but impose a **resonance condition**.

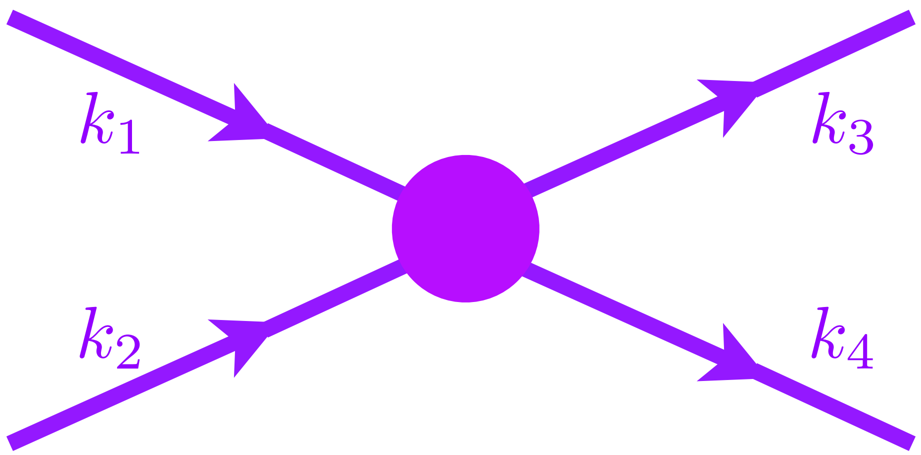
This leads to a solution which obeys the Planckian ansatz as $T \to 0$.

The resonance condition implies off-site interactions with correlations which decay with a power-law in space.

$$\overline{U(k_1,k_2,k_3,k_4)U^*(k_5,k_6,k_7,k_8)} = U^2 \Big[\delta(k_1+k_2-k_3-k_4-k_5-k_6+k_7+k_8)\Big] \times \Big[\delta(e_{k_1}+e_{k_2}-e_{k_3}-e_{k_4}) + \delta(e_{k_5}+e_{k_6}-e_{k_7}-e_{k_8})\Big]$$

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Resonant SYK model



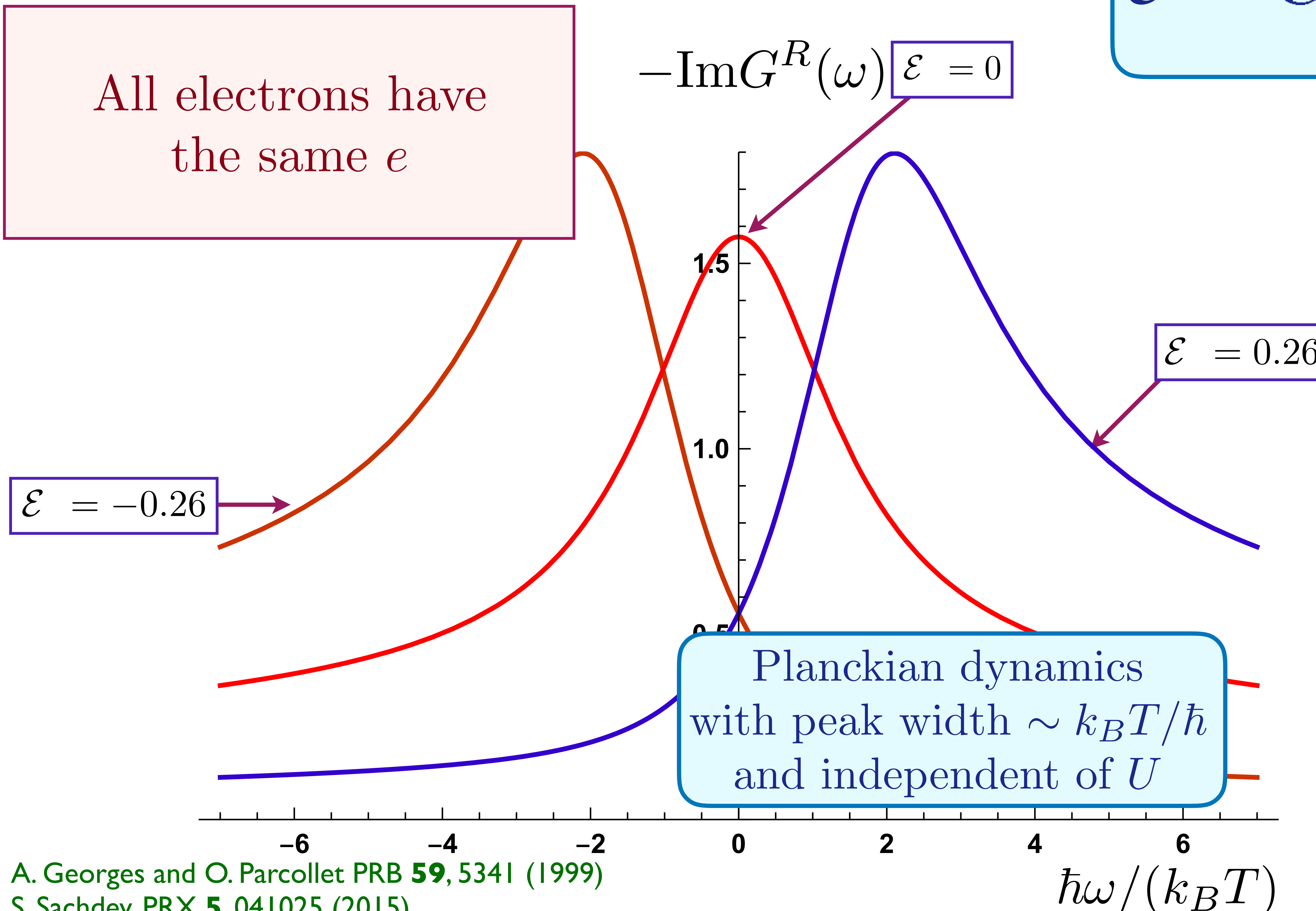
Interactions with $e_{k_1} + e_{k_2} \neq e_{k_3} + e_{k_4}$ are non-resonant: we "integrate these out" in a RG procedure, and assume that their main effect is a renormalization of the quasiparticle dispersion e_k, which we have already accounted for.

Keep only the
interactions resonant in the bare quasiparticle energy
with $e_{k_1} + e_{k_2} = e_{k_3} + e_{k_4}$ and account for them with a self-consistent SYK-like analysis.

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Flat band metal

$$\mathcal{E} = \mathbb{C}\,\frac{e}{U}$$

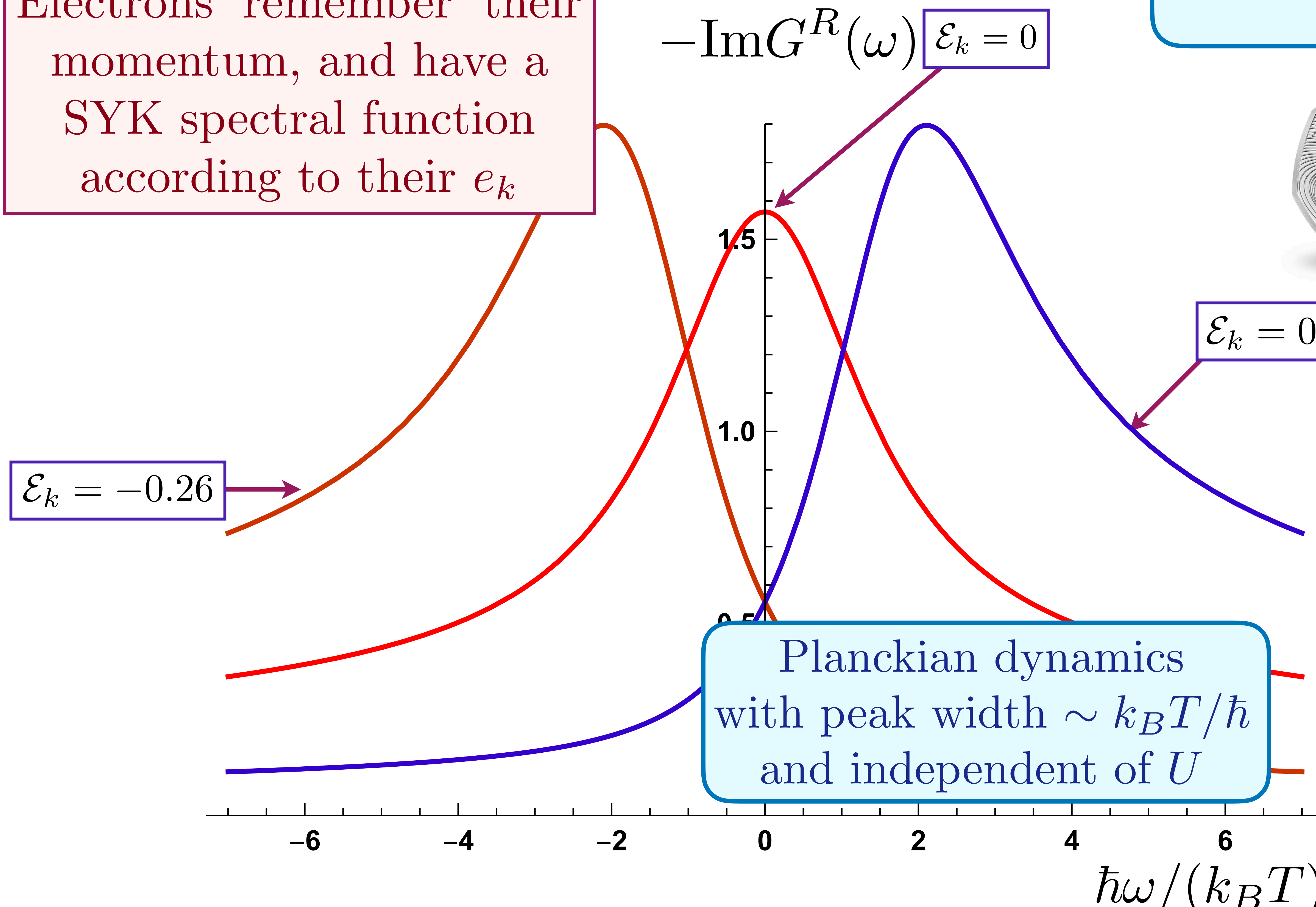


A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

Planckian metal with dispersion

$$\mathcal{E}_k = \mathbb{C}\frac{e_k}{U}$$

Electrons 'remember' their momentum, and have a SYK spectral function according to their e_k



Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of U

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

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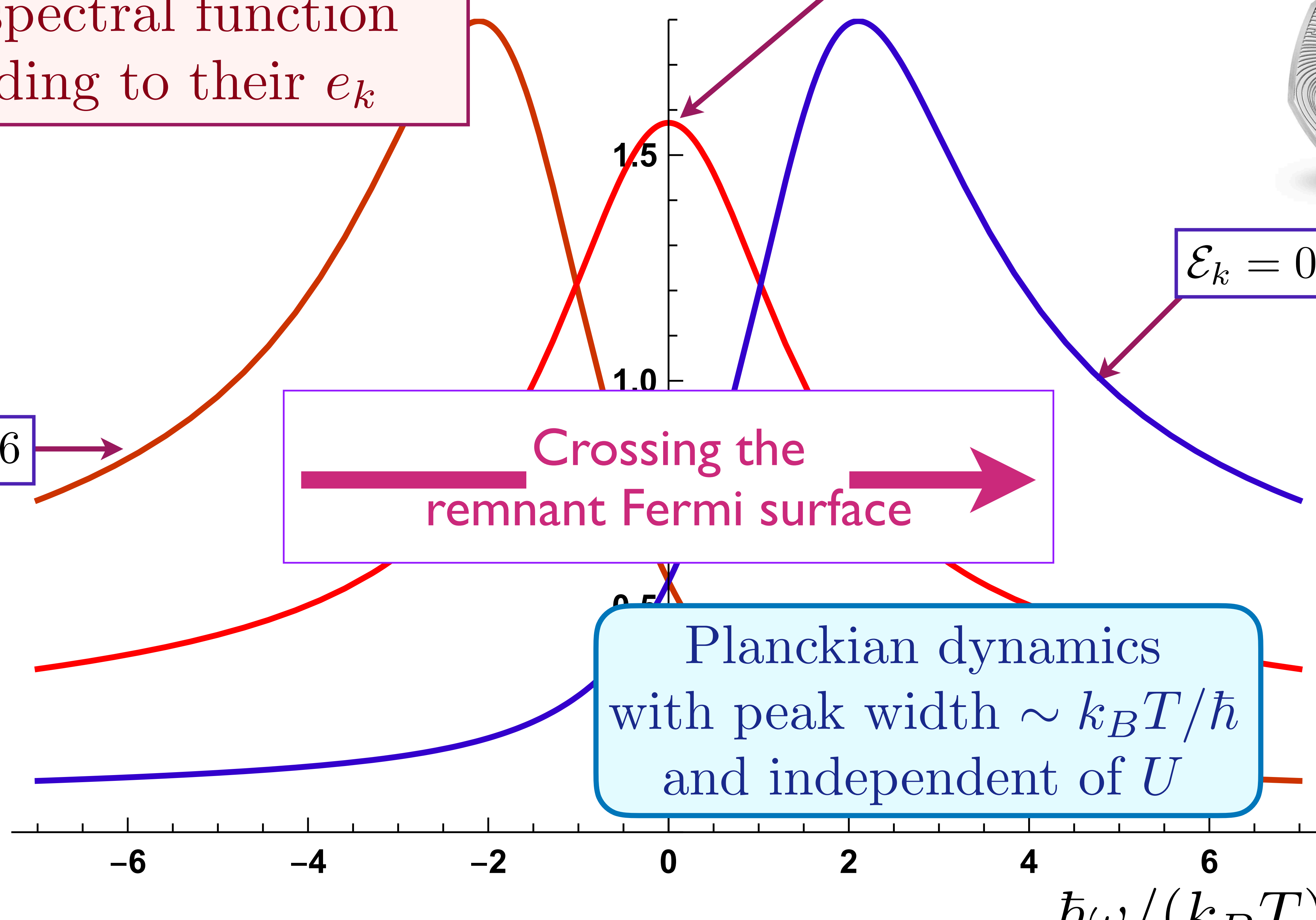
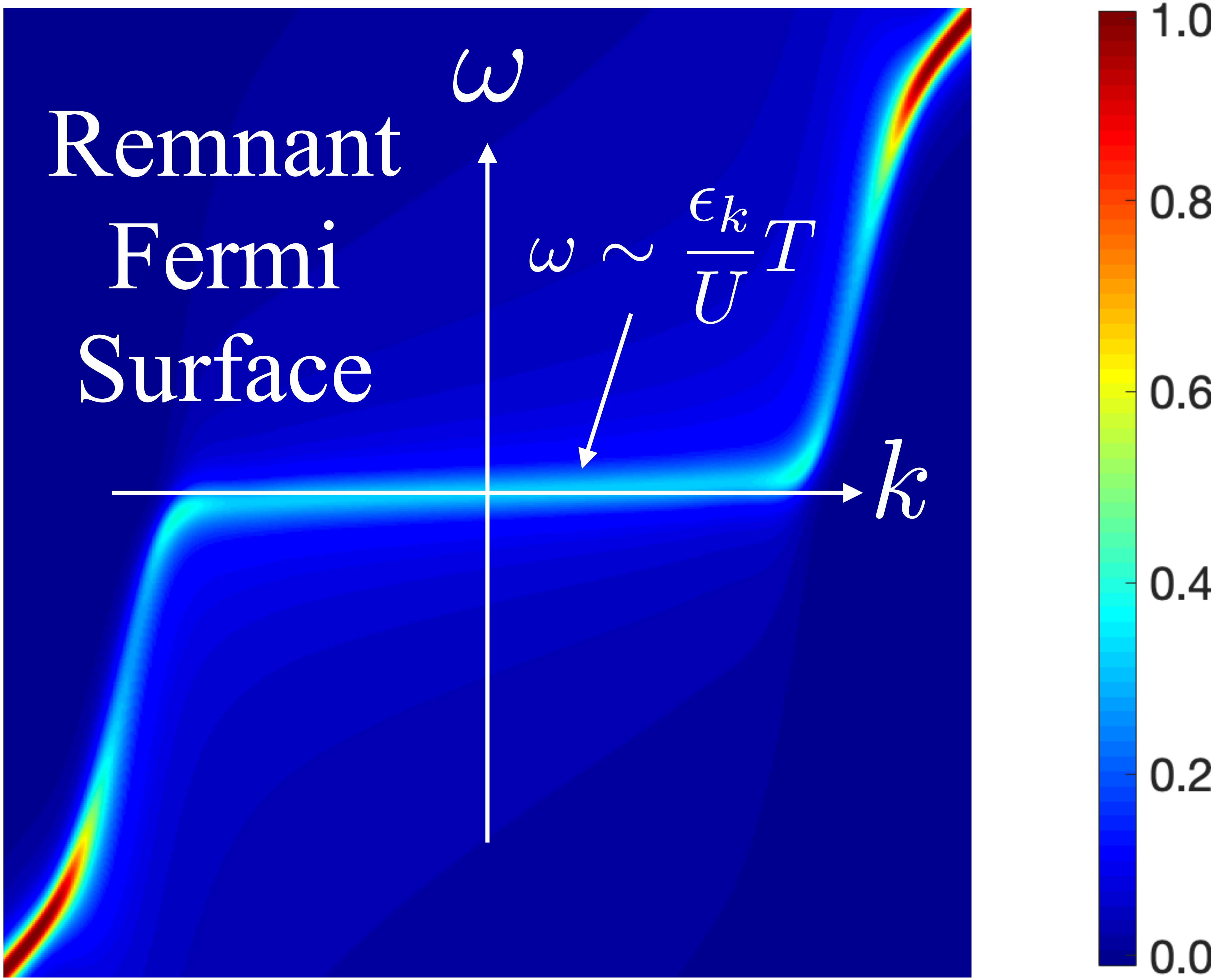


Crossing the remnant Fermi surface

Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of U

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Planckian metal with dispersion



A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Flat band metal

For a dispersionless SYK model

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim e^{-(e/U)2\pi \mathbb{C} T \tau} \times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)

Planckian metal with dispersion

For a strongly-interacting metal with underlying quasiparticle dispersion e_k (k is the momentum)

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle \sim e^{-(e_k/U) 2\pi \mathbb{C} T \tau} \times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

At $e_k = 0$ we have a 'remnant Fermi surface' with a particle-hole symmetric spectral function.

