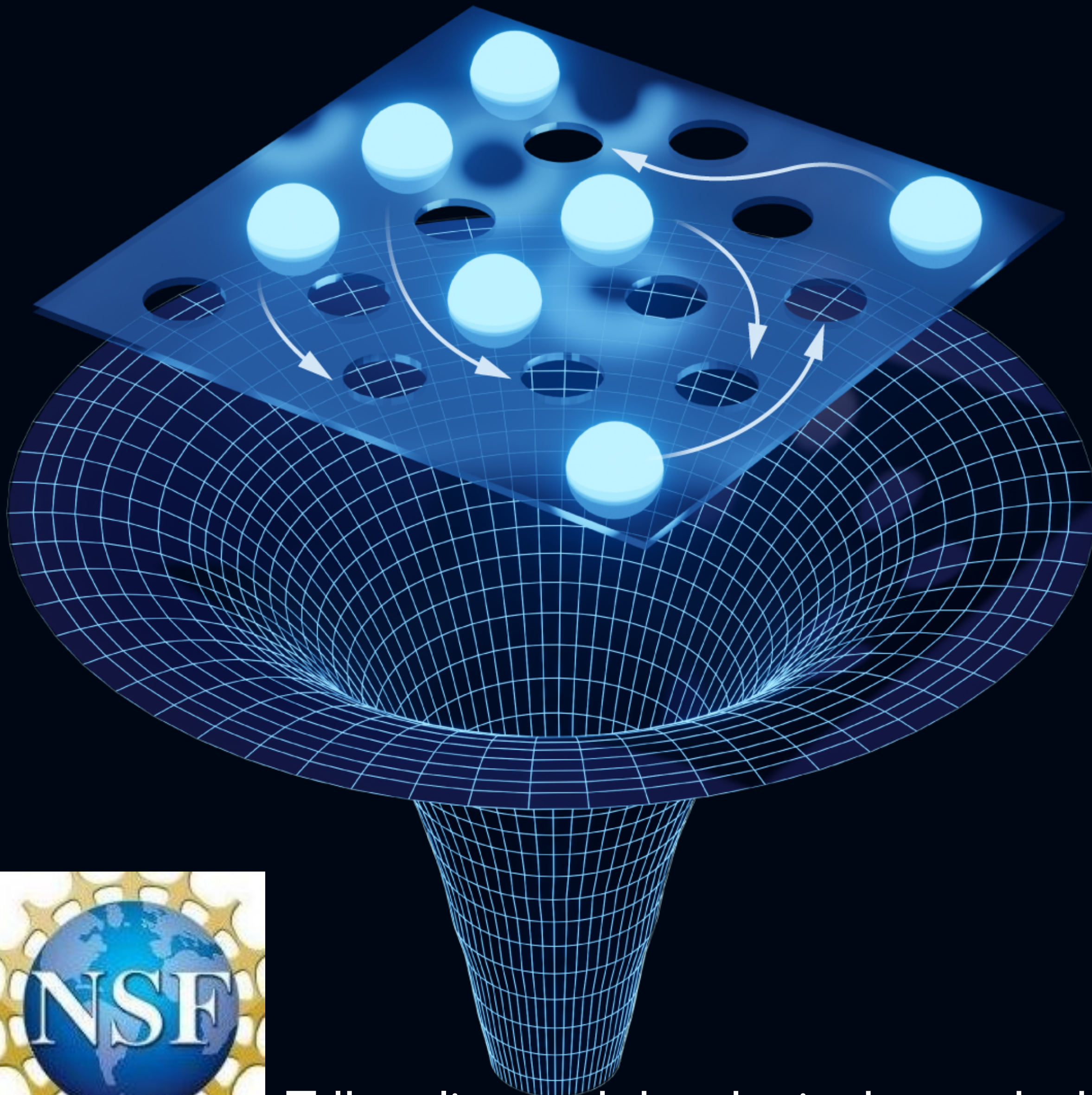


# Quantum entanglement in nature: superconductors and black holes



Indian Academy of Sciences Public Lecture  
National College, Bengaluru  
December 28, 2023

Subir Sachdev



Talk online: [sachdev.physics.harvard.edu](https://sachdev.physics.harvard.edu)



# Great discoveries in physics

Entropy (1870)

Superconductivity (1911)

Black holes (1916)

Quantum entanglement (1935)

What is entropy  
and temperature ?

Clausius (1865):

Second Law of Thermodynamics:

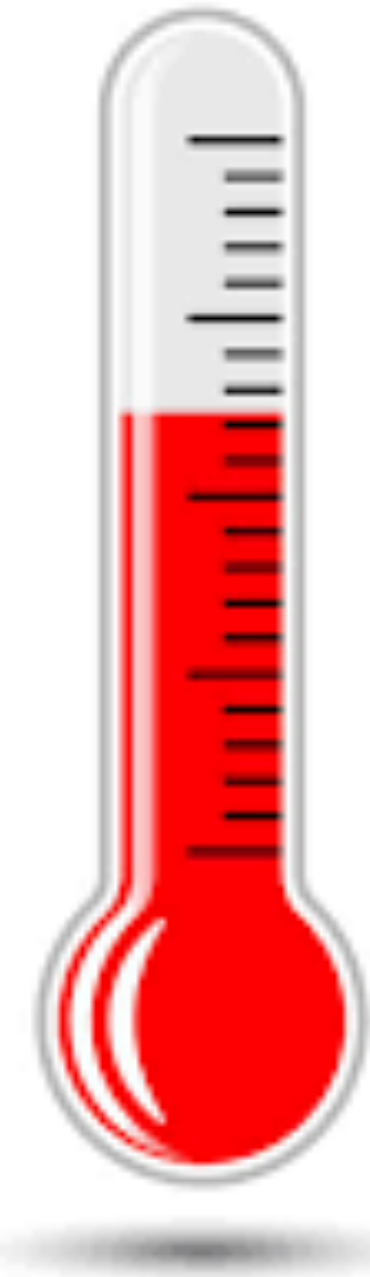
Every macroscopic system has an “entropy” which cannot decrease.

➔ No perpetual motion machines!



Clausius (1865):  
Second Law of Thermodynamics:  
Every macroscopic system has an  
“entropy” which cannot decrease.

➔ No perpetual motion machines!



Temperature

# Statistical interpretation of entropy (1870)

$$S = k_B \log W$$



Ludwig Boltzmann

20 February 1844 - September 5, 1906

Vienna, Austria

# Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

$$\frac{1}{T} = \frac{dS}{dE}$$



Ludwig Boltzmann

20 February 1844 - September 5, 1906

Vienna, Austria

Ordinary conductors  
(metals)  
and  
superconductors

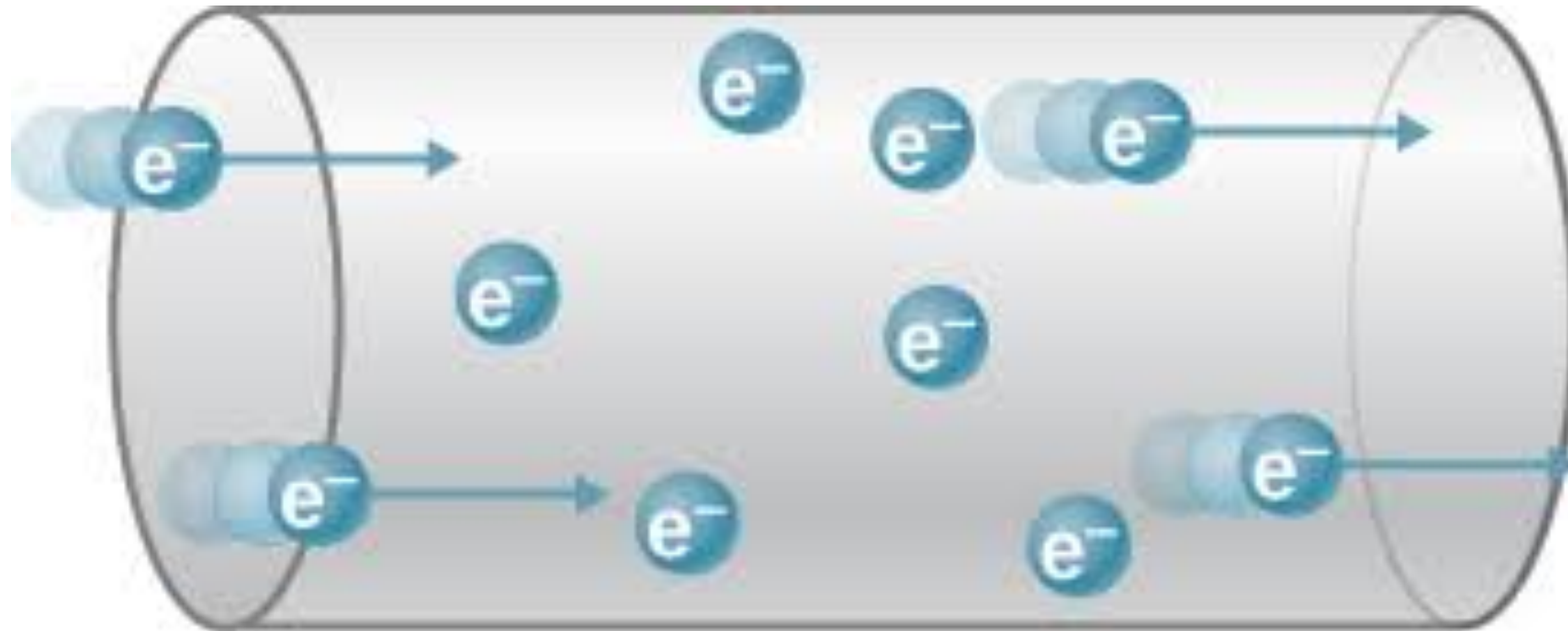


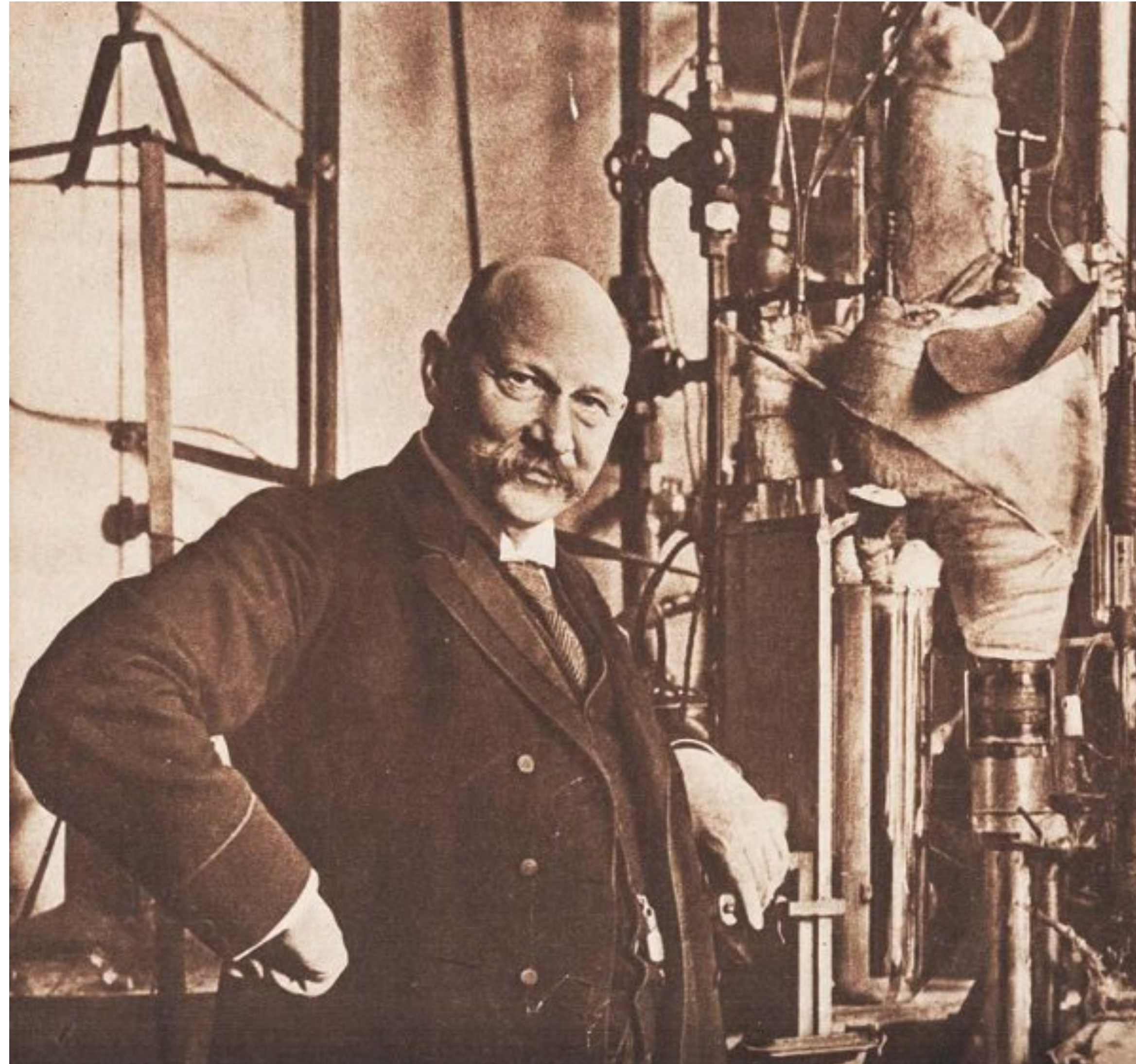
CONDUCTOR

SEMICONDUCTOR

SUPERCONDUCTOR

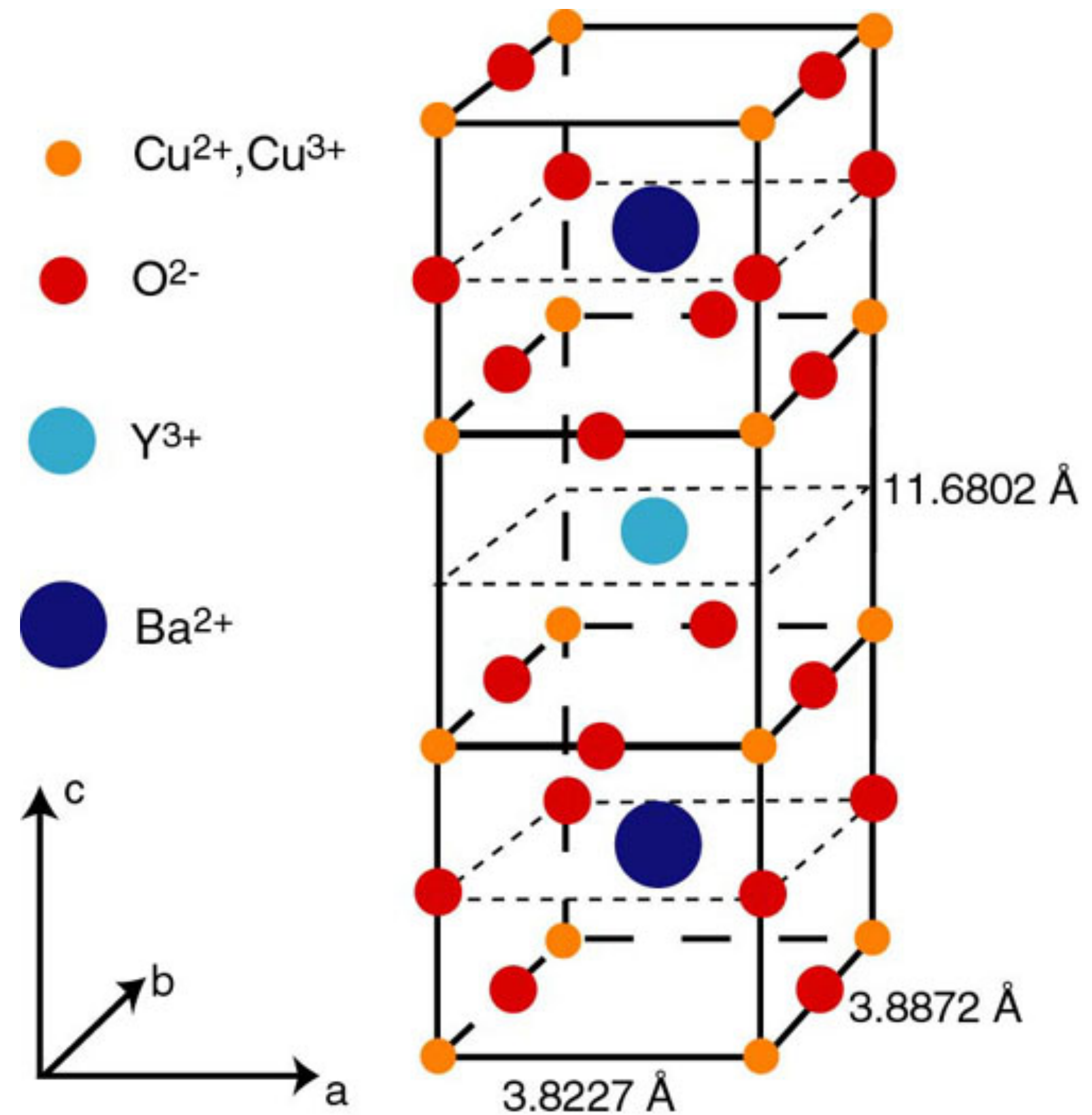
*Copper: ordinary metal quantum matter*

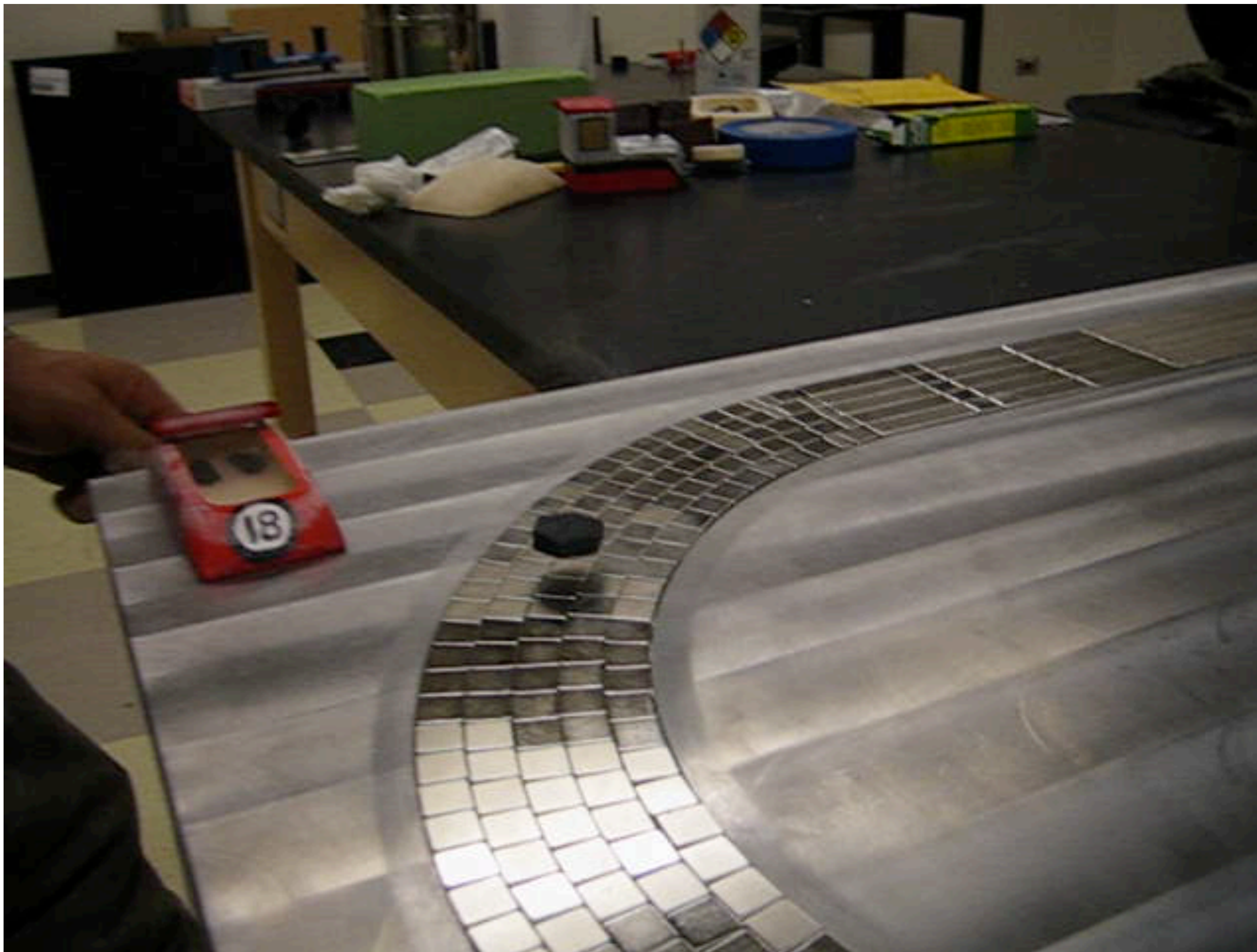




Kamerlingh Onnes 1911:  
Mercury is a superconductor below  $-269\text{ }^{\circ}\text{C}$

# Cuprate high temperature superconductors





Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

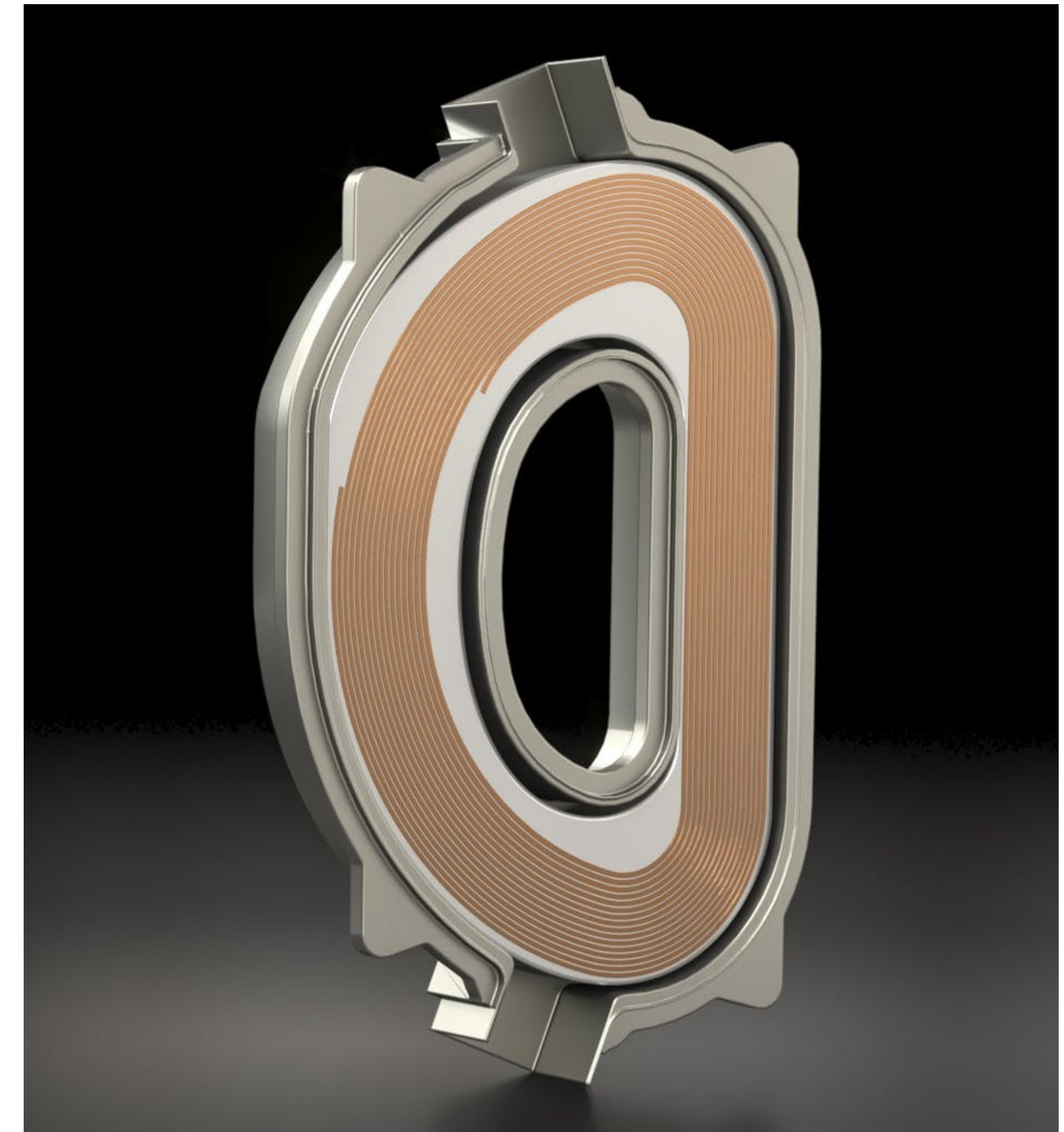
# HTS Magnets: Enabling Technology

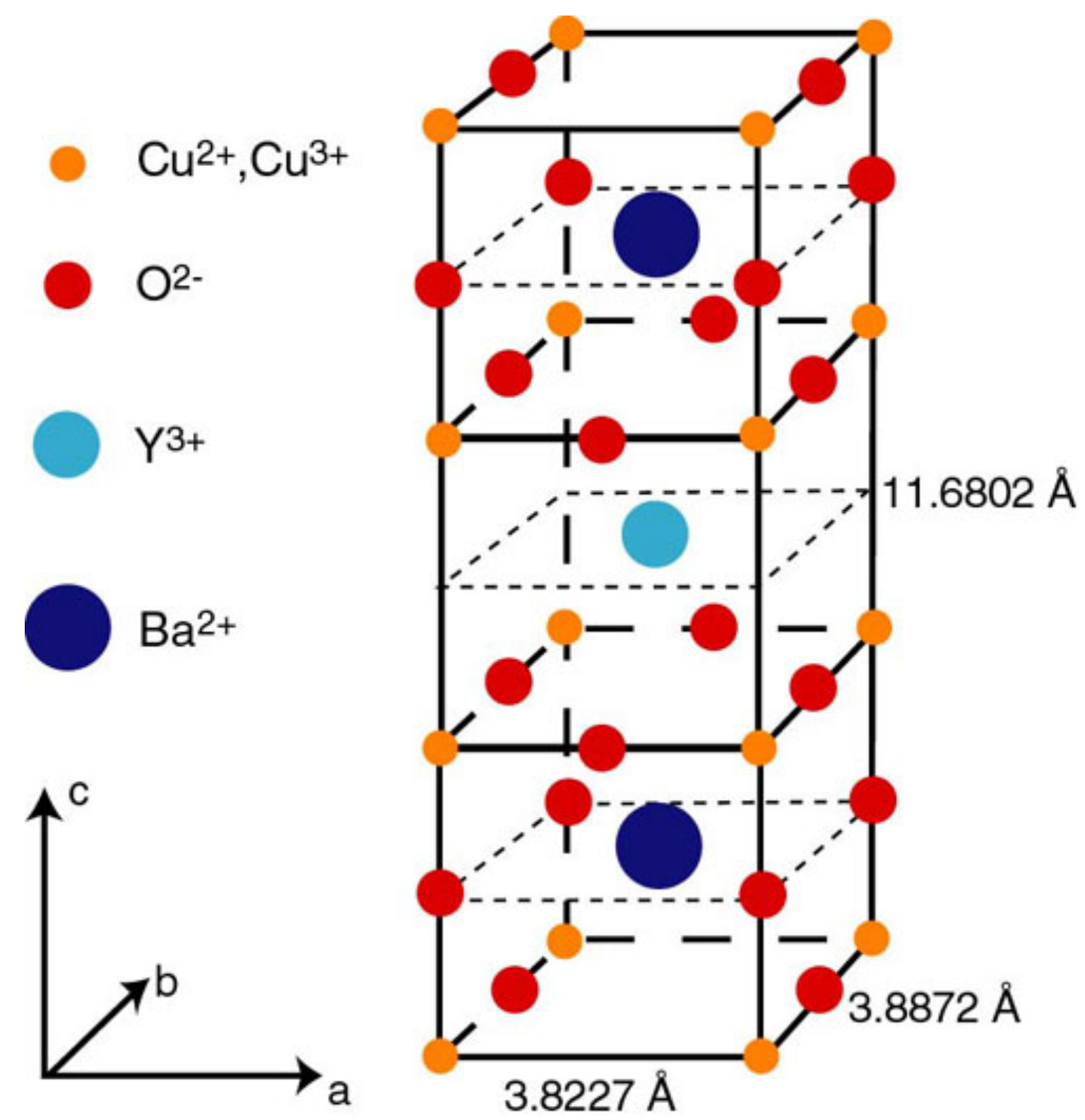
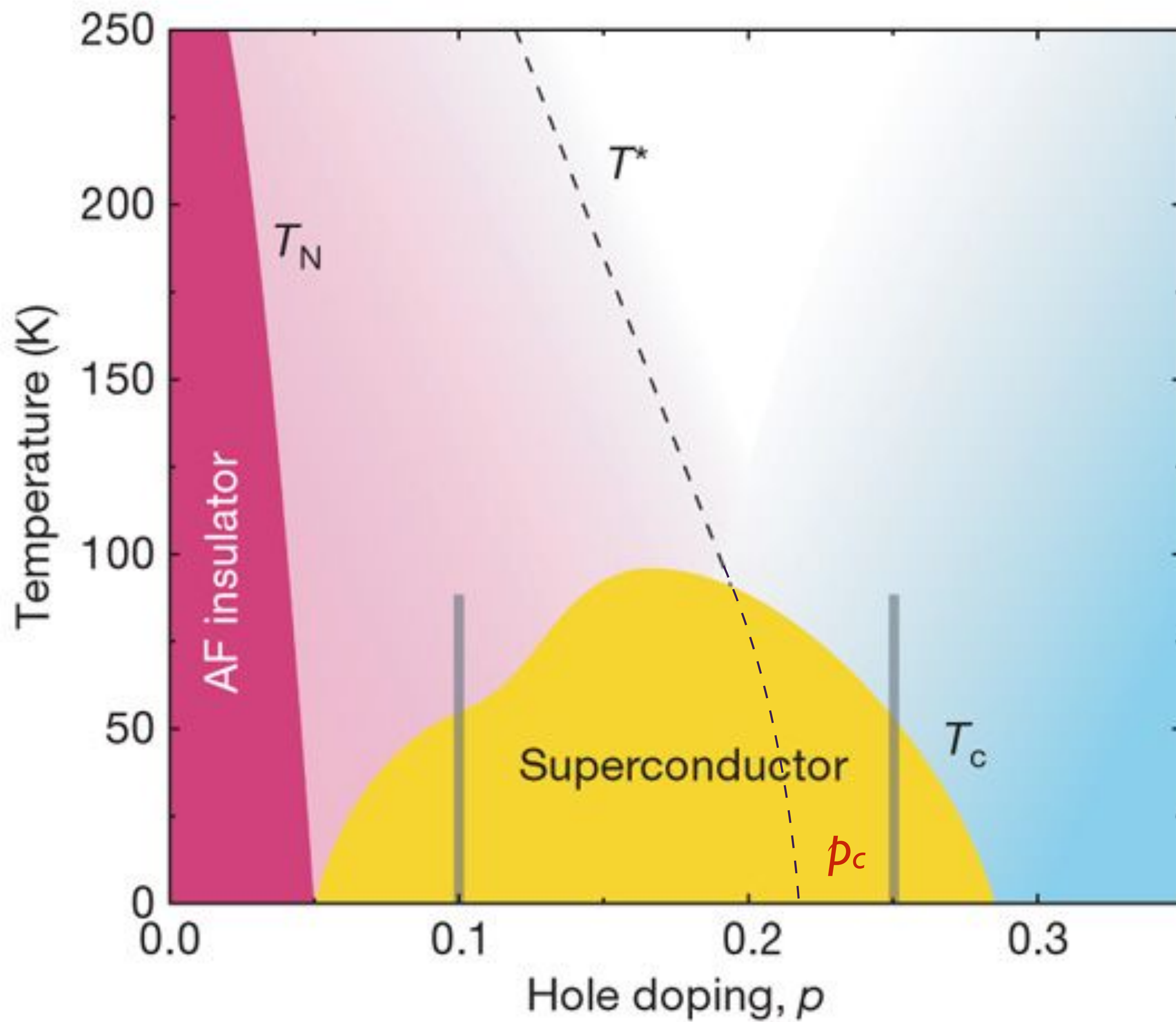
The surest path to limitless,  
clean, fusion energy

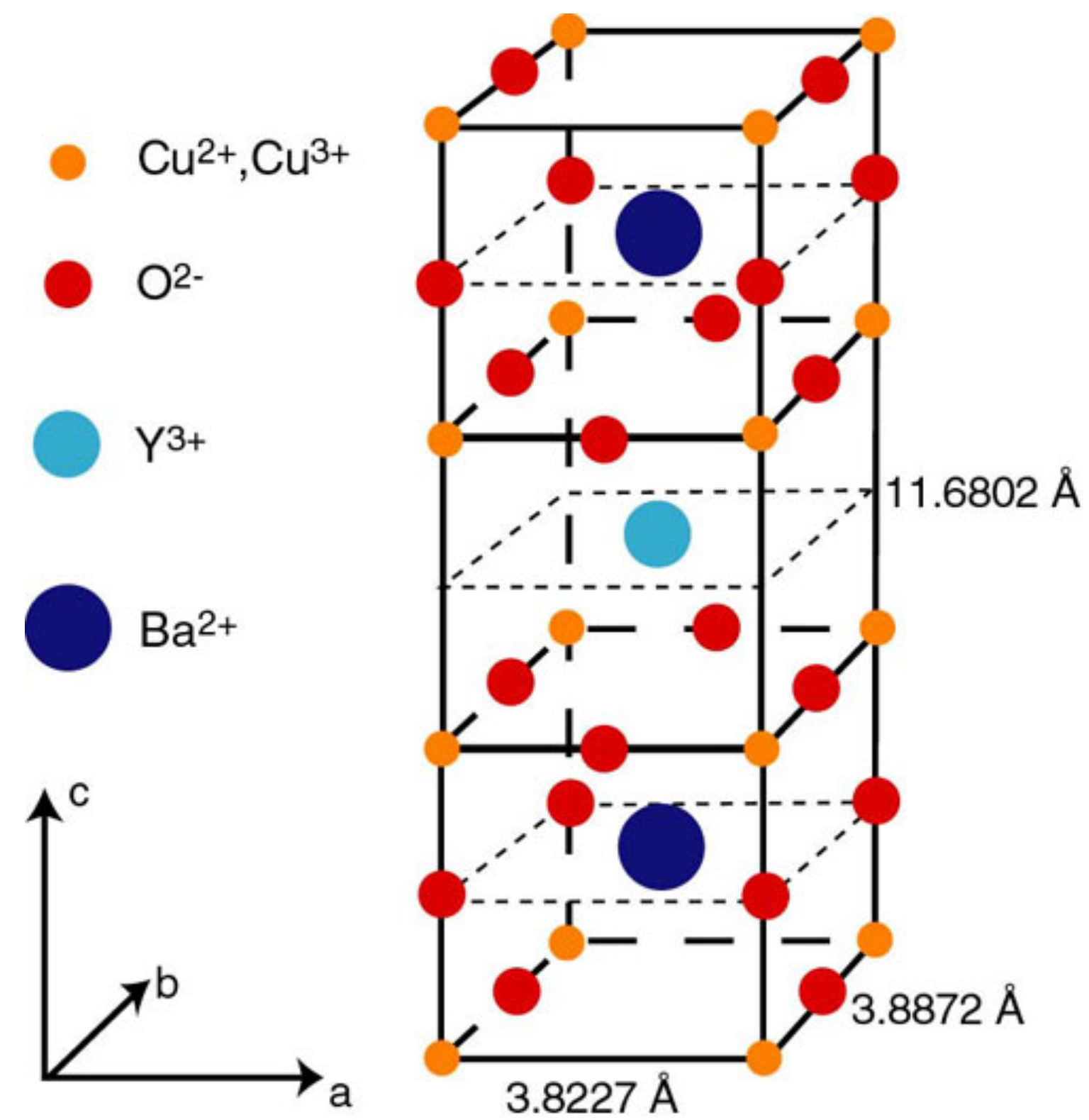
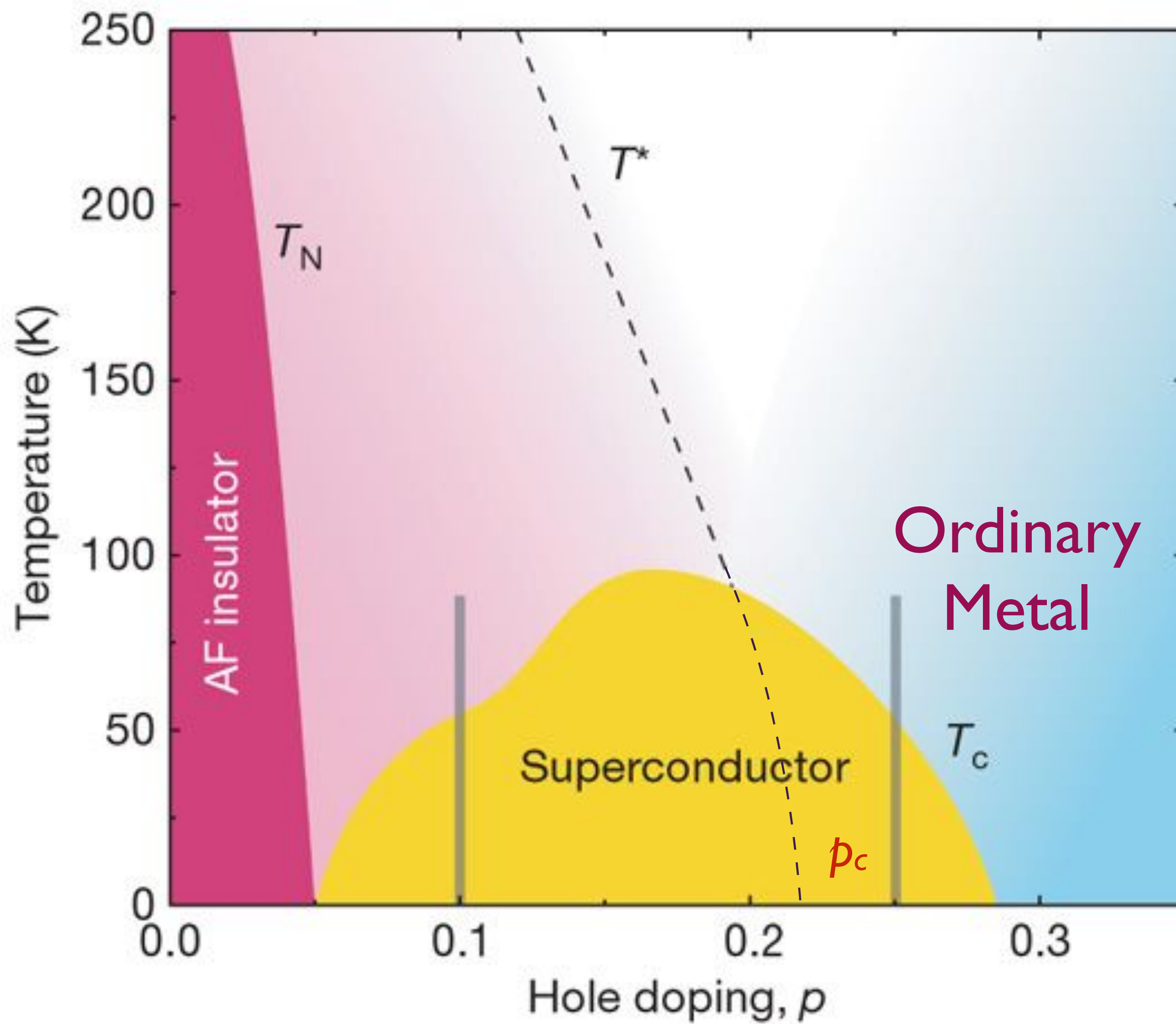
YBCO magnets allow for smaller,  
faster, and less expensive  
tokamaks for plasma fusion

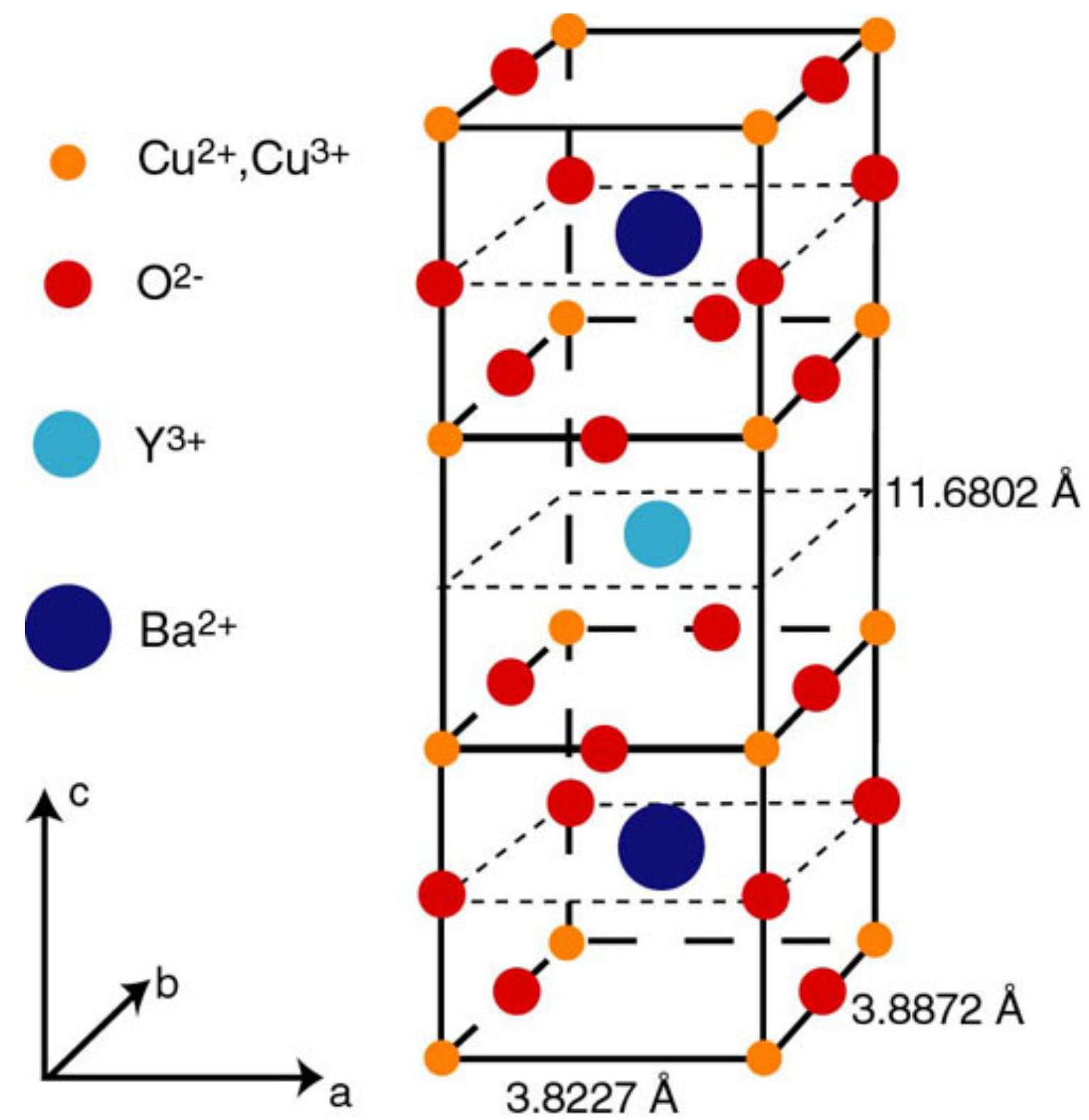
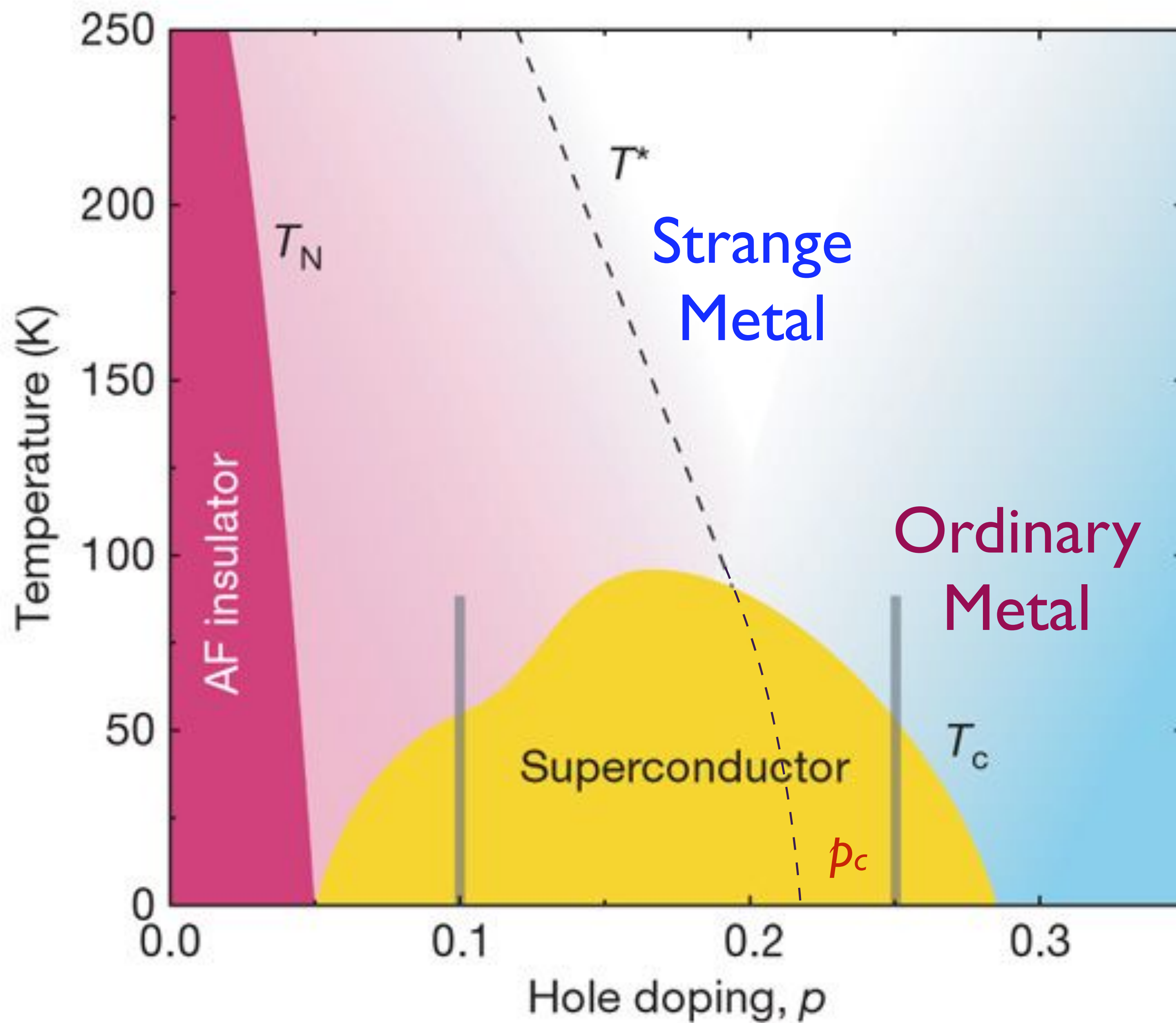


