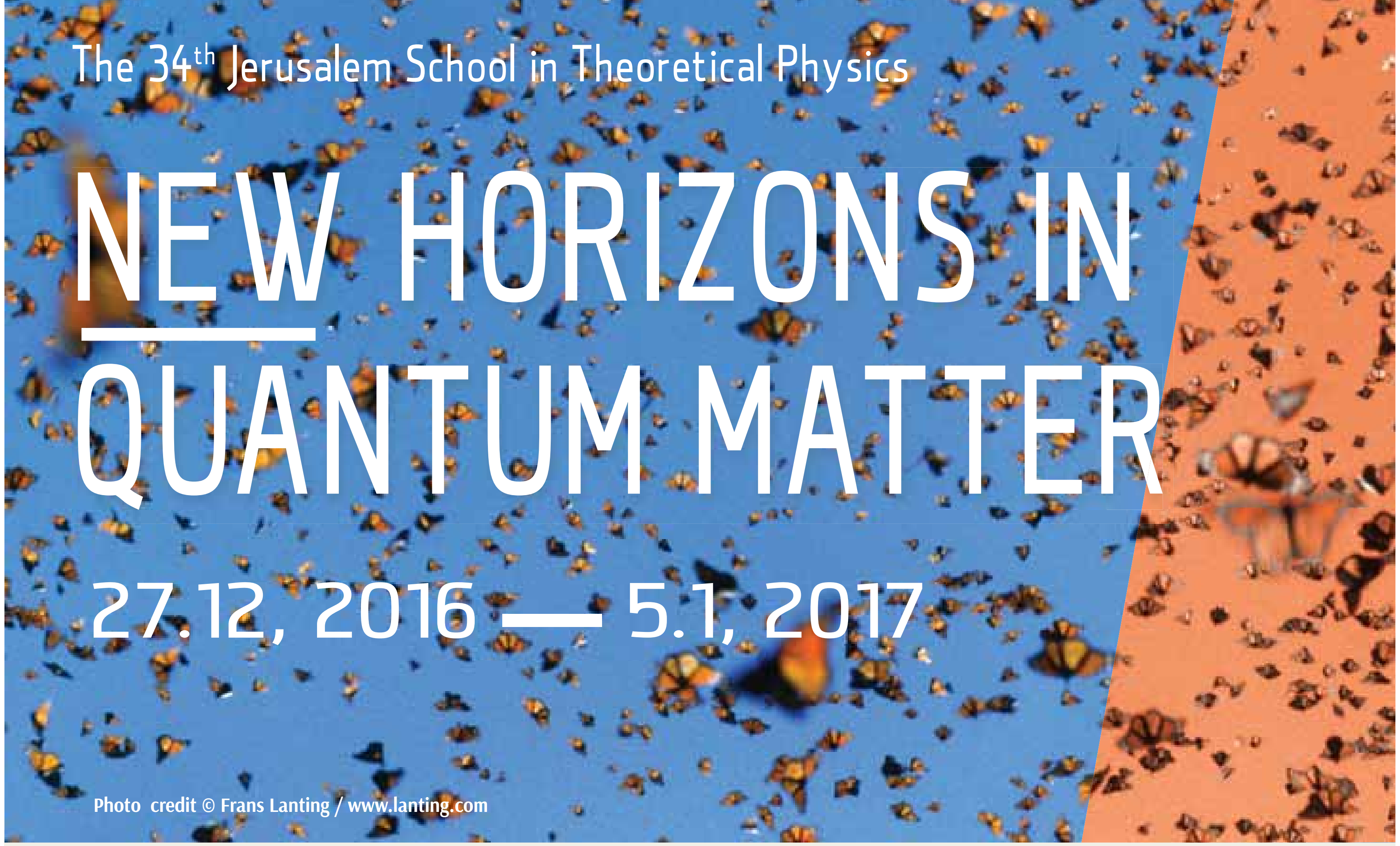

The 34th Jerusalem School in Theoretical Physics

NEW HORIZONS IN QUANTUM MATTER

27.12, 2016 — 5.1, 2017

Photo credit © Frans Lanting / www.lanting.com

Modern quantum materials realize a remarkably rich set of electronic phases. This school will explore the many new concepts and methods which have been developed in recent years, moving beyond the traditional paradigms of Fermi liquid theory and spontaneous symmetry breaking. In particular, long-range quantum entanglement appears in topological and quantum-critical states, and the school will discuss new techniques required to describe their observable properties.

For more details:
www.as.huji.ac.il/horizons-in-quantum

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Stanford University

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Harvard University

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THE HEBREW UNIVERSITY OF JERUSALEM

GORDON AND BETTY MOORE FOUNDATION

Strange metals, black holes, and graphene

Institute for Advanced Study
Princeton, October 17, 2016

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

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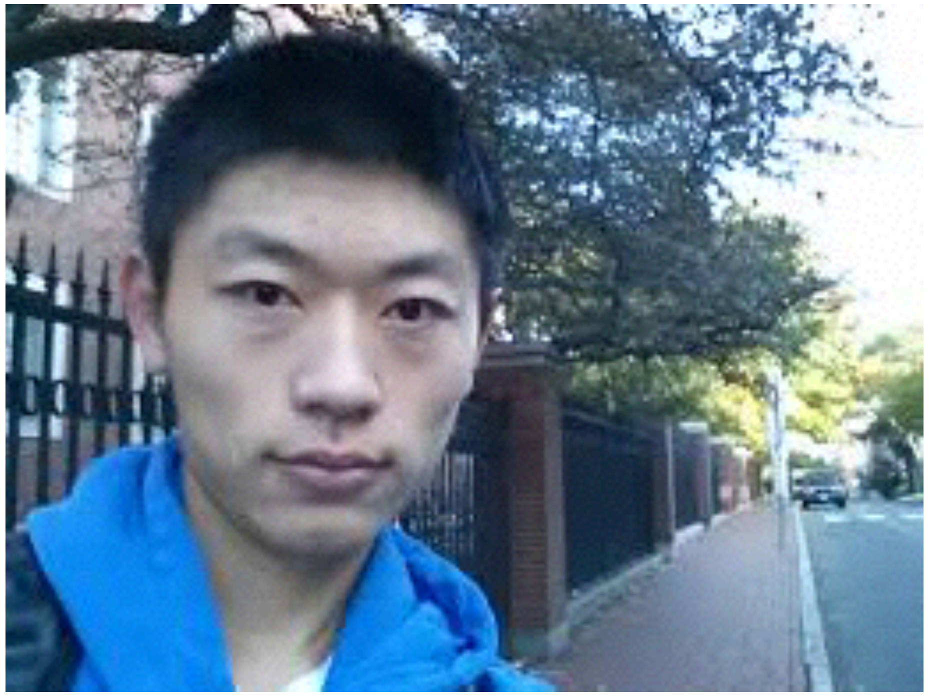
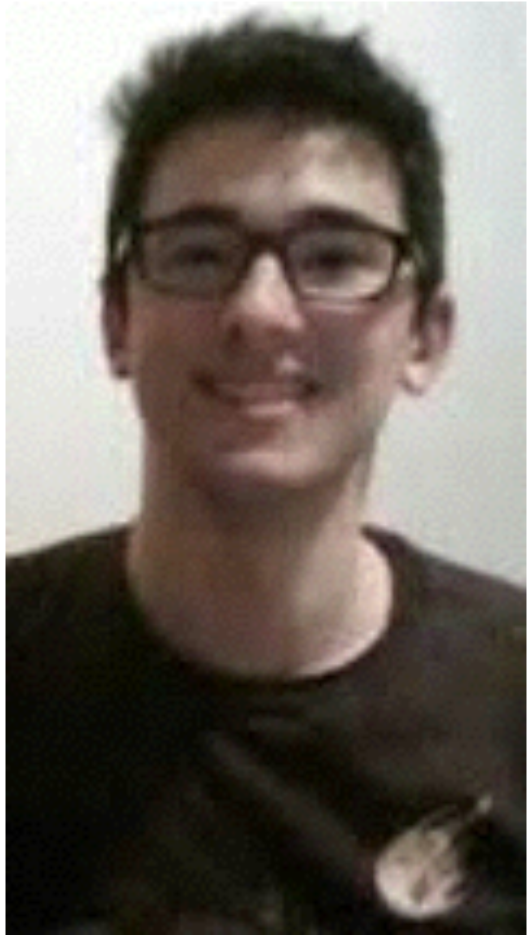
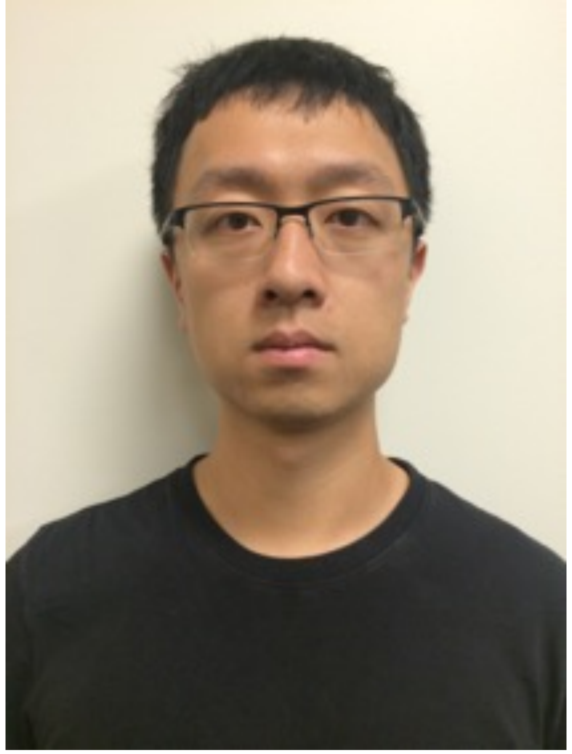
Aavishkar Patel, Harvard



Wenbo Fu, Harvard



Andrew Lucas, Stanford



Yingfei Gu, Stanford

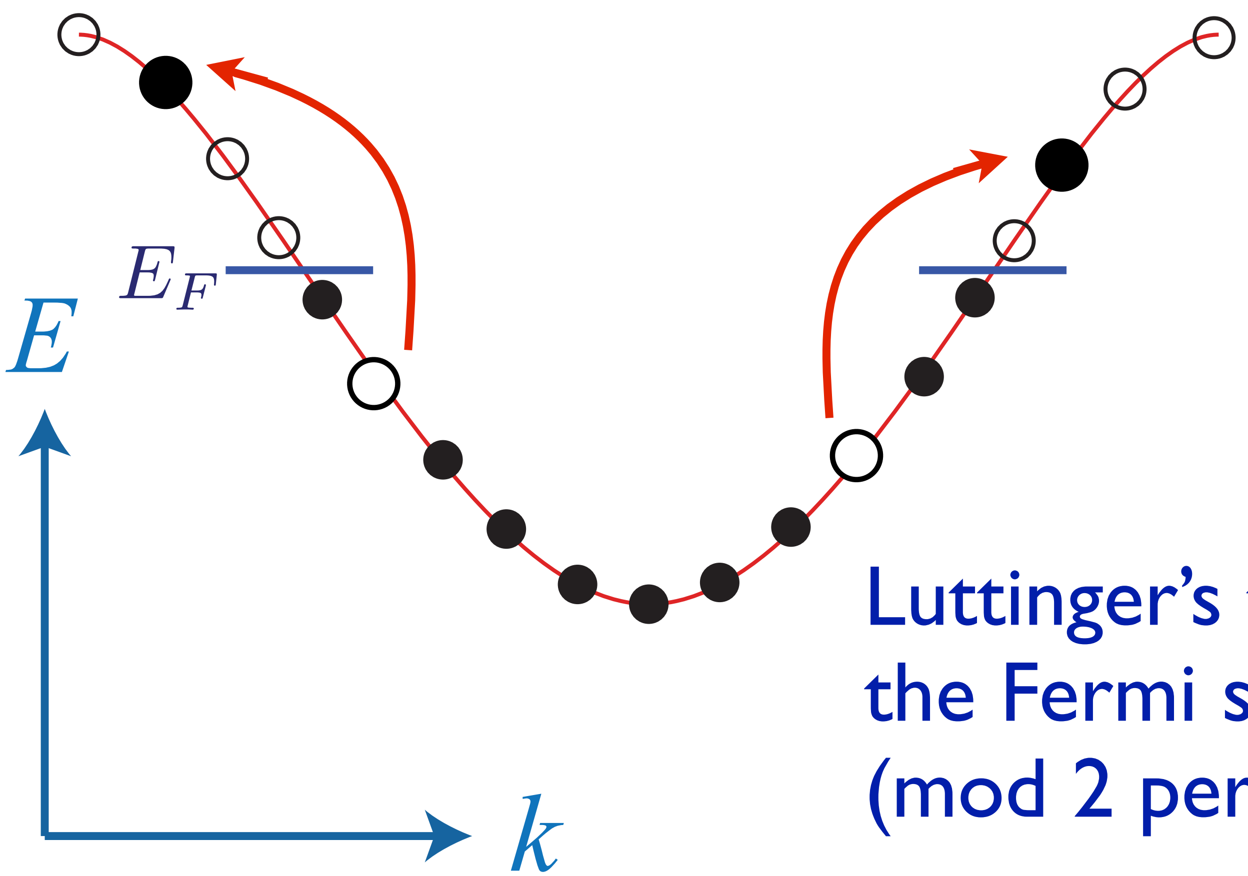
Richard Davison, Harvard

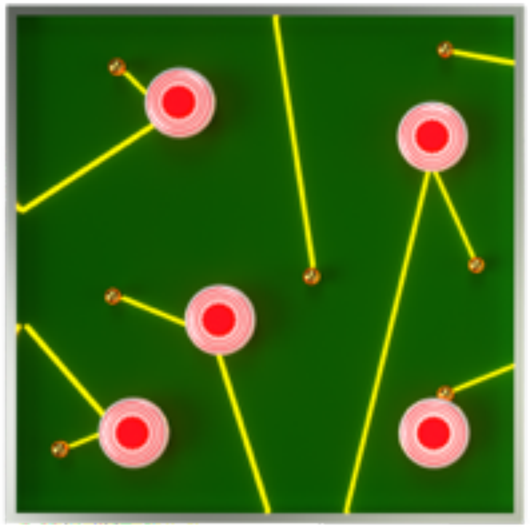
Conventional quantum matter:

1. Ground states connected adiabatically to independent electron states

2. Boltzmann-Landau theory of quasiparticles

Metals





Luttinger's theorem: volume enclosed by the Fermi surface = density of all electrons (mod 2 per unit cell).

Topological quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. Boltzmann-Landau theory of quasiparticles

(a) The fractional quantum Hall effect: the ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

(b) The pseudogap metal: proposed to have electron-like quasiparticles but on a "small" Fermi surface which does not obey the Luttinger theorem.

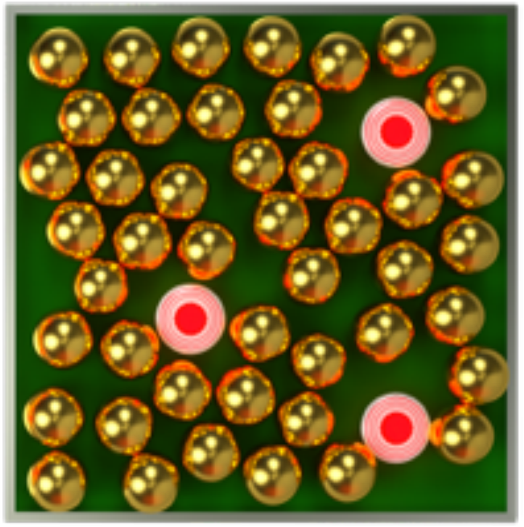
<u>*Quantum matter without quasiparticles:*</u>

1. Ground states <u>disconnected</u> from independent electron states: many-particle entanglement

2. No quasiparticles

<u>Strange metals:</u>

Such metals are found, most prominently, near optimal doping in the the cuprate high temperature superconductors.



Quantum matter without quasiparticles:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. No quasiparticles

Strange metals:

Such metals are found, most prominently, near optimal doping in the the cuprate high temperature superconductors.

But how can we be sure that no quasiparticles exist in a given system? Perhaps there are some exotic quasiparticles inaccessible to current experiments……..

Local thermal equilibration or phase coherence time, τ_φ:

- There is an *lower bound* on τ_φ in all many-body quantum systems of order $\hbar/(k_B T)$,

$$\tau_\varphi > C \frac{\hbar}{k_B T},$$

  and the lower bound is realized by systems *without* quasiparticles.

- In systems *with* quasiparticles, τ_φ is parametrically larger at low T;
  e.g. in Fermi liquids $\tau_\varphi \sim 1/T^2$,
  and in gapped insulators $\tau_\varphi \sim e^{\Delta/(k_B T)}$ where Δ is the energy gap.

