



# Quantum Criticality and Black Holes

*Sean Hartnoll, Chris Herzog, Pavel Kovtun,*

*Markus Müller, Subir Sachdev, Dam Son*

Talk online at <http://sachdev.physics.harvard.edu>



# Outline

## 1. CFT3s in condensed matter physics

*Superfluid-insulator and Neel-valence bond solid transitions*

## 2. Quantum-critical transport

*Collisionless- $t_0$ -hydrodynamic crossover of CFT3s*

## 3. SYM3 with $\mathcal{N} = 8$ supersymmetry

## 4. Nernst effect in the cuprate superconductors

*Quantum criticality and dyonic black holes*

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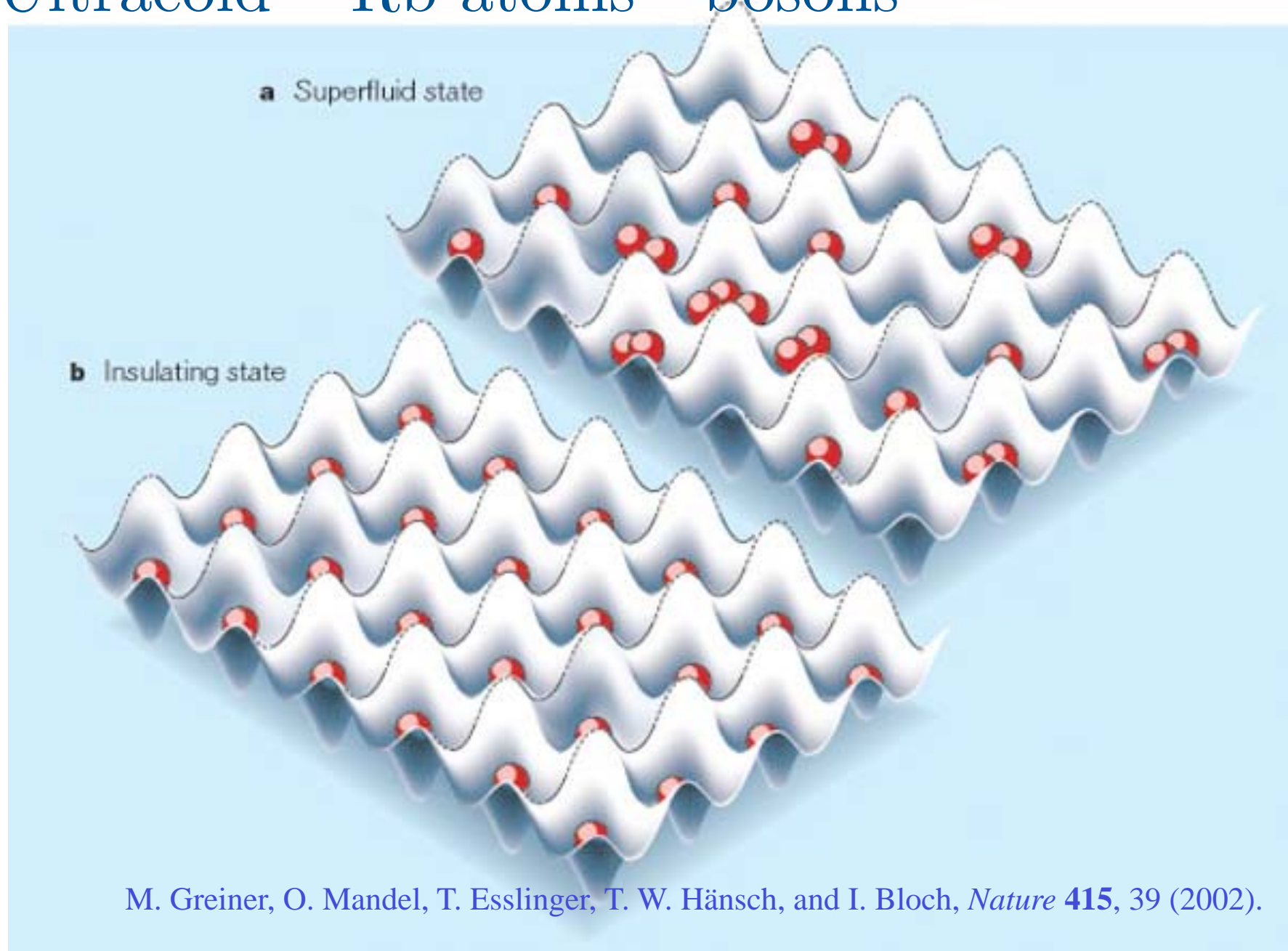
*Collisionless- $t_0$ -hydrodynamic crossover of CFT3s*

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# Ultracold $^{87}\text{Rb}$ atoms - bosons

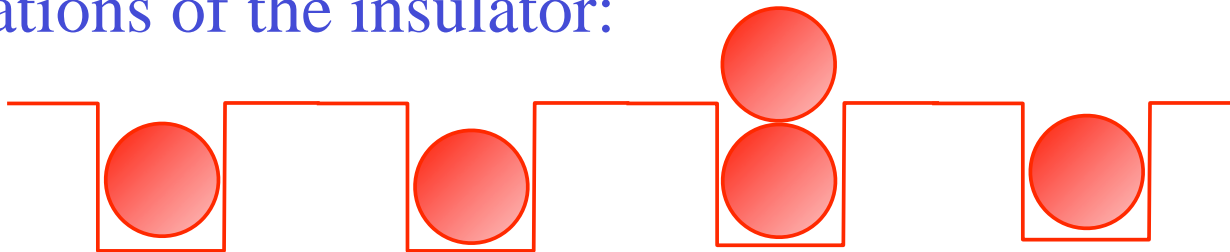


M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

The insulator:



Excitations of the insulator:

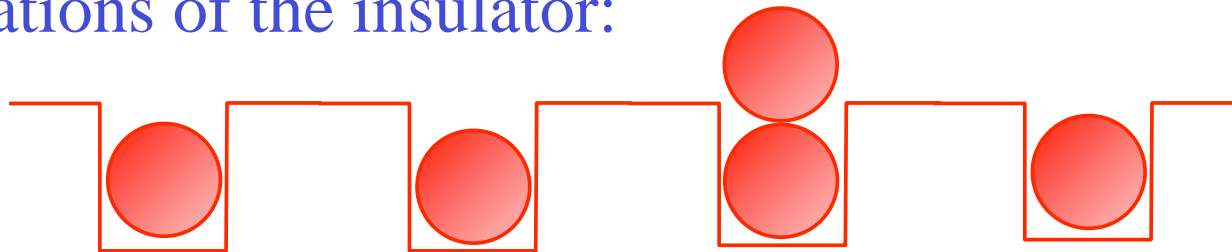


Particles  $\sim \psi^\dagger$



Holes  $\sim \psi$

Excitations of the insulator:



Particles  $\sim \psi^\dagger$



Holes  $\sim \psi$

Density of particles = density of holes  $\Rightarrow$

“relativistic” field theory for  $\psi$ :

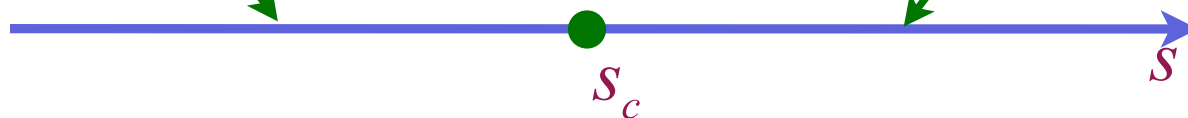
$$\mathcal{S} = \int d^2r d\tau \left[ |\partial_\tau \psi|^2 + c^2 |\vec{\nabla} \psi|^2 + s |\psi|^2 + \frac{u}{2} |\psi|^4 \right]$$

Insulator  $\Leftrightarrow \langle \psi \rangle = 0$

Superfluid  $\Leftrightarrow \langle \psi \rangle \neq 0$

Superfluid  
 $\langle \psi \rangle \neq 0$   
 $\sigma = \infty$

Insulator  
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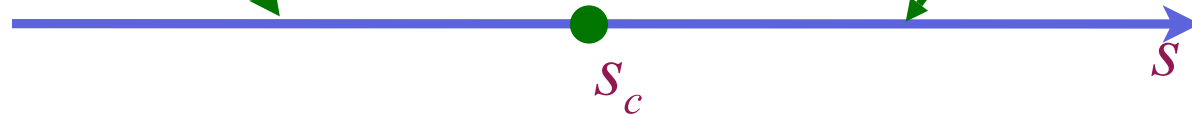


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Conformal field theory:  
Wilson-Fisher fixed point

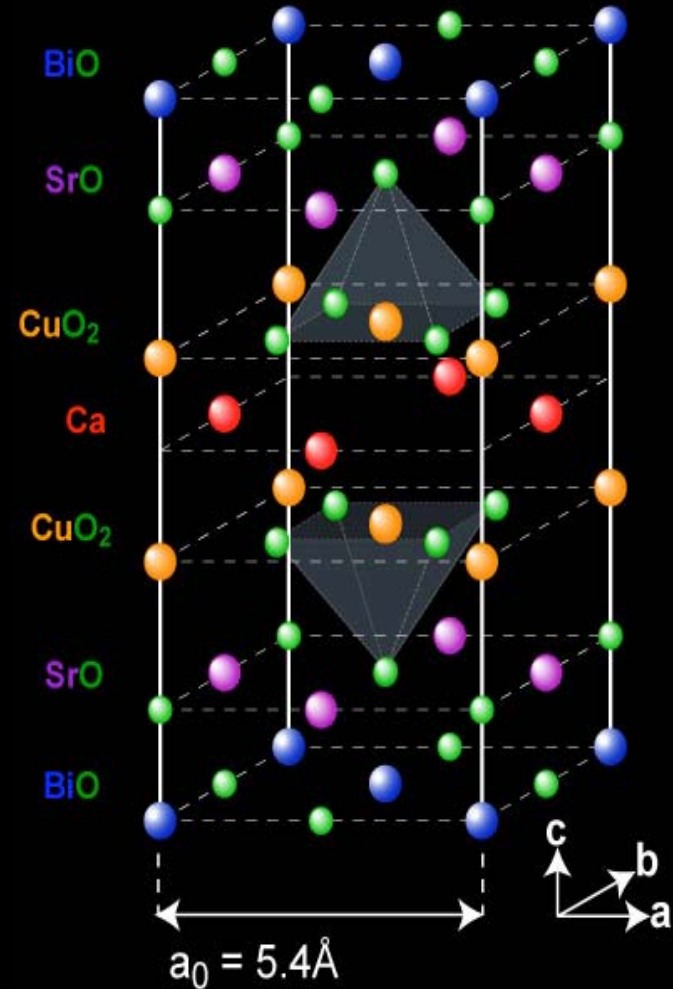
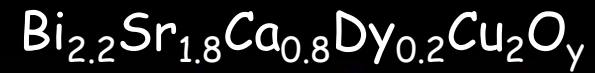
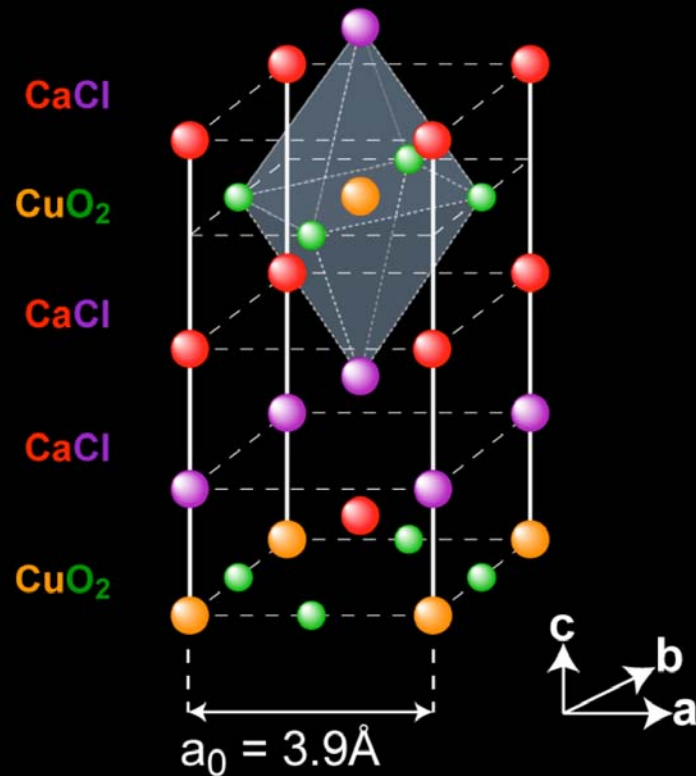
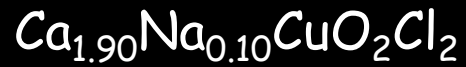
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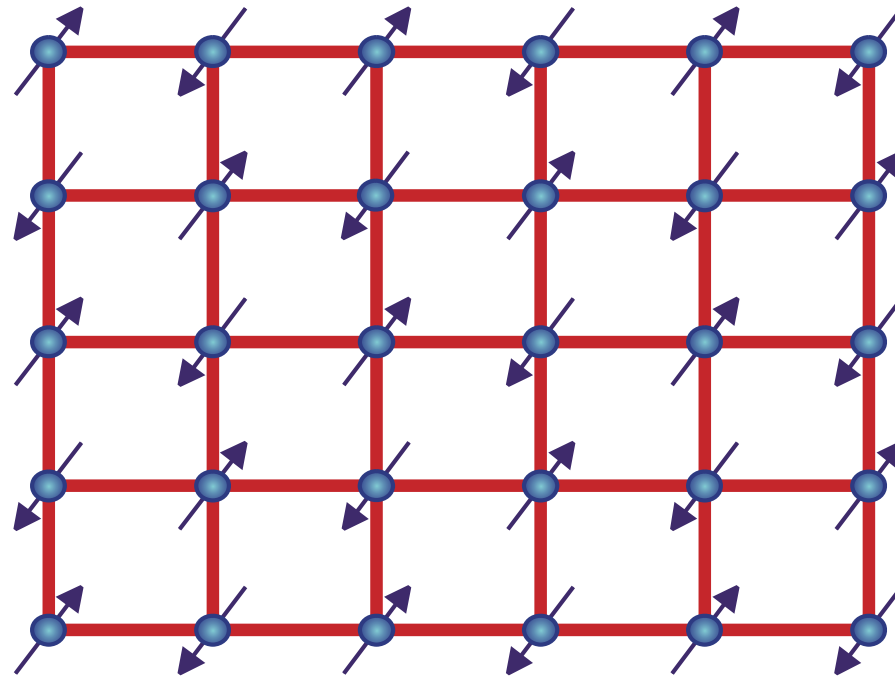
# Antiferromagnetism in the cuprate superconductors



## S=1/2 Heisenberg antiferromagnets on the square lattice

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left( \mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} \right) \left( \mathbf{S}_k \cdot \mathbf{S}_l - \frac{1}{4} \right)$$

For small  $Q$



$$\text{Order parameter } \Phi = (-1)^i \mathbf{S}_i$$

## S=1/2 Heisenberg antiferromagnets on the square lattice

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### Low energy field theory

Initial guess

$$\mathcal{S} = \int d^2r d\tau \left[ (\partial_\tau \Phi)^2 + c^2 (\vec{\nabla} \Phi)^2 + s \Phi^2 + u (\Phi^2)^2 \right]$$

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### Low energy field theory

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$$\mathcal{S} = \int d^2r d\tau \left[ (\partial_\tau \Phi)^2 + c^2 (\vec{\nabla} \Phi)^2 + s \Phi^2 + u (\Phi^2)^2 \right]$$

**Incorrect:** predicts a state for  $s > s_c$  with no broken symmetries, a gap to all excitations, and a low energy  $S = 1$  quasiparticle of  $\Phi$  quanta.

There is no such state for the  $S = 1/2$  antiferromagnet

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### Low energy field theory

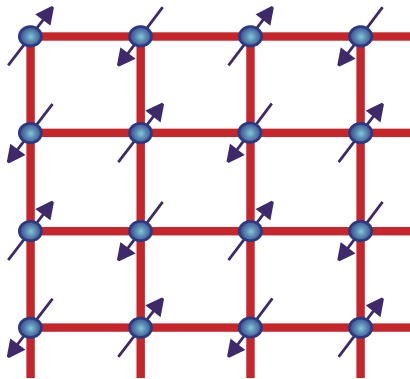
Use  $\mathbb{C}\mathbb{P}^1$  parameterization in terms of complex scalars  $z_\alpha$   
with  $\Phi = z_\alpha^* \sigma_{\alpha\beta} z_\beta$

$$\mathcal{S}_z = \int d^2r d\tau \left[ |(\partial_\mu - iA_\mu)z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right]$$

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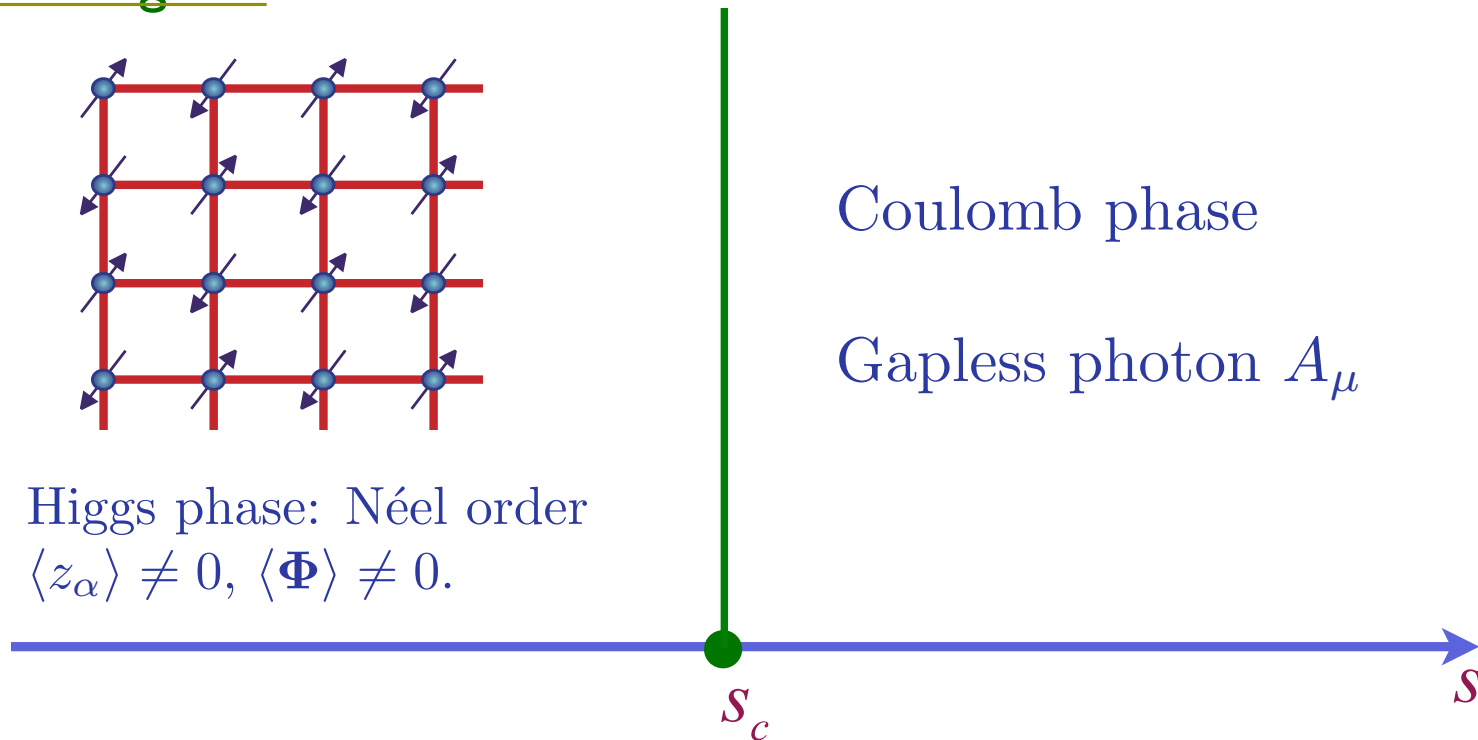
## Phase diagram



Higgs phase: Néel order  
 $\langle z_\alpha \rangle \neq 0, \langle \Phi \rangle \neq 0.$

