

Quantum spin liquids and the phases of the cuprates

International Centre for Theoretical Physics, Trieste

May 29, 2023

Subir Sachdev

Maine Christos, Zhu-Xi Luo, Henry Shackleton, Ya-Hui Zhang,
Mathias Scheurer, and S. S., PNAS **120**, e2302701120 (2023)
Alexander Nikolaenko, Jonas v. Milczewski, Darshan G. Joshi,
and S.S., arXiv:2211.10452

Talk online: sachdev.physics.harvard.edu



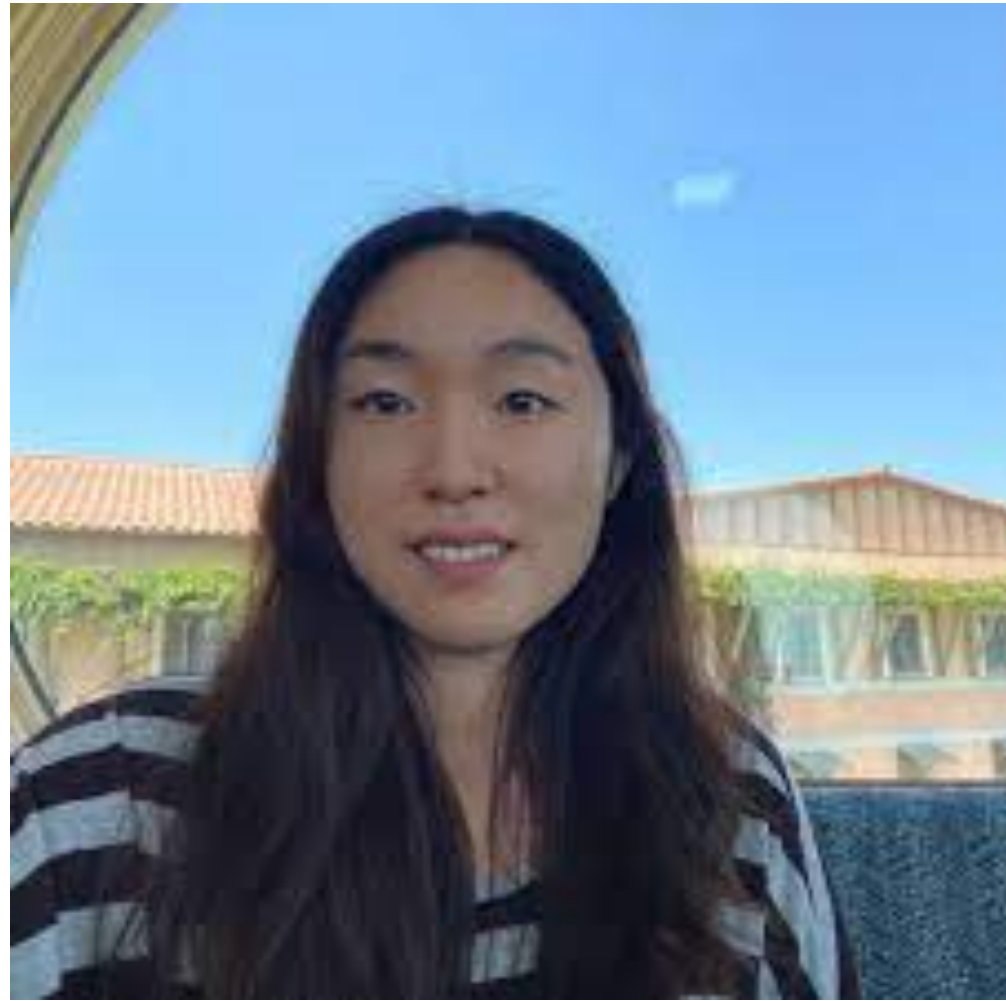
PHYSICS



HARVARD



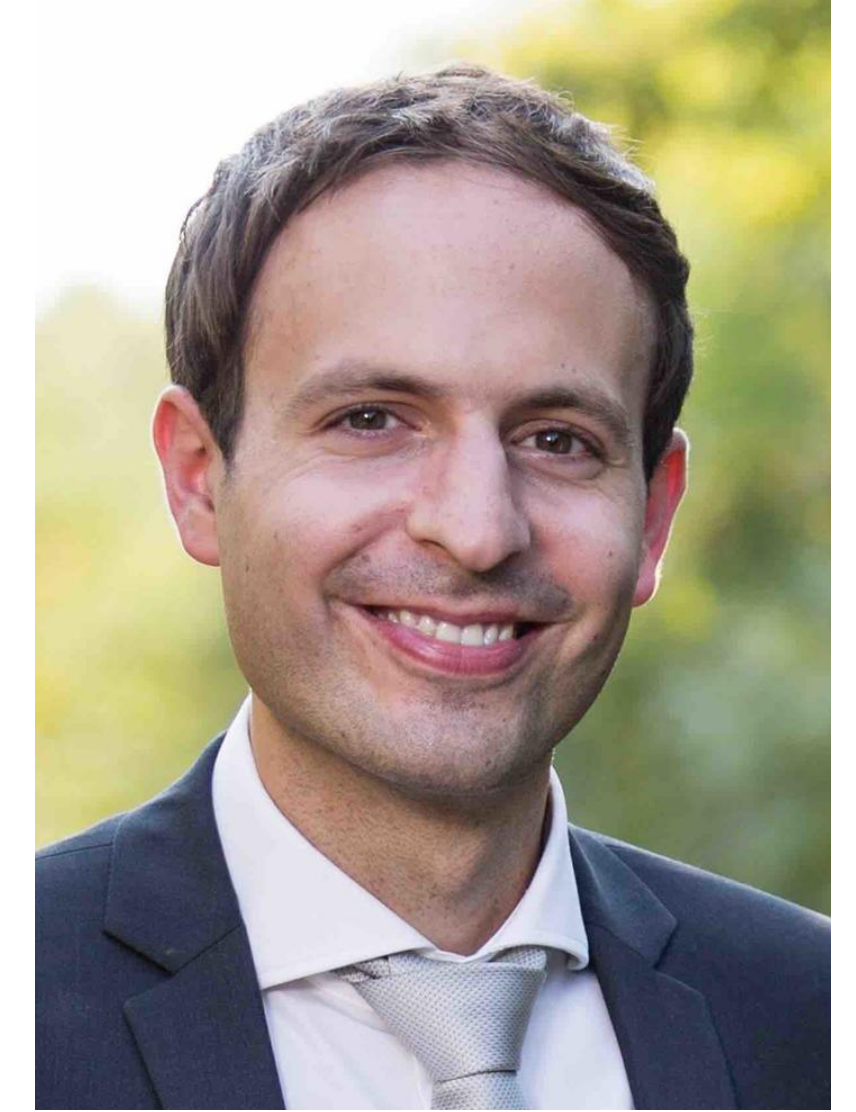
Maine Christos



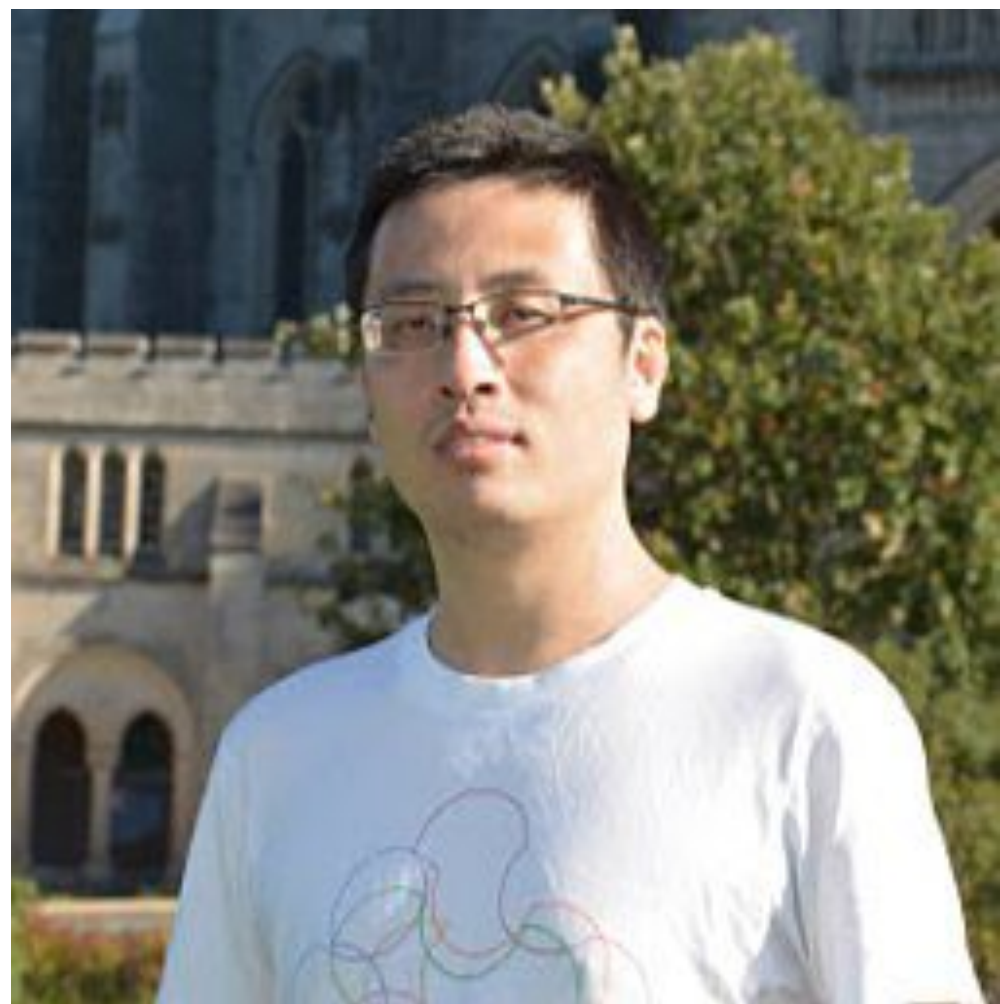
Zhu-Xi Luo
→GA Tech



Henry Shackleton



Mathias Scheurer
Innsbruck → Stuttgart



Ya-Hui Zhang
Johns Hopkins



Alexander Nikolaenko



Darshan Joshi
TIFR Hyderabad

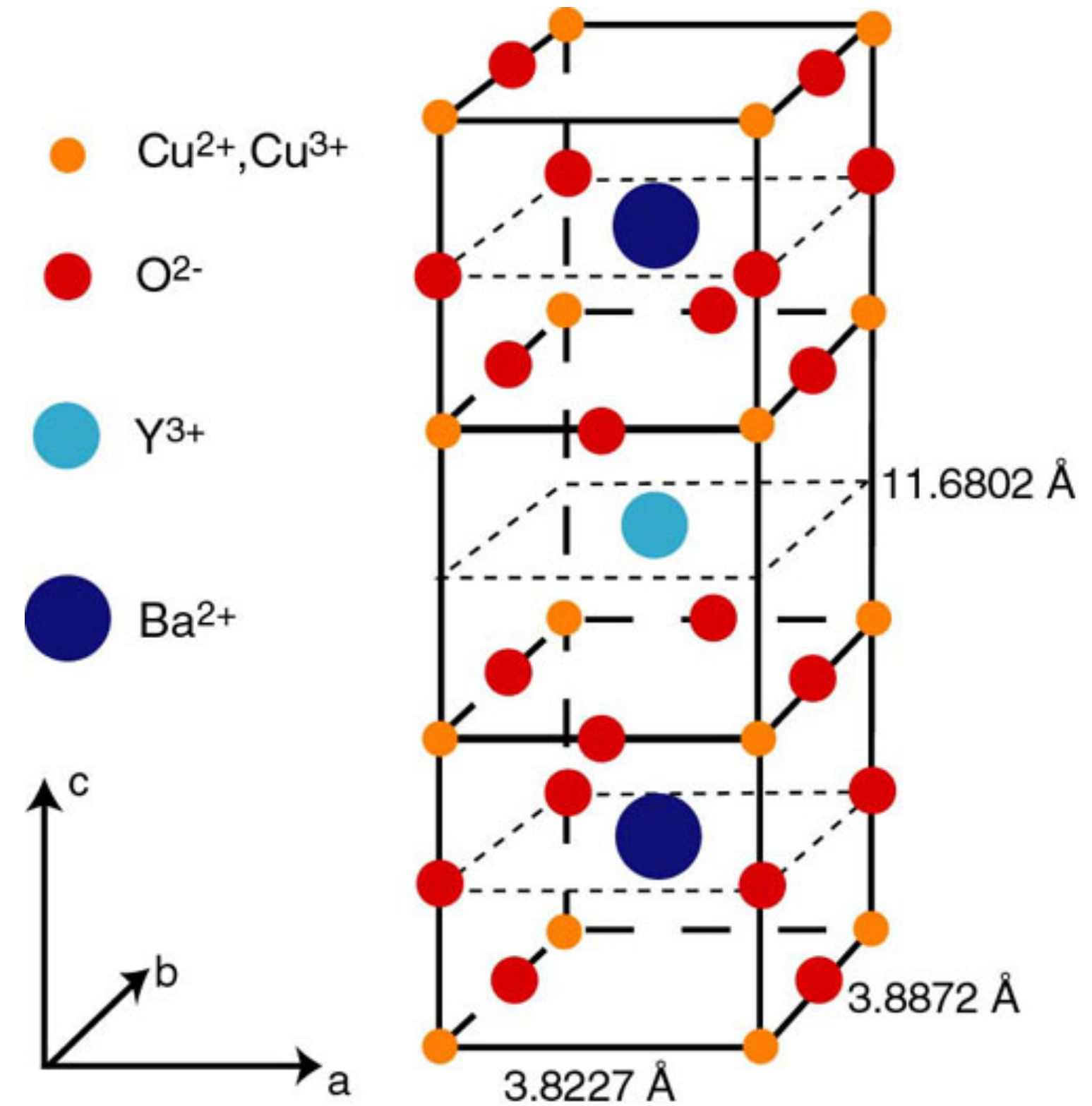
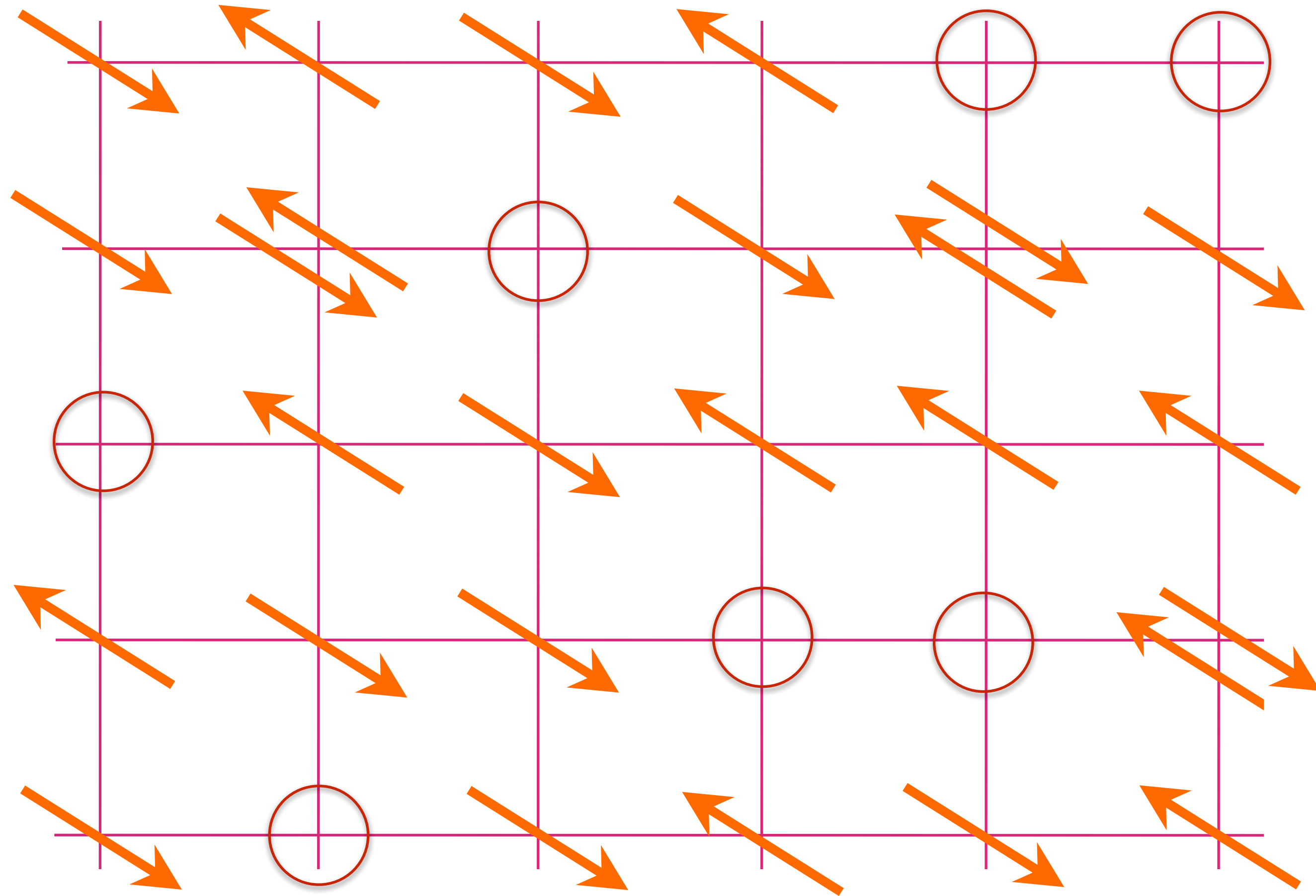


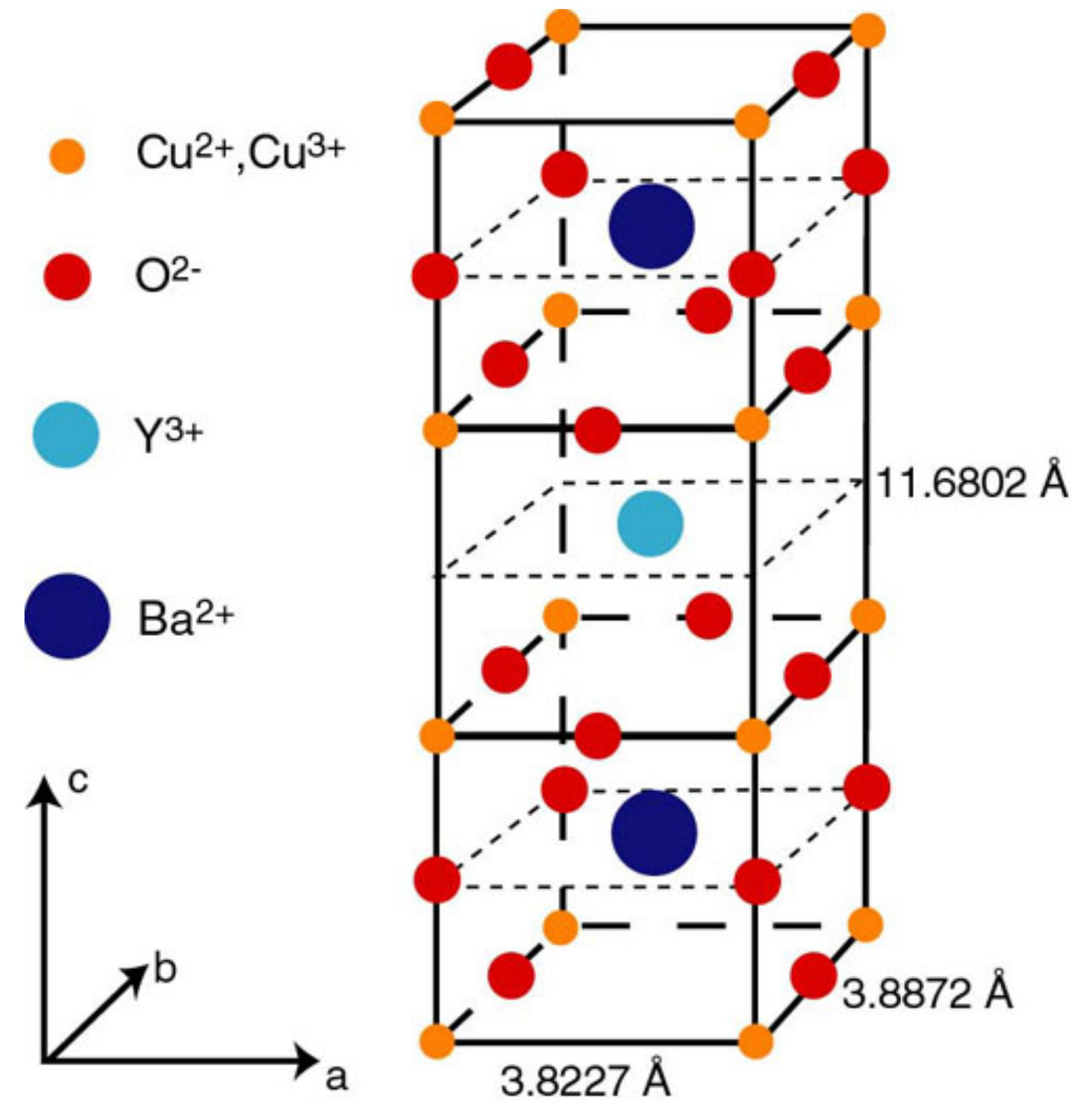
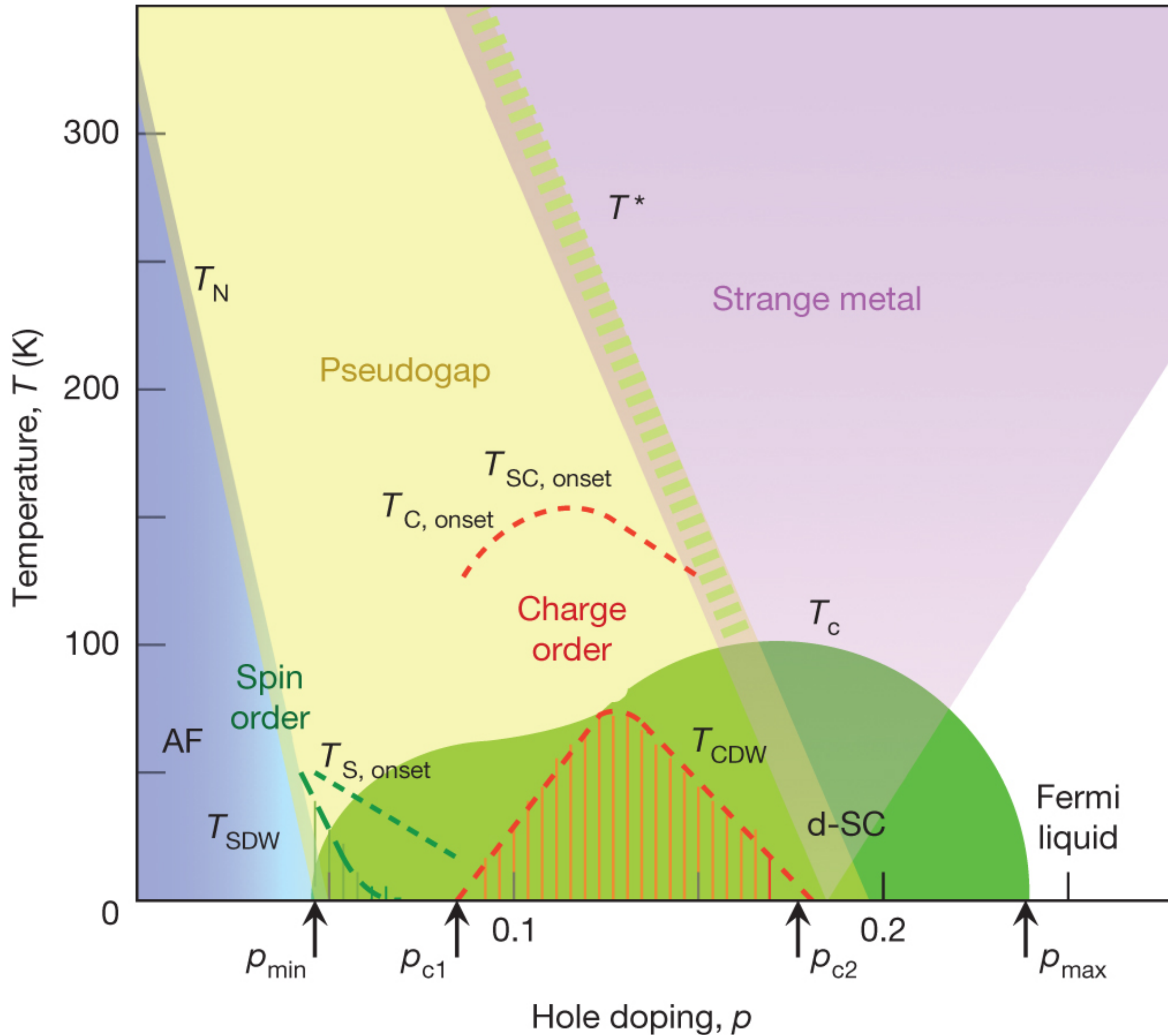
Jonas von Milczewski

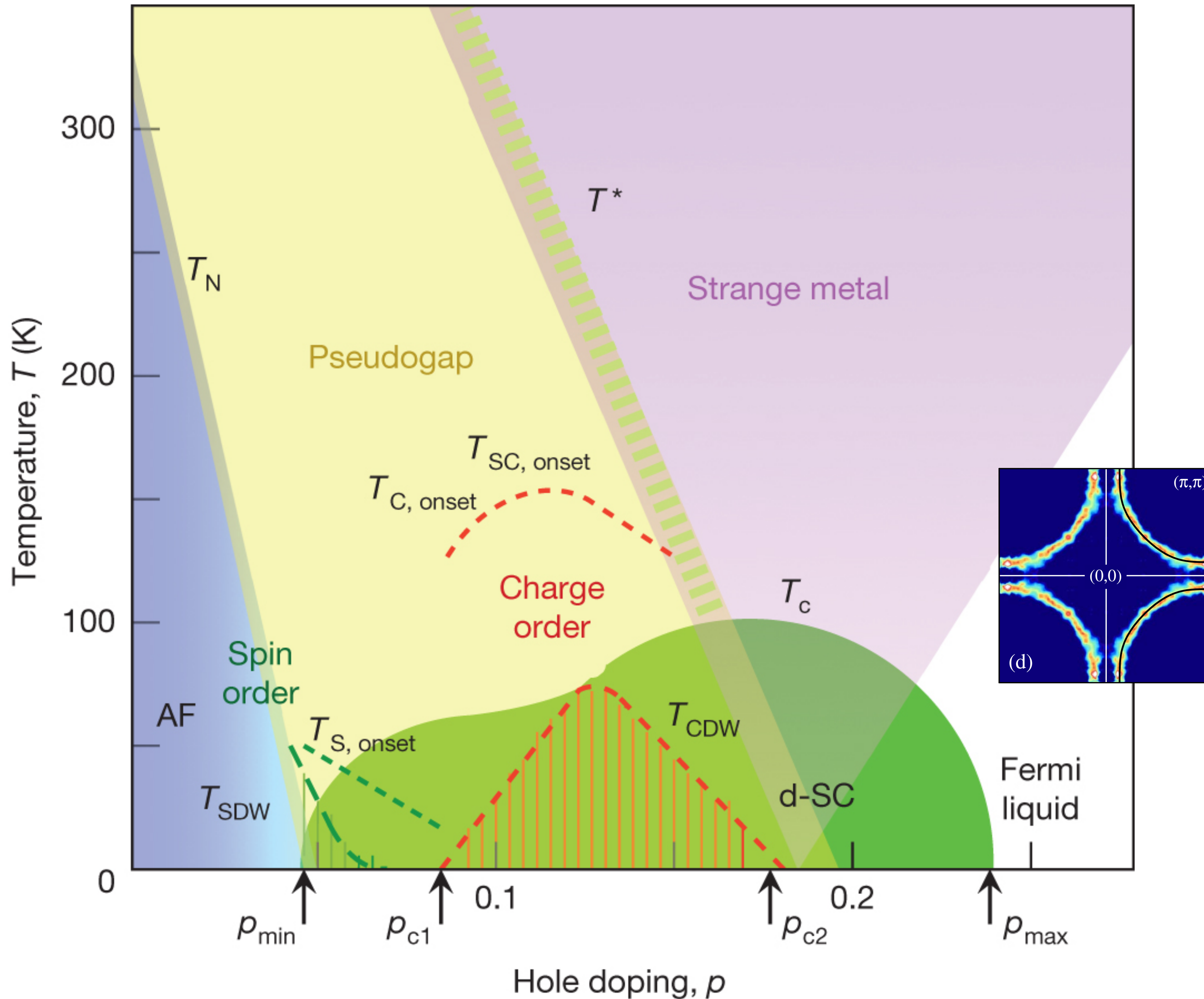
1. Open questions on the cuprate phase diagram
2. Theory of the pseudogap metal
3. The π -flux spin liquid
4. Confinement transitions of the π -flux spin liquid
5. Recap

Square lattice Hubbard model with electron density $1 - p$.

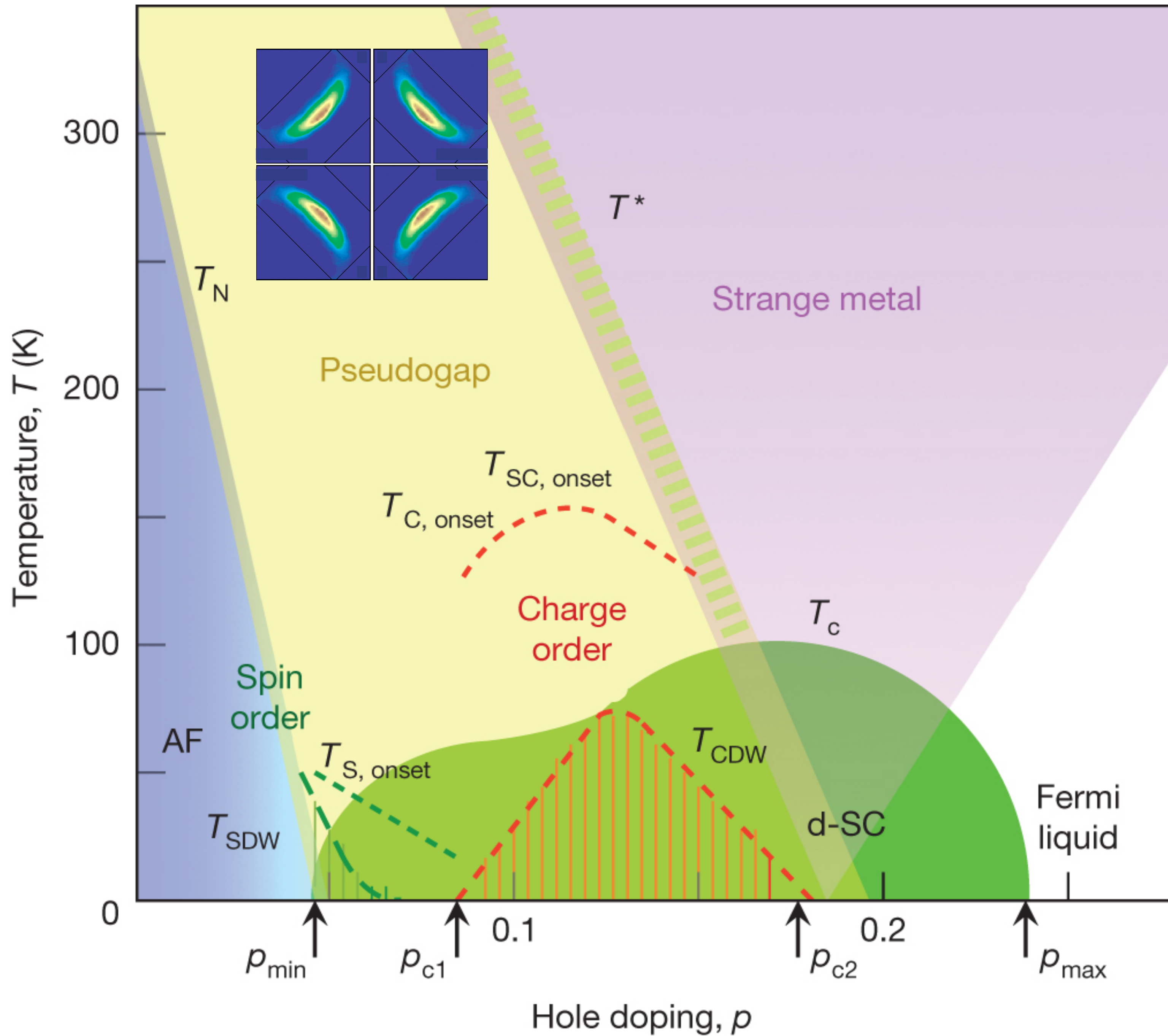
$$\mathcal{H}_{\text{Hubbard}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} + U \sum_i n_{i\uparrow} n_{i\downarrow} + \dots$$



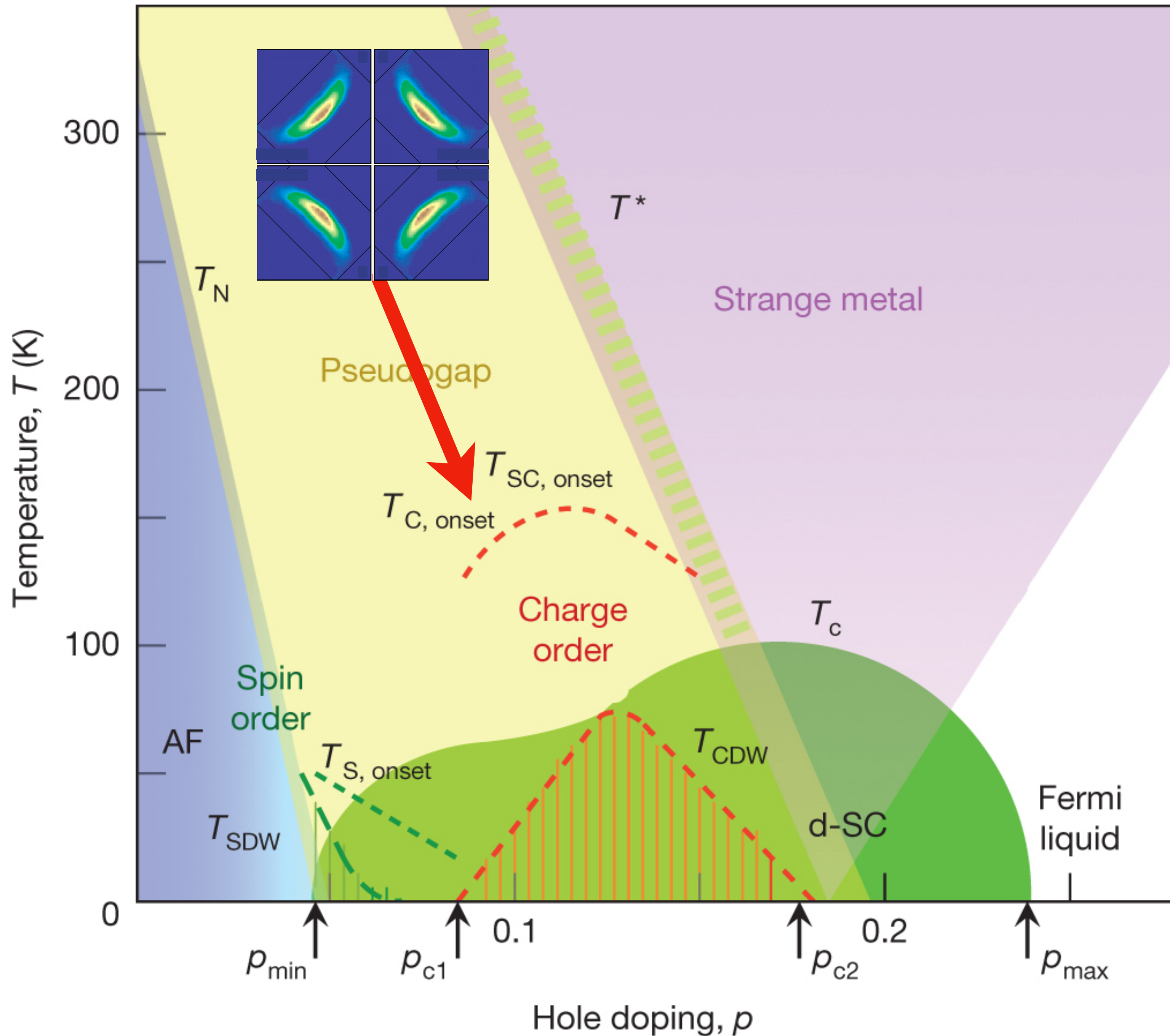




Fermi liquid
in the
overdoped metal



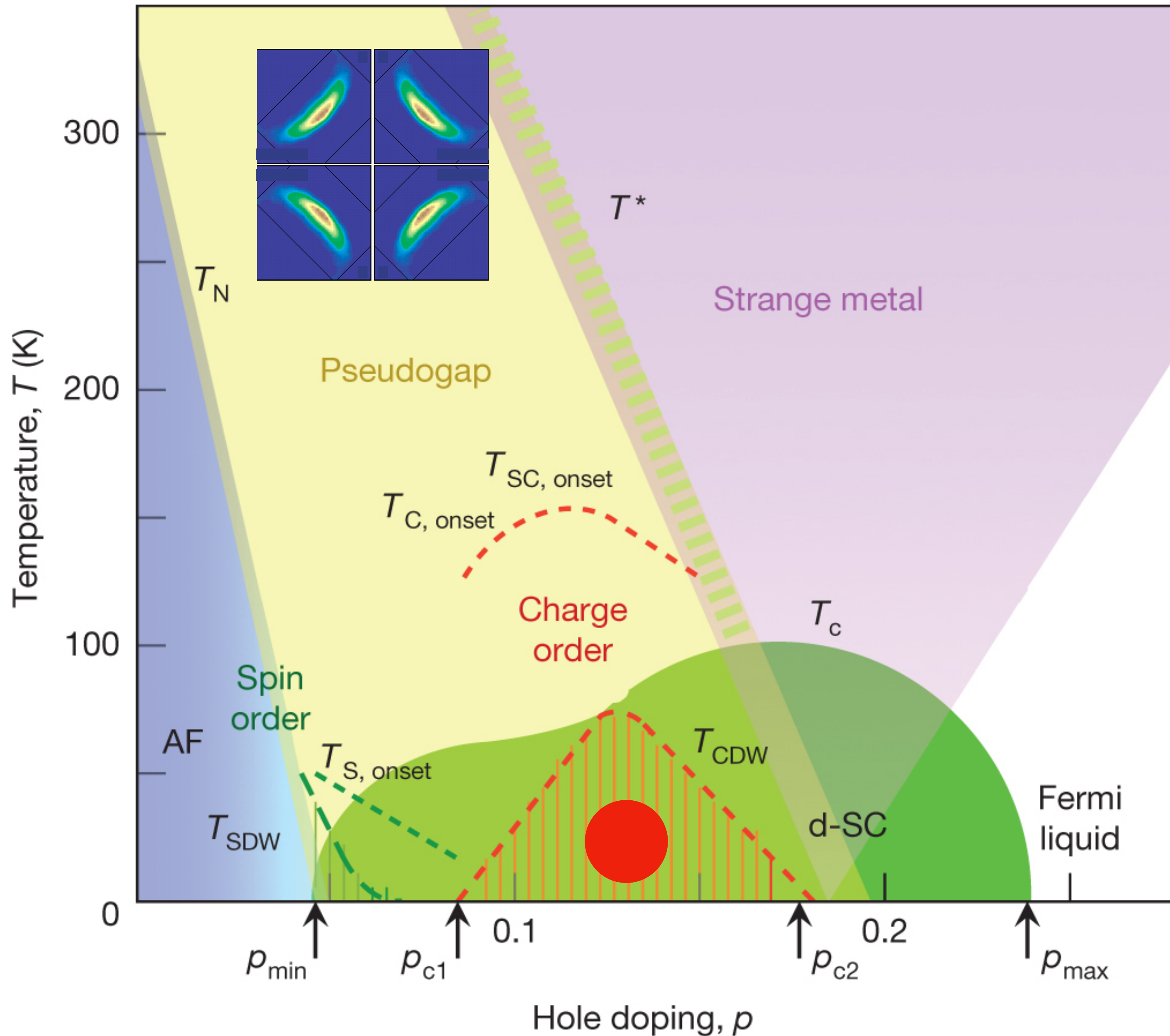
Theory for
“pseudogap metal”
with “Fermi arcs”?



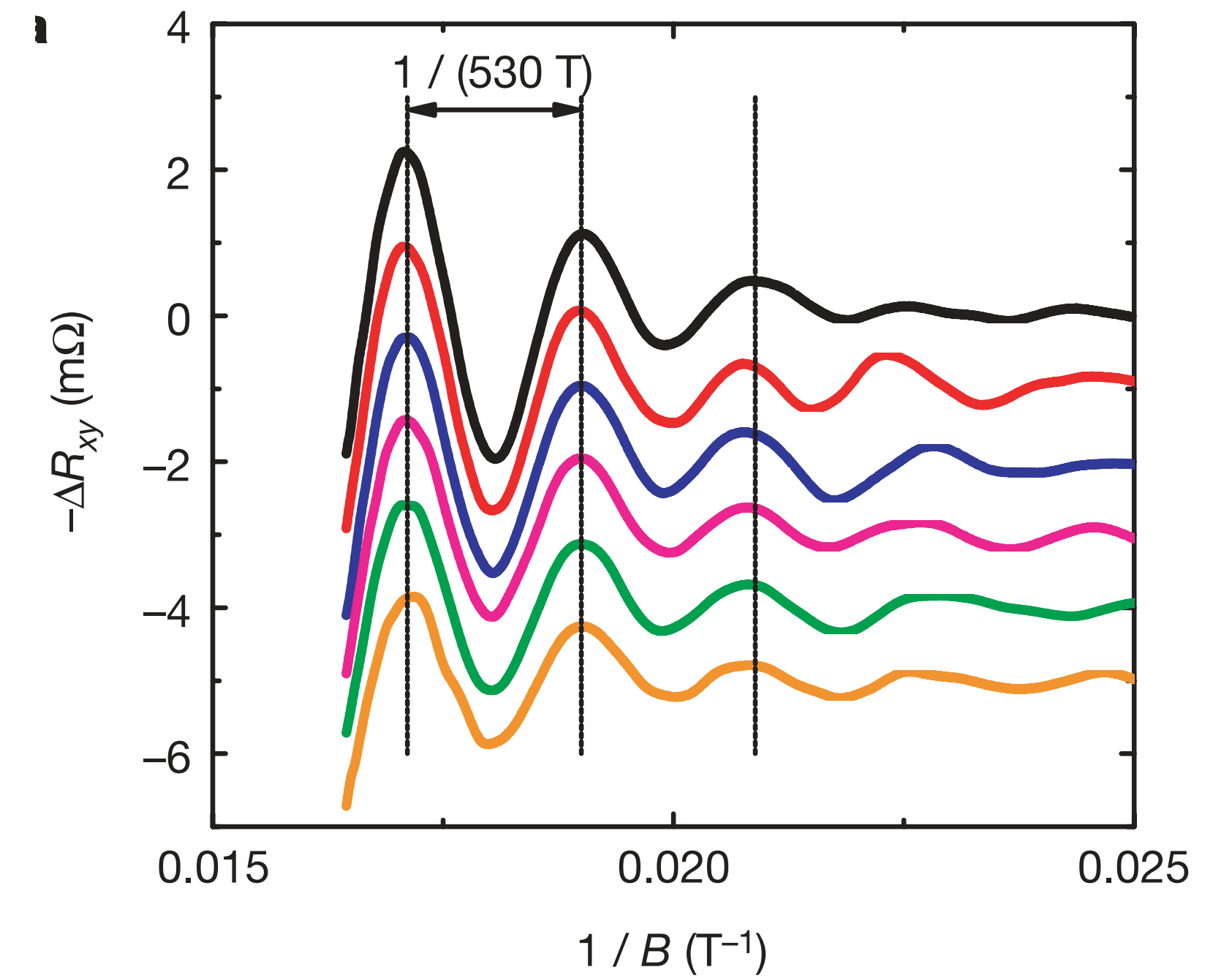
Needed: a theory for the onset of charge order and *d*-wave superconductivity from the pseudogap metal.

Why are T_c and T_{CDW} about the same?

B Keimer *et al.* *Nature* **518**, 179-186 (2015)

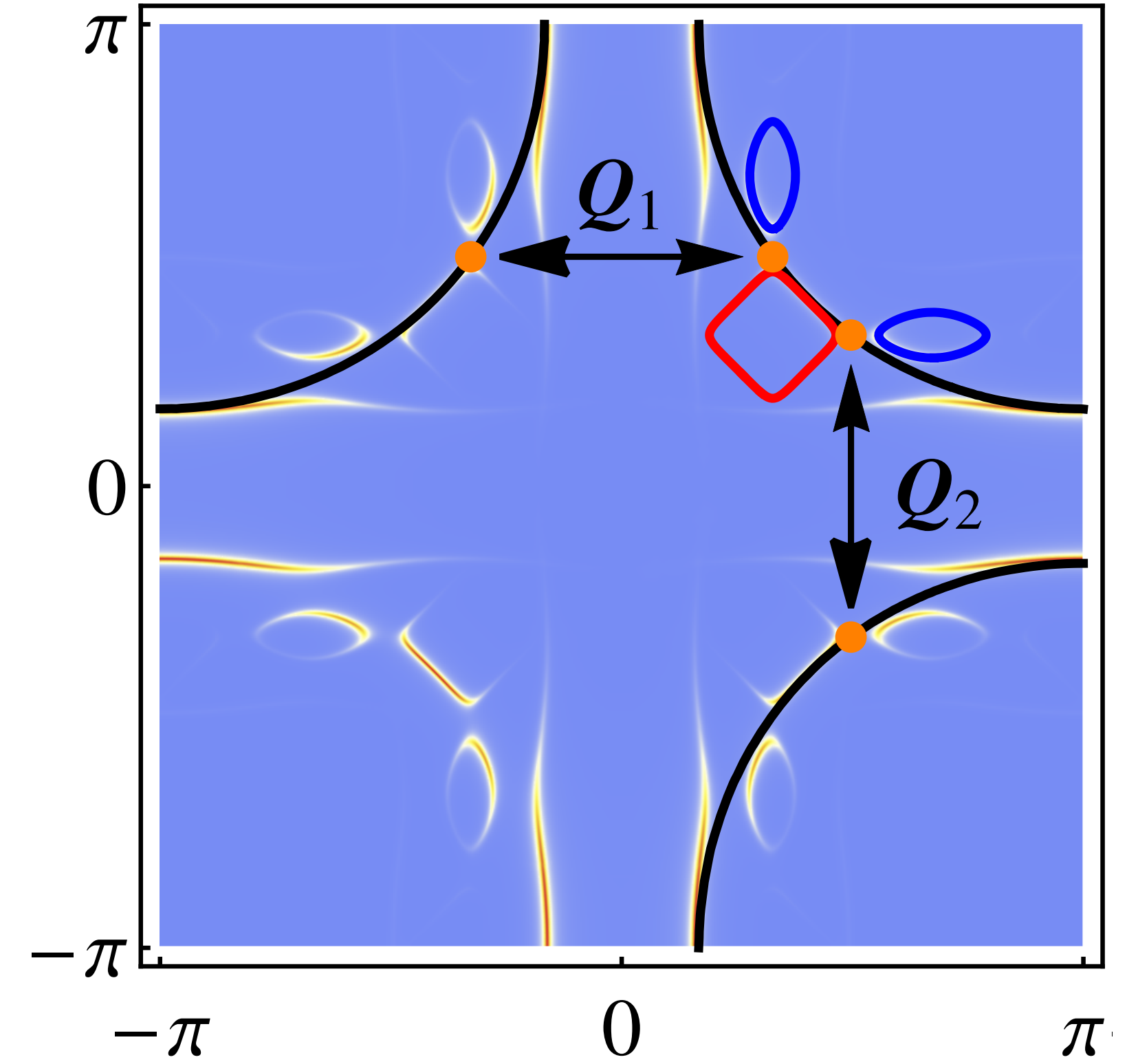
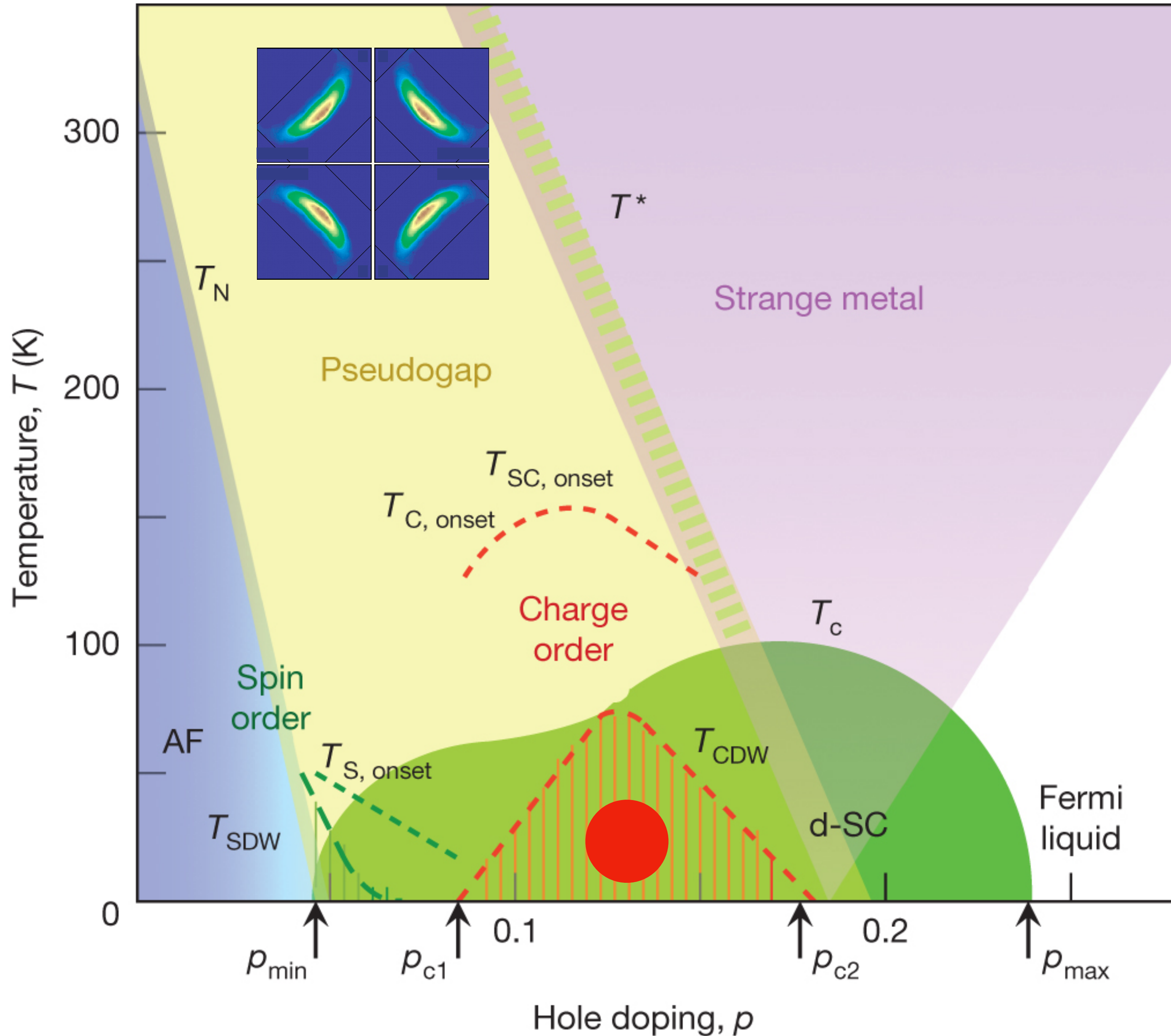


N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.B. Bonnemaïson, R. Liang, D.A. Bonn, W.N. Hardy, L. Taillefer, *Nature* **447**, 565 (2007)



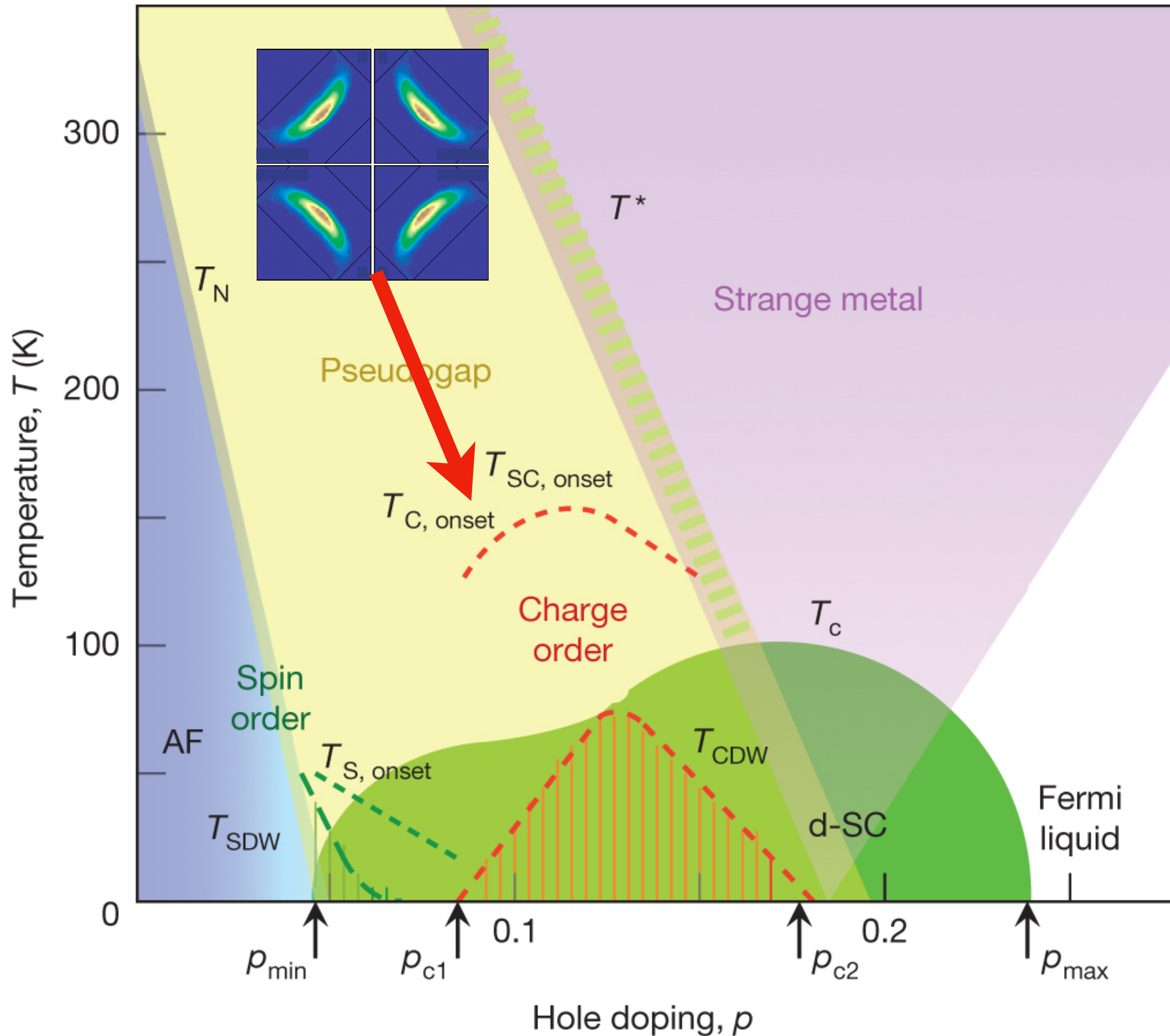
Quantum oscillations in the CDW phase at low T show only a single electron pocket of size p .

