Quantum entanglement at all distances

ICTP, Trieste
June 14, 2022

Subir Sachdev

INSTITUTE FOR ADVANCED STUDY

PHYSICS
HARVARD

Talk online: sachdev.physics.harvard.edu

Quantum statistical mechanics of black holes and strange metals

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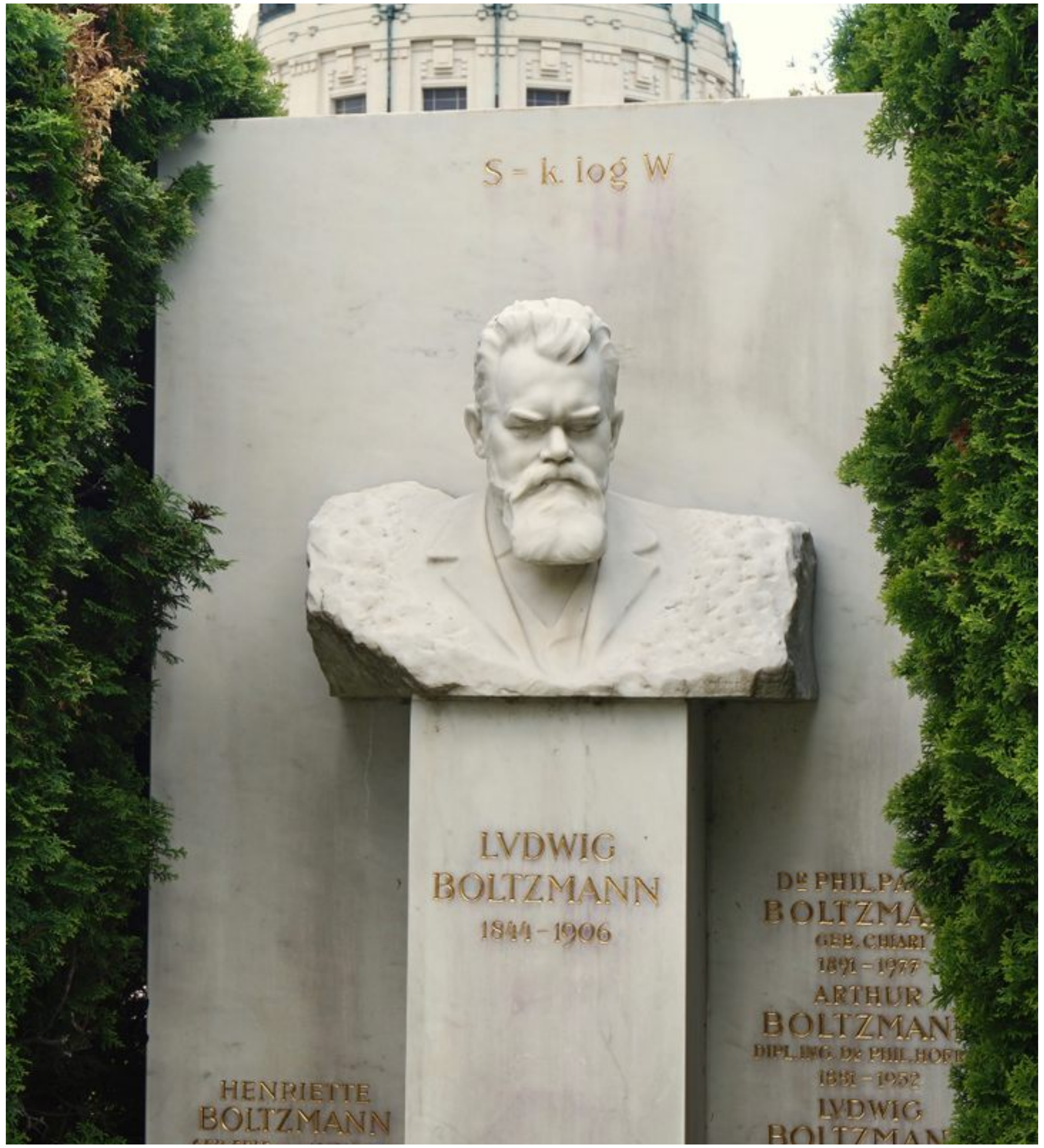
Foundations
by
Boltzmann



Ludwig Boltzmann
20 February 1844, Vienna -
September 5, 1906, Trieste

Statistical interpretation of entropy (1870)

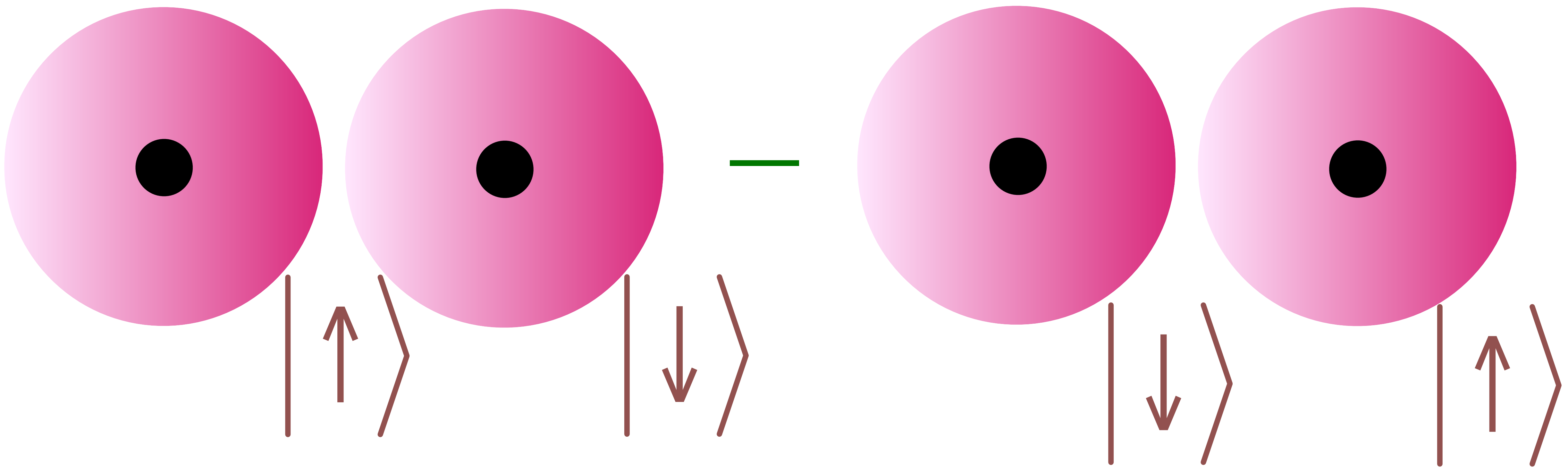
$$S = k_B \log W$$

Density of quantum states $D(E) = \exp(S(E)/k_B)$

Quantum Entanglement

Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



$$|\uparrow\rangle \, |\downarrow\rangle - |\downarrow\rangle \, |\uparrow\rangle$$

Quantum Entanglement

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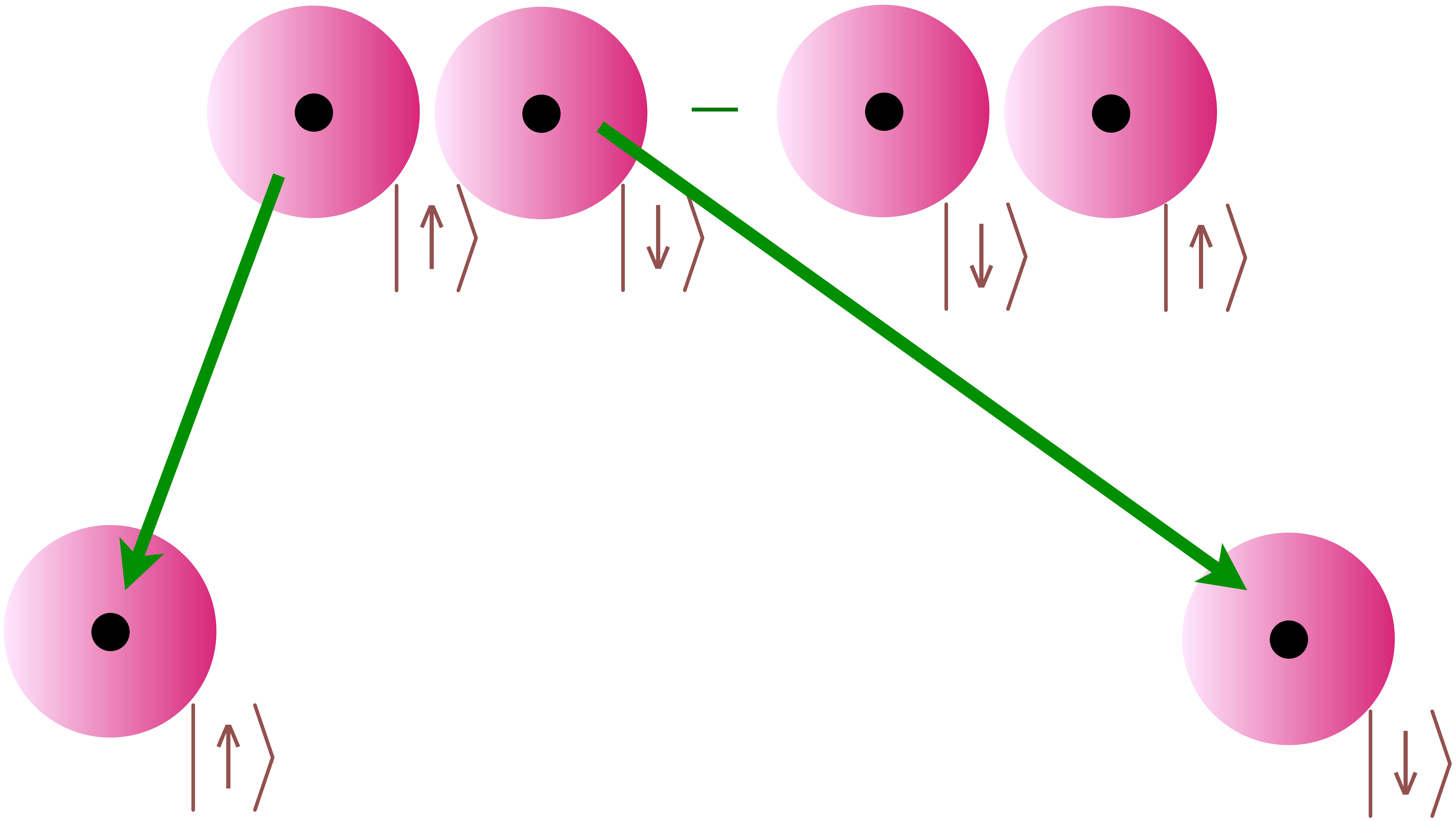

Quantum Entanglement

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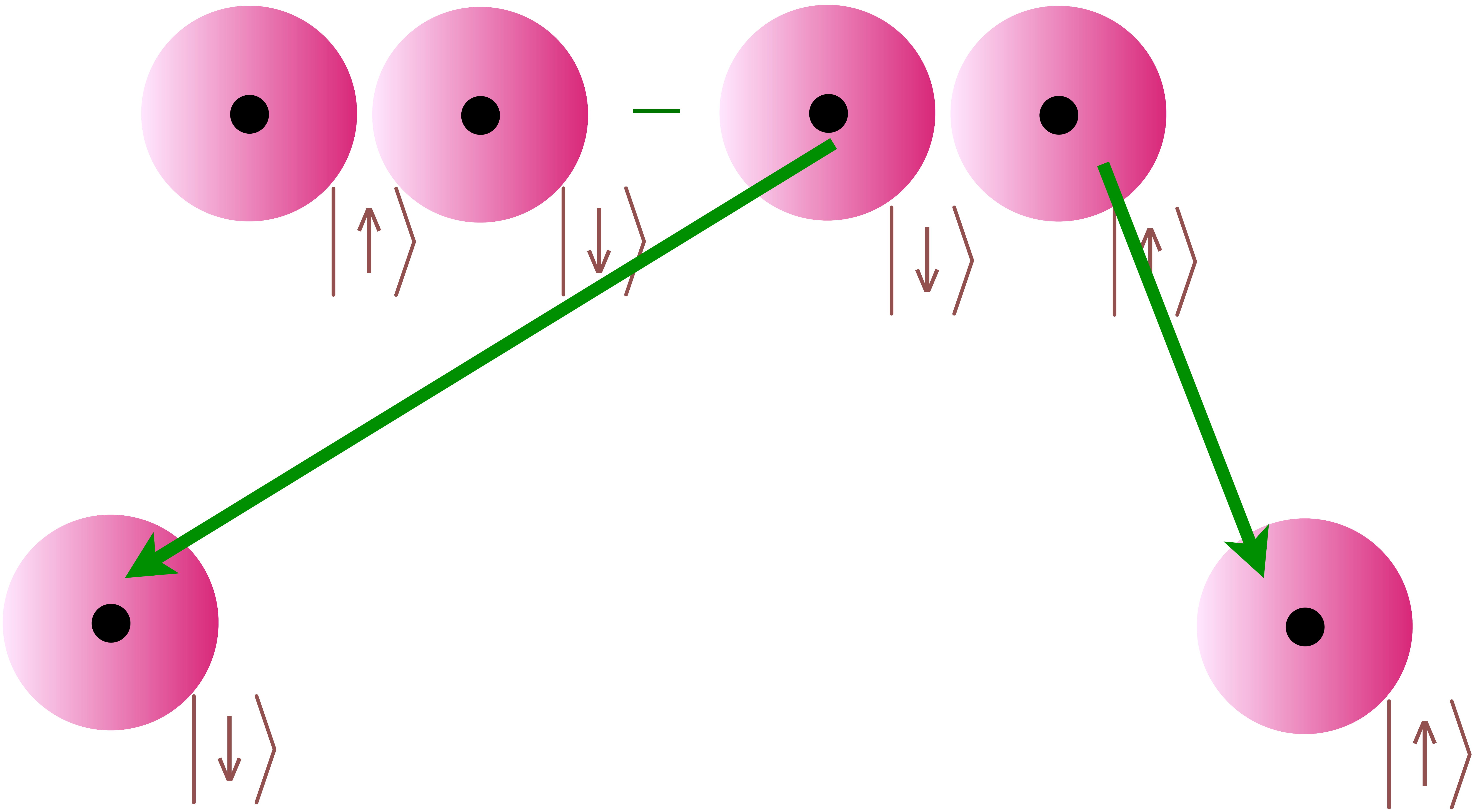

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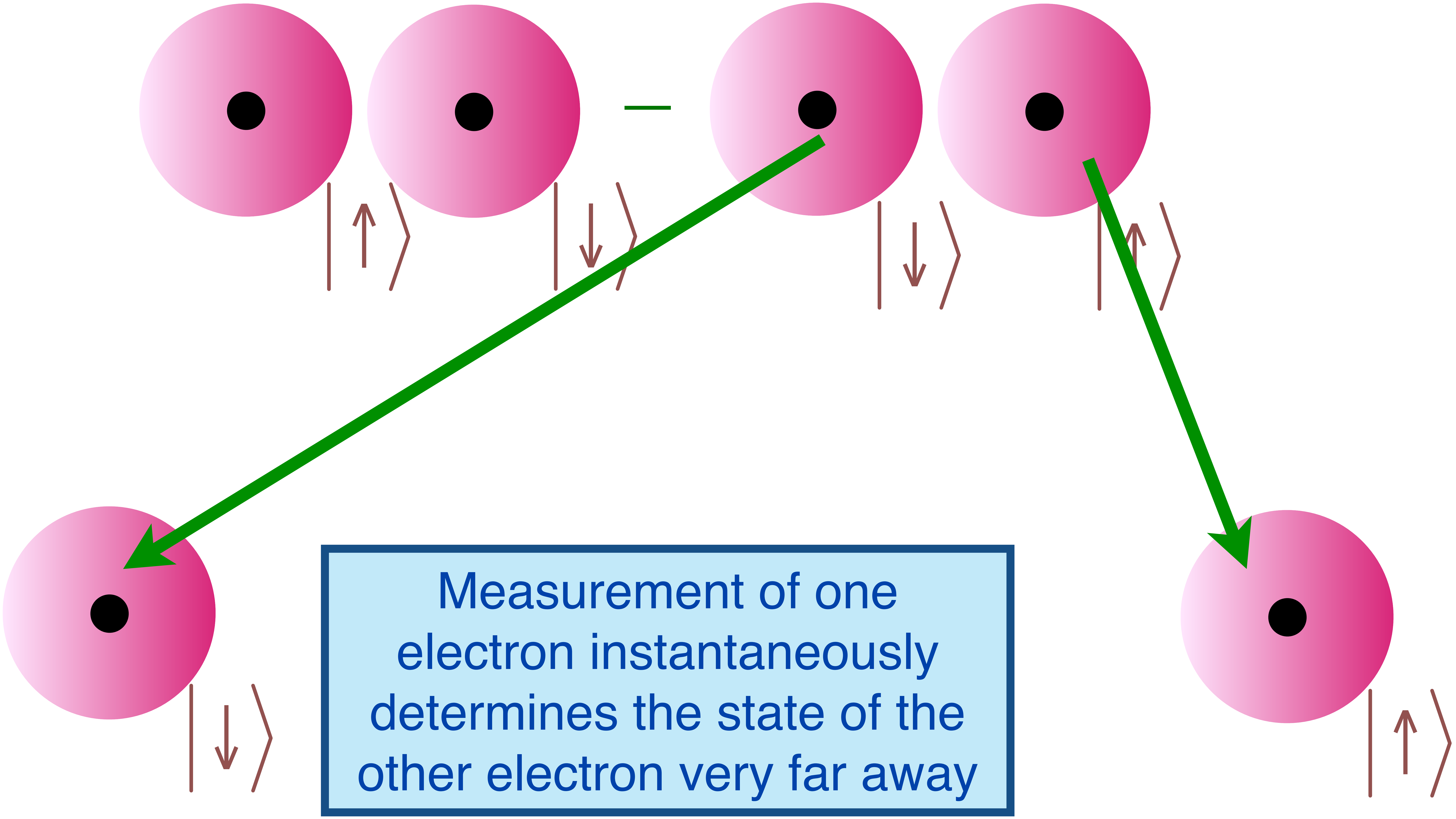

Measurement of one electron instantaneously determines the state of the other electron very far away

Quantum Entanglement

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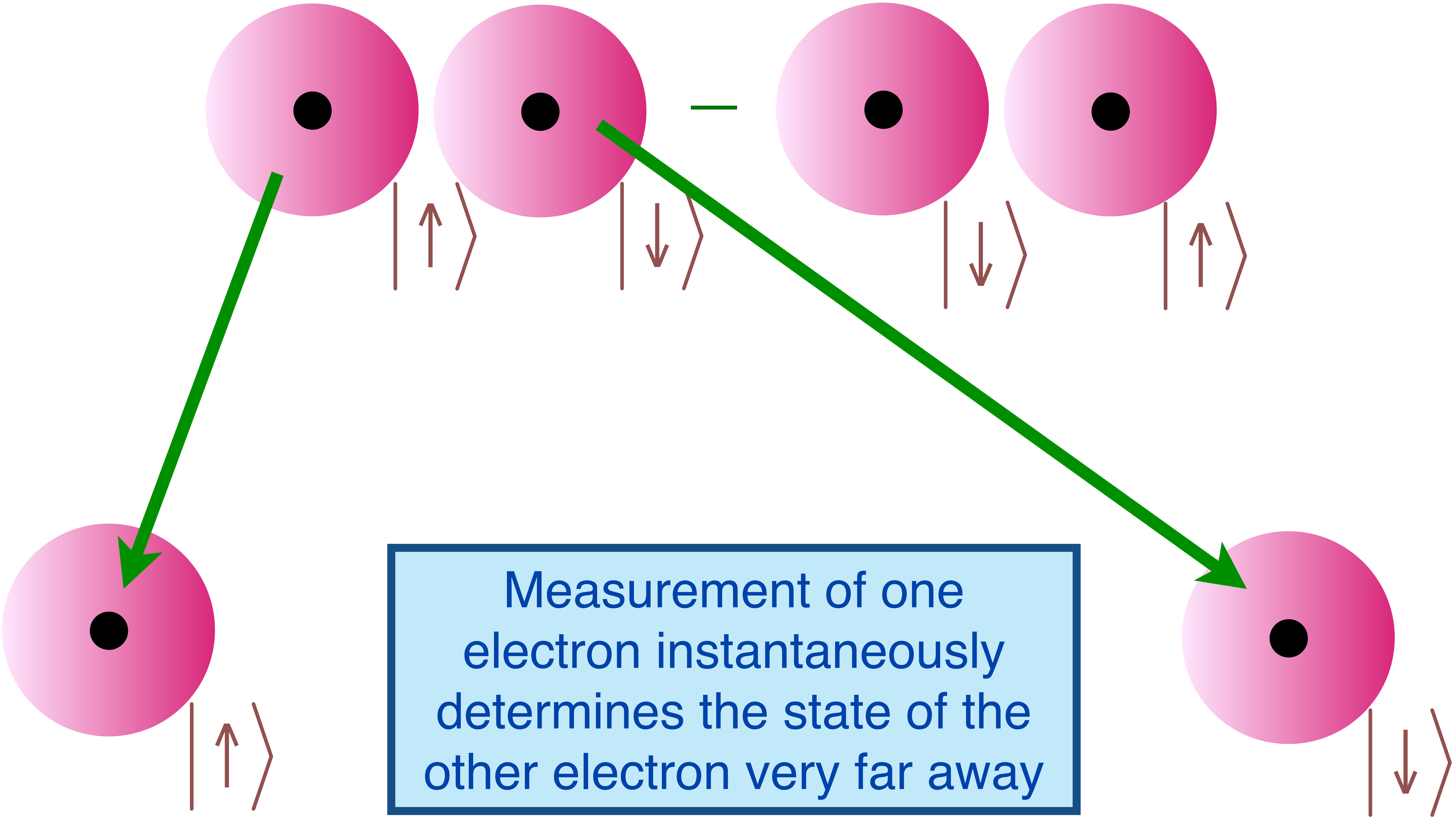

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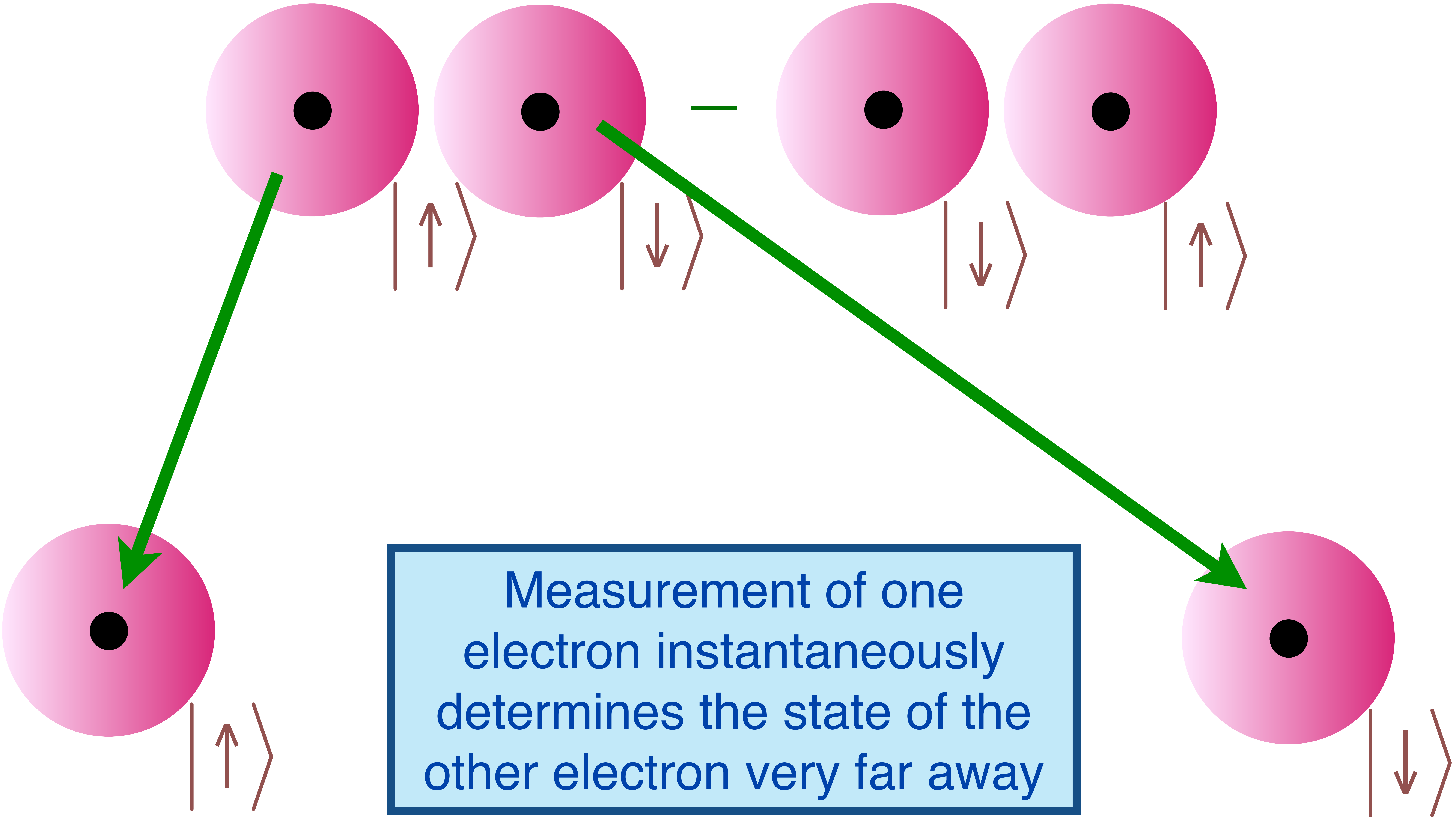
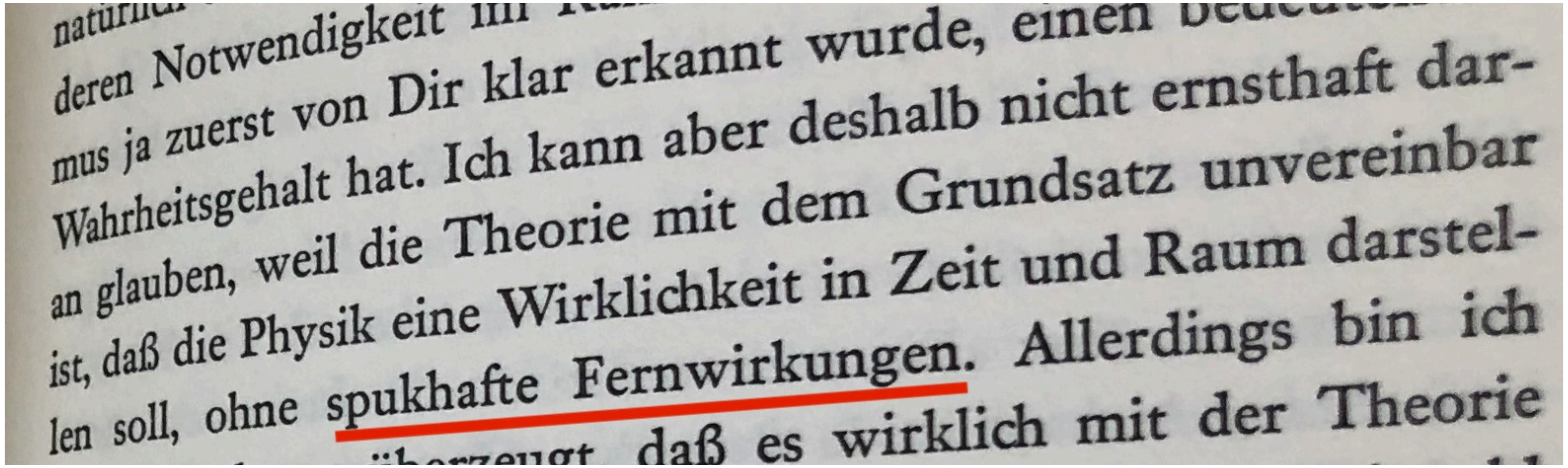


Measurement of one electron instantaneously determines the state of the other electron very far away

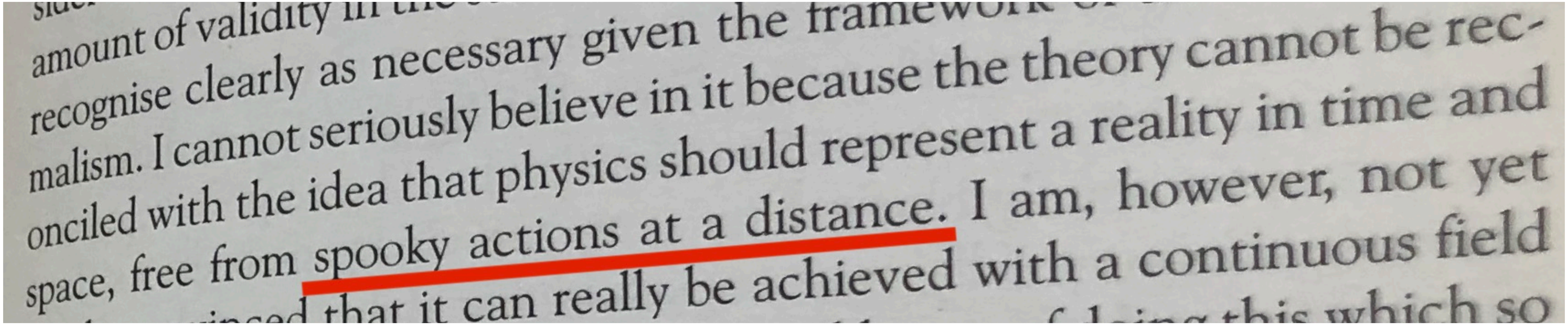
Spooky action at a distance !



naturli

deren Notwendigkeit im

mus ja zuerst von Dir klar erkannt wurde, einen bedeut Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft daran glauben, weil die Theorie mit dem Grundsatz unvereinbar ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstellen soll, ohne spukhafte Fernwirkungen. Allerdings bin ich überzeugt, daß es wirklich mit der Theorie



amount of validity in the

recognise clearly as necessary given the framework

malism. I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance. I am, however, not yet

vinced that it can really be achieved with a continuous field

this which so

I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

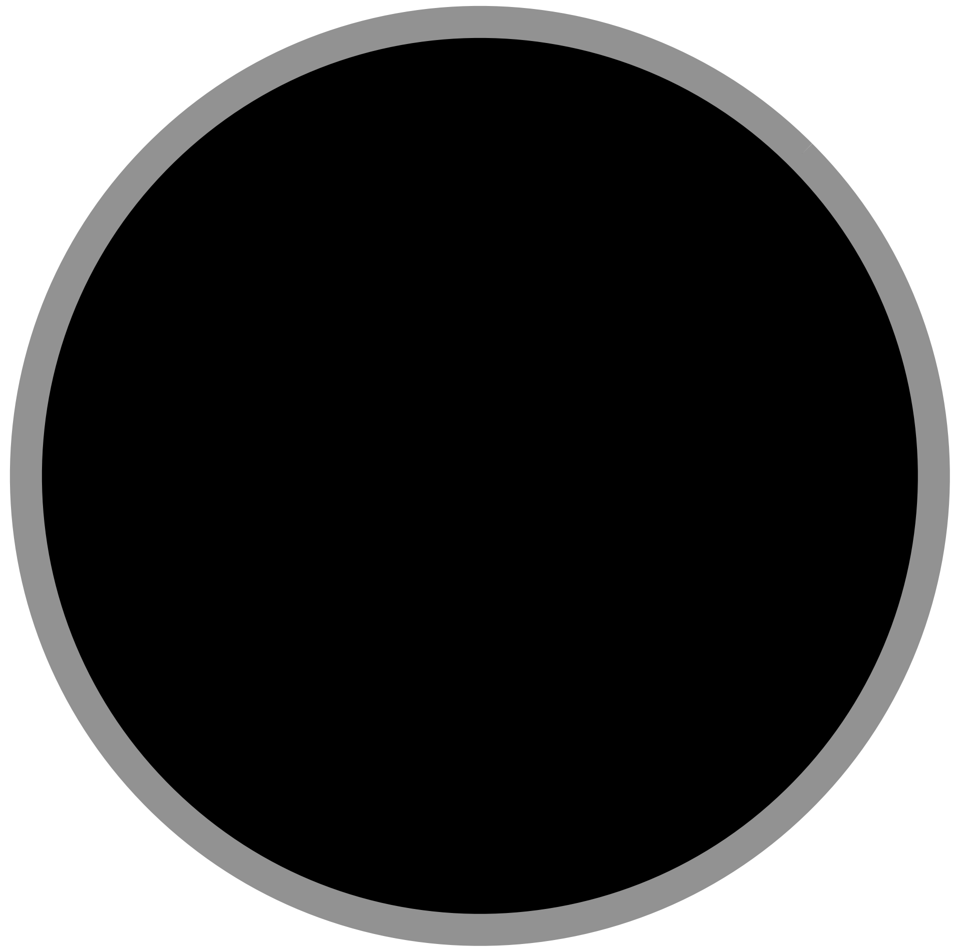
Albert Einstein to Max Born, 3 March 1947

Quantum black holes and holography

Black Holes

Objects so dense that light is gravitationally bound to them.

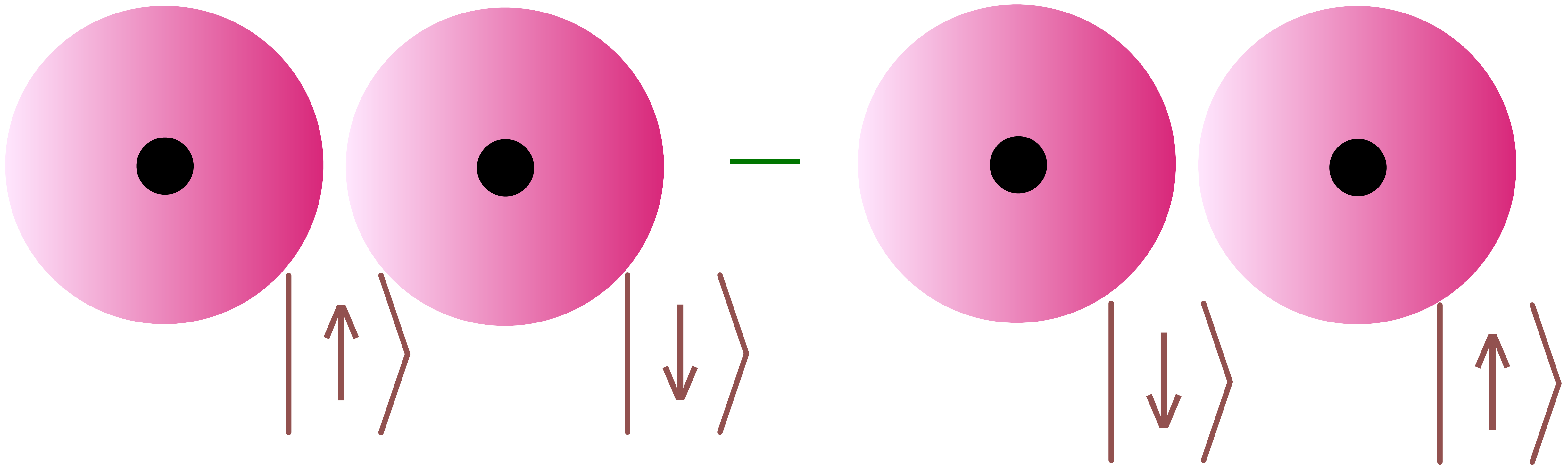
Horizon radius $R = \dfrac{2GM}{c^2}$



G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

Quantum Entanglement across a black hole horizon



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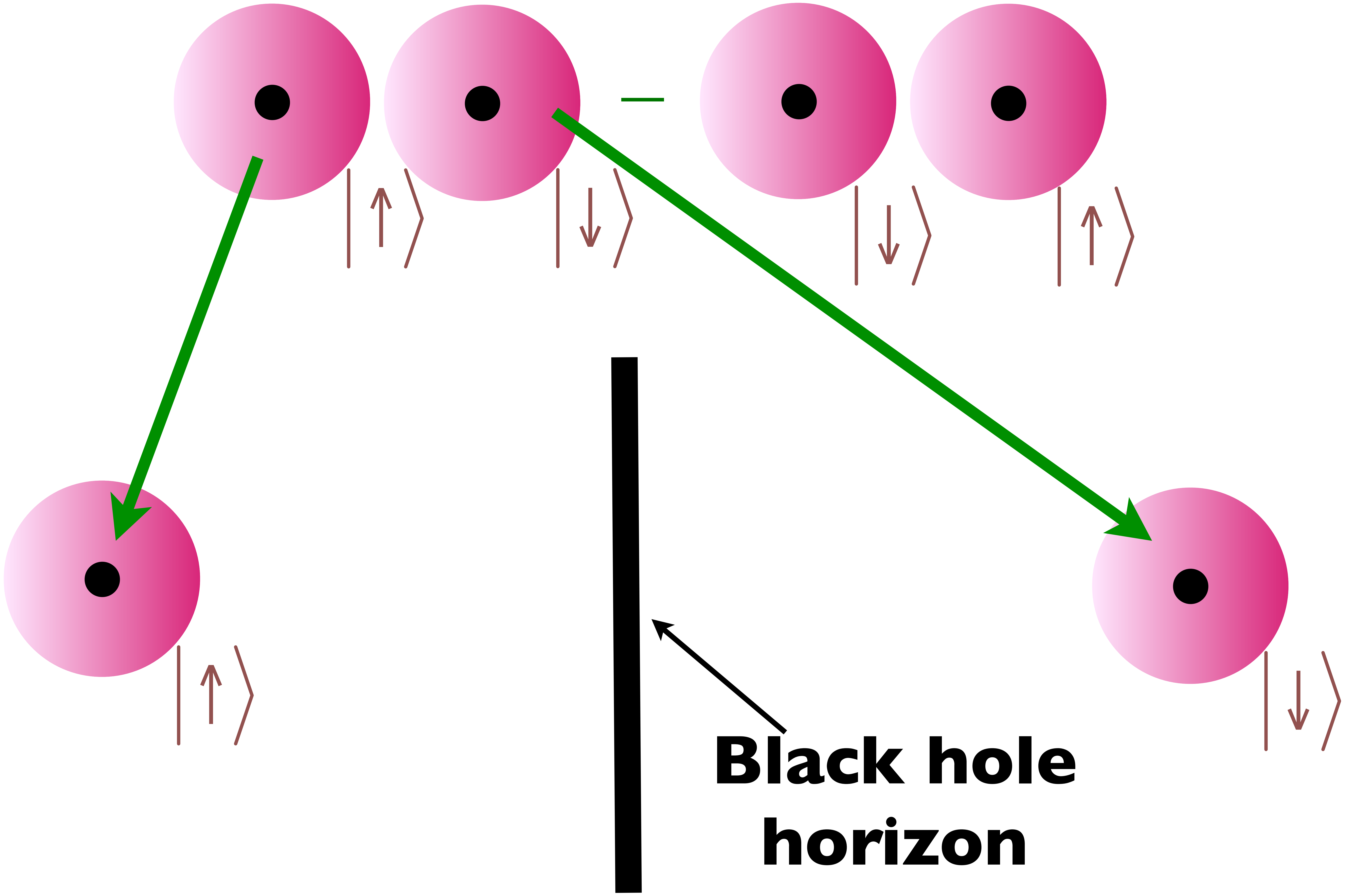