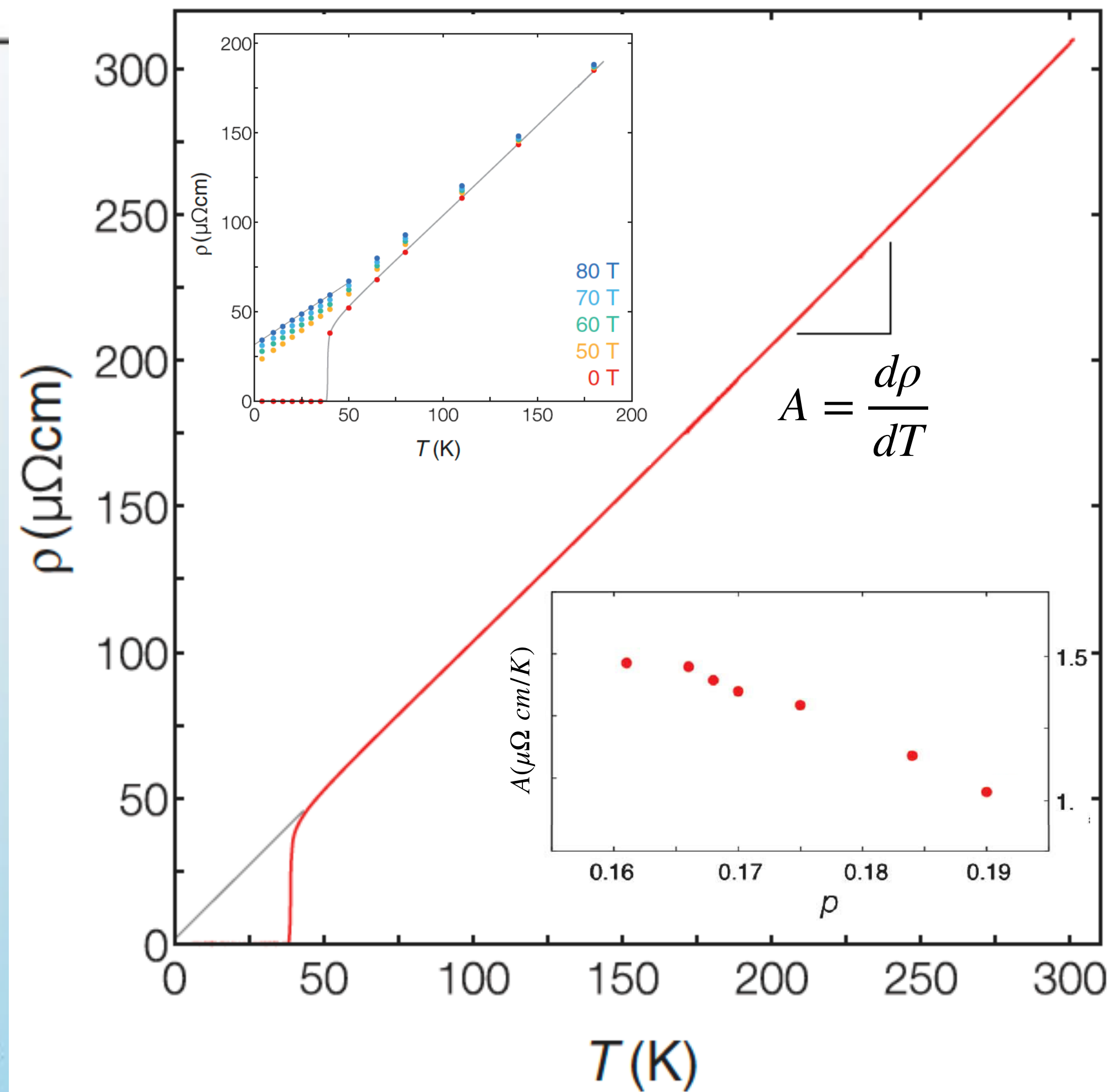
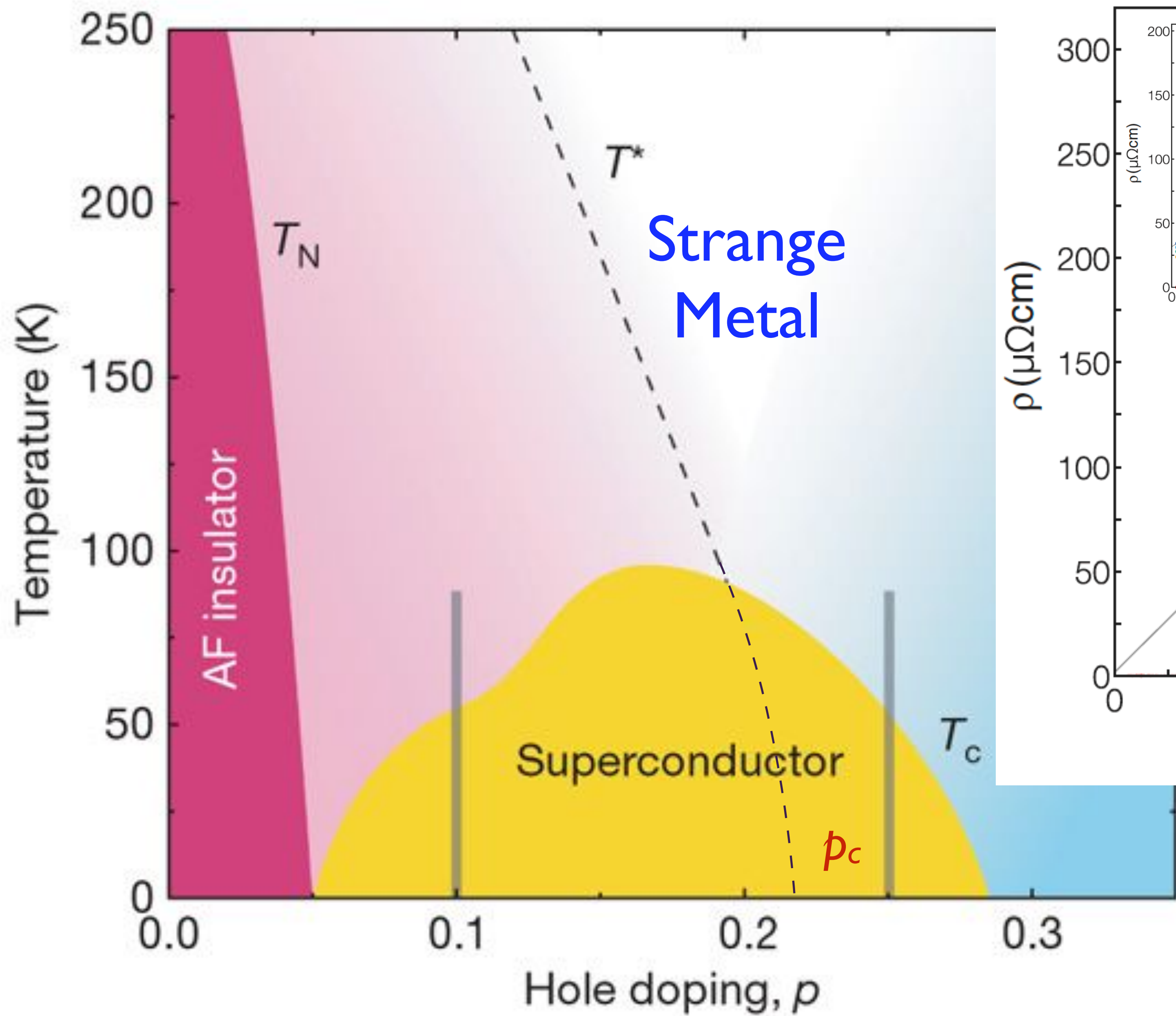
Commonwealth  
Fusion Systems











LSCO: Giraldo-Gallo et al. 2018

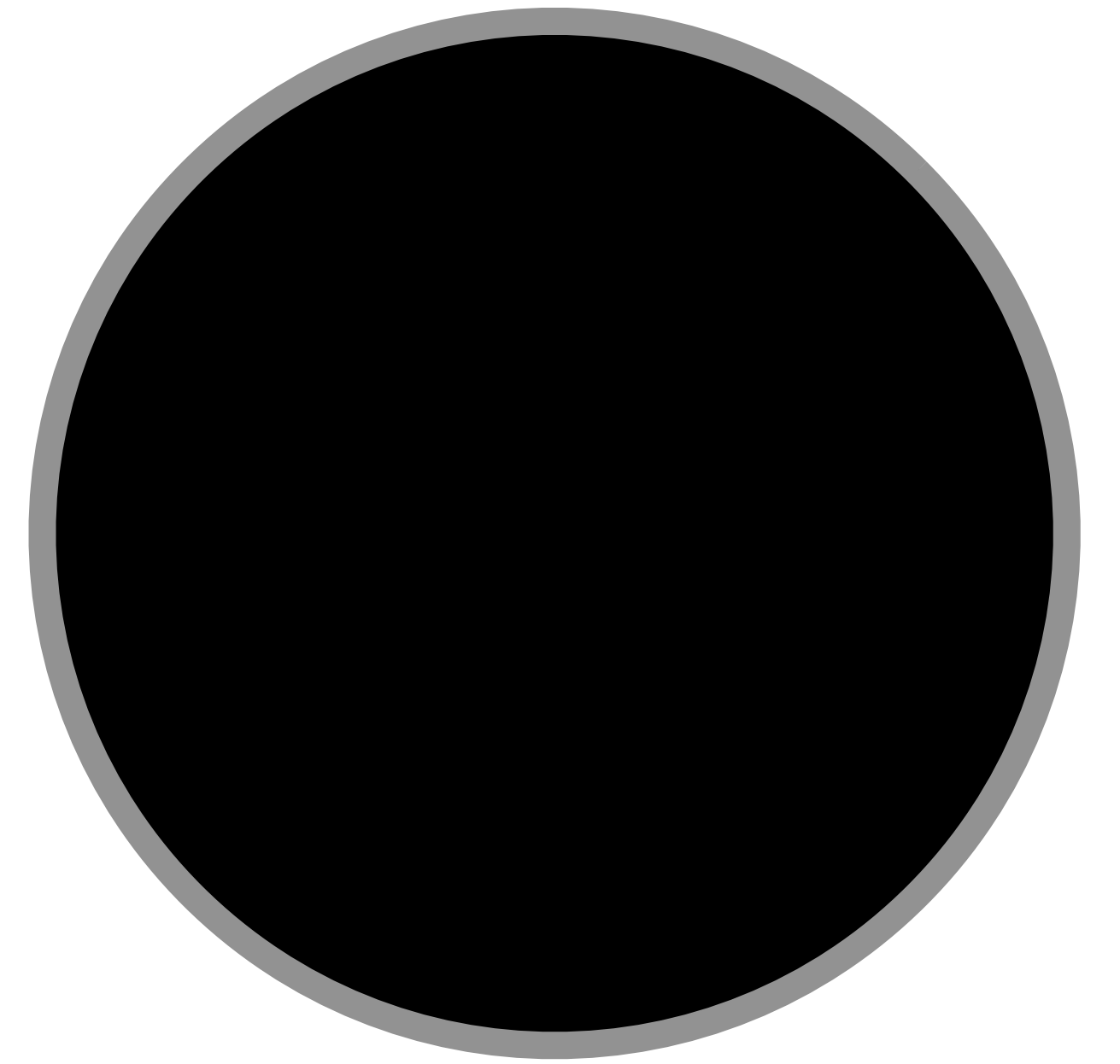
**Black  
holes**

# Black Holes

Objects so dense that light is gravitationally bound to them.



Horizon radius  $R = \frac{2GM}{c^2}$

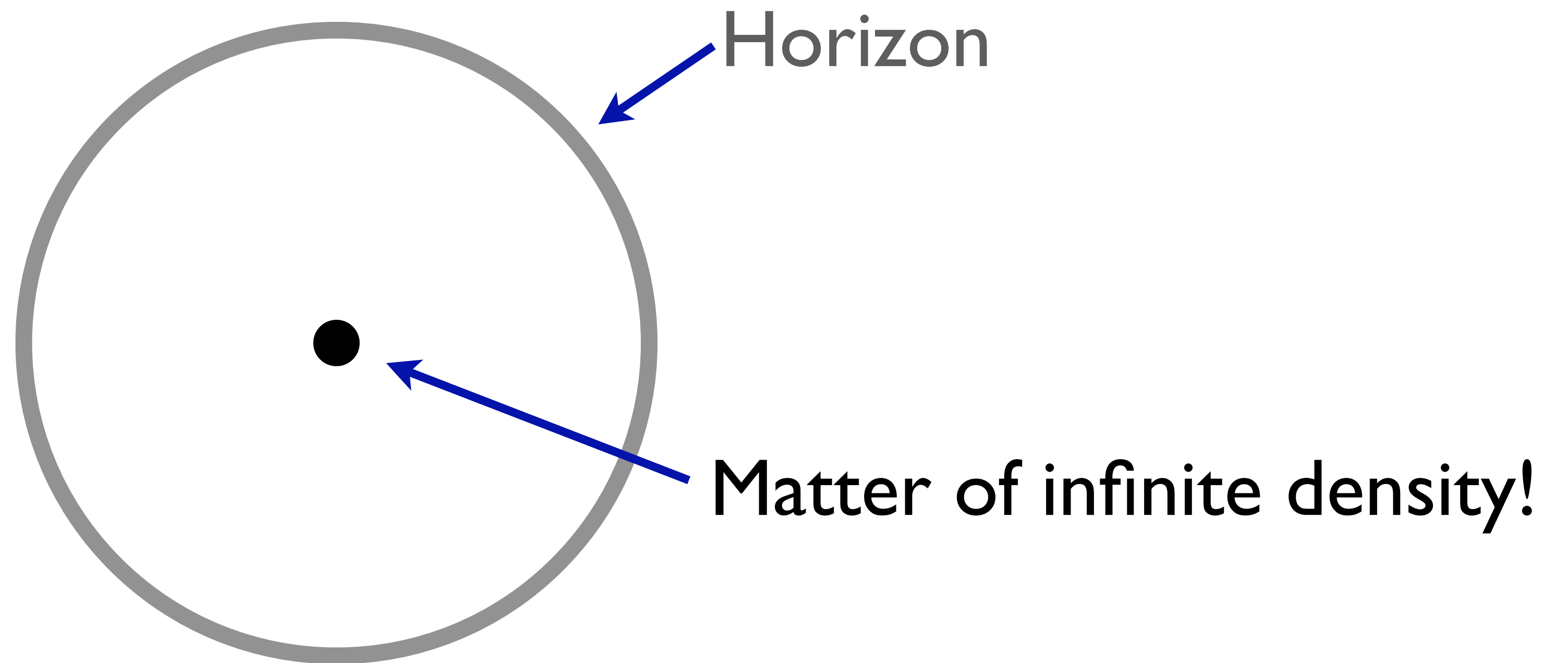


Karl Schwarzschild (1916)

$G$  Newton's constant,  $c$  velocity of light,  $M$  mass of black hole  
For  $M = \text{earth's mass}$ ,  $R \approx 9 \text{ mm!}$

# What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



# What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.

This singularity convinced many early on that black holes were unphysical solutions of Einstein's equations, and did not exist in our universe.



The supermassive black hole lurking at the heart of the Milky Way – Sagittarius A\* contains about 4.3 million solar masses

$$R = 1.3 \times 10^{11} \text{ m}$$

$\approx$  earth's orbit

Event Horizon Telescope  
May 12, 2022

# What is inside a black hole ???

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# What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.

This singularity convinced many early on that black holes were unphysical solutions of Einstein's equations, and did not exist in our universe.

In any case, it was clear that quantum theory should be applied to the collapsed matter, but no one knew how to.

# What is inside a black hole ???

Hawking (1975): when viewed from the outside, black holes have an entropy and a temperature, and slowly evaporate like any thermal object



$$T = \frac{\hbar c^3}{8\pi G M k_B}$$

What is  
quantum entanglement?

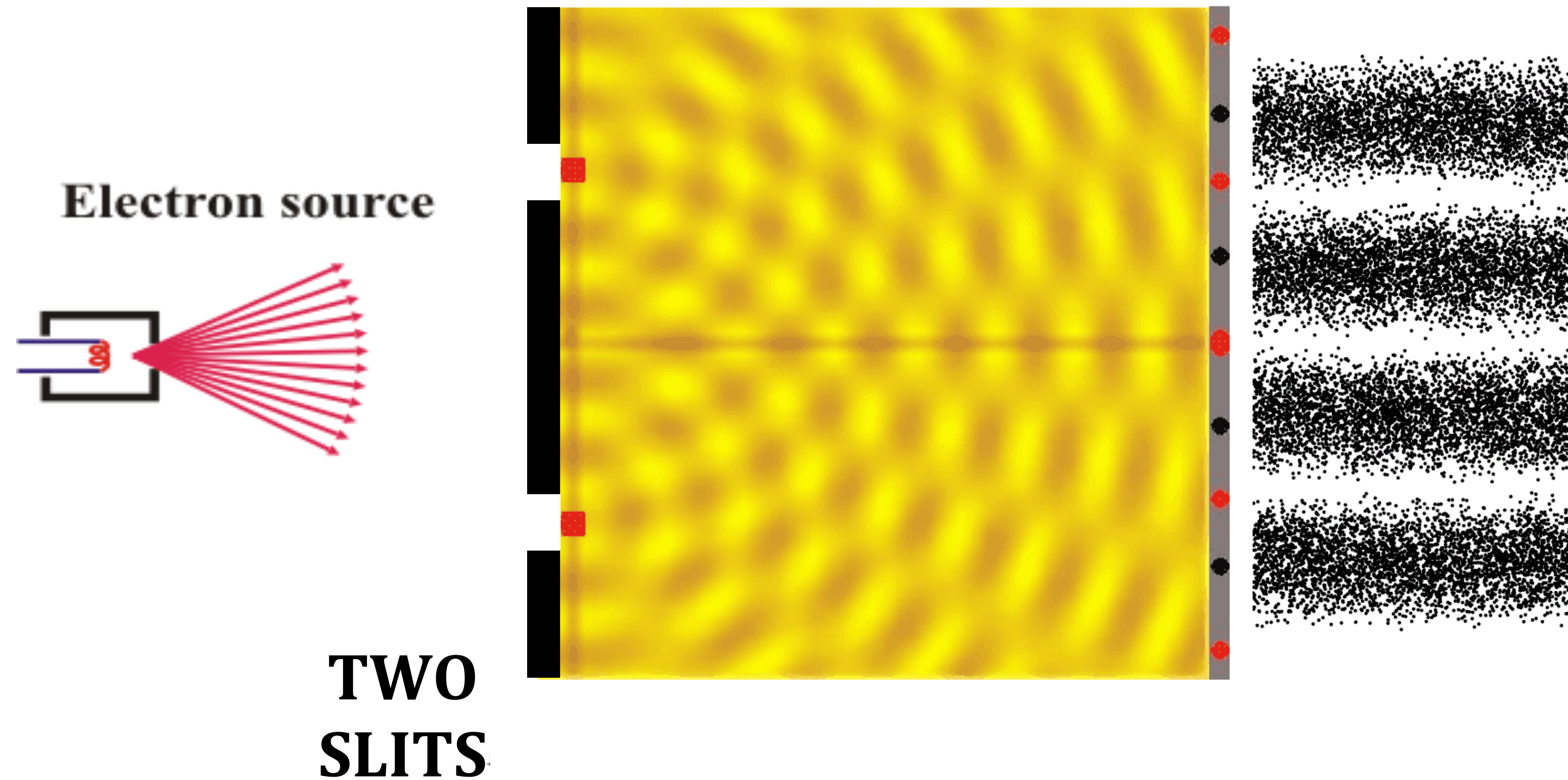


*“About your cat, Mr. Schrödinger—I have good news and bad news.”*

The most remarkable new idea in the quantum theory is the  
*principle of superposition*:  
a physical system can be in a  
superposition of two (or more) distinct states.

# Principles of Quantum Mechanics: I. Quantum Superposition

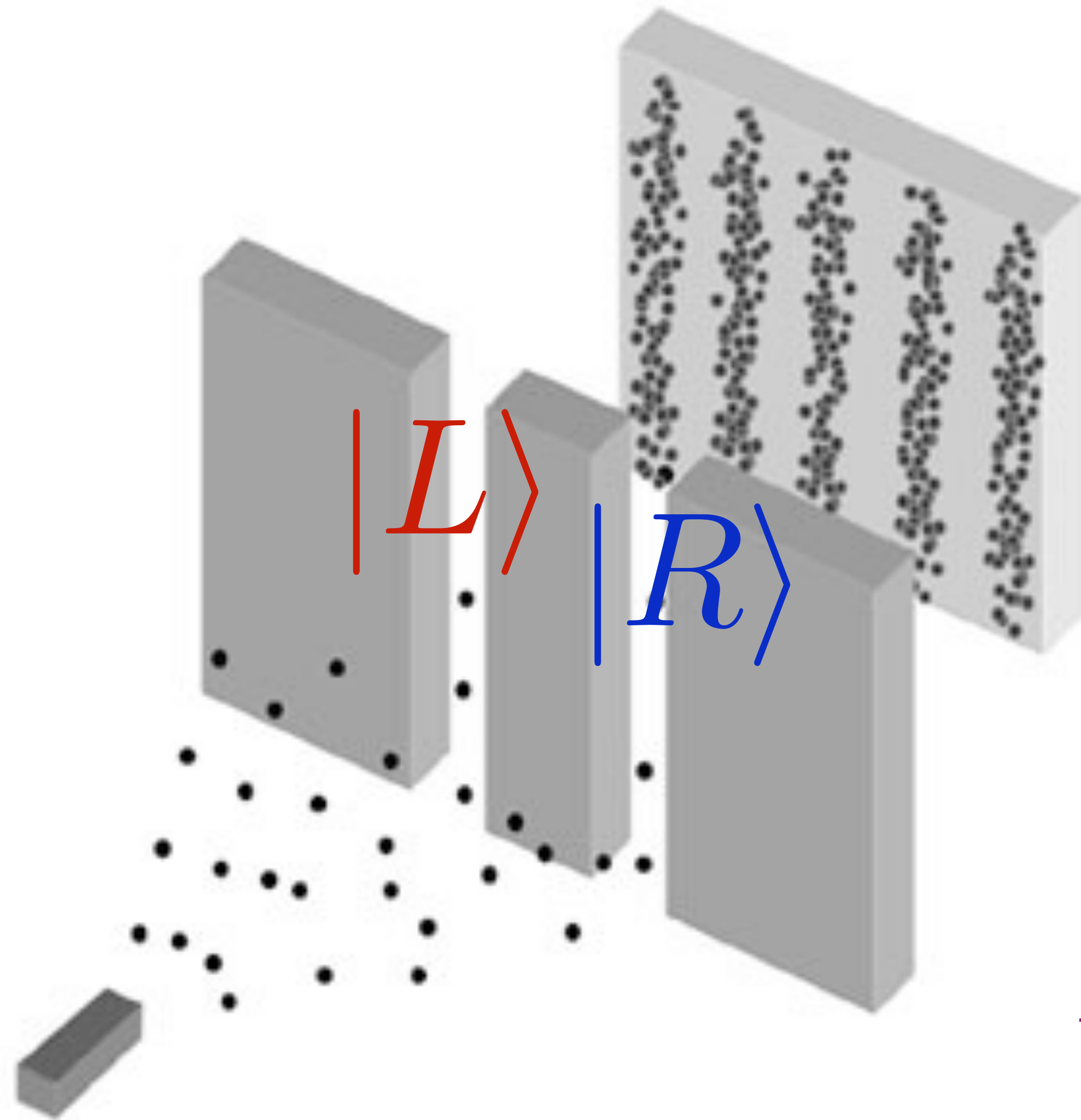
## The double slit experiment



Unlike water waves, electrons arrive one-by-one (so is it like a particle ?)

Interference of electrons

## The double slit experiment



Let  $|L\rangle$  represent the state with the electron in the left slit

And  $|R\rangle$  represents the state with the electron in the right slit

Actual state of *each* electron is

$$|L\rangle + |R\rangle$$

MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

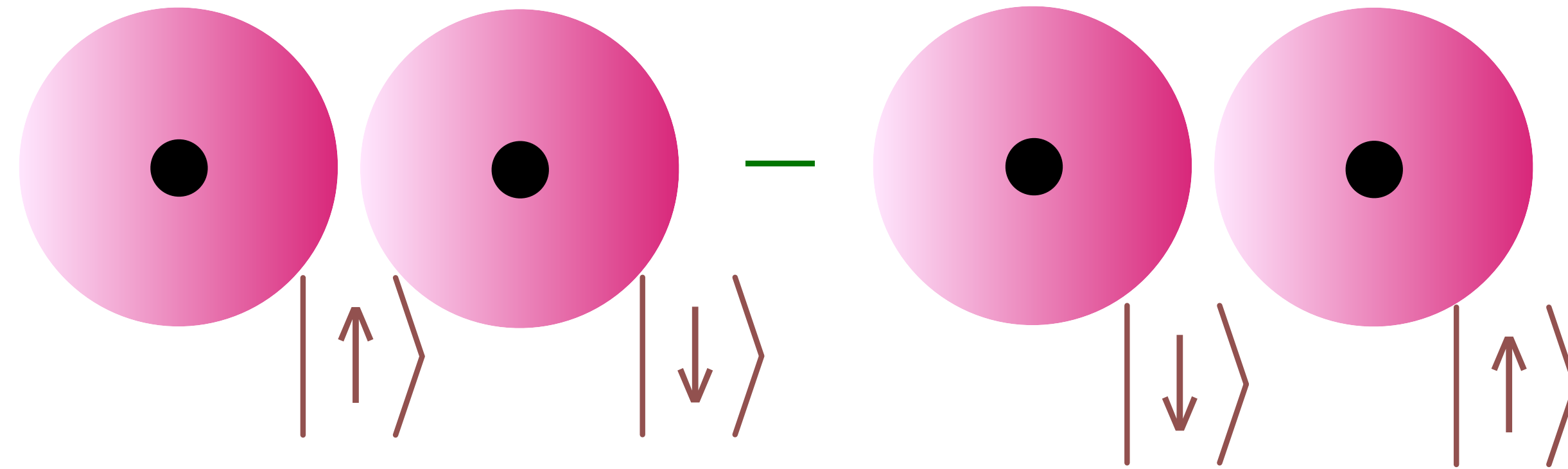
# Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

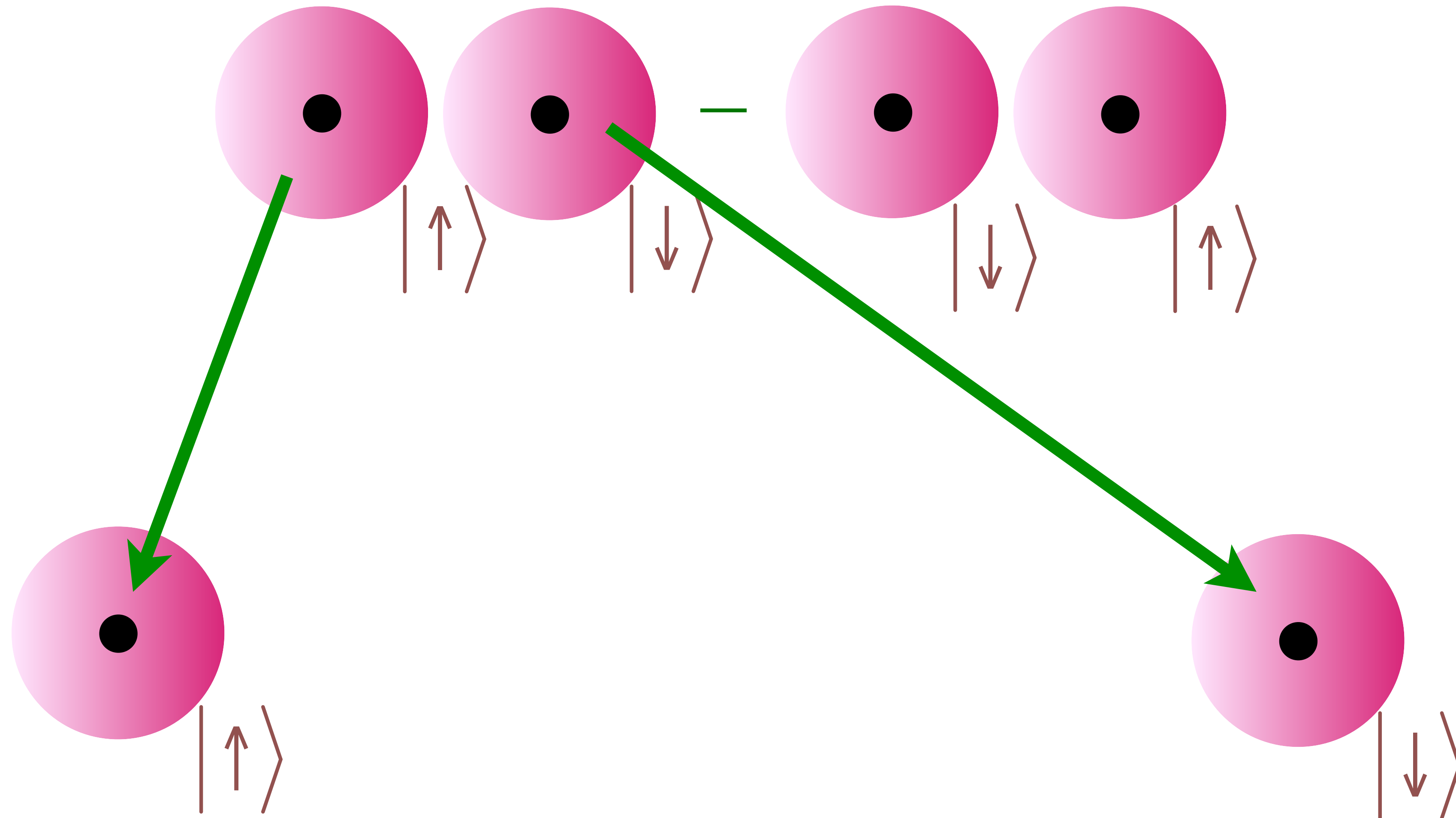
# Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



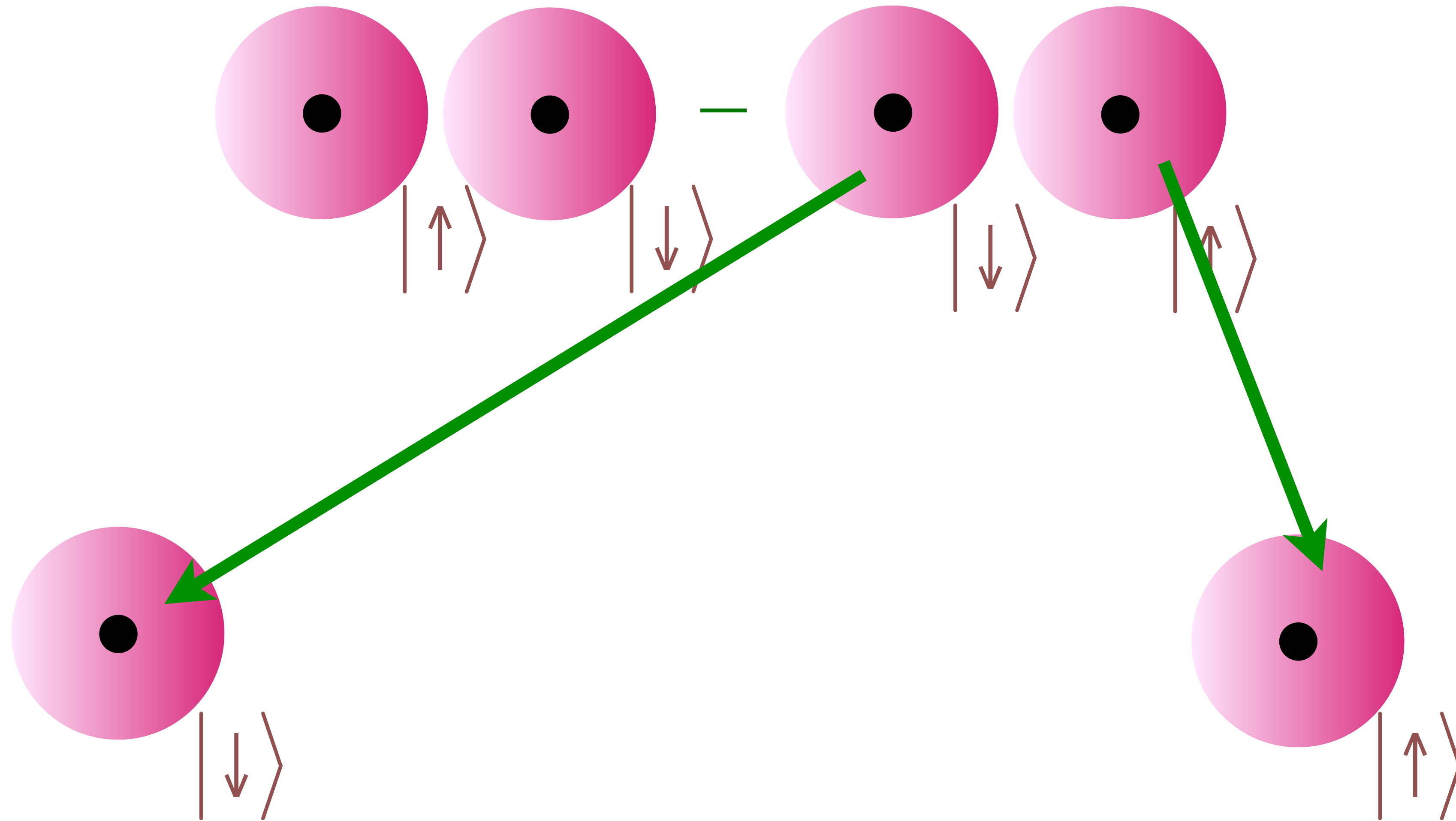
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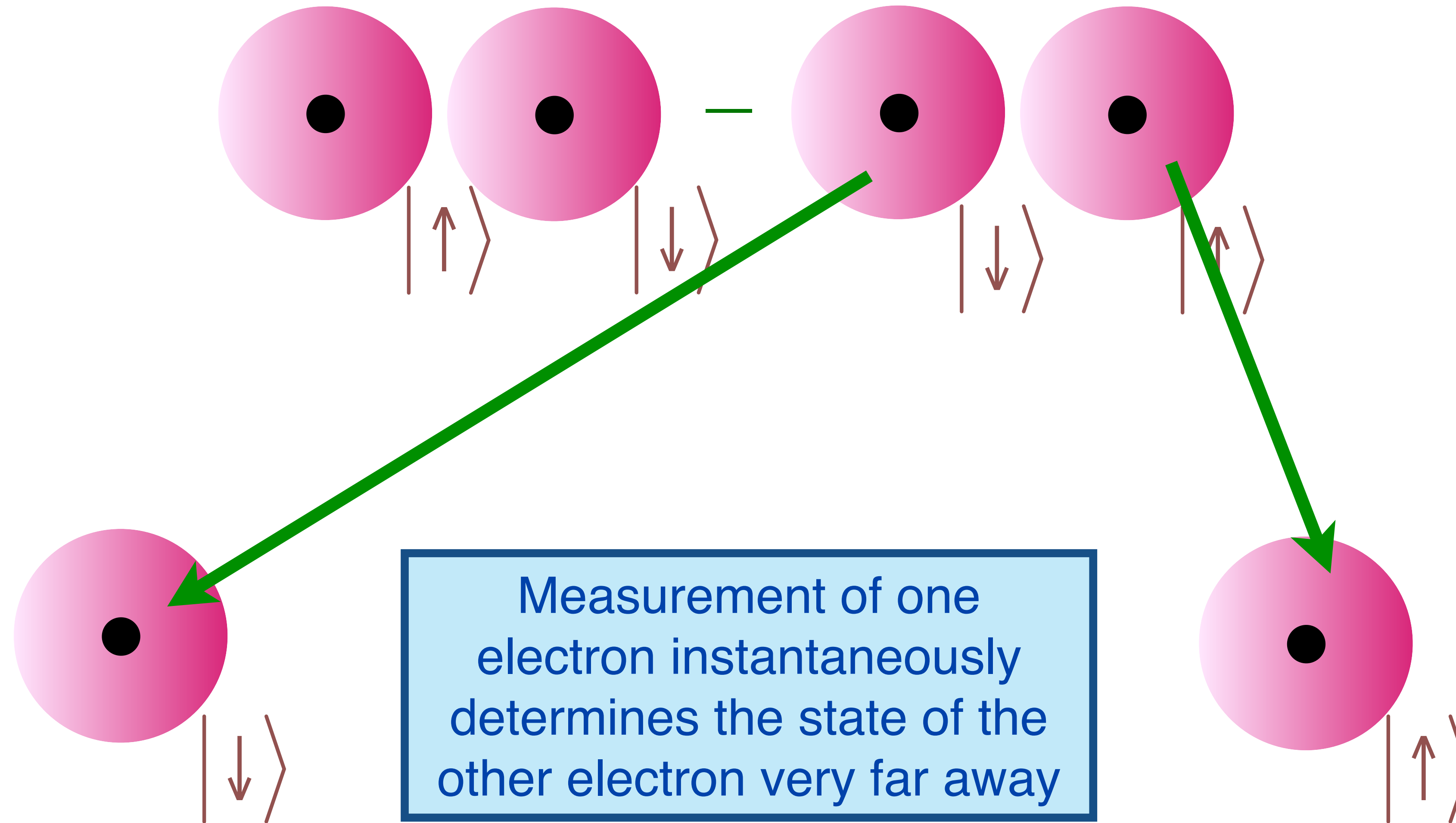
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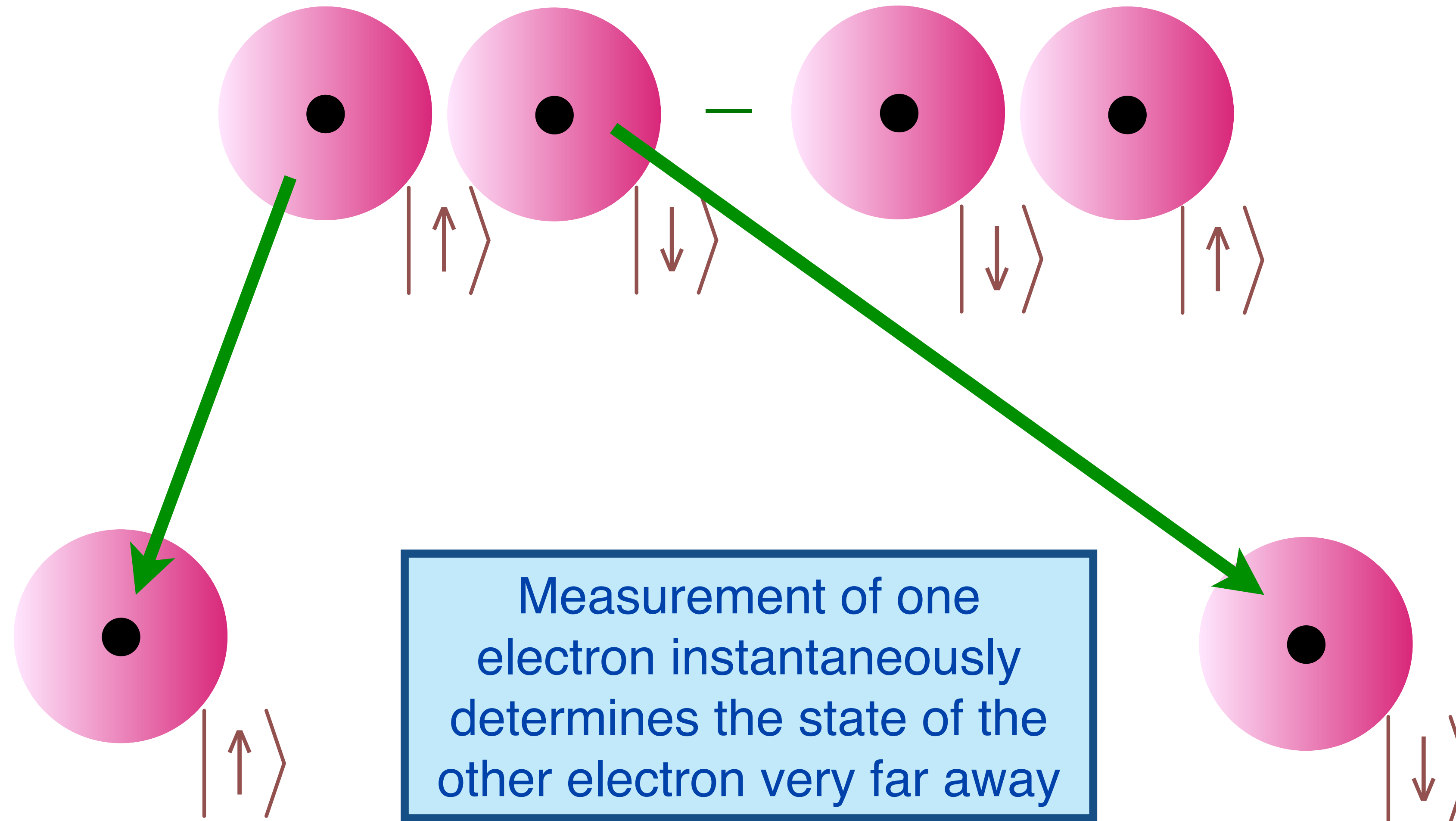
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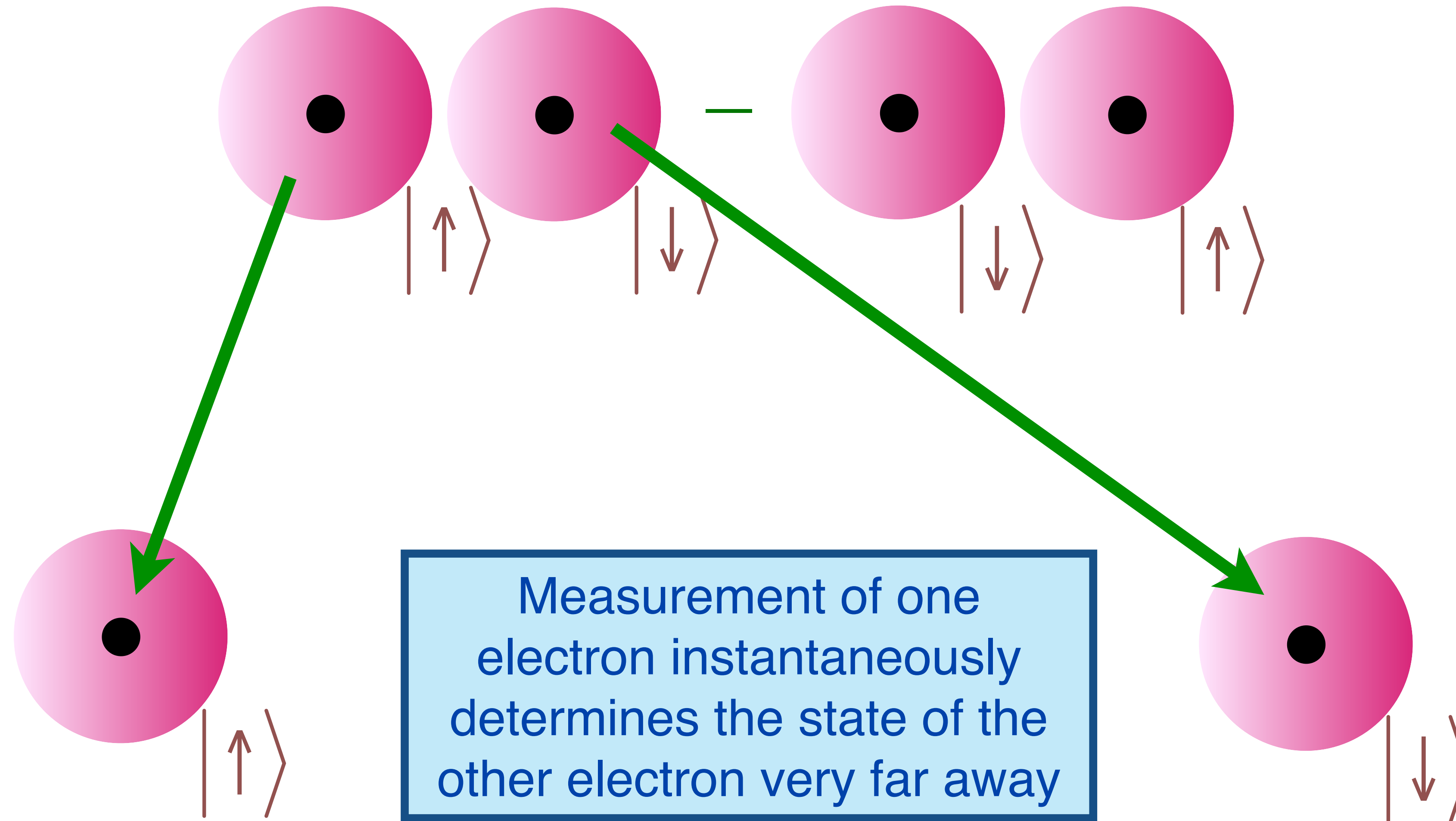
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# Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



**Spooky action at a distance !**

natürlicher  
deren Notwendigkeit im  
mus ja zuerst von Dir klar erkannt wurde, einen Bedeutung  
Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft dar-  
an glauben, weil die Theorie mit dem Grundsatz unvereinbar  
ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstel-  
len soll, ohne spukhafte Fernwirkungen. Allerdings bin ich  
überzeugt, daß es wirklich mit der Theorie