This cannot be obtained in the theory of CDWs in a Fermi liquid.



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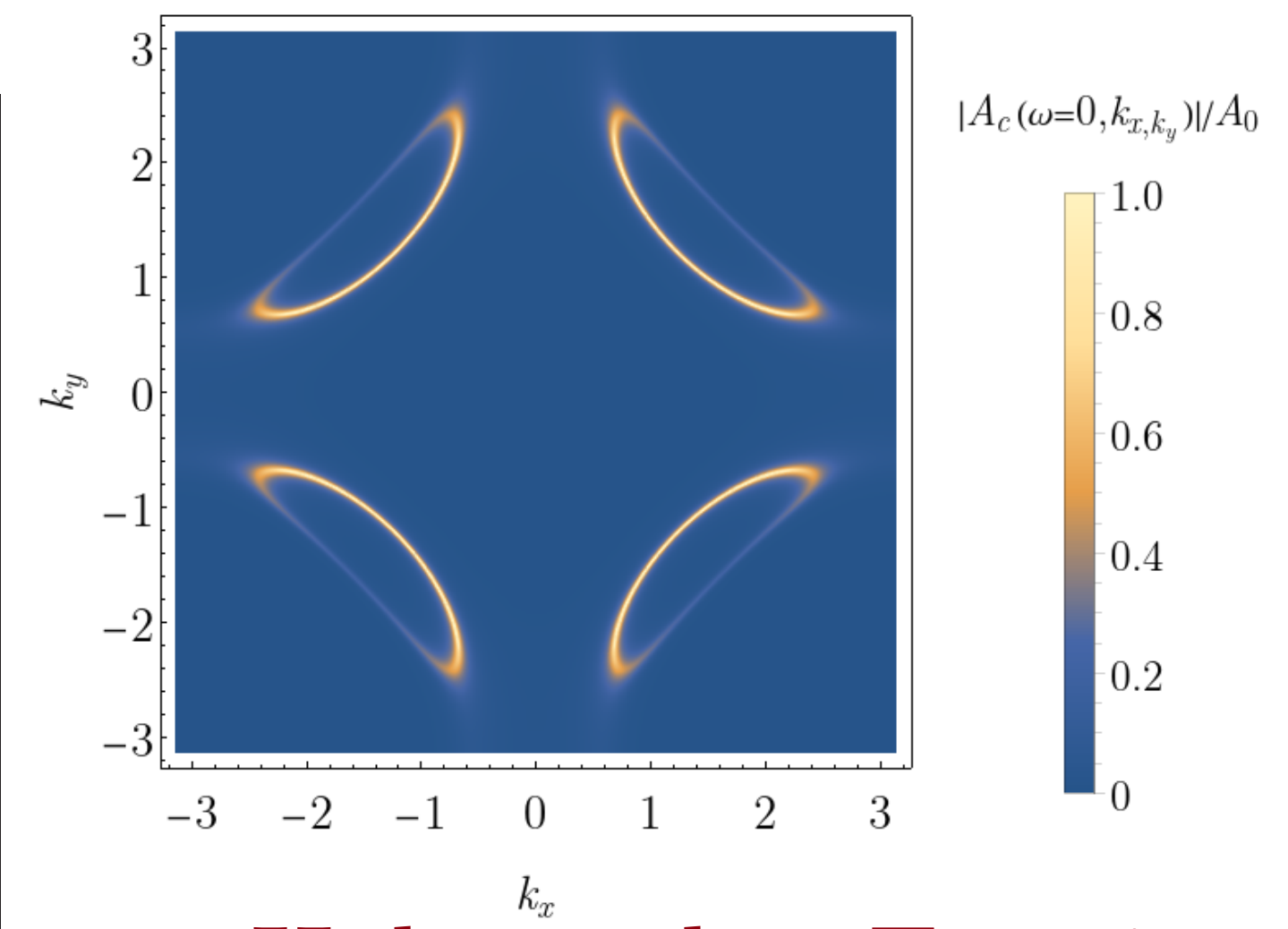
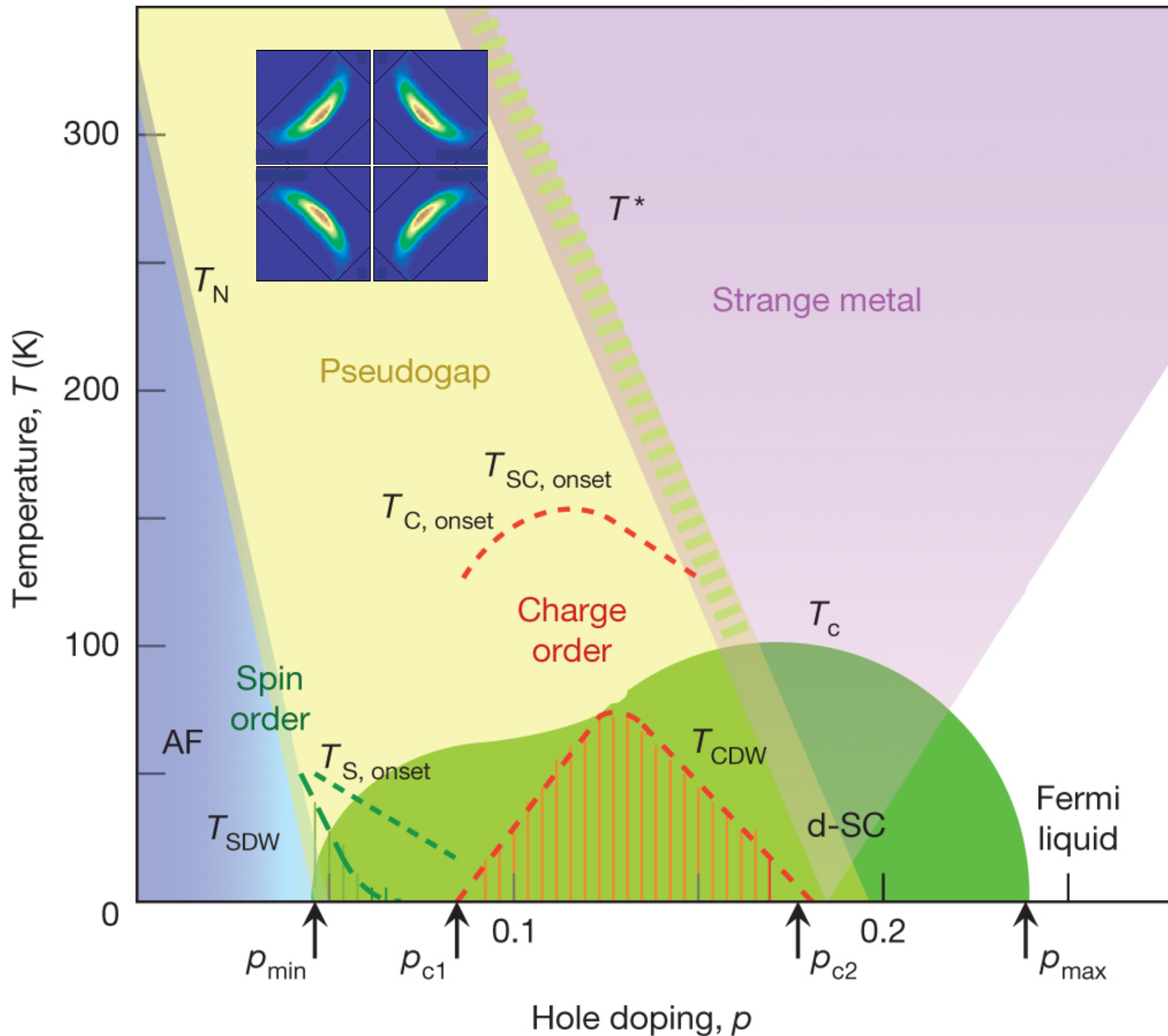


Theory for
“pseudogap metal”
with “Fermi arcs”?

Use the pseudogap metal
in place of the Fermi liquid
as the ‘parent’ to
conventional
d-wave superconductor,
charge density wave,
spin density wave,
pair density wave

...

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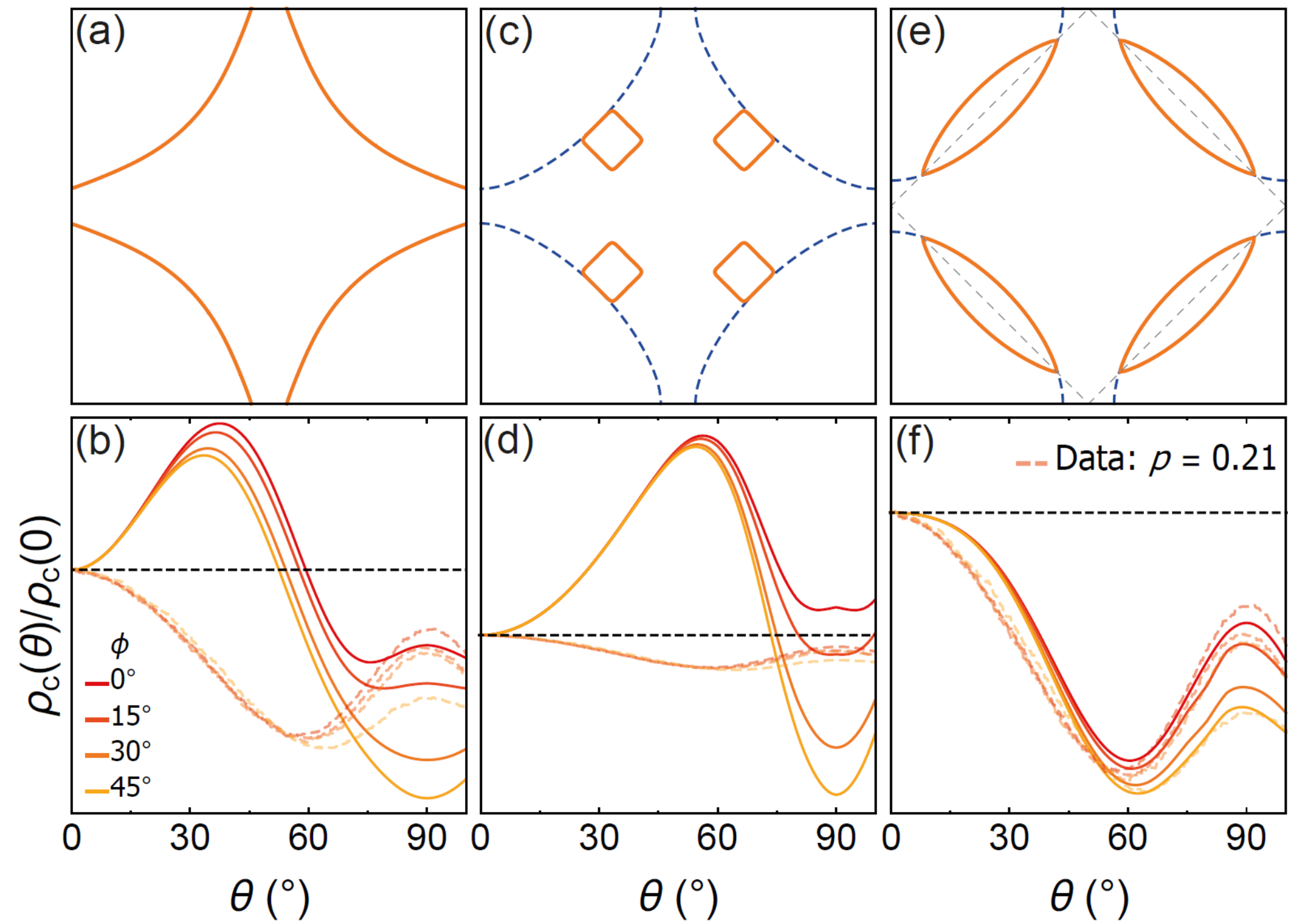
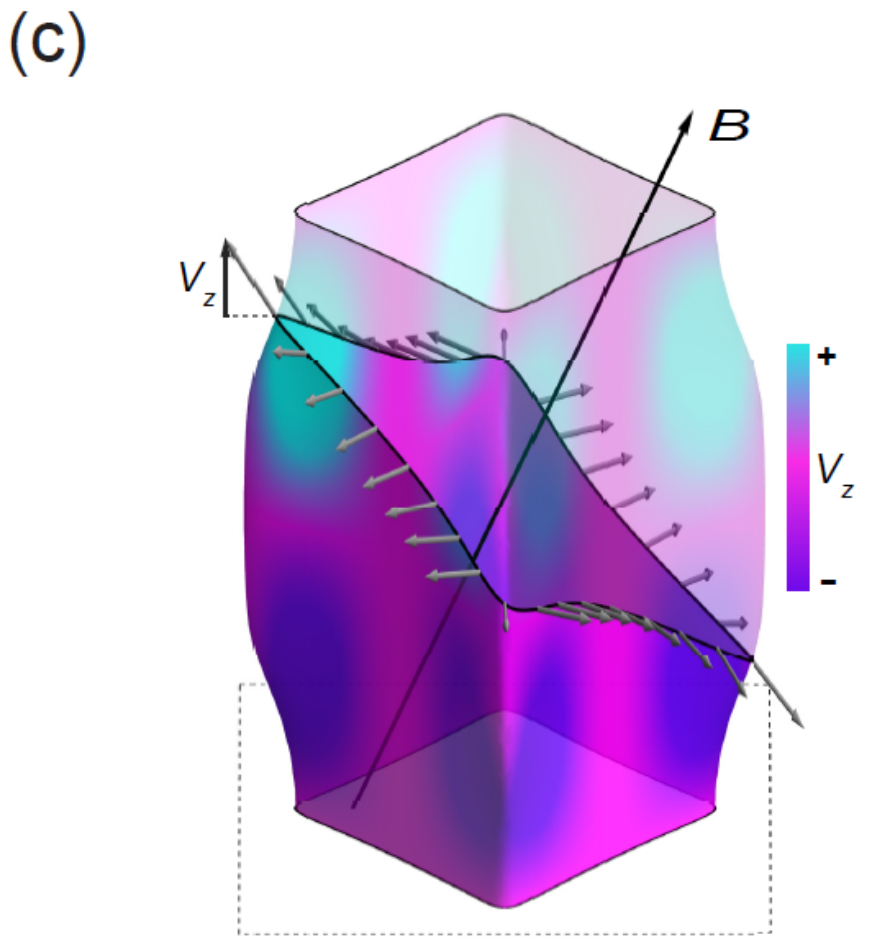
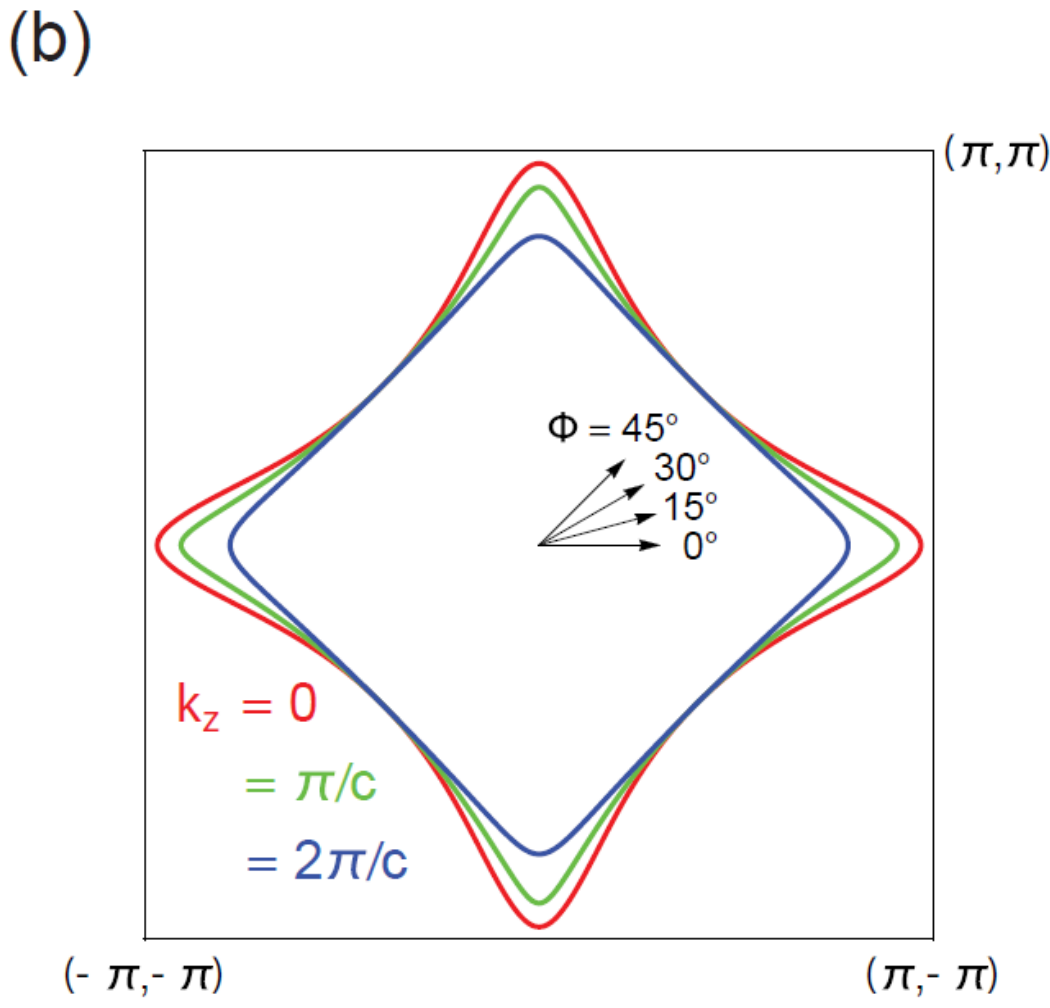
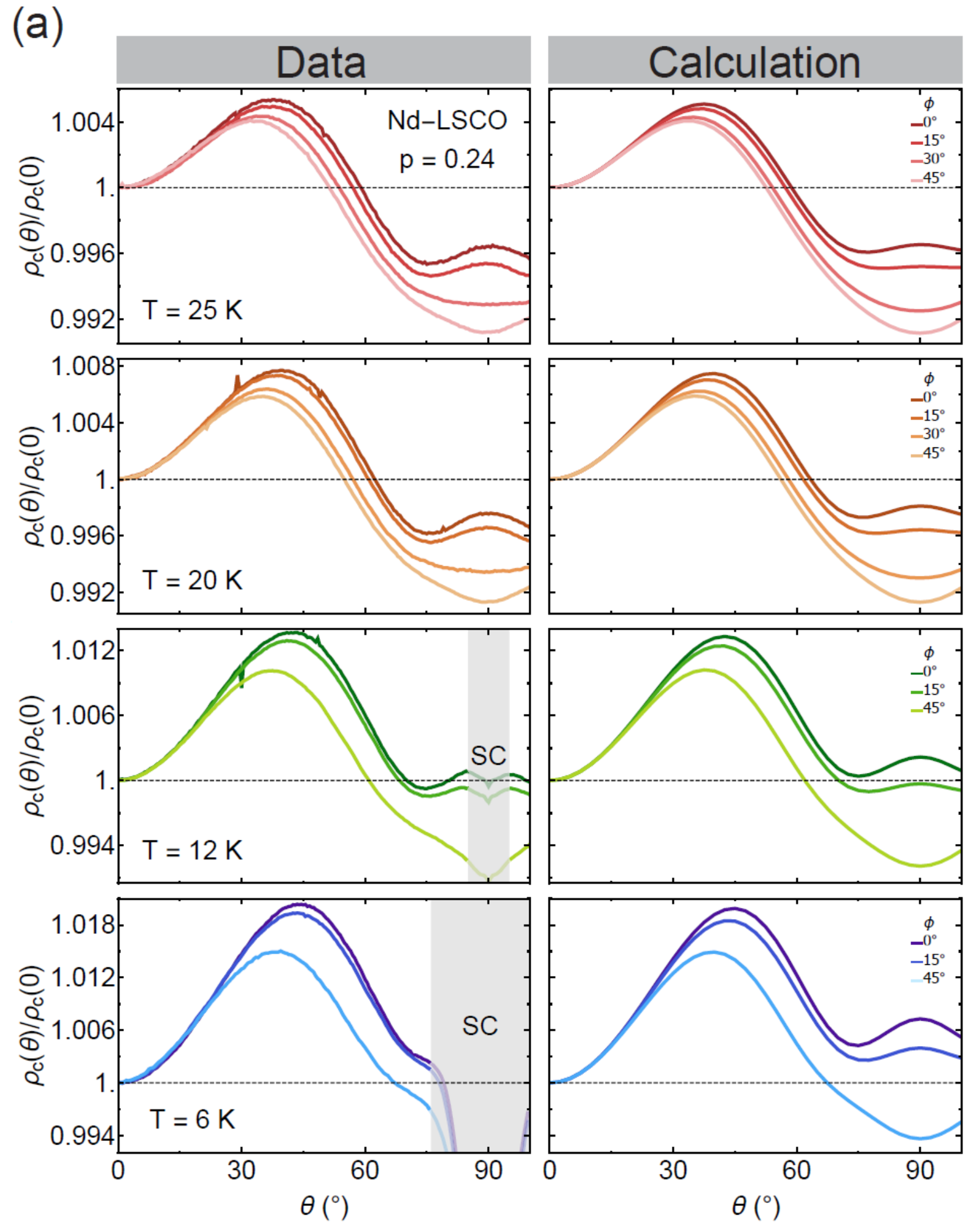
E. Mascot,
A. Nikolaenko,
M. Tikhanovskaya,
Ya-Hui Zhang,
D. K. Morr, and
S. S., PRB **105**,
075146 (2022)

Hole pocket Fermi surfaces
of size p with
charge e , spin-1/2 quasiparticles

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang,
PRB **73**, 174501 (2006).
T. D. Stanescu and G. Kotliar,
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PRL **97**, 136401 (2006).
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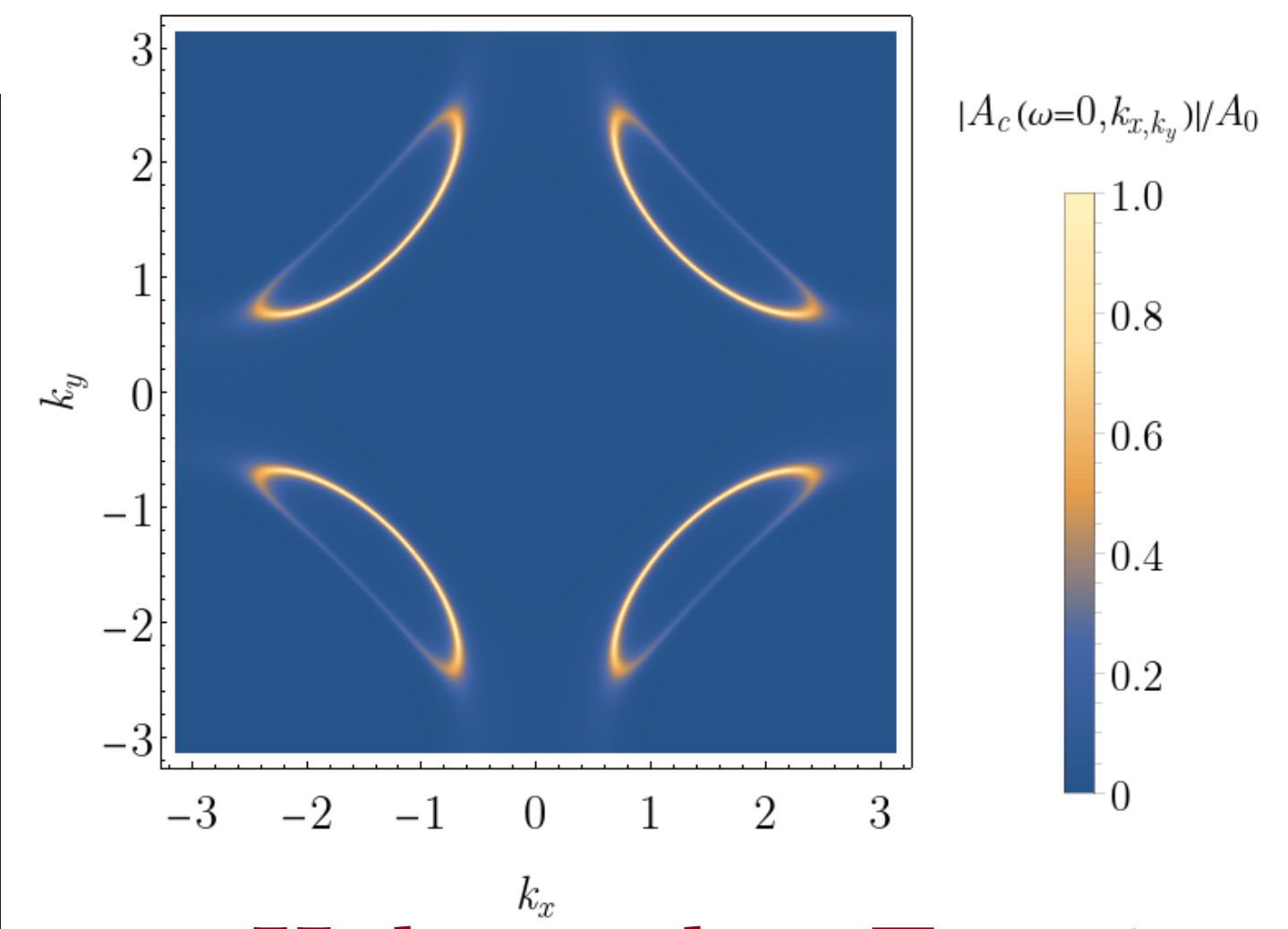
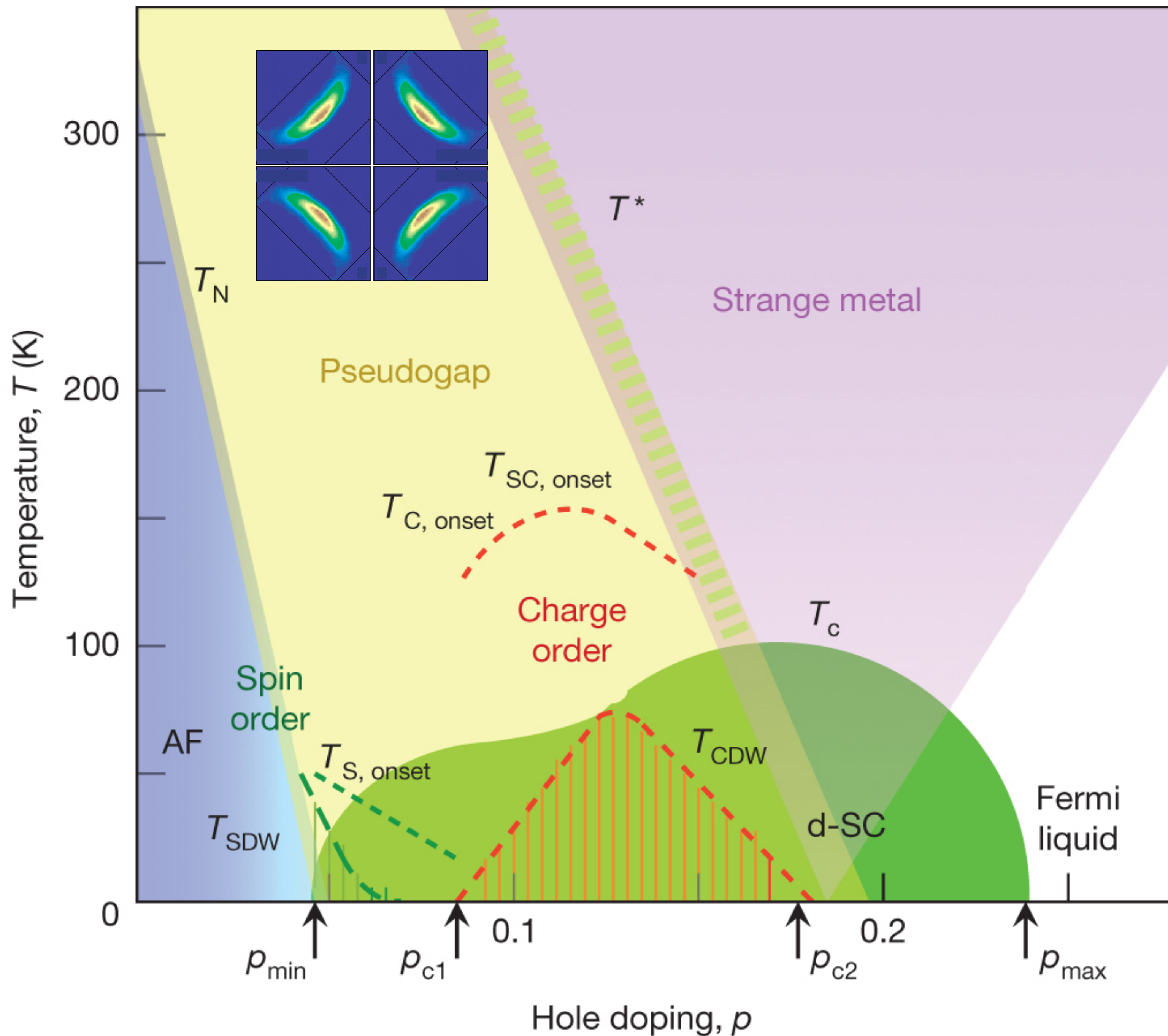
Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, Nature Physics **18**, 558 (2022)



$p < p_c$ Reconstructed Fermi surface

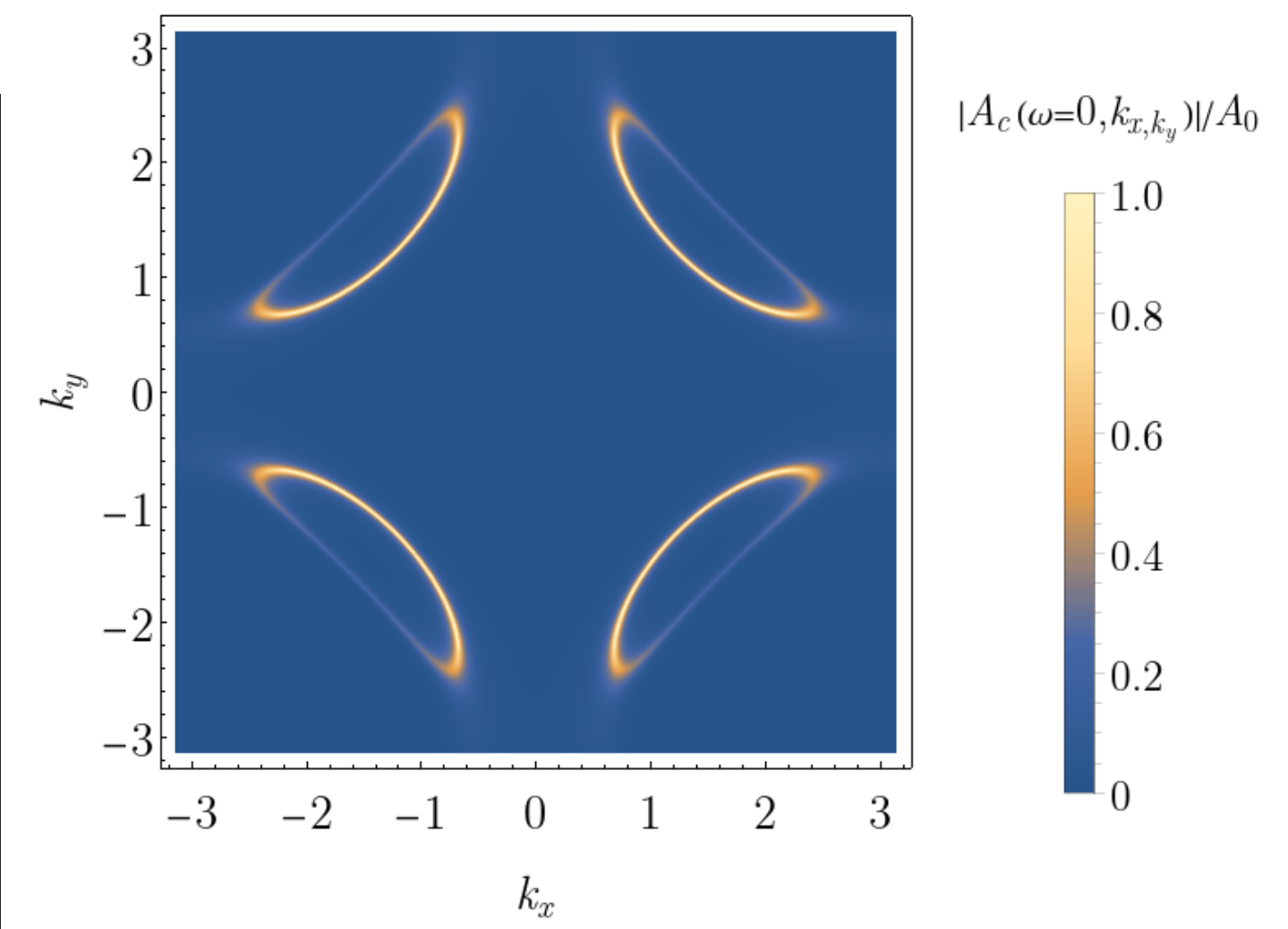
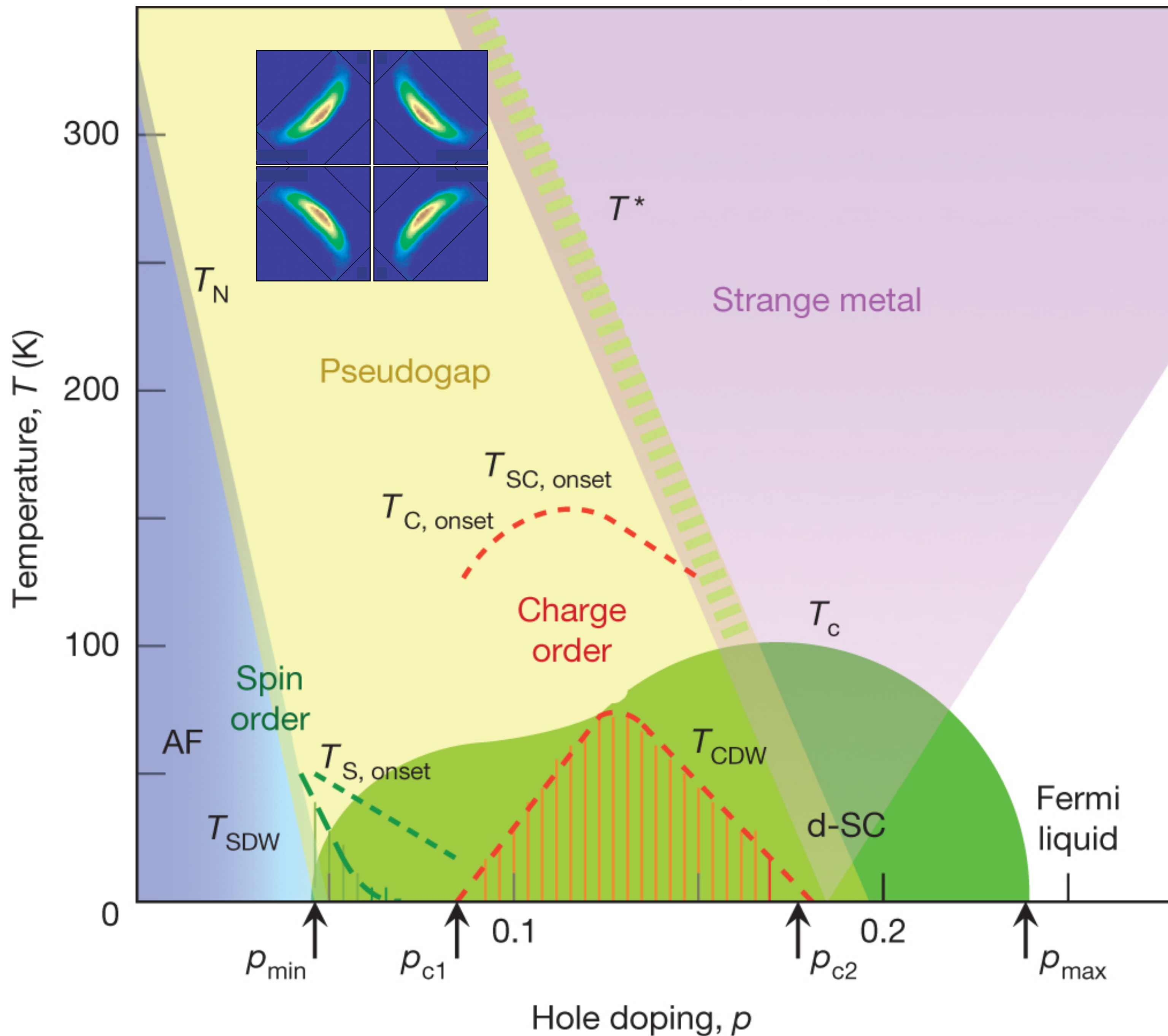
$p > p_c$ Large Fermi surface



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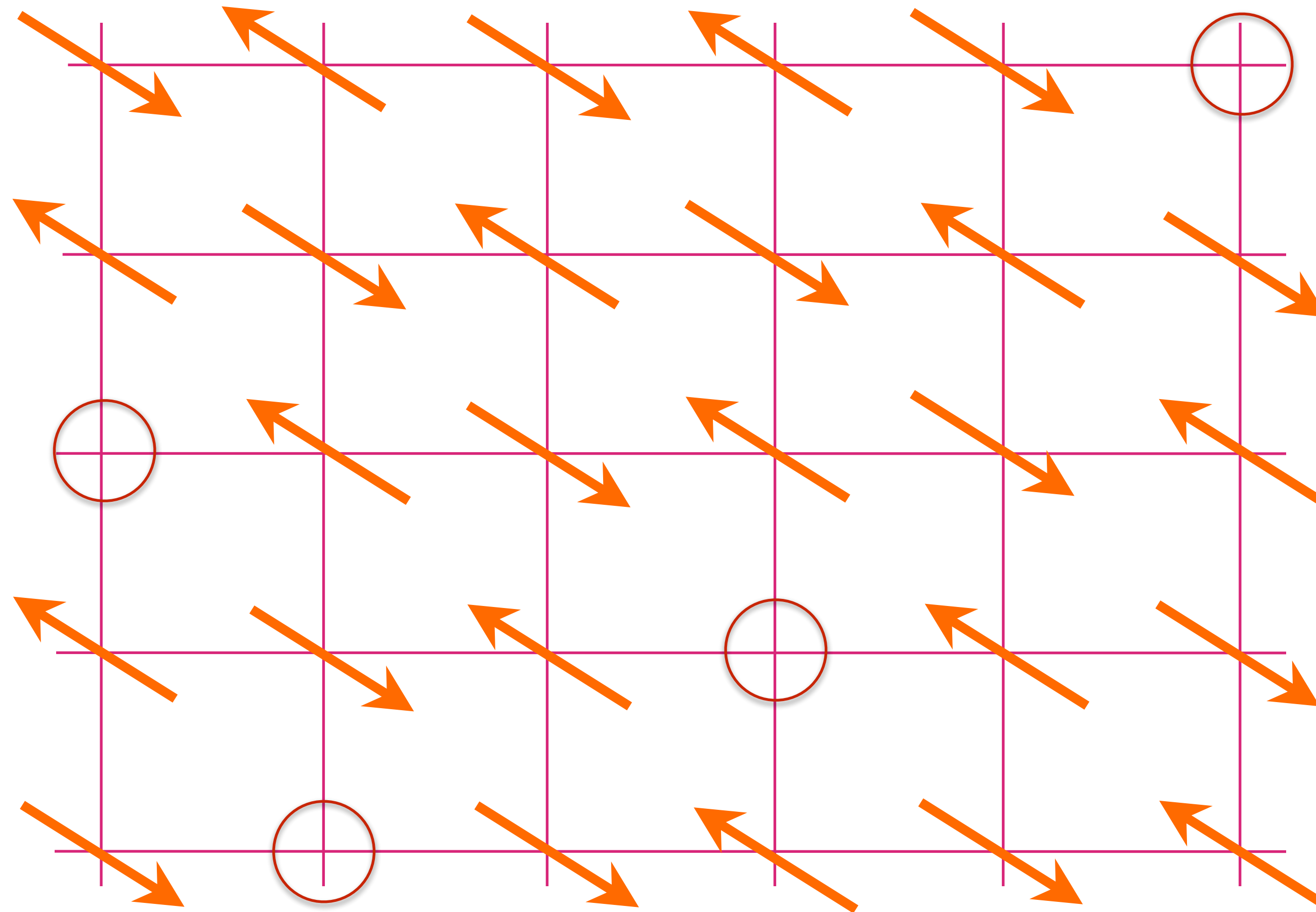
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Hole pocket Fermi surfaces
of size p with
charge e , spin-1/2 quasiparticles
+
'spectator'
square lattice spin liquid
at half-filling.

FL*: Spin liquid is *required* because
the Fermi surface does not enclose
the Luttinger volume $(1 + p)$.



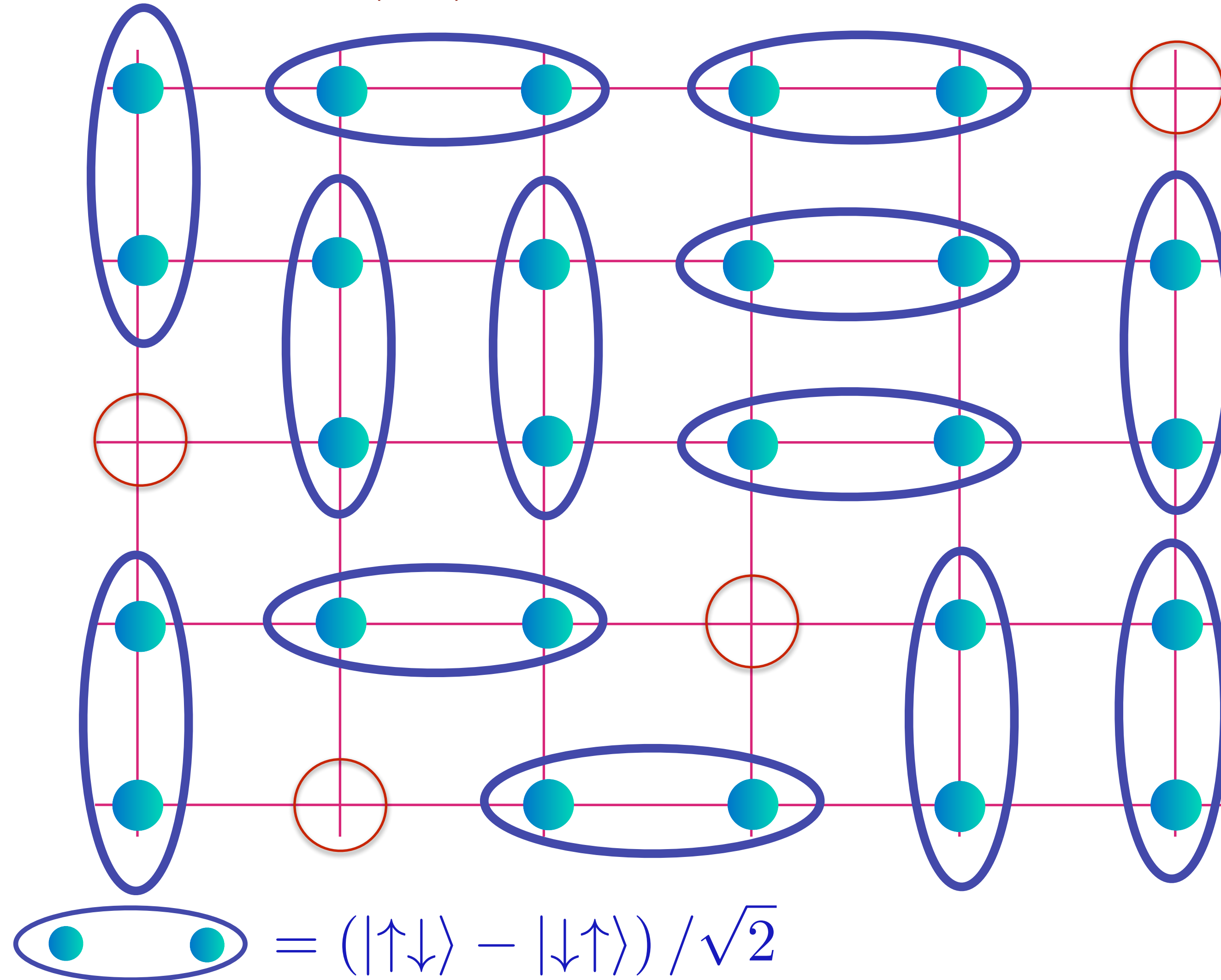
Anti-ferromagnet
with p holes
per square

Holon metal

G. Baskaran, Z. Zou, P.W.Anderson, Solid State Comm. **63**, 973 (1987)

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)