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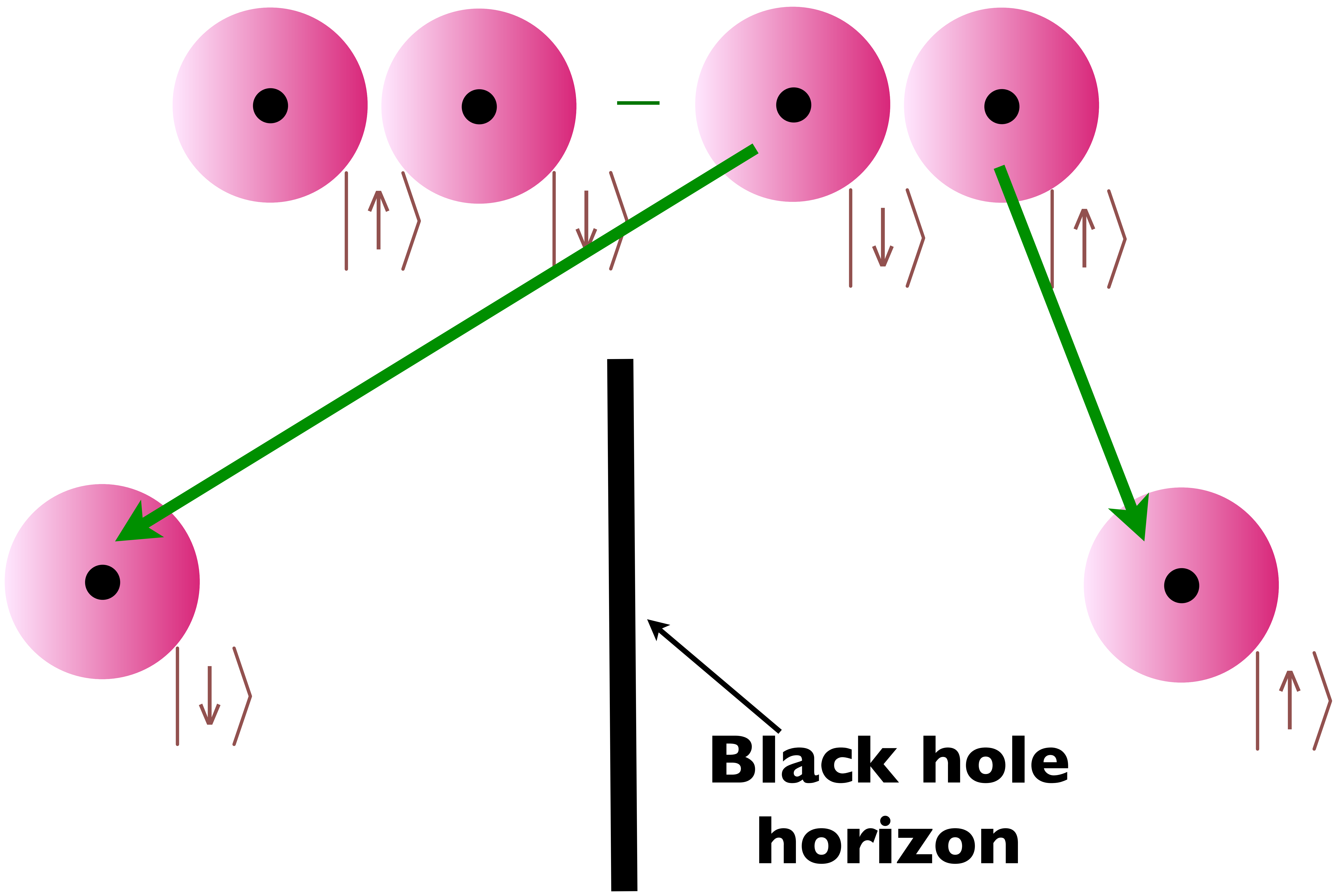
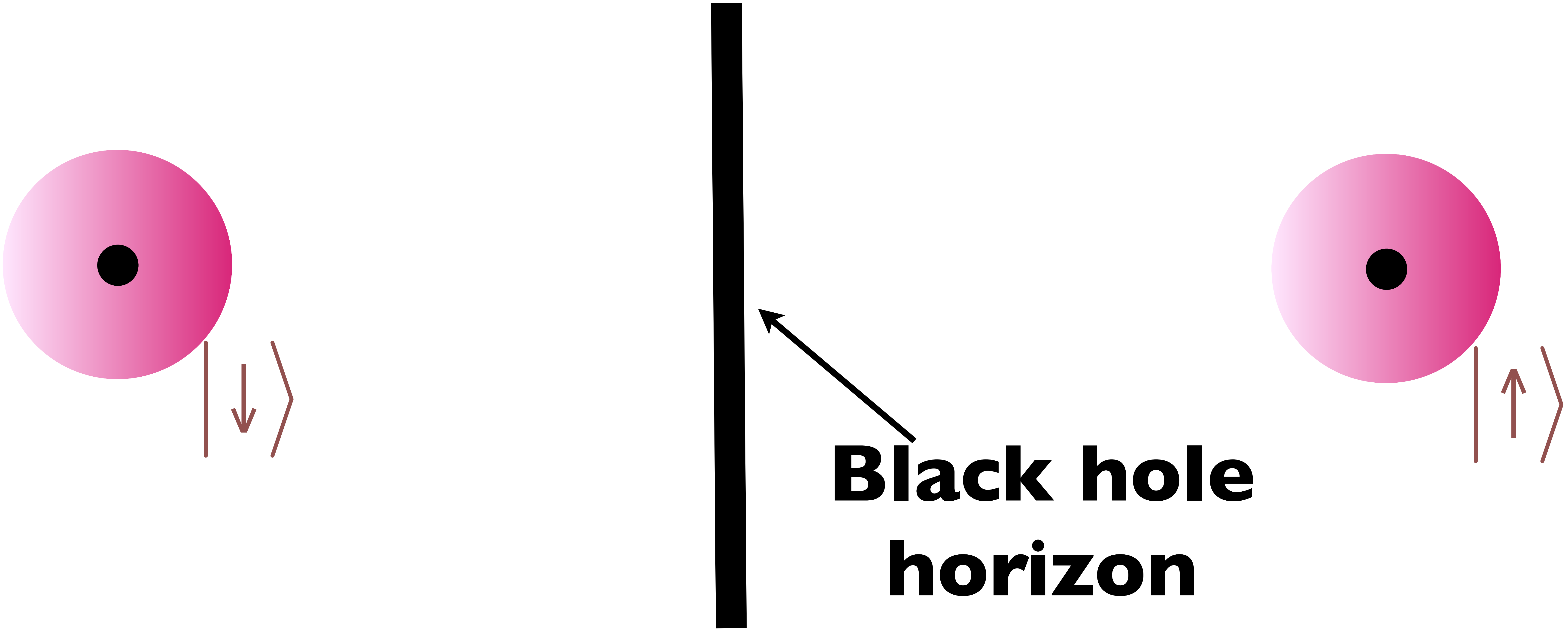


Quantum Entanglement across a black hole horizon

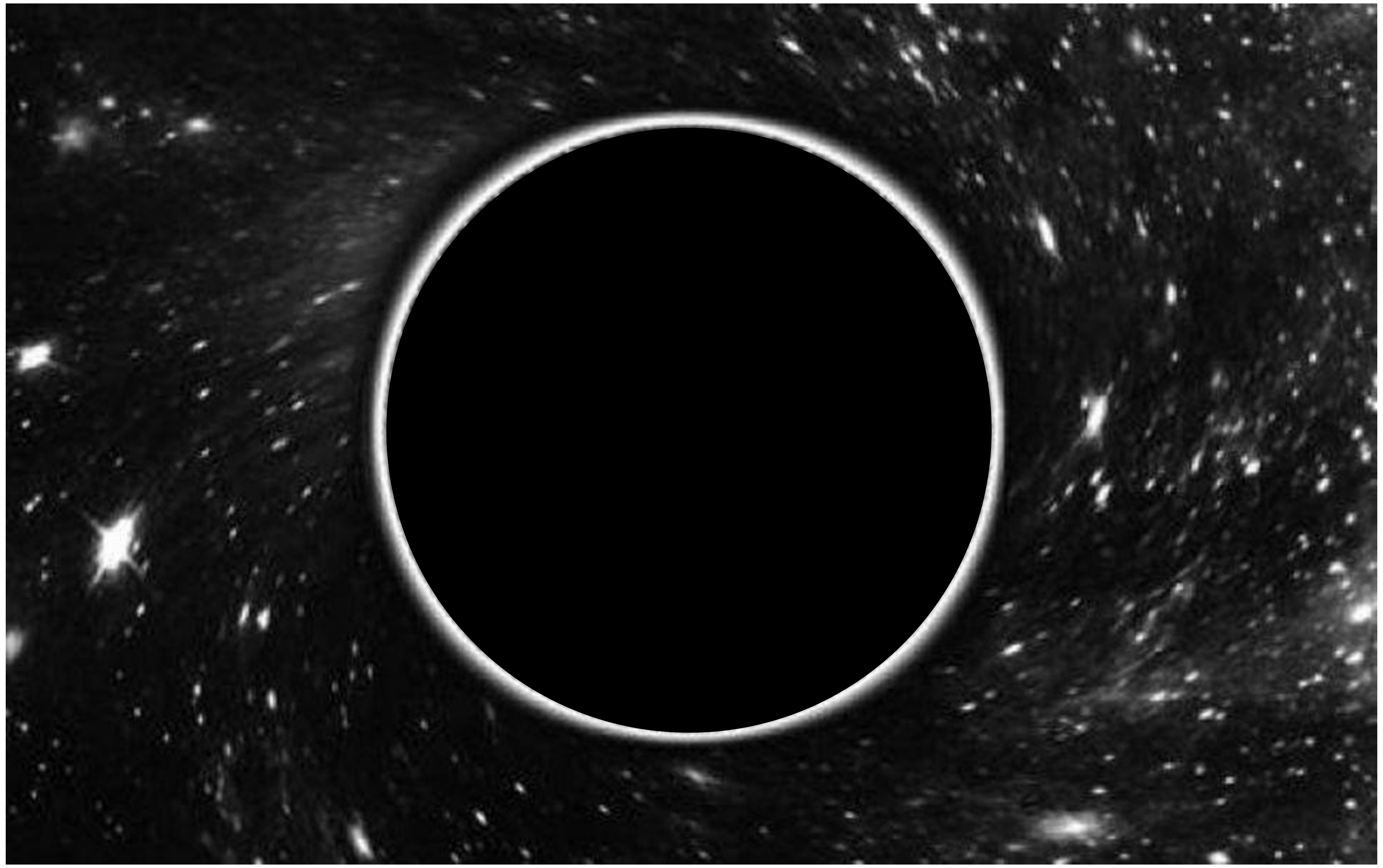
Hawking (1975) used other arguments to show that black hole horizons have a temperature

(The entanglement reasoning: to an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.)



Black hole horizon

Quantum Black holes



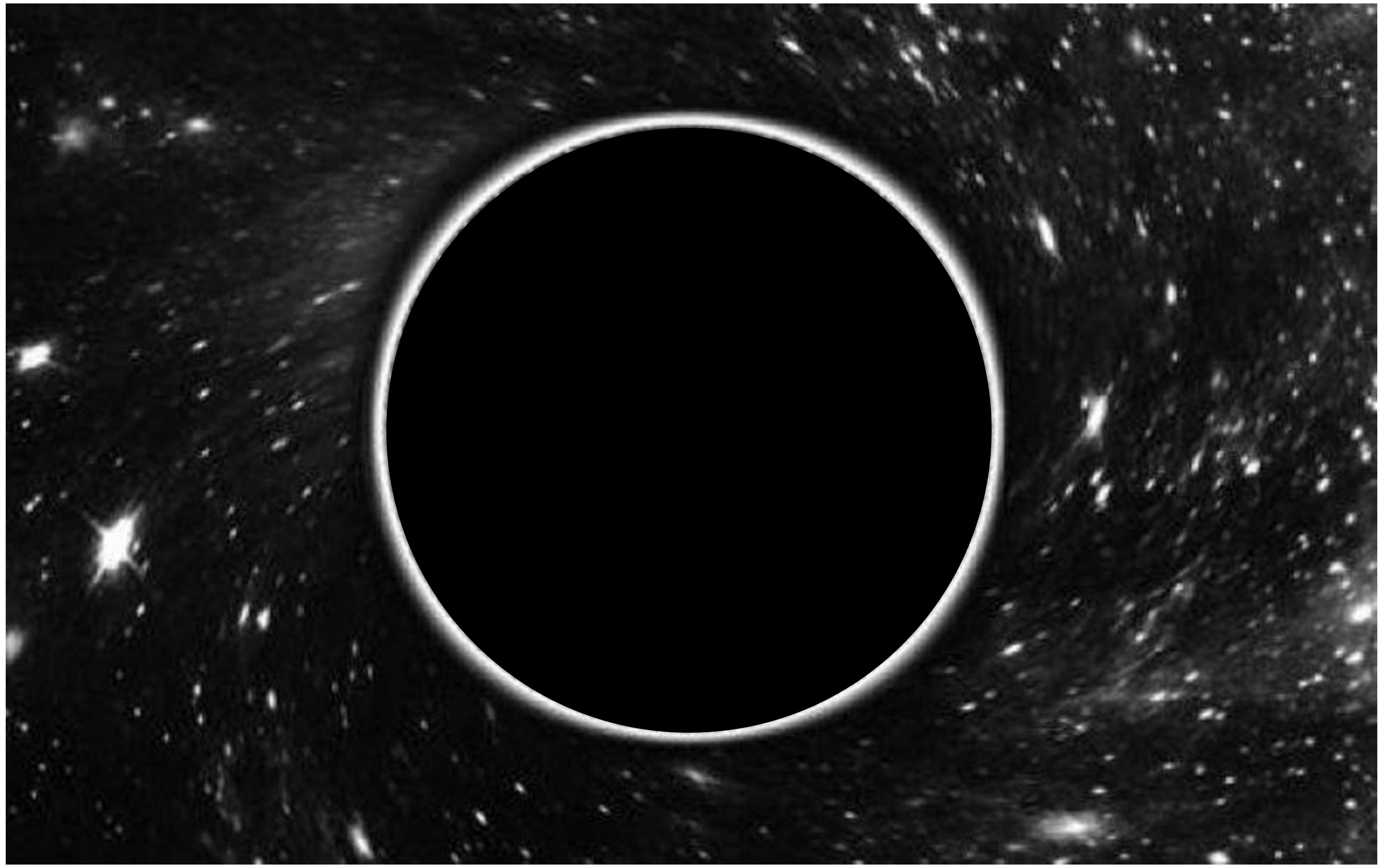
From Hawking, we learn that…

- Black holes have an entropy and a temperature $T = \dfrac{\hbar c^3}{8\pi G M k_B}$
- The entropy is proportional to their surface area $S = \dfrac{A k_B c^3}{4 G \hbar}$

Quantum Black holes

We need one more important fact…

- Black holes have an entropy and a temperature $T = \dfrac{\hbar c^3}{8\pi G M k_B}$
- The entropy is proportional to their surface area $S = \dfrac{A k_B c^3}{4 G \hbar}$
- They reach thermal equilibrium at the fastest possible rate $\sim \dfrac{\hbar}{k_B T}$!



T.B. BAKKER / DR. J.P. VAN DER SCHAAR

Black Holes Obey Information-Emission Limits

April 22, 2021 • *Physics* 14, s47 –Christopher Crockett

G. Carullo, D. Laghi, J. Veitch, W. Del Pozzo, Phys. Rev. Lett. **126**, 161102 (2021)

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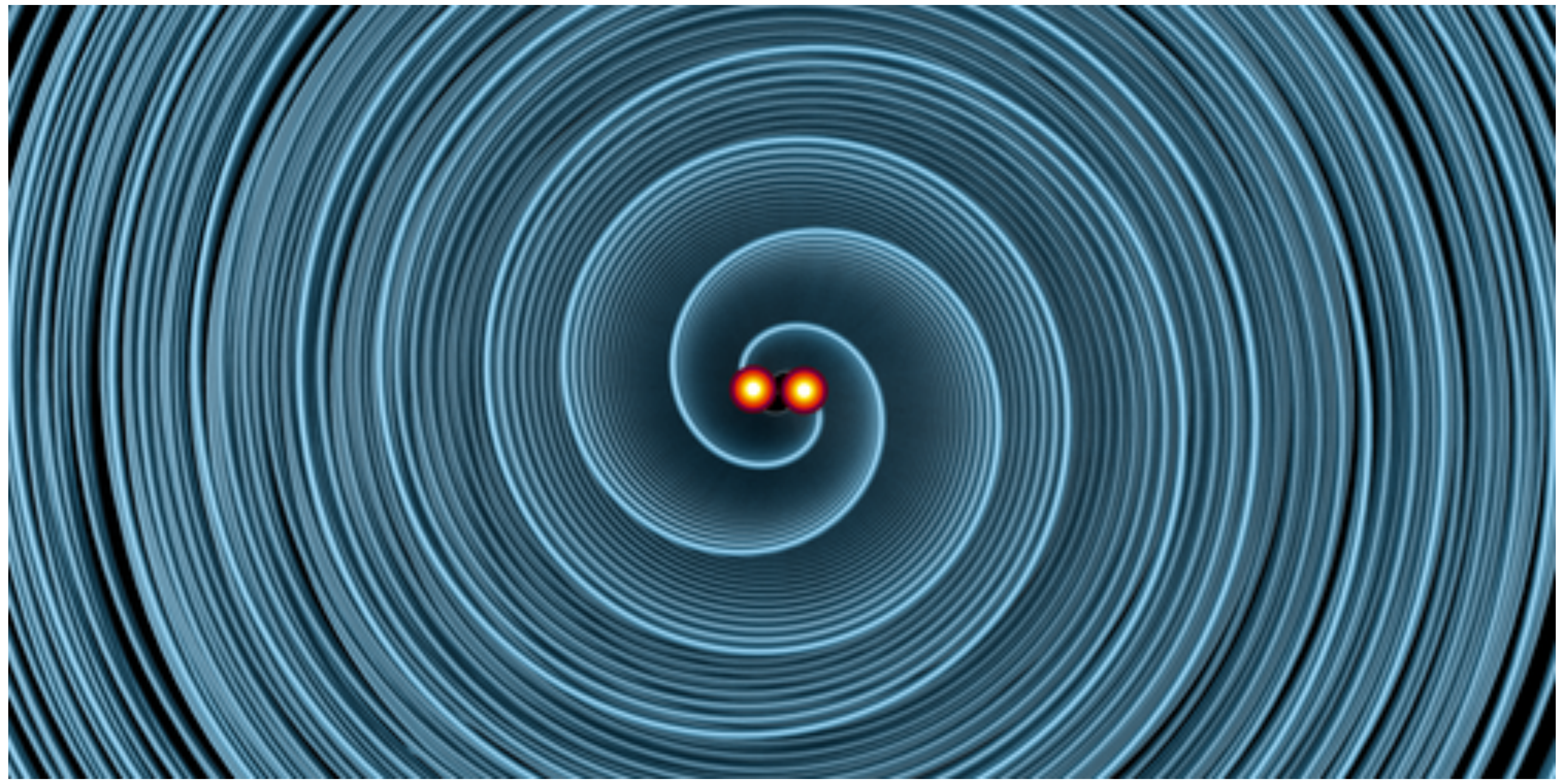

Gravity wave observations of 8 different black holes show a relaxation time

$$\tau \sim \frac{8\pi GM}{c^3} = \frac{\hbar}{k_B T}$$

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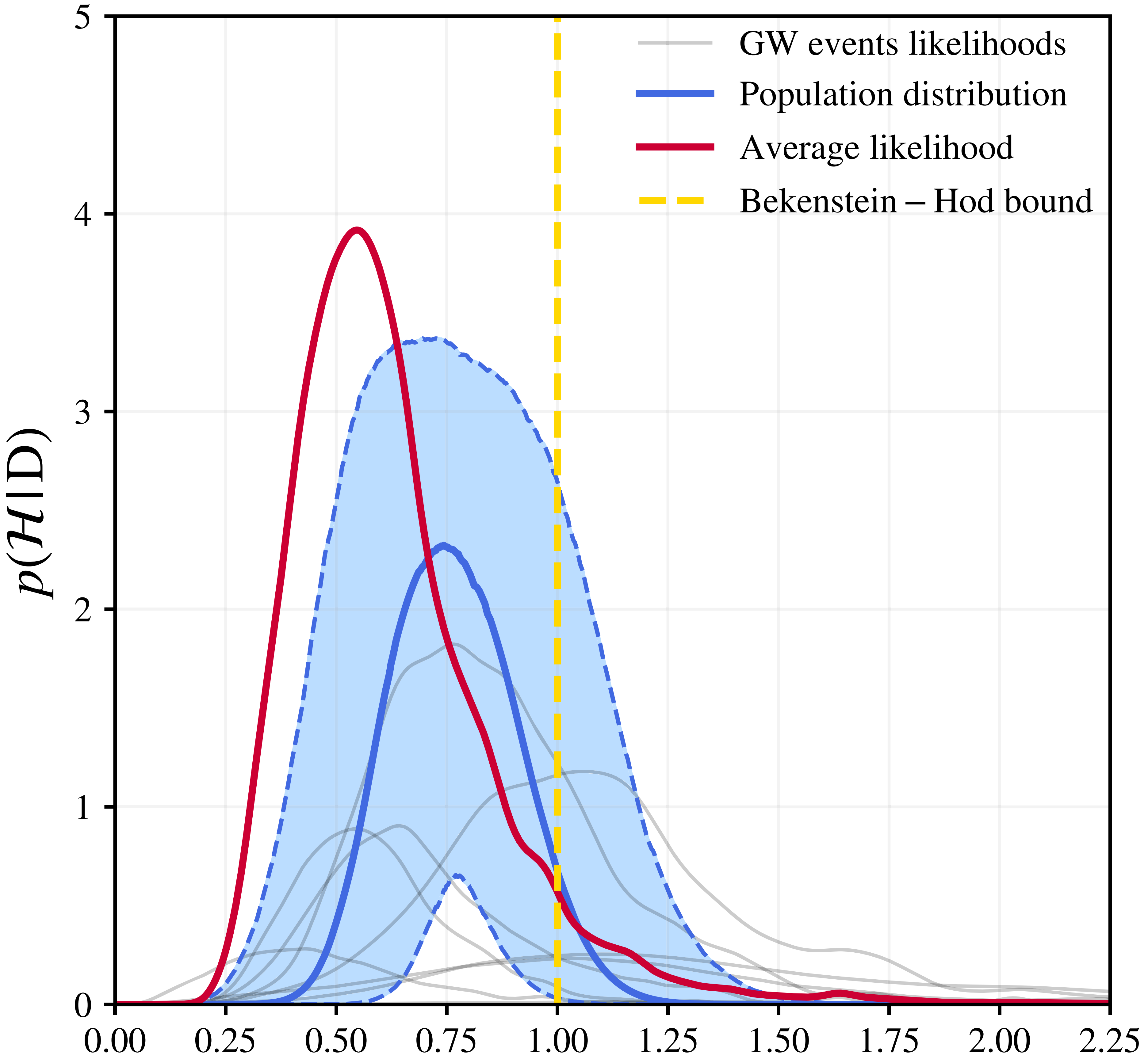
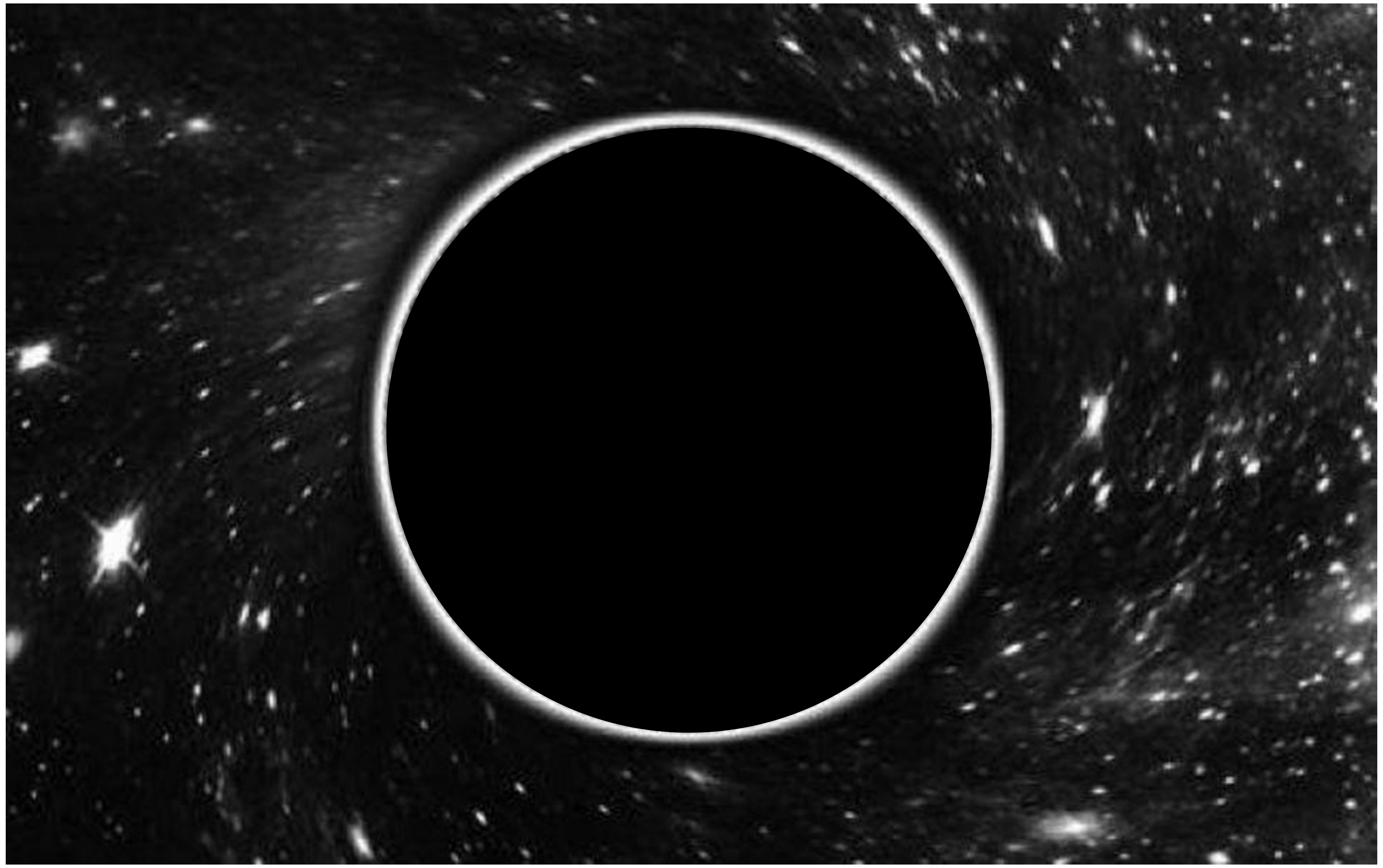


$$\mathcal{H} = \frac{1}{\pi}\frac{\hbar/\tau}{k_B T}$$

**Quantum
Black
holes**



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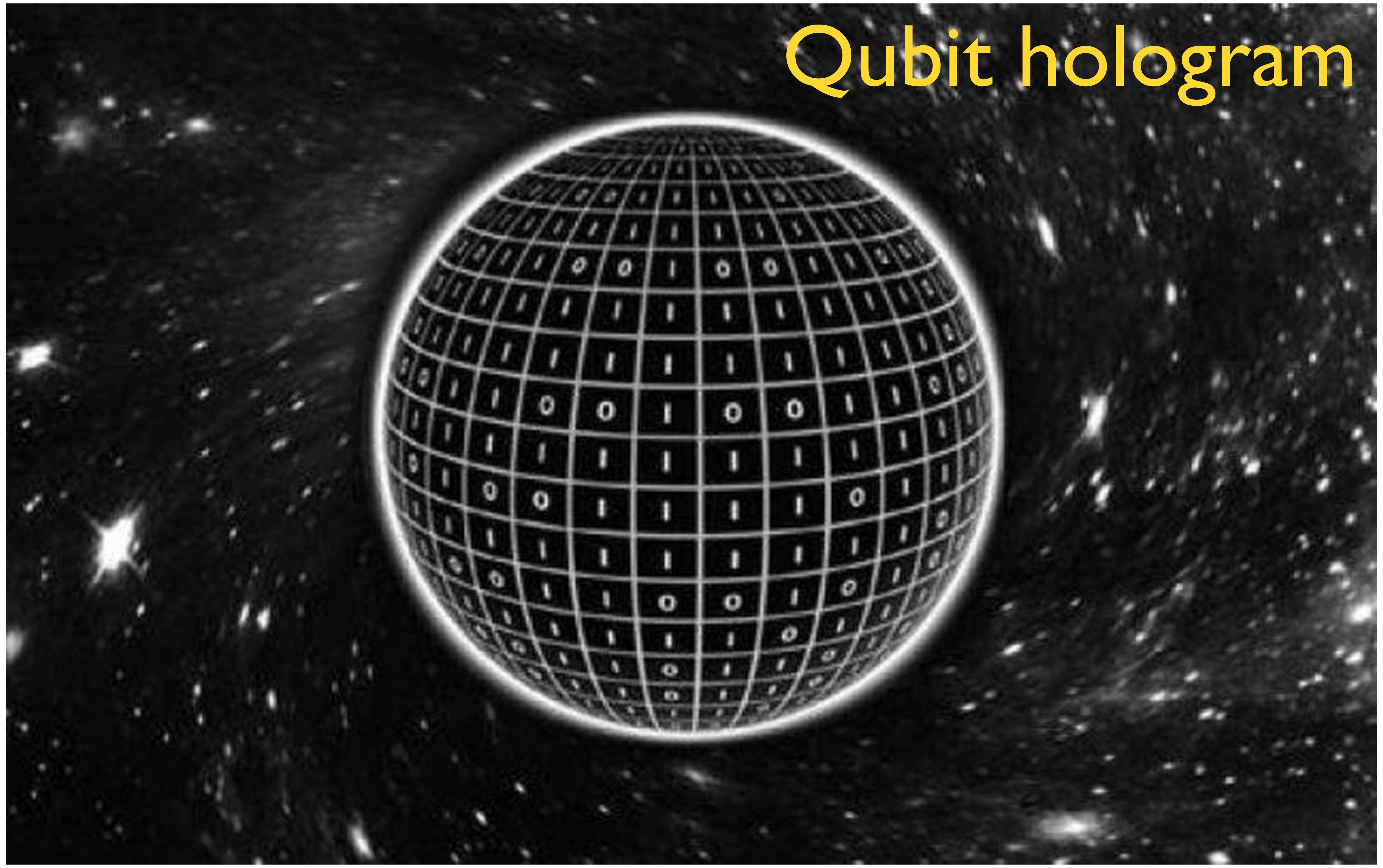
Quantum Black holes



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A quantum computer simulating a black hole must have:

- Number of 'qubits' proportional to the surface area
- Maximal, long-range quantum entanglement between the qubits

Spooky action at a distance!

Questions and Answers

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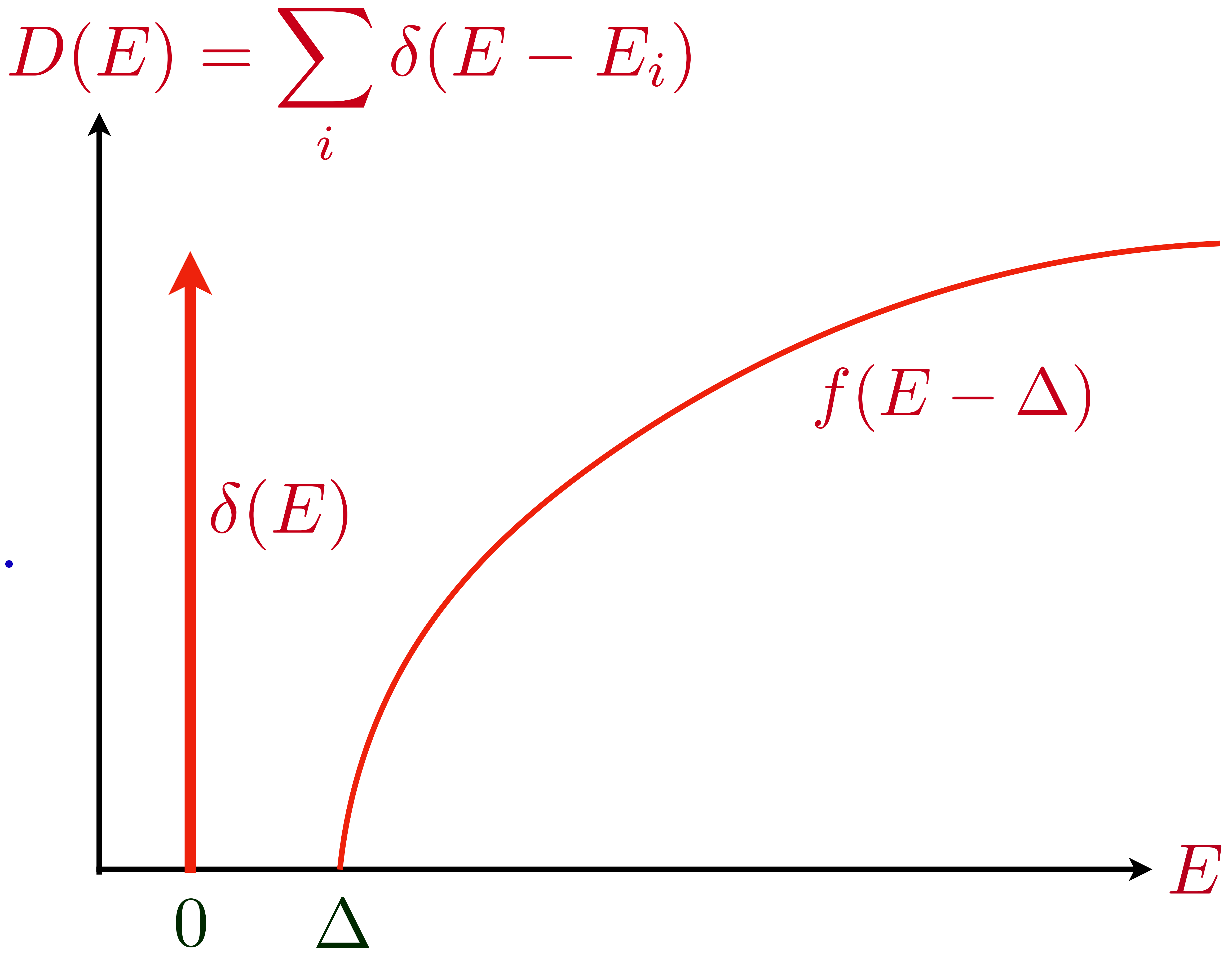
- With sufficient low energy supersymmetry, string theory yields:

$$D(E) = \exp\left(\frac{Ac^3}{4\hbar G}\right)\delta(E) + \theta(E-\Delta)f(E-\Delta) + \ldots$$

There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state.



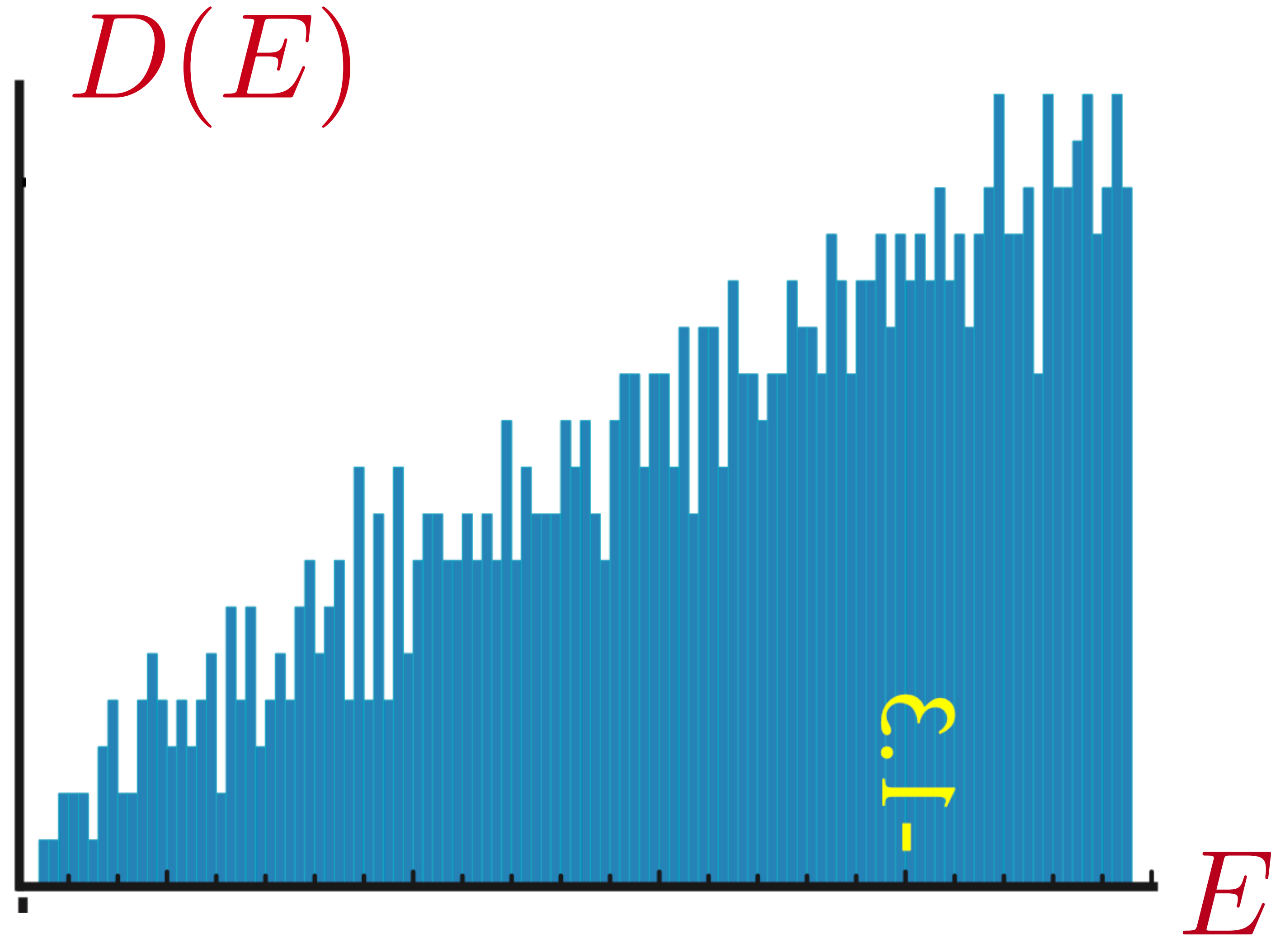
Questions and Answers

Can we find a quantum simulation of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy? Yes, for charged black holes:

- For generic black holes in 3+1 dimensions, the SYK model yields:

$$\boxed{D(E) \sim \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)}$$

where A_0 is the horizon area at $T = 0$. There is no degeneracy, but an exponentially small level spacing down to the ground state.



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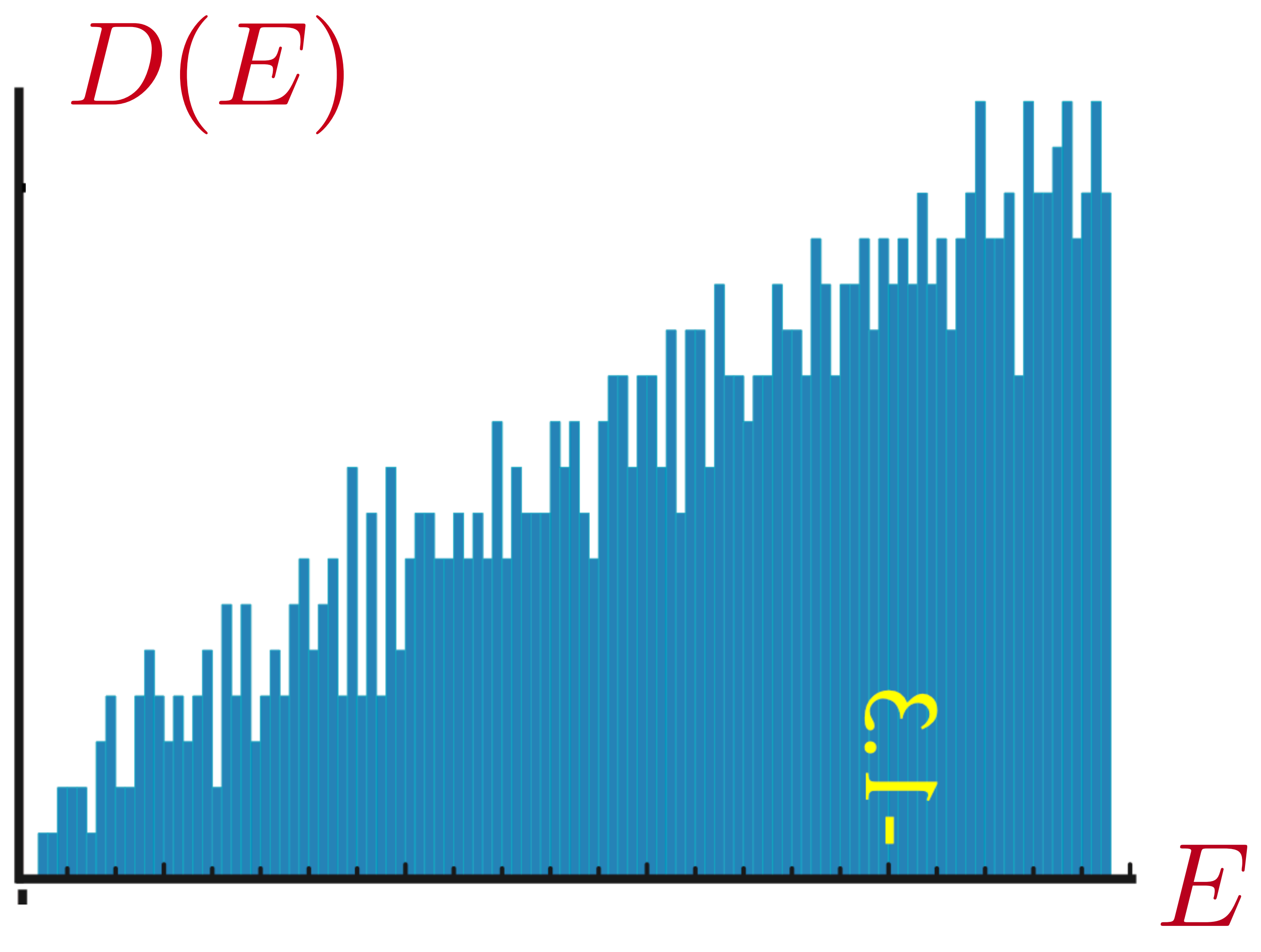
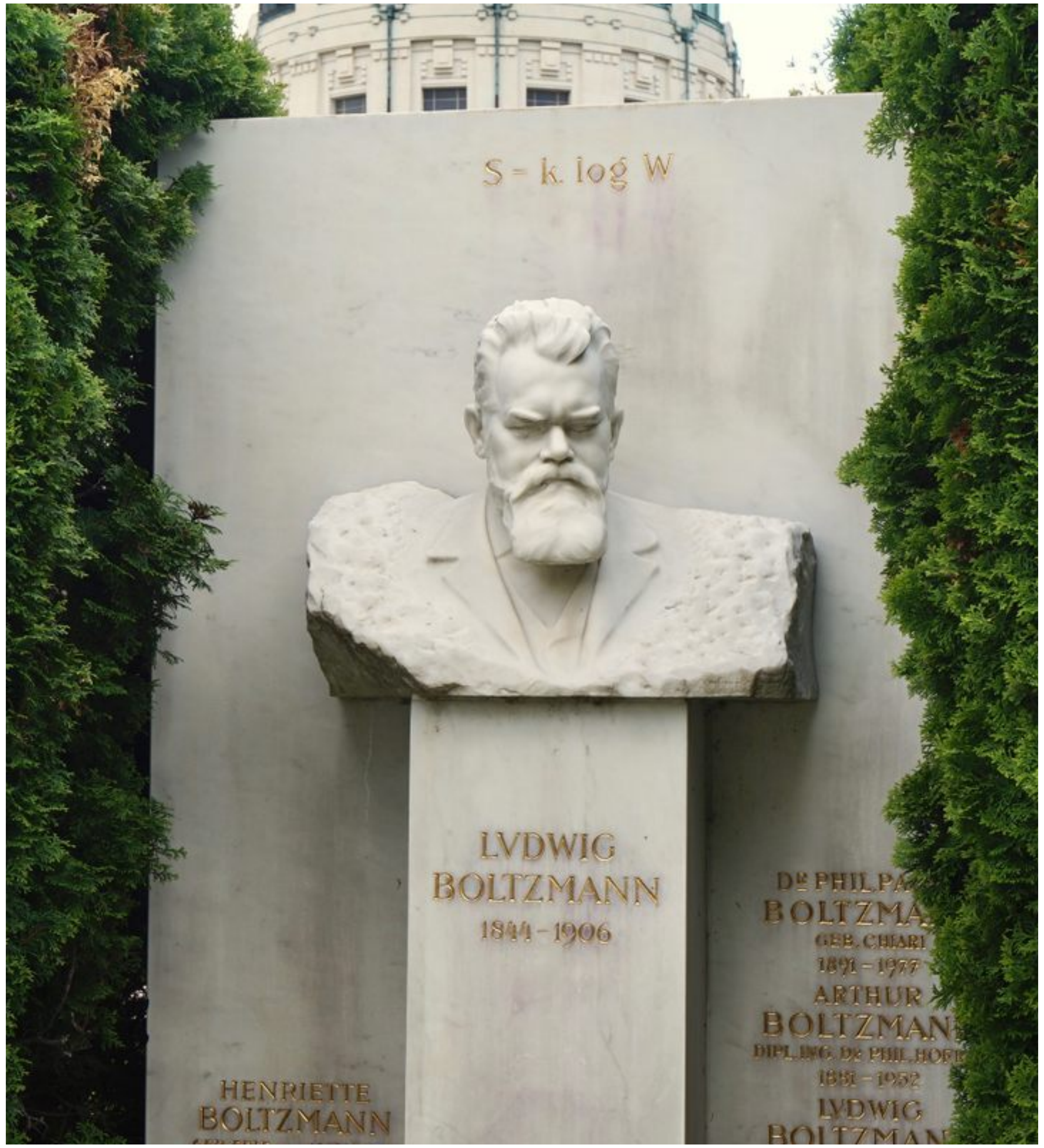


In more recent work, the SYK quantum simulation has also consistently described the evolution of the entropy for a black hole past the Page time.

