# Exotic metals in graphene and the high temperature superconductors

International Center for Theoretical Sciences, Bengaluru
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Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

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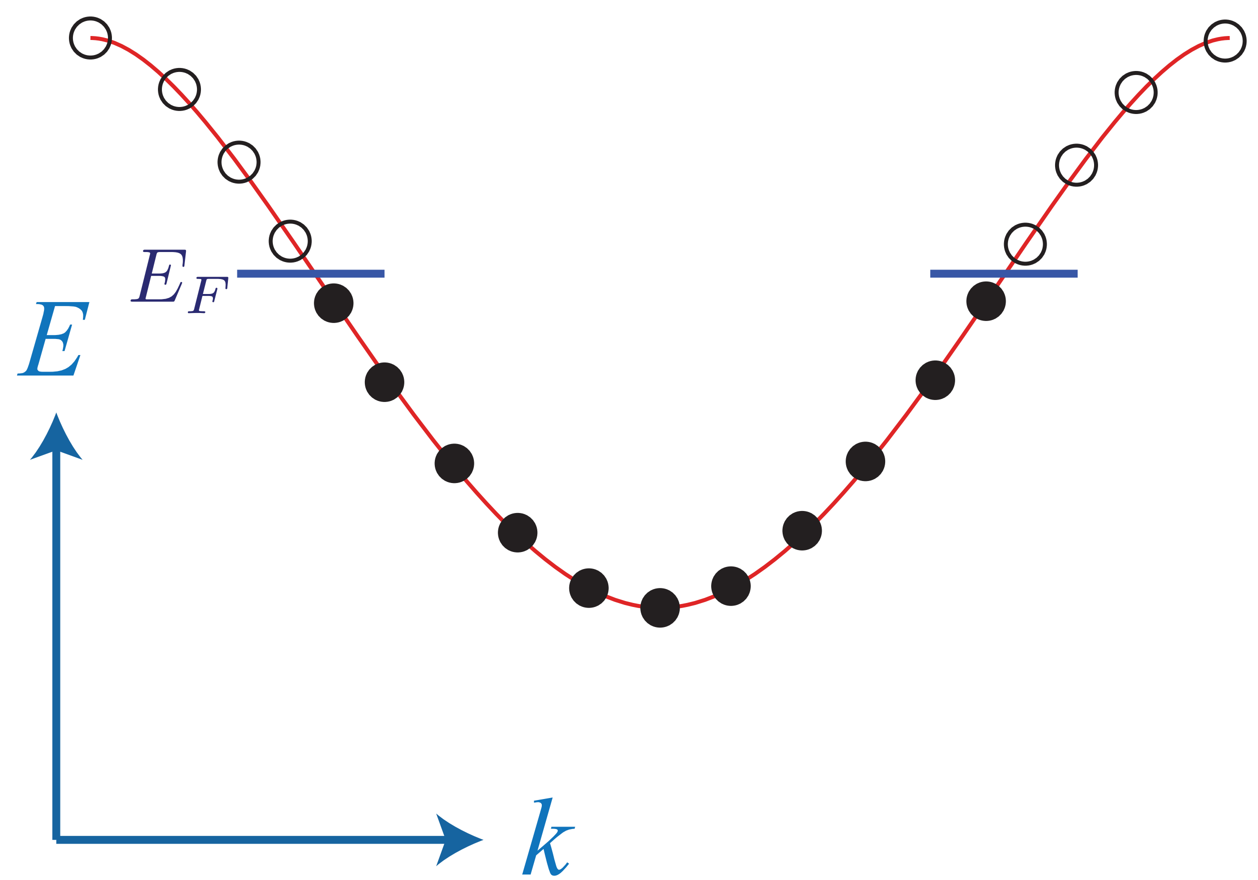
*Foundations of quantum many body theory:*

*1. Ground states connected adiabatically to independent electron states*

## Metals



*Foundations of quantum many body theory:*

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*2. Boltzmann-Landau theory of quasiparticles*

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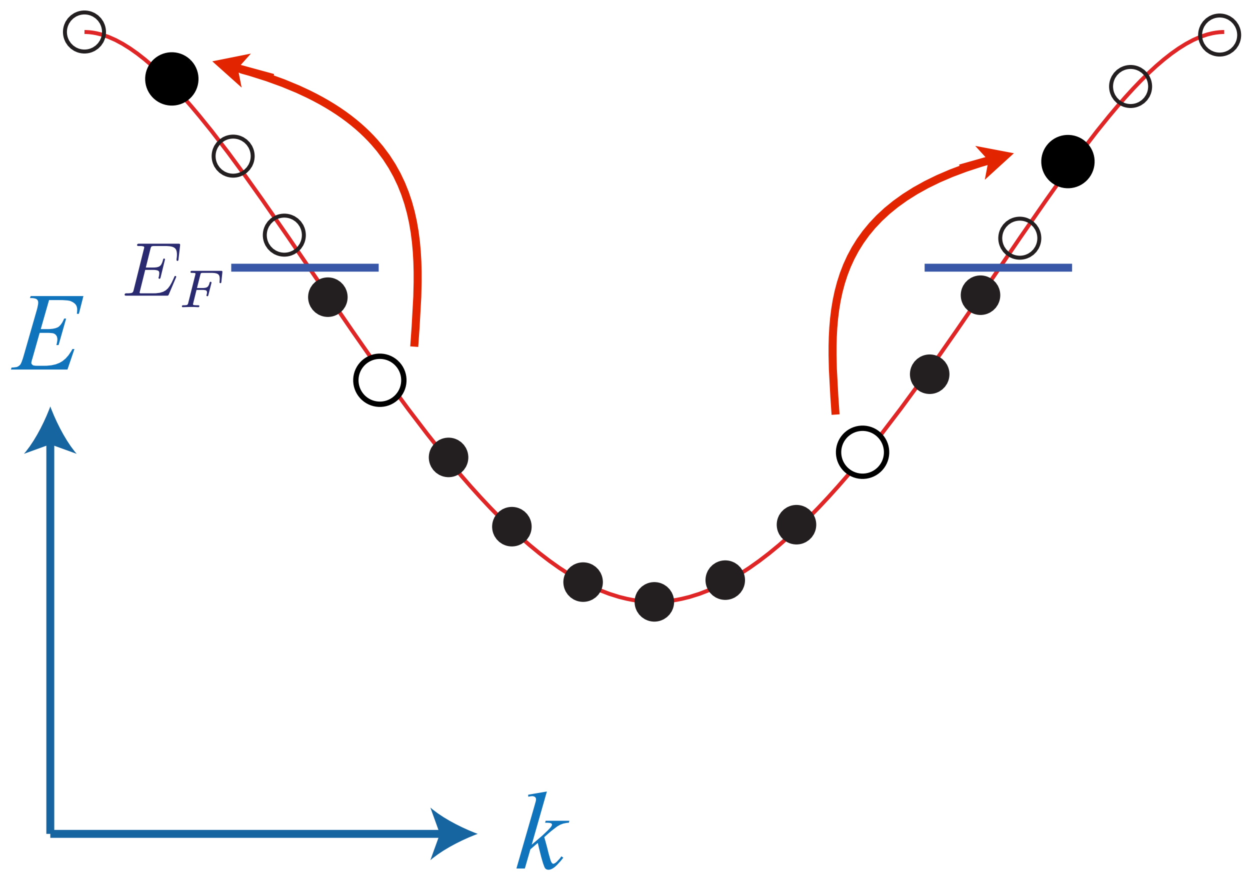

*Modern phases of quantum matter:*

1. *Ground states disconnected from independent electron states: many-particle entanglement*

2. *Boltzmann-Landau theory of quasiparticles*

## Famous example:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

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Field theory: topological quantum field theory

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Strange metals

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Field theory example: conformal field theory

1. Strange metal in graphene

*Experiments vs. theory combining hydrodynamic/holographic/Boltzmann/memory-function methods*

2. Strange metal in correlated electron superconductors

*Onset of spin density wave order in metals*

3. The pseudogap metal of the hole-doped cuprate superconductors

*Evidence for a metal with emergent gauge fields*

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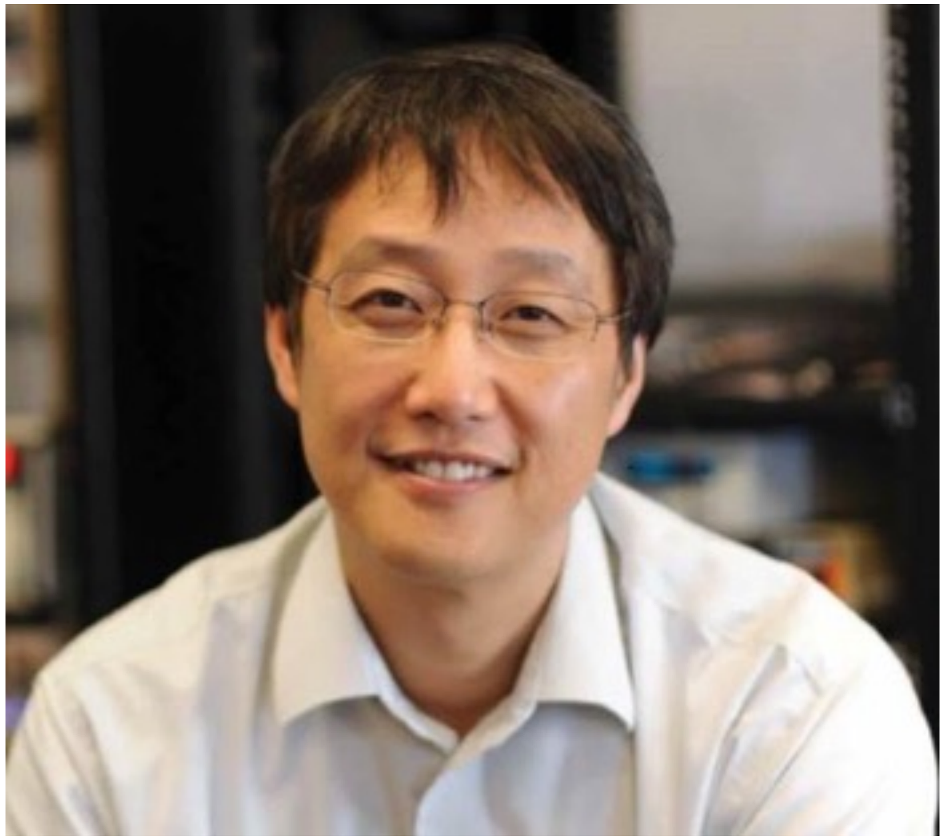
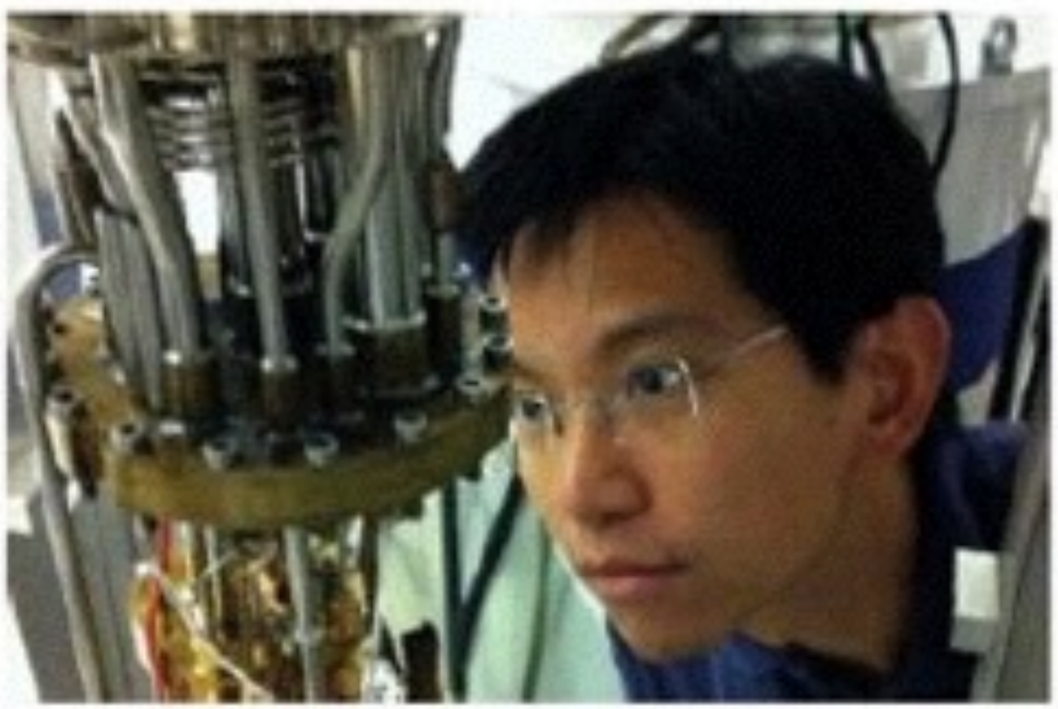
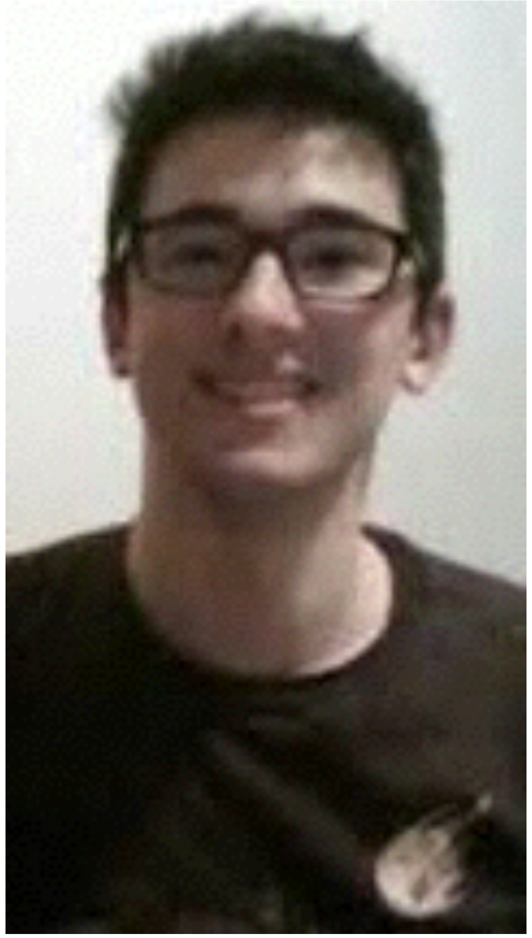


Philip Kim

Jesse Crossno

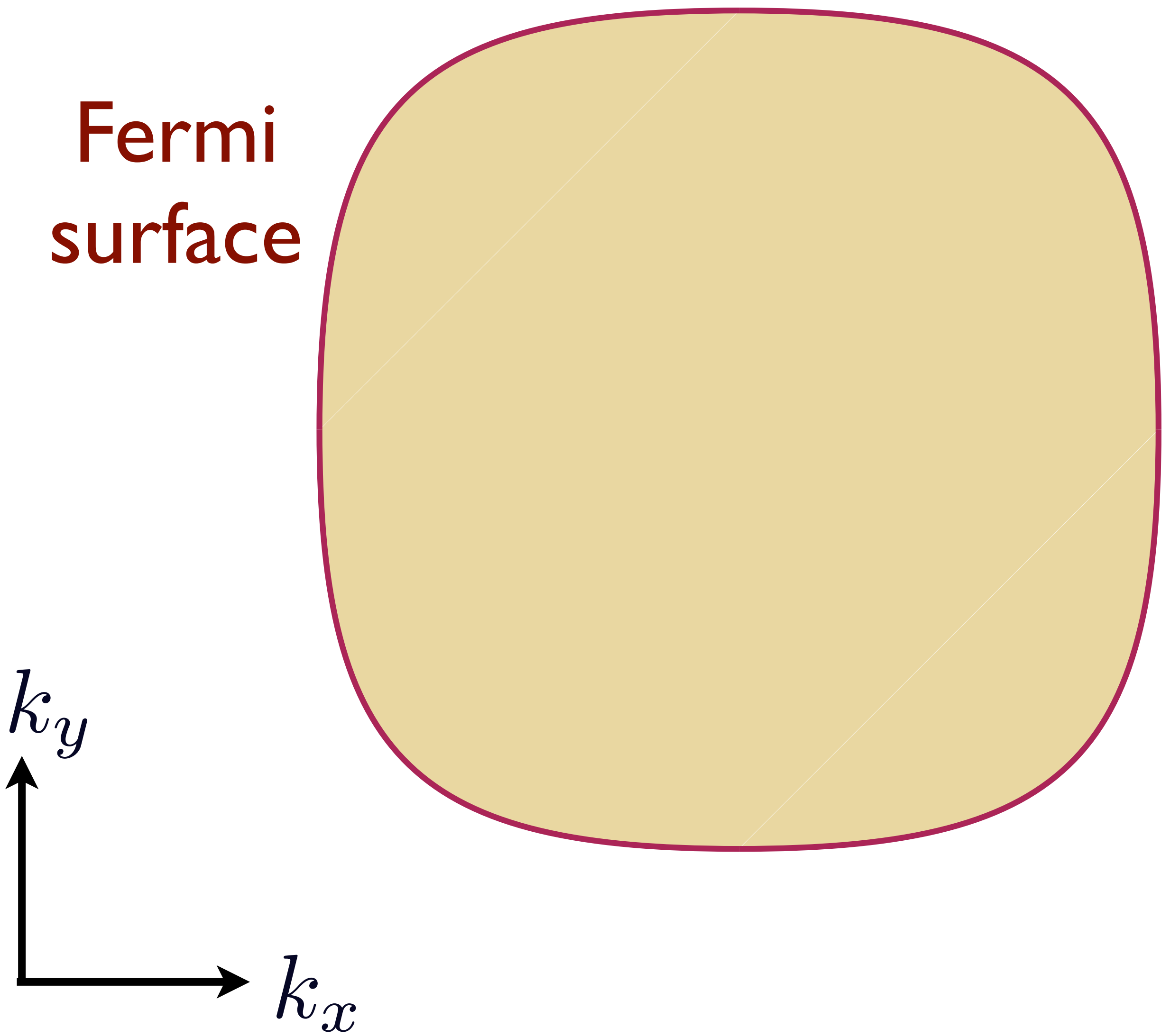


Kin Chung Fong



Andrew Lucas

# Ordinary metals: the Fermi liquid




- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface = $\mathcal{Q}$. Momenta of low energy excitations fixed by density of *all* electrons.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.

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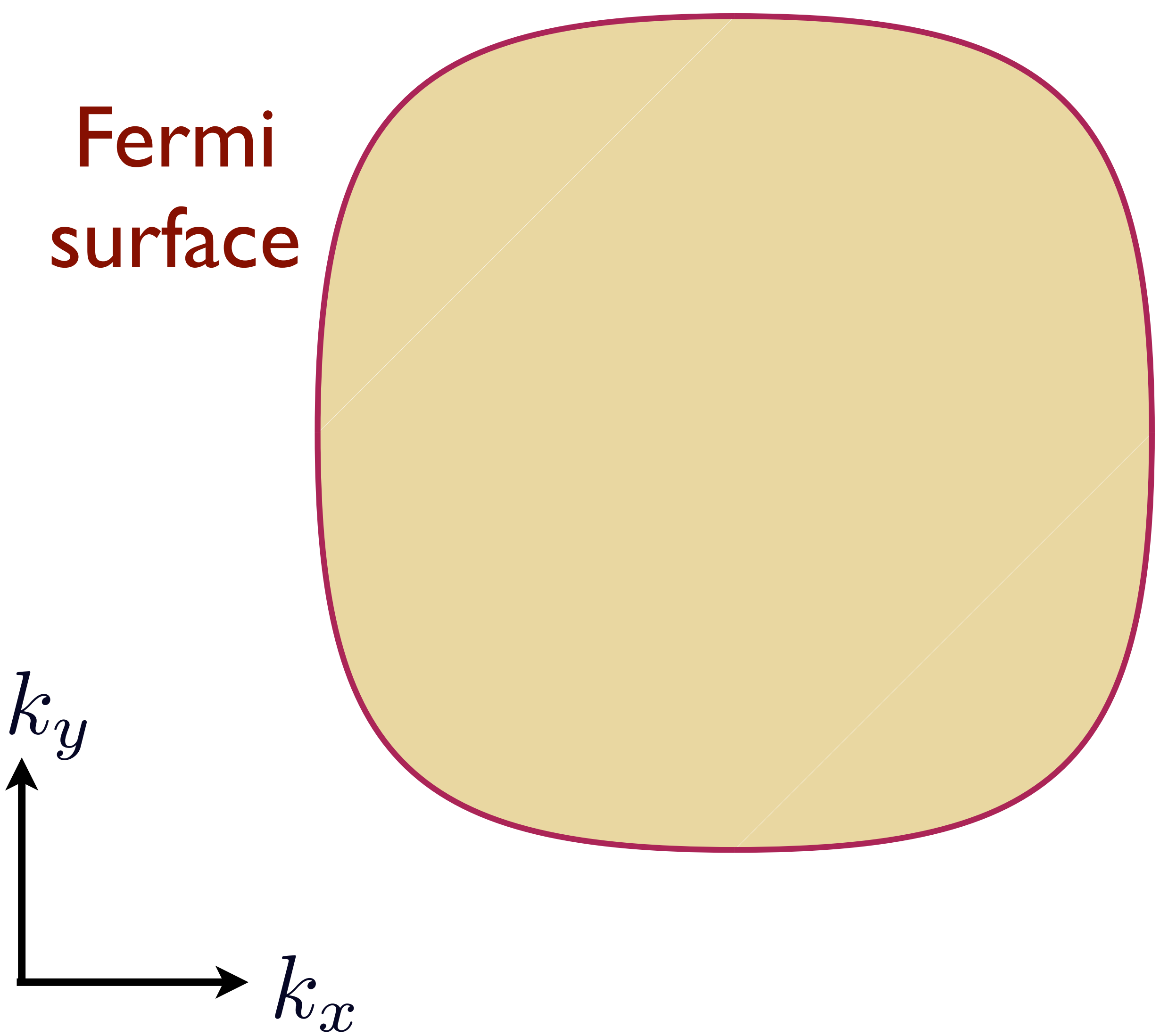


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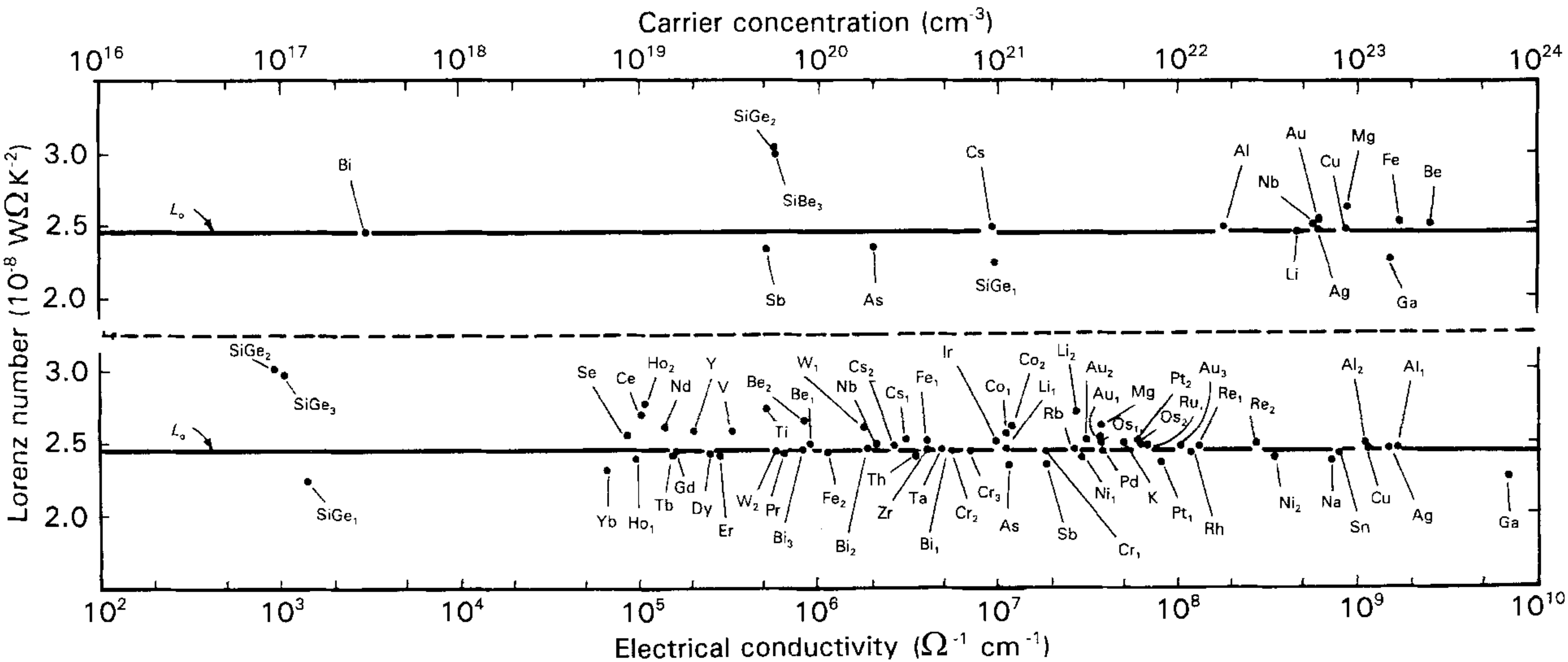
- $$\frac{\text{(Thermal conductivity)}}{T\,\text{(Electrical conductivity)}} = \frac{\pi^2 k_B^2}{3e^2}$$

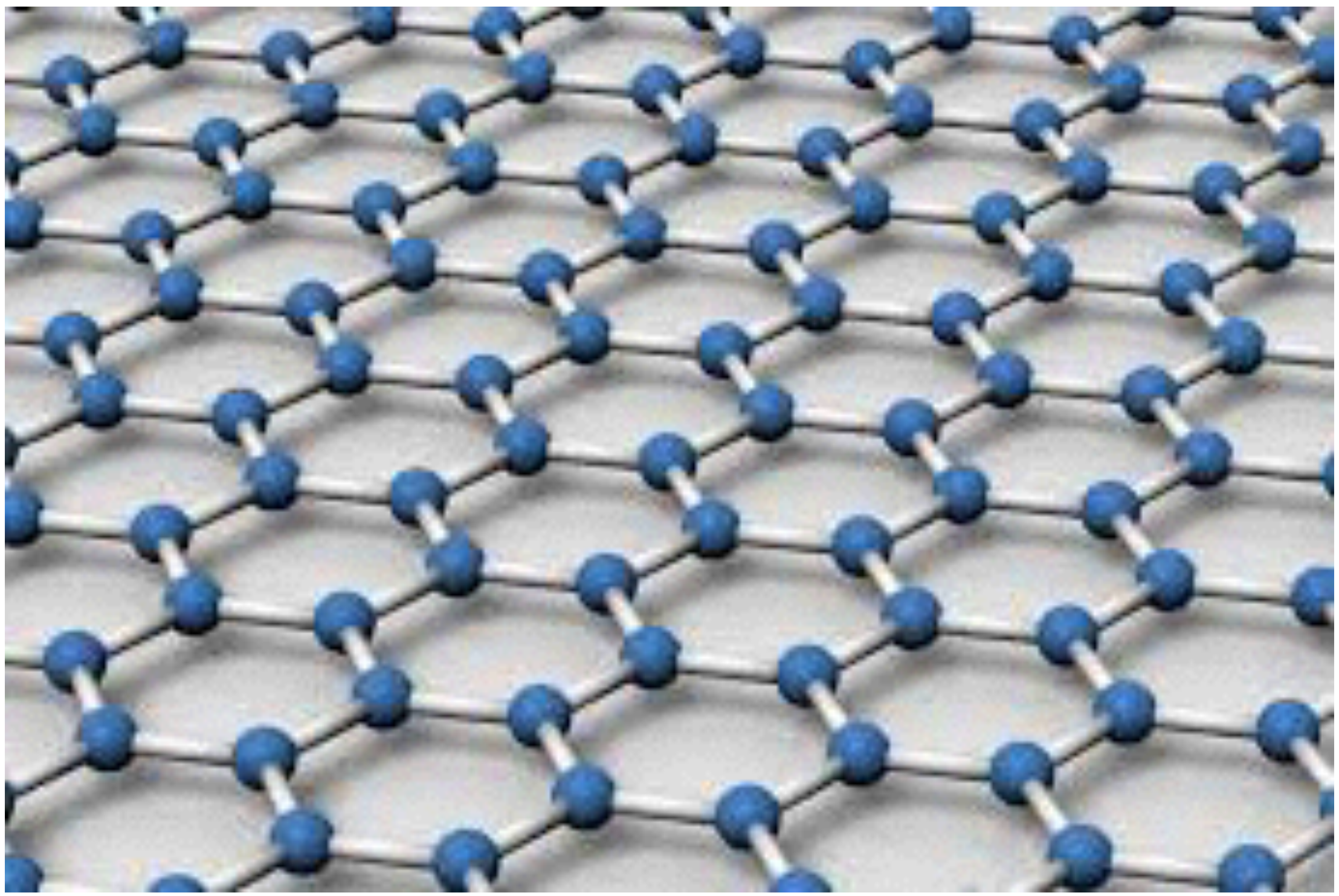
- Wiedemann-Franz law in a Fermi liquid:

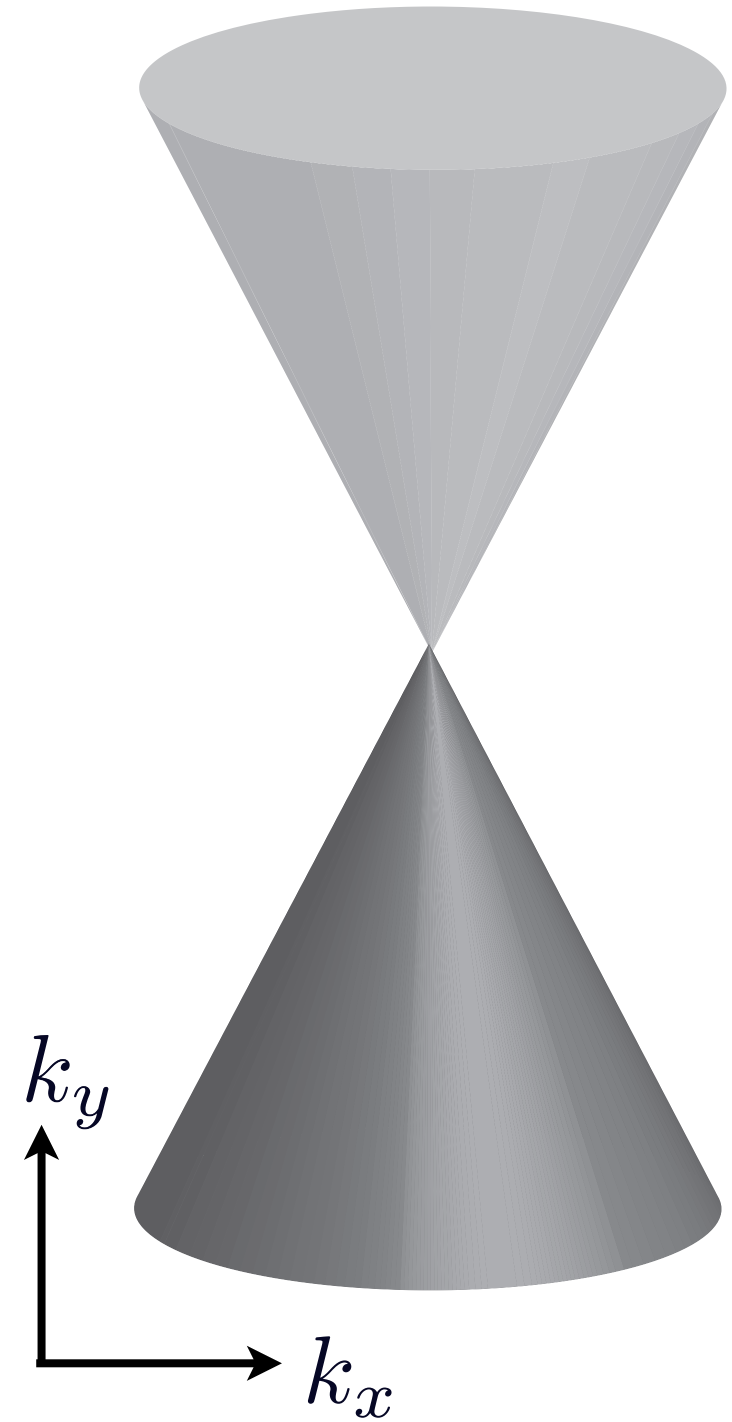
$$\frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_{\mathrm{B}}^2}{3e^2} \approx 2.45 \times 10^{-8} \, \frac{\mathrm{W} \cdot \Omega}{\mathrm{K}^2}.$$




G. S. Kumar, G. Prasad, and R.O. Pohl, J. Mat. Sci. **28**, 4261 (1993)

# Graphene





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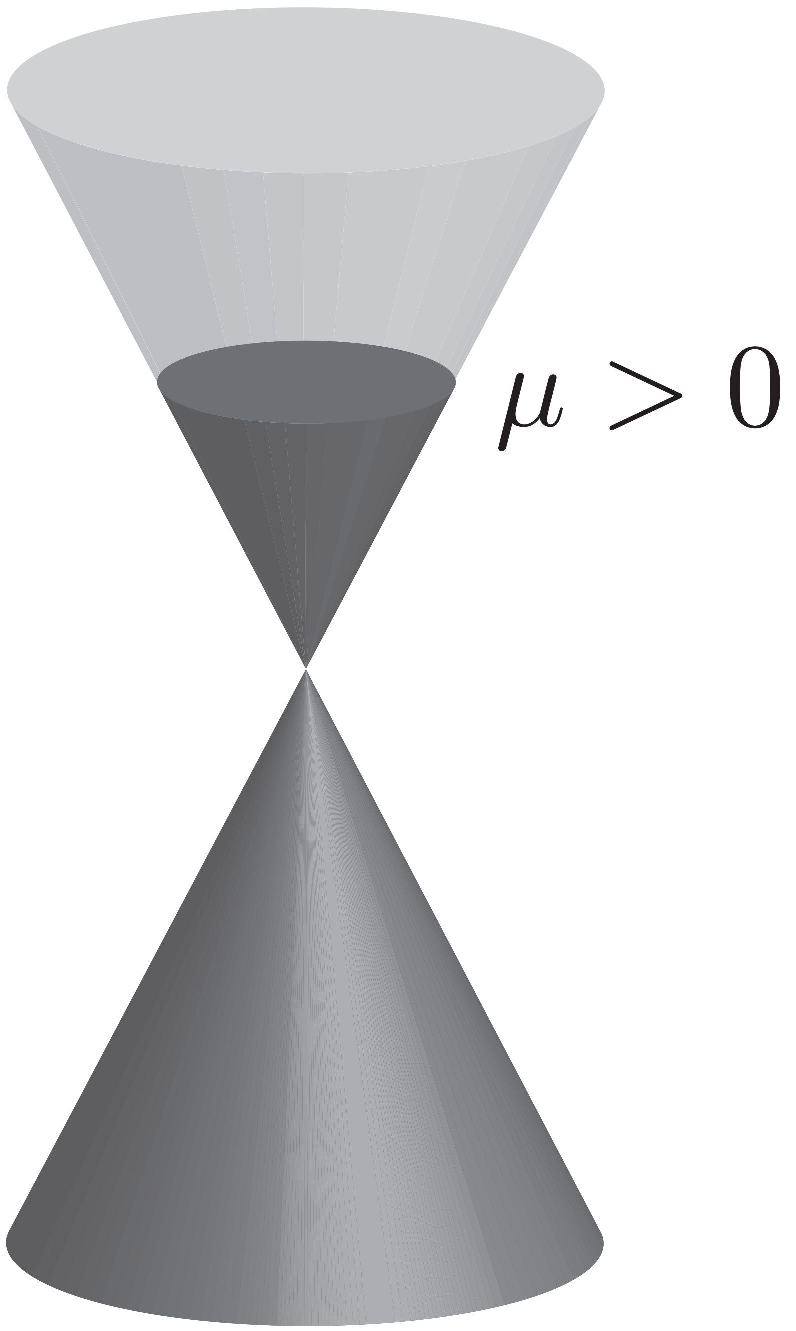


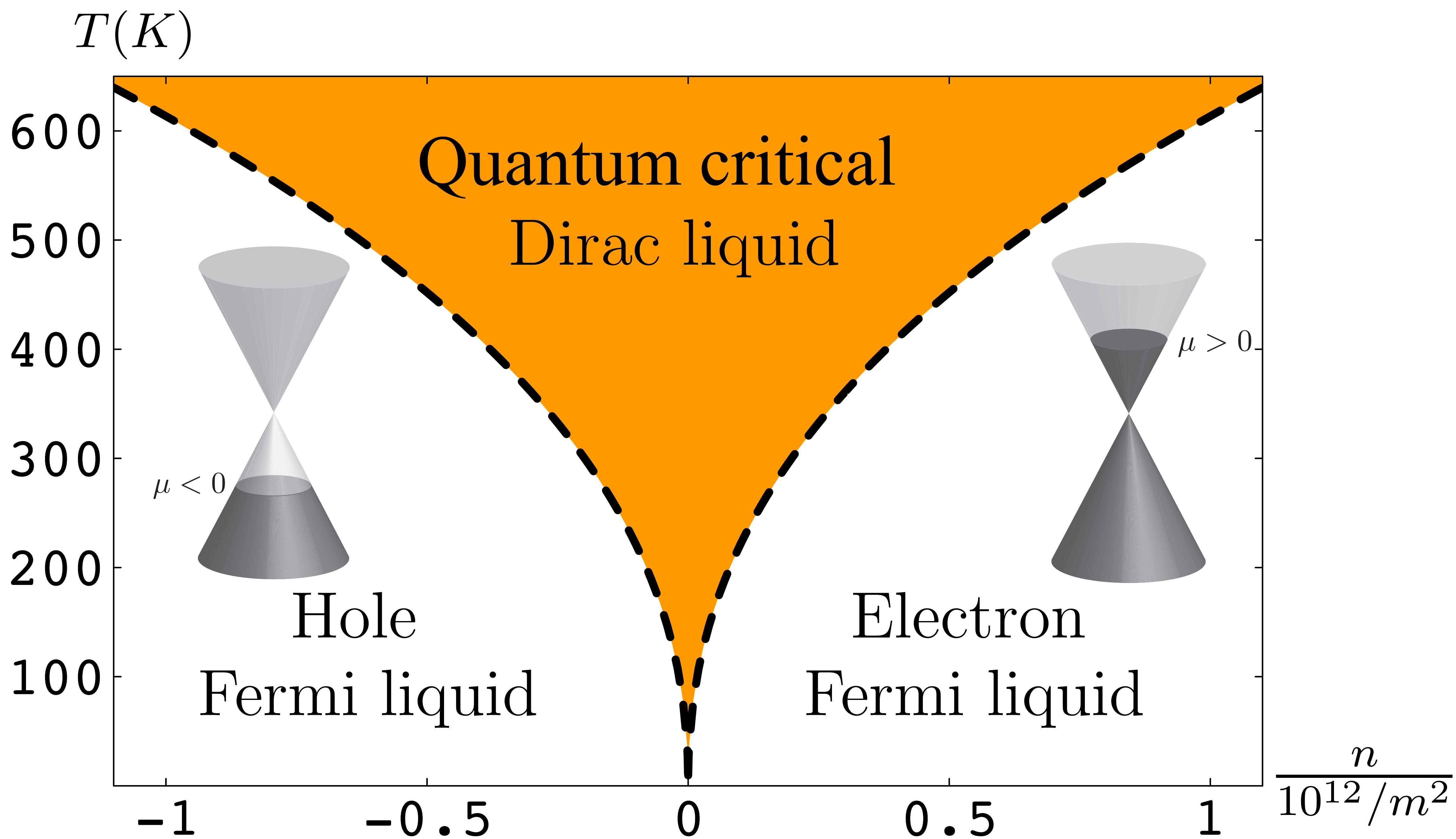

Electron
Fermi surface

# Graphene


Hole
Fermi surface

Electron
Fermi surface

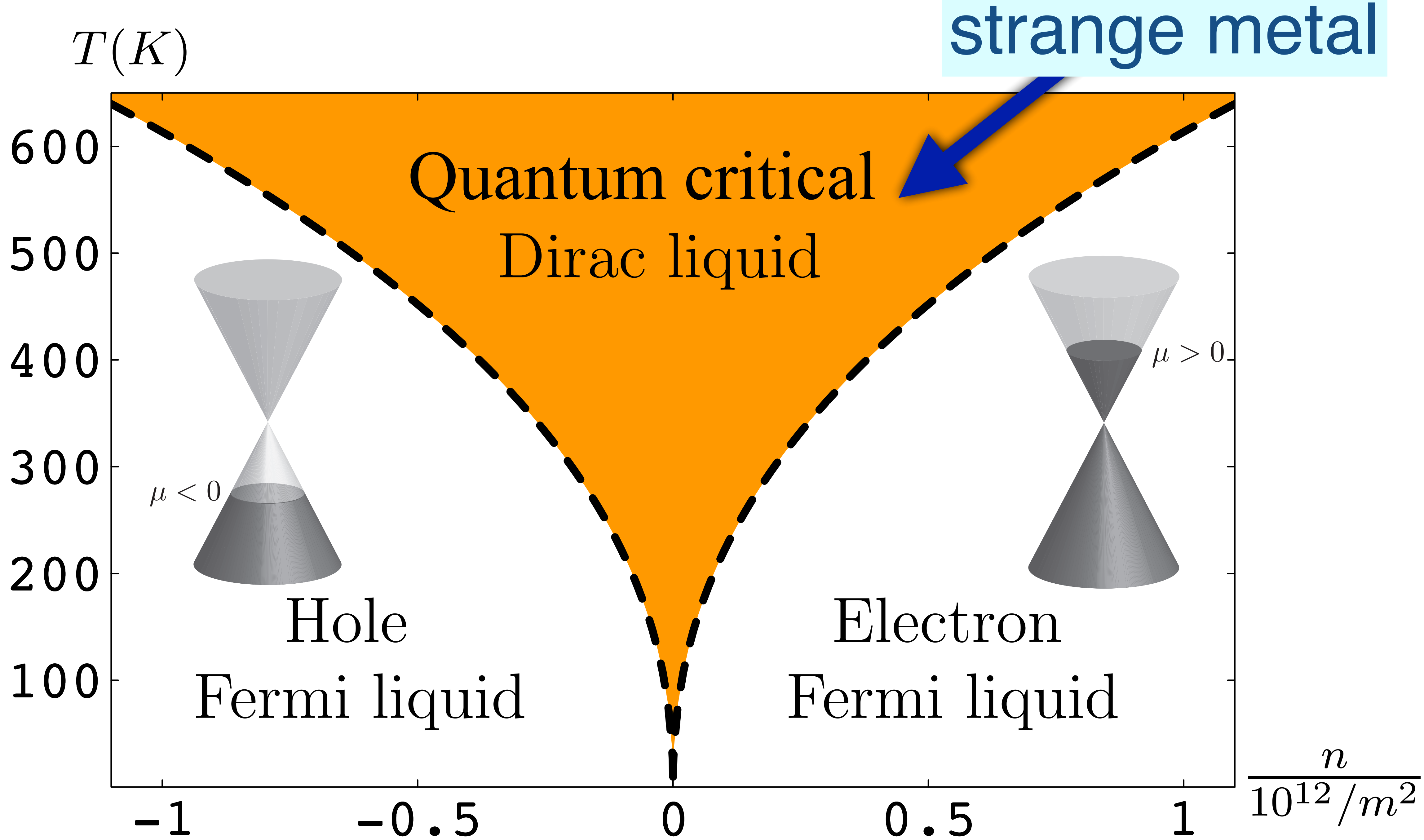
# Graphene




D. E. Sheehy and J. Schmalian, PRL **99**, 226803 (2007)
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

# Graphene

Predicted strange metal

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- Shortest possible "collision time", or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$

S. Sachdev, Quantum Phase Transitions (1999)

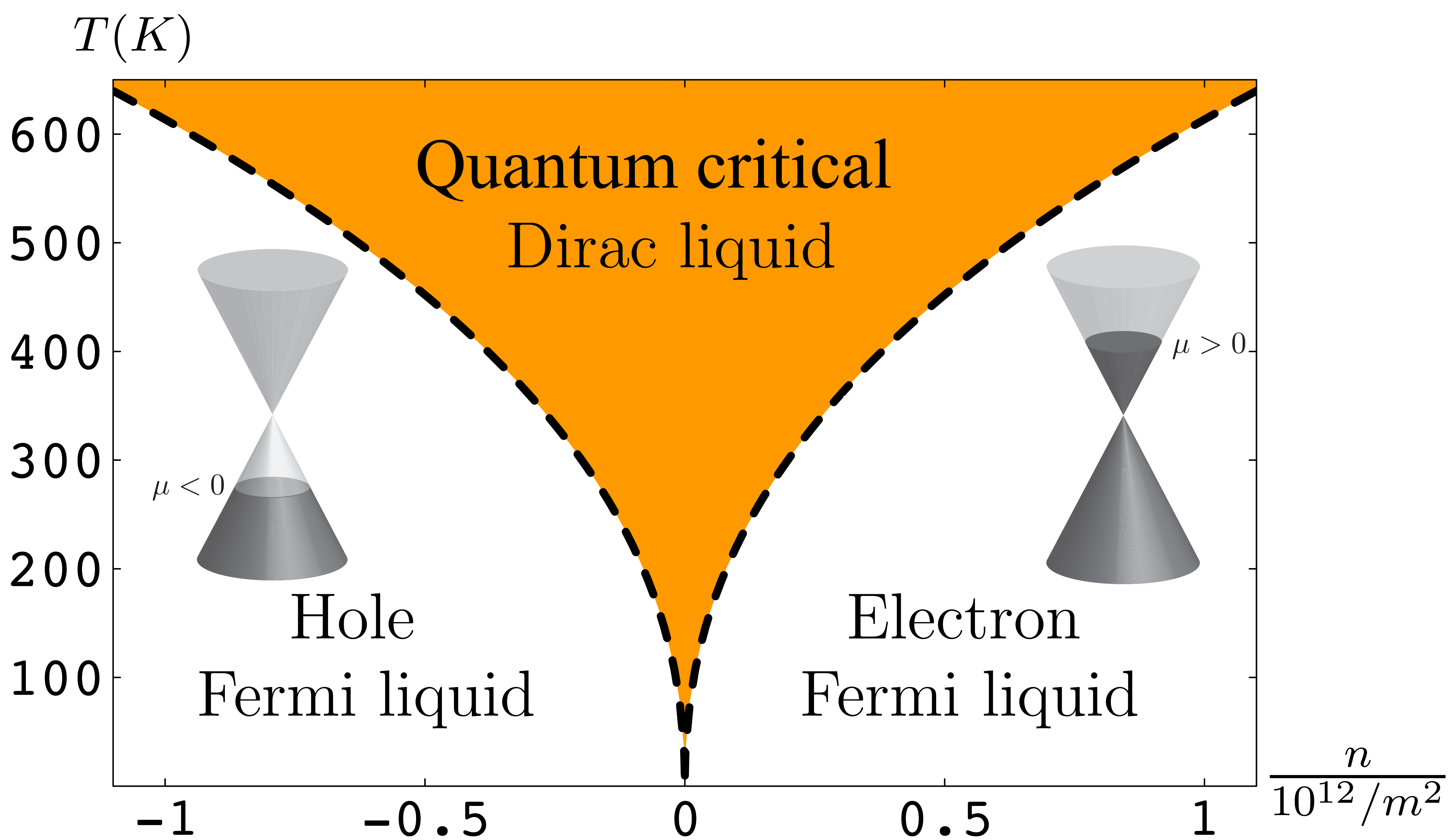
# Key properties of a strange metal

- No quasiparticle excitations

- Shortest possible "collision time", or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$

- Continuously variable density, $\mathcal{Q}$ (conformal field theories are usually at fixed density, $\mathcal{Q} = 0$)

# Graphene

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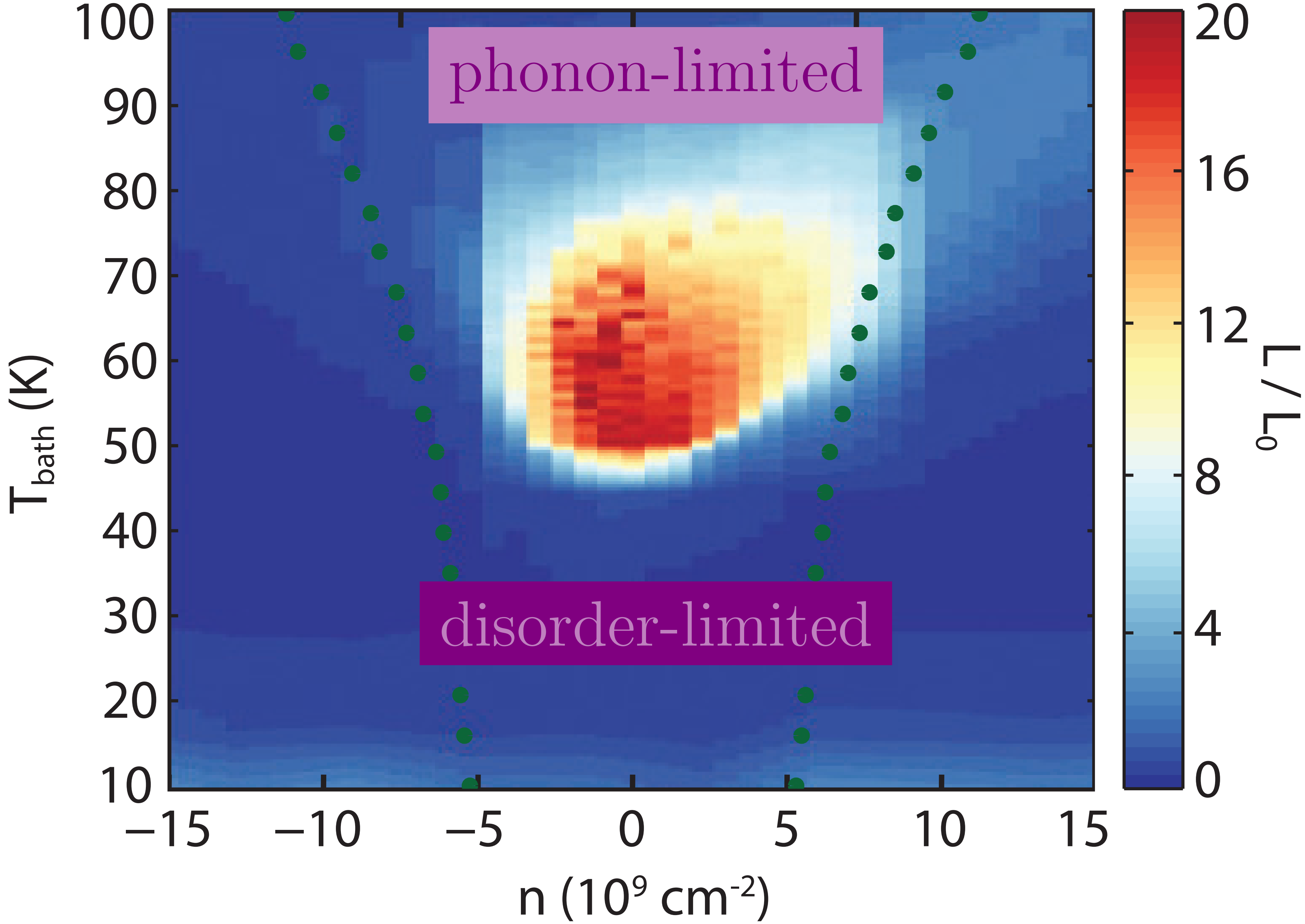
J. Crossno *et al.* arXiv:1509.04713; Science, to appear

# Strange metal in graphene



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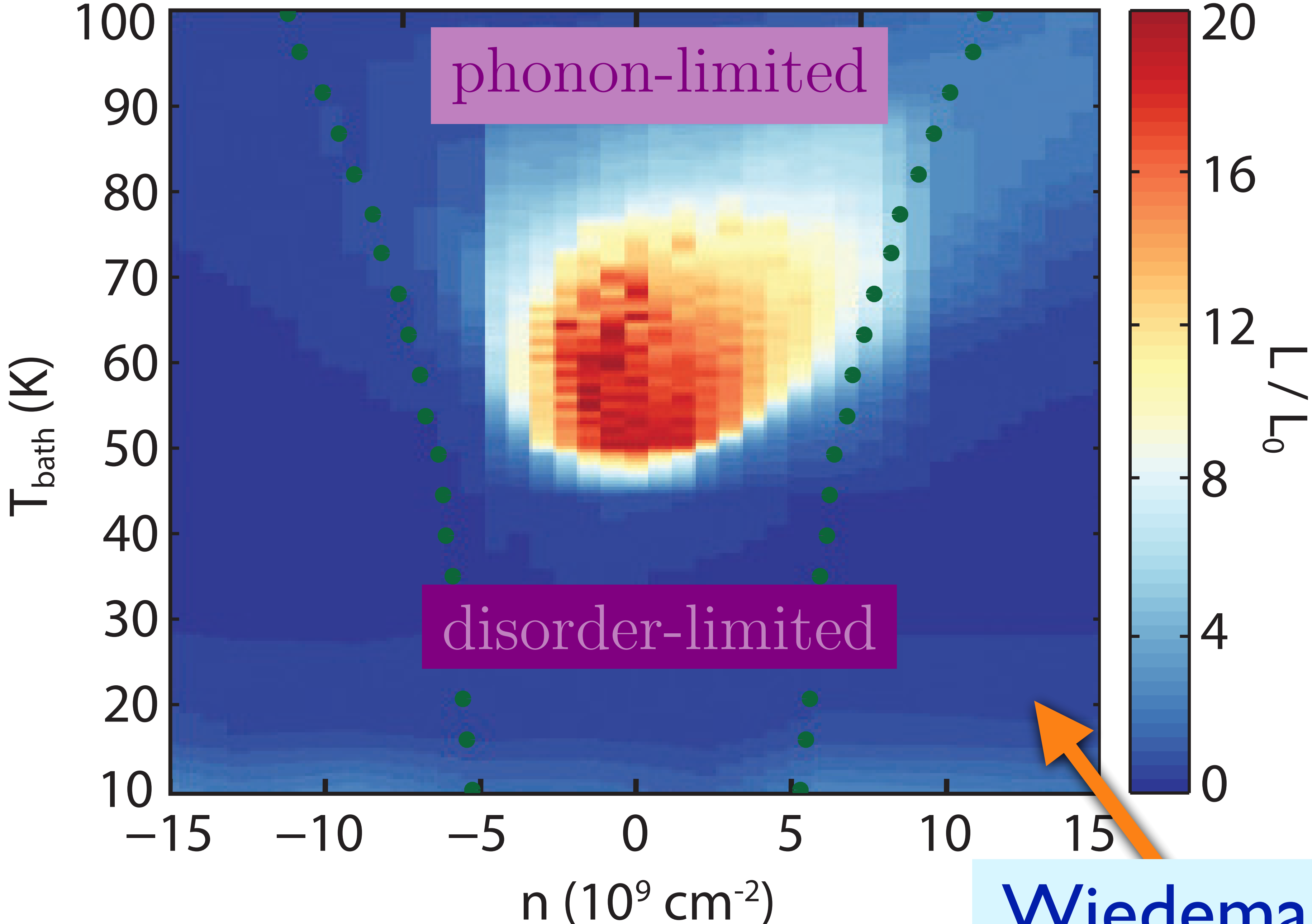


Wiedemann-Franz obeyed

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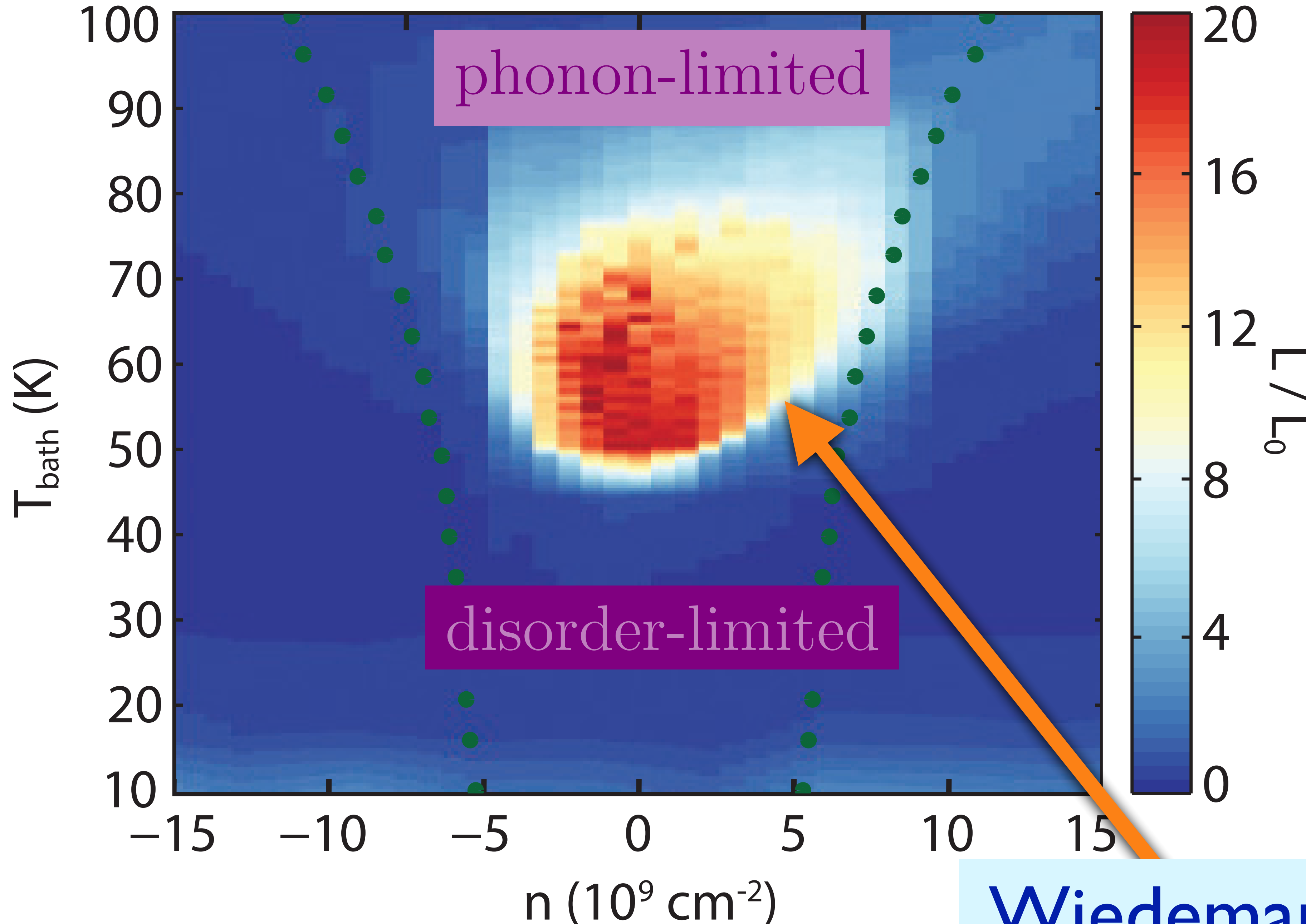
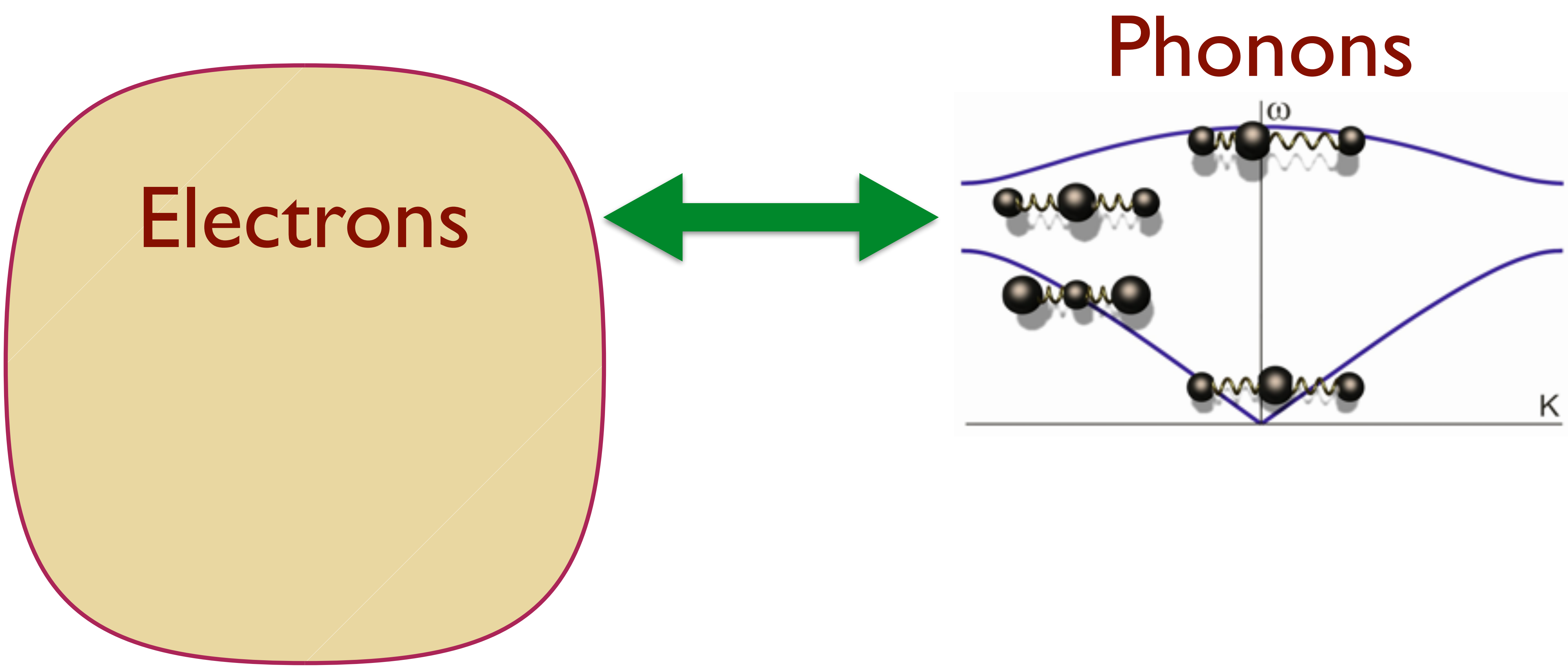



Wiedemann-Franz violated !

