Fractionalized Fermi liquids

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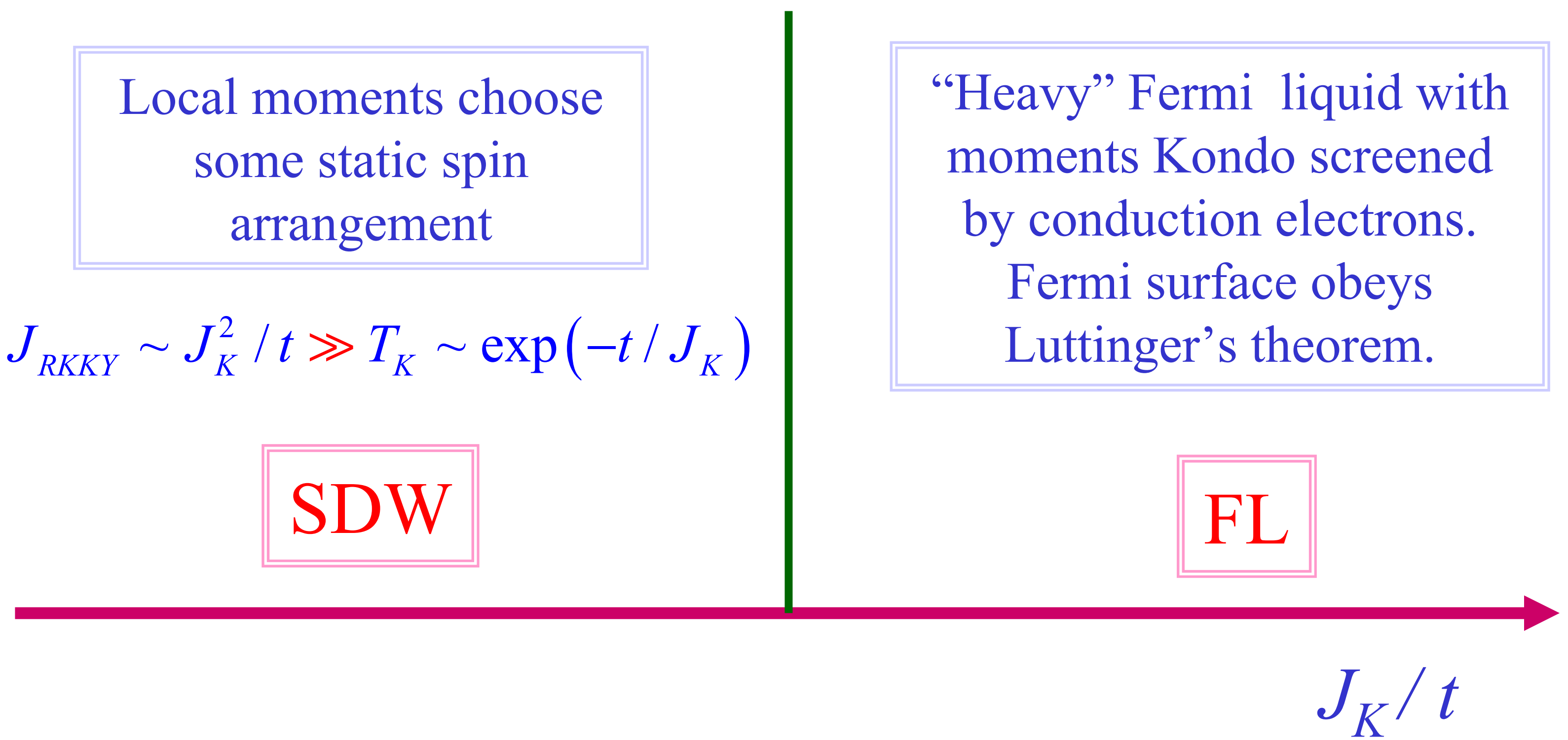
Doniach's $T=0$ phase diagram for the Kondo lattice

$$H = \sum_{i<j} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + \sum_{i} \left(J_K c^{\dagger}_{i\sigma} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right)$$

