Gauge theory for the cuprates near optimal doping

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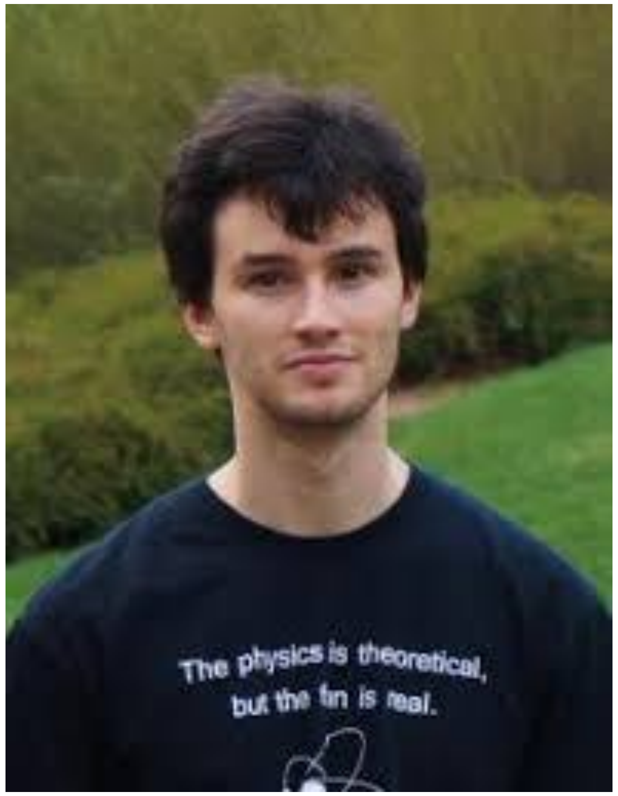
PHYSICS
HARVARD

Mathias Scheurer



Grigory Tarnopolsky

Harley Scammell

arXiv:1811.04930

PHYSICS

HARVARD

1. Emergent gauge fields and topological order in the 3D XY model

2. Electron doped cuprates

   (A) Simple models of metals with intrinsic topological order

   (B) SU(2) gauge theory of fluctuating antiferromagnetism

3. Hole doped cuprates

   SU(2) gauge theory with N_h adjoint Higgs fields

   Higgs-confinement transition to a Fermi liquid

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   Higgs-confinement transition to a Fermi liquid

$$\begin{aligned} \mathcal{Z}_{XY} &= \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-H_{XY}\right) \\ H_{XY} &= -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) \end{aligned}$$

- Non-zero *T* (classical) phase transitions of superfluids, magnets with \`easy-plane’ spins, …..in *D* spatial dimensions

- *T*=0 (quantum) phase transitions of bosons at integer filling between superfluid and insulator in *D-1* spatial dimensions

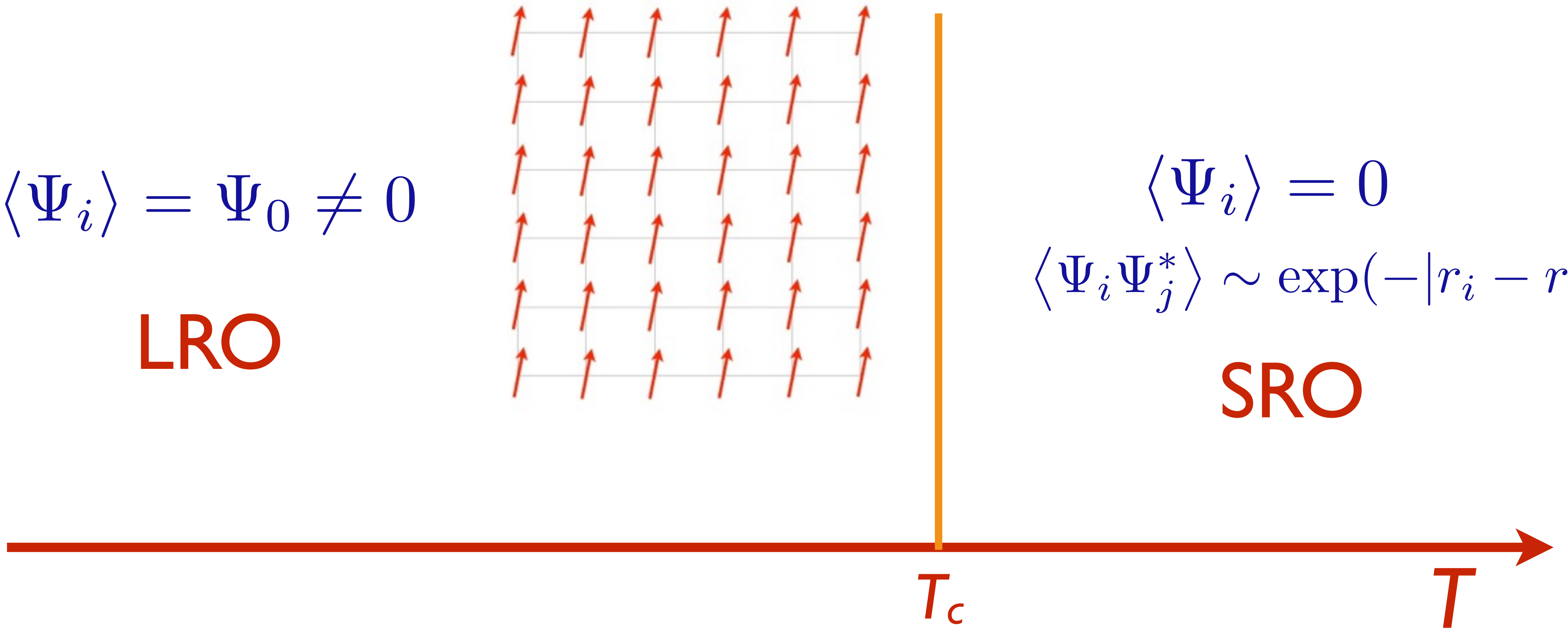
In dimension $\boxed{D=3,}$ in the low T phase, the symmetry $\theta_i \to \theta_i + c$ is "spontaneously broken". There is (off-diagonal) long-range order (LRO) characterized by ($\Psi_i \equiv e^{i\theta_i}$)

$$\lim_{|r_i - r_j| \to \infty} \left\langle \Psi_i \Psi_j^* \right\rangle = |\Psi_0|^2 \neq 0\,.$$

We break the symmetry by choosing an overall phase so that

$$\langle \Psi_i \rangle = \Psi_0 \neq 0$$

Wilson-Fisher theory



$\langle \Psi_i \rangle = \Psi_0 \neq 0$

LRO

$\langle \Psi_i \rangle = 0$

$\left\langle \Psi_i \Psi_j^* \right\rangle \sim \exp(-|r_i - r_j|/\xi)$

SRO

Kosterlitz-Thouless theory in *D*=2

In spatial dimension $\boxed{D = 2,}$ the symmetry $\theta_i \to \theta_i + c$ is preserved at all non-zero T. There is no LRO, and

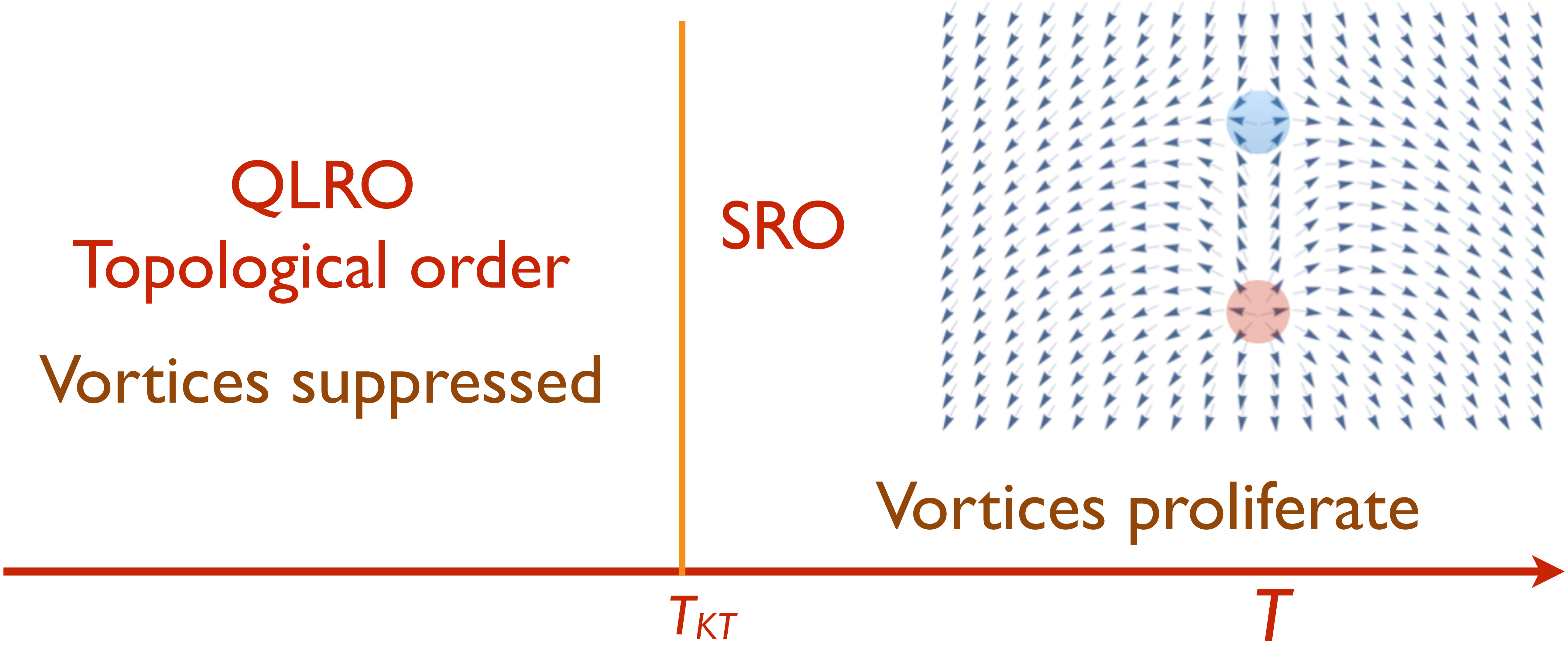
$$\langle \Psi_i \rangle = 0 \text{ for all } T > 0.$$

Nevertheless, there is a phase transition at $T = T_{KT}$, where the nature of the correlations changes

$$\lim_{|r_i - r_j| \to \infty} \left\langle \Psi_i \Psi_j^* \right\rangle \sim \begin{cases} |r_i - r_j|^{-\alpha}, & \text{for } T < T_{KT}, \text{ (QLRO)} \\ \exp(-|r_i - r_j|/\xi), & \text{for } T > T_{KT}, \text{ (SRO)} \end{cases}$$

KT theory

Kosterlitz-Thouless theory in D=2



QLRO
Topological order

Vortices suppressed

SRO

Vortices proliferate

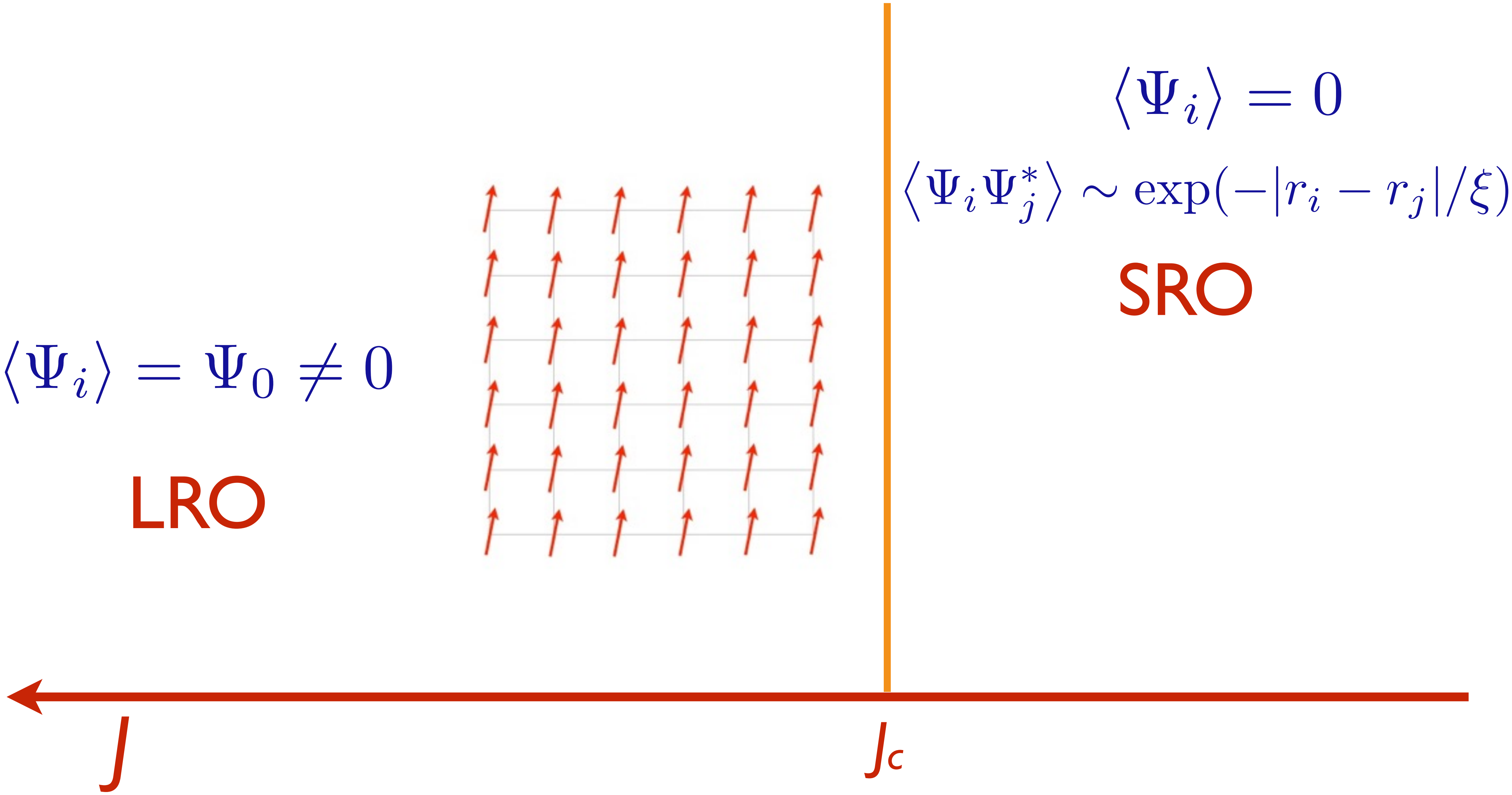
T_{KT}

T

The low T phase also has **topological order** associated with the suppression of vortices.

KT theory

Can we modify the XY model Hamiltonian to obtain a phase with "topological order" in *D=3* ?



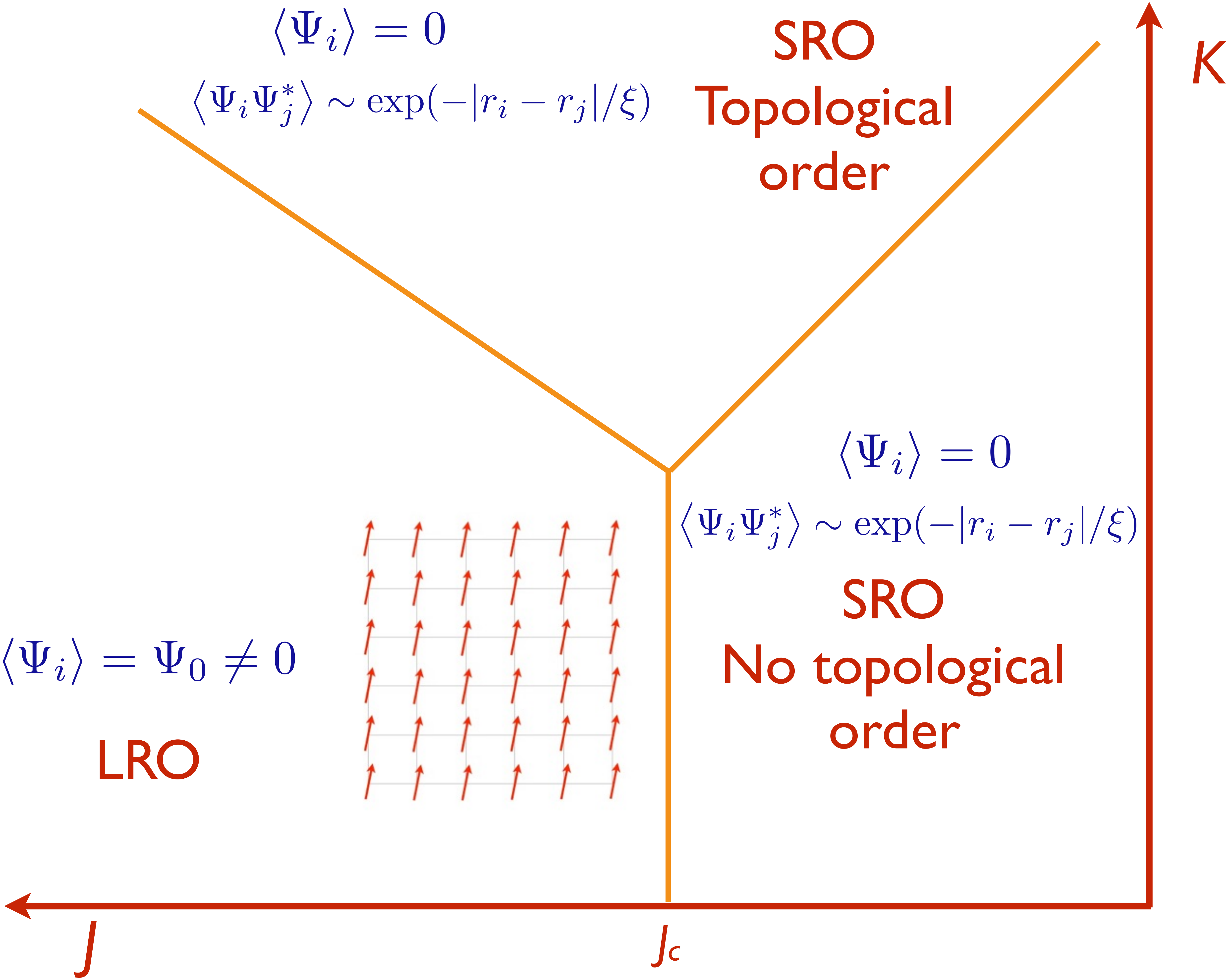
$$\langle \Psi_i \rangle = 0$$

$$\langle \Psi_i \Psi_j^* \rangle \sim \exp(-|r_i - r_j|/\xi)$$

SRO

$$\langle \Psi_i \rangle = \Psi_0 \neq 0$$

LRO



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$\langle \Psi_i \Psi_j^* \rangle \sim \exp(-|r_i - r_j|/\xi)$

SRO
Topological
order

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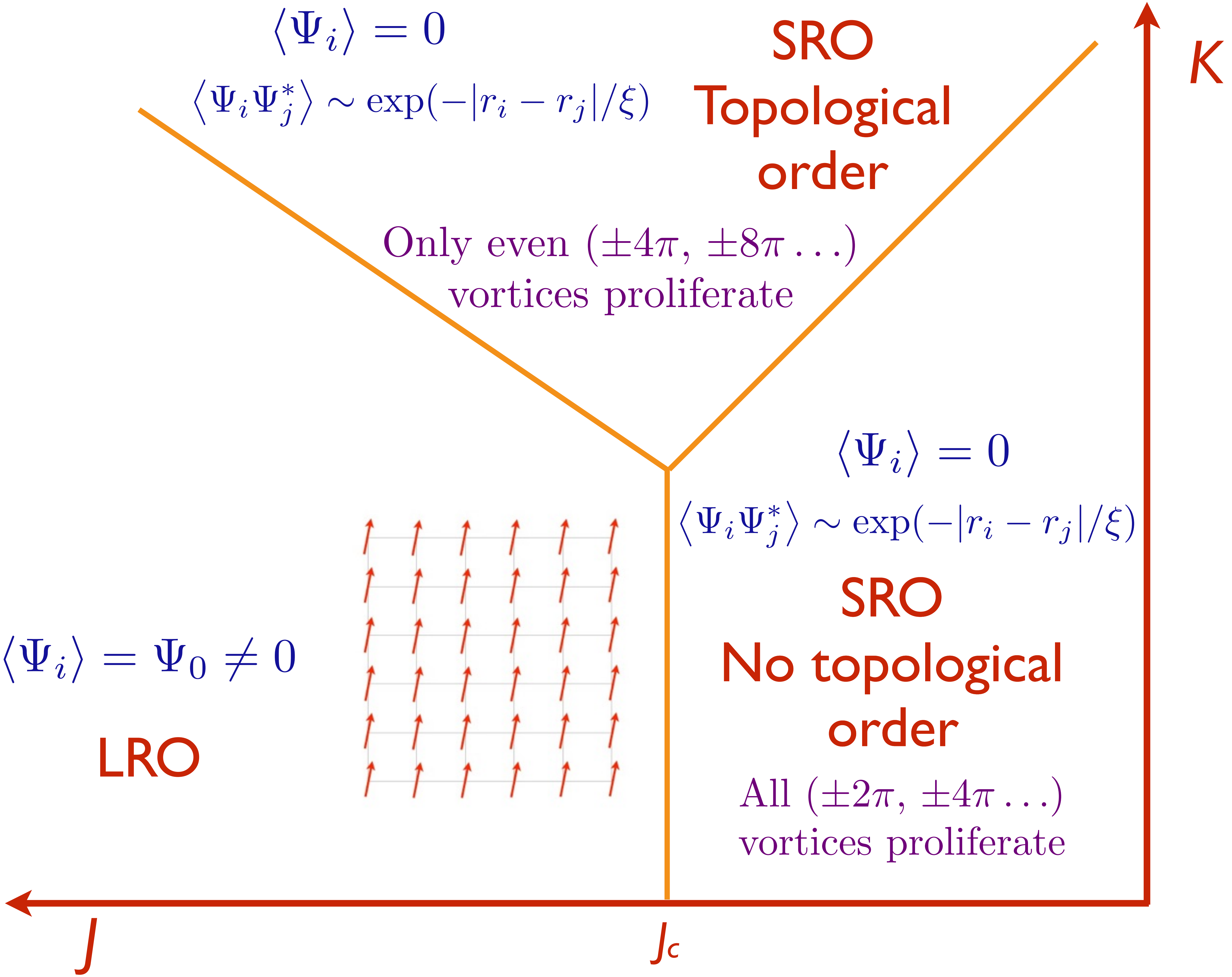
SRO
No topological
order



$\langle \Psi_i \rangle = 0$

$\langle \Psi_i \Psi_j^* \rangle \sim \exp(-|r_i - r_j|/\xi)$

SRO
Topological order

Only even ($\pm 4\pi, \pm 8\pi \ldots$) vortices proliferate

$\langle \Psi_i \rangle = 0$

$\langle \Psi_i \Psi_j^* \rangle \sim \exp(-|r_i - r_j|/\xi)$

SRO
No topological order

All ($\pm 2\pi, \pm 4\pi \ldots$) vortices proliferate

$\langle \Psi_i \rangle = \Psi_0 \neq 0$

LRO

$$\widetilde{\mathcal{Z}}_{XY} = \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}_{XY}[\theta]\right)$$

$$\widetilde{H}_{XY}[\theta] = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) + \sum_{ijk\ell} K_{ijk\ell} \cos(\theta_i + \theta_j - \theta_k - \theta_\ell) + \ldots\ldots$$

Add terms which suppress single but not double vortices…..

All allowed terms are invariant under a global U(1) symmetry ($\theta_i \rightarrow \theta_i + c$) and periodic in all the θ_i ($\theta_i \rightarrow \theta_i + 2\pi n_i$, n_i integers)

We rewrite $\widetilde{\mathcal{Z}}_{XY}$ using the decomposition

$$\Psi_i = H_i \phi_i^2$$

where

$$H_i \equiv e^{i\vartheta_i} \text{ and } \phi_i \equiv e^{i\varphi_i}$$

The idea is that single vortices in Ψ will appear as single vortices in H, while double vortices in Ψ will appear as single vortices in ϕ.

This decomposition now demands that any action be invariant under the U(1) gauge transformations

$$\vartheta_i \to \vartheta_i + 2\alpha_i \quad , \quad \varphi_i \to \varphi_i - \alpha_i$$

To obtain simple effective actions, we also introduce a U(1) gauge field $A_{i\mu}$ ($\mu = 1, 2, 3$) which transforms as

$$A_{i\mu} \to A_{i\mu} + \Delta_\mu \alpha_i$$

We now write down a U(1) gauge theory, $\mathcal{Z}_{U(1)}$ consistent the U(1) gauge invariance and the global symmetry

$$
\begin{aligned}
\mathcal{Z}_{U(1)} &= \prod_i \int_0^{2\pi} \frac{d\vartheta_i}{2\pi} \frac{d\varphi_i}{2\pi} \prod_\mu \frac{dA_{i\mu}}{2\pi} \exp\left(-H_U[\vartheta, \varphi, A_\mu]\right) \\
H_U[\vartheta, \varphi, A_\mu] &= -J_1 \sum_{i,\mu} \cos(\Delta_\mu \vartheta_i - 2A_{i\mu}) \\
&\quad -J_2 \sum_{i,\mu} \cos(\Delta_\mu \varphi_i + A_{i\mu}) \\
&\quad -K \sum_{\square} \cos(\epsilon_{\mu\nu\lambda} \Delta_\nu A_{i\lambda})
\end{aligned}
$$

Our claim is that this is the same theory as $\widetilde{\mathcal{Z}}_{XY}$; in particular

$$
\prod_{i,\mu} \int_0^{2\pi} \frac{dA_{i\mu}}{2\pi} \exp\left(-H_U[\vartheta, \varphi, A_\mu]\right) \approx \exp(-\widetilde{H}_{XY}[\vartheta + 2\varphi])
$$

This result follows from gauge invariance and the global U(1) symmetry, and can be explicitly established by performing the integrals over $A_{i\mu}$ order-by-order in K.

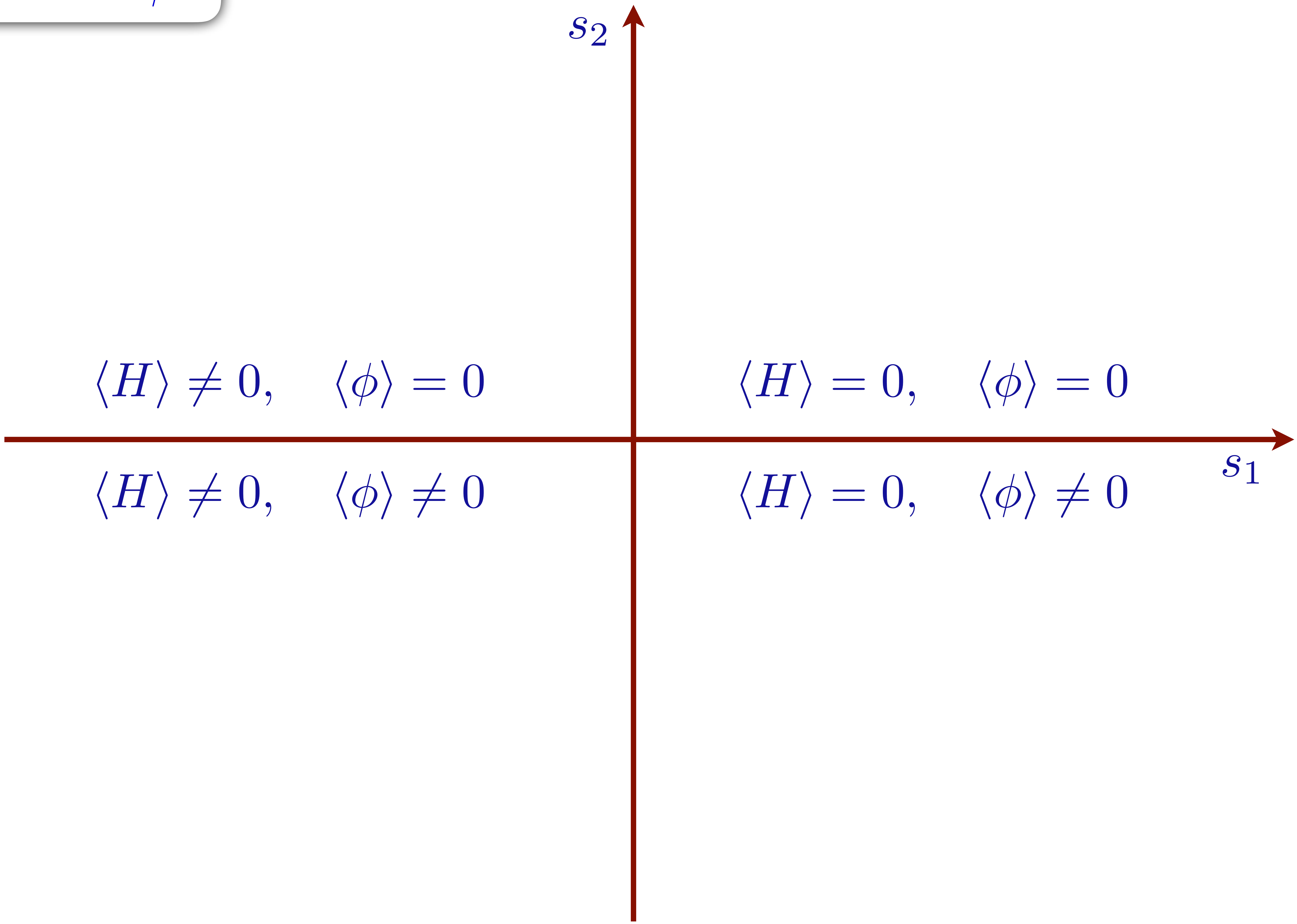
First we examine the phase diagram by taking a naive continuum limit of H_U, and studying the resulting mean-field theory

$$\begin{aligned}\mathcal{L} = & |(\partial_\mu - 2iA_\mu - ia_\mu^{\text{ext}})H|^2 + s_1|H|^2 + u_1|H|^4 \\ & + |(\partial_\mu + iA_\mu)\phi|^2 + s_2|\phi|^2 + u_2|\phi|^4 \\ & + K(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 + \mathcal{L}_{\text{monopoles}}\end{aligned}$$

We have included a fixed external field a_μ^{ext} which couples to the current of the global U(1) charge. The monopoles play a crucial role, similar to those of vortices in the 2D XY model, and they will strongly modify the mean-field phase diagram.

$$\Psi = H\phi^2$$

Mean field phase diagram



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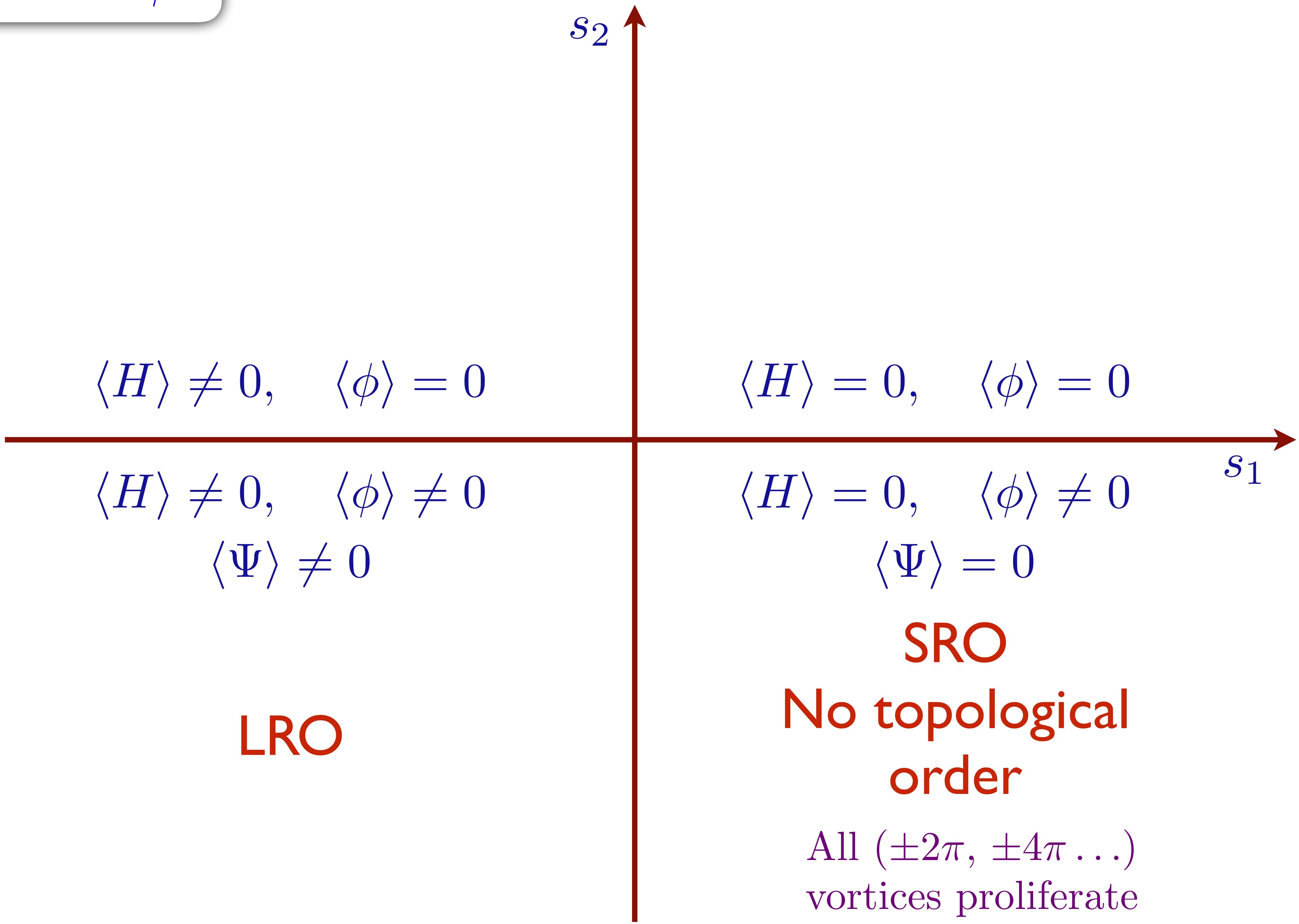
Mean field phase diagram



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$\langle \Psi \rangle \neq 0$

LRO

$\langle H \rangle = 0, \quad \langle \phi \rangle \neq 0$

$\langle \Psi \rangle = 0$

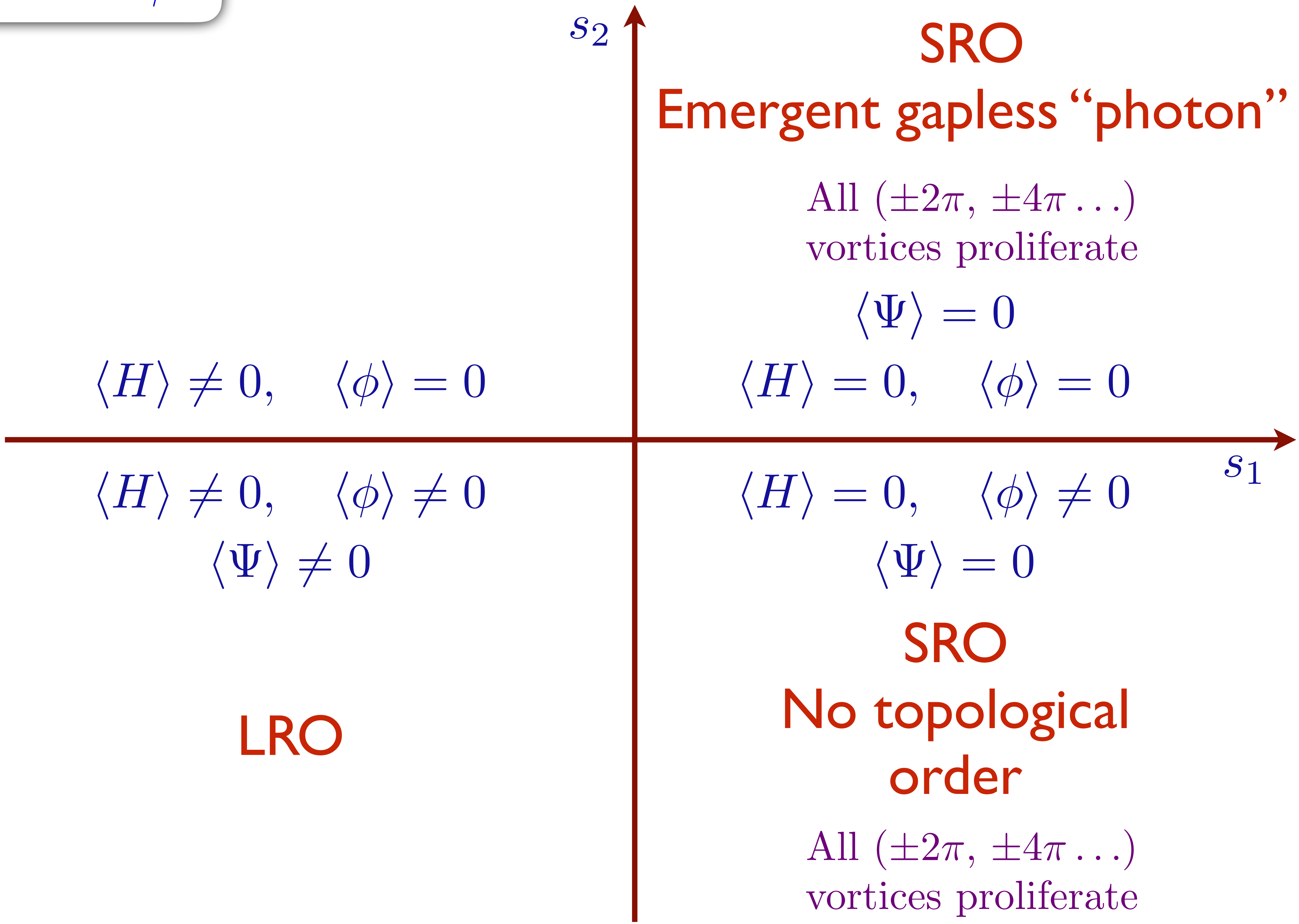
SRO

No topological order

All $(\pm 2\pi, \pm 4\pi \ldots)$ vortices proliferate

$\Psi = H\phi^2$

Mean field phase diagram



SRO

Emergent gapless “photon”

All $(\pm 2\pi, \pm 4\pi \ldots)$ vortices proliferate

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LRO

$\langle H \rangle = 0, \quad \langle \phi \rangle \neq 0$

$\langle \Psi \rangle = 0$

SRO

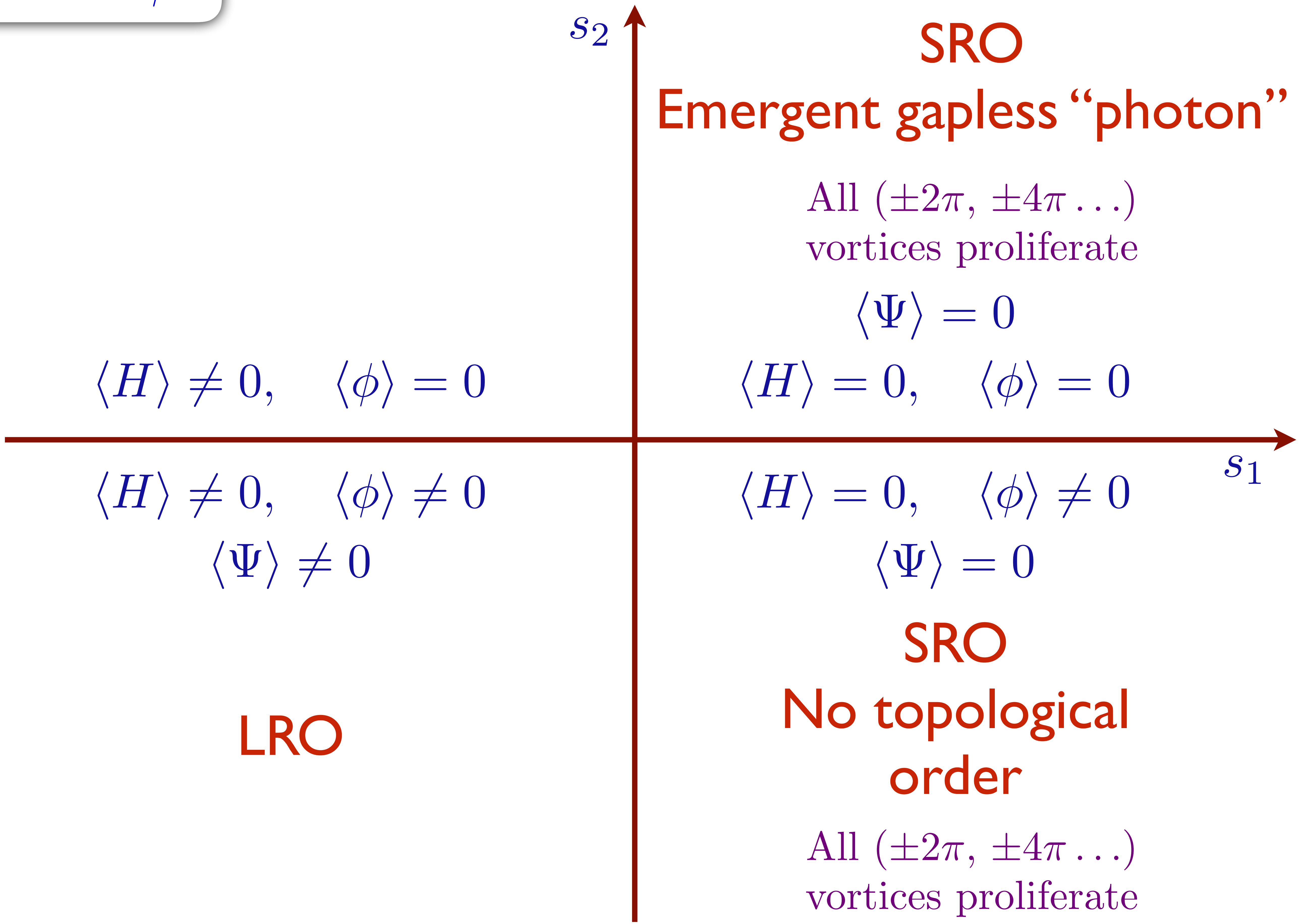
No topological order

All $(\pm 2\pi, \pm 4\pi \ldots)$ vortices proliferate

The emergent "photon" phase is unstable to the proliferation of monopoles. The monopoles form a Coulomb plasma with $1/r$ interactions in 3D, very similar to the Coulomb plasma of vortices with $\ln(r)$ interactions in 2D. However, unlike 2D, in 3D there is never a state where monopoles are bound to antimonopoles. The $1/r$ interactions are always Debye screened, and the monopoles are effectively free. This proliferation of monopoles implies that there is no emergent gapless photon.