Coulomb phase

Gapless photon  $A_\mu$

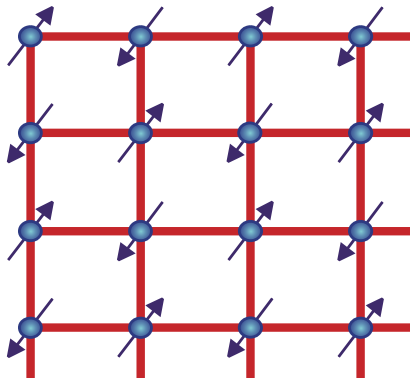


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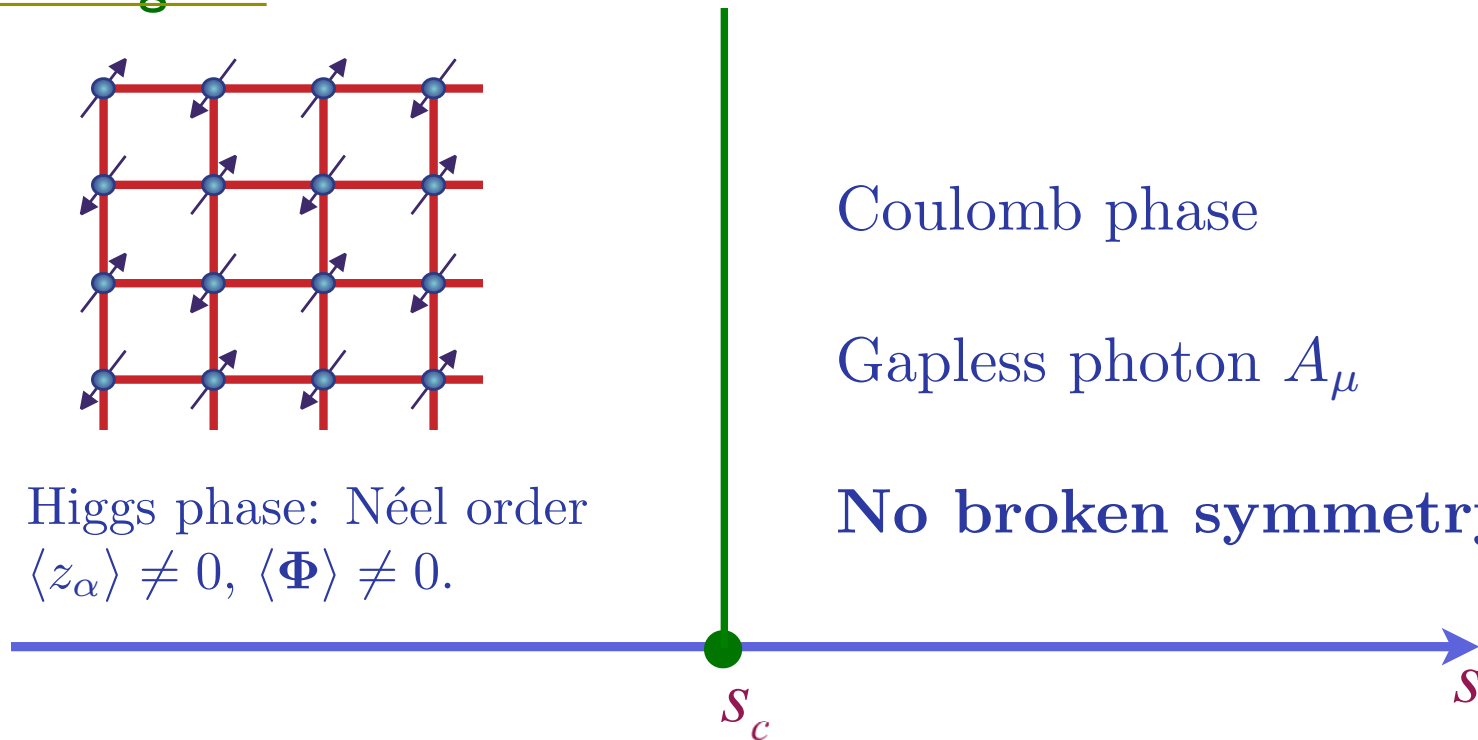


Higgs phase: Néel order  
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Gapless photon  $A_\mu$

**No broken symmetry ?**



$$\mathcal{S}_z = \int d^2 r d\tau \left[ |(\partial_\mu - iA_\mu)z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right]$$

The Coulomb phase has a conserved ‘topological’ current:

$$J_\mu = \epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda$$

We can write this current in terms of a real scalar  $\chi$

$$J_\mu = \partial_\mu \chi$$

The continuous shift symmetry  $\chi \rightarrow \chi + \text{constant}$  is *spontaneously* broken in the Coulomb phase, and the massless  $\chi$  particle (*i.e.* the photon) is the Goldstone boson of this shift symmetry.

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### Key observations:

- Berry phase (*i.e.* single spin Wess-Zumino) terms imply that the continuous shift symmetry is an enlargement of the  $Z_4$  spatial rotation symmetry of the square lattice. So this spatial rotation symmetry is spontaneously broken in the Coulomb phase.
- The monopole operator

$$V = \exp\left(i \frac{2\pi\chi}{e_0^2}\right)$$

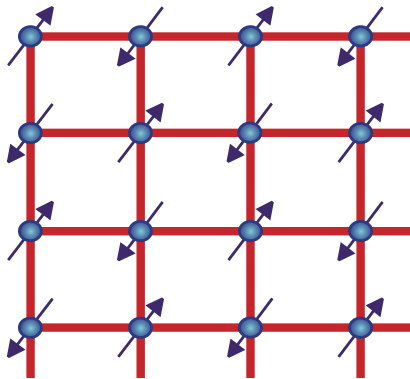
is equivalent to the valence bond solid (VBS) operator, and  $\langle V \rangle \neq 0$

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

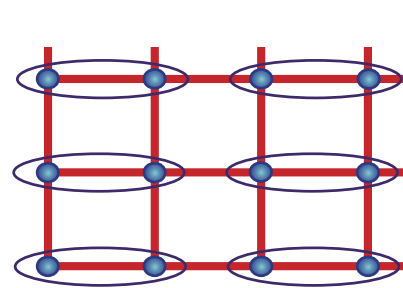
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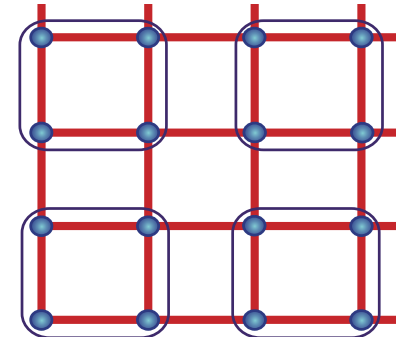
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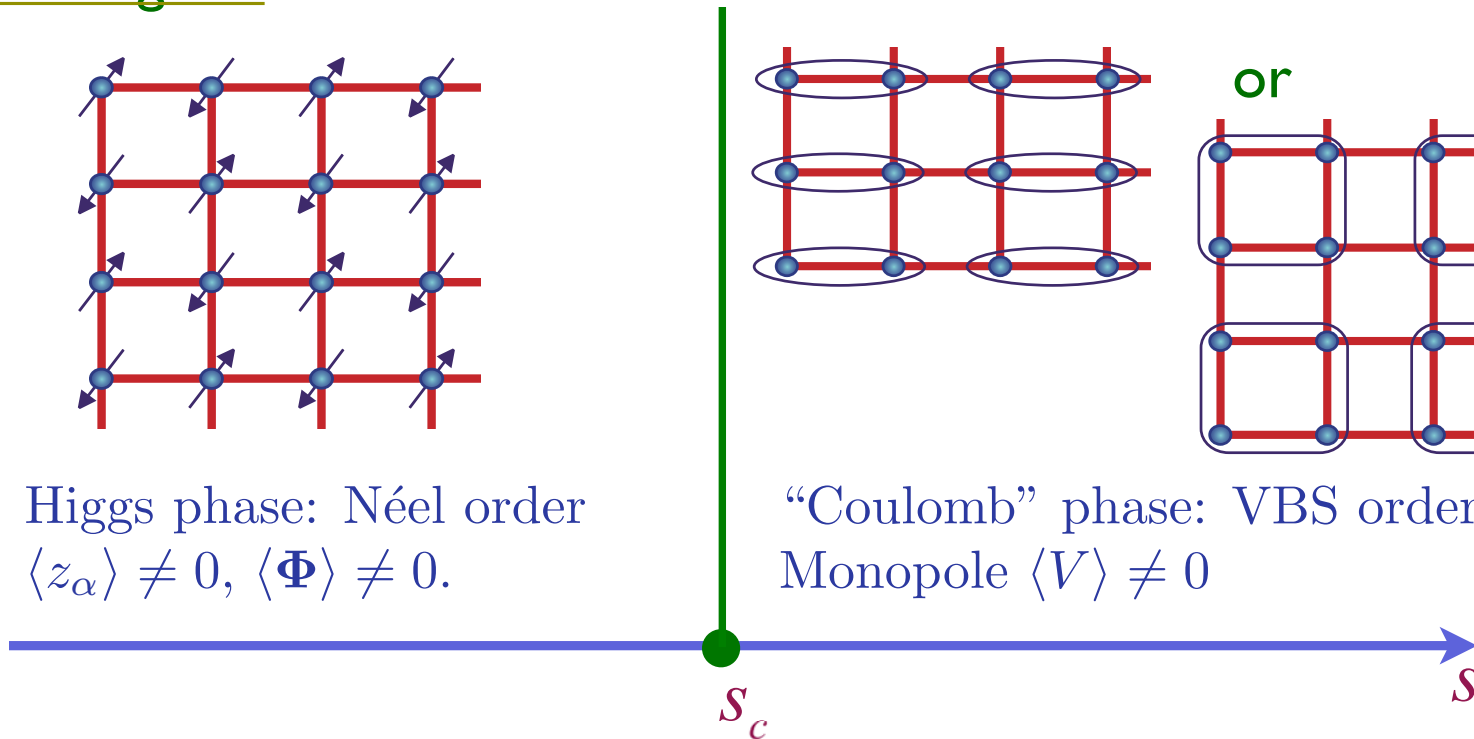
Higgs phase: Néel order  
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or



“Coulomb” phase: VBS order,  
 Monopole  $\langle V \rangle \neq 0$



$$\mathcal{S}_z = \int d^2 r d\tau \left[ |(\partial_\mu - iA_\mu)z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right]$$

Similar phase diagram for  $\mathcal{N} = 4$  SQED with 2 matter hypermultiplets

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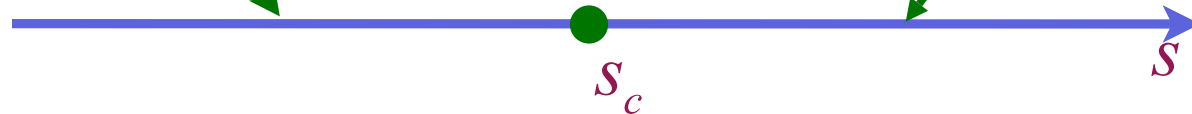
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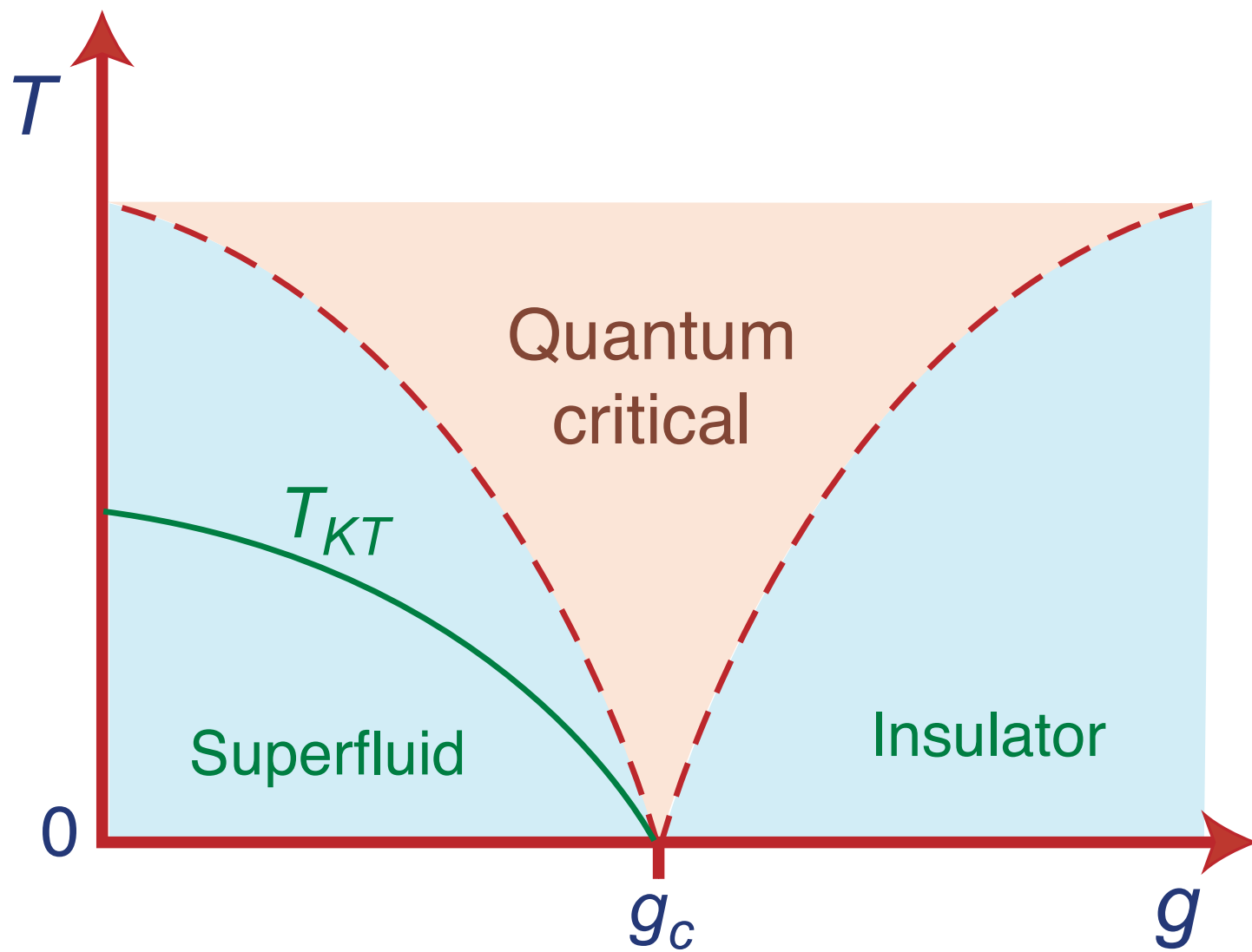
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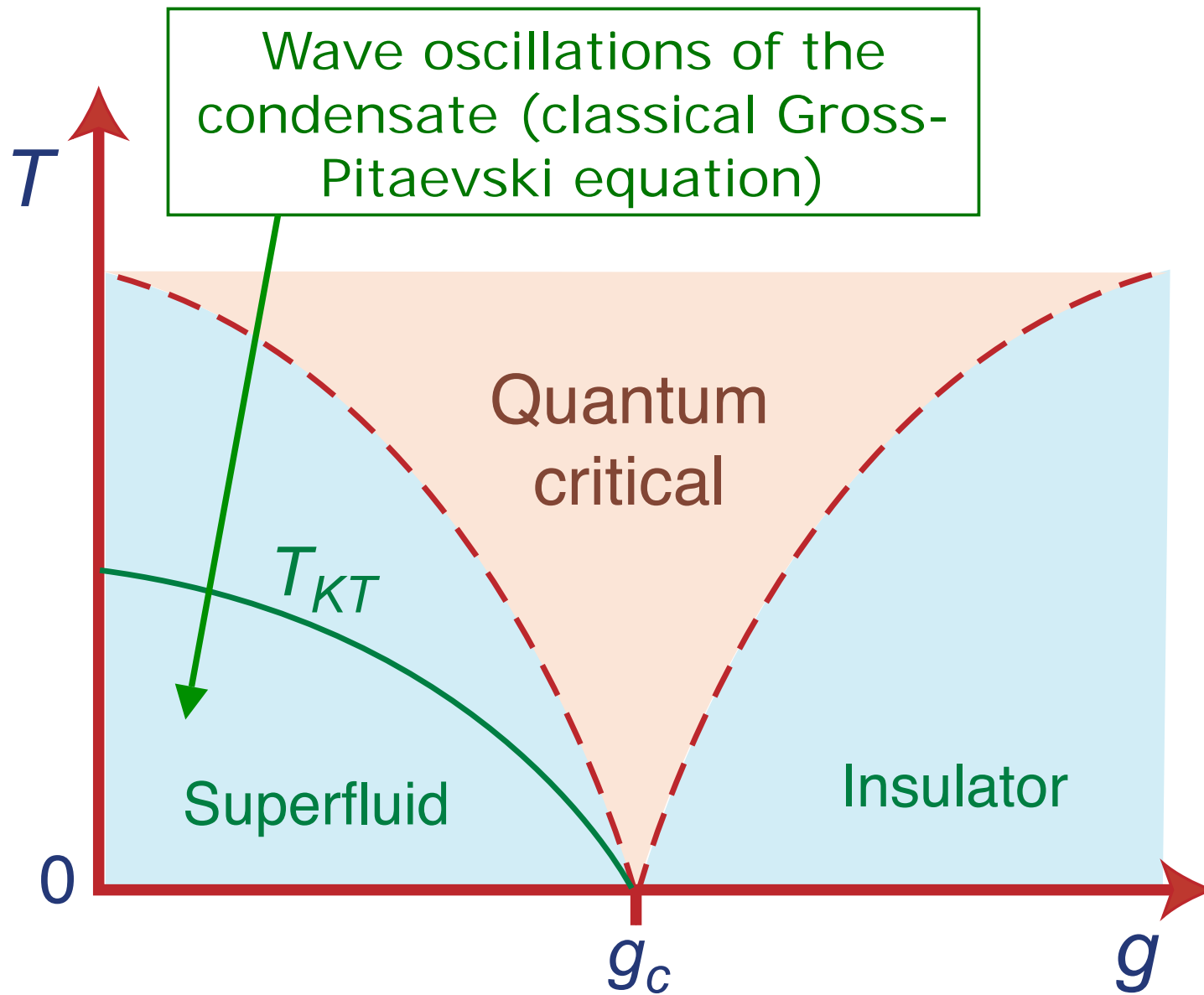
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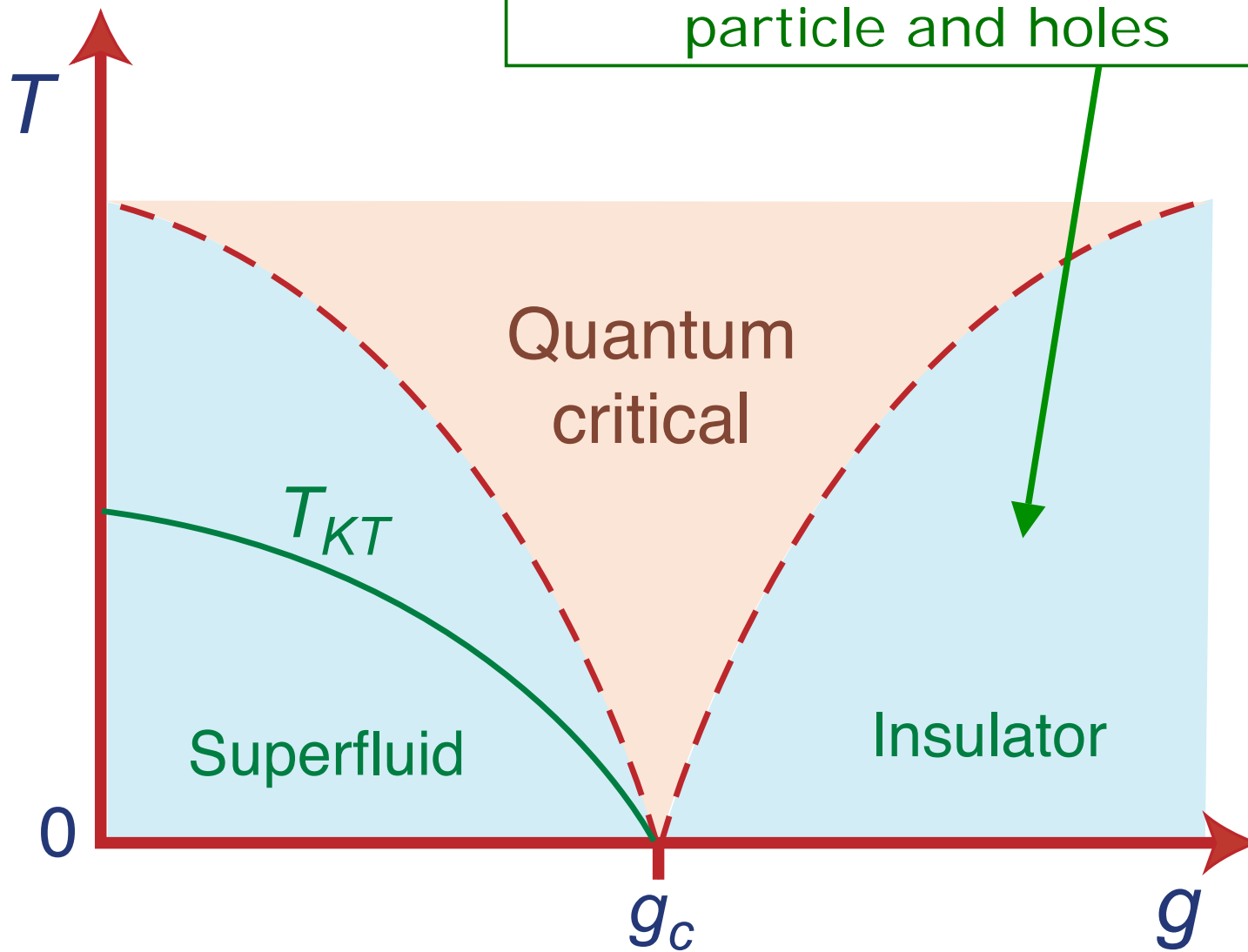