# Quasiparticle transport in metals:

- Compute the scattering rate of charged quasiparticles off phonons: this leads to Bloch's law (1930) : a resistivity $\rho(T) \sim T^5$.



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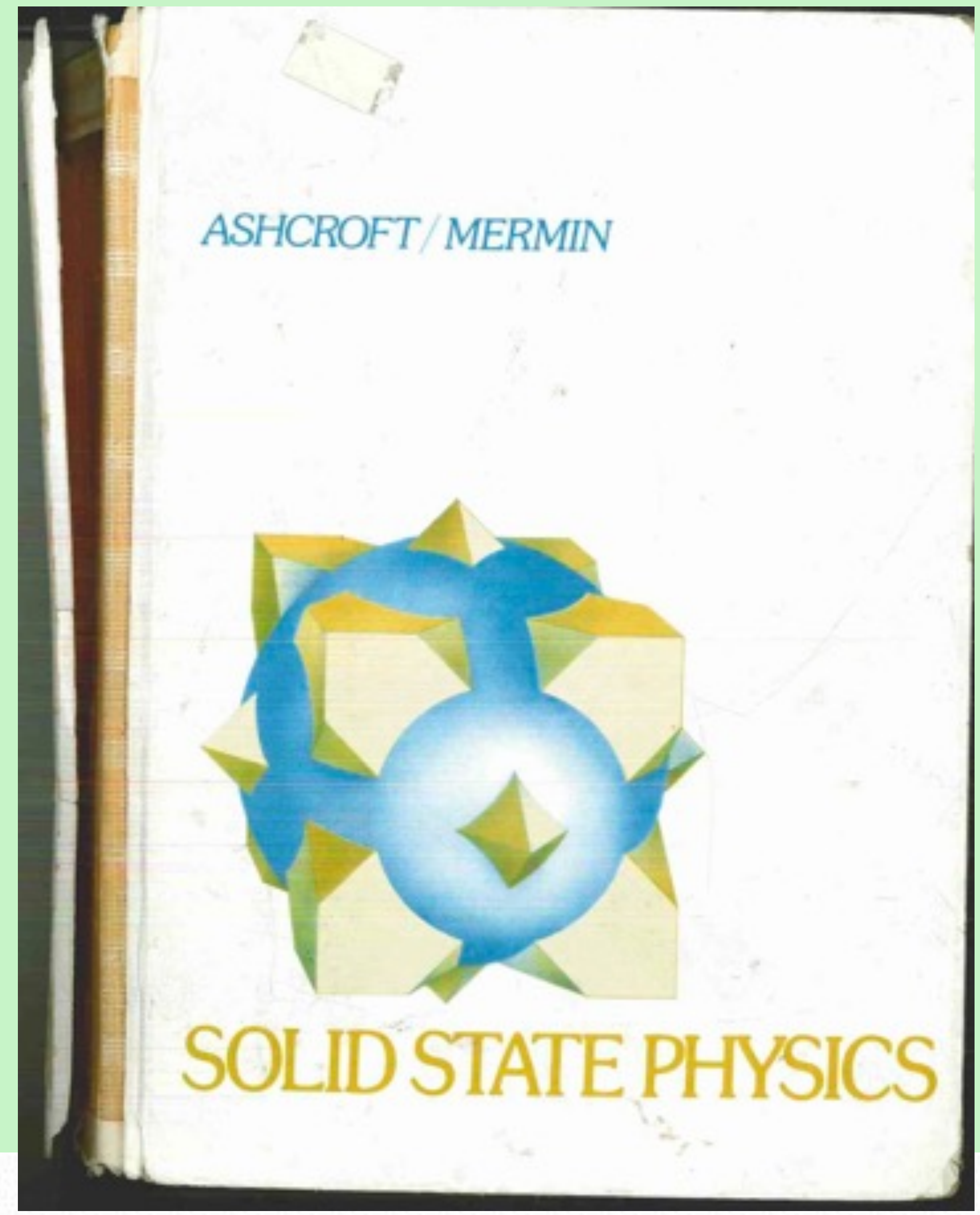


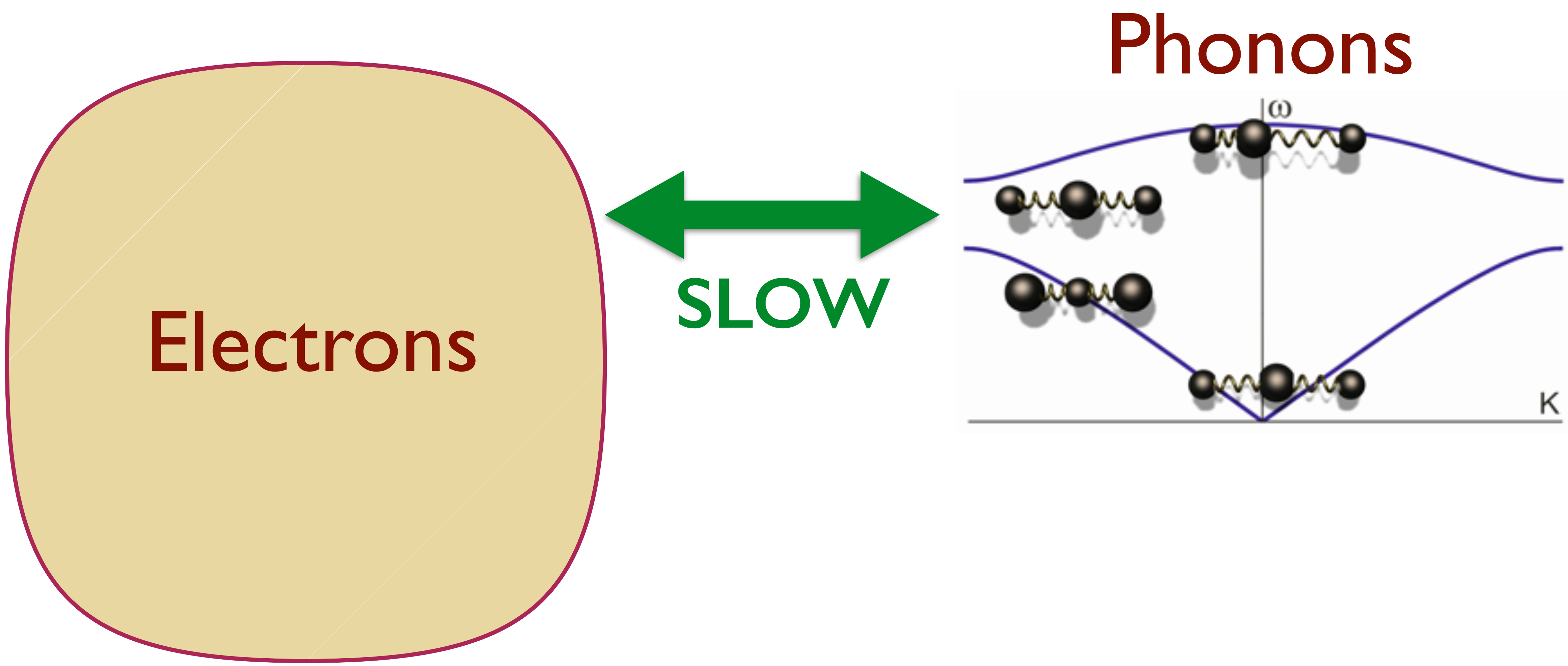

**However, this ignores "phonon drag"**

**PHONON DRAG**

Peierls[28] pointed out a way in which the low temperature resistivity might decline more rapidly than $T^5$. This behavior has yet to be observed,

[28] R. E. Peierls, *Ann. Phys.* (5) **12**, 154 (1932).

# Rates of Momentum Flow



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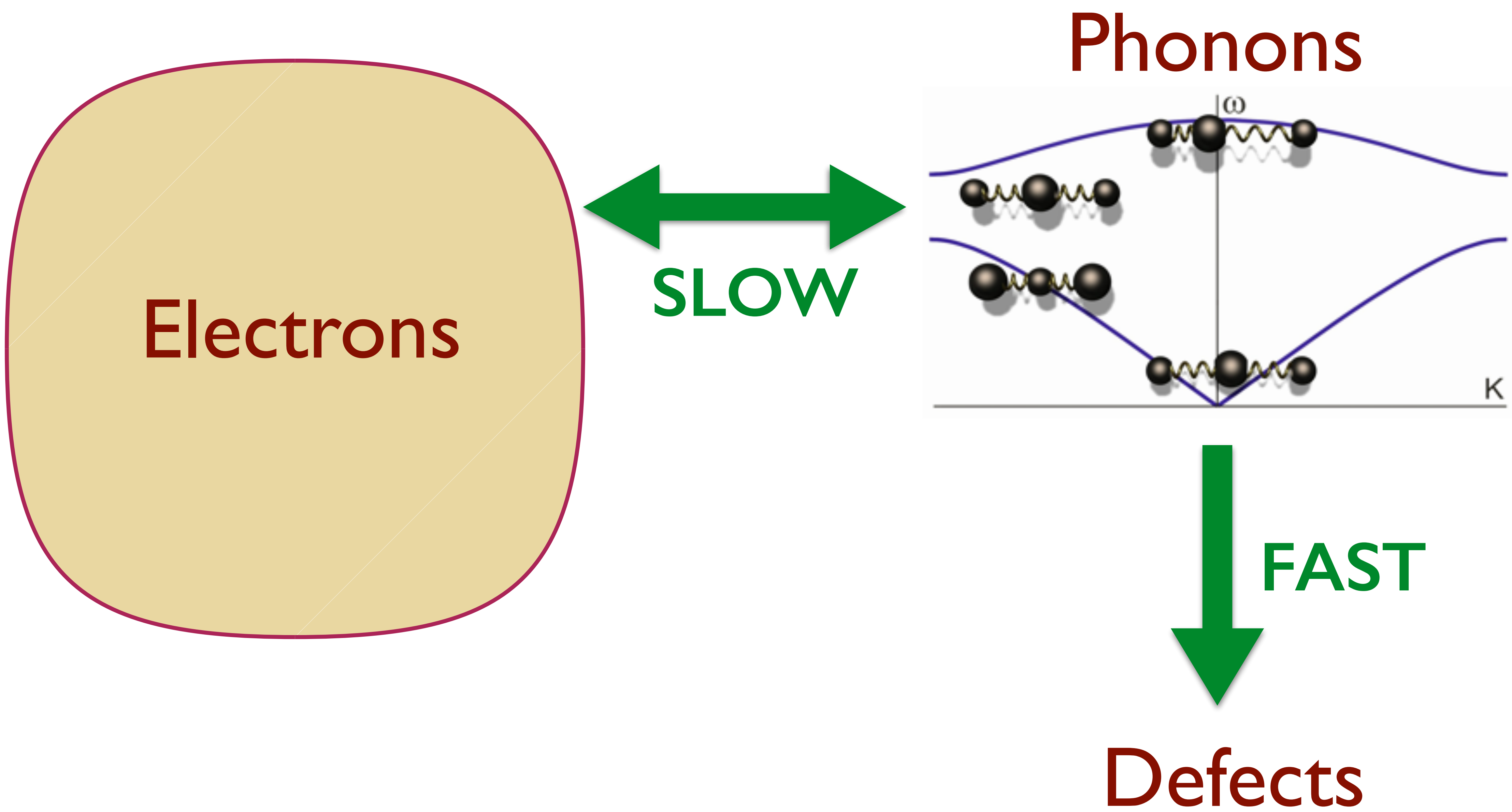

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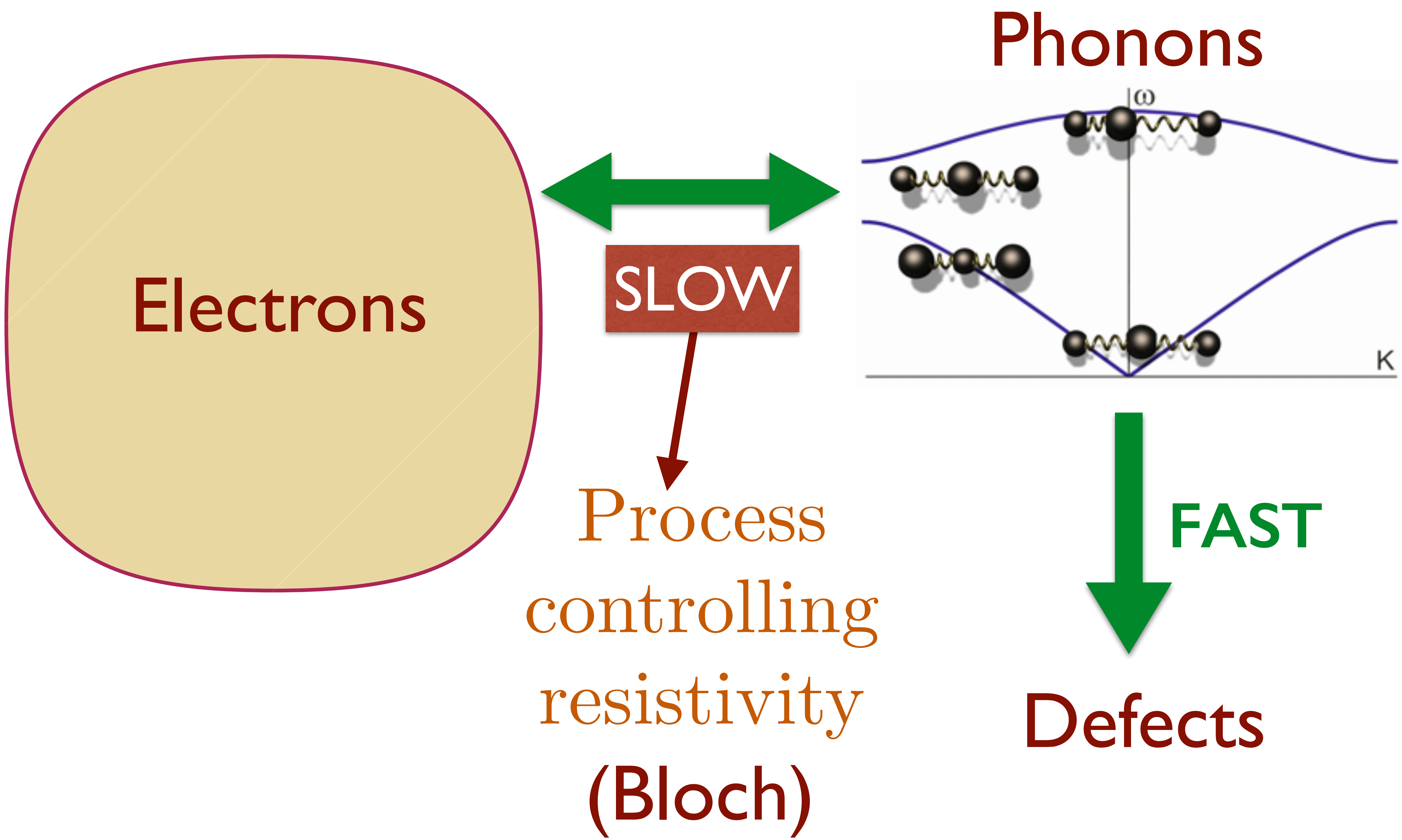

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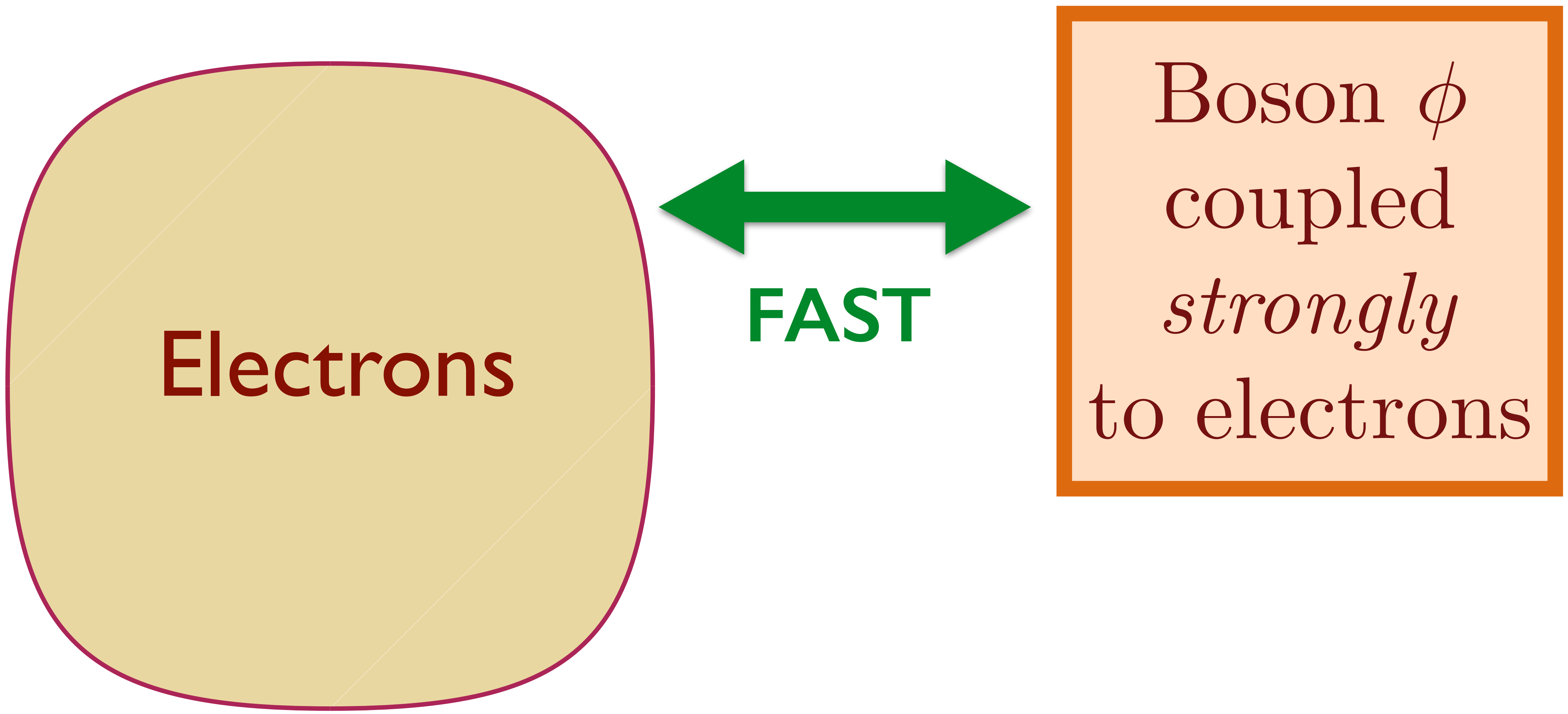

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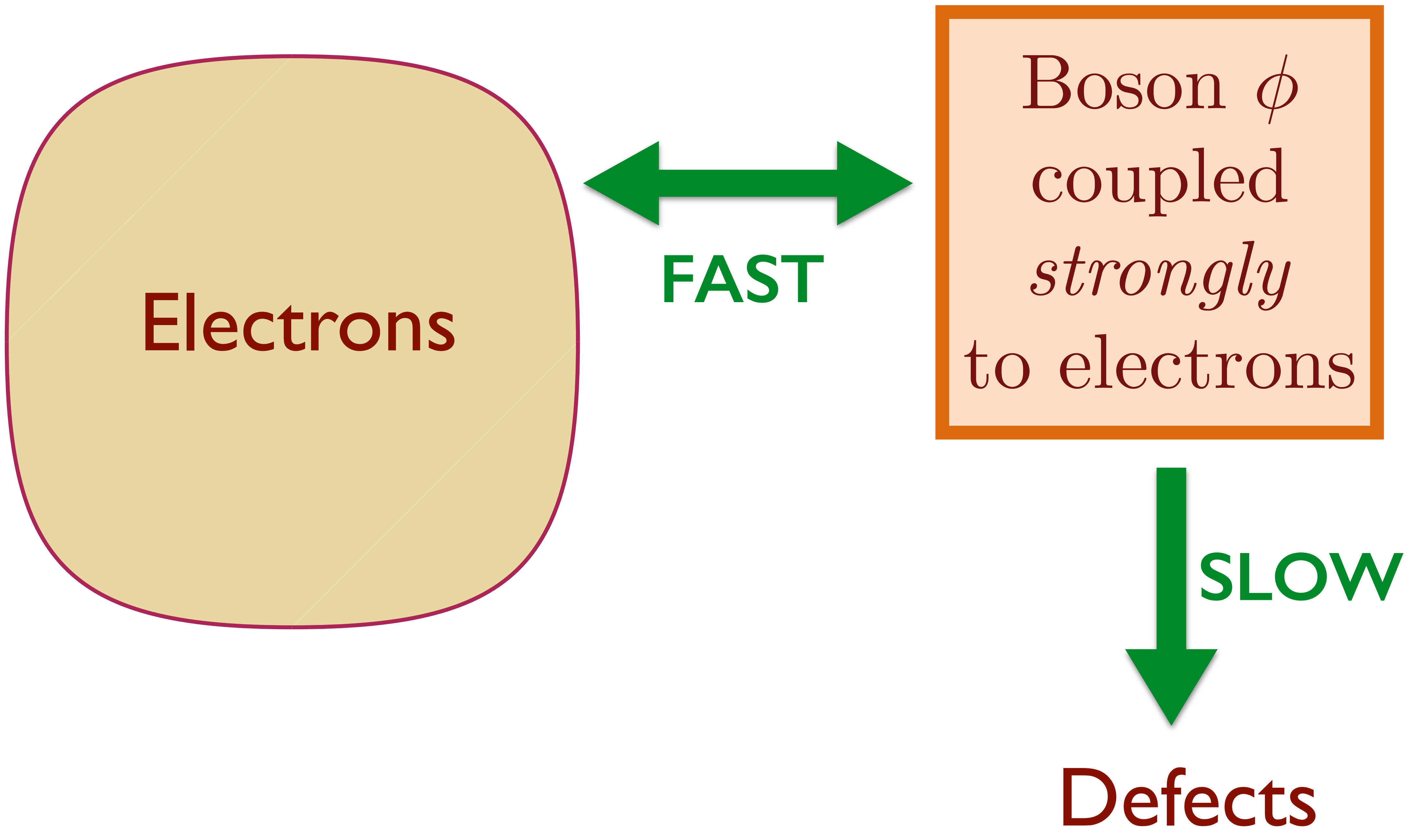

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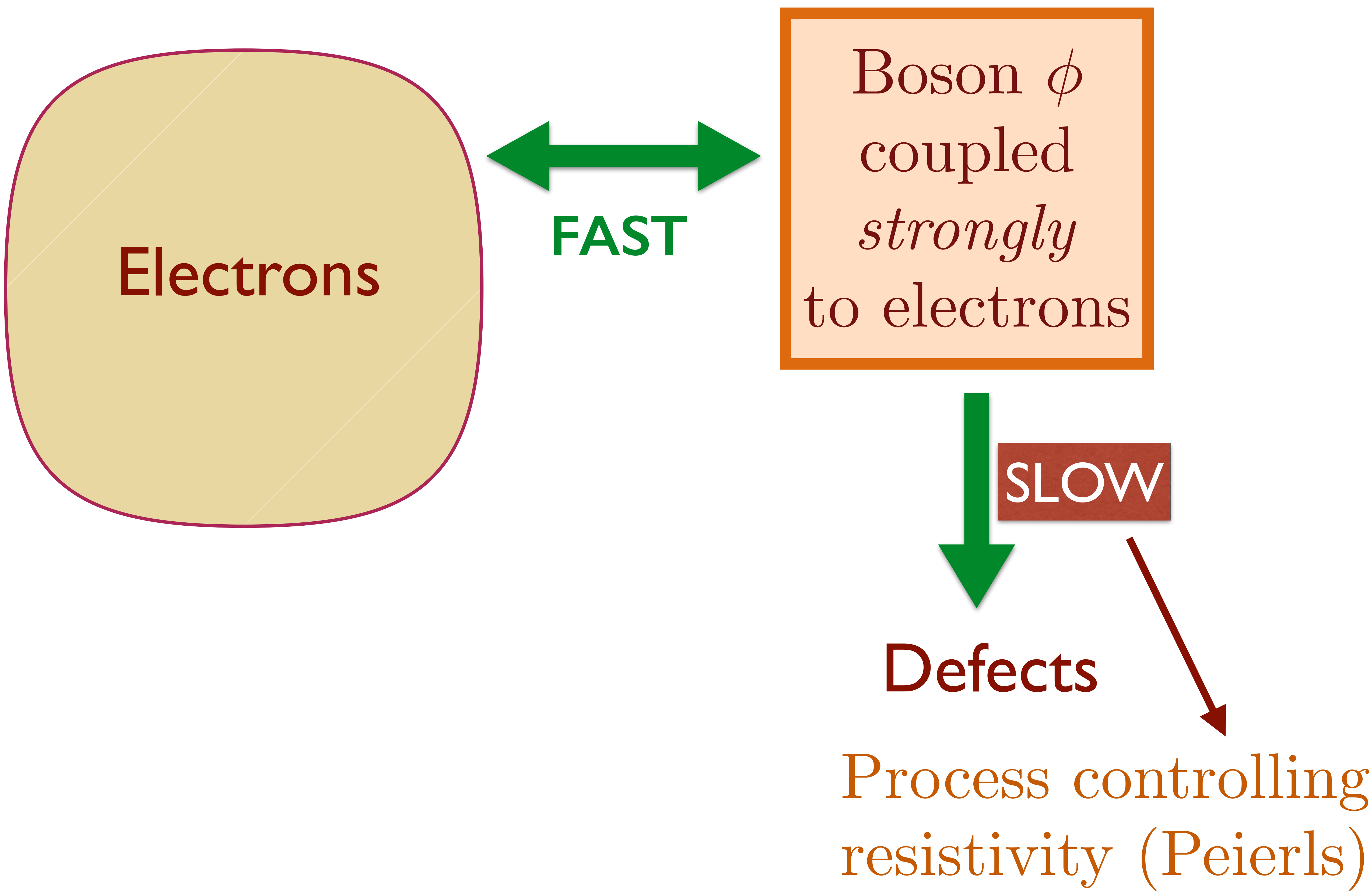

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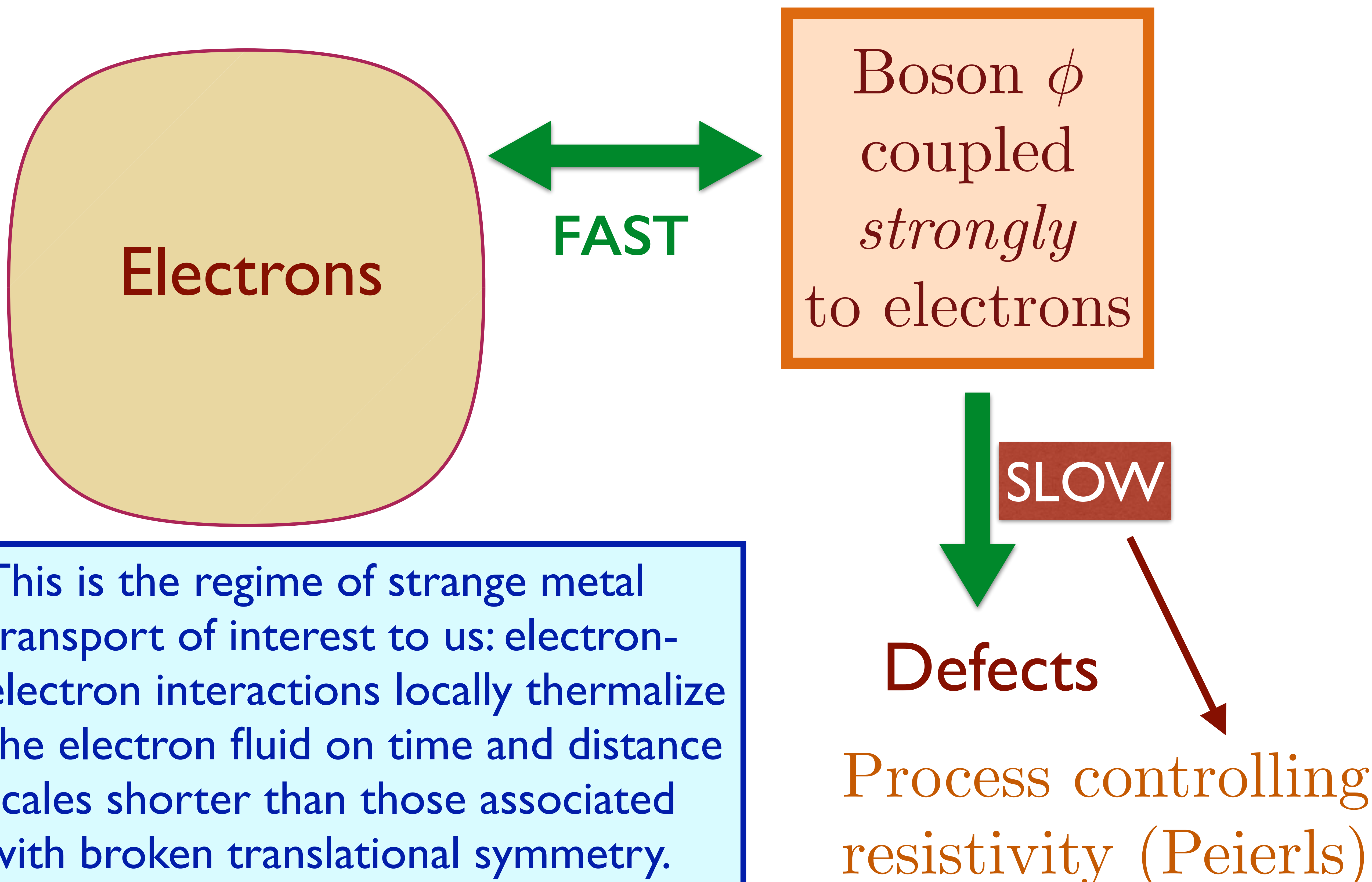


This is the regime of strange metal transport of interest to us: electron-electron interactions locally thermalize the electron fluid on time and distance scales shorter than those associated with broken translational symmetry.

# Transport in Strange Metals

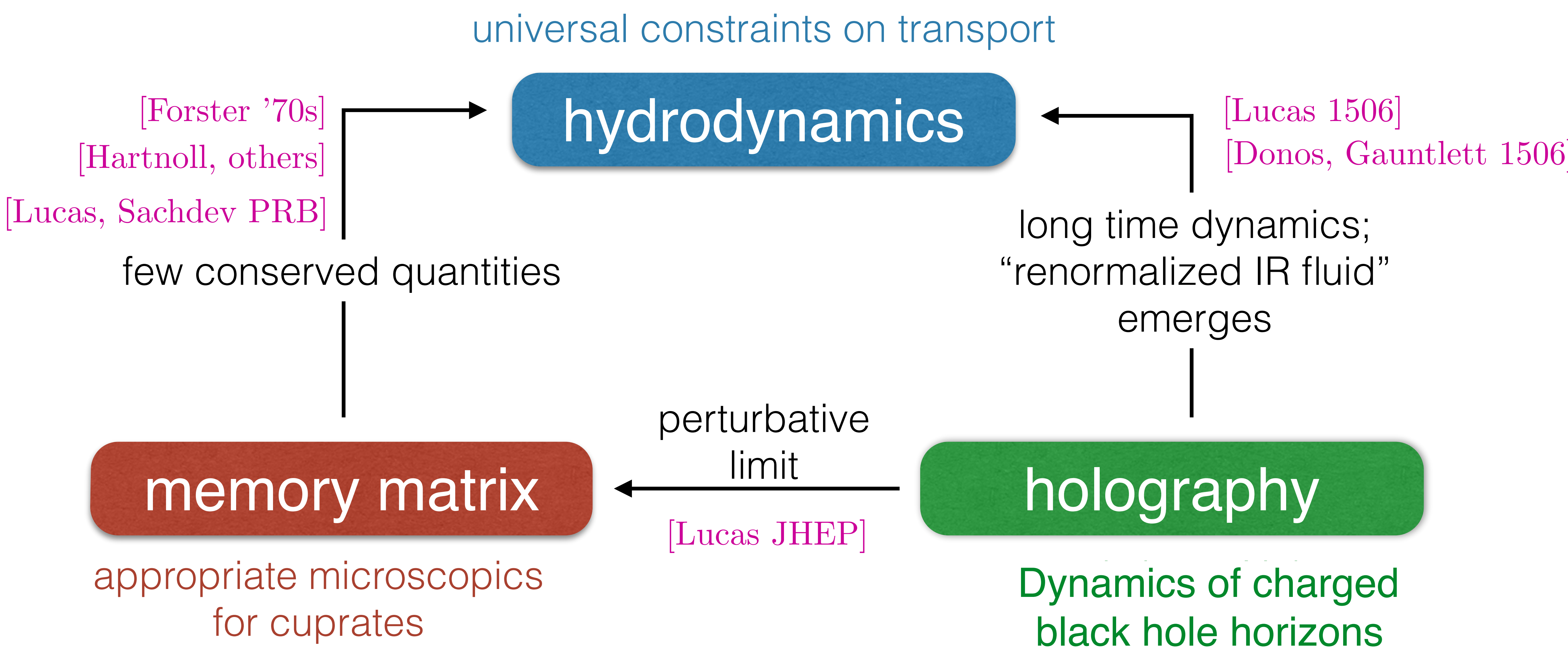


figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

# Relativistic hydrodynamics

- hydrodynamics when $l \gg l_{\mathrm{ee}}$, $t \gg t_{\mathrm{ee}}$
- long time dynamics governed by conservation laws:

$$\partial_\nu T^{\mu\nu} = J_\nu \left(F^{\mathrm{ext}}\right)^{\mu\nu}, \qquad \partial_\mu J^\mu = 0.$$

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dynamics of relaxation to equilibrium

- expand $T^{\mu\nu}$, $J^\mu$ in perturbative parameter $l_{\text{ee}}\partial_\mu$ :

$$T^{\mu\nu} = P\eta^{\mu\nu} + (\epsilon + P)u^\mu u^\nu$$

$$J^\mu = \mathcal{Q}u^\mu - \sigma_{\text{Q}}\mathcal{P}^{\mu\rho}\left(\partial_\rho\mu - \frac{\mu}{T}\partial_\rho T - u^\nu F^{\text{ext}}_{\rho\nu}\right) + \cdots,$$

$$\mathcal{P}^{\mu\nu} \equiv \eta^{\mu\nu} + u^\mu u^\nu,$$

$$Q^i = T^{ti} - \mu J^i$$

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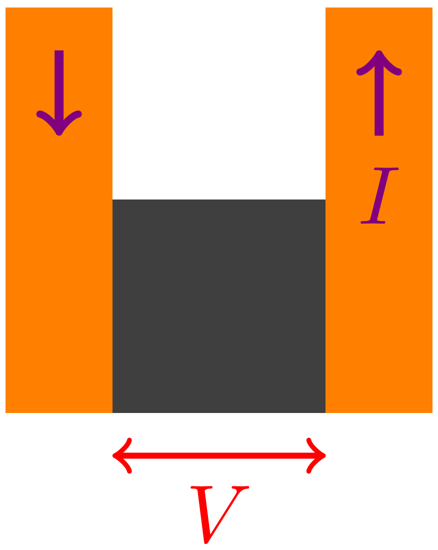
- New (and only) transport co-efficient, $\sigma_Q$: "quantum critical" conductivity at $\mathcal{Q} = 0$.

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)




$$V = IR \qquad R \sim \frac{1}{\sigma}$$

- more generally, measure thermoelectric transport:

$$\begin{pmatrix} \delta J_i \\ \delta Q_i \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & \alpha_{ij} \\ T\bar{\alpha}_{ij} & \bar{\kappa}_{ij} \end{pmatrix} \begin{pmatrix} \delta E_j \\ -\partial_j \delta T \equiv T\delta\zeta_j \end{pmatrix}.$$

- $\sigma$ = easy experiment; related to QFT correlators:

$$\sigma_{ij}(\omega) = \frac{\mathrm{i}}{\omega}\langle J_i(-\omega) J_j(\omega)\rangle, \quad \text{etc.}$$

# Thermoelectric transport coefficients

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

$$\sigma = \frac{\mathcal{Q}^2}{\mathcal{M}} \pi \delta(\omega) + \sigma_Q(\omega)$$

$$\alpha = \frac{\mathcal{S}\mathcal{Q}}{\mathcal{M}} \pi \delta(\omega) + \alpha_Q(\omega)$$

$$\bar{\kappa} = \frac{T\mathcal{S}^2}{\mathcal{M}} \pi \delta(\omega) + \bar{\kappa}_Q(\omega)$$

with entropy density $\mathcal{S}$, $\mathcal{Q} \equiv \chi_{J_x,P_x}$, and $\mathcal{M} \equiv \chi_{P_x,P_x}$.

Obtained in hydrodynamics, and by memory functions


S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

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In theories which are relativistic at high energies (including graphene), $T\alpha_Q(\omega) = -\mu\sigma_Q(\omega)$, $T\bar{\kappa}_Q(\omega) = \mu^2\sigma_Q(\omega)$, $\mathcal{M} = T\mathcal{S} + \mu\mathcal{Q} = \mathcal{H}$ the enthalpy density, and $\mathcal{Q} = n$ the electron density

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
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# Translational symmetry breaking

Momentum relaxation by an external source $h$ coupling to the operator $\mathcal{O}$

$$H = H_0 - \int d^d x\, h(x)\, \mathcal{O}(x).$$

Leads to an additional term in equations of motion:

$$\partial_\mu T^{\mu i} = \ldots - \frac{T^{it}}{\tau_{\rm imp}} + \ldots$$

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"Memory function" methods yield an explicit expression for $\tau_{\rm imp}$:

$$\frac{\mathcal{M}}{\tau_{\rm imp}} = \lim_{\omega \to 0} \int d^d q\, |h(q)|^2 q_x^2 \frac{\mathrm{Im}\left(G^{\rm R}_{\mathcal{O}\mathcal{O}}(q,\omega)\right)_{H_0}}{\omega} + \ldots$$

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# Thermoelectric transport coefficients

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

$$\sigma = \frac{\mathcal{Q}^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau_{\rm imp})} + \sigma_Q(\omega)$$

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# Magneto-electric transport

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

$$\sigma_{xx} = \frac{(\tau_{\rm imp}^{-1} - i\omega)\mathcal{M}\sigma_Q + \mathcal{Q}^2 + B^2\sigma_Q^2}{\mathcal{Q}^2 B^2 + ((\tau_{\rm imp}^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2}\mathcal{M}\left(\frac{1}{\tau_{\rm imp}} - i\omega\right),$$

$$\sigma_{xy} = \frac{2(\tau_{\rm imp}^{-1} - i\omega)\mathcal{M}\sigma_Q + \mathcal{Q}^2 + B^2\sigma_Q^2}{\mathcal{Q}^2 B^2 + ((\tau_{\rm imp}^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2}B\mathcal{Q}.$$

## Electrical and thermal magnetotransport in a magnetic field *B* with no additional parameters

(assuming $\sigma_Q$ is field-independent)

Obtained in hydrodynamics, and by memory functions

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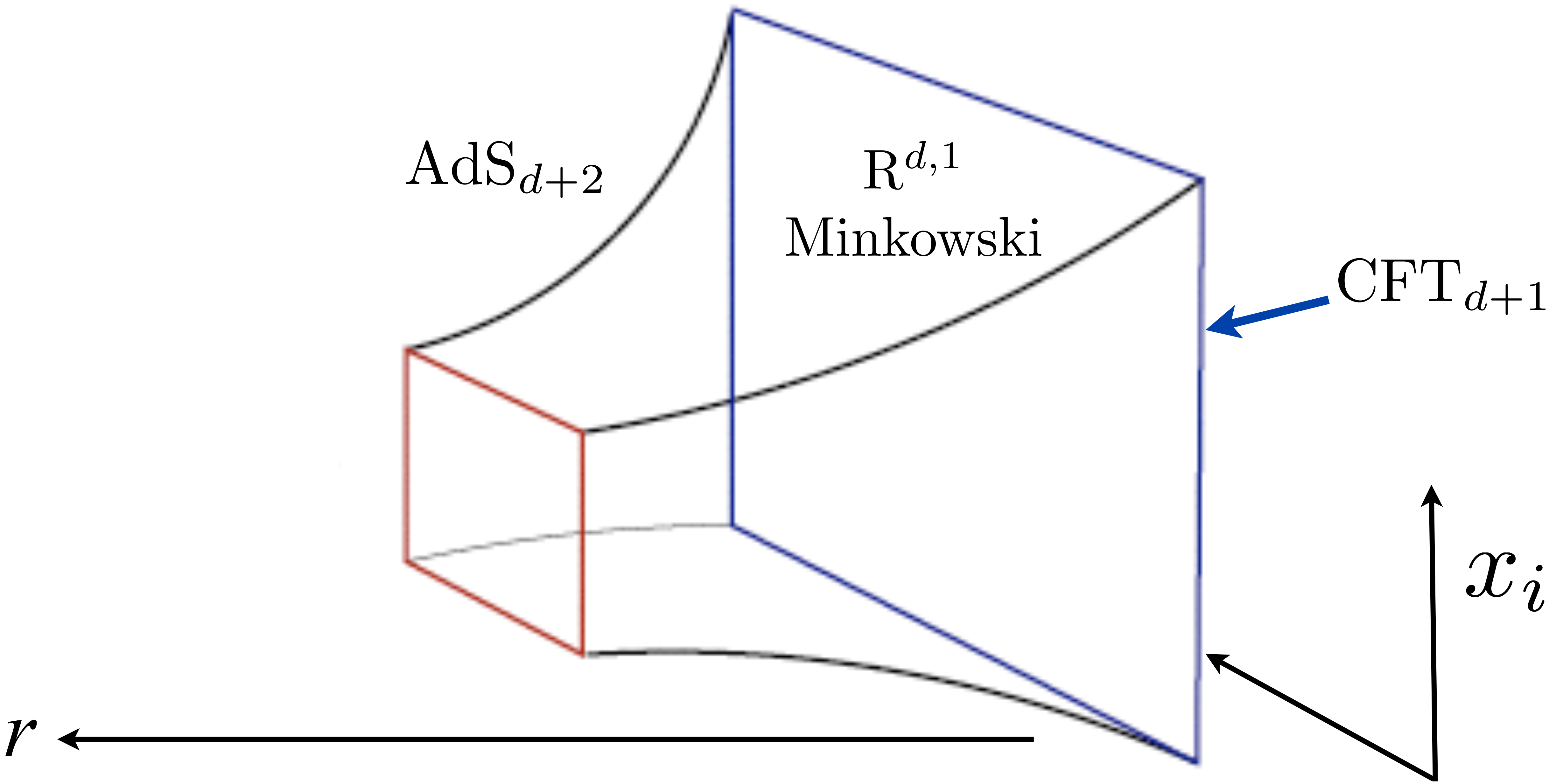
# AdS/CFT correspondence at zero temperature

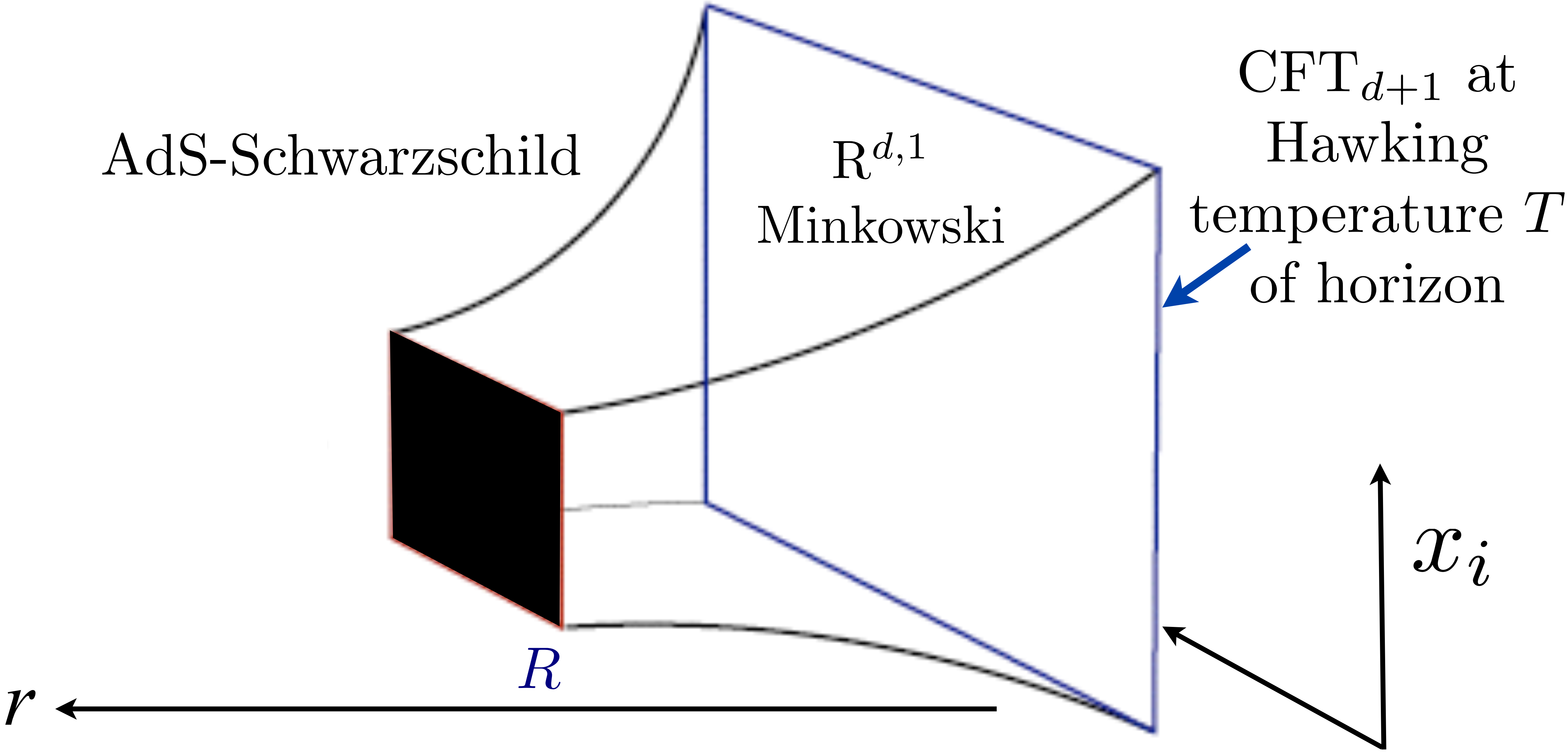
Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2}\right)\right]$




$$ds^2 = \left(\frac{L}{r}\right)^2 \left[dr^2 - dt^2 + d\vec{x}^2\right]$$

## AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( \mathcal{R} + \frac{d(d+1)}{L^2} \right) \right]$

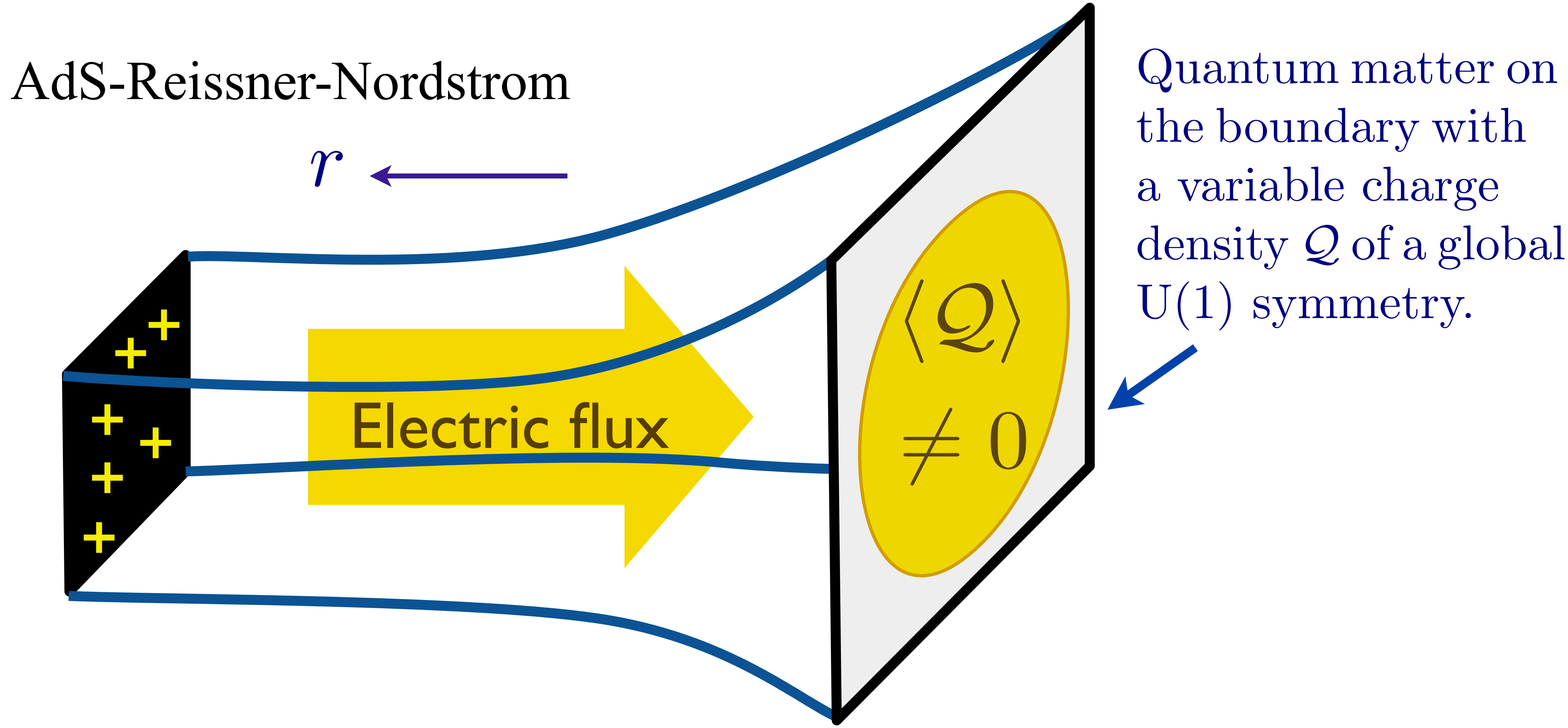



Entropy density of CFT$_{d+1}$, $\mathcal{S} \sim T^d$

Bekenstein-Hawking entropy density, $\mathcal{S}_{\rm BH} \sim T^d$

# Charged black branes

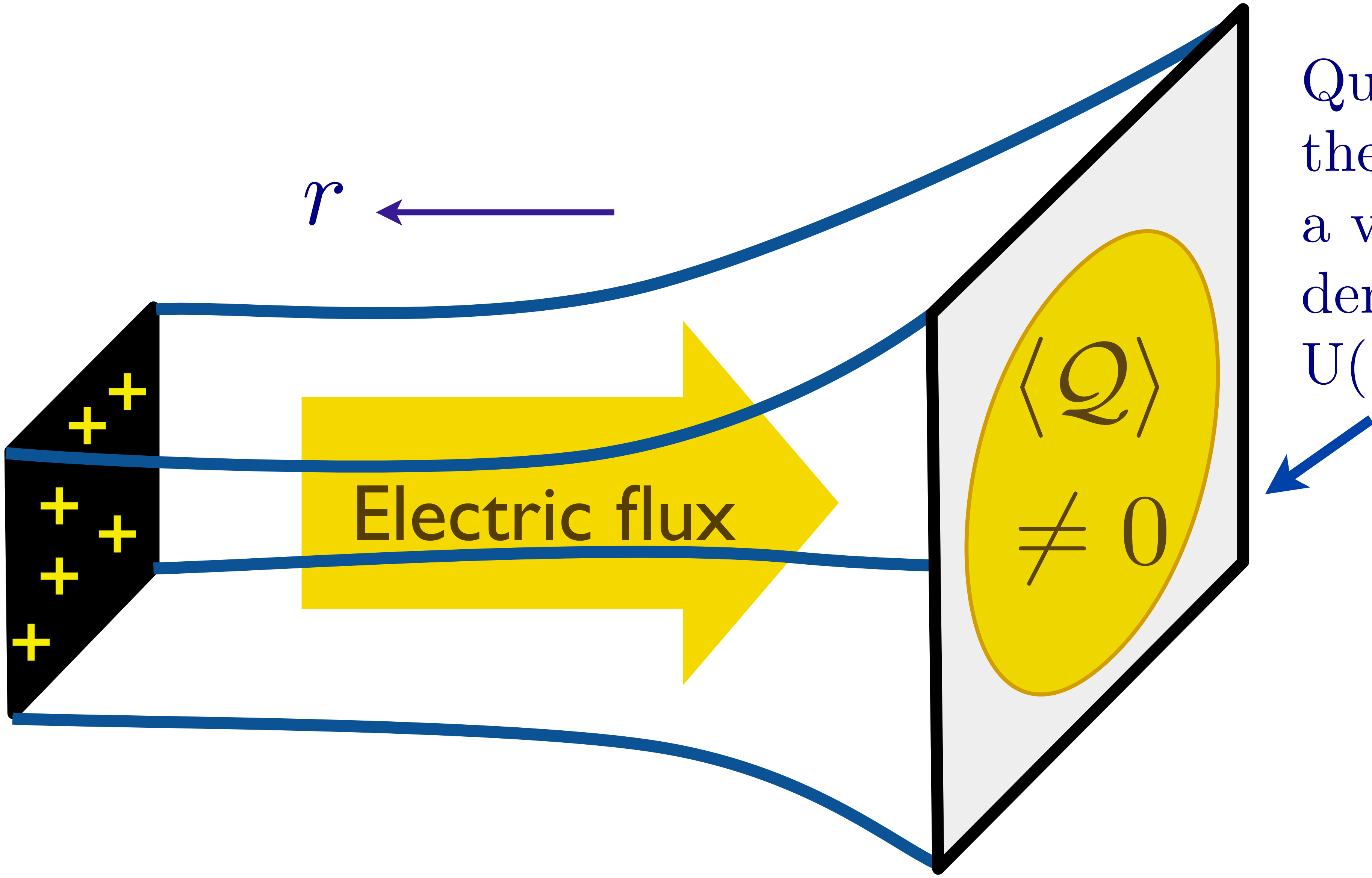
Einstein-Maxwell theory $\quad \mathcal{S}_{EM} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2}F^2\right)\right]$




Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, $\mathcal{Q}$, at $T = 0$ which does not have any quasiparticle excitations.