K. Damle and S. Sachdev, PRB **56**, 8714 (1997)
S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

A bound on quantum chaos:

- The time over which a many-body quantum system becomes "chaotic" is given by $\tau_L = 1/\lambda_L$, where λ_L is the "Lyapunov exponent" determining memory of initial conditions. This LYAPUNOV TIME obeys the rigorous lower bound

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

A. I. Larkin and Y. N. Ovchinnikov, JETP **28**, 6 (1969)

J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

A bound on quantum chaos:

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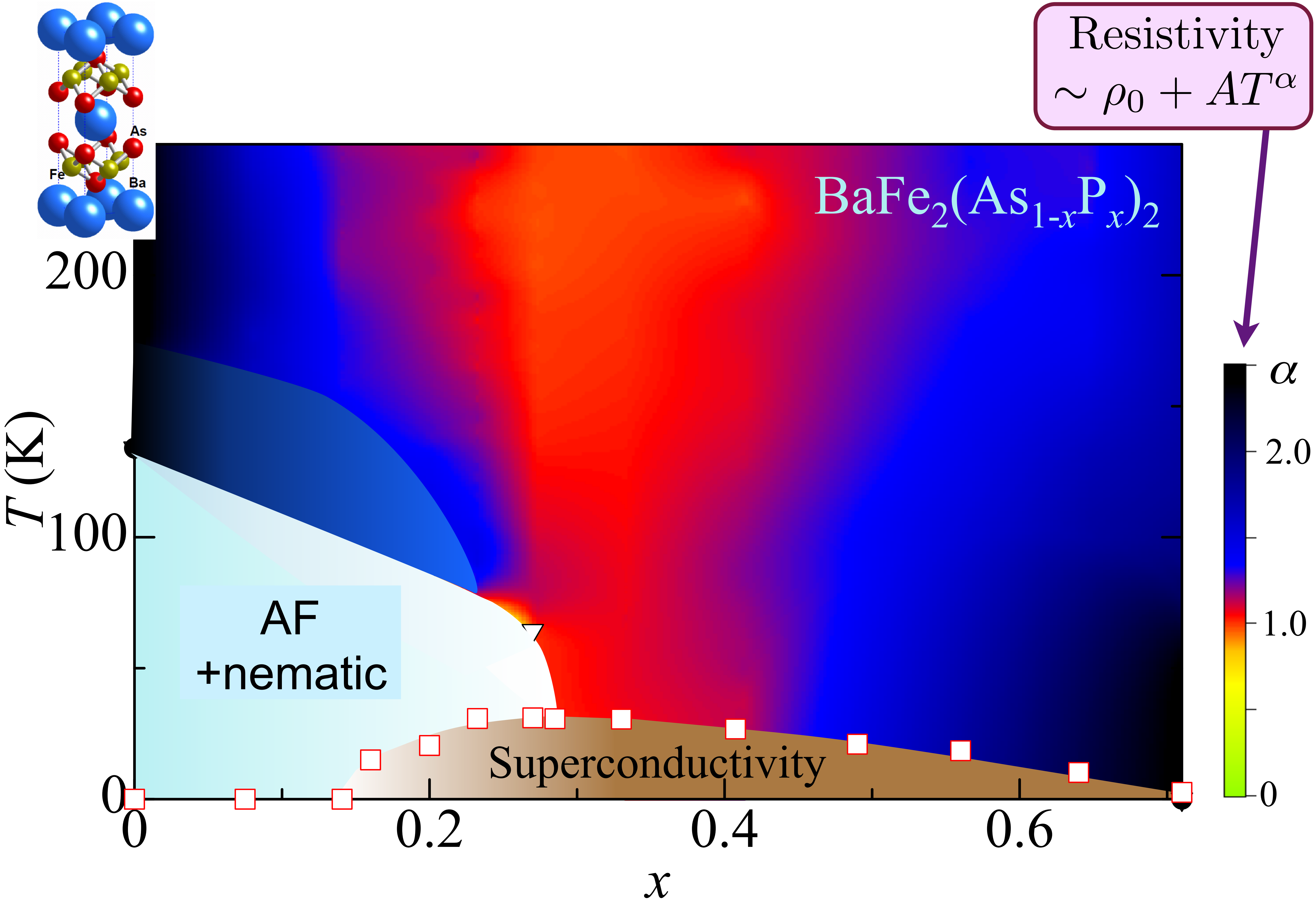
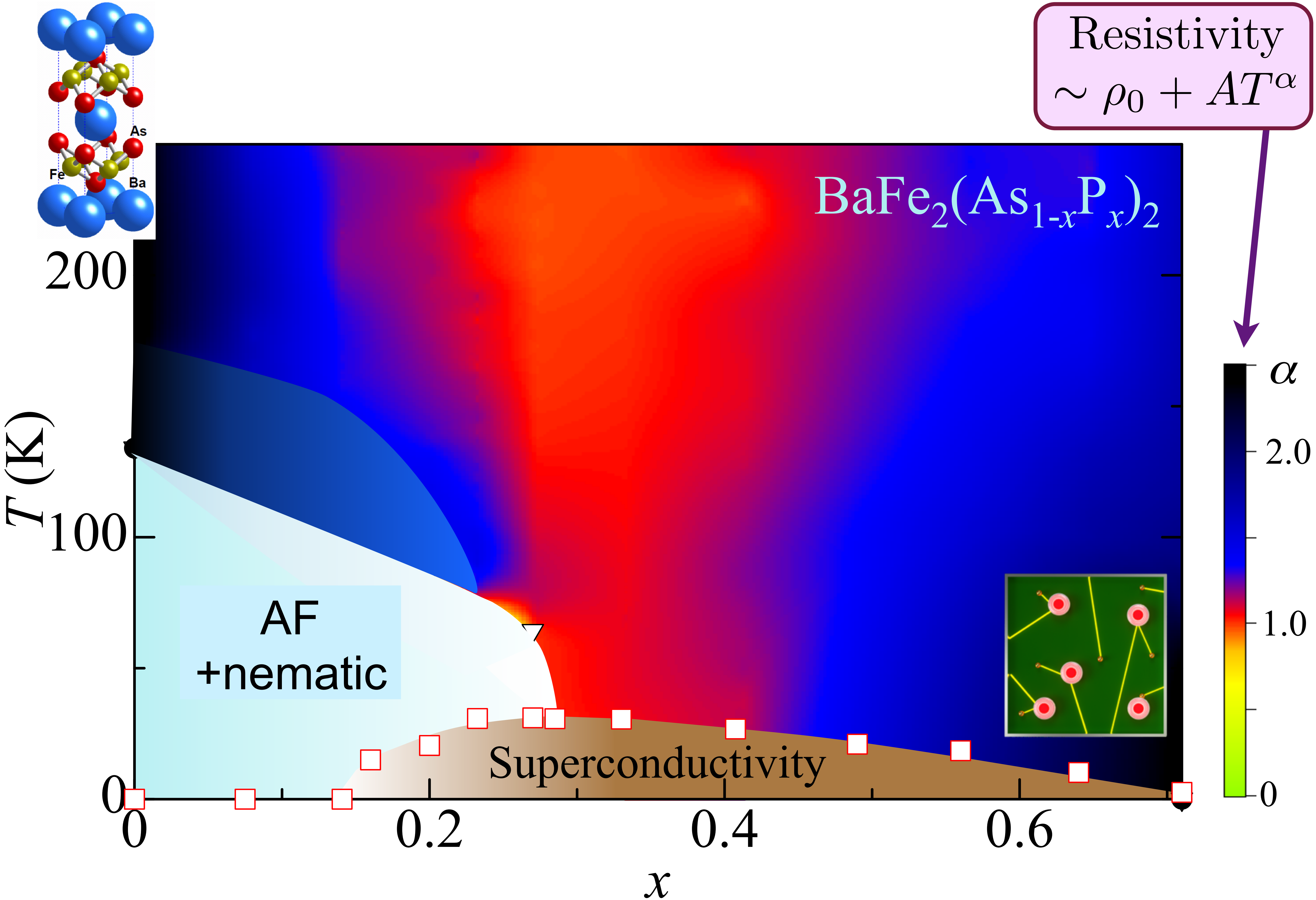
$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

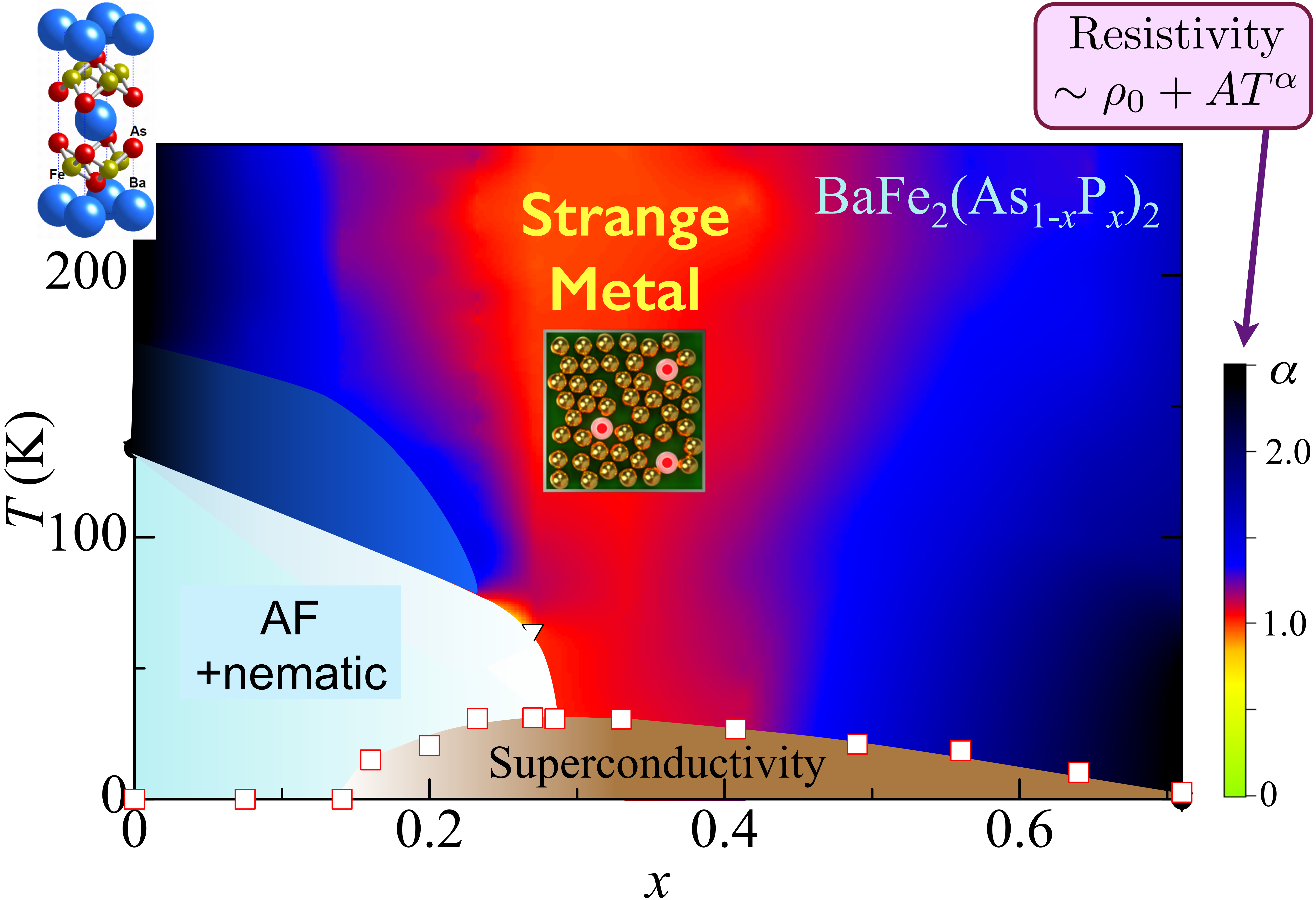
Quantum matter without quasiparticles
$\approx$ fastest possible many-body quantum chaos



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

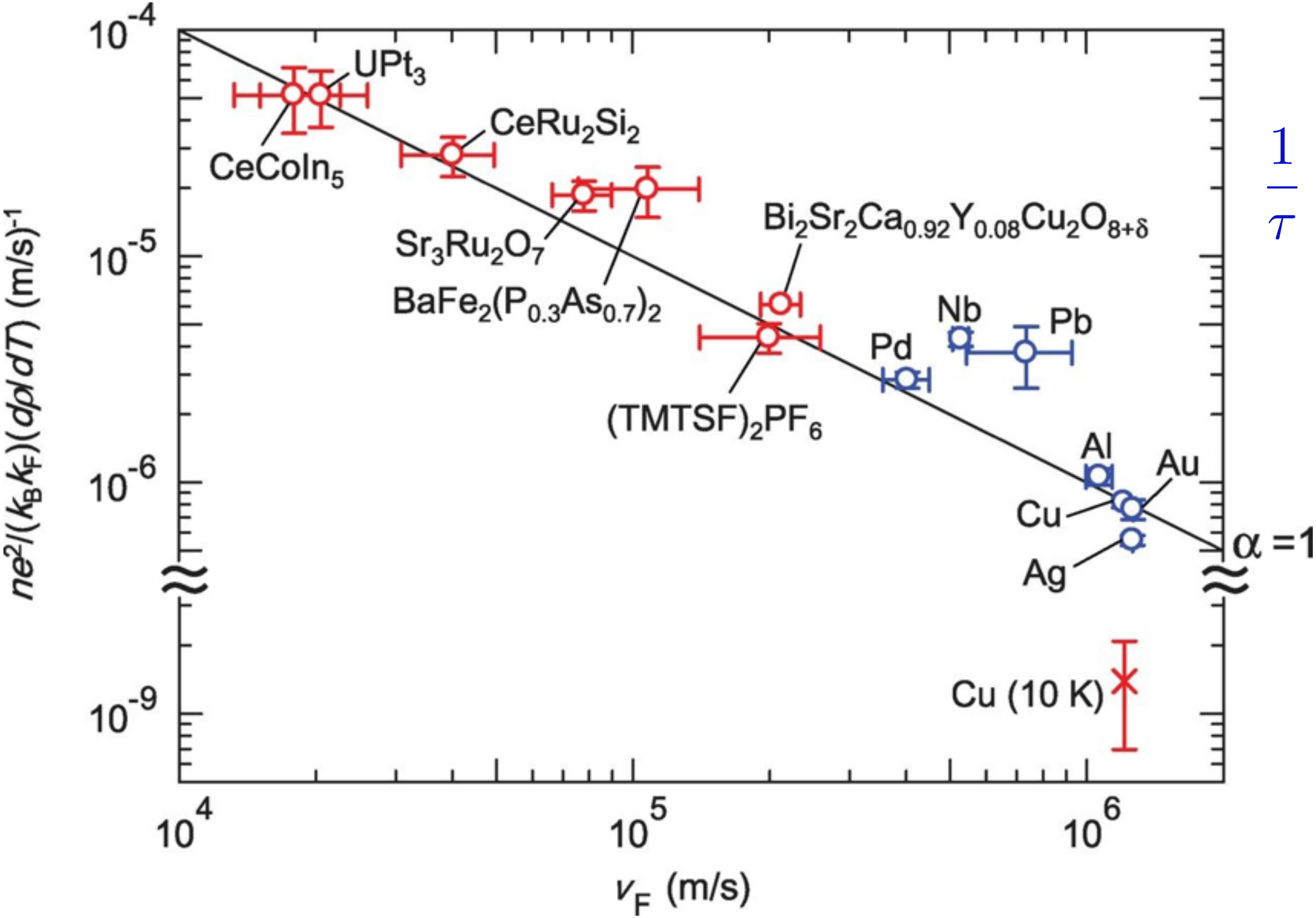


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



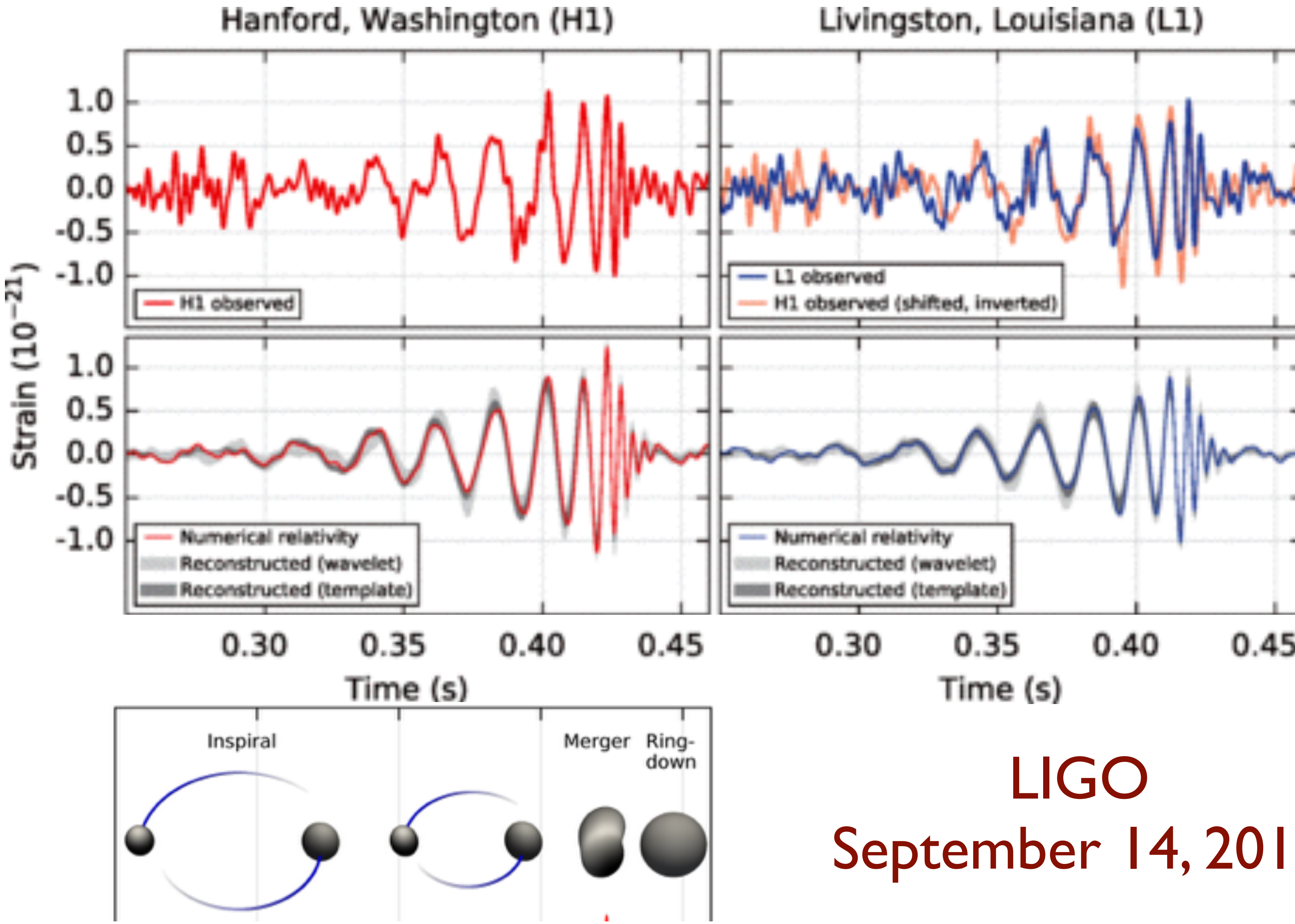
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Strange metals