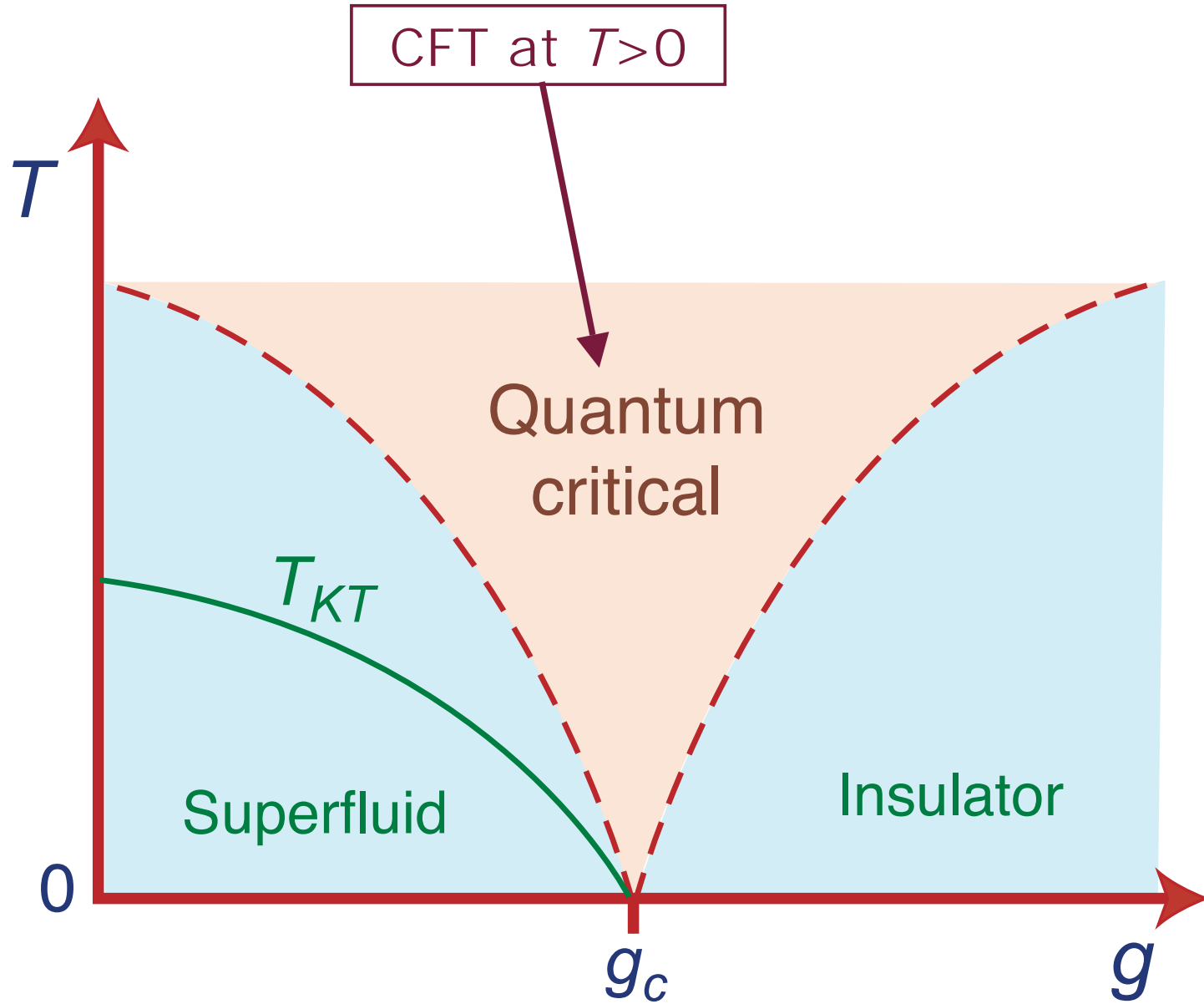
$$\mathcal{S} = \int d^2r d\tau \left[ |\partial_\tau \psi|^2 + c^2 |\vec{\nabla} \psi|^2 + s |\psi|^2 + \frac{u}{2} |\psi|^4 \right]$$





Dilute Boltzmann gas of  
particle and holes





# Resistivity of Bi films

## Conductivity $\sigma$

$$\sigma_{\text{Superconductor}}(T \rightarrow 0) = \infty$$

$$\sigma_{\text{Insulator}}(T \rightarrow 0) = 0$$

$$\sigma_{\text{Quantum critical point}}(T \rightarrow 0) \approx \frac{4e^2}{h}$$

D. B. Haviland, Y. Liu, and A. M. Goldman,  
*Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

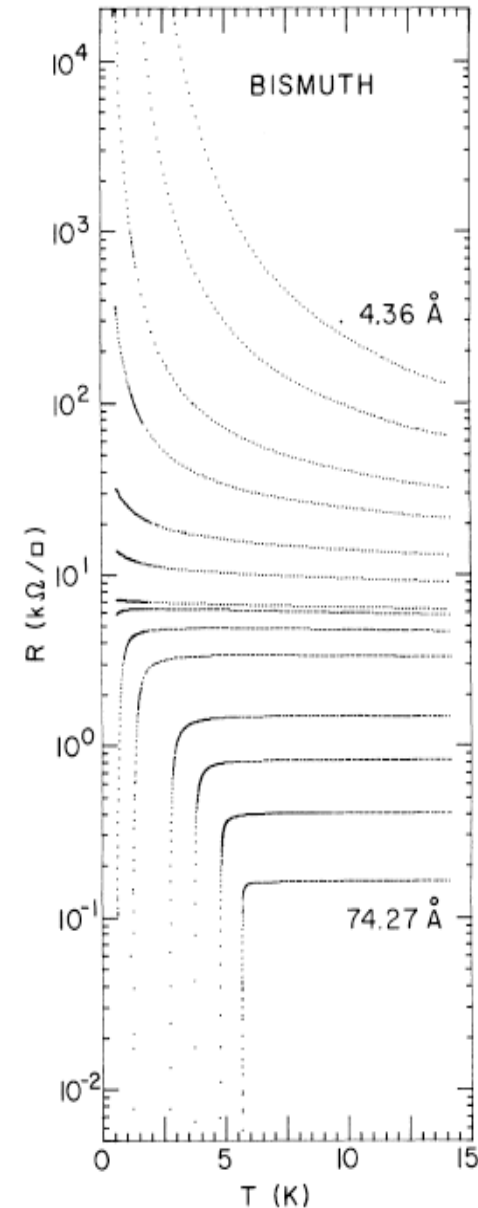


FIG. 1. Evolution of the temperature dependence of the sheet resistance  $R(T)$  with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

## Density correlations in CFTs at $T > 0$

Two-point density correlator,  $\chi(k, \omega)$

Kubo formula for conductivity  $\sigma(\omega) = \lim_{k \rightarrow 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

For *all* CFT2s, at all  $\hbar\omega/k_B T$

$$\chi(k, \omega) = \frac{4e^2}{h} K \frac{vk^2}{v^2k^2 - \omega^2} ; \quad \sigma(\omega) = \frac{4e^2}{h} \frac{Kv}{-i\omega}$$

where  $K$  is a universal number characterizing the CFT2 (the level number), and  $v$  is the velocity of “light”.

## Density correlations in CFTs at $T > 0$

Two-point density correlator,  $\chi(k, \omega)$

Kubo formula for conductivity  $\sigma(\omega) = \lim_{k \rightarrow 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

For *all* CFT3s, at  $\hbar\omega \gg k_B T$

$$\chi(k, \omega) = \frac{4e^2}{h} K \frac{k^2}{\sqrt{v^2 k^2 - \omega^2}} ; \quad \sigma(\omega) = \frac{4e^2}{h} K$$

where  $K$  is a universal number characterizing the CFT3, and  $v$  is the velocity of “light”.

## Density correlations in CFTs at $T > 0$

Two-point density correlator,  $\chi(k, \omega)$

Kubo formula for conductivity  $\sigma(\omega) = \lim_{k \rightarrow 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

**However**, for *all* CFT3s, at  $\hbar\omega \ll k_B T$ , we have the Einstein relation

$$\chi(k, \omega) = 4e^2 \chi_c \frac{Dk^2}{Dk^2 - i\omega} \quad ; \quad \sigma(\omega) = 4e^2 D \chi_c = \frac{4e^2}{h} \Theta_1 \Theta_2$$

where the **compressibility**,  $\chi_c$ , and the **diffusion constant**  $D$  obey

$$\chi = \frac{k_B T}{(h\nu)^2} \Theta_1 \quad ; \quad D = \frac{h\nu^2}{k_B T} \Theta_2$$

with  $\Theta_1$  and  $\Theta_2$  universal numbers characteristic of the CFT3

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

## Density correlations in CFTs at $T > 0$

In CFTs collisions are “phase” randomizing, and lead to relaxation to local thermodynamic equilibrium. So there is a crossover from collisionless behavior for  $\hbar\omega \gg k_B T$ , to hydrodynamic behavior for  $\hbar\omega \ll k_B T$ .

$$\sigma(\omega) = \begin{cases} \frac{4e^2}{h} K & , \quad \hbar\omega \gg k_B T \\ \frac{4e^2}{h} \Theta_1 \Theta_2 & , \quad \hbar\omega \ll k_B T \end{cases}$$

and in general we expect  $K \neq \Theta_1 \Theta_2$  (verified for Wilson-Fisher fixed point).

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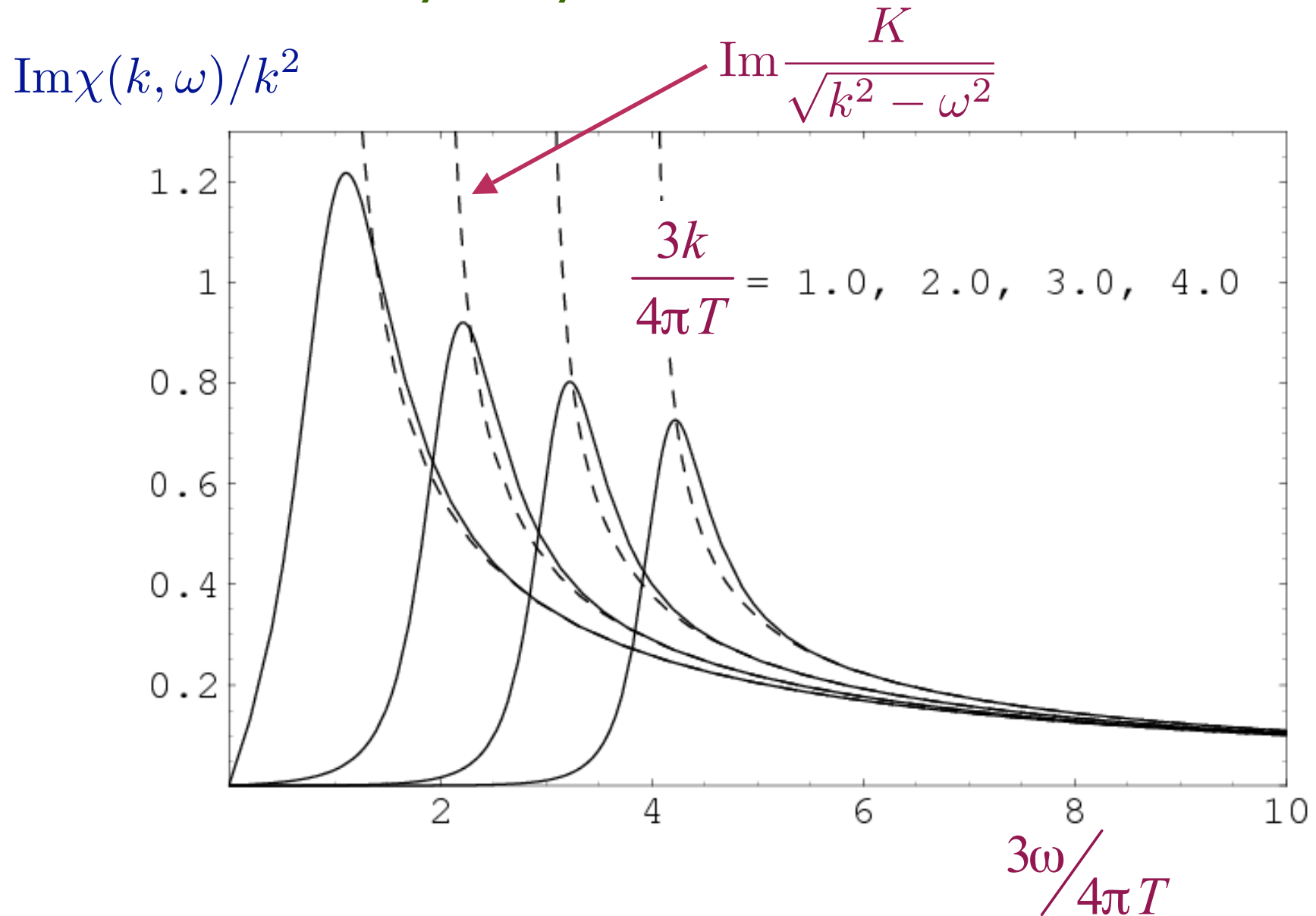
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## SU(N) SYM3 with $\mathcal{N} = 8$ supersymmetry

- Has a single dimensionful coupling constant,  $g$ , which flows to a strong-coupling fixed point  $g = g^*$  in the infrared.
- The CFT3 describing this fixed point resembles “critical spin liquid” theories.
- This CFT3 is the low energy limit of string theory on an M2 brane. The AdS/CFT correspondence provides a dual description using 11-dimensional supergravity on  $\text{AdS}_4 \times S_7$ .
- The CFT3 has a global SO(8) R symmetry, and correlators of the SO(8) charge density can be computed exactly in the large  $N$  limit, even at  $T > 0$ .

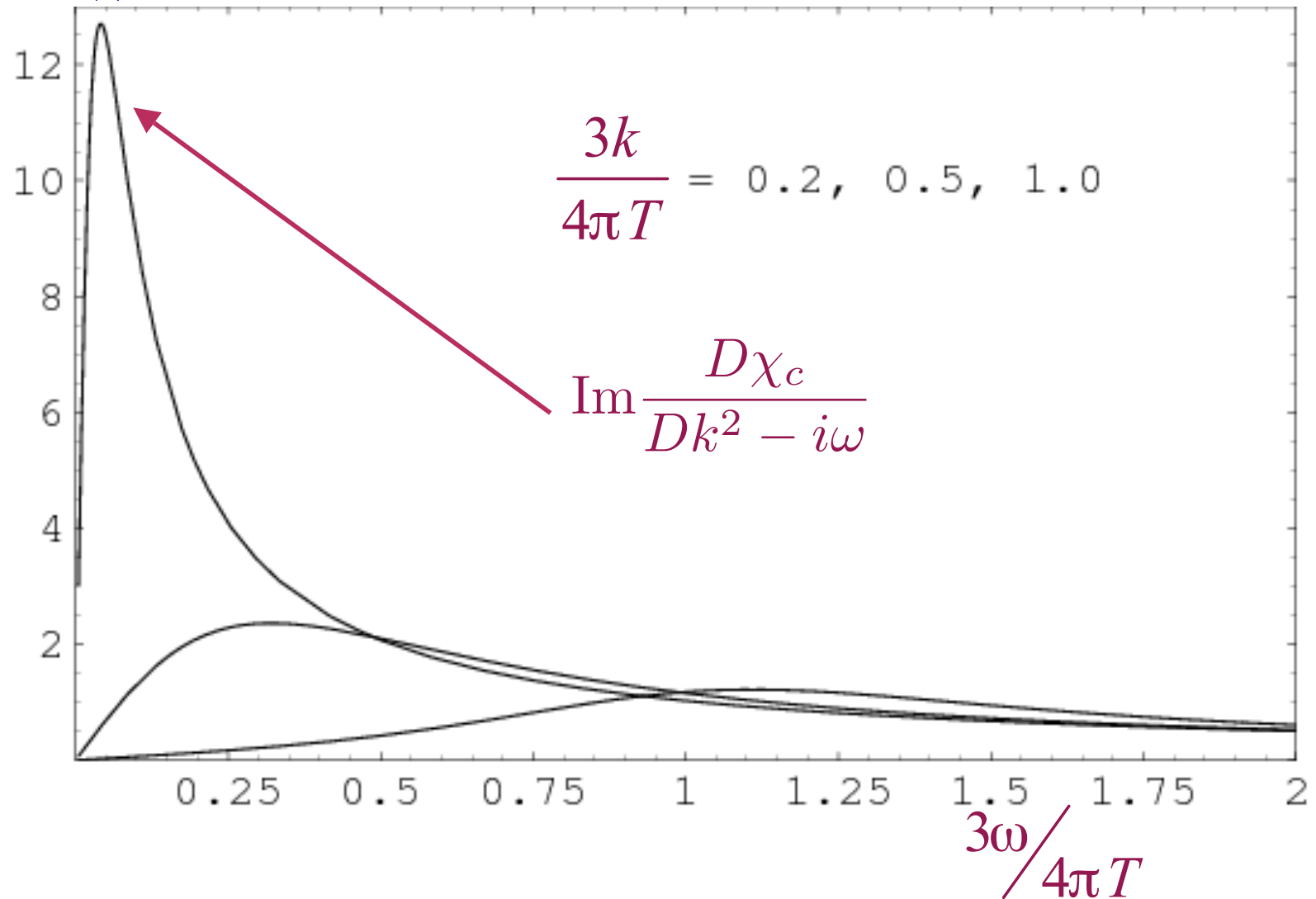
# Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

# Collisionless to hydrodynamic crossover of SYM3

$\text{Im}\chi(k, \omega)/k^2$



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

## Universal constants of SYM3

$$\chi_c = \frac{k_B T}{(h\nu)^2} \Theta_1$$
$$D = \frac{h\nu^2}{k_B T} \Theta_2$$
$$\sigma(\omega) = \begin{cases} \frac{4e^2}{h} K & , \quad \hbar\omega \gg k_B T \\ \frac{4e^2}{h} \Theta_1 \Theta_2 & , \quad \hbar\omega \ll k_B T \end{cases}$$

$$K = \frac{\sqrt{2} N^{3/2}}{3}$$
$$\Theta_1 = \frac{8\pi^2 \sqrt{2} N^{3/2}}{9}$$
$$\Theta_2 = \frac{3}{8\pi^2}$$

C. Herzog, JHEP **0212**, 026 (2002)

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

## Electromagnetic self-duality

- Unexpected result,  $K = \Theta_1 \Theta_2$ .
- This is traced to a *four*-dimensional electromagnetic self-duality of the theory on  $\text{AdS}_4$ . In the large  $N$  limit, the  $\text{SO}(8)$  currents decouple into 28  $\text{U}(1)$  currents with a Maxwell action for the  $\text{U}(1)$  gauge fields on  $\text{AdS}_4$ .
- This special property is not expected for generic CFT3s.
- Open question: Does  $K = \Theta_1 \Theta_2$  hold beyond the  $N \rightarrow \infty$  limit? In other words, does this “self-duality” survive in the full M theory.