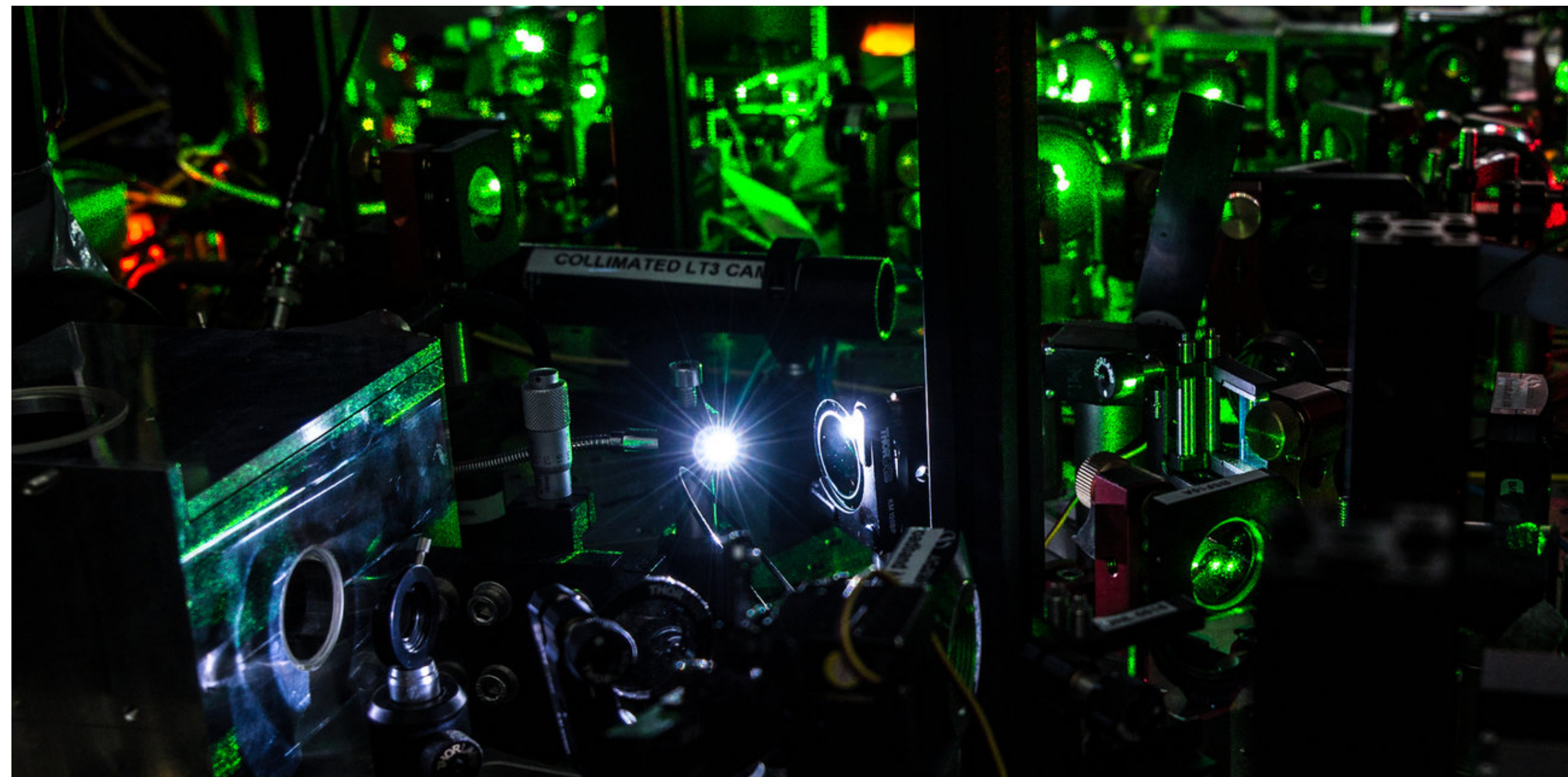
I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

Albert Einstein to Max Born, 3 March 1947

**The New York Times**

# Sorry, Einstein. Quantum Study Suggests 'Spooky Action' Is Real.

By **JOHN MARKOFF** OCT. 21, 2015



Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

# Great discoveries in physics

Entropy (1870)

Superconductivity (1911)

Black holes (1916)

Quantum entanglement (1935)

Needed,  
to solve open problems in the theory of  
superconductivity and black holes:

A solvable model of quantum entanglement  
of 3, 4, 5, ...  $\infty$  particles

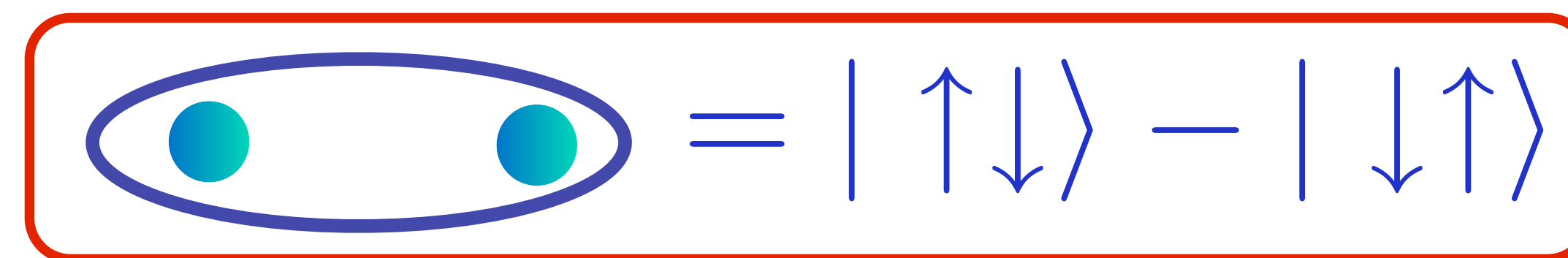
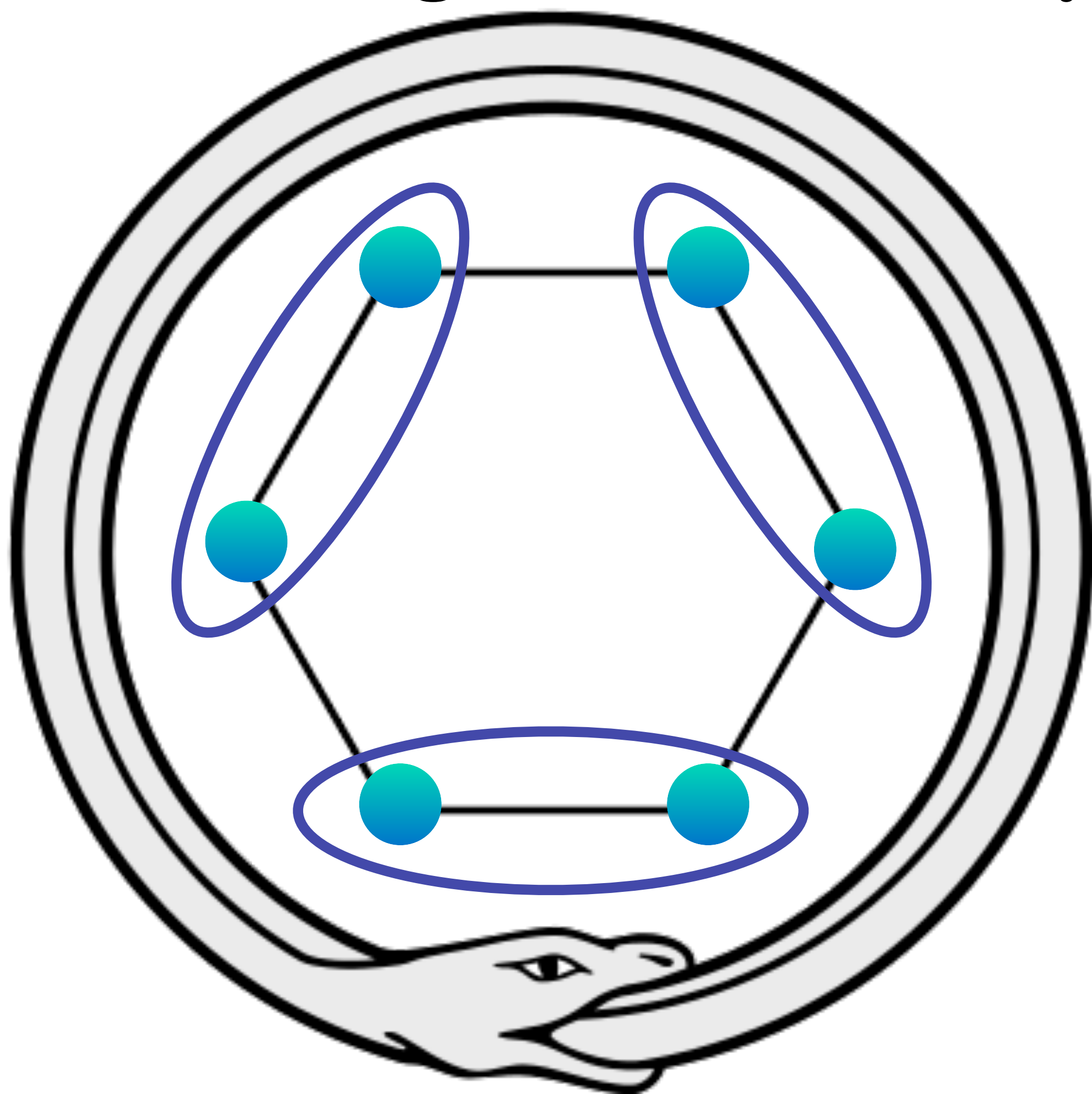
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**The Sachdev-Ye-Kitaev model  
of many-particle entanglement**

# Kekulé's spooky dream (1865)

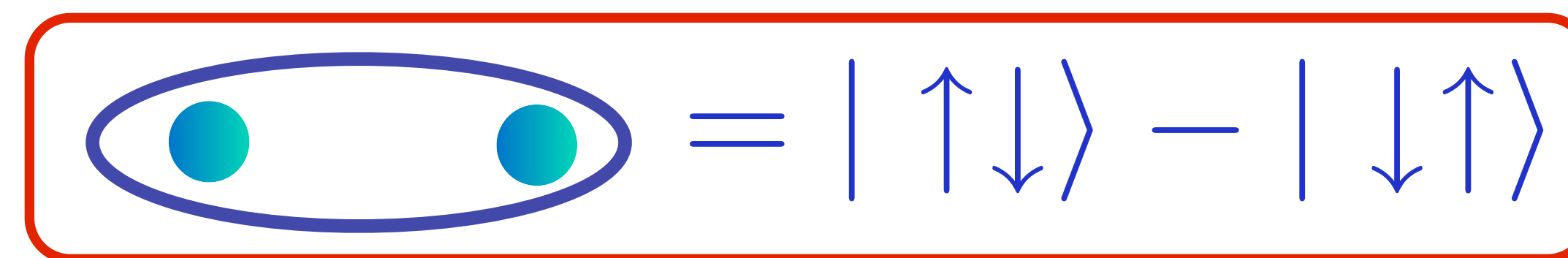
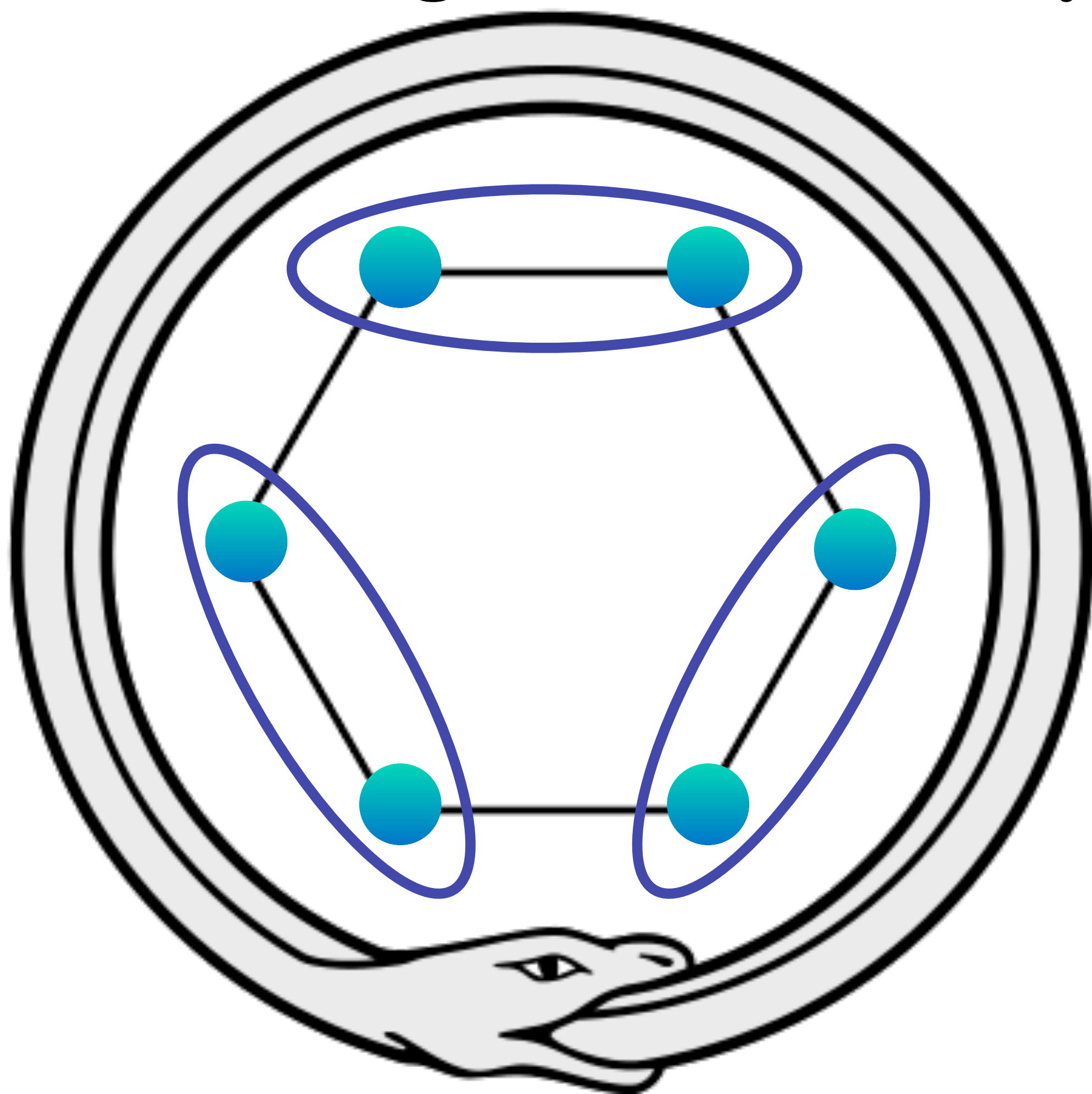
Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail\*



**Benzene**

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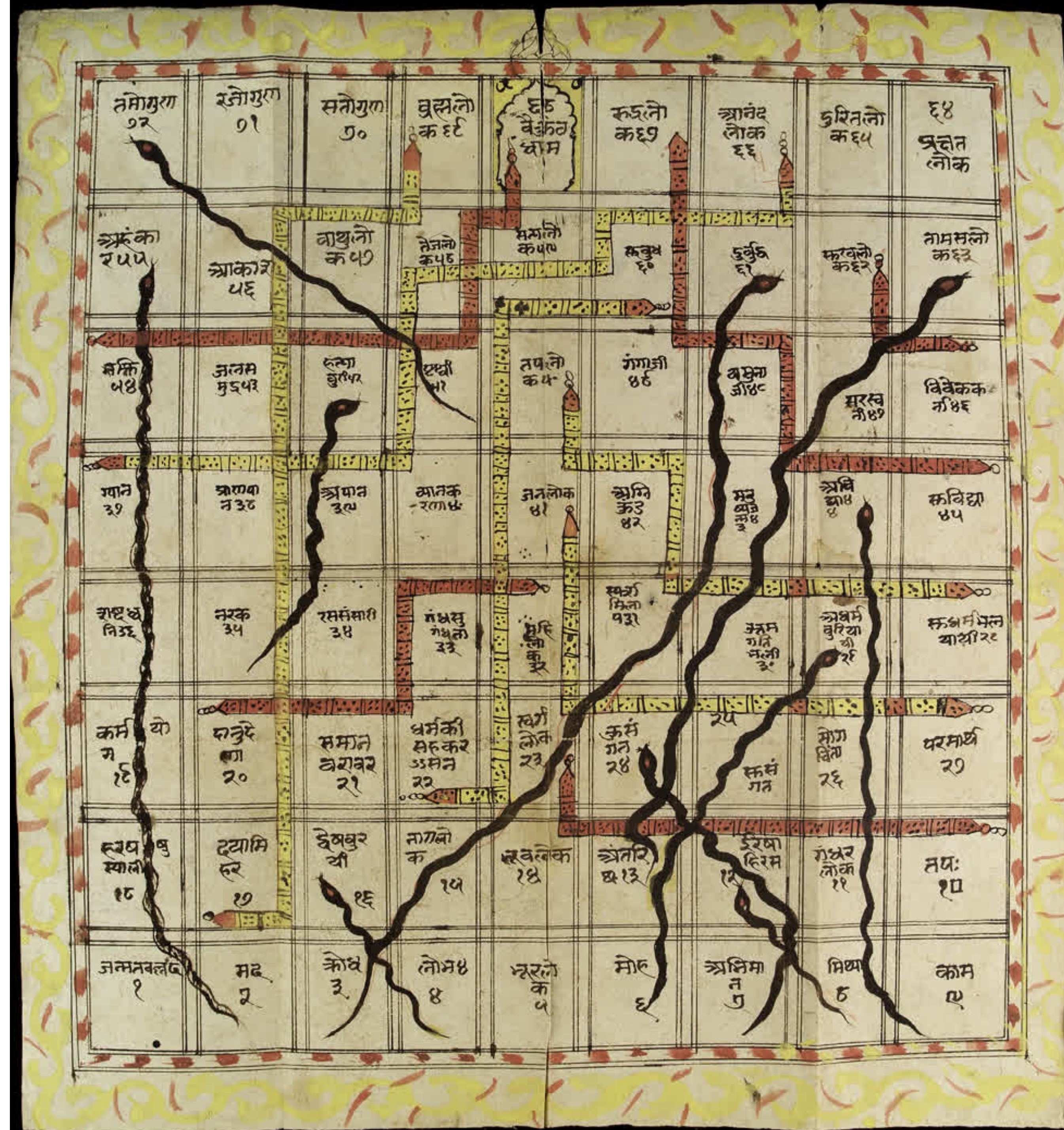


Benzene

My  
spooky  
dream\*

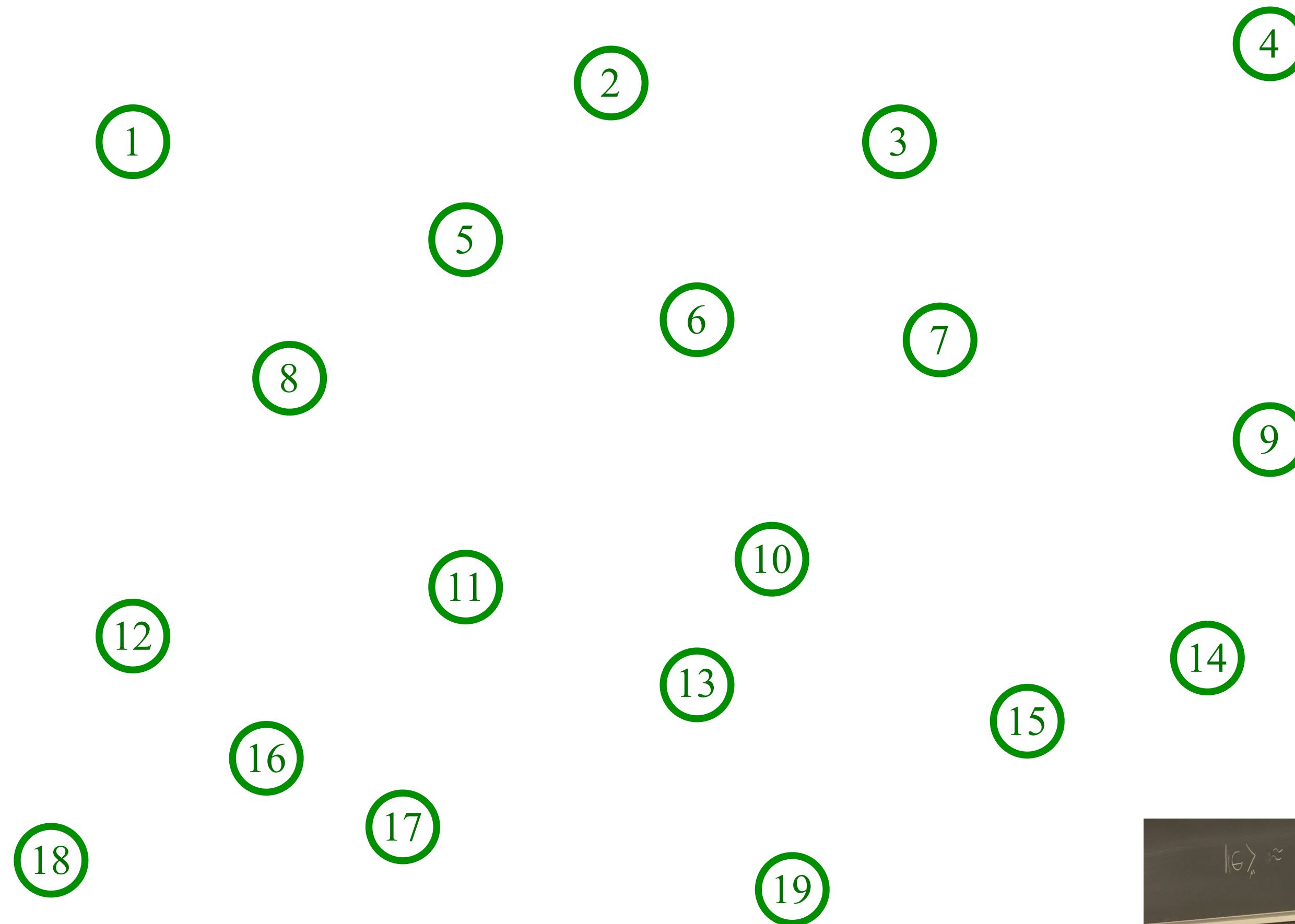
Ancient  
Indian  
game of  
Snakes  
and  
Ladders

\*Not true

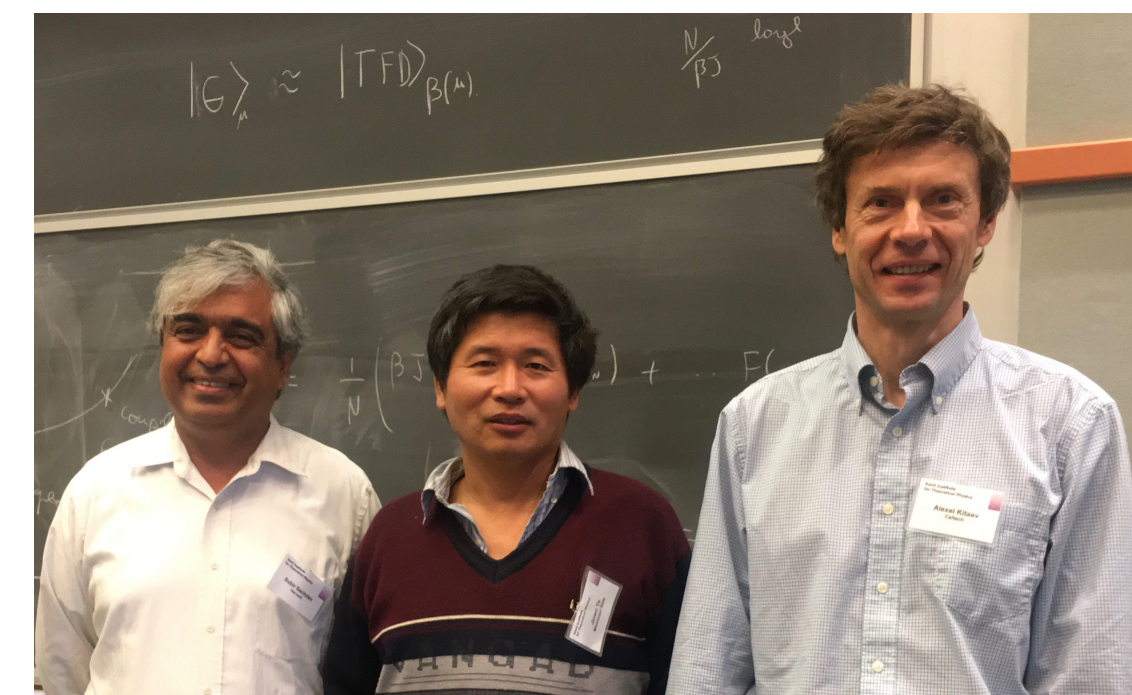


# The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

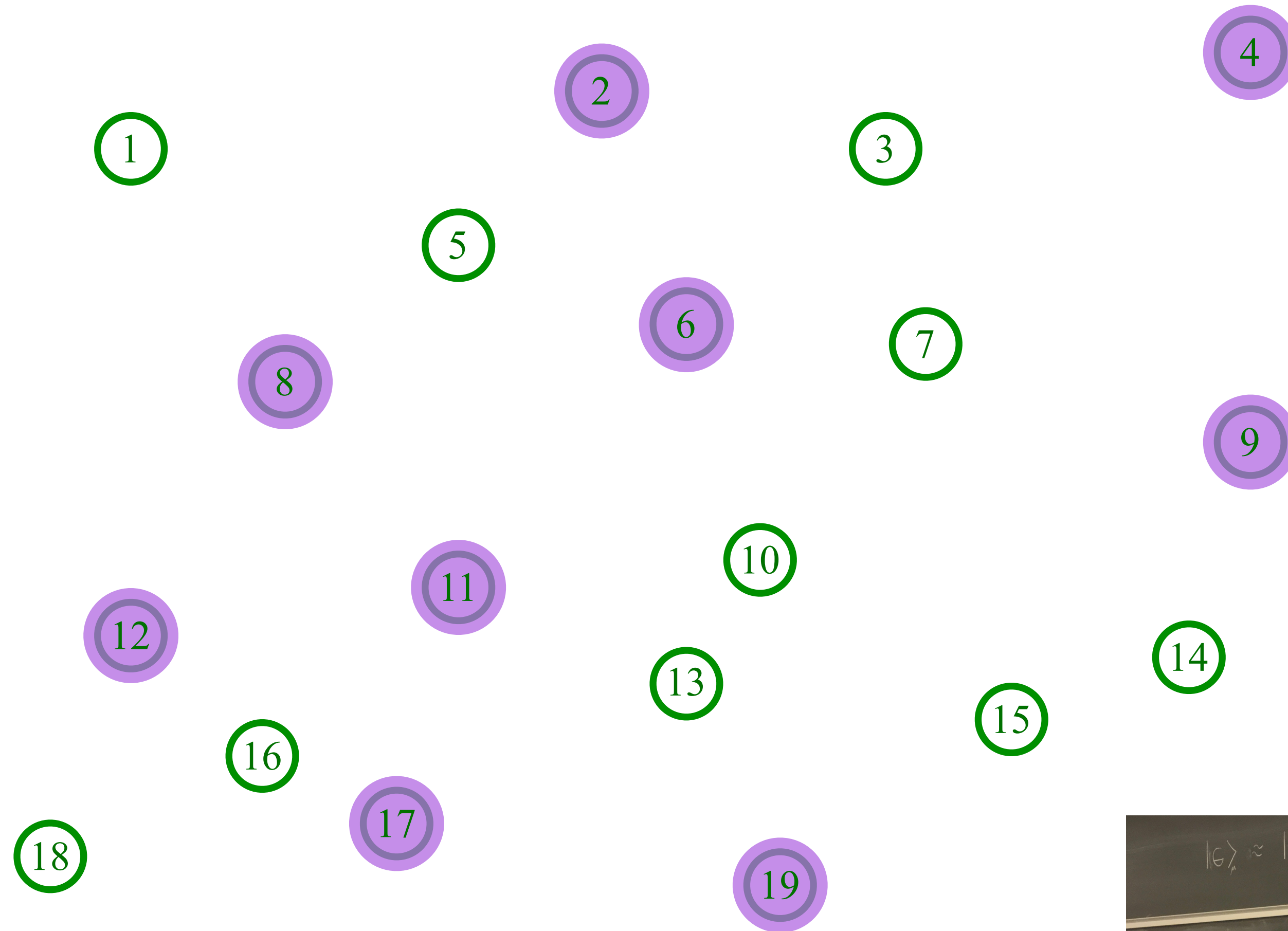


Pick a set of random positions

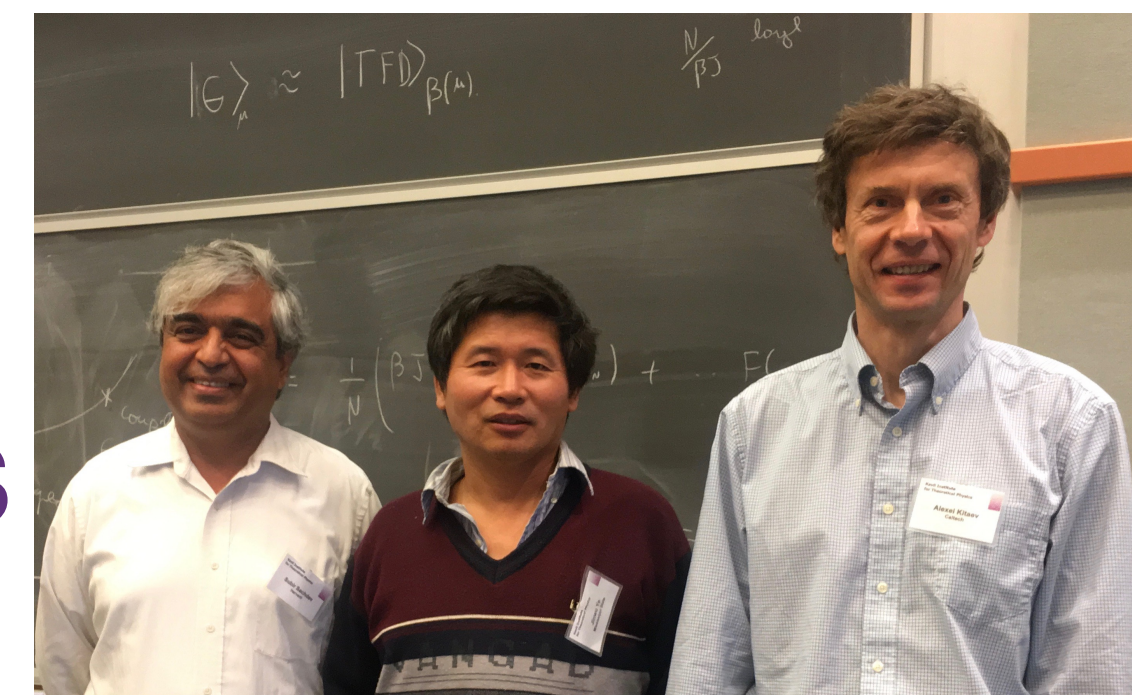


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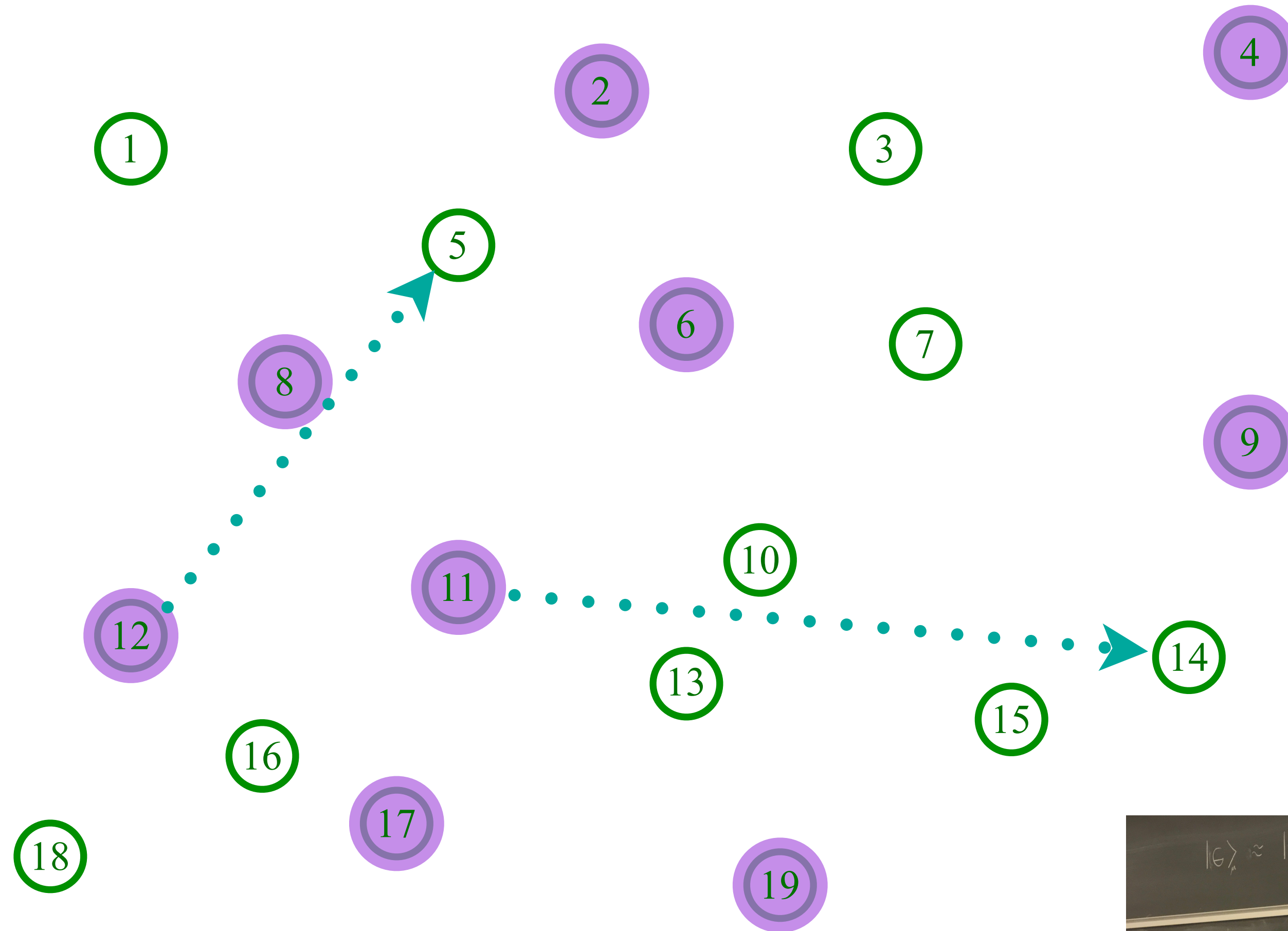
Place electrons randomly on some sites



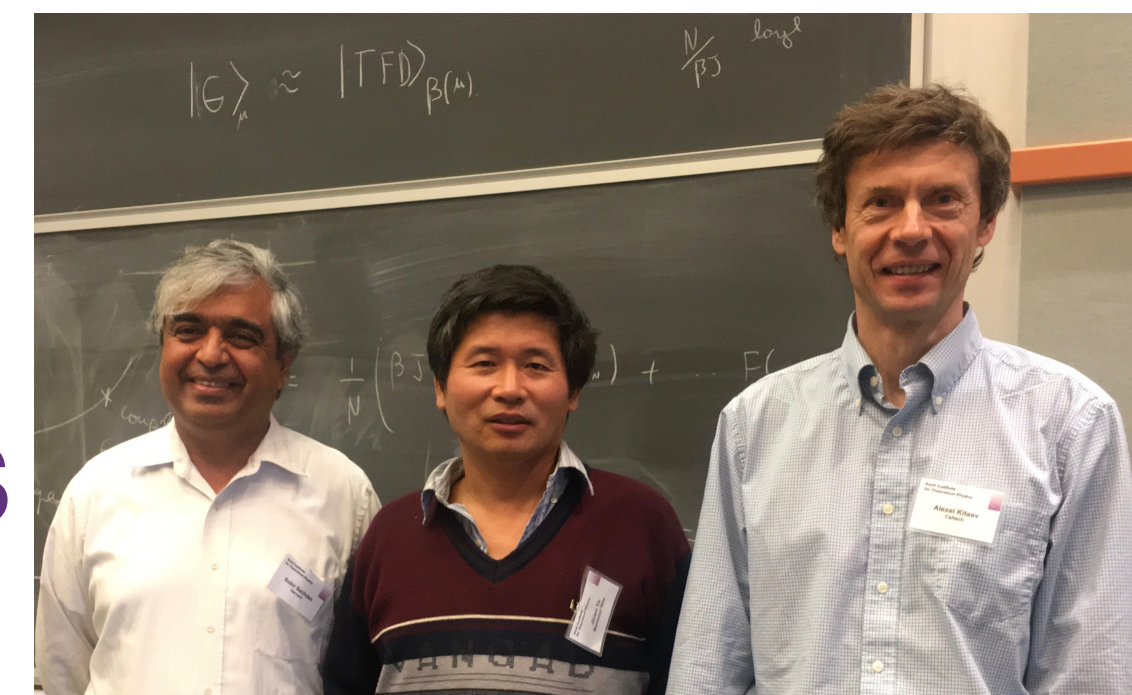
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$$U_{11,12;5,14}$$



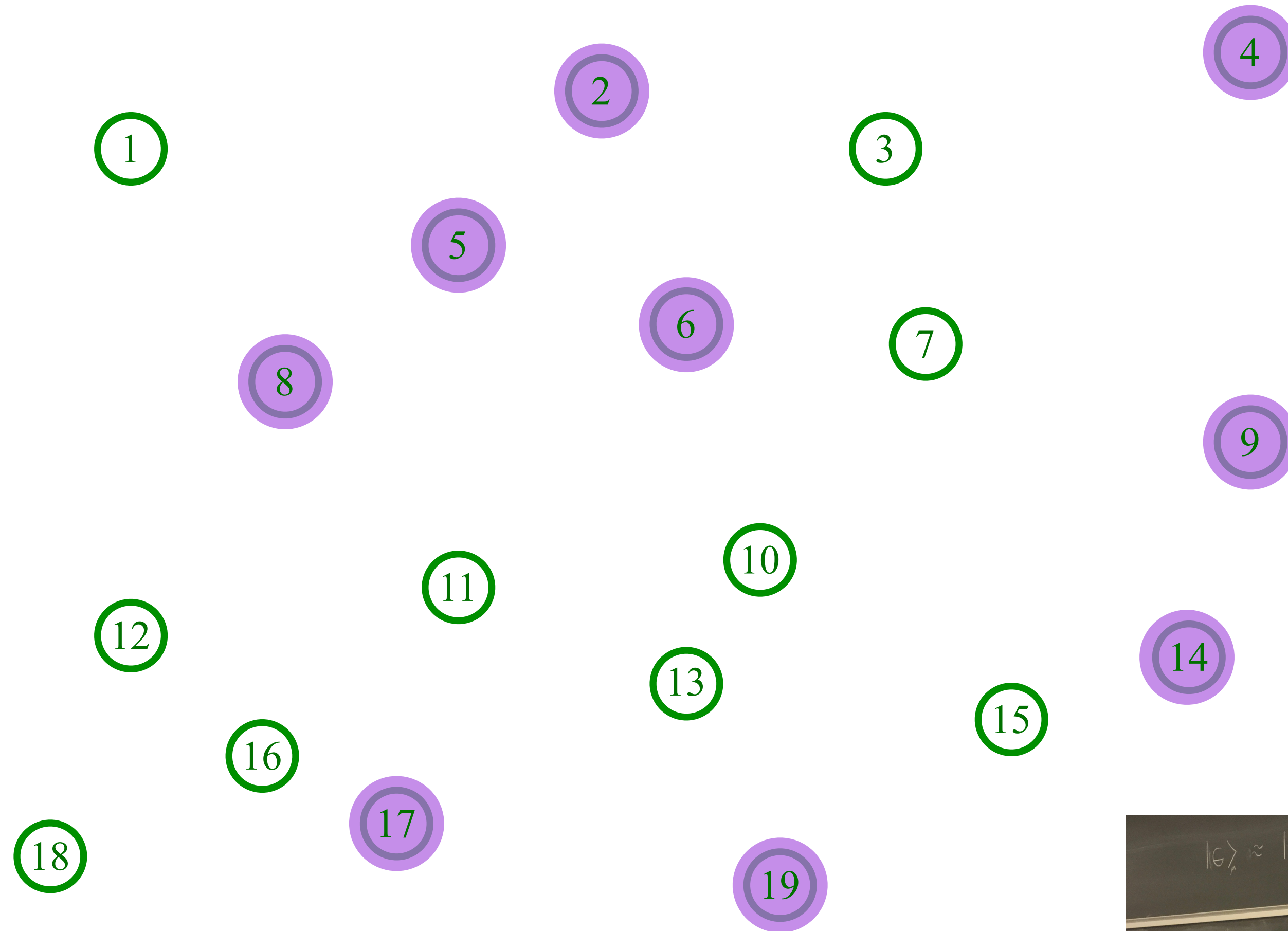
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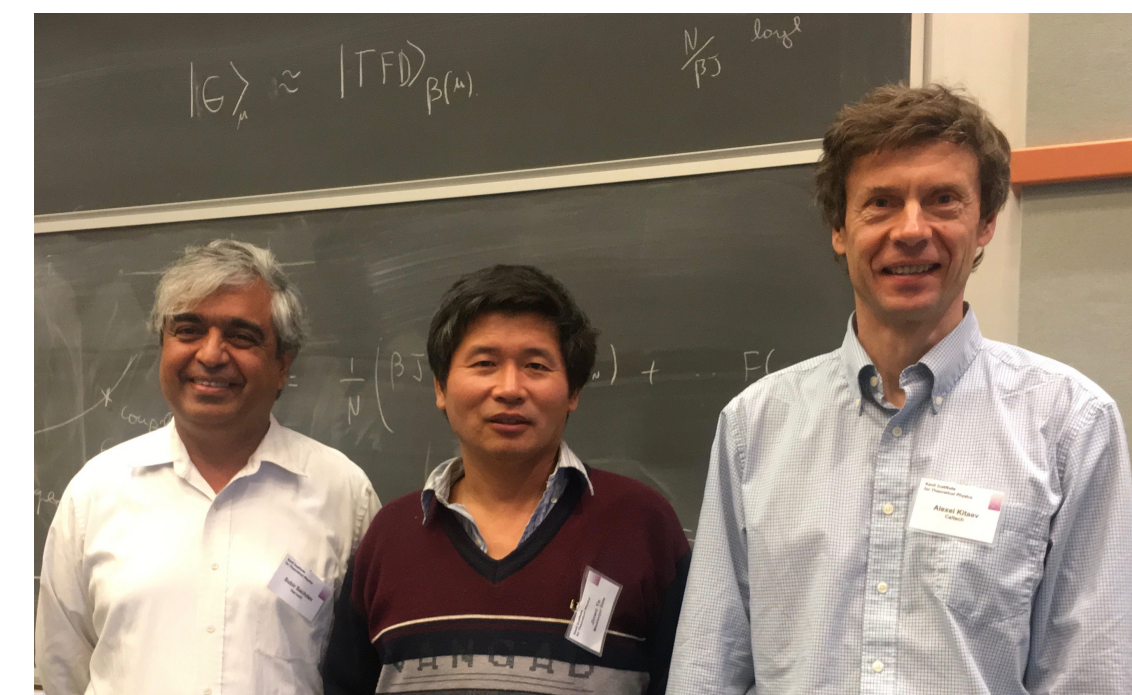
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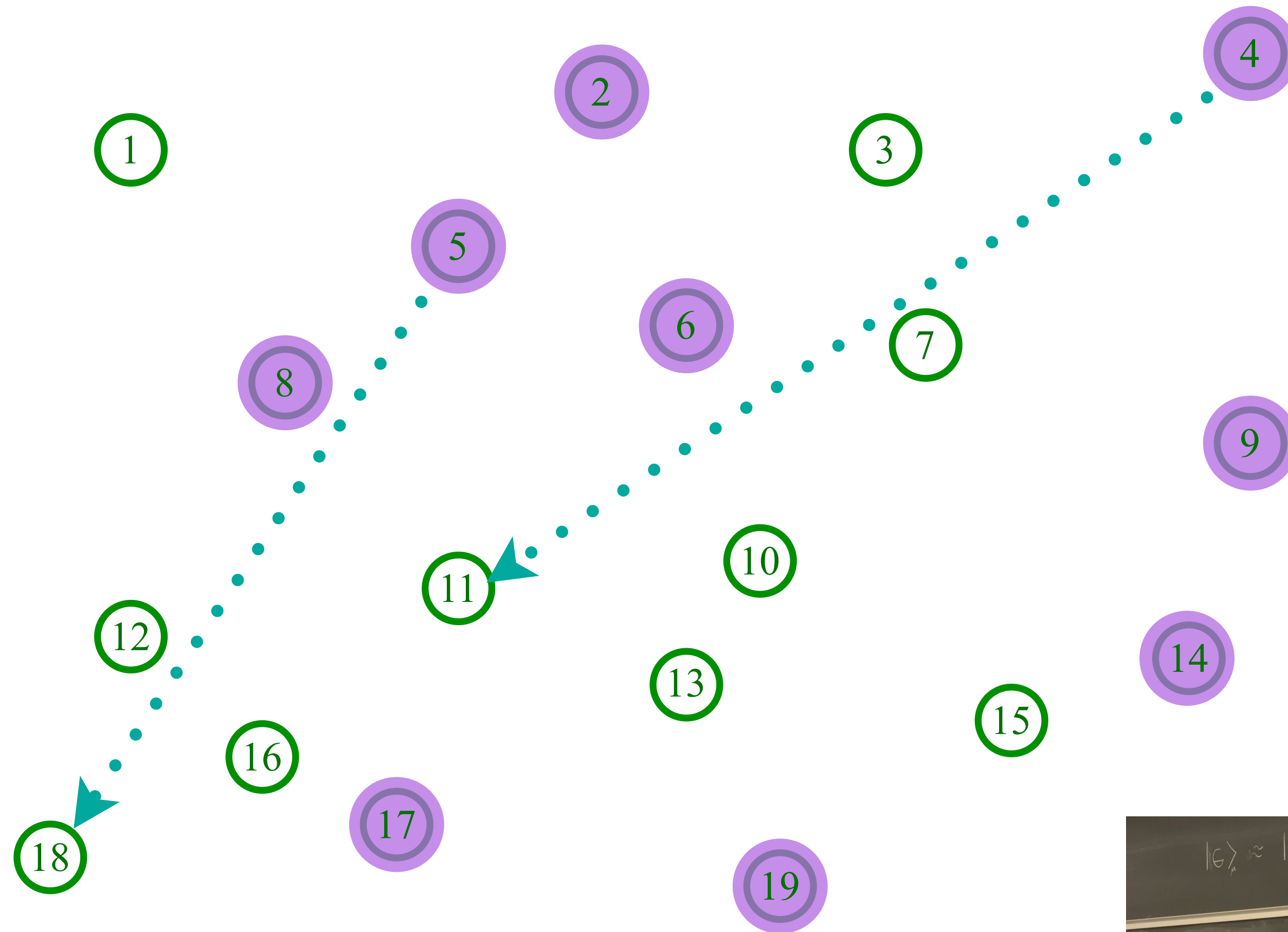
Entangle electrons pairwise randomly