$\Psi = H\phi^2$

Mean field phase diagram



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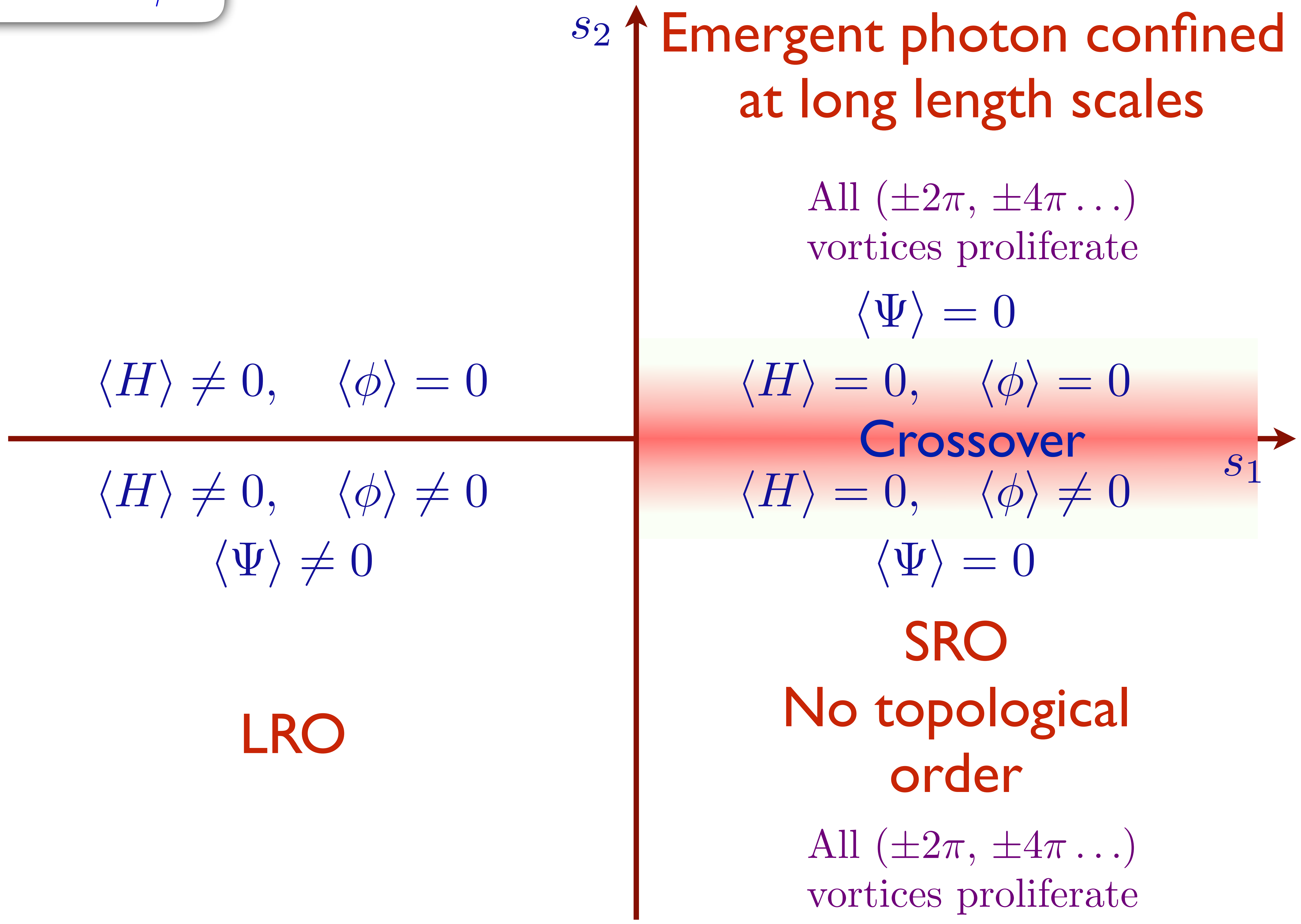
Phase diagram



s_2

SRO

Emergent photon confined at long length scales

All $(\pm 2\pi, \pm 4\pi \ldots)$ vortices proliferate

$\langle \Psi \rangle = 0$

$\langle H \rangle = 0, \quad \langle \phi \rangle = 0$

Crossover

$\langle H \rangle = 0, \quad \langle \phi \rangle \neq 0$

$\langle \Psi \rangle = 0$

s_1

SRO

No topological order

All $(\pm 2\pi, \pm 4\pi \ldots)$ vortices proliferate

$\langle H \rangle \neq 0, \quad \langle \phi \rangle = 0$

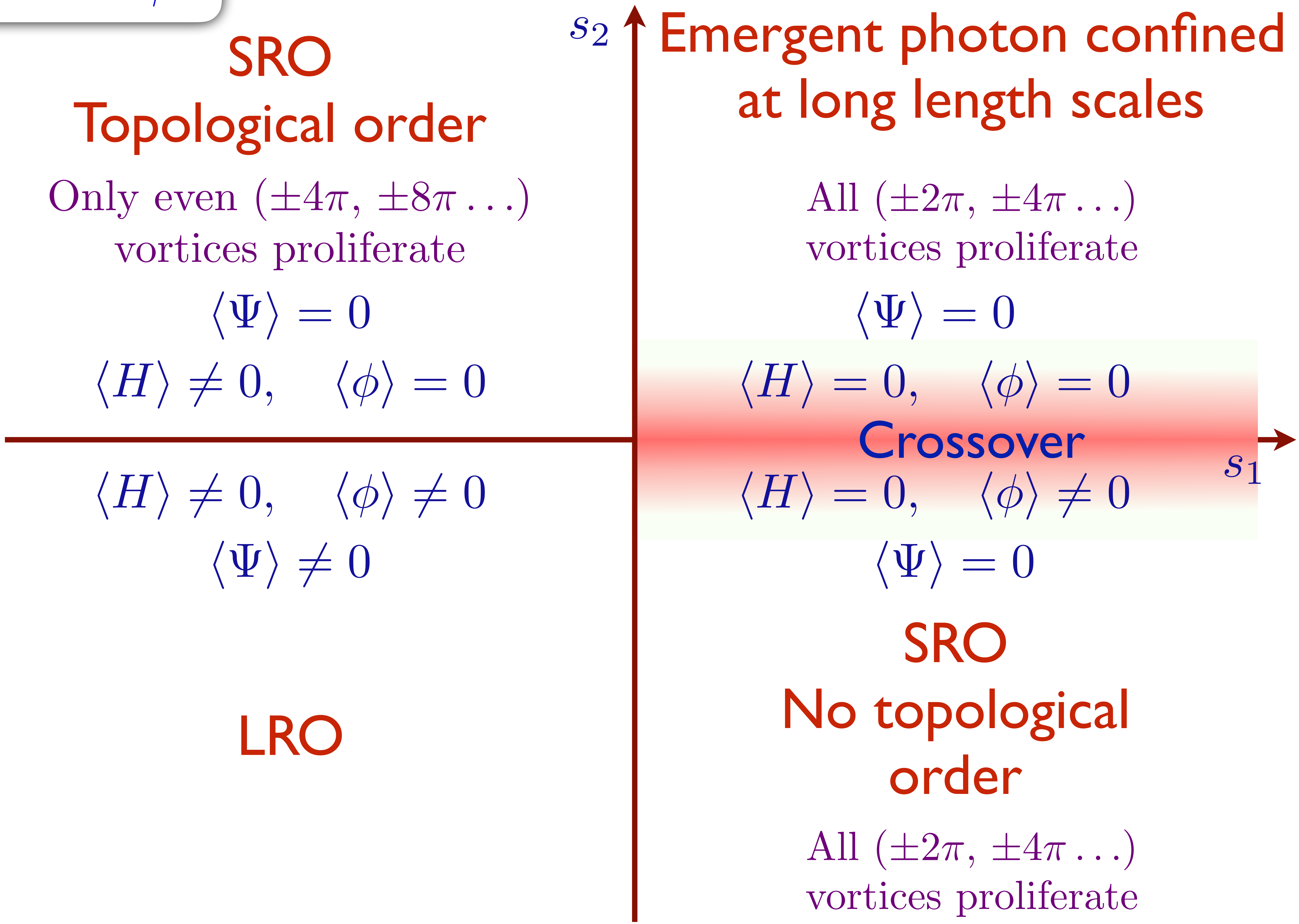
$\langle H \rangle \neq 0, \quad \langle \phi \rangle \neq 0$

$\langle \Psi \rangle \neq 0$

LRO

Phase diagram

$\Psi = H\phi^2$



SRO

Topological order

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LRO

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$$\langle \Psi \rangle = 0$$

SRO

No topological order

All $(\pm 2\pi, \pm 4\pi \ldots)$ vortices proliferate

Structure of the topological phase

The topological phase is in the regime $s_1 < 0$ and $s_2 > 0$ in the field theory

$$\begin{aligned}\mathcal{L} &= |(\partial_\mu - 2iA_\mu - ia_\mu^{\text{ext}})H|^2 + s_1|H|^2 + u_1|H|^4 \\ &\quad + |(\partial_\mu + iA_\mu)\phi|^2 + s_2|\phi|^2 + u_2|\phi|^4 + K(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 + \mathcal{L}_{\text{monopoles}}\end{aligned}$$

Perform a boson-boson (*i.e.* particle-vortex) duality on the boson H, while (temporarily) treating A_μ as a background field. This leads to a theory of a dual boson (vortex) ψ coupled to a dual emergent gauge field B_μ

$$\boxed{\begin{aligned}\mathcal{L}_{\text{dual}} &= |(\partial_\mu - iB_\mu)\psi|^2 + \widetilde{s}_1|\psi|^2 + \widetilde{u}_1|\psi|^4 + \frac{i}{\pi}\epsilon_{\mu\nu\lambda}B_\mu\partial_\nu A_\lambda + \frac{i}{2\pi}\epsilon_{\mu\nu\lambda}B_\mu\partial_\nu a_\lambda^{\text{ext}} \\ &\quad + |(\partial_\mu + iA_\mu)\phi|^2 + s_2|\phi|^2 + u_2|\phi|^4 + K(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 + \mathcal{L}_{\text{monopoles}}\end{aligned}}$$

Note that when $s_1 < 0$, then $\widetilde{s}_1 > 0$: so both the ψ and ϕ bosons are massive. Also, a monopole changes U(1) flux by 2π and this corresponds to inserting two ψ bosons (each is a vortex carrying π flux); therefore

$$\boxed{\mathcal{L}_{\text{monopoles}} = \lambda\left(\psi^2 + \psi^{*2}\right)}$$

Structure of the topological phase

- The topological phase is described by a TQFT:

$$\mathcal{L}_{TQFT} = \frac{i}{\pi}\epsilon_{\mu\nu\lambda}B_\mu\partial_\nu A_\lambda + \frac{i}{2\pi}\epsilon_{\mu\nu\lambda}B_\mu\partial_\nu a^{\text{ext}}_\lambda$$

- The gapped complex boson ϕ carries unit A_μ charge, and global U(1) charges $\pm 1/2$.

- The gapped real boson ψ carries unit B_μ charge.

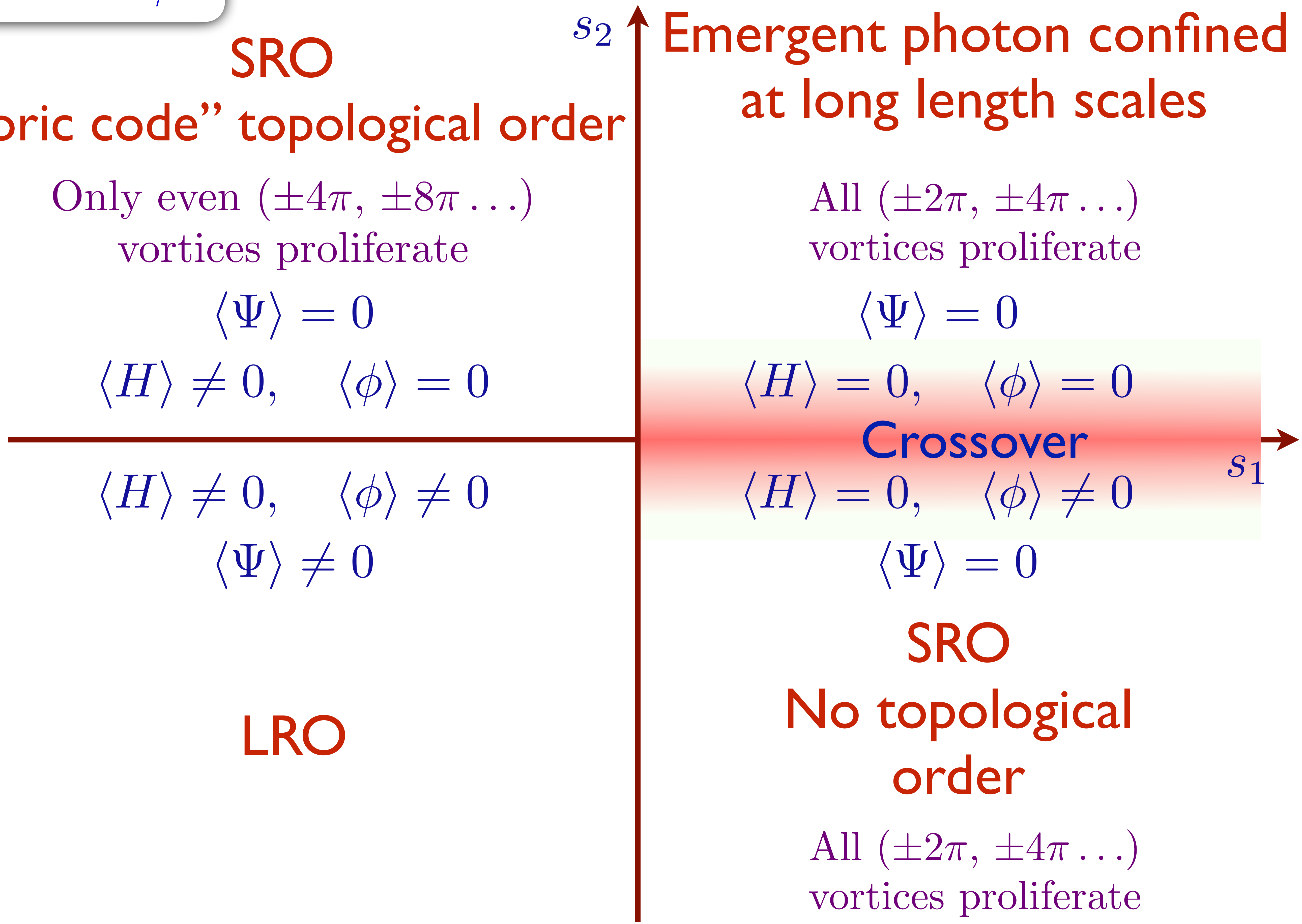
- The ϕ and ψ particles are mutual semions.

- The bound state of ϕ and ψ is a fermion, which also has mutual semionic statistics with individual ϕ and ψ particles.

- This topological order is the same as that of the 'toric code', the ϕ are e particles, and the ψ are m particles, and ϕ-ψ bound state is the ϵ particle. There is 4-fold ground state degeneracy on a large torus.

$$\boxed{\Psi = H\phi^2}$$

Phase diagram



SRO

“Toric code” topological order

Only even $(\pm 4\pi, \pm 8\pi \ldots)$ vortices proliferate

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SRO

Emergent photon confined at long length scales

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Crossover

$$\langle H \rangle = 0, \quad \langle \phi \rangle \neq 0$$

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SRO

No topological order

All $(\pm 2\pi, \pm 4\pi \ldots)$ vortices proliferate

$$\langle H \rangle \neq 0, \quad \langle \phi \rangle \neq 0$$

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LRO

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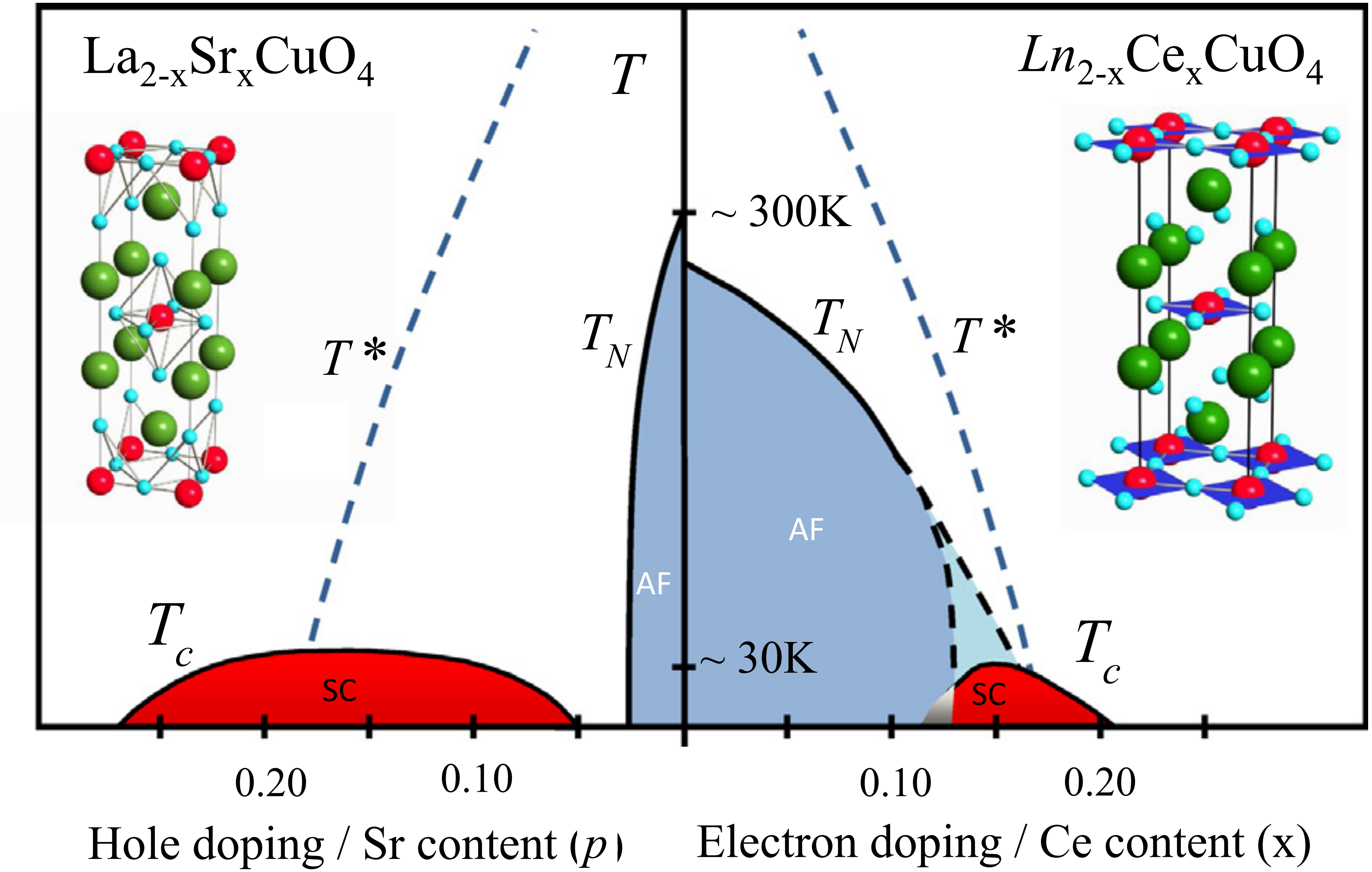
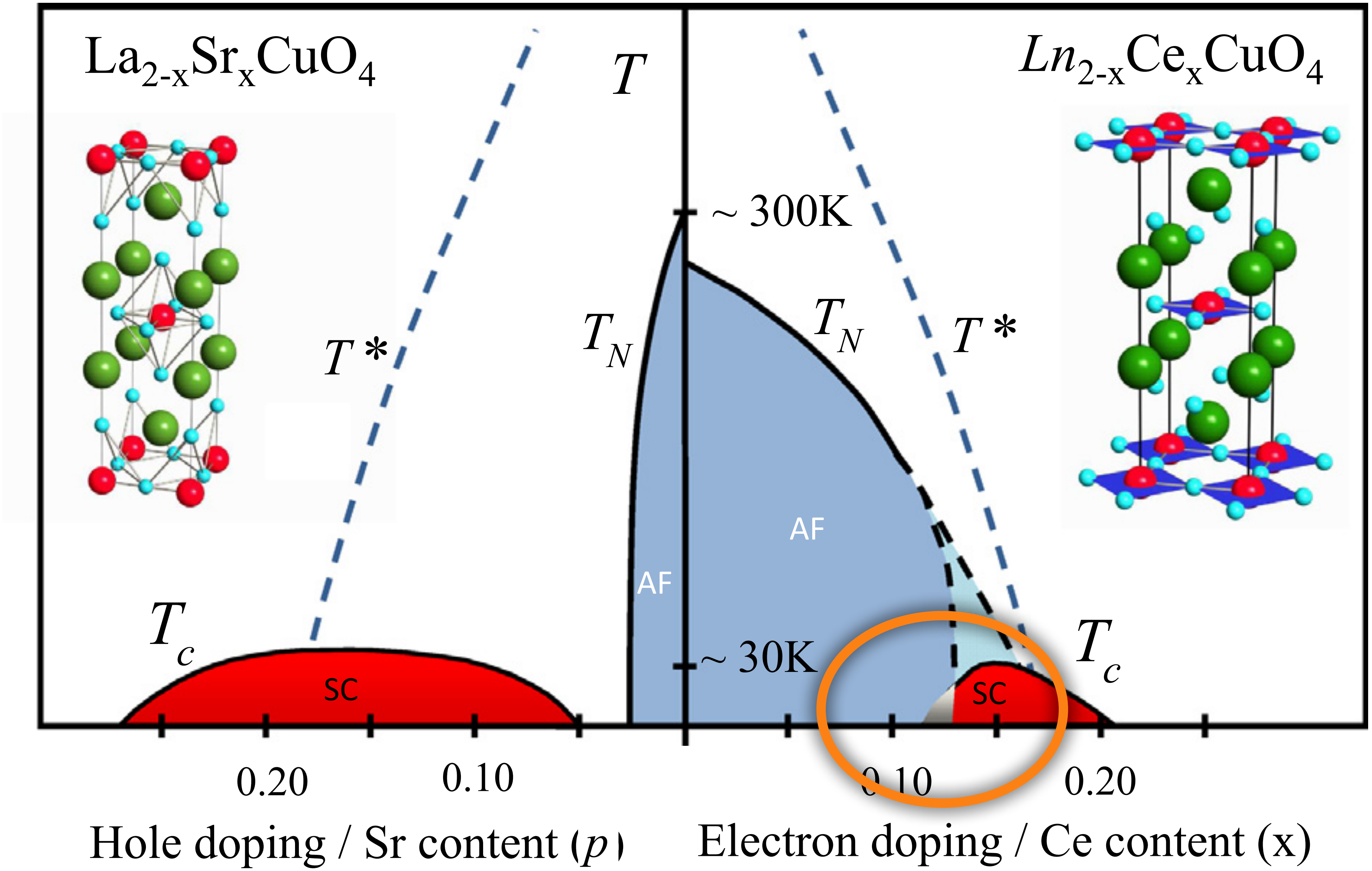

La2-xSrxCuO4
T
Ln2-xCexCuO4
~ 300K
T*
T*
TN
TN
AF
AF
~ 30K
Tc
Tc
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)


$La_{2-x}Sr_xCuO_4$
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AF
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T_c
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The Hubbard Model

$$H = -\sum_{i<j} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c_{i\alpha}^{\dagger} c_{i\alpha}$$

$t_{ij} \rightarrow$ “hopping”. $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

Spin index $\alpha = \uparrow, \downarrow$

$$n_{i\alpha} = c_{i\alpha}^{\dagger} c_{i\alpha}$$

$$\begin{gathered} c_{i\alpha}^{\dagger} c_{j\beta} + c_{j\beta} c_{i\alpha}^{\dagger} = \delta_{ij} \delta_{\alpha\beta} \\ c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0 \end{gathered}$$

Will study on the square lattice

Fermi surface+antiferromagnetism

We use the operator equation $S_i^a = (1/2) c_{i\alpha}^\dagger \sigma^a_{\alpha\beta} c_{i\beta}$ (valid on each site i):

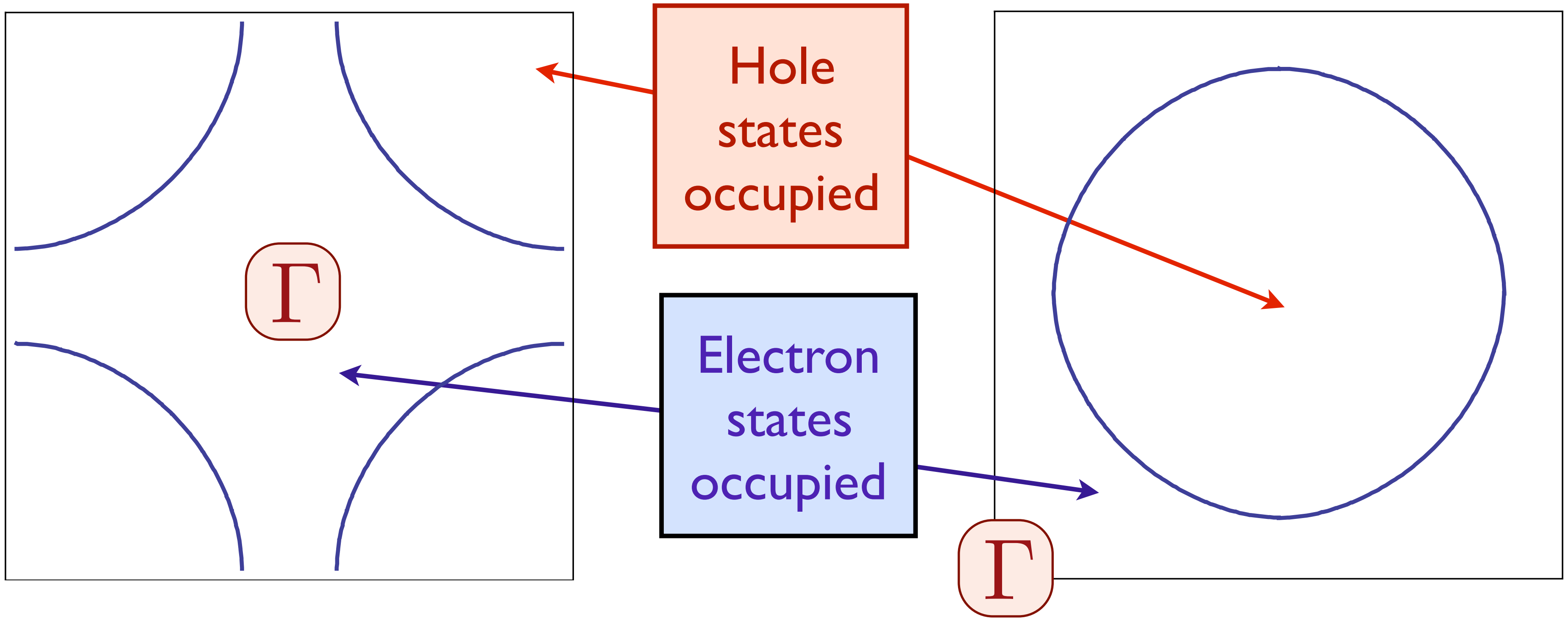
$$U\left(n_\uparrow - \frac{1}{2}\right)\left(n_\downarrow - \frac{1}{2}\right) = -\frac{2U}{3} S^{a2} + \frac{U}{4}$$

Then we decouple the interaction via

$$\exp\left(\frac{2U}{3}\sum_i \int d\tau S_i^{a2}\right) = \int \mathcal{D}\Phi_i^a(\tau) \exp\left(-\sum_i \int d\tau \left[\frac{3}{8U}\Phi_i^{a2} - \Phi_i^a S_i^a\right]\right)$$

We now integrate out the fermions, and look for the saddle point of the resulting effective action for Φ_i^a. At the saddle-point we find that the lowest energy is achieved when Φ^a has opposite orientations on the A and B sublattices.

Fermi surface+antiferromagnetism



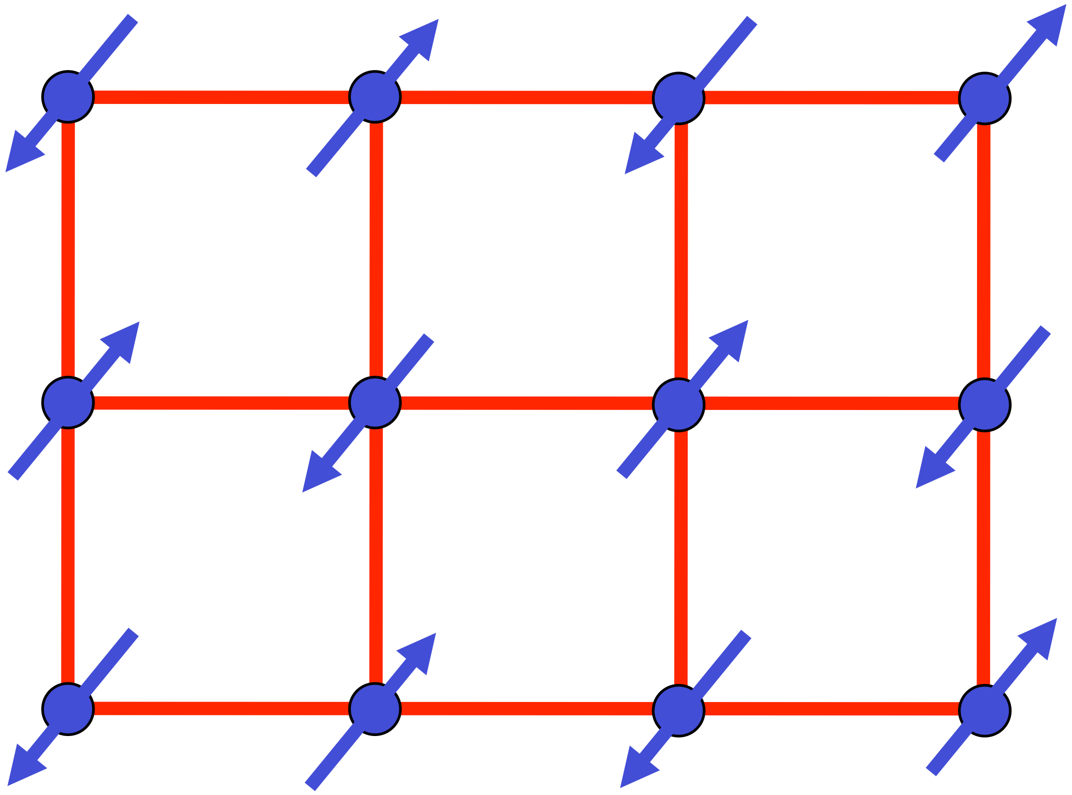
Hole states occupied

Electron states occupied

Γ

Γ

+



The electron spin polarization obeys

$$\langle S^a(\mathbf{r}, \tau) \rangle \sim \Phi^a(\mathbf{r})$$

where $\Phi^a \sim e^{i\mathbf{K}\cdot\mathbf{r}}$ with $\mathbf{K} = (\pi, \pi)$ the ordering wavevector.