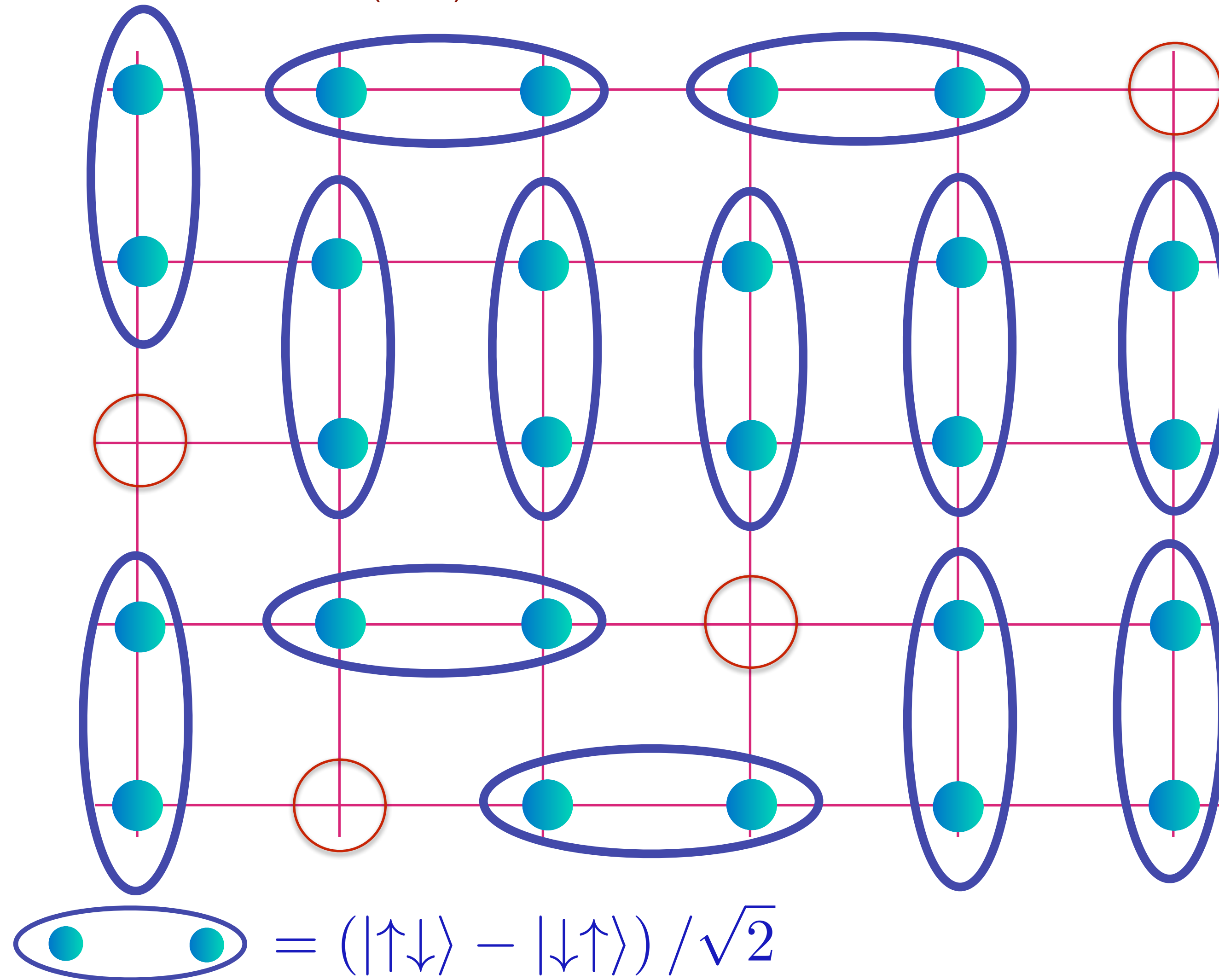
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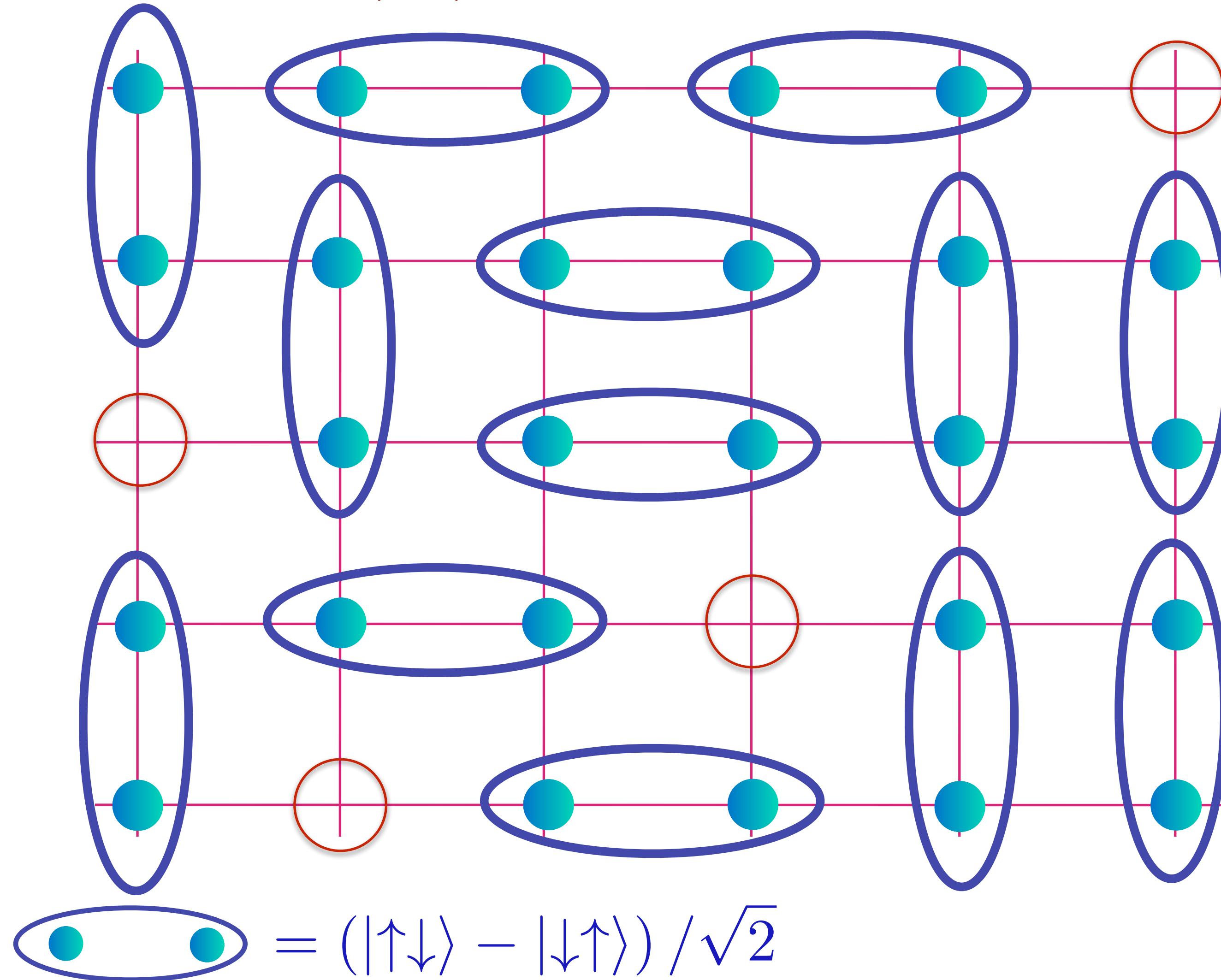
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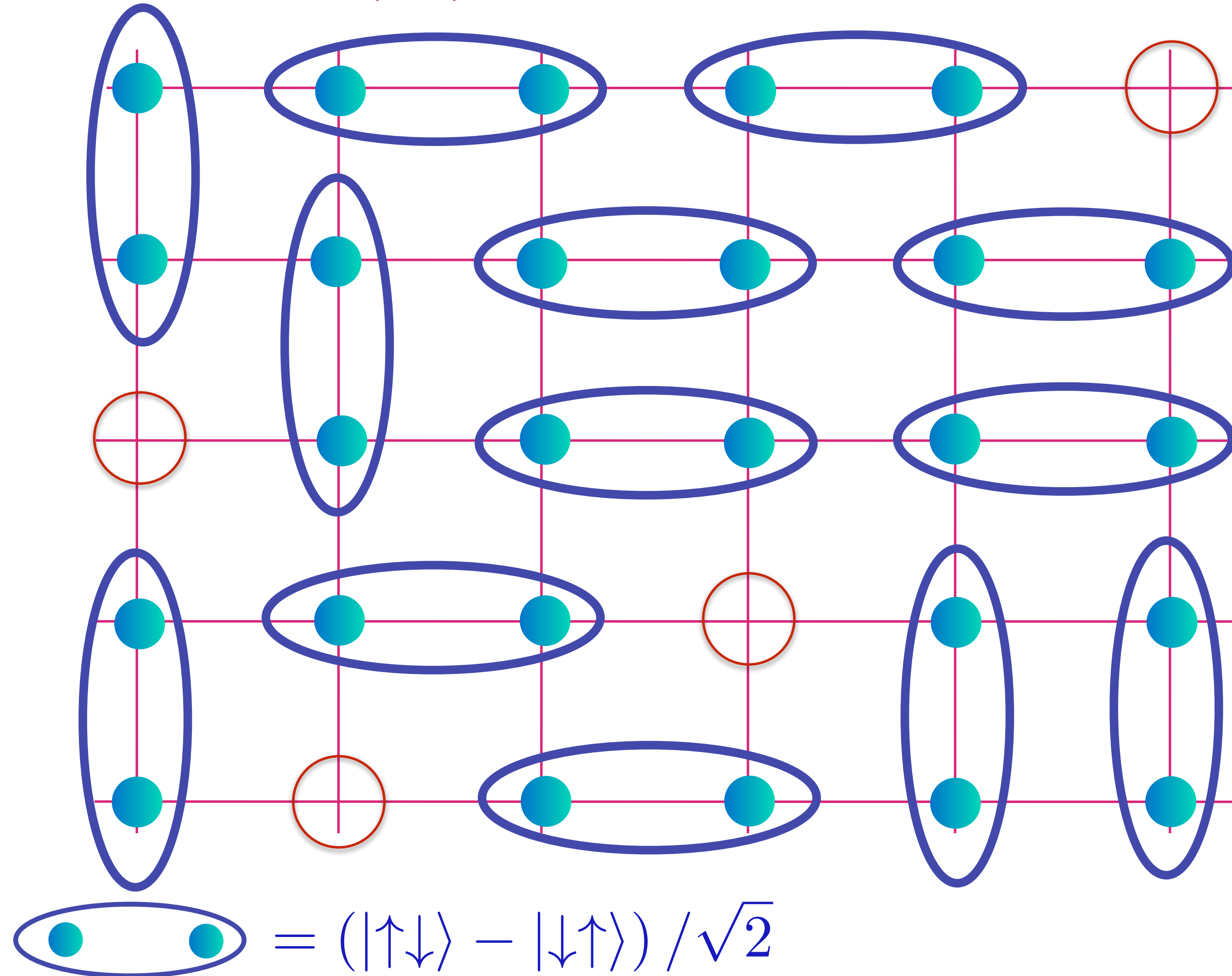
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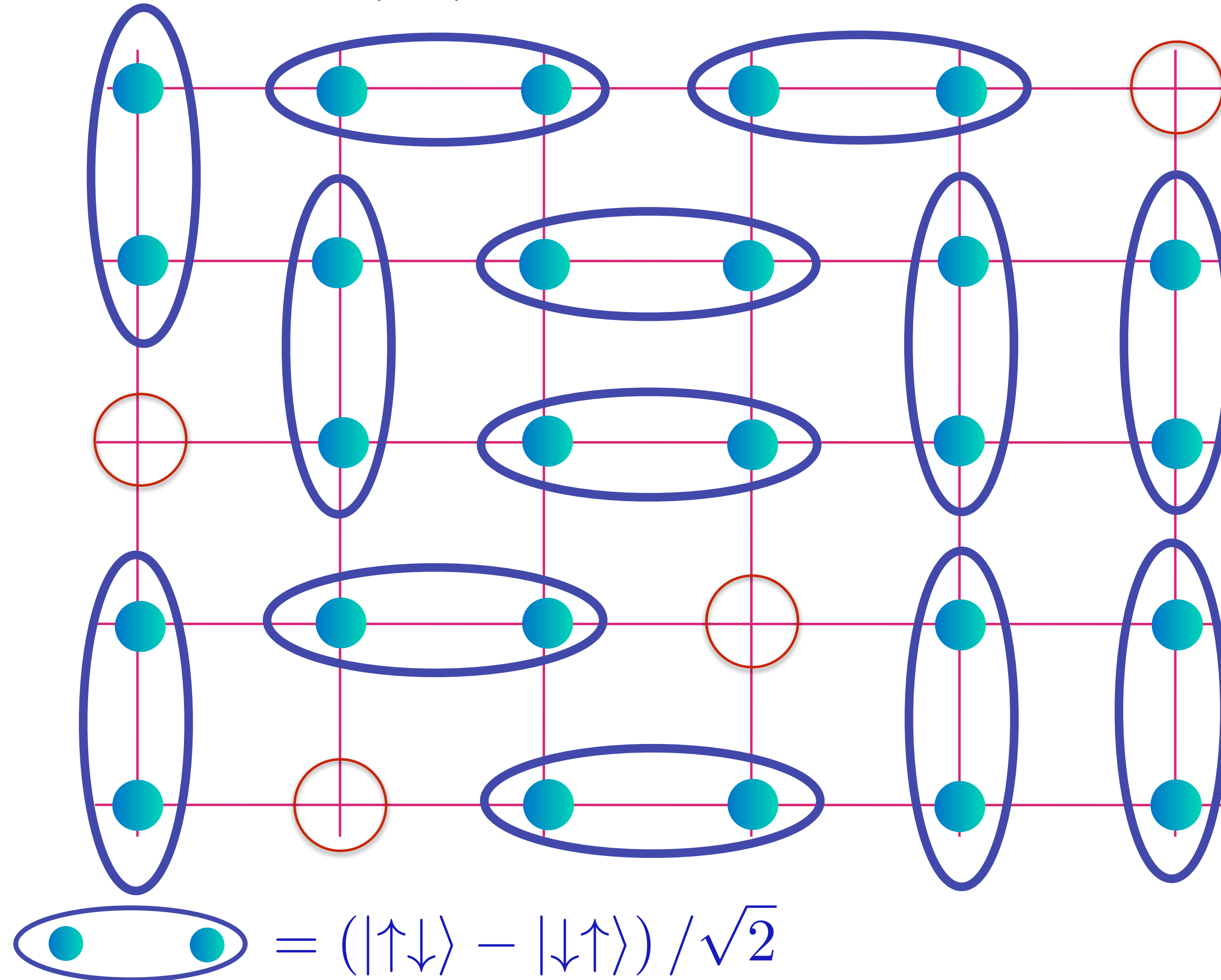
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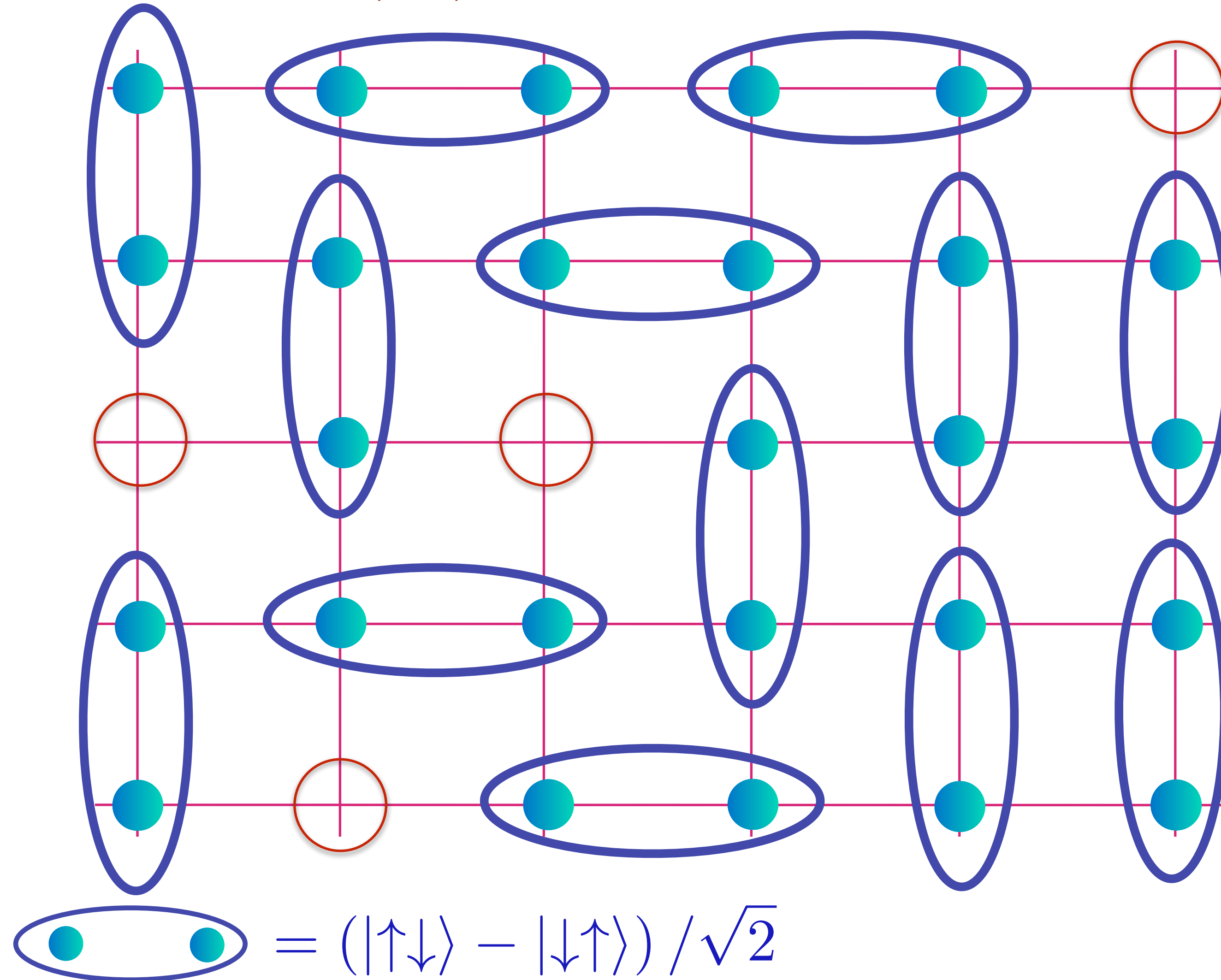
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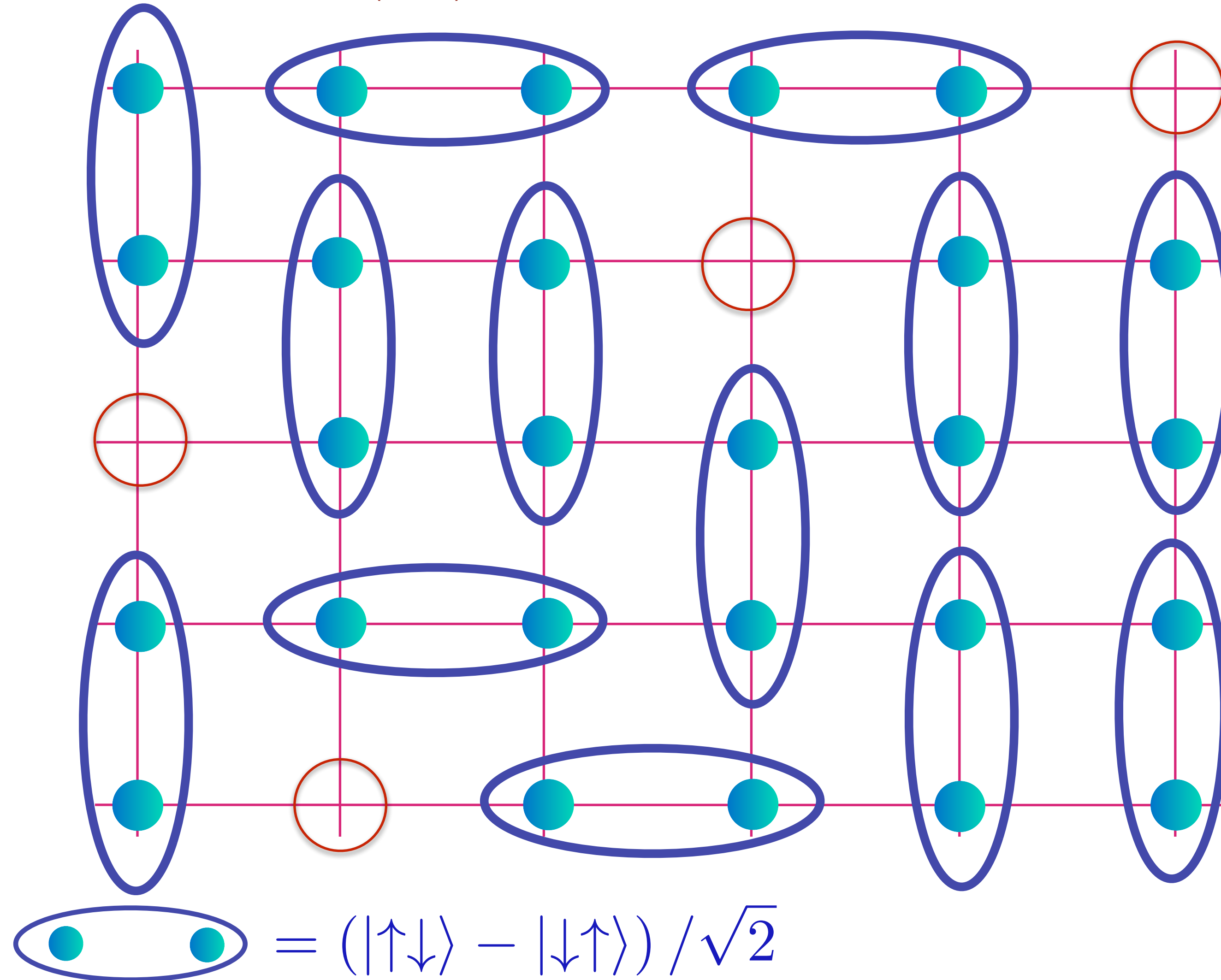
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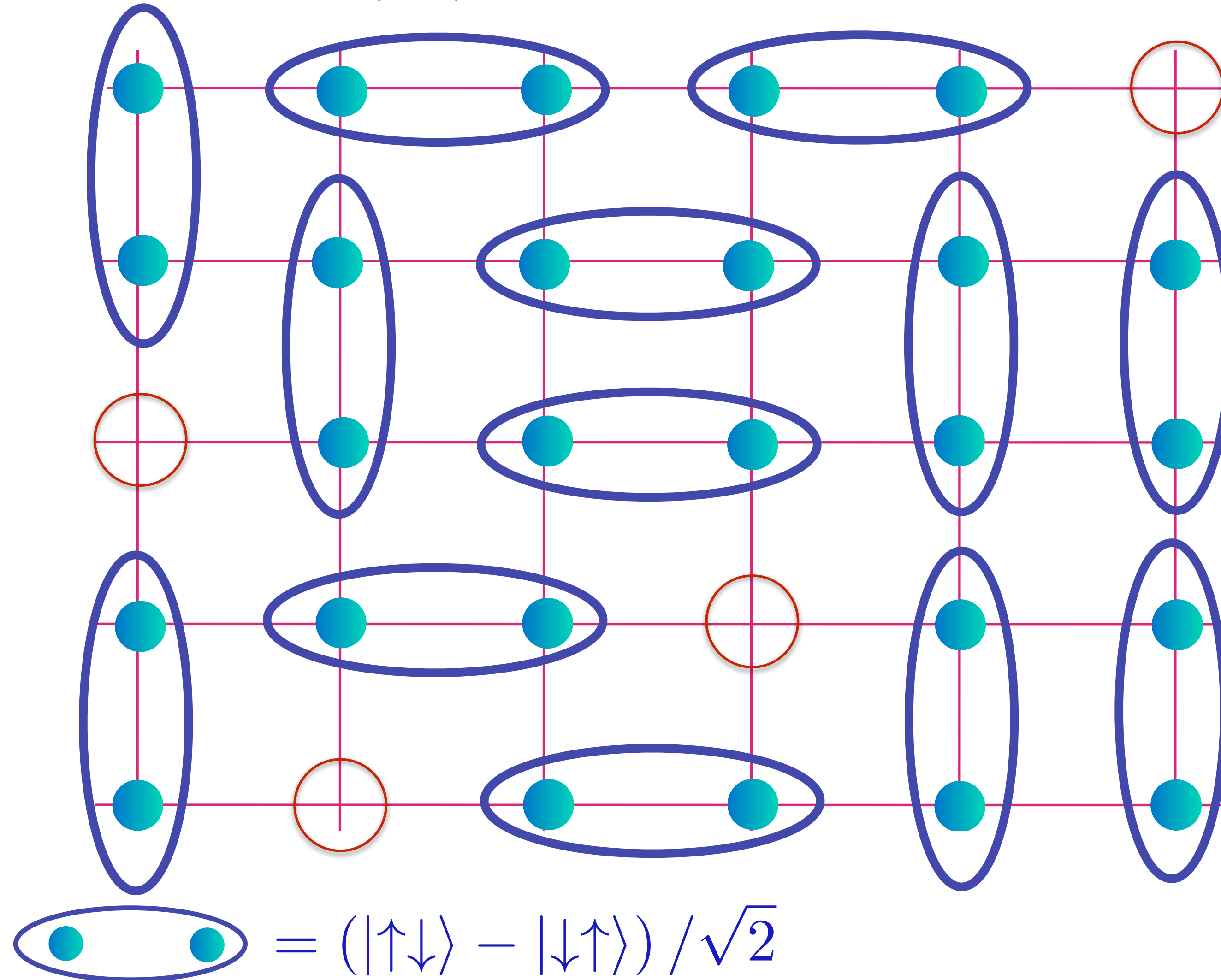
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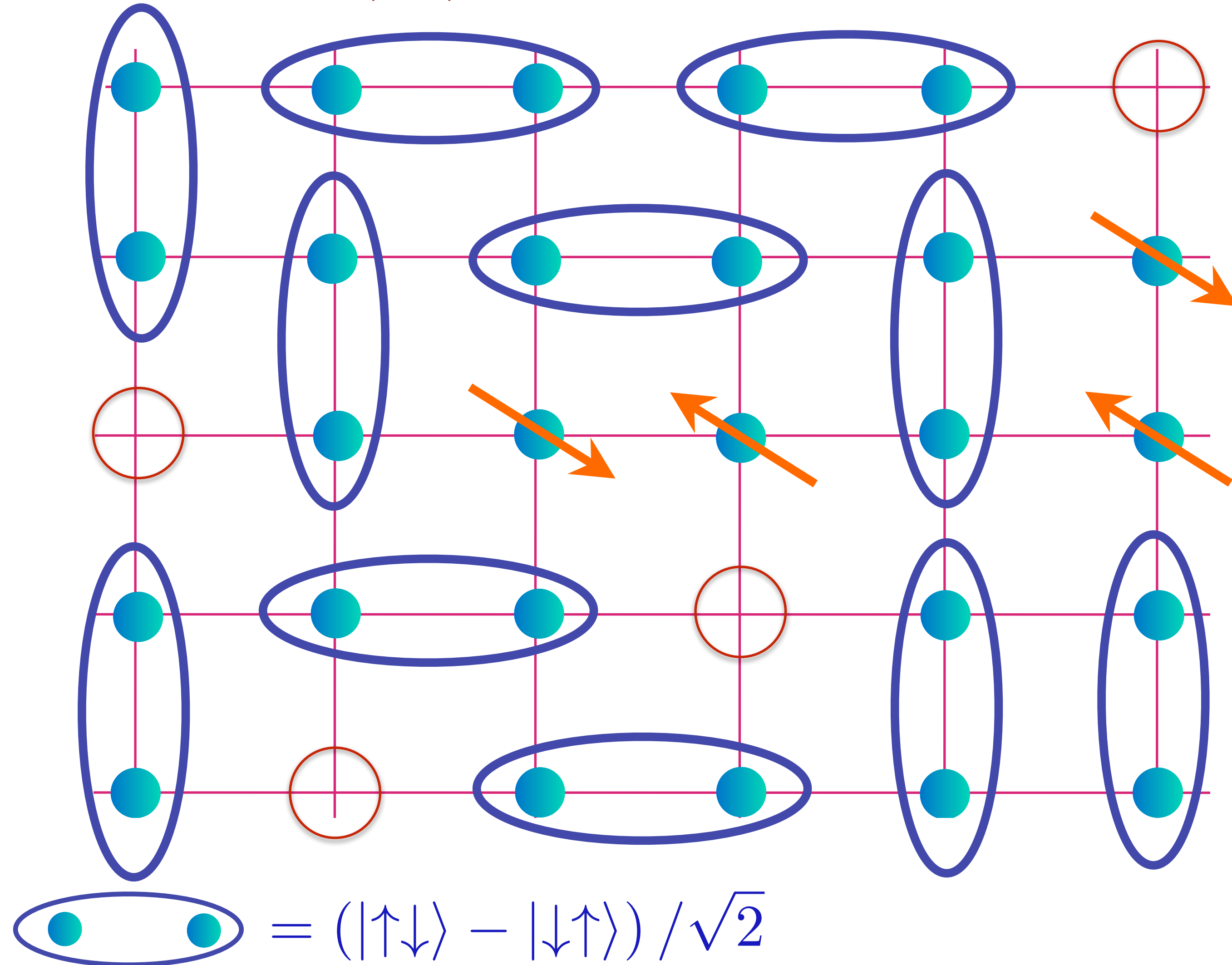
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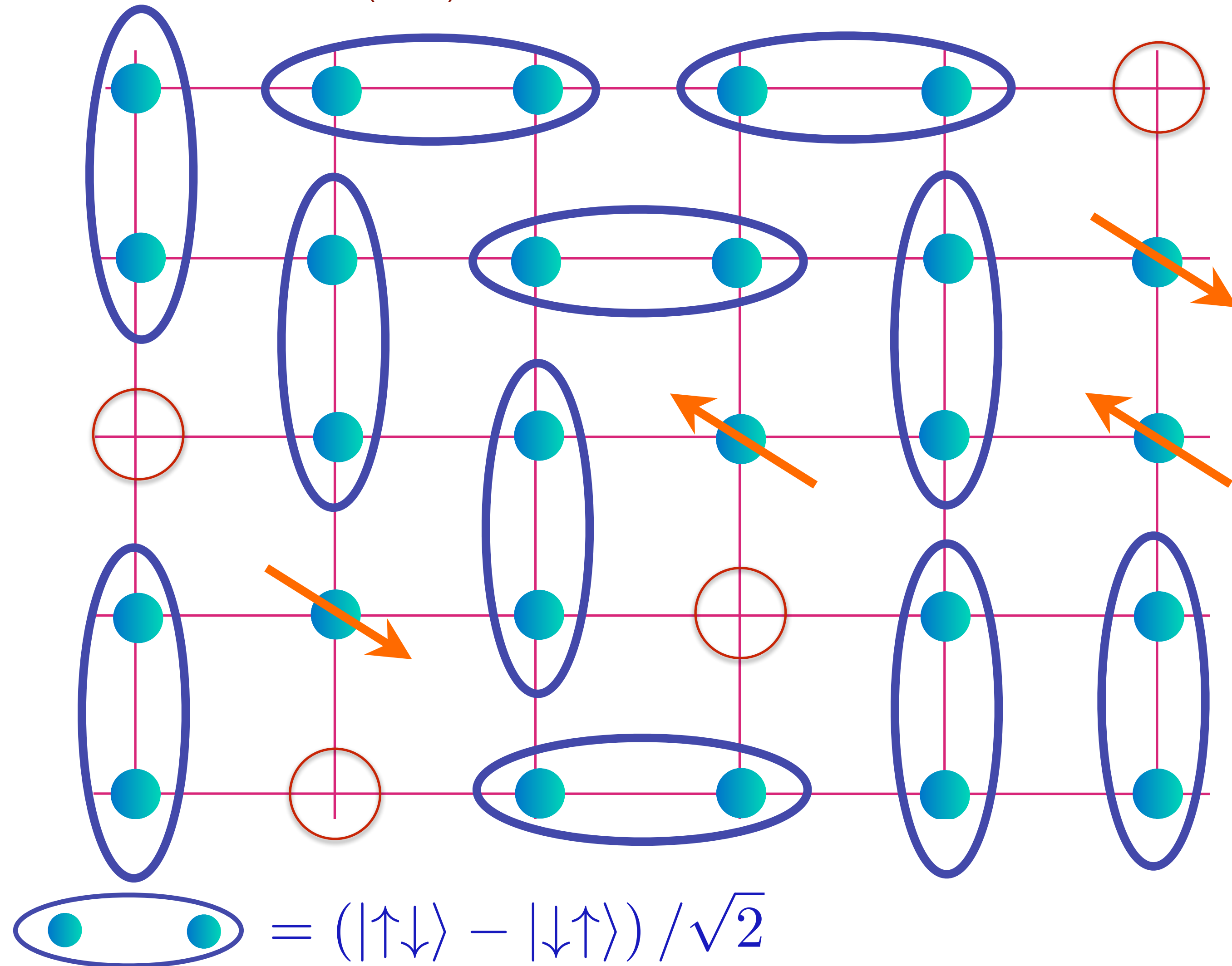
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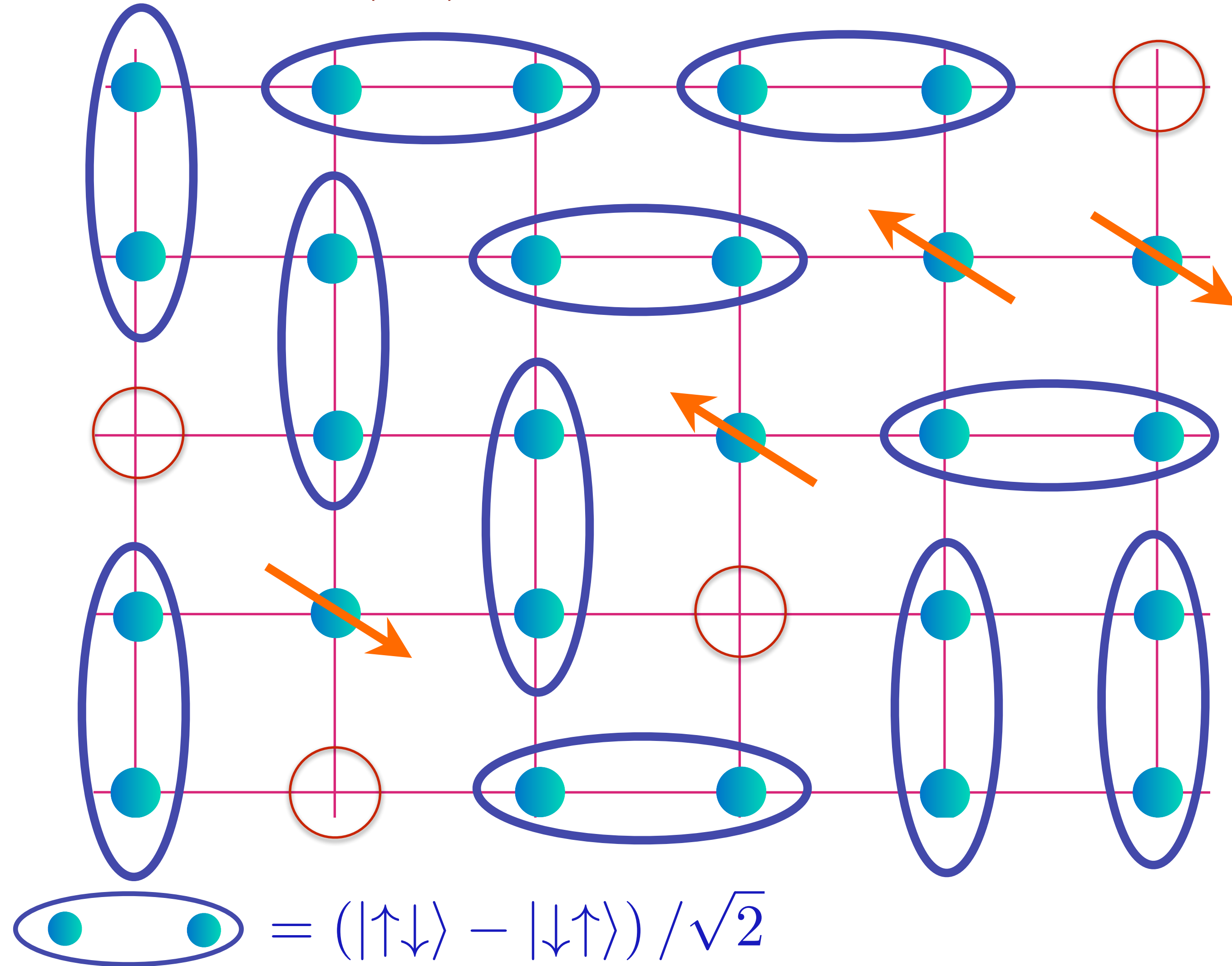
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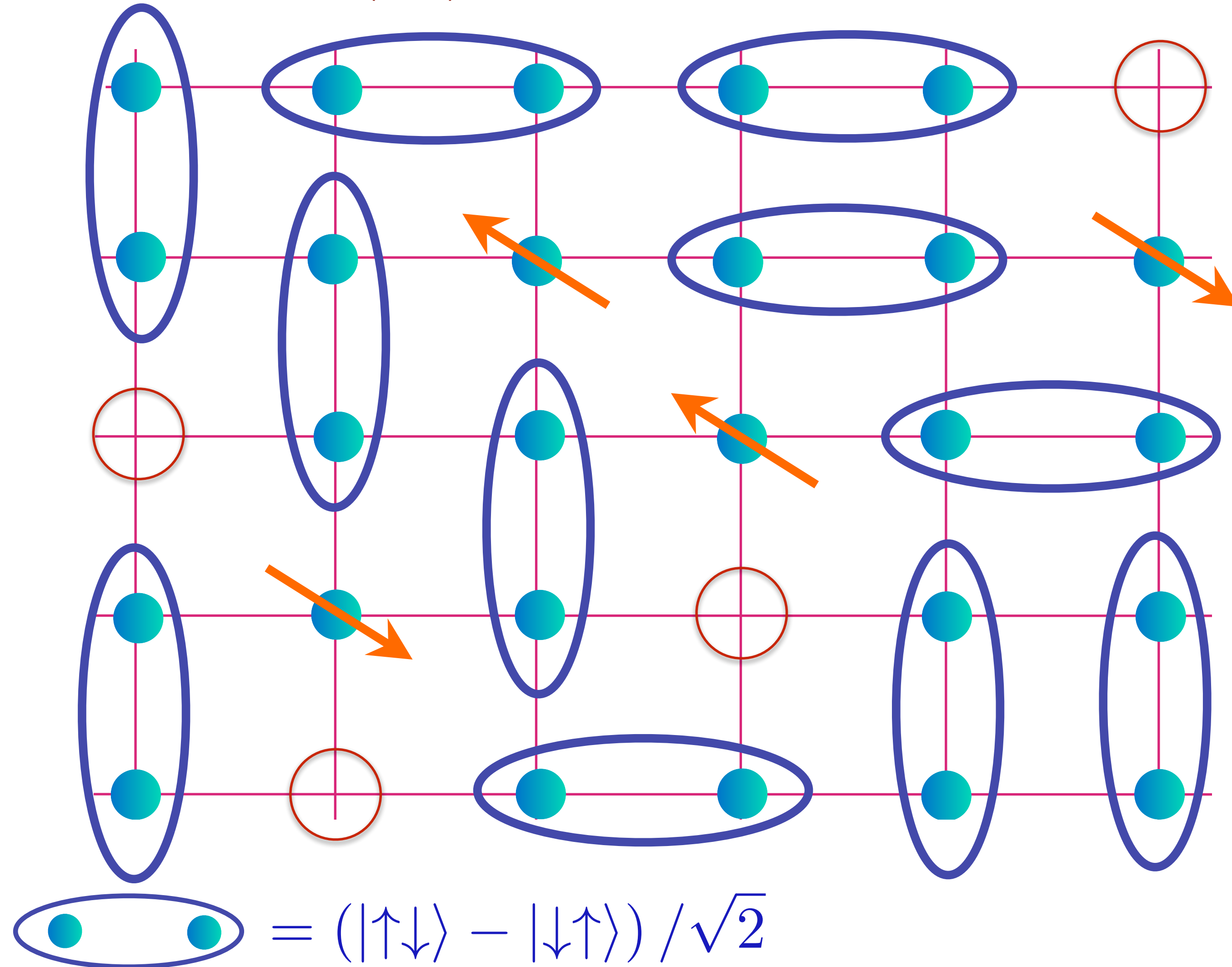
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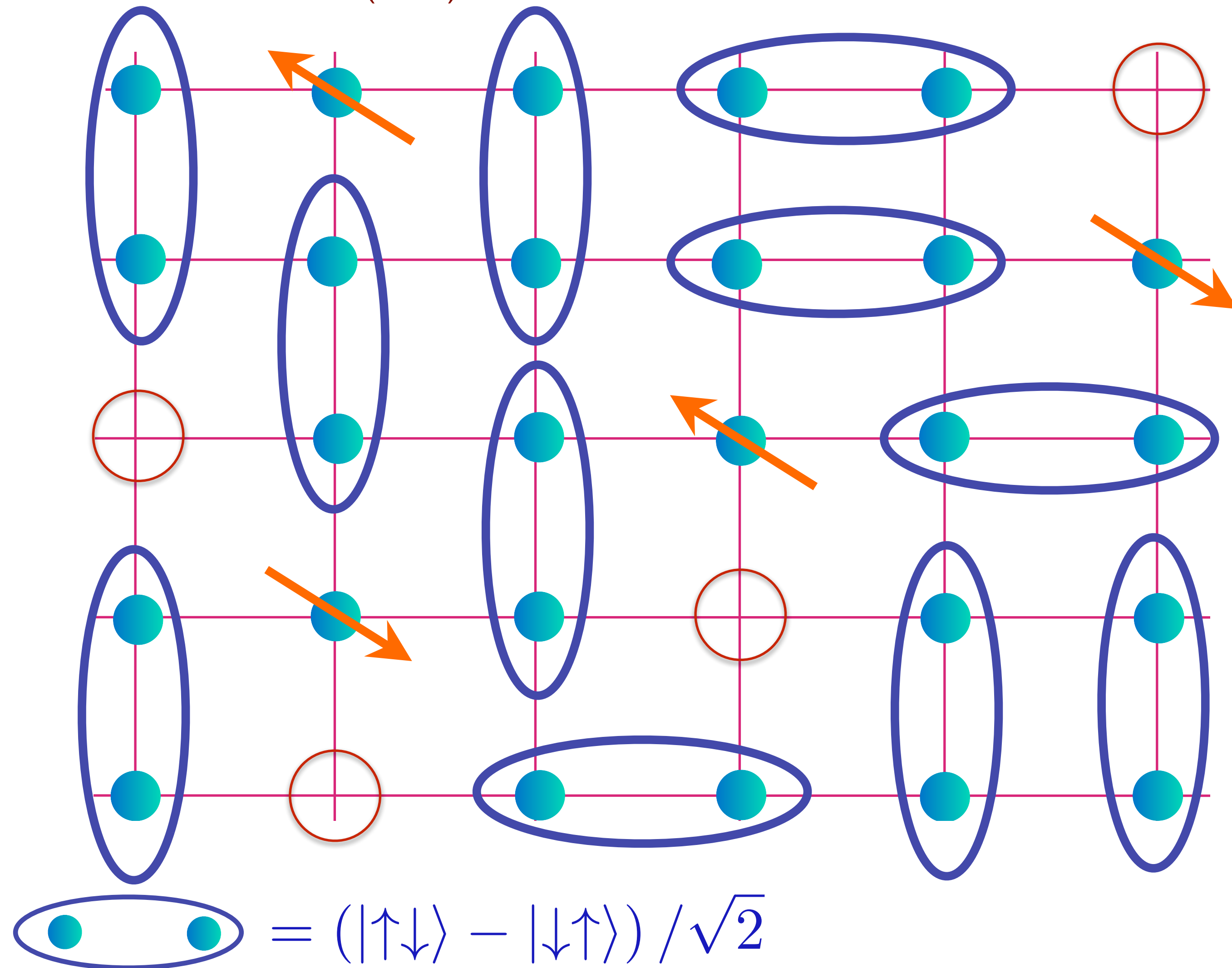
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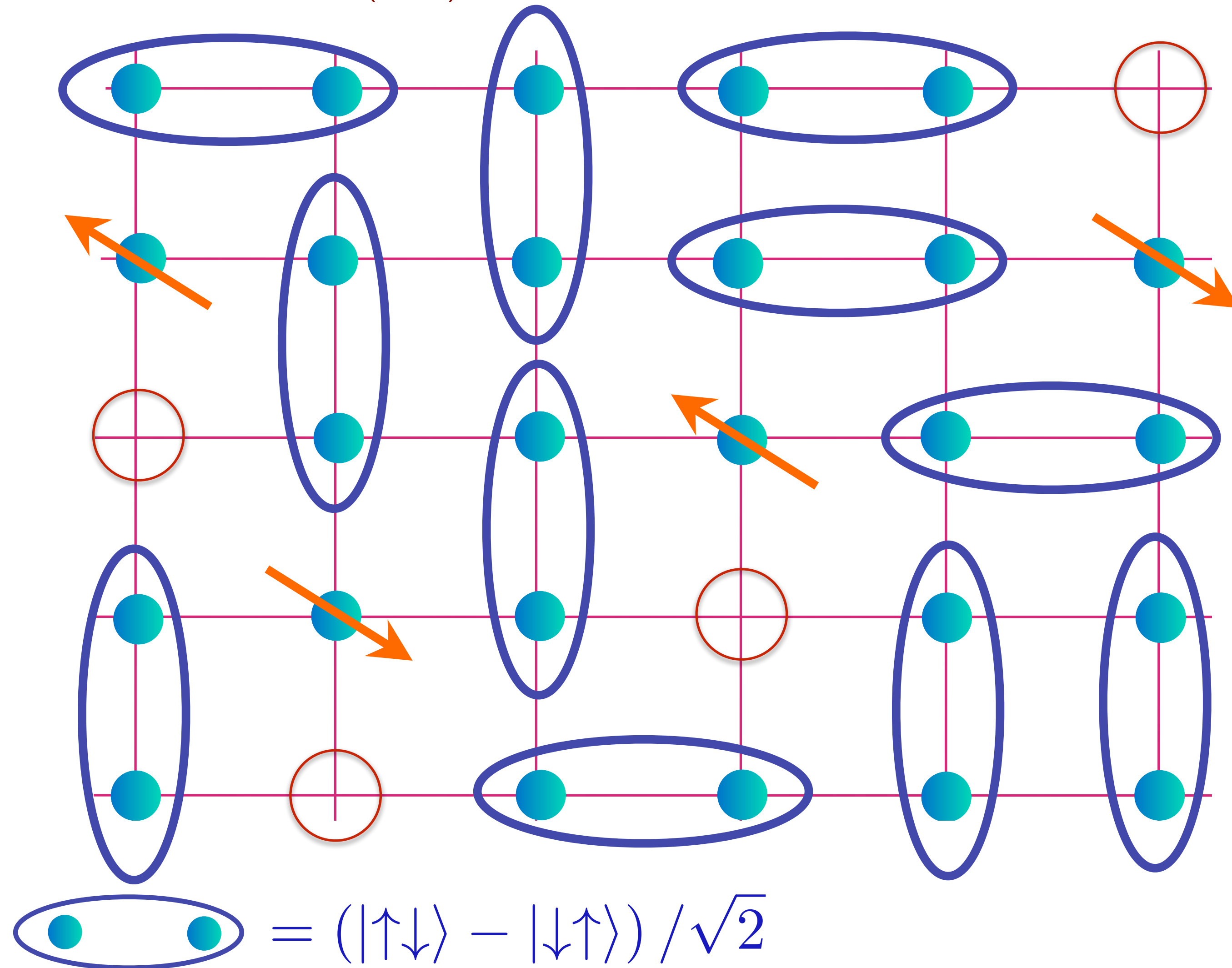
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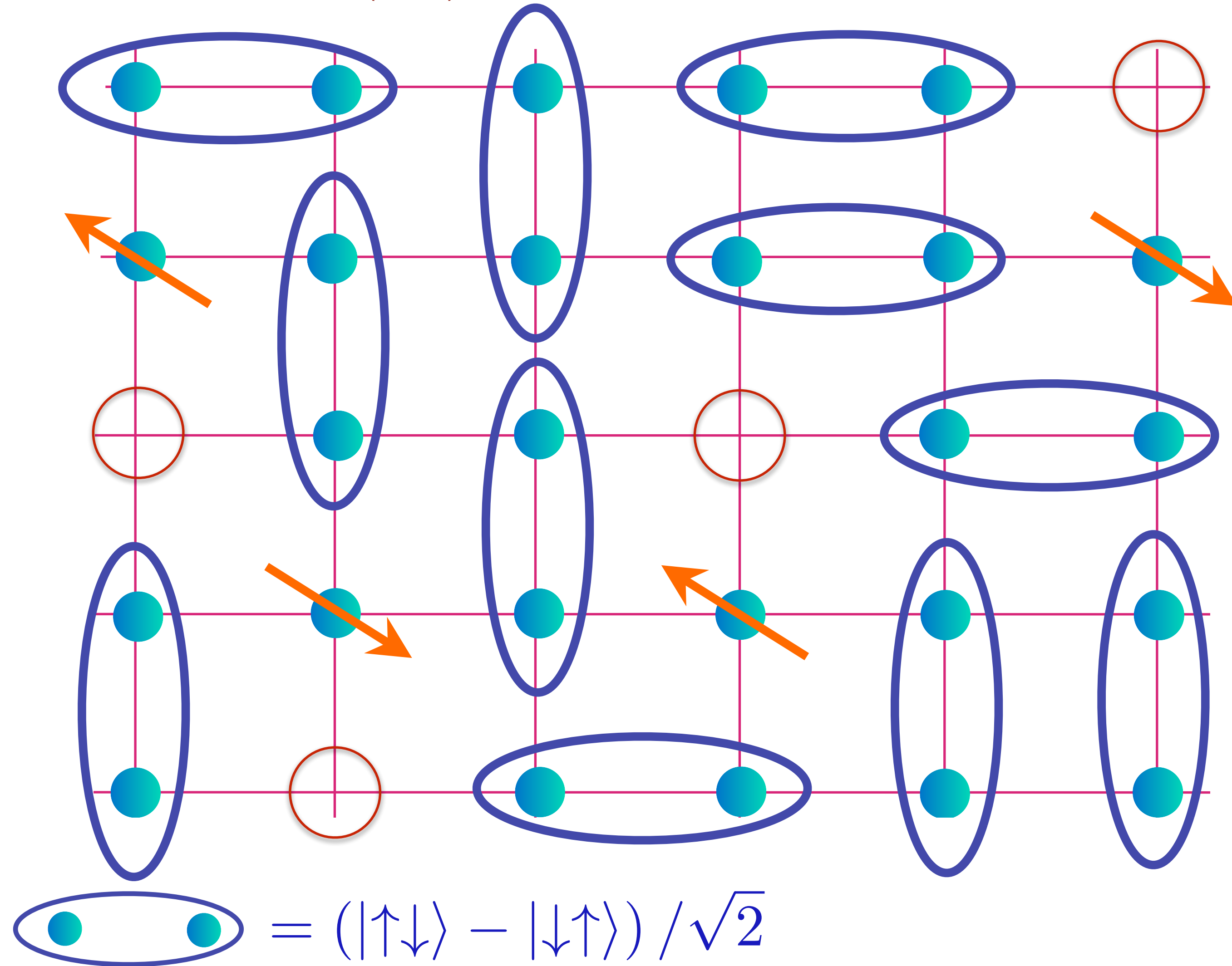
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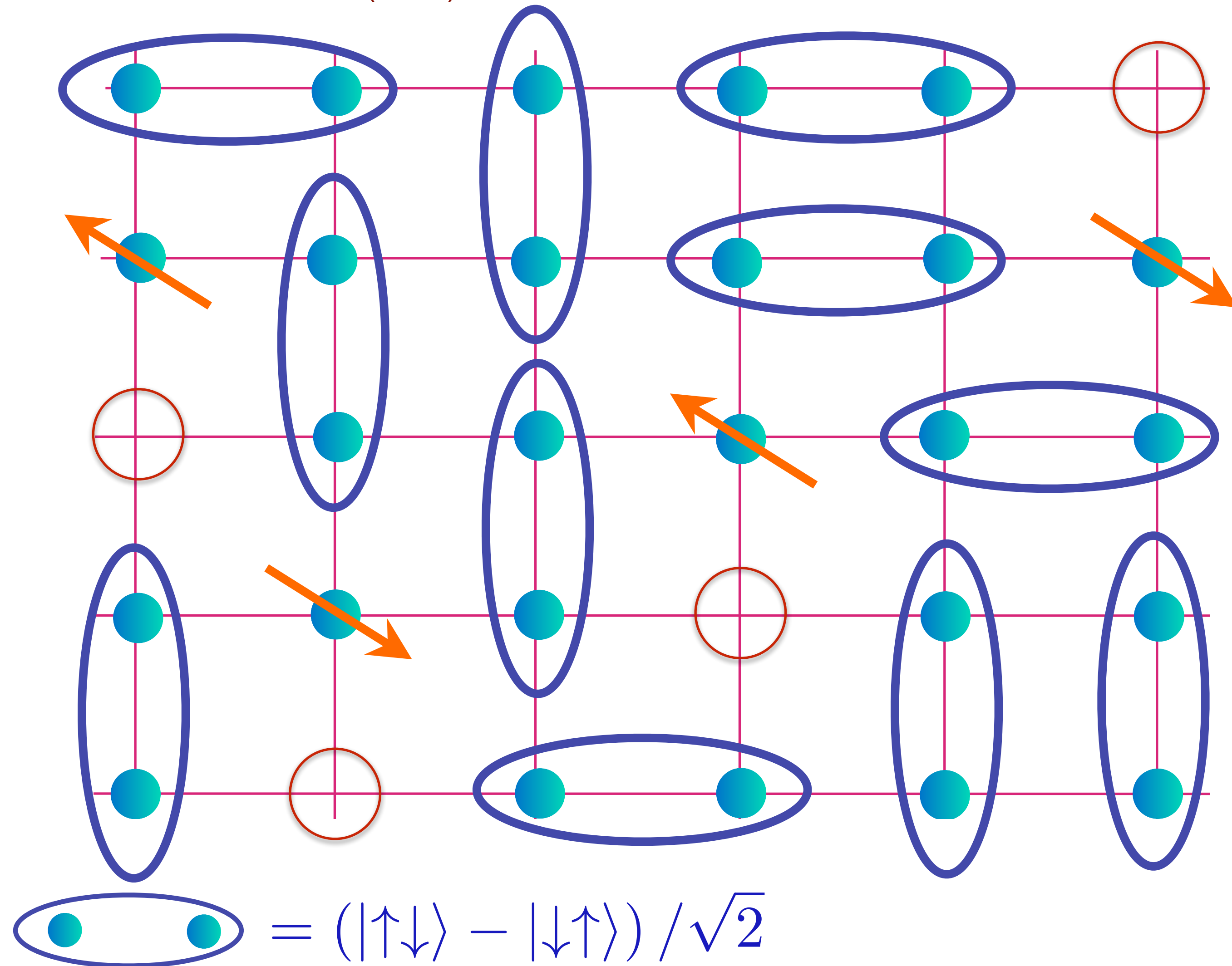
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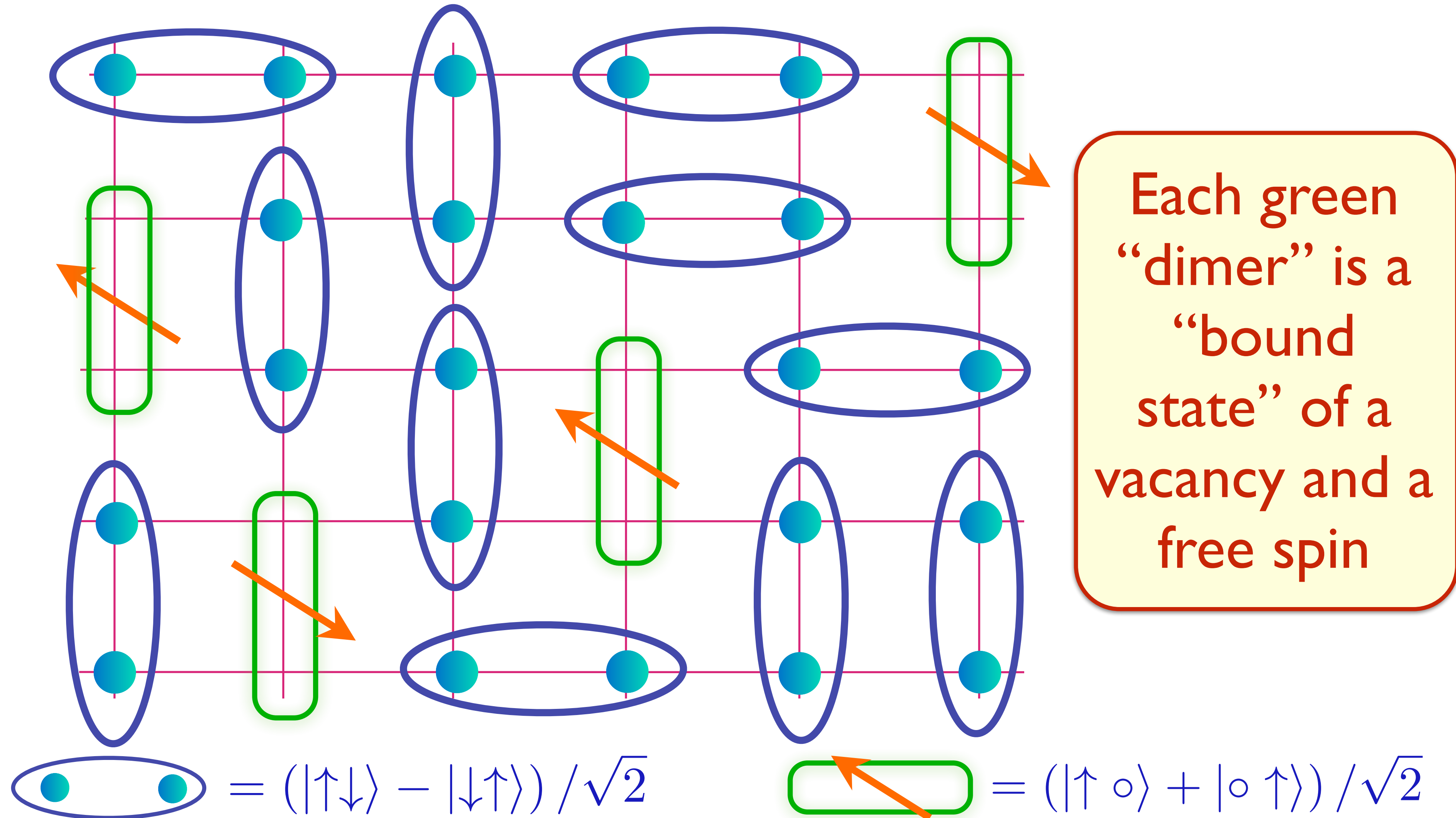