$c_{i\sigma} \rightarrow$ Conduction electrons;

$\vec{S}_{fi} \rightarrow$ localized $f_{i\sigma}$ moments (assumed S=1/2, for specificity)



Luttinger's theorem on a d-dimensional lattice for the FL phase

Let v_0 be the volume of the unit cell of the ground state,
n_T be the total number density of electrons per volume v_0.
(need not be an integer)

$$n_T = n_f + n_c = 1 + n_c$$

$$2 \times \frac{v_0}{(2\pi)^d} \left(\text{Volume enclosed by Fermi surface} \right)$$

$$= n_T \pmod{2}$$

Reconsider Doniach phase diagram

It is more convenient to analyze the Kondo-Heiseberg model:

$$H = \sum_{i<j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + \sum_i \left(J_K c^\dagger_{i\sigma} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H(i,j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Work in the regime $J_H > J_K$

Determine the ground state of the quantum antiferromagnet defined by J_H, and then couple to conduction electrons by J_K

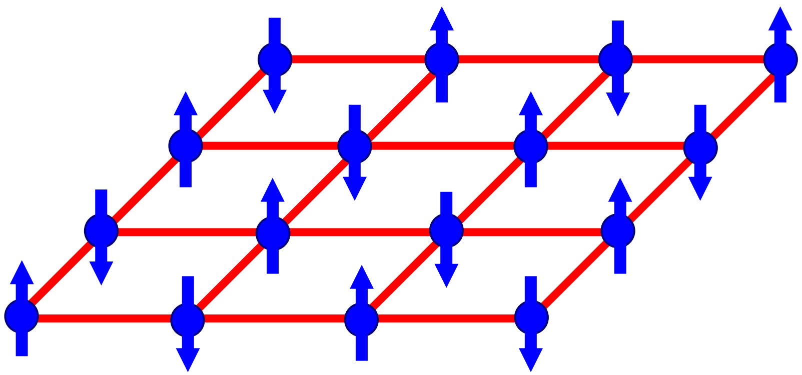
"*f* moments screen each other"

Ground states of quantum antiferromagnets

Begin with magnetically ordered states, and consider quantum transitions which restore spin rotation invariance, leading to a quantum paramagnet

Two classes of magnetically ordered states:

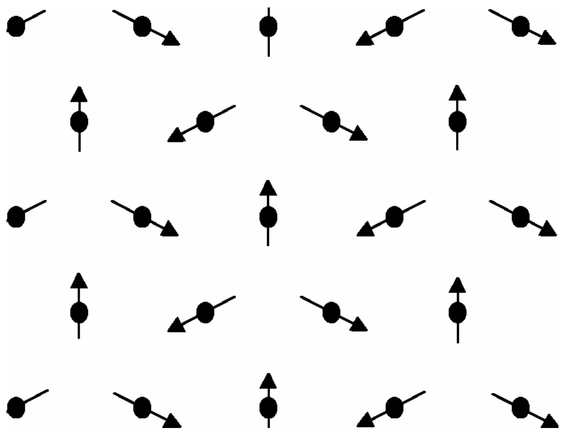
(α) Collinear spins



$$\left\langle \vec{S}(\boldsymbol{r}) \right\rangle \propto \vec{N} \cos(\boldsymbol{Q} \bullet \boldsymbol{r})$$

$$\boldsymbol{Q} = (\pi, \pi);\ \vec{N}^2 = 1$$

(β) Non-collinear spins



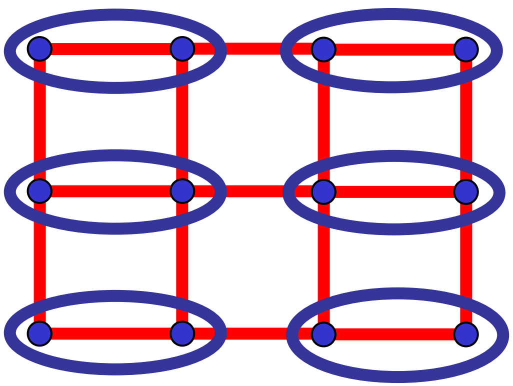
$$\left\langle \vec{S}(\boldsymbol{r}) \right\rangle \propto \vec{N}_1 \cos(\boldsymbol{Q} \bullet \boldsymbol{r}) + \vec{N}_2 \sin(\boldsymbol{Q} \bullet \boldsymbol{r})$$

