

The pseudogap phase of the cuprate superconductors

Quantum Entanglement in Macroscopic Matter
International Center for Theoretical Sciences, Bangalore
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Subir Sachdev



PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS



JOHN TEMPLETON
FOUNDATION

PHYSICS



HARVARD

Talk online: sachdev.physics.harvard.edu

Theorists at Harvard



Max Metlitski
(KITP, UCSB)



Andrea Allais



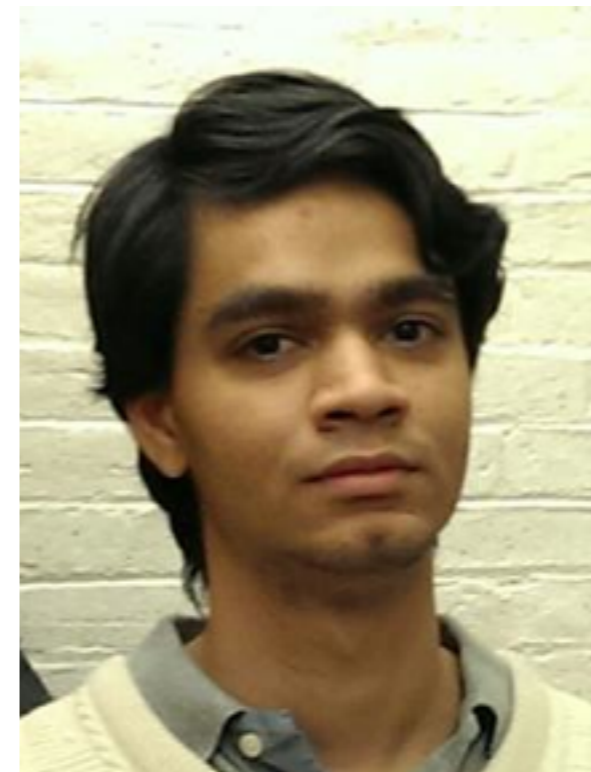
Matthias Punk
(Innsbruck)



Debanjan
Chowdhury



Alexandra
Thomson



Aavishkar Patel



Jay Sau
(Maryland)

Cornell



Kazuhiro Fujita
Cornell/ BNL



Mohammad Hamidian
Cornell / BNL



Stephen Edkins
Cornell / St Andrews



Michael Lawler

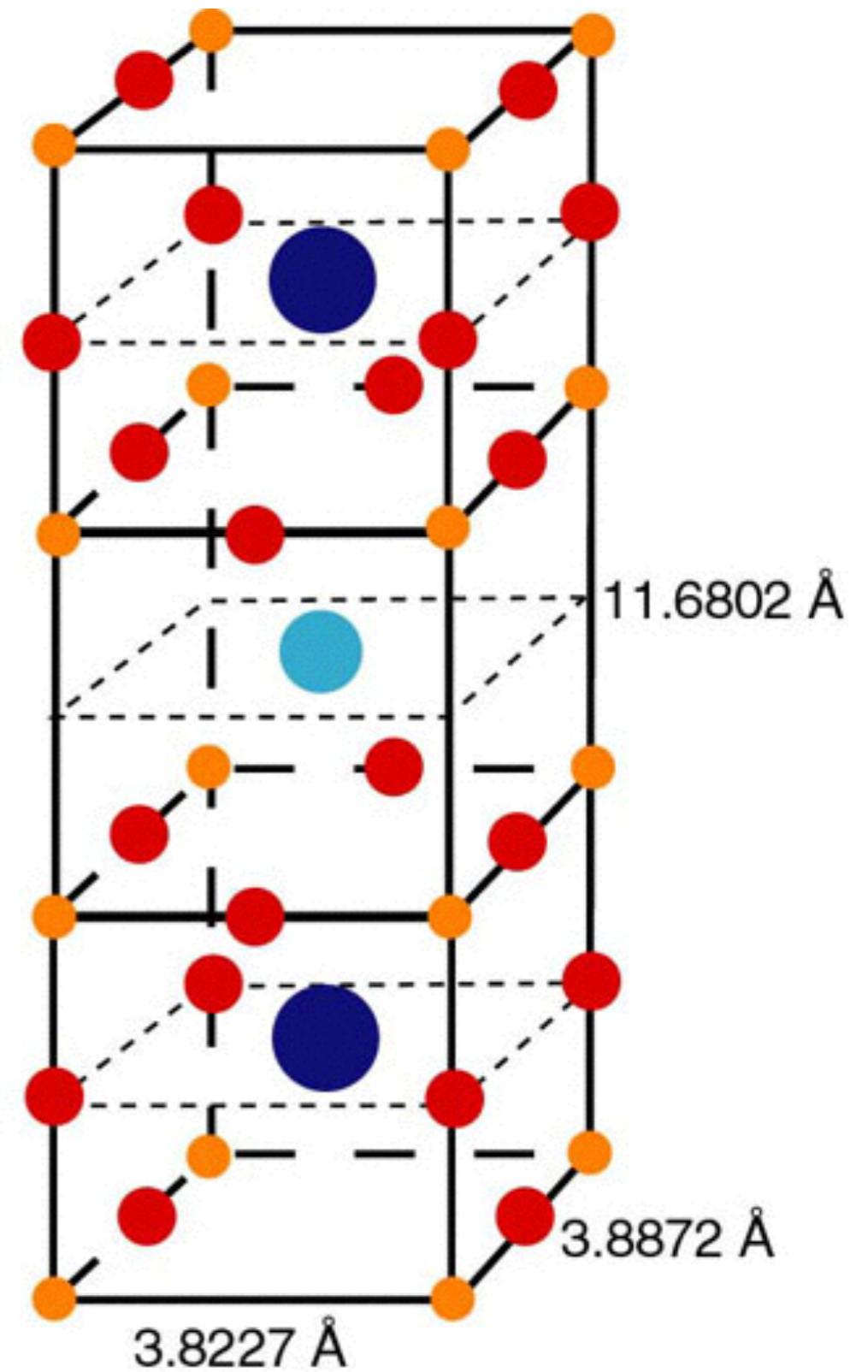
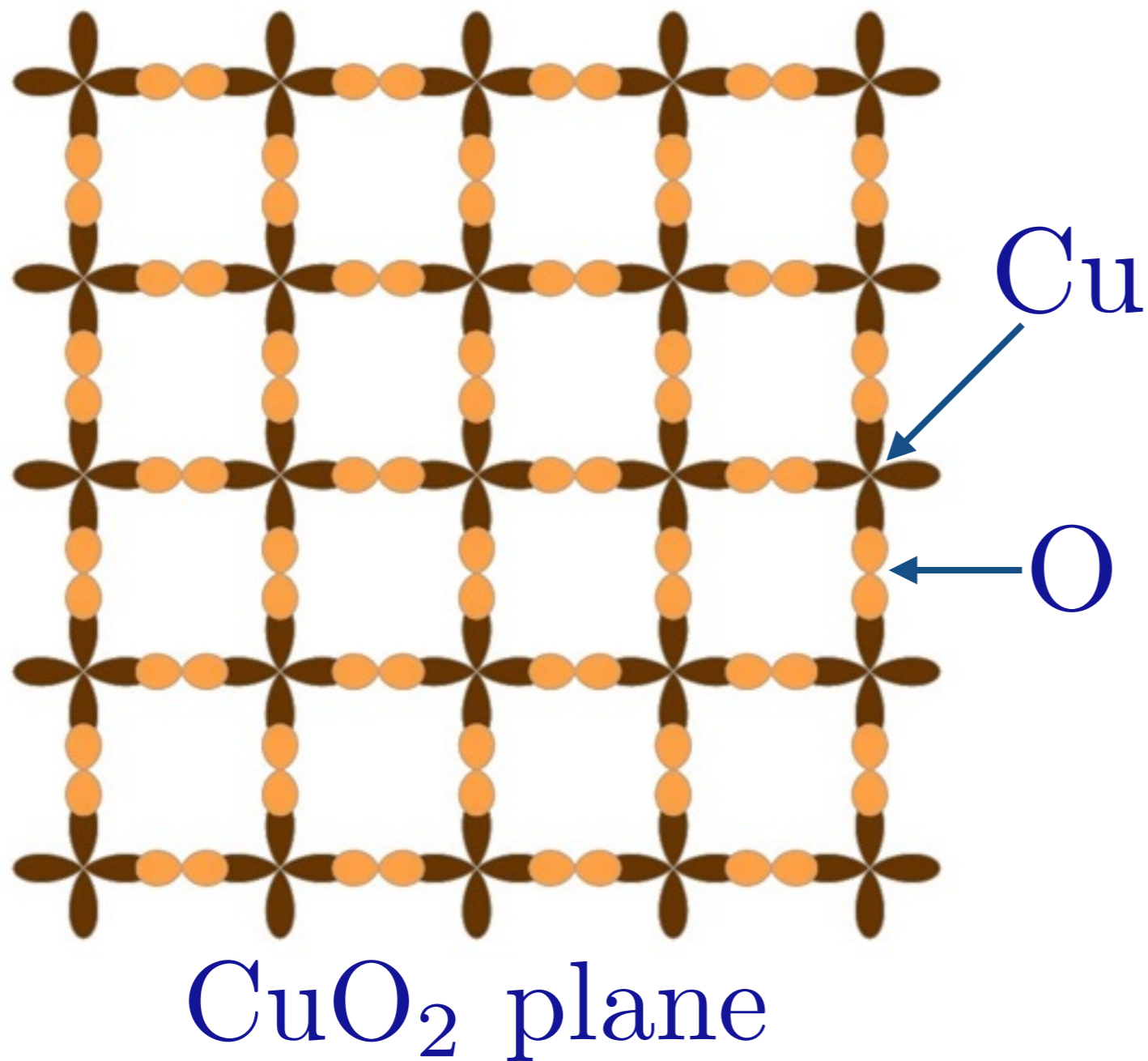


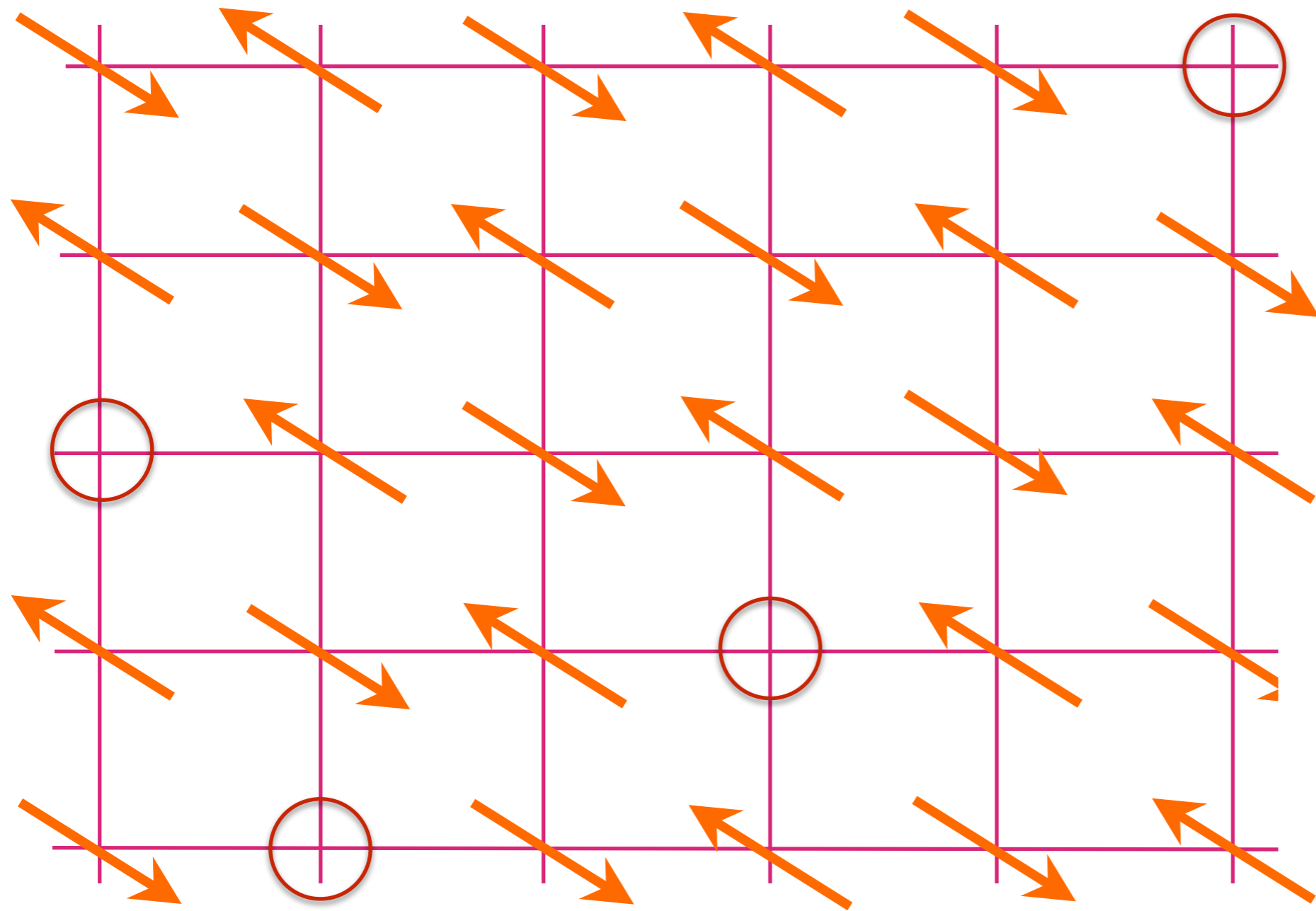
J. C. Seamus Davis



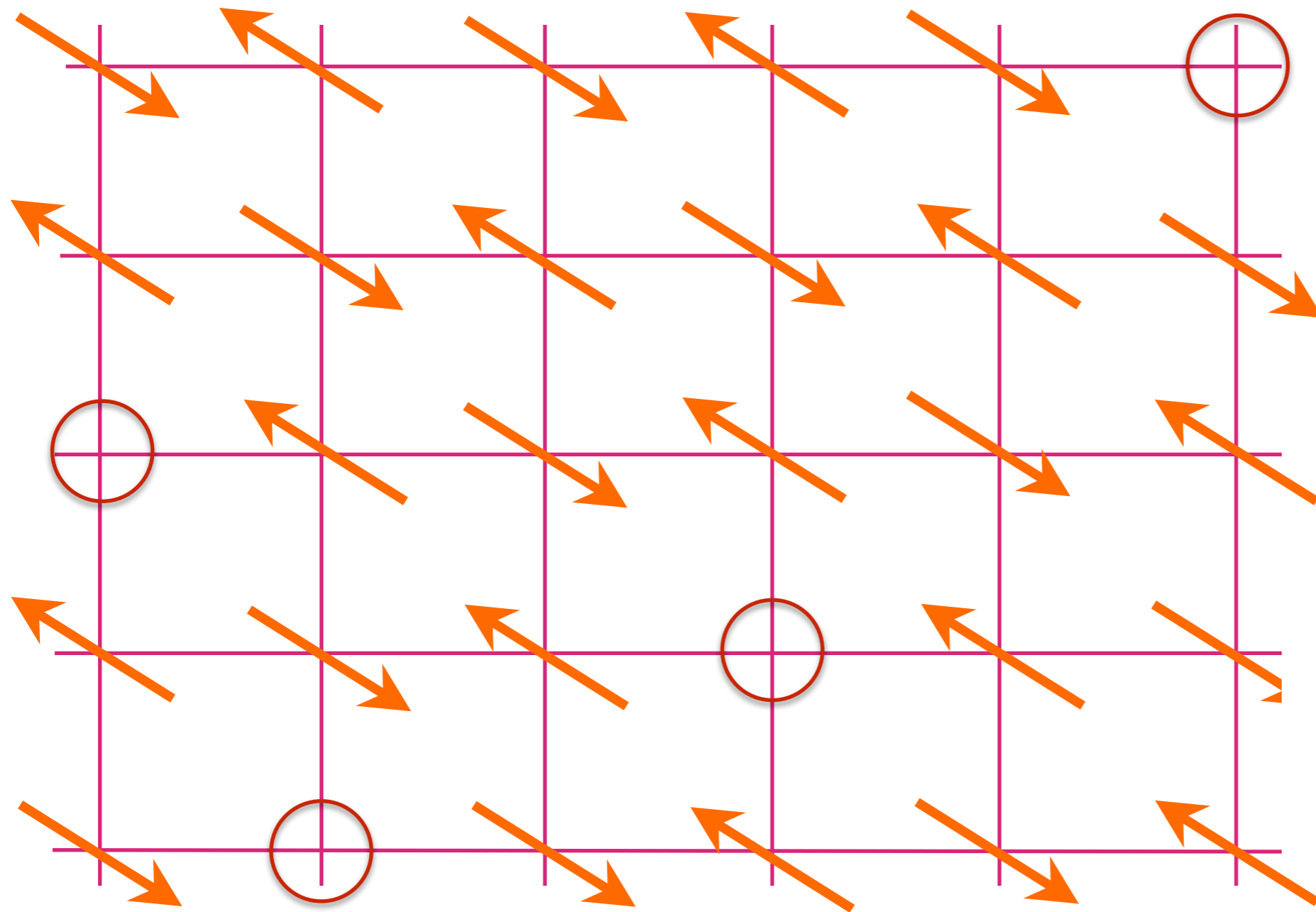
Eun-Ah Kim

High temperature superconductors



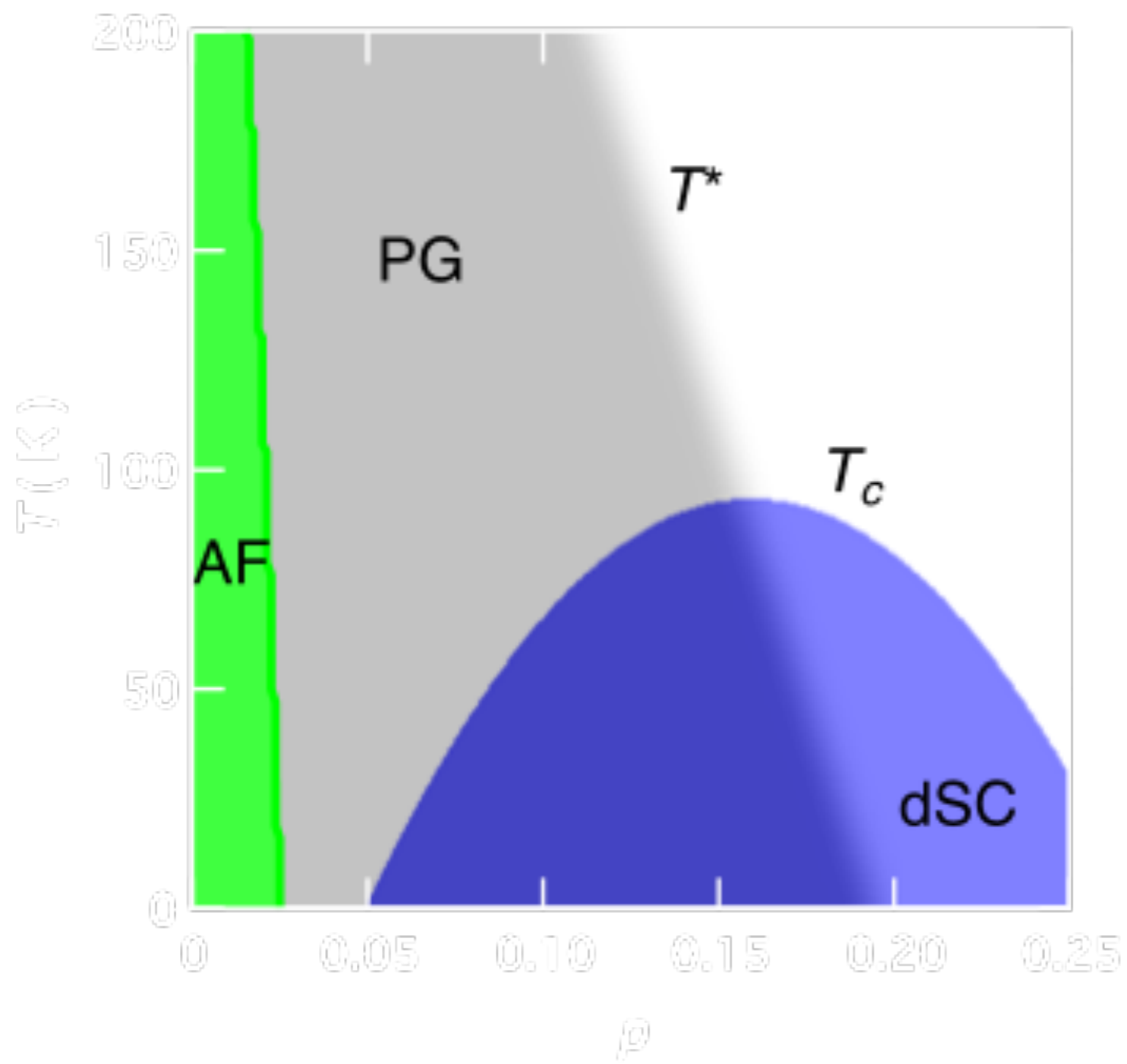


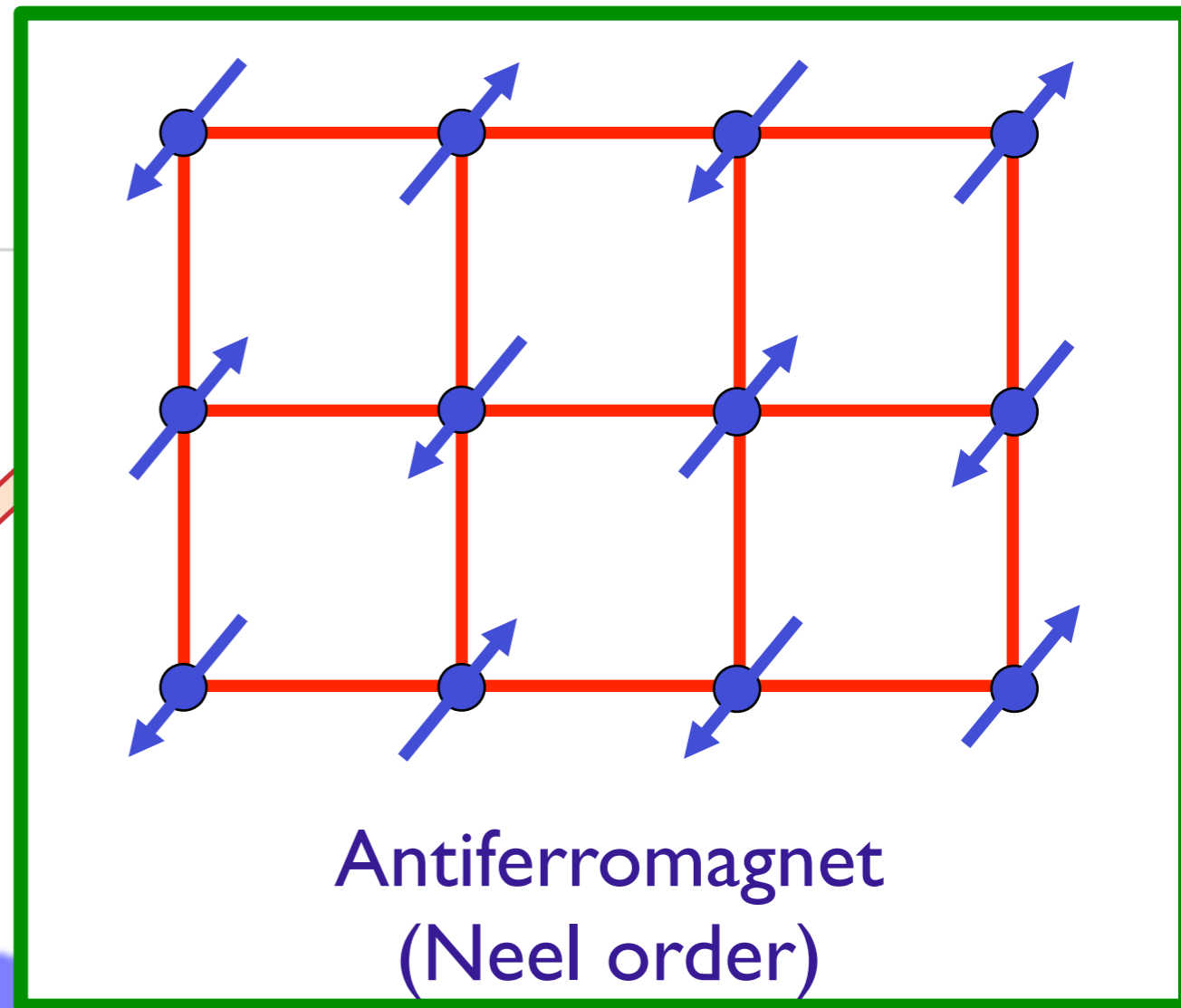
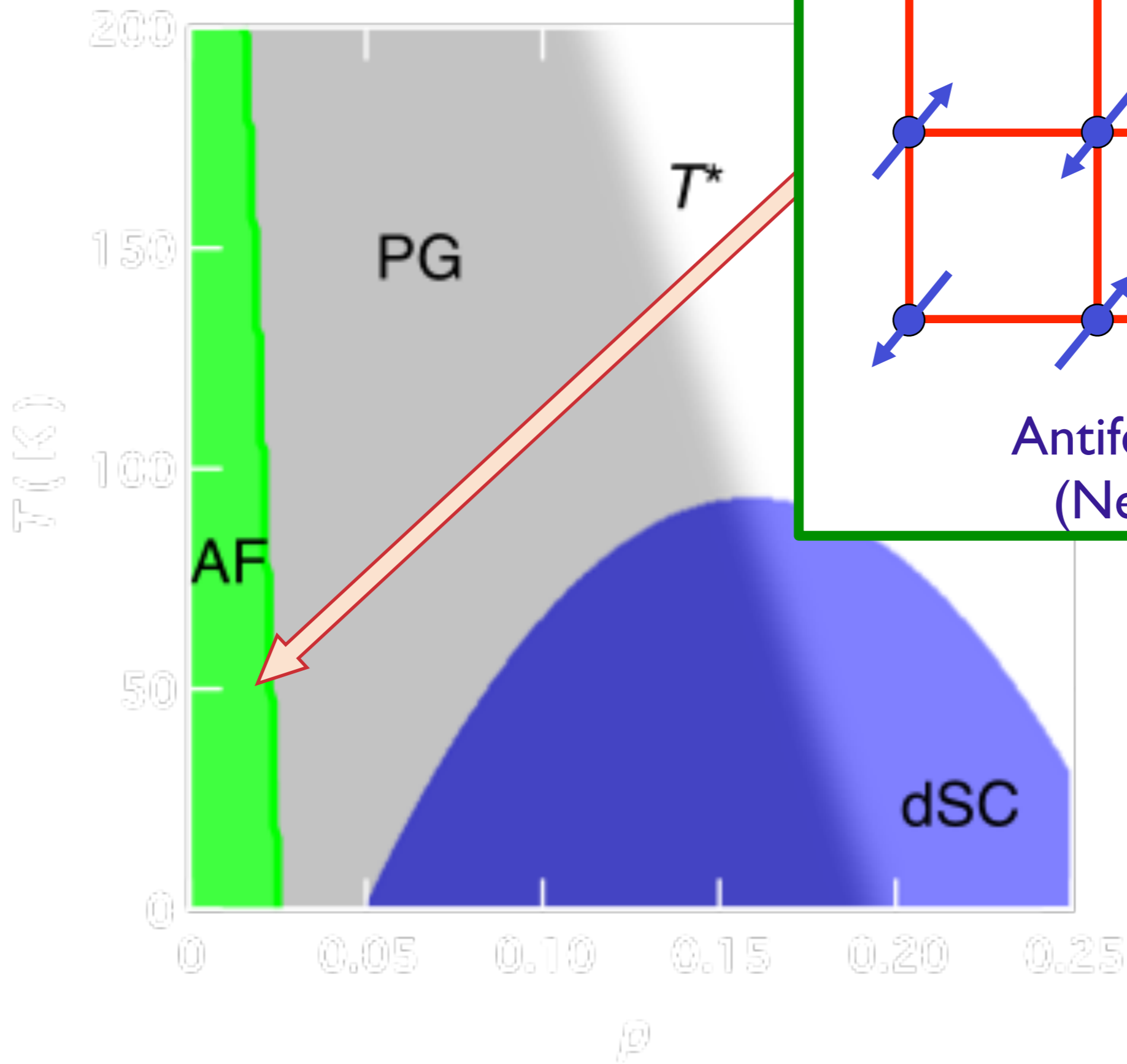
Anti-ferromagnet
with p holes
per square

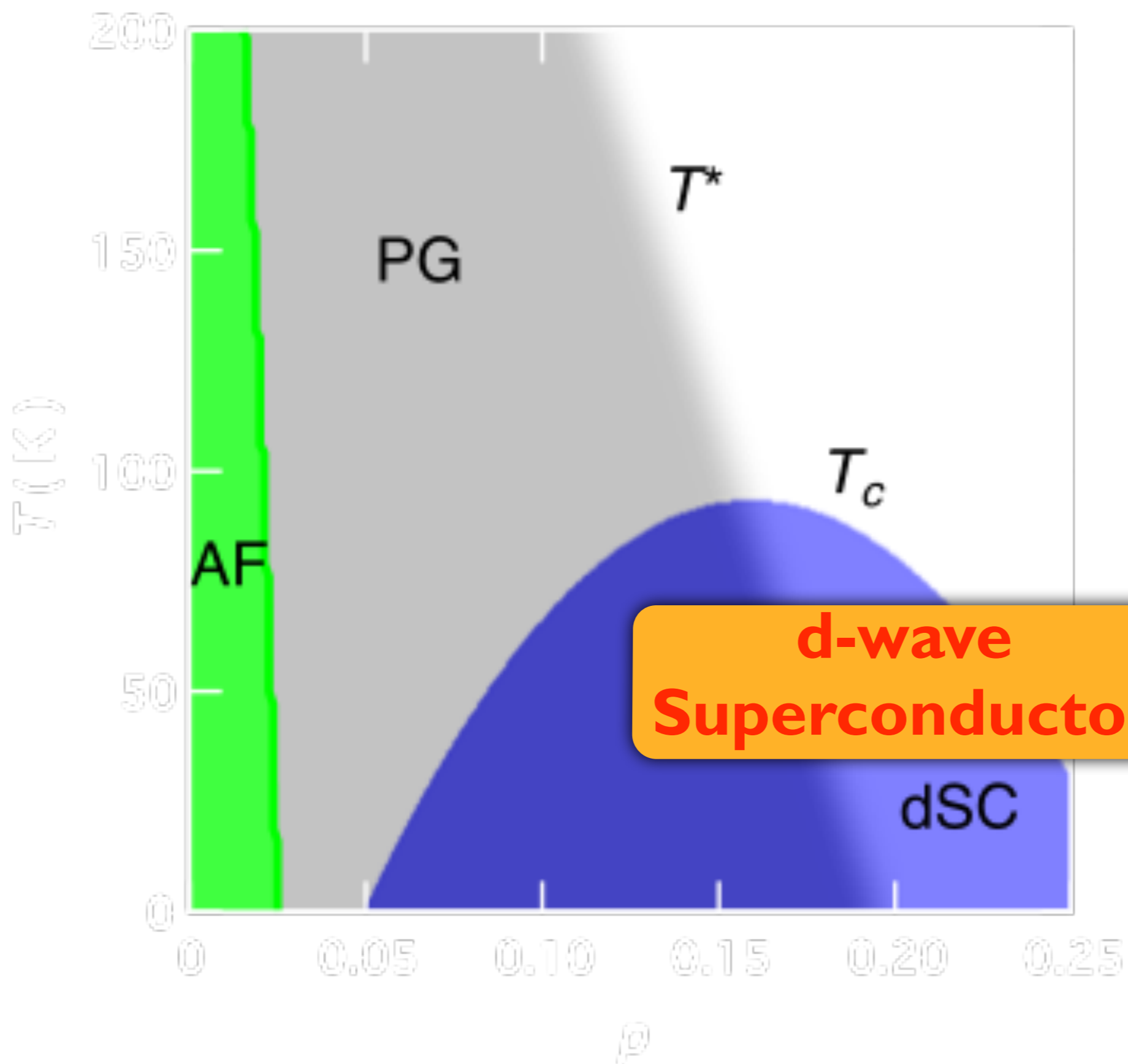


Anti-ferromagnet with p holes per square

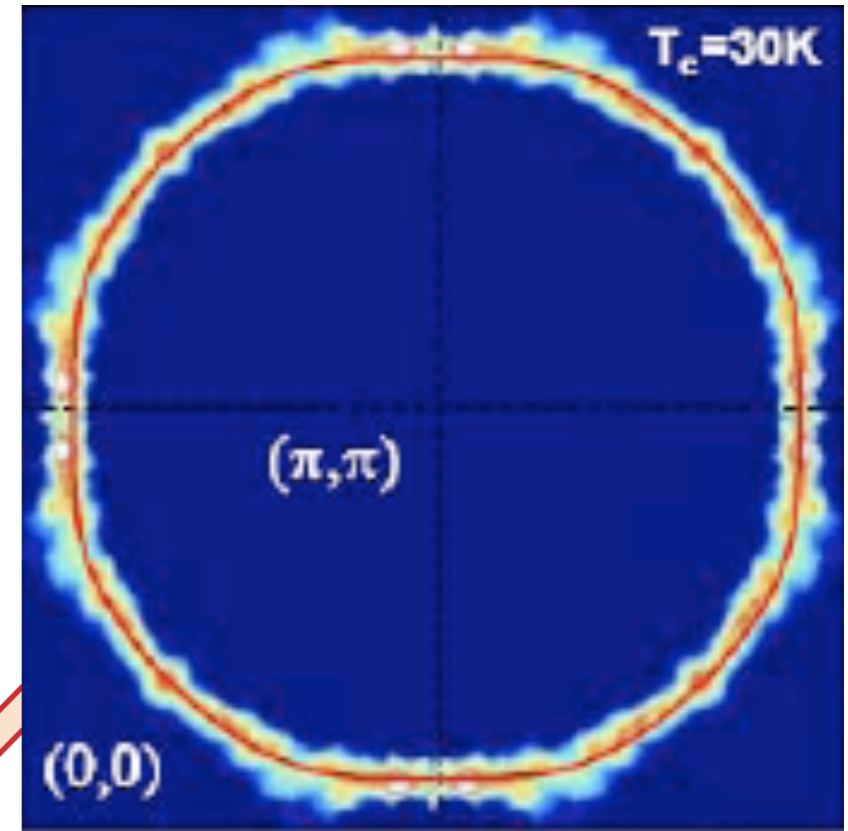
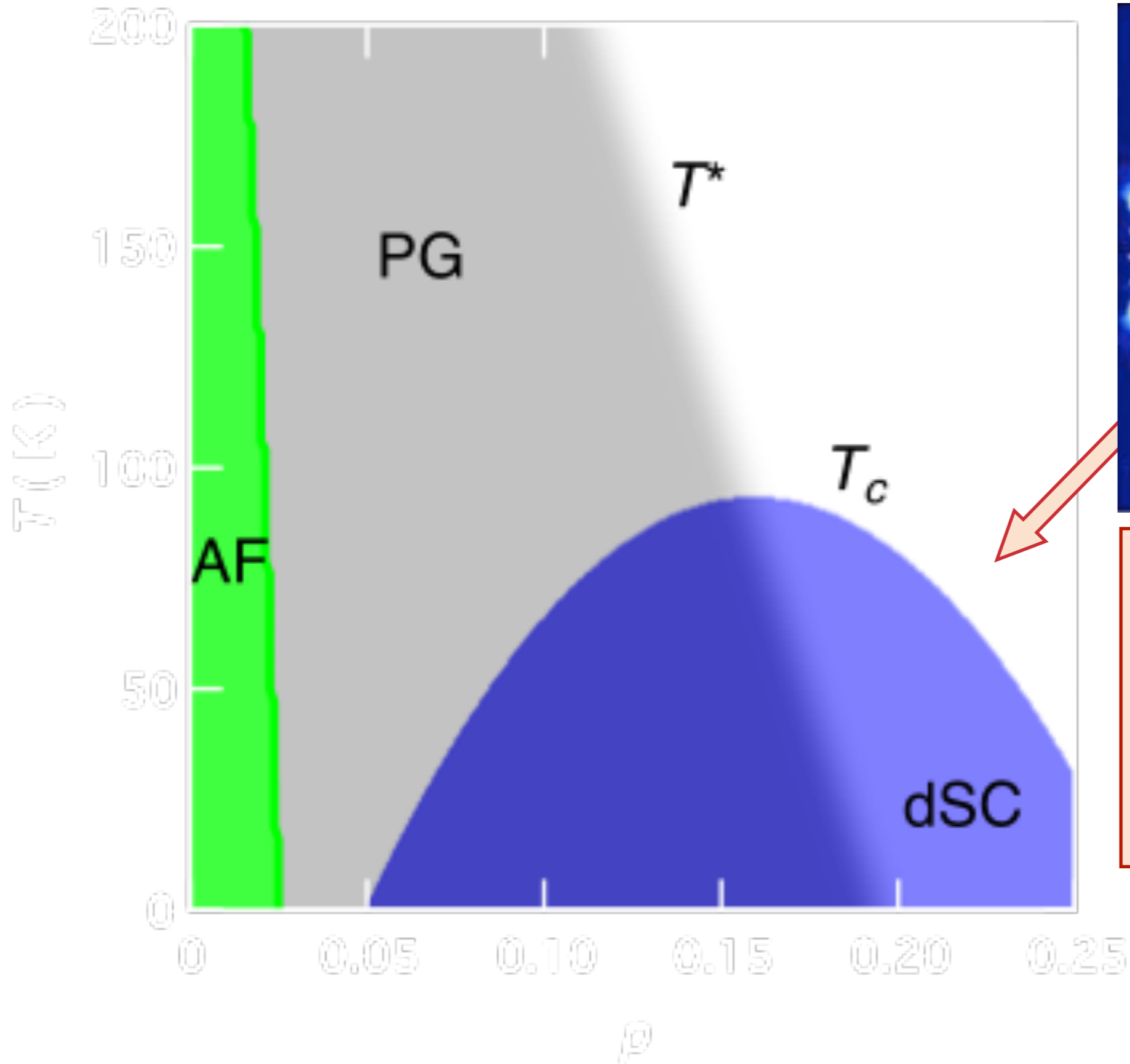
Note: relative to the fully-filled band insulator, there are $1+p$ holes per square





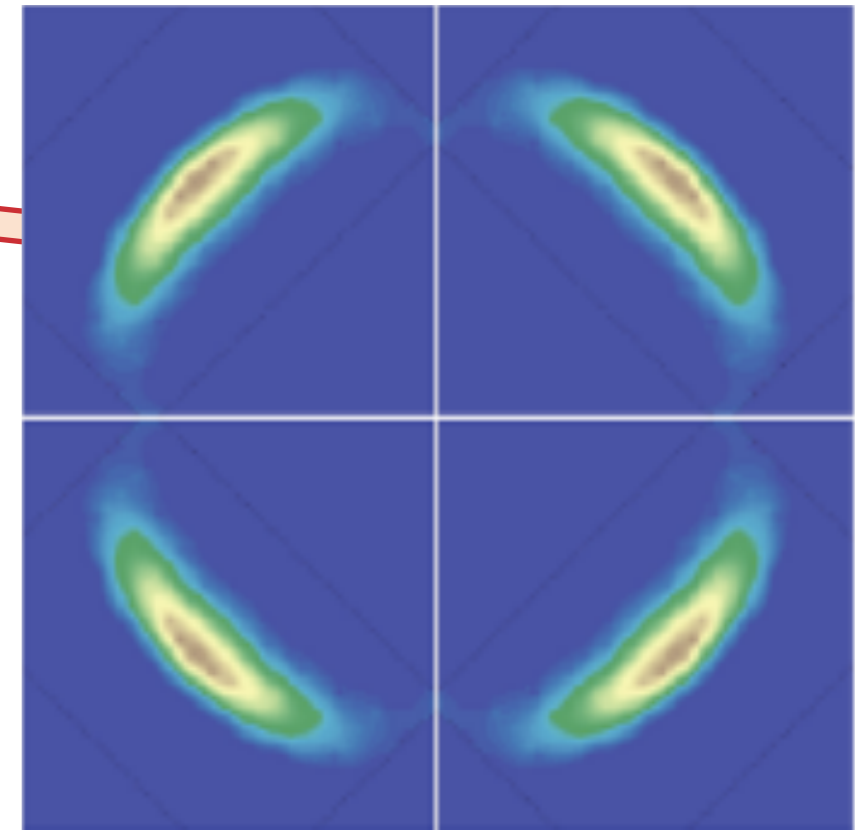
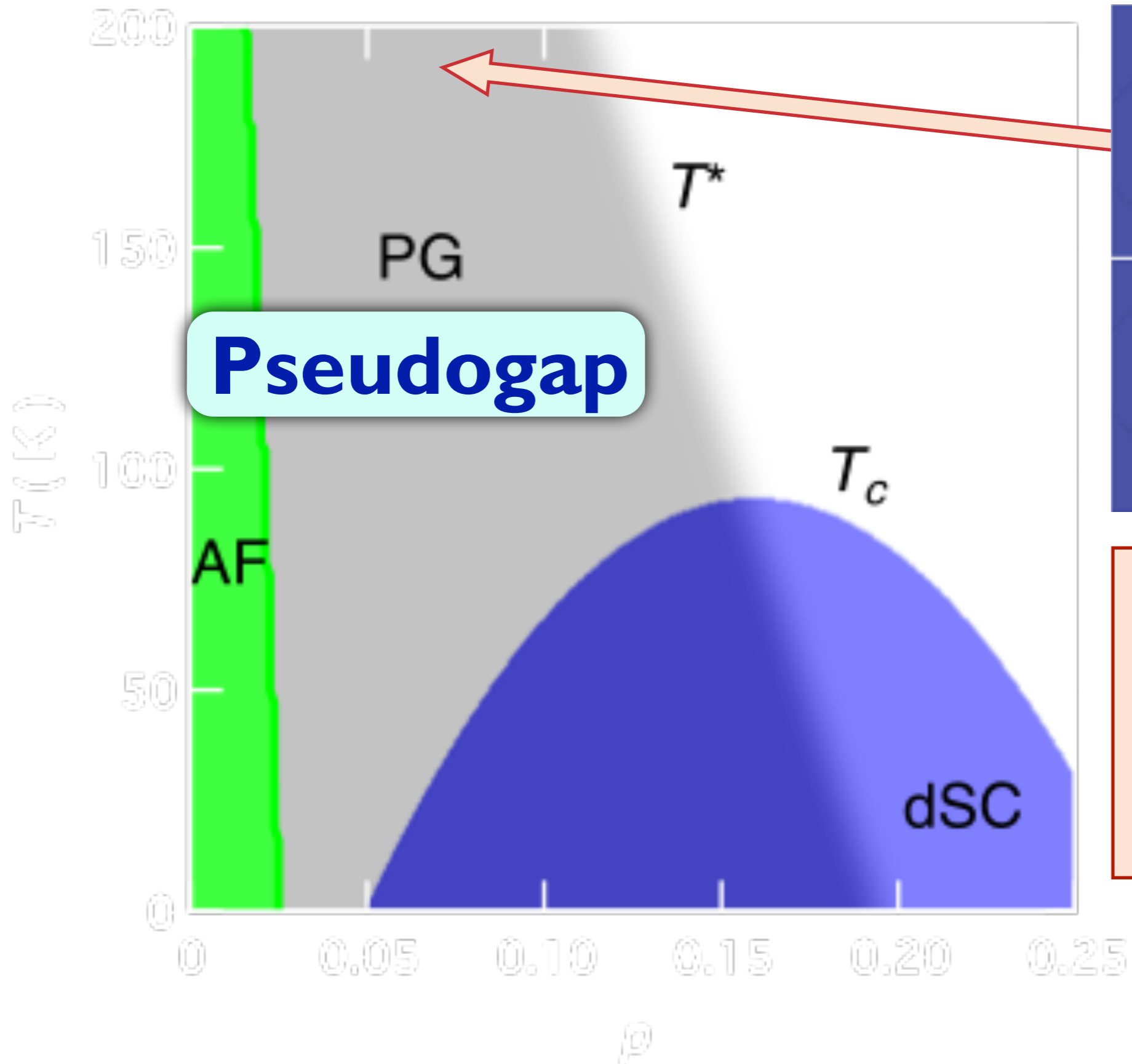


M. Platié, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

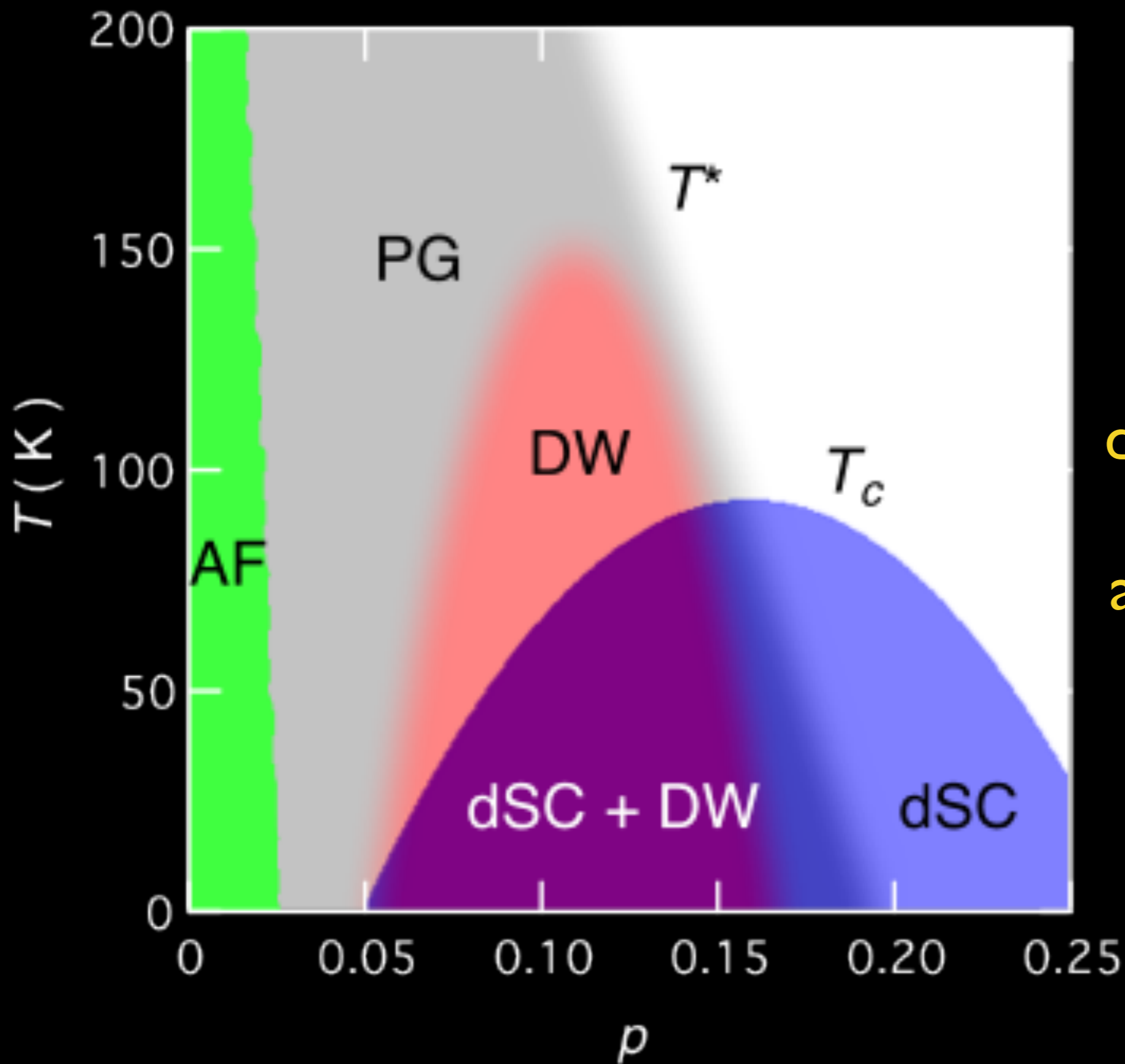


Fermi liquid:
Area enclosed
by Fermi surface
 $= | + p$

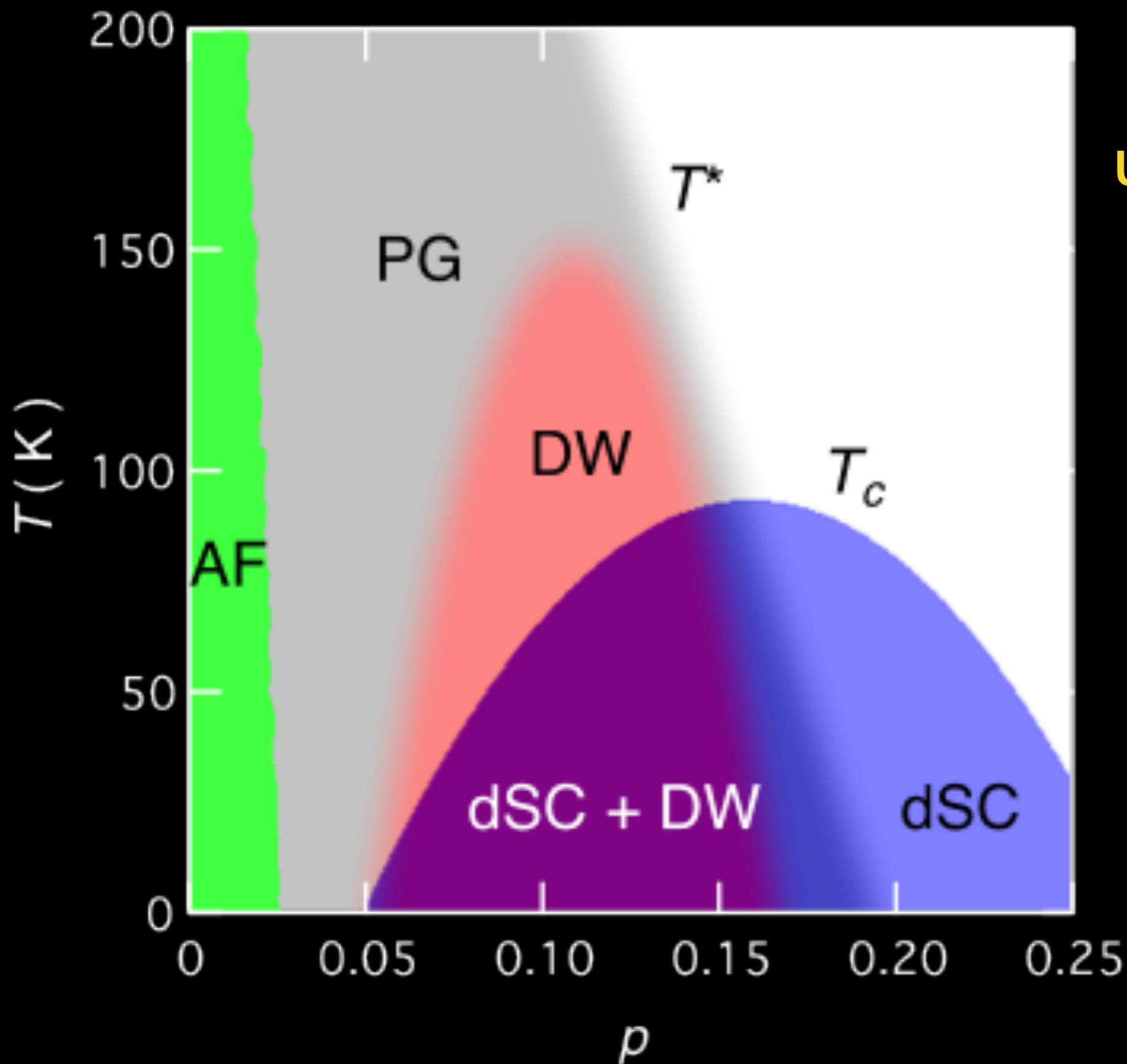
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, *Science* **307**, 901 (2005)



“Fermi arcs”
at
low p



Numerous recent experiments indicating density wave (DW) order at low T in all the underdoped cuprates



How do we understand the Fermi arc spectrum, and what is its relationship to the density wave (DW) order at lower T ?

Is the higher temperature pseudogap
(with ``Fermi arc'' spectra) described by

(A) Thermal fluctuations of the low
temperature orders (superconductivity,
density wave, antiferromagnetism...)

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(A) Thermal fluctuations of the low temperature orders (superconductivity, density wave, antiferromagnetism...)