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Planckian metal with dispersion

For a strongly-interacting metal with underlying quasiparticle dispersion e_k (k is the momentum)

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle \sim e^{-(e_k/U) 2\pi \mathbb{C} T \tau} \times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

No free parameters—everything is determined by the (underlying) quasiparticle dispersion e_k, and the interaction strength U.

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Resistivity of a Planckian metal as $T \to 0$

From the Kubo formula,

$$\sigma = \frac{e^2 m^* v_F^2}{2T} \int_{-\infty}^{\infty} \frac{de}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{4\pi} \left[\text{Im}\, G^R_{\text{SYK}}\left(e, \frac{\omega}{T}\right)\right]^2 \text{sech}^2\left(\frac{\omega}{2T}\right)$$

where the Fermi surface is defined by $e_k = 0$, $\boldsymbol{v}_F = \nabla_{\boldsymbol{k}} e_k$ on the Fermi surface, and

$$m^* = \frac{d\, V_{FS}}{\oint_{FS} |\boldsymbol{v}_F|},$$

with d the spatial dimensionality, and V_{FS} is the volume enclosed by the Fermi surface. For a circular Fermi surface, this is the usual m^*.

Evaluating the integrals, we find

$$\rho = \frac{m^*}{ne^2}\, 2.71 \mathbb{C}\, \frac{k_B T}{\hbar} \quad , \quad \text{using } \mathcal{E} = \mathbb{C} e/U \,,$$

where $n = V_{FS}/(2\pi)^d$ is the density.

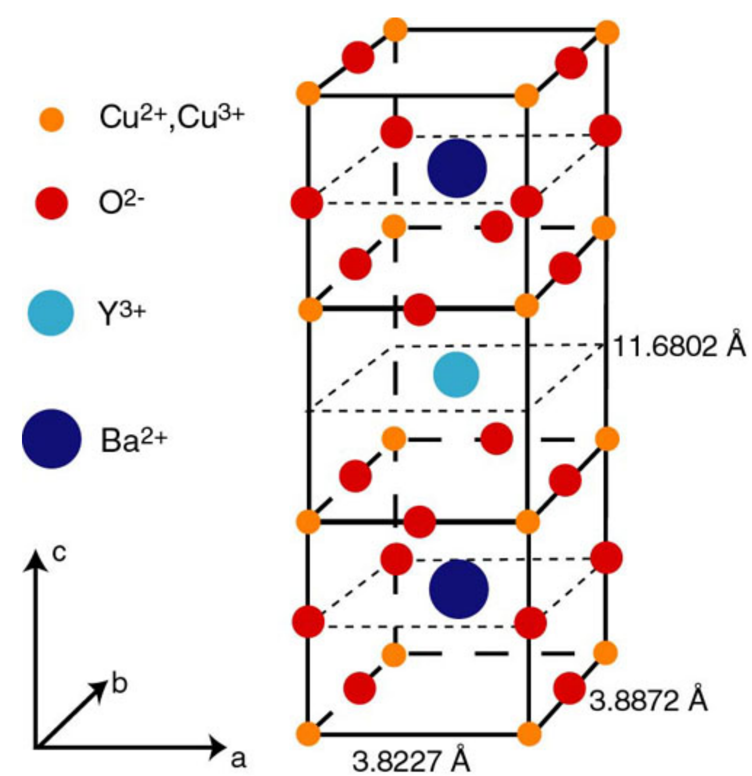
A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Resistivity of a Planckian metal as $T \to 0$

$$\rho = \frac{m^*}{ne^2}\, 2.71\mathbb{C}\, \frac{k_B T}{\hbar}$$

Note that all explicit dependence on U has cancelled out!

Choosing $\mathbb{C} = 0.41$ as in the SYK model, we have the prefactor $2.71\mathbb{C} = 1.11$.



Aavishkar Patel

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

- Resonant SYK models are compressible and dispersive quantum systems with $\hbar\omega/(k_B T)$ scaling as $T \to 0$.

- The resonance is a single 'fine-tuning' condition designed to obtain $\hbar\omega/(k_B T)$ scaling as $T \to 0$. However, then many other nice features follow: we obtain a Planckian metal with remnant large Fermi surface at $e_k = 0$, and an effective mass m^* defined by the dispersion of e_k, with a resistivity $\rho \sim (m^*/(ne^2))k_B T/\hbar$ independent of the strength of interactions.

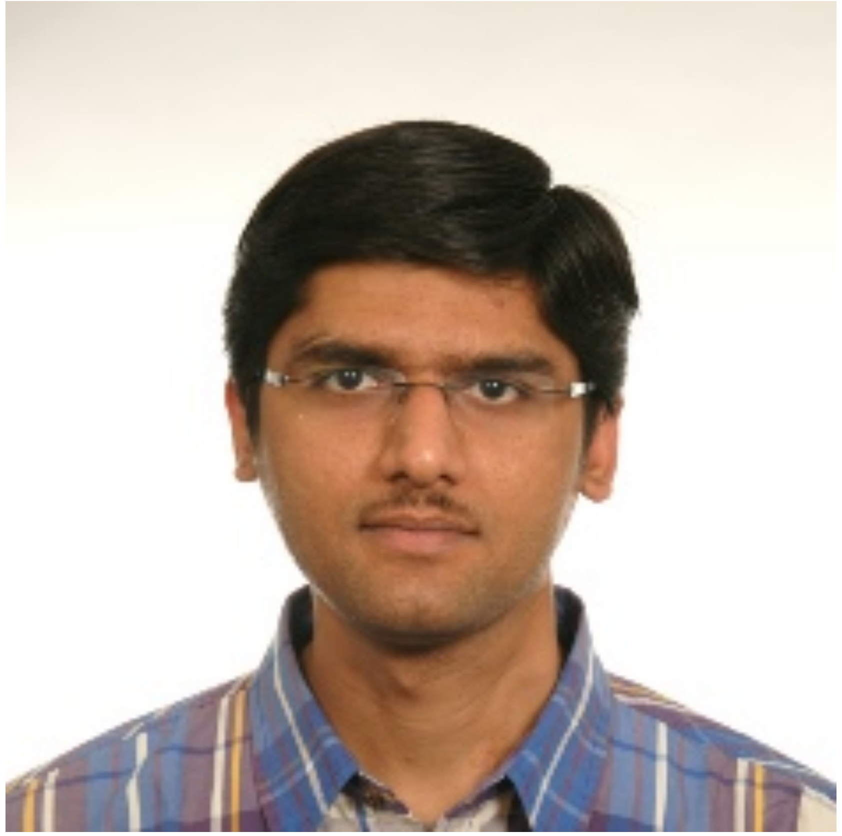
Aavishkar Patel (Harvard $\longrightarrow$ Miller Fellow at Berkeley)

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

1. SYK models

2. Resonant SYK models
and Planckian metals

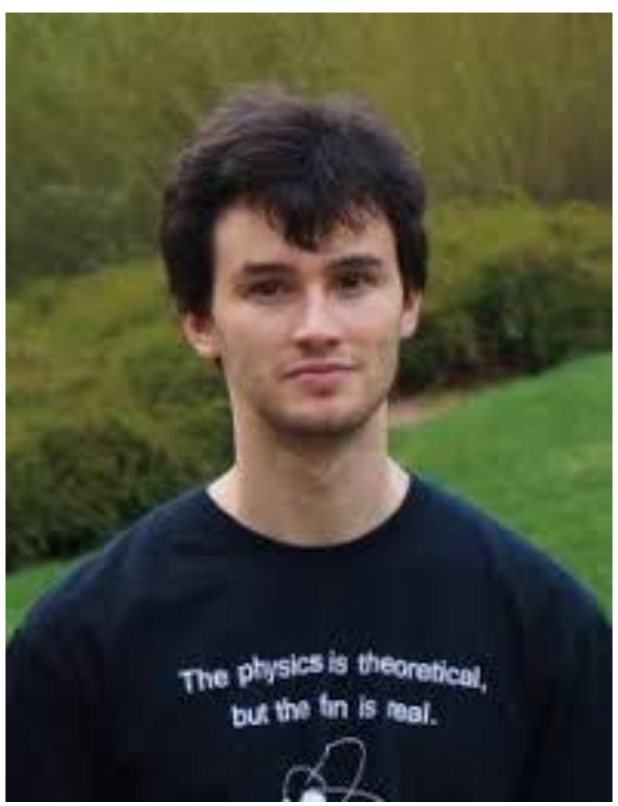
3. Deconfined quantum criticality of
random *t*-*J* models