“Spin-fermion” theory of the Hubbard model

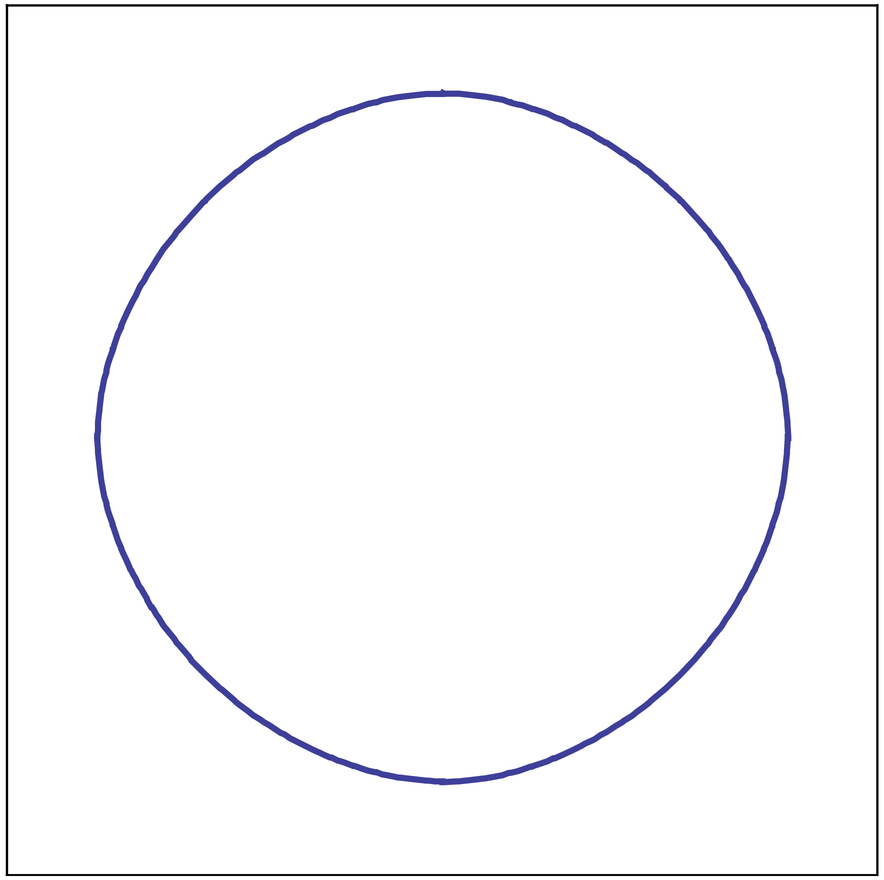
We can (exactly) transform the Hubbard model to the “spin-fermion” model:
electrons $c_{i\alpha}$ on the square lattice with dispersion

$$\begin{aligned} \mathcal{H}_c &= -\sum_{i,\rho} t_\rho \left(c_{i,\alpha}^\dagger c_{i+\boldsymbol{v}_\rho,\alpha} + c_{i+\boldsymbol{v}_\rho,\alpha}^\dagger c_{i,\alpha} \right) \\ &\quad -\mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} + \mathcal{H}_{\text{int}} \end{aligned}$$

are coupled to a magnetic moment order parameter $\Phi^a(i)$, $a = x, y, z$

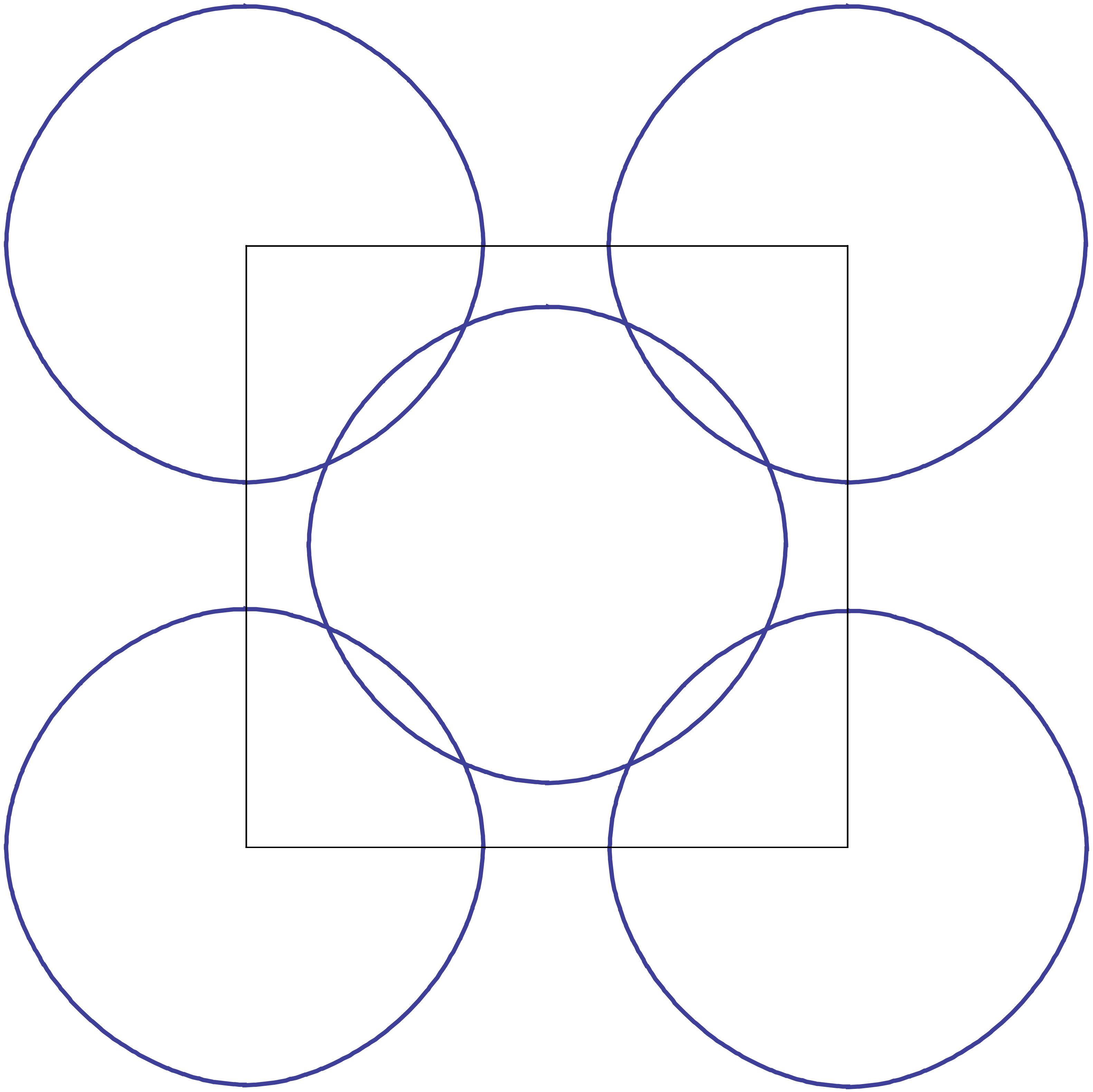
$$\begin{aligned} \mathcal{H}_{\text{int}} &= -\lambda \sum_i \Phi^a(i) c_{i,\alpha}^\dagger \sigma_{\alpha\beta}^a c_{i,\beta} + V(\Phi^a) \\ V(\Phi^a) &= s\Phi^a\Phi^a + u\Phi^a\Phi^a\Phi^b\Phi^b \end{aligned}$$

“Spin-fermion” theory of the Hubbard model



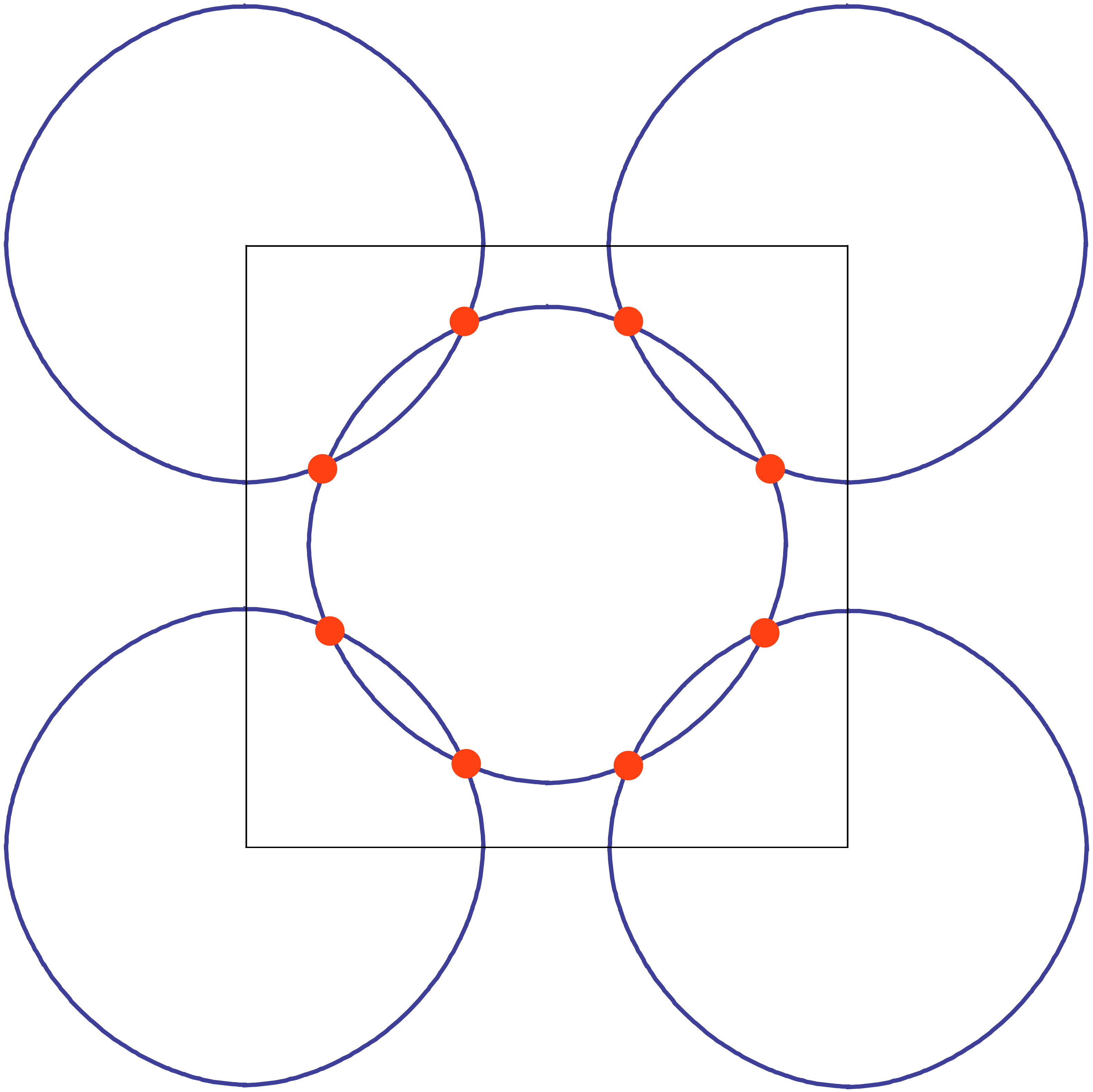
Metal with “large” Fermi surface

“Spin-fermion” theory of the Hubbard model



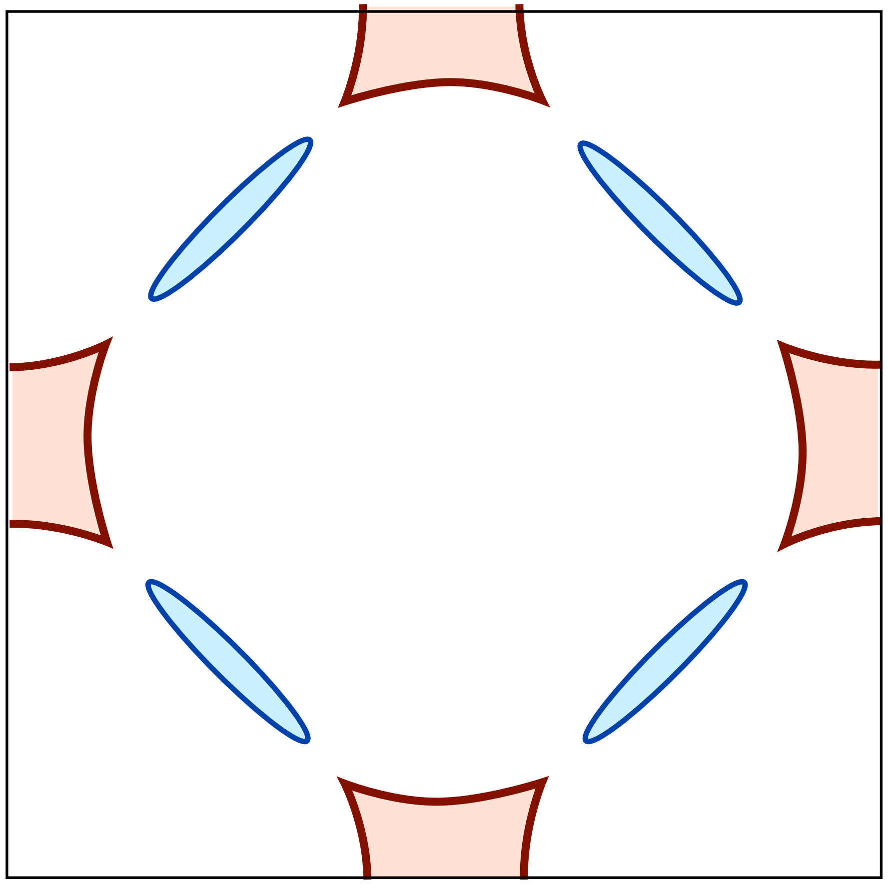
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

“Spin-fermion” theory of the Hubbard model



“Hot” spots

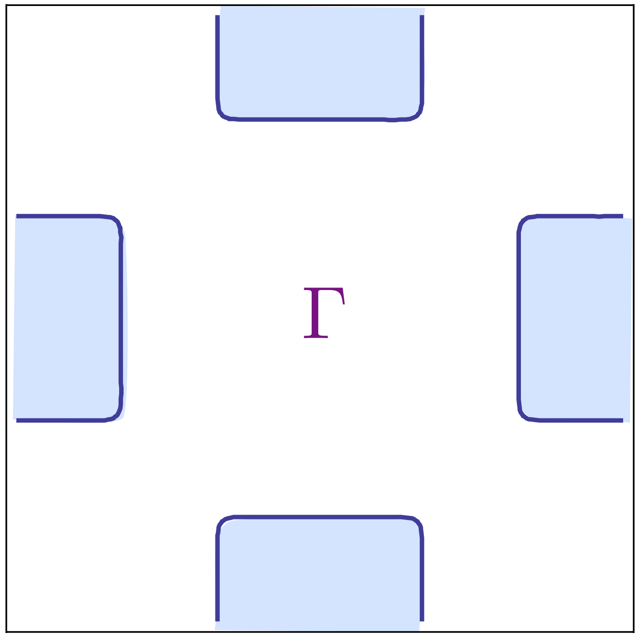
"Spin-fermion" theory of the Hubbard model



Electron and hole pockets in antiferromagnetic phase with $\langle \Phi^a \rangle \neq 0$

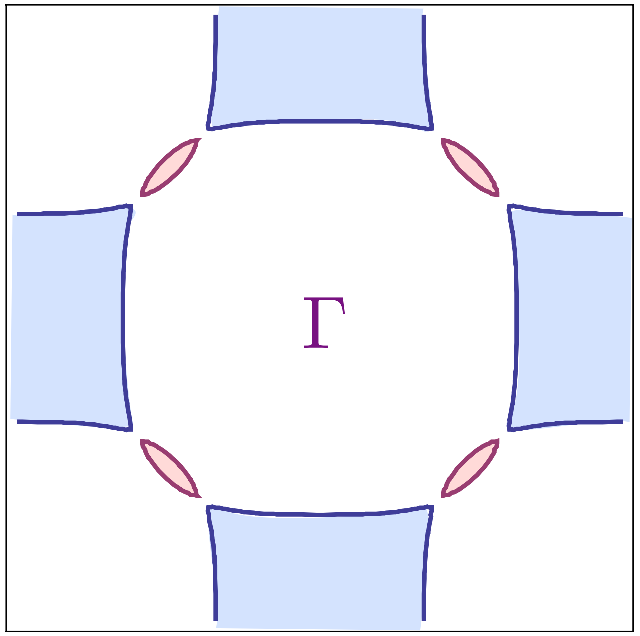
Square lattice Hubbard model with electron doping

$\langle \Phi^a \rangle \neq 0$
and large



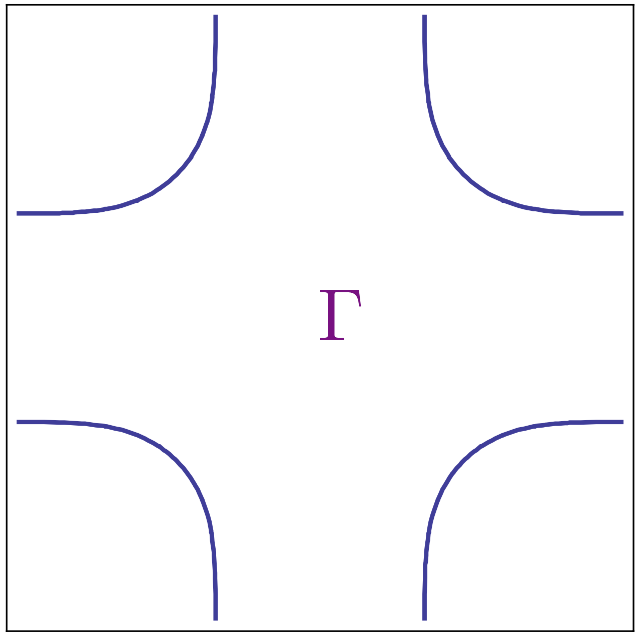
Metal with
electron pockets

$\langle \Phi^a \rangle \neq 0$
and small



Metal with
electron and
hole pockets

$\langle \Phi^a \rangle = 0$



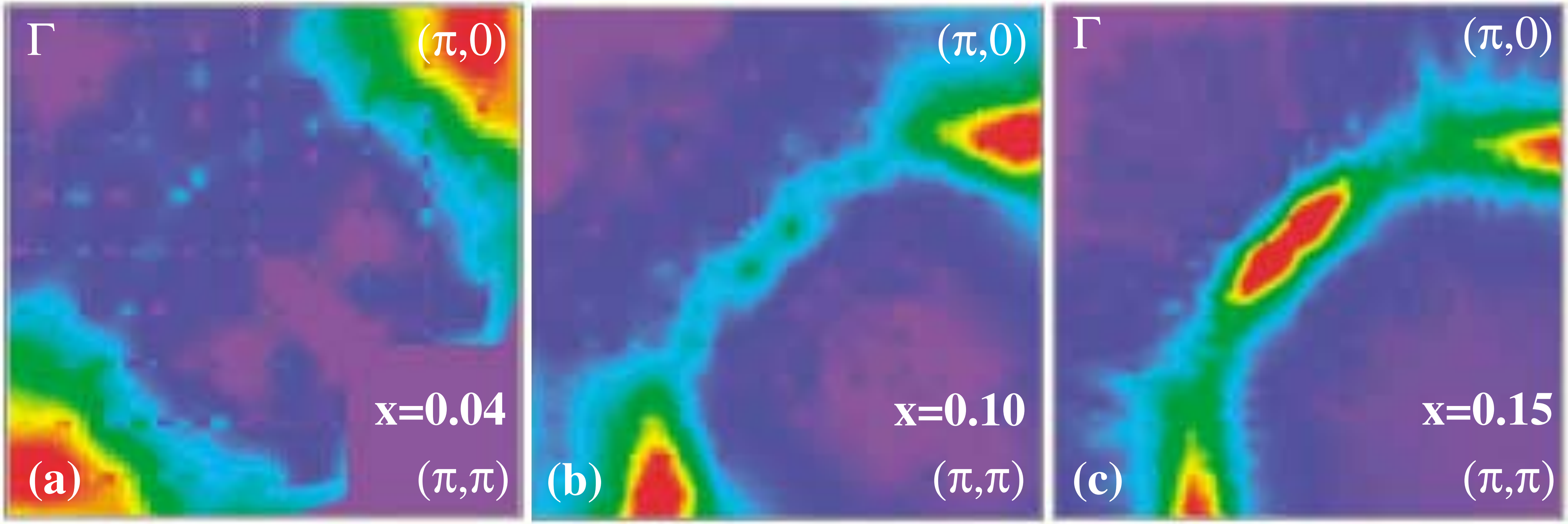
Metal with
“large” Fermi
surface

s

$\Phi^a \Rightarrow \text{Antiferromagnetism at } (\pi, \pi)$

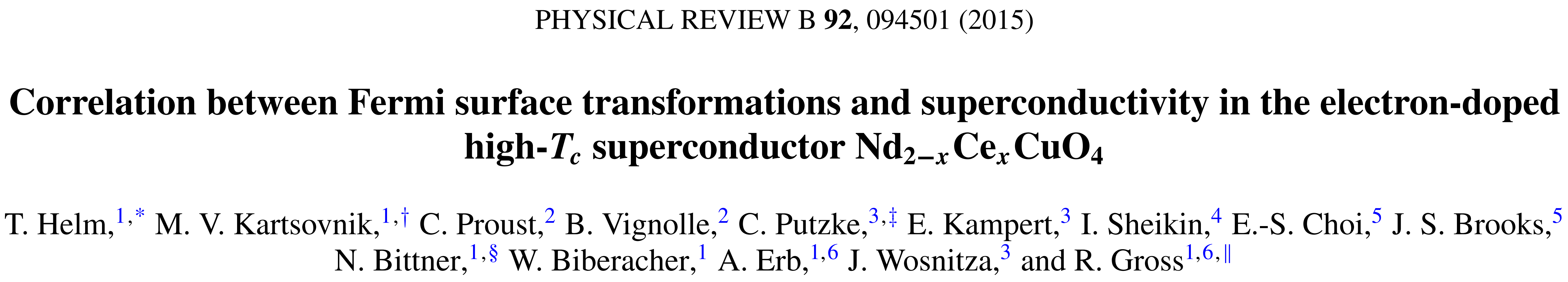
Electron doped cuprates

$\mathrm{Nd}_{2-x}\mathrm{Ce}_x\mathrm{CuO}_{4\pm\delta}$



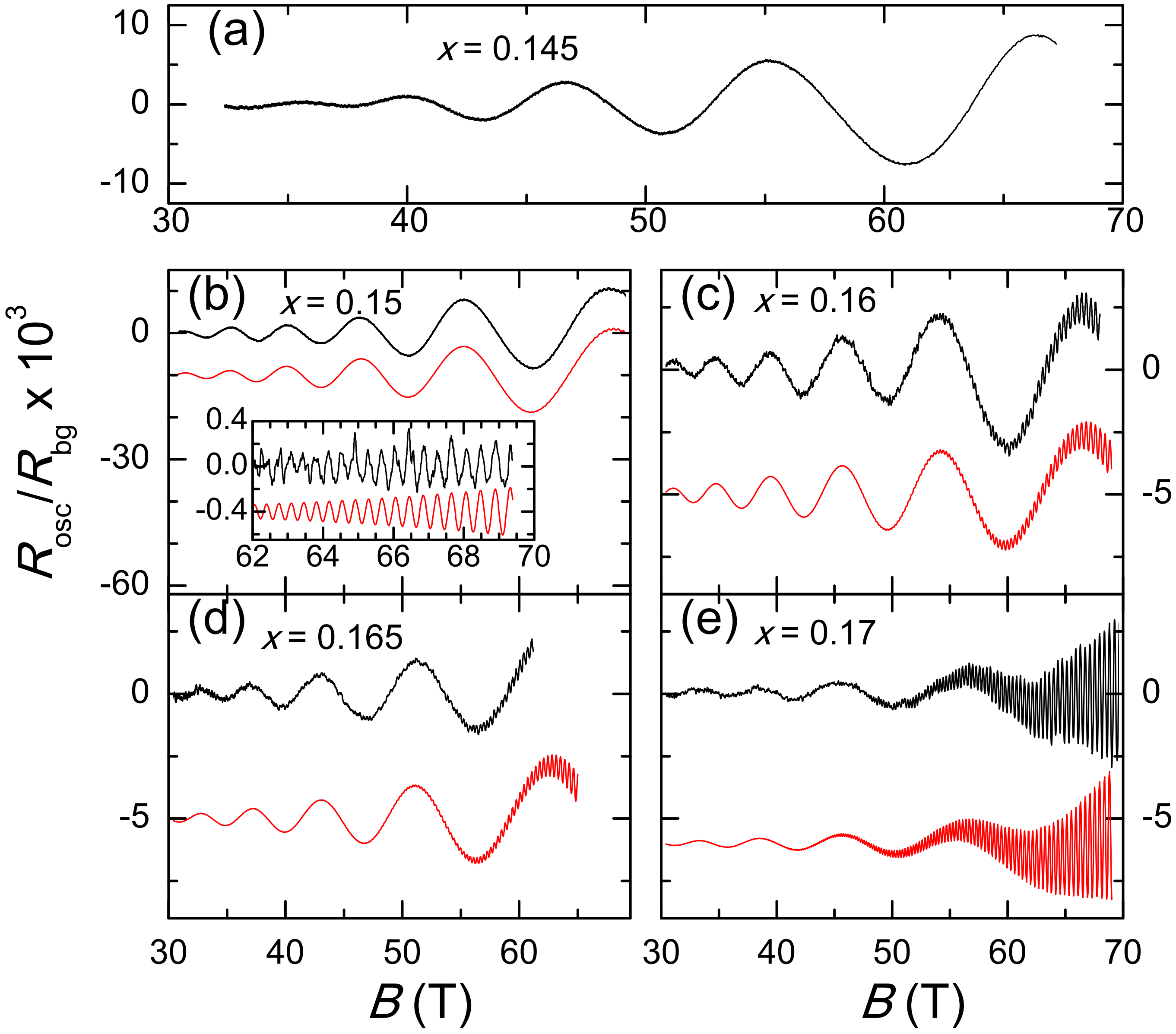
Doping Dependence of an n-Type Cuprate Superconductor Investigated by Angle-Resolved Photoemission Spectroscopy

N. P. Armitage, F. Ronning, D. H. Lu, C. Kim, A. Damascelli, K. M. Shen, D. L. Feng, H. Eisaki, Z.-X. Shen, P. K. Mang, N. Kaneko, M. Greven, Y. Onose, Y. Taguchi, and Y. Tokura
Phys. Rev. Lett. **88**, 257001 (2002)


PHYSICAL REVIEW B **92**, 094501 (2015)

Correlation between Fermi surface transformations and superconductivity in the electron-doped high-T_c superconductor $Nd_{2-x}Ce_xCuO_4$

T. Helm,[1,*] M. V. Kartsovnik,[1,†] C. Proust,[2] B. Vignolle,[2] C. Putzke,[3,‡] E. Kampert,[3] I. Sheikin,[4] E.-S. Choi,[5] J. S. Brooks,[5]
N. Bittner,[1,§] W. Biberacher,[1] A. Erb,[1,6] J. Wosnitza,[3] and R. Gross[1,6,‖]



- Quantum oscillations show the presence of small hole pockets up to a doping $x = 0.175$ although anti-ferromagnetism disappears near $x = 0.14$

arXiv:1811.04992

Fermi surface reconstruction in electron-doped cuprates without antiferromagnetic long-range order

Junfeng He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto, K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu, Y. S. Lee, T. P. Devereaux and Z.-X. Shen

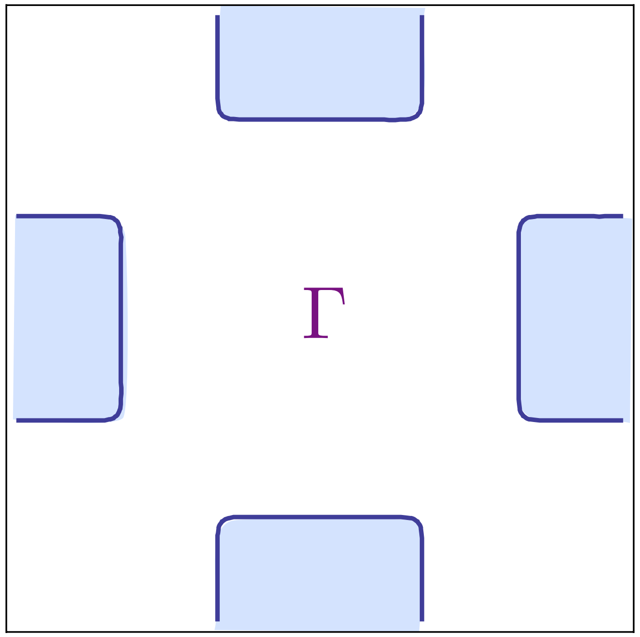
- New photoemission measurements at zero magnetic field show Fermi surfaces in quantitative agreement with quantum oscillation measurements.

- The energy gap between the electron and hole pockets collapses near $x = 0.17$ like an order parameter.

Square lattice Hubbard model with electron doping

$\langle \Phi^a \rangle \neq 0$

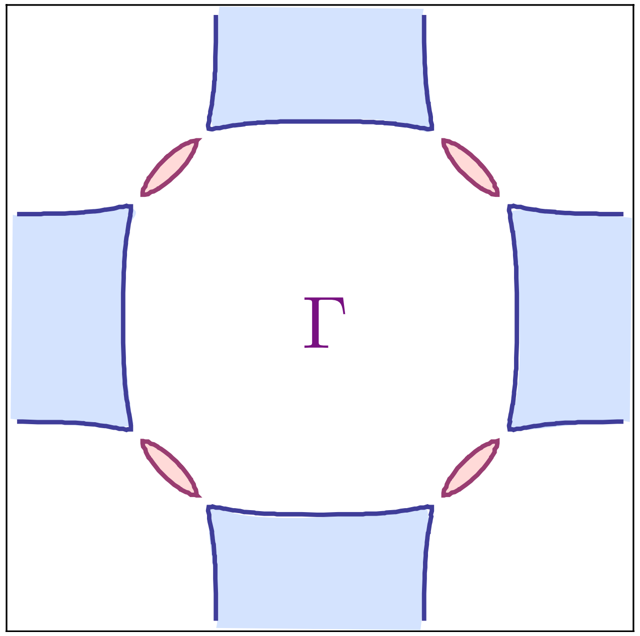
and large



Metal with
electron pockets

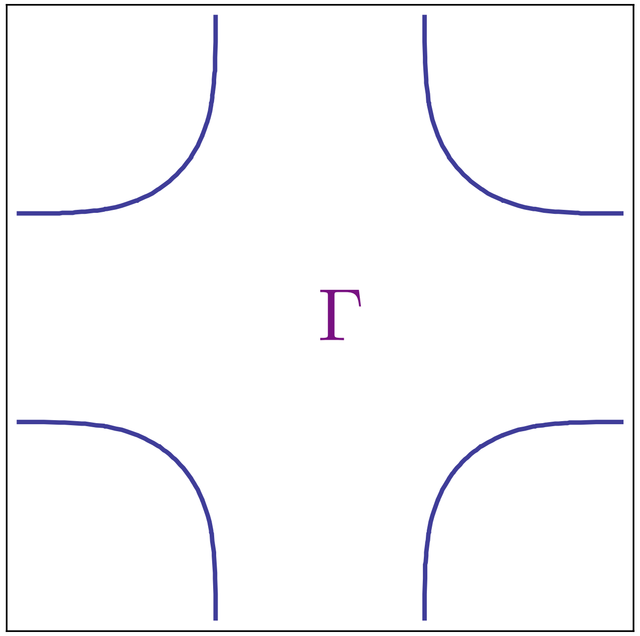
$\langle \Phi^a \rangle \neq 0$

and small



Metal with
electron and
hole pockets

$\langle \Phi^a \rangle = 0$

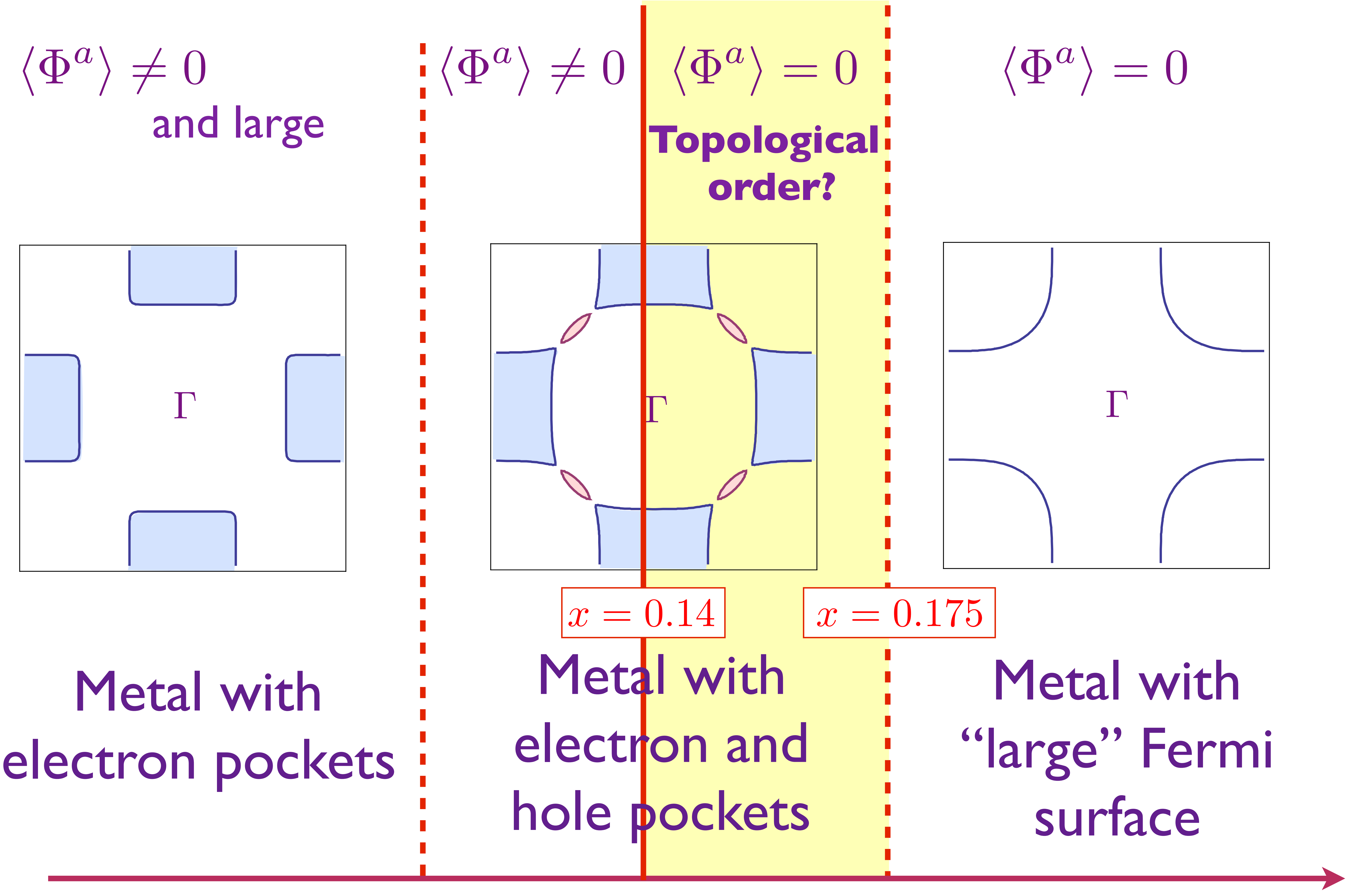


Metal with
"large" Fermi
surface

s

$\Phi^a \Rightarrow$ Antiferromagnetism at (π, π)

Square lattice Hubbard model with electron doping



$\langle \Phi^a \rangle \neq 0$ and large

$\langle \Phi^a \rangle \neq 0$

$\langle \Phi^a \rangle = 0$

Topological order?

$\langle \Phi^a \rangle = 0$

$x = 0.14$

$x = 0.175$

Metal with electron pockets

Metal with electron and hole pockets

Metal with "large" Fermi surface

s

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Fermi surface reconstruction in electron-doped cuprates without antiferromagnetic long-range order

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- New photoemission measurements at zero magnetic field show Fermi surfaces in quantitative agreement with quantum oscillation measurements.

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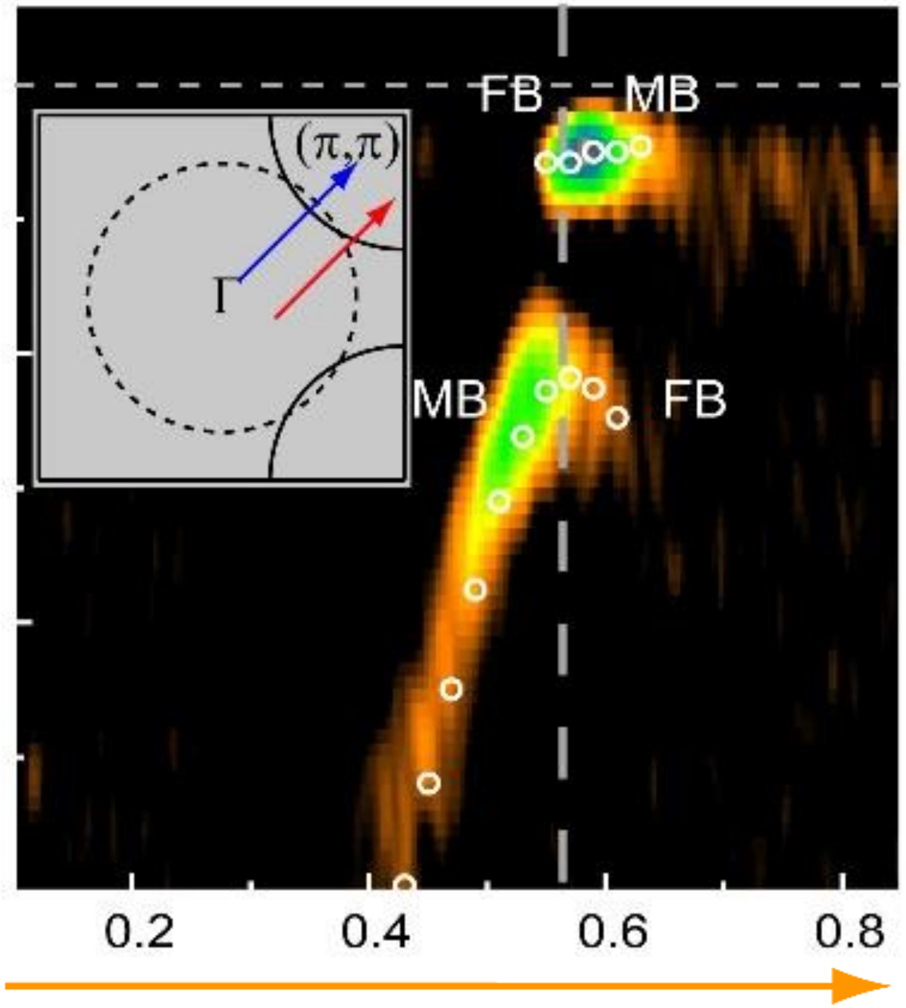
- "The totality of the data points to a mysterious order between $x = 0.14$ and $x = 0.17$, whose appearance favors the FS reconstruction and disappearance defines the quantum critical doping. A recent topological proposal provides an ansatz for its origin."