Foundations
by
Boltzmann



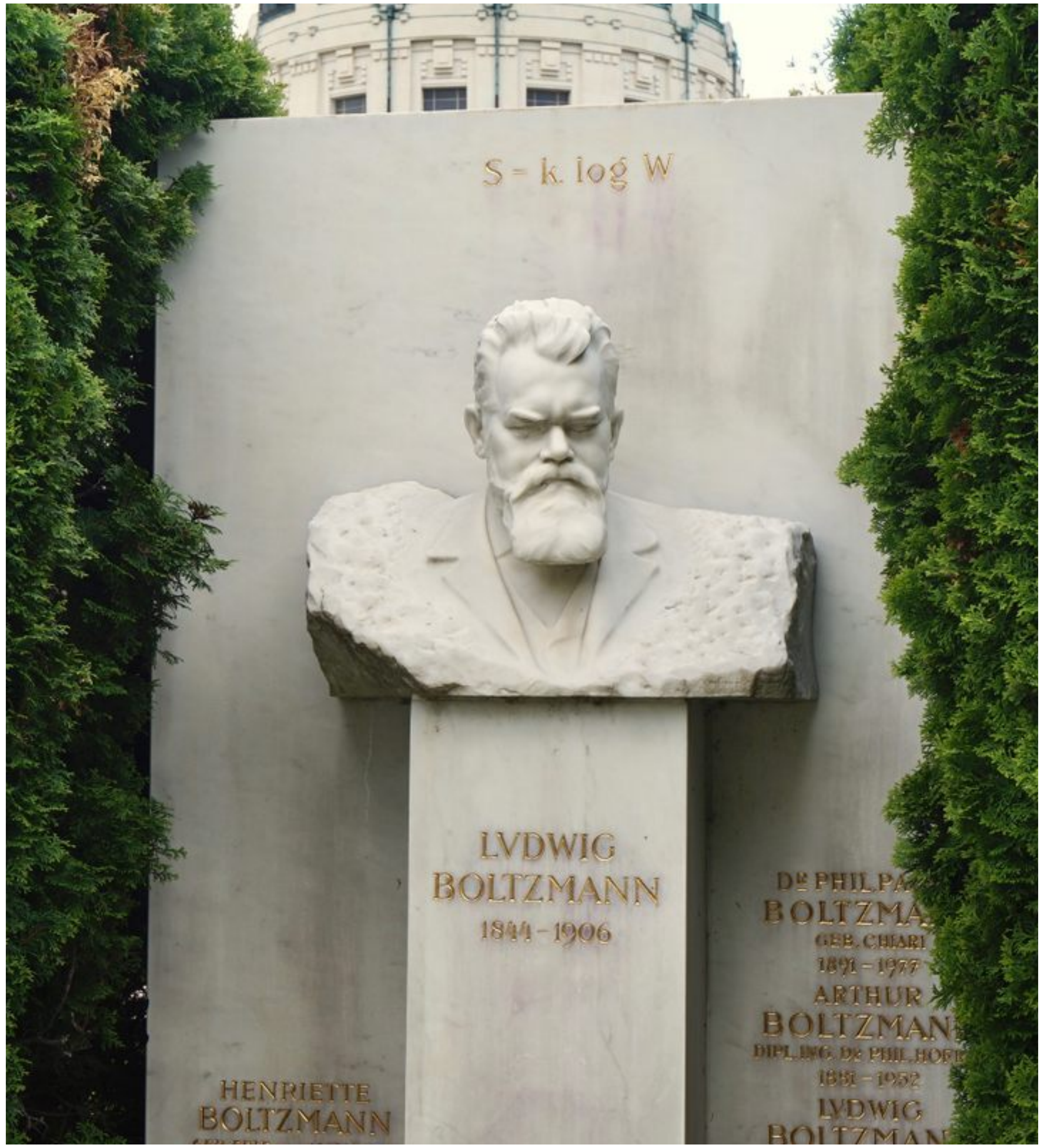
Ludwig Boltzmann
20 February 1844, Vienna -
September 5, 1906, Trieste

Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states $D(E) = \exp(S(E)/k_B)$

Boltzmann equation (1872)
Dilute classical gas



Ludwig Boltzmann
20 February 1844, Vienna -
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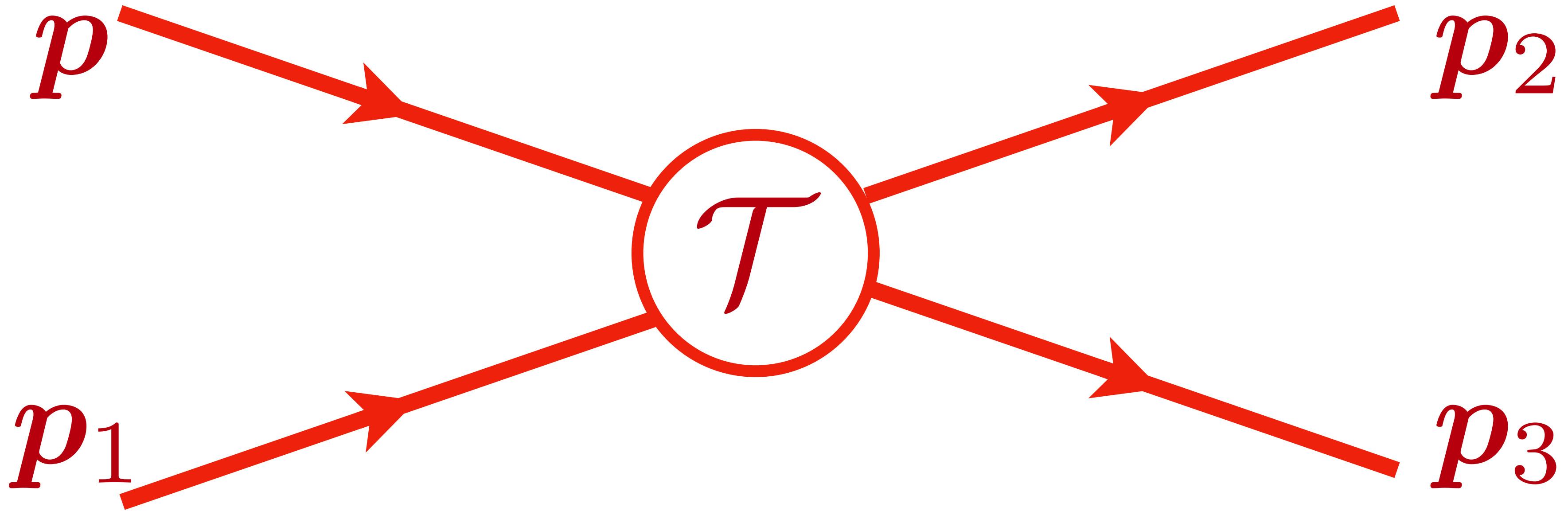
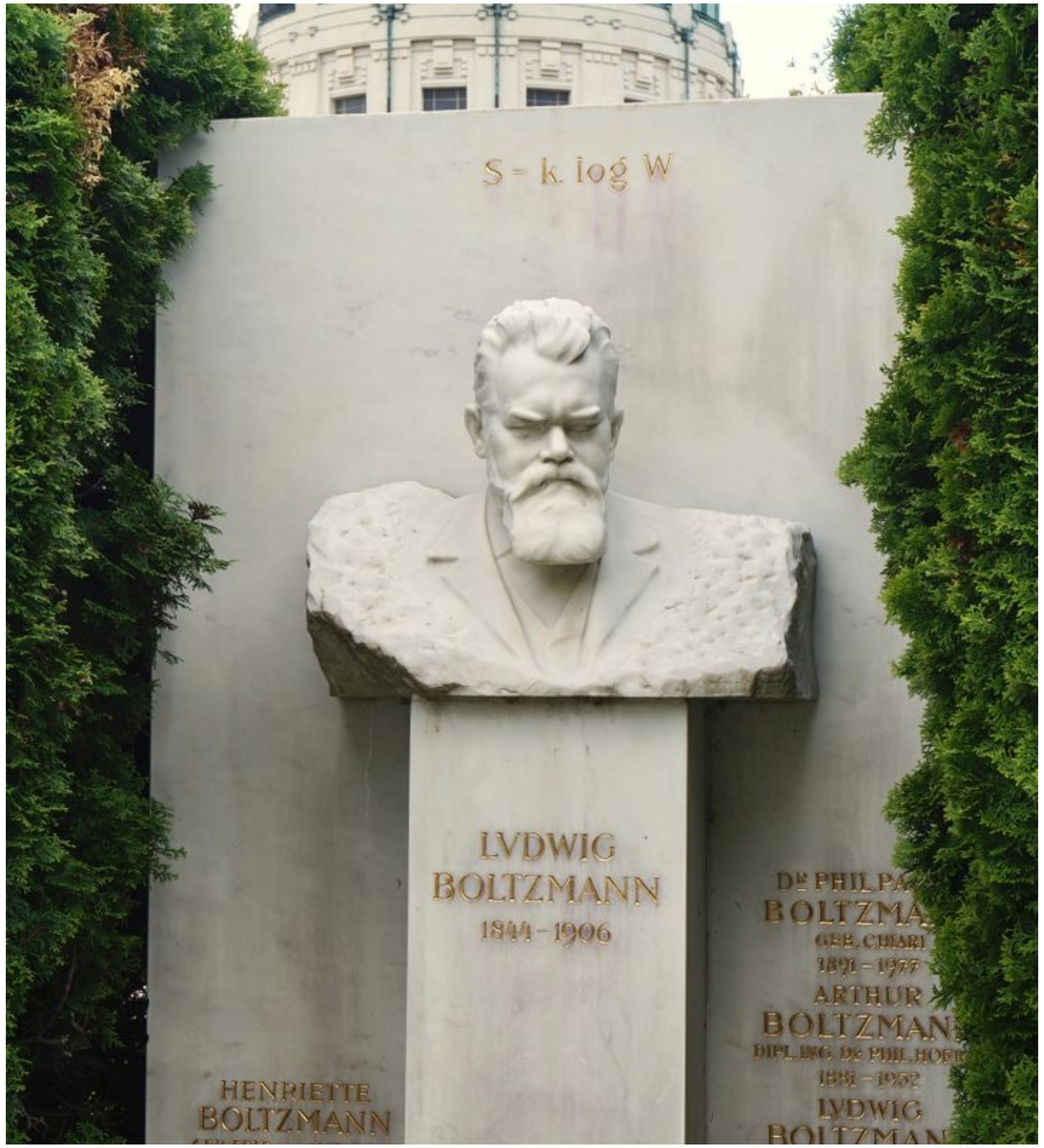
Molecular chaos: successive collisions are statistically independent

$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} = \mathcal{C}[f]$$

$$\mathcal{C}[f] \propto \int_{\boldsymbol{p}_{1,2,3}} \cdots [f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3}]$$



Quantum Boltzmann equation (Landau)
Dense gas of electrons

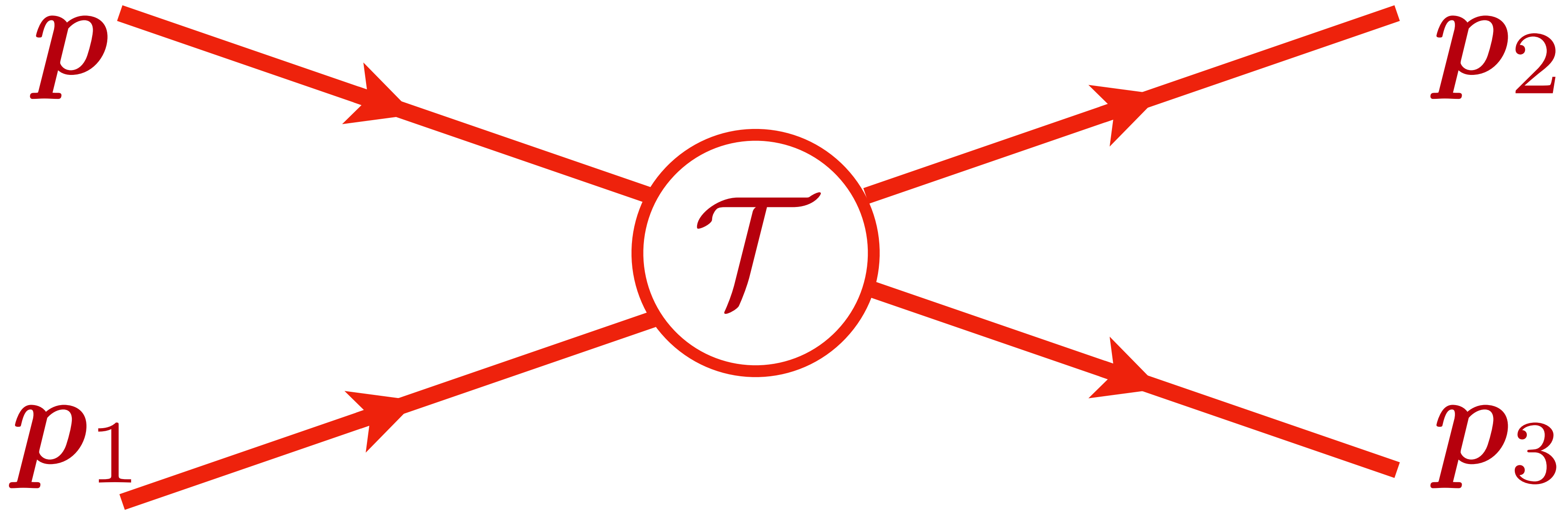


Ludwig Boltzmann
20 February 1844, Vienna -
September 5, 1906, Trieste

Neglects quantum interference (entanglement)
between successive collisions

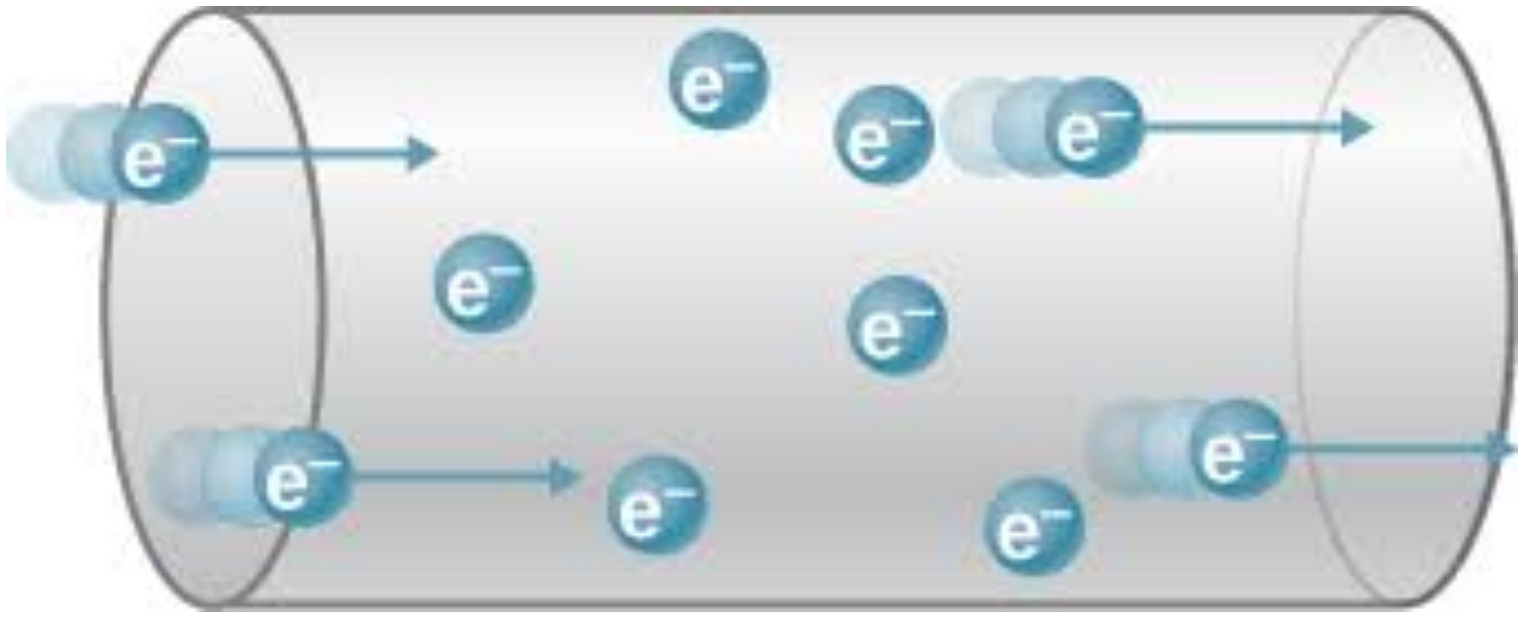
$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} = \mathcal{C}[f]$$

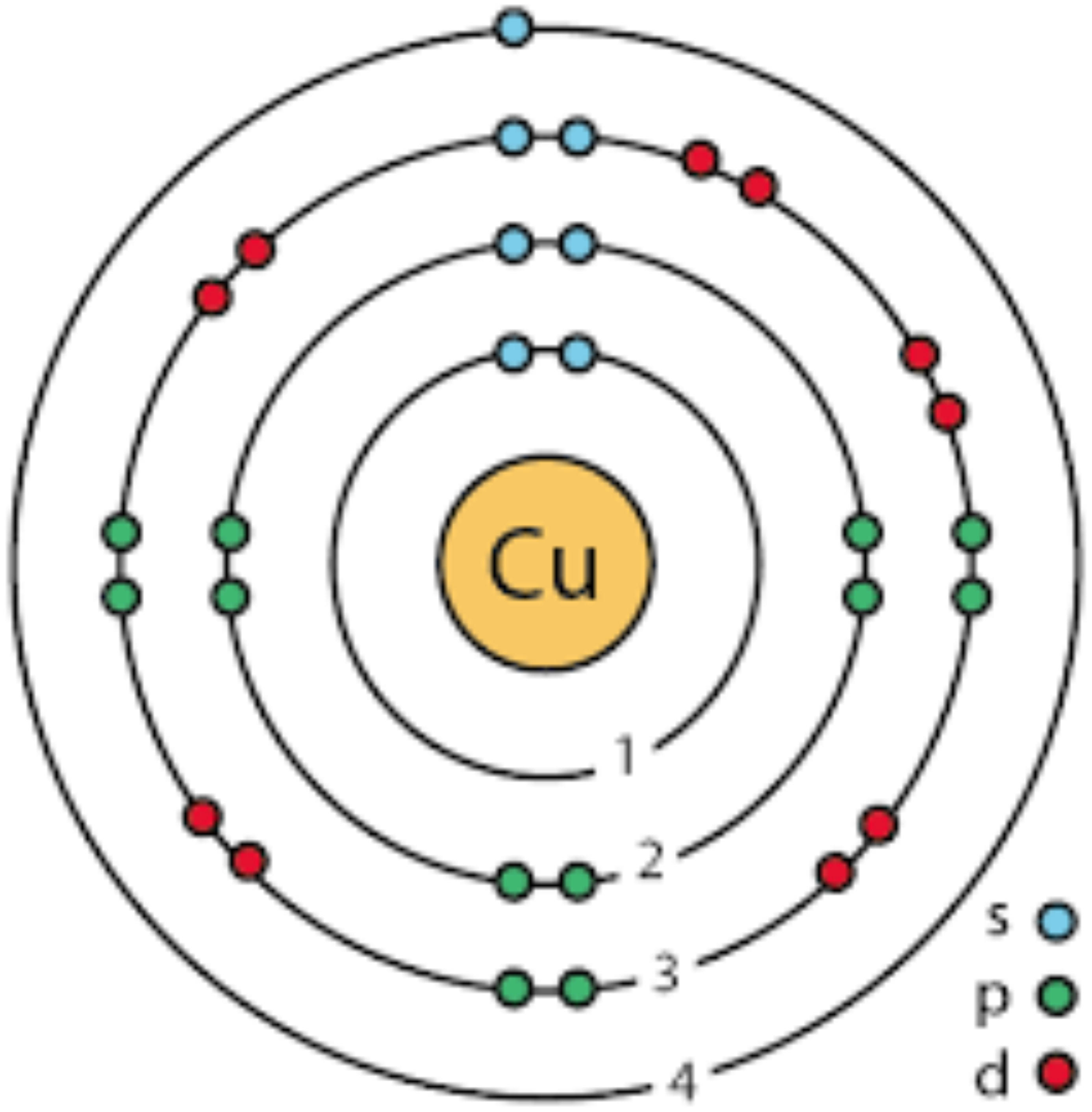
$$\mathcal{C}[f] \propto \int_{\boldsymbol{p}_{1,2,3}} \cdots [f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} (1 - f_{\boldsymbol{p}_2})(1 - f_{\boldsymbol{p}_3}) - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} (1 - f_{\boldsymbol{p}})(1 - f_{\boldsymbol{p}_1})]$$



Quantum theory of electrons: ordinary metals and strange metals

Copper

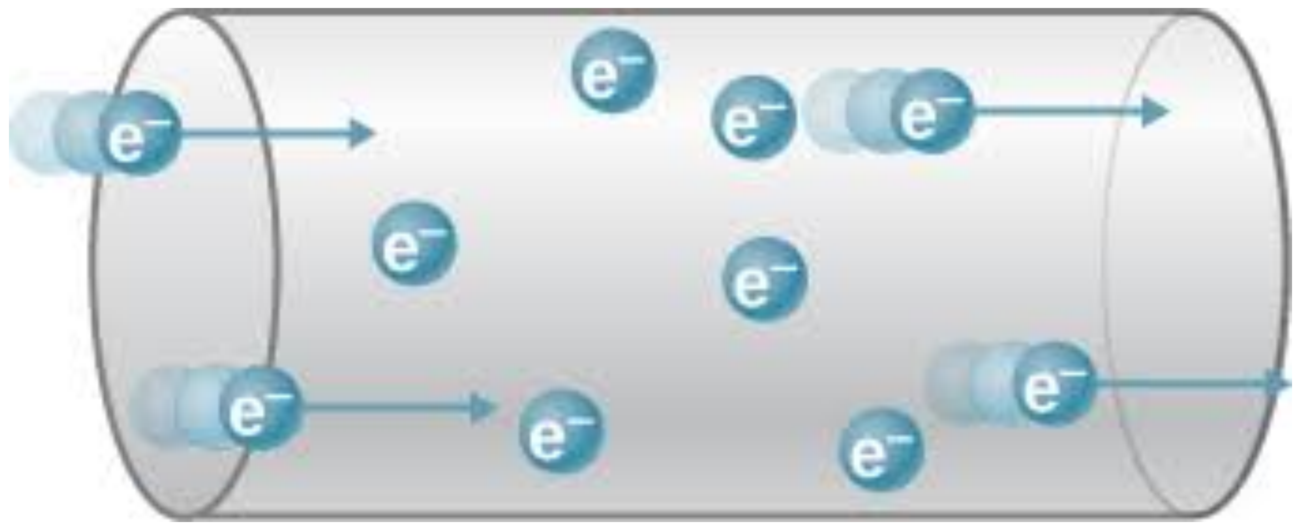




Each copper atom donates its outermost electron

These electrons move freely throughout the crystal and carry current

Current flow with electrons in Copper

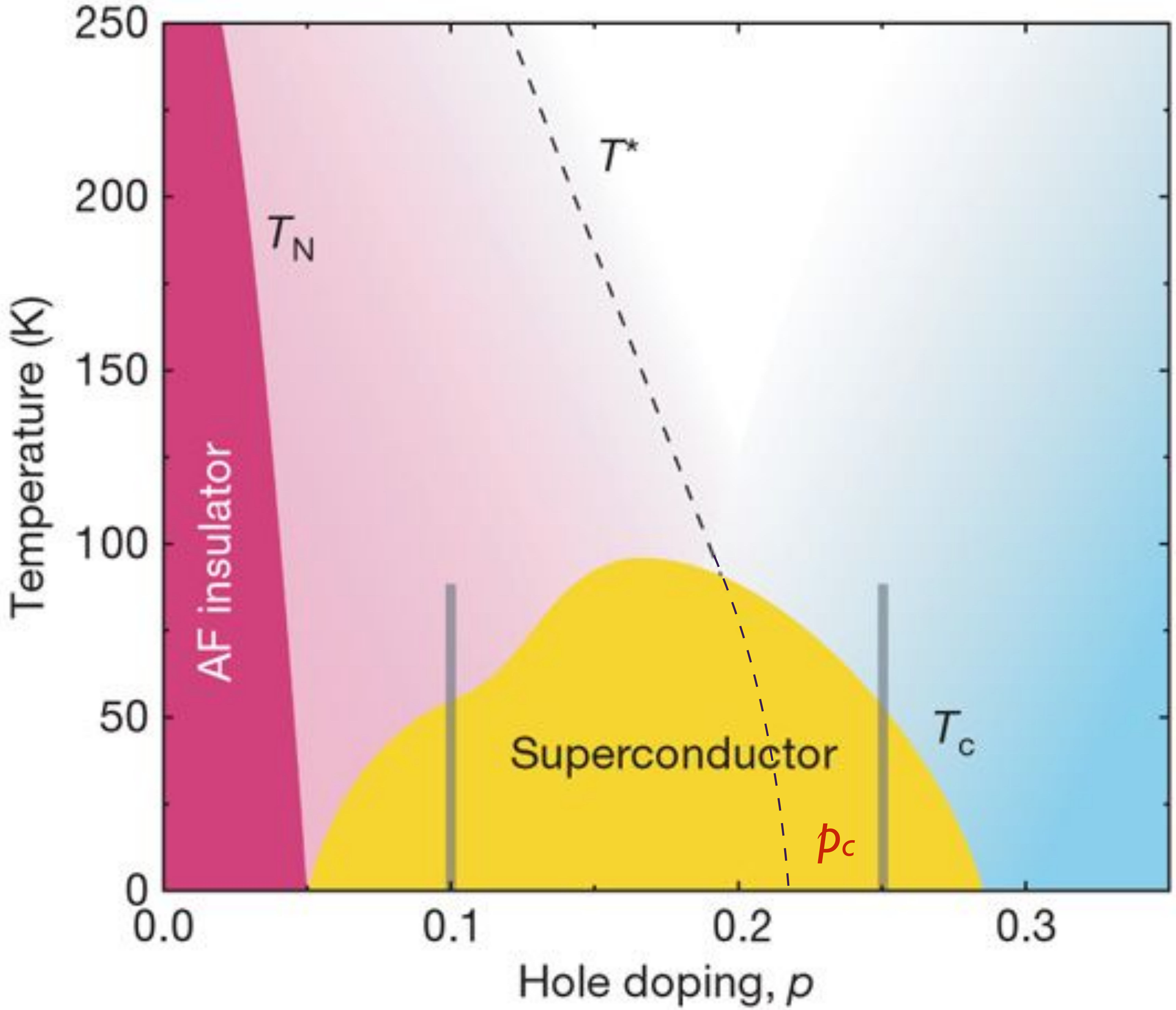


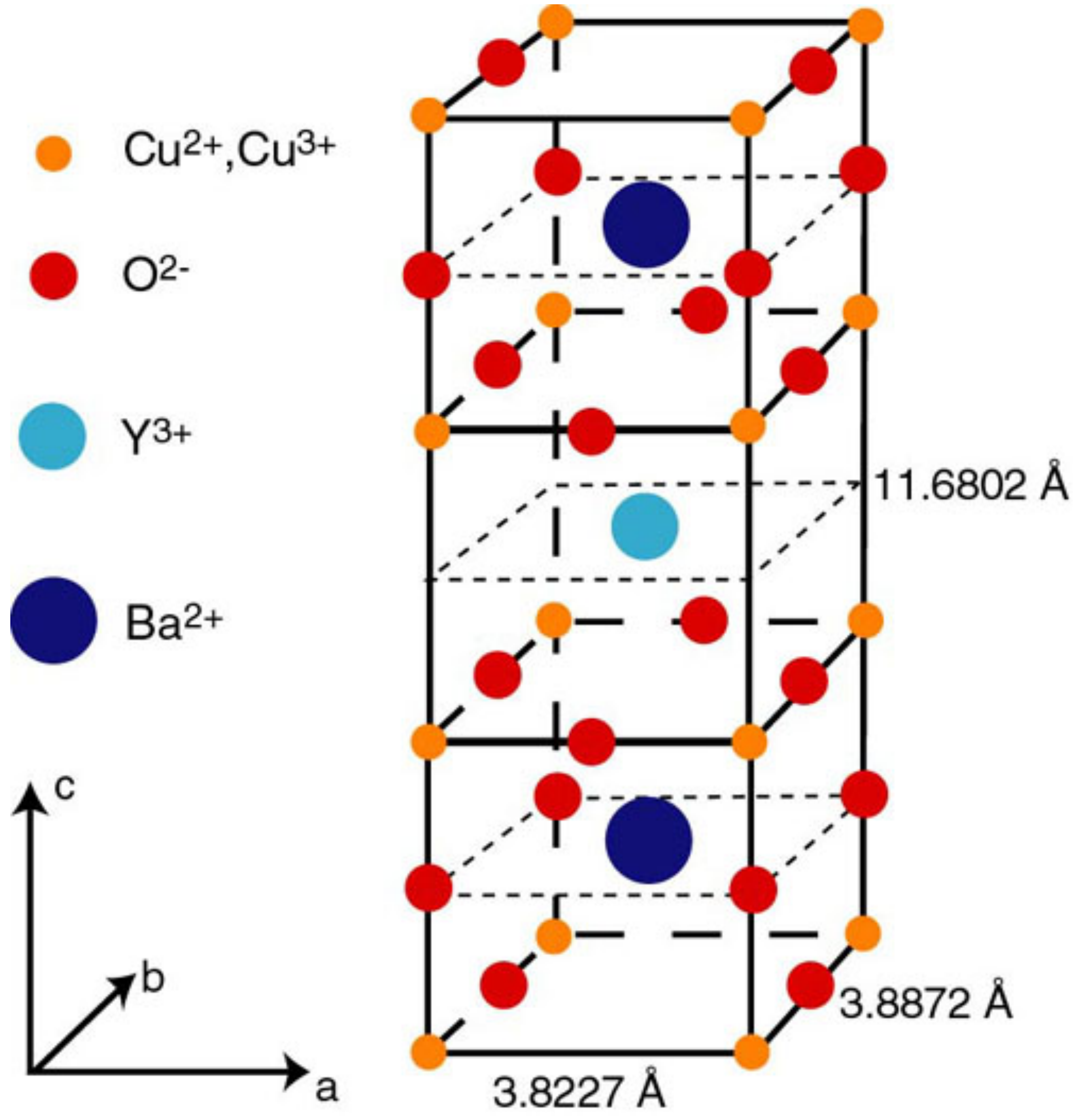
Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

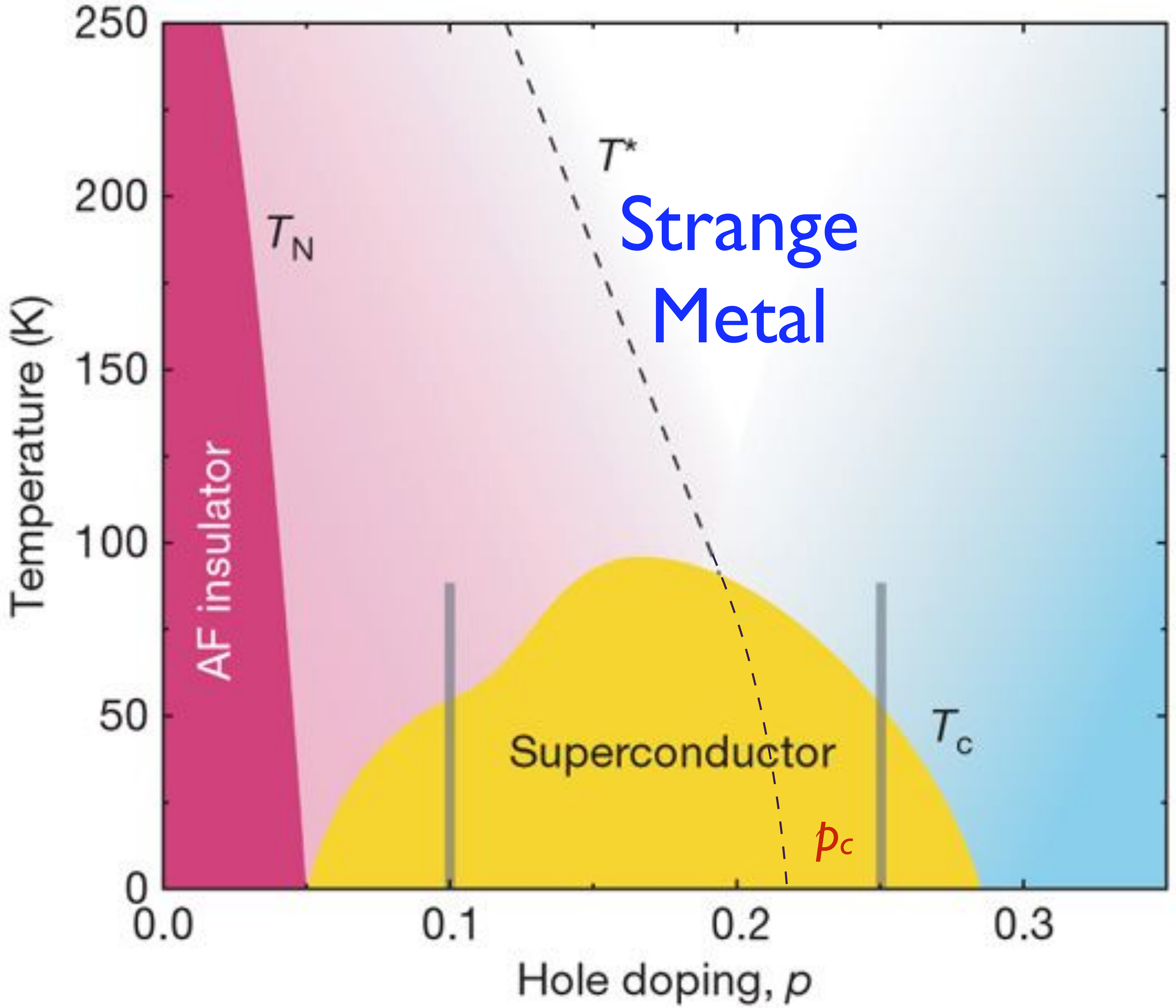
The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

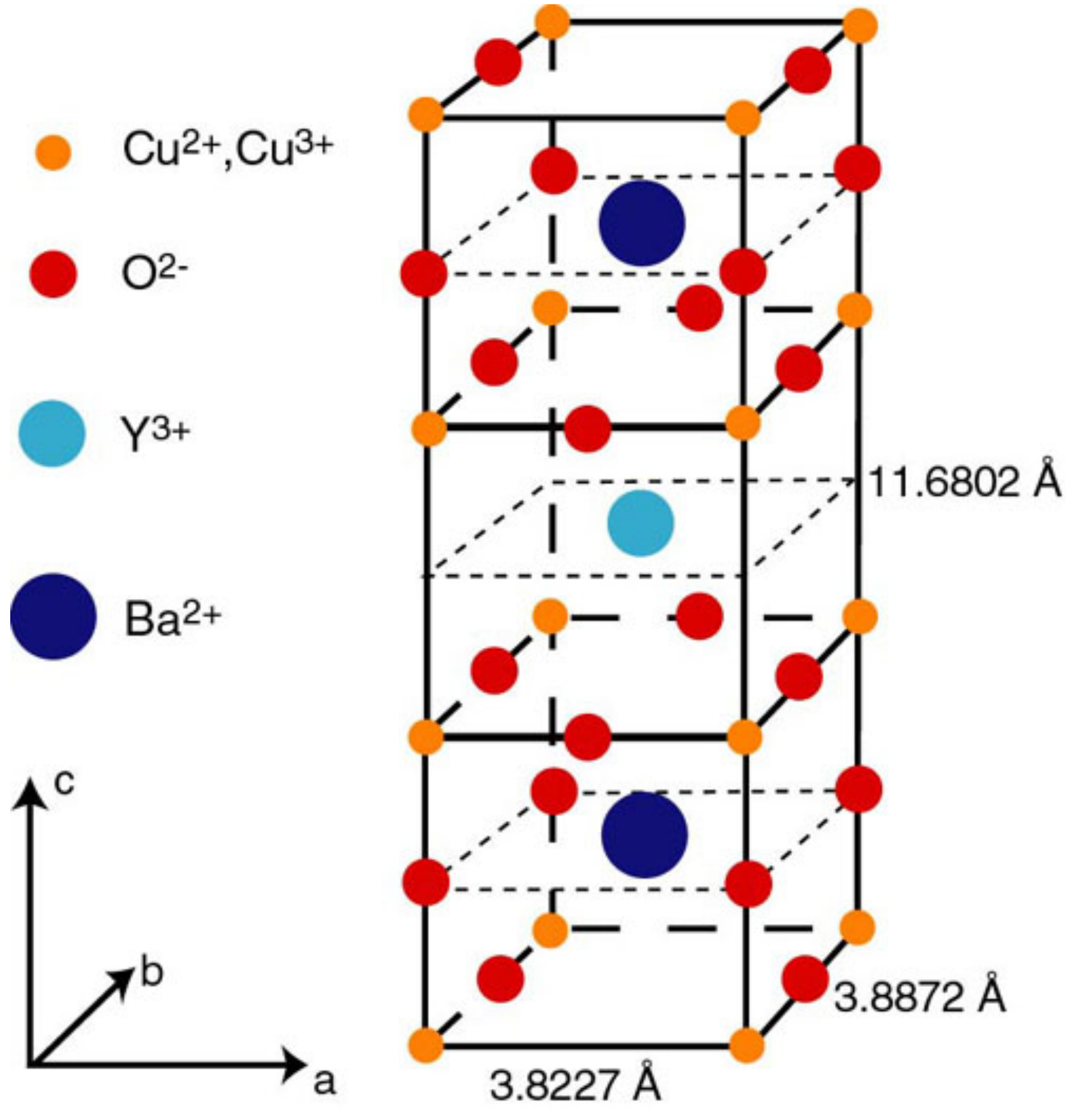
The long scattering time implies that individual electrons are well-defined.