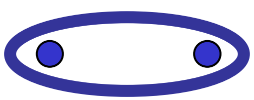
$$\boldsymbol{Q} = \left(\frac{4\pi}{3}, \frac{4\pi}{\sqrt{3}} \right);\ \vec{N}_1^2 = \vec{N}_2^2 = 1;\ \vec{N}_1 \bullet \vec{N}_2 = 0$$

Paramagnetic states with $\langle \boldsymbol{S}_j \rangle = 0$

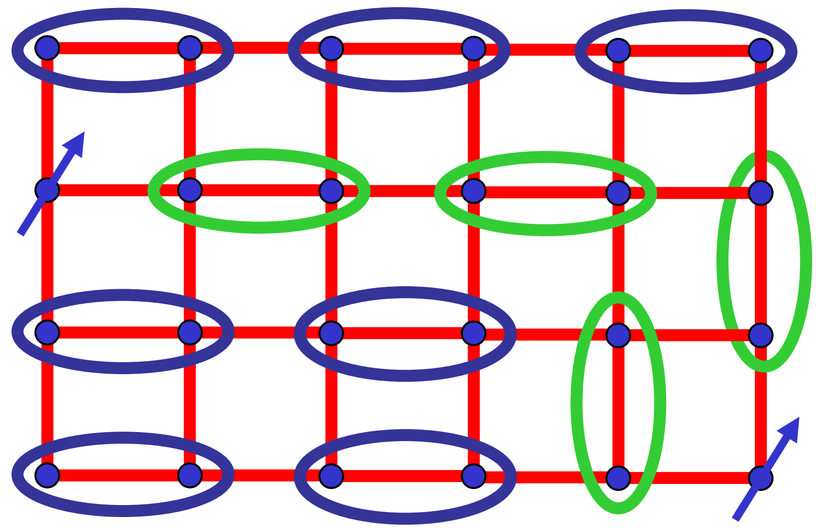
(α) Collinear spins

<u>Bond order and confined spinons</u>





$$=\frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle-\left|\downarrow\uparrow\right\rangle\right)$$