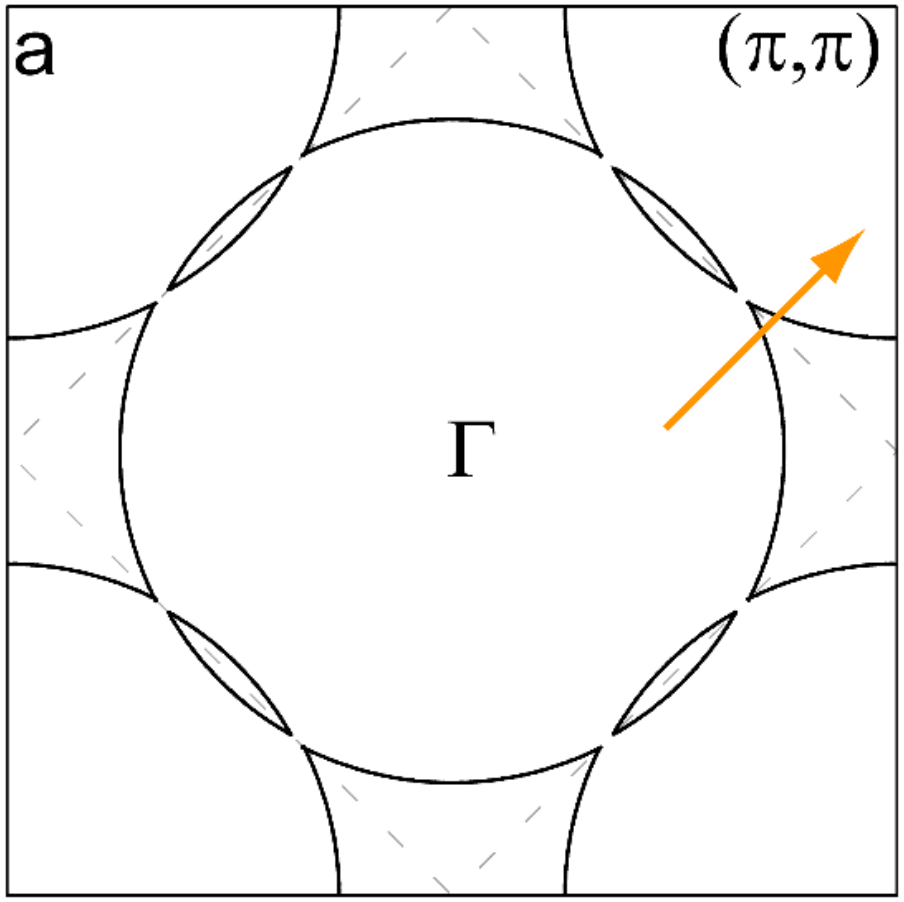
Mathias Scheurer

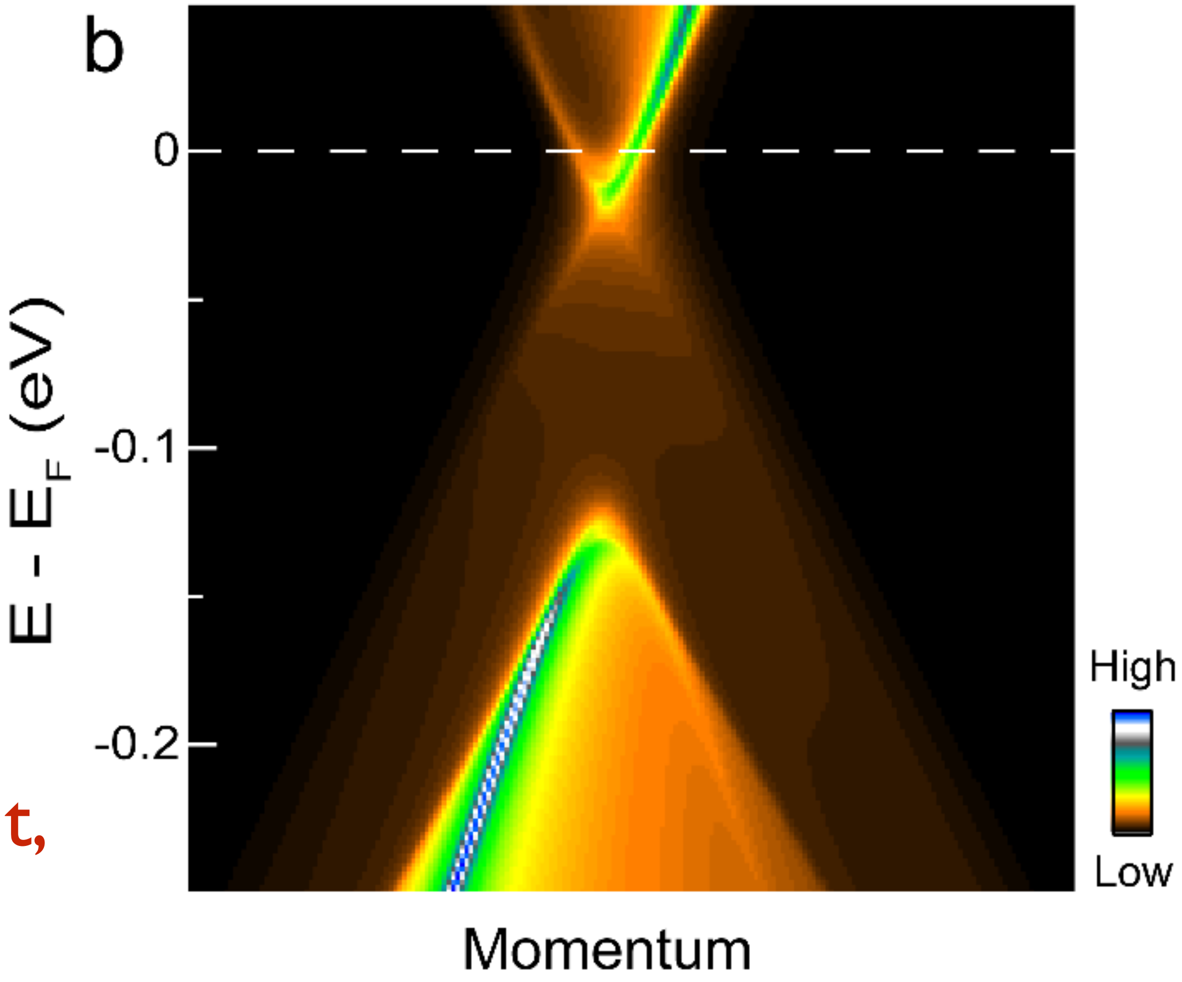
S. Sachdev, arXiv:1801.01125

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, Proceedings of the National Academy of Sciences **115**, E3665 (2018)

Junfeng He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto, K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu, Y. S. Lee, T. P. Devereaux and Z.-X. Shen, arXiv:1811.04992







SU(2) gauge theory of fluctuating antiferromagnetism compared with experiment, and with numerics on the Hubbard model

1. Emergent gauge fields and topological order in the 3D XY model

2. Electron doped cuprates

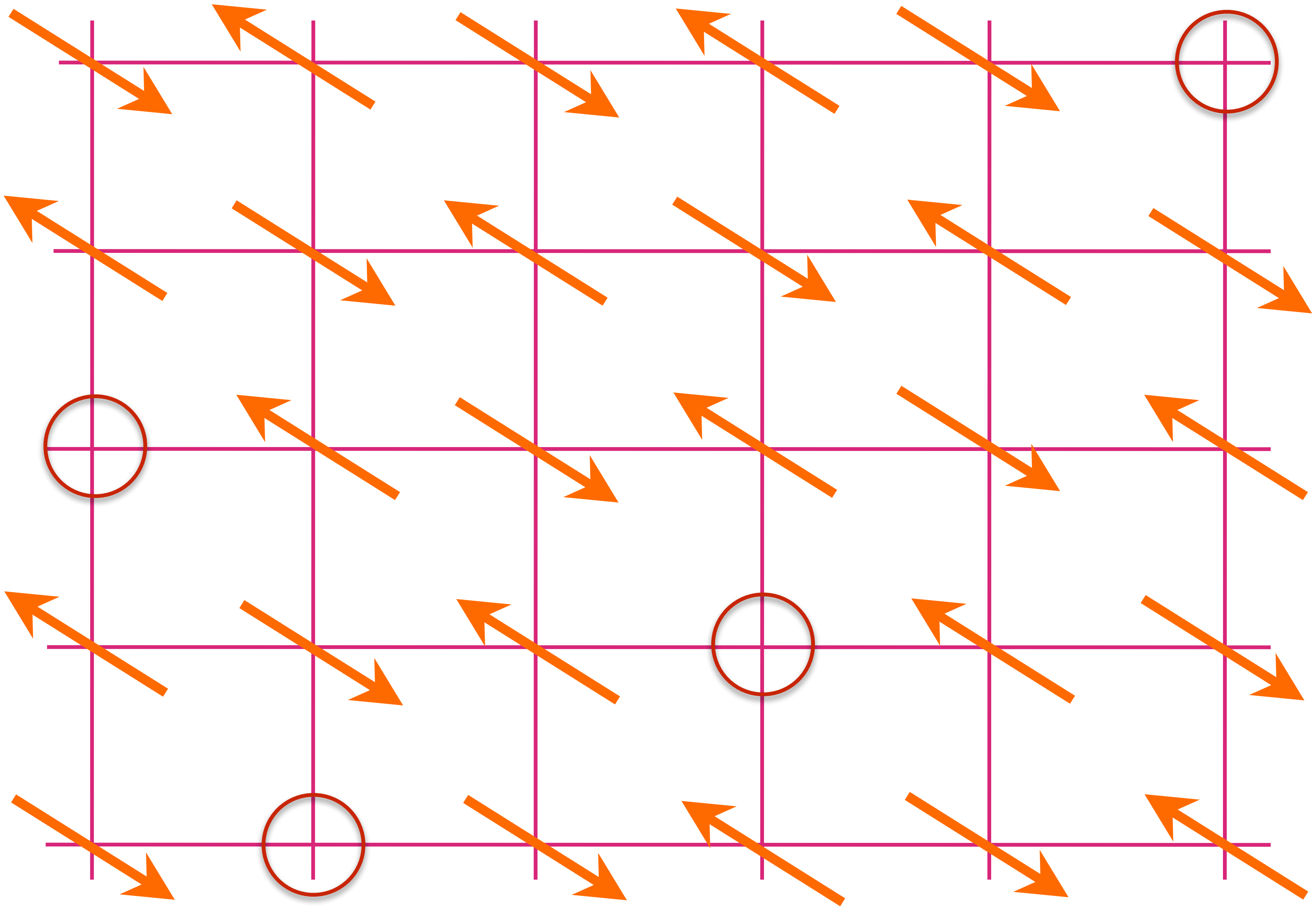
   (A) Simple models of metals with intrinsic topological order

   (B) SU(2) gauge theory of fluctuating antiferromagnetism

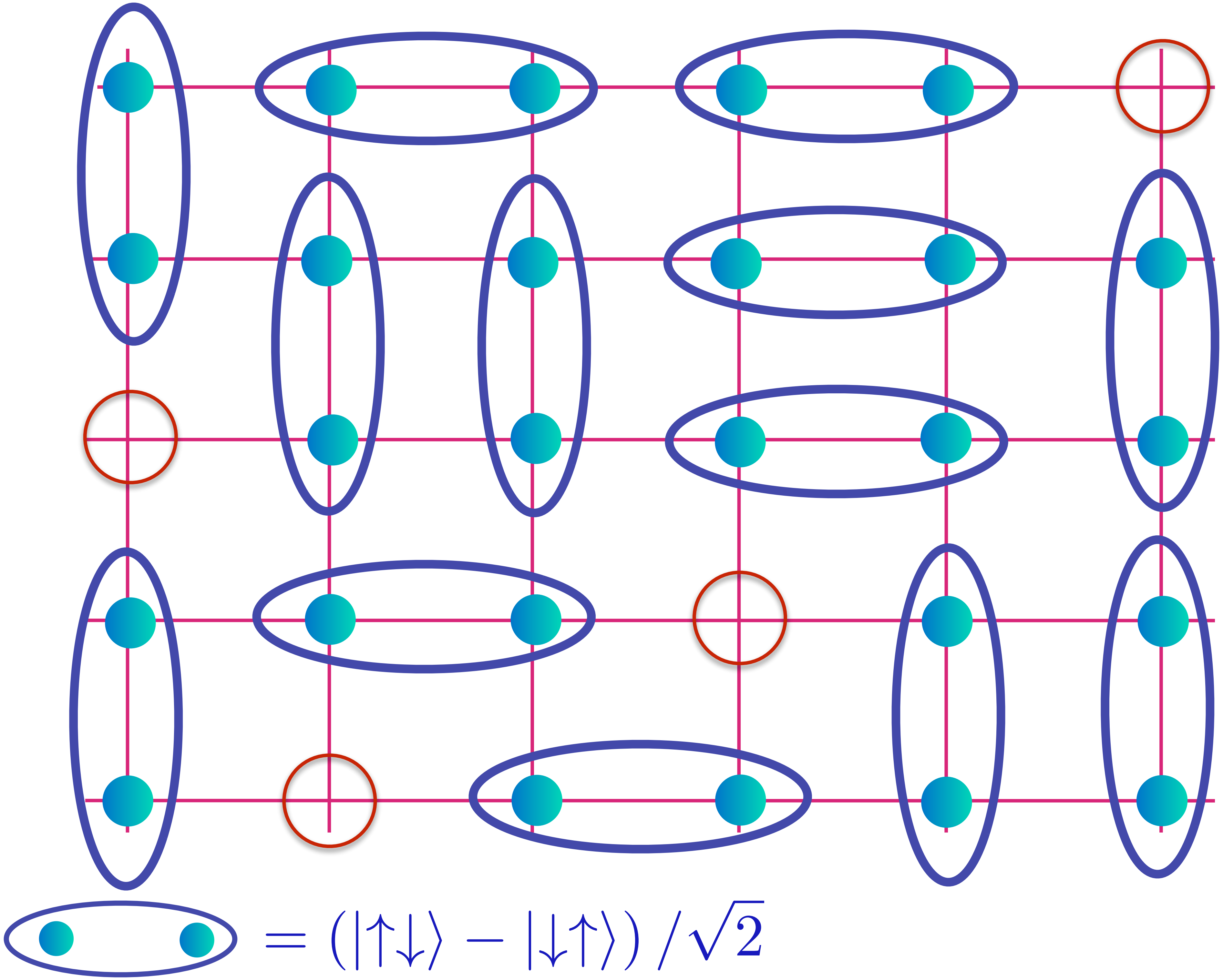
3. Hole doped cuprates

   SU(2) gauge theory with N_h adjoint Higgs fields

   Higgs-confinement transition to a Fermi liquid

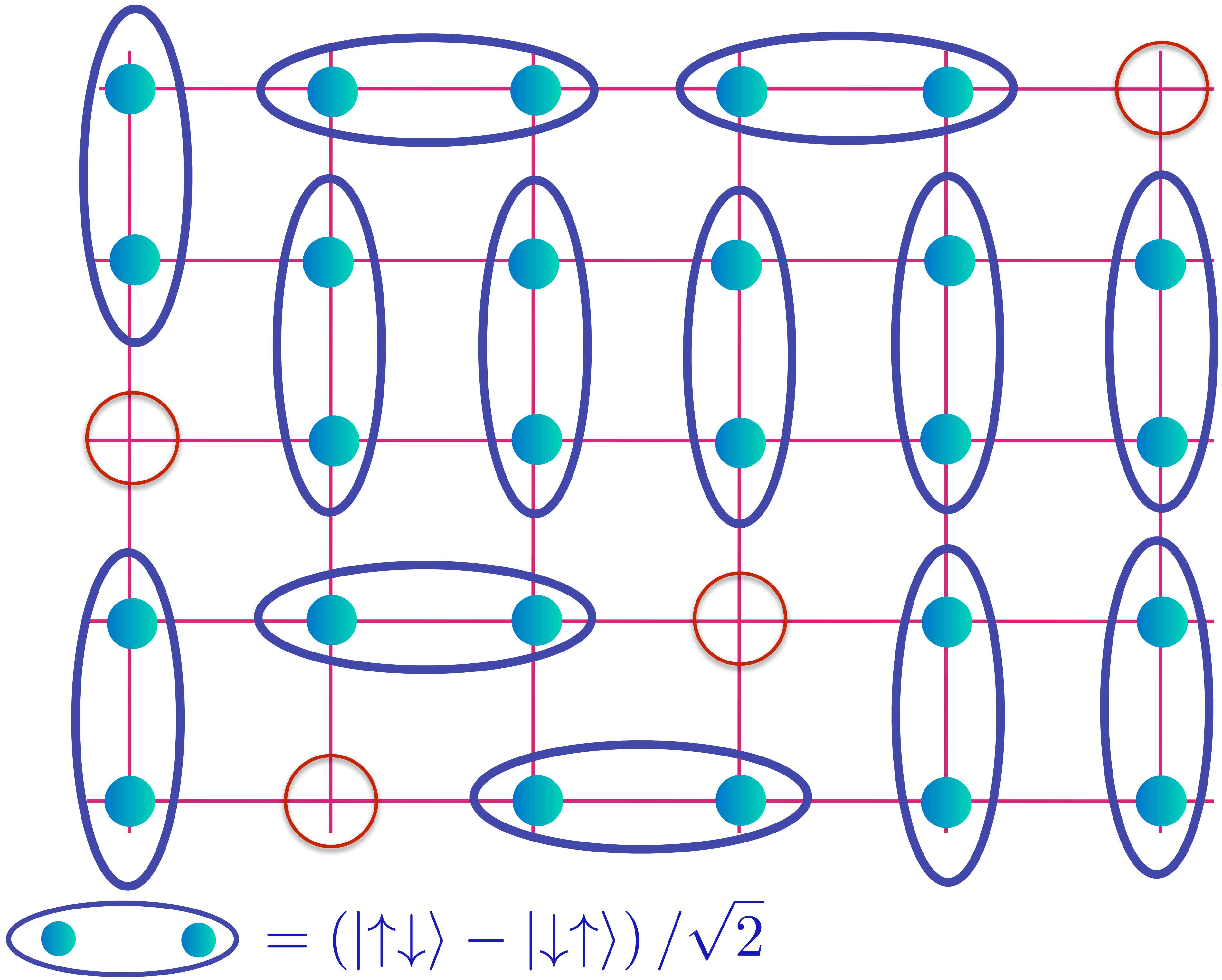


Anti-ferromagnet with p holes per square



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),



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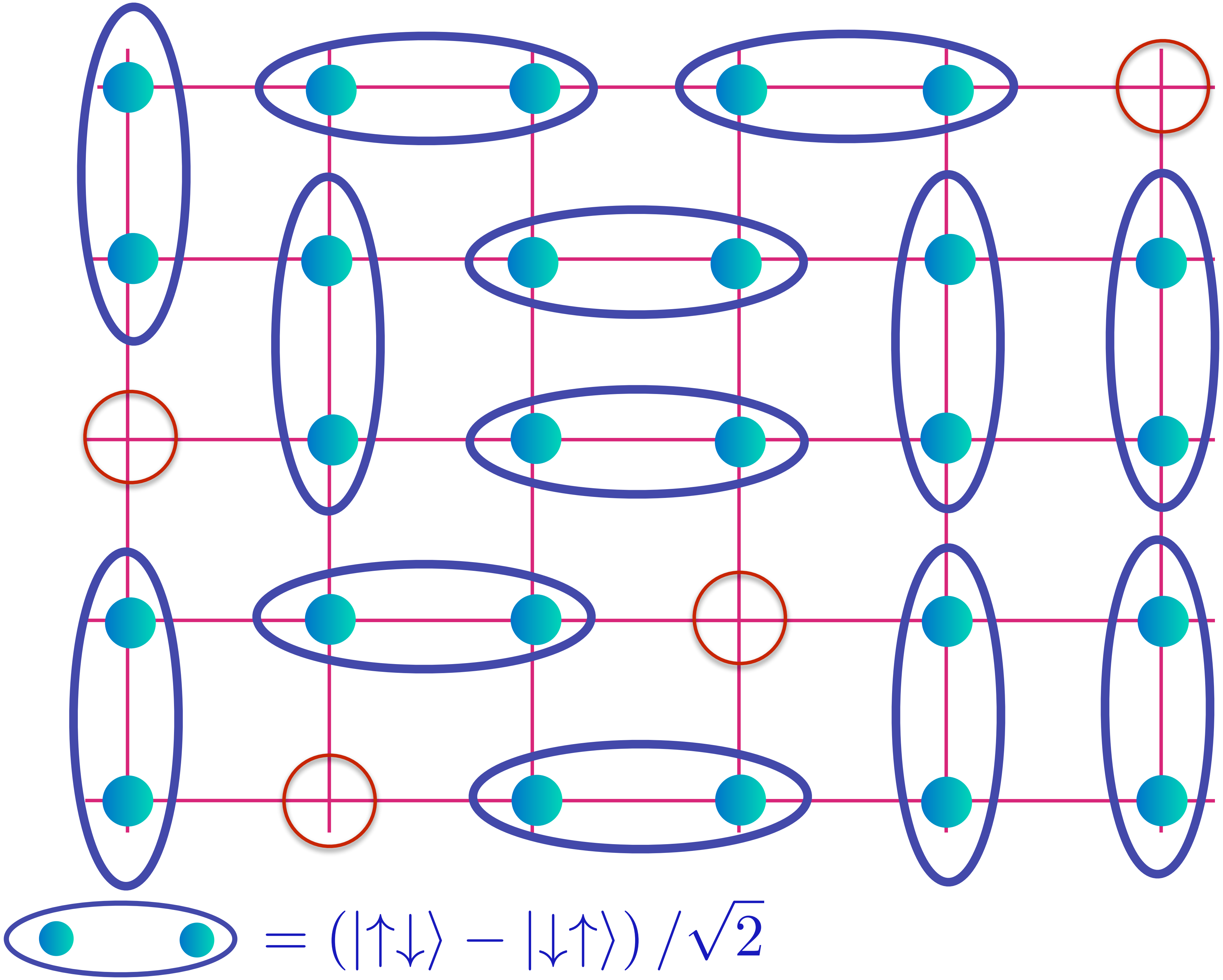
ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

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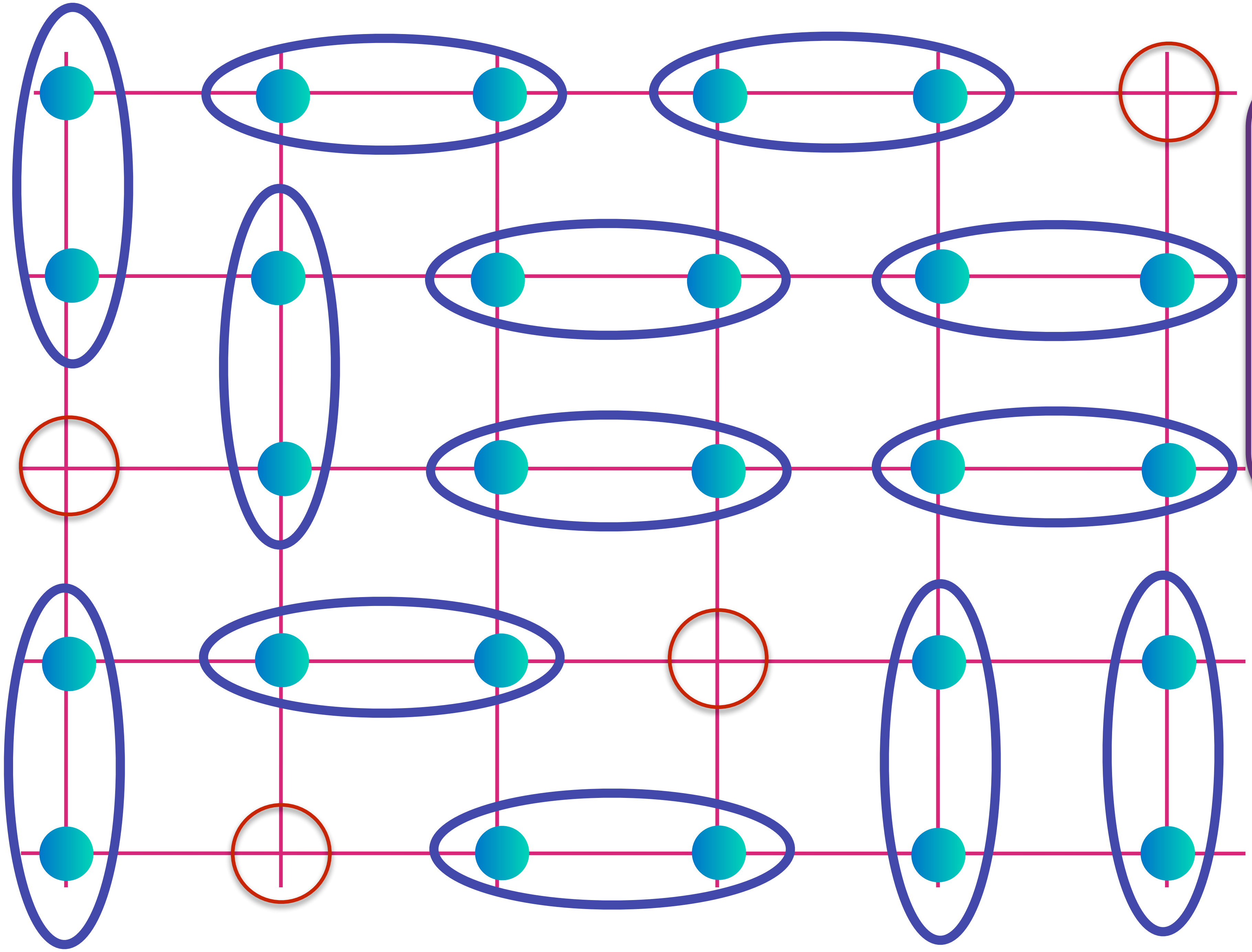
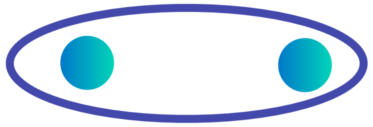
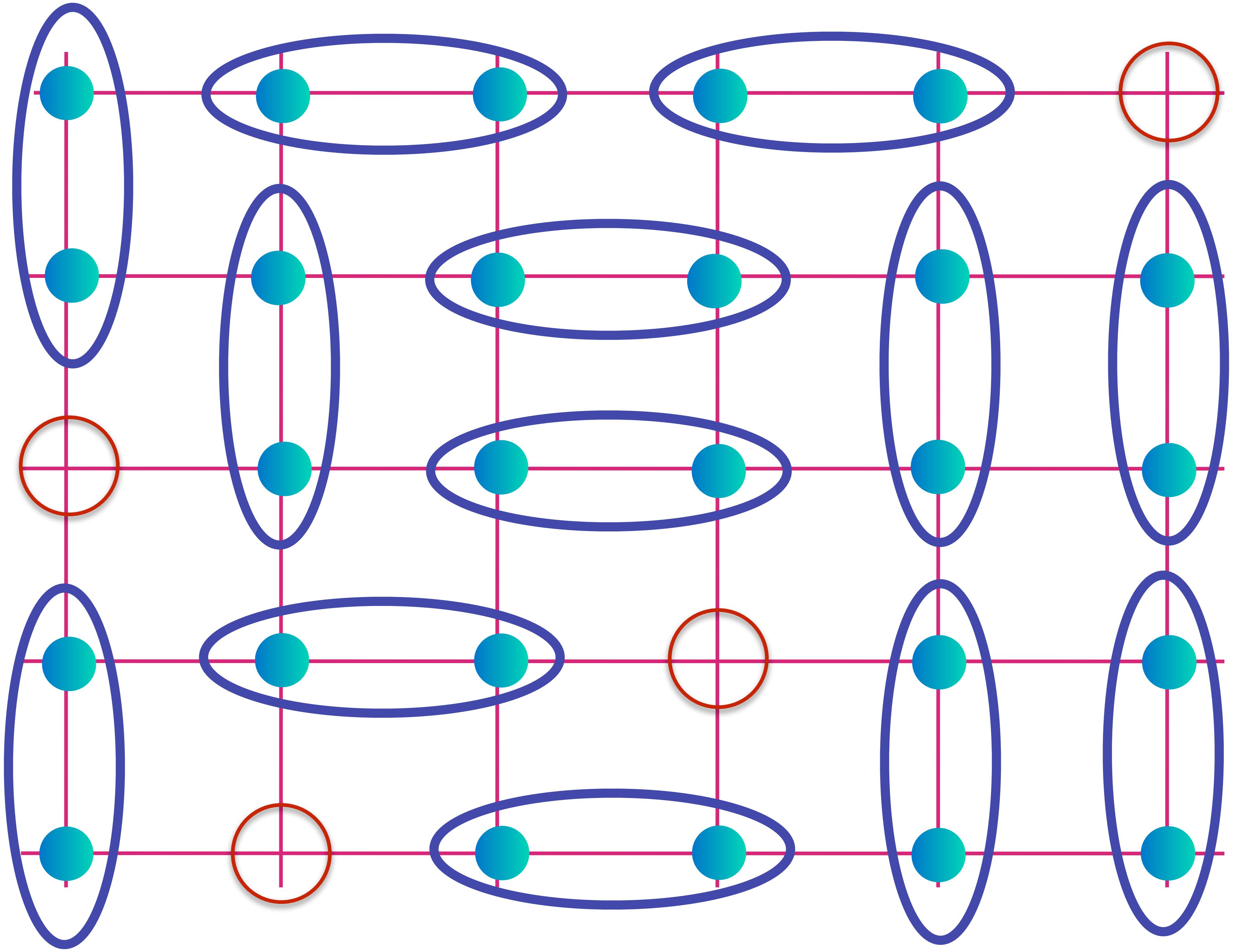


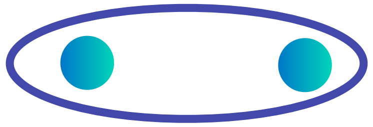
ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),



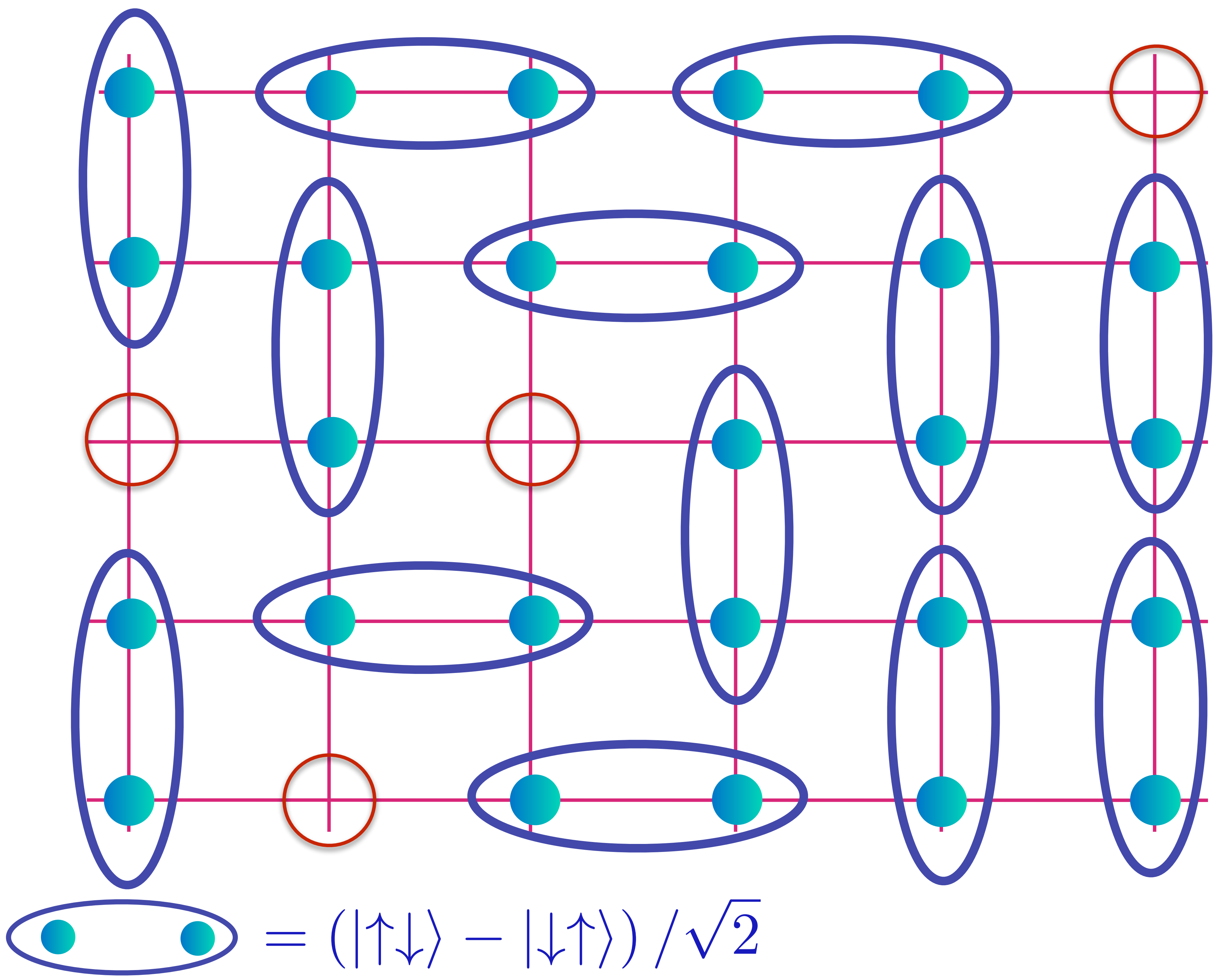
$$= \left(|{\uparrow\downarrow}\rangle - |{\downarrow\uparrow}\rangle \right) / \sqrt{2}$$



ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),

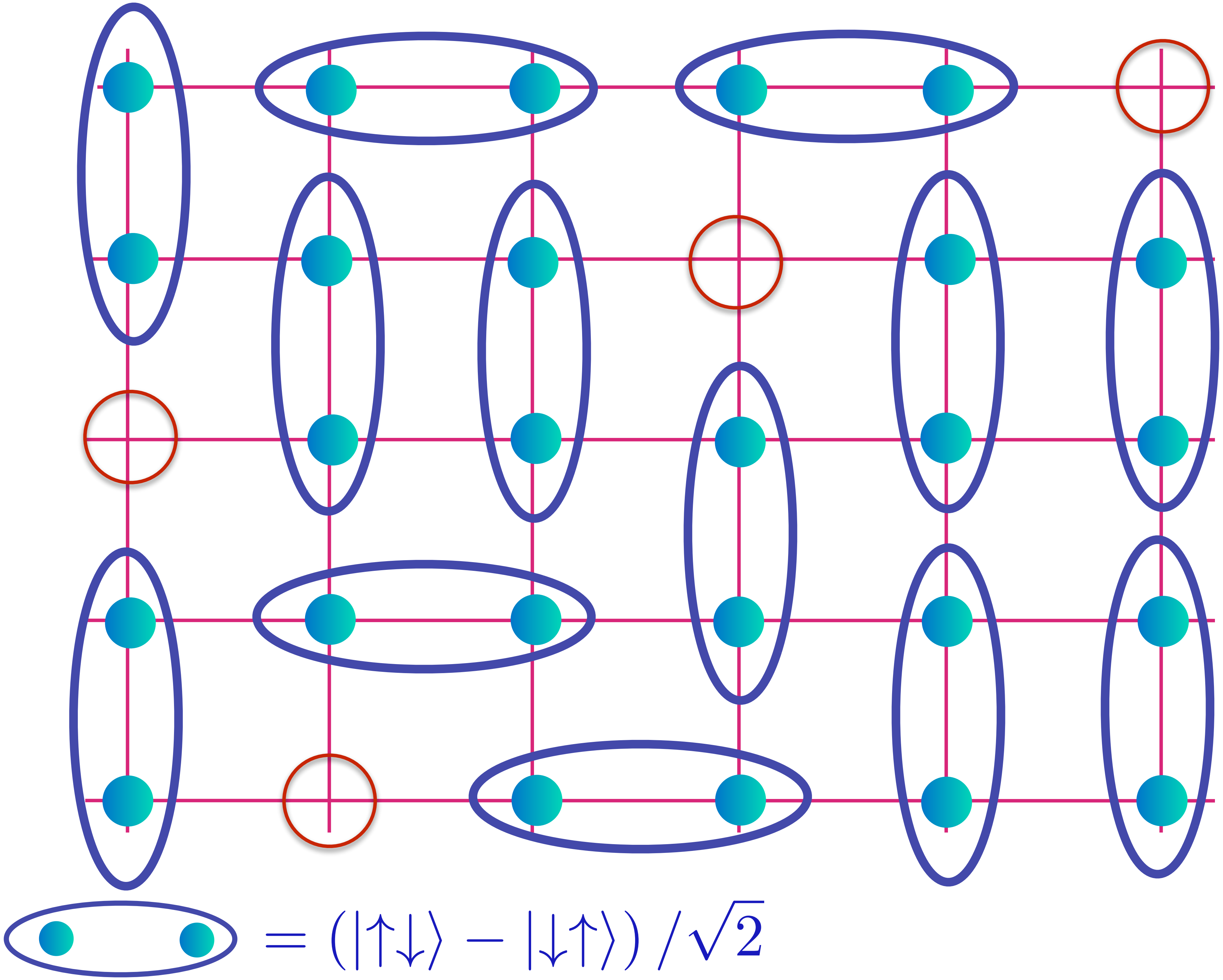


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$$



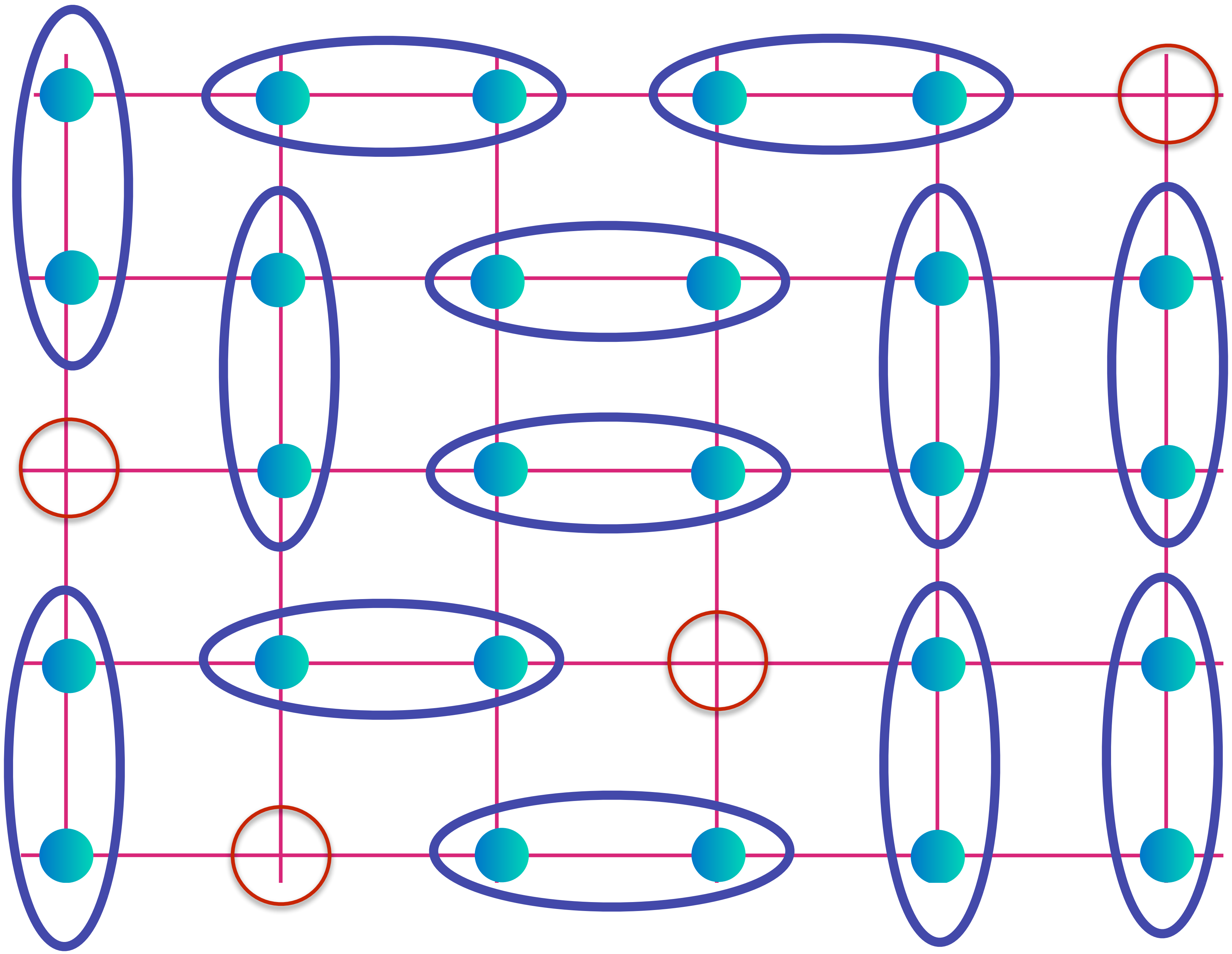
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

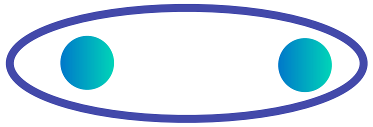
ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),