The motion of electrons is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
T_N
T^*
AF insulator
Superconductor
T_c
p_c


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Strange Metal
Superconductor
T_c
p_c


Cu^{2+},Cu^{3+}
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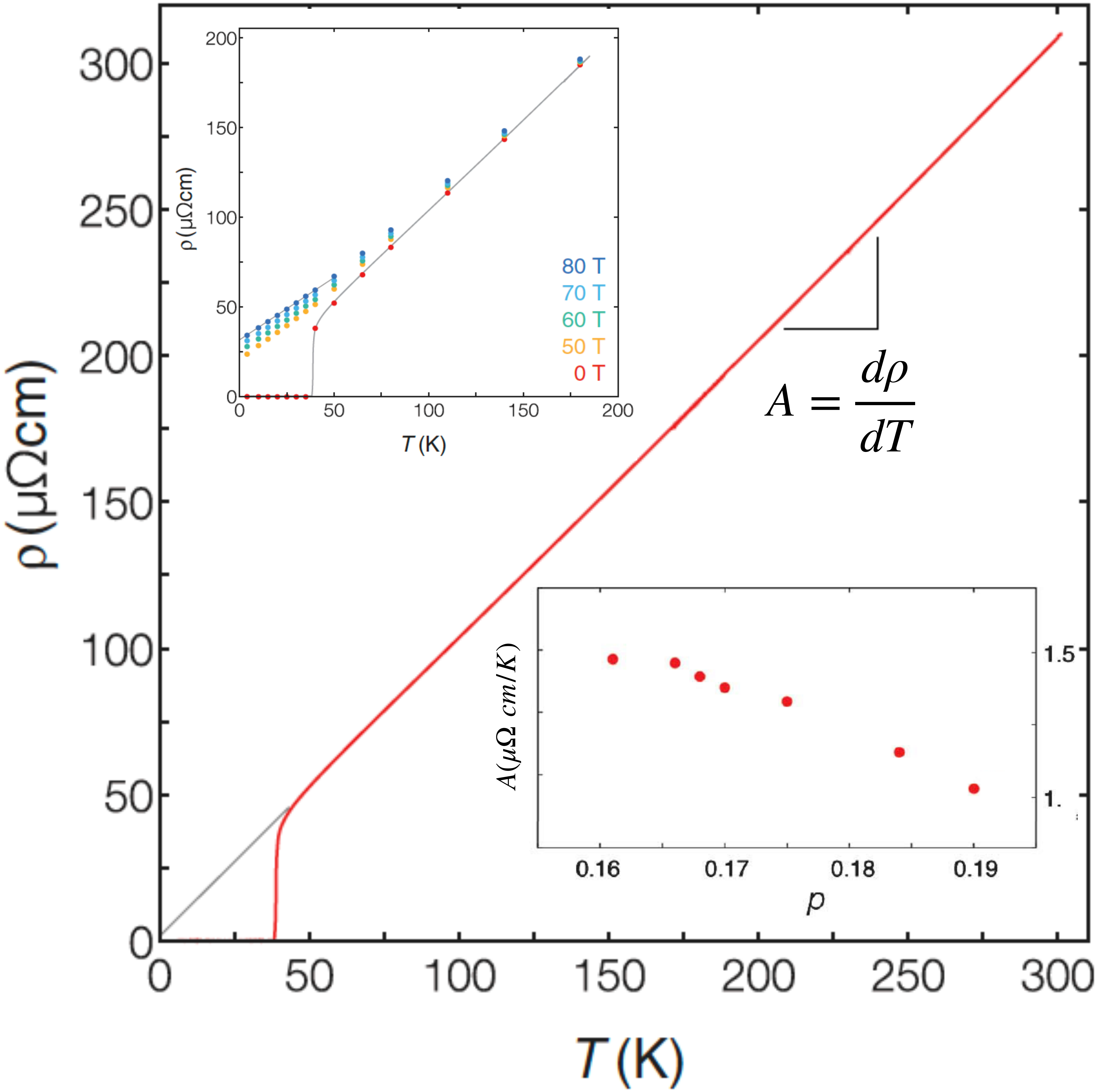
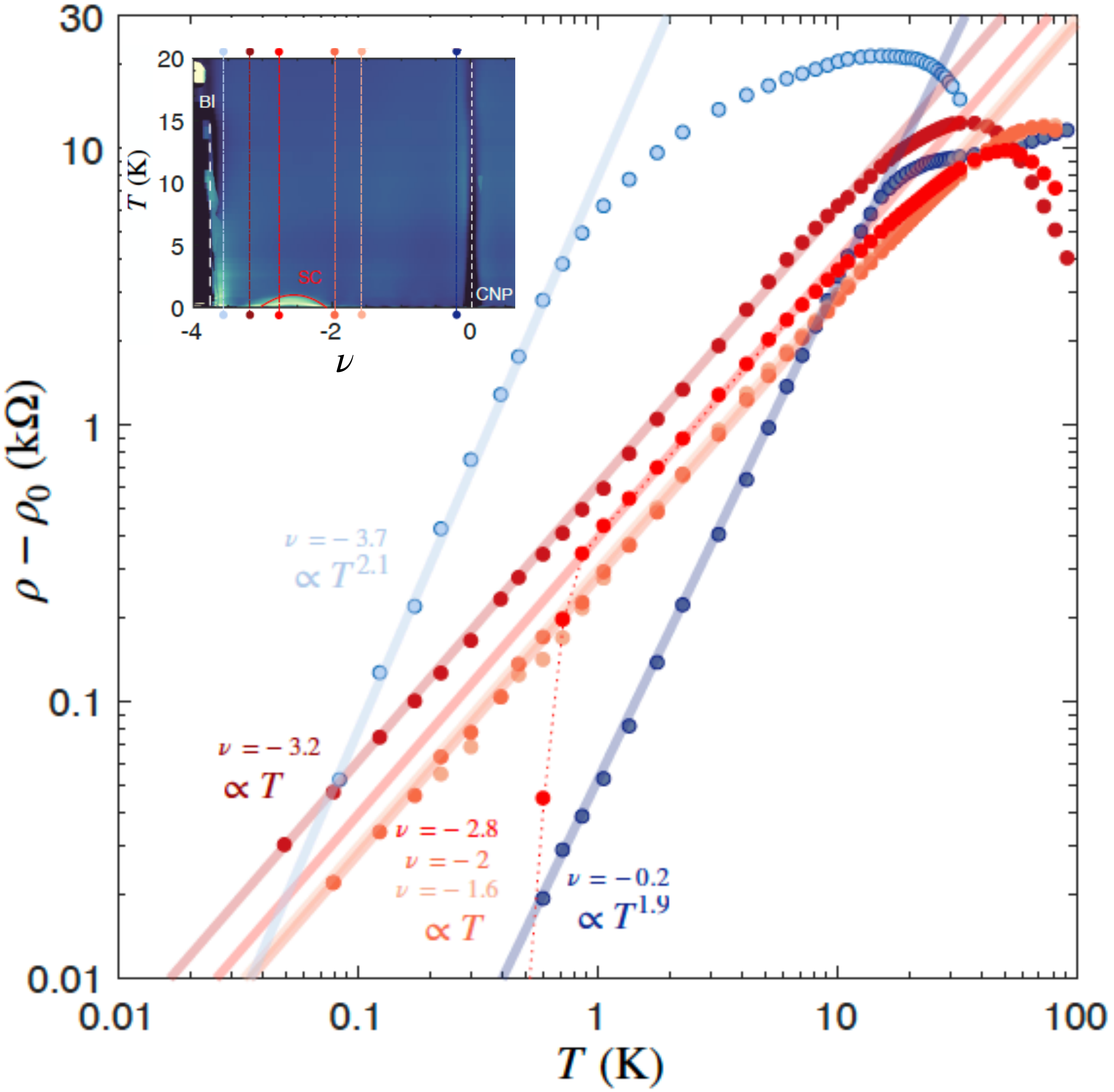


LSCO: Giraldo-Gallo et al. 2018

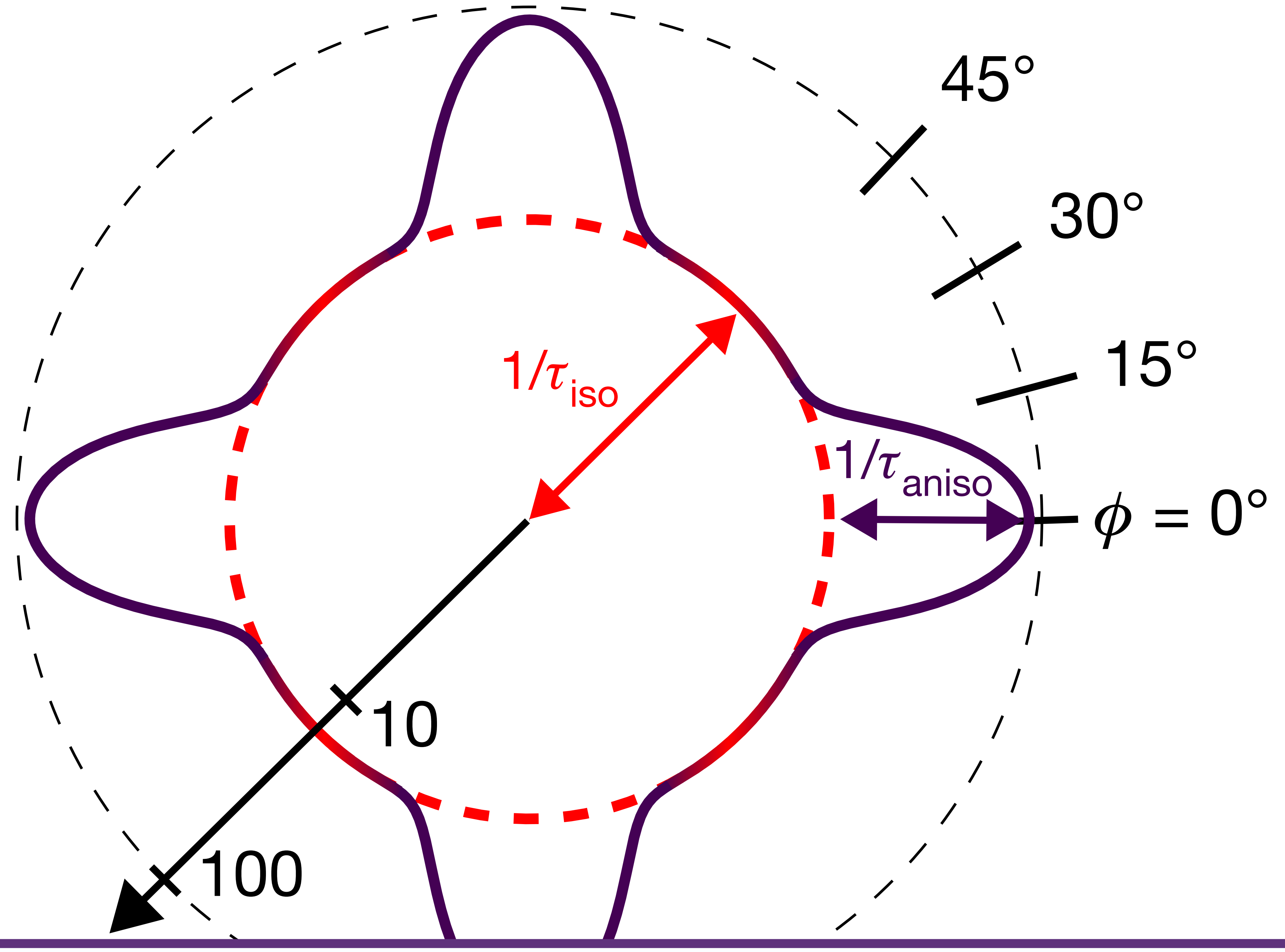


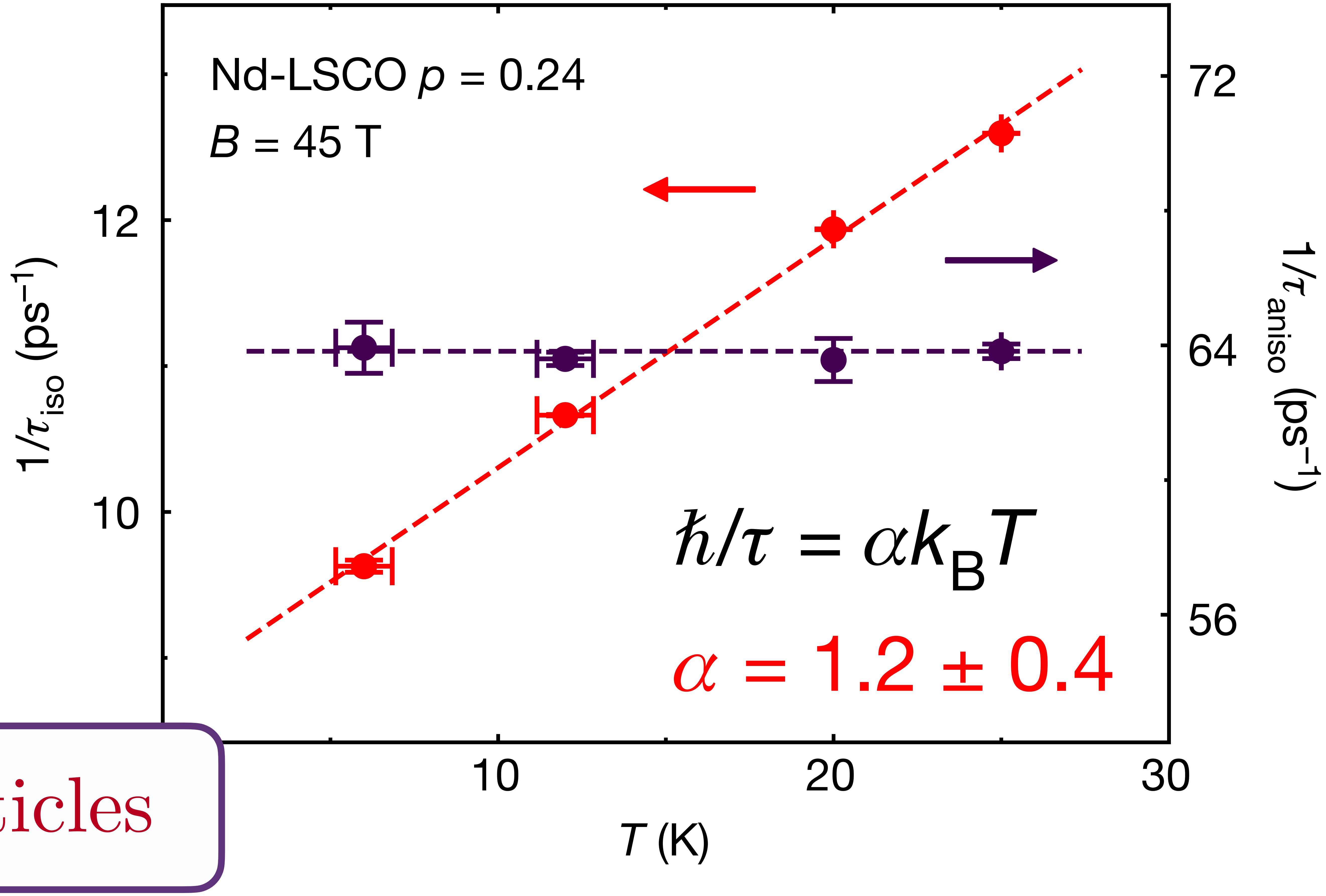
MATBG: Jaoui et al. 2021

Linear-in temperature resistivity from an isotropic Planckian scattering rate

Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw





Current flow without quasiparticles

Black Holes Obey Information-Emission Limits

April 22, 2021 • *Physics* 14, s47 –Christopher Crockett

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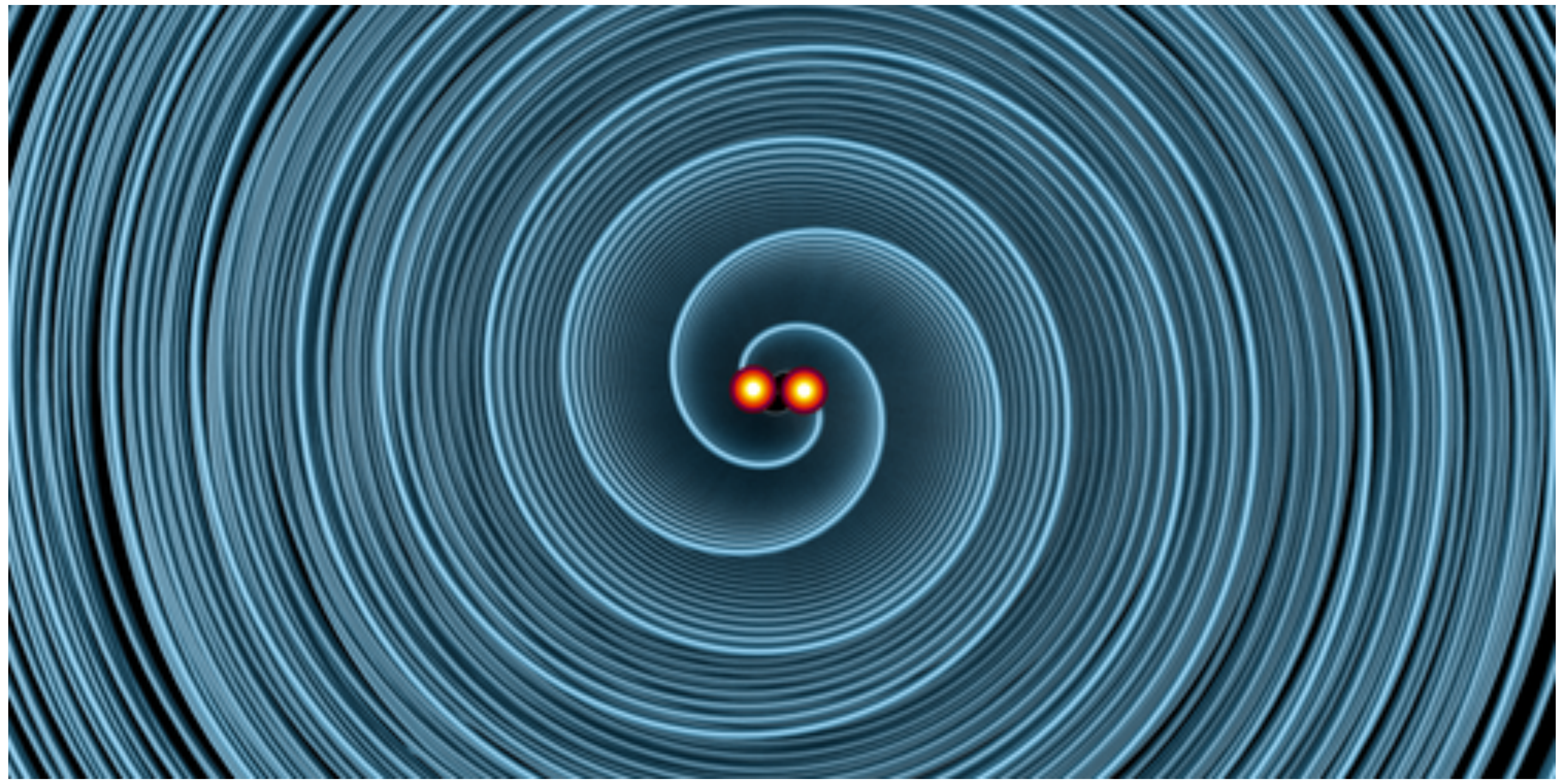

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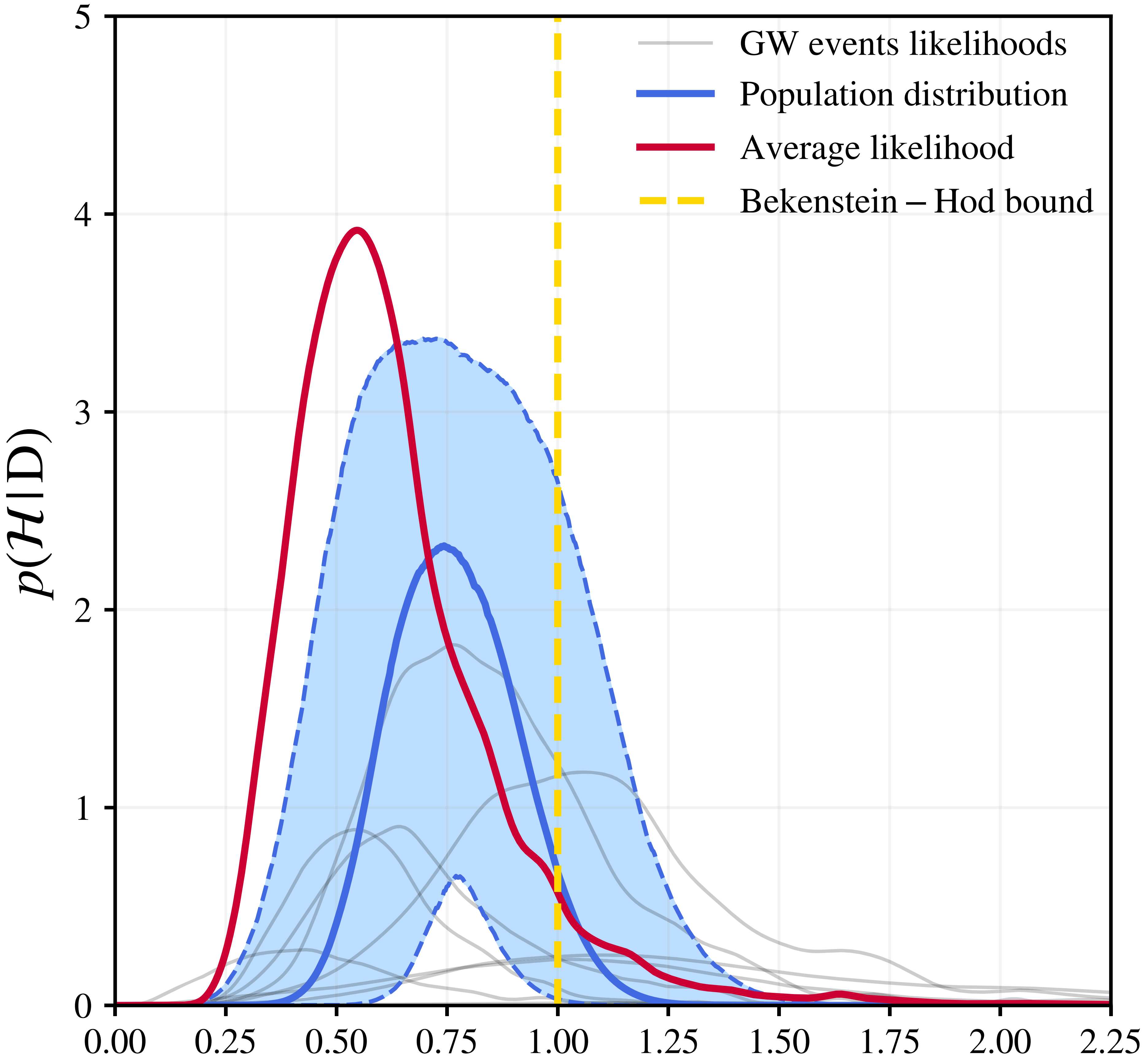

$$\mathcal{H} = \frac{1}{\pi}\frac{\hbar/\tau}{k_B T}$$

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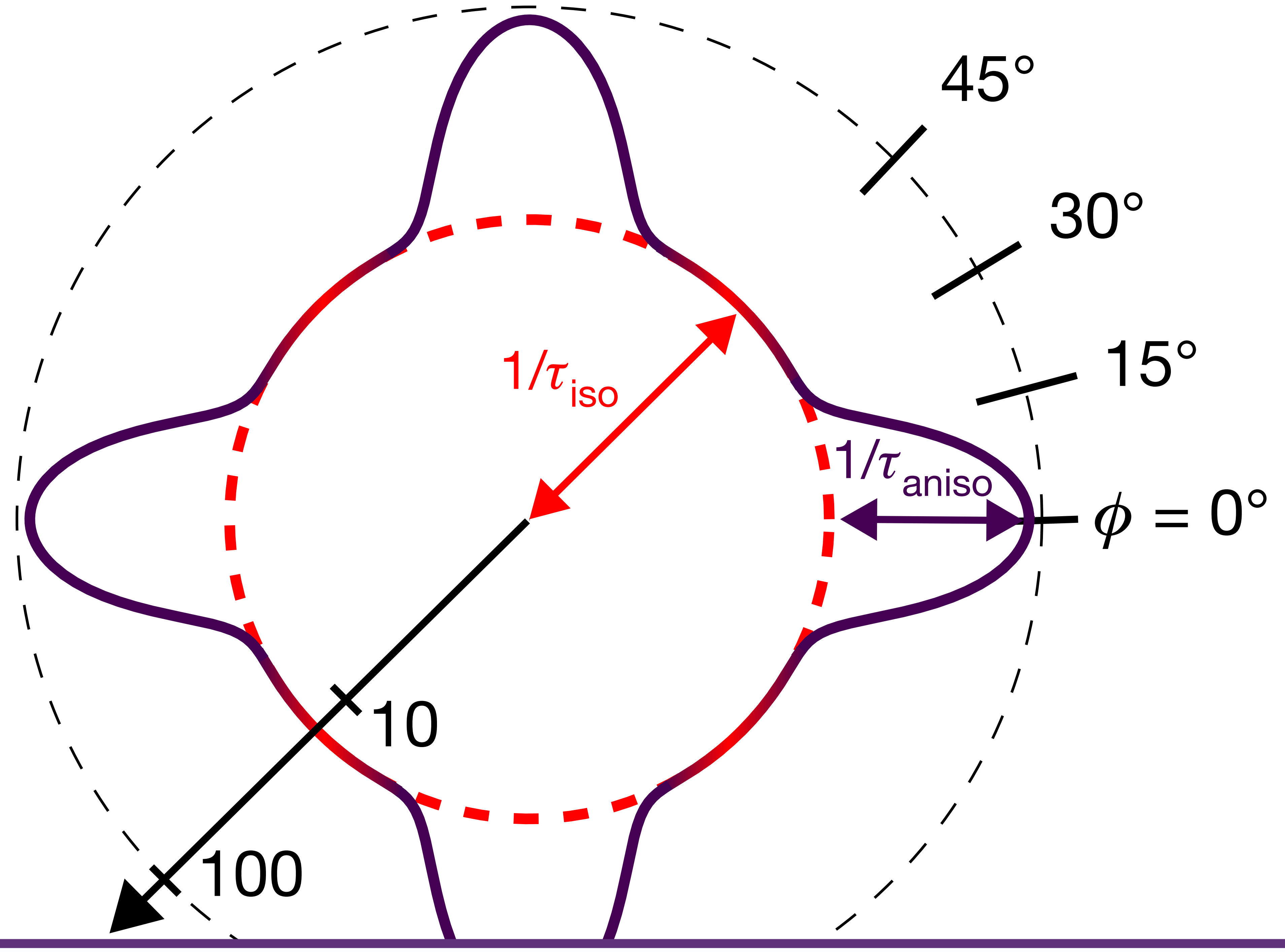
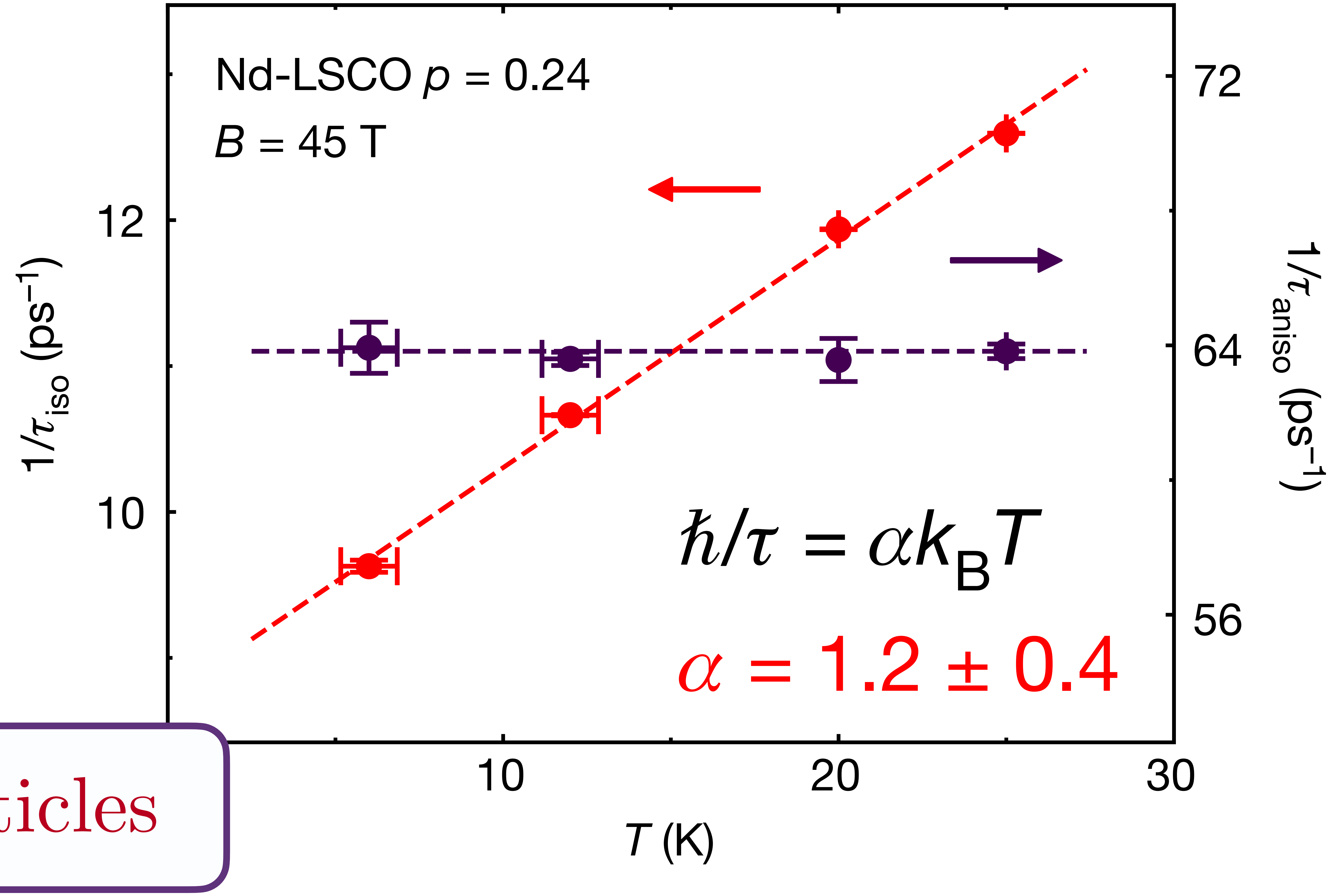




Current flow without quasiparticles

Questions

- Needed: A theory for current flow in a 'strange metal' with an entangled soup of electrons.
- Needed: theory for collision time in resistivity $\sim \hbar/(k_B T)$.
- Needed: theory for the appearance of superconductivity in such a 'strange metal'.

Sachdev-Ye-Kitaev Model

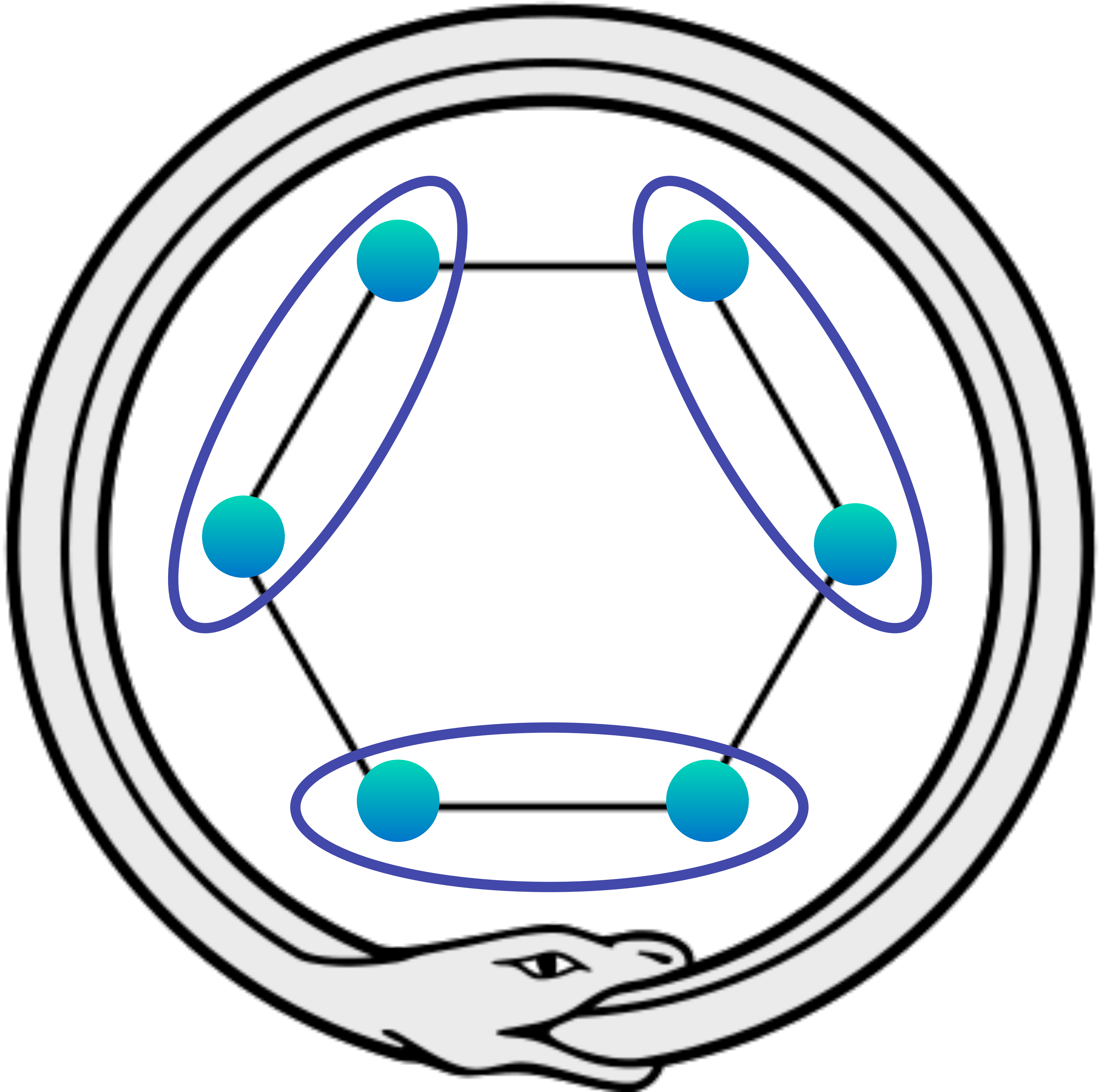
A solvable model of multi-particle entanglement which accounts for quantum interference between successive collisions:

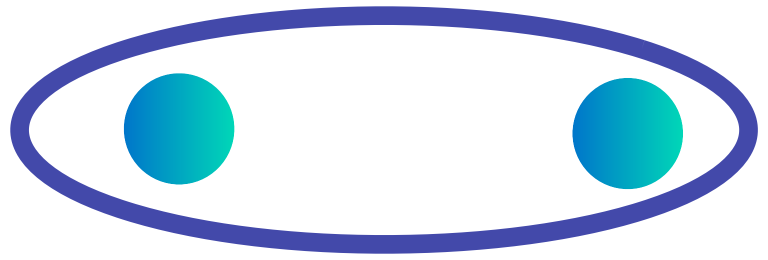
leading to a metal with no particle-like excitations

August Kekule, theory of the benzene molecule, 1865

Kekule's spooky dream

Here Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail[*]



 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

[*] Wikipedia

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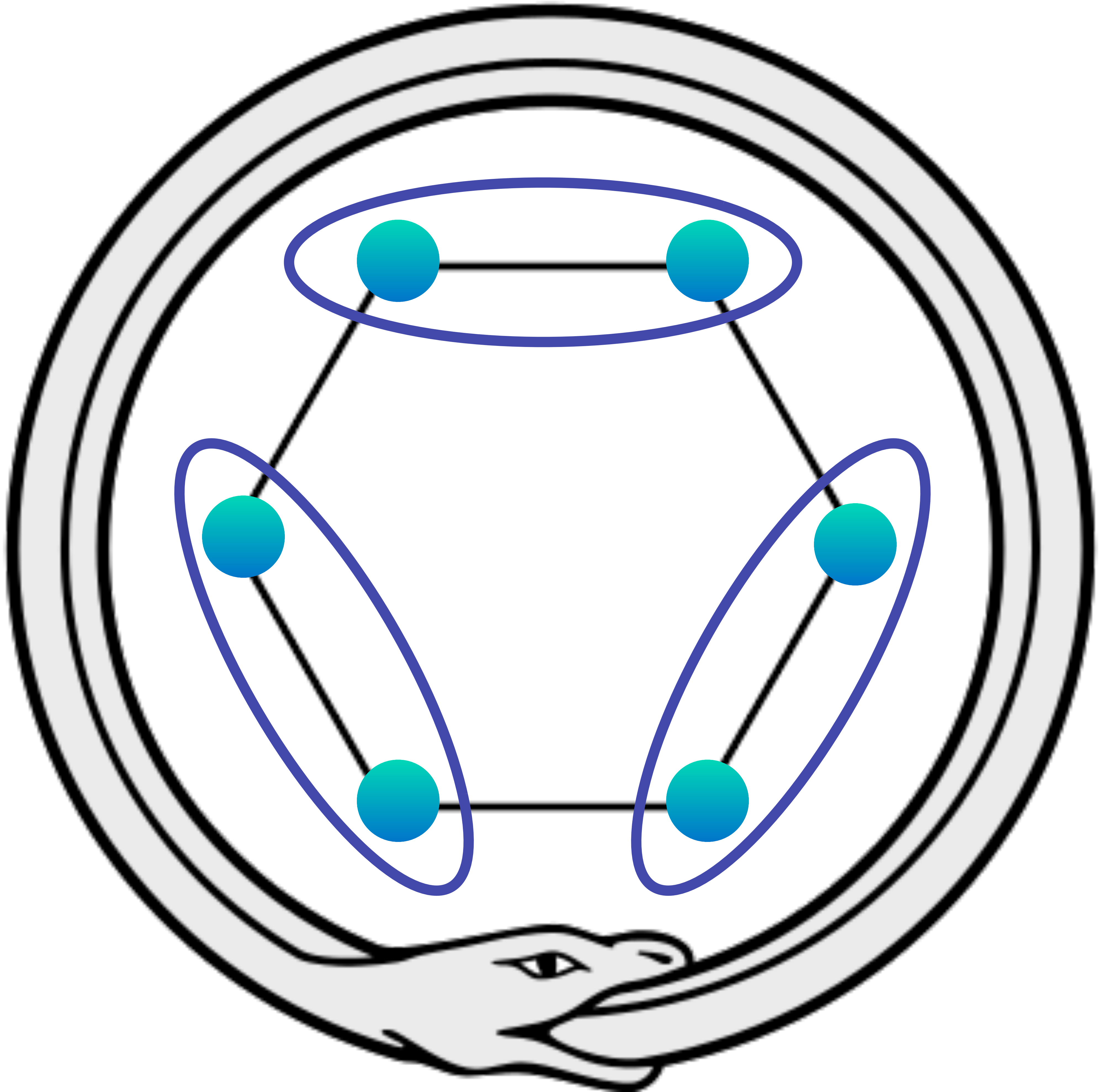