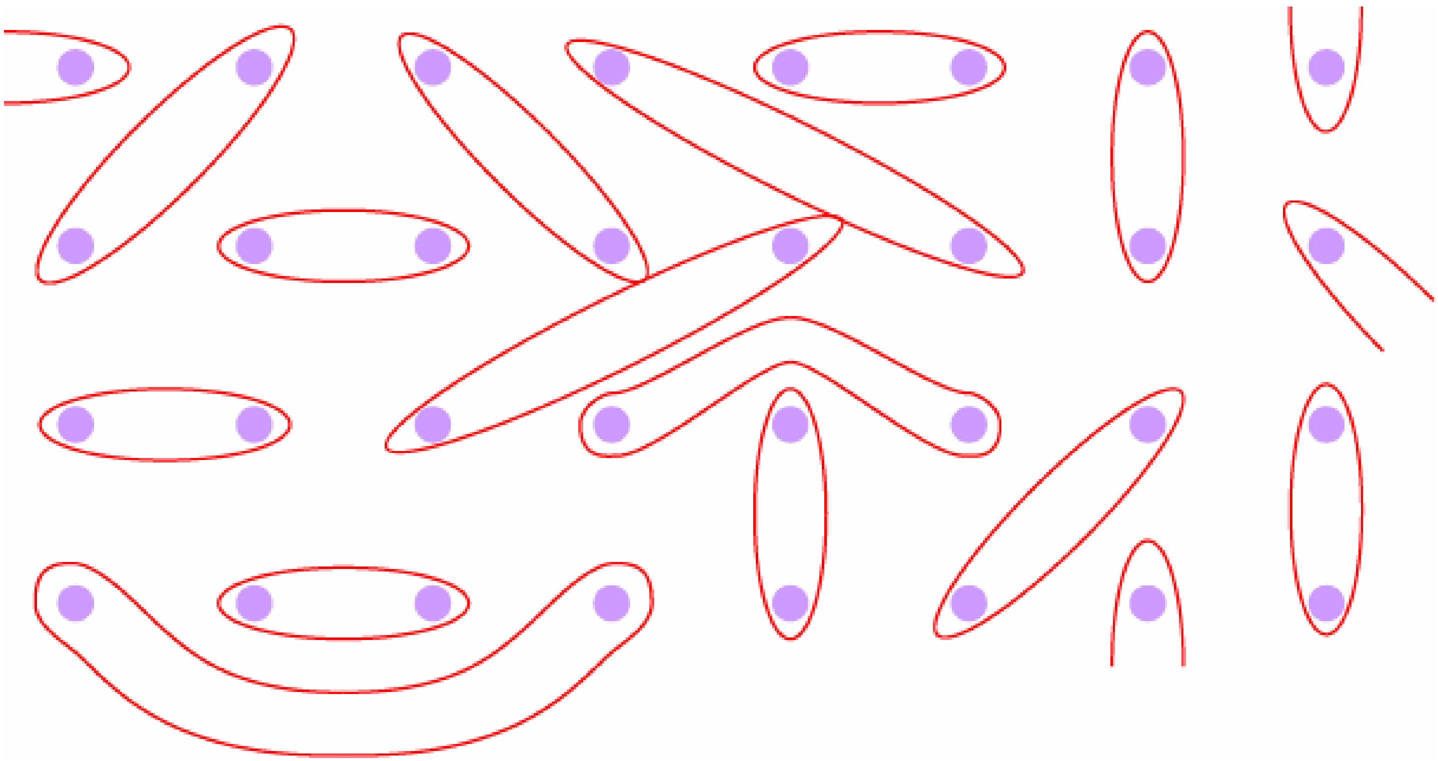
S=1/2 spinons are ***confined*** by a linear potential into a *S*=1 spin exciton