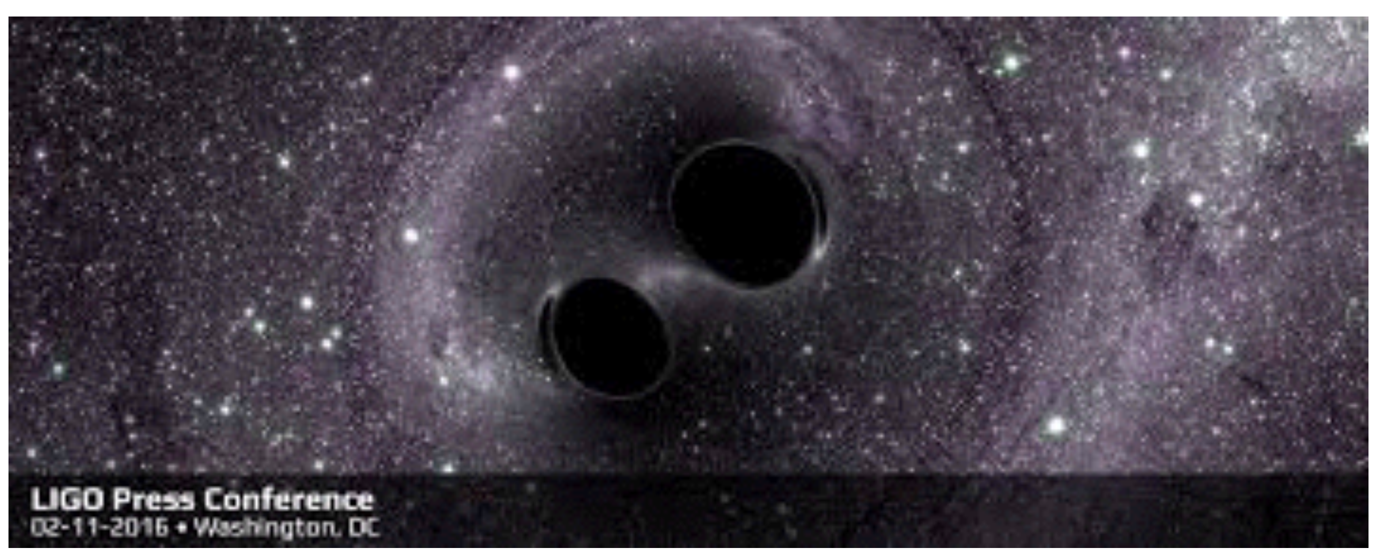


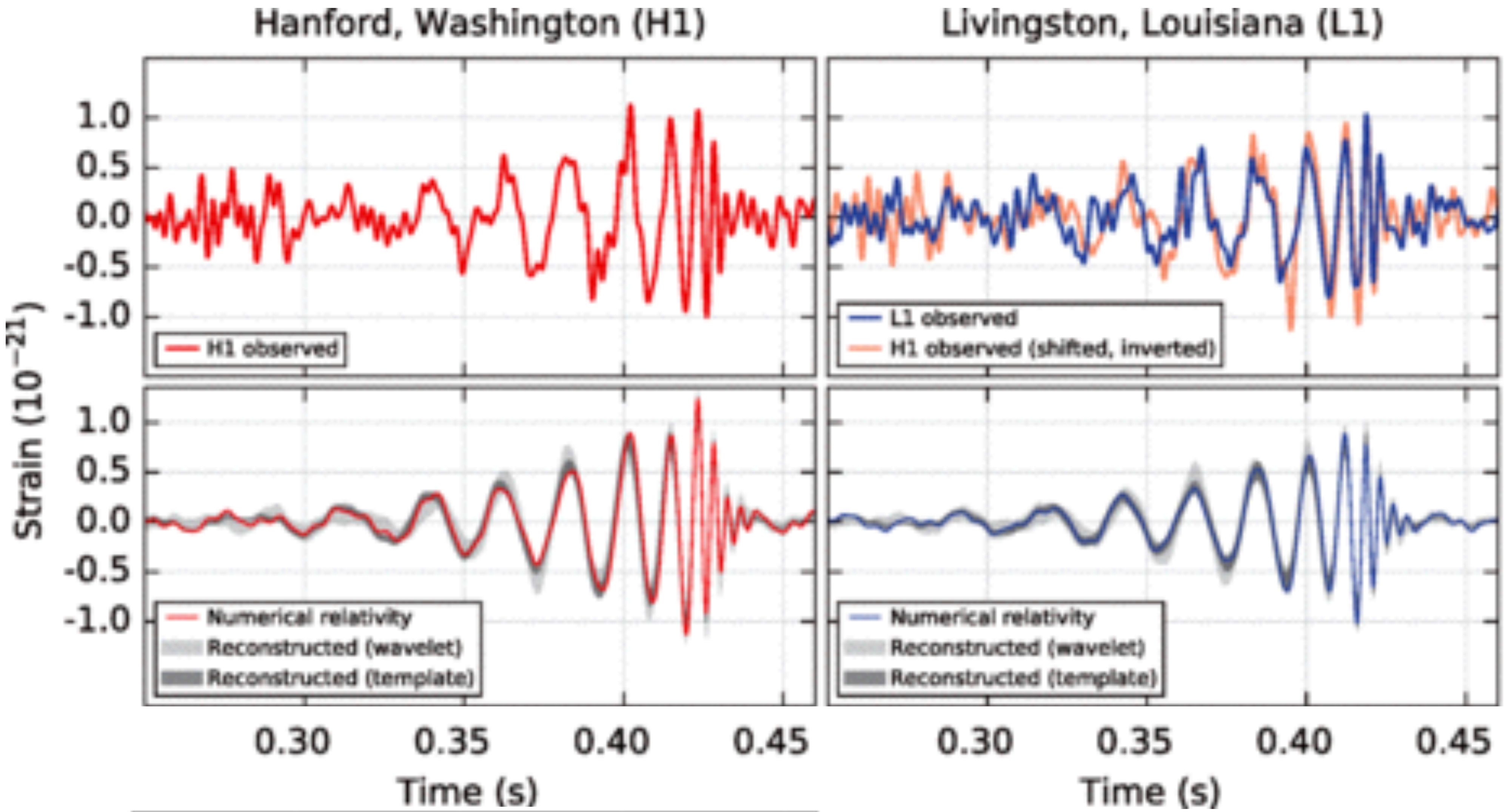
$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

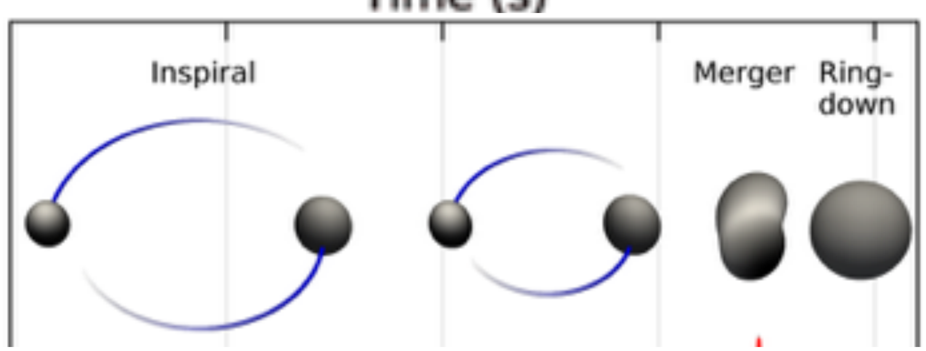
J. A. N. Bruin, H. Sakai, R. S. Perry, A. P. Mackenzie, *Science*. **339,** 804 (2013)



LIGO
September 14, 2015



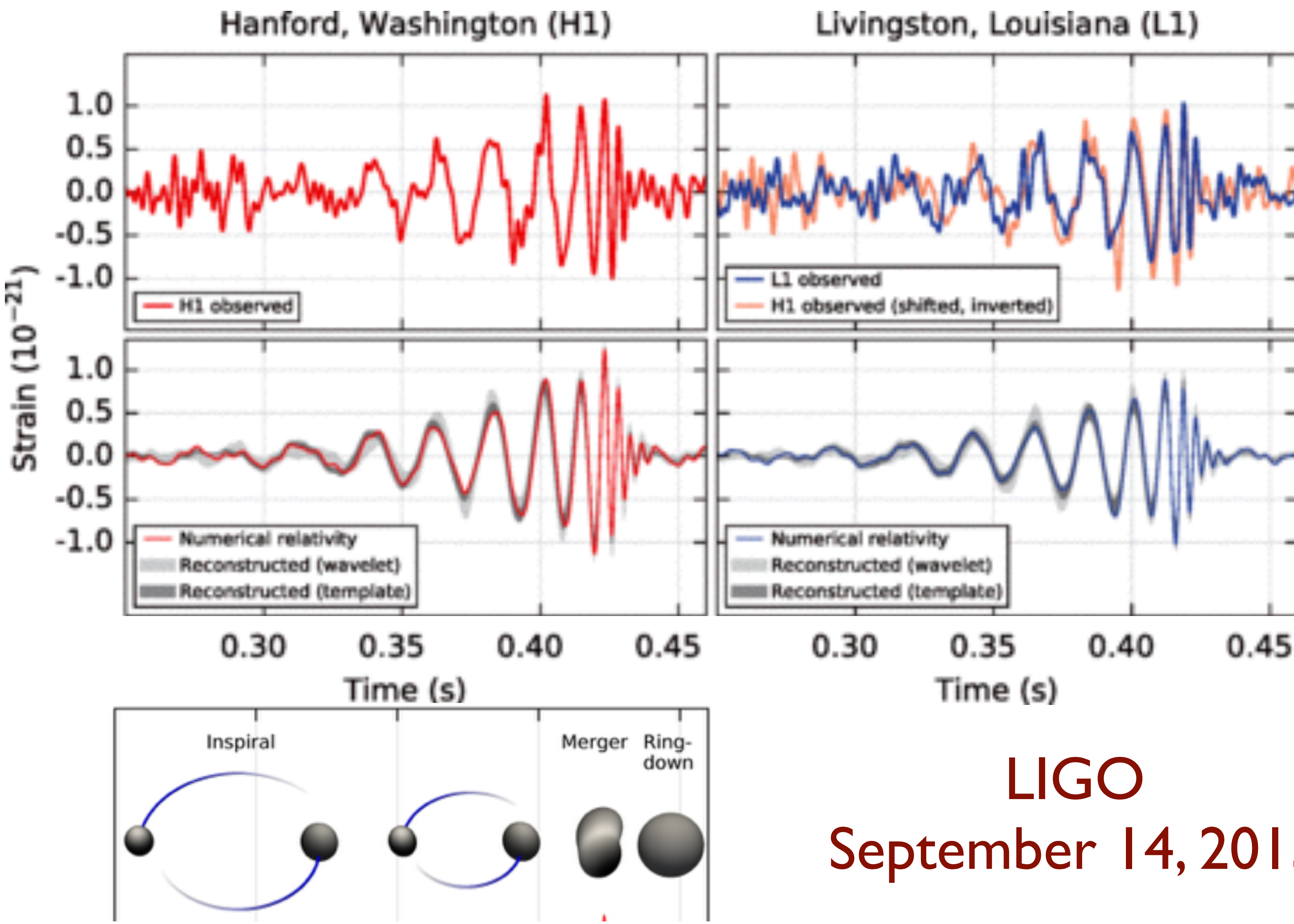




LIGO
September 14, 2015

- Black holes have a "ring-down" time, τ_r, in which they radiate energy, and stabilize to a 'featureless' spherical object. This time can be computed in Einstein's general relativity theory.

- For this black hole $\tau_r = 7.7$ milliseconds. (Radius of black hole $= 183$ km; Mass of black hole $= 62$ solar masses.)



LIGO
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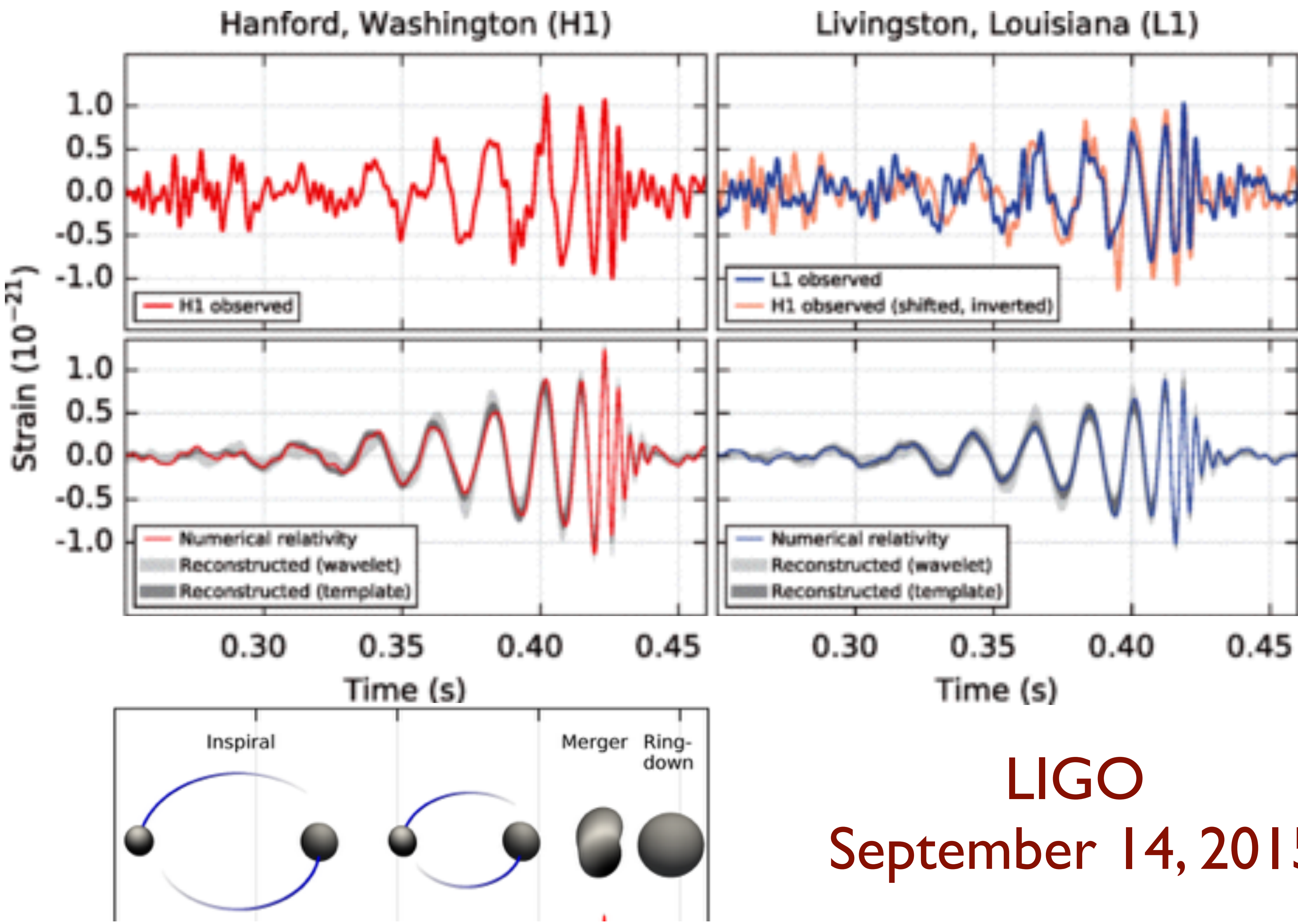
- 'Featureless' black holes have a Bekenstein-Hawking entropy, and a Hawking temperature, T_H.



LIGO
September 14, 2015

- Expressed in terms of the Hawking temperature, the ring-down time is $\tau_r \sim \hbar/(k_B T_H)$!

- For this black hole $T_H \approx 1$ nK.

Quantum matter without quasiparticles

- Model systems with weak momentum relaxation

  Fermi surface coupled to a gauge field

  Holography: Reissner-Nordstrom-AdS black hole of Einstein-Maxwell theory

- SYK models

  Diffusive metals without quasiparticles (and no momentum conservation).

  Holography: Einstein-Maxwell-axion theories with momentum dissipation

- Graphene

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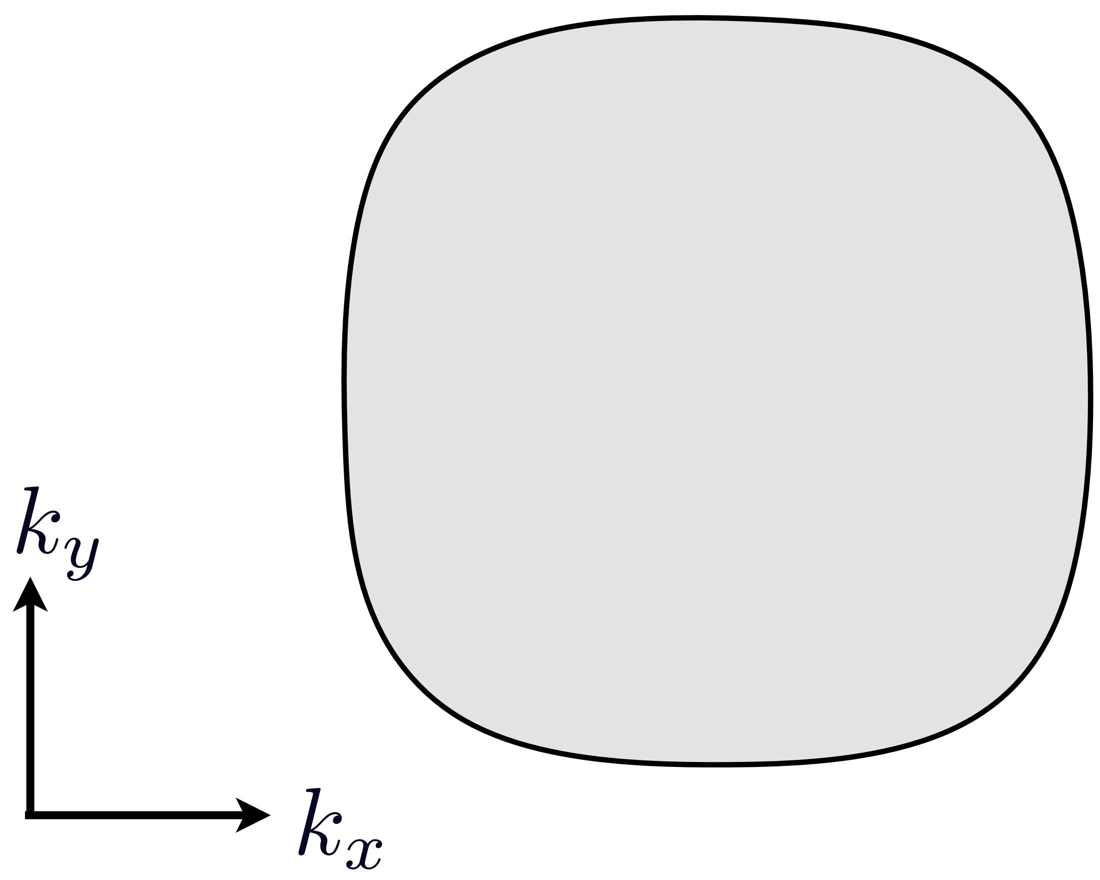
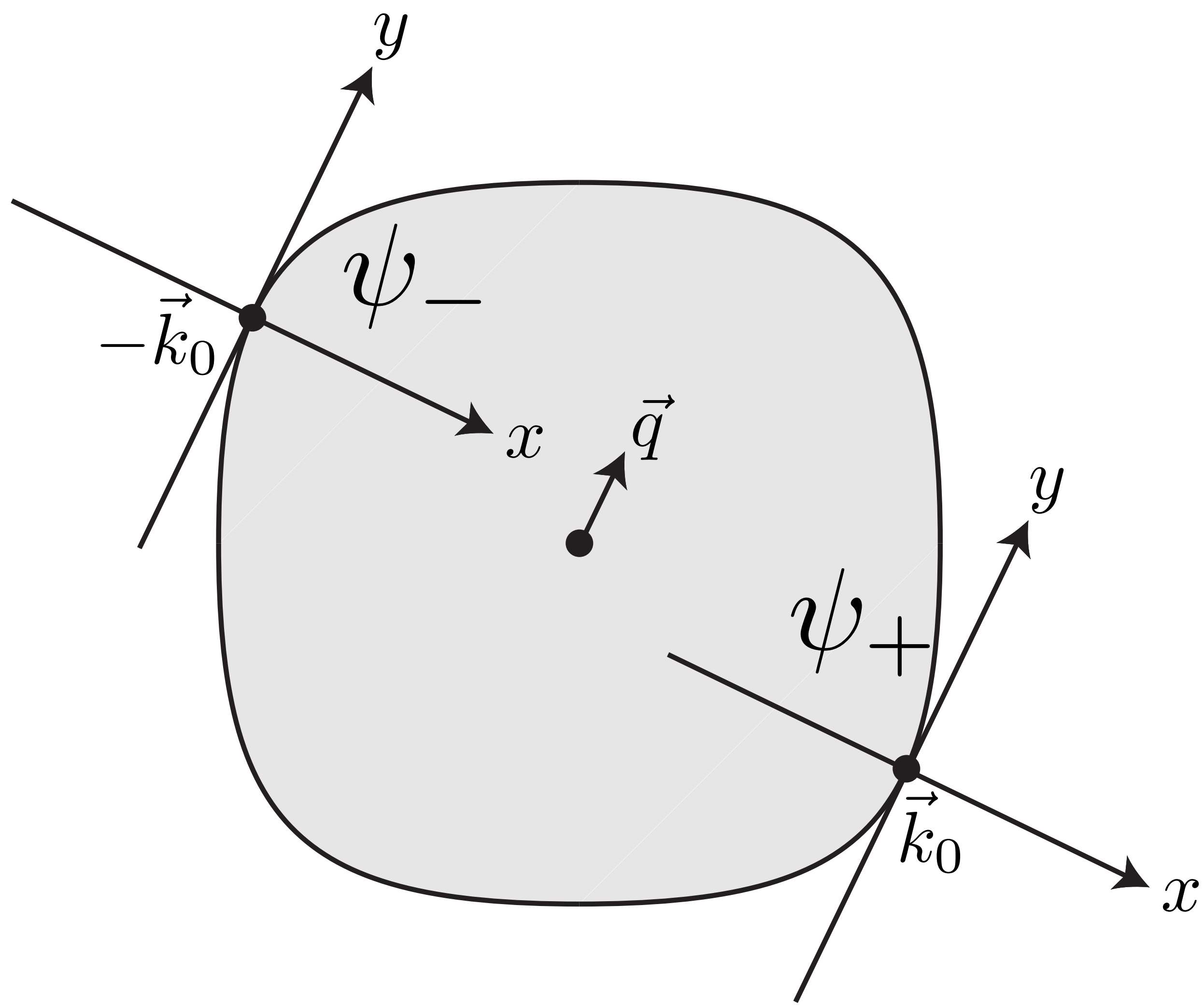