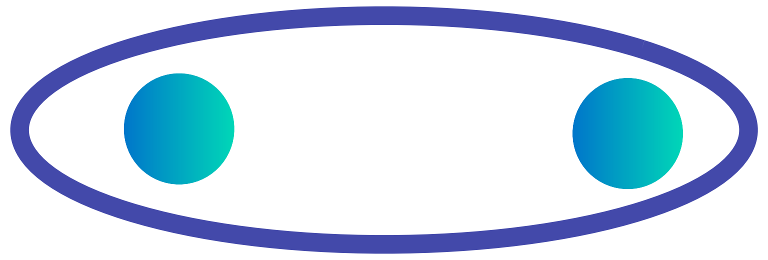

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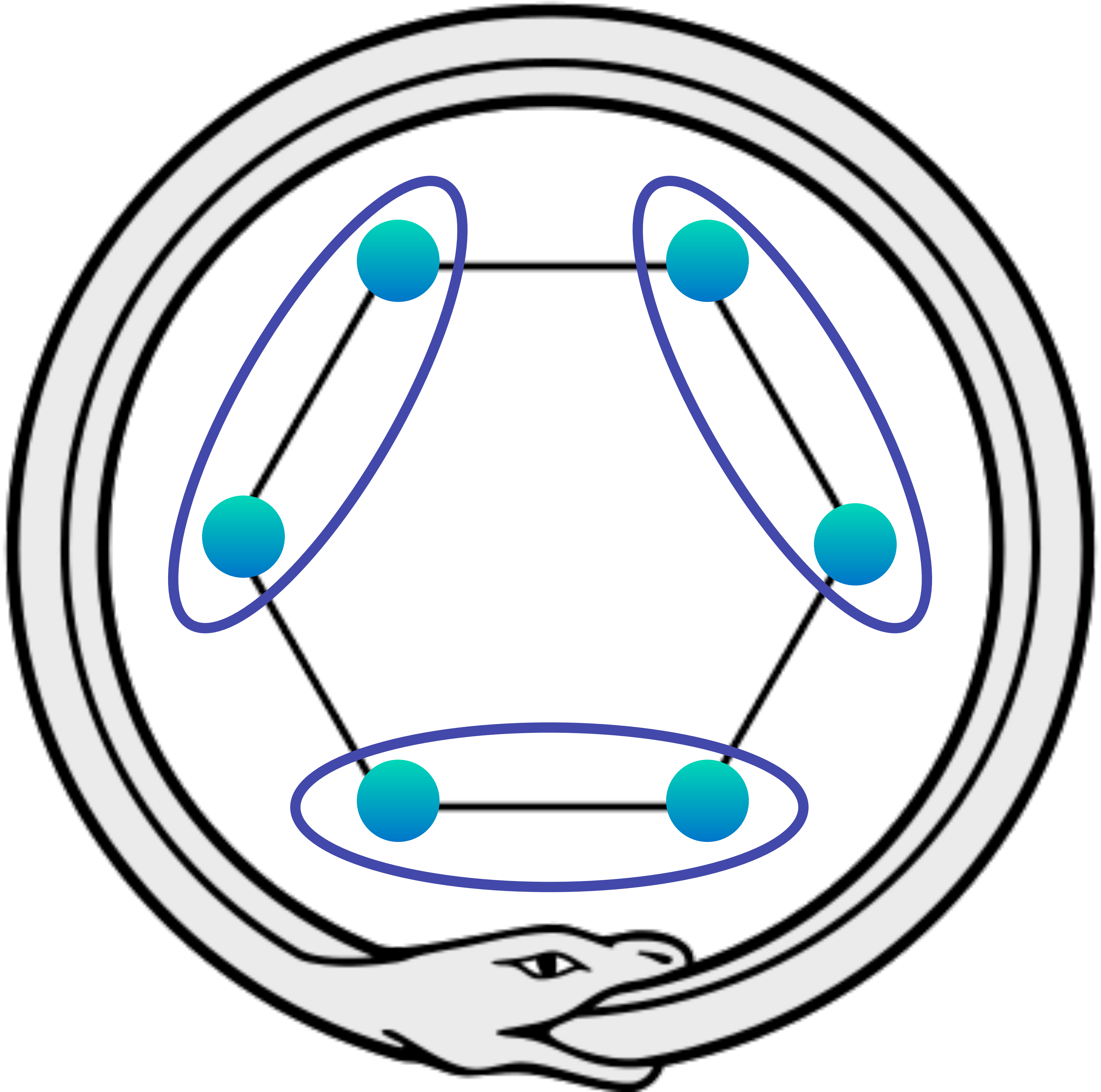

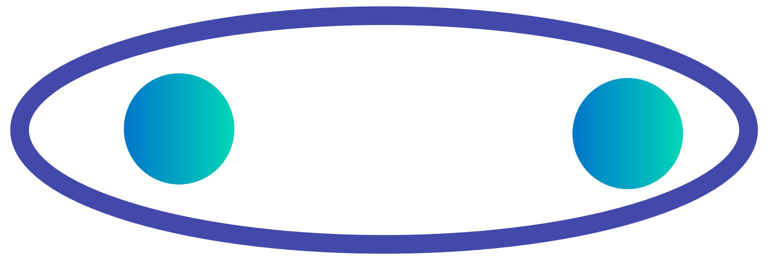

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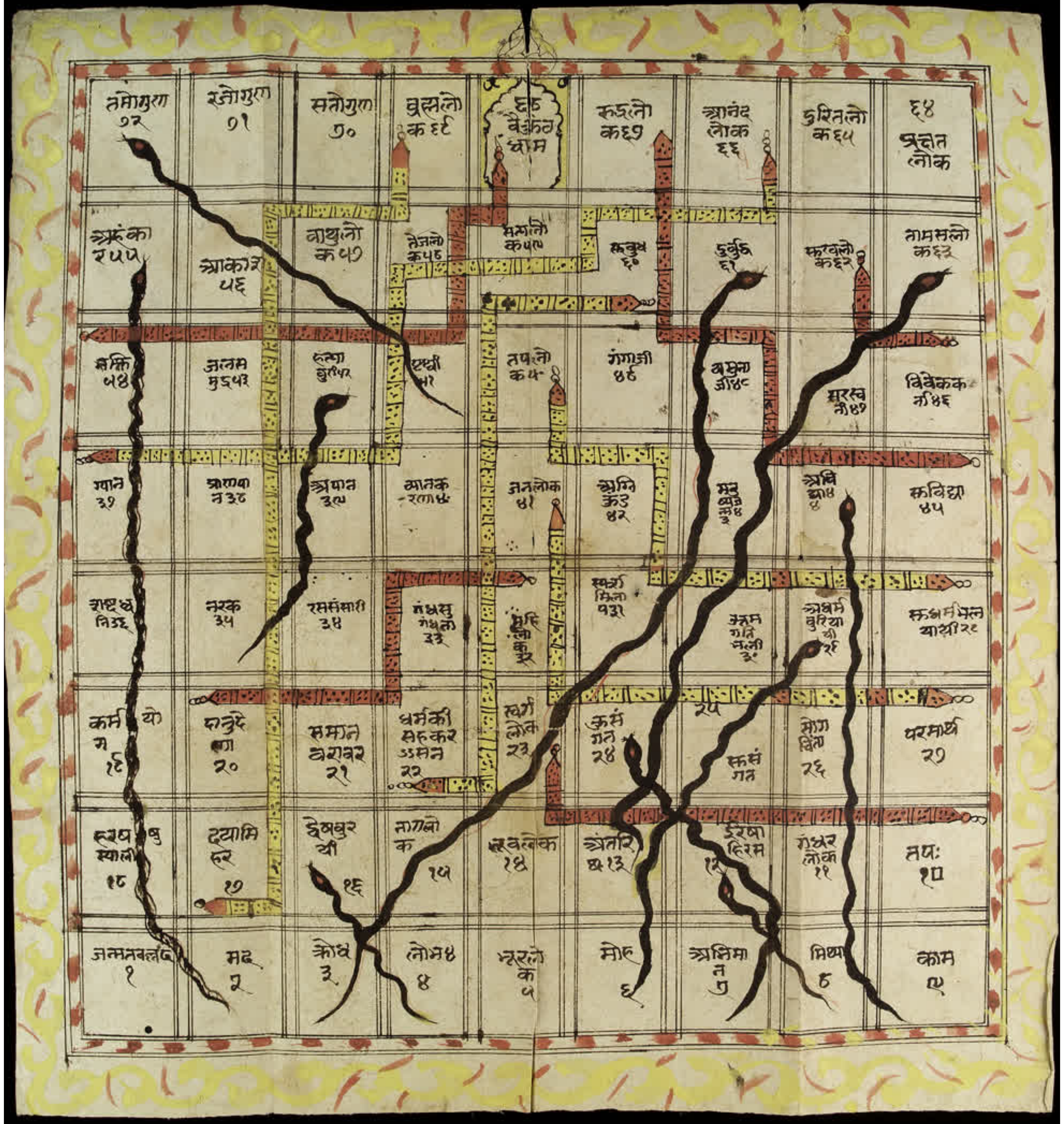


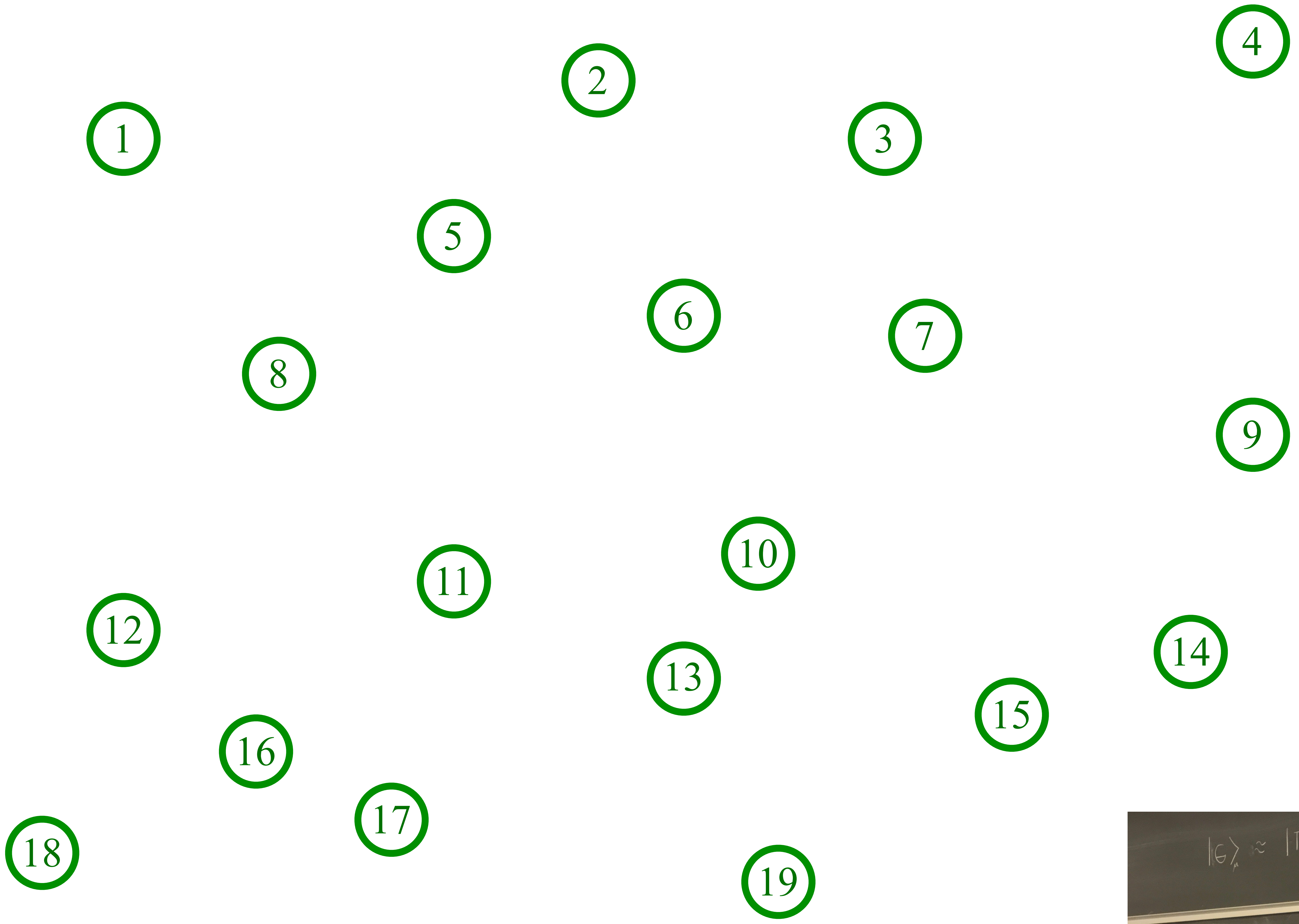
My spooky dream*

Ancient Indian game of Snakes and Ladders

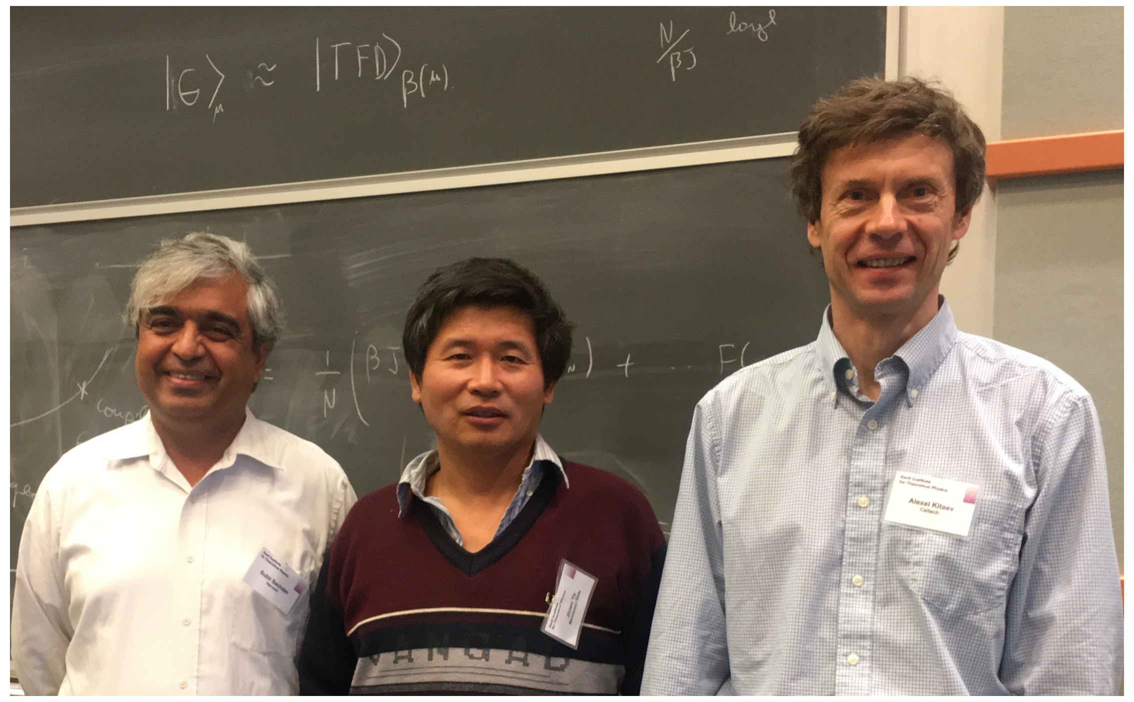
*Not true

The SYK model

Sachdev, Ye (1993); Kitaev (2015)



Pick a set of random positions



The SYK model

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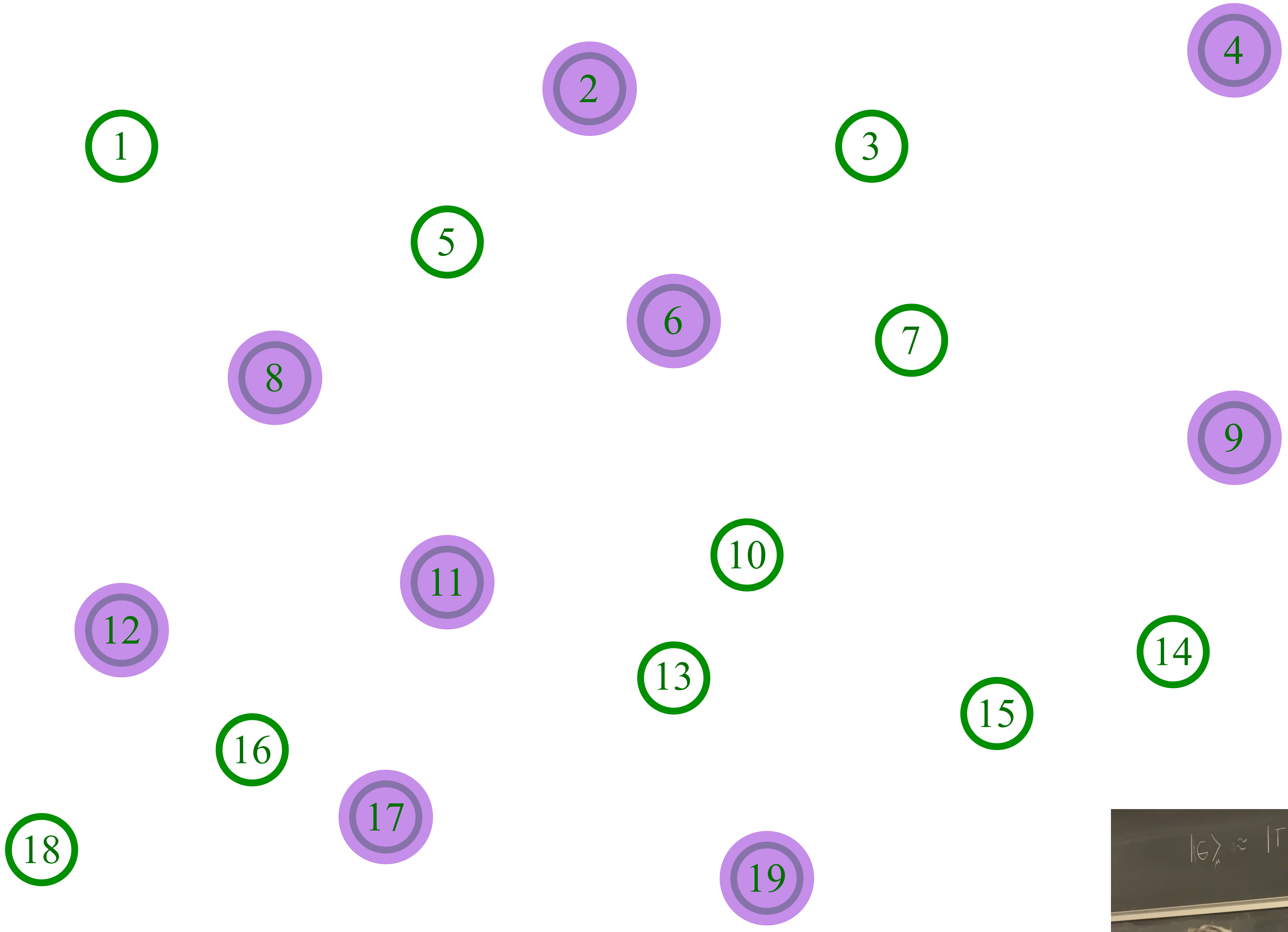
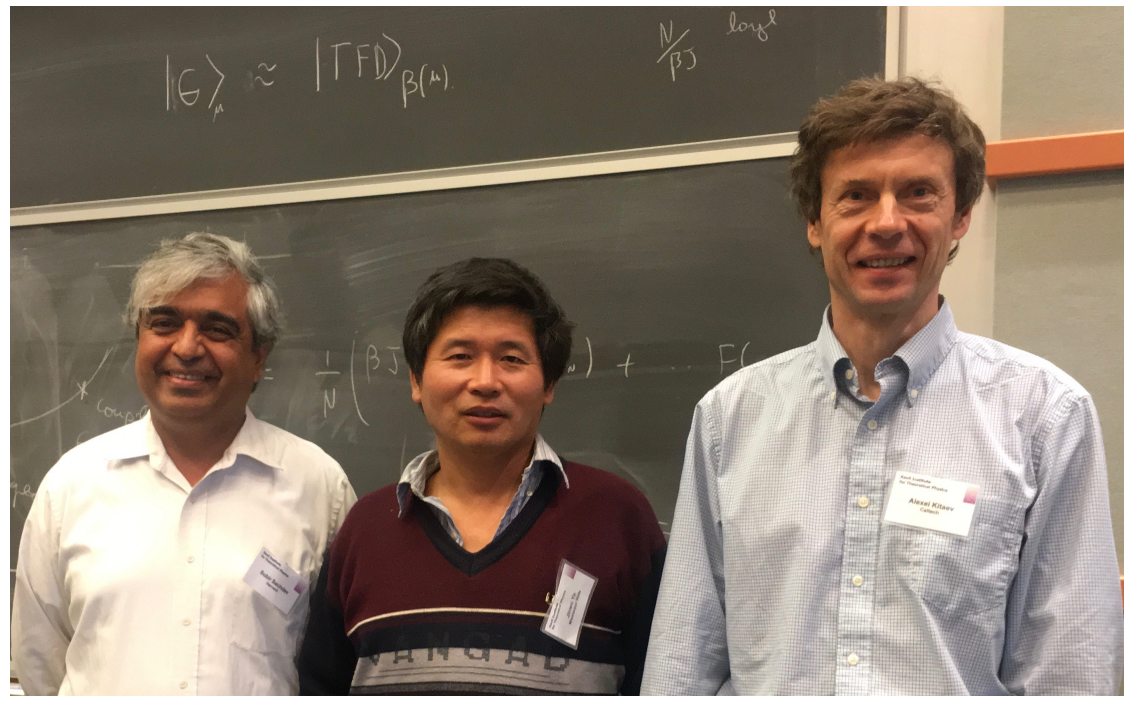


Place electrons randomly on some sites



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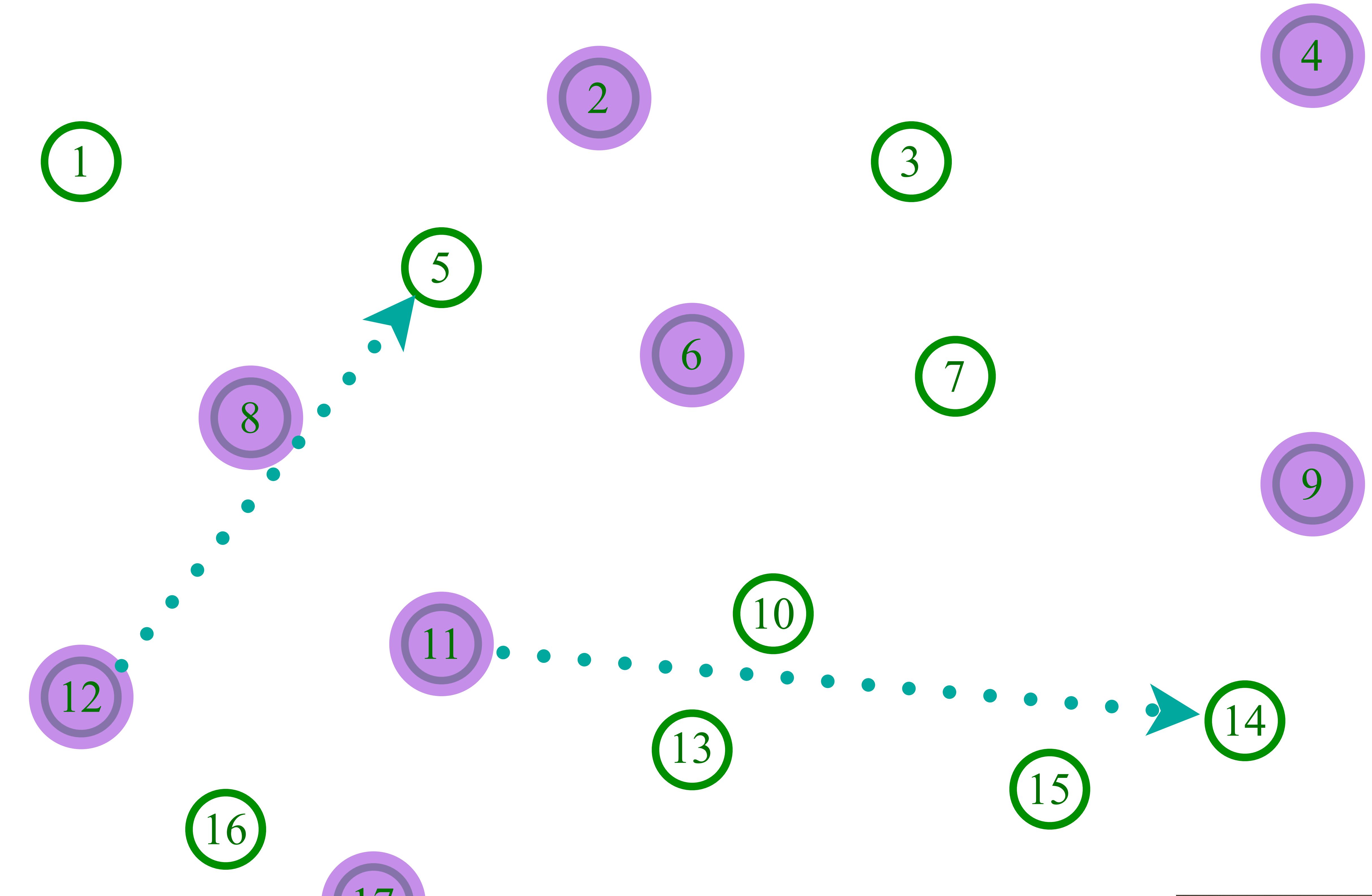
Sachdev, Ye (1993); Kitaev (2015)

$U_{11,12;5,14}$



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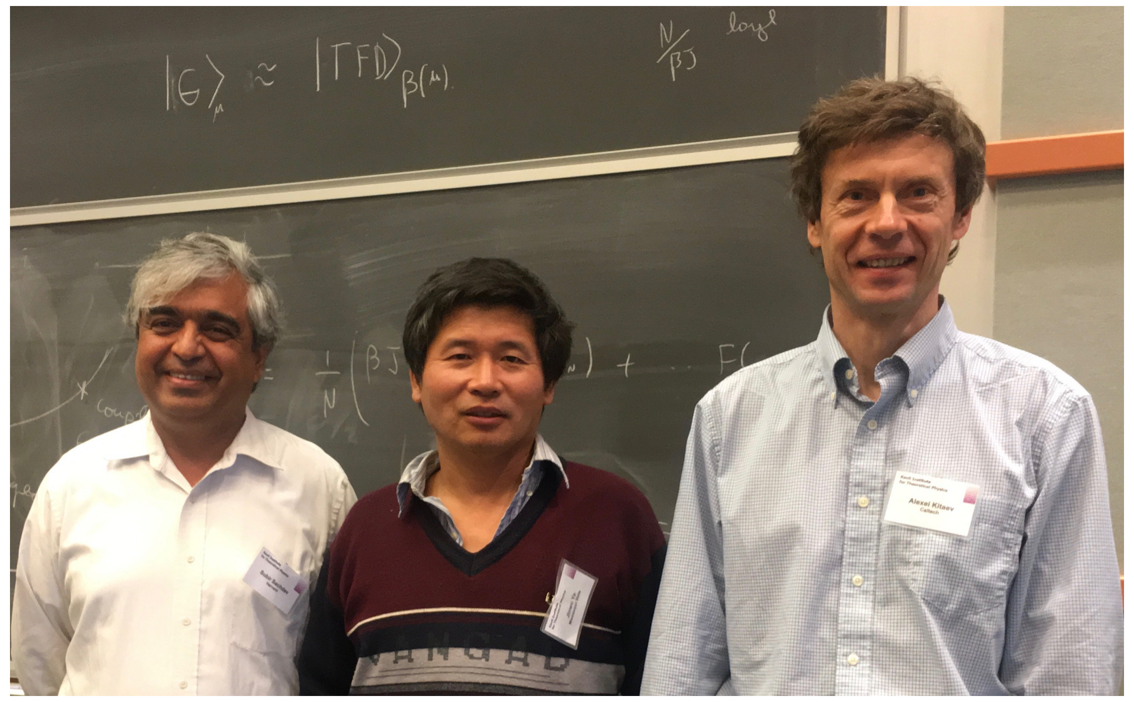

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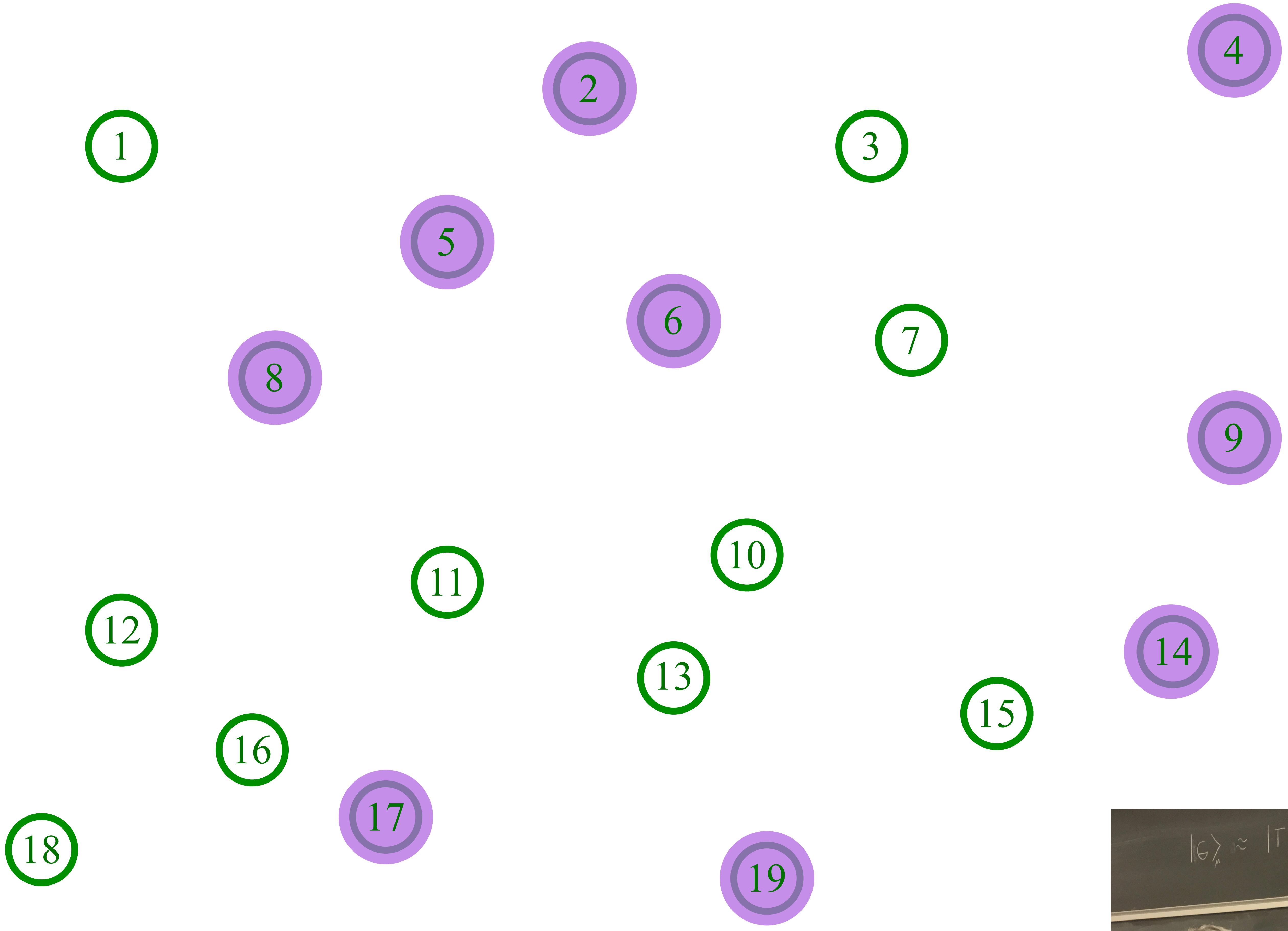
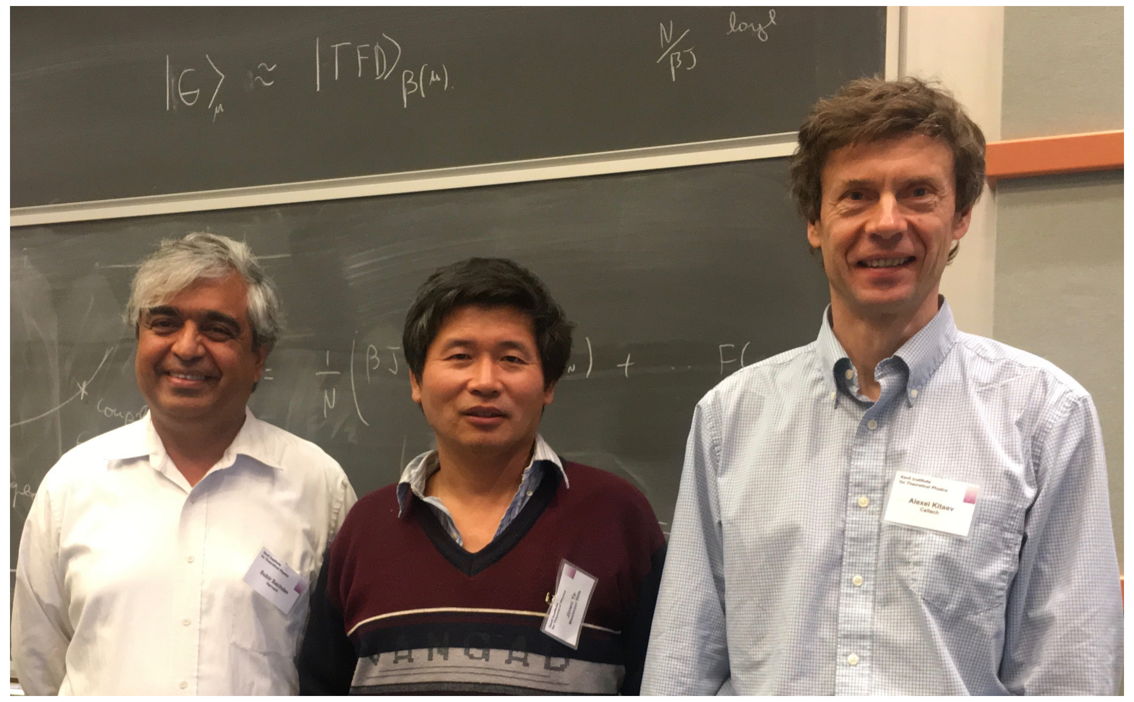


Entangle electrons pairwise randomly



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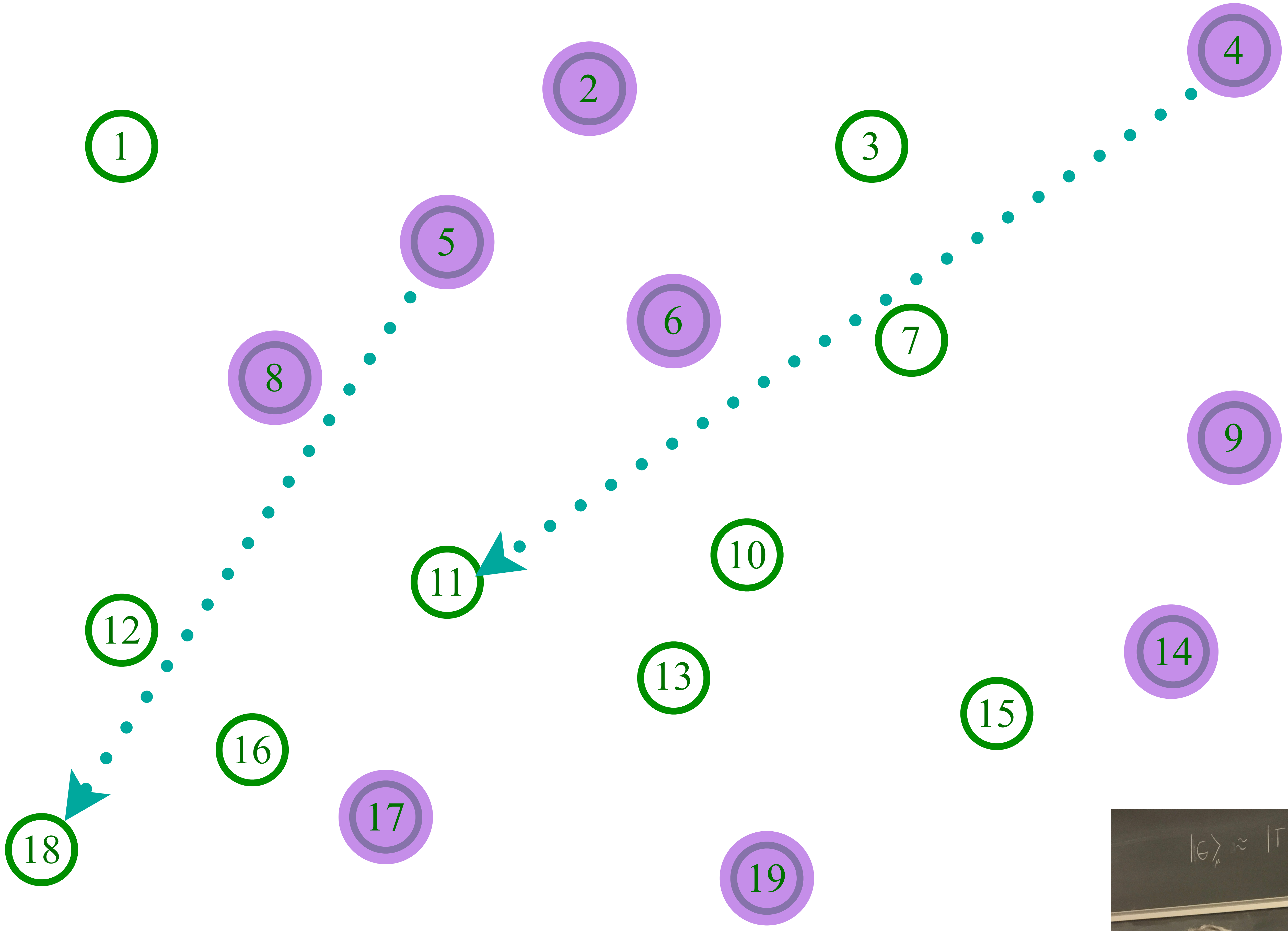
Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



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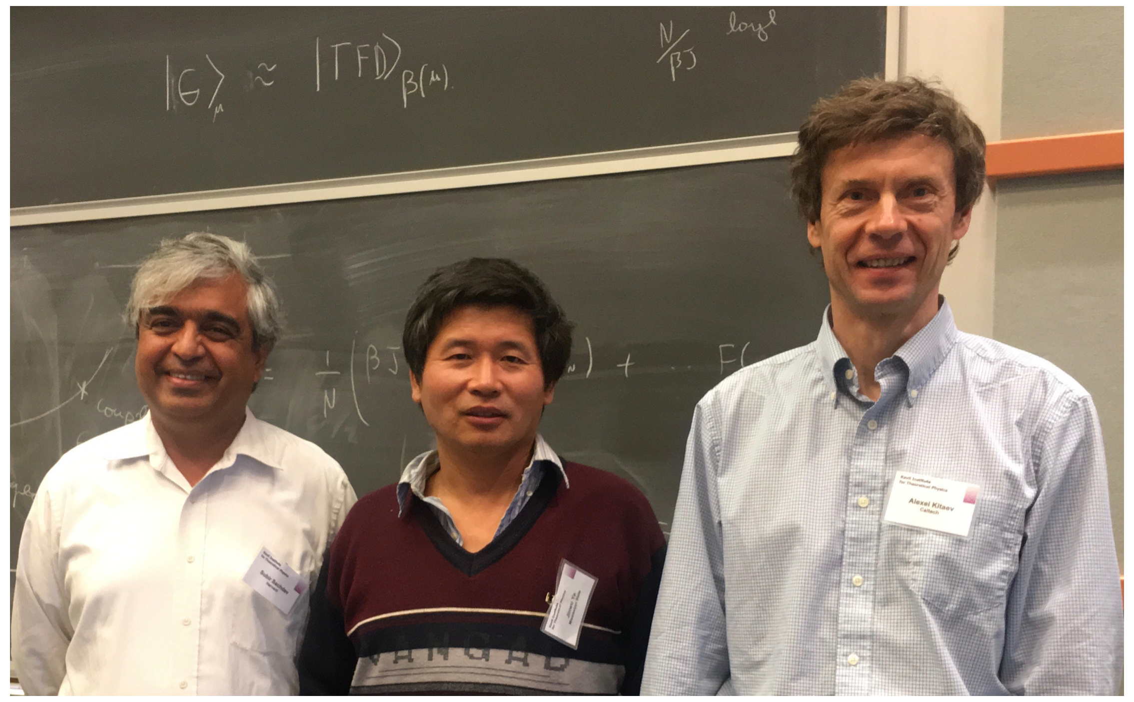

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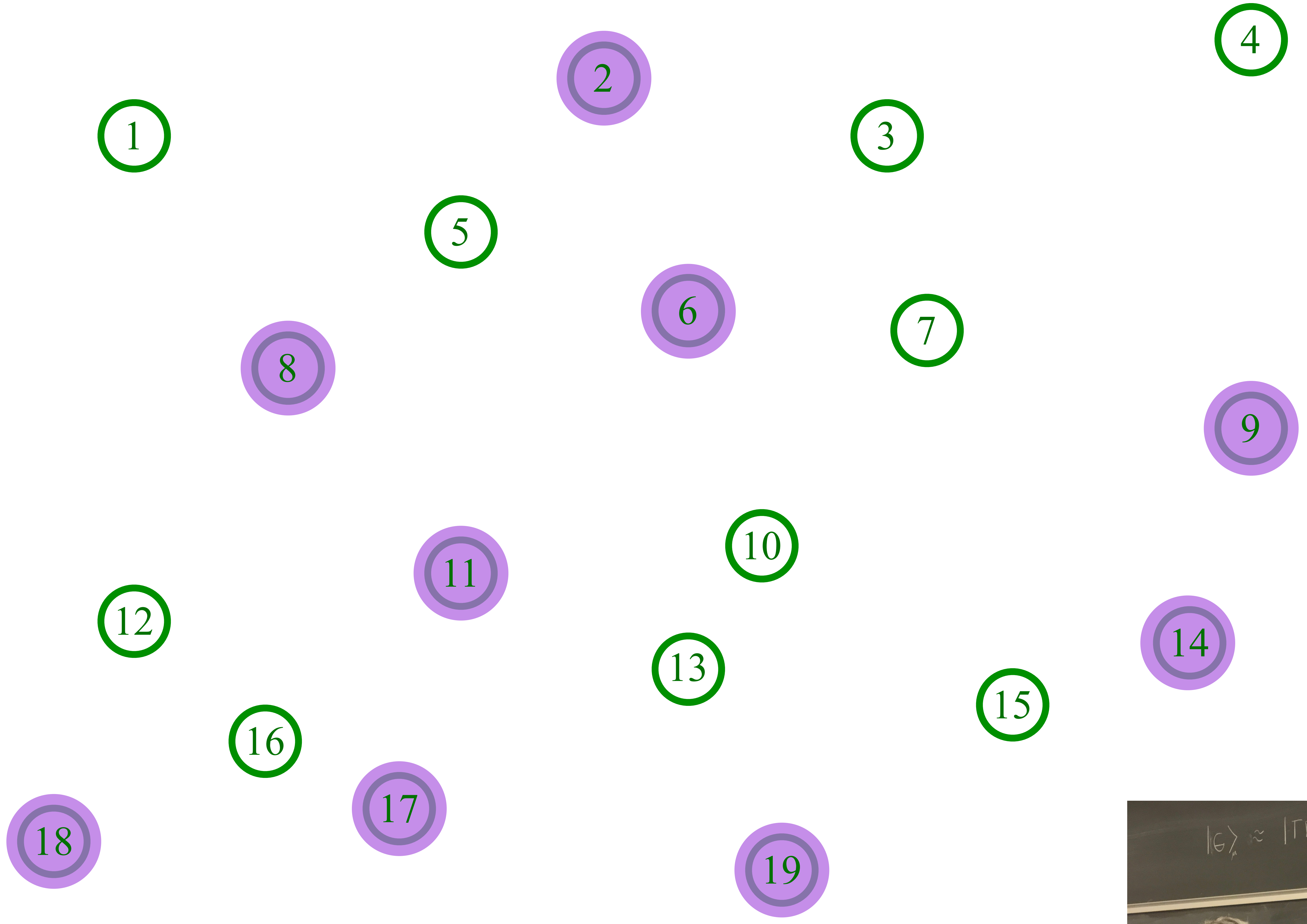