Generic behavior in *d*=2

Paramagnetic states with $\langle \boldsymbol{S}_j \rangle = 0$

(α) Collinear spins

U(1) spin liquid with deconfined spinons

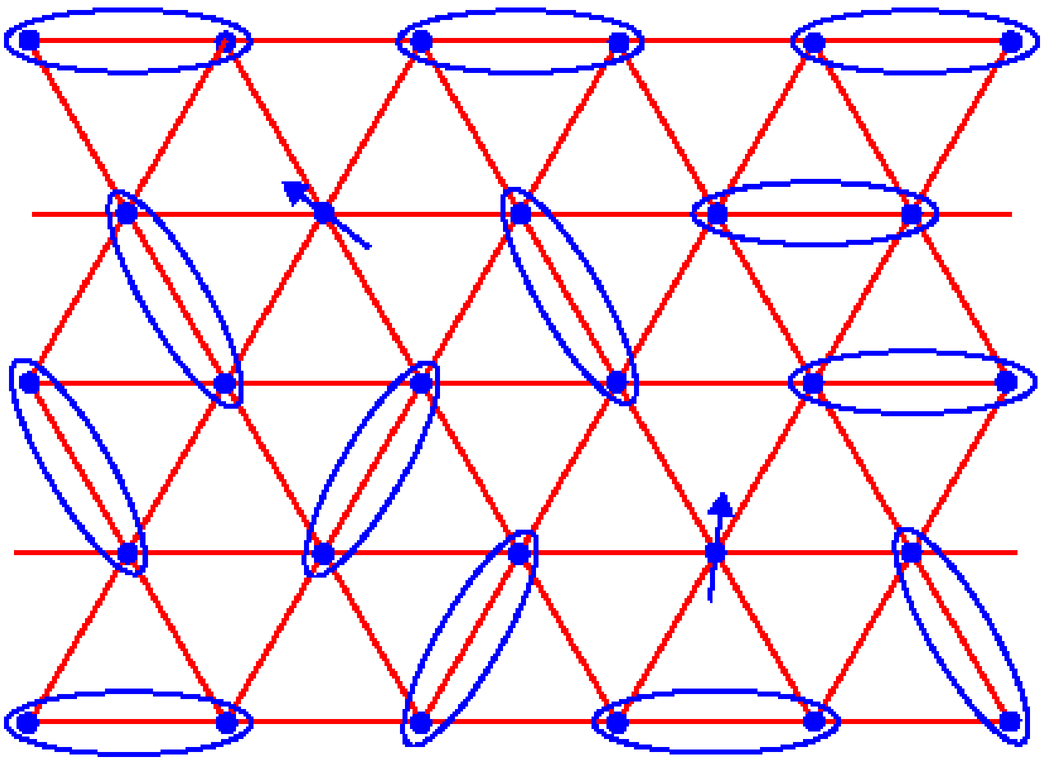


Possible ground state in d=3

Paramagnetic states with $\left\langle \boldsymbol{S}_j \right\rangle = 0$

(β) Non-collinear spins

Z_2 spin liquid with deconfined spinons



Can appear in d=2,3

N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991);
X. G. Wen, *Phys. Rev*. B **44**, 2664 (1991).
T. Senthil and M.P.A. Fisher, *Phys. Rev*. B **62**, 7850 (2000).
R. Moessner and S.L. Sondhi, *Phys. Rev. Lett*. **86**, 1881 (2001).

(α) Collinear spins, Berry phases, and bond-order

$S=1/2$ antiferromagnet on a bipartitie lattice

$$H = \sum_{i<j} J_{ij} \ \vec{S}_i \cdot \vec{S}_j$$

Include Berry phases after discretizing coherent state path integral on a cubic lattice in spacetime

$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \boldsymbol{n}_a \cdot \boldsymbol{n}_{a+\mu} - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

$\eta_{\mathrm{a}} \rightarrow \pm 1$ on two sublattices ;

$\boldsymbol{n}_a \sim \eta_a \vec{S}_a \rightarrow$ Neel order parameter;

$A_{a\mu} \rightarrow$ oriented area of spherical triangle

formed by $\boldsymbol{n}_a$, $\boldsymbol{n}_{a+\mu}$, and an arbitrary reference point $\boldsymbol{n}_0$

Small g $\rightarrow$ Spin-wave theory about Neel state receives minor modifications from Berry phases.

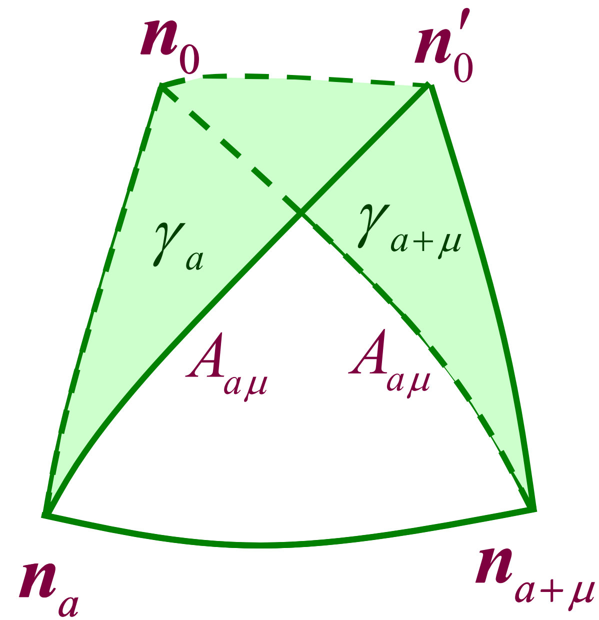
Large g $\rightarrow$ Berry phases are crucial in determining structure of "quantum-disordered" phase with $\langle \boldsymbol{n}_a \rangle = 0$

Integrate out $\boldsymbol{n}_a$ ***to obtain effective action for*** $A_{a\mu}$

Change in choice of $\boldsymbol{n}_0$ is like a “gauge transformation”

$$A_{a\mu} \rightarrow A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

(γ_a is the oriented area of the spherical triangle formed by $\boldsymbol{n}_a$ and the two choices for $\boldsymbol{n}_0$).