OR

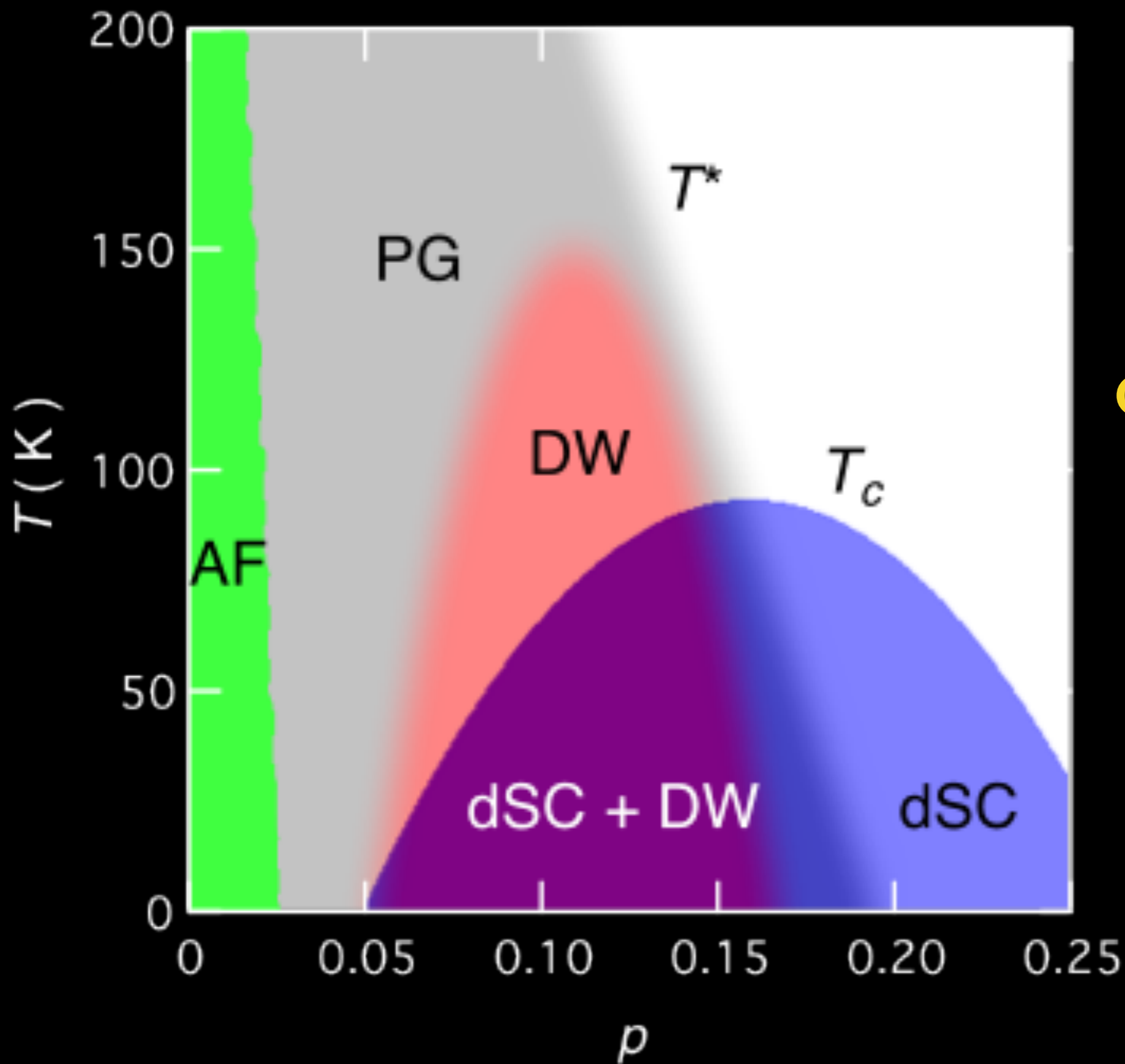
(B) A new type of metal with “topological order”, which can be stable (in principle) as a quantum ground state

Outline

1. The low T pseudogap:
*STM observation of predicted
 d -form factor density wave*
2. The high T pseudogap:
*A quantum dimer model for a metal
with topological order*
3. Connecting high and low T :
Density wave instabilities
4. Quantum critical point near optimal p :
A Higgs critical point

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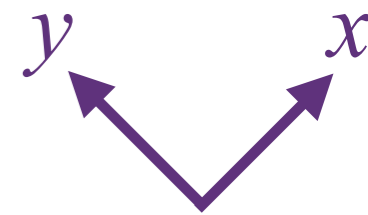
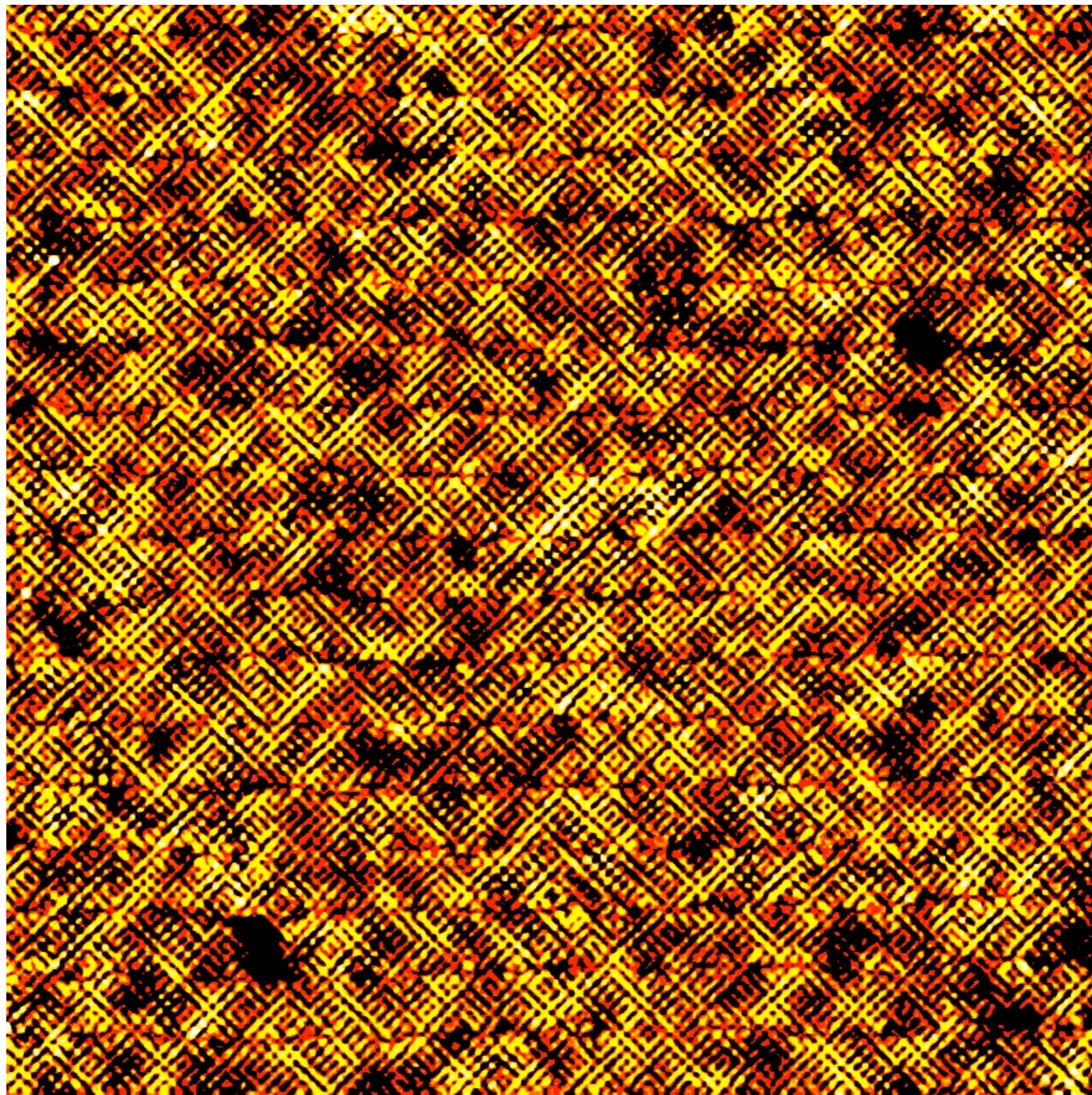
STM
of DW order
in non-La
materials

See also

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Nature Phys. **4**, 696
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“R-map” of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007). **Davis group has sub-angstrom resolution capabilities, with lattice drift corrections, which make sublattice phase-resolved STM possible.**

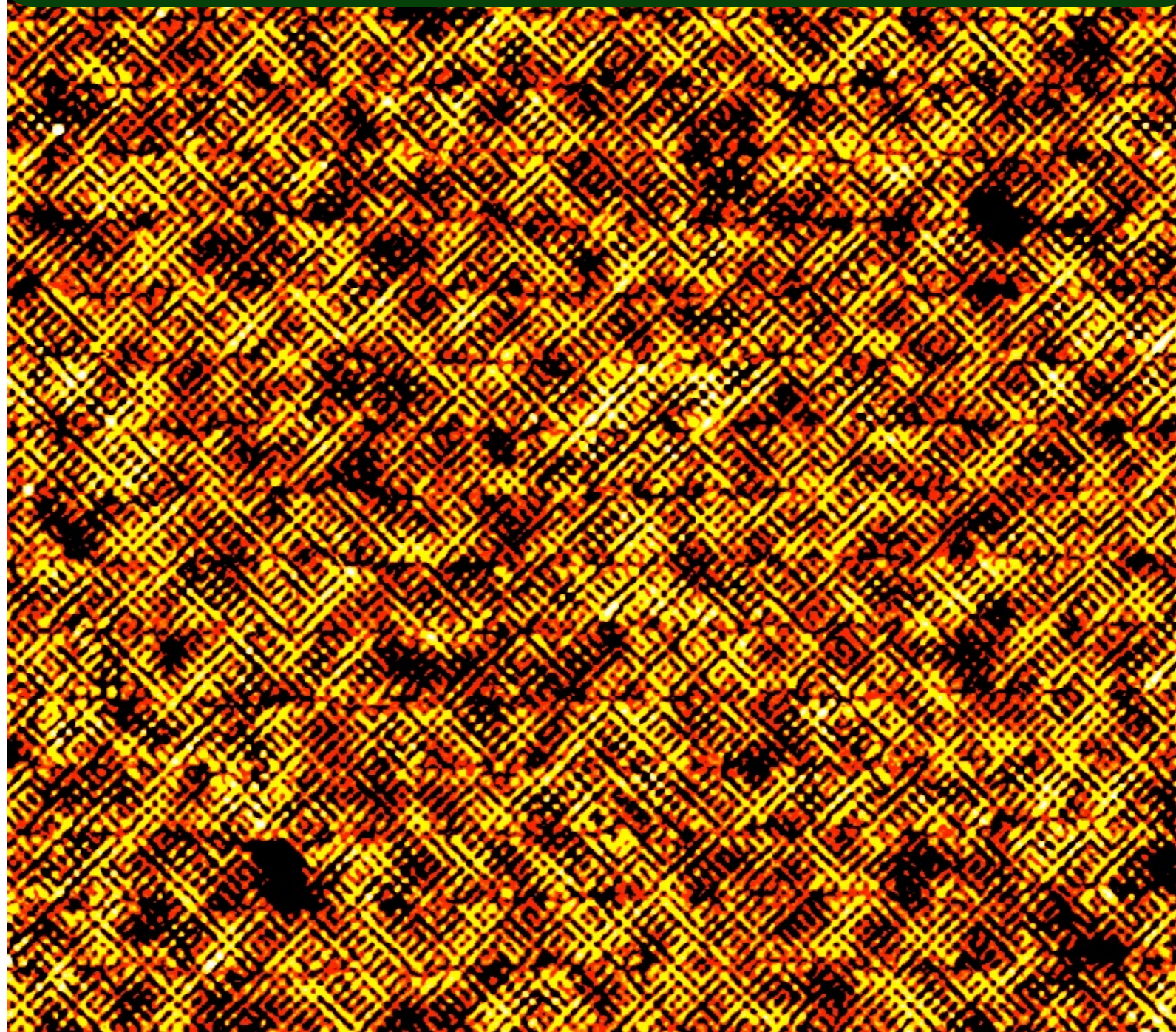
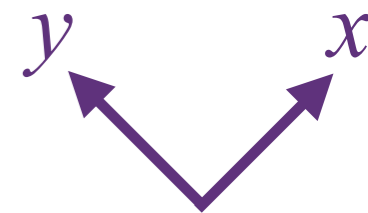
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Disordered uni-directional charge density waves
("stripes") with wavelength ≈ 4 lattice sites ?



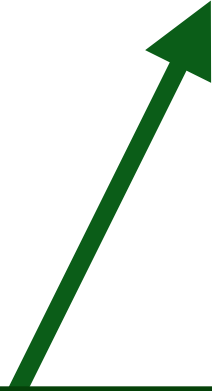
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Unconventional density wave (DW) :
Bose condensation of particle-hole pairs

$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1) c_{\alpha}(\mathbf{r}_2) \rangle$$
$$= \left[\mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2) / 2}$$

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Crucial “center-of-mass” co-ordinate.
(Not used in previous work)
Simplifies action of time-reversal

Unconventional density wave (DW) :
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$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1)c_{\alpha}(\mathbf{r}_2) \rangle = \left[\mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2}$$

Density wave form factor (internal particle-hole pair wavefunction)

$$\mathcal{P}(\mathbf{r}) = \int \frac{d^2k}{4\pi^2} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}}$$

Time-reversal symmetry requires $\mathcal{P}(\mathbf{k}) = \mathcal{P}(-\mathbf{k})$.

We expand (using reflection symmetry for \mathbf{Q} along axes or diagonals)

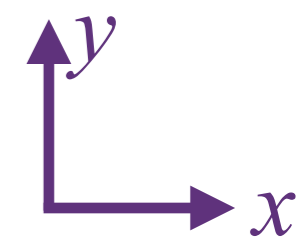
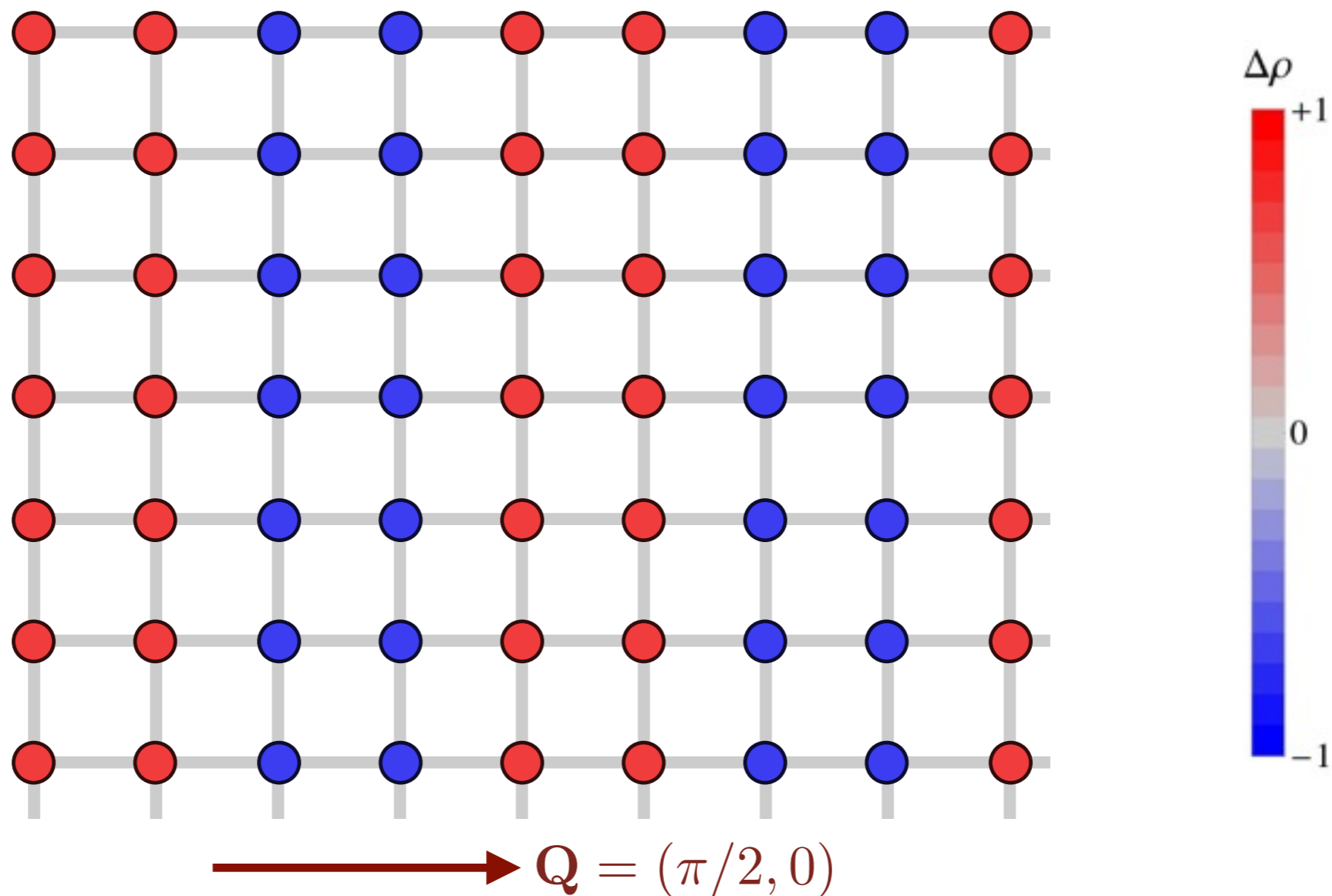
$$\mathcal{P}(\mathbf{k}) = \mathcal{P}_s + \mathcal{P}_{s'}(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y)$$

Conventional CDW order: s -form factor

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = 1 \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

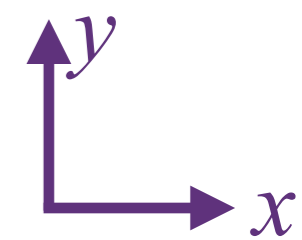
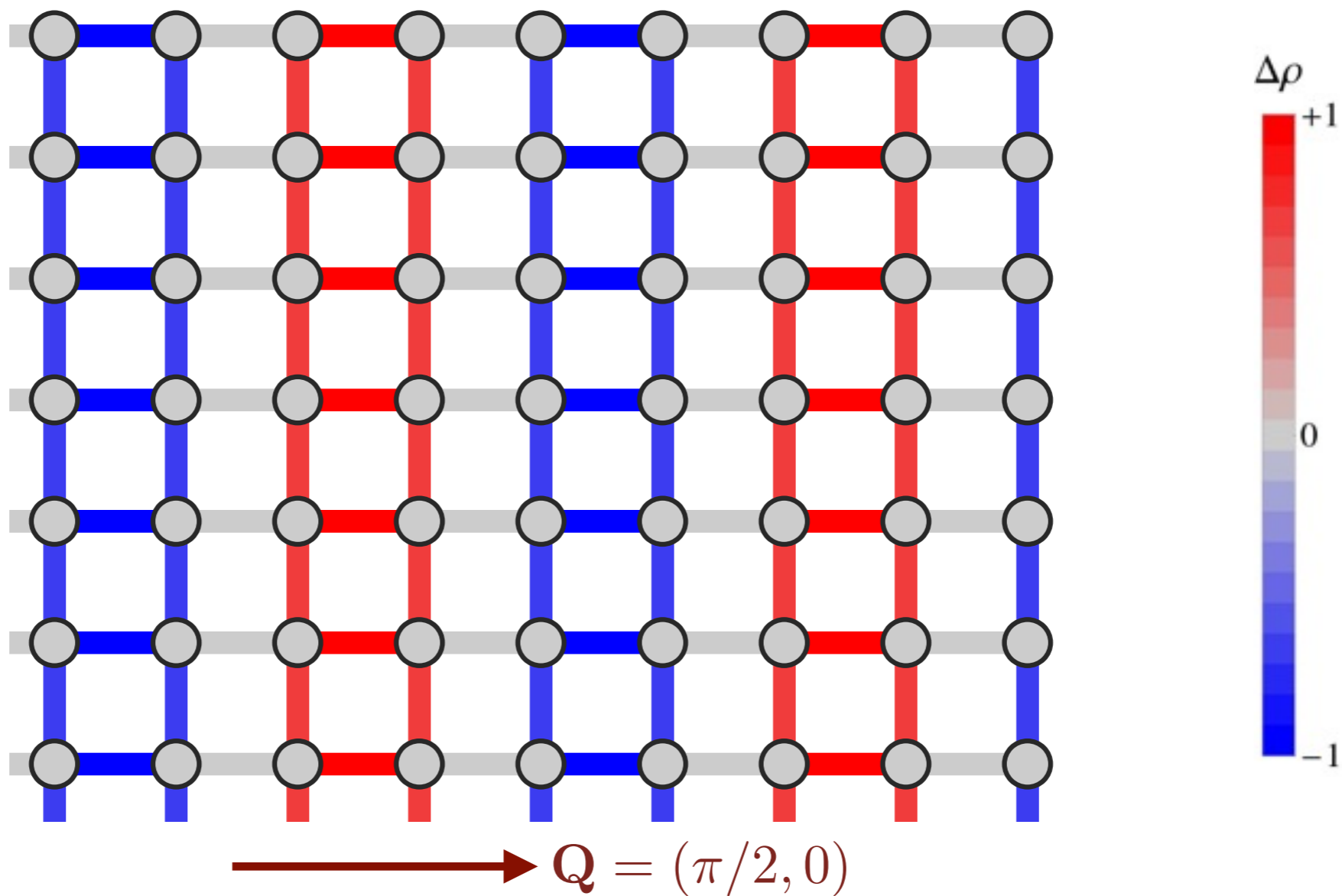


Unconventional DW order: s' -form factor

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

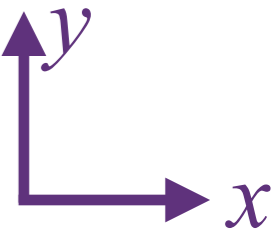
$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) + \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$



Unconventional DW order: $s + s'$ -form factor

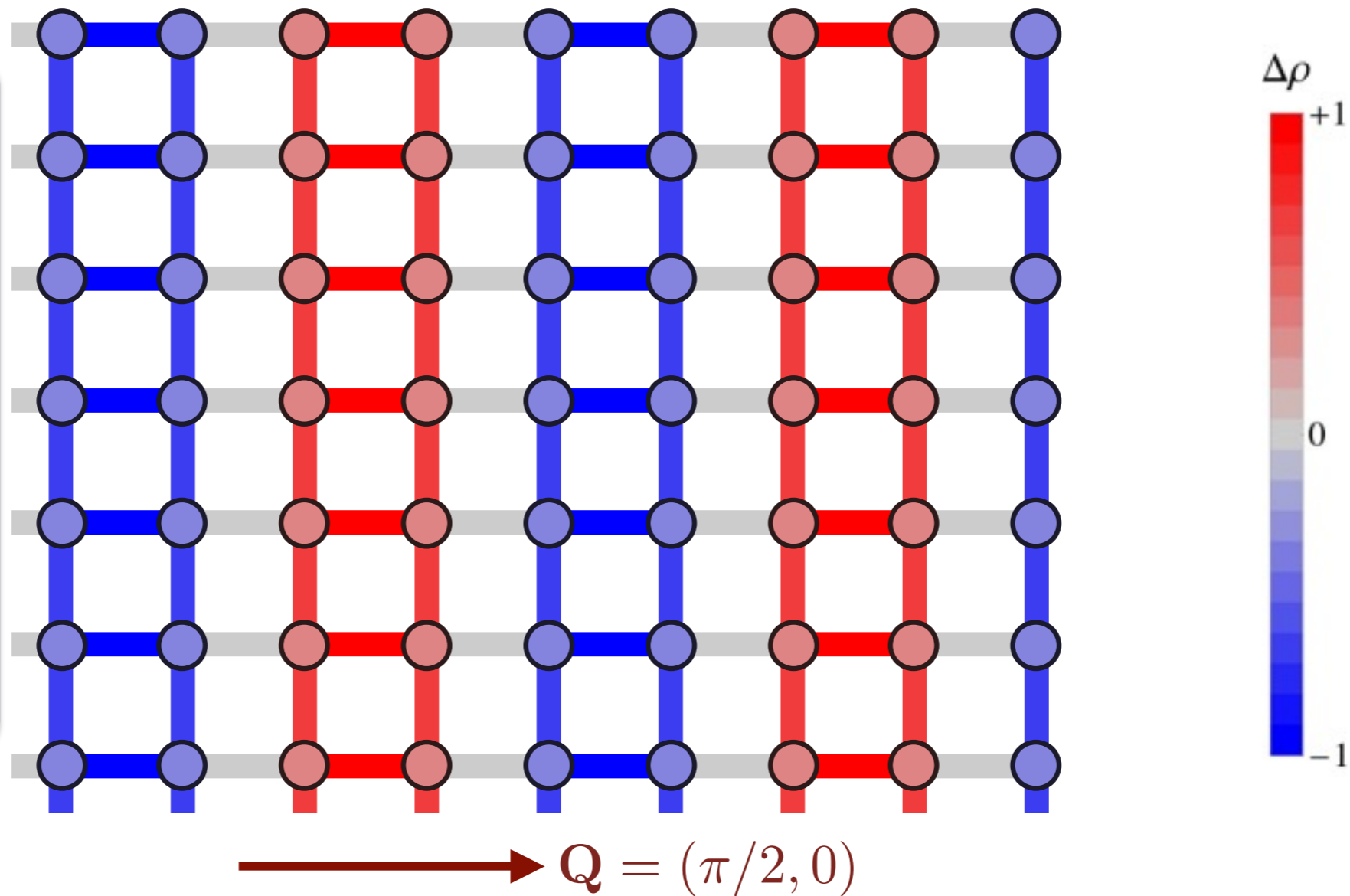
Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.



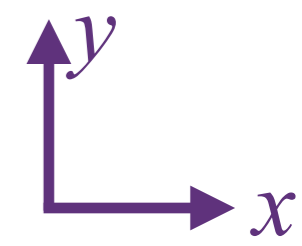
$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [1/2 + \cos(k_x) + \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

The pattern of charge densities and bond energies in the “stripe” model yields a combination of s and s' components



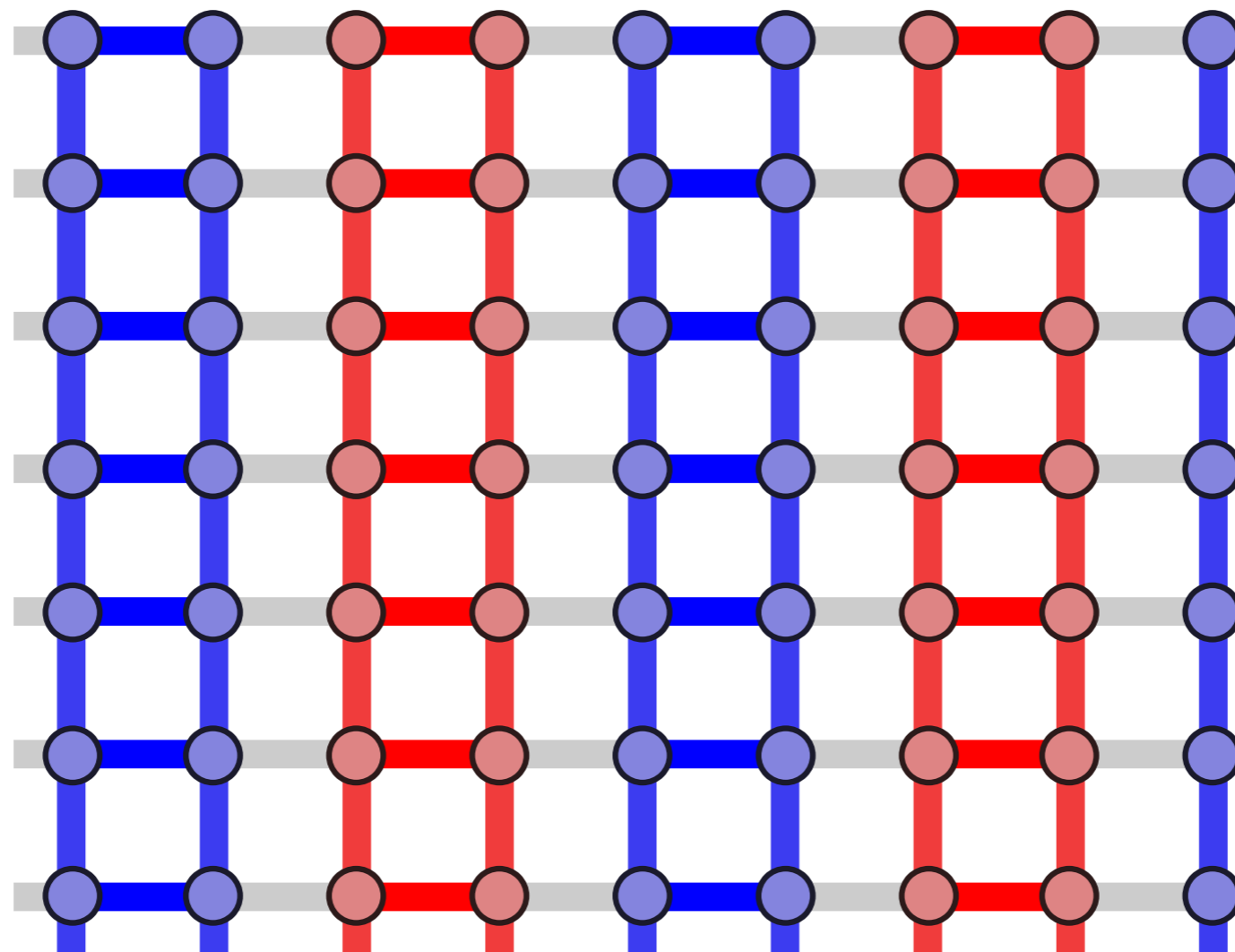
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→ $\mathbf{Q} = (\pi/2, 0)$



David Hawthorn, Waterloo

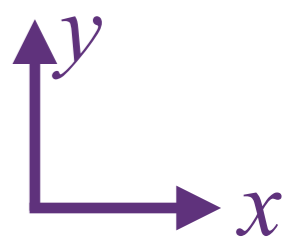
X-ray observations indicate strong s' component in LBCO

Unconventional DW order: d -form factor

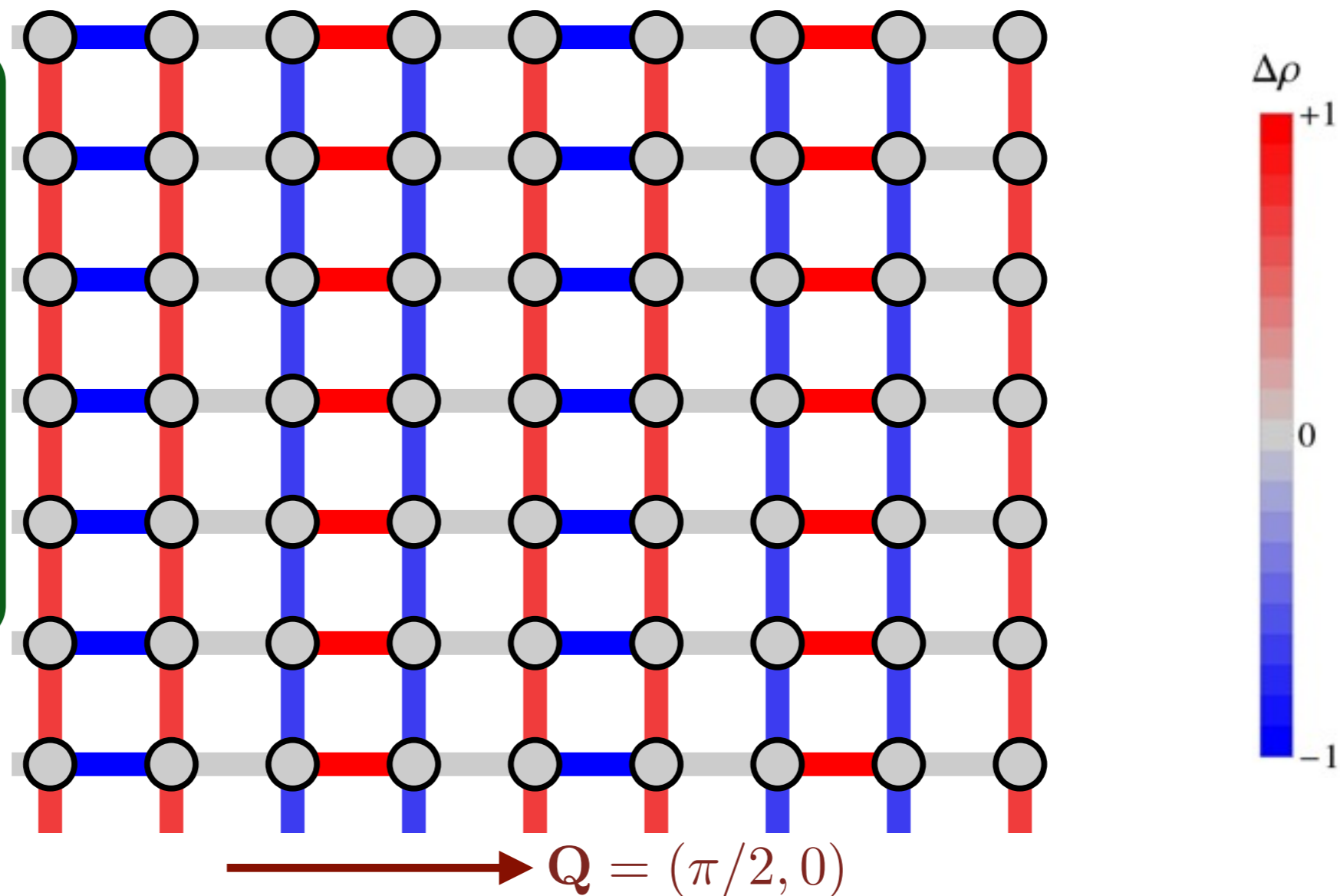
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$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$



Our prediction:
Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



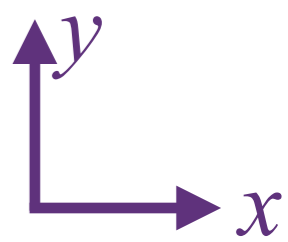
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).
S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

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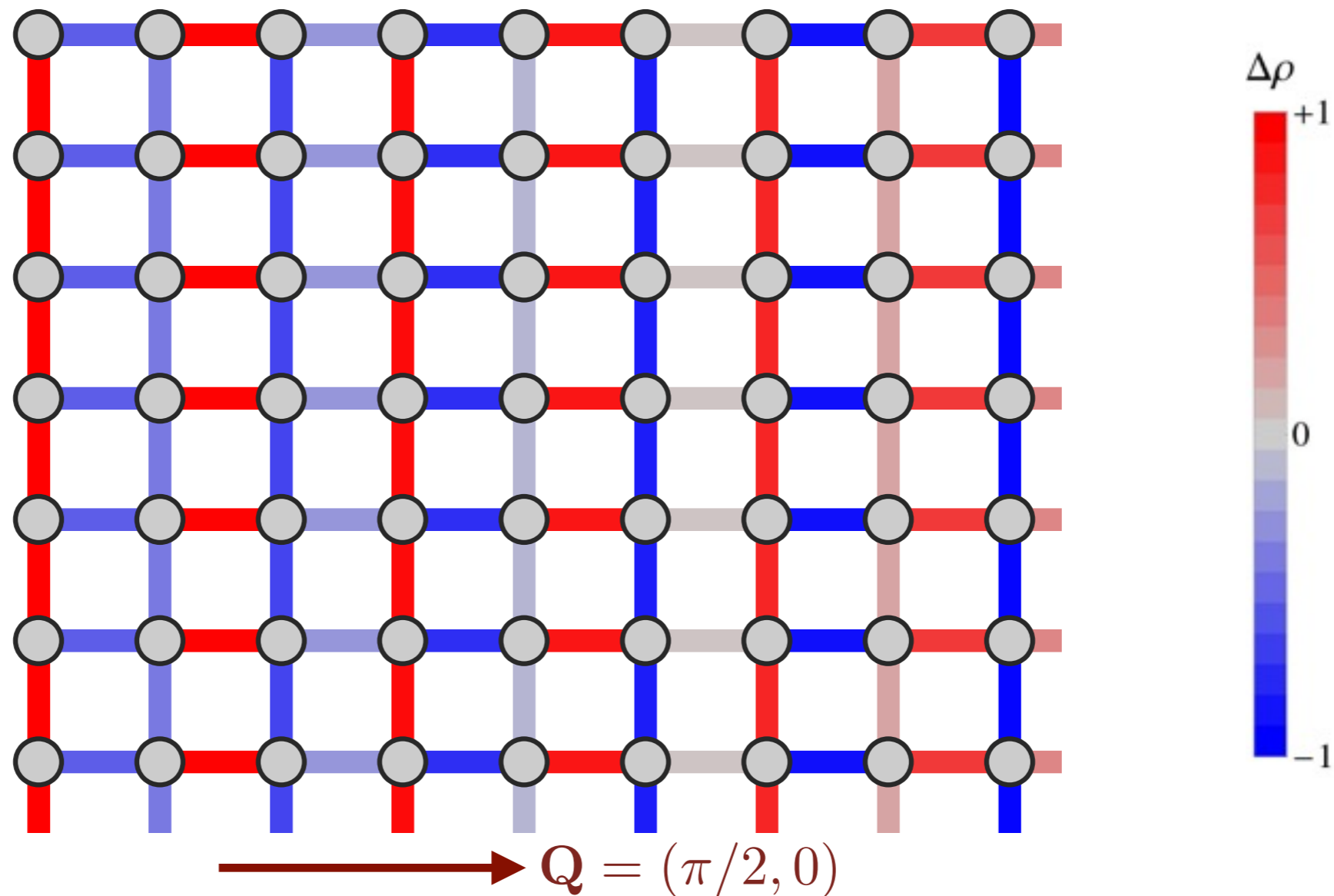
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Direct phase-sensitive identification of a d -form factor density wave in underdoped cuprates

Kazuhiro Fujita^{a,b,c,1}, Mohammad H. Hamidian^{a,b,1}, Stephen D. Edkins^{b,d}, Chung Koo Kim^a, Yuhki Kohsaka^e, Masaki Azuma^f, Mikio Takano^g, Hidenori Takagi^{c,h,i}, Hiroshi Eisaki^j, Shin-ichi Uchida^c, Andrea Allais^k, Michael J. Lawler^{b,l}, Eun-Ah Kim^b, Subir Sachdev^{k,m}, and J. C. Séamus Davis^{a,b,d,2}

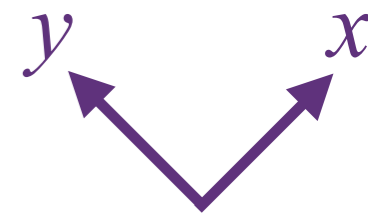
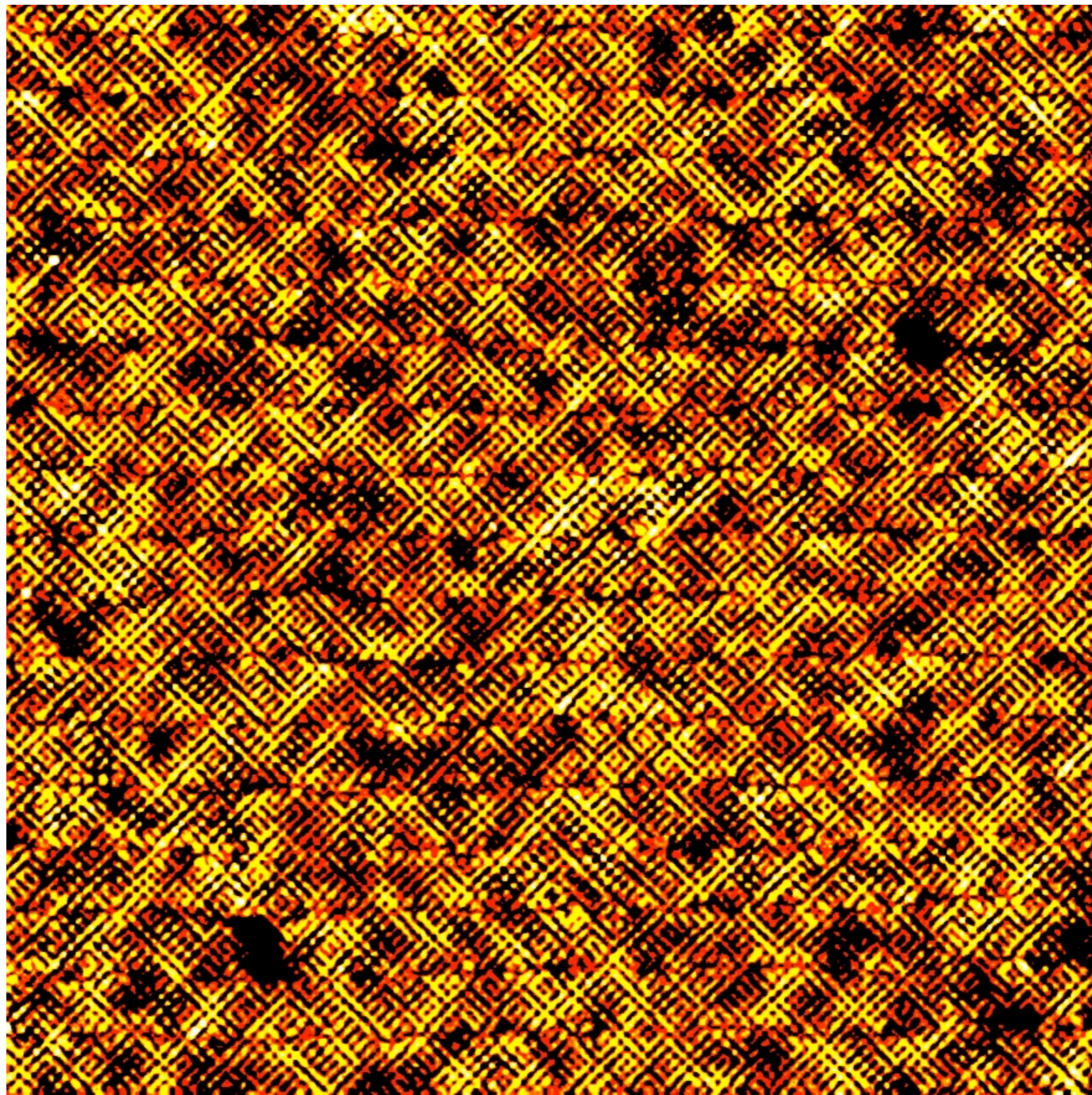
The identity of the fundamental broken symmetry (if any) in the underdoped cuprates is unresolved. However, evidence has been accumulating that this state may be an unconventional density wave. Here we carry out site-specific measurements within each CuO_2 unit cell, segregating the results into three separate electronic structure images containing only the Cu sites [$\text{Cu}(r)$] and only the x/y axis O sites [$\text{O}_x(r)$ and $\text{O}_y(r)$]. Phase-resolved Fourier analysis reveals directly that the modulations in the $\text{O}_x(r)$ and $\text{O}_y(r)$ sublattice images consistently exhibit a relative phase of π . We confirm this discovery on two highly distinct cuprate compounds, ruling out tunnel matrix-element and materials-specific systematics. These observations demonstrate by direct sublattice phase-resolved visualization that the density wave found in underdoped cuprates consists of modulations of the intraunit-cell states that exhibit a predominantly d -symmetry form factor.

See also

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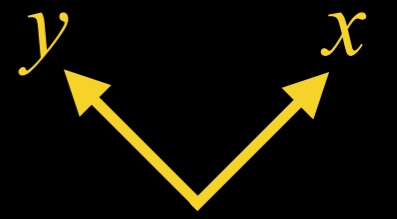
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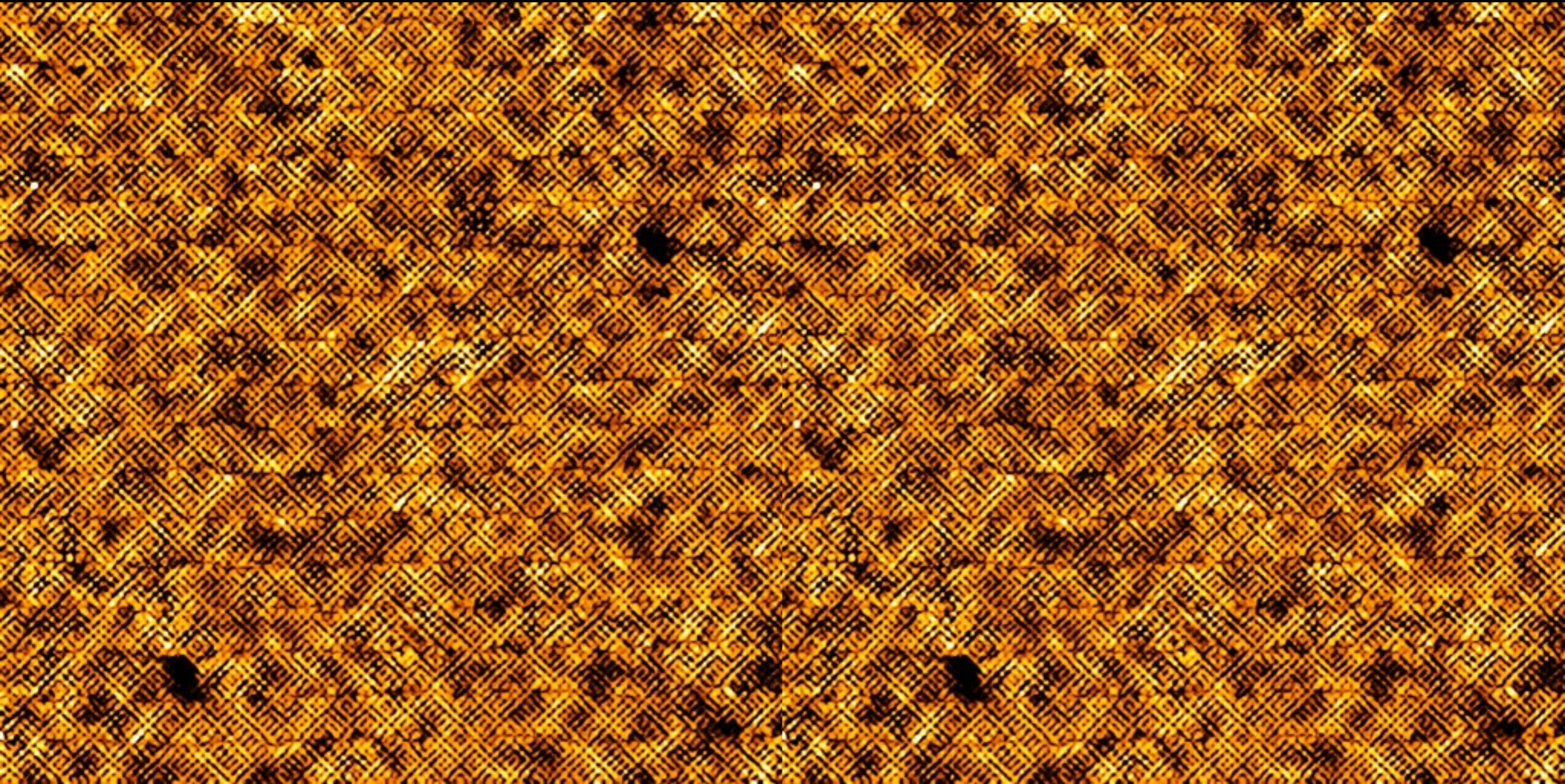
UD45K
BSCCO

$R(r, 150\text{mV})$



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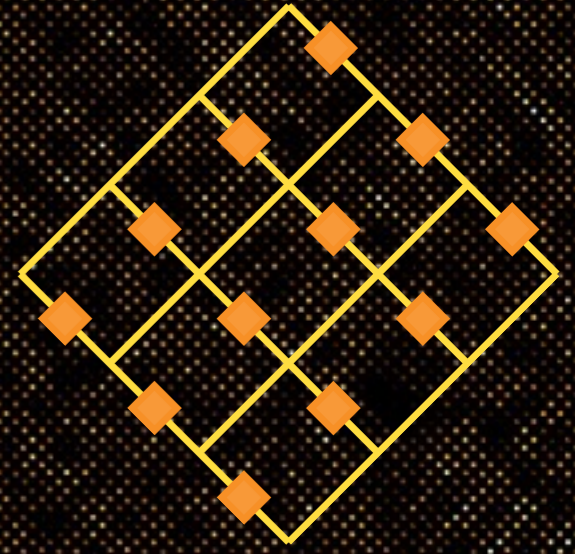
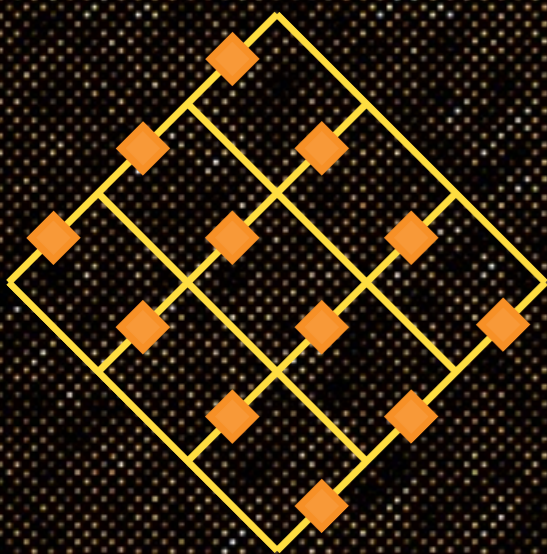
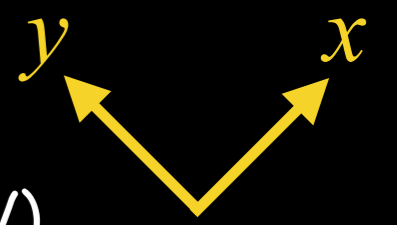
Note that these are identical images.