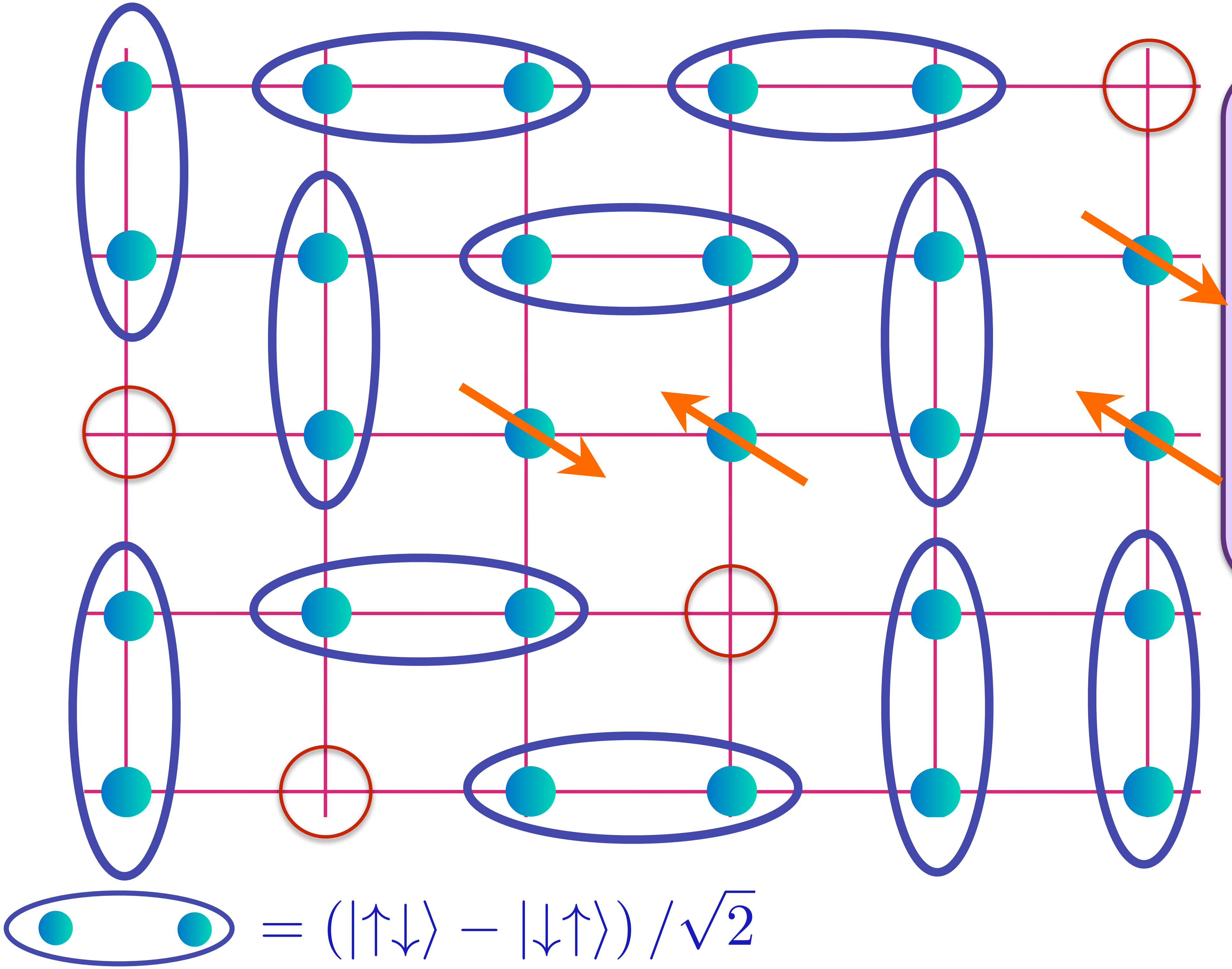
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),



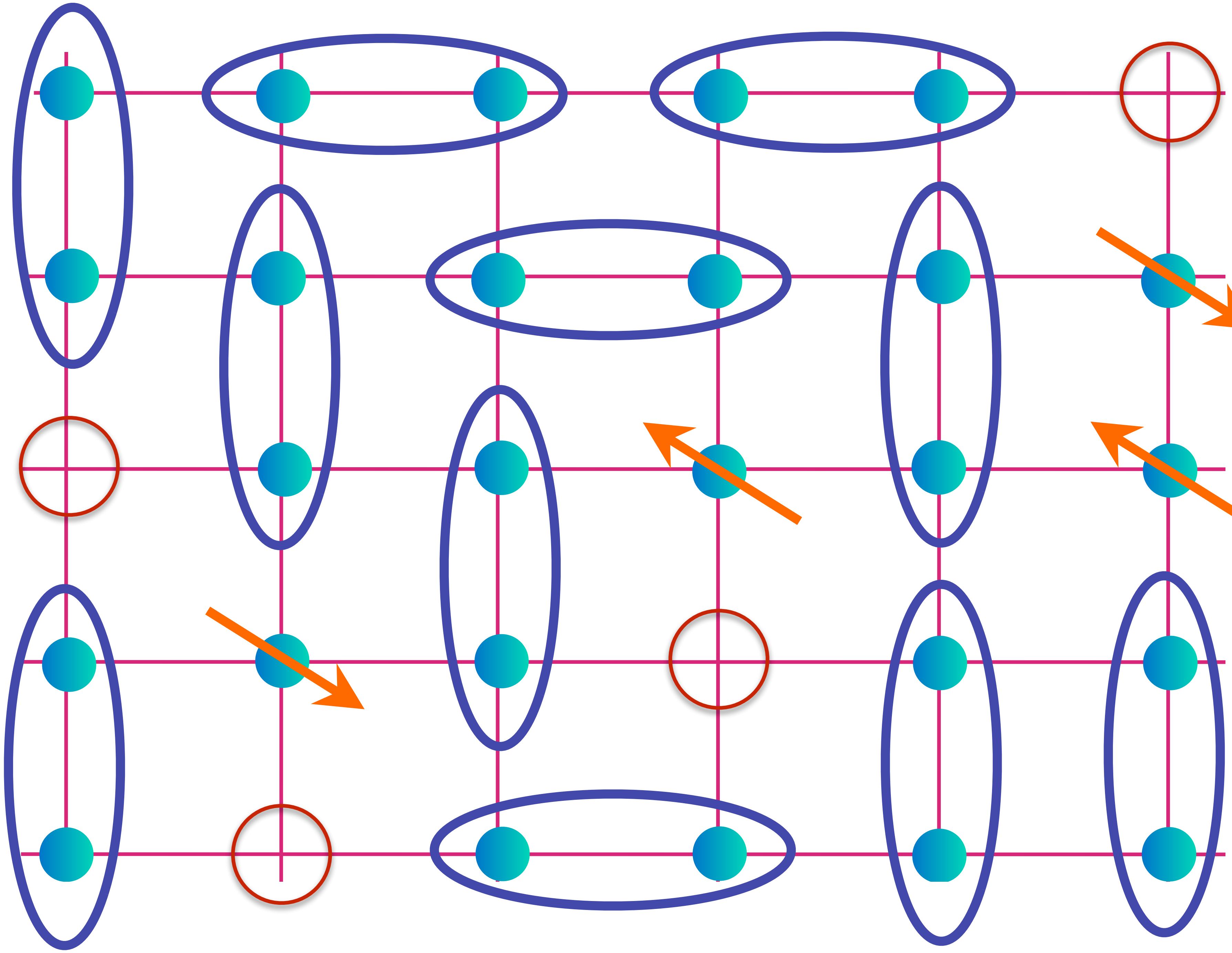
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),

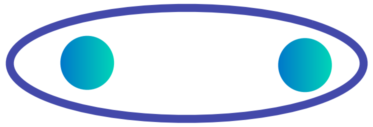


$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers), and spin $S = 1/2$, bosons R



ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers), and spin $S = 1/2$, bosons R



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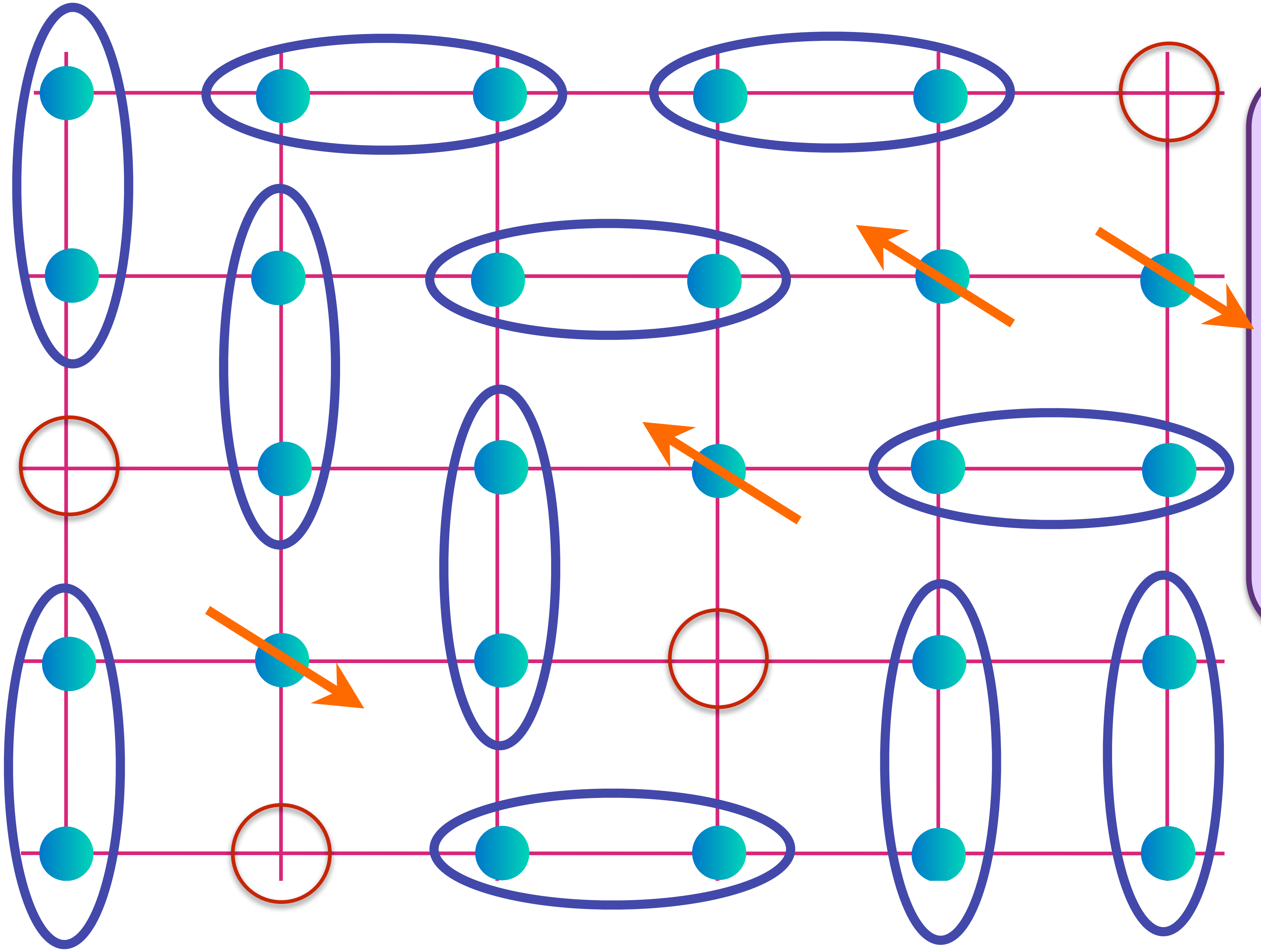
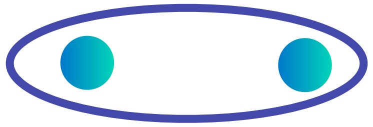


ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers), and spin $S = 1/2$, bosons R



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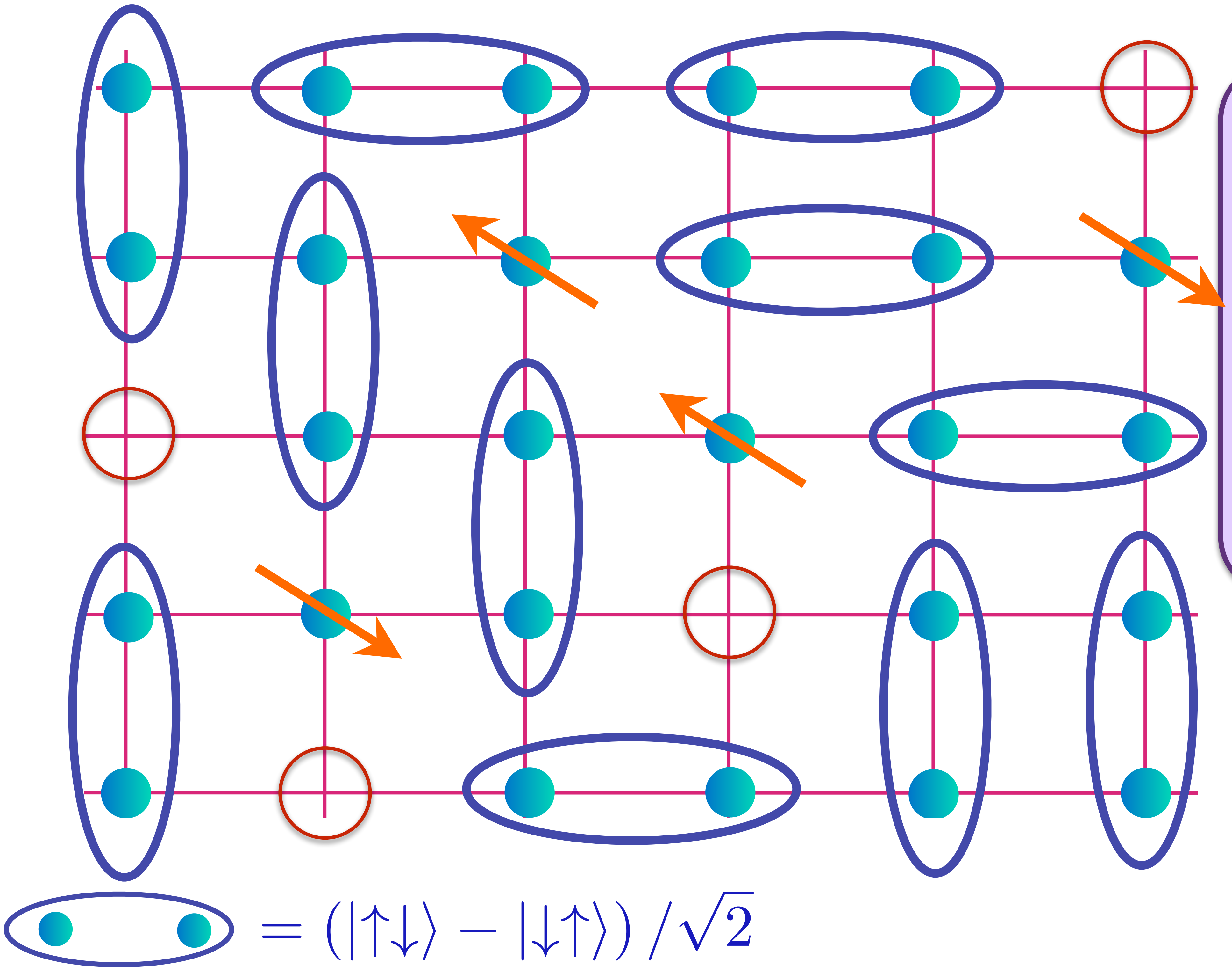

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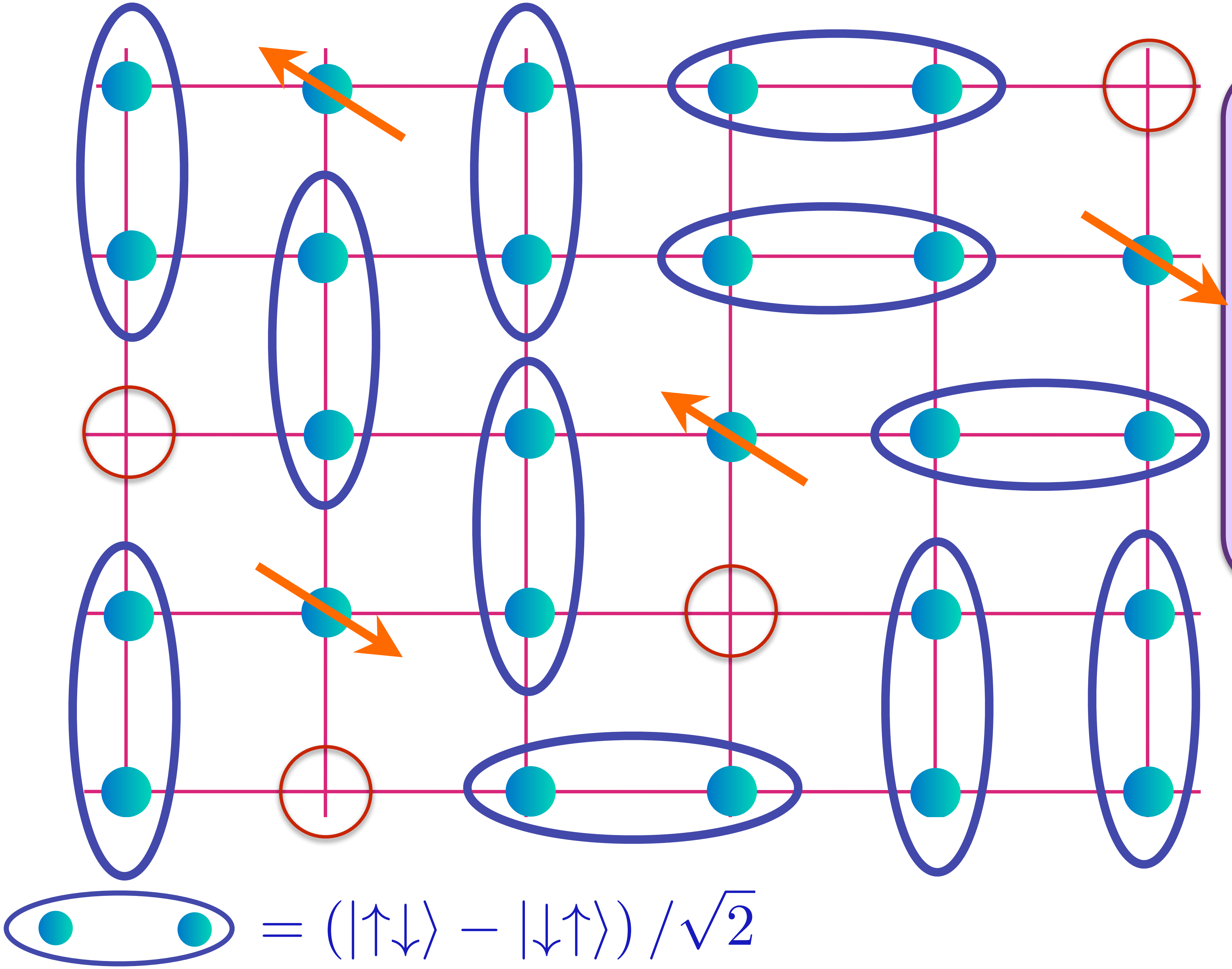

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ACL with density p
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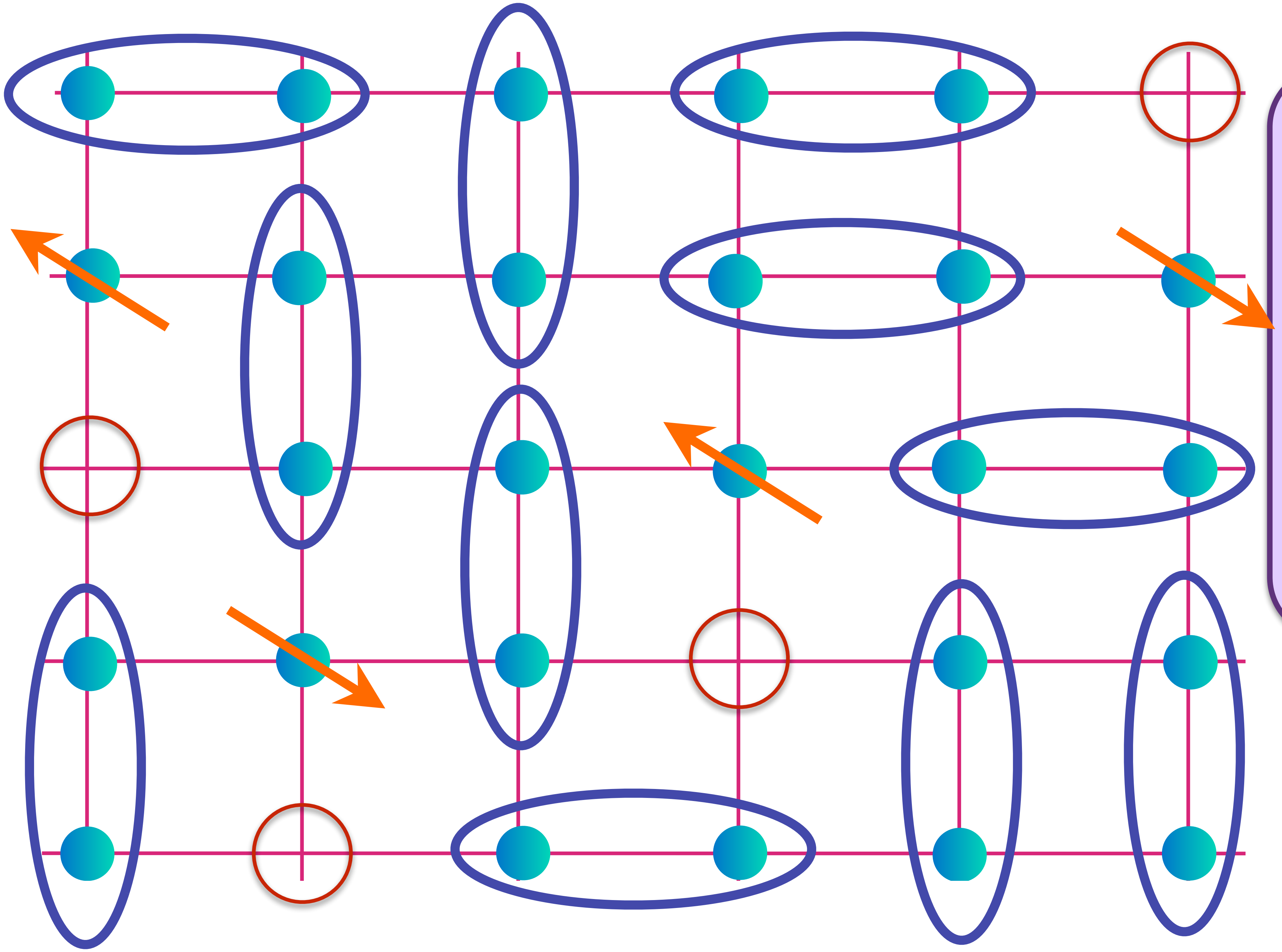
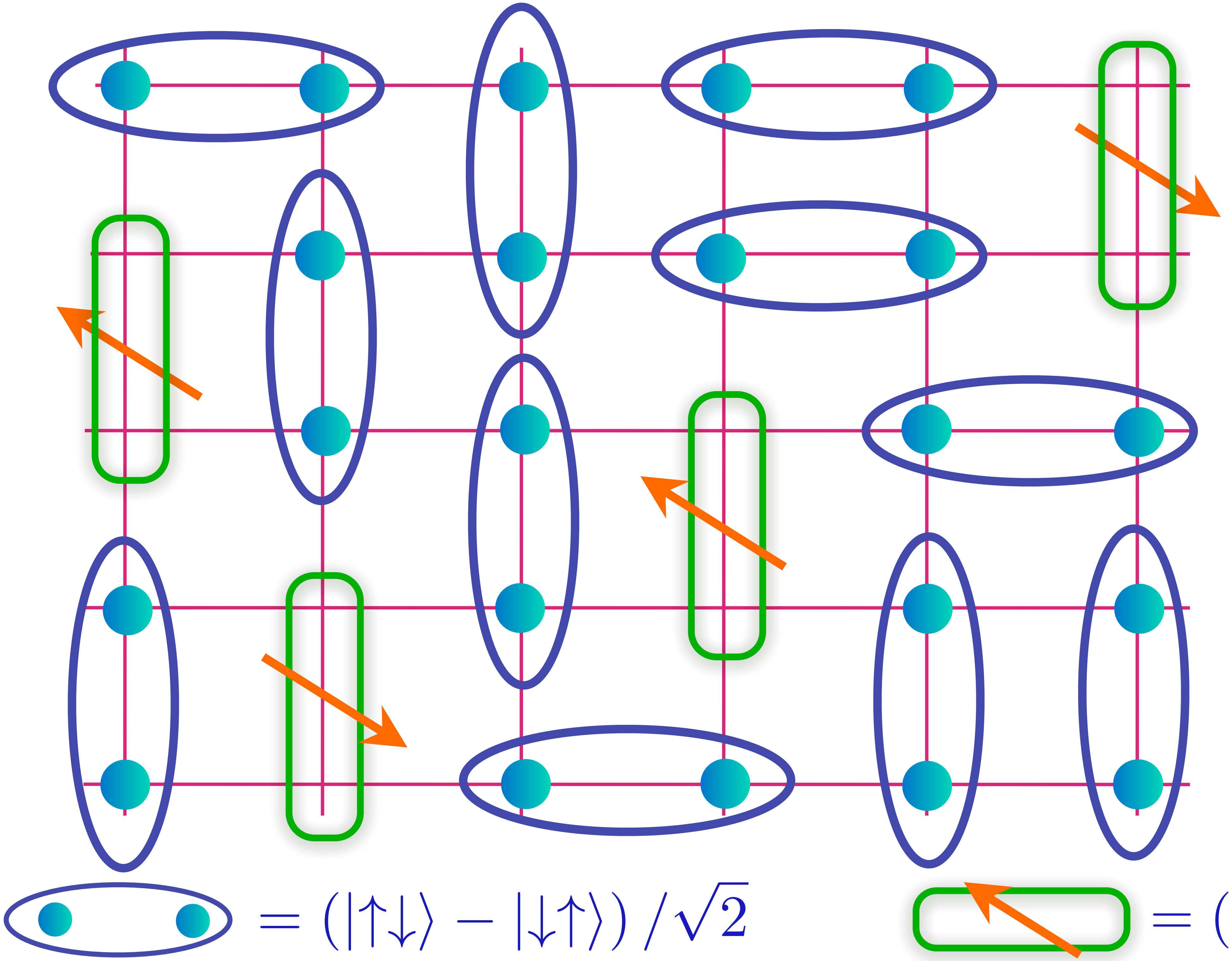
bosons R



ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers), and spin $S = 1/2$, bosons R

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

FL*



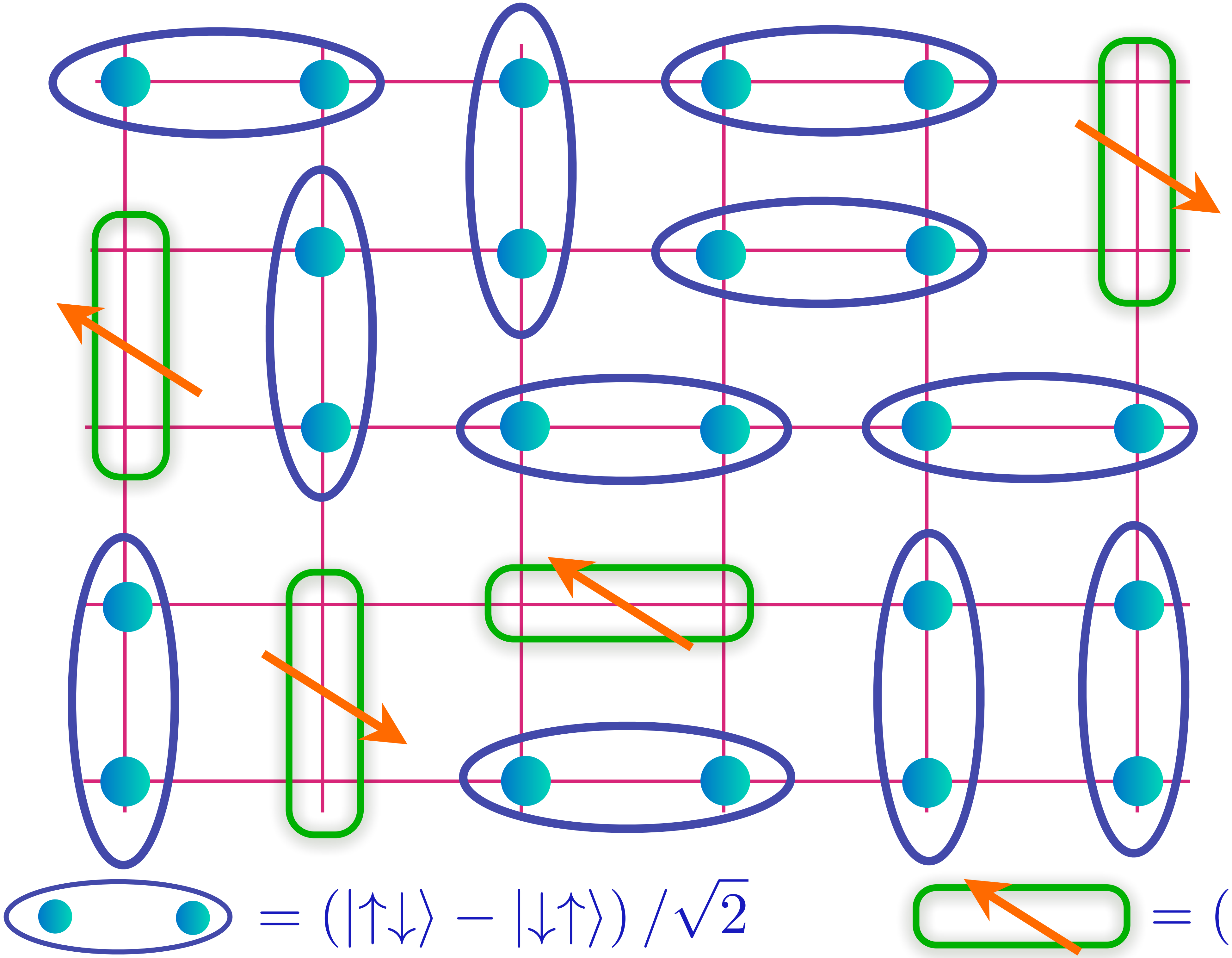
$$= \left(|{\uparrow\downarrow}\rangle - |{\downarrow\uparrow}\rangle\right)/\sqrt{2}$$

$$= \left(|{\uparrow \circ}\rangle + |{\circ \uparrow}\rangle\right)/\sqrt{2}$$

Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

$= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

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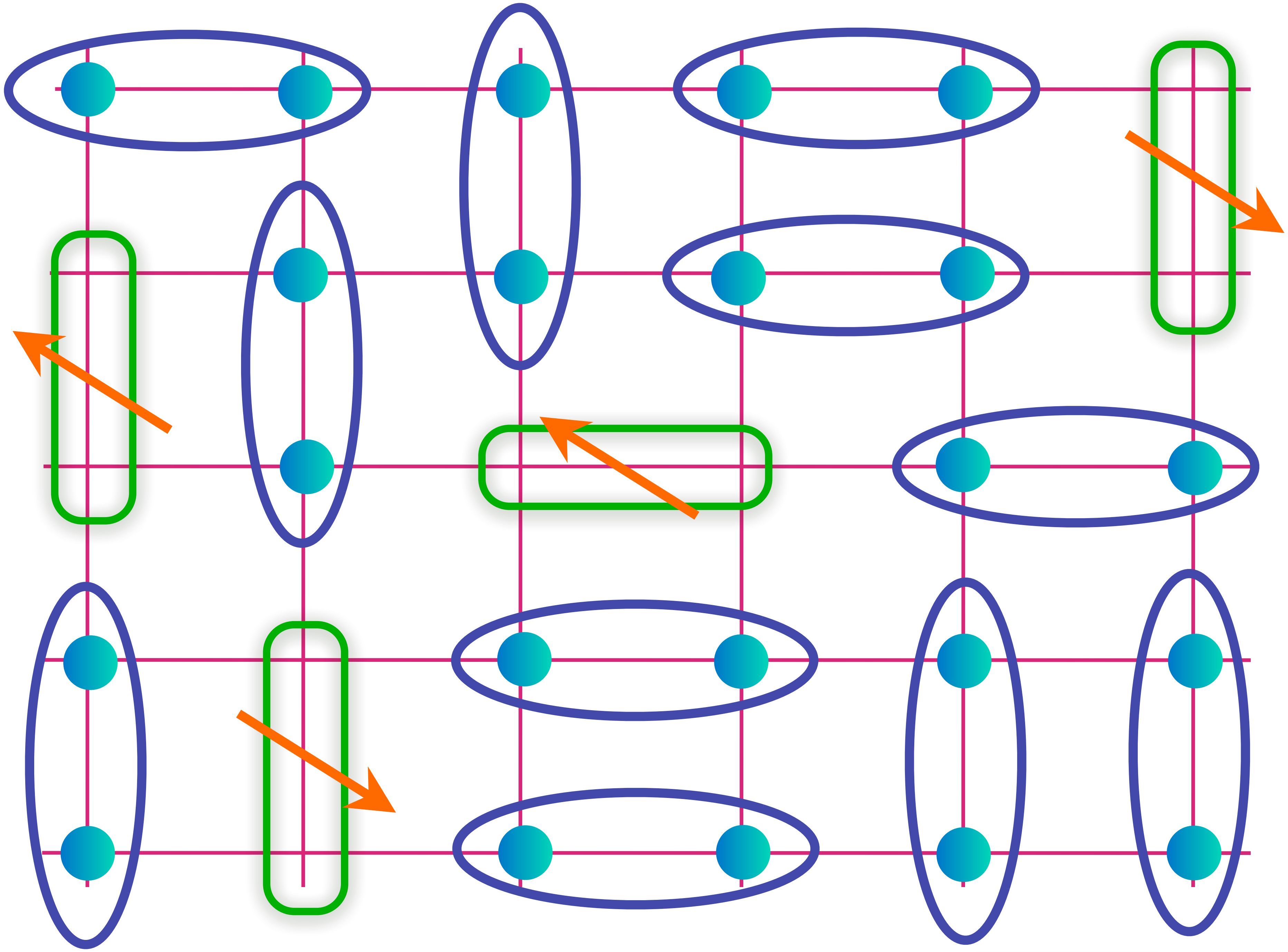
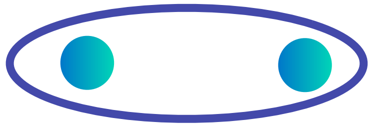
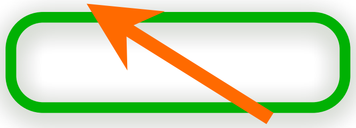


Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields



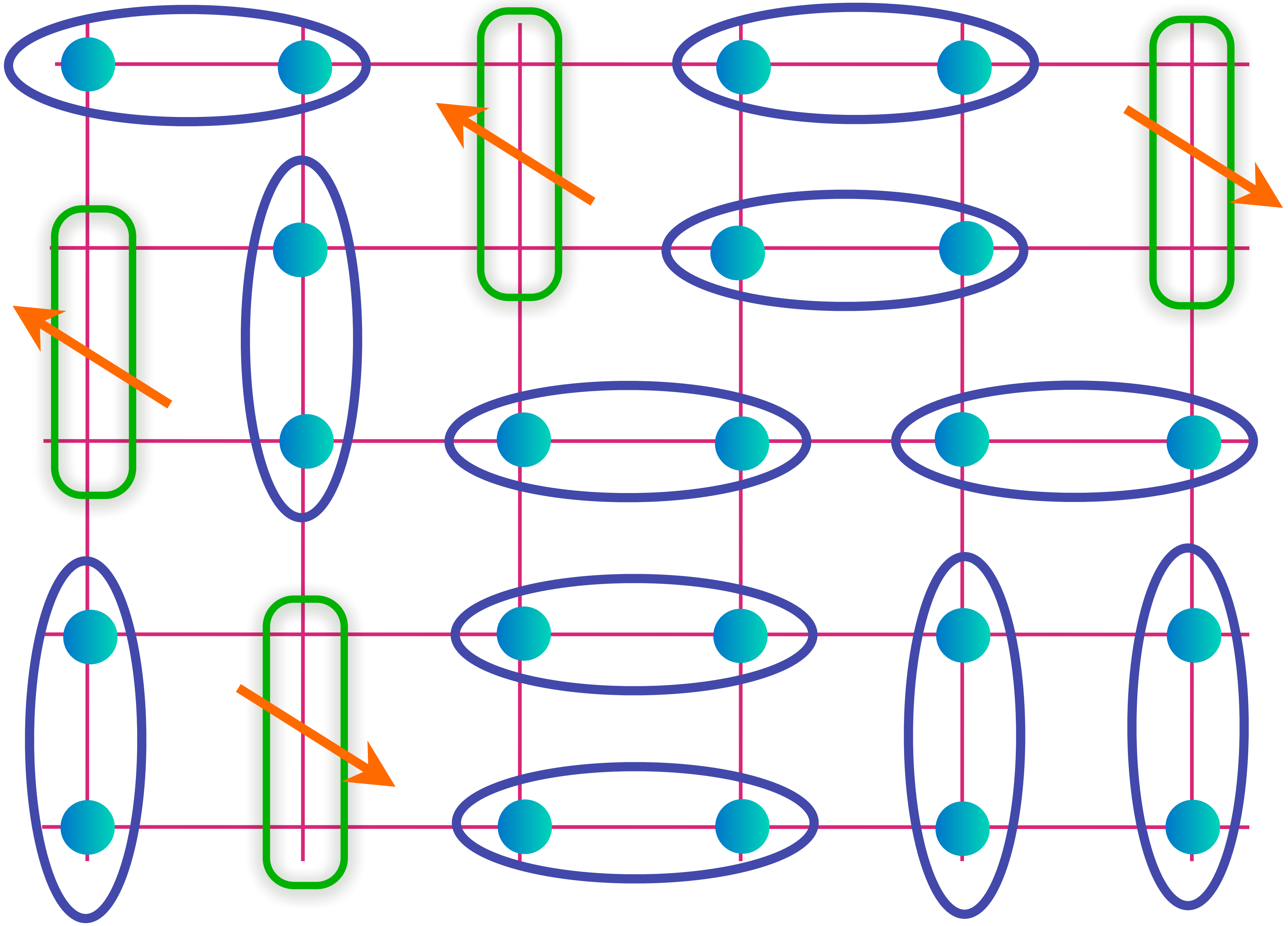
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



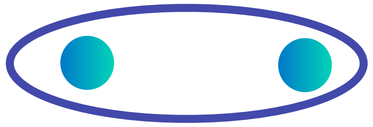
$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

