FL* theory of the pseudogap, and the transition to d-wave superconductivity in the cuprates

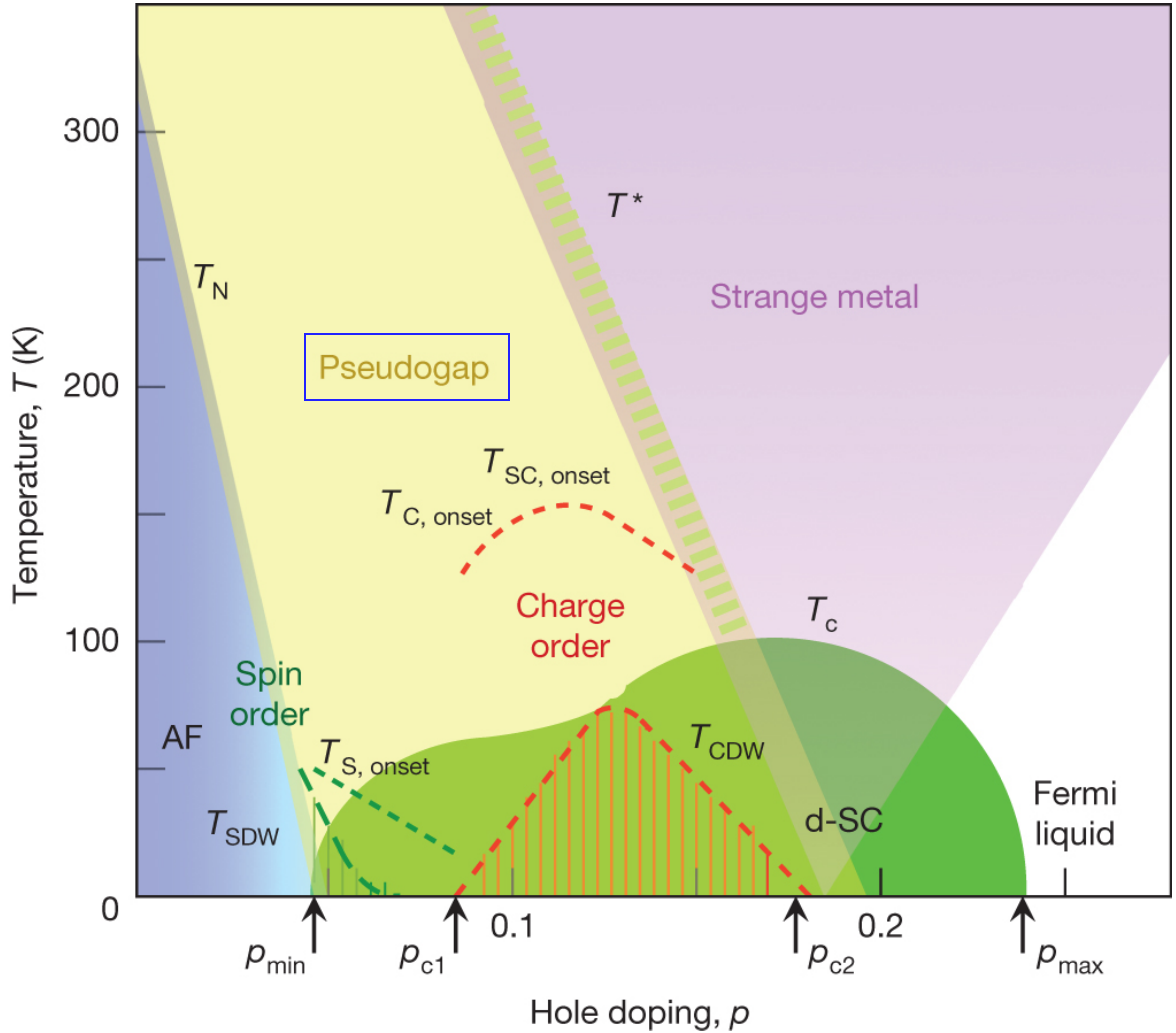
Discussion Meeting on Fractionalized Quantum Matter
International Centre for Theoretical Sciences, Bengaluru
July 29, 2025

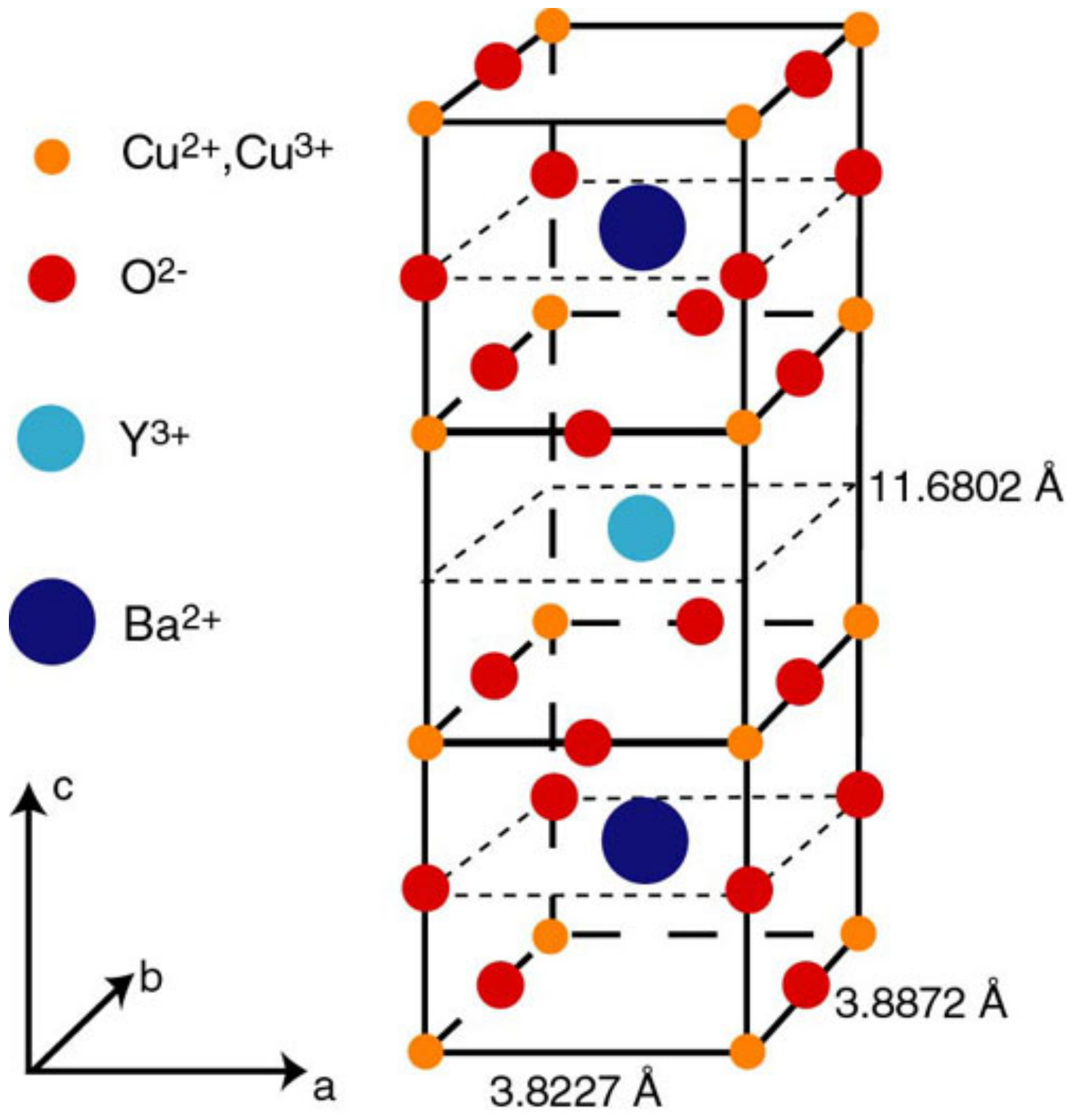
Subir Sachdev

PHYSICS
HARVARD

Experiments on the cuprate pseudogap phase

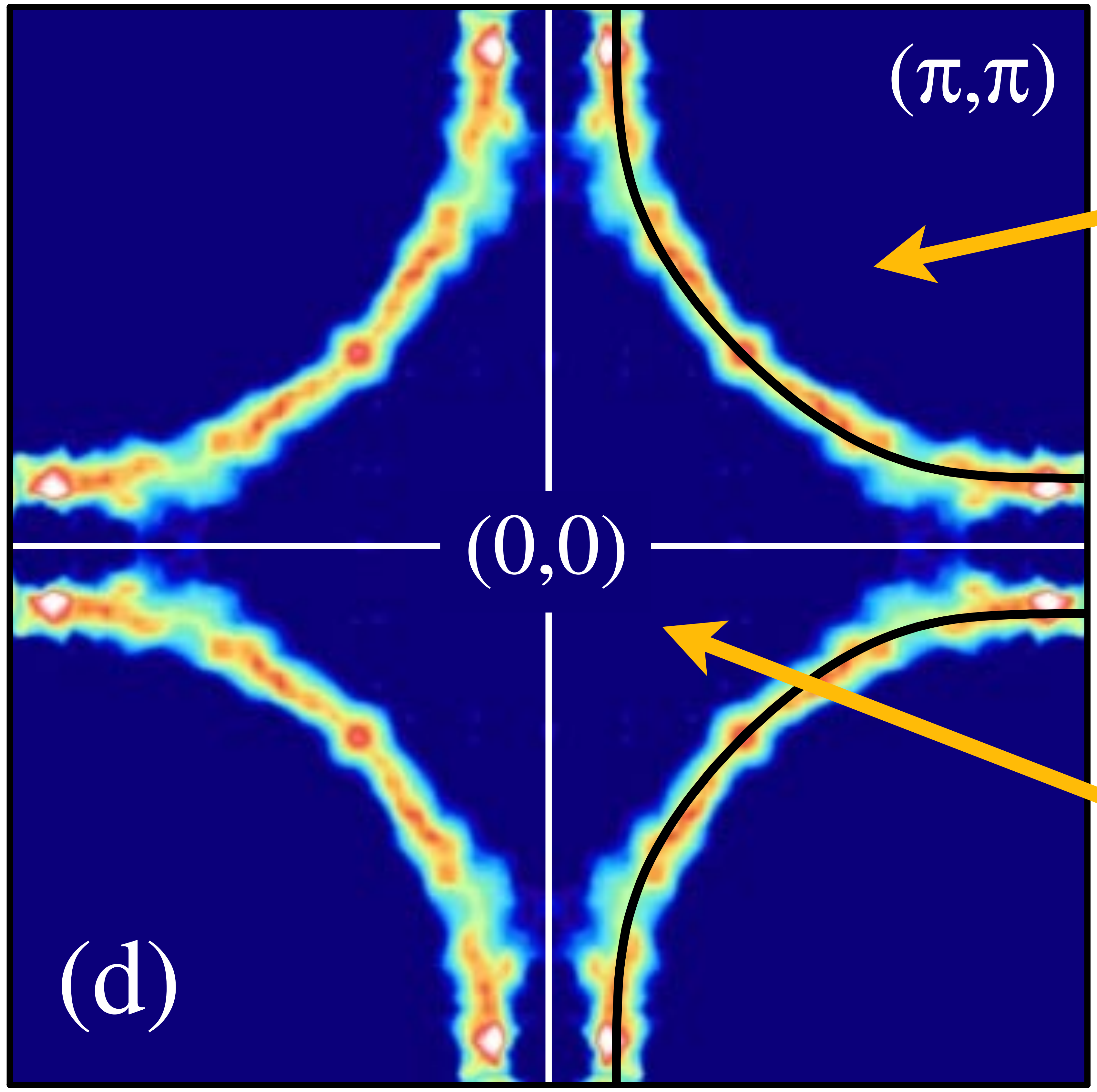
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





$YBa_2Cu_3O_{6+x}$

Photoemission at large p



$1{+}p$ holes

$1{-}p$ electrons

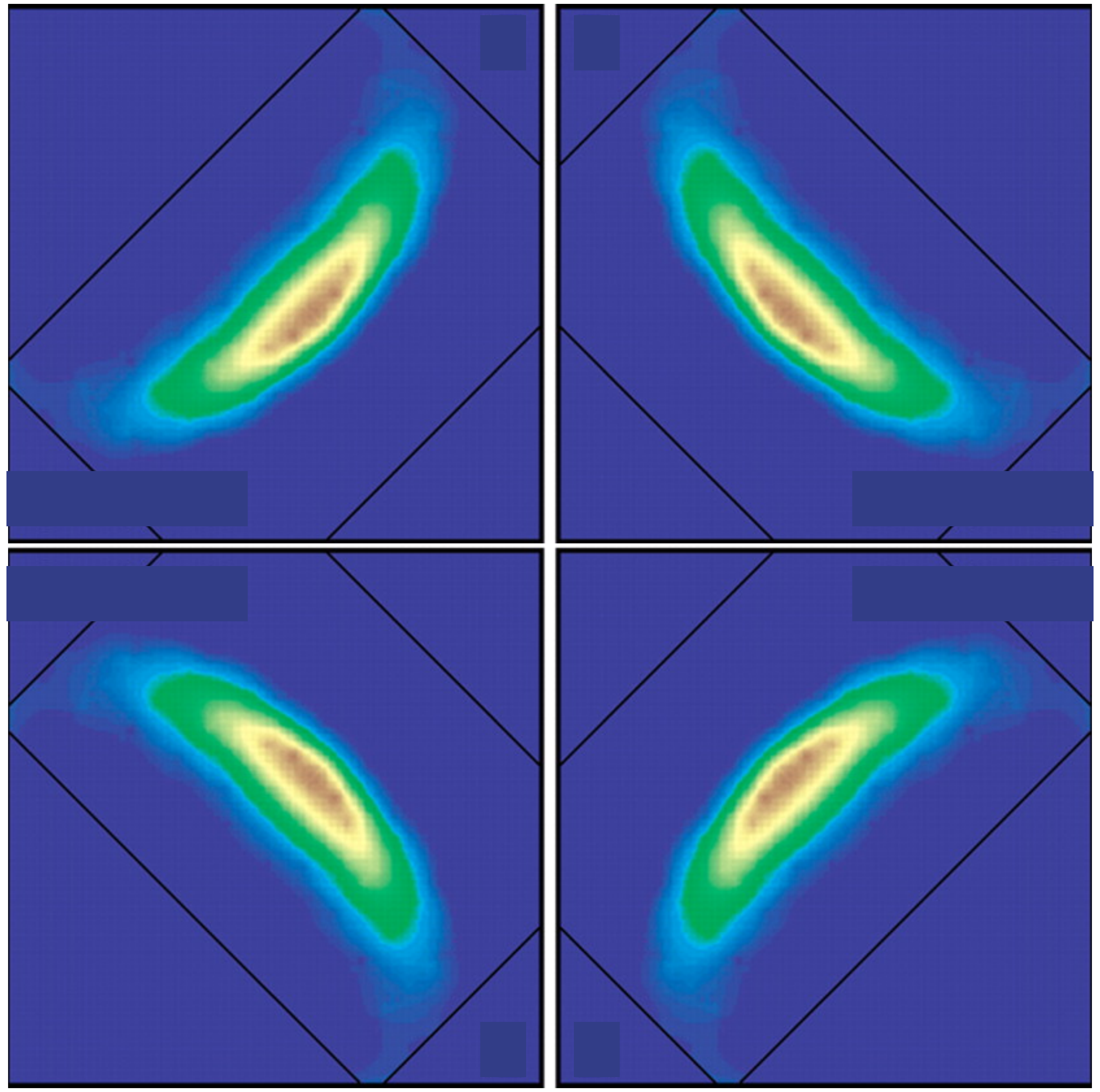
Overdoped $Tl_2Ba_2CuO_{6+\delta}$

$T_c = 30\text{K}$

$1{+}p$ mobile holes in a filled band

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

Photoemission at small p



$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

"Fermi arcs"

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

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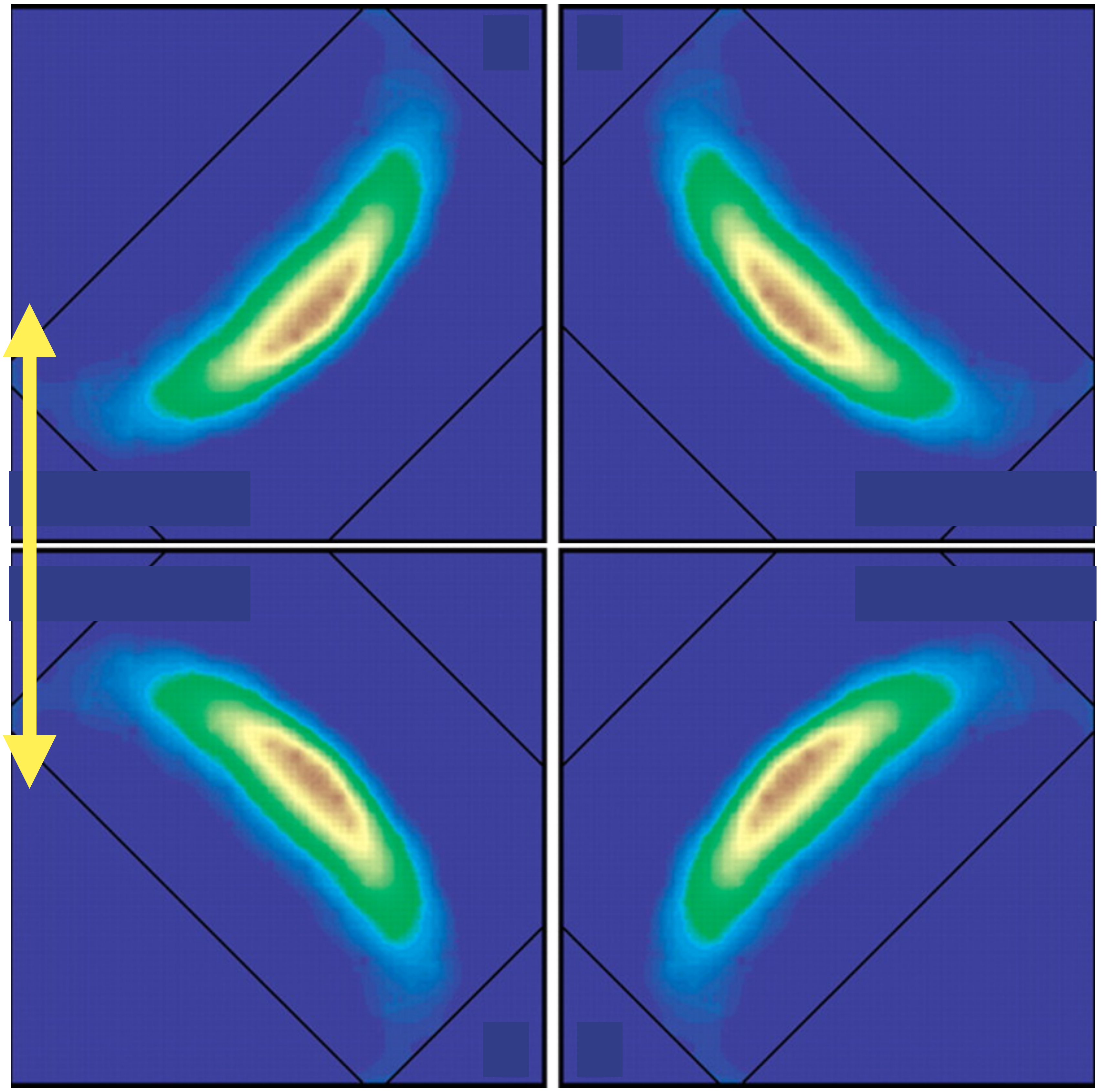

$Ca_{2-x}Na_xCuO_2Cl_2$
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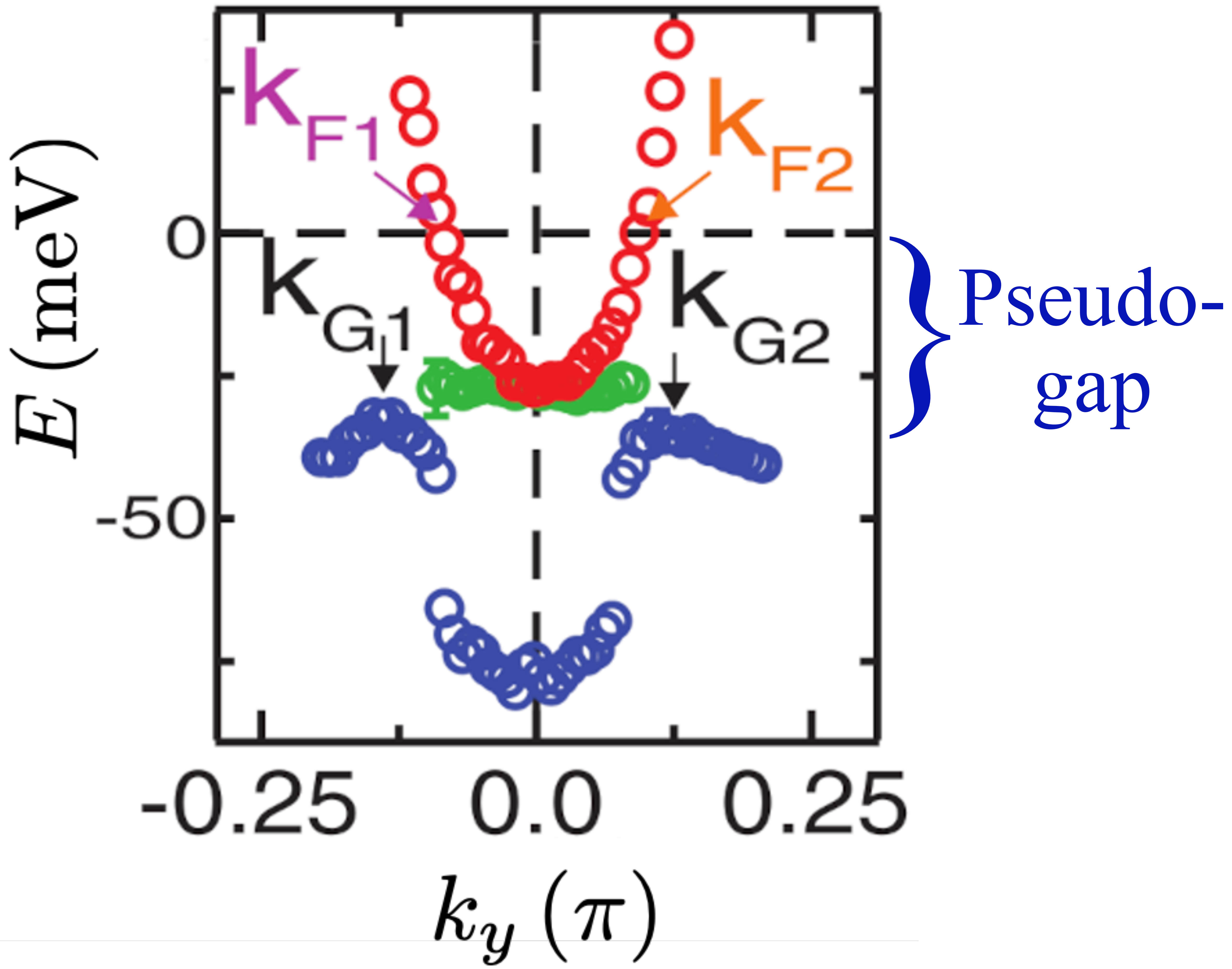
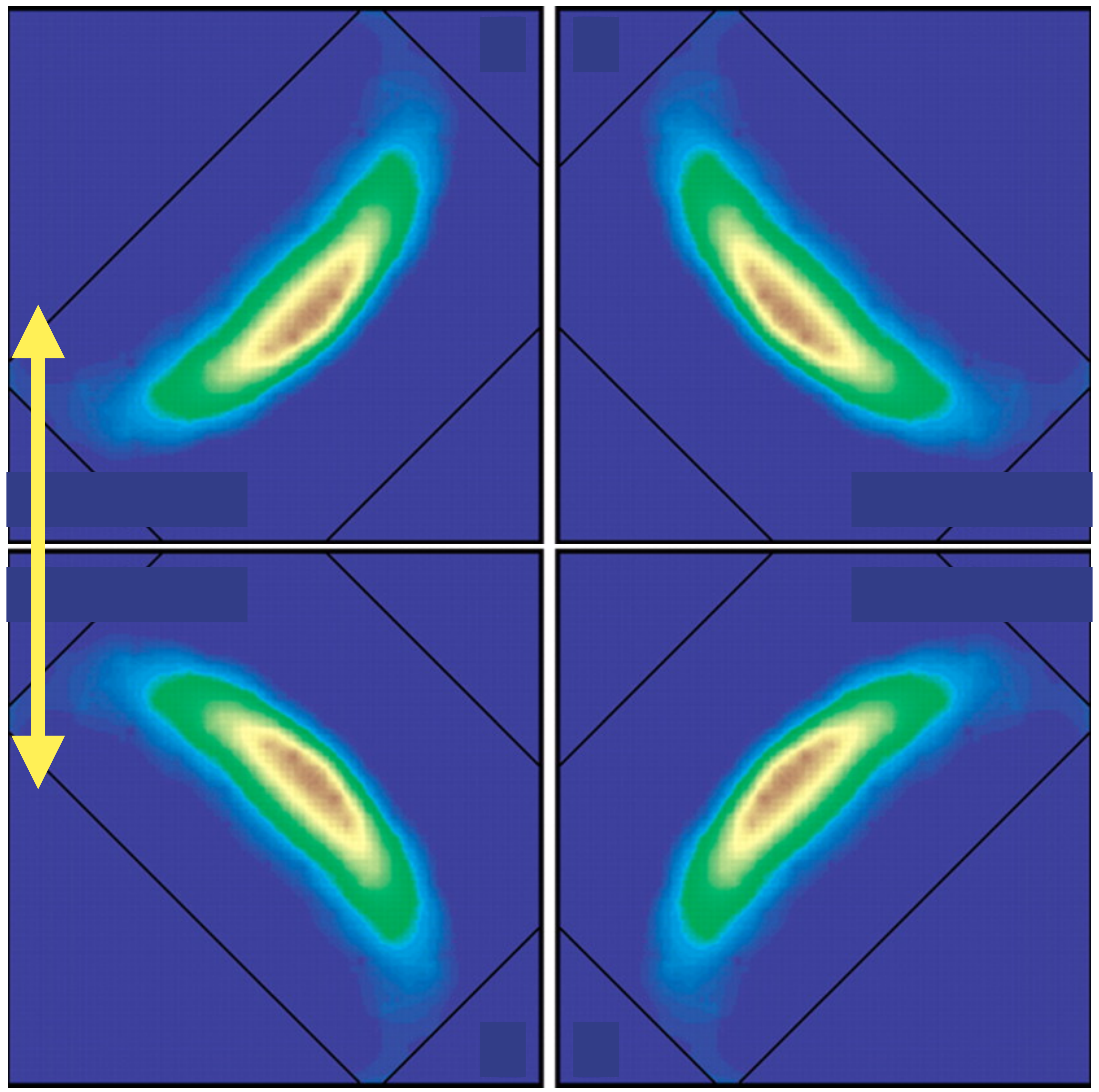
Photoemission at small p

$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

Bi2201



R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)



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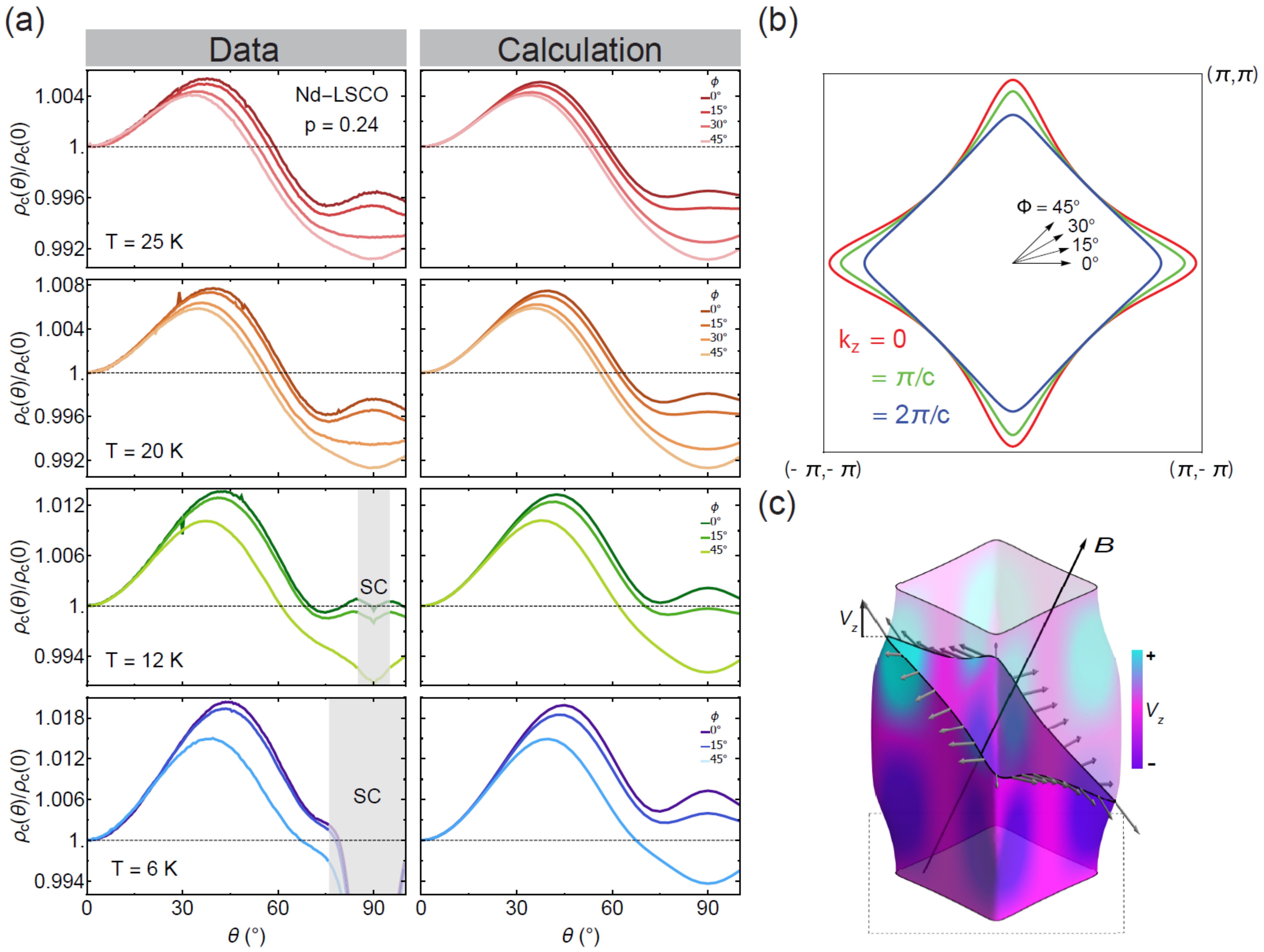
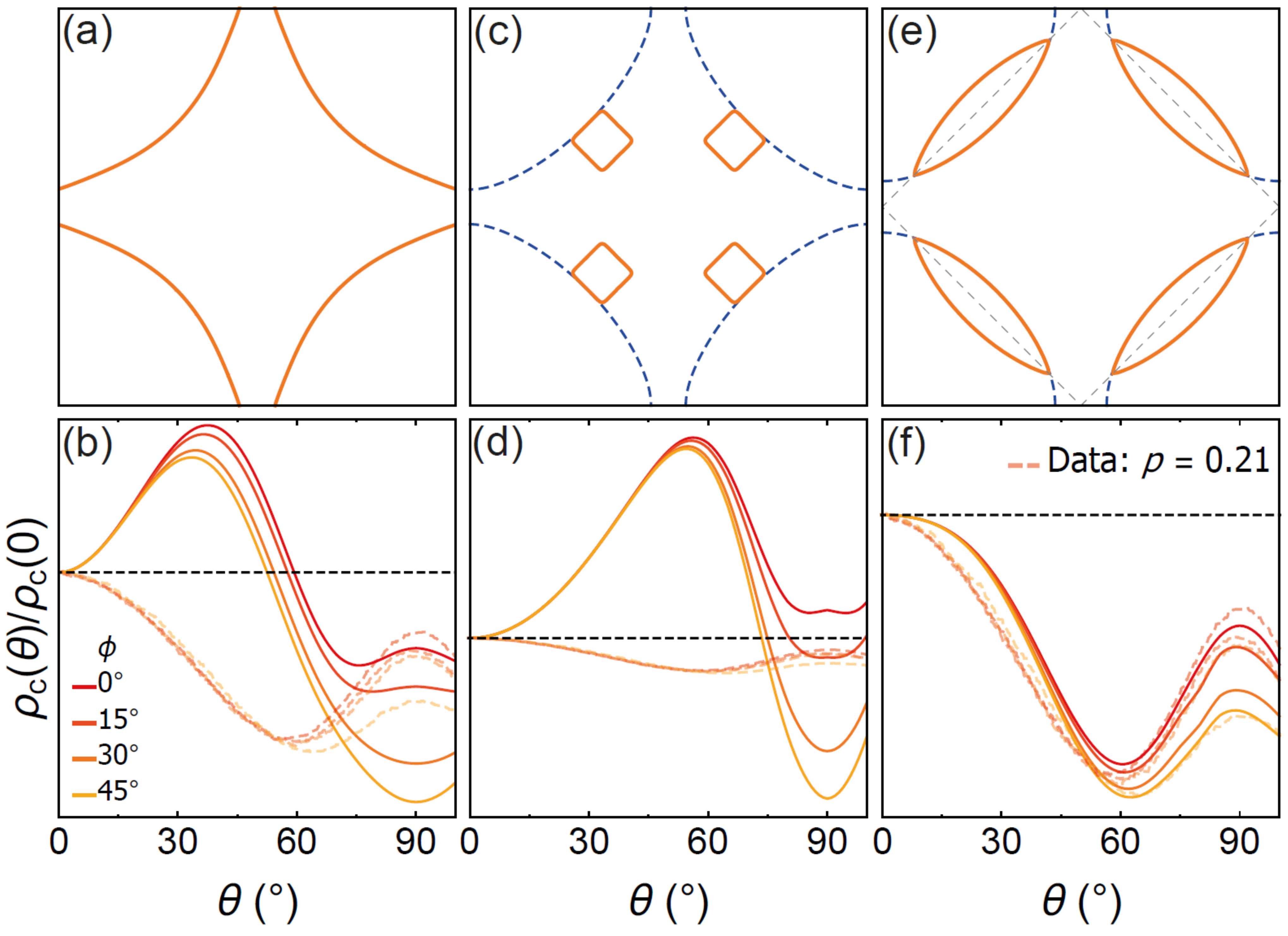
Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei,
Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, *Nature Physics* **18**, 558 (2022)

Angle-dependent magnetoresistance (ADMR) of $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$



$p > p_c$ Large Fermi surface

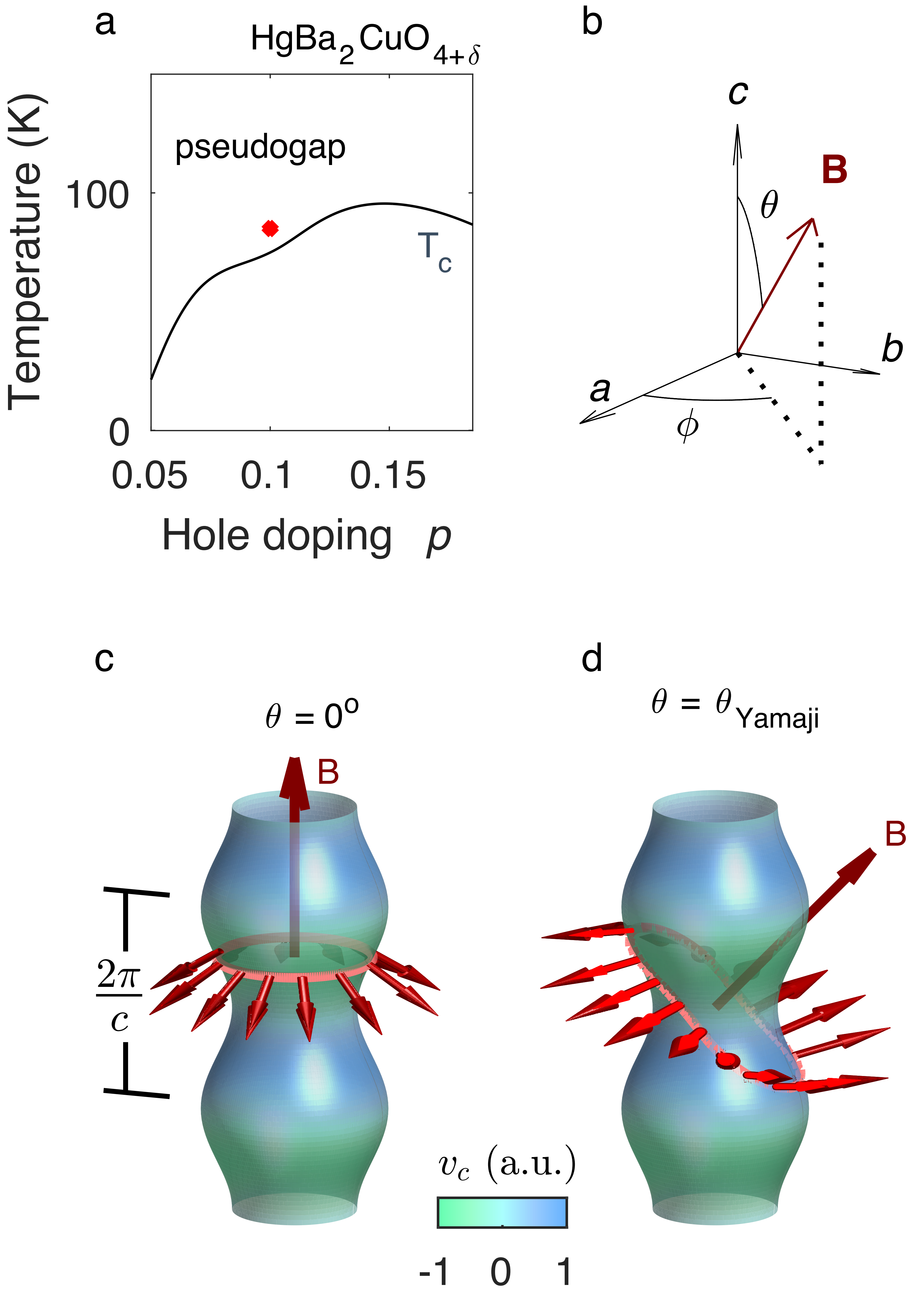


$p < p_c$ Reconstructed Fermi surface

Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan,[1,*] Katherine A. Schreiber,[1] Oscar E. Ayala-Valenzuela,[1]
Eric D. Bauer,[2] Arkady Shekhter,[1] and Neil Harrison[1]

arXiv:2411.10631



At the Yamaji angle, the orbits in the plane orthogonal to $\boldsymbol{B}$ have an area which is independent of momentum in the c direction, to first order in the hopping along the c direction.

K. Yamaji JPSJ **58**, 1520 (1989)

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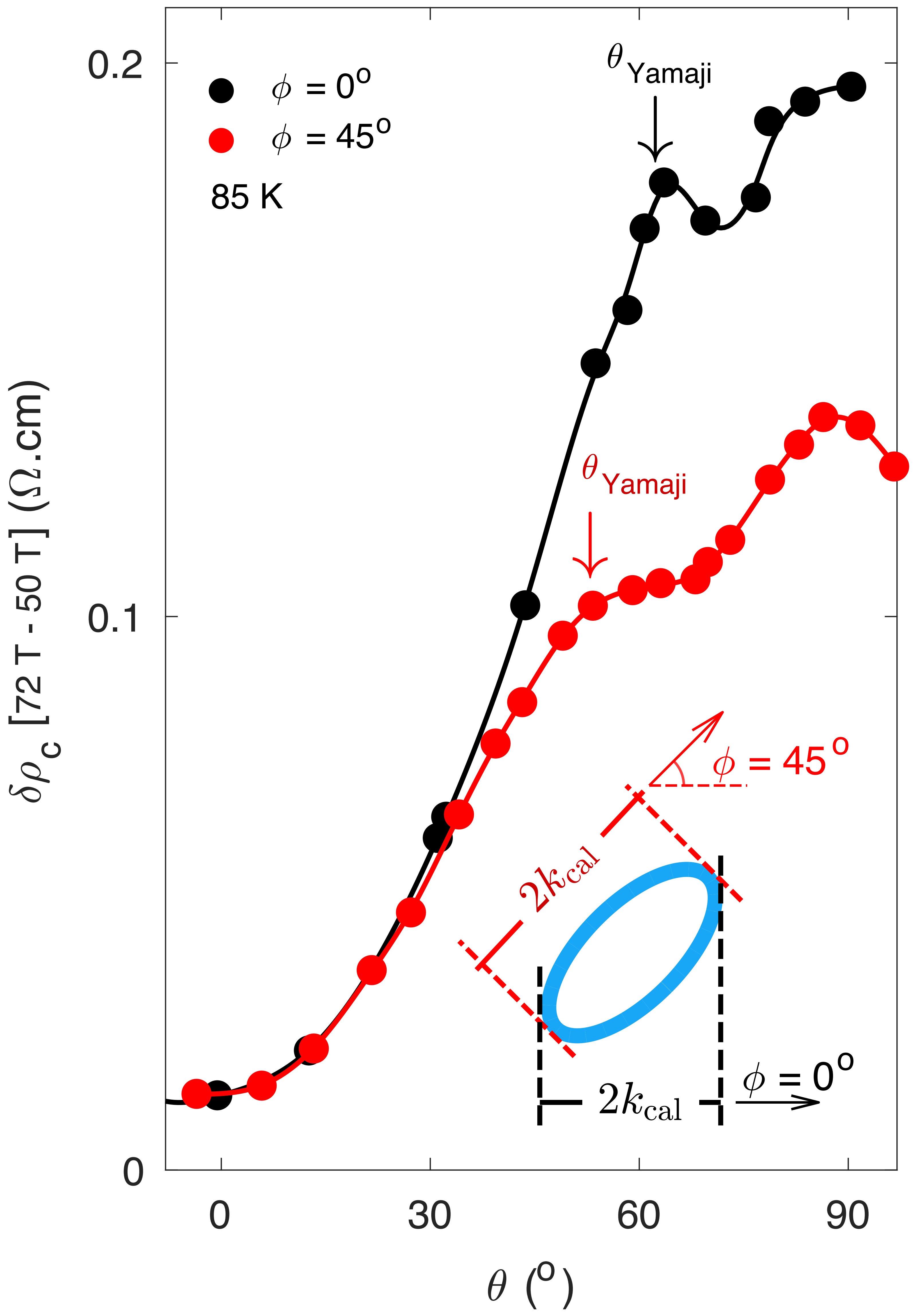
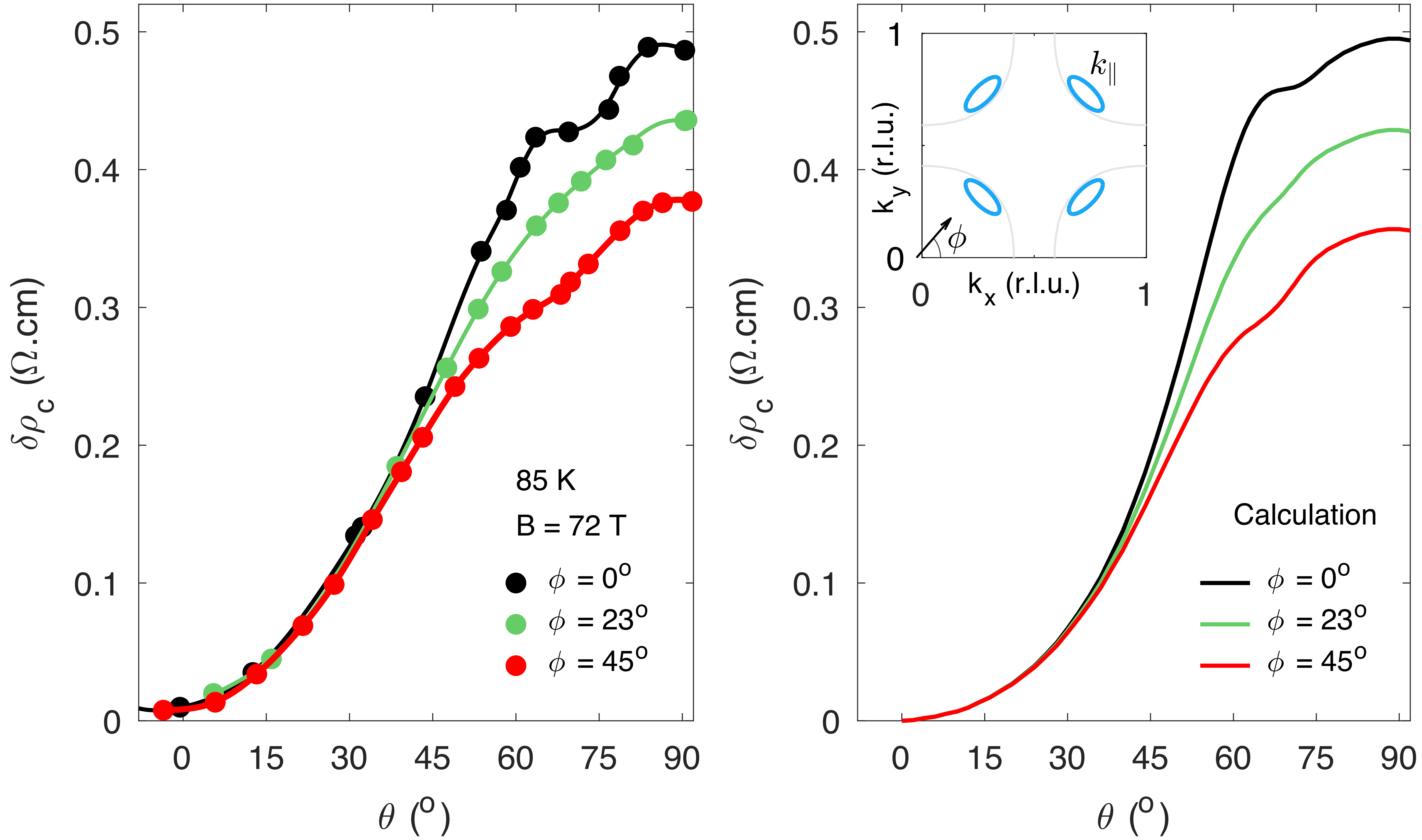
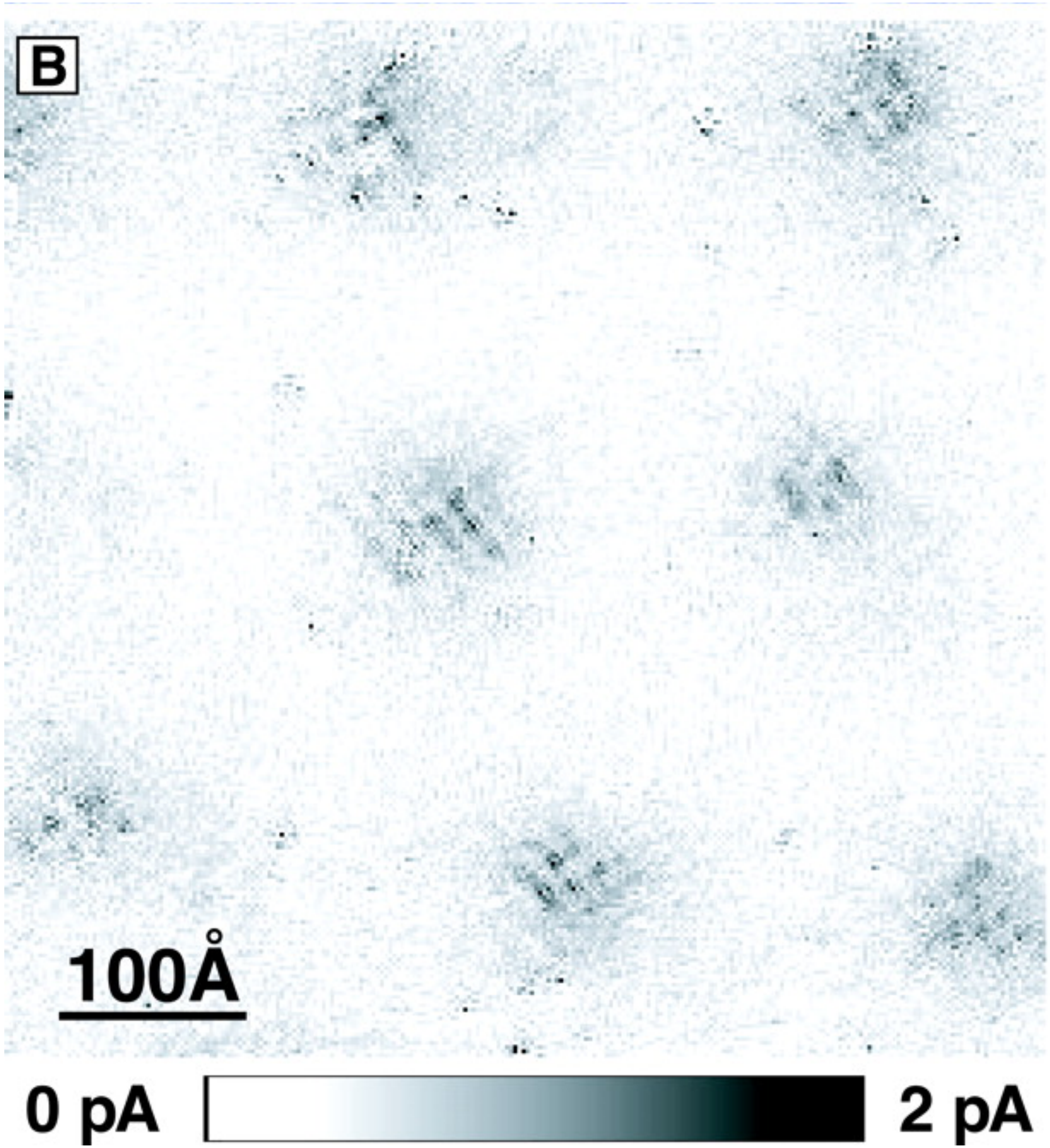




Doping
$p = 0.1$

"The small size of the pockets determined from the Yamaji effect is ...
approximately 1.3% of the Brillouin zone area"

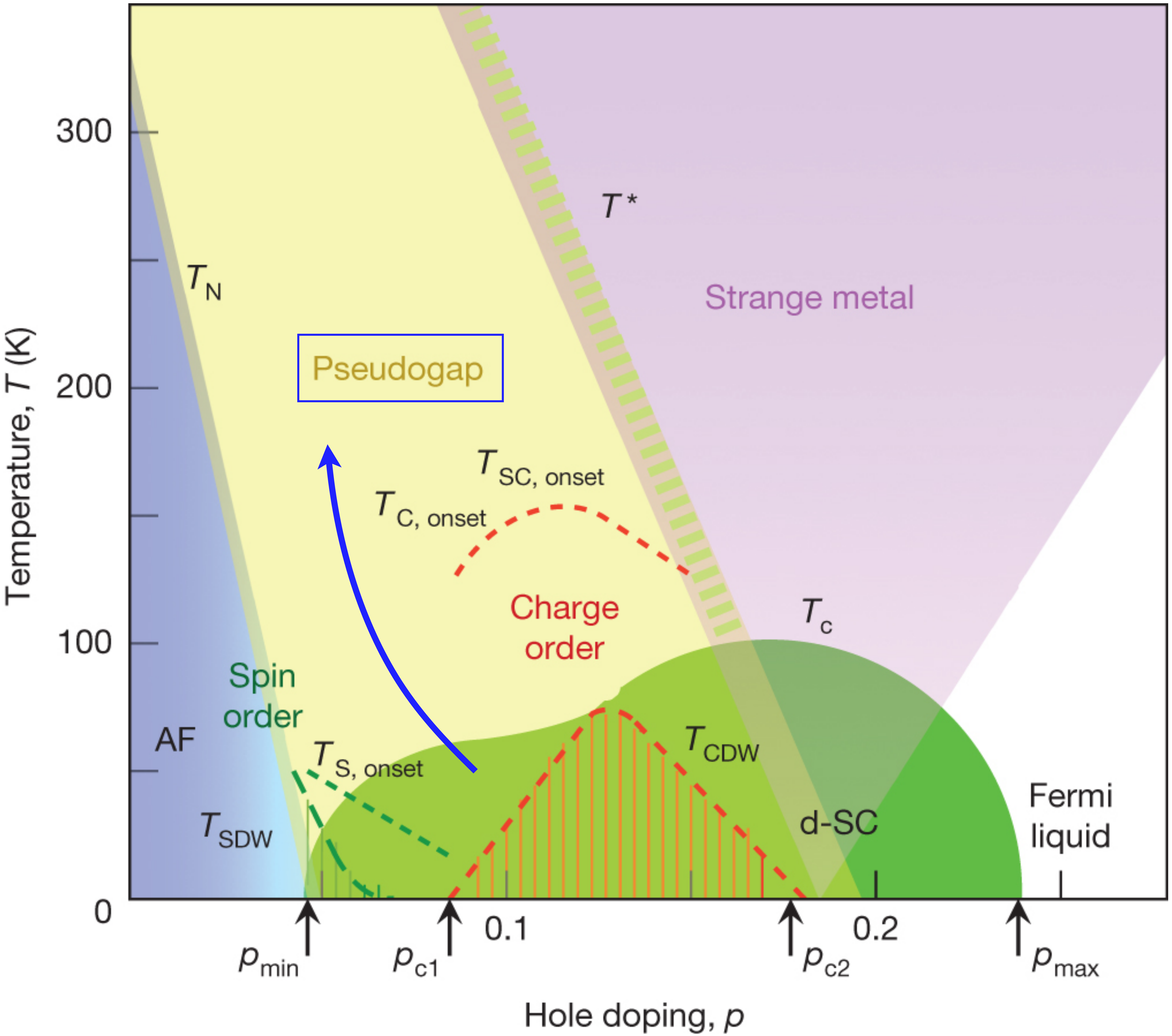


A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $Bi_2Sr_2CaCu_2O_{8+\delta}$

J. E. Hoffman, E. W. Hudson,
K. M. Lang, V. Madhavan,
H. Eisaki, S. Uchida, J.C. Davis
Science **295**, 466 (2002)

Fluctuating order theories

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Phase fluctuations of d-wave superconductor (Emery-Kivelson 1995)

- 'Fermi arc' spectra with classical, thermal XY model modulating pairing term in Bogoliubov Hamiltonian. Eckl, Scalapino, Arrigoni, Hanke, PRB 2002; Han, Li, Wang, PRB 2010

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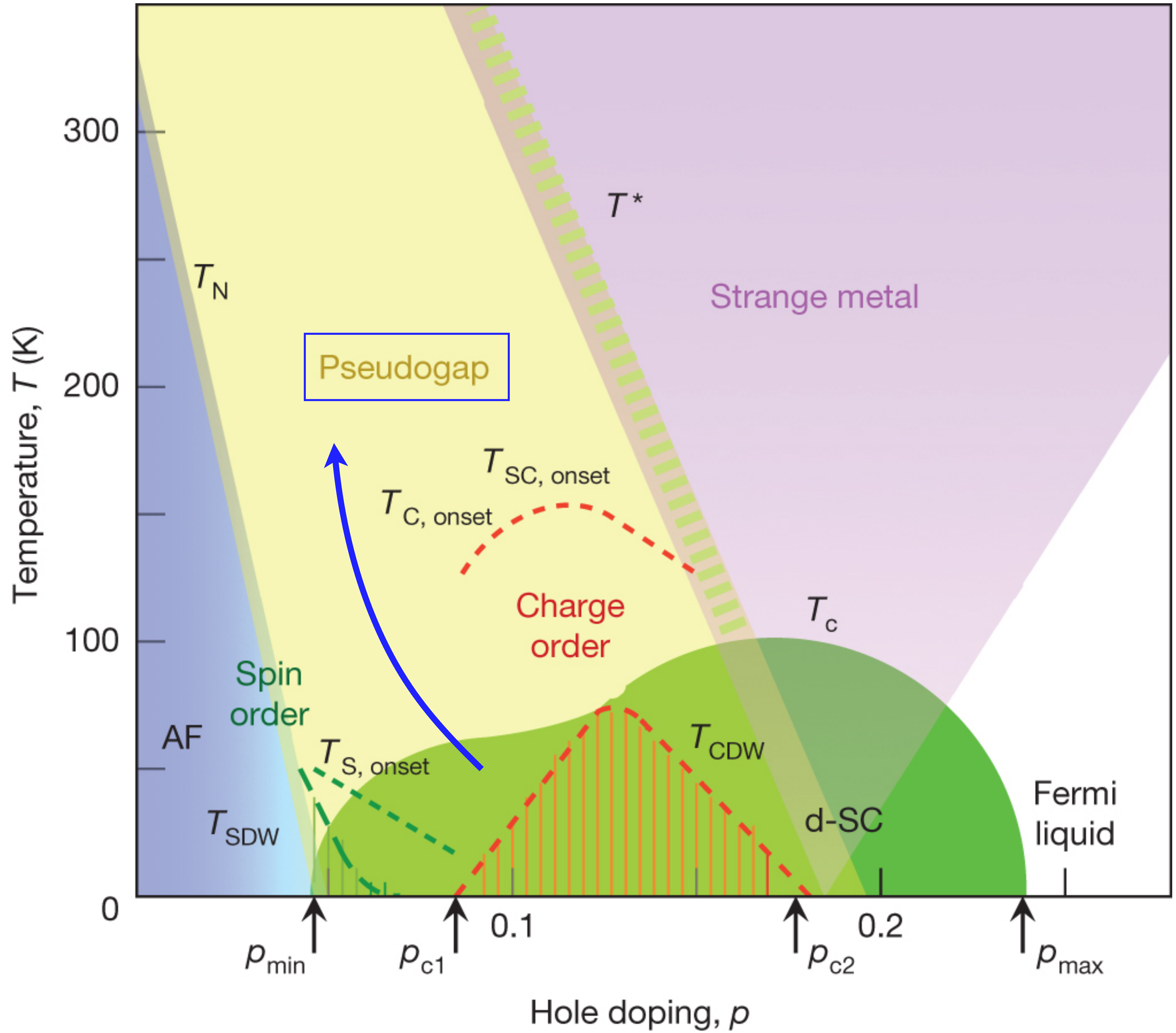

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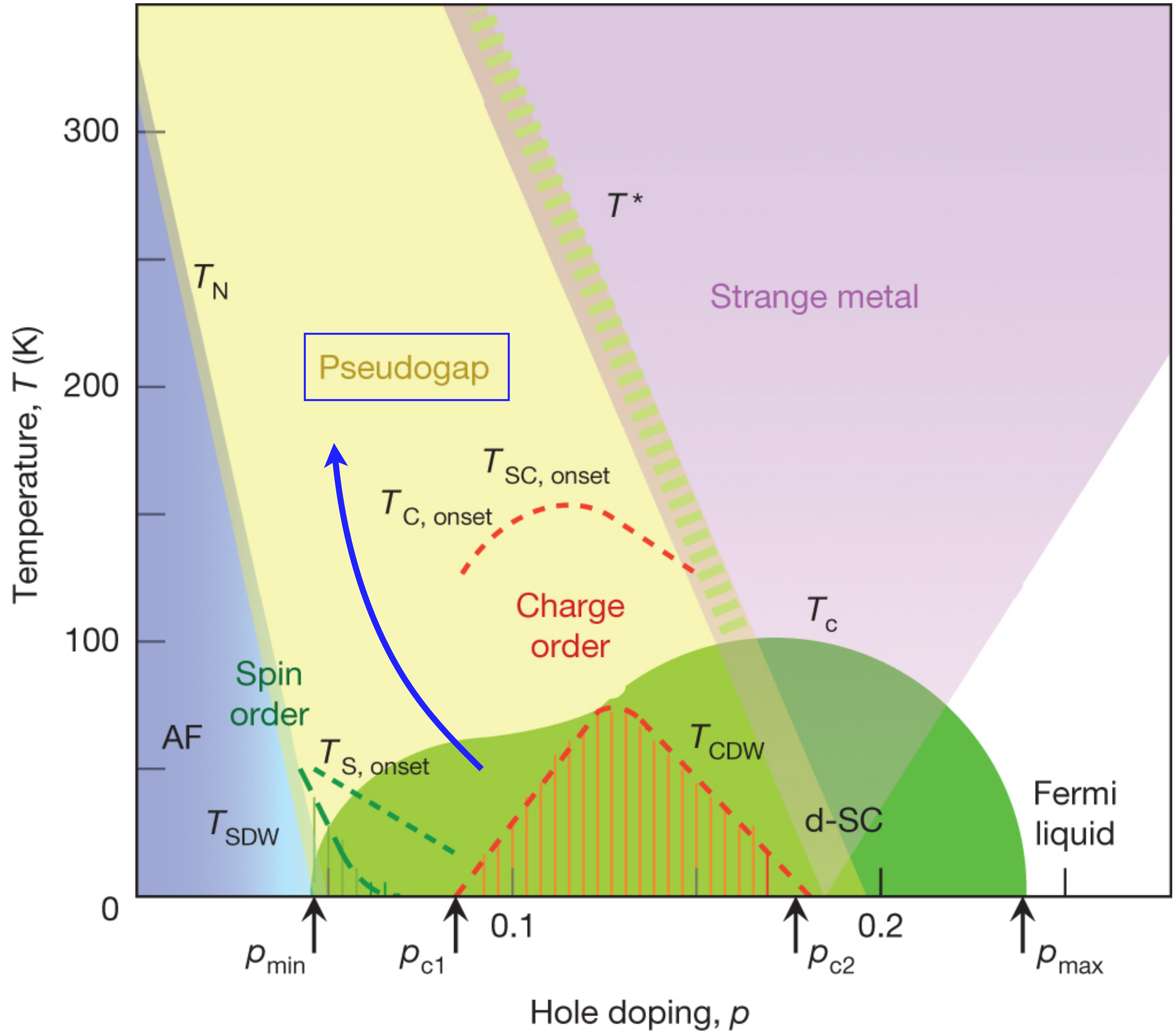
- But restores large Fermi surface above $T_{\rm SC,onset}$.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



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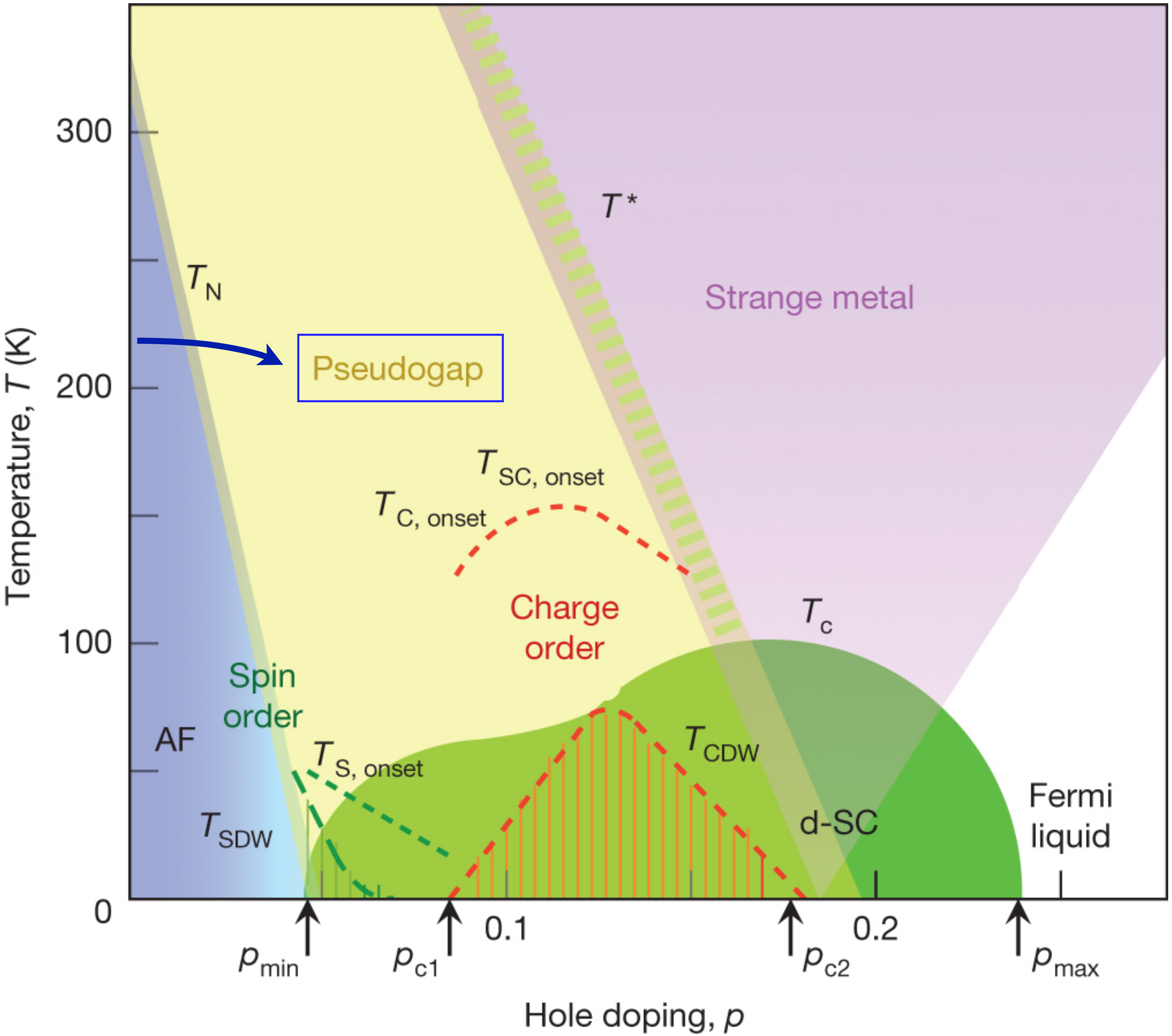
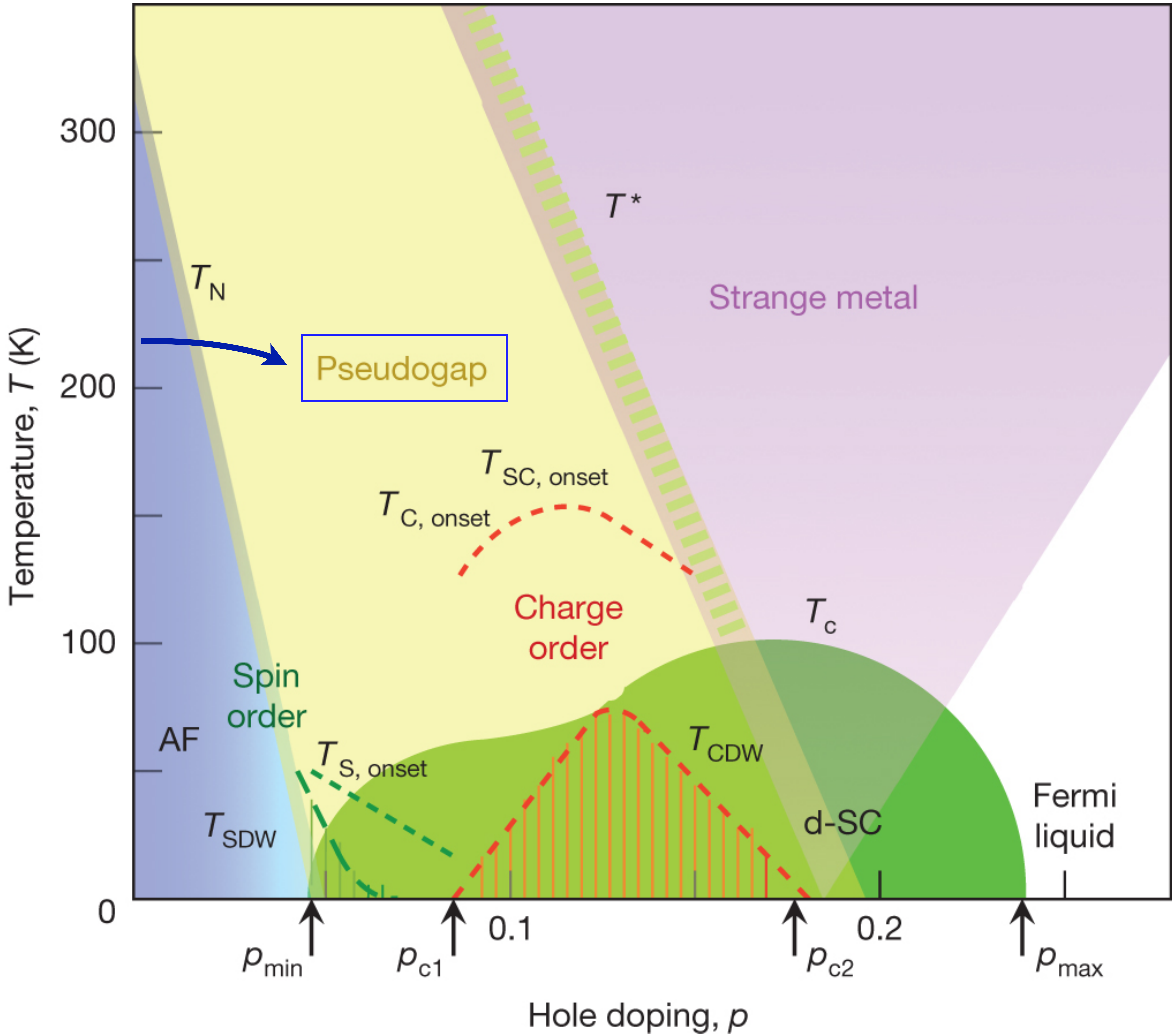
  Eckl, Scalapino, Arrigoni, Hanke, PRB 2002;
  Han, Li, Wang, PRB 2010
- But restores large Fermi surface above $T_{\rm SC,onset}$.
- Cannot explain recent magneto-transport evidence for hole pockets.
  Y. Fang . . . B. J. Ramshaw, Nature Physics 2022.
  M. K. Chan . . . N. Harrison, arXiv:2411.10631.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Thermally fluctuating,
Landau damped paramagnon
with spin $S = 1$
(Schmalian, Pines,
Stojkovic, 1998)

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Thermally fluctuating, Landau damped paramagnon with spin $S = 1$ (Schmalian, Pines, Stojkovic, 1998)

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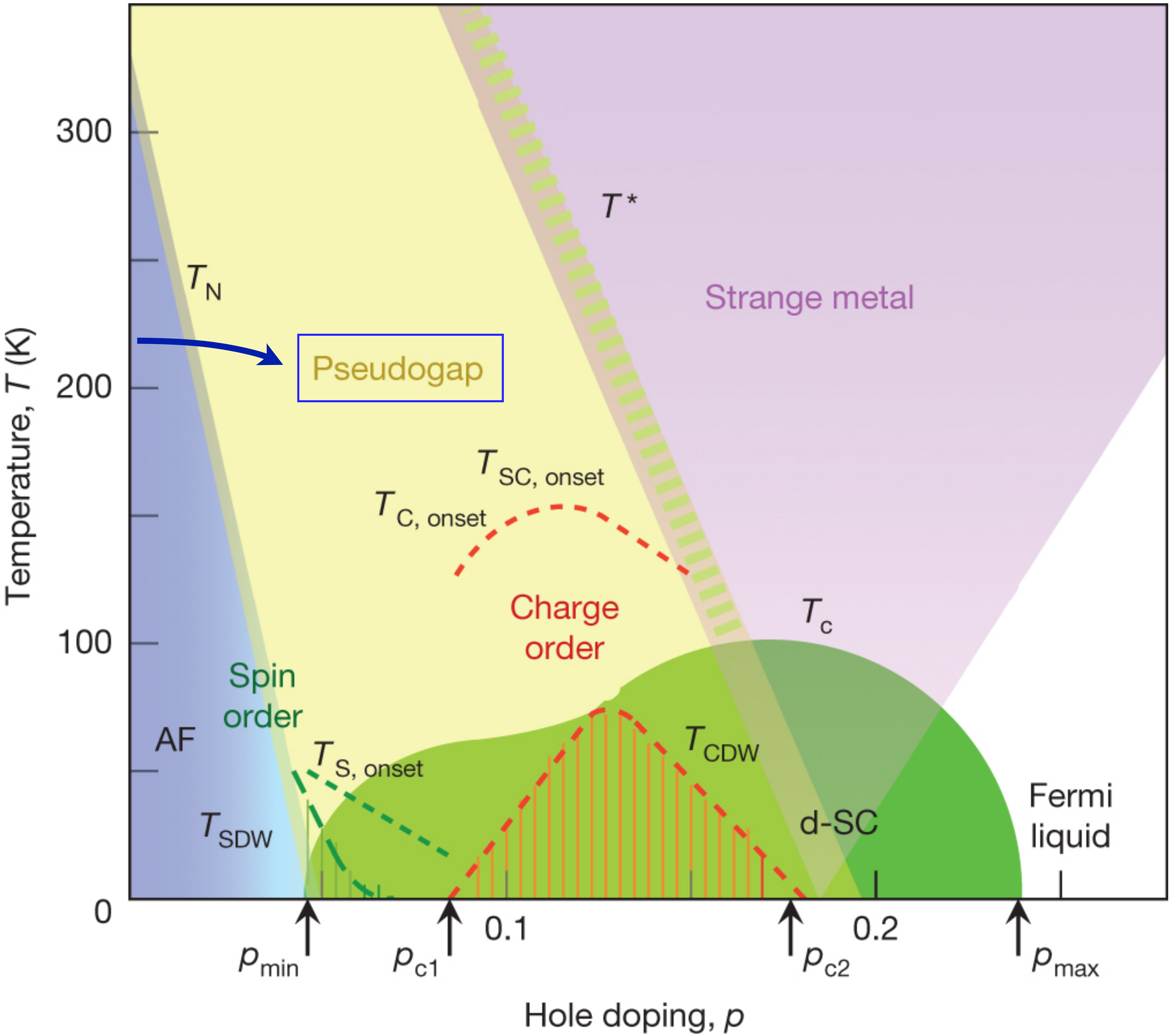
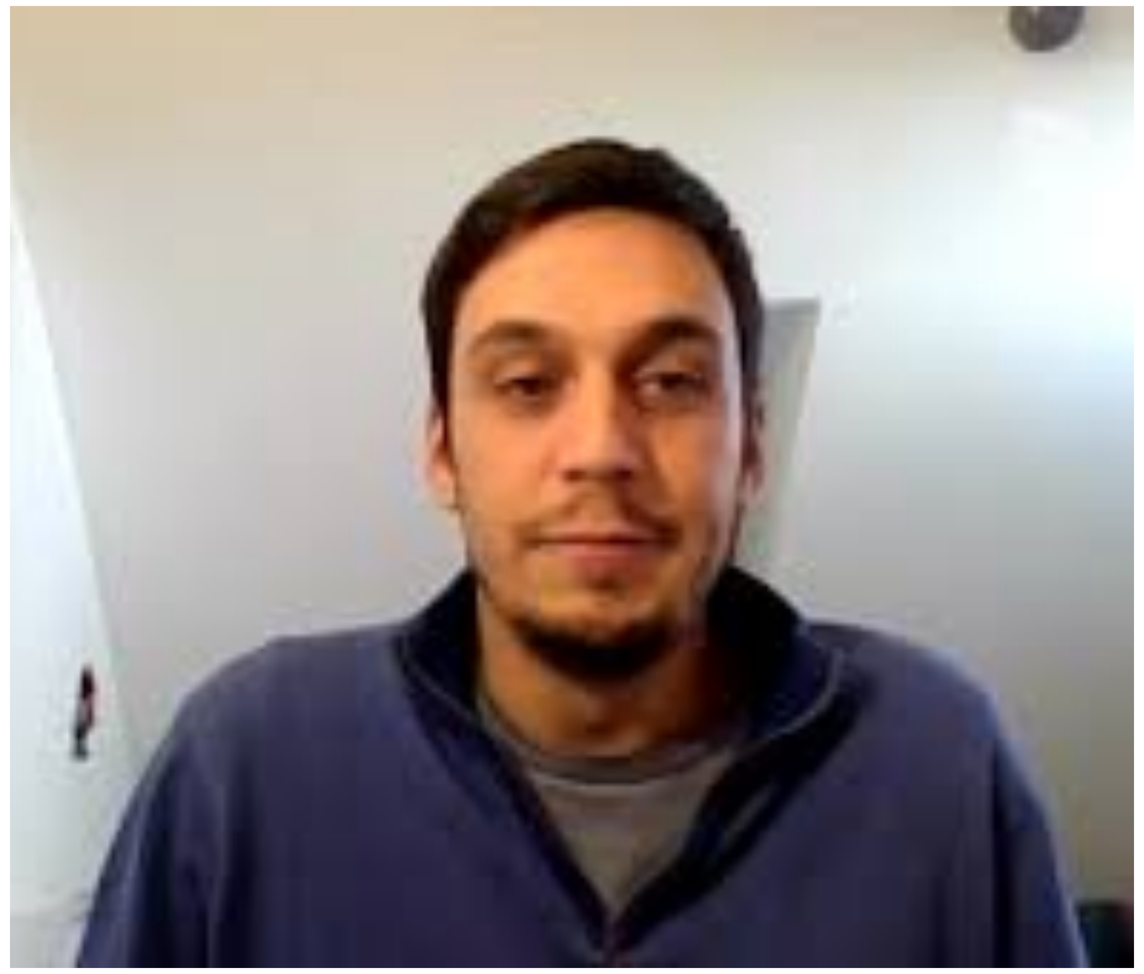


Thermally fluctuating, Landau damped paramagnon with spin $S = 1$ (Schmalian, Pines, Stojkovic, 1998)

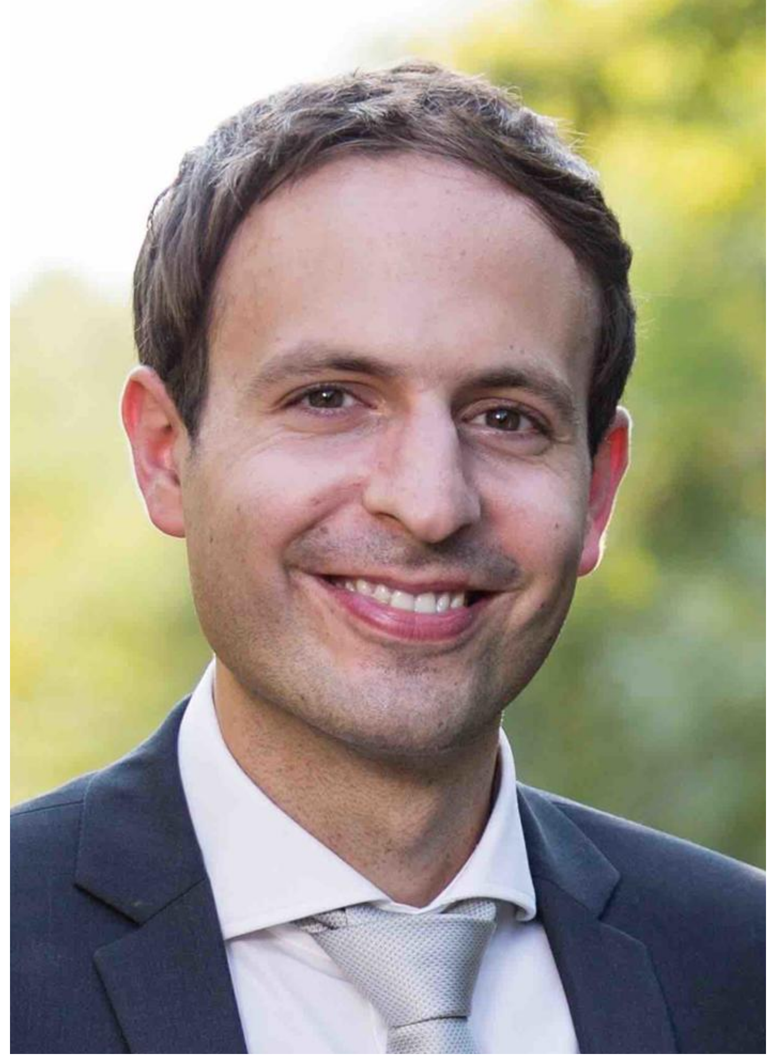
- Can yield hole pockets.
- But the area of the hole pockets does not agree with Yamaji effect, as we shall see . . .