UD45K

$R(r=0, 150\text{mV})$

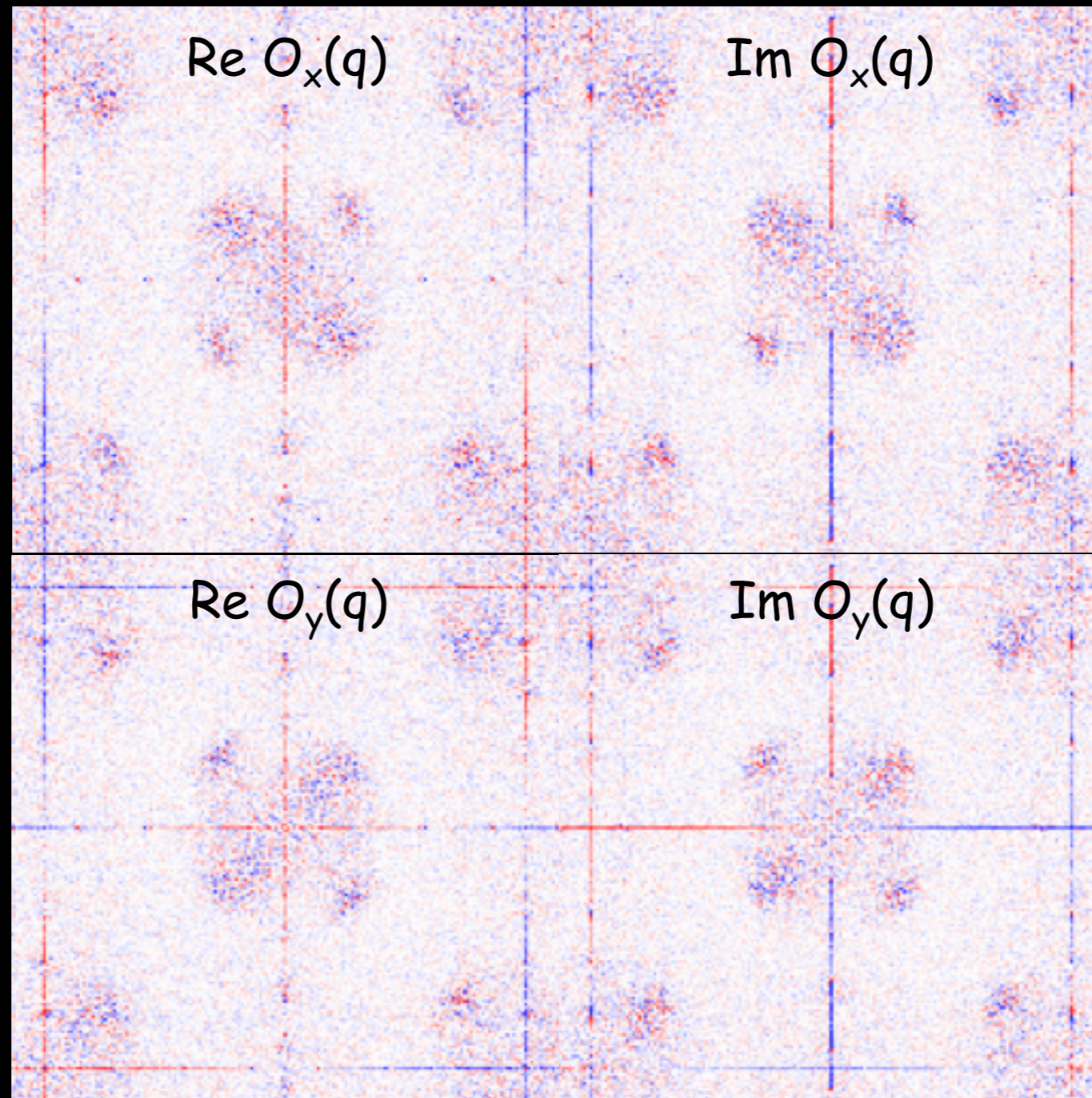
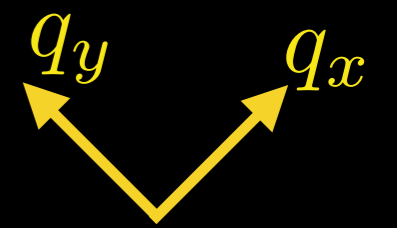
$R(r=O_x, 150\text{mV})$

$R(r=O_y, 150\text{mV})$

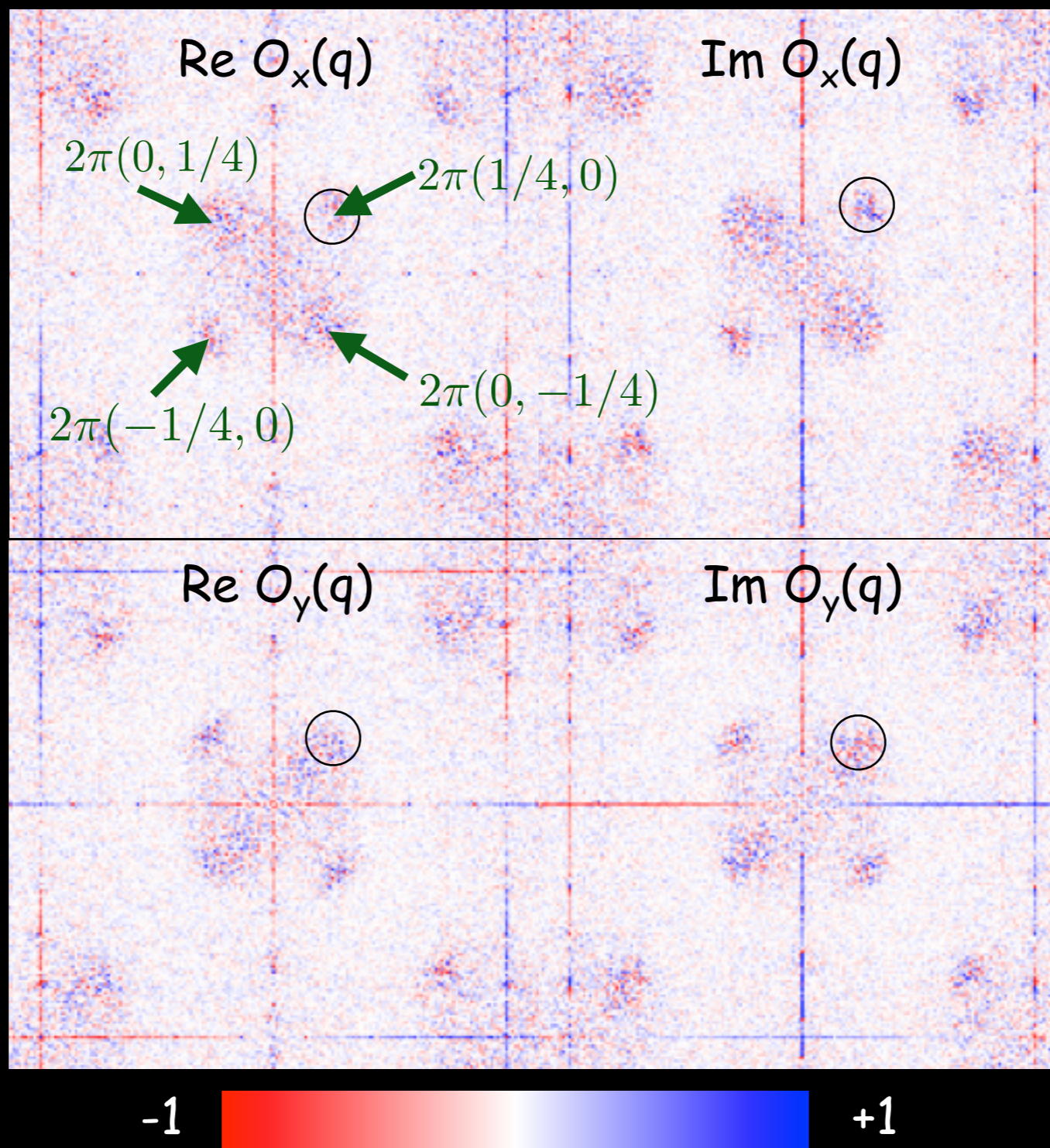
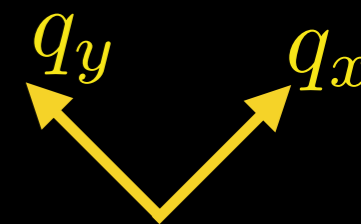


UD45K

Broad (0,Q) and (Q,0) DW Features

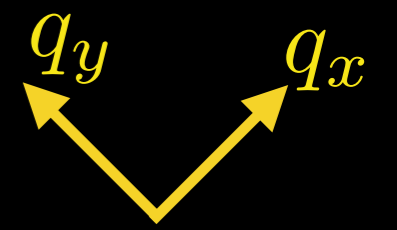


Broad (0,Q) and (Q,0) DW Features

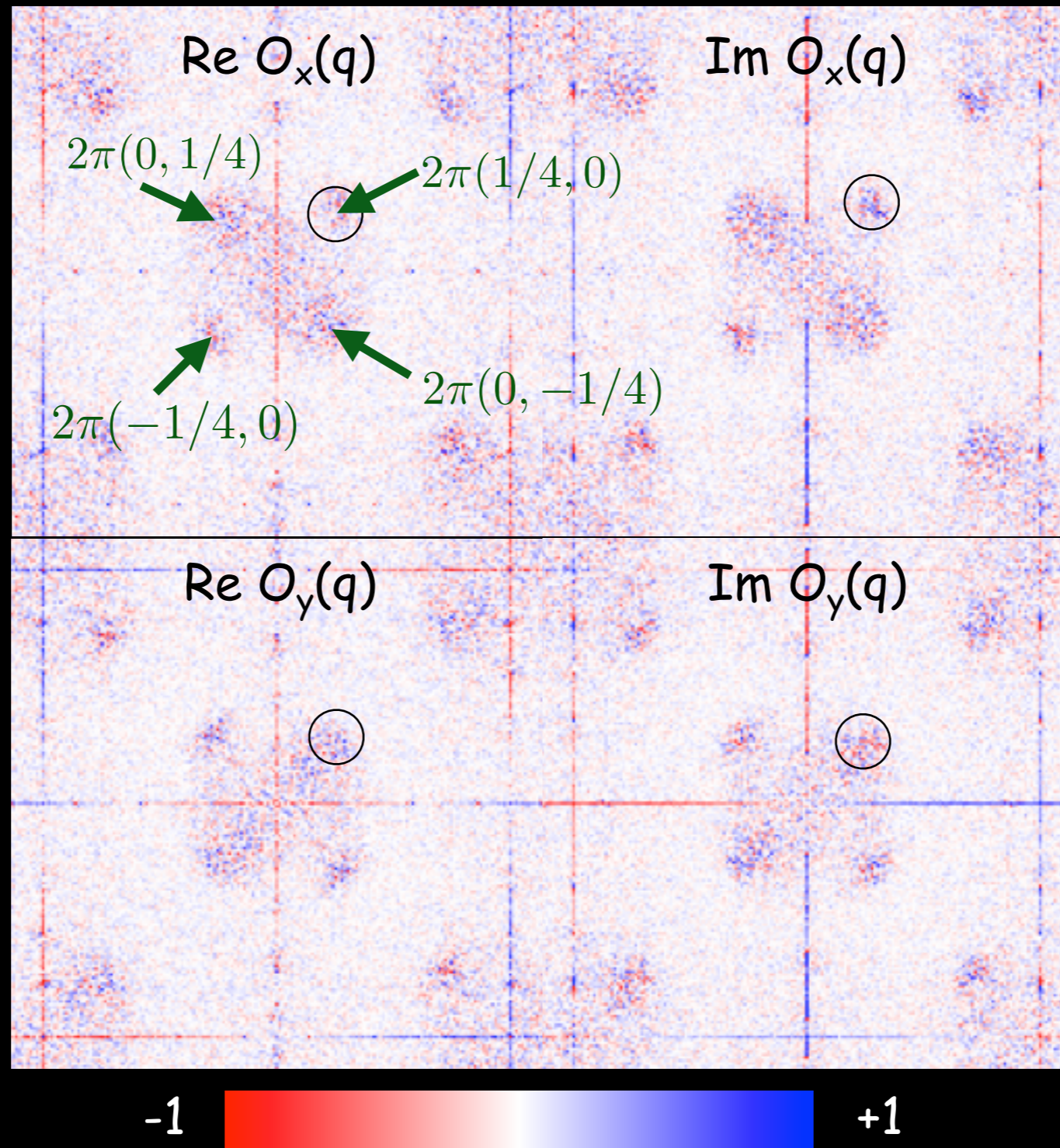


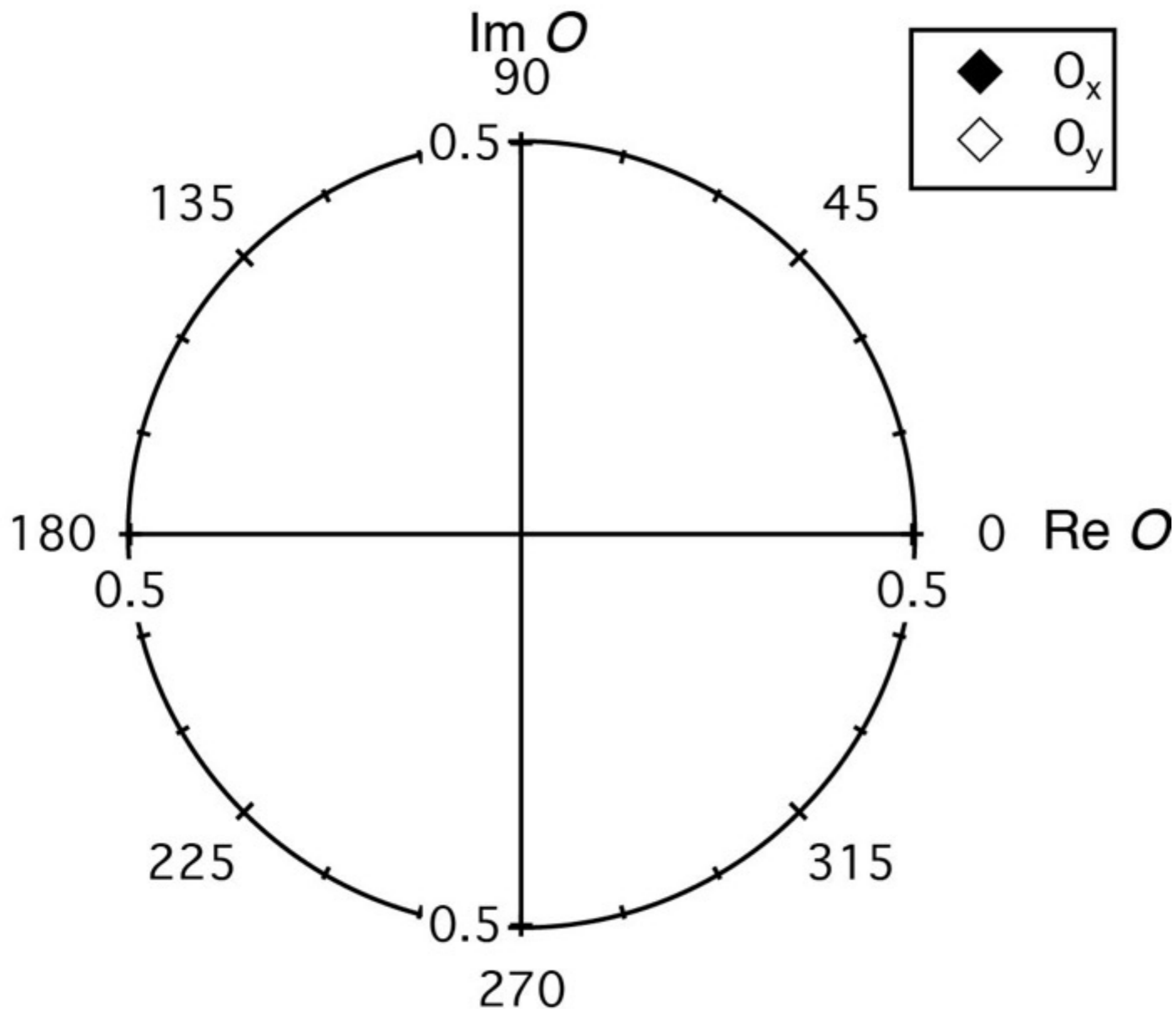
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Broad (0,Q) and (Q,0) DW Features

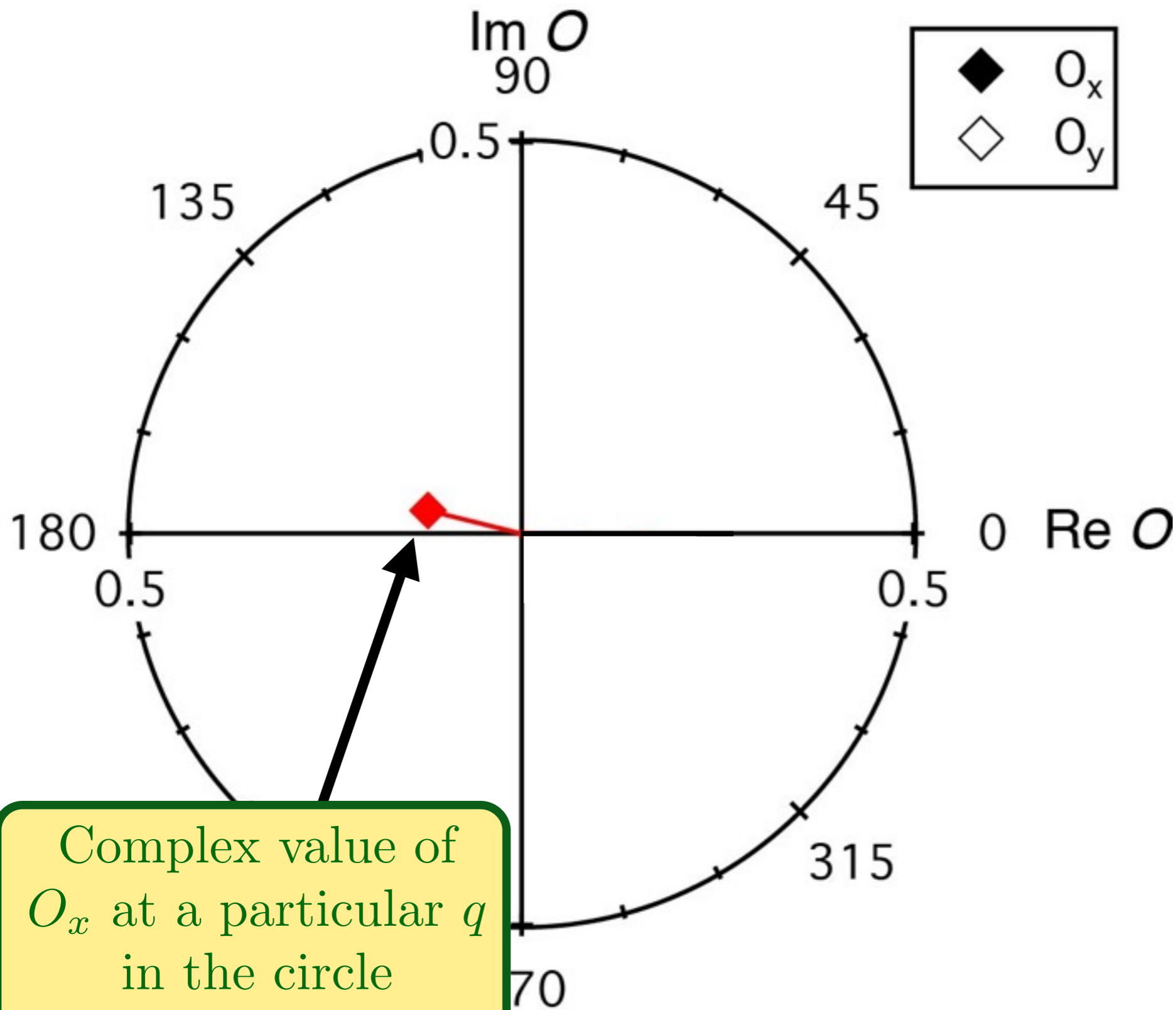


For each pixel in the circles, we obtain 2 complex numbers, $O_x(q)$ and $O_y(q)$.



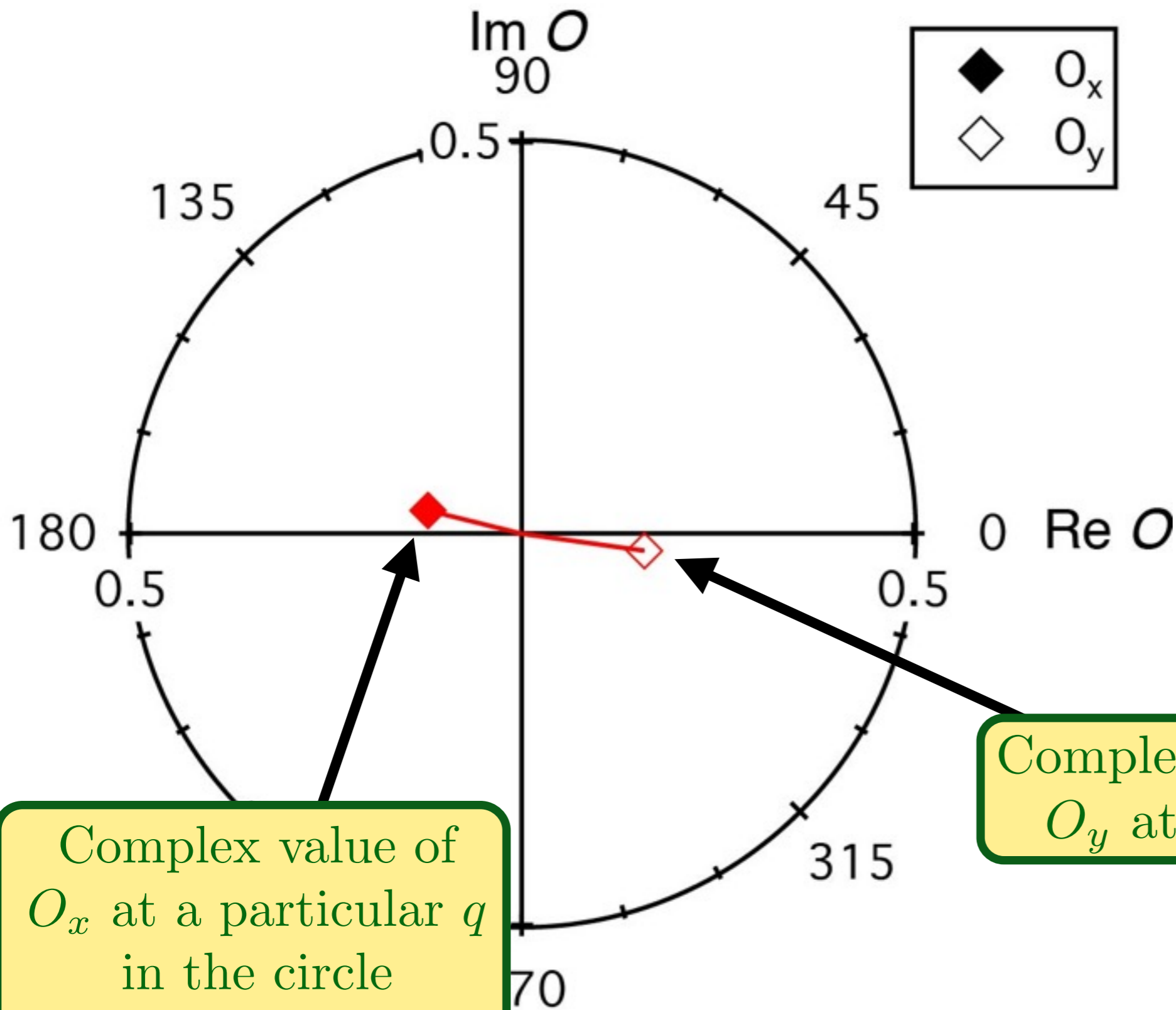


Phase-sensitive measurement of the d -form factor of density wave order



Phase-sensitive measurement of the d -form factor of density wave order

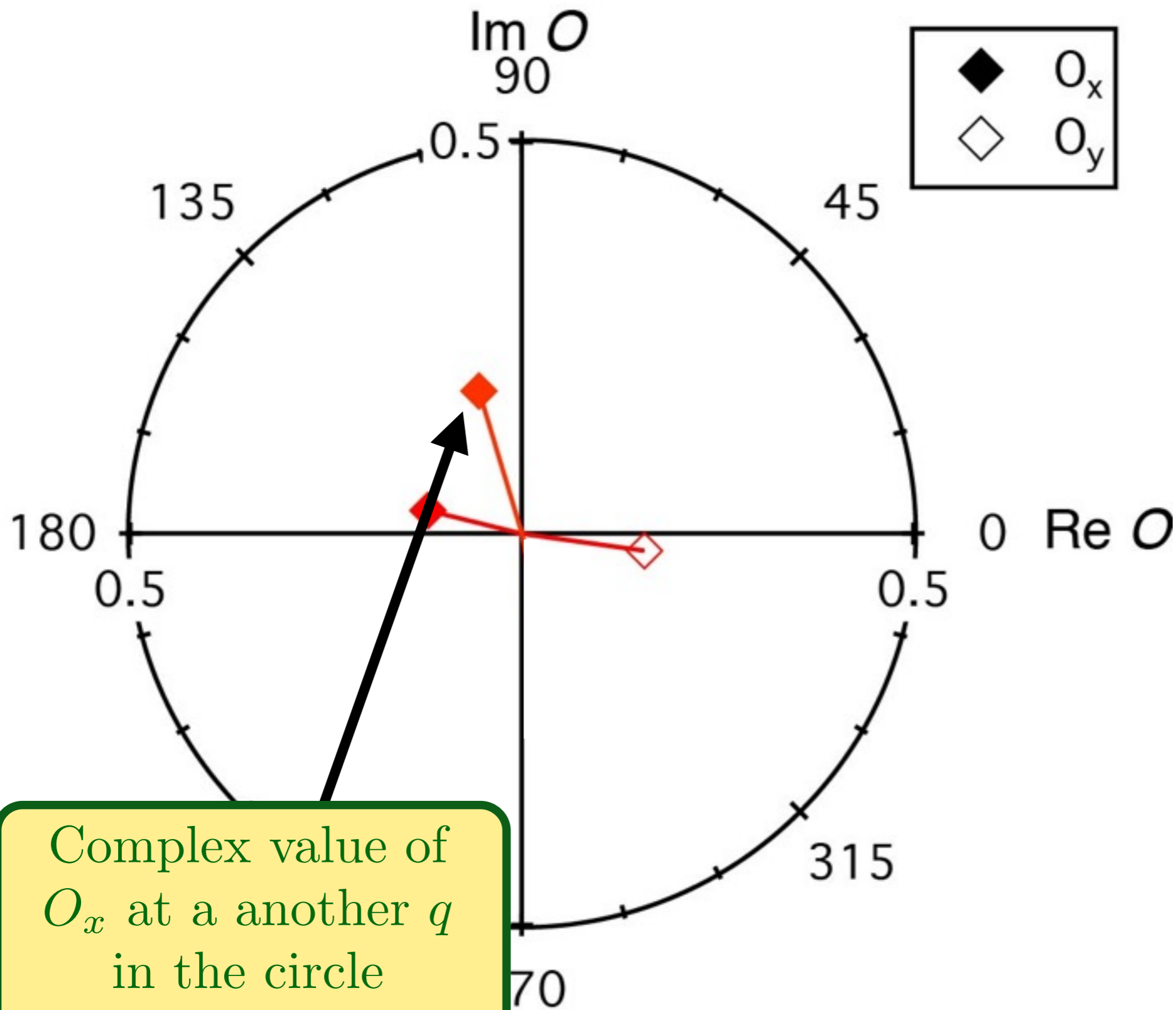
Complex value of O_x at a particular q in the circle around $2\pi(1/4, 0)$.



Phase-sensitive measurement of the d -form factor of density wave order

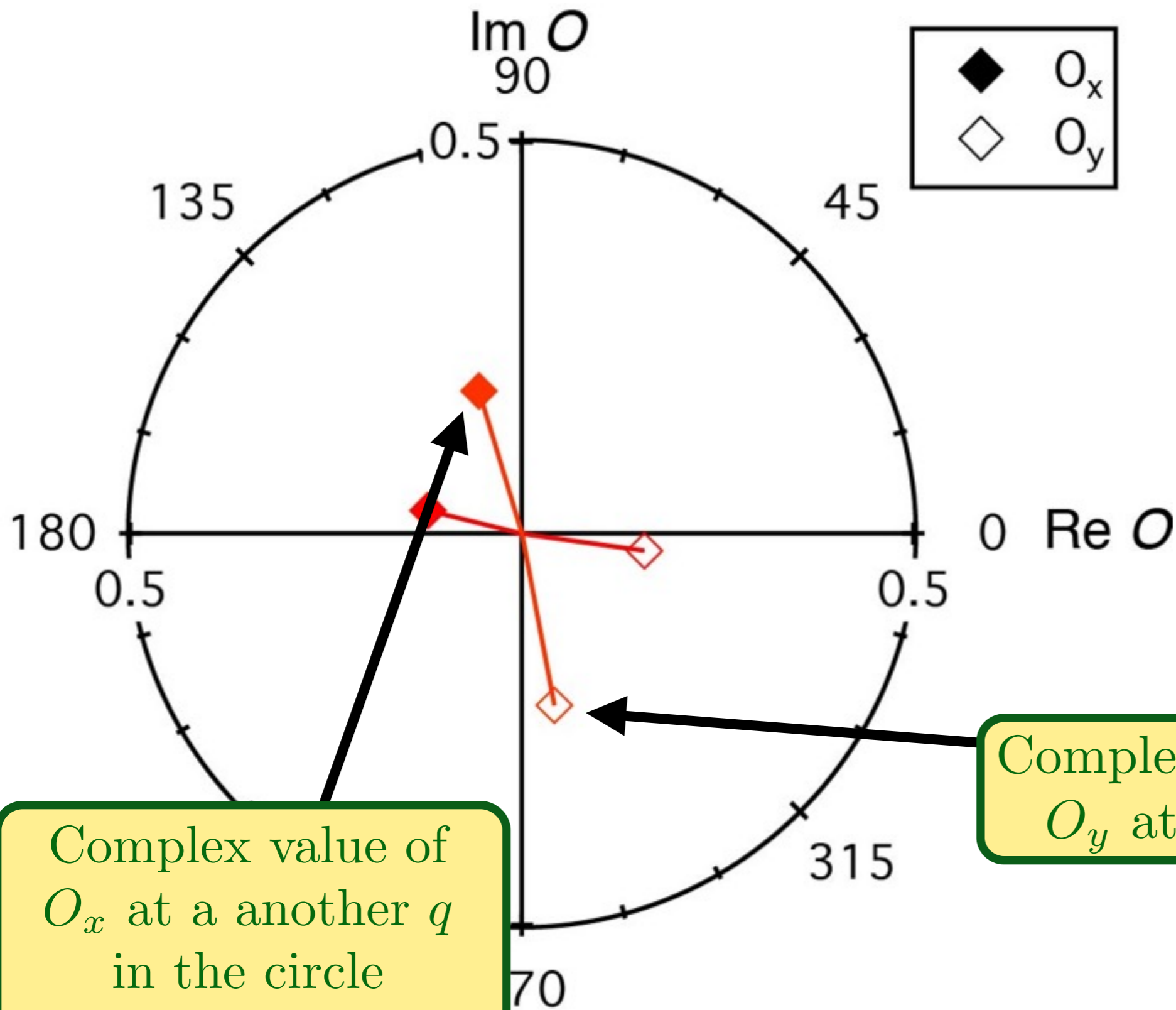
Complex value of O_x at a particular q in the circle around $2\pi(1/4, 0)$.

Complex value of O_y at same q



Phase-sensitive measurement of the d -form factor of density wave order

Complex value of O_x at a another q in the circle around $2\pi(1/4, 0)$.

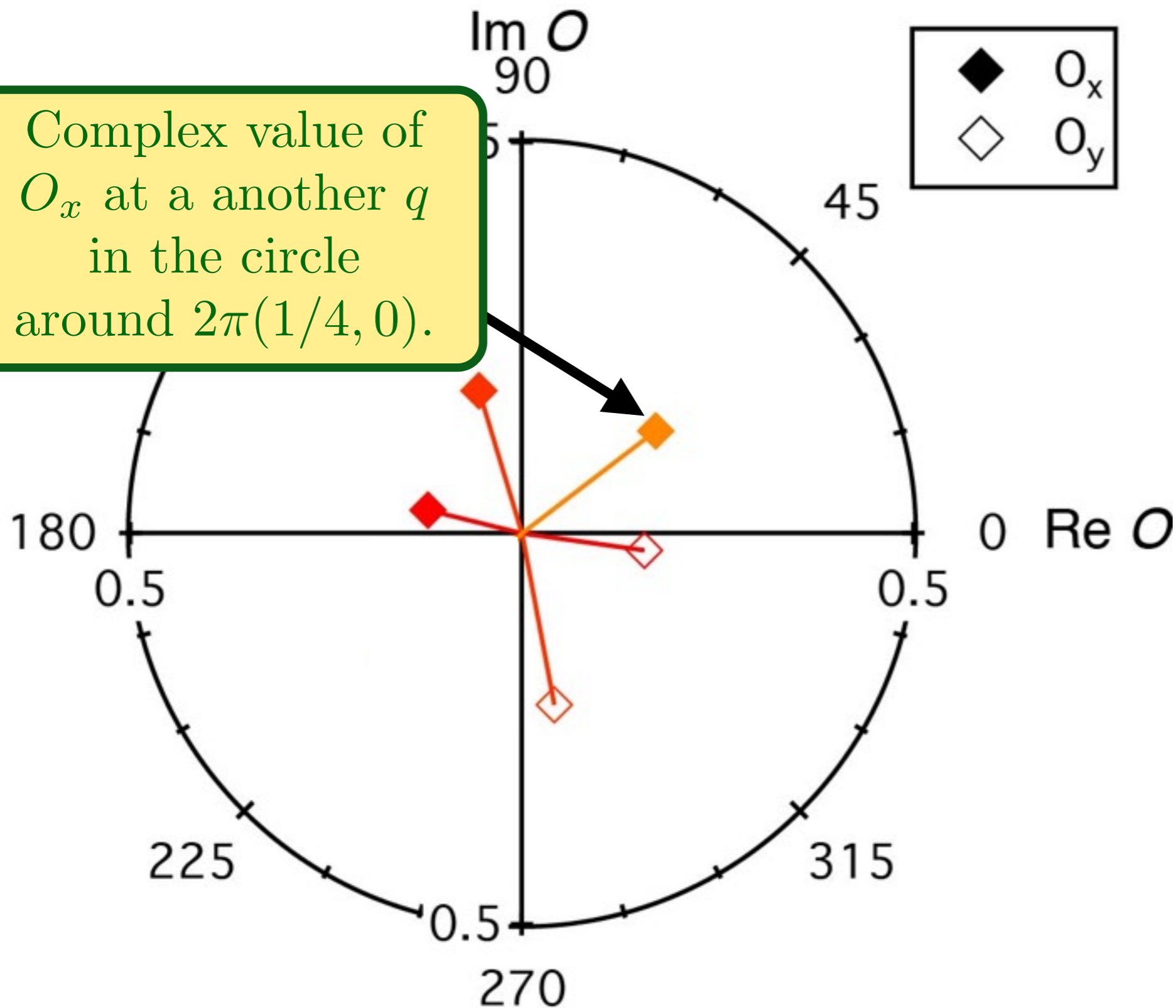


Phase-sensitive measurement of the d -form factor of density wave order

Complex value of O_x at a another q in the circle around $2\pi(1/4, 0)$.

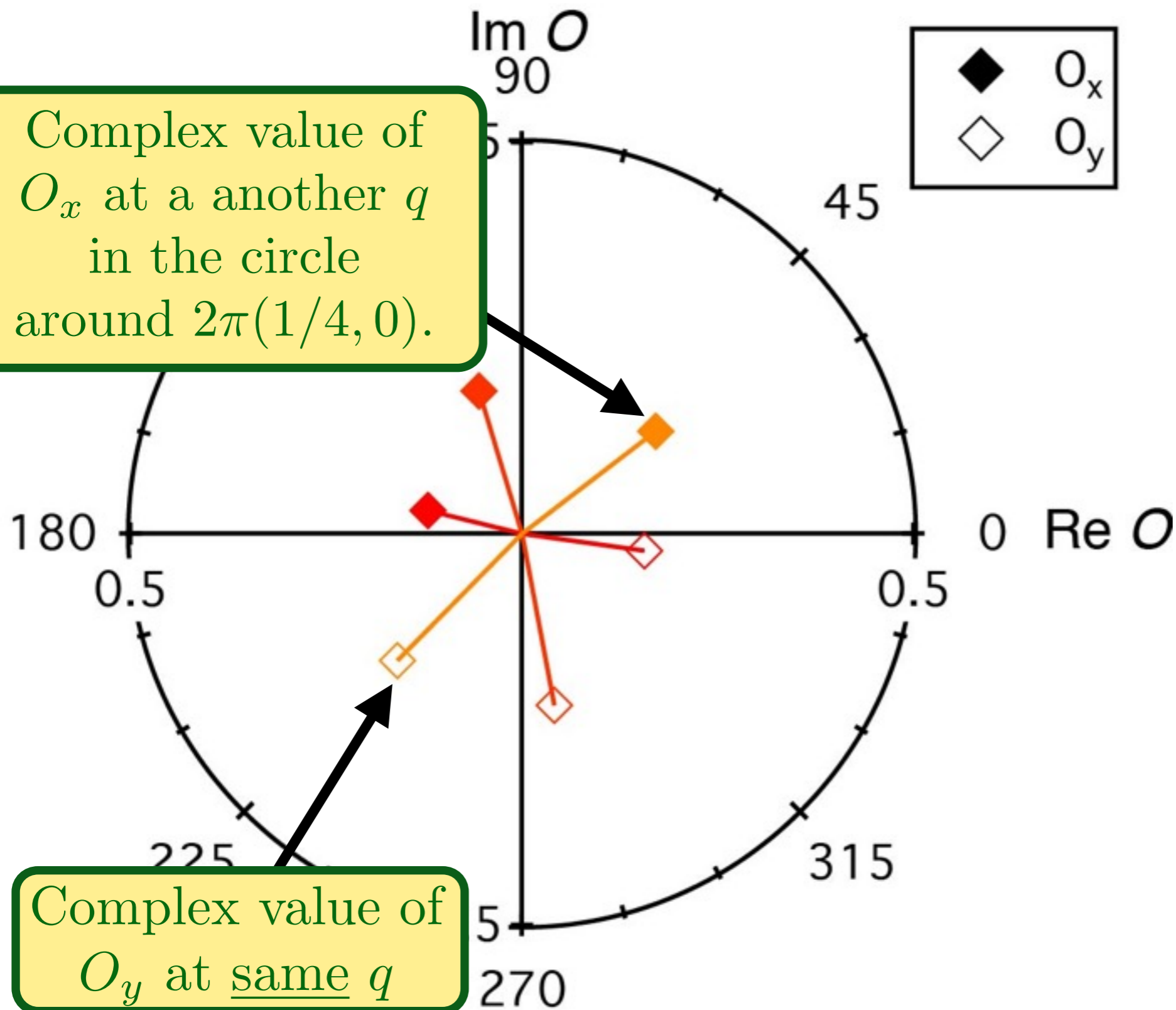
Complex value of O_y at same q

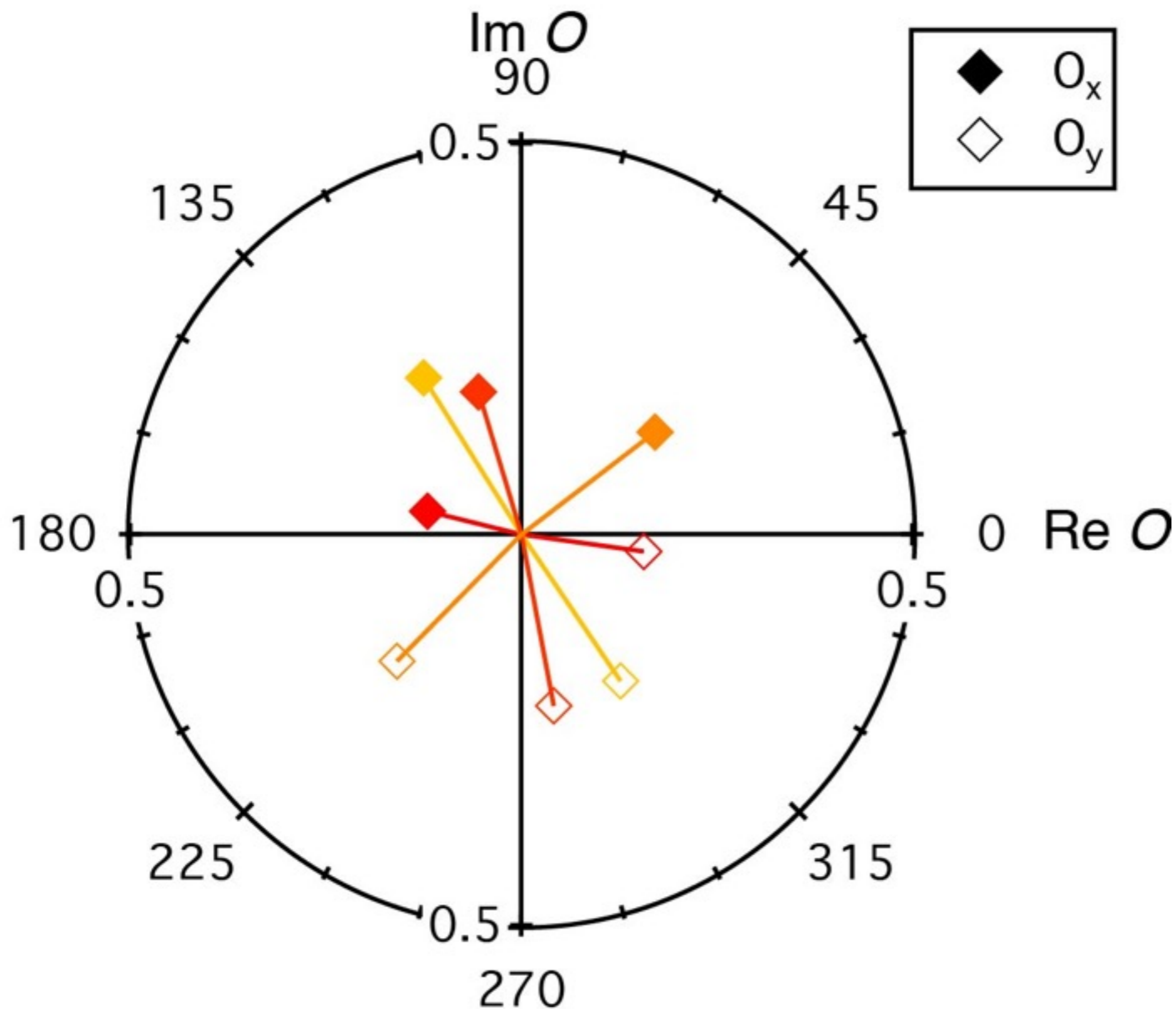
Complex value of O_x at a another q in the circle around $2\pi(1/4, 0)$.



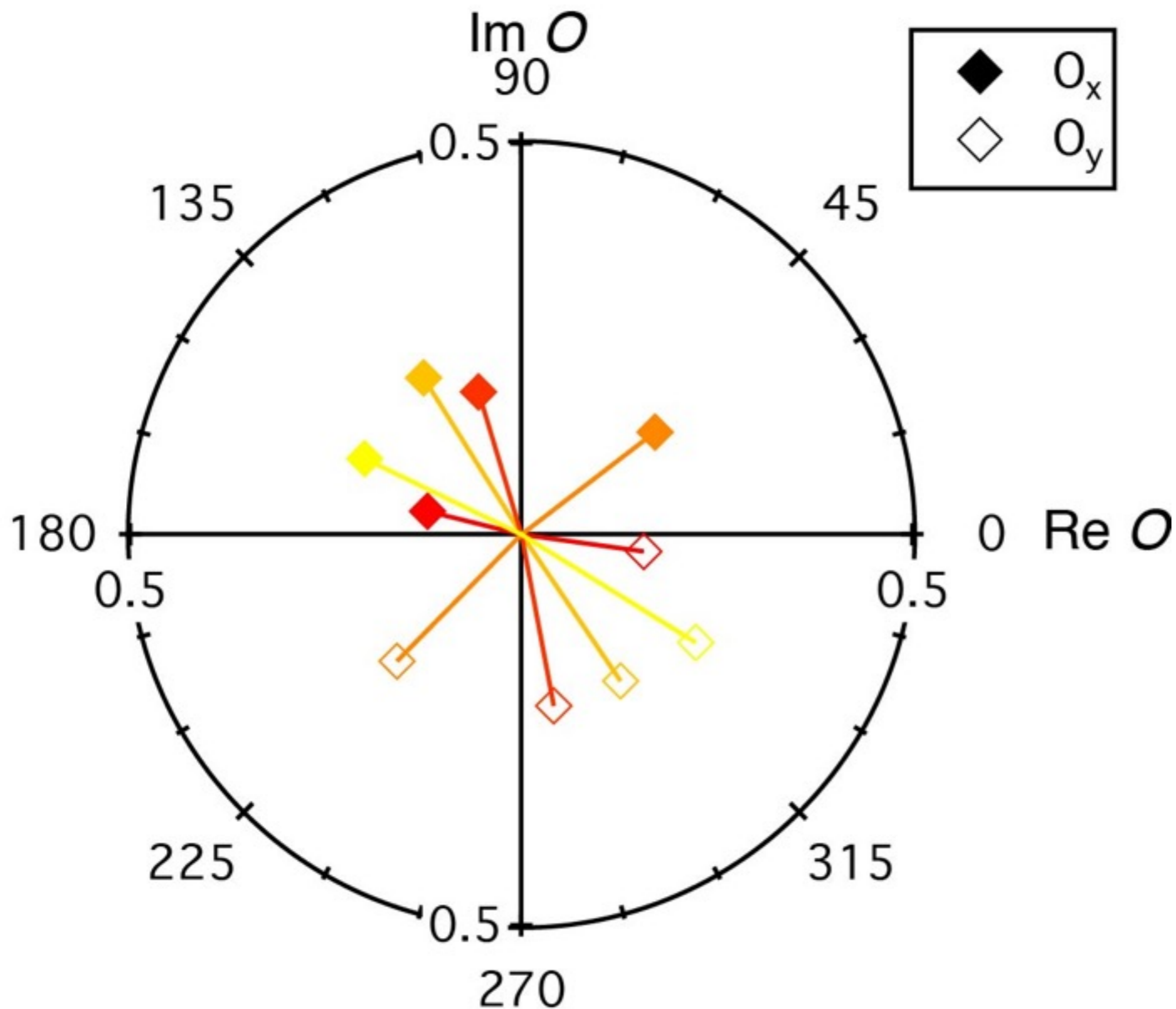
Phase-sensitive measurement of the d -form factor of density wave order

Phase-sensitive measurement of the d -form factor of density wave order

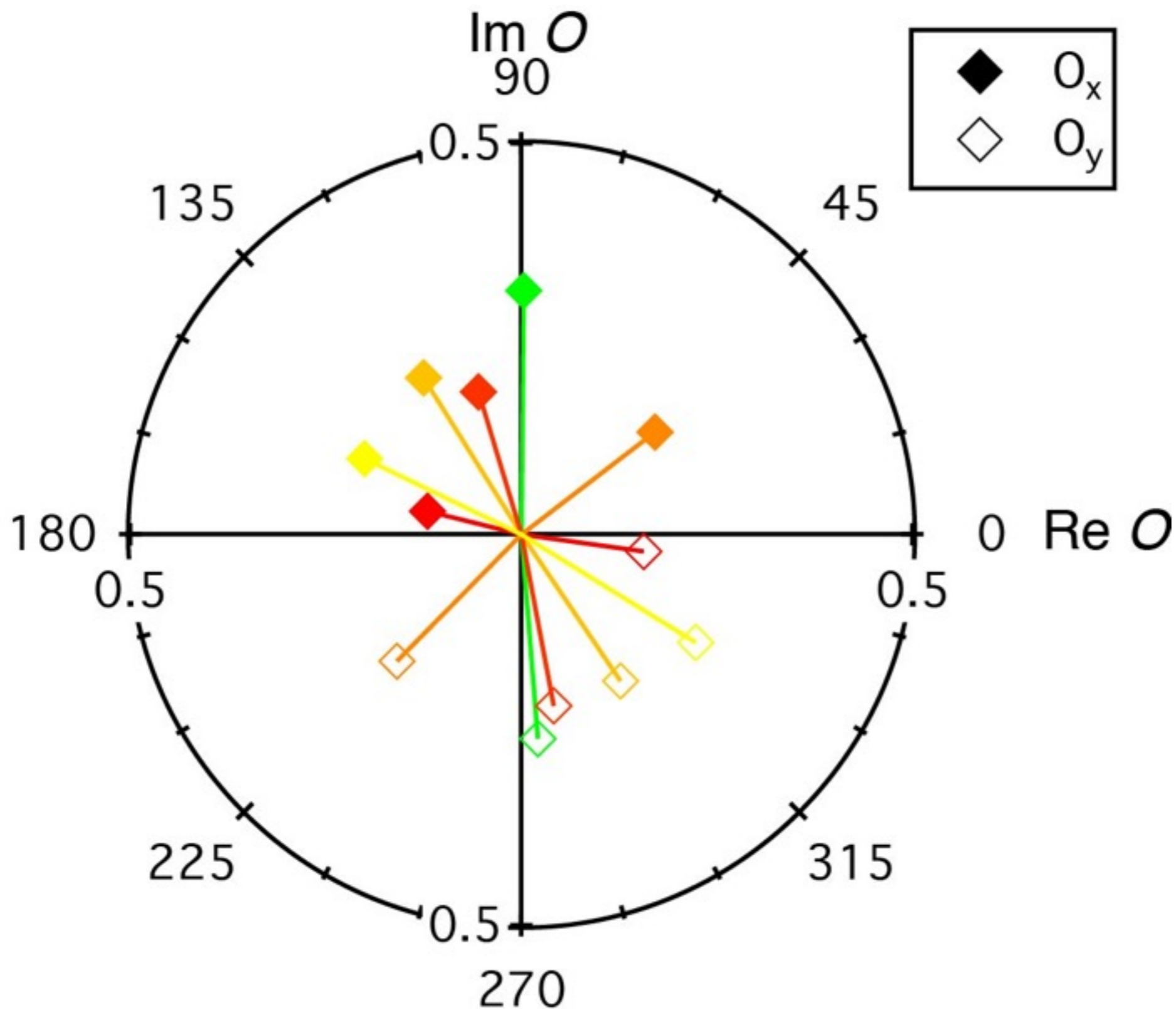




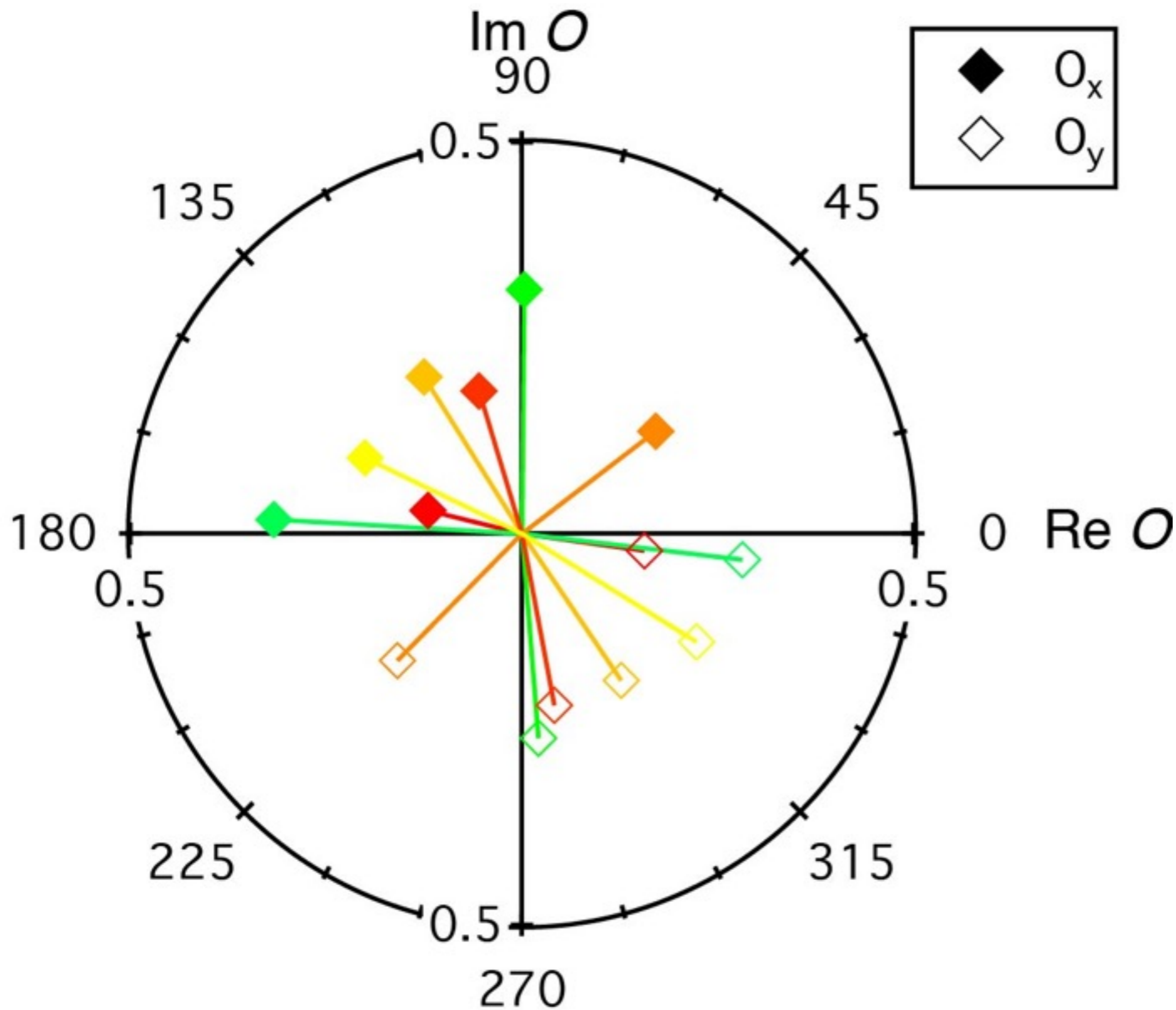
Phase-sensitive measurement of the d -form factor of density wave order