A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, 99

# Charged black branes



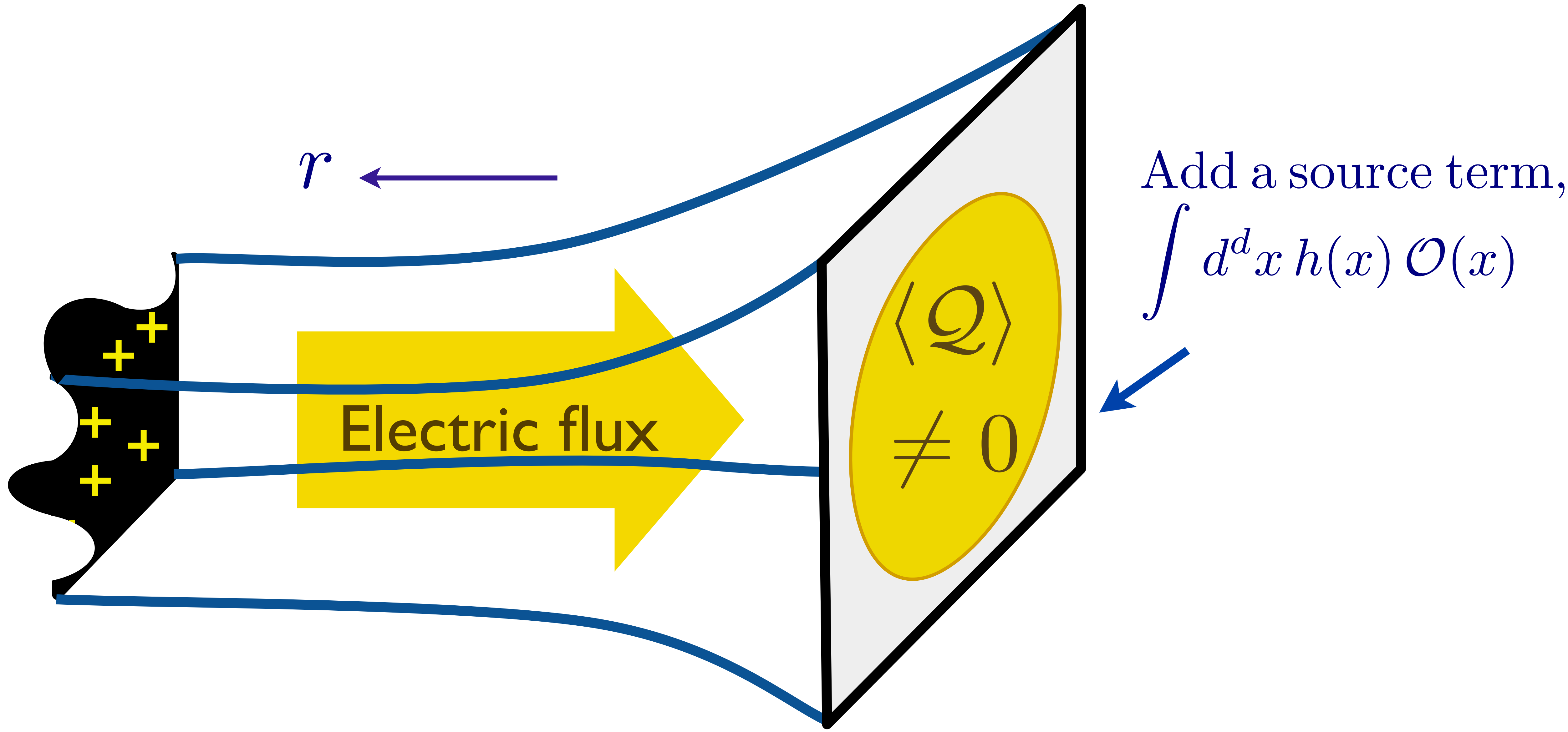

Quantum matter on the boundary with a variable charge density $\mathcal{Q}$ of a global U(1) symmetry.

More general theories have "hyperscaling violating metric"

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right) \quad \text{at } T=0$$


C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, JHEP **1201**, 94 (2012).
L. Huijse, S. Sachdev, B. Swingle, Phys. Rev. B **85**, 035121 (2012)

# Inhomogeneous charged black branes




**Weakly disordered charged black branes yield results identical to those obtained from memory functions and hydrodynamics**


• G.T. Horowitz, J.E. Santos, and D. Tong, JHEP **1207**, 168 (2012), JHEP **1211**, 102 (2012). • D. Vegh, arXiv:1301.0537. • M. Blake, D. Tong, and D. Vegh, PRL **112**, 071602 (2013). • M. Blake and D. Tong, PRD **88**, 106004 (2013). • A. Lucas, S. Sachdev, and K. Schalm, PRD 89, 066018 (2014). • A. Lucas, JHEP **1503**, 071 (2015). • R. A. Davison and B. Goutéraux, arXiv:1505.05092; arXiv:1507.07137. • M. Blake, arXiv:1505.06992; arXiv:1507.04870.

# Transport in Strange Metals

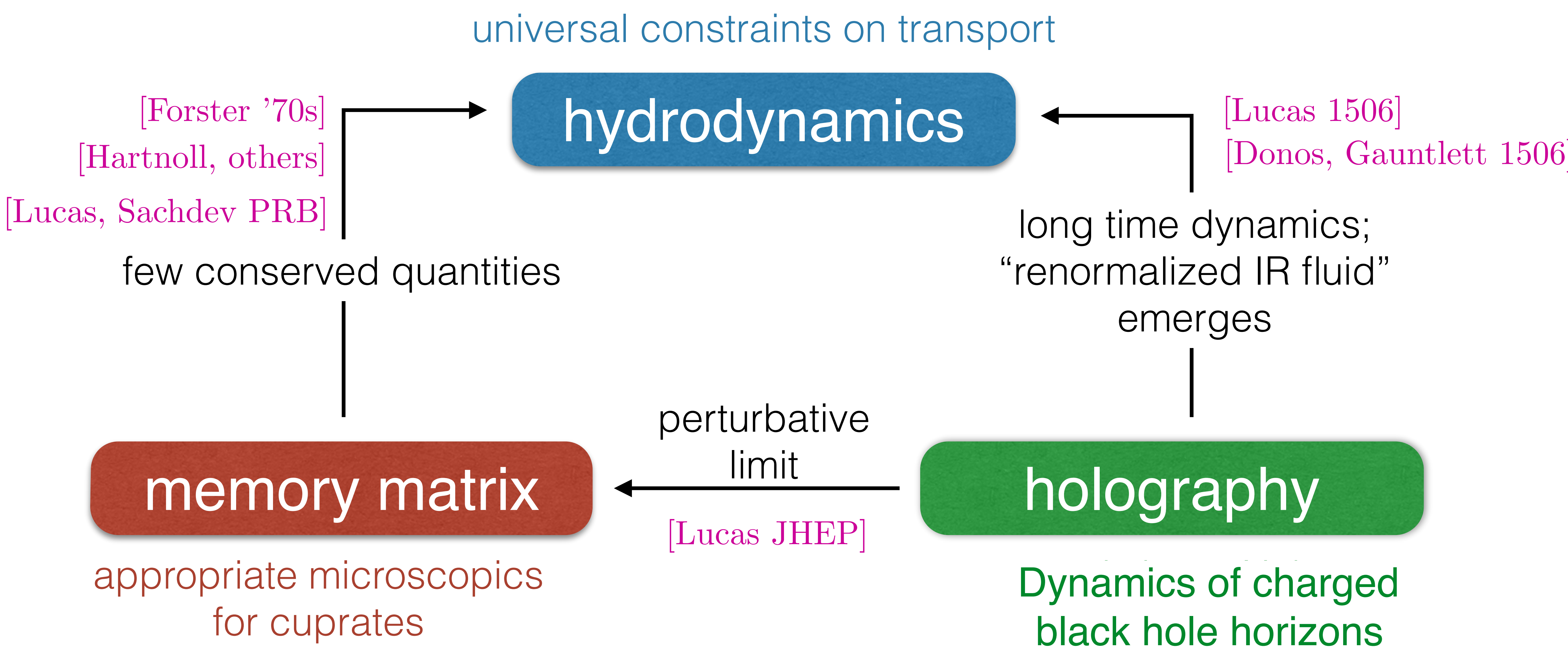



figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

# Prediction for transport in the graphene strange metal

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa/(T\sigma)$, where $\kappa$ is the thermal conductivity, and $\sigma$ is the conductivity, is given by $L = \pi^2 k_B^2/(3e^2)$.

# Prediction for transport in the graphene strange metal

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa/(T\sigma)$, where $\kappa$ is the thermal conductivity, and $\sigma$ is the conductivity, is given by $L = \pi^2 k_B^2/(3e^2)$.

For a strange metal with a "relativistic" Hamiltonian, hydrodynamic, holographic, and memory function methods yield

$$\sigma = \sigma_Q \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H}\sigma_Q}\right) \quad , \quad \kappa = \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H}\sigma_Q}\right)^{-1}$$

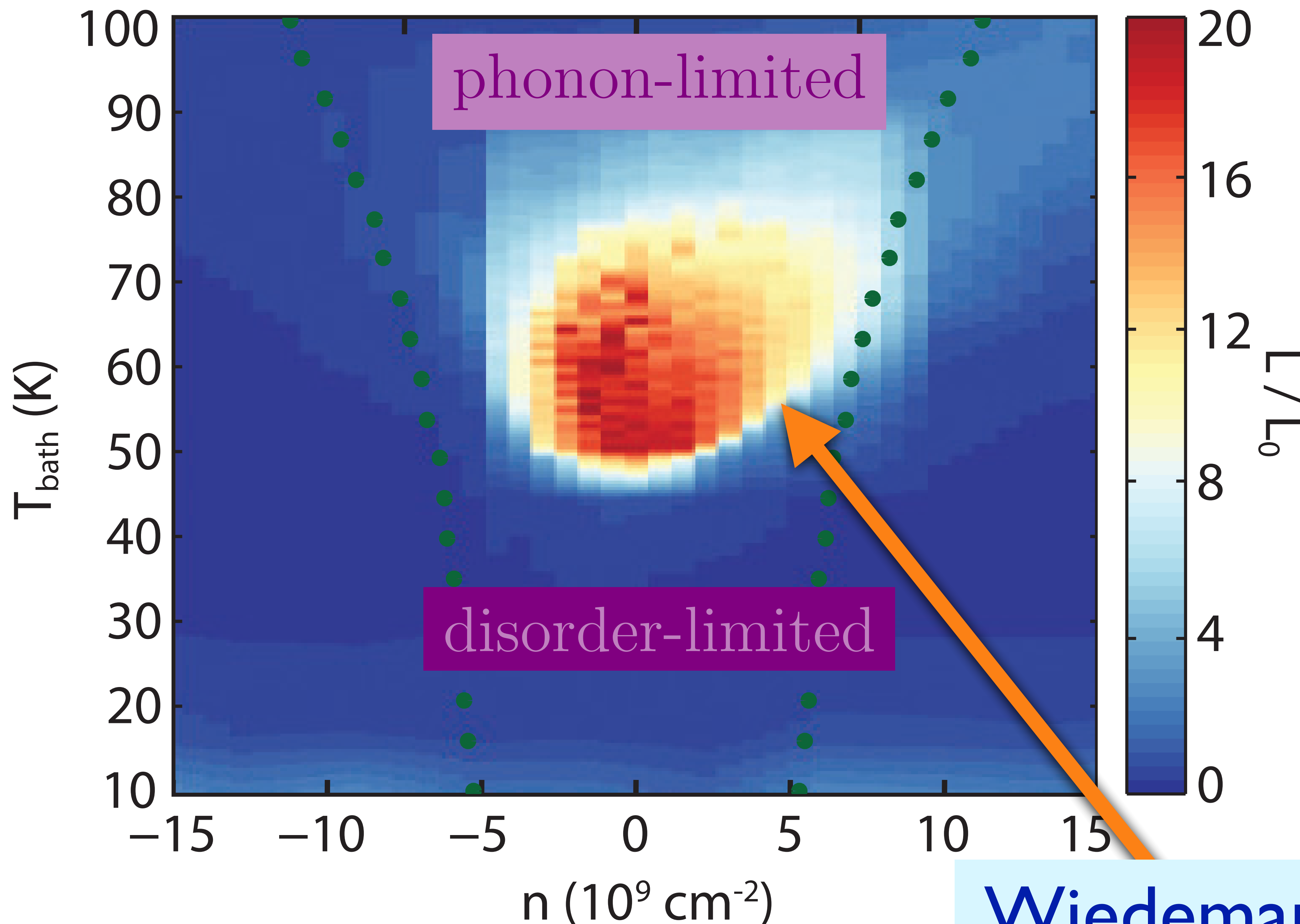
$$L = \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H}\sigma_Q}\right)^{-2},$$

where $\mathcal{H}$ is the enthalpy density, $\tau_{\rm imp}$ is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, "quantum critical" conductivity. Note that the limits $\mathcal{Q} \to 0$ and $\tau_{\rm imp} \to \infty$ do not commute.

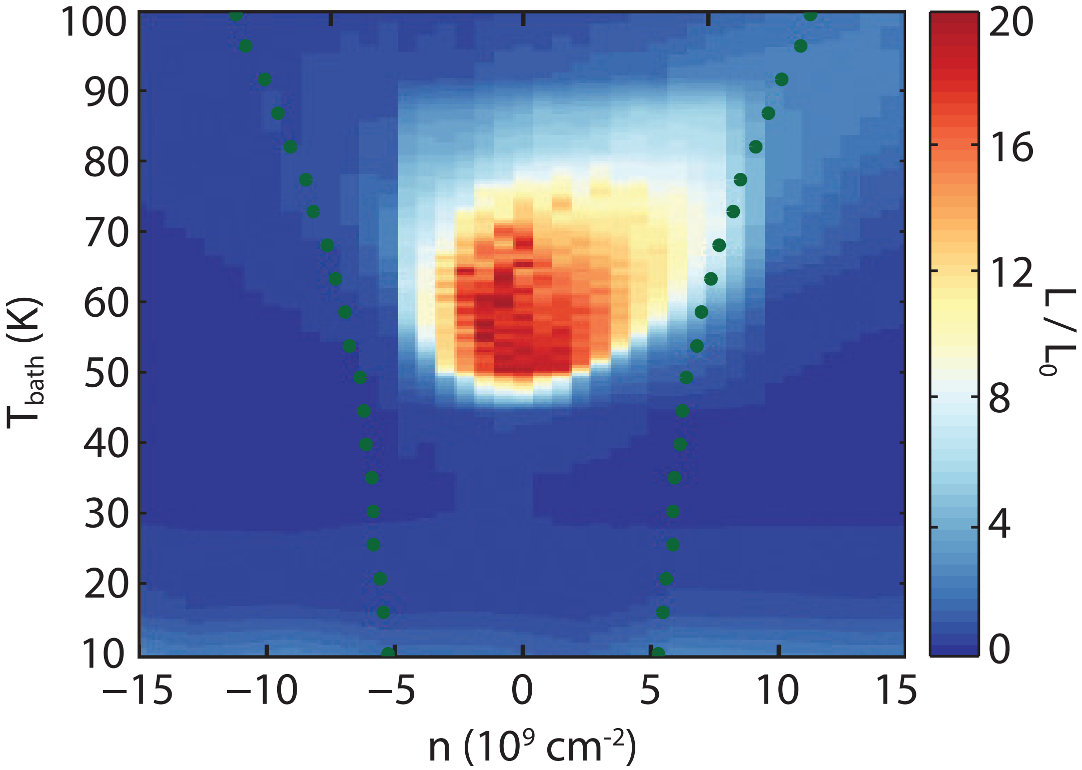
S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

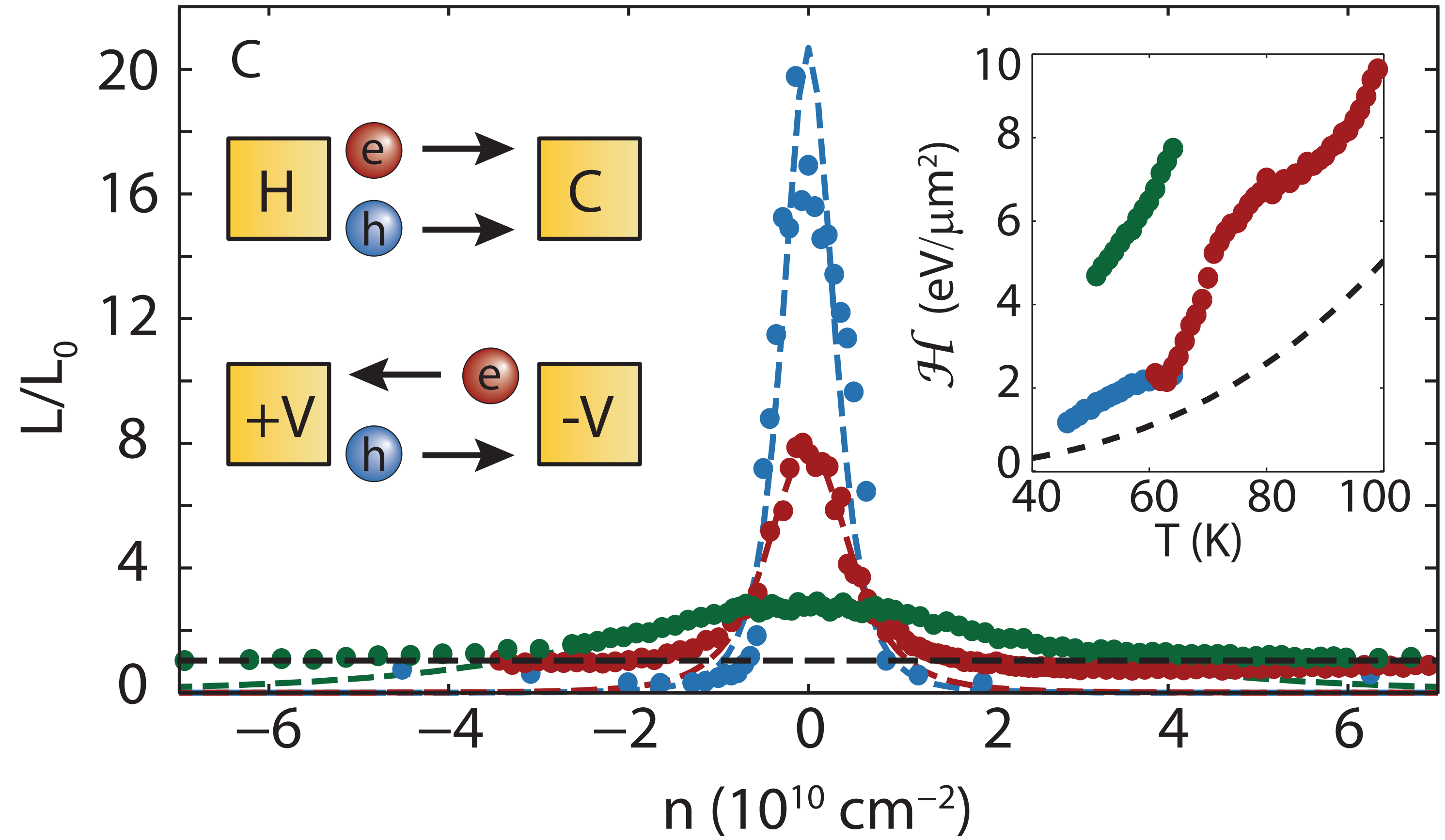
J. Crossno *et al.* arXiv:1509.04713; Science, to appear

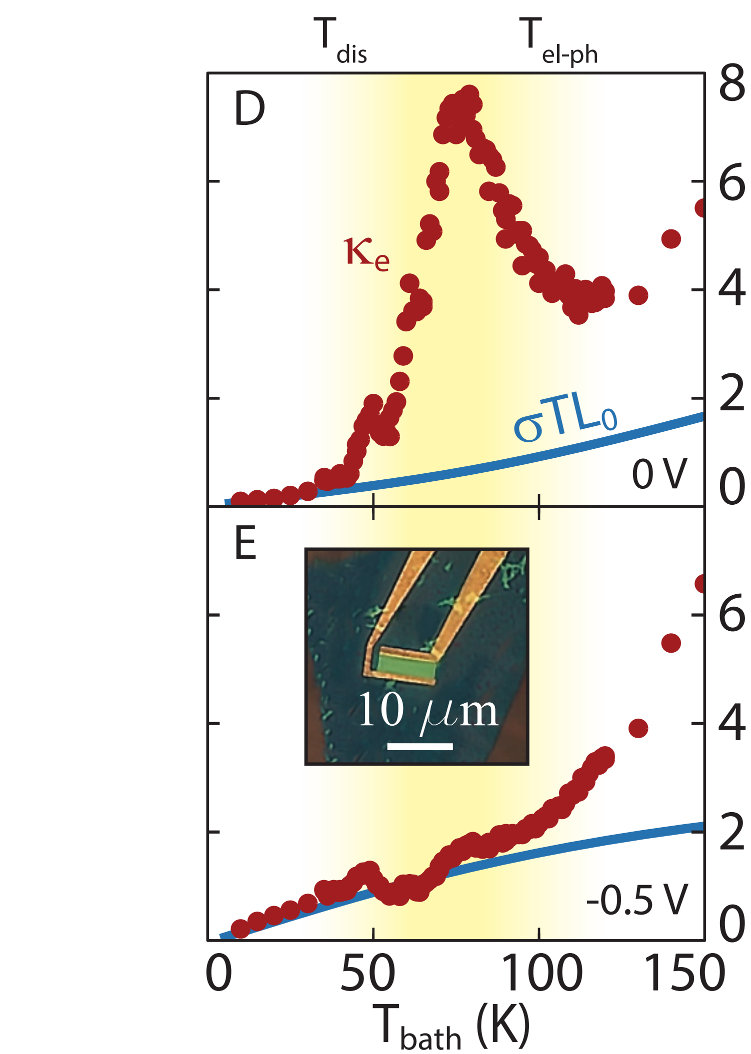
# Strange metal in graphene




Wiedemann-Franz violated !



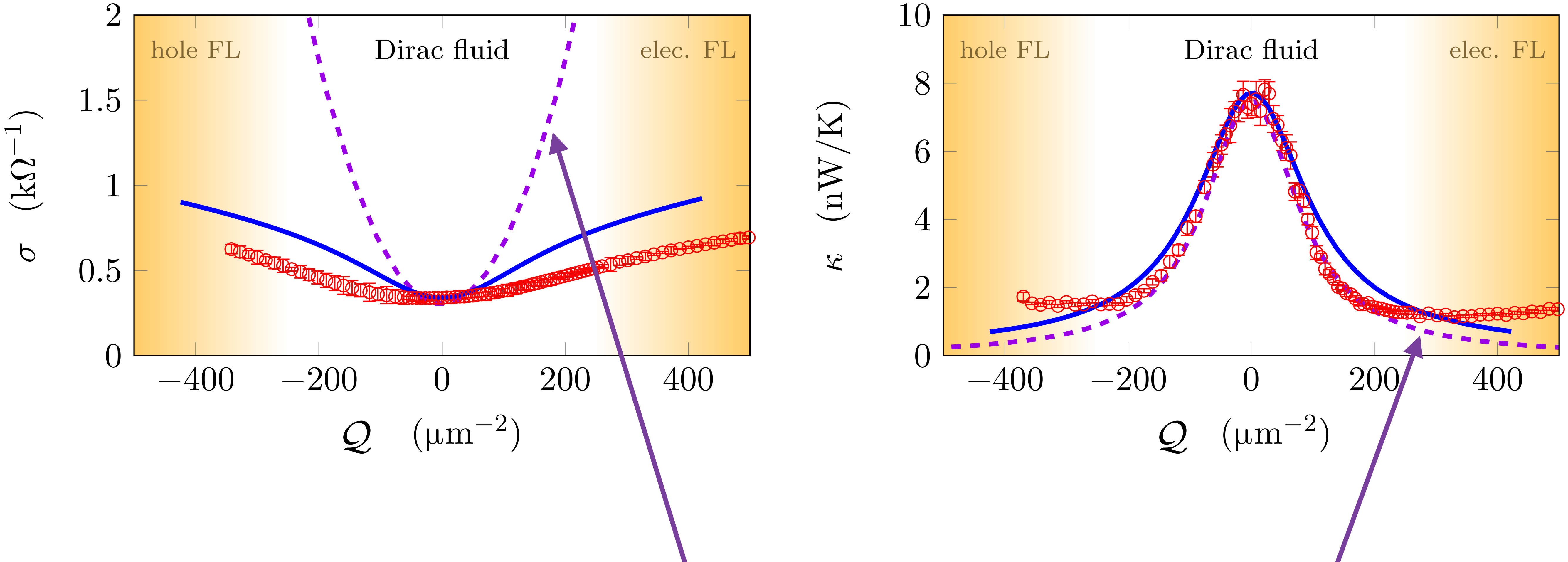




Lorentz ratio $L = \kappa/(T\sigma)$

$$= \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \frac{1}{\left(1 + e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}/(\mathcal{H}\sigma_Q)\right)^2}$$

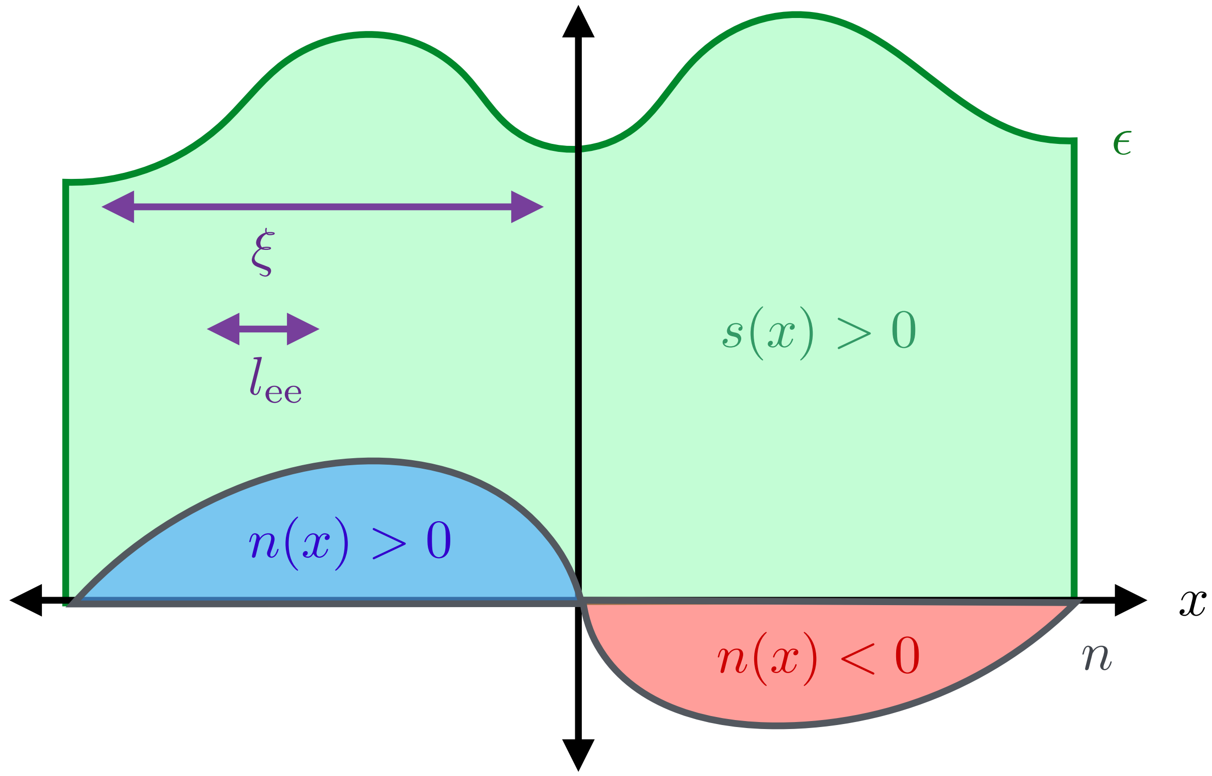
J. Crossno *et al.* arXiv:1509.04713; Science, to appear




Comparison to theory with a single momentum relaxation time $\tau_{\text{imp}}$.
Best fit of density dependence to thermal conductivity does not capture the density dependence of electrical conductivity

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738

# Non-perturbative treatment of disorder

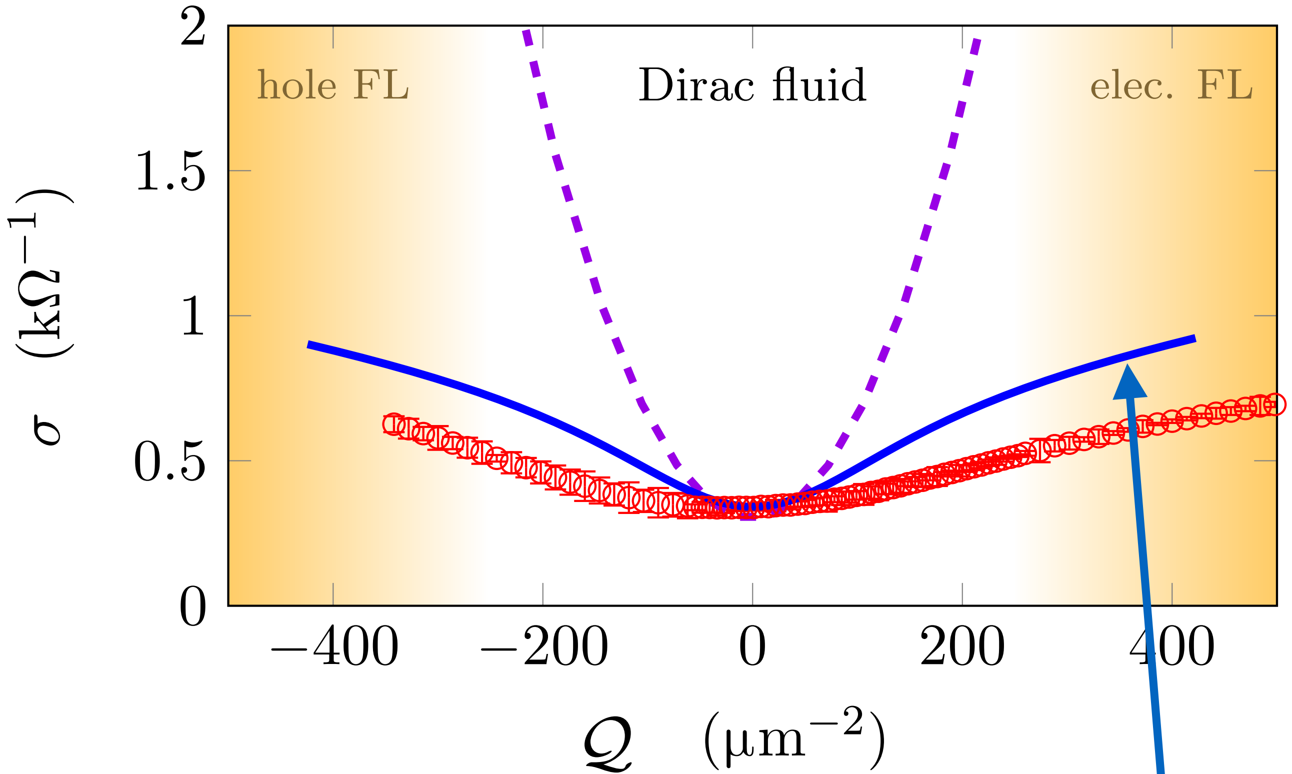


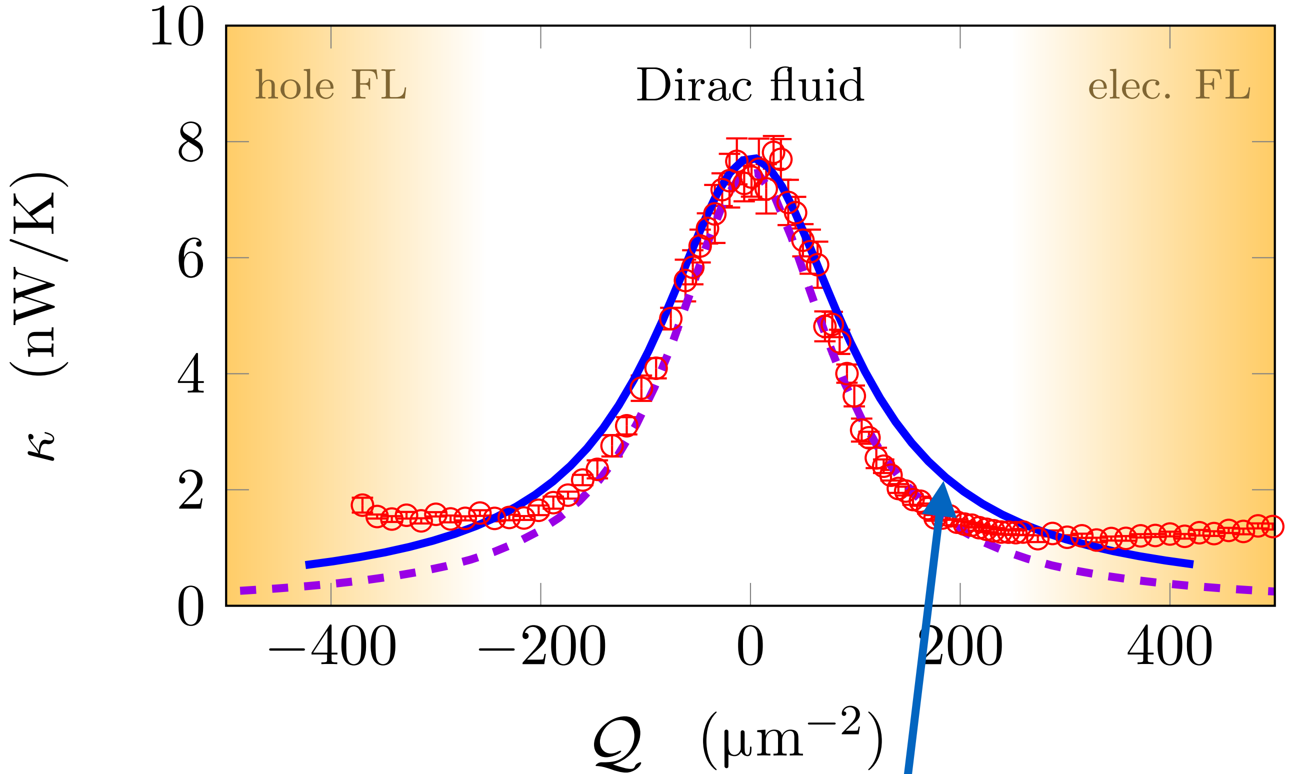

Note
$n \equiv \mathcal{Q}$

**Figure 3:** A cartoon of a nearly quantum critical fluid where our hydrodynamic description of transport is sensible. The local chemical potential $\mu(\mathbf{x})$ always obeys $|\mu| \ll k_{\mathrm{B}}T$, and so the entropy density $s/k_{\mathrm{B}}$ is much larger than the charge density $|n|$; both electrons and holes are everywhere excited, and the energy density $\epsilon$ does not fluctuate as much relative to the mean. Near charge neutrality the local charge density flips sign repeatedly. The correlation length of disorder $\xi$ is much larger than $l_{\mathrm{ee}}$, the electron-electron interaction length.

Numerically solve the hydrodynamic equations in the presence of a $x$-dependent chemical potential. The thermoelectric transport properties will then depend upon the value of the shear viscosity, $\eta$.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738




Solution of the hydrodynamic equations in the presence of a space-dependent chemical potential.
Best fit of density dependence to thermal conductivity now gives a better fit to the density dependence of the electrical conductivity (for $\eta/s \approx 10$). The $T$ dependencies of other parameters also agree well with expectation.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738

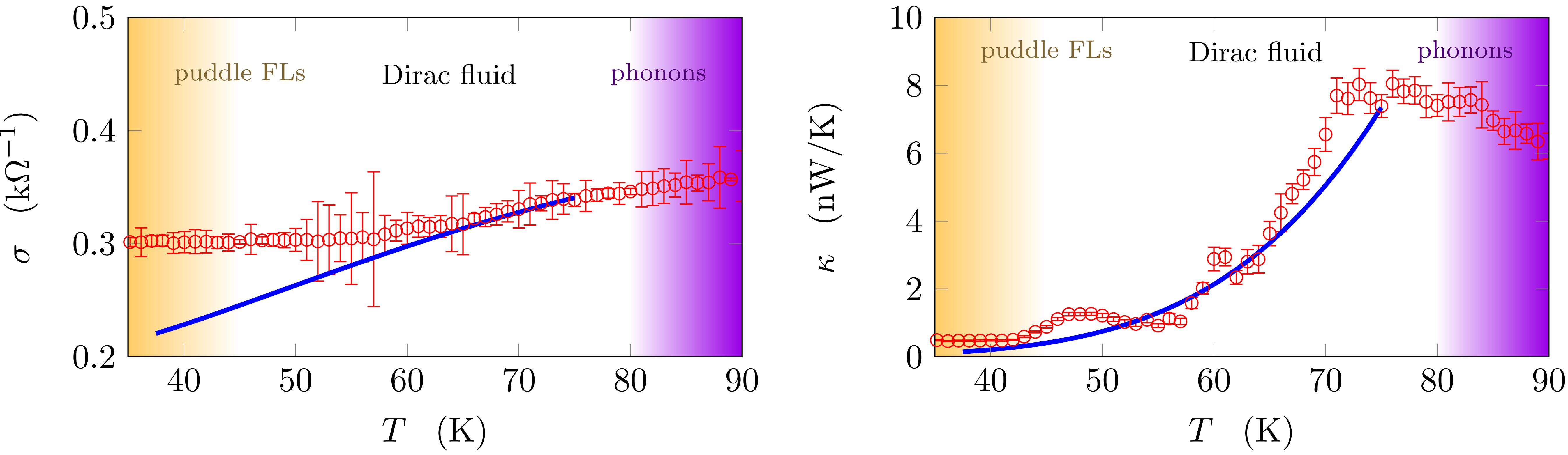



**Figure 2:** A comparison of our hydrodynamic theory of transport with the experimental results of [33] in clean samples of graphene at the charge neutrality point ($n = 0$). We use no new fit parameters compared to Figure 1. The yellow shaded region denotes where Fermi liquid behavior is observed; the purple shaded region denotes the likely onset of electron-phonon coupling.

Solution of the hydrodynamic equations in the presence of a space-dependent chemical potential.
Best fit of density dependence to thermal conductivity now gives a better fit to the density dependence of the electrical conductivity (for $\eta/s \approx 10$). The $T$ dependencies of other parameters also agree well with expectation.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738

# Quantum matter without quasiparticles

- No quasiparticle excitations
- Shortest possible "collision time", or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$
- Continuously variable density, $\mathcal{Q}$
(conformal field theories are usually at fixed density, $\mathcal{Q} = 0$)
- Theory built from hydrodynamics/holography
/memory-functions/strong-coupled-field-theory
- Exciting experimental realization in graphene.

1. Strange metal in graphene

*Experiments vs. theory combining hydrodynamic/holographic/Boltzmann/memory-function methods*

2. Strange metal in correlated electron superconductors

*Onset of spin density wave order in metals*

3. The pseudogap metal of the hole-doped cuprate superconductors

*Evidence for a metal with emergent gauge fields*

1. Strange metal in graphene

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3. The pseudogap metal of the hole-doped cuprate superconductors

*Evidence for a metal with emergent gauge fields*

Erez Berg

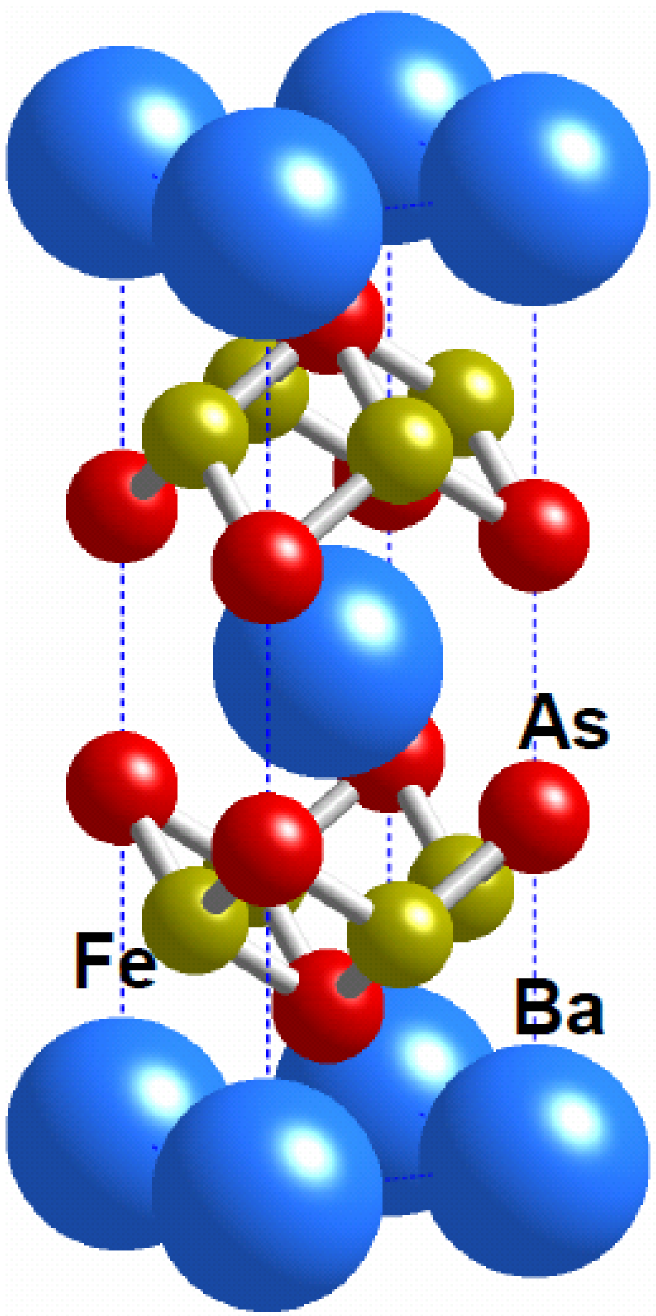
Max Metlitski

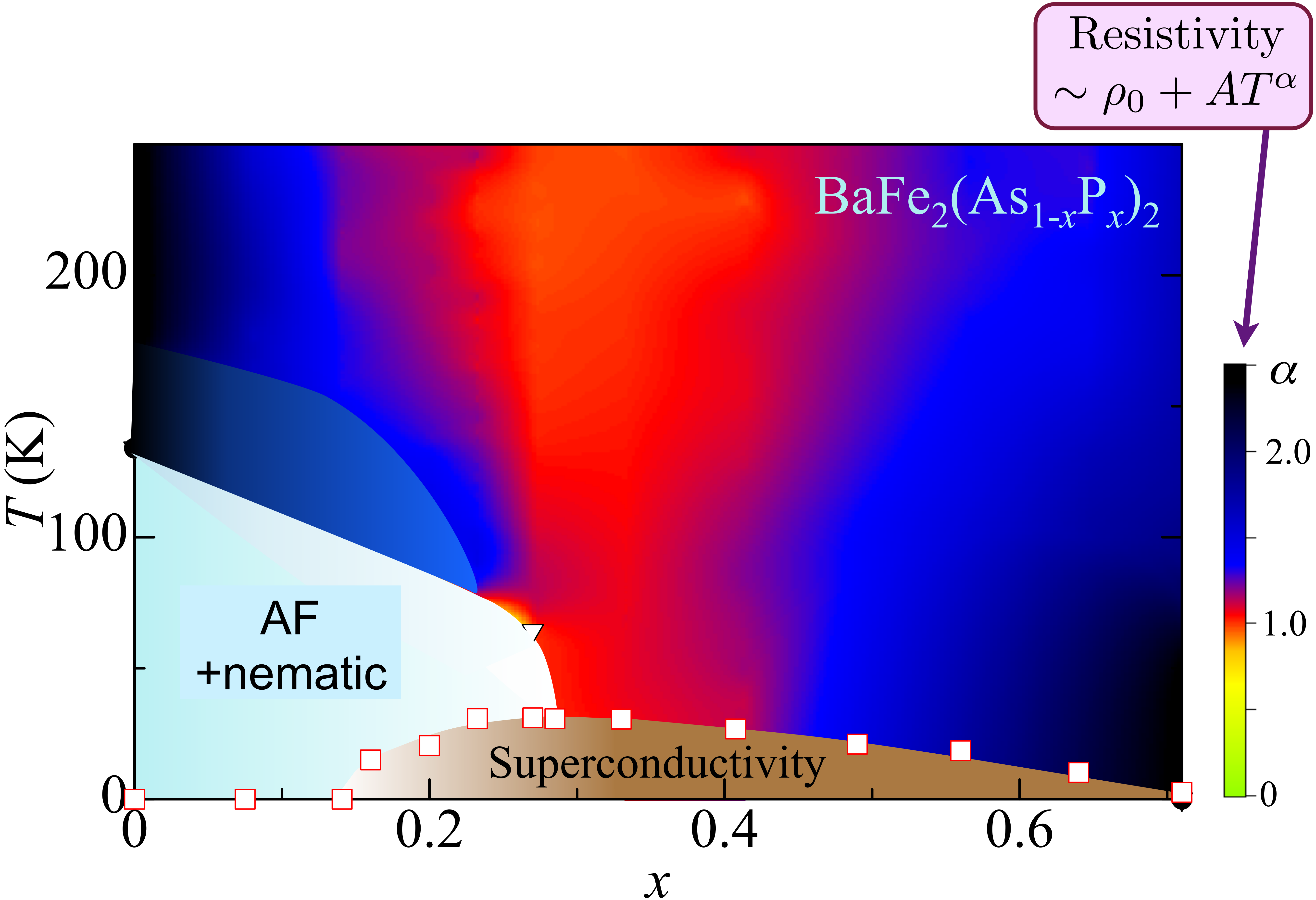
Aavishkar Patel

Philipp Strack

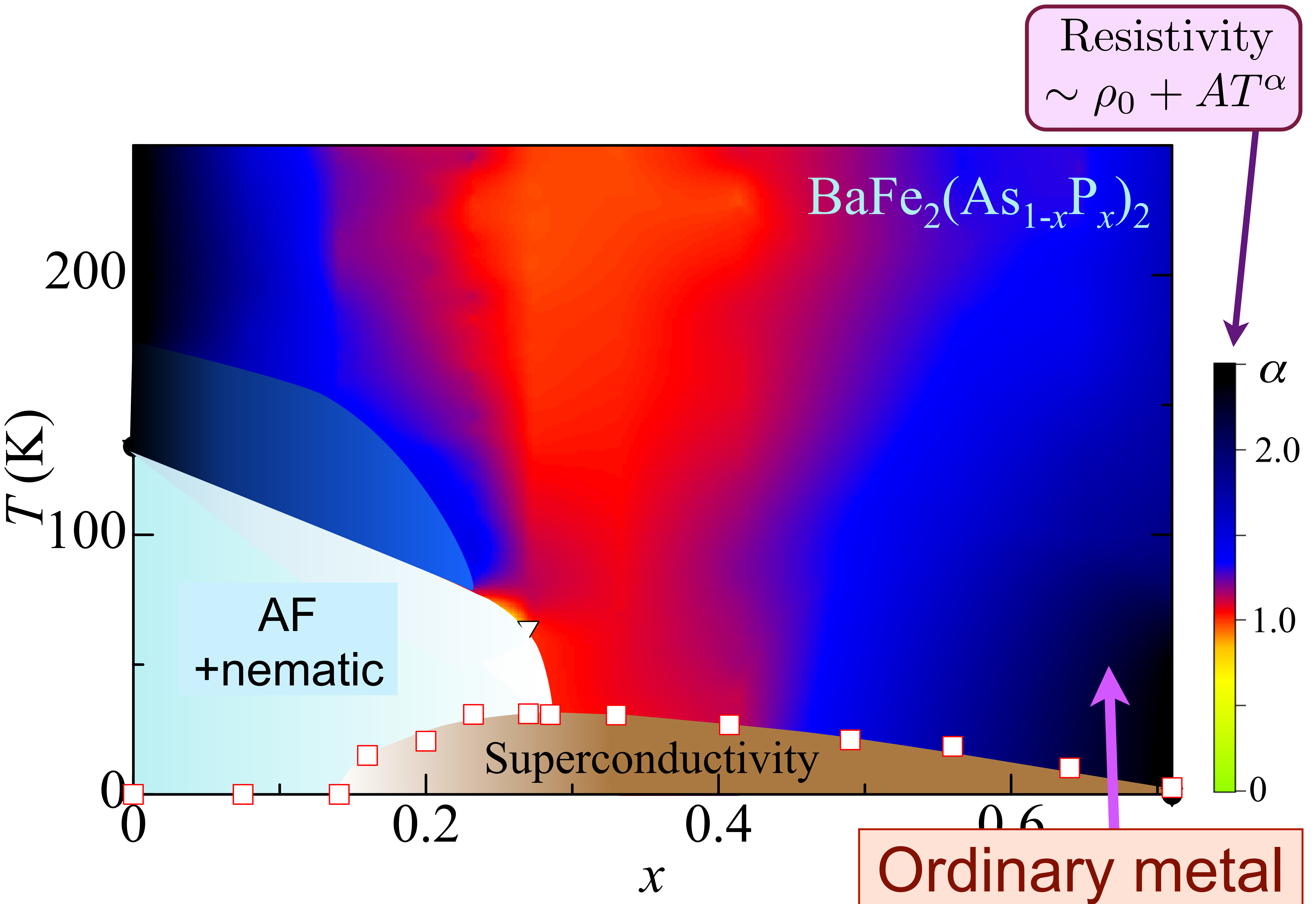
# Iron pnictides:

## a new class of high temperature superconductors

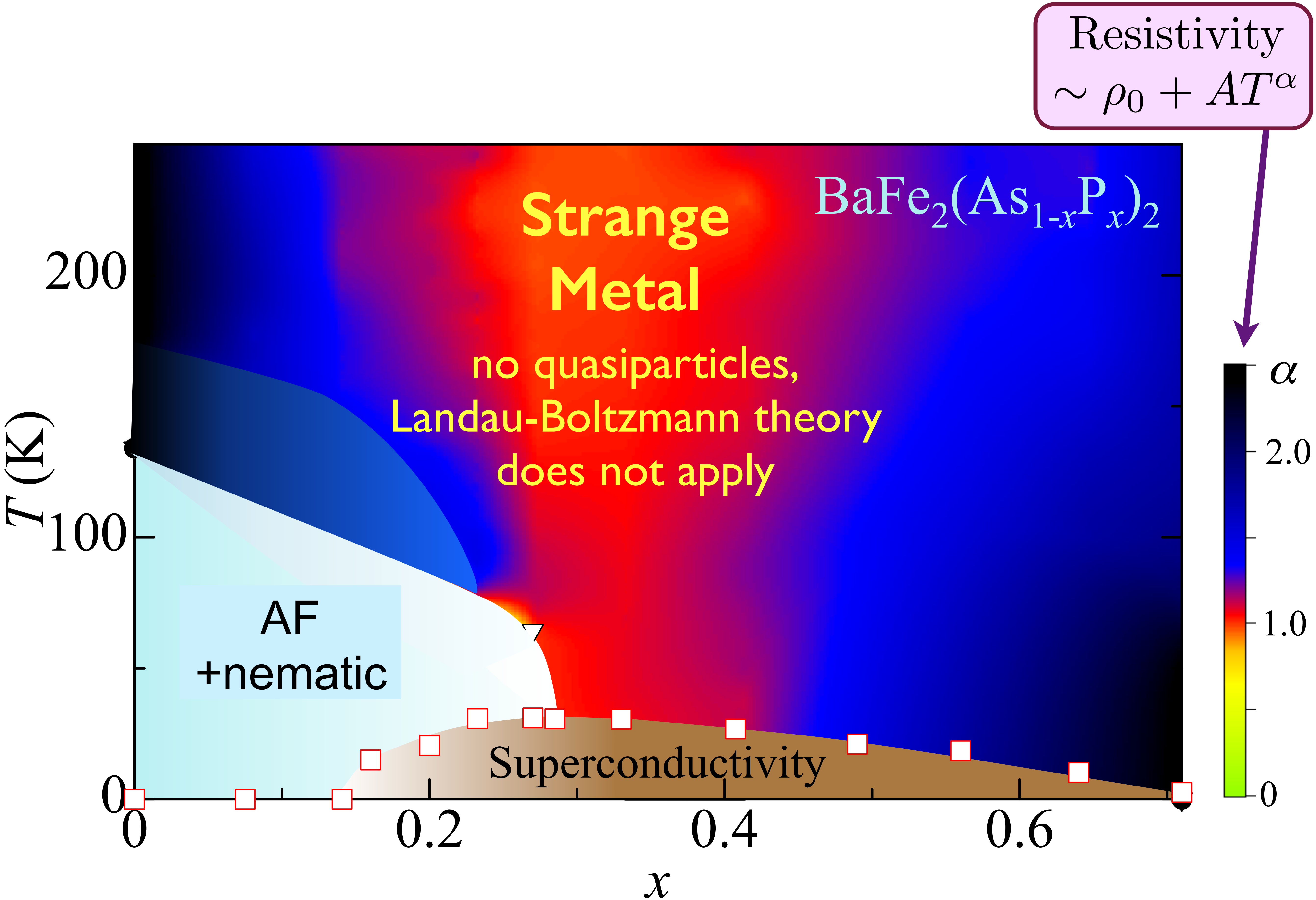





S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)




S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. O
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Mat
*Physical Review B* **81**, 184519 (2010)

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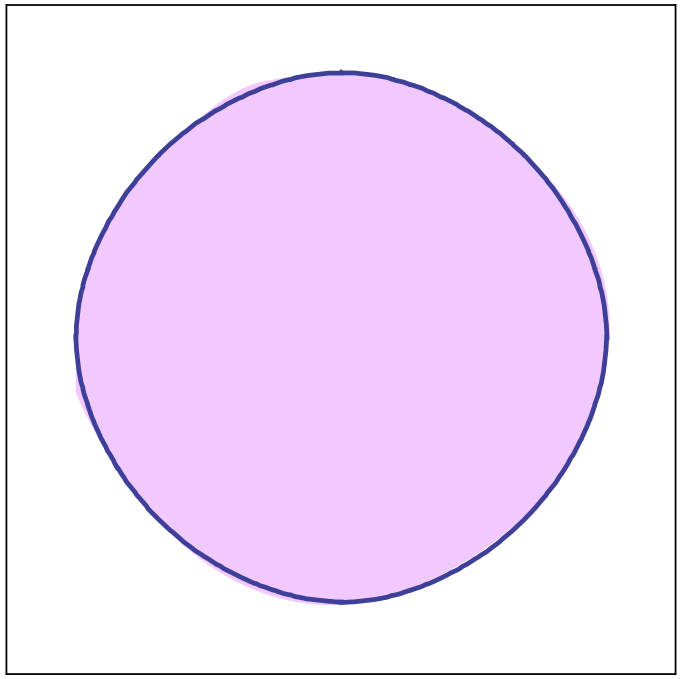
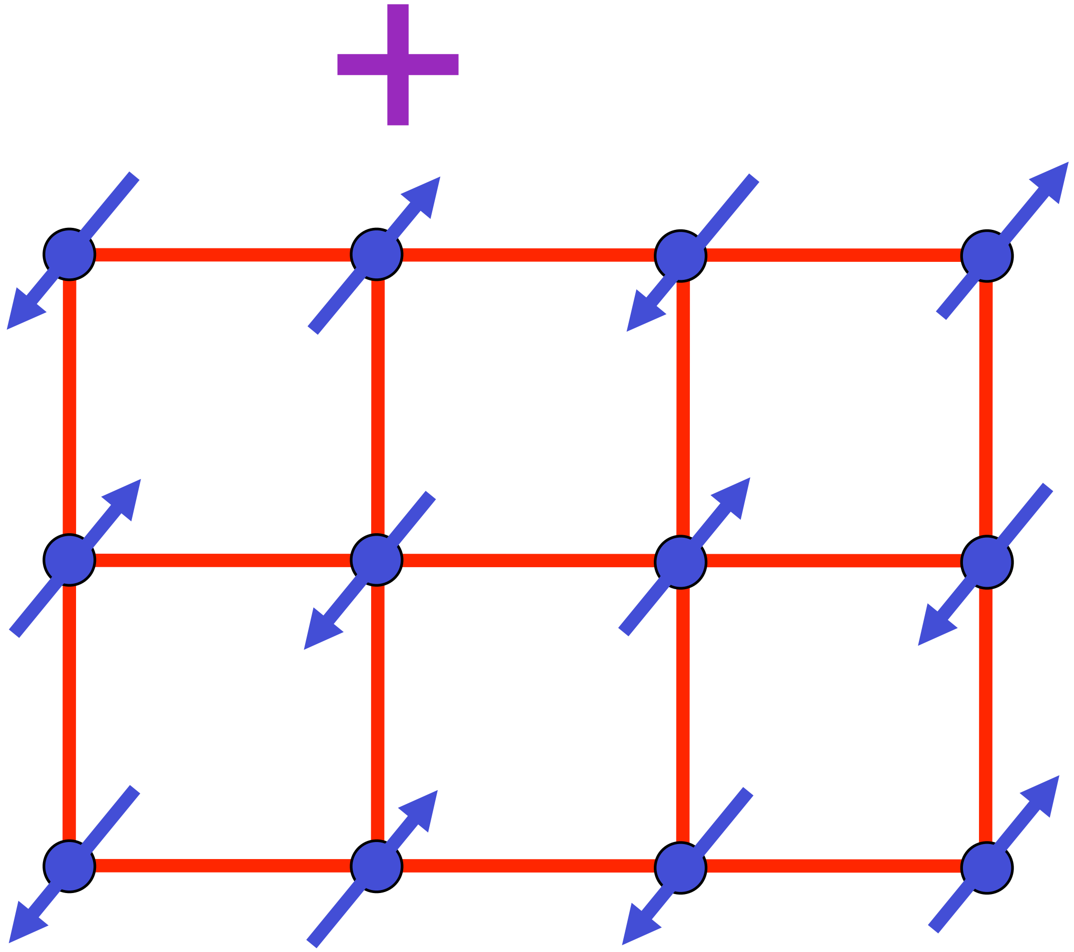
# Fermi surface+antiferromagnetism