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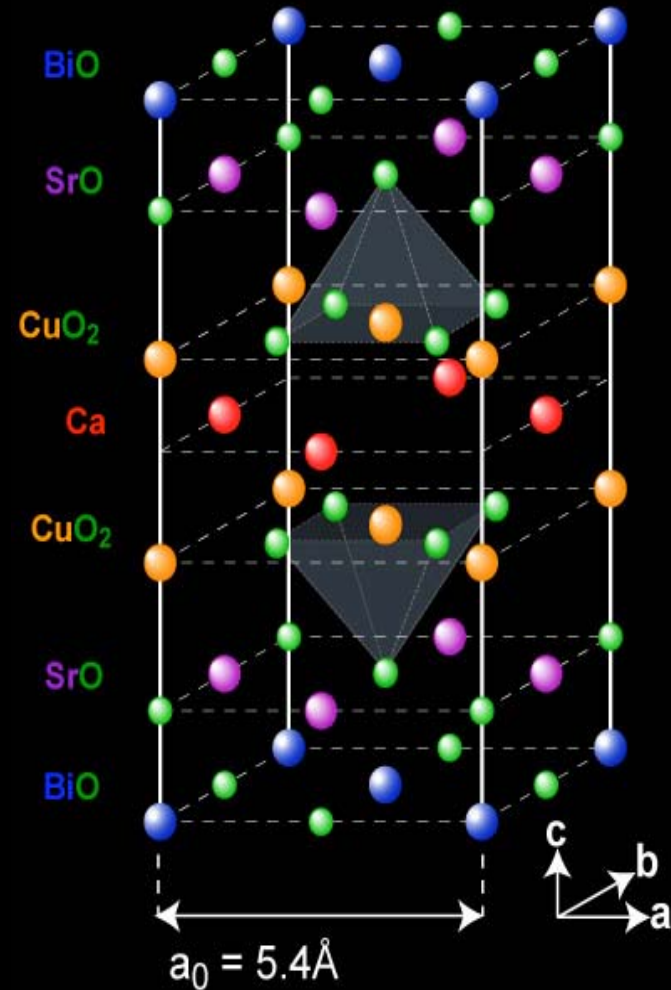
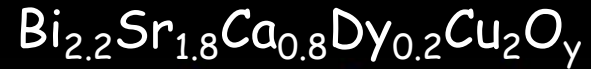
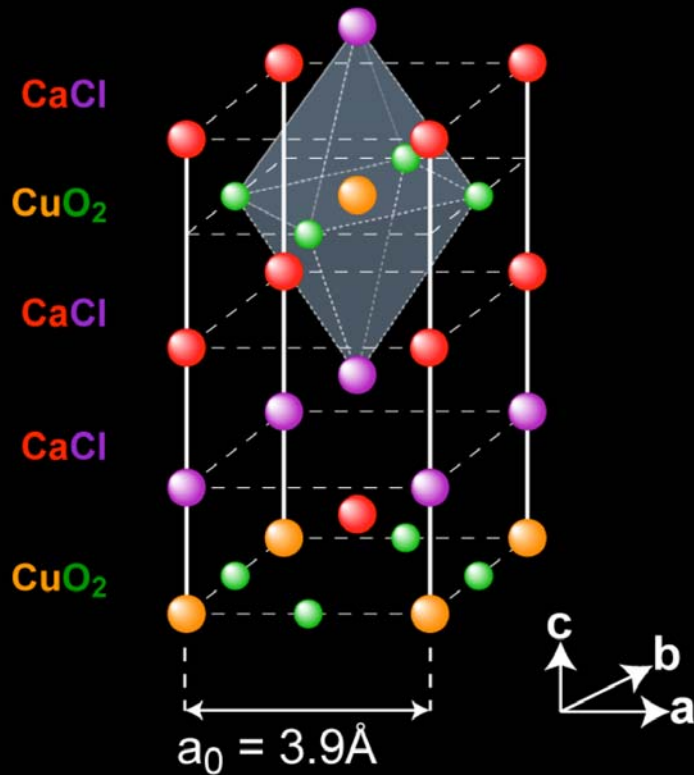
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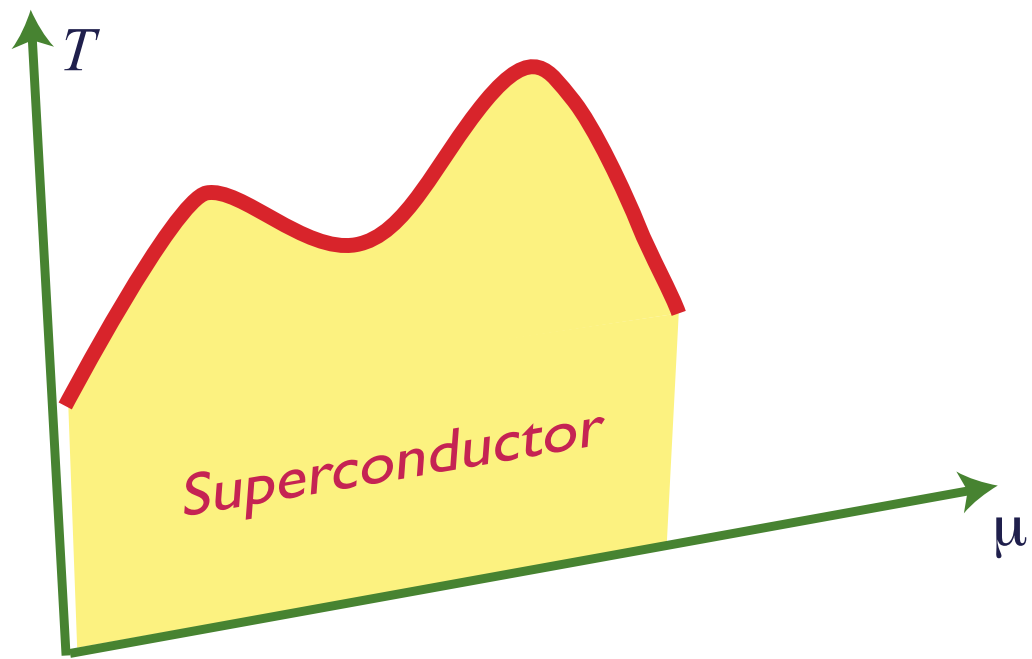
## 3. SYM3 with $\mathcal{N} = 8$ supersymmetry

## 4. Nernst effect in the cuprate superconductors

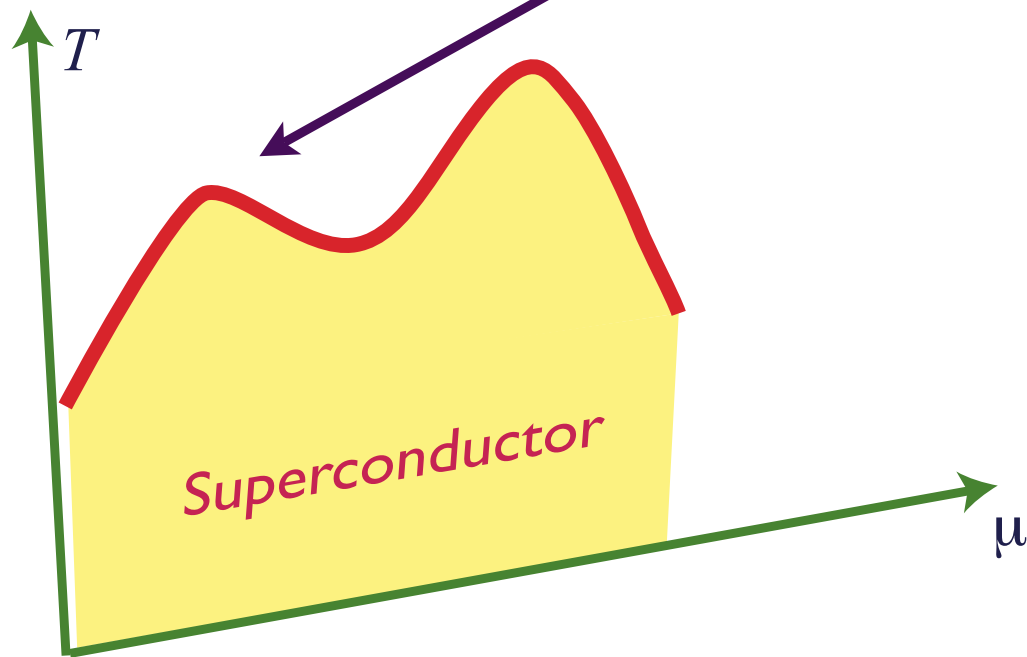
*Quantum criticality and dyonic black holes*

Dope the antiferromagnets with charge carriers of density  $x$  by applying a chemical potential  $\mu$

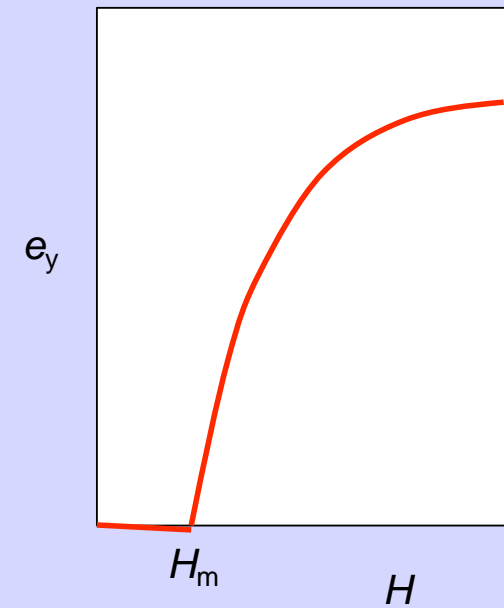
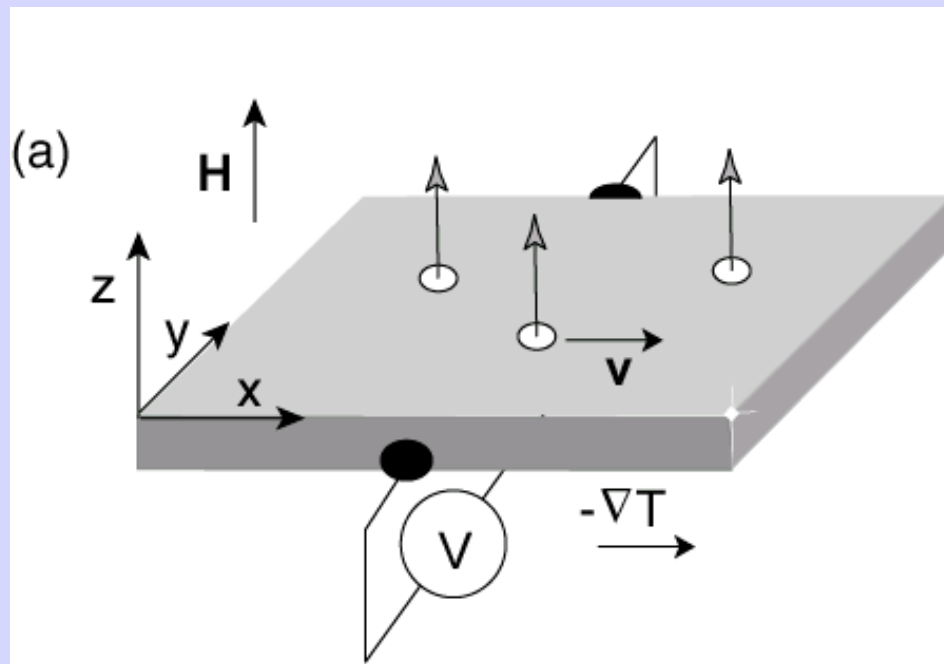




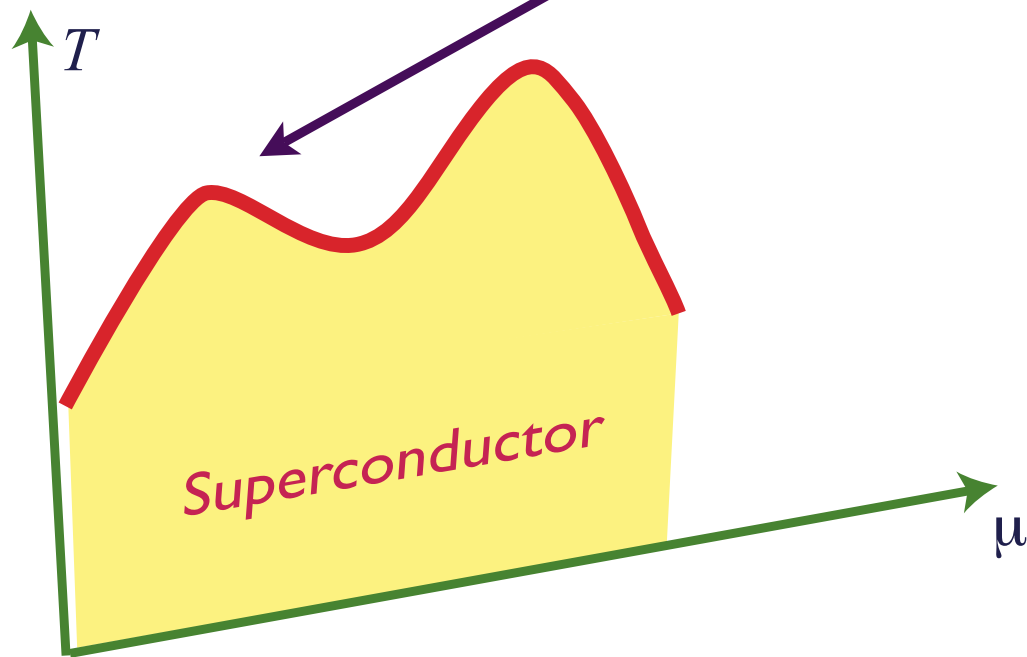
Nernst measurements

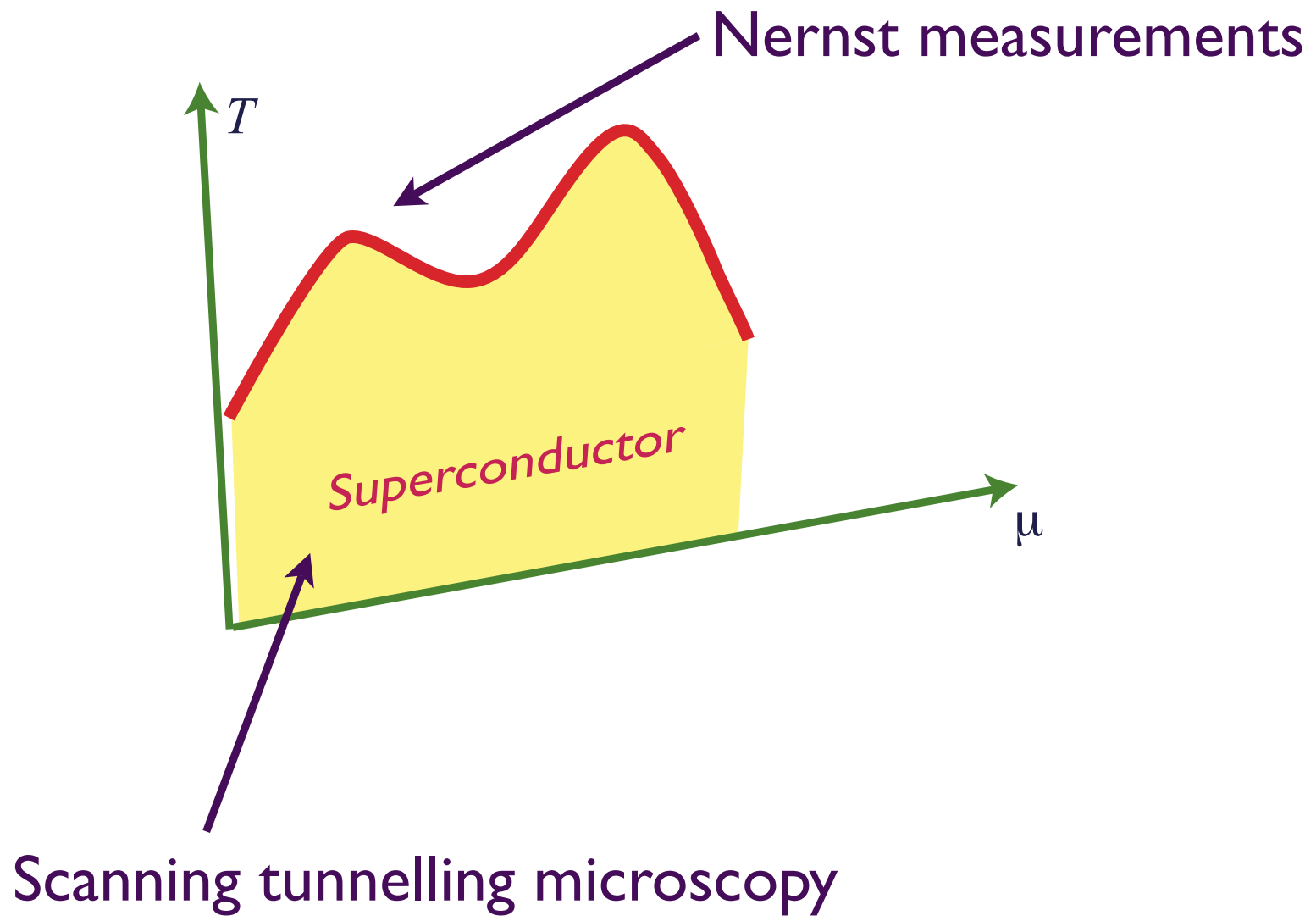


# Nernst experiment

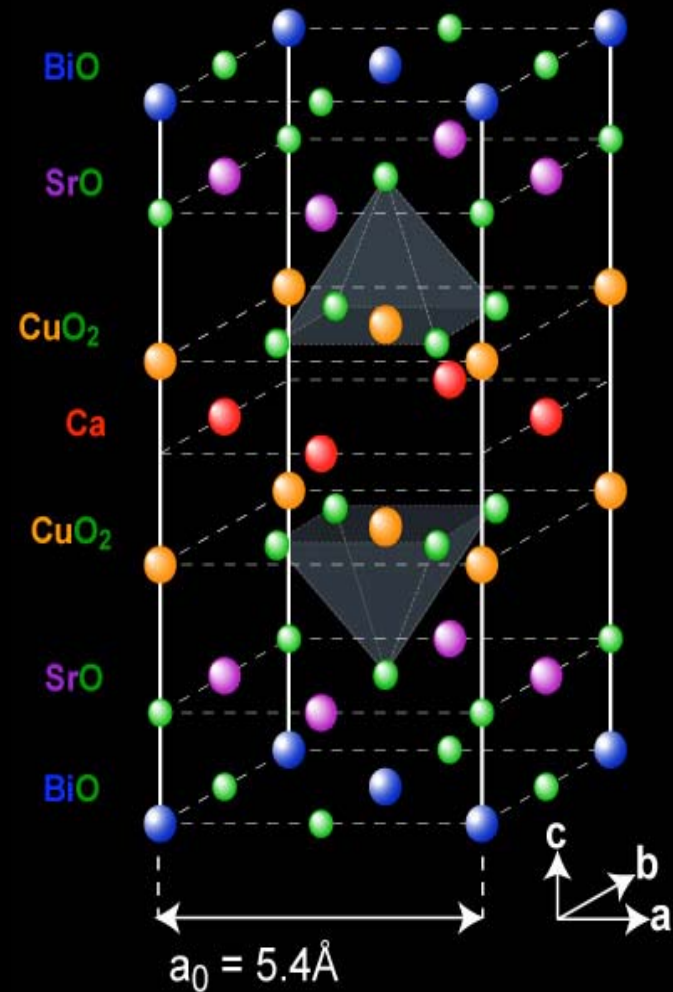
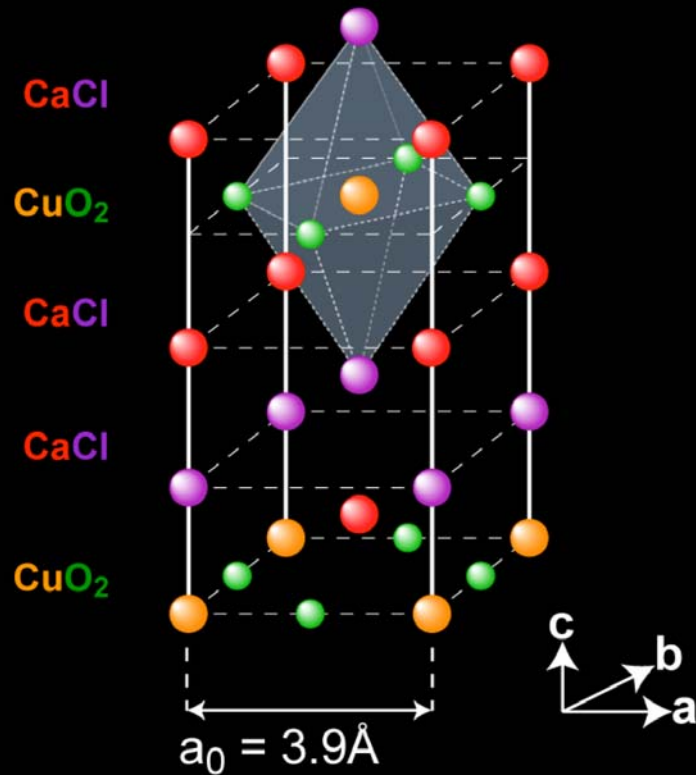
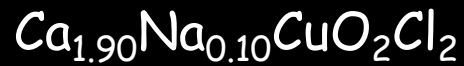