Maine Christos
→ Caltech



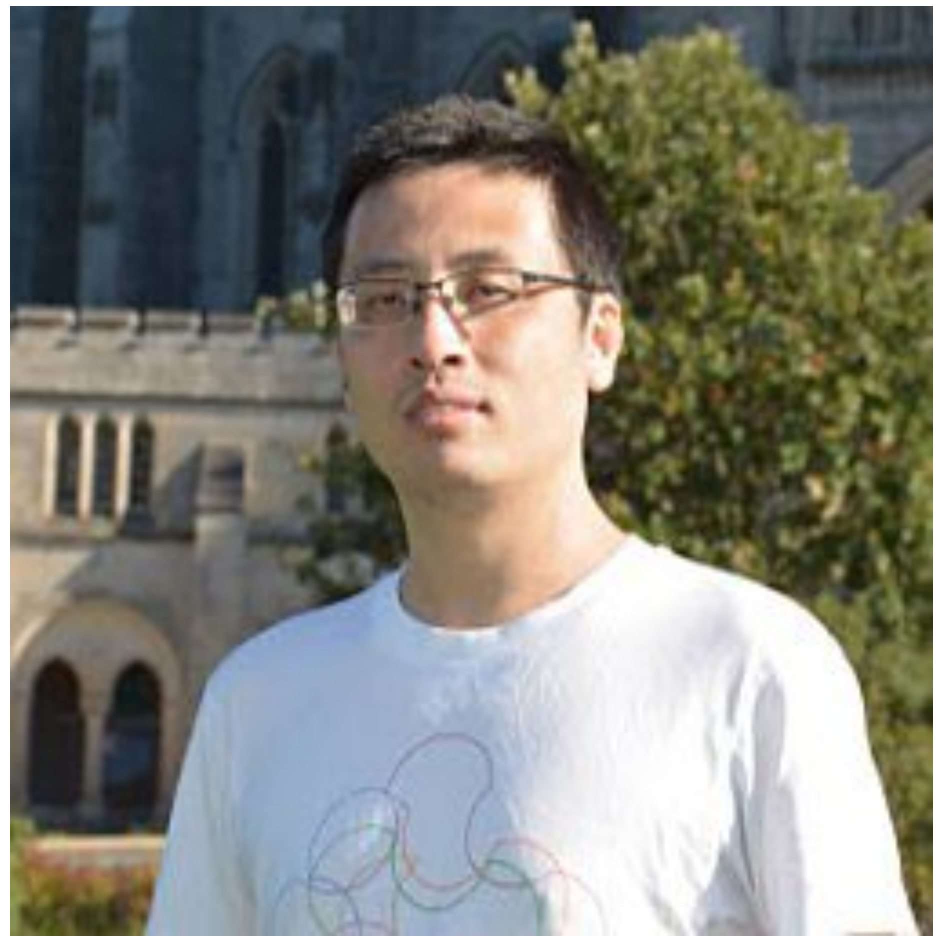
Pietro Bonetti

Maria
Tikhanovskaya
Allen Institute



Mathias Scheurer
Stuttgart

Jia-Xin Zhang
UCSB



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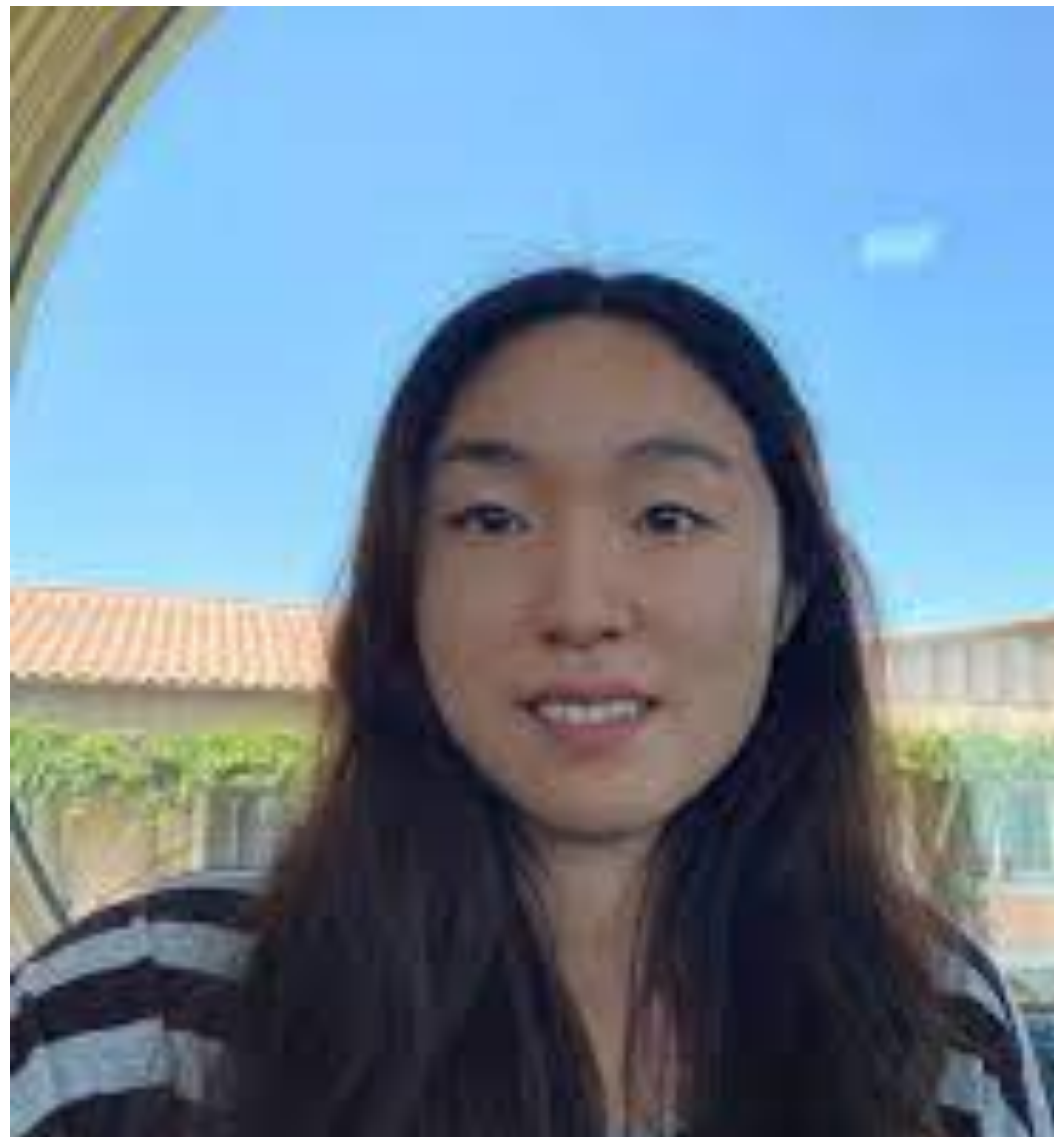
Johns Hopkins

Alexander
Nikolaenko



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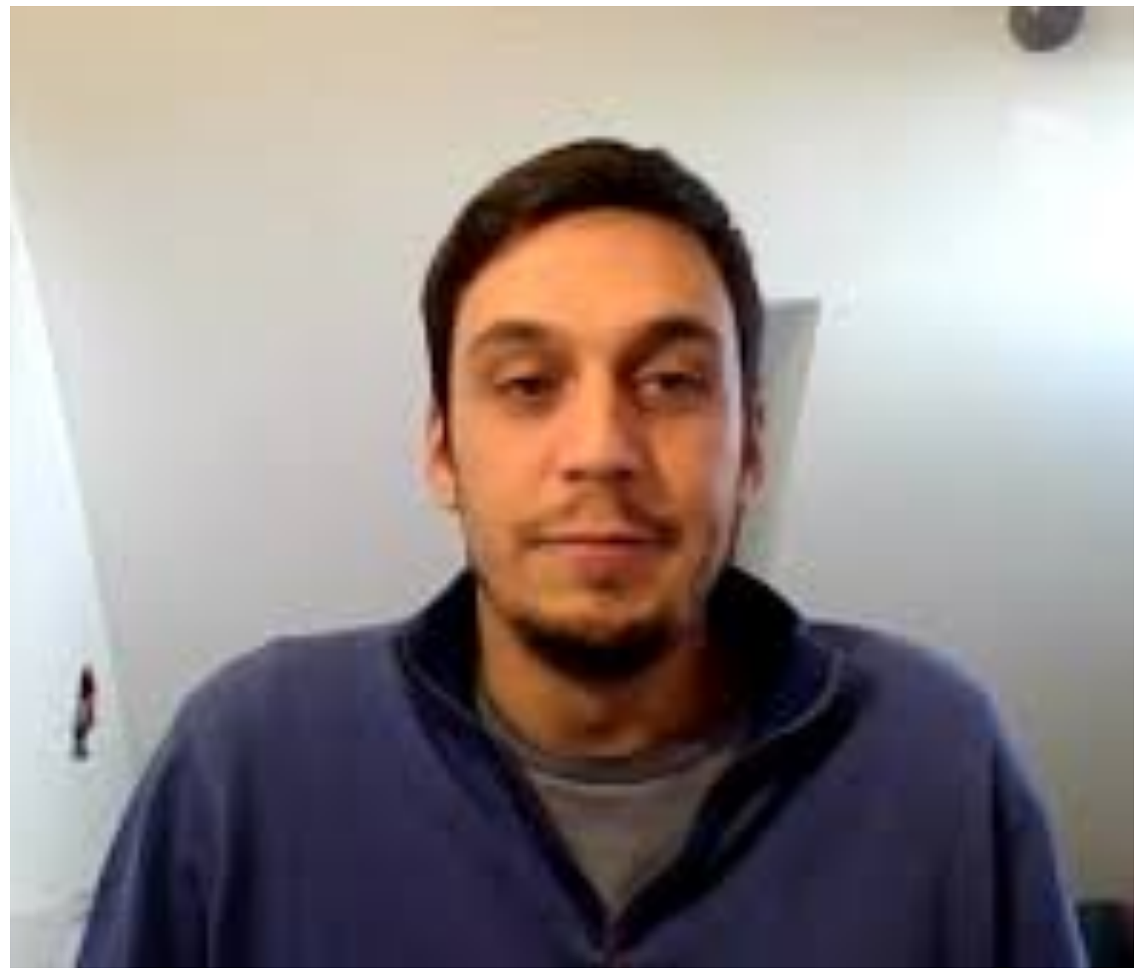
Henry Shackleton
MIT

Darshan Joshi
TIFR Hyderabad



H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

Pietro Bonetti

Maine Christos
Caltech

Harshit Pandey

Ravi Shanker

Sayantan Sharma

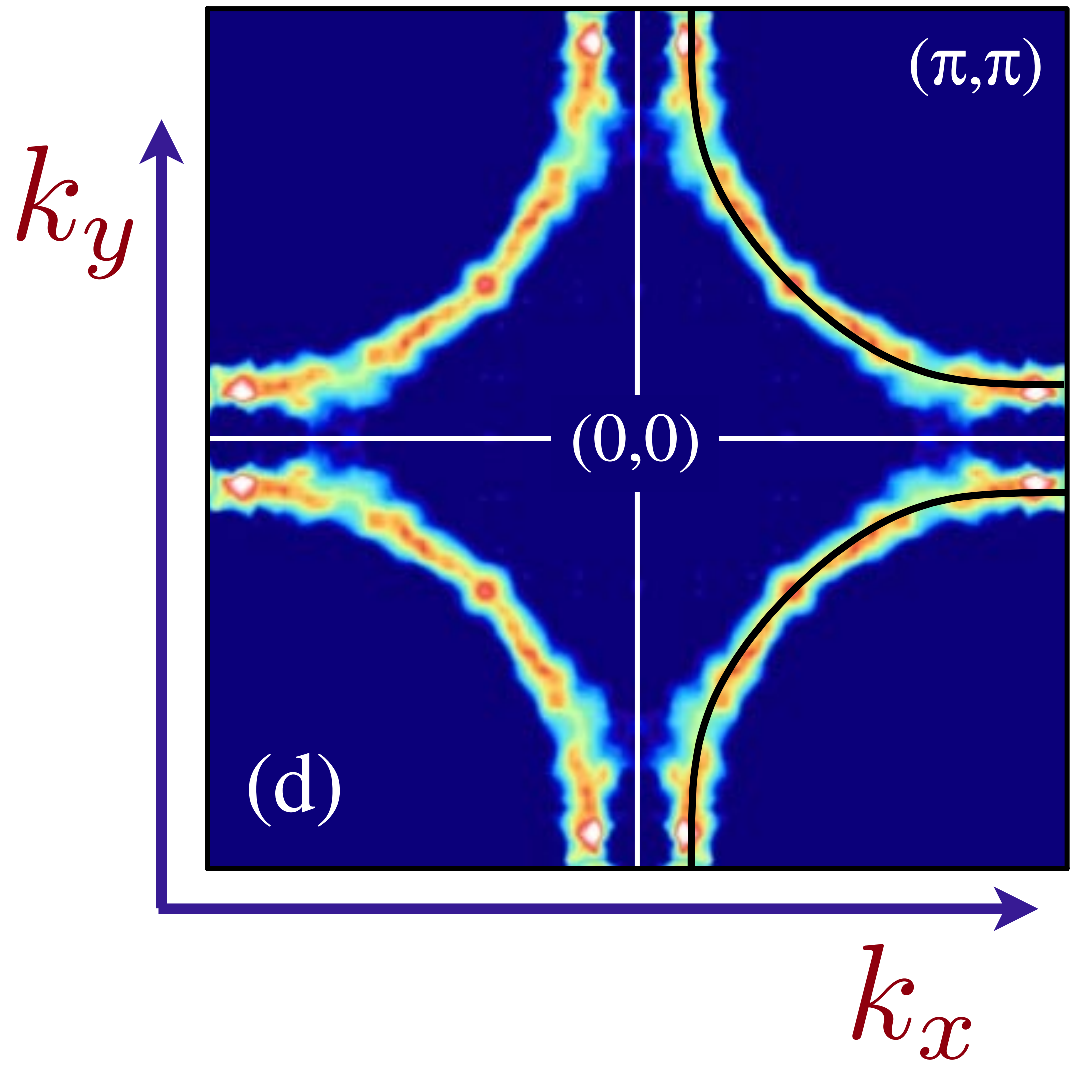
The Institute of Mathematical Sciences, Chennai

Introduction to FL* theory of the pseudogap

Ordinary metals

Nearly-free gas of fermions, with a Fermi surface between empty and occupied states (Sommerfeld, 1927).

Area enclosed by the Fermi surface is the same as that for free fermions with the same symmetry
Luttinger, 1960 - perturbative;
Oshikawa, 2000 - non-perturbative anomaly matching



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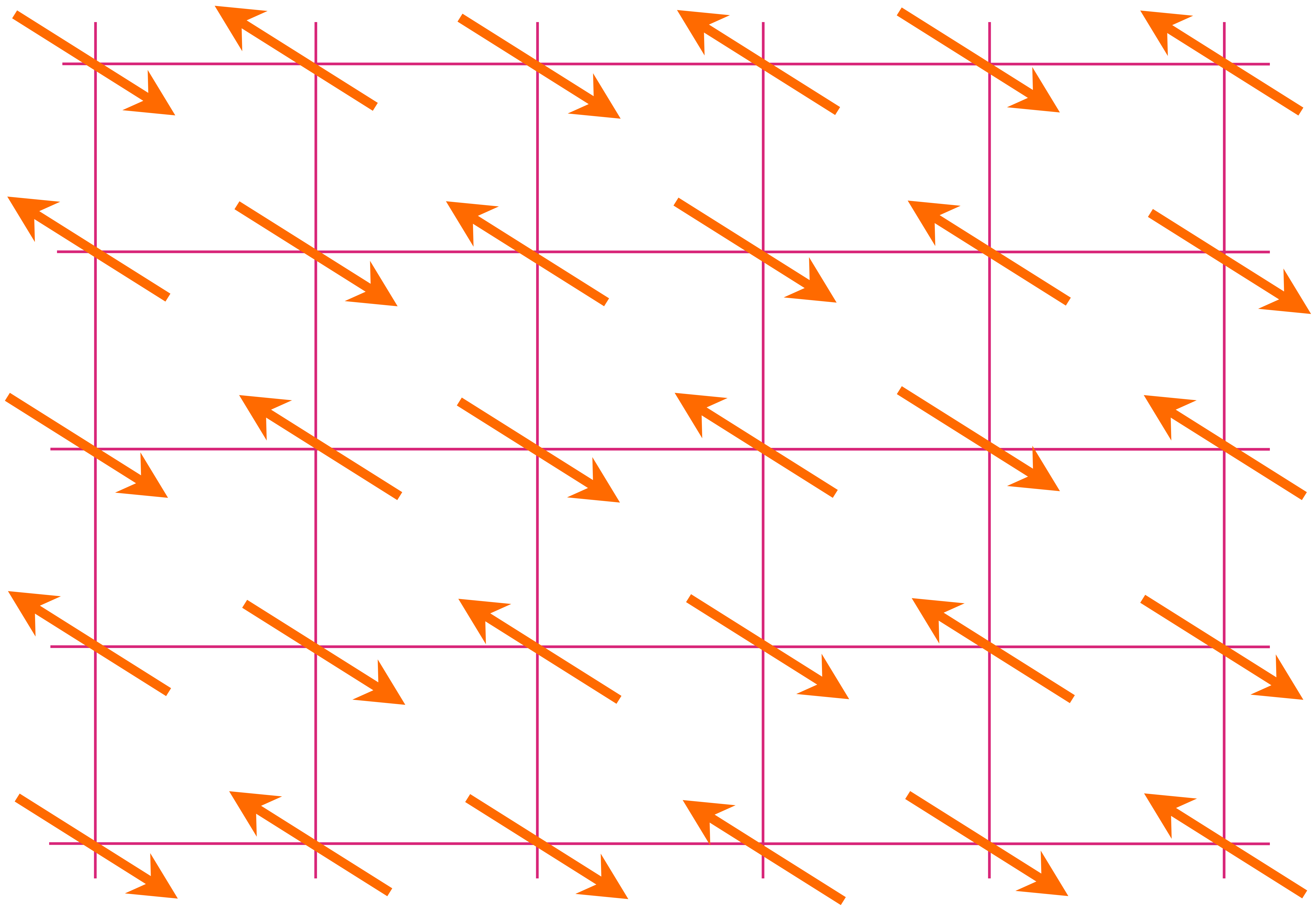
Fractionalized Fermi liquids (FL*)

There can be metals with Fermi surface area not equal to the Luttinger value provided the fractionalized excitations of a spin liquid are also present. The sum of the Fermi surface and spin liquid anomalies equals the Oshikawa anomaly.

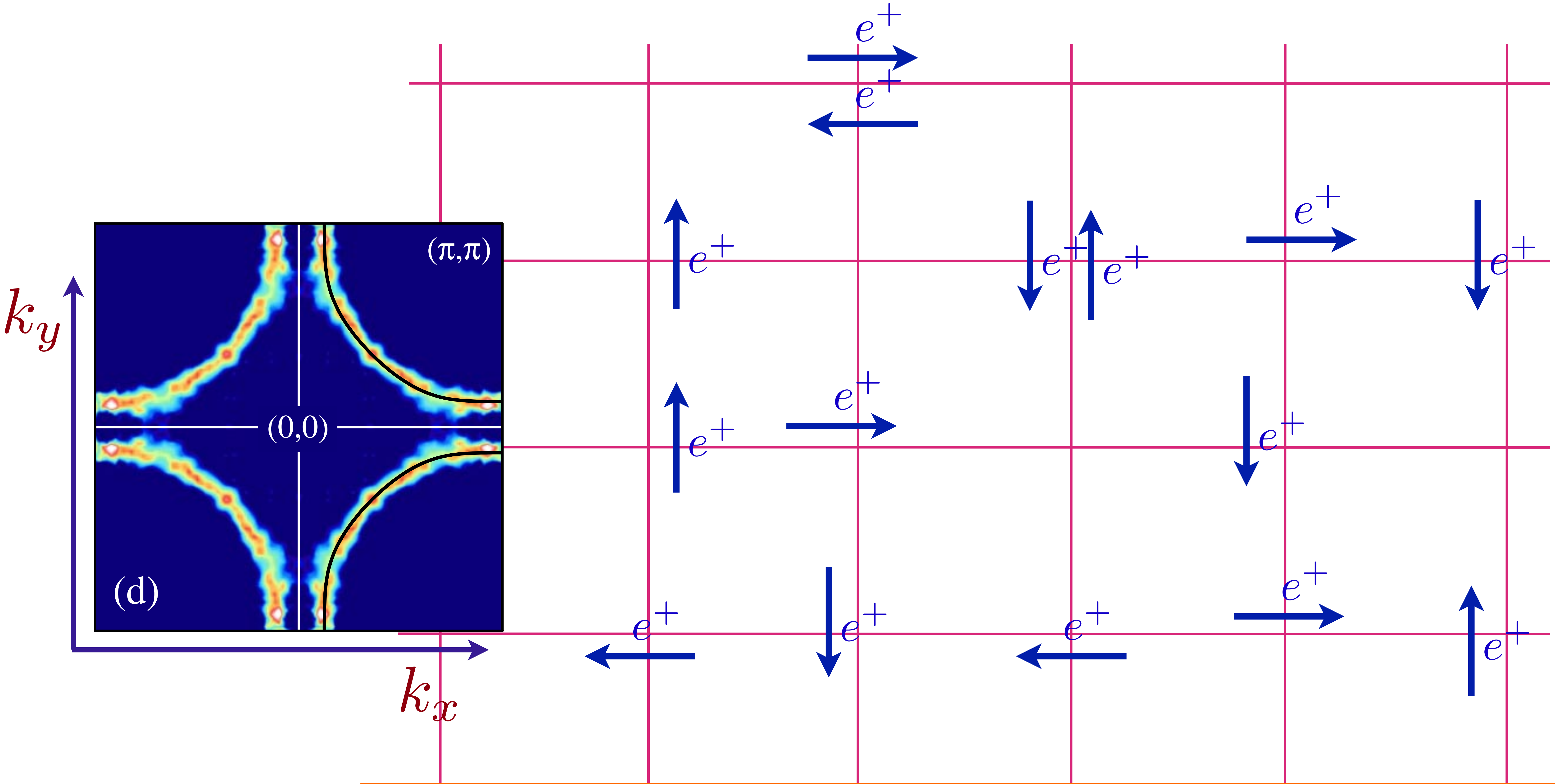
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, S.S., PRB **69**, 035111 (2004)

Insulating antiferromagnet

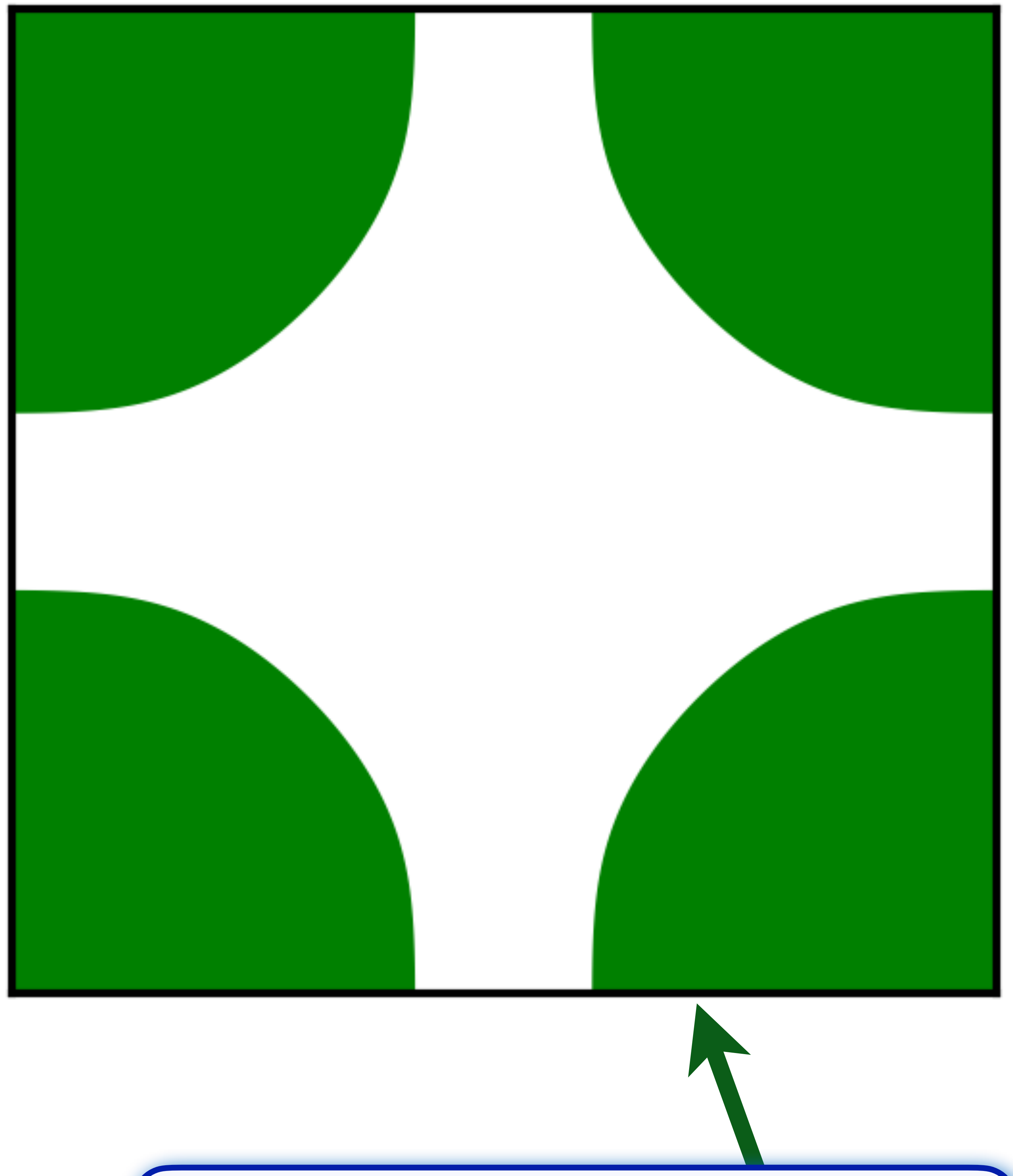


Doping an insulating antiferromagnet with holes of density p

Ordinary metal



Luttinger area.
No broken symmetry

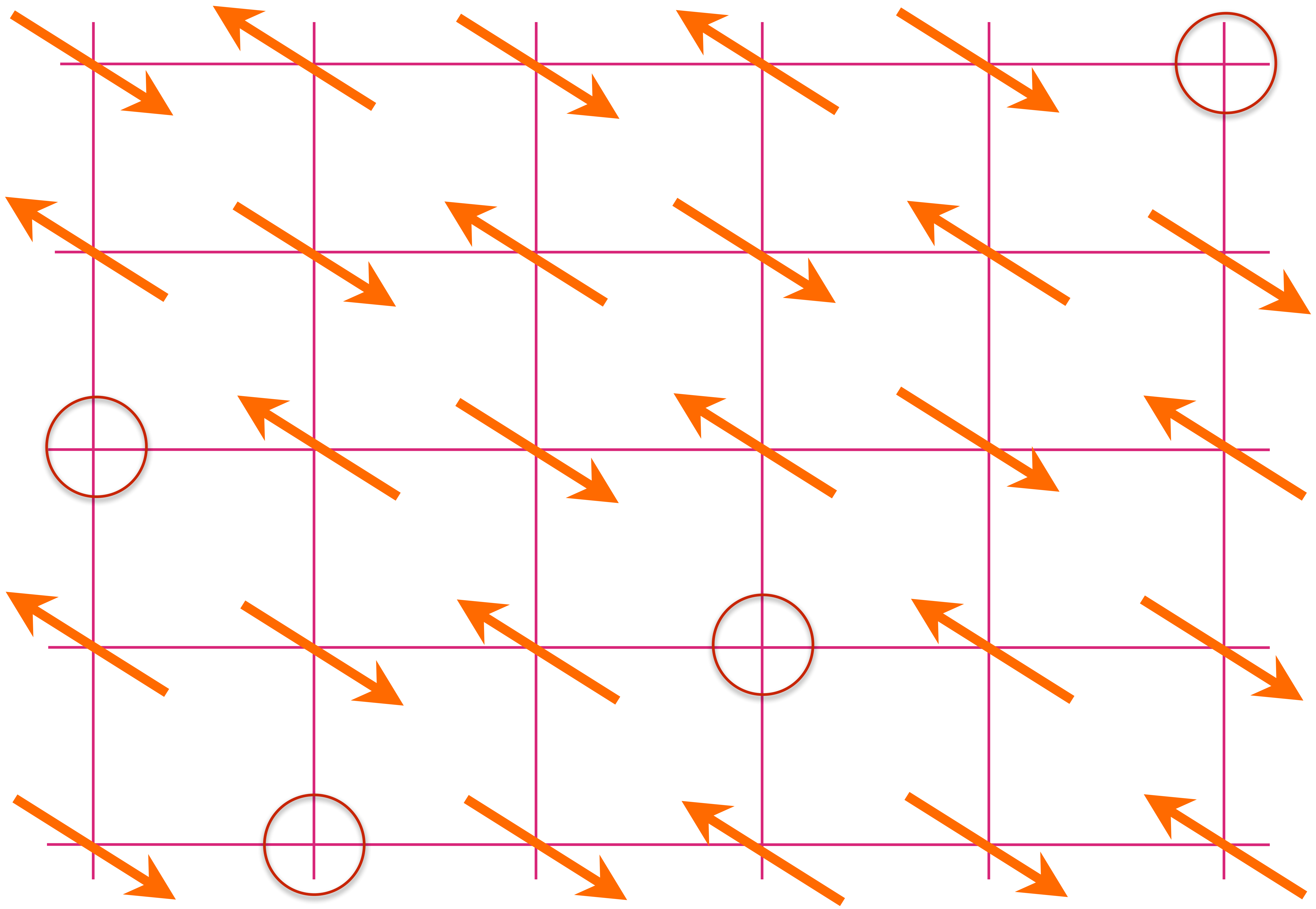


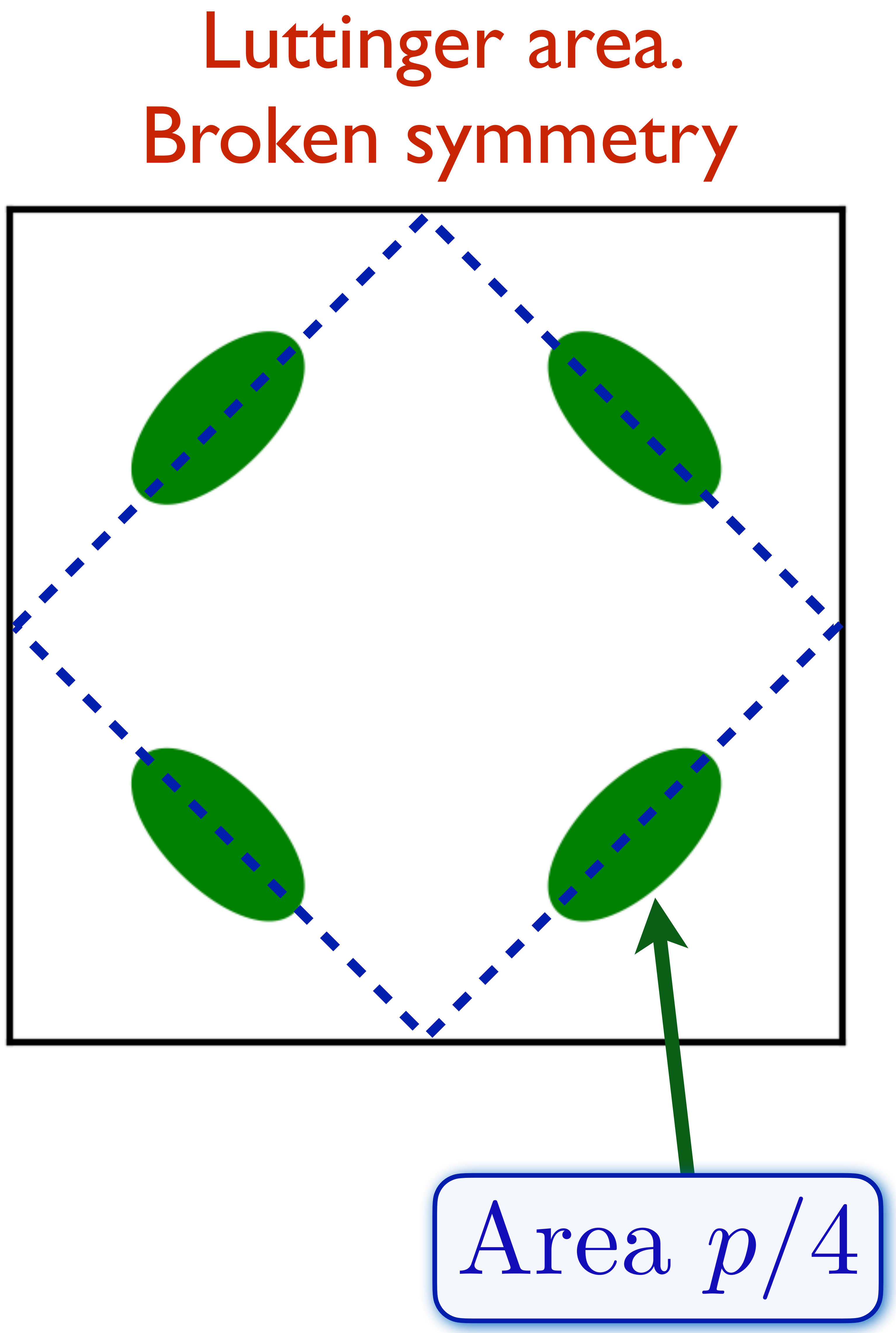
Area $(1+p)/2$

At large p, we obtain a gas of nearly free fermionic holes of density $1+p$ (relative to the filled band with 2 electrons per site)

Doping an insulating antiferromagnet with holes of density p

AF metal

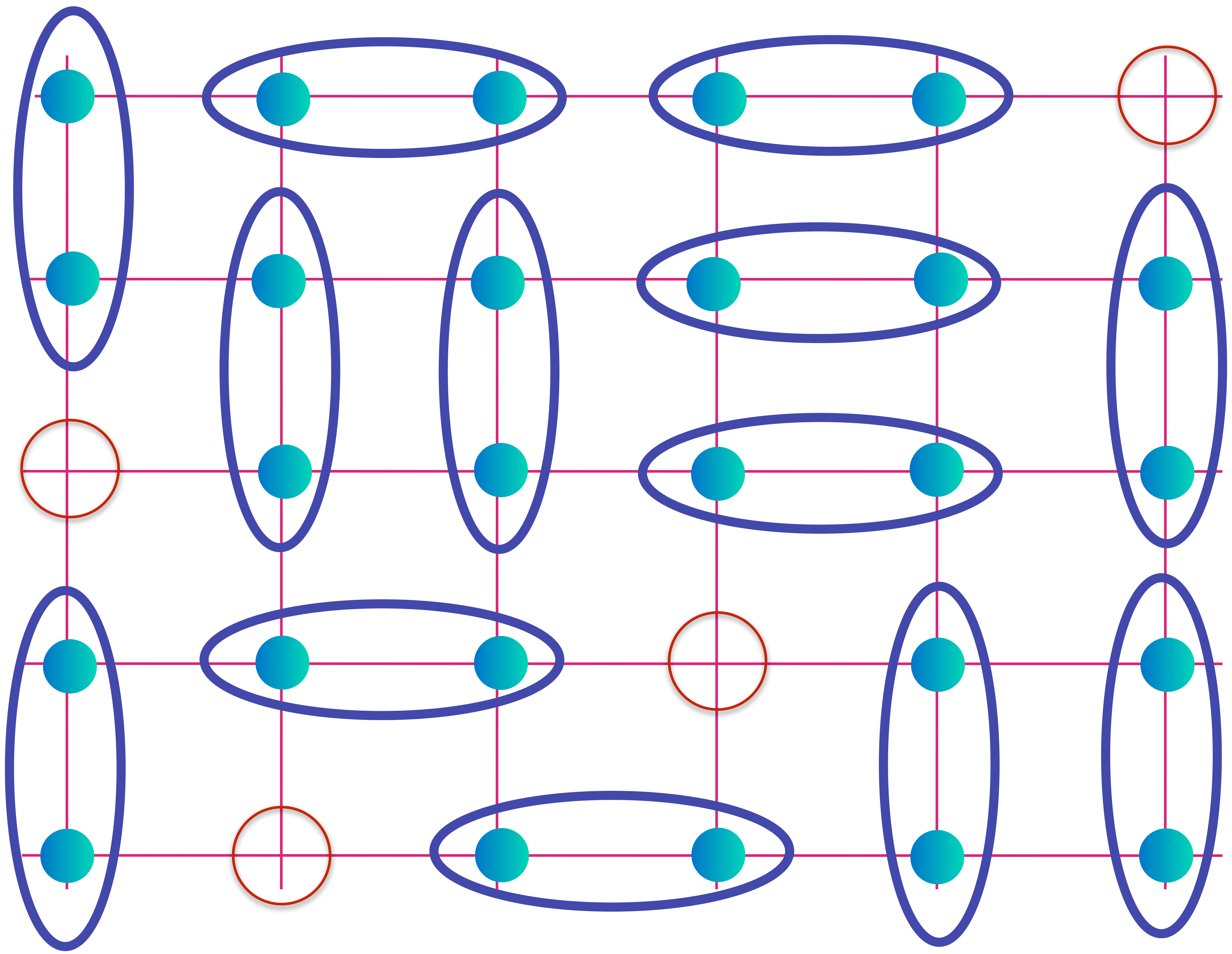


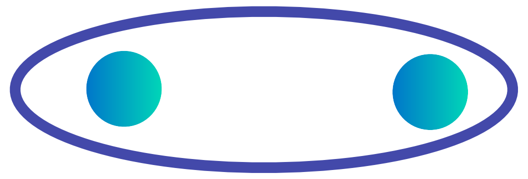


Doping an insulating antiferromagnet with holes of density p

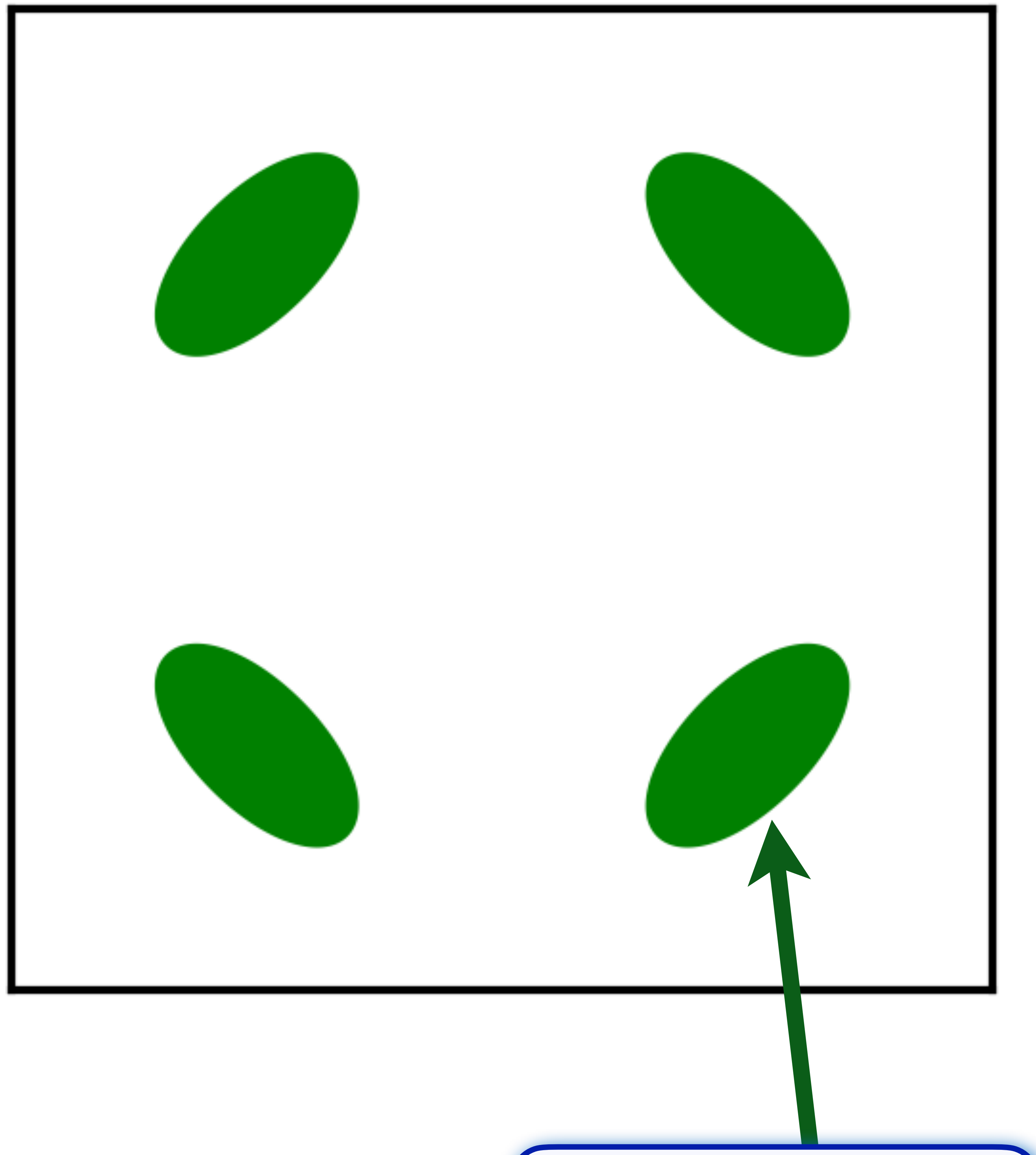
Holon metal

Spin liquid with density p of spinless, charge +e "holons".



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

non-Luttinger area.
Spin liquid



Area $p/4$

Doping an insulating antiferromagnet with holes of density p

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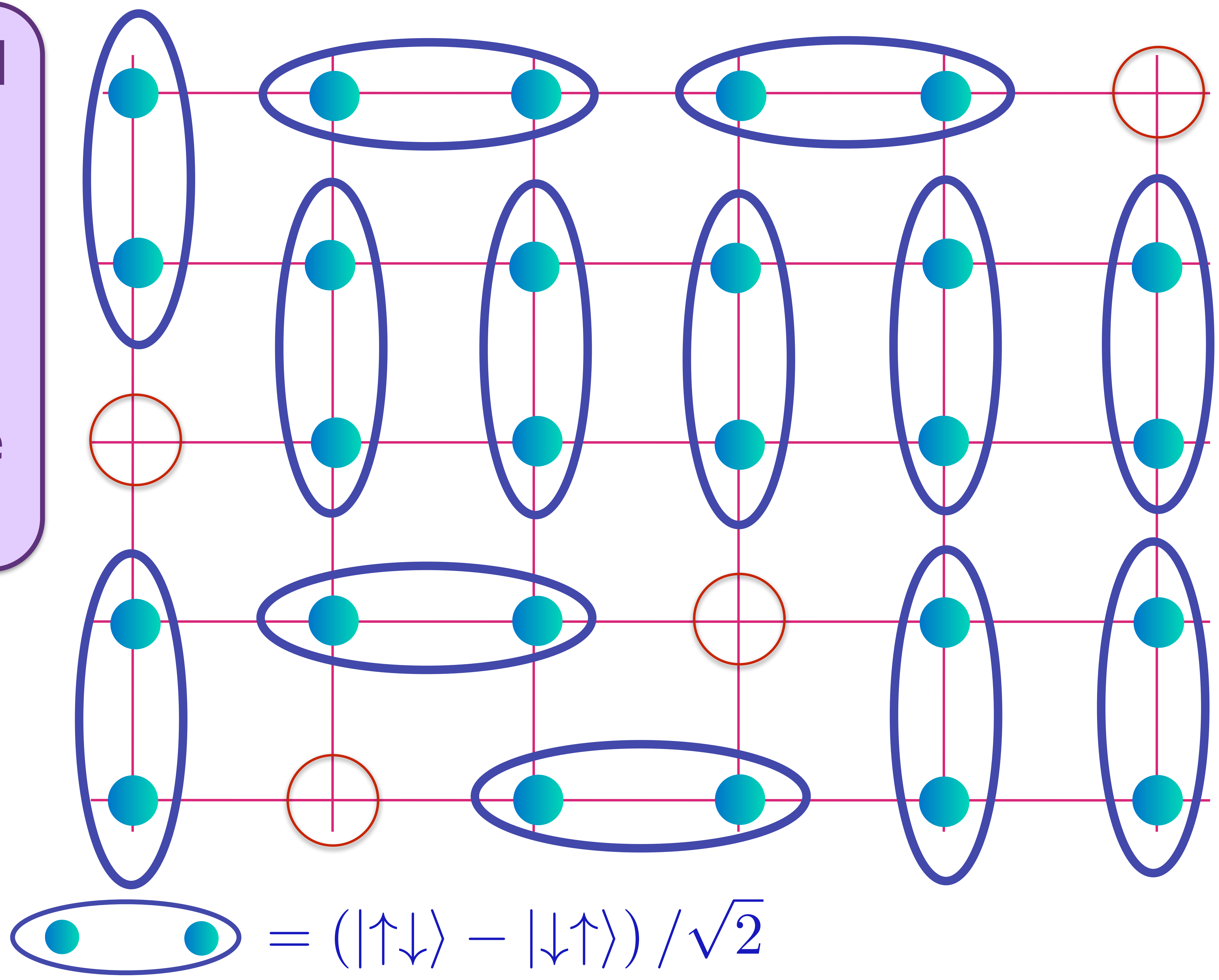

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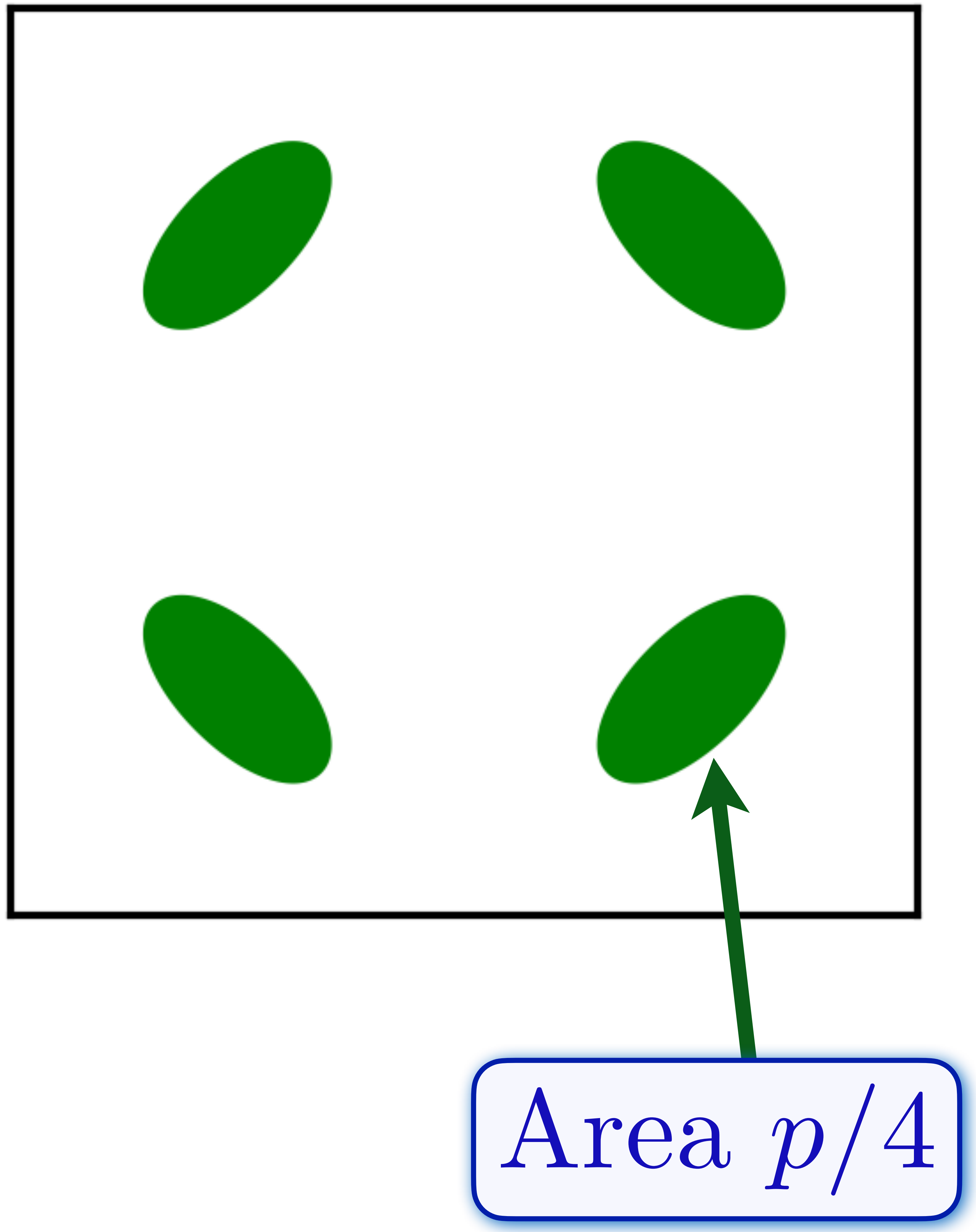

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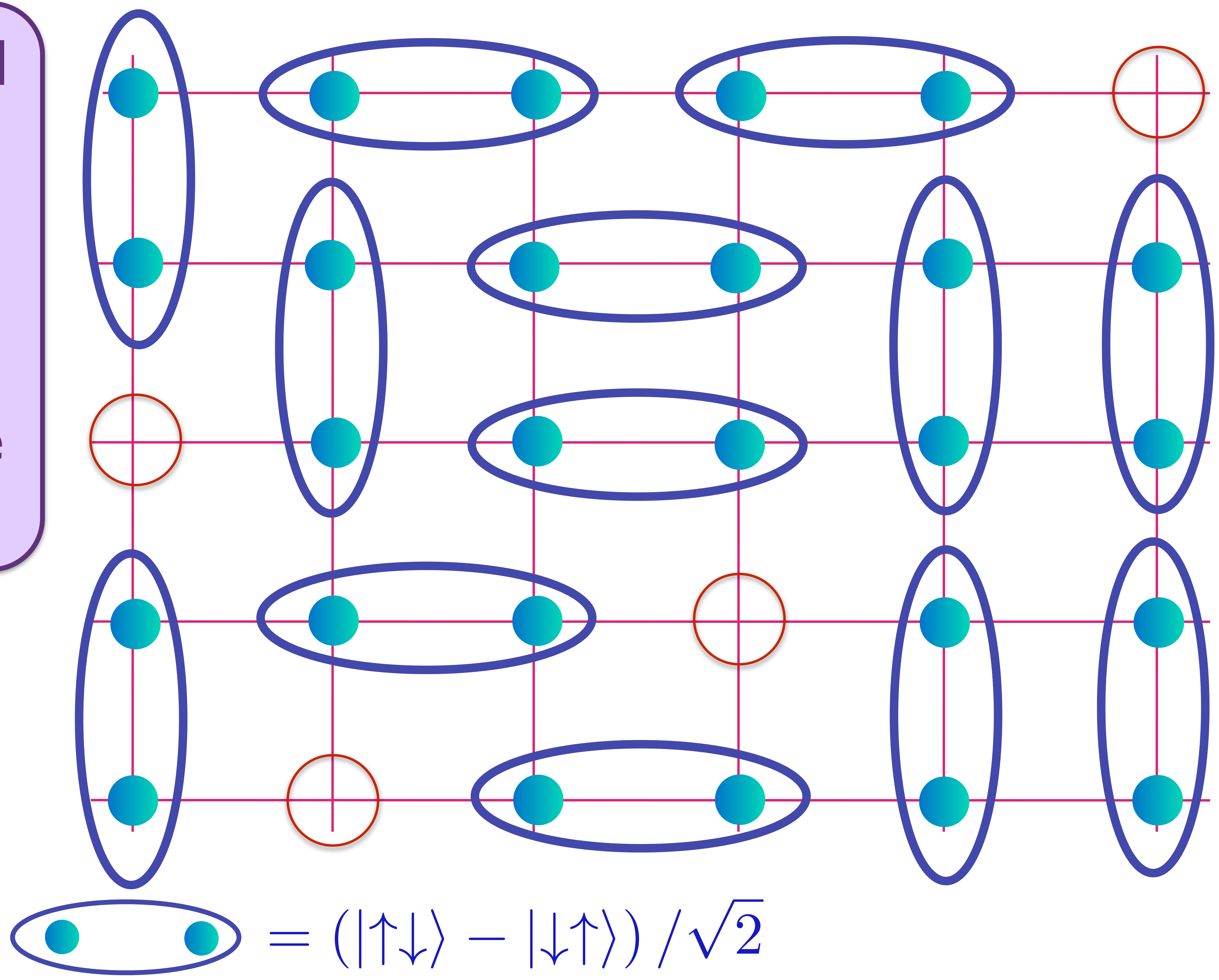

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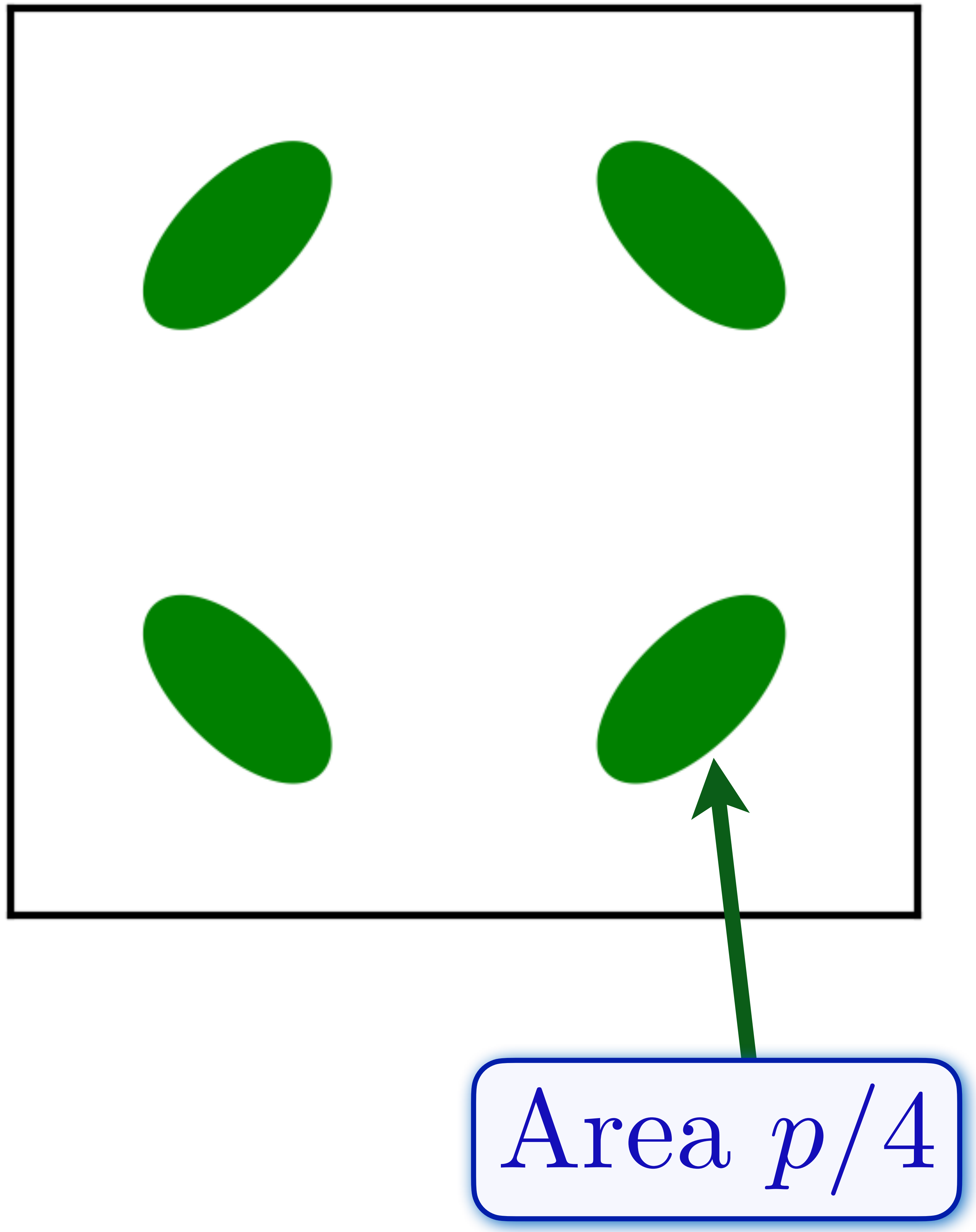

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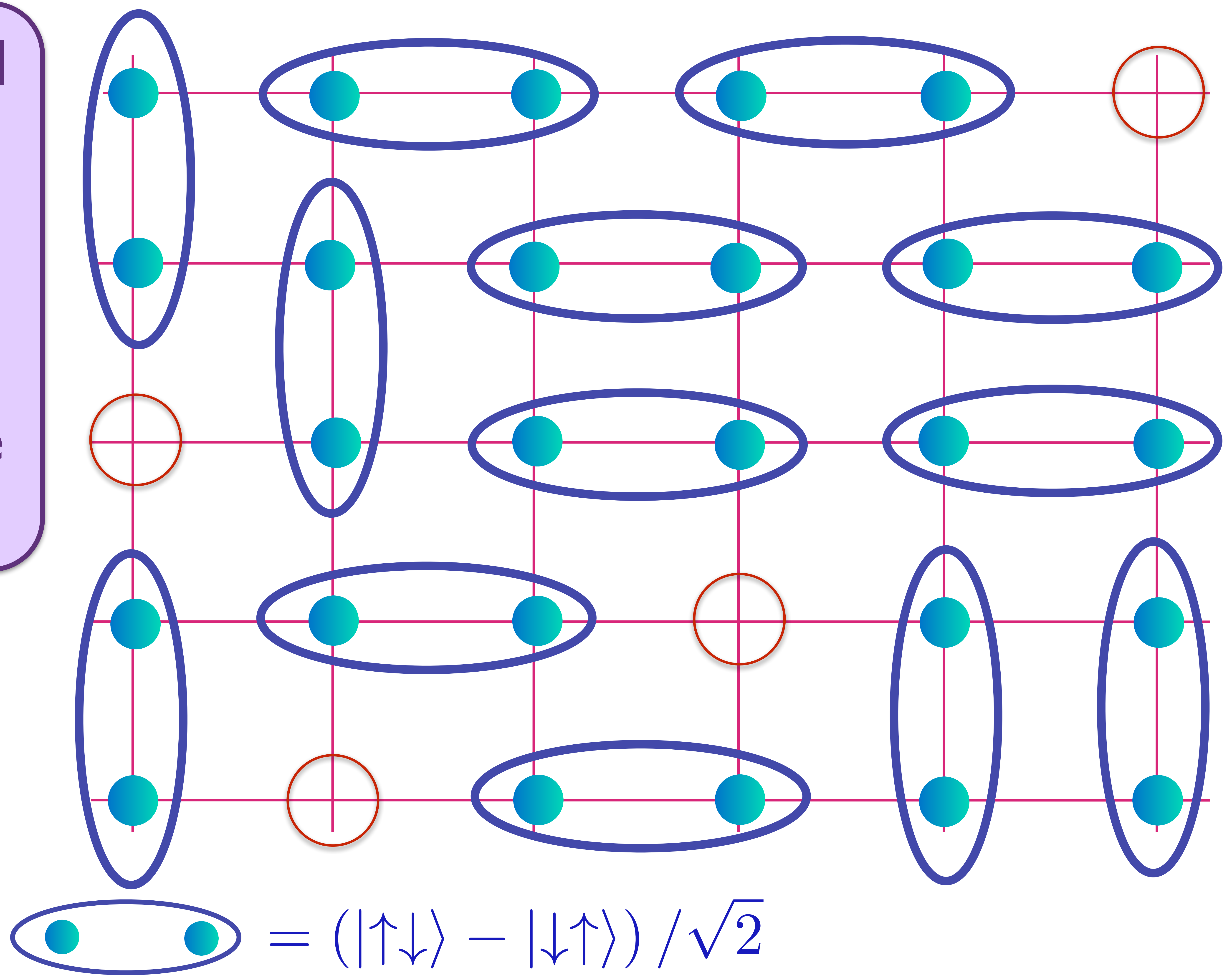

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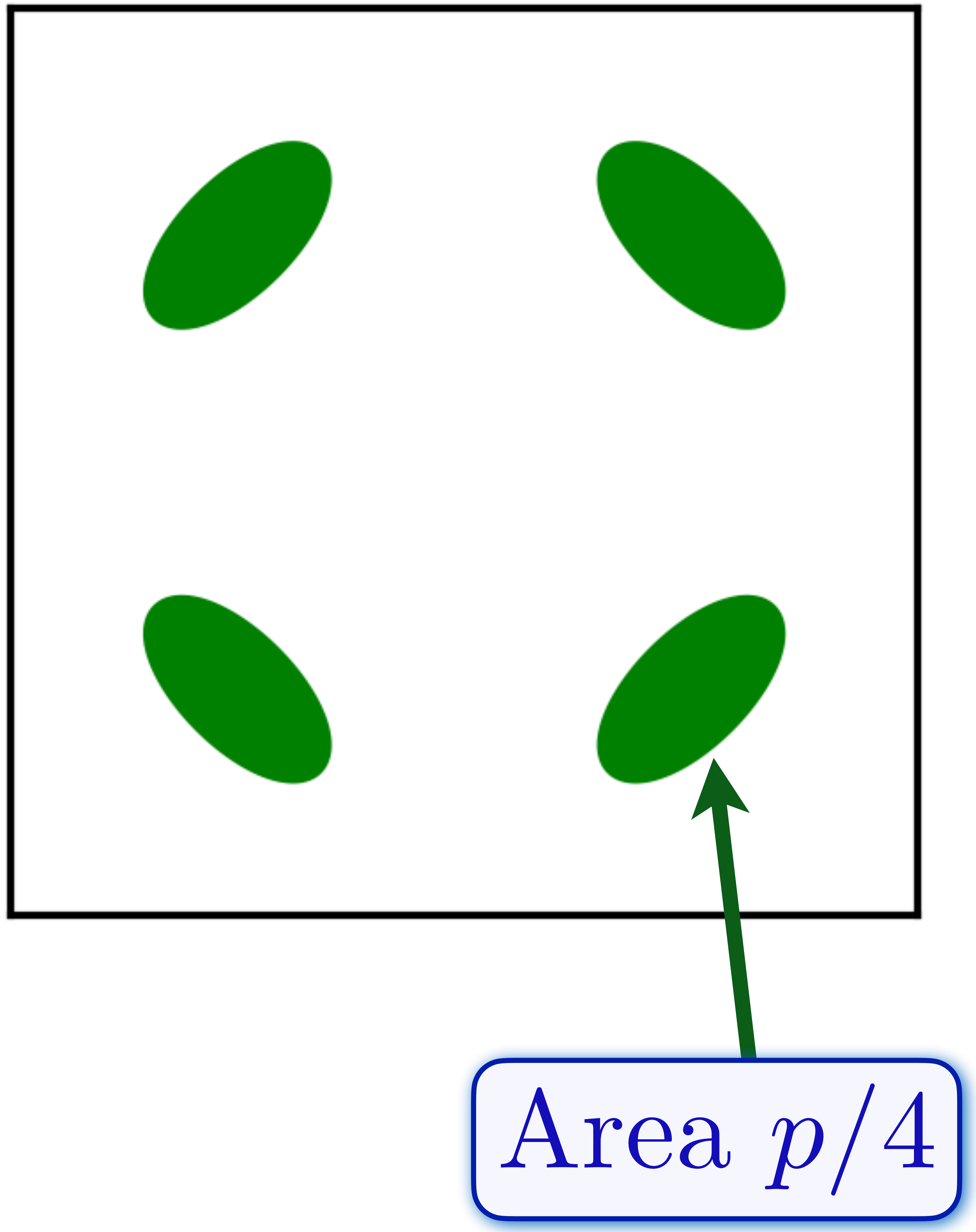