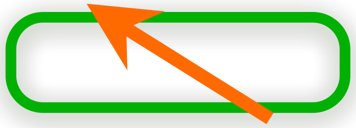
FL*



Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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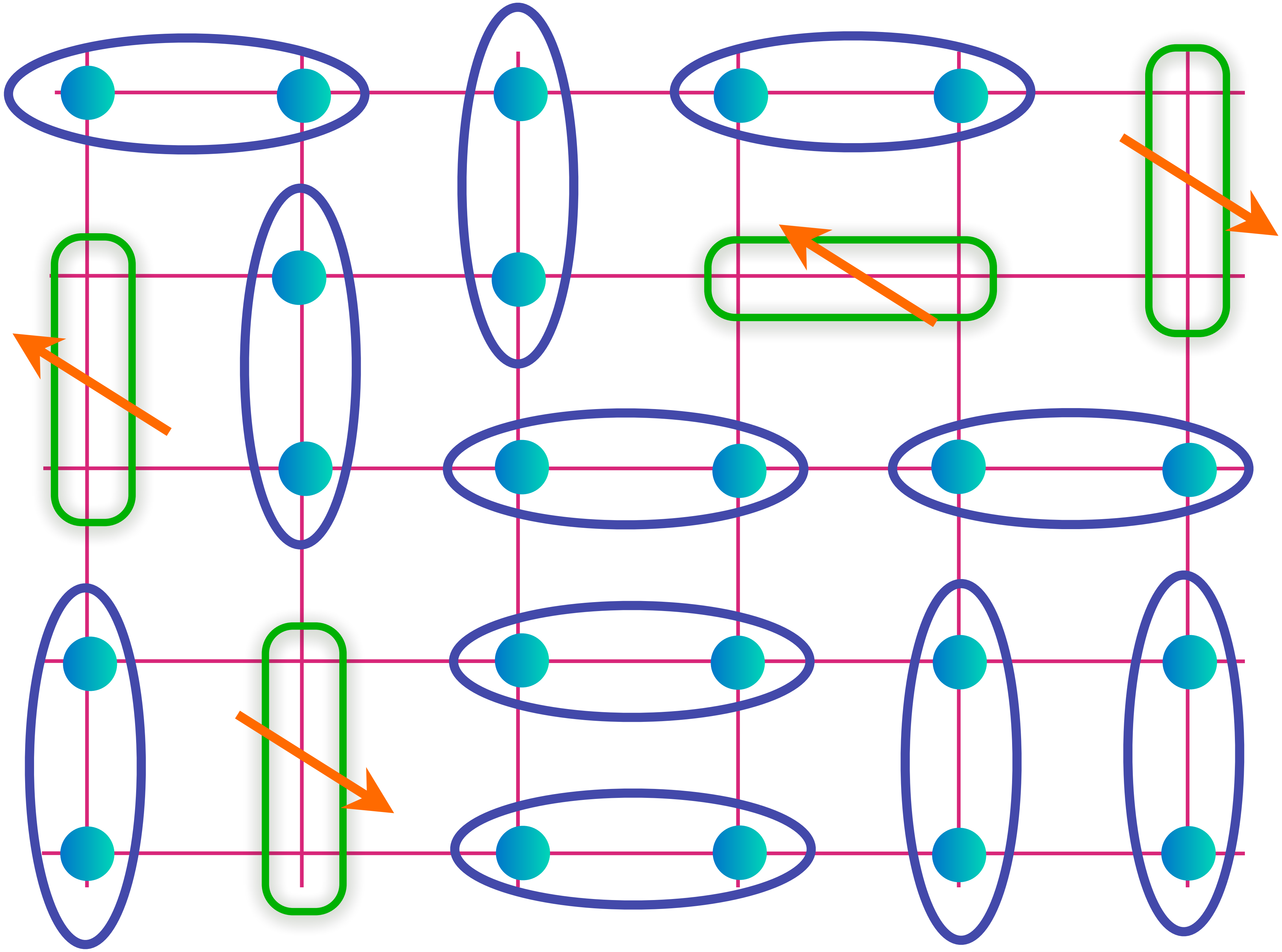
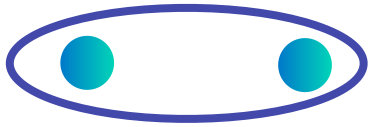
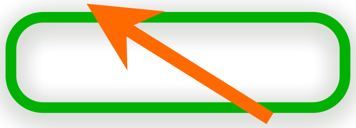


Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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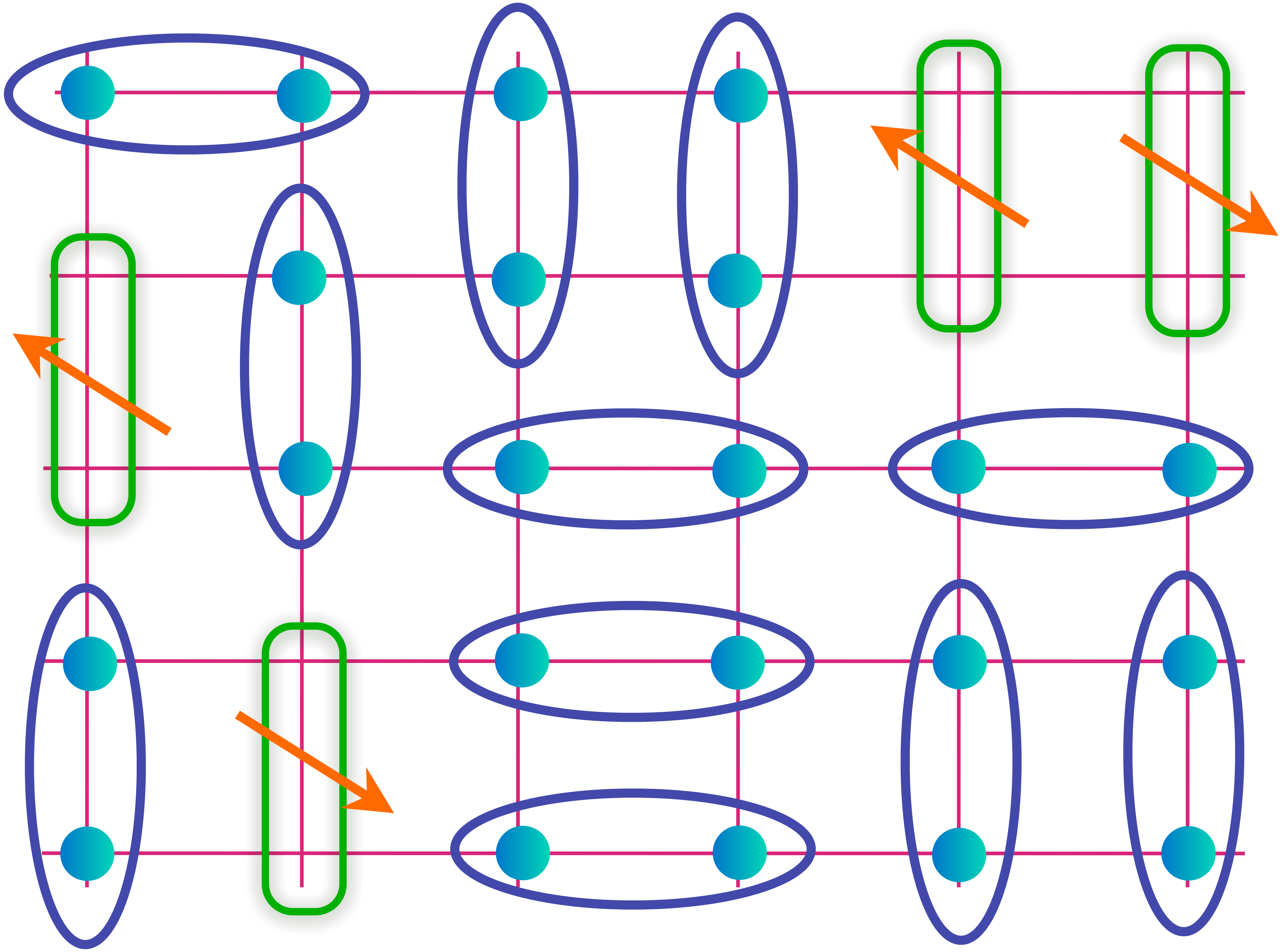
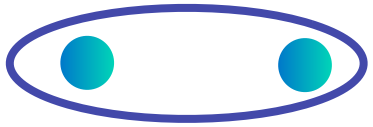
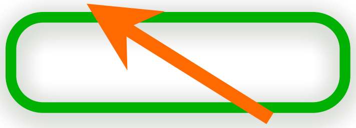


Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields



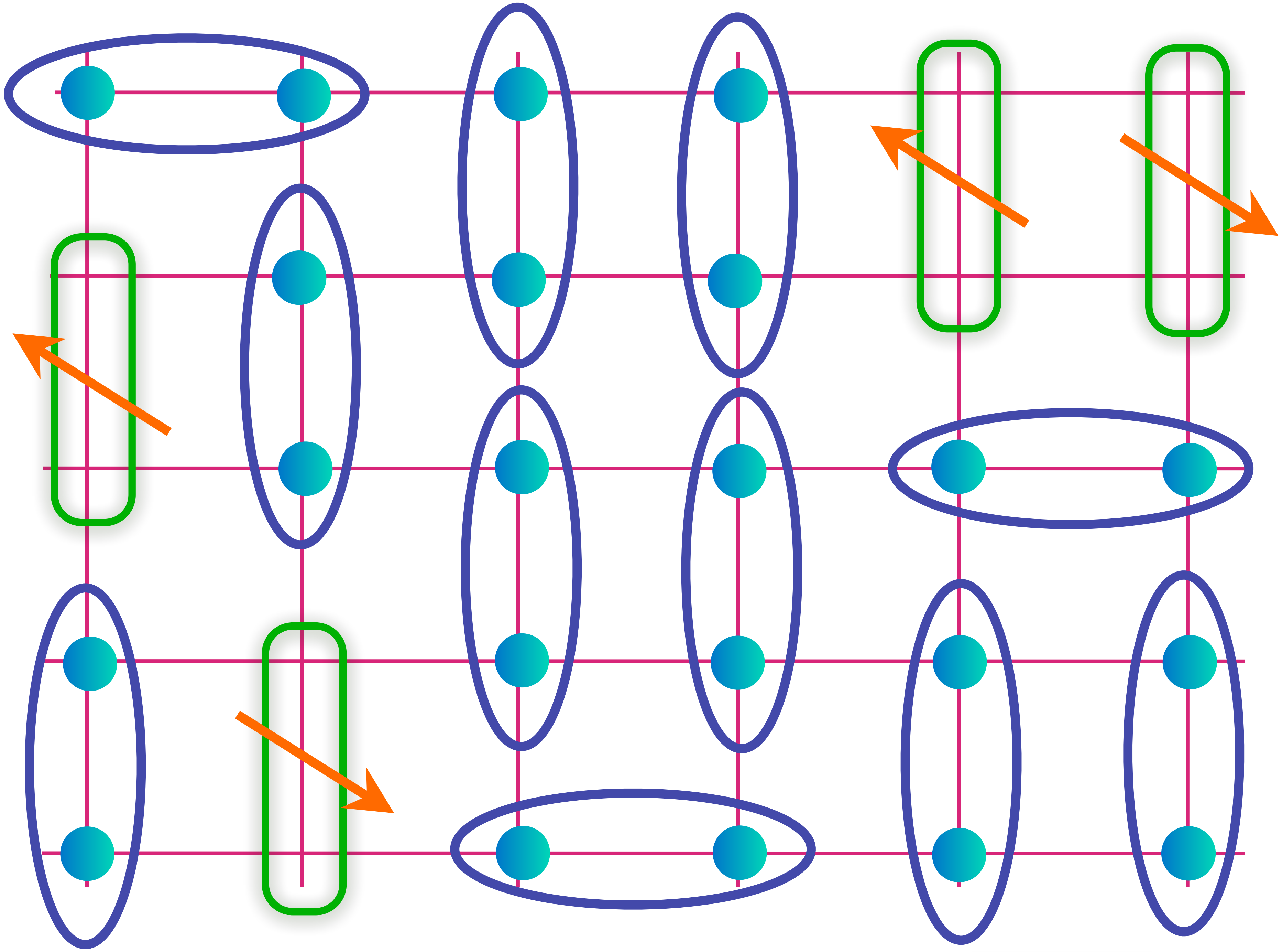
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



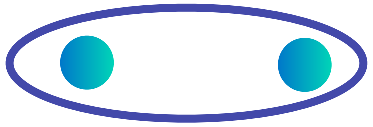
$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

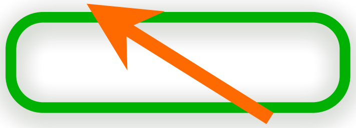
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*



Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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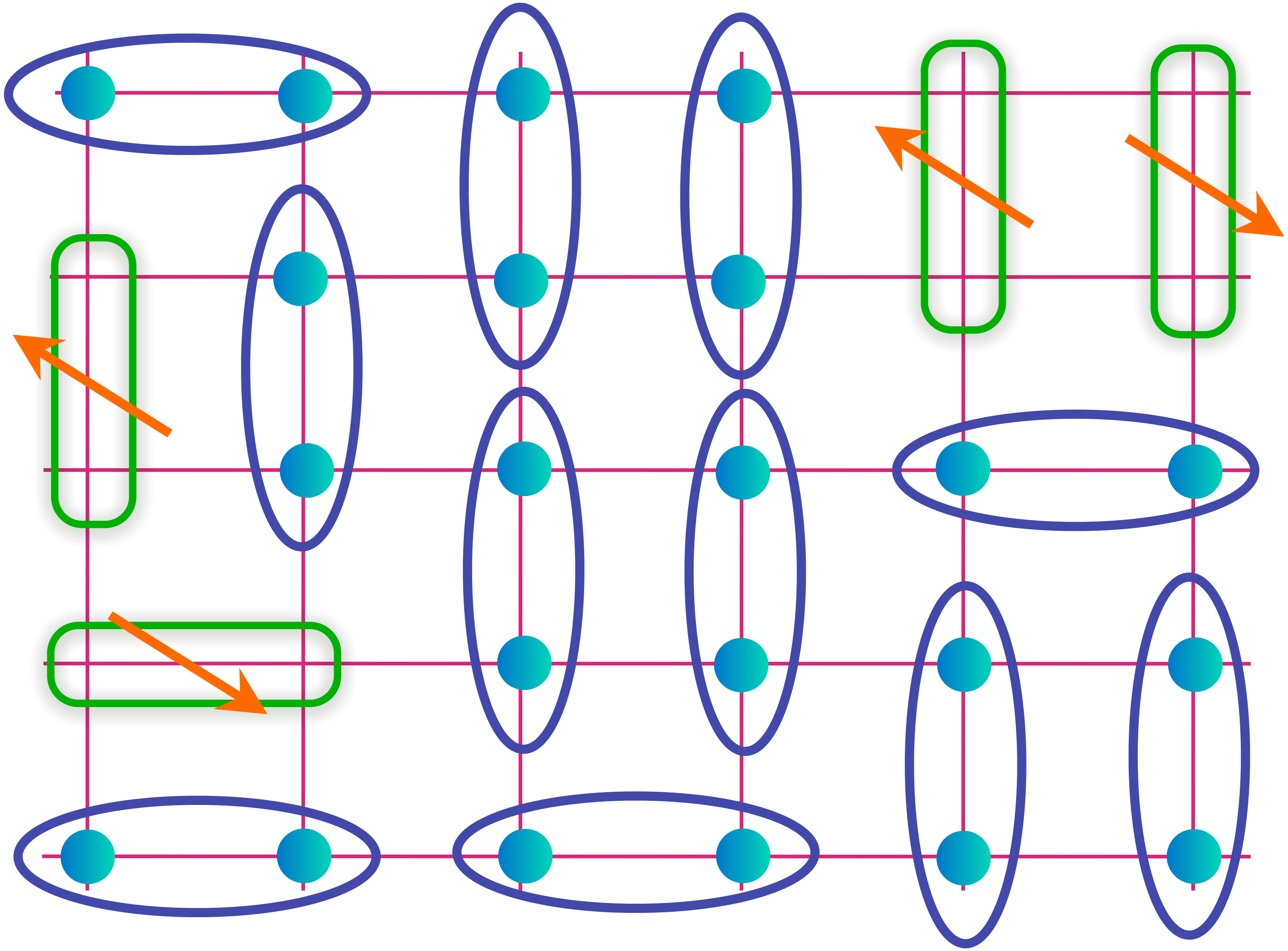

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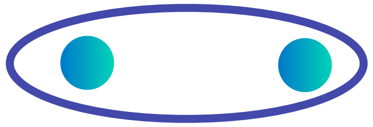
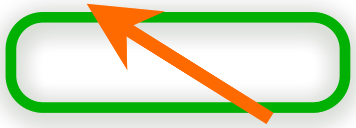


$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Constraints on volume enclosed by the Fermi surface

- In a conventional Fermi liquid state, Fermi volume must equal $(1\text{-}p)\ (\mathrm{mod}\ 2)$.
- When the unit cell is doubled by SDW order, total Fermi volume must equal $(1\text{-}p)\ (\mathrm{mod}\ 1)$.
- A state with Fermi volume $(-p)\ (\mathrm{mod}\ 2)$, but no translational symmetry breaking, must have non-quasiparticle excitations with vanishing energy on a torus *i.e.* emergent gauge fields (bulk topological order)

M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000)
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

1. Emergent gauge fields and topological order in the 3D XY model

2. Electron doped cuprates

   (A) Simple models of metals with intrinsic topological order

   (B) SU(2) gauge theory of fluctuating antiferromagnetism

3. Hole doped cuprates

   SU(2) gauge theory with N_h adjoint Higgs fields

   Higgs-confinement transition to a Fermi liquid

Transforming to a rotating reference frame

We can (exactly) transform the Hubbard model to the "spin-fermion" model:
electrons $c_{i\alpha}$ on the square lattice with dispersion

$$\begin{aligned} \mathcal{H}_c &= -\sum_{i,\rho} t_\rho \left(c^\dagger_{i,\alpha} c_{i+\boldsymbol{v}_\rho,\alpha} + c^\dagger_{i+\boldsymbol{v}_\rho,\alpha} c_{i,\alpha} \right) \\ &\quad -\mu \sum_i c^\dagger_{i,\alpha} c_{i,\alpha} + \mathcal{H}_{\text{int}} \end{aligned}$$

are coupled to a magnetic moment order parameter $\Phi^a(i)$, $a = x, y, z$

$$\begin{aligned} \mathcal{H}_{\text{int}} &= -\lambda \sum_i \Phi^a(i) c^\dagger_{i,\alpha} \sigma^a_{\alpha\beta} c_{i,\beta} + V(\Phi^a) \\ V(\Phi^a) &= s\Phi^a\Phi^a + u\Phi^a\Phi^a\Phi^b\Phi^b \end{aligned}$$

Transforming to a rotating reference frame

For fluctuating antiferromagnetism (spin density waves (SDW)), we transform to a rotating reference frame using the SU(2) rotation R_i

$$\left(\begin{array}{c} c_{i\uparrow} \\ c_{i\downarrow} \end{array} \right) = R_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right),$$

in terms of fermionic "chargons" ψ_s and a **Higgs field** $H^a(i)$

$$\sigma^a \Phi^a(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

The Higgs field is the SDW order in the rotating reference frame.

Transforming to a rotating reference frame

The Higgs field is the SDW order in the rotating reference frame.

$$\sigma^a \Phi^a(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

Note that this is the precise analog of the relationship in the XY model

$$\Psi = H \, \phi^2$$

The two relations take the same form if we write

$$\begin{aligned} \Phi^a &= (\Psi + \Psi^*, -i(\Psi - \Psi^*), 0)/2 \\ H^a &= (H + H^*, -i(H - H^*), 0)/2 \\ R &= \begin{pmatrix} \phi^* & 0 \\ 0 & \phi \end{pmatrix} \end{aligned}$$

Transforming to a rotating reference frame

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the SDW order replaced by the Higgs field.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi^\dagger_{i,s} \psi_{i+\boldsymbol{v}_\rho, s} + \psi^\dagger_{i+\boldsymbol{v}_\rho, s} \psi_{i,s} \right) - \mu \sum_i \psi^\dagger_{i,s} \psi_{i,s} + \mathcal{H}_{\rm int}$$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i H^a(i) \psi^\dagger_{i,s} \sigma^a_{ss'} \psi_{i,s'} + V(H^a)$$

IF we can transform to a rotating reference frame in which $H^a(i) =$ a constant independent of i and time, **THEN** the ψ fermions in the presence of (fluctuating) SDW SRO will inherit the small Fermi surfaces of the electrons in the presence of SDW LRO.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Gauge theory of fluctuating antiferromagnetism

Field	Symbol	Statistics	$\mathrm{SU}(2)_{\mathrm{gauge}}$	$\mathrm{SU}(2)_{\mathrm{spin}}$	$\mathrm{U}(1)_{\mathrm{e.m.charge}}$
Electron	c	fermion	**1**	**2**	-1
AF order	Φ	boson	**1**	**3**	0
Chargon	ψ	fermion	**2**	**1**	-1
Spinon	R or z	boson	$\bar{\mathbf{2}}$	**2**	0
Higgs	H	boson	**3**	**1**	0

Note that the transformation to a rotating reference frame is ambiguous up to a SU(2) gauge transformation, V_i

$$\begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix} \rightarrow V_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix}$$

$$R_i \rightarrow R_i V_i^\dagger$$

$$\sigma^a H^a(i) \rightarrow V_i \, \sigma^b H^b(i) \, V_i^\dagger .$$

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SU(2) gauge theory: fractionalize the SDW order parameter into the Higgs field (H) and the spinons (R); fractionalize the electron (c) into chargons (ψ) and spinons (R). When the Higgs field is condensed, the ψ fermions and R bosons are deconfined particles in an algebraic charge liquid (ACL) state, but with a strong residual attractive interaction from the t_{ij}.

Gauge theory of fluctuating antiferromagnetism

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SU(2) gauge theory: The ACL was used to model the photoemission and the cluster DMFT results in the intermediate phase.

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev,
Proceedings of the National Academy of Sciences **115**, E3665 (2018)

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Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

SU(2) gauge theory for FL^*: Assume the Ψ and R form an electron-like bound state, and develop an effective theory for the Higgs fields and the electrons to describe also the crossover/transition to the Fermi liquid state on the overdoped side.

Gauge theory of fluctuating antiferromagnetism

Taking the continuum limit for the Higgs field:

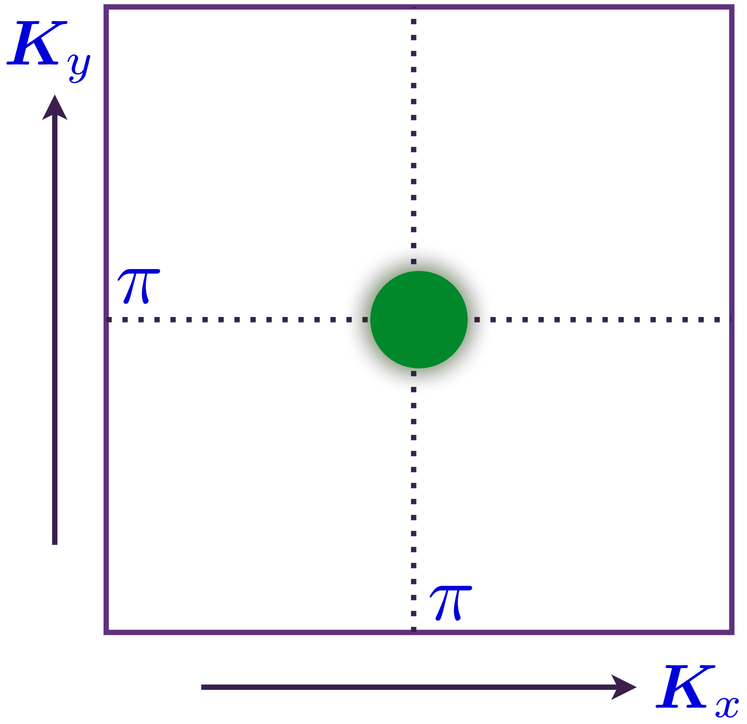
We obtain different numbers of adjoint Higgs scalars, N_h, depending upon the spatial dependence of the local spin correlations:

Neel correlations (electron doped cuprates):

$N_h = 1$,

$\boldsymbol{K} = (\pi, \pi)$,

$H^a(i) = H_1^a(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}_i}$



Optimal doping for electron-doped cuprates

SU(2) gauge theory

SU(2) gauge theory with $N_h = 1$ adjoint real Higgs fields $\mathcal{H}^a$ $(a = 1, 2, 3)$, and gauge-invariant, electron-like fermions c_α with a large Fermi surface.

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \mathcal{H}^a - \epsilon_{abc} A^b_\mu \mathcal{H}^c\right)^2 + \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + V(\mathcal{H}^a)$$

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \epsilon_{abc} A^b_\mu A^c_\nu \\ V(H^a) &= s\, H^a H^a + u_0\, H^a H^a H^b H^b \end{aligned}$$

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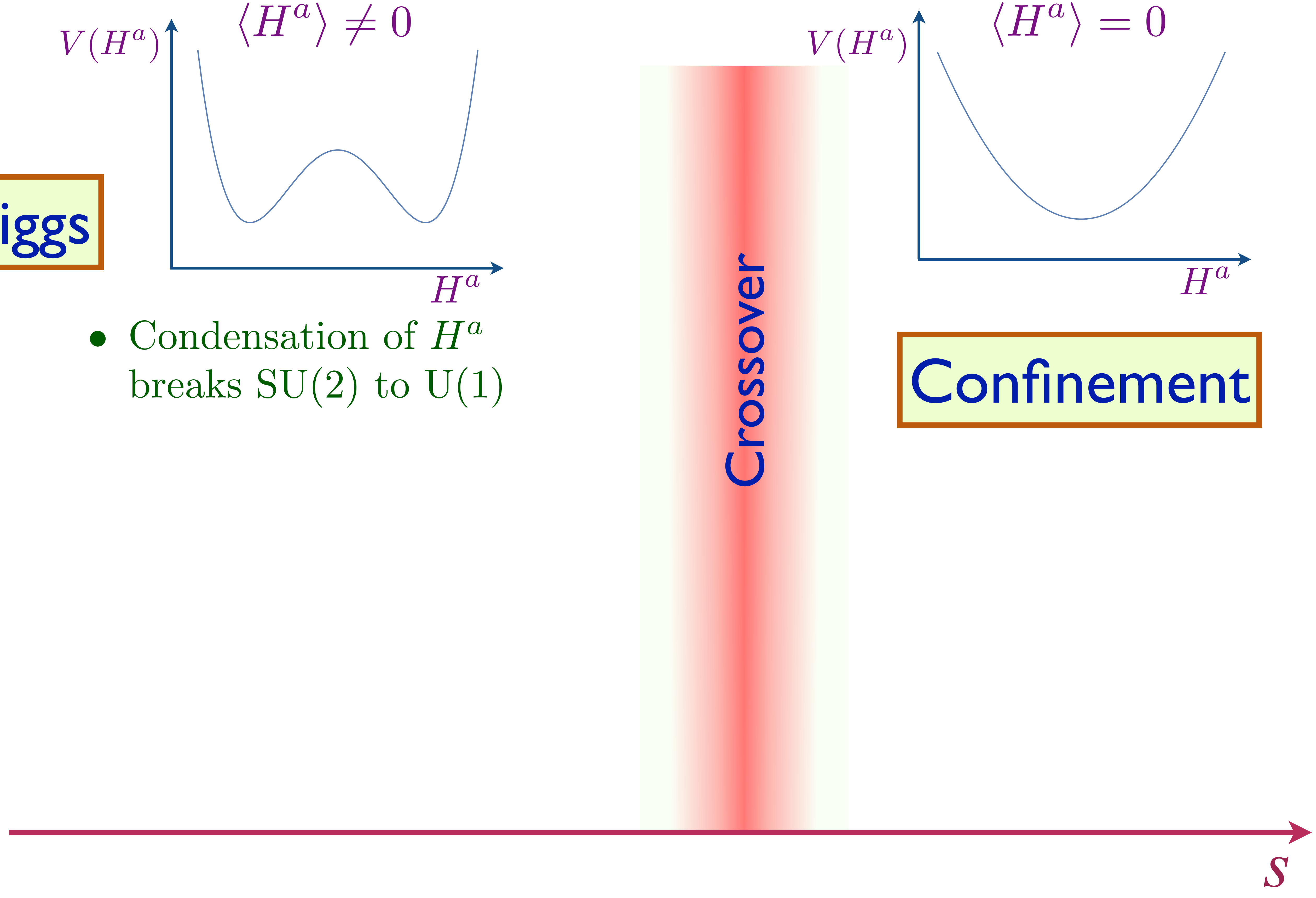
&\quad - \sum_{j,\rho} t_\rho \left(c^\dagger_{j,\alpha} c_{j+\boldsymbol{v}_\rho,\alpha} + c^\dagger_{j+\boldsymbol{v}_\rho,\alpha} c_{j,\alpha} \right) - \mu \sum_j c^\dagger_{j,\alpha} c_{j,\alpha} \\
&\quad + \lambda \sum_j c^\dagger_{j,\alpha} c_{j,\alpha} H^a(j) H^a(j) \\
F^a_{\mu\nu} \quad &= \quad \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \epsilon_{abc} A^b_\mu A^c_\nu \\
V(H^a) \quad &= \quad s\, H^a H^a + u_0\, H^a H^a H^b H^b
\end{aligned}$$

The fermions do not have Yukawa coupling to the Higgs fields, or a minimal coupling to the gauge fields: both are prohibited by gauge invariance. We treat the quartic coupling, λ perturbatively.

$N_h = 1$

Phase diagrams of SU(2) gauge theory



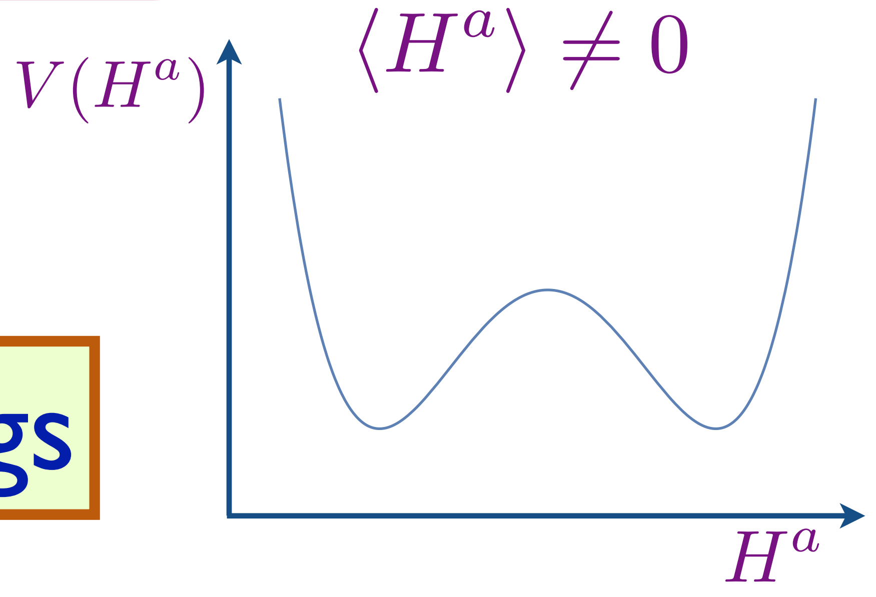
- Condensation of H^a breaks SU(2) to U(1)

A. C. Davis, A. Hart, T.W. B. Kibble, and A. Rajantie, Phys. Rev. D 65, 125008 (2002)

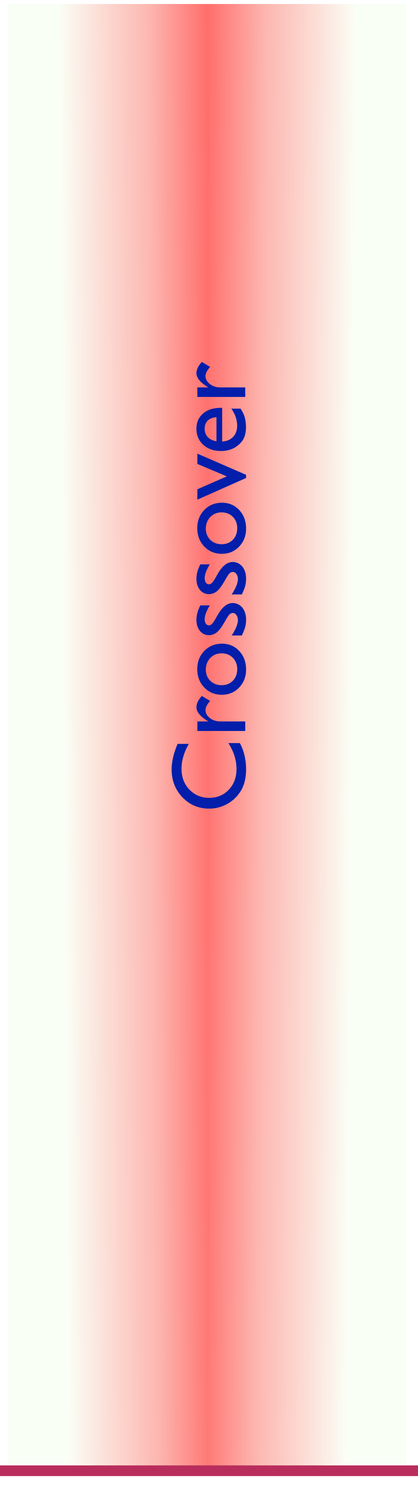
$N_h = 1$

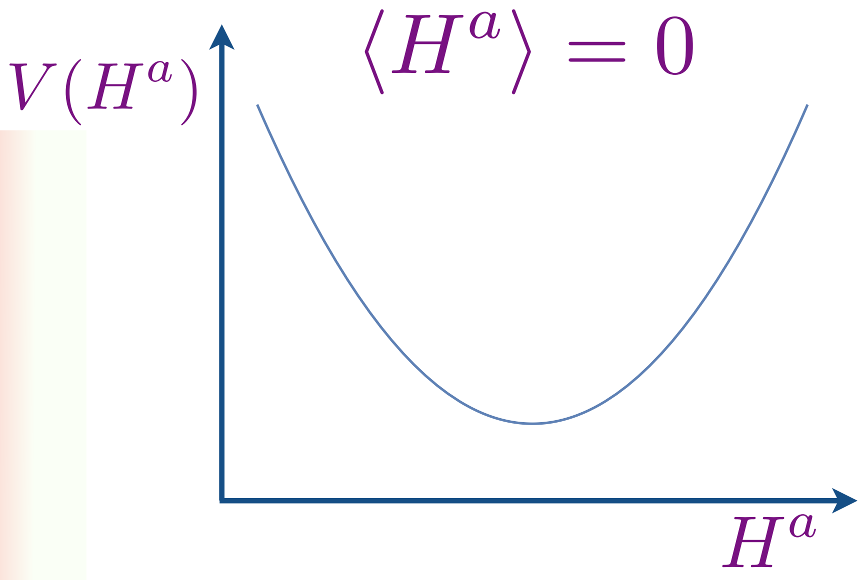
Phase diagrams of SU(2) gauge theory

Higgs



- Condensation of H^a breaks SU(2) to U(1)
- U(1) confines because of proliferation of 'tHooft-Polyakov monopoles





Confinement

s

A. C. Davis, A. Hart, T.W. B. Kibble, and A. Rajantie, Phys. Rev. D 65, 125008 (2002)

$N_h = 1$

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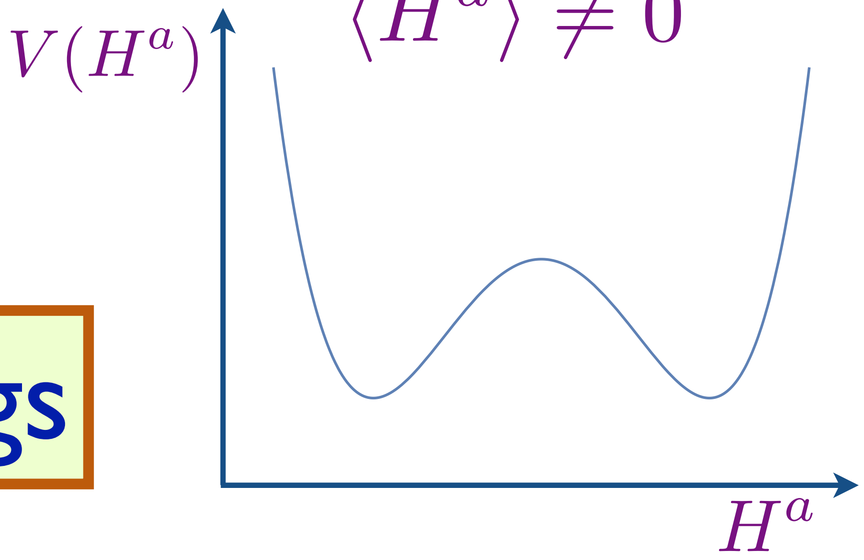
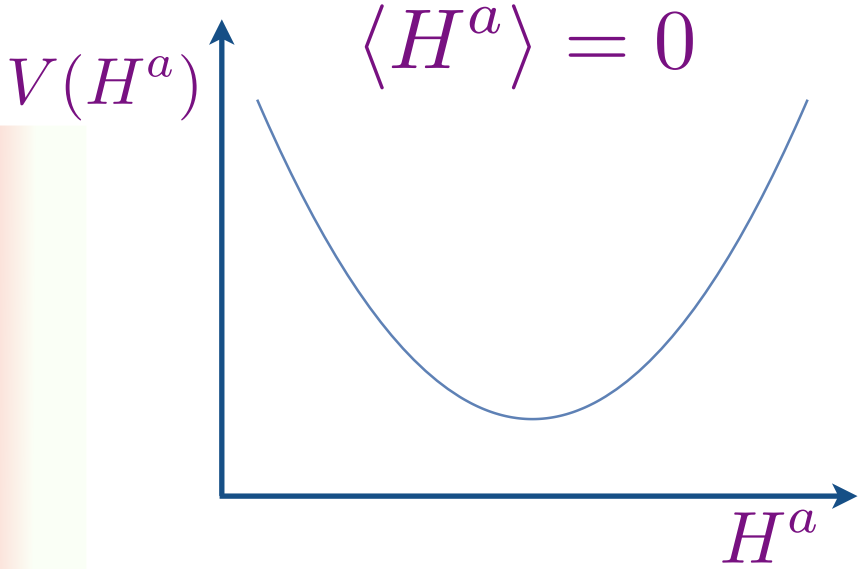


Higgs

- Condensation of H^a breaks SU(2) to U(1)
- U(1) confines because of proliferation of 'tHooft-Polyakov monopoles
- Monopole action $\sim \sqrt{-s}$, leading to an exponentially large confinement scale

Crossover



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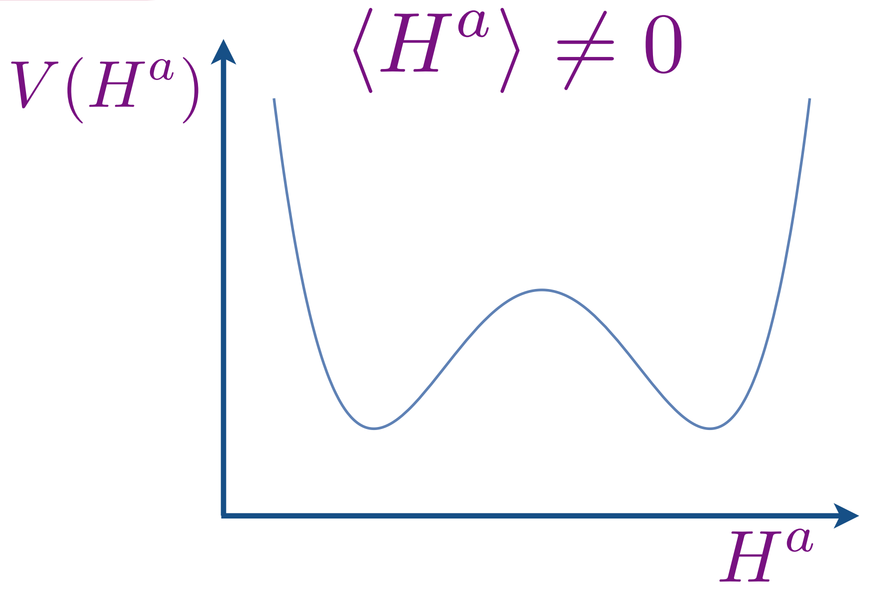