$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

Fermi surface coupled to a gauge field



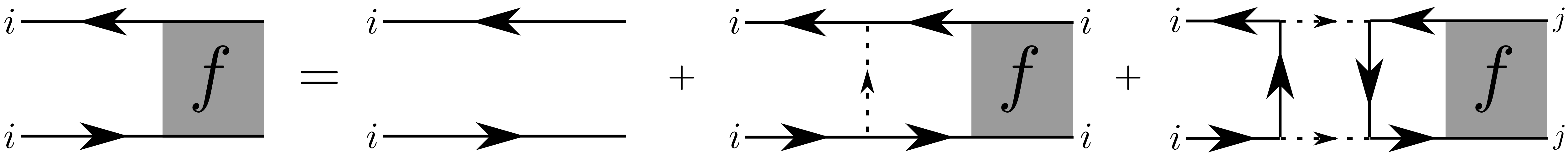
$$\mathcal{L}[\psi_\pm, a] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Fermi surface coupled to a gauge field



$$f(t) = \frac{1}{N^2}\theta(t)\sum_{i,j=1}^{N}\int d^2x\ \mathrm{Tr}\left[e^{-\beta H/2}\{\psi_i(x,t),\psi_j^\dagger(0)\}e^{-\beta H/2}\{\psi_i(x,t),\psi_j^\dagger(0)\}^\dagger\right]$$

$$= f_0 - \frac{f_1}{N}e^{t/\tau_L} + \ldots.$$

Strongly-coupled theory with no quasiparticles and fast scrambling:

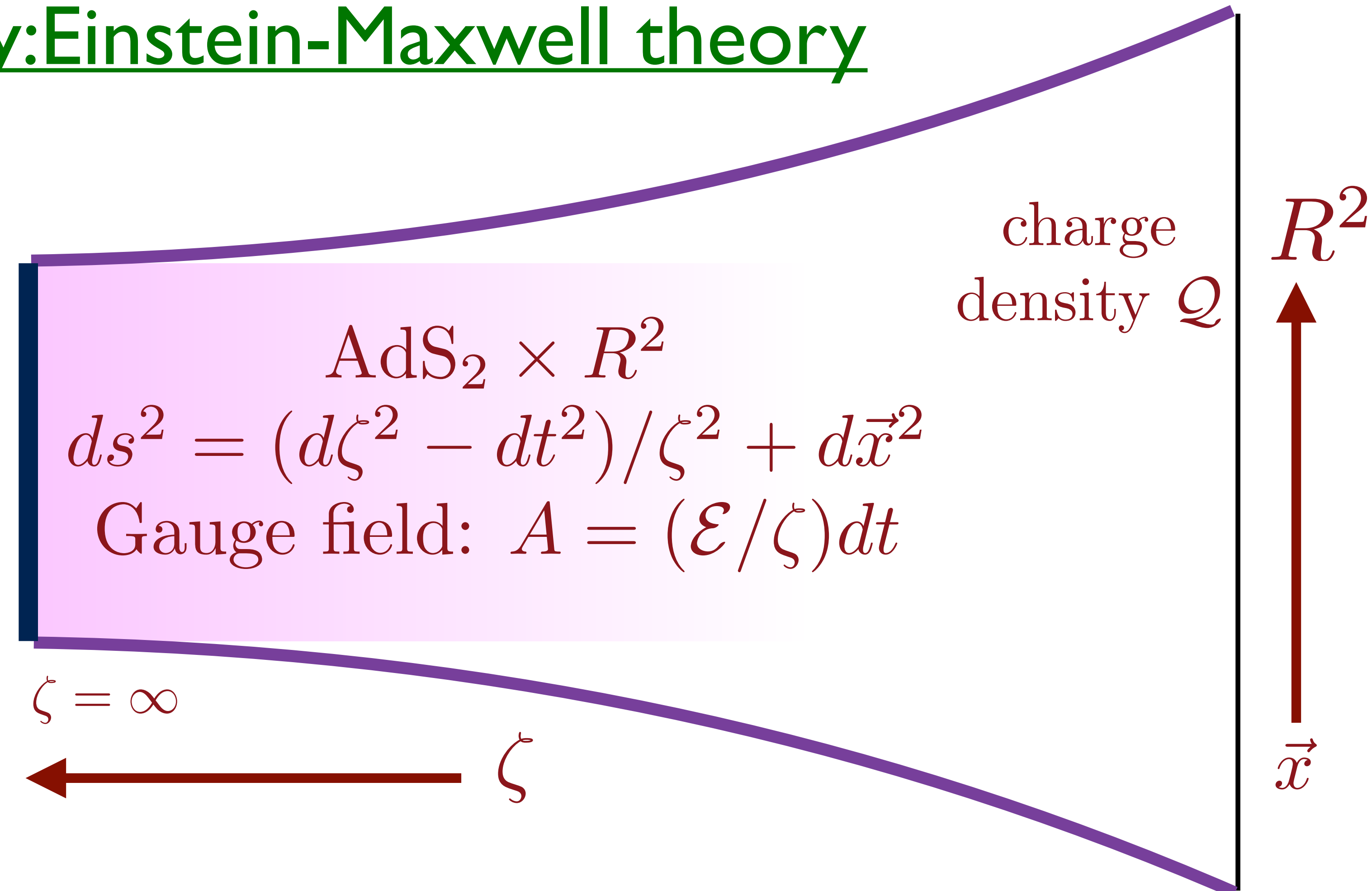
$$\tau_L \approx 0.40\frac{\hbar}{k_B T}$$

$$v_{\text{butterfly}} \sim NT^{1/3}$$

A. A. Patel and S. Sachdev,
Quantum chaos on a critical Fermi surface,
to appear

Note τ_L is independent of N, the number of fermion flavors.

Holography:Einstein-Maxwell theory

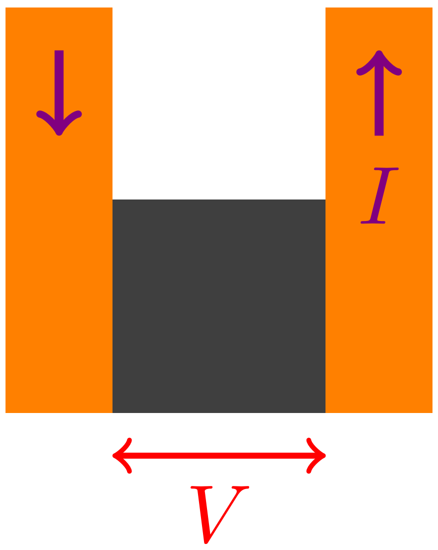


$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),$$

- Reissner-Nördstrom-AdS charged black hole, with a $\mathrm{AdS}_2 \times R^2$ near-horizon geometry describes a 'strange metal' with no quasiparticles.
- No Fermi surface: requires bulk monopole quantum fluctuations.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)
T. Faulkner and N. Iqbal arXiv: 1207.4208; S. Sachdev, PRD **86**, 126003 (2012)

Transport



$$V = IR \qquad R \sim \frac{1}{\sigma}$$

- more generally, measure thermoelectric transport:

$$\begin{pmatrix} \delta J_i \\ \delta Q_i \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & \alpha_{ij} \\ T\bar{\alpha}_{ij} & \bar{\kappa}_{ij} \end{pmatrix} \begin{pmatrix} \delta E_j \\ -\partial_j \delta T \equiv T\delta\zeta_j \end{pmatrix}.$$

Thermoelectric transport coefficients

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

$$\sigma = \frac{\mathcal{Q}^2}{\mathcal{M}} \frac{1}{(-i\omega)} + \sigma_Q(\omega)$$

$$\alpha = \frac{\mathcal{S}\mathcal{Q}}{\mathcal{M}} \frac{1}{(-i\omega)} + \alpha_Q(\omega)$$

$$\bar{\kappa} = \frac{T\mathcal{S}^2}{\mathcal{M}} \frac{1}{(-i\omega)} + \bar{\kappa}_Q(\omega)$$

with entropy density $\mathcal{S}$, $\mathcal{Q} \equiv \chi_{J_x,P_x}$, and $\mathcal{M} \equiv \chi_{P_x,P_x}$.

Obtained in hydrodynamics, memory functions, and holography

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

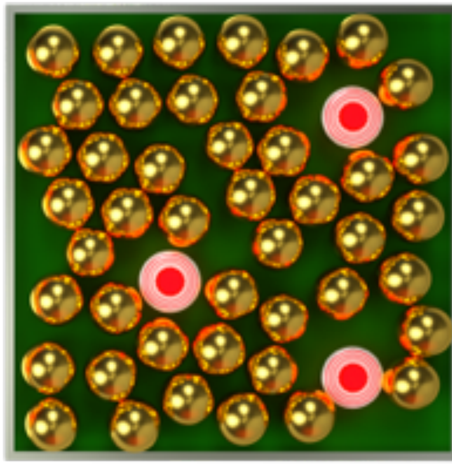
Thermoelectric transport coefficients

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

$$\sigma = \frac{\mathcal{Q}^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau_{\rm imp})} + \sigma_Q(\omega)$$

$$\alpha = \frac{\mathcal{S}\mathcal{Q}}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau_{\rm imp})} + \alpha_Q(\omega)$$

$$\bar{\kappa} = \frac{T\mathcal{S}^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau_{\rm imp})} + \bar{\kappa}_Q(\omega)$$



with entropy density $\mathcal{S}$, $\mathcal{Q} \equiv \chi_{J_x,P_x}$, and $\mathcal{M} \equiv \chi_{P_x,P_x}$.

Add a weak perturbation which relaxes momentum (impurities)
Obtained in hydrodynamics, and by memory functions

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

Quantum matter without quasiparticles

- Model systems with weak momentum relaxation

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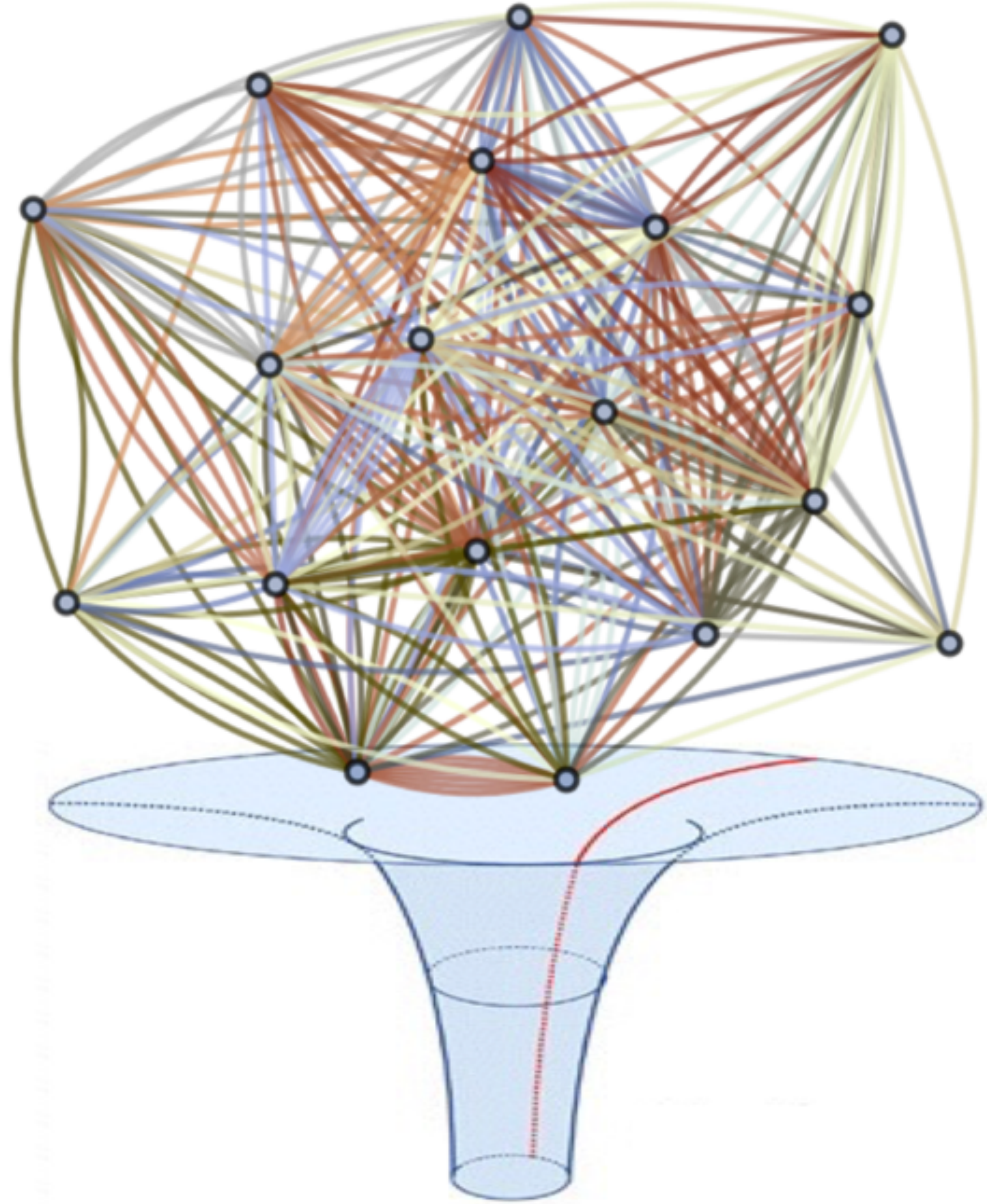
The Sachdev-Ye-Kitaev (SYK) model:

- A theory of a strange metal
- Dual theory of gravity on AdS_2

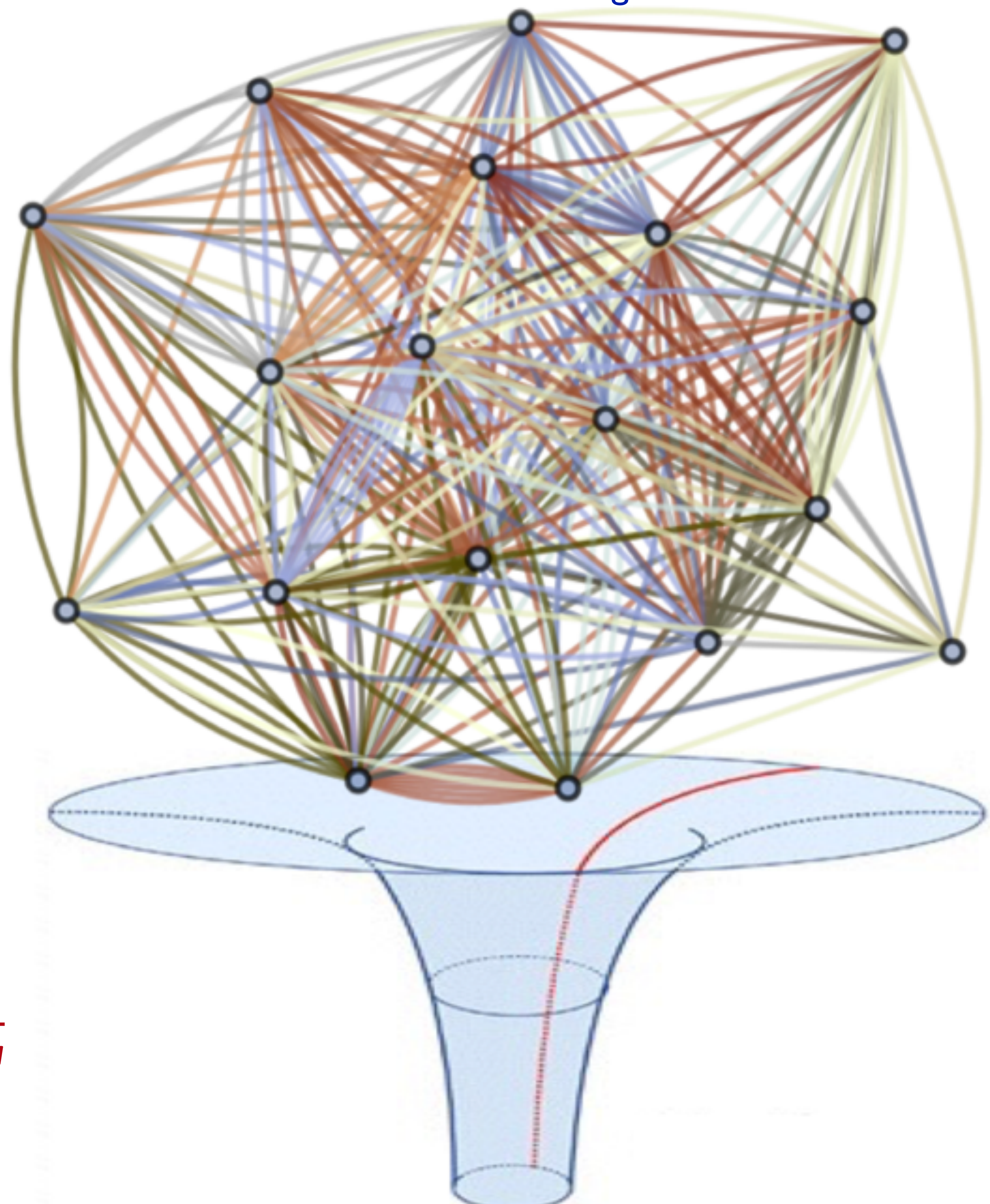


The Sachdev-Ye-Kitaev (SYK) model:

- A theory of a strange metal
- Dual theory of gravity on AdS_2
- Fastest possible quantum chaos with $\tau_L = \dfrac{\hbar}{2\pi k_B T}$