Darshan Joshi

Chenyuan Li

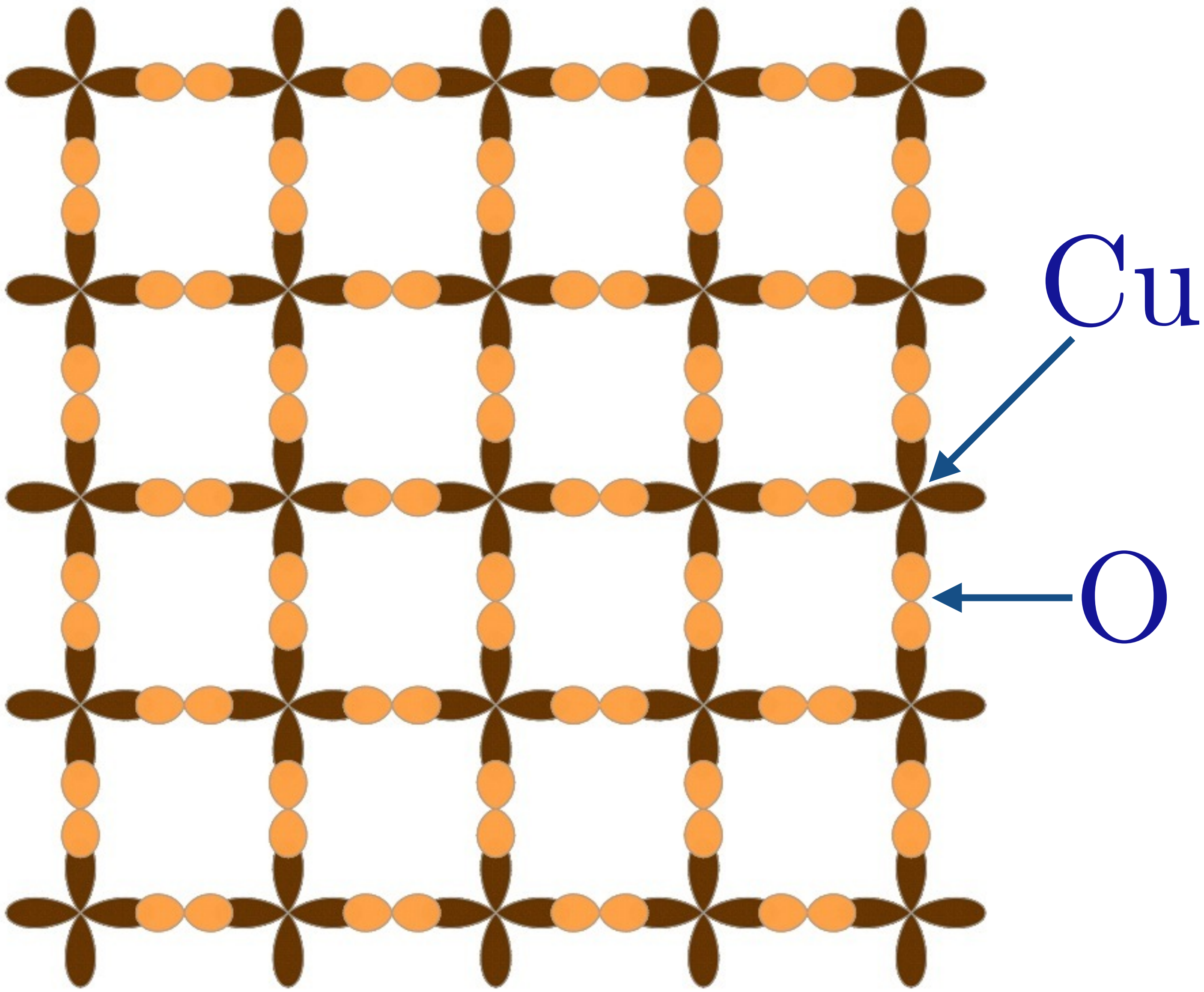
arXiv:1912.08822



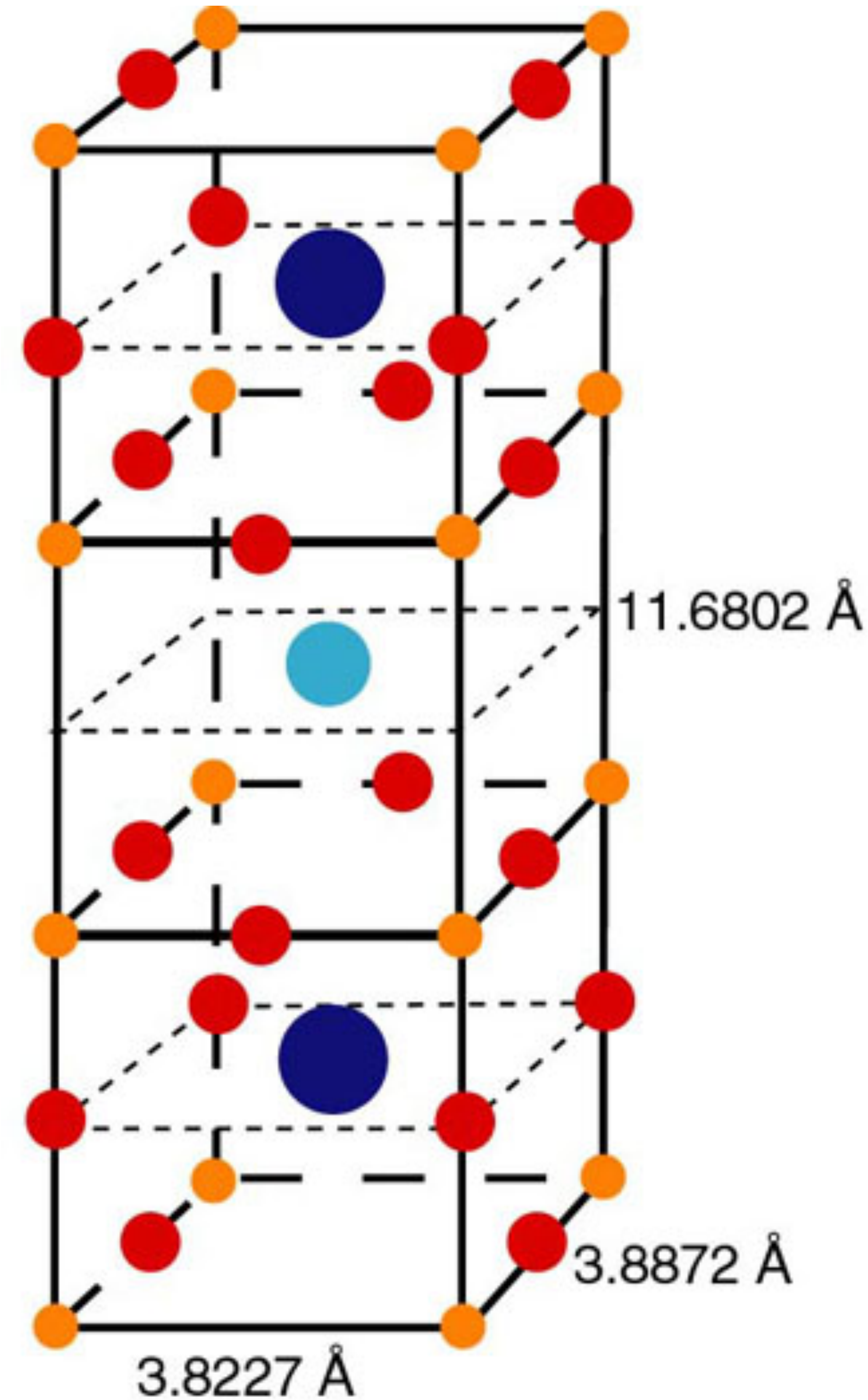
Grigory Tarnopolsky

Antoine Georges

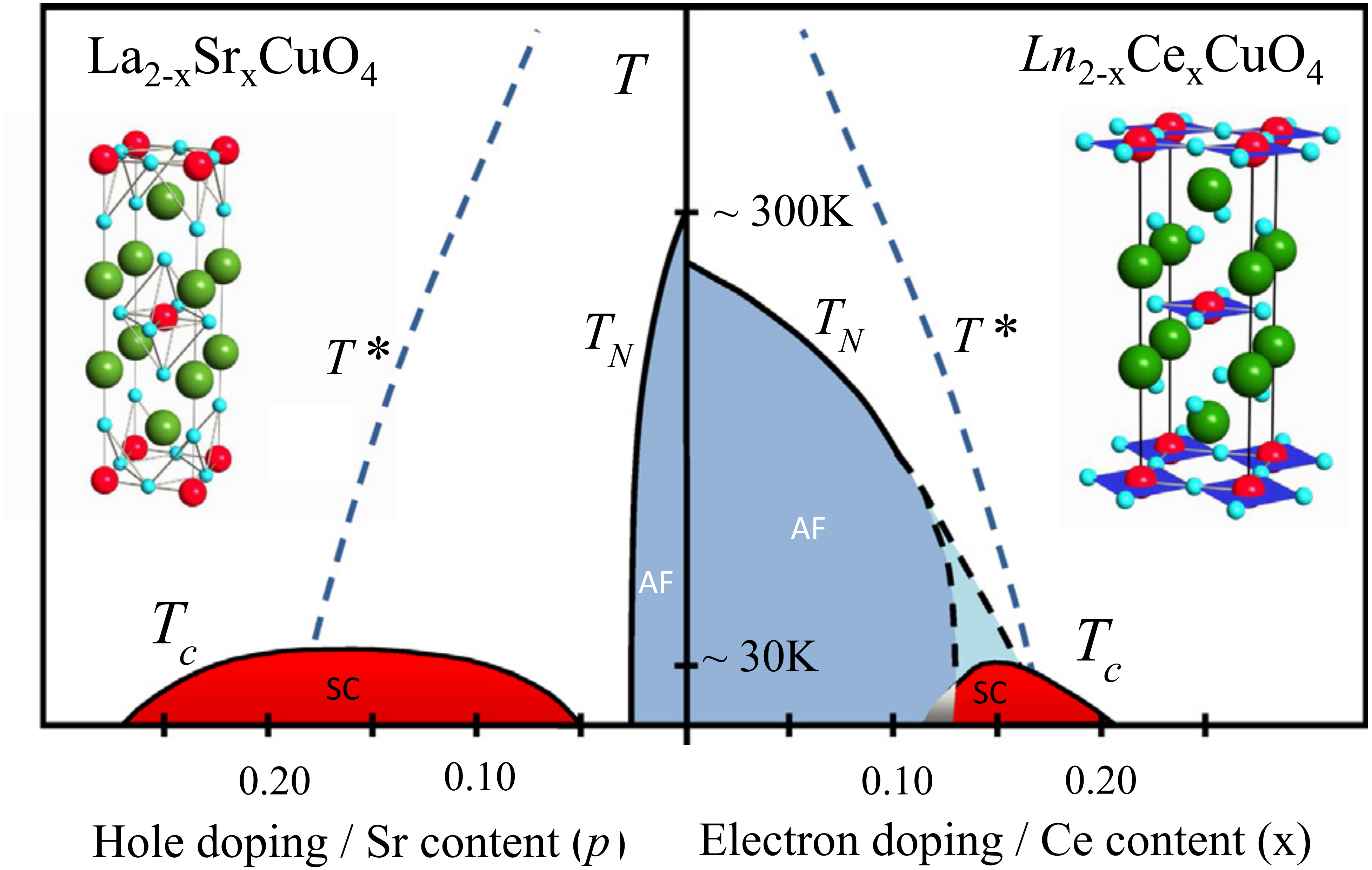
High temperature superconductors



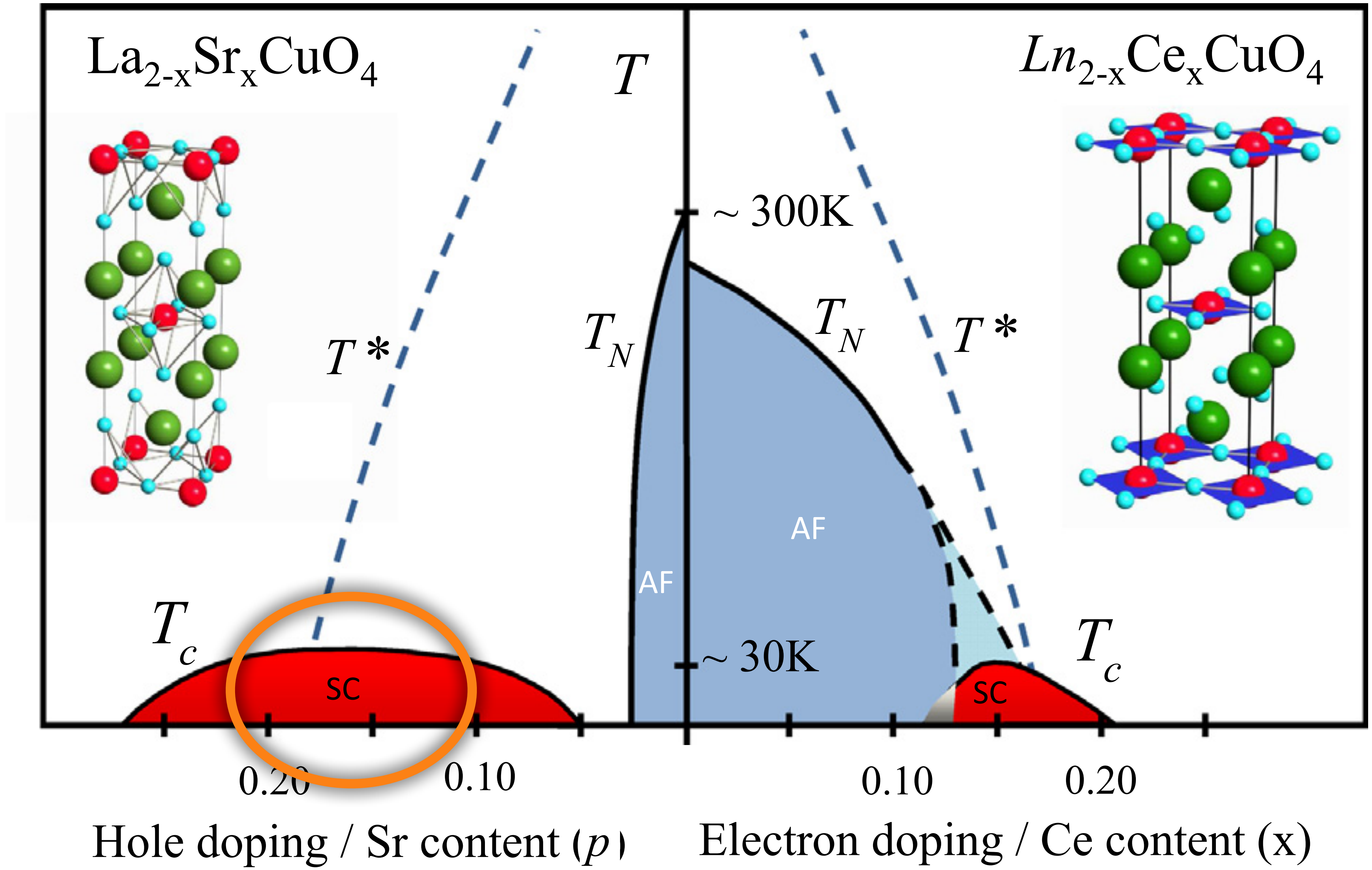
CuO_2 plane



$YBa_2Cu_3O_{6+x}$

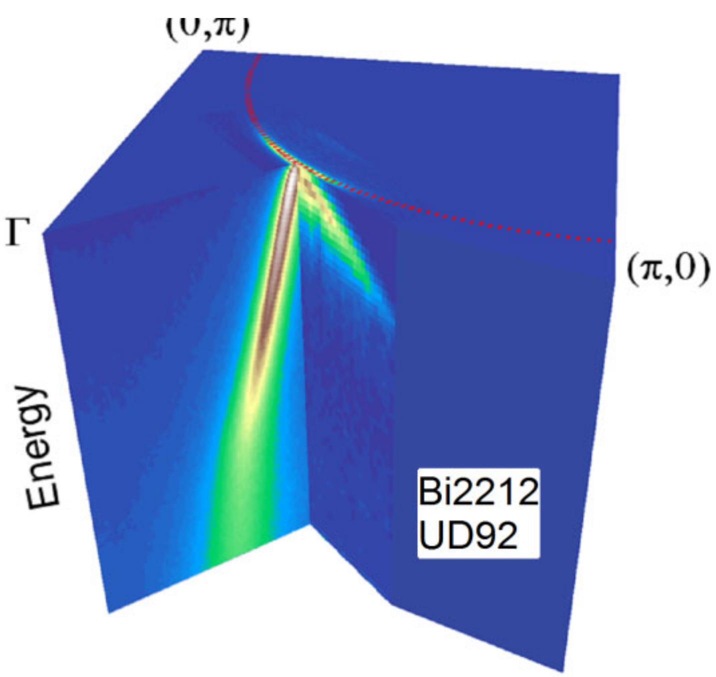


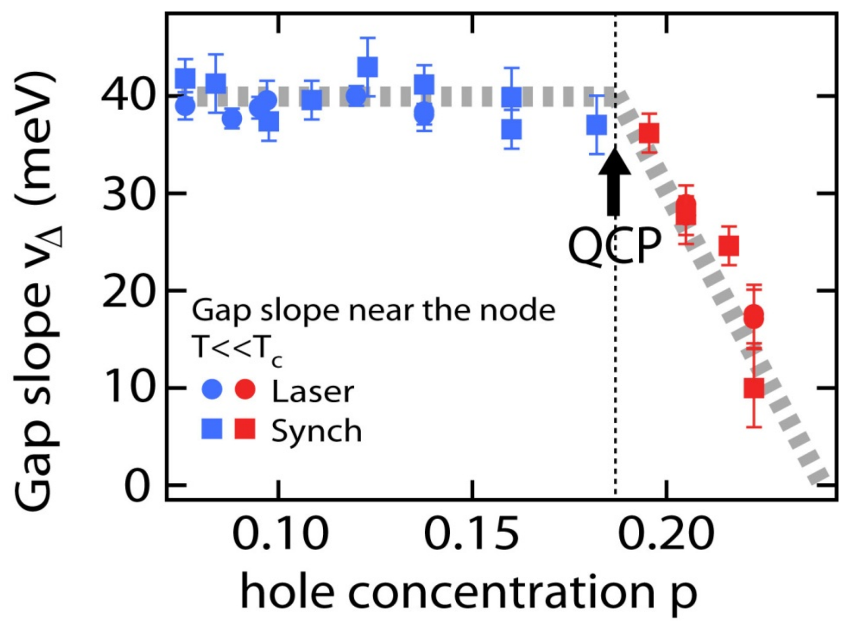
$La_{2-x}Sr_xCuO_4$
$Ln_{2-x}Ce_xCuO_4$
T
~ 300K
~ 30K
T_N
T_N
T^*
T^*
AF
AF
T_c
T_c
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)

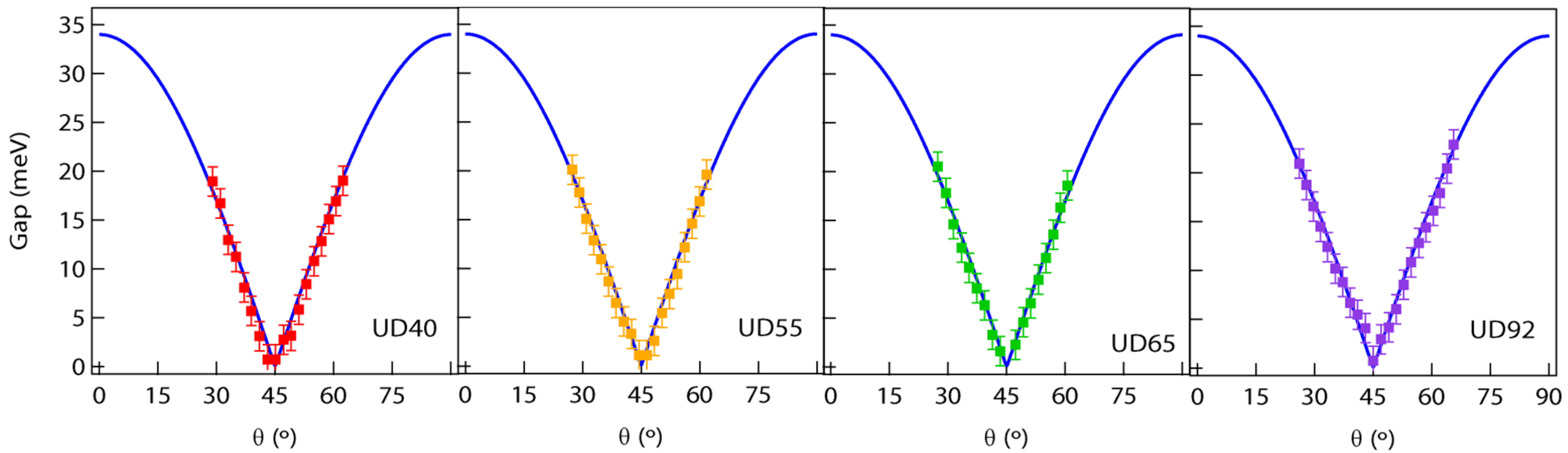


La2-xSrxCuO4
Ln2-xCexCuO4
T
~ 300K
~ 30K
T*
T*
TN
TN
AF
AF
Tc
Tc
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)

Precision Measurement of the Node





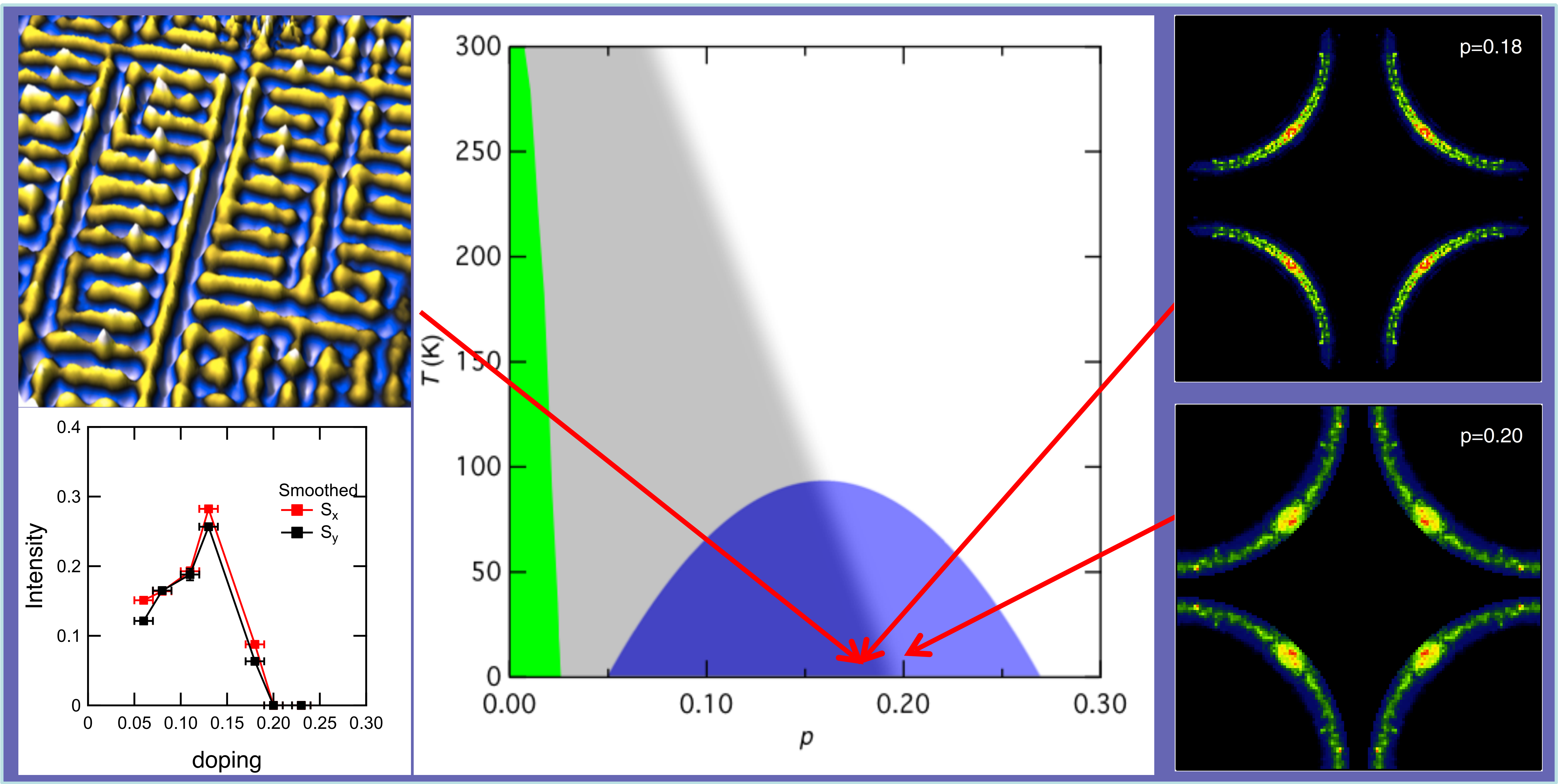


I. M. Vishik, M. Hashimoto, Rui-Hua He, Wei-Sheng Lee, Felix Schmitt, Donghui Lu, R. G. Moore, C. Zhang, W. Meevasana, T. Sasagawa, S. Uchida, Kazuhiro Fujita, S. Ishida, M. Ishikado, Yoshiyuki Yoshida, Hiroshi Eisaki, Zahid Hussain, Thomas P. Devereaux, and Zhi-Xun Shen, PNAS **109**, 18332 (2012)