Metal with "large" Fermi surface





The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

# The Hubbard Model

## Decouple *U* term by a Hubbard-Stratanovich transformation

$$\mathcal{S} = \int d^2r d\tau \left[\mathcal{L}_c + \mathcal{L}_\varphi + \mathcal{L}_{c\varphi}\right]$$

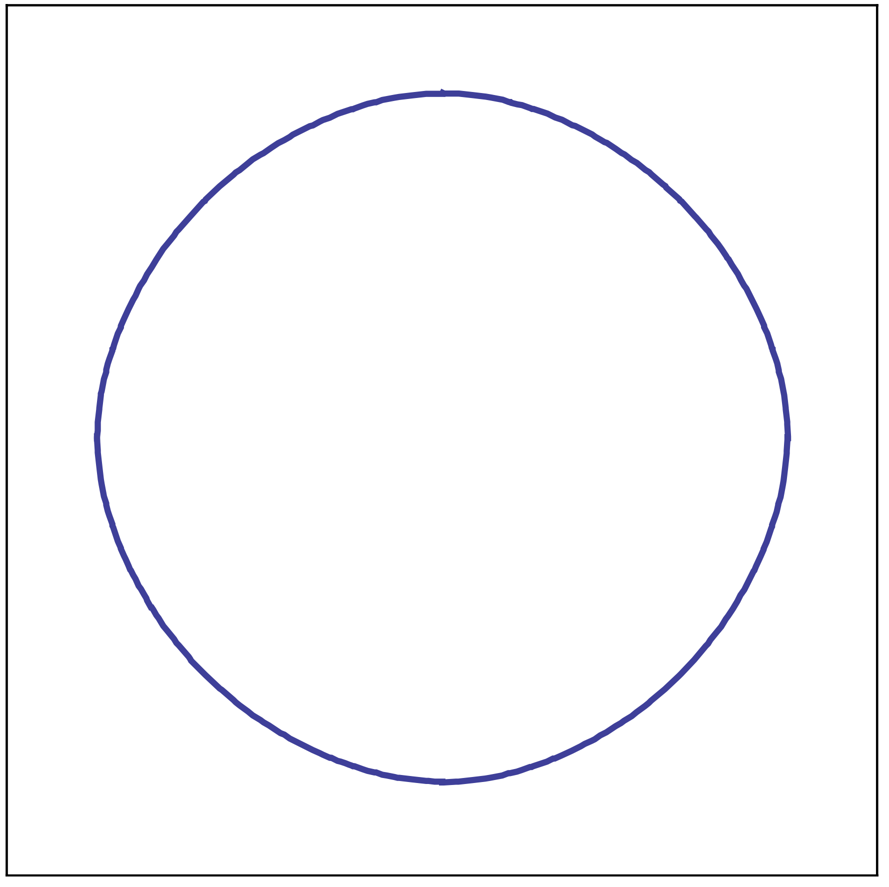
$$\mathcal{L}_c = c_a^\dagger \varepsilon(-i\boldsymbol{\nabla}) c_a$$

$$\mathcal{L}_\varphi = \frac{1}{2}(\boldsymbol{\nabla}\varphi_\alpha)^2 + \frac{r}{2}\varphi_\alpha^2 + \frac{u}{4}\left(\varphi_\alpha^2\right)^2$$

$$\mathcal{L}_{c\varphi} = \lambda\,\varphi_\alpha\, e^{i\mathbf{K}\cdot\mathbf{r}}\, c_a^\dagger\, \sigma_{ab}^\alpha\, c_b.$$

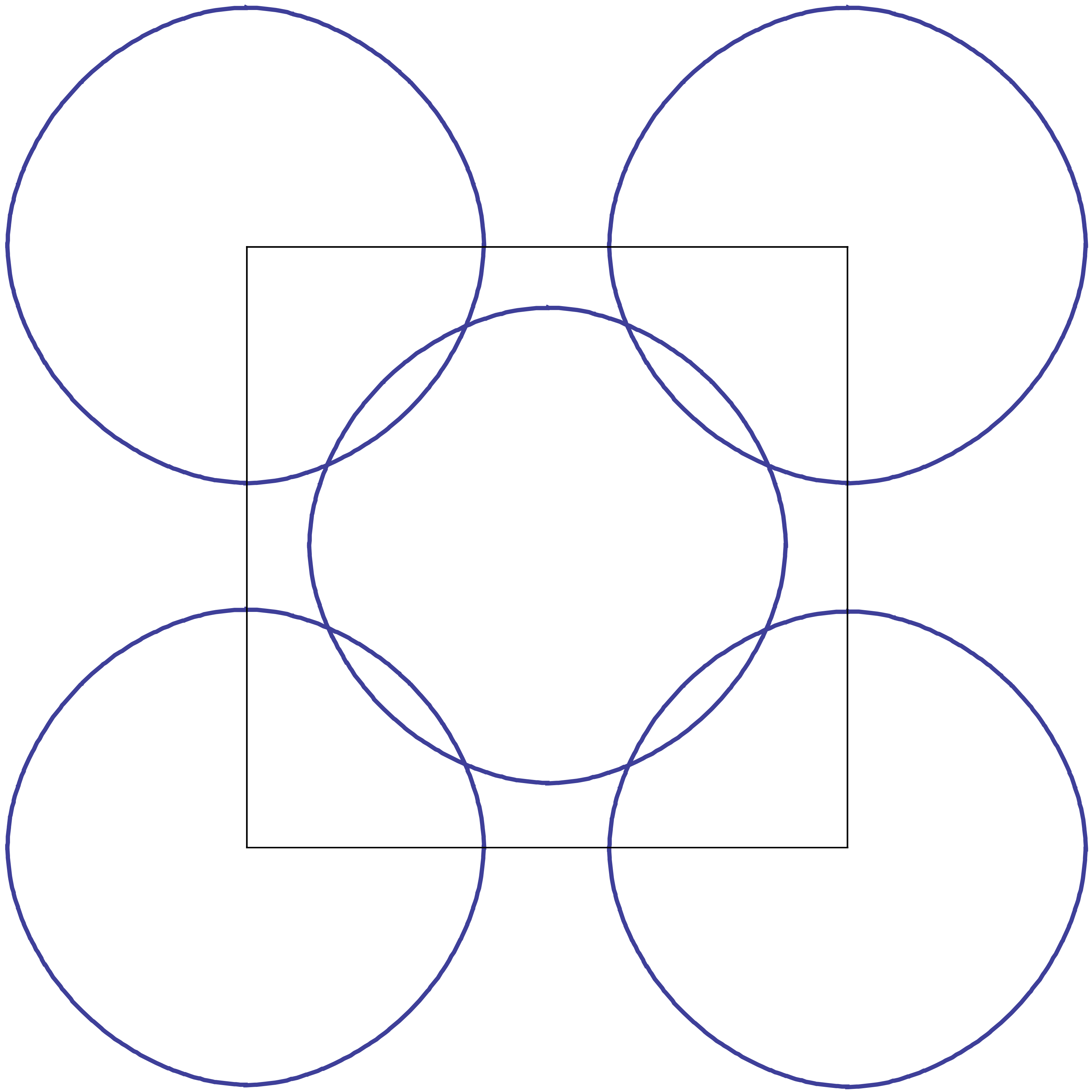
"Yukawa" coupling between fermions and antiferromagnetic order:
$\lambda^2 \sim U$, the Hubbard repulsion

# Fermi surface+antiferromagnetism



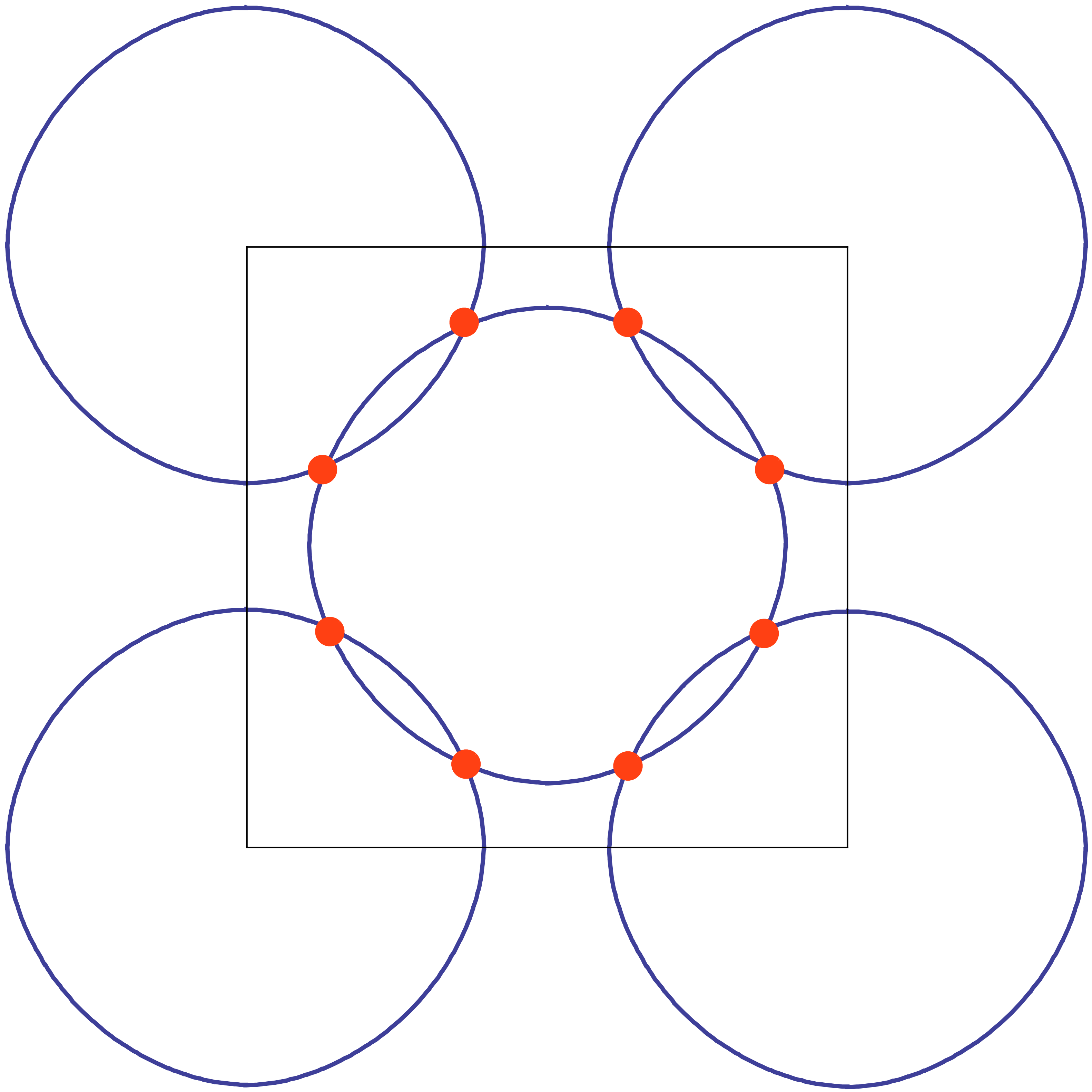
Metal with “large” Fermi surface

# Fermi surface+antiferromagnetism



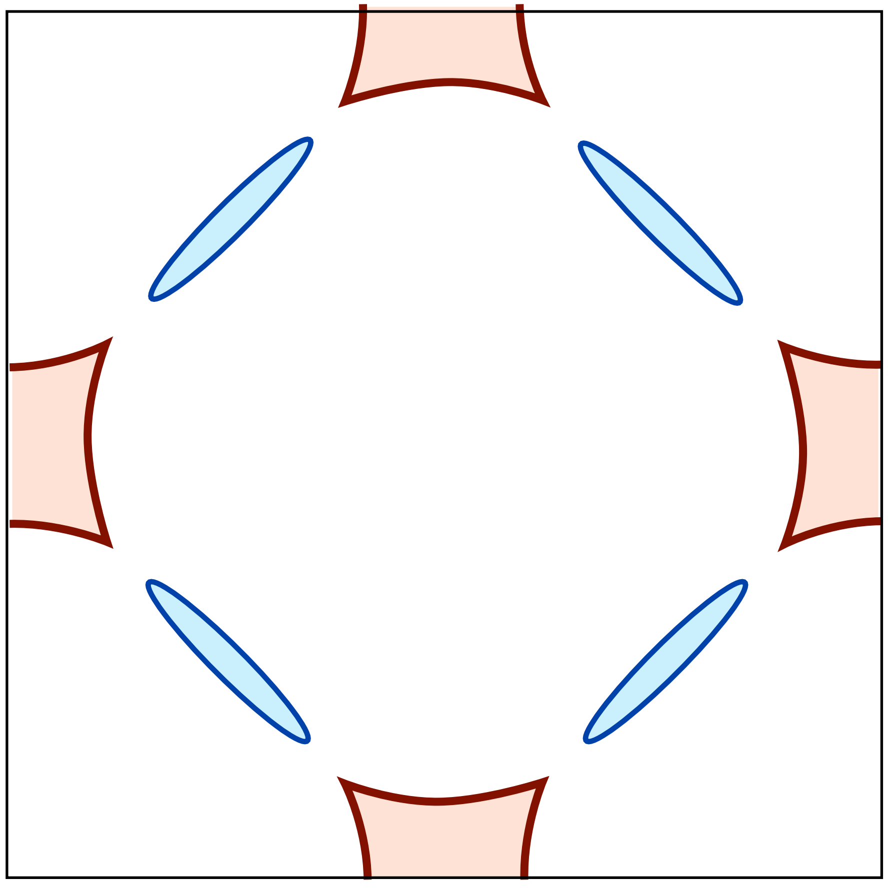
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

# Fermi surface+antiferromagnetism



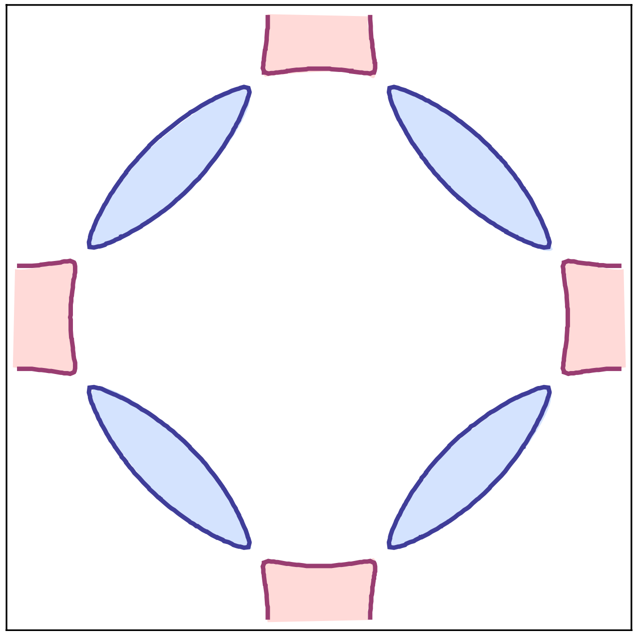
“Hot” spots

# Fermi surface+antiferromagnetism



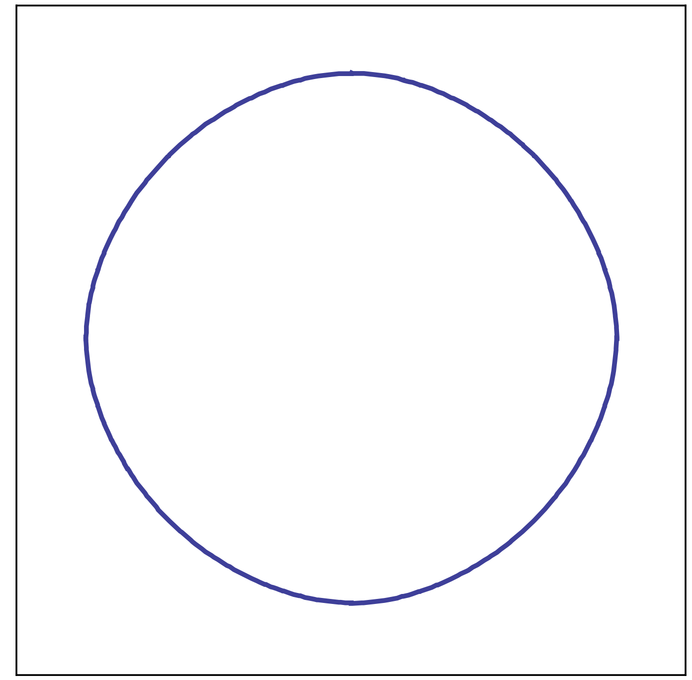
Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$

# Fermi surface+antiferromagnetism



$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets



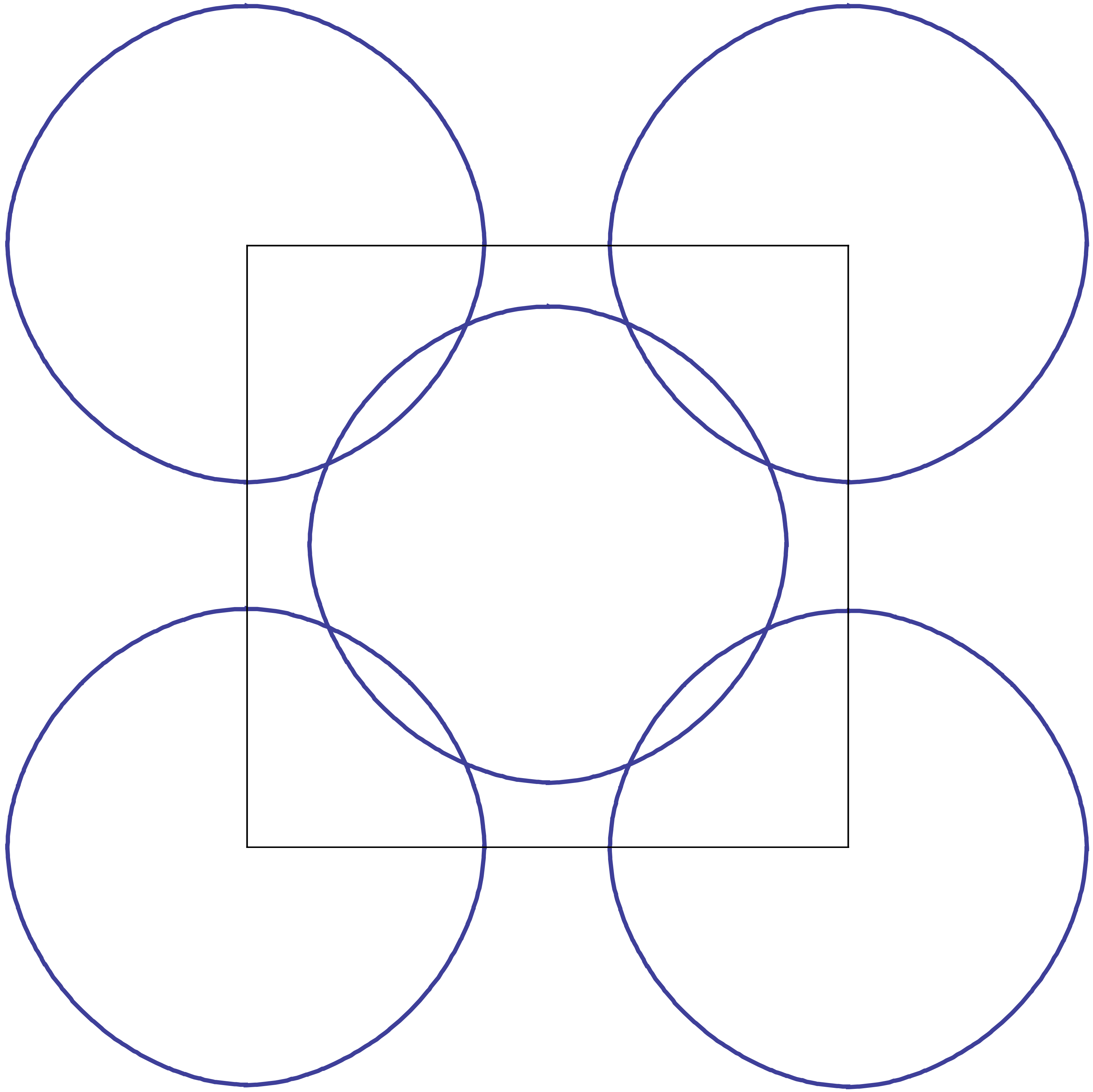
$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

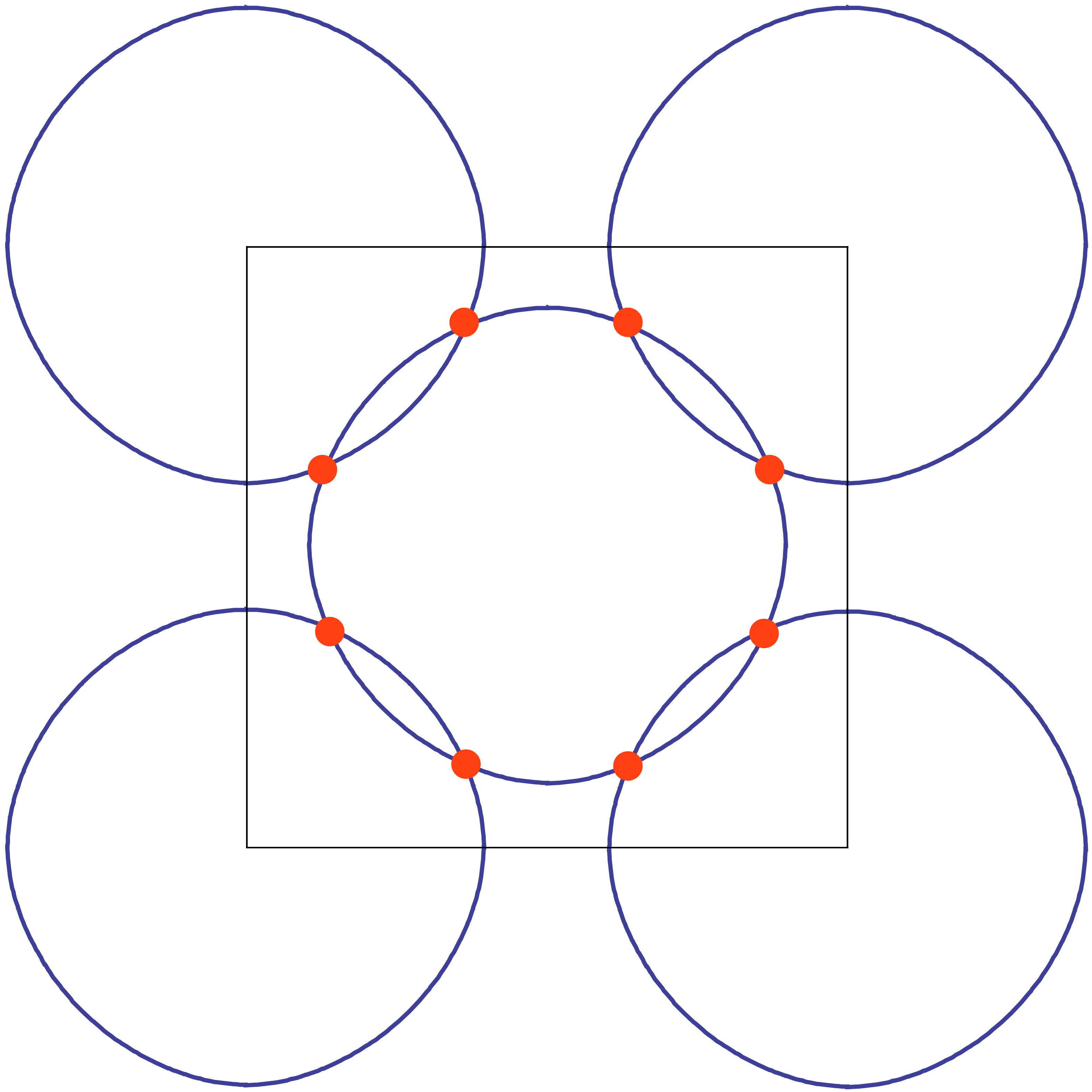
← Increasing interaction

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

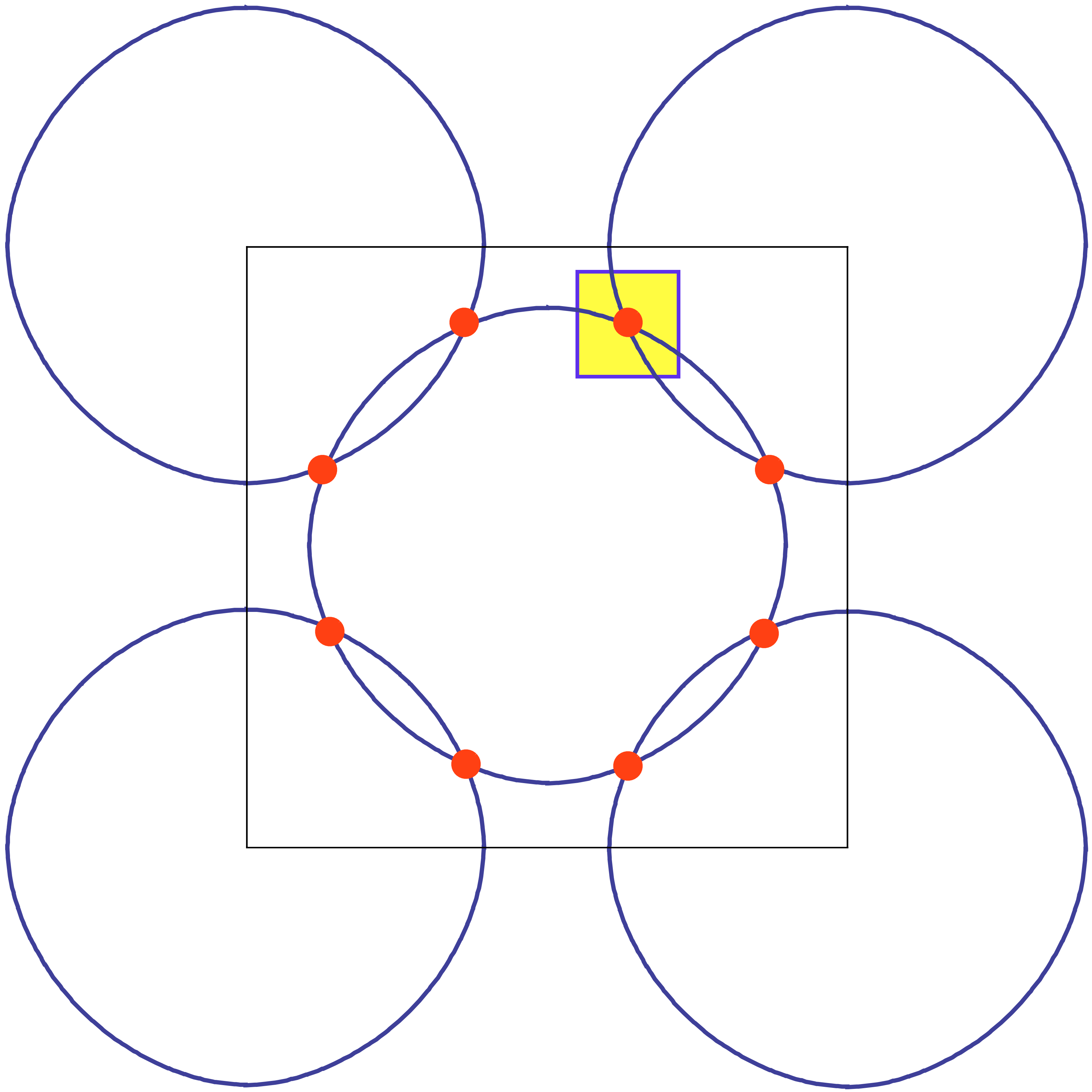
# Fermi surface+antiferromagnetism



Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

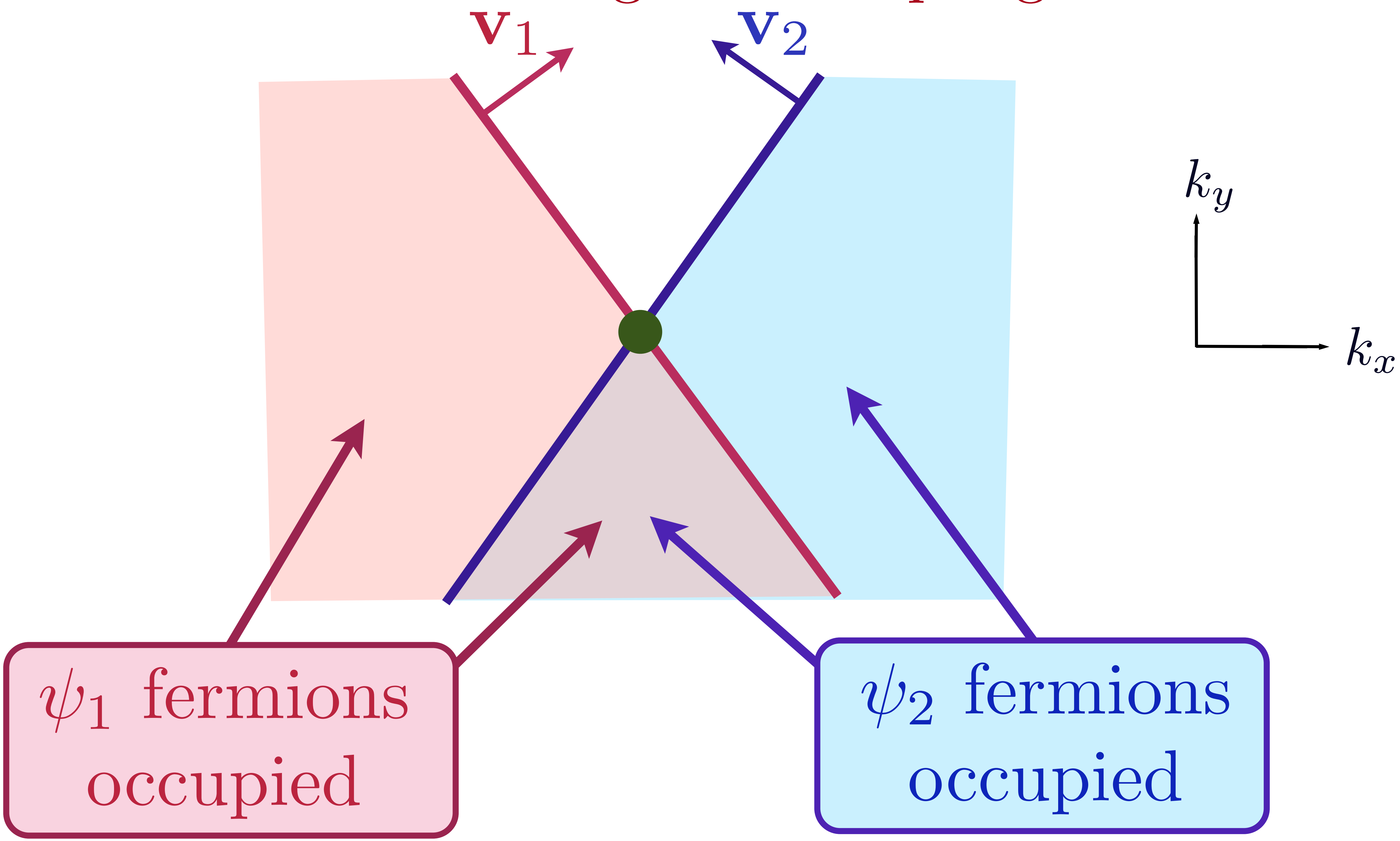


“Hot” spots



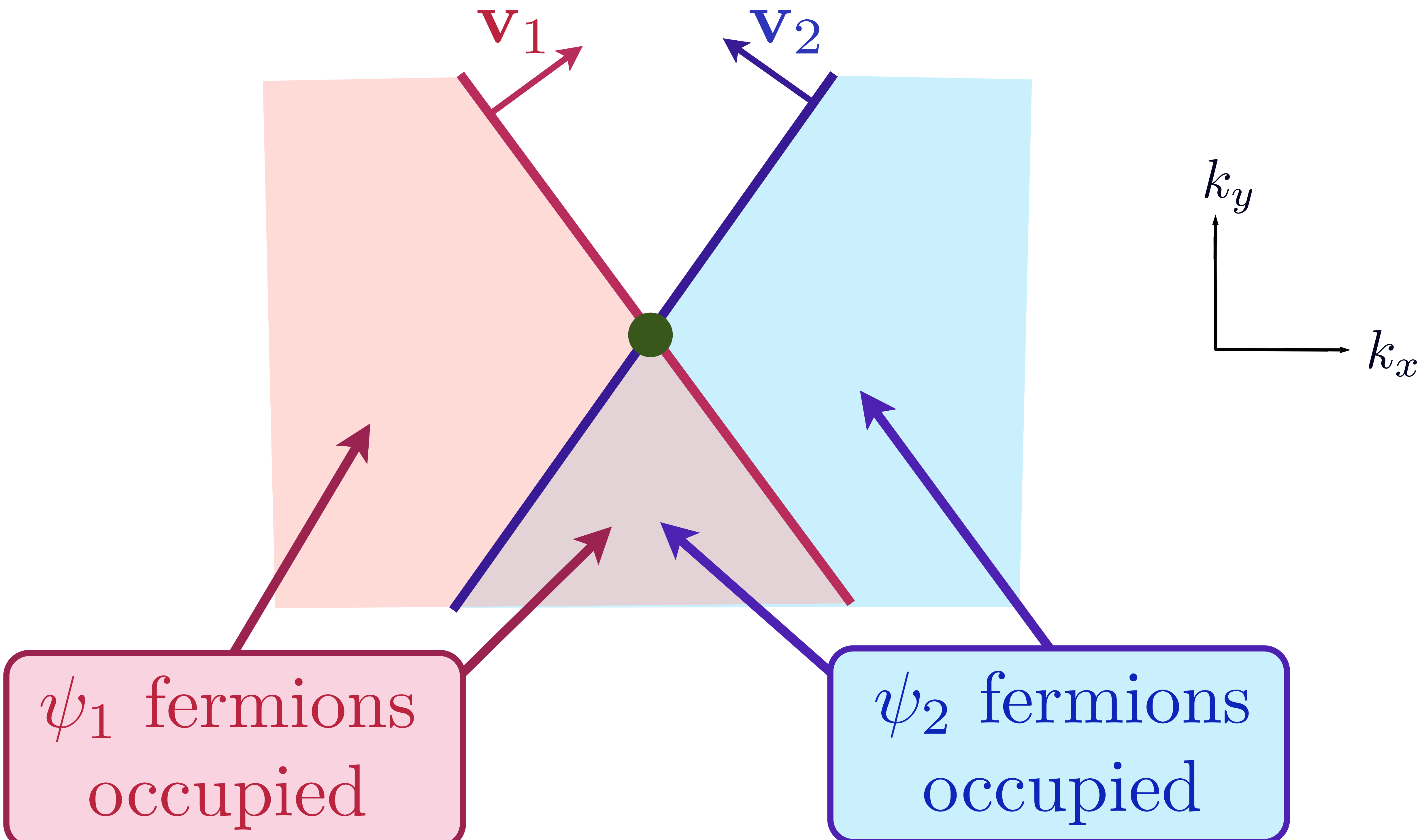
Low energy theory for critical point near hot spots

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling $\lambda$



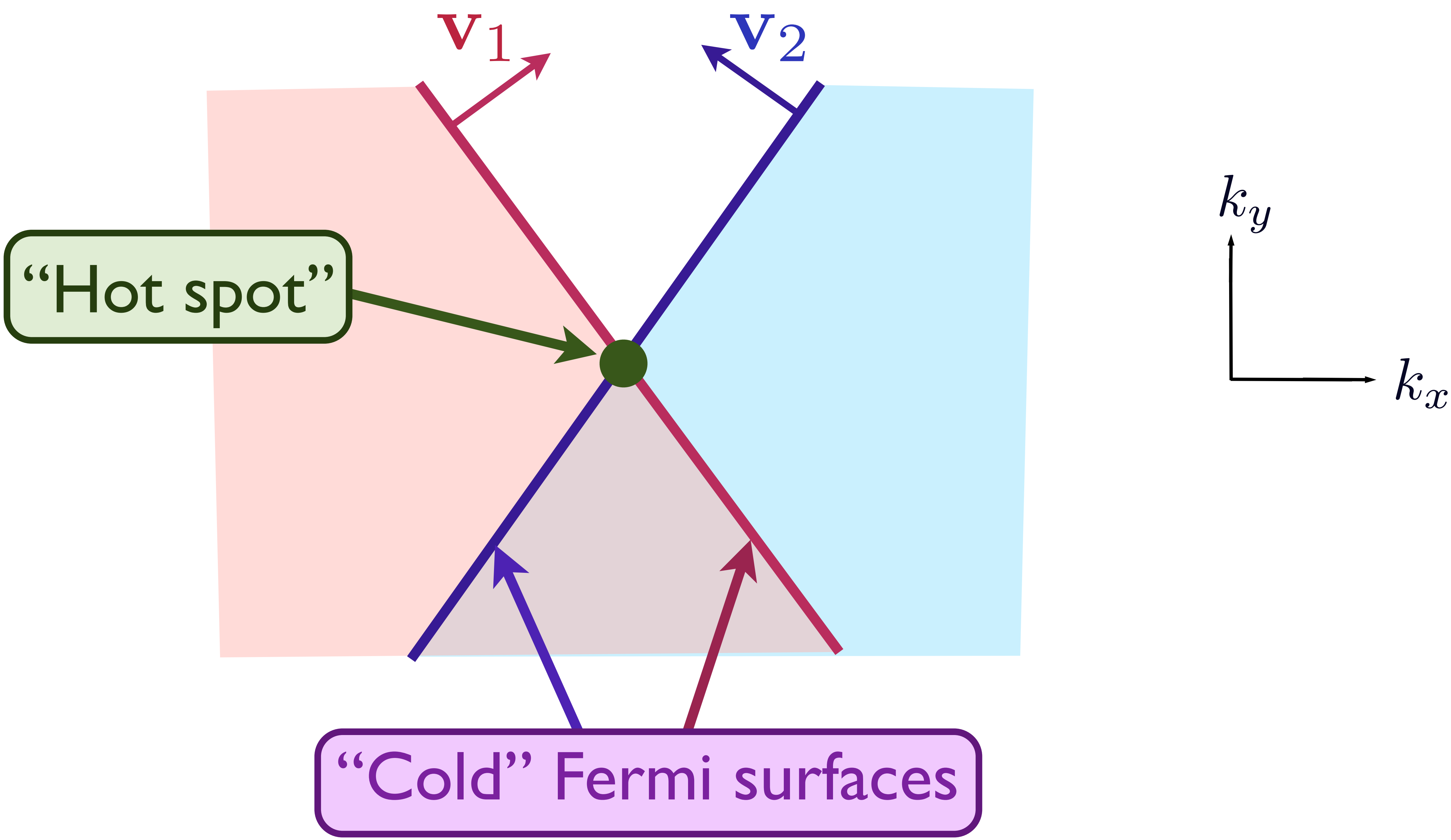

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

$$\mathcal{L}_f = \psi^\dagger_{1\alpha}\left(\partial_\tau - i\mathbf{v}_1\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha} + \psi^\dagger_{2\alpha}\left(\partial_\tau - i\mathbf{v}_2\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}$$




Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

$$\mathcal{L}_f = \psi^\dagger_{1\alpha}\left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha} + \psi^\dagger_{2\alpha}\left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}$$




Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

$$\mathcal{L}_f = \psi^\dagger_{1\alpha}\left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha} + \psi^\dagger_{2\alpha}\left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{1}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

$$\mathcal{L}_f = \psi^\dagger_{1\alpha}\left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha} + \psi^\dagger_{2\alpha}\left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}$$

Order parameter: $$\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{1}{2}\left(\partial_\tau \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$$

"Yukawa" coupling: $$\mathcal{L}_c = -\lambda\vec{\varphi} \cdot \left(\psi^\dagger_{1\alpha}\vec{\sigma}_{\alpha\beta}\psi_{2\beta} + \psi^\dagger_{2\alpha}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}\right)$$

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

$$\mathcal{L}_f = \psi_{1\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right) \psi_{1\alpha} + \psi_{2\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right) \psi_{2\alpha}$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{1}{2}\left(\partial_\tau \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

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This theory is strongly-coupled in $d = 2$.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **85**, 075127 (2010)

$$\mathcal{L}_f = \psi_{1\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right) \psi_{1\alpha} + \psi_{2\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right) \psi_{2\alpha}$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{1}{2}\left(\partial_\tau \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

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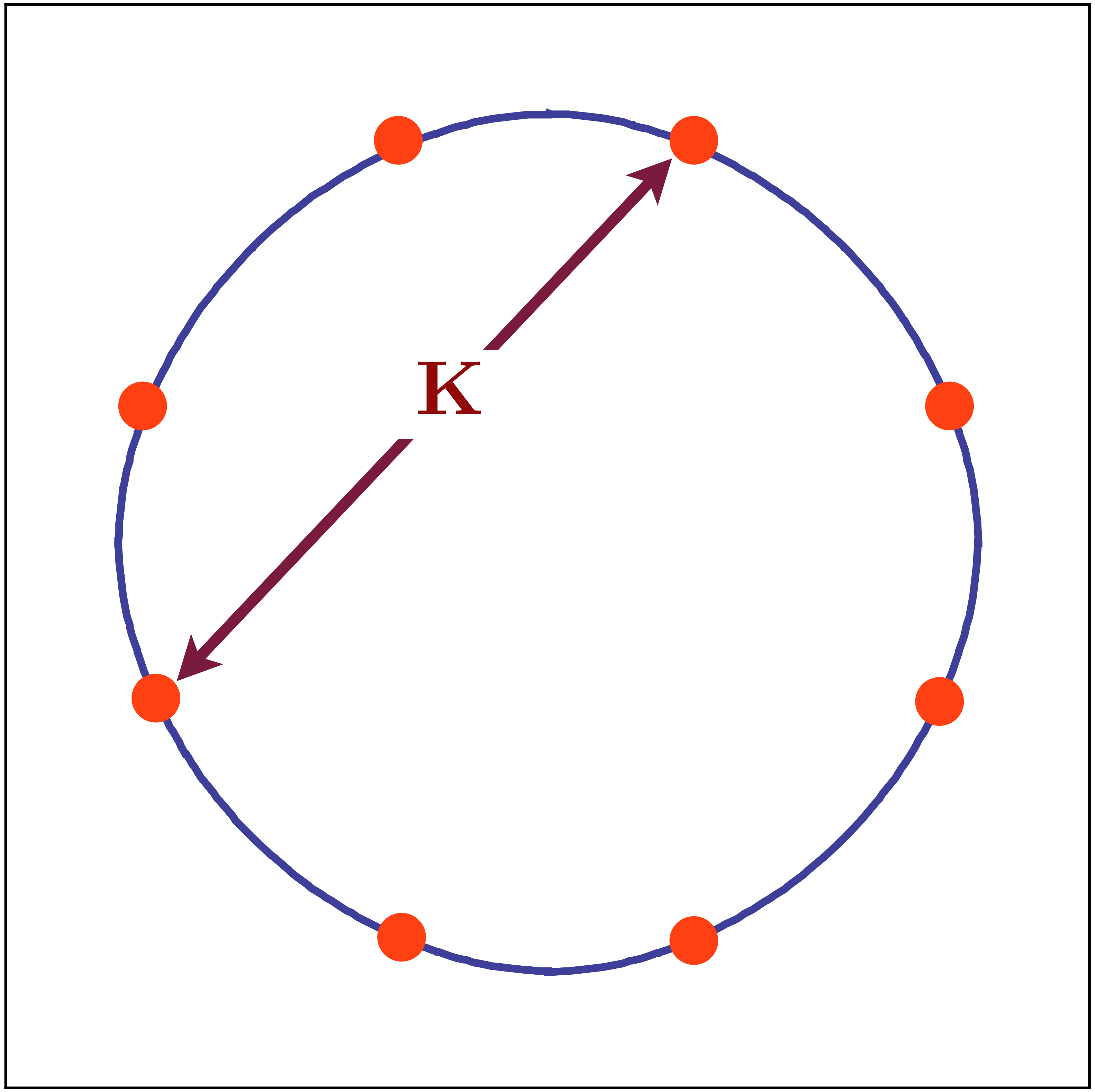
This theory is strongly-coupled in $d = 2$.

M.A. Metlitski and S. Sachdev, Phys. Rev. B **85**, 075127 (2010)

Controlled analysis possible by a novel dimensional regularization method.

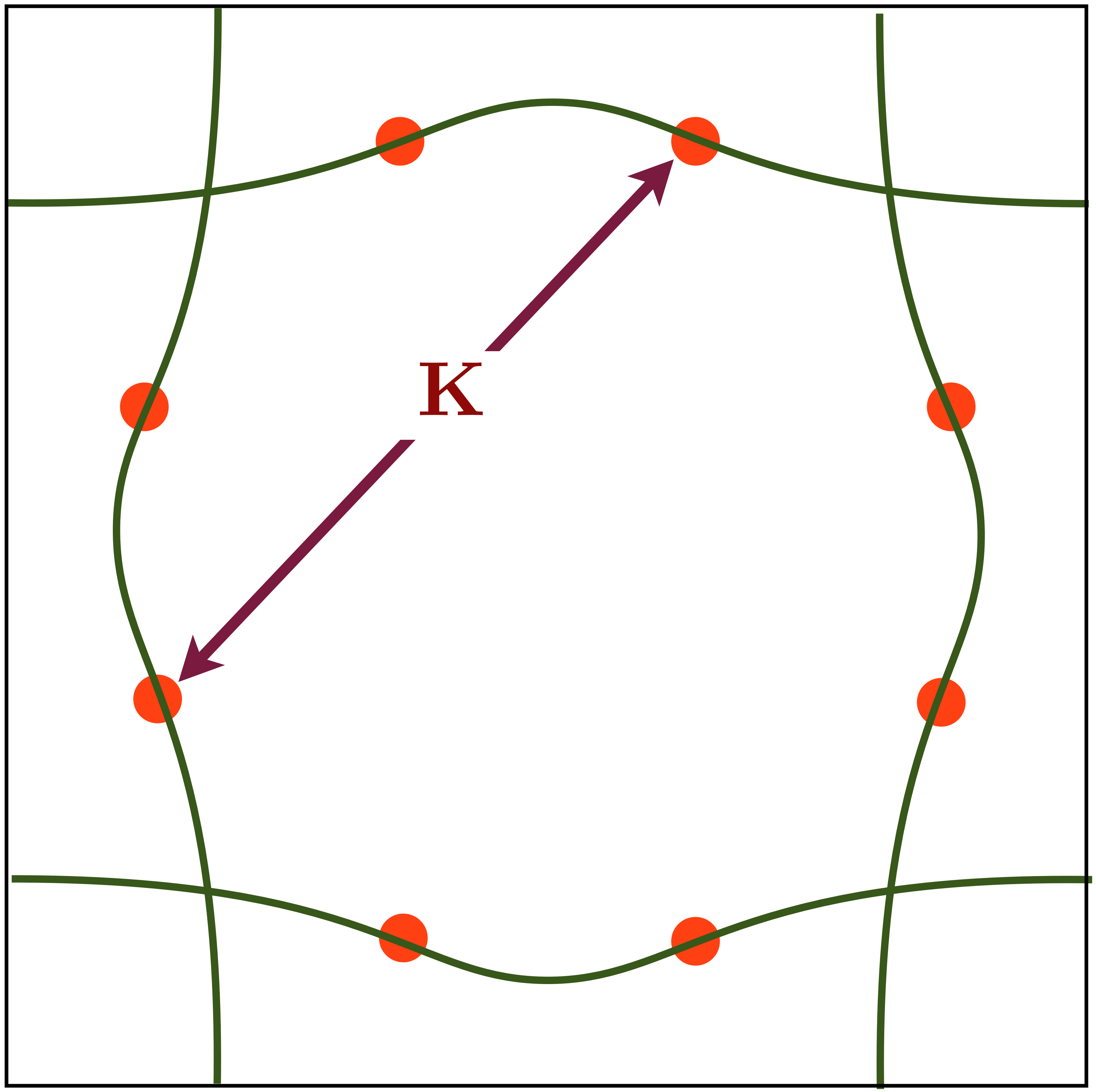
Shouvik Sur and Sung-Sik Lee, Phys. Rev. B **91**, 125136 (2015)
A.A. Patel, P. Strack, and S. Sachdev, Phys. Rev. B **92**, 165105 (2015)

# QMC for the onset of antiferromagnetism




Hot spots in a single band model

# QMC for the onset of antiferromagnetism

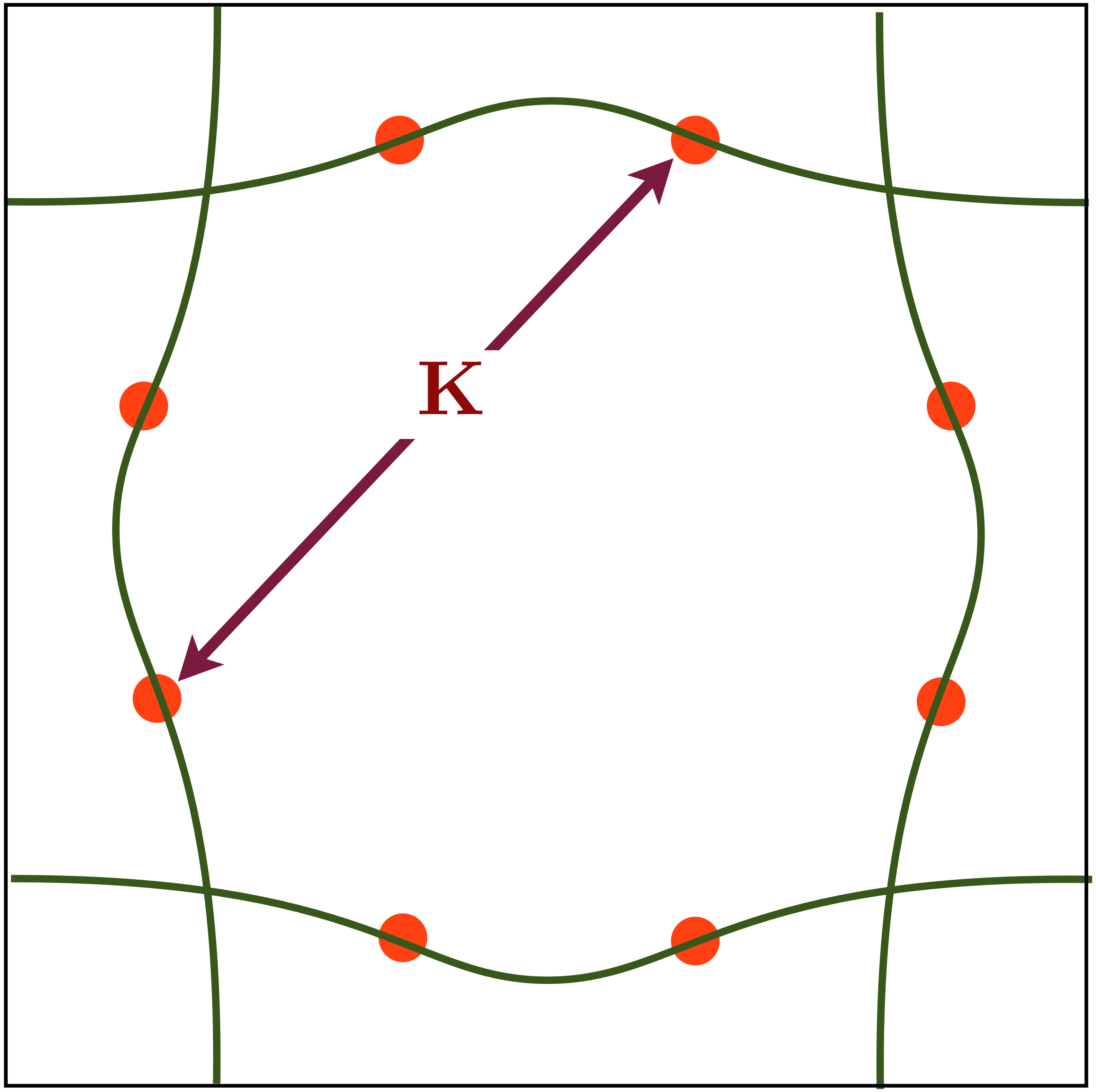



Hot spots in a two band model

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

# QMC for the onset of antiferromagnetism

Faithful realization of the *generic* universal low energy theory for the onset of antiferromagnetism.