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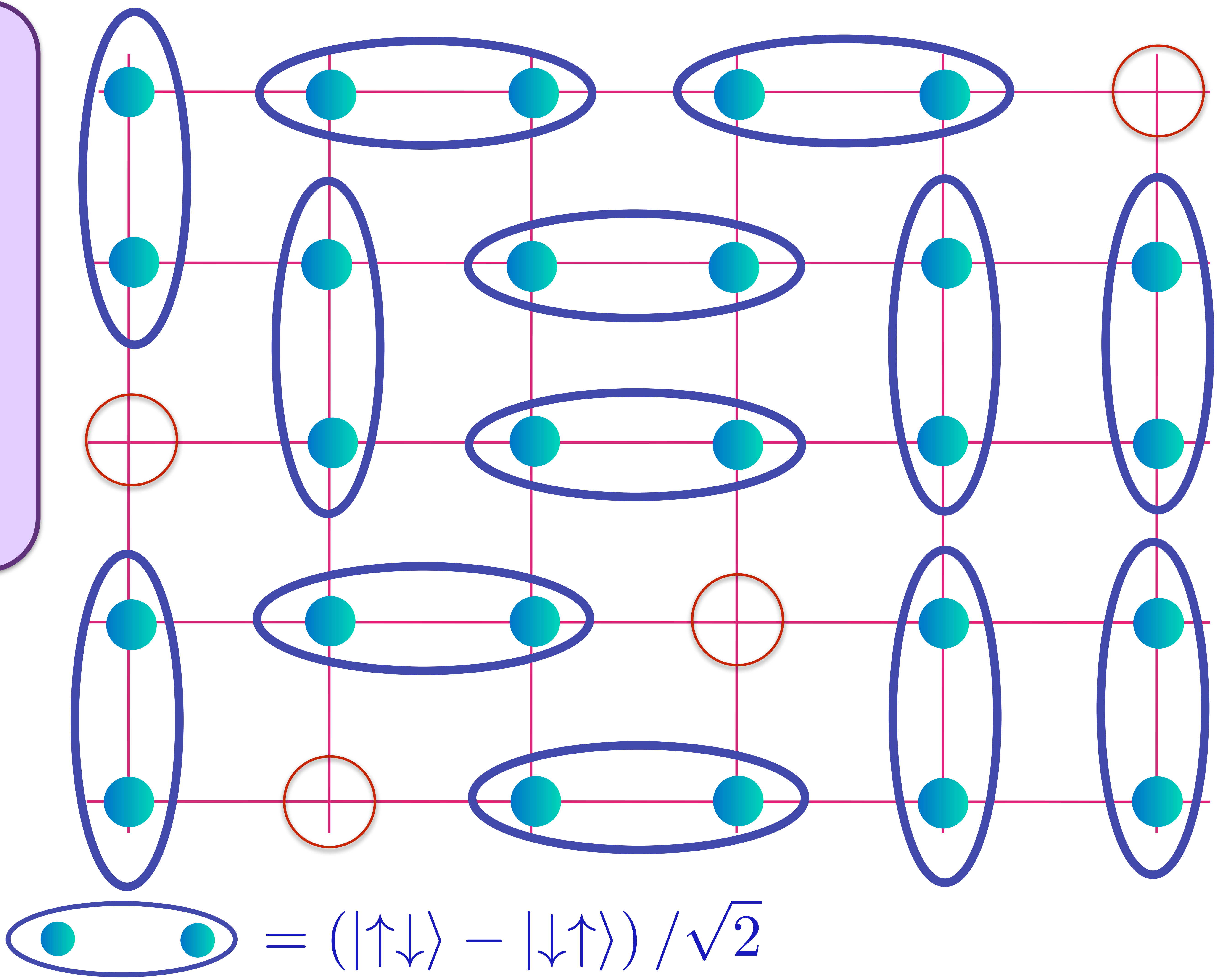

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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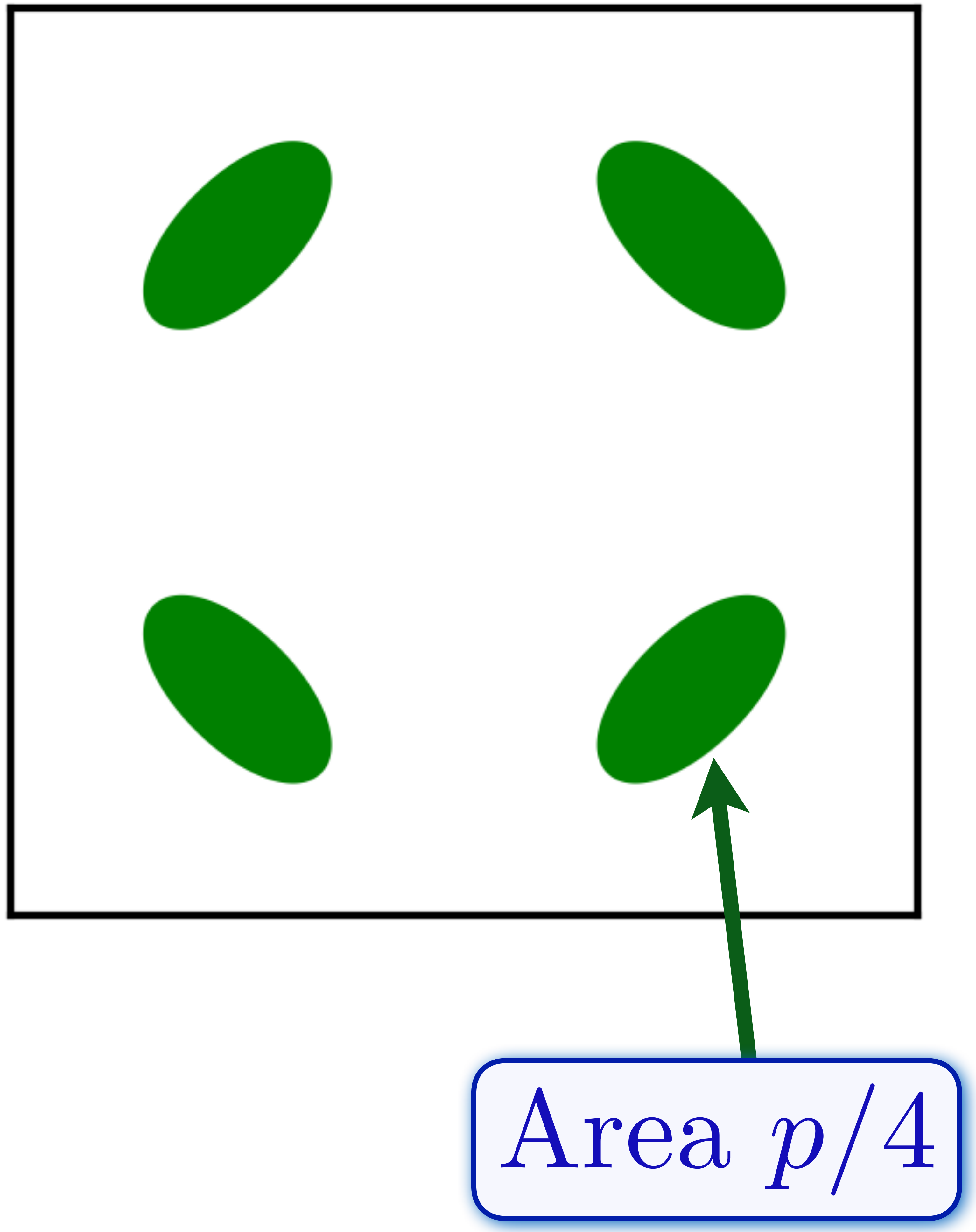

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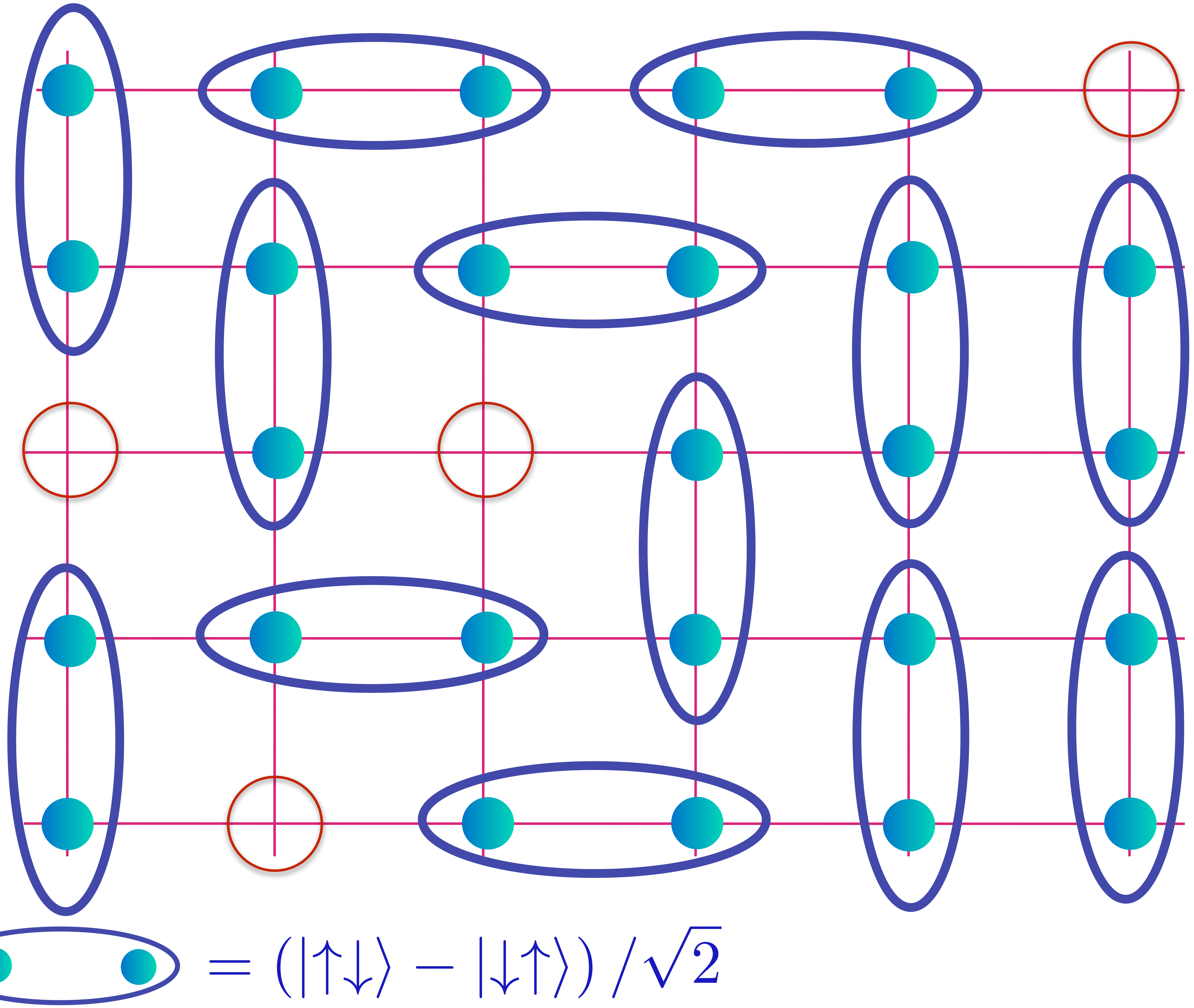

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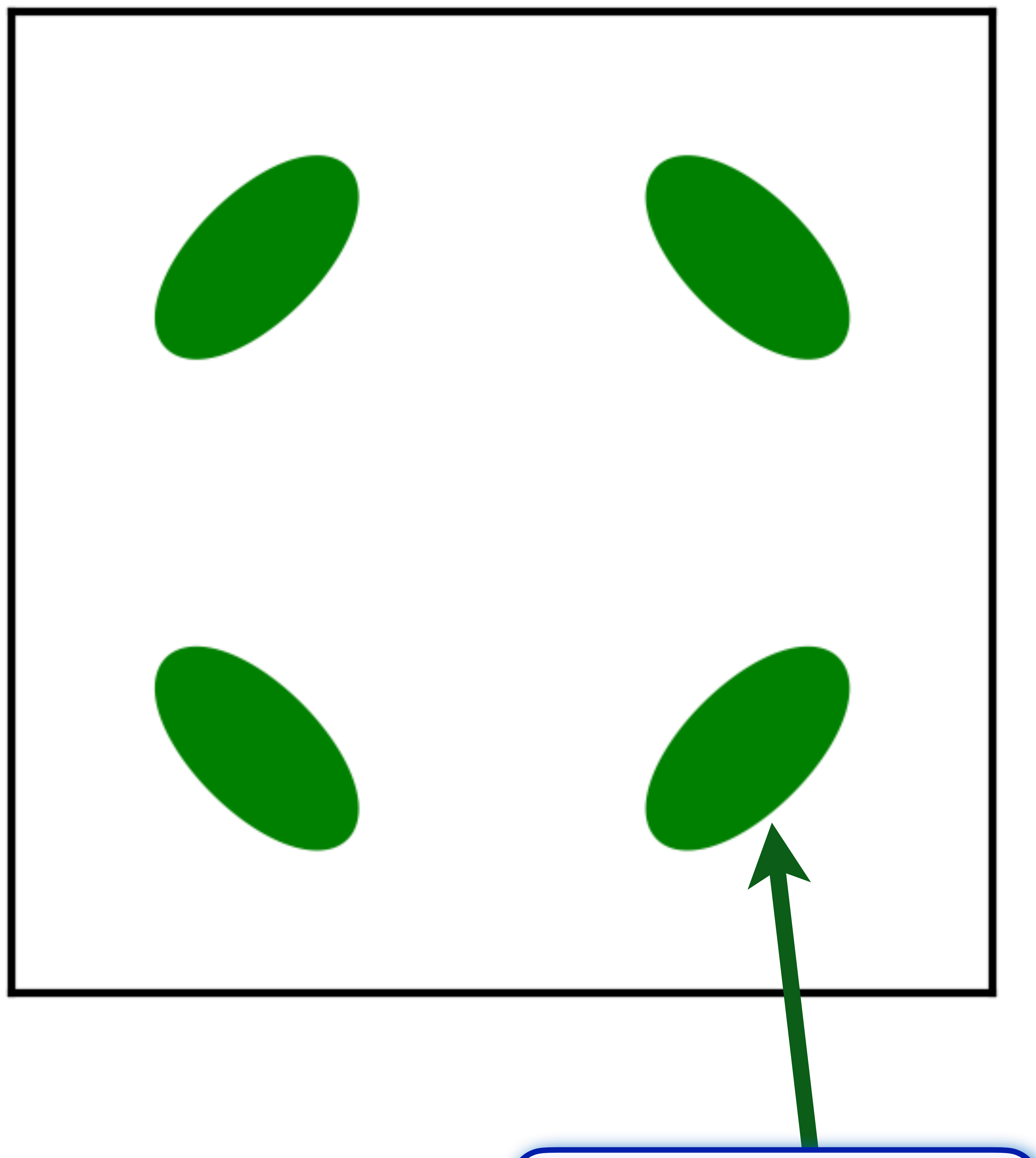

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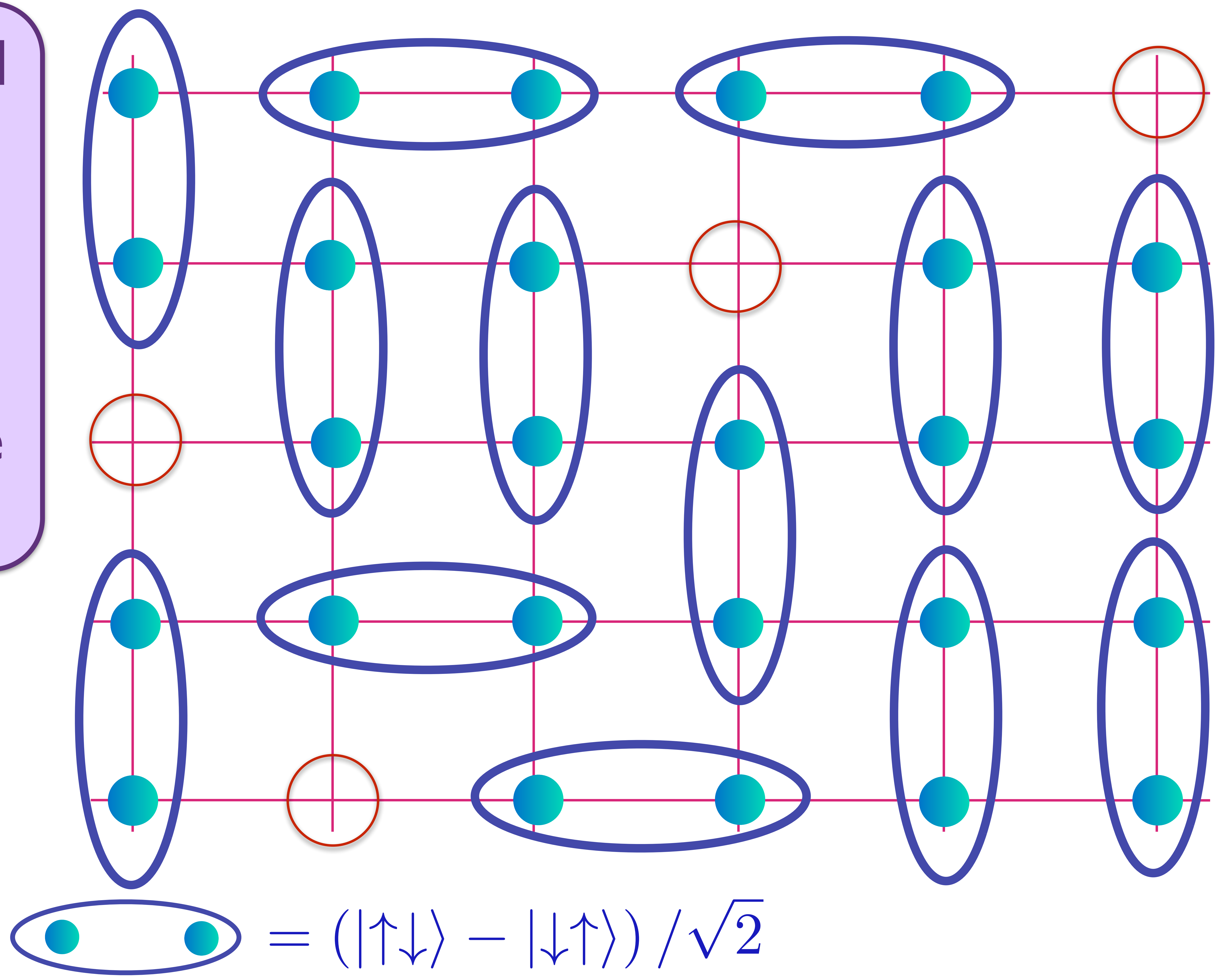

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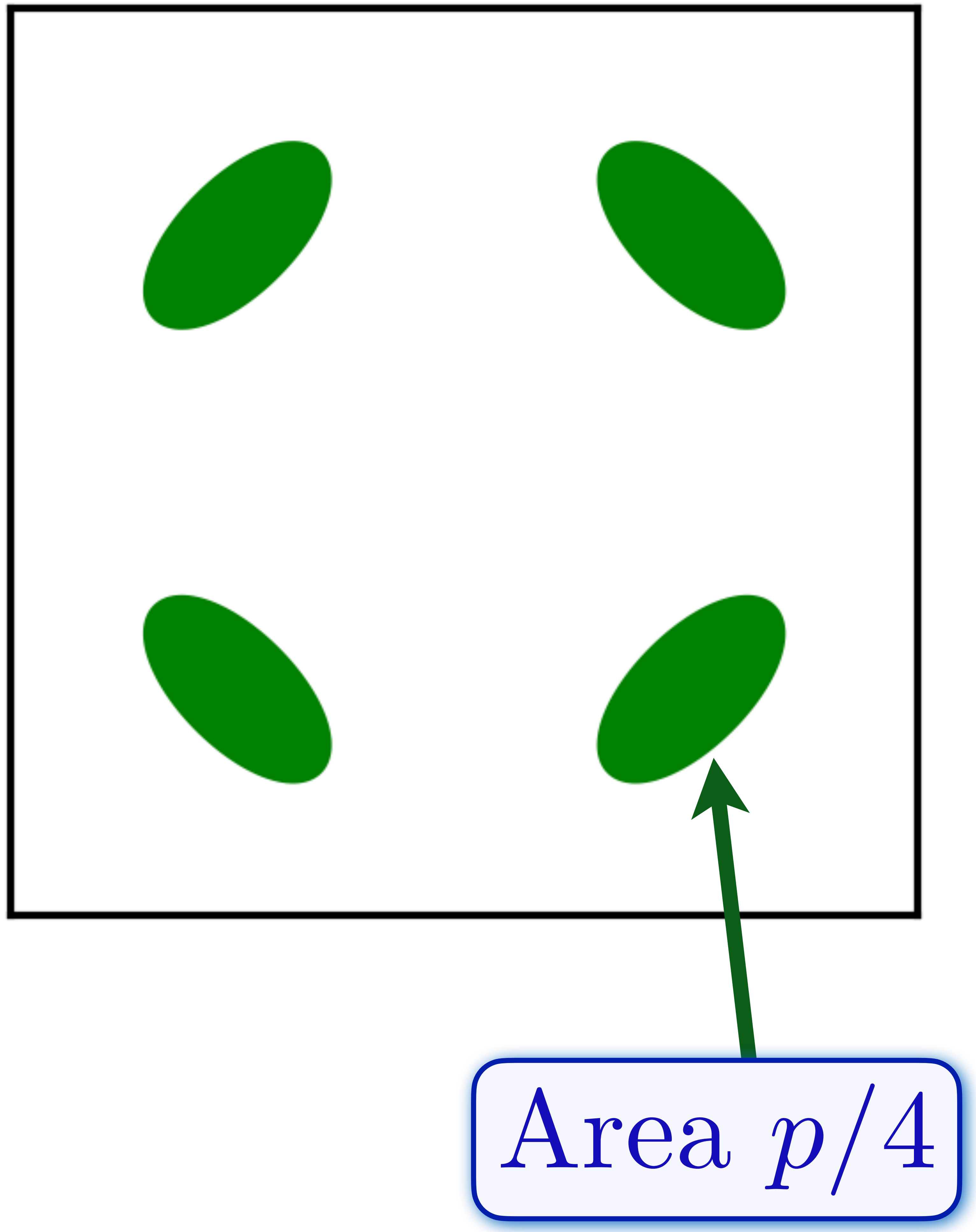

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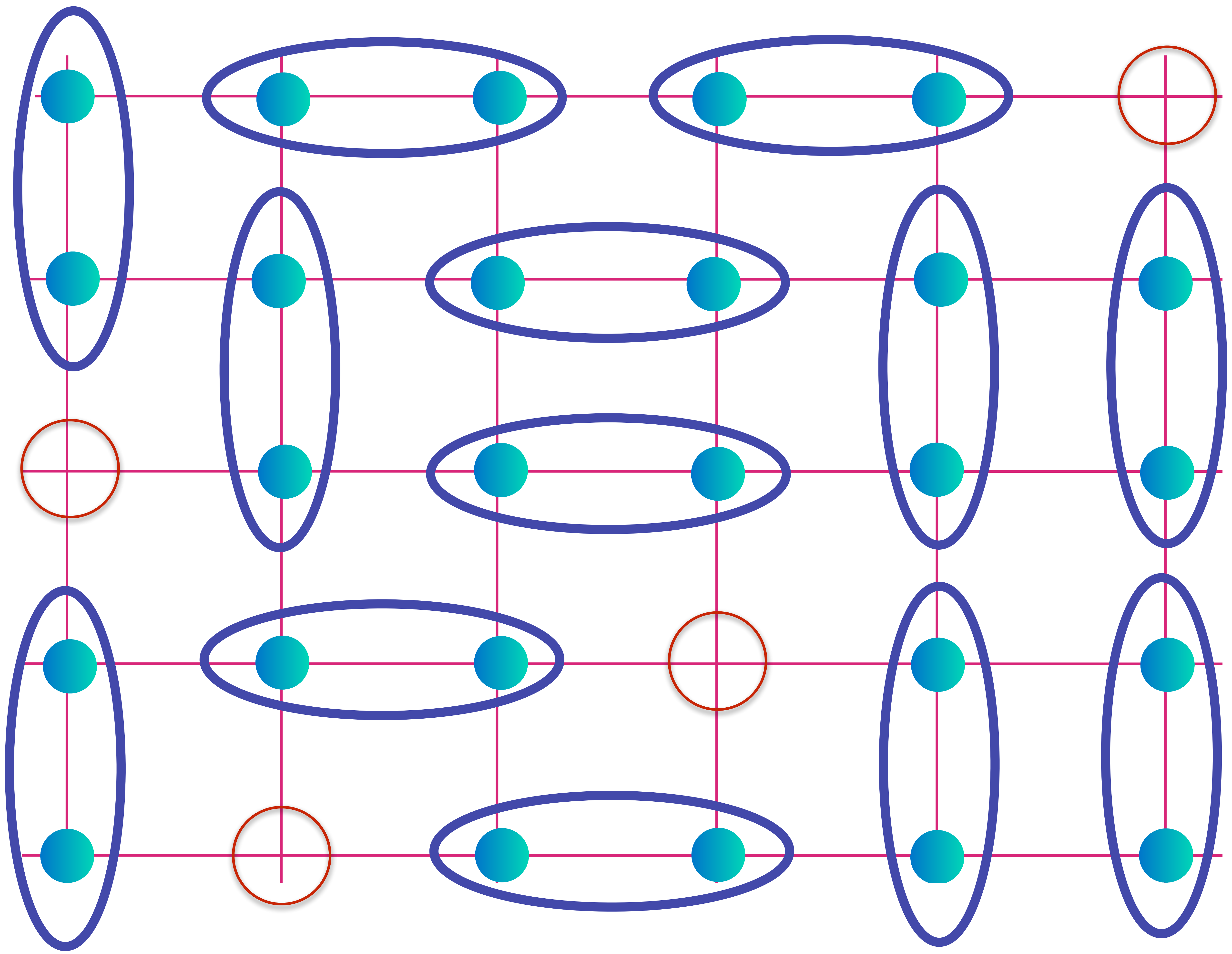

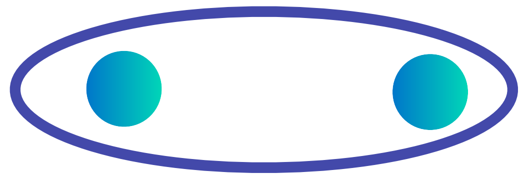 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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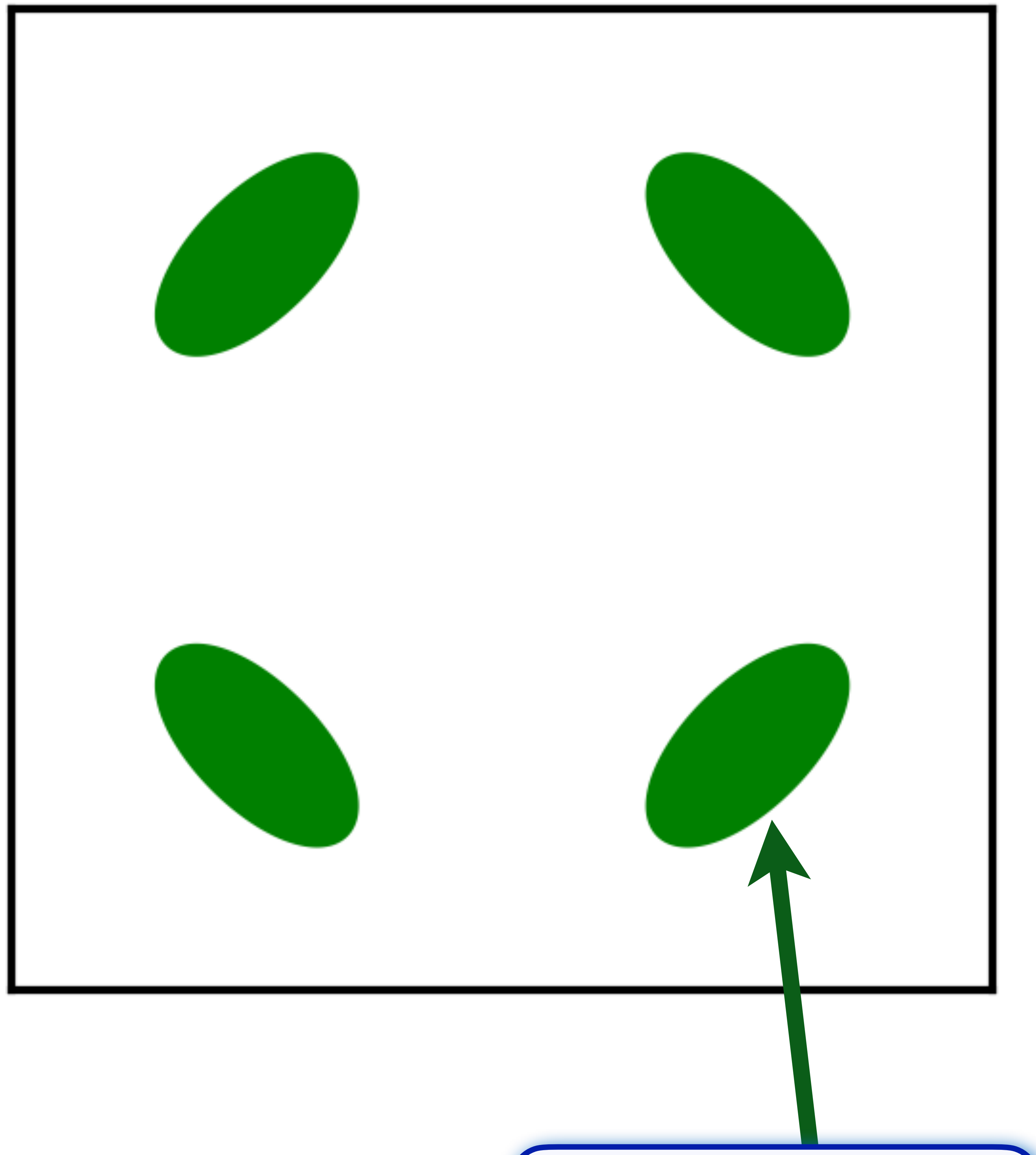
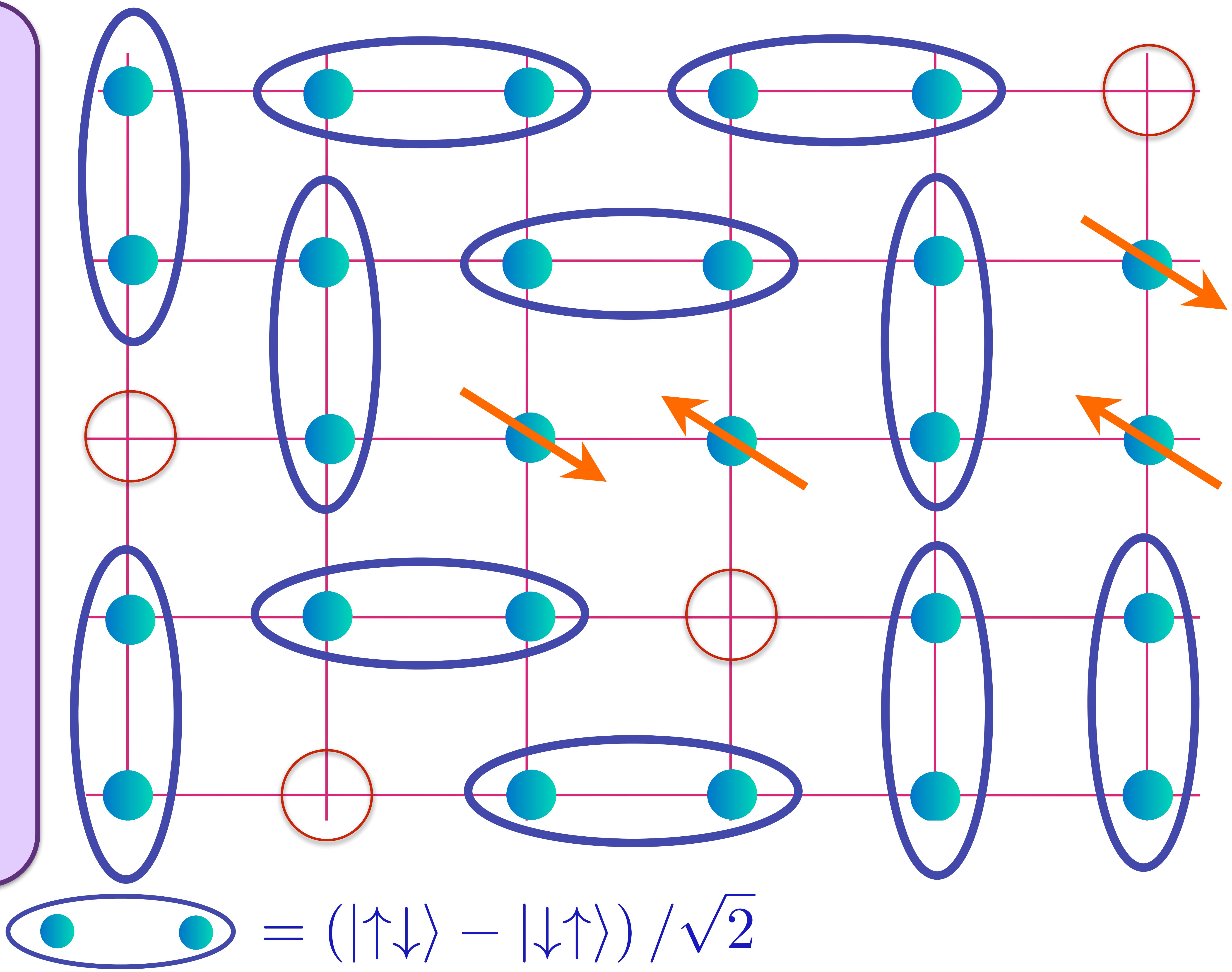


Area $p/4$

Doping an insulating antiferromagnet with holes of density p

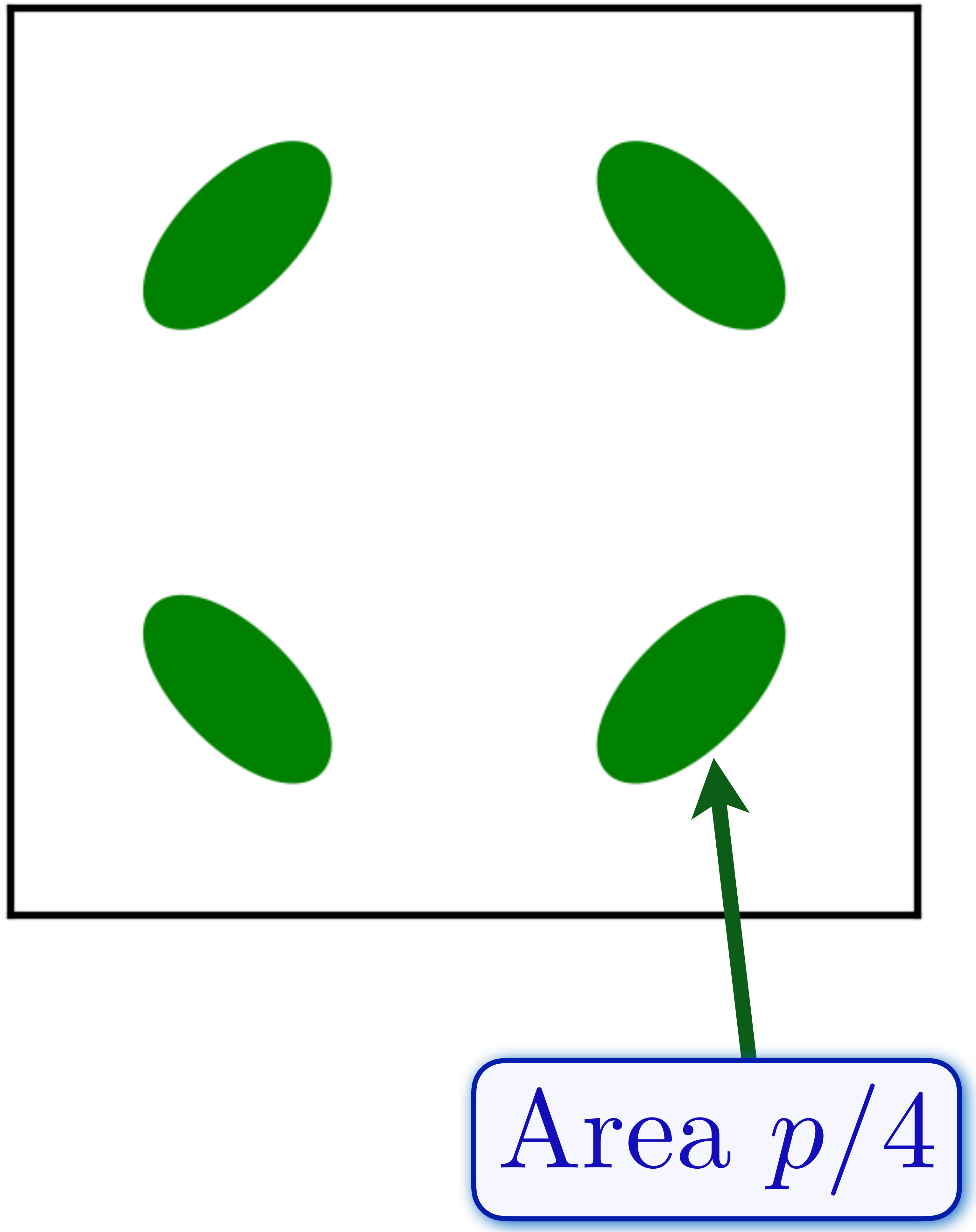
Holon metal

Spin liquid with density p of spinless, charge +e "holons" and charge 0 spin-1/2 "spinons".



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

non-Luttinger area.
Spin liquid



Doping an insulating antiferromagnet with holes of density p

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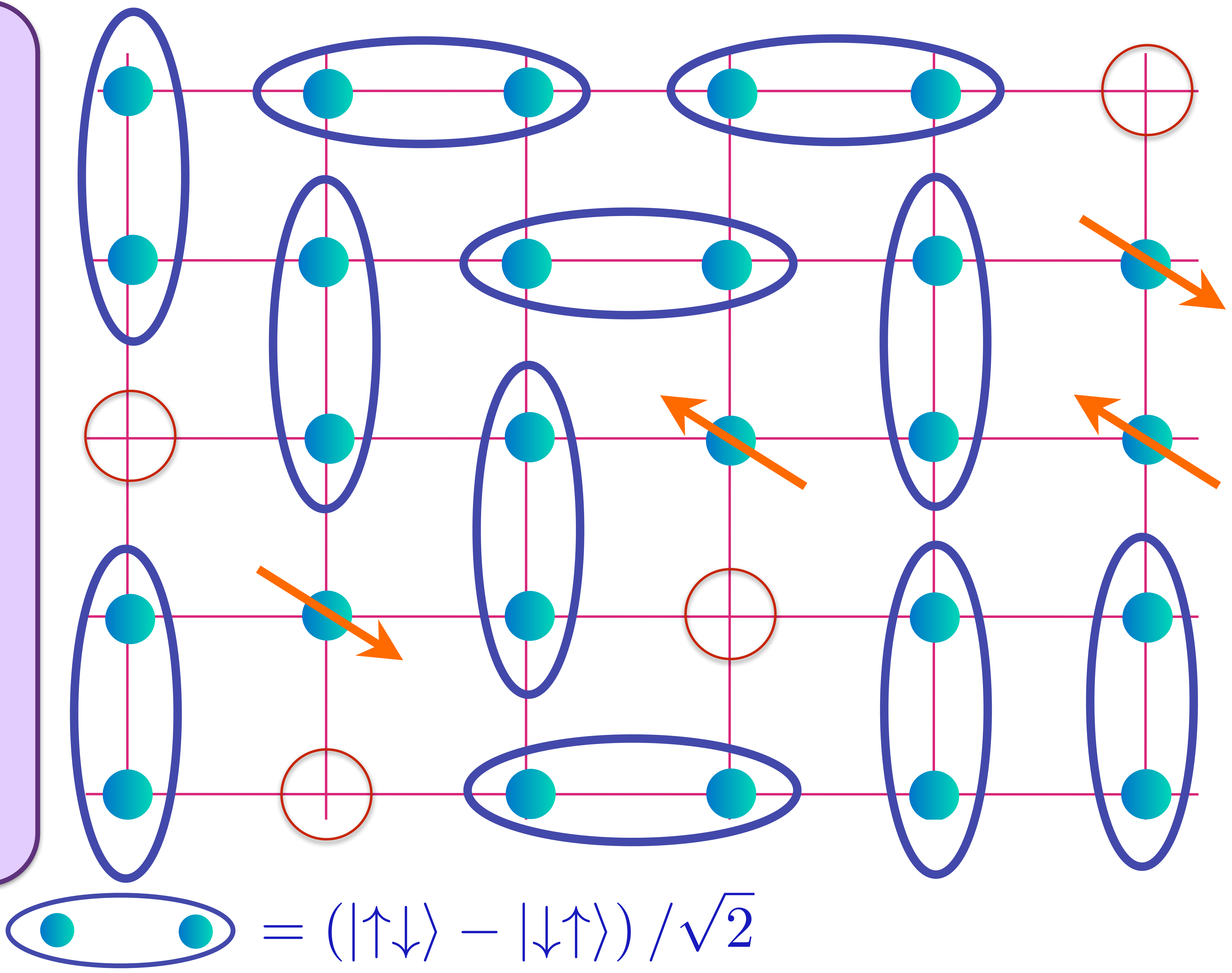
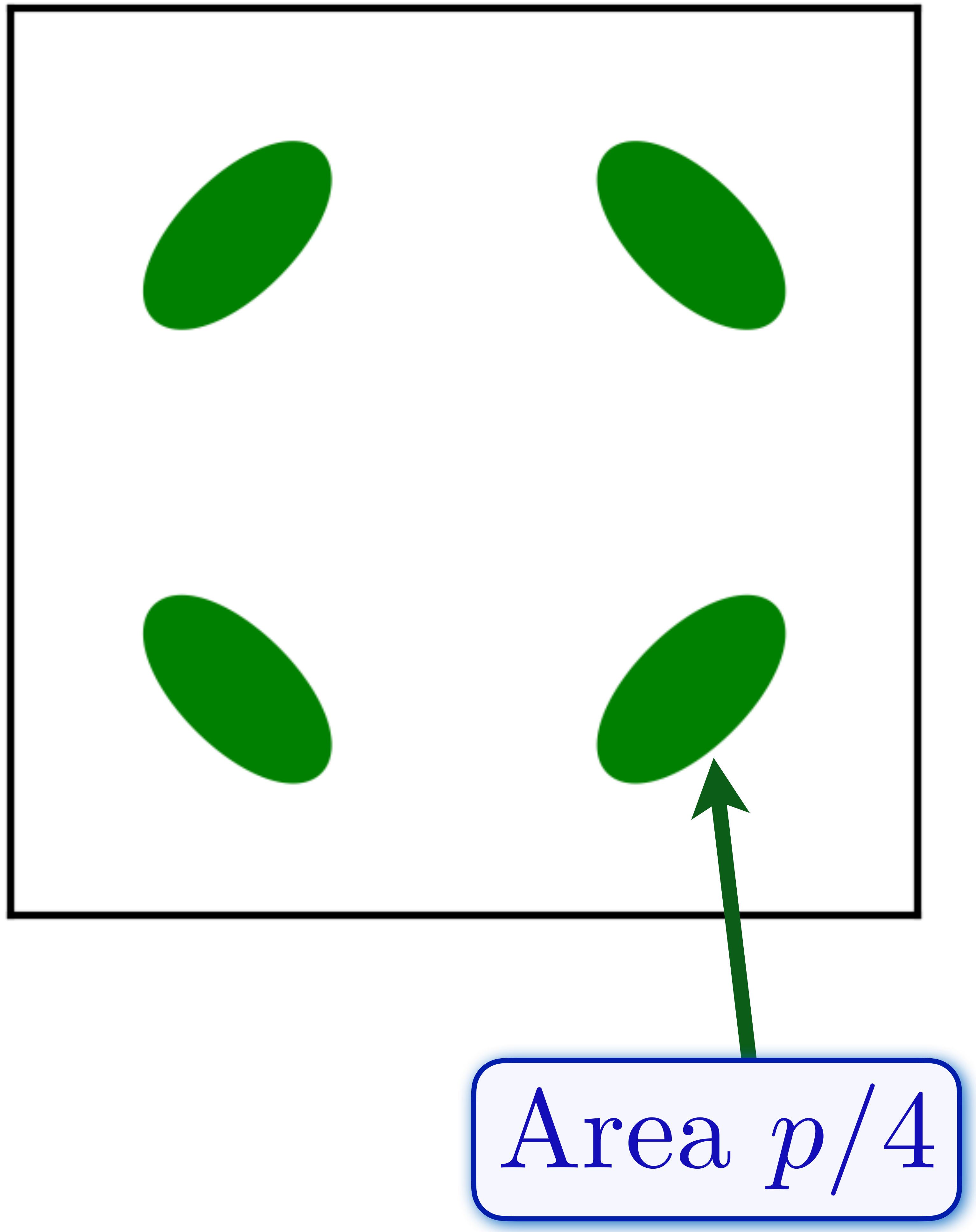


non-Luttinger area.
Spin liquid



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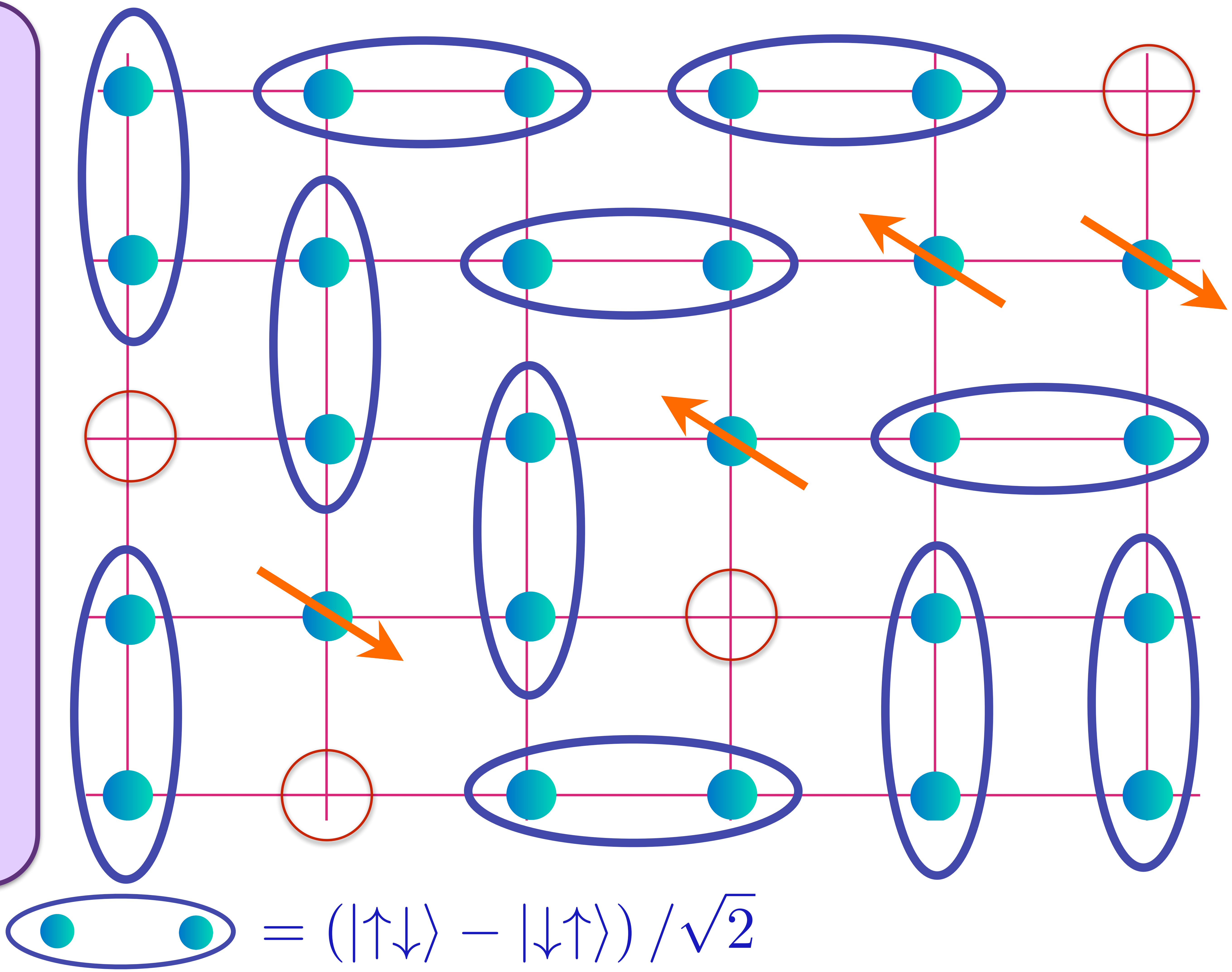
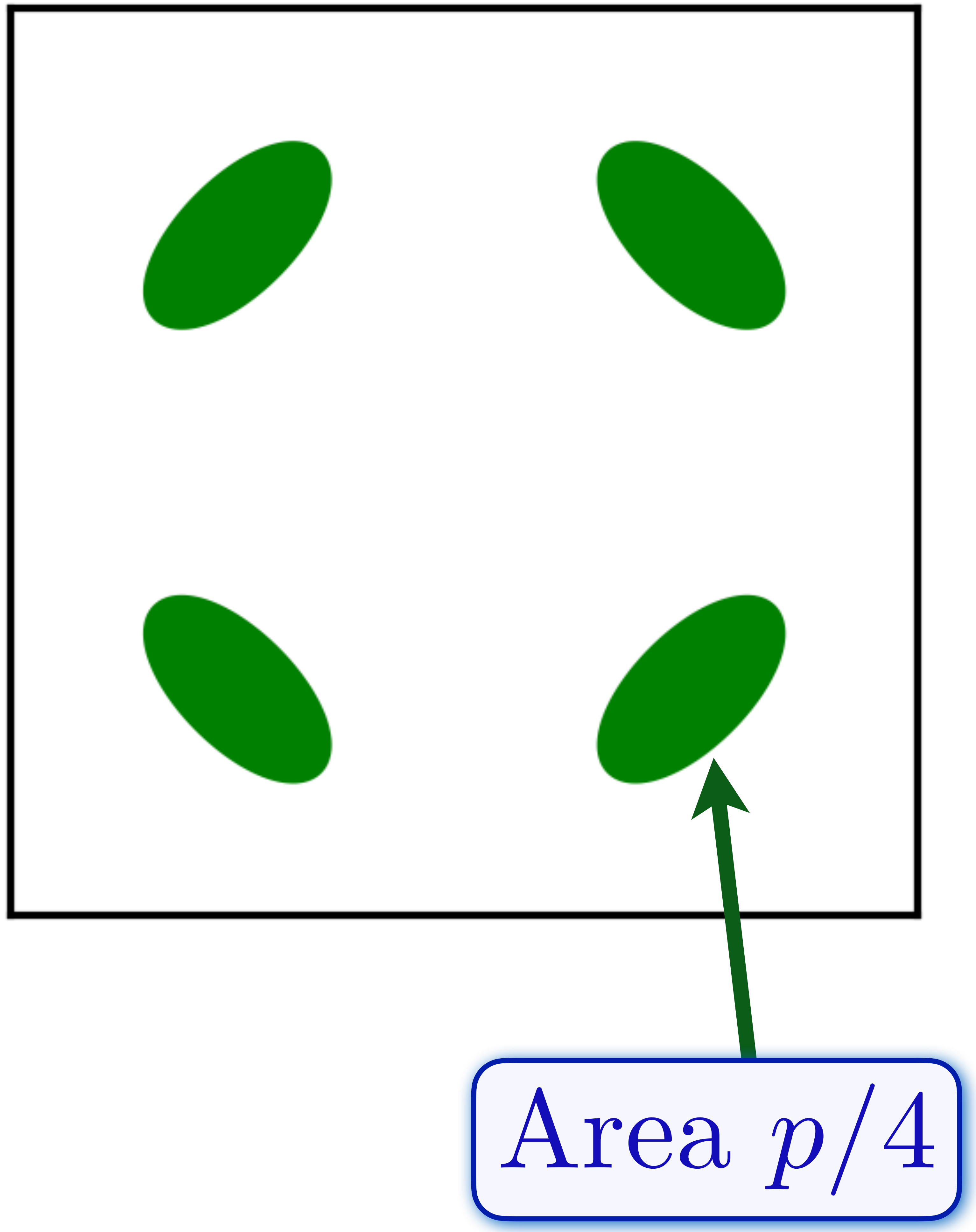


non-Luttinger area.
Spin liquid



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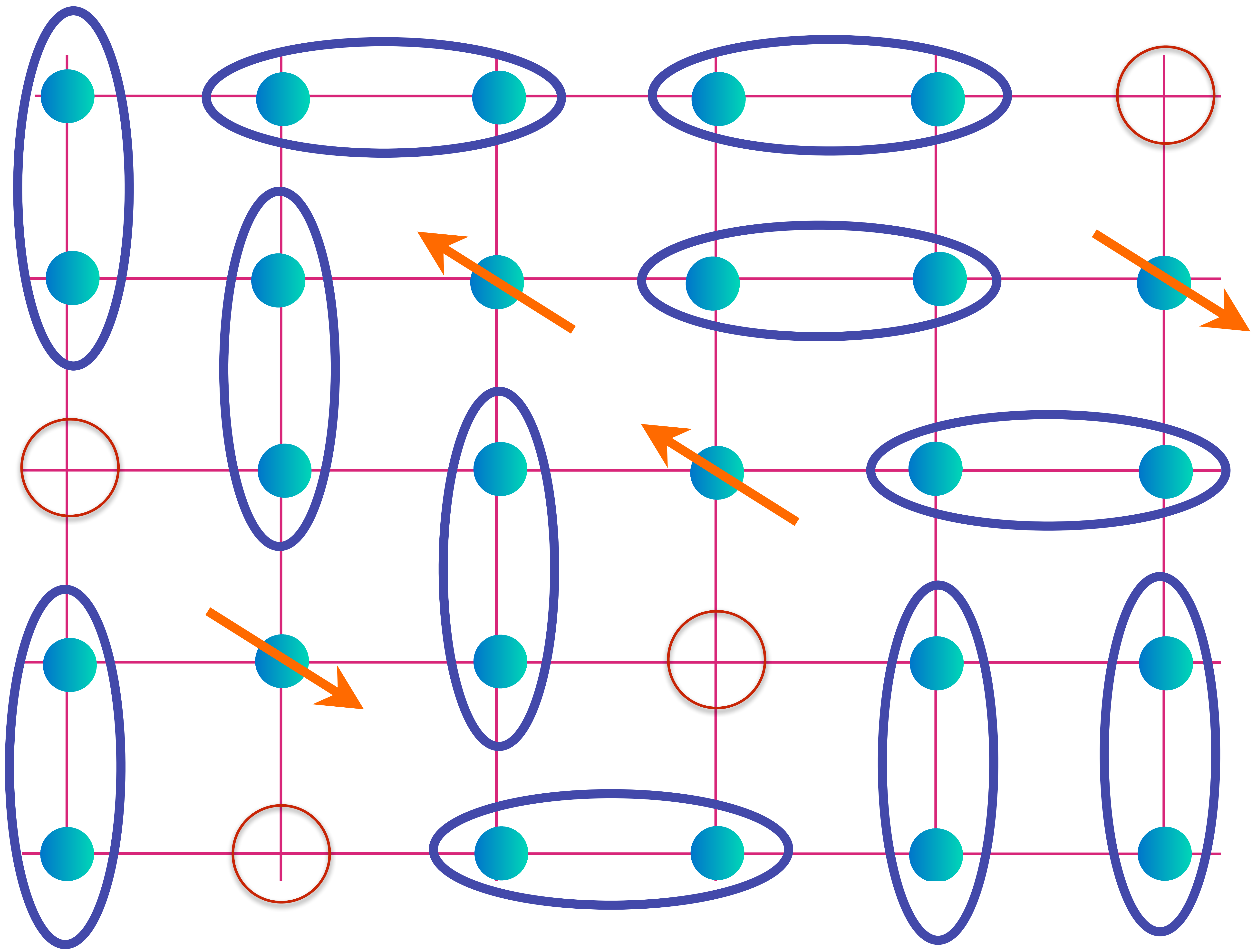
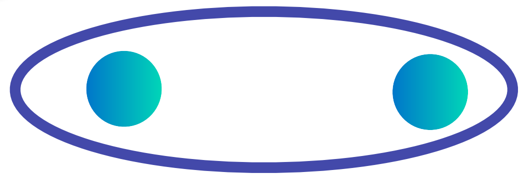
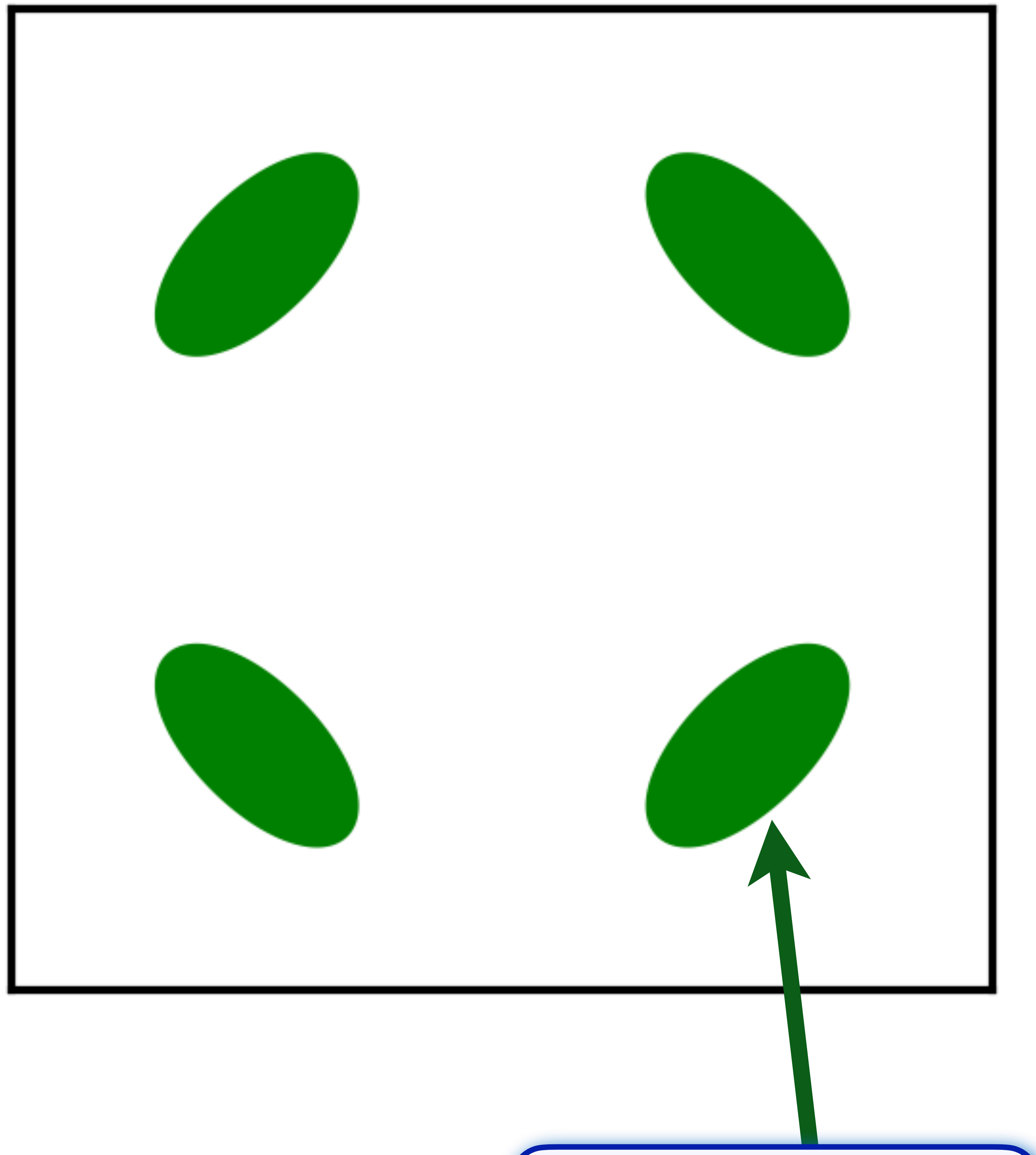


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

non-Luttinger area.
Spin liquid



Area $p/4$

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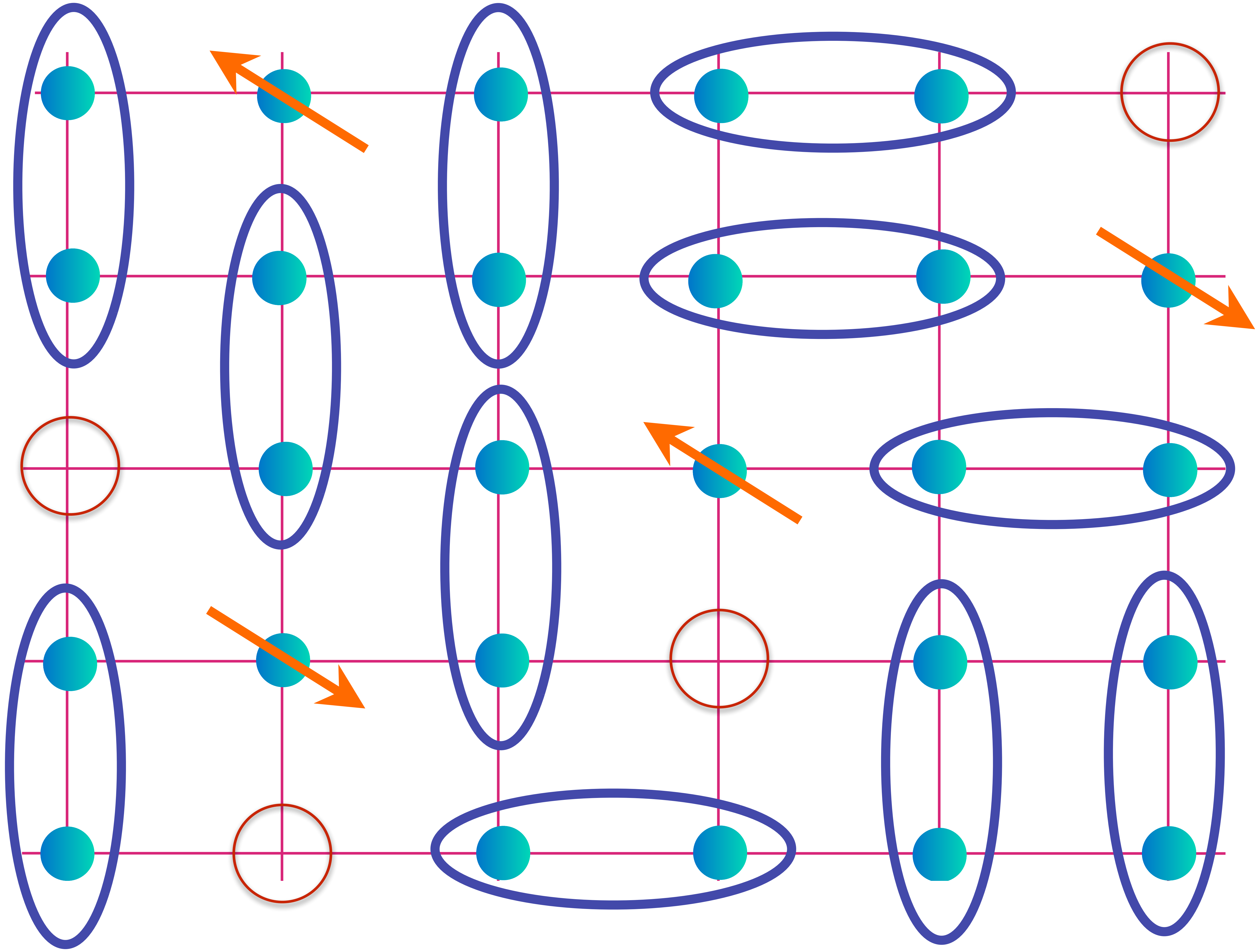

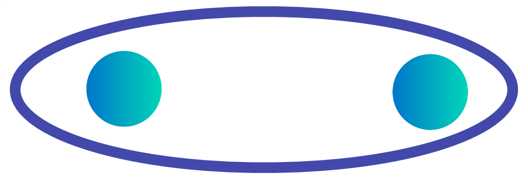 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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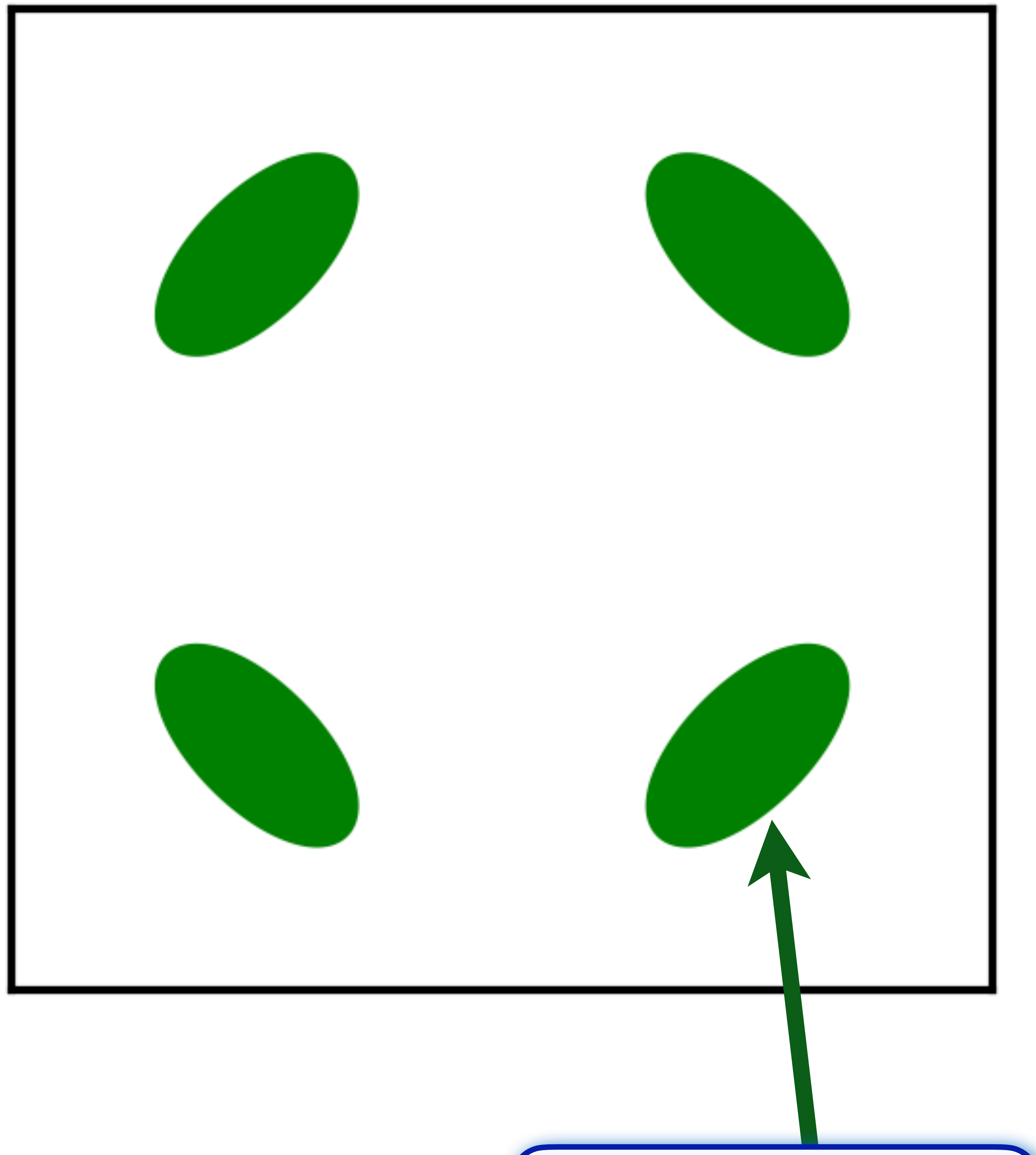
Spin liquid



Area $p/4$

Doping an insulating antiferromagnet with holes of density p

Holon metal

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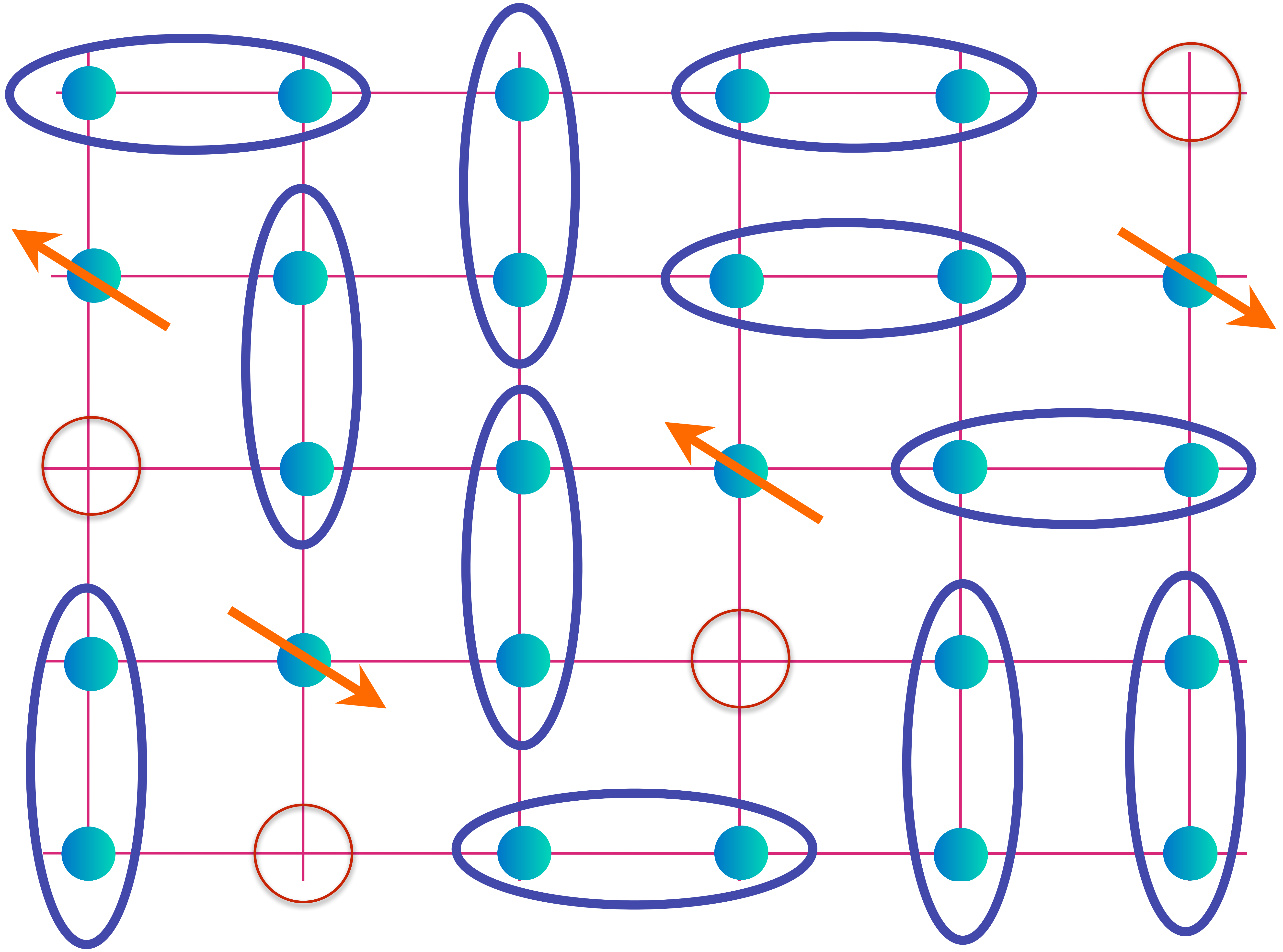
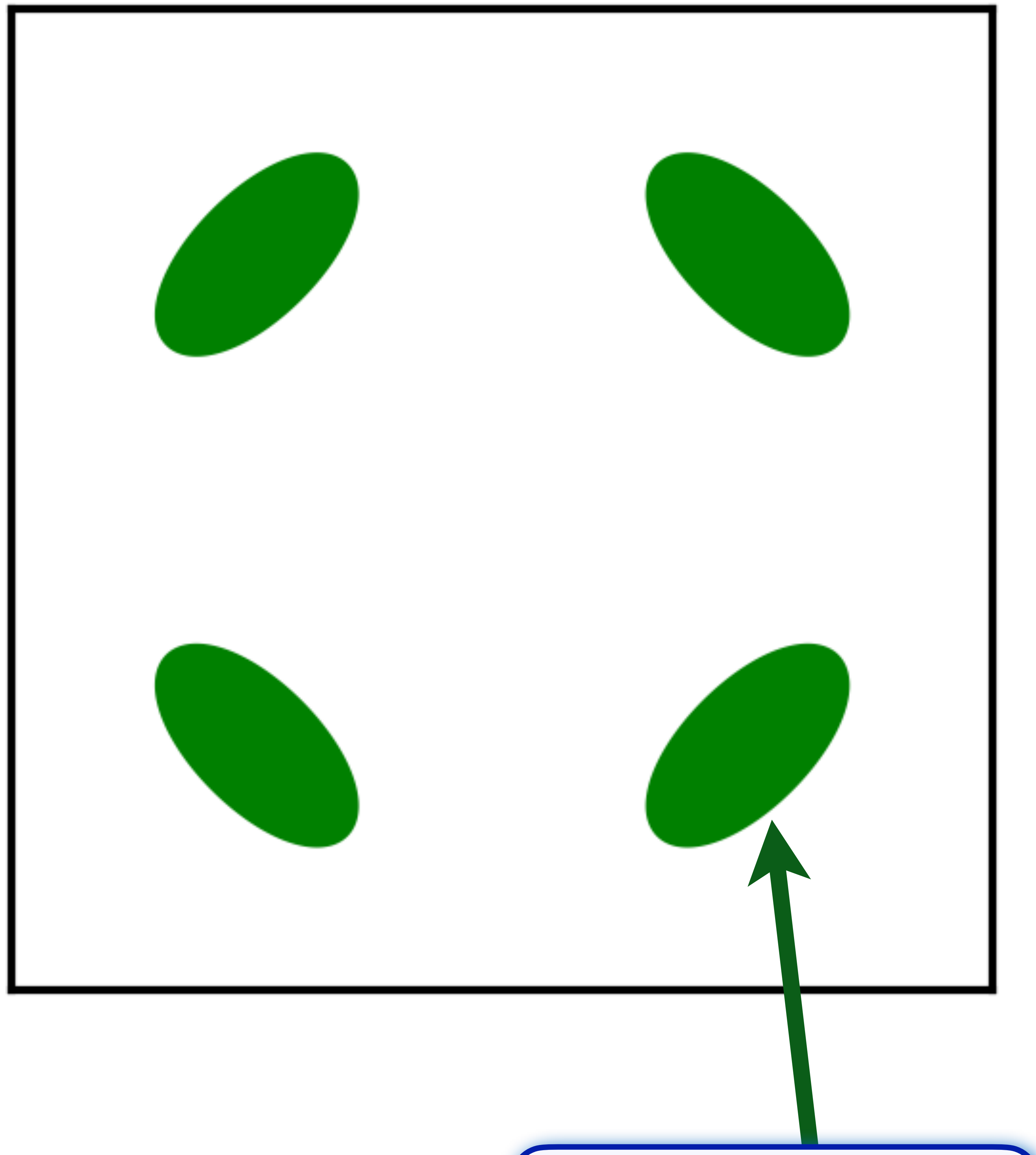