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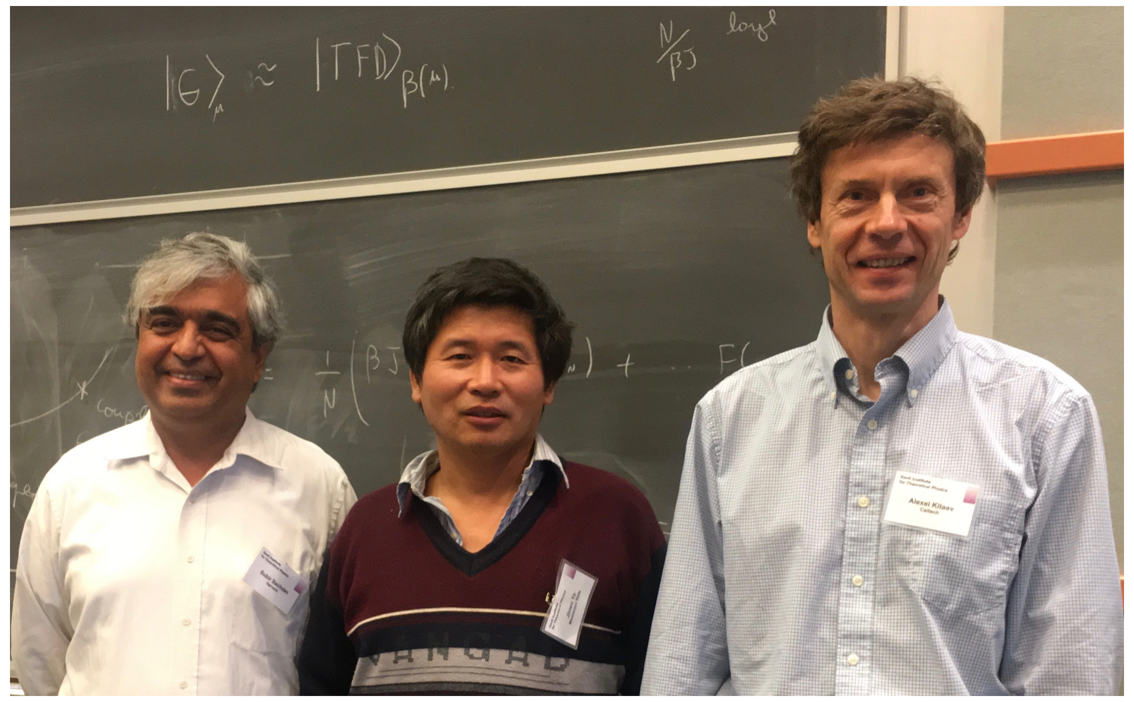

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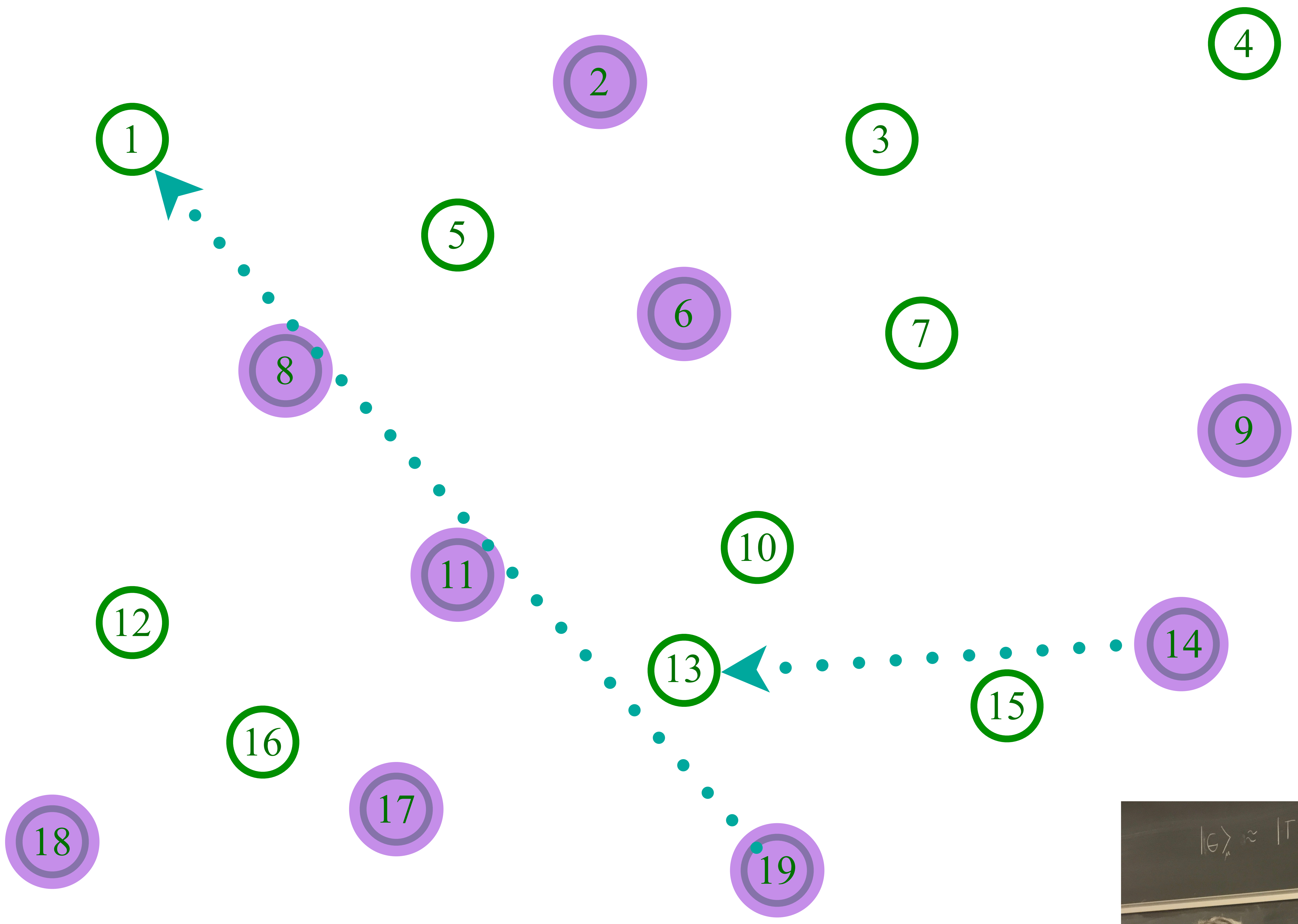
Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



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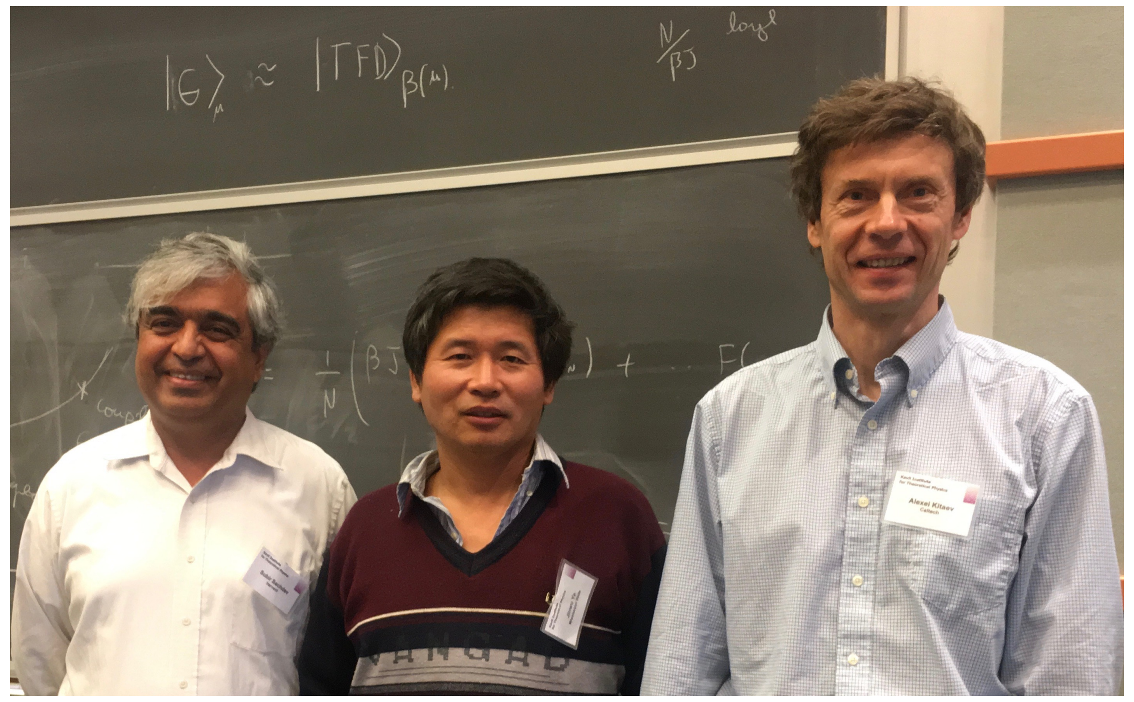

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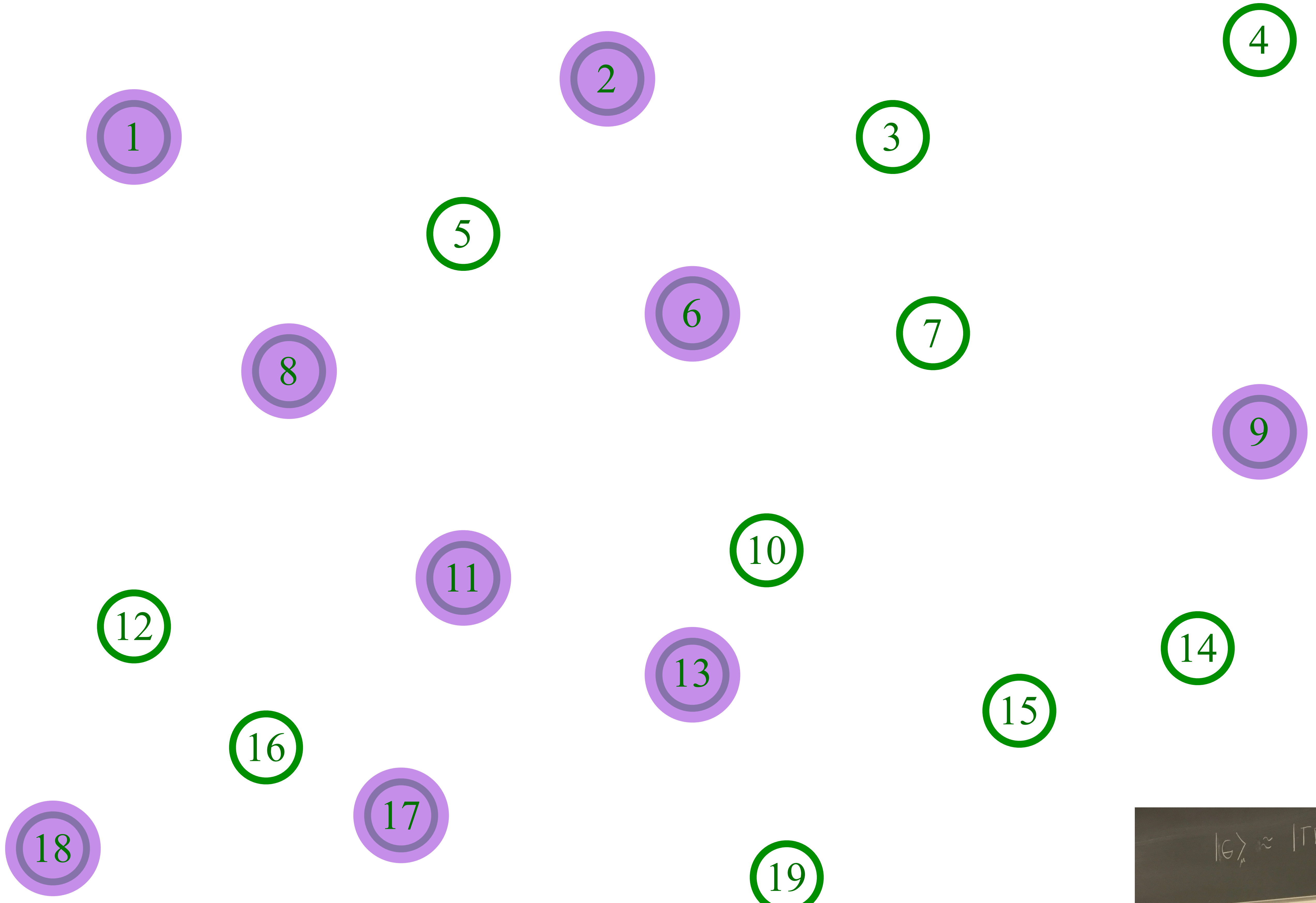

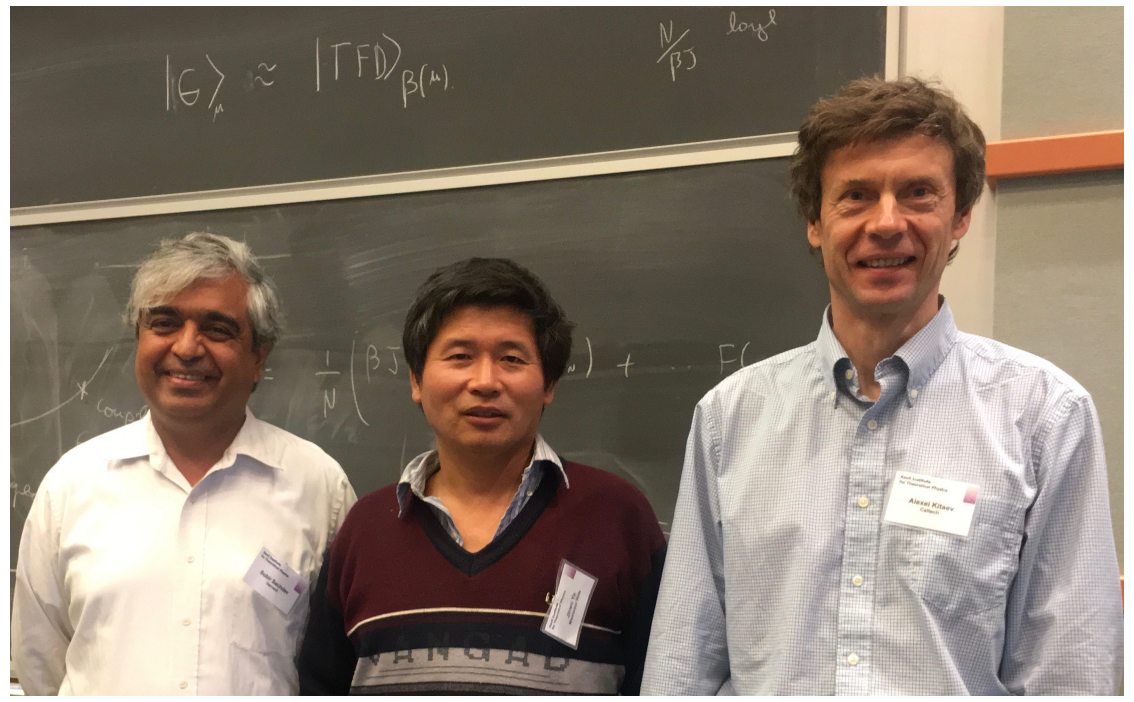

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The SYK model

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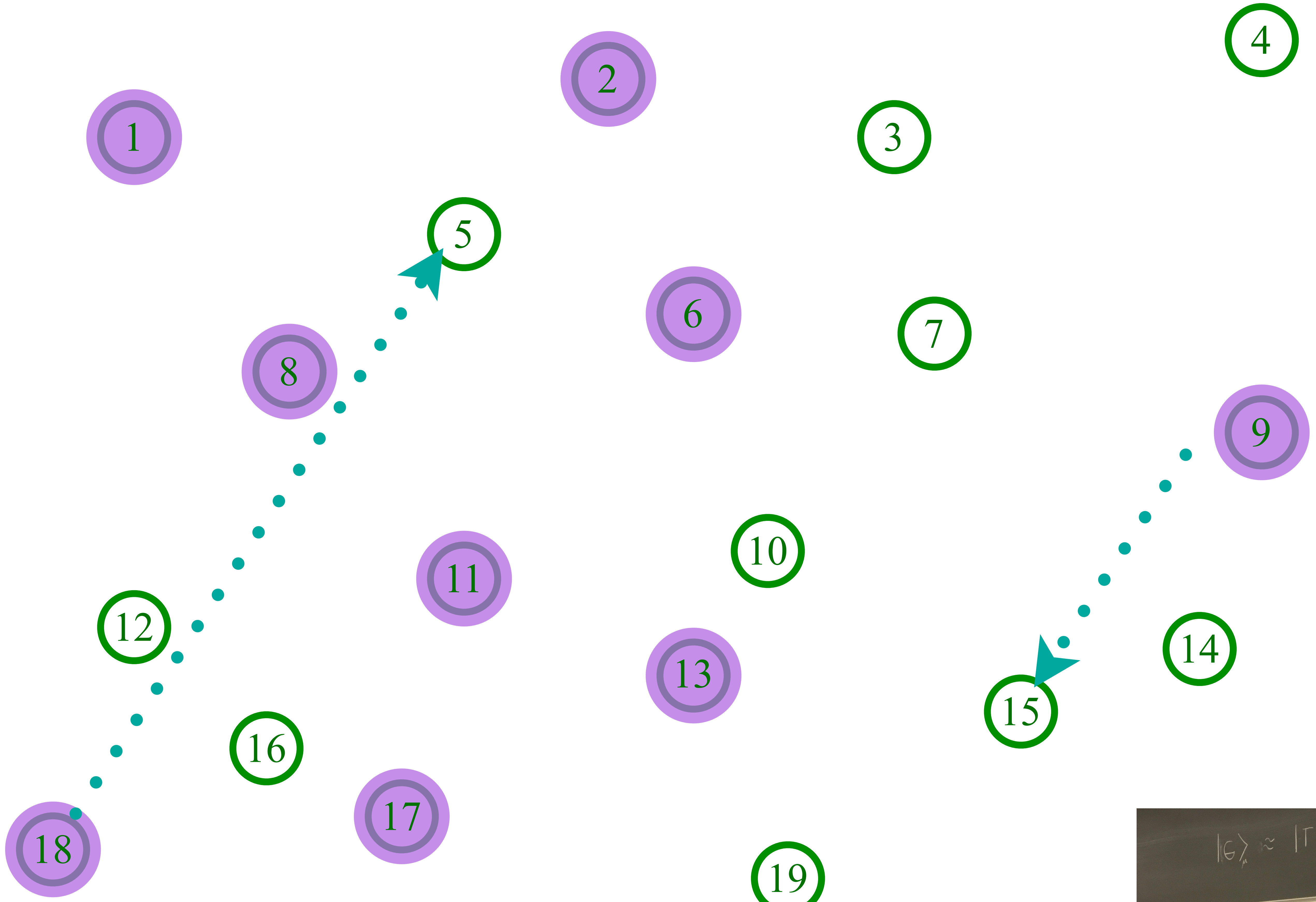
$$U_{9,18;5,15}$$



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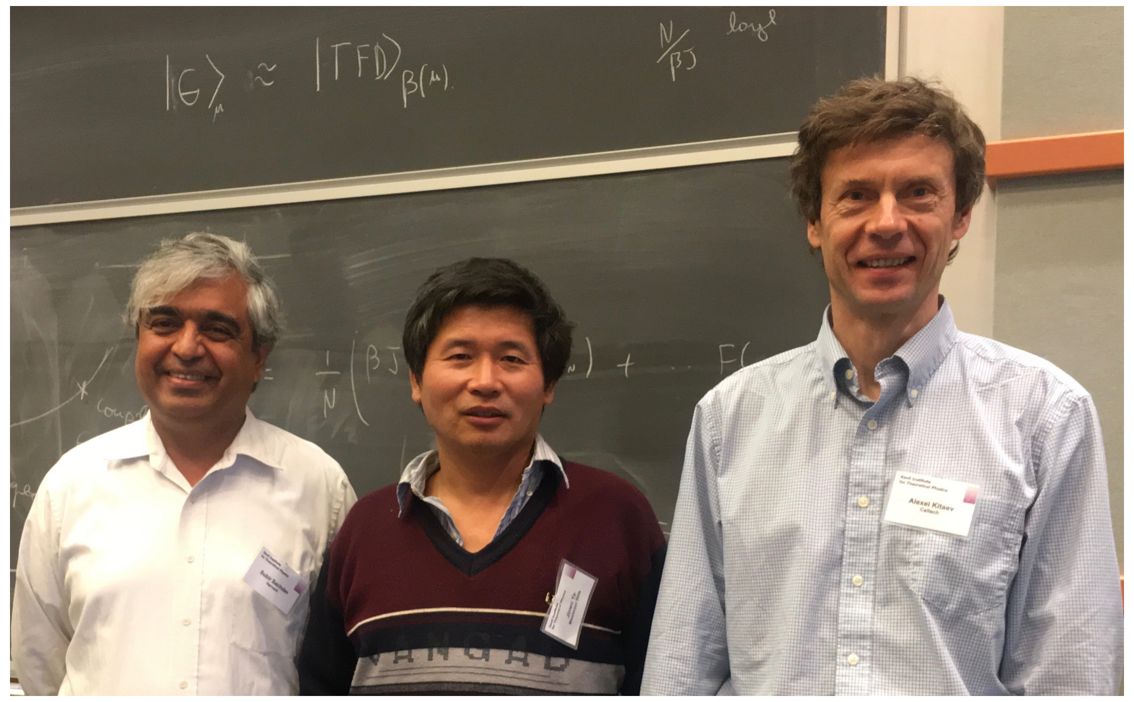

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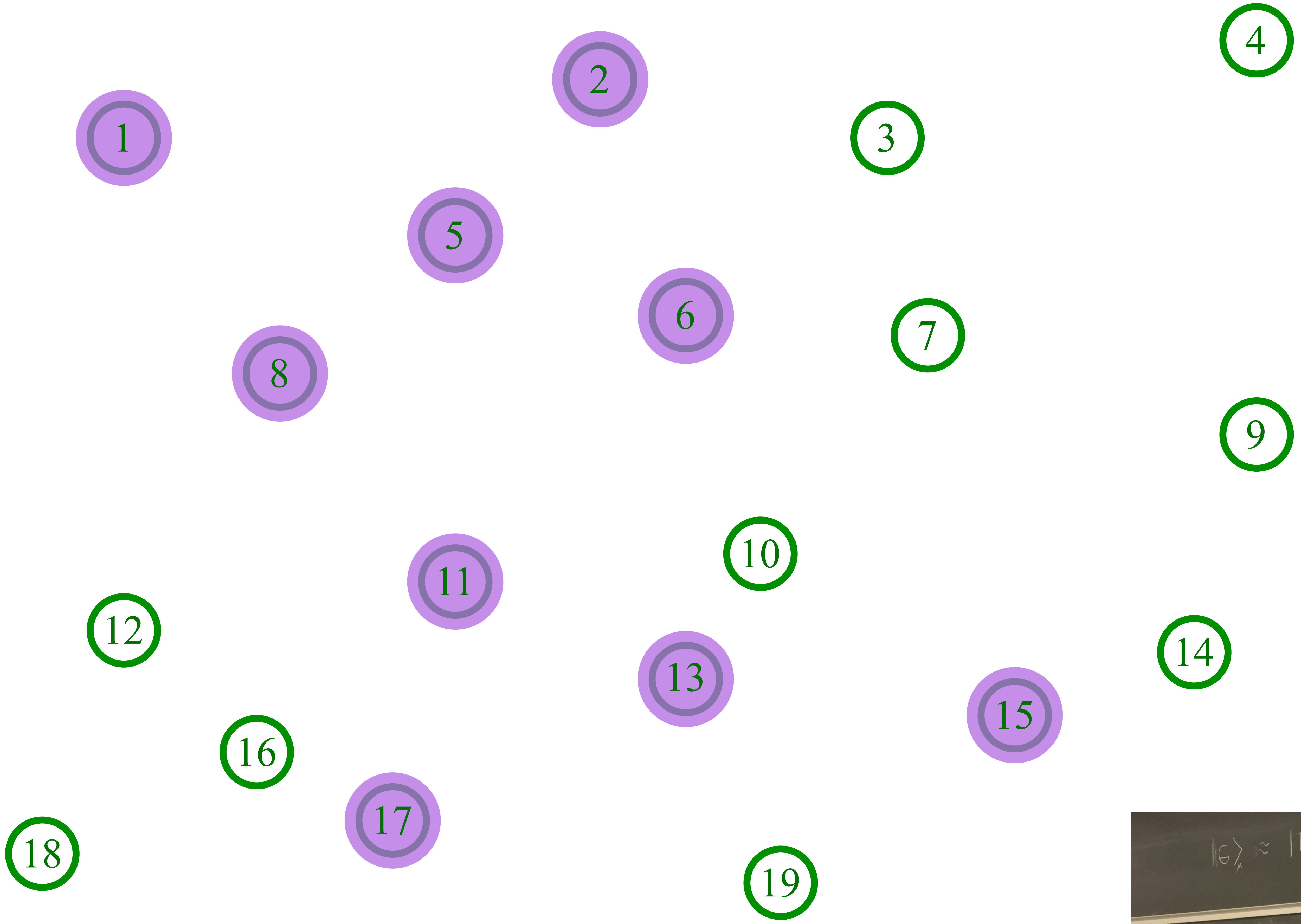
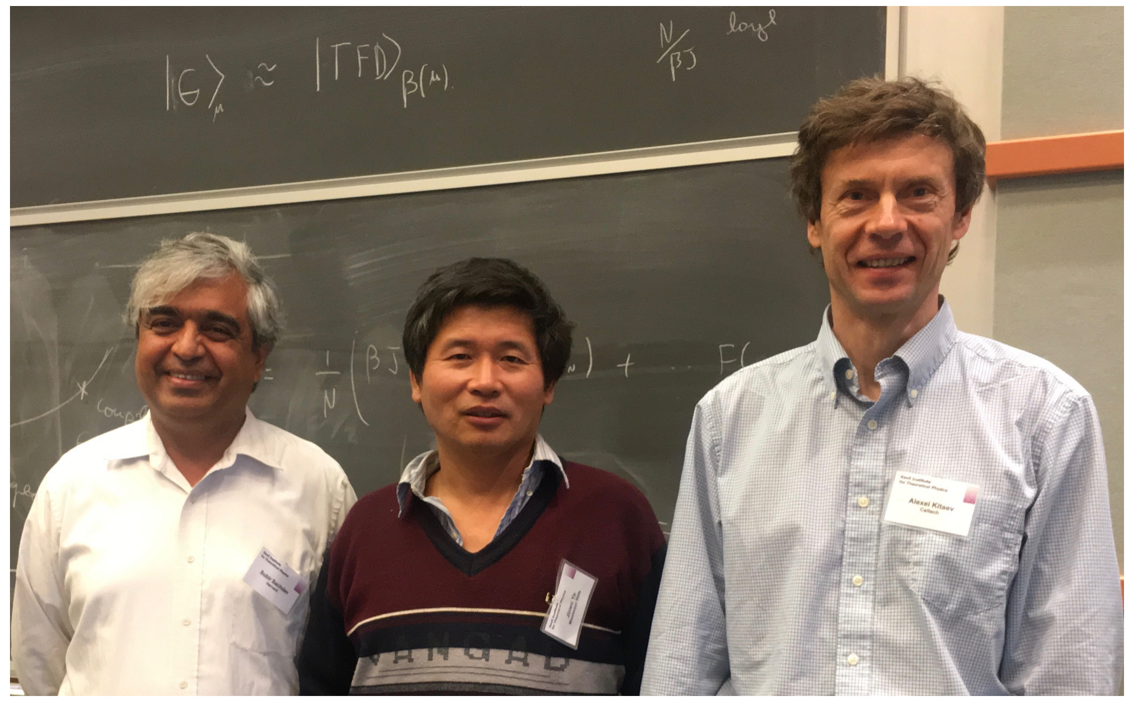


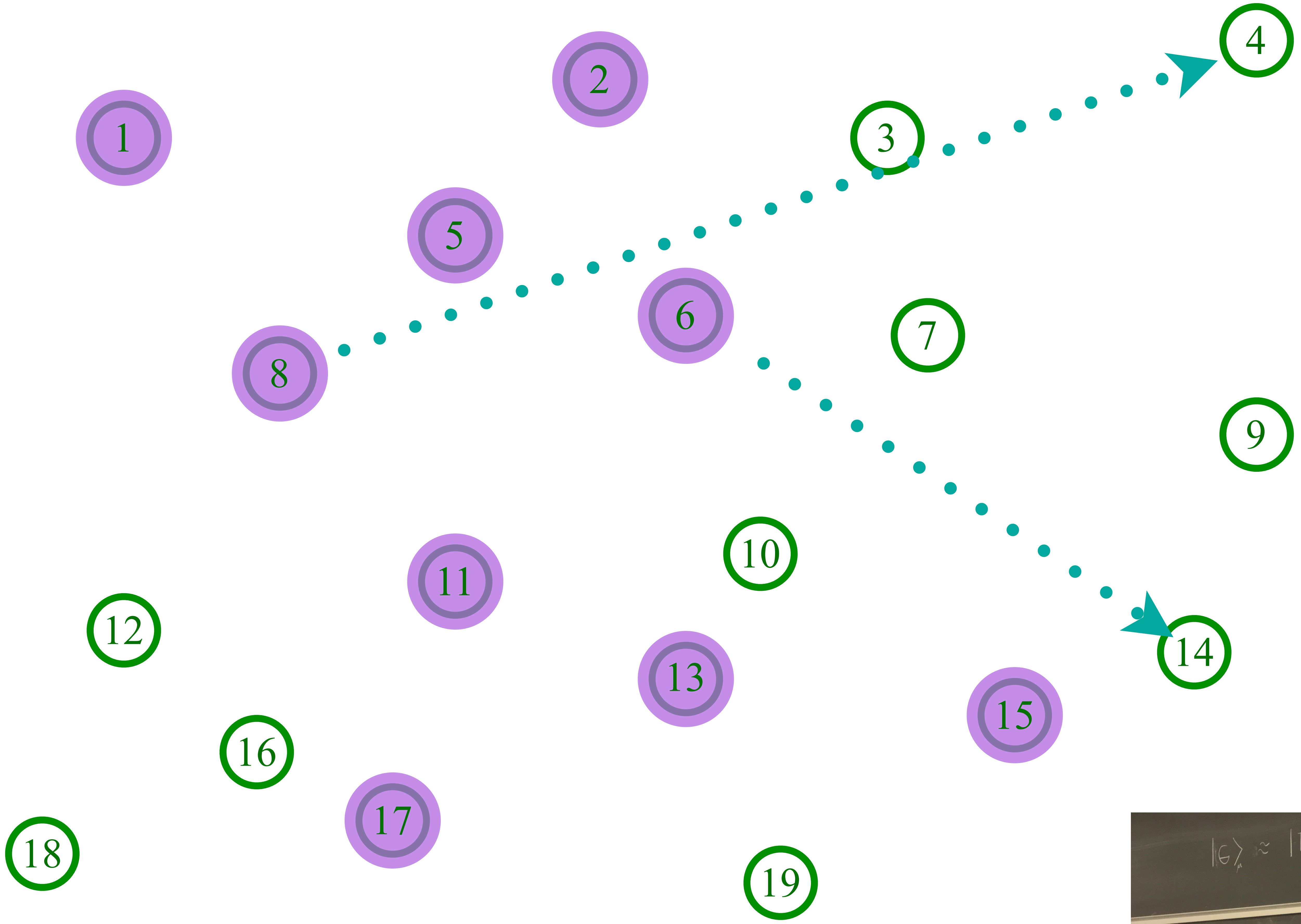


Entangle electrons pairwise randomly

The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



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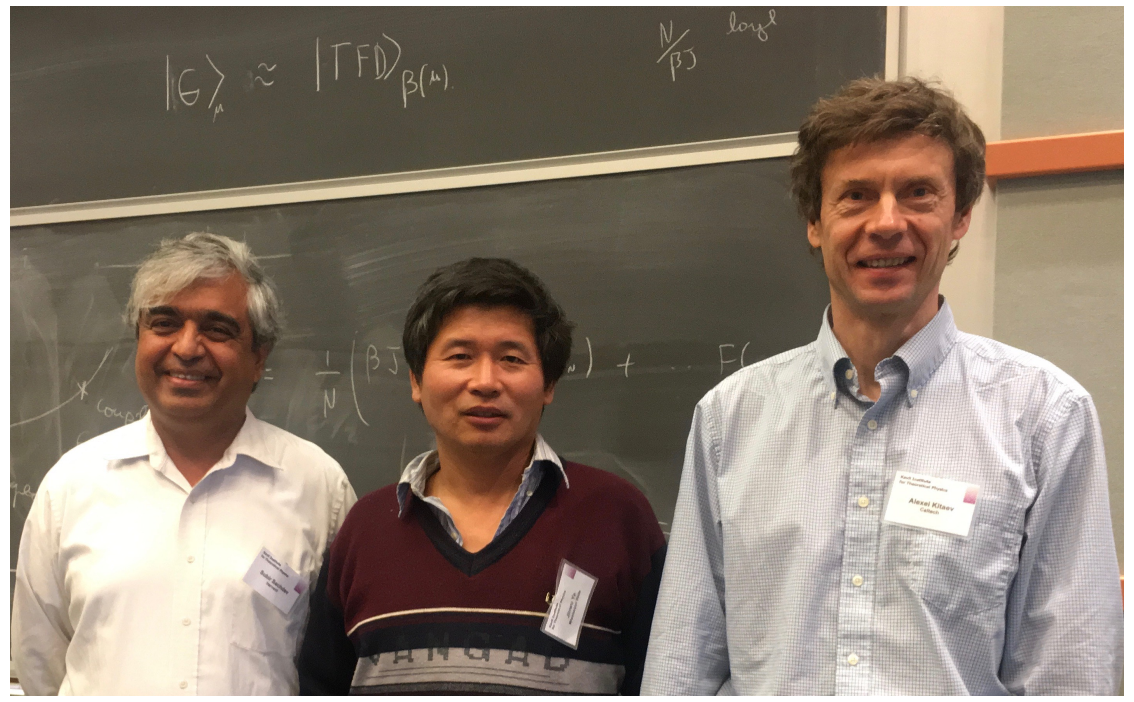

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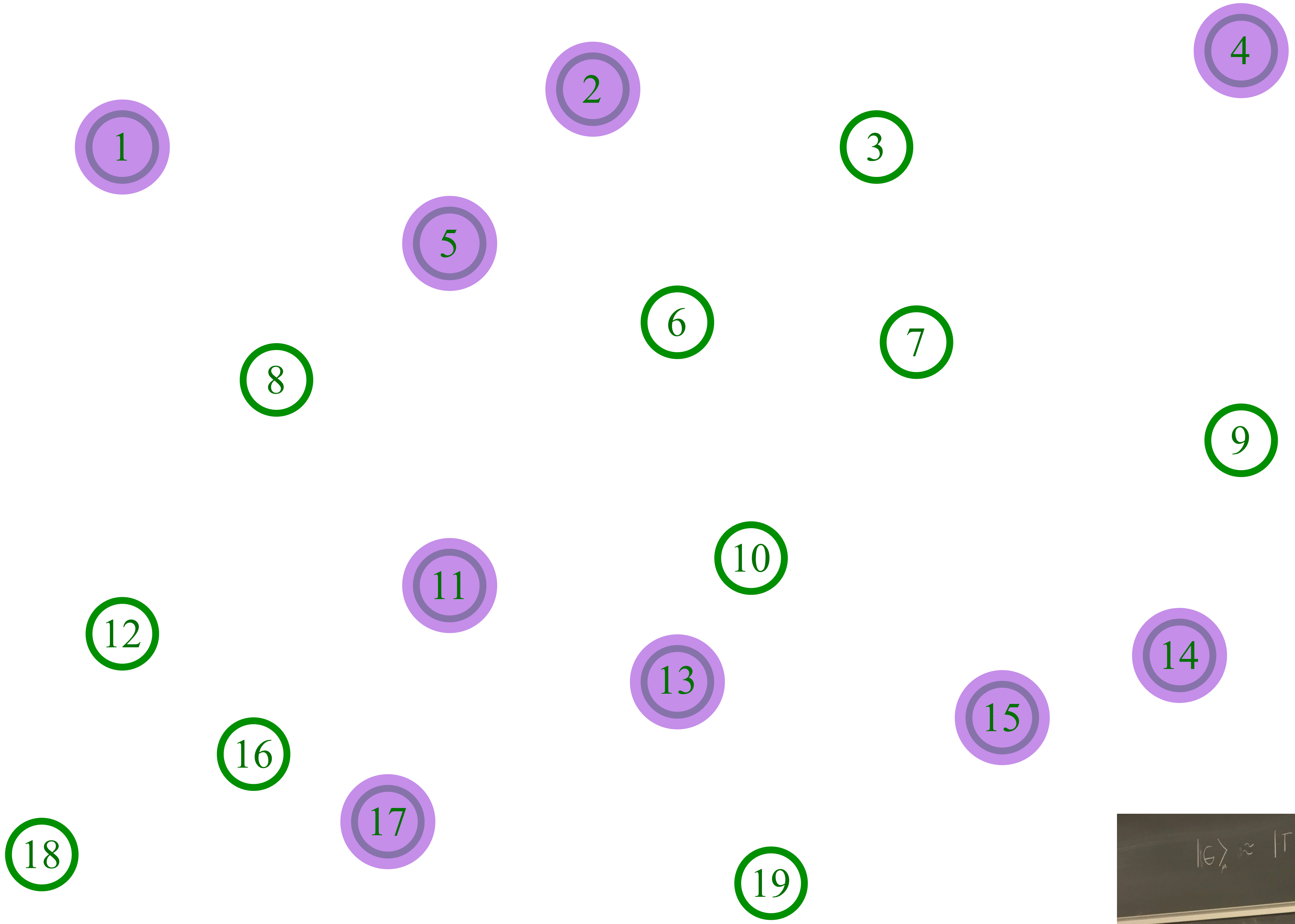

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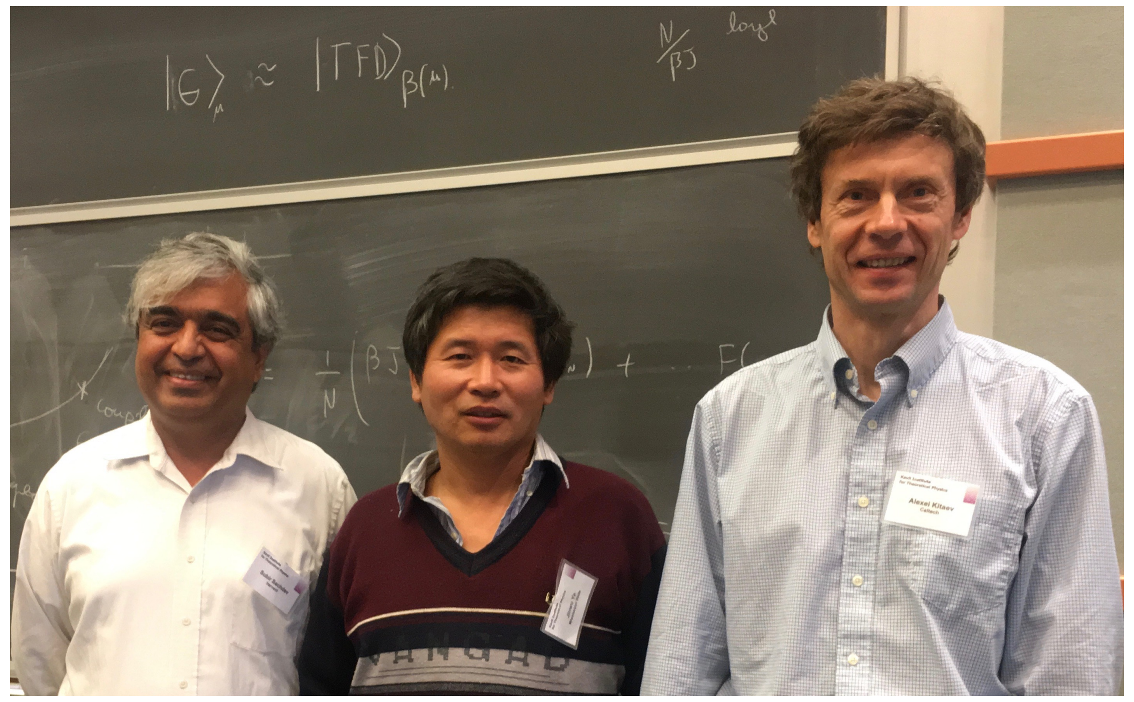