Nernst measurements

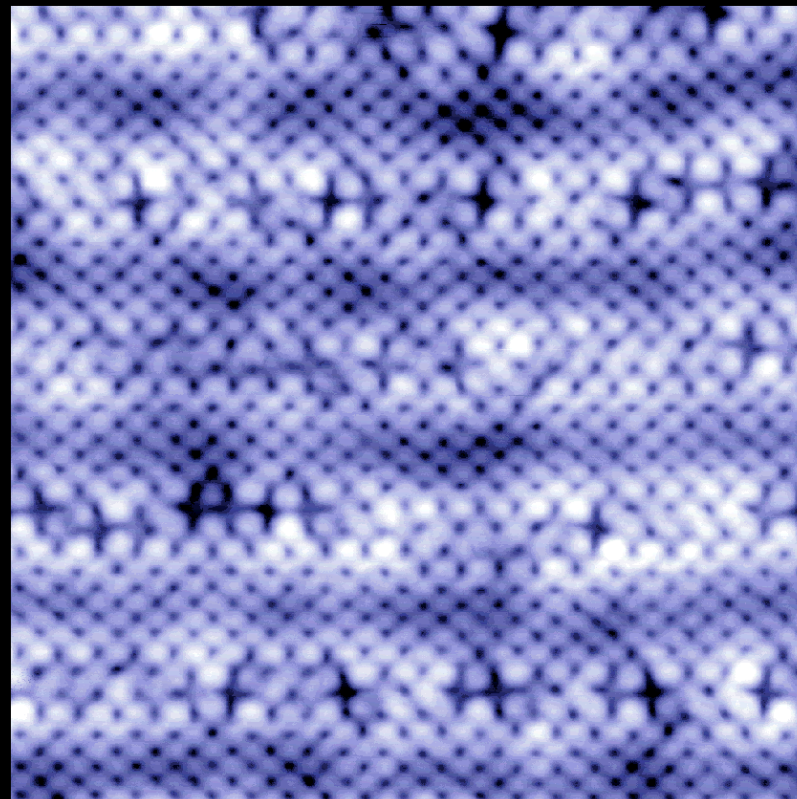
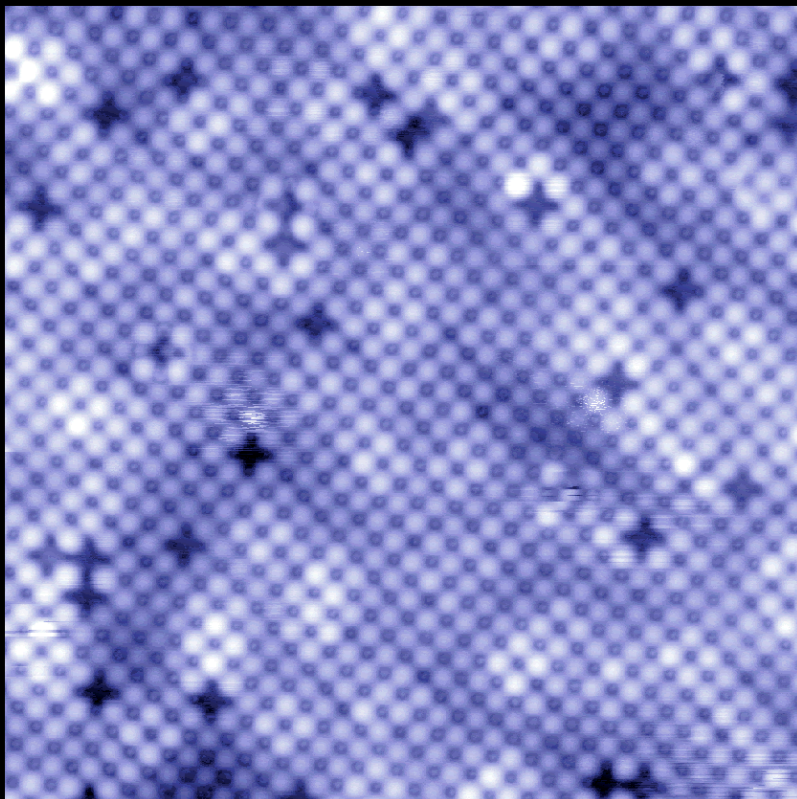
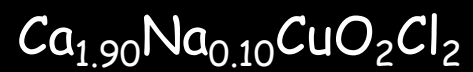




# STM studies of the underdoped superconductor

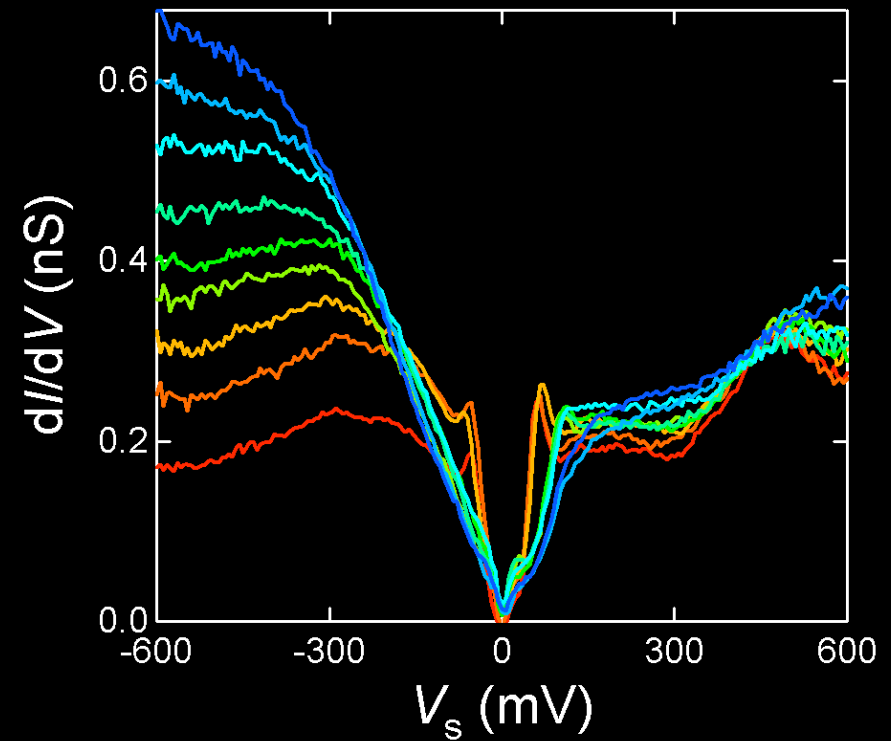
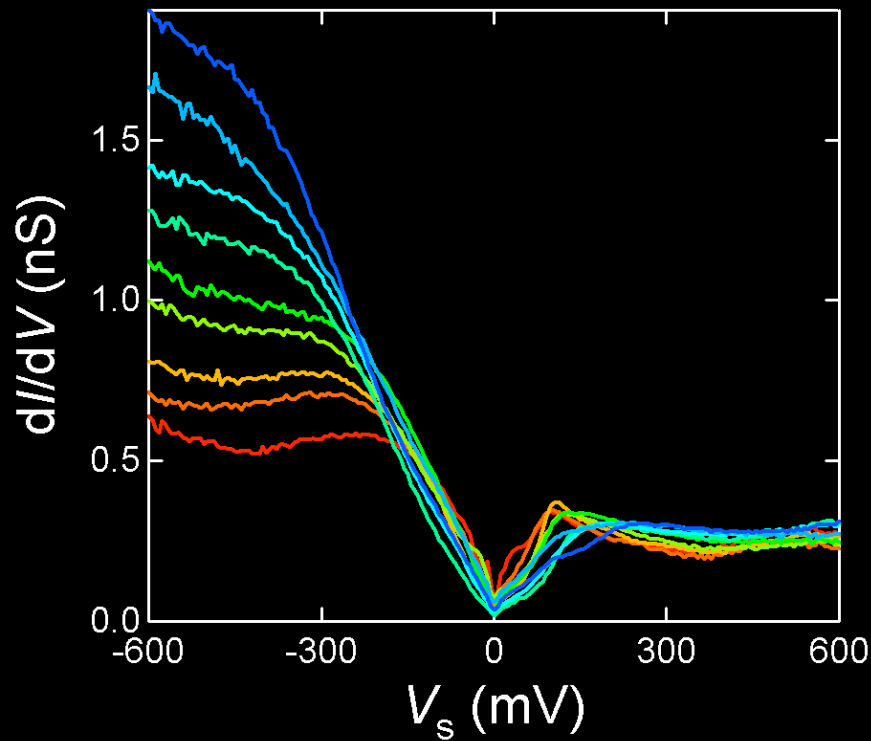
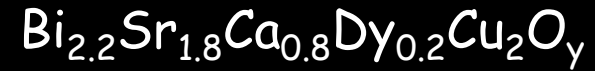
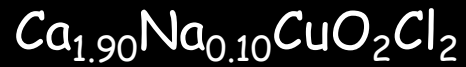


## Topograph



12 nm

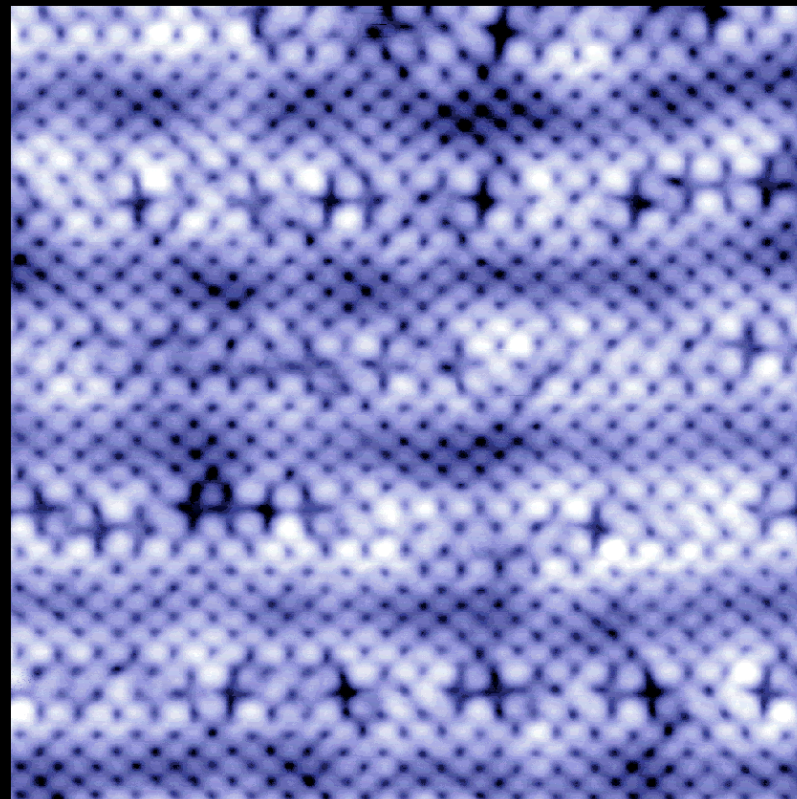
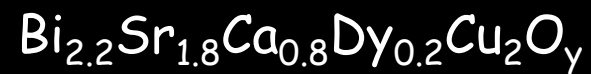
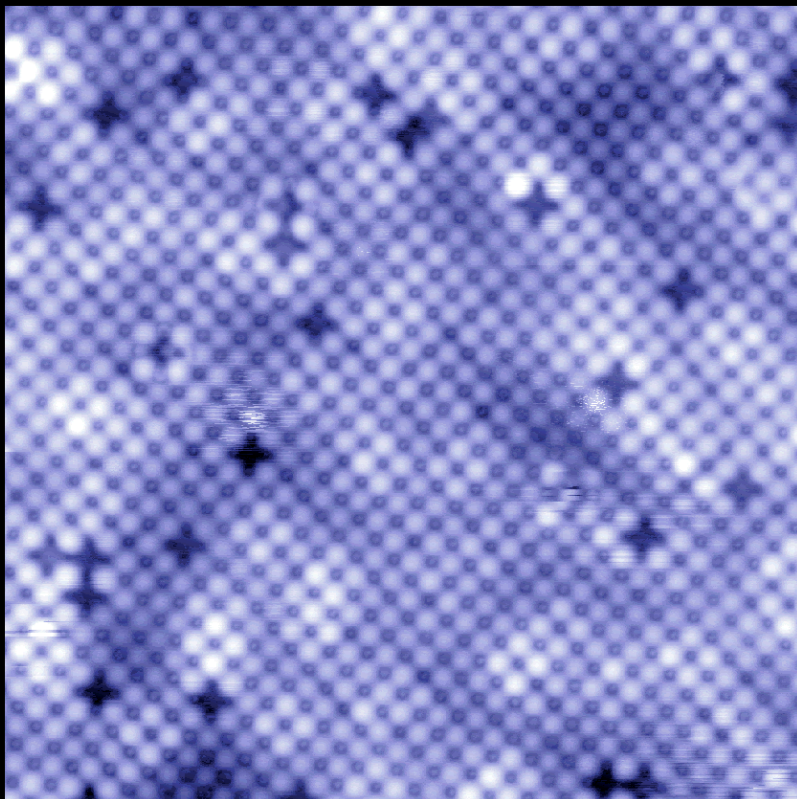
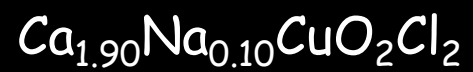
## dI/dV Spectra



Intense Tunneling-Asymmetry (TA)  
variation are highly similar

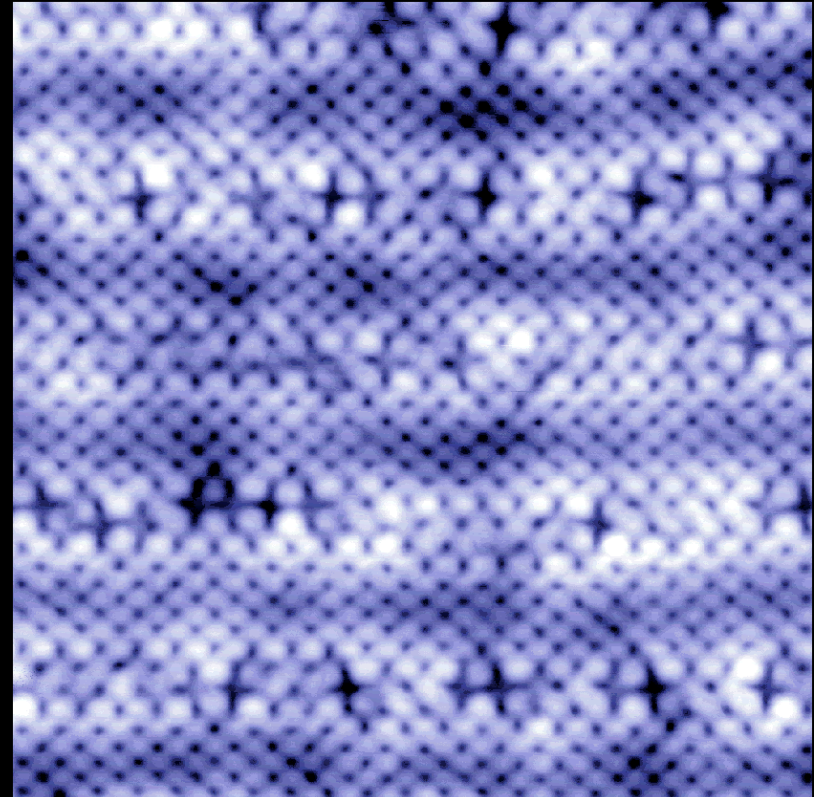
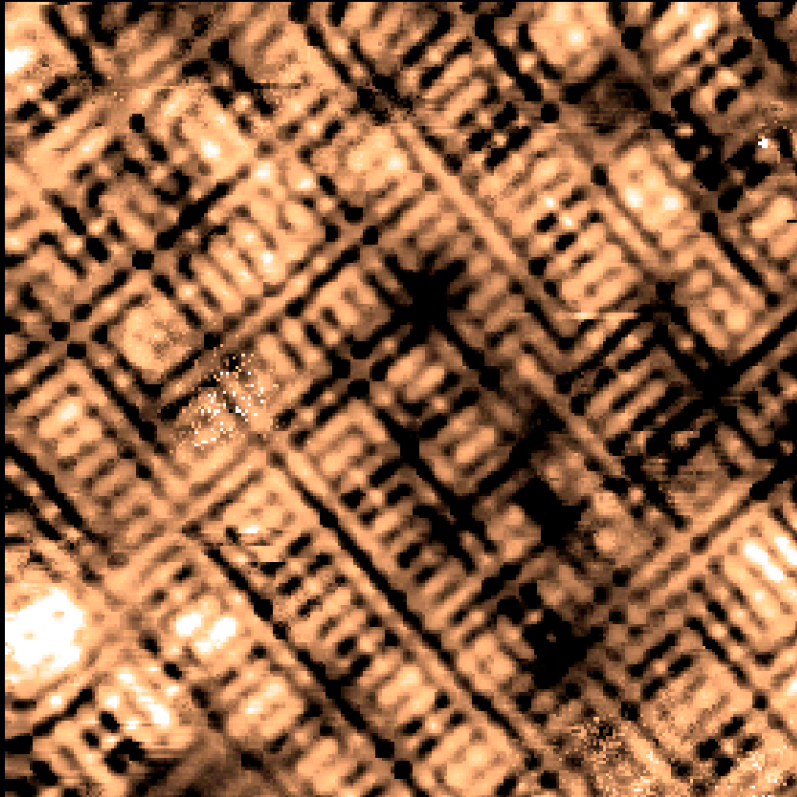
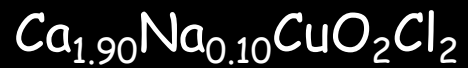
Y. Kohsaka et al. Science 315, 1380 (2007)