$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

non-Luttinger area.
Spin liquid



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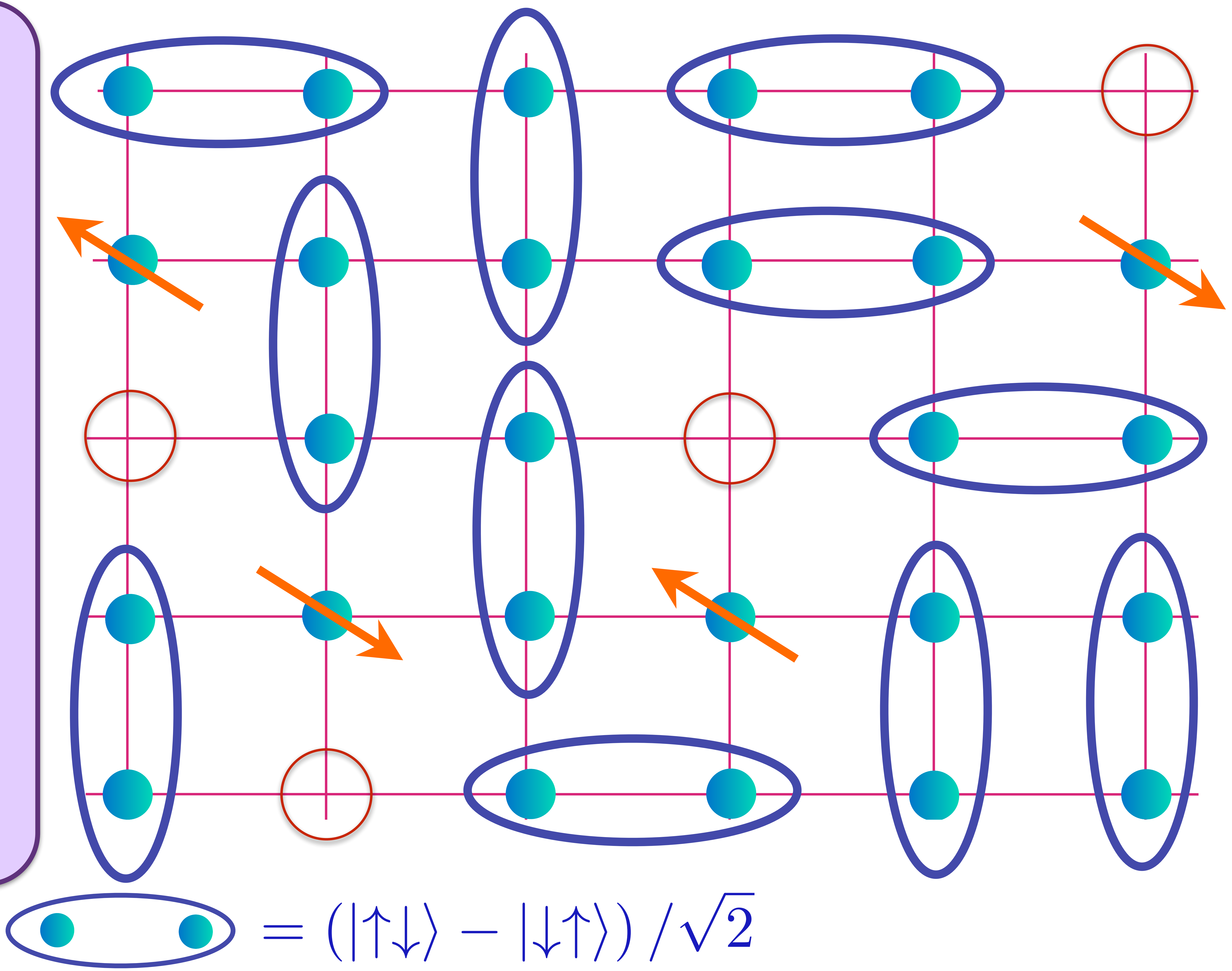
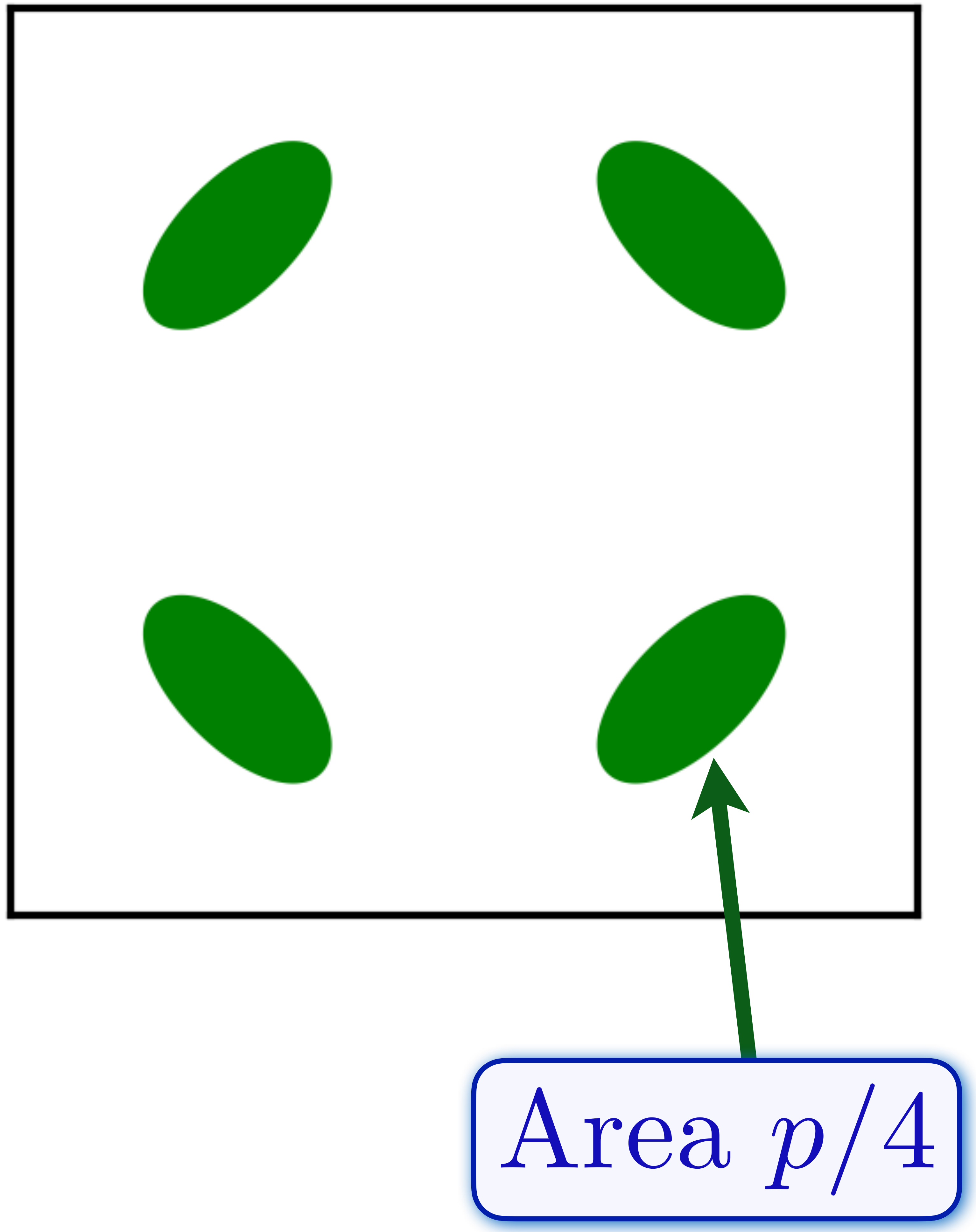


non-Luttinger area.
Spin liquid



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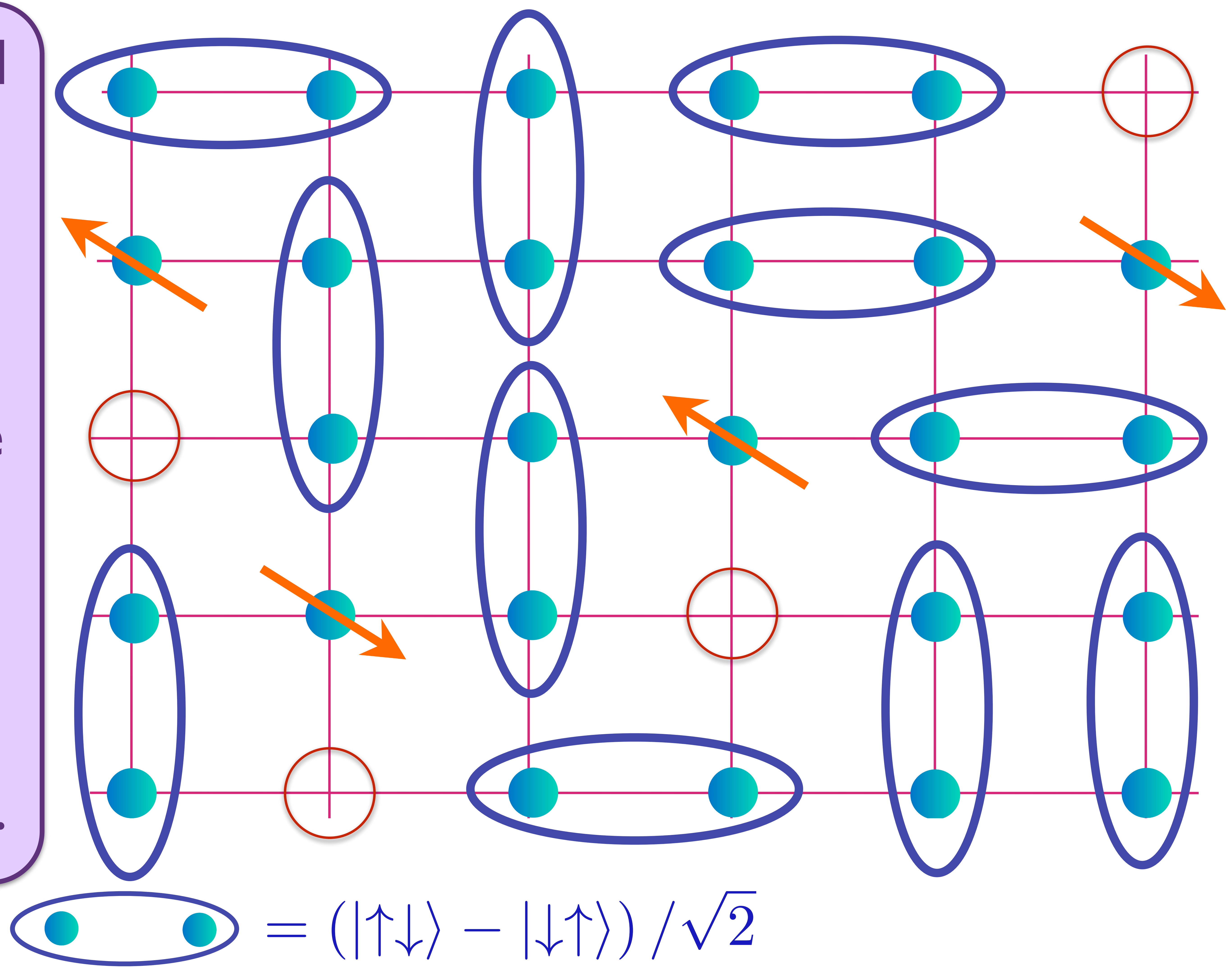
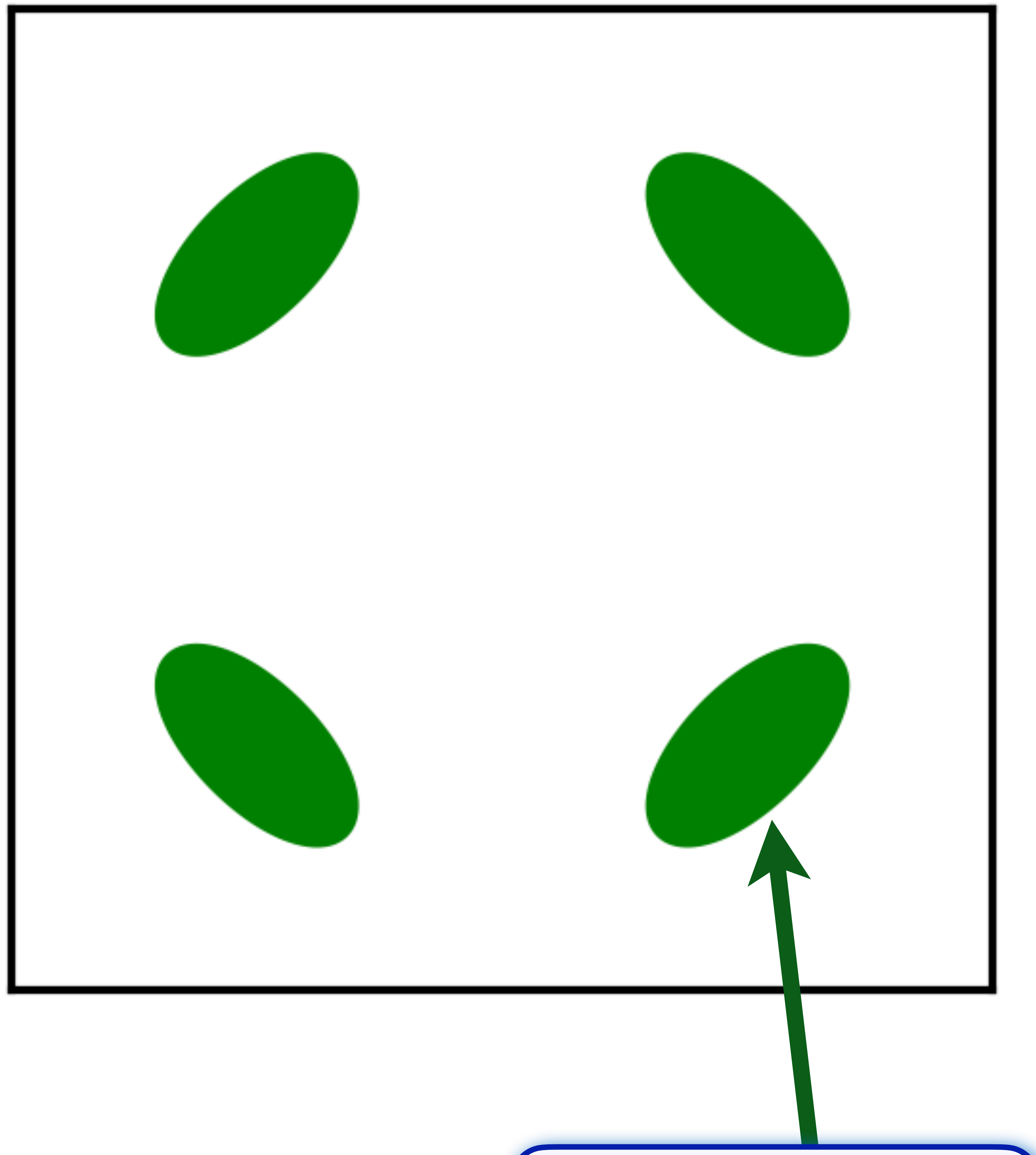


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

non-Luttinger area.
Spin liquid

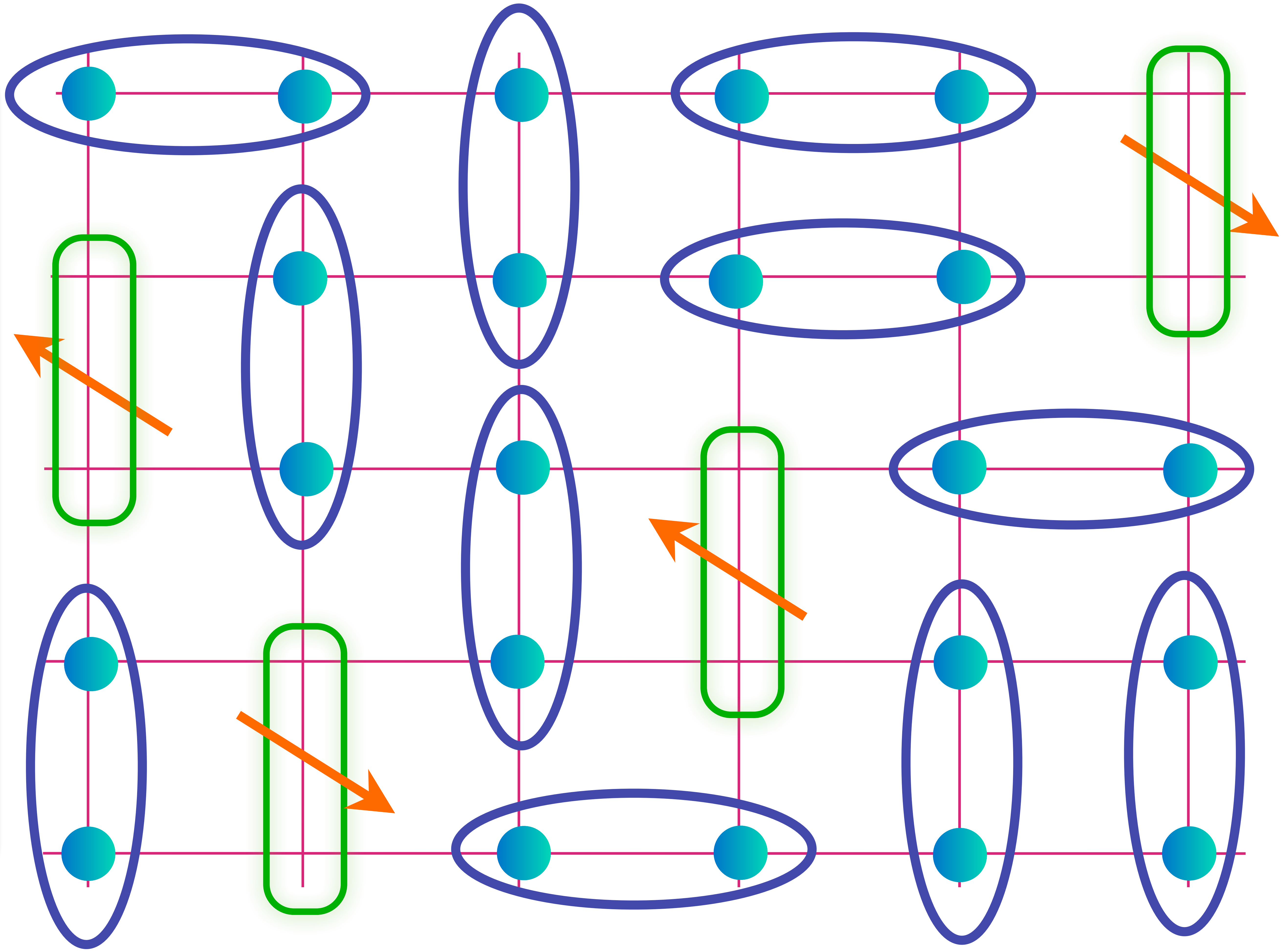


Area $p/4$

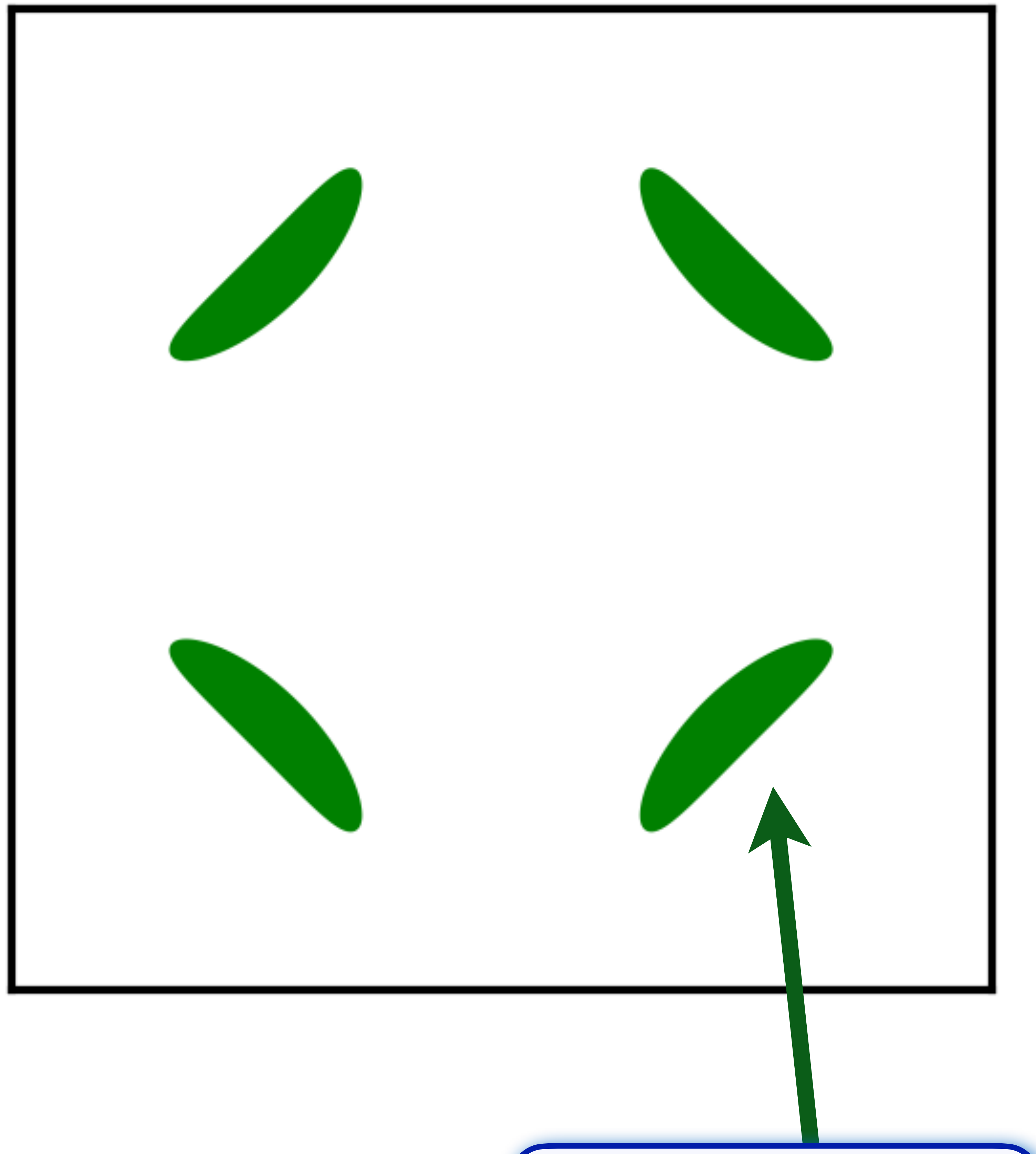
Doping an insulating antiferromagnet with holes of density p

FL*

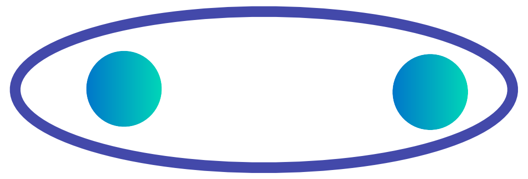
Each green "dimer" is a bound state (a "magnetic polaron") of a vacancy and a free spin

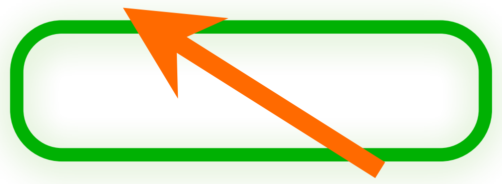


non-Luttinger area.
Spin liquid



Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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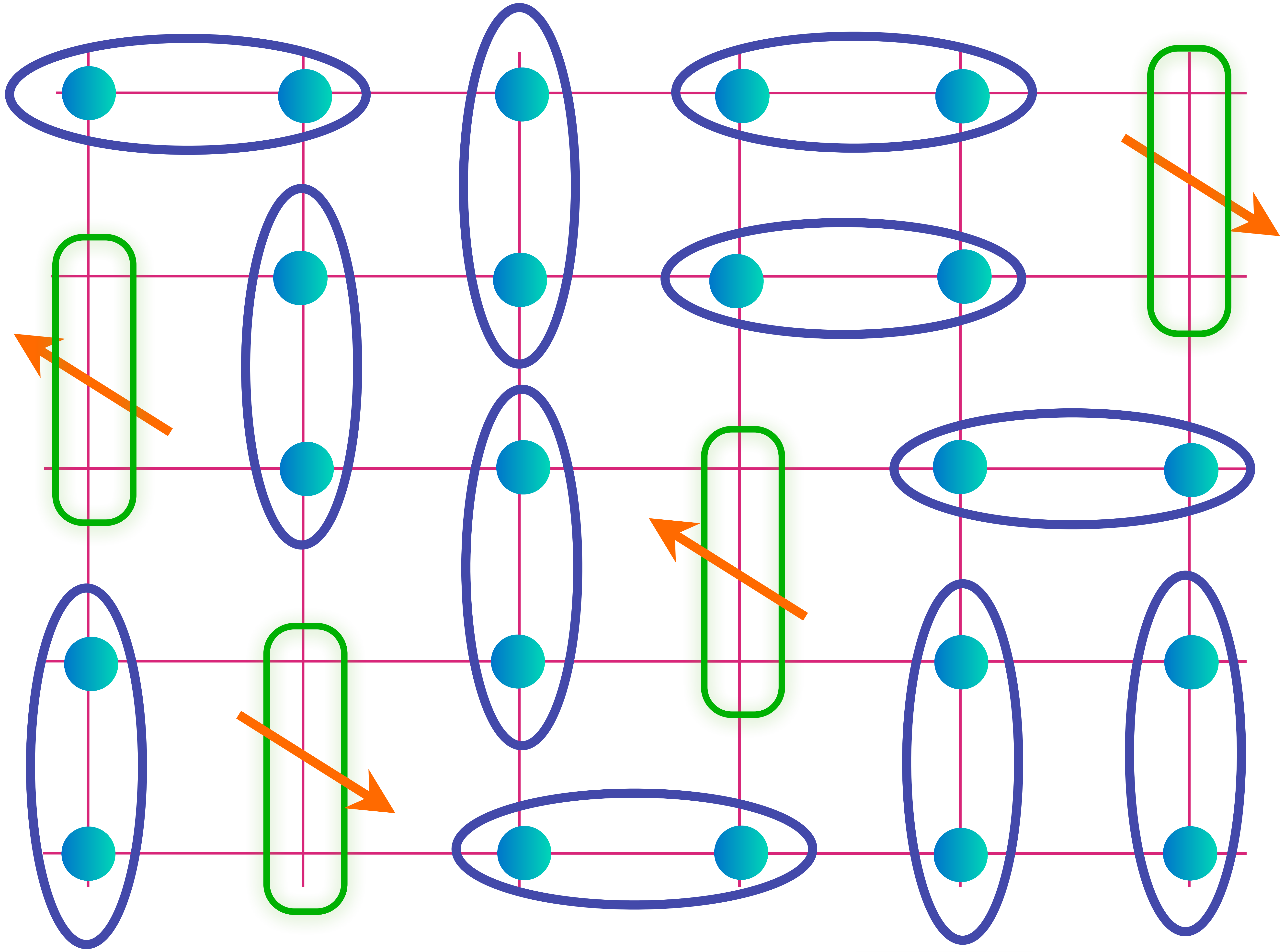
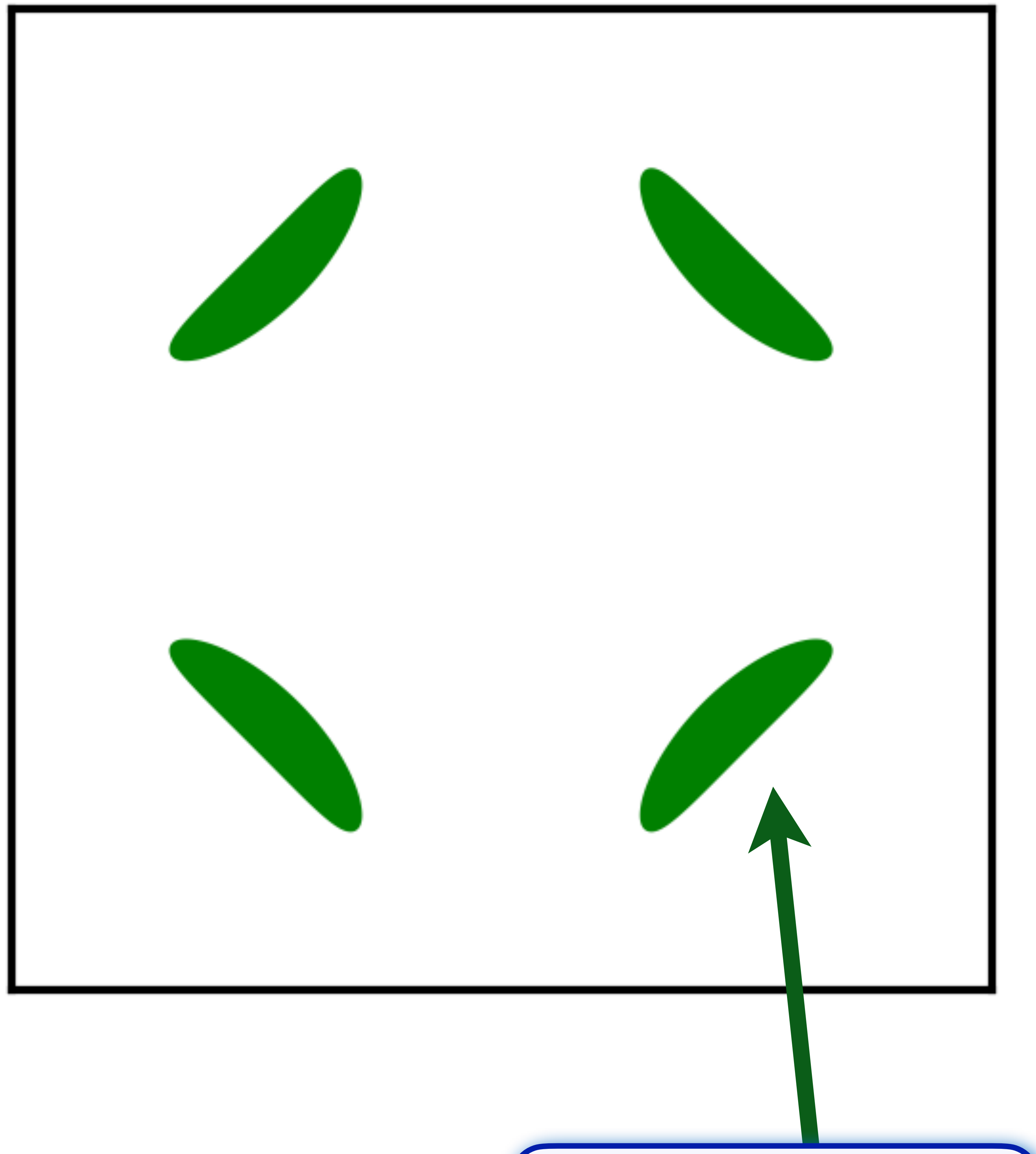
FL*

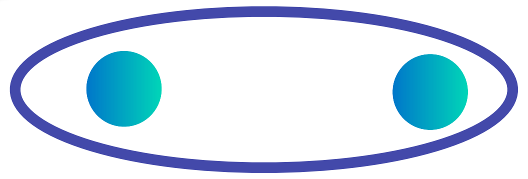
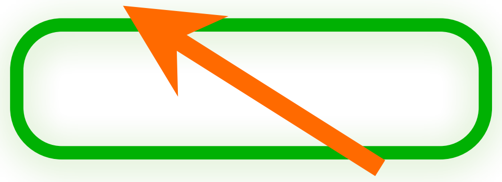
Metal with density p of spin-1/2, charge +e "holes" (or "magnetic polarons") and charge 0 spin-1/2 "spinons".



non-Luttinger area.
Spin liquid



Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$  $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
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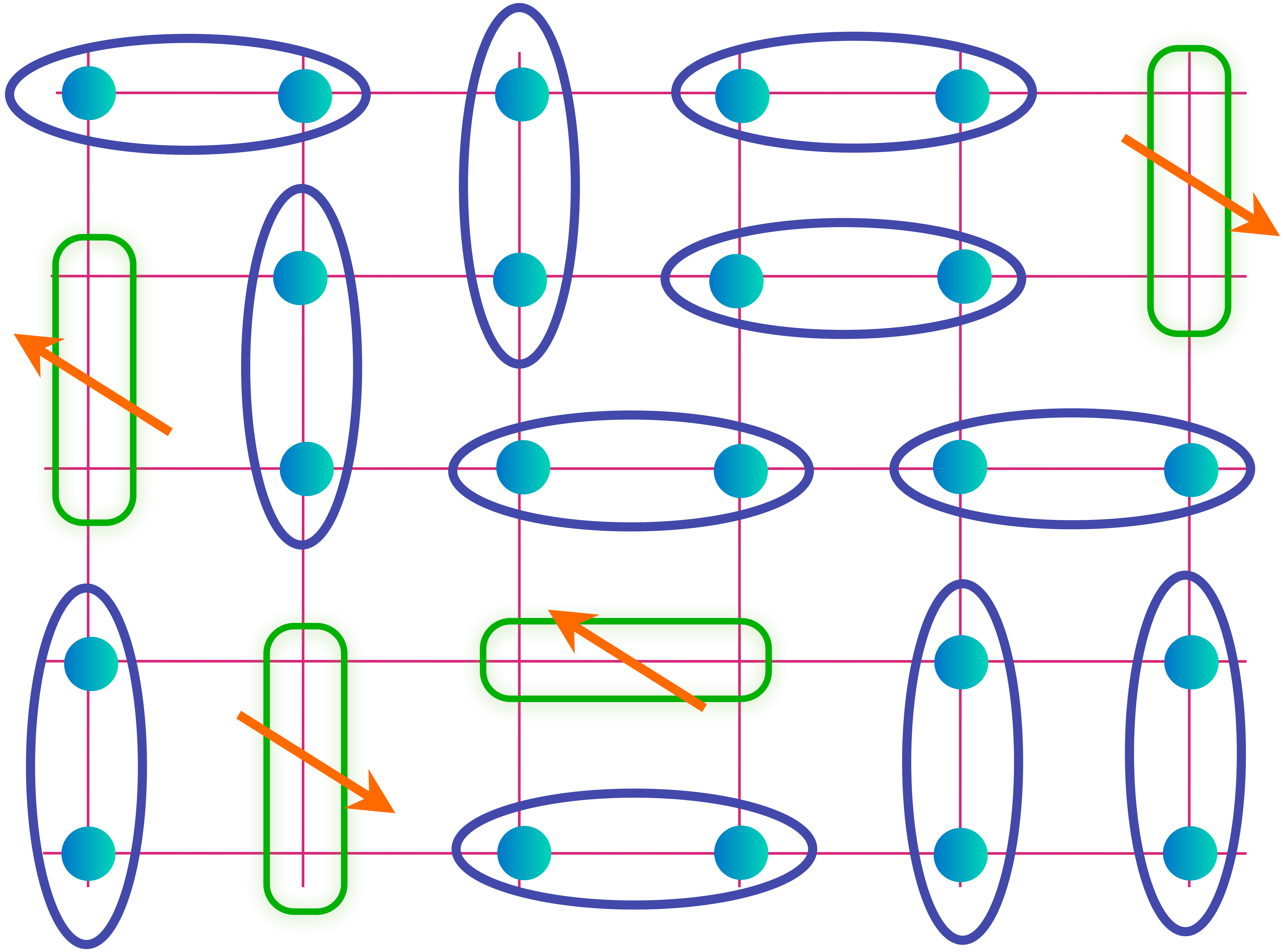
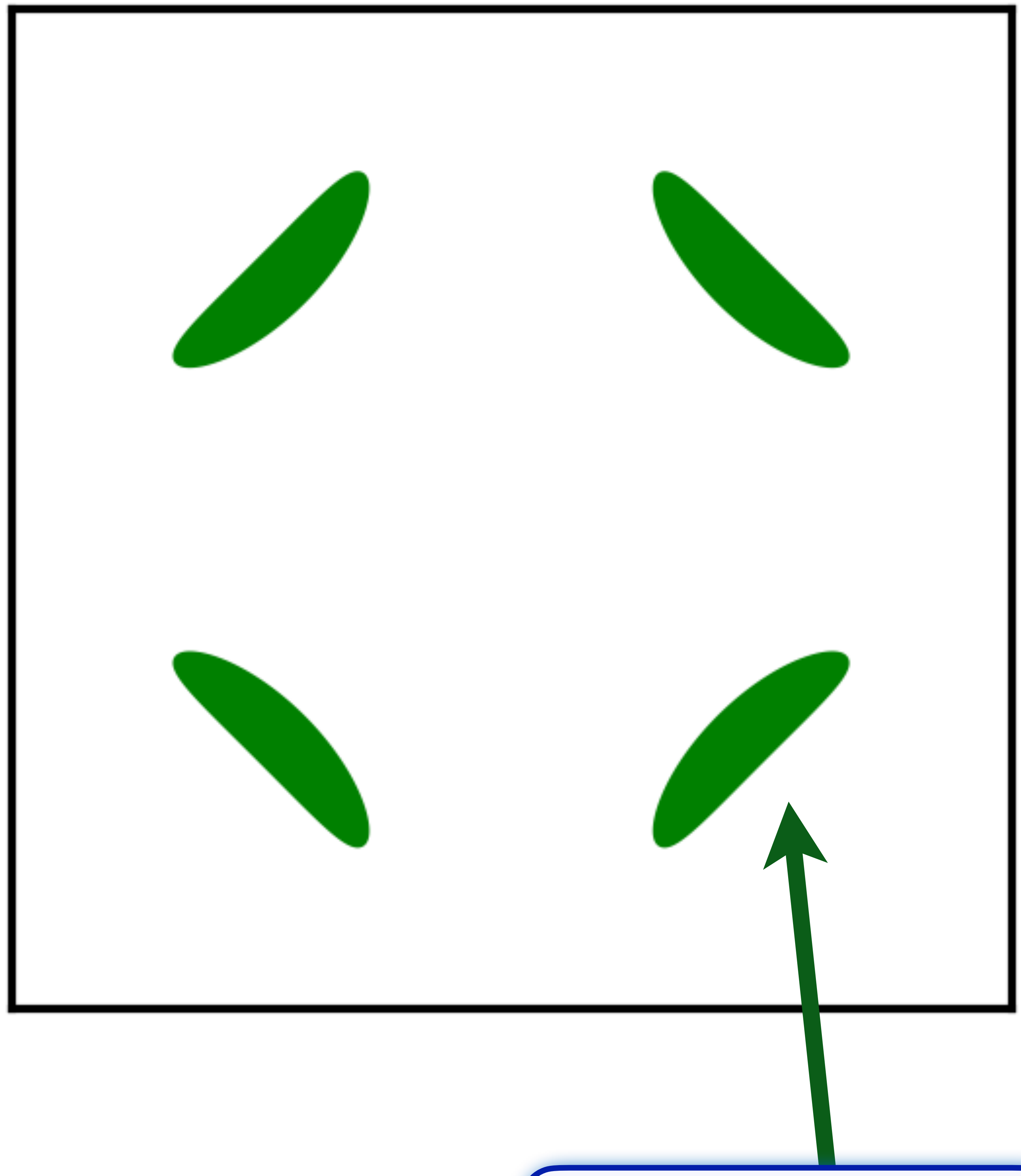
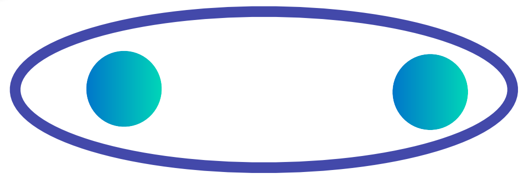
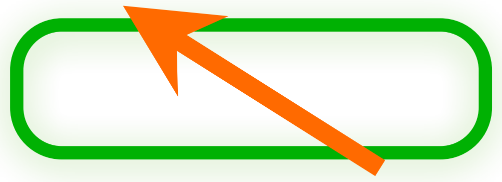


non-Luttinger area.
Spin liquid



Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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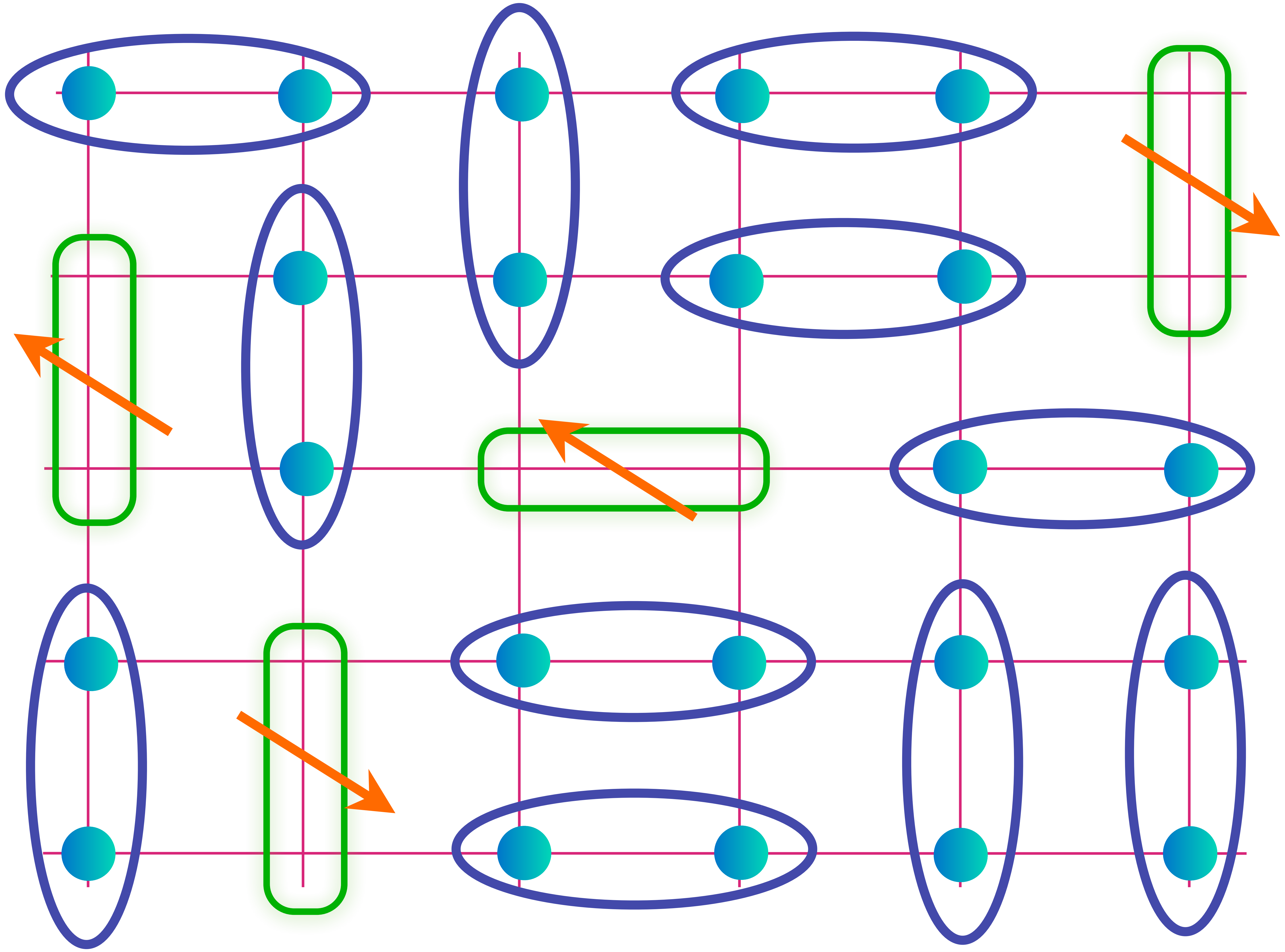
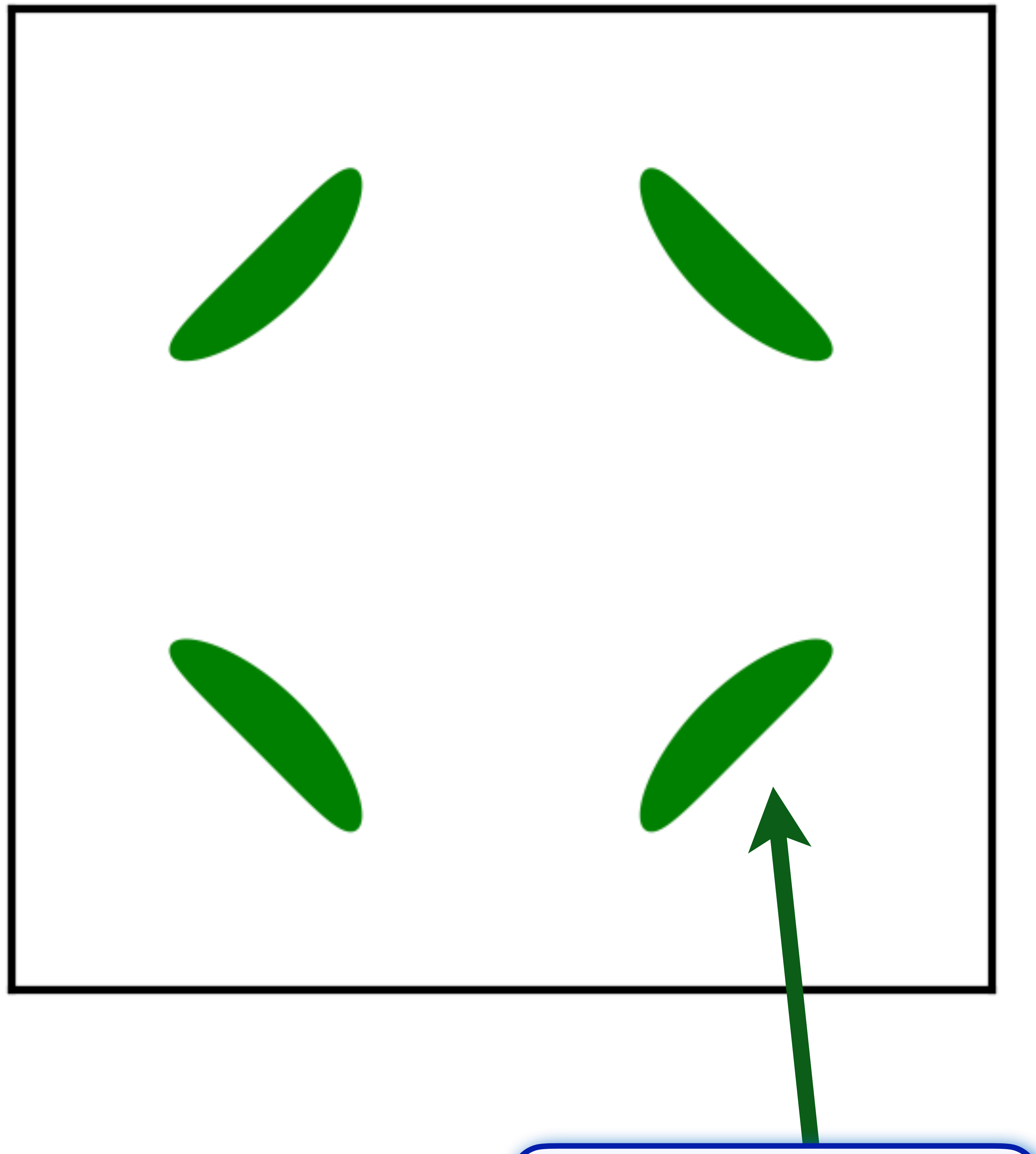


non-Luttinger area.
Spin liquid



Area $p/8$

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

$$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$$

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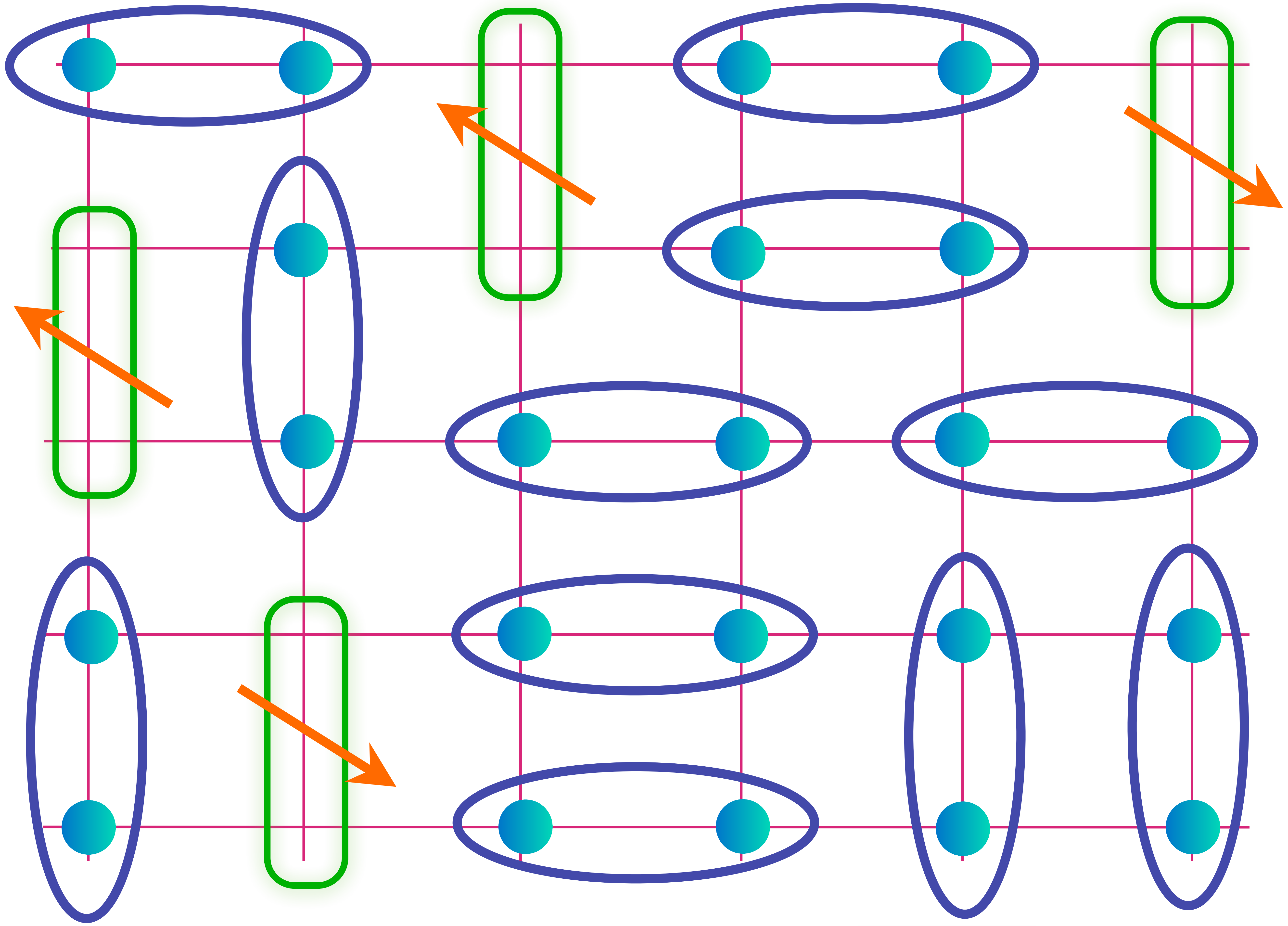
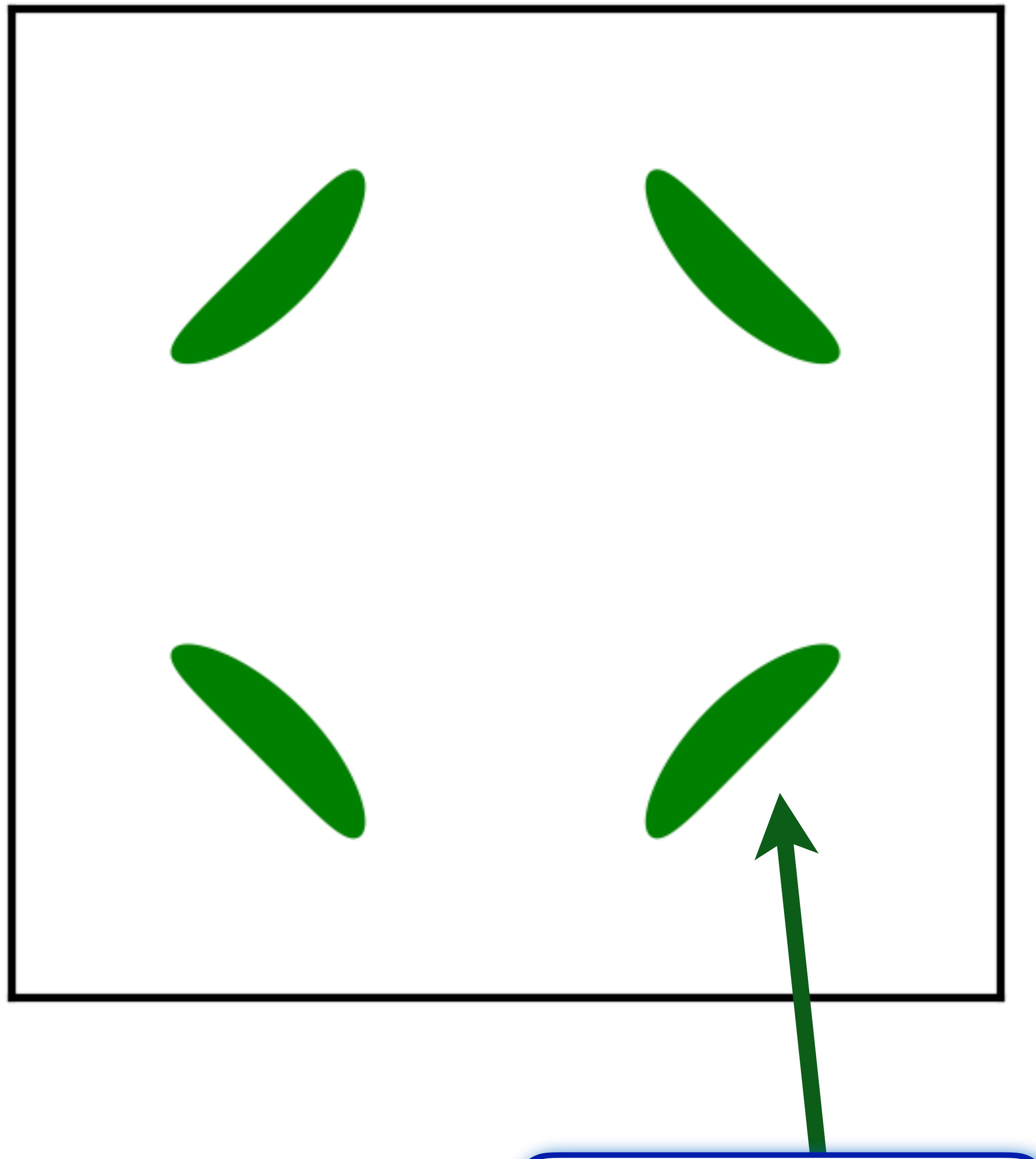
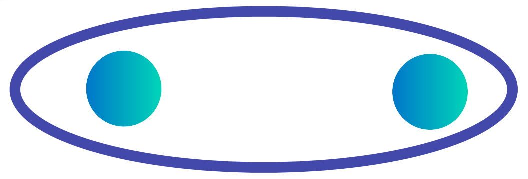
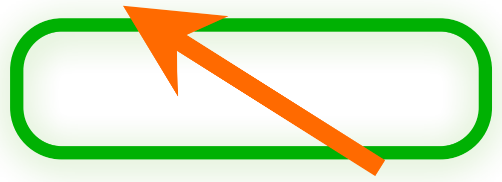


non-Luttinger area.
Spin liquid



Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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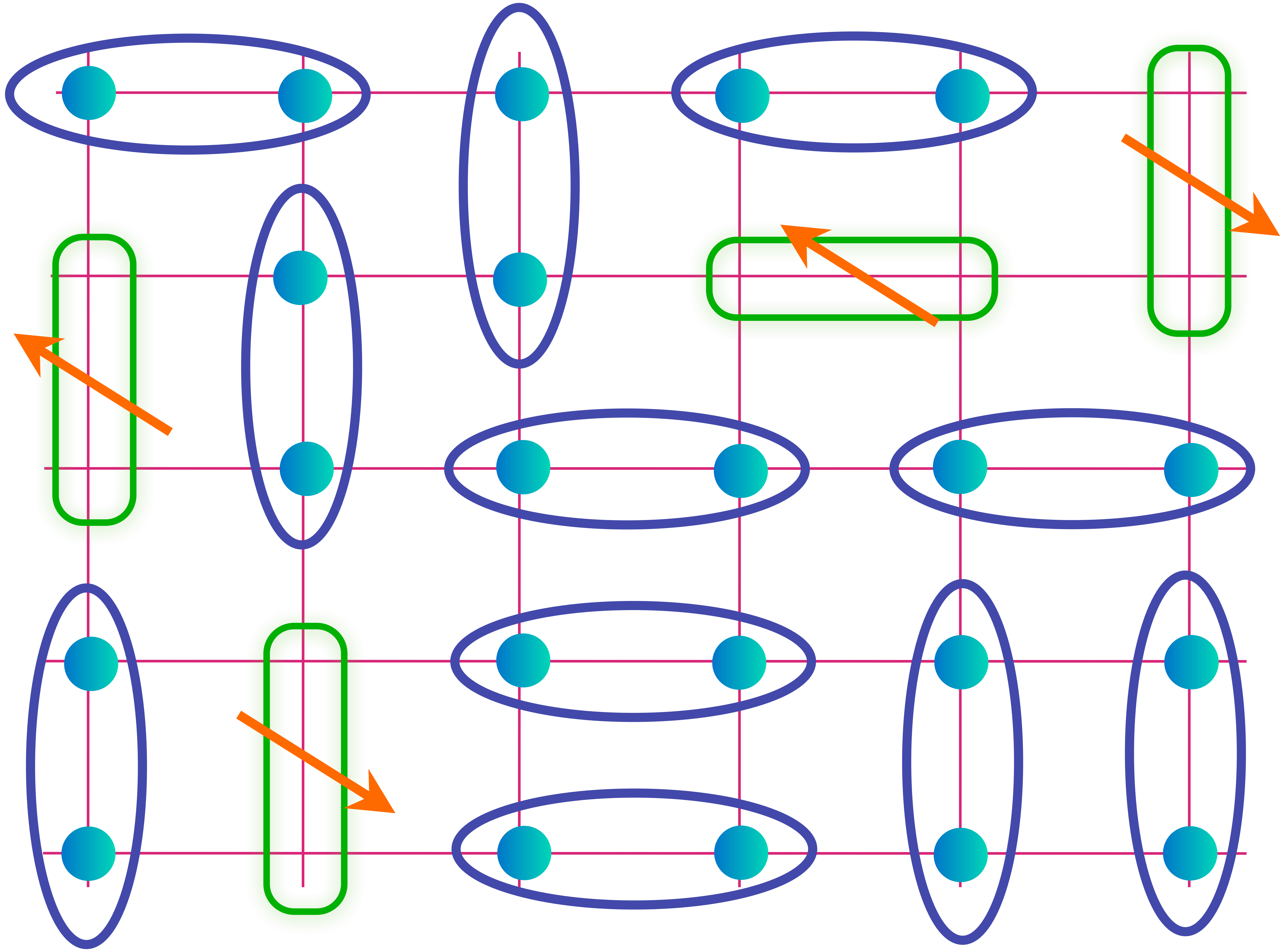
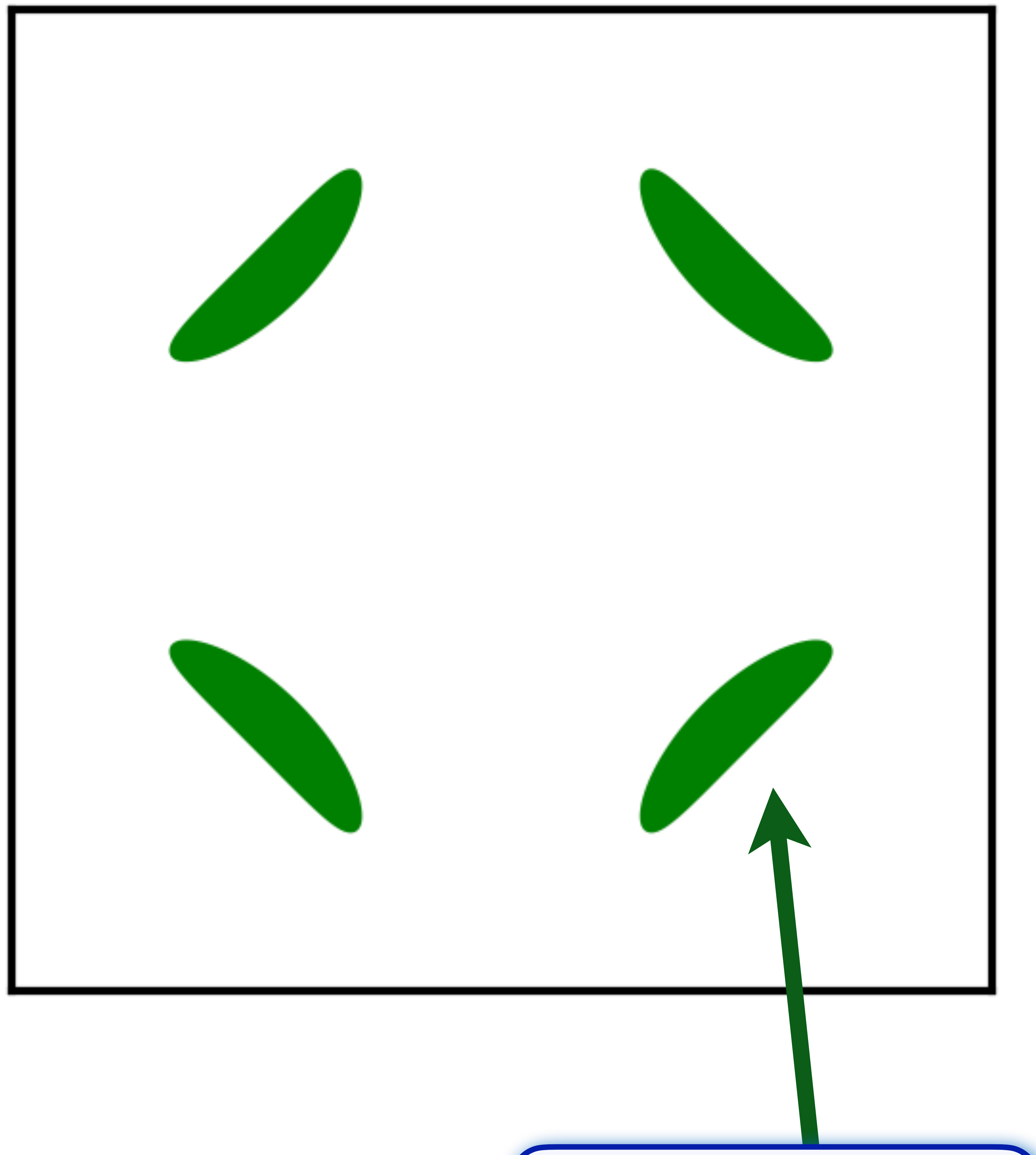
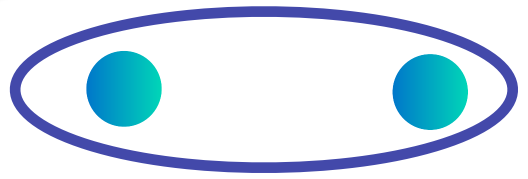
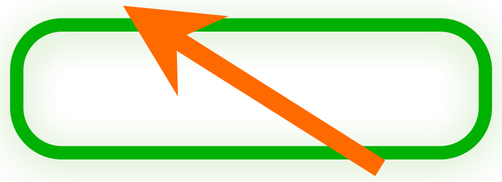


non-Luttinger area.
Spin liquid



Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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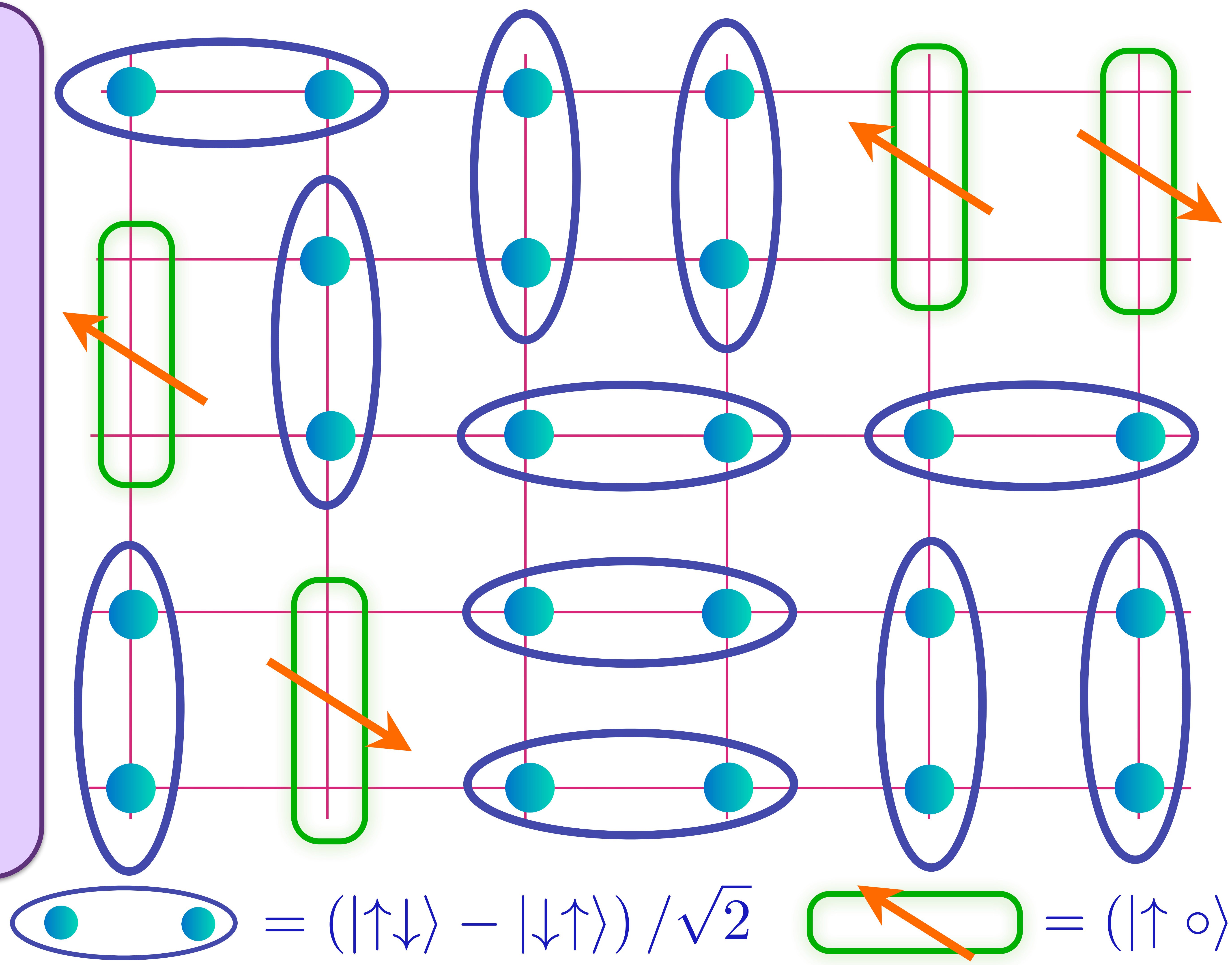

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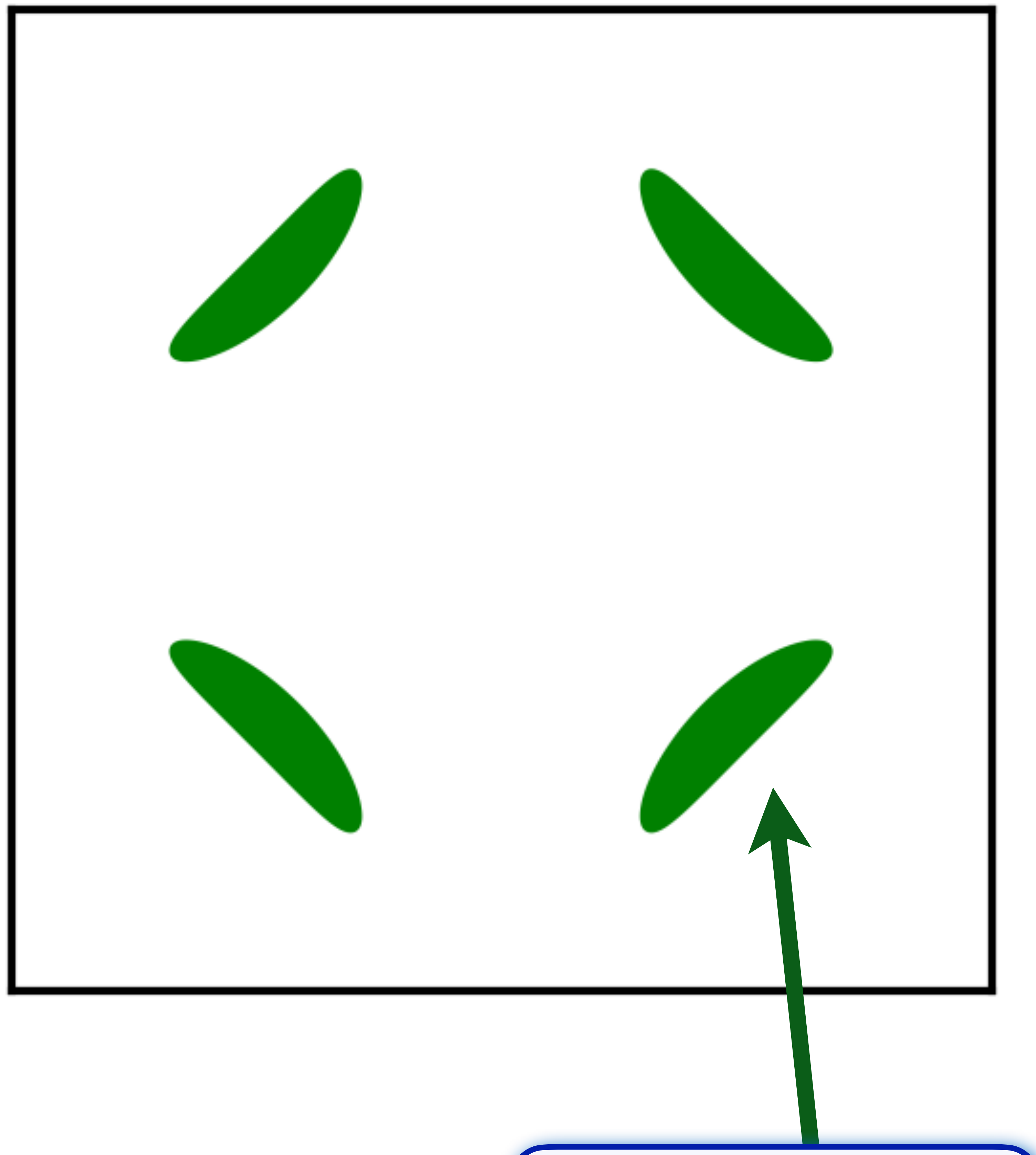
Spin liquid