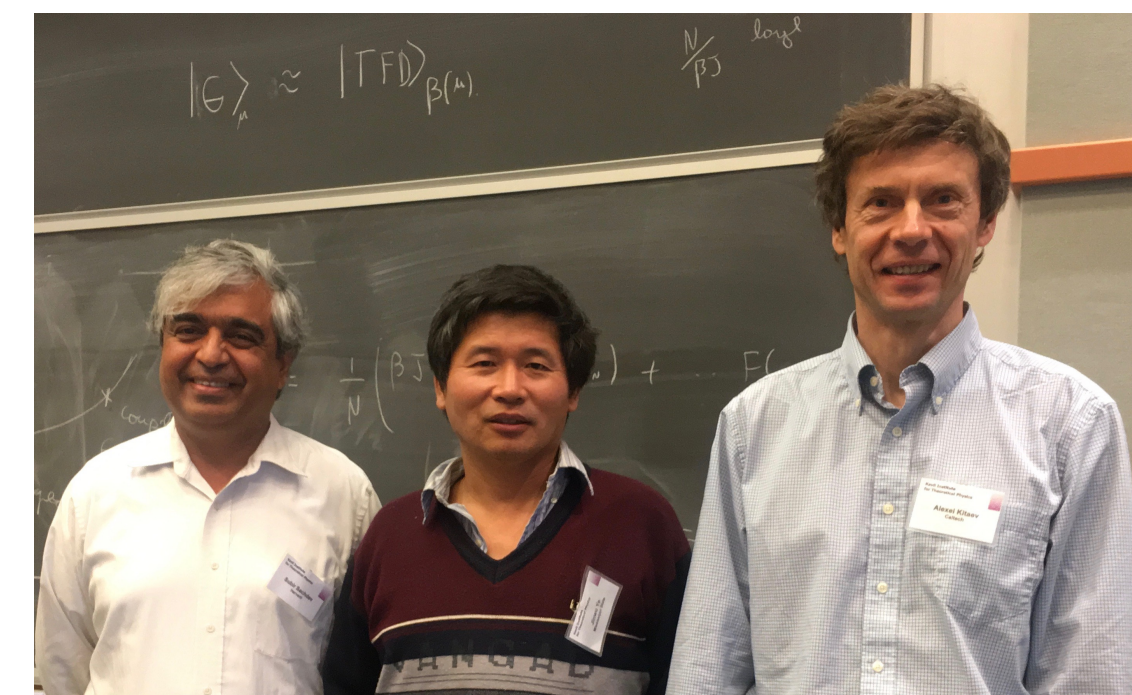
# The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



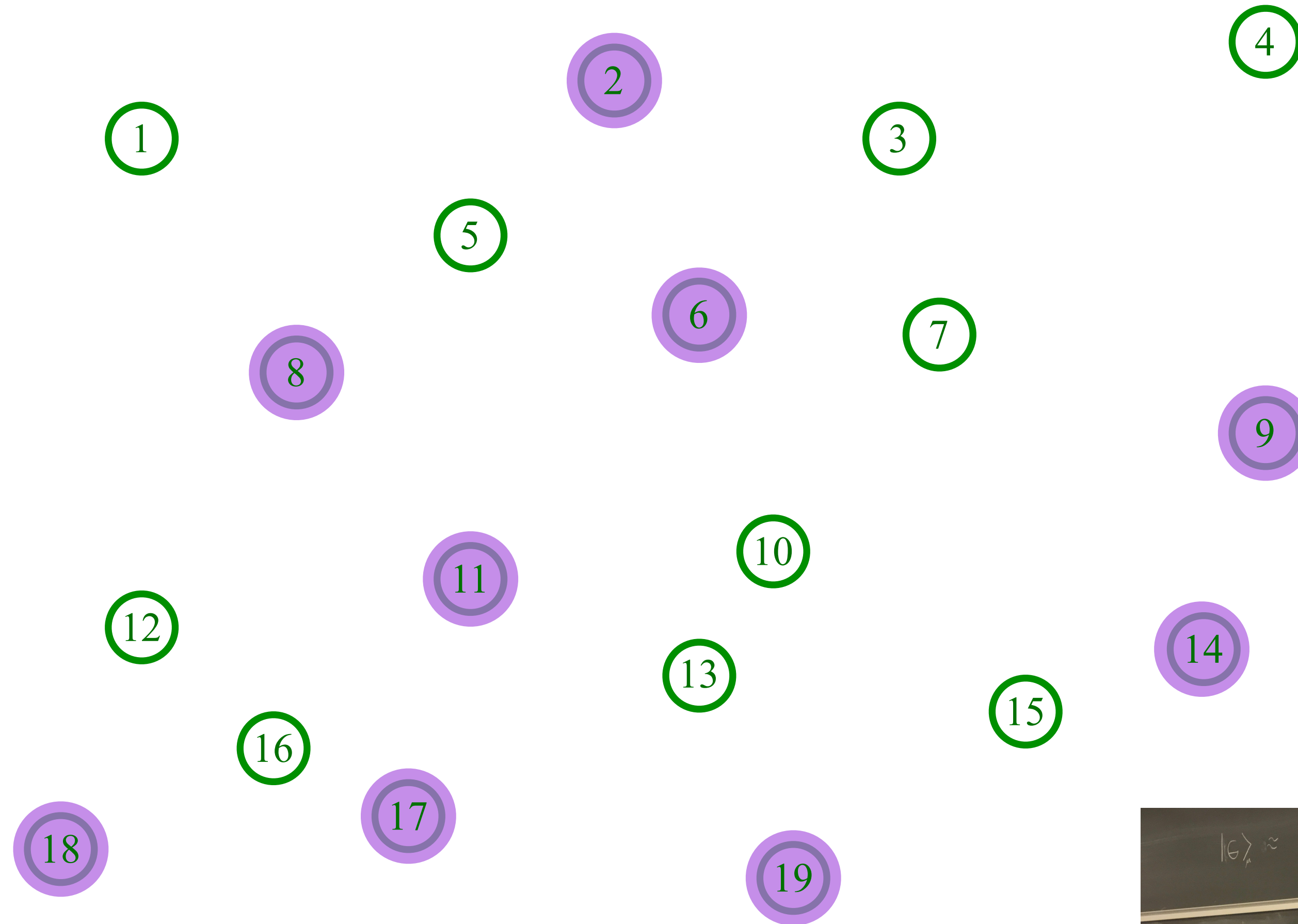
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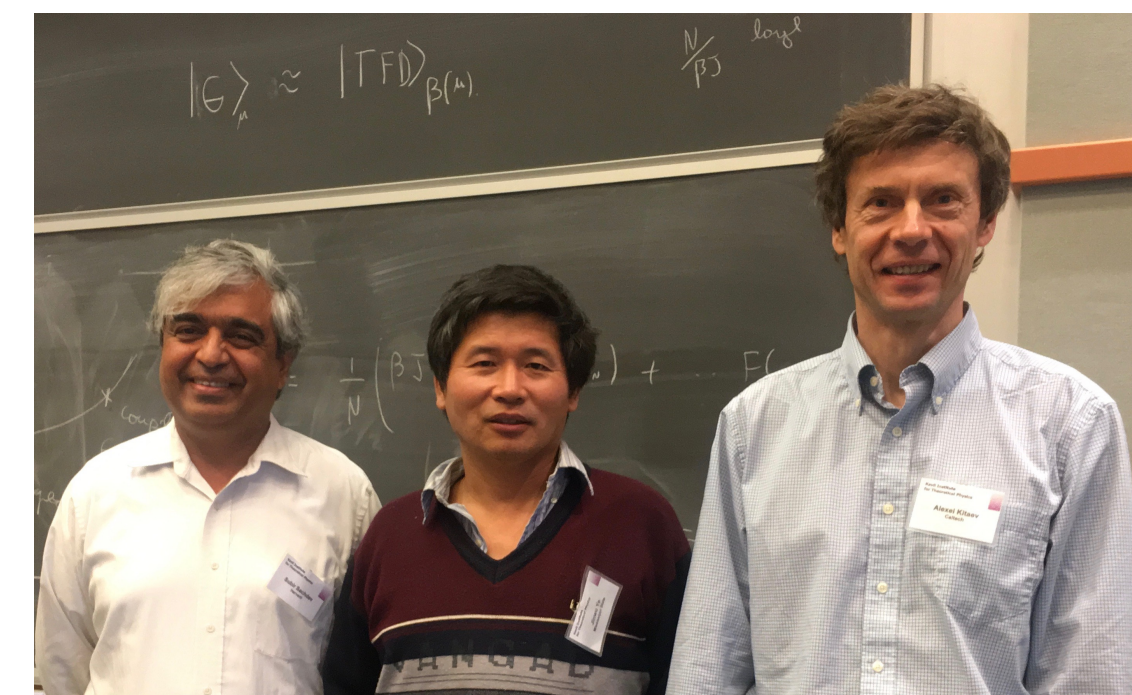
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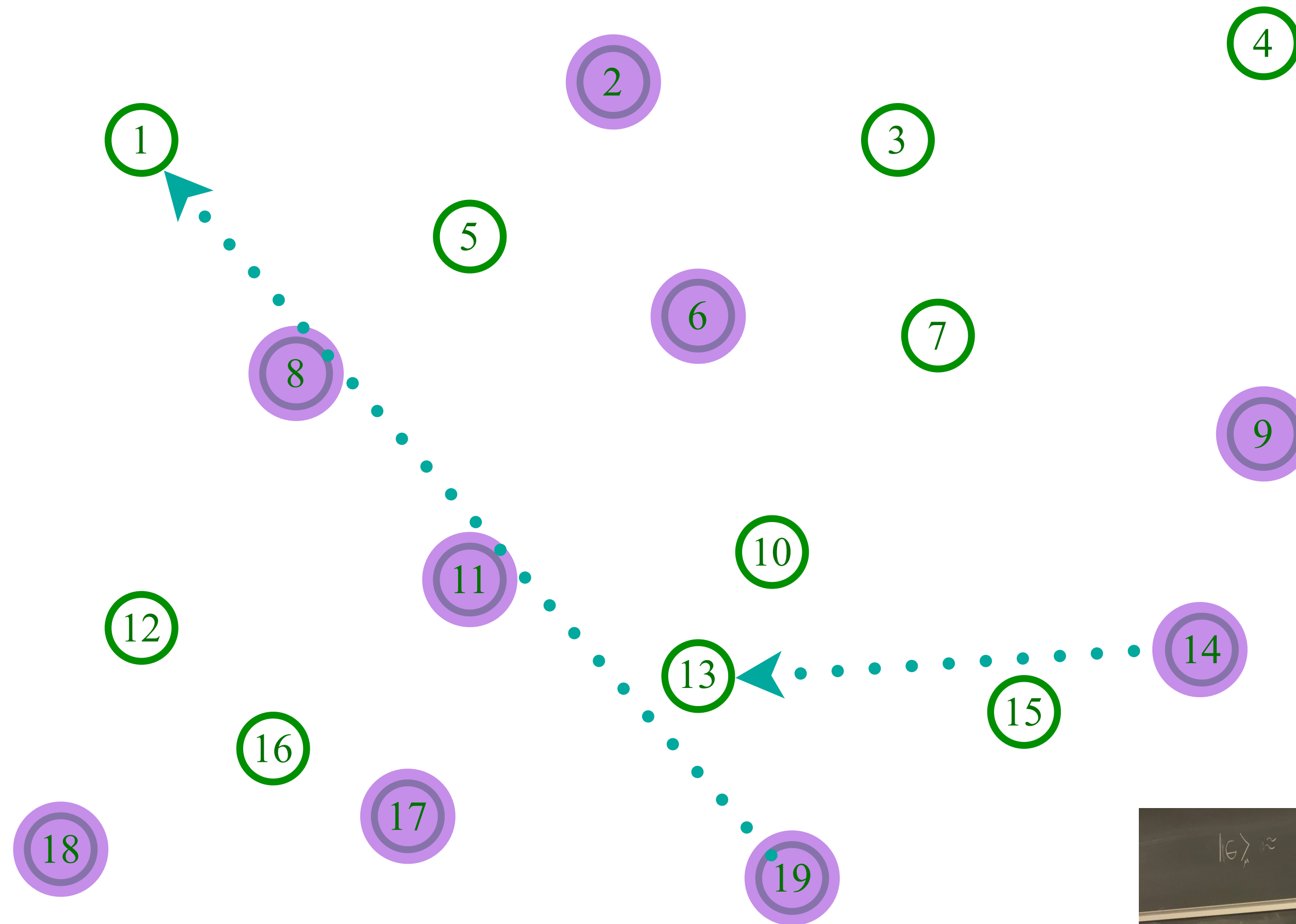
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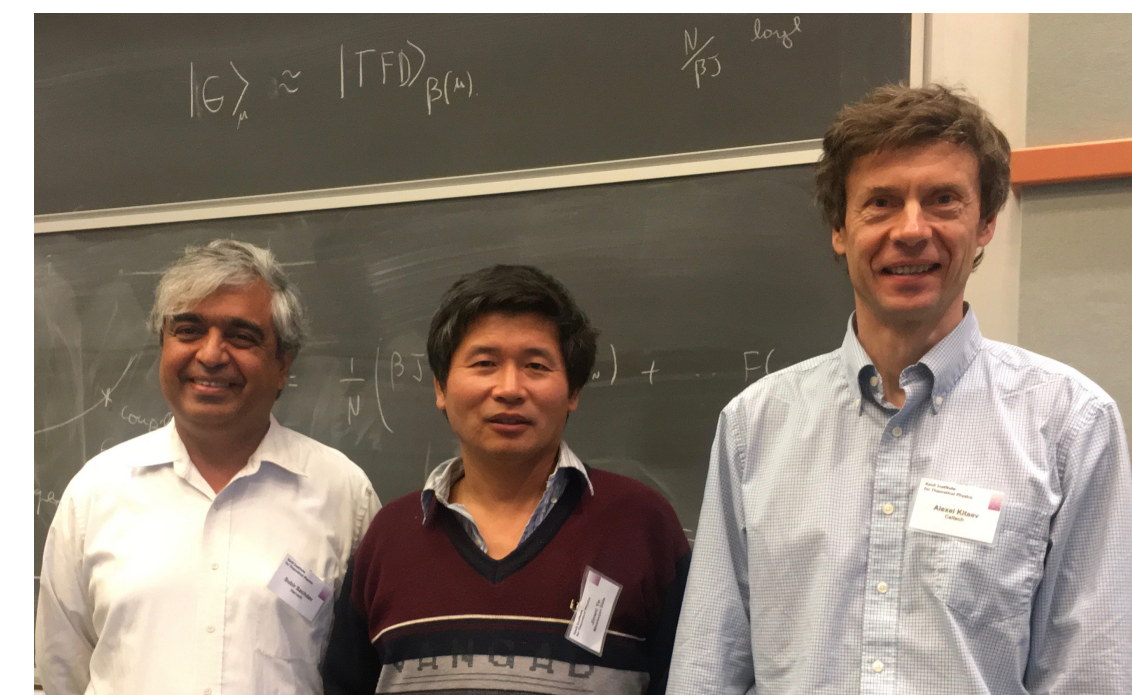
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Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



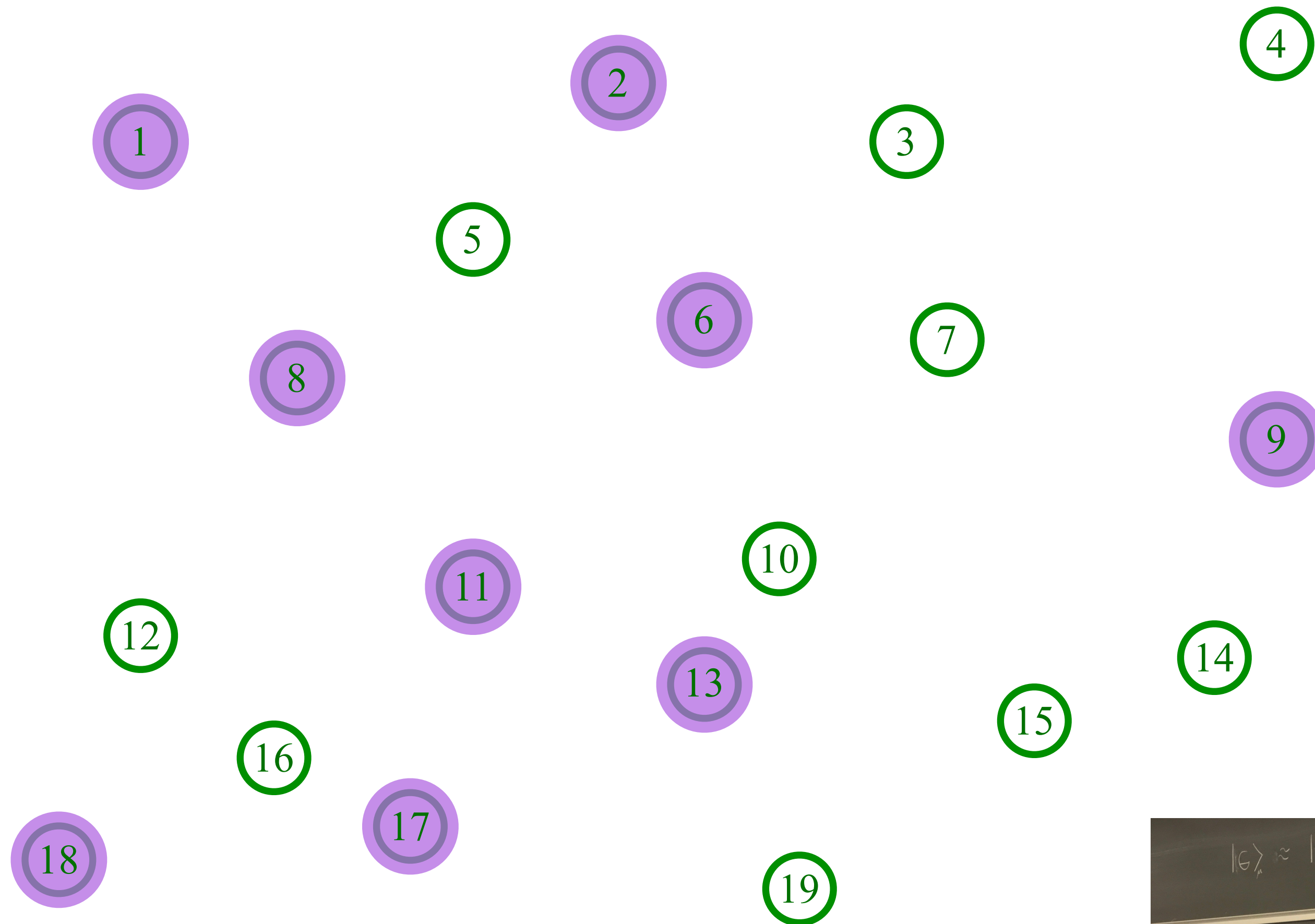
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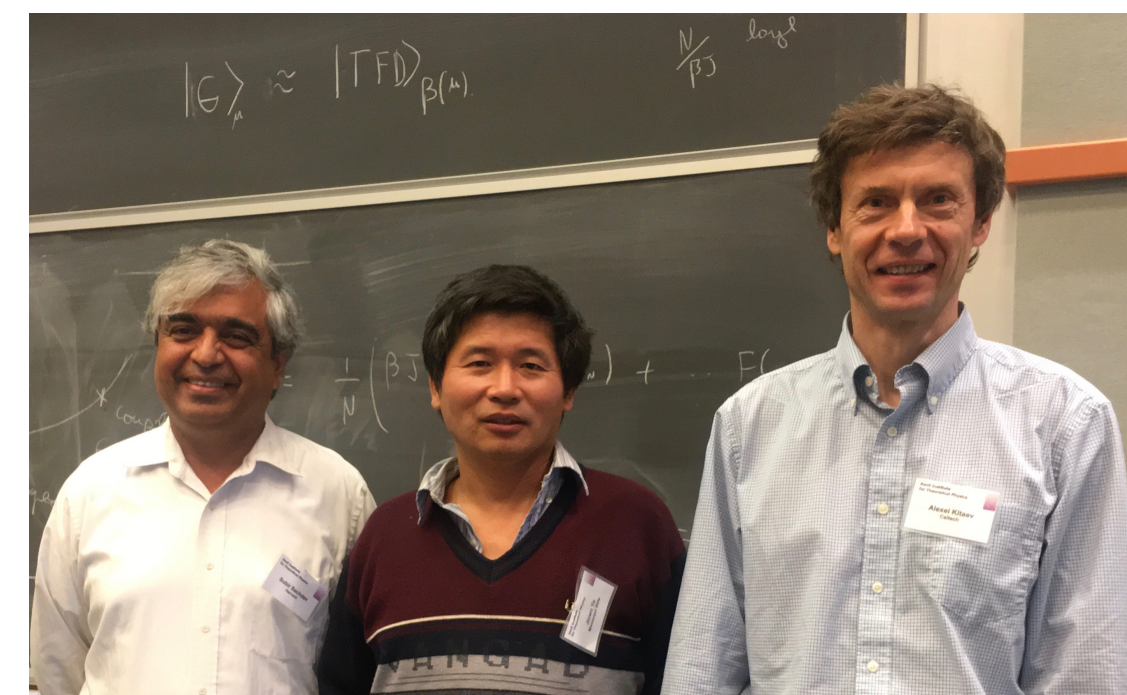
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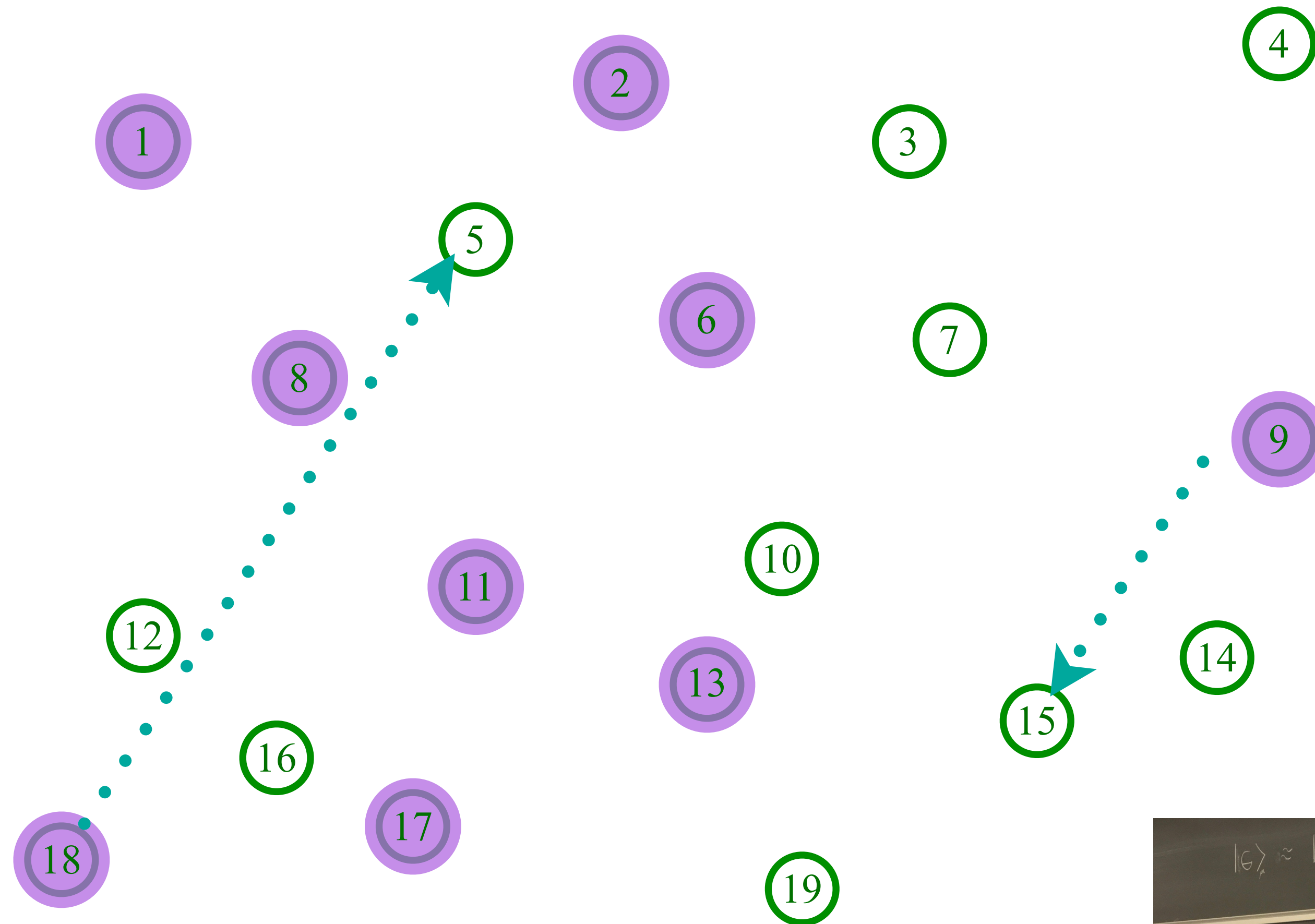
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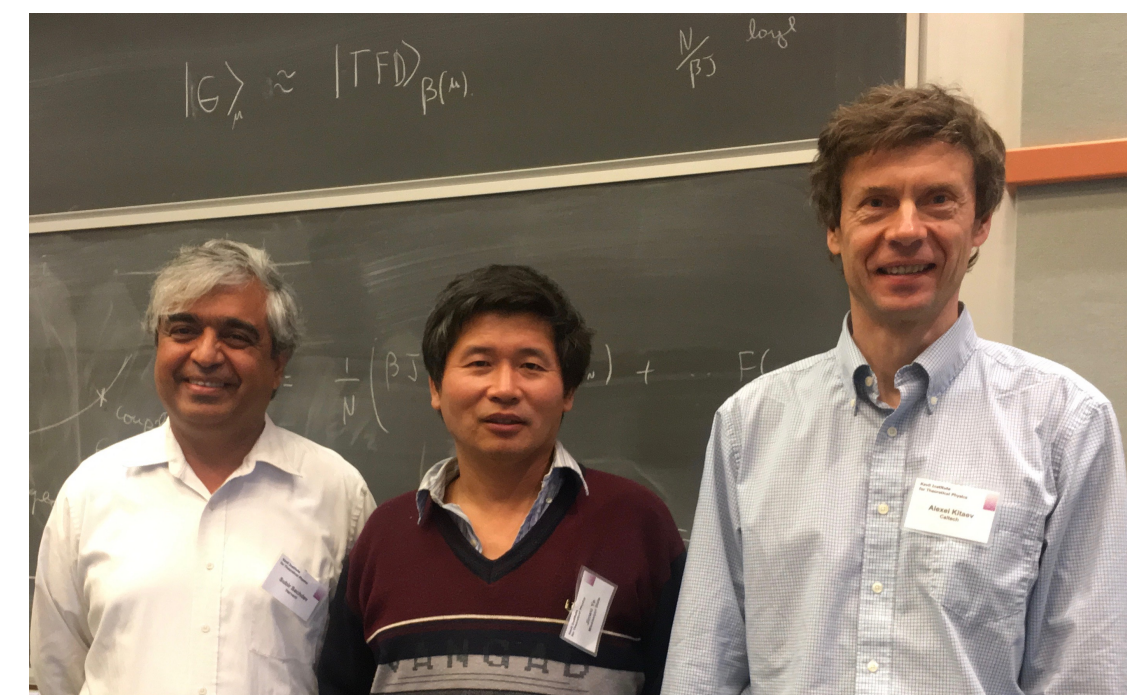
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$$U_{9,18;5,15}$$



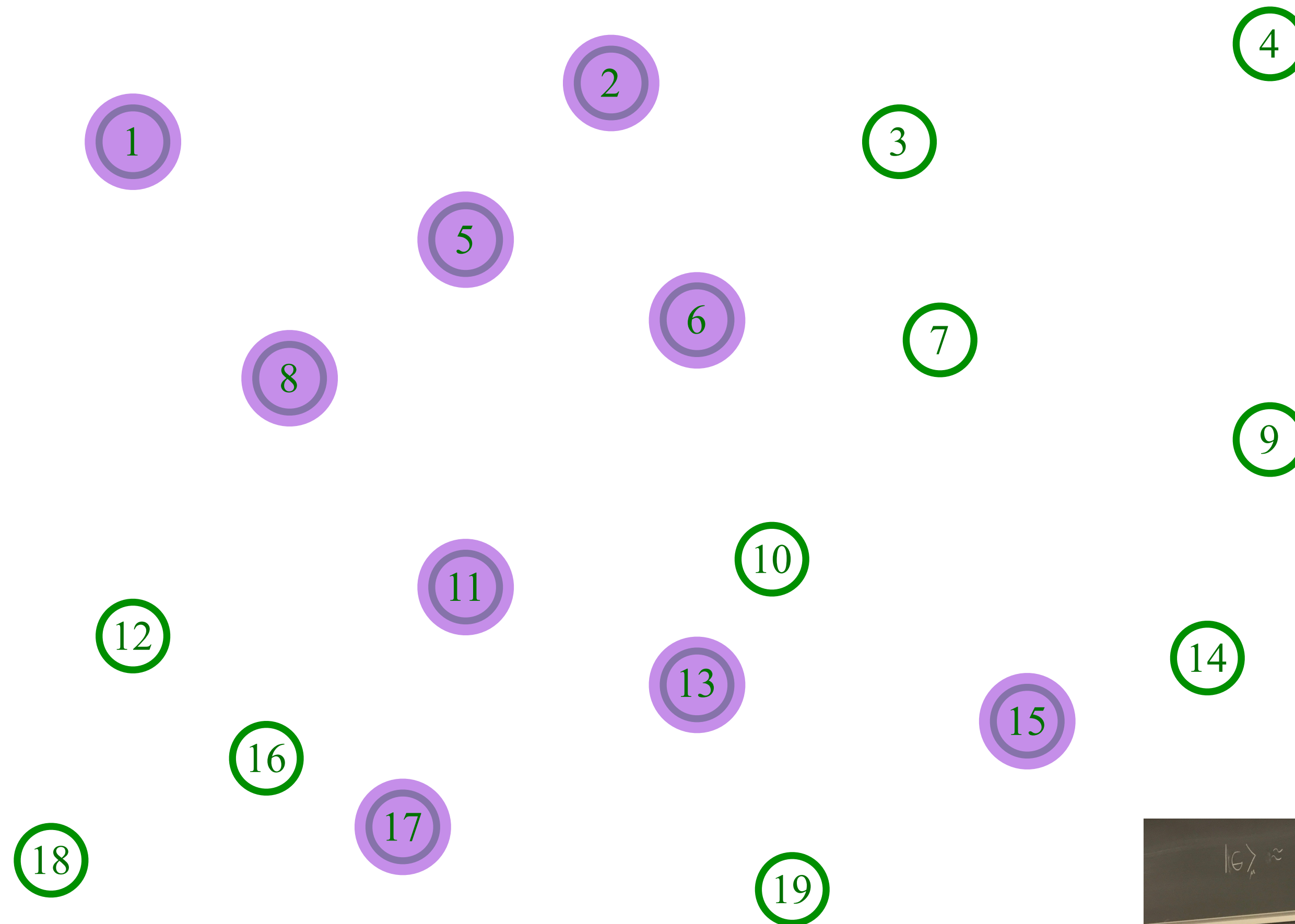
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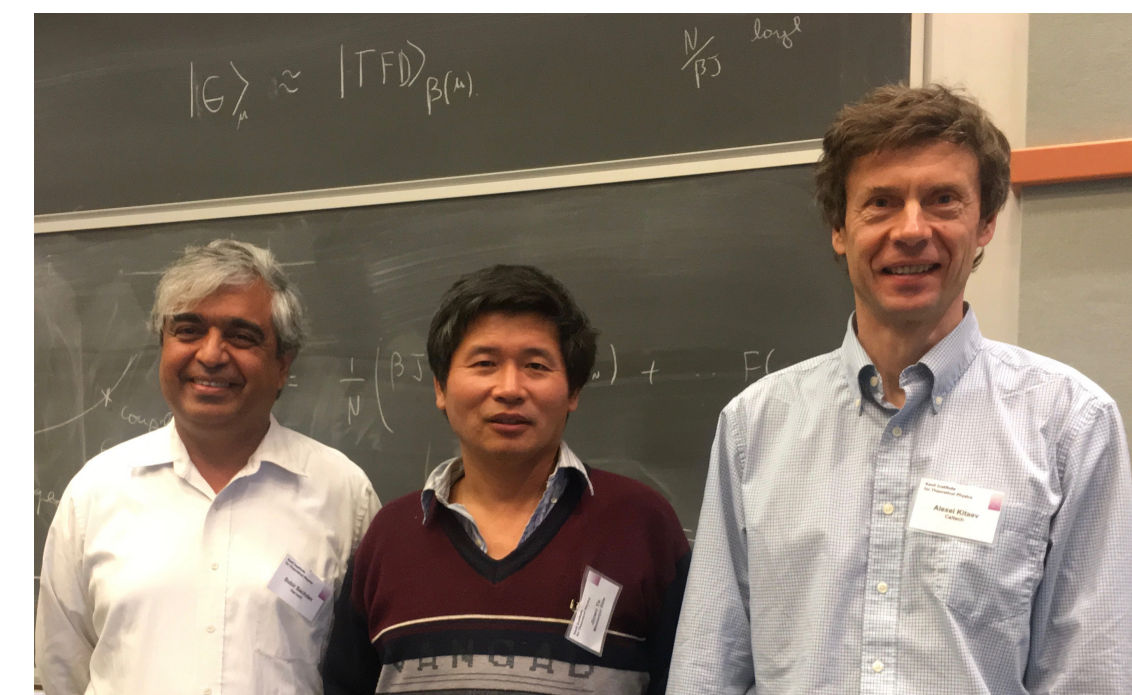
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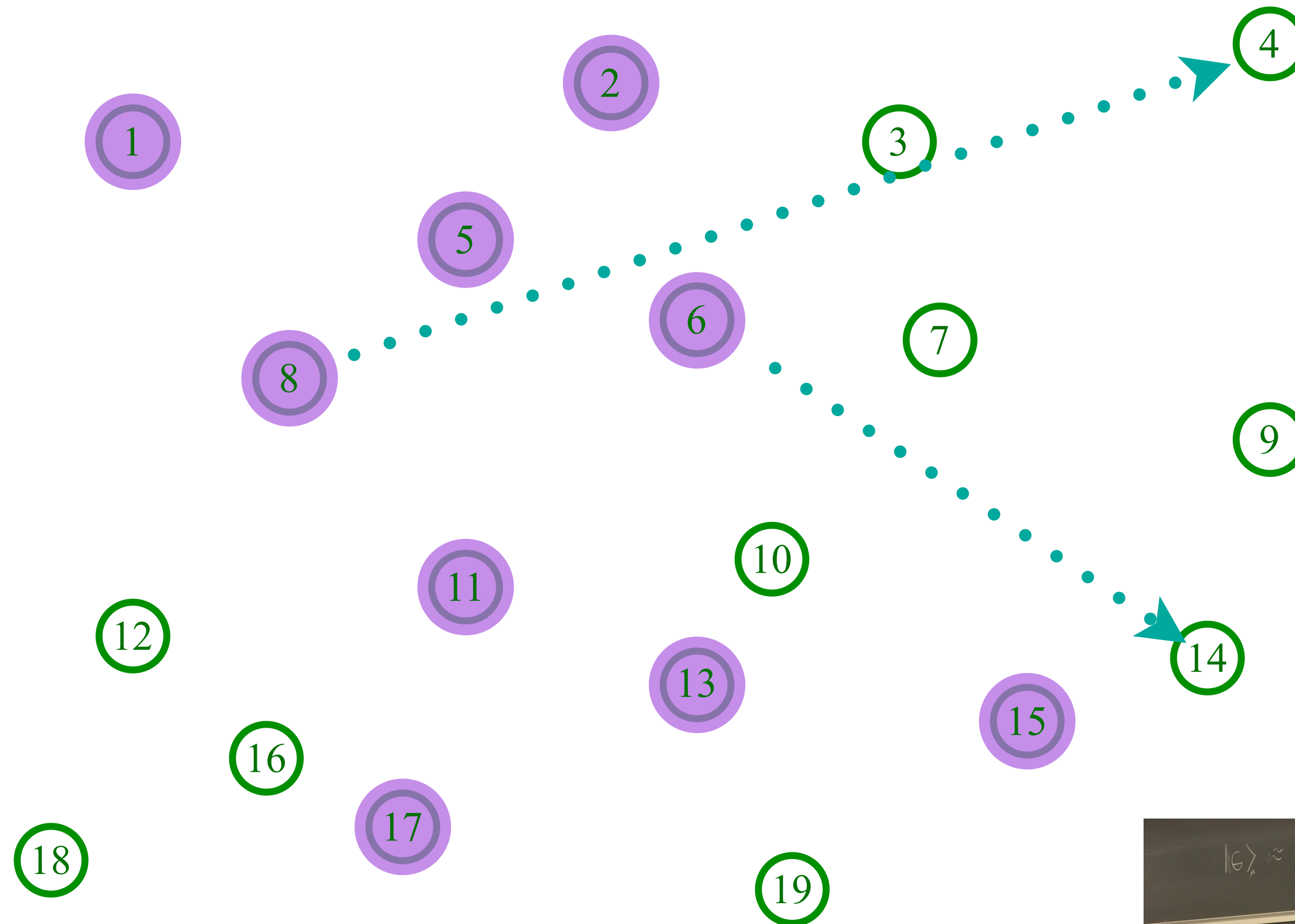
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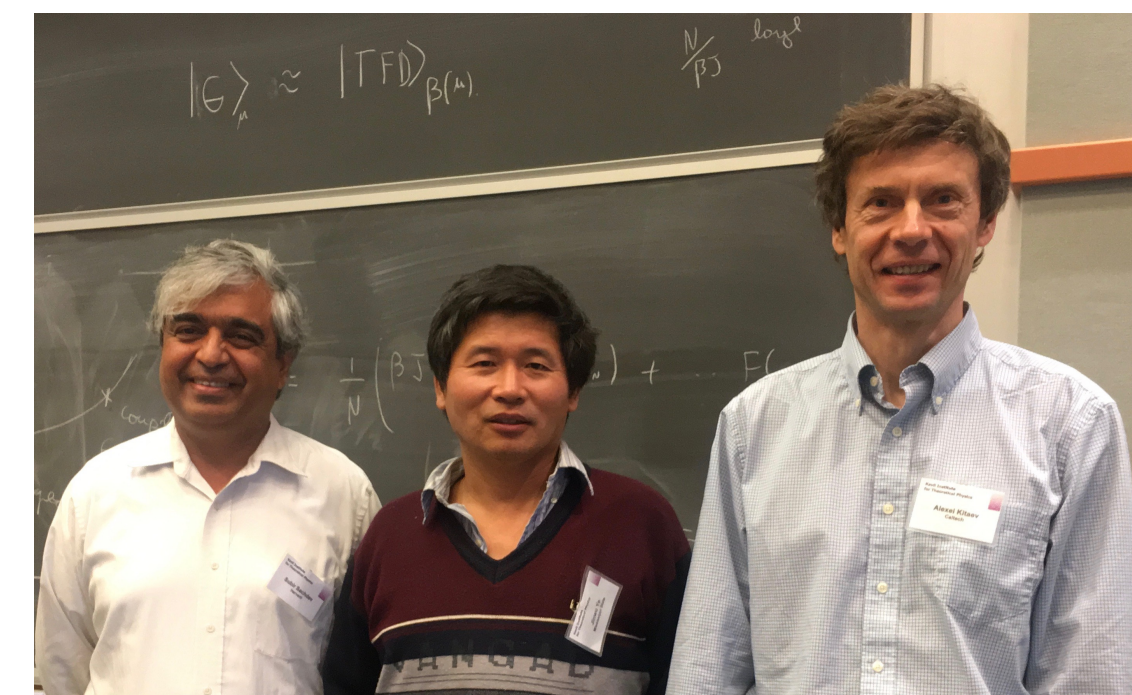
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Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