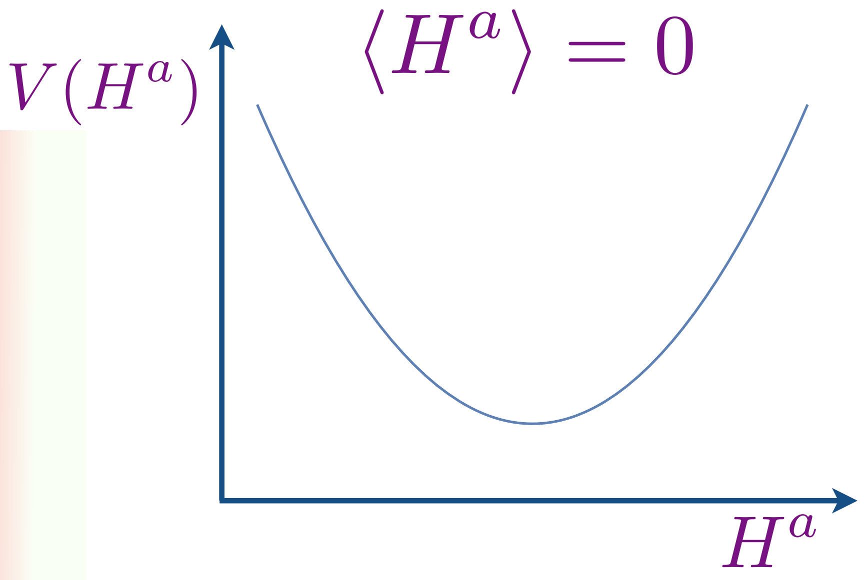

Higgs/U(1) confinement

Reconstructed (FL*) Fermi surfaces, with large length scale confinement in a U(1) gauge theory, leading to re-emergence of large Fermi surface

Crossover

Confinement

Fermi liquid with large Fermi surface

s

A. C. Davis, A. Hart, T. W. B. Kibble, and A. Rajantie, Phys. Rev. D 65, 125008 (2002)

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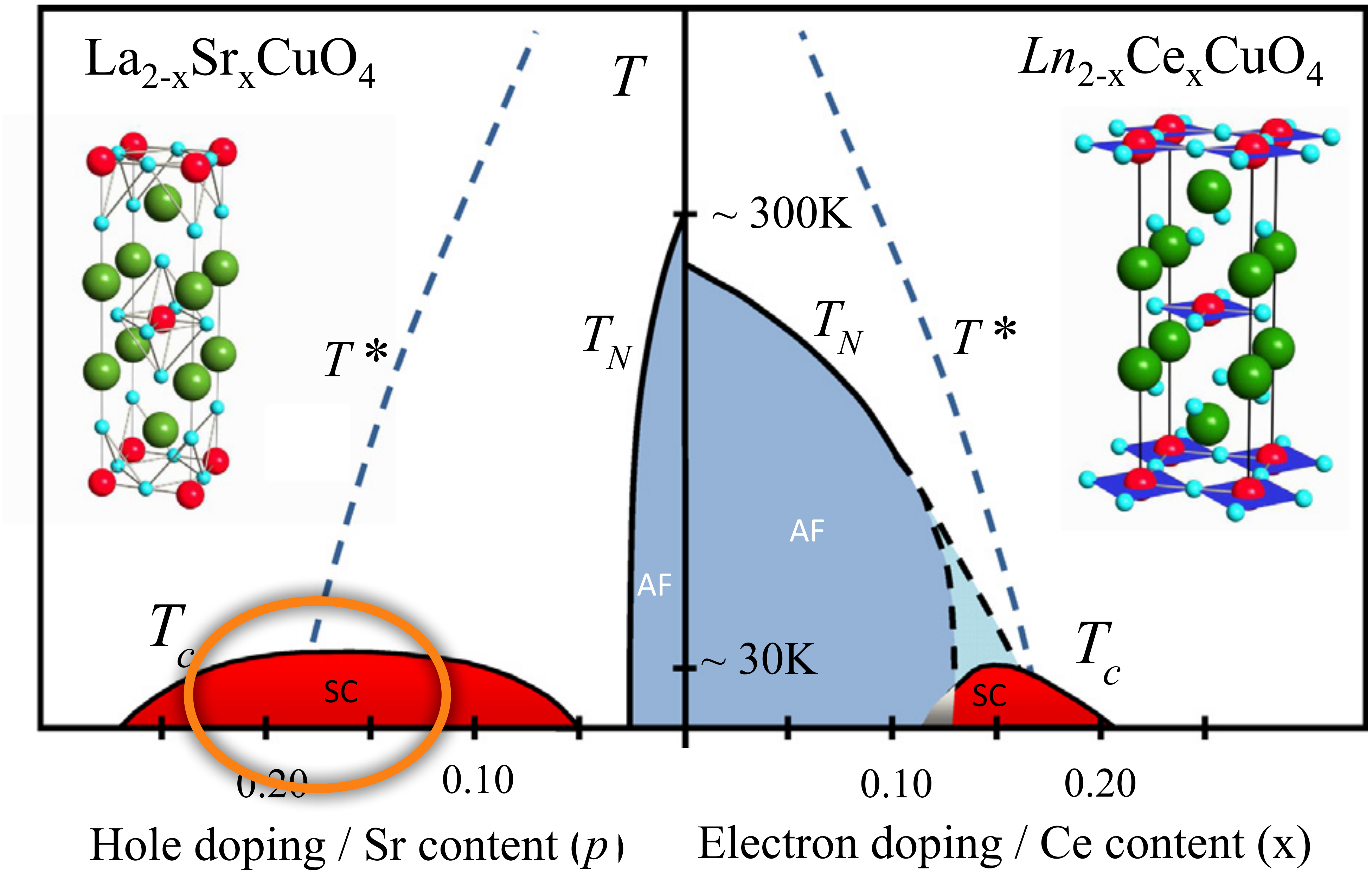
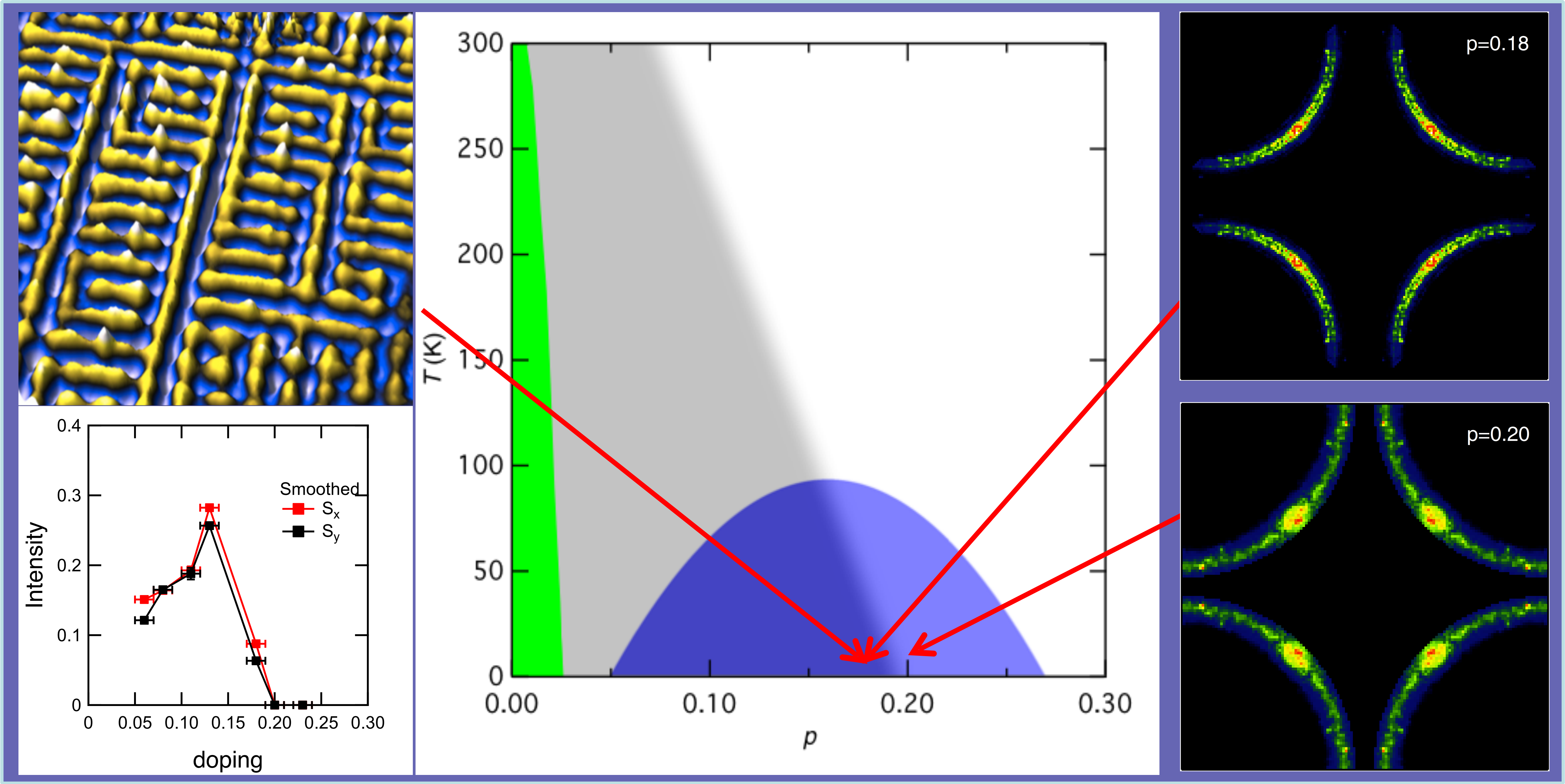

$La_{2-x}Sr_xCuO_4$
T
$Ln_{2-x}Ce_xCuO_4$
~ 300K
T^*
T_N
T_N
T^*
AF
AF
T_c
~ 30K
T_c
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)

Hole doped cuprates

Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, Science **344**, 608 (2014)

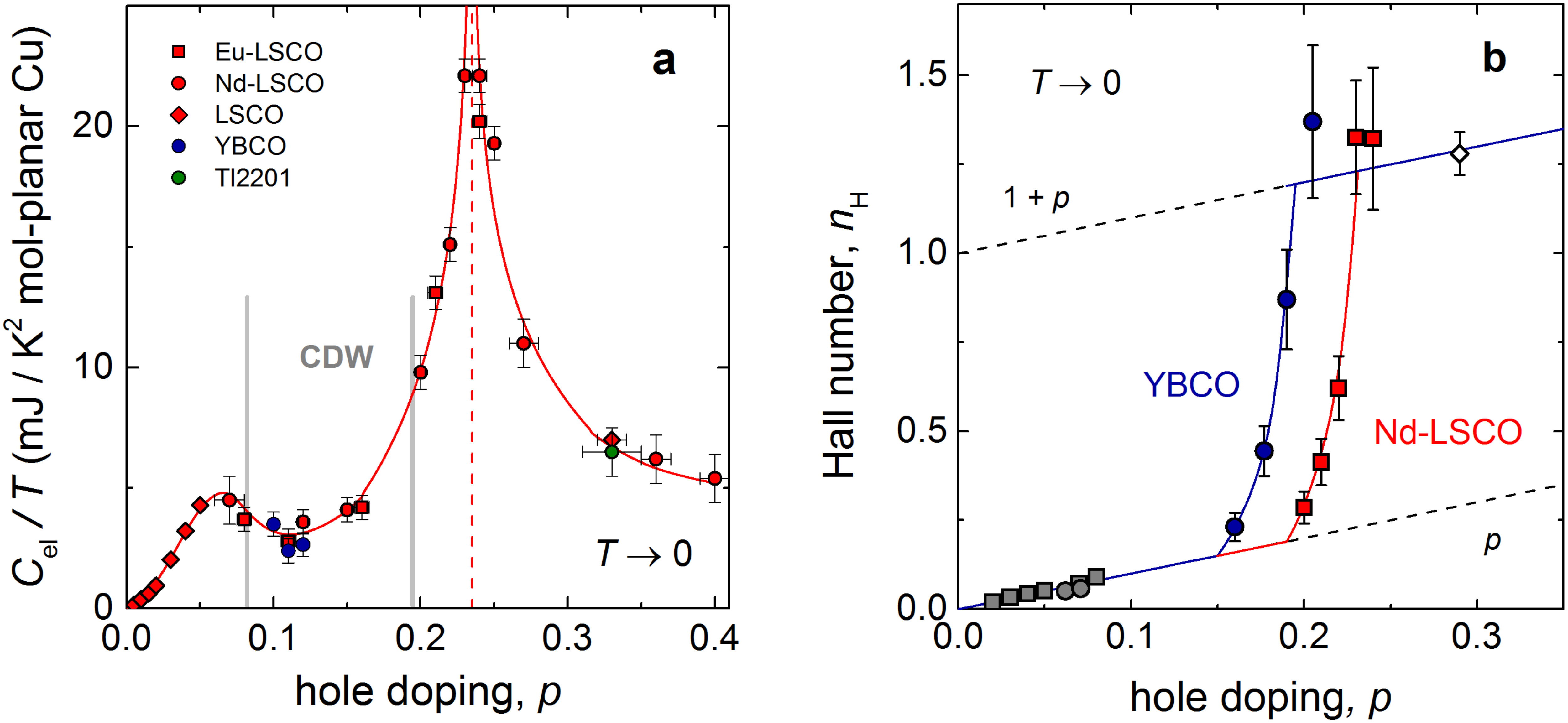
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, Science **344**, 612 (2014)



Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507



Gauge theory of fluctuating antiferromagnetism

Taking the continuum limit for the Higgs field:

We obtain different numbers of adjoint Higgs scalars, N_h, depending upon the spatial dependence of the local spin correlations:

Neel correlations (electron doped cuprates):

$N_h = 1$,

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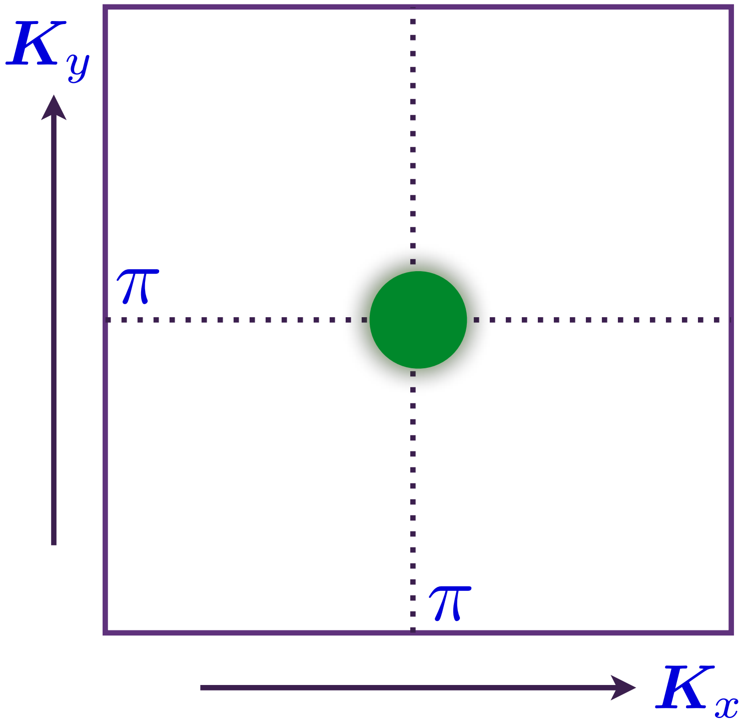

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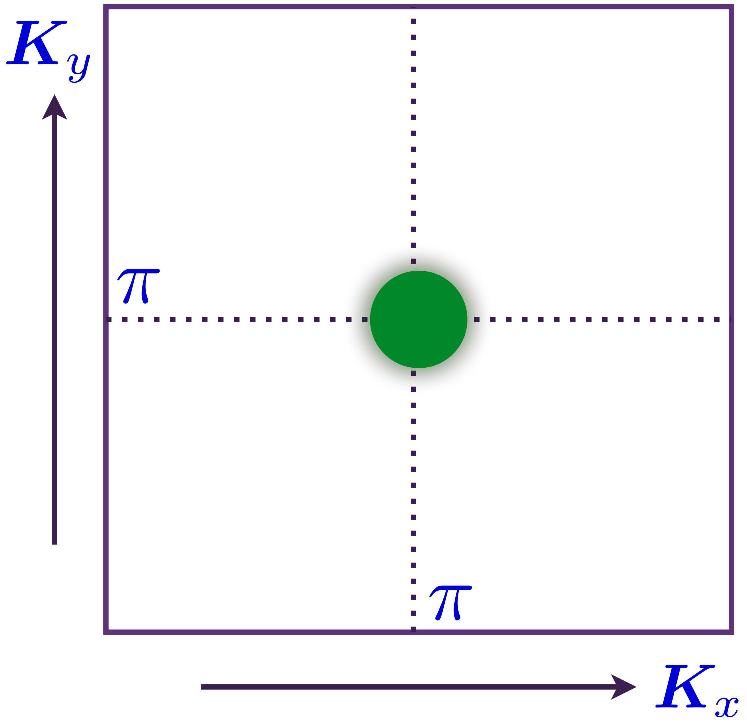

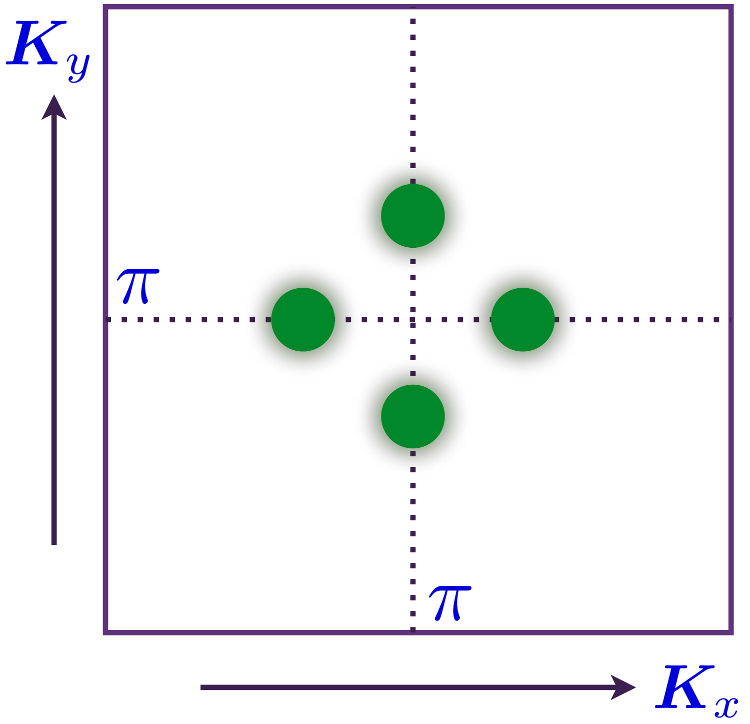

Bidirectional incommensurate correlations (hole doped cuprates):

$N_h = 4$,

$\boldsymbol{K}_y = (\pi, \pi - \delta)$, $\boldsymbol{K}_x = (\pi - \delta, \pi)$,

$$H^a(i) = \mathrm{Re}\left\{\left[H^a_1(\boldsymbol{r}) + iH^a_2(\boldsymbol{r})\right] e^{i\boldsymbol{K}_x\cdot\boldsymbol{r}_i} + \left[H^a_3(\boldsymbol{r}) + iH^a_4(\boldsymbol{r})\right] e^{i\boldsymbol{K}_y\cdot\boldsymbol{r}_i}\right\}$$

Optimal doping for hole-doped cuprates

SU(2) gauge theory

For the hole-doped cuprates, $N_h = 4$, we define complex Higgs fields

$$\mathcal{H}_x^a = H_1^a + i H_2^a \quad , \quad \mathcal{H}_y^a = H_3^a + i H_4^a \,.$$

The SU(2) gauge theory is

$$\mathcal{L} = \frac{1}{2}\left|\partial_\mu \mathcal{H}_x^a - \epsilon_{abc} A_\mu^b \mathcal{H}_x^c\right|^2 + \frac{1}{2}\left|\partial_\mu \mathcal{H}_y^a - \epsilon_{abc} A_\mu^b \mathcal{H}_y^c\right|^2 + \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a$$

$$F_{\mu\nu}^a \quad = \quad \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \epsilon_{abc} A_\mu^b A_\nu^c$$

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$$+ V(\mathcal{H}_{x,y}^a) + \text{coupling to electrons with large Fermi surface}$$

$$\begin{aligned} F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \epsilon_{abc} A_\mu^b A_\nu^c \\ V(\mathcal{H}_{x,y}^a) &= s\left(\mathcal{H}_x^{a*}\mathcal{H}_x^a + \mathcal{H}_y^{a*}\mathcal{H}_y^a\right) + u_0\left(\mathcal{H}_x^{a*}\mathcal{H}_x^a + \mathcal{H}_y^{a*}\mathcal{H}_y^a\right)^2 \\ & \frac{u_1}{4}\left(\mathcal{H}_x^{a*}\mathcal{H}_x^a - \mathcal{H}_y^{a*}\mathcal{H}_y^a\right)^2 + \frac{u_2}{2}\left[\left|\mathcal{H}_x^a\mathcal{H}_x^a\right|^2 + \left|\mathcal{H}_y^a\mathcal{H}_y^a\right|^2\right] \\ & + u_3\left(\left|\mathcal{H}_x^a\mathcal{H}_y^a\right|^2 + \left|\mathcal{H}_x^a\mathcal{H}_y^{a*}\right|^2\right) \, . \end{aligned}$$

SU(2) gauge theory (simplified)

SU(2) gauge theory with N_h adjoint Higgs fields H^a_ℓ $(a = 1, 2, 3,\ \ell = 1 \ldots N_h)$, with potential $V(H^a_\ell)$ and SU(2) gauge field A^a_μ.

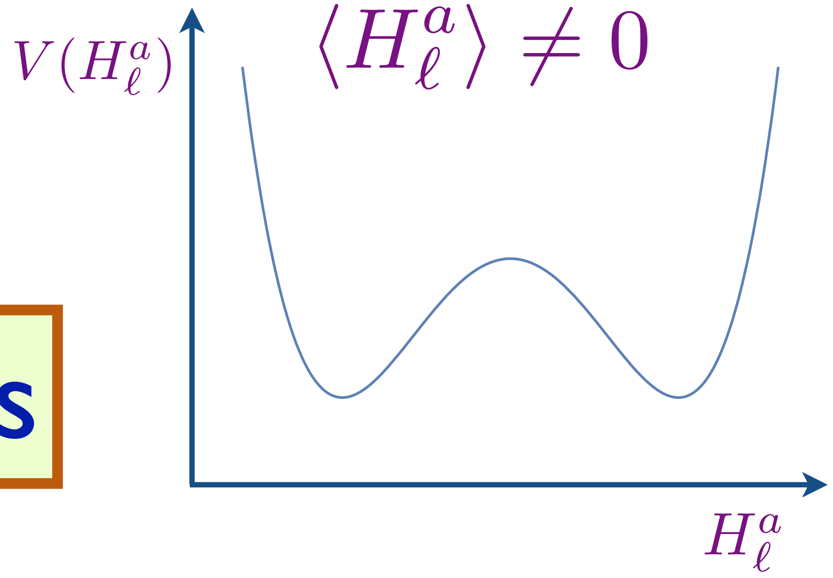
Consider the simpler case with O(N_h) global flavor symmetry.

$$\mathcal{L} = \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} \left(\partial_\mu H^a_\ell - \epsilon_{abc} A^b_\mu H^c_\ell \right)^2 + V(H^a_\ell)$$

$$\begin{aligned}
F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \epsilon_{abc} A^b_\mu A^c_\nu \\
V(H^a_\ell) &= s\, H^a_\ell H^a_\ell + u_0\, H^a_\ell H^a_\ell H^b_m H^b_m \\
&\quad + u_1 \left(H^a_\ell H^a_m H^b_\ell H^b_m - \frac{1}{N_h} H^a_\ell H^a_\ell H^b_m H^b_m \right)
\end{aligned}$$

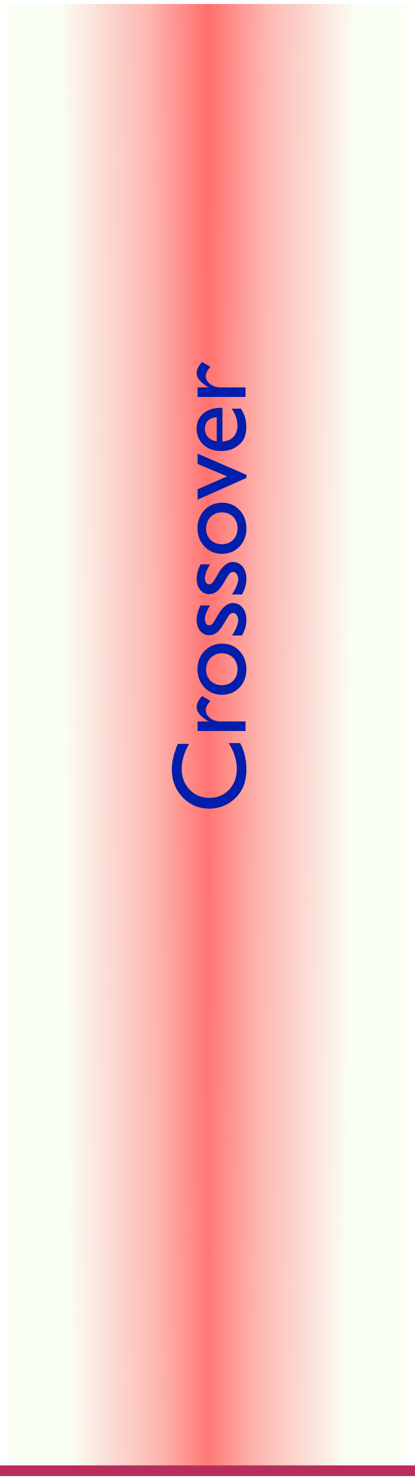
$N_h = 1$

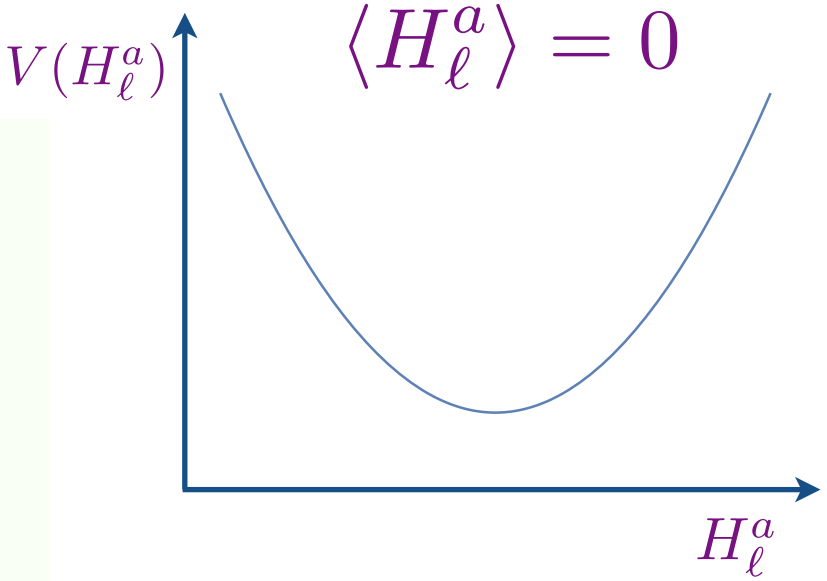
Phase diagrams of SU(2) gauge theory



Higgs

- Condensation of H^a breaks SU(2) to U(1)
- U(1) confines because of proliferation of 'tHooft-Polyakov monopoles
- Monopole action $\sim \sqrt{-s}$, leading to an exponentially large confinement scale





Confinement

s

A. C. Davis, A. Hart, T.W. B. Kibble, and A. Rajantie, Phys. Rev. D 65, 125008 (2002)

$N_h = 2$

- There is a global $\mathrm{O}(N_h)$ symmetry, and we can define a gauge-invariant order parameter

$$Q_{\ell m} = H^a_\ell H^a_m - \frac{\delta_{\ell m}}{N_h} H^a_n H^a_n$$

- For $u_1 < 0$, the Higgs condensate is of the form

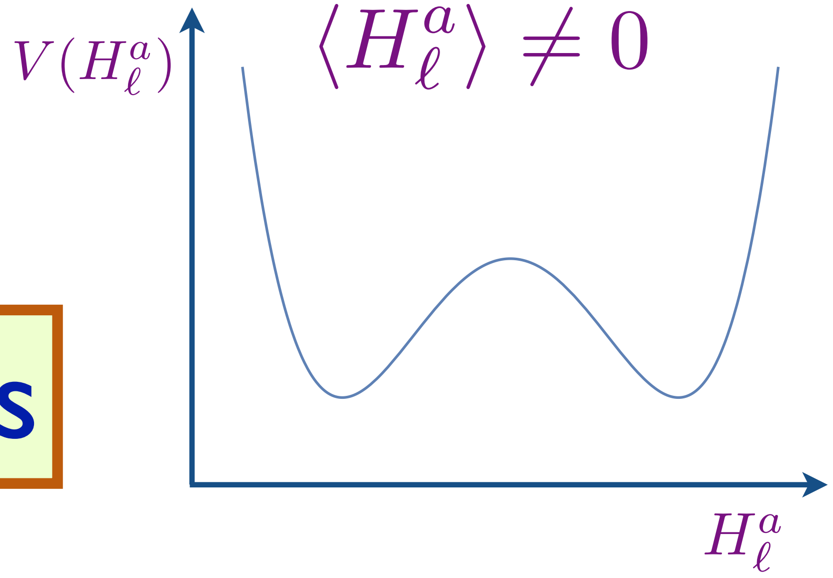
$$H^a_1 = (H_0, 0, 0) \quad , \quad H^a_2 = (0, 0, 0)$$

This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1$. But the global O(2) is broken because

$$\langle Q_{\ell m} \rangle \neq 0$$

$N_h = 2$

Phase diagrams of SU(2) gauge theory

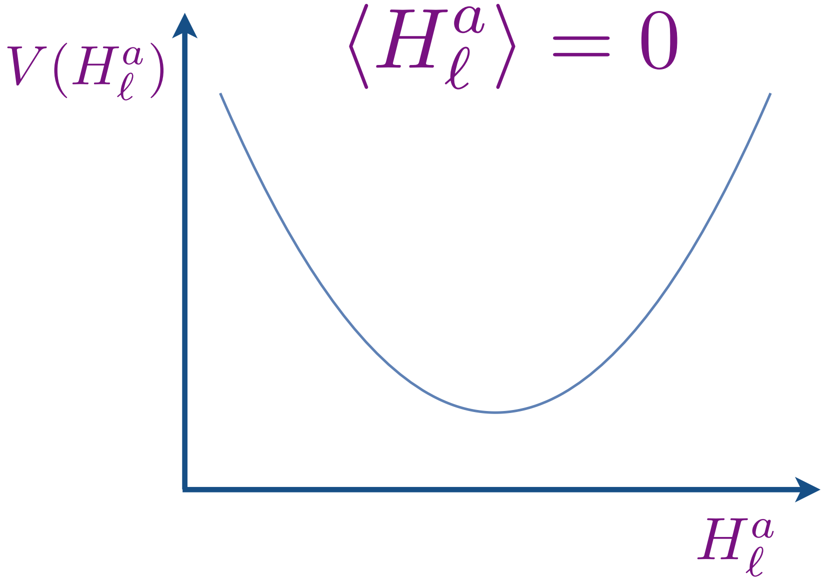


Higgs

$u_1 < 0$

- Condensation of H^a breaks SU(2) to U(1), which confines
- Broken O(2) symmetry, $\langle Q_{\ell m} \rangle \neq 0$.

Possible Deconfined critical SU(2) gauge theory



Confinement

s

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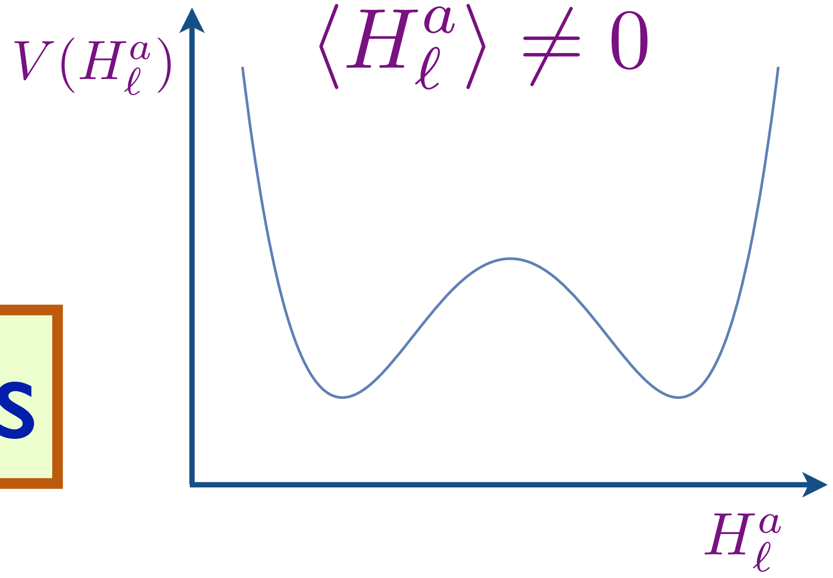
Phase diagrams of SU(2) gauge theory



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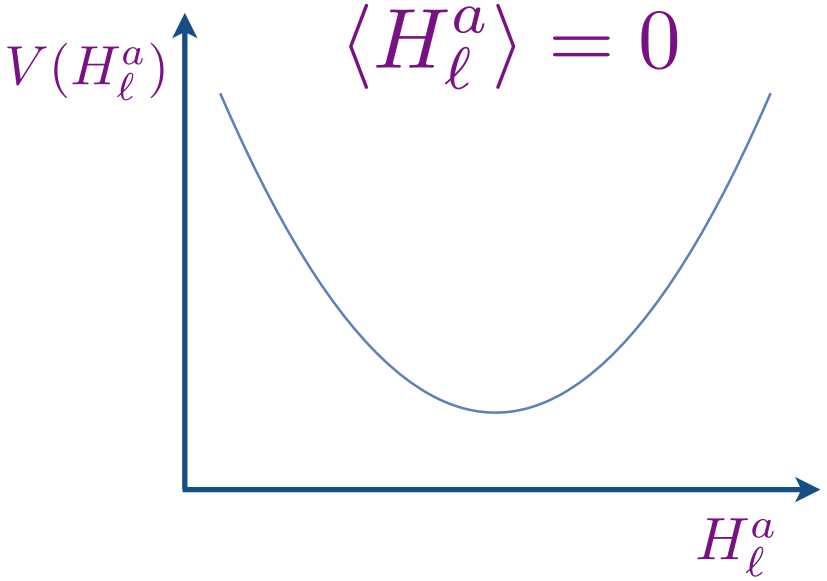

Confinement

Multi-universality of a Landau-allowed transition ?

s

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$$H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, H_0, 0)$$

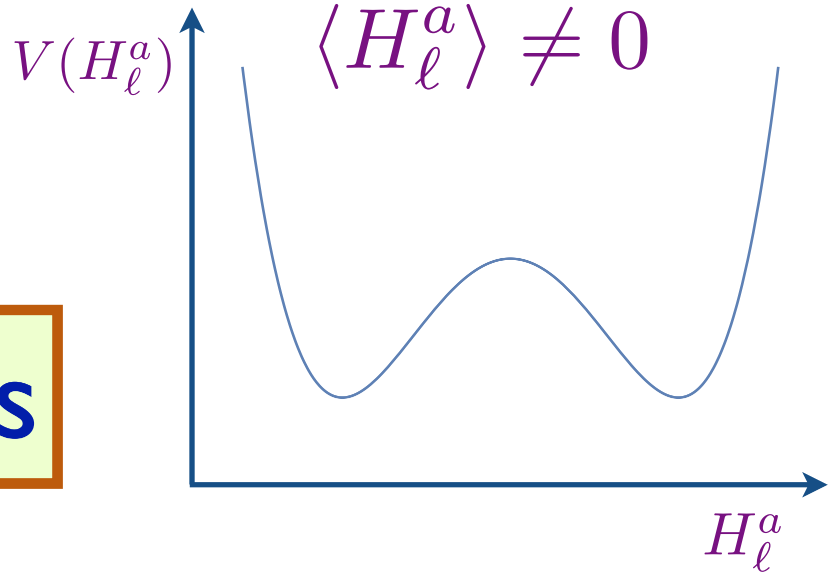
  This breaks gauge SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order (as in the ‘toric code’). But now the global O(2) is unbroken because

$$\langle Q_{\ell m} \rangle = 0$$

$N_h = 2$

Phase diagrams of SU(2) gauge theory

Higgs



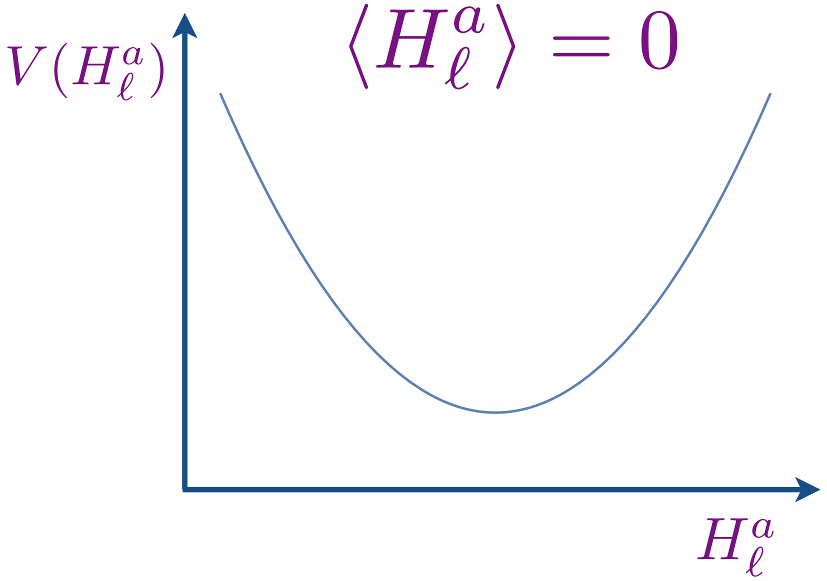
$\langle H^a_\ell \rangle \neq 0$

$u_1 > 0$

- Condensation of H^a breaks SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order.
- Unbroken O(2) symmetry, $\langle Q_{\ell m} \rangle = 0$.