Hole doped cuprates

Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, Science **344**, 608 (2014)

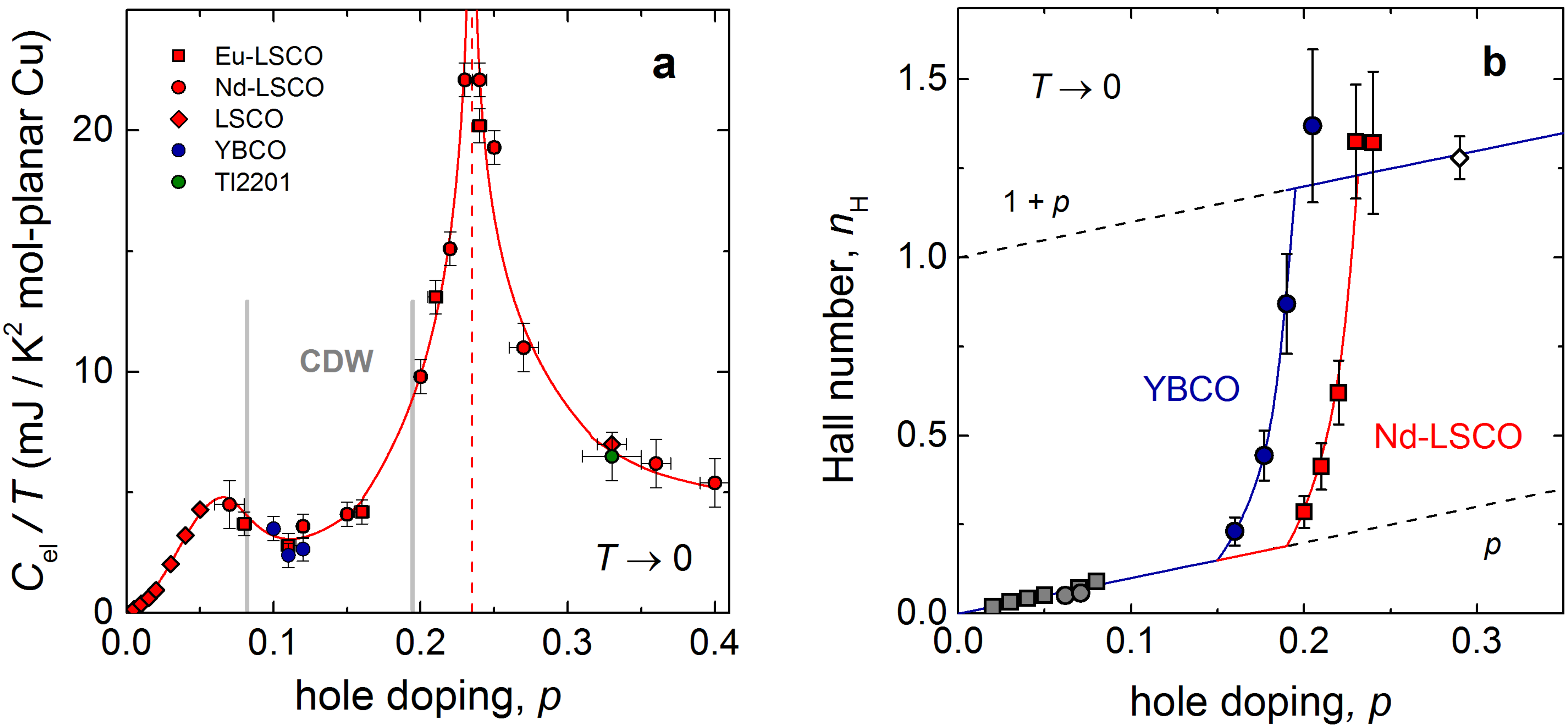
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, Science **344**, 612 (2014)



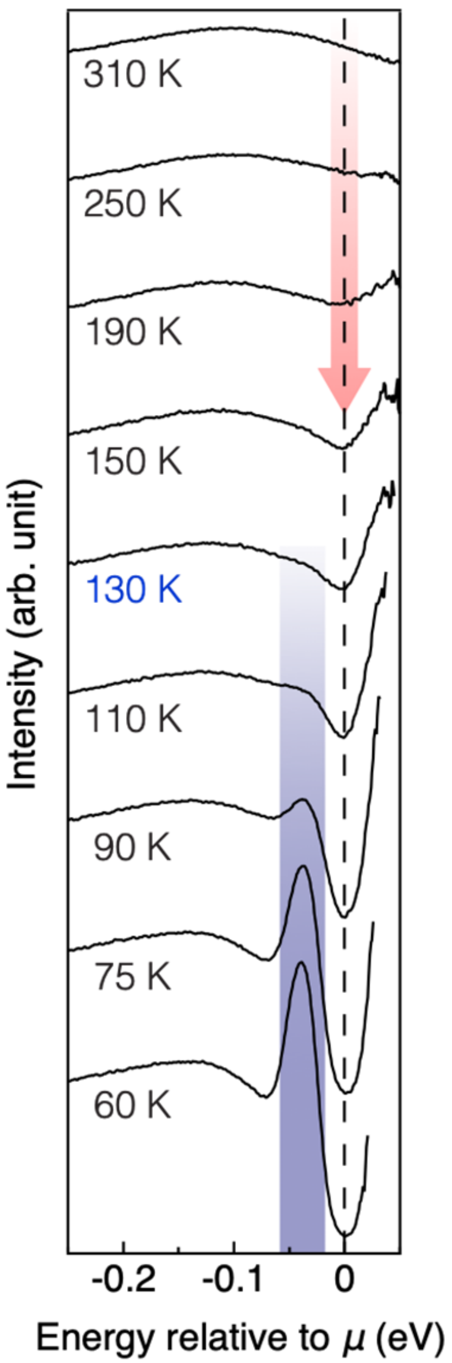
Hole doped cuprates

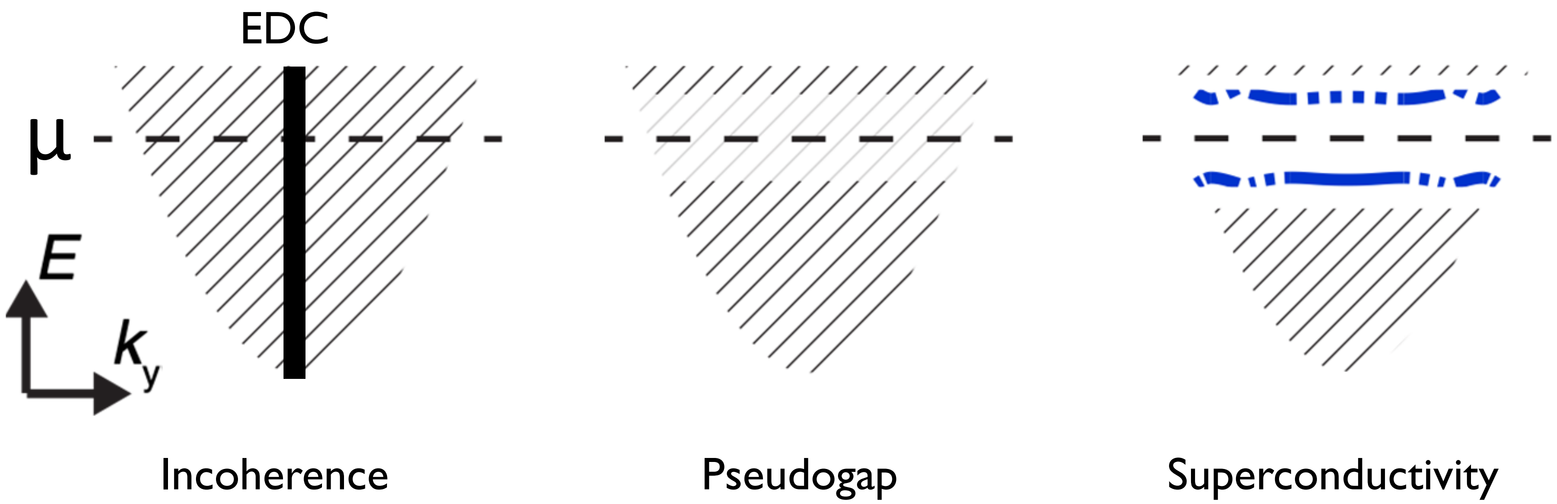
The remarkable underlying ground states of cuprate superconductors

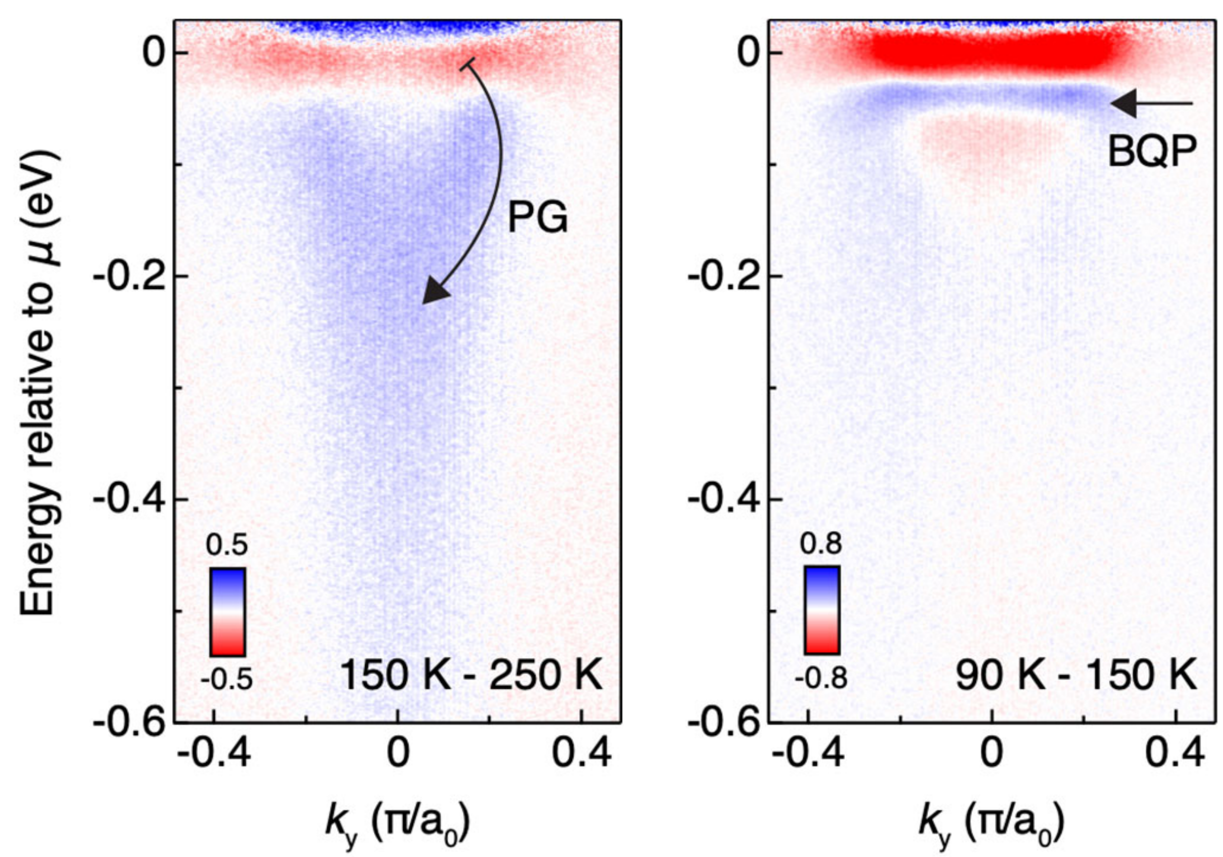
Cyril Proust and Louis Taillefer, arXiv:1807.0507



Two “gaps” for p < 0.19 (T_c~ 86 K)

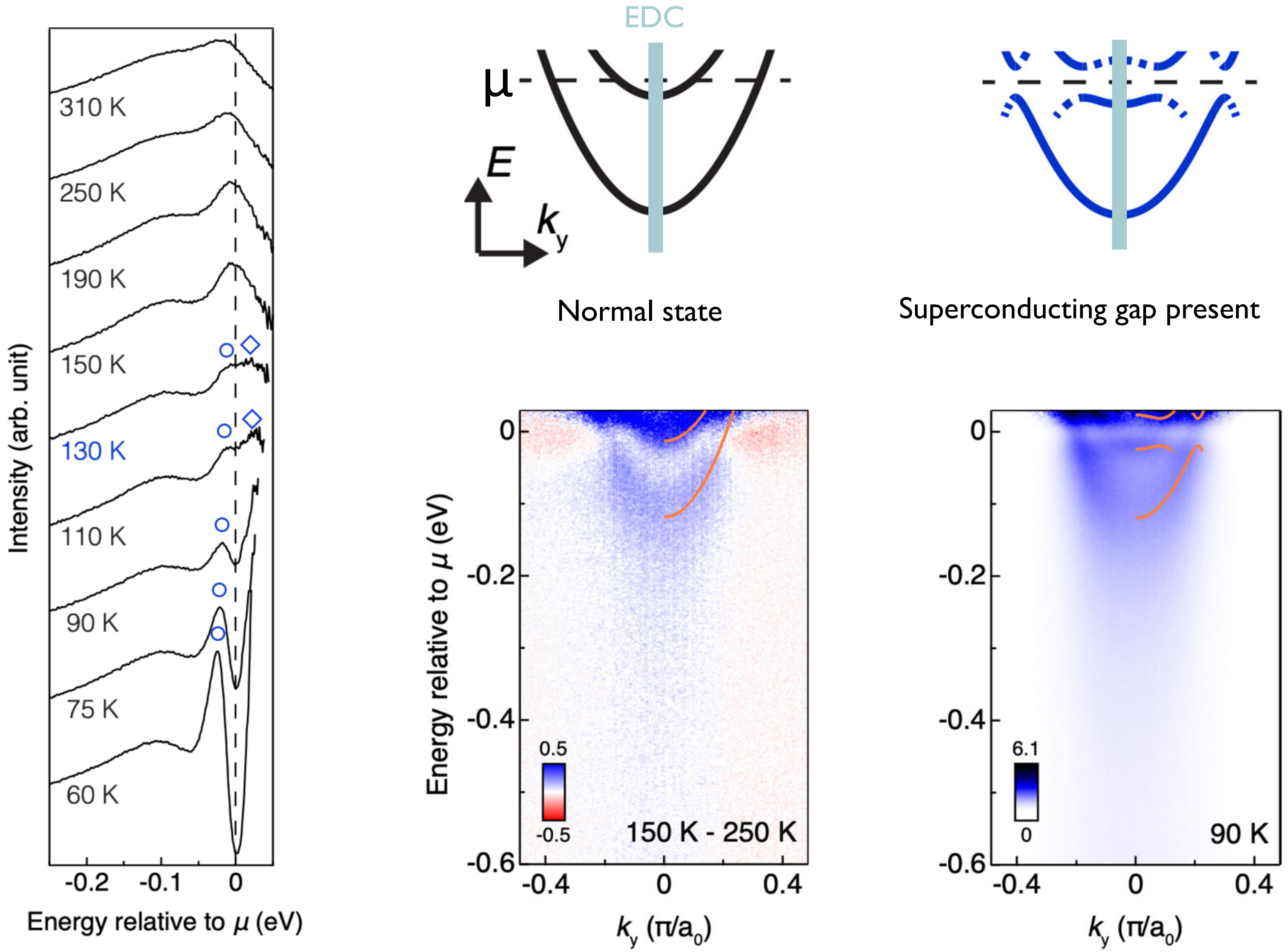






Su-Di Chen, Makoto Hashimoto, Yu He, Dongjoon Song, Ke-Jun Xu, Jun-Feng He, T. P. Devereaux, Hiroshi Eisaki, Dong-Hui Lu, J. Zaanen, Zhi-Xun Shen, *Science* **366**, 6469 (2019)