A. Kitaev, unpublished
J. Maldacena and D. Stanford, arXiv:1604.07818

Figure credit: L. Balents

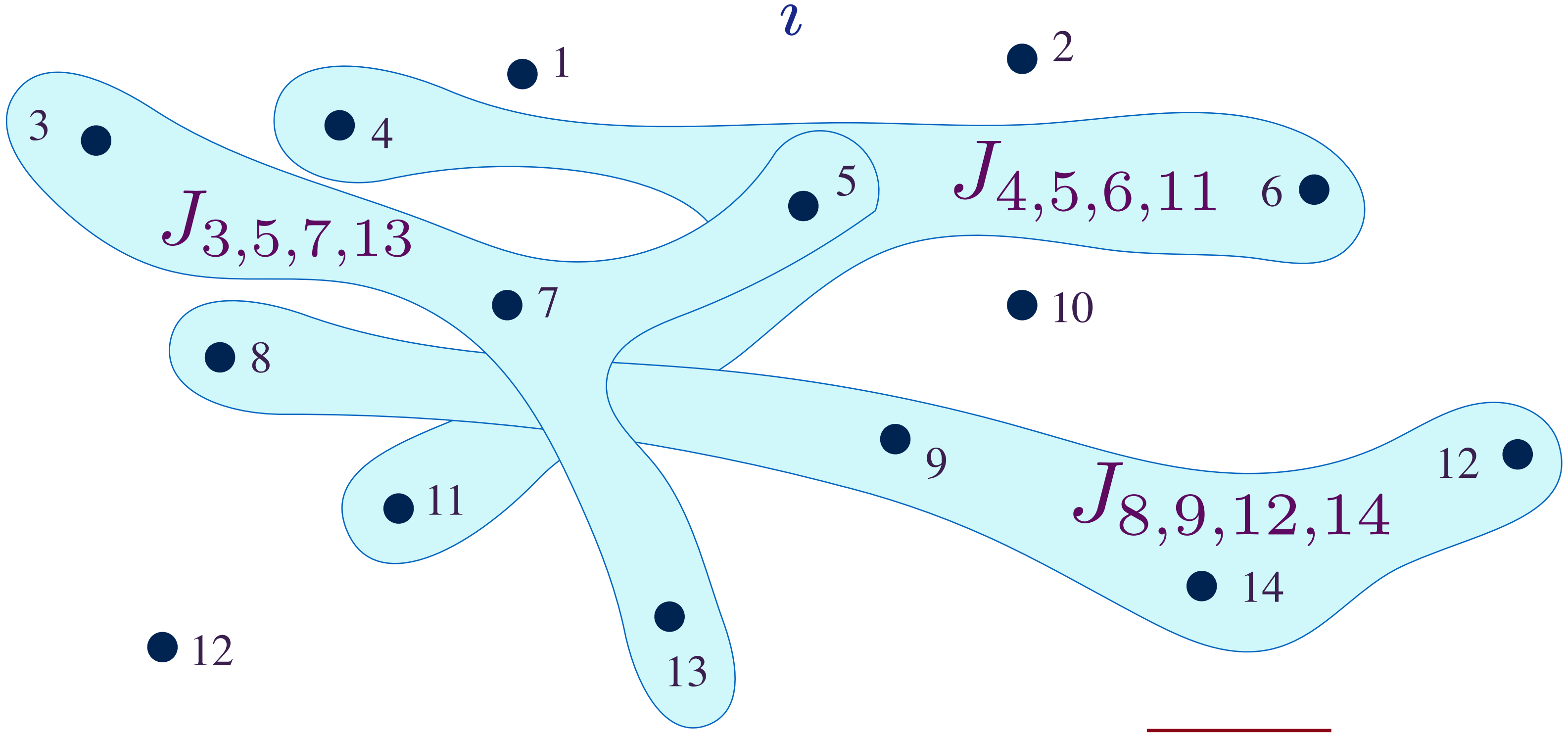


SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$

$N \to \infty$ yields critical strange metal.

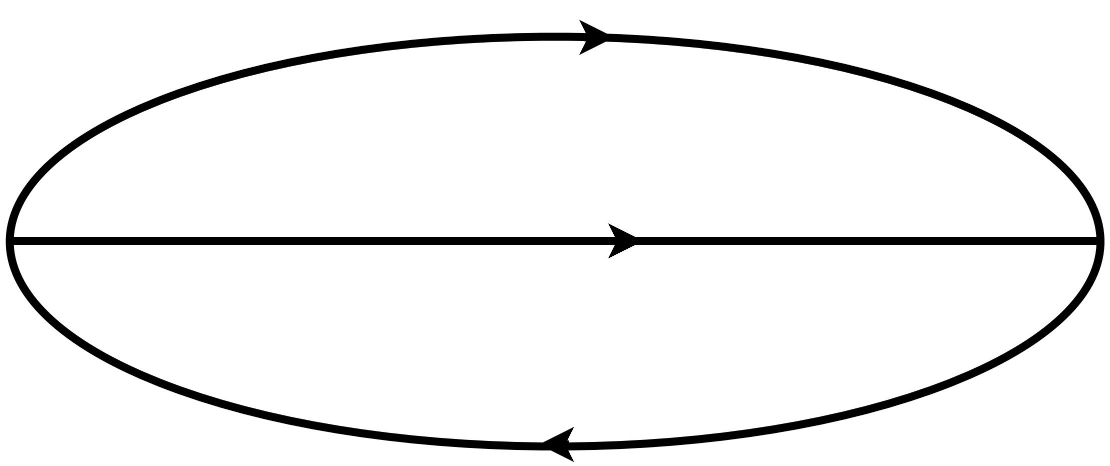
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$\Sigma =$ 

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

SYK model

- $T = 0$ Green's function $G \sim \begin{cases} -1/\sqrt{\tau} & \text{for } \tau > 0 \\ e^{-2\pi\mathcal{E}}/\sqrt{\tau} & \text{for } \tau < 0 \end{cases}$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

SYK model

- $T = 0$ Green's function $G \sim \left\{ \begin{array}{cc} -1/\sqrt{\tau} & \text{for } \tau > 0 \\ e^{-2\pi\mathcal{E}}/\sqrt{\tau} & \text{for } \tau < 0 \end{array} \right.$

- $T > 0$ Green's function has conformal invariance
  $G \sim e^{-2\pi\mathcal{E}T\tau}/(\sin(\pi T\tau))^{1/2}$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

SYK model

- $T = 0$ Green's function $G \sim \begin{cases} -1/\sqrt{\tau} & \text{for } \tau > 0 \\ e^{-2\pi\mathcal{E}}/\sqrt{\tau} & \text{for } \tau < 0 \end{cases}$

- $T > 0$ Green's function has conformal invariance
  $G \sim e^{-2\pi\mathcal{E}T\tau}/(\sin(\pi T\tau))^{1/2}$

- Non-zero GPS entropy as $T \to 0$, $S(T \to 0) = NS_0 + \ldots$

A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001)

SYK model

- $T = 0$ Green's function $G \sim \begin{cases} -1/\sqrt{\tau} & \text{for } \tau > 0 \\ e^{-2\pi\mathcal{E}}/\sqrt{\tau} & \text{for } \tau < 0 \end{cases}$

- $T > 0$ Green's function has conformal invariance $G \sim e^{-2\pi\mathcal{E}T\tau}/(\sin(\pi T\tau))^{1/2}$

- Non-zero GPS entropy as $T \to 0$, $S(T \to 0) = NS_0 + \ldots$

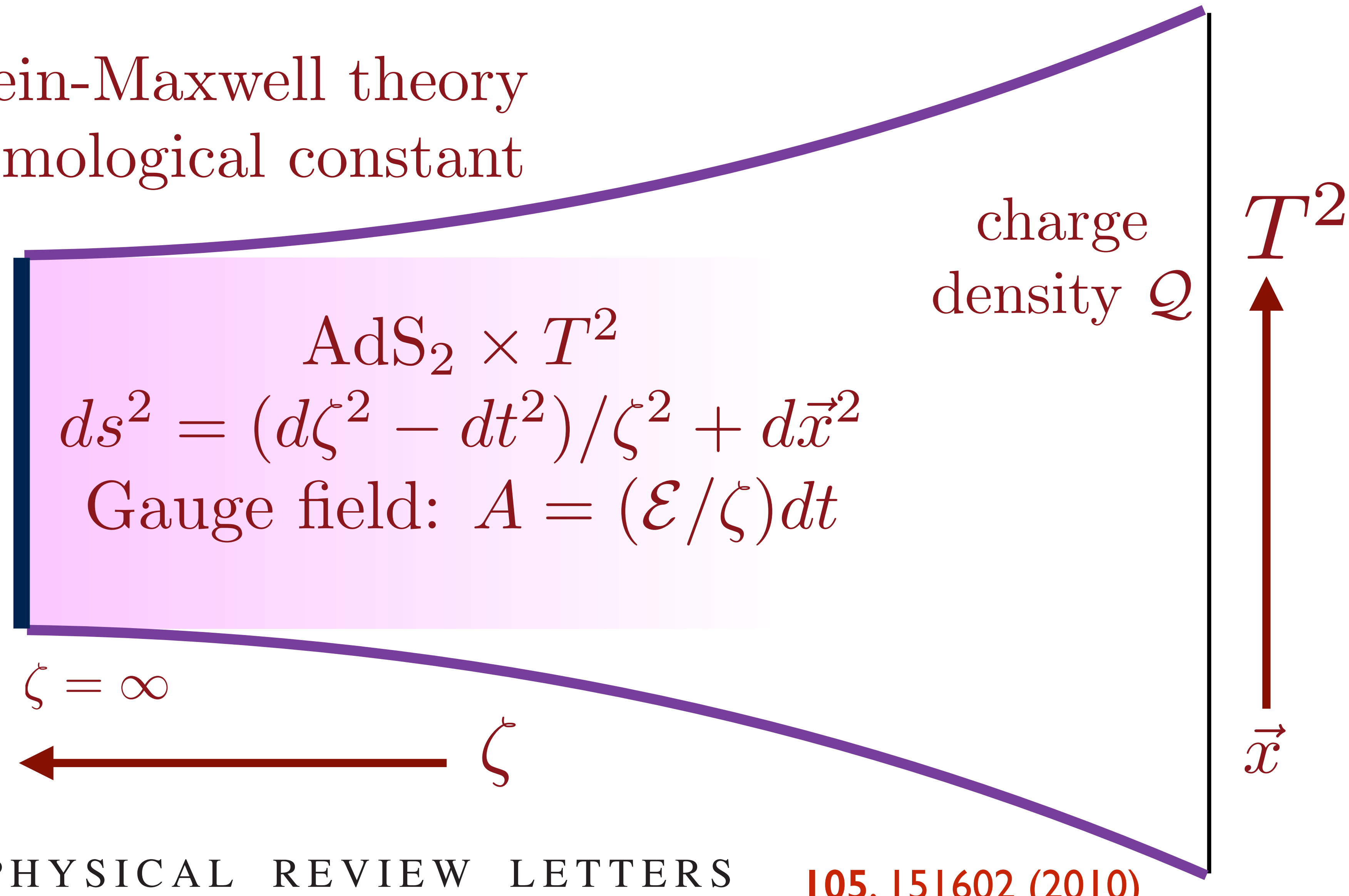
- SYK models are "are states of matter at non-zero density realizing the near-horizon, $\text{AdS}_2 \times R^2$ physics of Reissner-Nördstrom black holes". The Bekenstein-Hawking entropy is NS_0 (GPS = BH).

S. Sachdev, PRL **105**, 151602 (2010)

- $\mathcal{E}$ is identified with the electric field on AdS_2. The relationship $(\partial S/\partial \mathcal{Q})_T = 2\pi\mathcal{E}$ is obeyed both by the GPS entropy, and by the BH entropy of AdS_2 horizons in a large class of gravity theories.

S. Sachdev, PRX **5**, 041025 (2015)

SYK and AdS$_2$



PHYSICAL REVIEW LETTERS 105, 151602 (2010)

Holographic Metals and the Fractionalized Fermi Liquid

Subir Sachdev

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti–de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a "small" Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, AdS$_2 \times$ R^2 physics of Reissner-Nordström black holes.

SYK model

After integrating the fermions, the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$

$$S = \ln\det\left[\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)\right]$$

$$+ \int d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2)\left[G(\tau_2, \tau_1) + (J^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)\right]$$

A. Kitaev, unpublished

SYK model

After integrating the fermions, the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1,\tau_2)\mathcal{D}\Sigma(\tau_1,\tau_2)\exp(-NS)$$

$$S = \ln\det\left[\delta(\tau_1-\tau_2)(\cancel{\partial_{\tau_1}} + \cancel{\mu}) - \Sigma(\tau_1,\tau_2)\right]$$
$$+ \int d\tau_1 d\tau_2 \Sigma(\tau_1,\tau_2)\left[G(\tau_2,\tau_1) + (J^2/2)G^2(\tau_2,\tau_1)G^2(\tau_1,\tau_2)\right]$$

At frequencies $\ll J$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1,\tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-1/4}\frac{g(\sigma_1)}{g(\sigma_2)}G(\sigma_1,\sigma_2)$$

$$\Sigma(\tau_1,\tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-3/4}\frac{g(\sigma_1)}{g(\sigma_2)}\Sigma(\sigma_1,\sigma_2)$$

S. Sachdev, PRX **5**, 041025 (2015)

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Kitaev, unpublished

SYK model

Let us write the large N saddle point solutions of S as

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2} \quad , \quad \Sigma_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-3/2}.$$

These are not invariant under the reparametrization symmetry but are invariant only under a SL(2,R) subgroup under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken.

A. Kitaev, unpublished

SYK model

Let us write the large N saddle point solutions of S as

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So ... en.

Connections of SYK to gravity and AdS$_2$ horizons

- Reparameterization and gauge invariance are the ‘symmetries’ of the Einstein-Maxwell theory of gravity and electromagnetism

- SL(2,R) is the isometry group of AdS$_2$.

A. Kitaev, unpublished

SYK model

Let us write the large N saddle point solutions of S as

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2} \quad , \quad \Sigma_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-3/2}.$$

These are not invariant under the reparametrization symmetry but are invariant only under a SL(2,R) subgroup under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken.

Reparametrization zero mode

Expand about the saddle point by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1) f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2))$$

(and similarly for Σ) and obtain an effective action for $f(\tau)$. This action does not vanish because of the time derivative in the determinant which is not reparameterization invariant.

J. Maldacena and D. Stanford, arXiv:1604.07818

See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

SYK model

With $g(\tau) = e^{-i\phi(\tau)}$, the action for $\phi(\tau)$ and $f(\tau) = \dfrac{1}{\pi T}\tan(\pi T(\tau + \epsilon(\tau))$ fluctuations is

$$S_{\phi,f} = \frac{K}{2}\int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T)\partial_\tau \epsilon)^2 - \frac{\gamma}{4\pi^2}\int_0^{1/T} d\tau\, \{f,\tau\},$$

where $\{f,\tau\}$ is the Schwarzian:

$$\{f,\tau\} \equiv \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2 .$$

The couplings are given by thermodynamics: Ω is the grand potential, S_0 is the GPS entropy, and $\mathcal{Q}$ is the density.

$$K = -\left(\frac{\partial^2 \Omega}{\partial \mu^2}\right)_T \quad , \quad \gamma + 4\pi^2 \mathcal{E}^2 K = -\left(\frac{\partial^2 \Omega}{\partial T^2}\right)_\mu$$

$$2\pi\mathcal{E} = \frac{\partial S_0}{\partial \mathcal{Q}}$$

In holography: the γ term in the action has been obtained from theories on AdS_2; $\mathcal{E}$ is the electric field, and has the same relationship to S_0.

J. Maldacena and D. Stanford, arXiv:1604.07818; Wenbo Fu, Yingfei Gu, S. Sachdev, unpublished;
S. Sachdev, PRX **5**, 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

SYK model

With $g(\tau) = e^{-i\phi(\tau)}$, the action for $\phi(\tau)$ and $f(\tau) = \dfrac{1}{\pi T}\tan(\pi T(\tau + \epsilon(\tau))$ fluctuations is

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where $\{f,\tau\}$ is the Schwarzian:

$$\{f,\tau\} \equiv \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2 .$$

The correlators of the density fluctuations, $\delta\mathcal{Q}(\tau)$, and the energy fluctuations $\delta E - \mu\delta\mathcal{Q}(\tau)$ are time independent and given by

$$\left(\begin{array}{cc} \langle \delta\mathcal{Q}(\tau)\delta\mathcal{Q}(0)\rangle & \langle (\delta E(\tau) - \mu\delta\mathcal{Q}(\tau))\delta\mathcal{Q}(0)\rangle / T \\ \langle (\delta E(\tau) - \mu\delta\mathcal{Q}(\tau))\delta\mathcal{Q}(0)\rangle & \langle (\delta E(\tau) - \mu\delta\mathcal{Q}(\tau))(\delta E(0) - \mu\delta\mathcal{Q}(0))\rangle / T \end{array} \right) = T\chi_s$$

where χ_s is the static susceptibility matrix given by

$$\chi_s \equiv \left(\begin{array}{cc} -(\partial^2\Omega/\partial\mu^2)_T & -\partial^2\Omega/(\partial T\partial\mu) \\ -T\partial^2\Omega/(\partial T\partial\mu) & -T(\partial^2\Omega/\partial T^2)_\mu \end{array} \right).$$

Wenbo Fu, Yingfei Gu, S. Sachdev, unpublished

Coupled SYK models

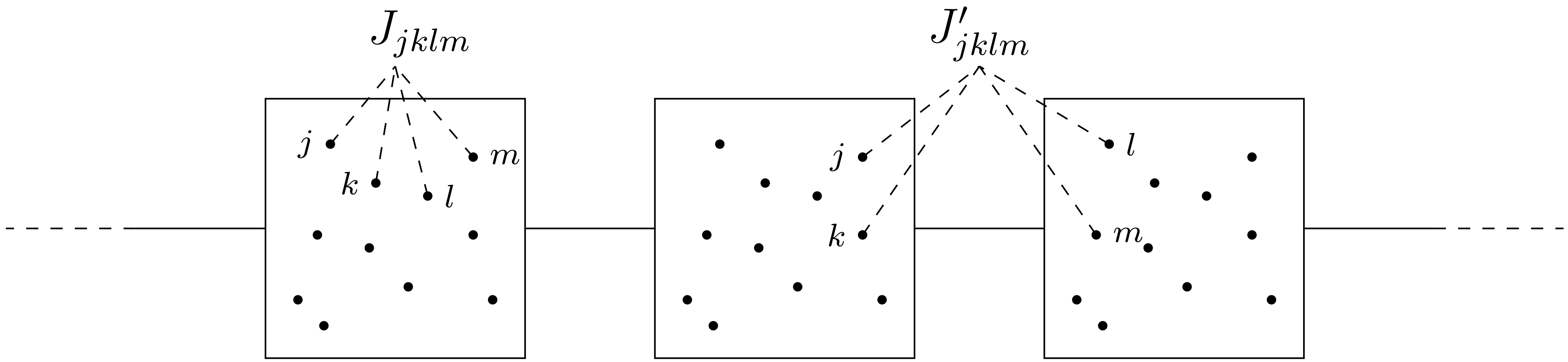


Figure 1: A chain of coupled SYK sites: each site contains $N \gg 1$ fermion with SYK interaction. The coupling between nearest neighbor sites are four fermion interaction with two from each site.