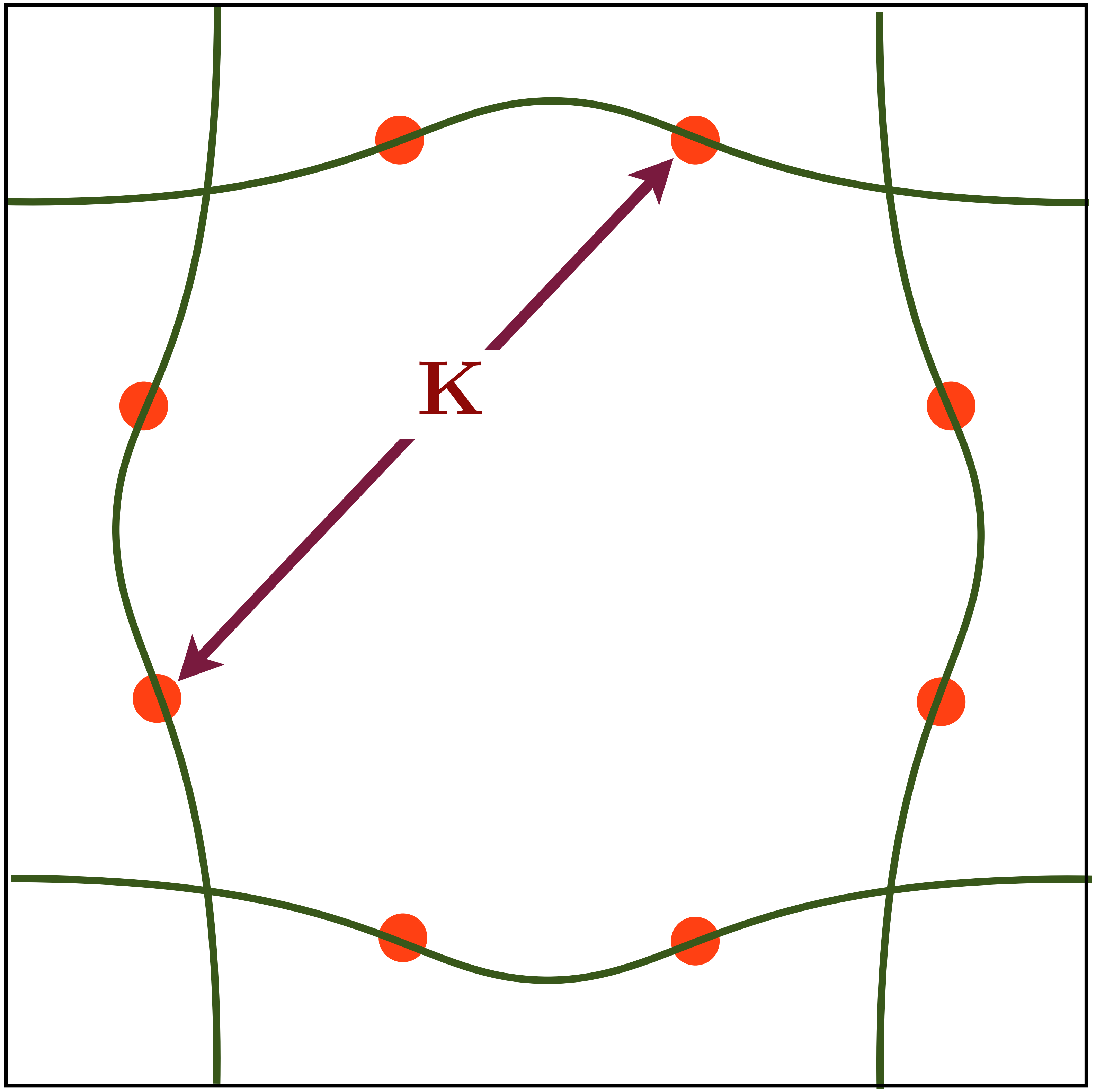



Hot spots in a two band model

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

# QMC for the onset of antiferromagnetism

Sign problem is absent as long as *K* connects hotspots in distinct bands

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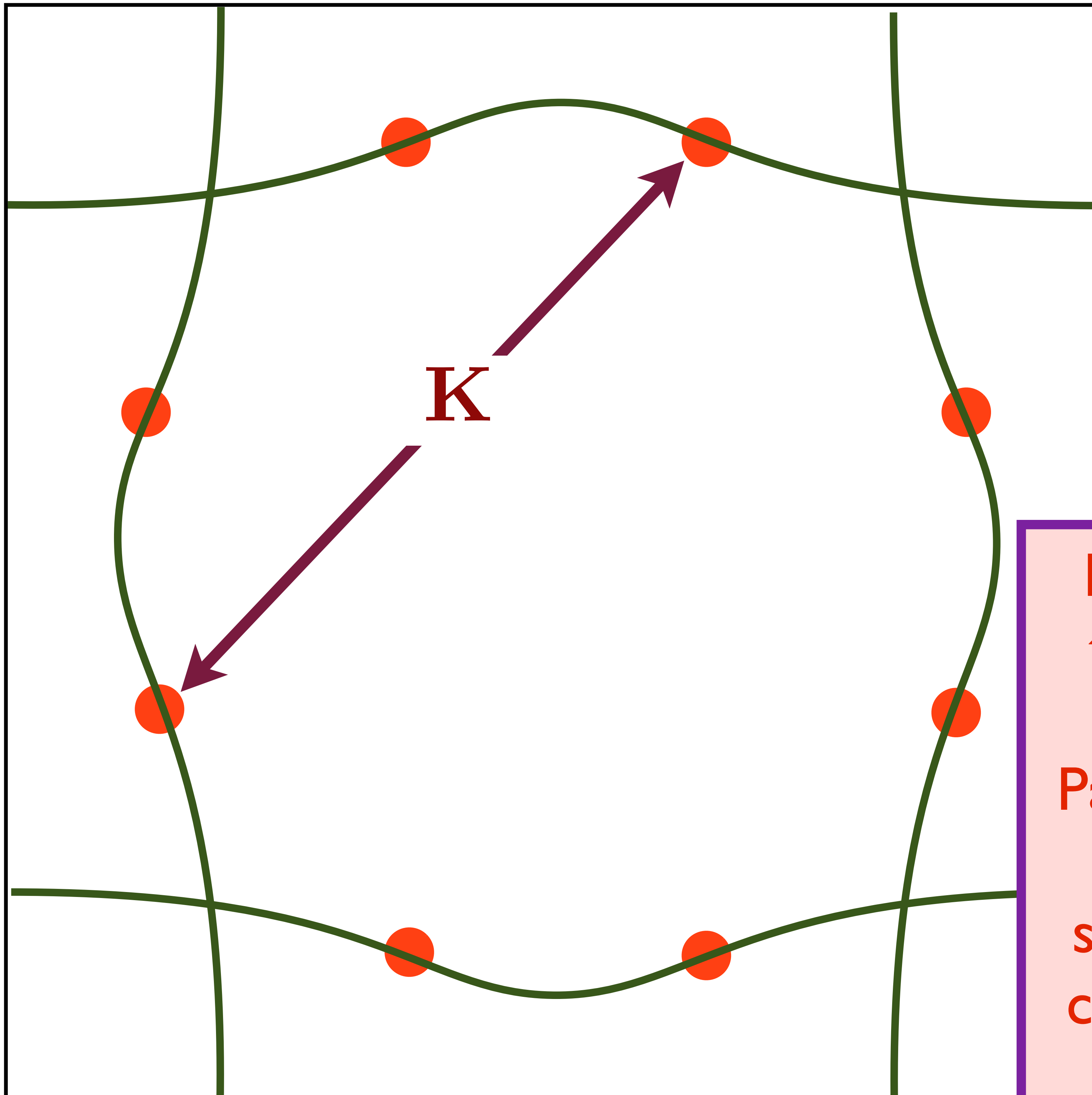


Hot spots in a two band mod

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Requires only time-reversal symmetry. Particle-hole or point-group symmetries or commensurate densities ***not*** required !

# QMC for the onset of antiferromagnetism

Electrons with dispersion $\varepsilon_{\mathbf{k}}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left( \frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$+ \int d\tau d^2x \left[ \frac{1}{2} \left(\mathbf{\nabla}_x \vec{\varphi}\right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

# QMC for the onset of antiferromagnetism

Electrons with dispersions $\varepsilon_{\mathbf{k}}^{(x)}$ and $\varepsilon_{\mathbf{k}}^{(y)}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha^{(x)} \mathcal{D}c_\alpha^{(y)} \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(x)\dagger} \left( \frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(x)} \right) c_{\mathbf{k}\alpha}^{(x)}$$

$$+ \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(y)\dagger} \left( \frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(y)} \right) c_{\mathbf{k}\alpha}^{(y)}$$

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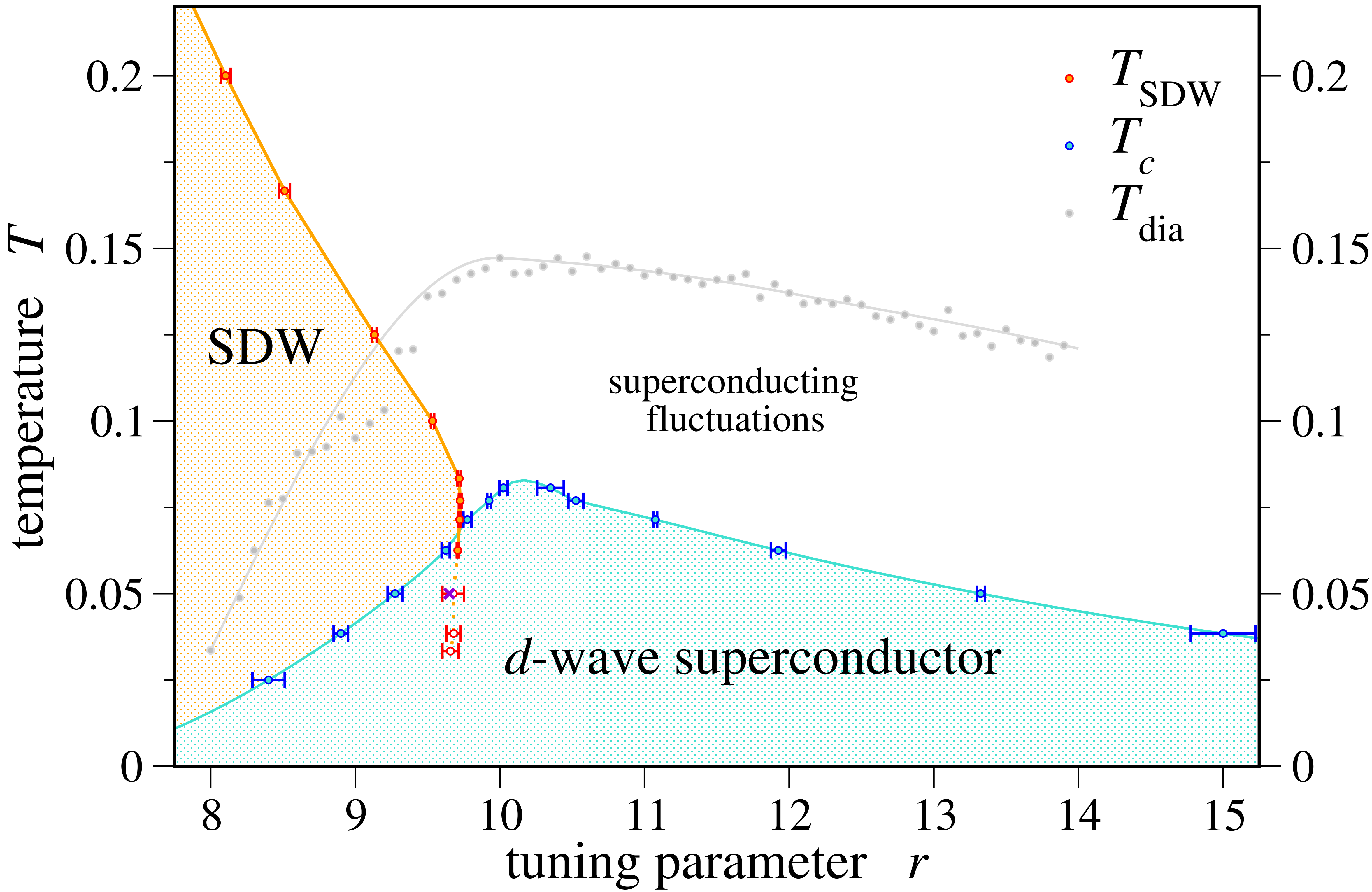
$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c_{i\alpha}^{(x)\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}^{(y)} + \text{H.c.}$$

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

No sign problem !

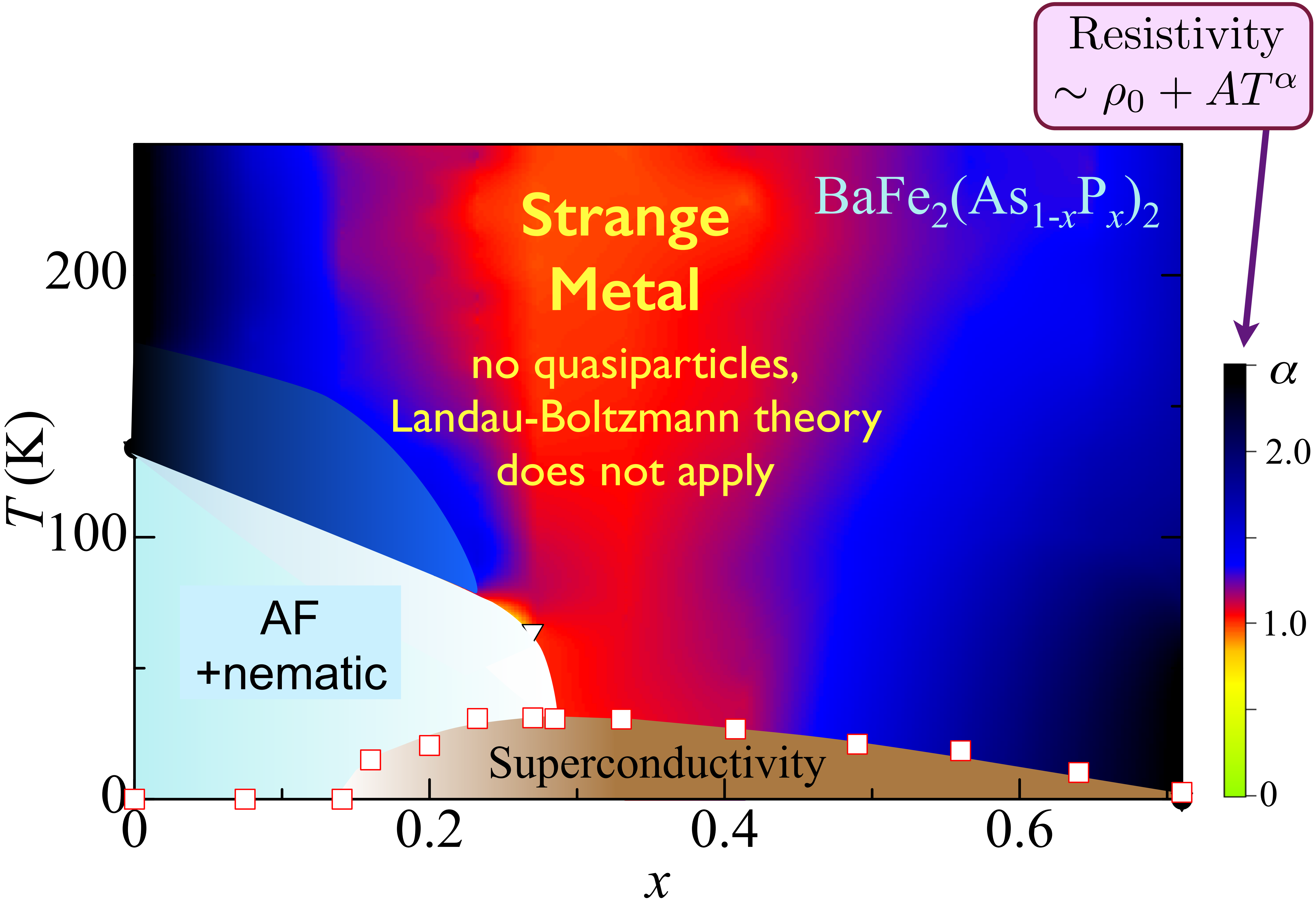
# QMC for the onset of antiferromagnetism




Yoni Schattner, Max H. Gerlach, Simon Trebst, and Erez Berg, arXiv:1512.07257

Zi-Xiang Li, Fa Wang, Hong Yao, and Dung-Hai Lee, arXiv:1512.04541, arXiv:1512.06179

Philipp T. Dumitrescu, Maksym Serbyn, Richard T. Scalettar, and Ashvin Vishwanath, arXiv:1512.08523




S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

# Transport near SDW critical point

- Assume excitations around the full Fermi surface locally thermalize via interactions with excitations of the SDW boson $\varphi_\alpha$. These interactions conserve a (suitably defined) total momentum.

S. A. Hartnoll, D. M. Hofman, M. A. Metlitski and S. Sachdev, PRB **84**, 125115 (2011)

A. A. Patel and S. Sachdev, PRB **90**, 165146 (2014)

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- Momentum relaxation occurs via disorder perturbations which change the local position of the quantum critical point.

$$\begin{aligned} H &= H_0 - \int d^d x \, h(x) \, \varphi_\alpha^2(x) \\ \overline{h(x)h(x')} &= h_0^2 \, \delta^d(x - x') \end{aligned}$$

A. A. Patel and S. Sachdev, PRB **90**, 165146 (2014)

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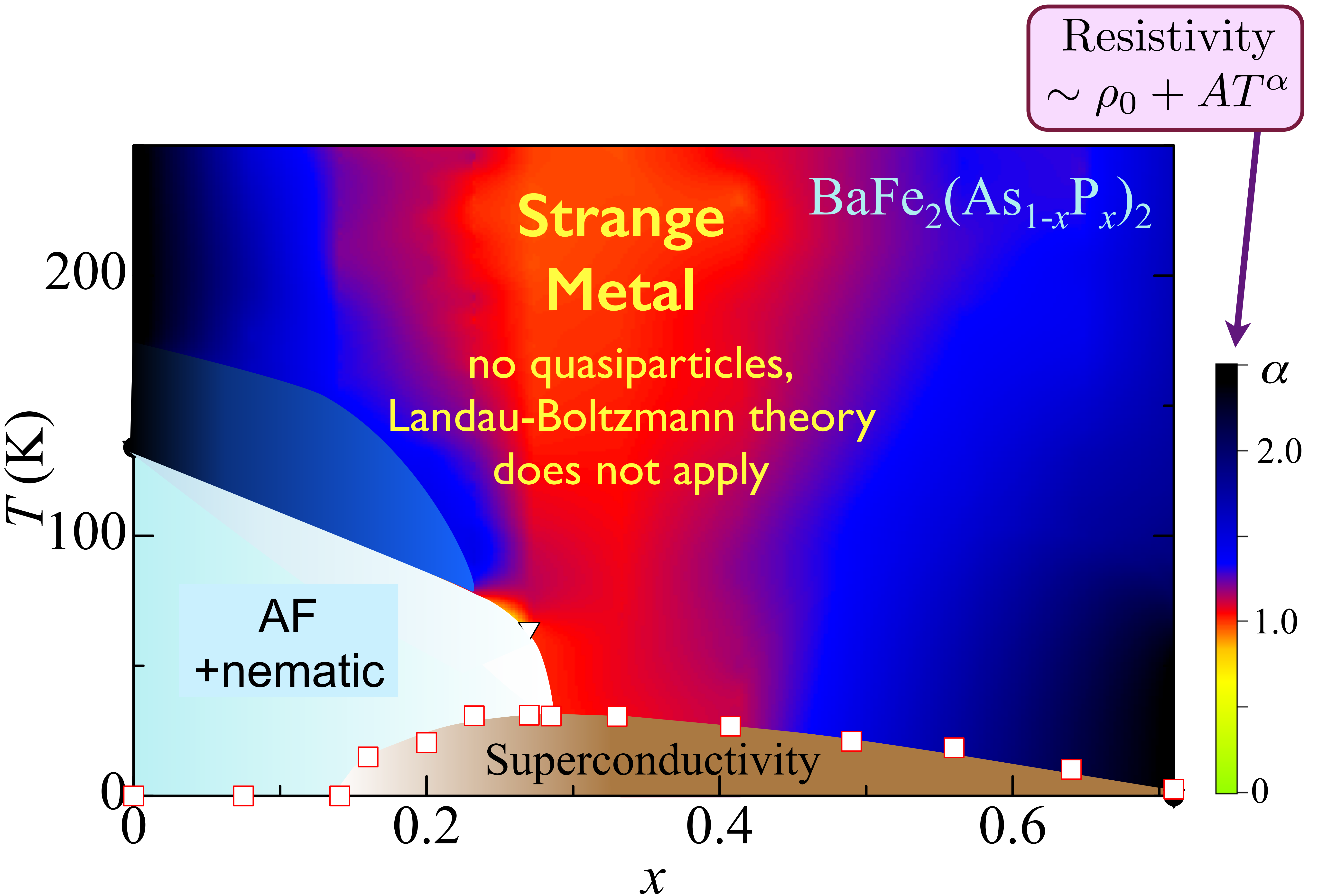
\overline{h(x) h(x')} &= h_0^2 \, \delta^d(x - x')
\end{aligned}
$$

- Memory function methods yield

$$
\begin{aligned}
\frac{1}{\tau_{\text{imp}}} &\sim \lim_{\omega \to 0} h_0^2 \int d^2 q \, q_x^2 \frac{\text{Im} \left( G^{\text{R}}_{\varphi_\alpha^2, \varphi_\alpha^2}(q, \omega) \right)_{H_0}}{\omega} \\
&\sim h_0^2 \, T \quad \text{(up to logarithms)}
\end{aligned}
$$

A. A. Patel and S. Sachdev, PRB **90**, 165146 (2014)

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1. Strange metal in graphene

*Experiments vs. theory combining hydrodynamic/holographic/Boltzmann/memory-function methods*

2. Strange metal in correlated electron superconductors

*Onset of spin density wave order in metals*

3. The pseudogap metal of the hole-doped cuprate superconductors

*Evidence for a metal with emergent gauge fields*

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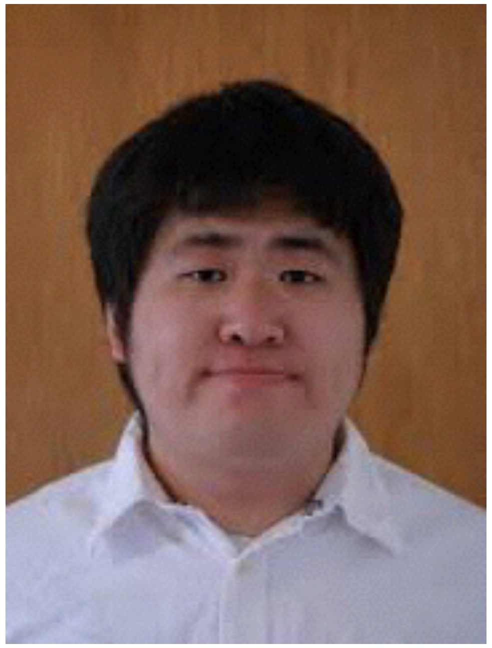
*Evidence for a metal with emergent gauge fields*

Debanjan Chowdhury

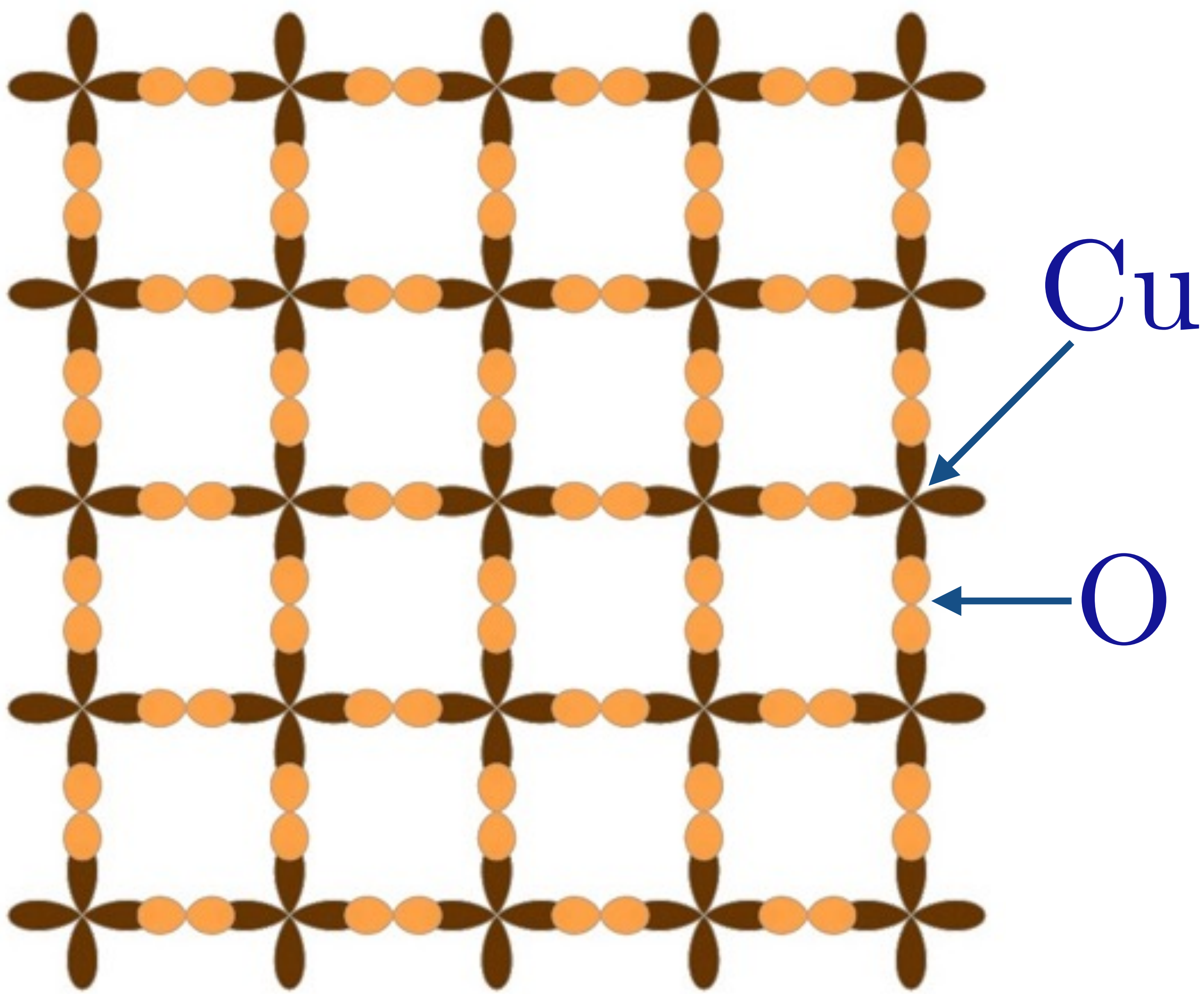
Andrea Allais



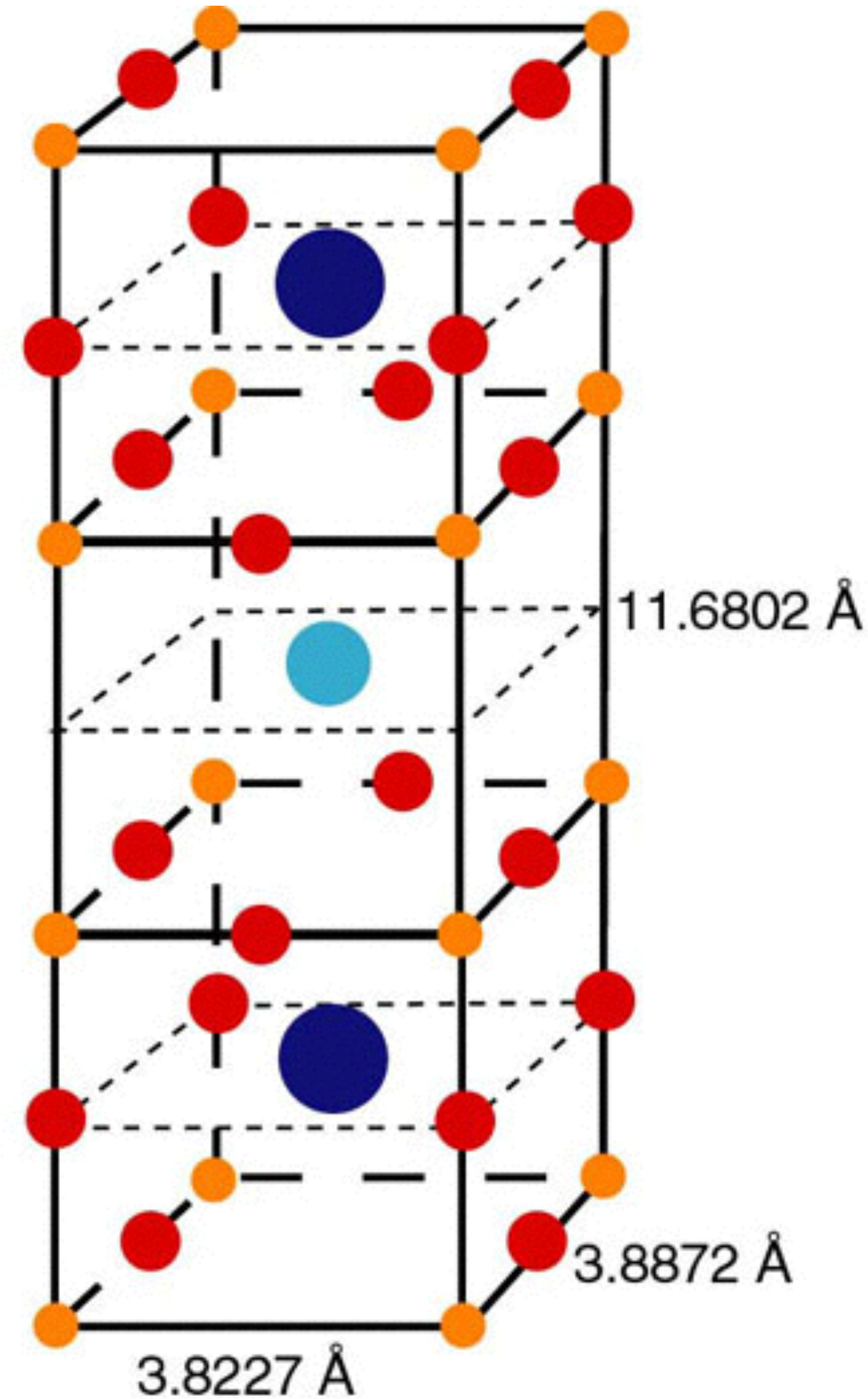
Yang Qi

Matthias Punk

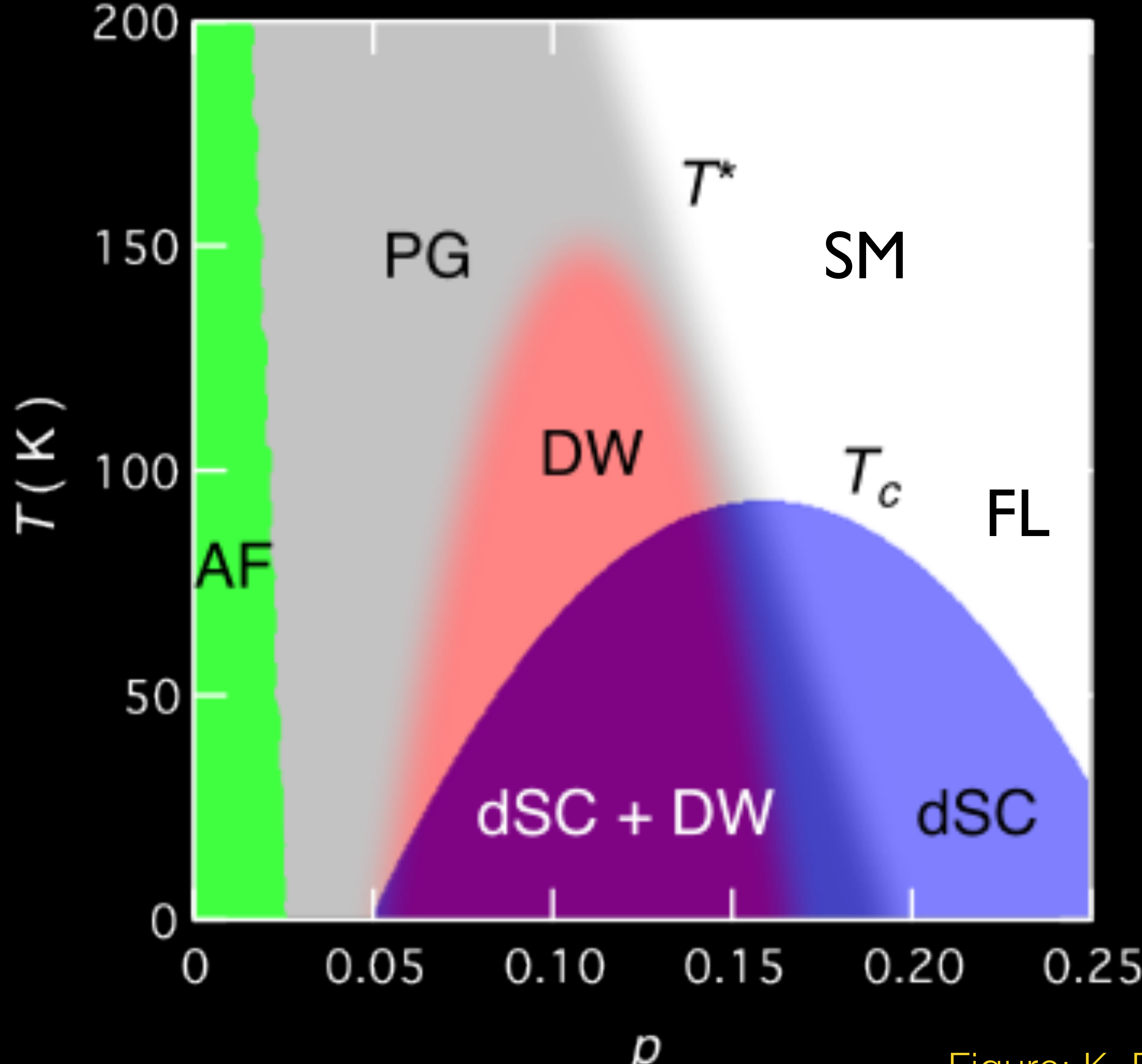
# High temperature superconductors

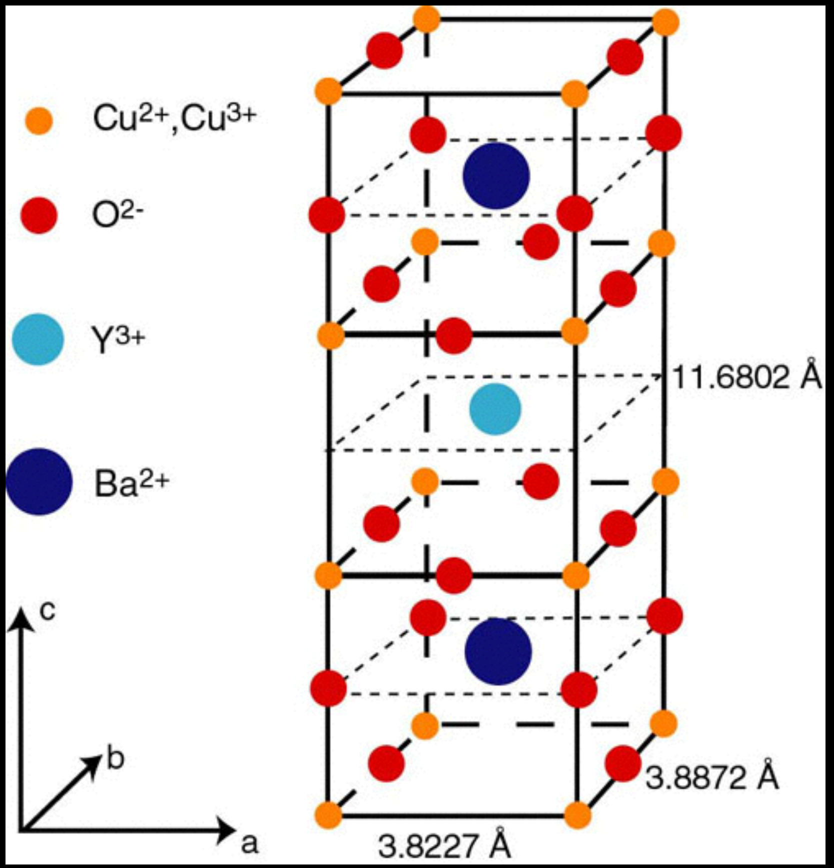



$CuO_2$ plane




$YBa_2Cu_3O_{6+x}$




$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

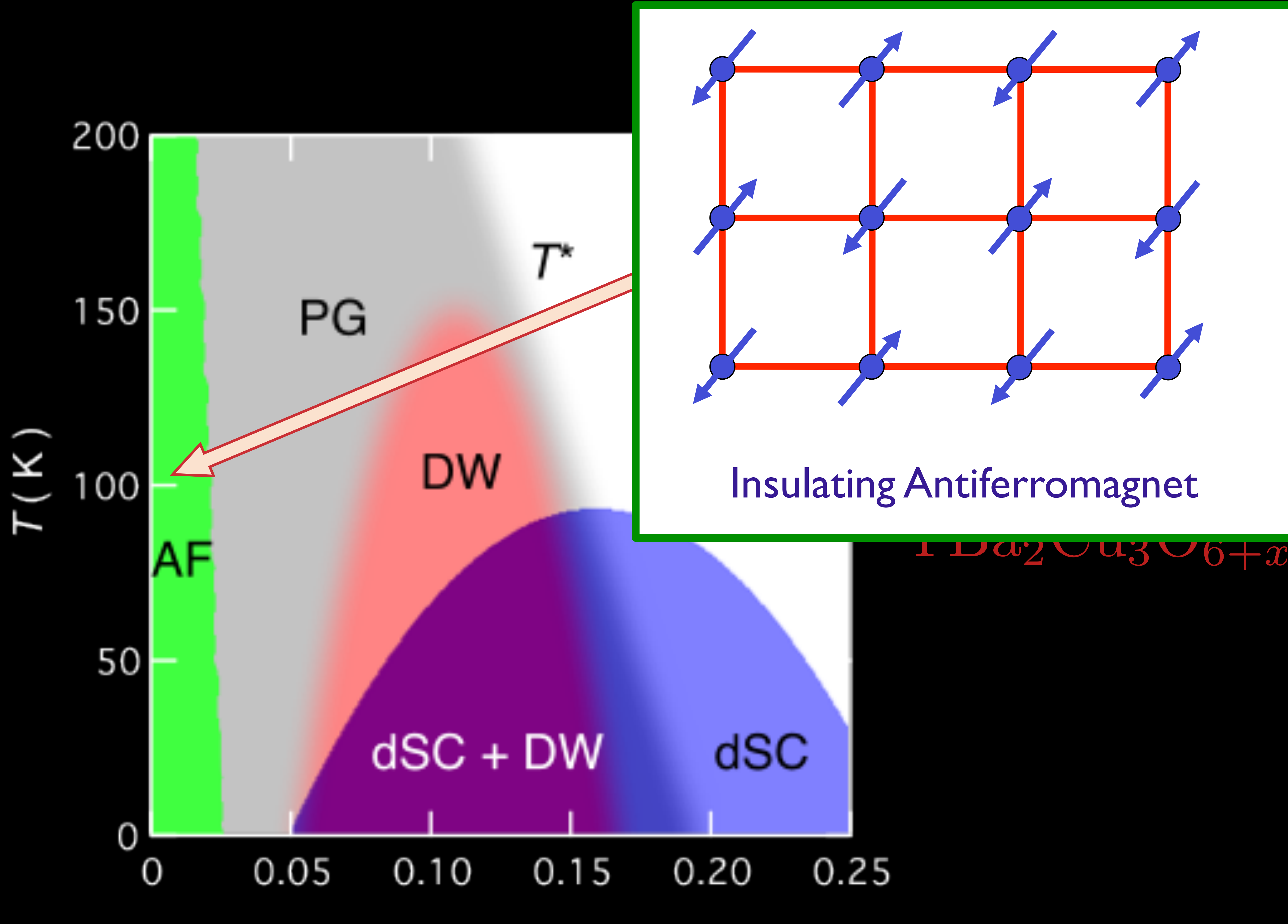
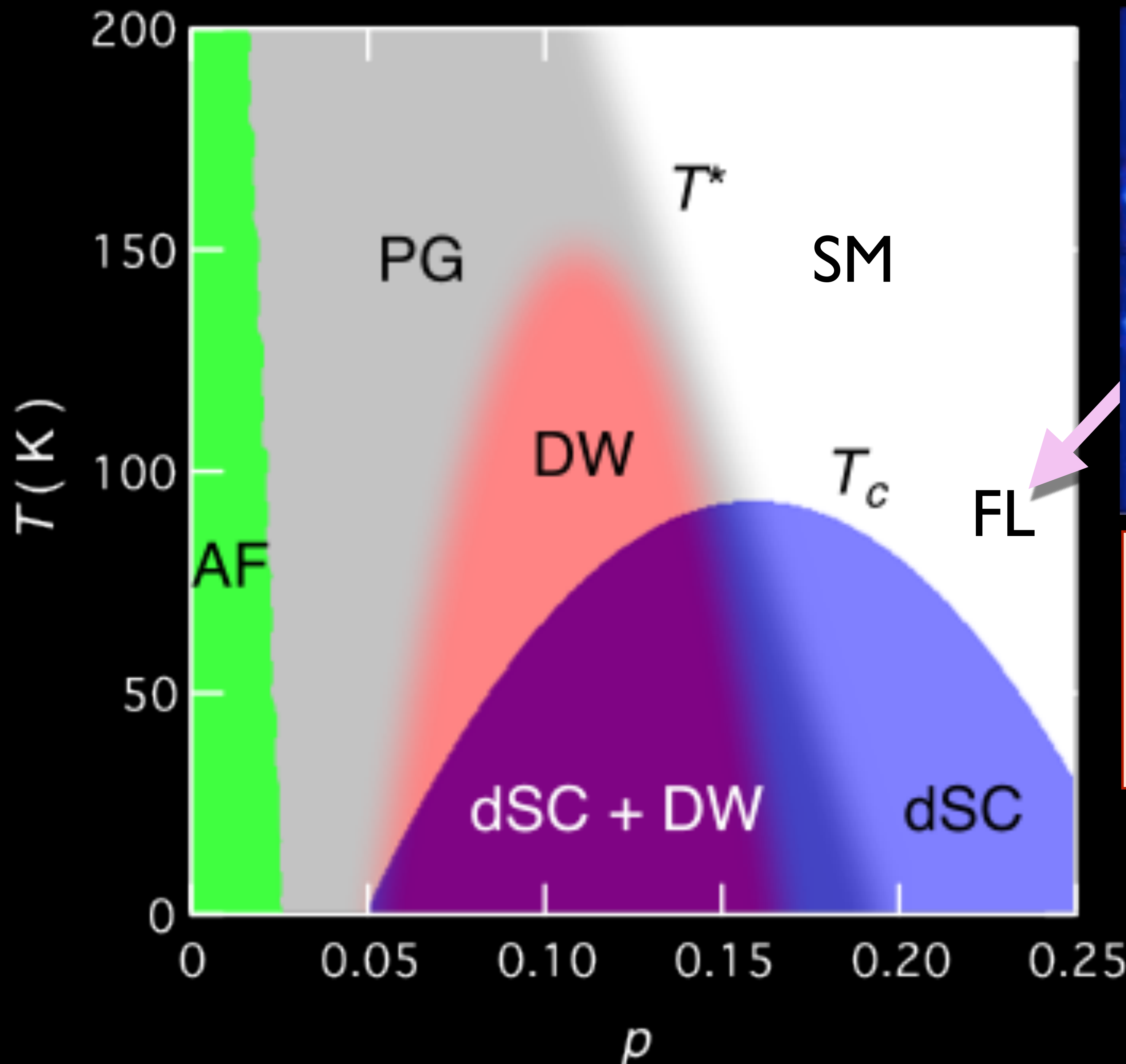
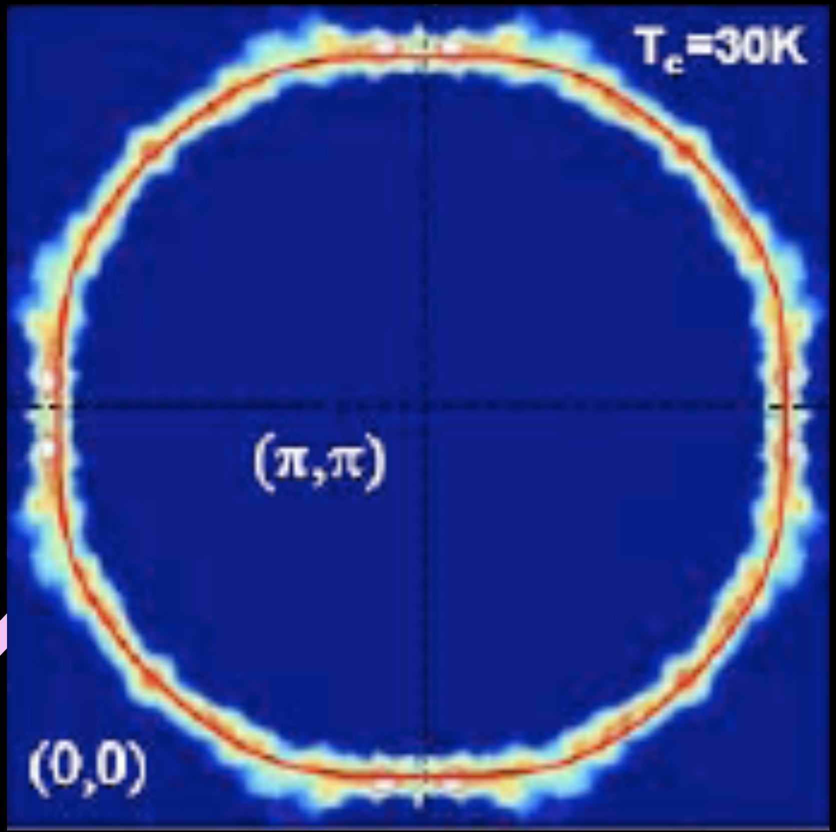


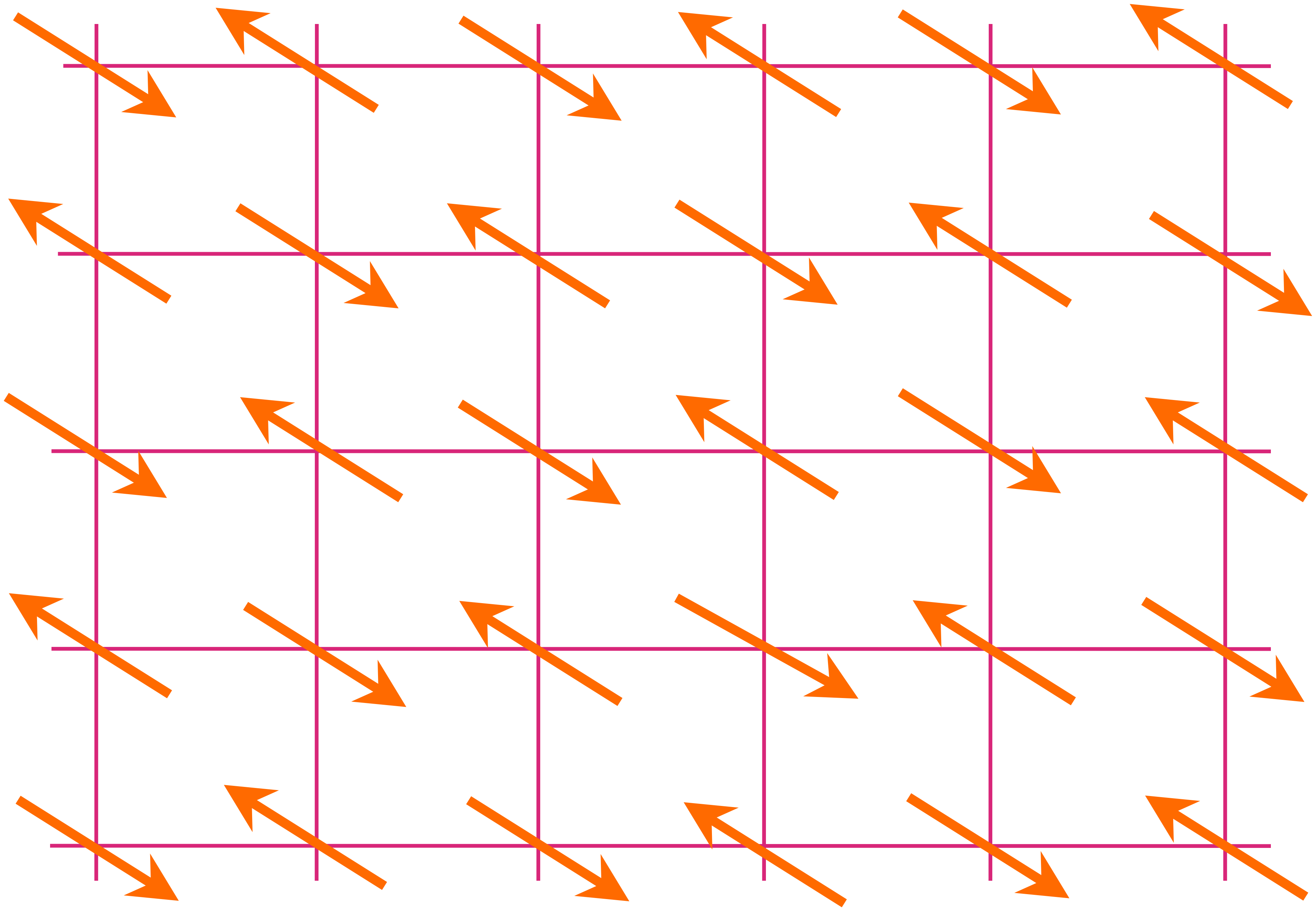

Figure: K. Fujita and J. C. Seamus Davis

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

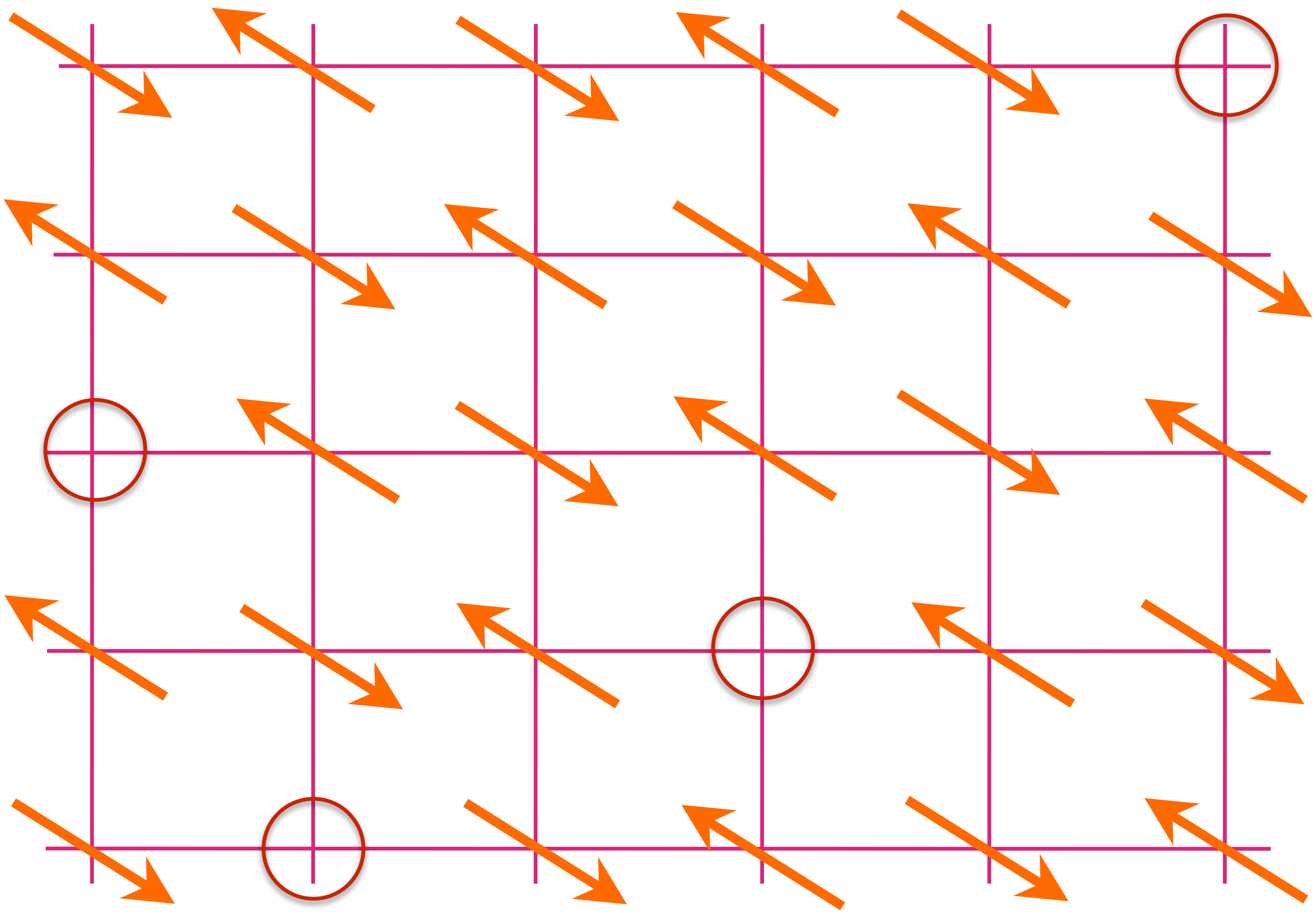


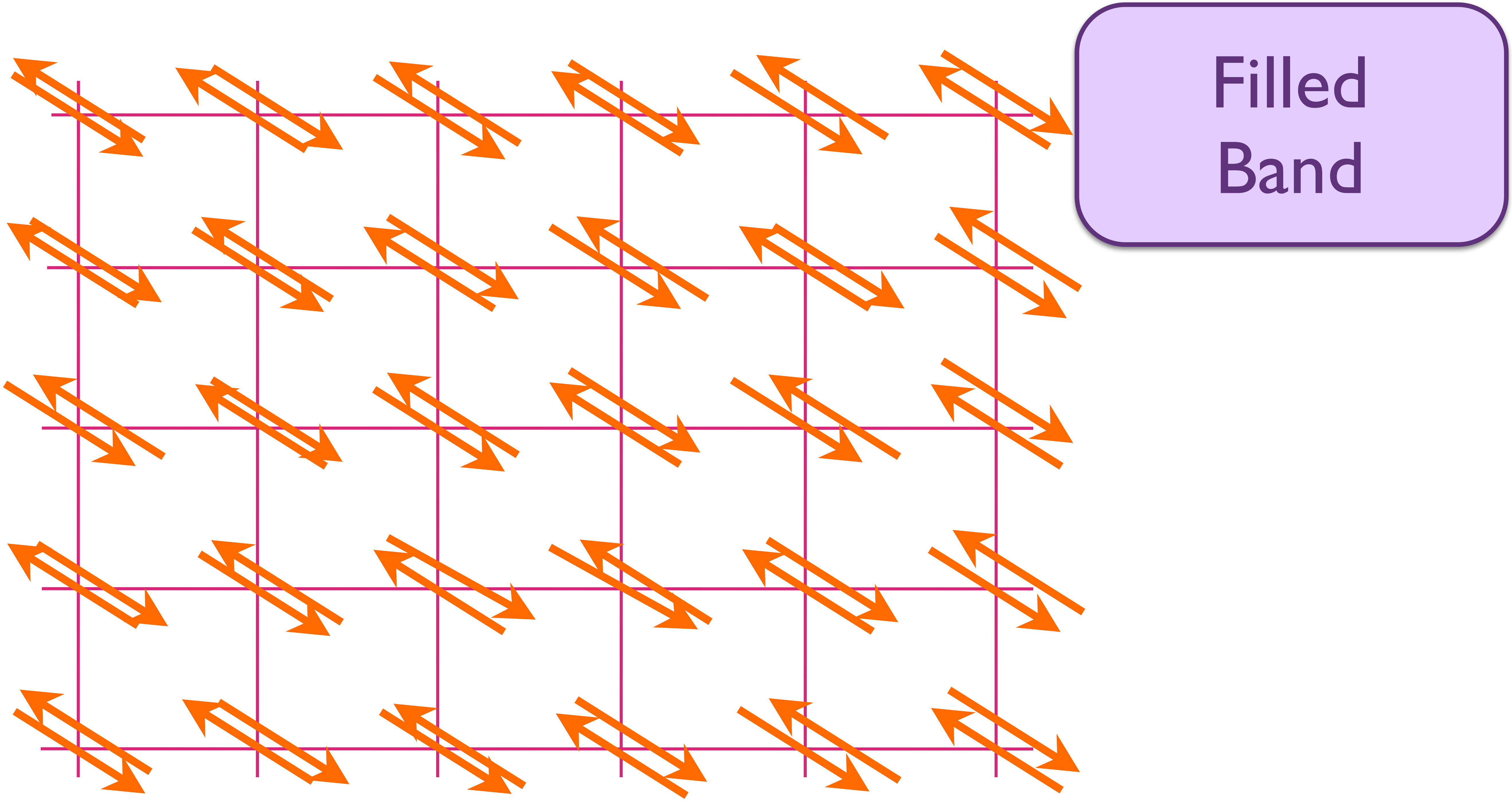

A conventional metal: the Fermi liquid

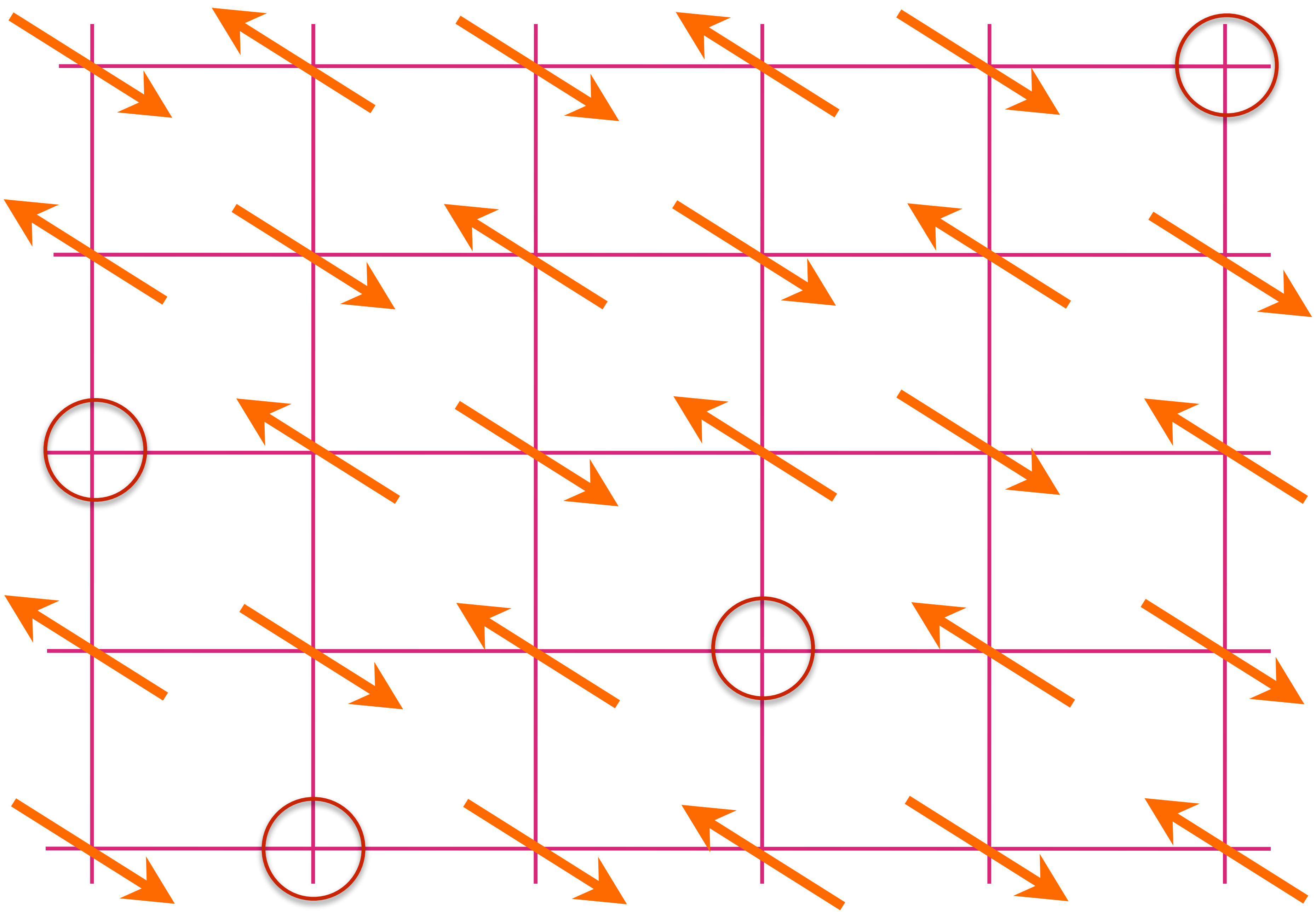


“Undoped” Anti-ferromagnet



Anti-ferromagnet with $p$ holes per square

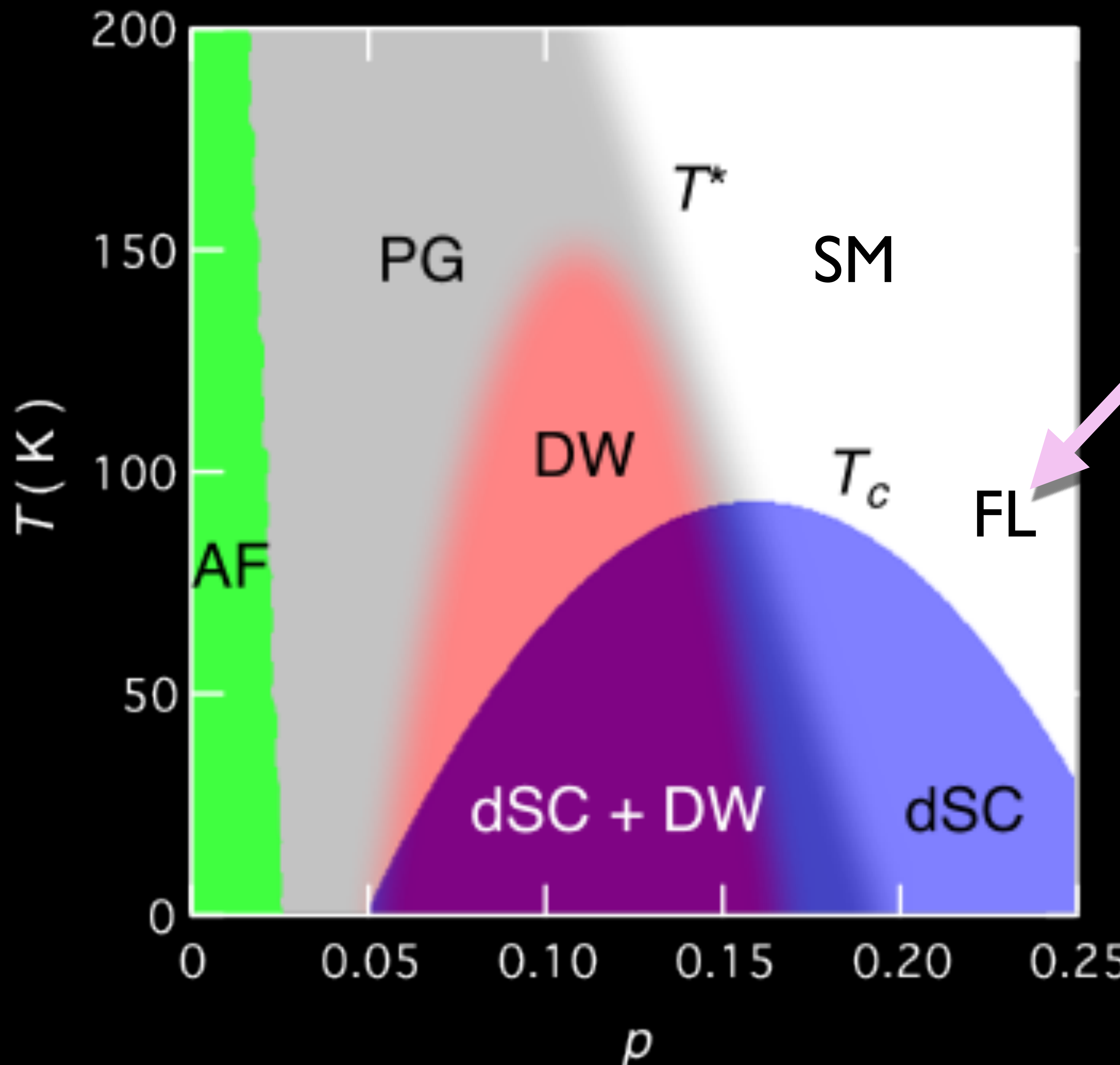

Filled
Band

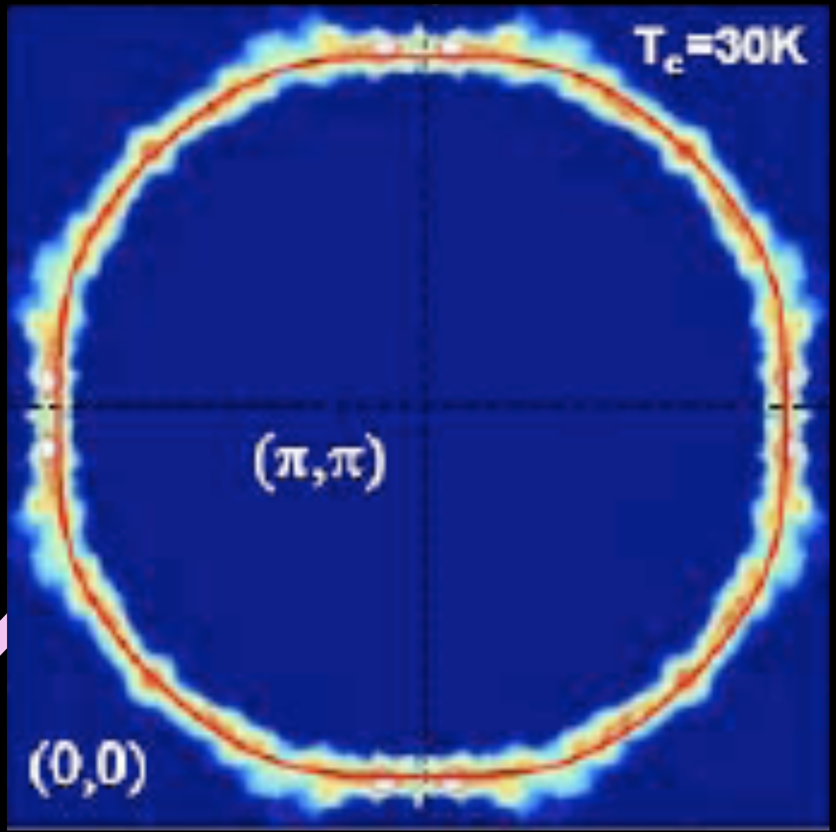


Anti-ferromagnet with $p$ holes per square

But relative to the band insulator, there are $1+p$ holes per square, and so a Fermi liquid has a Fermi surface of size $1+p$