Area $p/8$

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ $\quad$ $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

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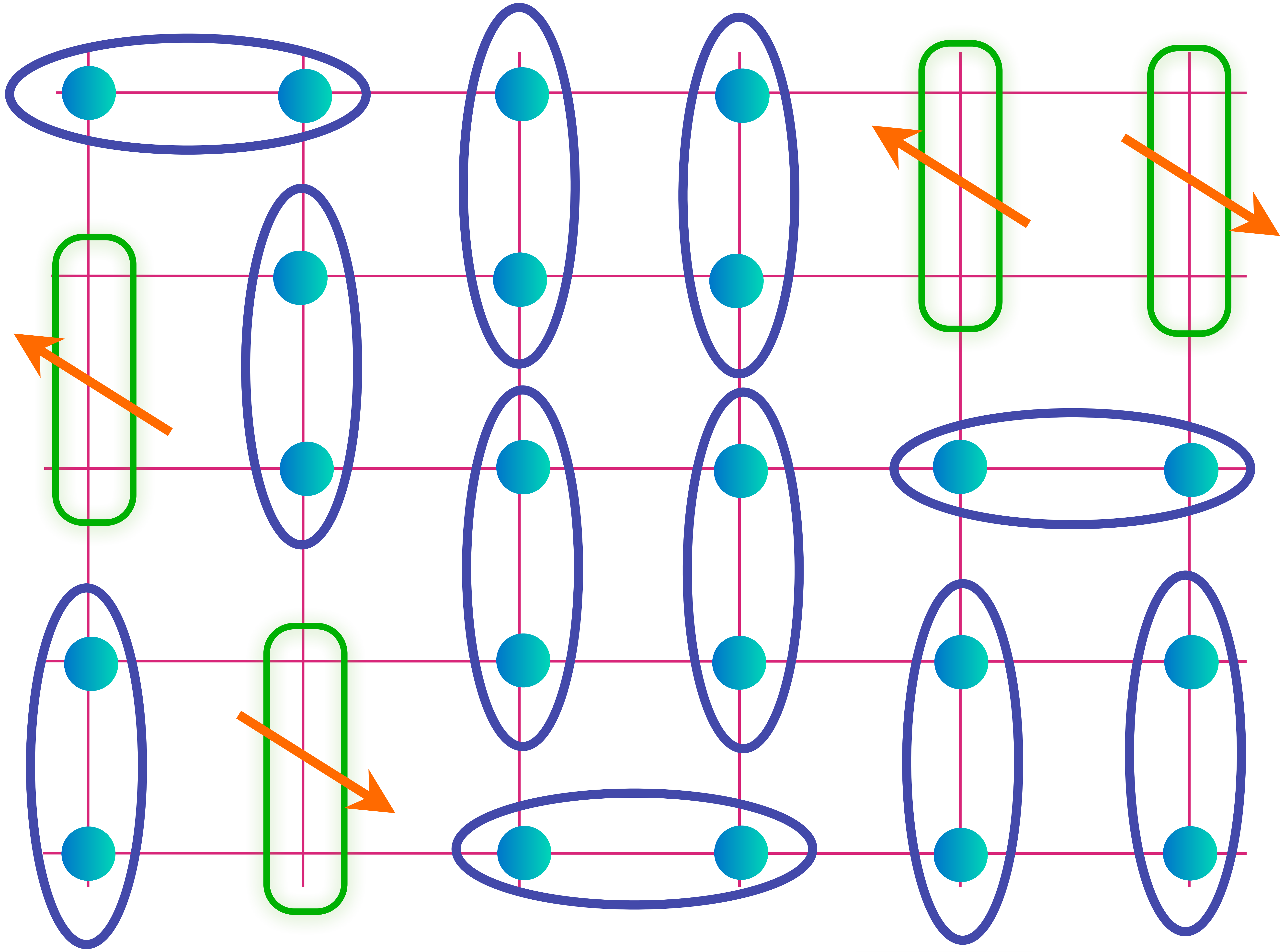

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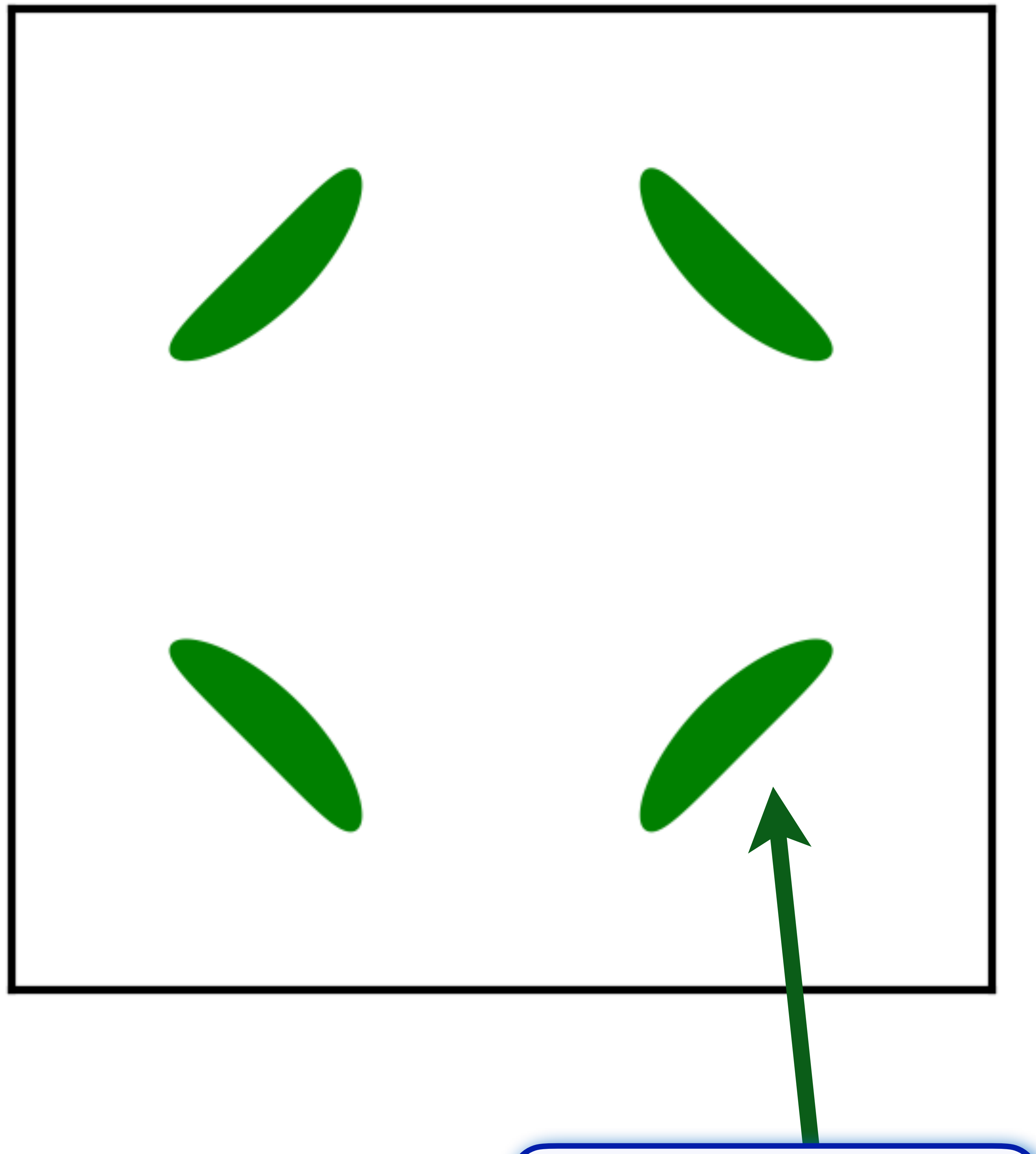

Area $p/8$

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

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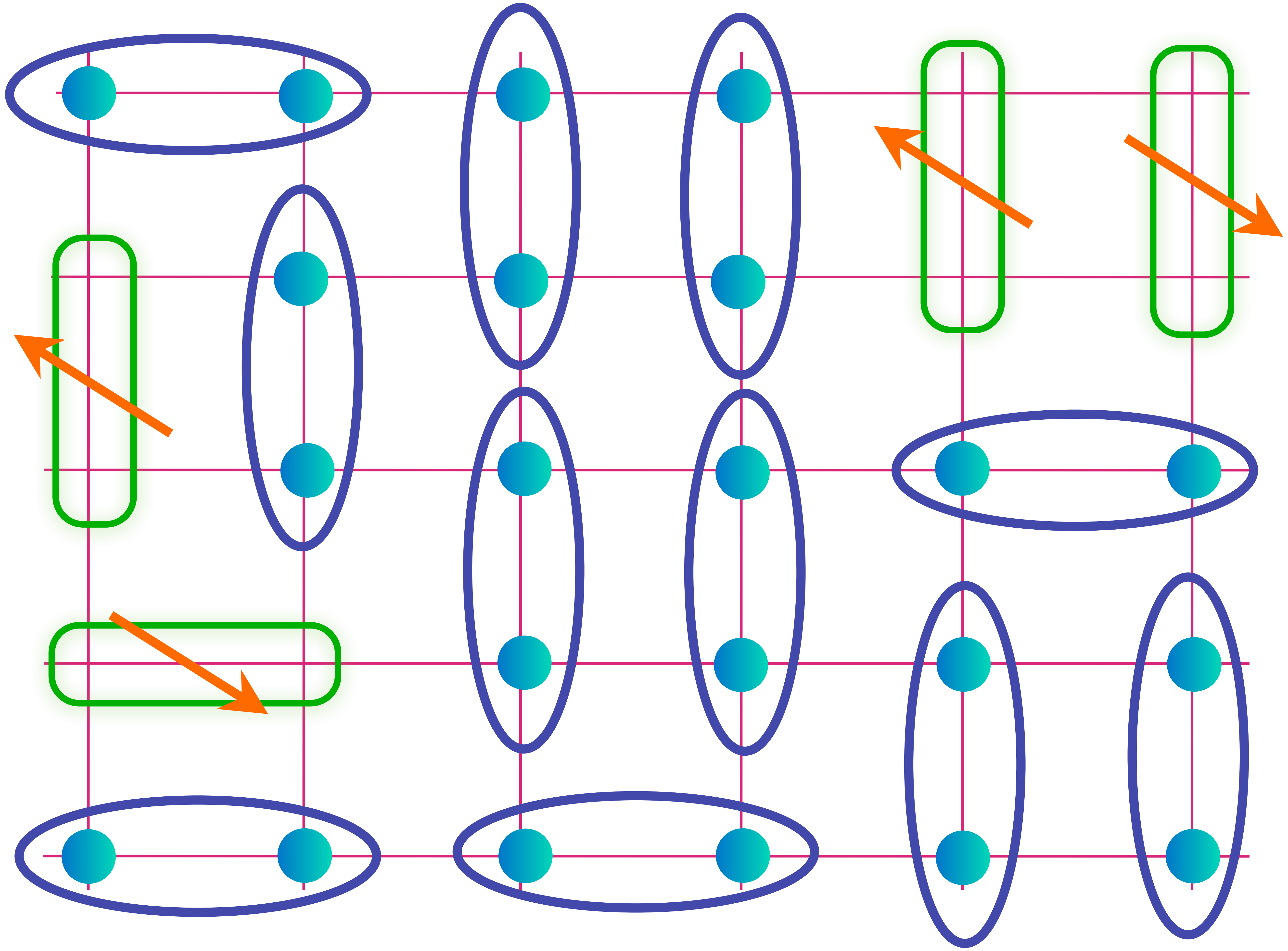

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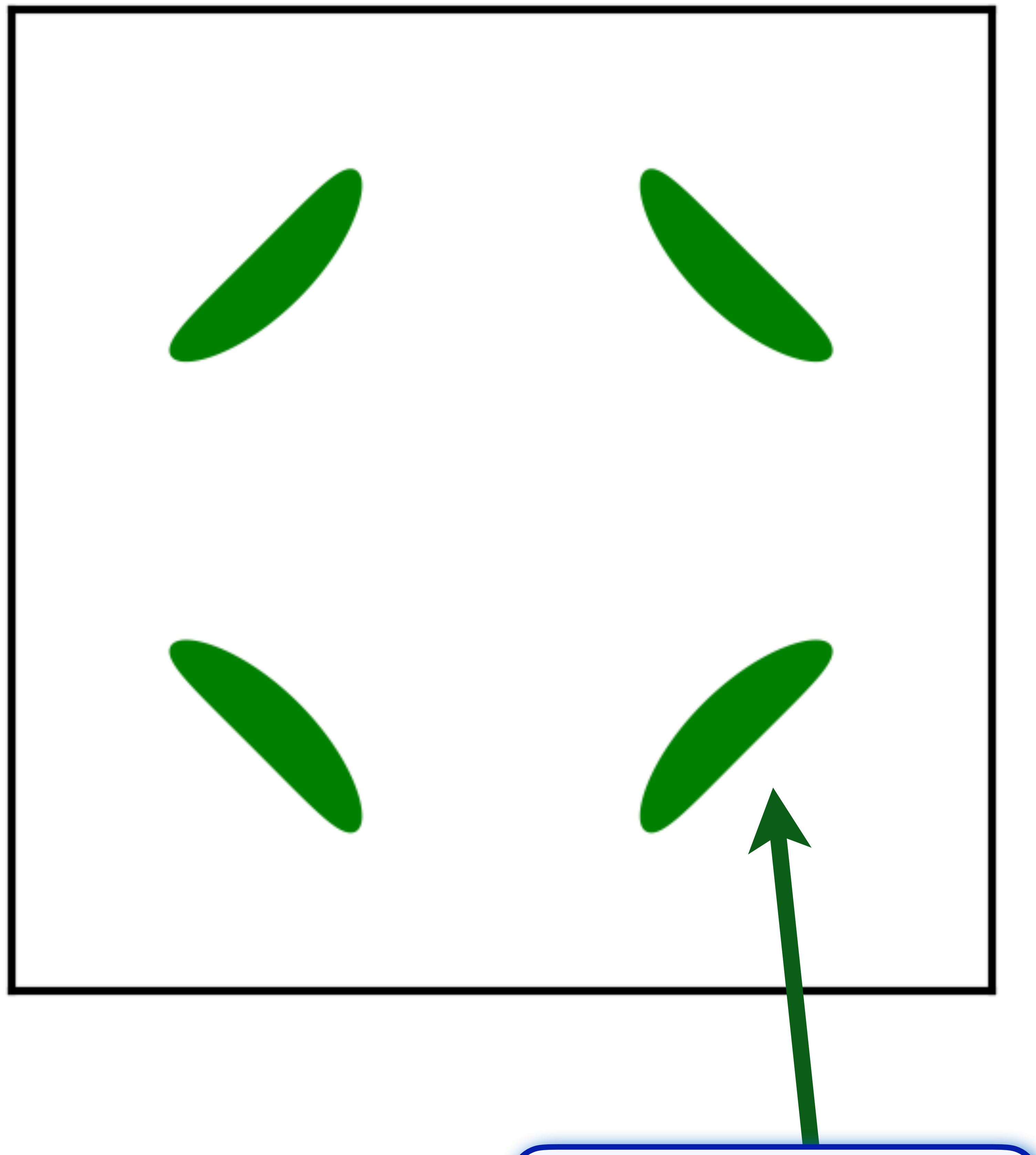
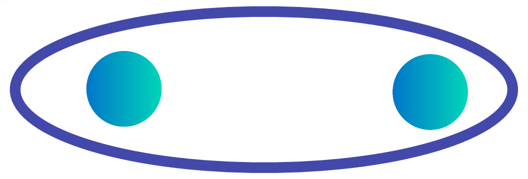
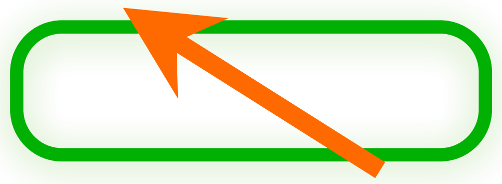
Spin liquid



Area $p/8$

 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$  $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

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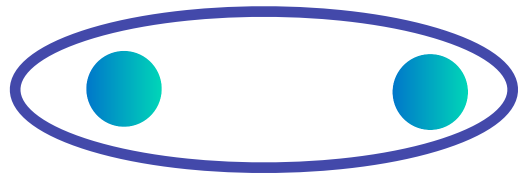
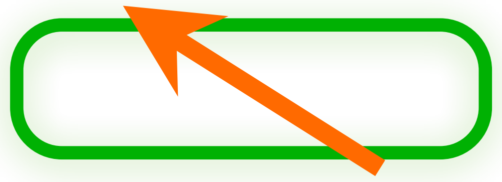
Doping an insulating antiferromagnet with holes of density p

FL*

non-Luttinger area.
Spin liquid



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



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non-Luttinger area.
Spin liquid

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Doping an insulating antiferromagnet with holes of density p

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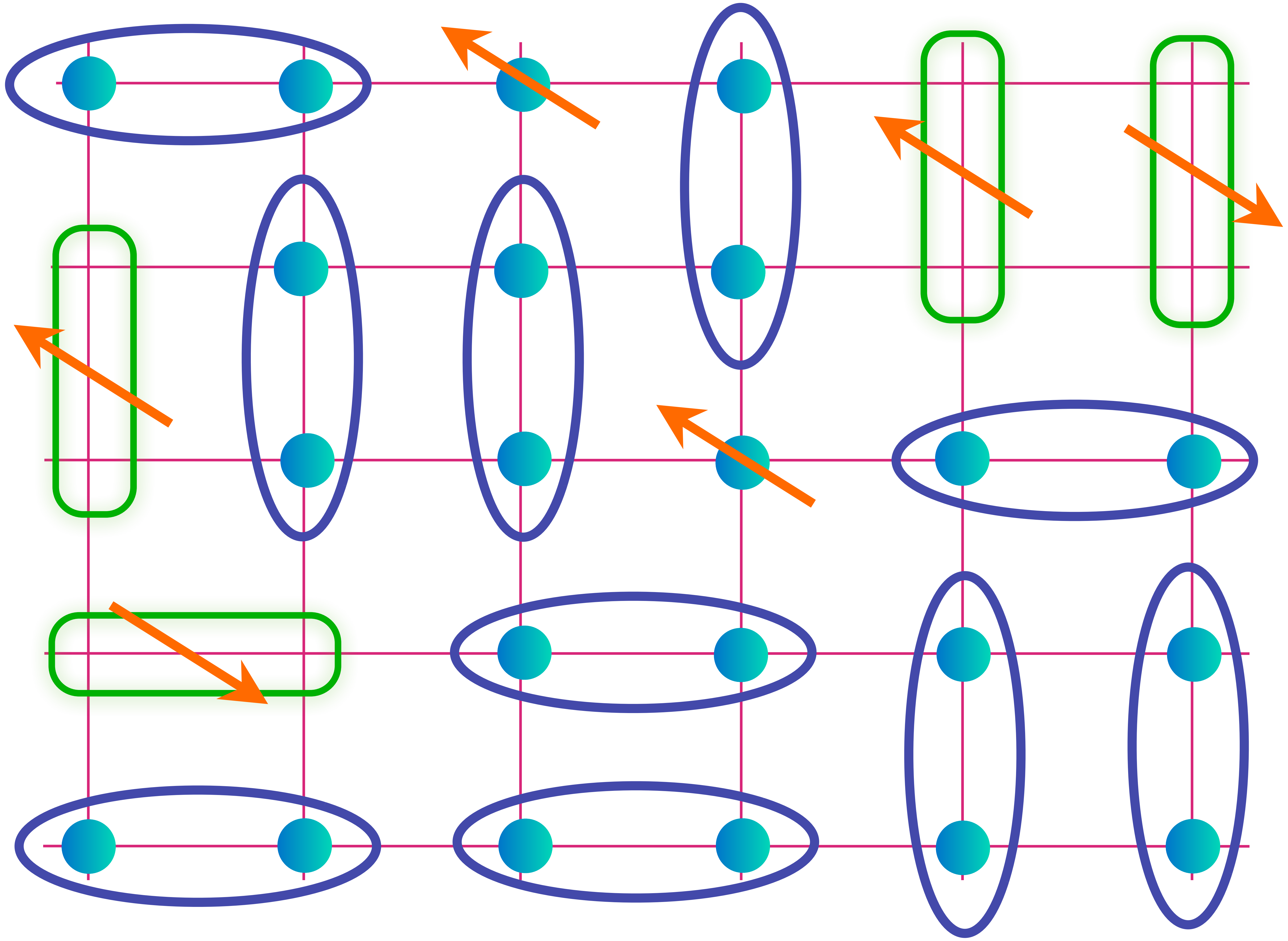
The FL* state retains the spinon excitations



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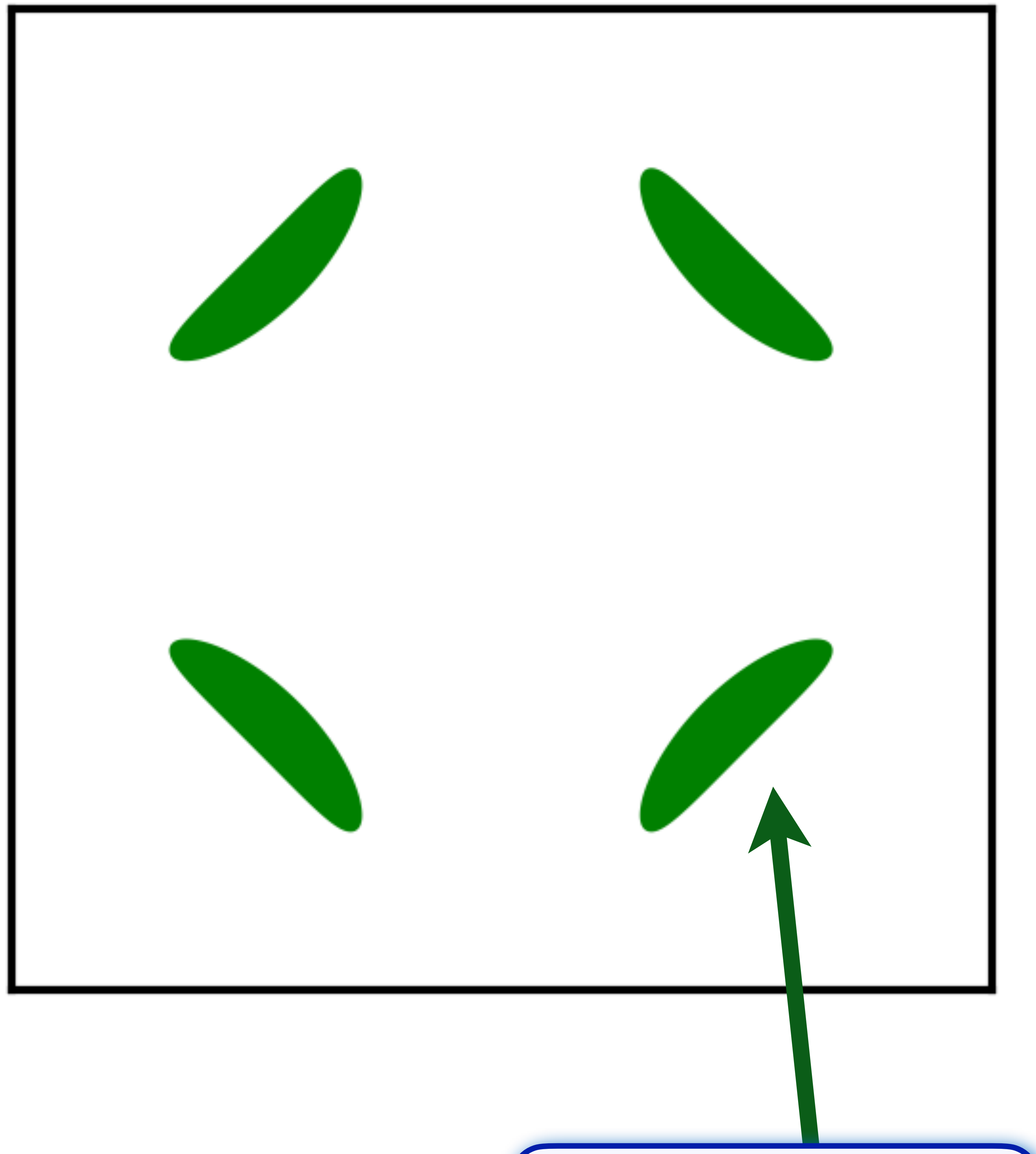
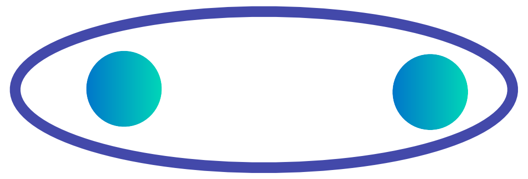
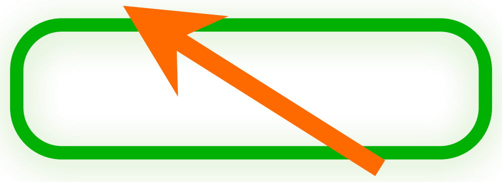


Area $p/8$

 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

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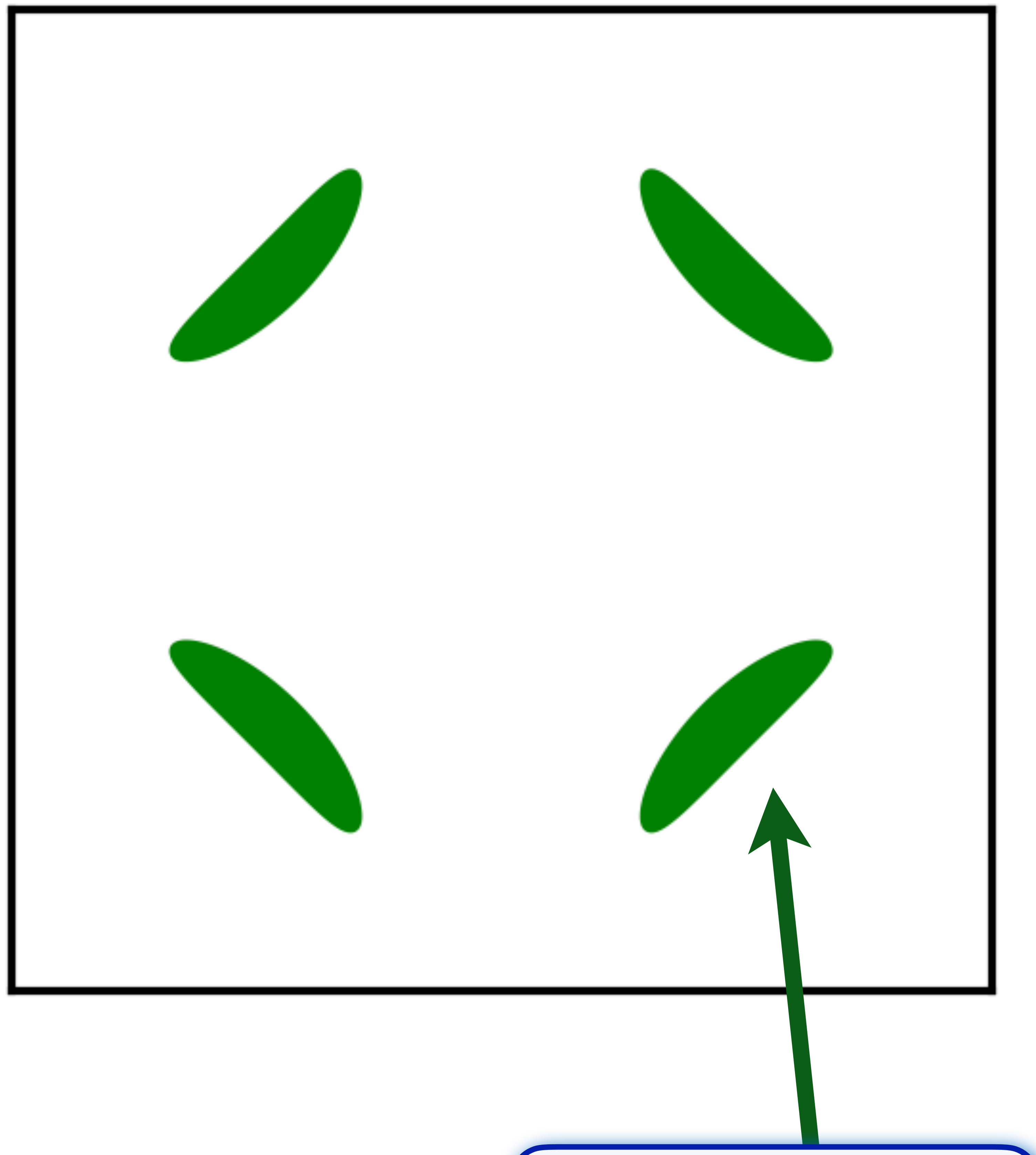

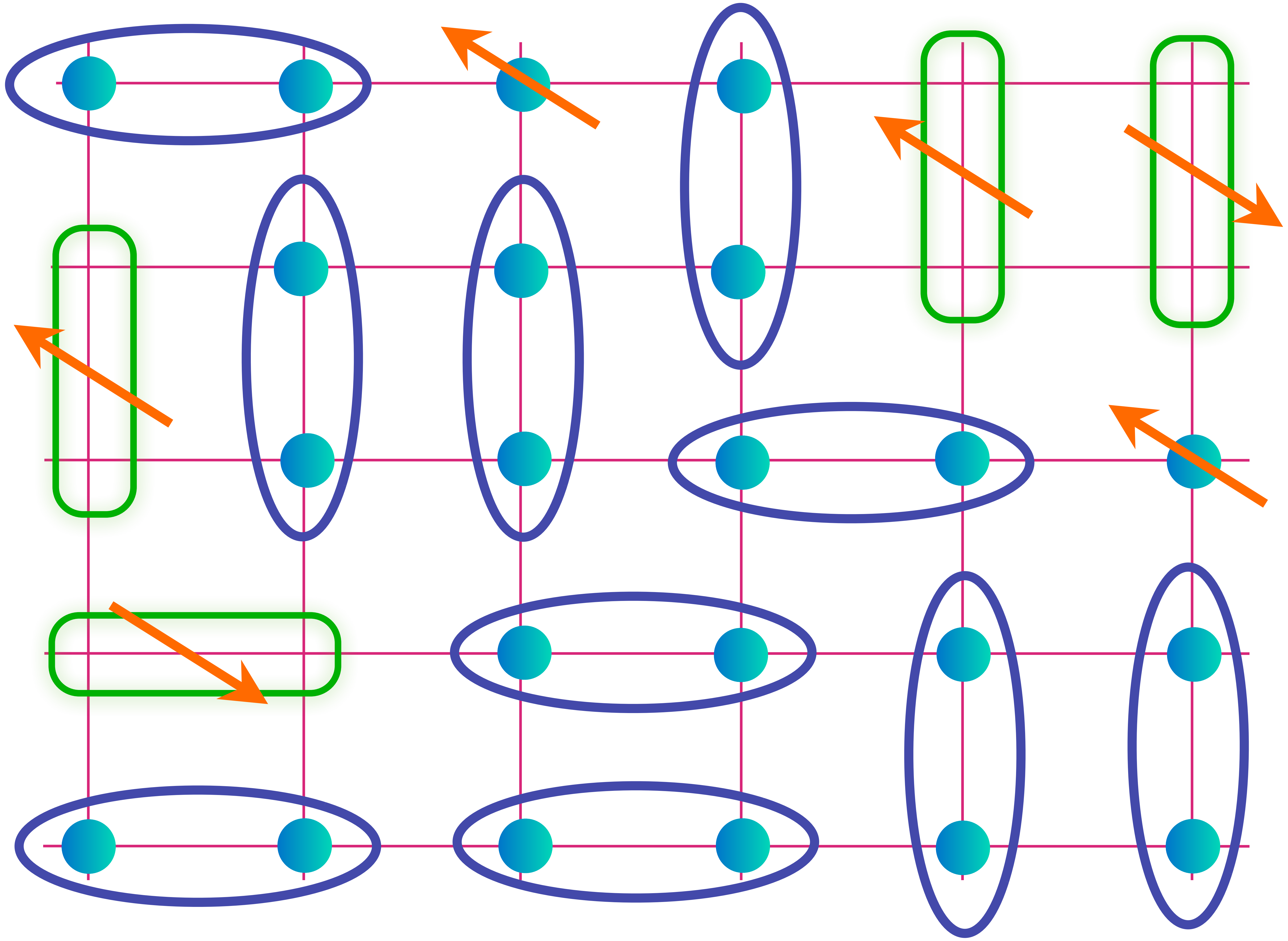

The FL* state retains the spinon excitations

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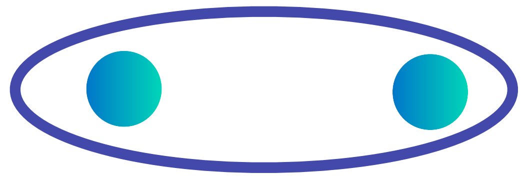
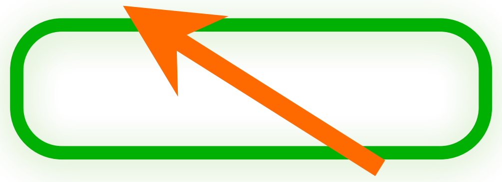


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



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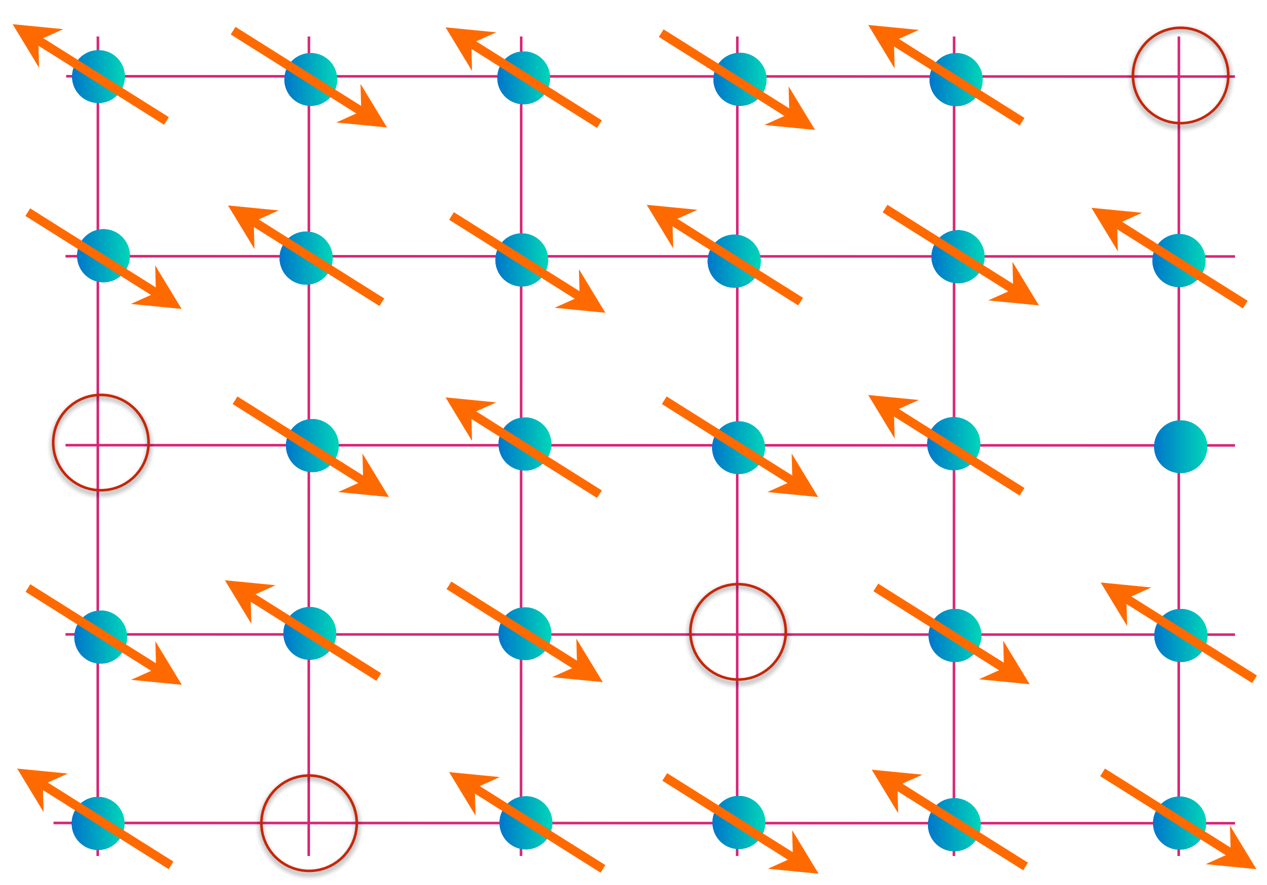
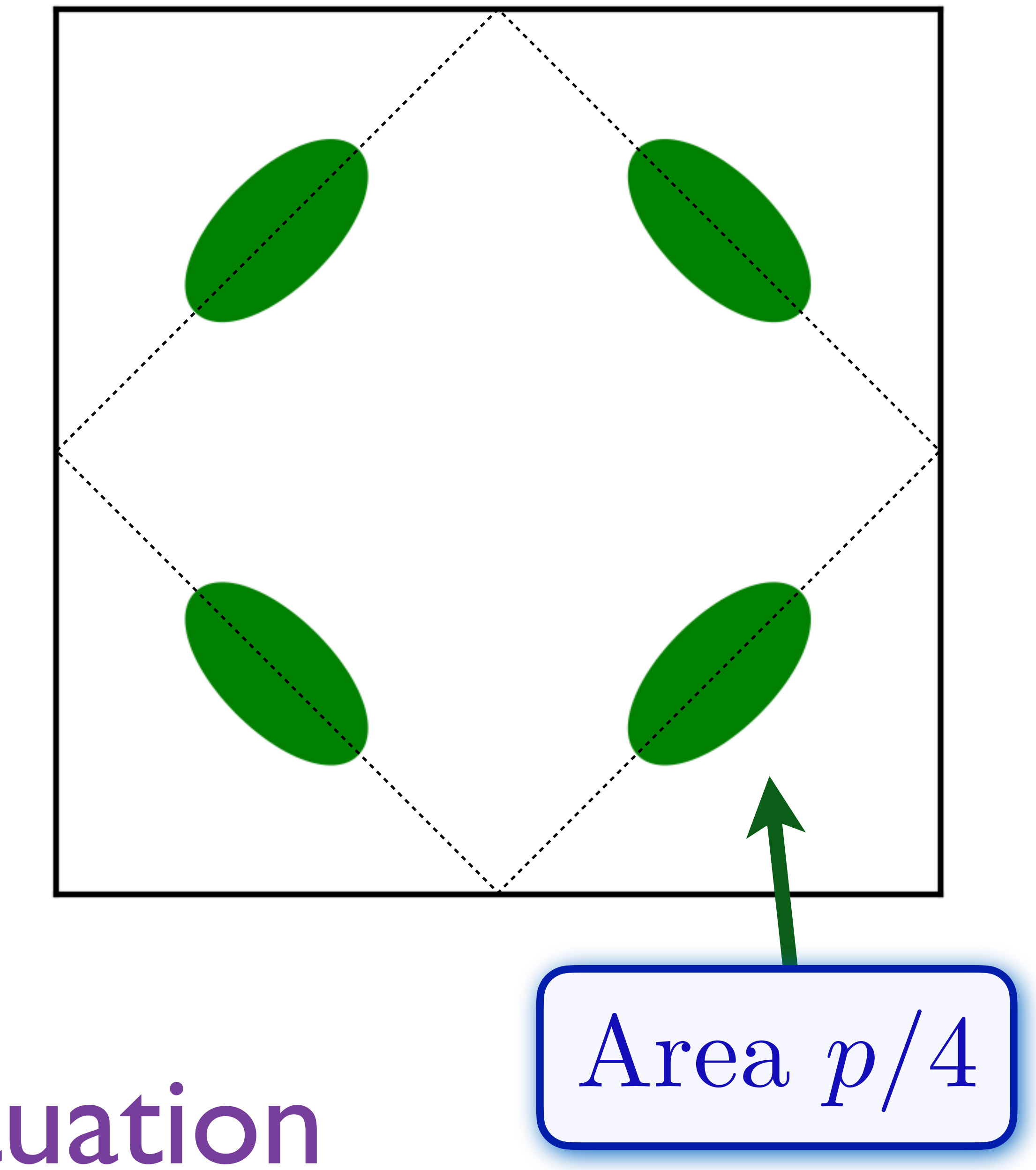
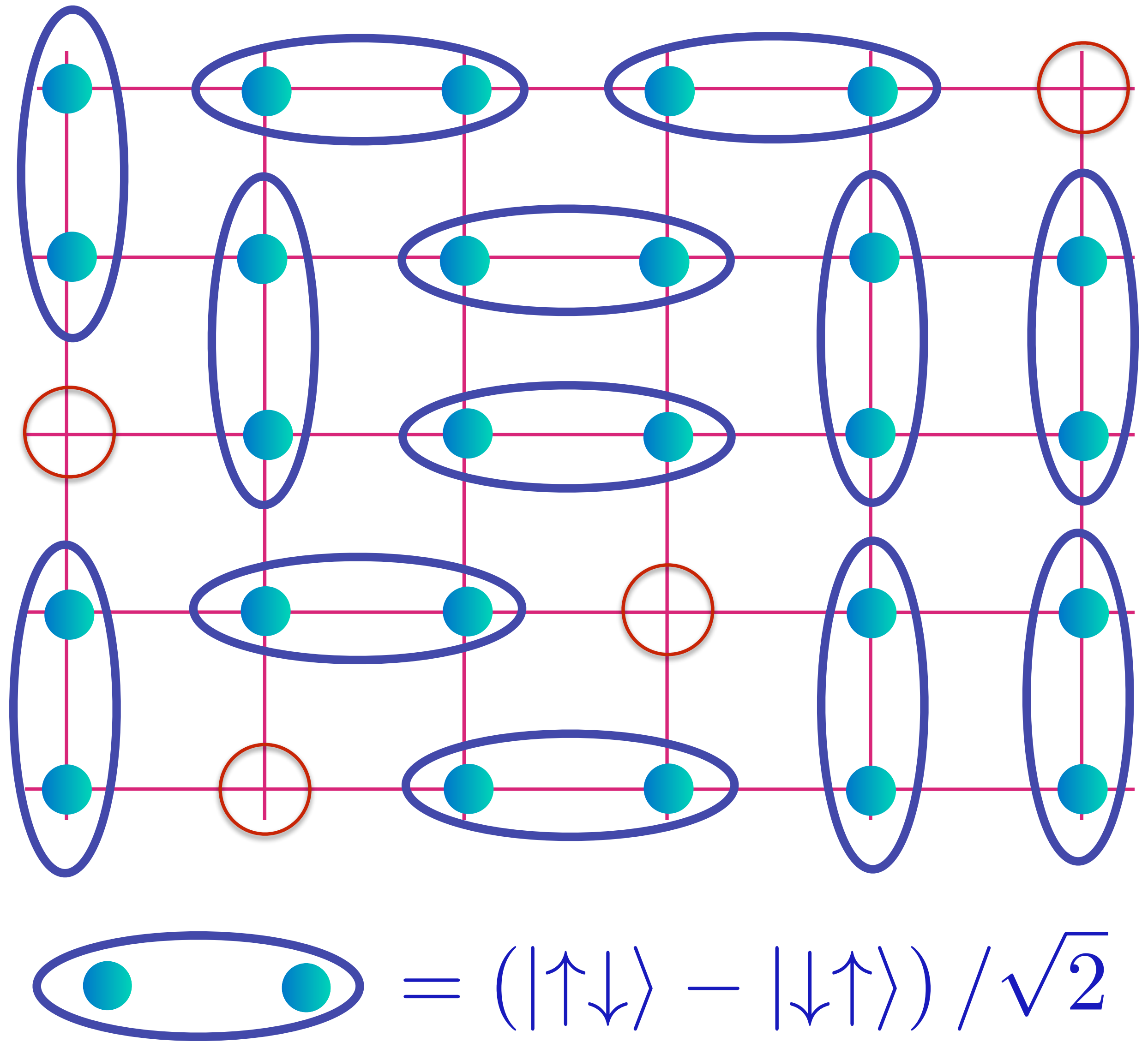
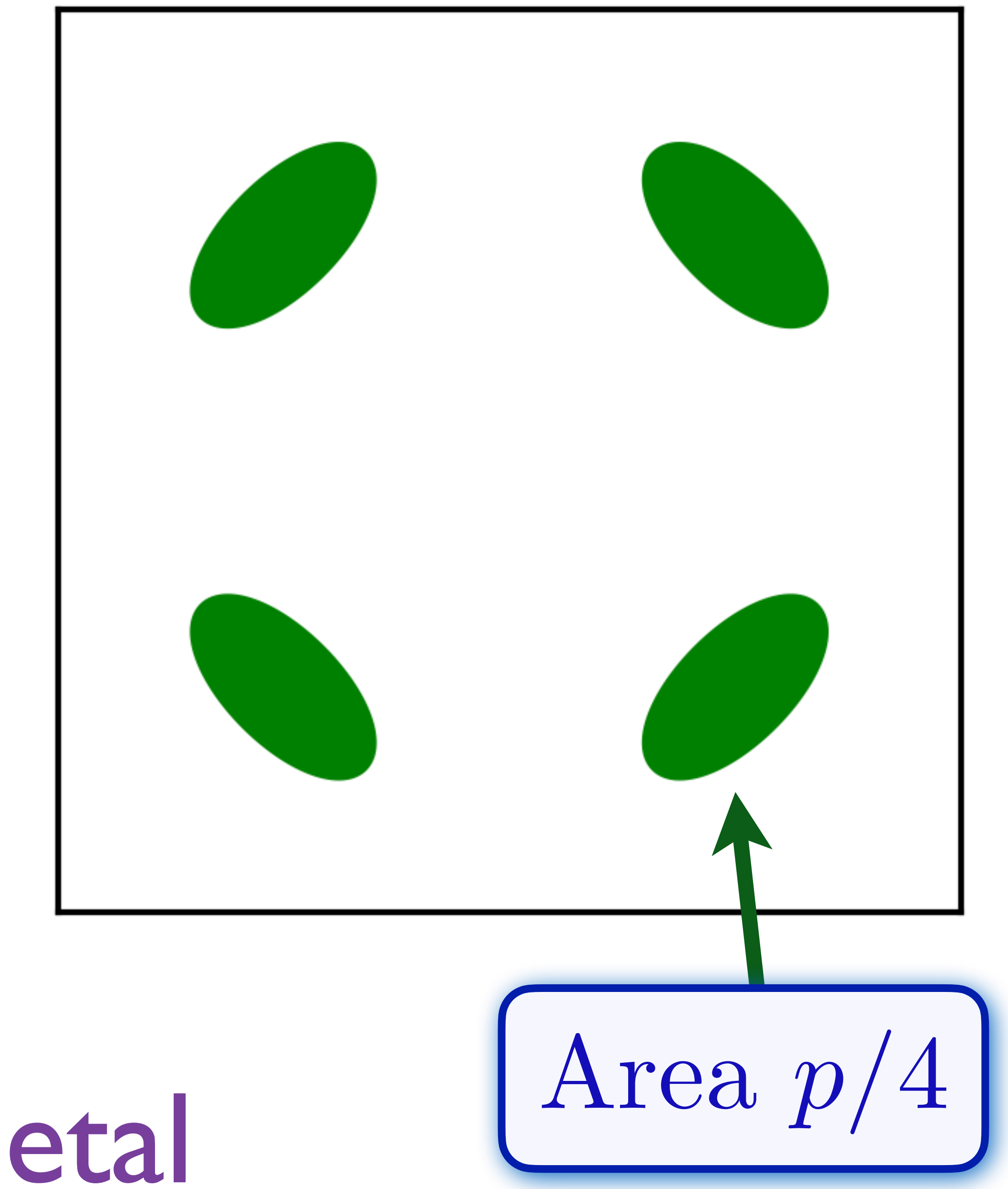




AF metal and SDW fluctuation





Holon metal

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003);
R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
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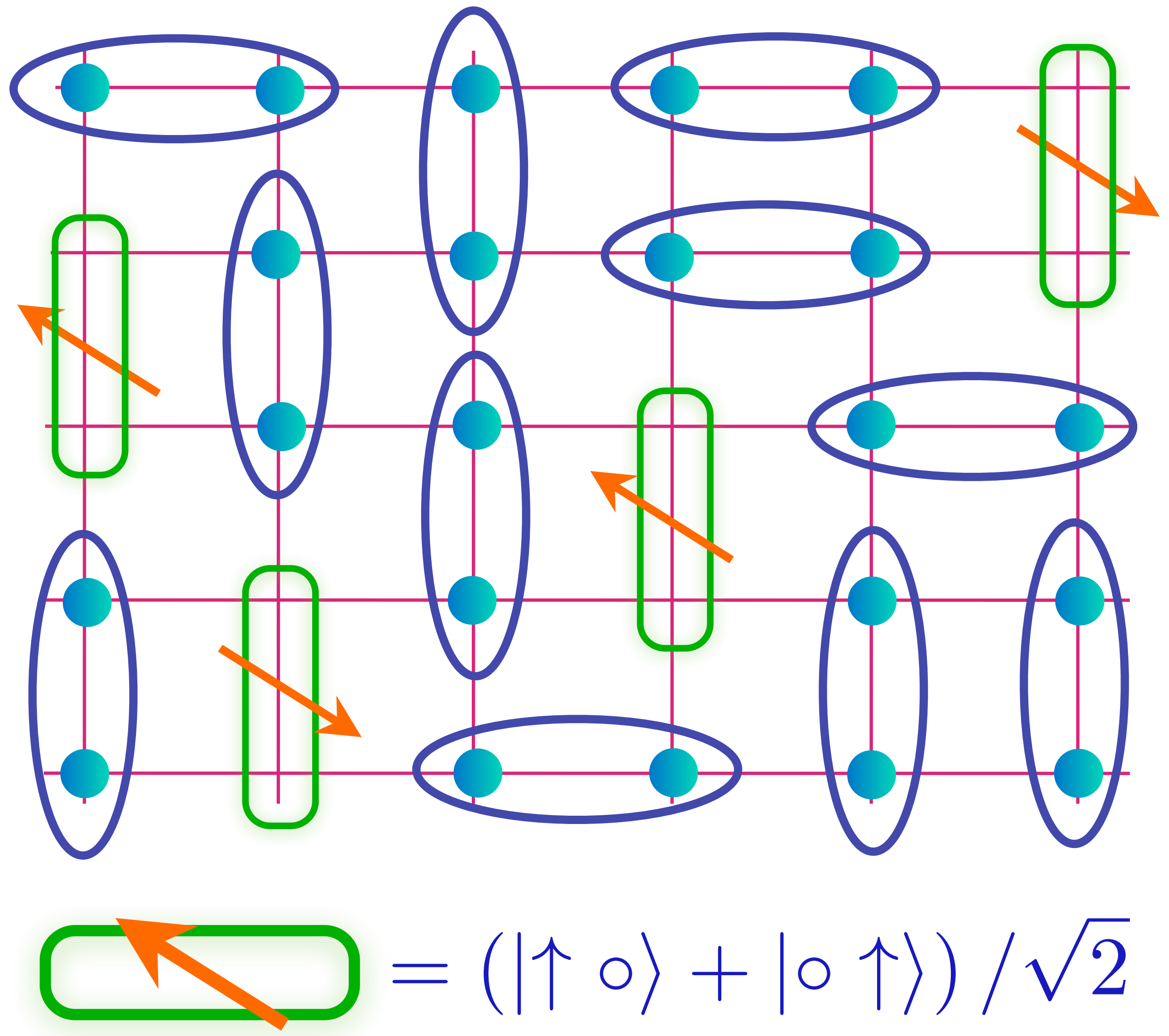
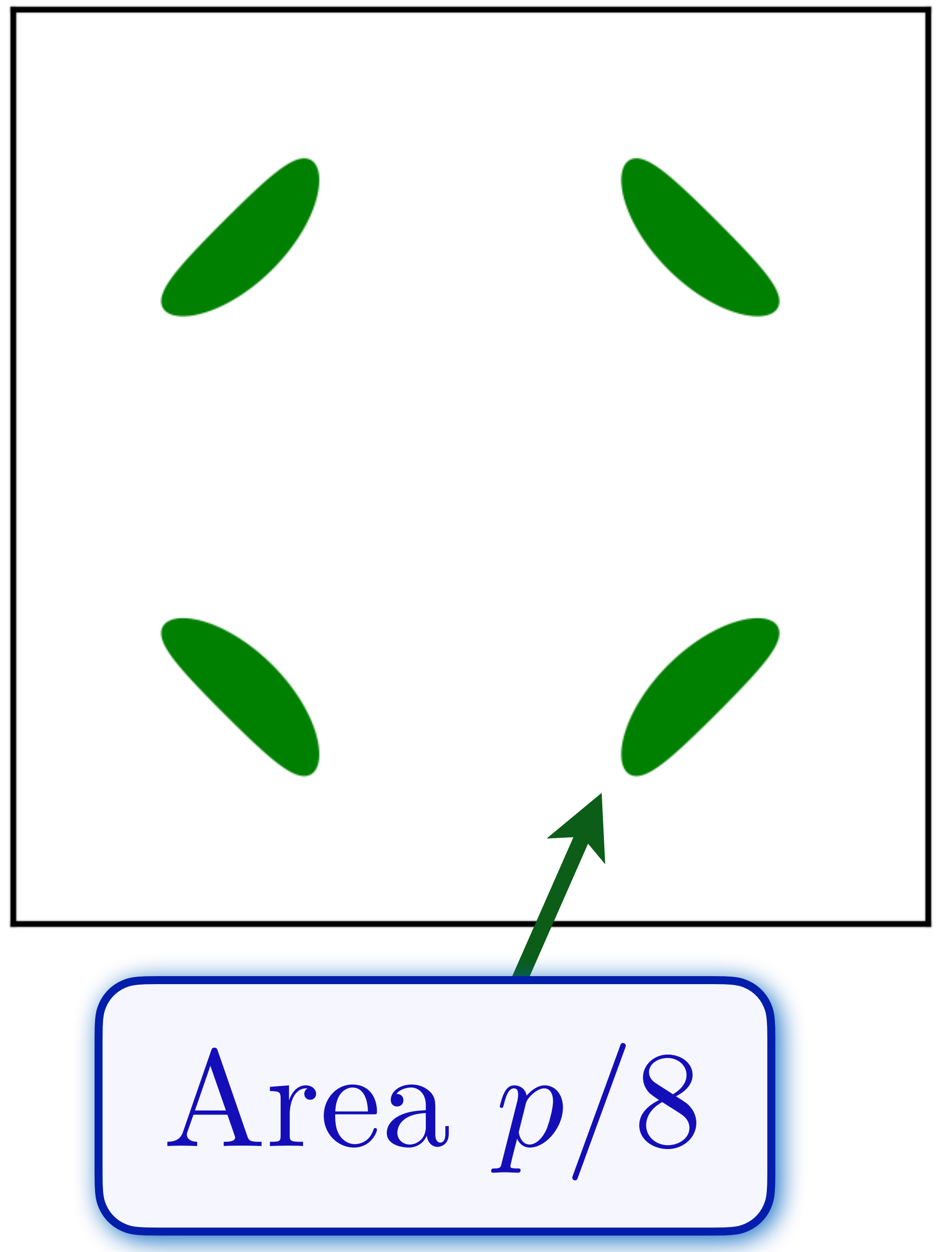


FL*

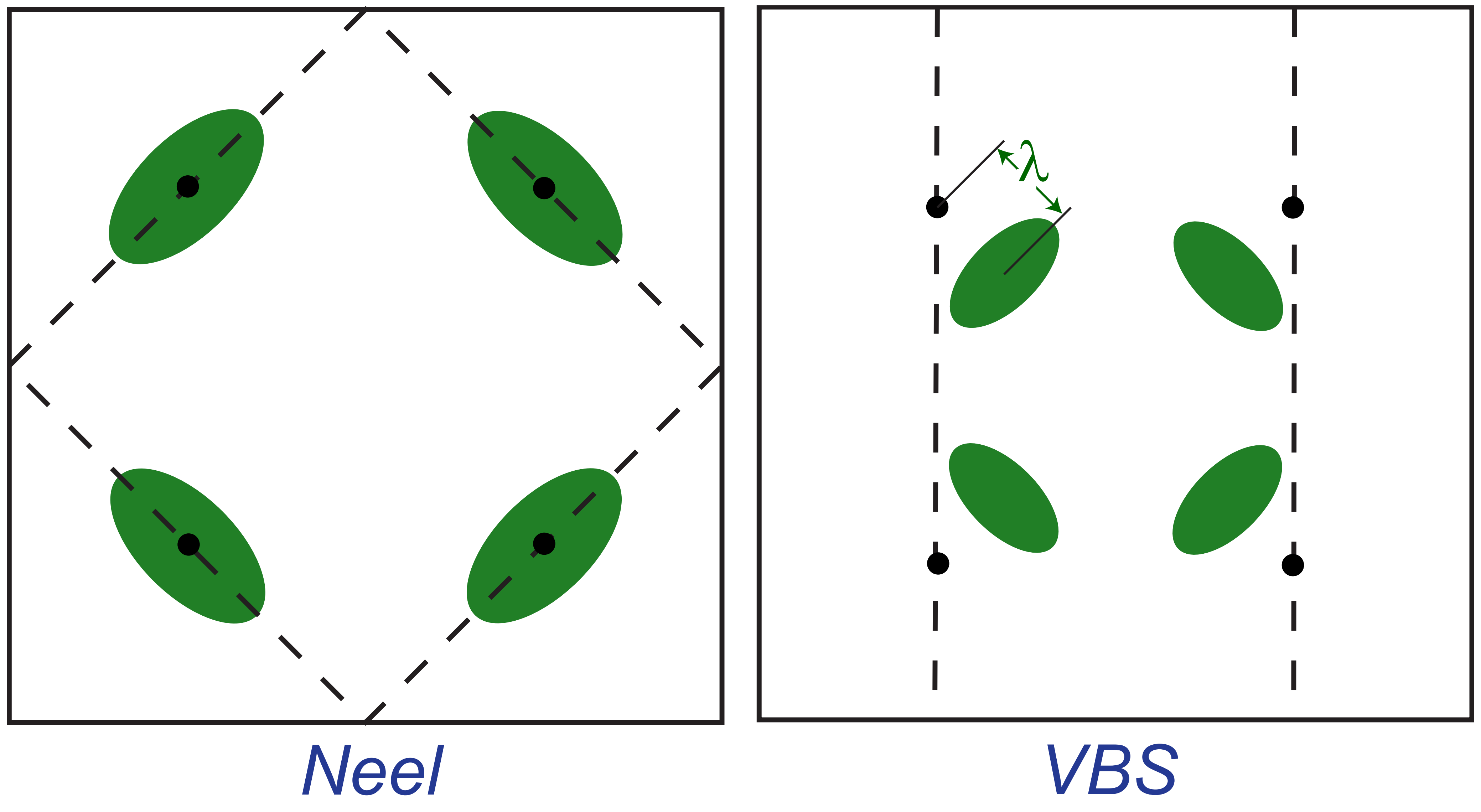


Fermi surface areas are robust to all corrections!
Factor of 2 between SDW fluctuation and FL*

PHYSICAL REVIEW B **75**, 235122 (2007)

Hole dynamics in an antiferromagnet across a deconfined quantum critical point

Ribhu K. Kaul,[1] Alexei Kolezhuk,[1,2] Michael Levin,[1] Subir Sachdev,[1] and T. Senthil[3,4]



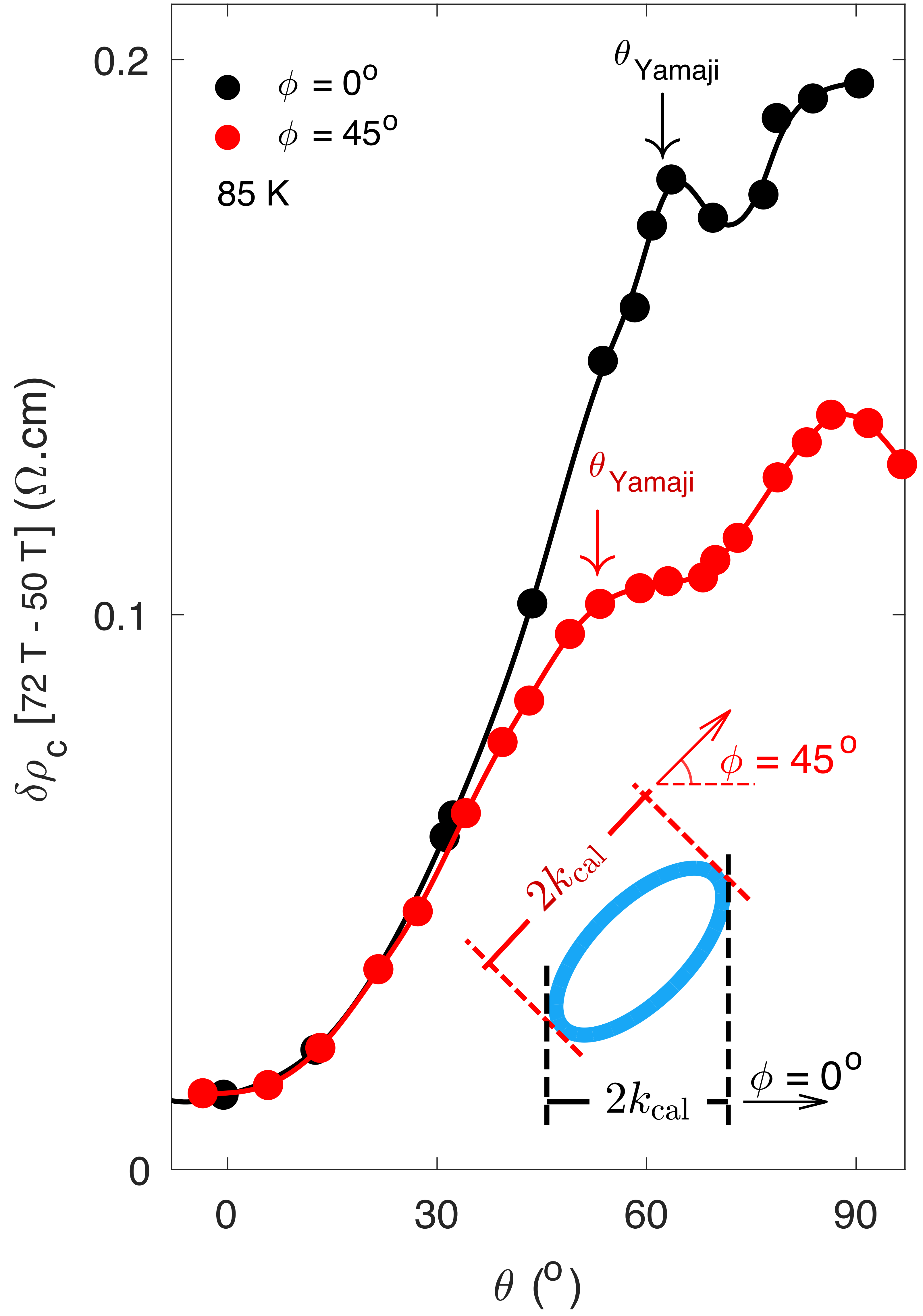
The dashed line in the Néel phase indicates the boundary of the magnetic Brillouin zone. Only the Fermi surfaces within this zone contribute to the Luttinger counting, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/4$. In the VBS phase, all four pockets are inequivalent, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/8$.

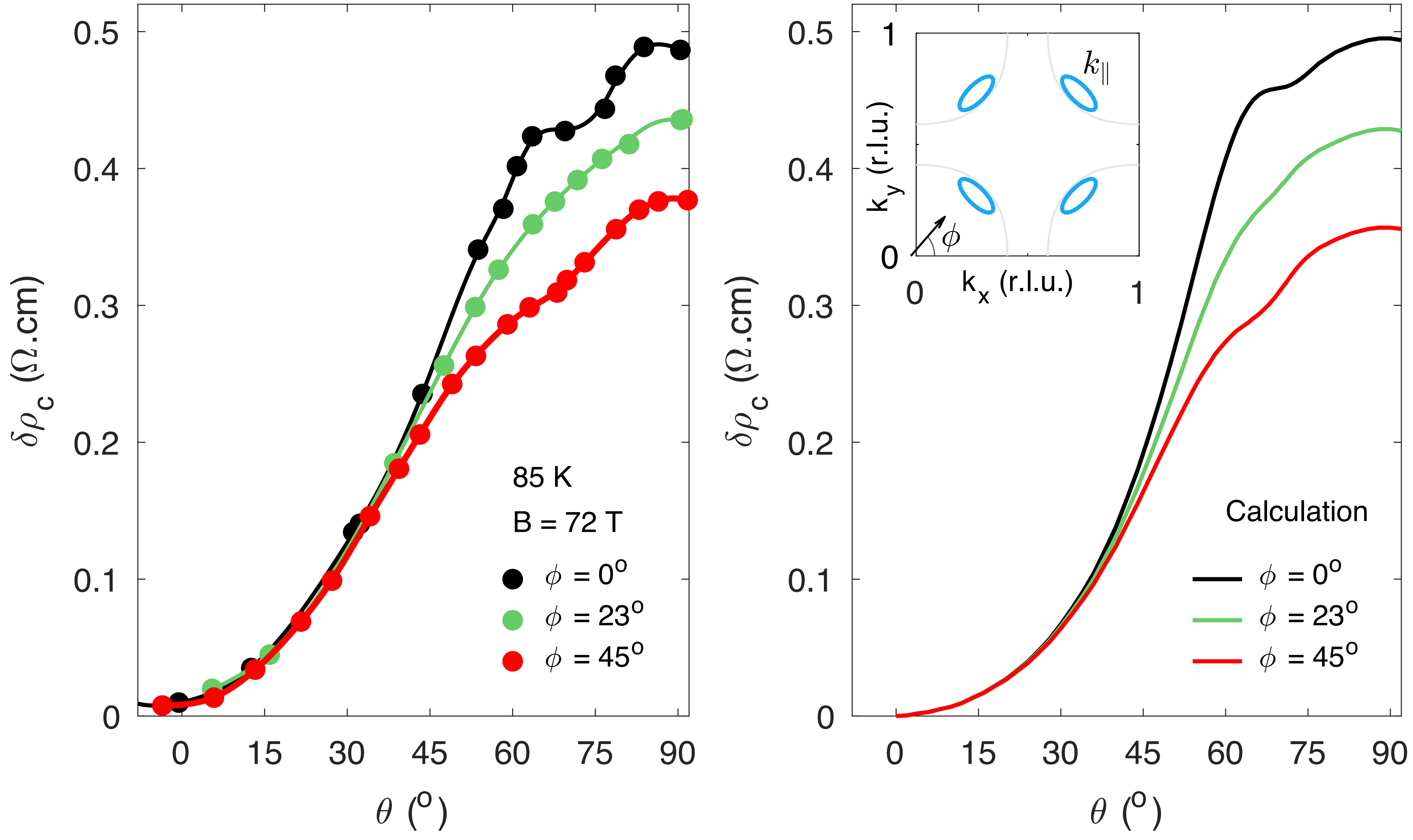
Fermi surface areas are robust to all corrections! Factor of 2 between SDW fluctuation and FL*

Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan,[1,*] Katherine A. Schreiber,[1] Oscar E. Ayala-Valenzuela,[1]
Eric D. Bauer,[2] Arkady Shekhter,[1] and Neil Harrison[1]

arXiv:2411.10631





Doping
$p = 0.1$

"The small size of the pockets determined from the Yamaji effect is … approximately 1.3% of the Brillouin zone area"

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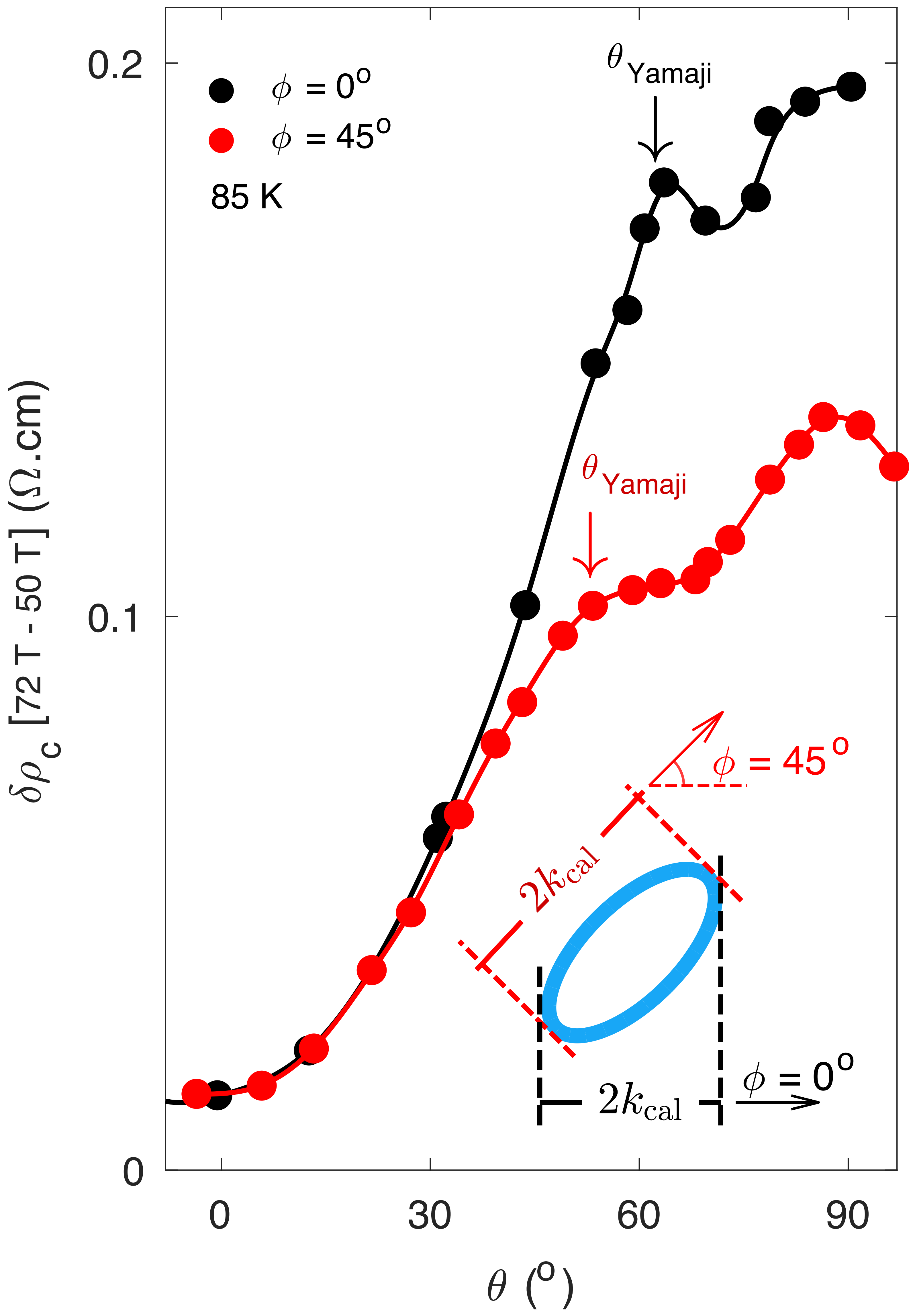

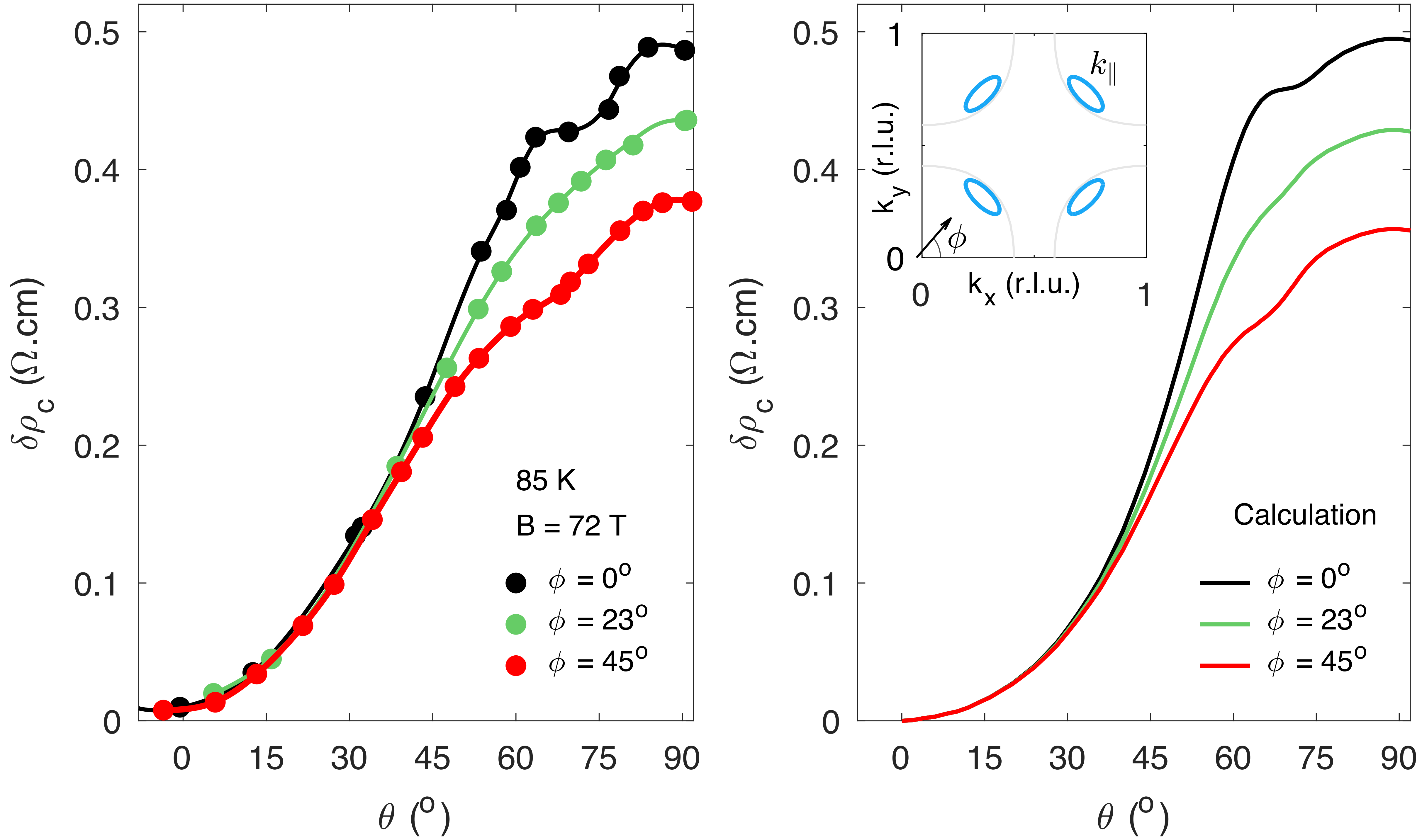

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FL* pocket fraction $= p/8 = 1.25\%$!
Fluctuating AF metal fraction $= p/4 = 2.5\%$.