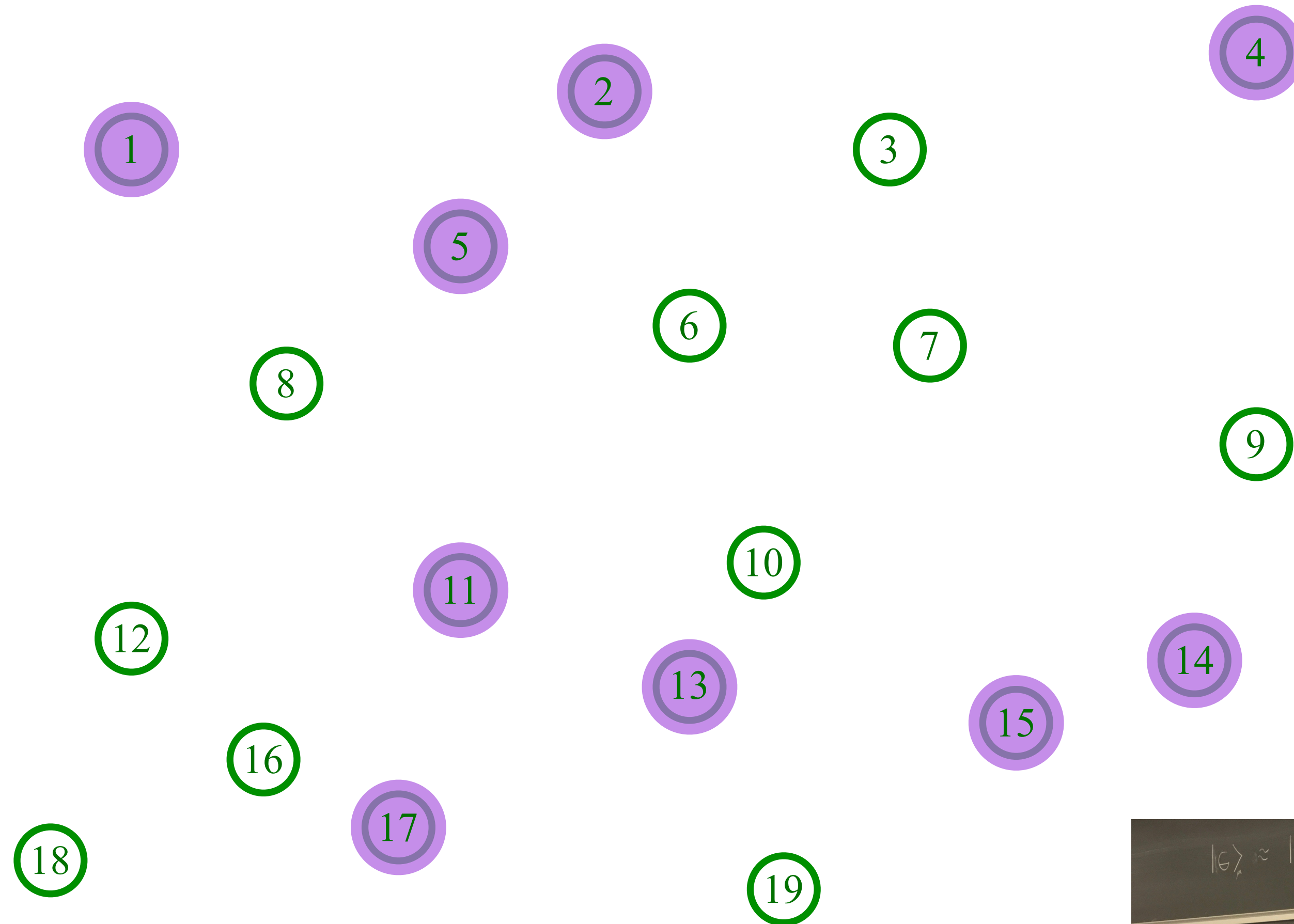
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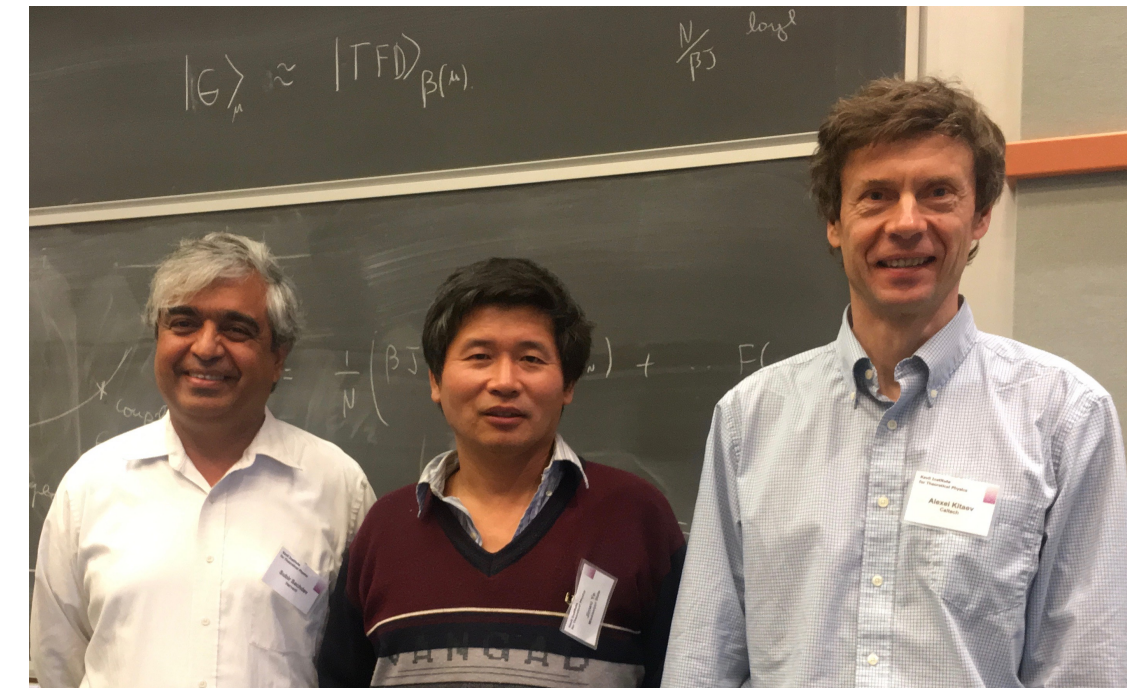
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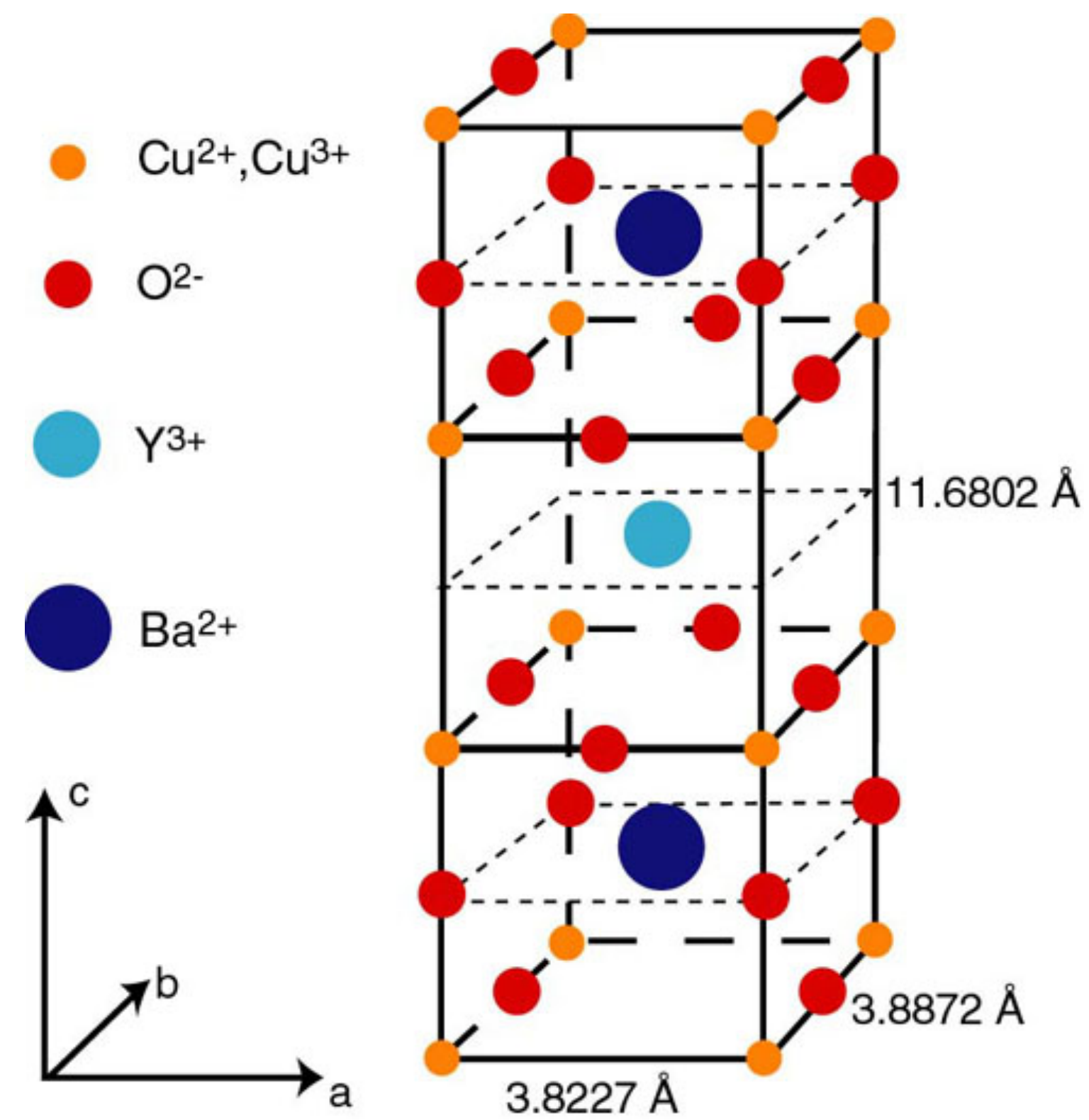
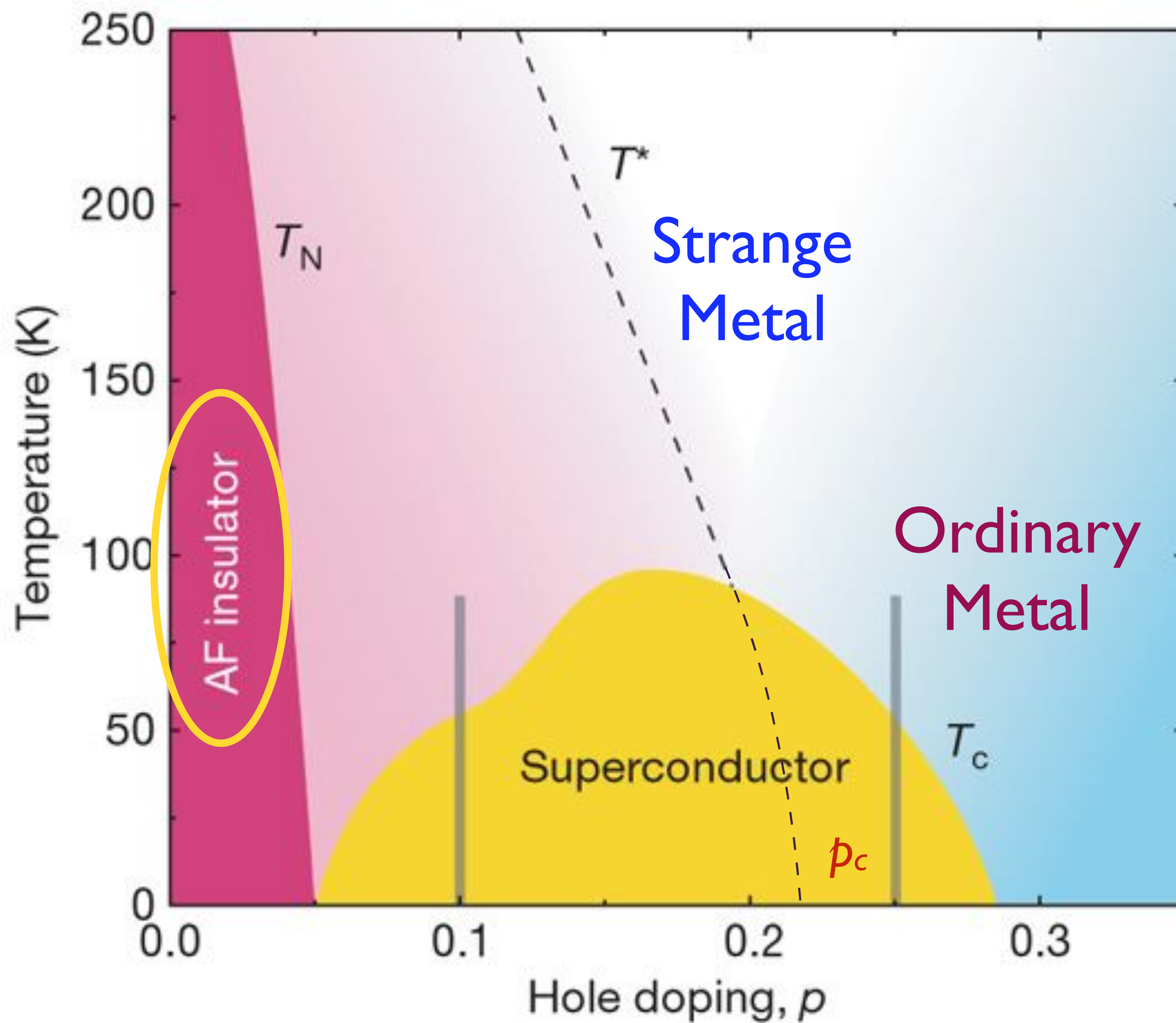
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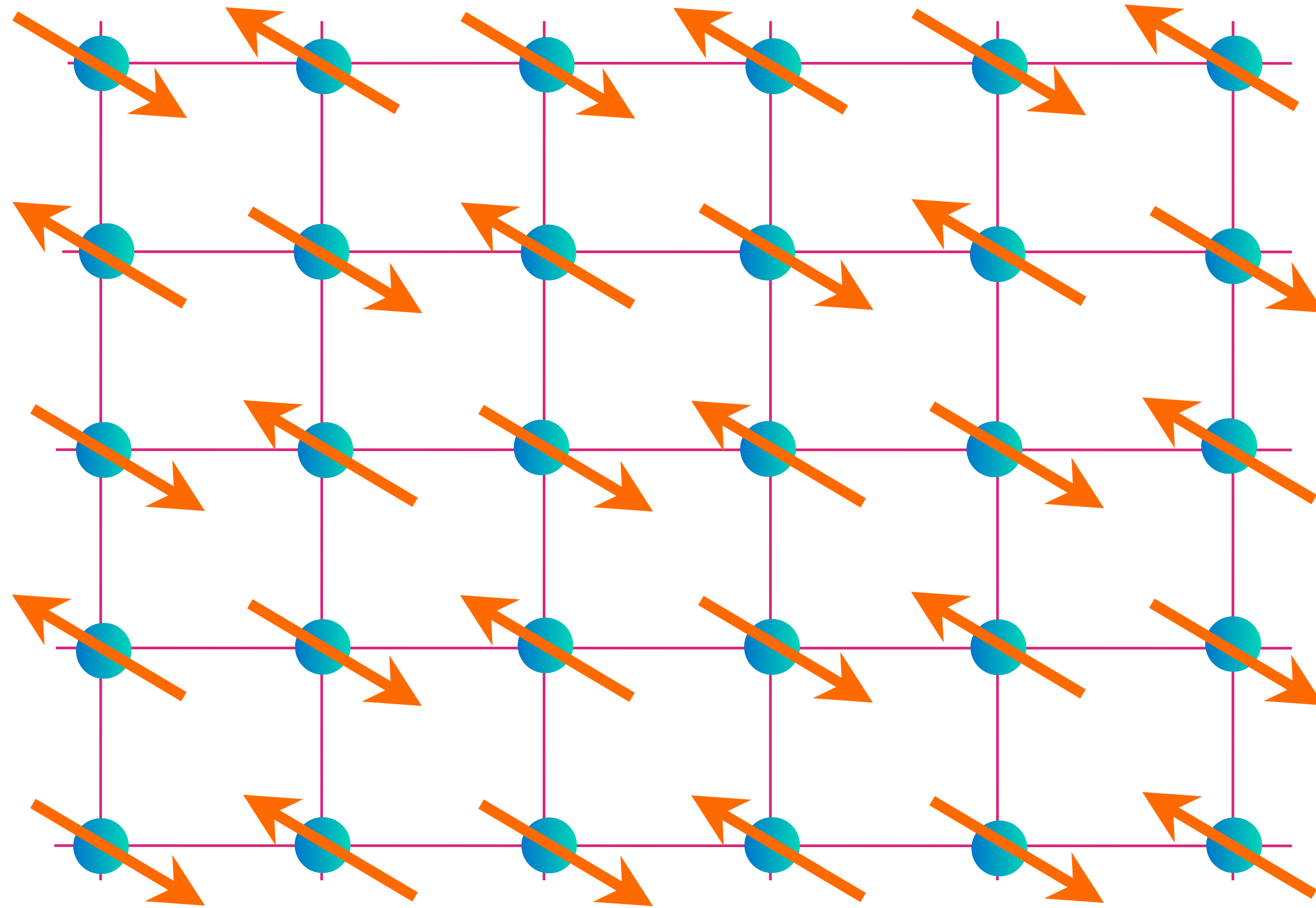
A solvable model of multi-particle  
quantum entanglement.

Yields a metal in which current is carried  
not by individual electrons,  
but by an entangled “quantum soup”

# Quantum entanglement and superconductivity

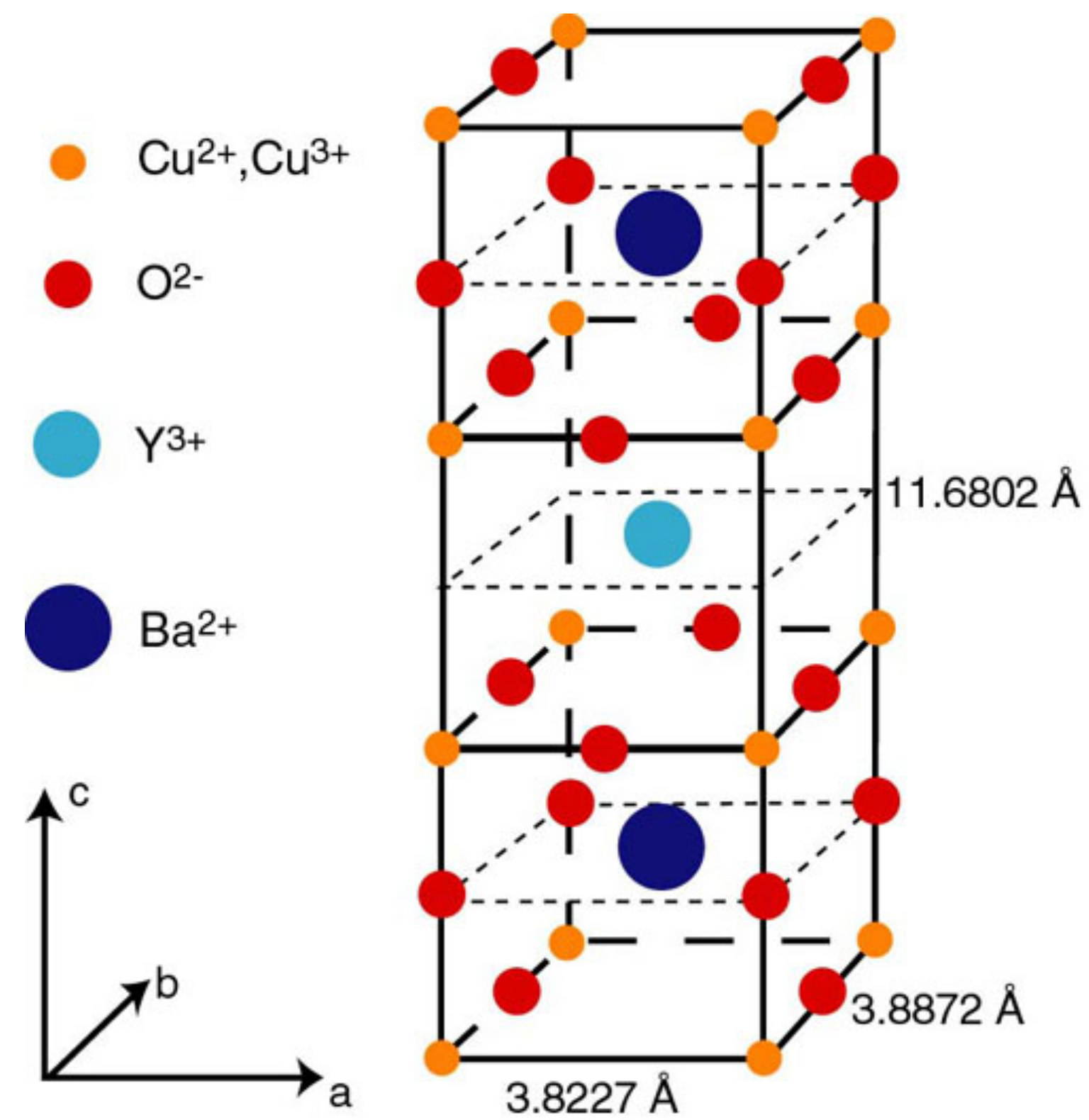
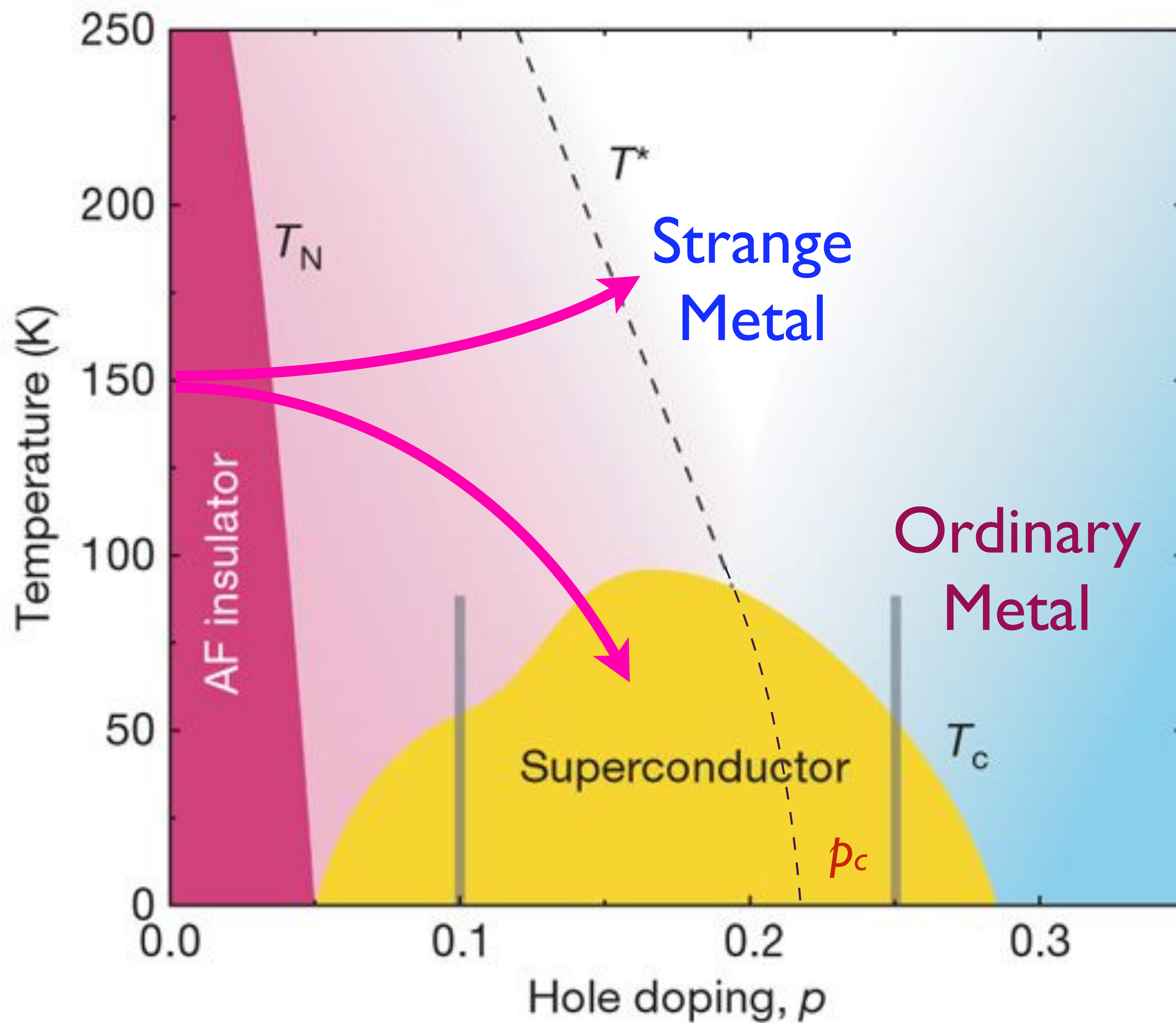


# The dance of electrons on Cu atoms in YBCO

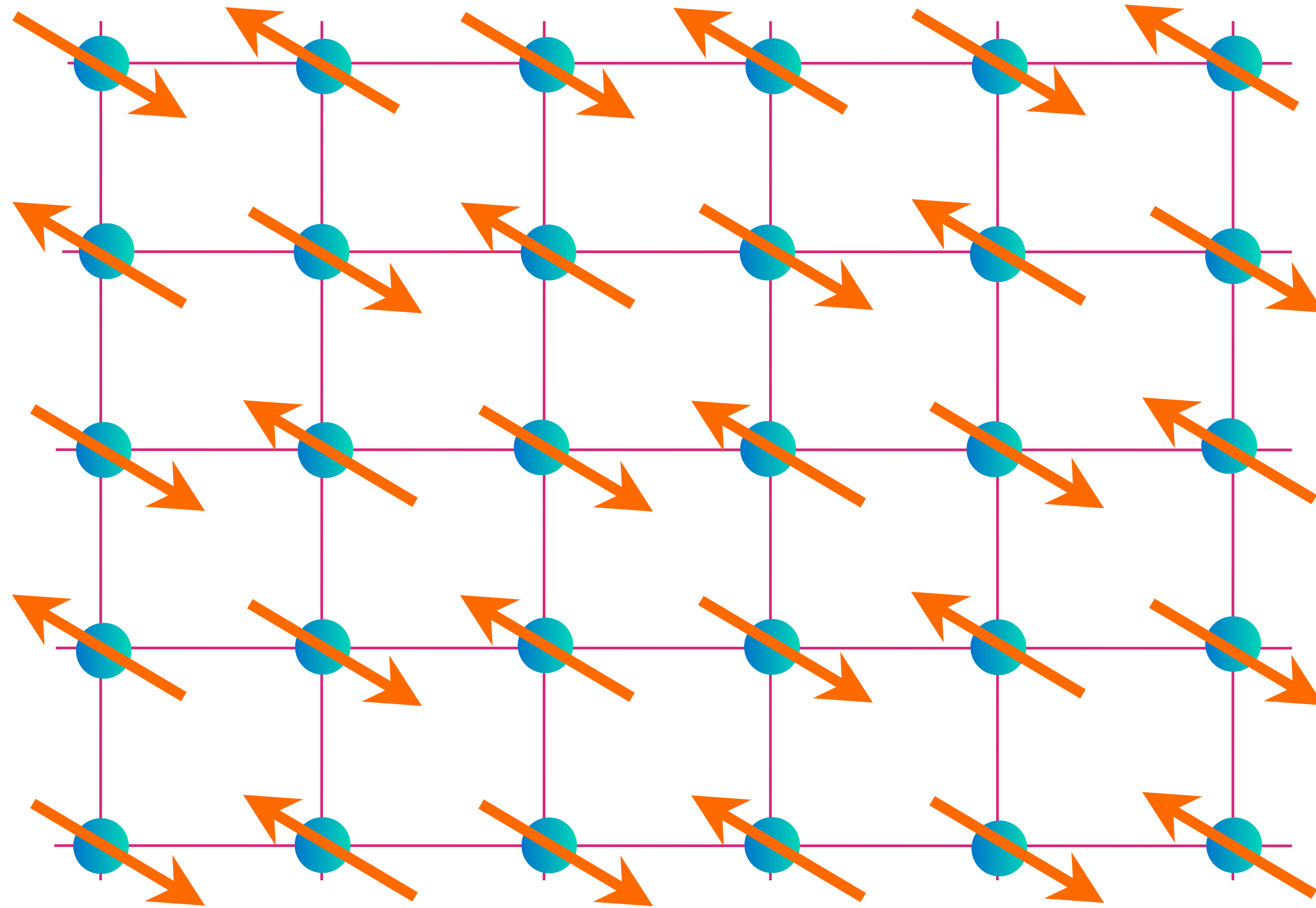


**Antiferromagnetism**

All nearest-neighbor pairs of electrons have opposite spins



# The dance of electrons on Cu atoms in YBCO



**Antiferromagnetism**

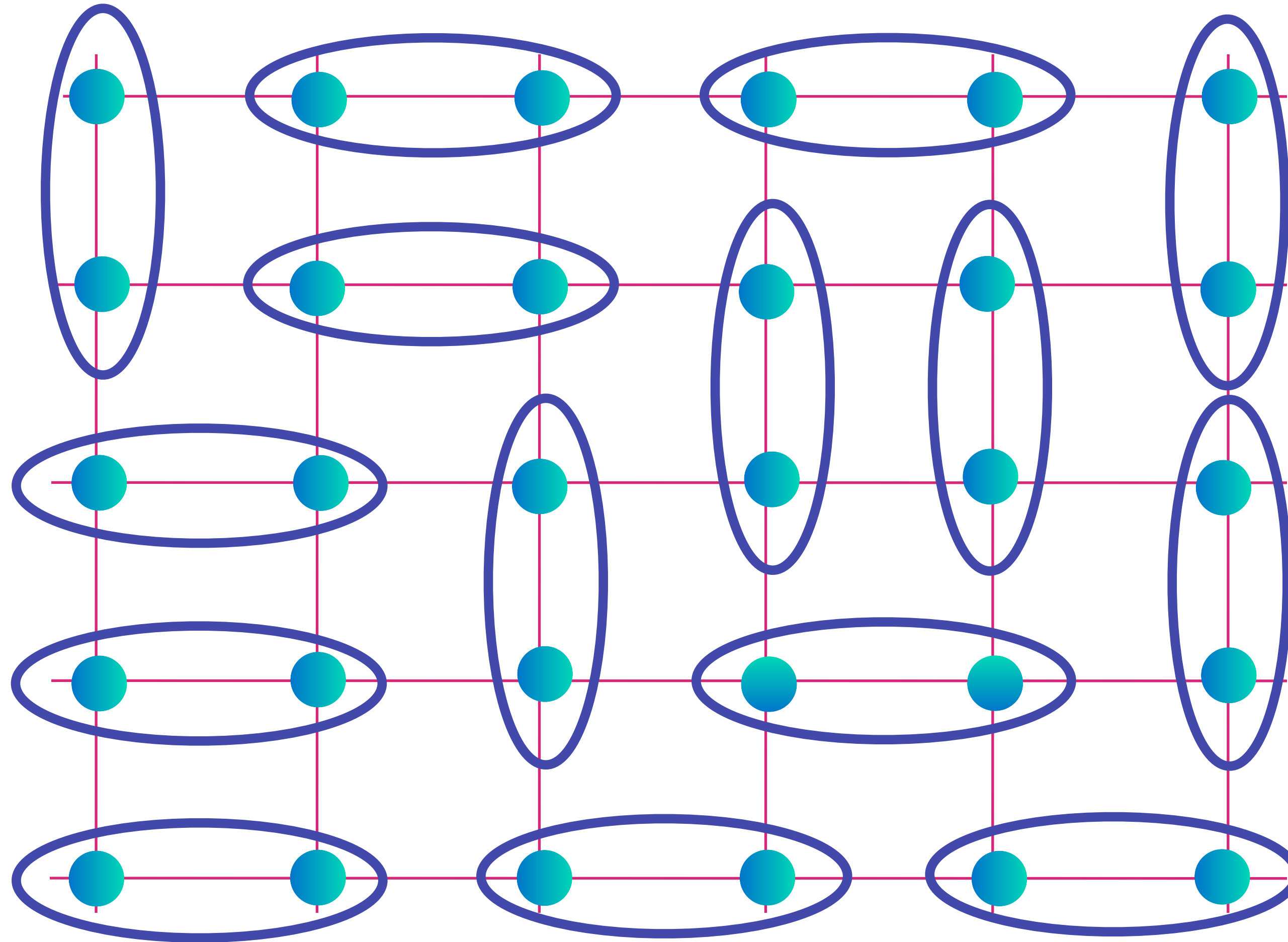
All nearest-neighbor pairs of electrons have opposite spins

# The dance of electrons on Cu atoms in YBCO

Baskaran+Anderson (1987)

**Spin liquid**

Electrons form entangled pairs, and the pairs entangle across the entire sample



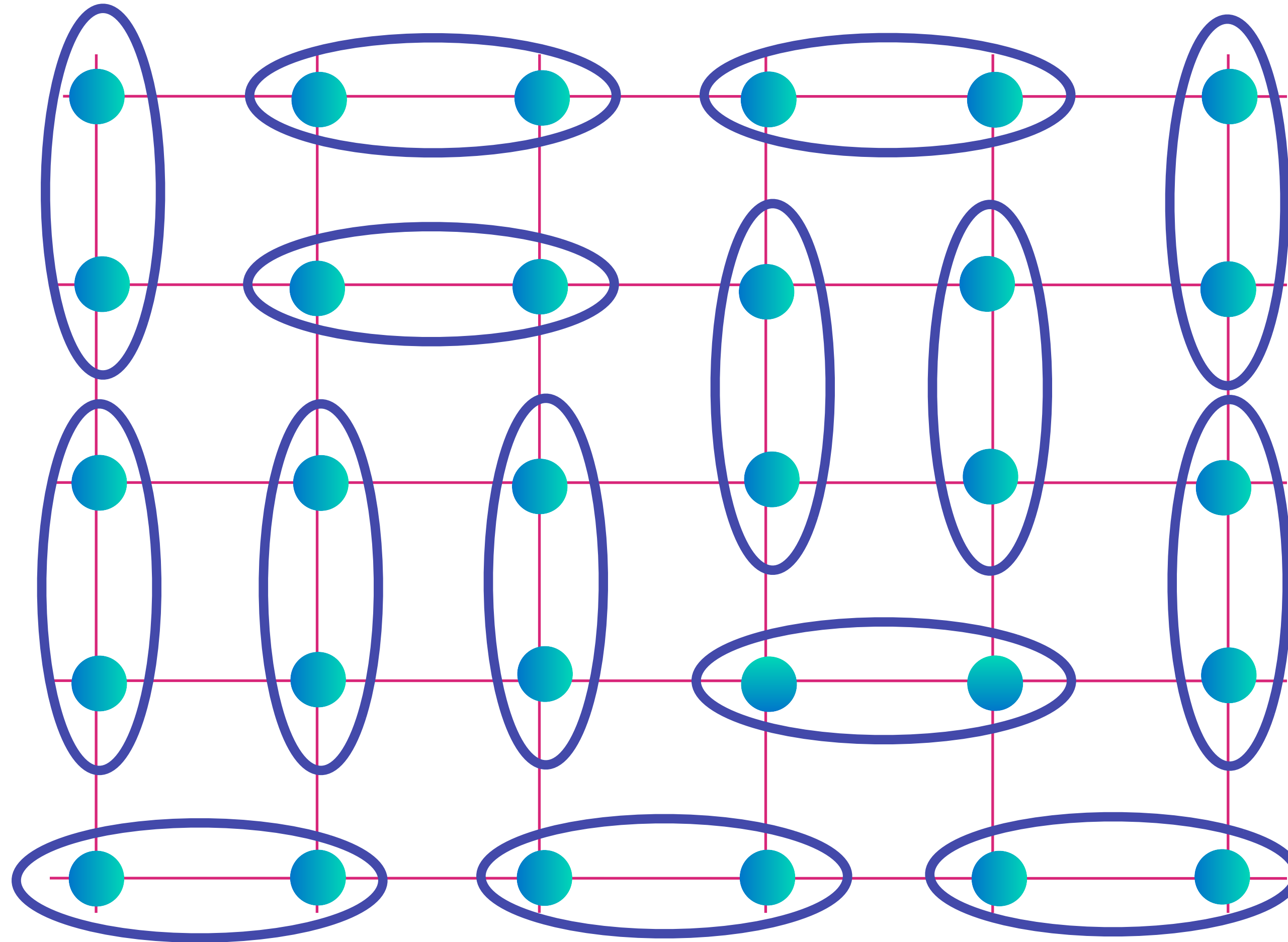
$$\text{[Diagram of two cyan dots in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

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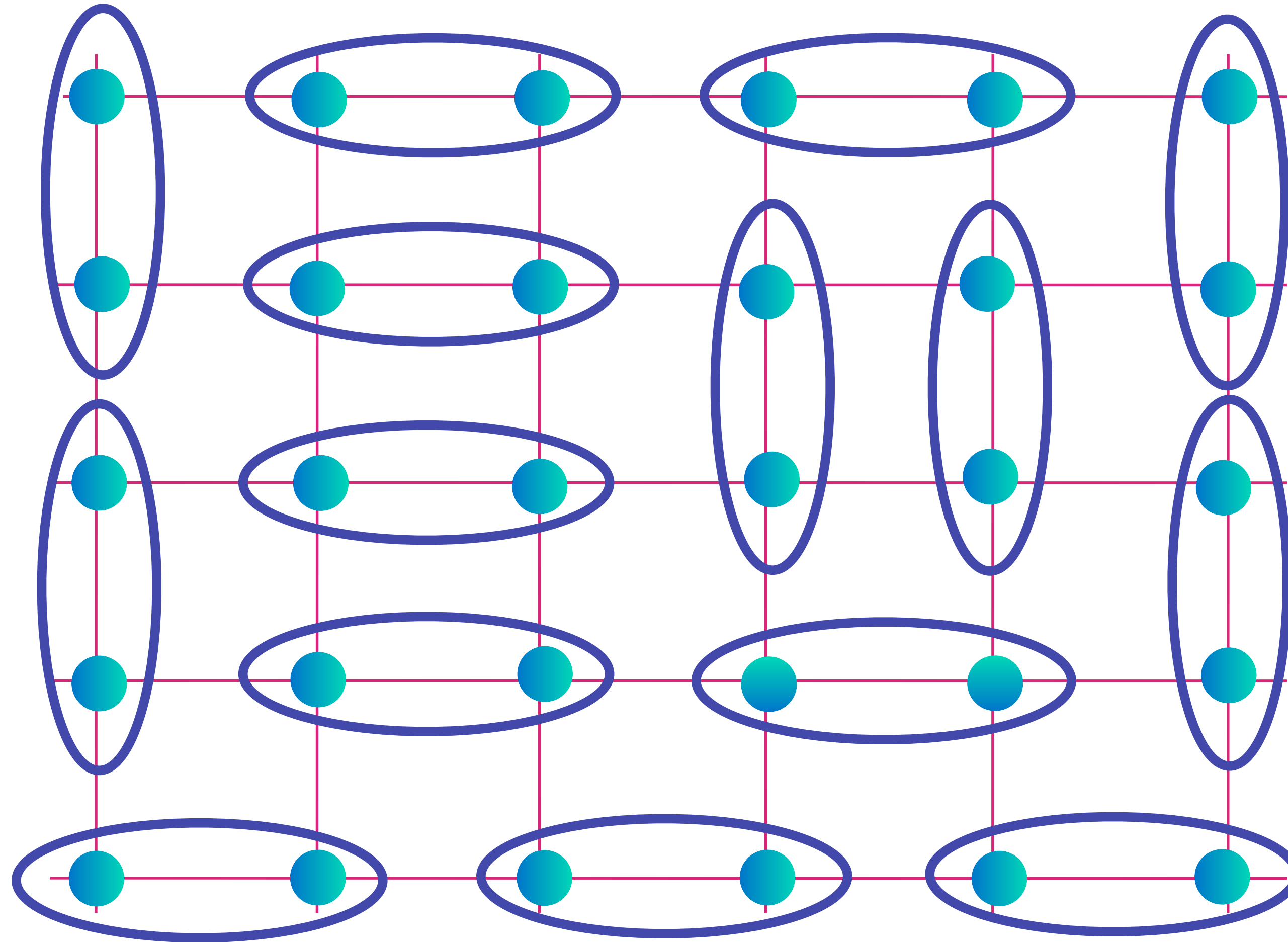
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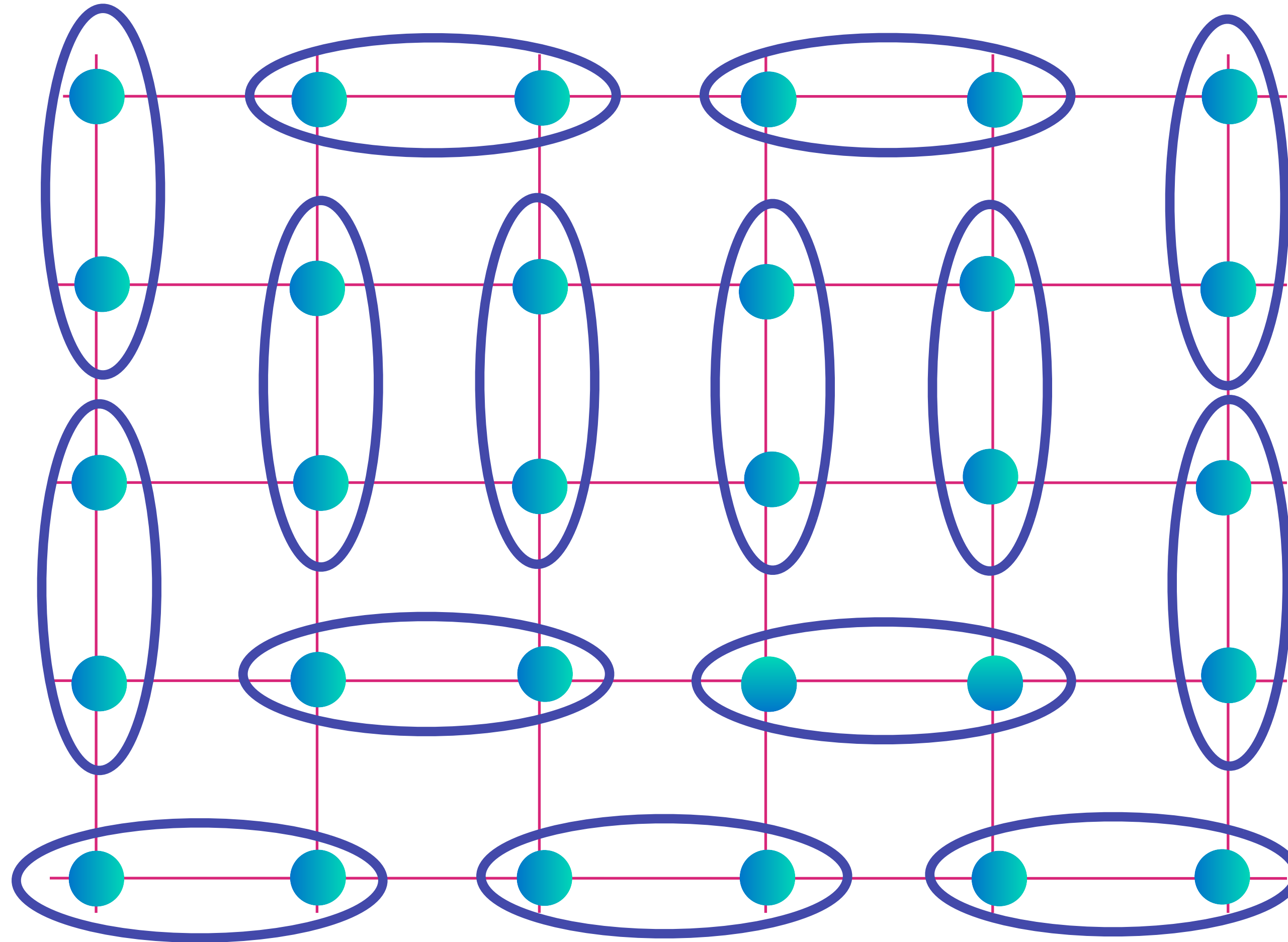
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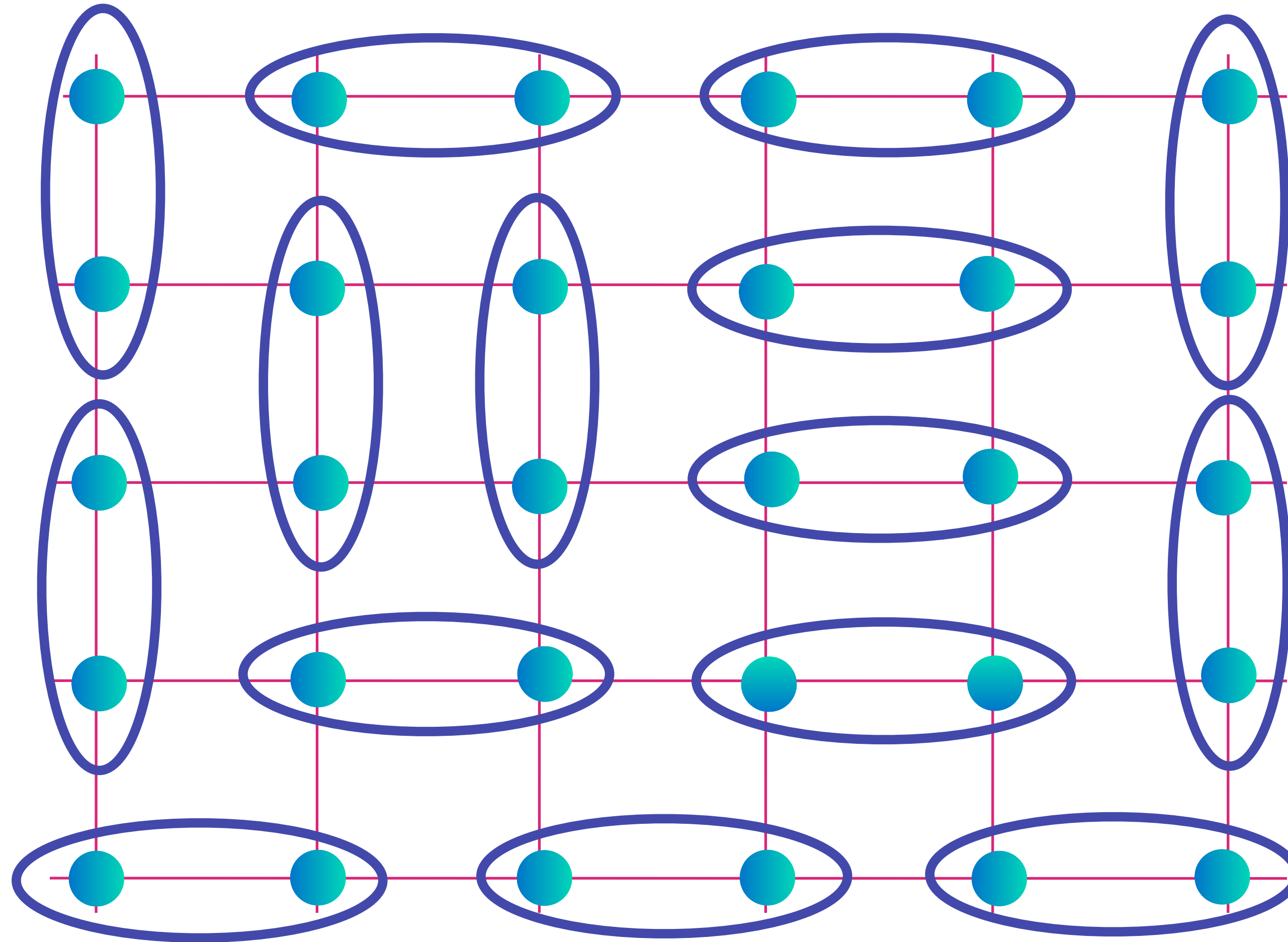
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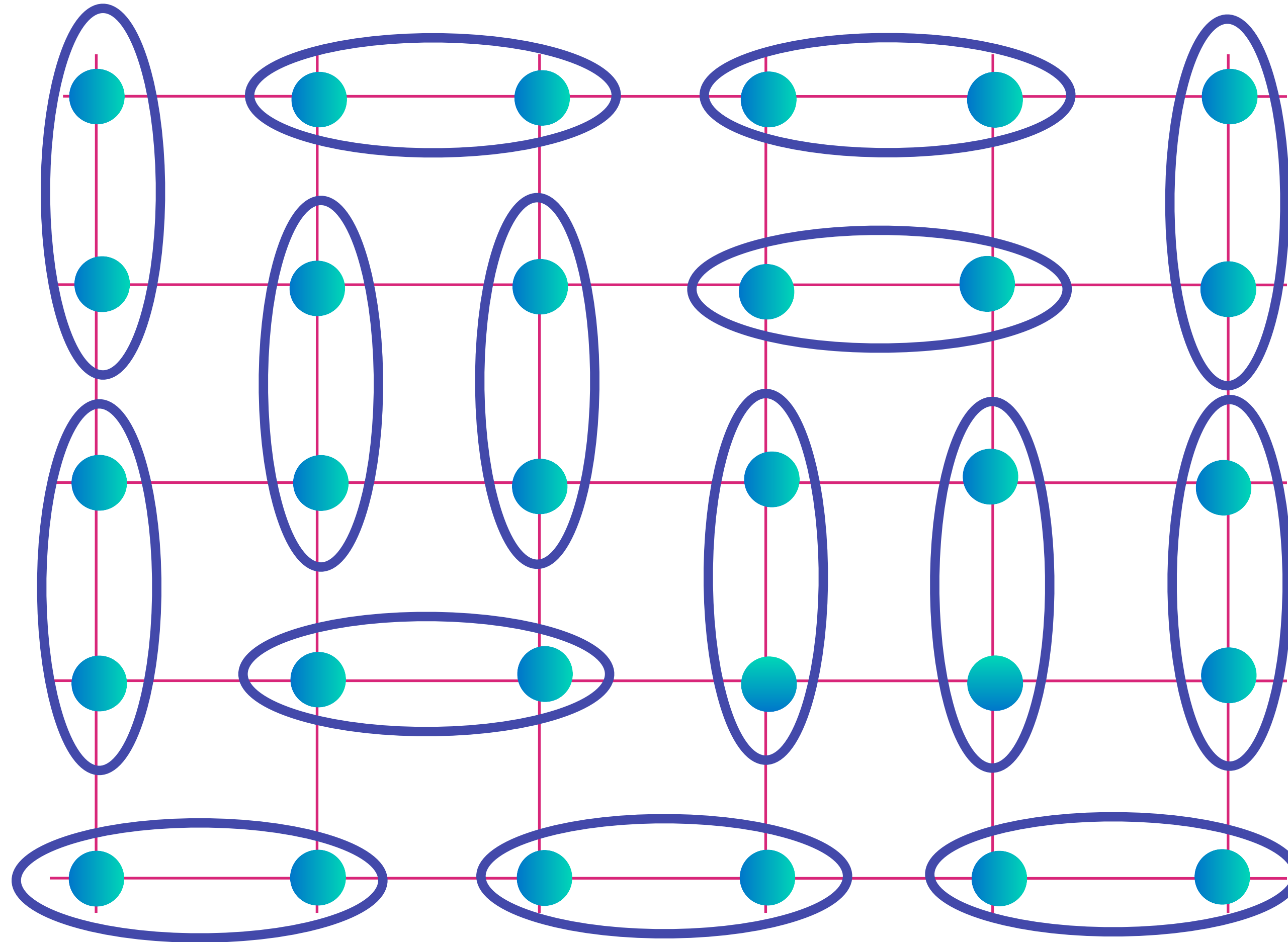
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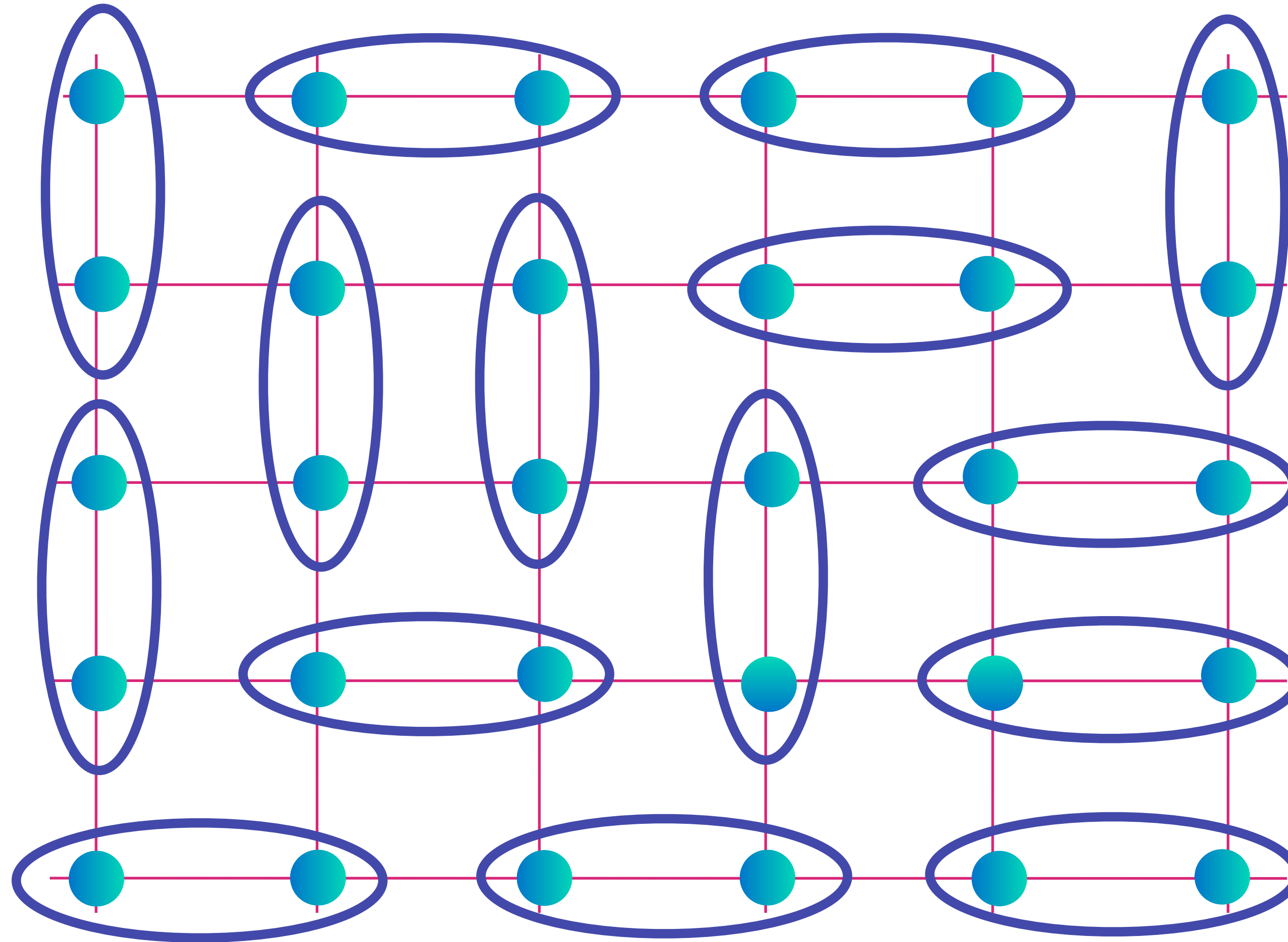
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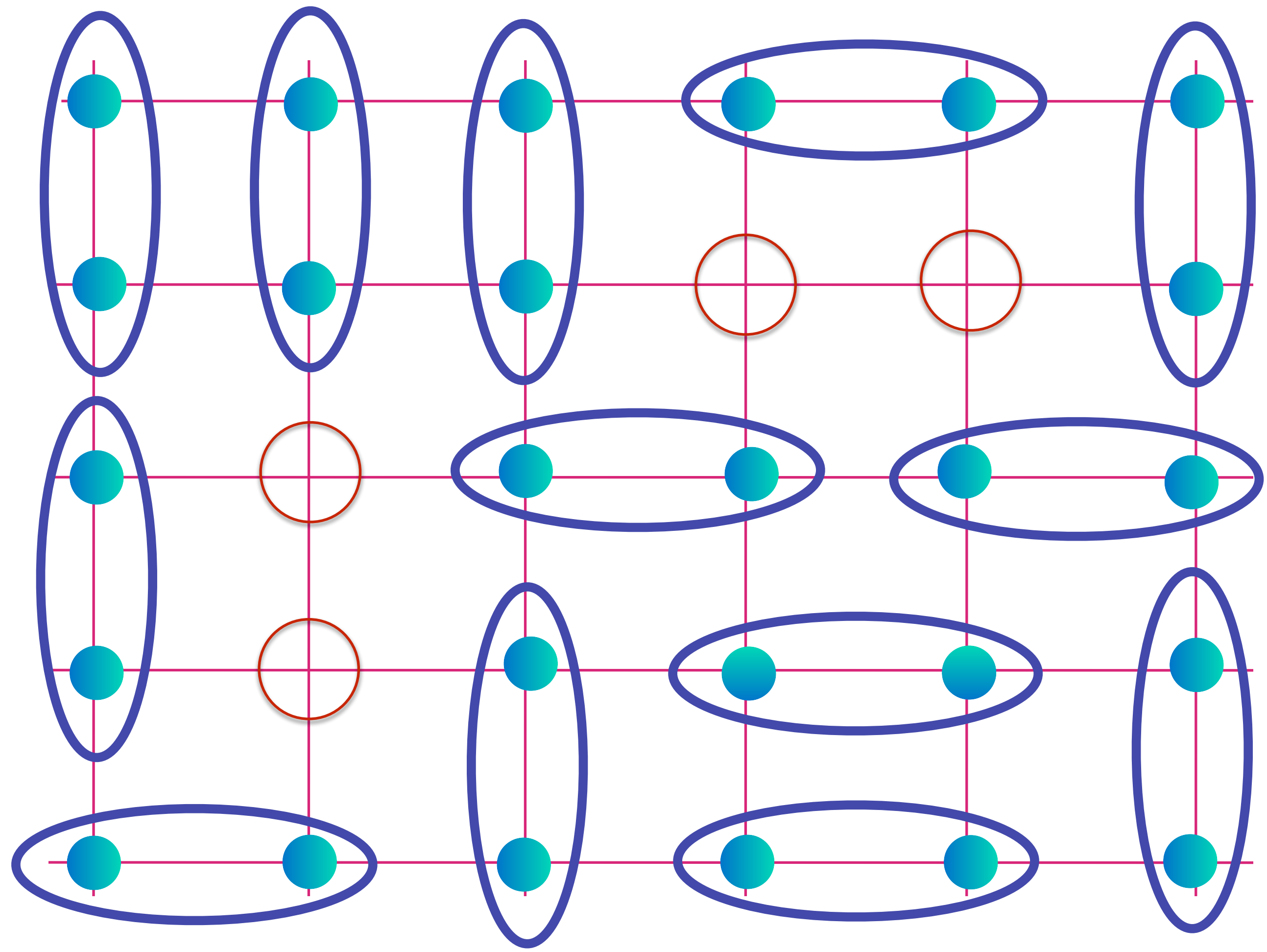
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Superconductivity