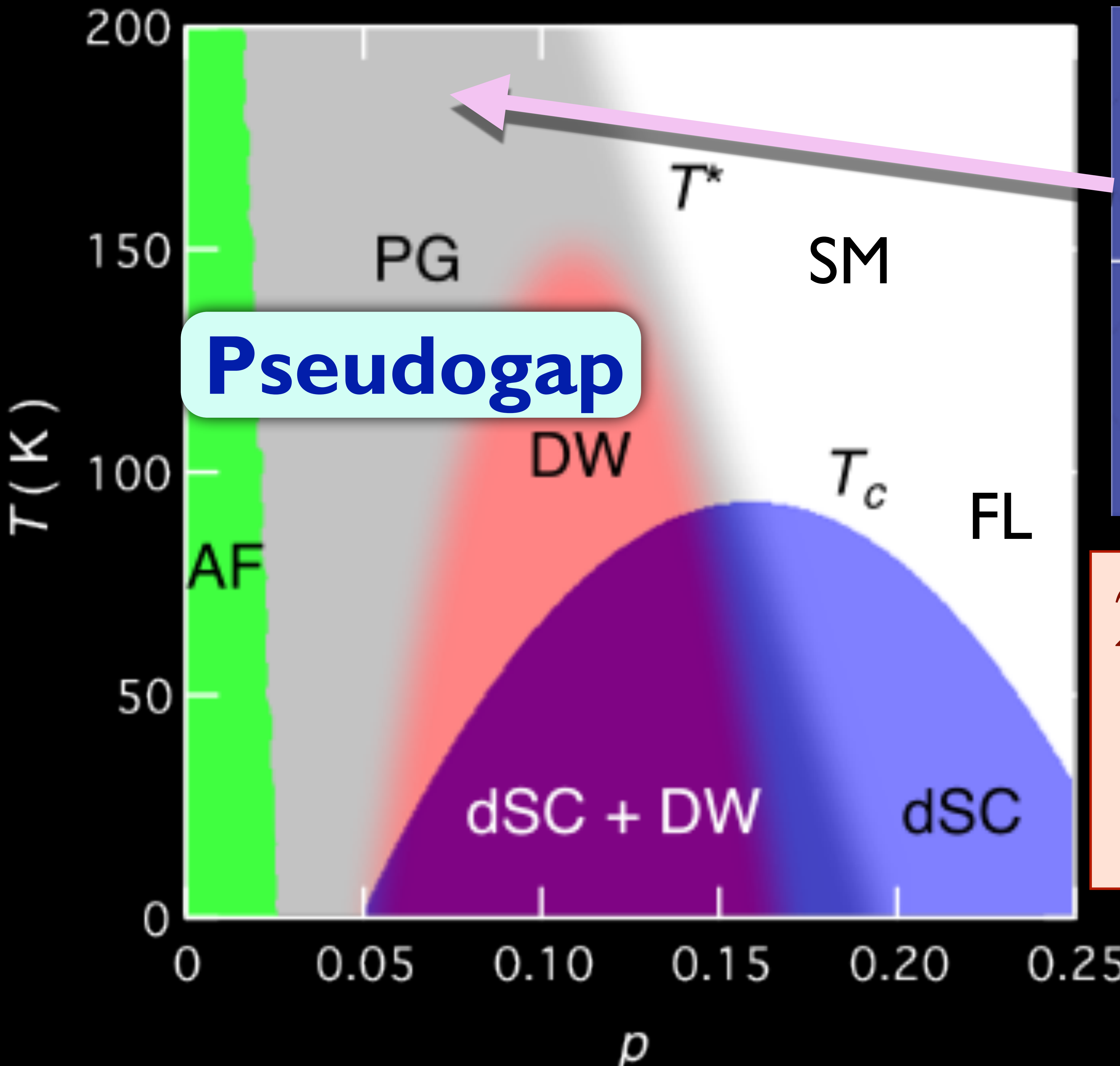
M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

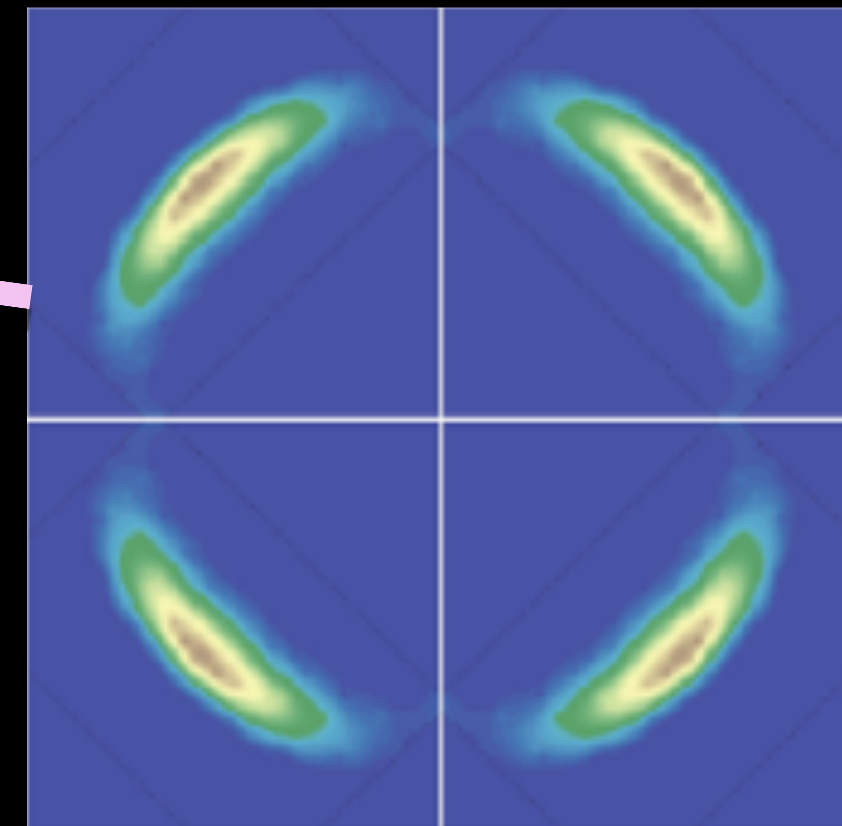


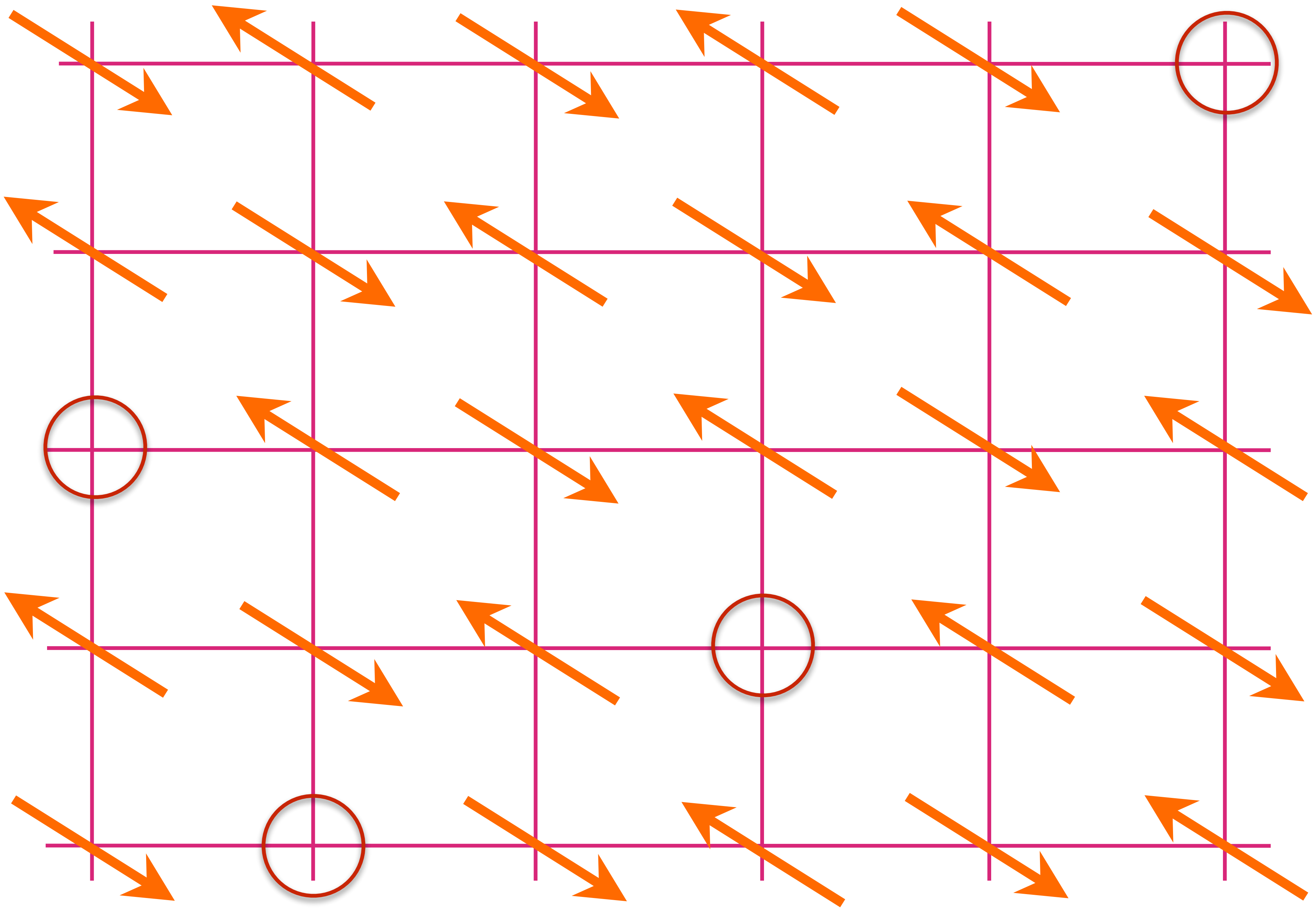

Fermi liquid
Area enclosed by
Fermi surface =1+$p$

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

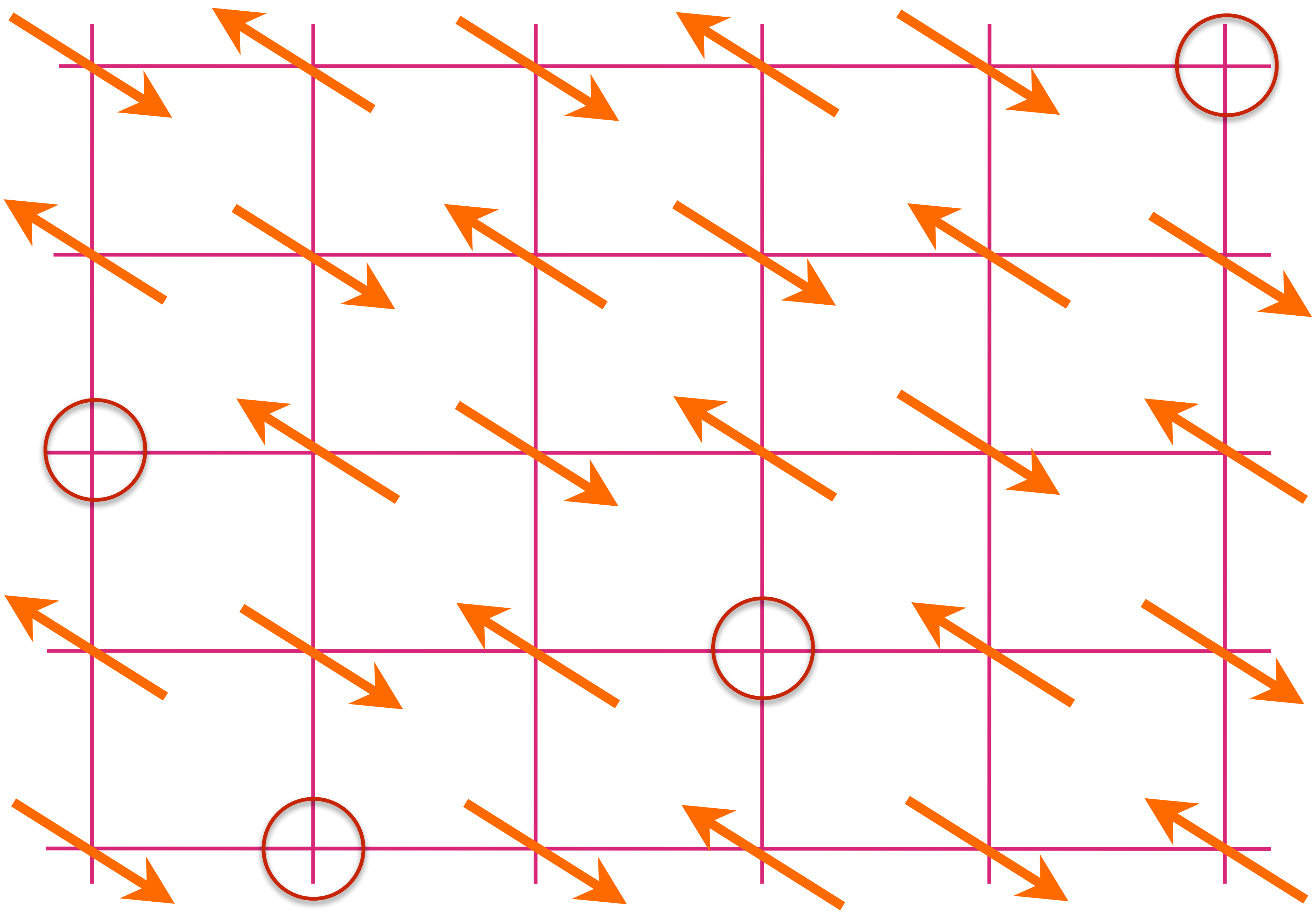


2. <u>Pseudogap metal</u> at low $p$



Anti-ferromagnet with $p$ holes per square



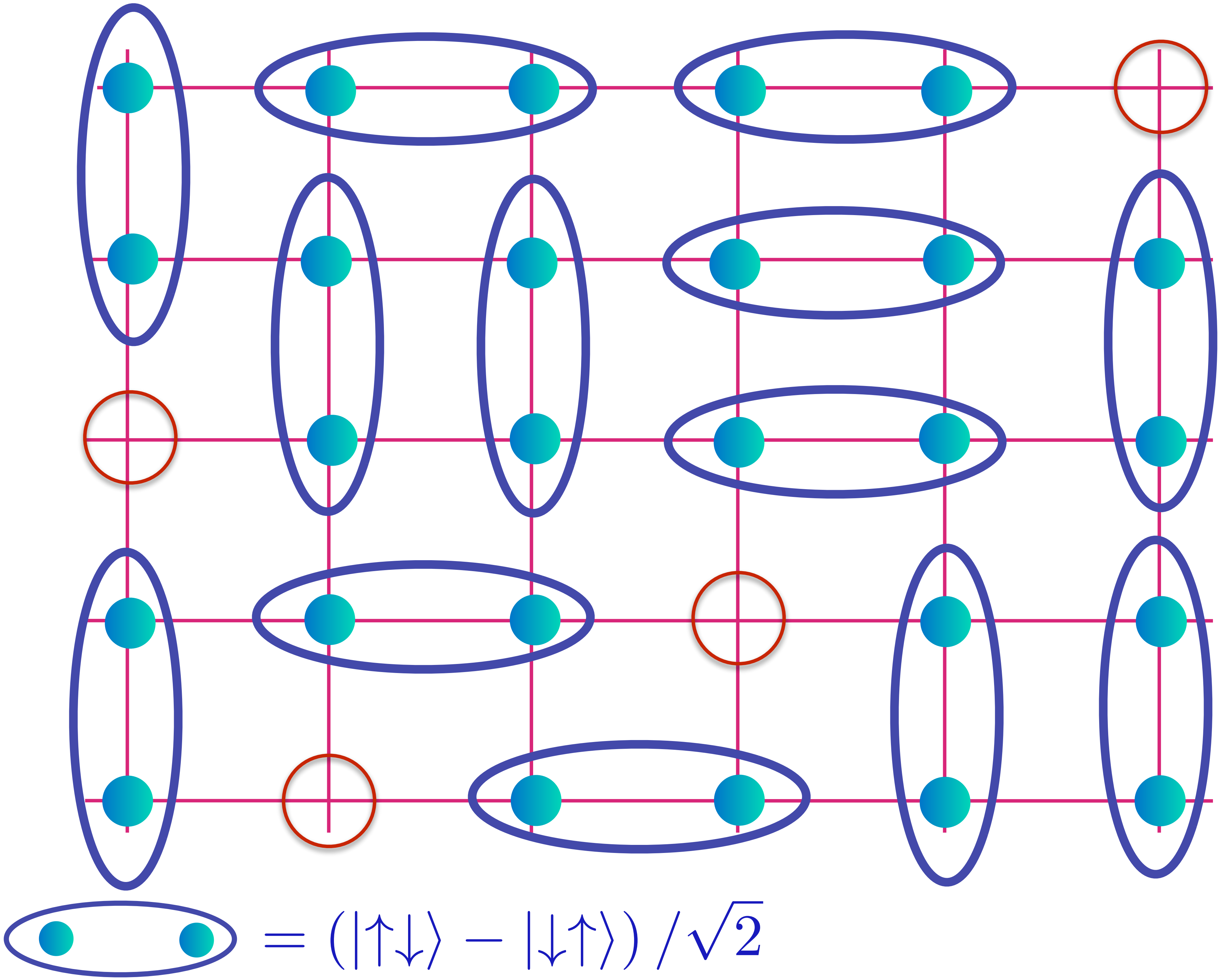
Anti-ferromagnet with $p$ holes per square

Can we get a Fermi surface of size $p$?

(and full square lattice symmetry)

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

N. Read and B. Chakraborty, PRB **40**, 7133 (1989)



$$= \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

Spin liquid with density *p* of spinless, charge +e "holons". These can form a Fermi surface of size p, but this is not visible in electron photo-emission

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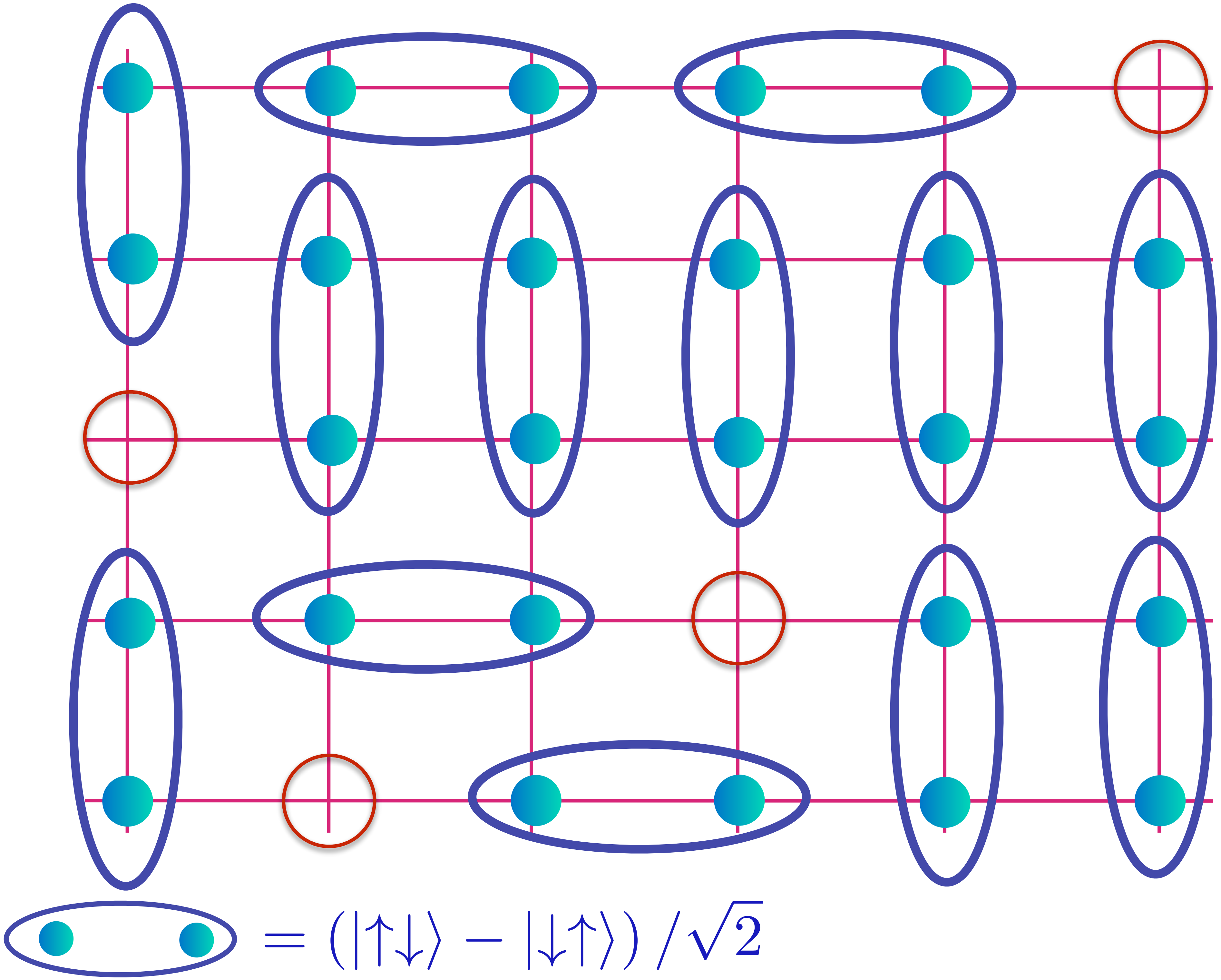

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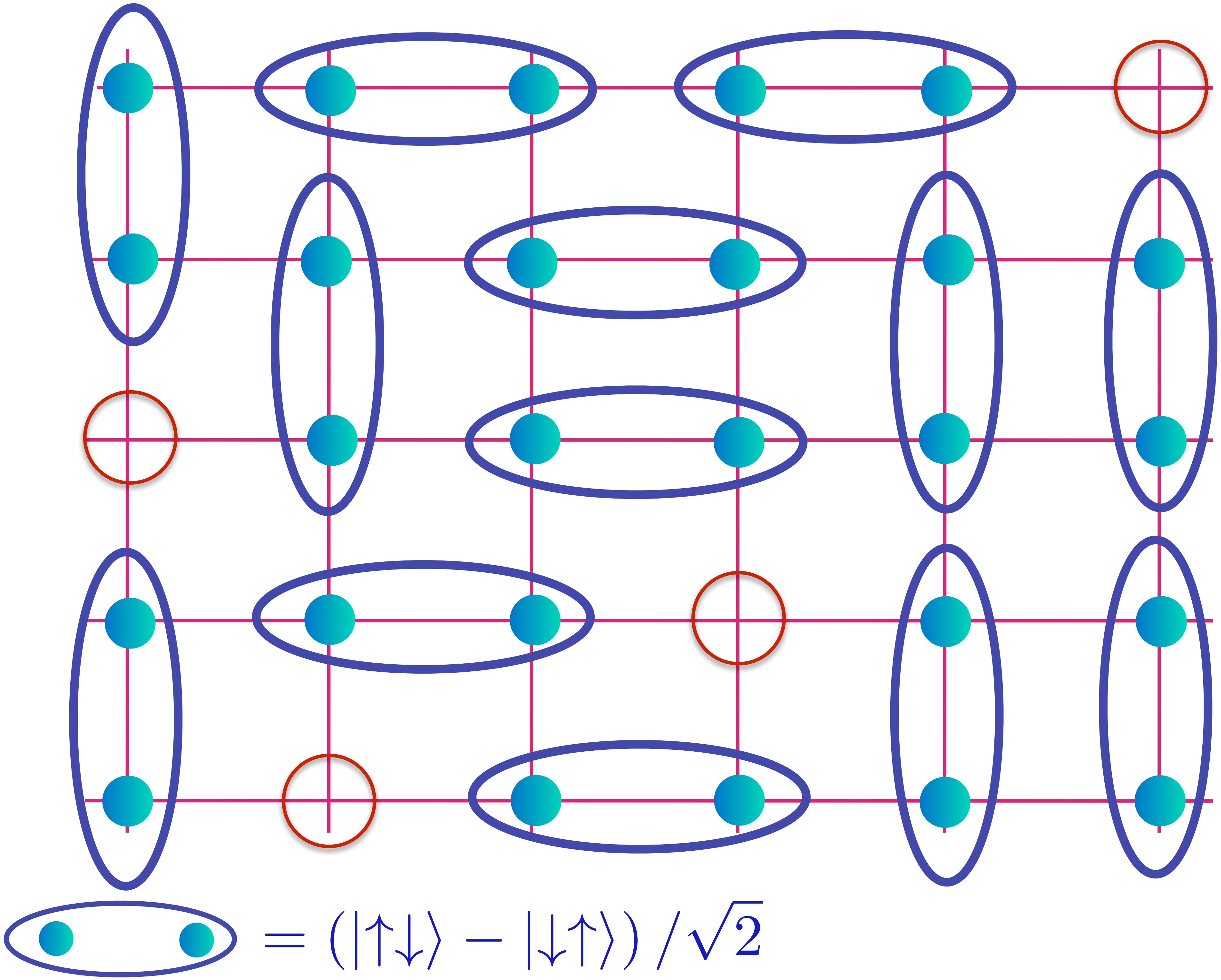

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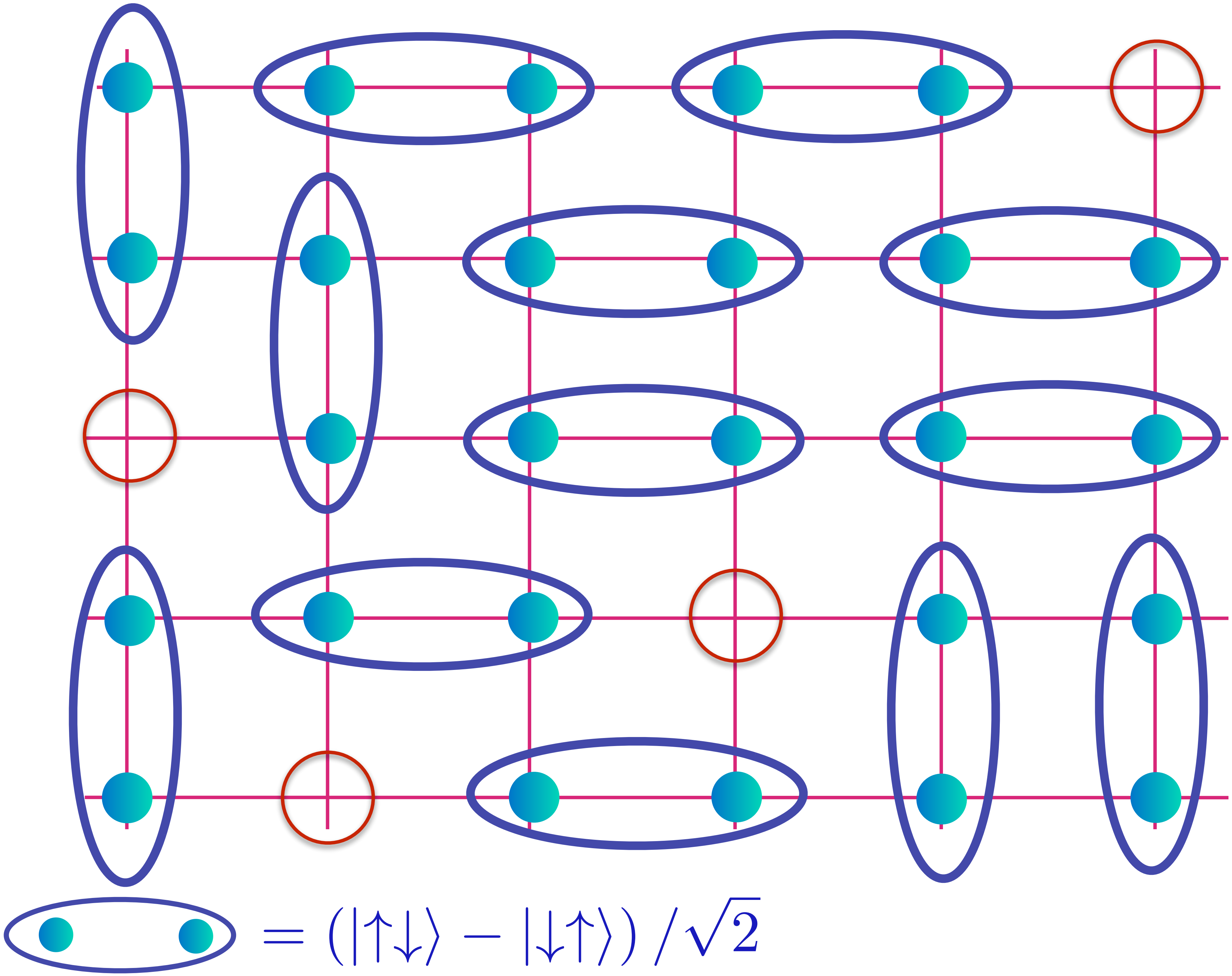

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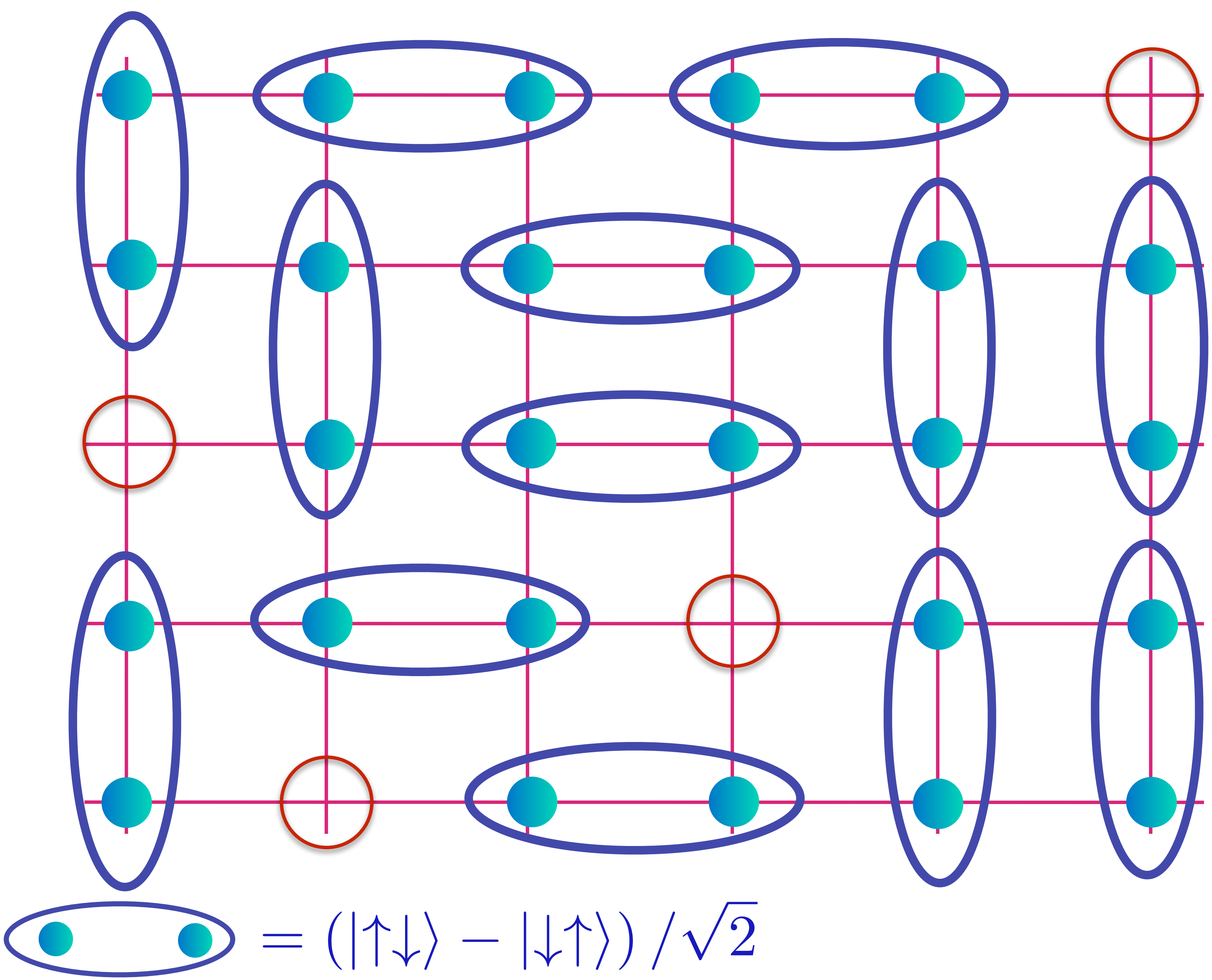

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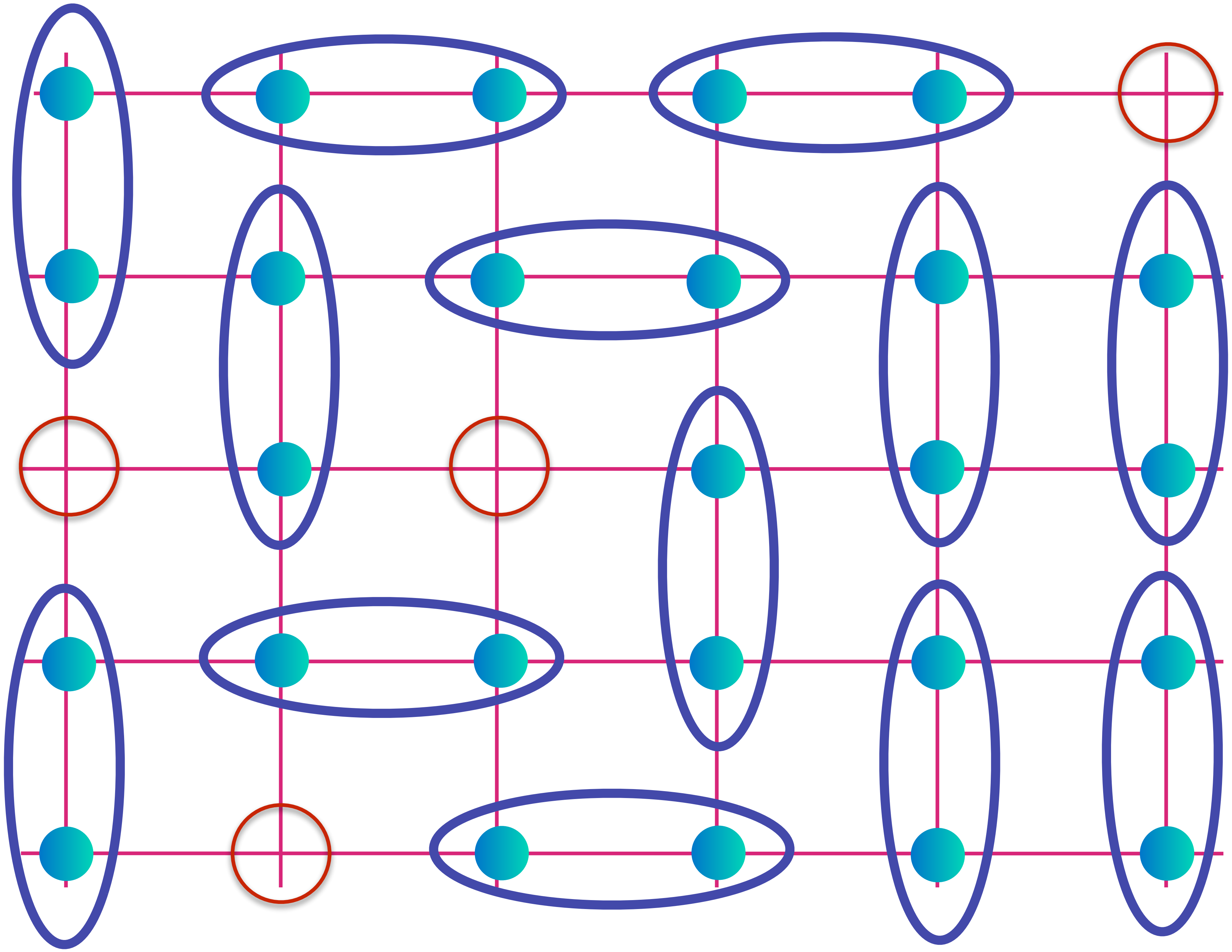

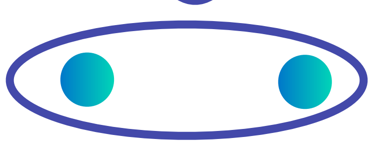 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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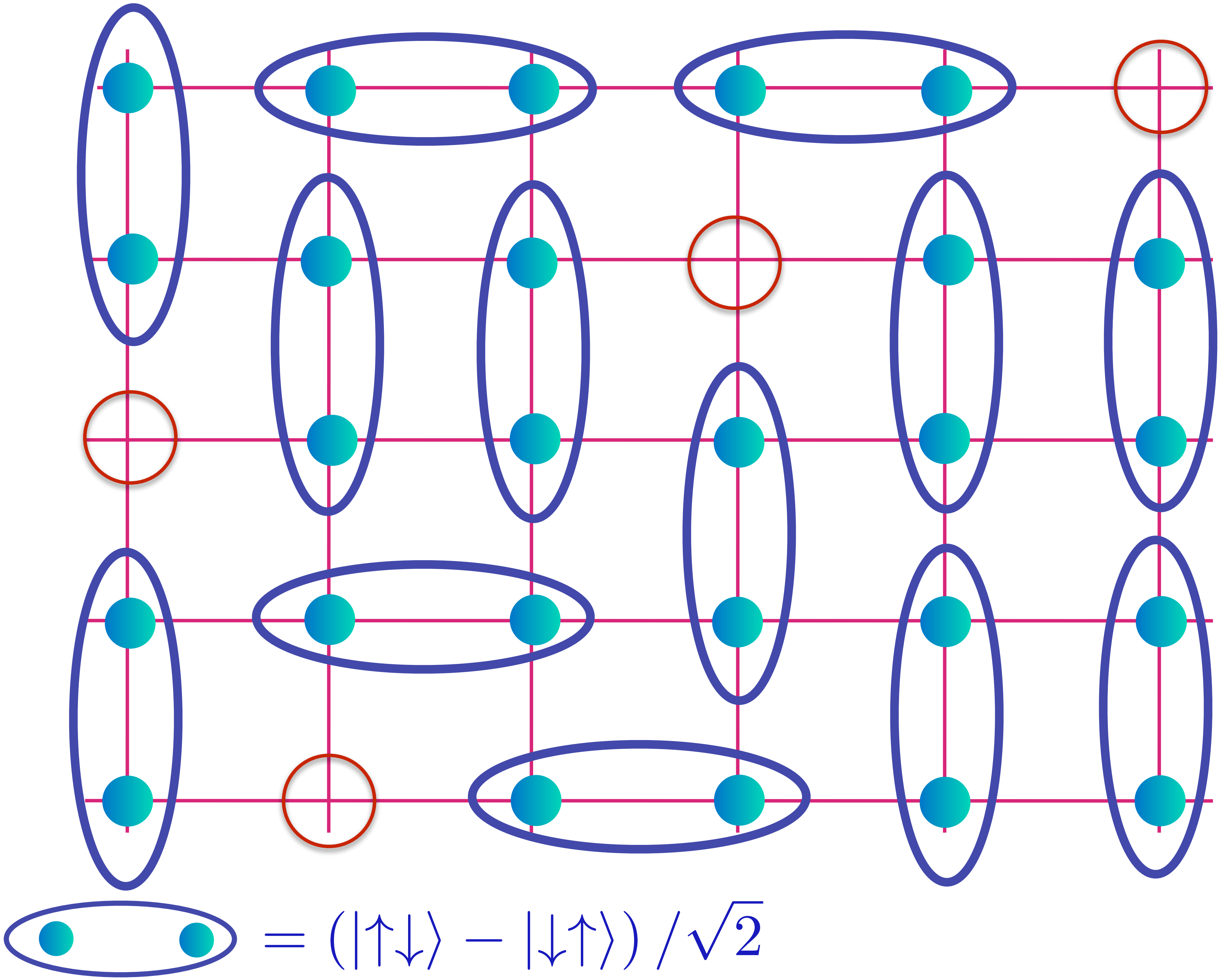

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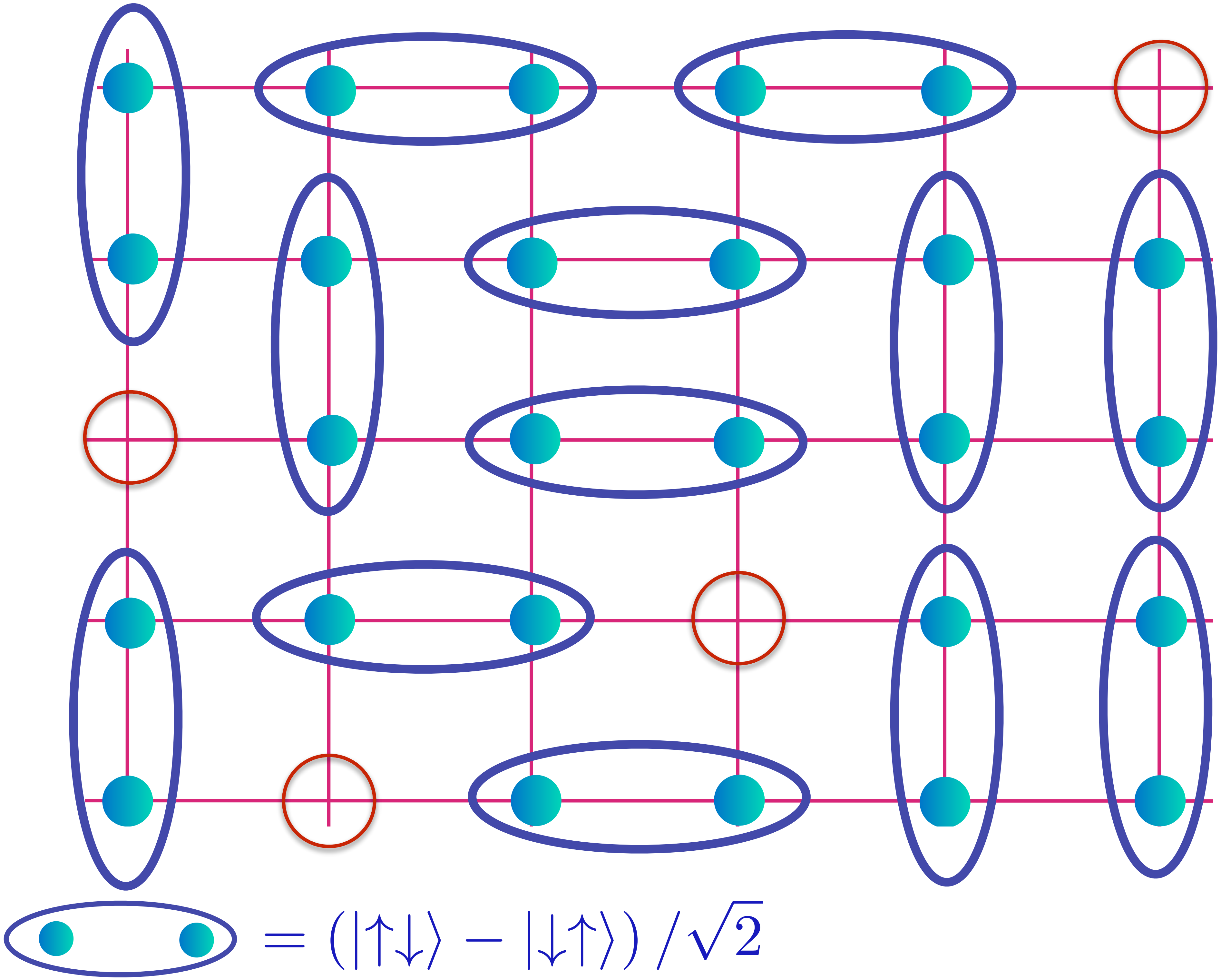

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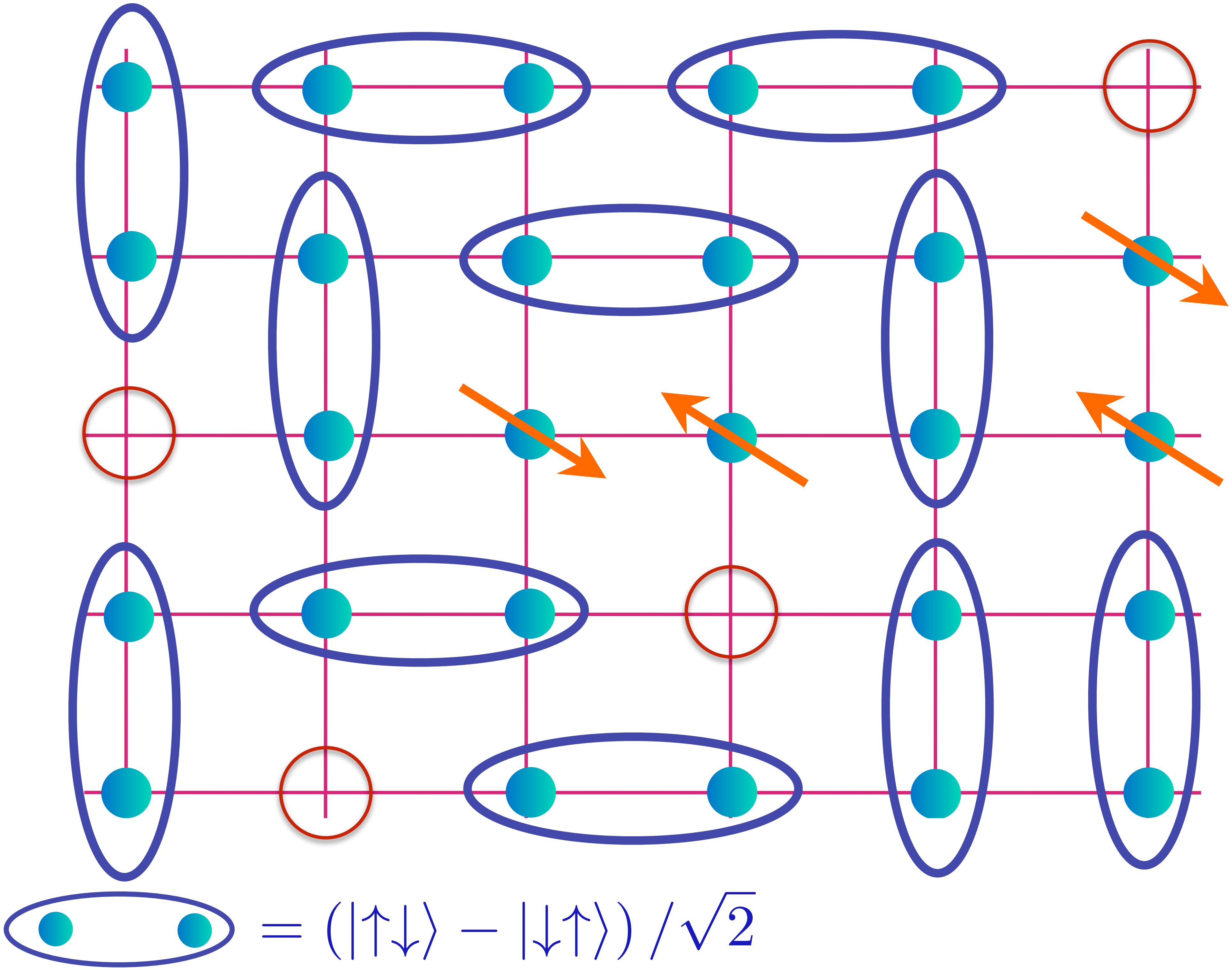
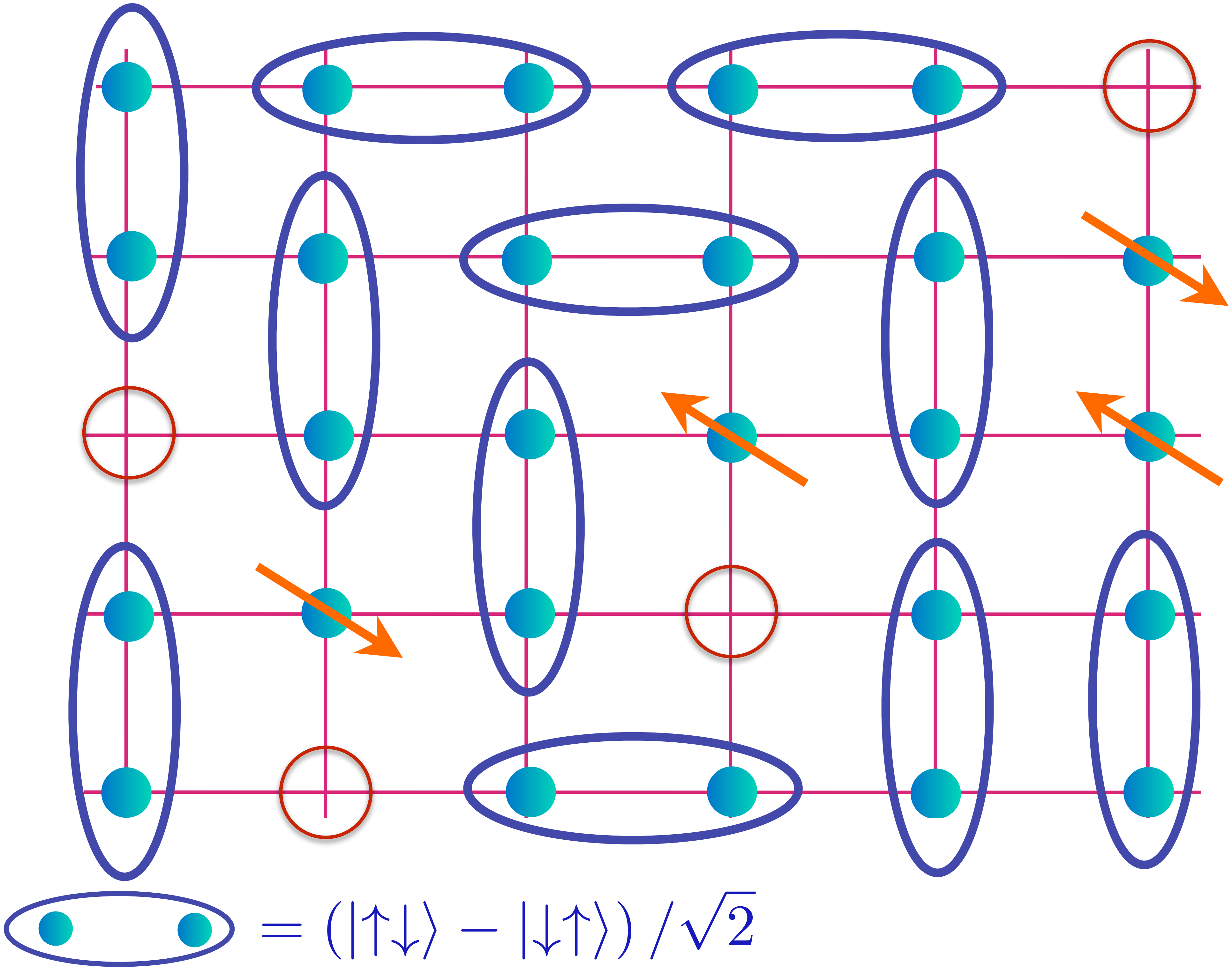