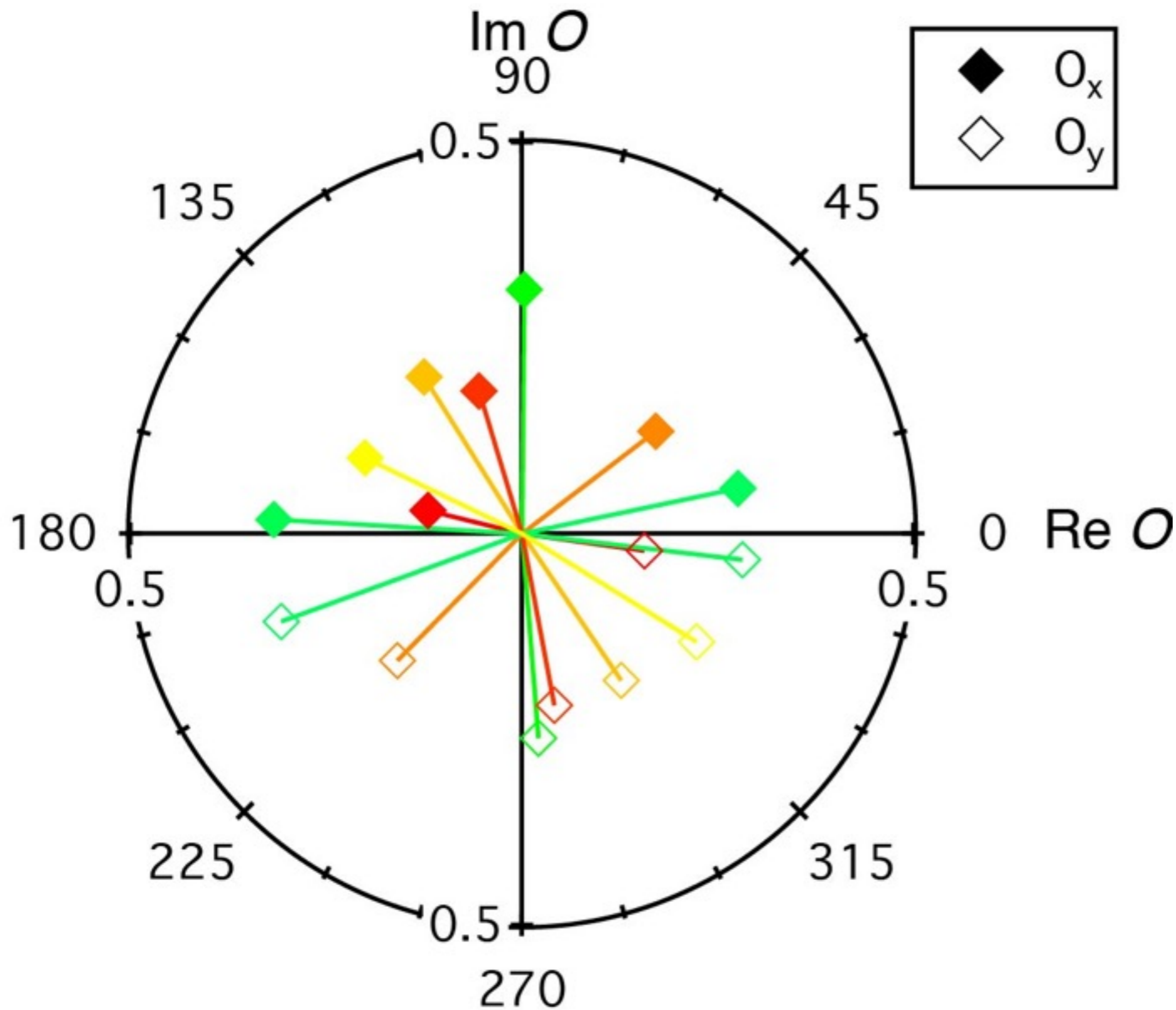
Phase-sensitive measurement of the d -form factor of density wave order



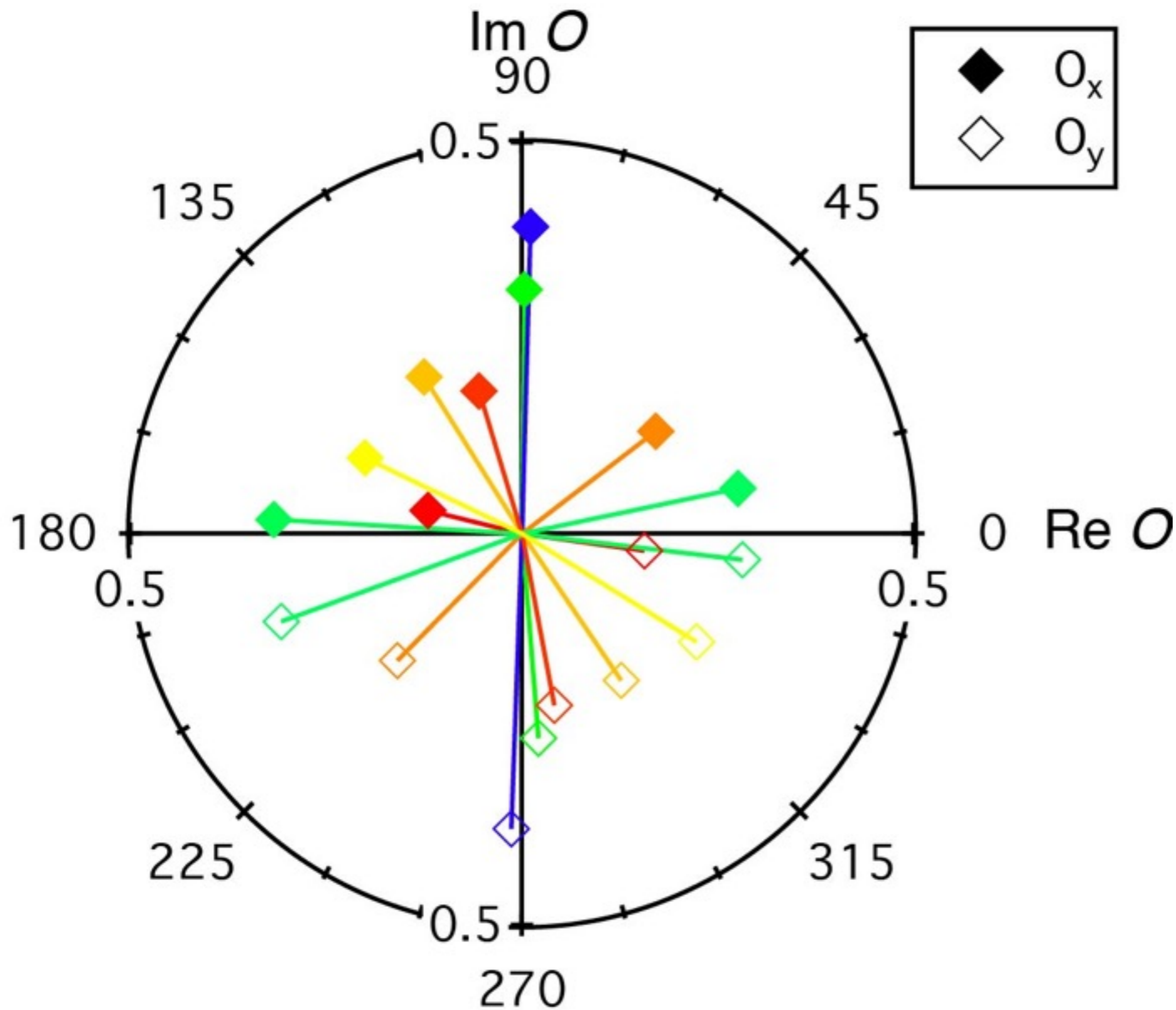
**Phase-sensitive
measurement of
the d -form factor
of density wave
order**



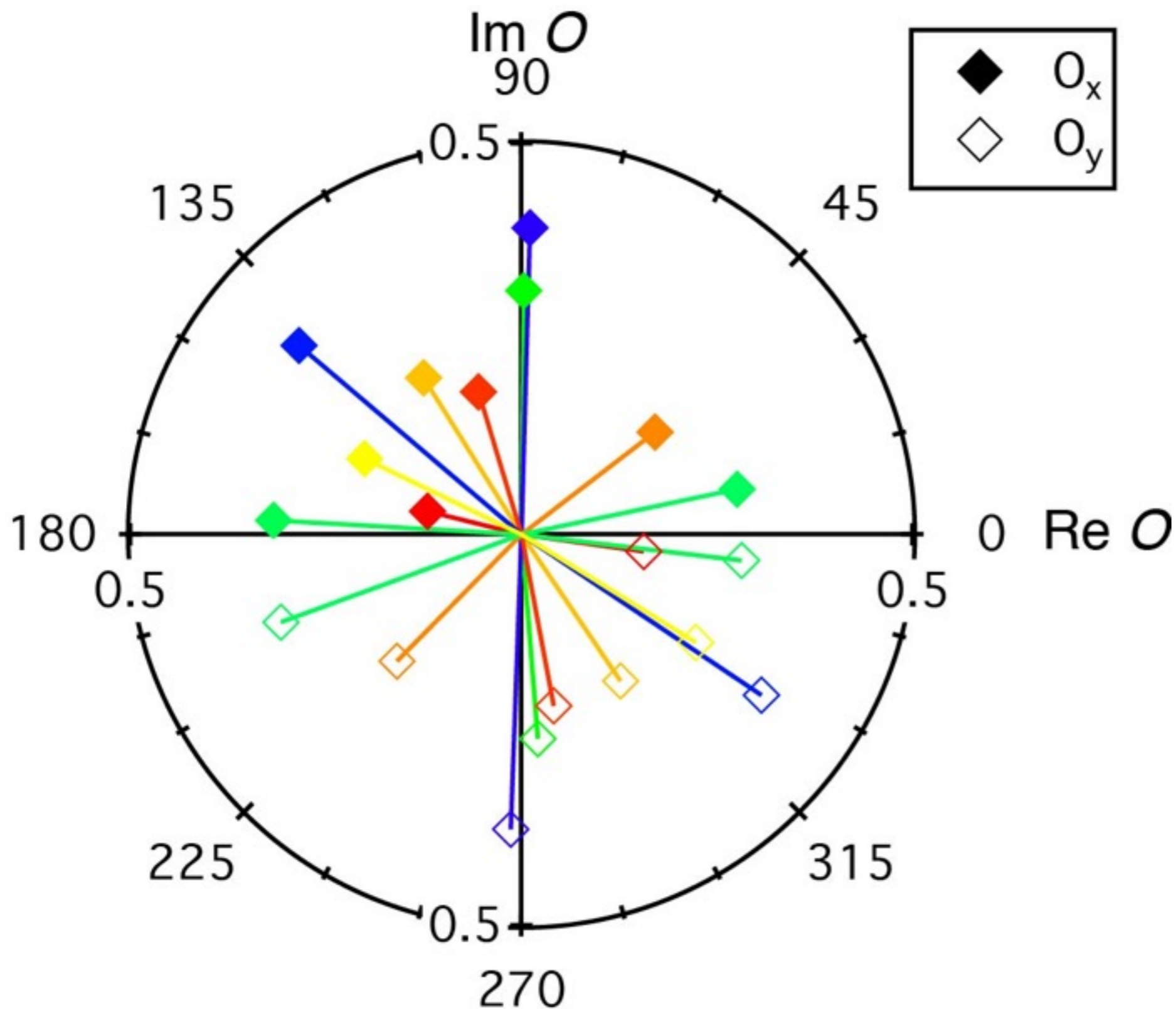
Phase-sensitive measurement of the *d*-form factor of density wave order



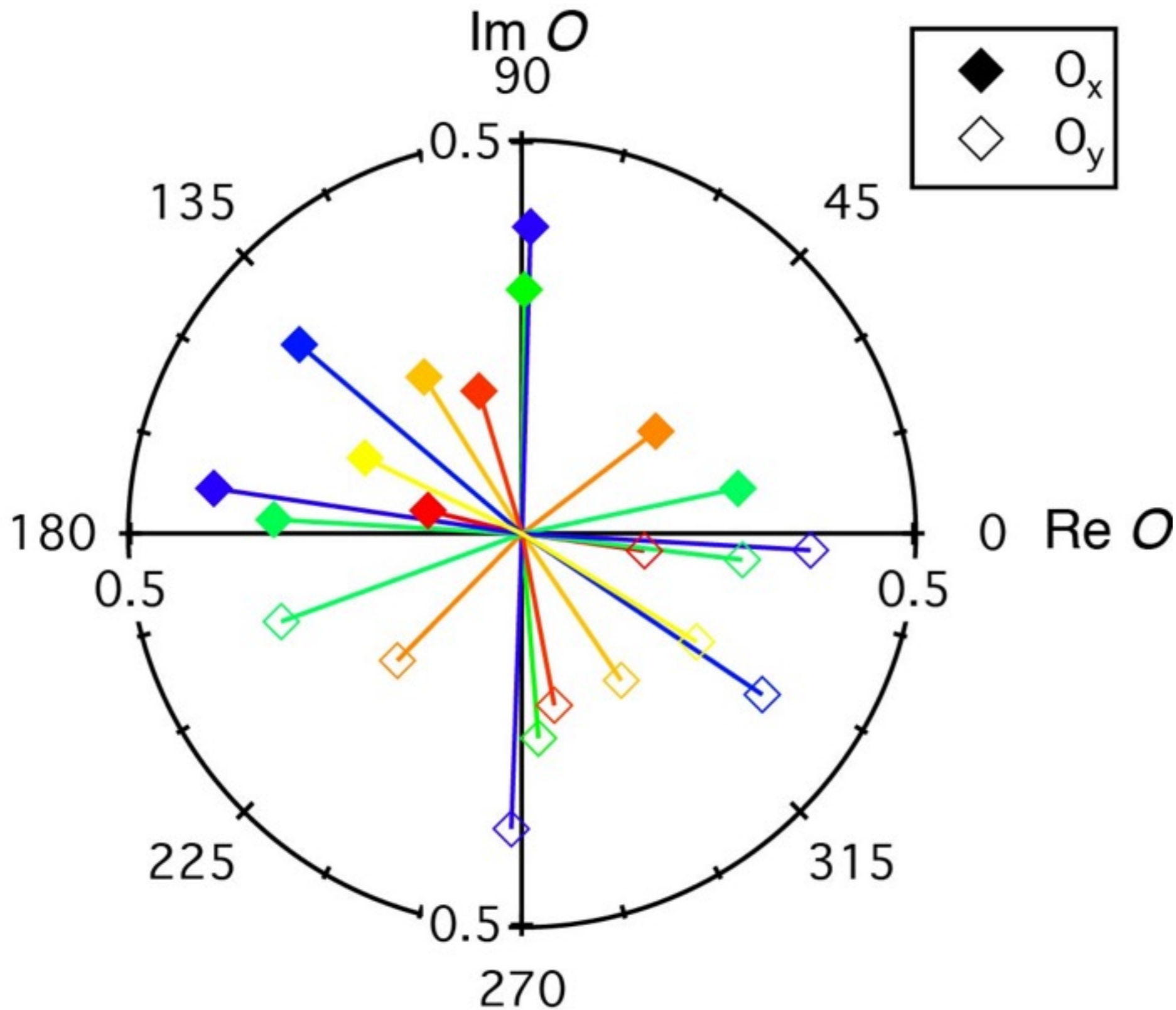
Phase-sensitive measurement of the d -form factor of density wave order



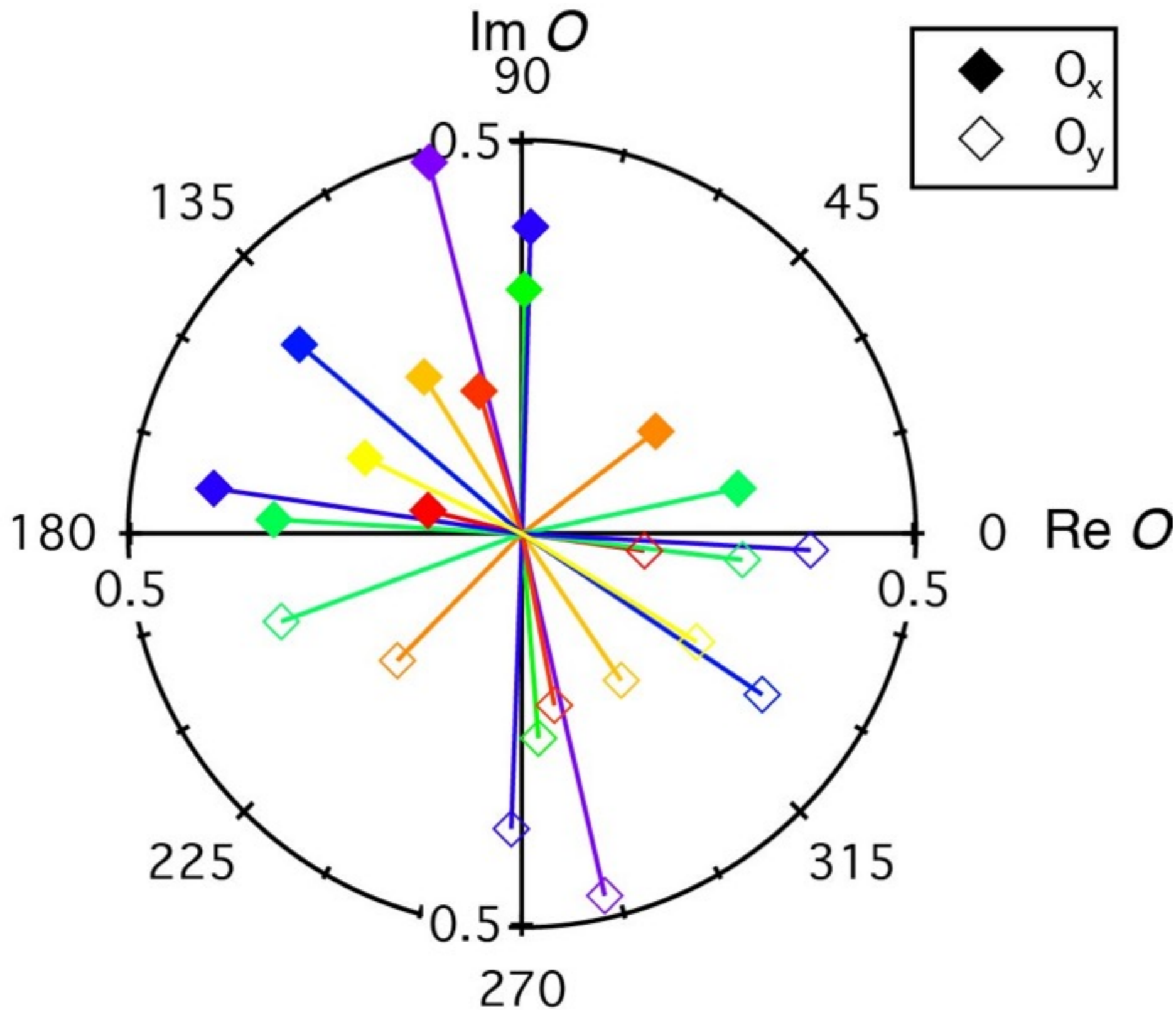
Phase-sensitive measurement of the d -form factor of density wave order



**Phase-sensitive
measurement of
the d -form factor
of density wave
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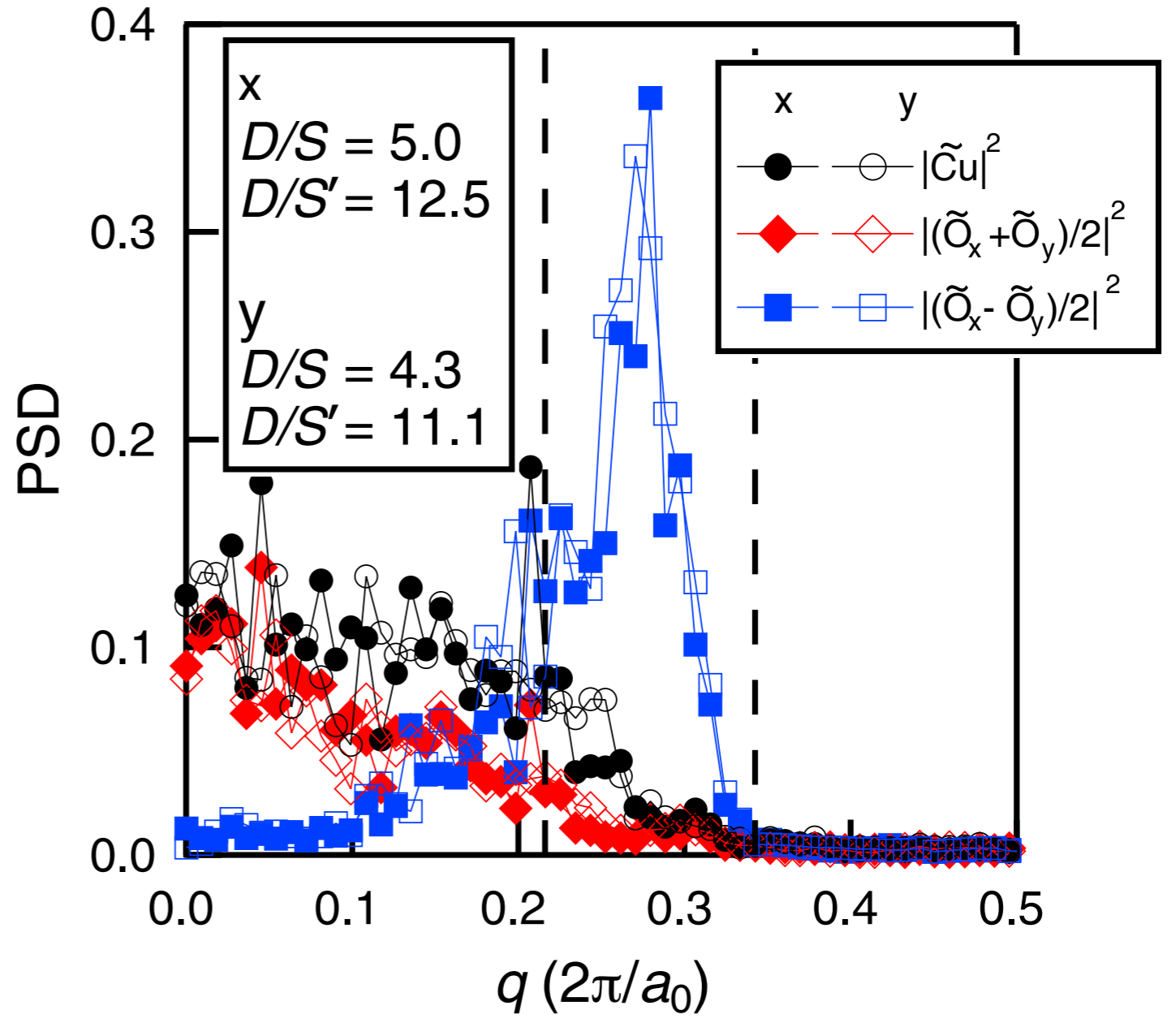
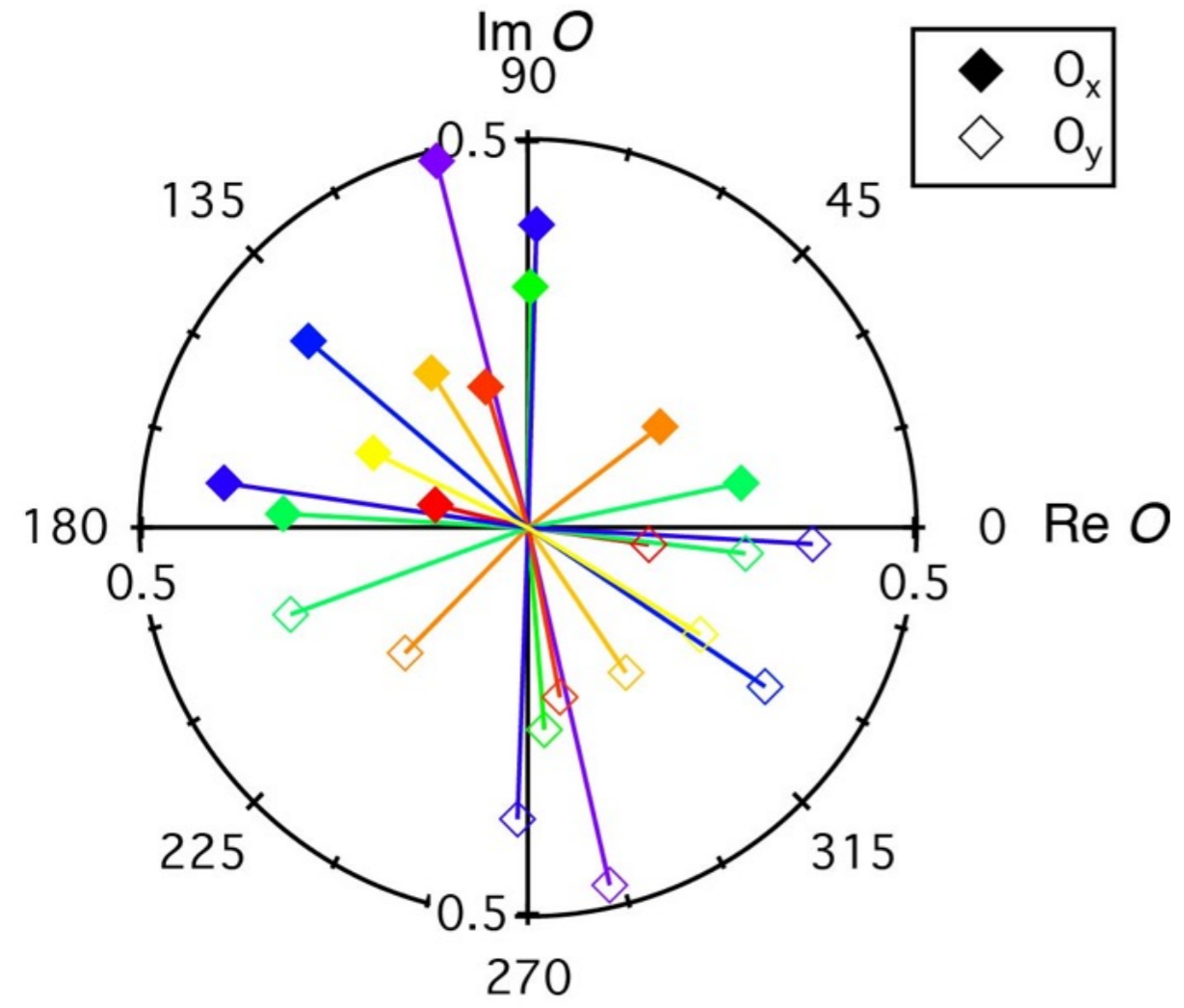


Phase-sensitive measurement of the *d*-form factor of density wave order



Phase-sensitive measurement of the *d*-form factor of density wave order

Phase-sensitive measurement of the d -form factor of density wave order



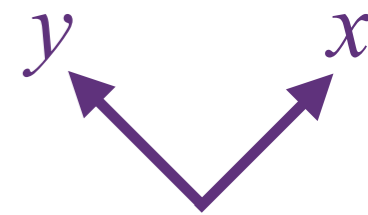
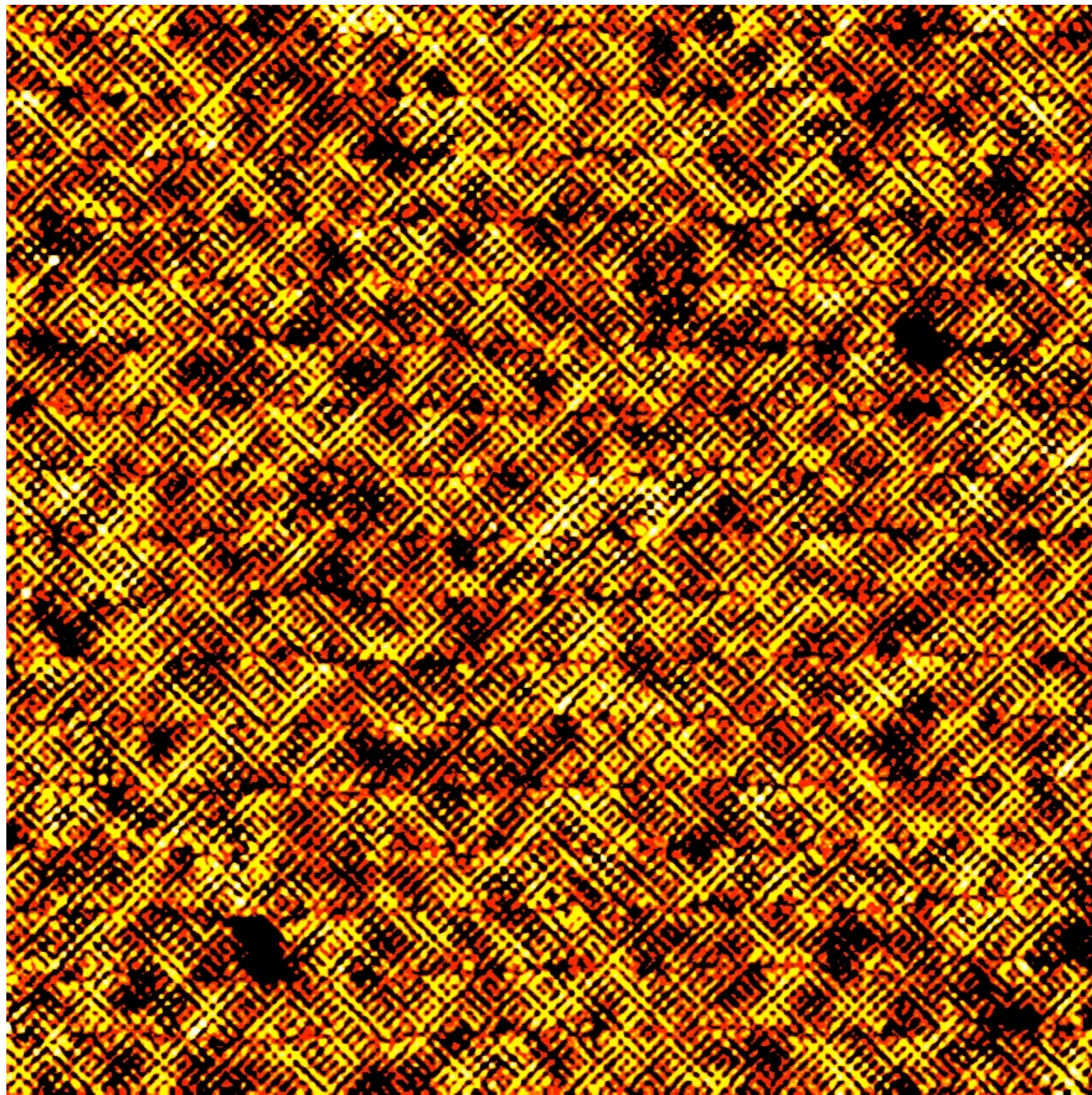
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)

See also

C. Howald, H. Eisaki,
N. Kaneko, M. Greven,
and A. Kapitulnik,
Phys. Rev. B **67**,
014533 (2003);

M. Vershinin, S. Misra,
S. Ono, Y. Abe, Yoichi
Ando, and
A. Yazdani, *Science*
303, 1995 (2004).

W. D. Wise, M. C. Boyer,
K. Chatterjee, T. Kondo,
T. Takeuchi, H. Ikuta,
Y. Wang, and
E. W. Hudson,
Nature Phys. **4**, 696
(2008).



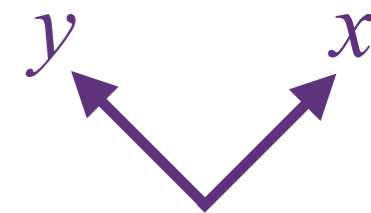
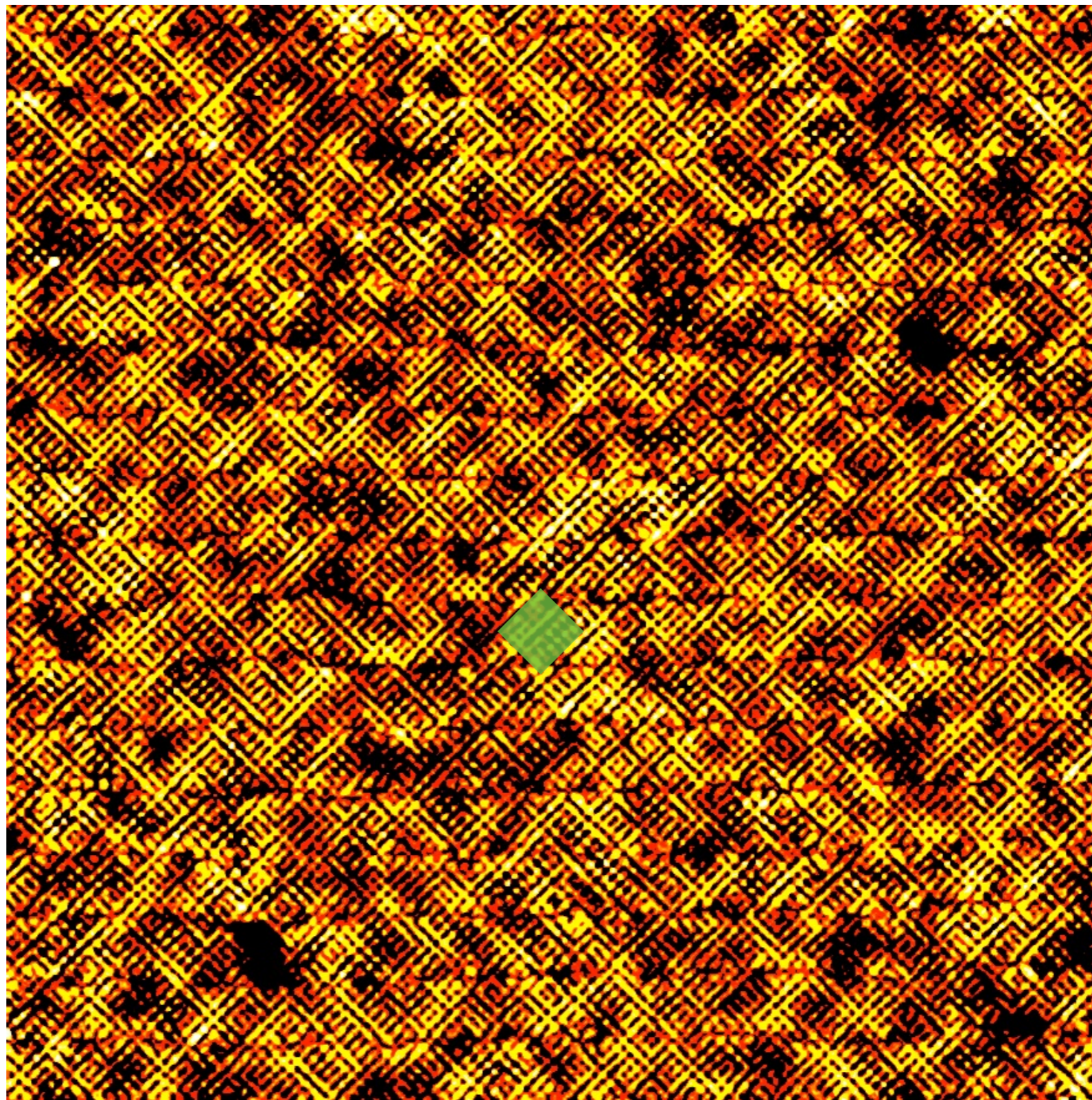
“R-map” of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007). **Davis group has sub-angstrom resolution capabilities, with lattice drift corrections, which make sublattice phase-resolved STM possible.**

See also

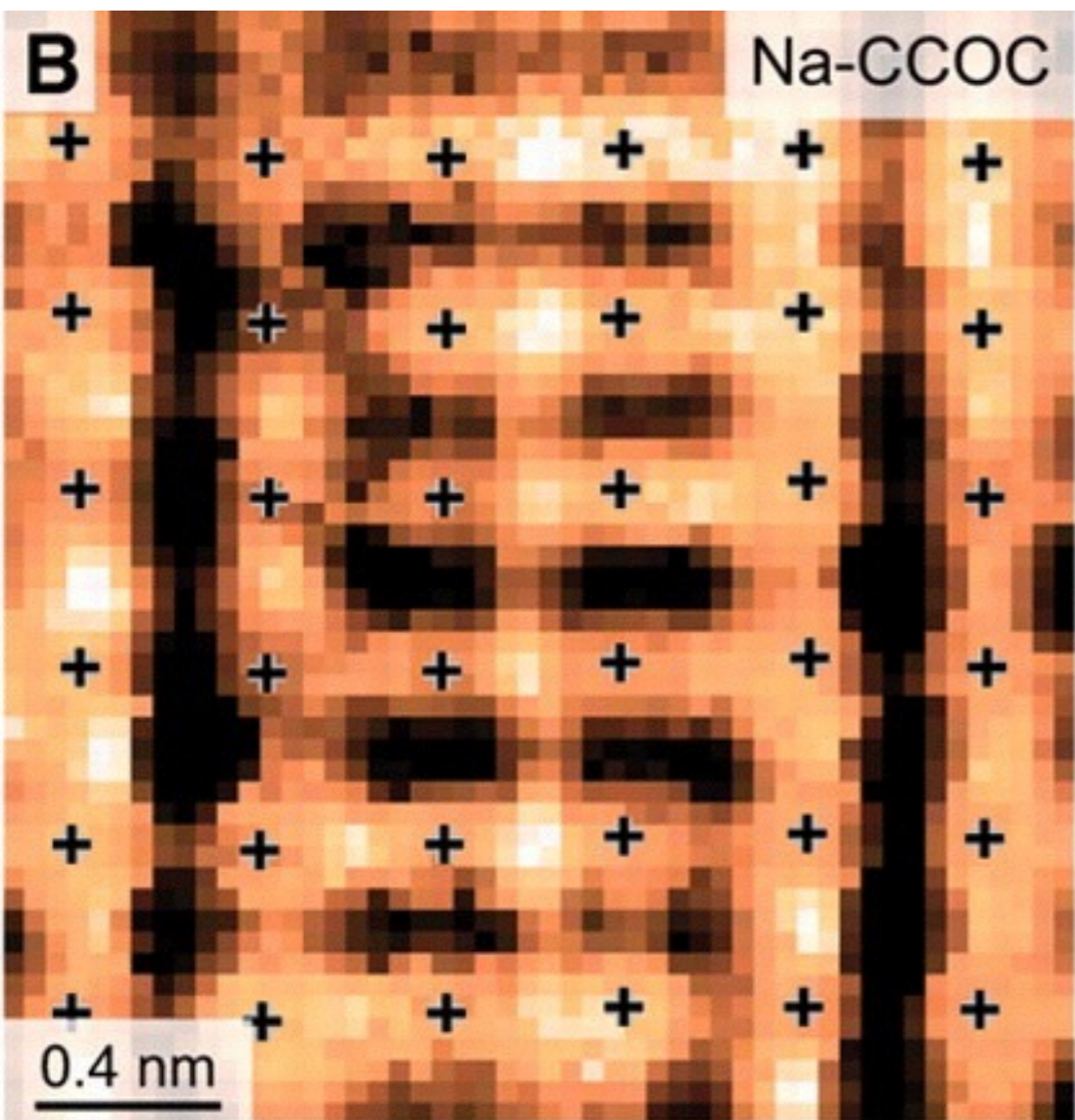
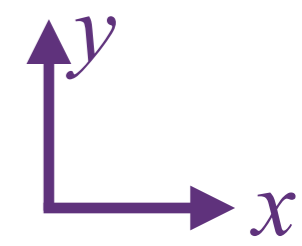
C. Howald, H. Eisaki,
N. Kaneko, M. Greven,
and A. Kapitulnik,
Phys. Rev. B **67**,
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M. Vershinin, S. Misra,
S. Ono, Y. Abe, Yoichi
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A. Yazdani, *Science*
303, 1995 (2004).

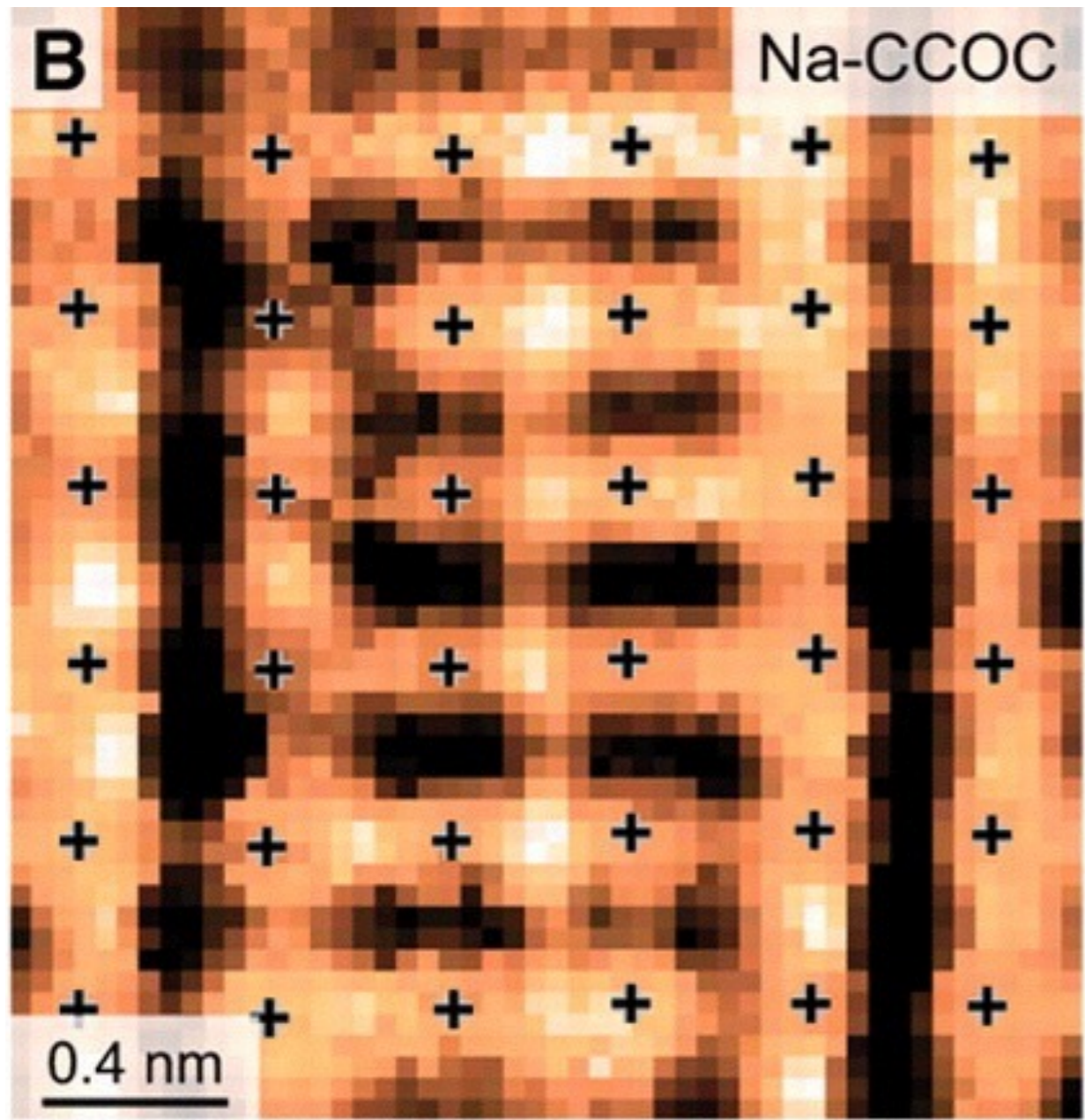
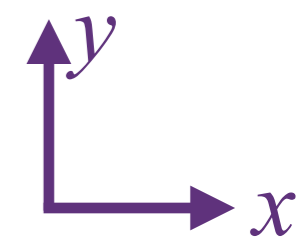
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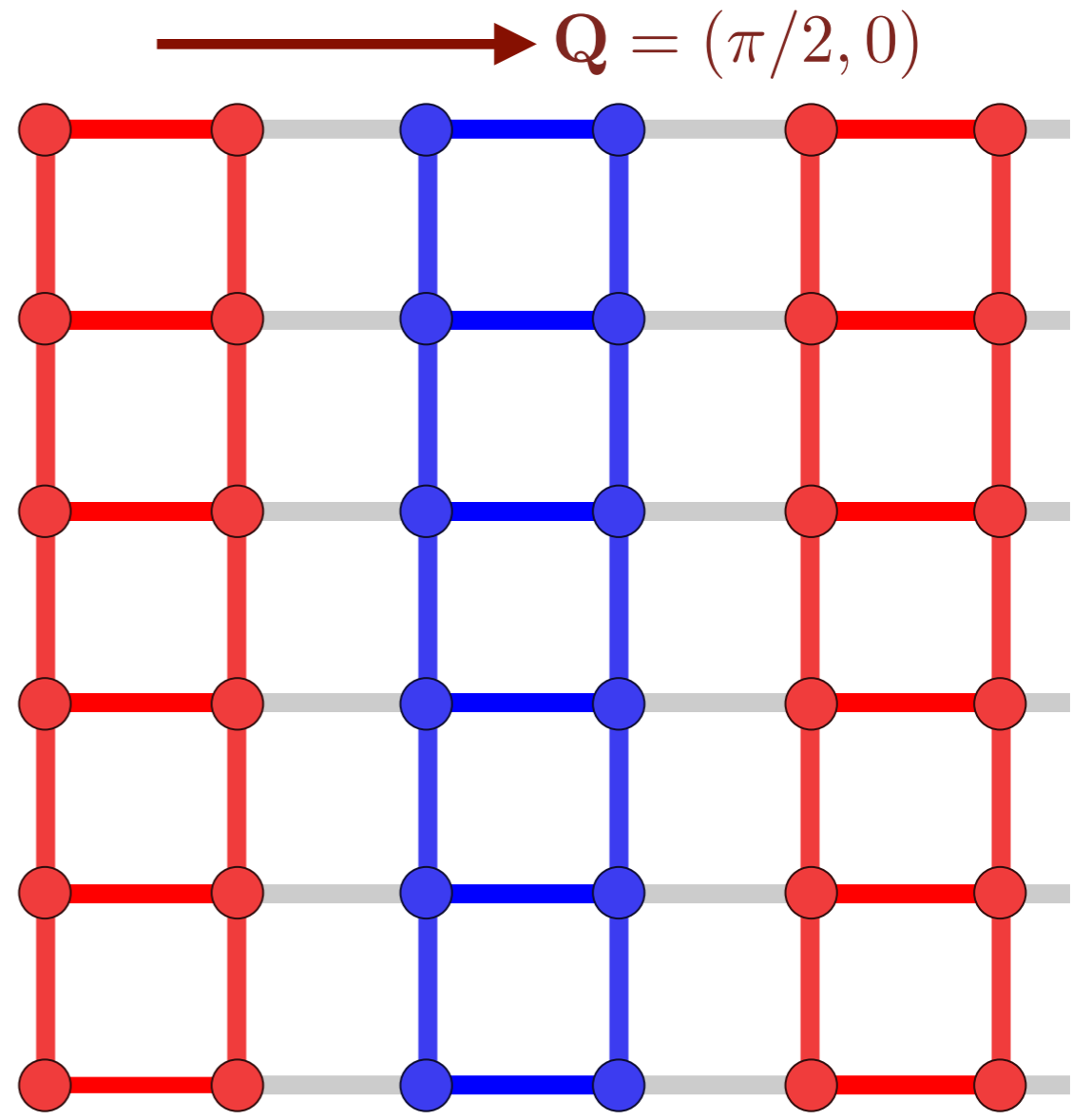
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Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

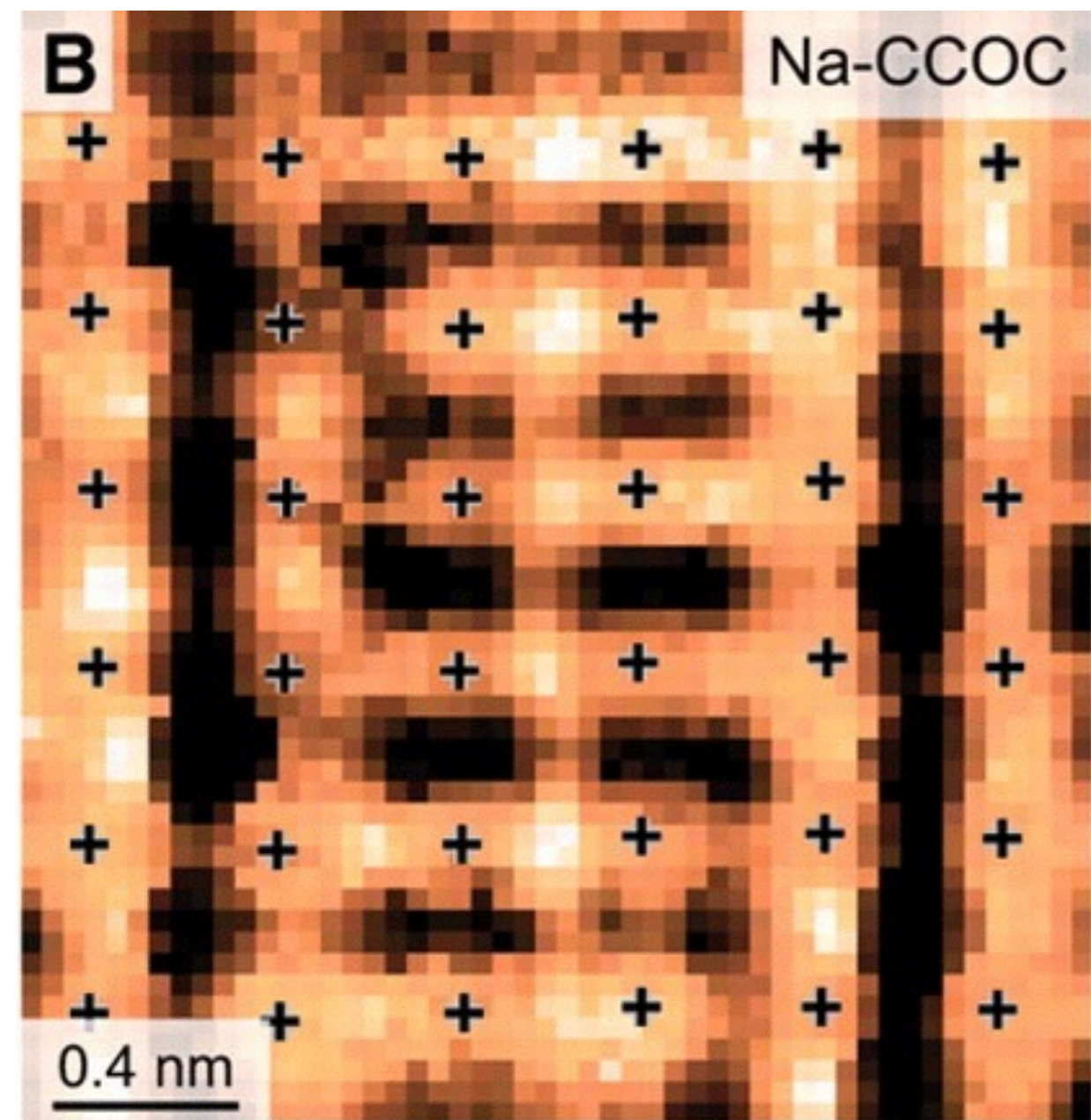


Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

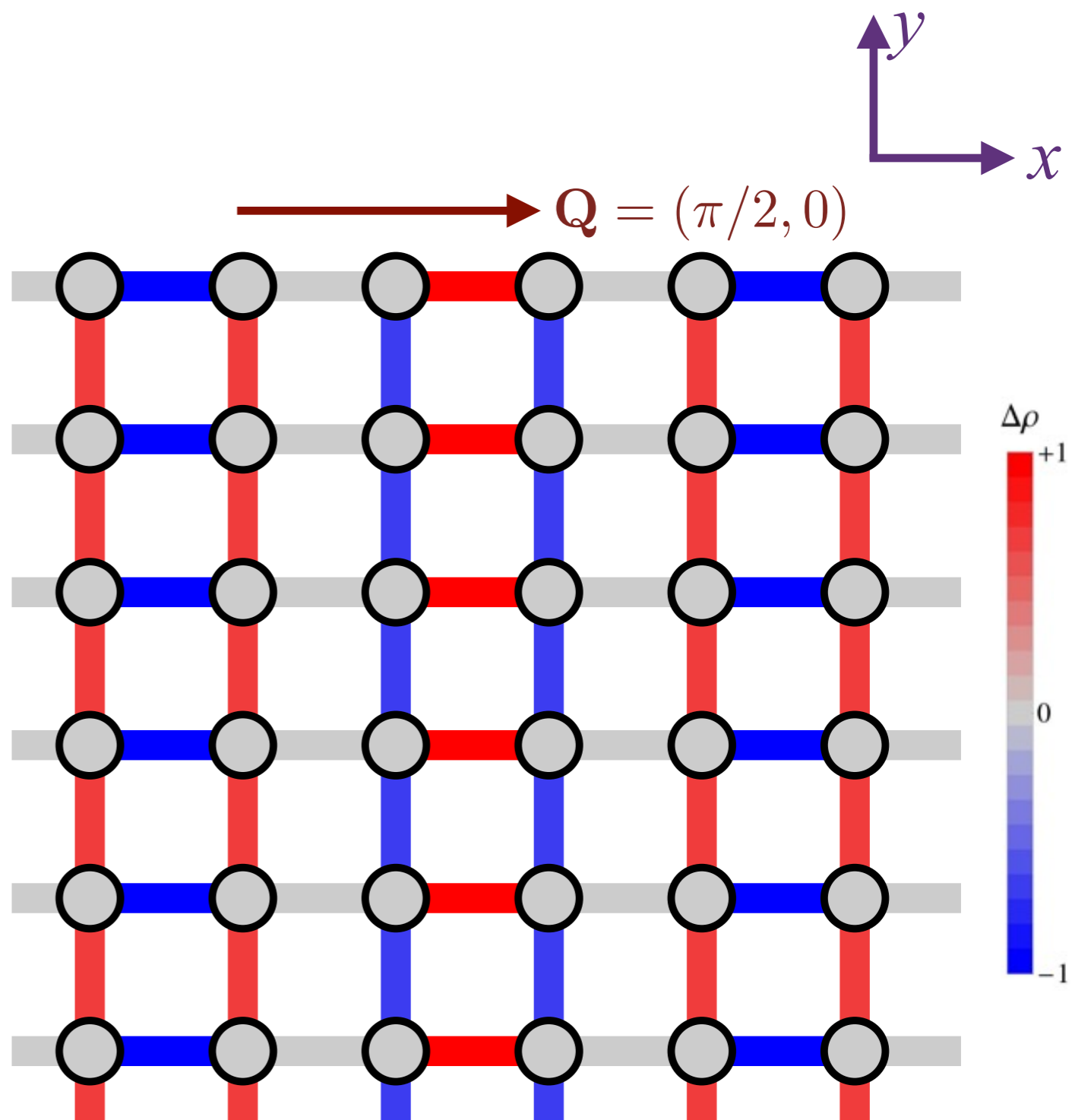


$s + s'$ -form factor density wave

$s + s'$ form factor (stripe model) does not match STM measurements on BSCCO, Na-CCOC.