Yingfei Gu, Xiao-Liang Qi, and D. Stanford, arXiv:1609.07832

SYK models

The correlators of the density fluctuations, $\delta\mathcal{Q}(\tau)$, and the energy fluctuations $\delta E - \mu\delta\mathcal{Q}(\tau)$ are time independent and given by

$$\left(\begin{array}{cc} \langle \delta\mathcal{Q}(\tau)\delta\mathcal{Q}(0)\rangle & \langle (\delta E(\tau) - \mu\delta\mathcal{Q}(\tau))\delta\mathcal{Q}(0)\rangle /T \\ \langle (\delta E(\tau) - \mu\delta\mathcal{Q}(\tau))\delta\mathcal{Q}(0)\rangle & \langle (\delta E(\tau) - \mu\delta\mathcal{Q}(\tau))(\delta E(0) - \mu\delta\mathcal{Q}(0))\rangle /T \end{array} \right) = T\chi_s$$

where χ_s is the static susceptibility matrix given by

$$\chi_s \equiv \left(\begin{array}{cc} -(\partial^2\Omega/\partial\mu^2)_T & -\partial^2\Omega/(\partial T\partial\mu) \\ -T\partial^2\Omega/(\partial T\partial\mu) & -T(\partial^2\Omega/\partial T^2)_\mu \end{array} \right).$$

Coupled SYK models

$$\left(\begin{array}{cc} \langle \mathcal{Q};\mathcal{Q}\rangle_{k,\omega} & \langle E-\mu\mathcal{Q};\mathcal{Q}\rangle_{k,\omega}/T \\ \langle E-\mu\mathcal{Q};\mathcal{Q}\rangle_{k,\omega} & \langle E-\mu\mathcal{Q};E-\mu\mathcal{Q}\rangle_{k,\omega}/T \end{array} \right) = \left[i\omega(-i\omega + Dk^2)^{-1} + 1 \right] \chi_s$$

where the diffusivities are related to the thermoelectric conductivities by the Einstein relations

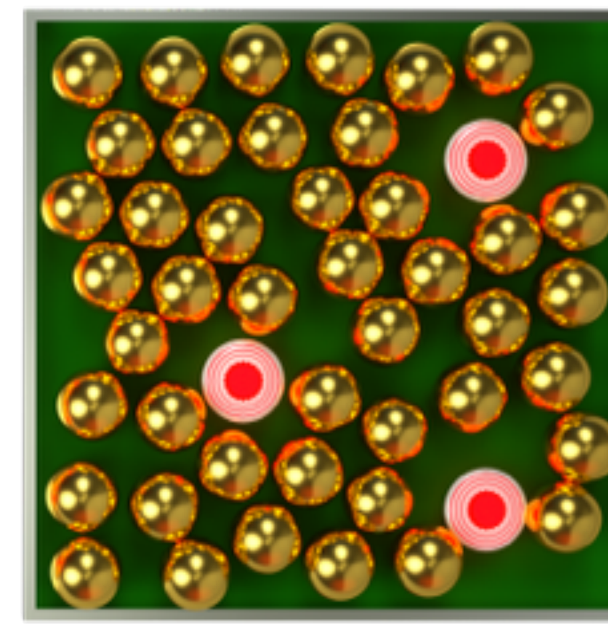
$$D = \left(\begin{array}{cc} \sigma & \alpha \\ \alpha T & \overline{\kappa} \end{array} \right) \chi_s^{-1}.$$

Coupled SYK models

$$\begin{pmatrix} \langle \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} / T \\ \langle E - \mu \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu \mathcal{Q}; E - \mu \mathcal{Q} \rangle_{k,\omega} / T \end{pmatrix} = \left[i\omega(-i\omega + Dk^2)^{-1} + 1 \right] \chi_s$$

where the diffusivities are related to the thermoelectric conductivities by the Einstein relations

$$D = \begin{pmatrix} \sigma & \alpha \\ \alpha T & \overline{\kappa} \end{pmatrix} \chi_s^{-1}.$$

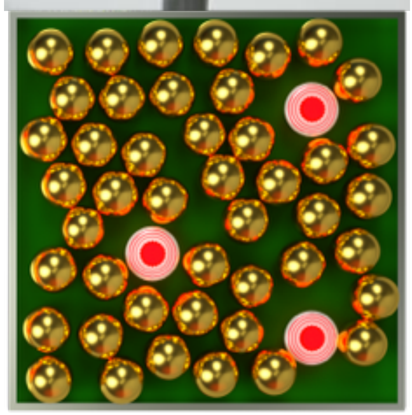


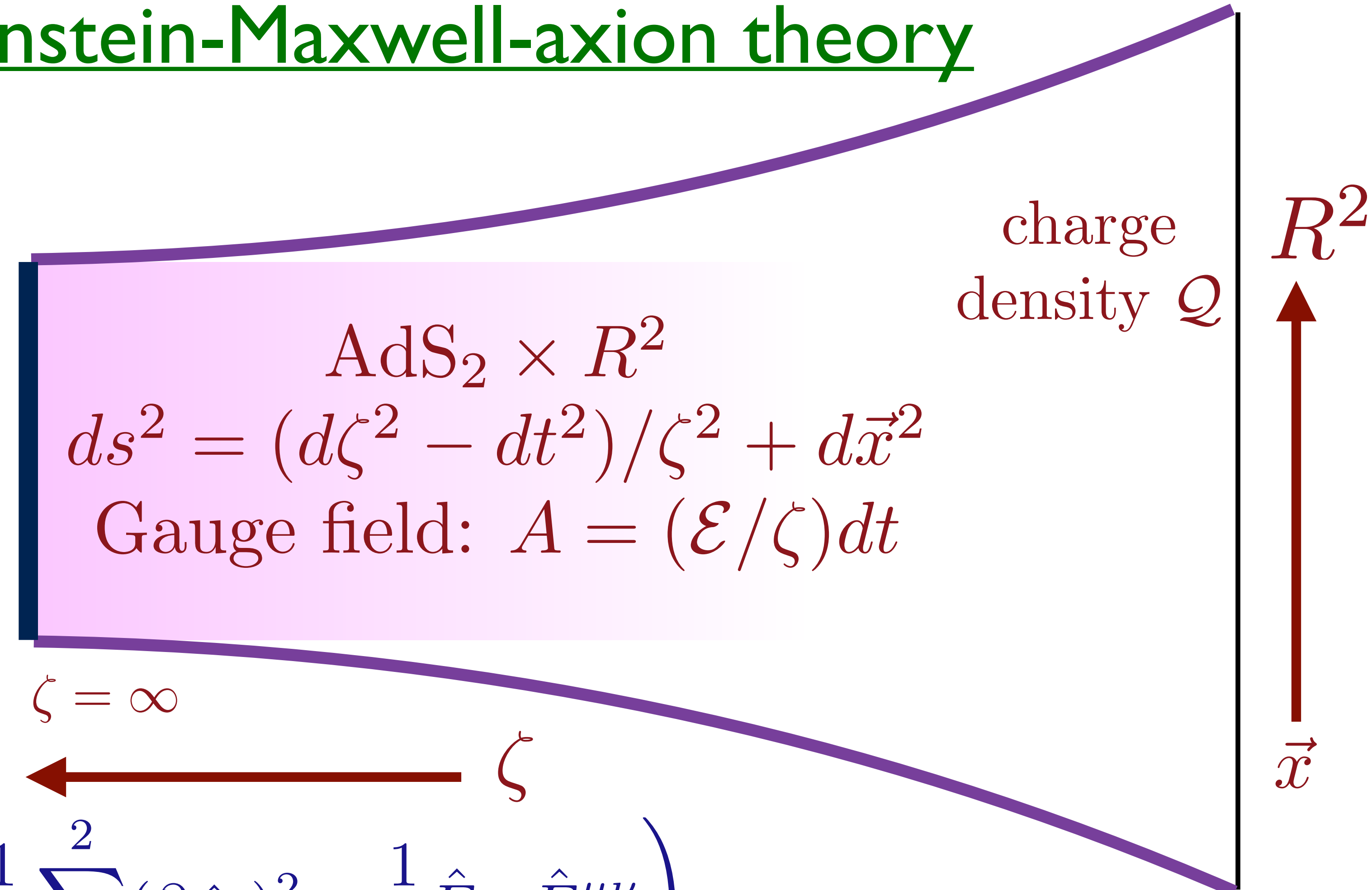
$$\boxed{D \sim \frac{\hbar v_{\rm butterfly}^2}{k_B T}}$$

M. Blake, PRL **117**, 091601 (2016)
Yingfei Gu, Xiao-Liang Qi, and D. Stanford, arXiv:1609.07832

The coupled SYK models realize a diffusive metal with no quasiparticle excitations.
(a "strange metal")

Holography: Einstein-Maxwell-axion theory





$$S = \int d^4x\sqrt{-\hat{g}}\left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{2}\sum_{i=1}^{2}(\partial\hat{\varphi}_i)^2 - \frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}\right),$$

- For $\hat{\varphi}_i = 0$, we obtain the Reissner-Nördstrom-AdS charged black hole, with a near-horizon $AdS_2 \times R^2$ near-horizon geometry.

- For $\hat{\varphi}_i = kx_i$, we obtain a similar solution but with momentum dissipation (a bulk massive graviton). This yields the same diffusive metal correlators as the coupled SYK models, and the same relationship between D and $v_{\rm butterfly}$.

Y. Bardoux, M. M. Caldarelli, and C. Charmousis, JHEP 05 (2012) 054; D. Vegh, arXiv:1301.0537;
R. A. Davison, PRD 88 (2013) 086003; M. Blake and D. Tong, PRD 88 (2013), 106004;
T. Andrade and B. Withers, JHEP 05 (2014) 101; M. Blake, PRL **117**, 091601 (2016).