$= \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$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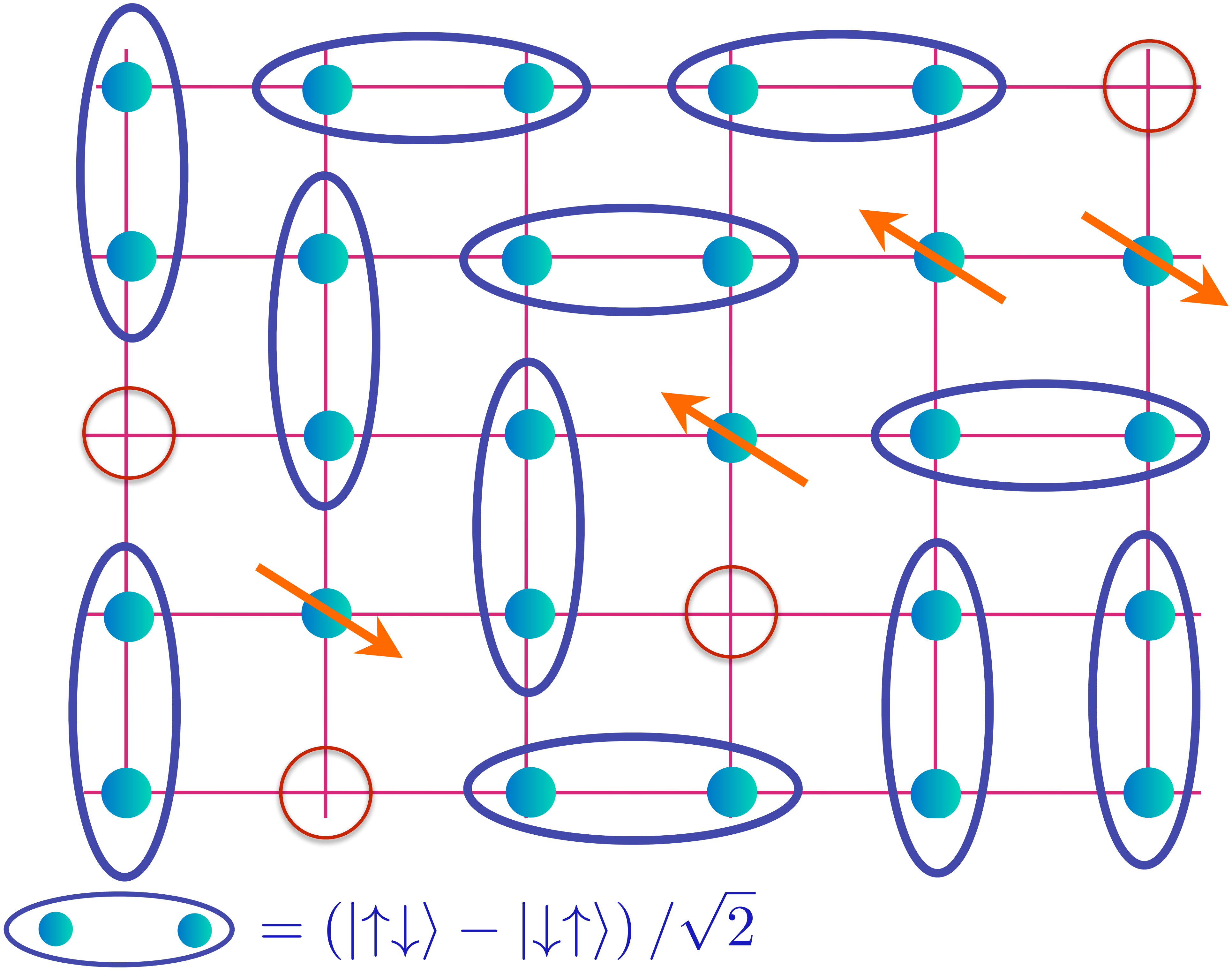

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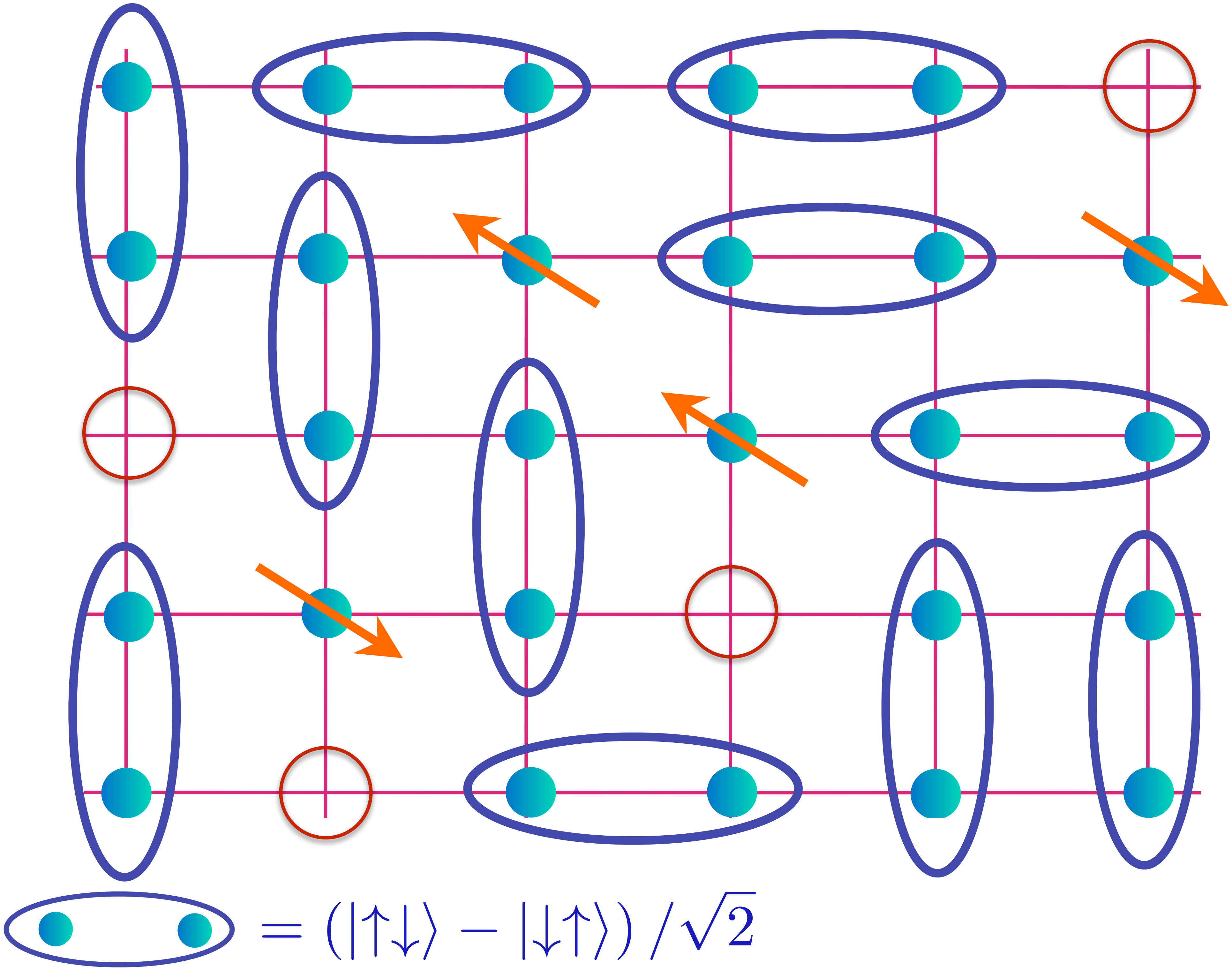

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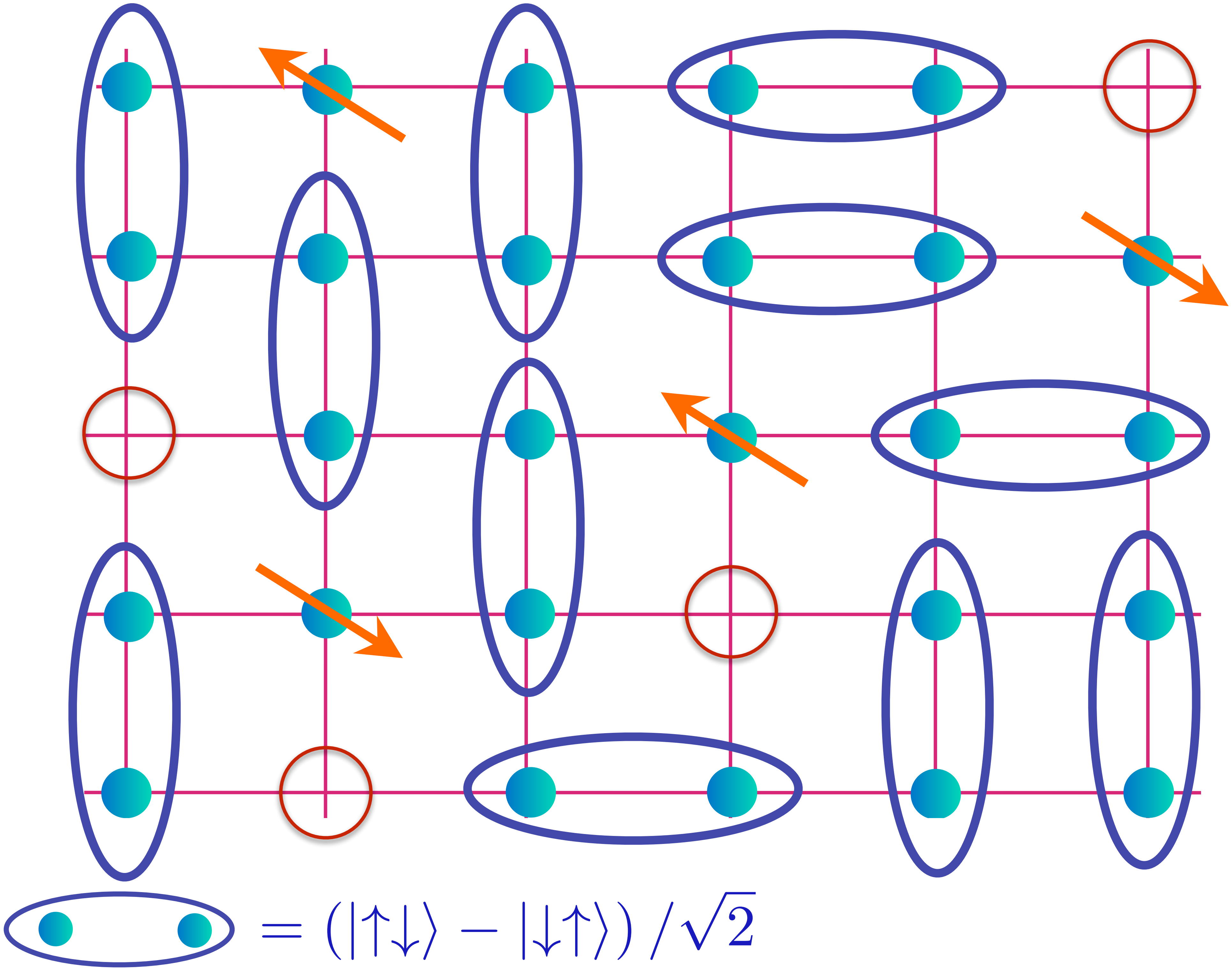
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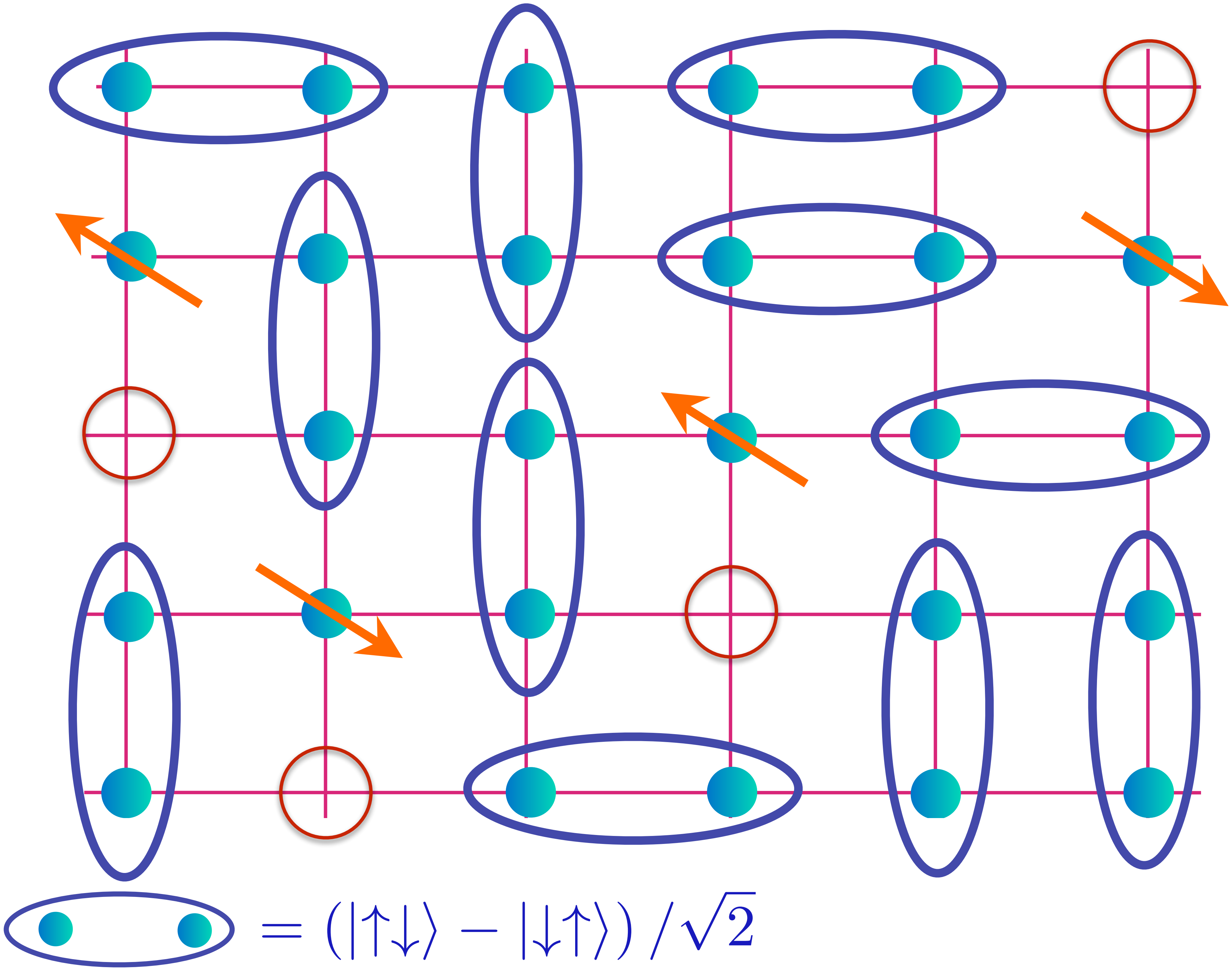
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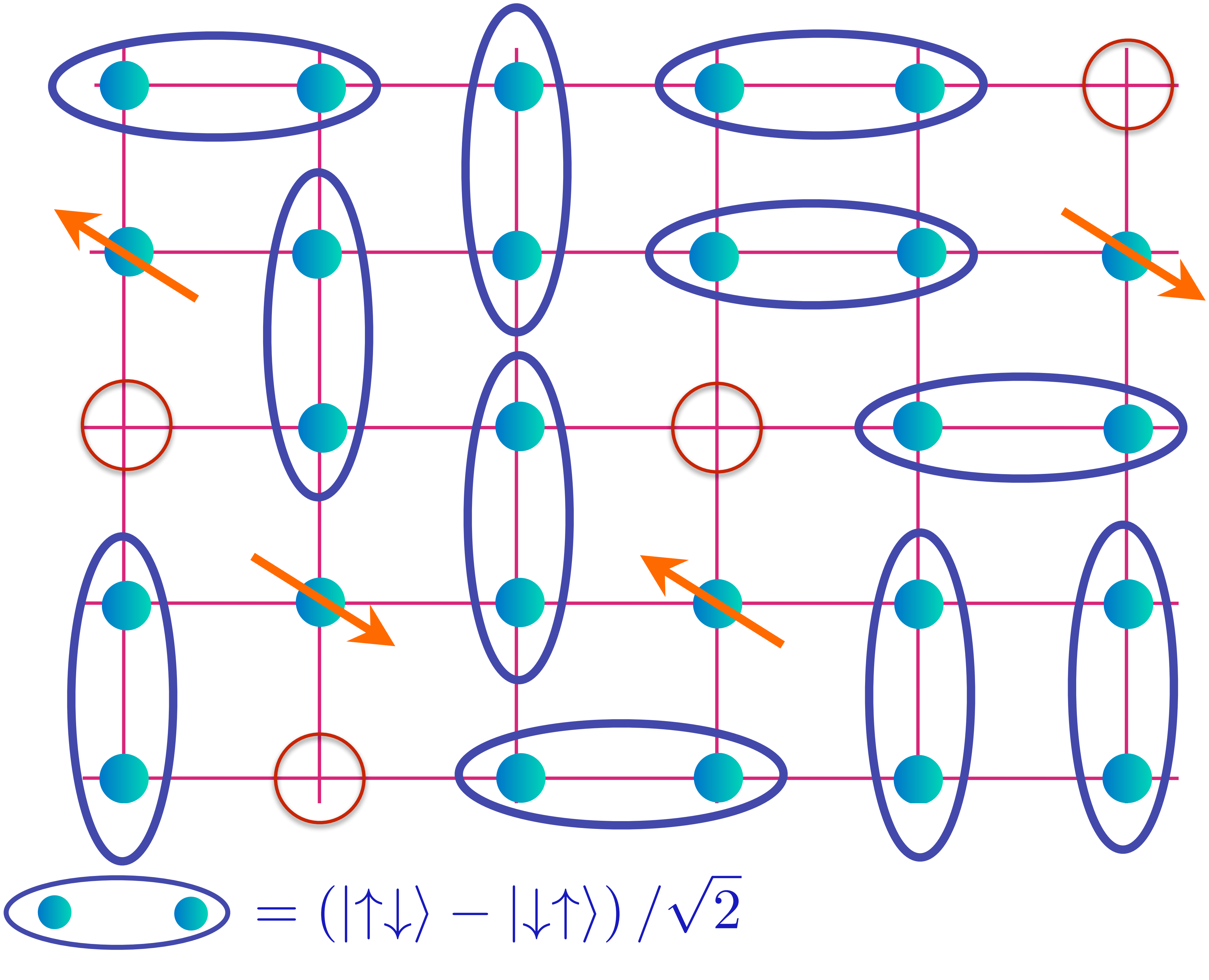
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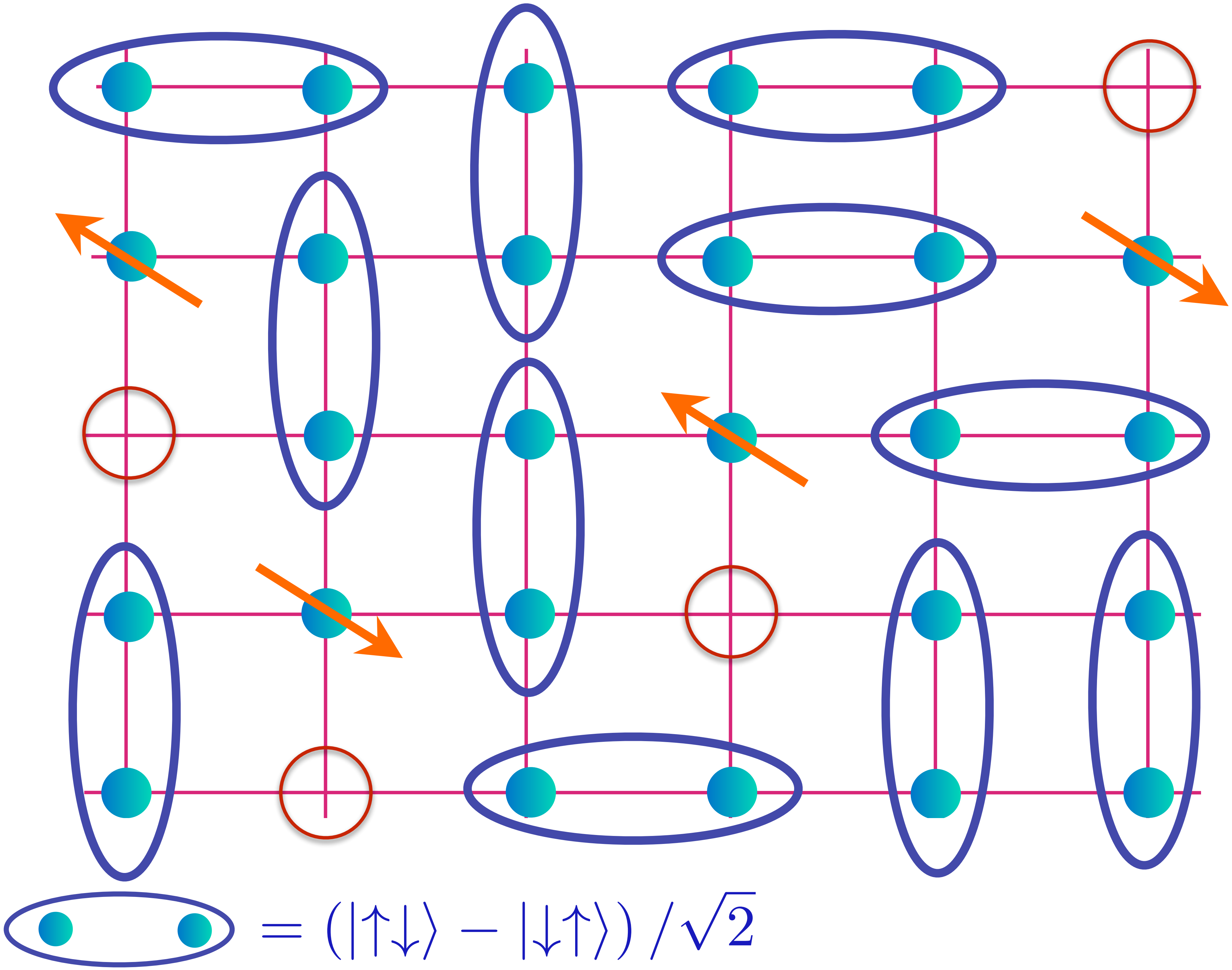
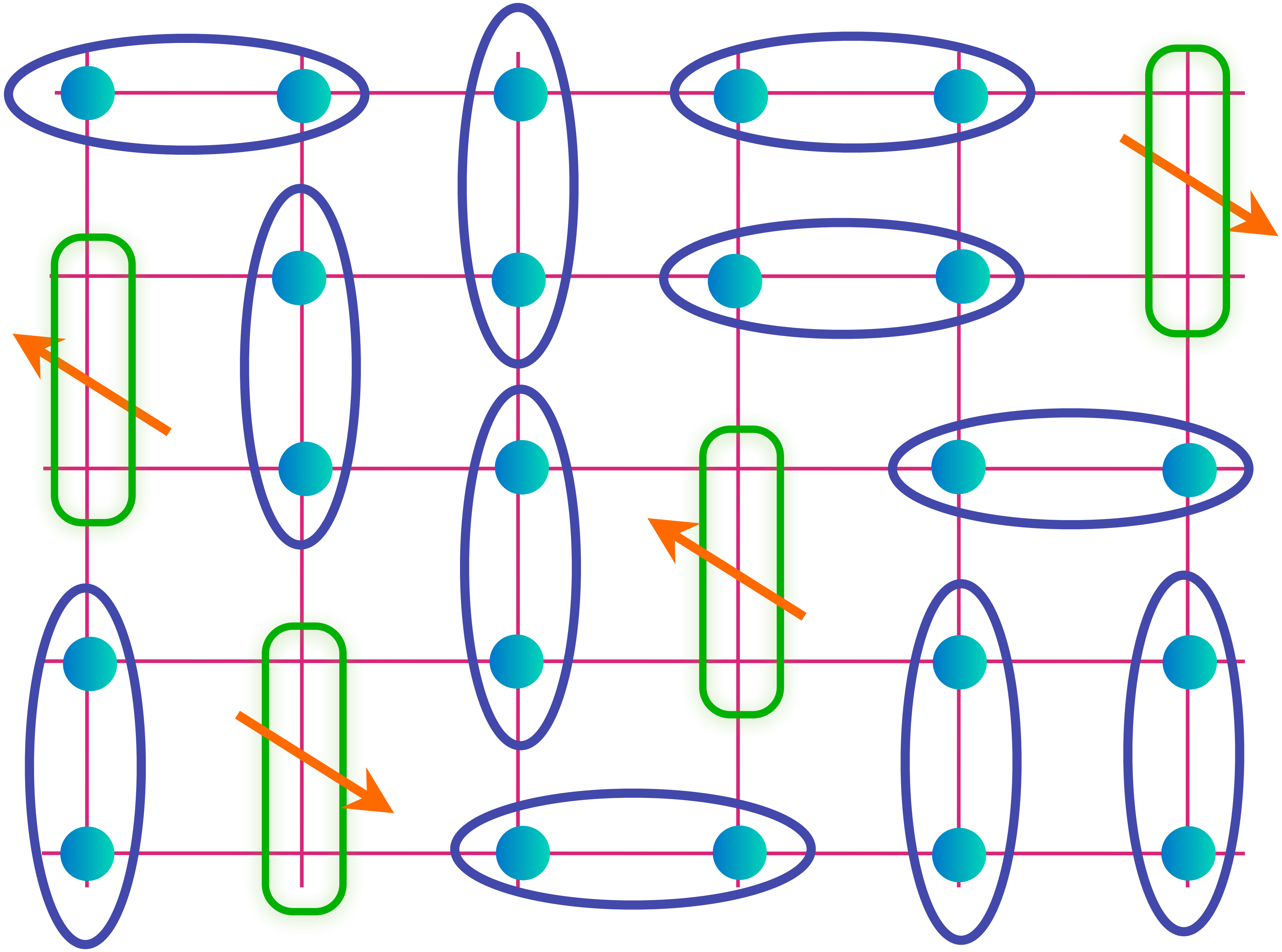

$= \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

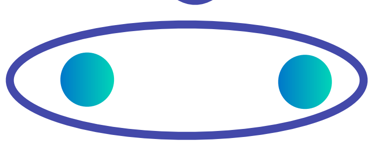
# Fractionalized Fermi liquid (FL*)

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

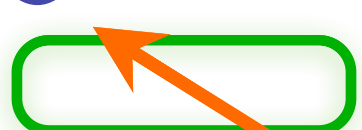
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size *p* visible in electron photo-emission



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

# Fractionalized Fermi liquid (FL*)

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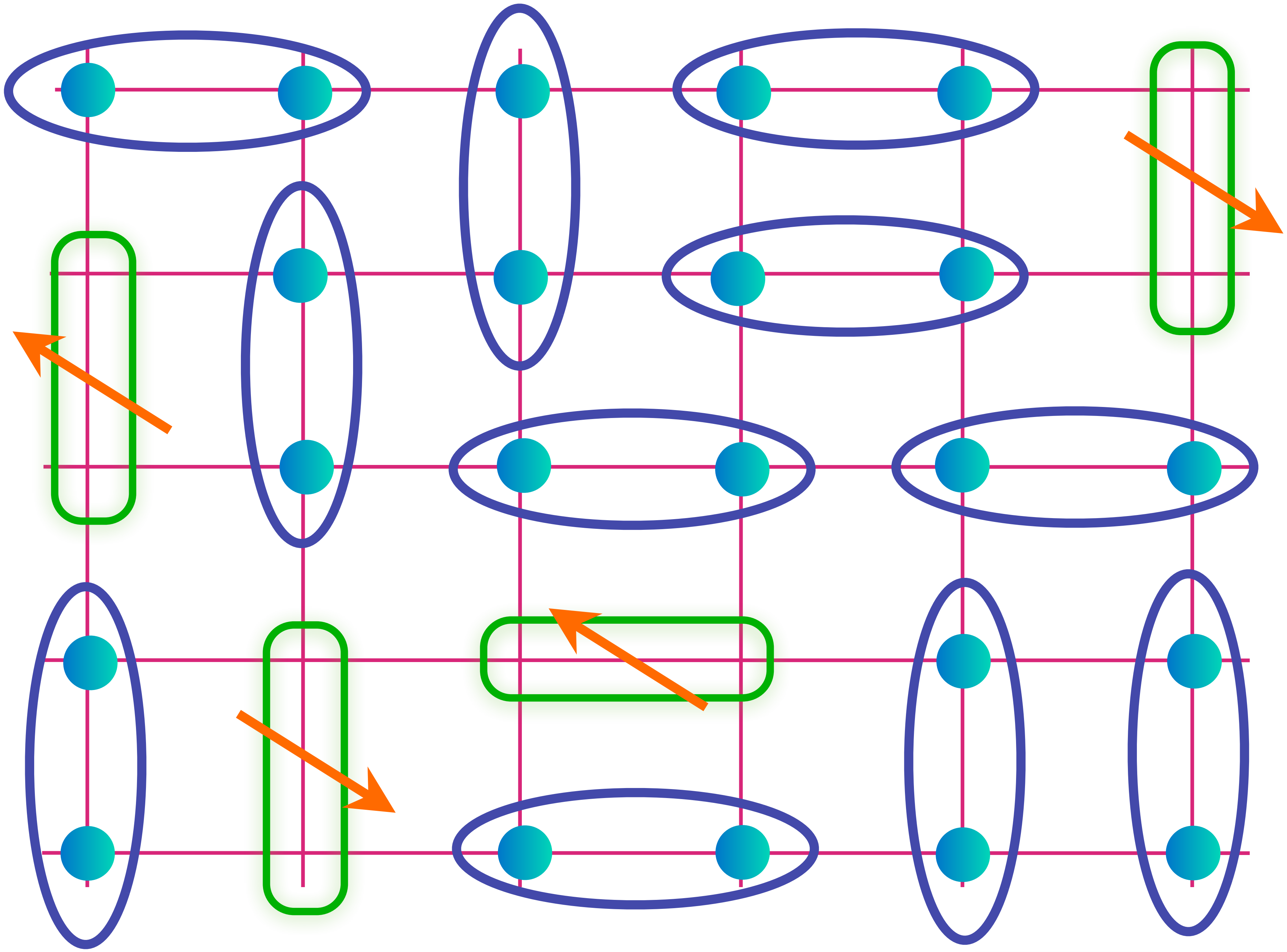

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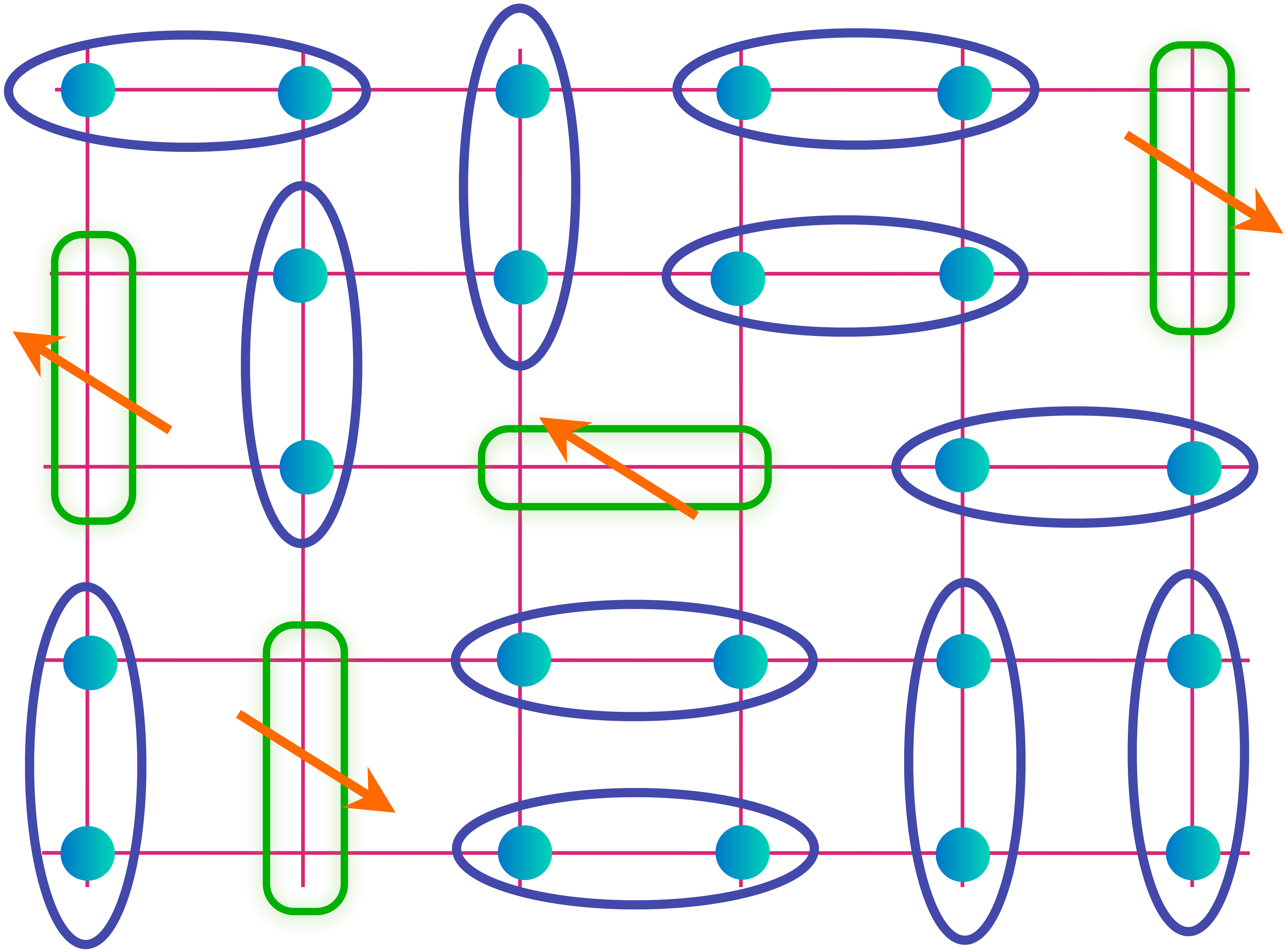

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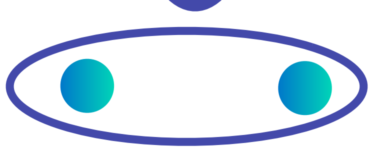
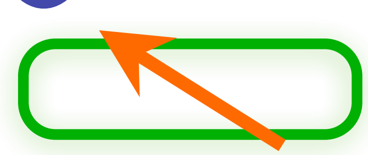
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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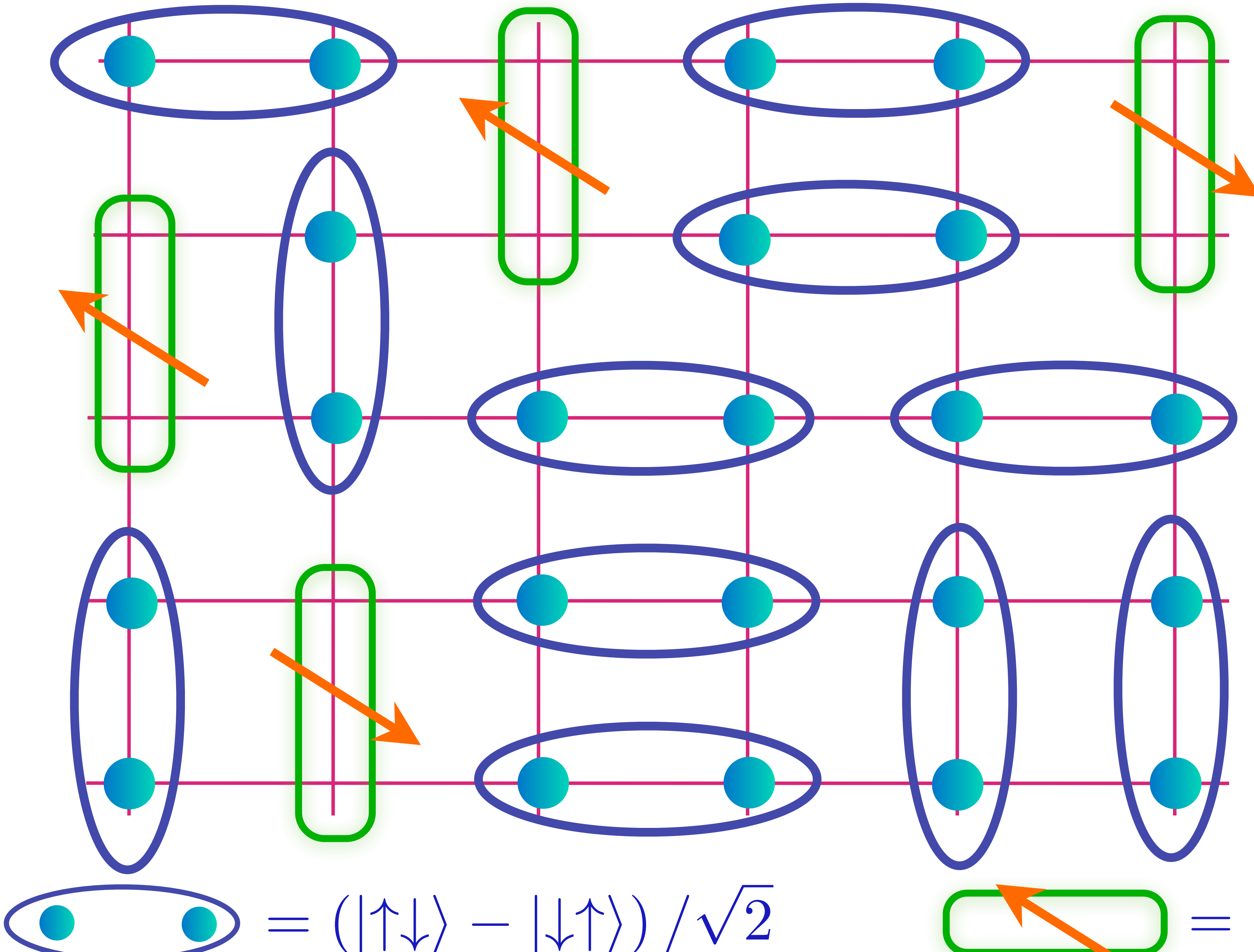

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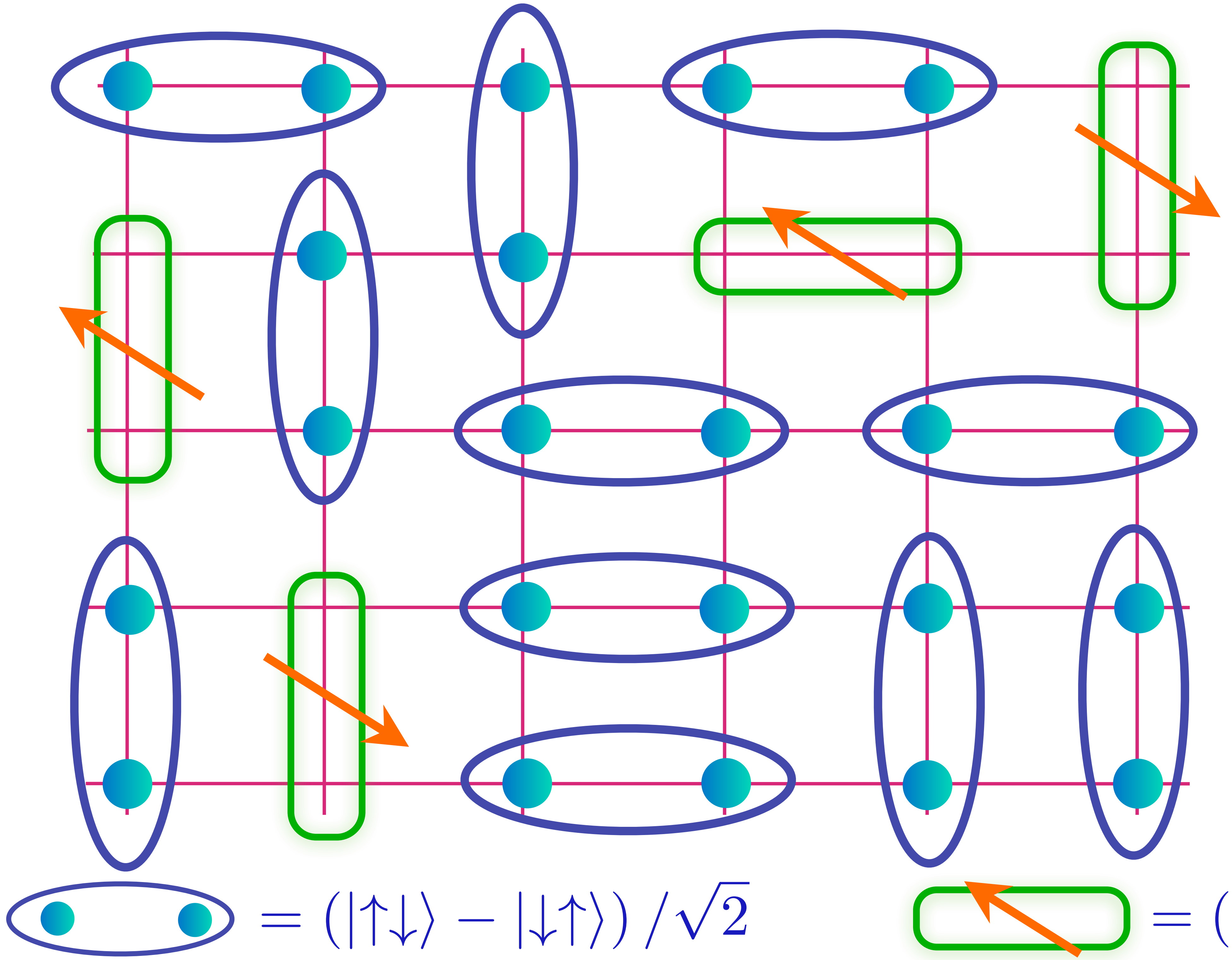

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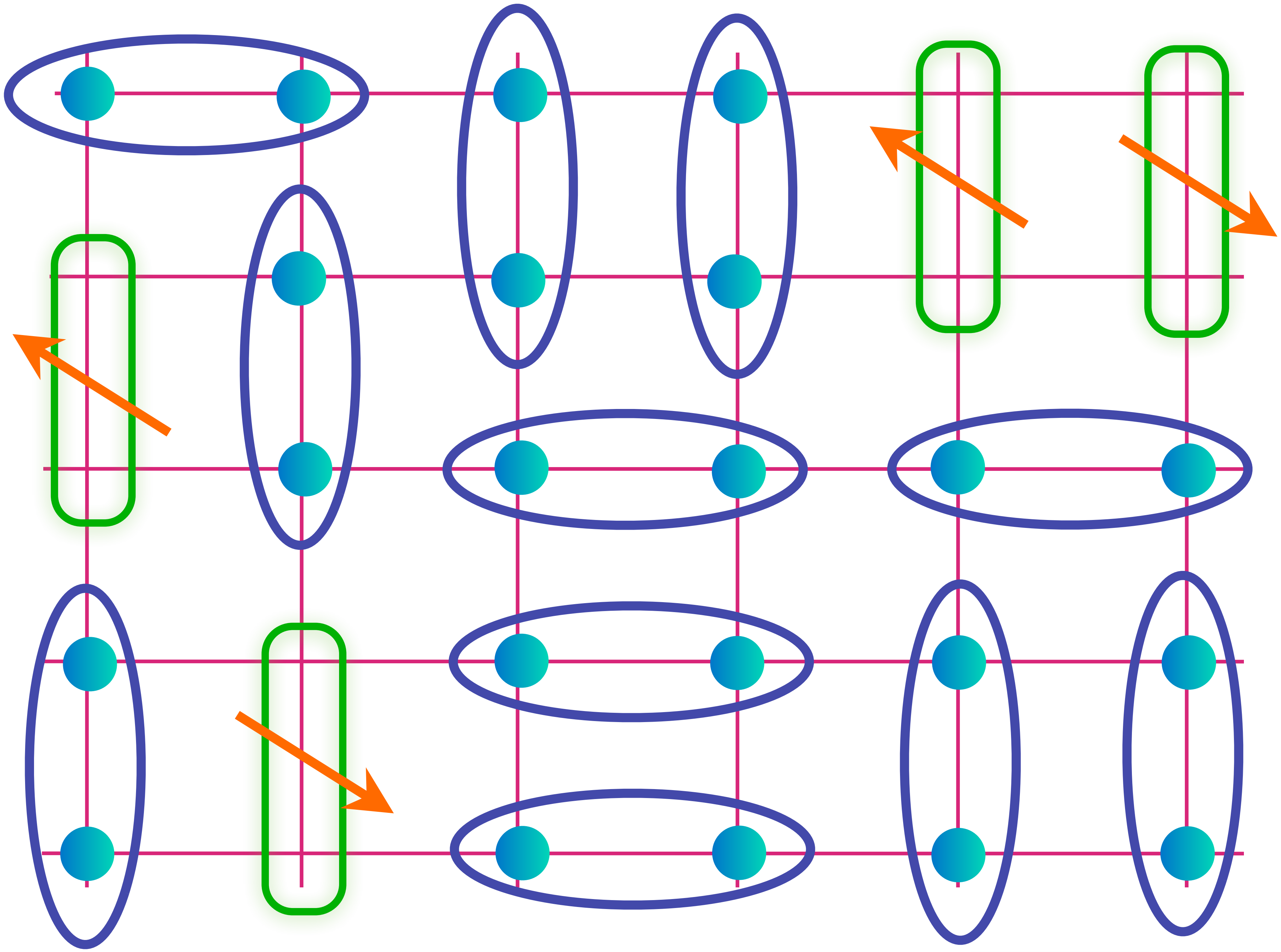

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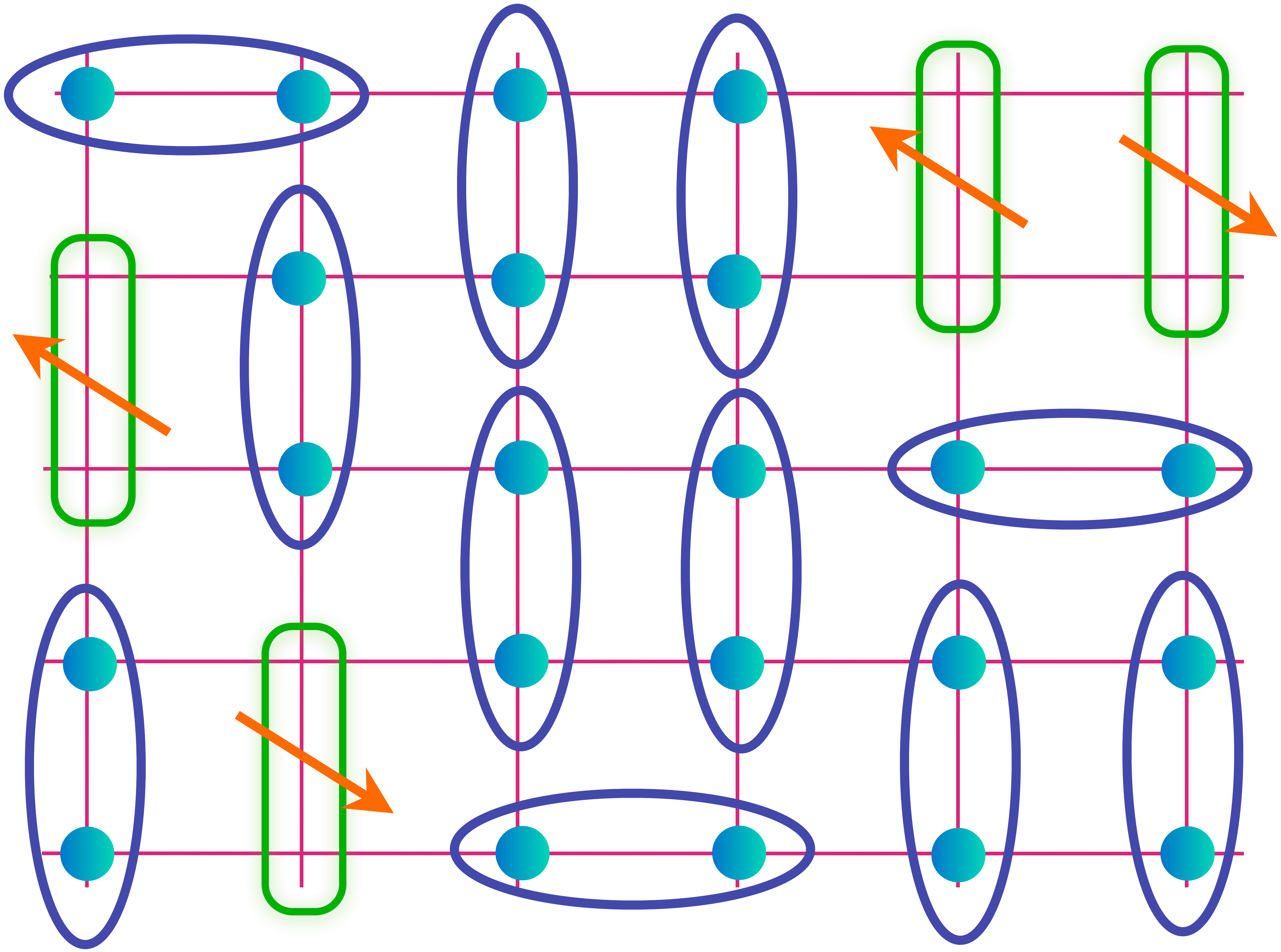

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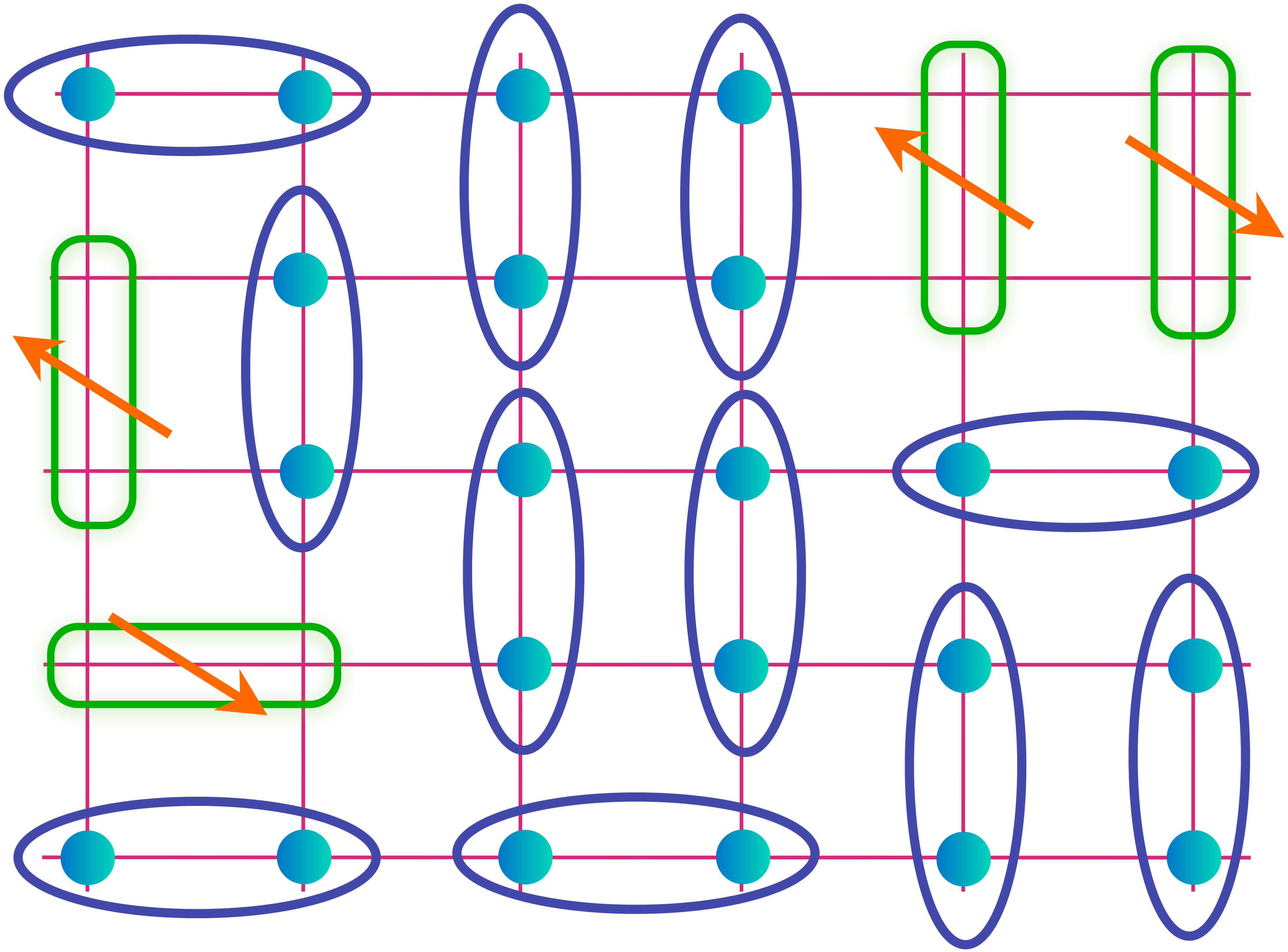

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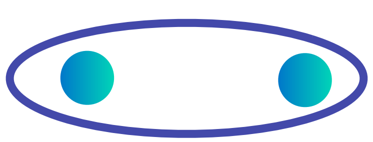
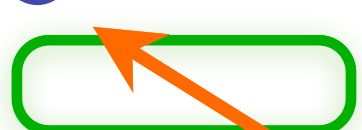


$= \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$



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# Ground state degeneracy

Place FL*
on a torus:

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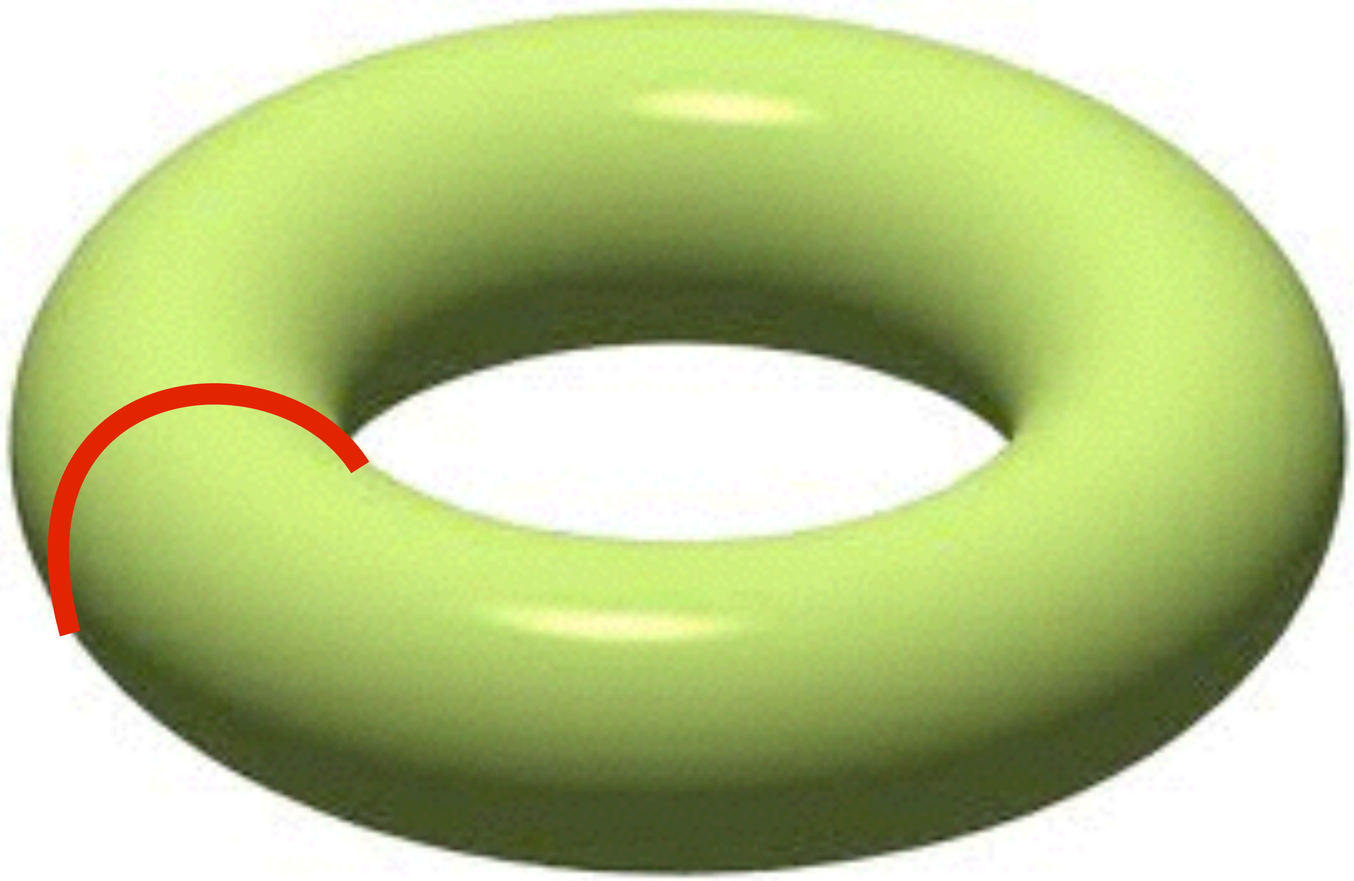

**Place FL* on a torus:**

obtain "topological" states nearly degenerate with quasiparticle states: number of dimers crossing red line is conserved modulo 2