(p/8 also in YRZ ansatz, Peter Johnson photoemission,
and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar?)

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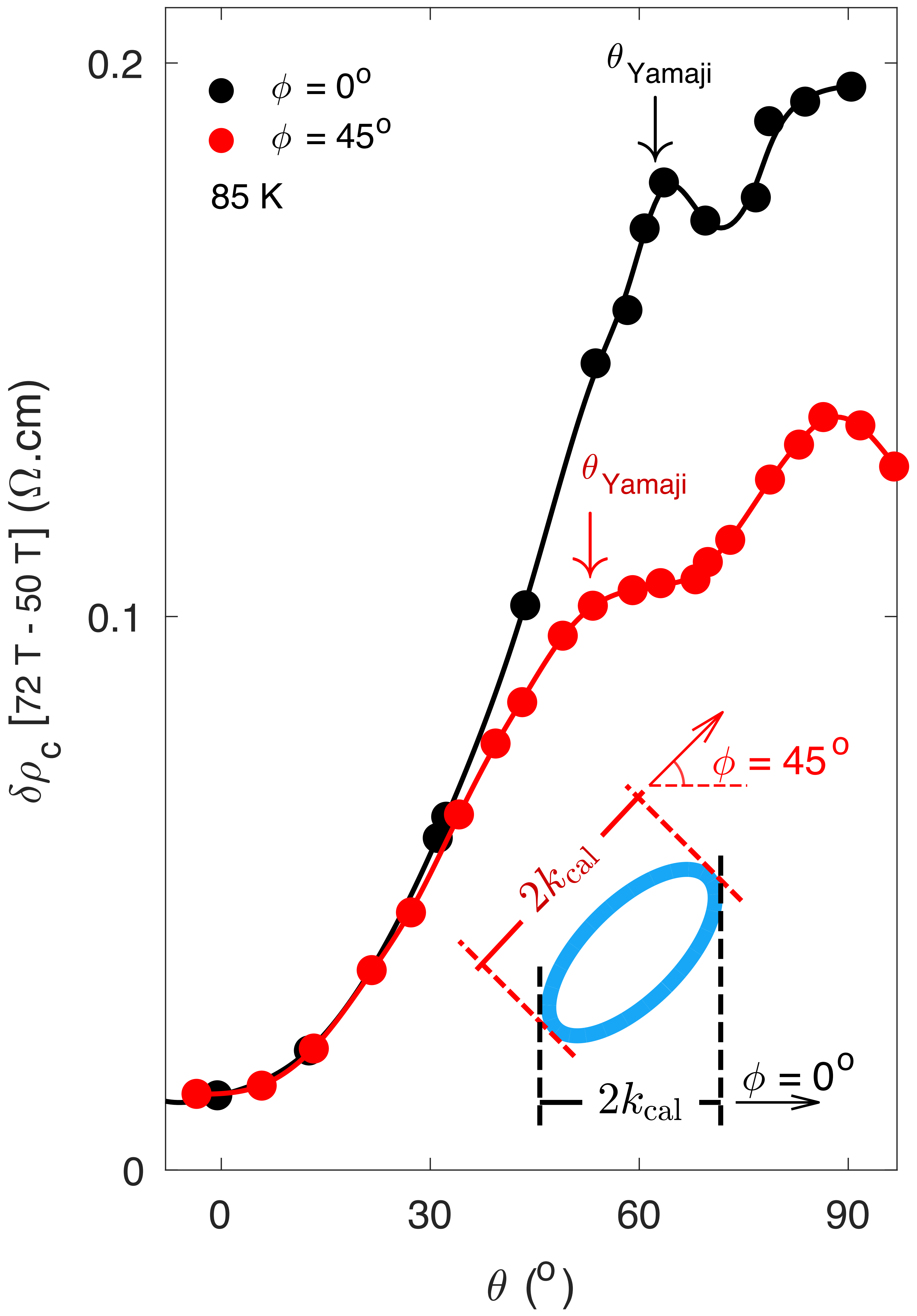

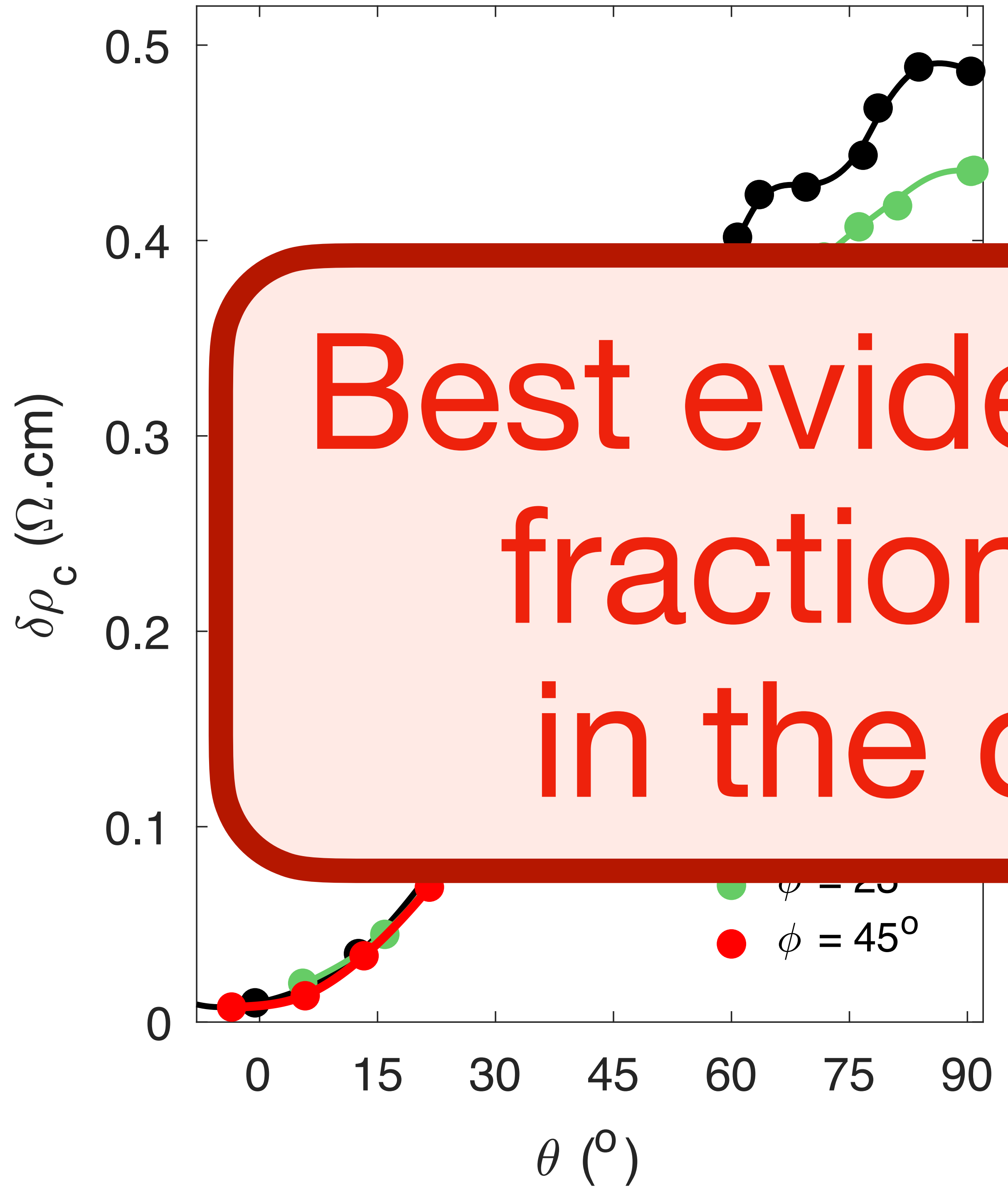

Best evidence yet for fractionalization in the cuprates

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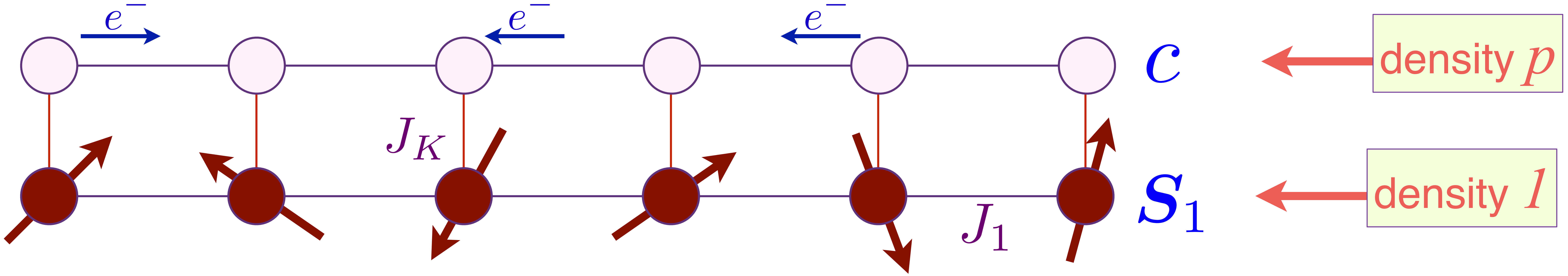
Theory of FL*
in a Kondo lattice

Fermi-volume-changing QPT in the Kondo lattice

$$\mathcal{H}_{\mathrm{KL}} = \sum_{\boldsymbol{i}<\boldsymbol{j}} J_{1,\boldsymbol{ij}} \, \boldsymbol{S}_{1\boldsymbol{i}} \cdot \boldsymbol{S}_{1\boldsymbol{j}} - \sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{ij}} c_{\boldsymbol{i}\alpha}^{\dagger} c_{\boldsymbol{j}\alpha} + \sum_{\boldsymbol{i}} \frac{J_K}{2} \boldsymbol{S}_{1i} \cdot c_{\boldsymbol{i}\alpha}^{\dagger} \boldsymbol{\sigma}_{\alpha\beta} c_{\boldsymbol{i}\beta}$$



Assume J_1 is chosen so that at $J_K = 0$
the $\boldsymbol{S}_{1\boldsymbol{i}}$ spins have a fractionalized spin liquid ground state.

Fermi-volume-changing QPT in the Kondo lattice

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, S.S., PRB **69**, 035111 (2004)

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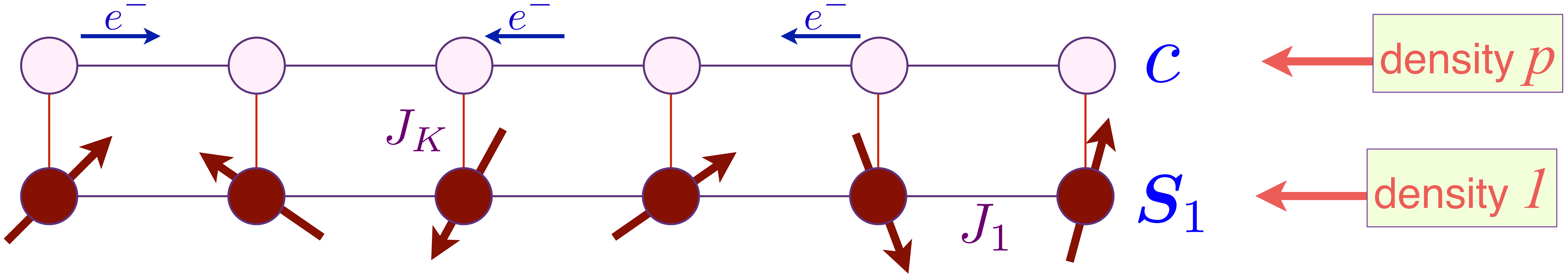

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Represent $\boldsymbol{S}_{1\boldsymbol{i}}$ by fermionic spinons: $\boldsymbol{S}_{1\boldsymbol{i}} = \frac{1}{2} f^\dagger_{1\boldsymbol{i}\alpha} \frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} f_{1\boldsymbol{i}\beta}$

$$\sum_\alpha f^\dagger_{1\boldsymbol{i}\alpha} f_{1\boldsymbol{i}\alpha} = 1 \text{ for all } i.$$

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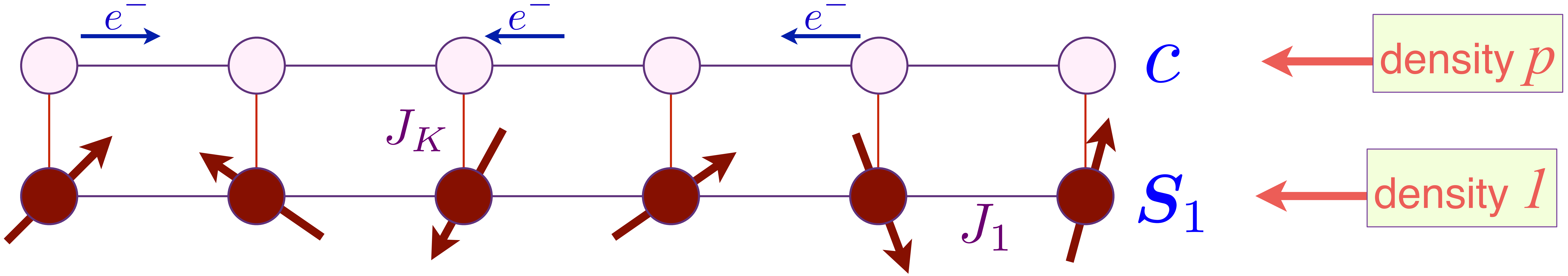

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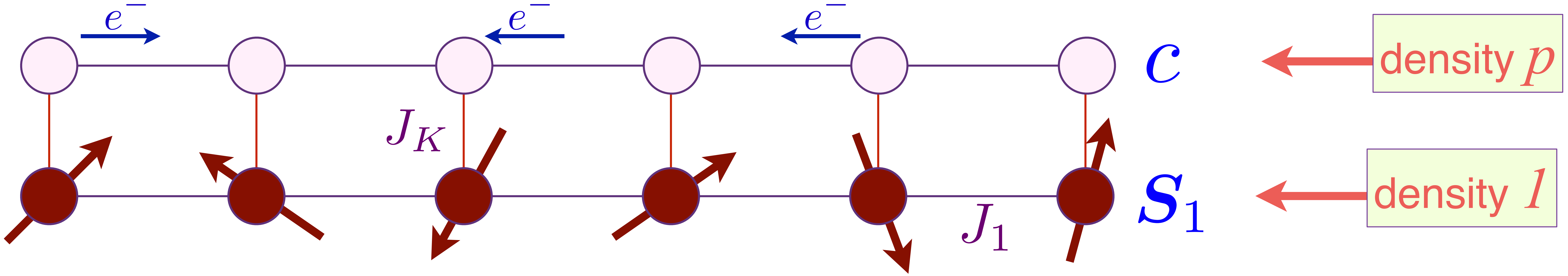
Fermi-volume-changing QPT in the Kondo lattice

$$\mathcal{H}_{\rm KL} = \sum_{i<j} J_{1,ij}\, \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{1j} - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_i \frac{J_K}{2} \boldsymbol{S}_{1i} \cdot c_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} c_{i\beta}$$



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$$\sum_\alpha f_{1i\alpha}^\dagger f_{1i\alpha} = 1 \text{ for all } i.$$

$$\mathcal{H}_{\rm KLmf} = \sum_{i,j} \left[-t_{ij} c_{i\alpha}^\dagger c_{j\alpha} - t_{1,ij} f_{1i\alpha}^\dagger f_{1j\alpha} \right] - \sum_i \left(\Phi\, c_{i\alpha}^\dagger f_{1i\alpha} + \Phi^*\, f_{1i\alpha}^\dagger c_{i\alpha} \right).$$

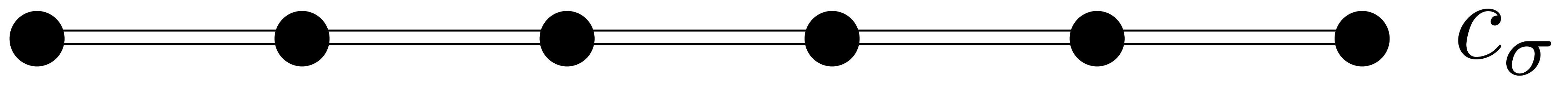
Fermi-volume-changing QPT in the Kondo lattice

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, S.S., PRB **69**, 035111 (2004)

FL* pseudogap metal in single-band models using ancillas

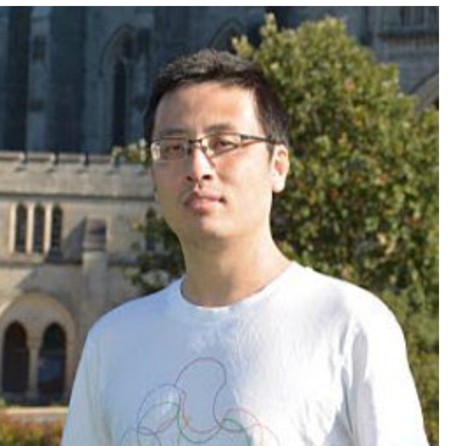
Ancilla theory of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)



Hubbard model of hole density 1+p

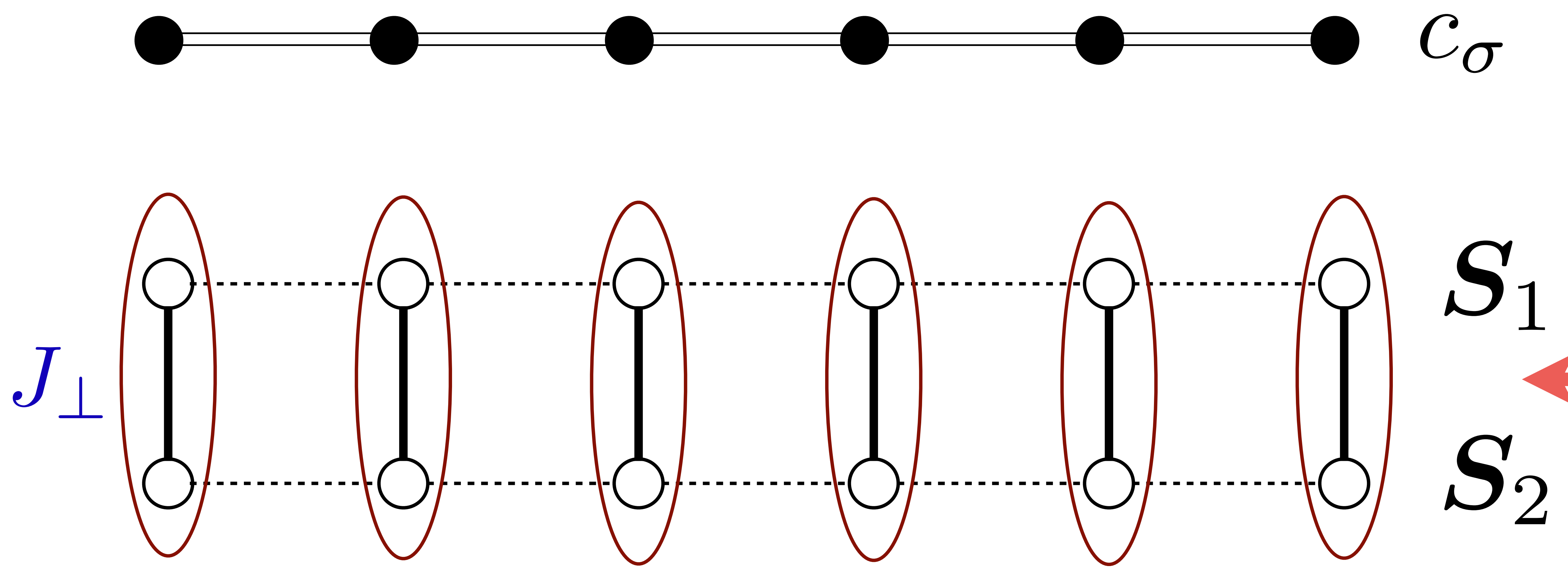
Ya-Hui Zhang



Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla theory of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)

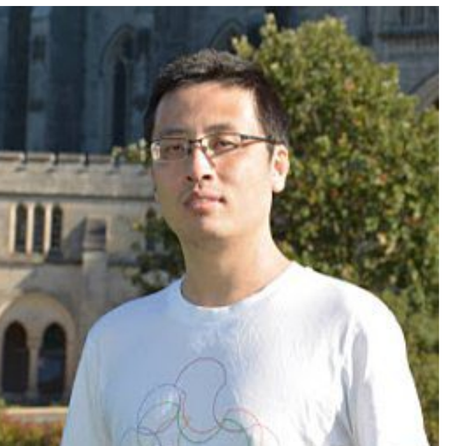


Hubbard model of hole density 1+p

Ancilla qubits in a "trivial" gapped insulator

Integrating out
$\Rightarrow$ Schrieffer-Wolff
Integrating in
$\Rightarrow$ Hubbard-Stratonovich

Ya-Hui Zhang



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Ancilla theory of the Hubbard model

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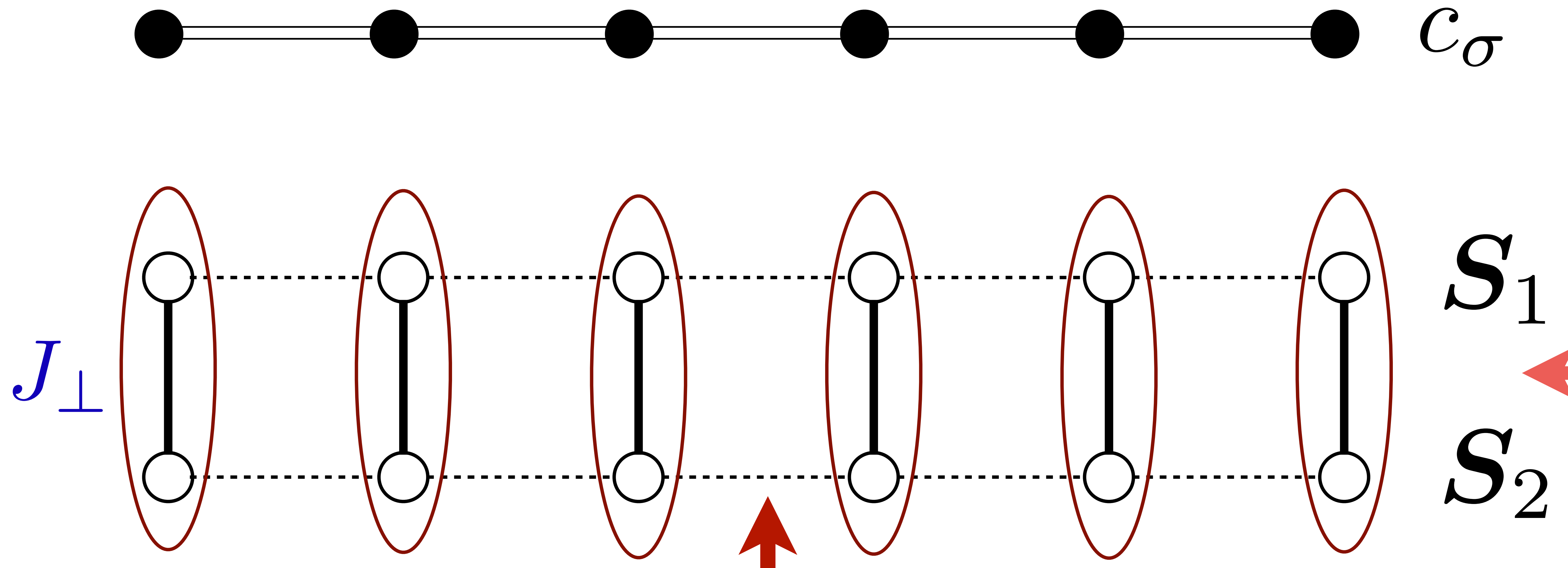
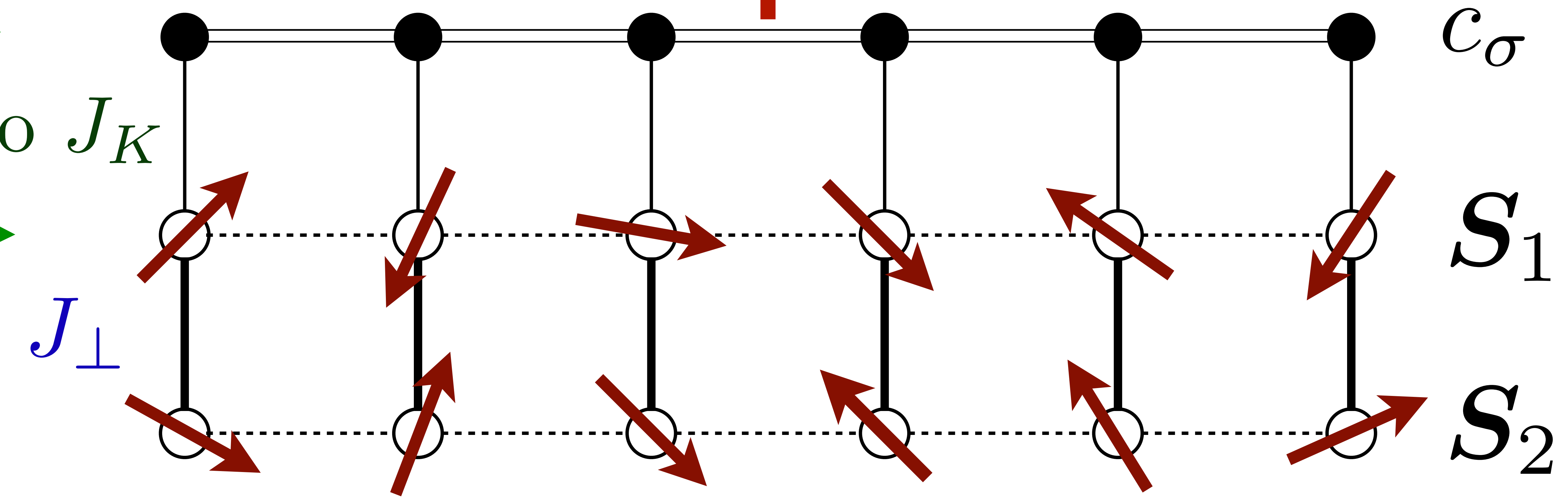


Hubbard model of hole density 1+p

Ancilla qubits in a "trivial" gapped insulator

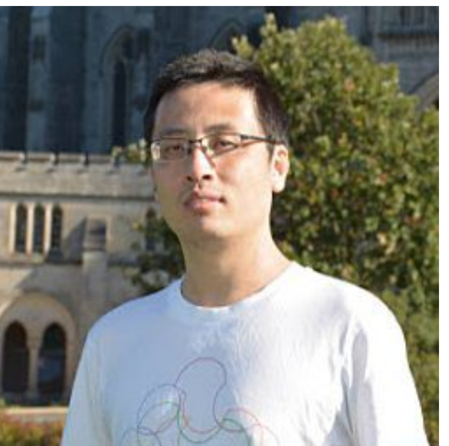
Schrieffer-Wolff (large $J_\perp$): $U = \dfrac{3J_K^2}{8J_\perp} + \dfrac{3J_K^3}{16J_\perp^2} + \ldots$



Antiferromagnetic Kondo J_K

Free holes of density 1+p

Ya-Hui Zhang



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Ancilla theory of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)

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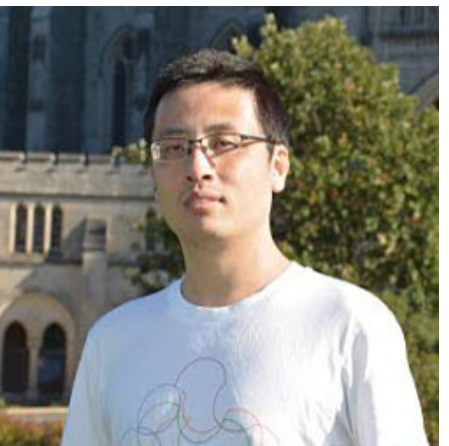

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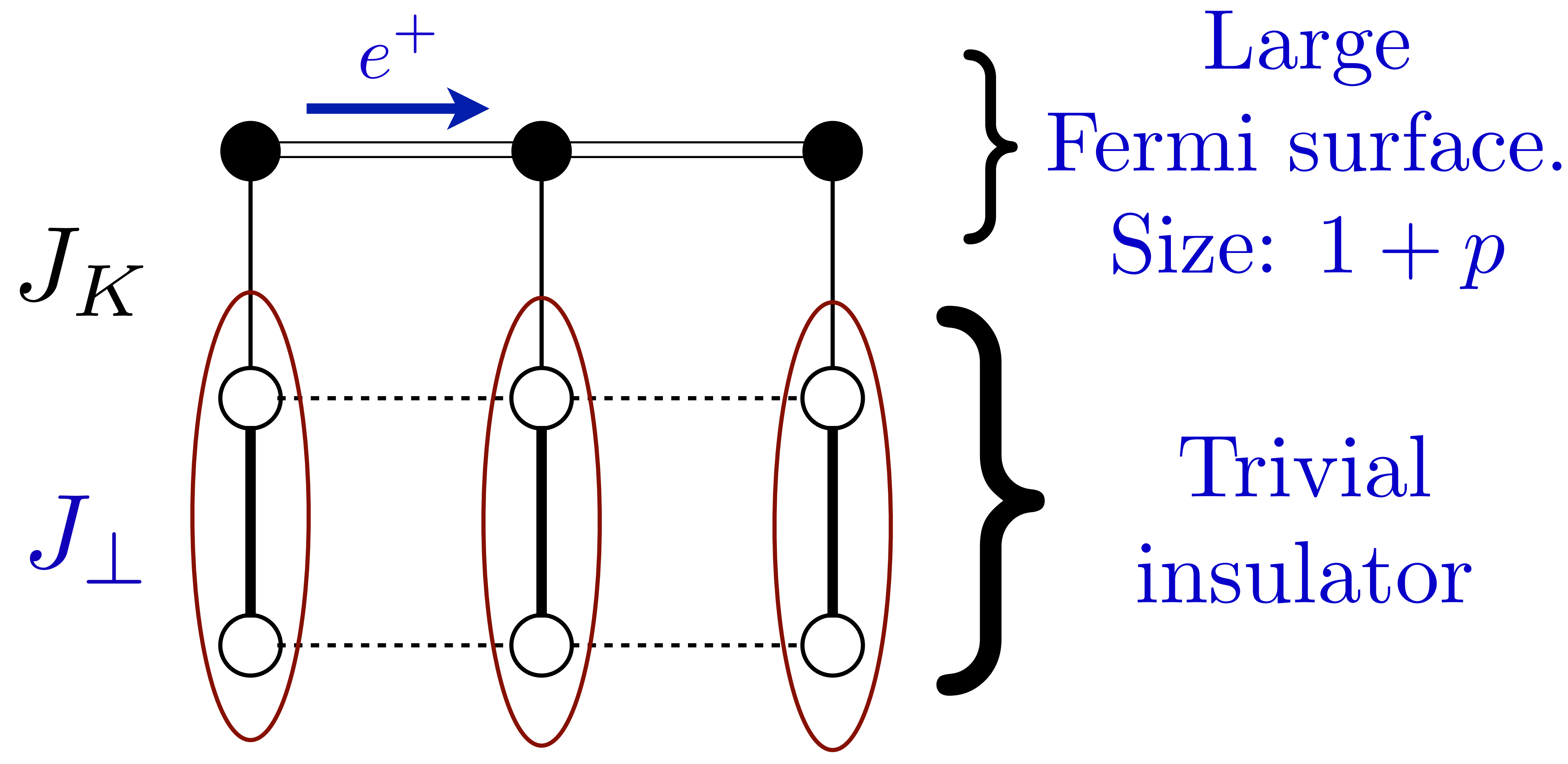
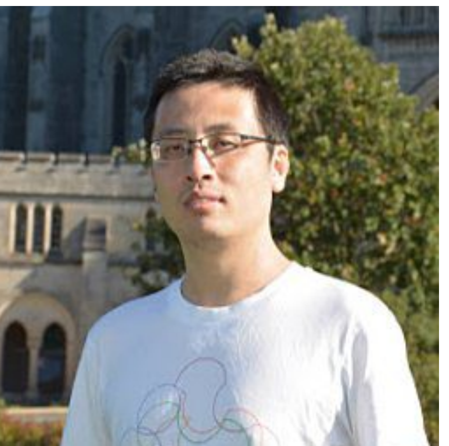


FL

J_K

doping p

Ya-Hui Zhang



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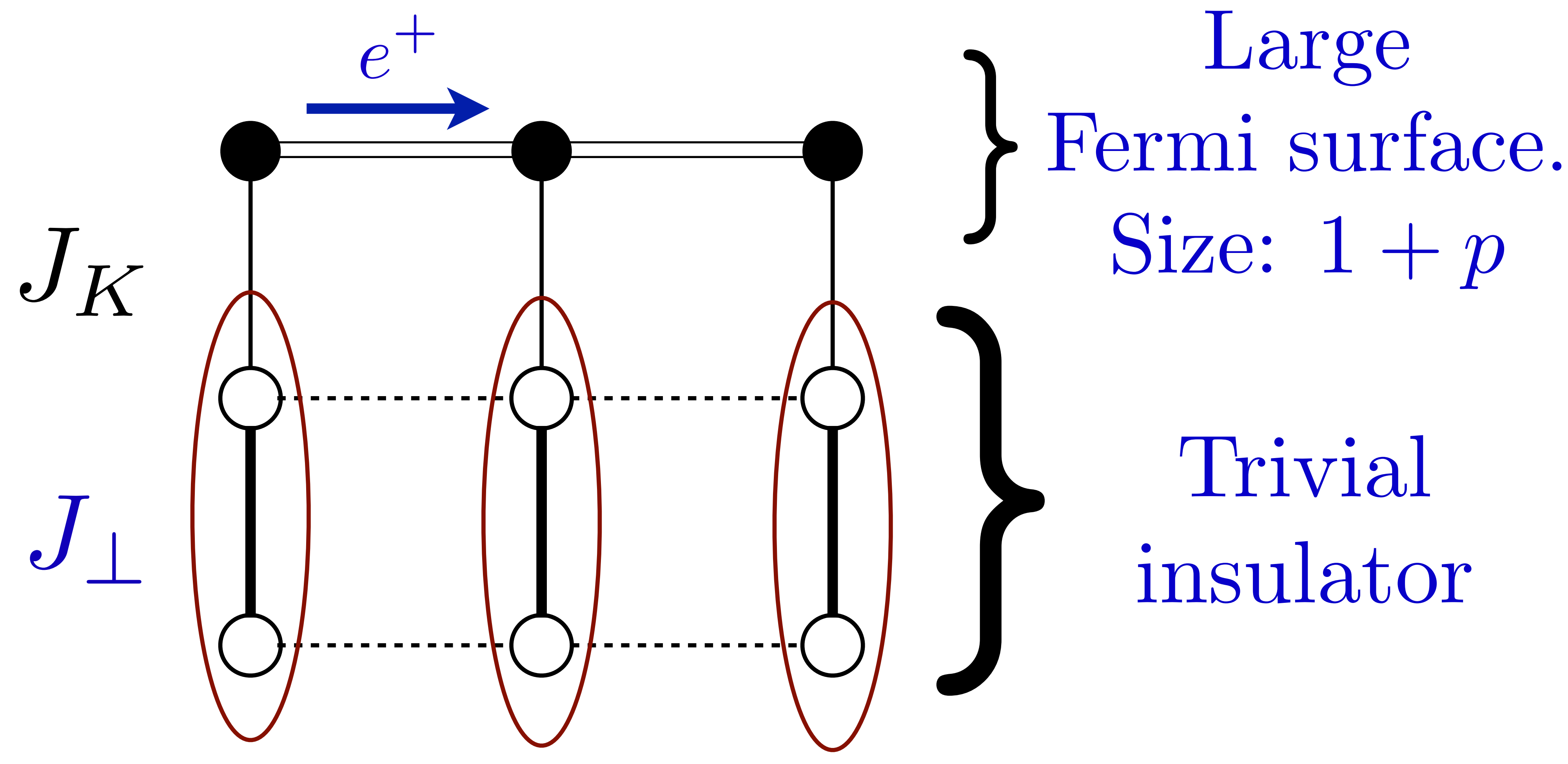
Ancilla theory of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)

- Antiferromagnetic J_K renormalizes to larger values
- Need additional gauge fields to project to rung singlets



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J_K

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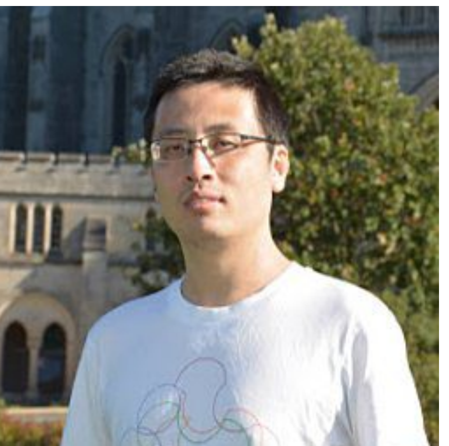

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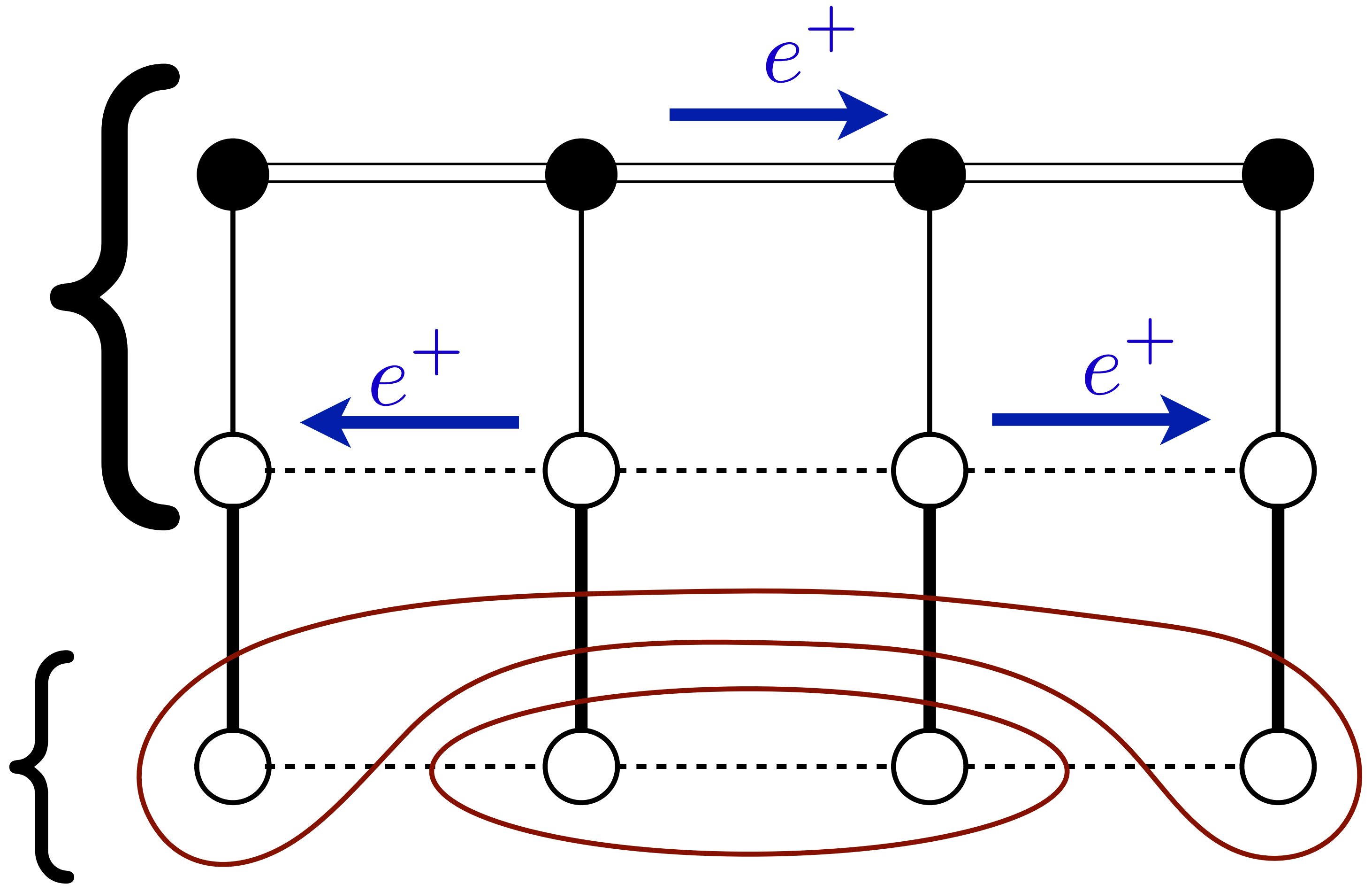
Ancilla theory of the Hubbard model

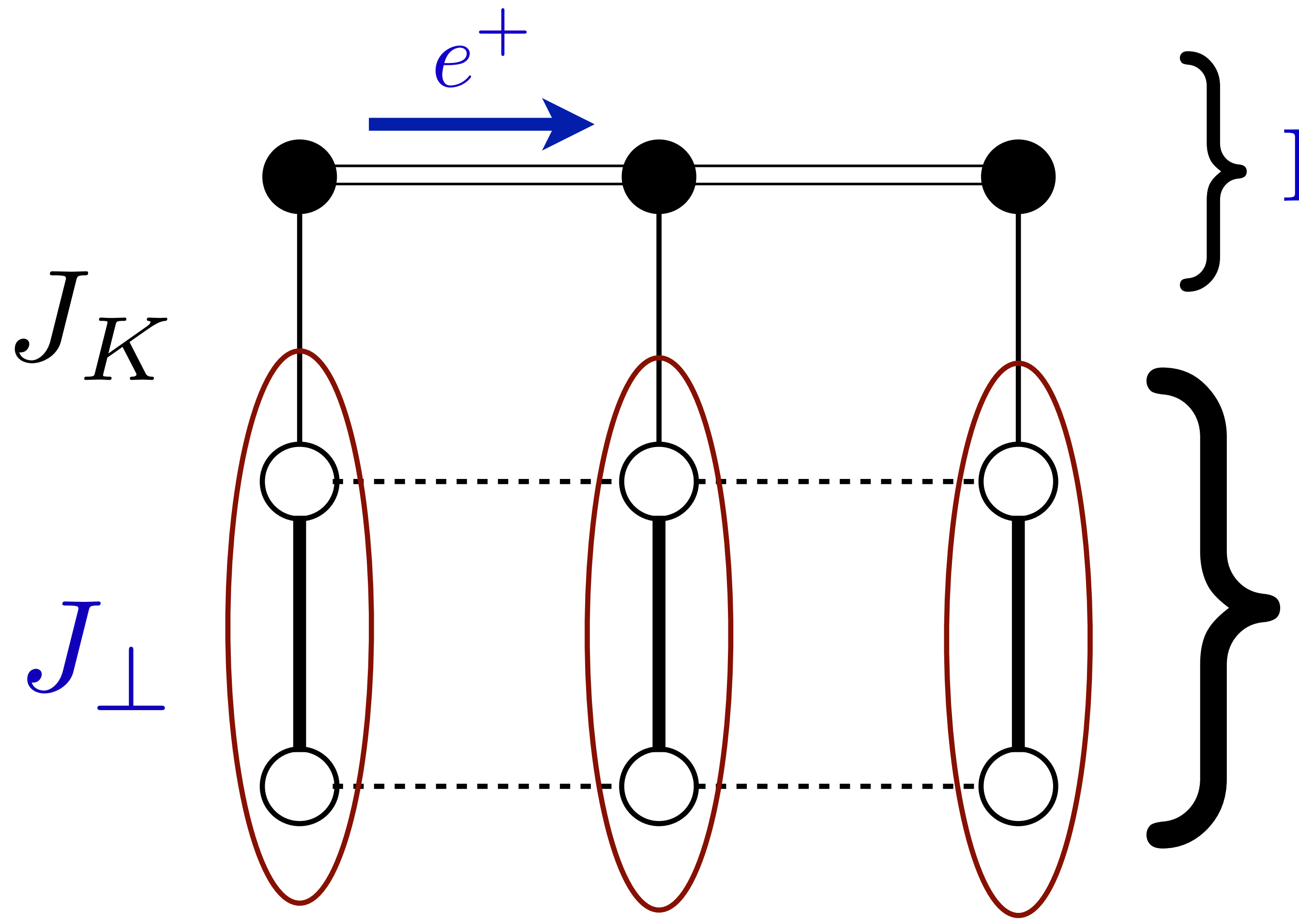
(Foolproof method to satisfy the Oshikawa anomaly)

Kondo lattice heavy Fermi liquid. Size $1 + p + 1$ $= p$ (mod 2). *Small* Fermi surface!

Your favorite spin liquid



FL*



Large Fermi surface. Size: $1 + p$

Trivial insulator

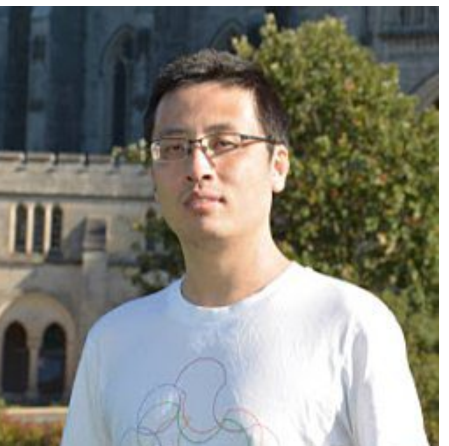
FL

J_K

doping p

Pseudogap metal = Kondo Lattice Heavy Fermi Liquid $\oplus$ Spin Liquid

Ya-Hui Zhang



Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla theory of FL*

$$H_{\rm Kondo\,lattice} = \sum_{\boldsymbol{i},\boldsymbol{j}} \left[-t_{\boldsymbol{ij}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} - t_{1,\boldsymbol{ij}} f^\dagger_{1\boldsymbol{i}\alpha} f_{1\boldsymbol{j}\alpha} \right] - \sum_{\boldsymbol{i}} \Phi \left(c^\dagger_{\boldsymbol{i}\alpha} f_{1\boldsymbol{i}\alpha} + f^\dagger_{1\boldsymbol{i}\alpha} c_{\boldsymbol{i}\alpha} \right)$$

Heavy Fermi liquid of electrons c, f_1

$\boldsymbol{S}_1 \sim f^\dagger_{1\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{1\beta}$

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

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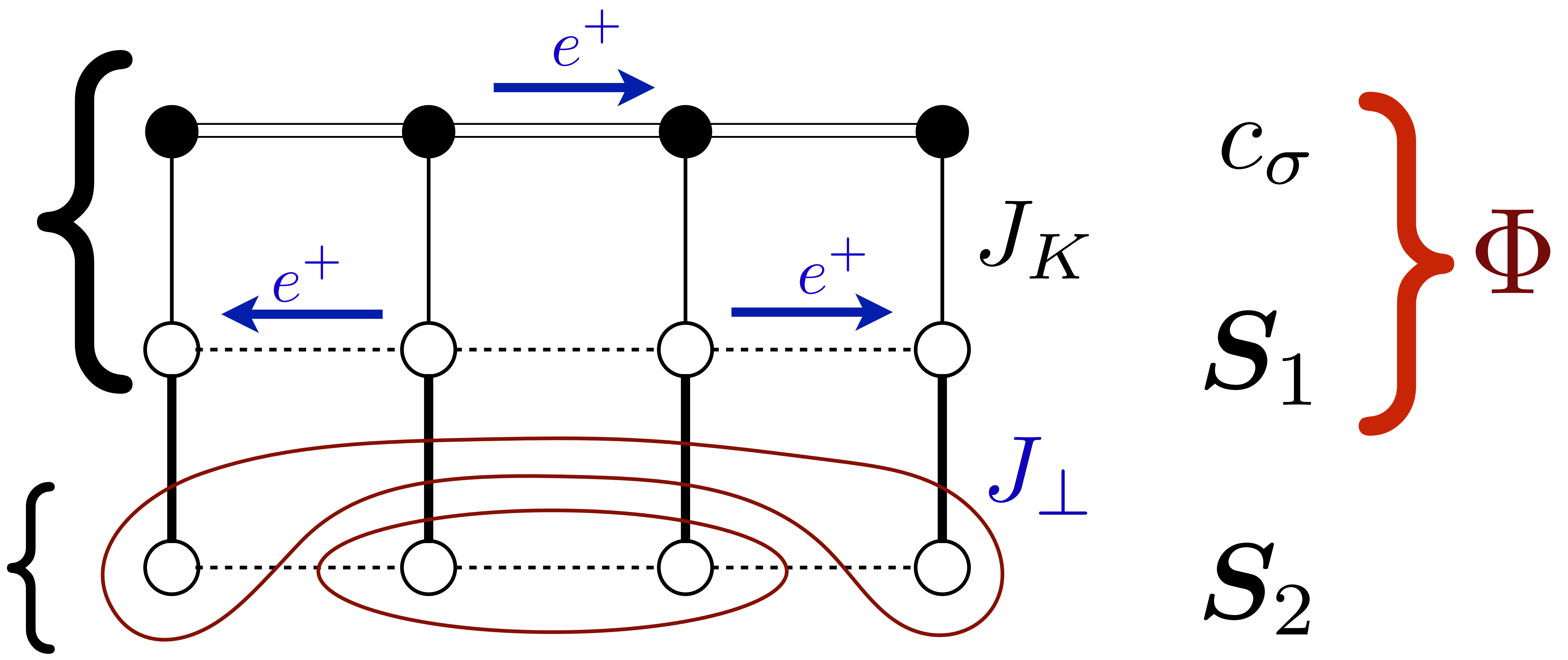
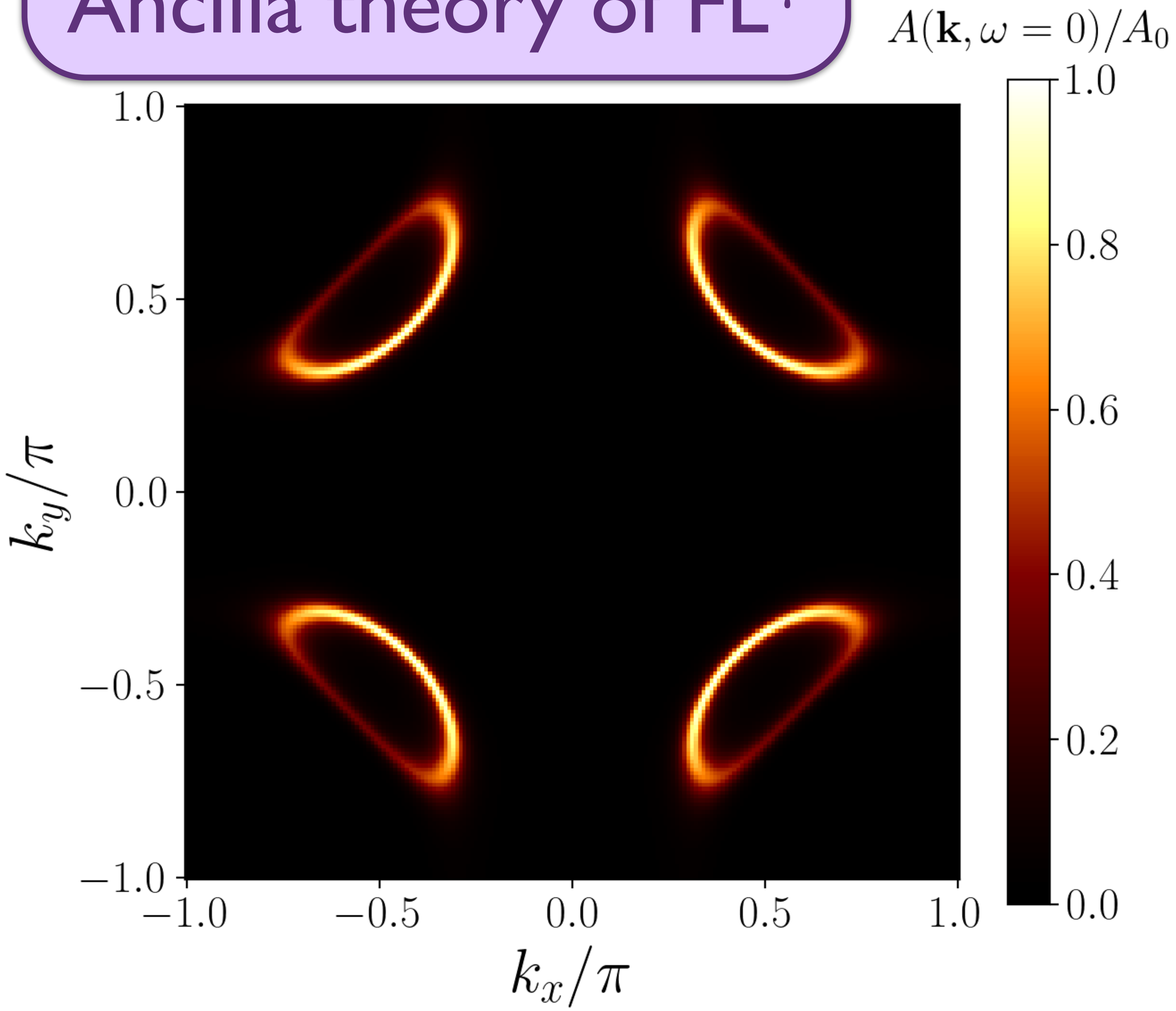
Size $1 + p + 1 = p$ (mod 2).
Small Fermi surface!

Your favorite spin liquid



E. Mascot,
A. Nikolaenko,
M. Tikhanovskaya,
Ya-Hui Zhang,
D. K. Morr,
and S. S.,
PRB **105**,
075146 (2022)

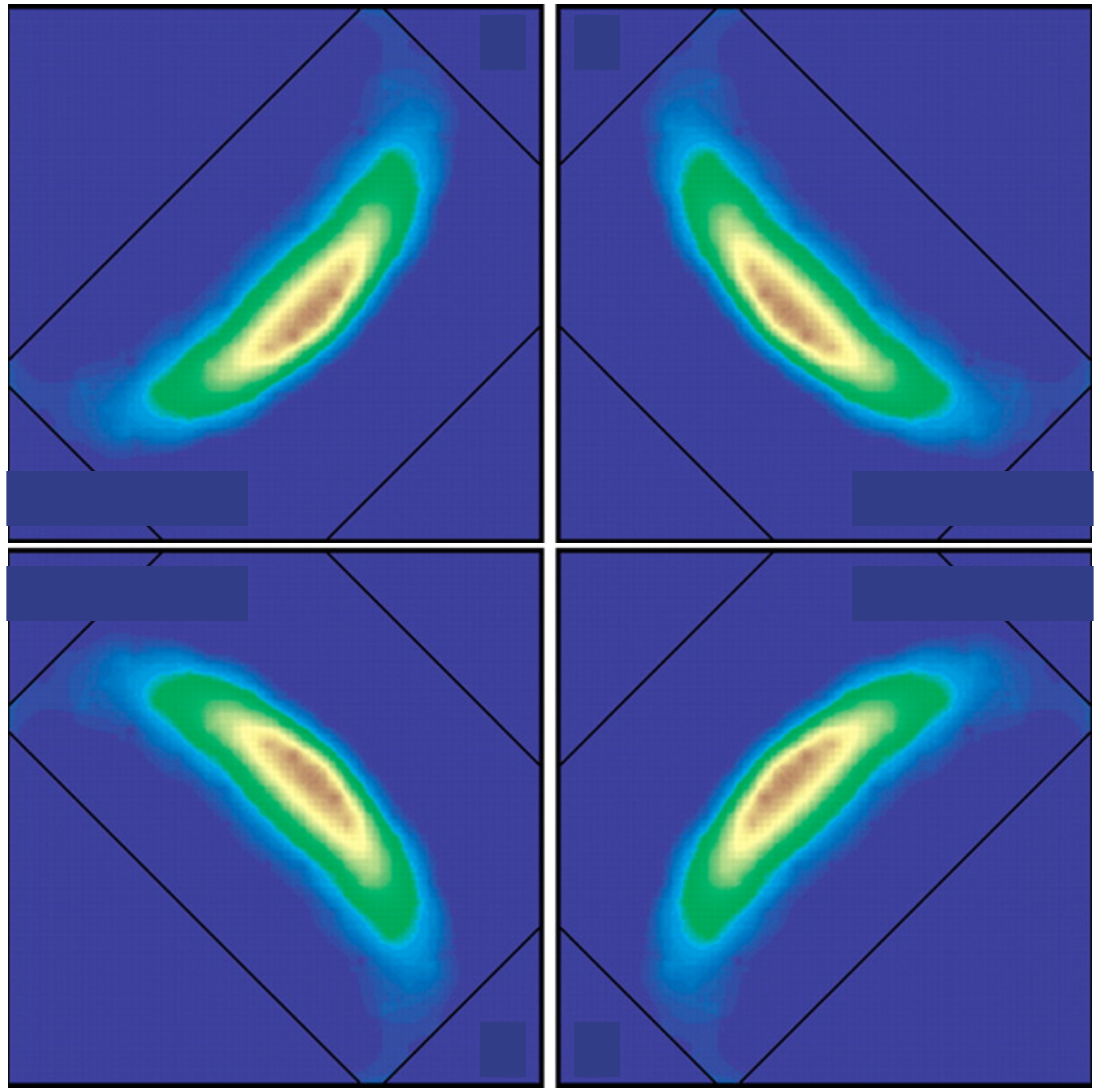
Ancilla theory of FL*



Decoupled Kondo lattice and spin liquid yields pockets of area $\underline{p/8}$.

Photoemission expts

$Ca_{2-x}Na_xCuO_2Cl_2$ at $x = 0.10$



Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

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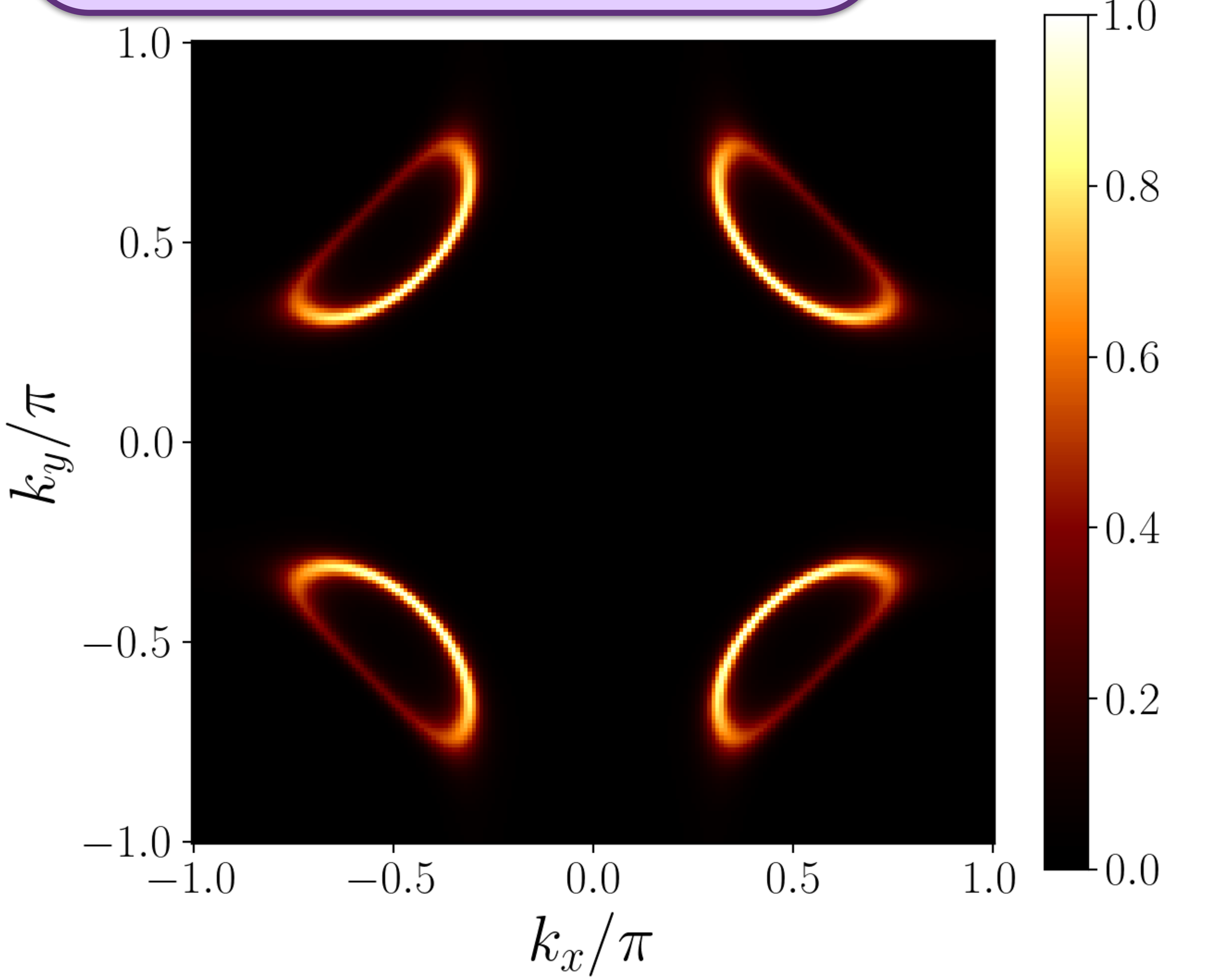
Ancilla theory of FL*

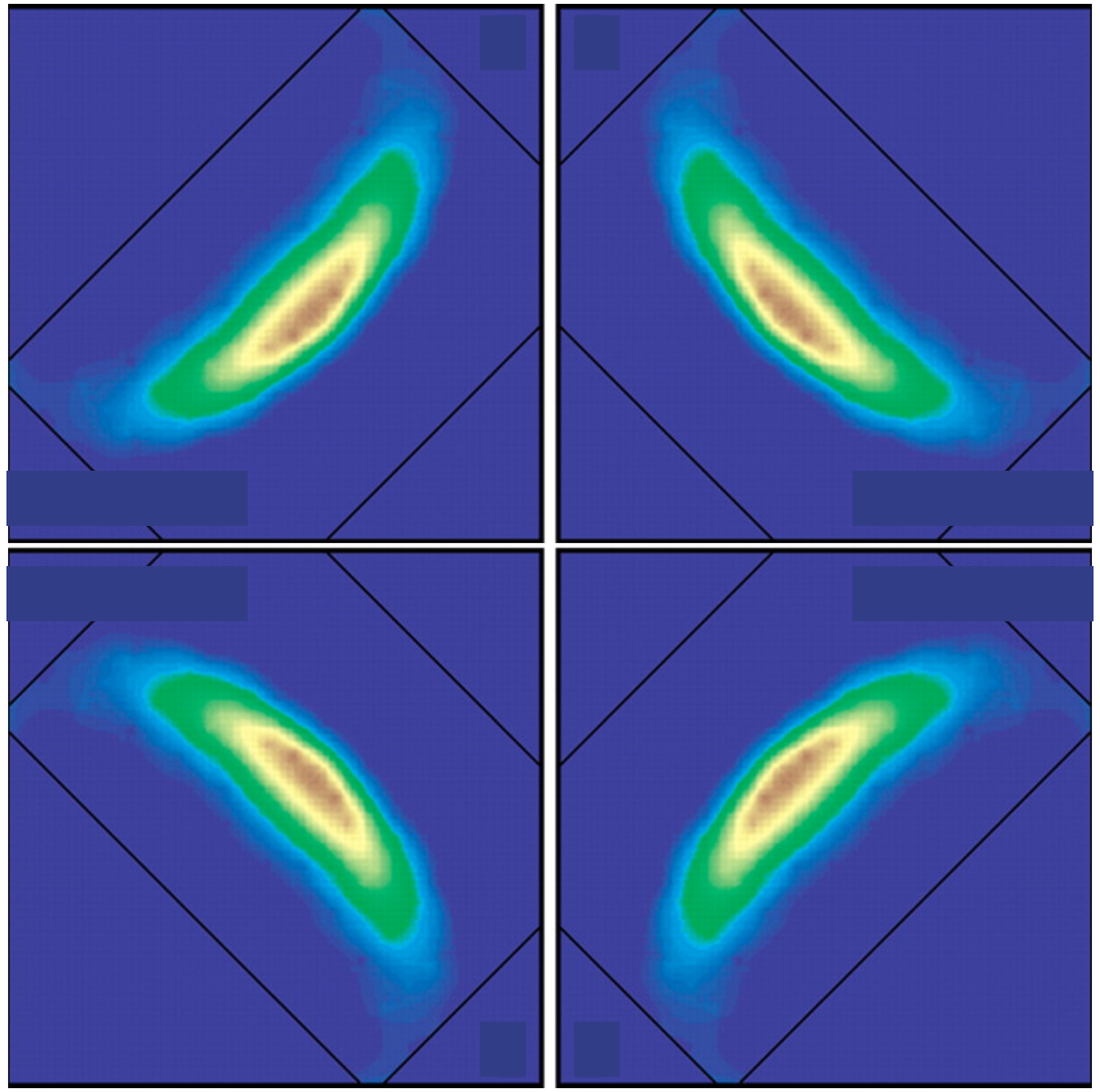


Decoupled Kondo lattice and spin liquid yields pockets of area $p/8$.

Hybridization of Fermi surfaces of size $1 + p$ and 1: imposed by "rigidity" of bottom layer spin liquid. (SDW theory has 2 Fermi surfaces of size $1 + p$)

Photoemission expts

$Ca_{2-x}Na_xCuO_2Cl_2$ at $x = 0.10$



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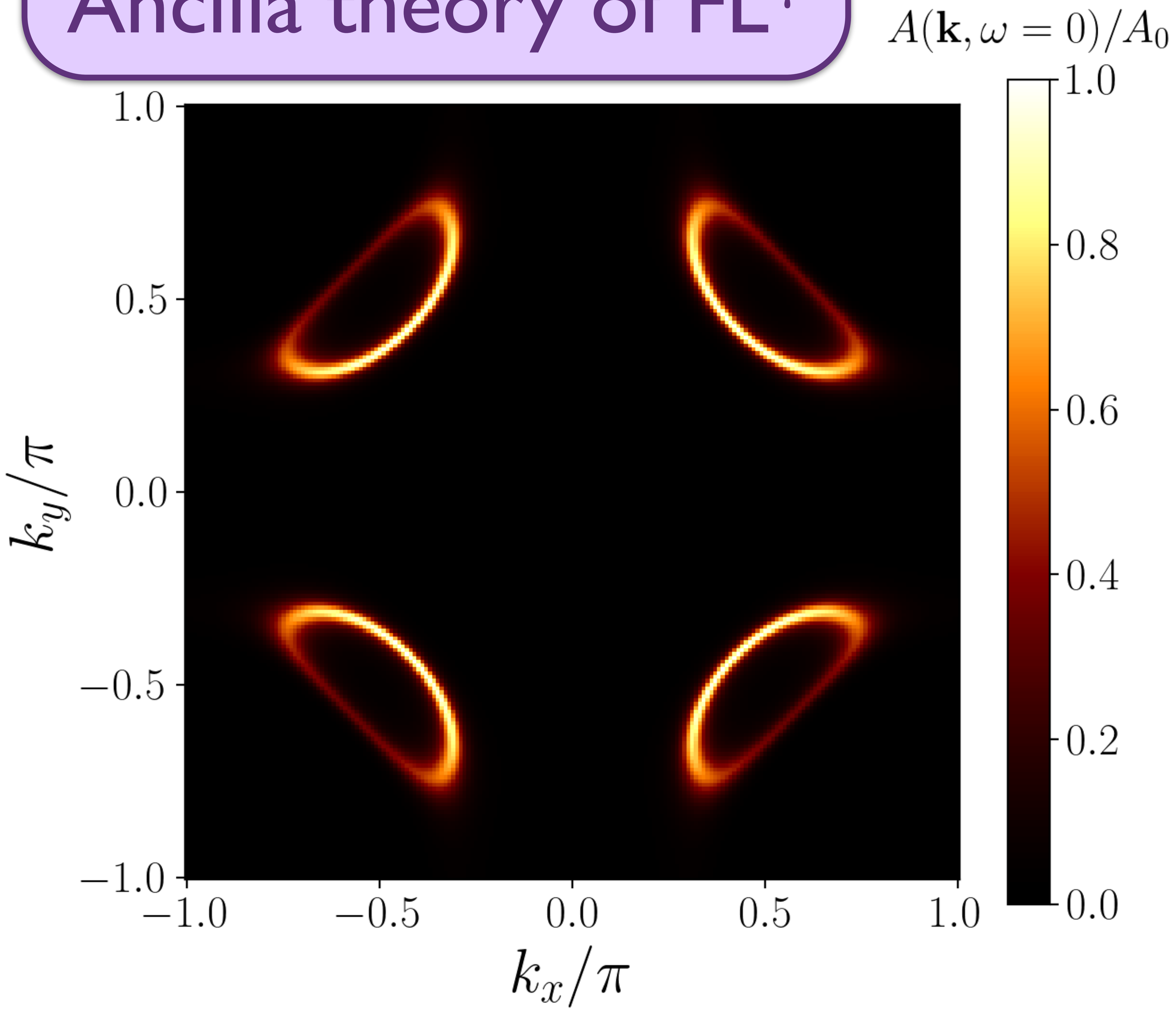
Ancilla theory of FL*

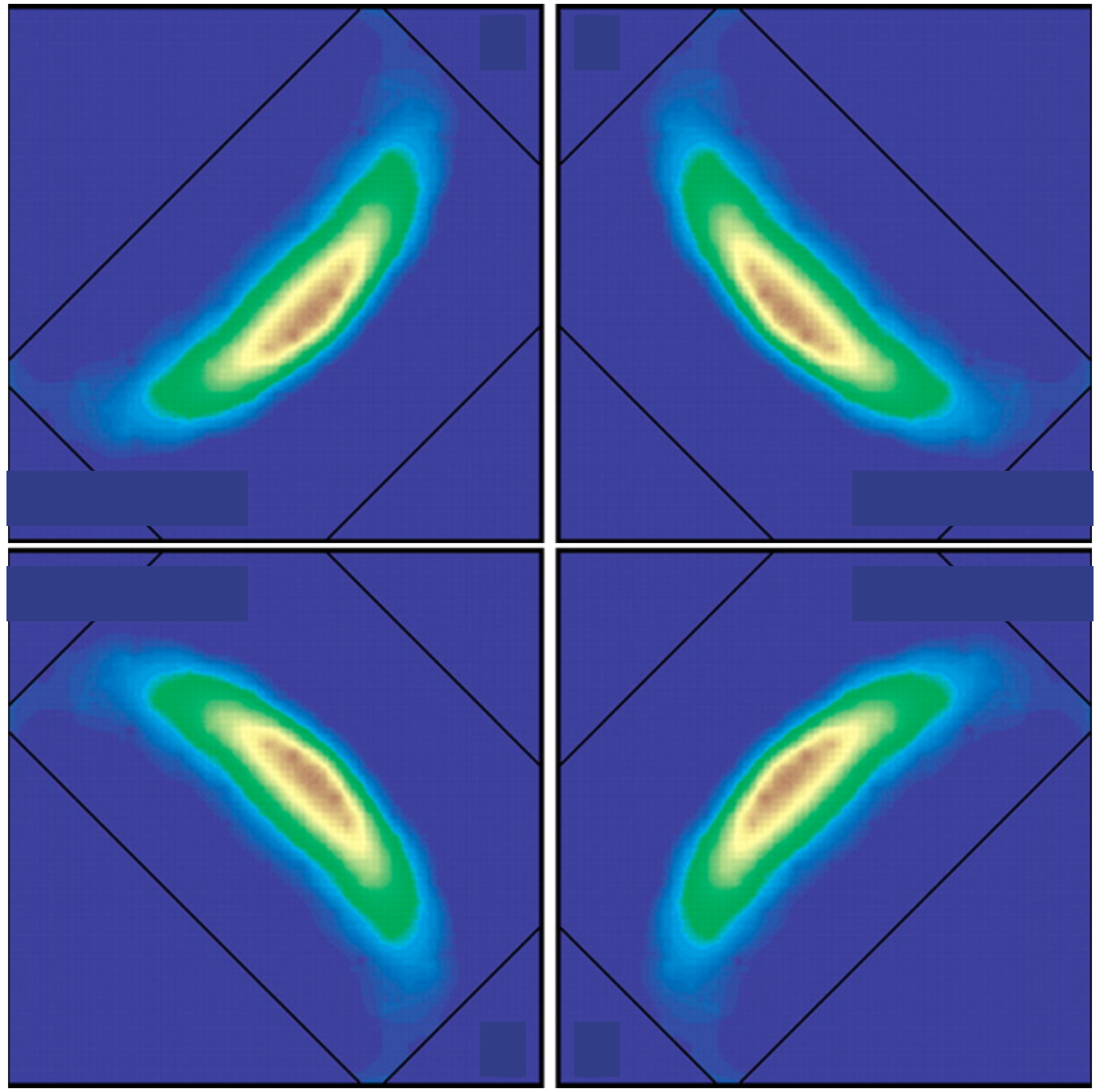


Decoupled Kondo lattice and spin liquid yields pockets of area $\underline{p/8}$.