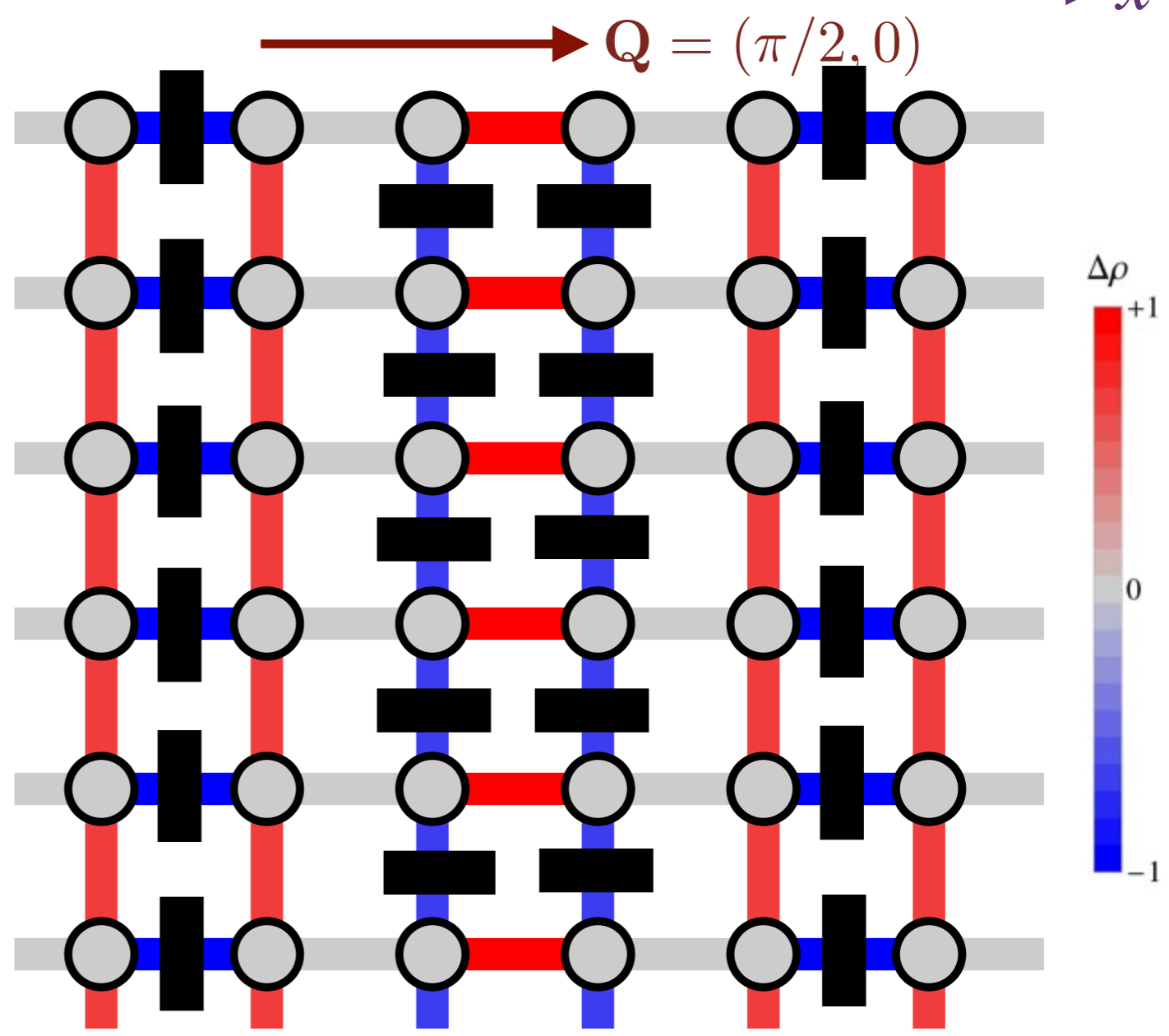
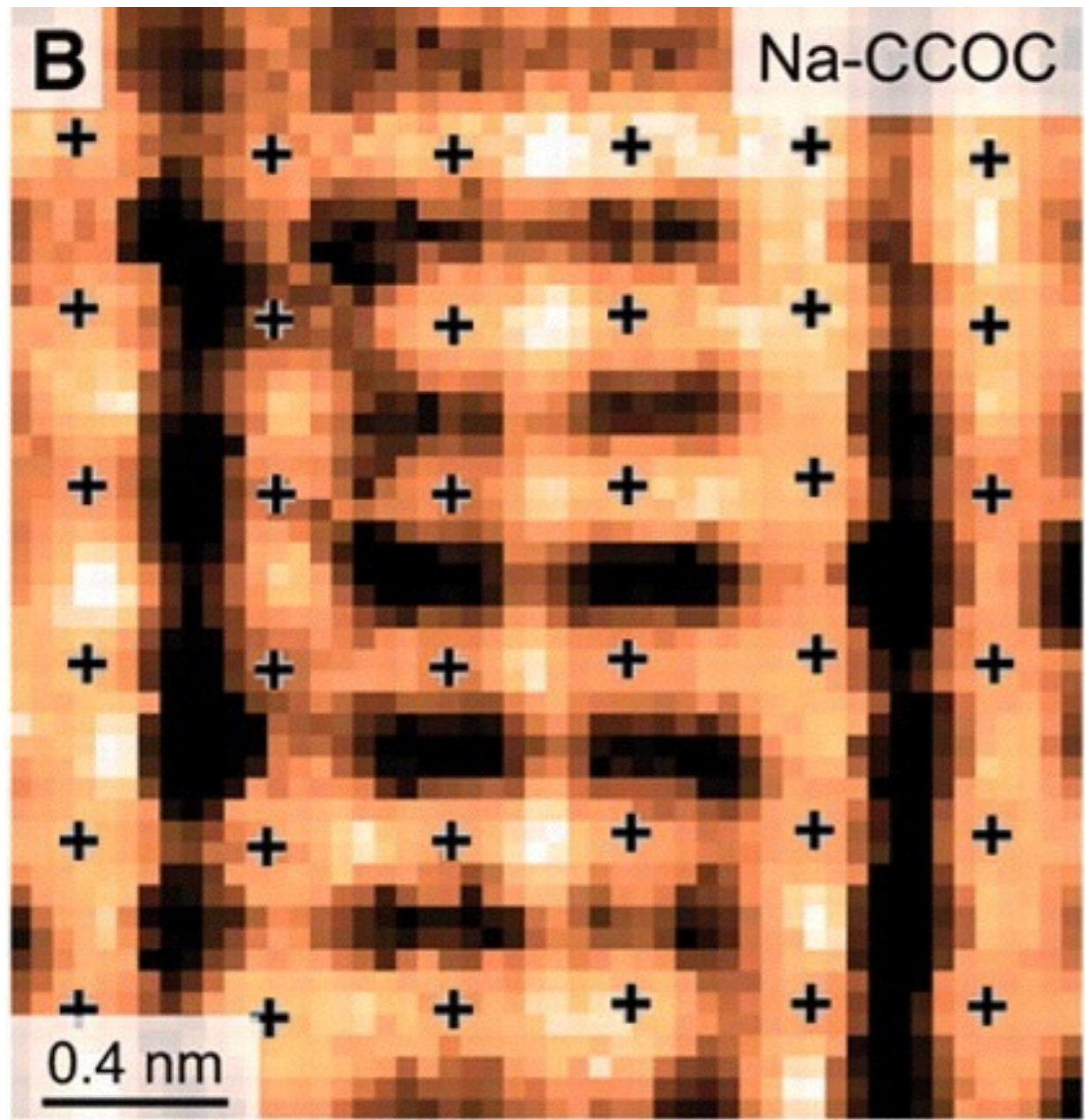
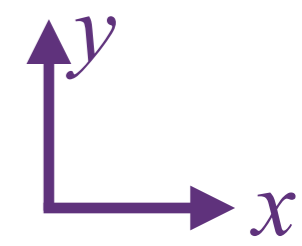


Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



d-form factor density wave order

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).
 S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).



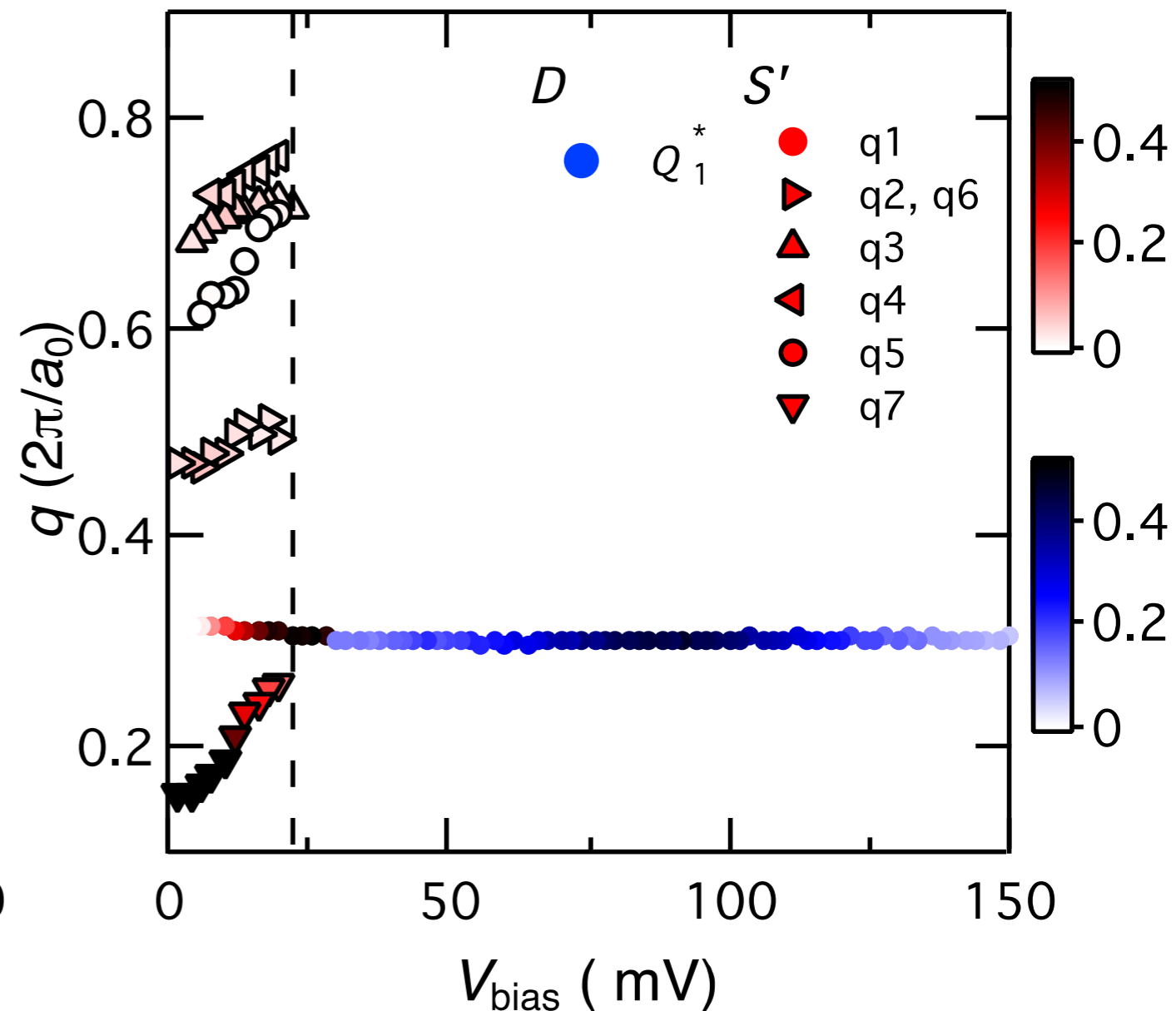
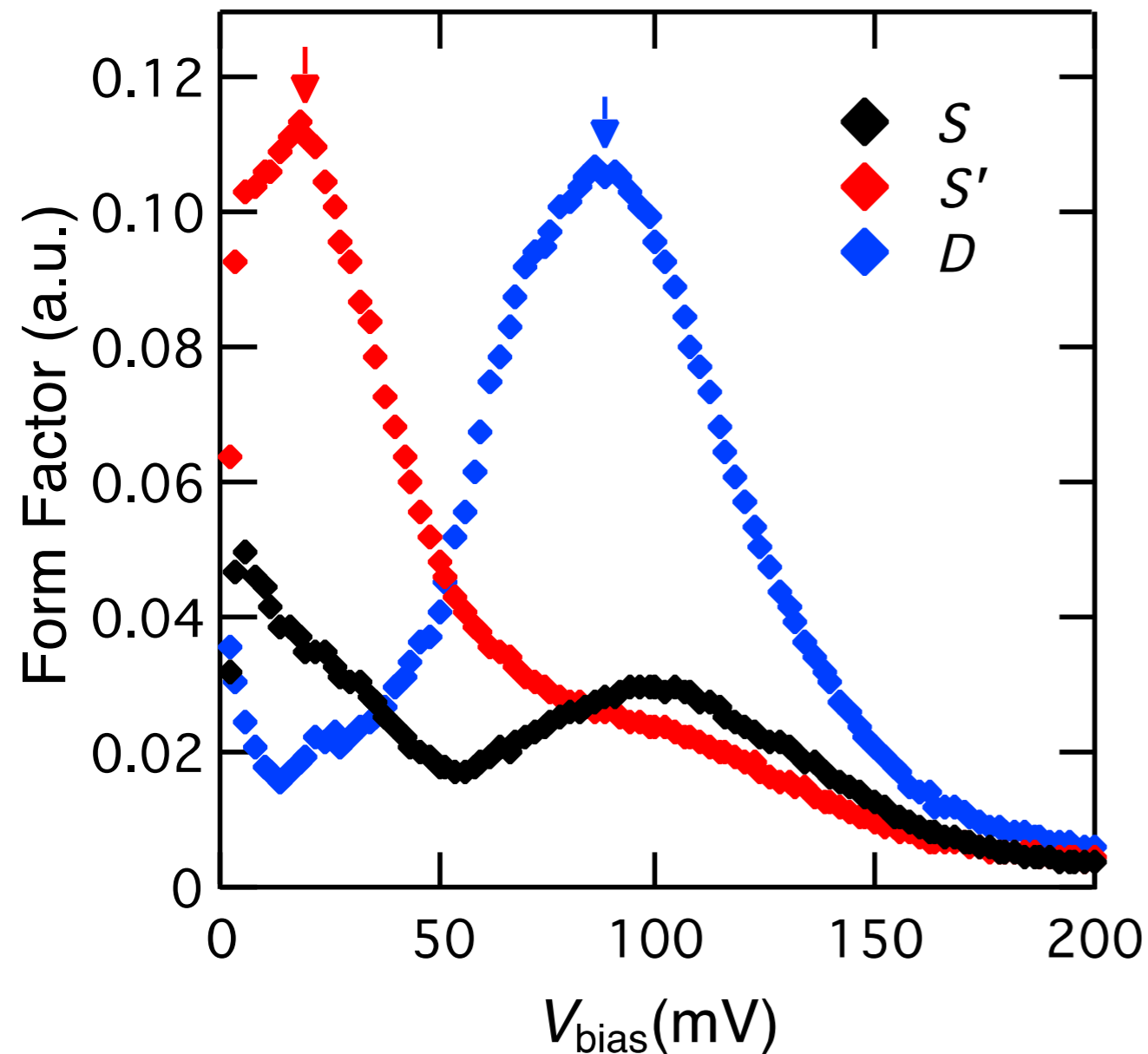
Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

d-form factor density wave order

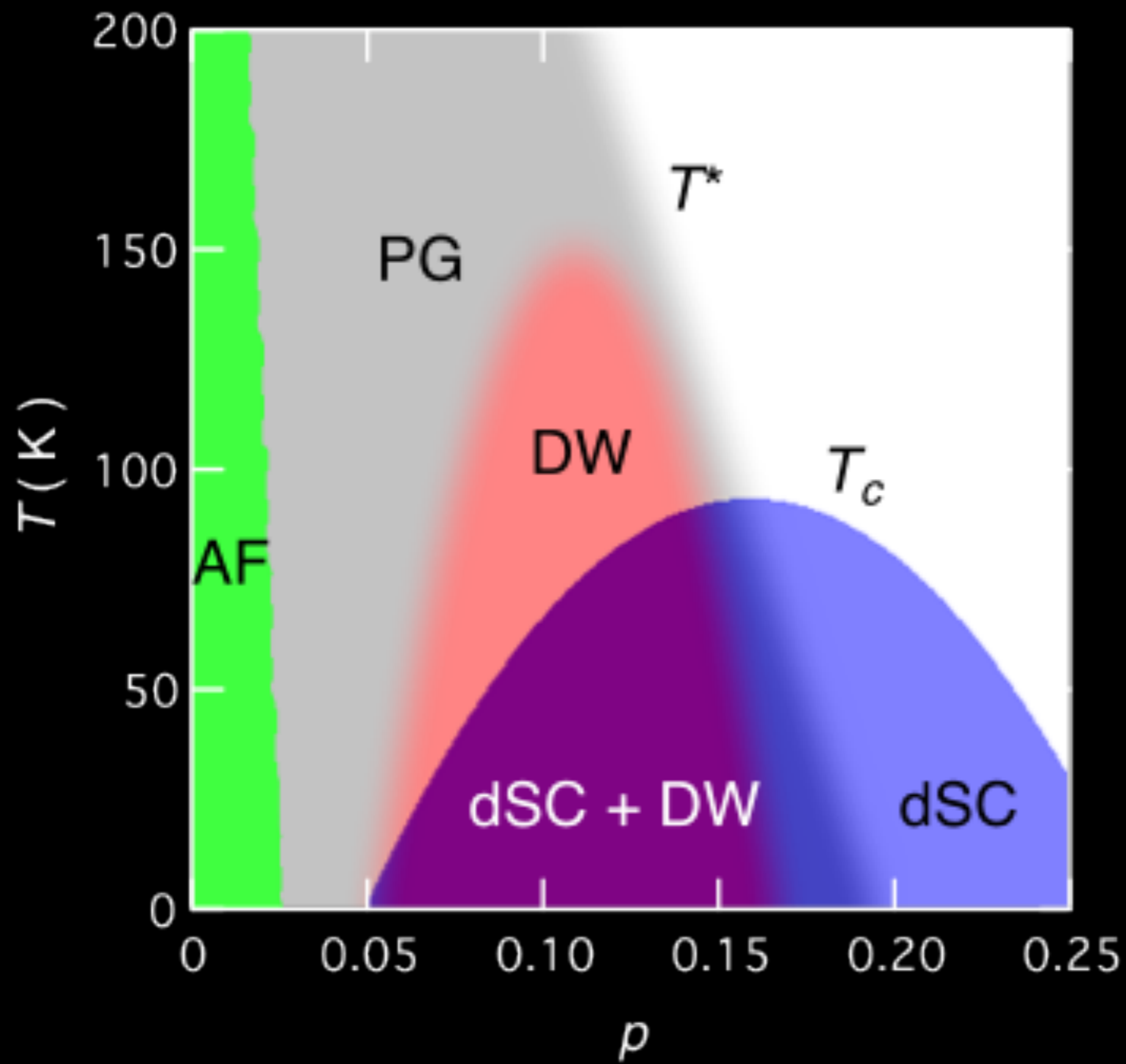
Predicted *d* form factor observed
in STM measurements on BSCCO, Na-CCOC !

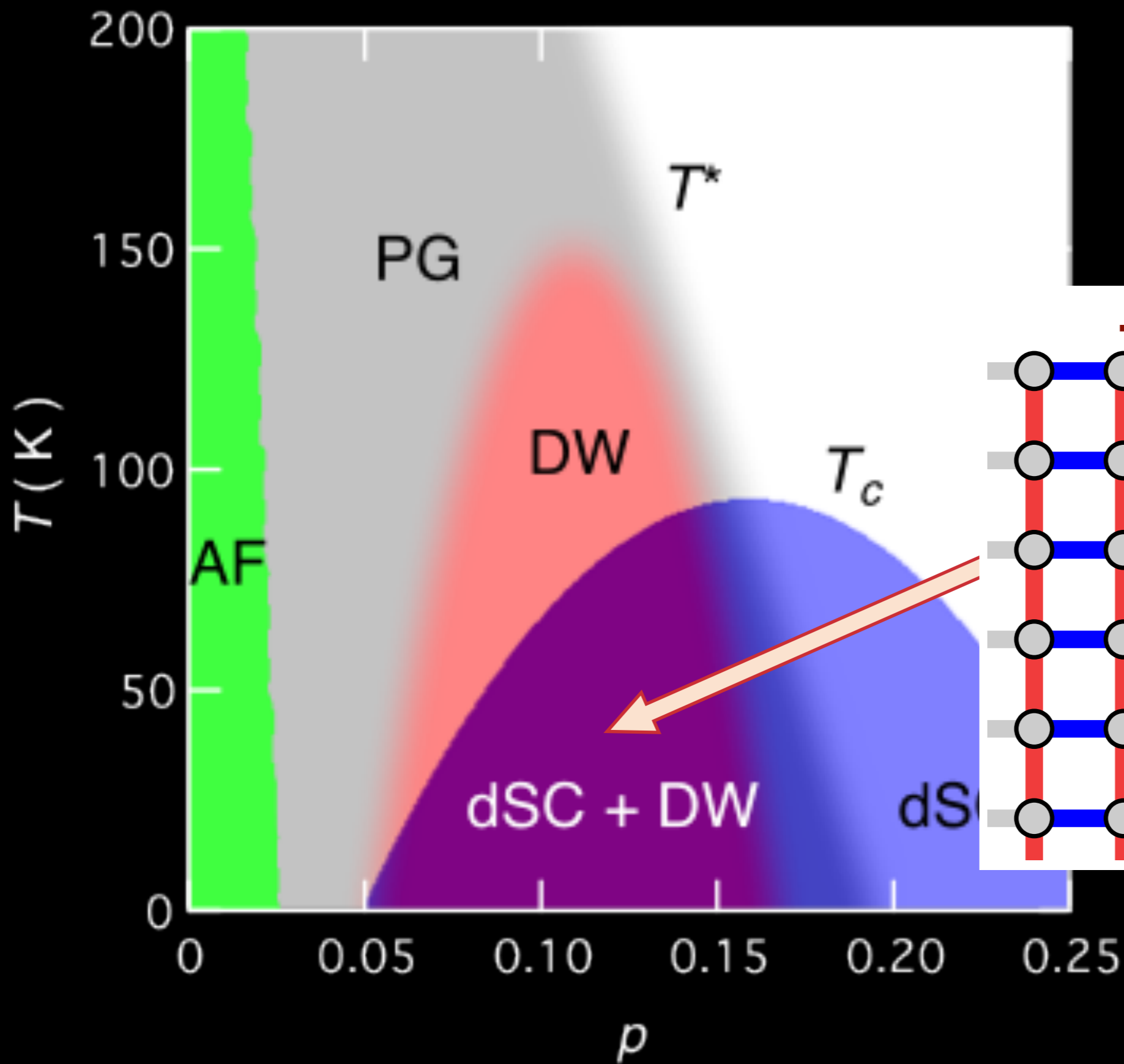
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).
S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

d -form factor is peaked at the pseudogap energy, and does not disperse as a function of wavevector

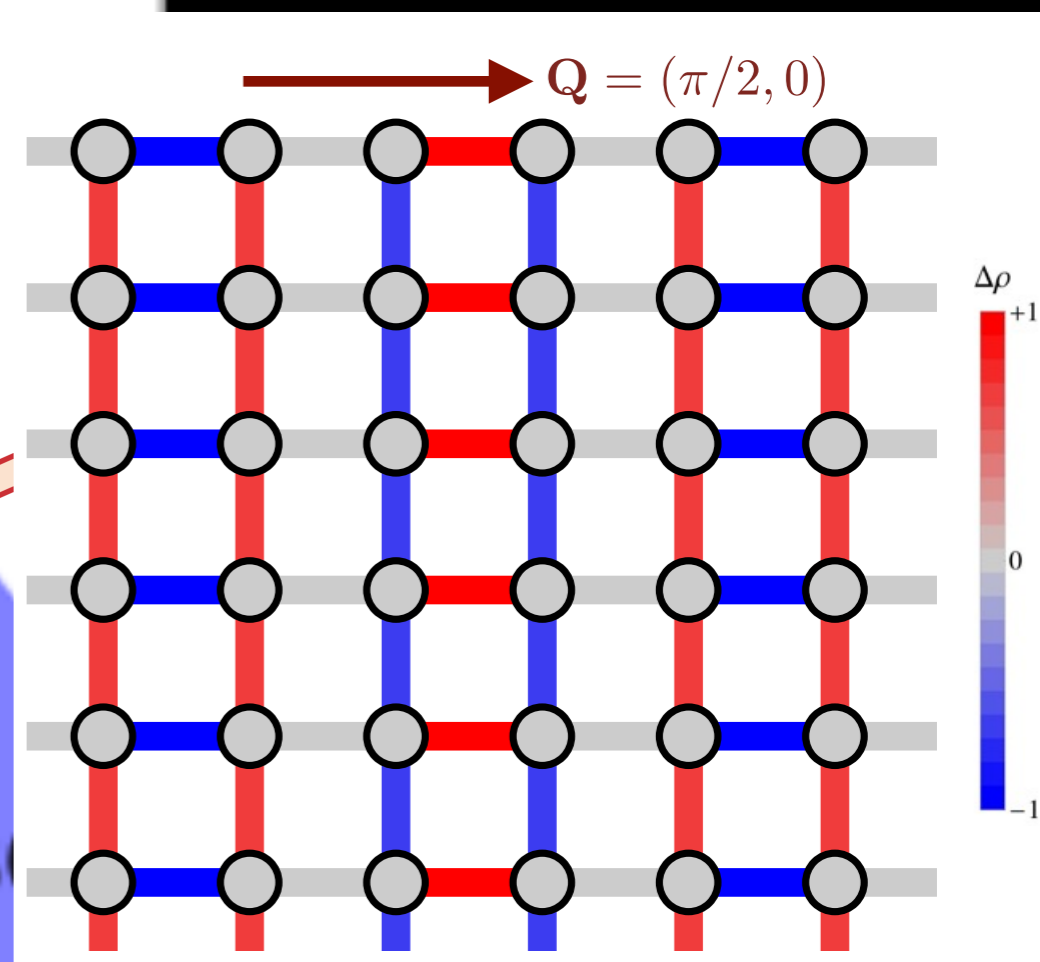


K. Fujita, M. H. Hamidian, S. D. Edkins, Chung Koo Kim, A. P. MacKenzie, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, to appear





***d*-form
factor
density
wave**



Outline

1. The low T pseudogap:

*STM observation of predicted
d-form factor density wave*

2. The high T pseudogap:

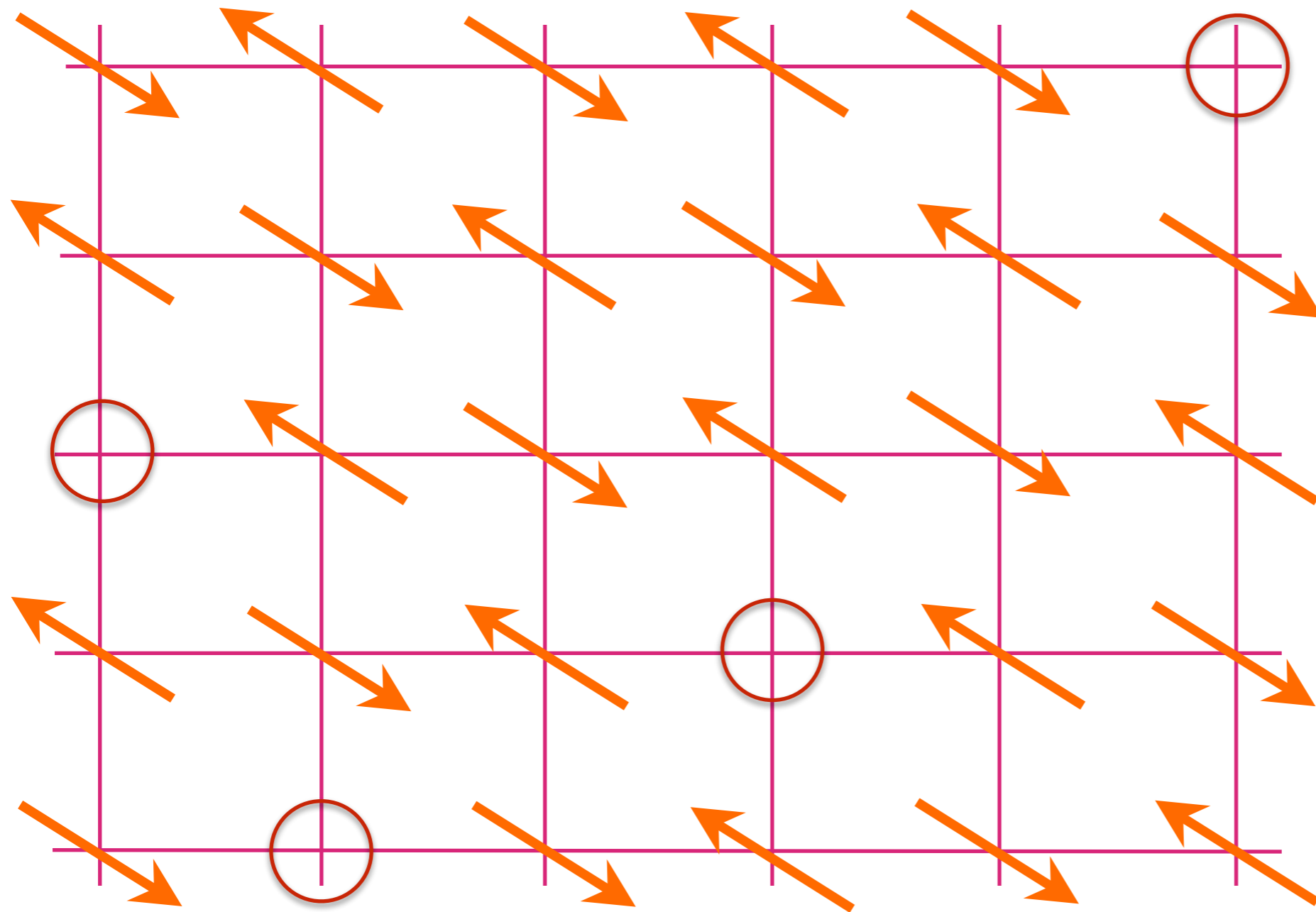
*A quantum dimer model for a metal
with topological order*

3. Connecting high and low T :

Density wave instabilities

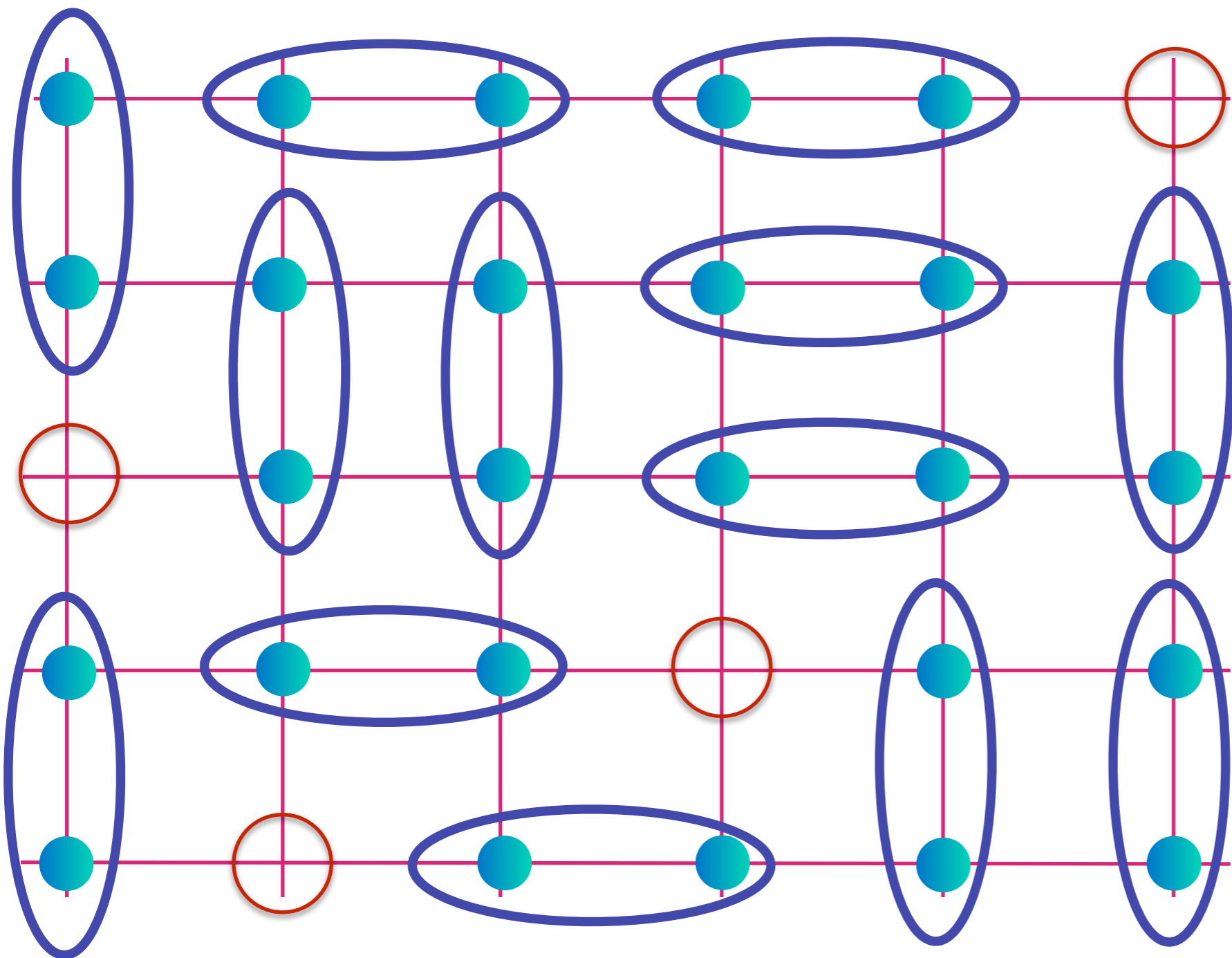
4. Quantum critical point near optimal p :

A Higgs critical point



Anti-ferromagnet with p holes per square

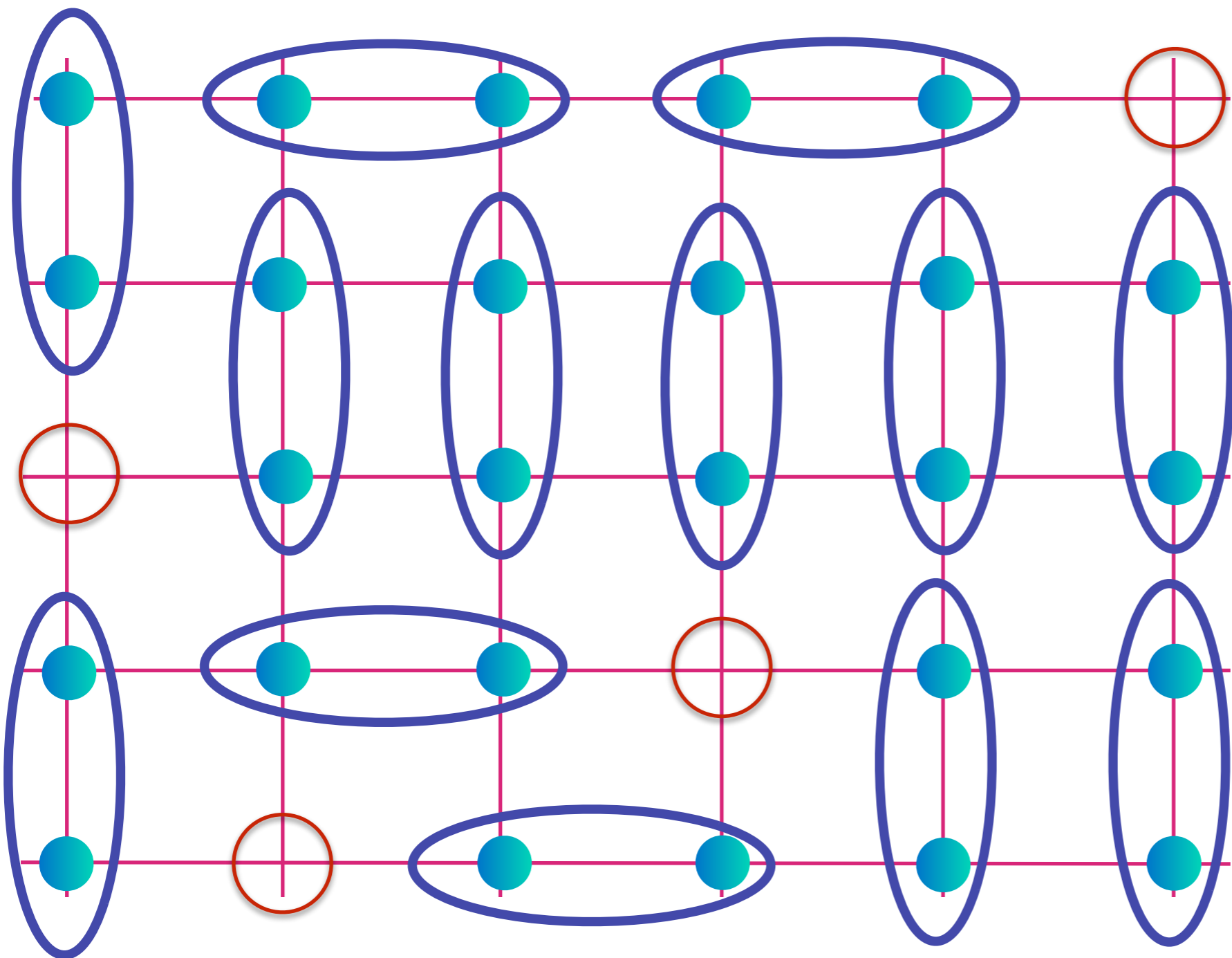
Note: relative to the fully-filled band insulator, there are $1+p$ holes per square




Spin liquid
with emergent
gauge field
and
 p "holons"
(gauge-charged,
spinless,
charge $+e$
quasiparticles)
per square

$$\text{[Blue oval with two teal dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

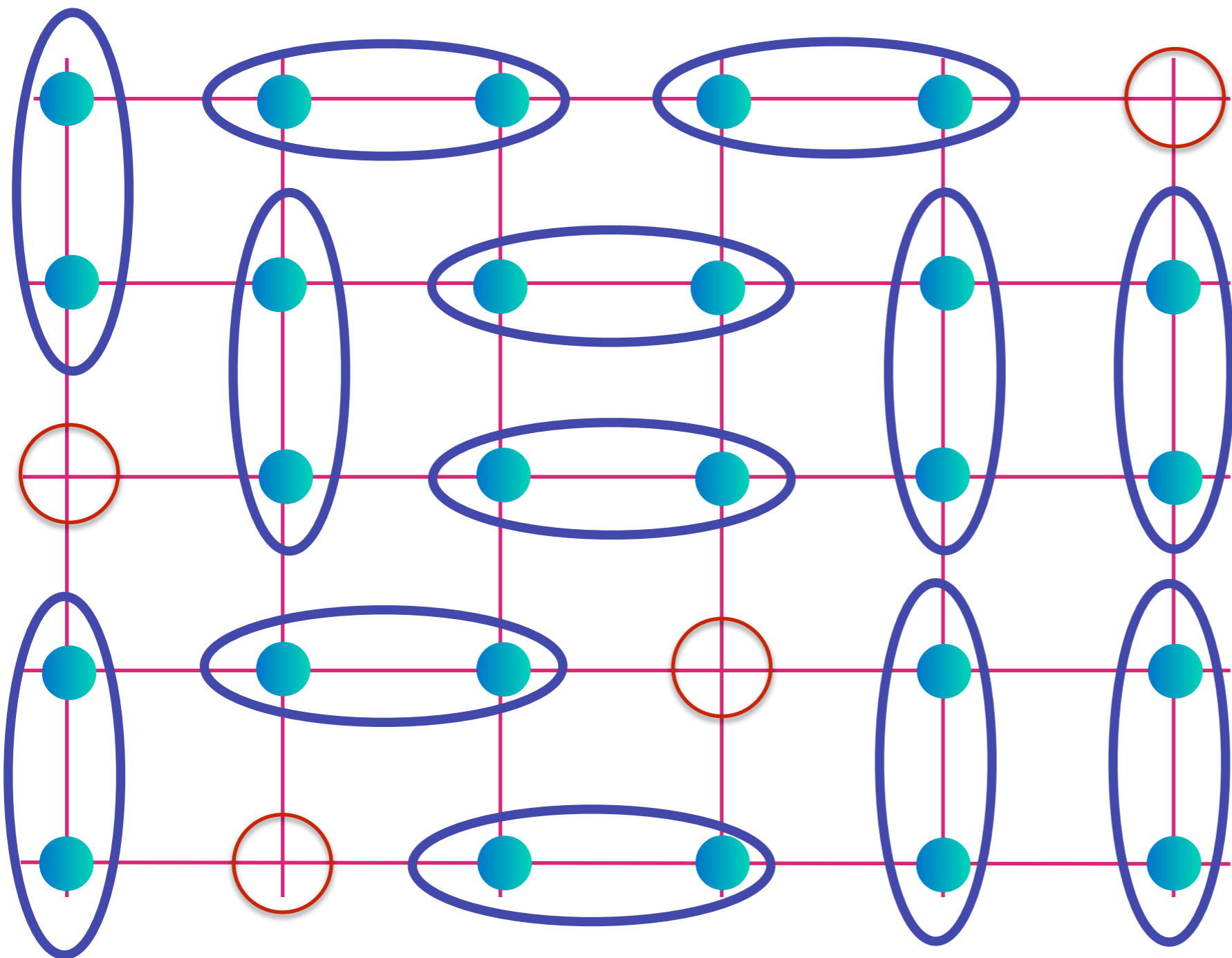
Baskaran, Zou, Anderson, Fradkin, Kivelson...



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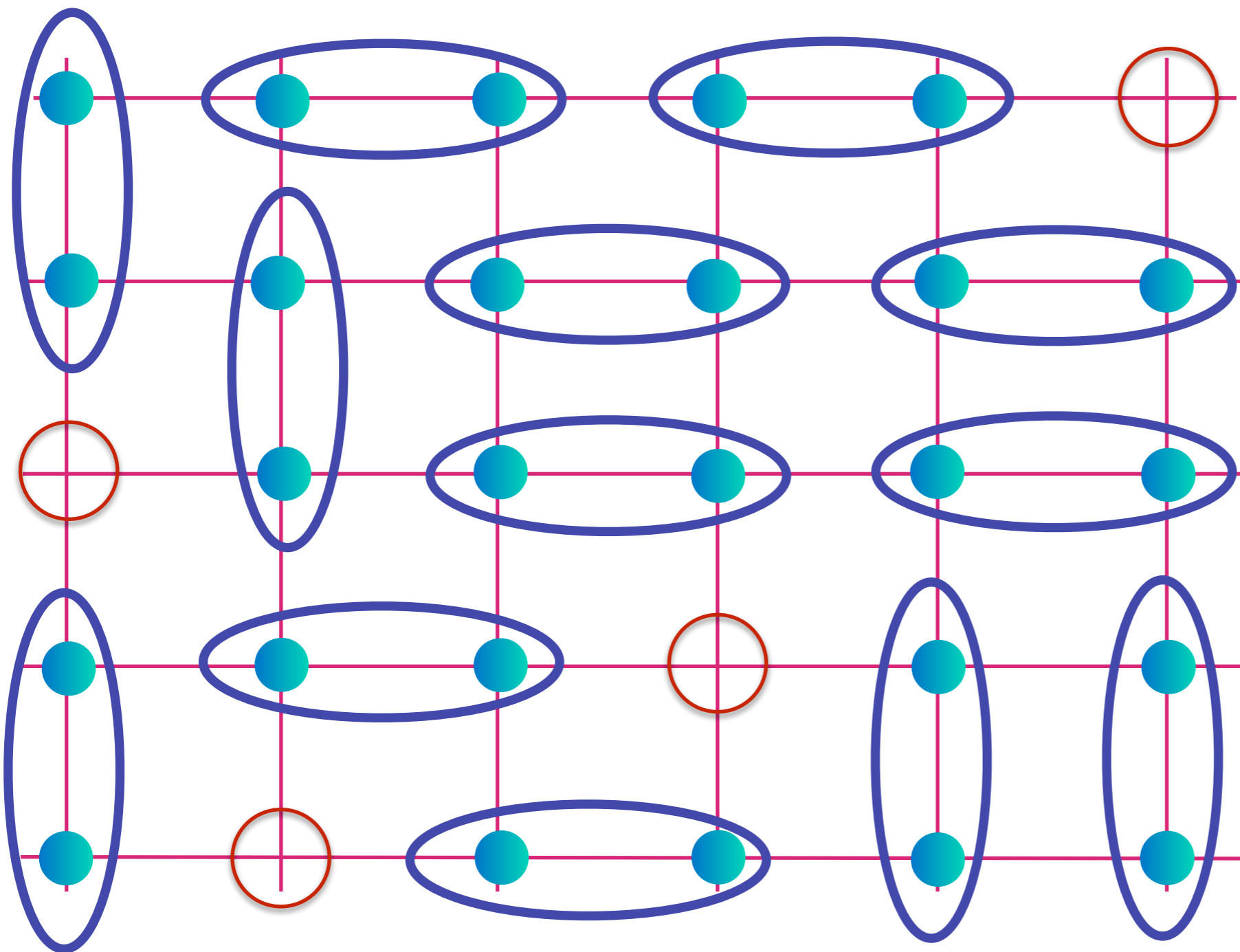
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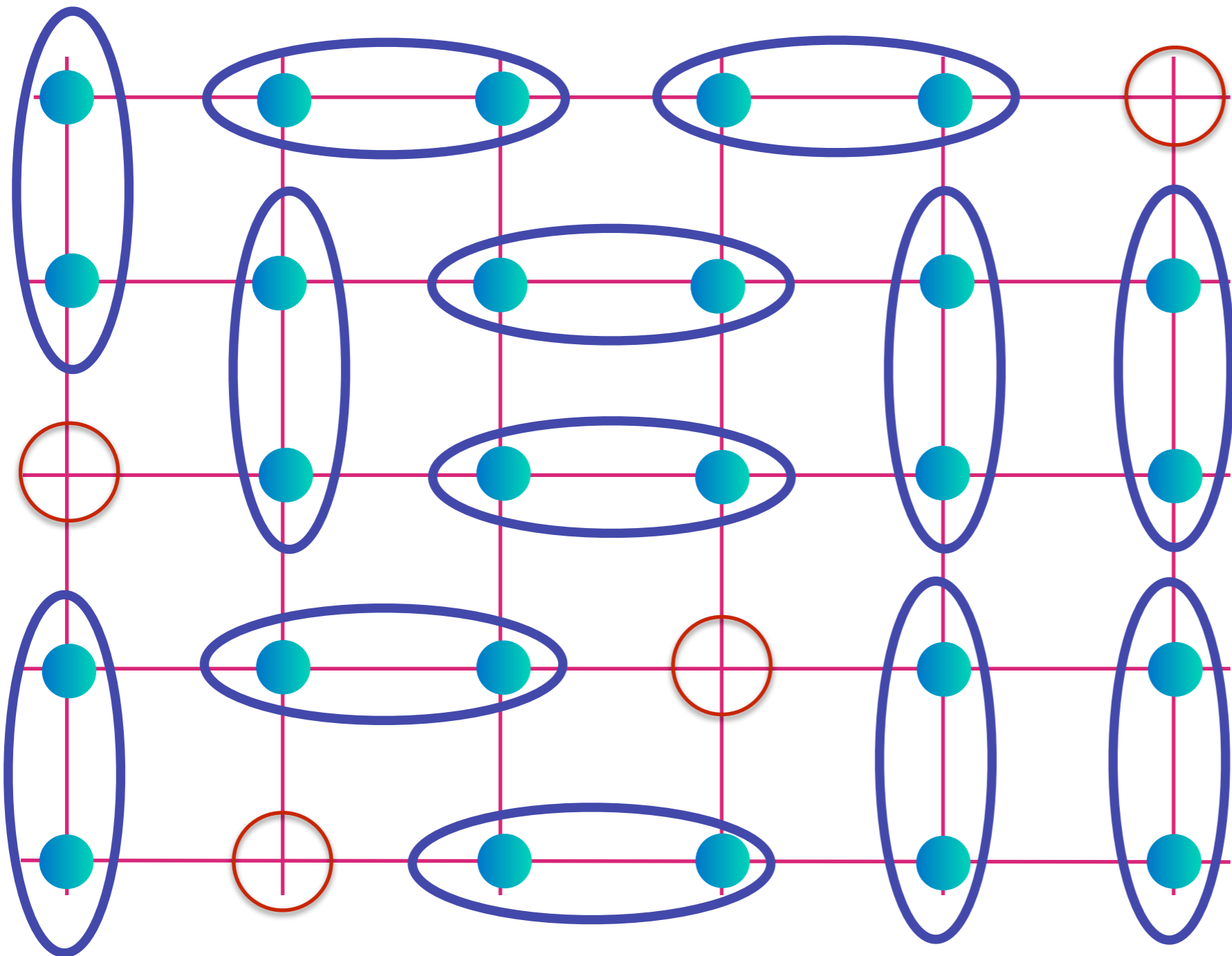
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$$\text{[blue oval with two cyan dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