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

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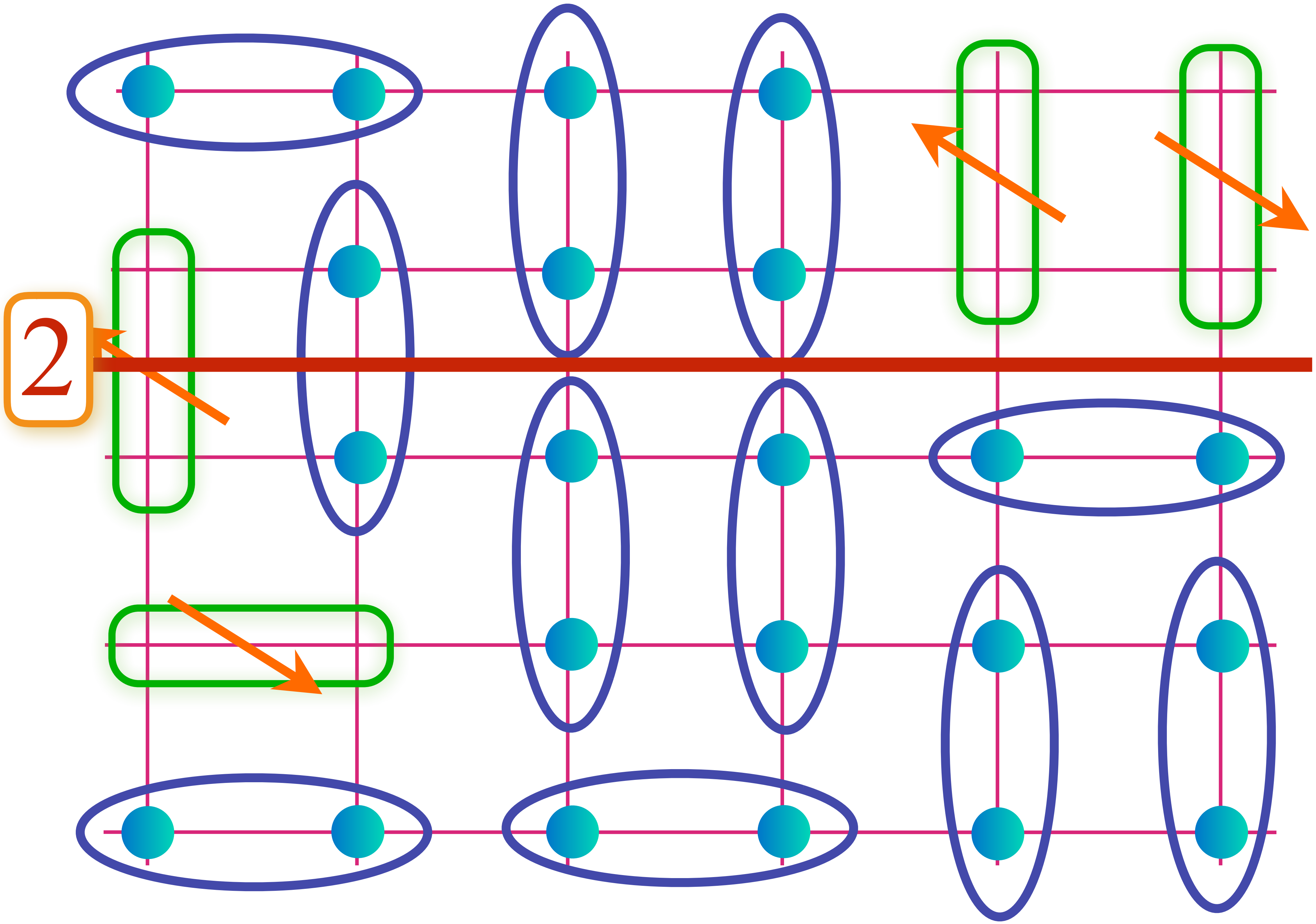


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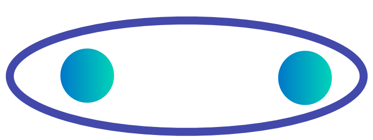
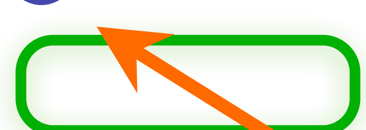
 $= \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left( |\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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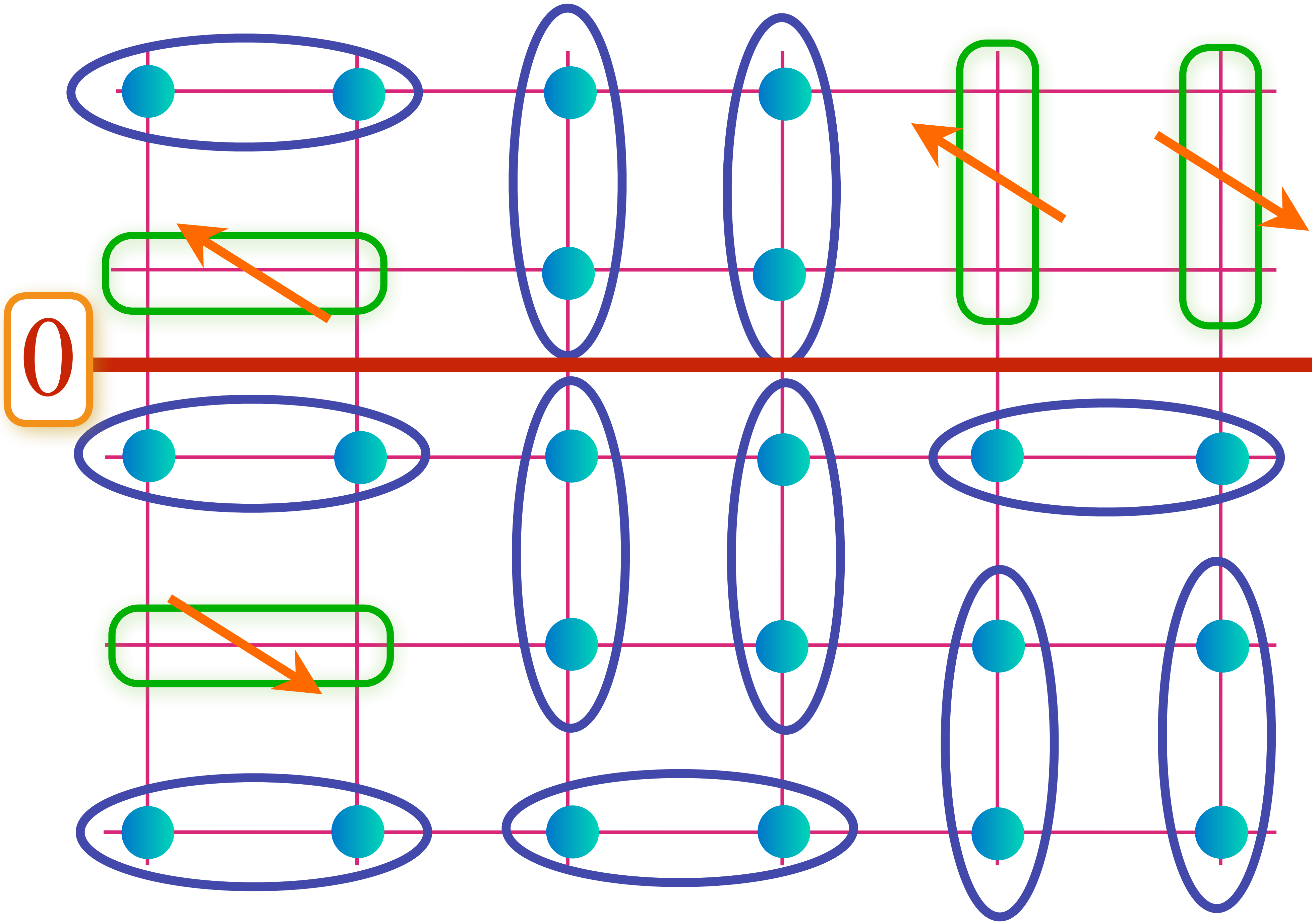


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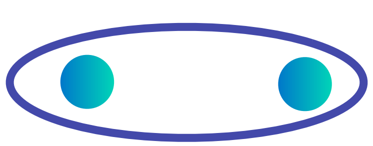
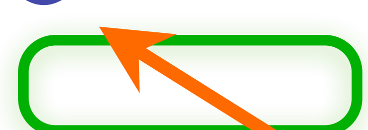
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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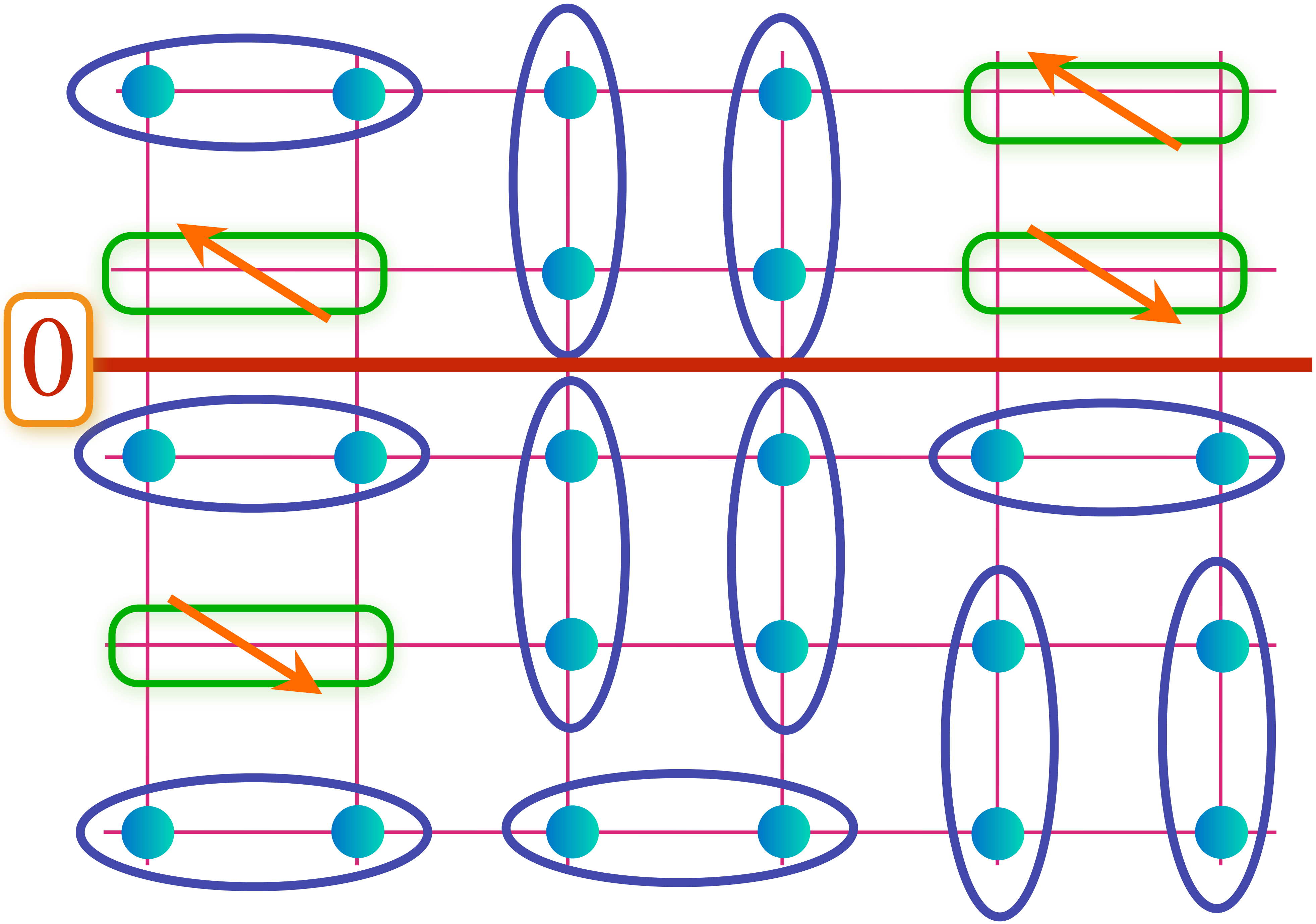


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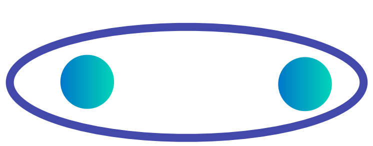

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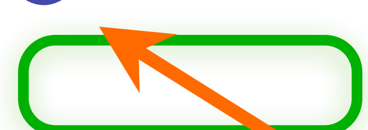

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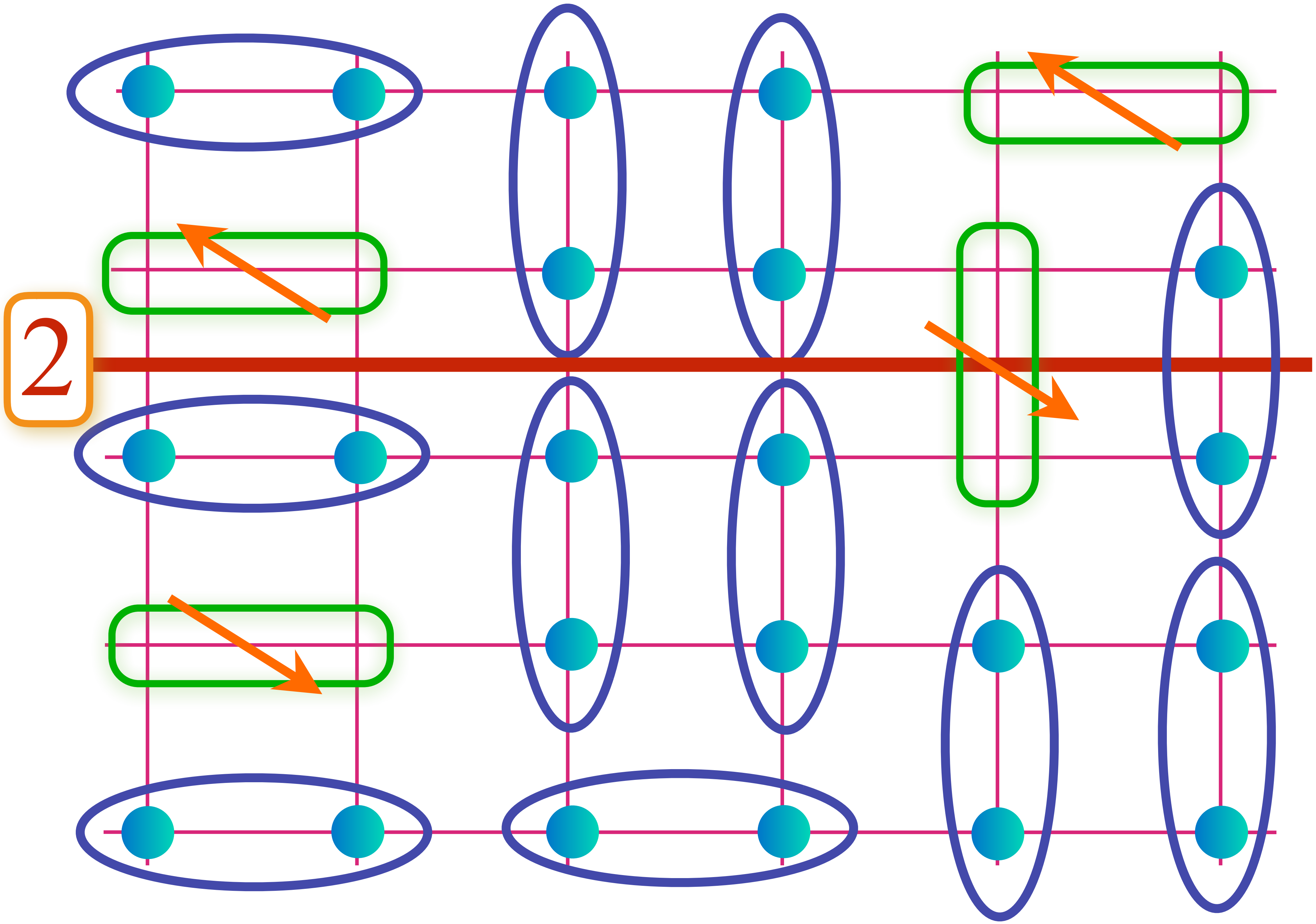


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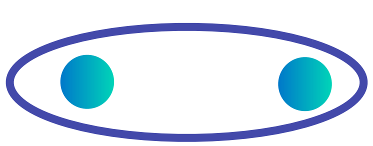

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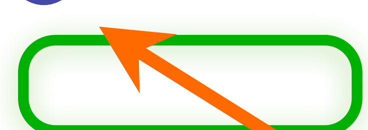

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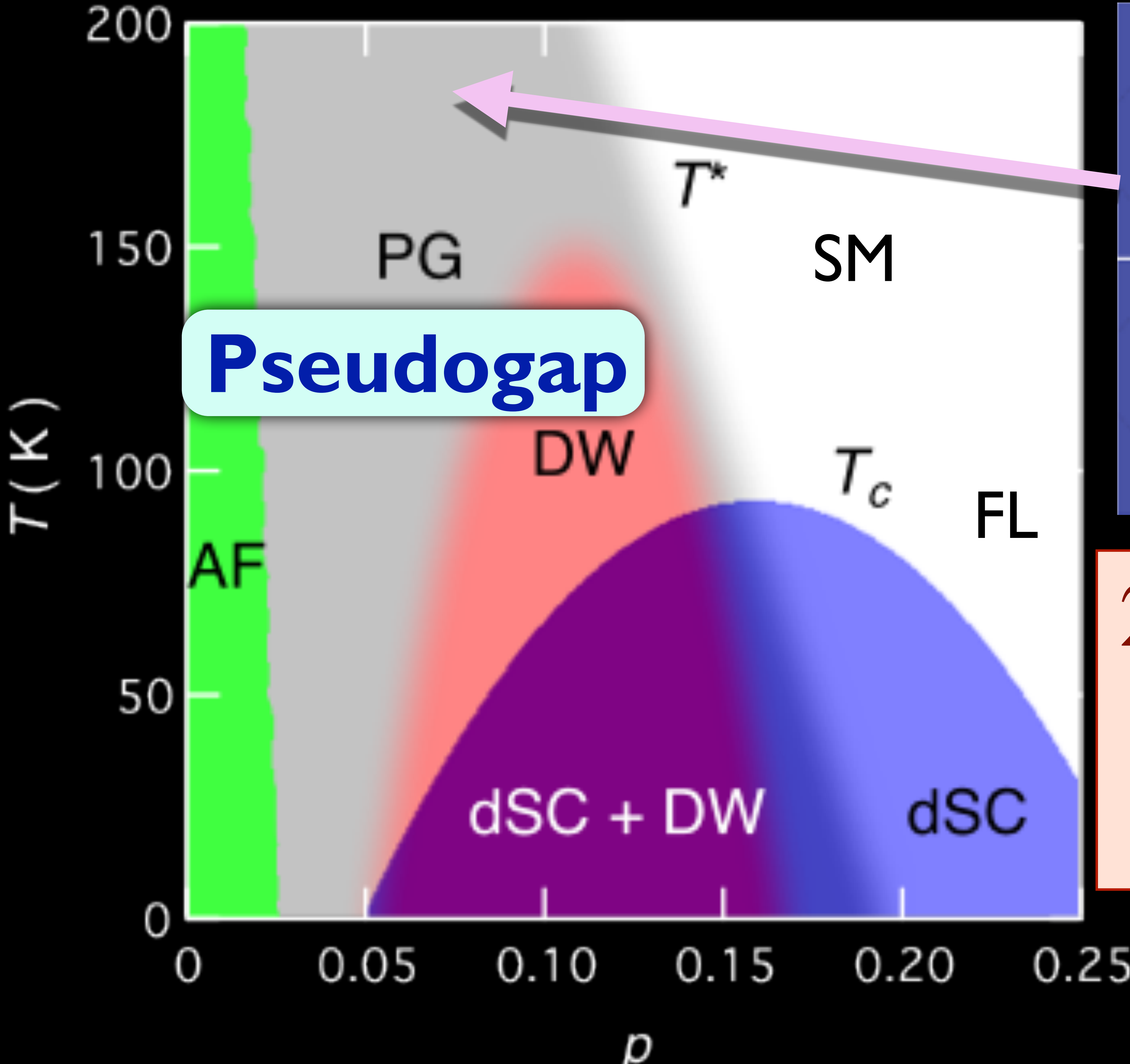
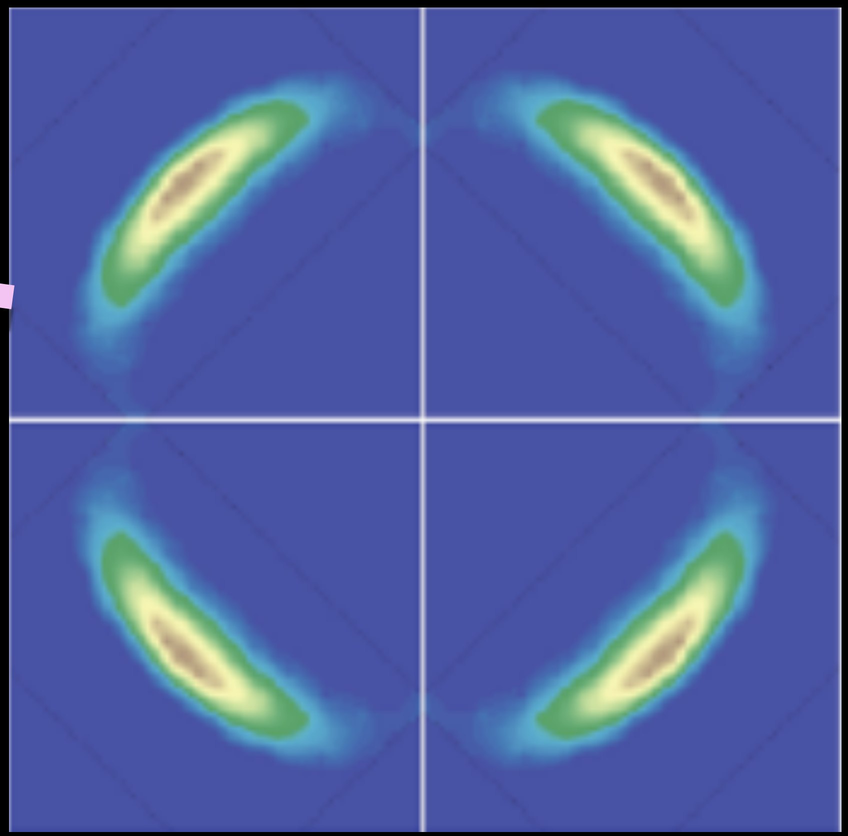
M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000)
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

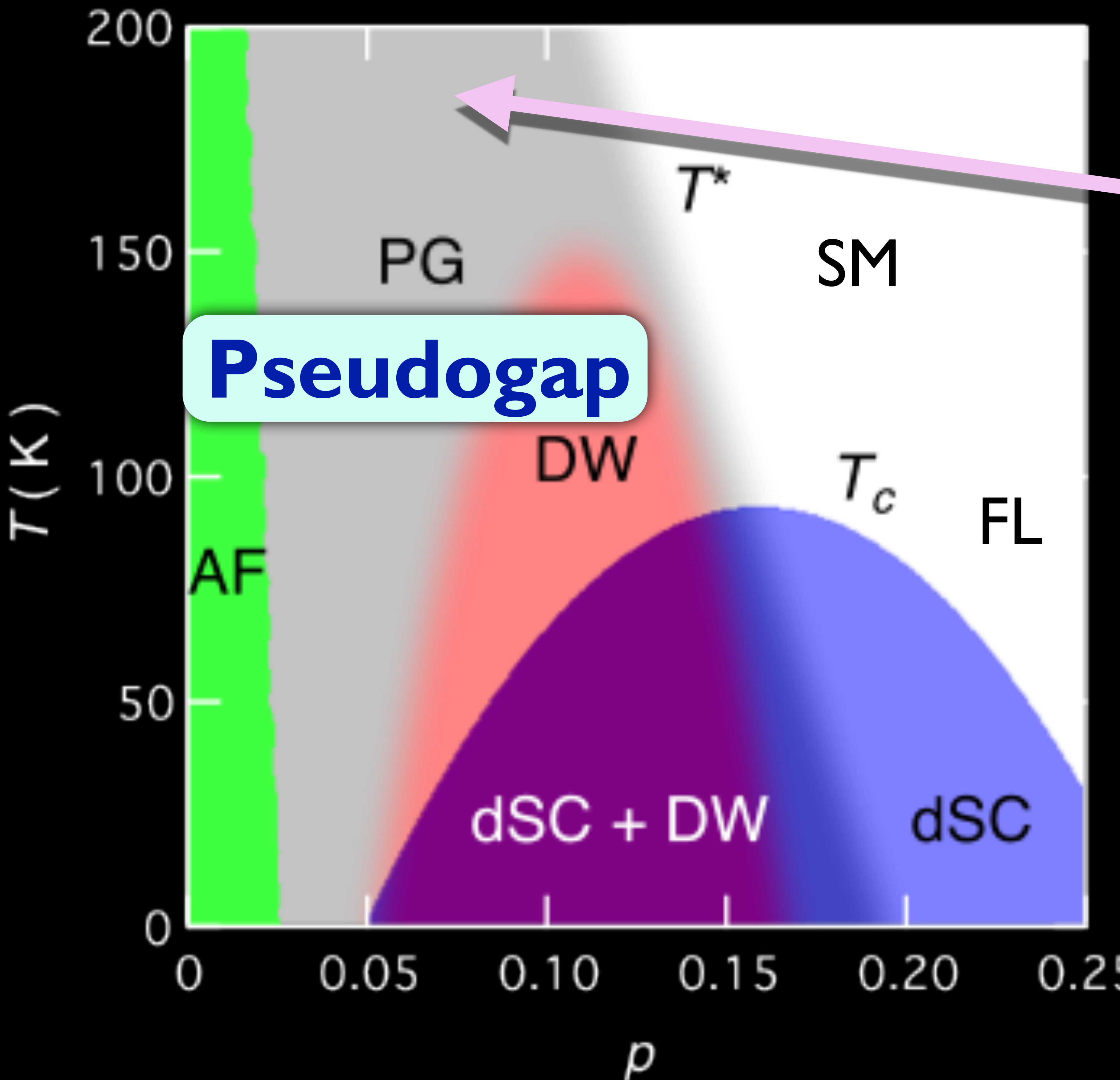
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

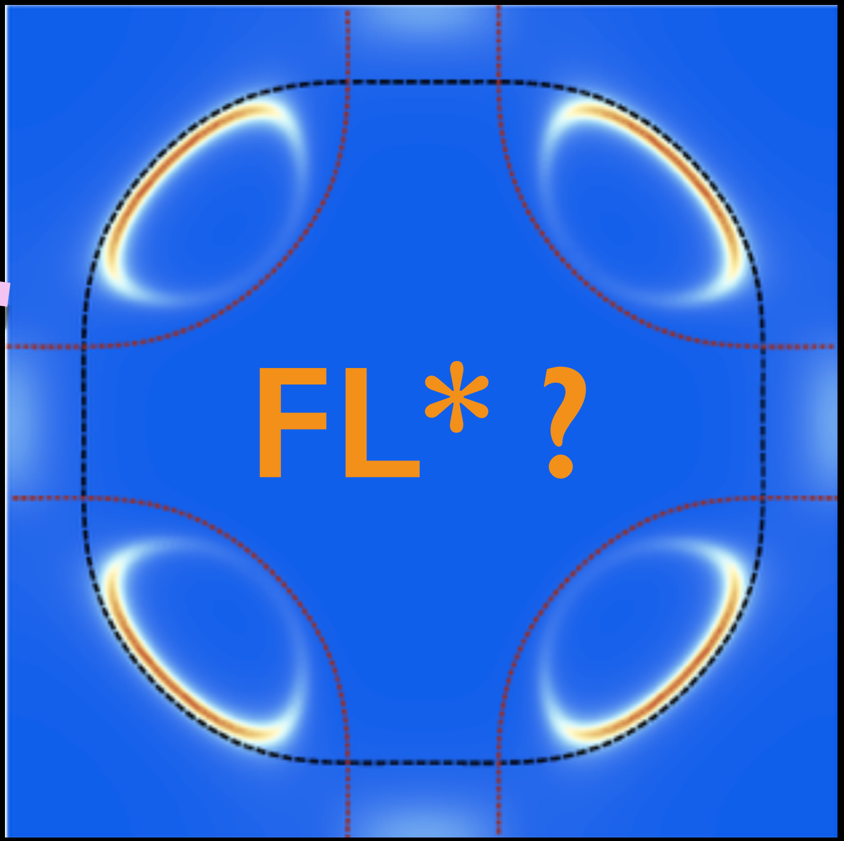


2. Pseudogap metal at low $p$

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)




A new metal — a fractionalized Fermi liquid (FL*) — with electron-like quasiparticles on a Fermi surface of size $p$

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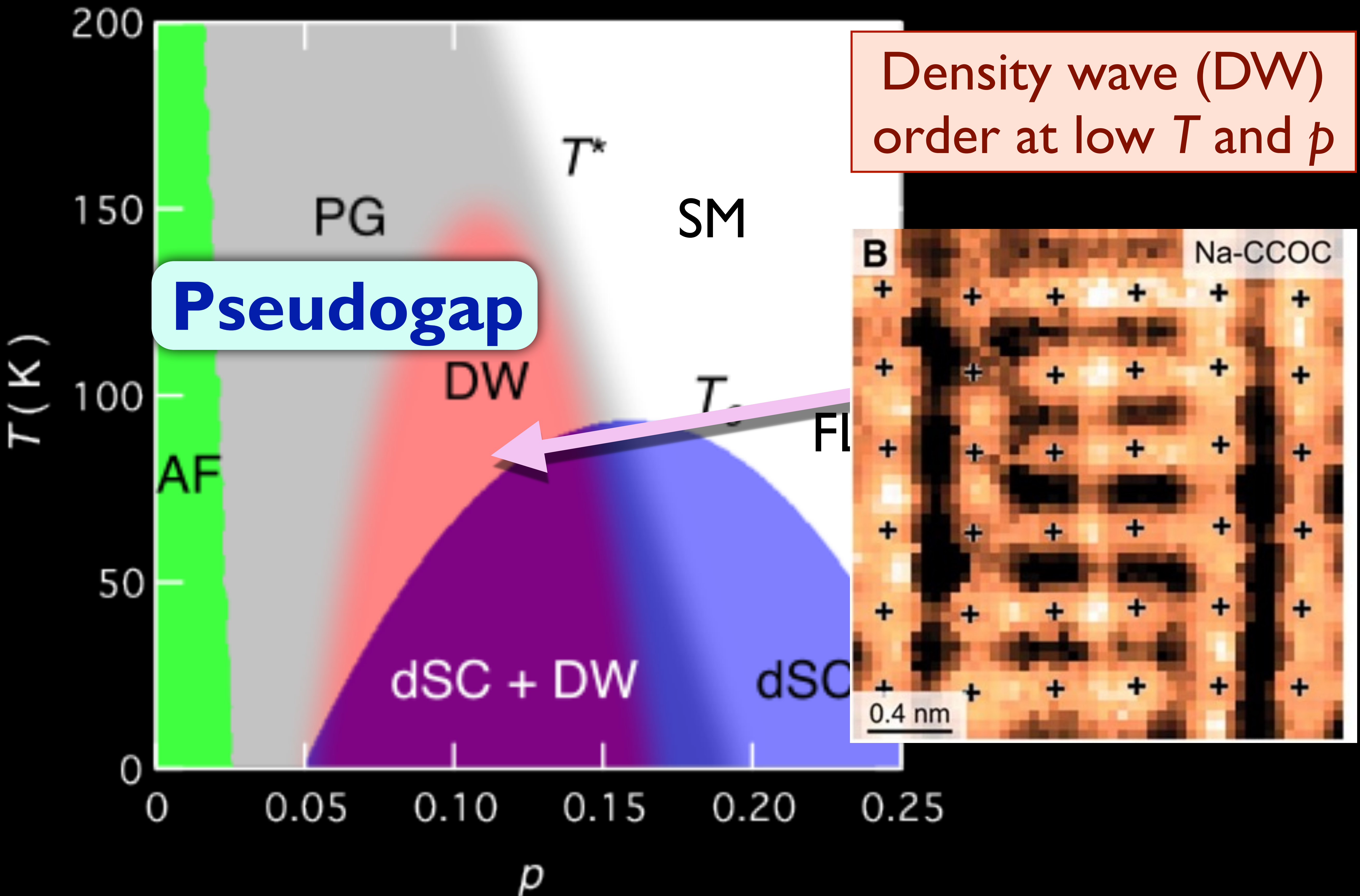
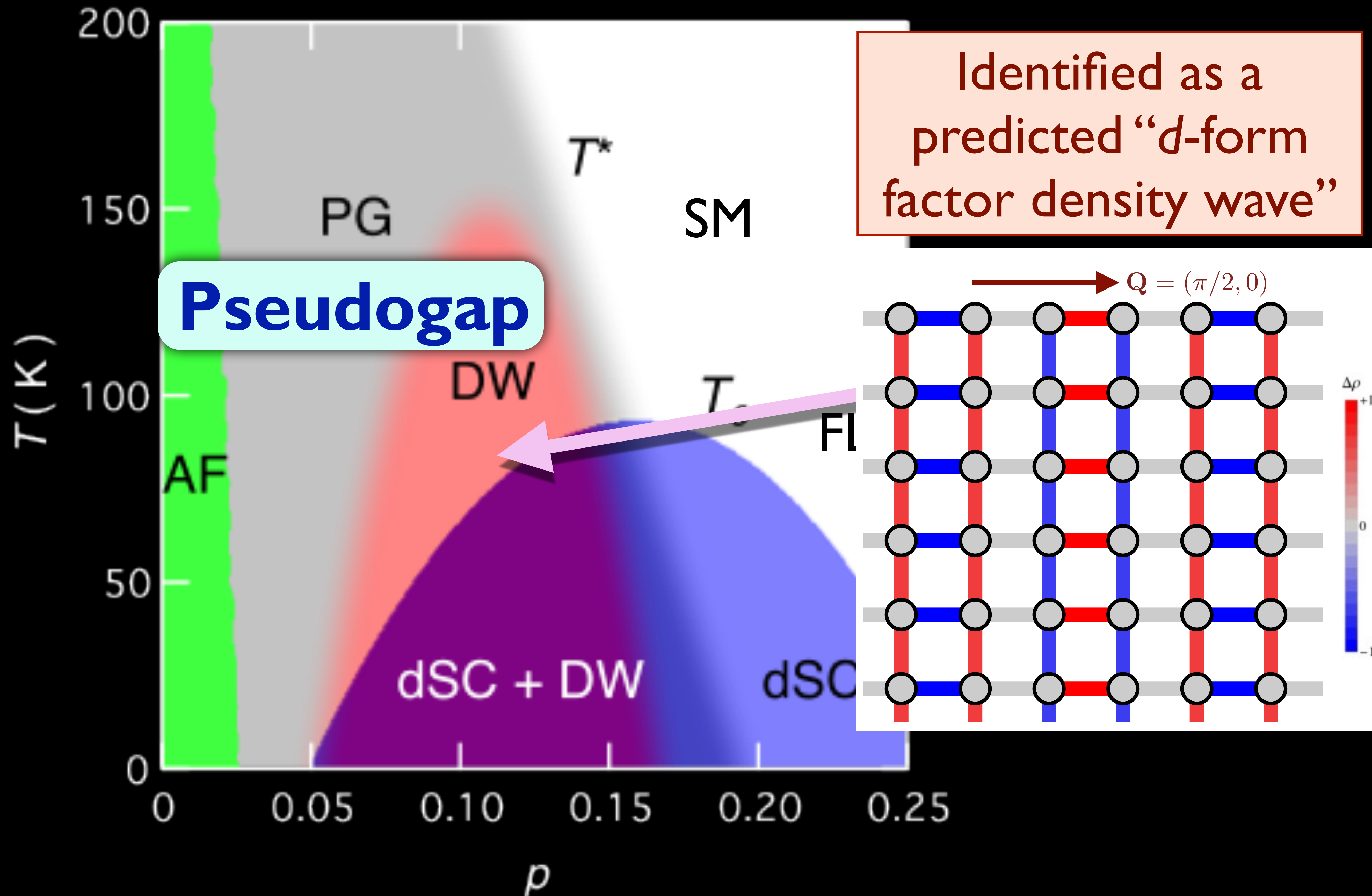
Y. Kohsaka *et al.*, Science **315**, 1380 (2007)



M. A. Metlitski and S. Sachdev, PRB **82**, 075128 (2010). S. Sachdev R. La Placa, PRL **111**, 027202 (2013).
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi,
H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)

Identified as a predicted "*d*-form factor density wave"



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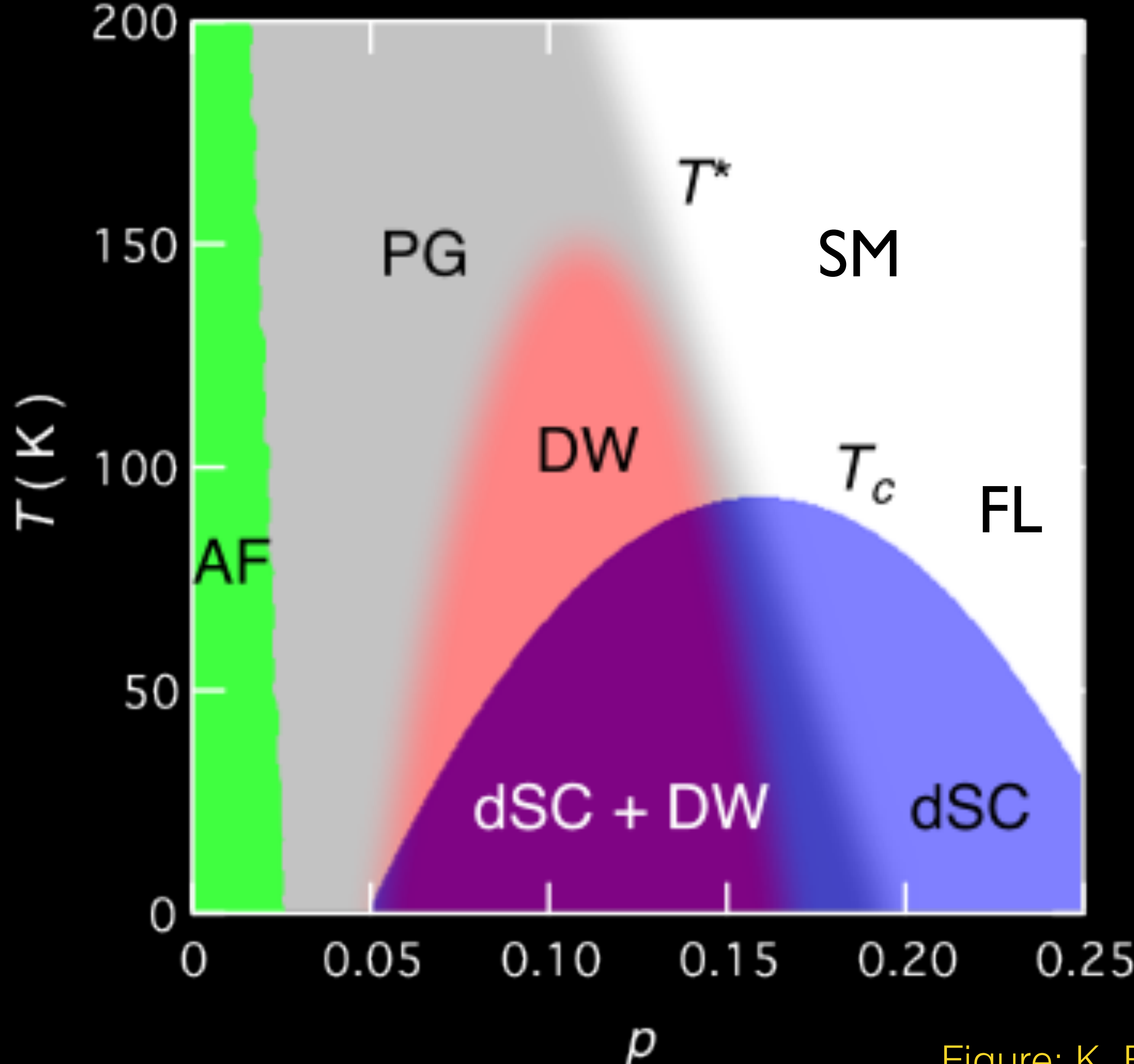
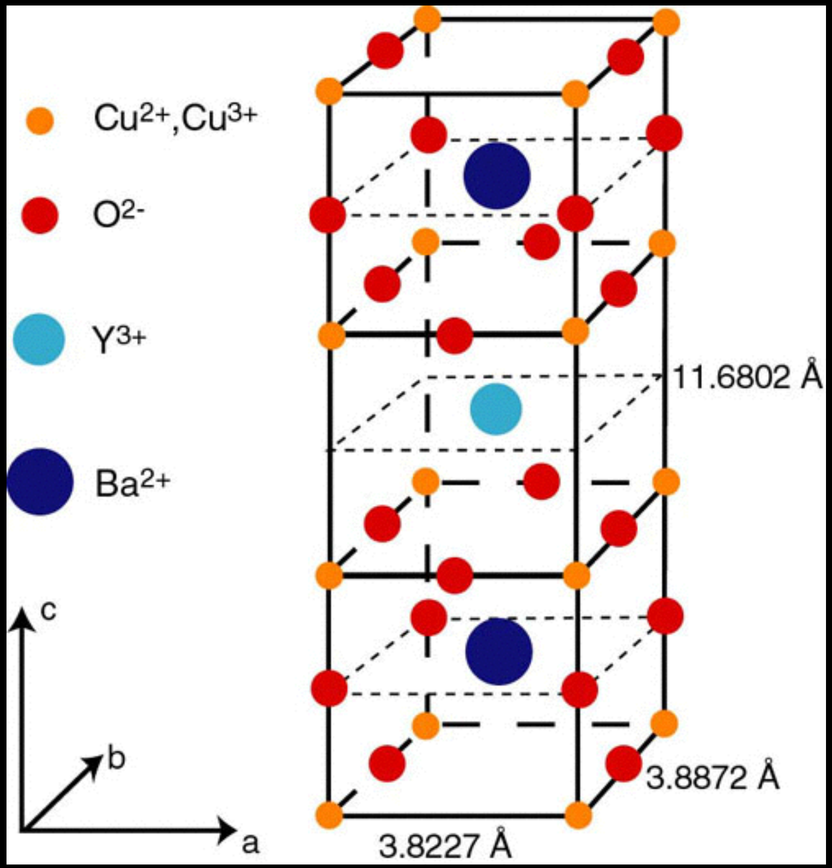
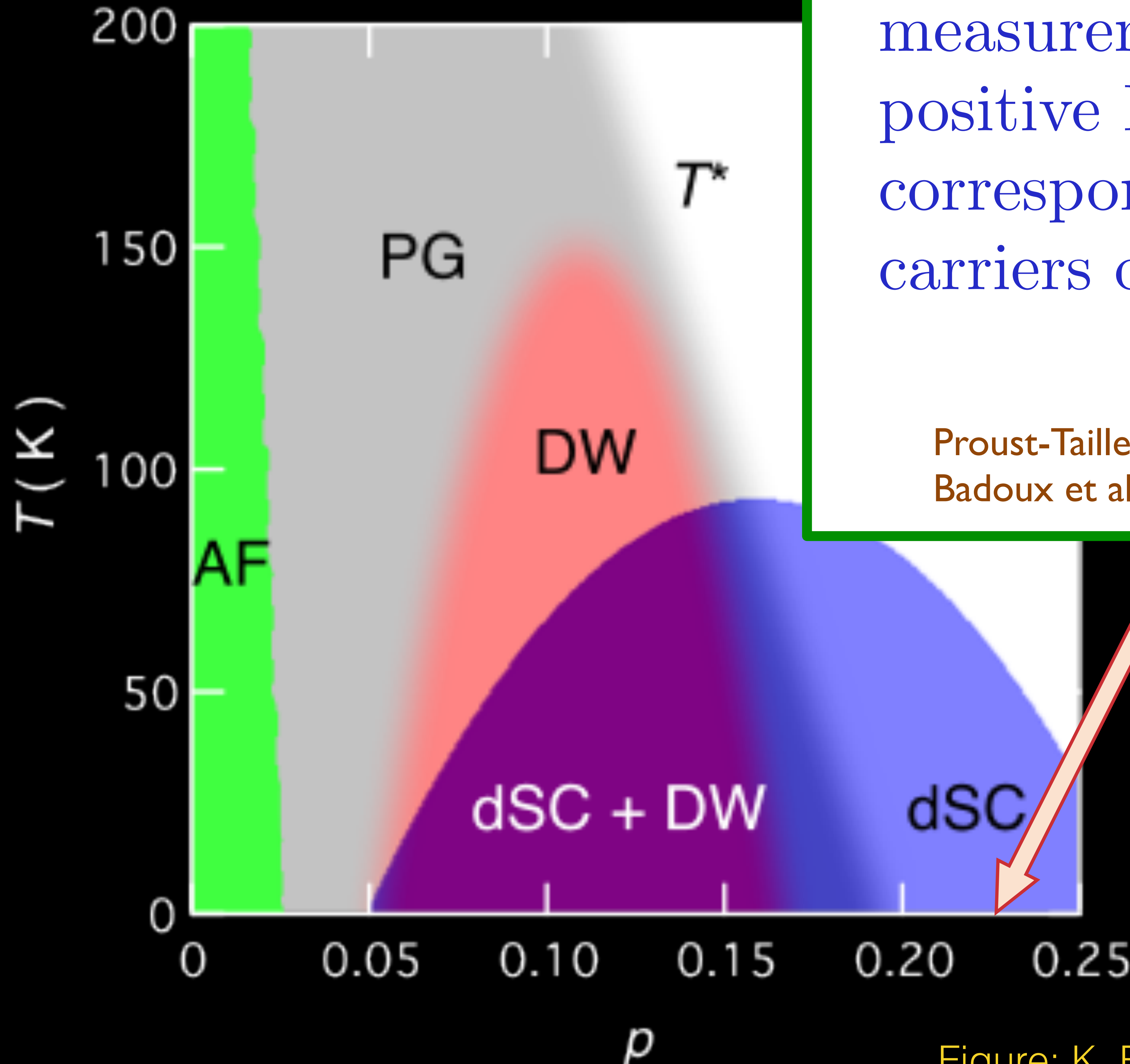
- $T$-independent positive Hall co-efficient, $R_H$, corresponding to carrier density $p$ in the higher temperature pseudogap (Ando *et al.*, PRL **92**, 197001 (2004)) *and* in recent measurements at high fields, low $T$, and around $p \approx 0.16$ in YBCO (Proust-Taillefer-UBC collaboration, Badoux *et al.*, arXiv:1511.08162).




$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis




High field, low $T$ measurements show a positive Hall co-efficient corresponding to carriers of density $1 + p$

Proust-Taillefer-UBC collaboration,
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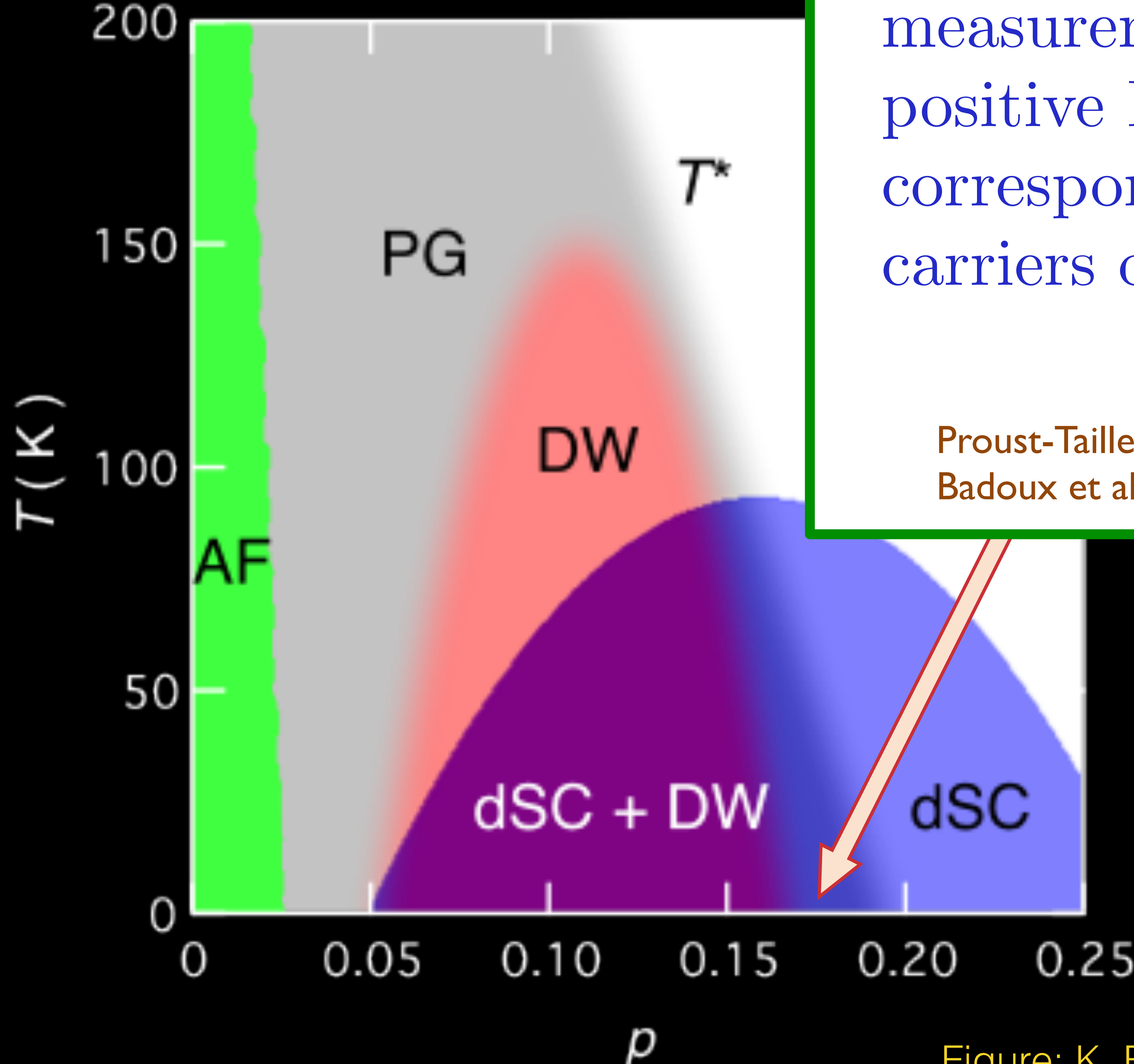


Figure: K. Fujita and J. C. Seamus Davis

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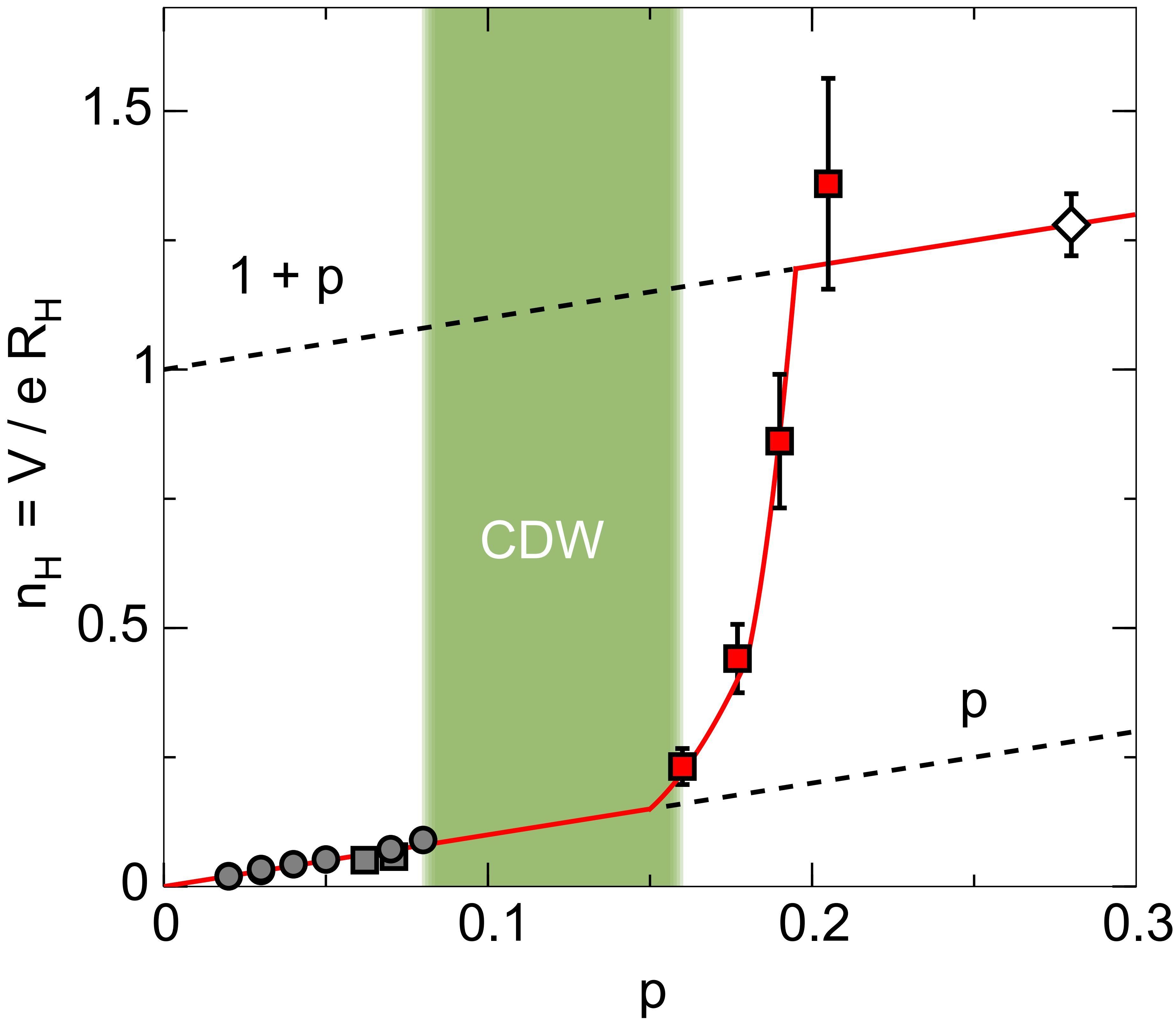


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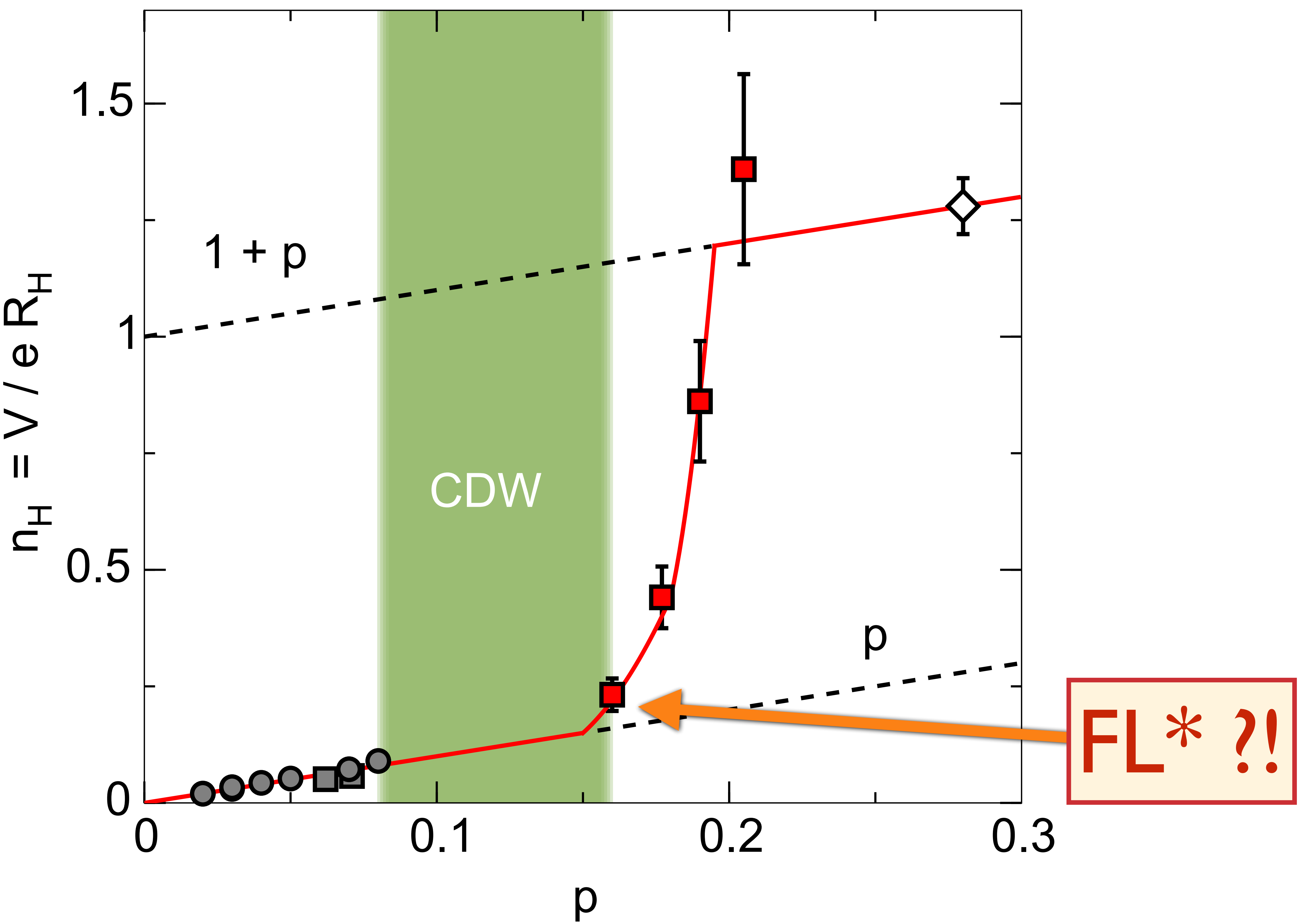


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