The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

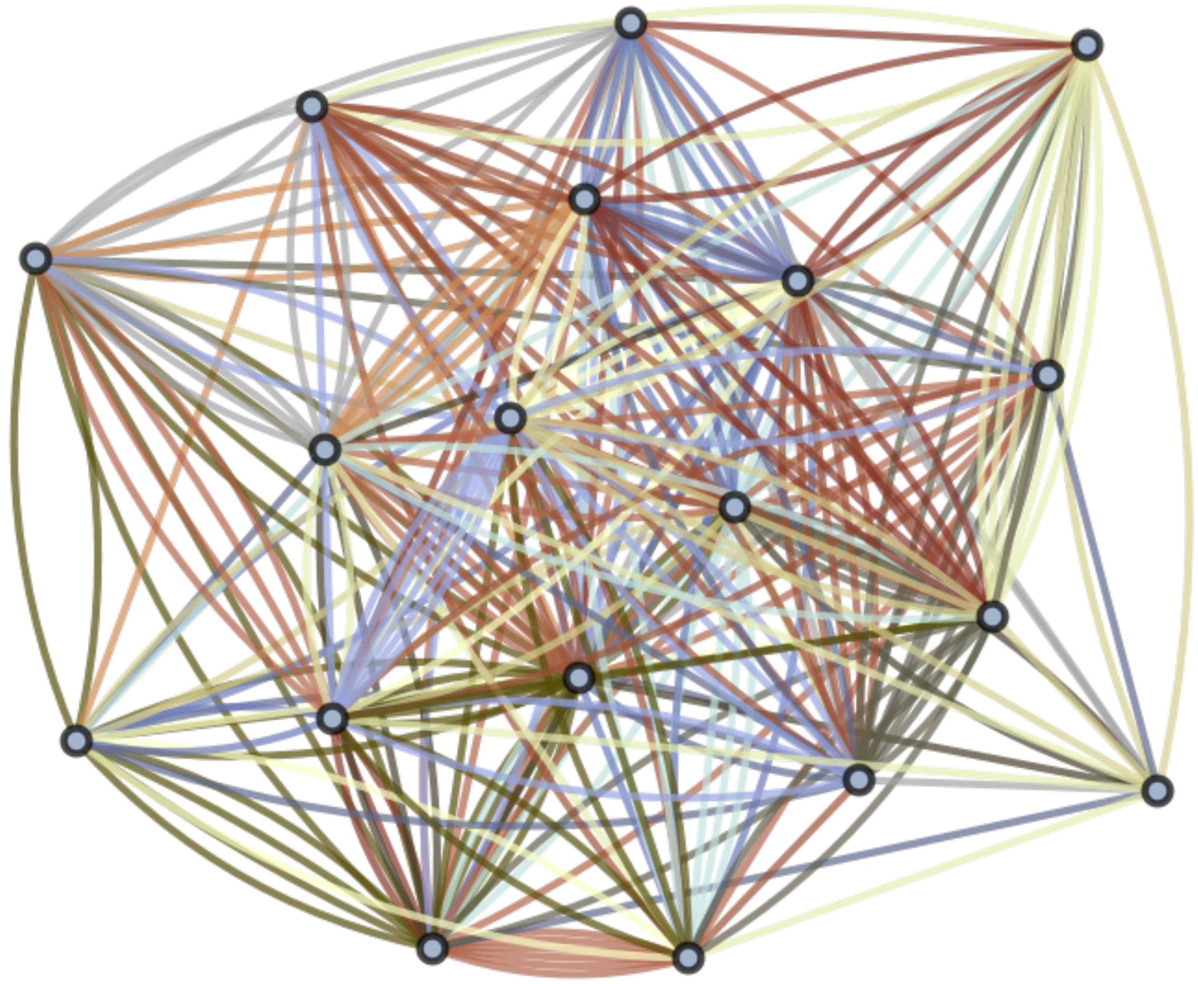
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

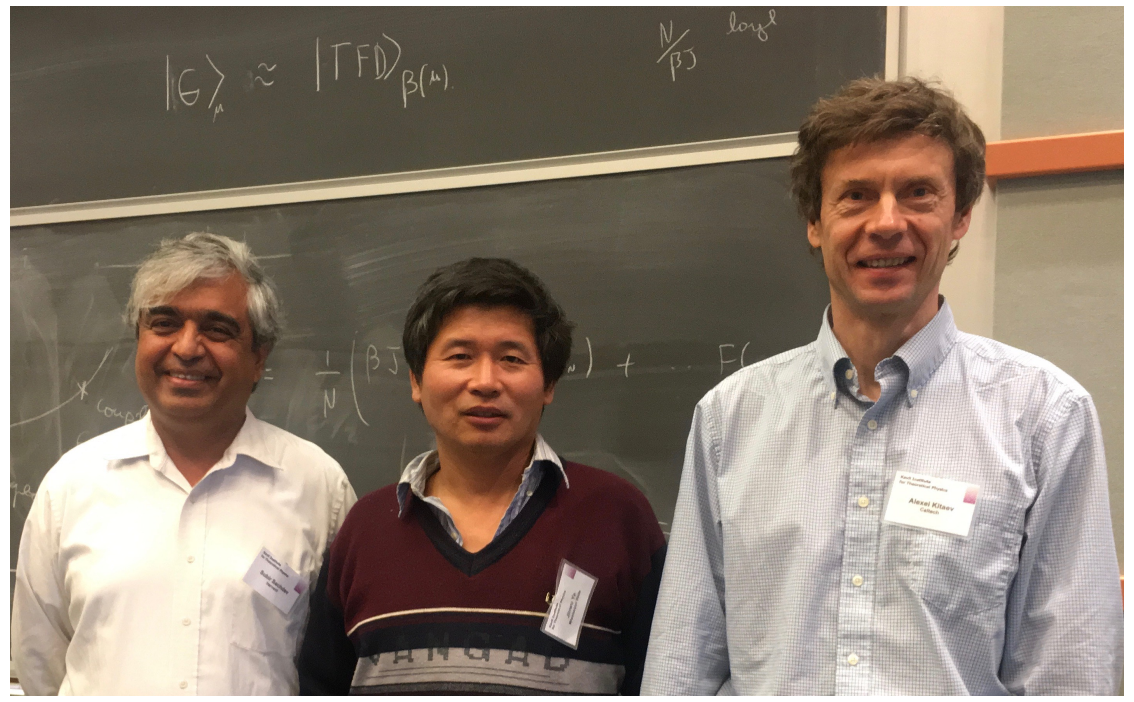
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



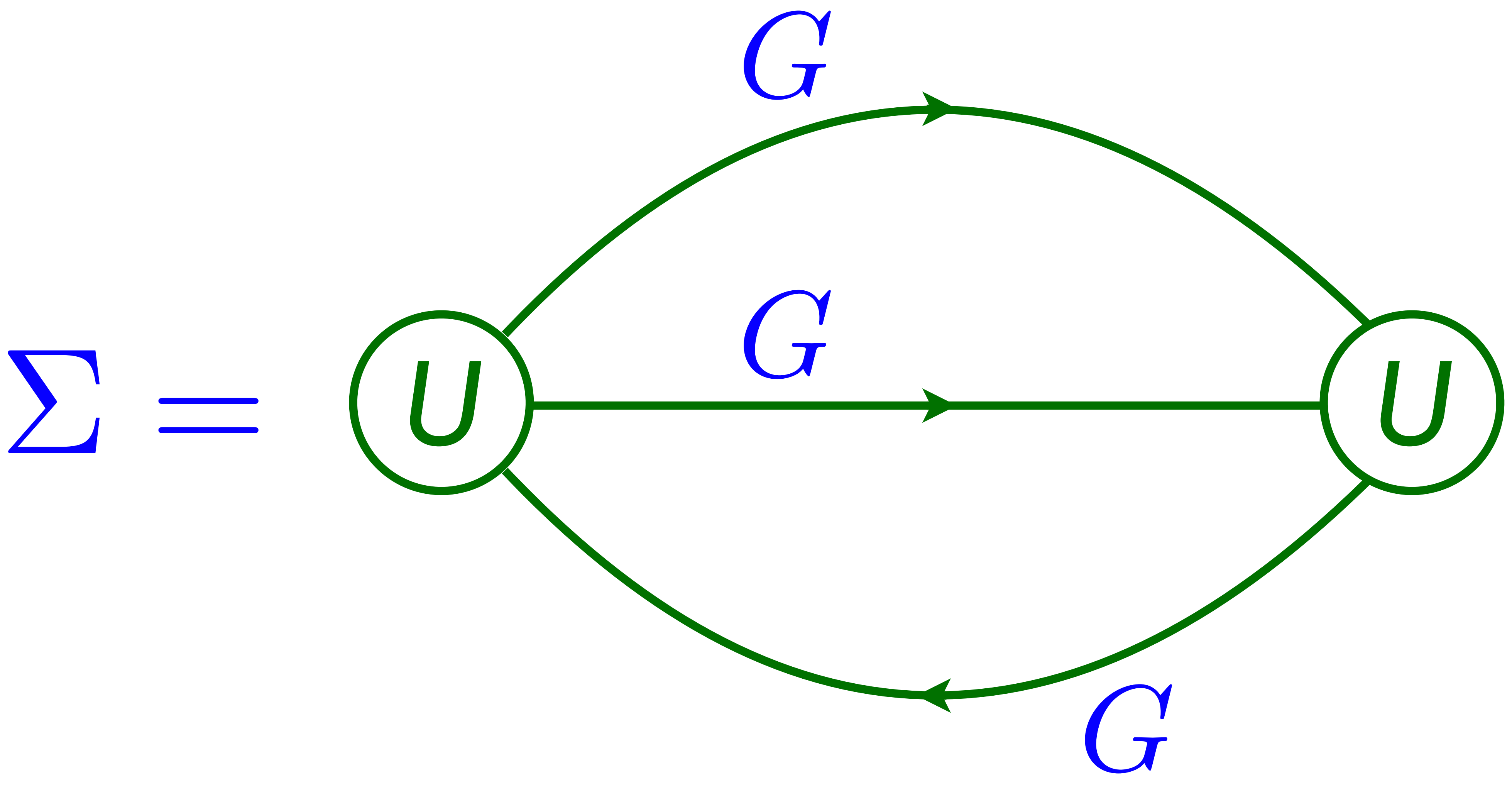


Complex SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$



Conformal solution at $\mu = 0$, $G(\tau) \sim \frac{\text{sgn}(\tau)}{\sqrt{|\tau|}}$.

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

The SYK model

- Planckian time dynamics without quasiparticles with a relaxation time $\sim \hbar/(k_B T)$ when $k_B T \ll U$.

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, review article

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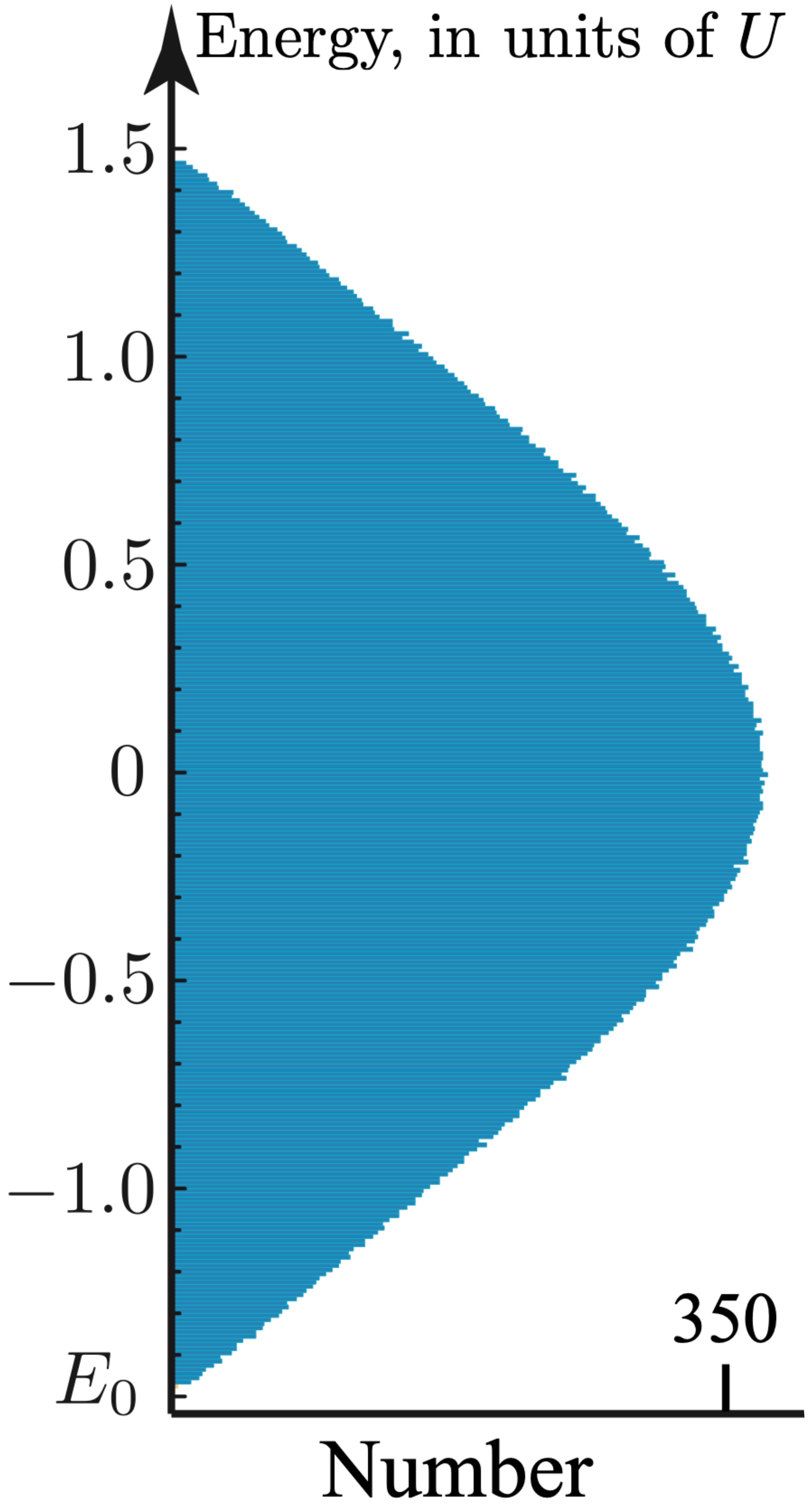
D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, review article

The SYK model

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



Many-body density of states

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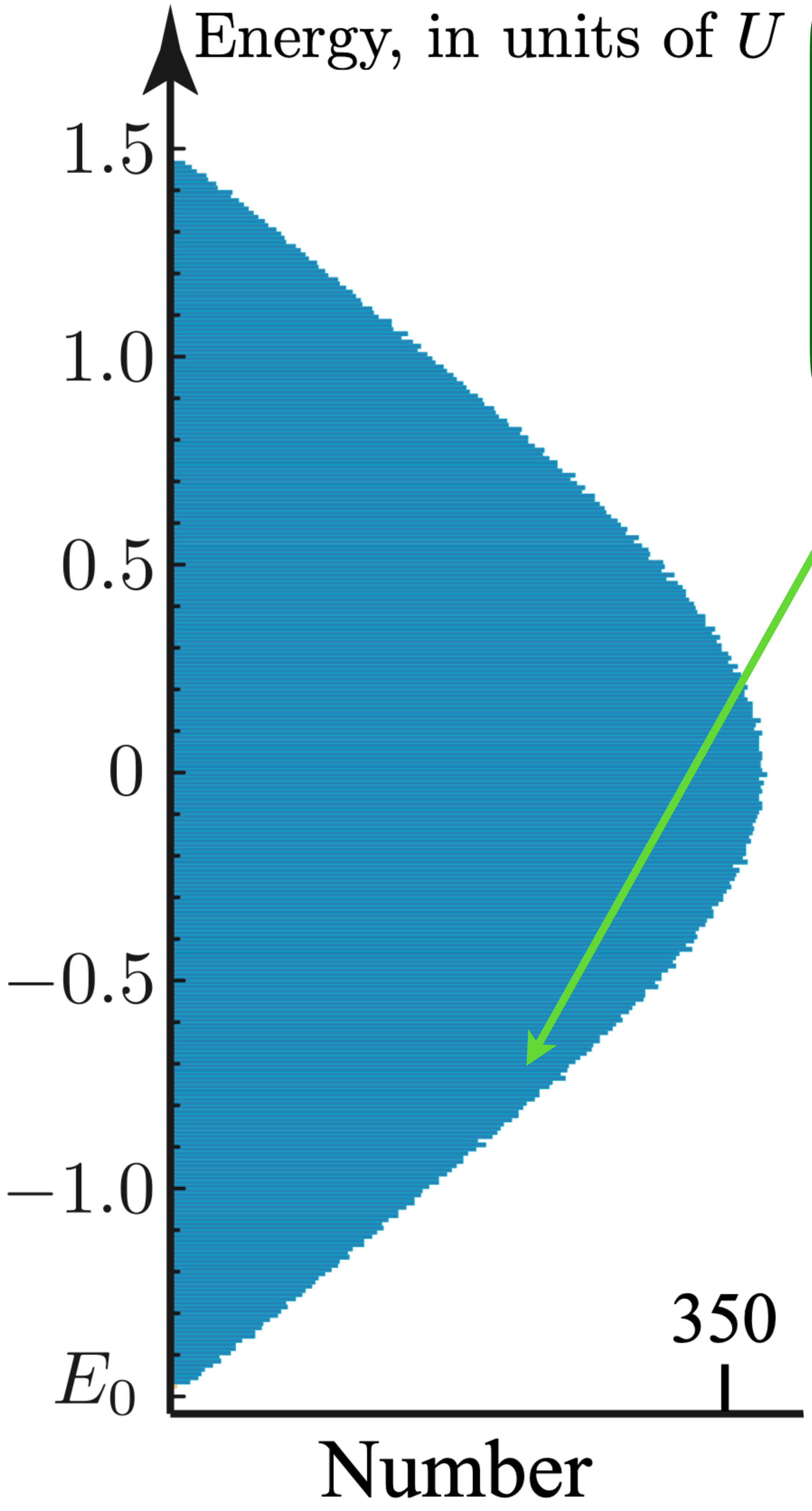

$$D(E) \sim e^{S(E)}$$

$$= e^{Ns_0 + \sqrt{2N\gamma E}}$$

$$S(T \to 0) = N(s_0 + \gamma T)$$

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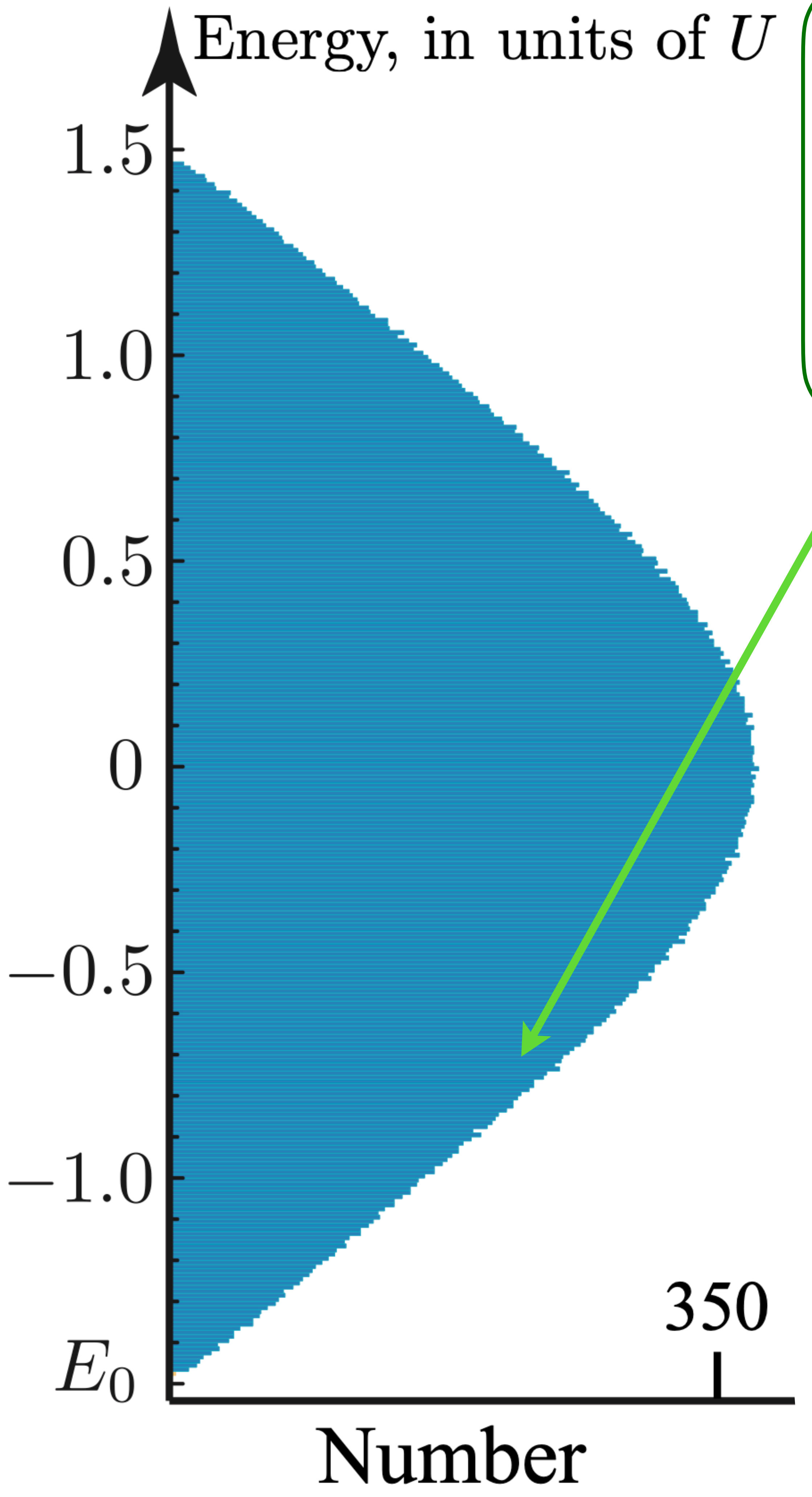

$$\begin{aligned} D(E) &\sim e^{S(E)} \\ &= e^{Ns_0 + \sqrt{2N\gamma E}} \end{aligned}$$

$$S(T \to 0) = N(\boxed{s_0} + \gamma T)$$

$$s_0 = 0.464848\ldots$$

A. Georges, O. Parcollet, and
S. Sachdev,
PRB **63**, 134406 (2001)

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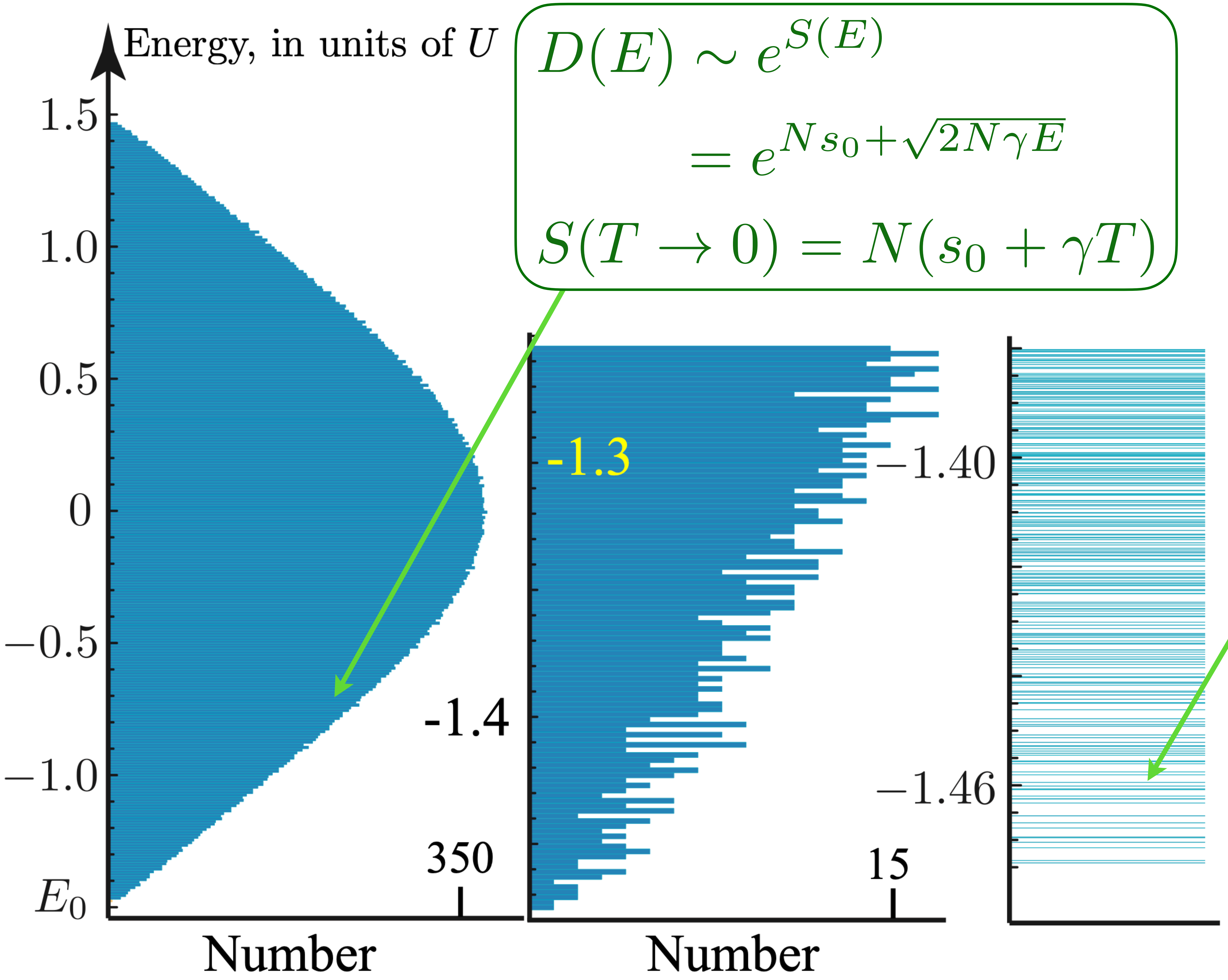

$$D(E) \sim e^{S(E)}$$
$$= e^{Ns_0 + \sqrt{2N\gamma E}}$$
$$S(T \to 0) = N(s_0 + \gamma T)$$

$$D(E) \sim 2\, e^{Ns_0} \sqrt{2N\gamma E}$$

No particle-like decomposition: wavefunctions change chaotically from one state to the next.

$$s_0 = 0.464848\ldots$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Maldacena, Stanford (2016); Bagrets *et al.* (2017); Cotler *et al.* (2017); Kitaev, Suh (2017); Stanford, Witten (2017)

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- The $D(E)$ is determined by a time-reparameterization $\tau \to f(\tau)$ mode (similar to the graviton being fluctuations of the spacetime metric), and a phase mode $\phi(\tau)$:

$$\mathcal{Z}_{SYK} = e^{N s_0} \int \mathcal{D} f \mathcal{D}\phi \exp\left(-\frac{1}{\hbar} \int_0^{\hbar/(k_B T)} d\tau \, \mathcal{L}_{SYK}[f, \phi] \right)$$

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, review article

From the SYK model to charged black holes

Quantum Black holes

Bohr-Sommerfeld semiclassical quantum theory of a black hole in d spatial dimensions

$$\mathcal{Z} = \int \mathcal{D}g_{\mu\nu}\mathcal{D}a_\mu \exp\left(-\frac{1}{\hbar}\int d^d x \int_0^{\hbar/(k_B T)} d\tau \sqrt{g}\,\mathcal{L}_d[g_{\mu\nu}, a_\mu]\right)$$

$g_{\mu\nu} \Rightarrow$ spacetime metric, $g = \det(g_{\mu\nu})$

$a_\mu \Rightarrow$ Electromagnetic gauge field

$\mathcal{L}_d \Rightarrow$ *Classical* Einstein-Maxwell action

$$\mathcal{U}(t) = \exp\left(-i\mathcal{H}t/\hbar\right) \quad \Leftrightarrow \quad \mathcal{Z} = \mathrm{Tr}\exp\left(-\mathcal{H}/(k_B T)\right)$$

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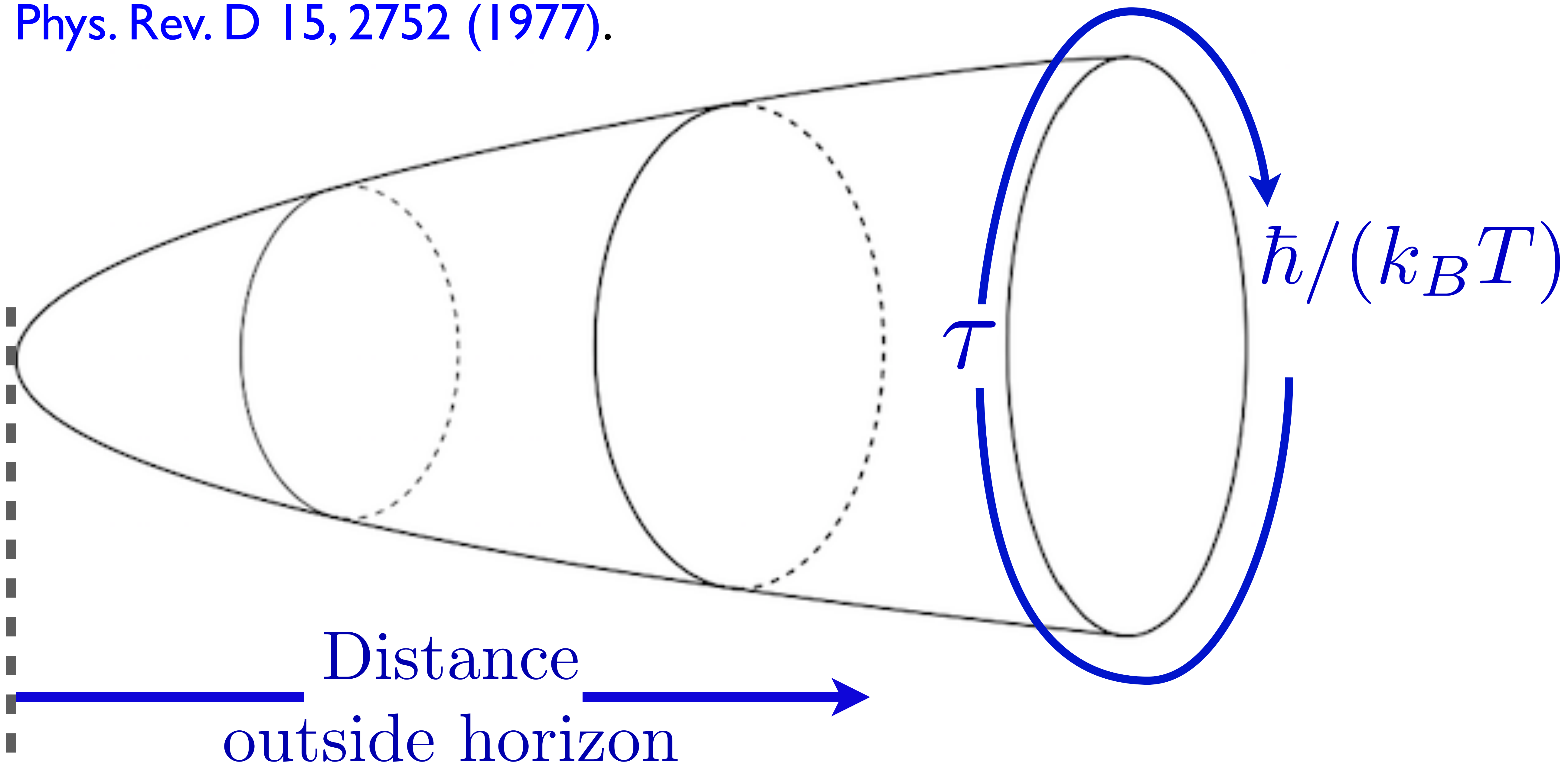
G.W. Gibbons and S.W. Hawking, Phys. Rev. D 15, 2752 (1977).