## Topograph



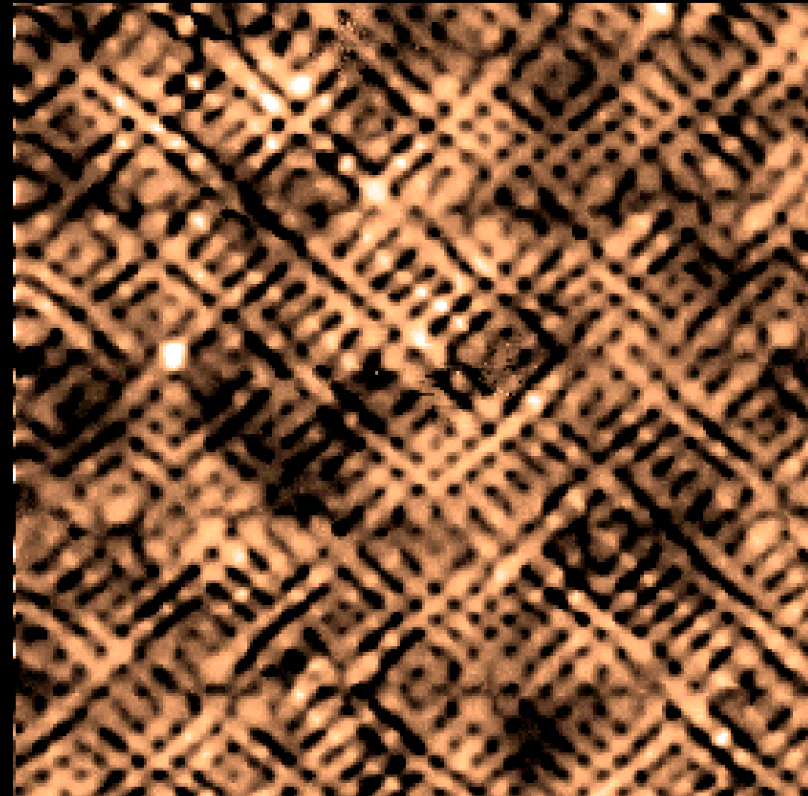
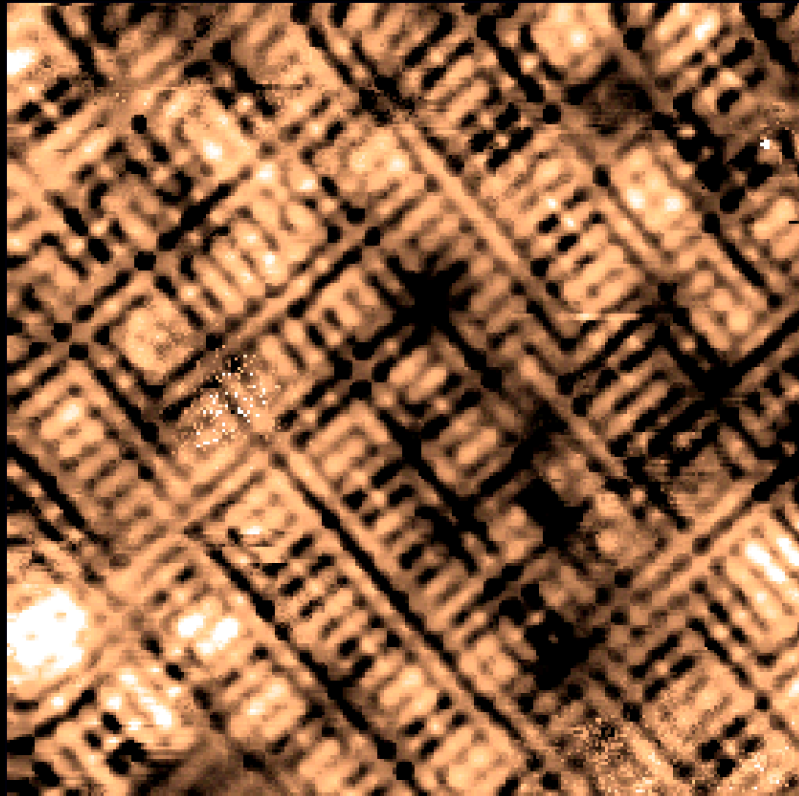
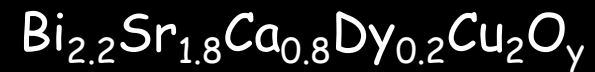
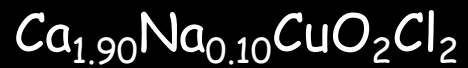
12 nm

# Tunneling Asymmetry (TA)-map at $E=150\text{meV}$



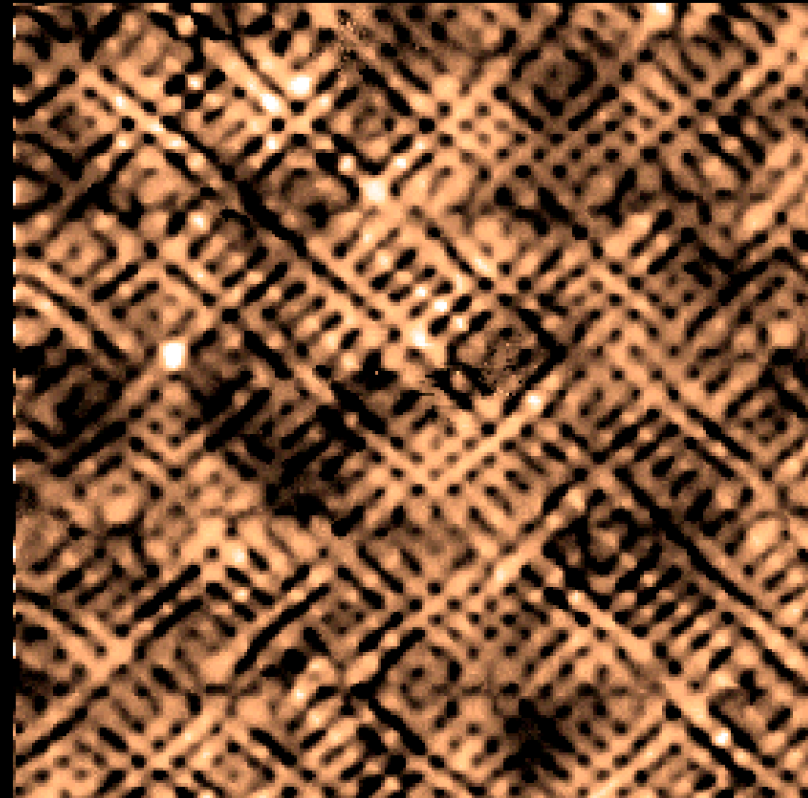
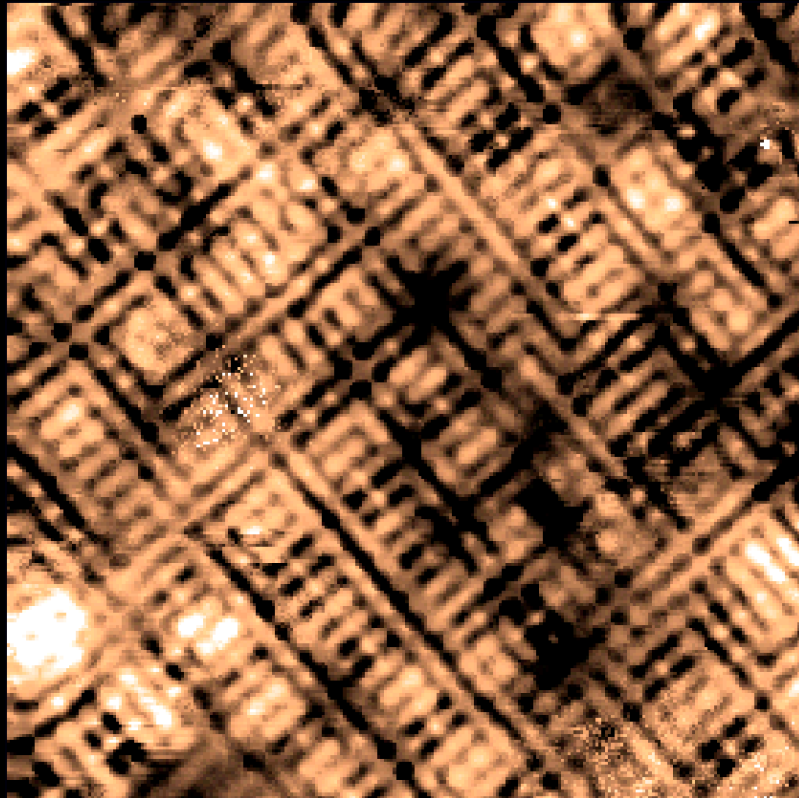
12 nm

## Tunneling Asymmetry (TA)-map at $E=150\text{meV}$



12 nm

## Tunneling Asymmetry (TA)-map at $E=150\text{meV}$



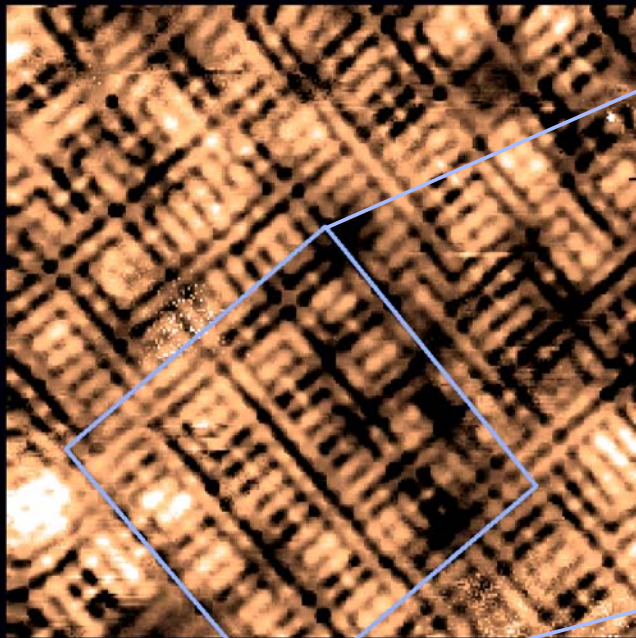
12 nm

Indistinguishable bond-centered TA contrast  
with disperse  $4a_0$ -wide nanodomains

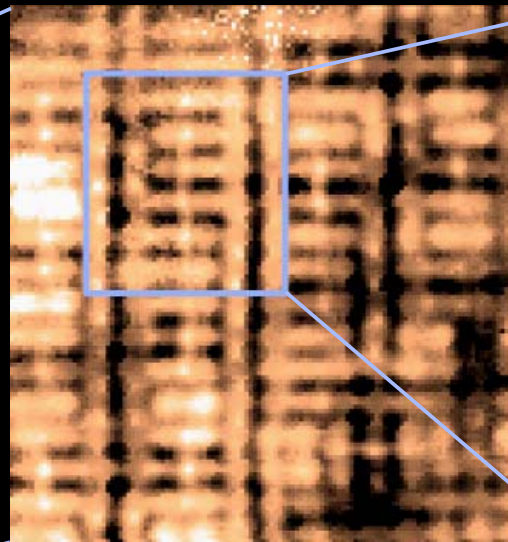
Y. Kohsaka et al. Science 315, 1380 (2007)

# TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

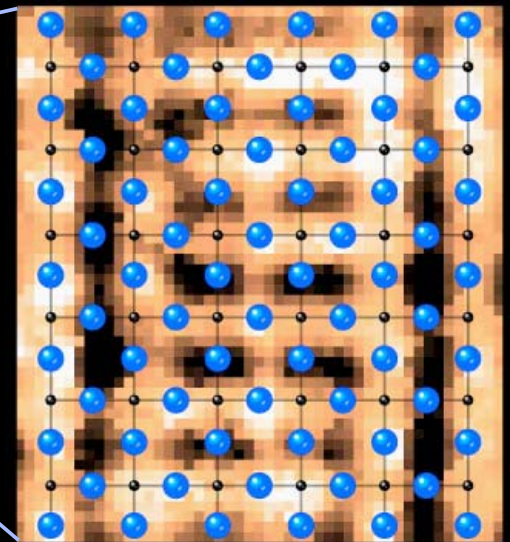
R map (150 mV)



12 nm



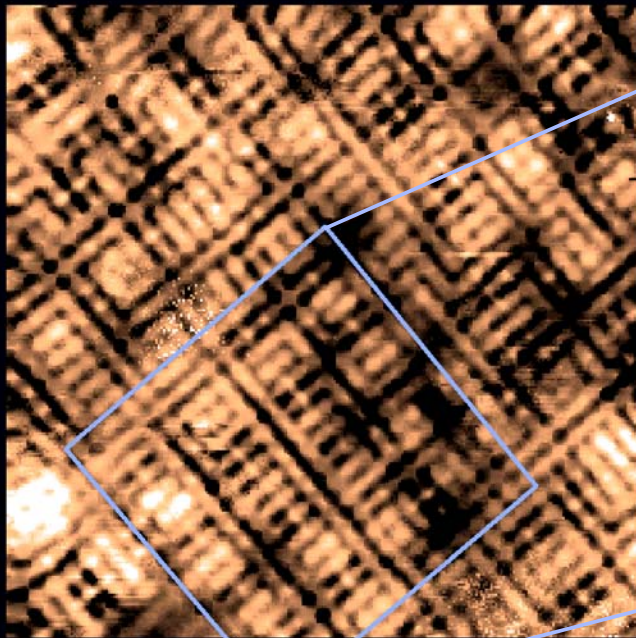
$\text{Ca}_{1.88}\text{Na}_{0.12}\text{CuO}_2\text{Cl}_2$ , 4 K



$4a_0$

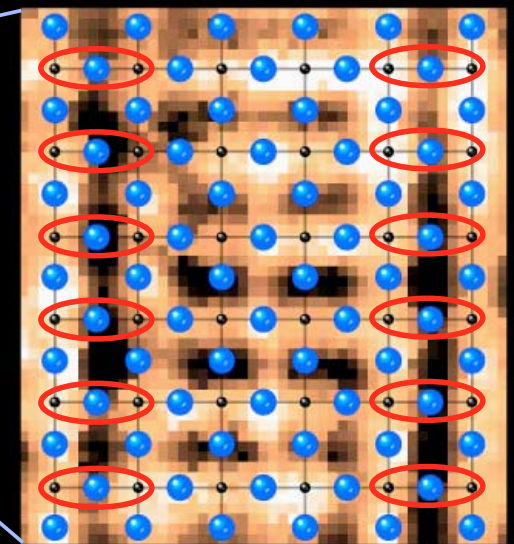
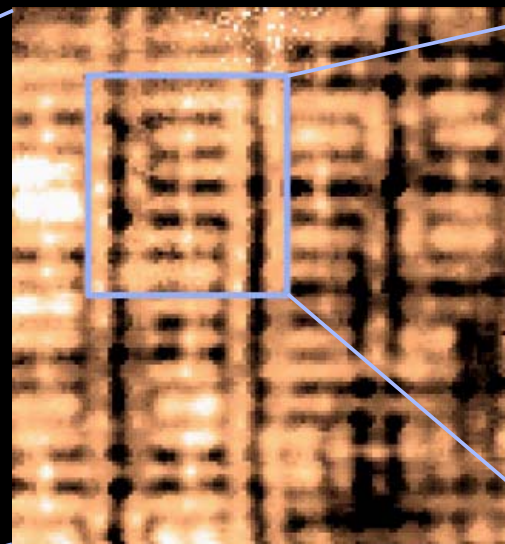
# TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

R map (150 mV)



12 nm

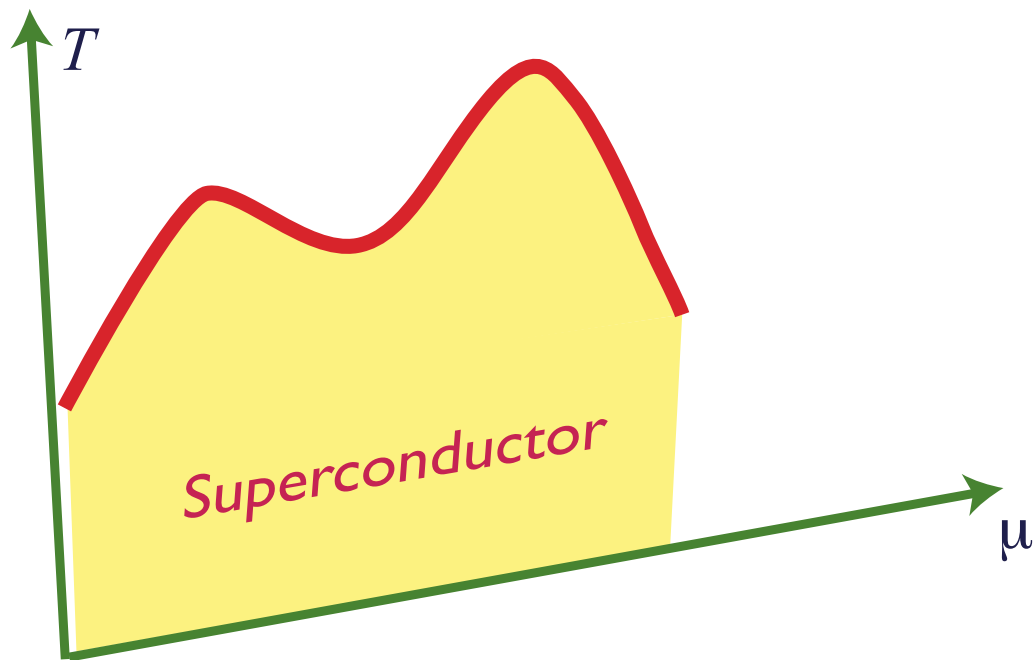
$\text{Ca}_{1.88}\text{Na}_{0.12}\text{CuO}_2\text{Cl}_2$ , 4 K

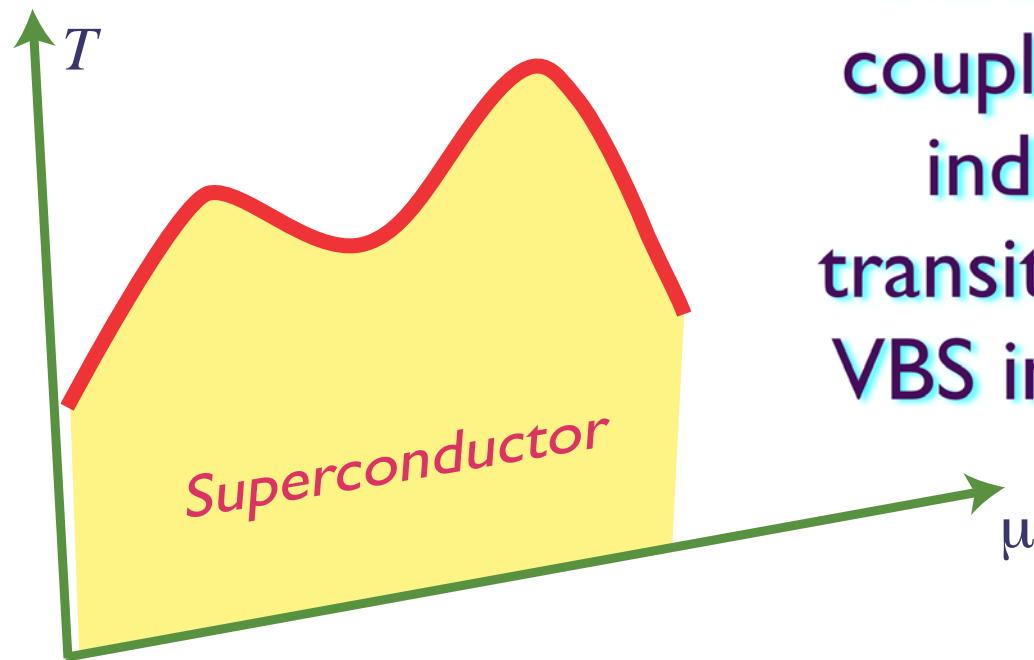


$4a_0$

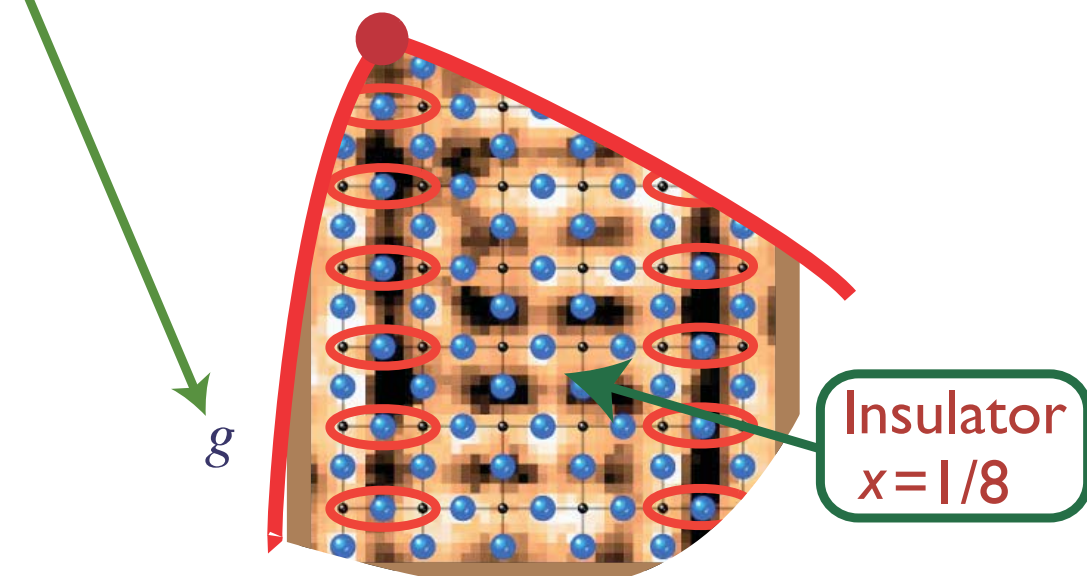
## Evidence for VBS order - a valence bond supersolid