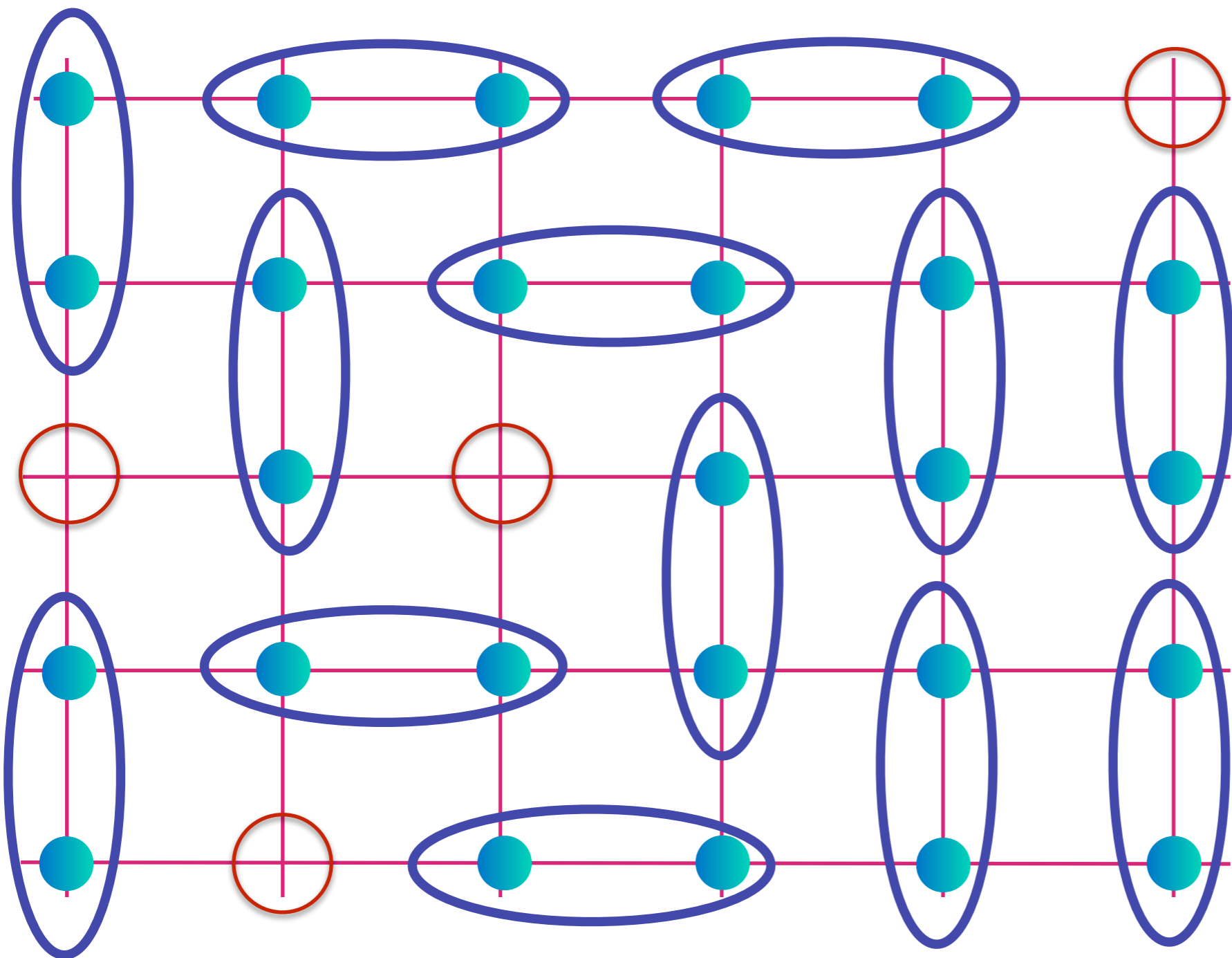
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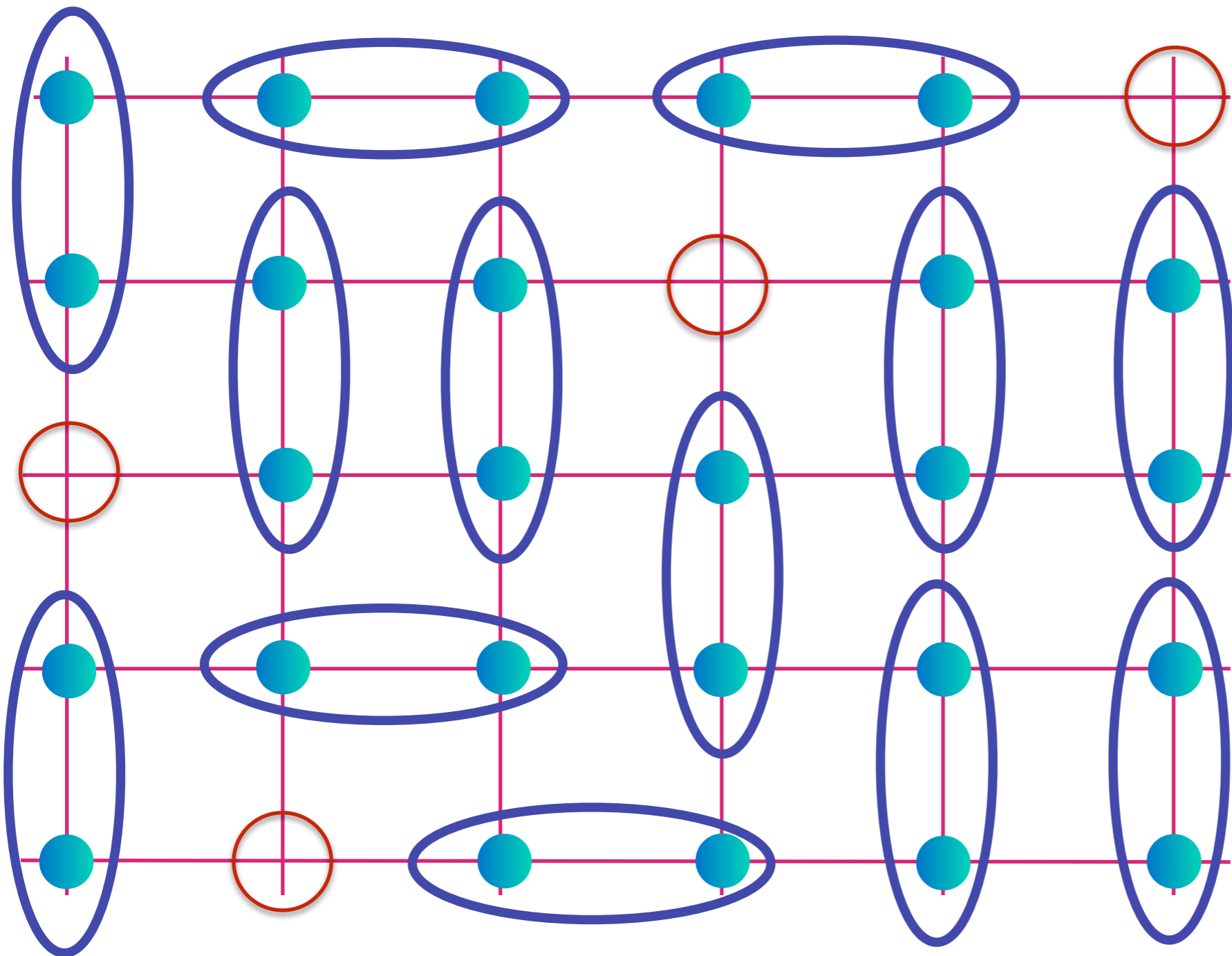
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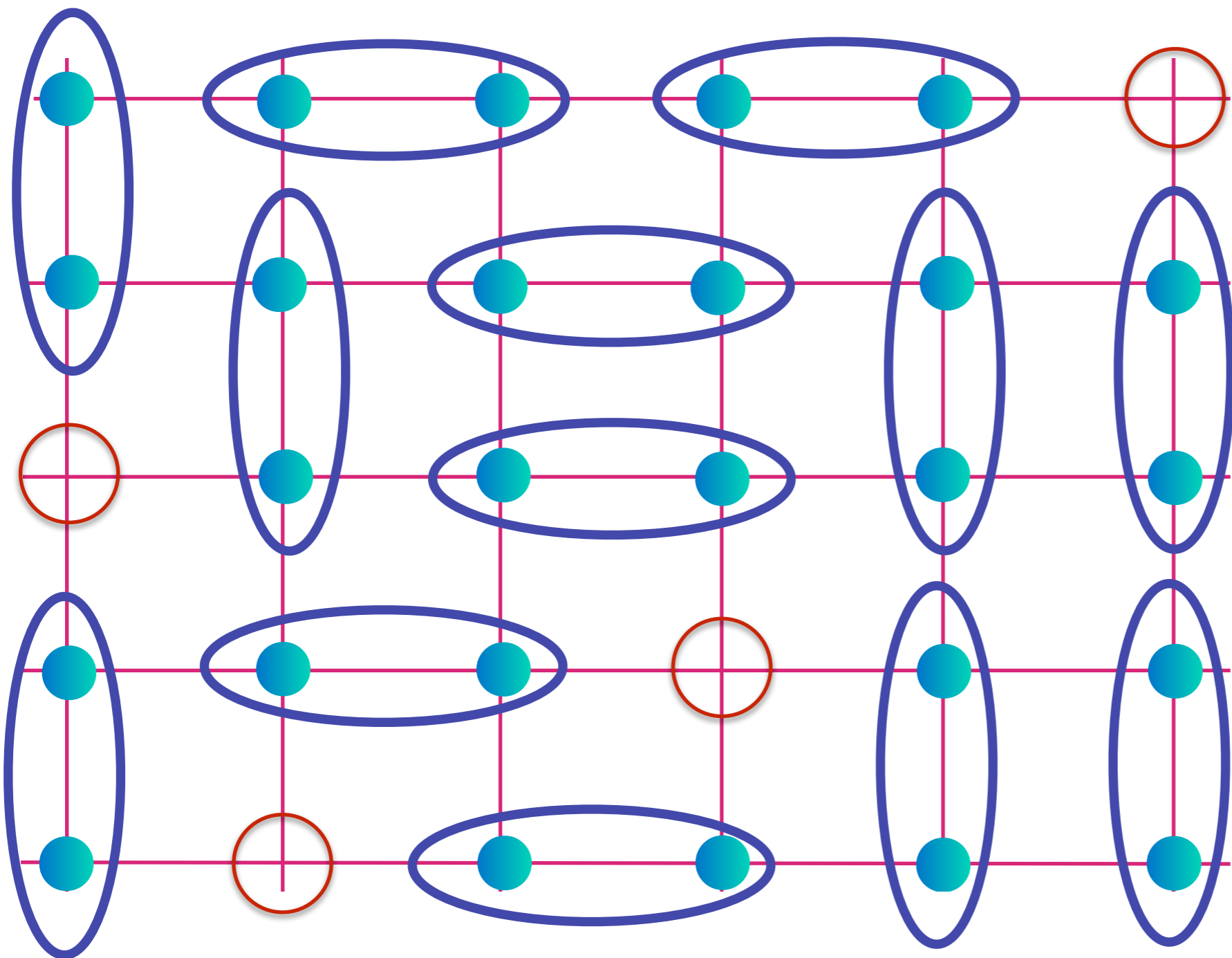
Baskaran, Zou, Anderson, Fradkin, Kivelson...



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Baskaran, Zou, Anderson, Fradkin, Kivelson...

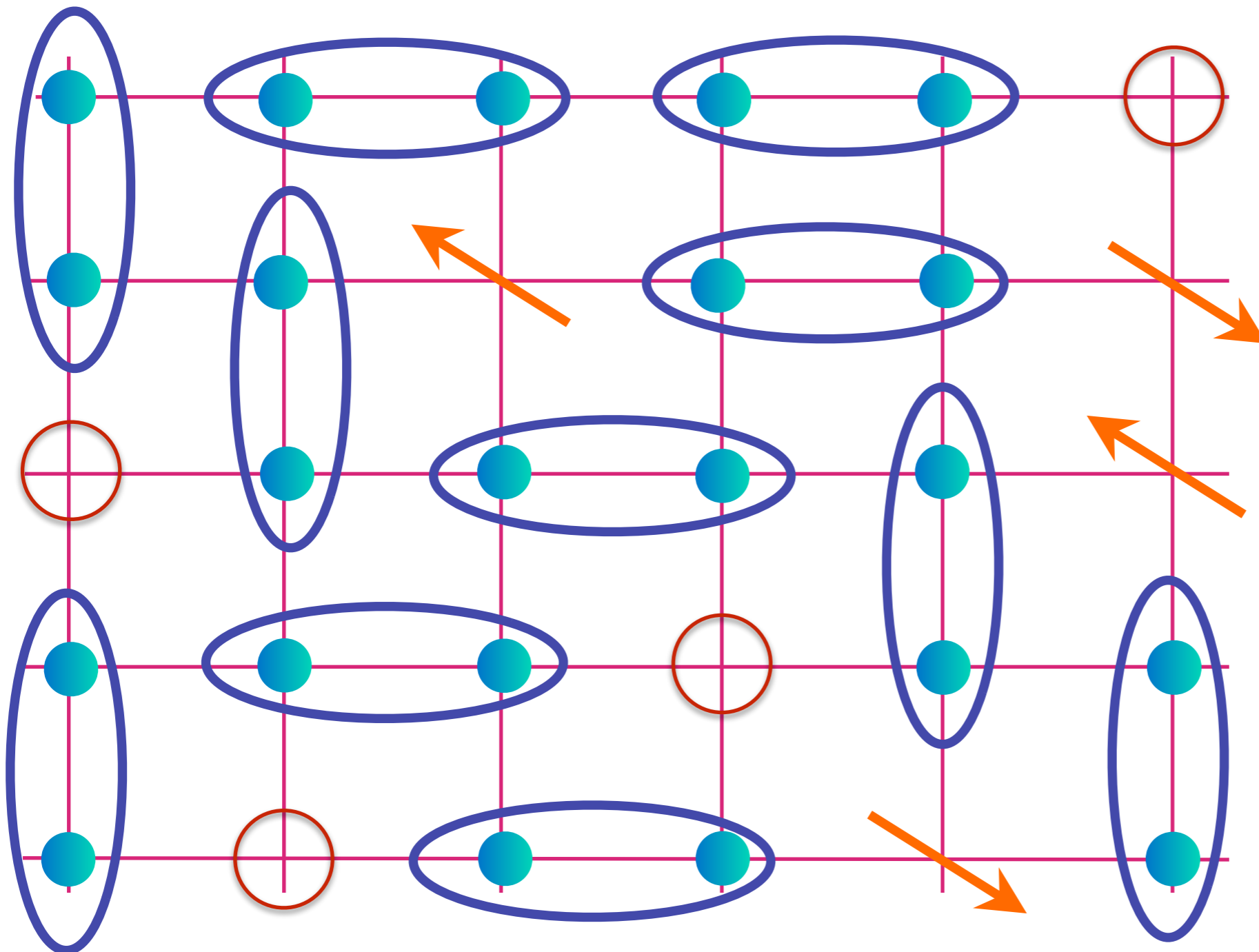


Spin liquid
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per square

$$\text{[Pair of sites]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

Gauge-charged, spin $S=1/2$, neutral “spinon” excitations

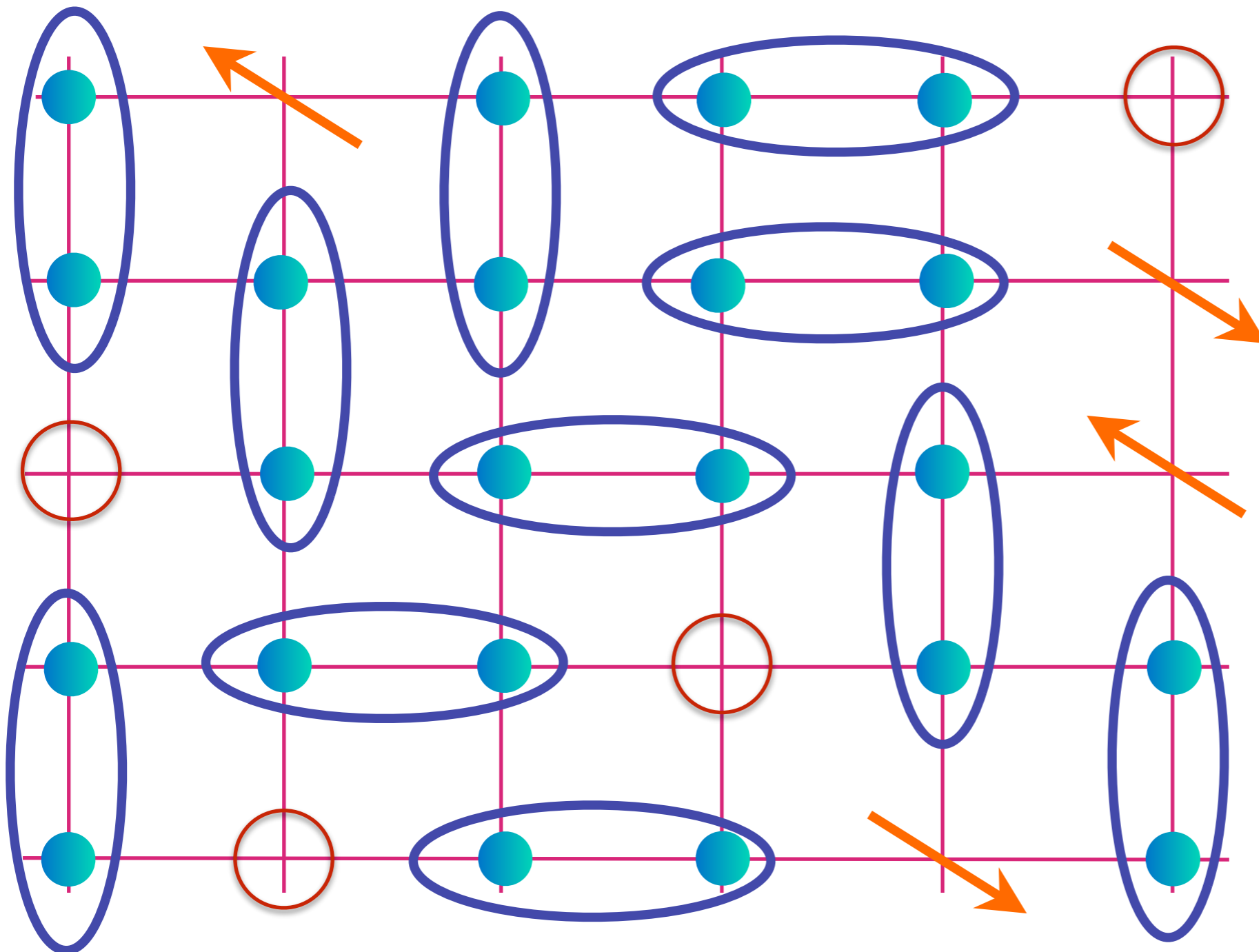


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Baskaran, Zou, Anderson, Fradkin, Kivelson...

Gauge-charged, spin $S=1/2$, neutral “spinon” excitations

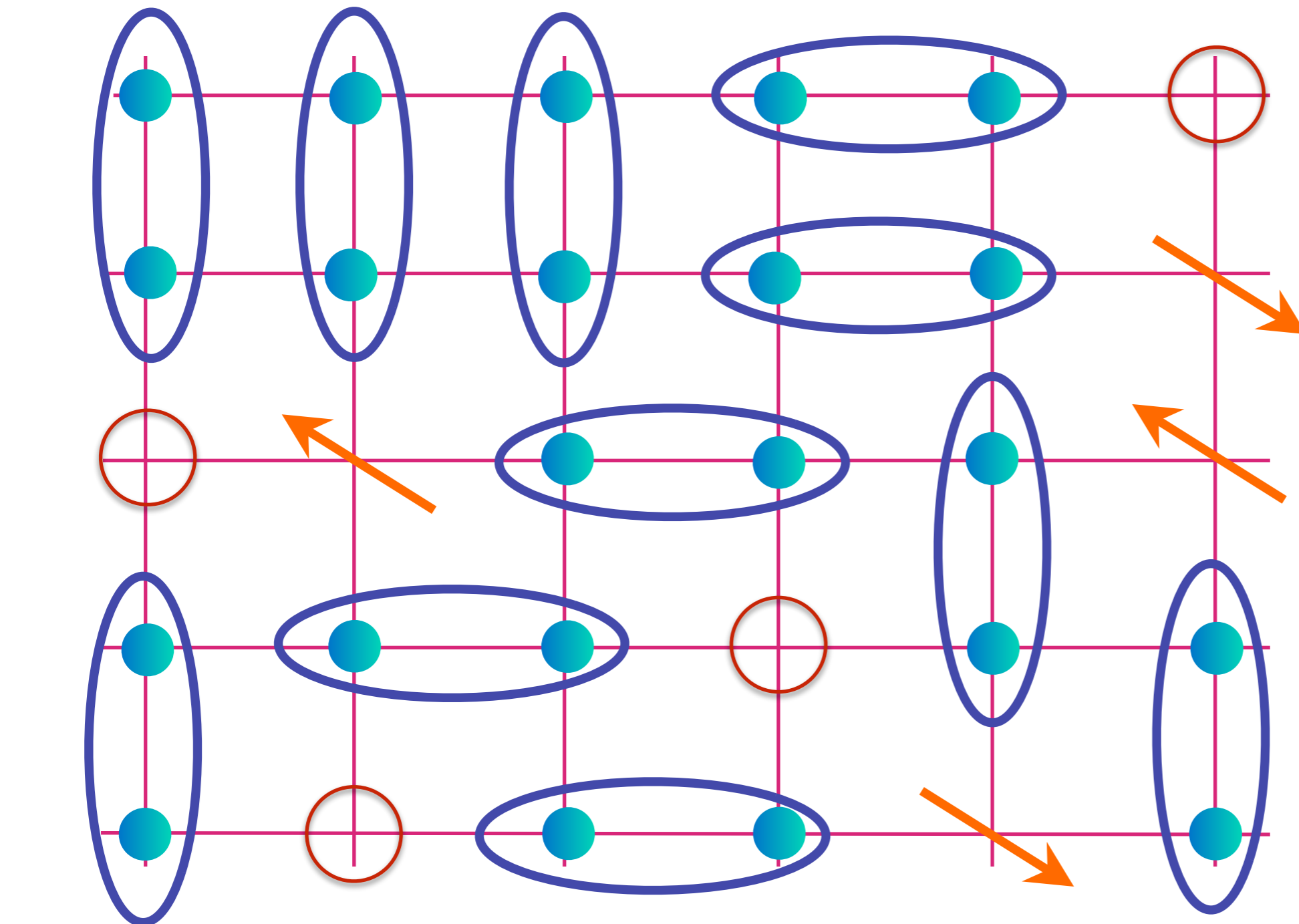


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Baskaran, Zou, Anderson, Fradkin, Kivelson...

Gauge-charged, spin $S=1/2$, neutral “spinon” excitations

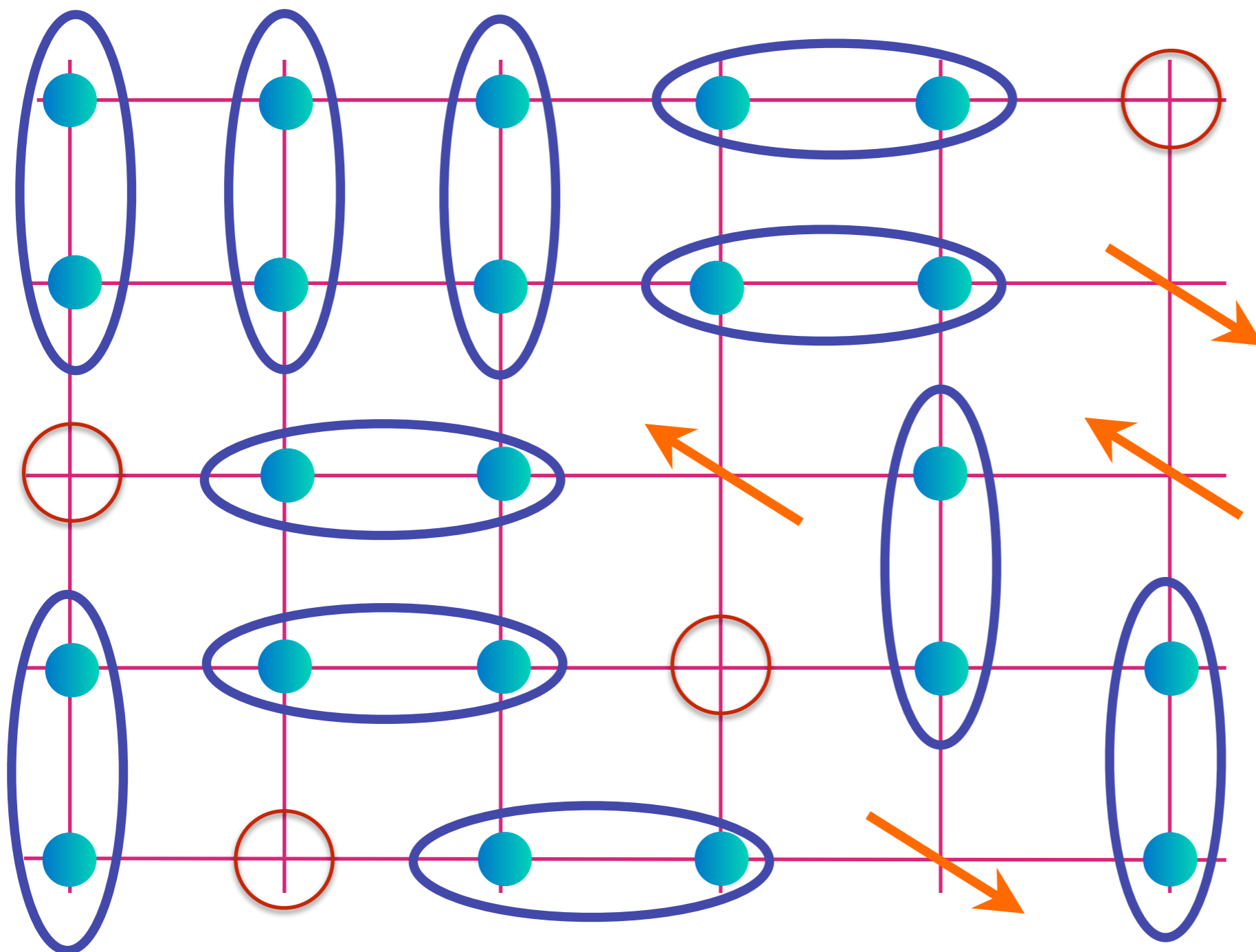


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$$\text{[Blue oval with two teal dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

Gauge-charged, spin $S=1/2$, neutral “spinon” excitations

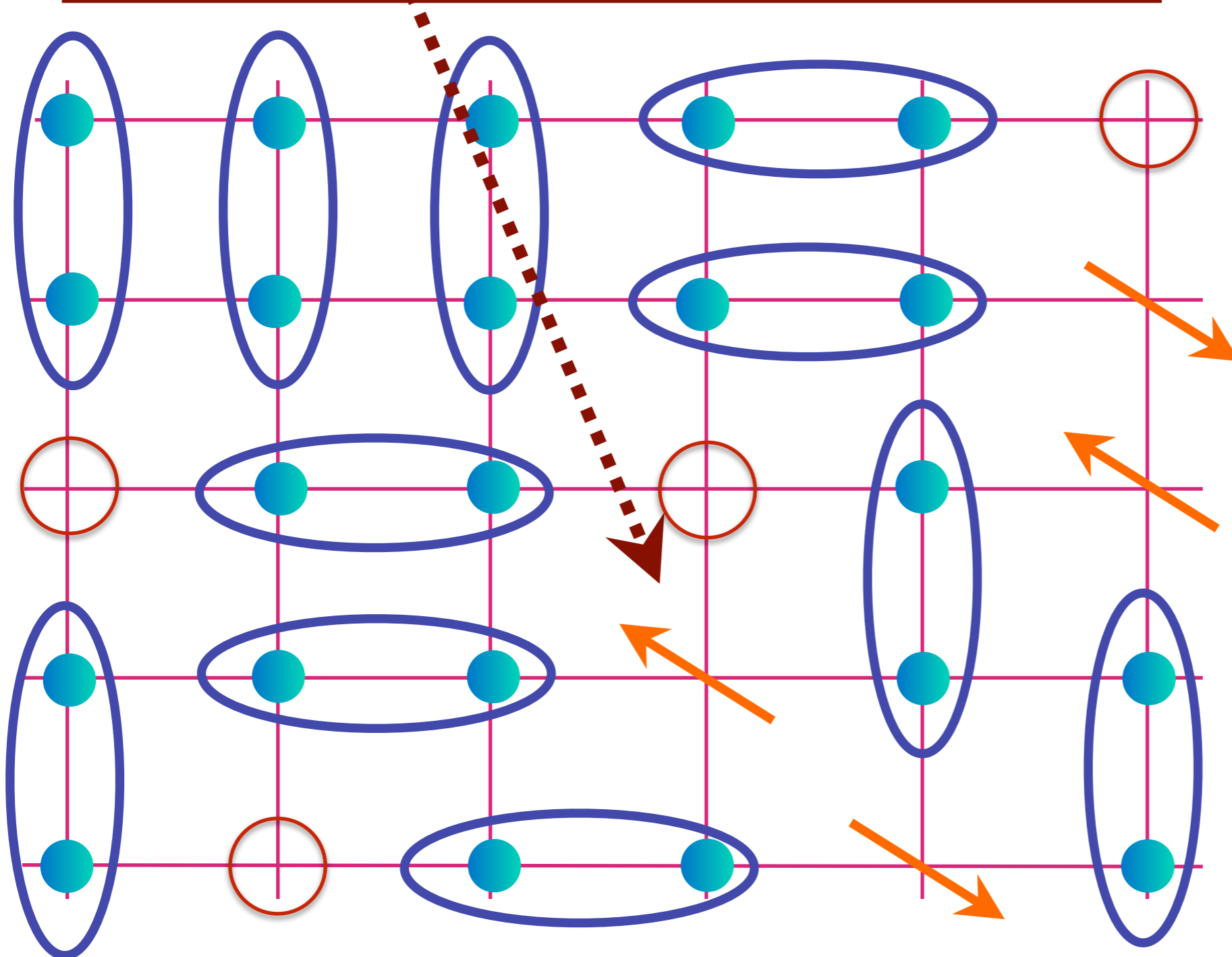


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per square

$$\text{[Pair of spinons]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

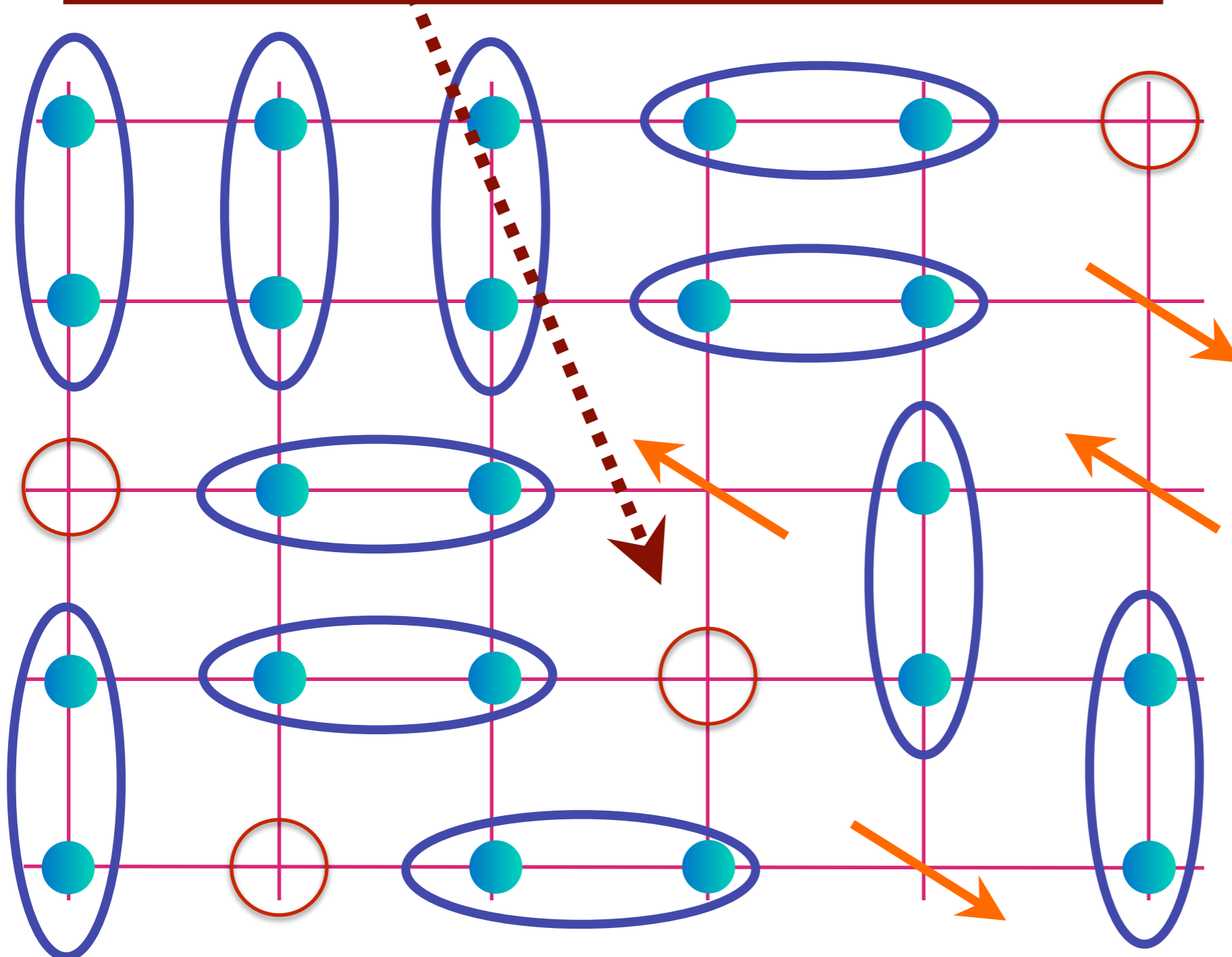
Nearest-neighbor hopping leads to attraction between holon and spinon, which can pay for the energy needed to create the spinon



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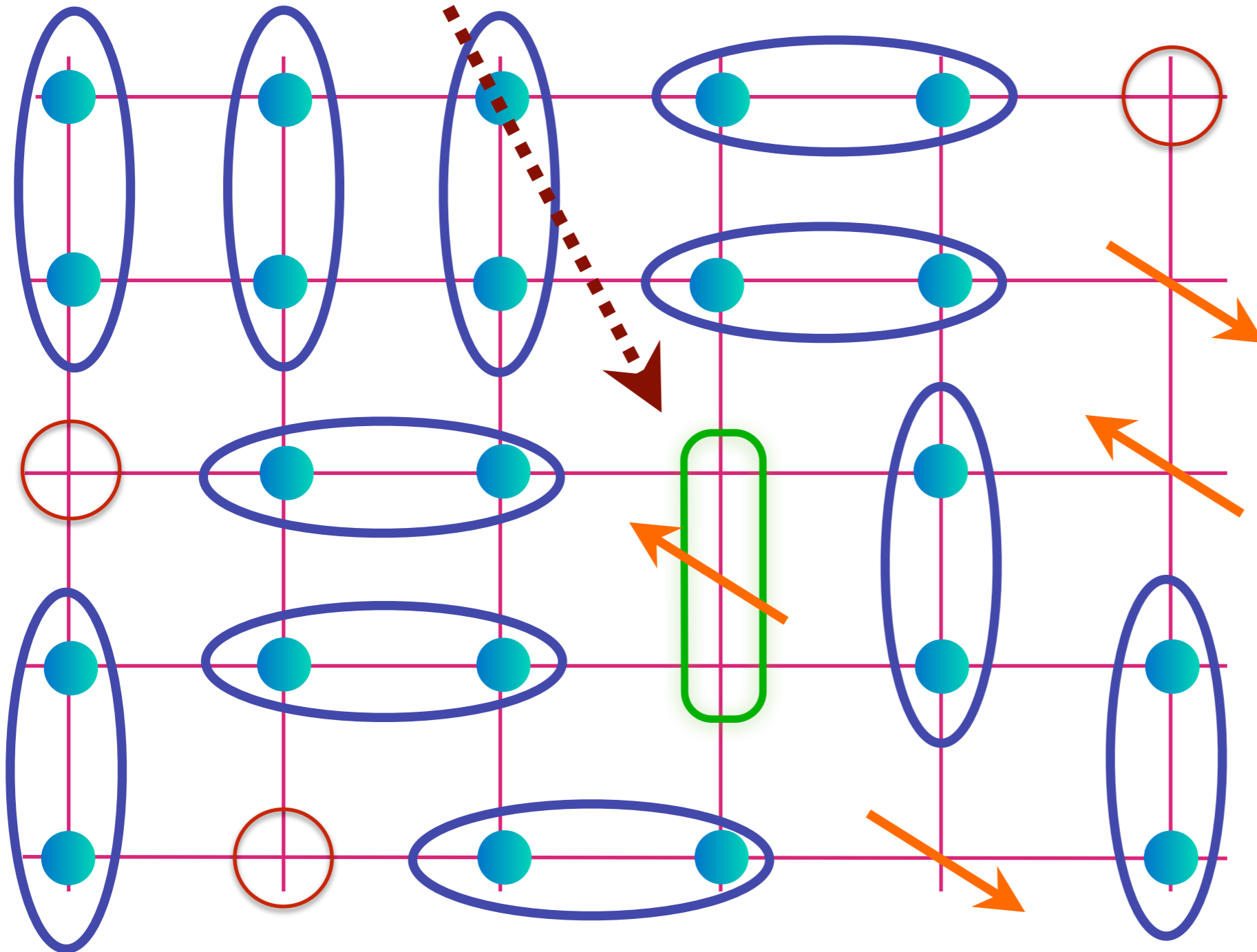
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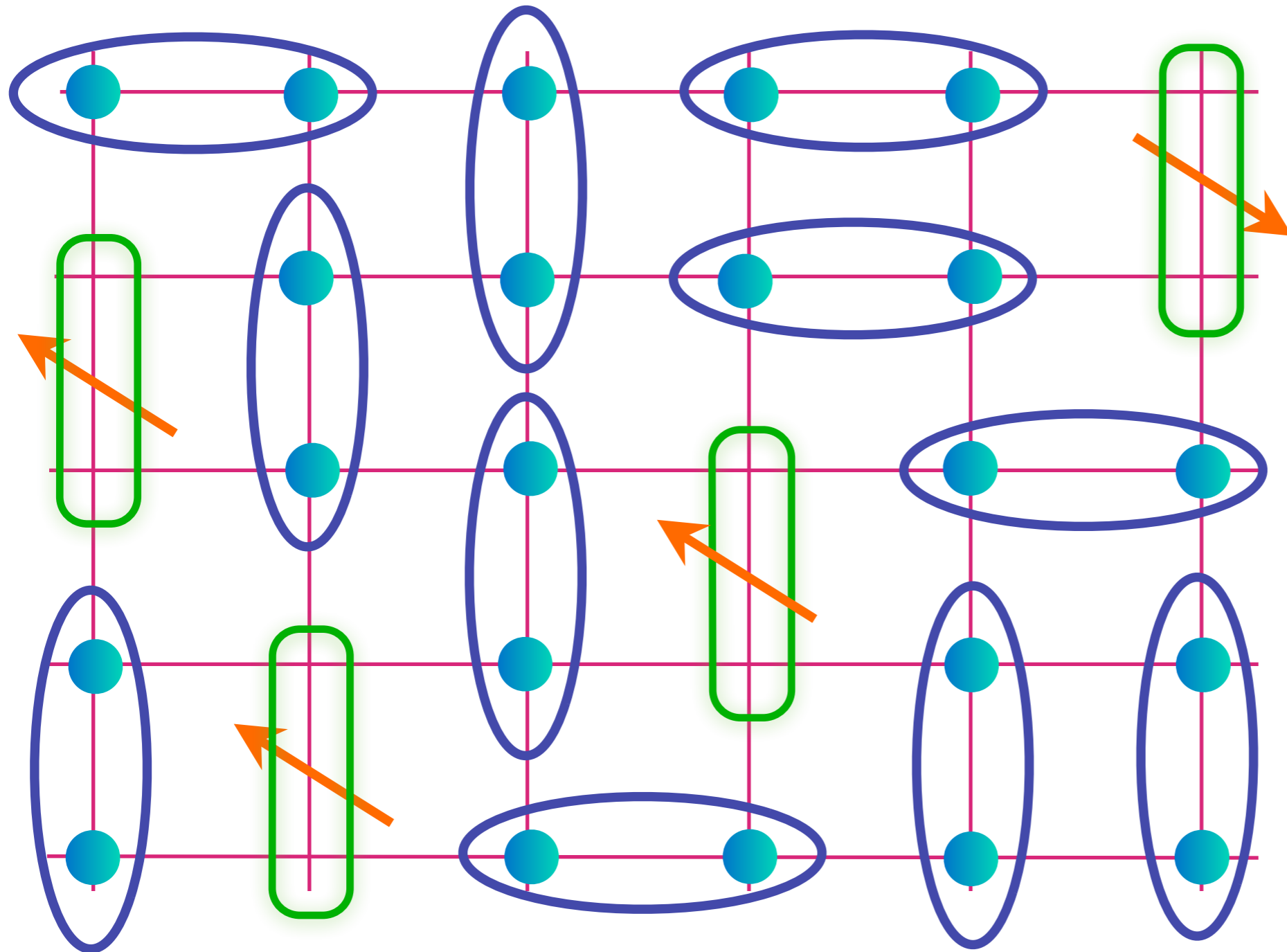
Electron (or hole) resides on a
“bonding” orbital between two sites



Spin liquid
with emergent
gauge field
and
 p “holons”
(gauge-charged,
spinless,
charge $+e$
quasiparticles)
per square

$$\text{[Blue oval with two cyan dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Fractionalized Fermi liquid (FL*)

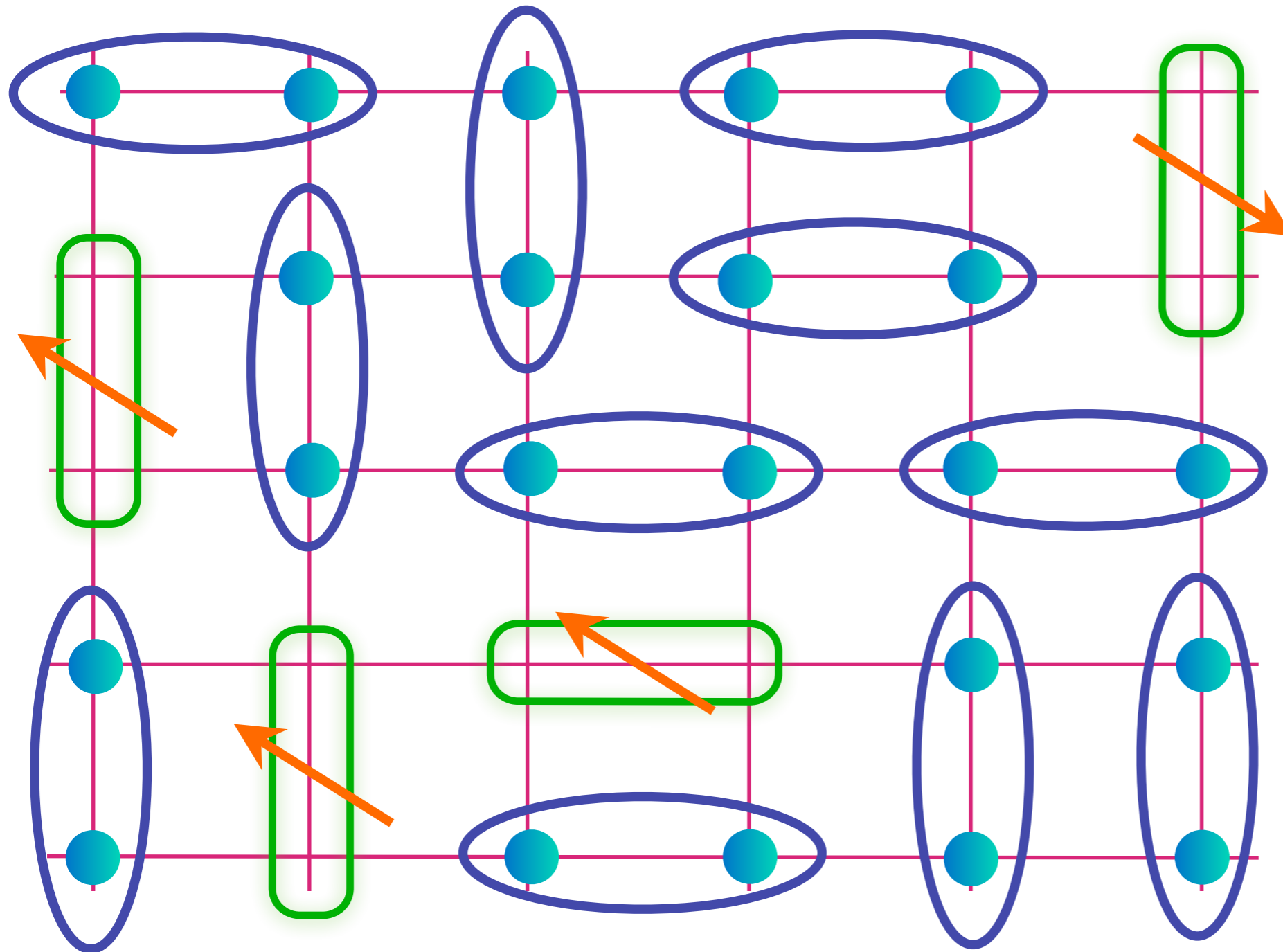


Emergent
gauge field
and
gauge-neutral,
spin $S=1/2$,
charge $+e$
fermions
of density p

T. Senthil, S. Sachdev, M. Vojta *Phys. Rev. Lett.* **90**, 216403 (2003)

M. Punk, A. Allais, and S. Sachdev, arXiv:1501.00978.

Fractionalized Fermi liquid (FL*)

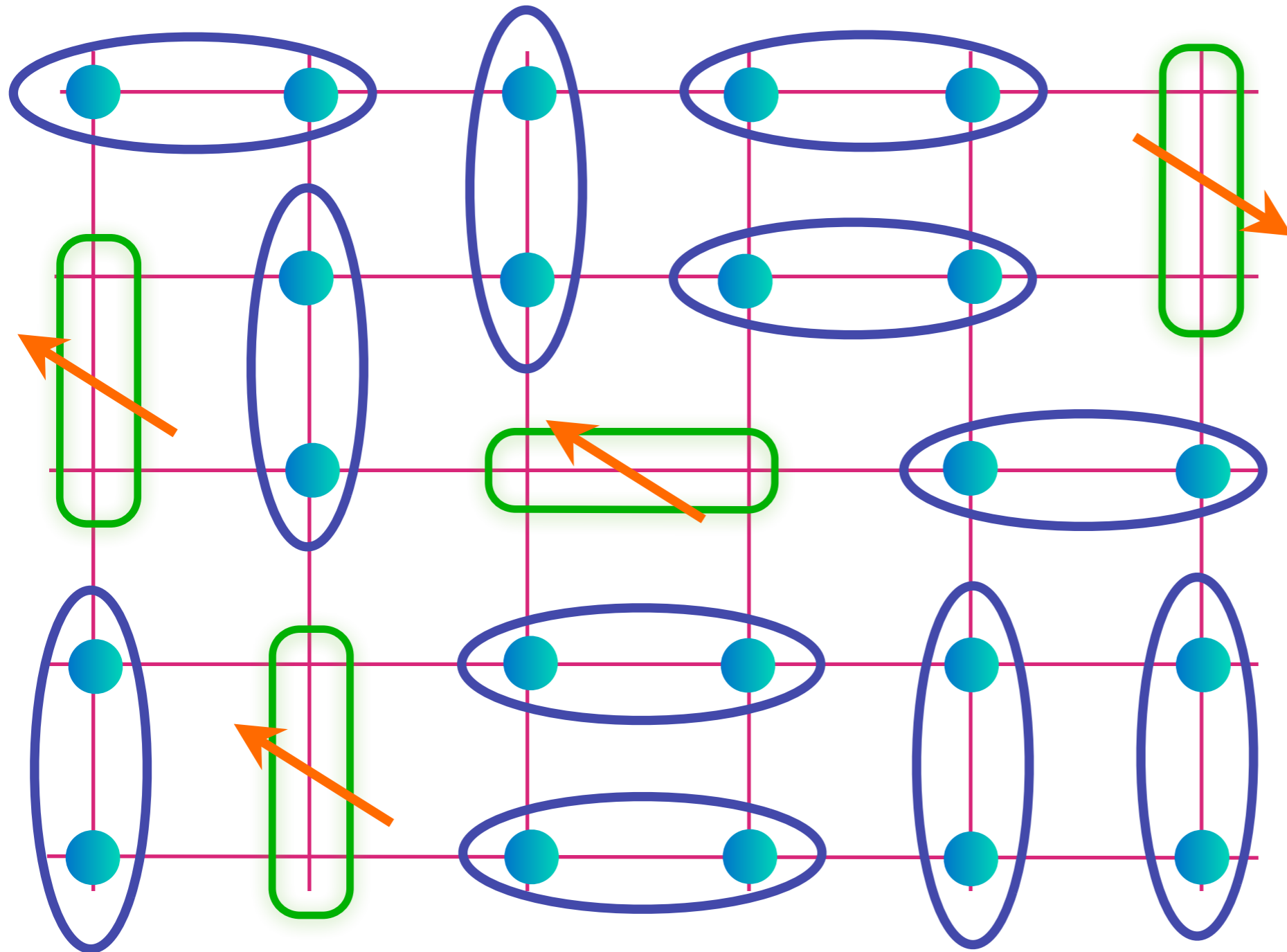


Emergent
gauge field
and
gauge-neutral,
spin $S=1/2$,
charge $+e$
fermions
of density ρ

T. Senthil, S. Sachdev, M. Vojta *Phys. Rev. Lett.* **90**, 216403 (2003)

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Fractionalized Fermi liquid (FL*)

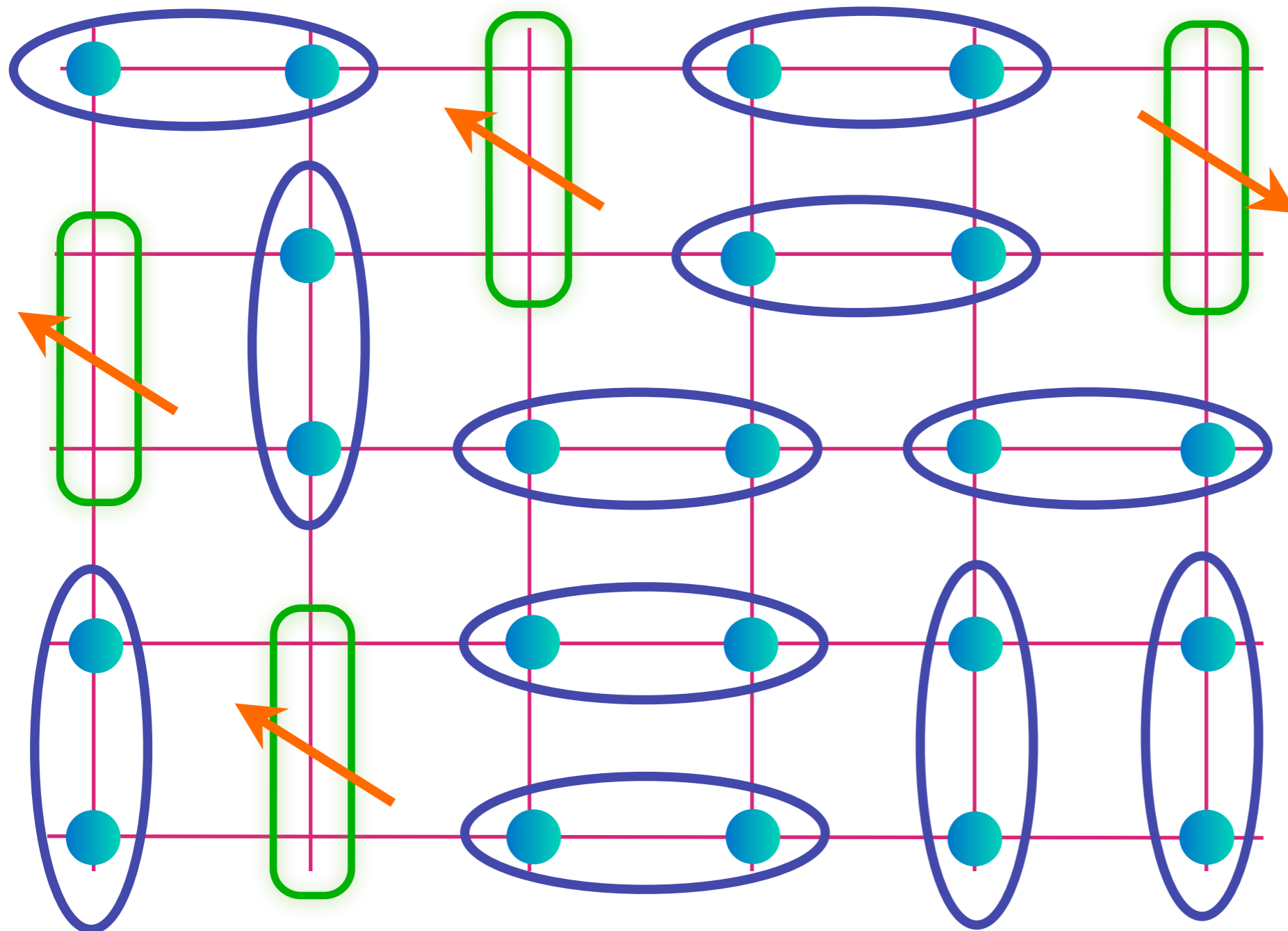


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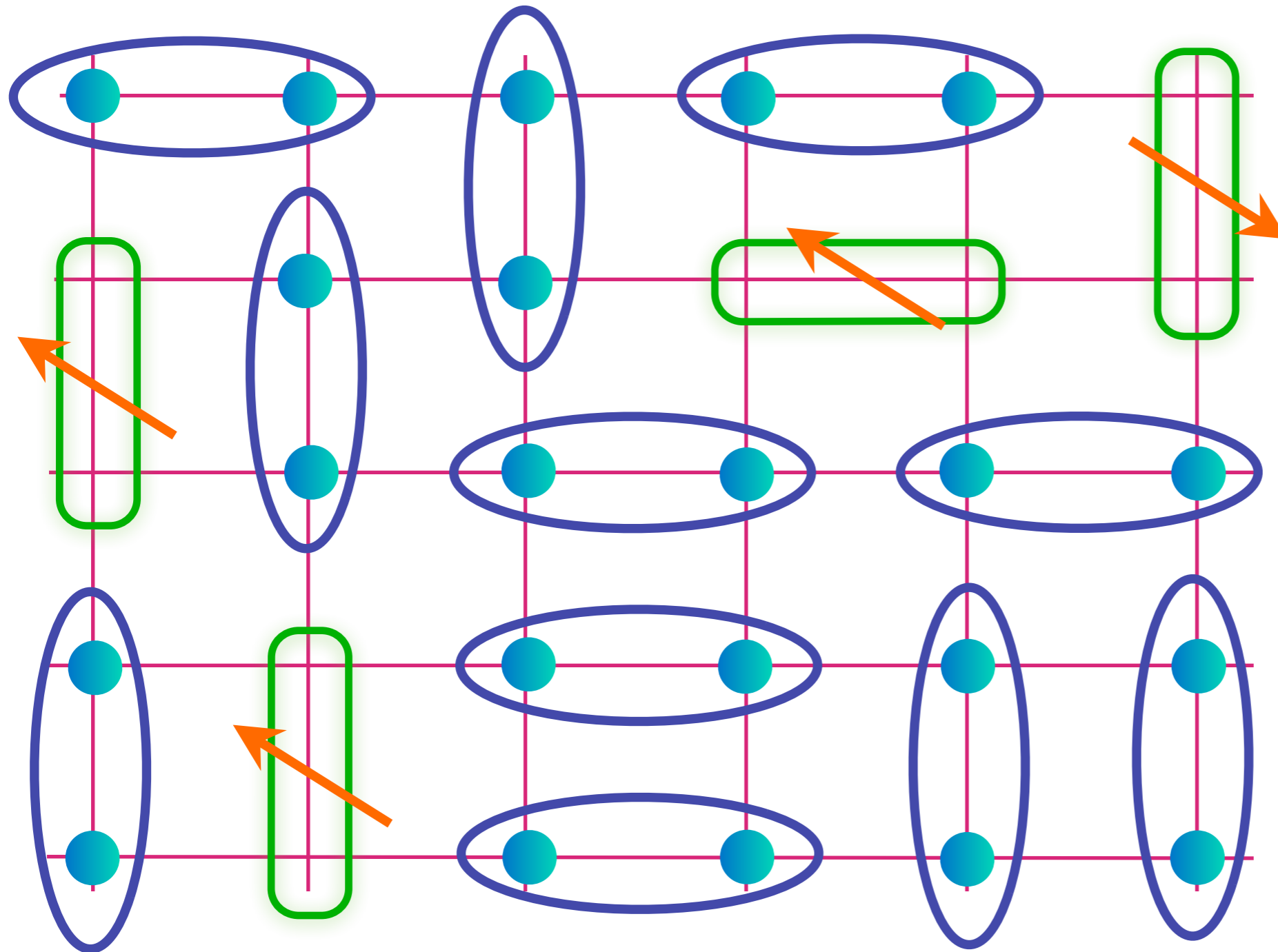