The area of the triangle is uncertain modulo 4π, and the action is invariant under

$$A_{a\mu} \rightarrow A_{a\mu} + 4\pi$$

$\boxed{\text{These principles strongly constrain the effective action for } A_{a\mu}}$

Simplest large g effective action for the $A_{a\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left(-\frac{1}{2e^2} \sum_{\square} \cos\left(\frac{1}{2} \varepsilon_{\mu\nu\lambda} \Delta_\nu A_{a\lambda} \right) - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in d+1 dimensions with Berry phases.

This theory can be reliably analyzed by a duality mapping.

(I) *d=2:*

The gauge theory is always in a confining phase. There is an energy gap and the ground state has bond order (induced by the Berry phases).

(II) *d=3:*

An additional spin liquid ("Coulomb") phase is also possible. There are deconfined spinons which are minimally coupled to a gapless U(1) photon.

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990).
K. Park and S. Sachdev, *Phys. Rev.* B **65**, 220405 (2002).

Reconsider Doniach phase diagram

It is more convenient to analyze the Kondo-Heiseberg model:

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \sum_{i} \left(J_K c_{i\sigma}^{\dagger} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H(i,j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Work in the regime $J_H > J_K$

Determine the ground state of the quantum antiferromagnet defined by J_H, and then couple to conduction electrons by J_K

Nature of small J_K expansion depends upon the paramagnetic ground state obtained at $J_K = 0$

Consider, first the case J_K=0 and J_H chosen so that the spins form a *bond ordered* paramagnet

This system has a Fermi surface of conduction electrons with volume n_c (mod 2)

However, because n_f=2 (per unit cell of ground state)
$n_T = n_f + n_c = n_c$(mod 2), and
"small" Fermi volume="large" Fermi volume
(mod Brillouin zone volume)

These statements apply also for a finite range of J_K

Conventional Luttinger Theorem holds

Consider, next the case J_K=0 and J_H chosen so that the spins form a *spin liquid* paramagnet

This system has a Fermi surface of conduction electrons with volume n_c (mod 2)

Now n_f=1 (per unit cell of ground state)

$$n_T = n_f + n_c \neq n_c \left(\text{mod } 2\right)$$

$$2 \times \frac{v_0}{\left(2\pi\right)^d}\left(\text{Volume enclosed by Fermi surface}\right)$$

$$= \left(n_T - 1\right)\left(\text{mod } 2\right)$$

Perturbation theory is J_K is free of infrared divergences, and the topological order in the ground state is protected.

This state, and its Fermi volume, survive for a finite range of J_K

A Fractionalized Fermi Liquid (FL*)

Doping spin liquids

A likely possibility:

Added electrons do *not* fractionalize, but retain their bare quantum numbers.
Spinon, "photon", and vison states of the insulator survive unscathed.

There is a Fermi surface of *sharp electron-like* quasiparticles, enclosing a volume determined by the dopant electron alone.