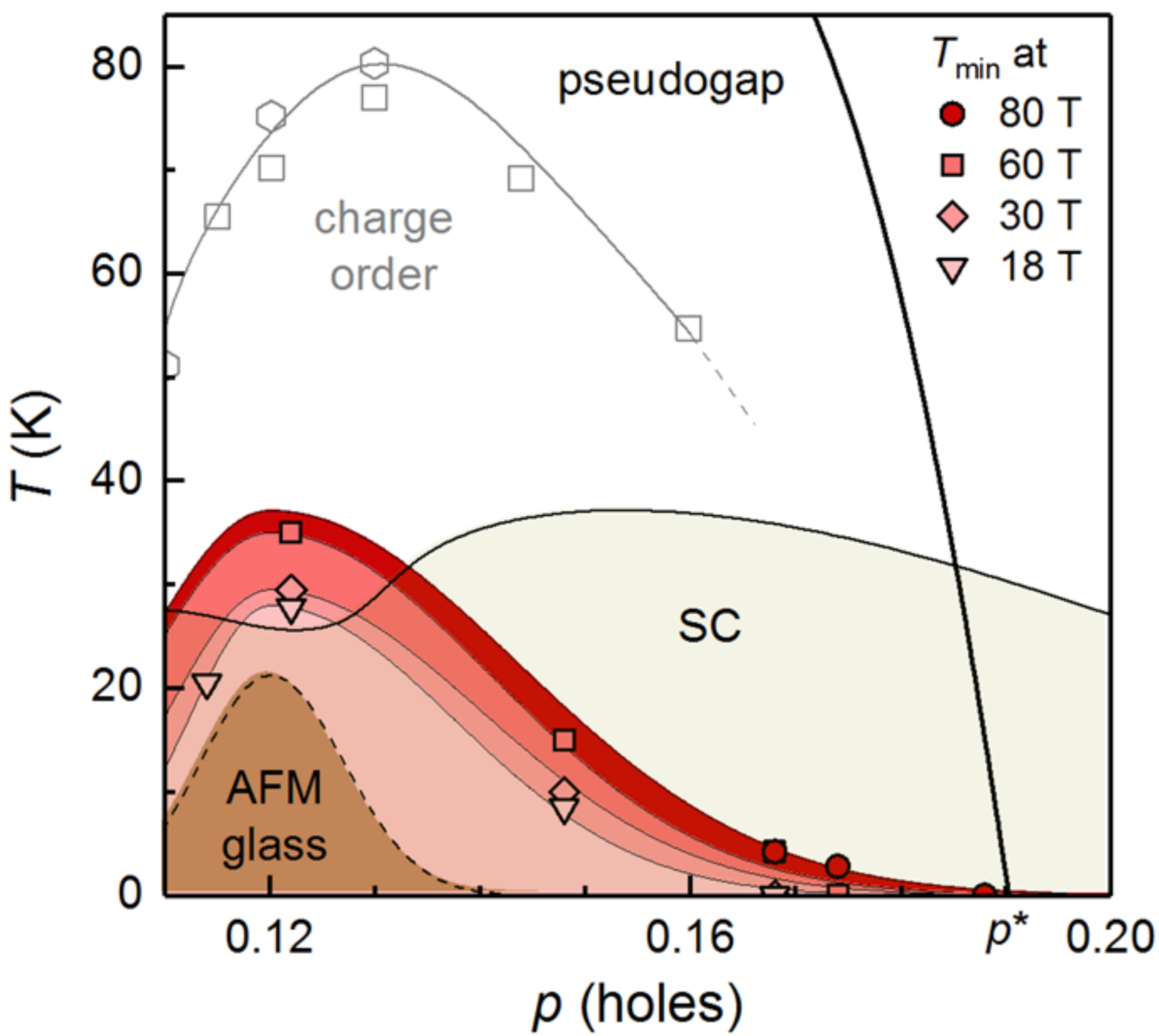
One gap for p > 0.19 (T_c ~ 81 K)



Su-Di Chen, Makoto Hashimoto, Yu He, Dongjoon Song, Ke-Jun Xu, Jun-Feng He, T. P. Devereaux, Hiroshi Eisaki, Dong-Hui Lu, J. Zaanen, Zhi-Xun Shen, *Science* **366**, 6469 (2019)

Hidden magnetism at the pseudogap critical point of a high temperature superconductor

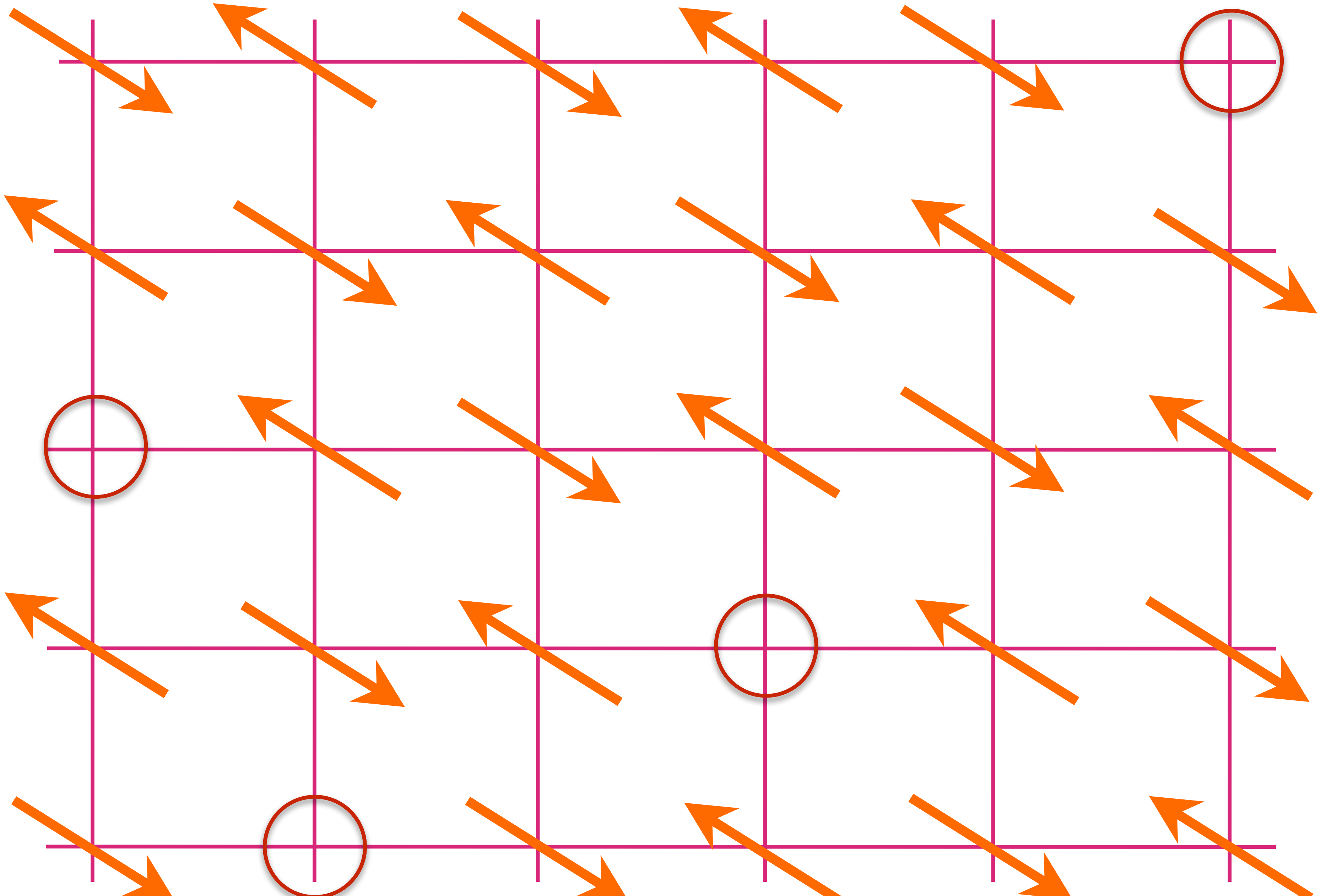
Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1*], Marc-Henri Julien[1*]



arXiv:1909.10258

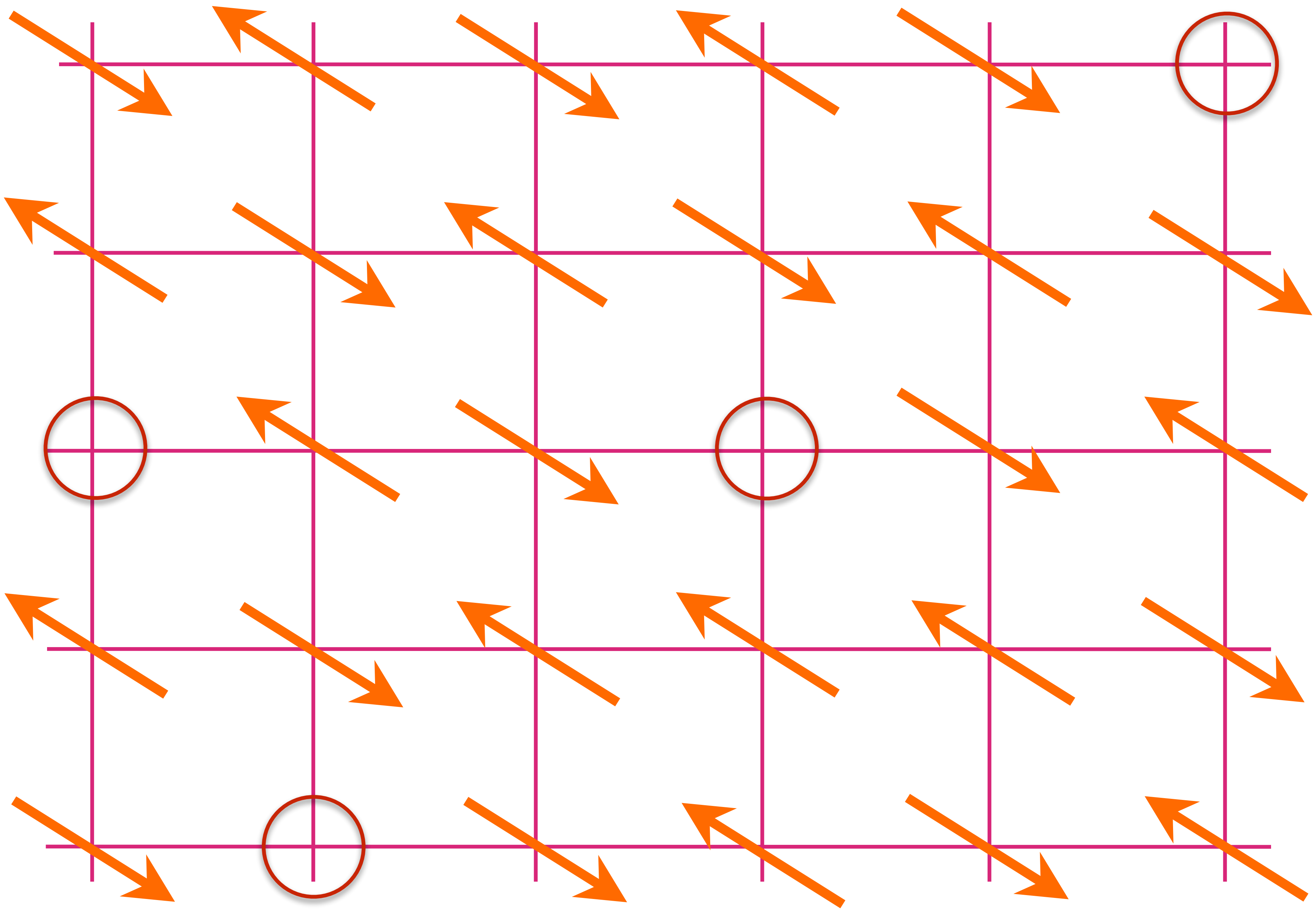
Quasi-static magnetism in the pseudogap state of $La_{2-x}Sr_xCuO_4$. Temperature – doping phase diagram representing T_{min}, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of T_{min} in zero-field, the dashed line (brown area) represents the extrapolated $T_{min}(B=0)$. While not exactly equal to the freezing temperature T_f (see Fig. 2), T_{min} is closely tied to T_f and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).

Real-space view at small p



p mobile holes in a background of fluctuating spins

Real-space view at small p



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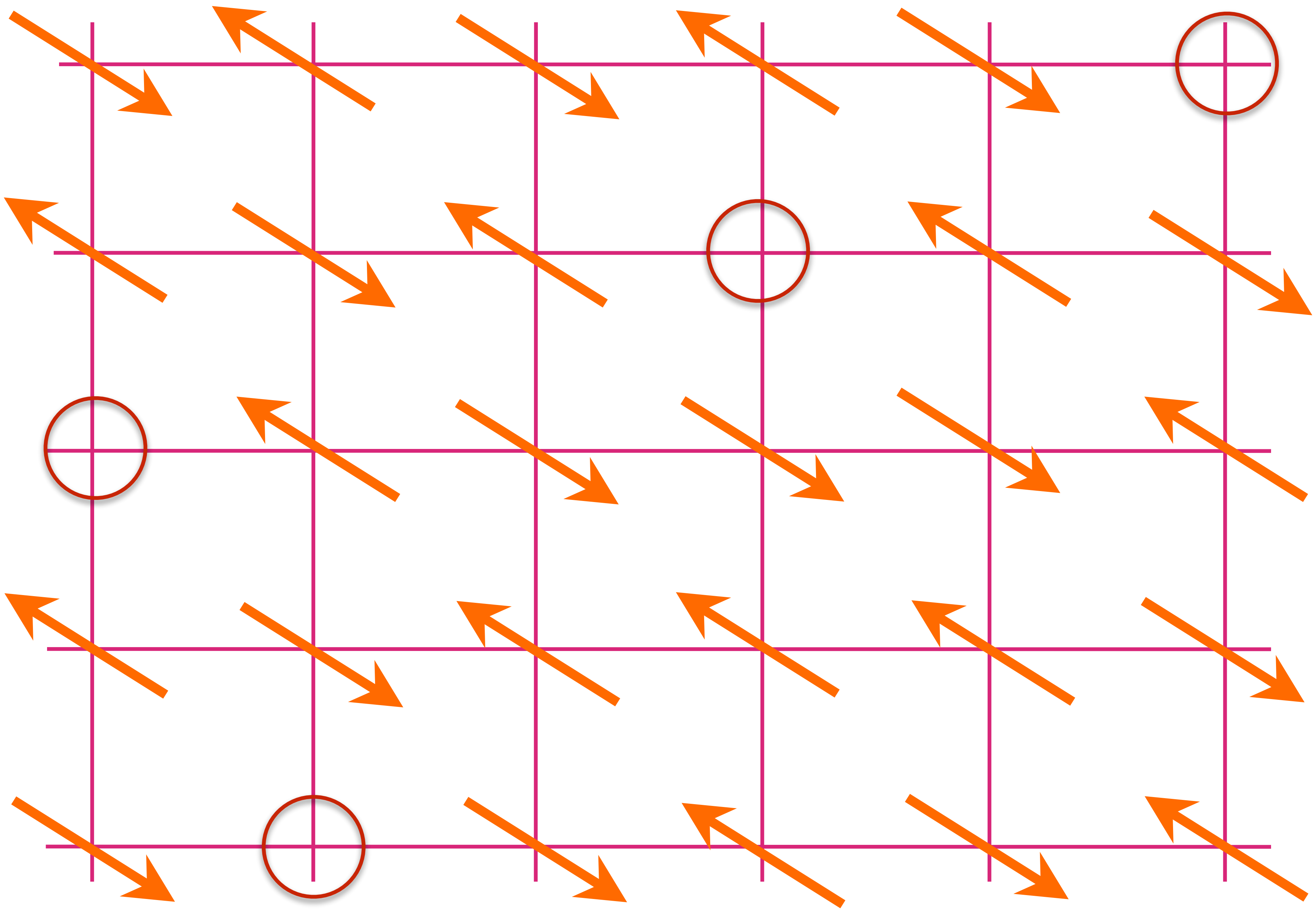
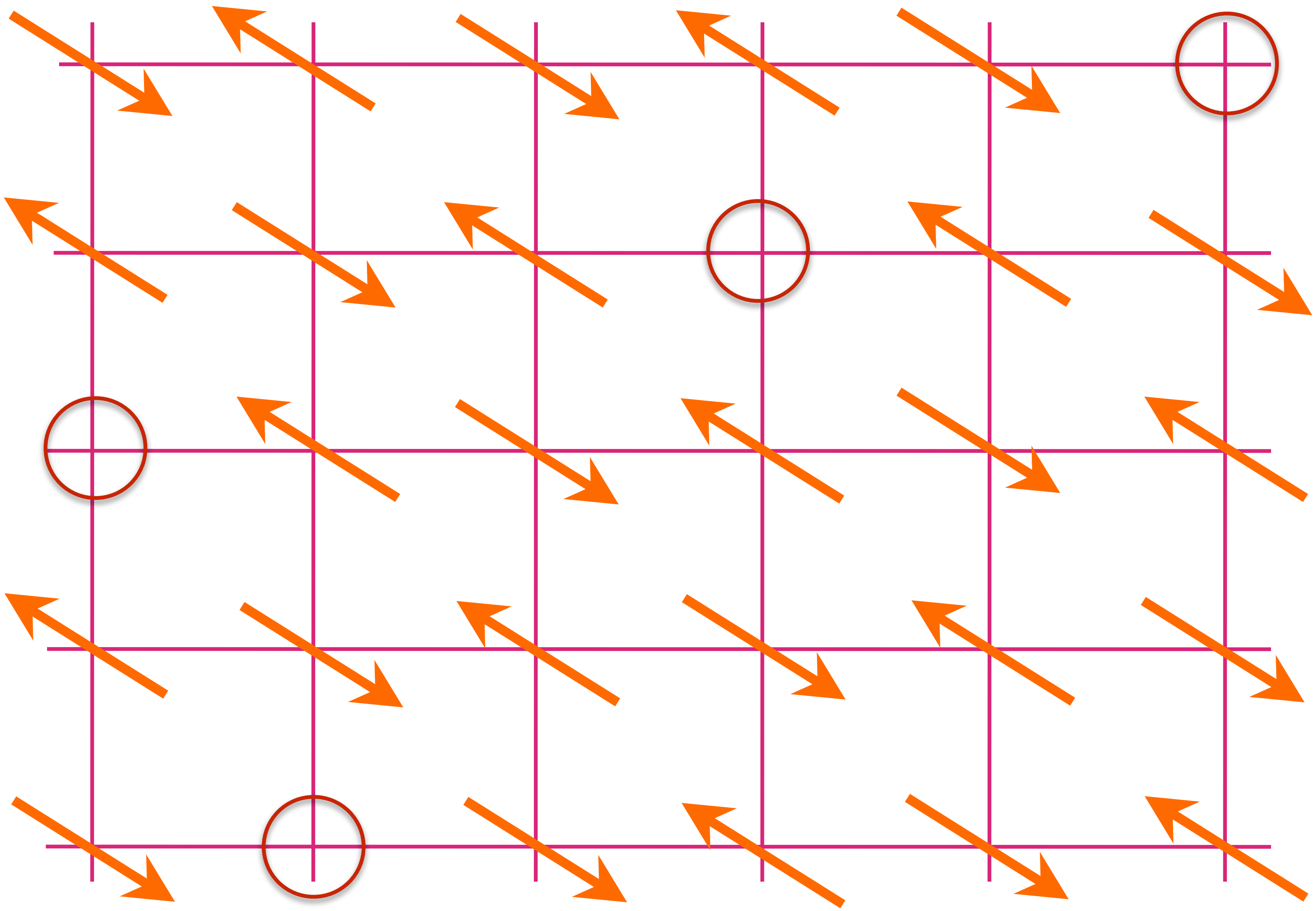
Real-space view at small p