But pocket backsides are not observed!

Photoemission expts

$Ca_{2-x}Na_xCuO_2Cl_2$ at $x = 0.10$



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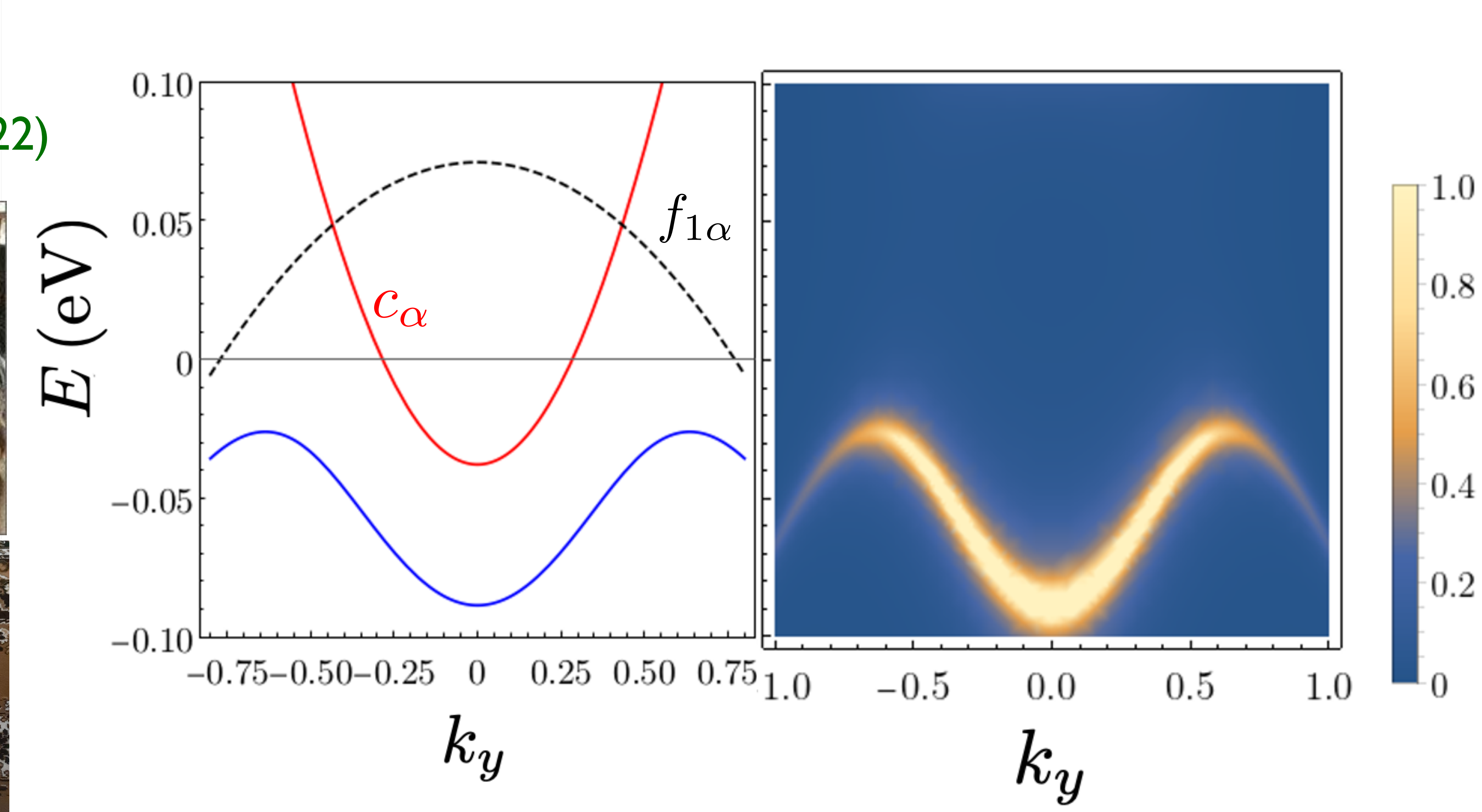
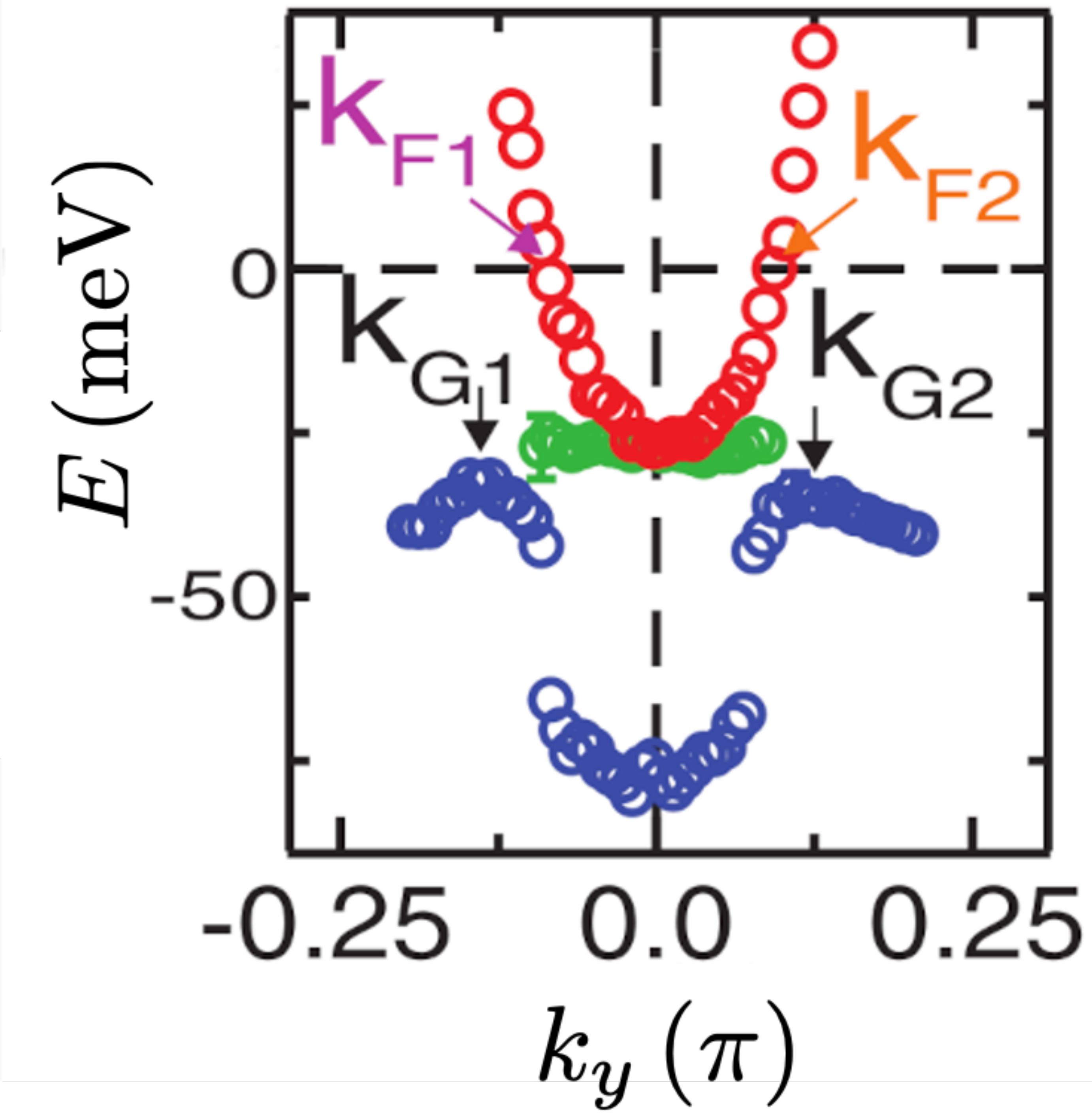
Ancilla theory of FL*



Decoupled Kondo lattice and spin liquid

Shift in k_F also related to
Hybridization of Fermi surfaces of size $1 + p$ and 1:
(SDW theory has 2 Fermi surfaces of size $1 + p$)

Photoemission expts



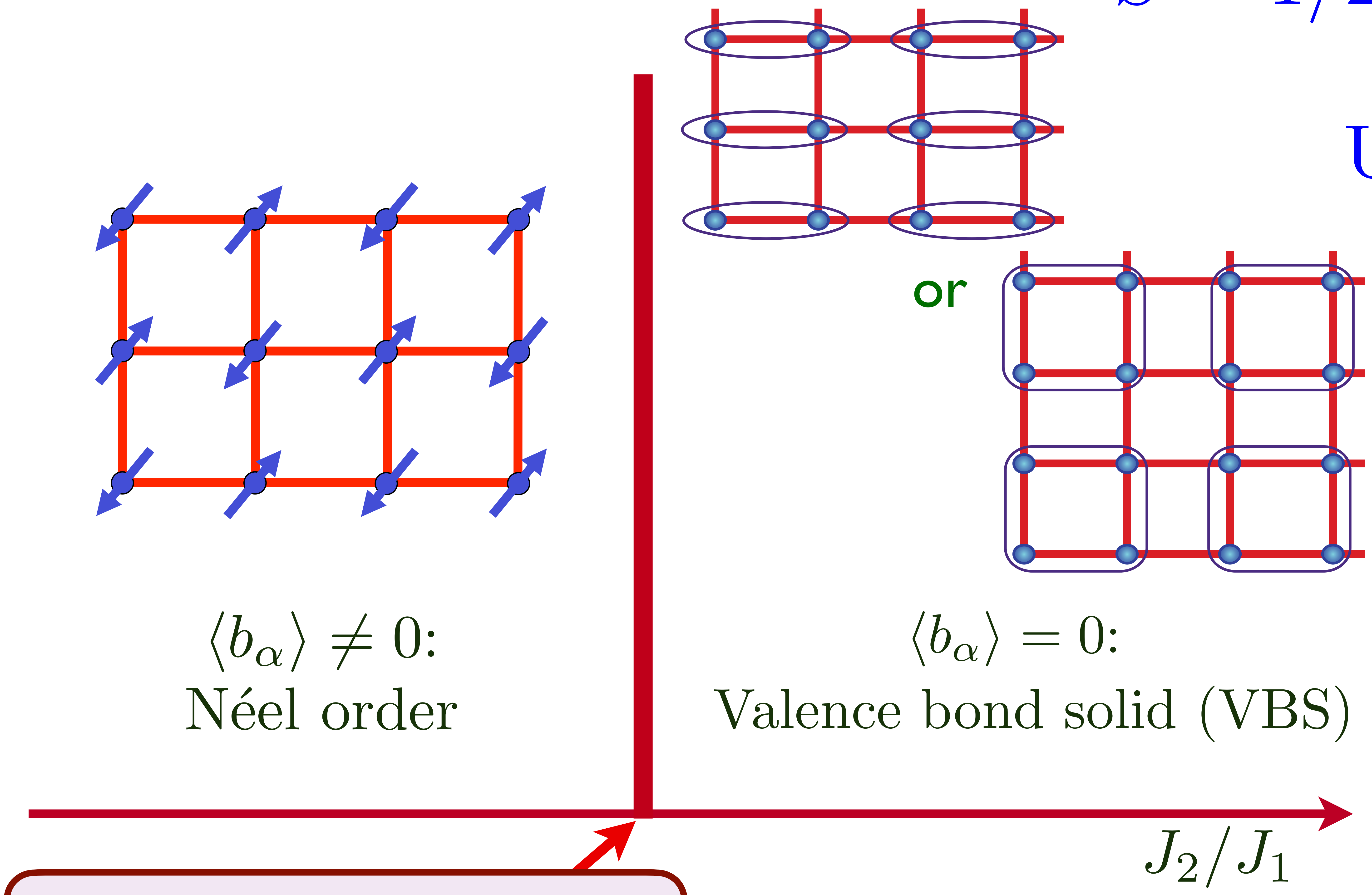
Bi2201

R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

Nearly-critical quantum spin liquid on the square lattice

S=1/2 square lattice

Represent spins in terms of
$S = 1/2$ bosonic spinons $\boldsymbol{S} \sim b_\alpha^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_\beta$



U(1) gauge symmetry: $b \to b e^{i\theta}$

$\mathbb{CP}^1$ U(1) gauge theory.

’t Hooft/Lieb-Schultz-Mattis anomalies realized by monopole Berry phases.

Critical spin liquid without quasiparticles?

$$\mathcal{L}_z = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Science **303**, 1490 (2004)

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989)
N. Read and S. Sachdev, Phys. Rev. B **42**, 4568 (1990)

S=1/2 square lattice

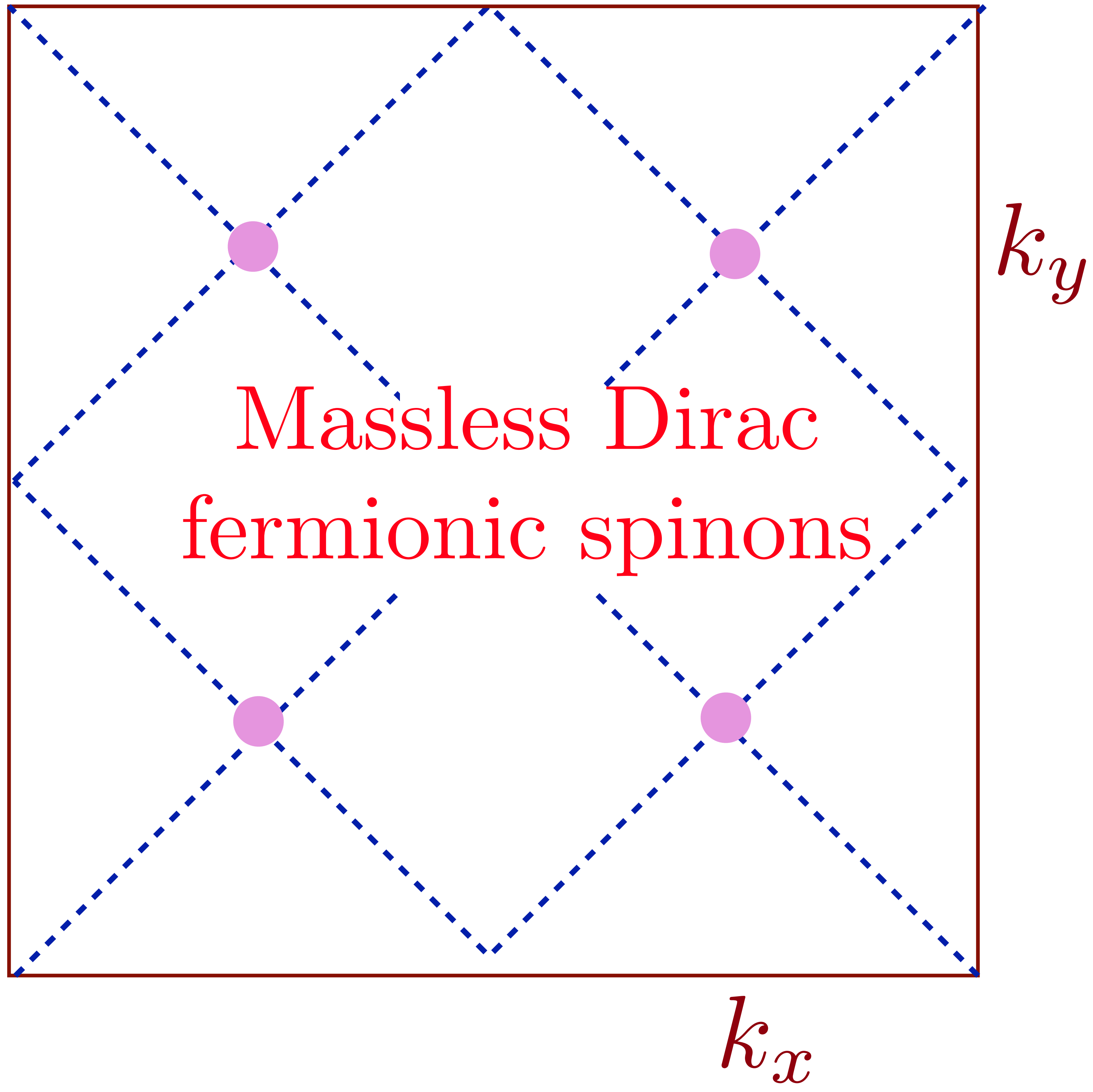
Represent spins in terms of
$S = 1/2$ fermionic spinons $\boldsymbol{S} \sim f_\alpha^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_\beta$

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

J_2/J_1

$$\mathcal{L} = i\overline{\psi}\gamma_\mu D_\mu \psi.$$

$N_f = 2$ SU(2) QCD



S=1/2 square lattice

Represent spins in terms of
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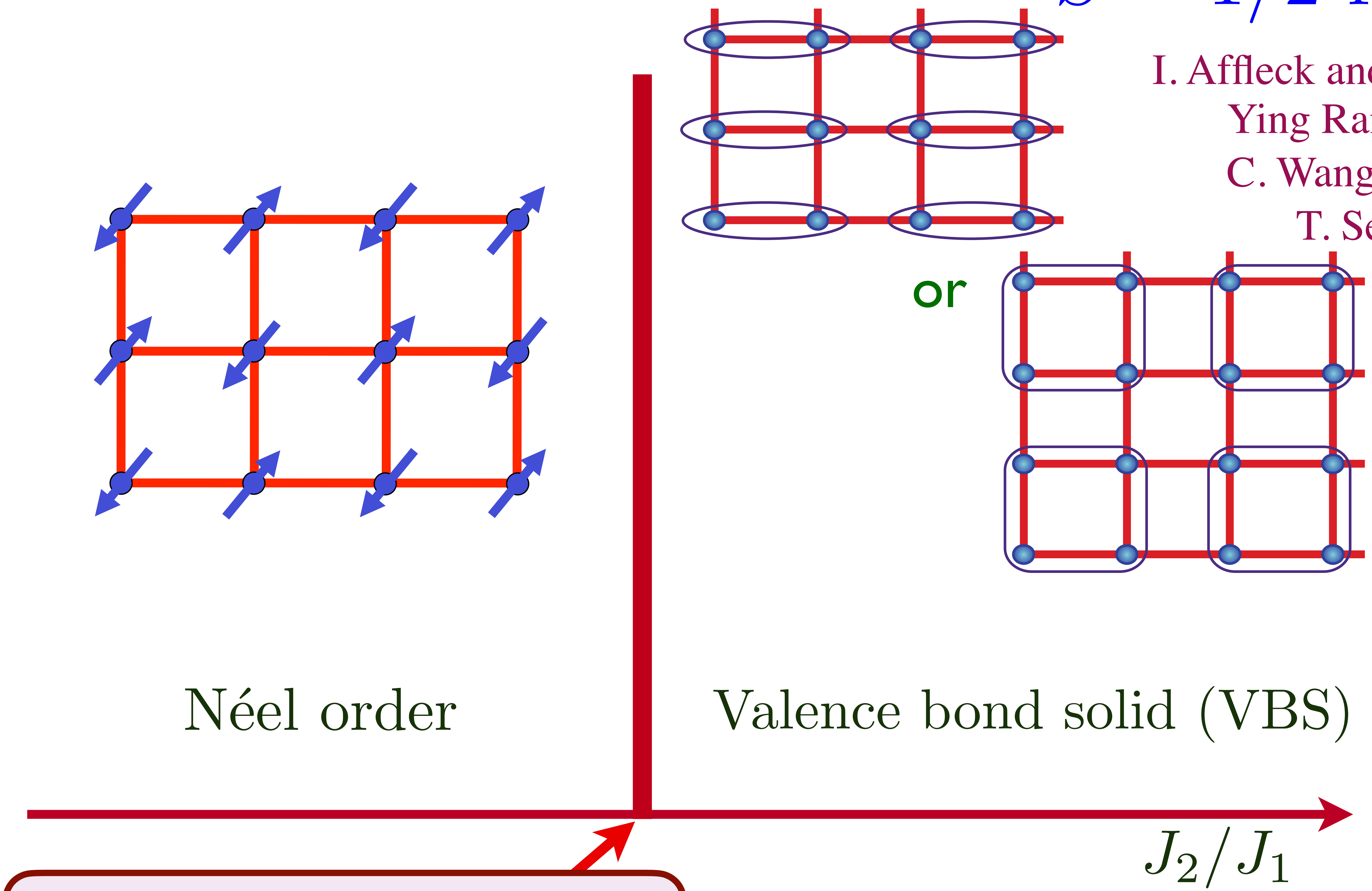
I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)
Ying Ran and X.-G. Wen, cond-mat/0609620
C. Wang, A. Nahum, M. A. Metlitski, C. Xu,
T. Senthil, *Phys. Rev. X* **7**, 031051 (2017)



Critical spin liquid
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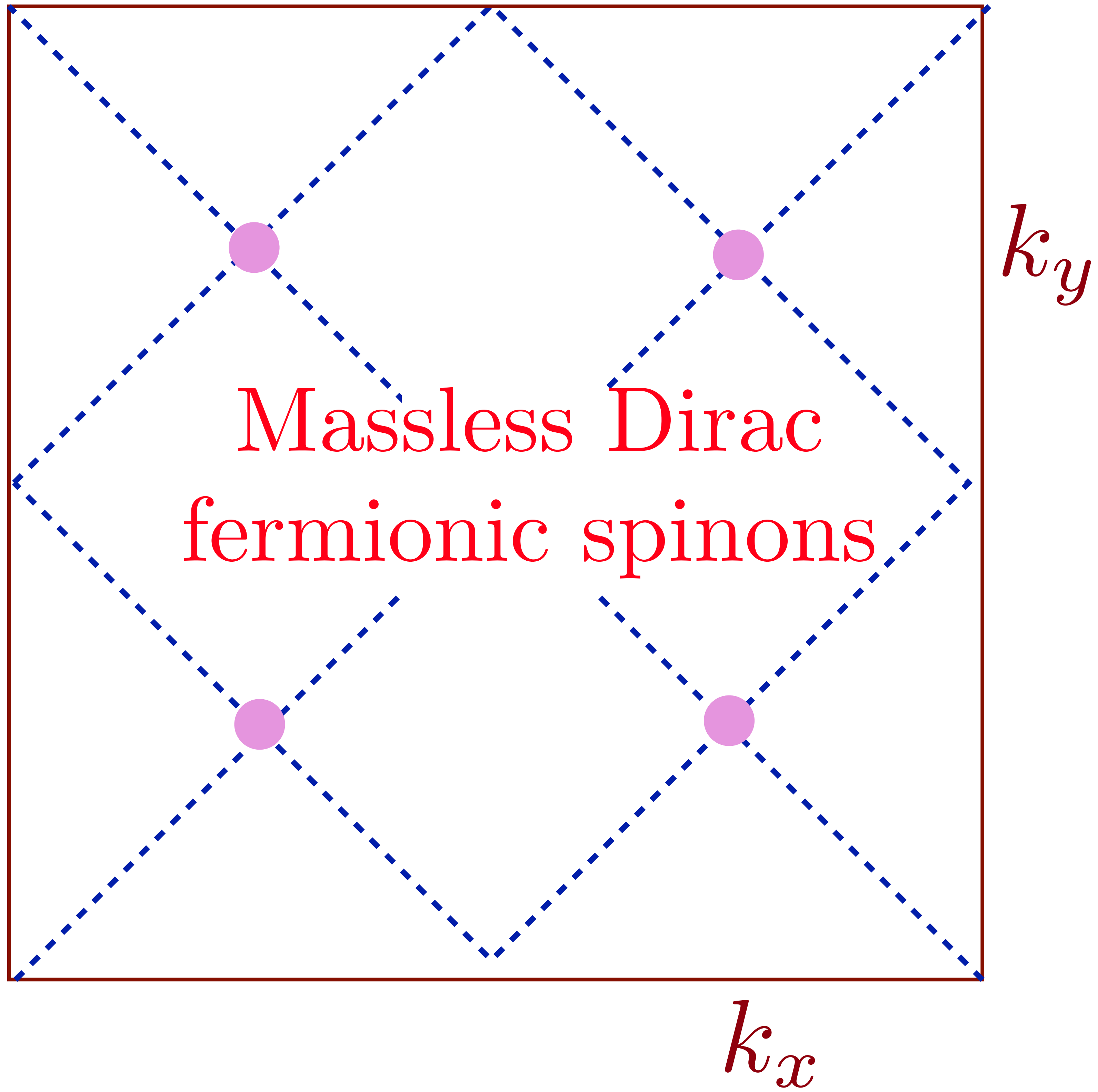
$N_f = 2$ SU(2) QCD

$$\mathcal{L} = i\overline{\psi}\gamma_\mu D_\mu \psi.$$



Confining instability also to Néel and VBS.

S=1/2 square lattice

Bosonic spinons:
$\mathbb{CP}^1$ U(1) gauge theory

Nearly-critical Néel/VBS spin liquid without quasiparticles obeying 't Hooft/LSM anomalies

SU(2) gauge theory of $N_f = 2$ fundamental, massless, Dirac fermions.

Obtained from a saddle-point of fermionic spinons moving in π-flux.

SO(5) non-linear σ-model of Néel/VBS orders with $k = 1$ WZW term

Many numerical works show that deconfined critical theory applies over a substantial length scale, but ultimately confines at the longest distances.

A. Tanaka and X. Hu, *Phys. Rev. Lett.* **95**, 036402 (2005); T. Senthil and M.P.A. Fisher *Phys. Rev. B* **74**, 064405 (2006); C. Wang, A. Nahum, M. A. Metlitski, C. Xu, T. Senthil, *Phys. Rev. X* **7**, 031051 (2017); Zheng Zhou, Liangdong Hu, Wei Zhu, Yin-Chen He, PRX **14**, 021044 (2024); S. M. Chester N. Su, PRL **132**, 111601 (2024). B.-B. Chen, X. Zhang, Y. Wang, K. Sun, Z. Y. Meng, PRL **132**, 246503 (2024); J. Takahashi, H. Shao, B. Zhao, W. Guo, A. W. Sandvik, arXiv:2405.06607.

SU(2) gauge theory of FL* pseudogap metal in single-band models using ancillas

Ancilla theory of FL*

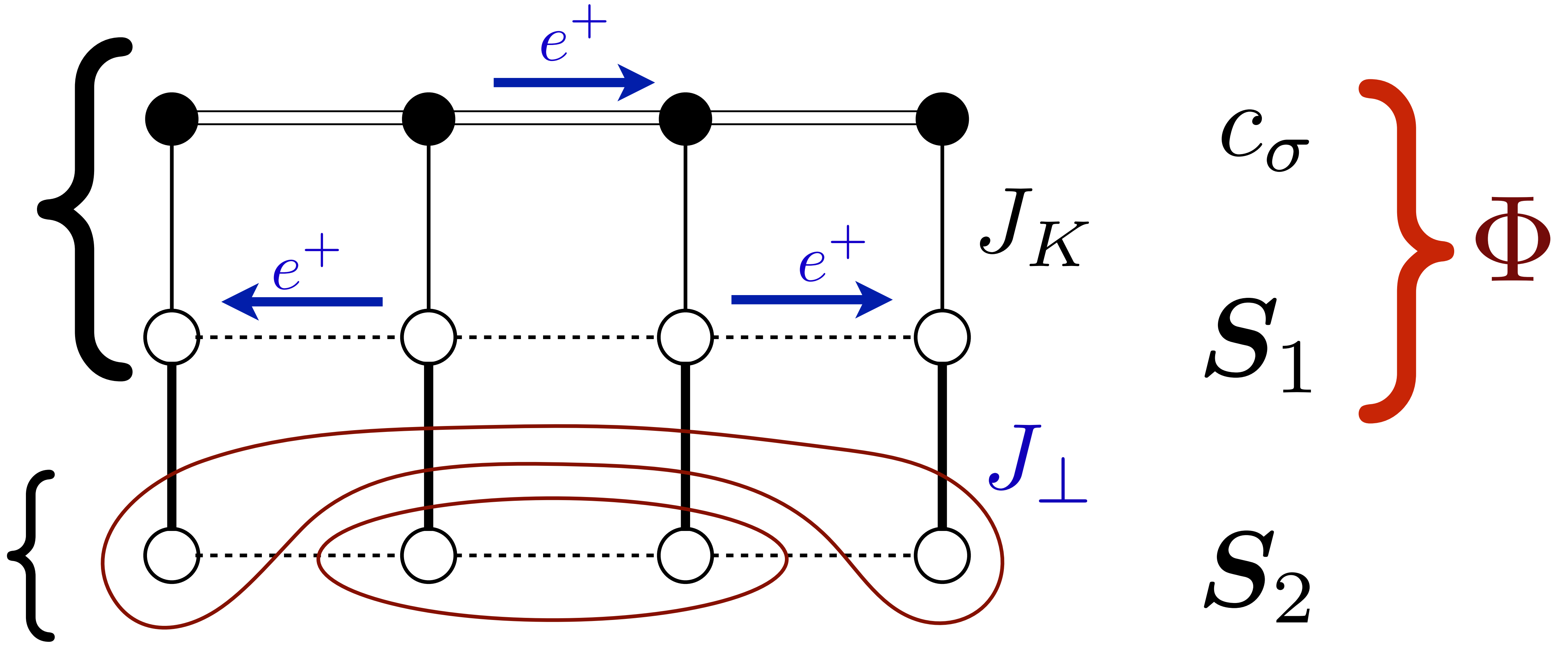
$$H_{\text{Kondo lattice}} = \sum_{\boldsymbol{i},\boldsymbol{j}} \left[-t_{\boldsymbol{ij}} c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} - t_{1,\boldsymbol{ij}} f_{1\boldsymbol{i}\alpha}^\dagger f_{1\boldsymbol{j}\alpha} \right] - \sum_{\boldsymbol{i}} \Phi \left(c_{\boldsymbol{i}\alpha}^\dagger f_{1\boldsymbol{i}\alpha} + f_{1\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right)$$

Heavy Fermi liquid of electrons c, f_1
$\boldsymbol{S}_1 \sim f_{1\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{1\beta}$

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Kondo lattice heavy Fermi liquid.
Size $1 + p + 1 = p$ (mod 2).
Small Fermi surface!

Your favorite spin liquid



Ancilla theory of FL*

$$H_{\text{Kondo lattice}} = \sum_{\boldsymbol{i},\boldsymbol{j}} \left[-t_{\boldsymbol{ij}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} - t_{1,\boldsymbol{ij}} f^\dagger_{1\boldsymbol{i}\alpha} f_{1\boldsymbol{j}\alpha} \right] - \sum_{\boldsymbol{i}} \Phi \left(c^\dagger_{\boldsymbol{i}\alpha} f_{1\boldsymbol{i}\alpha} + f^\dagger_{1\boldsymbol{i}\alpha} c_{\boldsymbol{i}\alpha} \right)$$

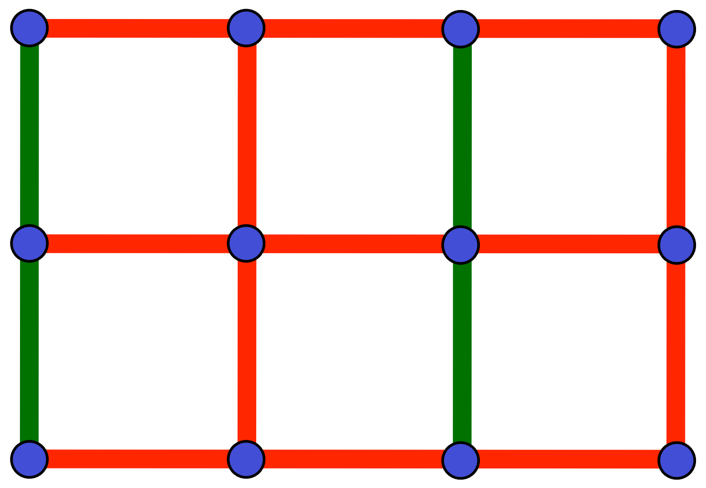
Heavy Fermi liquid of electrons c, f_1

$\boldsymbol{S}_1 \sim f^\dagger_{1\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{1\beta}$

$$H_{\text{spin liquid}} = iJ \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(\Psi^\dagger_{\boldsymbol{i}} U_{\boldsymbol{ij}} \Psi_{\boldsymbol{j}} - \Psi^\dagger_{\boldsymbol{j}} U_{\boldsymbol{ji}} \Psi_{\boldsymbol{i}} \right) ; \quad \Psi_{\boldsymbol{i}} = \begin{pmatrix} f_{\boldsymbol{i}\uparrow} \\ f^\dagger_{\boldsymbol{i}\downarrow} \end{pmatrix}$$

π-flux $\boldsymbol{S}_2$ spin liquid.

$\boldsymbol{S}_2 = f^\dagger_\alpha \boldsymbol{\sigma}_{\alpha\beta} f_\beta$



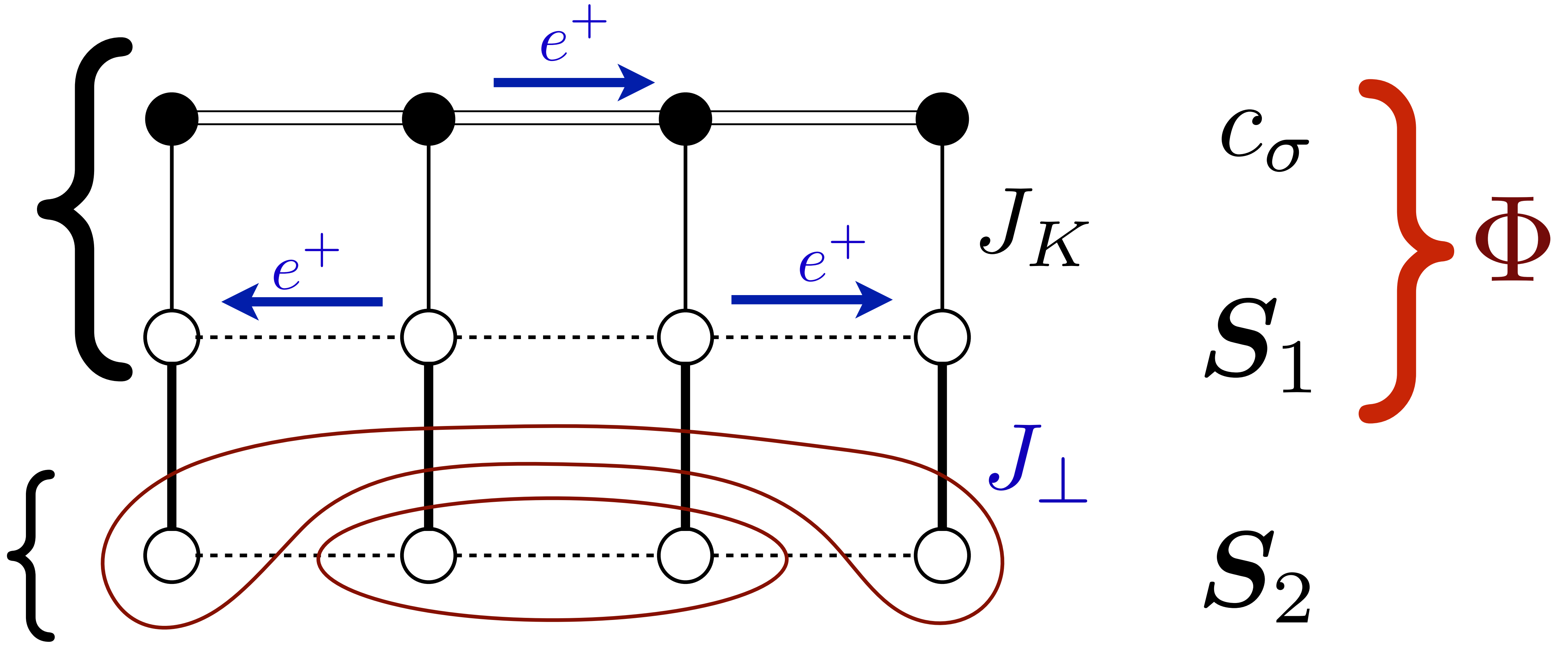
$e_{\boldsymbol{ij}} = 1$

$e_{\boldsymbol{ij}} = -1$

Fermionic spinons f moving in π-flux and an emergent SU(2) gauge field U

Kondo lattice heavy Fermi liquid.
Size $1 + p + 1 = p$ (mod 2).
Small Fermi surface!

π-flux spin liquid



Ancilla theory of FL*

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Heavy Fermi liquid of electrons c, f_1
$\boldsymbol{S}_1 \sim f^\dagger_{1\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{1\beta}$

$$H_{\text{spin liquid}} = iJ \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(\Psi^\dagger_{\boldsymbol{i}} U_{\boldsymbol{ij}} \Psi_{\boldsymbol{j}} - \Psi^\dagger_{\boldsymbol{j}} U_{\boldsymbol{ji}} \Psi_{\boldsymbol{i}} \right) ; \quad \Psi_{\boldsymbol{i}} = \begin{pmatrix} f_{\boldsymbol{i}\uparrow} \\ f^\dagger_{\boldsymbol{i}\downarrow} \end{pmatrix}$$

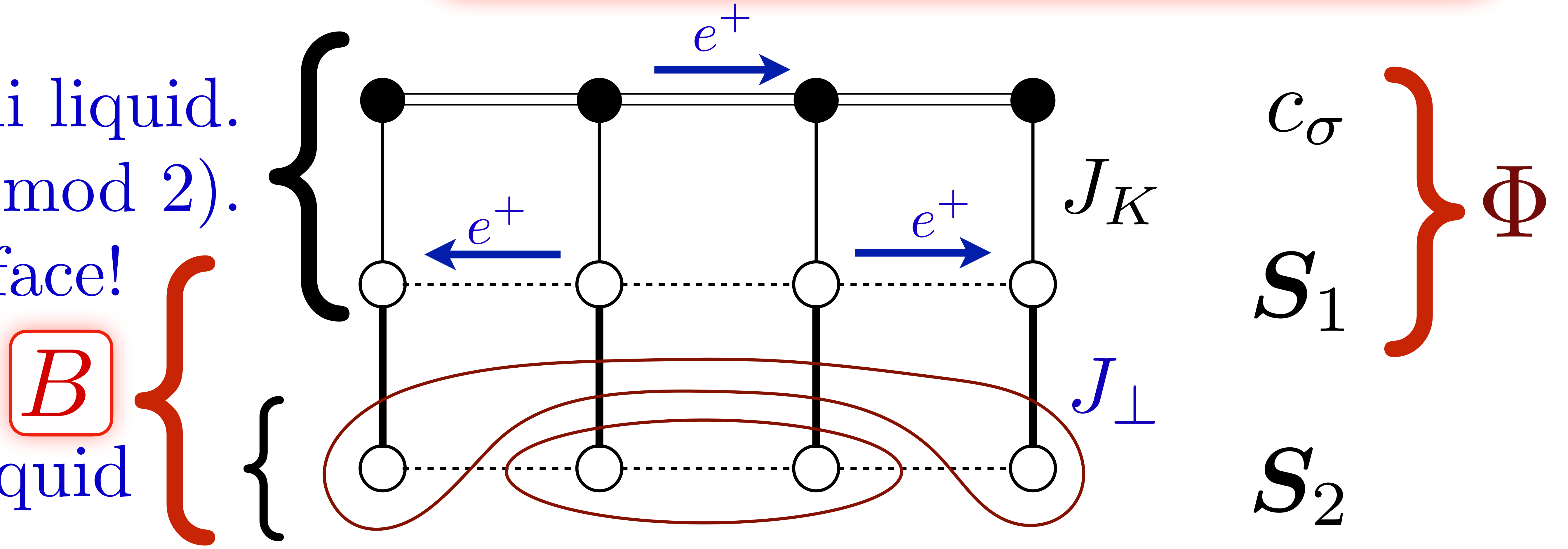
π-flux $\boldsymbol{S}_2$ spin liquid.
$\boldsymbol{S}_2 = f^\dagger_\alpha \boldsymbol{\sigma}_{\alpha\beta} f_\beta$

$$H_{\text{coupling}} = \sum_{\boldsymbol{i}} \left(B^*_{1\boldsymbol{i}} f^\dagger_{1\boldsymbol{i}\alpha} f_{\boldsymbol{i}\alpha} + B^*_{2\boldsymbol{i}} \varepsilon_{\alpha\beta} f^\dagger_{1\boldsymbol{i}\alpha} f^\dagger_{\boldsymbol{i}\beta} + \text{H.c.} \right)$$

Couple Kondo lattice and spin liquid by charge e, SU(2) fundamental Higgs boson B

Kondo lattice heavy Fermi liquid.
Size $1 + p + 1 = p$ (mod 2).
Small Fermi surface!

π-flux spin liquid



Ancilla theory of FL*

$$H_{\text{Kondo lattice}} = \sum_{i,j} \left[-t_{ij} c^\dagger_{i\alpha} c_{j\alpha} - t_{1,ij} f^\dagger_{1i\alpha} f_{1j\alpha} \right] - \sum_i \Phi \left(c^\dagger_{i\alpha} f_{1i\alpha} + f^\dagger_{1i\alpha} c_{i\alpha} \right)$$

Heavy Fermi liquid of electrons c, f_1
$\boldsymbol{S}_1 \sim f^\dagger_{1\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{1\beta}$

$$H_{\text{spin liquid}} = iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi^\dagger_i U_{ij} \Psi_j - \Psi^\dagger_j U_{ji} \Psi_i \right) ; \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f^\dagger_{i\downarrow} \end{pmatrix}$$

π-flux $\boldsymbol{S}_2$ spin liquid.
$\boldsymbol{S}_2 = f^\dagger_\alpha \boldsymbol{\sigma}_{\alpha\beta} f_\beta$

$$H_{\text{coupling}} = \sum_i \left(B^*_{1i} f^\dagger_{1i\alpha} f_{i\alpha} + B^*_{2i} \varepsilon_{\alpha\beta} f^\dagger_{1i\alpha} f^\dagger_{i\beta} + \text{H.c.} \right)$$

$$V_{\text{Higgs}} = \mathcal{E}_2[B, U] + \mathcal{E}_4[B, U]$$

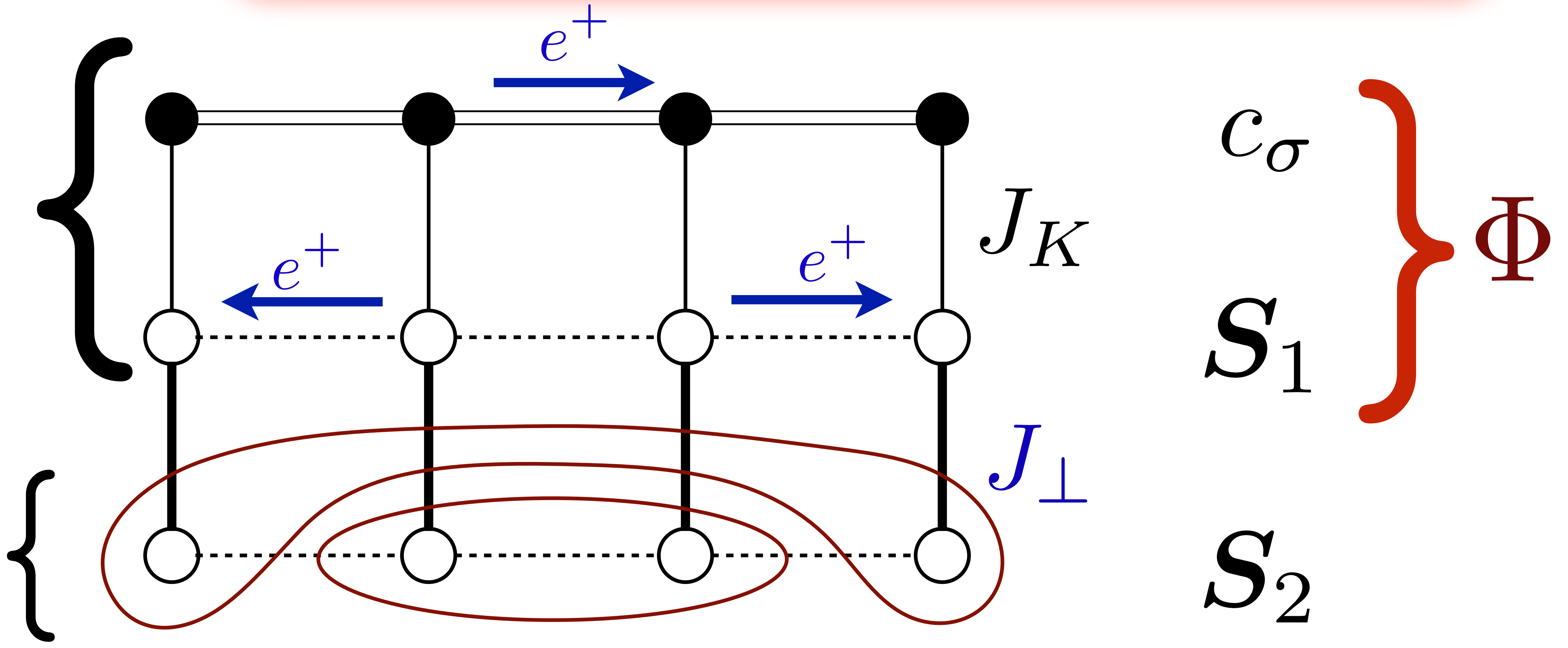
Couple Kondo lattice and spin liquid by charge e, SU(2) fundamental Higgs boson B

V_{Higgs} dictated by symmetry of spin liquid

Kondo lattice heavy Fermi liquid.
Size $1 + p + 1 = p \pmod 2$.
Small Fermi surface!

B

π-flux spin liquid



Adding charge fluctuations to the π-flux spin liquid

$$\mathcal{E}_2[B,U] = r\sum_{\boldsymbol{i}} B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}} + iw\sum_{\langle \boldsymbol{ij}\rangle} e_{\boldsymbol{ij}}\left(B_{\boldsymbol{i}}^\dagger U_{\boldsymbol{ij}} B_{\boldsymbol{j}} - B_{\boldsymbol{j}}^\dagger U_{\boldsymbol{ji}} B_{\boldsymbol{i}}\right)$$

$$\mathcal{E}_4[B,U] = \frac{u}{2}\sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}}^2 + V_1\sum_{\boldsymbol{i}}\rho_{\boldsymbol{i}}\left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{y}}}\right) + g\sum_{\langle \boldsymbol{ij}\rangle}\left|\Delta_{\boldsymbol{ij}}\right|^2 + J_1\sum_{\langle \boldsymbol{ij}\rangle} Q_{\boldsymbol{ij}}^2 + K_1\sum_{\langle \boldsymbol{ij}\rangle} J_{\boldsymbol{ij}}^2$$

$$+ V_{11}\sum_{\boldsymbol{i}}\rho_{\boldsymbol{i}}\left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}+\hat{\boldsymbol{y}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}-\hat{\boldsymbol{y}}}\right) + V_{22}\sum_{\boldsymbol{i}}\rho_{\boldsymbol{i}}\left(\rho_{\boldsymbol{i}+2\hat{\boldsymbol{x}}+2\hat{\boldsymbol{y}}} + \rho_{\boldsymbol{i}+2\hat{\boldsymbol{x}}-2\hat{\boldsymbol{y}}}\right)$$

site charge density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{i}\alpha}\right\rangle \sim \rho_{\boldsymbol{i}} = B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}}$

bond density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha}\right\rangle \sim Q_{\boldsymbol{ij}} = Q_{\boldsymbol{ji}} = \mathrm{Im}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}}\right)$

bond current: $i\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} - c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha}\right\rangle \sim J_{\boldsymbol{ij}} = -J_{\boldsymbol{ji}} = \mathrm{Re}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}}\right)$

Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta}\rangle \sim \Delta_{\boldsymbol{ij}} = \Delta_{\boldsymbol{ji}} = \varepsilon_{ab} B_{a\boldsymbol{i}} e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{b\boldsymbol{j}}\,.$

M. Christos, Zhu-Xi Luo, H. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120**, e2302701120 (2023)

Born-Oppenheimer theory of FL* pseudogap

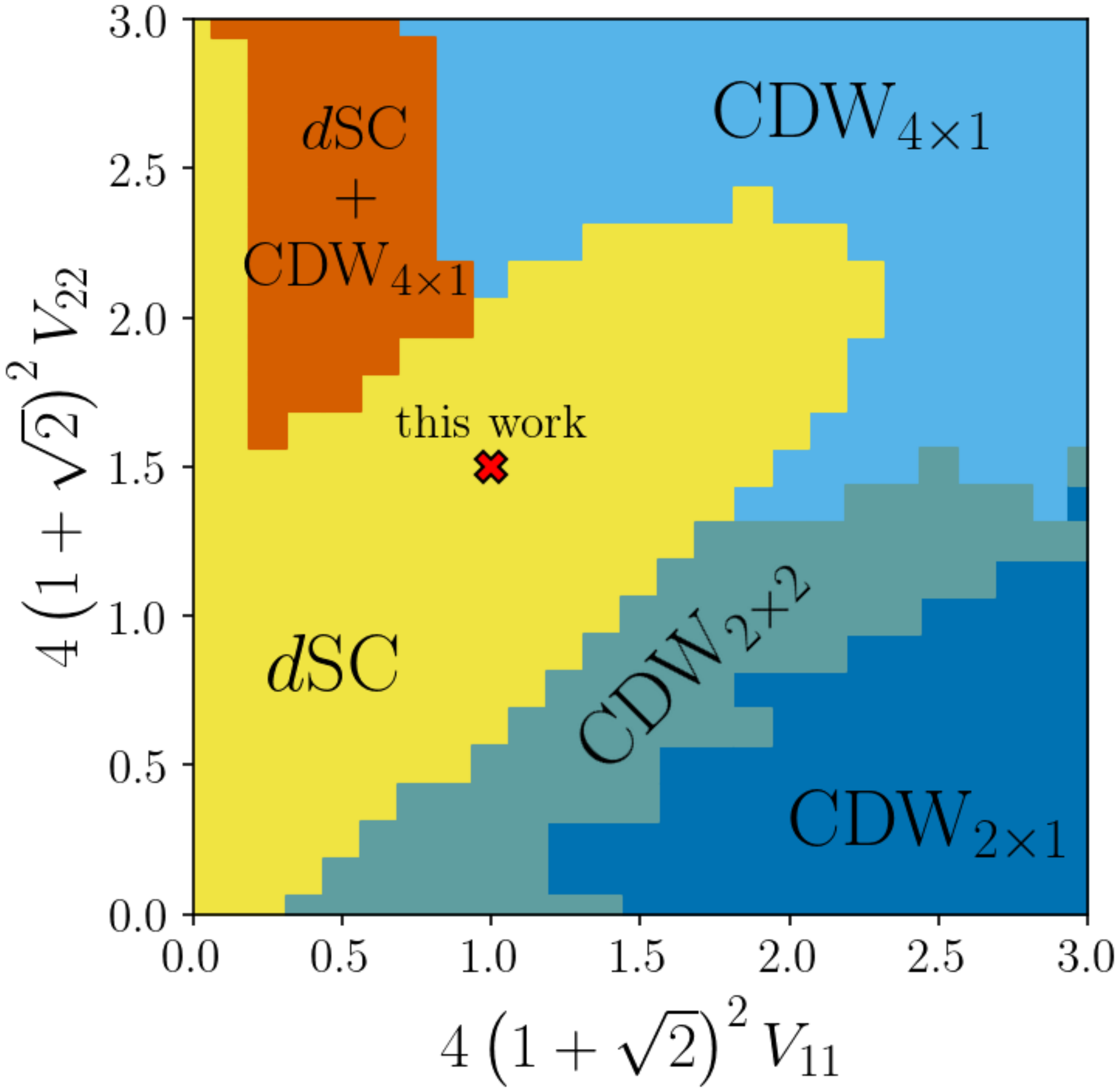
$$\mathcal{Z}_{2+0} = \int \prod_{i} \mathcal{D}B_{\boldsymbol{i}} \int \prod_{\langle \boldsymbol{ij} \rangle} \mathcal{D}U_{\boldsymbol{ij}} \exp\left[- \left(V_{\rm Higgs}[B,U] - \kappa \sum_{\square} {\rm Re Tr} \prod_{\boldsymbol{ij} \in \square} U_{\boldsymbol{ij}} \right) / T \right]$$

- Simulation of classical, thermal theory for bosons B, U defined by $\mathcal{Z}_{2+0}$
- Diagonalize 3-layer fermion Hamiltonian for c, f_1, f for each snapshot of B, U, and average.

Born-Oppenheimer theory of FL* pseudogap

$T = 0$ phase diagram

V_{Higgs} chosen so that the ground state is a d-wave superconductor, and second best state is a period-4 stripe.

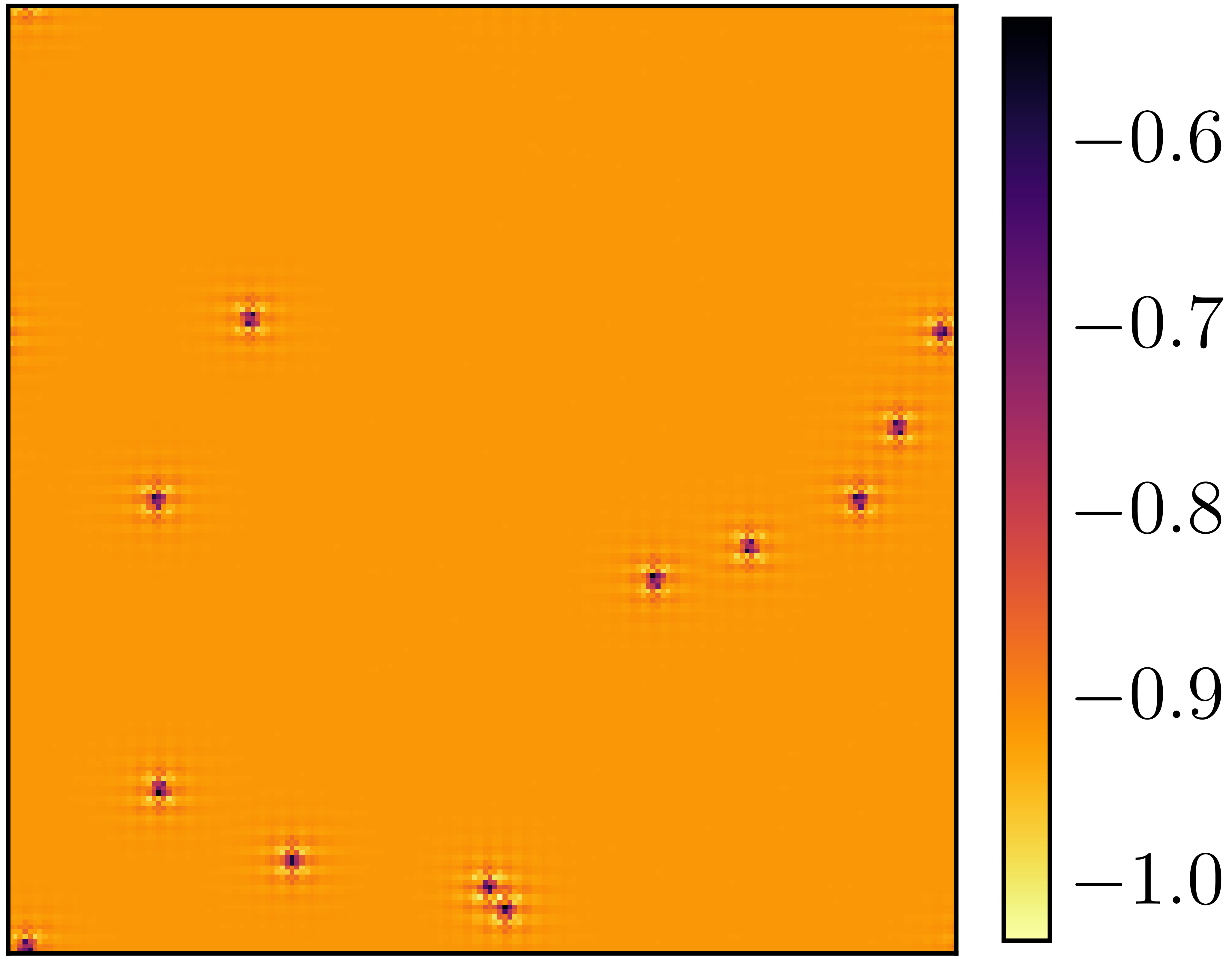


H. Pandey,
M. Christos,
P.M. Bonetti,
R. Shanker,
S. Sharma,
S.S.,
arXiv:2507.05336

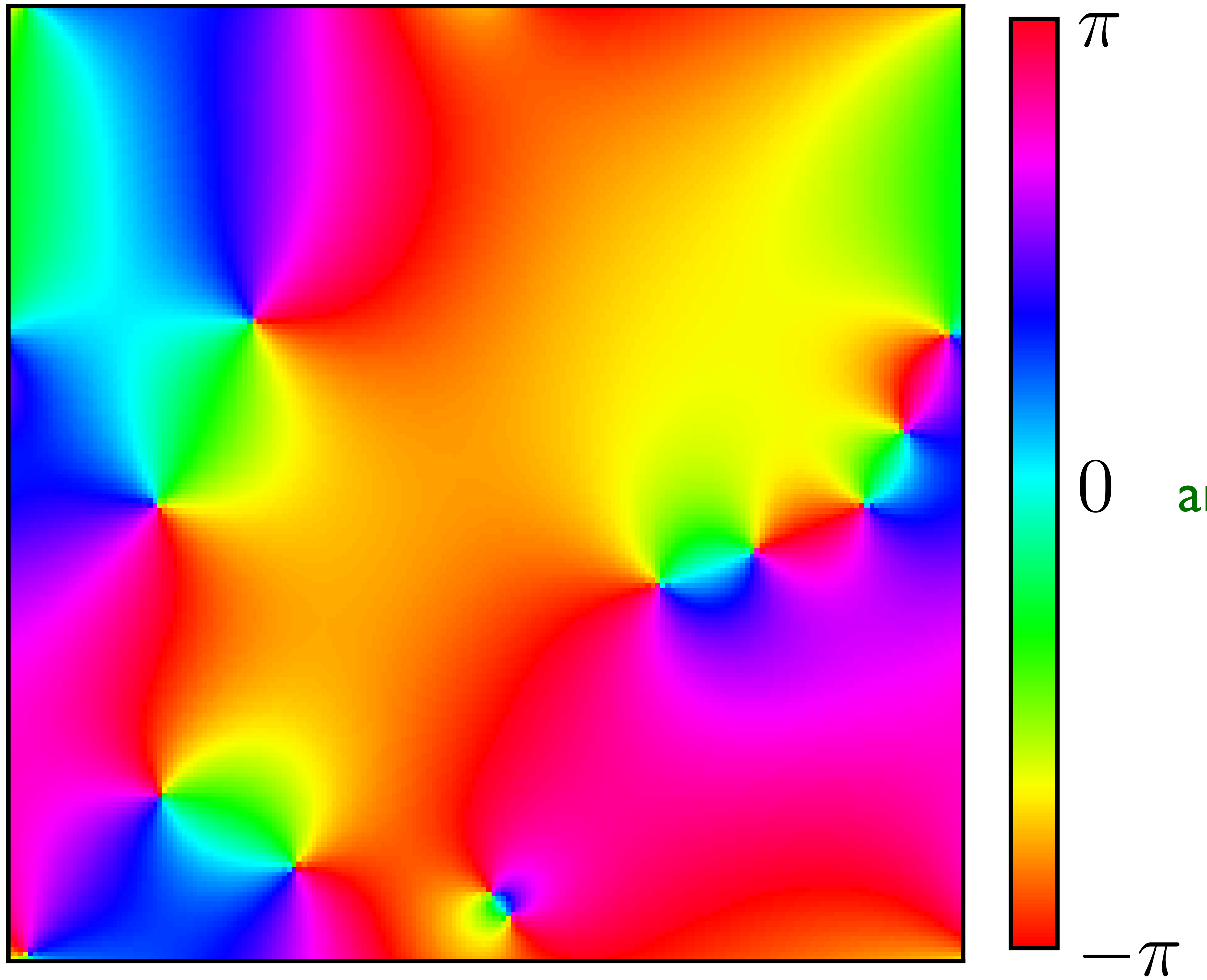
Minimize V_{Higgs} w.r.t. $\left\langle \begin{pmatrix} B_{1i} \\ B_{2i} \end{pmatrix} \right\rangle$

Set $U_{ij} = 1$.

Born-Oppenheimer theory of FL* pseudogap



Bond density

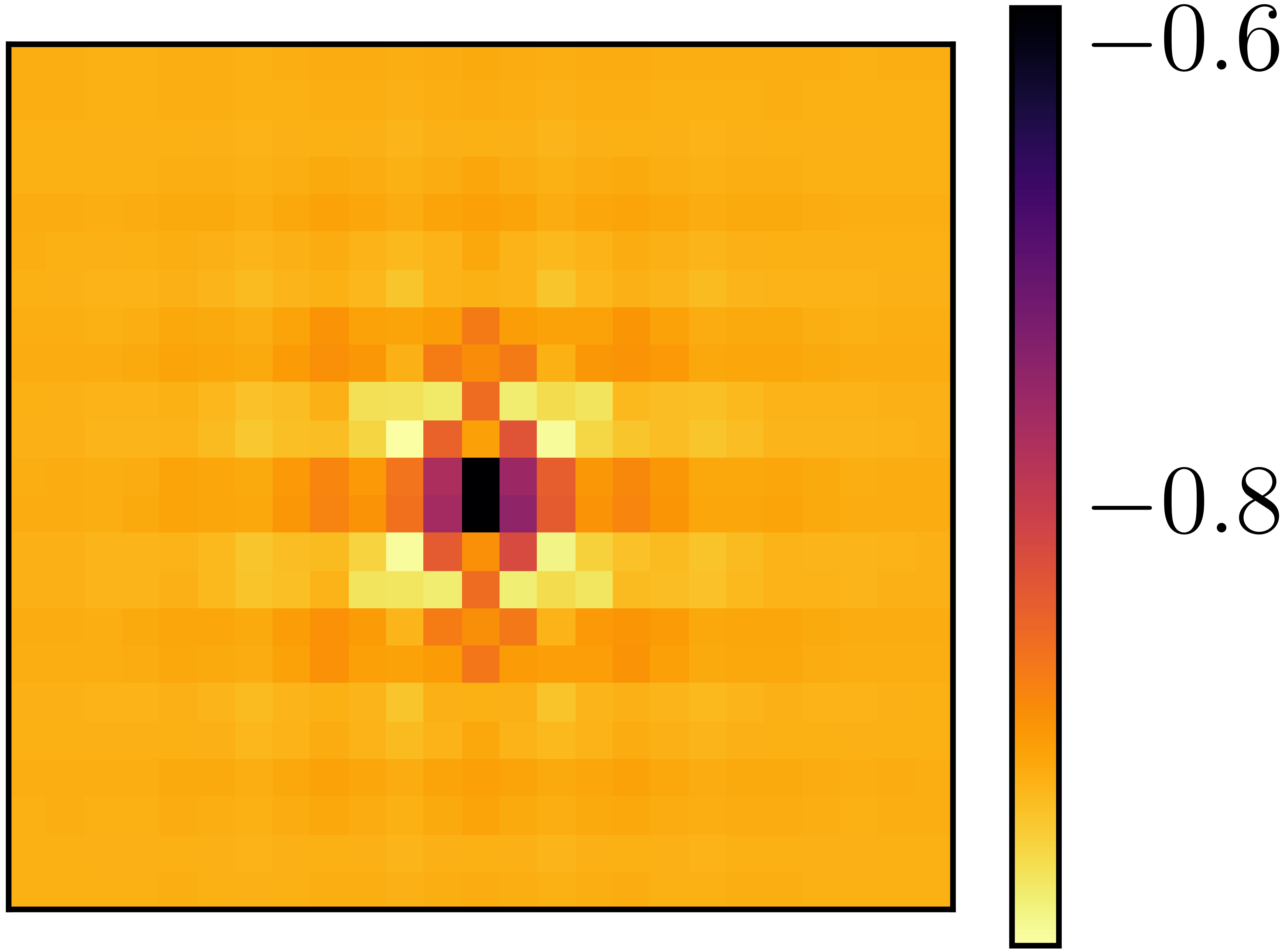


Phase of pairing amplitude

H. Pandey,
M. Christos,
P.M. Bonetti,
R. Shanker,
S. Sharma,
S.S.,
arXiv:2507.05336

See also
Jia-Xin Zhang
and S. S.,
PRB **110**,
235120
(2024)

Born-Oppenheimer theory of FL* pseudogap



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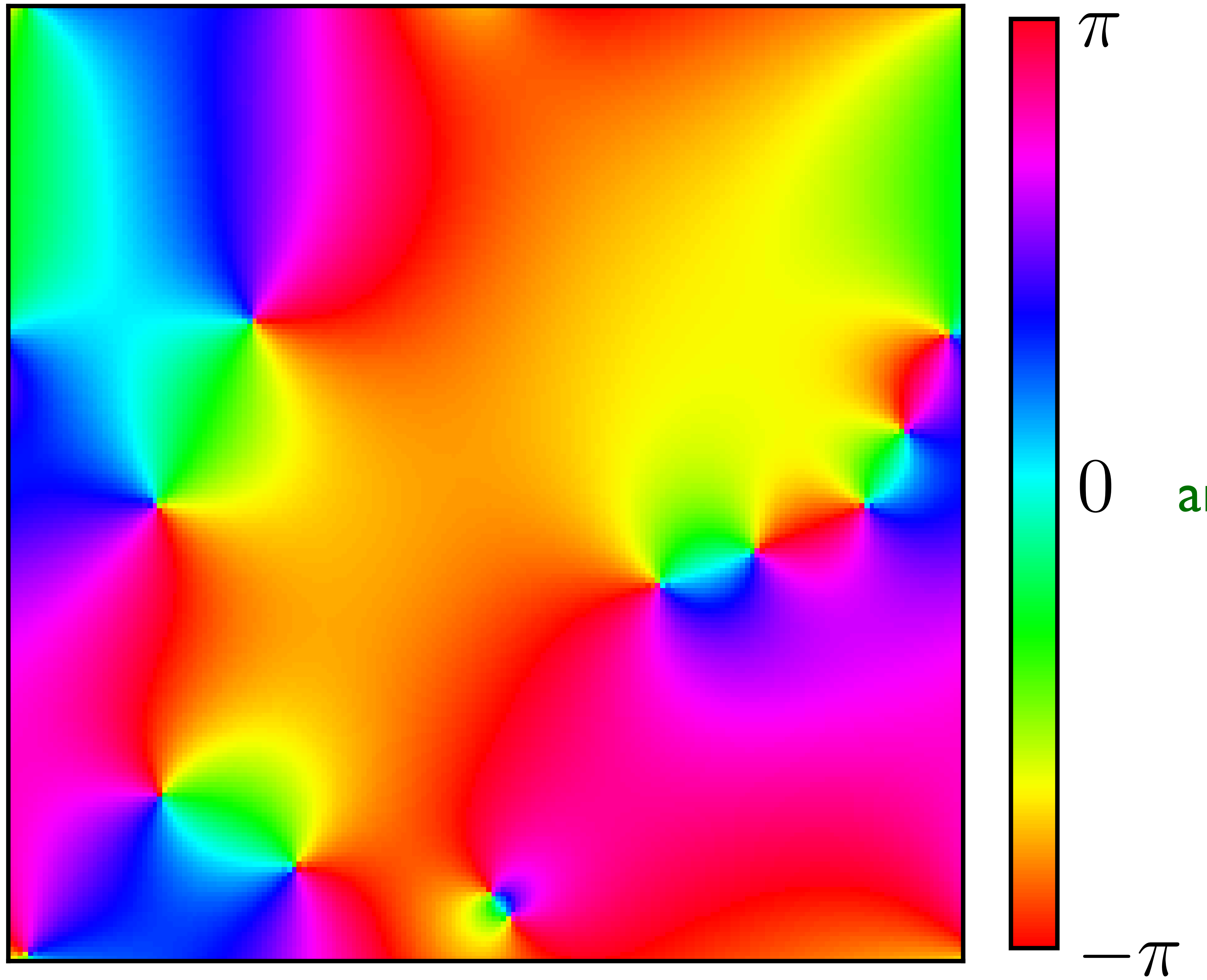
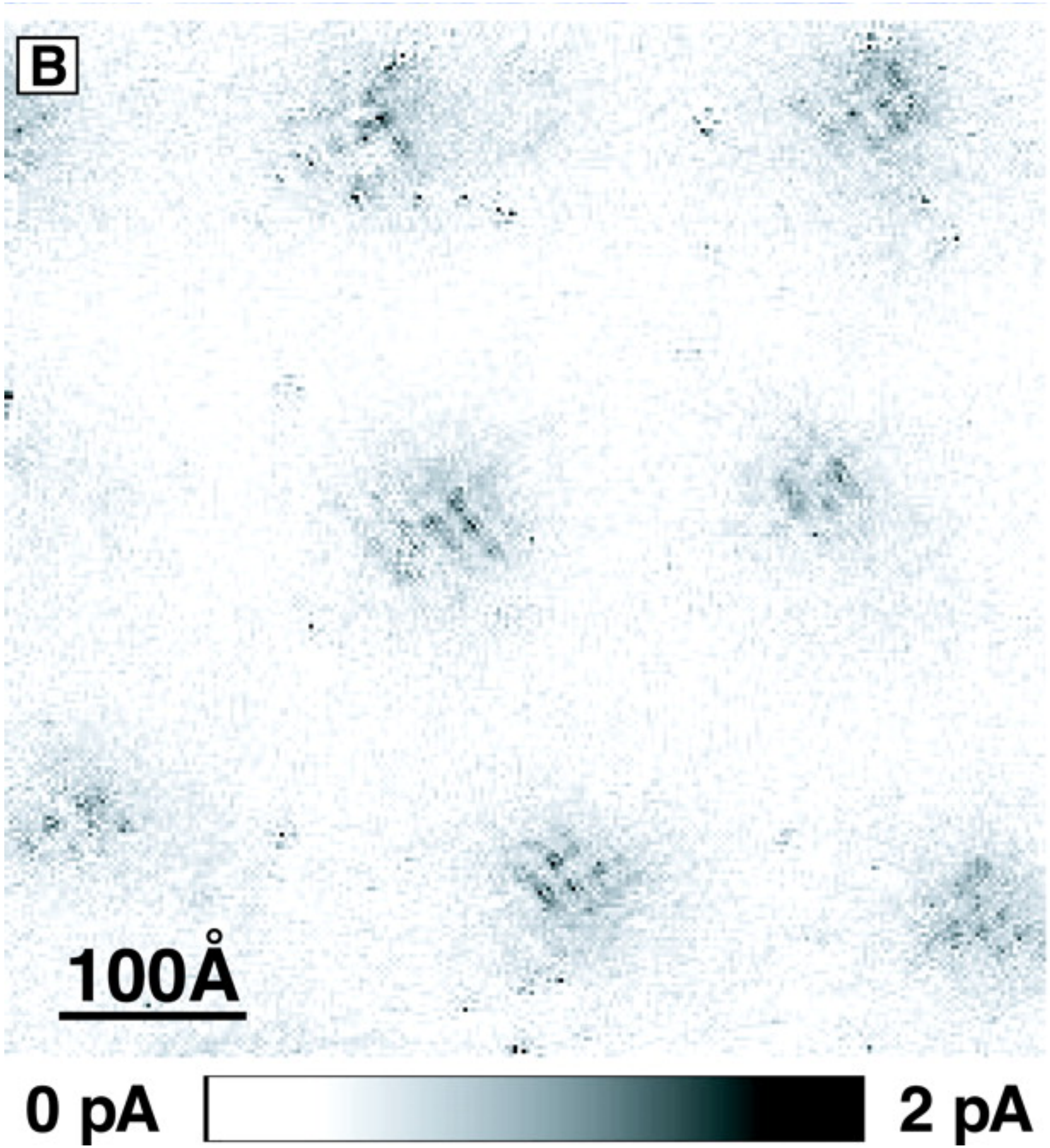


Phase of pairing amplitude

H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

See also Jia-Xin Zhang and S. S., PRB **110**, 235120 (2024)

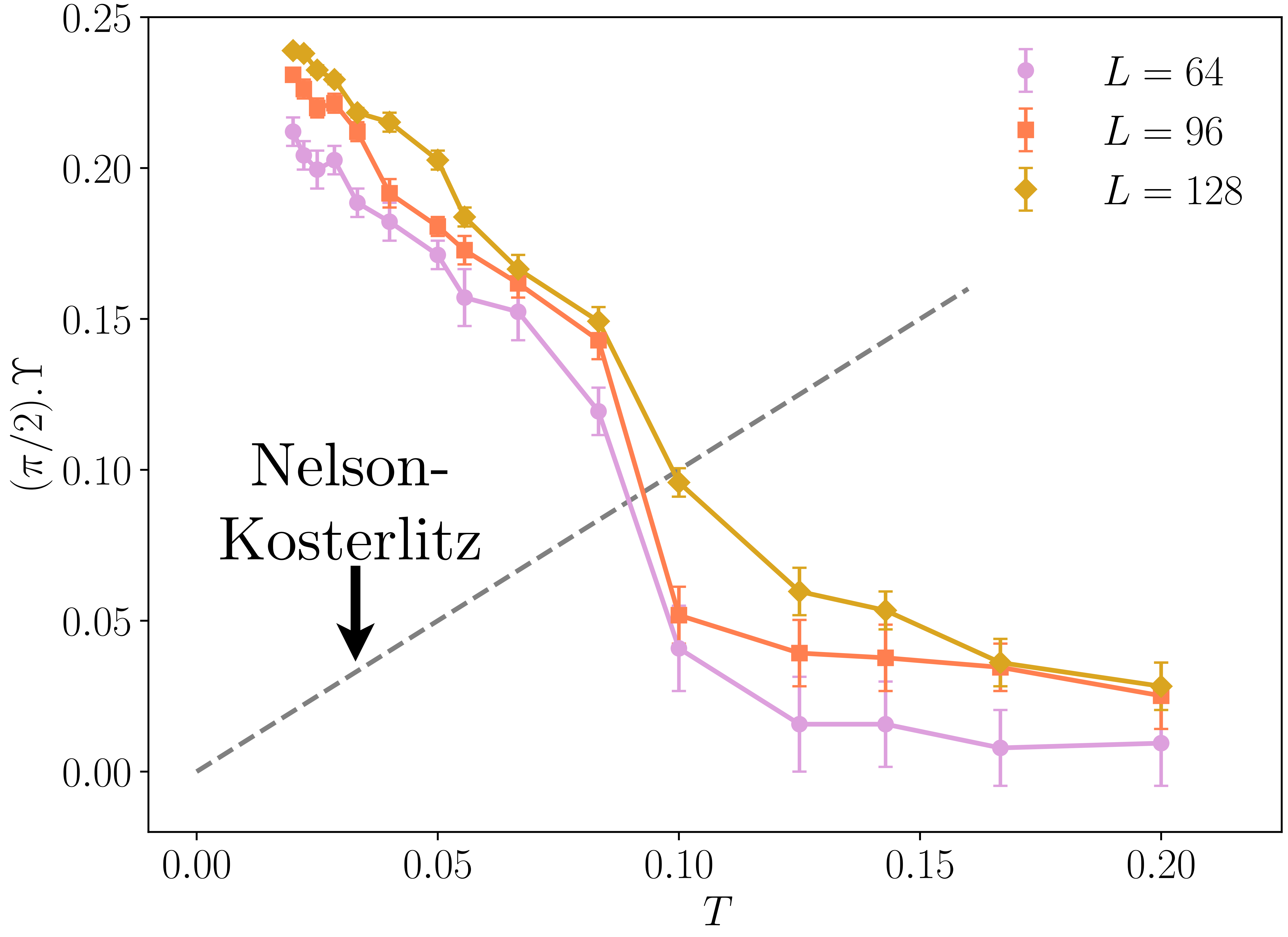


A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $Bi_2Sr_2CaCu_2O_{8+\delta}$

J. E. Hoffman, E. W. Hudson,
K. M. Lang, V. Madhavan,
H. Eisaki, S. Uchida, J.C. Davis
Science **295**, 466 (2002)

Born-Oppenheimer theory of FL* pseudogap

$\Upsilon =$
Helicity Modulus



H. Pandey,
M. Christos,
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arXiv:2507.05336

Born-Oppenheimer theory of FL* pseudogap

Consequences for fermion spectrum

- At $T = 0$, the d-wave superconductor obtained by condensing B has 4 nodal points with $v_F \gg v_\Delta$ (earlier spin liquid theories had $v_F \approx v_\Delta$).
- For $T > T_c$, with thermal fluctuations of B and U, Fermi pockets turn into Fermi arcs.
- Quantum oscillations associated with pocket size $p/8$ can survive when photoemission shows only Fermi arcs.