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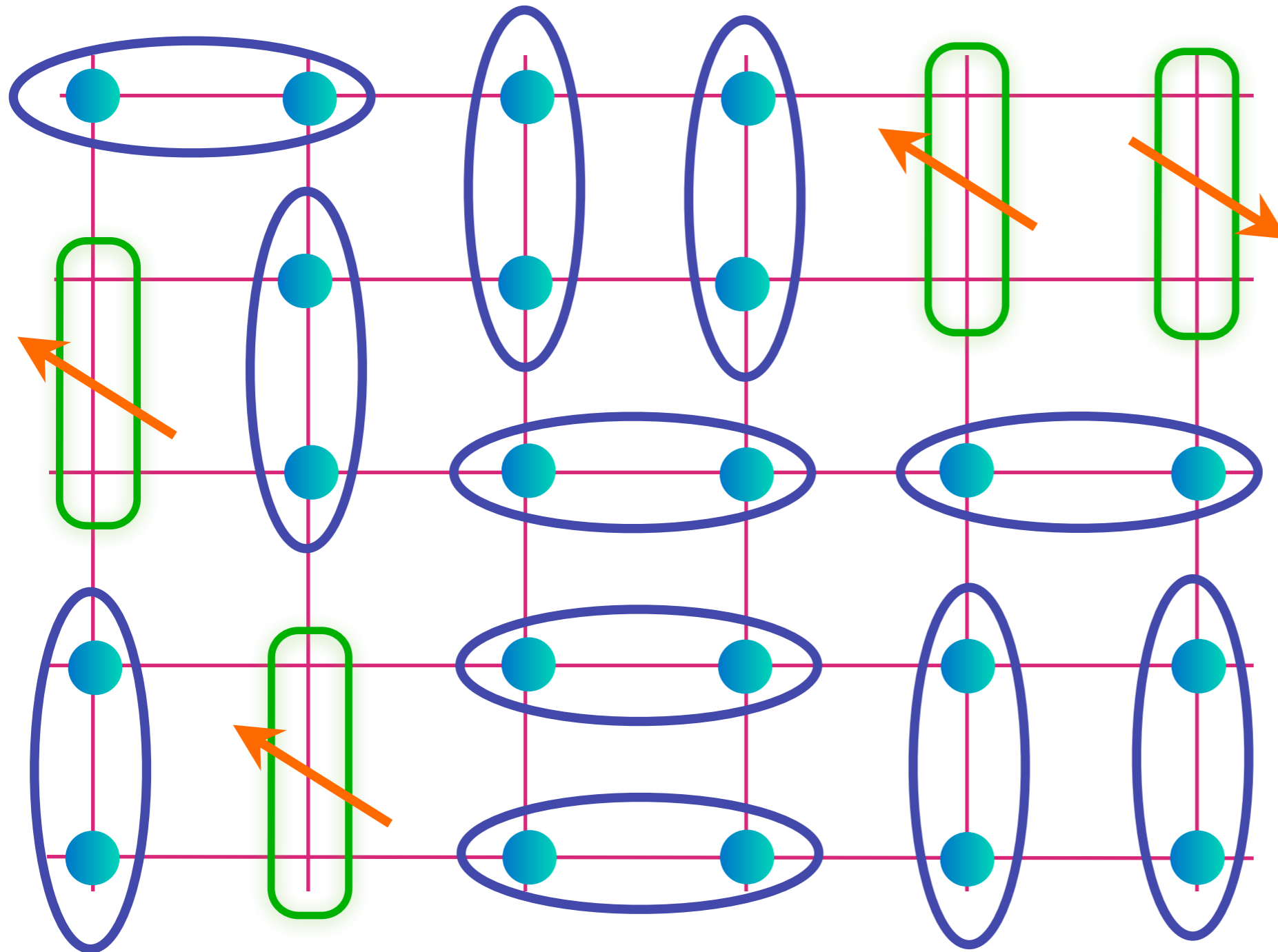


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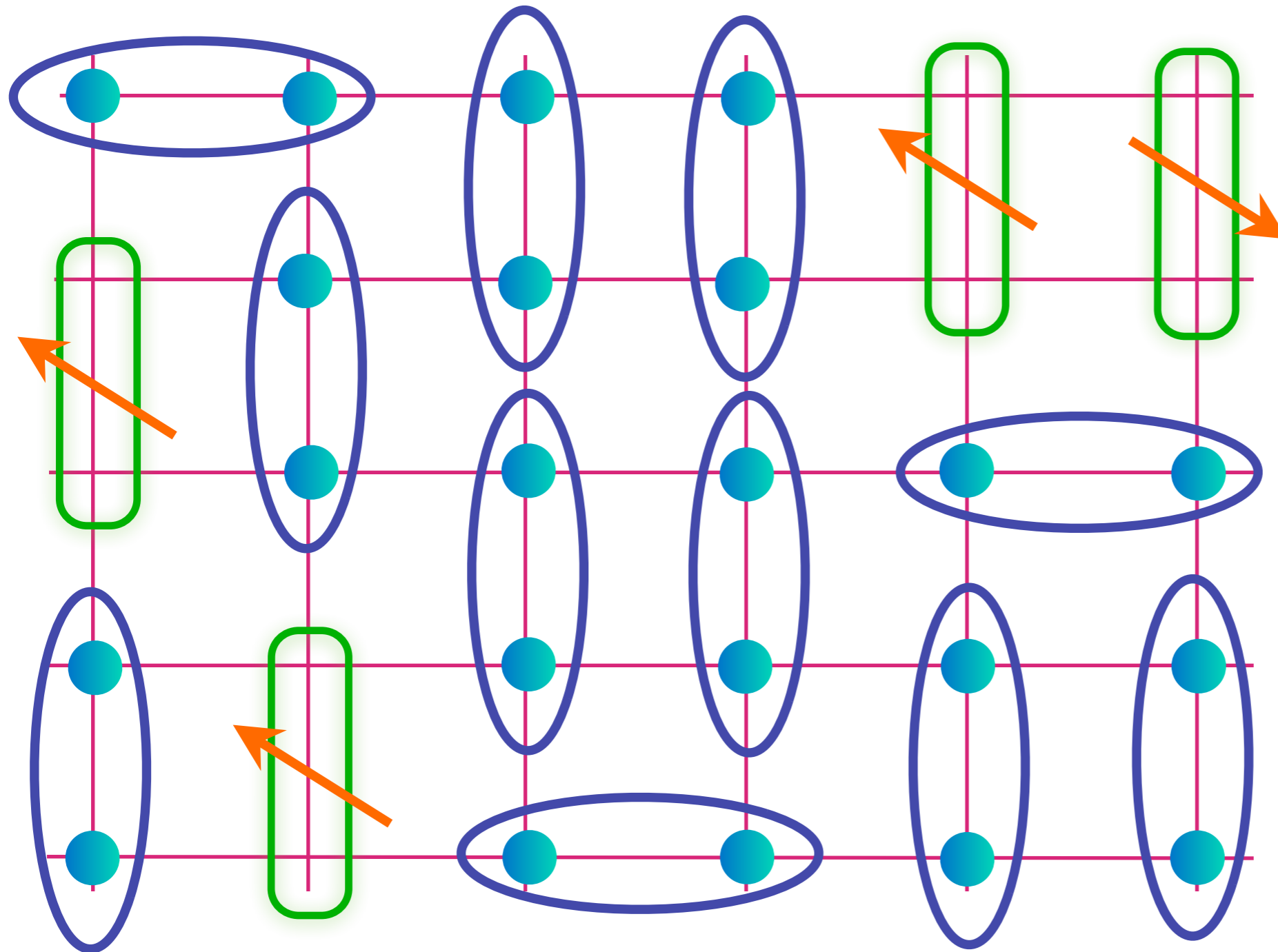


Emergent
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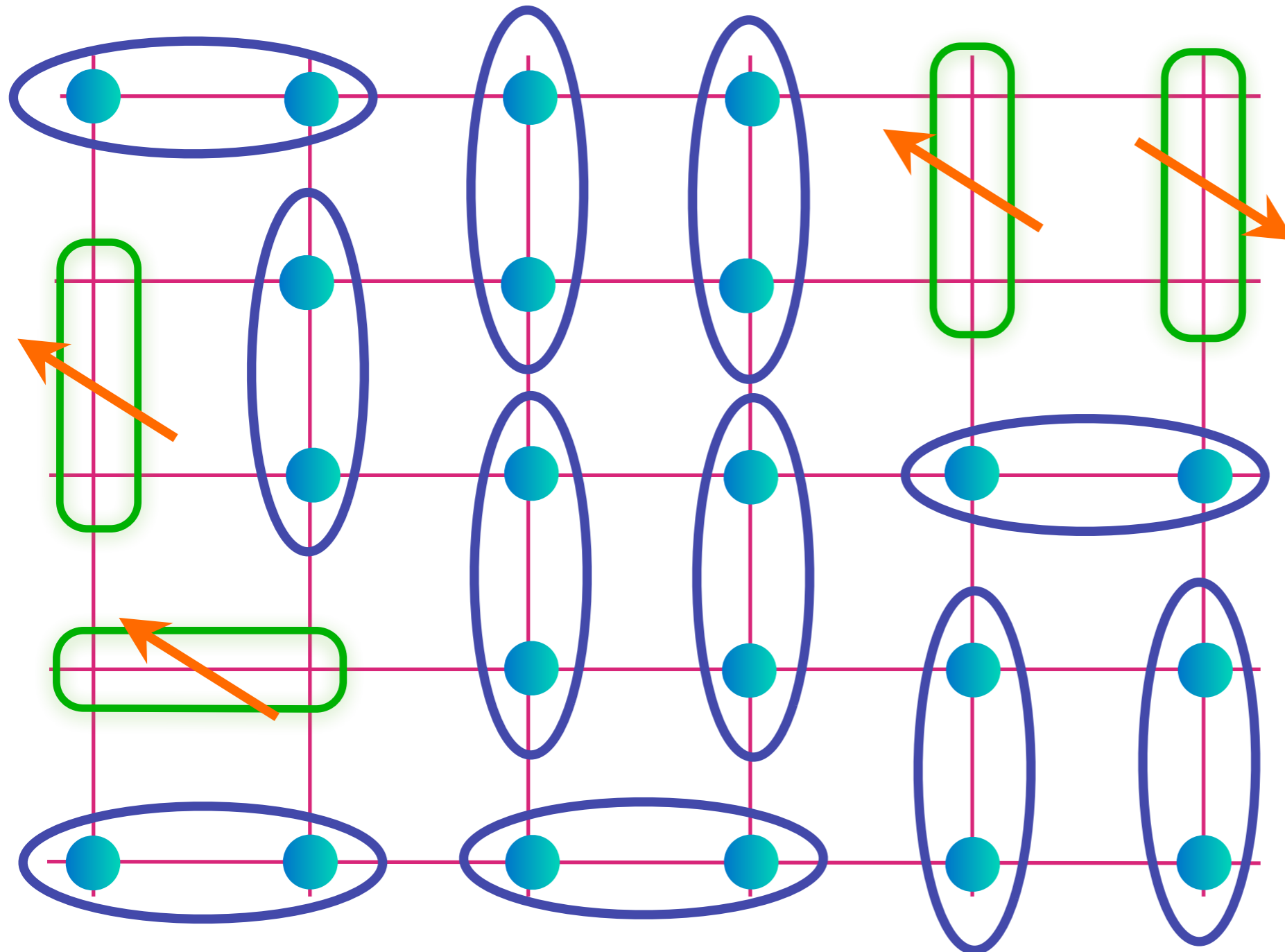


Emergent
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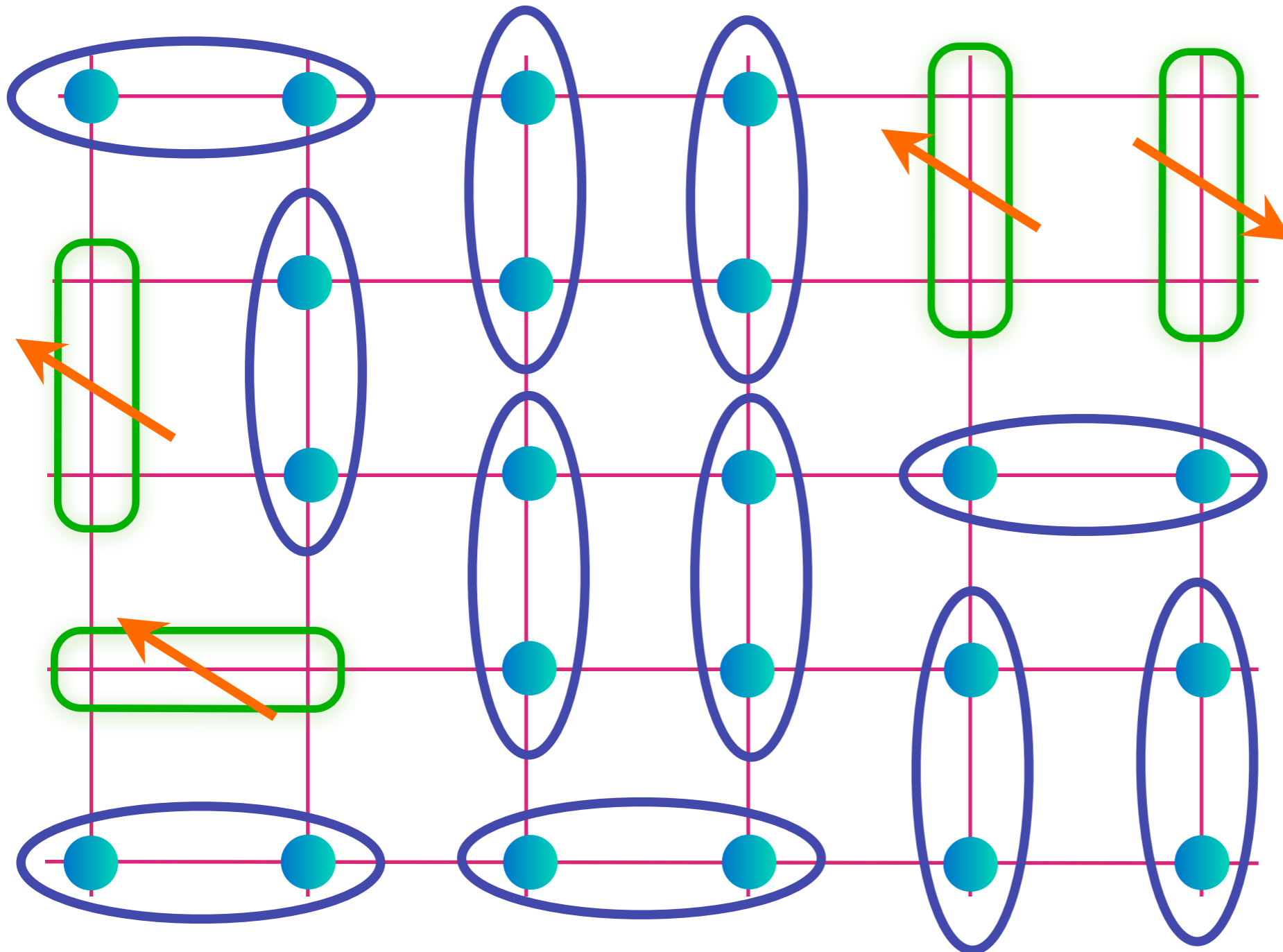


Emergent
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and
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T. Senthil, S. Sachdev, M. Vojta *Phys. Rev. Lett.* **90**, 216403 (2003)

M. Punk, A. Allais, and S. Sachdev, arXiv:1501.00978.

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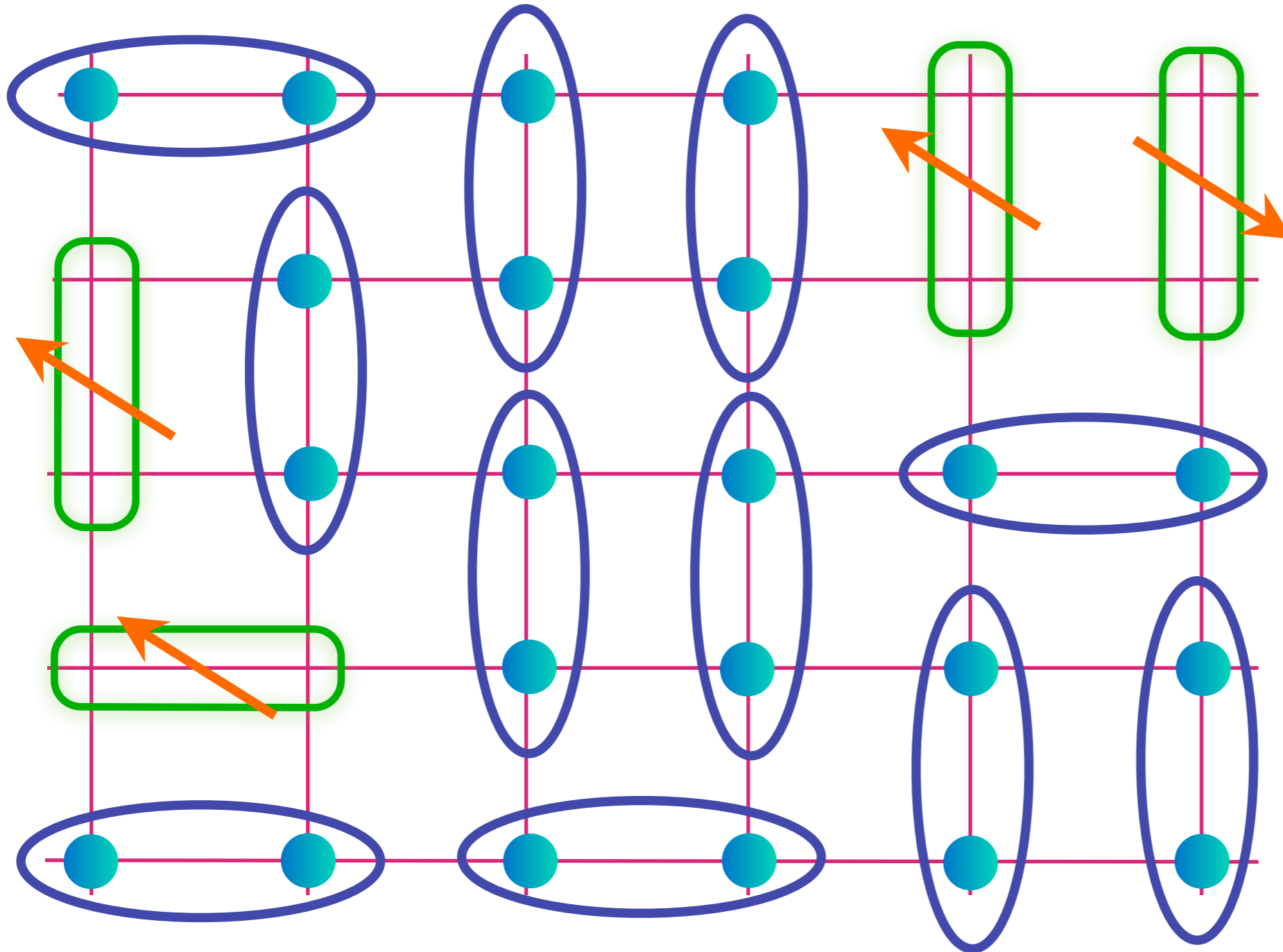


Realizes a metal with a Fermi surface of area p , and “topological order”

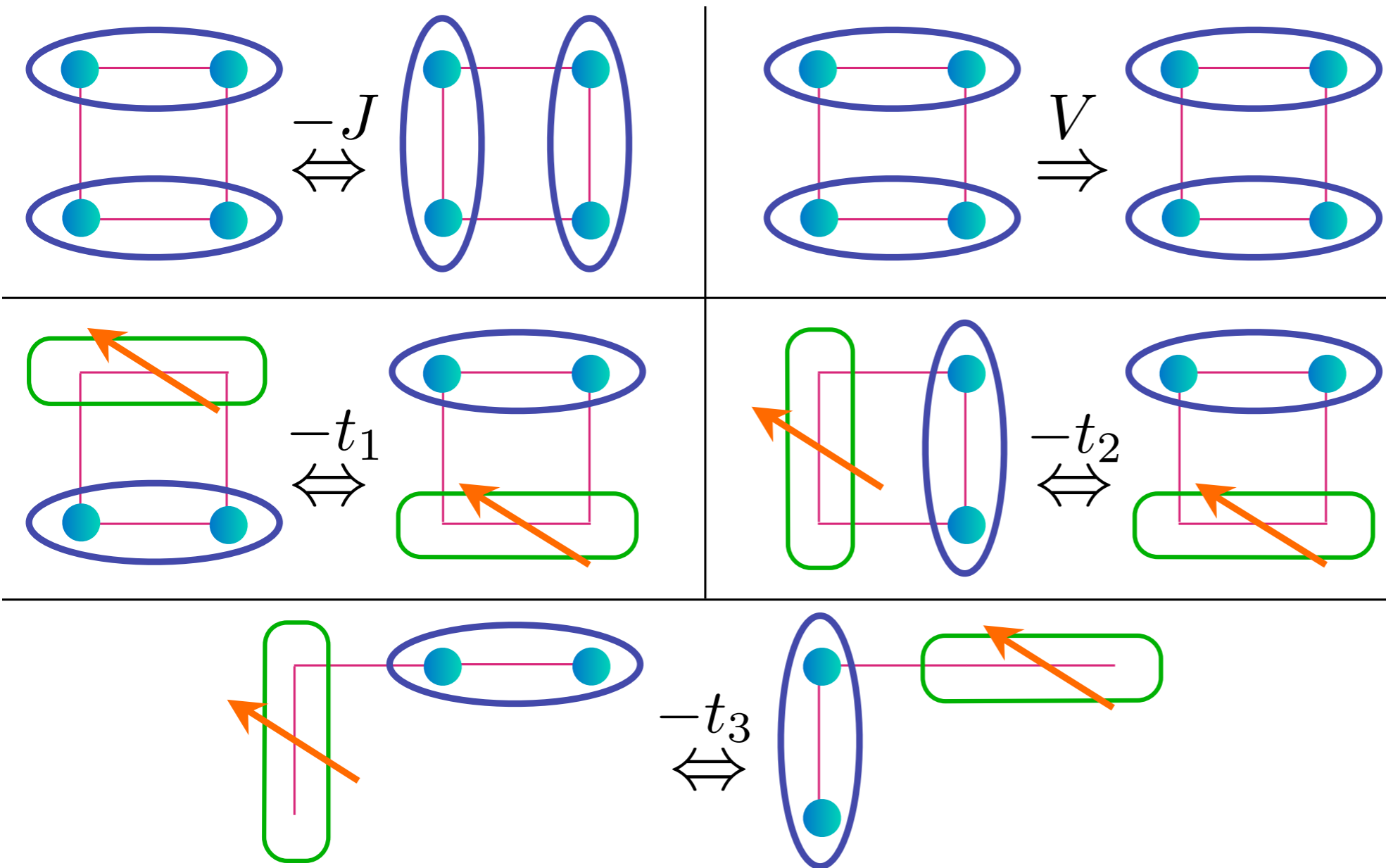
T. Senthil, S. Sachdev, M. Vojta *Phys. Rev. Lett.* **90**, 216403 (2003)

M. Punk, A. Allais, and S. Sachdev, arXiv:1501.00978.

Quantum dimer model with bosonic and fermionic dimers

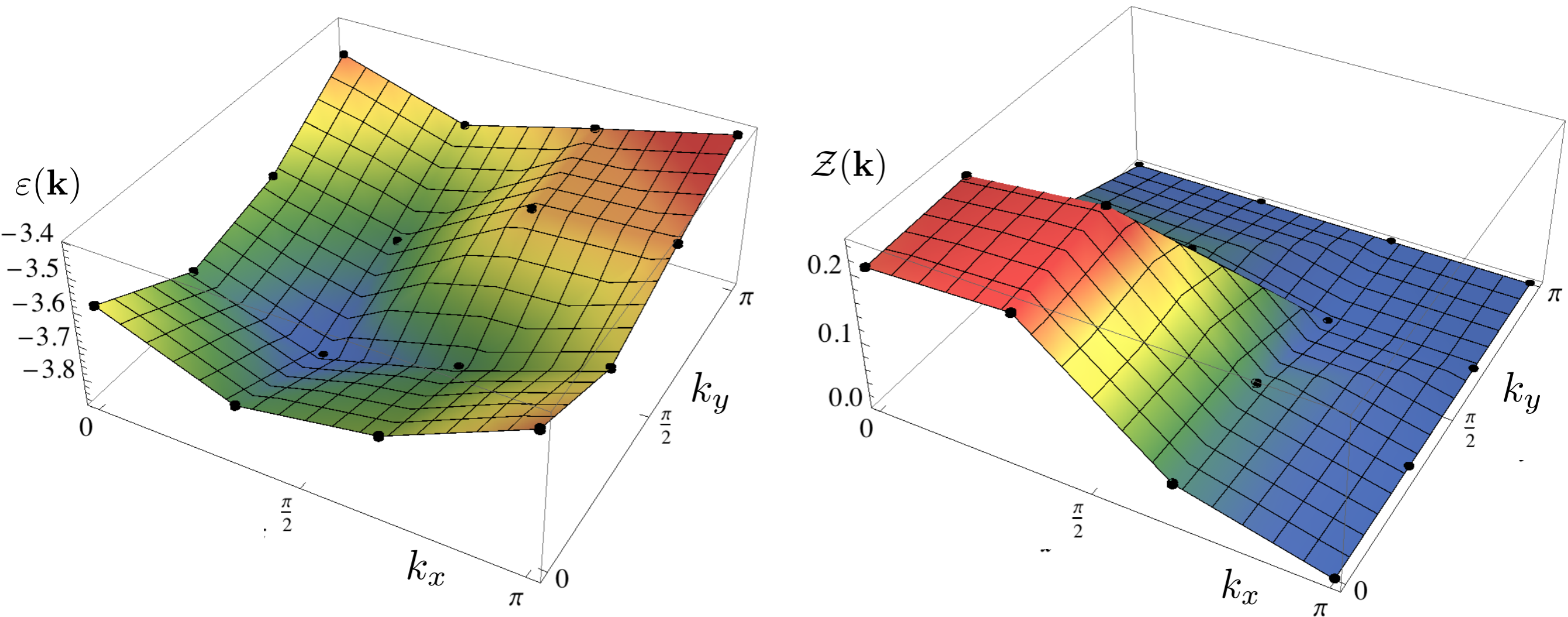


Quantum dimer model with bosonic and fermionic dimers



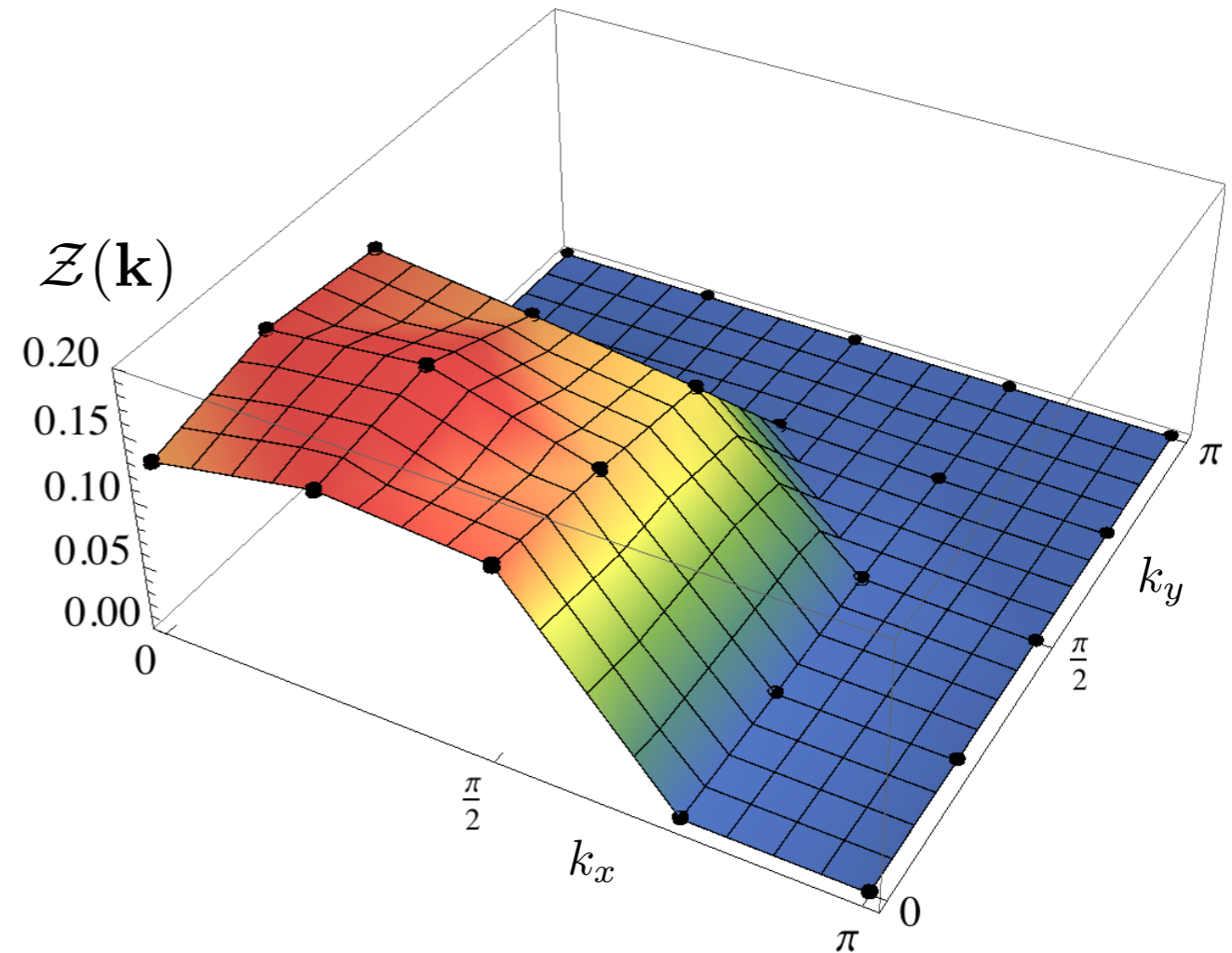
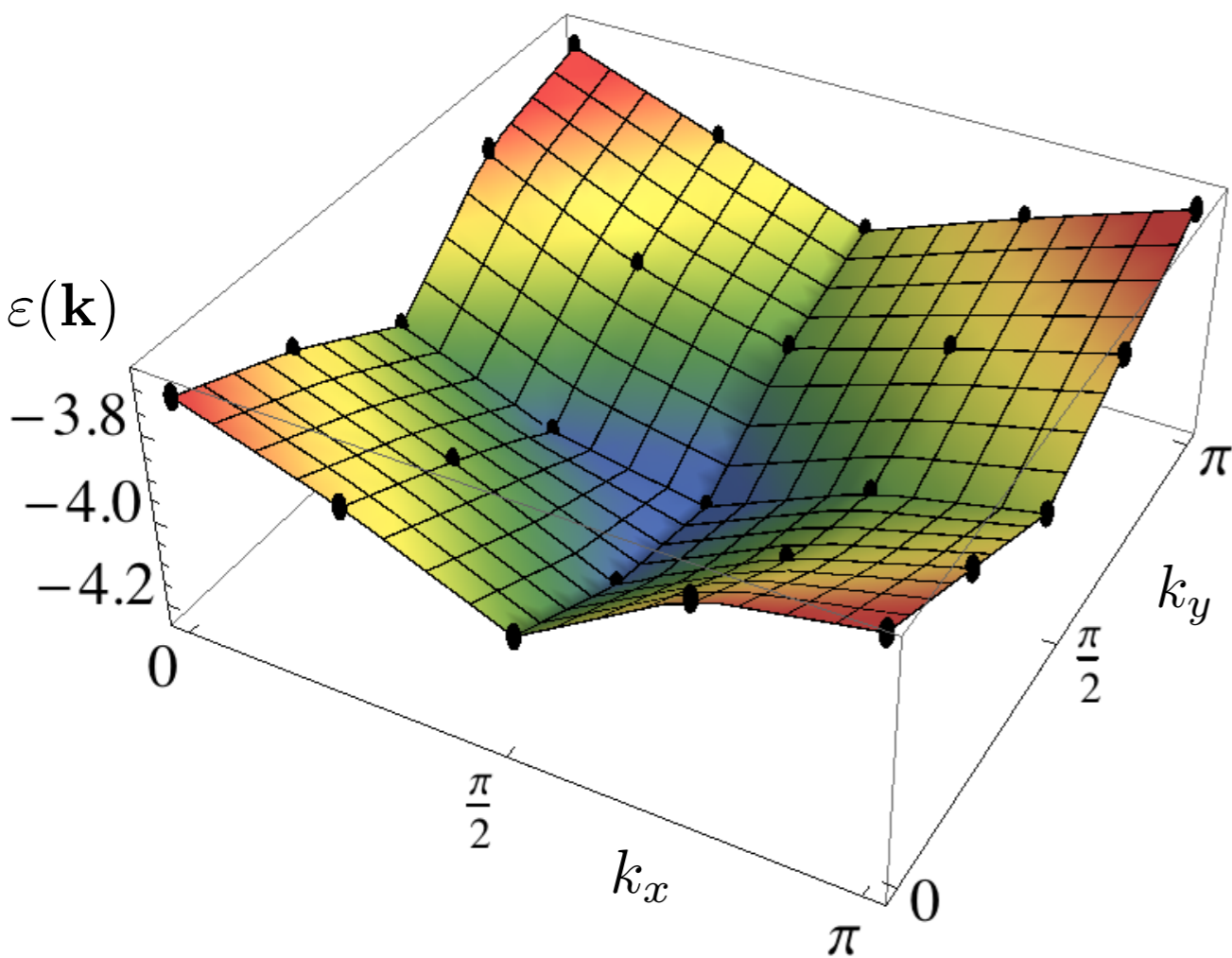
Connection to the $t-t'-t''-J$ model:
 $t_1 = -(t + t')/2$
 $t_2 = (t - t')/2$
 $t_3 = -(t + t' + t'')/4$

Quantum dimer model with bosonic and fermionic dimers

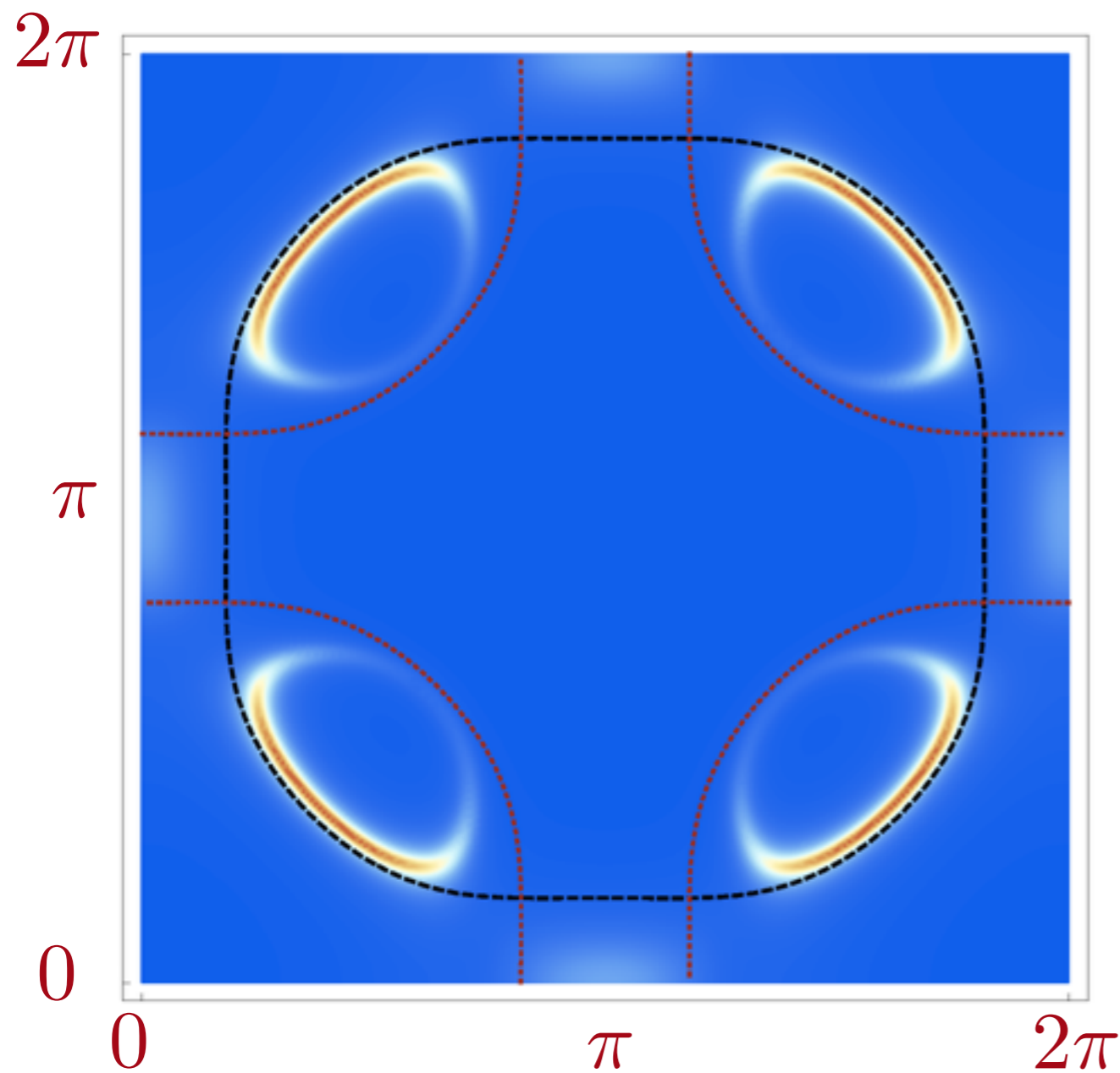


Dispersion and quasiparticle residue for $J = V = 1$, and hopping parameters obtained from the t - J model for the cuprates, $t_1 = -1.05$, $t_2 = 1.95$ and $t_3 = -0.6$, on a 6×6 lattice.

Quantum dimer model with bosonic and fermionic dimers



Dispersion and quasiparticle residue for $J = V = 1$, and hopping parameters $t_1 = 1$, $t_2 = 1$ and $t_3 = -1$, on a 8×8 lattice.

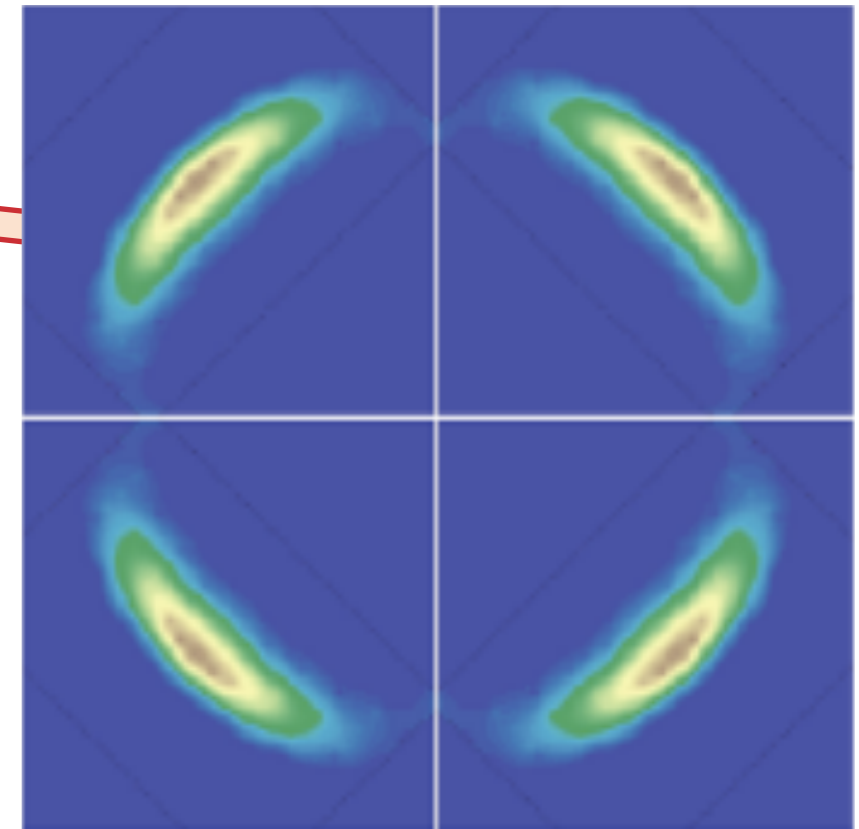
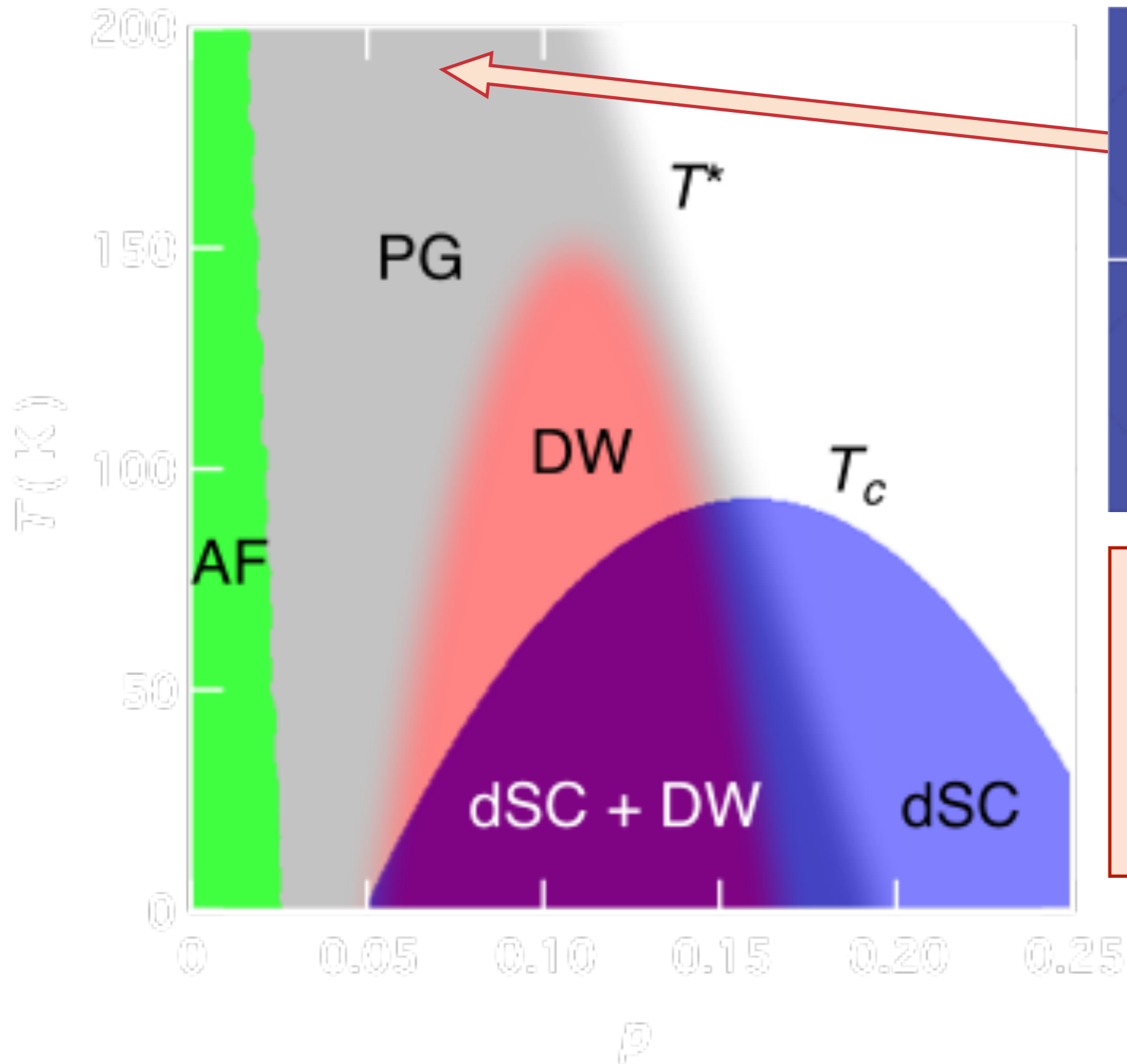


Electron spectral
function of FL*

Semi-phenomenological theory of a FL* state with hole pockets of volume p , along with a background spin liquid with an emergent U(1) gauge field. Note that the quasiparticle excitations around the Fermi surface do not carry U(1) gauge charges.

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
M. Punk, A. Allais, and S. Sachdev, arXiv:1501.00978

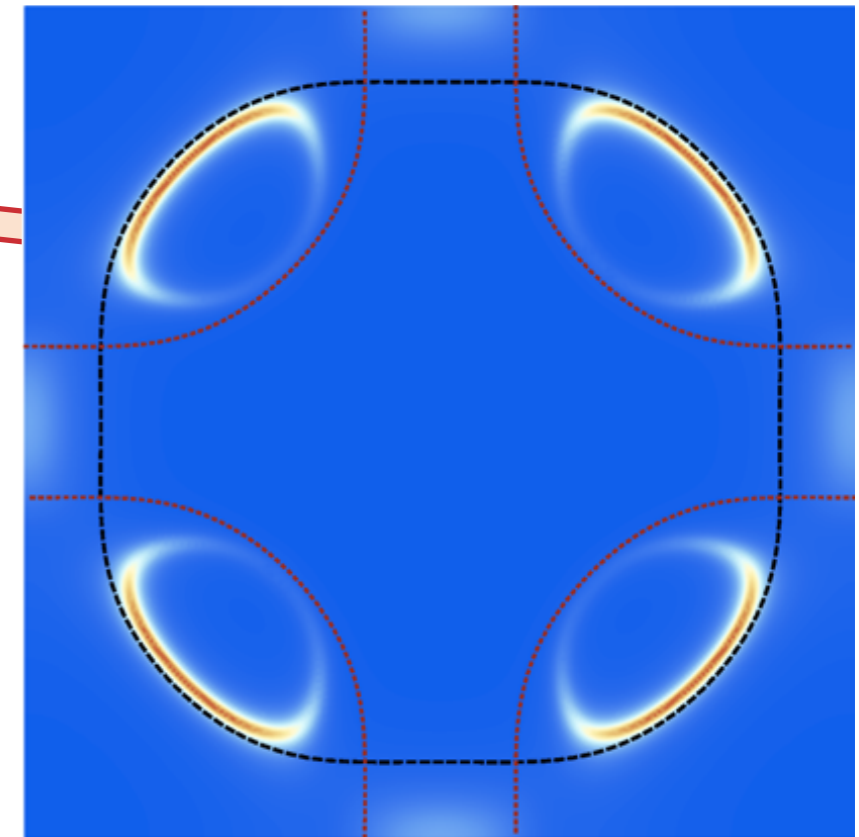
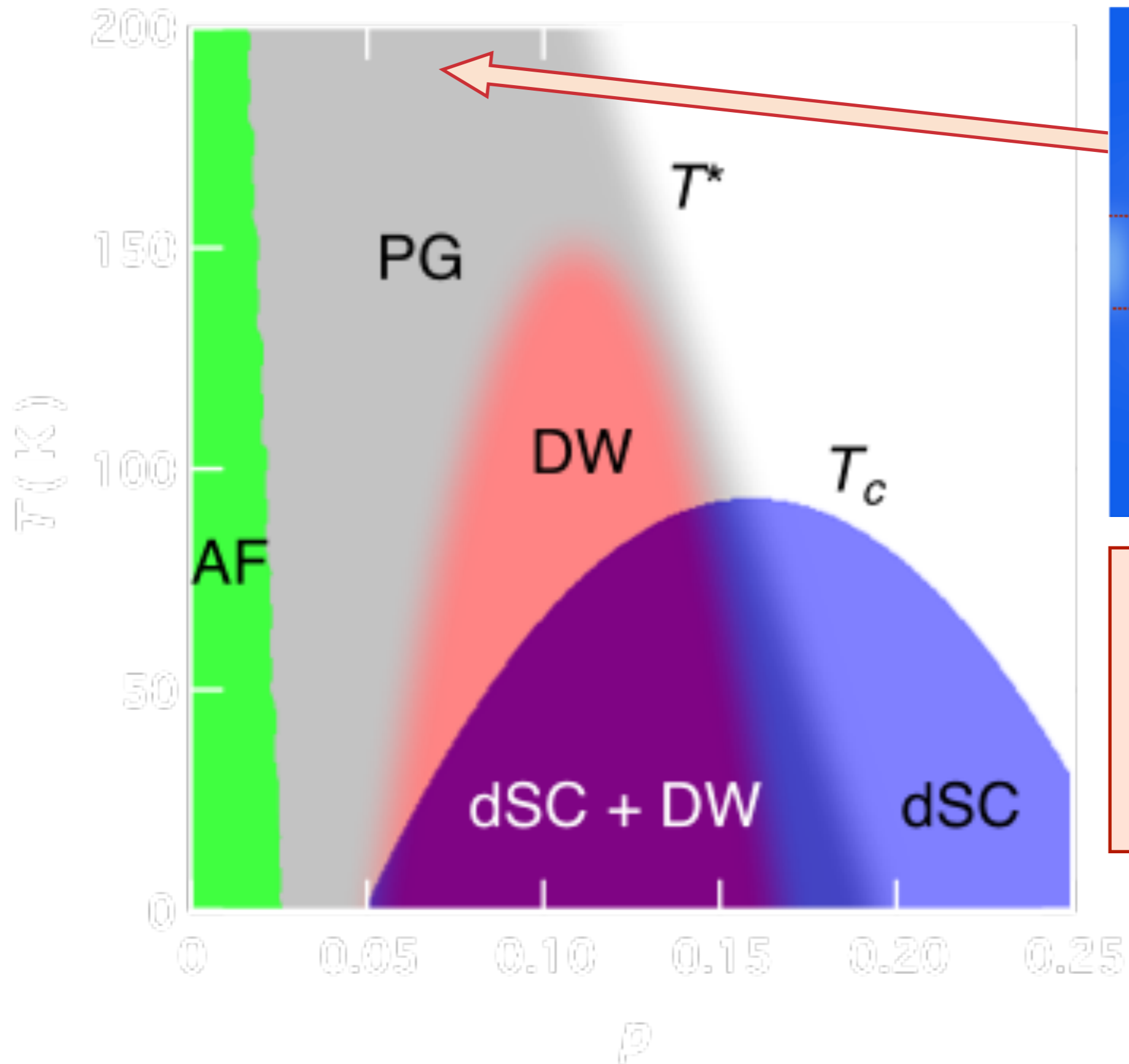
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, *Science* **307**, 901 (2005)



“Fermi arcs”
at
low p

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)

M. Punk, A. Allais, and S. Sachdev, arXiv:1501.00978



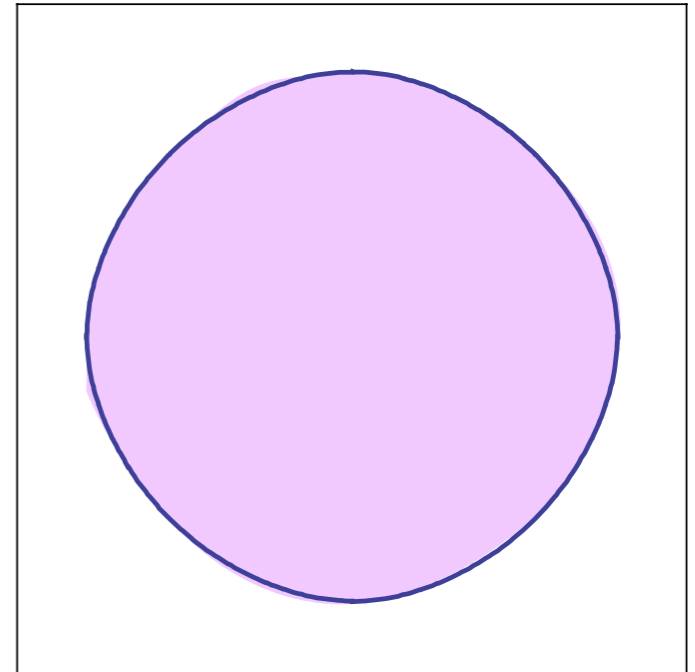
FL* described
by a quantum
dimer model

Outline

1. The low T pseudogap:
*STM observation of predicted
 d -form factor density wave*
2. The high T pseudogap:
*A quantum dimer model for a metal
with topological order*
3. Connecting high and low T :
Density wave instabilities
4. Quantum critical point near optimal p :
A Higgs critical point

Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface

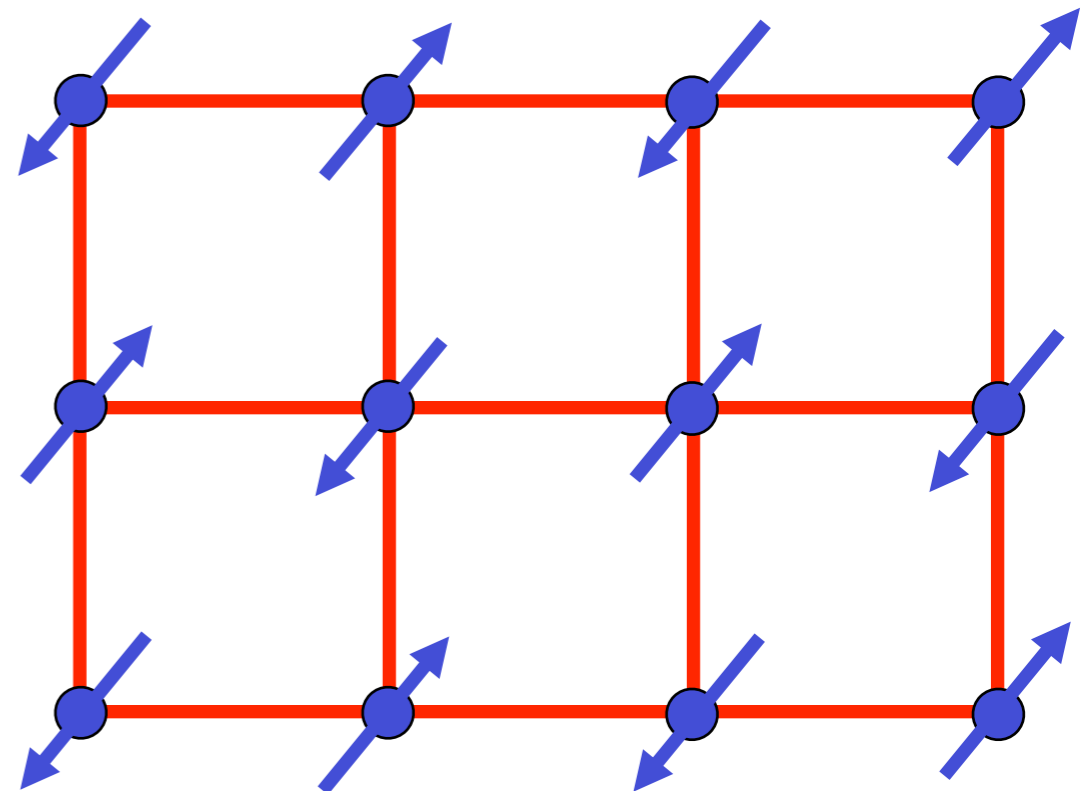


+

The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

where $\mathbf{K} = (\pi, \pi)$ is the ordering
wavevector.