p mobile holes in a background of fluctuating spins

Real-space view at small p



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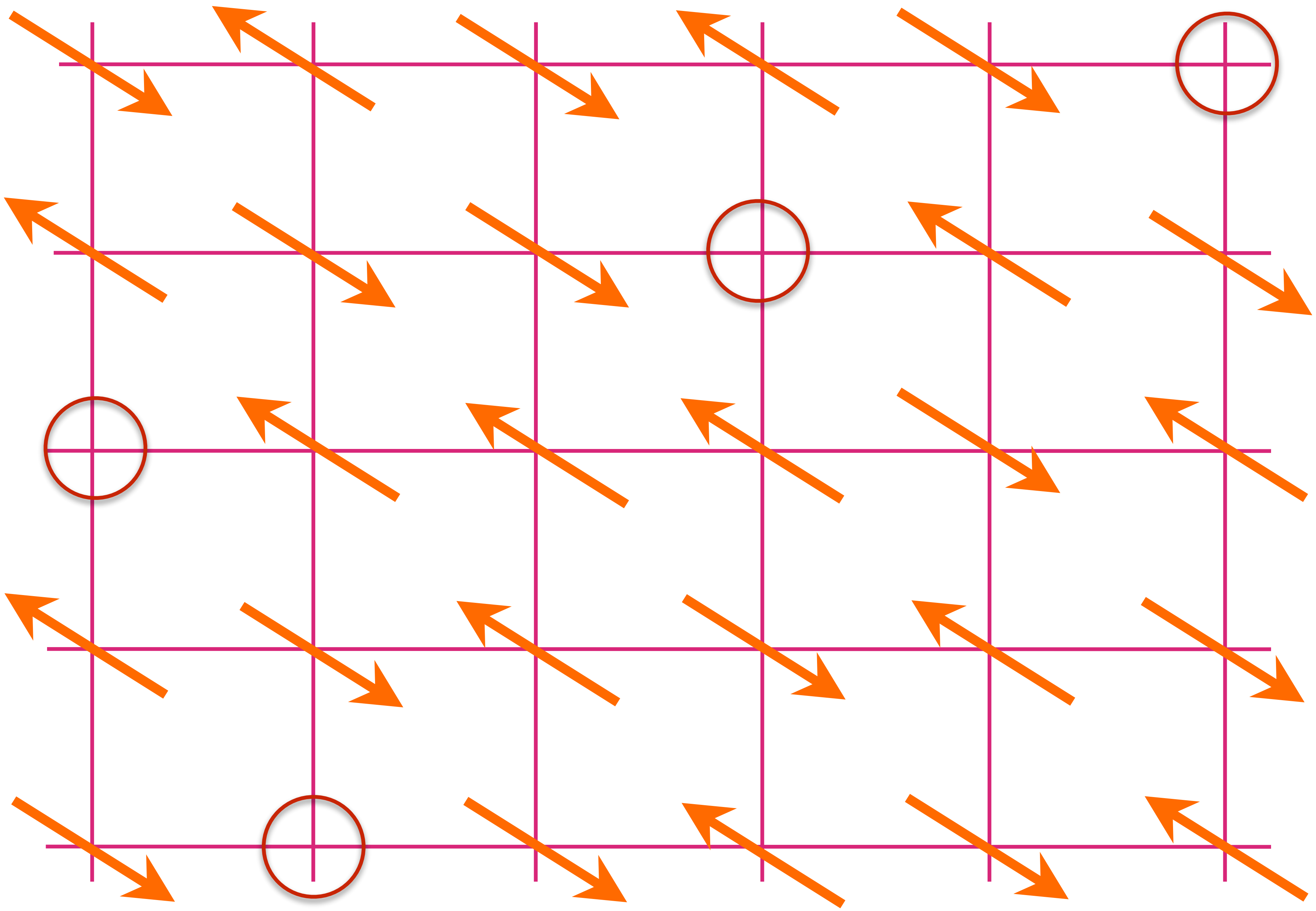
Real-space view at small p



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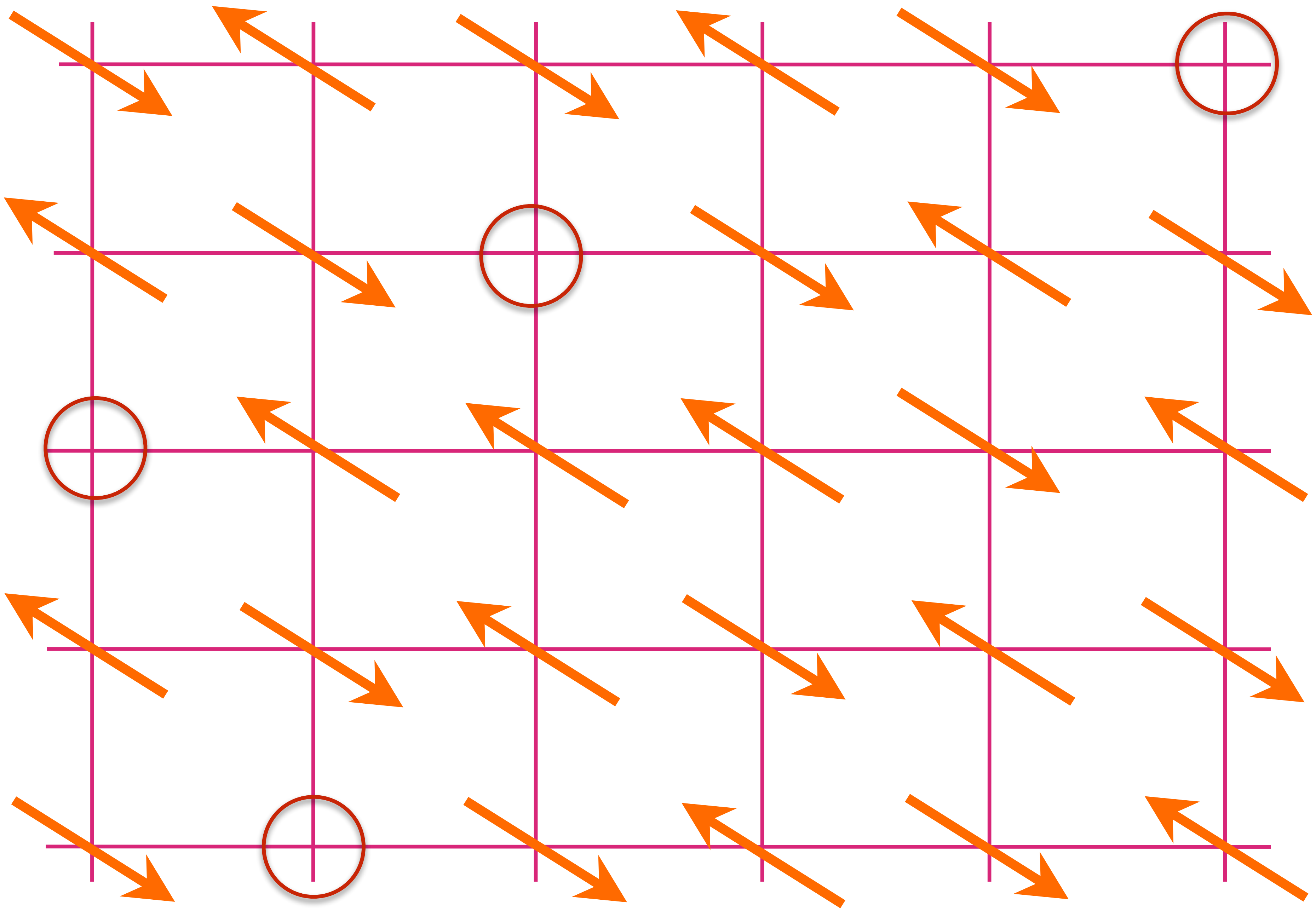
Real-space view at small p



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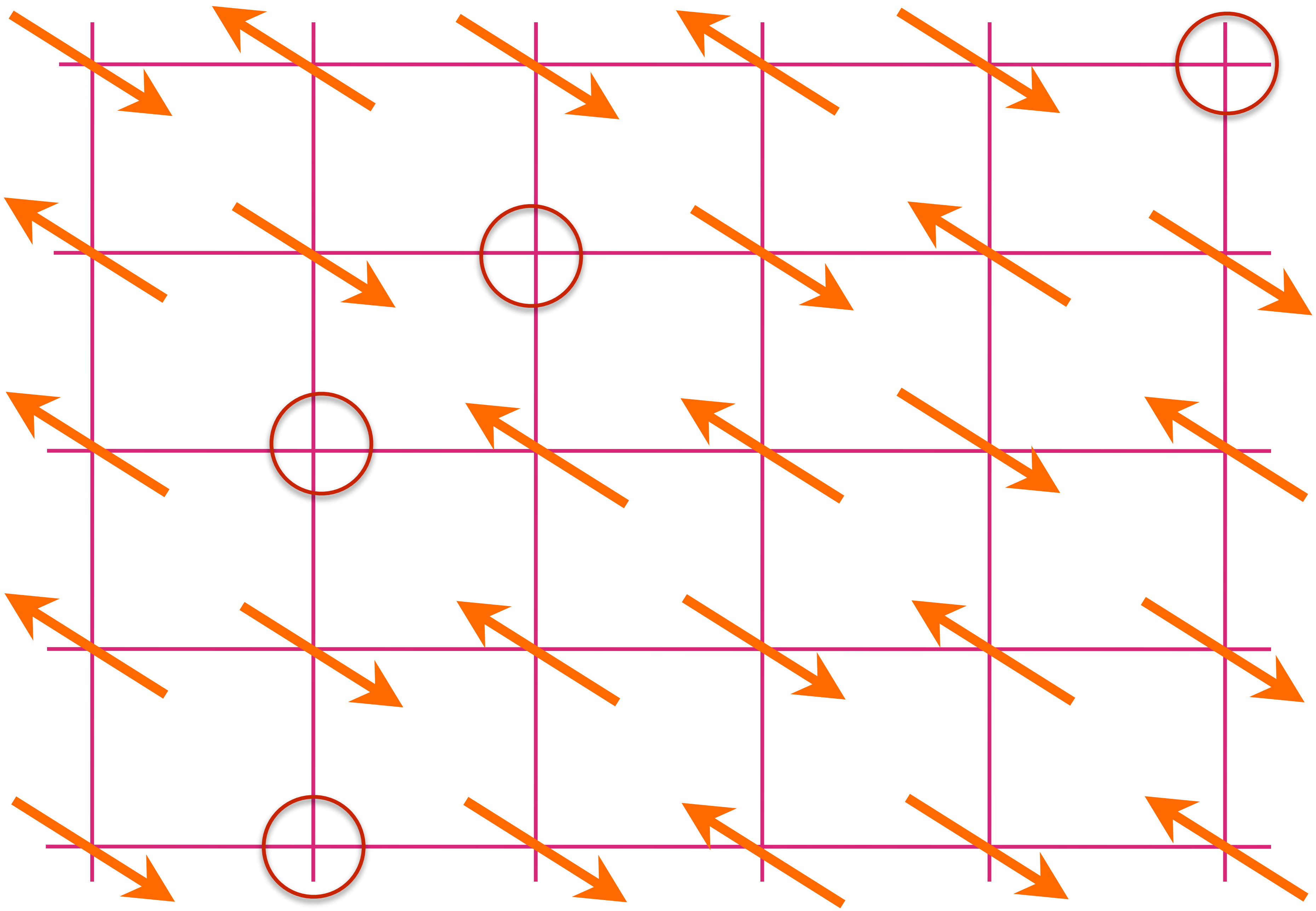
Real-space view at small p



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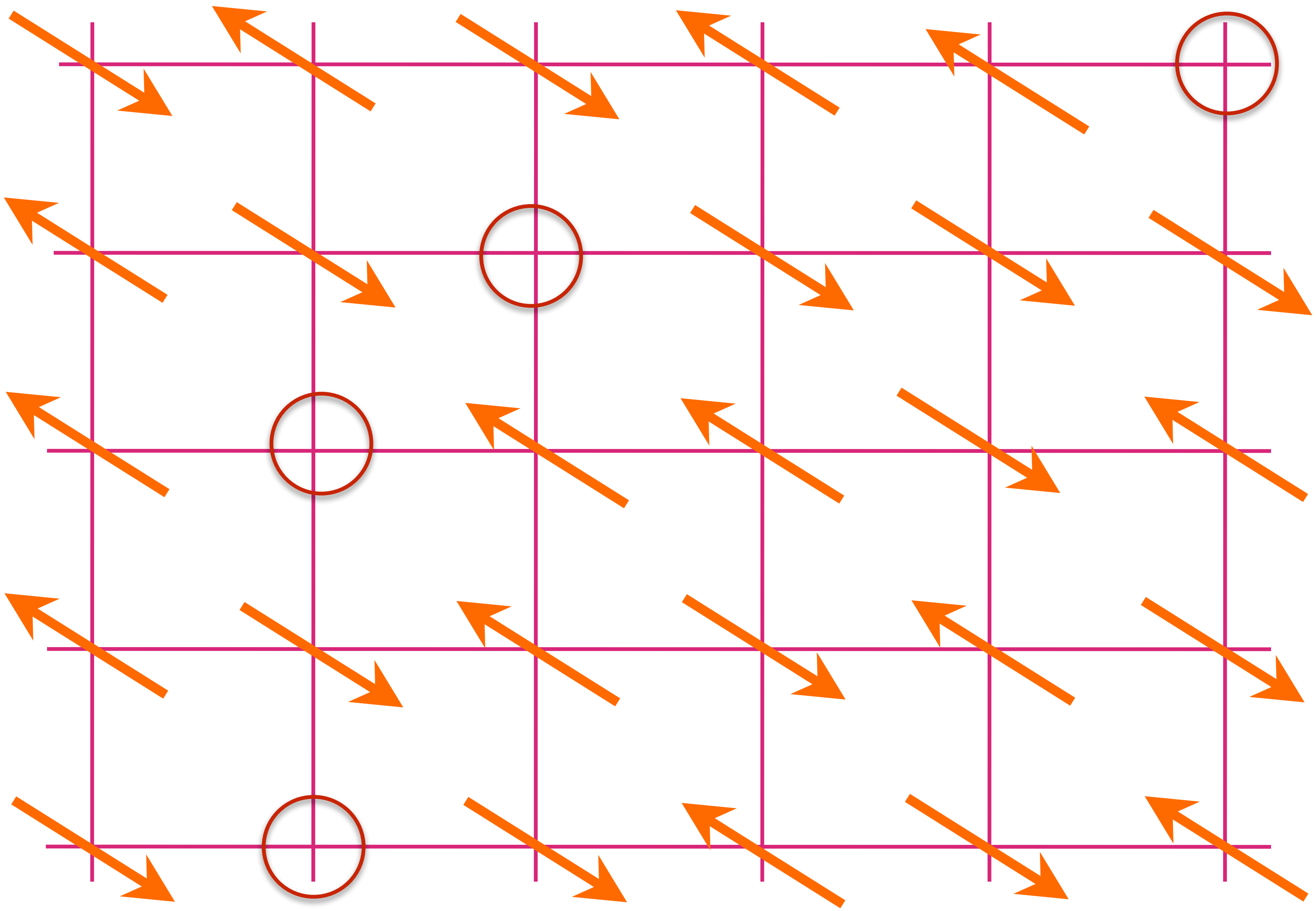
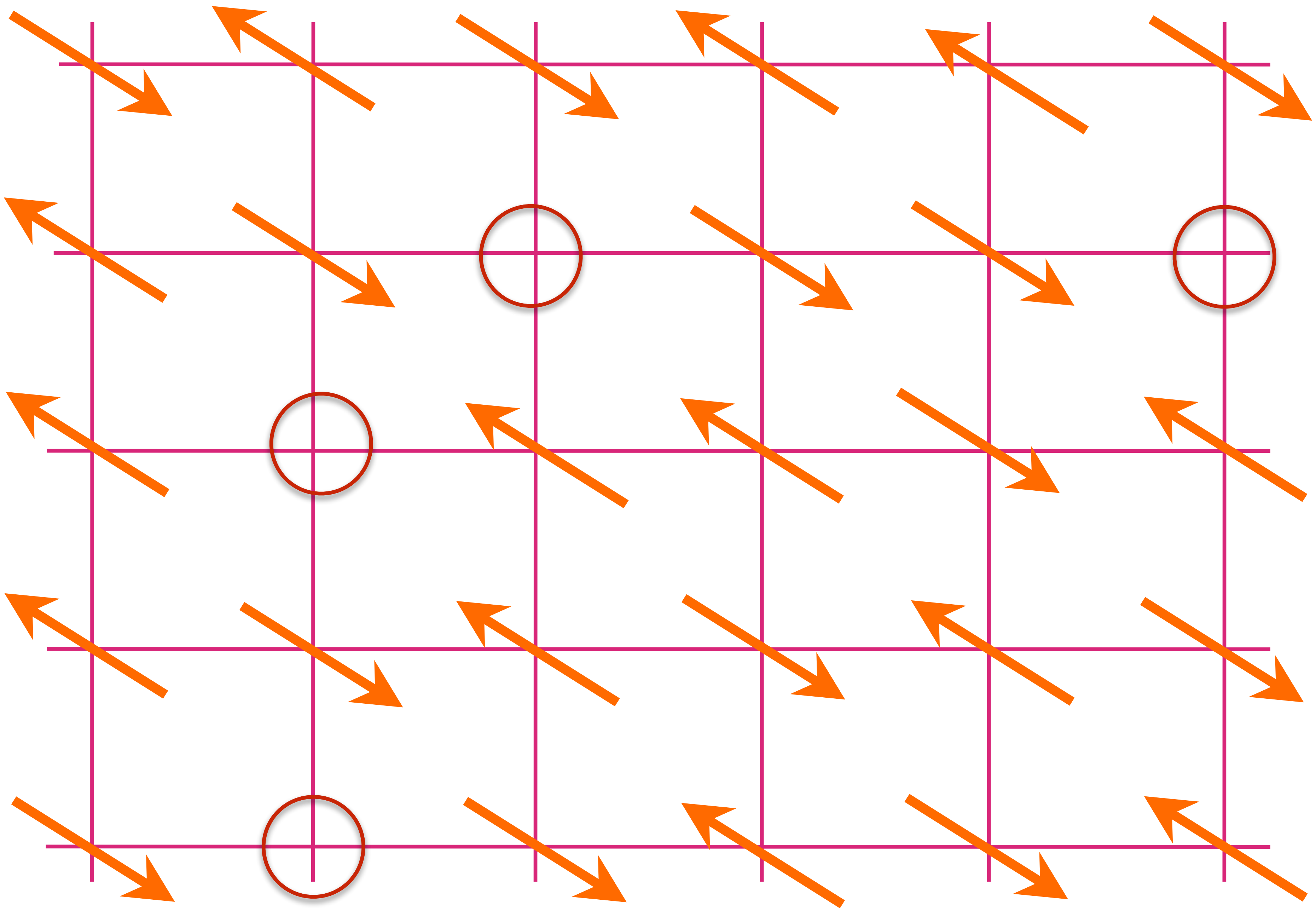
Real-space view at small p