Born-Oppenheimer theory of FL* pseudogap

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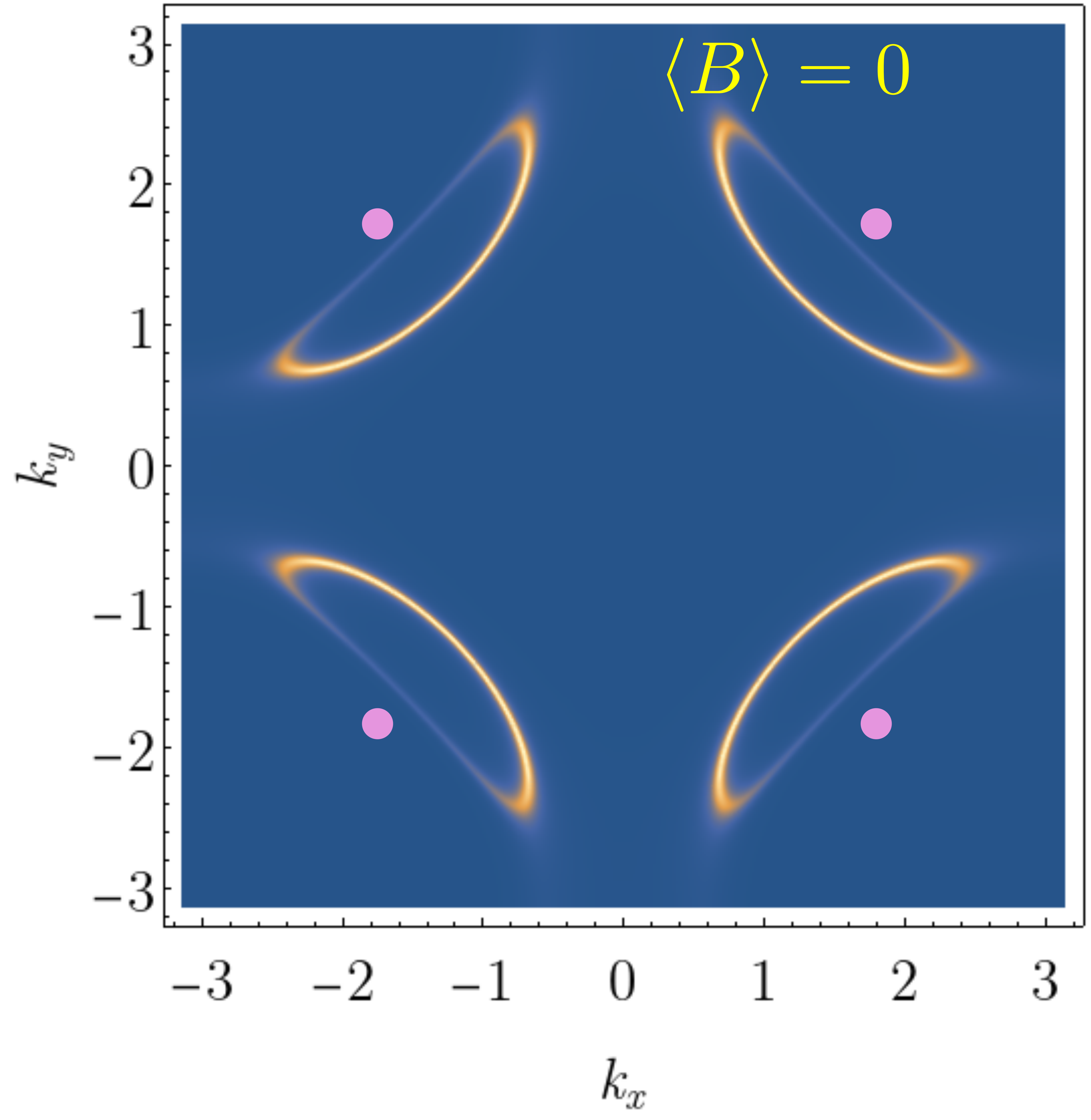
FL*

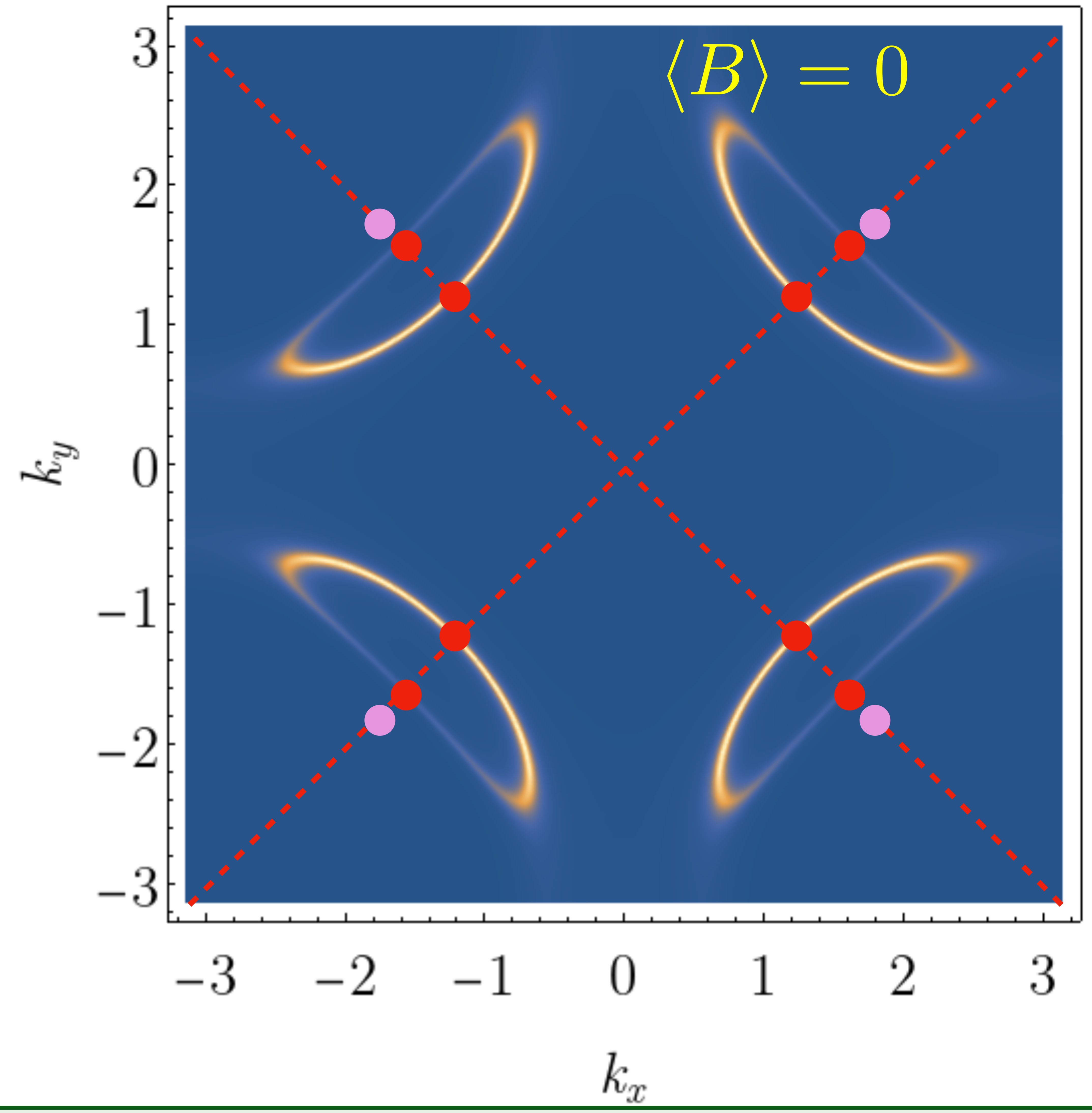


FL* $\Rightarrow$ d-SC:

Cooper pairing of the Fermi surface?

FL* has 4 electron-like pockets
and 4 nodal spinons
of the π-flux spin liquid

FL*→d-SC*



FL* ⇒ d-SC:

Cooper pairing of the Fermi surface?

$$E_{\boldsymbol{k}} = \left(\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2\right)^{1/2}$$

$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$$

No!

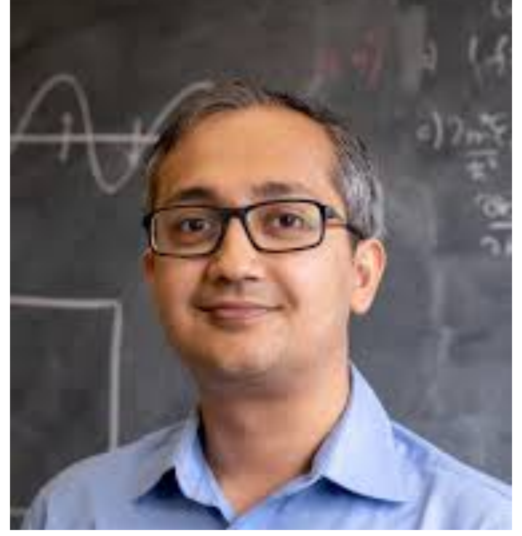
Leads to 8 nodal points of Bogoliubov quasiparticles and 4 nodal spinons of π-flux spin liquid.

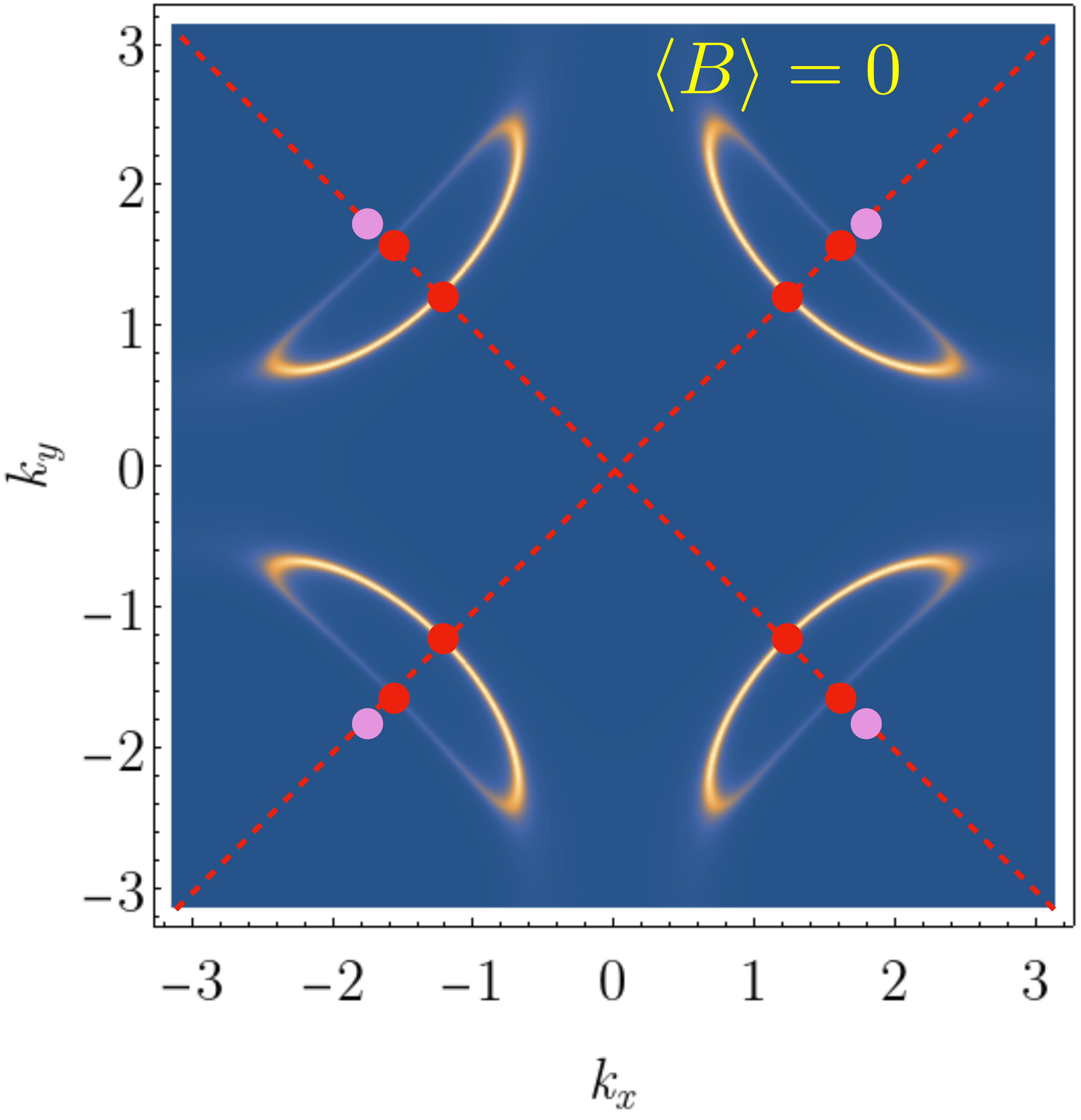
FL* ⇒ d-SC*

BCS mechanism applied to FL* pseudogap leads to non-BCS superconductor!

FL*→d-SC

Shubhayu Chatterjee and S. S.,
PRB **94**, 205117 (2016)
Maine Christos and S.S.,
npj Quantum Materials **9**, 4 (2024)

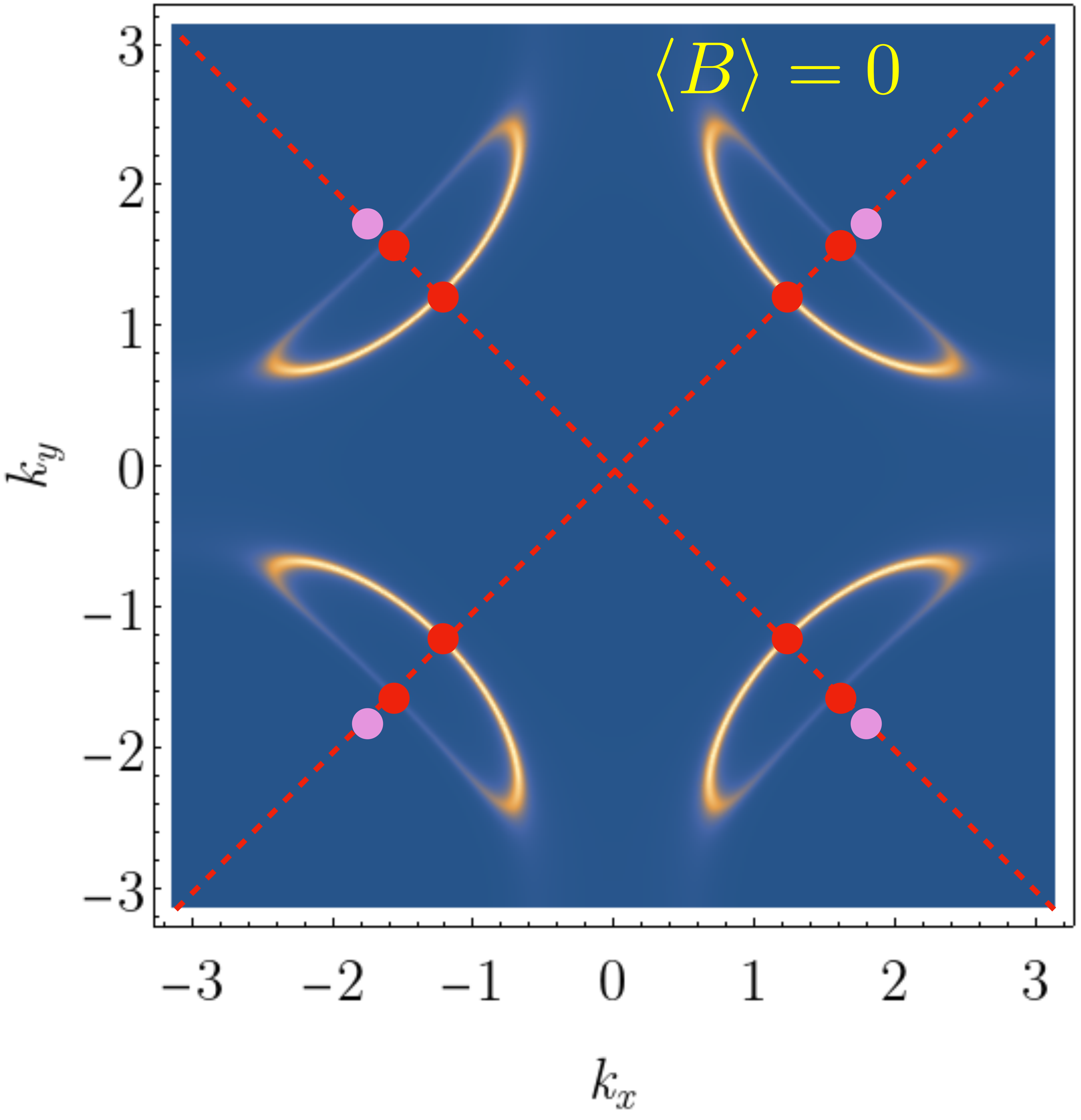




Alternative route to d-wave superconductivity:

Use the pre-existing pairing of the underlying spin liquid and confine the spin liquid!

FL*→d-SC



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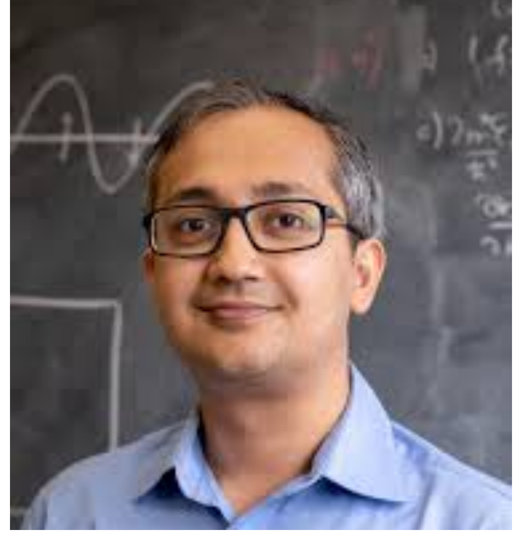
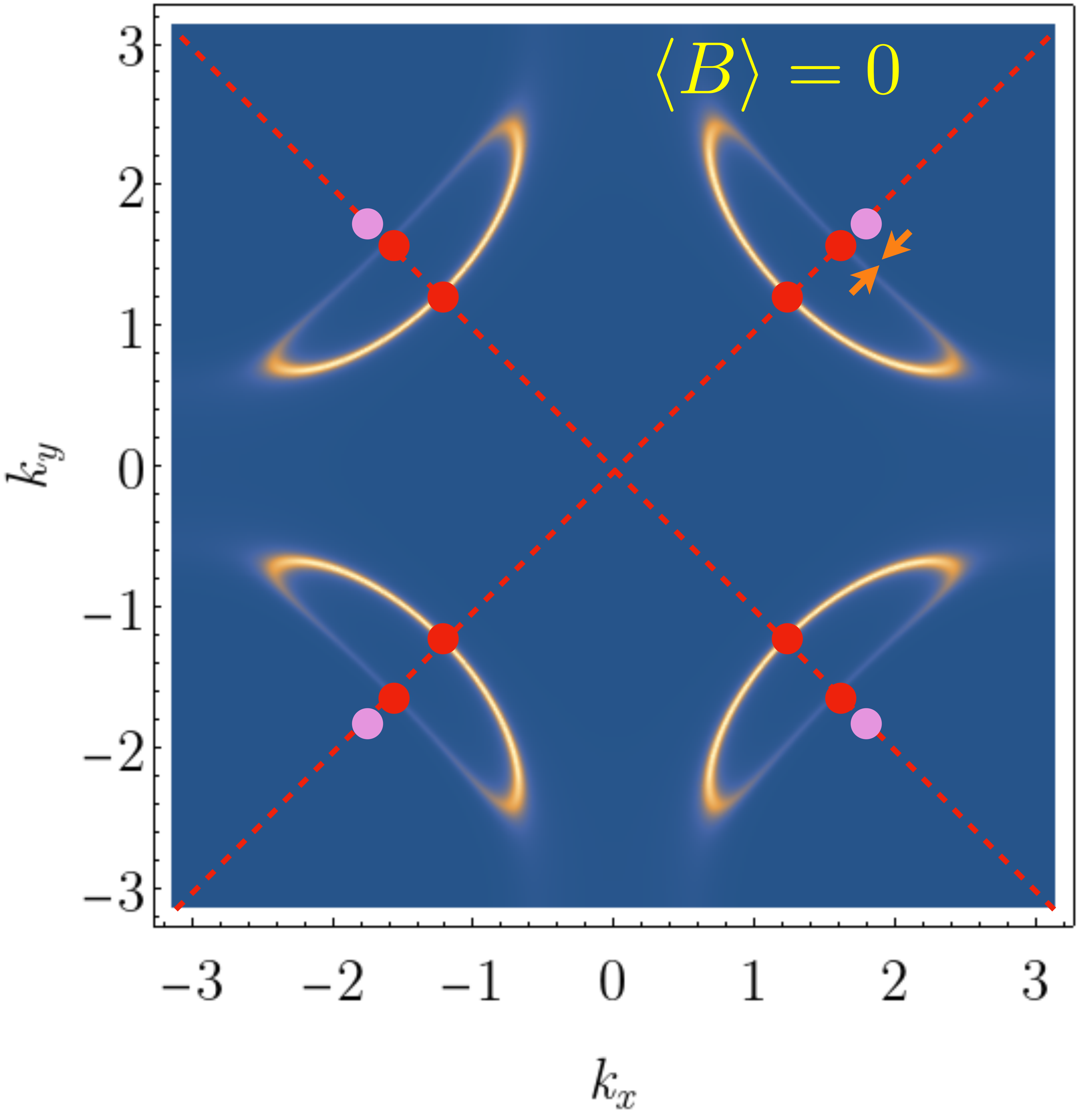


Alternative route to d-wave superconductivity:

Confine the π-flux spin-liquid by a condensate of B, $\langle B \rangle \neq 0$ for a suitable Higgs potential $\mathcal{E}_4(B)$. This leads to a d-wave superconductor with 4 nodal points and $v_F \gg v_\Delta$!

FL*→d-SC



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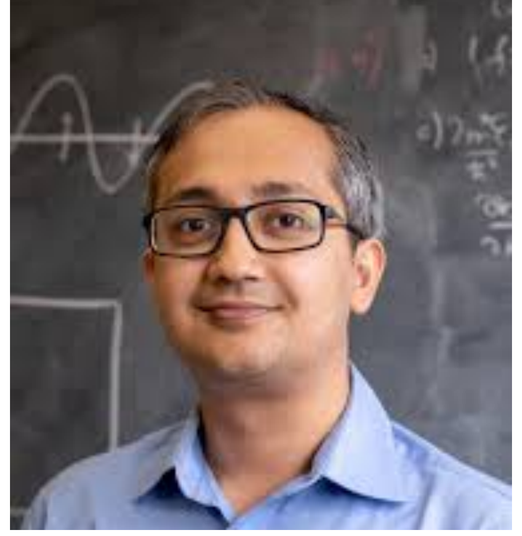

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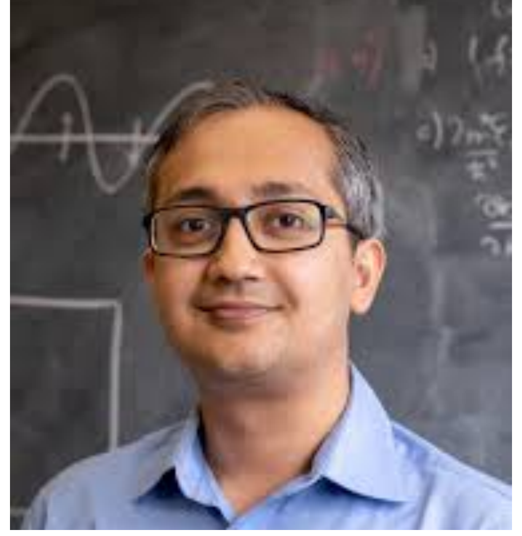

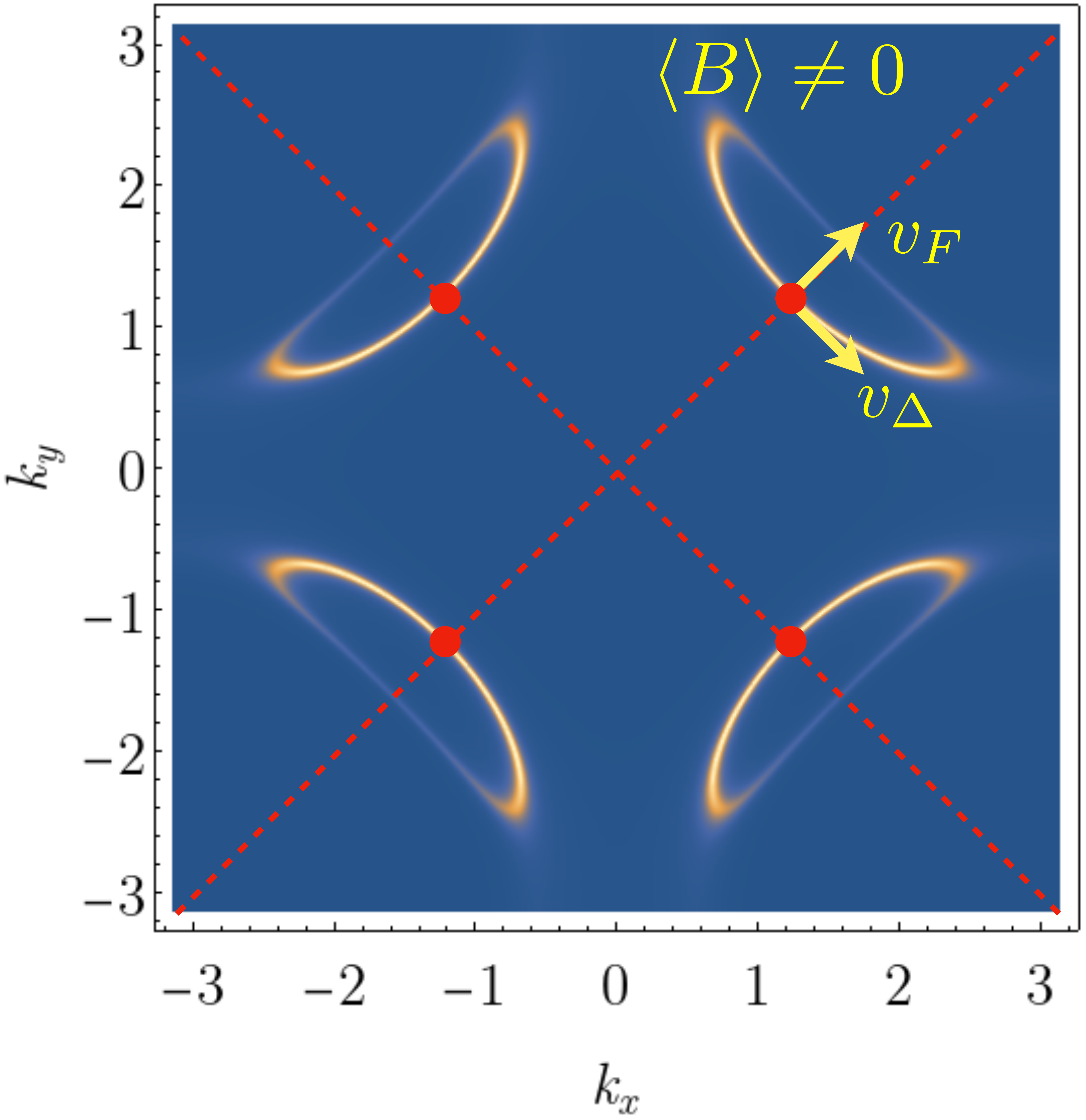

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Resolves difficulty in early theories of d-SC from spin liquids (Zhang, Gros,Rice, Shiba (1988); Kotliar, Liu (1988)), in which spinons turned into Bogoliubov quasiparticles with $v_F \approx v_\Delta$.

FL*→d-SC

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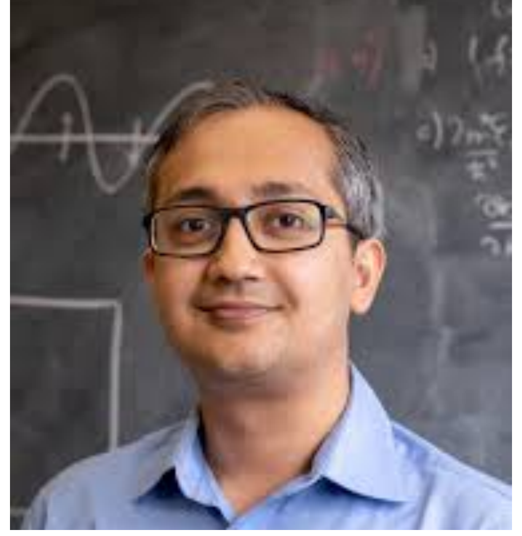

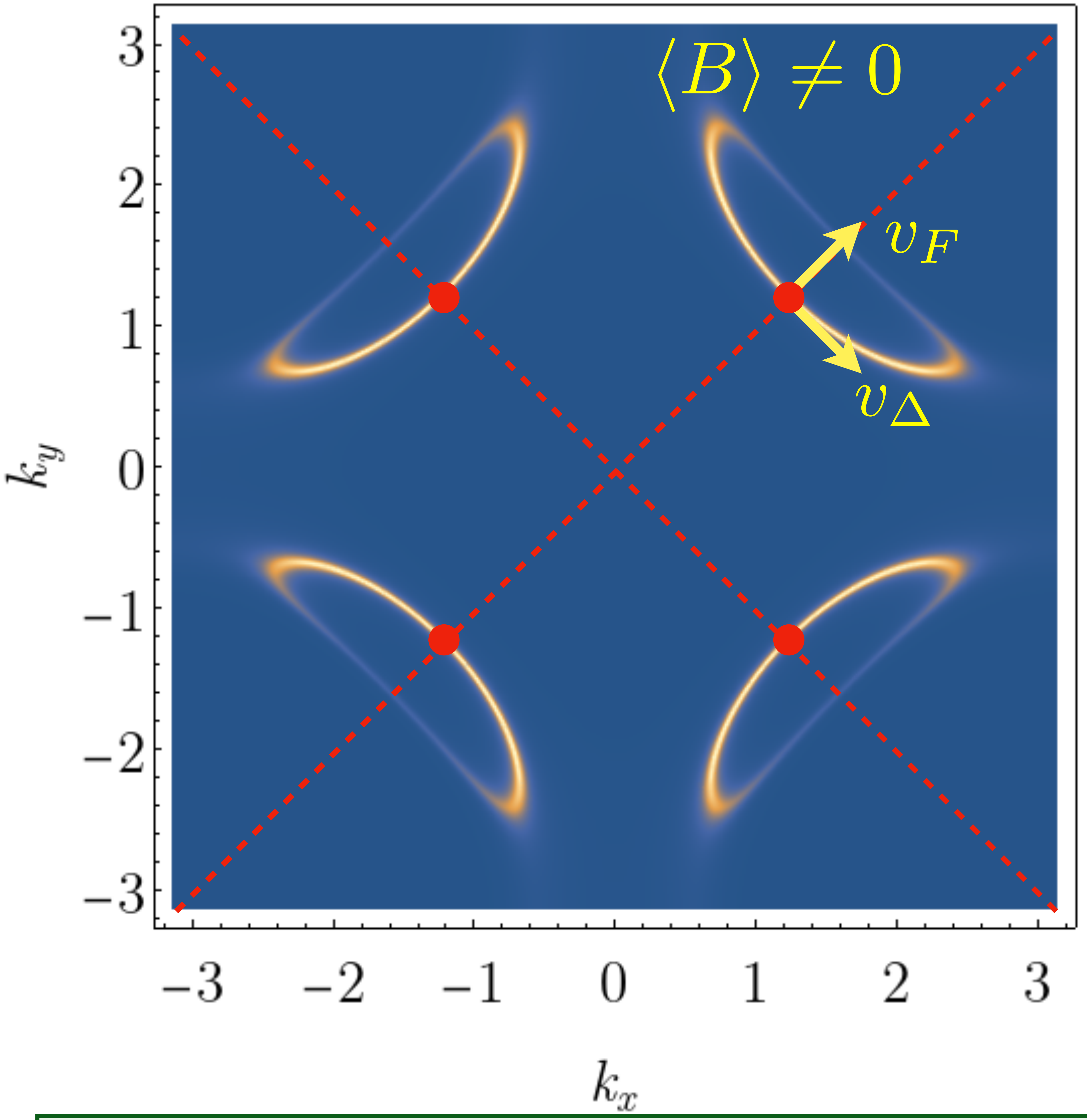

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Non-BCS mechanism applied to pseudogap leads to BCS superconductor!

Born-Oppenheimer theory of FL* pseudogap

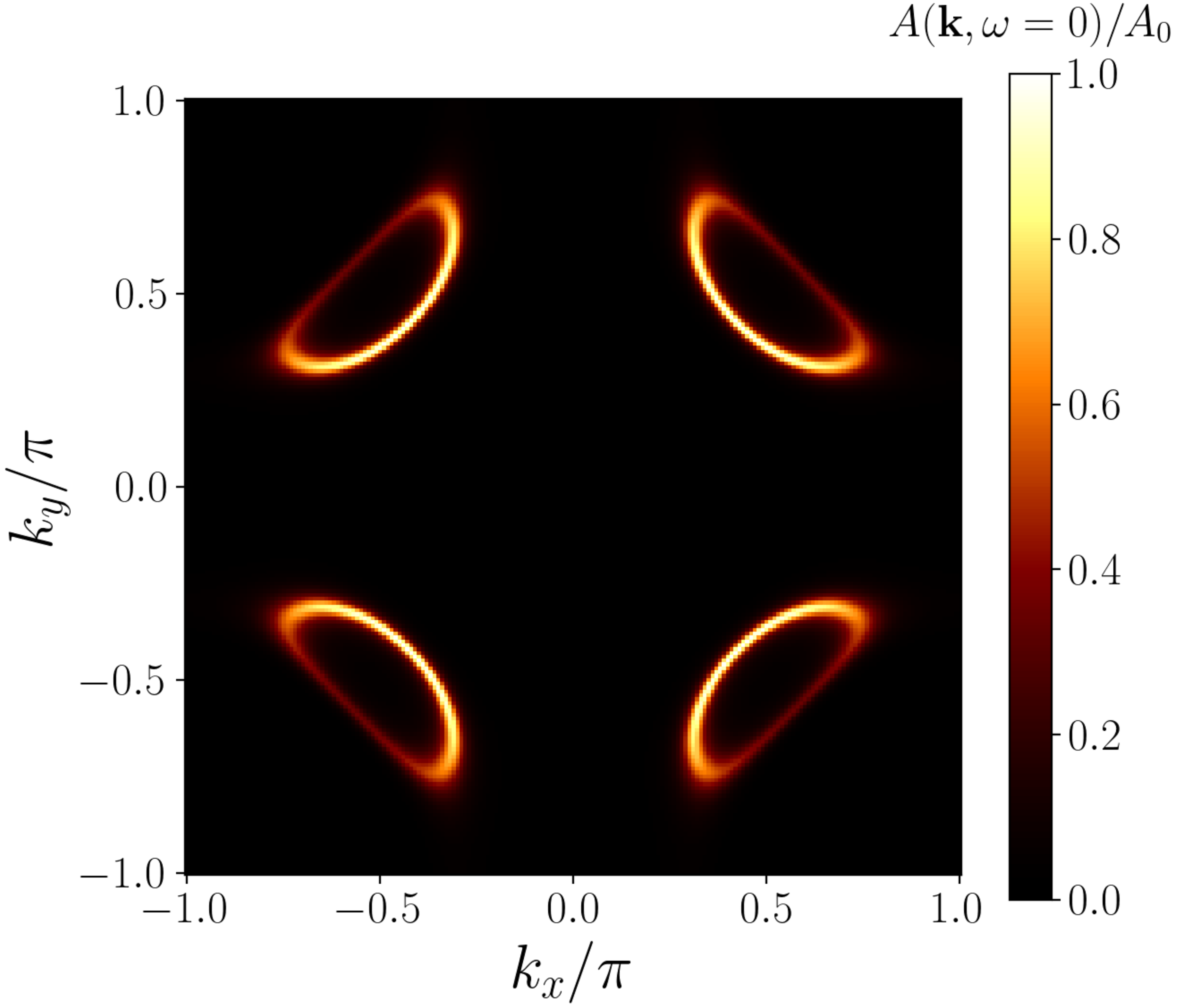
Consequences for fermion spectrum

- At $T = 0$, the d-wave superconductor obtained by condensing B has 4 nodal points with $v_F \gg v_\Delta$ (earlier spin liquid theories had $v_F \approx v_\Delta$).

- For $T > T_c$, with thermal fluctuations of B and U, Fermi pockets turn into Fermi arcs.

- Quantum oscillations associated with pocket size $p/8$ can survive when photoemission shows only Fermi arcs.

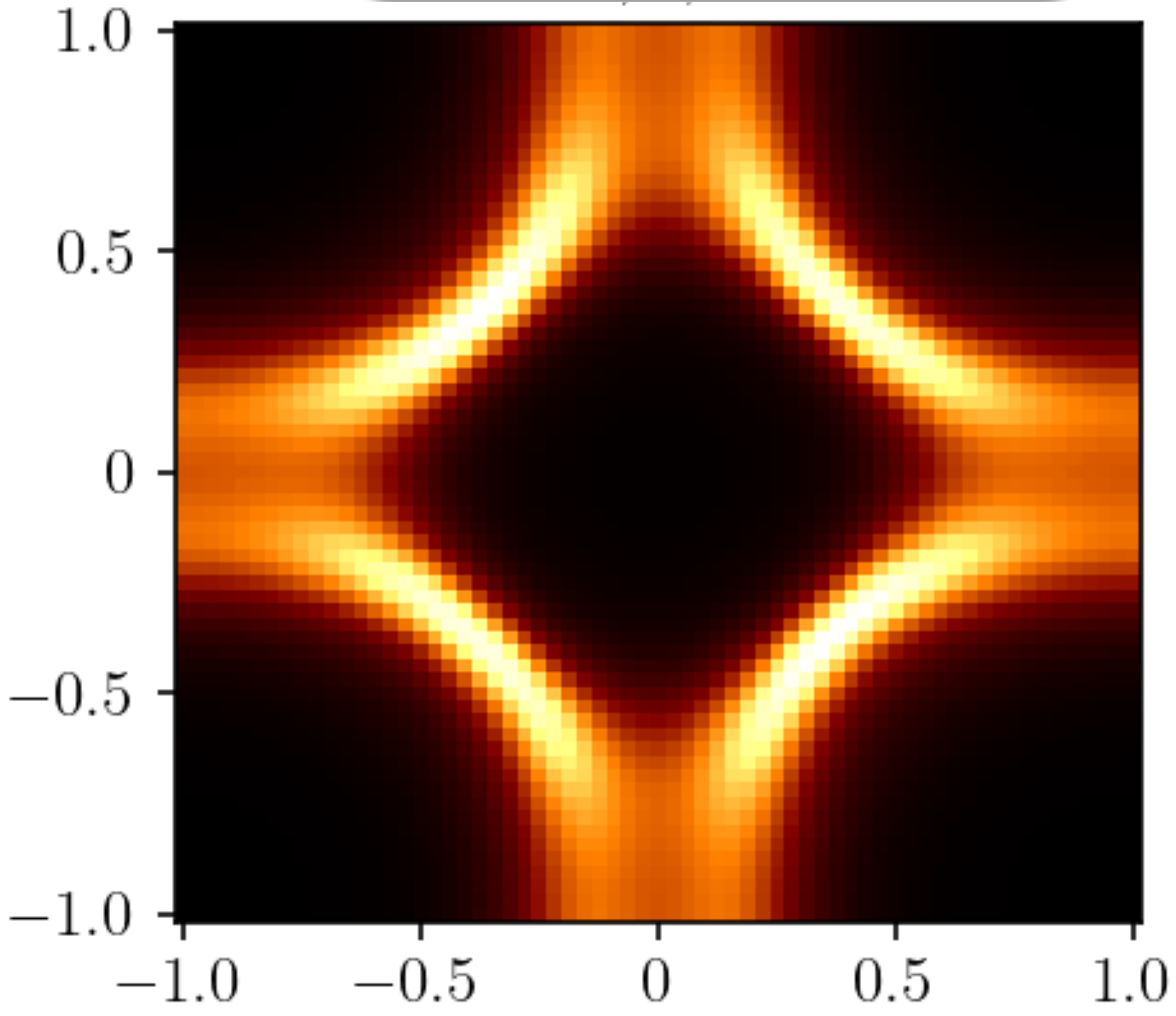
Ancilla theory of FL* and its confinement



$B = 0$

Decoupled Kondo lattice and spin liquid

Photoemission theory



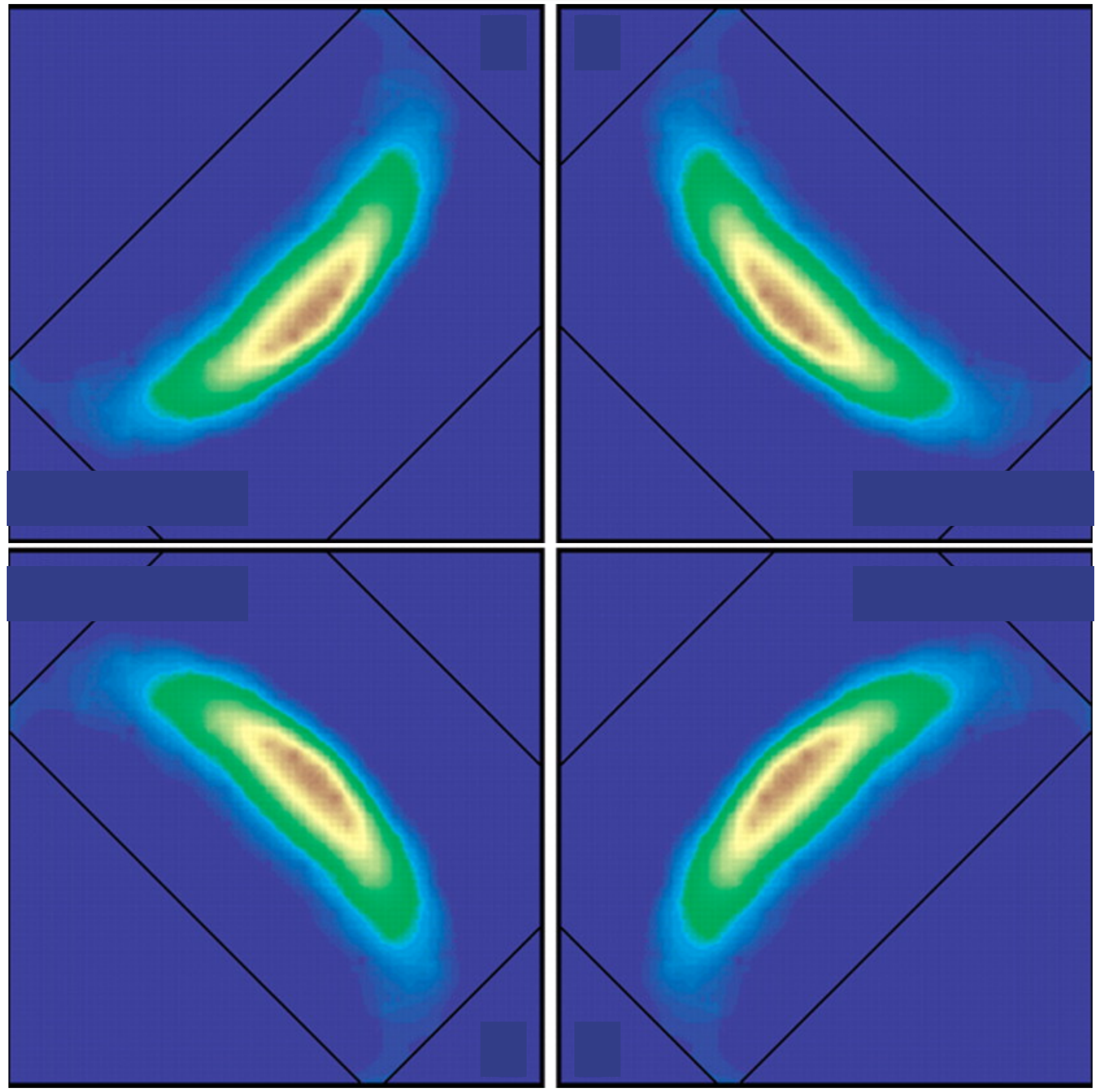
Born-Oppenheimer theory

$$\mathcal{Z}_{2+0} = \int \prod_i \mathcal{D}B_i \int \prod_{\langle ij \rangle} \mathcal{D}U_{ij} \exp\left[-\left(V_{\rm Higgs}[B,U] - \kappa \sum_{\Box} {\rm ReTr} \prod_{ij \in \Box} U_{ij} \right) /T \right]$$

H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

Ancilla theory of FL* and its confinement

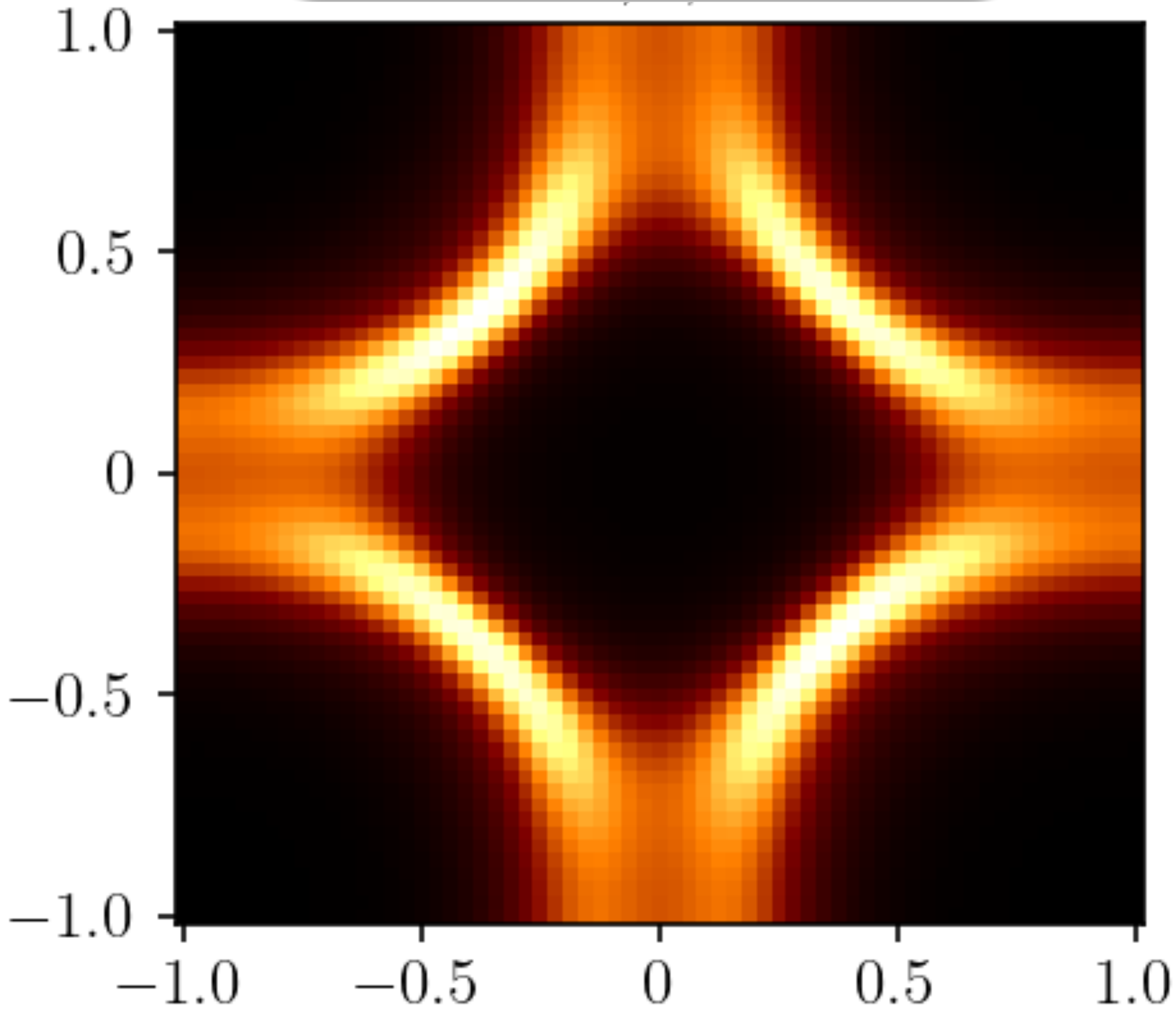
Photoemission expts



$Ca_{2-x}Na_xCuO_2Cl_2$ at $x = 0.10$

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

Photoemission theory



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Ancilla theory of FL* and its confinement

Photoemission expts

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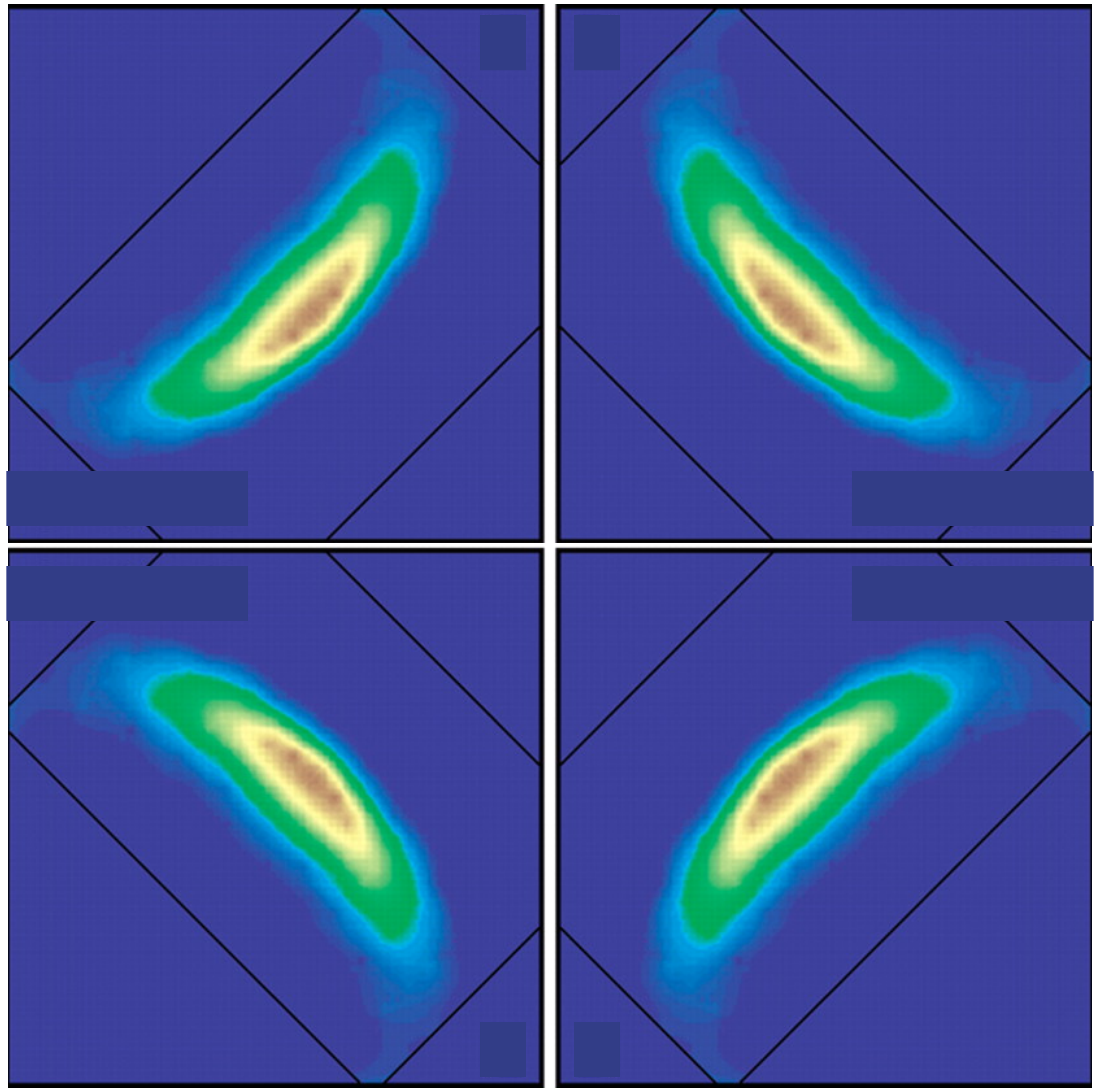

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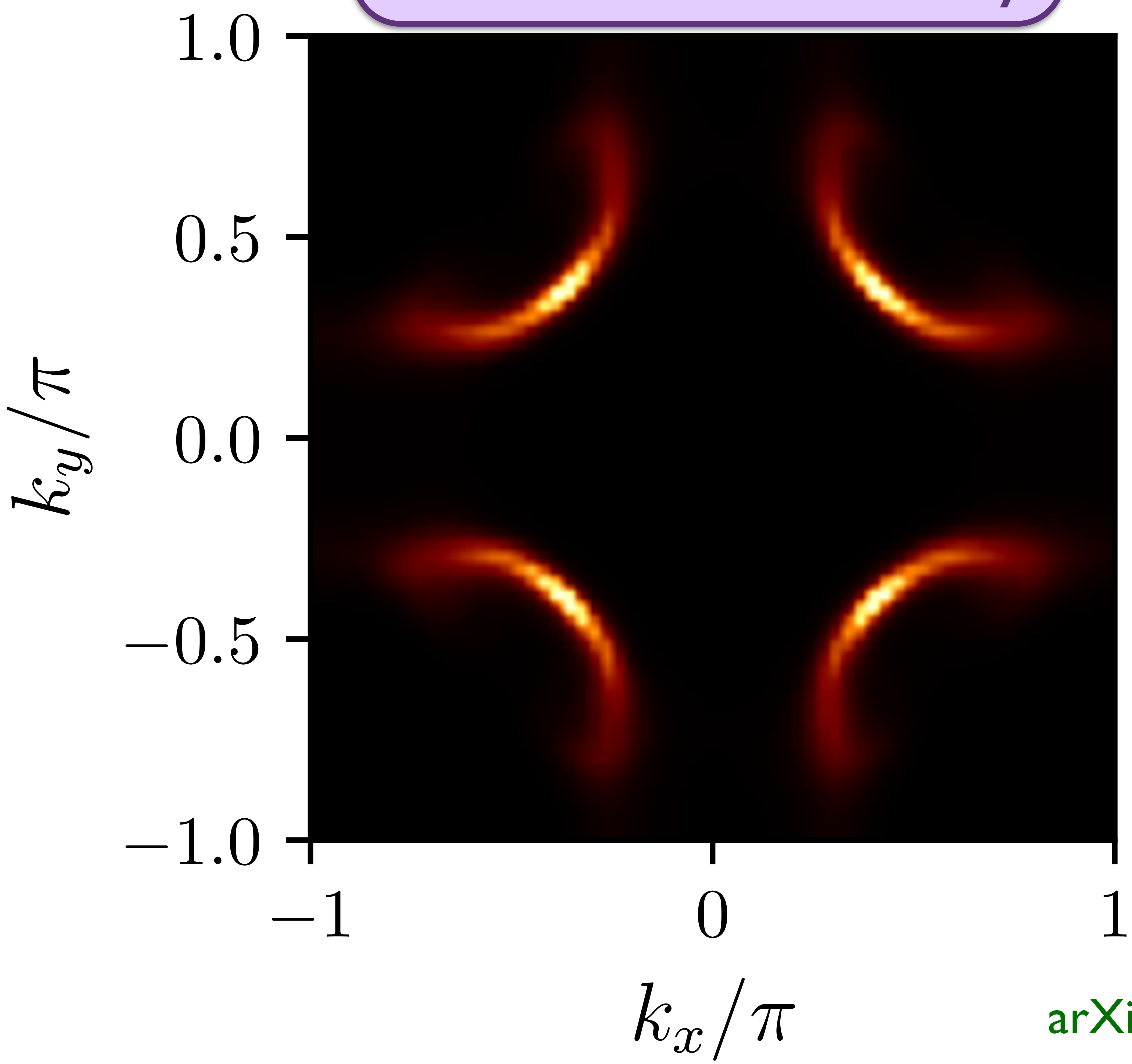
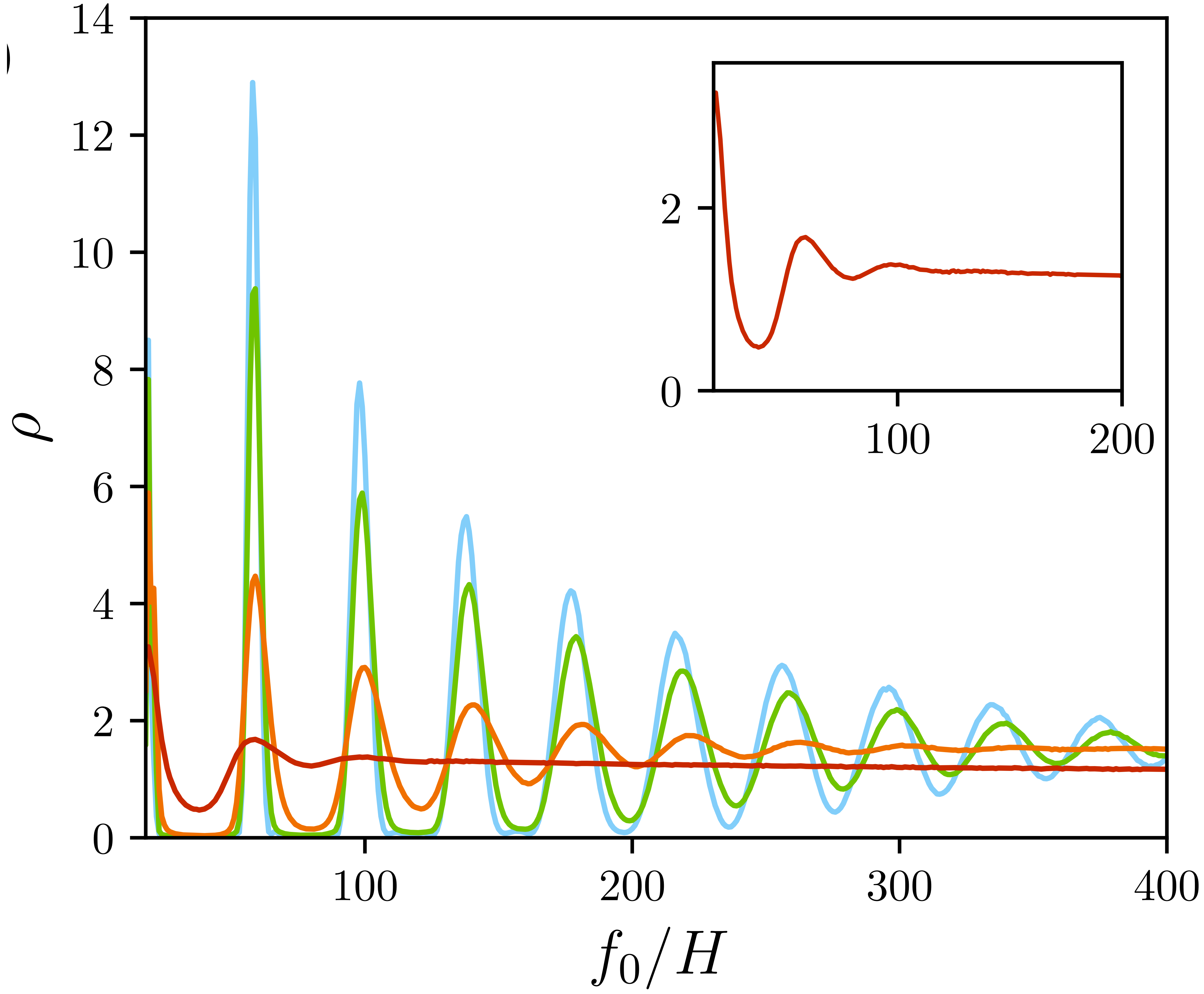


H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

Born-Oppenheimer theory— $U = 1$ and Gaussian fluctuations of B

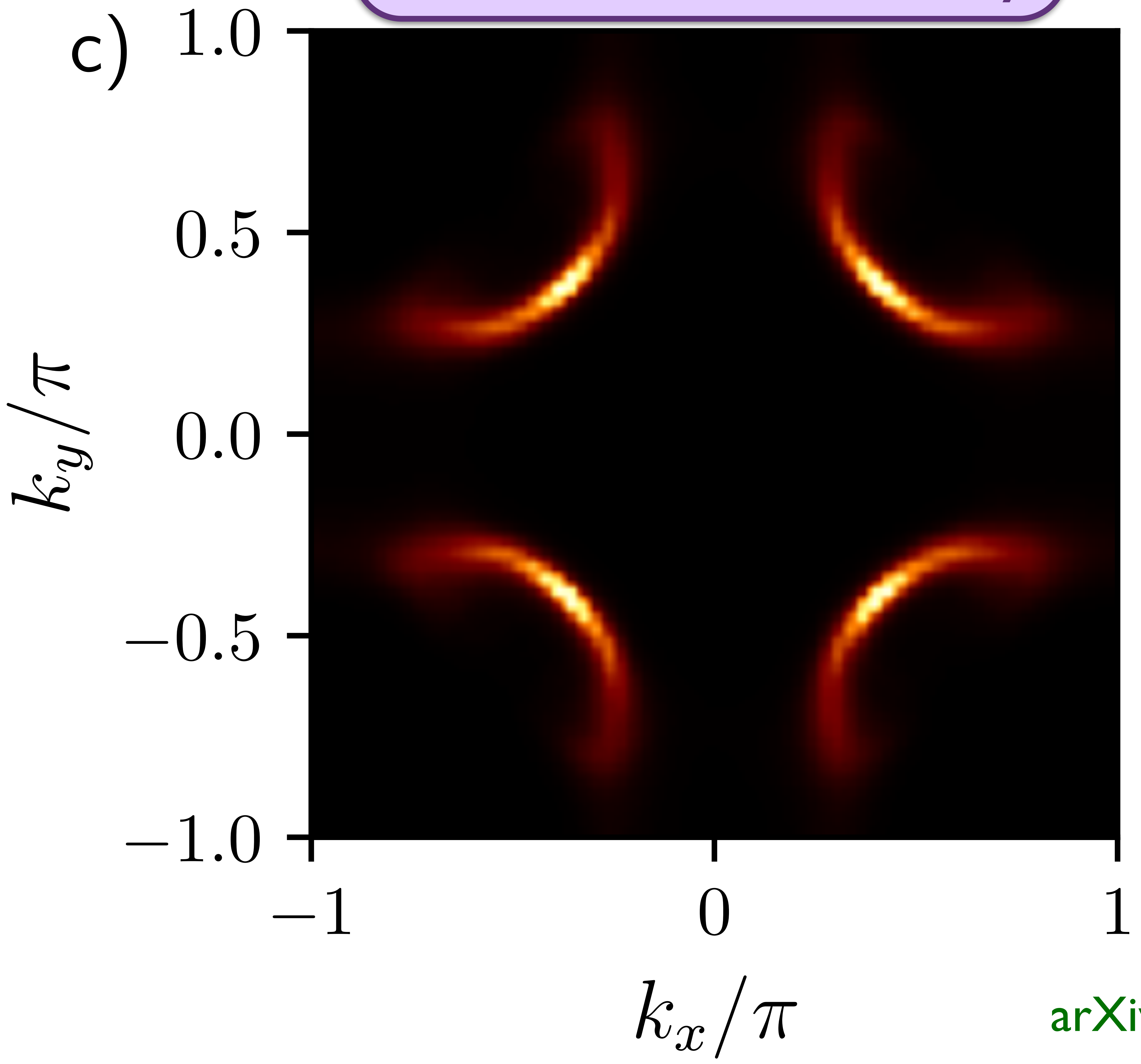
Ancilla theory of FL* and its confinement

Quantum oscillations theory



Born-Oppenheimer theory—
$U = 1$ and Gaussian fluctuations of B

Photoemission theory



Born-Oppenheimer theory—
$U = 1$ and Gaussian fluctuations of B

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Temperature, T (K)
300
200
100
0
0.1
0.2
Hole doping, p
p_{min}
p_{c1}
p_{c2}
p_{max}
T_{N}
T^*
Strange metal
$\langle \Phi \rangle \neq 0$
$\langle B \rangle = 0$
Pseudogap
$T_{\text{SC, onset}}$
$T_{\text{C, onset}}$
Charge order
T_{c}
Spin order
AF
$T_{\text{S, onset}}$
$\langle B \rangle \neq 0$
T_{CDW}
T_{SDW}
d-SC
$\langle \Phi \rangle = 0$
Fermi liquid
Kondo lattice heavy Fermi liquid when $\langle \Phi \rangle \neq 0$.
Size $1 + p + 1 = p \pmod 2$.
Small Fermi pockets!
e^+
c_σ
J_K
$\boldsymbol{S}_1$
Φ
B
$J_\perp$
$\boldsymbol{S}_2$
π-flux spin liquid with SU(2) gauge field U