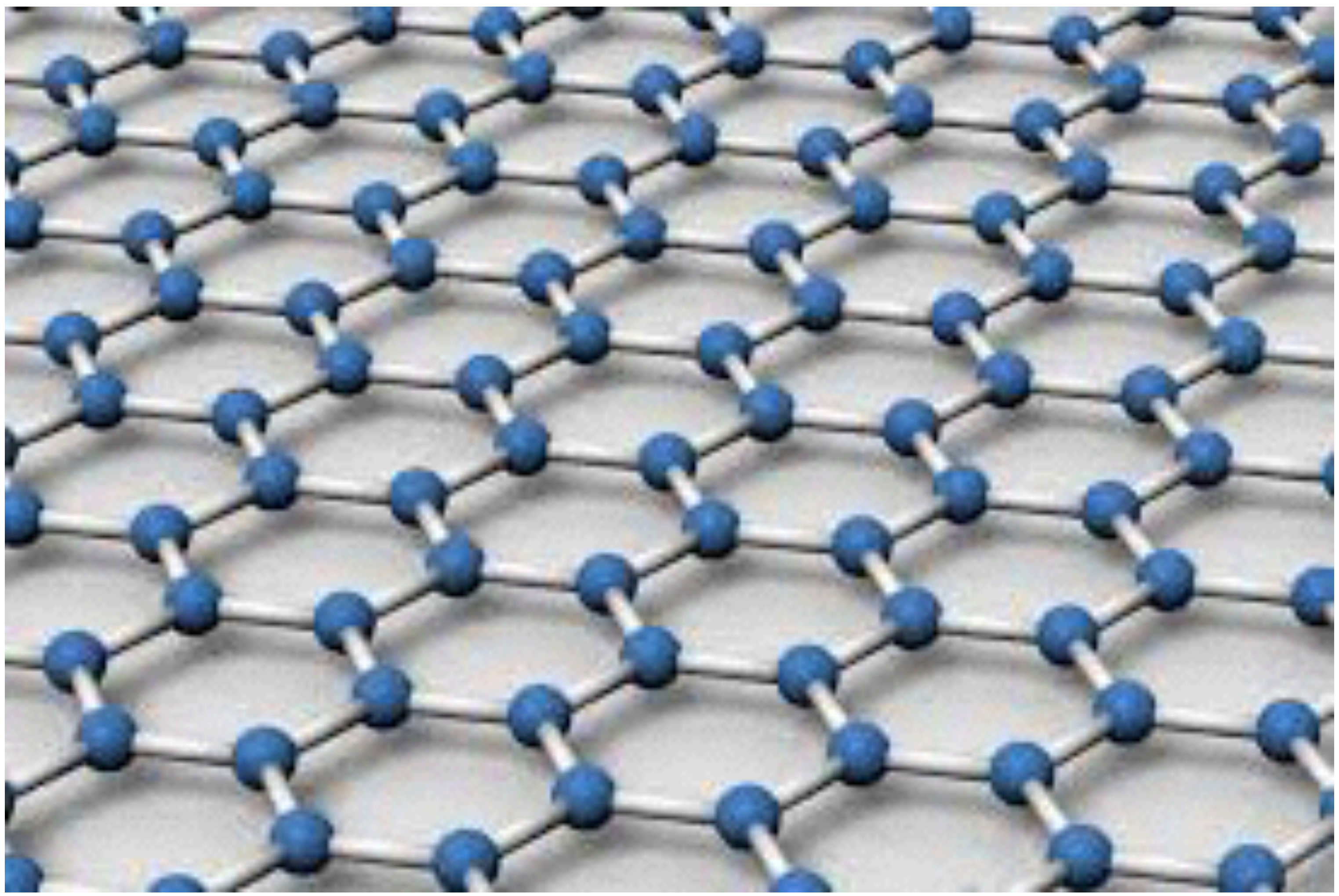
- Graphene

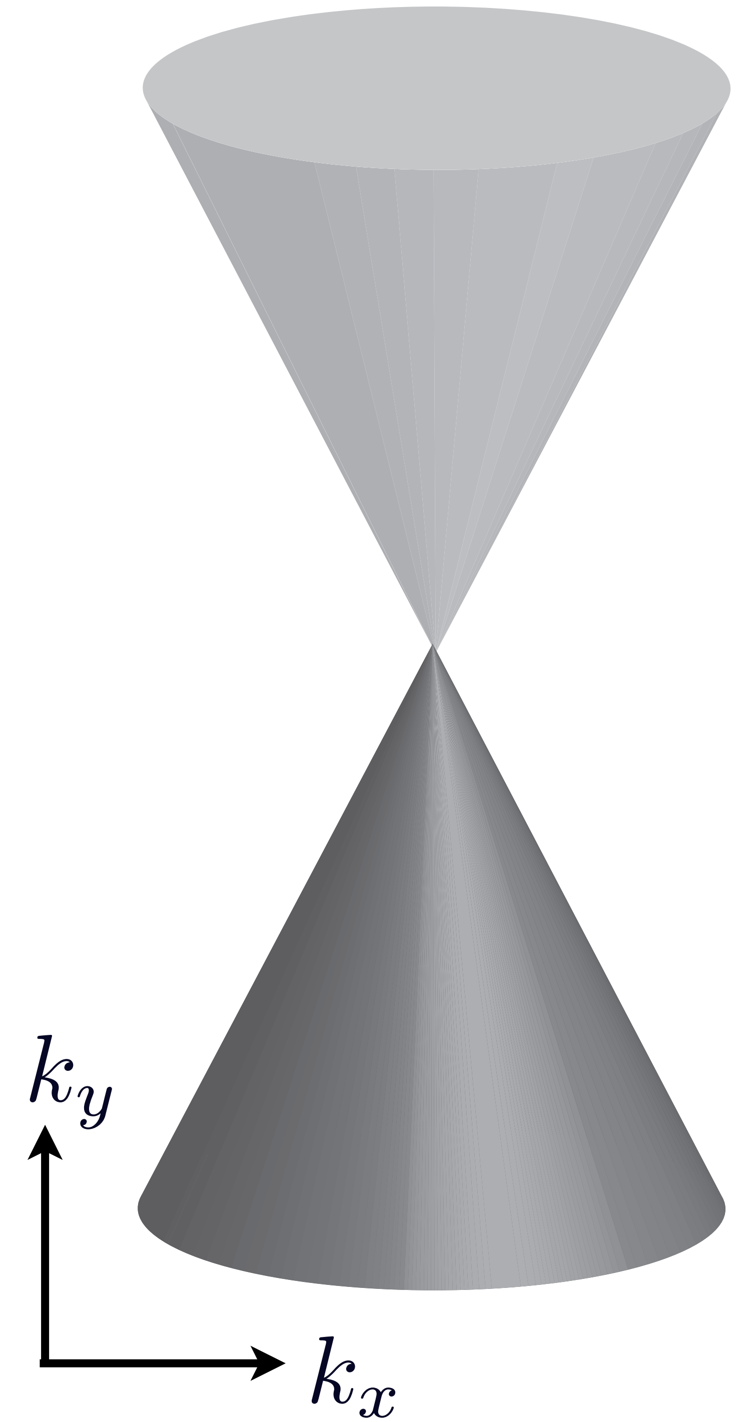
Strange metal transport

Theoretical predictions inspired by holography

Comparison with experiments

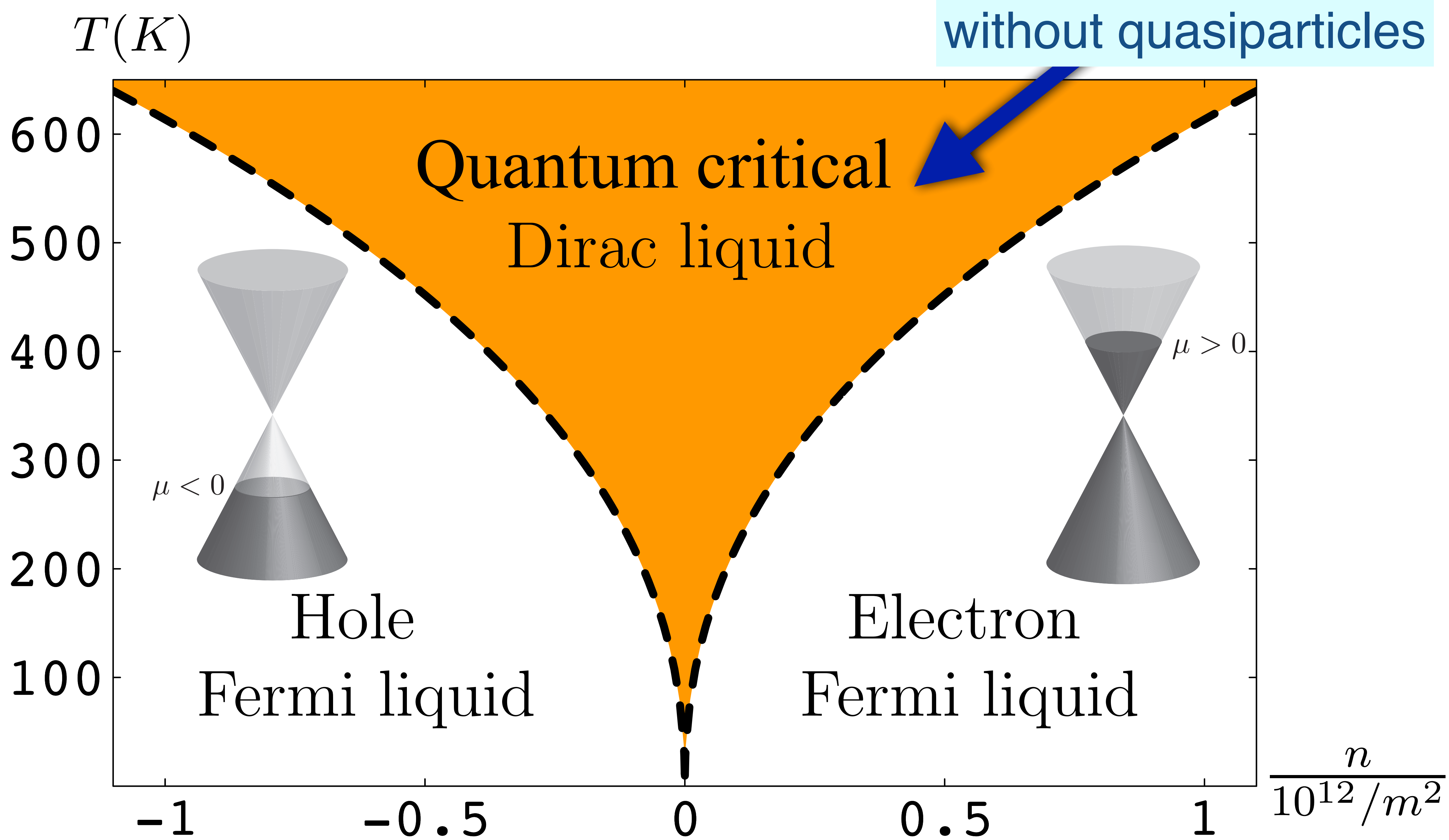
Graphene





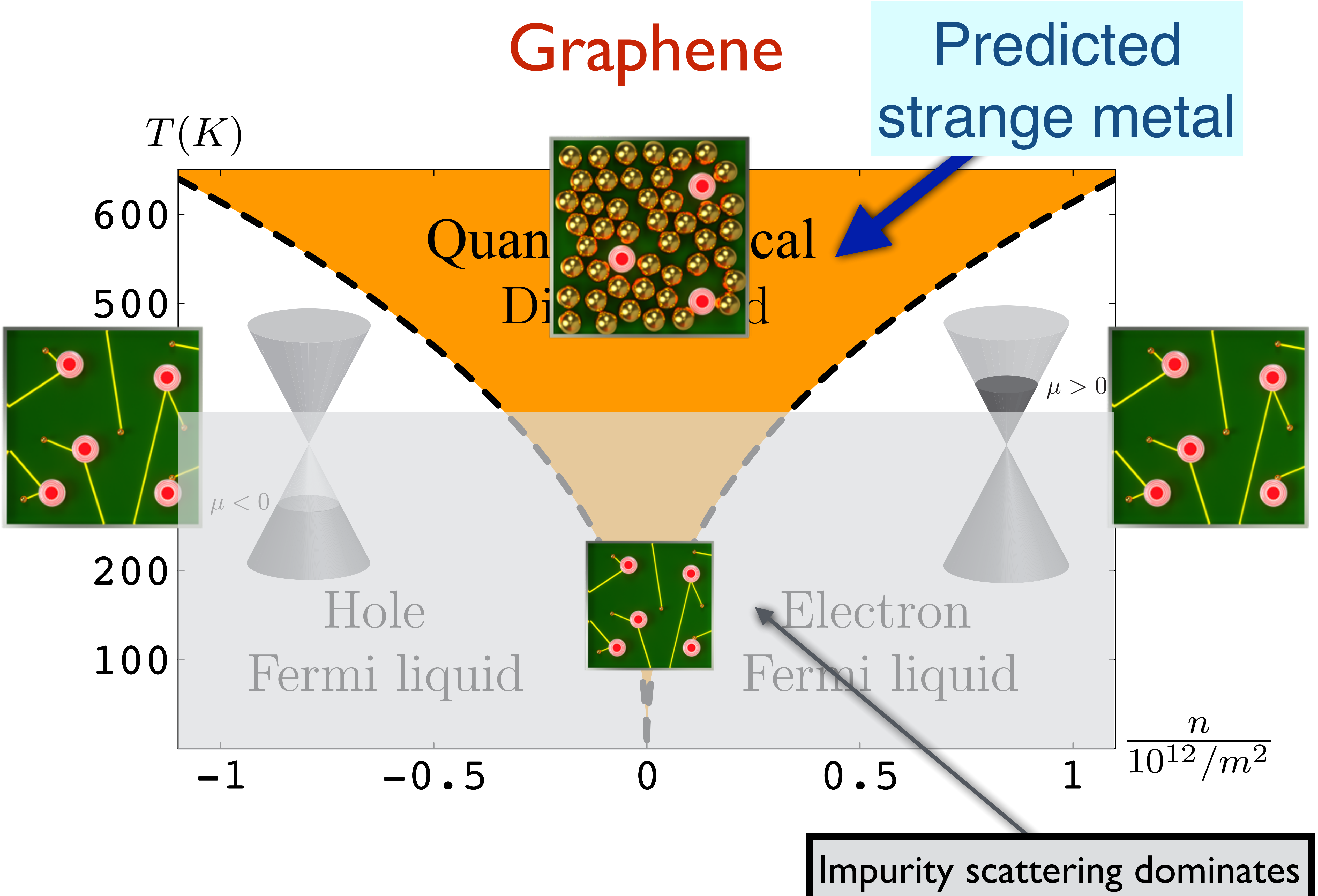
Graphene

Predicted
"strange metal"
without quasiparticles



K. Damle and S. Sachdev, PRB **56**, 8714 (1997)
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Graphene



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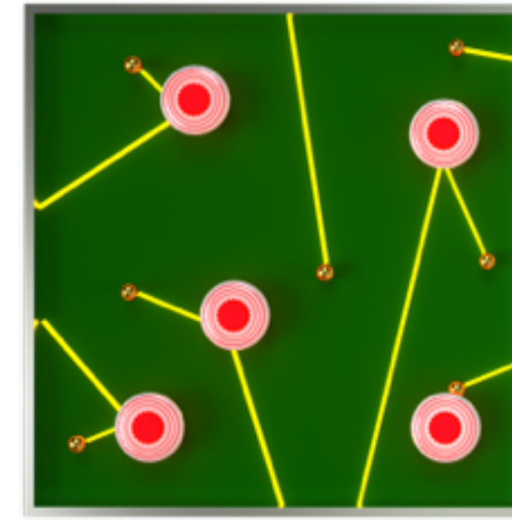
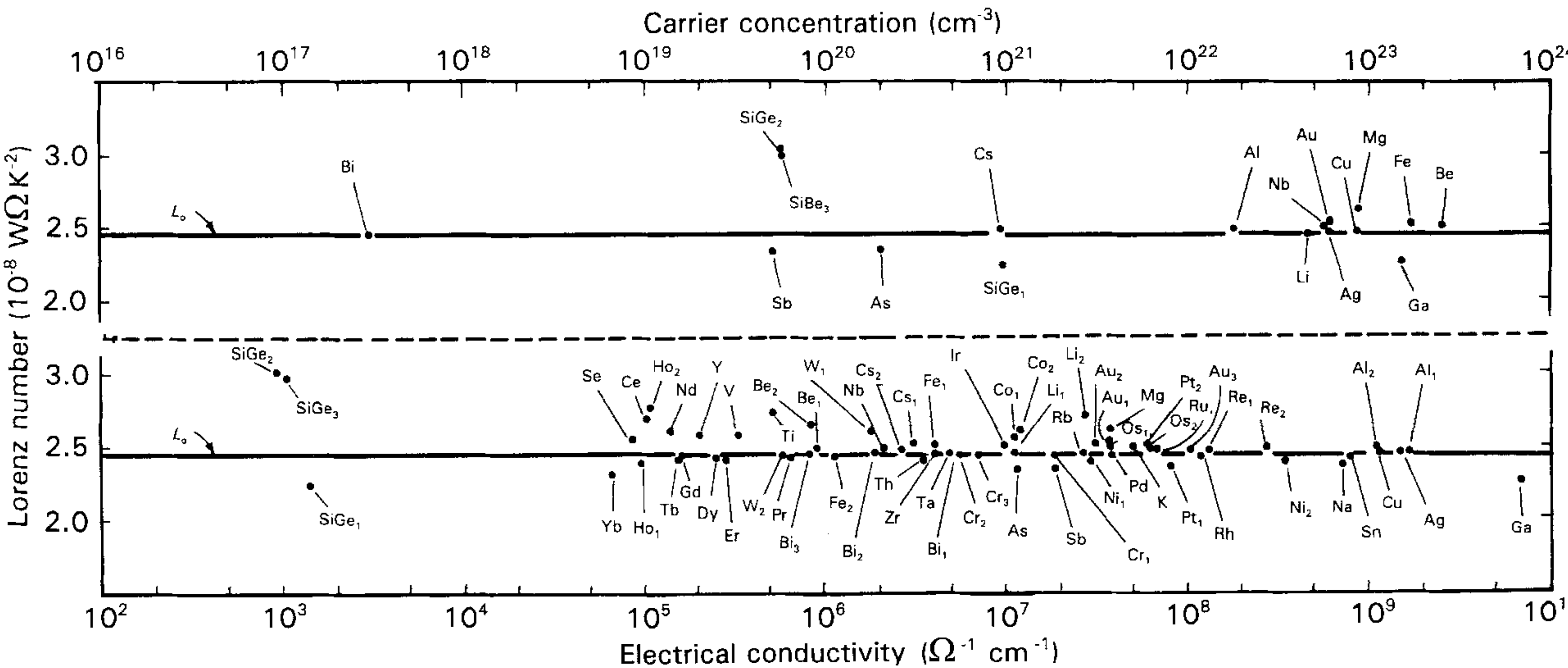
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Thermal and electrical conductivity with quasiparticles



- Wiedemann-Franz law in a Fermi liquid:

$$L_0 = \frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \, \frac{\mathrm{W} \cdot \Omega}{\mathrm{K}^2}.$$



G. S. Kumar, G. Prasad, and R.O. Pohl, J. Mat. Sci. **28**, 4261 (1993)

Prediction for transport in the graphene strange metal

For a strange metal with a "relativistic" Hamiltonian, hydrodynamic, holographic, and memory function methods yield for the Lorentz ratio $L = \kappa/(T\sigma)$

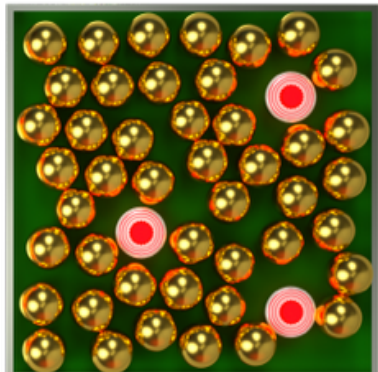
$$\sigma = \sigma_Q \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\text{imp}}}{\mathcal{H}\sigma_Q}\right) \quad , \quad \kappa = \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\text{imp}}}{\mathcal{H}\sigma_Q}\right)^{-1}$$

$$L = \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\text{imp}}}{\mathcal{H}\sigma_Q}\right)^{-2},$$

where $\mathcal{H}$ is the enthalpy density, τ_{imp} is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, "quantum critical" conductivity.

- For $\mathcal{Q} = 0$, as $\tau_{\text{imp}} \to \infty$, σ remains finite, while $\kappa \to \infty$, and so $L \to \infty$.
- For $\mathcal{Q} \neq 0$, as $\tau_{\text{imp}} \to \infty$, $\sigma \to \infty$, while κ remains finite, and so $L \to 0$.

Prediction: L diverges as $1/\mathcal{Q}^4$ near $\mathcal{Q} = 0$ in clean graphene.



S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
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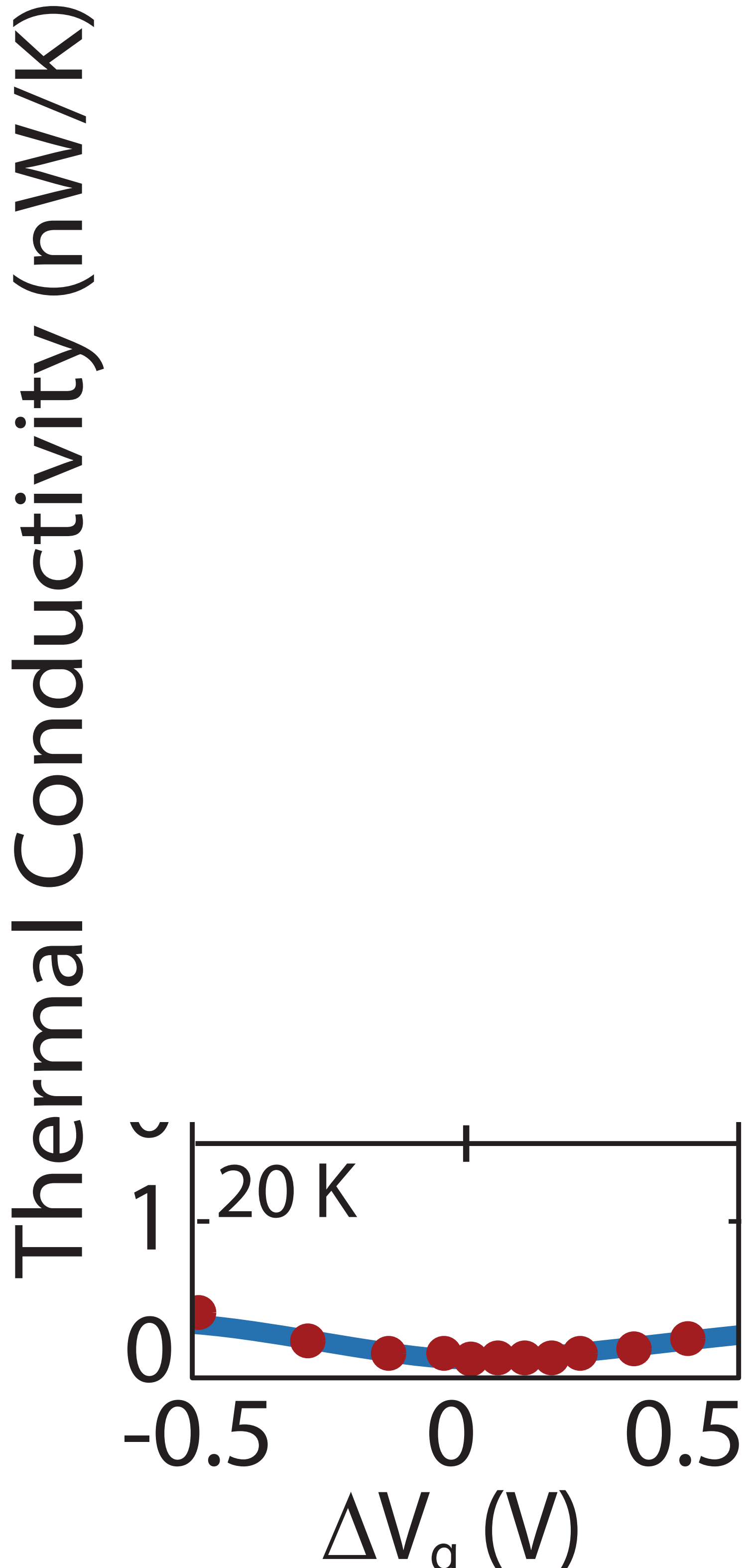
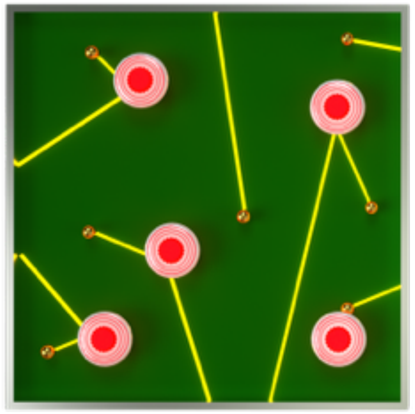
J. Crossno, Jing K. Shi, Ke Wang, Xiaomeng Liu, A. Harzheim, A. Lucas, S. Sachdev, Philip Kim, Takashi Taniguchi, Kenji Watanabe, T. A. Ohki, and Kin Chung Fong, Science **351**, 1058 (2016)



Red dots: data

Blue line: value for $L = L_0$



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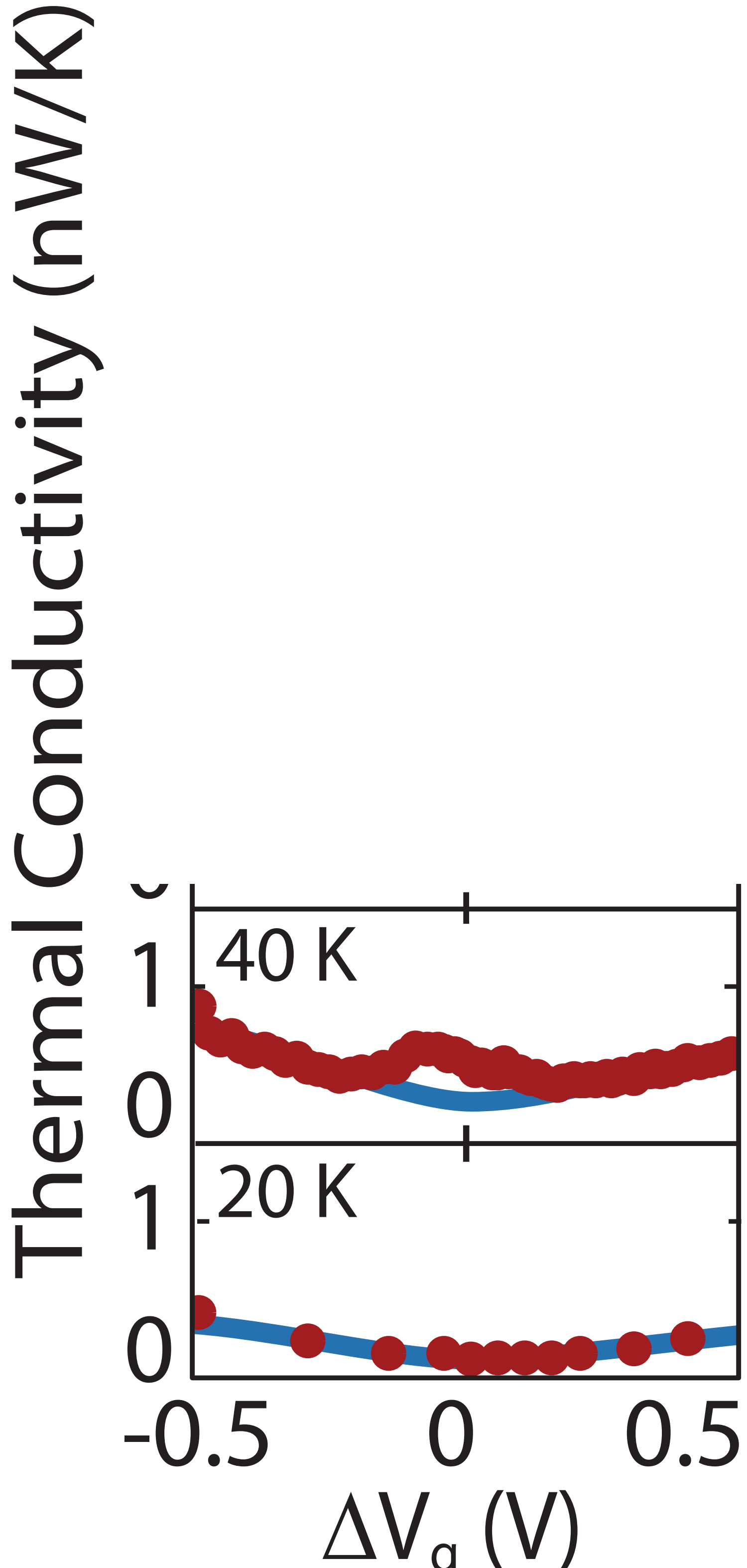

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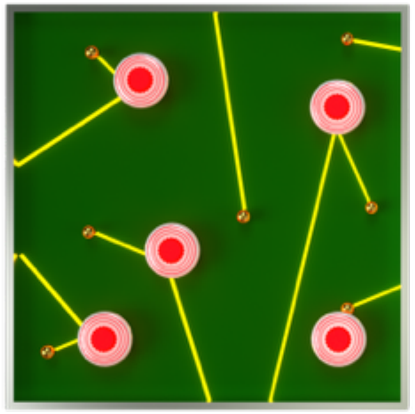