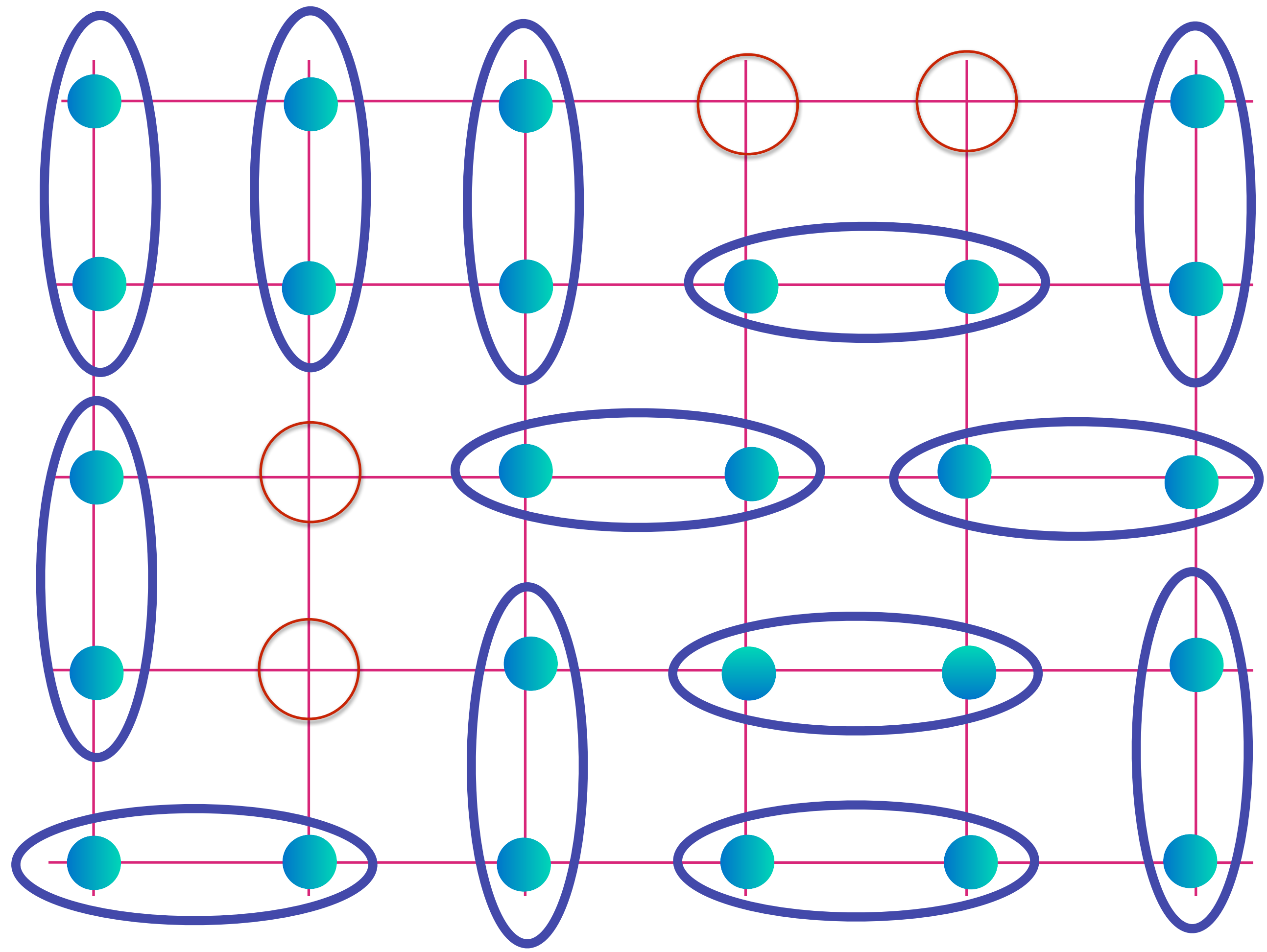
Bose  
condensation of  
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S.N. Bose (1924)

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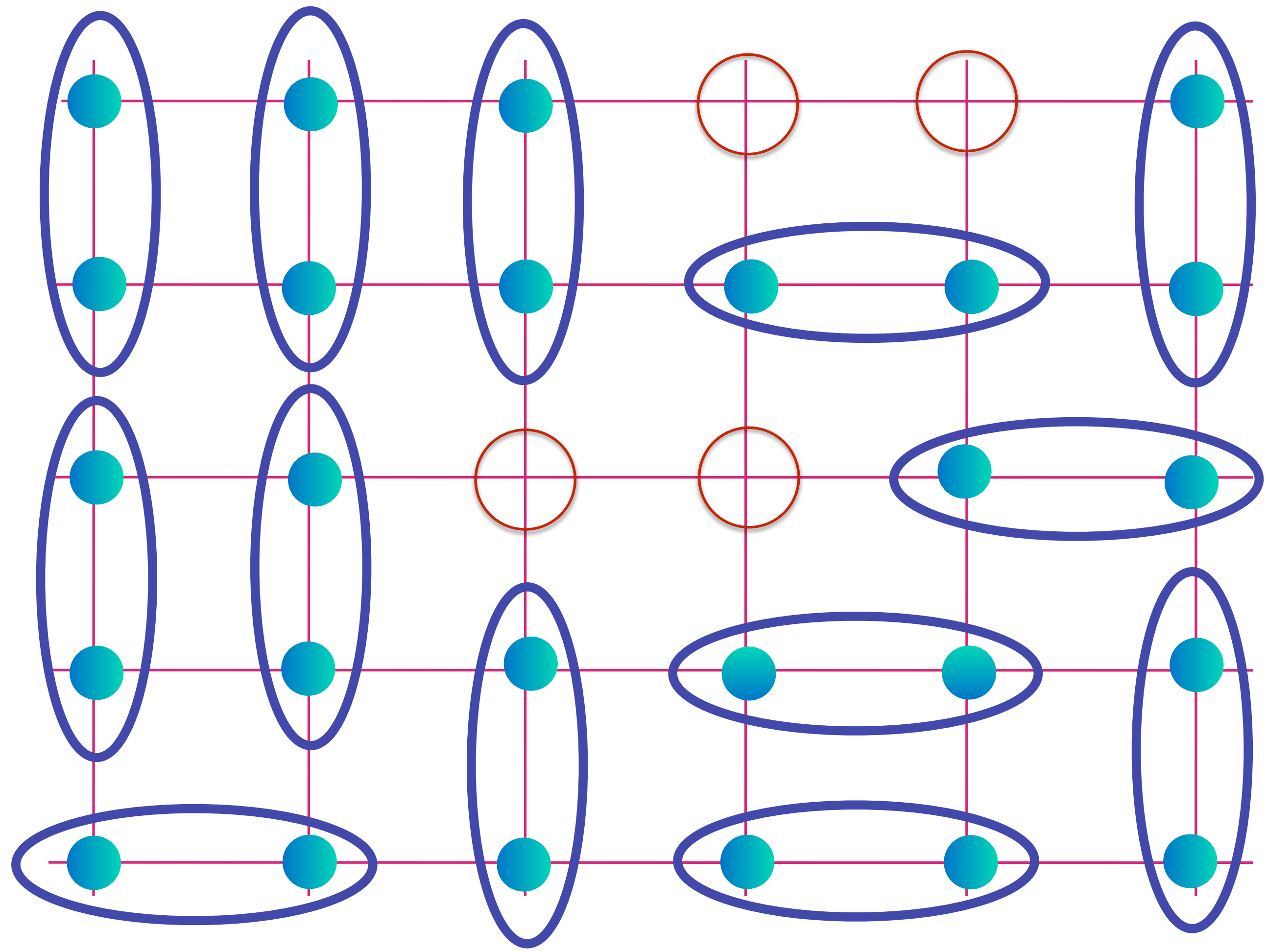
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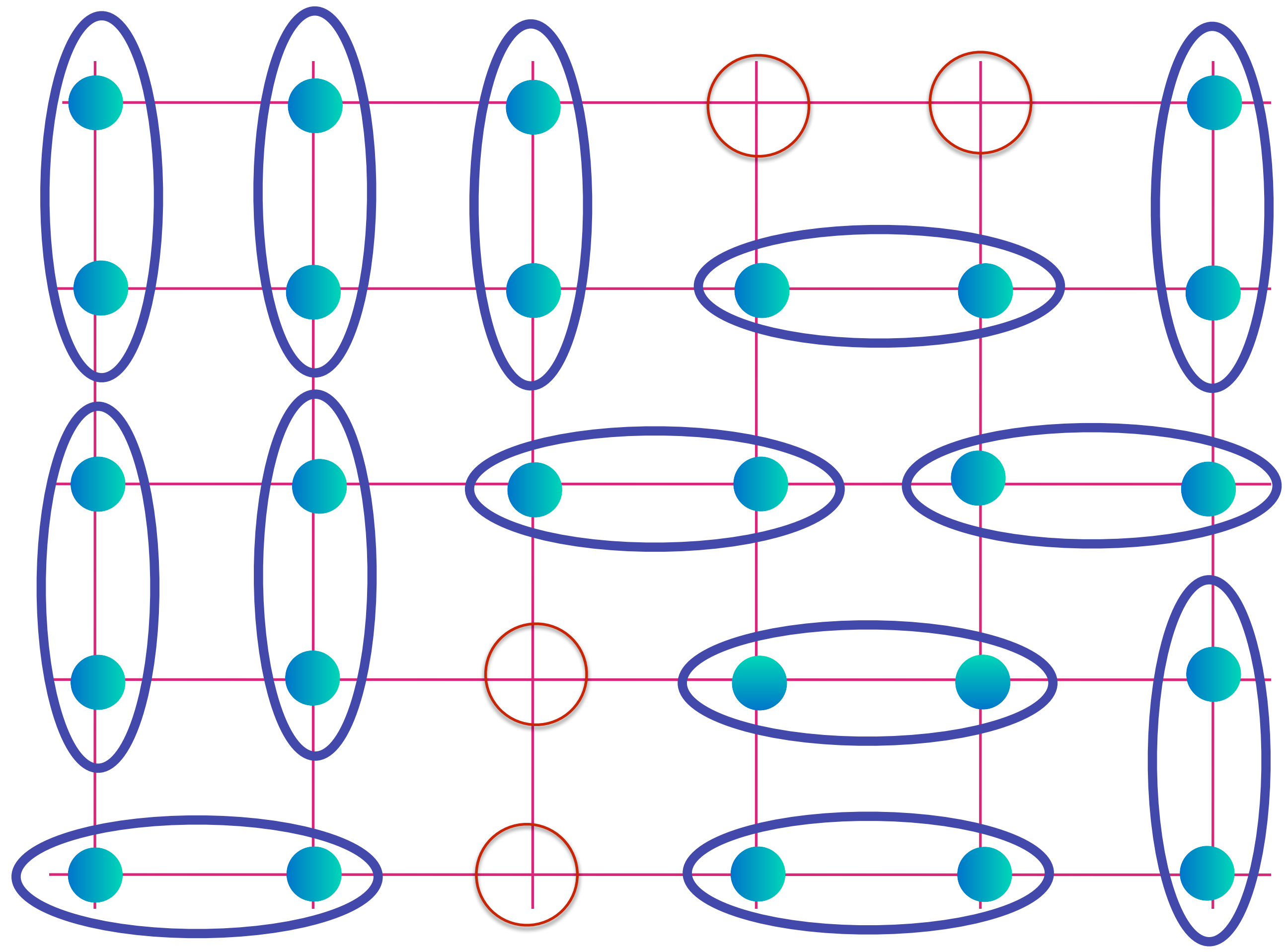
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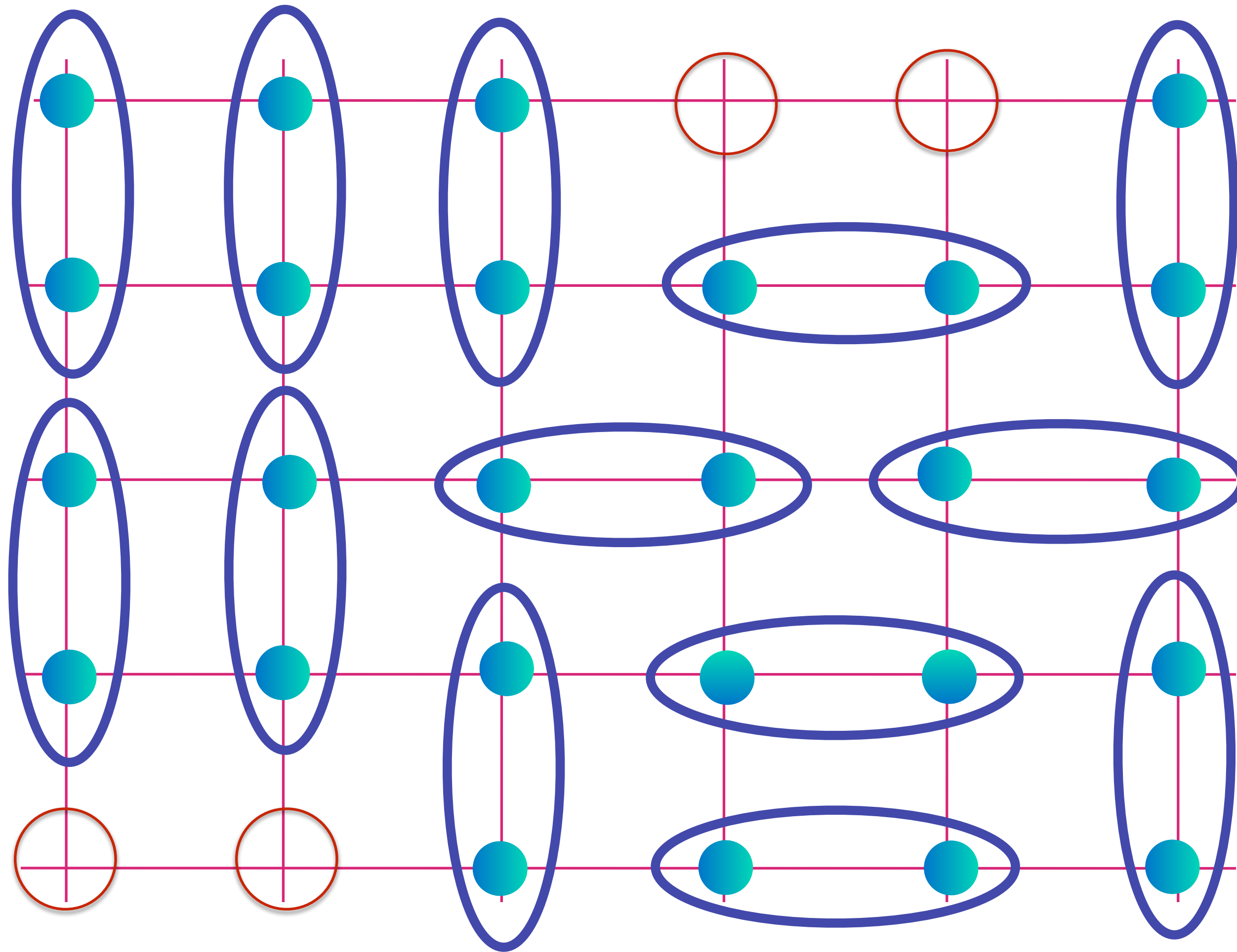
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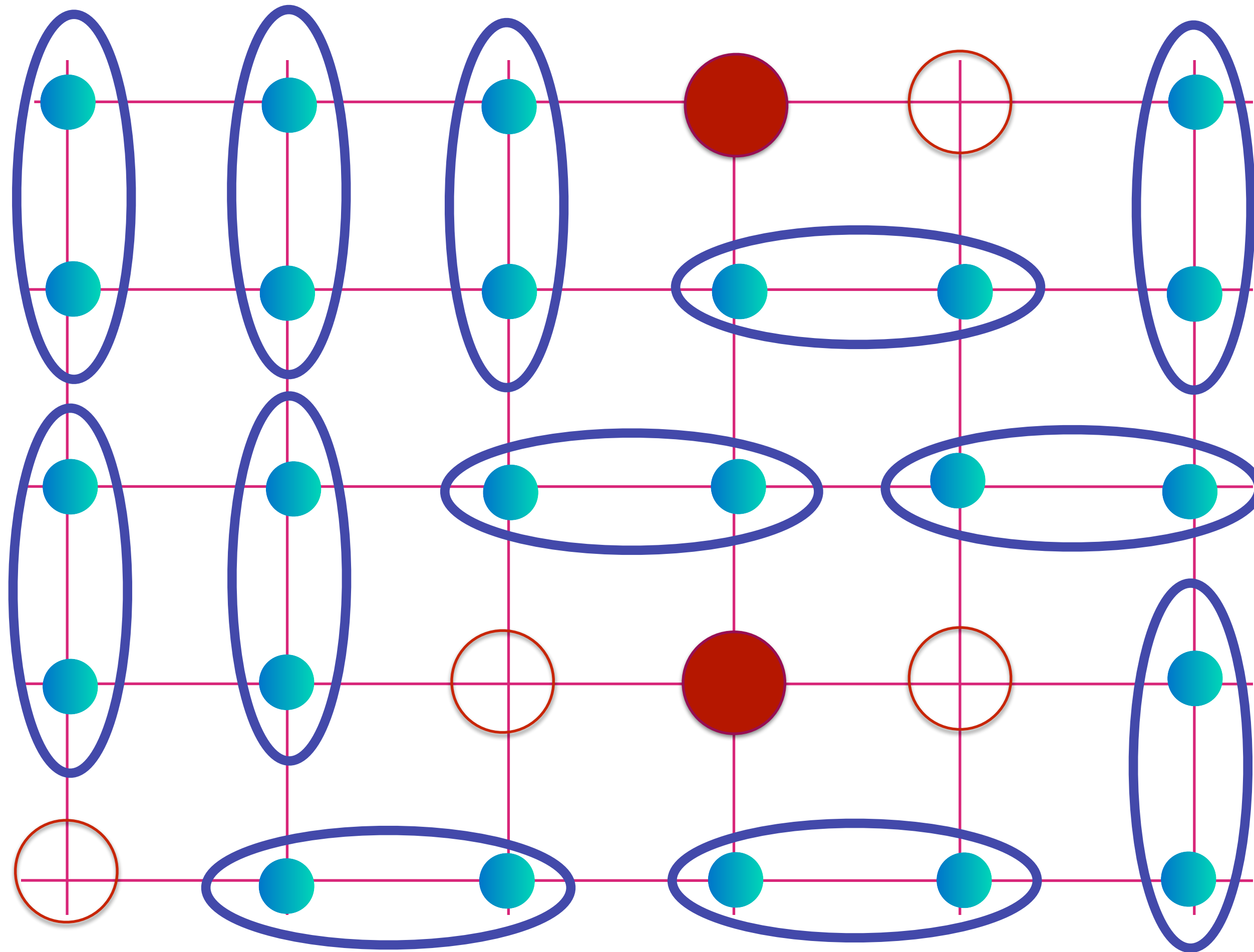
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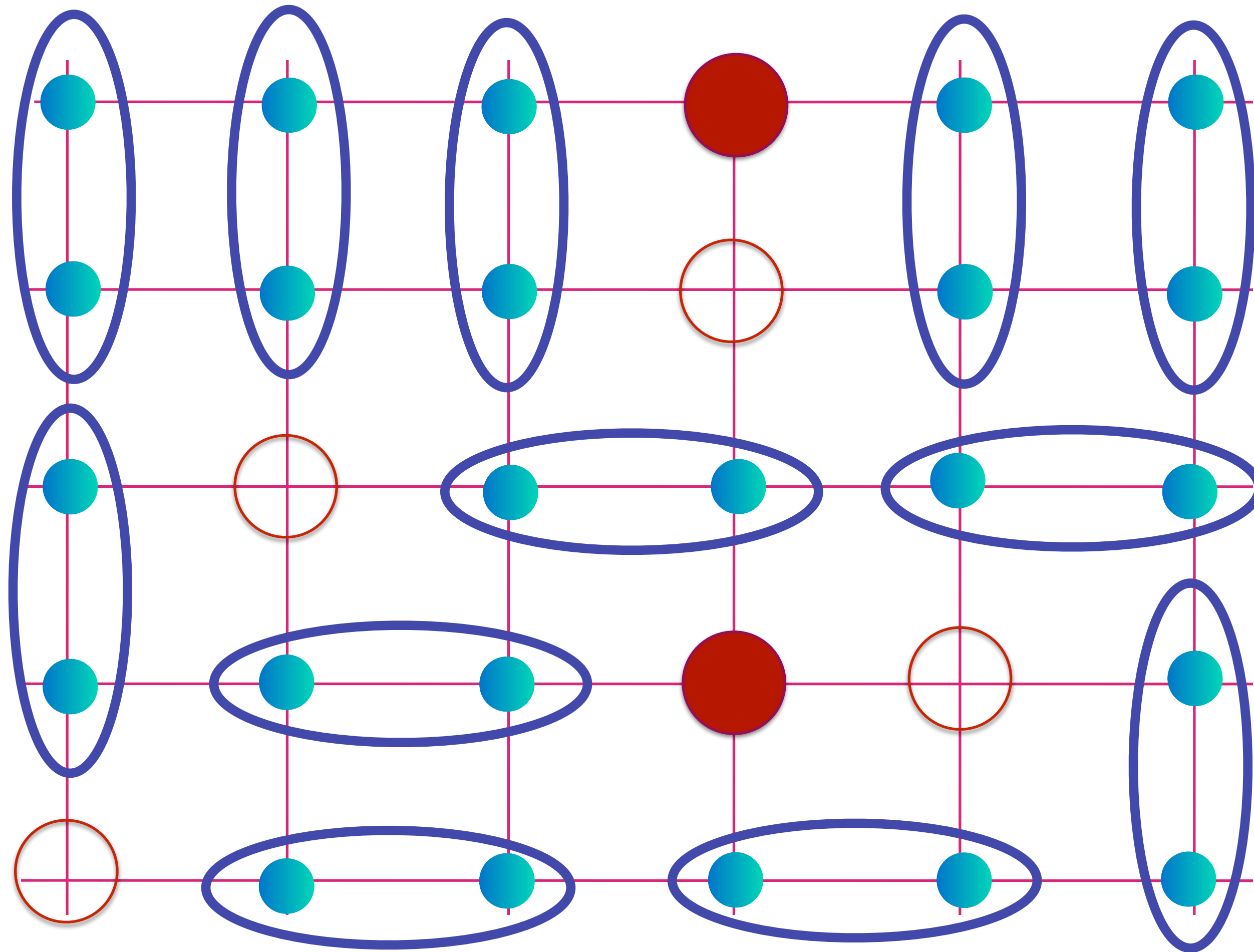


Strange metal

Complex entanglement in the presence of impurities, similar to that in the SYK model

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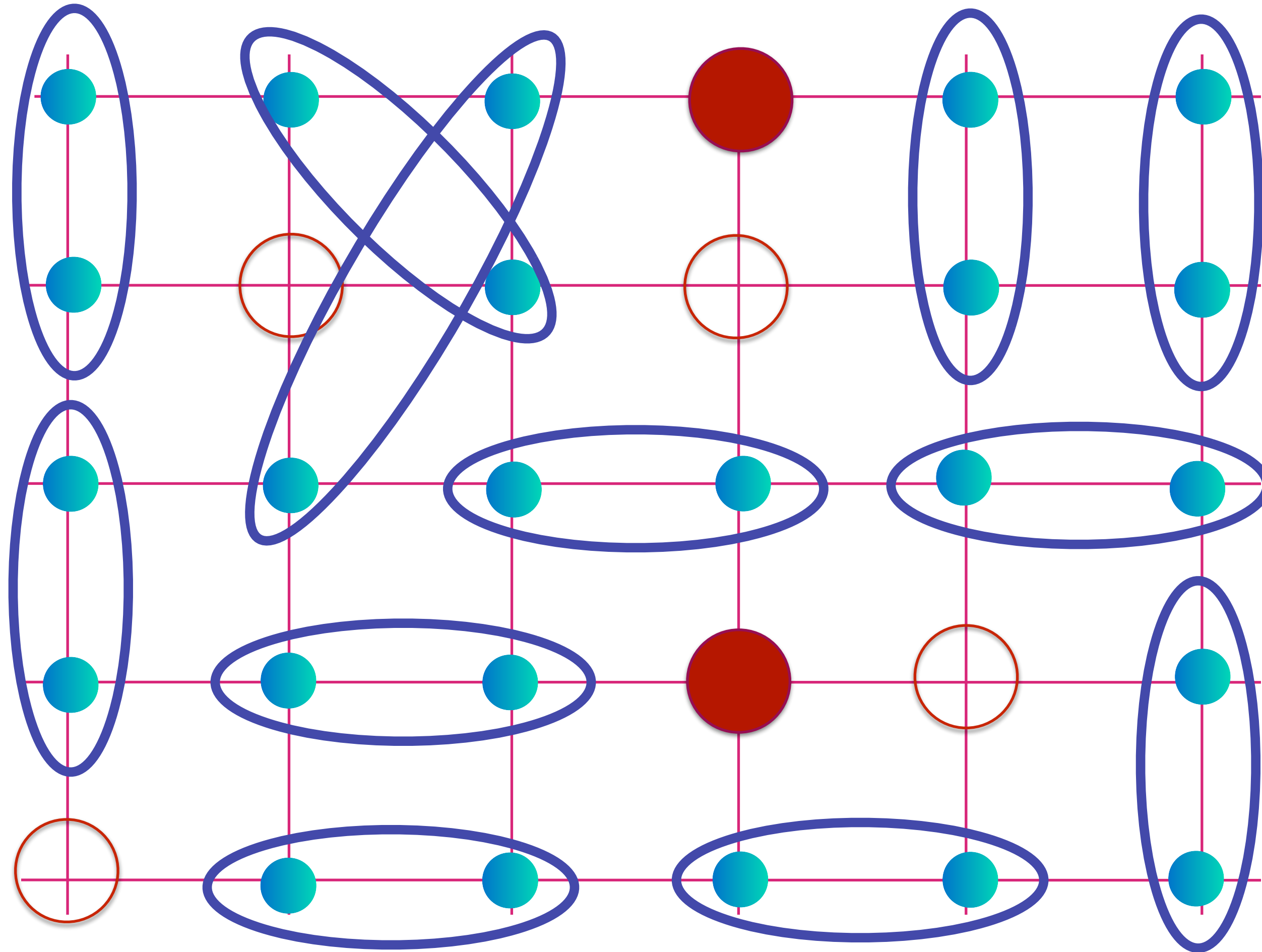


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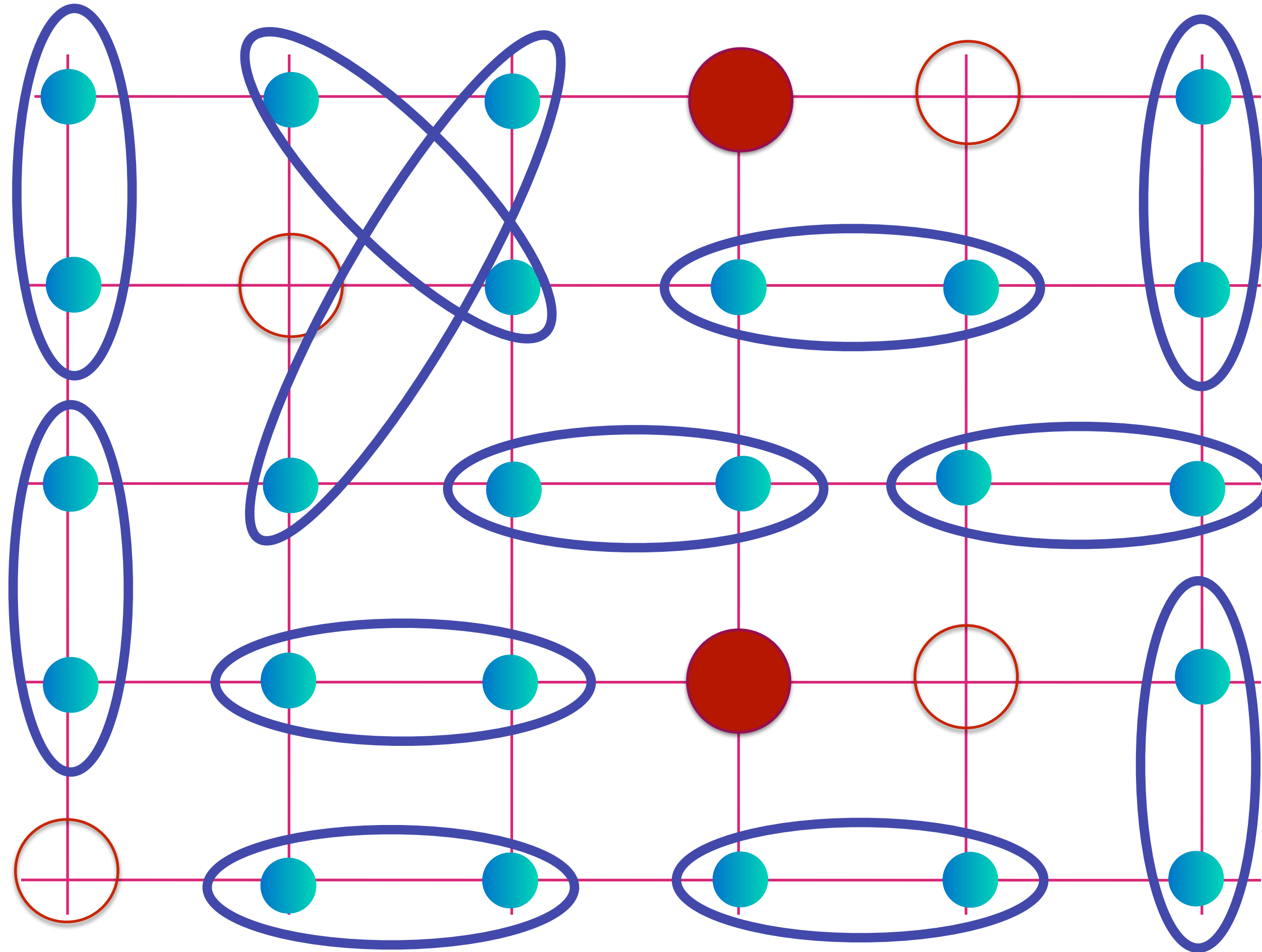


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**Strange metal**

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$$\text{Diagram of two electrons in an orbital} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



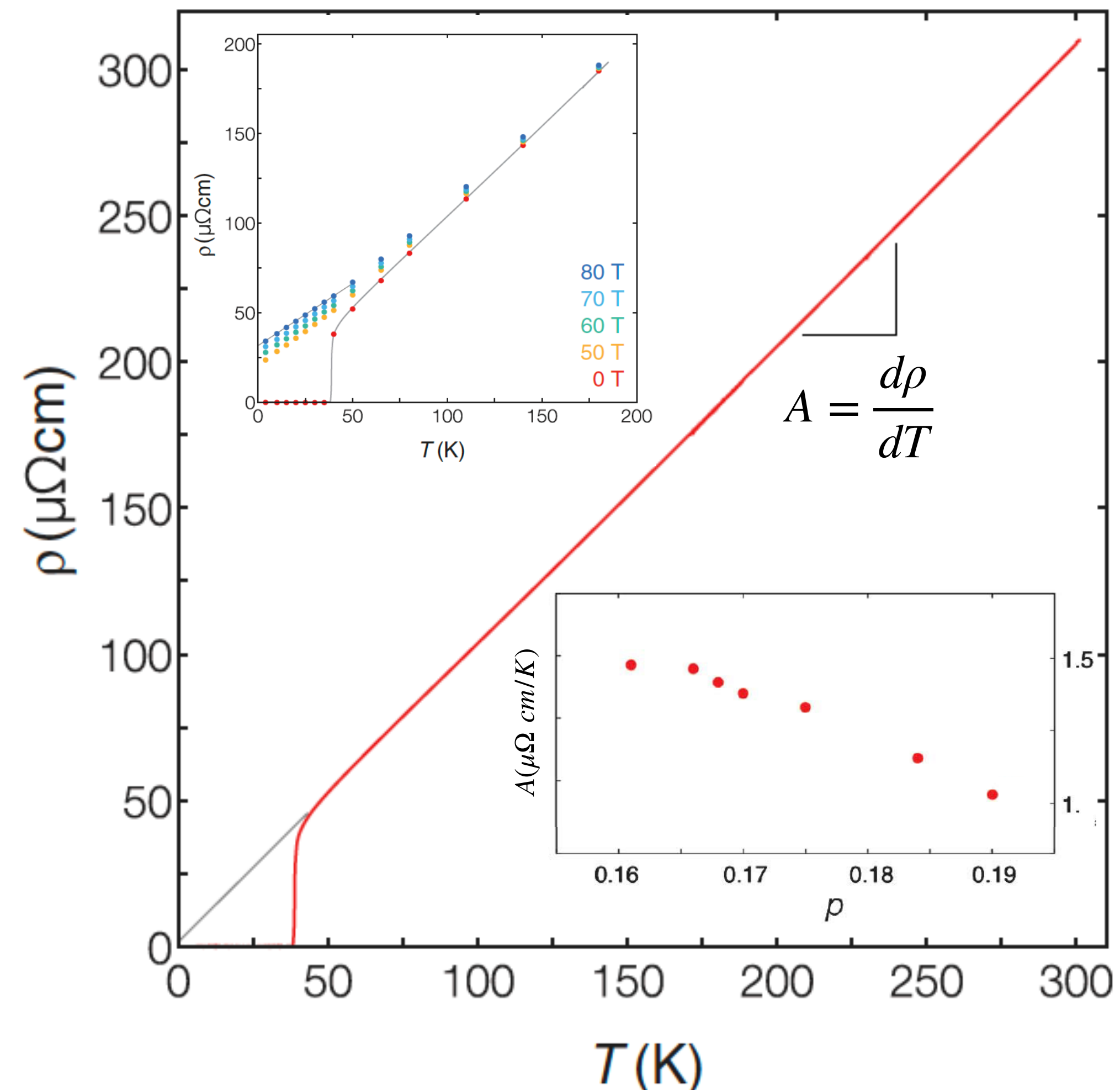
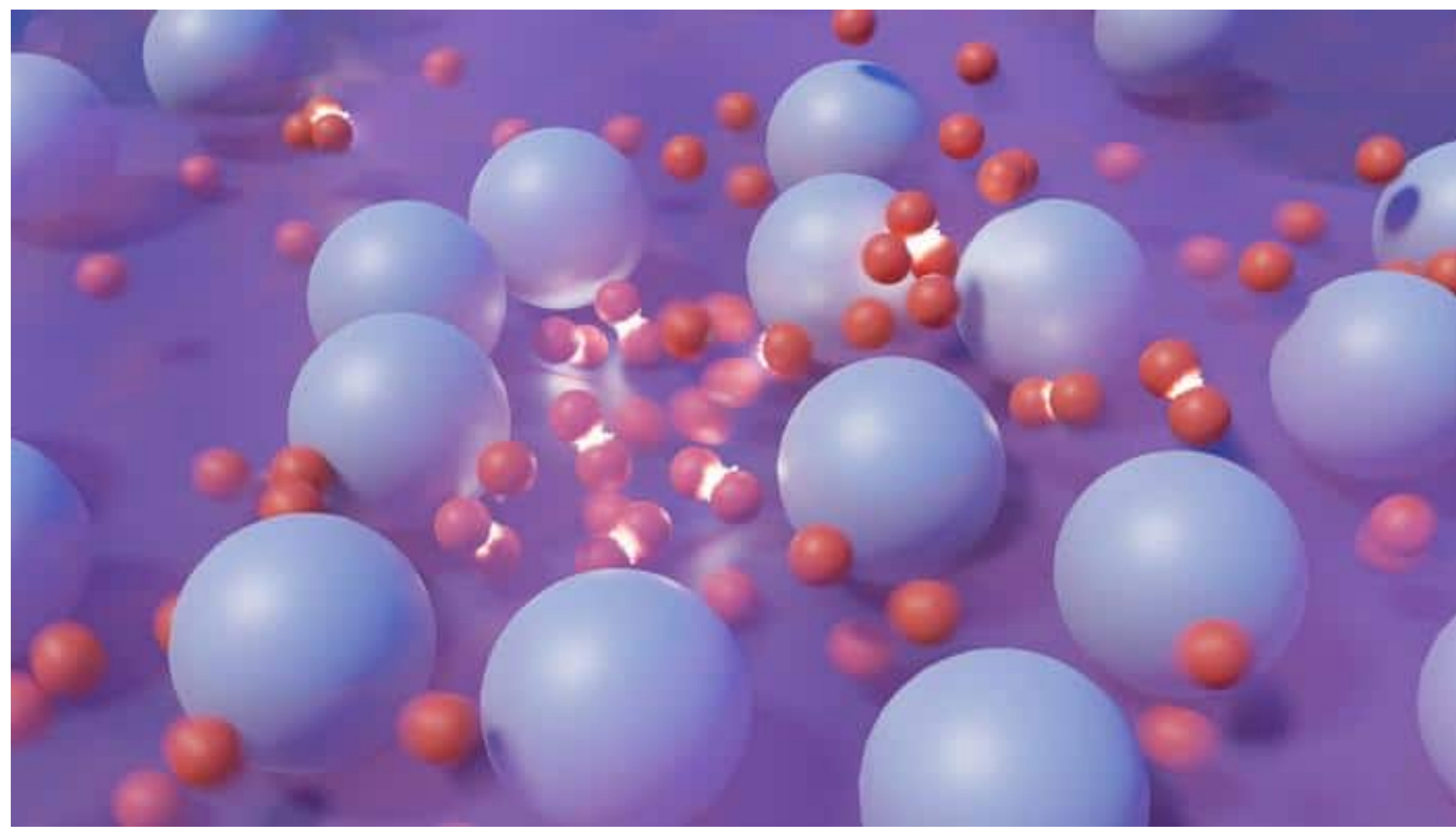
Aavishkar Patel  
Flatiron Institute