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FL* in a **one-band** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)

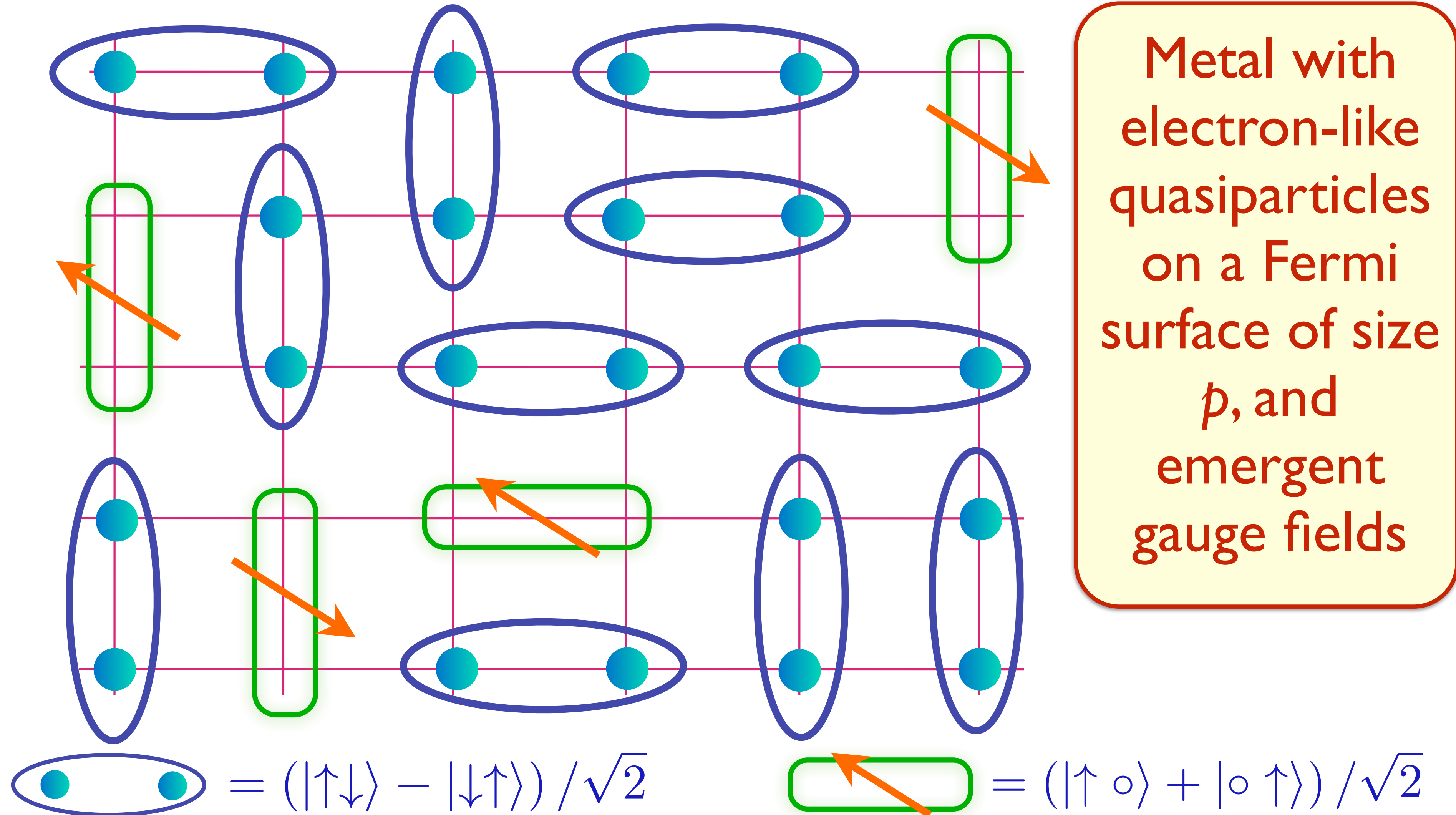


E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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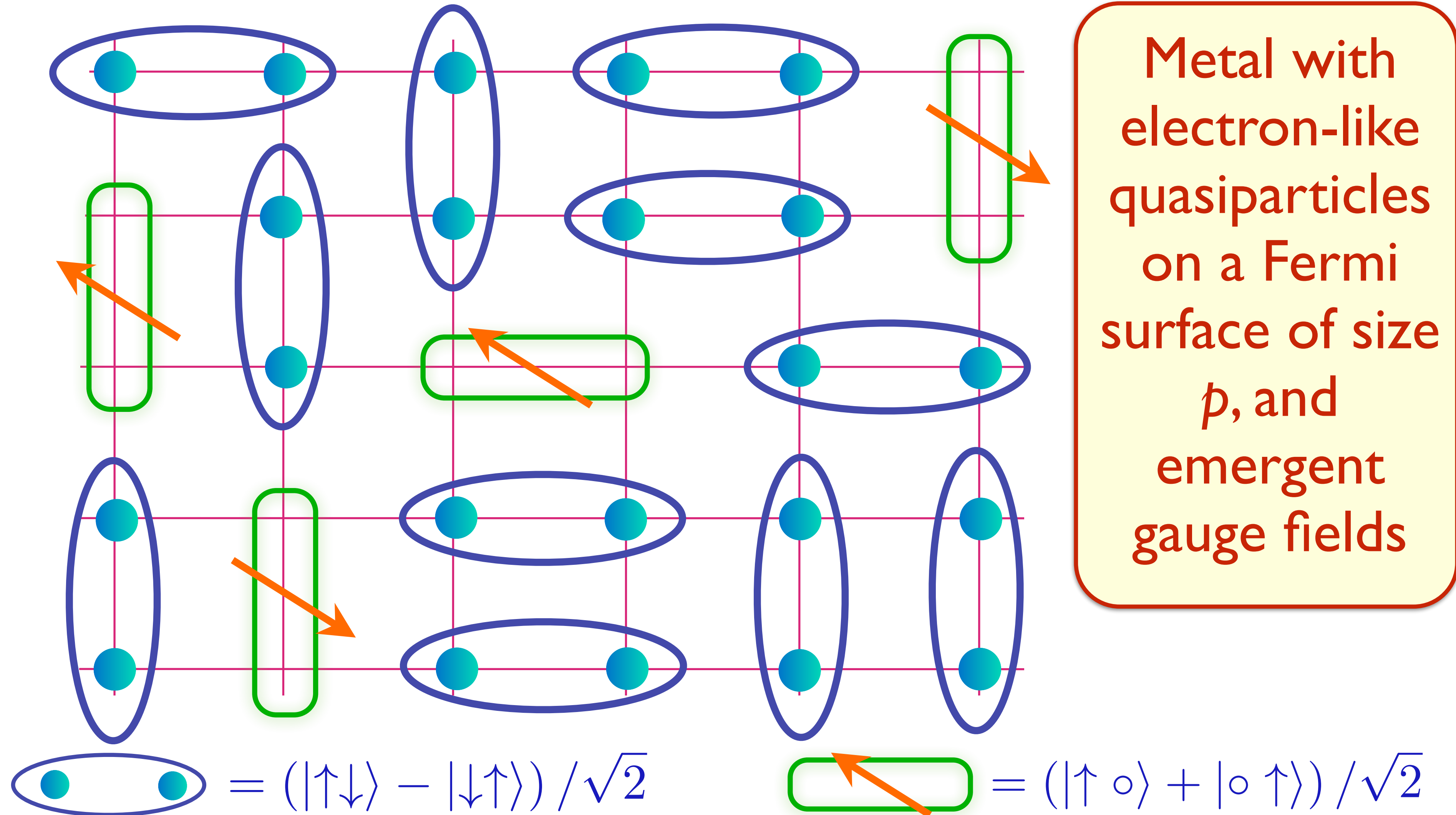


Metal with
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 surface of size
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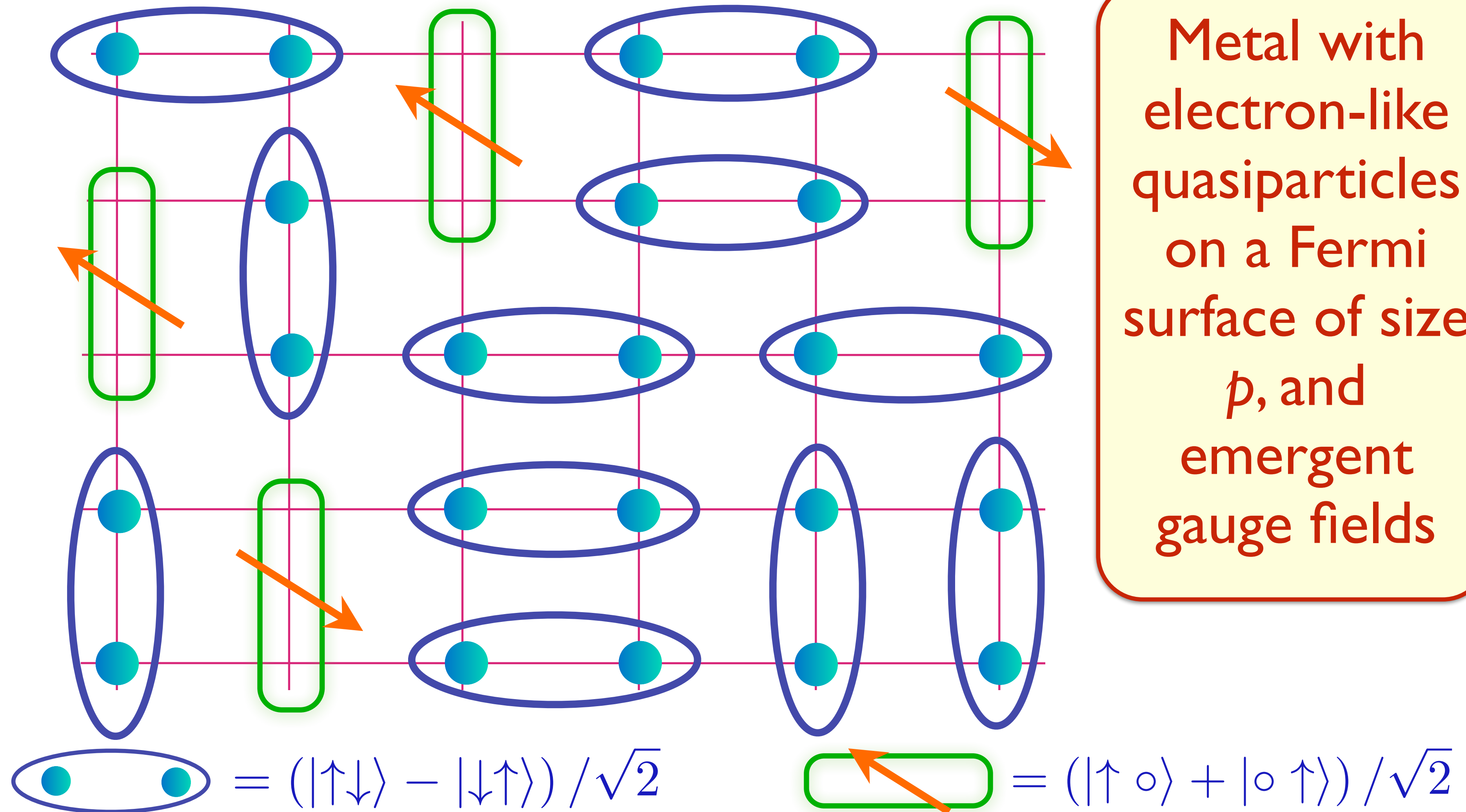


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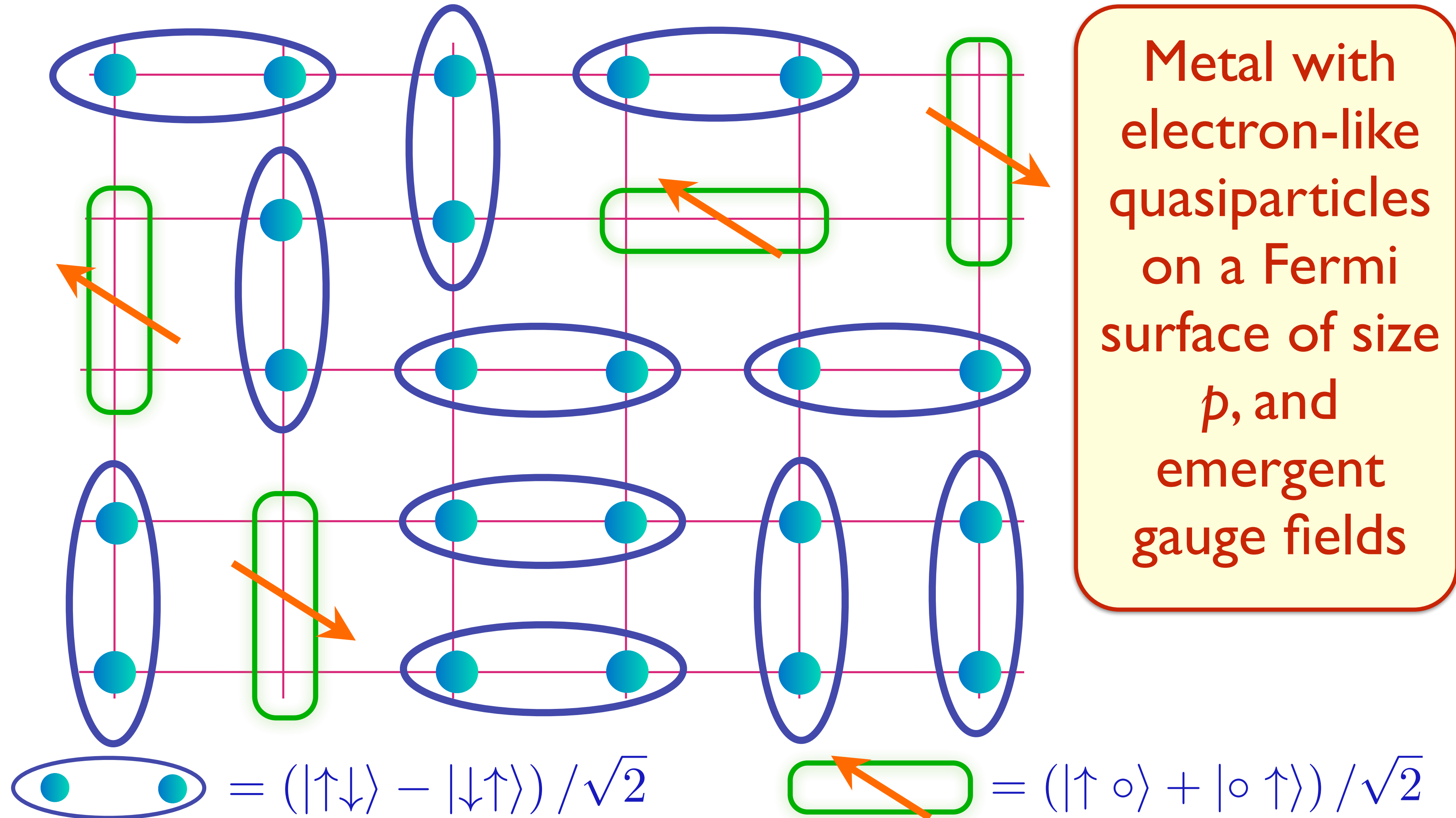


Metal with electron-like quasiparticles on a Fermi surface of size p , and emergent gauge fields

FL* in a **one-band** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)

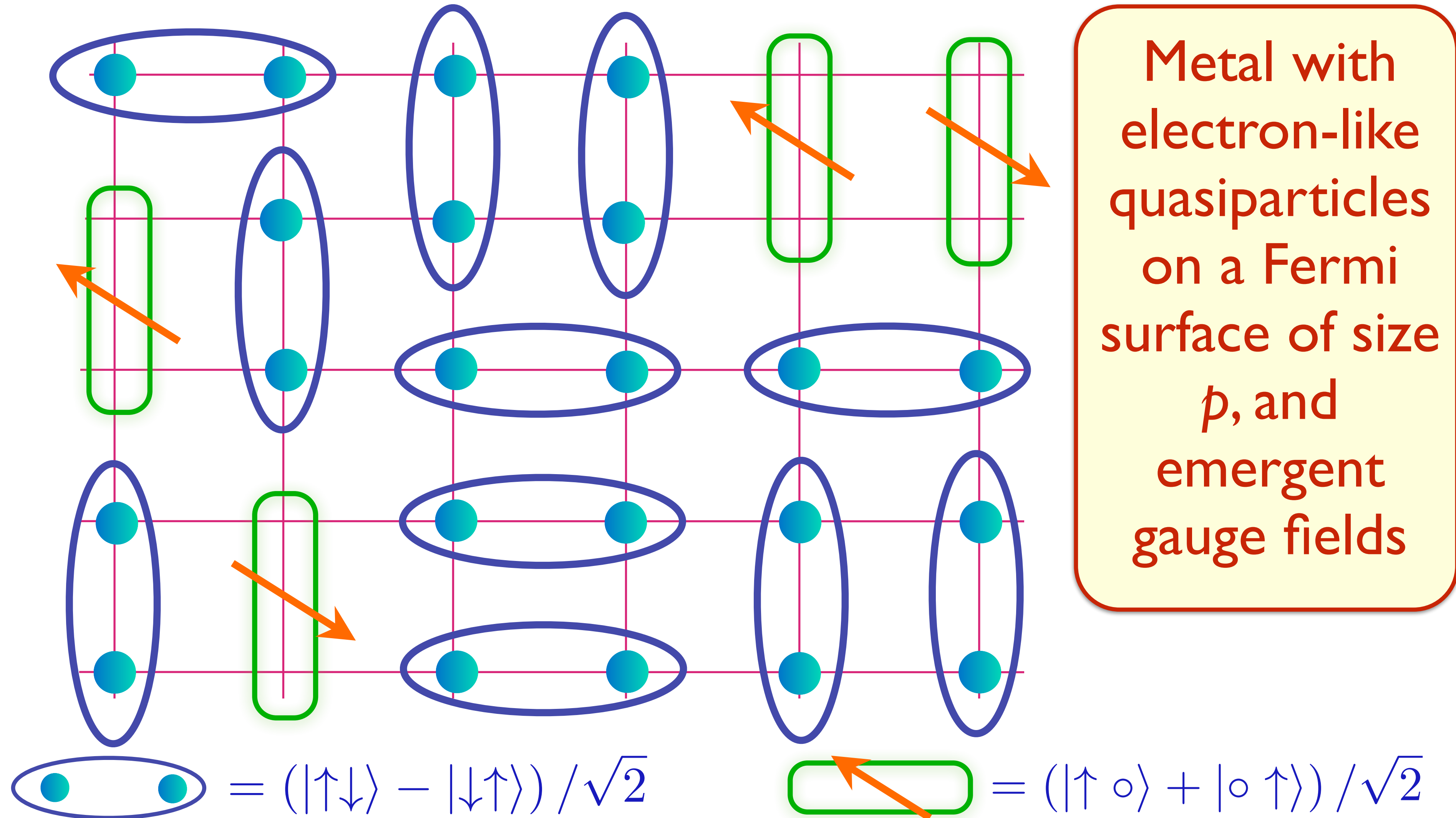


E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL* in a **one-band** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

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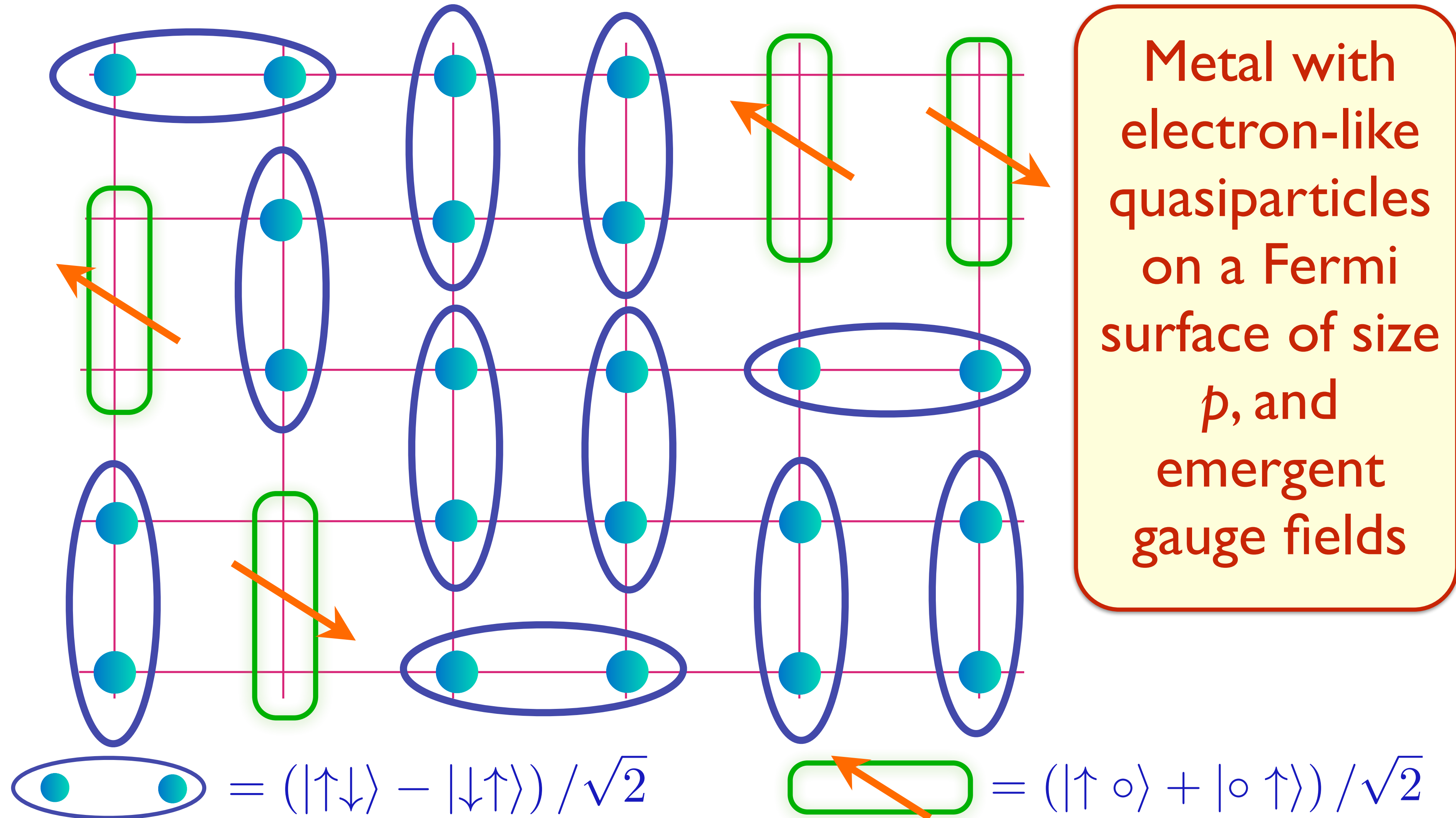


Metal with
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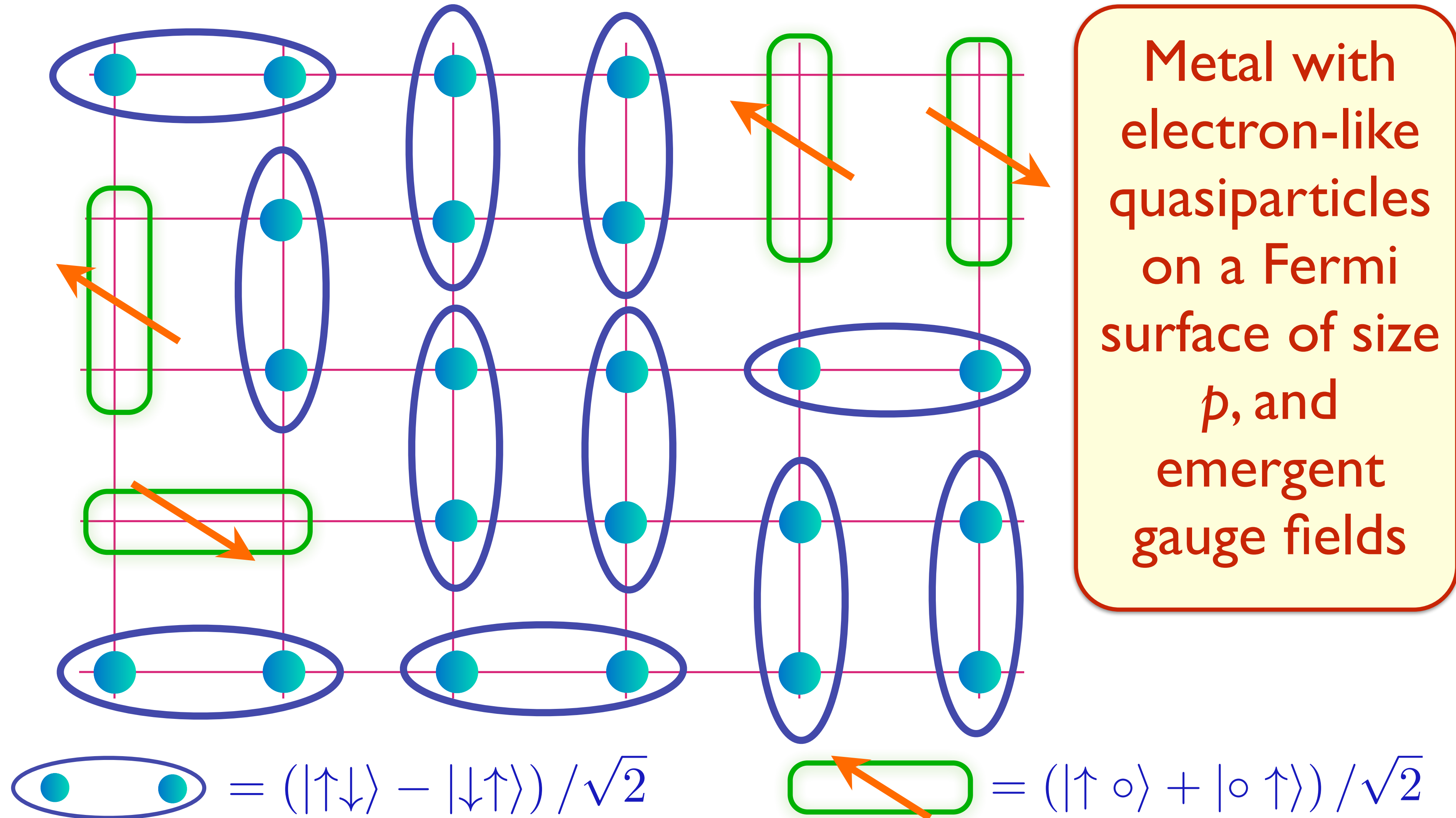


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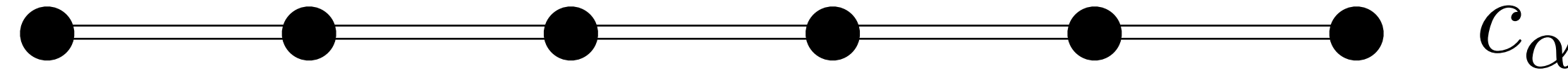
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Ancilla theory of the Hubbard model



Hubbard
model of
hole density
 $1+p$

Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S.,
PRR **2**, 023172 (2020)

Ya-Hui
Zhang

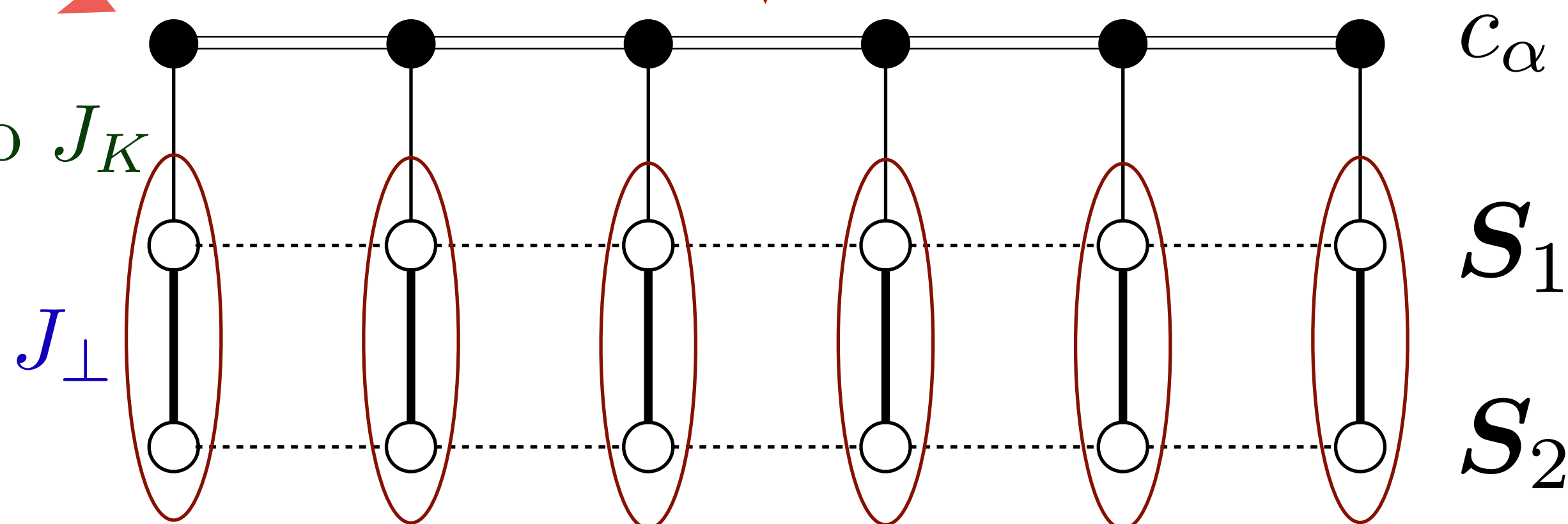
Free
holes of
density
 $1+p$

Schrieffer-Wolff transformation at large J_{\perp} yields $U \sim J_K^2/J_{\perp}$

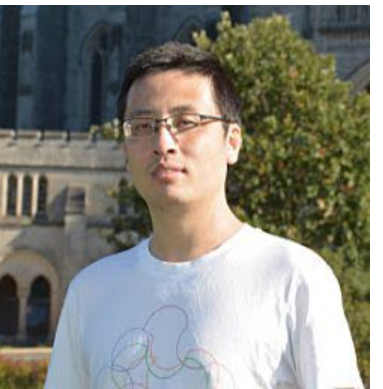
Hubbard
model of
hole density
 $1+p$

Antiferromagnetic Kondo J_K

Ancilla
qubits



$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^{\dagger} c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^{\dagger} \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \mathbf{S}_{1i} + J_{\perp} \sum_i \mathbf{S}_{1i} \cdot \mathbf{S}_{2i}$$



Ya-Hui Zhang

Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S.,
PRR 2, 023172 (2020)

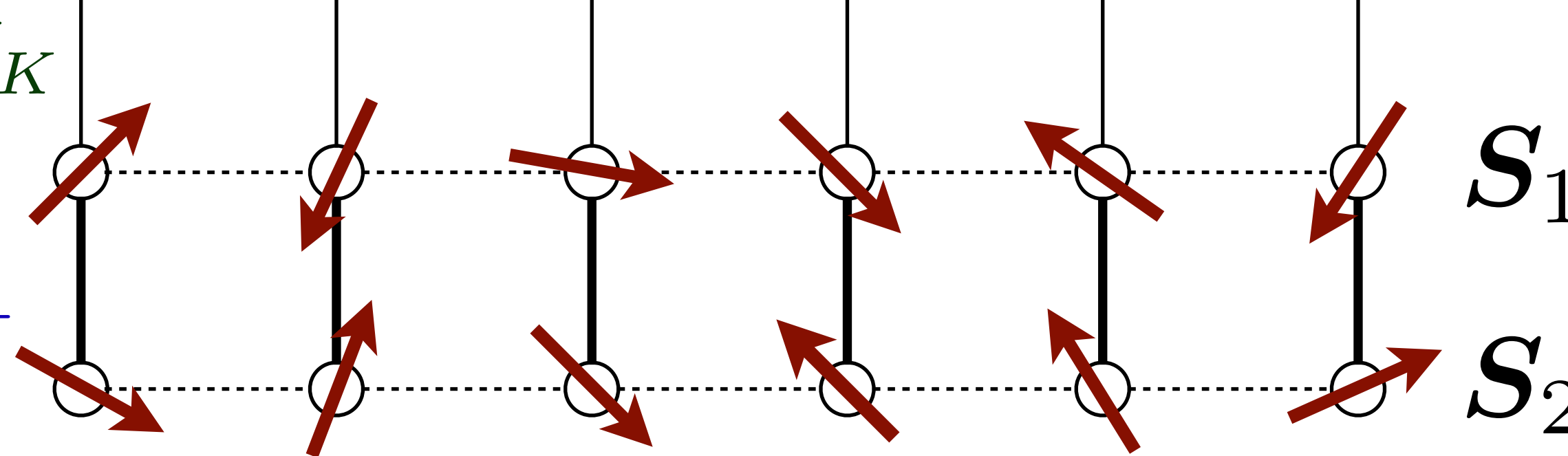
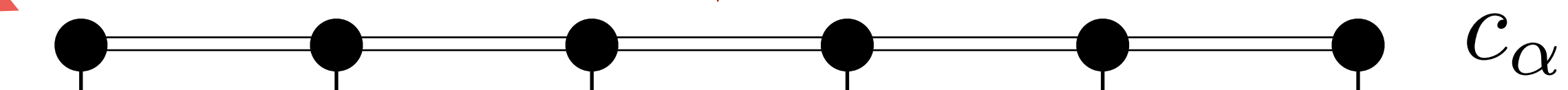
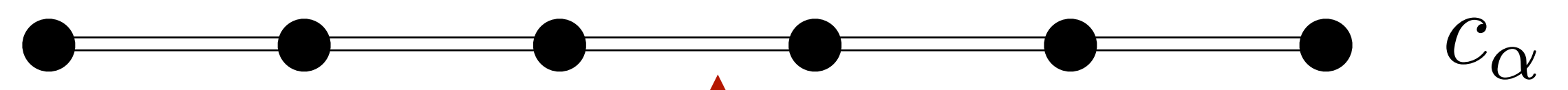
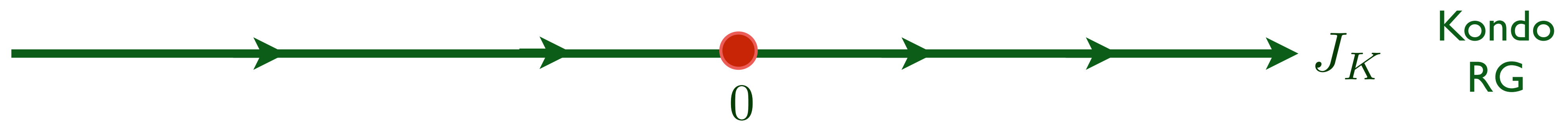
Free holes of density $1+p$

Hubbard model of hole density $1+p$

Antiferromagnetic Kondo J_K

Ancilla qubits

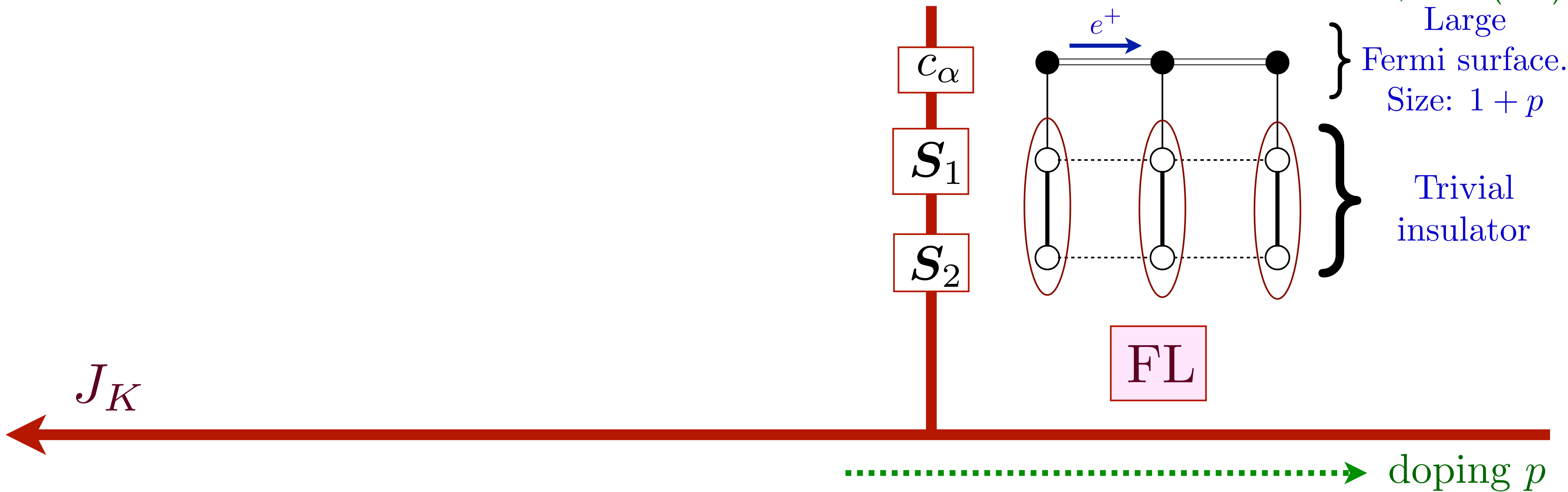
Ferromagnetic Kondo \tilde{J}_K



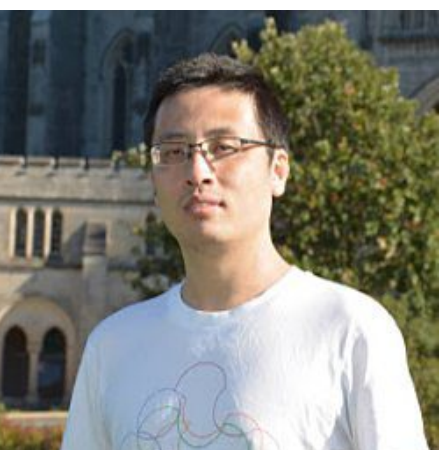
$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \epsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^\dagger \frac{\sigma_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \mathbf{S}_{1i} + J_\perp \sum_i \mathbf{S}_{1i} \cdot \mathbf{S}_{2i}$$

Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S.,
PRR **2**, 023172 (2020)

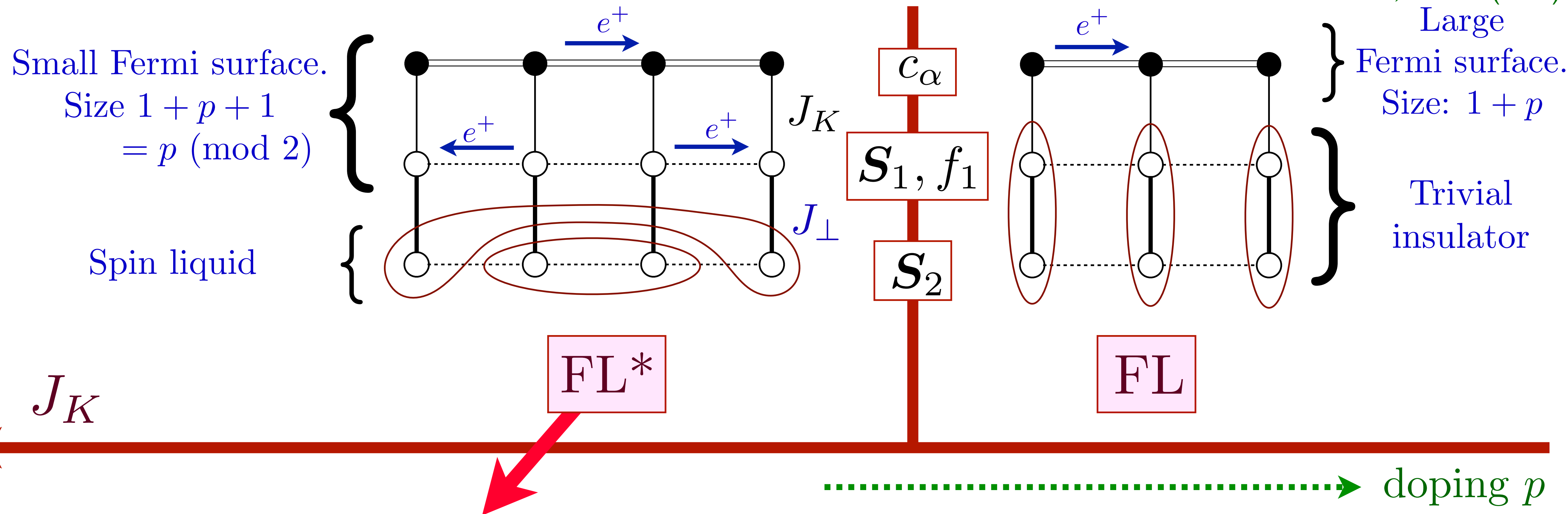


Ya-Hui
Zhang



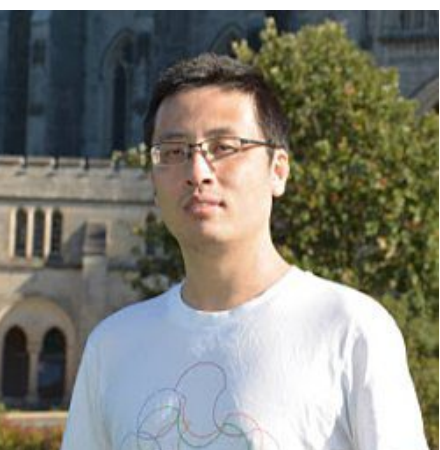
Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S.,
PRR **2**, 023172 (2020)

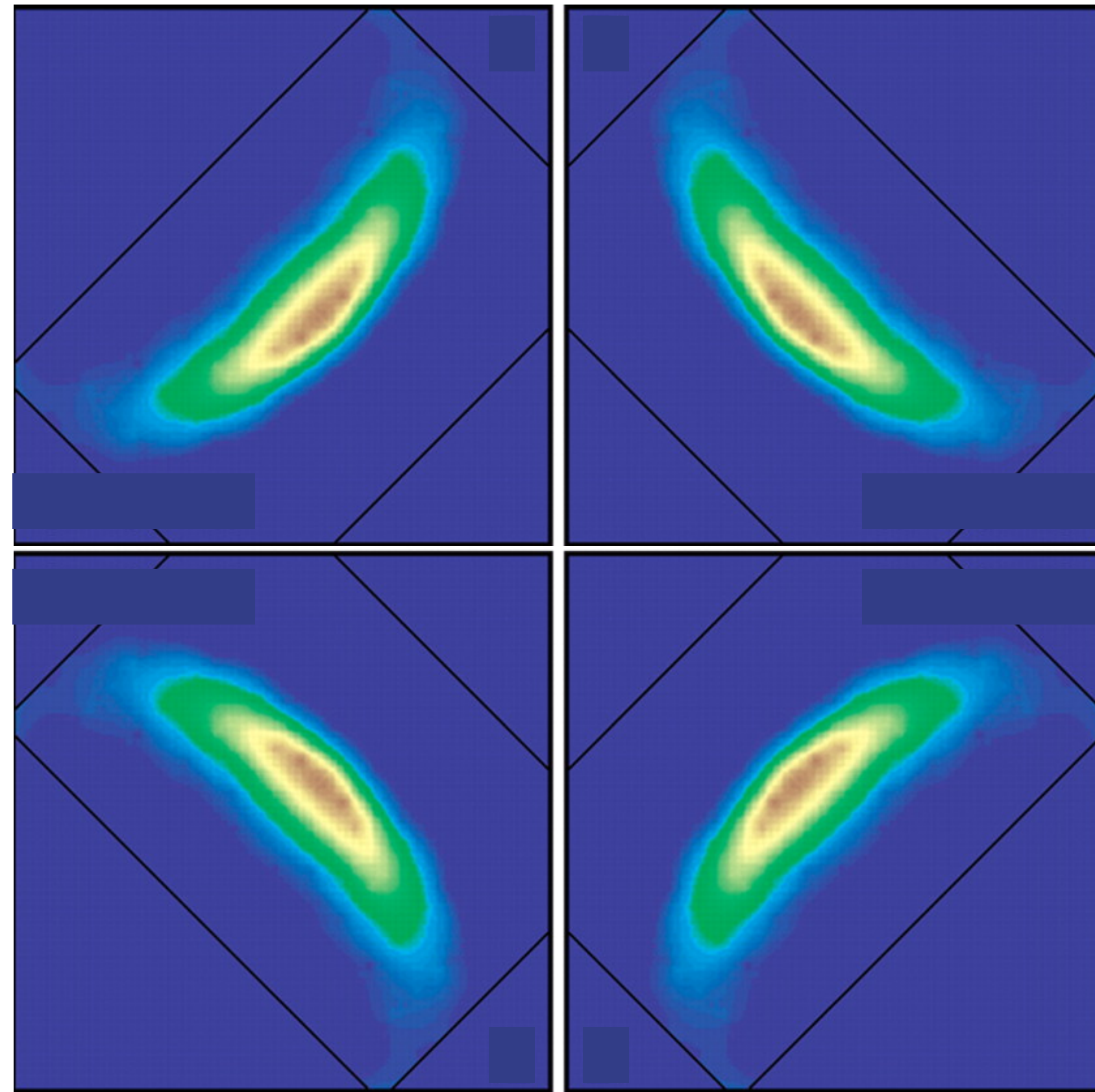


Pseudogap metal =
Kondo Lattice Heavy
Fermi Liquid $\langle c_\alpha^\dagger f_{1\alpha} \rangle \neq 0$
 \oplus
Spin Liquid

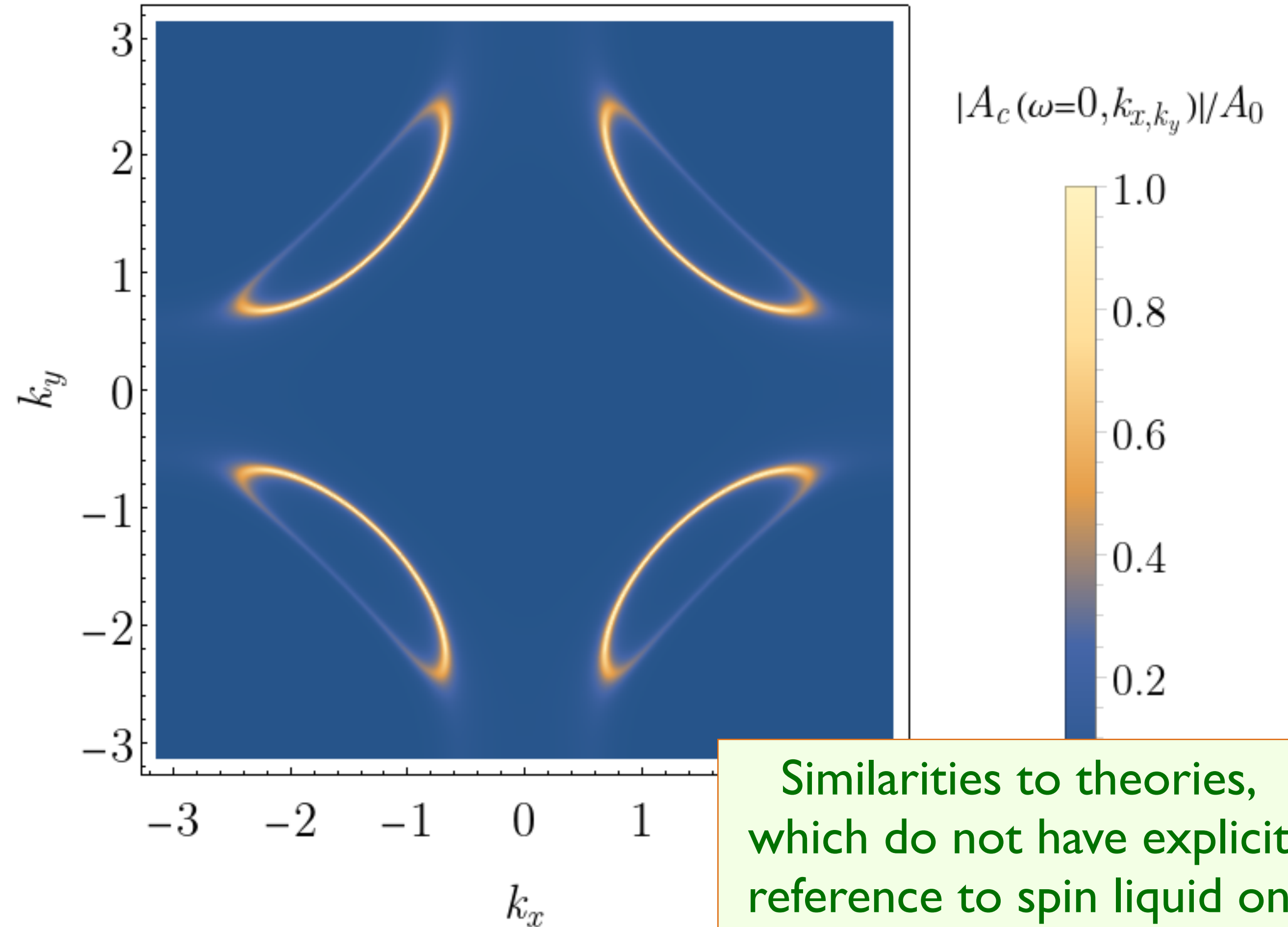
Ya-Hui
Zhang



Photoemission at small p



$\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$
at $x = 0.10$



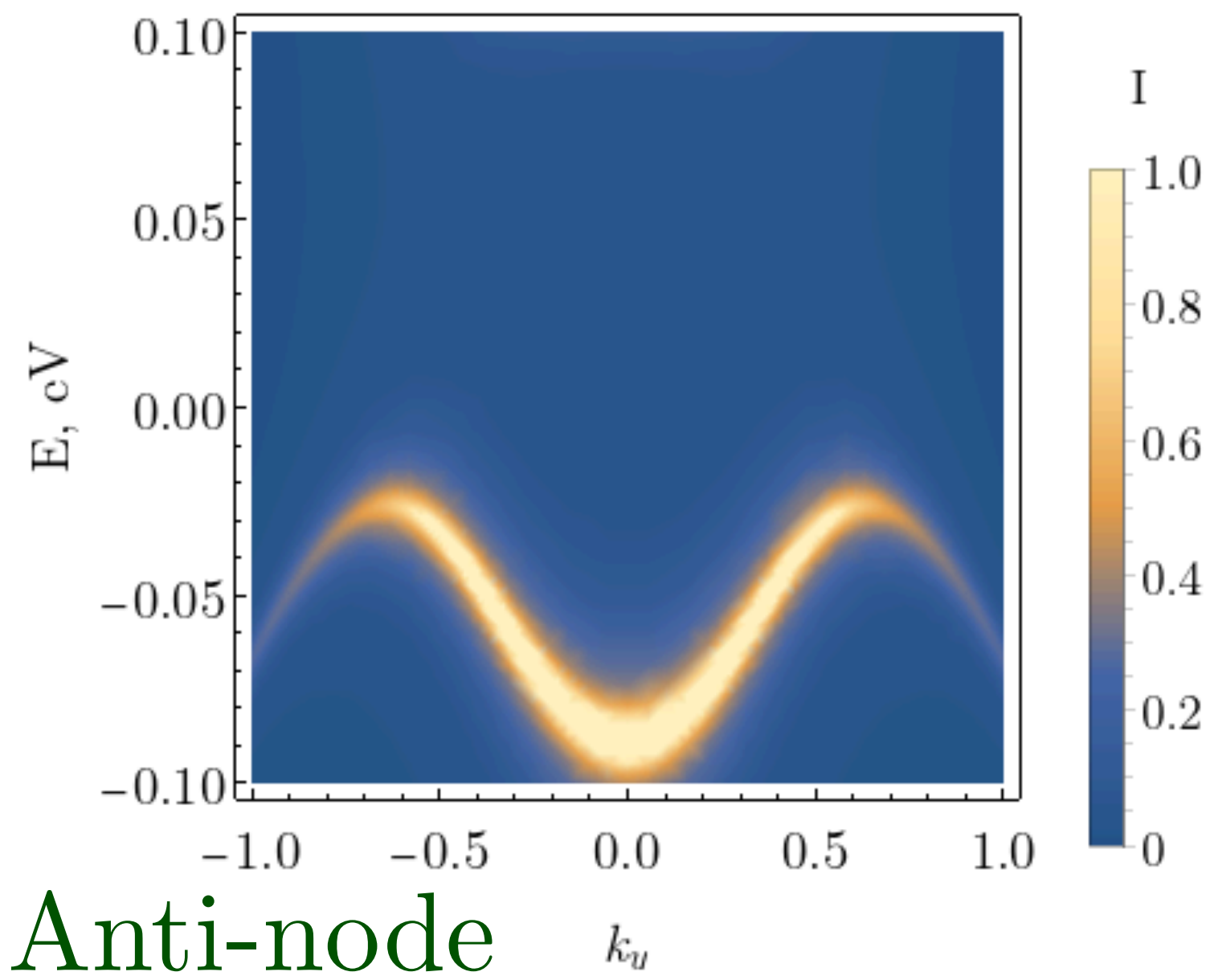
“*Fermi arcs*”