Pairing “glue” for d-wave superconductivity from antiferromagnetic fluctuations



- V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
P. Monthoux, A. V. Balatsky, and D. Pines, *Phys. Rev. Lett.* **67**, 3448 (1991)

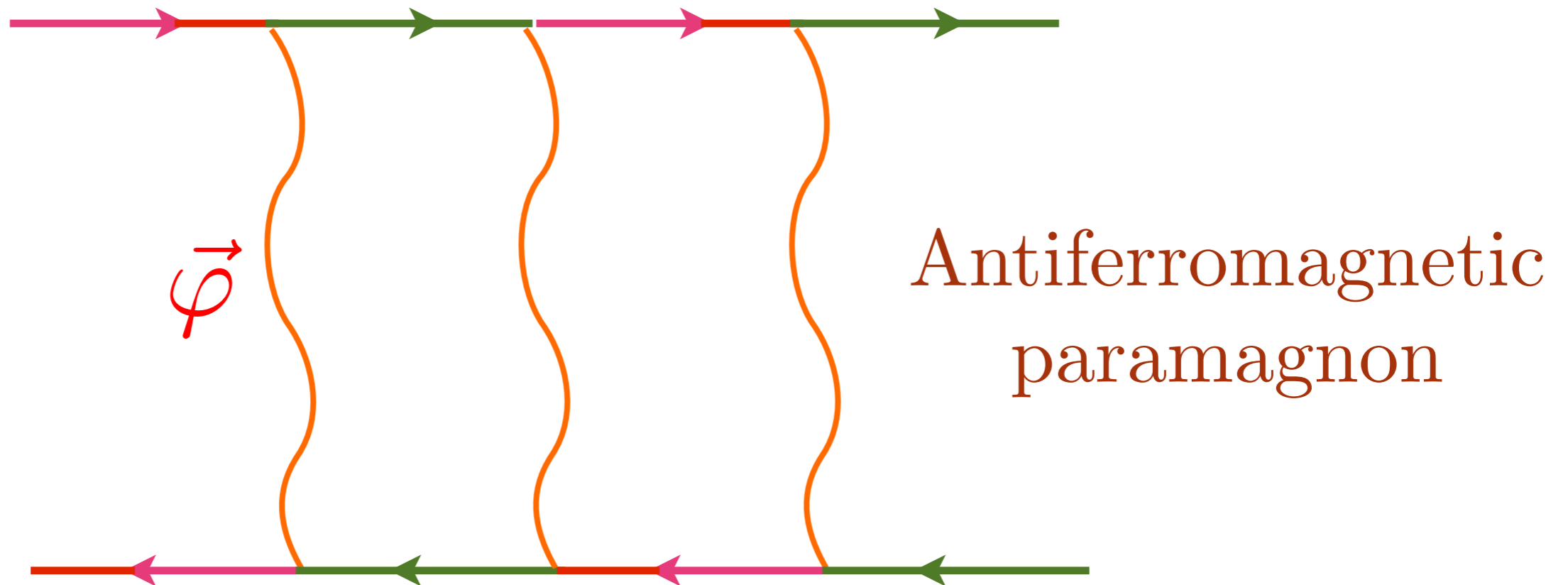
Pairing “glue” for d-wave superconductivity from antiferromagnetic fluctuations



Leads to $\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$

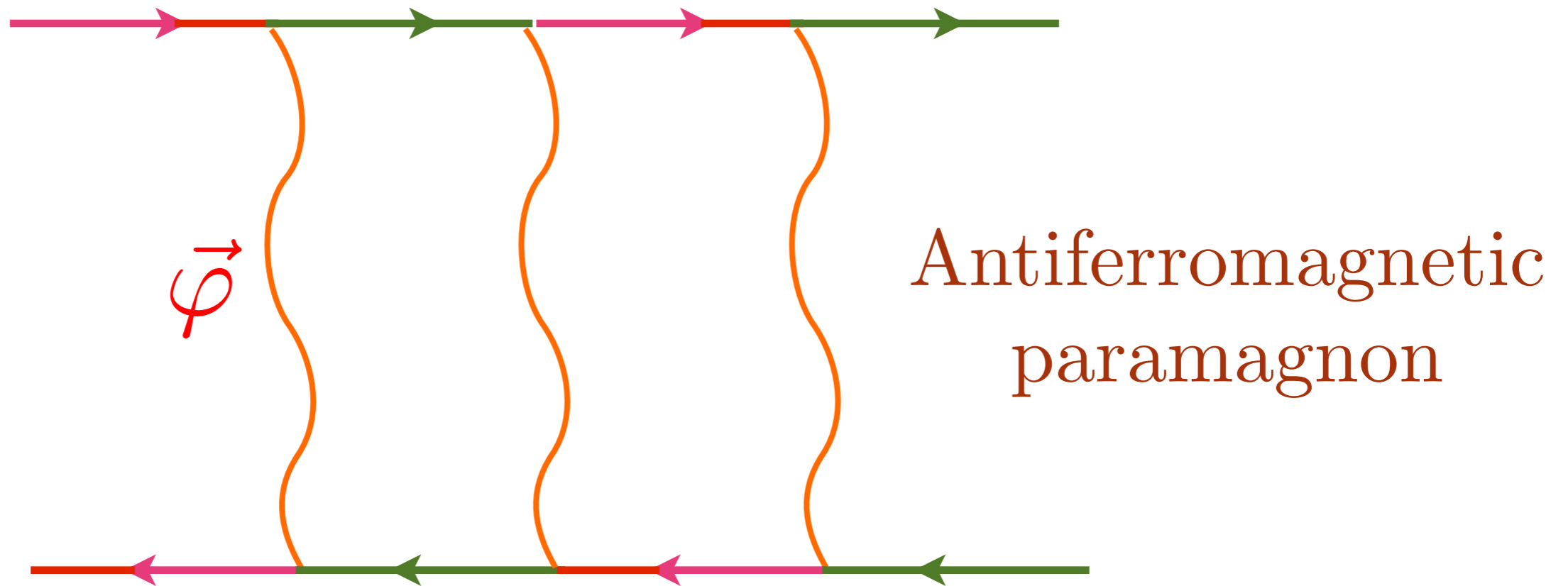
- V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
 D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
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 P. Monthoux, A. V. Balatsky, and D. Pines, *Phys. Rev. Lett.* **67**, 3448 (1991)

Same glue can lead to “d-wave” particle-hole pairing

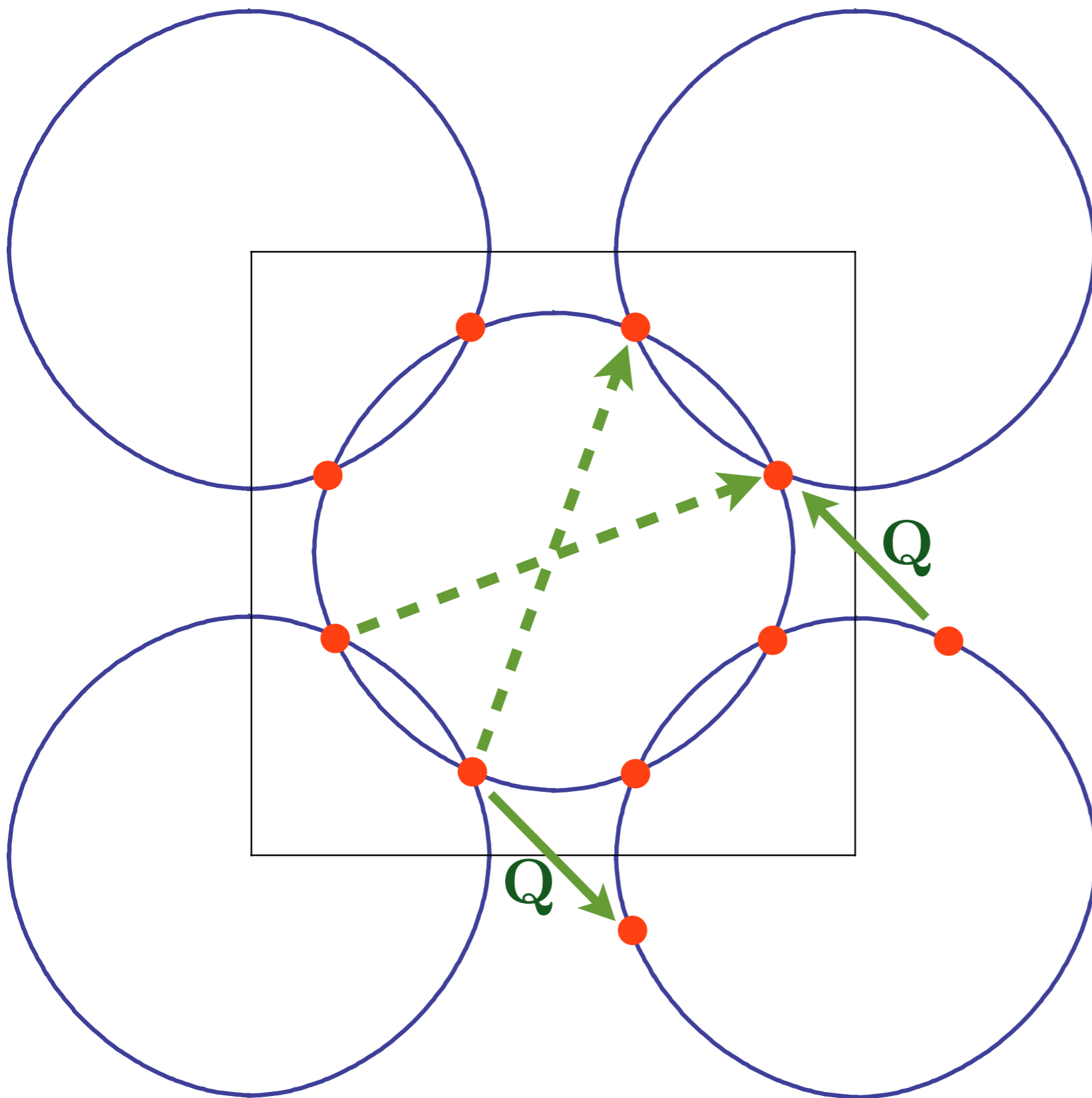


- M. A. Metlitski and S. Sachdev, Phys. Rev. B **85**, 075127 (2010)
T. Holder and W. Metzner, Phys. Rev. B **85**, 165130 (2012)
M. Bejas, A. Greco, and H. Yamase, Phys. Rev. B **86**, 224509 (2012)
S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013)
K. B. Efetov, H. Meier, and C. Pépin, Nat. Phys. **9**, 442 (2013)
J. D. Sau and S. Sachdev, Phys. Rev. B **89**, 075129 (2014)
Y. Wang and A. V. Chubukov, Phys. Rev. B **90**, 035149 (2014)

Same glue can lead to “d-wave” particle-hole pairing

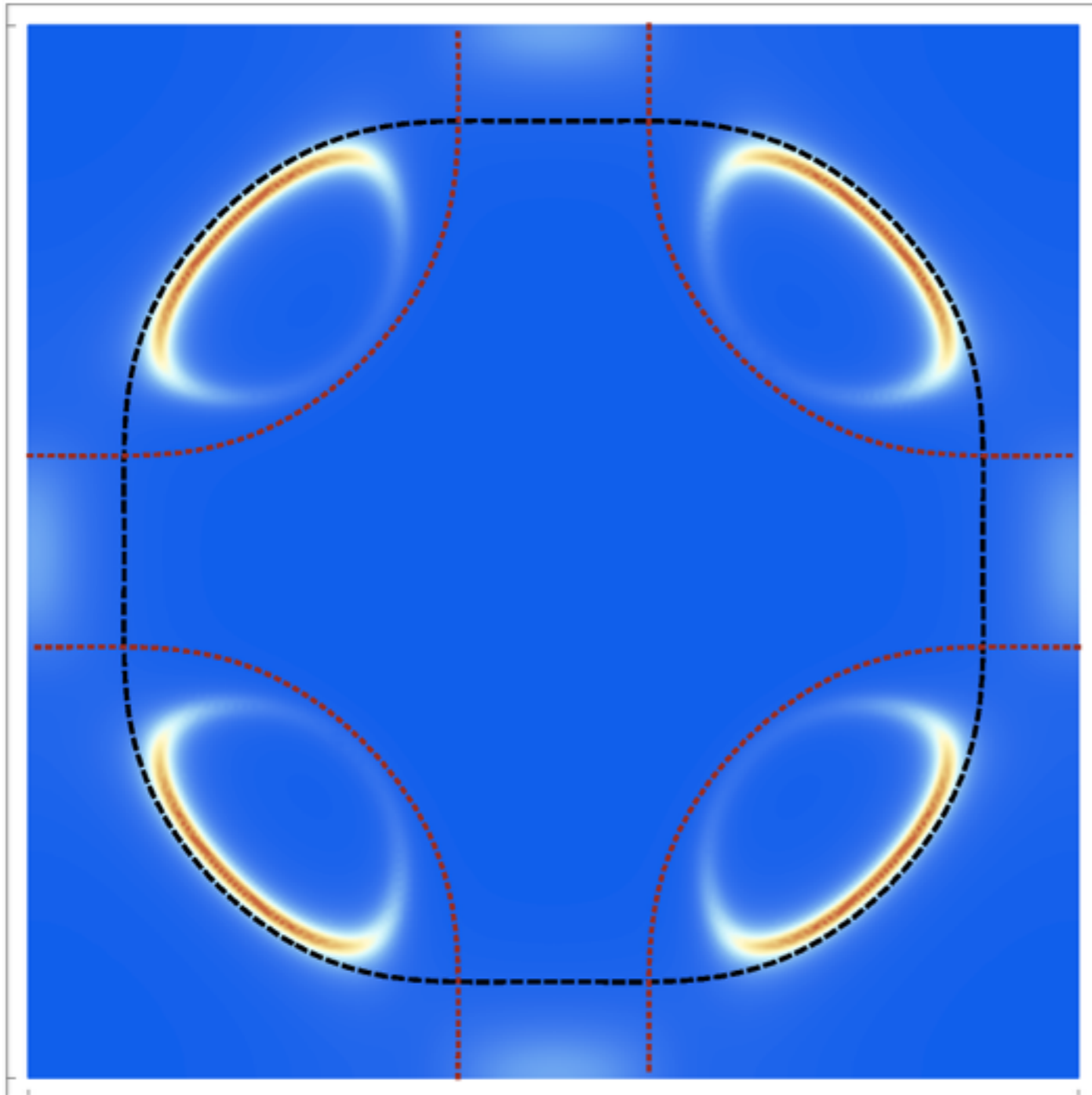


Leads to $\langle c_{\mathbf{k}+\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{Q}/2,\alpha} \rangle =$
 $\mathcal{P}_s + \mathcal{P}_{s'}(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y)$
with \mathcal{P}_d dominant.

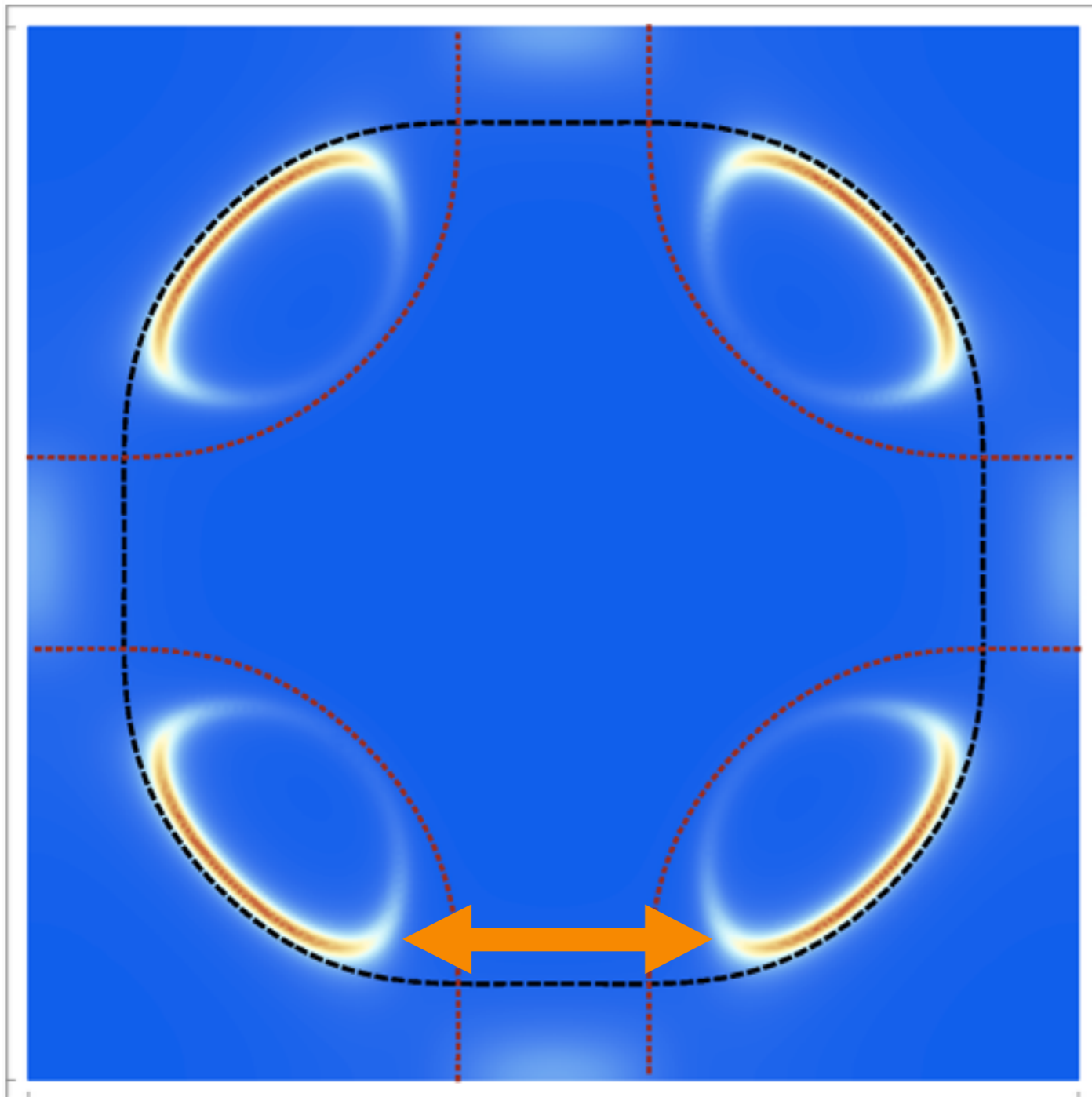


Density wave instability of large Fermi surface leads to an incorrect “diagonal” wavevector

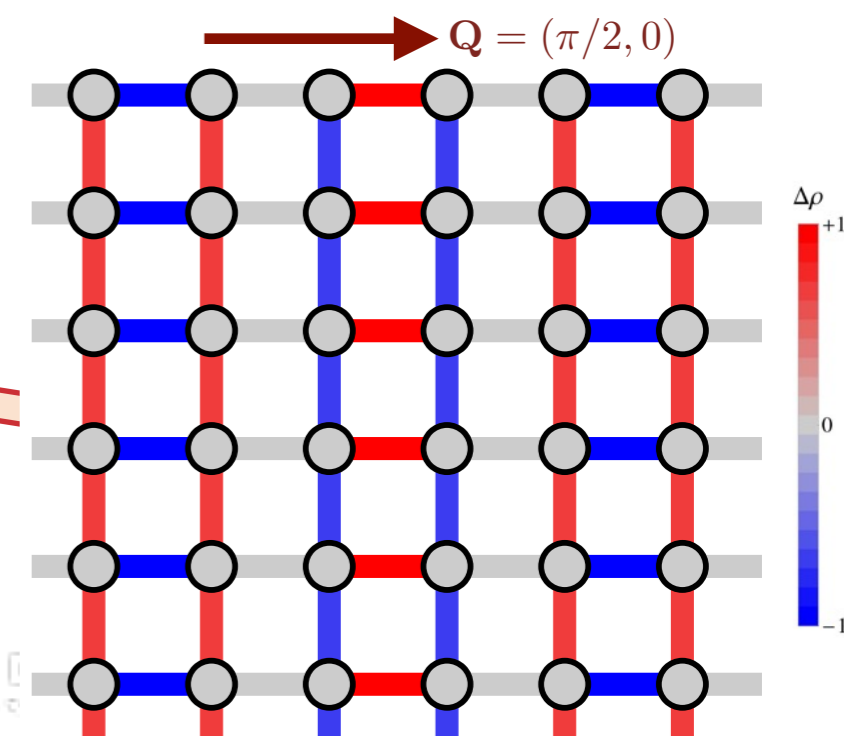
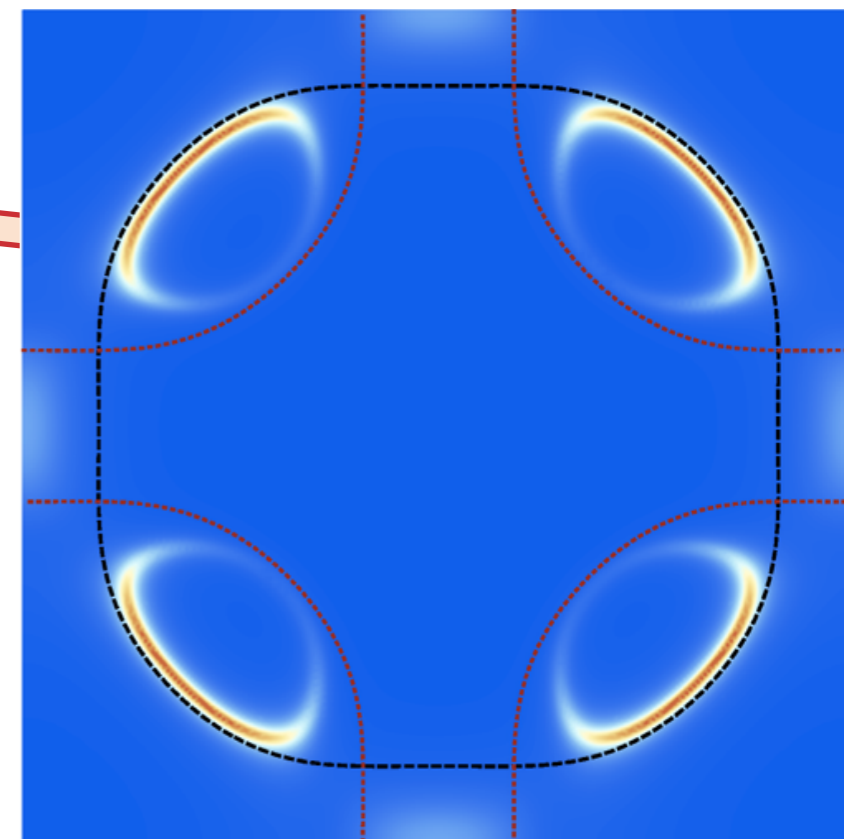
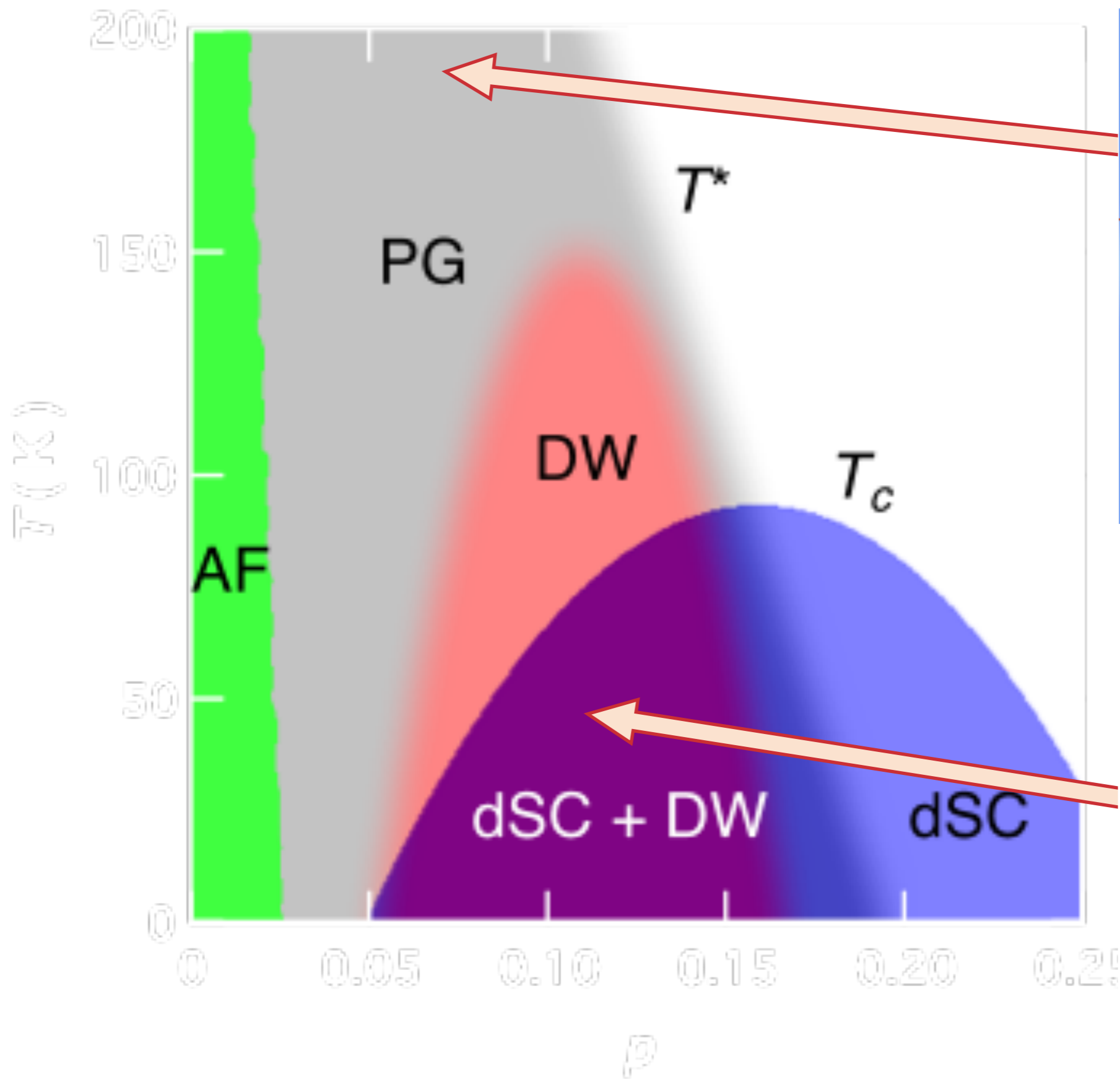
$$\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle = \mathcal{P}_d(\cos k_x - \cos k_y)$$

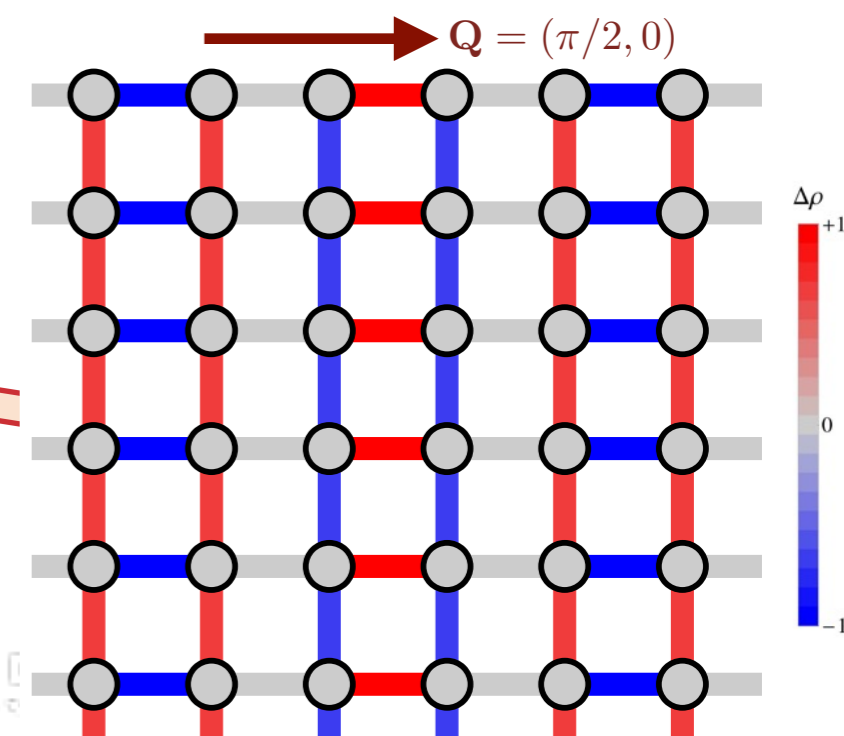
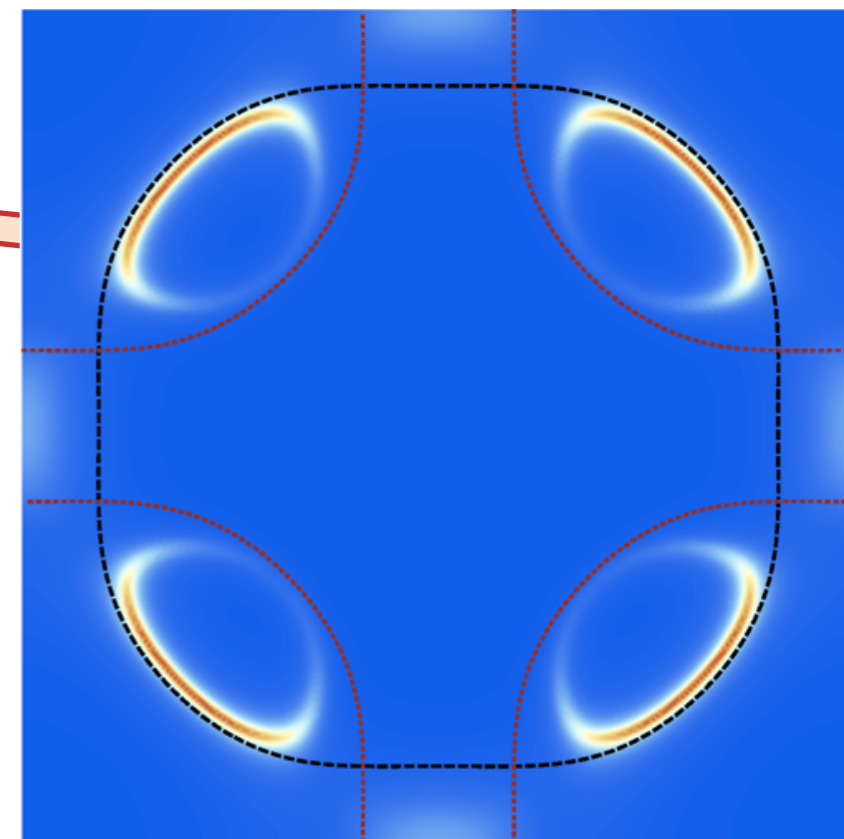
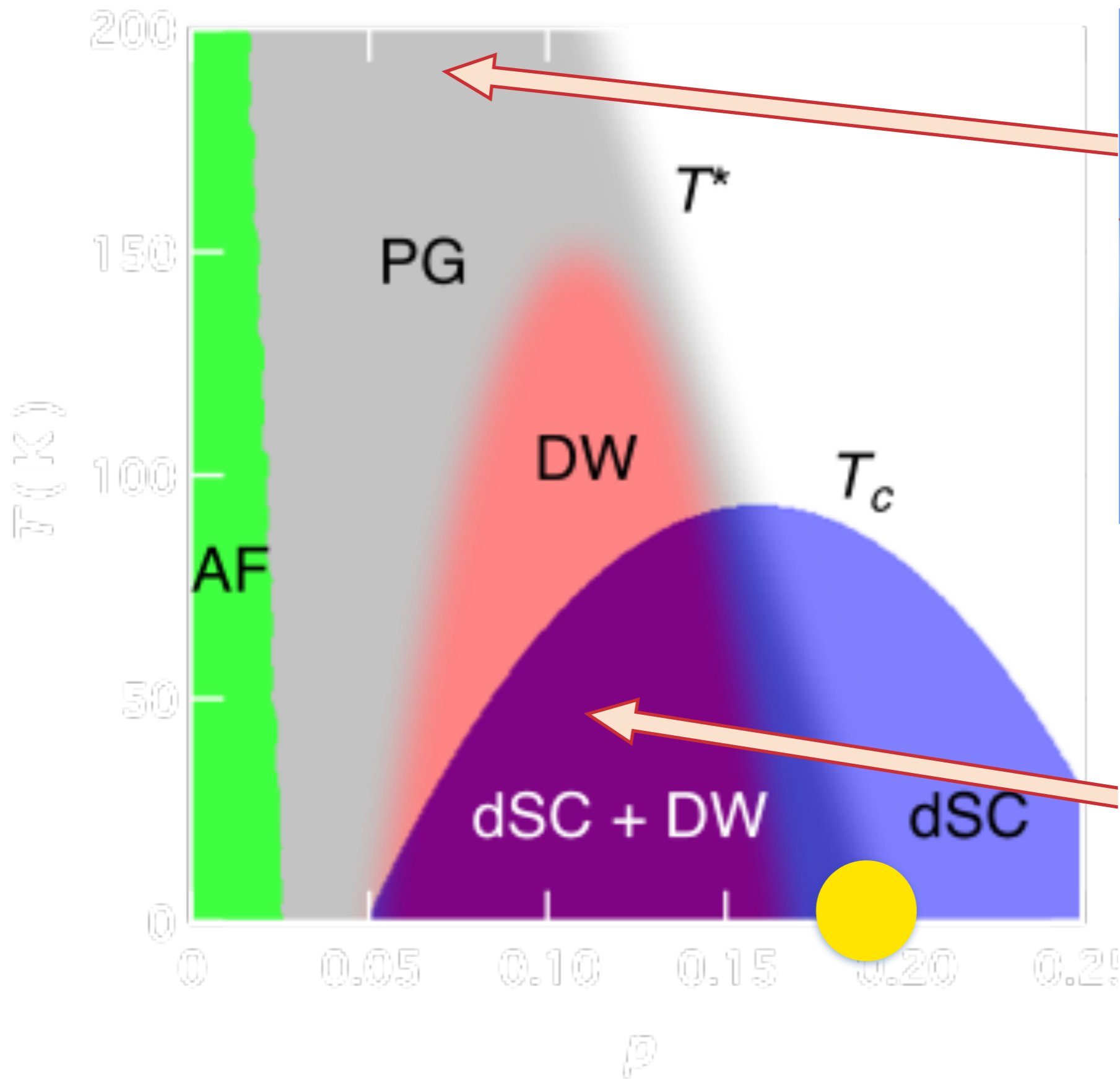


Fermi
surface
of FL*



Density wave
instability of
 FL^* leads to the
observed
wavevector
and form-factor





Outline

1. The low T pseudogap:

*STM observation of predicted
d-form factor density wave*

2. The high T pseudogap:

*A metal with topological order:
the Fractionalized Fermi liquid: FL**

3. Connecting high and low T :

Density wave instabilities

4. Quantum critical point near optimal p :

A Higgs critical point

SU(2) gauge theory for underlying quantum critical point

Write the electron operator c_α ($\alpha = \uparrow, \downarrow$ are spin indices) as

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = R \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

where R is a SU(2) matrix which determines the orientation of the local antiferromagnetic order, and ψ_\pm are spinless fermions which carry the global electron U(1) charge.

This parameterization is invariant under a SU(2) *gauge* transformation

$$\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \rightarrow U \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} ; \quad R \rightarrow RU^\dagger$$

SU(2) gauge theory for underlying quantum critical point

Assume field R is non-critical.

- Fermion ψ , transforming as a gauge SU(2) fundamental, with dispersion $\varepsilon_{\mathbf{k}}$ from the band structure, at a non-zero chemical potential: has a “large” Fermi surface.
- A SU(2) gauge boson.

SU(2) gauge theory for underlying quantum critical point

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- A SU(2) gauge boson.

The electrons, c_{α} move in the presence of local antiferromagnetic order \vec{n} . In the rotating reference frame, this is equivalent to a Yukawa coupling between the ψ fermions and a spin-singlet, gauge-SU(2) adjoint *Higgs field* H^a given by

$$H^a = \frac{1}{2} \text{Tr} [\sigma^{\ell} R \sigma^a R^{\dagger}] n_{\ell}$$

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)

D. Chowdhury and S. Sachdev, arXiv:1412.1086

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- A SU(2) gauge boson.
- A real Higgs field, H , transforming as a gauge SU(2) adjoint, carrying lattice momentum (π, π) . Condensation of the Higgs breaks $SU(2) \rightarrow U(1)$, and transforms the large Fermi surface to a small Fermi surface.

SU(2) gauge theory for underlying quantum critical point

- The quantum critical theory is the Higgs transition where the gauge “symmetry” breaks from SU(2) down to U(1), in the presence of a Fermi surface of fermions carrying fundamental SU(2) charges.

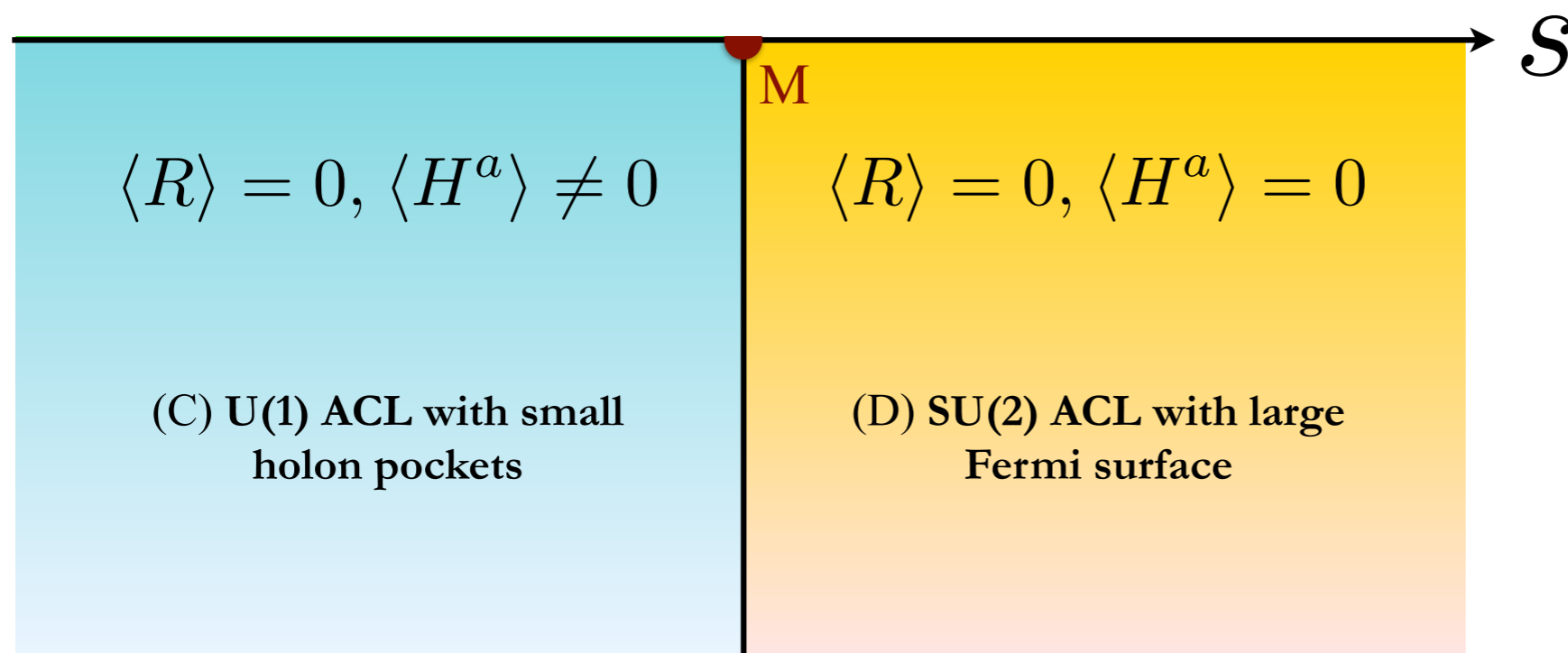
SU(2) gauge theory for underlying quantum critical point

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- The Higgs condensation does not give the fermions a “mass”; instead it reconstructs the Fermi surface from *large* to *small*.

SU(2) gauge theory for underlying quantum critical point

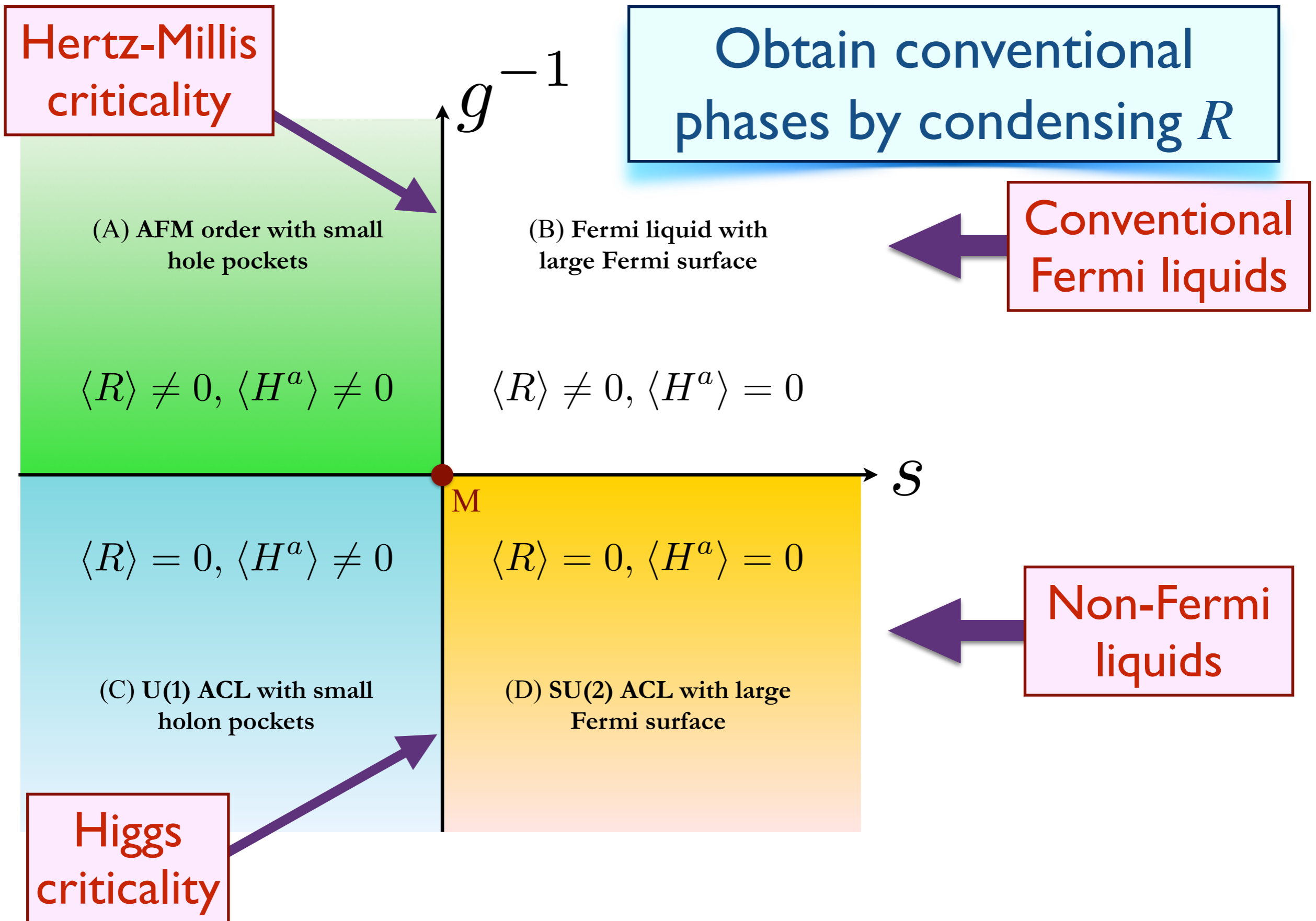
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- The Higgs condensation does not give the fermions a “mass”; instead it reconstructs the Fermi surface from *large* to *small*.
- The quantum phase transition has no gauge-invariant “order parameter”, and it does not break any global symmetries.

Higgs transition between non-Fermi liquid metals with large and small Fermi surfaces

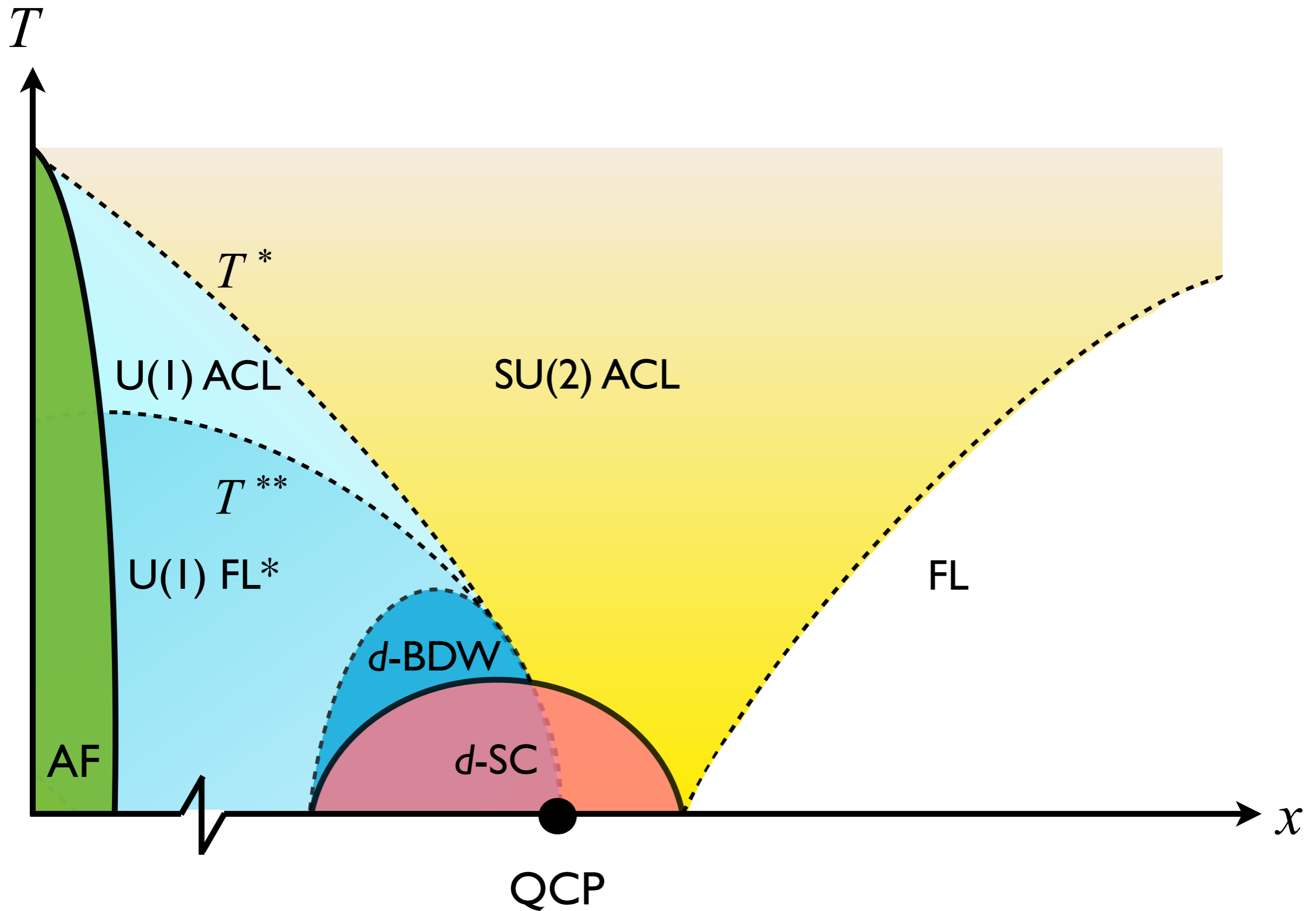


S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)

D. Chowdhury and S. Sachdev, arXiv:1412.1086



SU(2) gauge theory for underlying quantum critical point



Transport in QC regime:

All points on the Fermi surface have a rapid relaxation to local thermal equilibrium by processes which conserve a suitably defined momentum.

Relaxation of the momentum occurs at a slower rate.

SU(2) gauge theory for underlying quantum critical point

The resistivity of this strange metal is *not* determined by the scattering rate of charged excitations near the Fermi surface, but by the dominant rate of momentum loss by *any* excitation, whether neutral or charged, or fermionic or bosonic.

There is a dominant contribution $\rho(T) \sim T$ by the coupling of long-wavelength disorder to the gauge-invariant operator $\mathcal{O} \sim H^2$.

Conclusions

1. Predicted d -form factor density wave order observed in the non-La hole-doped cuprate superconductors.
2. The “electron becomes a dimer” in the pseudogap metal: proposed a quantum dimer model.
3. The d -form factor is an unexpected window into the electronic structure of the pseudogap, with evidence for a fractionalized Fermi liquid (FL*) model.