Haoyu Guo  
Cornell



Ilya Esterlis  
Wisconsin

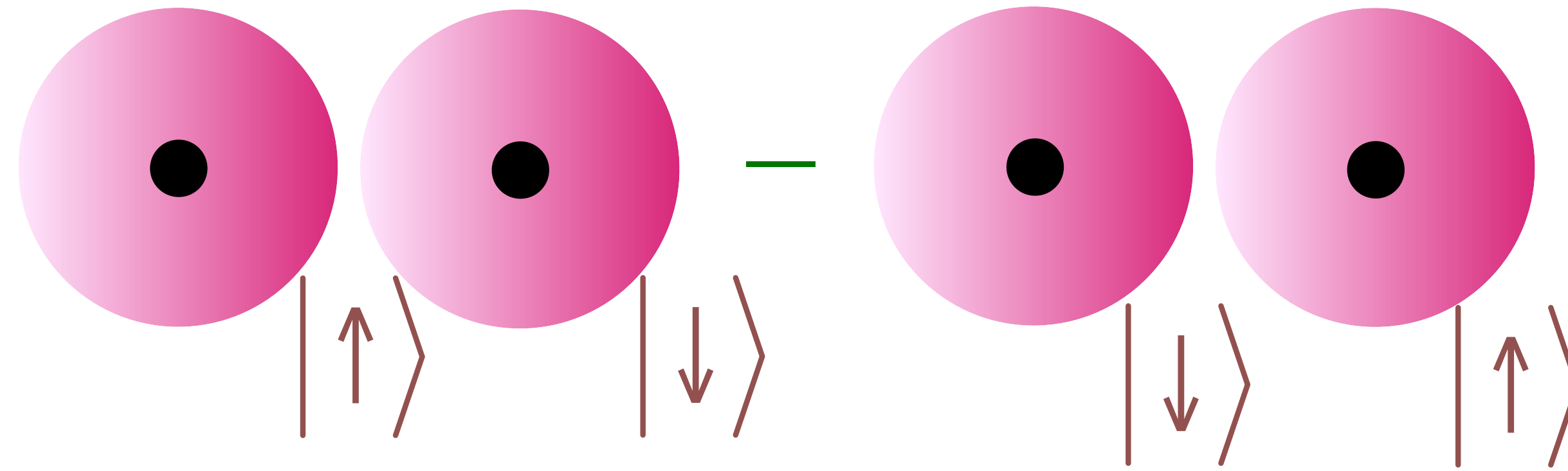


LSCO: Giraldo-Gallo et al. 2018

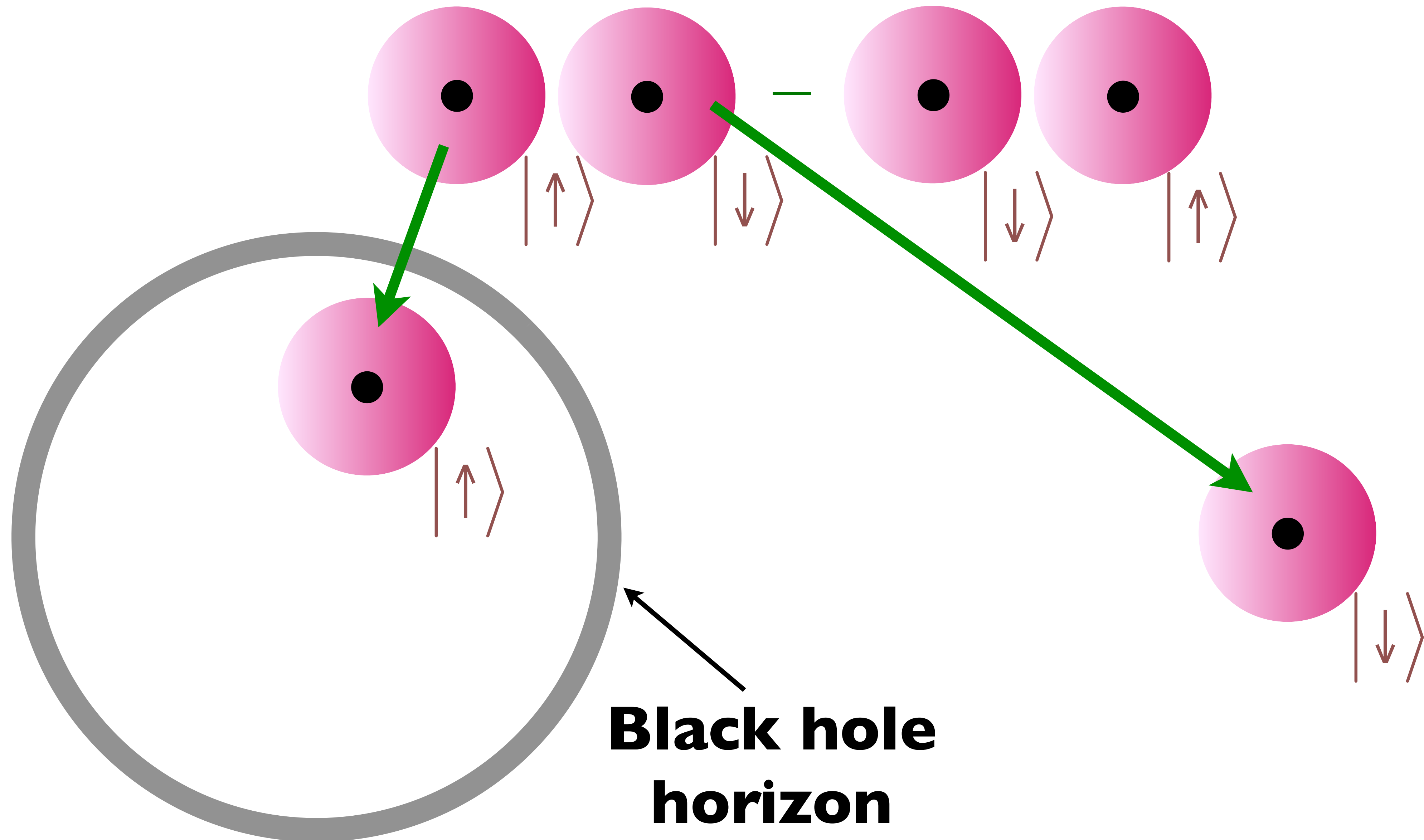
Universal theory of strange metals from  
spatially random interactions,  
Aavishkar A. Patel, Haoyu Guo,  
Ilya Esterlis, and S. Sachdev,  
*Science* **381**, 790 (2023)

Quantum entanglement,  
the SYK model,  
and black holes  
holes

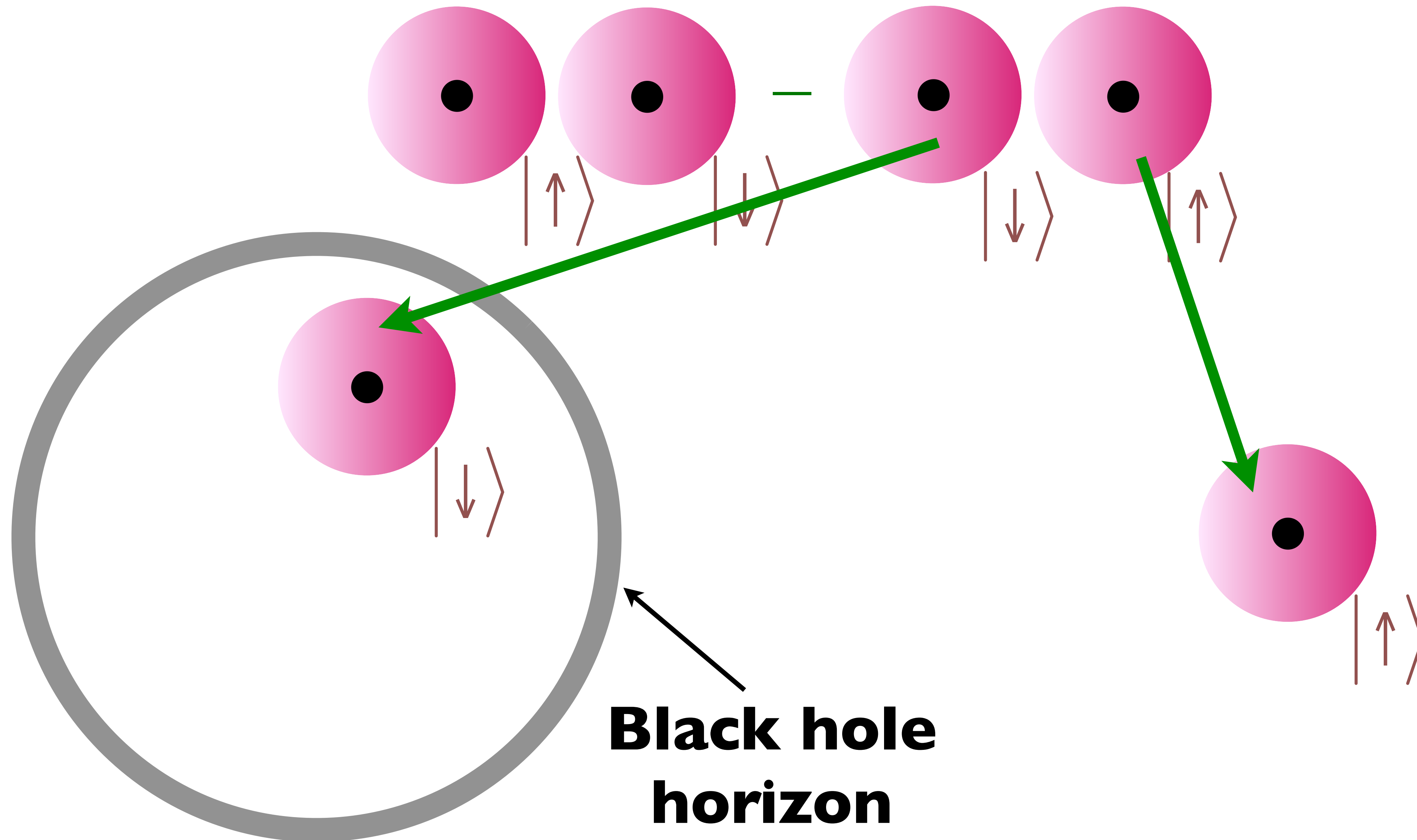
# Quantum Entanglement across a black hole horizon



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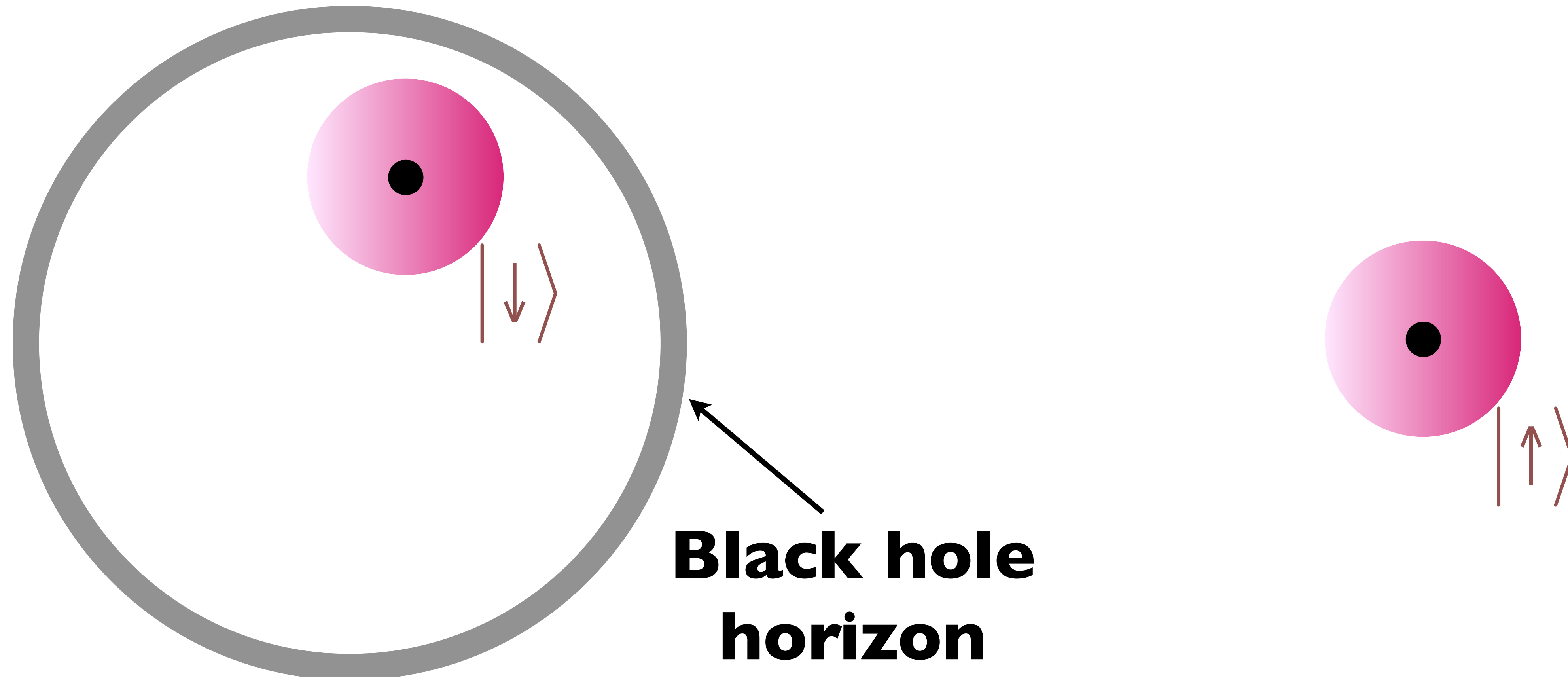


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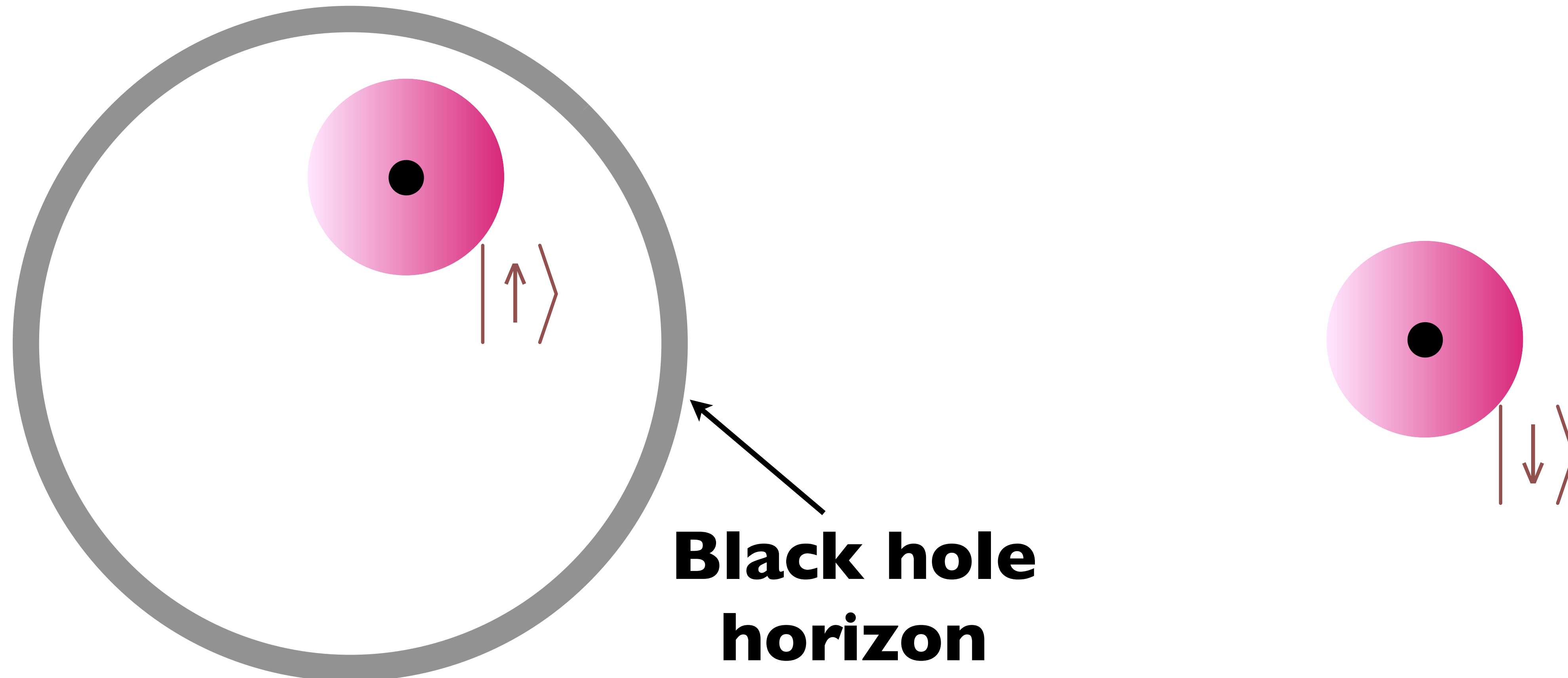
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There is quantum entanglement between the inside and outside of a black hole



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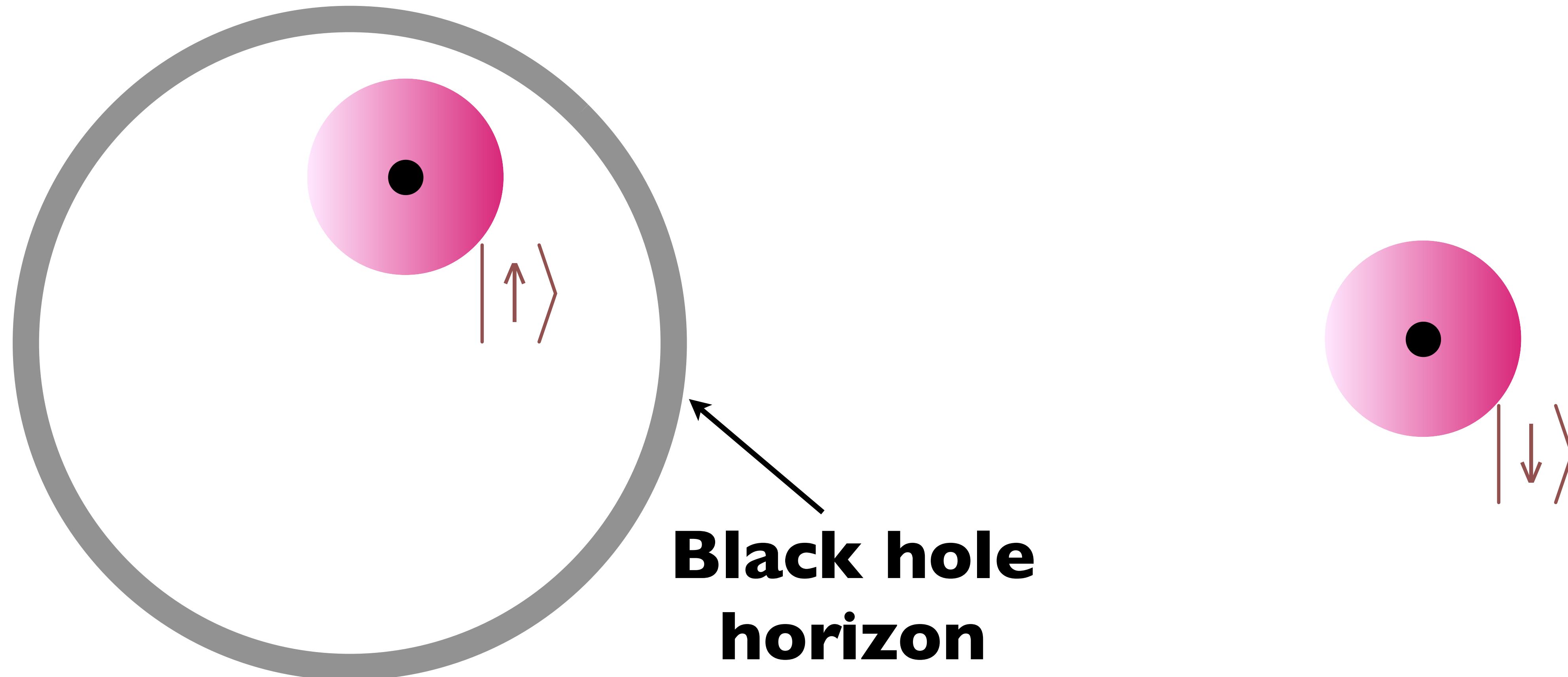
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# Quantum Entanglement across a black hole horizon

*Hawking (1975): Black holes have a temperature and an entropy!*

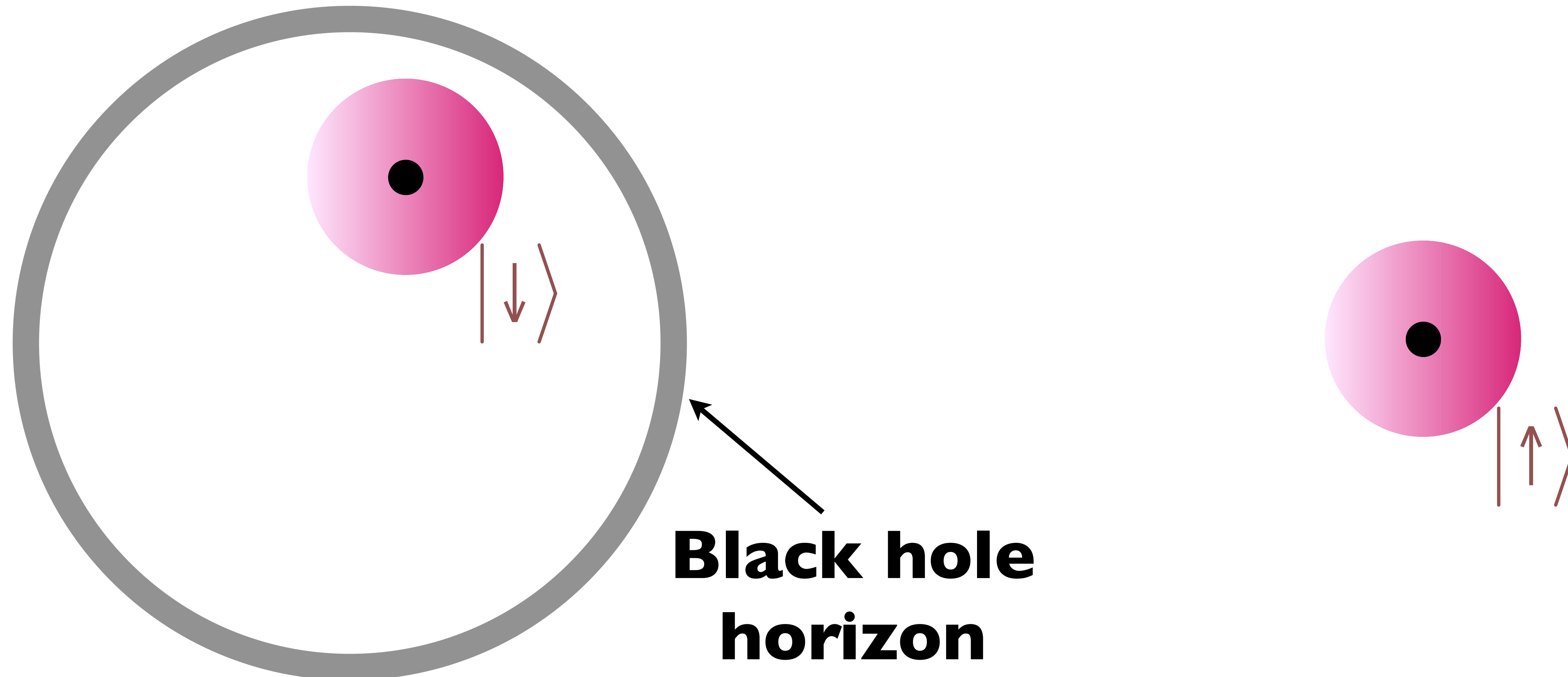
To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.



# Quantum Entanglement across a black hole horizon

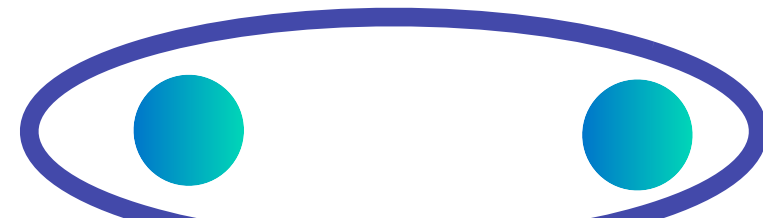
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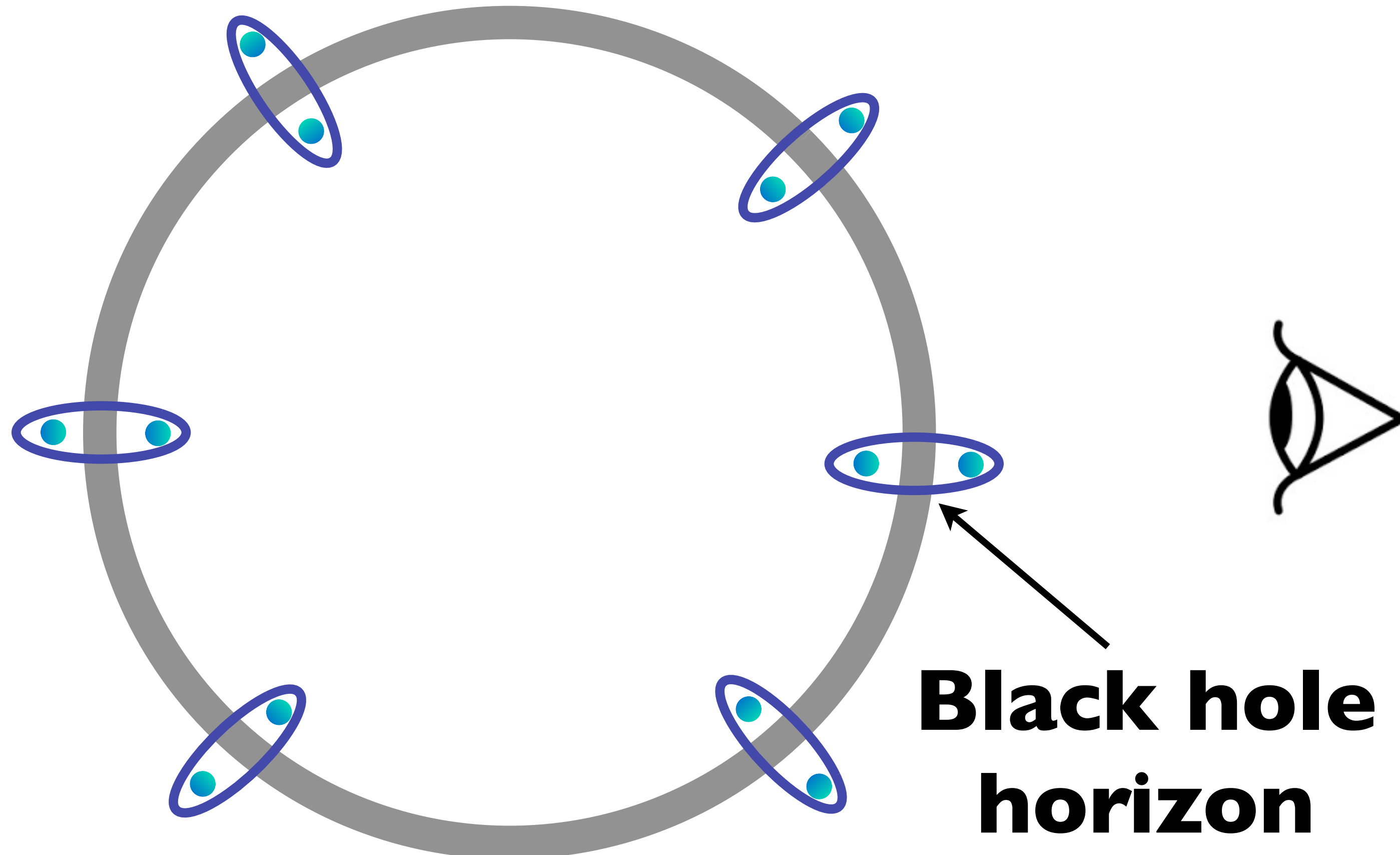
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# Quantum Entanglement across a black hole horizon

Quantum entanglement  
on the surface


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



By computations *outside*  
the black hole,  
Hawking obtained

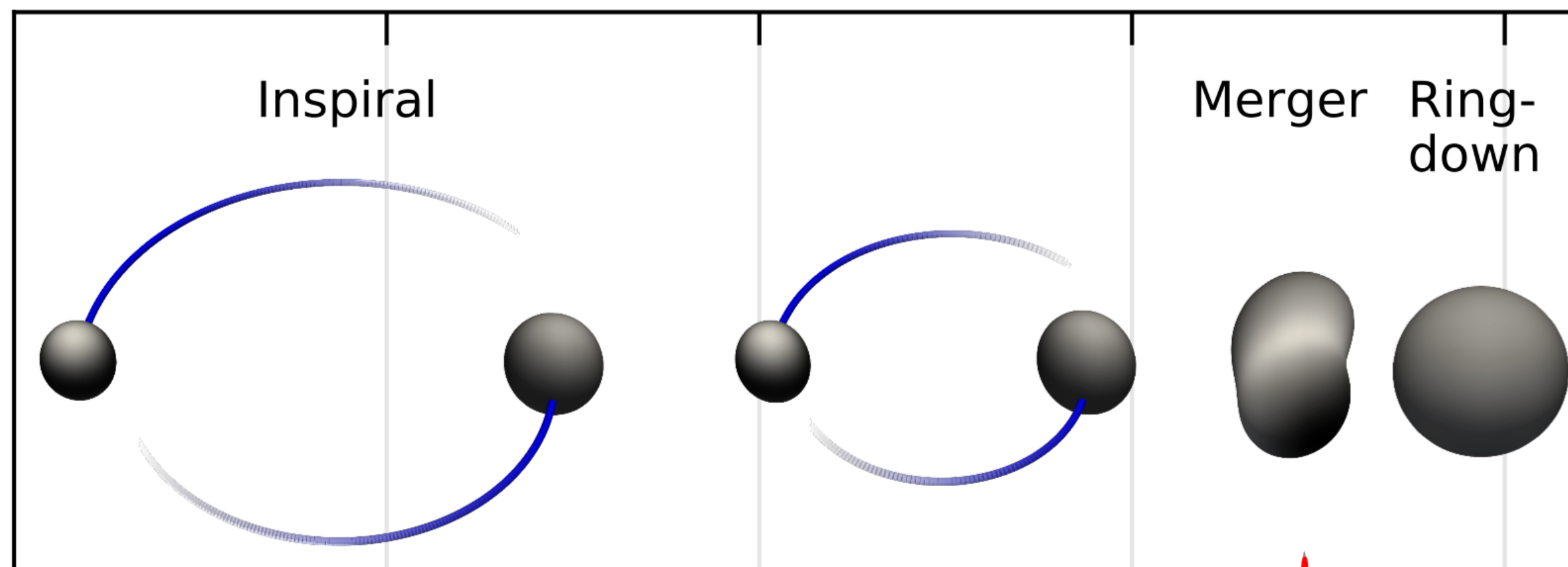
$$S = \frac{Ac^3}{4G\hbar}$$

where  $A$  is area of the  
black hole horizon.

All other systems have  
entropy proportional to  
their volume.

# Quantum black holes

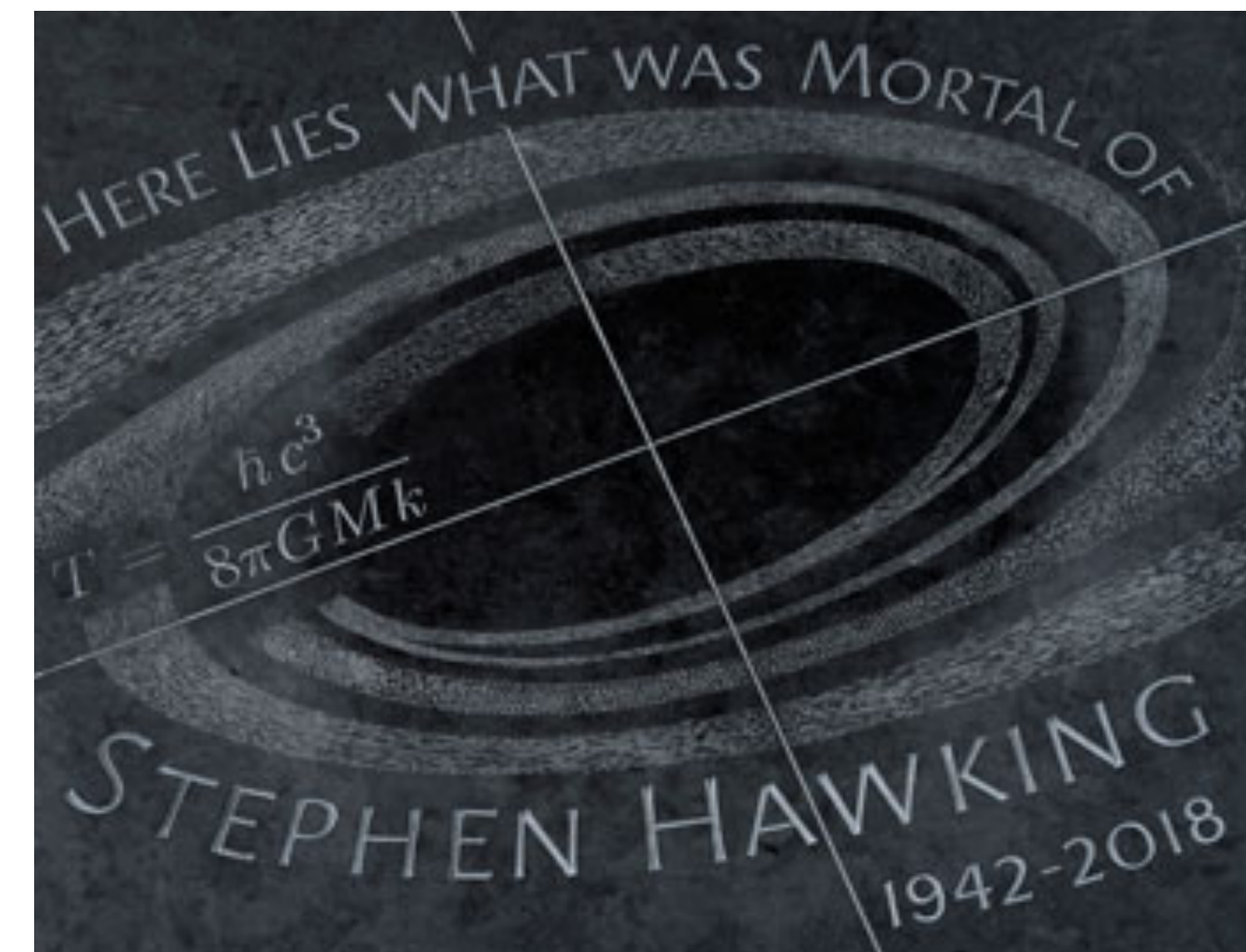
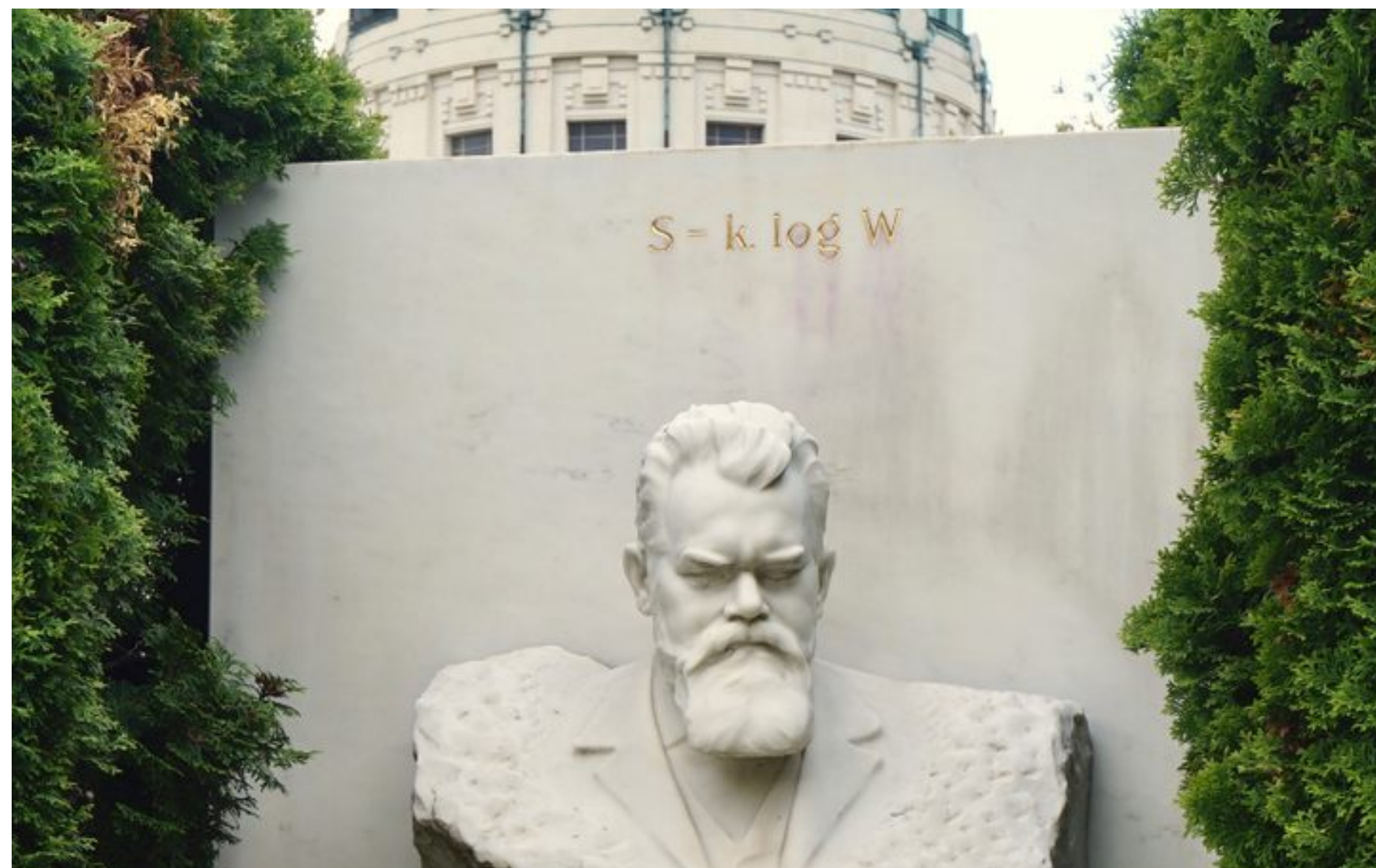
- Black holes have an entropy and a temperature,  $T_H = \hbar c^3 / (8\pi G M k_B)$ .
- The entropy is proportional to their surface area.  $S = A k_B c^3 / (4G\hbar)$ .
- They relax to thermal equilibrium in a time  $\sim 8\pi G M / c^3 = \hbar / (k_B T_H)$ , the Planckian time also found in strange metals, which suggests a connection to the SYK model!



J. D. Bekenstein, PRD **7**, 2333 (1973)  
S.W. Hawking, Nature **248**, 30 (1974)  
C.V. Vishveshwara, Nature **227**, 936 (1970)

# Quantum Black Holes

- Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states*  $D(E)$  [the quantum analog of Boltzmann's  $W$ ] matches Hawking's entropy, in accordance with Boltzmann's principles of statistical mechanics,  $S(E) = k_B \log D(E)$  ?



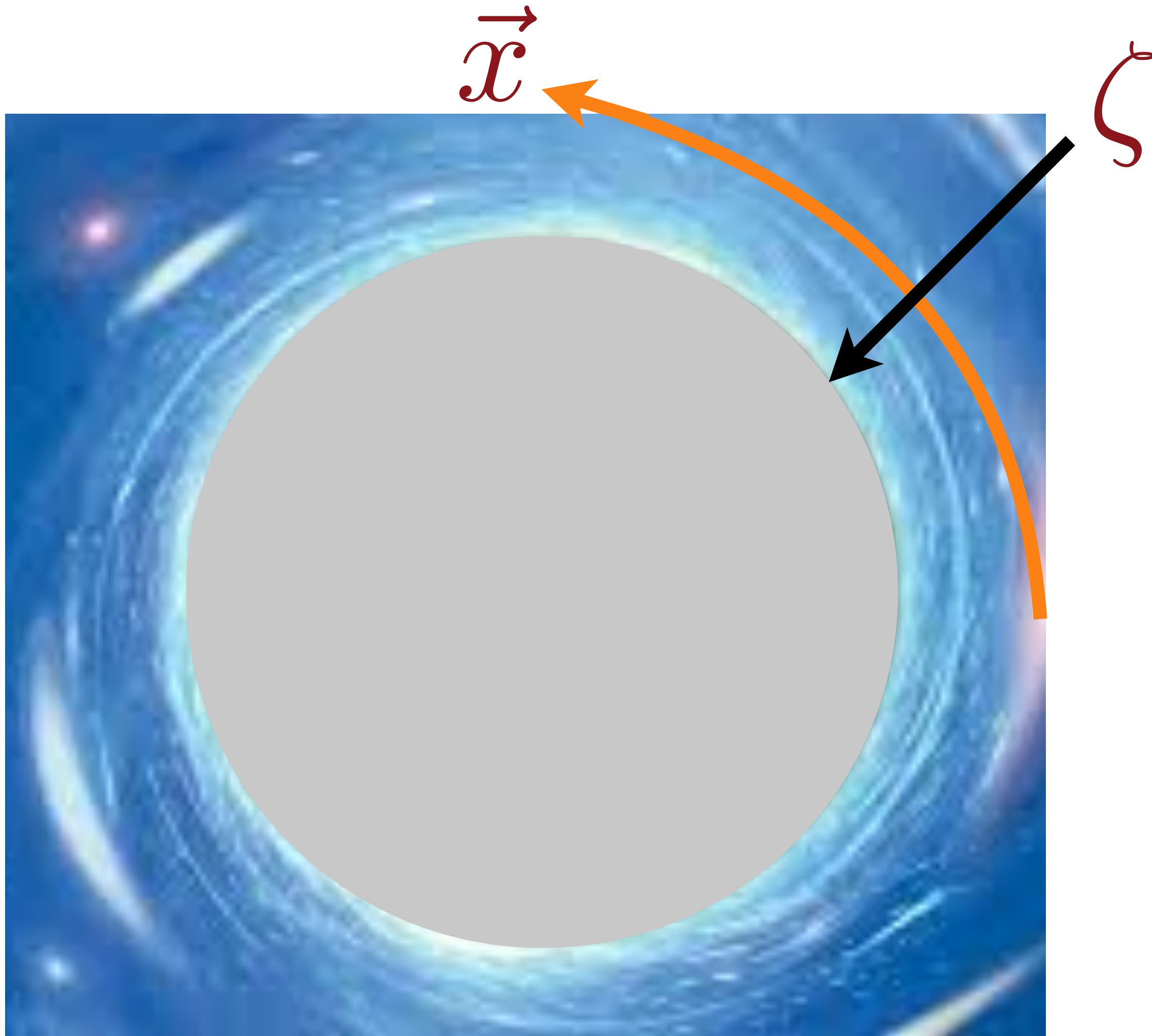
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- Answer from string theory for 'supersymmetric' charged black holes:  $D(E) = e^S \delta(E)$  *i.e.* all the states required by Hawking's entropy have exactly the same energy.