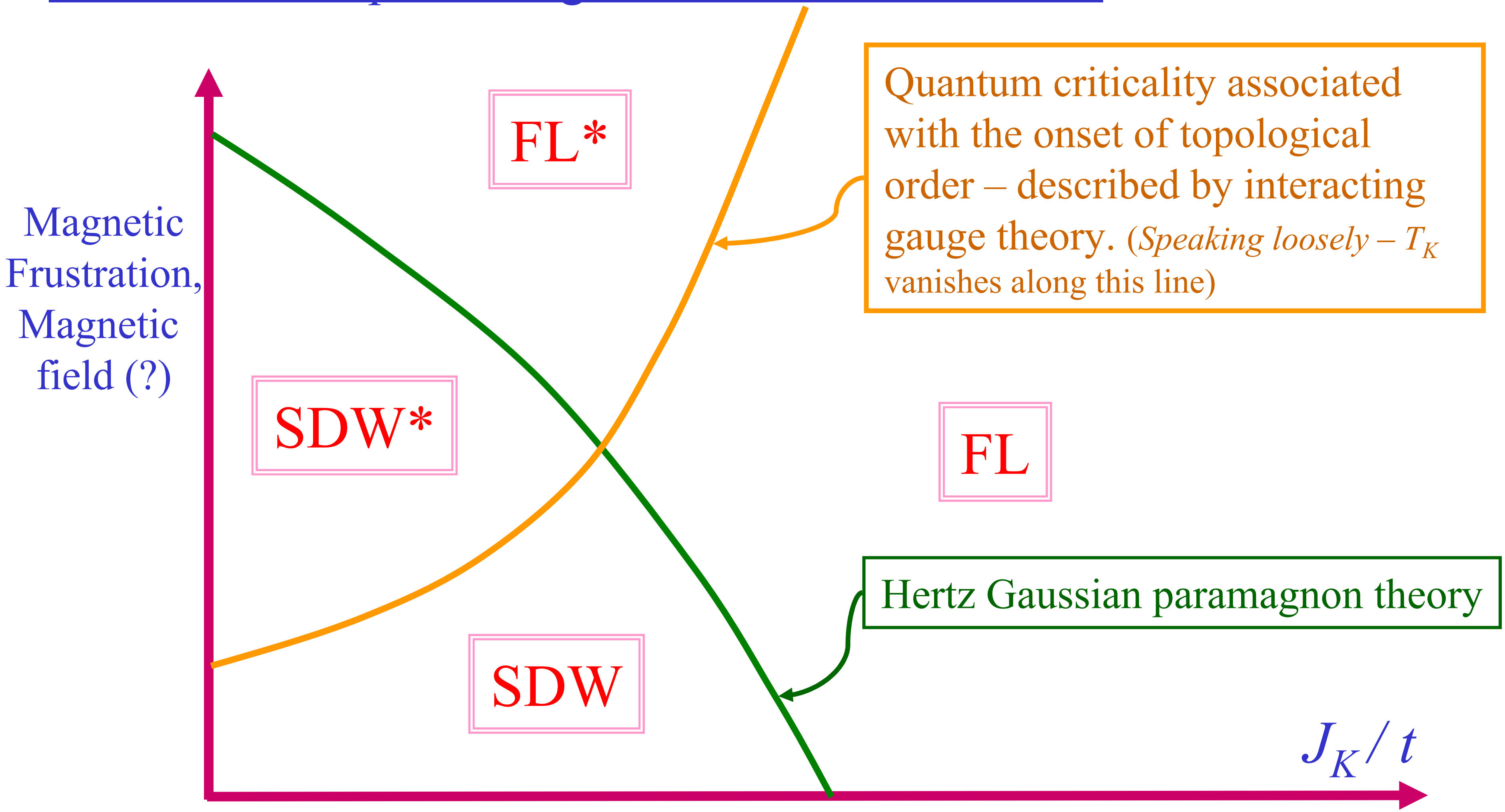
This is a "Fermi liquid" state which violates Luttinger's theorem

A Fractionalized Fermi Liquid (FL*)