Similarities to theories,
which do not have explicit
reference to spin liquid on
second ancilla layer

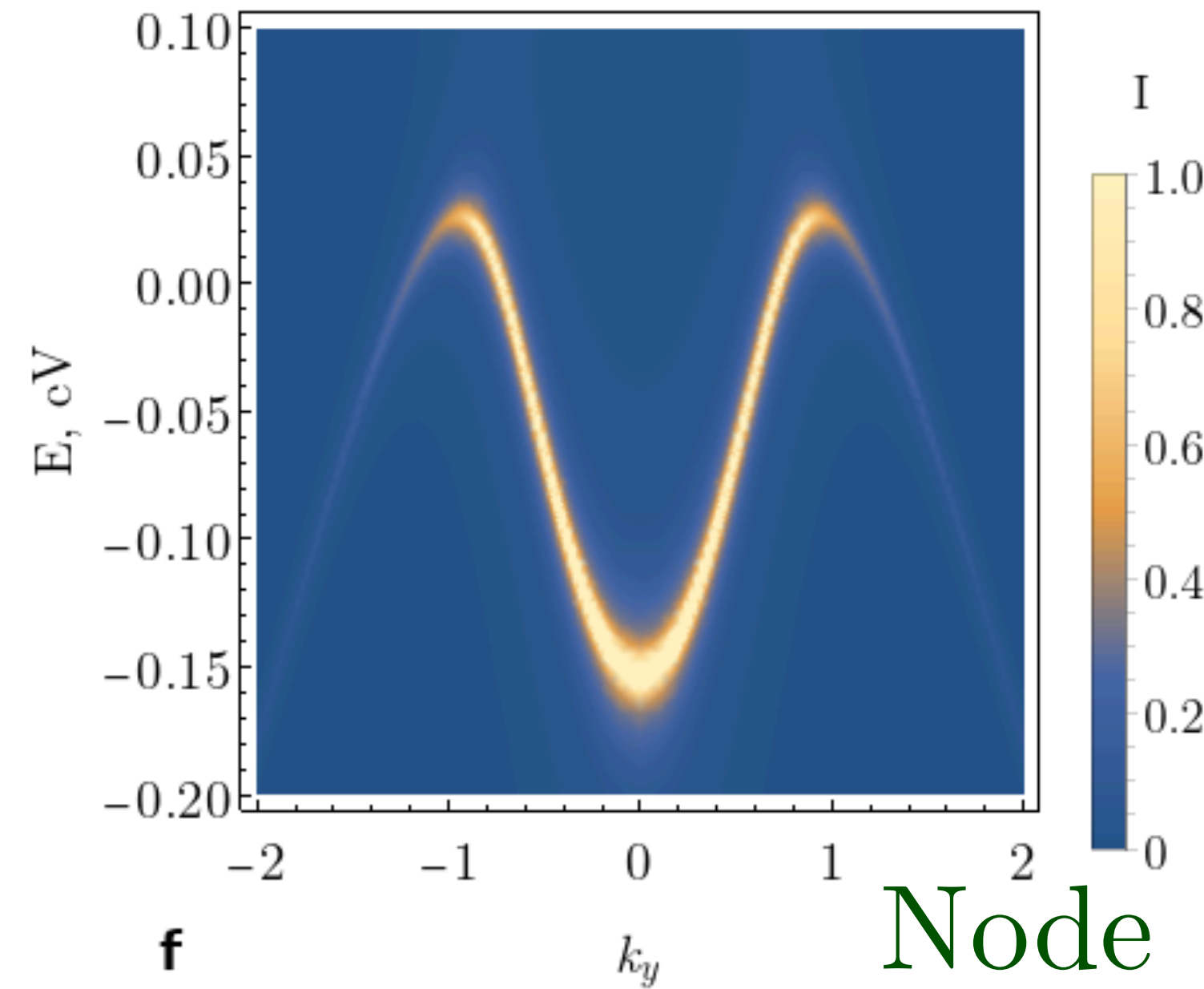
Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang,
PRB **73**, 174501 (2006)
S. Sakai, Y. Motome, M. Imada,
PRL **102**, 056404 (2009)

**FL* in a
one-band model**

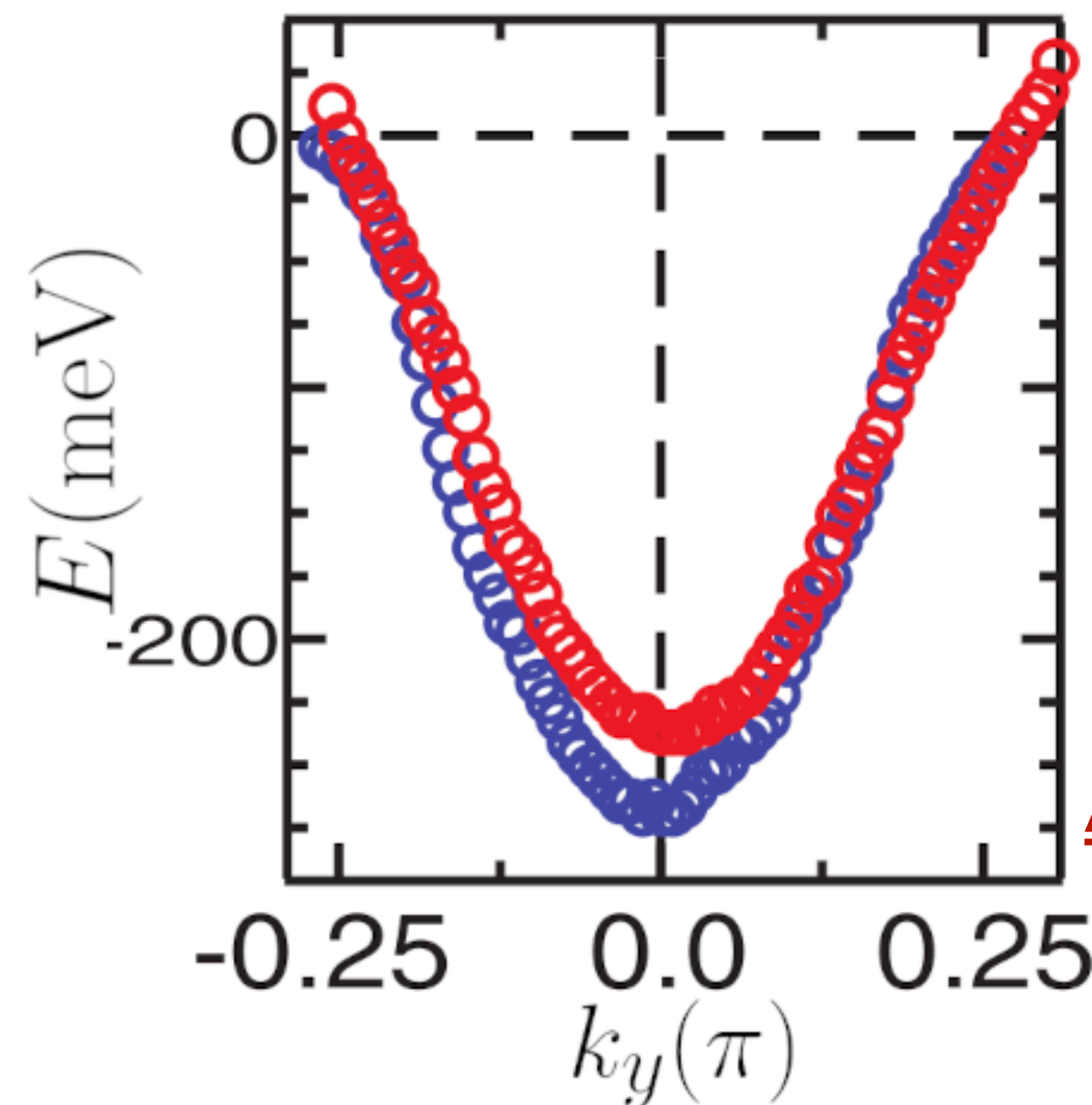
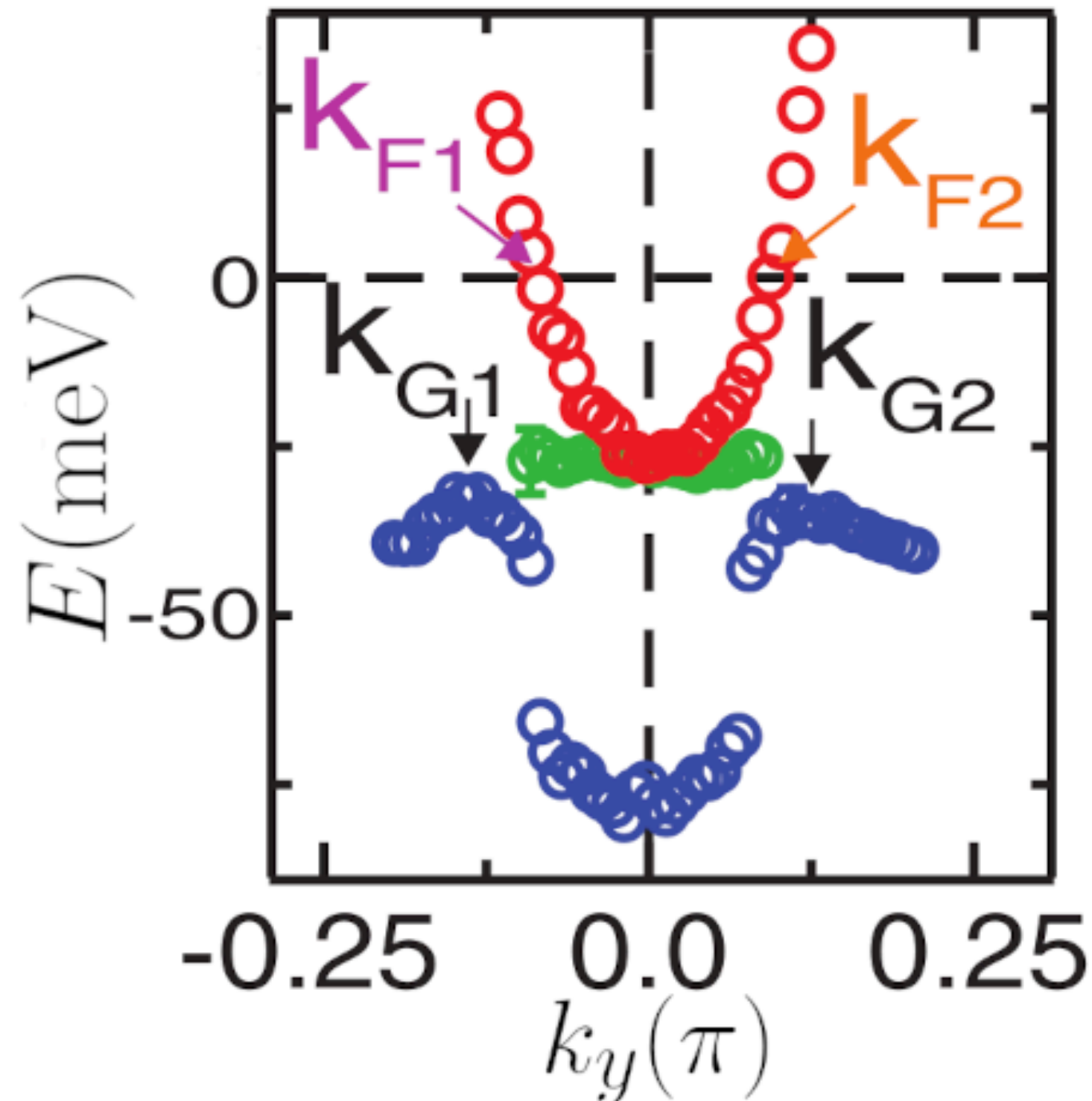
Second ancilla layer is needed
to describe MDC and EDC



Anti-node



Node

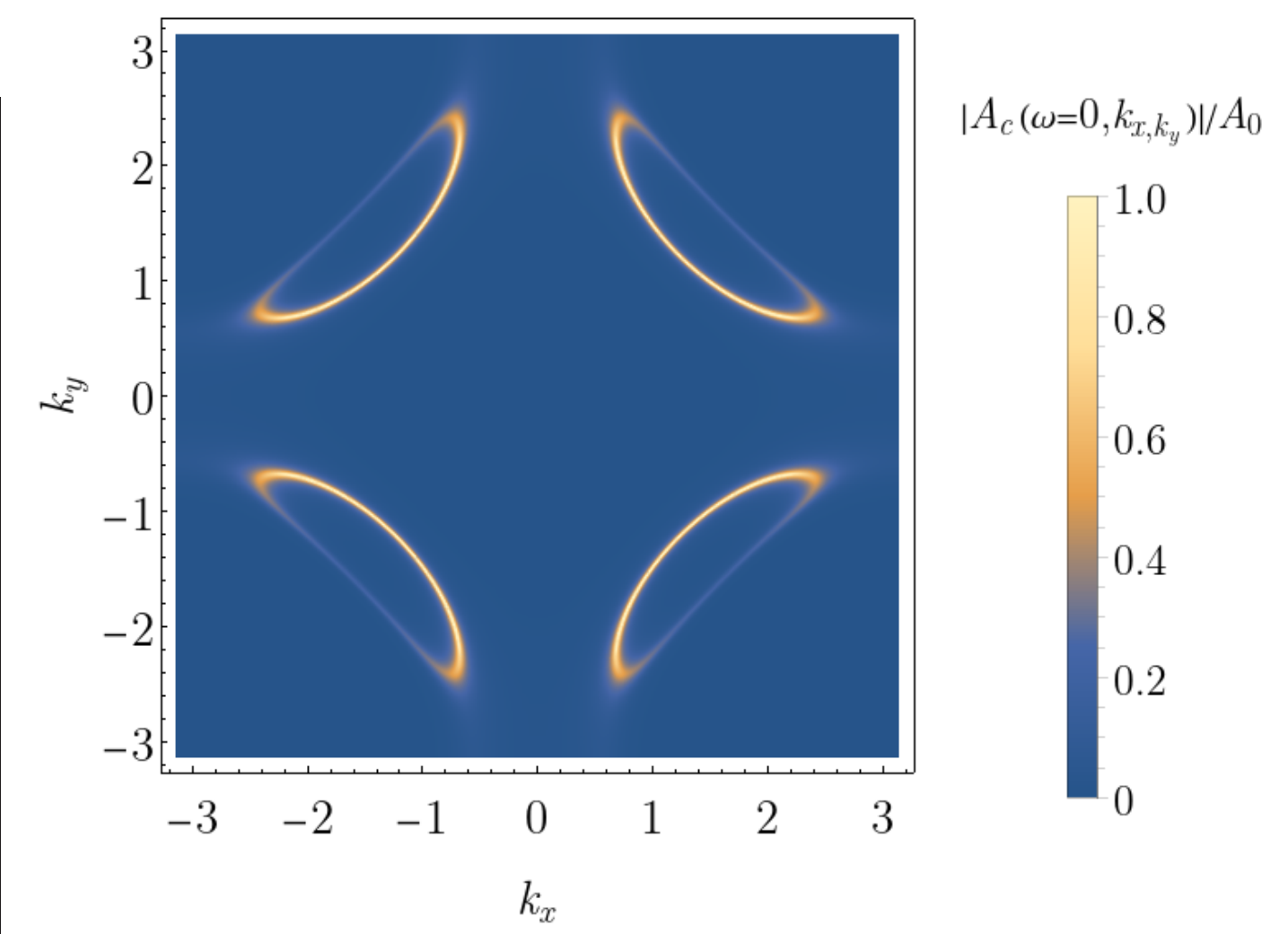
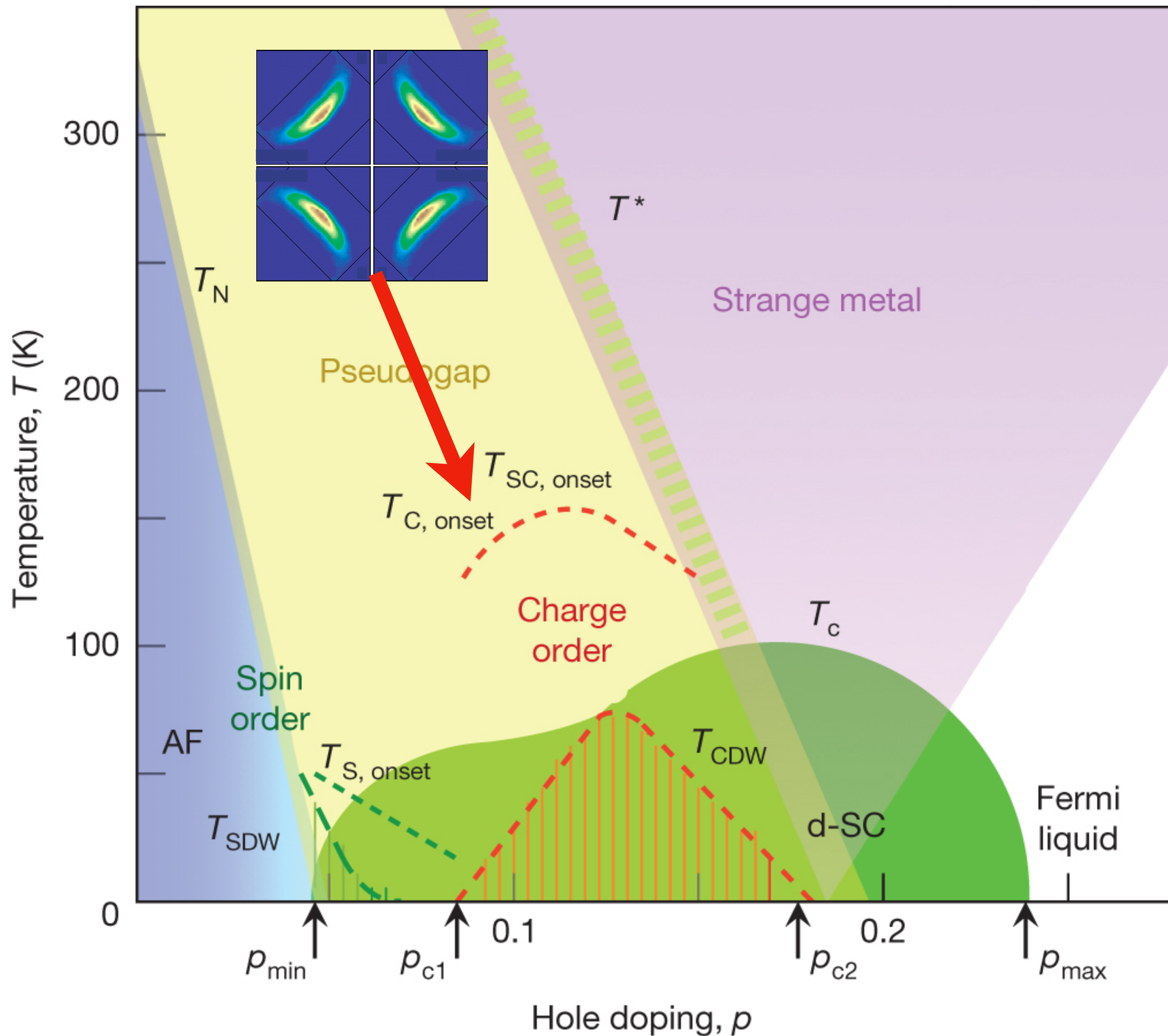


**ARPES on
Bi2201**

R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

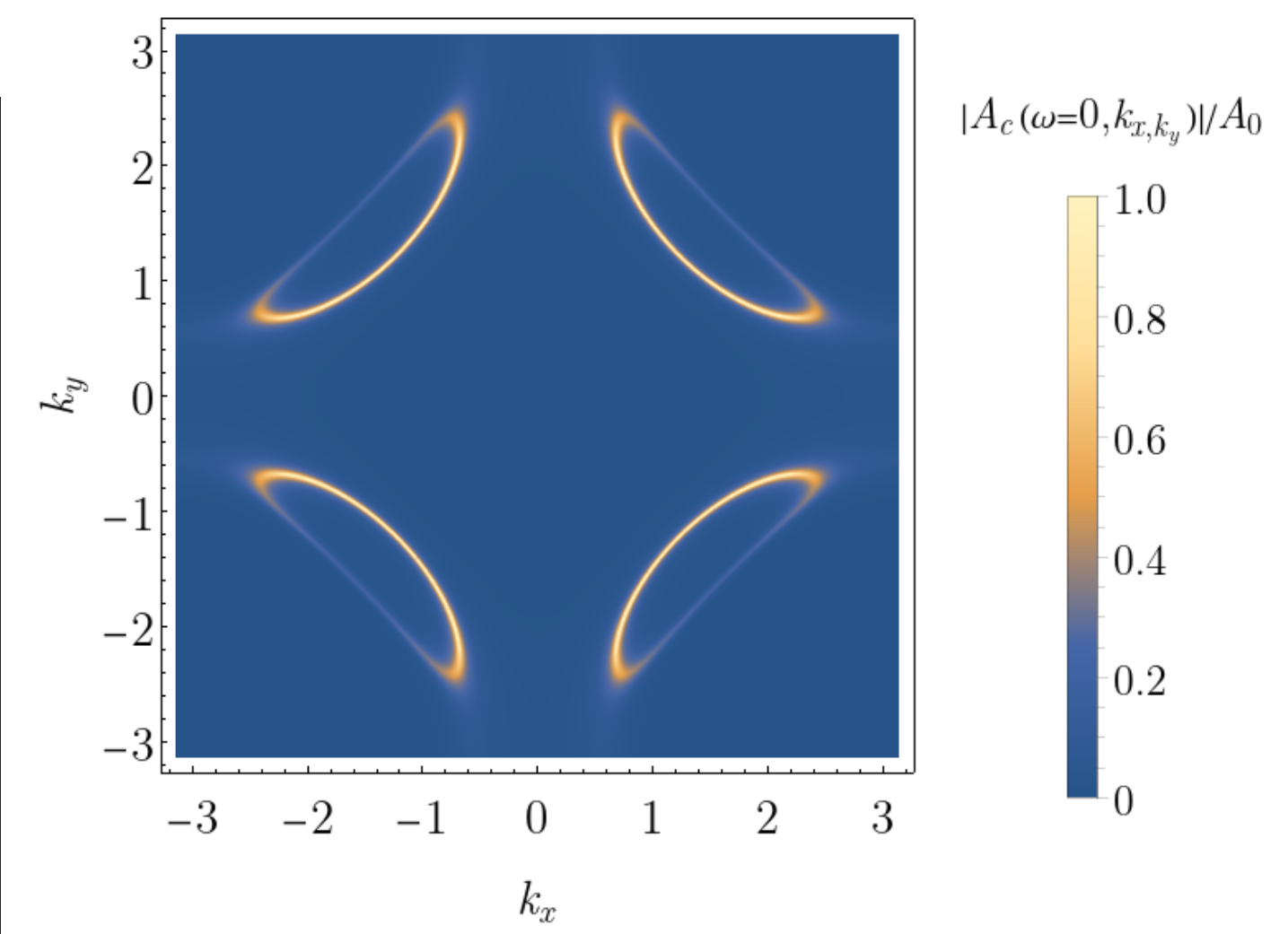
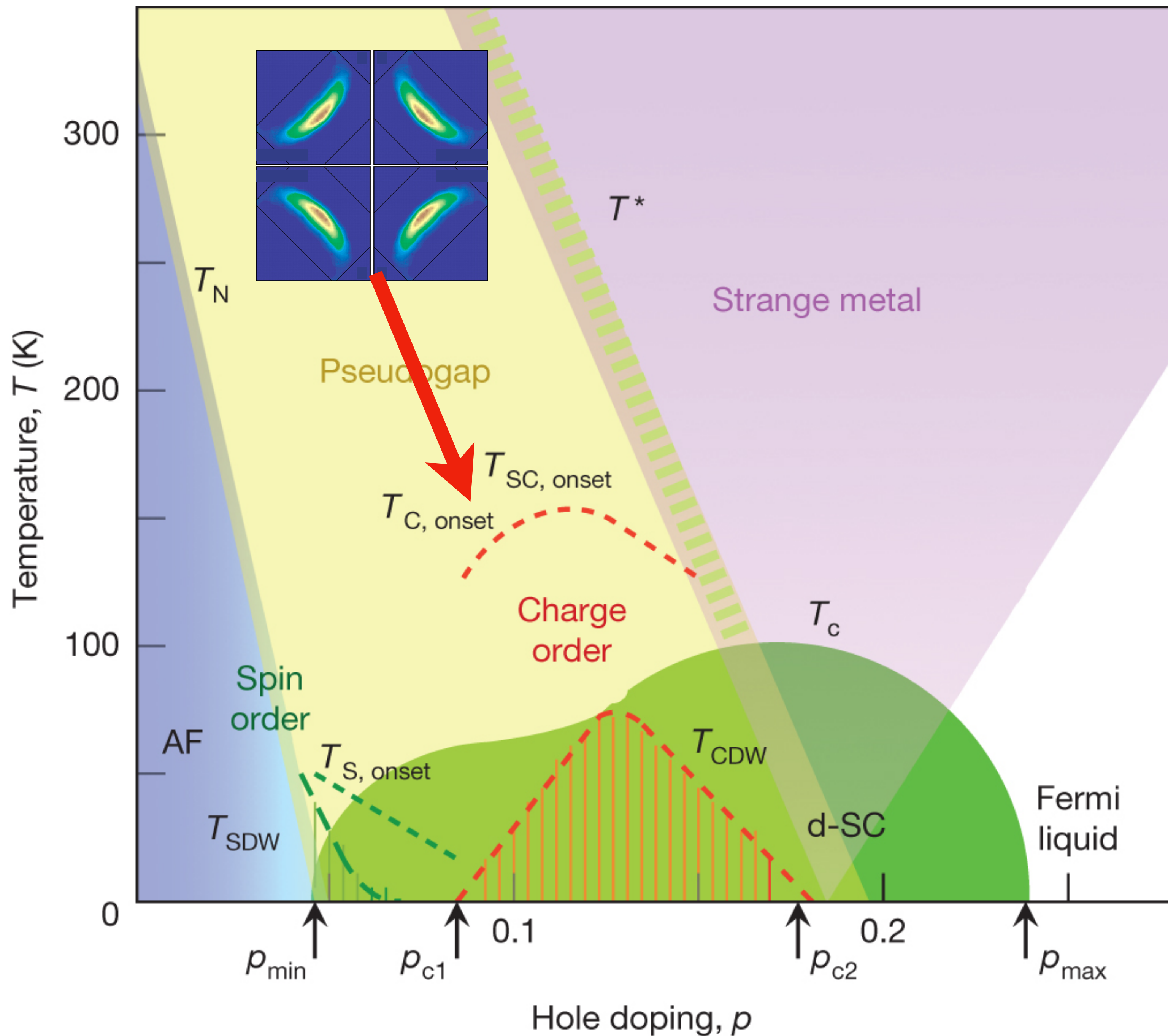
Similarities to theories,
which do not have explicit
reference to spin liquid on
second ancilla layer

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang,
PRB **73**, 174501 (2006)
S. Sakai, Y. Motome, M. Imada,
PRL **102**, 056404 (2009)



E. Mascot,
 A. Nikolaenko,
 M. Tikhonovskaya,
 Ya-Hui Zhang,
 D. K. Morr, and
 S. S., *PRB* **105**,
 075146 (2022)

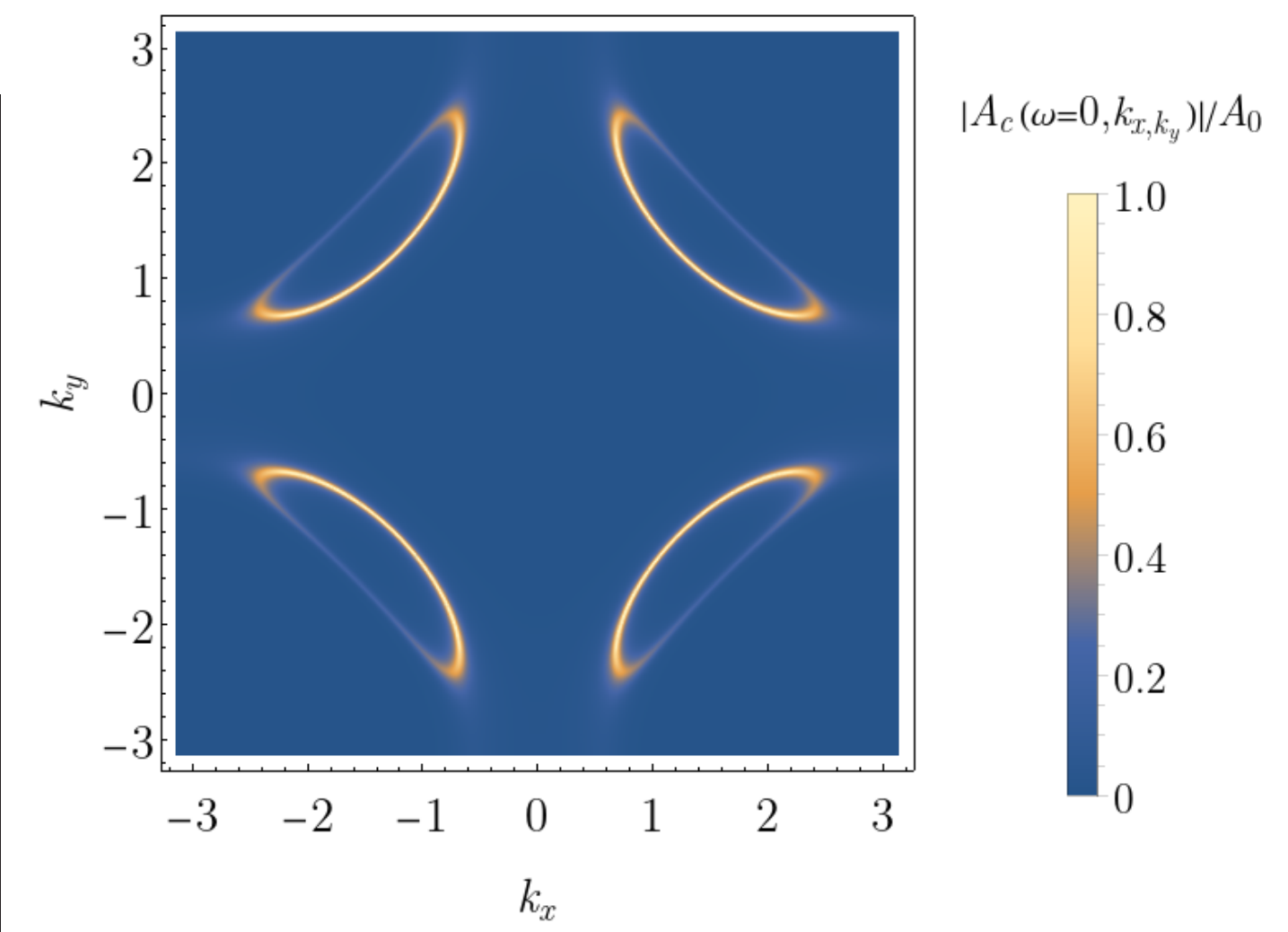
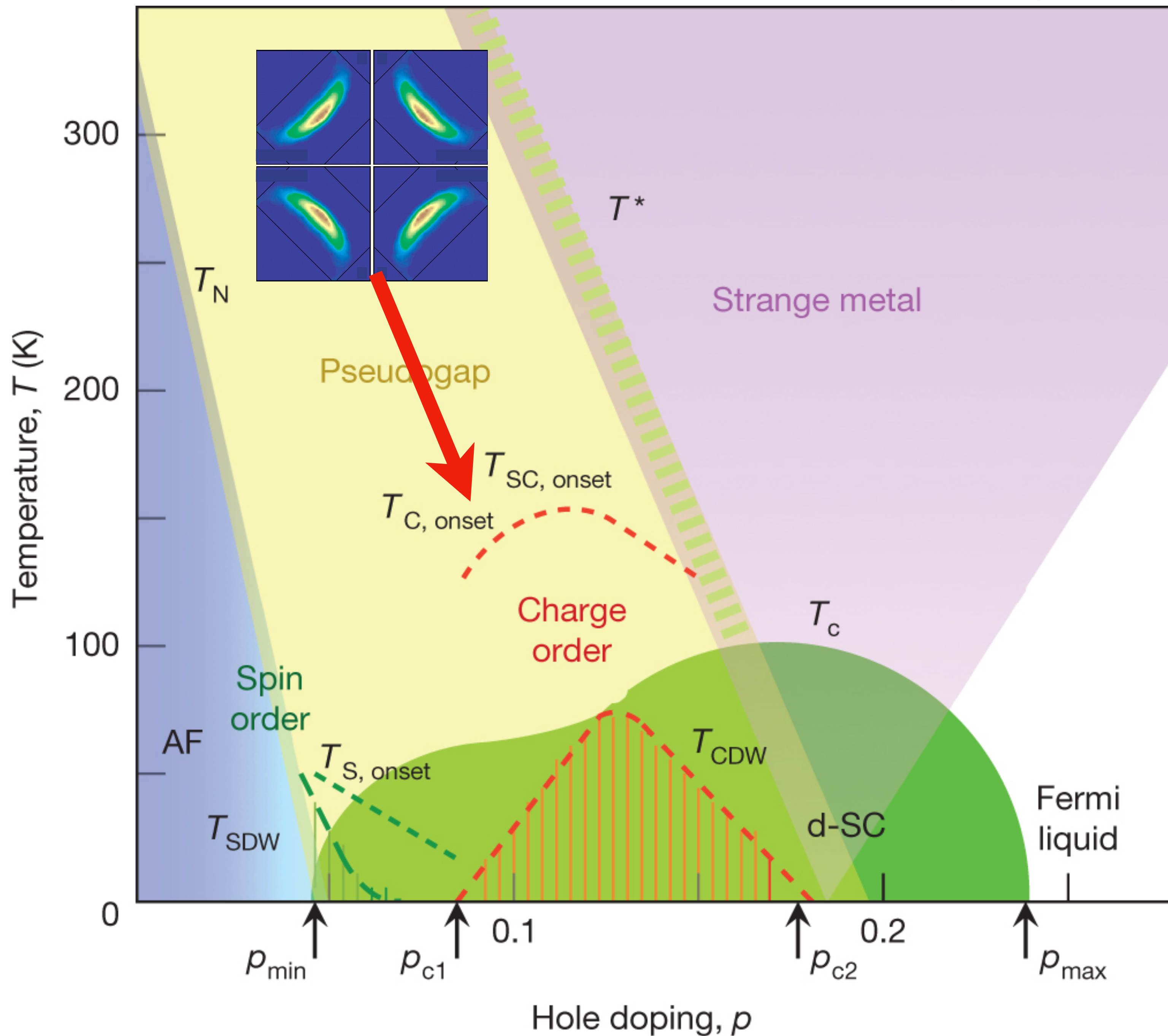
Hole pocket Fermi surfaces
 of size p with
 charge e , spin-1/2 quasiparticles
 +
 ‘spectator’
 square lattice spin liquid
 at half-filling.



E. Mascot,
A. Nikolaenko,
M. Tikhanovskaya,
Ya-Hui Zhang,
D. K. Morr, and
S. S., PRB **105**,
075146 (2022)

Hole pocket Fermi surfaces
of size p with
charge e , spin-1/2 quasiparticles
+
'spectator'
square lattice spin liquid
at half-filling.

But which spin liquid?



E. Mascot,
A. Nikolaenko,
M. Tikhonovskaya,
Ya-Hui Zhang,
D. K. Morr, and
S. S., *PRB* **105**,
075146 (2022)

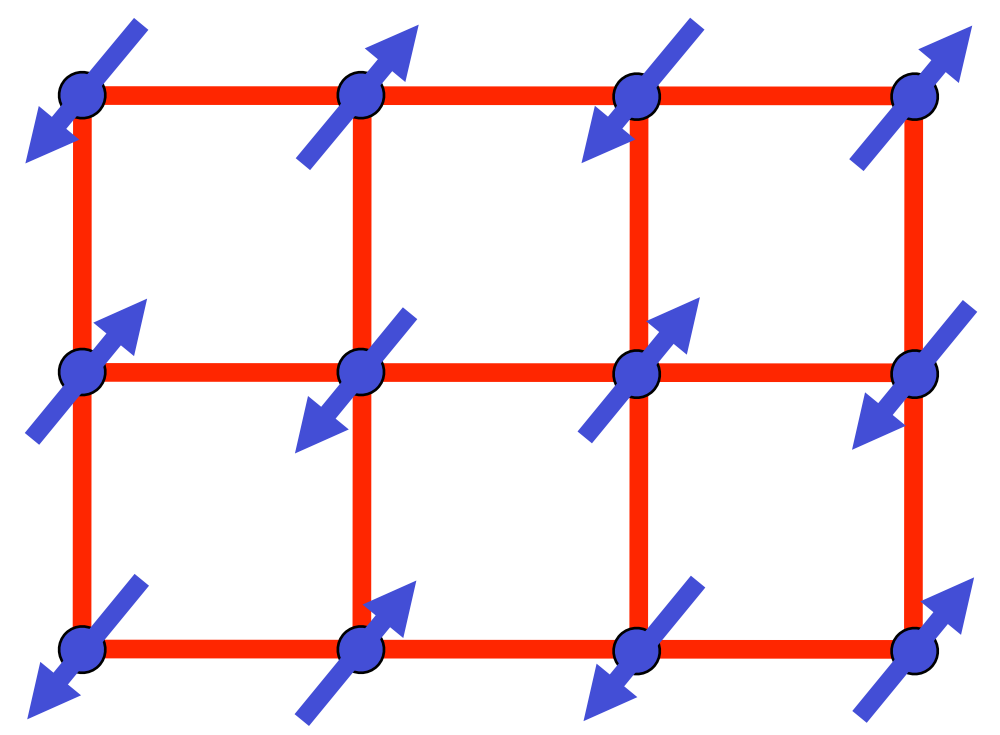
Hole pocket Fermi surfaces
of size p with
charge e , spin-1/2 quasiparticles
+
'spectator'
square lattice spin liquid
at half-filling.

But which spin liquid?

A spin liquid which is ultimately
IR-unstable to confinement

1. Open questions on the cuprate phase diagram
2. Theory of the pseudogap metal
3. The π -flux spin liquid
4. Confinement transitions of the π -flux spin liquid
5. Recap

Insulating $S=1/2$ antiferromagnet



Spin liquid

$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Schwinger bosons

$$\mathbf{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}, \quad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^\dagger b_{i\alpha} = 1$$

Mean-field spin liquid
with gapped bosonic spinons.

D.P. Arovas and A. Auerbach, PRB **38**, 316 (1988)

Insulating $S=1/2$ antiferromagnet

$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

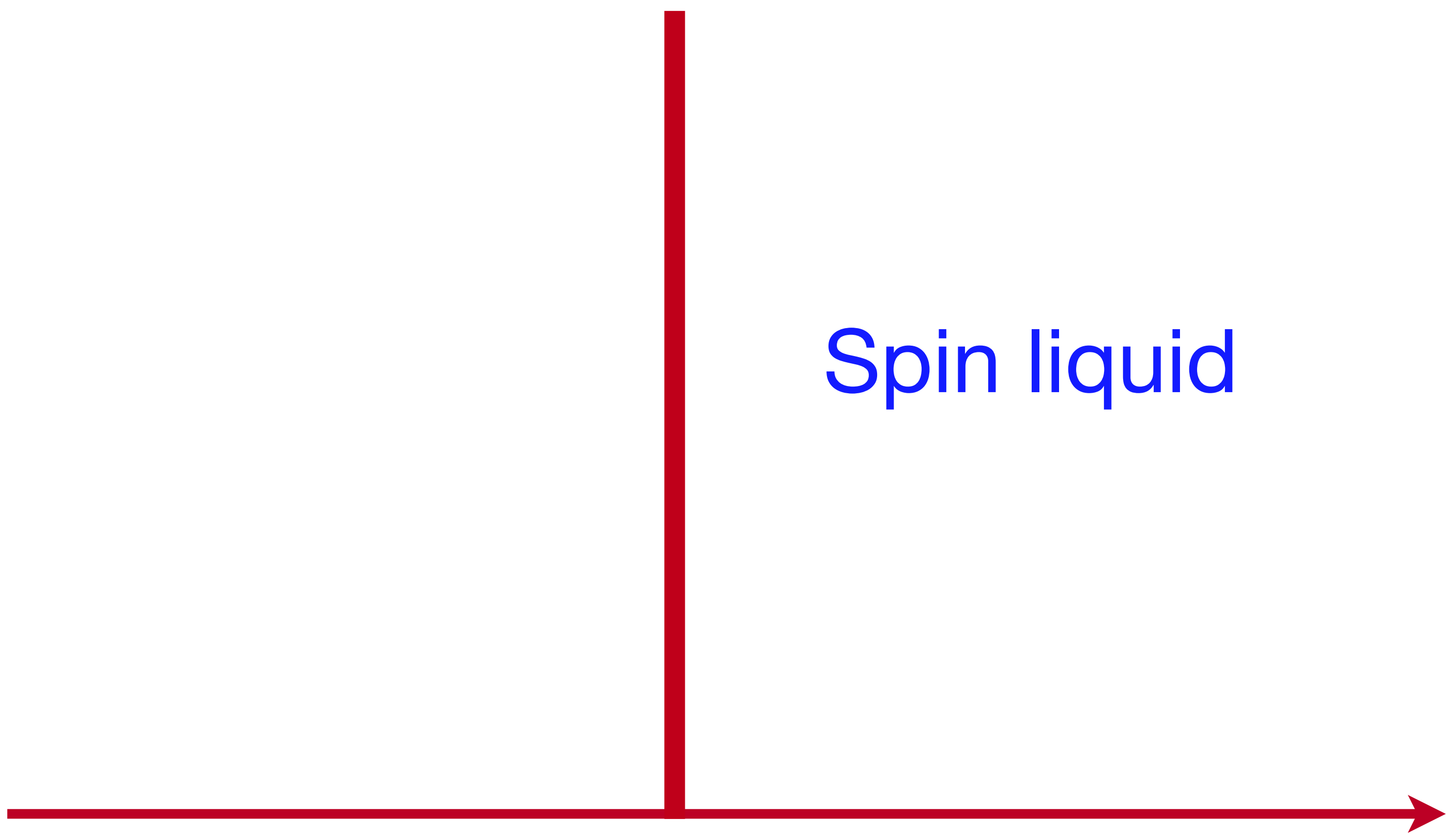
Schwinger fermions

$$\mathbf{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta}, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

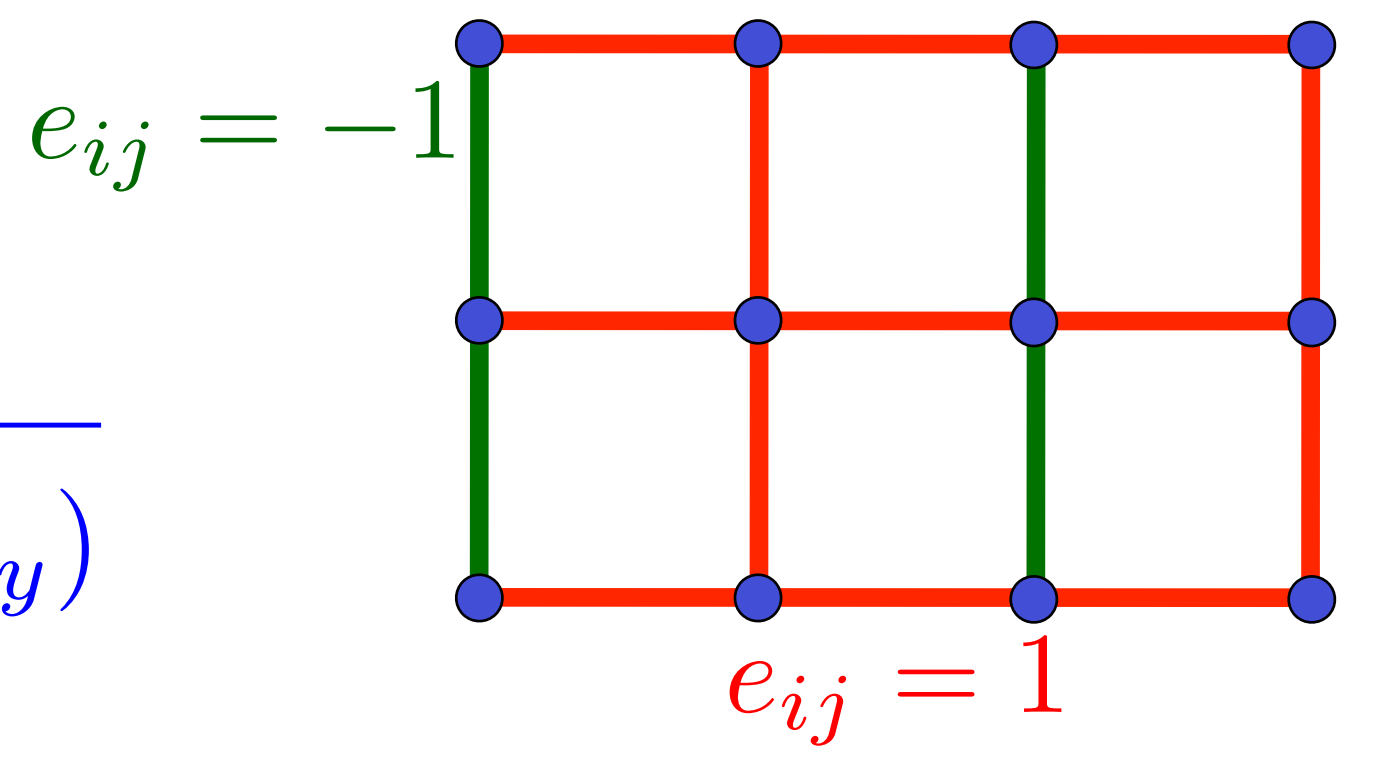
π -flux mean-field theory
with gapless spinons at 2 Dirac points.