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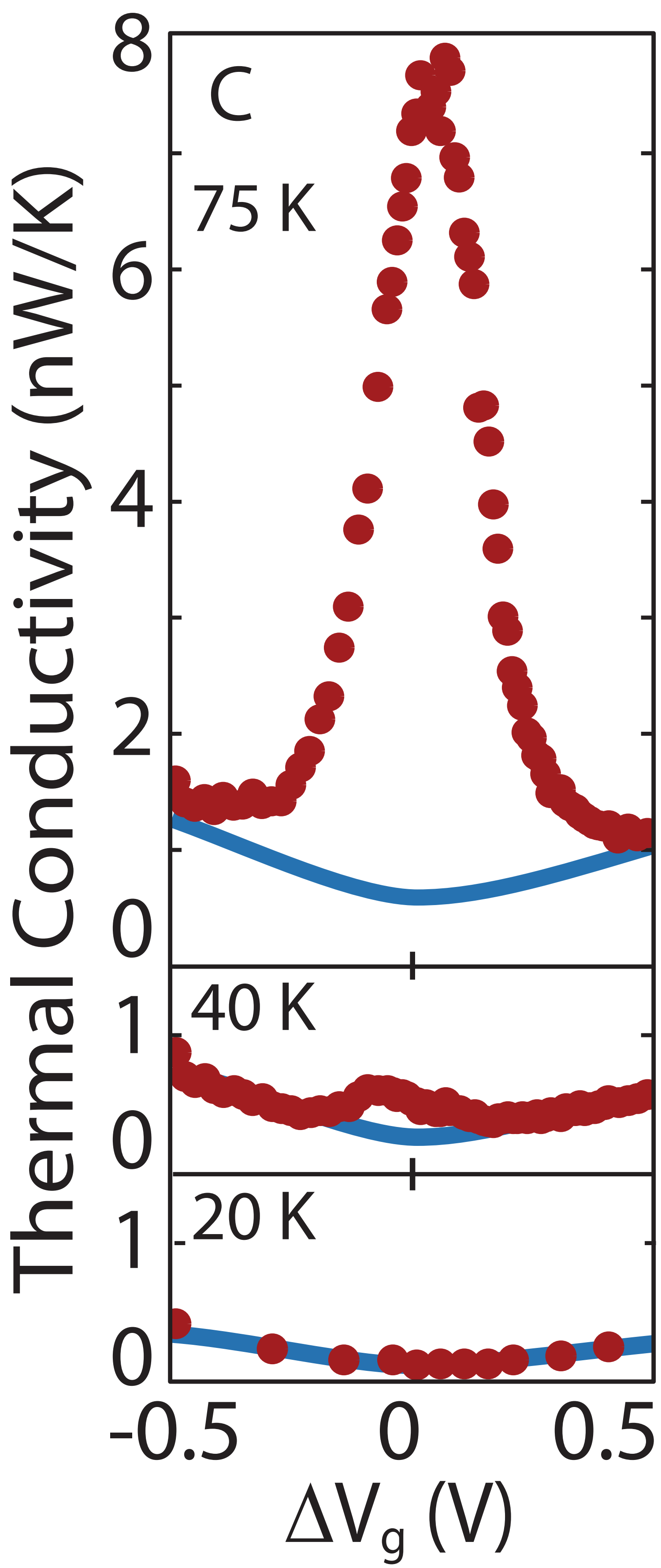

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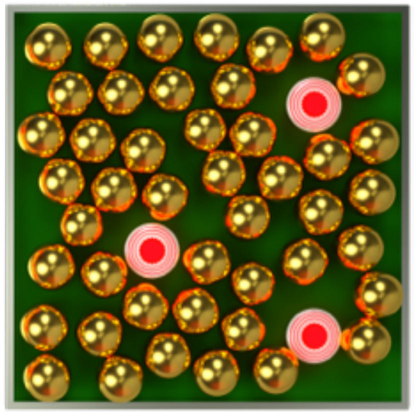

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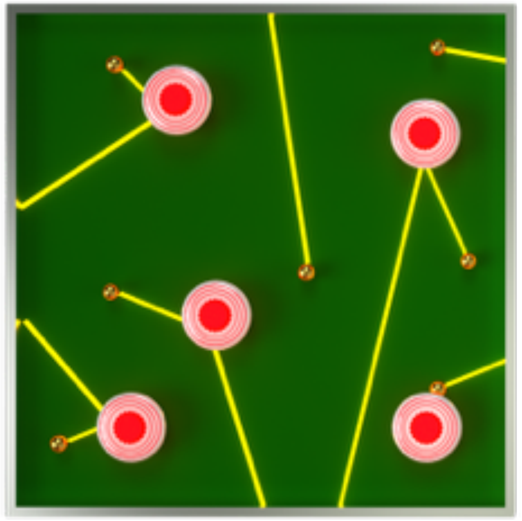
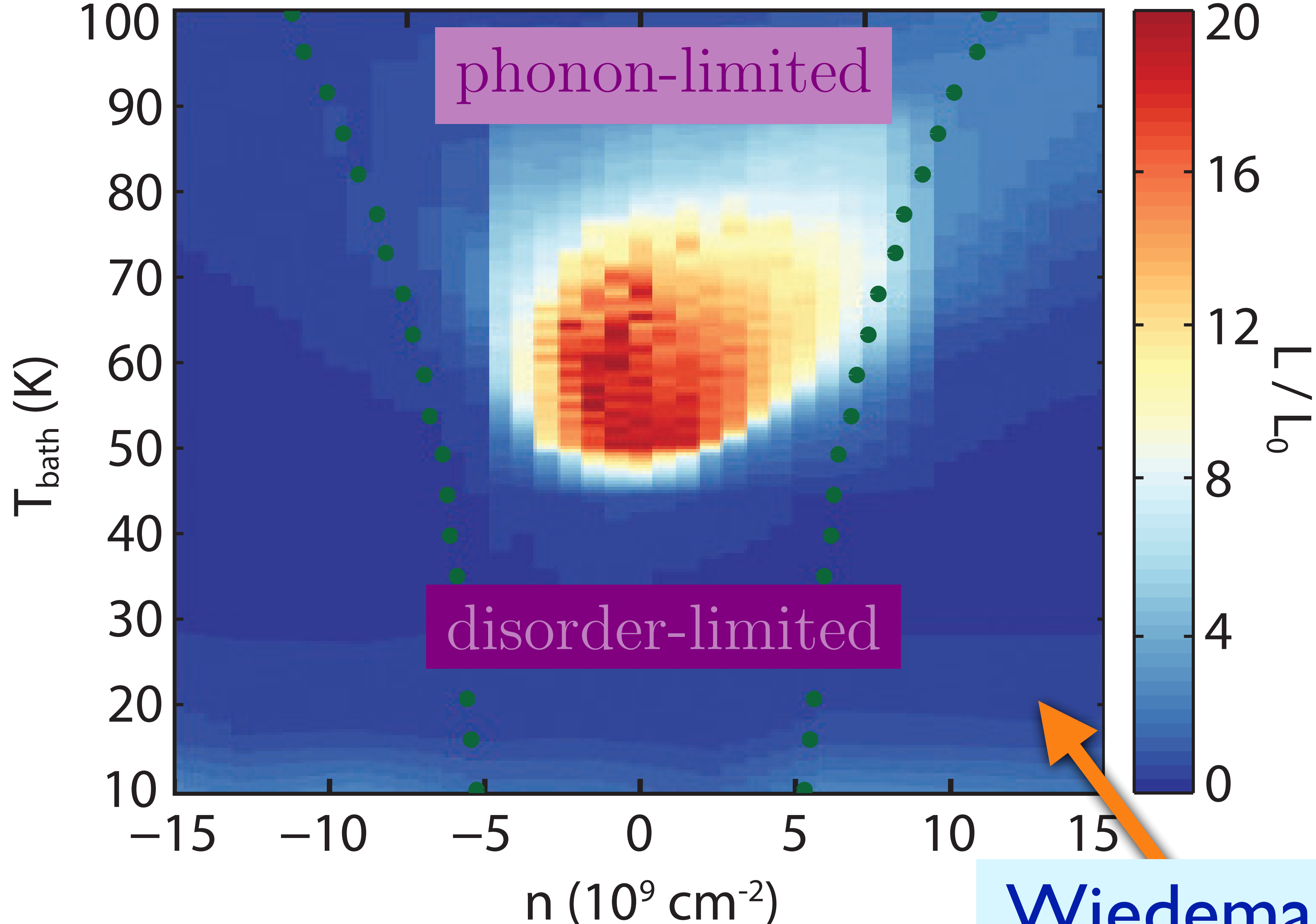
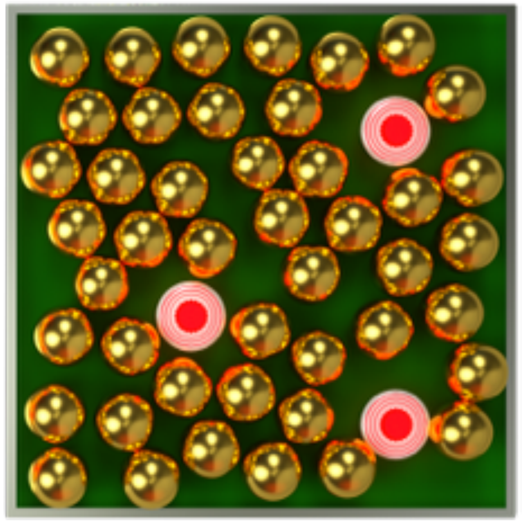


Strange metal in graphene

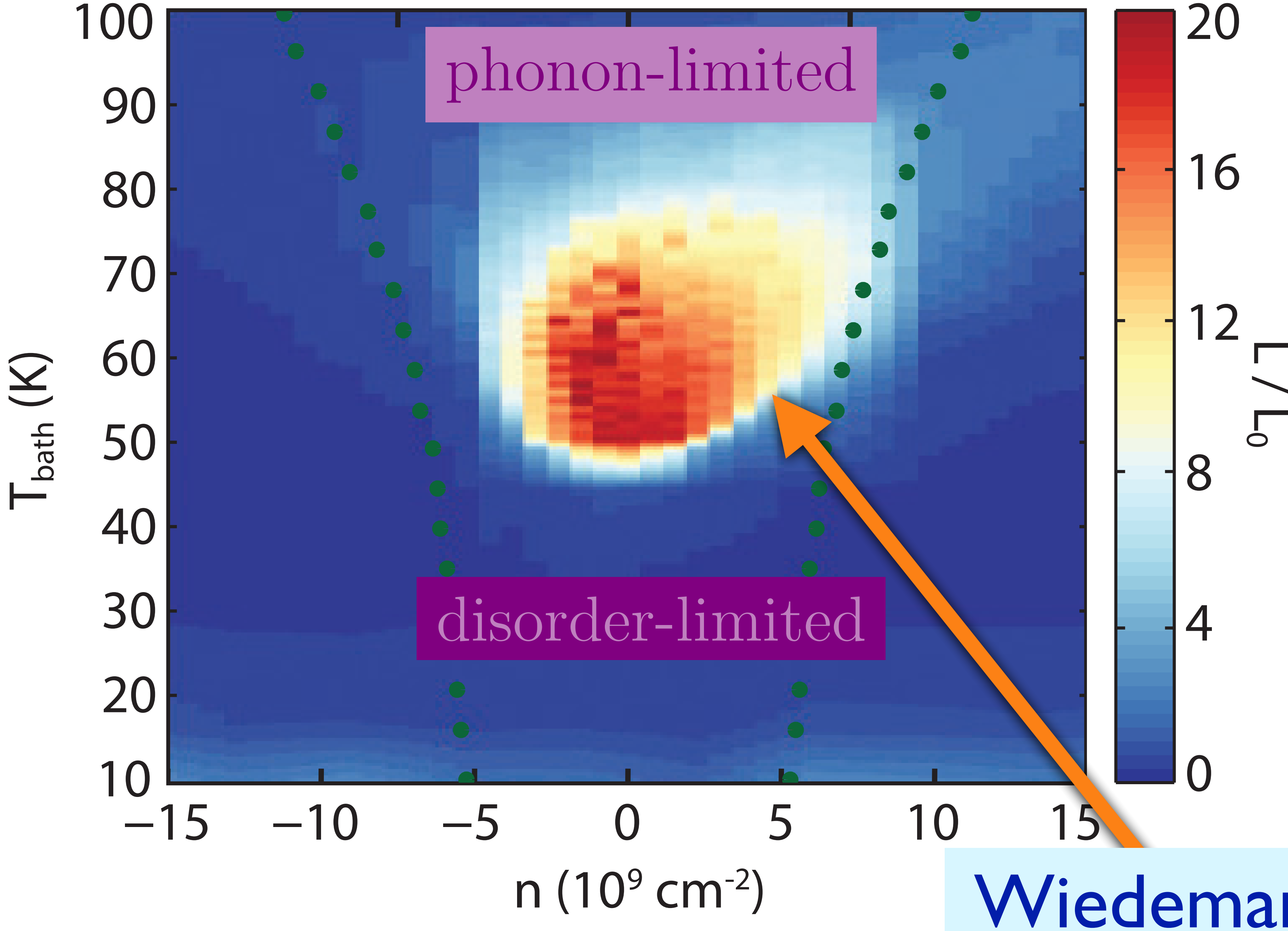


Wiedemann-Franz obeyed

J. Crossno, Jing K. Shi, Ke Wang, Xiaomeng Liu, A. Harzheim, A. Lucas, S. Sachdev, Philip Kim, Takashi Taniguchi, Kenji Watanabe, T. A. Ohki, and Kin Chung Fong, Science **351**, 1058 (2016)



Strange metal in graphene



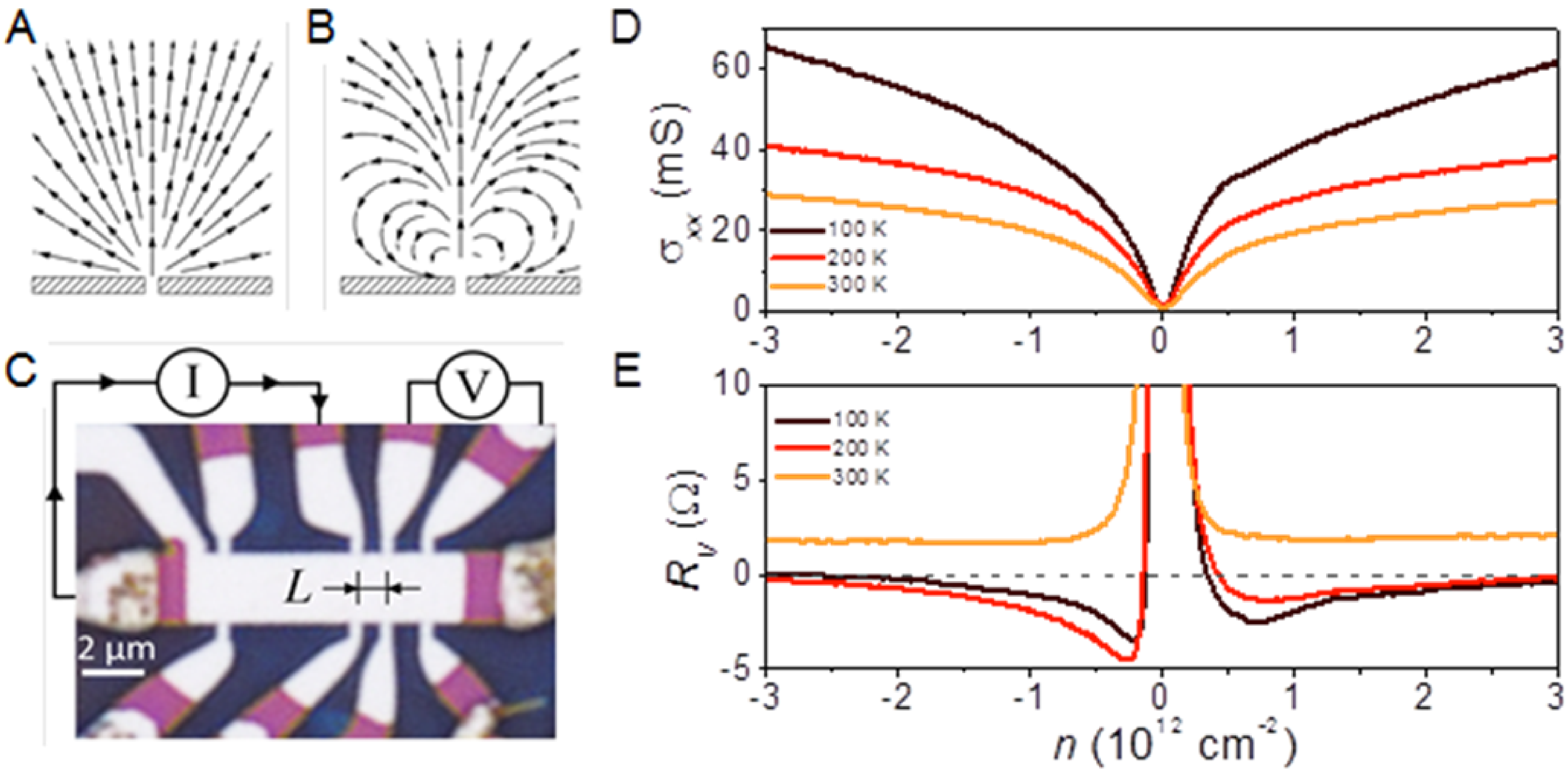
Wiedemann-Franz violated !

Science **351**, 1055 (2016)

Strange metal in graphene

Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin[1], I. Torre[2,3], R. Krishna Kumar[1,4], M. Ben Shalom[1,5], A. Tomadin[6], A. Principi[7], G. H. Auton[5], E. Khestanova[1,5], K. S. Novoselov[5], I. V. Grigorieva[1], L. A. Ponomarenko[1,4], A. K. Geim[1], M. Polini[3,6]



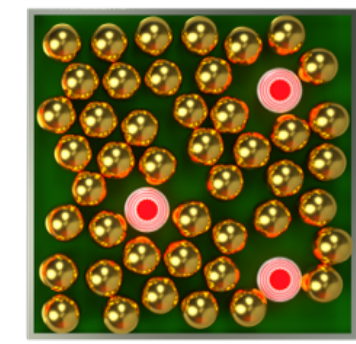


Figure 1. Viscous backflow in doped graphene. (**a,b**) Steady-state distribution of current injected through a narrow slit for a classical conducting medium with zero ν (a) and a viscous Fermi liquid (b). (**c**) Optical micrograph of one of our SLG devices. The schematic explains the measurement geometry for vicinity resistance. (**d,e**) Longitudinal conductivity σ_{xx} and R_{V} for this device as a function of n induced by applying gate voltage. $I = 0.3$ μA; $L = 1$ μm. For more detail, see Supplementary Information.

See also L. Levitov and G. Falkovich, *Nature Physics* **12**, 672 (2016)

Entangled quantum matter without quasiparticles

- Is there a connection between strange metals and black holes?
  Yes, *e.g.* the SYK model.

- Why do they have the same equilibration time $\sim \hbar/(k_B T)$?
  Strange metals don't have quasiparticles and thermalize rapidly;
  Black holes are "fast scramblers".

- Theoretical predictions for strange metal transport in graphene agree well with experiments