S. Sachdev and N. Read, *Int. J. Mod. Phys. B* 5, 219 (1991).

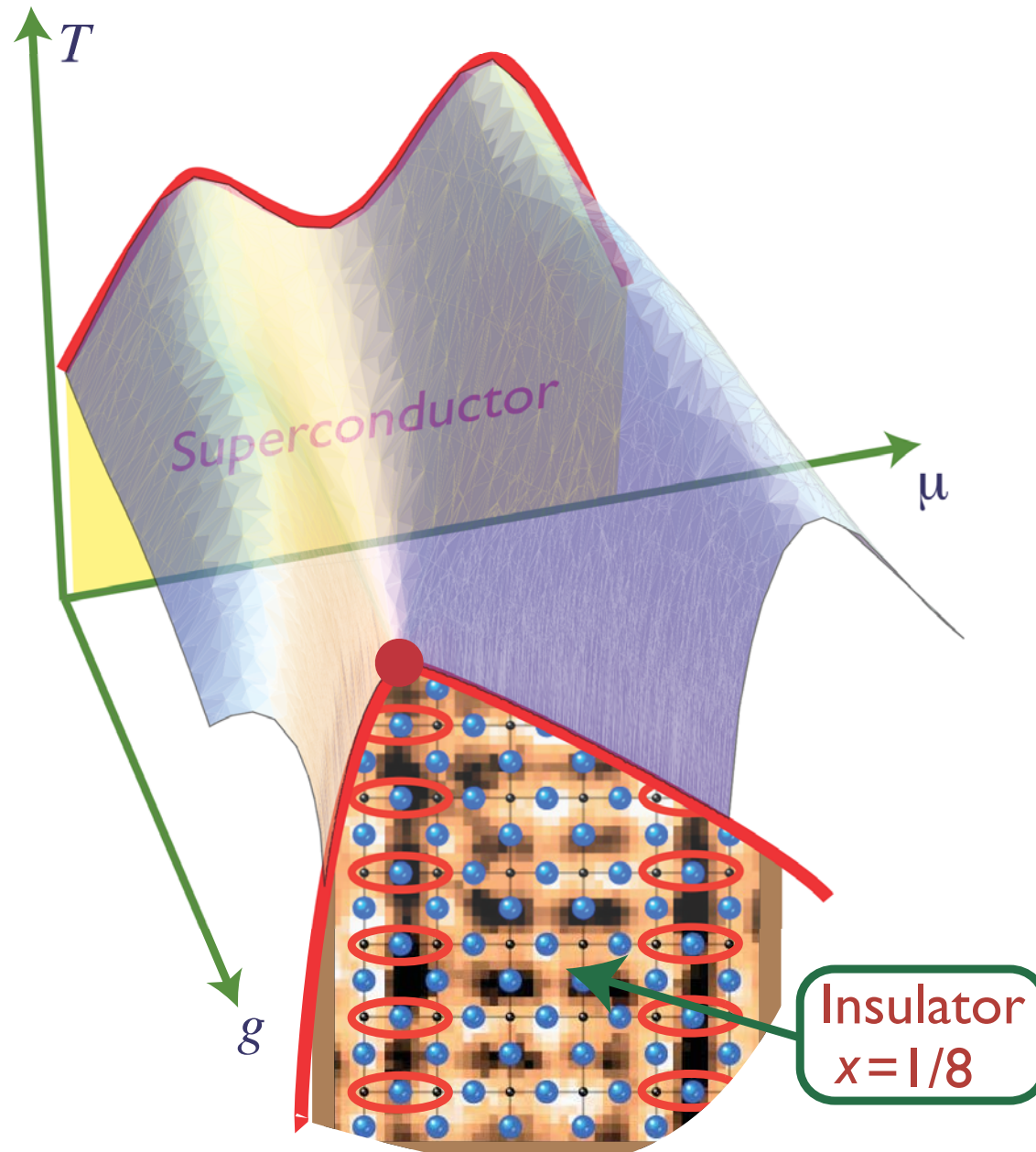




Introduce a coupling  $g$  to induce a transition to a VBS insulator



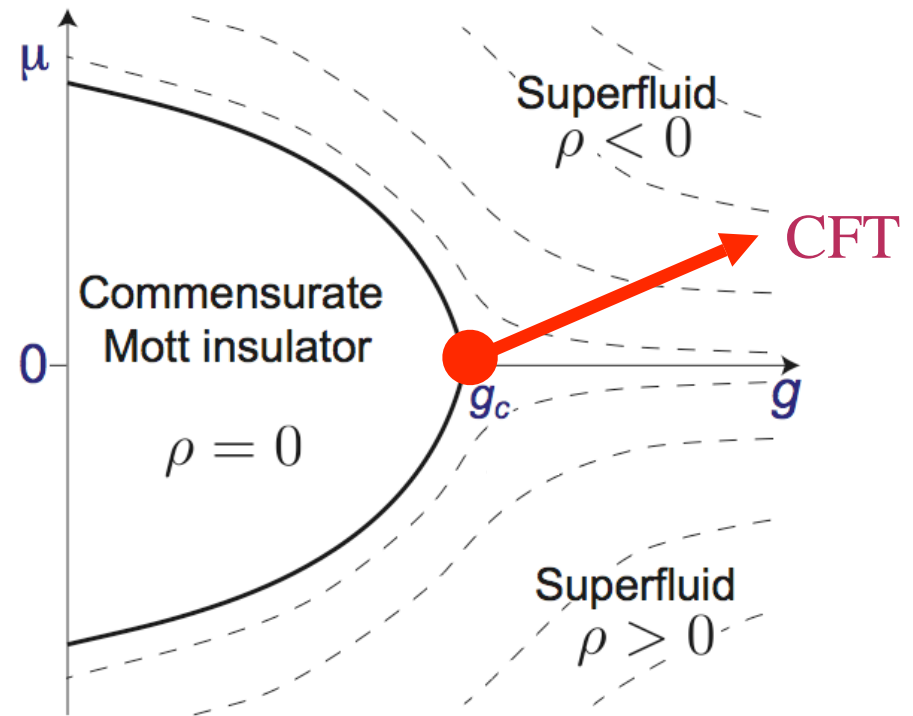
# Proposed generalized phase diagram



For experimental applications, we must move away from the ideal CFT

- A chemical potential  $\mu$
- A magnetic field  $B$

In the gravity dual theory, these perturbations correspond to electric and magnetic charges on the black hole



e.g.

$$\mathcal{S} = \int d^2r d\tau \left[ |(\partial_\tau - \mu)\psi|^2 + v^2 |(\vec{\nabla} - i\vec{A})\psi|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B$$

In the hydrodynamic regime,  $\hbar\omega \ll k_B T$ , we can use classical principles involving relaxation to local equilibrium to understand these perturbations.

The variables entering the hydrodynamic theory are

- the external magnetic field  $F^{\mu\nu}$ ,

$$F^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{pmatrix},$$

- $T^{\mu\nu}$ , the stress energy tensor,
- $J^\mu$ , the current,
- $\rho$ , the local number density,
- $\varepsilon$ , the local energy density,
- $P$ , the local pressure,
- $u^\mu$ , the local velocity, and
- $\sigma_Q$ , a universal conductivity, which is the **single transport co-efficient**.

The dependence of  $\varepsilon$ ,  $P$ ,  $\sigma_Q$  on  $T$  and  $v$  follows from simple scaling arguments

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu$$

Conservation laws/equations of motion



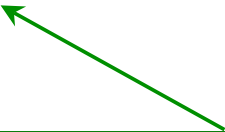
Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu$$

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu}$$

$$J^\mu = \rho u^\mu$$



Constitutive relations which follow from Lorentz transformation to moving frame

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu$$

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu}$$

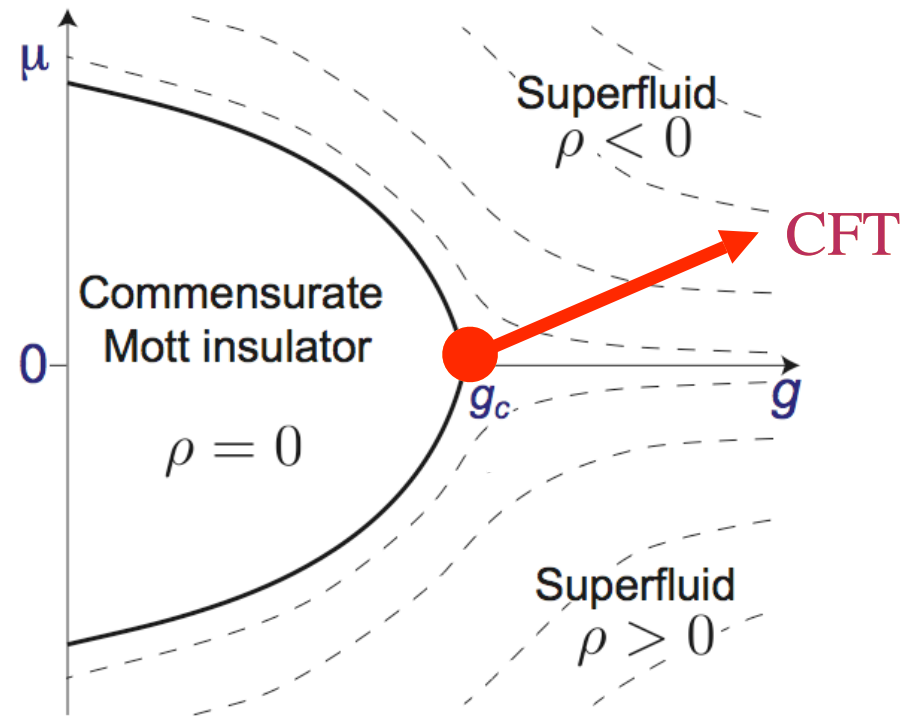
$$J^\mu = \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[ (-\partial_\nu \mu + F_{\nu\lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T} \right]$$

Single dissipative term allowed by requirement of positive entropy production. There is only one independent transport co-efficient

For experimental applications, we must move away from the ideal CFT

- A chemical potential  $\mu$
- A magnetic field  $B$

In the gravity dual theory, these perturbations correspond to electric and magnetic charges on the black hole



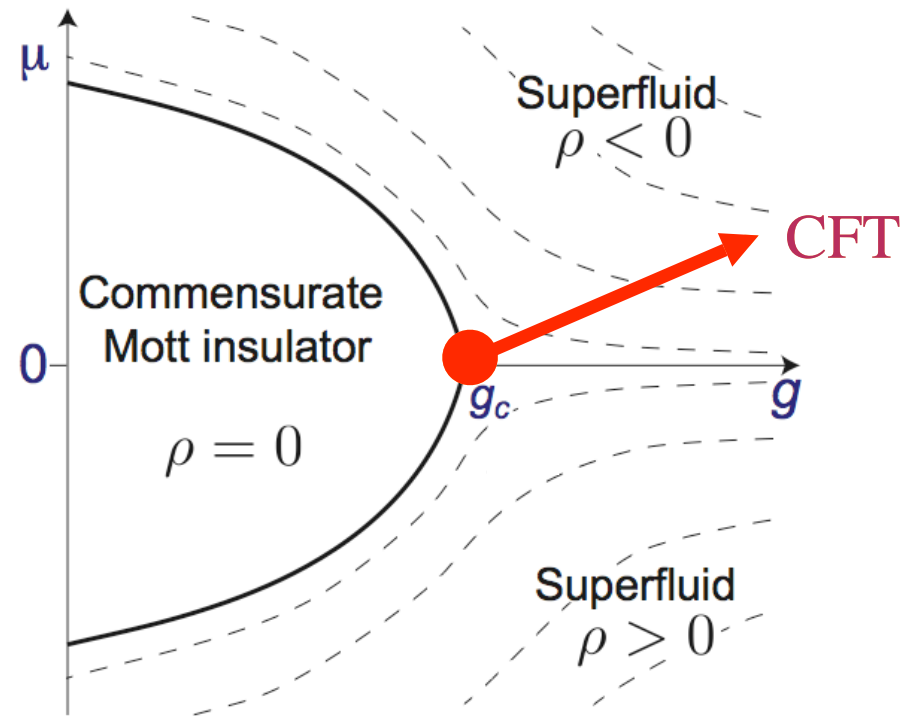
e.g.

$$\mathcal{S} = \int d^2r d\tau \left[ |(\partial_\tau - \mu)\psi|^2 + v^2 |(\vec{\nabla} - i\vec{A})\psi|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B$$

For experimental applications, we must move away from the ideal CFT

- A chemical potential  $\mu$
- A magnetic field  $B$
- An impurity scattering rate  $1/\tau_{\text{imp}}$  (its  $T$  dependence follows from scaling arguments)



e.g.

$$\mathcal{S} = \int d^2r d\tau \left[ |(\partial_\tau - \mu)\psi|^2 + v^2 |(\vec{\nabla} - i\vec{A})\psi|^2 - g|\psi|^2 + V(r)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B \quad , \quad \overline{V(r)} = 0 \quad , \quad \overline{V(r)V(r')} = V_{\text{imp}}^2 \delta^2(r - r')$$

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu + \frac{1}{\tau_{\text{imp}}} (\delta_\nu^\mu + u^\mu u_\nu) T^{\nu\gamma} u_\gamma$$

$$T^{\mu\nu} = (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu}$$

$$J^\mu = \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[ (-\partial_\nu \mu + F_{\nu\lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T} \right]$$

From these relations, we obtained results for the transport co-efficients, expressed in terms of a “cyclotron” frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Longitudinal conductivity

$$\sigma_{xx} = \sigma_Q \left[ \frac{(\omega + i/\tau_{\text{imp}})(\omega + i\gamma + i\omega_c^2/\gamma + i/\tau_{\text{imp}})}{(\omega + i\gamma + i/\tau_{\text{imp}})^2 - \omega_c^2} \right] .$$

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Hall conductivity

$$\begin{aligned} \sigma_{xy} &= -\frac{2e\rho c}{B} \left[ \frac{\gamma^2 + \omega_c^2 - 2i\gamma\omega + 2\gamma/\tau_{\text{imp}}}{(\omega + i\gamma + i/\tau_{\text{imp}})^2 - \omega_c^2} \right] \\ &= B \left[ \sigma_Q \frac{4e\rho v^2}{(\varepsilon + P)(1/\tau_{\text{imp}} - i\omega)} + \frac{8e^3\rho^3 v^4}{(\varepsilon + P)^2(1/\tau_{\text{imp}} - i\omega)^2} \right] \\ &\quad \text{as } B \rightarrow 0 \end{aligned}$$

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Thermal conductivity

$$\begin{aligned} \kappa_{xx} &= \sigma_Q \left( \frac{k_B^2 T}{4e^2} \right) \left( \frac{\varepsilon + P}{k_B T \rho} \right)^2 \left[ \frac{(\omega_c^2/\gamma)(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2} \right] \\ &= \frac{1}{\sigma_Q} k_B^2 T \left( \frac{c(\varepsilon + P)}{k_B T B} \right)^2 \left[ \frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2} \right] \end{aligned}$$

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Thermal conductivity

$$\begin{aligned} \kappa_{xx} &= \sigma_Q \left( \frac{k_B^2 T}{4e^2} \right) \left( \frac{\varepsilon + P}{k_B T \rho} \right)^2 \boxed{\rightarrow 1 \text{ as } B \rightarrow 0} \\ &= \frac{1}{\sigma_Q} k_B^2 T \left( \frac{c(\varepsilon + P)}{k_B T B} \right)^2 \left[ \frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2} \right] \end{aligned}$$

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From these relations, we obtained results for the transport co-efficients, expressed in terms of a “cyclotron” frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Nernst signal

$$e_N = \left( \frac{k_B}{2e} \right) \left( \frac{\varepsilon + P}{k_B T \rho} \right) \left[ \frac{\omega_c / \tau_{\text{imp}}}{(\omega_c^2 / \gamma + 1 / \tau_{\text{imp}})^2 + \omega_c^2} \right]$$
$$\frac{k_B}{2e} = 43.086 \mu\text{V/K}$$

From these relations, we obtained results for the transport co-efficients, expressed in terms of a “cyclotron” frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

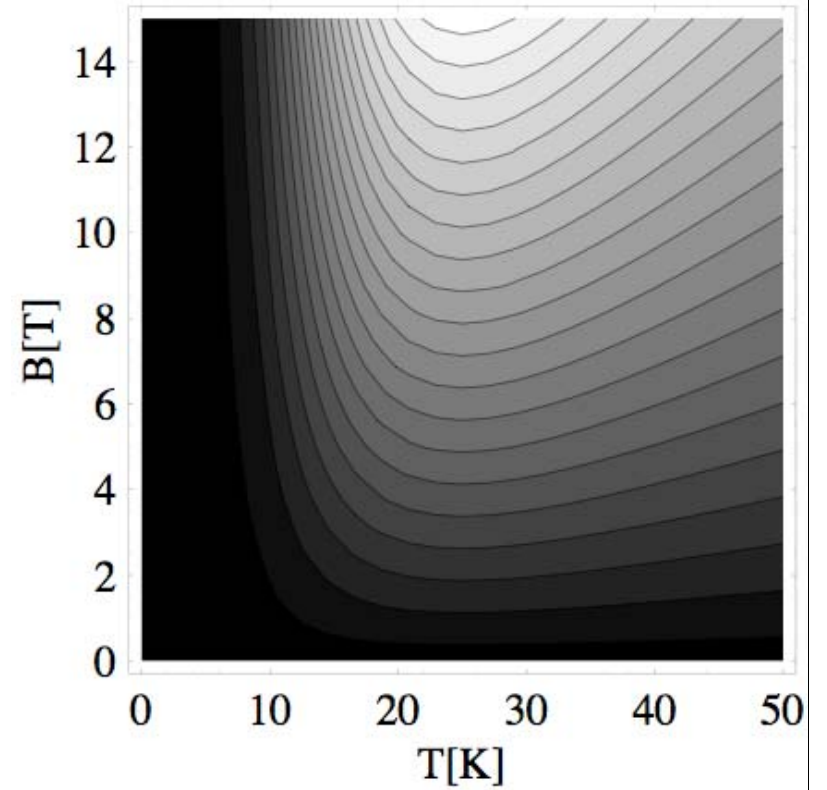
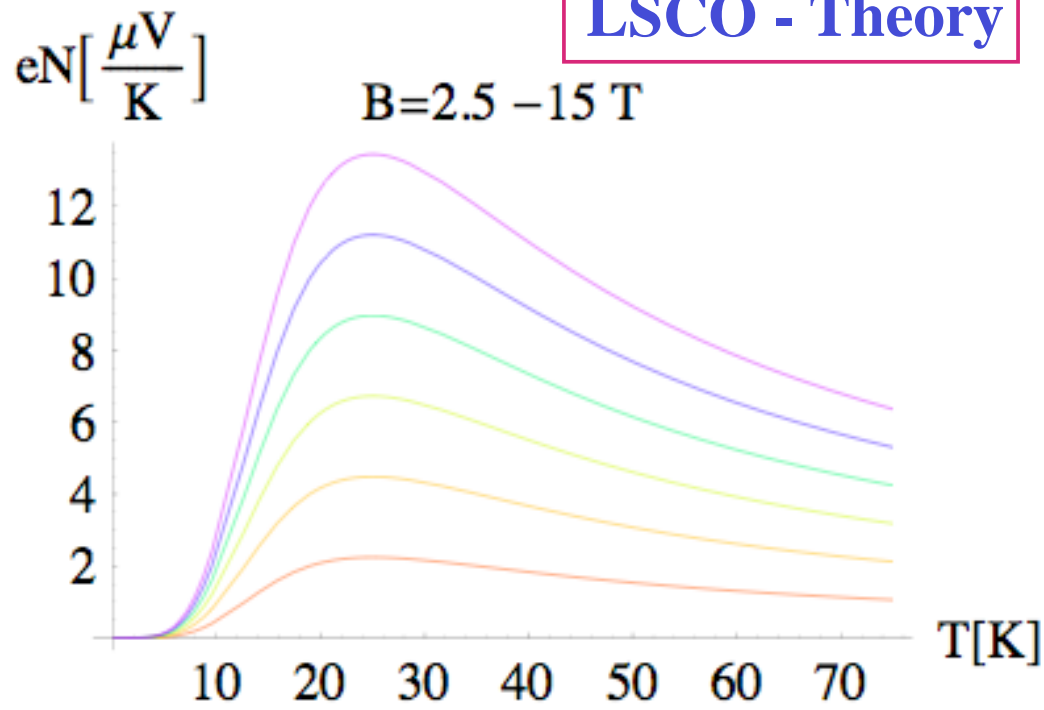
Transverse thermoelectric co-efficient