Possible Deconfined critical SU(2) gauge theory

Confinement



$\langle H^a_\ell \rangle = 0$

s

$N_h = 3$

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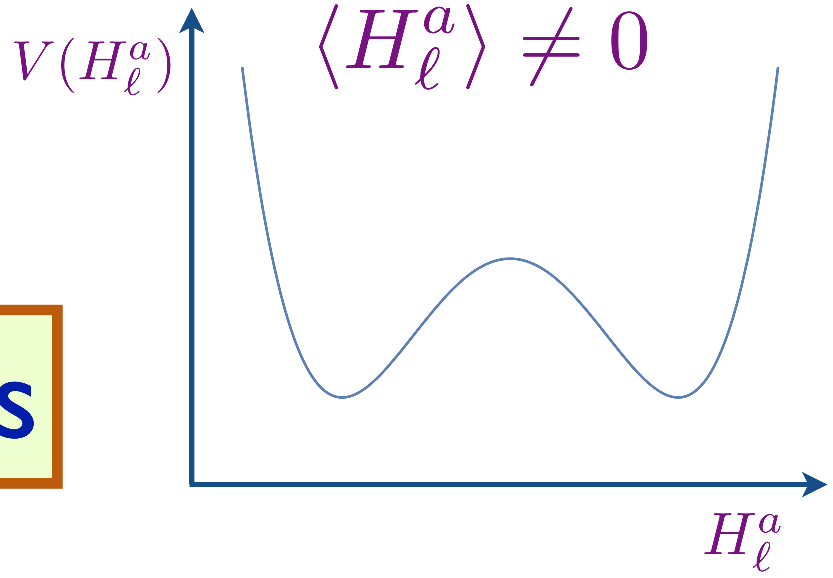
Phase diagrams of SU(2) gauge theory



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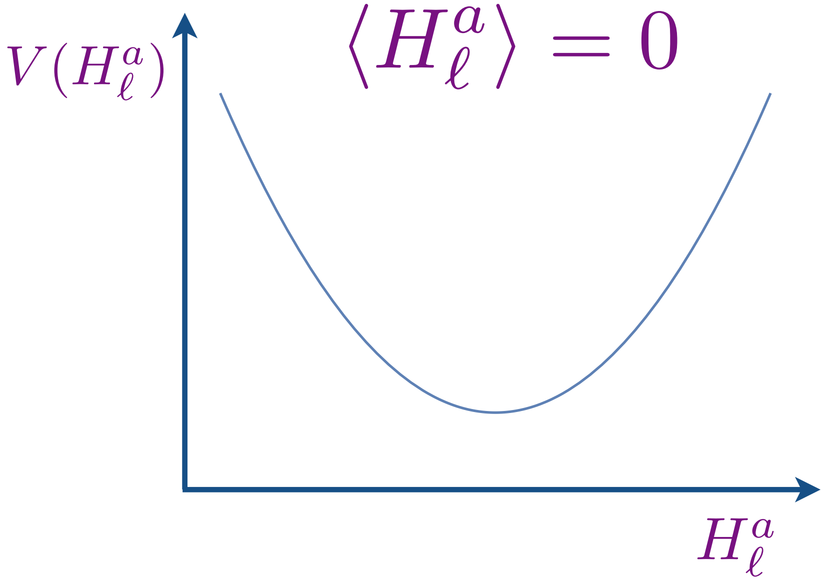

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Confinement

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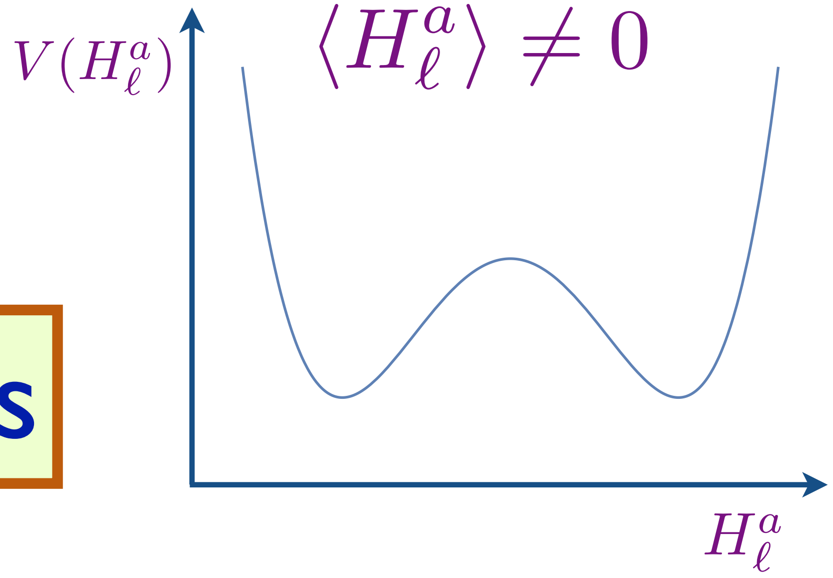
Phase diagrams of SU(2) gauge theory



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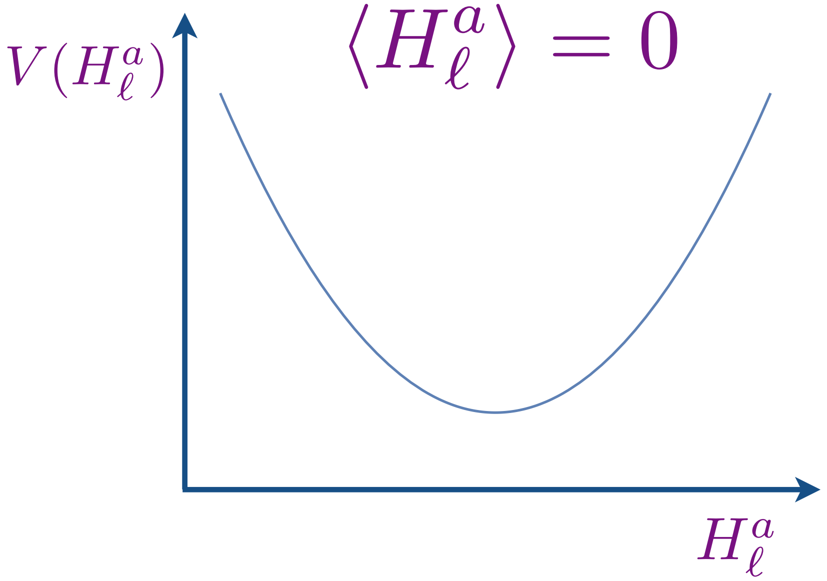

Confinement

s

$\boxed{N_h \geq 4}$

$N_h \geq 4$

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  This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1, 2$. The global O(N_h) is broken to O($N_h - 1$) and

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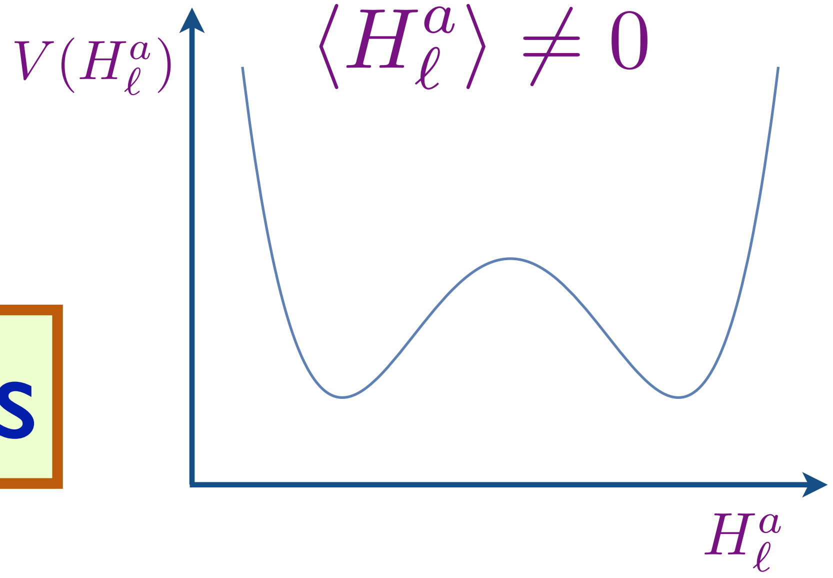
$N_h \geq 4$

Phase diagrams of SU(2) gauge theory

Higgs



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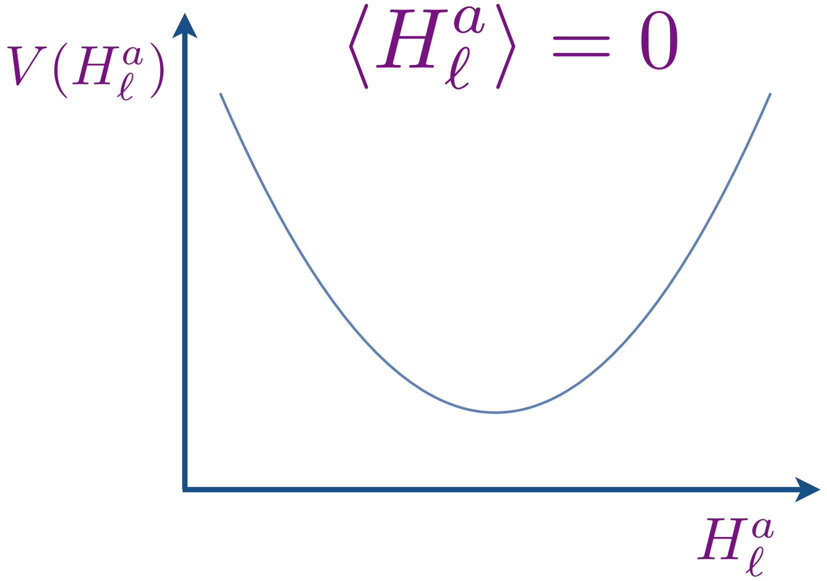
Confinement



Possible Deconfined critical SU(2) gauge theory

Multi-universality of a Landau-allowed transition ?

s

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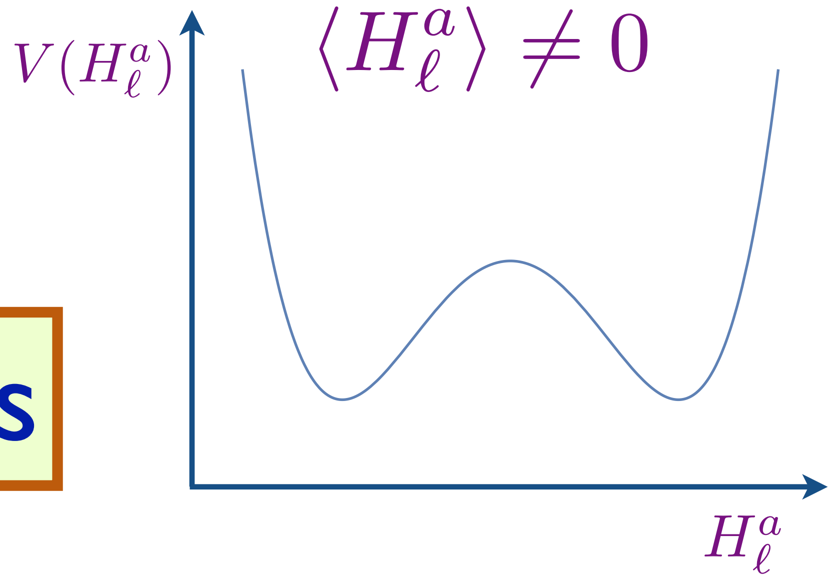
Phase diagrams of SU(2) gauge theory

Higgs



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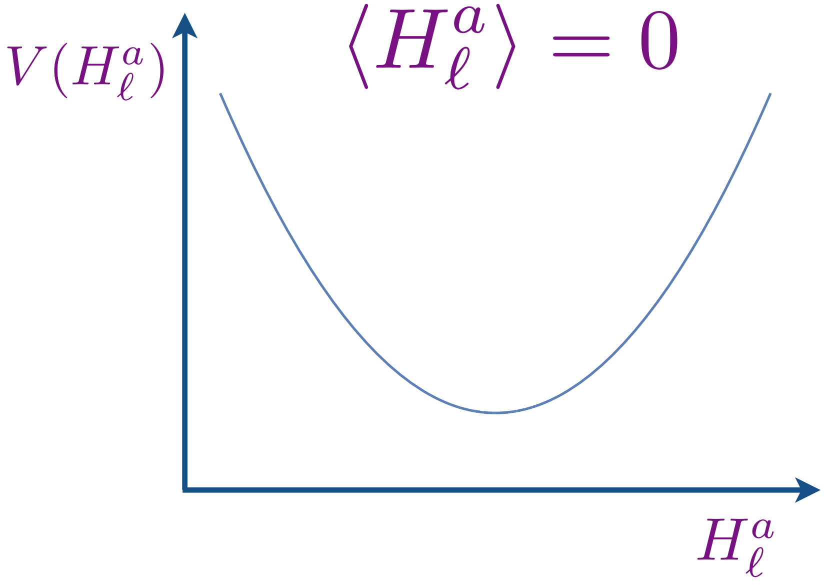

Confinement

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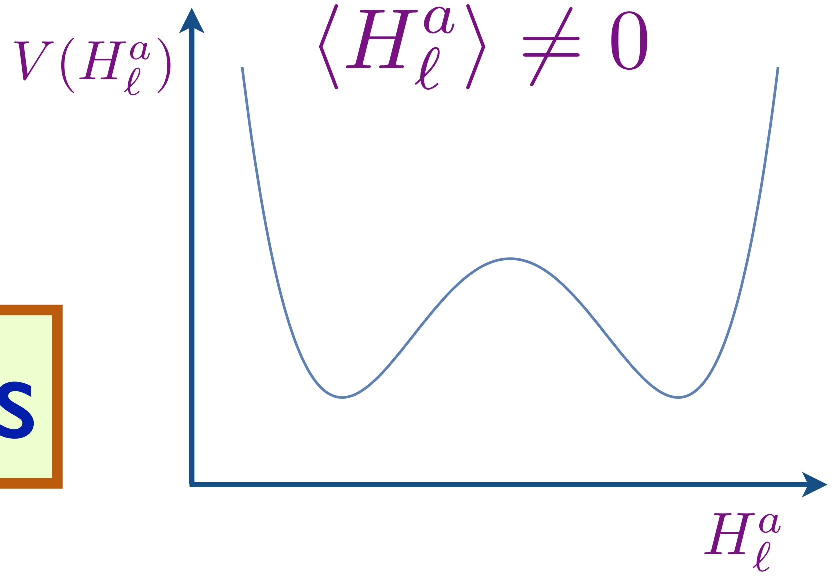
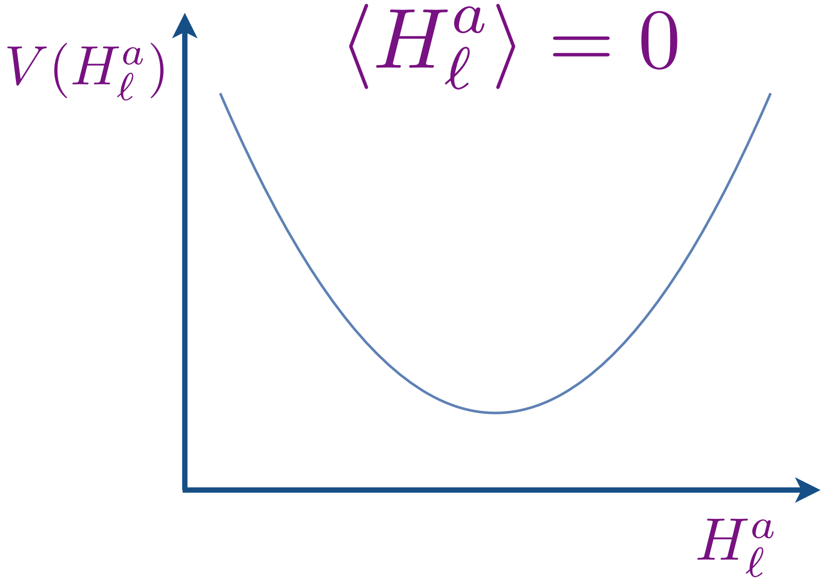
$N_h = 4$

Phase diagrams of SU(2) gauge theory

Higgs



- Condensation of H^a breaks $\mathrm{SU}(2)$ to $\mathrm{U}(1)$ or $\mathbb{Z}_2$.

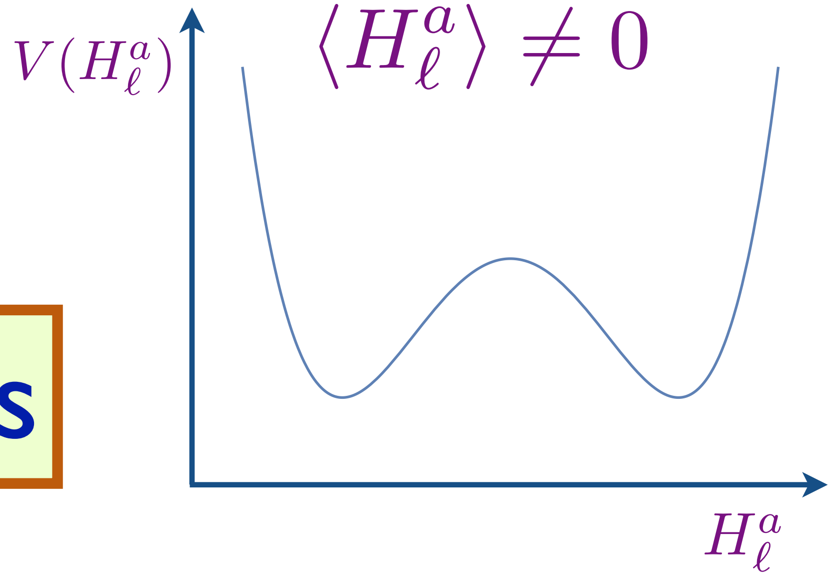


Confinement

Possible Deconfined critical SU(2) gauge theory

$N_h = 4$

Phase diagrams of SU(2) gauge theory



Higgs

- Condensation of H^a breaks $\mathrm{SU}(2)$ to $\mathrm{U}(1)$ or $\mathbb{Z}_2$.
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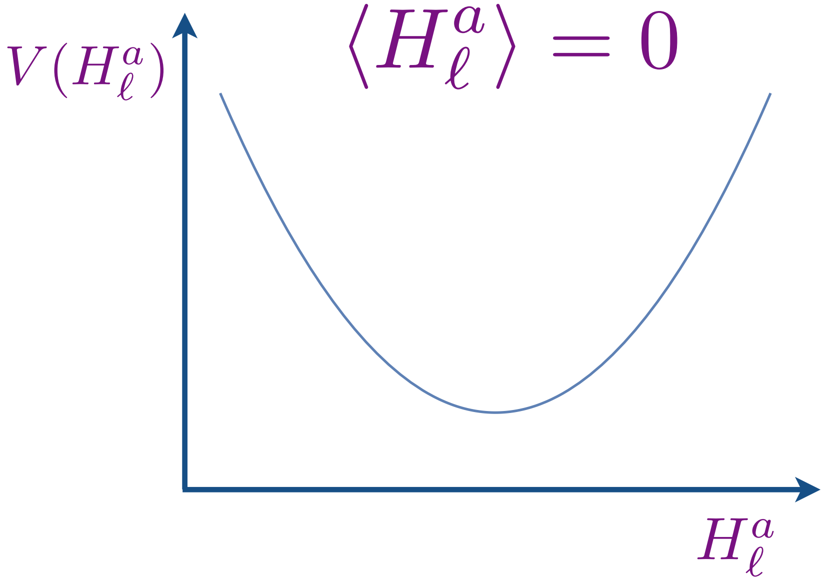
Possible Deconfined critical SU(2) gauge theory

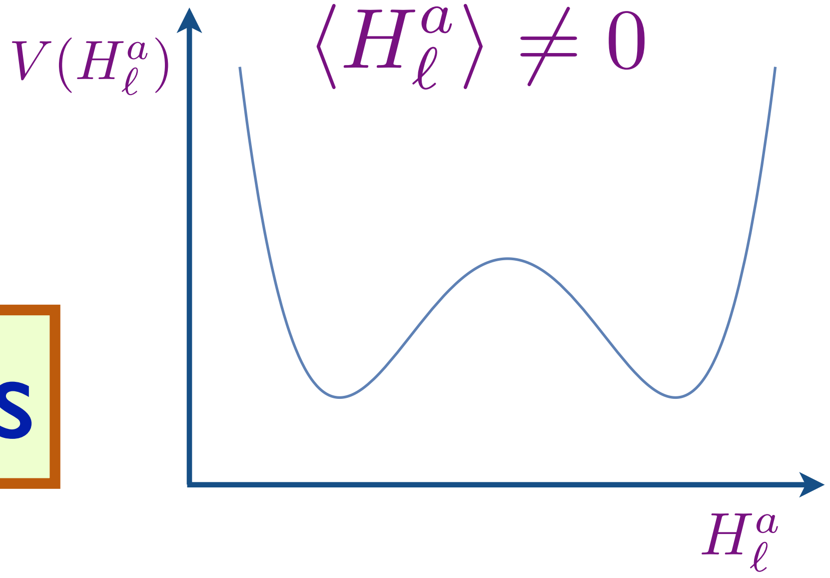


Confinement

s

$N_h = 4$

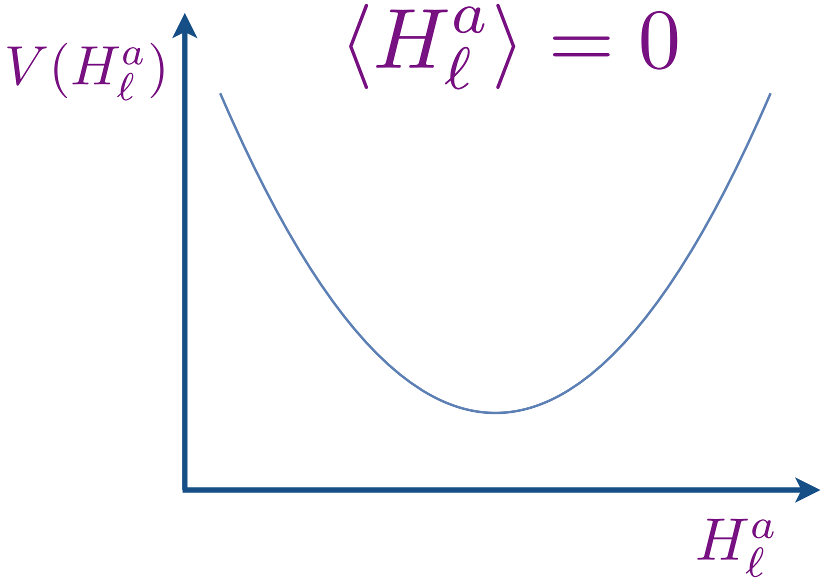
Phase diagrams of SU(2) gauge theory



Higgs

- Condensation of H^a breaks SU(2) to U(1) or $\mathbb{Z}_2$.
- The U(1) cases confine, while the $\mathbb{Z}_2$ cases have $\mathbb{Z}_2$ (toric code) topological order.
- One or more global symmetries are broken in all cases.

Possible Deconfined critical SU(2) gauge theory



Confinement

s

Optimal doping for hole-doped cuprates

There are multiple order parameters for different broken symmetries (Note: spin rotations are preserved and there is no SDW order)

- Ising nematic order

$$\phi = \mathcal{H}_x^{a*}\mathcal{H}_x^a - \mathcal{H}_y^{a*}\mathcal{H}_y^a$$

- Charge density wave (CDW) order at wavevectors $2\boldsymbol{K}_{x,y}$

$$\Phi_x = \mathcal{H}_x^a\mathcal{H}_x^a \quad , \quad \Phi_y = \mathcal{H}_y^a\mathcal{H}_y^a$$

- Charge density wave (CDW) order at wavevectors $\boldsymbol{K}_x \pm \boldsymbol{K}_y$

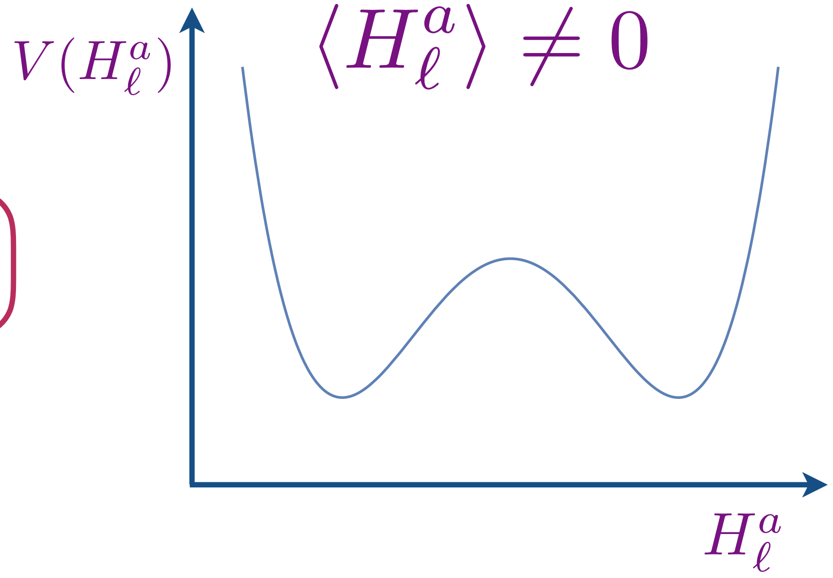
$$\Phi_+ = \mathcal{H}_x^a\mathcal{H}_y^a \quad , \quad \Phi_- = \mathcal{H}_x^a\mathcal{H}_y^{a*}$$

- (Modulated) scalar spin chirality

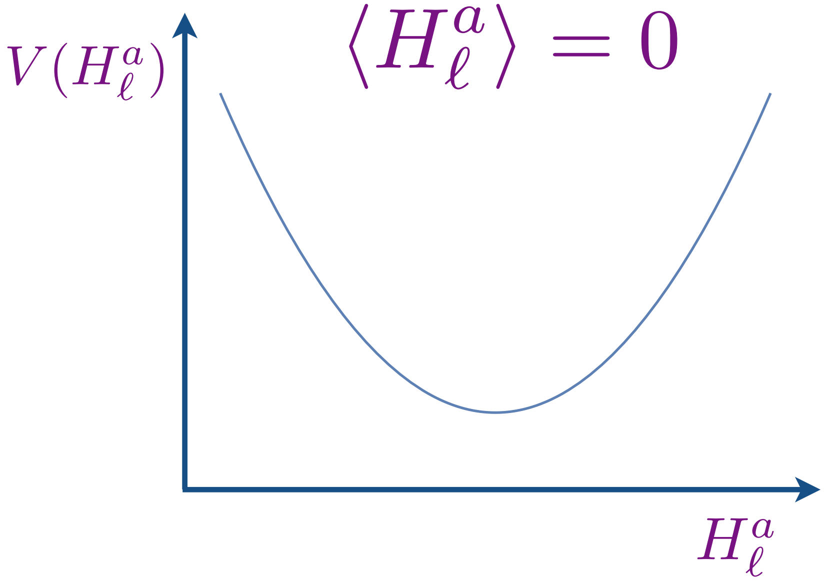
$$\chi_{ijk} = \epsilon_{abc}H^a(\boldsymbol{r}_i)H^b(\boldsymbol{r}_j)H^c(\boldsymbol{r}_k)$$

Phase diagram of SU(2) gauge theory for hole-doped cuprates

$N_h = 4$



$\langle H_\ell^a \rangle \neq 0$



$\langle H_\ell^a \rangle = 0$

Higgs/U(1) confinement /Z_2 deconfined

One or more of Ising-nematic, CDW, scalar spin chirality, and $\mathbb{Z}_2$ topological orders

Reconstructed (FL*) Fermi surfaces, with large length scale confinement in the U(1) cases

Possible Deconfined critical SU(2) gauge theory

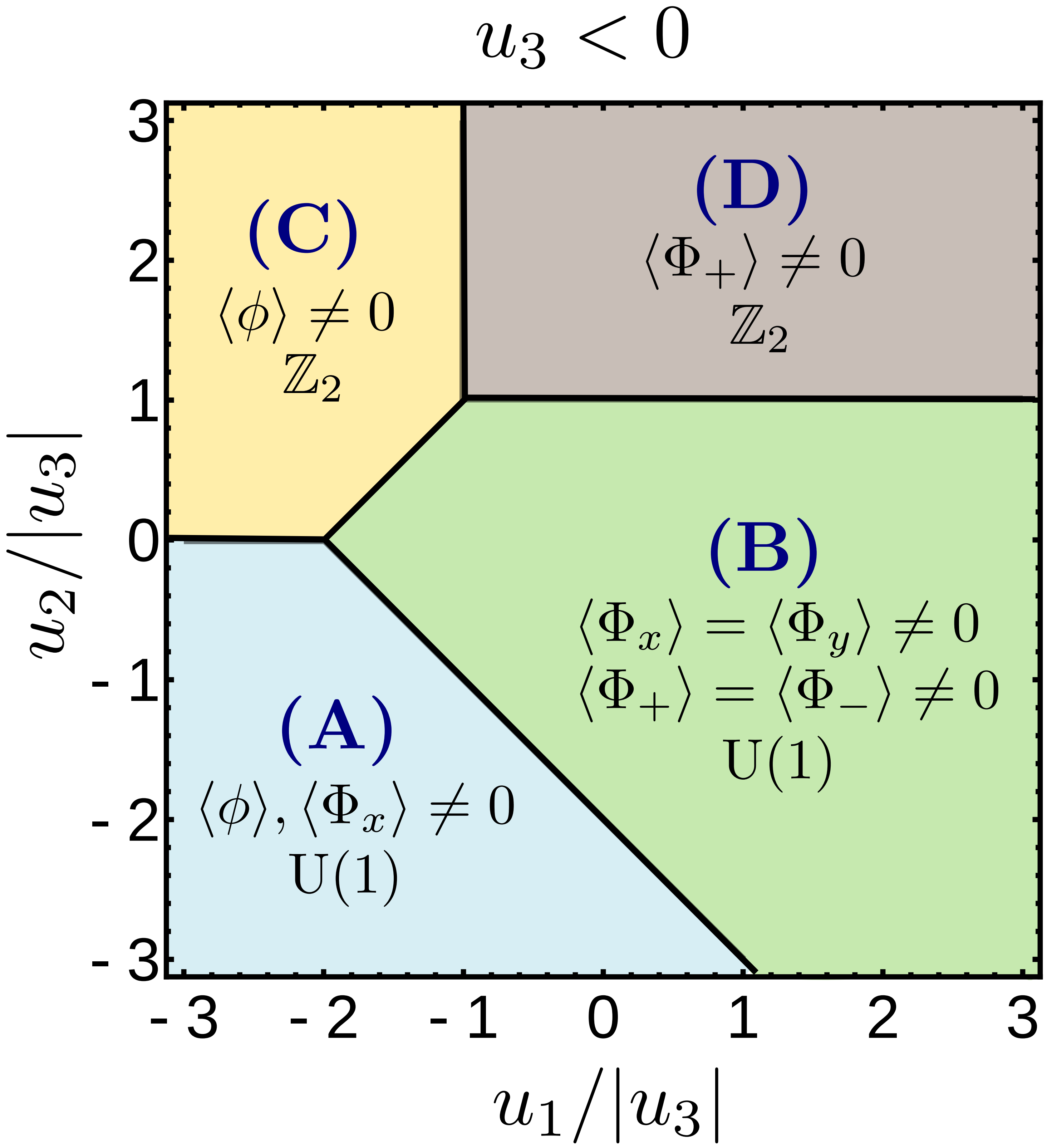
Confinement

Fermi liquid with large Fermi surface

s

Optimal doping for hole-doped cuprates

Broken symmetries and topological order in the Higgs phase



Optimal doping for hole-doped cuprates

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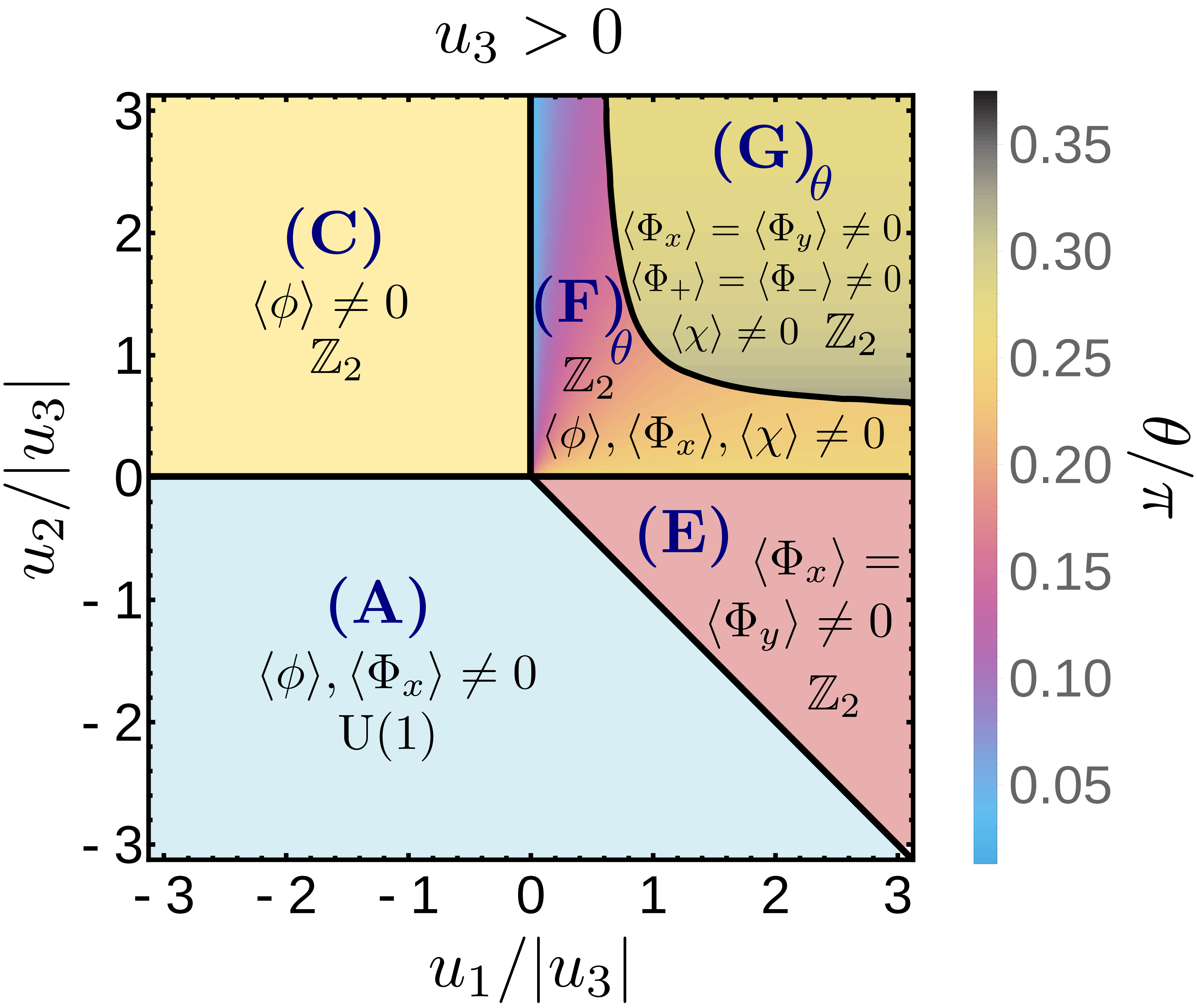
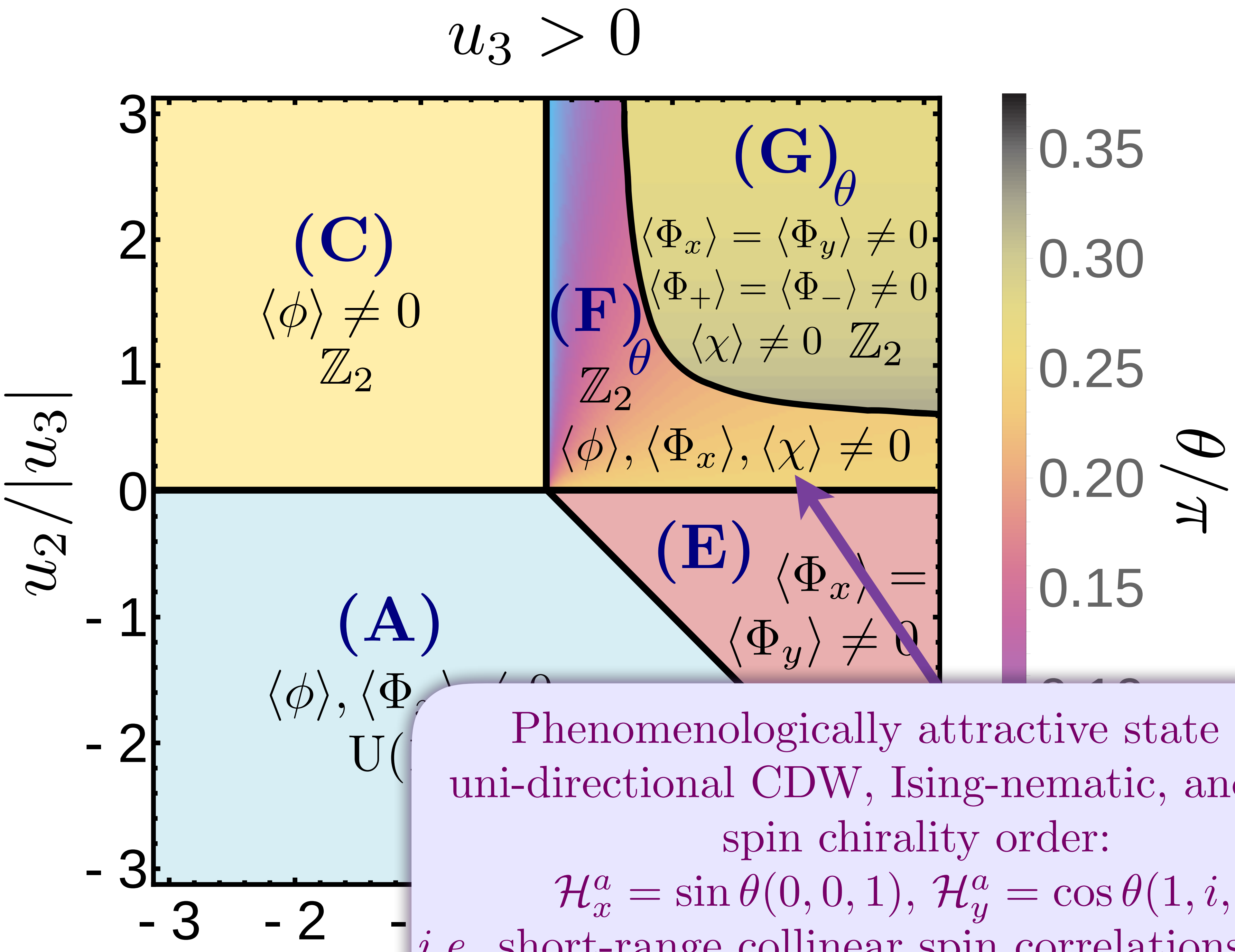


Optimal doping for hole-doped cuprates

Broken symmetries and topological order in the Higgs phase



Phenomenologically attractive state with uni-directional CDW, Ising-nematic, and scalar spin chirality order:

$\mathcal{H}_x^a = \sin\theta(0,0,1)$, $\mathcal{H}_y^a = \cos\theta(1,i,0)$

i.e. short-range collinear spin correlations along x, and short-range spiral spin correlations along y.

- Cuprates are described across optimal doping by the Higgs-to-confinement crossover/transition of a SU(2) gauge theory with N_h adjoint Higgs fields coupled to a large Fermi surface of gauge-neutral electrons.

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- Electron doped cuprates are described by $N_h = 1$. In this case, there is no phase transition, only a crossover. The underdoped cuprates are described by the Higgs regime, while the overdoped Fermi liquid is the confinement regime.

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- Hole doped cuprates are described by $N_h = 4$. In this case, a phase transition must occur, at least in the absence of disorder. The Higgs phase is characterized by:
  - Stable $\mathbb{Z}_2$ topological order, or U(1) topological up to an exponentially large length scale
  - One or more broken symmetries involving Ising-nematic, CDW, and scalar spin chirality.