Strominger, Vafa (1996)

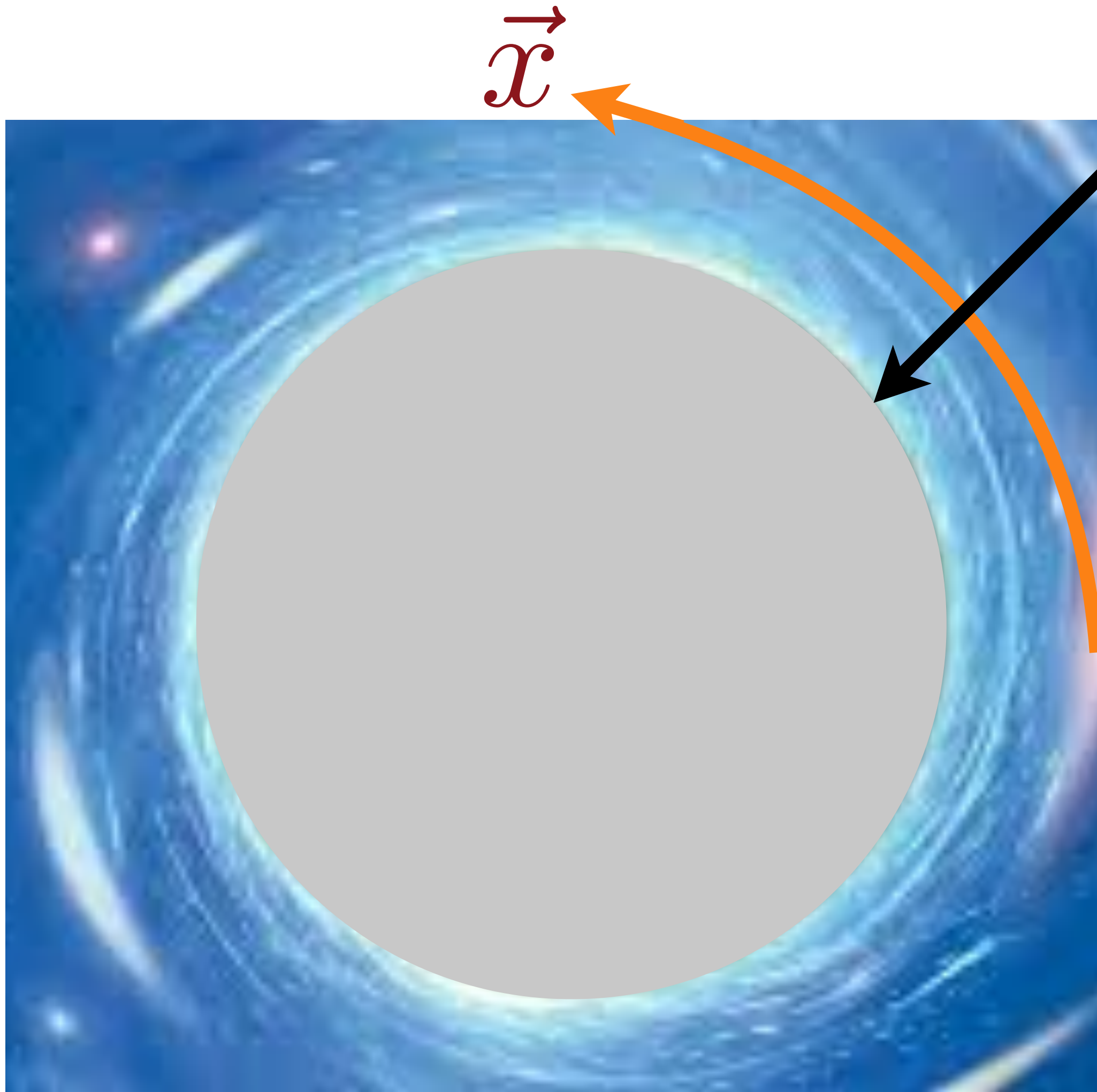


Maxwell's electromagnetism  
and Einstein's general relativity  
allow black hole solutions with a net charge





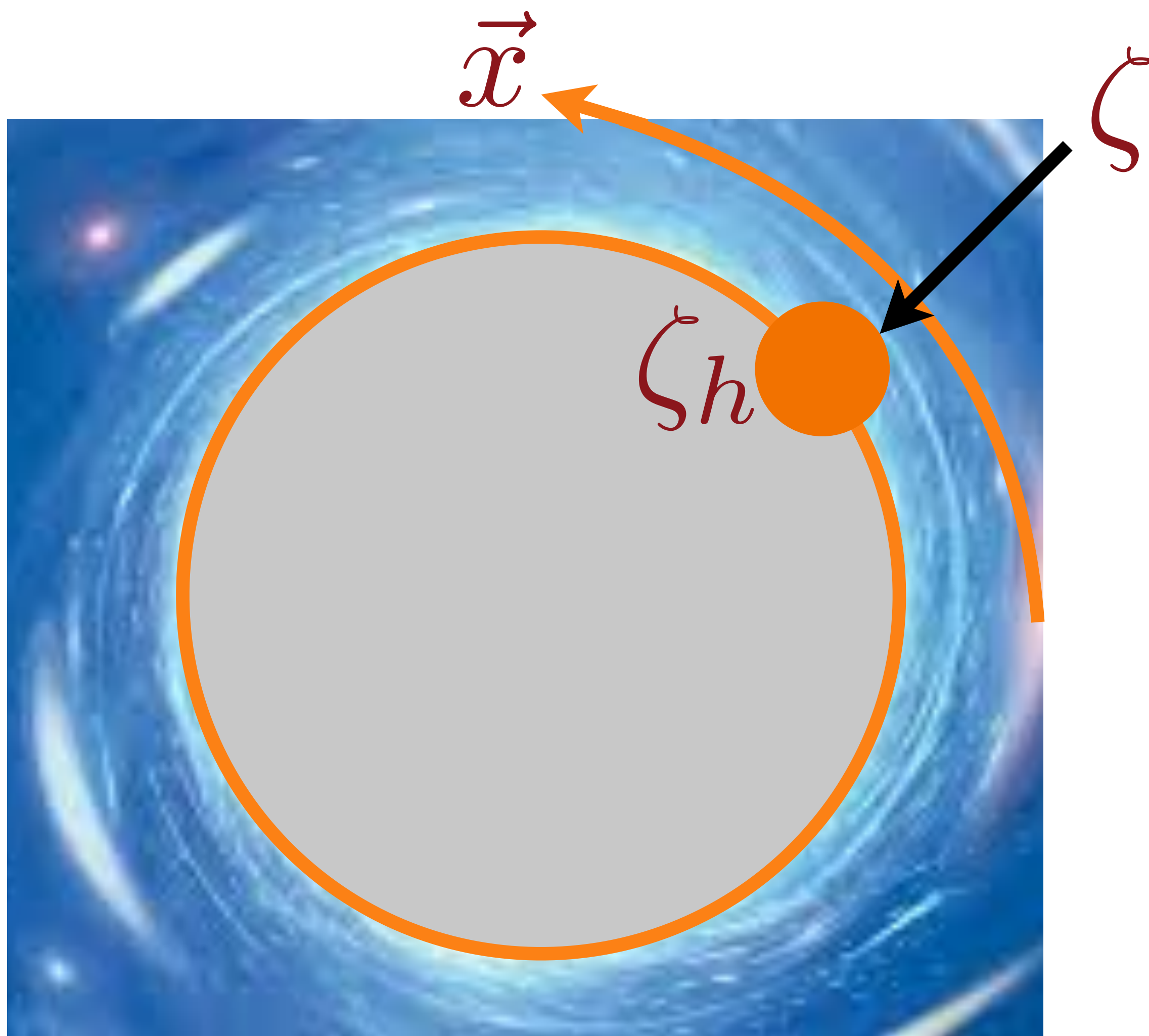
Maxwell's electromagnetism  
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Zooming into the  
near-horizon region  
of a charged black hole  
at low temperature,  
yields a theory  
in one space ( $\zeta$ ) and  
one time dimension



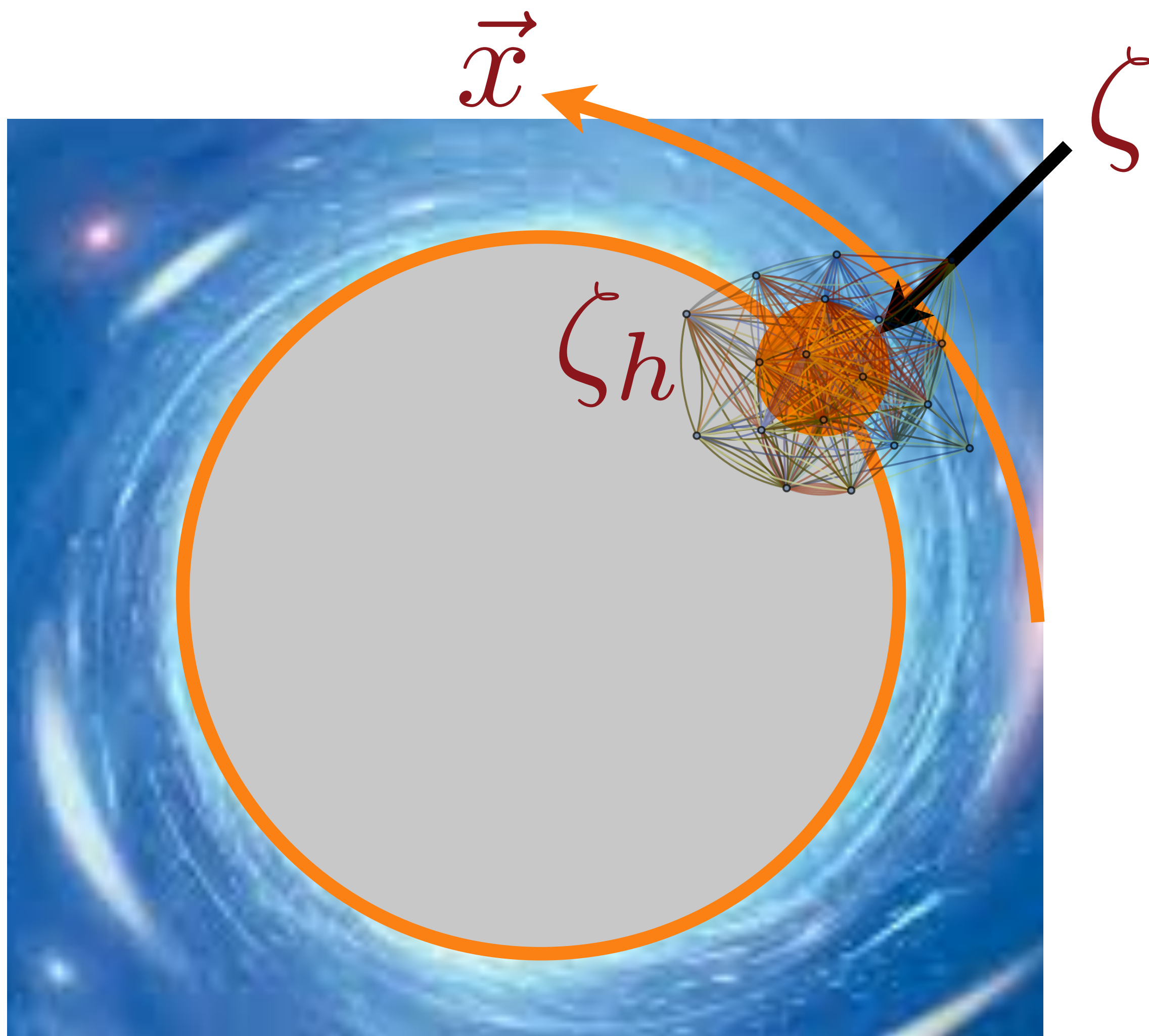
Maxwell's electromagnetism  
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So we need only consider  
complex entanglement at  
one spatial “point”  
on the horizon ( $\zeta = \zeta_h$ ),  
just as is described  
by the SYK model



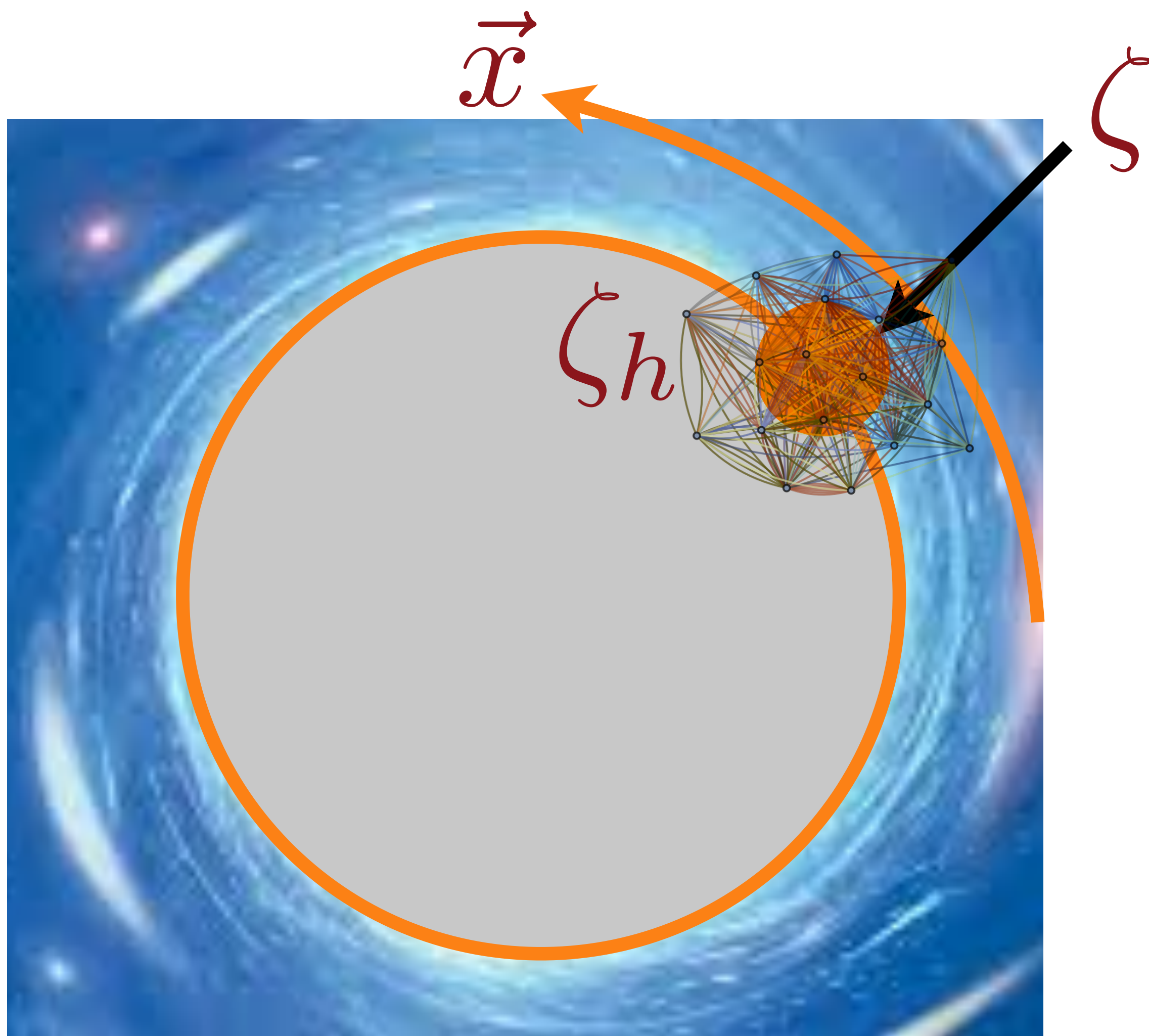
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The quantum versions of  
Maxwell's and Einstein's  
equations in this  
two-dimensional spacetime are  
also the equations describing  
electron entanglement in the  
SYK model!



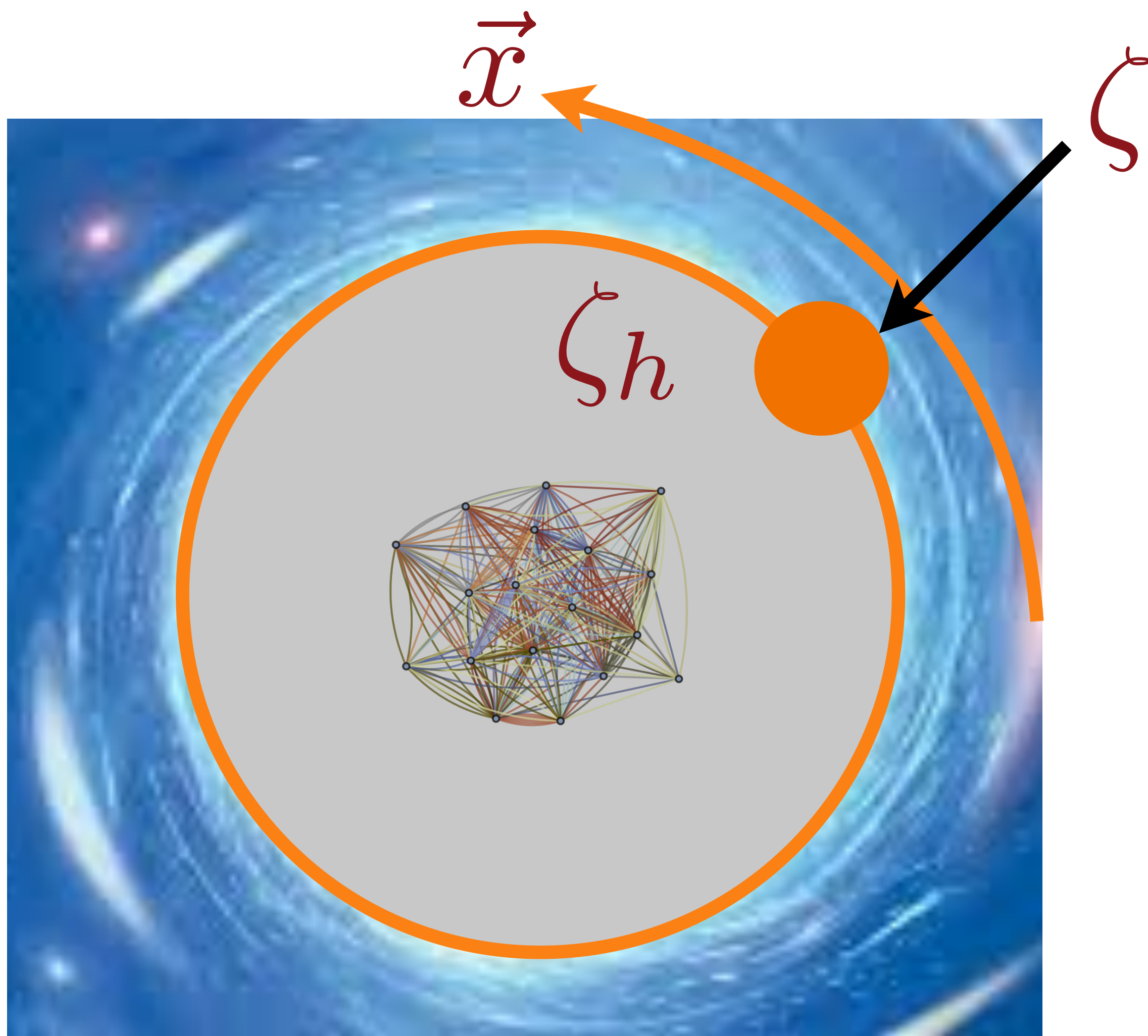
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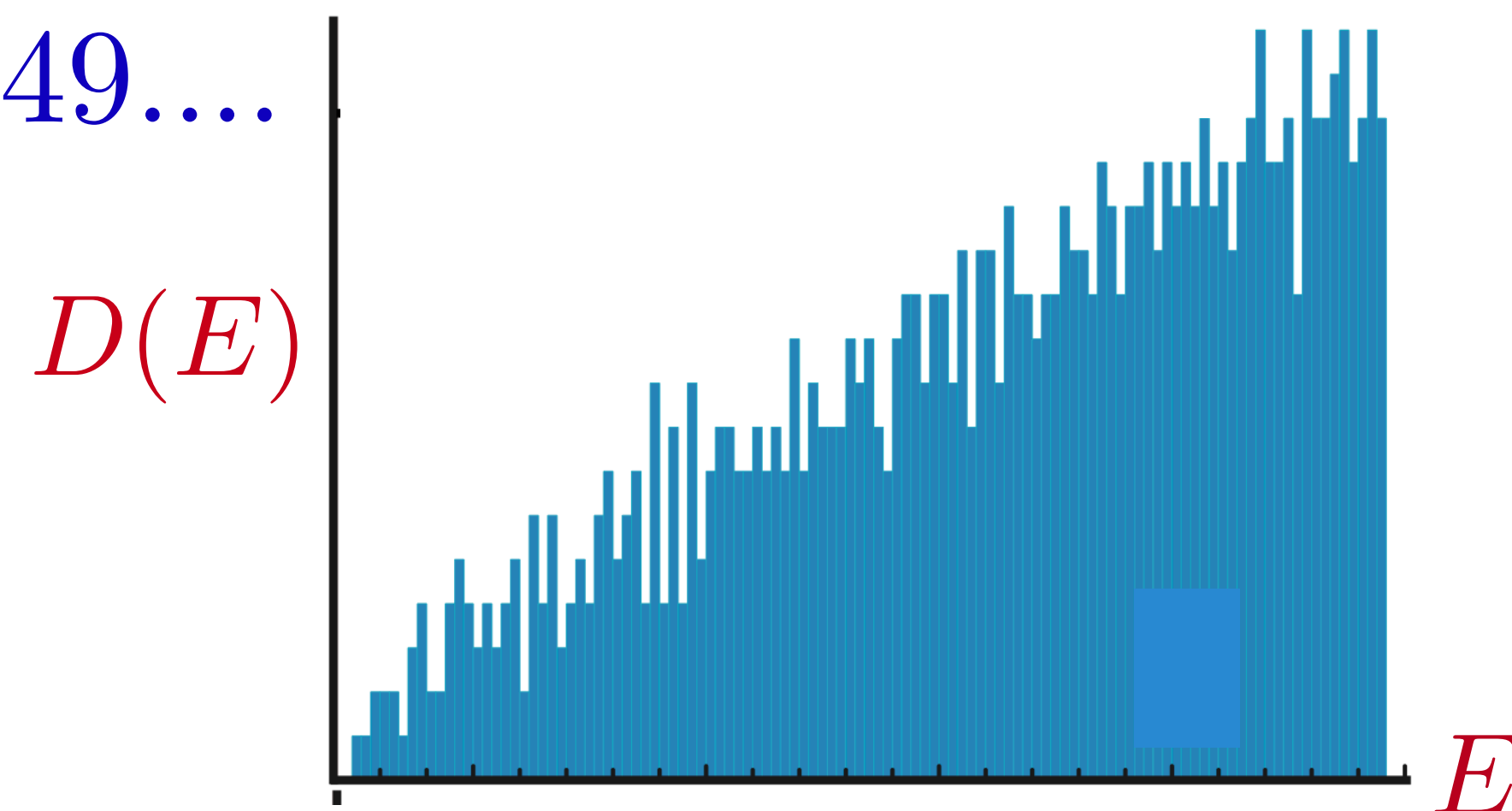
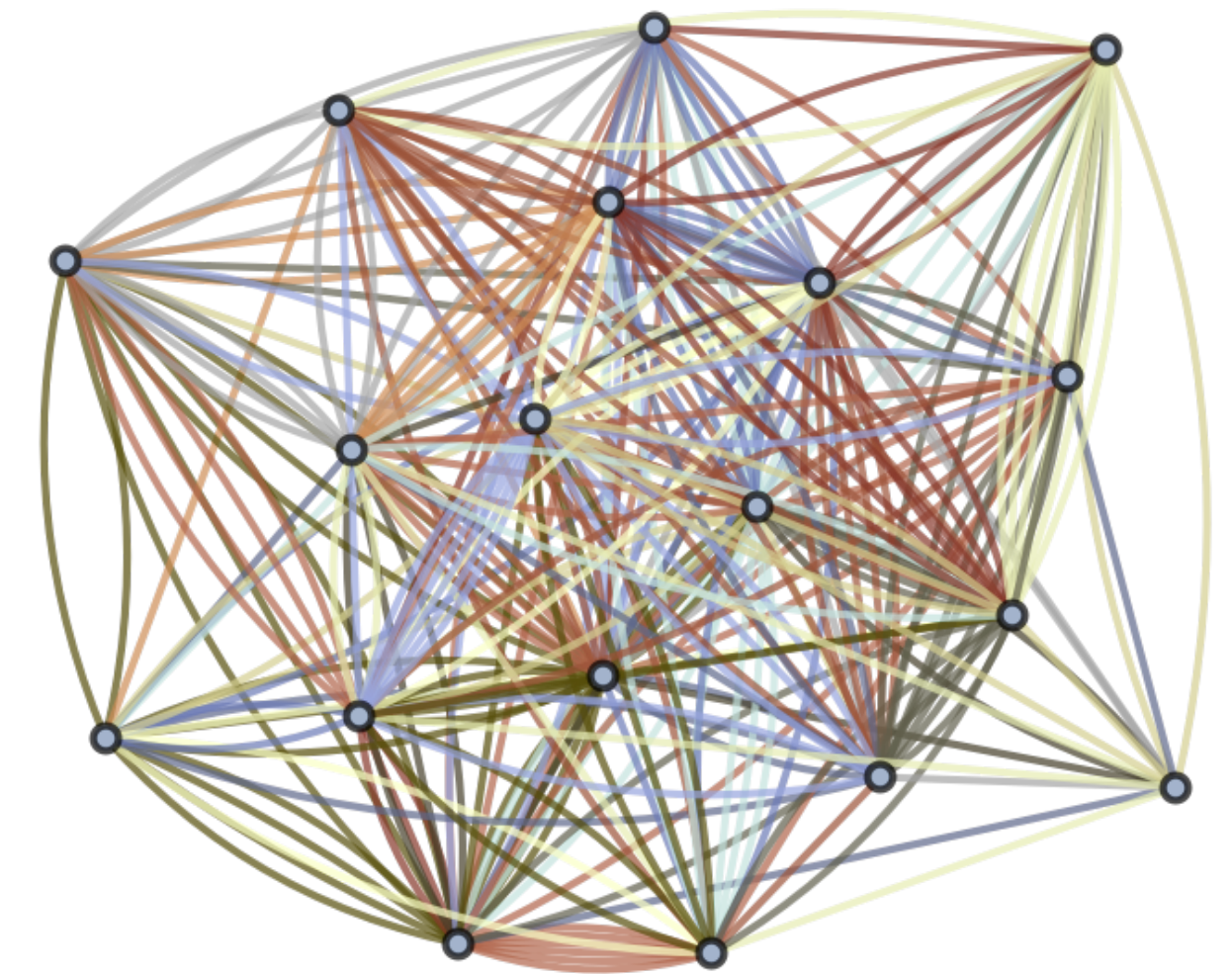
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# The Sachdev-Ye-Kitaev (SYK) model

- Density of quantum states of the SYK model with  $N$  sites

$$D(E) \sim \frac{1}{N} \exp(N s_0) \sinh\left([2N\gamma E]^{1/2}\right)$$

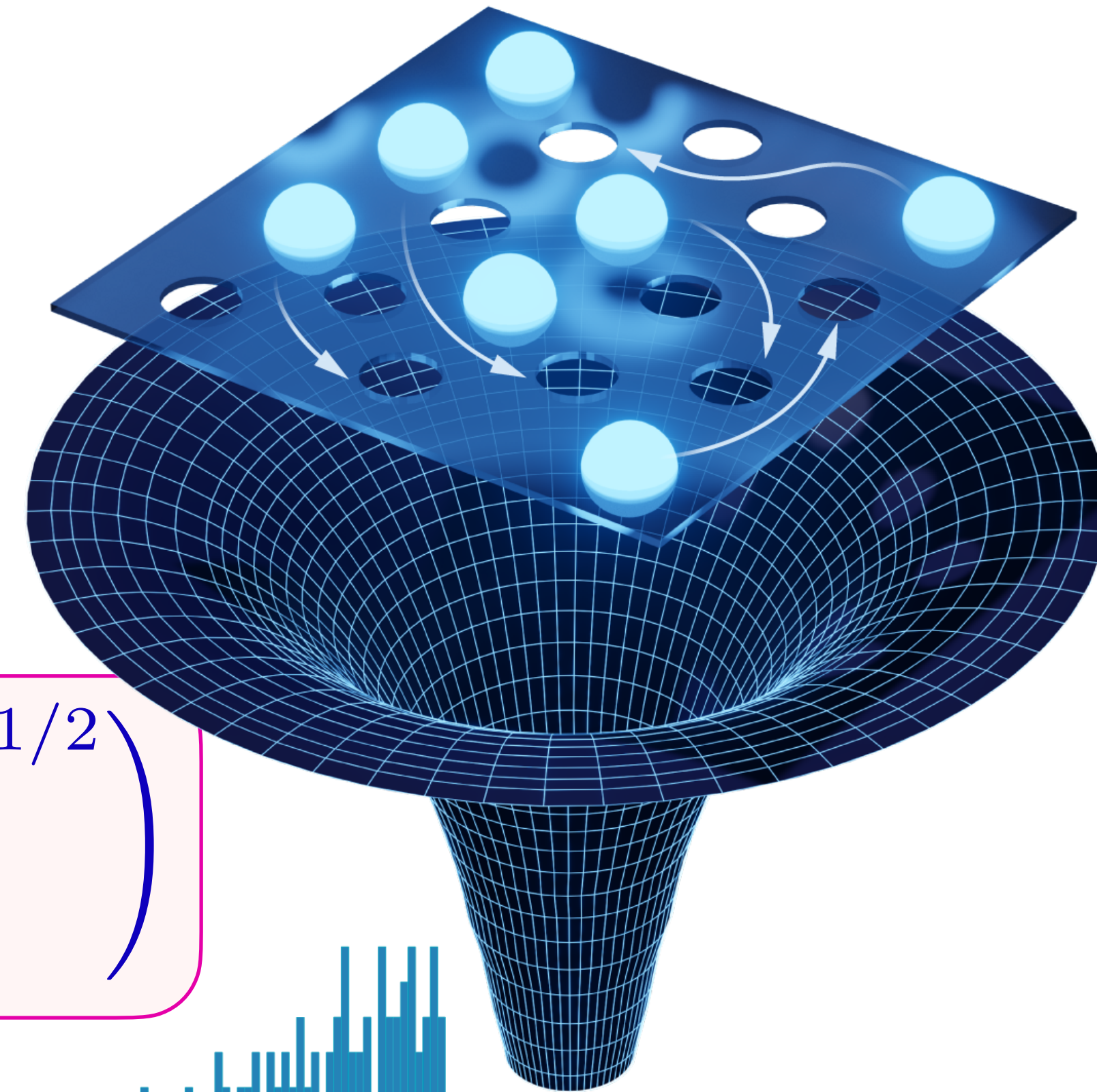
where  $s_0 = 0.46484769917080510749\dots$



# D(E) of charged black holes from the SYK model

- For generic charged black holes in 3+1 dimensions with horizon area  $A$  at  $T = 0$  and fixed charge  $Q$  ( $A = 2GQ^2/c^4$ ), the density of quantum states at small energy  $E$  is

$$D(E) \sim \left( \frac{Ac^3}{\hbar G} \right)^{-347/90} \exp \left( \frac{Ac^3}{4\hbar G} \right) \sinh \left( \left[ \frac{\sqrt{\pi} A^{3/2} c^2}{\hbar^2 G} E \right]^{1/2} \right)$$

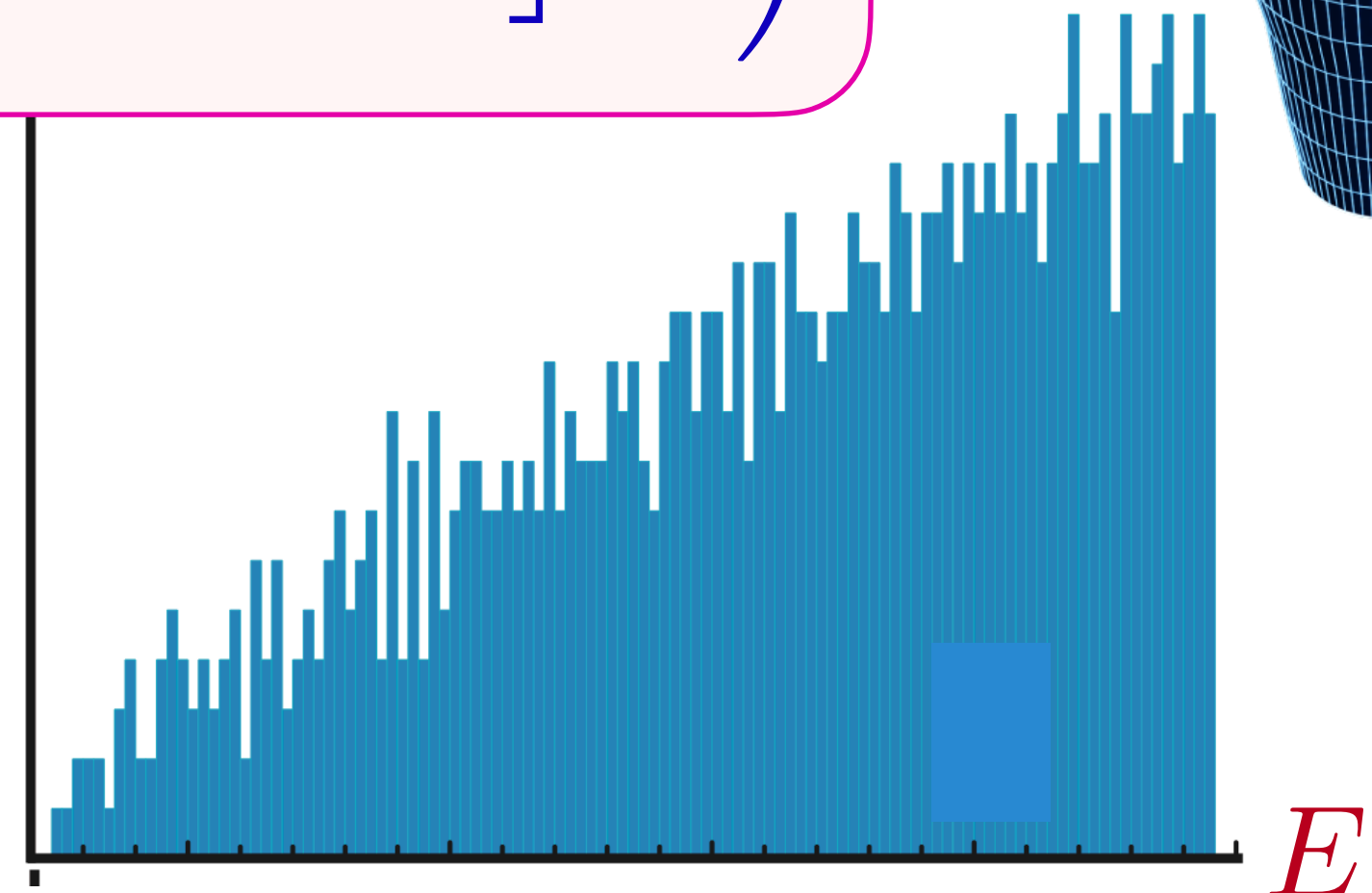


Bekenstein-Hawking

Iliesiu, Murthy, Turiaci (2022)

Developments from the SYK model

$D(E)$



Recap

# Great discoveries in physics

Entropy (1870)

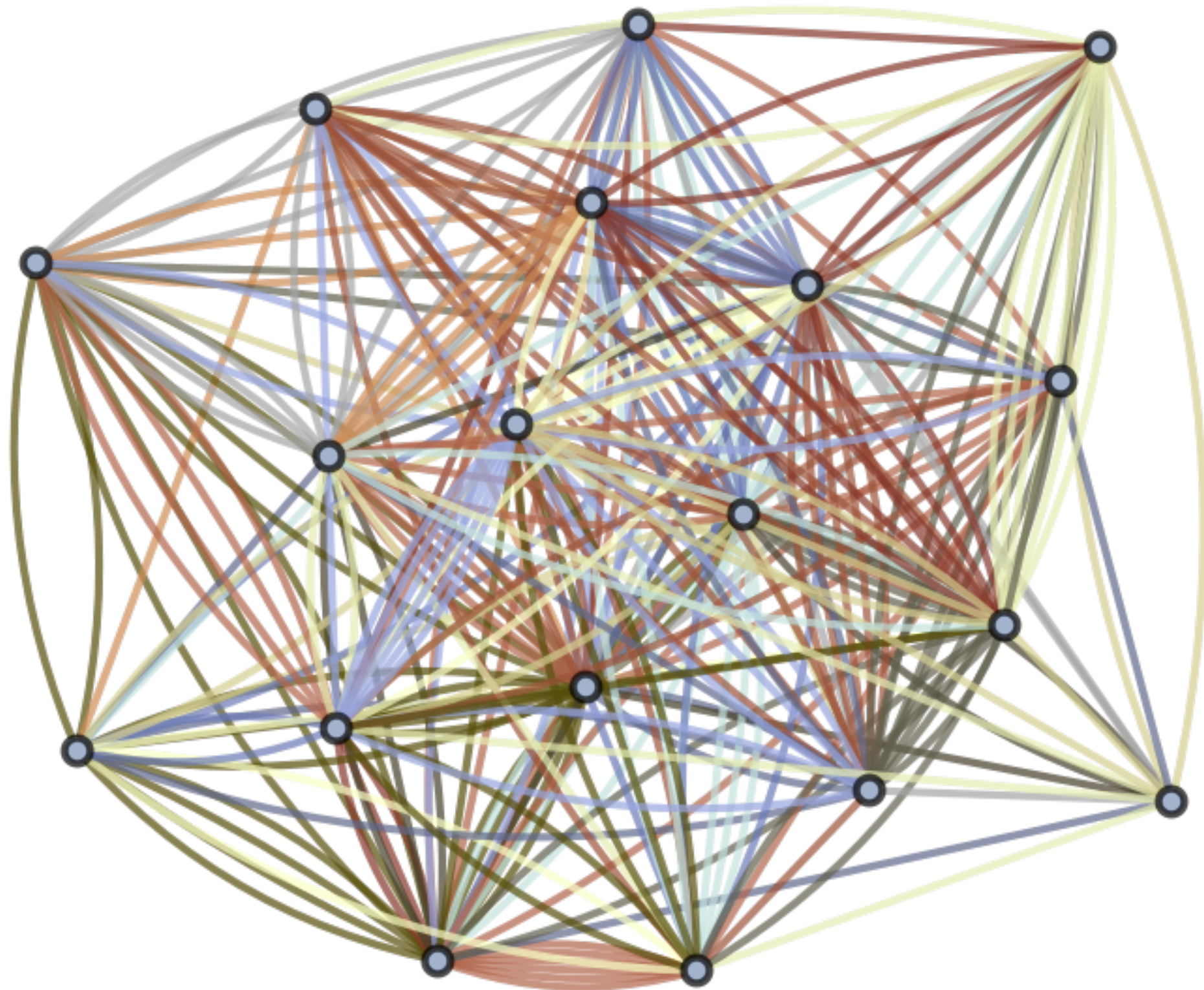
Superconductivity (1911)

Black holes (1916)

Quantum entanglement (1935)

# The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

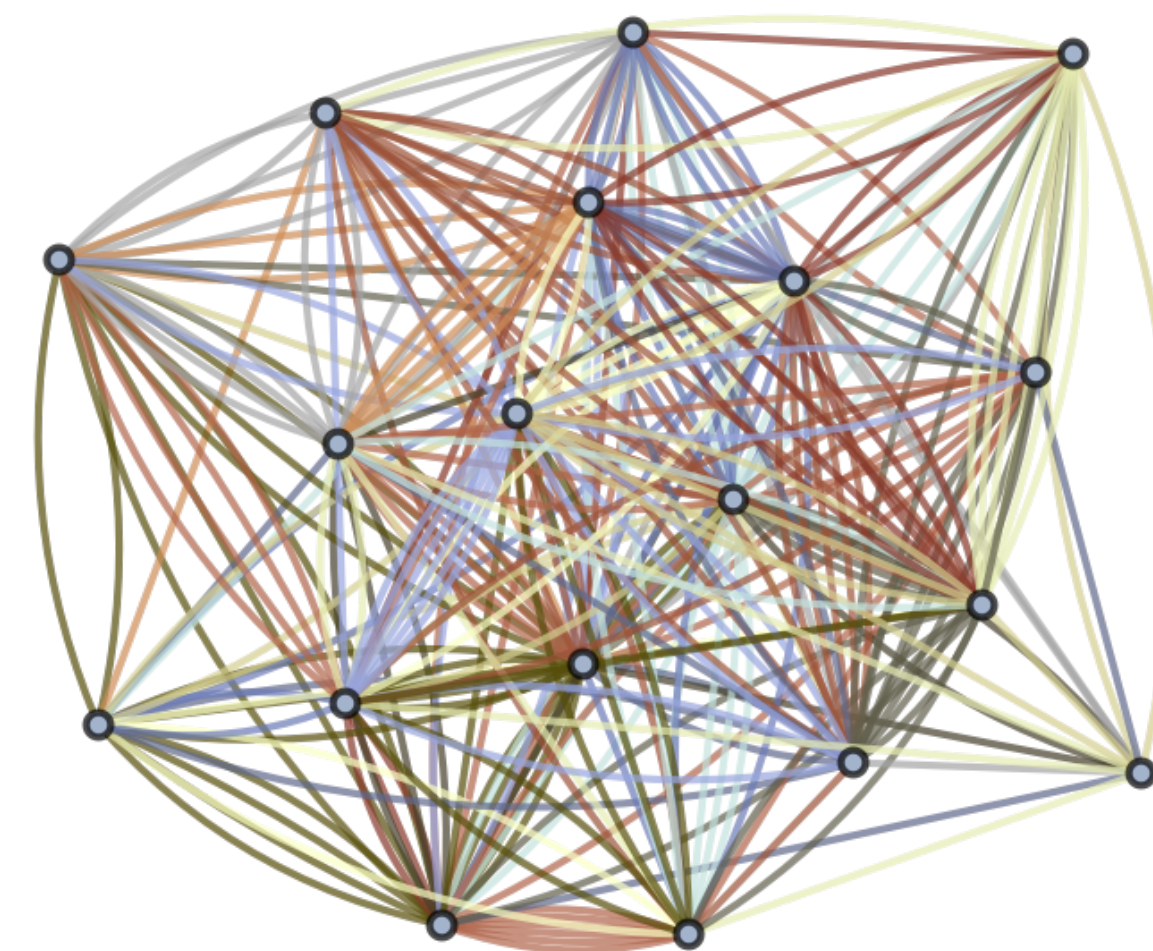
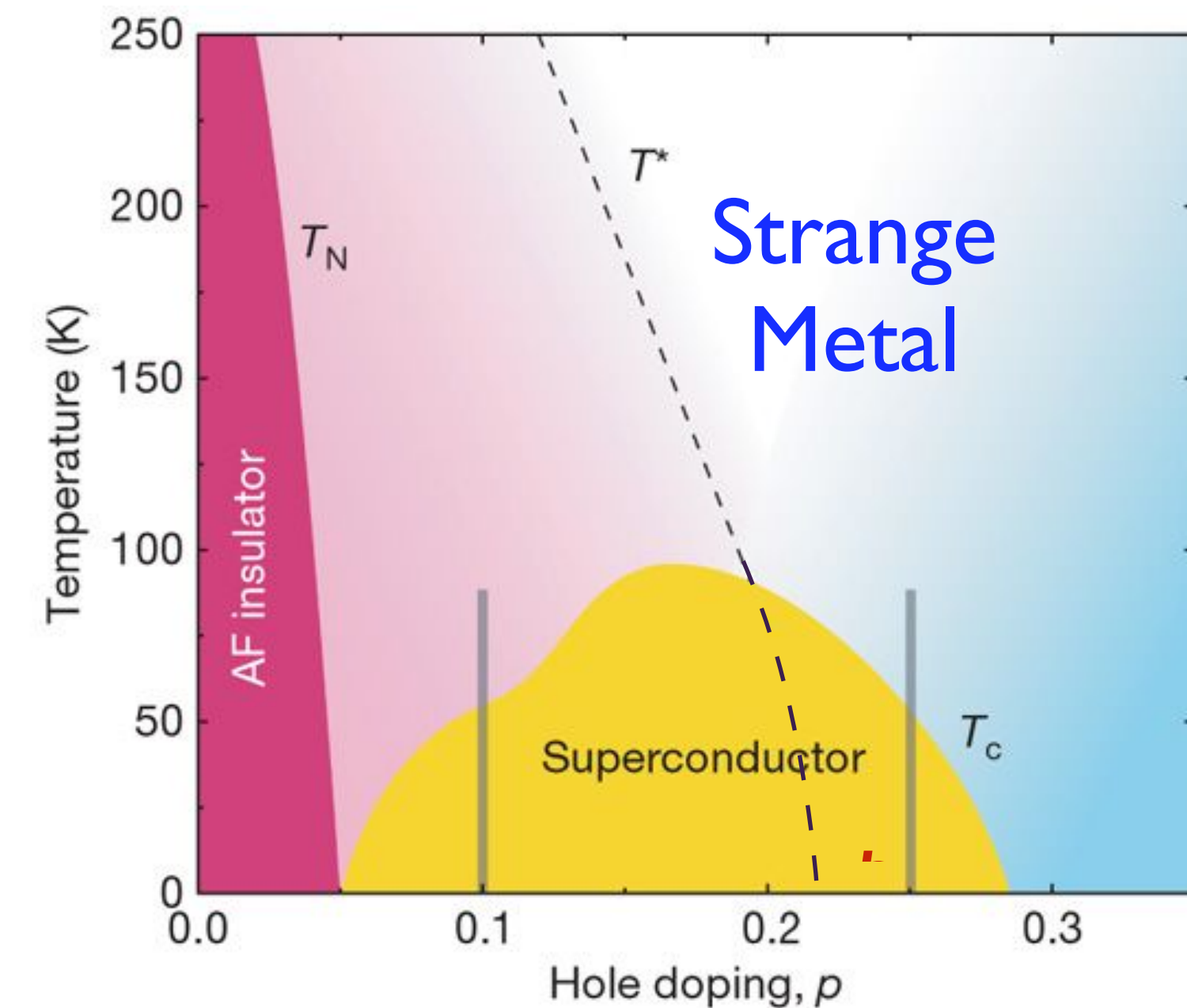


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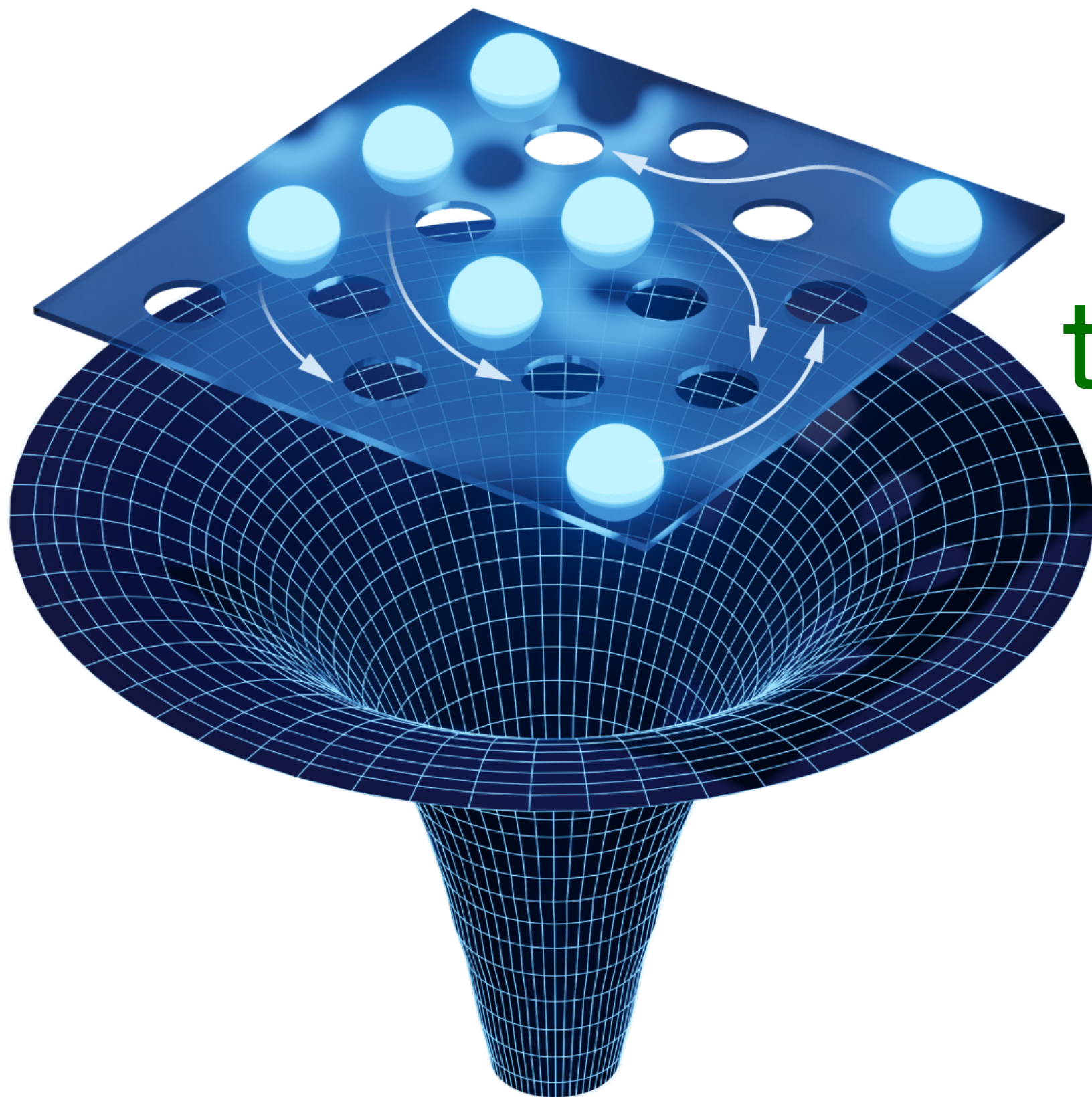
In one set of variables, it helps describe the *strange* electrical properties of YBCO

Sachdev, Ye (1993)



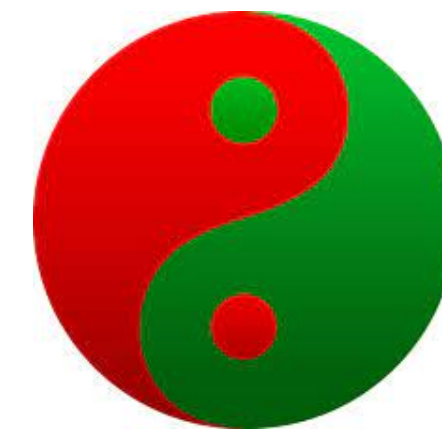
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In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)



In a ***dual*** set of variables it describes the interior of ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)