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

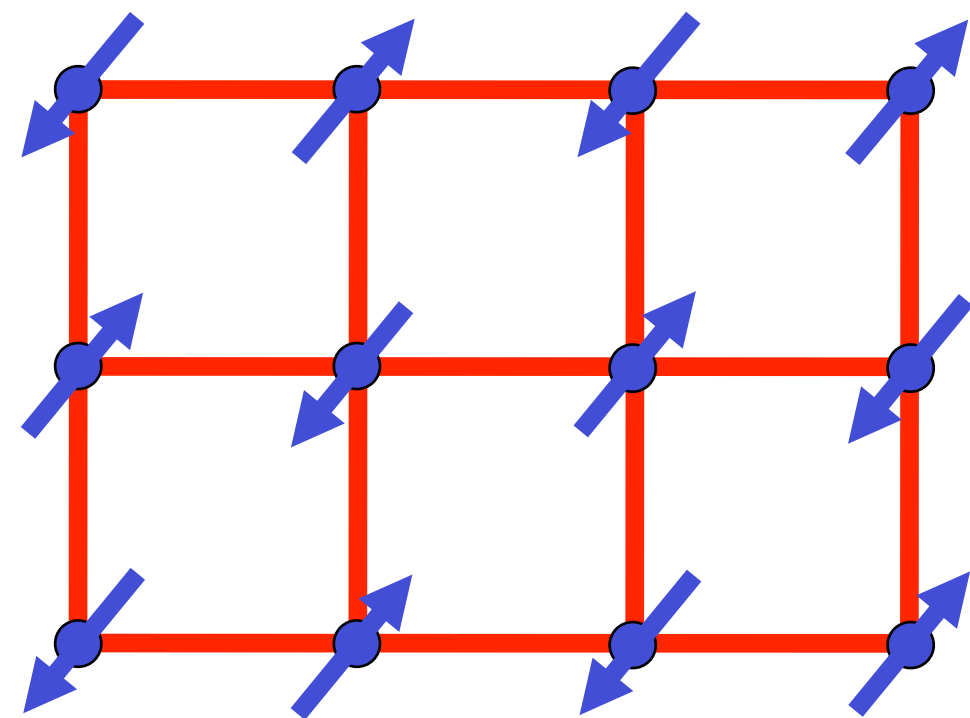
Spin liquid



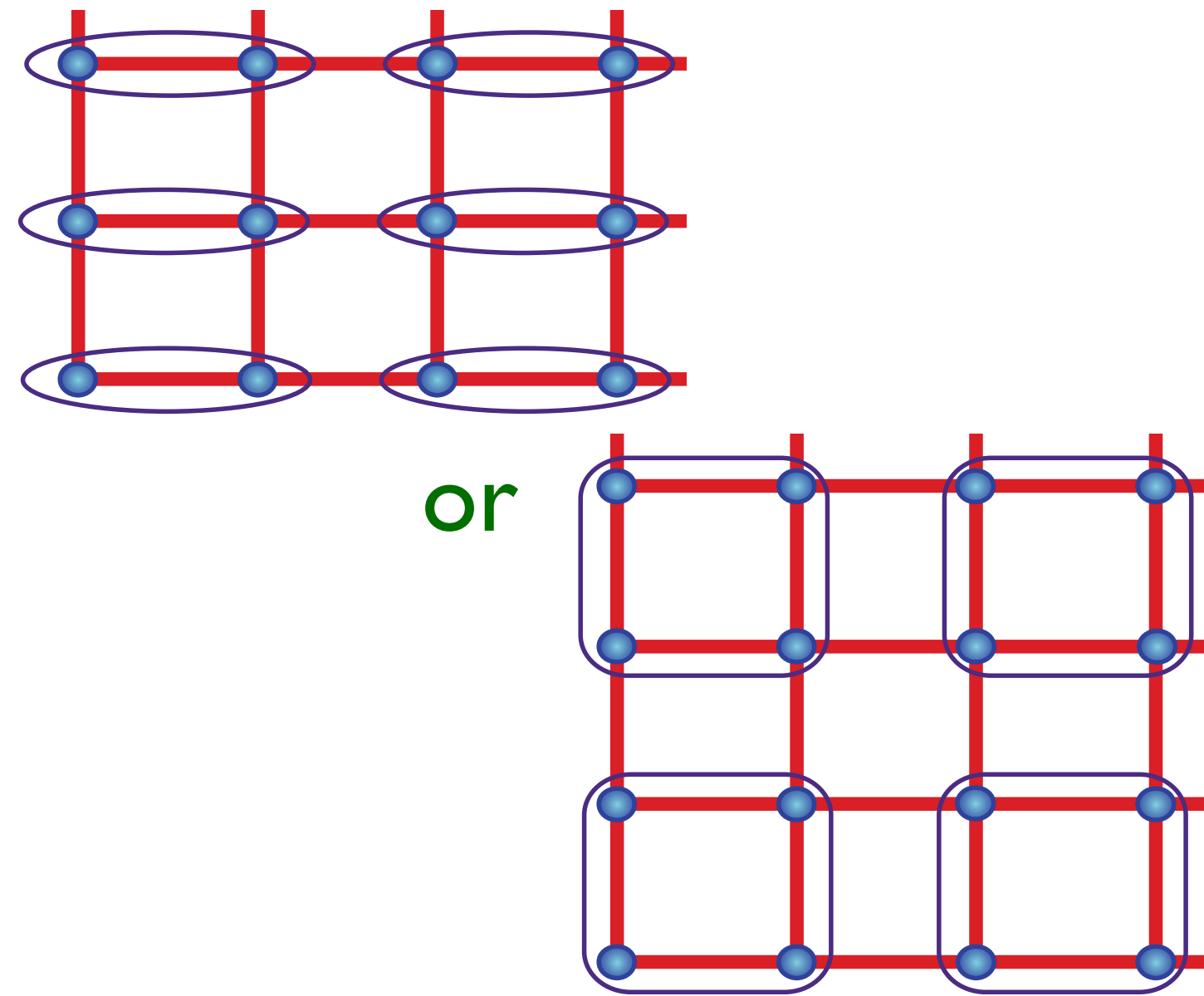
$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right), \quad \epsilon_{\mathbf{k}} = 2J \sqrt{\sin^2(k_x) + \sin^2(k_y)}$$



Insulating $S=1/2$ antiferromagnet



Higgs phase, $\langle z_\alpha \rangle \neq 0$:
Néel order



Confining phase, $\langle z_\alpha \rangle = 0$:
VBS order

s

$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Schwinger bosons

$$\mathbf{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}, \quad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^\dagger b_{i\alpha} = 1$$

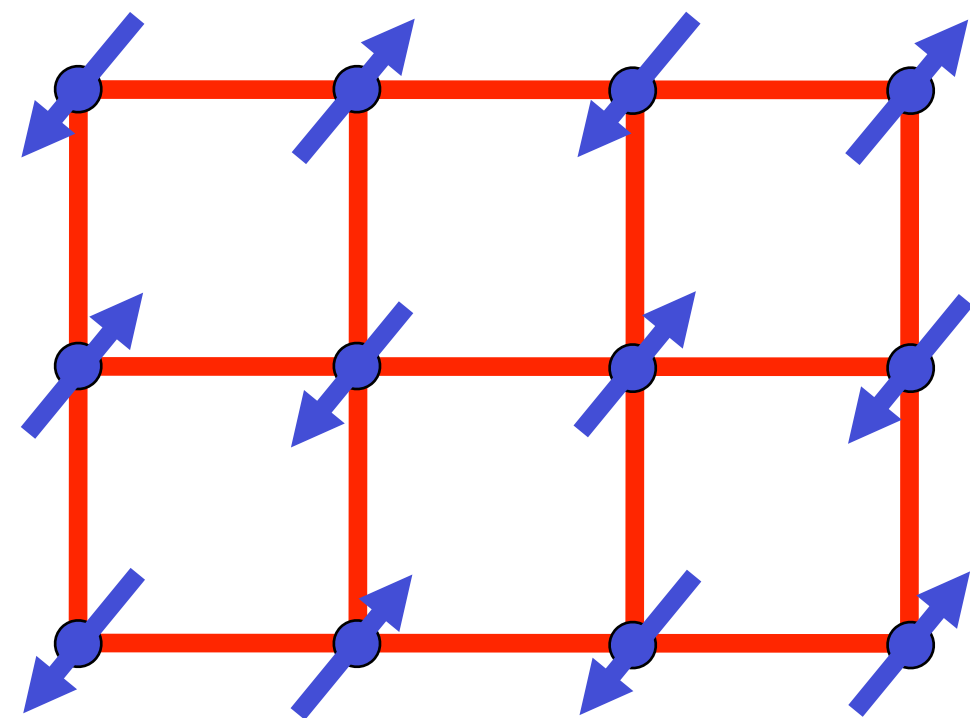
Mean-field spin liquid
with gapped bosonic spinons.

Low energy $\mathbb{C}\mathbb{P}^1$ U(1) gauge theory

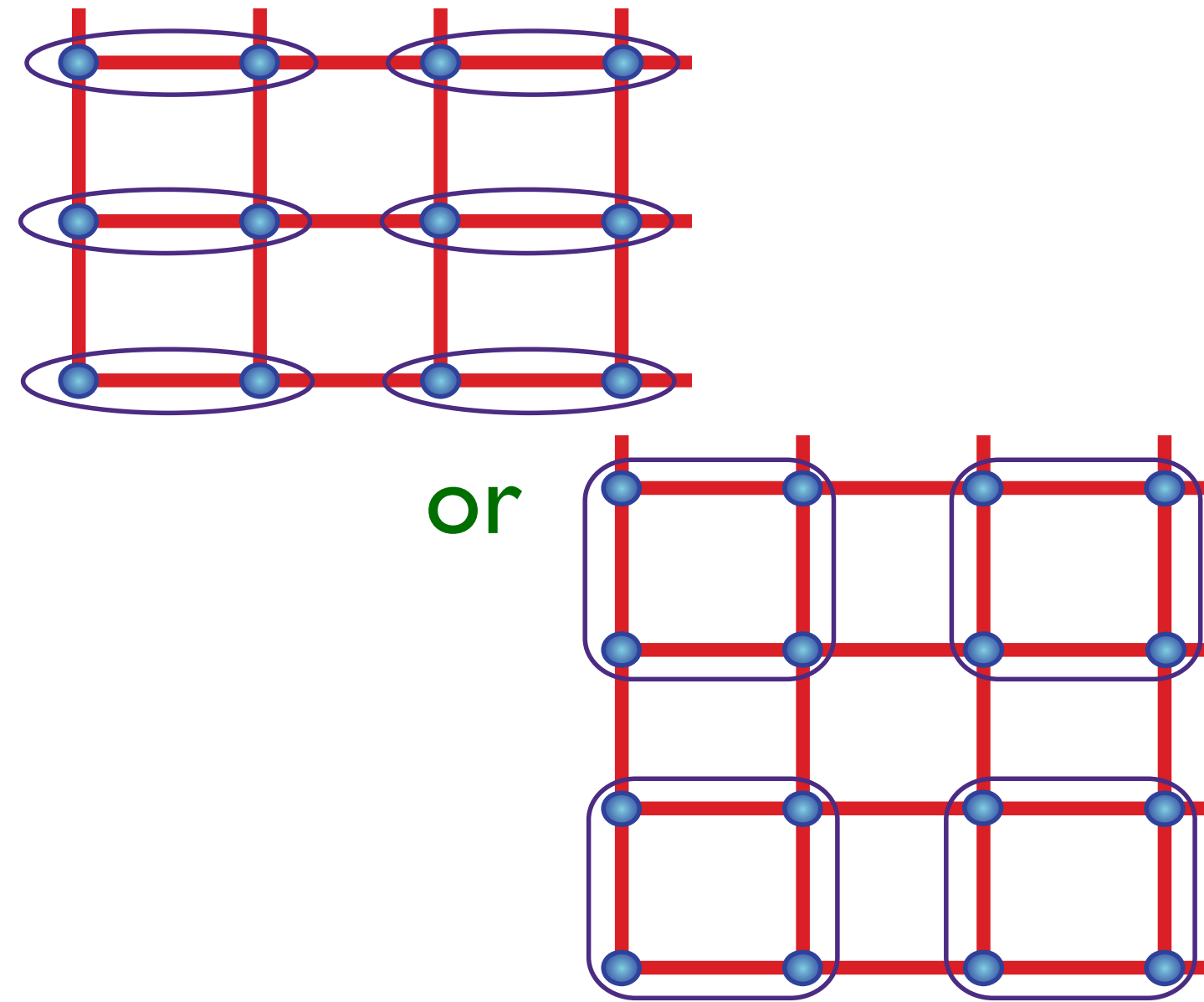
$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}$$

$$\mathcal{L} = |(\partial_\mu - ia_\mu)z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

Insulating $S=1/2$ antiferromagnet



Confining phase:
Néel order



Confining phase:
VBS order

$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

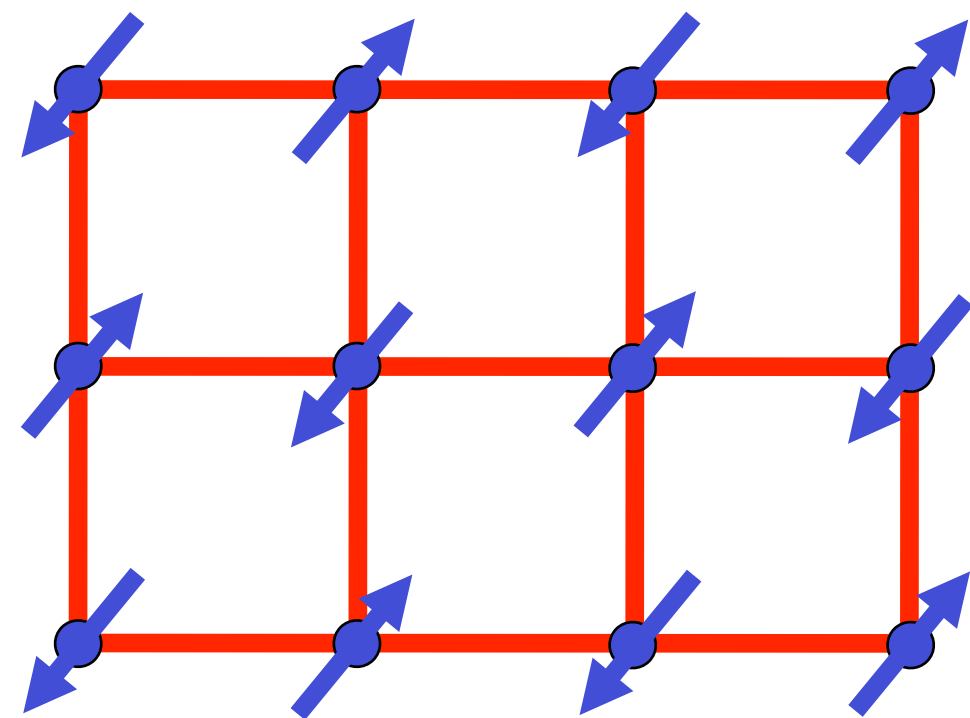
Schwinger fermions

$$\mathbf{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta}, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

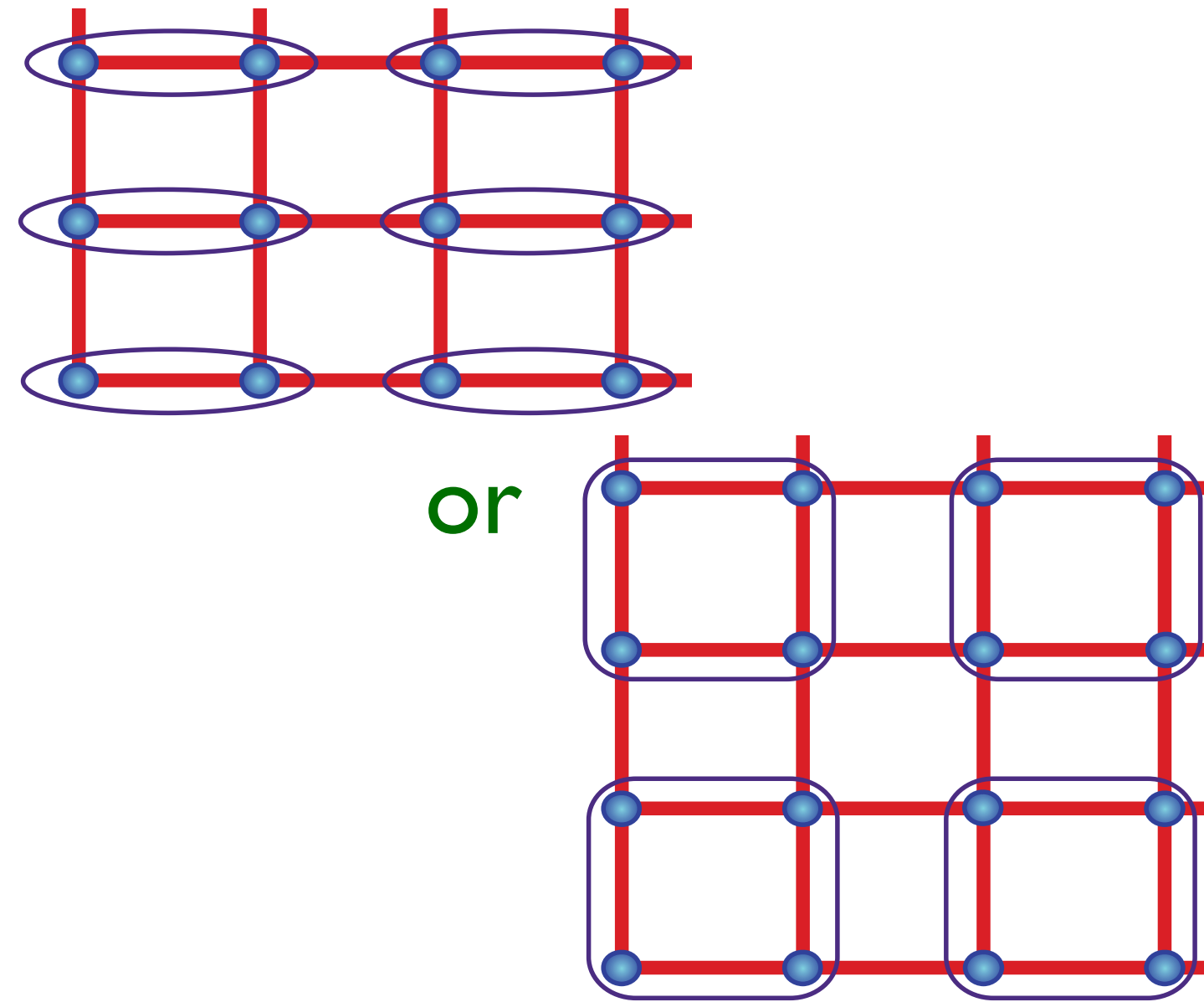
π -flux mean-field theory
with gapless spinons at 2 Dirac points.
Low energy theory of $N_f = 2$
Dirac fermions Ψ_s coupled to
an emergent $SU(2)_N$ gauge field.
Confining order parameters
are Néel and VBS states,
with a global $SO(5)_f$ symmetry!

$$\mathcal{L} = i\bar{\Psi}_s \gamma_\mu D_\mu \Psi_s + \dots$$

Insulating $S=1/2$ antiferromagnet



Confining phase:
Néel order



Confining phase:
VBS order

$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Schwinger fermions

$$\mathbf{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta}, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

π -flux mean-field theory

with gapless spinons at 2 Dirac points.

Low energy theory of $N_f = 2$

Dirac fermions Ψ_s coupled to an emergent $SU(2)_N$ gauge field.

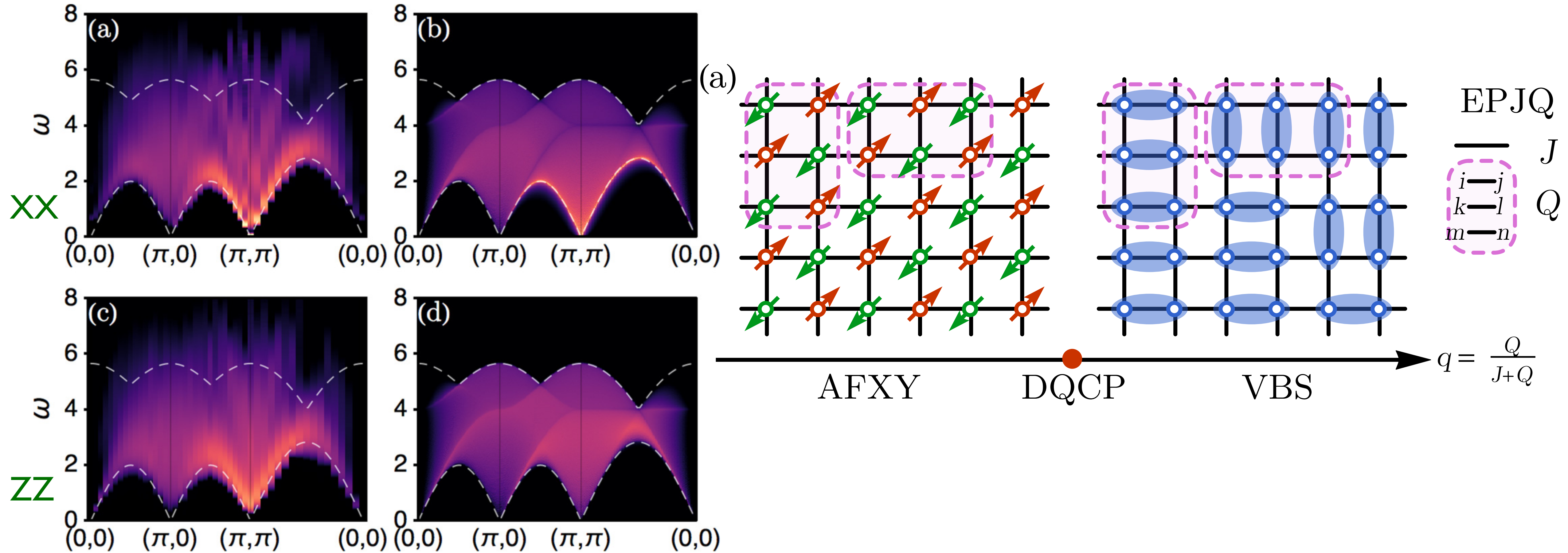
Confining order parameters are Néel and VBS states, with a global $SO(5)_f$ symmetry!

$$\mathcal{L} = i\bar{\Psi}_s \gamma_\mu D_\mu \Psi_s + \dots$$

Dual to $\mathbb{C}P^1$ U(1) gauge theory.

QMC

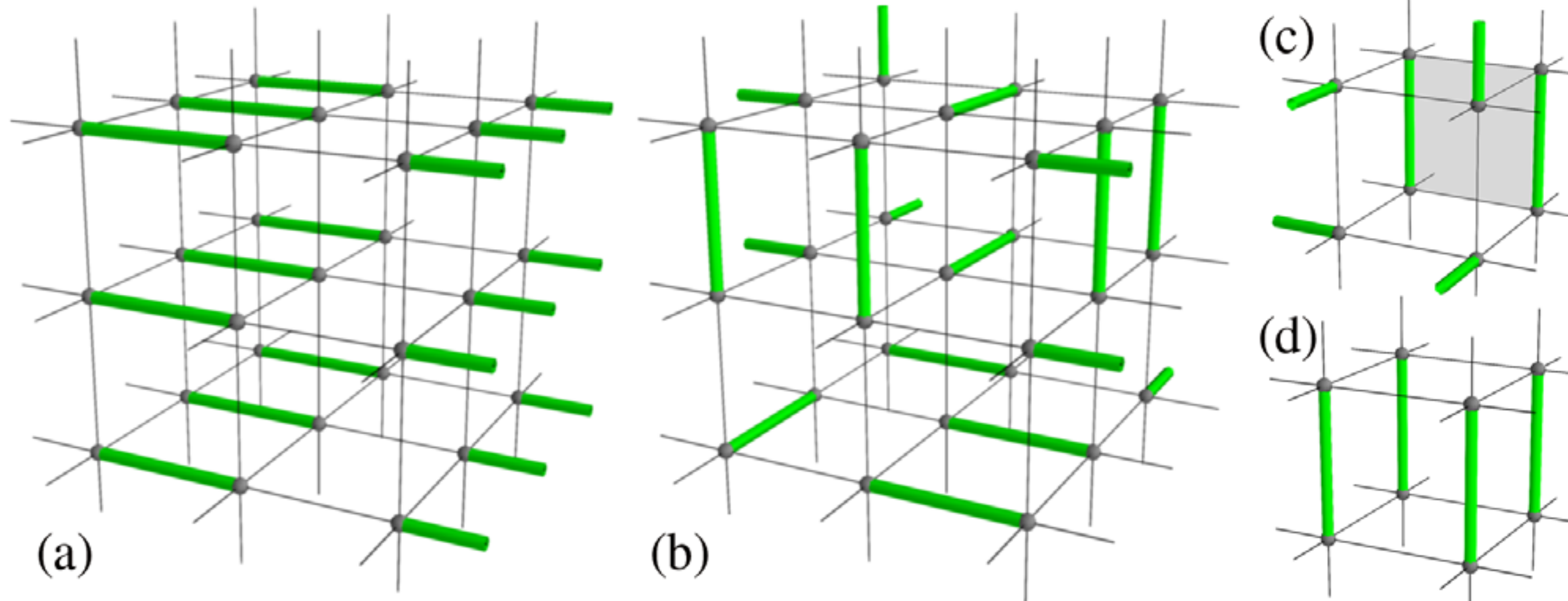
Free fermion spinons in π -flux



Emergent $SO(5)$ Symmetry at the Columnar Ordering Transition in the Classical Cubic Dimer Model

PRL **122**, 080601 (2019)

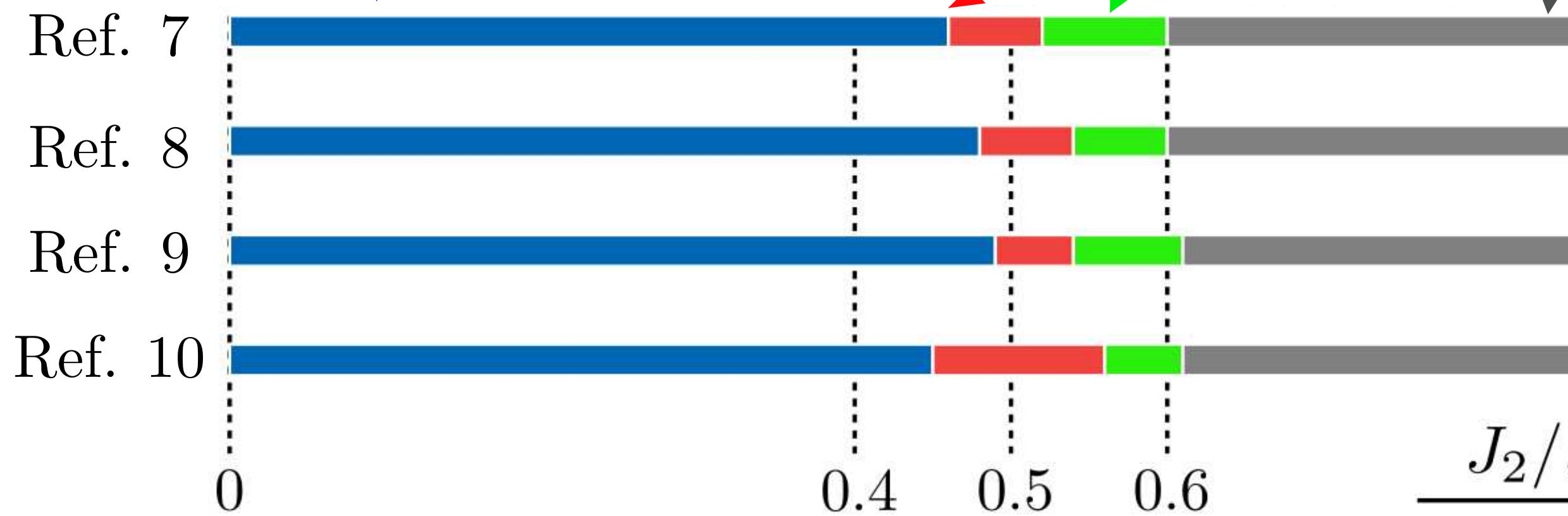
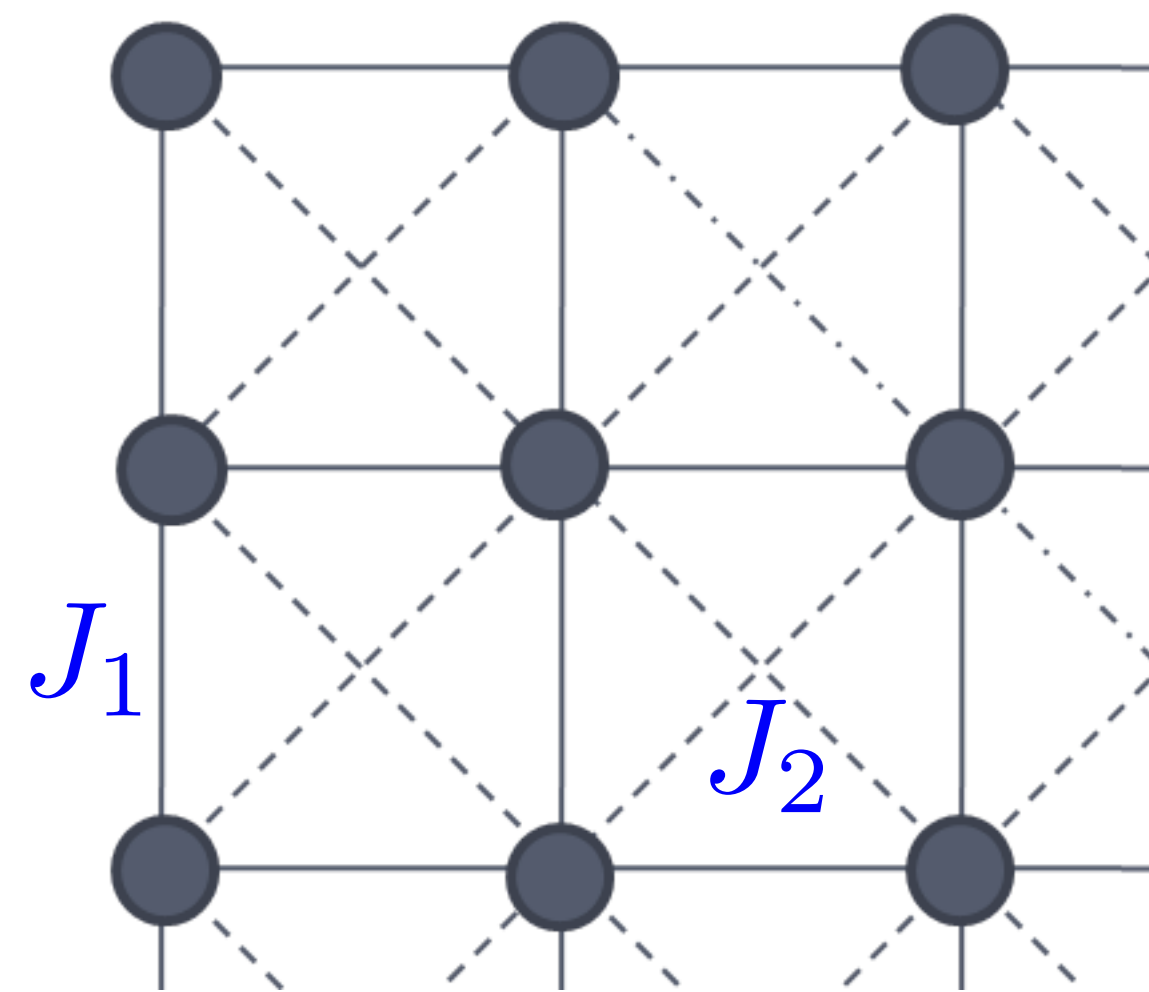
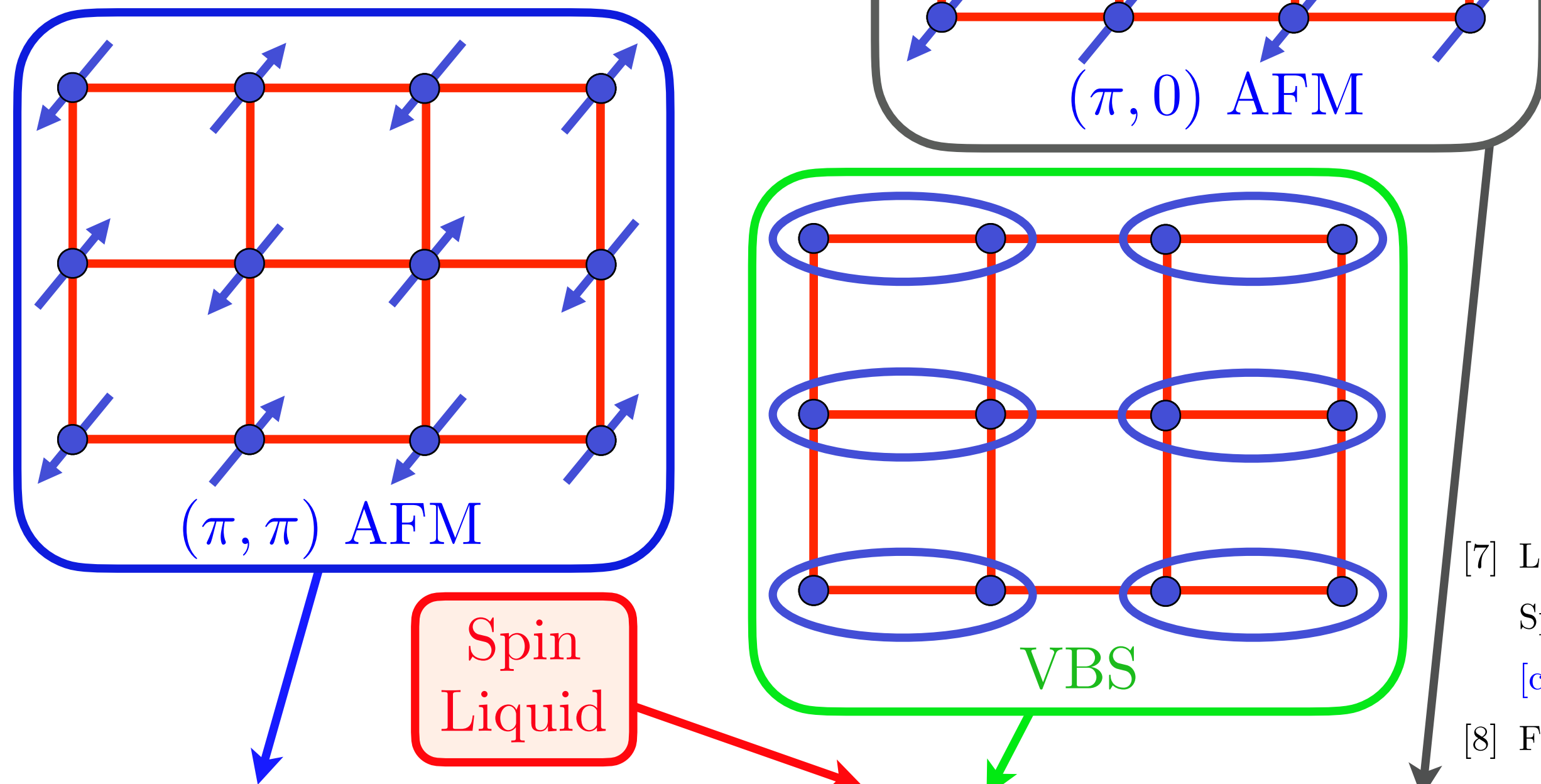
G. J. Sreejith,¹ Stephen Powell,² and Adam Nahum³



Using Monte Carlo simulations, we show that this transition has emergent $SO(5)$ symmetry relating quantities characterizing the two phases. While the low-temperature phase has a conventional order parameter, the defining property of the Coulomb liquid on the high-temperature side is deconfinement of monomers, and so $SO(5)$ relates fundamentally different types of objects. Studying linear system sizes up to $L=96$, we find that this symmetry applies with an excellent precision, consistently improving with system size over this range. It is remarkable that $SO(5)$ emerges in a system as basic as the cubic dimer model, with only simple discrete degrees of freedom. Our results are important evidence for the generality of the $SO(5)$ symmetry that has been proposed for the noncompact CP^1 field theory.

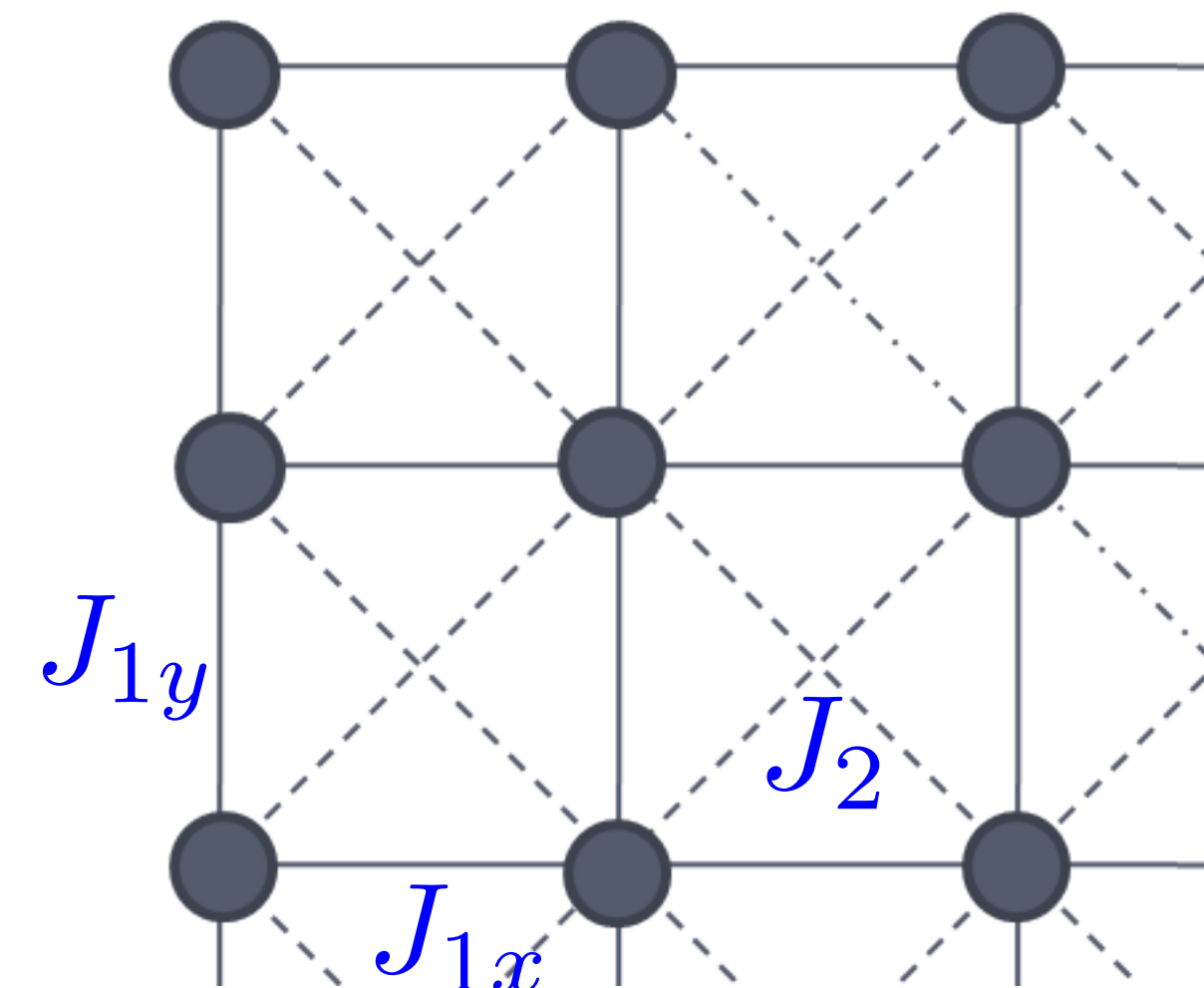
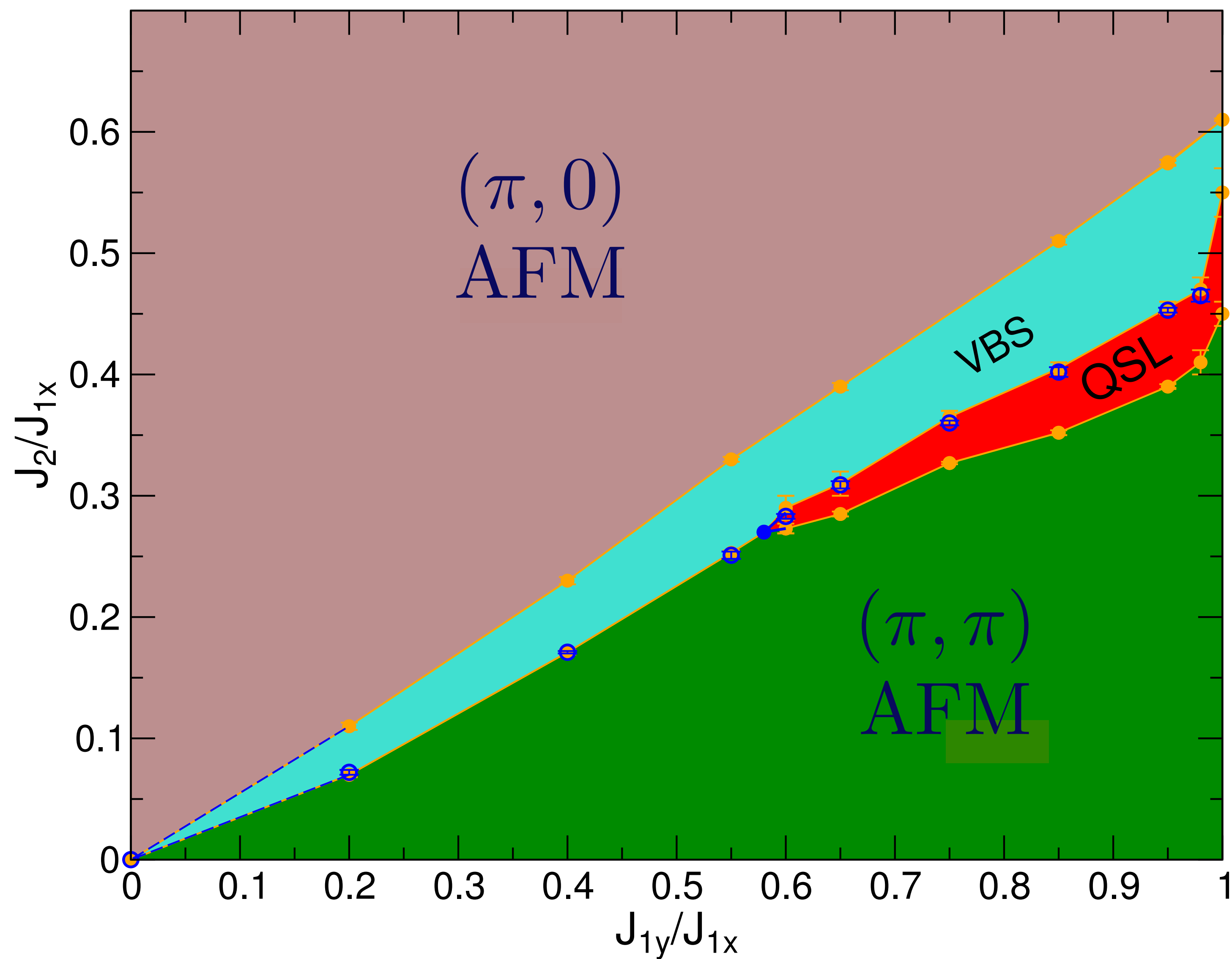
$$\mathcal{L} = \frac{\kappa}{2} |\vec{\nabla} \times \vec{A}|^2 + |(\vec{\nabla} - i\vec{A})\mathbf{z}|^2 + s|\mathbf{z}|^2 + u(|\mathbf{z}|^2)^2,$$

$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



- [7] L. Wang and A. W. Sandvik, “Critical Level Crossings and Gapless Spin Liquid in the Square-Lattice Spin-1/2 J_1 - J_2 Heisenberg Antiferromagnet,” *Phys. Rev. Lett.* **121**, 107202 (2018), [arXiv:1702.08197 \[cond-mat.str-el\]](#).
- [8] F. Ferrari and F. Becca, “Gapless spin liquid and valence-bond solid in the J_1 - J_2 Heisenberg model on the square lattice: Insights from singlet and triplet excitations,” *Phys. Rev. B* **102**, 014417 (2020), [arXiv:2005.12941 \[cond-mat.str-el\]](#).
- [9] Y. Nomura and M. Imada, “Dirac-Type Nodal Spin Liquid Revealed by Refined Quantum Many-Body Solver Using Neural-Network Wave Function, Correlation Ratio, and Level Spectroscopy,” *Phys. Rev. X* **11**, 031034 (2021), [arXiv:2005.14142 \[cond-mat.str-el\]](#).
- [10] W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, “Gapless quantum spin liquid and global phase diagram of the spin-1/2 J_1 - J_2 square antiferromagnetic Heisenberg model,” (2020), [arXiv:2009.01821 \[cond-mat.str-el\]](#).

$$H = J_{1x} \sum_{\langle i,j \rangle_x} \mathbf{S}_i \cdot \mathbf{S}_j + J_{1y} \sum_{\langle i,j \rangle_y} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j.$$

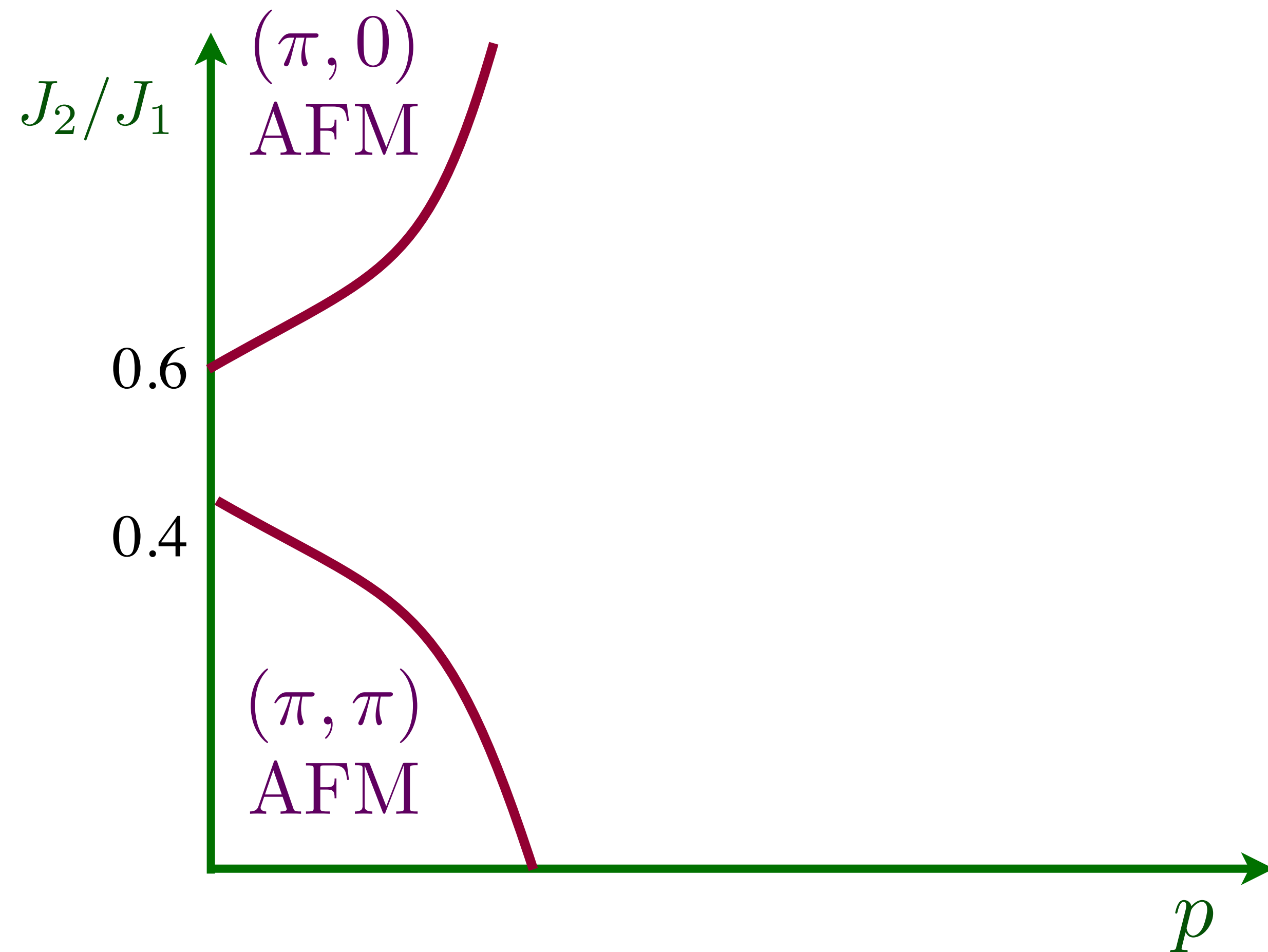


Wen-Yuan Liu,
 Shou-Shu Gong,
 Wei-Qiang Chen,
 and Zheng-Cheng Gu,
 arXiv:2212.00707

High Temperature Superconductivity in a Lightly Doped Quantum Spin Liquid

Hong-Chen Jiang^{1,*} and Steven A. Kivelson²

PHYSICAL REVIEW LETTERS **127**, 097002 (2021)



Superconducting valence bond fluid in
lightly doped 8-leg t - J cylinders

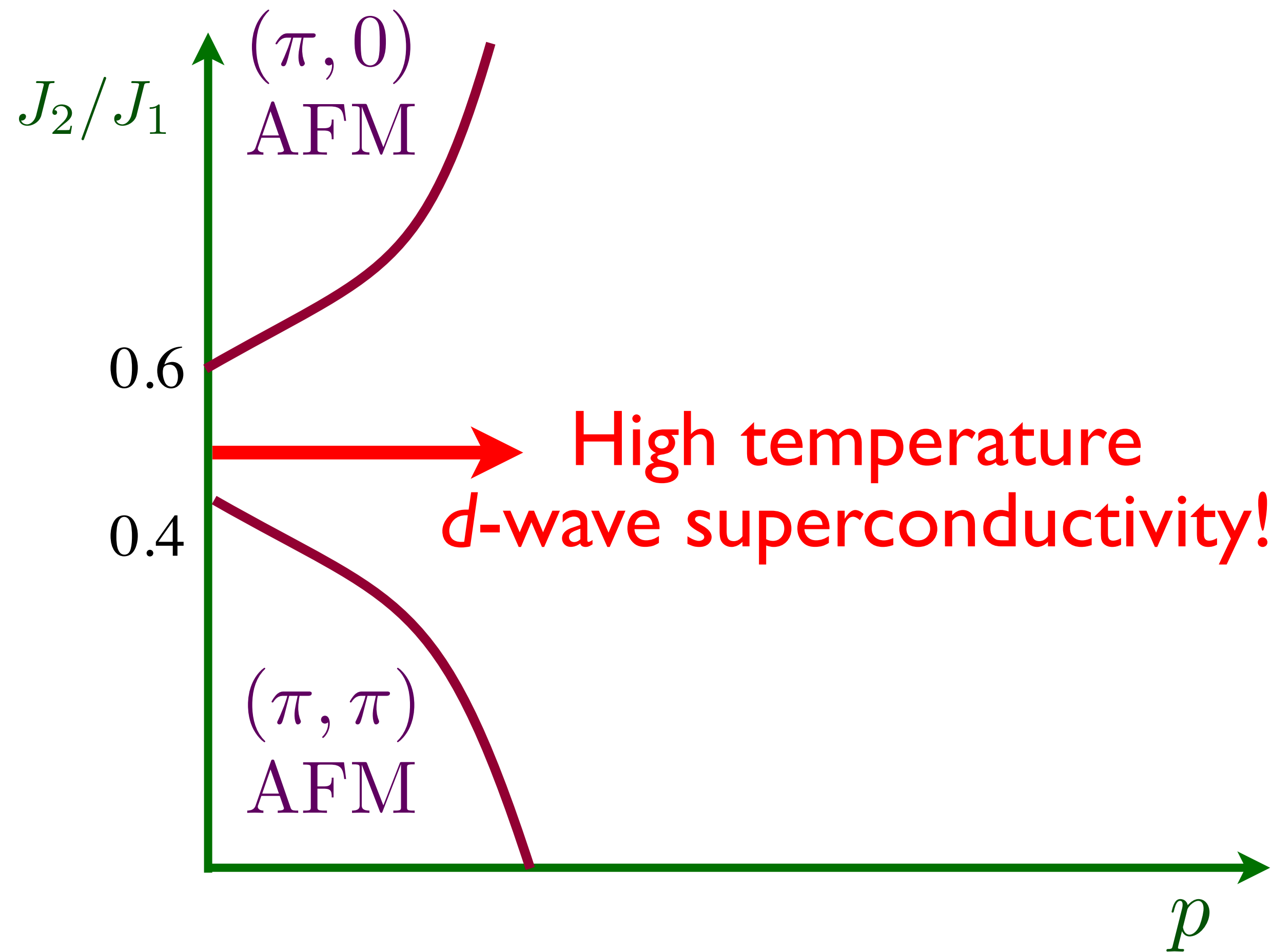
Hong-Chen Jiang, Steven A. Kivelson, and
Dung-Hai Lee, arXiv:2302.11633

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High Temperature Superconductivity in a Lightly Doped Quantum Spin Liquid

Hong-Chen Jiang^{1,*} and Steven A. Kivelson²

PHYSICAL REVIEW LETTERS **127**, 097002 (2021)

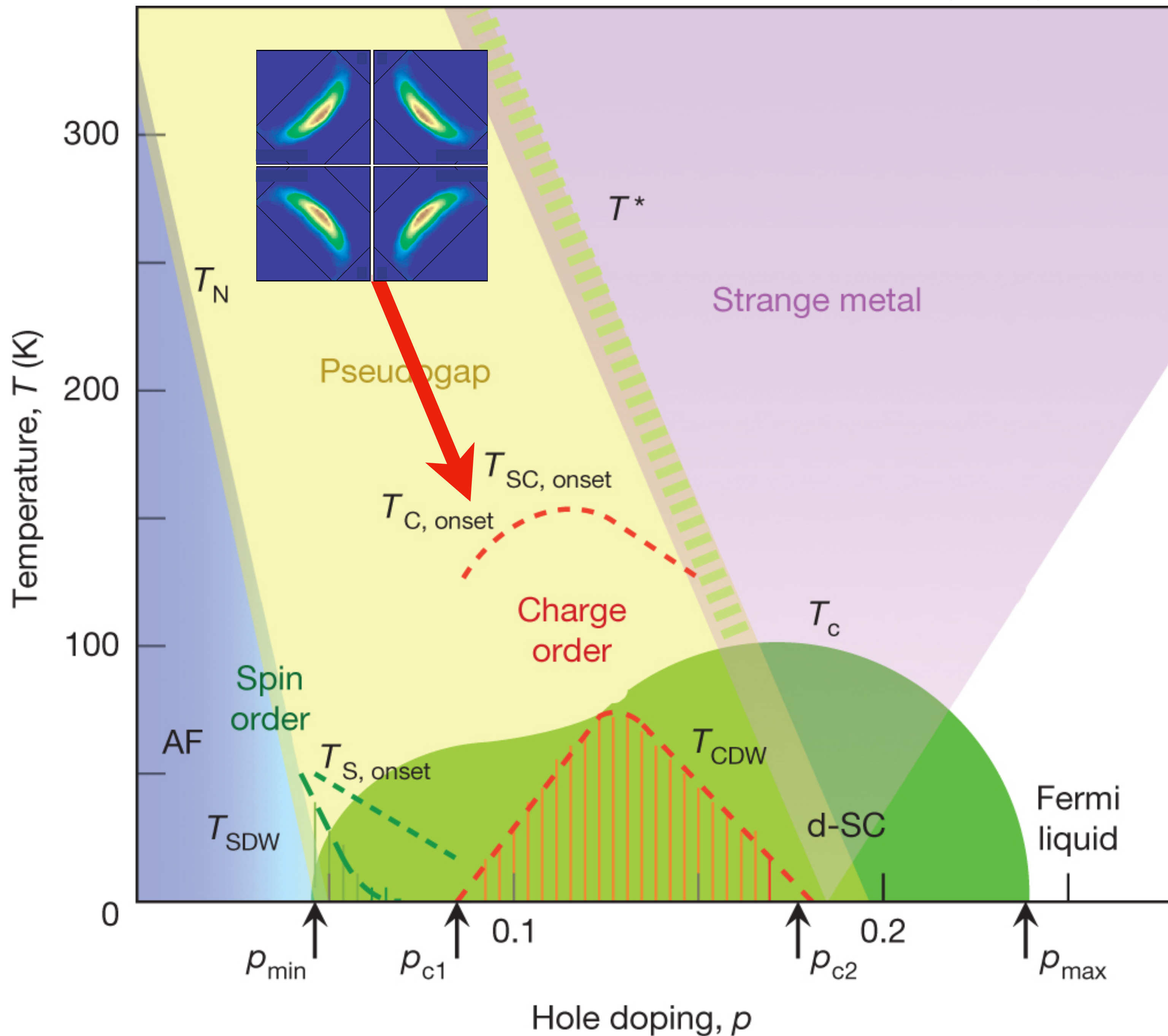


Superconducting valence bond fluid in
lightly doped 8-leg t - J cylinders

Hong-Chen Jiang, Steven A. Kivelson, and
Dung-Hai Lee, arXiv:2302.11633

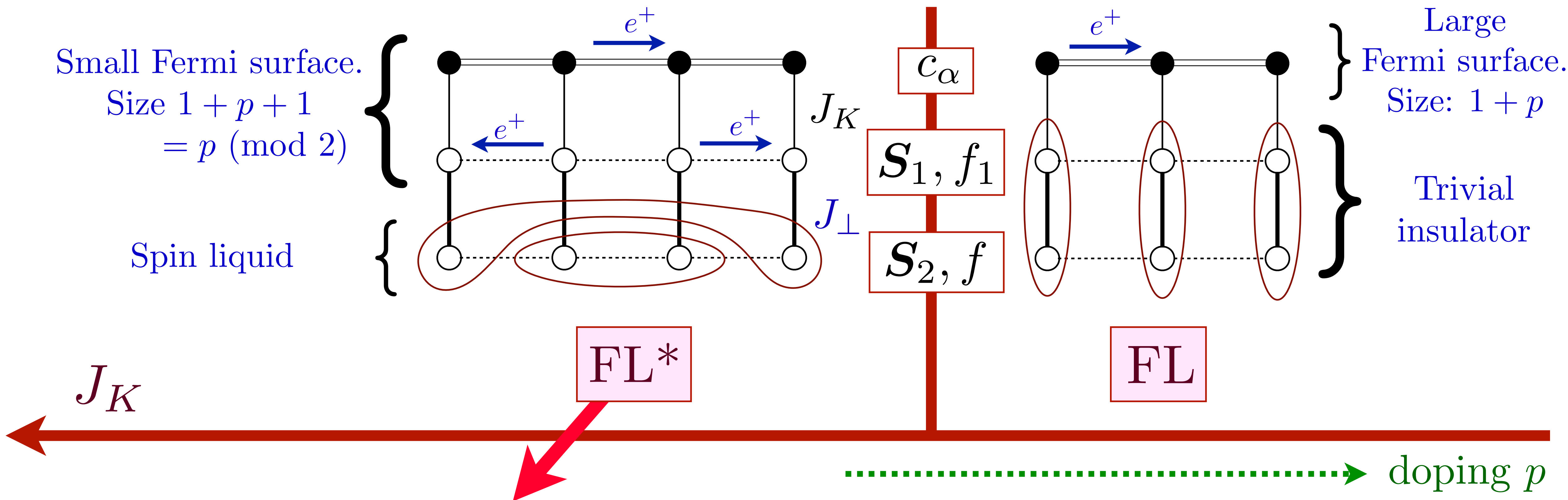
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1. Open questions on the cuprate phase diagram
2. Theory of the pseudogap metal
3. The π -flux spin liquid
4. Confinement transitions of the π -flux spin liquid
5. Recap



A theory for the confinement of fractionalized excitations in the π -flux spin liquid with fermionic spinons dual to the CP^1 spin liquid with bosonic spinons from electrically charged excitations.