T. Senthil, S. Sachdev, and M. Vojta, cond-mat/0209144

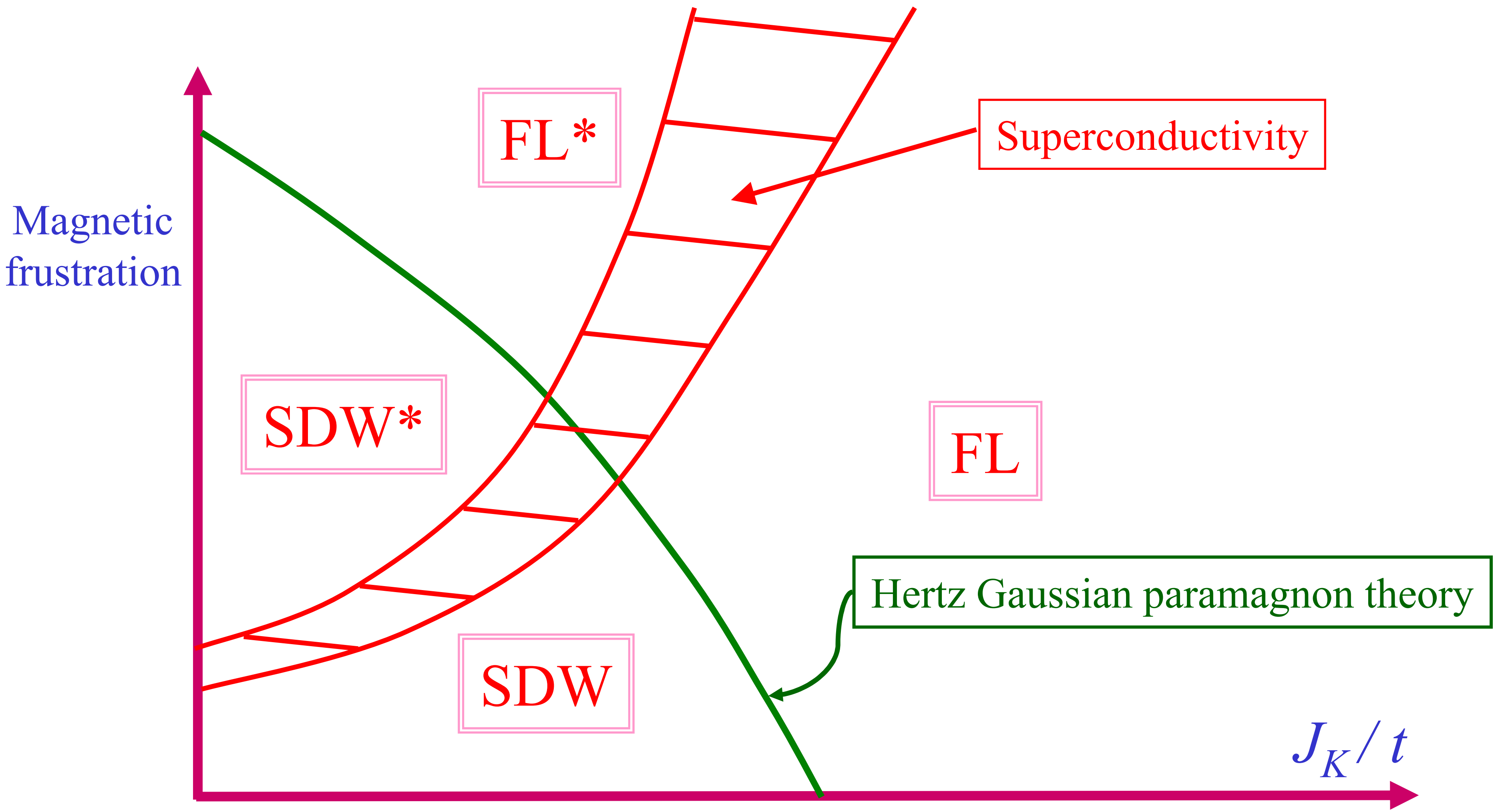
Precursors: L. Balents and M. P. A. Fisher and C. Nayak, *Phys. Rev.* B **60**, 1654, (1999);
T. Senthil and M.P.A. Fisher, Phys. Rev. B **62**, 7850 (2000);
S. Burdin, D. R. Grempel, and A. Georges, *Phys. Rev.* B **66**, 045111 (2002).

Extended T=0 phase diagram for the Kondo lattice



- $*$ phases have spinons with Z_2 (d=2,3) or U(1) (d=3) gauge charges, and associated gauge fields.
- magnetic field can induce a generic second-order transition to a * phase.
- Fermi surface volume does *not* distinguish SDW and SDW* phases.

Z_2 fractionalization



- **Superconductivity is generic between FL and Z_2 FL* phases.**