$$\left(\frac{h}{2ek_B}\right) \alpha_{xy} = \Phi_s \bar{B} (k_B T)^2 \left(\frac{2\pi\tau_{\text{imp}}}{\hbar}\right)^2 \frac{\bar{\rho}^2 + \Phi_\sigma \Phi_{\varepsilon+P} (k_B T)^3 \hbar / 2\pi\tau_{\text{imp}}}{\Phi_{\varepsilon+P}^2 (k_B T)^6 + \bar{B}^2 \bar{\rho}^2 (2\pi\tau_{\text{imp}}/\hbar)^2},$$

where

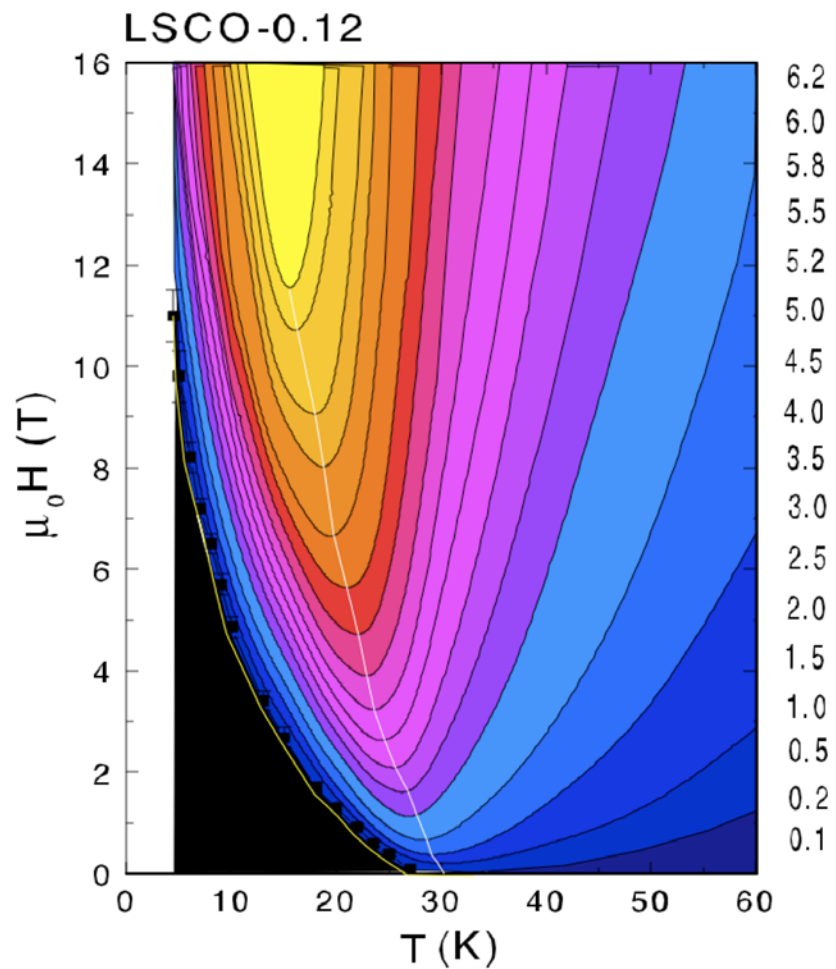
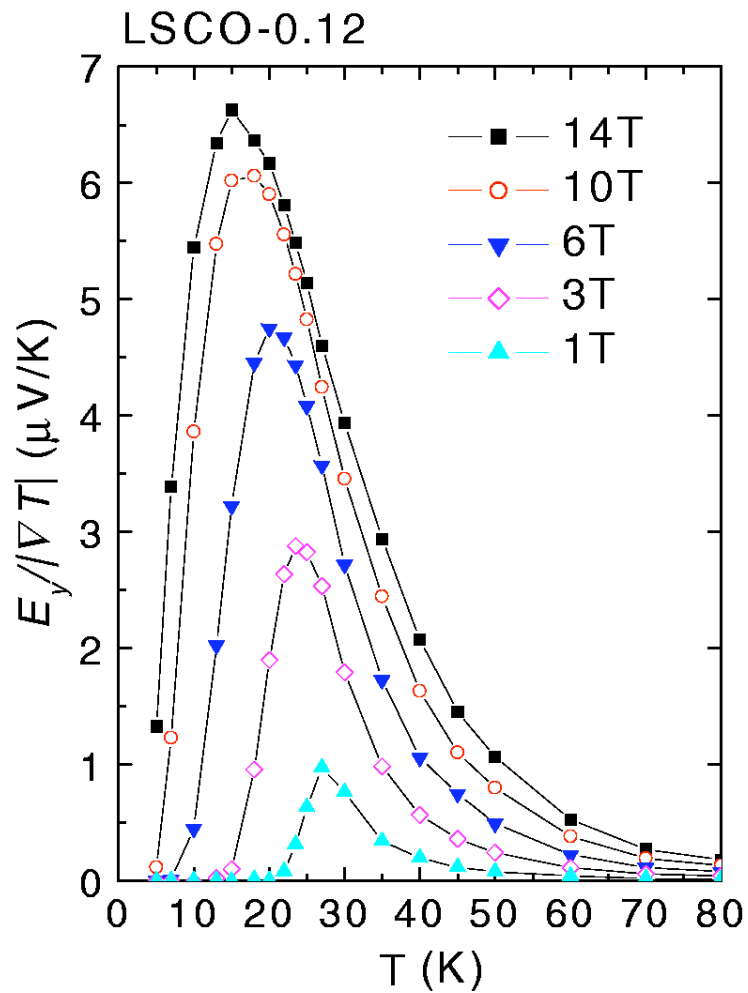
$$B = \bar{B}\phi_0/(\hbar v)^2 \quad ; \quad \rho = \bar{\rho}/(\hbar v)^2.$$

LSCO - Theory



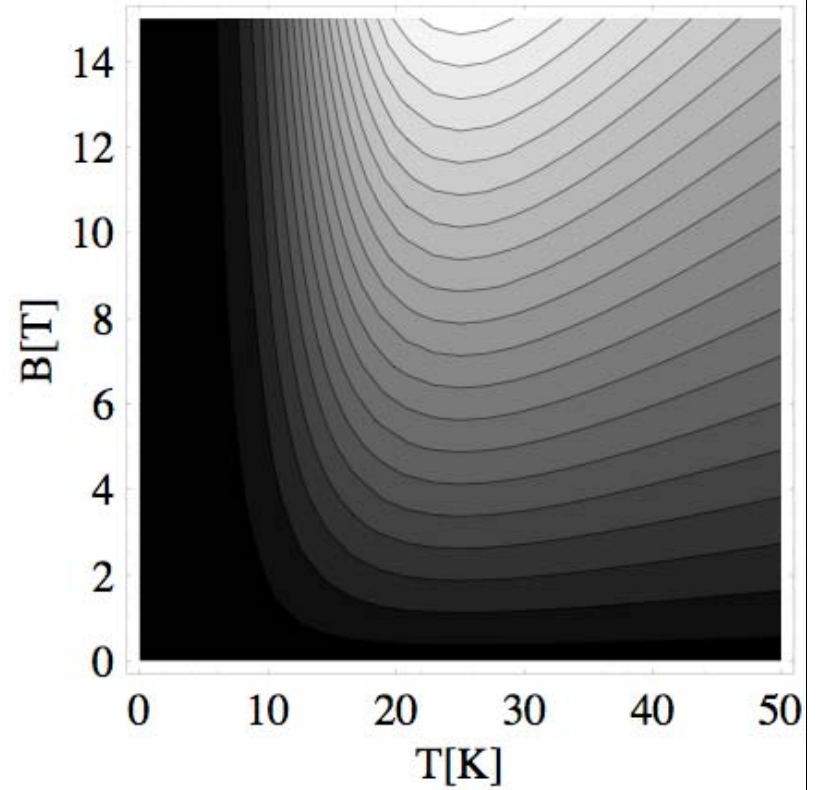
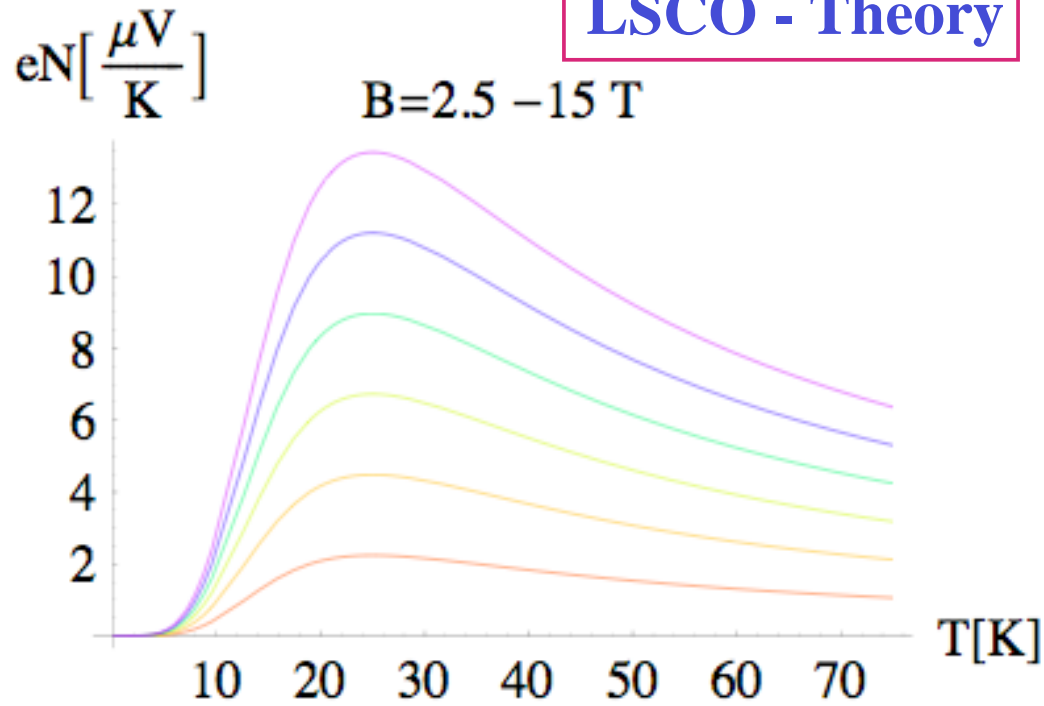
S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev. B* **76** 144502 (2007)

# LSCO - Experiments



N. P. Ong *et al.*

## LSCO - Theory



Only input parameters

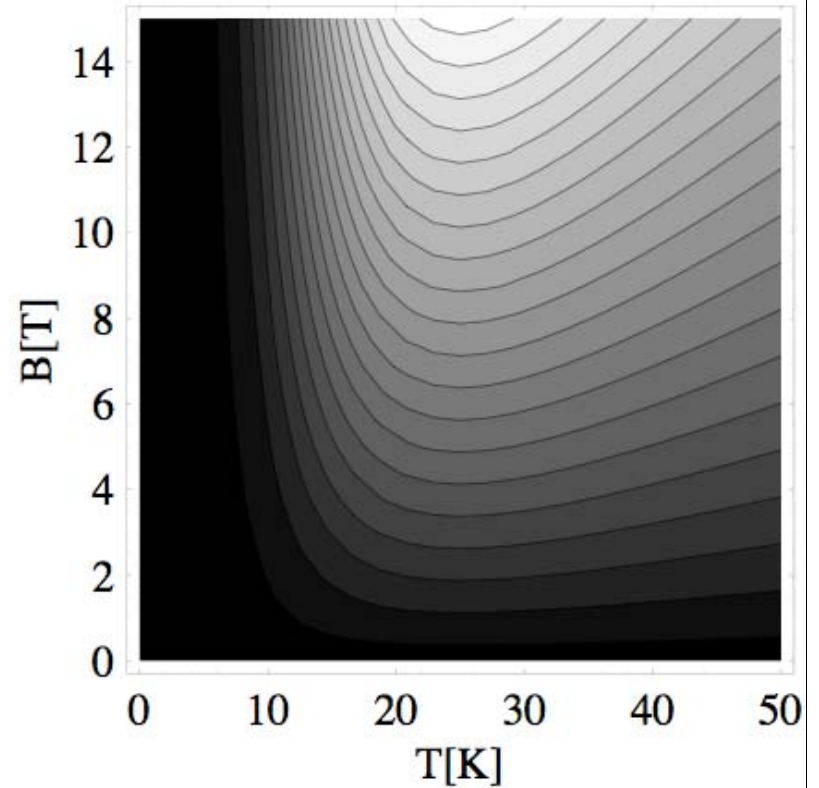
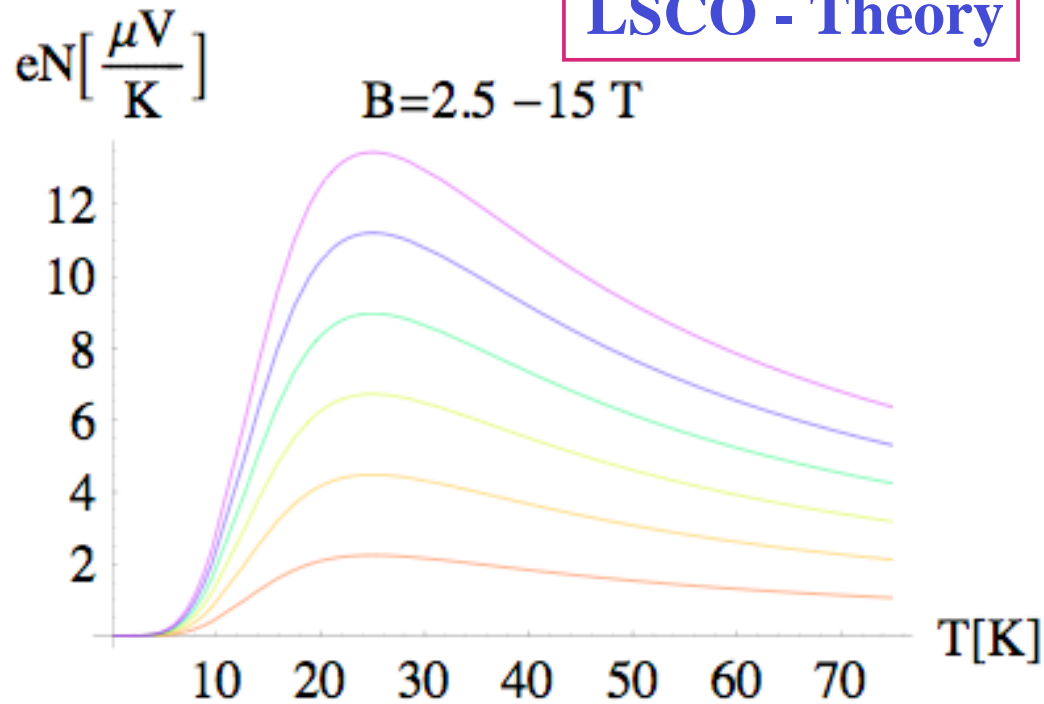
$$\hbar v = 47 \text{ meV } \text{\AA}$$

$$\tau_{\text{imp}} \approx 10^{-12} \text{ s}$$

Output

$$\omega_c = 6.2 \text{ GHz} \cdot \frac{B}{1 \text{ T}} \left( \frac{35 \text{ K}}{T} \right)^3$$

## LSCO - Theory



Only input parameters

$$\hbar v = 47 \text{ meV } \text{\AA}$$

$$\tau_{\text{imp}} \approx 10^{-12} \text{ s}$$

Output

$$\omega_c = 6.2 \text{ GHz} \cdot \frac{B}{1 \text{ T}} \left( \frac{35 \text{ K}}{T} \right)^3$$

Similar to velocity estimates by

A.V. Balatsky and Z-X. Shen, *Science* **284**, 1137 (1999).

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev. B* **76** 144502 (2007)

To the solvable supersymmetric, Yang-Mills theory CFT, we add

- A chemical potential  $\mu$
- A magnetic field  $B$

After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

- An electric charge
- A magnetic charge

The exact results are found to be in *precise* accord with *all* hydrodynamic results presented earlier

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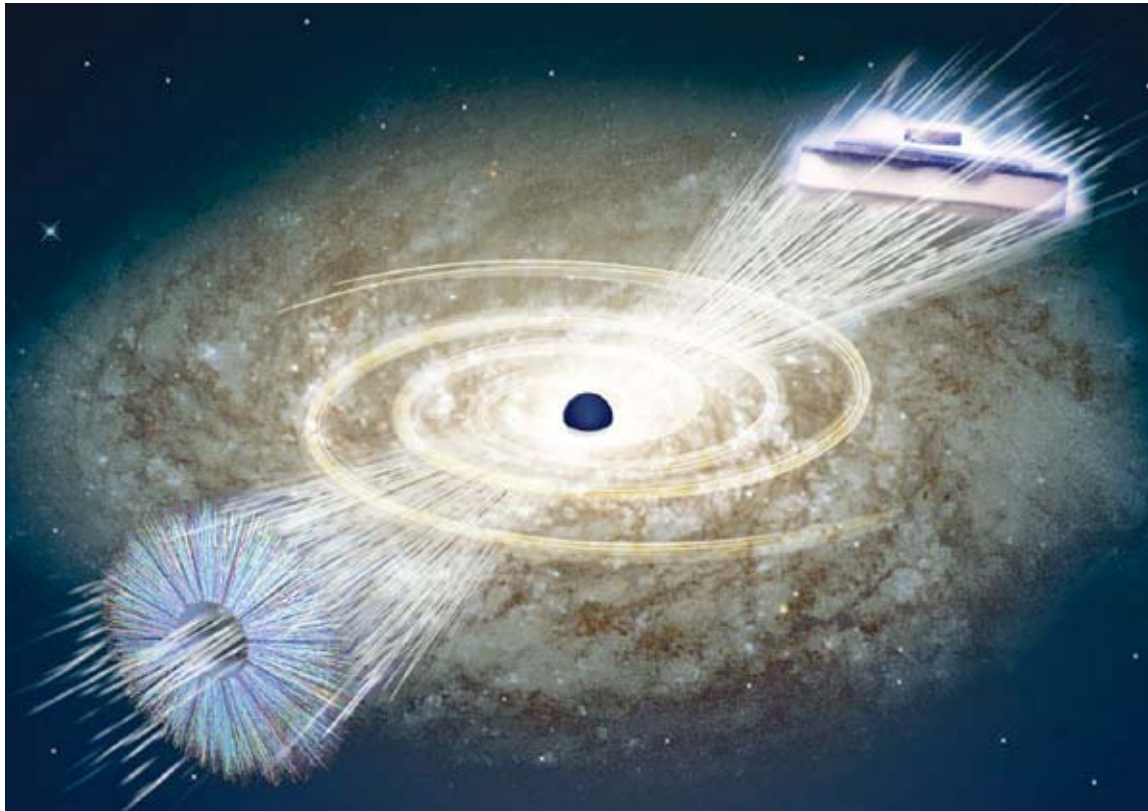
**THEORETICAL PHYSICS**

# A black hole full of answers

Jan Zaanen

**A facet of string theory, the currently favoured route to a 'theory of everything', might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.**

NATURE|Vol 448|30 August 2007



## Conclusions

- Condensed matter systems realize several interesting CFT3s.
- Collisionless-to-hydrodynamic crossover in CFT3s at  $T > 0$ .
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Theory of VBS order and Nernst effect in cuprates.