Spacetime geometry of a black hole in imaginary time τ

Evaluate path integral at black hole saddle point

Reissner-Nordstrom black hole of Einstein-Maxwell theory

Horizon

$\mathrm{AdS}_2 \times S^2$

1+1 dimensional spacetime

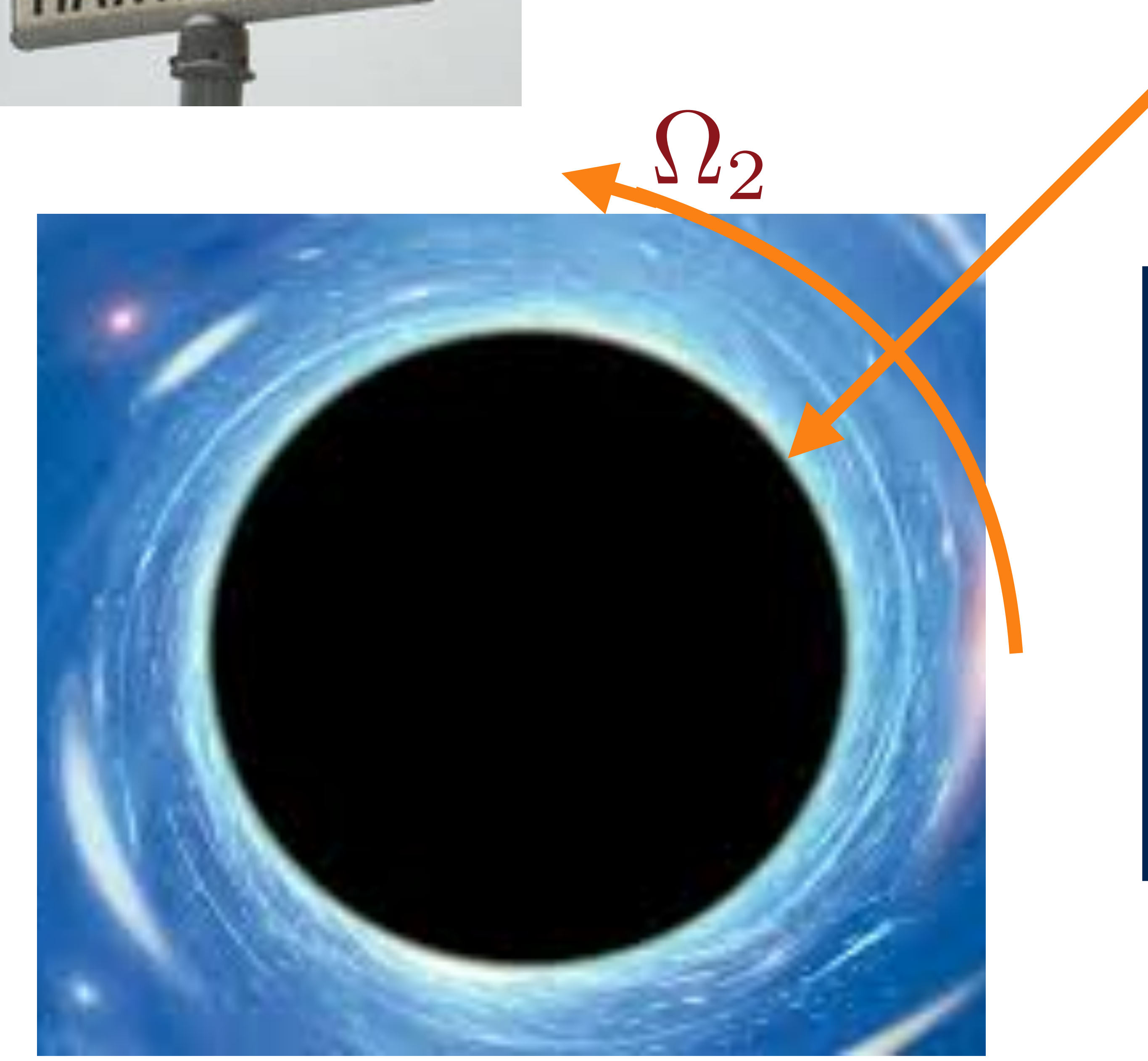
Boundary graviton

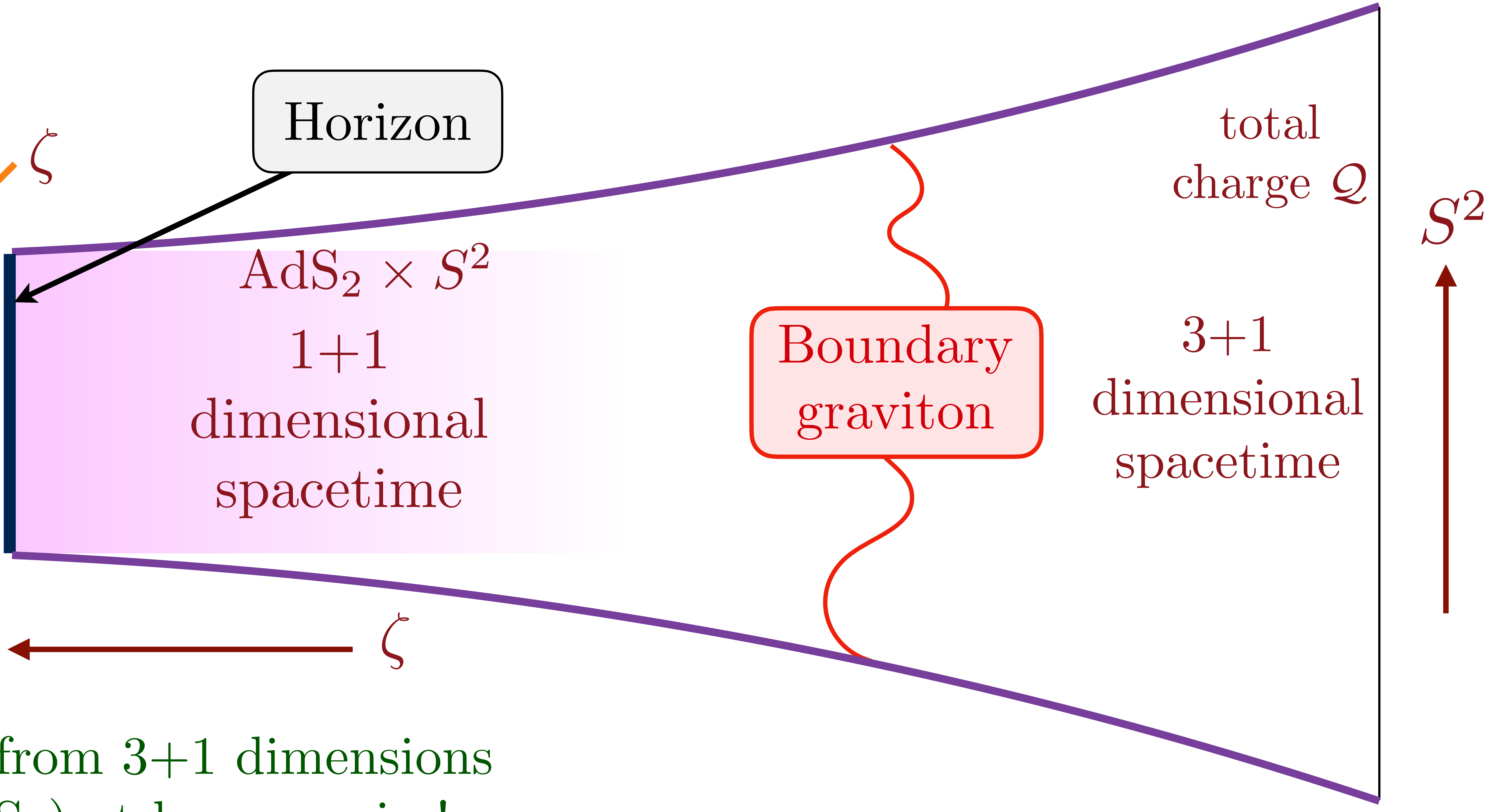
total charge $\mathcal{Q}$

3+1 dimensional spacetime

Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

Reissner-Nordstrom black hole of Einstein-Maxwell theory





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$\mathrm{AdS}_2 \times S^2$

1+1 dimensional spacetime

Boundary graviton

total charge $\mathcal{Q}$

3+1 dimensional spacetime

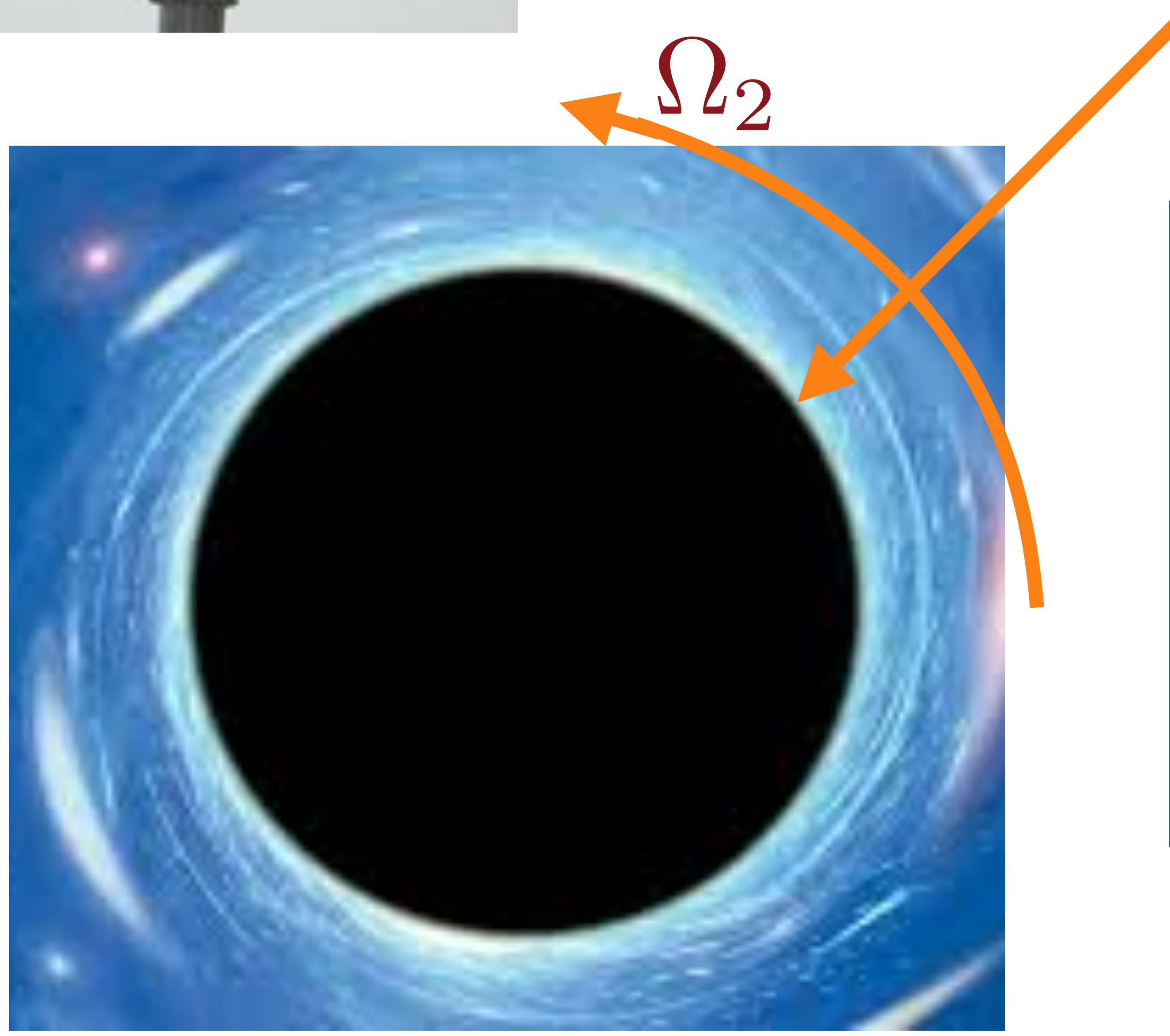
S^2

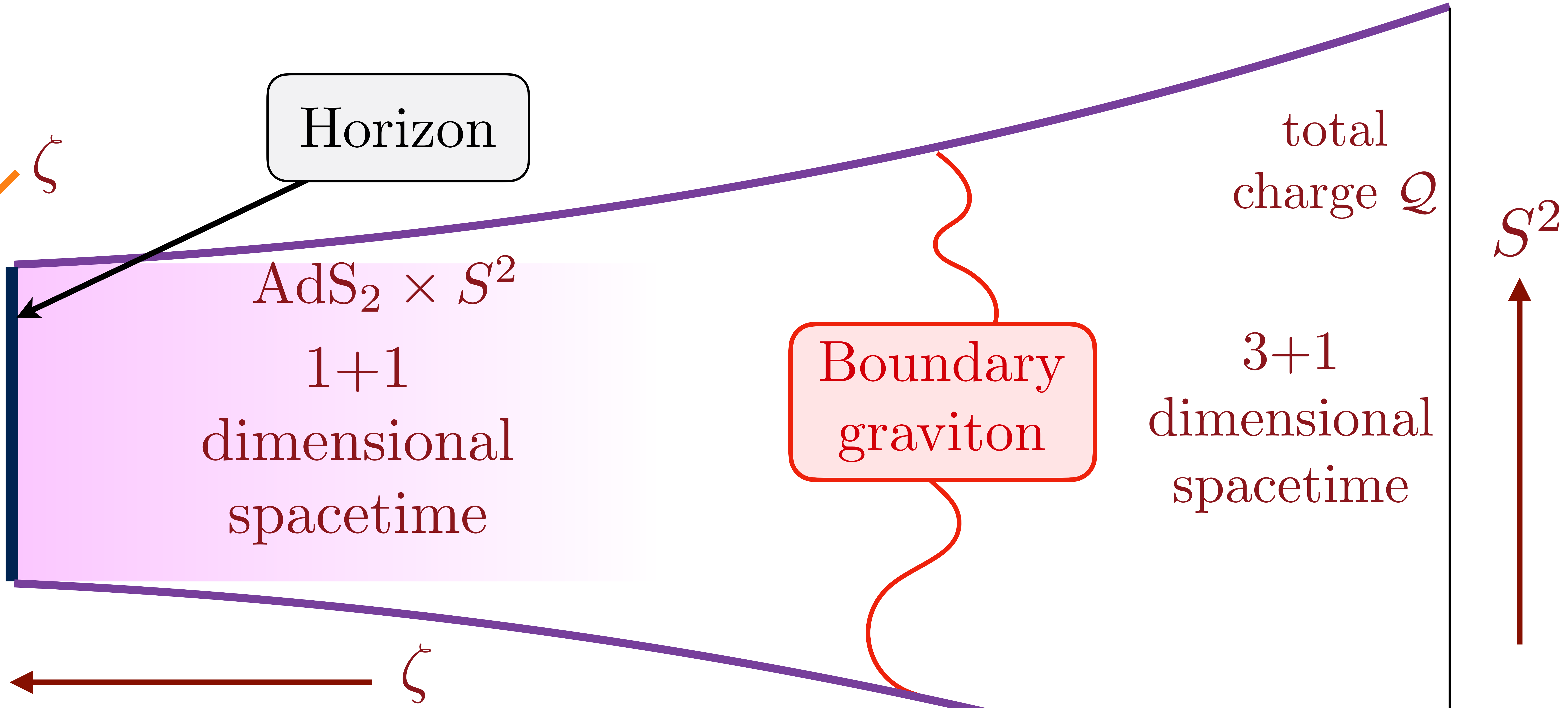
ζ

Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

Is there a mapping to a quantum system with Planckian dynamics in 0+1 dimensions?

Reissner-Nordstrom black hole of Einstein-Maxwell theory





Horizon

$\mathrm{AdS}_2 \times S^2$

1+1 dimensional spacetime

Boundary graviton

total charge $\mathcal{Q}$

3+1 dimensional spacetime

S^2

ζ

Yes!

The low energy theory of a charged black hole is *exactly* the low energy theory of time reparameterizations of the SYK model.

Quantum theory of charged black holes

The near-horizon 1+1 dimensional theory of a charged black hole

$$\mathcal{Z}_{\mathcal{Q}} = e^{S_H(0)} \int \mathcal{D}g_{\mu\nu}\mathcal{D}a_\mu \exp\left(-\frac{1}{\hbar} \int d\zeta \int_0^{\hbar/(k_B T)} d\tau \sqrt{g}\, \mathcal{L}_{JT}[g_{\mu\nu}, a_\mu] \right)$$

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$$\Updownarrow$$

$$\mathcal{Z}_{SYK} = e^{N s_0} \int \mathcal{D}f\mathcal{D}\phi \exp\left(-\frac{1}{\hbar}\int_0^{\hbar/(k_B T)} d\tau\, \mathcal{L}_{SYK}[f, \phi]\right)$$

after relating the boundary component of $g_{\mu\nu}$ to f,
and the boundary value of a_τ to ϕ.

Sachdev (2010); Kitaev (2015); Sachdev (2015); Maldacena, Stanford, Yang (2016);
Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019; Iliesiu, Turiaci (2020)

Quantum theory of charged black holes

The near-horizon 1+1 dimensional theory of a charged black hole

$$\mathcal{Z}_{\mathcal{Q}} = e^{S_H(0)} \int \mathcal{D}g_{\mu\nu}\mathcal{D}a_\mu \exp\left(-\frac{1}{\hbar}\int d\zeta \int_0^{\hbar/(k_B T)} d\tau \sqrt{g}\, \mathcal{L}_{JT}[g_{\mu\nu}, a_\mu]\right)$$

$$\Updownarrow$$

$$\mathcal{Z}_{SYK} = e^{N s_0} \int \mathcal{D}f\mathcal{D}\phi \exp\left(-\frac{1}{\hbar}\int_0^{\hbar/(k_B T)} d\tau\, \mathcal{L}_{SYK}[f, \phi]\right)$$

after relating the boundary component of $g_{\mu\nu}$ to f,
and the boundary value of a_τ to ϕ.

$$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{A_0 c^3}{4} + \frac{\sqrt{\pi} A_0^{3/2} c^2}{2}\frac{k_B T}{\hbar}\right) \longleftarrow \boxed{\text{Gibbons Hawking}}$$

where A_0 is the area of the black hole horizon at $T = 0$.

Quantum theory of charged black holes

The near-horizon 1+1 dimensional theory of a charged black hole

$$\mathcal{Z}_{\mathcal{Q}} = e^{S_H(0)} \int \mathcal{D}g_{\mu\nu}\mathcal{D}a_\mu \exp\left(-\frac{1}{\hbar}\int d\zeta \int_0^{\hbar/(k_BT)} d\tau \sqrt{g}\,\mathcal{L}_{JT}[g_{\mu\nu},a_\mu]\right)$$

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after relating the boundary component of $g_{\mu\nu}$ to f,
and the boundary value of a_τ to ϕ.

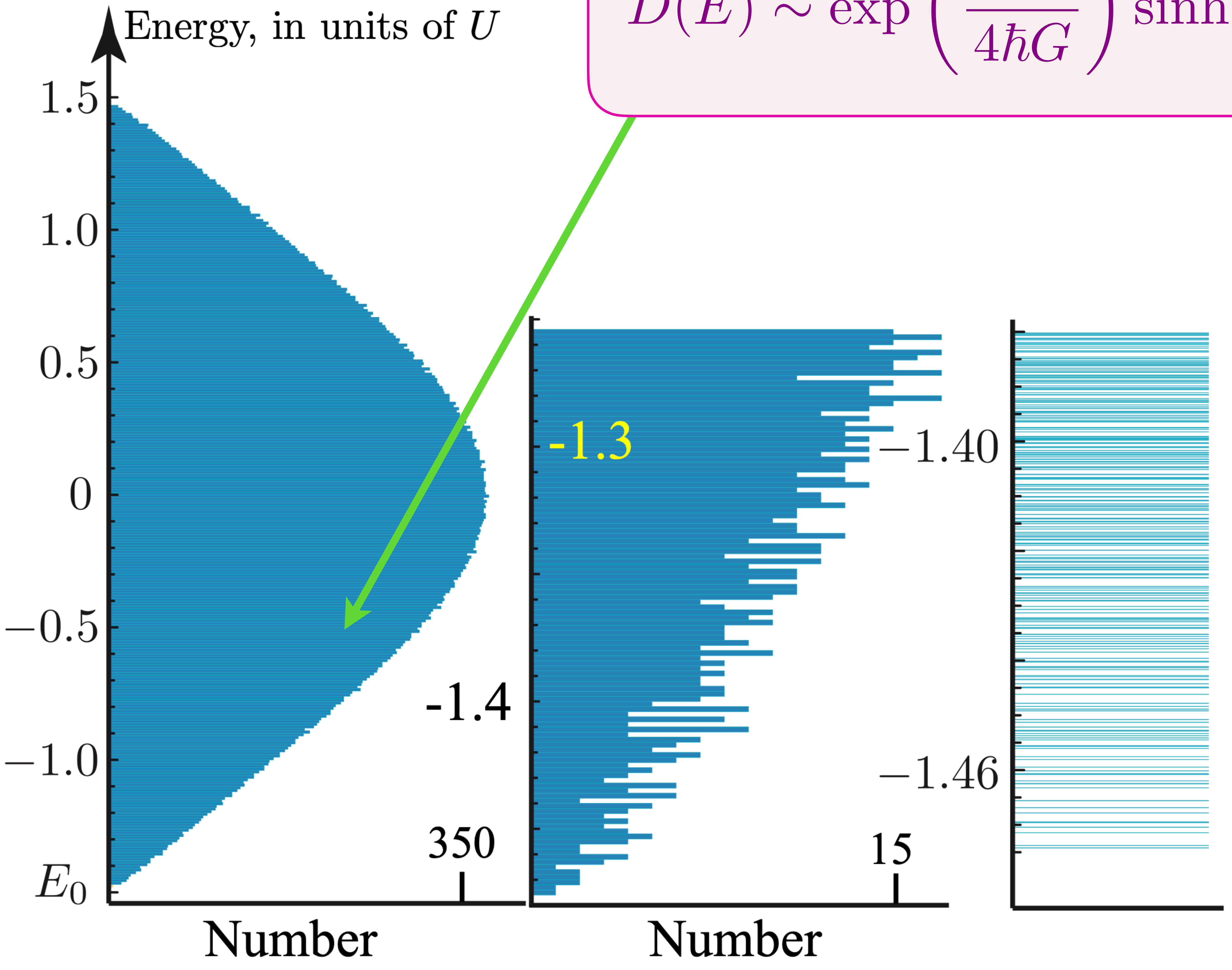
SYK

$$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{A_0c^3}{4} + \frac{\sqrt{\pi}A_0^{3/2}c^2}{2}\frac{k_BT}{\hbar}\right) - \frac{3}{2}\ln\left(\frac{\sqrt{c^5/\hbar G}}{k_BT/\hbar}\right) + \ldots .$$

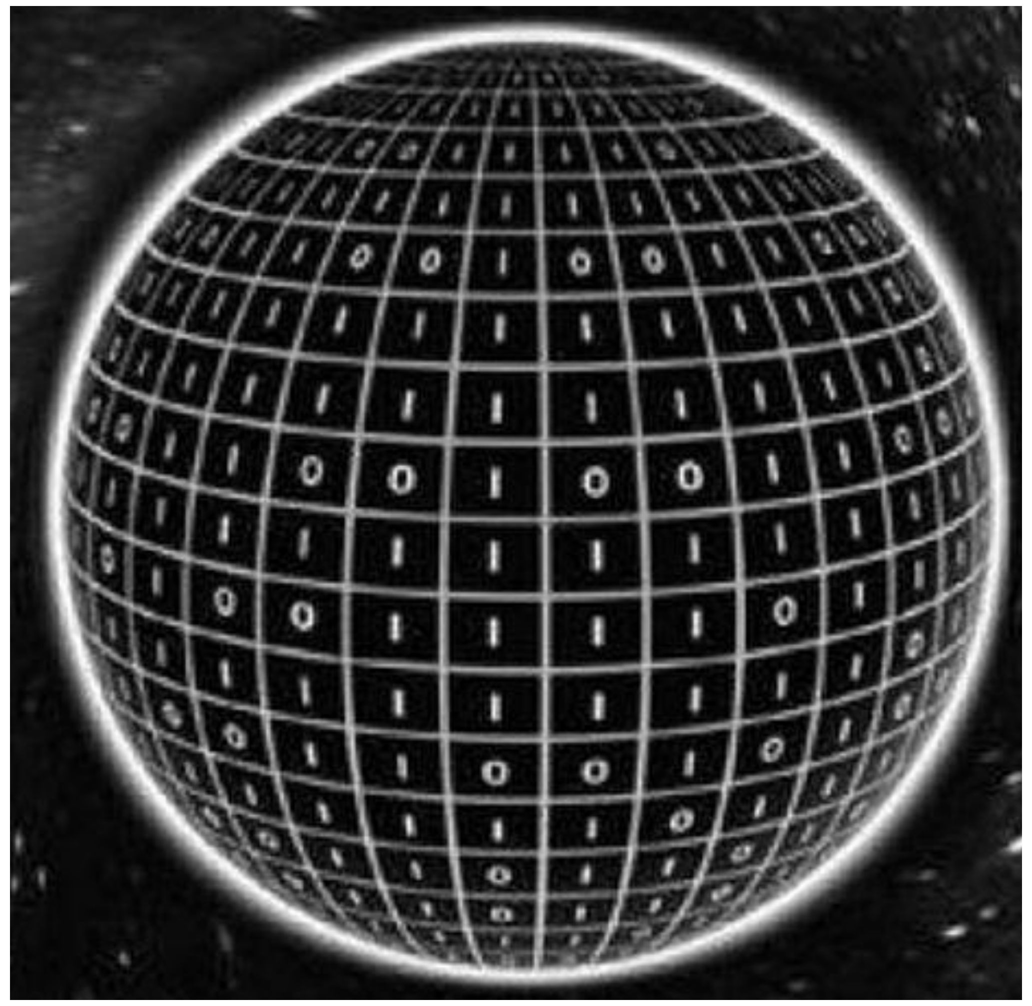
where A_0 is the area of the black hole horizon at $T=0$.

Quantum theory of charged black holes

$$D(E) \sim \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$



Same lower energy coarse-grained density of states in a model of interacting (fermionic) qubits with a discrete spectrum!



Yukawa-SYK models
and
strange metals

Yukawa-SYK models

$$H = \sum_{ij} t_{ij} \psi_i^\dagger \psi_j + \sum_\ell \frac{1}{2} \left(\pi_\ell^2 + \omega_\ell^2 \phi_\ell^2 \right) + \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

Leads to fully self-consistent Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$ in a SYK-like large N limit.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_+ \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N saddle point equations are

$$\boxed{\begin{aligned} G(i\omega_n) &= \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)} \\ \Sigma(\tau) &= g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau) \end{aligned}}$$

Make the low frequency ansatz

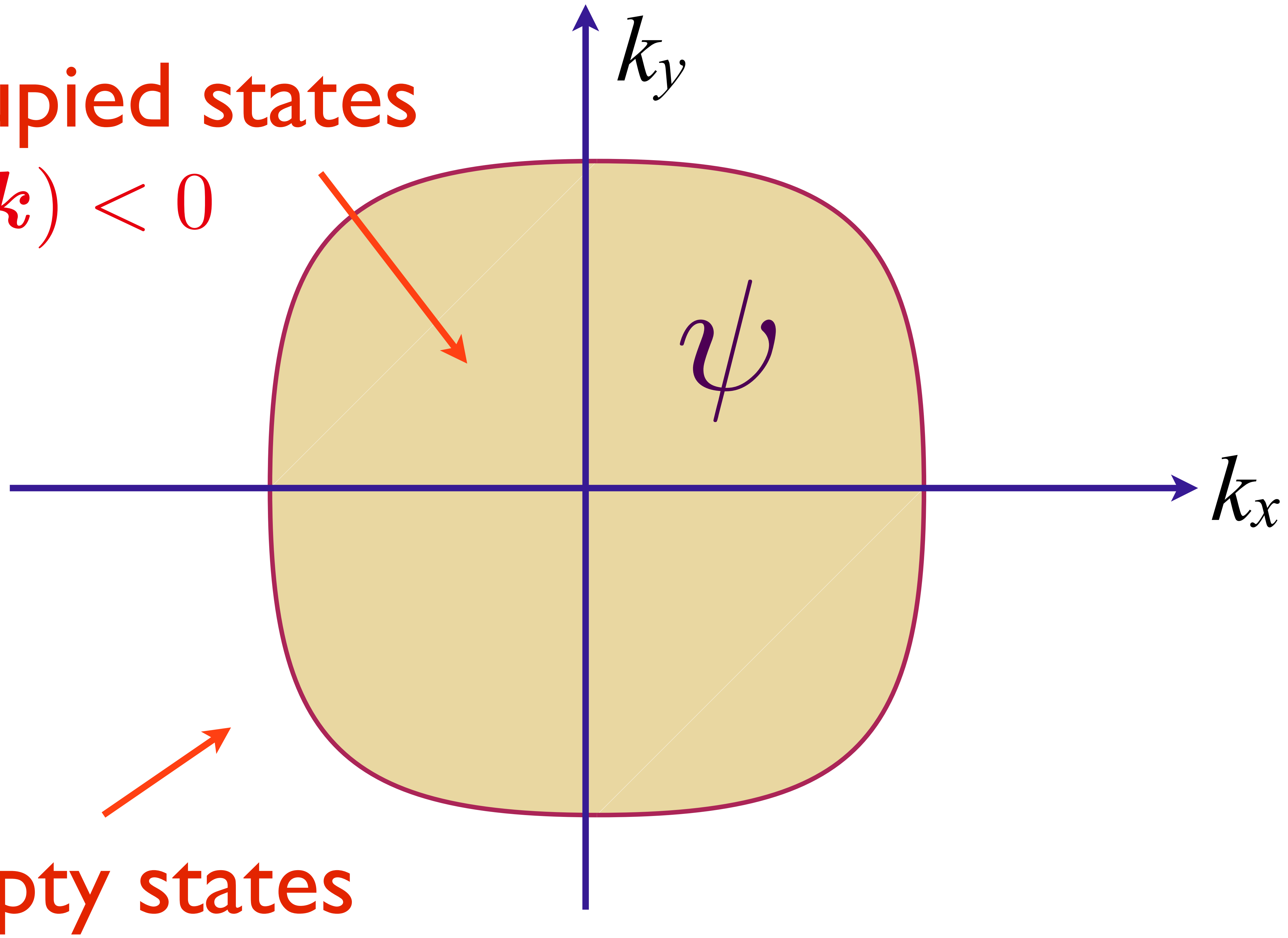
$$G(i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

Fermi surface coupled to a critical boson



Occupied states

$\varepsilon(\boldsymbol{k}) < 0$

Empty states

$\varepsilon(\boldsymbol{k}) > 0$

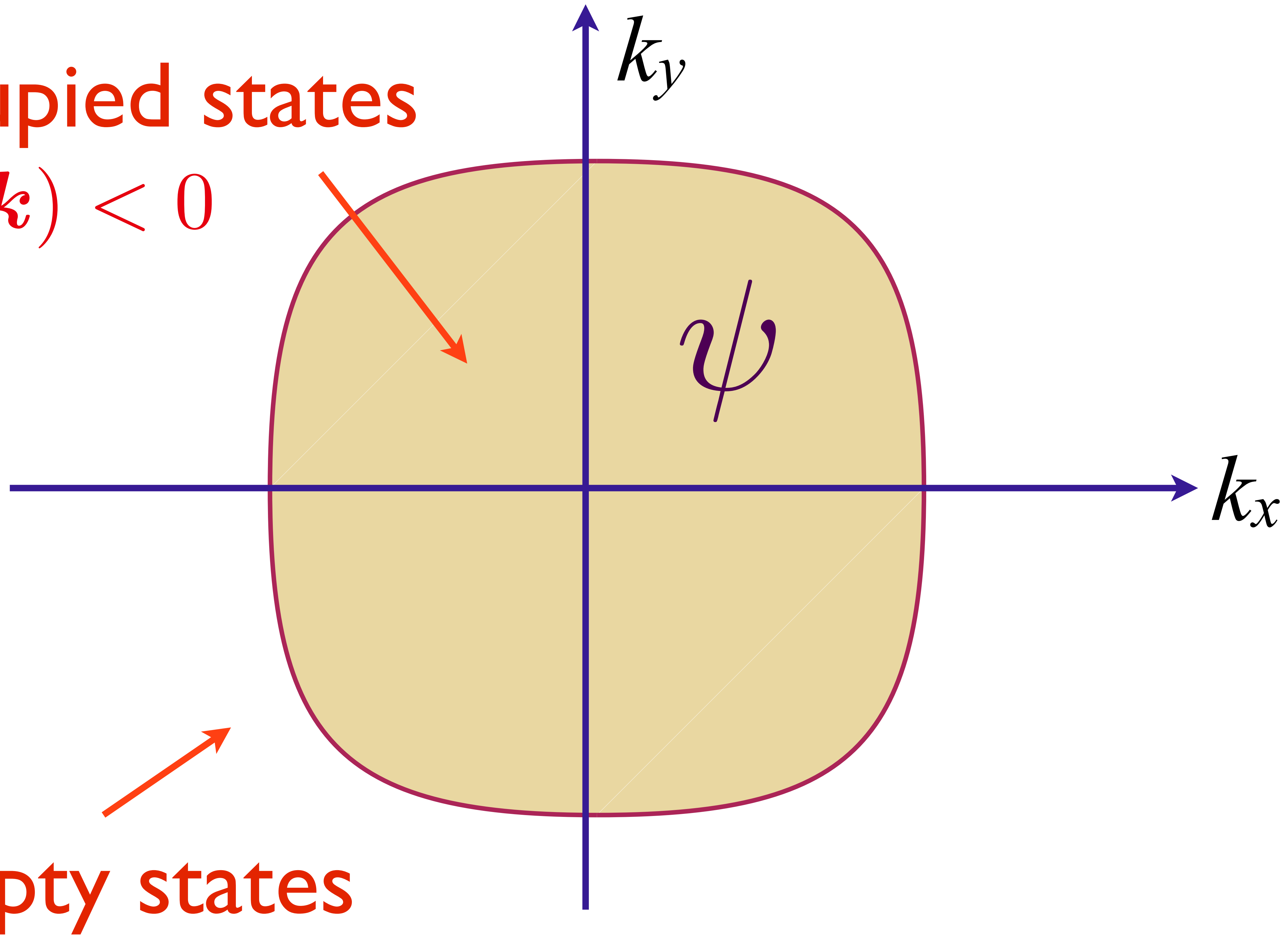
$+$

a critical boson

ϕ

- Nematic order
- Ferromagnetic order
- Transverse component of abelian or non-abelian gauge field

Fermi surface coupled to a critical boson



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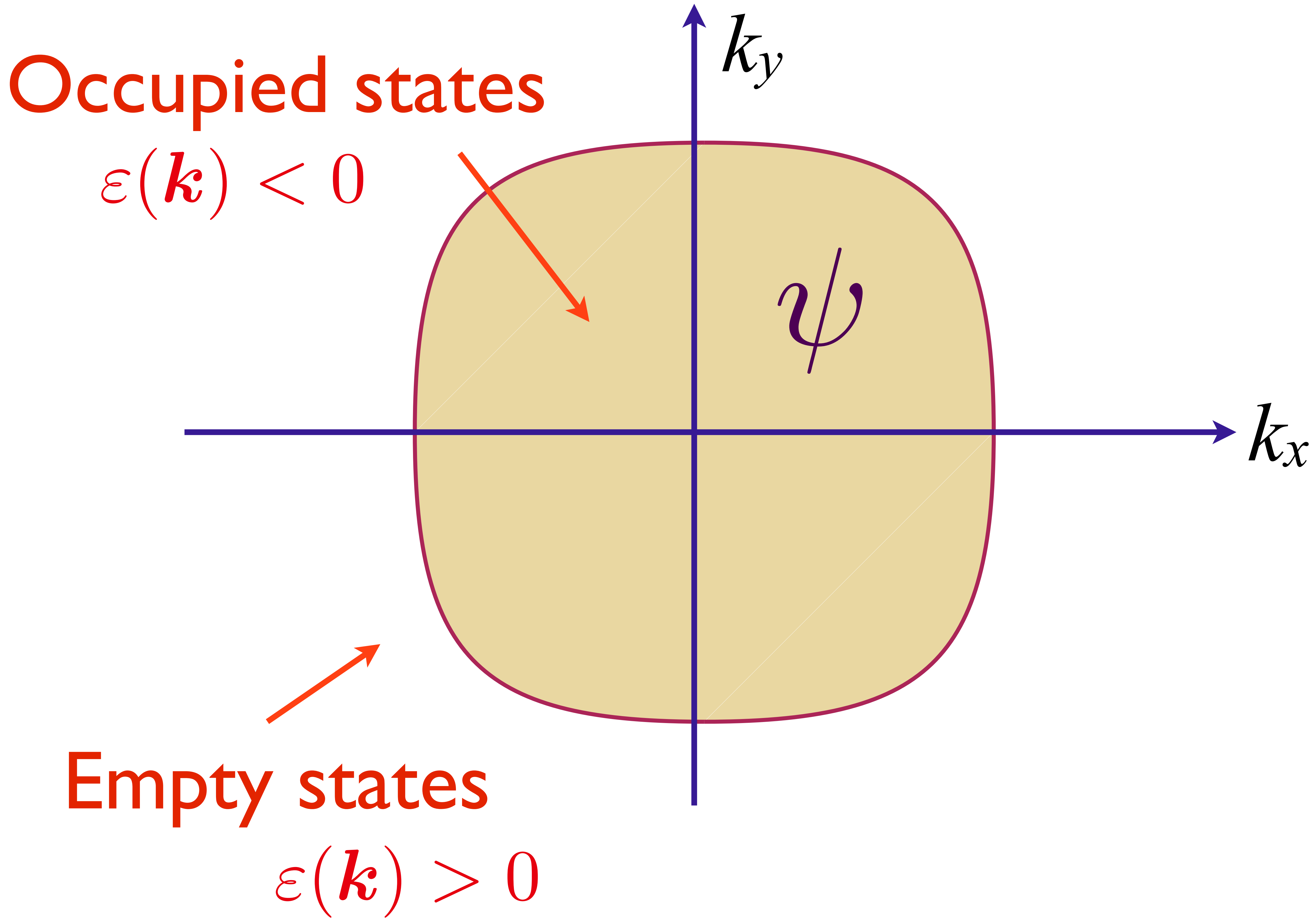
"Yukawa" coupling: $g \int d^2 r d\tau \, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

Yields a non-Fermi liquid without quasiparticles, but with zero resistivity (due to boson "drag")!

Fermi surface coupled to a critical boson



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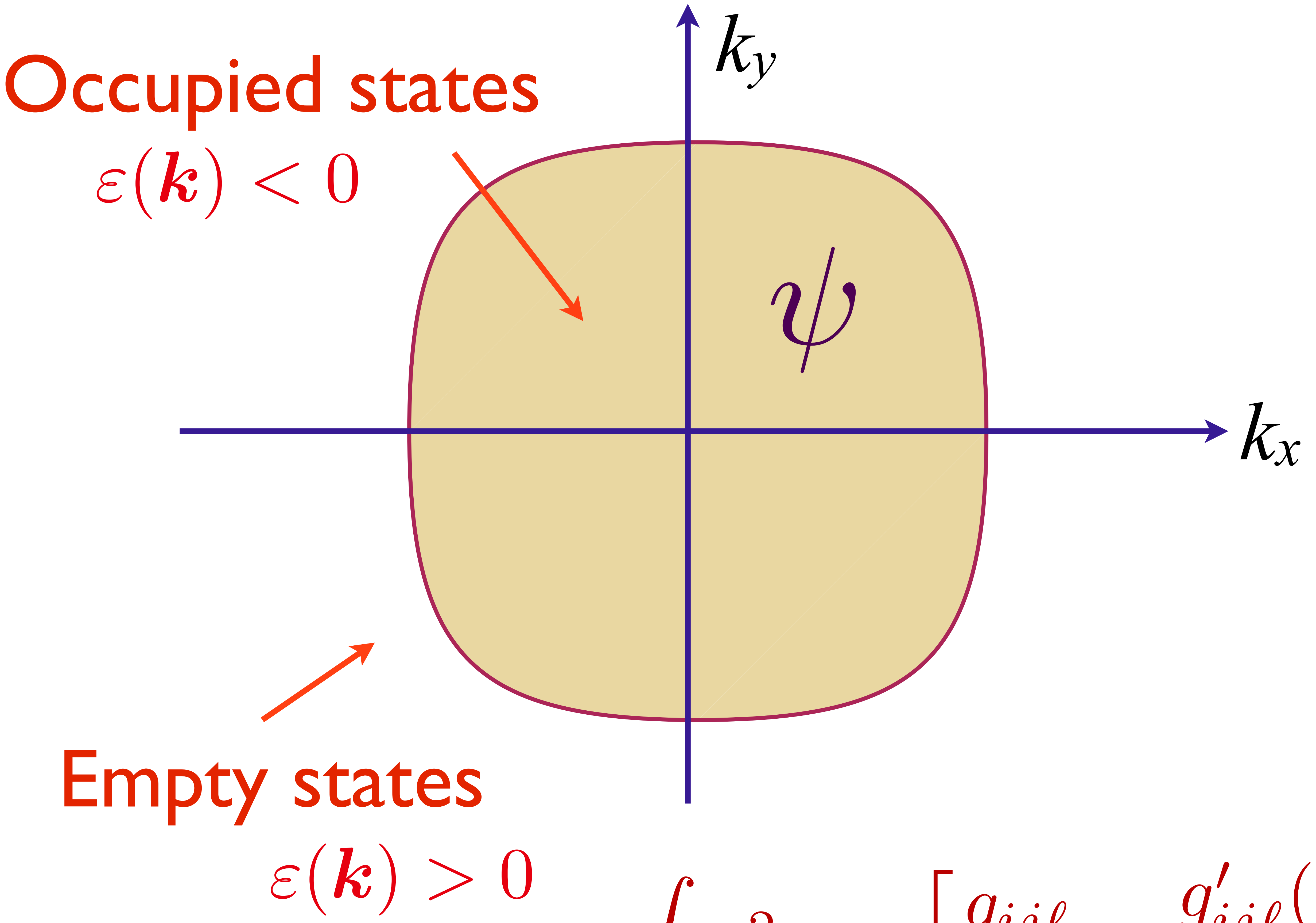
"Yukawa" coupling: $\dfrac{g_{ij\ell}}{N} \displaystyle\int d^2r d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

Random couplings in flavor space lead to large N theory of a strange metal, with zero resistivity

Fermi surface coupled to a critical boson



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$$\int d^2 r d\tau \left[\frac{g_{ij\ell}}{N} + \frac{g'_{ij\ell}(r)}{N} \right] \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

Random couplings in flavor *and* position space leads to large N theory of a strange metal, with linear-T resistivity

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990

Summary

- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

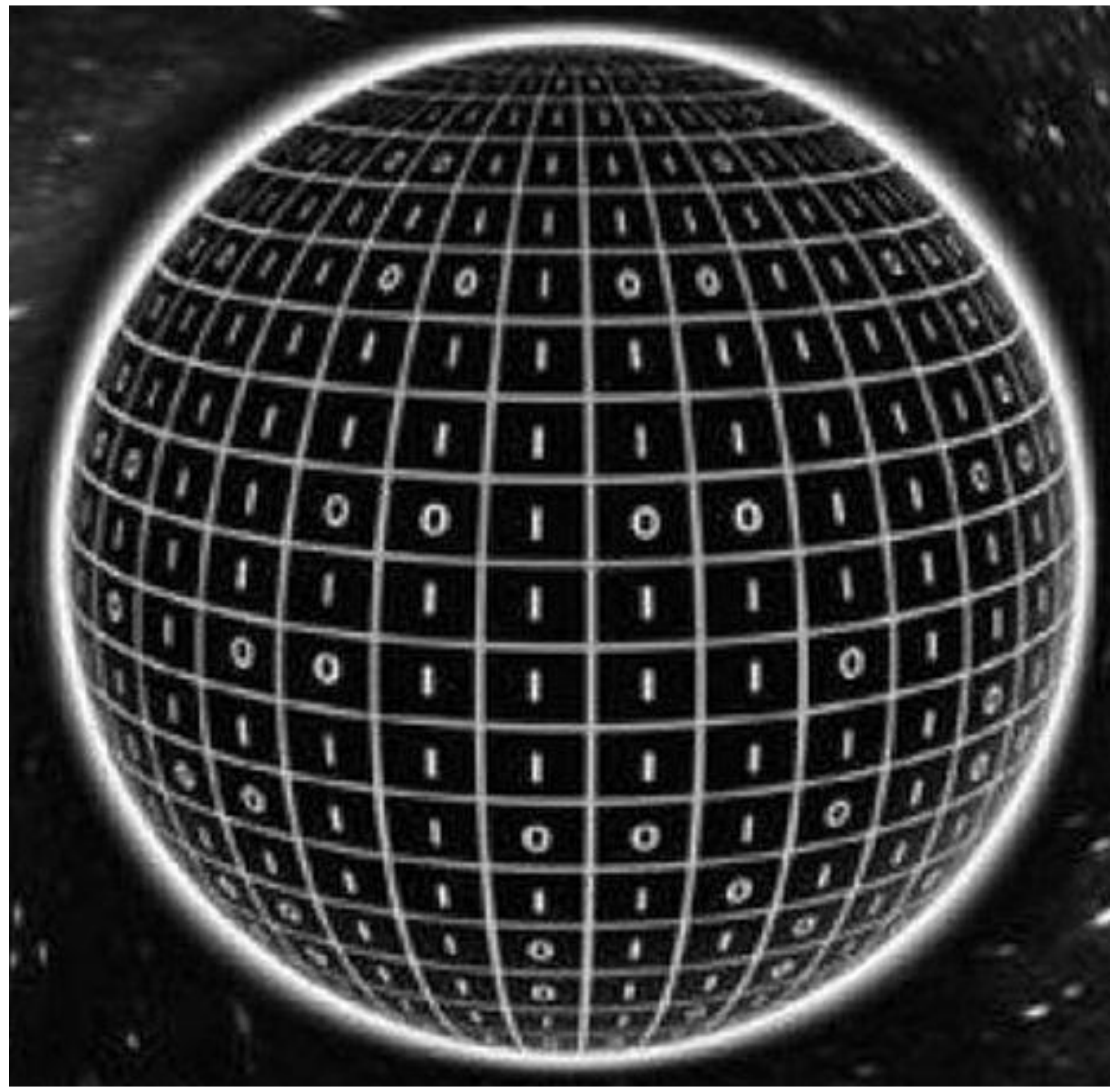
Summary

- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

- Low energy theory of time reparameterizations is the theory of the boundary graviton in 1+1 dimensional quantum gravity on AdS$_2$.

Summary

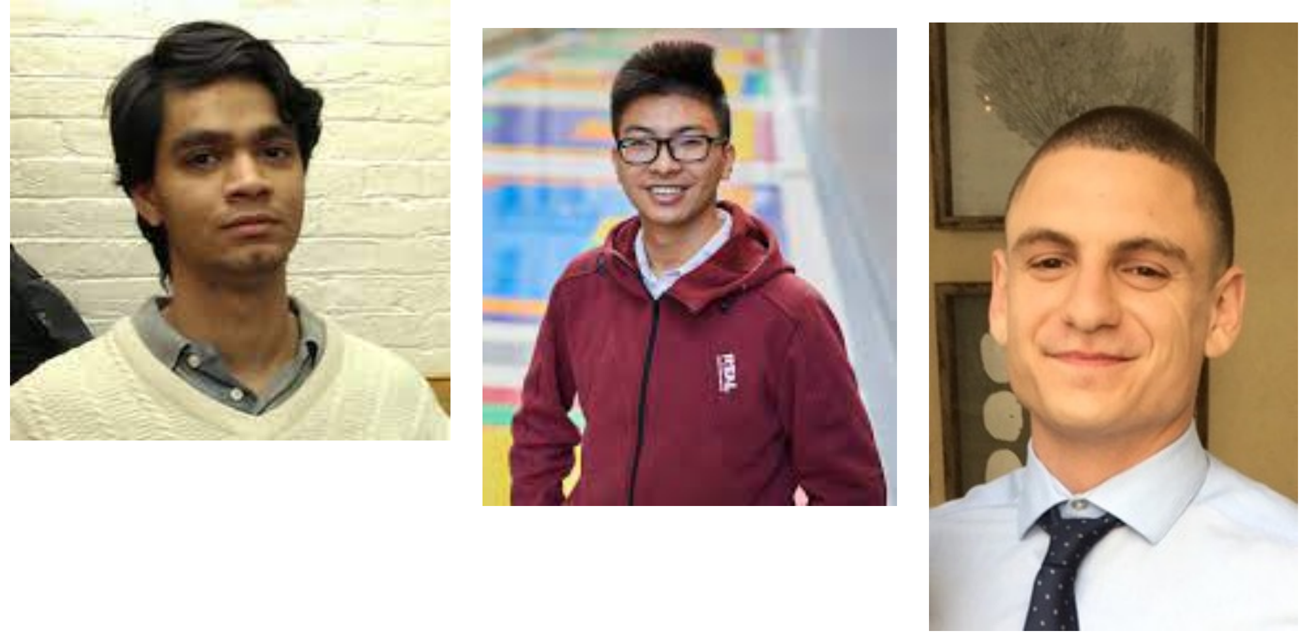
- The density of states of a charged black holes in Einstein gravity is reproduced by a unitary quantum system with a discrete spectrum. Further work along these lines has led to progress on the Page curve describing the time evolution of the entropy of an evaporating black hole.



Summary

- The density of states of a charged black holes in Einstein gravity is reproduced by a unitary quantum system with a discrete spectrum. Further work along these lines has led to progress on the Page curve describing the time evolution of the entropy of an evaporating black hole.

- Linear-T resistivity arises from spatially random interactions in a two-dimensional quantum-critical metal.



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