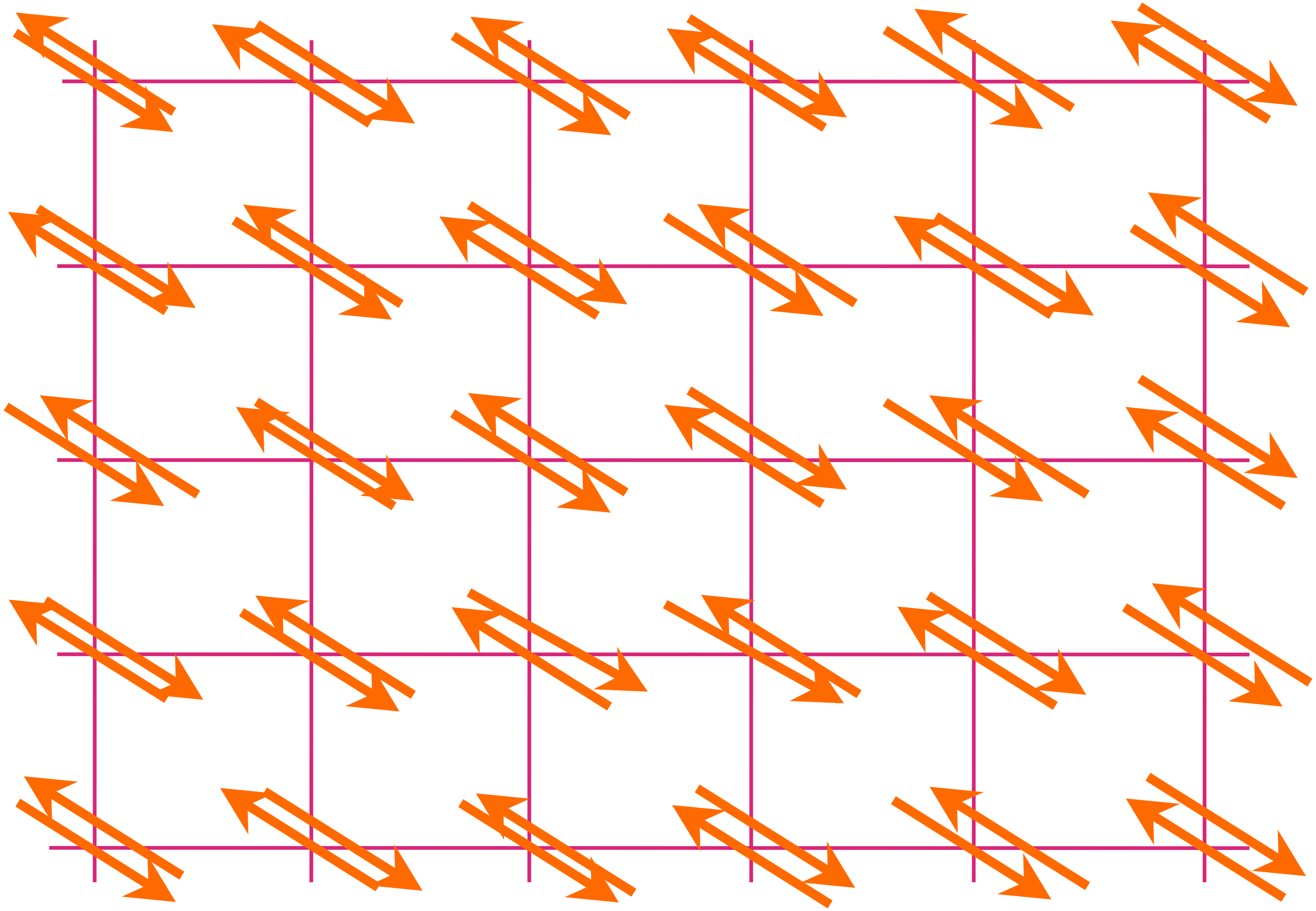
p mobile holes in a background of fluctuating spins

Real-space view at small p



p mobile holes in a background of fluctuating spins

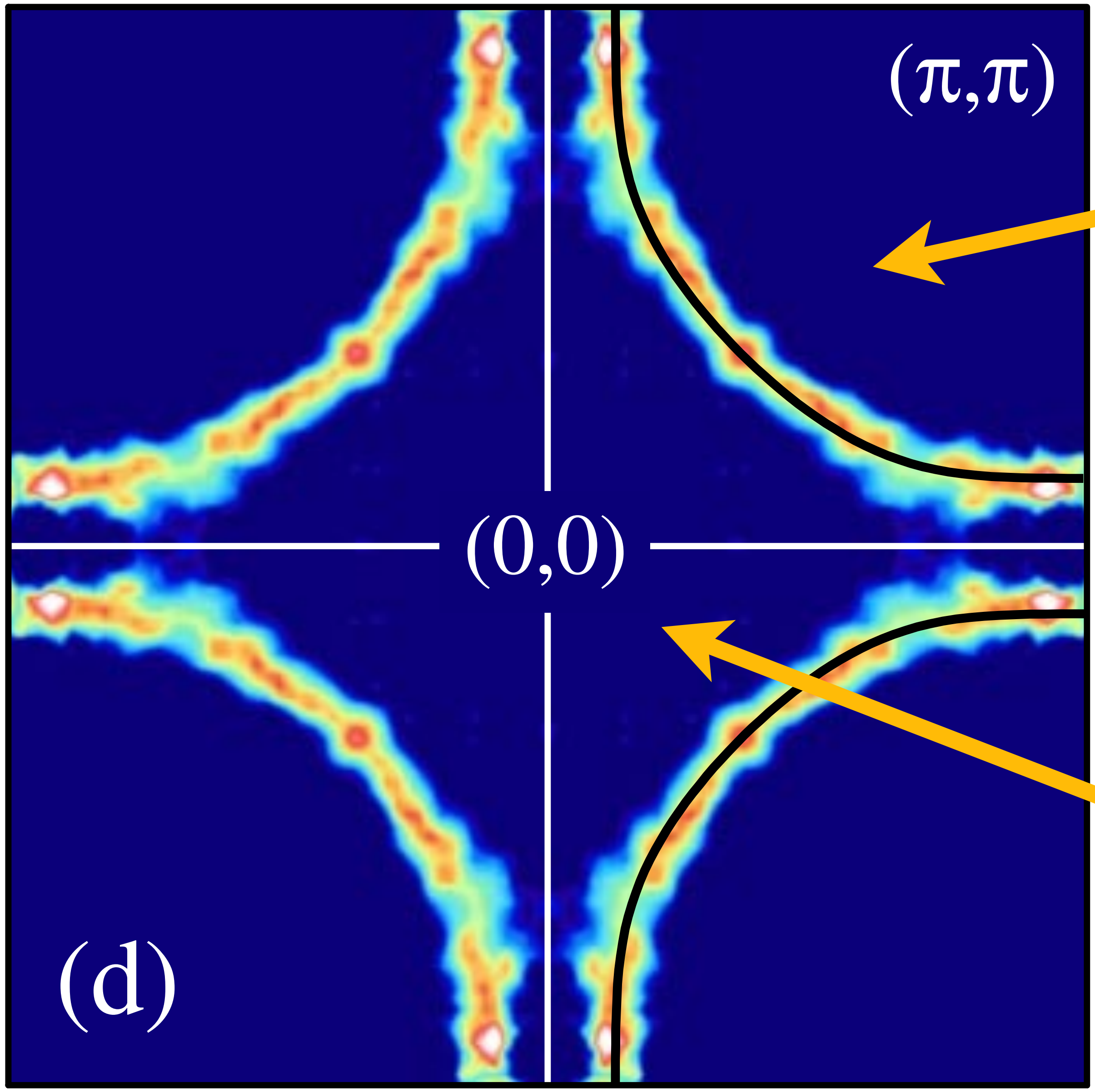
Momentum-space view at large p



Filled Band

$1+p$ mobile holes in a filled band

Momentum-space view at large p



$1+p$ holes

$1-p$ electrons

$1+p$ mobile holes in a filled band

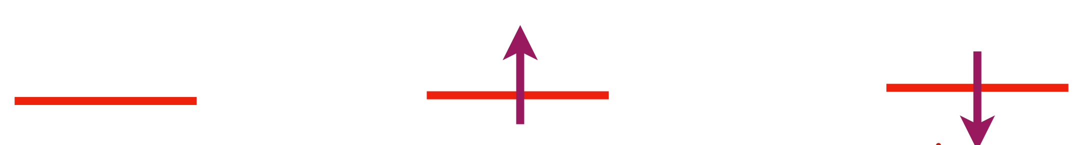
t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1$$

$$J_{ij} \text{ random}, \overline{J_{ij}} = 0, \overline{J_{ij}^2} = J^2$$
$$t_{ij} \text{ random}, \overline{t_{ij}} = 0, \overline{t_{ij}^2} = t^2$$



$|0\rangle$ $\qquad c_{\uparrow}^{\dagger} |0\rangle$ $\qquad c_{\downarrow}^{\dagger} |0\rangle$

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):

$$\begin{aligned} |0\rangle \Rightarrow b^\dagger |v\rangle \quad &, \quad c_\alpha^\dagger |0\rangle \Rightarrow f_\alpha^\dagger |v\rangle \\ c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\ f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \\ \text{U(1) gauge invariance,} \quad & b \to b e^{i\phi}, \quad f_\alpha \to f_\alpha e^{i\phi} \end{aligned}$$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):

$$|0\rangle \Rightarrow \mathfrak{f}^{\dagger}\,|v\rangle \qquad , \qquad c_{\alpha}^{\dagger}\,|0\rangle \Rightarrow \mathfrak{b}_{\alpha}^{\dagger}\,|v\rangle$$

$$c_\alpha = \mathfrak{b}_\alpha \mathfrak{f}^{\dagger}$$

$$\vec{S} = \frac{1}{2}\mathfrak{b}_{\alpha}^{\dagger}\sigma_{\alpha\beta}\mathfrak{b}_{\beta}$$

$$\mathfrak{b}_{\alpha}^{\dagger}\mathfrak{b}_{\alpha} + \mathfrak{f}^{\dagger}\mathfrak{f} = 1$$

$$\text{U(1) gauge invariance,} \qquad \mathfrak{f} \to \mathfrak{f}e^{i\phi}\,, \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi}$$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):

$$|0\rangle \Rightarrow \mathfrak{f}^{\dagger}\,|v\rangle \quad , \quad c_\alpha^{\dagger}\,|0\rangle \Rightarrow \mathfrak{b}_\alpha^{\dagger}\,|v\rangle$$

$$c_\alpha = \mathfrak{b}_\alpha \mathfrak{f}^{\dagger}$$

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$$\mathfrak{b}_\alpha^{\dagger}\mathfrak{b}_\alpha + \mathfrak{f}^{\dagger}\mathfrak{f} = 1$$

U(1) gauge invariance, $\mathfrak{f} \rightarrow \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \rightarrow \mathfrak{b}_\alpha e^{i\phi}$

$\mathrm{SU}(1|2) \equiv \mathrm{SU}(2|1)$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} \, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

Differences from SYK models:

- The interaction term J_{ij} has 2-index randomness, in contrast to the 4-index randomness in the SYK models

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Differences from SYK models:

- The interaction term J_{ij} has 2-index randomness, in contrast to the 4-index randomness in the SYK models
- There is a *local* constraint. Consequently *both* the t_{ij} and J_{ij} are 4-particle terms, when expressed in terms of SU(1|2) particles f_α, b, or the SU(2|1) particles $\mathfrak{b}_\alpha$, $\mathfrak{f}$.

t-J model

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Differences from SYK models:

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- There is competition between the hopping and exchange terms, controlled by the values J/t and p, leading to a deconfined quantum critical point (DQCP) separating two distinct phases with Fermi-liquid-like behavior as $T \to 0$.

t-J model

$$\mathcal{Z} = \int \mathcal{D}\mathcal{P}(\tau) e^{-\mathcal{S}_B - \mathcal{S}_{tJ}}$$

$$\mathcal{S}_B = i \int_0^1 du \int d\tau \, \mathrm{Tr} \left(\mathcal{P} \partial_\tau \mathcal{P} \partial_u \mathcal{P} \right)$$

$$\mathcal{S}_{tJ} = \int d\tau d\tau' \, \mathrm{Tr} \left(\mathcal{P}(\tau) \mathcal{Q}(\tau - \tau') \mathcal{P}(\tau') \right) + \int d\tau \mathrm{Tr} \left(s_0 \, \mathcal{P}(\tau) \right) \, .$$

Path integral over a superspin $\mathcal{P}(\tau)$ with a self-consistent self-interaction $\mathcal{Q}(\tau)$ and a ‘Zeeman superfield’ s_0.