Ancilla theory of the Hubbard model



Pseudogap metal: $\langle c_\alpha^\dagger f_{1\alpha} \rangle \neq 0$

f_α form π -flux spin liquid with $SU(2)_N$ gauge field

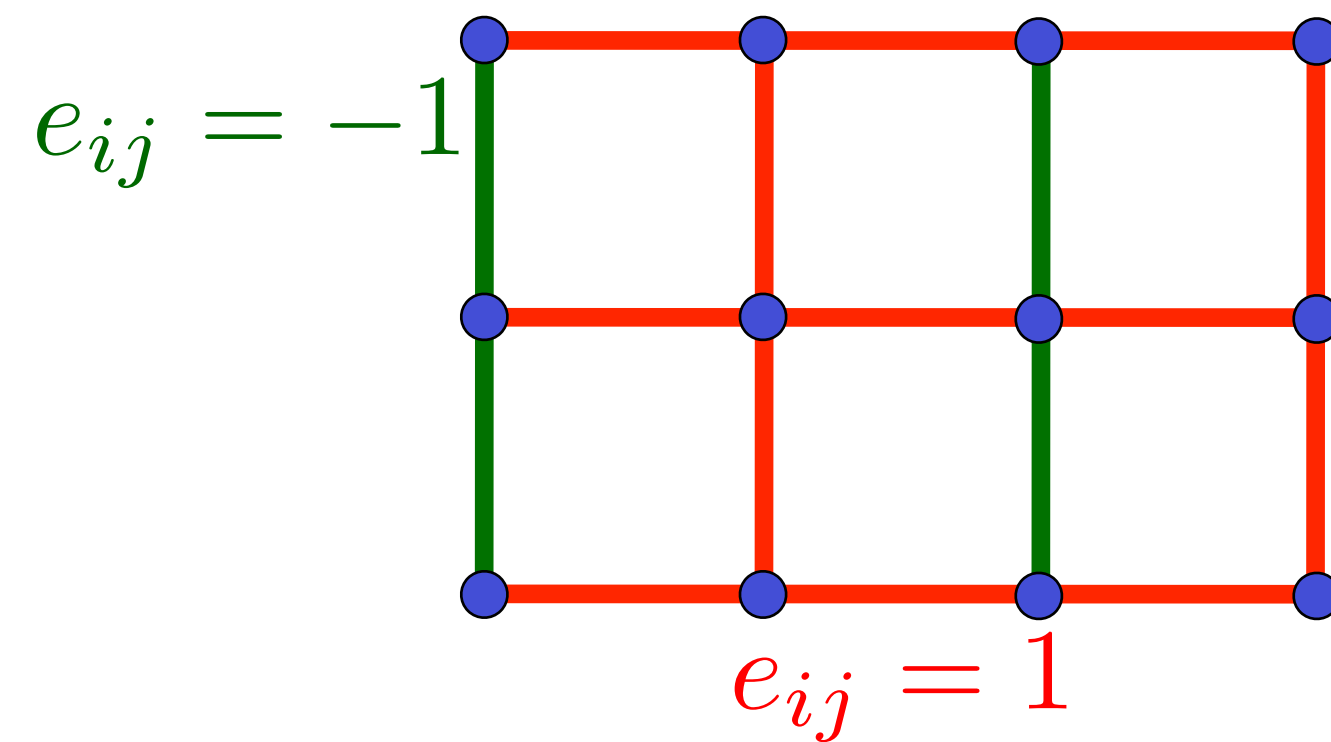
Charge e , $SU(2)_N$ fundamental, Higgs boson $B \sim \begin{pmatrix} f_{1\alpha}^\dagger f_\alpha \\ \varepsilon_{\alpha\beta} f_{1\alpha}^\dagger f_\beta^\dagger \end{pmatrix}$

Boson with same quantum numbers in X.-G. Wen and P.A. Lee, PRL **76**, 503 (1996)

Confinement of $SU(2)_N$ gauge theory by charge fluctuations

- Begin with the π -flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right)$$

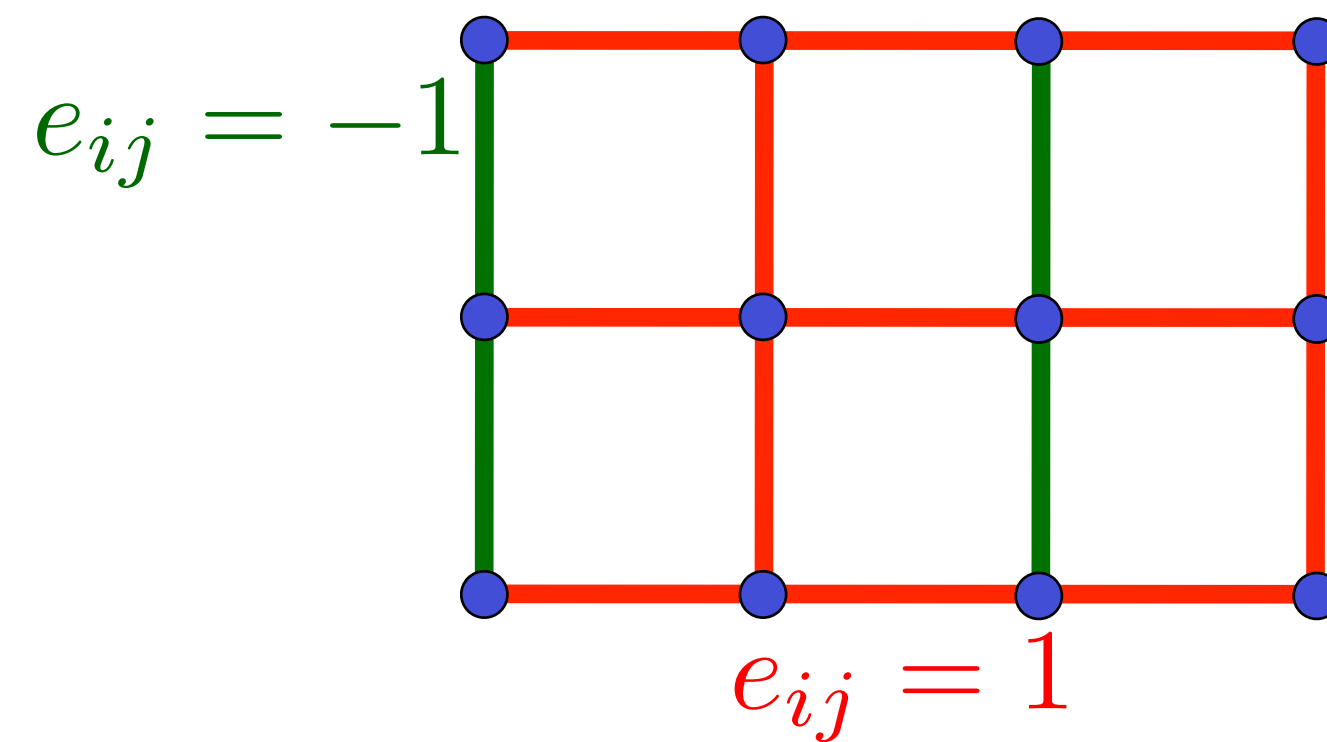


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H_f is invariant under $SU(2)$ rotations in spin and $SU(2)_N$ rotations in Nambu space; U_{ij} is the $SU(2)_N$ gauge field.

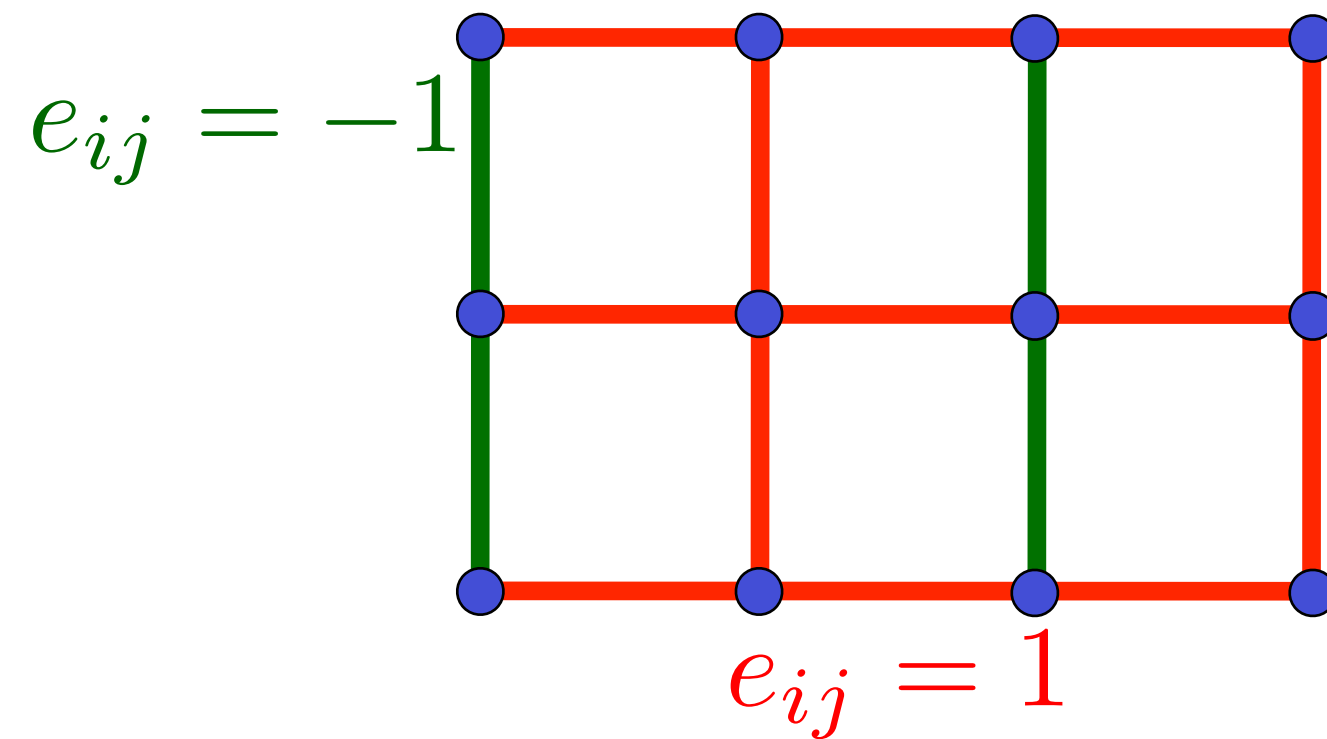


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H_f is invariant under $SU(2)$ rotations in spin and $SU(2)_N$ rotations in Nambu space; U_{ij} is the $SU(2)_N$ gauge field.



- The nearest-neighbor effective Hamiltonian for charge e , $SU(2)_N$ fundamental boson B_i is constrained by the fact that the composite of B_i and Ψ_i is an electron:

$$H_B = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B_i^\dagger U_{ij} B_j - B_j^\dagger U_{ji} B_i \right) + \dots$$

Confinement of $SU(2)_N$ gauge theory by charge fluctuations

$$\mathcal{L}(B) = H_B + \frac{u}{2} \sum_i \rho_i^2 + V_1 \sum_i \rho_i (\rho_{i+\hat{x}} + \rho_{i+\hat{y}}) + g \sum_{\langle ij \rangle} |\Delta_{ij}|^2$$

$$+ J_1 \sum_{\langle ij \rangle} Q_{ij}^2 + K_1 \sum_{\langle ij \rangle} J_{ij}^2.$$

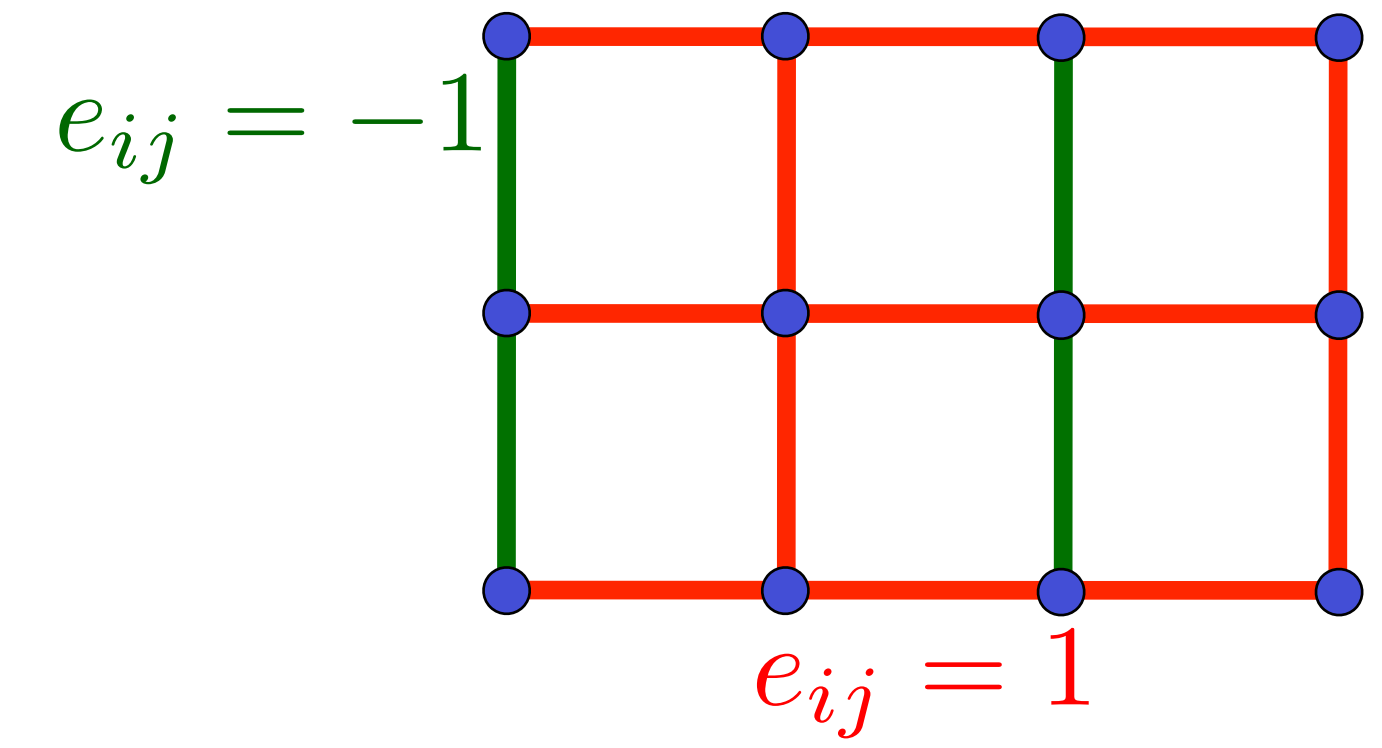
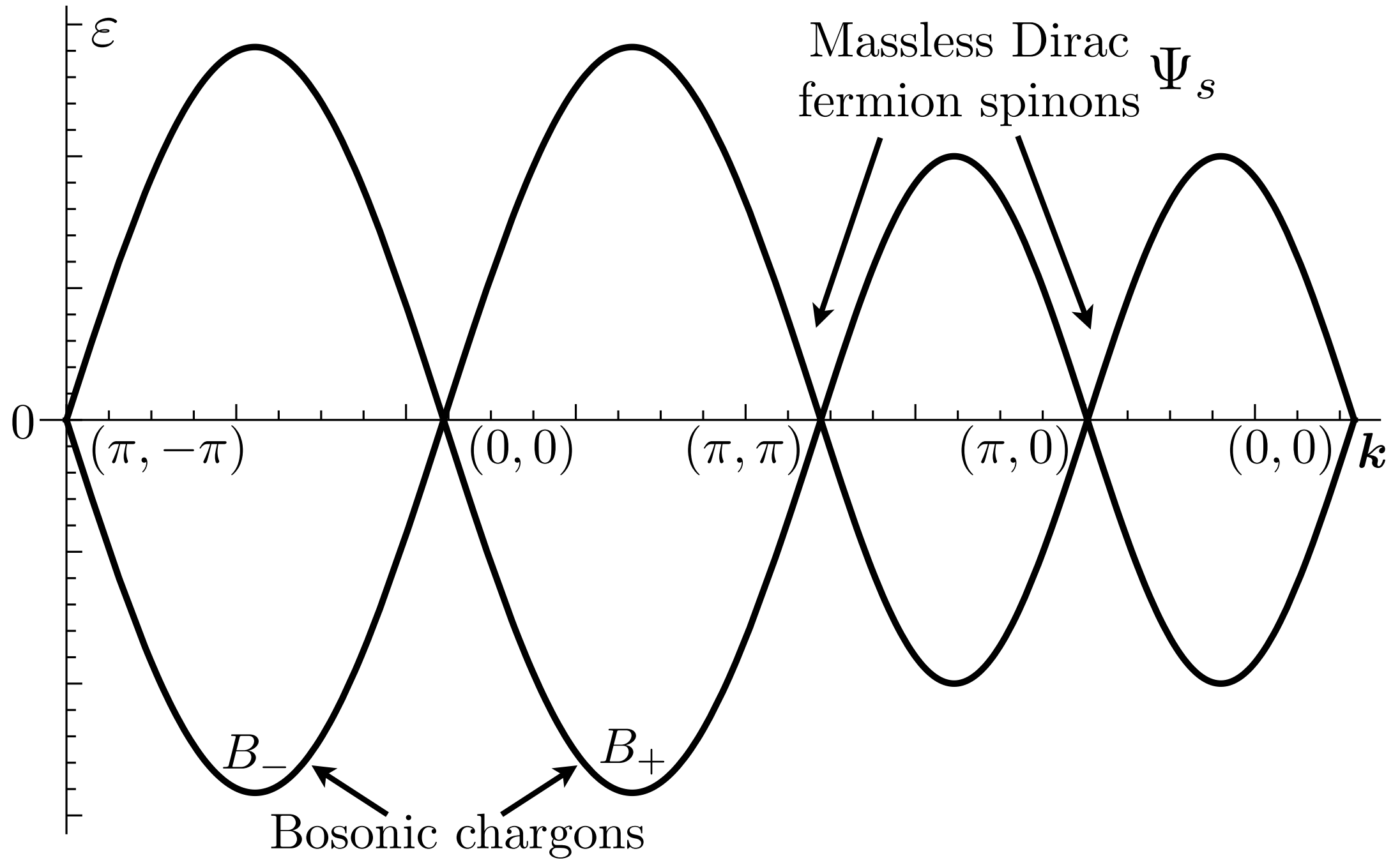
site charge density: $\langle c_{i\alpha}^\dagger c_{i\alpha} \rangle \sim \rho_i = B_i^\dagger B_i$

bond density: $\langle c_{i\alpha}^\dagger c_{j\alpha} + c_{j\alpha}^\dagger c_{i\alpha} \rangle \sim Q_{ij} = Q_{ji} = \text{Im} \left(B_i^\dagger e_{ij} U_{ij} B_j \right)$

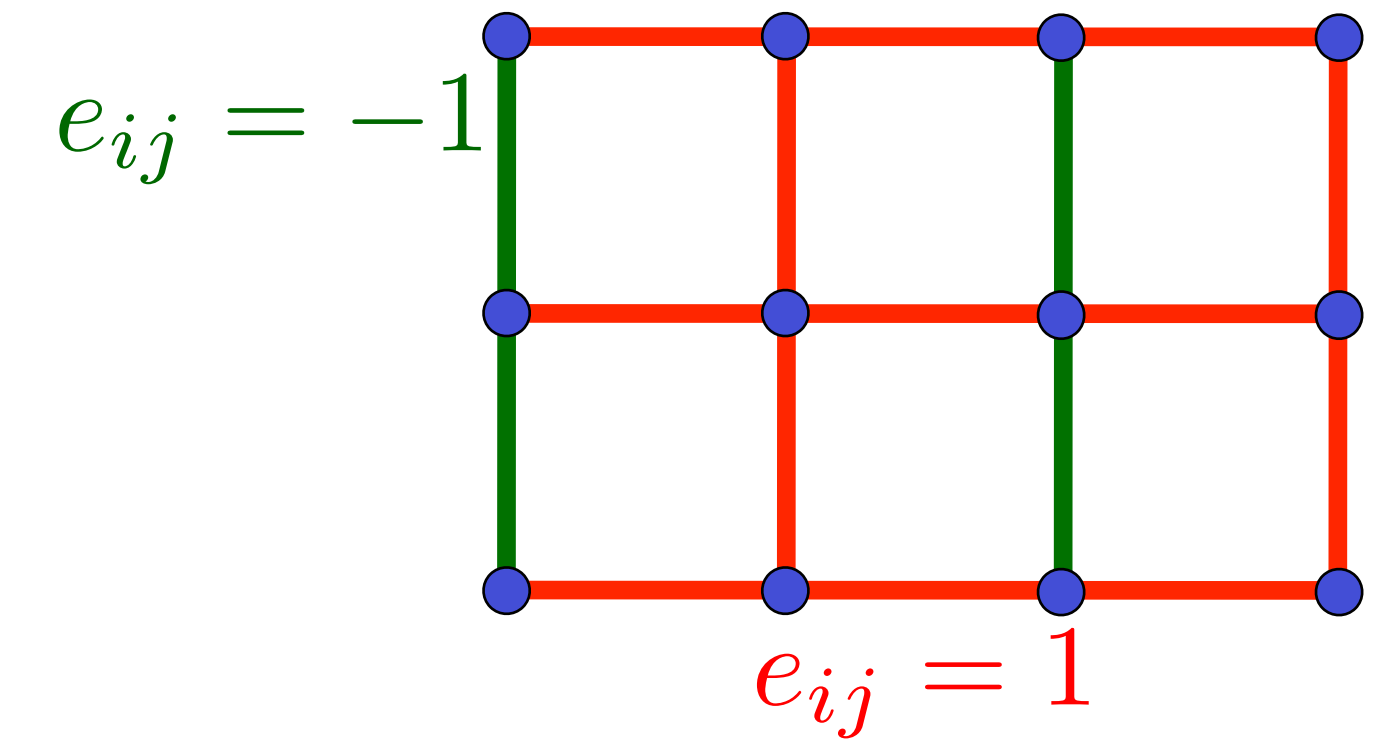
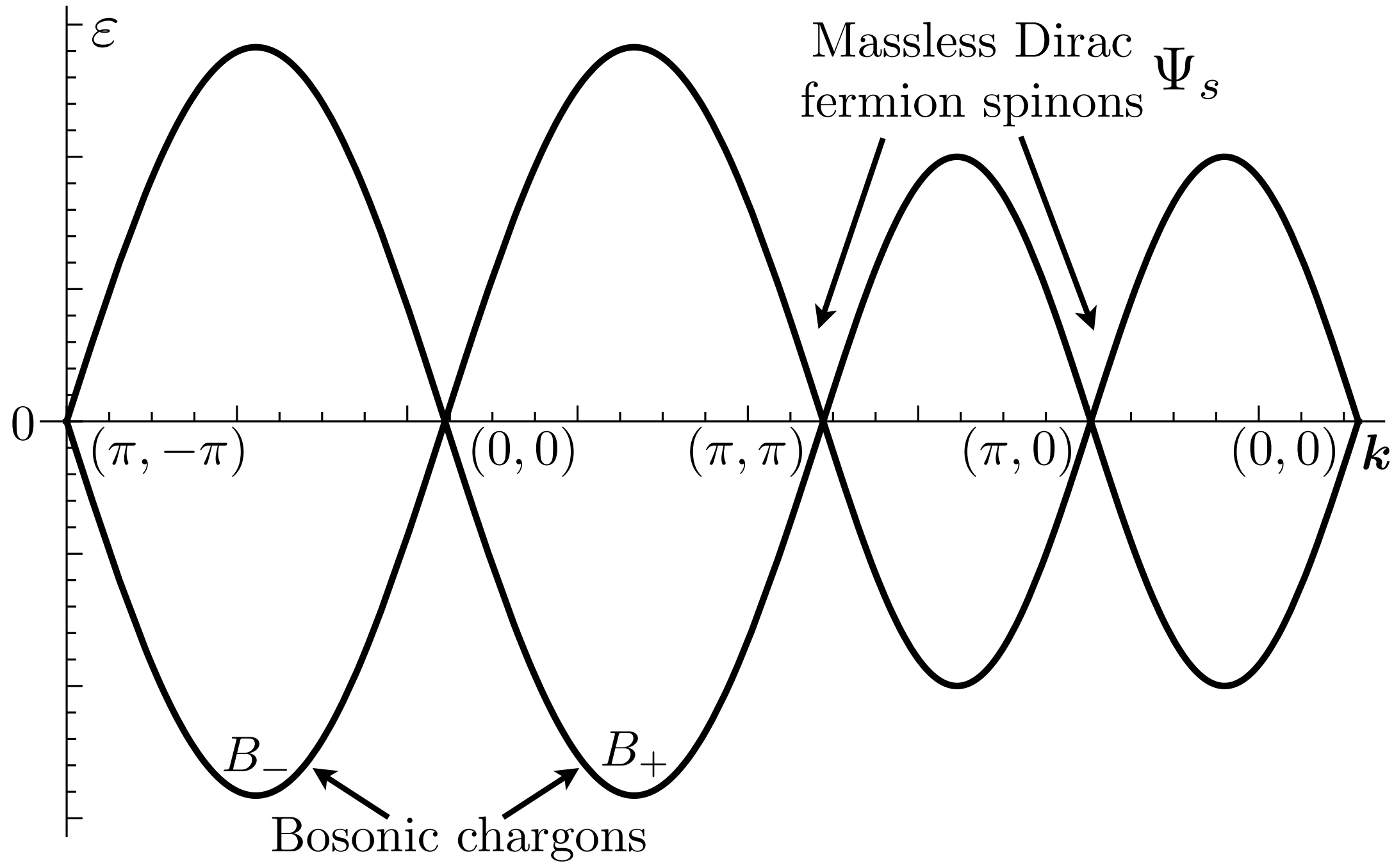
bond current: $i \langle c_{i\alpha}^\dagger c_{j\alpha} - c_{j\alpha}^\dagger c_{i\alpha} \rangle \sim J_{ij} = -J_{ji} = \text{Re} \left(B_i^\dagger e_{ij} U_{ij} B_j \right)$

Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim \Delta_{ij} = \Delta_{ji} = \varepsilon_{ab} B_{ai} e_{ij} U_{ij} B_{bj}.$

Confinement of $SU(2)_N$ gauge theory by charge fluctuations



Confinement of $SU(2)_N$ gauge theory by charge fluctuations



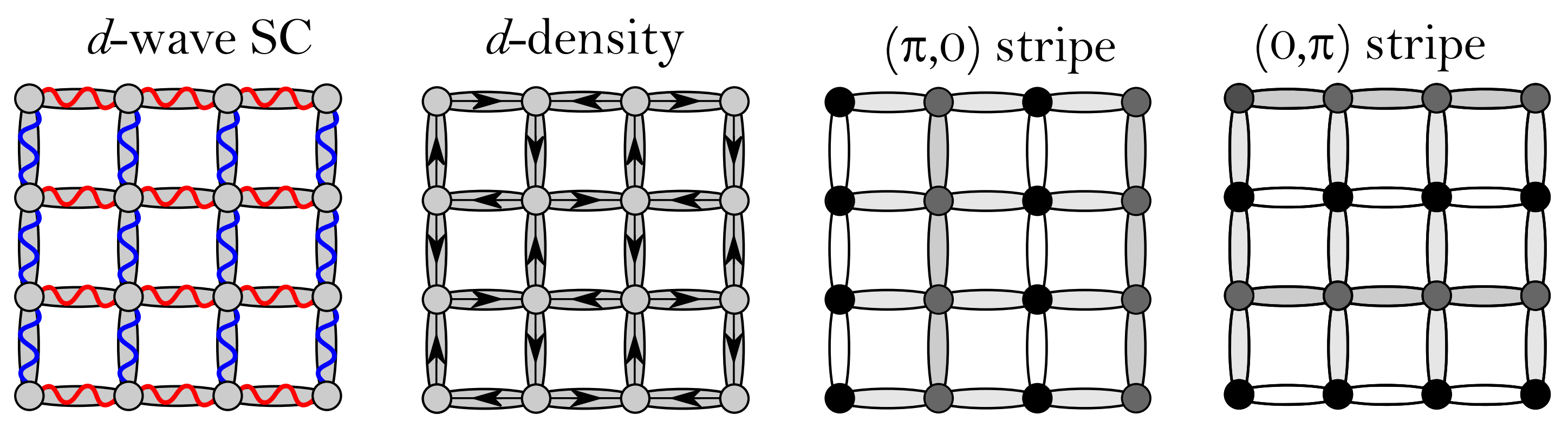
$SU(2)_N$ gauge-invariant and $SU(2)$ spin invariant order parameters of Higgs phases:

x -CDW : $\rho_{(\pi,0)} = B_{a+}^* B_{a+} - B_{a-}^* B_{a-}$

y -CDW : $\rho_{(0,\pi)} = B_{a+}^* B_{a-} + B_{a-}^* B_{a+}$

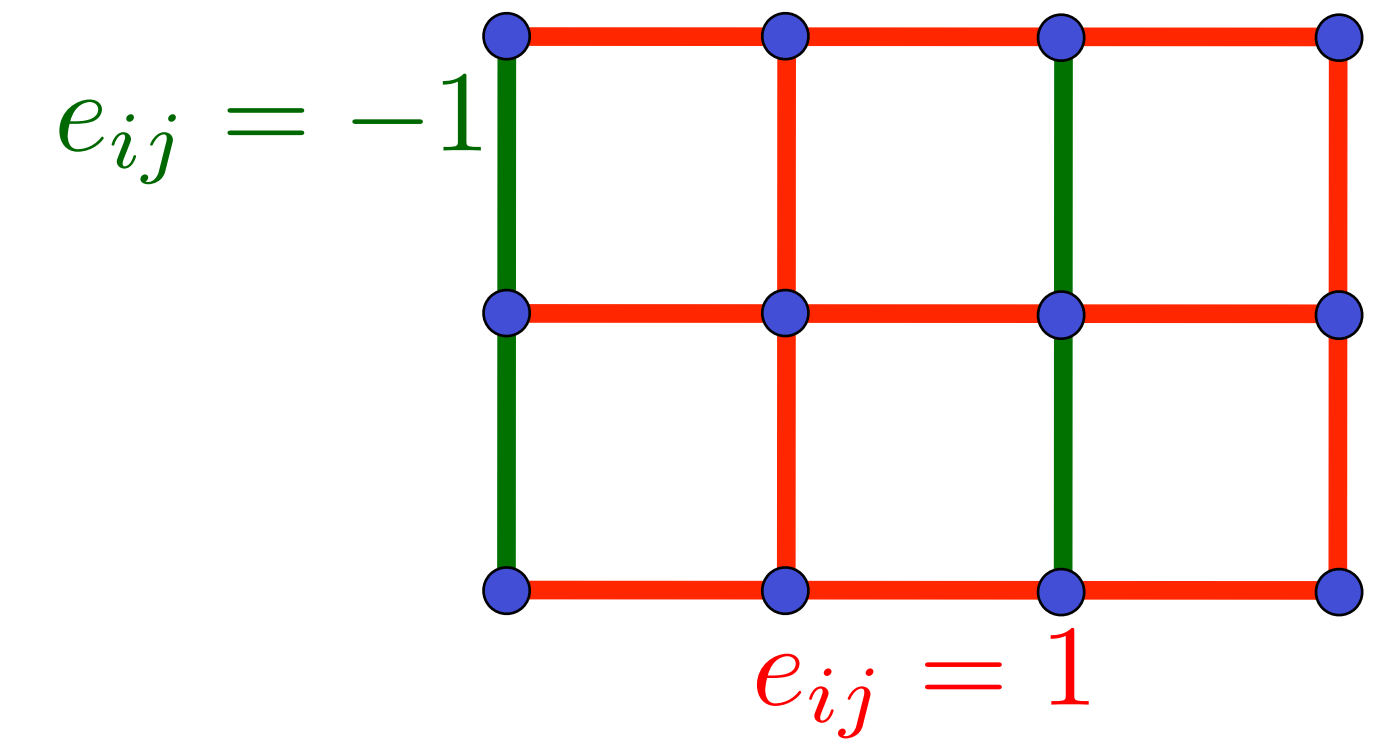
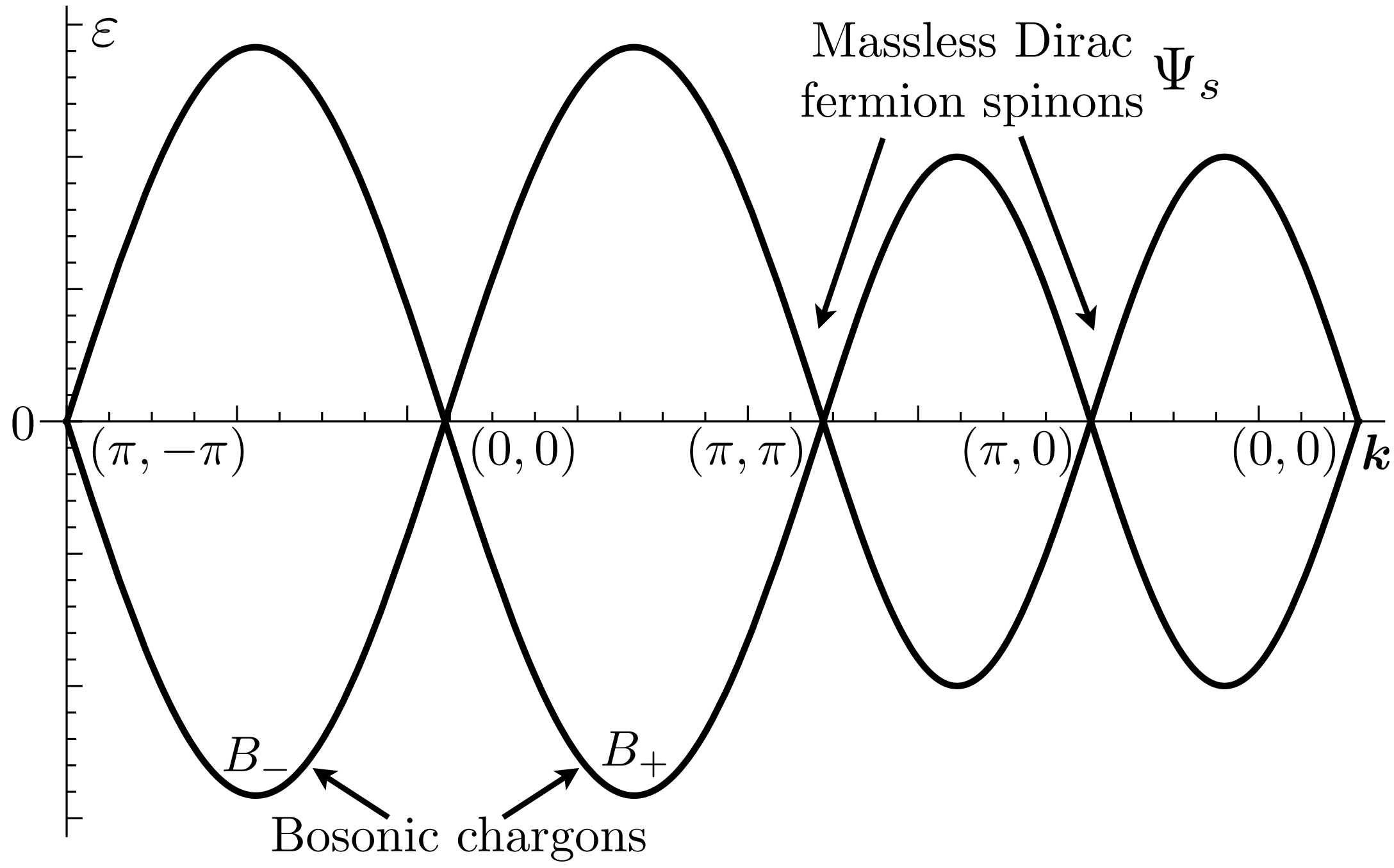
d -density wave : $D = i (B_{a+}^* B_{a-} - B_{a-}^* B_{a+})$

d -wave superconductor : $\Delta = \varepsilon_{ab} B_{a+} B_{b-}$

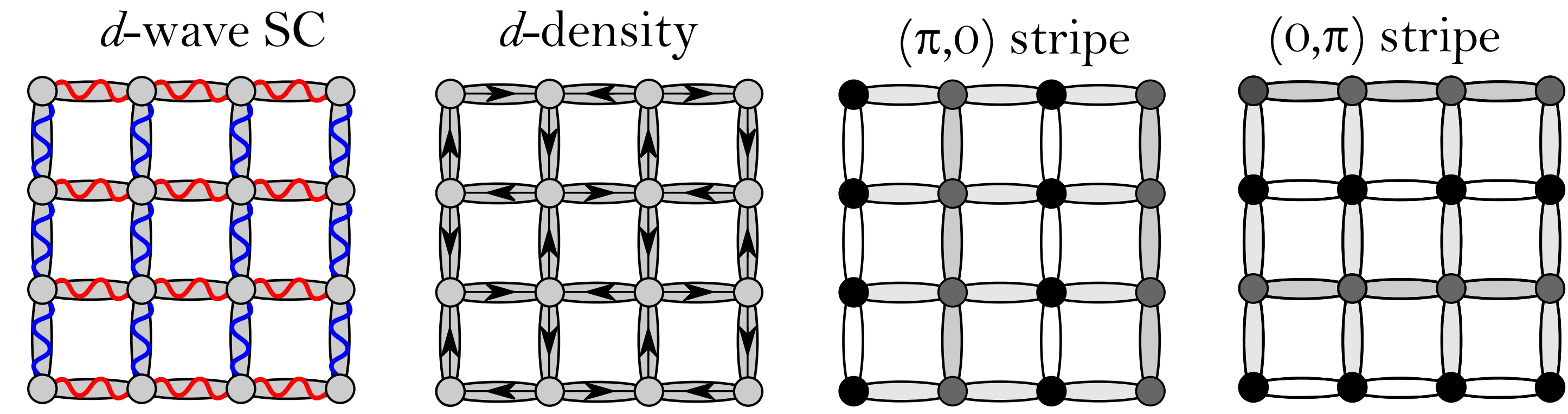


The $\mathcal{O}(B_{a\pm}^2)$ terms in the energy have a $SO(5)_b$ rotation symmetry between these orders.

Confinement of $SU(2)_N$ gauge theory by charge fluctuations

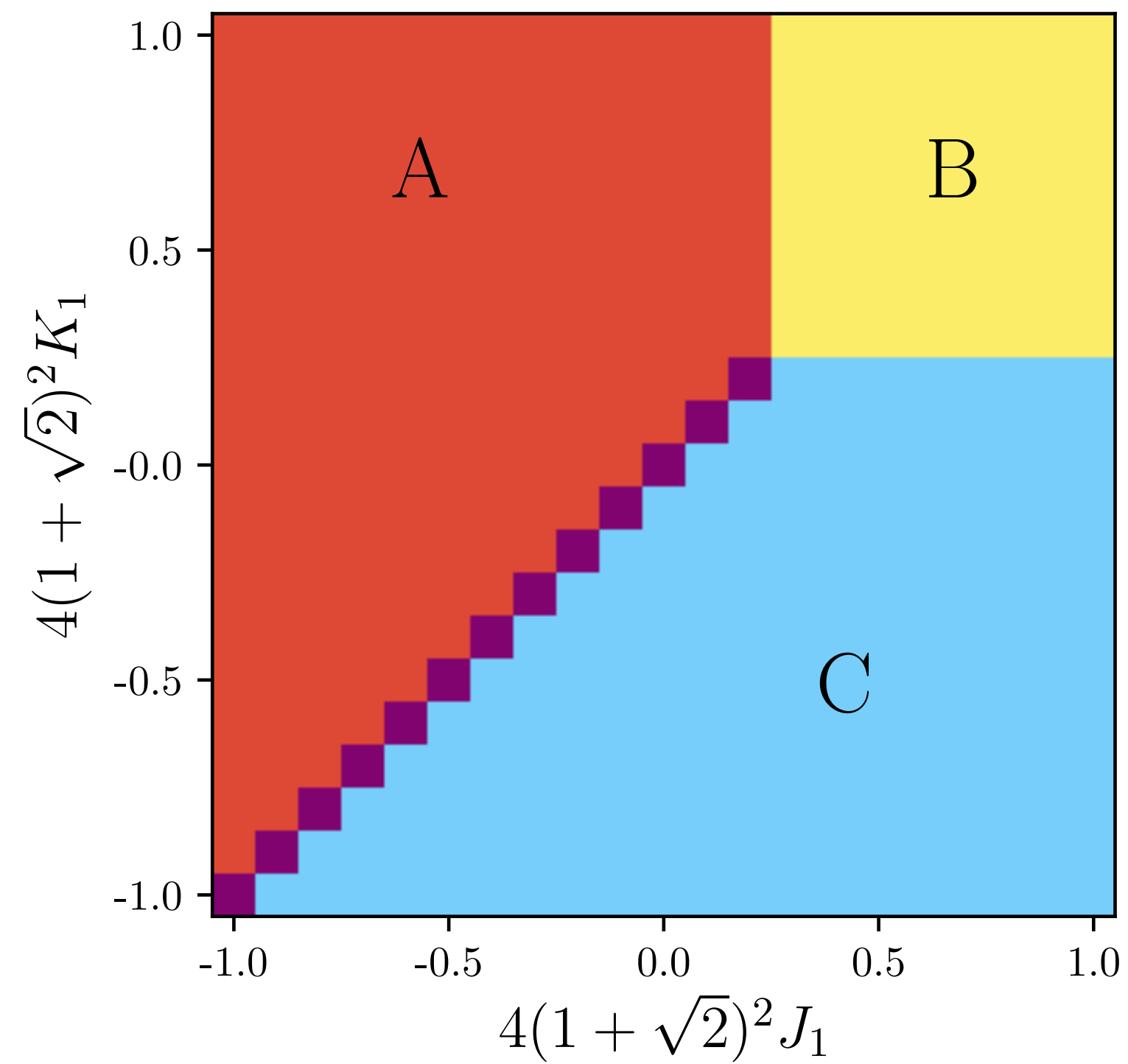


The B_{av} ($a \rightarrow SU(2)_N$ gauge, $v \rightarrow$ valley) are the “square roots” of conventional *d*-wave superconductor, charge density wave, pair density wave
...