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model

$$\mathcal{Z} = \int \mathcal{D}f_\alpha(\tau)\mathcal{D}b(\tau)\mathcal{D}\lambda(\tau)e^{-\mathcal{S}_B-\mathcal{S}_{tJ}}$$

$$\mathcal{S}_B = \int d\tau \left[f_\alpha^\dagger(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)f_\alpha(\tau)+b^\dagger(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)b(\tau)-i\lambda\right]$$

$$\mathcal{S}_{tJ} = \int d\tau\, s_0 f_\alpha^\dagger(\tau)f_\alpha(\tau)+t^2\int d\tau d\tau' R(\tau-\tau')c_\alpha^\dagger(\tau)c_\alpha(\tau') - \frac{J^2}{2}\int d\tau d\tau' Q(\tau-\tau')\vec{S}(\tau)\cdot\vec{S}(\tau')\,.$$

From this action we determined the correlators

SU(1|2) theory

$$\overline{R}(\tau-\tau') = -\left\langle c_\alpha(\tau)c_\alpha^\dagger(\tau')\right\rangle_{\mathcal{Z}}$$

$$\overline{Q}(\tau-\tau') = \frac{1}{3}\left\langle\vec{S}(\tau)\cdot\vec{S}(\tau')\right\rangle_{\mathcal{Z}}$$

and finally impose the self-consistency conditions

$$R(\tau)=\overline{R}(\tau)\quad,\quad Q(\tau)=\overline{Q}(\tau).$$

D. Joshi,
Chenyuan Li,
G. Tarnopolsky,
A. Georges,
S. Sachdev,
arXiv:1912.08822

t-J model

$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}\mathfrak{b}_\alpha(\tau)\mathcal{D}\mathfrak{f}(\tau)\mathcal{D}\lambda(\tau)e^{-\mathcal{S}_B-\mathcal{S}_{tJ}} \\
\mathcal{S}_B &= \int d\tau \left[\mathfrak{b}^\dagger_\alpha(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)\mathfrak{b}_\alpha(\tau)+\mathfrak{f}^\dagger(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)\mathfrak{f}(\tau)-i\lambda\right] \\
\mathcal{S}_{tJ} &= \int d\tau\, s_0\mathfrak{b}^\dagger_\alpha(\tau)\mathfrak{b}_\alpha(\tau)+t^2\int d\tau d\tau' R(\tau-\tau')c^\dagger_\alpha(\tau)c_\alpha(\tau') \\
&\quad -\frac{J^2}{2}\int d\tau d\tau' Q(\tau-\tau')\vec{S}(\tau)\cdot\vec{S}(\tau')\,.
\end{aligned}
$$

From this action we determined the correlators

SU(2|1) theory

$$
\begin{aligned}
\overline{R}(\tau-\tau') &= -\left\langle c_\alpha(\tau)c^\dagger_\alpha(\tau')\right\rangle_{\mathcal{Z}} \\
\overline{Q}(\tau-\tau') &= \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(\tau')\right\rangle_{\mathcal{Z}}
\end{aligned}
$$

and finally impose the self-consistency conditions

$$
R(\tau)=\overline{R}(\tau) \quad , \quad Q(\tau)=\overline{Q}(\tau).
$$

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A. Georges,
S. Sachdev,
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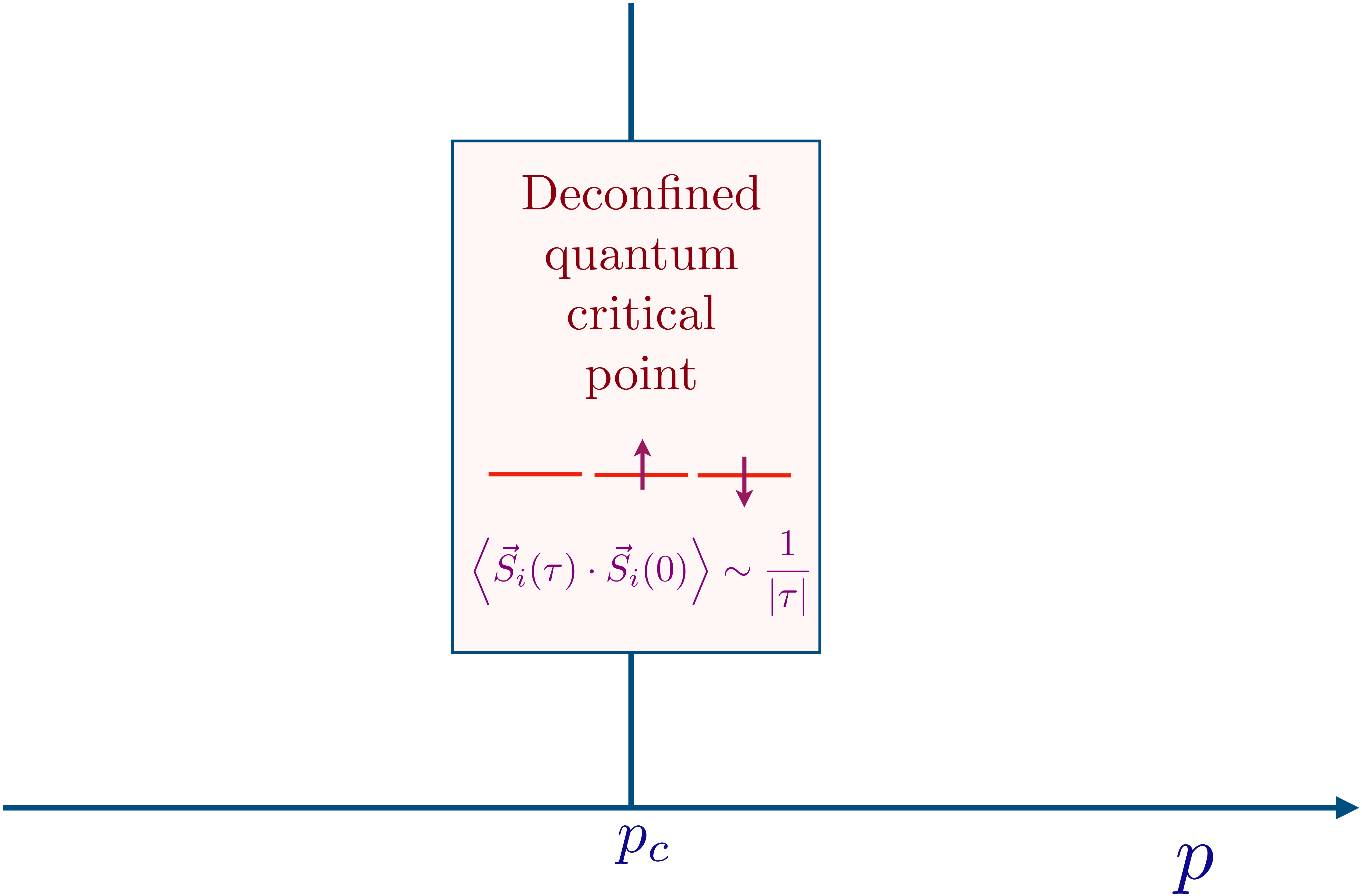
t-J model phase diagram



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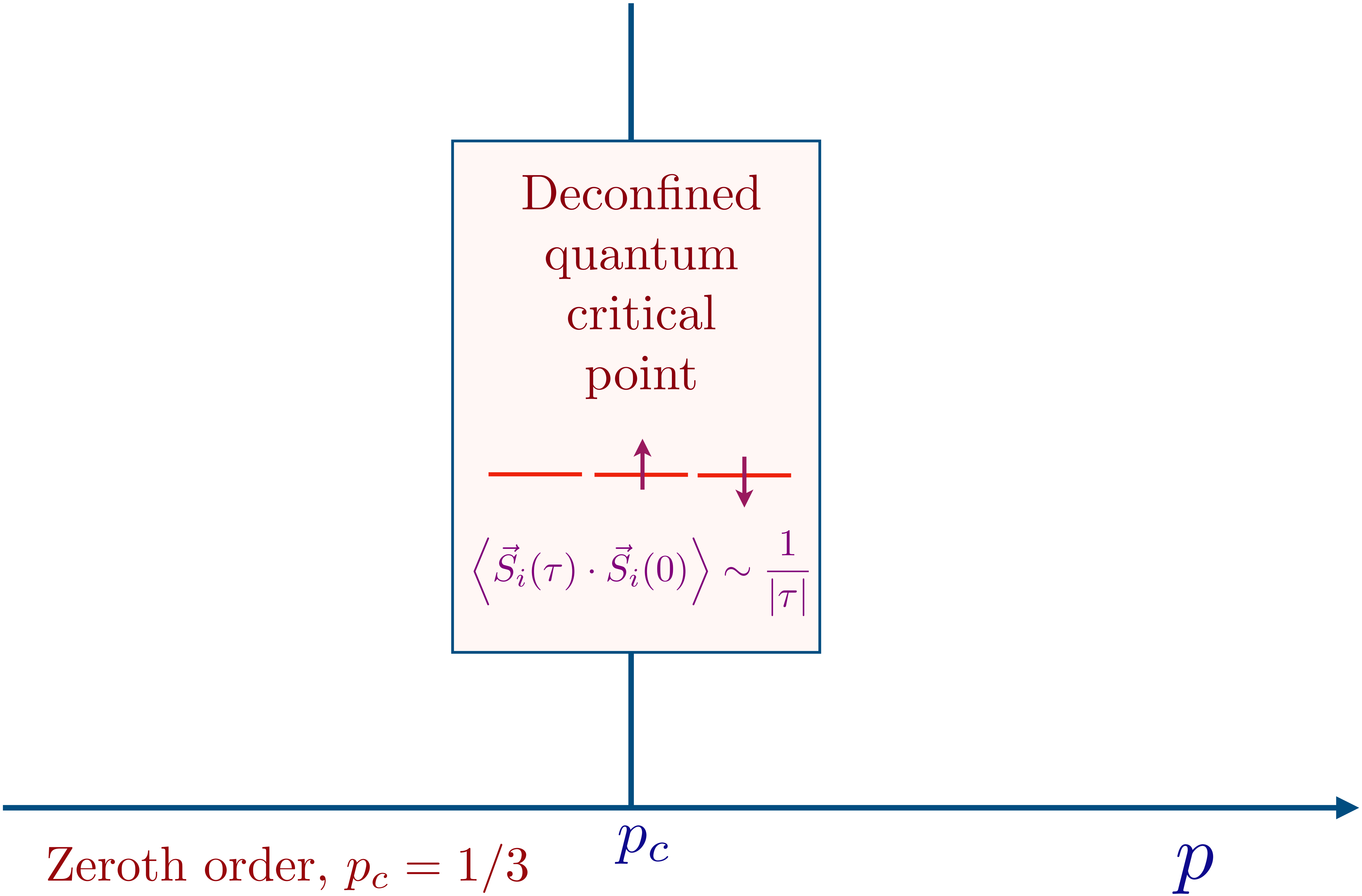
t-J model phase diagram



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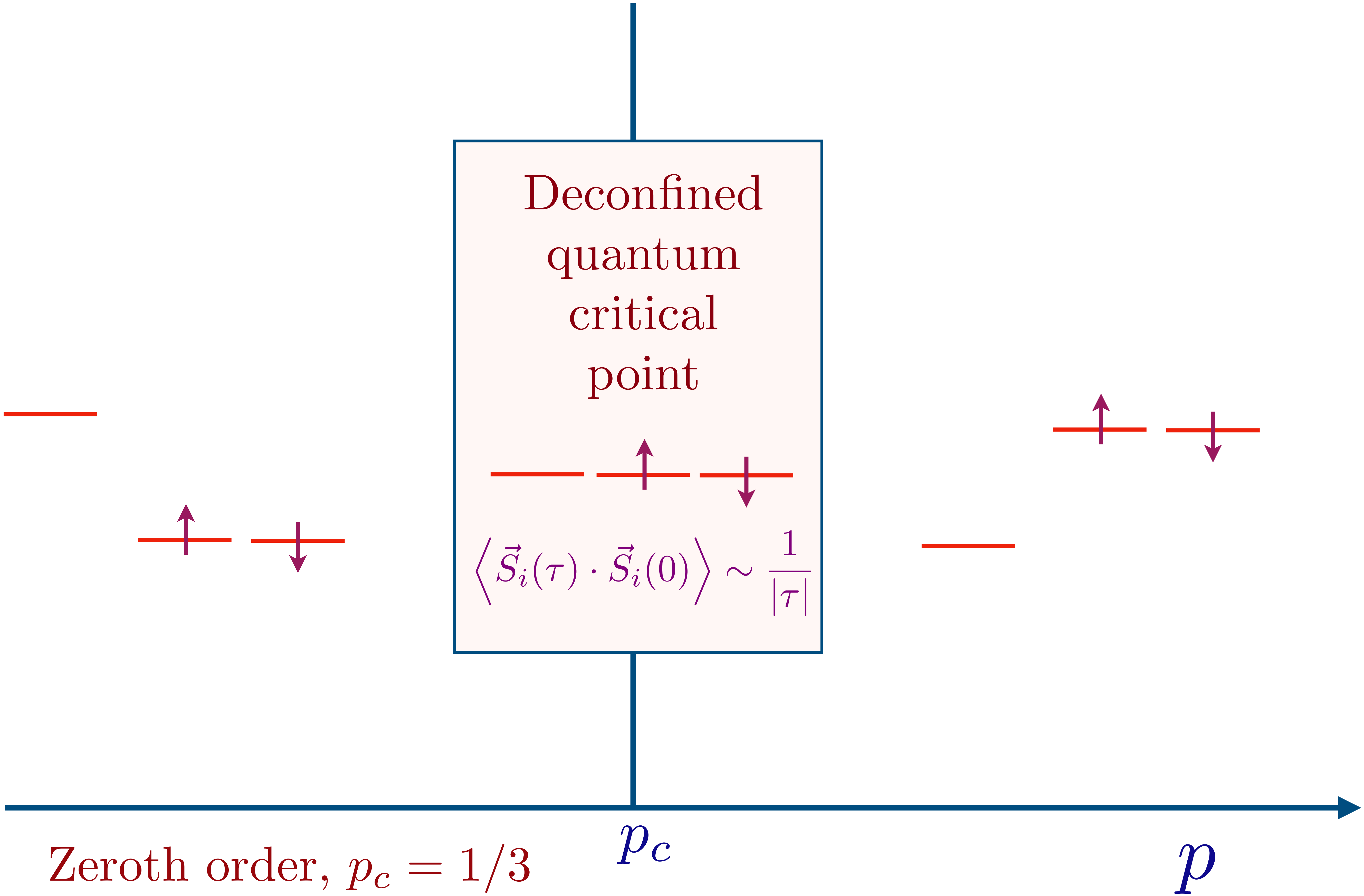

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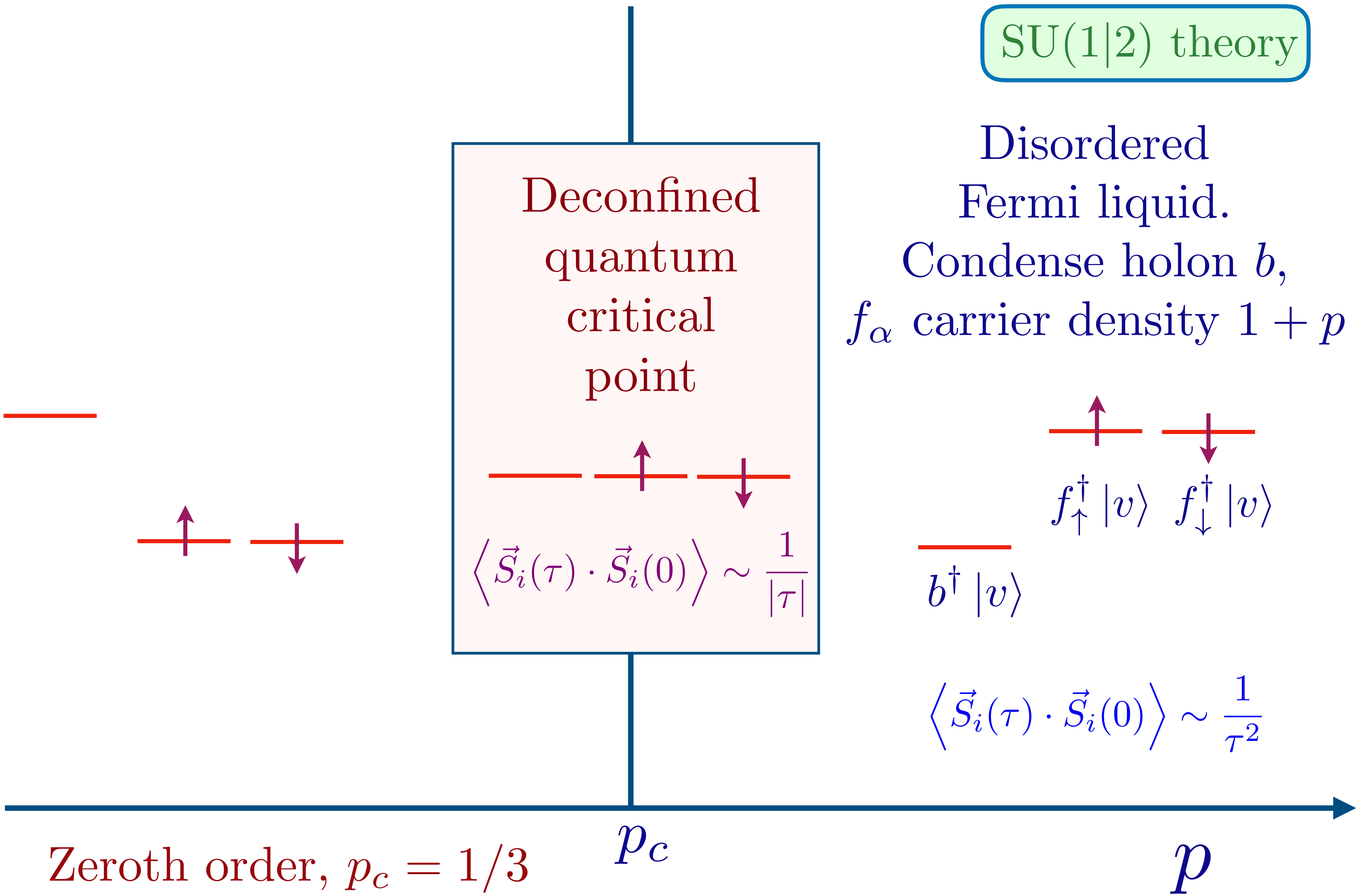

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t-J model phase diagram

SU(2|1) theory

SU(1|2) theory

Metallic spin glass. Condense spinon $\mathfrak{b}_\alpha$, $\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger |v\rangle$

$\mathfrak{b}_\uparrow^\dagger |v\rangle$ $\mathfrak{b}_\downarrow^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

Deconfined quantum critical point

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

Disordered Fermi liquid. Condense holon b, f_α carrier density $1 + p$

$f_\uparrow^\dagger |v\rangle$ $f_\downarrow^\dagger |v\rangle$

$b^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

p_c

p

Zeroth order, $p_c = 1/3$

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- t-J models with random and all-to-all interactions exhibit a deconfined critical point at a non-zero doping $p = p_c$, flanked by conventional confining phases with carrier densities p and $1 + p$.

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