Confinement of $SU(2)_N$ gauge theory by charge fluctuations

$$\langle B \rangle \neq 0$$

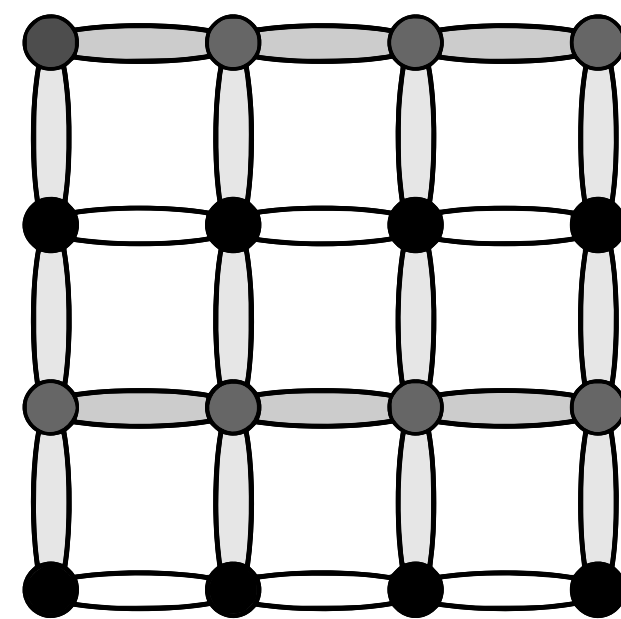
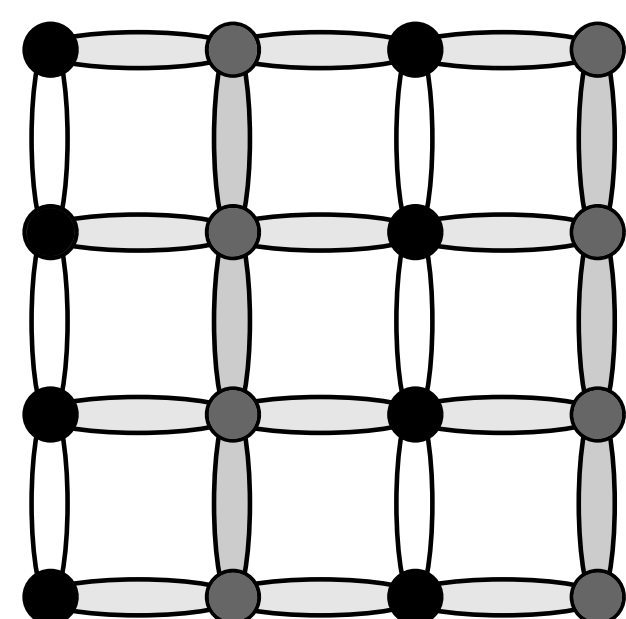
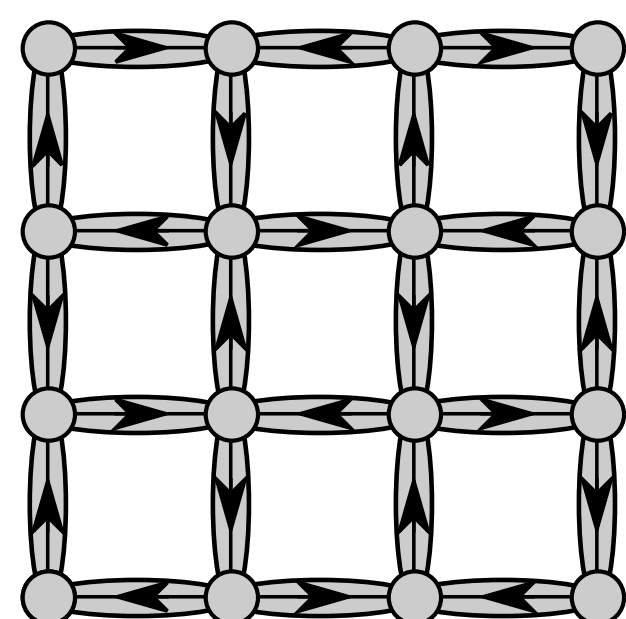
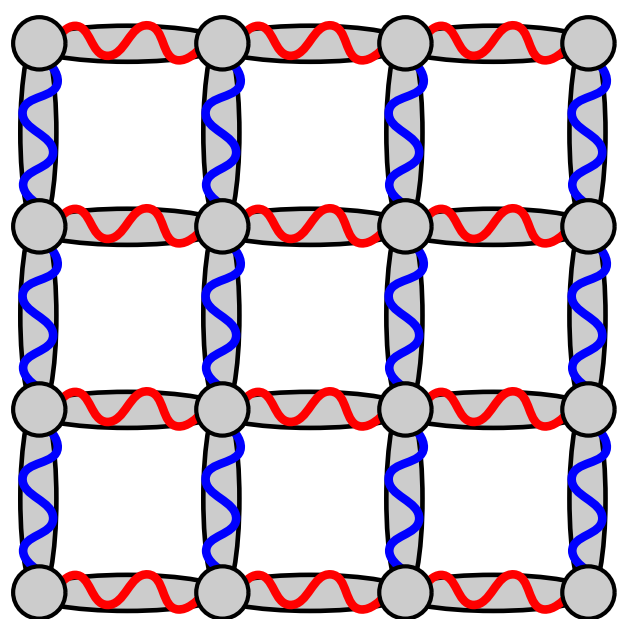


Phase B
d-wave SC

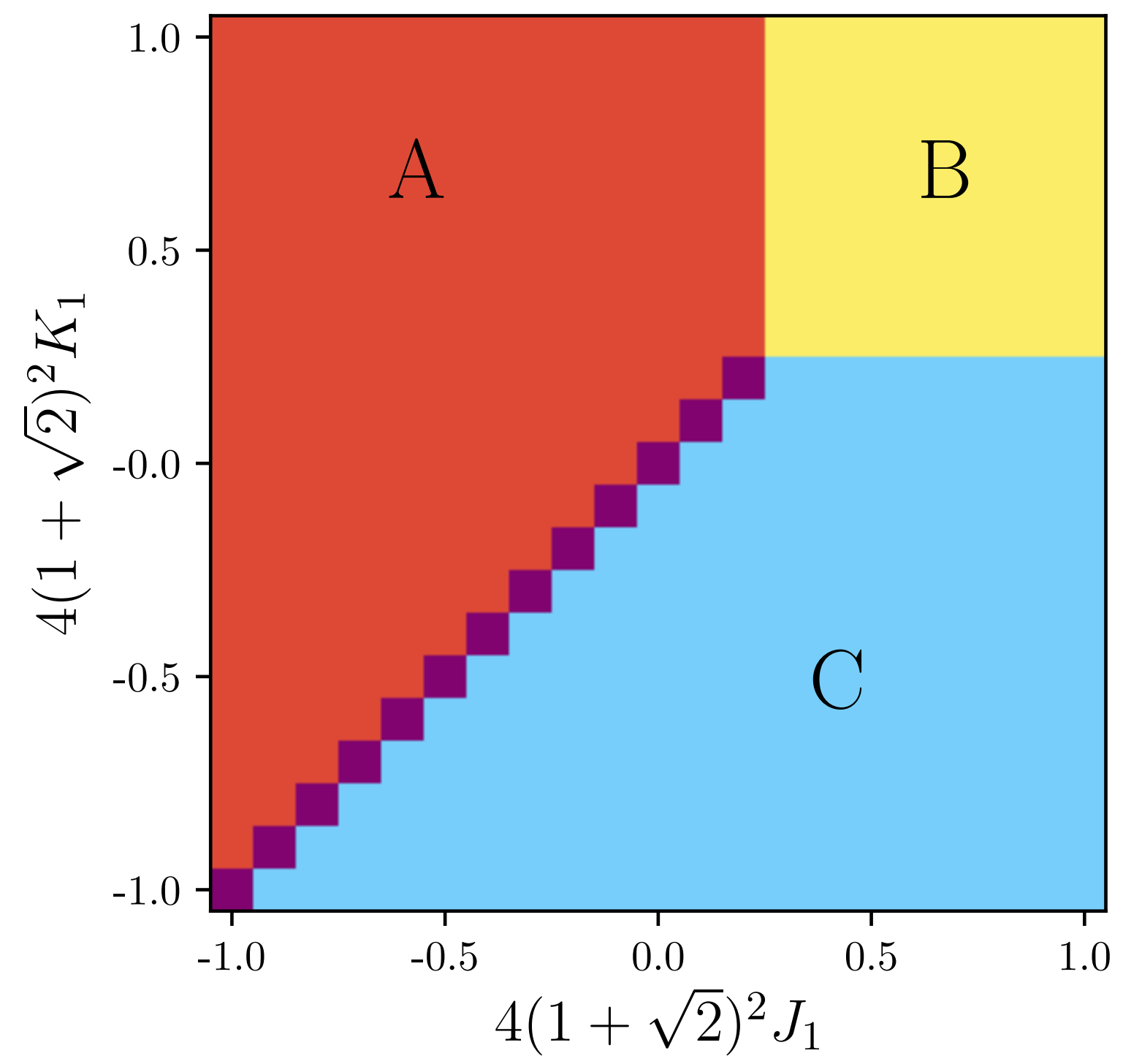
Phase C
d-density

Phase A
 $(\pi, 0)$ stripe

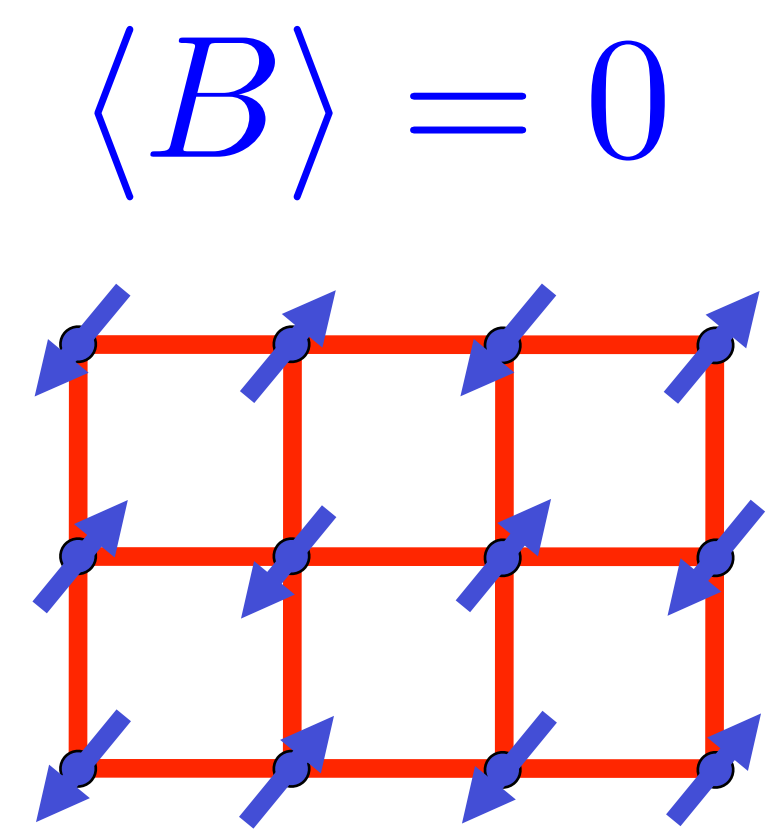
Phase A
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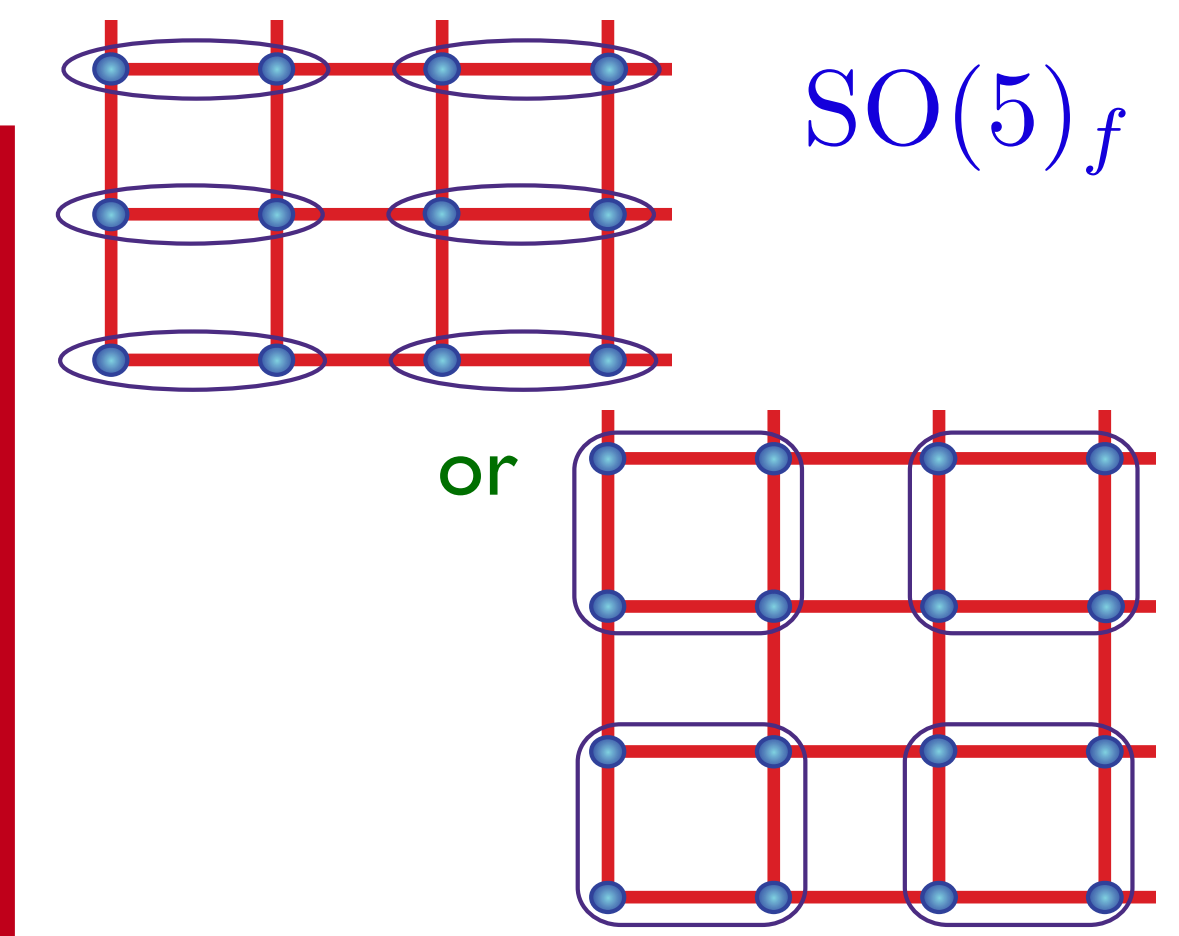
Global phase diagram of $SU(2)_N$ gauge theory



$\langle B \rangle \neq 0$
 $SO(5)_b$



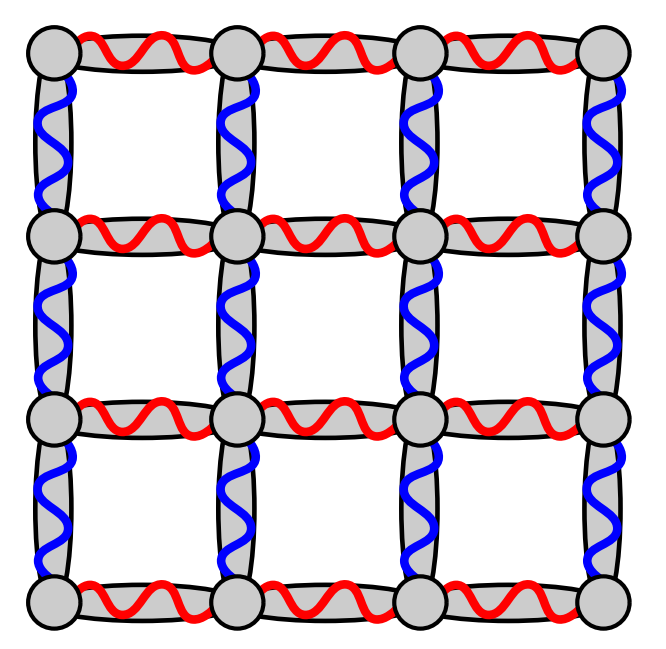
Confining phase:
Néel order



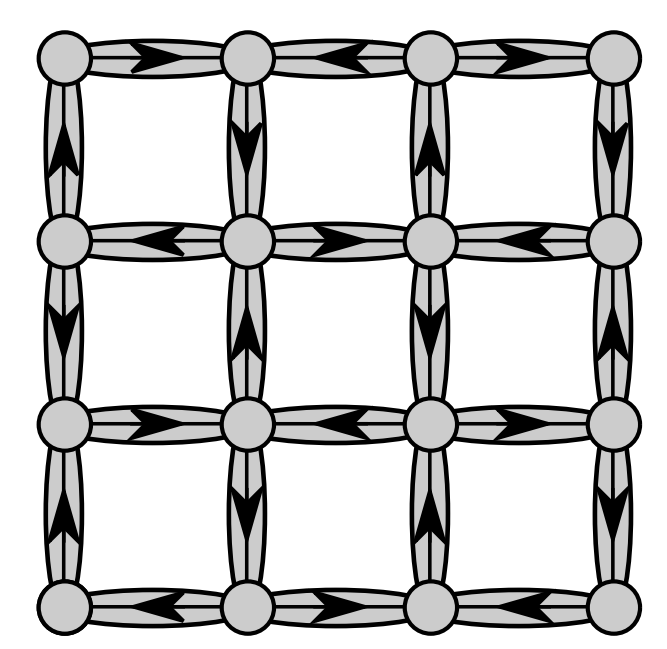
Confining phase:
VBS order



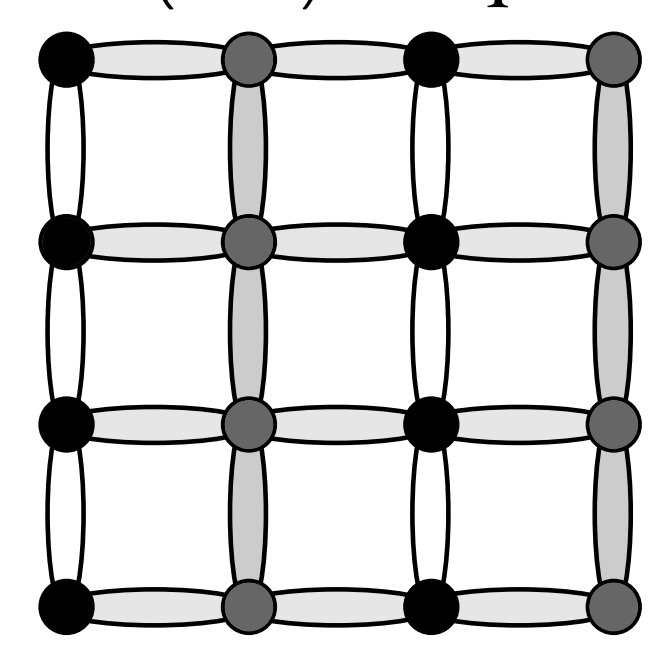
Phase B
d-wave SC



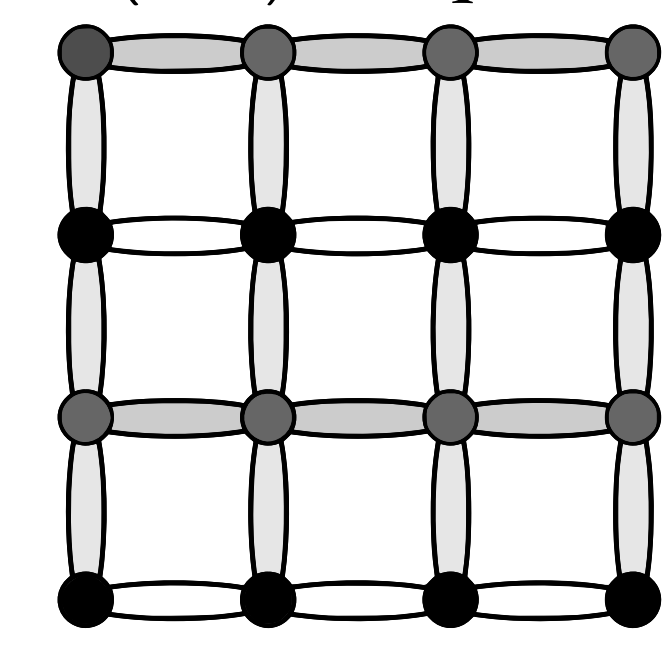
Phase C
d-density



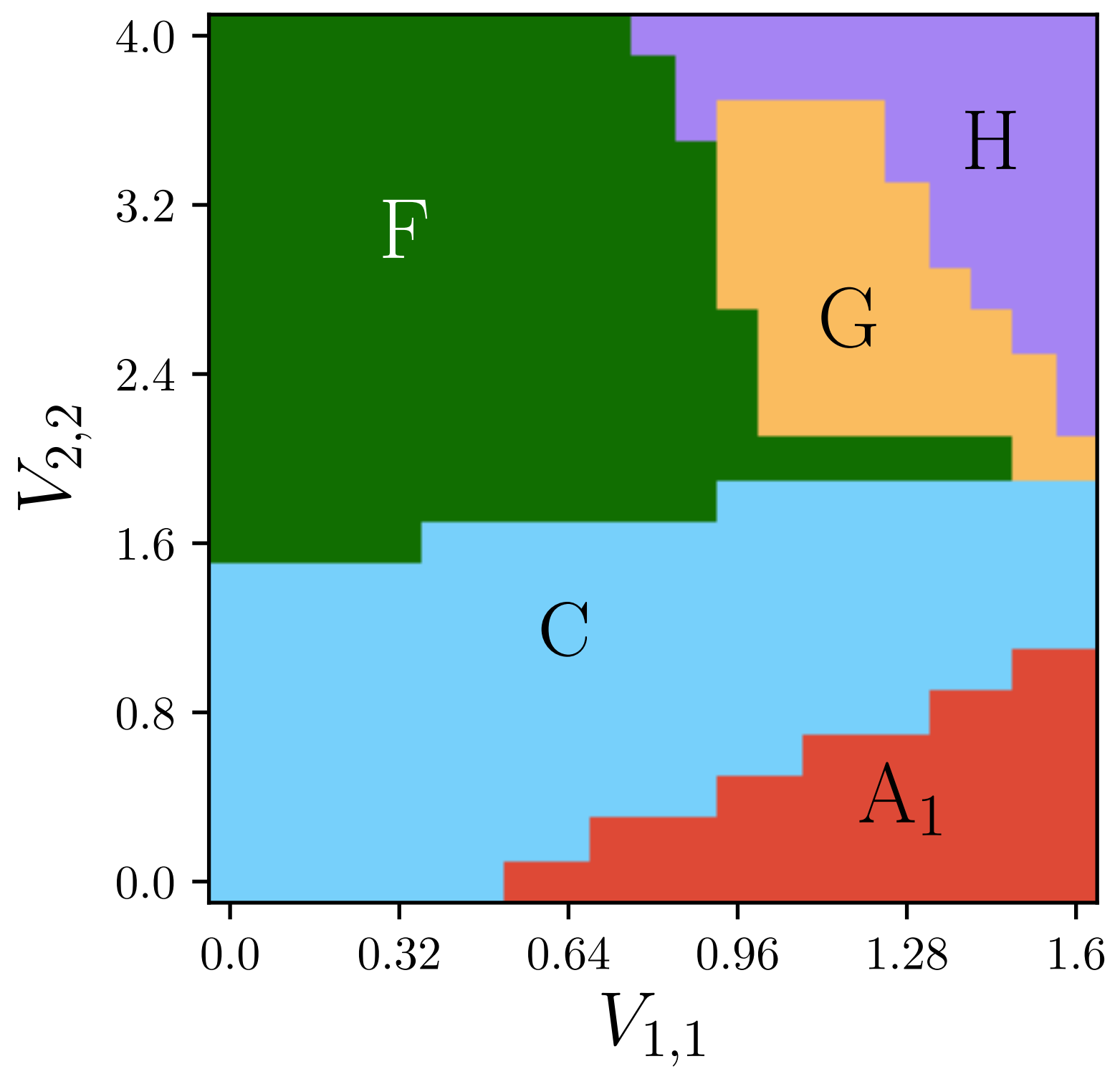
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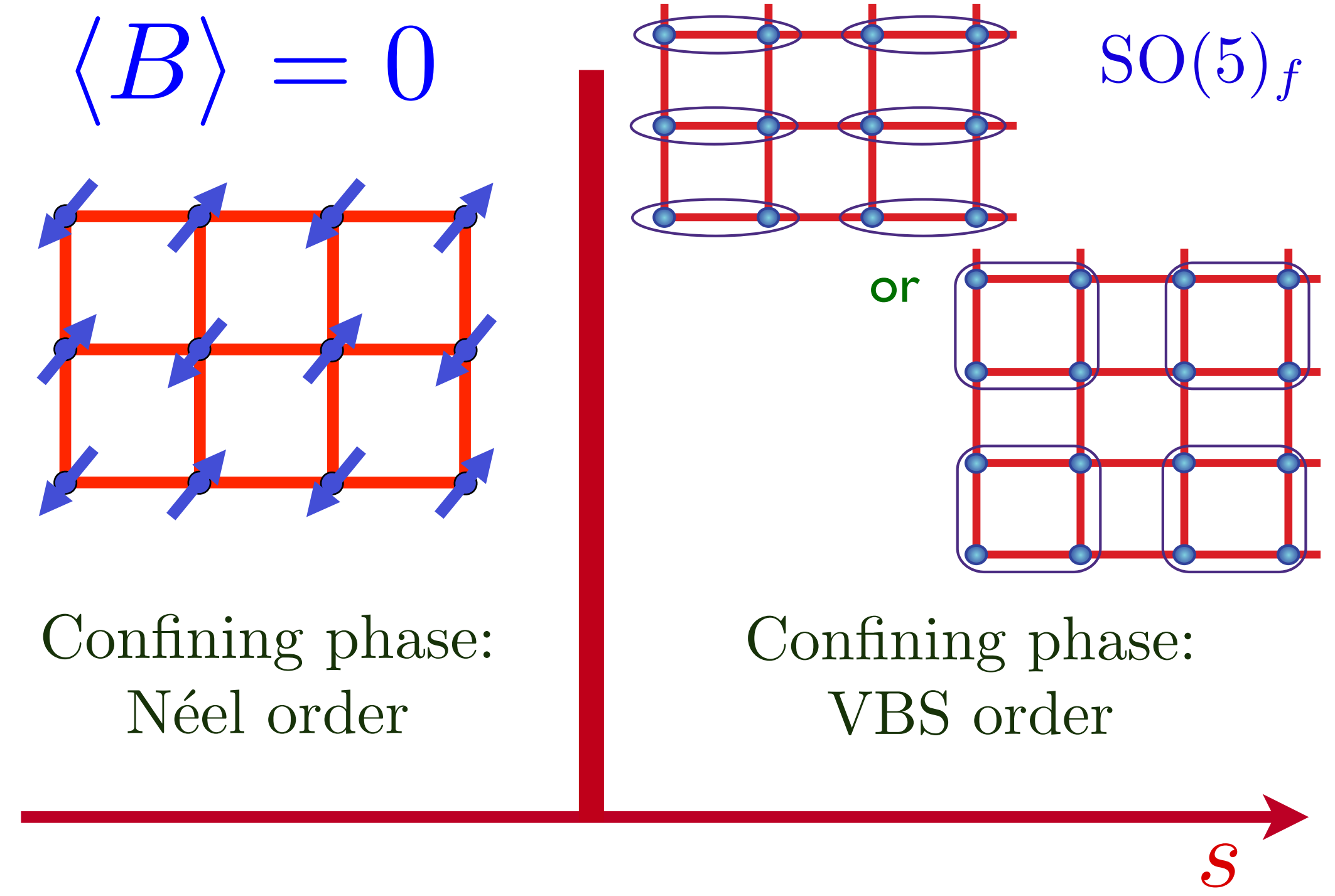


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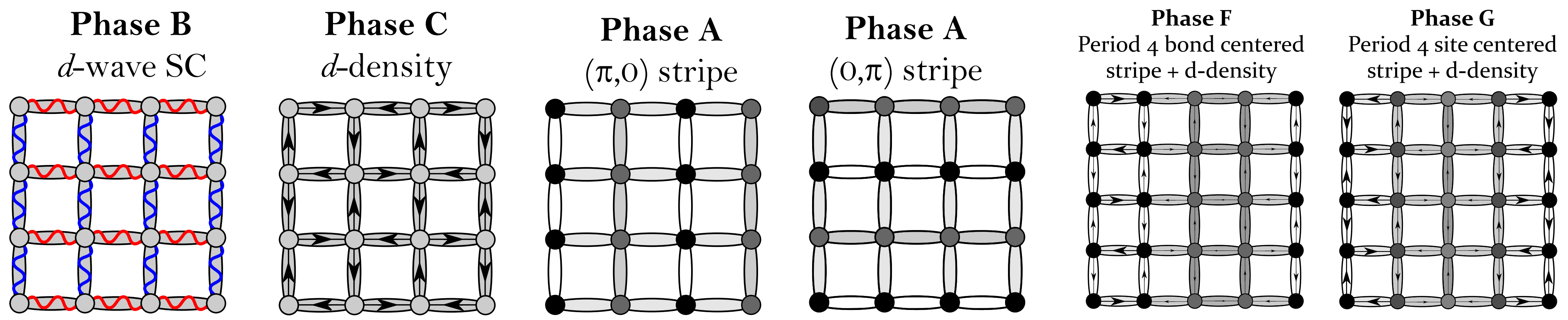


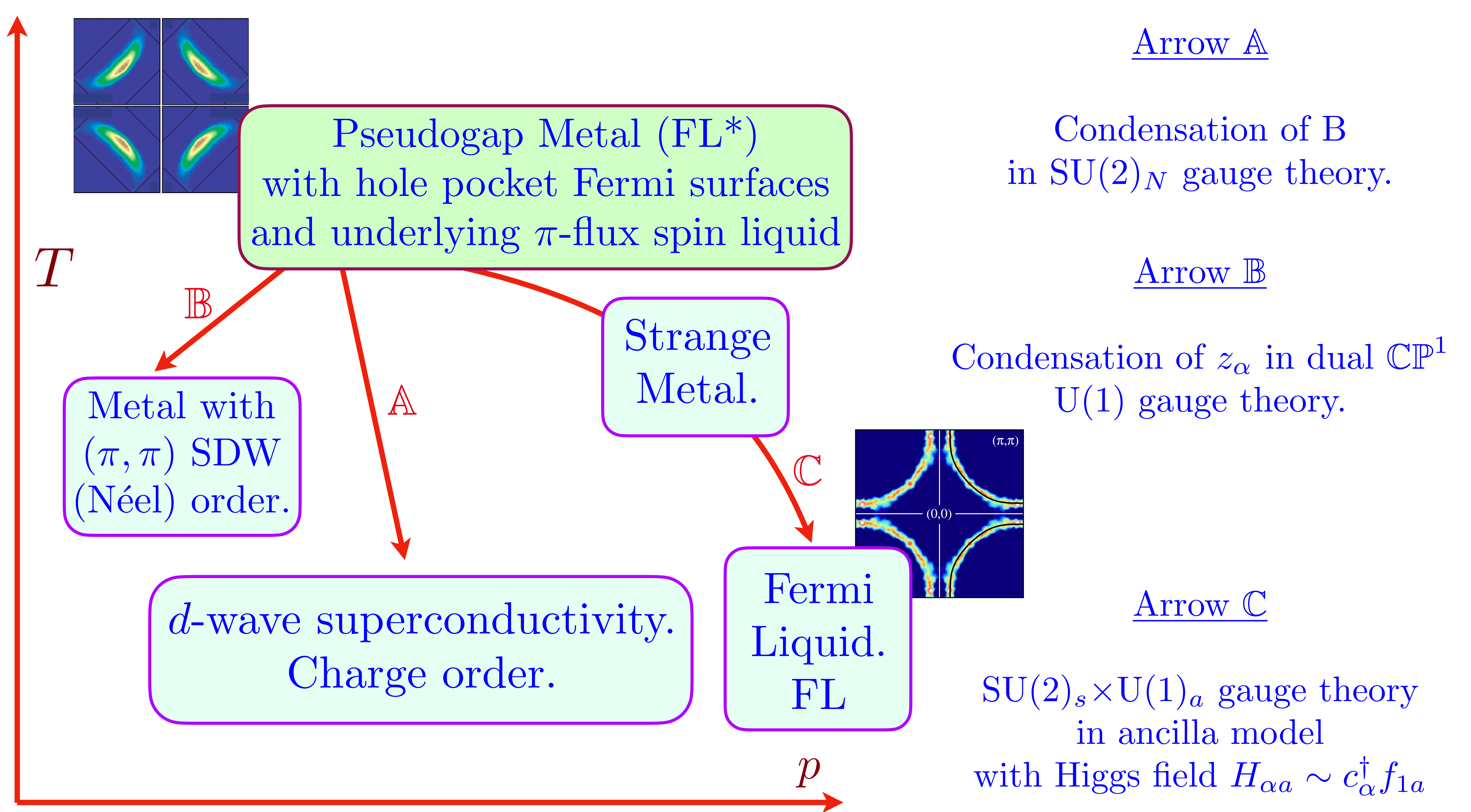
$\langle B \rangle \neq 0$

Including
further-neighbor
couplings in B



r





Can the underlying spin liquid help understand experimental observations of spin dynamics ?

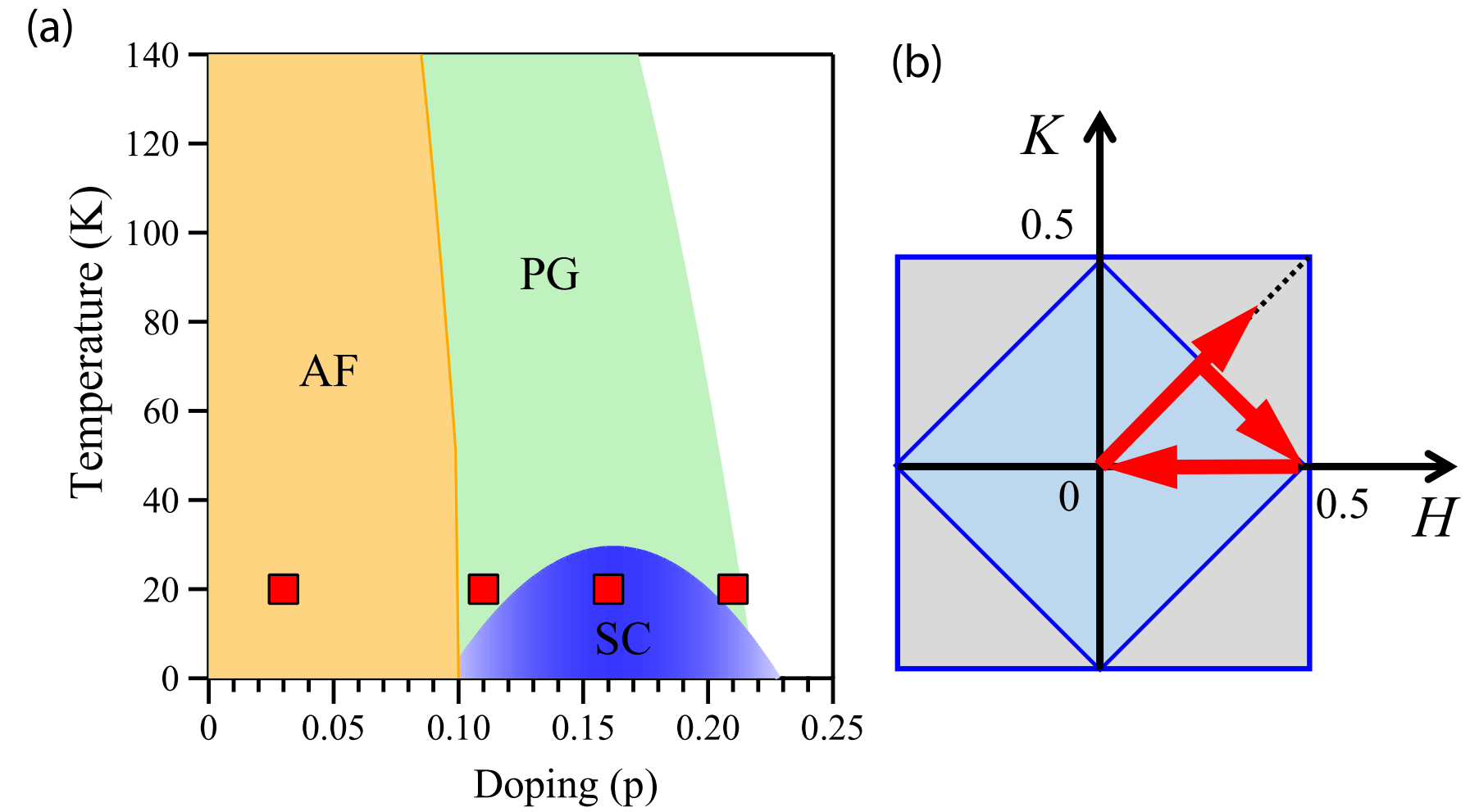


FIG. 1. (a) Schematic temperature-doping phase diagram of $(\text{Bi,Pb})_2(\text{Sr,La})_2\text{CuO}_{6+\delta}$. It shows the antiferromagnetic (AF), superconducting (SC), and the pseudogap (PG) regions. Here we study four doping levels as indicated by the solid red squares. (b) 2D reciprocal lattice for the pseudotetragonal structure and the first Brillouin zones (structural in light grey, magnetic in light blue). Coordinates H and K are in r.l.u.. The path followed for the measurements is indicated by the red arrows, starting at $(0.25, 0.25)$ and ending around $(0.30, 0.30)$ via $(0.5, 0)$ and $(0, 0)$.

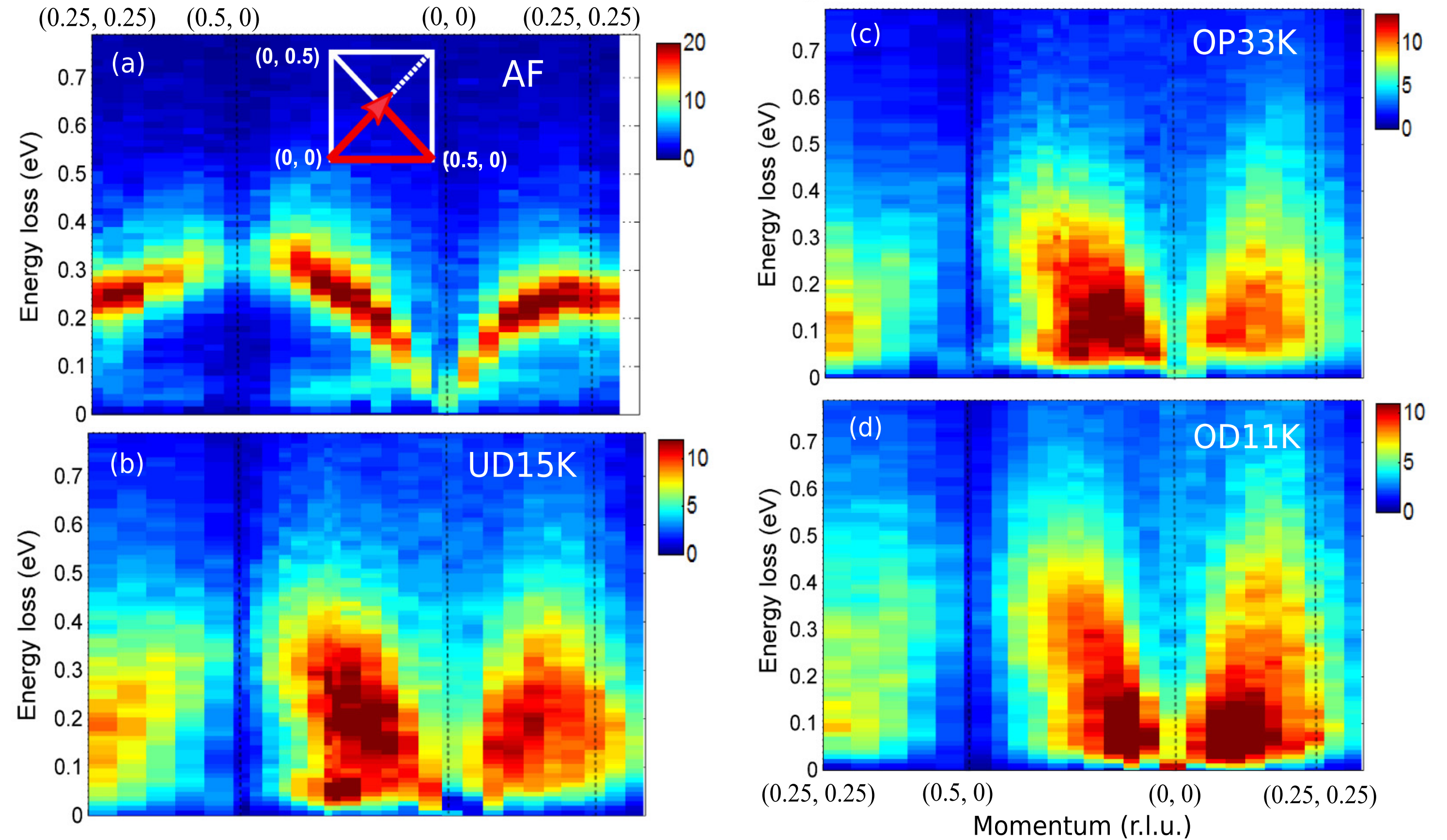
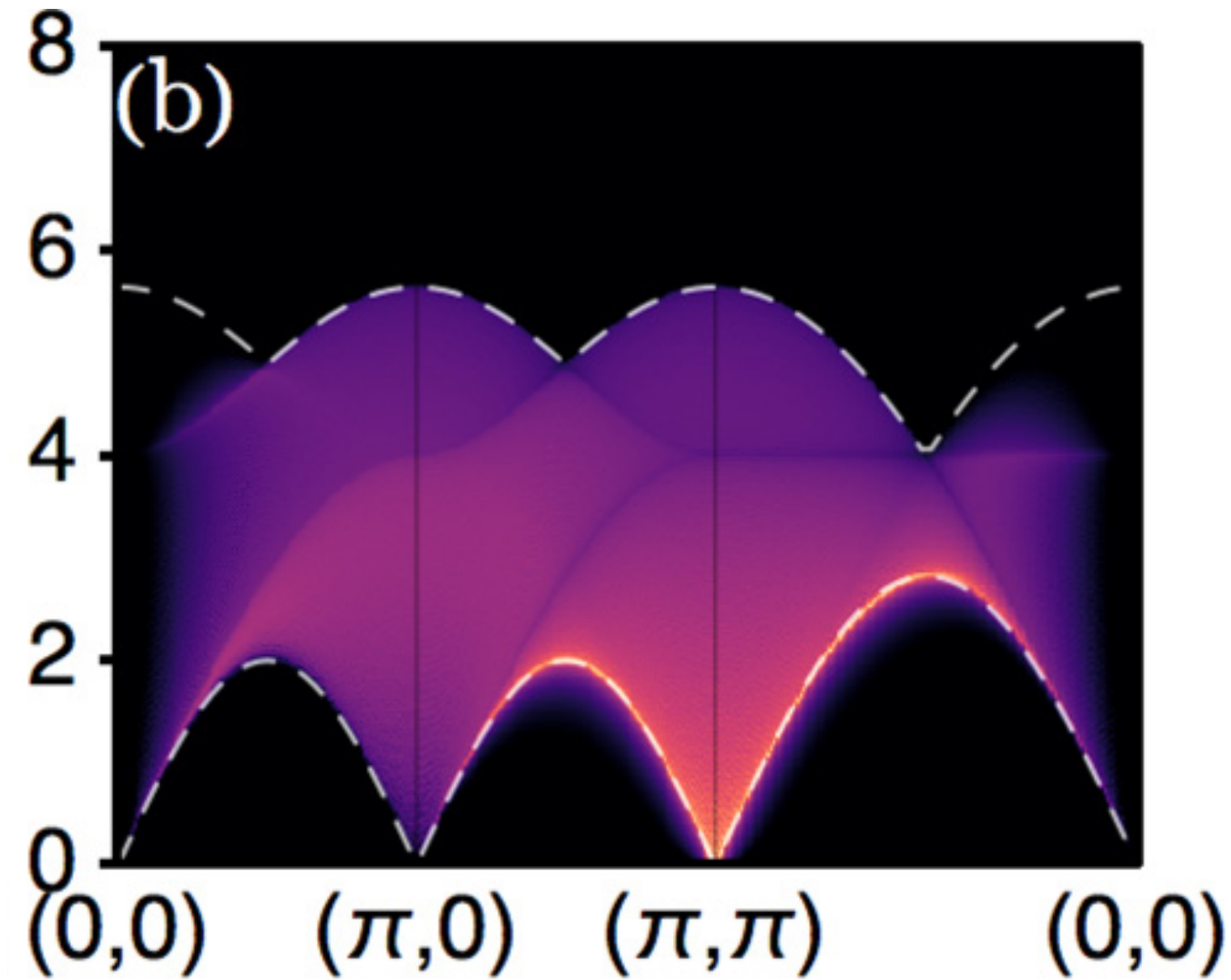


FIG. 2. Energy/momentum intensity maps of RIXS spectra for (a) AF ($p \simeq 0.03$), (b) UD15K ($p \simeq 0.11$), (c) OP33K ($p \simeq 0.16$), and (d) OD11K ($p \simeq 0.21$) along the high-symmetry momentum trajectory indicated in Fig. 1(b) and in the inset of (a). The intensity is in unit of photons/s/eV. Data were taken with π -polarized incident light at 20 K. Elastic peaks were subtracted for a better visualization of the low energy features.

Dispersion, damping, and intensity of spin excitations in the monolayer $(\text{Bi,Pb})_2(\text{Sr,La})_2\text{CuO}_{6+\delta}$ cuprate superconductor family, Y.Y. Peng *et al.*, PRB **98**, 144507 (2018)

Can the underlying spin liquid help understand experimental observations of spin dynamics ?



Free fermion spinons in π -flux

Nvsen Ma, Guang-Yu Sun, Yi-Zhuang You, Cenke Xu, Ashvin Vishwanath, Anders W. Sandvik, and Zi Yang Meng, PRB **98**, 174421 (2018)

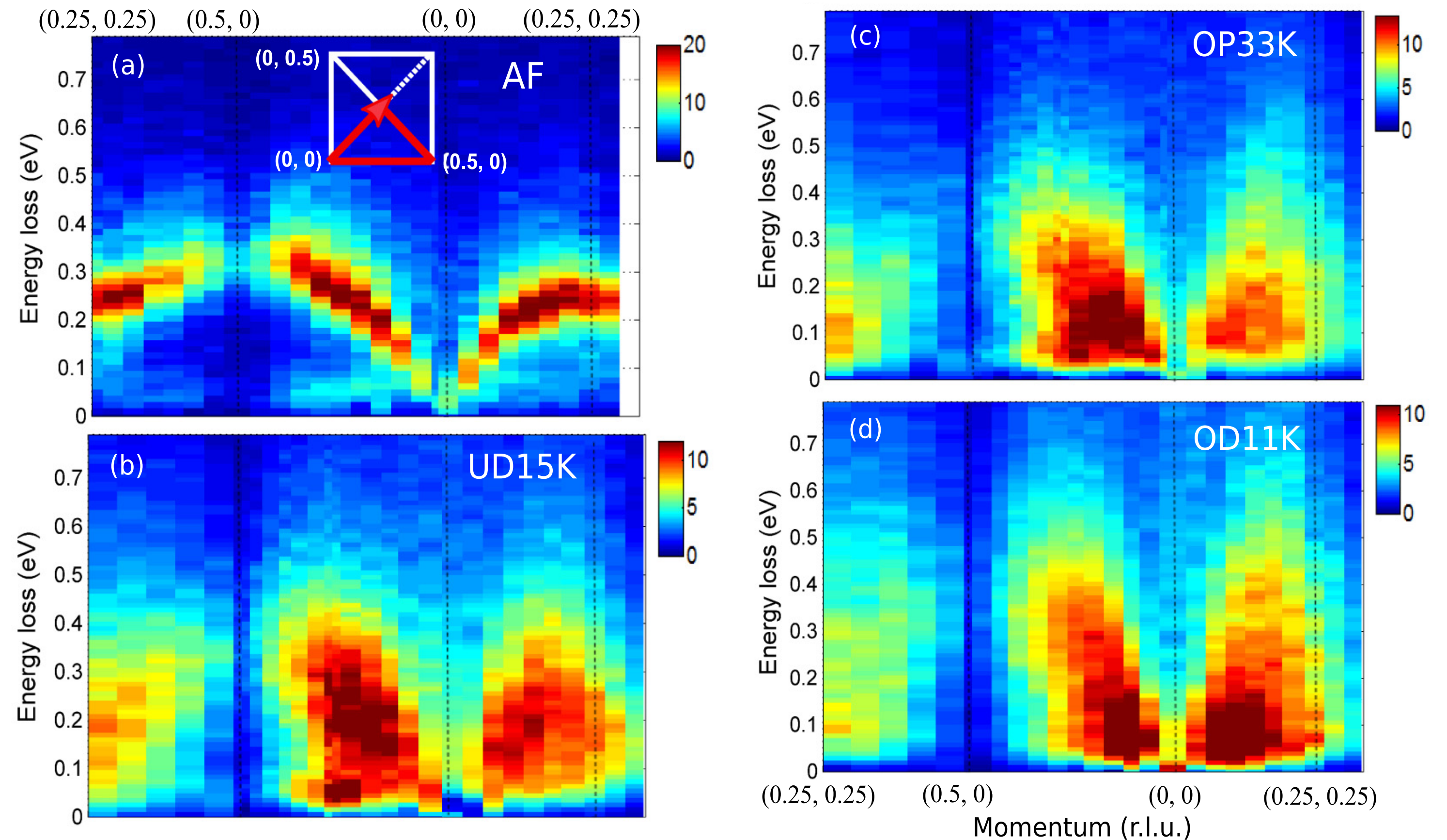


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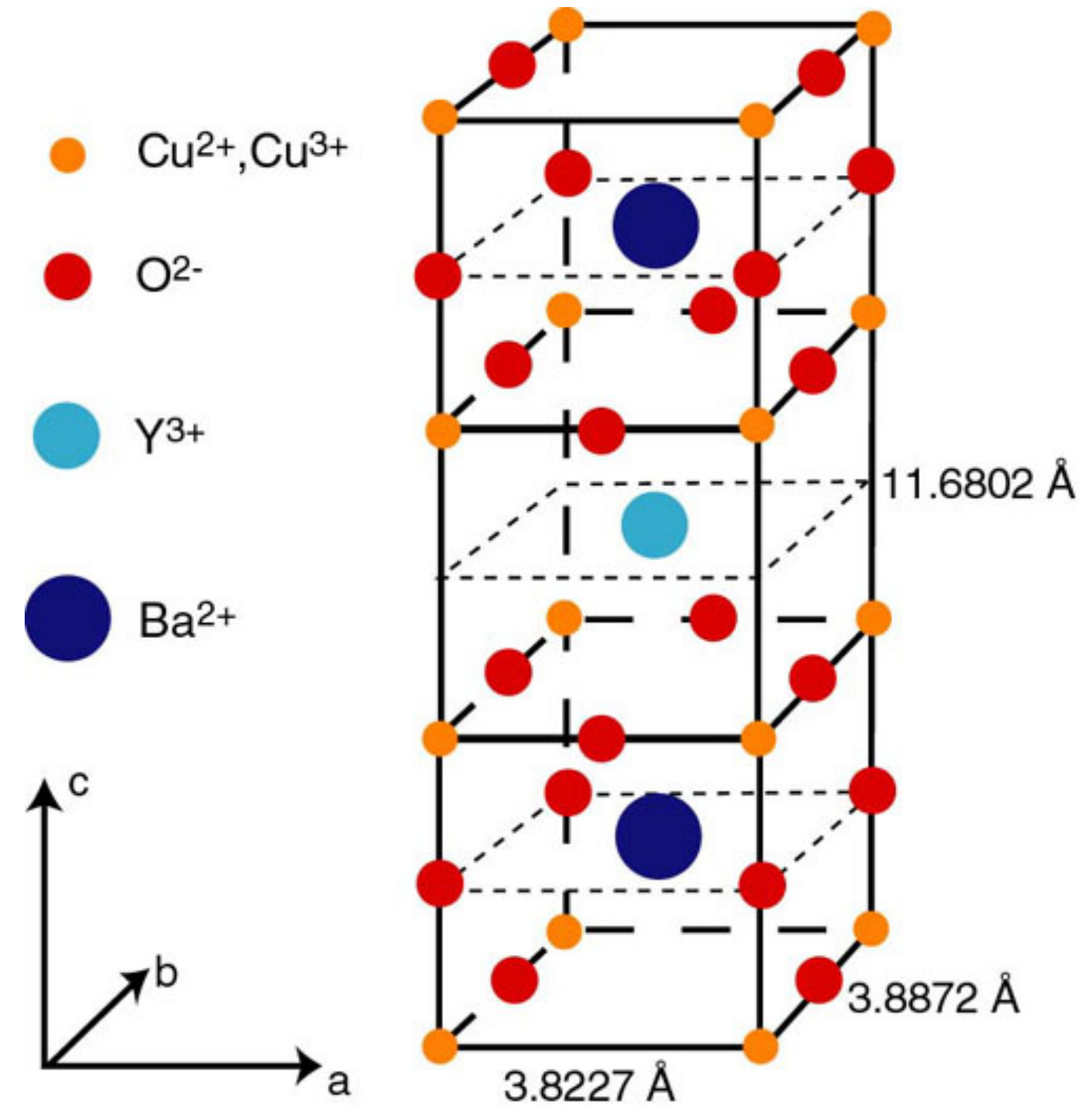
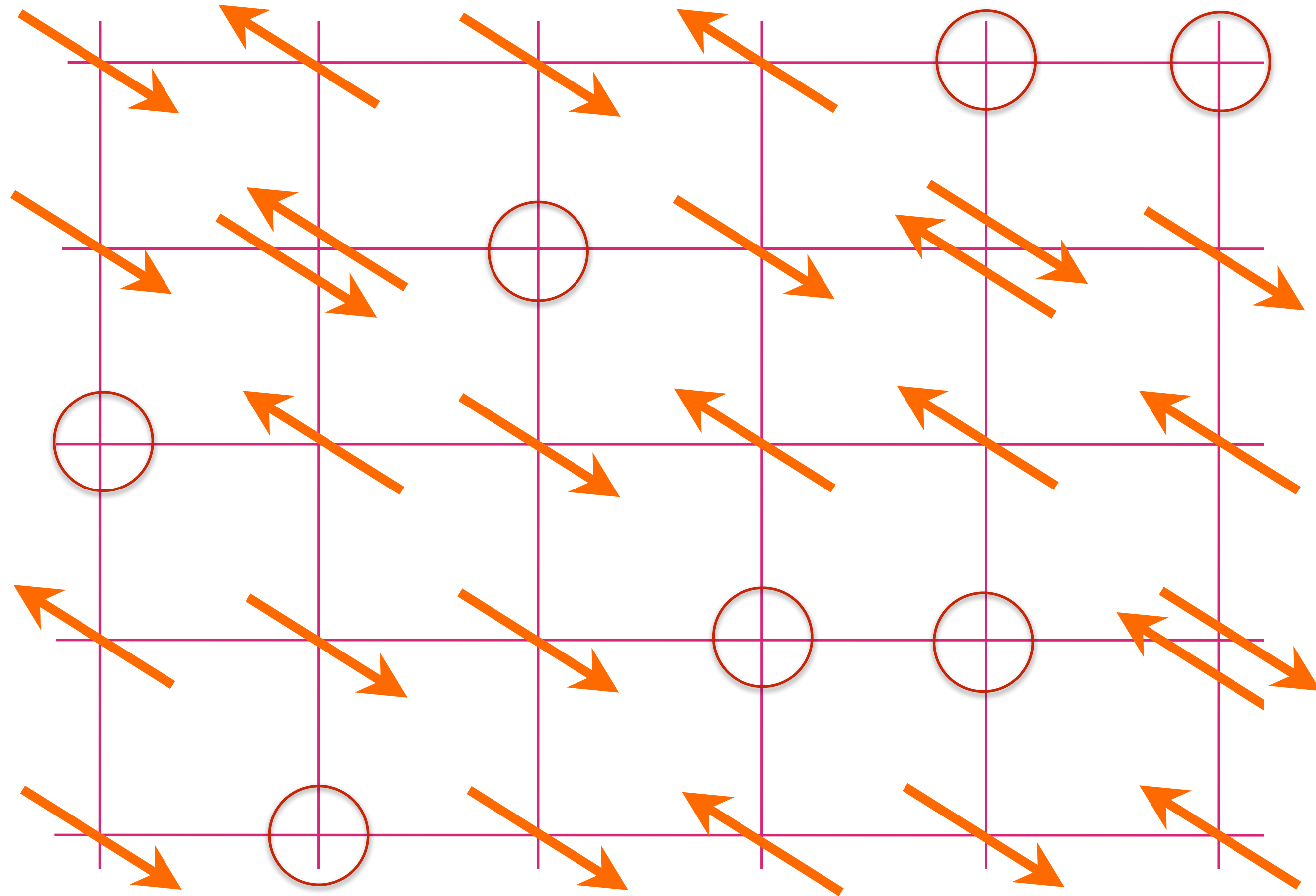
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5. Recap

Square lattice Hubbard model with electron density $1 - p$.

$$\mathcal{H}_{\text{Hubbard}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} + U \sum_i n_{i\uparrow} n_{i\downarrow} + \dots$$



Unified $SU(2) \times U(1)$ gauge theory of spinons, electrons and Higgs bosons: uncanny similarities to the Salam-Weinberg-Glashow theory of weak interactions

- The electromagnetic $U(1)$ is effectively global, because $\alpha \ll 1$.

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where U_{ij} is the (lattice) $SU(2)$ gauge field. The spinons are the analog of the neutrinos

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- The Higgs sector has a boson B_i which is fundamental of $SU(2)$

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- The hole pockets in the nodal region of the Brillouin zone are described by electron $\bar{c}_{i\alpha}$ which have a Yukawa coupling to the spinons and the Higgs field $B_i = (B_{1i}, B_{2i})$:

$$H_Y = \sum_{ij} \bar{t}_{ij} \bar{c}_{i\alpha}^\dagger \bar{c}_{j\alpha} + i \sum_i \left(B_{1i} f_{i\alpha}^\dagger \bar{c}_{i\alpha} - B_{2i} \varepsilon_{\alpha\beta} f_{i\alpha} \bar{c}_{i\beta} \right) + \text{H.c.}$$